Minimizing Manipulator Performance Variability by Selecting Control Parameters and Tolerances for Optimal Robust Designs of Manipulators

THESIS

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By

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CERTIFICATE

This is to certify that the thesis entitled "**Minimizing Manipulator Performance Variability by Selecting Control Parameters and Tolerances for Optimal Robust Designs of Manipulators**" and submitted by Bijay Kumar Rout ID.No. 1999PHXF411 for award of Ph.D. Degree of the Institute embodies original work done by him under my supervision.

Signature in full of the Supervisor ________________________

Name in capital block letters: **Prof. R.K. MITTAL**

Designation: Professor of Mechanical Engineering Group & Dean, Academic Registration & Counselling Division

Date: _______________

DEDICATED TO MY PARENTS

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ABSTRACT

The capability of a robot to position its end effector with high accuracy and repeatability is an important attribute for industrial automation technology. A robotic arm must manipulate objects with a high degree of accuracy and repeatability. It has been well recognized that quality can be significantly improved during the early design stage if appropriate quality engineering methods are used. While much research has been done in field of robotics and robust design, but attempt to combine robotic manipulator design concepts with robust design techniques are rare. This deficiency significantly limits the performance of manipulator due to lack of proper selection of design parameters and tolerance. The motivation of this thesis is to develop an integrated methodology to reduce performance variations of manipulator by selecting optimal design parameters and tolerances.

Like every physical system, there are number of factors, which cause performance variations of a manipulator. The performance variations of manipulators are attributed to control factors and noise factors. The factors whose values can be changed or controlled easily by designer are called control factors and which are difficult to control are called noise factors. Development of approach to simulate real life performance of manipulator incorporating effect of noise is a challenge to robotic system designer. Therefore, control and noise factors of the problem are identified and a probabilistic approach has been proposed to simulate the performance incorporating effect of noise. In order to investigate the effect of these control factors and noise factors and to obtain a better insight into the manipulator performance, the manipulator kinematics and dynamics models are used to simulate the performance of manipulator.

Conducting physical experiments on manipulator by changing its parameters are very tedious, time consuming and uneconomical. To solve this problem, help of simulation is taken where simulation experiments are conducted by varying the values of control and noise factors. To explore the effects systematically, design of experiments technique is applied and the manipulator performance is made insensitive to effect of noise factors. This approach is also called parameter design technique, where the values of the control

factors i.e. design parameters that minimize the effect of noise factors on the functional characteristics of the product are found.

As identified control factors and noise factors are large in number, conducting simulation experiment becomes computationally expensive. Therefore, to screen the important parameters from identified parameters, fraction factorial design of experiments approach is used. Using this design of experiments technique and proposed probabilistic approach performance of the manipulator are simulated and analyzed. ANOVA technique has been used to screen the parameters, and identify those, which have significant influence on performance variability. To investigate the influence of time law of the trajectory for performing the task, above investigation is carried out.

To get the optimal design parameters, response surface methodology (RSM) has been used. RSM is used to develop the relationship between independent control and noise parameters with the performances. As the mathematical relations relating the manipulator performances and the design and noise parameters are not available, therefore, application of RSM technique to this problem becomes pertinent. The design matrix obtained from central composite design (CCD), performance of manipulator is simulated using the proposed probabilistic approach. Then response equations relating the control and noise factors to response are developed. Subsequently response equations relating the control factors and noise factors to response for mean and variance of performance have been developed. To optimize the parameters, which deliver optimum performance, an optimization problem is formulated. The objective function of formulated optimization problem minimizes the performance while achieving decided value of performance variability. As the optimization problem formulated is nonlinear in nature, it is solved using an optimization subroutine. From this subroutine, optimal design parameters of manipulator are obtained while performing task following different trajectories.

To reduce the variability in performance further and to determine the statistically significant parameter tolerances, further investigations are carried out. For parameter tolerance design, cross array design of experiment approach has been used. For simulation of performance same probabilistic approach is utilized. Statistical analysis of simulated results has been carried out to obtain the significant parameters for which tolerance can be tightened. Along with this analysis, optimal parameter tolerances which deliver optimum performance and insensitive to noise of control factor variation have been obtained. To validate the results obtained in cross array design of experiment approach, Monte Carlo simulation method has been utilized. This investigation is carried out for the manipulator for performing different tasks following different trajectories. The results obtained are compared with the results of cross array approach.

An attempt has been made to obtain robust manipulator parameter design, using non-conventional optimization method. A novel hybrid evolutionary optimization method has been proposed. In this approach, hybrid of orthogonal array available in the Taguchi method and Differential Evolution Technique are used. The proposed method has been used to select optimal design parameters for the manipulator for performing different tasks following different trajectories.

In addition to above investigations, parametric sensitivity has also been explored. To determine the impact of change in parameter values on performance of manipulator, two dimensionless parameter indices are proposed. By changing these indices individually and simultaneously, investigations have been carried out and the performance sensitivity of manipulator to these changes has been analyzed to understand their influence. Similarly, the design parameter tolerances sensitivity has also been investigated for manipulator performing a task following cubic and quintic trajectories.

In past research in robotic manipulator design primarily focused on system level design metrics based on geometry and dynamics. However, to achieve improved performance and to design better robotic systems, the effect of design and noise parameters on the capabilities of these systems has to be understood. The work done on the design of manipulators can be used for an array of applications and better complex manipulators can be designed.

Key Words: *Control Factor, Noise Factor, Performance, Positional Error, Mean Positional Error, SN Ratio, Manipulator Reliability, Optimal manipulator deign, Tolerance design, Fractional Factorial Design, Response Surface Methodology, Cross Array Design of Experiment, Hybrid Differential Evolution Technique.*

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CHAPTER-1

PERFORMANCE OF MANIPULATOR

1.1 INTRODUCTION

Robot is a mechanical device, which performs programmed physical tasks, either according to a pre-defined program or, a set of general guidelines using artificial intelligence techniques. Robots are typically used to perform tasks that are too dirty, dangerous, difficult, repetitive, or dull for humans. Different types of robotic manipulators are used in the industrial settings for performing a variety of tasks, such as spray-painting, welding, pick-n-place, material handling and many other tasks. Certain operations, such as laser cutting, high-pressure water jet machining etc., require high accuracy and precision in robot's end-effector path tracking. Operations, such as spot welding, part assembly and measuring require high accuracy in terms of robot end-effector pose accuracy.

A robotic manipulator is characterized by its degree of freedom, number of joints, type of joints, joint placements and joint motions; link lengths and shapes, which influence its performances, namely, the workspace, manipulability, speed of operation positioning accuracy, etc. The speed of operation significantly depends on the complexities of the kinematic and dynamic equations and their computations. The manipulators with different architectures will have different kinematic and dynamic equations with varying computational complexities.

The performance of a robotic manipulator is estimated by simulating experiments. The performance evaluation problem is formulated as "for a given desired motion task, determine suitable design parameters that would help the robot to have motion as close as possible to the desired destination". The performance of the robotic manipulator is mitigated by proper selection of optimal design parameters, process parameters and tolerances. To be as close as possible to the desired target point consistently indicates a better quality of performance. Improving quality of performance of a manipulator is of particular concern for the industry and hence has been taken up as the focus of the research.

The mathematical models are not accurate representations of the robot kinematics and dynamics, and therefore, in reality there are significant discrepancies between the actual and the expected performance. Therefore, under the prevailing conditions designers should look for alternative approaches to obtain the desired quality performance. This thesis investigates possibilities to enhance the performance measure using parameter and tolerance based design strategies and offers algorithms and techniques based on design parameters and their tolerances approach to realize the desired performance quality.

1.2 ROBOTICS TODAY

Robotics is a system created by humans to create substitutes that would be able to mimic their behaviors in various aspects of interaction with the environment. The term *robot* is coined from the Slav word *rabota*, which means executive labour. The writer of science fiction literature Isaac Asimov introduced the term *robotics* to describe the science devoted to the study of robots that represent automation of human appearance but have no feelings. Today's robotics covers many aspects of human living, although it is still far from the models anticipated by human imagination and science fiction. Some of the areas where robotics is present are industrial manufacturing, military, sea and space exploration, medicine, rescue services, entertainment.

A robot represents a flexibly programmable manipulation system with multiple degrees of freedom, able to perform different tasks. Regardless of particular tasks, any robot should be capable to perform motions in a multidimensional working space to realize the desired position and orientation either of its complete structure (e.g. mobile robots) or of a part of its structure which is called the terminal device or the end-effector. One robot may contain more than one terminal device. Its multiple degrees of freedom must be controlled in a coordinated way in order to achieve a functional movement. The robot's capacity for movement is provided by a mechanical system which in general consists of the locomotion and manipulation subsystems. The purpose of the locomotion subsystem is to move the robot within the environment, while the manipulation subsystem is used to operate neighboring objects. The robot's capacity for perception is provided by a sensory system which can acquire data on the internal status of the mechanical system, as well as on the external status of the environment. The robot's capacity for connecting perception to movement in a functional way is provided by a control system which can decide the execution of the action in respect of the constraints imposed by the mechanical system and the environment.

Robots are not versatile enough, to accomplish different duties without significant human interference in the instances of path planning, trajectory generation, control and programming. This limited capability is a major handicap and may be attributed to the actual mechanical designs (e.g. low number of degrees of freedom and parasitic effects, such as flexibilities and friction) and the challenges of intelligent connection of robot perception with robot action (needs for arrays of sensors, efficient processing of sensory information, large computational power of control systems, effective but safe control strategies, etc.).

Significant efforts have been spent to overcome these limitations. Various aspects of robotics are under permanent research and many contributions are still appearing. The subject of research reported in this thesis is optimal parameter and tolerance design of robotic manipulator by which better quality of performance is delivered. The objective is to develop parameter and tolerance design strategies that can improve in realizing the desired quality of performance. The quality of performance in this case refers to higher position accuracy and repeatability with low sensitivity to noises. Another emphasis is on the efficiency of robot motions, i.e., the thesis considers quality of performance improvement when the desired motions or tasks are followed. The motives for such considerations can be illustrated by an example from industrial manufacturing. If a robot accurately accomplishes its task when manufacturing a product, then the steps on the product post processing are reduced. The next two sections summarize theoretical issues of interest for robot performance and discuss the concepts that will be used later on in the thesis.

1.3 PERFORMANCE OF A MANIPULATOR

The performance of robotic manipulators are crucial to their commercial viability and widespread use in industry. Nowadays, robotic manipulators are required to achieve high positional accuracy and orientation at high speeds and should have the ability to interact with their environment in the desired manner. This increases the range of tasks for which they could be suitable but the idea of selecting a robot on basis of over all performance in workspace is rarely addressed by the research fraternity. Several measures of performance like reliability, positional error, SN ratio and performance index etc. are proposed by many researchers. For computation of performance measure, manipulator dynamics is generally neglected, even though it is widely used for control and simulation of robots.

Traditionally, a solution to the performance problem requires the knowledge of mathematical models representing the kinematics and dynamics of the robot. Kinematics is modeled by applying the well-known physical principles describing the motion of a body in the multidimensional working space. A dynamic model is typically obtained from the basic physical laws governing the robot dynamics. When the models are available, some strategy is needed to act on them, in order to realize the desired task. This strategy requires capabilities of simulating the real life performance and means to act on the design variables, is called uncertainty modeling and robust design.

This thesis develops strategies to minimize robot performance variability. The strategy includes development of modeling and simulation technique using robot kinematic and dynamic models to investigate the effect of different design and process parameters on performance variability. These strategies are off-line approaches, which select optimal design parameter using different optimization approaches. The strategies used do not essentially bring new theories, but contributes by merging of various concepts available in the literature. Such merging is not found elsewhere in the literature.

1.3.1 Static and Dynamic Performance of Manipulator

An industrial robot is a multifunctional and computer-controlled mechanical manipulator exhibiting a complex and highly nonlinear behavior. Even though most current robots have anthropomorphic configurations, they have far inferior manipulating abilities compared to humans. A great deal of research effort is presently being directed towards improving their overall performance by using optimal mechanical structures and control strategies. The optimal design of robot manipulators can include kinematic performance characteristics such as workspace, accuracy, repeatability, and redundancy.

Static performance refers to point-wise performance measures that do not involve motion or control. Dynamic performance refers to integrable performance measures, such as motion time, energy, and tracking accuracy. Static performance is characteristic of a point while dynamic performance is characteristics of a direction. In designing robotic systems for desired performance, it is necessary to select a path, or a set of paths, representative of tasks for which the system is to be designed.

Motion accuracy while performing a task is an important criterion that needs the designer's attention. For the purpose of this study, a pose describes the location and orientation of an object relative to a reference frame. While performing a task, various poses are obtained. The quality of performance of the robot motion, final position and orientation achieved, exertion of force or torque, or other general output is described by three distinct and measurable groups: resolution, accuracy and repeatability. Industry-wide standards for measuring these parameters are not yet fully established; however, improved methods for analyzing robot performance are being developed. The measures of performances representing the precision of the robot movement are:

(i) Spatial Resolution,

(ii) Accuracy,

(iii) Repeatability.

The above measures of performances are defined with the following assumptions. First these terms are defined in the context of worst case condition and point-to-point motion of robot. It is easier to define the various precision features in the static context rather than a dynamic context. It is considerably more difficult to define and measure the robot's capacity to achieve a defined motion path in space because it would be complicated by speed and other factors.

(i) Spatial Resolution: The spatial resolution of a robot is the smallest incremental movement into which the robot can divide its work volume. Spatial resolution depends on two factors: the system's control resolution and robot's mechanical inaccuracies. The control resolution is determined by the robot's position control system and its feed back measurement system. It is the controller's ability to divide the total range of movement for the joint into individual increments that can be addressed in the controller. The increments are some times referred to as "addressable points". The ability to divide the joint range into increments depends on the bit storage capacity in the control memory. The number is separate, identifiable increments (addressable point) for a particular axis is given by

$$\text{Number of increments} = 2^n \tag{1.1}$$

where n is the number of bits in the control memory.

The control resolution is defined as the total motion range divided by the number of increments. It is assumed that the system designer will make all the increments equal. This definition is applicable for robots with one joint. A robot with several degrees of freedom would have a control resolution for each joint of motion. To obtain the control resolution of the entire robot, component resolution for each joint would have to be summed up vectorially. The total control resolution would depend on the wrist motions as well as the arm and body motions. Since some of the joints of robots are combinations of rotary and prismatic joints the robots control resolution is difficult to determine.

Mechanical inaccuracies in the robot's link and joint components and its feed back measurement system constitute the other factor that contributes to spatial resolution. The mechanical inaccuracies can be listed as:

(a) Elastic deflection in the structural members,

(b) Gear backlash,

(c) Stretching pulley cord,

(d) Leakage of hydraulic fluids,

(e) Other imperfections in mechanical systems.

These inaccuracies tend to be worse for the larger robots simply because the errors are magnified by the larger components. The spatial resolution of the robot is the control resolution degraded by these mechanical inaccuracies. Spatial resolution can be improved by increasing the bit capacity of the control memory.

(ii) Accuracy: Accuracy refers to a robot's ability to position its wrist end at a desired target point within the work volume. The accuracy of a robot can be defined in terms of spatial resolution because the ability to achieve a given target point depends on how closely the robot can define the control increments for each of its joint motions. In the worst case the desired point would lie in the middle between two adjacent control increments. Ignoring the mechanical inaccuracies that reduce the robot's accuracy, under worst case assumption, the accuracy becomes half of the control resolution. This relationship is illustrated in Fig. 1.1. In fact the mechanical inaccuracies would affect the ability to reach the target position. Accordingly the robot's accuracy can be defined as one-half of its spatial resolution as shown in Fig. 1.2

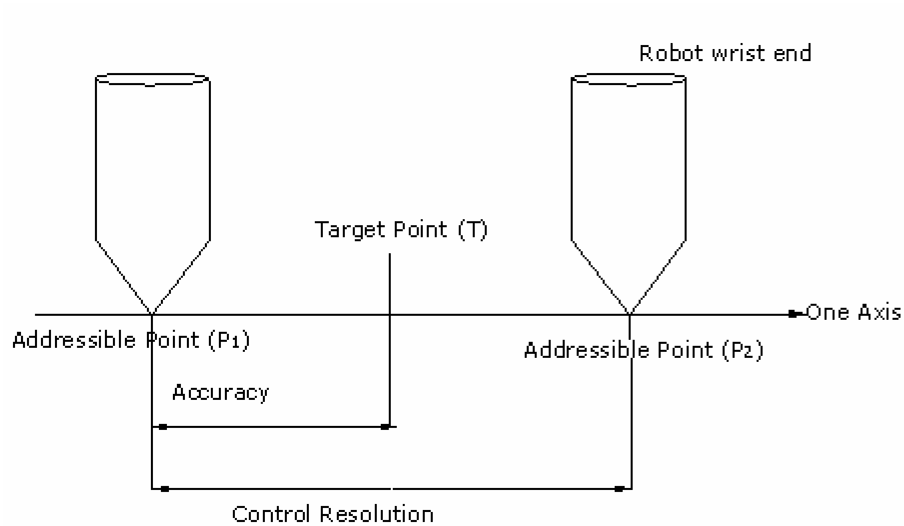

Fig. 1.1 Accuracy of Robot in Absence of Mechanical Inaccuracies

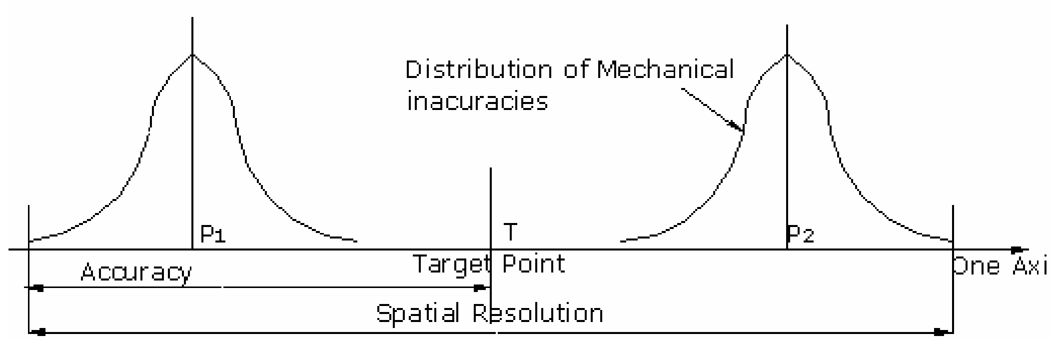

Fig. 1.2 Accuracy and Spatial Resolution with Statistical Distribution of Mechanical Inaccuracies

The definition of accuracy applies to the worst case, where the target point is directly between the two control points. This definition implies that the accuracy is the same anywhere in the robot's work volume. In fact accuracy of a robot is affected by several factors. First the accuracy varies within the work volume, tending to be worse when the arm is in outer range of its work volume and better when the arm is closer to its base. The reason for this is that the mechanical inaccuracies are magnified with the robot's arm fully extended. The mechanical errors will tend to be reduced when the robot is exercised through a restricted range of motions. The robot's ability to reach a particular reference point within the limited workspace is called its local accuracy. When the accuracy is assessed within the robot's full work volume, the term global accuracy is used.

(iii) Repeatability: Repeatability is concerned with the robot's ability to position its wrist or an end effector attached to its wrist at a point in space that had previously been taught to the robot. Repeatability and accuracy refer to two different aspect of the robot's precision. Accuracy relates to the robot's capacity to be programmed to achieve a given target point. The actual programmed point will probably be different from the target point due to limitations of control resolution. Repeatability refers to the robot's ability to return to the programmed point i.e. the point (P) reached by manipulator because of spatial resolution, when commanded to do so. These concepts are illustrated in Fig. 1.3.

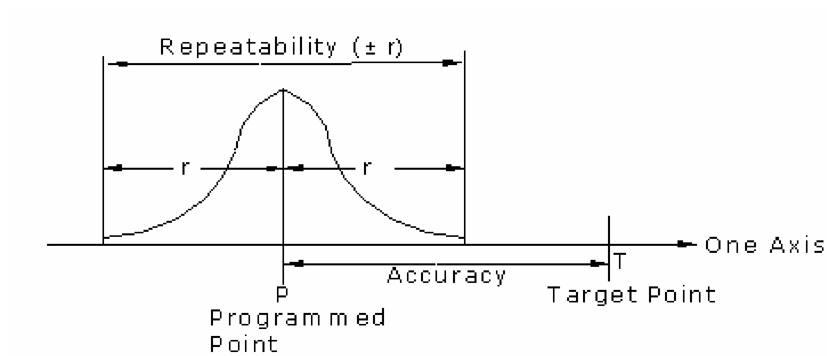

Fig. 1.3 Repeatability and Accuracy

The desired target point is denoted by T, because of the limitations on its accuracy, the programmed point becomes point P. Subsequently the robot is instructed to return to the programmed point P; however it does not return to the exact same position. Instead it returns to position R. The difference between P and R is a result of limitations on the robot's repeatability. The robot will not return always to the same position R on subsequent repetitions of the motion cycle. Instead it will form a cluster of points on both sides of the position P in Fig. 1.3. Repeatability errors form a random variable and constitute a statistical distribution as shown in the figure. It would be convenient if the repeatability errors formed a nice bell shaped curve, suggesting a normally distributed random variable. What is closer to the reality is that for each joint, the mechanical in accuracies that are principally responsible for repeatability errors do not form nice symmetric bell-shaped distribution shown in the figure. However, when the errors from several axes of motion are combined together the resulting aggregate error is influenced by the central limit theorem in probability. The central limit theorem states that the "sums of random variables tend to form a normally distributed variable, even though the individuals come from a distribution other than normal". Accordingly, inference that the repeatability error of a robot with five or six axes is approximately normal, even if the error due to each axis is not normal.

An analogy to some of the performance parameters of a robotic system can be obtained by examining the performance of an archer or marksman who is attempting to place a projectile on a target, as represented in Fig. 1.4. The archer shoots his arrows from a fixed location in space and that the target is at a fixed location. The dynamic performance measures describe the quality of the path of the arrow, while for the static performance measures, the path of the arrow is not important. The static performance of the archer is measured by the placement of the arrow in the target. Accuracy and repeatability describe the ability of a robot to move to a desired location without any deviation and to follow a desired trajectory with little or no variance.

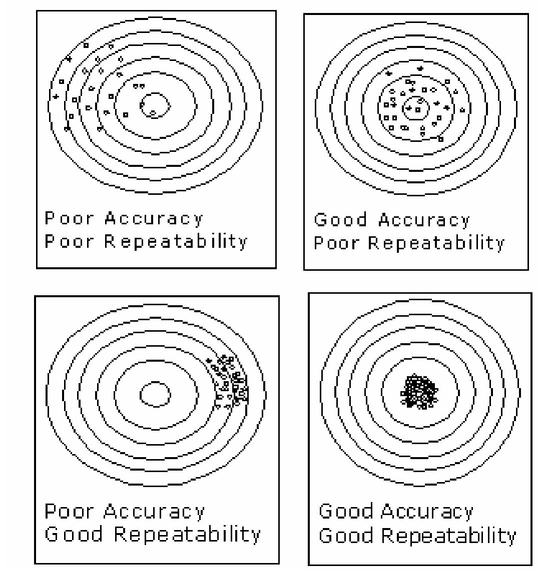

Fig. 1.4 Illustrations of Repeatability and Accuracy

1.4 IMPORTANCE OF ROBUST ROBOTIC MANIPULATOR DESIGN

A robust product is one that works as intended regardless of variation in a product's manufacturing process, variation resulting from deterioration, and variation in use. Taguchi's robust design method may achieve this goal desensitizing a product's performance characteristic(s) to variation in critical product and process design parameters.

Three of the most common types of design parameters that influence quality characteristics are signal, noise, and control. Signal parameter levels are set by the user to produce the desired functional response, such as speed, thrust, and frequency. Noise parameters are difficult or expensive to control. They deviate from target values through unit-to-unit variation resulting from manufacturing processes and through deterioration from aging and wear. Control parameters are specified by the designer to minimize noise parameters that may or may not change the cost of the product. Tolerance factors are control parameters that affect costs.

Orthogonal array techniques reduce the number of parametric variations and combinations required to determine the most optimum control parameter adjustment. Tolerance design may be necessary to bring the performance to target. Tolerance design trades off quality loss due to variations in performance with increased cost to tighten manufacturing tolerance and cost of higher grade materials and components. Parameters independent of quality characteristics are adjusted to cost, reliability, operations, or other quality considerations. Robust design aims for the widest operating environment, low grade components and materials, and widest manufacturing tolerances and variations for least product cost and operation.

By selection of optimal parameters of manipulator following benefits will be achieved. The benefits will be in terms of quality of the manufactured product and the improvement in robot performance during the operation. The robot will be capable of performing the task with less variation. In processes that require a minimum of variations while performing task, robots will be able to perform these operations with greater uniformity than human workers. The processes can be fine tuned to operate at optimum condition, and these conditions contribute to higher quality. For processes that require consistency in the motion pattern e.g. spot welding, spray painting, robots will accomplish these tasks with more repeatability than humans.

1.5 PROBLEM FORMULATION

As mentioned in the preceding section, robots are commanded to perform a task in workspace. For performing a task within workspace, different type of trajectories are needed to be selected depending on the requirement. But the desired performance is not achieved because of presence of various forms of inaccuracies in parameters of manipulator. These inaccuracies are very difficult though not impossible to quantify and model.

Therefore, available dynamic model using Lagrange-Euler method is used to simulate the performance of manipulator. The performance obtained from simulation would be deterministic in nature and devoid of effect of inaccuracies of parameters. Therefore, to simulate real life performance without help of prototype become a challenge in itself. To assuage this difficulty it is decided to develop a mathematical approach by which real life performance can be simulated and modify the available techniques to analyze and optimize the performance of manipulator. While attempting this, available literatures are thoroughly reviewed to obtain the gaps and to avoid any duplication. It is observed that because of non-availability of proper mathematical approach for simulation of real life performance, the effects of kinematic and dynamic parameters and their tolerances on performance have not been studied. The use of design of experiment techniques to analyze, interpret correctly and optimize the performance has been observed to be nonexistent. The possible reason for no attempt could be technical difficulty and lack of awareness of the robot system designer.

Among the performance of the manipulator accuracy and repeatability are most important performance. There are many literatures available in the direction of accuracy reductions of manipulators. Techniques like robot calibration and compensation are used to reduce this type of error. But there are few attempts in the direction of repeatability reduction of manipulator. Except identifying the sources of poor repeatability there are almost no attempt to minimize this type of performance variations. Present thesis challenges the prevailing idea and attempts to devise methods by which performance variations can be reduced. This advocates additional effort in the direction of segregating the contributions of performance variations. In this pursuit, the various performance measures proposed by researchers are introduced to investigate the contribution of various design parameters and its tolerances on manipulator performance variations.

There are almost no attempts to reduce the performance variations of manipulator by selecting optimal kinematic and dynamic parameters in the published literature which has been reviewed in next chapter. The major hindrances come in this direction because of non-availability of suitable simulation methodology to simulate real life performance of manipulator. Likewise another difficulty arises in terms of suitable optimization strategy to select optimal kinematic and dynamic parameters. Since the use of simulation and optimization based approach, prior to costly manufacture of manipulator will reduce the overall cost, present thesis attempts to develop a suitable simulation method and modifies available optimization method to provide suitable tools to the designers. There have been several performance measures proposed by researchers to take care of kinematic and dynamic performances but none addresses accuracy and repeatability while performing task. This does not allow critical analysis of performance of manipulator within the workspace. Therefore, the present work develops approach to simulate the performance of manipulator and uses design of experiments technique to find statistically significant parameters which contributes the most to performance variations, and finds optimum combinations which is insensitive to effect of noise parameters. The work explores the effect of various parameters on performance. Design parameter tolerances also contribute to the performance variations of the manipulator, therefore design of experiment approach is used to investigate the statistical significance of parameter tolerances. Apart from above approaches there are almost no attempt to find optimal parameters based on performance of manipulator using conventional and non-conventional optimization approaches. As it is difficult to model the performance of manipulator in terms of design parameters, to obtain the optimal design parameters non-conventional (evolutionary) optimization technique is applied. The proposed approach is another off-line simulation optimization based approach which compliments the discussed design of experiment approach and searches for optimal solution with in the parameter bounds rather than at the corner points.

Investigations for optimal design which demands the introduction of performance variations reduction in manipulator design are almost nonexistent. The present work divides the investigation of effects of parameters and tolerance of manipulator on performance measure and optimal design in following steps:

- Attempt to simulate the performance using heuristic search based approach and finding optimal solutions.
- Development of probabilistic approach to simulate the performance of manipulator.
- Use of Combined array and Fractional factorial design of experiments approach to screen the parameters.
- Classification of parameters i.e. parameters for which investigations need to be pursued and no further investigation should be pursued.
- Development of dimensionless indices and exploration of effect of change of these indices on performance measure.
- Exploration of the optimal dimensionless number combination for optimal performance measure.
- Exploration for the optimal design parameter for robust design using response surface methodology (RSM).

- Development of second order response equation for manipulator for performing a task following a particular trajectory.
- Development of mean and variance of performance equation in parametric form.
- Formulation of optimization problem using both mean and variance of performance equation.
- Use of MATLAB *fmincon* optimization routine to obtain the optimal design parameters.
- Use of cross array design of experiment approach to obtain optimal control factor tolerance and validation using Monte Carlo simulation approach.
- Exploration of parametric tolerance sensitivity.
- Development of hybrid optimization technique which takes real life performance into consideration in optimization process.
- Conclusions and future perspective.

1.5.1 Thesis Outline

Conventional optimization paradigms are focused on maximizing or minimizing a certain performance measure of a system subject to some design constraints. In this, traditional view of optimization, the system design process is implicitly assumed to be deterministic in that is for a given input, the system will always produce the same output. In reality, however, this is not the case. There is a lot of uncertainty and variability involved in the design process that affects the system either directly or indirectly. Because of this uncertainty, the output performance of the system will be different from that predicted deterministically, which translates into an inferior system. So the problem is how to optimize a system given all the uncertainties?

There are many sources of uncertainties that can influence a system. The variability in the inputs to the system is an obvious source of uncertainty. For an example an automotive power train system, this may include dimensional variation in the power train components, variation in the material properties, etc. The environment and operating condition under which the system is used is also another source of uncertainty. Weather (snow, rain, summer time, etc.) and road condition (city roads, desert, mud, etc.) are examples of this type of uncertainty. In a simulation-based design process, the discrepancy between the mathematical models and the real world system is an important source of uncertainty. These issues are highly relevant to real-world engineering design practice, and their applications will have a positive impact on the product design and development strategy.

A model of product metric is simply a representation, simplification or estimation of a product's realization to aid in making product decisions. Trial and error approaches to improve quality of product are too costly and risky. Effective models must be developed during the product development process so that experiments can be conducted. With the use of quantitative model it is easier to understand how a product will perform under all circumstances and operating conditions.

The task of robust design is to select a best set of nominal configuration parameters that satisfies the performance specifications with minimum deviation due to manufacture, material, or use variations. A product is considered as a black box input-output model. The product uses material, information, and energy to produce output.

In Chapter 2, the reviews are organized chronologically in two parts. The first part presents the review on approaches, techniques and design procedures developed to improve the performance of robot and the second part discusses robust design techniques. This chapter highlights the development of methodology and approaches, over the years to improve robot performance. In later stage this helped finding the gap and formulating the problem.

In Chapter 3, the initial attempt to develop a heuristics search based method to simulate the performance incorporating effect of noise on manipulator is discussed. For optimal parameter design full factorial design of experiment approach has been applied. Statistical analysis of simulated results has been carried out to identify parameters responsible for performance variations. Lastly limitations of the proposed method have been discussed.

Taking identified control and noise parameters and clue from Chapter 3, a probabilistic approach to simulate the performance is proposed in Chapter 4. The developed approach is used to screen the parameters, which have statistically significant influence on performance variability. To investigate the influence of time law of the trajectory, cubic and quintic trajectories are considered for performing different tasks. In addition to this investigation, impact of two dimensionless indices on performance of manipulator has been attempted. The performance sensitivity of manipulator to parameter change has been presented. To complement above investigation, manipulator control parameter tolerance sensitivity to performance measure has also been carried out for task following both of the time law. This investigation is similar to conducting experiments by changing one factor tolerance at a time.

In Chapter 5, identified statistically significant parameters in Chapter 4 have been used to get the optimal design parameters using response surface methodology. The response equation for mean and variance of performance are developed. Using these fitted models optimal solutions are obtained.

To reduce the variability in performance further, optimal tolerance of design factors of manipulator are searched for. For manipulator parameter tolerance design, cross array design of experiment approach has been applied in Chapter 6. The method to simulate the performance of manipulator has been discussed. Statistical analysis of simulated results has been carried out and the optimal parameter tolerances, which deliver optimum performance, have been provided. To validate the results obtained in cross array design of experiment approach help of Monte Carlo simulation method is taken. The results have been compared and presented.

To achieve similar objective as discussed in Chapter 5, an attempt has been made to obtain the optimal design parameters of manipulator using a non-conventional optimization method. In Chapter 7, differential evolution optimization approach to find the optimal parameters of manipulator is discussed. However, this method has inherent deficiencies to handle simulated performance of a system, therefore a hybrid evolutionary optimization method has been proposed. The proposed method is applied to select optimal design parameters of manipulator. The results obtained have been presented for different tasks following different trajectories.

In Chapter 8, the conclusions made in all the above chapters are summarized. Then the future perspective of the investigations has been high lighted.

1.6 CONTRIBUTION OF THESIS

The main contribution of this thesis is the development of a method for simulating the performance of manipulator and application of design of experiment technique to find optimal parameters and tolerances and development of hybrid evolutionary optimization technique. Applications of design of experiments techniques to mathematical models are rare. This thesis illustrates how design of experiments techniques is applied to mathematical models and used to determine statistically significant parameters and optimal parameters. The design of experiment and non-traditional optimization method presented are theoretically rigorous computational technique. The methods are fully applicable to any system which has coupled and nonlinear dynamic model.

The thesis maintains that design of experiments and evolutionary techniques are important concepts that guarantee optimal solution with minimal number of computation. This application uses all good aspects of design of experiments, Taguchi method and evolutionary optimization method towards robot parameter design and optimization systems.

To achieve higher performance and better design of robotic systems, the effect of the actuator's parameters on the capabilities of these systems has to be understood. Significant work has been done in this thesis to develop design strategies for manipulator designs for use in an array of applications.

CHAPTER-2

LITERATURE REVIEW

2.1 INTRODUCTION

The literature review in this chapter is organized in two parts; first is the review on techniques and design procedures to improve the performance of robot and the second is on various robust design techniques. Research trends to improve positioning accuracy and repeatability of robot as well as the techniques for their improvement through calibration process for robots are reviewed. Research in the direction of measurement and identification of parameters of robot mechanisms, as well as to compensation of positioning errors introduced by the differences between the actual robot and the idealized (nominal) kinematic robot model are surveyed. Research attempts to analyze and improve dynamic accuracy and design optimization approach have also been systematically reviewed.

It is known that successful products rely on the best possible design and its performances. These designs are usually produced using the tools of engineering design optimization in order to meet design targets. However, conventional design optimization may not always satisfy the desired targets due to the significant uncertainty that exists in material, geometrical and process parameters. Therefore, ways to minimize the effect of uncertainty on product design and performance are of paramount concern to researchers and practitioners. The objective of robust design is to optimize the mean and minimize the variability that results from uncertainty represented by noise factors. The various objective functions and analysis techniques used for the Taguchi based approaches and optimization methods are reviewed. Robust optimal design methods are reviewed extensively and application of the robust design methods to different areas have been discussed. Most applications of robust design have been concerned with static performance in mechanical engineering and process systems, and applications in robotics are rare. The robust design of manipulators with kinematic and dynamic parameter uncertainty in the robotic system is used to obtain the optimal design.

This chapter is organized in five sections. In section 2.2 different approaches to improve performance of manipulators are discussed. This section is discussed with the help of subsections. Sub-Section 2.2.1 discusses research results on robot calibrations. In this section kinematic model suitable for calibration, procedures for measurement and identification of kinematic parameters, and techniques for compensation of positioning errors introduced by idealized kinematic models are discussed. Sub-section 2.2.2 discusses outlines of local calibration approaches other than discussed above available to improve positioning accuracy are presented. Along with kinematic calibration, precise identification of parameters of the dynamic robot model has received considerable emphasis in recently published literature. The research results are reviewed in sub-section 2.2.3. Different procedures for evaluating the effect of inaccuracies in dynamic parameters on the internal accuracy of robot trajectory tracking are reviewed in sub-section 2.2.4. Similarly different research attempts in the direction of design parameter optimization to improve performance of manipulator are presented.

2.2 METHODS TO IMPROVE PERFORMANCE OF MANIPULATOR

Robotic applications have been expanding since the introduction of robots in industries in the last century. The potential increase in the productivity and quality with deployment of robots in place of human operators is often the major reason for their use in many industrial applications. The ever increasing applications not only include the manufacturing processes, but also include applications in other fields, such as space, medicine and nuclear science. One of the serious bottlenecks for newer robots is the requirement that robots must be both accurate and repeatable. The performance characteristics like accuracy and repeatability required depends on the application. The question is, can new technology be utilized that will allow one to improve the performance characteristics of robots related to the positioning accuracy and repeatability? The positioning accuracy and repeatability are characterized by the error between the tool frame and the goal frame which has been a technological barrier in the robotics industry and the industry is searching for solutions for its reduction or "elimination".

The measures to improve performance of manipulators, over the years to improve performance of manipulators, which is a very vast concept, is the research area that has received considerable attention including a publication of a specialized monograph by Moorings et al. [Mooring 1991]. Robot accuracy is influenced by a number of factors. Kochekali et al. [Kochekali 1991] classified the factors that influence accuracy into six categories:

- Environmental (for example, temperature changes),
- Parametric (variation of kinematic parameters, joint zero-reference offsets,
- Influence of dynamic parameters (drive-train compliance, friction and other nonlinearities, including hysteresis and backlash),
- Measurement (resolution and nonlinearity of joint position sensors),
- Computational (computer round-off and steady-state servo errors), and
- Application (installation errors, and workpiece position and geometry errors).

Analysis of their influence and elimination of causes is a subject of active research aimed at improving both kinematic and dynamic performance of a manipulator leading to better accuracy and repeatability.

The research trends to improve performance have been categorized by different approaches as given below.

(i) Kinematic Calibration,
(ii) Local calibration approaches to improve positioning accuracy of manipulator,
(iii) Identification of dynamic parameters,
(iv) Influence of dynamic parameters to trajectory tracking accuracy,
(v) Design optimization of manipulator parameter.

2.2.1 Kinematic Calibration

Robot calibration is a process by which robot accuracy can be improved by modifying the robot positioning software rather than altering the design of the robot or its control system. The term calibration assumes a set of procedures aimed at improvement of robot accuracy by software without changing the mechanical structure or robot control system [Roth 1991]. For precise identification of robot parameter values, the procedure involves developing a model whose parameters accurately represent the real robot. Next, specifically chosen features of the robot are accurately measured. This step is followed by identification procedure to compute those parameter values, which when input in the robot model accurately reflect the measurements made.

Robot calibration makes possible the implementation of off-line designed computer integrated systems. Large time-saving is possible and costly mistakes can be avoided when the robot task can be planned and simulated off-line. Another important utility of robot calibration is the calibration for improvement in motion control and simulation. Precise identification of geometric and inertial parameter values is important for accurate control and simulation of the robot motion.

Kinematic calibration is a process which involves identification of functional relationship $f(.)$ between joint sensor readings $q = [q_1, q_2, ... q_n]$ and the transformation T_E that describes the actual position and the orientation of the robot end-effector. Application of calibration procedure is discrete event, contrary to control schemes where model identification is carried continuously. A significant number of researchers have published their work in this last few years, were mostly devoted to analysis of models suitable for calibration purposes.

Generally kinematic model-based calibration is considered as a global calibration method that improves robots accuracy across the whole volume of robot space. Kinematic calibration consists of four sequential steps: Modeling, Measurement, Identification, Compensation or Correction.

Modeling - A kinematic model is a mathematical description of the geometry and motion of a robot. It is primarily definition and selection of a suitable relationship $f(.)$.

Measurement - The measurement phase involves workspace sensing of positions of the end effector or tool of the robot e.g. physical data collection for selected set of measuring configurations,

Identification - Parameter identification involves numerical methods. The methods must be reliable enough so that a solution can be reached while maintaining a reasonable level of confidence in the resulting identified parameter values e.g. determination of model parameters so that the error between the measured and the modeled pose is minimized.

Error compensation - This is the final and decisive step in robot kinematic calibration, which is the implementation of the new model in the position control software of the robot. Sometimes referred as the correction step e.g. implementation of the identified model in the robot positioning software.

High positioning accuracy with low robot velocities is important, so to solve this problem adequate static robot model had to be selected for calibration [Duelen 1991]. Roth had proposed a hierarchical classification in which the three modeling levels, e.g. three calibration levels were separated (1) Joint level calibration, (2) Kinematic model calibration and (3) Non-geometric (non-kinematic) calibration. Among the enumerated factors, the maximum attention was given to calibration of geometric parameters, so that the static robot model is often simply called the kinematic model.

Whitney [Whitney 1986] had presented a forward calibration method for serial link manipulators. Authors used a model whose parameters represent link lengths, joint encoder offsets, the relative orientation of consecutive axes, and experimentally found the effects of joint compliance, backlash and gear transmission errors. Later Veitschegger and Wu [Veitschegger 1988] had developed a method to select a set of independent errors for modeling geometric errors in a manipulator’s structure. A method for calibration of a robot to correct position and orientation errors due to manufacturing has been presented by Boderick and Cipra [Boderick 1988]. It was based on the shape matrix of robot kinematic model description where each joint was individually and successively moved in order to explicitly calculate the shape matrix of each link.

The goal of modeling is to develop the simplest possible model which describes the phenomenon under study with desired level of accuracy. Effects of the model complexity on resulting robot accuracy are analyzed by a number of researchers. The methods of calibrating and compensating for the kinematic errors in robot manipulators have been discussed. Ha et al. [Ha 1989] had proposed an approach to identify model parameters in which neither prior knowledge of the geometric parameters nor restrictive robot motions were required. The number of the model parameters to be identified was minimized through a regrouping procedure for the Lagrangian functions of robotic manipulators. Vira and Shiferaw [Vira 1989] had presented a higher-order approximation of the "generalized" kinematic error compensation model to enhance position accuracy and repeatability of robotic manipulators. The "generalized" model originally proposed by Driels and Pathre was successfully extended to include non-linear coupling effects among all error parameters. Judd have presented models to calibrate industrial robots [Judd 1990]. The models developed, corrects problems with robot accuracy resulting from errors in the link and joint parameters, imperfection in the main spur and encoder pinion gears and structural deformations. Driels [Driels 1993] had attempted to improve the accuracy of a robot manipulator arm through the use of a calibration process. Subsequently Zak et al. [Zak 1993] had provided the necessary tools for the optimization of the calibration process, where a computer simulation of robot calibration and a systematic method for the evaluation of this calibration had been developed. They presented the generalized method for simulating the robot calibration procedure and evaluating its performance in terms of the robot's expected accuracy after the calibration. Goswami and Bosnik [Goswami 1993] expressed end-effector pose of a robotic manipulator with respect to a reference coordinate frame in terms of nonlinear mathematical functions involving the link parameters and joint variables. Karan and Vukobratovic [Karan 1994] had discussed recent research results in the field of calibration and accuracy of kinematic and dynamic models for manipulation robots. Driels and Swayze [Driels 1994] had reported research related to methods used to provide partial pose data for robot calibration tests. Rather than focusing on traditional precision measurement techniques, they discussed calibration using end point motion constraint of various kinds. Kaizerman et al. [Kaizerman 1994] had proposed a new inverse kinematic calibration method, model based method where a first order relationship was obtained between the errors in the joint encoder readings and the necessary corrections to the kinematic parameters of the robot model.

The basis of every calibration model is an adequate kinematic model, which should be able on one side to express deviations due to errors in construction and assembly of the robot links and the errors in setup of the zero-reference readings of the joint encoders, on the other side to describe propagation of errors due to nonlinear phenomena. A systematic methodology to perform the error analysis of serial link manipulators had been proposed by Mavroidis et al. [Mavroidis 1998] and its application to the patient positioning system was described. Experimental measurements that verified the validity of the method were shown. Abderrahim and Whittaker [Abderrahim 2000] have presented methods to improve the off-line programming capability of industrial robots by improving their accuracy. Rather than imposing more strict manufacturing tolerances, it was widely accepted that a method of identifying kinematic parameters specific to each individual robot provides a cost effective way of improving accuracy. Gong et al. [Gong 2000] had investigated the effect of geometric errors, link compliance and temperature variation on robot positioning accuracy. The comprehensive error model was derived for combining geometric errors, position–dependent compliance errors and time–variant thermal errors. A general methodology was also developed to identify these errors simultaneously. Ouafi et al. [Ouafi 2000] had presented a new approach designed to improve the accuracy of multi-axis CNC machines through software compensation of geometric, thermal and dynamic errors. Based on a multi-sensor monitoring system, the proposed compensation scheme was built to ensure error prediction. Sultan and Wager [Sultan 2001] had used the independent-axis technique for the analysis with new mathematical models proposed to overcome the drawbacks of the existing methods. Moreover, the techniques employed here result in the prediction of transmission error functions and the modeling of the joint motion dependencies. Jang et al. [Jang 2001] had pointed that inaccurate positioning of the robot end effector causes joint deformation as well as geometric errors when an industrial robot has a payload at its end effector. Veryha and Kurek [Veryha 2003] had presented a method of robot end-effector pose accuracy improvement using joint error mutual compensation. The developed method had allowed locating special robot configurations with the highest robot end-effector pose accuracy using joint error maximum mutual compensation.

Robot calibration technique is usually employed for evaluating the positioning accuracy of robots. In this method, one measurement is recorded each time the movement stops at a target position along the axis under test. As can be noted, this calibration technique does not consider the real mode of operation of the machine. The accuracy performance of the controller and measuring system of the machine depend on the way in which the machine moves to the target positions. Dynamic effects which are inherent during machine working affect the positional errors of the slide. Furthermore, this technique may be very time-consuming and labour intensive and may make the whole calibration process costly.

Calibration itself is costly, and it causes the additional loss due to stopped production. On the other hand, any delay in idling the production process in order to perform calibration increases the probability of producing nonconforming units. Finally, it is difficult to judge which method is better since the relative contribution of geometric and non-geometric errors to robot accuracy vary from one robot to another.

2.2.2 Local Calibration Approaches to Improve the Positioning Accuracy

The described kinematic calibration techniques can be classified as global techniques. Although they are suitable in situations where it is anticipated that the robot operation will be performed in a wide workspace, they are generally characterized by the long duration of the calibration procedure and the high complexity of the compensation model.

An alternative approach is to utilize local calibration techniques. In these techniques, rather than modeling various robot error sources explicitly, positioning errors were measured in localized regions, relative to workpieces, and such measurements were used for creating local inaccuracy distribution equations by using regression or weighted models. The local models are less complex than the global models, and they can provide a good basis for automatic calibration.

Vertutt and Liegeois [Vertutt 1981] had presented an approach to analyze the essential qualities of manipulators such as position, orientation, force, acceleration and stiffness. These properties were displayed graphically in a condensed form, which allowed the designer to evaluate easily the influence of the various design parameters upon the capabilities of a projected arm to perform expected classes of tasks. Kumar and Waldron [Kumar 1981] developed a mathematical model of the random positioning errors of mechanical manipulators. This model was applied to a computer program to plot the generating curves of equal error surfaces for given manipulator geometries. Veitschegger and Wu [Veitschegger 1986] had presented detailed model that was applied to consecutive parallel joints and included the second-order terms. By comparing the results of the linear model and the second-order model, the accuracy of the linear model was evaluated for a given manipulator and range of input kinematic errors. Colson and Perreira [Colson 1985] had presented generalized set of performance criteria, not biased towards any particular robotic system and described the relationship of these measures to satisfactorily develop online and offline programs for a robotic system to perform an operation. Chen and Chao [Chen 1987] had identified and parameterized the sources that contribute to positioning accuracy and estimated the values of the parameters. Benhabib et al. had presented task-space tolerances as geometric pseudo-volumes which can easily be adopted by any industrial robot [Benhabib 1987]. The algorithms developed for the “direct” and “inverse” error analysis was used as independent software modules in any computer- aided design package for robots. The feasible joint-tolerance domain concept introduced herein provides the designer with the flexibility of specifying different tolerance requirements for the joints and selecting an optimal set of tolerances. Azadivar [Azadivar 1987] had studied the effect of manufacturing errors on the accuracy of the operations using a stochastic model and suggested a procedure for determining the optimum position error in a given manufacturing situation. Chen and Chao [Chen 1988] had examined error sources that contribute to positioning accuracy of robots with rotary joints. The effects of errors were parameterized and measurement data were fitted to obtain the values of these parameters. It was concluded that with sufficient but not exhaustive detail in the error modeling the differences can be reduced significantly from 5.9 mm mean error with nominal model down to 0.28 mm mean error after error compensation. Bhatti and Rao [Bhatti 1988] had introduced reliability as a probabilistic measure of manipulator kinematic performance.

A complementary area to the identification of kinematic error parameters is study of the statistical performance of robot positioning and statistical analysis of relationship between the errors in parameters of the model and the performance of the robot. Stone and Sanderson [Stone 1988] had applied Monte Carlo simulation techniques to investigate the influence of encoder calibration errors and machining and assembly errors to different robot performance characteristics. Starting from a linear error model and assuming independent and normally distributed geometric errors, Menq and Borm [Menq 1989] had shown that the contour of equal probability of position errors (positioning error envelope) forms an ellipsoid. The probability with which the position error lies within the ellipsoid was governed by the two distributions. They proposed five error measure indices to statistically quantify the error ellipsoid.

Jiang et al. [Jiang 1989] had outlined robot process capability problem and defined the terms. Two methods were used to analyze robot process capability data: the randomized complete block design method and the Taguchi based experimental design methods. The purposes of the study was to compare the results obtained by these two methods and determine which method was better for analyzing the data to be used in determining/optimizing a robot's process capability. Wang and Roth [Wang 1989] had studied the effect of the clearances on the position errors. The study was conducted to find influence of joint errors on manipulator and mechanisms accuracy. Lee and Woo [Lee 1989] had investigated the effect of geometrical uncertainty and the probabilistic tolerance volume due to joint errors. For this purpose, linear mapping from Δq space to Δd space through Jacobian matrix was analyzed probabilistically. Gosselin and Angeles [Gosselin 1991] had presented a novel performance index for the kinematic optimization of robotic manipulators. The index was based on the condition number of the Jacobian matrix of the manipulator, which was known to be a measure of the amplification of the errors due to kinematic and static transformations between the joint in Cartesian spaces. Lee and Gilmore [Lee 1991] had presented a probabilistic model and methods to determine the means and variances of the velocity and acceleration within stochastically defined planar pin jointed kinematic chains. The presented model considered the effects of tolerances on link length and radial clearance and uncertainty of pin location as a net effect on the link's effective length. Shing and Loon [Shing 1992] had performed experiment and statistical analysis to evaluate the repeatability of a SCARA robot and proposed an alternative form of specification for repeatability. Gosselin [Gosselin 1992] had presented new dexterity indices that were applied to planar and spatial manipulators. These indices were based on the condition number of the Jacobian matrix of the manipulators, which was known to be a measure of their kinematic accuracy. Rastegar and Singh [Rastegar 1994] had developed a probabilistic method that can be used to solve a number of manipulator analysis, optimal synthesis, and task placement problems. This method was used to check whether the task requirements were met for a given manipulator and it was extended to the problems of determining kinematic parameters for which the task requirements were most closely met. Liu and Wang [Liu 1994] had assessed positioning accuracy of robot end-effectors using reliability approaches. The reliability provided a statistical means for representing robot accuracy. Both linear regression and iterative Taylor series methods were used to deal with limit states in reliability calculation. Kamrani et al. [Kamrani 1995] had developed a graphic simulation system for robotics operations, in conjunction with a kinematic error model to assist management in making investment decisions. Taguchi-type experimental design was used to predict the robot process capability. Vukobratovic and Borovac [Vukobratovic 1995] had developed accuracy portrait for specified deviations of links parameters, showing the gripper deviation over the whole working volume. In addition, by changing links parameters deviations, whole families of portraits with the accompanying deviations were obtained. This information was used in the stage of the mechanism assembly to select links with appropriate deviations to ensure a desired accuracy portrait. Singh and Rastegar [Singh 1995] had presented a method to represent and measure the global manipulability of manipulators. The method was based on a new concept, referred to as the global velocity ellipsoid, which represents the global motion manipulability or the velocity transmission characteristics of a manipulator. Doel and Pai [Doel 1996] had introduced formalism for systematic construction of performance measures of robot manipulators in a unified framework based on differential geometry. Dhillon and Fashandi [Dhillon 1997] presented an overview of the most suitable robot safety and reliability assessment techniques.

Ouezdou and Regnier [Ouezdou 1997] had dealt with the kinematic synthesis of manipulators. This method was based on distributed solving and used to determine the dimensional parameters of a general manipulator which will be able to reach a set of given tasks specified by orientation and position. Jiang et al. [Jiang 1997] had compared the results of accuracy from measurements of two systems and determined the degree of agreement and used the Taguchi method, as a basis of comparison analyses. In doing so, the accuracy of the robot under various operating conditions (load, speed, distance, direction, orientation, starting point and height) can also be determined. Edan et al. [Edan 1998] had developed a three dimensional statistical evaluation framework for performance measurement of robotic systems. A specific experimental setup was designed, implemented and evaluated for different robot characteristics (velocities and target locations) and for a specific task. A statistical analysis method to evaluate the performance was demonstrated. Riemer and Edan [Riemer 2000] had evaluated the influence of target location on robot repeatability. An experiment was set up to analyze the effect of the three-dimensional target location on robot repeatability. Zhu and Ting [Zhu 2000] had established a general probability density function (p.d.f.) of the endpoint of planar robots. The p.d.f. of the endpoint of a planar robot was equivalent to that of endpoint of a string of planar joint deviation vectors. Carrerasa and Walker [Carrerasa 2000] had applied interval methods to obtain significantly improved robot reliability estimates via fault trees for the case of uncertain and time-varying input reliability data. In this paper different parameterization strategies were evaluated in order to gain a complete understanding of the potential benefits of the approach. Kalanas and Kota, [Kalanas 2001] had proposed a method in which intermediate precision positions were expressed as a distribution. This method expanded the resulting set of acceptable solutions by adding an extra dimension to Burmester solutions i.e., Burmester surfaces instead of Burmester lines.

2.2.3 Identification of Dynamic Parameters

While considering the case of trajectory tracking, new factors appear, influencing primarily the internal trajectory tracking accuracy. An important source of the internal tracking inaccuracy is an imprecise knowledge of the dynamic robot model: inertia parameters of the robot links and the workpiece held by the robot, actuator parameters, and deformation of the mechanism links caused by dynamic forces. By separating the mechanism dynamics and actuator parameters, influencing the internal trajectory tracking accuracy, it can be said as "dynamic accuracy", i.e. the accuracy of the dynamic robot model. On the other hand, accurate knowledge of dynamic parameters is of importance only when a high-quality tracking of fast trajectories is demanded, and this is more and more the case in robotic practice today. However, the assessment of the inaccuracy in dynamic parameters is an important factor in evaluation of the quality of robust control strategies. Namely, the degree of influence of the accuracy of the dynamic robot model to the tracking accuracy depends on both the mechanism structure and the applied control algorithm.

There exist several methods, in the scope of which the problem of robot manipulator accuracy is solved by using various control laws. Various control systems are introduced enabling good quality of system functioning under the action of significant uncertainties in the system itself and the working environment, hereby enabling the compensation of the influences of large irregularities during system work. The goal of the synthesis of intelligent control systems is similar to the case of conventional adaptive control algorithms. Similarity lies in the fact that the knowledge about the system is acquired directly during system operation by means of the learning processes, and the difference is that with intelligent control systems the uncertainty degree can be reasonably higher than that which can be tolerated in the case of adaptive control algorithms. Working requirements set in advance, demand supplementary functions of the control system, as a perception of the working environment, associative reasoning under the action of uncertainty, learning, knowledge generalization and using experience, decision making process on several levels, etc.

For dynamic control of a robot the central issue is to determine generalized forces that should be applied at robot joints in order to compensate dynamic forces in the robot system. Dynamic forces τ are considered as functions of joint sensors readings of positions q, velocities $\dot{q}$ and accelerations $\ddot{q}$:

$$\tau = f(q, \dot{q}, \ddot{q}, \alpha, \beta) \qquad \textbf{\textit{(2.1)}}$$

where α is the vector of kinematic parameters and β is vector of dynamic parameters.

Most of the effort in calibrating dynamic robot models has been devoted to identification of inertial parameters of robot links. The degree of uncertainty in dynamic parameters, especially in inertial parameters of links, is an important factor in judging the robustness of model-based control strategies. Besides off-line estimation procedures of inertial parameters of robot links should be preferred over adaptive control schemes, since parameters of links do not change once the manipulator is fabricated.

The problems of generating persistently exciting trajectories for parameter estimation have been addressed by Khosla et al. [Khosla 1989]. To achieve the goal they had proposed an algorithm that categorizes the dynamic parameters of manipulator into three classes: uniquely identifiable, identifiable in linear combination form and unidentifiable.

Wiens et al. [Wiens 1992] had defined a global measure of the indices and correlated quantities such as workspace volume and its reach. A complete investigation of how the scalar indices characterize inertial changes due to changes in geometric parameters (such as twist angles that may be caused by misalignment or bent shaft, etc) was presented. The study was based on the Eigen values, and their derivatives (sensitivities), of the generalized inertia matrix as indicators of performance measures. Deck and Dubowsky [Deck 1994] had presented work on the development and experimental verification of analytical models for the design of dynamic systems. Nenchev et al. [Nenchev 1997] had analyzed motion at direct kinematics singularities for a broad class of parallel manipulators based on the singularity-consistent parameterization framework. The aim was to present a motion feasibility study at and around direct kinematics singularities which were relevant to a broad class of parallel manipulators. Reyes and Kelly [Reyes 1997] described the experimental evaluation of three identification schemes to determine the dynamic parameters of a two degrees of freedom direct-drive robot. Samak and Gupta [Samak 1998] had presented an efficient dynamic formulation for modeling and control of realistic six degrees of freedom robot manipulators. The basic dynamic formulation was based upon Kane's approach. Tu and Rastegar [Tu 1999] had developed a method using the trajectory pattern to determine the inherent characteristics of the nonlinear dynamics of open-loop chain robot manipulators with rigid links. The method was based on the selection of a basic trajectory pattern and examining the corresponding inverse dynamics model.

Antonelli et al. [Antonelli 1999] had aimed at setting up a complete and systematic procedure for the identification of dynamic parameters of open-chain rigid manipulators. The procedure was articulated in the main points needed for the identification for a generic open-chain manipulator: model derivation, design of the identification trajectories, estimation algorithm and model simplification. Vukobratovic and Filipovic [Vukobratovic 2000] had developed the error model of tracking trajectories using a dynamic control law. By using the inverse dynamics method a control law was formed, into which the robot dynamics model was included. The sensitivity functions, for the analyses of the variations influence on the dynamic robot parameters of the trajectory tracking accuracy were given. Chan [Chan 2001] had proposed an efficient identification method for estimating the lumped parameters of robot manipulators including the characteristics of drive dynamics. The nonlinear and coupled dynamics of the robot were formulated into a form, which was linear in terms of a suitably selected set of equivalent manipulator parameters. Poignet et al. [Poignet 2003] had dealt with the application of interval analysis for outer bounding the physical parameters of parallel robots. In this paper, the problem of dynamic robot parameter estimation was expressed with a model which was linear with respect to the physical parameters. Burkan and Uzmay [Burkan 2003] had presented a new robust control law for robot manipulators subjected to uncertainties. Stability of the system was established by the Lyapunov function, and a control law that guaranteed the system stability was derived as a result of analytical solution.

Castro and Burdekin [Castro 2003] described a method for evaluating the positioning accuracy of machine tools and coordinate measuring machines under dynamic conditions. It was based on the Hewlett Packard 5519A laser interferometer, which was capable of performing dynamic calibration. This method used the A-quad-B pulses from the machine encoder as the position trigger signals, thus enabling to make measurements "on-the-fly". Stoenescu and Marghitu [Stoenescu 2004] had investigated the effect of prismatic joint inertia on dynamics of planar kinematic chains with friction. The mathematical model of a planar kinematic chain was developed using Lagrange's equations.

The directions of problem solving by means of very precise robots realization belongs to another area of research. Such robots are specific in the sense that their mechanical configuration is simpler, but that means they are more capable of producing higher accuracy in trajectory tracking.

These approaches to solve parameter identification and control problem are limited by several reasons. Technological possibilities to produce precise robot parts are among the most difficult challenge. Therefore, another group is the methods which model the causes of poor robot performance and analyze the influence of the deviations of the real values of the model parameters from the nominal onto the accuracy of robot trajectory tracking.

2.2.4 Influence of Dynamic Parameters to Trajectory Tracking Accuracy

Opposite to kinematic calibration, precise identification of parameters of the dynamic robot model received considerably less concern in the recent literature. Lack of interest in dynamic calibration can be described partly because of the fact that high positioning accuracy is usually required only in static or quasi-static conditions. On the other hand, accurate knowledge of dynamic parameters is of importance only when a high-quality tracking of fast trajectories is demanded, and this is still rarely the case in industrial practice today. The works concerning to modeling the sensitivity of trajectory tracking to variations of links inertial parameters and actuator parameters have been discussed. Although the results cannot be used directly to calibrate the robot, these can give an insight into the influence of the dynamic model parameters.

Shiller and Dubowsky [Shiller 1991] had presented a method for computing the time optimal motions of robotic manipulator that considered the nonlinear manipulator dynamics, actuator constraints, joint limits and obstacles. Muthuswamy and Manoochehri [Muthuswamy 1992] had dealt with a computer based methodology for the synthesis of an optimal tool path for robot manipulators in the presence of obstacles and singularities of the workspace. Paredis and Khosla [Paredis 1993] had dealt with two important issues in relation to modular reconfigurable manipulators, namely, the determination of modular assembly configuration optimally suited to perform a specific task and the synthesis of fault tolerant systems. Lu et al. [Lu 1993] had presented a practical method for determining the dynamic parameters of robotic arms as a closed chain mechanism. Tu and Rastegar [Tu 1993] had used Monte Carlo method to solve a number of manipulator link shape design, task space, and obstacle placement problems. Using this procedure shape of links of manipulators to operate within geometrically specified enclosures was determined. Shiller and Chang [Shiller 1995] presented a method for reducing the tracking errors of articulated systems, moving along specified paths at high speeds. It consists of preshaping the reference trajectory to account for the dynamics of the feedback controller. The trajectory was assumed to be feasible, satisfying the known manipulator dynamics and the actuator constraints. The correction term, added to the nominal trajectory, was computed by filtering the nominal control inputs through the inverse of the feedback controller.

Madrid and Badan [Madrid 1997] had proposed an on-line heuristic search method to solve the robot continuous-path tracking control problem. The numerical technique did not require inverse kinematic modeling of the robot and gave accurate tracking with good performance in the presence of disturbances. Pons et al. [Pons 1997] had presented a tool for identifying and quantifying nonlinear effects appearing during the motion of any manipulator, the nonlinear performance index. The index takes into account not only the geometrical parameters defining the manipulator but also its structural dynamics through the use of inertial parameters like mass, inertia, centre of mass. The index can be used in the design stage for analyzing and reducing these undesirable nonlinear effects in any general motion or in the trajectory planning looking for paths along which more precise control was expected. Duleba [Duleba 1997] had considered the minimum cost trajectory-planning problem with fixed time in robot manipulators. The task was solved by transforming the problem to a set of free right-end time optimal problems leading to a sub-optimal solution. Wredenhagen and Belanger [Wredenhagen 1998] had examined robot performance about a target point where the linearized robot dynamics was used. The method introduced a task based index characterized in terms of a linear quadratic cost criterion for a class of robots. Galicki and Ucinski [Galicki 2000] had presented an approach to planning time-optimal collision-free motions of robotic manipulators. It was based on using a negative formulation of the Pontryagin Maximum Principle which handles efficiently various controls and/or state constraints imposed on the manipulator motions. Constantinescu and Croft [Constantinescu 2000] had presented a method for determining smooth and time-optimal path constrained trajectories for robotic manipulators and investigated the performance of these trajectories through both simulations and experiments. The desired smoothness of the trajectory was imposed through limits on the torque rates. The third derivative of the path parameter with respect to time, the pseudo-jerk, was the controlled input. The limits on the actuator torques translate into state-dependent limits on the pseudo-acceleration. Rao and Bhatti [Rao 2001] had presented techniques to compute the kinematic and dynamic reliabilities of the manipulator. The effects of tolerances associated with the various manipulator parameters on the reliabilities were studied. Szkodny [Szkodny 2001] had focused on sensitivity analyses of position and orientation coordinates of manipulator gripper to errors of kinematics parameters. In addition, it highlighted the sensitivity analyses of mass forces to errors of dynamics parameters. Zribiy et al. [Zribiy 2001] had developed a variable structure control scheme for constrained robots. This control law ensured the asymptotic convergence of the position errors to zero and the boundedness of the force tracking error. The control scheme required a few online computations and thus can be easily implemented. In this manner, the number of model parameters to be identified was greatly reduced. Sharma and Mittal [Sharma 2001] had investigated the effect of design models on control performance of the manipulator. Different designs model were obtained by choosing different dimensional combination of link lengths in a two-degree of freedom robotic arm. Furukawa [Furukawa 2002] had found the control inputs, which lead the system from the initial state to a desired terminal state without determining the path. The performance of the system was dictated by the governing equations of motion. Therefore, if a certain cost functional was defined, it was advantageous to derive the trajectory that can result in an optimal value for that cost functional, without defining, (i.e. restricting) the path.

Khoukhi et al. [Khoukhi 2002] had considered the problem of optimal control design for off-line programming in educational assembly robotics. A new general algorithmic based simulation tool for the computer aided design of control systems and off-line programming for a large class of educational and industrial manipulators had been developed. Sergaki [Sergaki 2002] had proposed an innovative method, which achieves robot speed-control requirements, with simultaneous minimization of total electromechanical losses, while the drives follow the desired speed profiles of the robot joints under various loads and random load disturbances. Saramago and Ceccarelli [Saramago 2002] had presented a general methodology for the off-line planning of optimal trajectory of robot manipulators by taking into account the grasping forces in the manipulator gripper. Sergaki and Stavrakakis [Sergaki 2002] had considered the control problem of a robotic manipulator with separately excited dc motor drives as actuators. An innovative method was proposed which achieves robot speed-control requirements, with simultaneous minimization of total electromechanical losses, while the drives follow the desired speed profiles of the robot joints under various loads and random load disturbances. Saha [Saha 2003] had proposed a decomposition method for the generalized inertia matrix of an n-link serial manipulator for the simulation of industrial manipulators. Rodriguez and Weisbin [Rodriguez 2003] had summarized a new analytical method to conduct quantitative analysis of human-robot systems. The method was applicable to a broader class of systems whose performance needs to be evaluated.

Antonelli et al. [Antonelli 2003] had proposed technique based on second order inverse kinematic algorithm, which enables the handling of velocity and acceleration constraints while the desired end effector path. A new closed loop inverse kinematic algorithm for real time kinematic control of robot manipulators had been presented, which pursued end-effector path tracking capability in the presence of joint velocity and acceleration limits.

Simoesb et al. [Simoesb 2003] had presented an approach that characterizes the capabilities of the robotic system to produce plaster moulds and models by milling operations. Bhangale et al. [Bhangale 2004] had attempted to introduce a criterion based on the dynamics of a robot manipulator called generalized inertia matrix. The index influences both the control and simulation algorithms significantly by significantly enhancing the speed, precision, and stability of the robots. Saramago and Ceccarelli [Saramago 2004] had presented a study about the effect of numerical parameters on an optimal path planning of robot manipulators taking into account robot actuating energy and grasping forces in manipulator gripper. Shi et al. [Shi 2005] had constructed a probabilistic model for the deviation of the actual path generated by a coupler point from the desired one and presented a robust synthesis procedure of the path generating mechanism. Both the structural and mechanical errors were incorporated in the presented approach. It overcame the disadvantages of the previous works that usually dealt with these two kinds of errors. A four-bar path generating linkage was selected for numerical illustration. Waiboer et al. [Waiboer 2005] had presented the application of a perturbation method for the closed-loop dynamic simulation of a rigid-link manipulator with joint friction. In this method, the perturbed motion of the manipulator was modeled as a first-order perturbation of the nominal manipulator motion. Sharma and Mittal [Sharma 2005] had investigated about the existence of a better model among a set of model choices available for design, which gives more consistent performance than others without using a sophisticated control strategy and observed that there was a significant effect on control performance due to type of movement.

In recent times, there are increasing demands for high-quality fast trajectory tracking, and this will unavoidably introduce a need for using model-based dynamic control strategies, in which the accurate knowledge of dynamic parameters and effects of their tolerances plays one of dominant roles. Besides a systematic approach to analysis of the accuracy of kinematic and dynamic robot models it is not only a necessary prerequisite for control synthesis and evaluation of candidate control strategies, but it also offers a possibility to prescribe parameter tolerances in the design stage of the robotic mechanism, as well as to anticipate attainable accuracy, which is important in the selection of the appropriate robot for particular industrial application.

2.2.5 Design Optimization of Robotic Manipulator Parameters

To improve the performance of manipulator in various ways several available design optimization techniques are reviewed. The task that manipulator can perform, will vary greatly with the particular design. Its performance is dependent on load capacity, speed, size of workspace, and repeatability. Although robots are nominally programmable machine capable of performing wide variety of tasks, economy and practicality dictate different manipulators for particular types of tasks [Craig 1995]. The literatures, which dealt with the design of robot parameters using optimization methods are discussed below.

Vukobratovi and Kiranski [Vukobratovi 1984] had developed a computer-oriented method for sensitivity model construction of open-chain mechanism. It comprised two different calculations: (a) the sensitivity of generalized forces acting at mechanism joints and (b) the sensitivity of actuator inputs, variable parameters (mass and moments of inertia) considered to be the dynamic parameters. Tourassis and Neuman [Tourassis 1985] had proposed a novel approach, which reinforced the need to integrate the mechanical and controller designs of robotic manipulators. They proposed a conceptual framework leading to design guidelines for simplifying and reducing the nonlinear kinematic and dynamic coupling of robot dynamics. The framework was applied to illustrate the properties and structural characteristics of industrial robots. Akeel [Akeel 1985] had described an approach to design robots for high performance in automotive painting. The approach had aimed at optimizing performance within process constraints and tolerances of its parameters. Manoochehri and Seireg [Manoochehri 1990] dealt with development of a generalized computer based methodology for the form synthesis and optimal design of robot manipulators. The methodology developed was implemented and operated in two modes. Wu et al. [Wu 1991] had implemented a methodology based on

Taguchi methods to determine/optimize robot process capability for path following. The methodology consisted of the characterization of the robot path data, the experimental design, the data analysis procedure, and the verification of the results. Offodile and Ugwu [Offodile 1991] had investigated how various process variables such as speed of the tool center point and payload affect robot repeatability. The study was a simulated multi-station assembly operation done on a flat tabletop. Various combinations of the process variables were used for the study. The results of the investigation showed that these process variables affect robot performance in varying degrees, with higher speeds and weights having the most significant effect.

Kota and Chiou [Kota 1993] had used experimental design techniques that were based on statistically designed orthogonal arrays and suggested an alternate method to solve mechanism design tasks. It will also be beneficial to use this method to obtain a good starting point for the traditional optimal synthesis procedures. Chou and Sadler [Chou 1993] had proposed an alternative approach based on reducing the required level of the actuator torques so that the performance of a robot can be improved without increasing the size of the actuators. An optimization technique was developed and applied to solve the problem of the optimum placement of a robotic manipulator based on minimizing actuator torque requirements.

Shiller and Sundar [Shiller 1993] had presented a design methodology for the selection of the actuator sizes and links lengths of multi degree of freedom mechanisms for minimum and near-minimum time motions along specified paths and between given points. Lee et al. [Lee 1994] developed a method to represent the kinematic and kinetic performance of the mechanism in such a way that the performance characteristics were quantifiable analytically and visible graphically to the designer in their entirety at the conceptual design stage. Sundar and Shiller [Sundar 1994] had presented a method to design multi-degree of freedom mechanisms for near time optimal motions. The design objective was to select system parameters, such as link lengths and actuator sizes that will minimize the optimal motion time of the mechanisms along a given path. Ibrahim [Ibrahim 1996] had presented the basic idea of computer-aided design of manipulation robots based on adopted optimization criteria and on set constraints of strengths, as well as on actuator capabilities. Using complete models of manipulator dynamics, a simulation programme was derived, giving at its output the optimal design parameters of manipulator. A study was conducted to investigate the effect of a robot's geometrical parameters on its dynamic performance. Therefore, a performance measure indicator was introduced to give a quantifiable measure of the dynamic performance's response to changes in the links' geometrical parameters. Paredis et al. [Paredis 1997] had developed a rapidly deployable manipulator system combining the flexibility of reconfigurable modular hardware with modular programming tools, allowing the user to rapidly create and program a manipulator, which is custom-tailored for a given task. This article describes the main building blocks of such a system: a reconfigurable modular manipulator system, modular and reusable control software, and a novel agent-based approach to task-based design of modular manipulators. Lio [Lio 1997] dealt with the optimal design of linkages, i.e. linkages with minimum sensitivity to variations in dimensions which may be induced by environment, operating conditions, manufacturing defects, aging, deterioration, etc.

Agrawal and Veeraklaew [Agrawal 1998] had proposed a technique to identify the optimal parameters of a robot for a motion sequence between two given states in a prescribed time such that a cost functional was minimized. Coello Coello et al. [Coello Coello 1998] had presented a hybrid approach to optimize the counterweight balancing of a robot arm. A new technique that combines an artificial intelligence technique called the genetic algorithm (GA) and the weighted min-max multiobjective optimization method was proposed. Rastegar et al. [Rastegar 1999] had proposed a task-specific optimal simultaneous kinematic, dynamic and control design approach for high performance robots. This design approach was based on the trajectory pattern method and a fundamentally new design philosophy that robots, in general, and ultrahigh-performance machines, in particular, must only be designed to perform a class or classes of motions effectively. Berner and Snyman [Berner 1999] had proposed a general optimization methodology to the design of a three link revolute-joint planar manipulator performing a complicated prescribed task. In particular the end effector follows a "figure-of-eight" path. The minimization of average torque required for execution of the task was addressed and the optimization was carried out with the link lengths and base coordinates taken as the five design variables. Zhu and Ting [Zhu 2001] had presented the theory of performance sensitivity distribution and a novel robust parameter design technique. In this technique, a Jacobian matrix was used to describe the effect of the component tolerance to the system performance and the performance distribution was characterized in the variation space by a set of Eigen values and Eigen vectors. Bi and Zhang [Bi 2001] had presented a new optimization design methodology that was applicable to modular systems. This new methodology was called concurrent optimization design method. A modular robot was taken as a case study. The method was superior to the existing methods for modular robot configuration designs in the sense that traditional type synthesis and dimensional synthesis was treated once. Shiakolas et al. [Shiakolas 2002] had studied four-bar mechanism synthesis by combining Differential Evolution (DE) an evolutionary optimization scheme that can search outside the initial defined bounds for the design variables, and a newly developed novel technique called the geometric centroid of precision points (GCPP) and the distant precision point in defining the initial bounds for the design variables. Shiakolas et al. [Shiakolas 2002] had discussed optimum robot design based on task specification using evolutionary optimization approaches. These evolutionary approaches were used for the optimum design of SCARA and articulated type manipulators based on kinematic, dynamic and structural analyses.

Banka and Lin [Banka 2003] had proposed an effective top-down design approach for the mechanical design for assembly of a four degree of freedom revolute jointed robotic arm. The design process begins by specifying top-level design criteria and passing down these criteria from the top level of the manipulator's structure to all of the subsequent components. Feng et al. [Feng 2004] had presented a new optimization method for dynamic design of planar linkage with clearances at joints. The general consideration was to optimize the mass distribution of links to reduce the change of joint forces. The center position of mass and the moment of inertia of moving links were taken as the optimizing variables. Zhang et al. [Zhang 2004] had developed a method for the optimum design of parallel kinematic tool heads using genetic algorithm considering global stiffness and workspace volume of the tool heads. Lastly, Rout and Mittal [Rout 2005(v)] had attempted to design and optimize the performance of 2-degree of freedom manipulator using artificial neural network technique. Developed model was utilized for simulating the performance of manipulator.

2.3 ROBUST DESIGN TECHNIQUES

Quality as customers perceive, has many quality elements such as performance, durability, reliability, service, delivery, etc. Among these quality elements, which directly influences engineering activities is the performance. It is known that quality of a product is in-built into it at the design stage.

In the past twenty years or so, various non-deterministic methods have been developed to deal with design uncertainties. These methods can be classified into two approaches, namely reliability-based methods and robust design based methods. The reliability based methods estimate the probability distribution of the system's response assuming a known probability distribution of the random parameters, and is predominantly used for risk analysis by computing the probability of failure of a system. However, the variation is not minimized in the reliability approaches [Siddall 1984], which concentrate on the rare events at the tails of the probability distribution [Doltsinis 2004]. The robust design based methods improve the quality of a product by minimizing the effect of the causes of variation without eliminating these causes. The objective is to optimize the mean performance and minimize its variation, while maintaining feasibility with probabilistic constraints. This is achieved by optimizing the product and process design to make the performance minimally sensitive to the various causes of variation. Hence, robust design concentrates on the probability distribution near to the mean values. In this section the available robust design methods and alternative approaches are explained in brief and the work done by different researchers in this area has been classified and reviewed.

The concepts of robust design and robust design techniques are discussed in sub-section 2.3.1. The Taguchi method and other implications are discussed in sub-section 2.3.2 and 2.3.3 respectively. Taxonomical reviews of methods developed by various researchers to obtain robust design are presented in section 2.4. In this section, reviews of research using design of experiment techniques, response surface methodology, nonlinear optimization method, stochastic optimization method, multi-criteria optimization to obtain robust design are presented in sub-sections 2.4.1 to 2.4.5 respectively. Finally, conclusions drawn from this review are presented in section 2.5.

2.3.1 The Concept of Robust Design

The robust design method can be traced back to the early 1920s when Fisher and Yates [Fisher 1951] developed the statistical design of experiments (DOE) approach to improve the yield of agricultural crops in England. Their methodology had little impact on manufacturing industry. The explanation partly lies in poor communication between statisticians and engineers and partly due to non-availability of proper study and research materials [Grove 1998]. In the late 1950s and early 1960s, Taguchi developed the foundations of robust design to meet the challenge of producing high-quality products. The arrival of the "quality movement" in 1980's gave renewed impetus to industrial applications of a range of well established statistical methods including design and analysis of experiments, as well as techniques which have come to be known as statistical process control (SPC). Taguchi method is in common use in developed countries and is often applied very loosely to any industrial experiment with a statistical basis. The fundamental definition of robust design is described as a combination of parameters of product, which is insensitive to the effects of sources of variability, even through the sources themselves have not been eliminated [Fowlkes 1995]. In the design process, a number of parameters can affect the quality characteristic or performance of the product. Parameters within the system may be classified as signal factors, noise factors and control factors. Signal factors are those parameters that determine the range of configurations to be considered by the robust design. Noise factors are parameters that cannot be controlled by the designer, or are difficult and expensive to control, and constitute the source of variability in the system. Control factors are the specified parameters that the designer has to optimize to give the least sensitivity of the response to the effect of the noise factors. A P-diagram [Phadke 1989] may be used to represent different types of parameters and their relationships. The aim of robust design is to make the system response close to the target with low variations, without eliminating the noise factors in the system. The key step in the robust design problem is the specification of the objective function, and once this has been developed the tools of statistics (such as the analysis of variance) and the design of experiments (such as orthogonal arrays) may be used to obtain the solution.

2.3.2 The Taguchi Method

Taguchi method has become increasingly popular as a method for developing engineering products. It promises and delivers an ability to increase the quality of an engineered product via simple changes in the method of design. Taguchi's definition of quality is the loss imparted to society from the time the product is shipped. That is, when a product fails to perform correctly, or when it breaks down, or when some of its parts do not conform to specifications, it creates undesirable cost to society. This definition represents a significant change in the way of thinking about quality. Instead of trying to define all the good things as quality, Taguchi emphasizes on losses that a product can create.

For translating the abstract concept of 'loss to society' into an operational concept, Taguchi defined a "loss function" which assigns measurable penalties that are proportional to the distance a quality characteristic is away from its desired target value.

The loss $L(y)$ for a given product or process quality characteristic, Y, with particular a value y is defined as:

$$L(y) = k(y - T)^2 \qquad \textbf{(2.1)}$$

where k is the loss function coefficient and T is the target value or desired value.

Taguchi has proposed methods for selecting design variables so that the effect of noise parameters is minimized. By making a design more tolerant to variation, it is possible to reduce the number of rejected parts. Such a design is called "robust design". This methodology allows decisions based on total societal loss that is, to prefer only those solutions, which minimize variance in society's preferences, minimizing cost to society as a whole.

Taguchi defined robust product as one which displays low functional variability despite the influence of noise. To achieve this Taguchi' suggested division of design process into three stages: system design, parameter design and tolerance design [Phadke 1989]. System design is the design stage where the system configuration is developed, and engineering knowledge comes into play. Parameter design, sometimes called robust design, identifies factors that reduce the system sensitivity to noise, thereby enhancing the system's robustness. Tolerance design specifies the allowable deviations in the parameter values, loosening tolerances, if possible and tightening tolerances, if necessary [Fowlkes 1995]. Taguchi's objective functions for robust design arise from quality measures using quadratic loss functions. In the extension of this definition to design optimization, Taguchi suggested the signal-to-noise ratio, $(\text{SN ratio} = -10\log_{10}(\text{MSD}))$, as a measure of the mean squared deviation (MSD) in the performance. The use of SN ratio in system analysis provides a quantitative value for response variation comparison. Maximizing the SN ratio results in the minimization of the response variation and more robust system performance is obtained. Suppose there is only one response variable y and only one configuration of the system (so the signal factor may be neglected). Then for any set of control factors, x, the noise factors are represented by n sets of parameters, leading to the n responses, y_i. Although there are many possible SN ratios, only two are considered here. These SN ratios depend on the type of responses are dealt with i.e. Nominal the better and Smaller the better. The nominal the better SN ratio is discussed below.

This SN ratio quantifies the deviation of the response from the target, t, and is expressed as

$$\text{SN ratio} = -10\log_{10}(\text{MSD}) = -10\log_{10}\left(\frac{1}{n}\sum_i (y_i - t)^2\right) = -10\log_{10}\left(S^2 + (\bar{y} - t)^2\right) \qquad \textbf{\textit{(2.2)}}$$

where, S is the population standard deviation. Equation (2.2) is essentially a sampled version of the general optimization criteria given in equation (2.1). It can be noted that the second form indicates that the MSD is the summation of population variance and the deviation of the population mean from the target. If the control parameters are so chosen such that, $\bar{y} = t$ (the population mean is the target value), then the MSD is just the population variance. If the population standard deviation is related to the mean, then the MSD may also be scaled by the mean to give

$$\text{SN ratio} = -10\log_{10}(\text{MSD}) = -10\log_{10}\left(\frac{S^2}{\bar{y}^2}\right) = 10\log_{10}\left(\frac{\bar{y}^2}{S^2}\right) \qquad \textbf{\textit{(2.3)}}$$

The smaller the better SN ratio considers the deviation from zero and as the name suggests, penalizes large responses. Thus

$$\text{SN ratio} = -10\log_{10}(\text{MSD}) = -10\log_{10}\left(\frac{1}{n}\sum_i y_i^2\right) \tag{2.4}$$

This is equivalent to saying that the target is the best SN ratio with t = 0. The most important task in Taguchi's robust design method is to test the effect of the variability in different experimental factors using statistical tools. The requirement to test multiple factors means that a full factorial experimental design that describes all possible conditions would result in a large number of experiments. Taguchi solved this difficulty by using orthogonal arrays (OA) to represent the range of possible experimental conditions. After conducting the experiments, the data from all experiments are evaluated using the analysis of variance (ANOVA) and the analysis of mean (ANOM) of the SN ratio to determine the optimum levels of the design variables. The optimization process consists of two steps; maximizing the SN ratio to minimize the sensitivity to the effects of noise, and adjusting the mean response to the target response.

2.3.3 Taguchi's Method and its other Implication to Design Problems

Initially people were apprehensive about the Taguchi method and thought cannot be used in upstream engineering processes and used to think robust engineering is the use of orthogonal array to identify the level of process design parameters that improves quality. In 1990's the need of an upstream tool to eliminate down stream problems was felt. This is when focus shifted to robust engineering. Robust engineering seeks to rapidly optimize the performance of products and processes, while lowering or maintaining costs, especially in R&D and other allied activities. Robust Engineering philosophy requires the engineer to concentrate on the function of design. As we discuss its implication will become clearer. It was commented that inherent lack of robustness in product design is a primary driver of superfluous manufacturing expenses [Taguchi 1990].

The product performance metrics that directly or indirectly cause loss to manufacturer or customer are warranty costs, scrap costs or rework cost, number of customer complaints. These post design and postproduction measures come into the picture only after the product is designed, manufactured and placed in the hands of customers.

To address all these issues Taguchi method proposed to begin asking questions at the initial stage of design, such as: What is the design supposed to do? What is the performance expected by the customer? How does the design utilize energy? Taguchi used the phrase ideal function to describe what is wanted from an engineering system. Ideal function is the transfer of energy from input to a desired output form, without diversion into unwanted output states. The relative amount of energy in wanted and unwanted states can be measured by a signal to noise ratio, though it is not clear how this is justified.

The task on hand is to ensure that energy being transferred to the desired state is maximized while energy left over to cause problems is minimized. It must be noted that minimizing energy in the unwanted state is no guarantee that energy in wanted state is maximized. The unwanted energy may get diverted into another state to cause another problem. The above concept is aptly applied to rectify design flaws in car windshield wiper system by using Taguchi method [Wilkins Jr 2000].

2.4 REVIEWS ON ROBUST DESIGN TECHNIQUES

Although Taguchi's contributions to the philosophy of robust design are almost unanimously considered to be of fundamental importance, there are certain limitations and inefficiencies associated with his methods. To reduce performance variation of a product several approaches have been developed over the years after Taguchi proposed his method. These can be classified as: design of experiments technique, response surface methodology, optimization methods, non-linear programming technique, stochastic optimization, multi-criteria optimization technique. In this chapter the research trends on each category is reviewed.

2.4.1 Design of Experiments Technique

A design of experiment is a test or series of tests in which purposeful changes are made to the input variables of system so that we may observe and identify the reasons for changes in the output response. For robust product design mostly fractional factorial design is used where number of experiments to be conducted is less and the Taguchi method tries to do the same.

The Taguchi method for robust design has been criticized by Box for the use of two part orthogonal array for experimental design and SN ratio in robust optimization criterion. Box and Fung [Box 1986] pointed out that the orthogonal array method does not always yield the optimal solution and suggested that non-linear optimization techniques should be employed when a computer model of the design exists. Significant improvement over the results predicted by Taguchi was observed while studying a wheatstone bridge circuit design problem. Montgomery [Montgomery 1999] had demonstrated that the inner array used for the control factors in the Taguchi's approach and the outer array used for noise factors, is often unnecessary and results in a large number of experiments. Tsui [Tsui 1992] had shown that the Taguchi method did not necessarily give an accurate solution for design problems with highly non-linear behavior. An excellent survey of these controversies was the panel discussion edited by Nair [Nair 1992].

It has earlier been discussed that the Taguchi method is nothing but an approach to engineering design optimization. Its application was illustrated in the design optimization [Unal 1993] study of aerospace propulsion system and found the Taguchi method can offer simultaneous improvement in quality, performance, cost, and engineering productivity. Steinberg and Bursztyn [Steinberg 1994] had shown that robust design experiments are effective when it is possible to build some variation directly into the experiment including noise factors. Nair [Nair 1993] had reviewed methods for analyzing data from robust parameter design experiments and discussed some new developments in parameter design and outlined their advantages. Taguchi's approach has been extended in a number of ways. Yu and Ishii [Yu 1993] used the fractional quadrature factorial method for systems with significant nonlinear effects.

Pledger [Pledger 1996] had observed the uncontrollable factors, explicitly introduced into experiment and established that some of the uncontrollable factors are observable during production. The extra information can enhance the choice of values for the controllable factors to keep, both, the mean and response on target and reduce the variance. Chipman [Chipman 1998] had considered use of Bayesian methods in fitting robust design experiment models and the subsequent optimization by incorporation of reliable assessment of uncertainty into the analysis of data. Borror and Montgomery [Borror 2000] had presented a combined array design as an alternative to standard Taguchi design. The method was better suited to robust design problem. The mixed resolution design was illustrated in an example involving control and noise variables. Sexton et al. [Sexton 2000] had used semi controlled experiments to improve mechanical design where procedure for searching a good plan using an algorithm was developed that can take account of key derived factors arising from component assembly.

The most successful applications of robust design are found in the fields of mechanical design engineering (static performance) and process systems, and there have been few applications to the robustness of dynamic performance. Seki and Ishii [Seki 1997] applied the robust design concept to the dynamic design of an optical pick-up actuator focusing on shape synthesis using computer models and design of experiments. The response in the first bending and torsion modes were selected as measures of undesirable vibration energy. The objective functions were defined as the signal-to-noise ratios of response frequencies and the sensitivities were derived from the design of experiments using an orthogonal array. Hwang et al. [Hwang 2001] optimized the vibration displacements of an automobile rear view mirror system for robustness, defined by the Taguchi concept. Rout and Mittal [Rout 2003(ii)] had applied the Taguchi method to find the optimum parameters of manipulator for reduced performance variations, thereby increasing positional accuracy of a manipulator. Rout and Mittal [Rout 2005(iii)] investigated the statistical significance of manipulator kinematic parameters using design of experiment approach.

2.4.2 Response Surface Methodology

The response surface methodology (RSM) is a set of statistical techniques used to construct an empirical model of the relationship between a response and the levels of some input variables, and to find the optimal responses. Lin and Tu [Lin 1995] had used dual response approach to achieve the goals of Taguchi's philosophy. They highlighted some deficiencies of Taguchi method. Khatree [Khatree 1996] had tried to present an alternative approach to Taguchi's robust parameter design. Lin et al. had tried to point out the limitation of goal formulations for approximation–based robust design. Based on different philosophies and mathematical deduction, they proposed three new methods to formulate robust design goals. Using a single variable function, they concluded that Kriging models perform better than RSM in a large design space with a high degree of non-linearity [Lin 1999]. Where Kriging model is an interpolation model comprised of two parts: a polynomial and a functional departure from that polynomial. Lucas [Lucas

1994] and Myers et al. [Myers 1992] considered the RSM as an alternative to Taguchi's robust design method.

Monte Carlo simulation generates instances of random variables according to their specified distribution types and characteristics and although accurate response statistics may be obtained, the computation is expensive and time consuming. Mavris and Bandte [Mavris 1997] had combined the response model with a Monte Carlo simulation to construct cumulative distribution functions and probability density functions for the objective function and constraints. All these methods depend on the sampling statistics, whereby the probabilistic distributions of the stochastic input sets are required. One concern was that the response surface approximations might not generate the accurate sensitivities required for robust design [Su 1997].

2.4.3 Nonlinear Programming Optimization Method

Terms or groups of terms that involve intrinsically nonlinear functions characterize a nonlinear programming problem. Methods that are derived to solve the broad set of problems that make up this functional classification will be referred as Nonlinear Programming Method.

The optimization procedure aims to minimize/maximize the objective functions. The uncertainty in the noise factors means that the system performance is a random variable. One option in robust optimization is to minimize both the deviation in the mean value, $\mu - t$, and the variance, σ^2, f of the performance function, subject to the constraints. The quantities of mean and the standard deviation of system performance, for given signal and control factors, may be calculated if the joint probability density function (PDF) of the noise factors is known. For most practical applications these PDFs are unknown, but often it is assumed that all variables have independent normal distributions. In this case, the joint PDF becomes a product of the individual PDFs. However, evaluating objective function is extremely time consuming and computationally expensive and approximations using Taylor's series expansions about the mean may be used. If only the linear terms are retained in the expansion, then the mean and variance of the response are readily computed in terms of the mean and variance of the noise factors. The constraints must also be satisfied.

The noise factors are assumed to have stochastic nature, whereas the control factors are used to optimize the system and therefore must remain as parameters in the constraints. Parkinson et al. [Parkinson 1993] proposed a general approach for robust optimal design and addressed two main issues. The first issue was design feasibility, where the procedures are developed to account for tolerances during design optimization such that the final design will remain feasible despite variations in parameters or variables. The second issue was the control of the transmitted variation by minimizing sensitivities or by trading of controllable and uncontrolled tolerances. The calculation of the transmitted variation was based on well-known results developed for the analysis of tolerances [Bjorke 1989]. Ramakrishnan and Rao had formulated the robust design problem as a nonlinear optimization problem with Taguchi loss function as the objective [Ramakrishnan 1991]. Other significant publications in this area are by Belegundu and Zhang [Belegundu 1992] that extended Taguchi's parameter design to the notion of conceptual robustness. Sundaresan et al. [Sundaresan 1993] had employed a single objective function that utilizes weighing factors for target performance and variance represented by sensitivity index. Yu and Ishii [Yu 1993] had also proposed nonlinear programming methods for robust design. Otto and Antonsson [Otto 1993] had illustrated the procedure to apply the Taguchi method for product design and applied to air tank design for determining which of two designs to pursue-an air tank with hemisphere heads or an air tank with flat heads. Cagan and Williams [Cagan 1993] had described a rigorous approach for robust optimal design, which allows a designer to explicitly consider and control the effects of variability in design variables and parameters on design. Castillo and Montgomery [Castillo 1993] had presented method to achieve same goals using standard non-linear programming technique; specifically the generalized reduced gradient algorithm. The proposed method is more flexible and easier to use than the dual response approach. Otto and Antonsson [Otto 1993] addressed robust design optimization with constraints, using constrained optimization methods.

A number of engineering examples showed that the method works well if the tolerances are small. For larger tolerances or for strongly non-linear problems, higher order Taylor series expansions must be used. A second order worst-case model was described by Emch and Parkinson [Emch 1994] and a second order model using statistical analysis was presented by Lewis and Parkinson [Lewis 1994]. The disadvantage of higher order models was that the formulae to evaluate the response became quite complicated and computationally expensive. Parkinson [Parkinson 1995] had discussed the feasibility robustness and sensitivity robustness for robust mechanical design using engineering models. Variability was defined in terms of tolerances. For a worst-case analysis, a tolerance band was defined, whereas for a statistical analysis the $\pm 3\sigma$ limits for the variable or parameter were used. Chen et al. [Chen 1996] had developed a robust design methodology to minimize variations caused by the noise and control factors, by setting the factors to zero in turn.

2.4.4 Stochastic Optimization Method

A slightly different strategy for robust design optimization is based on stochastic optimization. The stochastic nature of the optimization arises from incorporating uncertainty into the procedure, either as the parameter uncertainty through the noise factors, or because of the stochastic nature of the optimization procedure.

The earliest work on stochastic optimization can be traced back to the 1950s [Beale 1955] and detailed information may be obtained from recent books [Kall 1994]. The objective of stochastic optimization is to minimize the expectation of the sample performance as a function of the design parameters and the randomness in the system. Chakraborty and Dey [Chakraborty 1998] proposed a stochastic finite element method in the frequency domain for analysis of structural dynamic problems involving uncertain parameters. The uncertain structural parameters are modeled as homogeneous Gaussian stochastic fields and discretized by the local averaging method. Numerical examples were presented to demonstrate the accuracy and efficiency of the proposed method. Chen [Chen 2003] used Monte Carlo simulation and quadratic programming. Schueller [Schueller 2001] had given a recent review on structural stochastic analysis. Optimization approaches that are inherently stochastic include techniques such as simulated annealing, neural networks and evolutionary algorithms (EA) (genetic algorithms, evolutionary programming and evolution strategies (ES)), and these have been applied to multiobjective optimization problems [Hajela 1999]. These techniques do not require the computation of gradients, which is important if the objective function relies on estimating moments of the response random variables. Gupta and Li [Gupta 2000] applied mathematical programming and neural networks to robust design optimization, and showed that the approach is fruitful in solving highly non-linear design optimization problems in mechanical and structural design. Sandgren and Cameron [Sandgren 2002] used a hybrid combination of a genetic algorithm and non-linear programming for robust design optimization of structures with variations in loading, geometry and material properties. Parkinson [Parkinson 2000] employed a genetic algorithm for robust design to directly obtain a global minimum for the variability of a design function by varying the nominal design parameter values. The method proved effective and more efficient than conventional optimization algorithms. Additional studies are required before these methods are suitable for application to large-scale optimization problems.

2.4.5 Multi-Criteria Optimization Method

In design there may be situations, where it would be desirable to achieve a solution that is the “best” with respect to a number of different (competing) criteria. The mathematical tools necessary to formulate and solve such multi objective or multi criteria problems are called multi-criteria optimization method. A robust design is one that attempts to optimize both the mean and variance of the performance, and is therefore a multi-objective and non-deterministic problem. Optimization of the mean often conflicts with minimizing the variance, and a trade-off decision between them is needed to choose the best design. Chen et al. [Chen 1999] had used a combination of multi-objective mathematical programming methods and the principles of decision analysis to address the multi-objective optimization in robust design. The compromise programming (CP) approach, that was the Tchebyche for min–max method, replaced the conventional Weighted Sum (WS) method. The advantages of the CP method over the WS approach in locating the efficient multi-objective robust design solution (Pareto points) were illustrated both theoretically and through example problems. Chen et al. [Chen 2000] made the bi-objective robust design optimization perspective more powerful by using a physical programming approach, where each objective was controlled with more flexibility than by using CP. Discussions on inequality constraint satisfaction were given by Parkinson et al. [Parkinson 1993], Du and Chen [Du 2001], Lee and Park [Lee 2001]. Research on equality constraint problems are limited to three approaches; to relax the equality constraint, to satisfy the equality constraint in a probabilistic sense [Sunderasan

1991], or to remove the equality constraint through substitution [Das 2000]. Bras and Mistree introduced the compromise decision support problem (DSP) [Bras 1995]. Their approach is especially useful in design problems where there are no closed-form solutions and system performance is computationally expensive.

To effectively address performance with respect to multiple measures in product design, a methodical approach that integrates multi criteria optimization concepts with statistical robust design technique was used. The method includes a systematic treatment of constraints, and the results are presented as a set of non-inferior design solutions [Kunjur 1995]. These solutions utilized ANOVA results to quantify the relative dominance and significance of design factors. Yu and Ishii [Yu 1998] have addressed the impact of manufacturing errors on design robustness and constraint activity, and have developed a systematic procedure to identify a variation pattern for typical processes, approximate performance based on pattern. A method was presented that makes use of the concepts of robust design and the techniques of Multi-criteria optimization for simultaneous optimization of many quality characteristics and illustrated the method using a case study gleaned from literature [Song 1995].

Thurston et al. [Thurston 1994] had presented design method whose basic premise was that the process of design should be driven from the very beginning by consideration of how the artifact will ultimately be evaluated. An algorithm was developed by Korngold and Gabriele [Korngold 1997] to efficiently optimize multidisciplinary coupled, non-hierarchic systems with discrete variables. Parkinson [Parkinson 2000] demonstrated that potential means of improving assembly quality by the use of deterministic optimization process based on variable limits. Zhang et al. [Zhang 2000] had suggested a method for deriving a utility function as a local approximation of efficient frontier and investigated at different locations of candidate solutions, with different range of interest, in problems with convex and non-convex behavior.

A single criterion for robust design had been presented by using Taguchi's SN ratio, as designers were not able to explicitly address the trade off between achieving the design performance and its robustness. To this effect, Chen et al. [Chen 1999] had developed a multiple objective approach to robust design, which offers more flexibility in addressing the multiple aspect of robust design. Robust design procedure had been applied to achieve improved vehicle handling performance as an integral part of simulation based vehicle design. The proposed procedure was effective for preventing the roll over of ground vehicle as well as for identifying a design that was not only optimal against worst maneuver inputs. Chen et al. [Chen 1999] had compared the statistical approach and a bi-level optimization approach in terms of their effectiveness in solving robust design problems. Kalsi et al. [Kalsi 2001] had proposed concepts of robust design to reduce the effects of decisions made during the design of one subsystem on the performance of rest of the system. Concepts are demonstrated by considering a design of passenger aircraft. Chou and Chang [Chou 2000] had developed a relationship between the cost and tolerances of two characteristics of lock wheel and obtained as a bivariate cost tolerance function. Renaud and Tappeta [Renaud 1997] gave a comprehensive overview and provided a rigorous optimization strategy for multi-objective collaborative optimization. Rout and Mittal [Rout 2000(i)] presented an extensive review of the methodologies to obtain a robust design that have less performance variation due to the variations of control factors and noise factors.

2.5 EPILOGUE

This chapter reviewed the development of methods to improve the performance of manipulator since 1981 with a view to identify the need for investigation on robot performance variations. By increased industrialization and development in automation, the need to improve robotic performance has revived interest. The review presents the citation of developments on methodology to improve robot performance.

The majority of present industrial robots are relatively inaccurate. Robot positioning accuracy, expressed in task-related coordinates, is often worse than the repeatability by higher magnitude. This reduces significantly the possibilities of practical application of such robots in some classes of industrial tasks. A bulk of research efforts has been devoted to this problem, resulting in practical calibration procedures that can be efficiently used to improve the robot positioning accuracy in static or quasi-static operating conditions. The area of calibration of robot models, local calibration, dynamic parameter identification and problems related to the improvement of accuracy of fast trajectory tracking, are reviewed. Besides, a systematic approach to analyze the accuracy of kinematic and dynamic robot models it is not only a necessary prerequisite for design synthesis and evaluation of design strategies, but it also offers a possibility to prescribe parameter tolerances in the design stage of the robotic mechanism, as well as to anticipate attainable accuracy, which is important in the selection of the appropriate robot for particular industrial application.

The later part of the chapter contained a review of major approaches and techniques to robust design. It is noted that robust design is a multi-objective and non-deterministic problem. The objective is to optimize the mean and minimize the variability in the performance response that results from uncertainty represented through noise variables. The robust design approaches can generally be classified as statistical-based methods and optimization methods. Mostly the Taguchi method use direct experimentation and the objective functions for the optimization are expressed as the signal to noise ratio (SN ratio). Using the orthogonal array technique, the analysis of variance and analysis of mean of the SN ratio are used to evaluate the optimum design variables to ensure that the system performance is insensitive to the effects of noise, and to tune the mean response to the target. The optimization approaches for robust design are based on non-linear programming methods. The objective functions simultaneously optimize both the mean performance and the variance in performance. A trade-off decision must be made, to choose the best design with the maximum robustness. Recently, novel techniques such as simulated annealing and the field of evolutionary algorithms have been applied to solve the resulting multi-objective optimization problem.

Based on the above study following conclusions can be drawn. There has been relatively little or not much literature available to handle the effects of noise in design optimization of robotic manipulator. Therefore, renewed attempts must be made to systematically underline ways and means to minimize the effect of uncertainty on manipulator performance. These aspects are of paramount concern to researchers and practitioners. There have been few attempts to compare the technique with other methods either analytically or experimentally, except for comparison with experimental design techniques.

CHAPTER-3

SIMULATION OF MANIPULATOR PERFORMANCE USING SEARCH BASED METHOD AND OPTIMAL PARAMETER DESIGN

3.1 INTRODUCTION

Mostly industrial manipulators are required to perform tasks with a higher precision and speed than human beings. To perform task a manipulator is commanded to move its end-effector to a specified position but the actual position reached may be quite different from the desired one. This difference in the actual and desired position for the end-effector is termed as "positional error" of a manipulator and the average positional error is termed as its positioning accuracy. The parameters whose values can be changed or controlled by designer are called "control factors". The performance variations of manipulators are attributed to "noise factors", where noise factor are parameters which are difficult and costly to control.

Conducting physical experiments on a manipulator to find out its positioning accuracy by changing its parameters is very tedious, time consuming and uneconomical. To assuage this problem, help of simulation is taken where experiments are conducted by varying values of parameters. Therefore, development of simulation method to obtain real life performance without conducting experiment becomes quite a challenge. To simulate the performance of manipulator a heuristic-search based simulation methodology has been developed. This methodology helps in incorporating effect of noise in dynamic model of manipulator to simulate real life performance. This methodology is based on parametric design of manipulator using design of experiment (DOE) approach, to select a combination of control factors of a product or process in such a way that the performance becomes insensitive to noise factors. DOE techniques are fairly standard approach and commonly used in statistical quality control, but their application to robotic parameter design is rare. Experiments conducted using above technique helps in understanding the combined effect of factors on performance. In this chapter the procedure to apply DOE technique to manipulator parameter design, to identify parameters responsible for performance variation and to find optimal combination, that deliver optimal performance is presented. Such an investigation is offline strategy, which is novel and helps designer to select the parameters to reduce the performance variation, prior to actual manufacturing.

This chapter is organized in seven sections. In section 3.2 steps for parameter design optimization have been discussed. The kinematic and dynamic models used for simulation of performance have been presented in section 3.3. The application of DOE technique to 2-DOF RR planar manipulator is discussed in section 3.4. In section 3.5 proposed simulation method to incorporate effect of noise factors and to simulate the performance has been presented. The assumed data for simulation and analysis of results of experiment are discussed in section 3.6 and 3.7 respectively. The limitations of the proposed method are discussed in section 3.8.

3.2 PARAMETER DESIGN USING DESIGN OF EXPERIMENT (DOE) TECHNIQUE

Traditional optimization techniques used for determination of parameters for optimal performance are known for their inefficiency in handling uncertainties and nonlinearities of physical systems and the solutions obtained using these techniques may not be practical or may require high investments.

DOE technique overcomes the drawbacks of conventional optimization techniques and has been used successfully to optimize processes and designs for diverse systems. This method identifies the optimal factor combination for optimal performance and factors that have significant effect on the performance. DOE technique allows the effect of a factor to be estimated at several levels of the factor yielding conclusions that are valid over a range of experimental conditions [Montgomery 2001]. The steps described below are used for applying DOE technique:

Step 1. Statement of the problem,

Step 2. Choice of factors and levels,

Step 3. Selection of the response variable,

Step 4. Choice of experimental design,

Step 5. Perform the experiment, this involves following:

- (a) Conduct of the experiment,
- (b) Obtain the performance measure,

Step 6. Data analysis.

3.3 KINEMATIC AND DYNAMIC MODELS OF 2-DOF RR PLANAR MANIPULATOR

The parameter design using DOE is applied to get optimum design of robotic manipulators. A 2-DOF RR planar manipulator is considered to establish the application of DOE technique to manipulator design. The mathematical model to simulate the performance of manipulator and compute the position reached is developed first and then the step-by-step application of DOE technique to a robot manipulator design is presented.

The kinematic and dynamic models of 2-DOF RR planar manipulator used in this thesis are discussed briefly in this section. The detailed derivations of these are available in many textbooks [Mittal 2003].

Consider the 2-DOF RR planar manipulator shown in Fig. 3.1 having link lengths l_1 and l_2, and joint angles θ_1 and θ_2. Let $P(x_f, y_f)$ be the target position of end-effector in the Cartesian workspace of the manipulator, the point of interest for which the performance in terms of positional accuracy has to be modeled and optimized. The coordinates of P in Cartesian coordinates for joint angles θ_1 and θ_2 are given by

$$x_f = l_1 C_1 + l_2 C_{12} \tag{3.1}$$

$$y_f = l_1 S_1 + l_2 S_{12} \tag{3.2}$$

where, $C_i = \cos\theta_i$, $S_i = \sin\theta_i$, $C_{ij} = \cos(\theta_i + \theta_j)$ and $S_{ij} = \sin(\theta_i + \theta_j)$ with $i, j = 1, 2$ for the two links, link 1 and link 2, respectively.

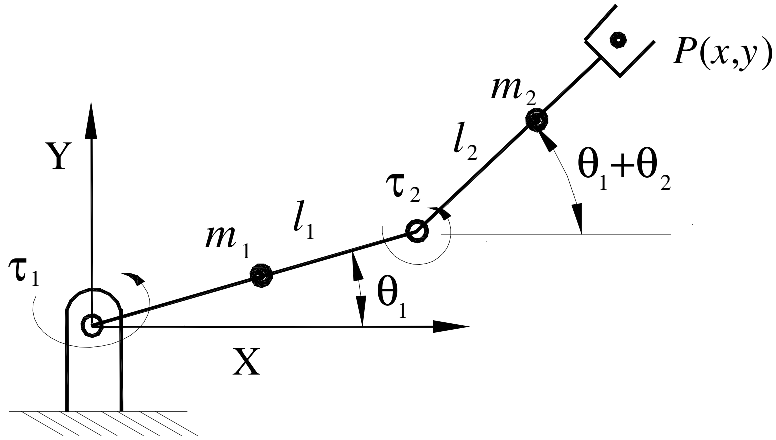

Fig. 3.1 A 2-DOF RR Planar Manipulator and its Parameters

From equations (3.1) and (3.2), joint variable θ_1 is obtained as:

$$\theta_1 = \tan^{-1}\left[\frac{y_f(l_1 + l_2C_2 - x_f l_2 S_2}{x_f(l_1 + l_2C_2) + y_f l_2 S_2}\right] \tag{3.3}$$

Assuming, $D = \dfrac{x_f^2 + y_f^2 - l_1^2 - l_2^2}{2l_1 l_2}$, joint variable θ_2 is obtained as

$$\theta_2 = \tan^{-1}\left[\frac{\sqrt{1-D^2}}{D}\right] \tag{3.4}$$

Differentiating equations (3.1) and (3.2) and solving for joint velocities $\dot{\theta}_1$ and $\dot{\theta}_2$ gives

$$\dot{\theta}_1 = \frac{\dot{x}C_{12} + \dot{y}S_{12}}{l_1 S_2} \tag{3.5}$$

and

$$\dot{\theta}_2 = -\frac{\dot{x}\, x_f + \dot{y}\, y_f}{l_1 l_2 S_2} \tag{3.6}$$

where $(\dot{x}, \dot{y})$ represent end-effector velocity v_e with $\dot{x} = v_e \cos\alpha$ and $\dot{y} = v_e \sin\alpha$, and α is angle made by v_e with positive *x*-axis of base frame. Assuming link masses as: m_1, m_2 and joint torques as τ_1, τ_2, respectively and assuming links as slender members with mass concentrated at center of gravity for each link, the dynamic model of 2-DOF RR planar manipulator is given by

$$\begin{aligned}\tau_1 = &\left[\left(\frac{m_1}{3} + m_2\right)l_1^2 + \frac{m_2}{3}l_2^2 + m_2 l_1 l_2 C_2\right]\ddot{\theta}_1 + m_2\left(\frac{l_2^2}{3} + \frac{l_1}{2}l_2 C_2\right)\ddot{\theta}_2 - m_2 l_1\, l_2 S_2 \dot{\theta}_1\dot{\theta}_2 \\ &- \frac{m_2}{2}l_1 l_2 S_2 \dot{\theta}_2^2 + \left(\frac{m_1}{2} + m_2\right)g l_1 C_1 + \frac{m_2}{2} g l_2 C_{12}\end{aligned} \tag{3.7}$$

$$\tau_2 = \left[\frac{m_2}{3}l_2^2 + \frac{m_2}{2}l_1 l_2 C_2\right]\ddot{\theta}_1 + \frac{m_2}{3}l_2^2\ddot{\theta}_2 + \frac{m_2}{2}l_1 l_2 S_2 \dot{\theta}_1^2 + \frac{m_2}{2} g l_2 C_{12} \tag{3.8}$$

where g is acceleration due to gravity, $\ddot{\theta}_1$ and $\ddot{\theta}_2$ are joint accelerations. Equations (3.7) and (3.8) are solved for $\ddot{\theta}_1$ and $\ddot{\theta}_2$ and are written in a compact form as:

$$\ddot{\theta}_1 = \frac{be - cd}{ad - b^2} \tag{3.9}$$

and

$$\ddot{\theta}_2 = \frac{bc - ae}{ad - b^2} \tag{3.10}$$

where a, b, c, d and e are given as:

$$a = \left(\frac{m_1}{3} + m_2\right) l_1^2 + \frac{1}{3} m_2 l_2^2 + m_2 l_1 l_2 C_2 \tag{3.11}$$

$$b = m_2 \left[\frac{l_2^2}{3} + \frac{1}{2} l_1 l_2 C_2\right] \tag{3.12}$$

$$c = \left(\frac{m_1}{2} + m_2\right) g l_1 C_1 + \frac{m_2}{2} g l_2 C_{12} - m_2 l_1 l_2 S_2 \dot{\theta}_1 \dot{\theta}_2 - \frac{m_2}{2} l_1 l_2 S_2 \dot{\theta}_2^2 - \tau_1 \tag{3.13}$$

$$d = \frac{m_2}{3} l_2^2 \tag{3.14}$$

$$e = \frac{m_2}{2} l_1 l_2 S_2 \dot{\theta}_1^2 + \frac{m_2}{2} g l_2 C_{12} - \tau_2 \tag{3.15}$$

Equations (3.1)-(3.15) have been used to identify significant parameters (factors) and compute the performance measures for the robotic manipulator using the DOE technique.

3.4 APPLICATION OF DOE TECHNIQUE TO MANIPULATOR DESIGN

The manipulator parameter design using DOE is carried out next according to the steps given in section 3.2.

3.4.1 Statement of the Problem

The problem of improving positional accuracy of the robot manipulator has been addressed in the past by considering optimal performance only with one prototype with fixed parameters and if the performance variation was found to be large then it was reduced by adoption of suitable calibration and control strategy.

The main function for a robot manipulator is to accurately reach the commanded position. For the 2-DOF RR planar manipulator, the target position is in the work-plane of the manipulator and is described by point $P(x_f, y_f)$ assuming the work-plane of the manipulator is *xy*-plane.

3.4.2 Identification of Factors and Levels

The robot parameters like link dimensions, configuration, inertias, actuators, etc. play a vital role in its performance. Robotic system designer often comes across a situation where decision is to be made regarding these parameters. Except in few specific applications, designer uses a particular parameter combination by choice or by convenience, overlooking available alternatives that may give optimal performance.

Various parameters involved in the manipulator design are identified with the help of the mathematical models developed and using the parameter diagram (P-Diagram) for manipulator as shown in Fig. 3.2. The parameters other than input and output are classified as control factors or noise factors.

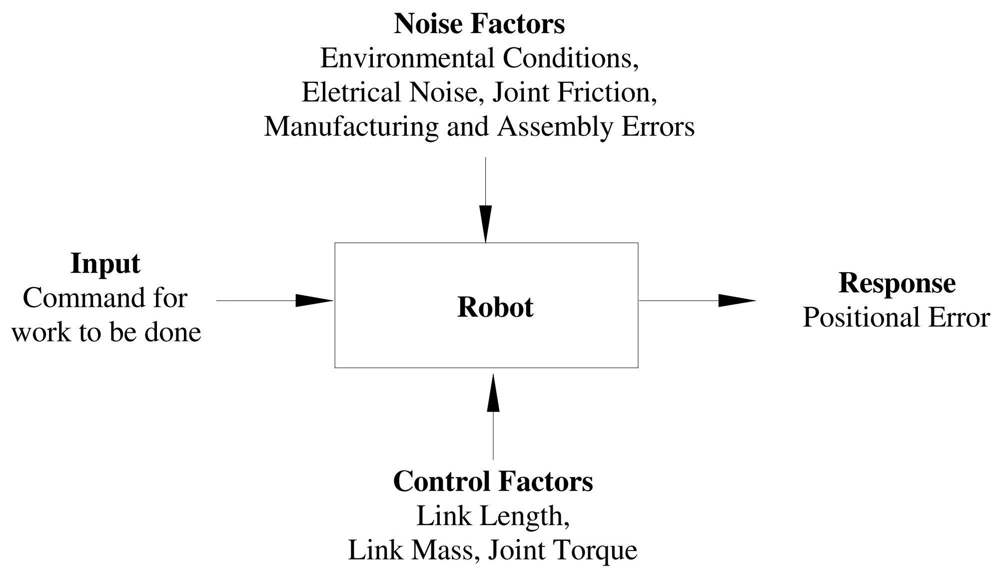

Fig. 3.2 Parameter Diagram (P-Diagram) for Manipulator Performance.

3.4.2.1 The Control Factors

The control factors for manipulator are identified from the kinematic and dynamic equations (3.9) and (3.10). It is observed that $\ddot{\theta}_1$ and $\ddot{\theta}_2$ depends on following six independent parameters apart from the process variables $\theta_1, \theta_2, \dot{\theta}_1$ and $\dot{\theta}_2$:

1. Length of link 1, l_1;
2. Length of link 2, l_2;
3. Mass of link 1, m_1;
4. Mass of link 2, m_2;
5. Torque applied at joint 1, τ_1;
6. Torque applied at joint 2, τ_2.

These six parameters are therefore, identified as control factors because designer can choose their values easily.

For optimal parameter design using DOE technique, control factors can be considered at several levels. In this investigation, control factors are considered at two levels: “high” and “low”. Therefore, six control factors at two levels, results in a set of 2^6 (64) different combinations of six control factors.

3.4.2.2 Sources of Noise in the Manipulator –The Noise Factors

The function of the manipulator is to move its end-effector to a desired point accurately. However, the discrepancy in the desired and actual point reached can be attributed to presence of noise factors. The noise factors cause the end-effector to deviate from its target point. Noise factors are those factors that are difficult, expensive or hard to control during production or operation. Some of the noise factors that have direct influence over the performance of a manipulator are:

(a) Environmental conditions in which manipulator operates,
(b) Errors in manufacture and assembly,
(c) Fluctuations in electricity supply, causing deviation in the joint actuator torques,
(d) Friction at the manipulator joints, and
(e) Joint compliance between the joint encoder and the actual angular output.

There can be several other noise factors, which may have influence over performance of the manipulator. These noise factors are very difficult to quantify, their effects on performance are still difficult to compute. To simulate the performance a heuristics based search approach has been proposed to include the effect of noise factors in the DOE to obtain an optimal design of robotic manipulator. This approach is explained in next section.

3.4.3 Selection of Performance Measures for Manipulator –The Response variable

To investigate the impact of different parameters on performance variation of manipulator, several performance measures have been proposed by researchers. Following four, performance measures have been considered in this work.

(a) Positional Error:

For a robotic manipulator positional performance measures are accuracy, repeatability and resolution. A combination of these measures are defined as positional error ε_i as distance between the actual point reached by the end-effector $P_i(x_a, y_a, z_a)$ in the i^{th} experiment and desired point $P(x_f, y_f, z_f)$ in 3-D space, that is,

$$\varepsilon_i = \sqrt{\left[(x_a - x_f)^2 + (y_a - y_f)^2 + (z_a - z_f)^2\right]} \tag{3.16}$$

For the 2-DOF RR planar manipulator, with *xy*-plane as the workplane, the positional error ε_i in equation (3.16) will reduce to

$$\varepsilon_i = \sqrt{\left[(x_a - x_f)^2 + (y_a - y_f)^2\right]} \tag{3.17}$$

The objective function to be optimized is therefore, to minimize the positional error considering the uncertainties involved.

(b) Mean Positional Error:

Mean positional error $\bar{\varepsilon}$ is defines as the average positional error for large number of experiments performed, that is,

$$\bar{\varepsilon} = \frac{1}{n}\sum_{i=1}^{n}\varepsilon_i \tag{3.18}$$

where n is number of experiments or replications and ε_i is positional error for i^{th} experiment.

(c) Signal to Noise Ratio:

The signal to noise ratio (SN ratio) proposed by Taguchi, has been used as the data transformation method to consolidate the repetitive data into one value, which reflects the mean value and amount of variation present in the data. For the robotic manipulator design objective is to minimize the positional error hence, it is always desired that it should be as small as possible. Therefore, as per Taguchi method, the quality characteristics (performance) is of 'smaller-the-better' type and for this case target is zero. SN ratio is given as [Park 1998]

$$\text{SN ratio} = -10\log_{10}(MSD) \tag{3.19}$$

where *MSD* denotes the mean squared deviation from target value of the quality characteristics. Therefore, for manipulator design SN ratio becomes

$$\text{SN ratio} = -10\log_{10}\frac{1}{n}\left(\sum_{i=1}^{n}(\varepsilon_i - 0)^2\right) = -10\log_{10}\frac{1}{n}\sum_{i=1}^{n}\varepsilon_i^2 \tag{3.20}$$

SN ratio is an essential indicator of the ability of the system to perform well in relation to the effect of noise and measure to carryout the analysis of experiment. Clausing stated that the SN ratio is a good performance measure of robustness against noise [Clausing 1988].

(d) Reliability:

The reliability (*R*) of a manipulator is defined by Bhatti and Rao [Bhatti 1988] as the probability of the end-effector reaching a point or in a close vicinity of it within specified range. If the end-effector reaches a point within the specified range, it is considered as a successful experiment. The reliability *R* is given as,

$$R = \frac{\text{Number of sucessful Experiments}}{\text{Total number of Experiments}} \tag{3.21}$$

The specified range around a target point is called the permissible error region and its shape and size depends on the intended use of the manipulator. The reliability as performance measure has been used to evaluate the overall performance of all control factor combinations.

3.4.4 Design the Experiment – Choice of Experimental Design

The next step is to design the experiment. Experiment is defined as a test in which purposeful changes are made to the input variables of a system so that reasons for changes in the output response observed can be identified. There are several factors of interest in an experiment; therefore, to deal with these factors a factorial experiment strategy is used. Factorial experimental is an experimental strategy in which the design factors are varied and effects of all possible combinations of the levels of factors for the experiment are investigated. For example, if there are *a* levels of factor A, *b* levels of factor B, and *c* levels of factor C, then each replicate contains all $a \times b \times c$ experimental combinations. One of the special case is that of *k* factors, each at two levels. Usually these levels are indicated by “high” and “low” levels of a factor. A complete replicate of such design requires $2 \times 2 \times 2 ...... \times 2 = 2^k$ combinations and is called a 2^k factorial design. Number of combinations of 2^k design depends on the value of the *k* i.e. number of factors. To conduct the experiments and strategies adopted to simulate performance are discussed next.

3.4.5 Performing and Analyzing the Experiment

To perform experiment factorial combinations are required. Assuming that for an investigation there are three factors A, B, and C each at two levels. The design is called a 2^3

factorial design and eight combinations can now be displayed with the help of – and + notations to represent the "low" and "high" levels of the factors respectively. The eight combinations of the 2^3 design are shown in Table 3.1. This representation scheme is called the design matrix.

Table 3.1 Design Matrix

Combination Number	**Factor**		
	A	**B**	**C**
1	–	–	–
2	+	–	–
3	–	+	–
4	+	+	–
5	–	–	+
6	+	–	+
7	–	+	+
8	+	+	+

This design matrix helps experimenter to conduct experiment to investigate the joint effect of factors on a response. Commonly to represent the design matrix better, high and low level values of factors are used in place of – and + notations. Each factorial combination is run for finite number of replications to capture the effect of noise. The developed methodology and data utilized to simulate the performance is provided in sections 3.5 and 3.6. It is important to mention that while simulating the performance for the experiment no constraints such as maximum velocity and acceleration of links of manipulator are imposed.

For each replication, outcome of the experiment is obtained as the positional error ε_i and thereafter the performance measure mean positional error $\bar{\varepsilon}$ and SN ratio are computed for each control factor combination. The relevance of the SN ratio equation is tied to interpreting the signal or numerator of the ratio as the ability of the product to perform correctly. By including the impact of the noise factors on the product as denominator, then SN ratio can be adopted as the barometer of the ability of the system to perform well in relation to the effect of noise. By applying this concept to experimentation control factor settings of the product that delivers both best performance (high signal) and minimizes the effect of noise factor influences (low noise) can easily be determined. When both the conditions are satisfied SN ratio becomes highest. This means that reduction in mean response or improvement in consistency in response one data to the next. Therefore, to obtain optimal parameter combination, SN ratio values are compared after the conduct of experiment.

Statistical analyses of performance of experiment have been carried out using analysis of variance (ANOVA) technique, which is a powerful tool for understanding complex physical phenomenon. ANOVA is used to subdivide the total variation into variation due to control factors, variation due to interacting control factors and variation due to error. After this, statistical tests like F-test are used to investigate statistically significant control factors and interacting factors, which help in screening many factors to discover the vital few and how they interact. For this study statistically significant control factors and interacting factors are determined using ANOVA, and its results are discussed in section 3.7.

3.5 STRATEGY TO INCORPORATE EFFECTS OF NOISE FOR PARAMETER DESIGN

For given set of control factor values and target point $P(x_f, y_f)$ in workspace, following six parameters are computed from the equations (3.3), (3.4), (3.5), (3.6), (3.9) and (3.10)

1. Angular displacements θ_1, θ_2
2. Angular velocities $\dot{\theta}_1$, $\dot{\theta}_2$
3. Angular accelerations $\ddot{\theta}_1$, $\ddot{\theta}_2$

The computed values of these six parameters are free from the effects of noise. To incorporate the effect of noise in the above parameters, individual errors in form of noise for the six control factors are generated randomly. The randomly generated errors are assumed to follow normal distribution with zero mean and a specified standard deviation. Using set of values of control factors with noise, the above six parameters with noise incorporated are obtained as: angular displacements (θ_1^n, θ_2^n) from equations (3.3) and (3.4); angular velocities ($\dot{\theta}_1^n$, $\dot{\theta}_2^n$) from equations (3.5) and (3.6); and angular accelerations ($\ddot{\theta}_1^n$, $\ddot{\theta}_2^n$) from equations (3.9) and (3.10), where superscript '*n*' is used to indicate the presence of noise in the parameter. To compute the point actually reached by the end-effector with the presence of noise in control factors, a search technique has been developed and is described in following section.

3.5.1 Locating Target Point Reached

To compute the actual target point reached by the manipulator the angular displacements of links θ_1^a and θ_2^a are required where superscript '*a*' indicates actual values with noise present. Taking computed values of $\ddot{\theta}_1^n$, $\ddot{\theta}_2^n$ as input and control factors at nominal values (without noise) θ_1^a and θ_2^a are obtained. Since equations are nonlinear transcendental equations, the values of θ_1^a and θ_2^a are obtained using a heuristic based search algorithm. The steps of the algorithm are given below:

Algorithm 1: Search algorithm

Step 1. Read nominal level values of six control factors l_1, l_2, m_1, m_2, τ_1 and τ_2.

Step 2. Read standard deviations $\sigma_{l_1}, \sigma_{l_2}, \sigma_{m_1}, \sigma_{m_2}, \sigma_{\tau_1}$ and σ_{τ_2} for the six control factors.

Step 3. Read the range and step size for θ_1, θ_2 variations and permissible error e.

Step 4. Read the manipulator target point $P(x_f, y_f)$.

Step 5. Obtain θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$, $\ddot{\theta}_1$ and $\ddot{\theta}_2$ for input nominal values of control factors from equations (3.3), (3.4), (3.5), (3.6), (3.9) and (3.10), respectively.

Step 6. Generate random errors based on standard deviations for six control factors and obtain control factor values with noise. Using these values of control factors, compute θ_1^n, θ_2^n, $\dot{\theta}_1^n$, $\dot{\theta}_2^n$, $\ddot{\theta}_1^n$, and $\ddot{\theta}_2^n$ from equations (3.3), (3.4), (3.5), (3.6), (3.9) and (3.10), respectively.

Step 7. Make starting guess for θ_1^a and θ_2^a, which is equal to θ_1 and θ_2 in step 5

Step 8. Compute $\ddot{\theta}_1^a$, $\ddot{\theta}_2^a$ from equations (3.9) and (3.10).

Step 9. Compare the values of $\ddot{\theta}_1^a, \ddot{\theta}_2^a$ obtained in step 8 with values of $\ddot{\theta}_1^n, \ddot{\theta}_2^n$ obtained in step 6. If differences are within the specified permissible error e go to step 10, else increment θ_1^a and θ_2^a by chosen step size within the range and go to step 8.

Step 10. Terminate the search and return θ_1^a and θ_2^a.

For each factorial combination simulations are performed to obtain the individual performance of the experiment. The simulation is also run for the decided number of replications to compute defined performance measure. Likewise, for all the factorial combinations, simulations are carried out for desired number of replications to compute the performance measures.

3.6 SIMULATION

To simulate the performance of manipulator computer programme is developed using the approach discussed in section 3.5. For the computer programme, MATLAB software and its commands are used.

The numerical values used to simulate the performance of 2-DOF RR planar manipulator are given below:

(a) Number of levels for each control factor = 2,

(b) Nominal values of six control factors at two levels and standard deviations are given in Table 3.2. These standard deviations are used for simulation of noises. It can also be observed that the torque available at joint one and two have negative (–) signs. This indicates that the torque available at the joints is of retarding type, which is required for stationary end-effector at the target point.

Table 3.2 Level Values of Control Factors and Standard Deviations

Control Factor	Low Level	High Level	Standard Deviation
l_1 (m)	0.40	0.50	0.0001
l_2 (m)	0.30	0.40	0.0001
m_1 (kg)	5.5	6.5	0.01
m_2 (kg)	4.0	5.0	0.01
τ_1 (Nm)	–500	–800	0.1
τ_2 (Nm)	–100	–105	0.1

(c) Number of combinations in factorial design = $2^6 = 64$

(d) Design matrix of few control factor combinations is shown in Table 3.3. All the 64 combinations of design matrix are given in Appendix A1.

Table 3.3 Design Matrix for Control Factor Combinations

Combination Number	l_1 (m)	l_2 (m)	m_1 (kg)	m_2 (kg)	τ_1 (Nm)	τ_2 (Nm)
1	0.40	0.30	7	5	–500	–100
2	0.40	0.30	7	5	–500	–105
3	0.40	0.30	7	5	–800	–100
.	.	.	.	.	.	.
	.	.	.	.	.	
.	.	.	.	.	.	.
62	0.50	0.40	8	6	–500	–105
63	0.50	0.40	8	6	–800	–100
64	0.50	0.40	8	6	–800	–105

(e) The step size of increment for search, range of search and permissible error value e for the search algorithm have been chosen as

Increments: $\theta_{1(incr)} = 0.01\theta_1$, $\theta_{2(incr)} = 0.01\theta_2$,

Ranges: $0.5\,\theta_1 \le \theta_1 \le 1.5\,\theta_1$ and $0.5\,\theta_2 \le \theta_2 \le 1.5\,\theta_2$,

Permissible error value: $e \approx 0.005$.

(f) Tolerances chosen for target point for computation of reliability are:

$$\Delta x = \pm 0.0005 \text{ m}, \;\; \Delta y = \pm 0.0005 \text{ m}.$$

(g) Chosen number of replications for each combination of factorial design = 8 and for reliability = 200

(h) To establish the proposed method, simulation is carried out for four different target points, Coordinates of four target points in workspace are given in Table 3.4

Table 3.4 Task to be performed by Manipulator

Case	Cartesian Coordinate of Target point (x_f, y_f)
(i)	(0.40 m, 0.30 m)
(ii)	(0.50 m, 0.40 m)
(iii)	(0.40 m, 0.40 m)
(iv)	(0.30 m, 0.30 m)

Using above numerical values simulations are run. To determine the performance for each factor combination, simulation is run for 8 replications for the four cases i.e. target points. Since each control factor used for experiment has two levels, 8 replications of six-factor experiment required 512 simulations to run. From each simulation, positional error (ε_i) is obtained as response and taking these responses, analysis of experiment are carried out using ANOVA. The performances for four cases are analyzed by ANOVA technique and results are provided in Tables 3.5, 3.6, 3.7 and 3.8, respectively.

As discussed already ANOVA helps in comparing different sources of variations and making inferences about their relative importance. In ANOVA table, sum of squares (SS) indicate measure of the variability due to a source and the mean square (MS) of a source of variation is computed by dividing SS by its associated degrees of freedom. Finally computed F statistic F_o for the ANOVA is computed by dividing the MS of source by the MS of residual. This computed F_o statistic is compared with the tabulated F statistic to draw inferences i.e. whether the factor has significant influence on performance variations or not.

Analyses of all four cases are carried out separately and results are discussed below. During analysis the level of significance is assumed to be 0.10. Statistically significant parameters are those for which F_o statistic is greater than tabulated F statistic. For assumed level of significance F tabulated is 2.71 i.e. $F_{0.10,\nu_1,\nu_2} = F_{0.10,1,478} = F_{0.10,1,\infty} \approx 2.71$ [Montegomery 2001]. The observed statistic F_o values have been provided in respective tables. To represent the ANOVA results in compact form, results of individual factors and few significant interacting factors are presented in the table.

Table 3.5 ANOVA of Full Factorial Design for case (i)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
l_1	11.52×10^{-4}	1	11.52×10^{-4}	3.67	Significant
l_2	47.33×10^{-4}	1	47.33×10^{-4}	58.72	Significant
m_1	3.154×10^{-5}	1	3.154×10^{-5}	3.913×10^{-5}	–
m_2	30.19×10^{-4}	1	30.19×10^{-4}	37.46	Significant
τ_1	1.02×10^{-4}	1	1.02×10^{-4}	1.26	–
τ_2	2.56×10^{-4}	1	2.56×10^{-4}	3.18	Significant
$l_1 l_2$	1.13×10^{-4}	1	1.13×10^{-4}	1.040	–
$m_1\tau_1$	3.53×10^{-4}	1	3.53×10^{-4}	4.38	Significant
$m_2\tau_1$	4.03×10^{-4}	1	4.03×10^{-4}	5.00	Significant
$l_1 l_2 m_1 \tau_1 \tau_2$	3.56×10^{-4}	1	3.56×10^{-4}	4.41	Significant
Residual	385.271×10^{-4}	478	0.81×10^{-4}		
Corrected Total	506.35×10^{-4}	511			

Table 3.6 ANOVA of Full Factorial Design for case (ii)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
l_1	193.54×10^{-4}	1	193.54×10^{-4}	46.22	Significant
l_2	415.76×10^{-4}	1	415.76×10^{-4}	99.29	Significant
m_1	1.134×10^{-5}	1	1.134×10^{-5}	3.913×10^{-5}	–
m_2	44.59×10^{-4}	1	44.59×10^{-4}	10.65	Significant
τ_1	39.04×10^{-4}	1	39.04×10^{-4}	9.32	Significant
τ_2	1.56×10^{-4}	1	1.56×10^{-4}	3.18	Significant
$l_1 l_2$	47.08×10^{-4}	1	47.08×10^{-4}	11.24	Significant
$l_1\tau_1$	147.30×10^{-4}	1	147.30×10^{-4}	35.18	Significant
$l_2\tau_1$	11.03×10^{-4}	1	11.03×10^{-4}	2.63	–
$l_1 l_2 \tau_1$	80.47×10^{-4}	1	80.47×10^{-4}	19.22	Significant
Residual	2106.19×10^{-4}	501	4.19×10^{-4}		
Corrected Total	3085.01×10^{-4}	511			

Table 3.7 ANOVA of Full Factorial Design for case (iii)

Source	**Sum of Squares**	**Degrees of Freedom**	**Mean Square**	F_o **Value**	**Remark**
l_1	7.936×10^{-4}	1	7.936×10^{-4}	14.42	Significant
l_2	0.021	1	0.021	101.30	Significant
m_1	4.178×10^{-4}	1	4.178×10^{-4}	1.99	–
m_2	3.452×10^{-3}	1	3.452×10^{-3}	16.47	Significant
τ_1	1.770×10^{-4}	1	1.770×10^{-4}	8.44	Significant
τ_2	2.865×10^{-4}	1	2.865×10^{-4}	13.66	Significant
l_1l_2	9.332×10^{-4}	1	9.332×10^{-4}	4.45	Significant
l_1m_1	3.716×10^{-4}	1	3.716×10^{-4}	1.77	–
l_1m_2	1.410×10^{-3}	1	1.410×10^{-3}	6.72	Significant
l_2m_2	6.054×10^{-7}	1	6.054×10^{-7}	2.887×10^{-3}	–
$l_2\tau_2$	1.653×10^{-3}	1	1.653×10^{-3}	7.88	Significant
$l_1l_2m_2$	1.379×10^{-3}	1	1.379×10^{-3}	6.57	Significant
Residual	0.10	499	2.097×10^{-4}		
Corrected Total	0.14	511			

Table 3.8 ANOVA of Full Factorial Design for case (iv)

Source	**Sum of Squares**	**Degrees of Freedom**	**Mean Square**	F_o **Value**	**Remark**
l_1	1.759×10^{-4}	1	1.759×10^{-4}	1.43	
l_2	5.125×10^{-4}	1	5.125×10^{-4}	4.17	Significant
m_1	1.112×10^{-3}	1	1.112×10^{-3}	9.05	Significant
m_2	4.990×10^{-4}	1	4.990×10^{-4}	4.06	Significant
τ_1	0.021	1	0.021	167.61	Significant
τ_2	1.022×10^{-3}	1	1.022×10^{-3}	8.32	Significant
$\tau_1\tau_2$	5.762×10^{-4}	1	5.762×10^{-4}	46.91	Significant
Residual	0.062	504	1.228×10^{-4}		
Corrected Total	0.092	511			

The results of ANOVA for case (i) are provided in Table 3.5. The F_o value of individual factor and interacting factors with tabulated F value are compared. It is observed that control factors l_1, l_2, m_2, τ_2 and interacting factors $m_1\tau_1$, $m_2\tau_1$ and $l_1l_2m_1\tau_1\tau_2$ are statistically significant. The results of ANOVA for case (ii) are provided in Table 3.5. The F_o value of individual factor and interacting factors with tabulated F value are compared and observed that control factors l_1, l_2, m_2, τ_1 and τ_2 and interacting factors l_1l_2, $l_1\tau_1$ and $l_1l_2\tau_1$ are significant.

ANOVA results of case (iii) are given in the Table 3.7. On similar lines, analysis and comparison is carried out. For case (iii), it is observed that factors l_2, m_2, τ_1 and τ_2 and interacting factors l_1l_2, l_1m_2, $l_2\tau_2$ and $l_1l_2m_2$ are significant. ANOVA results for case (iv), are provided in the Table 3.8. By comparing F_o value with the tabulated F value factors l_2, m_1, m_2, τ_1 and τ_2 and interacting factors $\tau_1\tau_2$ are observed to be significant.

The performance measures i.e. $\bar{\varepsilon}$ and SN ratio are computed for each parameter combination. To indicate the trends of performance measure against the combination number, results are shown in graphs. The graph showing trend of mean positional error against combination number for all four cases are presented in Figs 3.3, 3.4, 3.5 and 3.6.

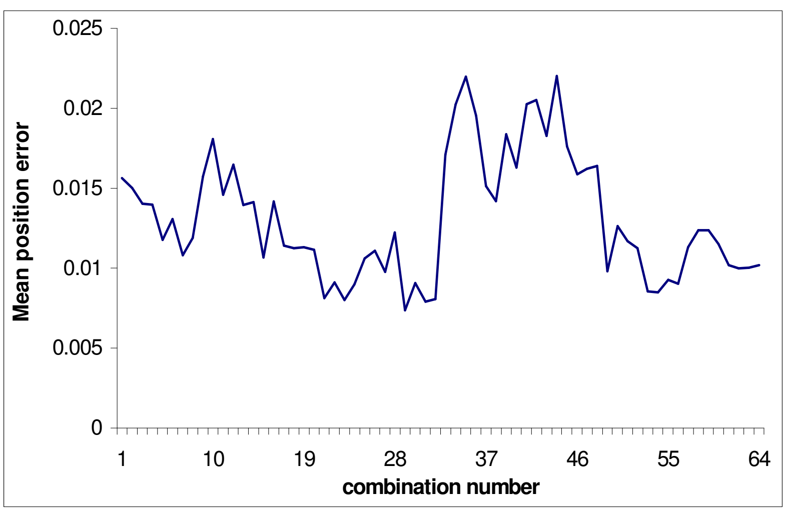

Fig 3.3 Mean Positional Error for case (i)

For case (i), it is observed from Fig. 3.3, that mean positional error is maximum (0.02203 m) at combination number 42 and is minimum (0.007355 m) at combination number 29.

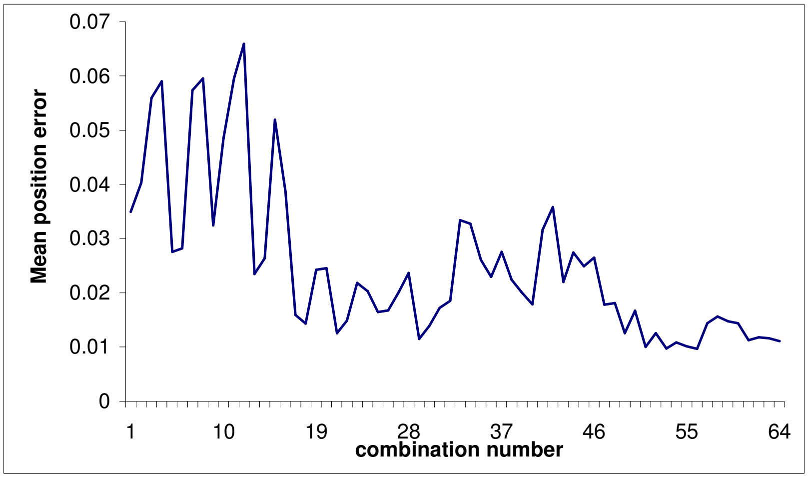

Fig 3.4 Mean Positional Error for case (ii)

For case (ii) mean positional error is maximum (0.0683 m) at combination number 12 and is minimum (0.00964 m) at combination number 56 as observed in Fig. 3.4. It is also observed that mean positional error is less than 0.02 m from combination number 47 to 64.

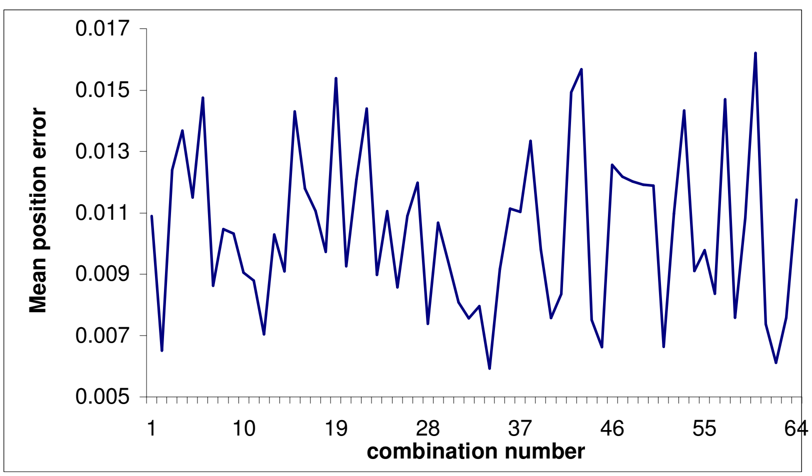

Fig 3.5 Mean Positional Error for case (iii)

Likewise for case (iii) it is observed that mean positional error is maximum $(0.01586\ \text{m})$ at combination number 43 and is minimum $(0.00593\ \text{m})$ at combination number 34, as shown in Fig. 3.5. It is also observed that mean positional error is less than $0.007\ \text{m}$.

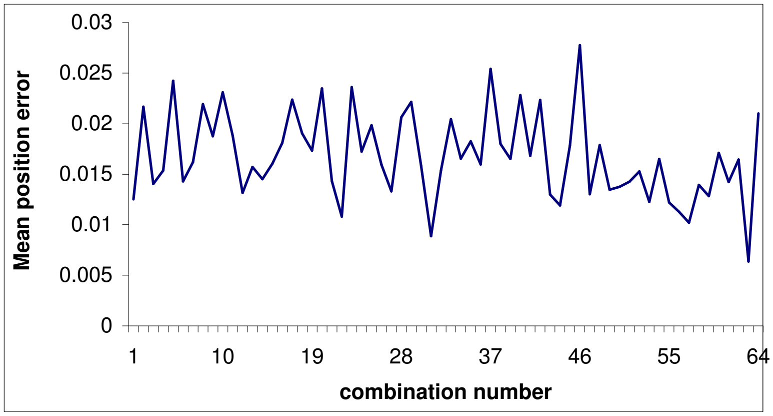

Fig 3.6 Mean Positional Error for case (iv)

For case (iv) mean positional error is maximum $(0.02653\ \text{m})$ at combination number 46 and is minimum $(0.0078\ \text{m})$ at combination number 63 as observed in Fig. 3.6(a). It is observed that mean positional error is less than $0.01\ \text{m}$ in few combinations.

Similarly using the simulation results performance measure SN ratio is computed. The change of SN ratio against the combination number is presented with the help of graphs. The results of cases (i), (ii), (iii) and (iv) are presented in Figs 3.7, 3.8, 3.9 and 3.10 respectively. As this investigation utilizes all combinations of control factors into consideration these graphs summarizes the over all behavior of manipulator to control parameter change.

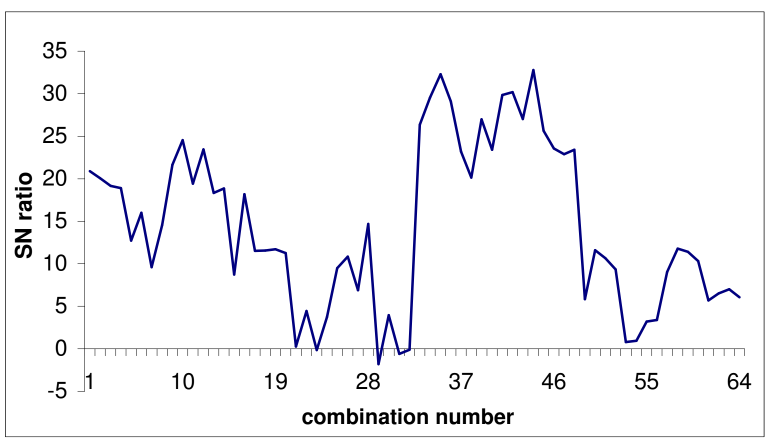

Fig 3.7 SN ratio for case (i)

In Fig.3.7, it is seen that SN ratio of different combinations are increasing from combination number 34th to 44th. The SN ratio is observed to vary between –1.78dB and 32.76 dB. Similarly in Fig. 3.8, for case (ii) it is observed that performance vary between 5.86 dB and 70.50 dB. Combination number 1 to 17 and 33 to 47 have performance above 35 dB. Rest of the factor combinations show poor performance.

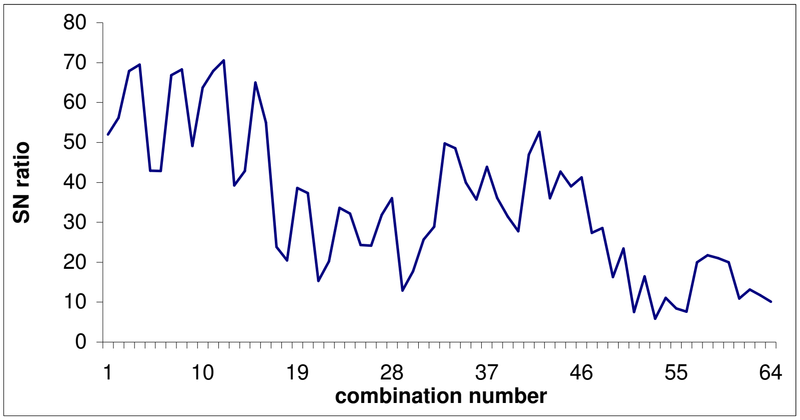

Fig 3.8 SN ratio for case (ii)

It is seen that for case (iii) in Fig. 3.9, SN ratio vary between 35 dB to 42 dB. The maximum SN ratio is 42.57 dB at combination number 2. Interestingly most of the performance is observed to be above 34 dB for all combinations.

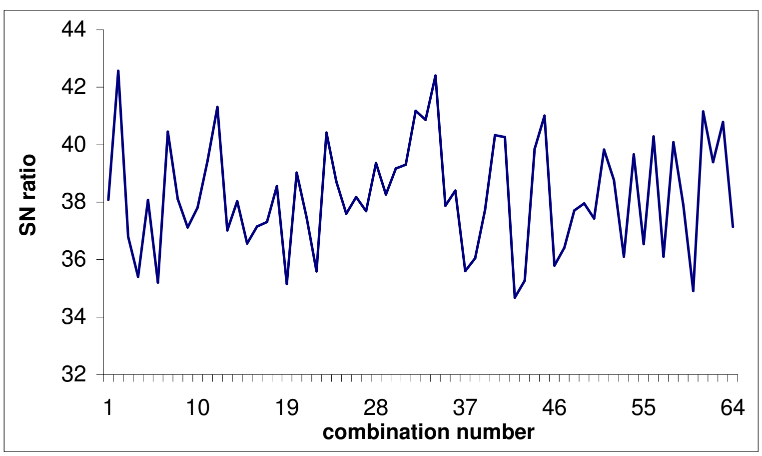

Fig 3.9 SN ratio for case (iii)

In Fig. 3.9, it is observed that maximum SN ratio is 42.57 dB at combination number 2 and minimum 34.67 dB at combination number 41. Most of the performance fluctutae on and around 38 dB. Lastly for case (iv) results are plotted in Fig. 3.10 and it is observed that maximum SN ratio is 47.65 dB at combination number 55 and minimum 24.2 dB at combination number 37. Most of the performance fluctuate on and around 32 dB.

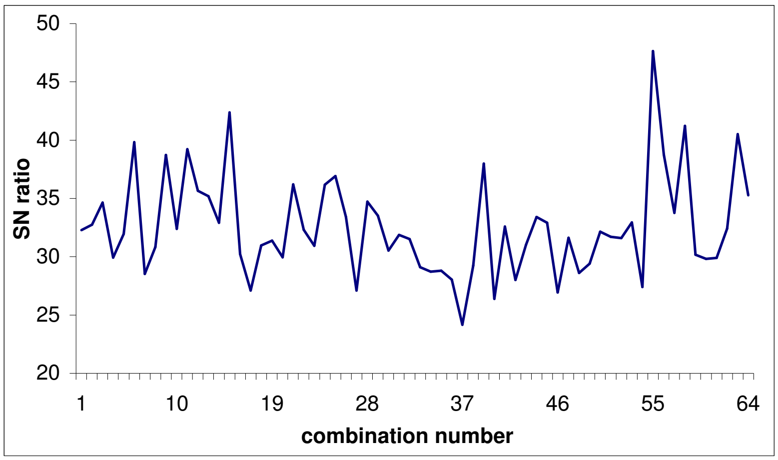

Fig 3.10 SN ratio for case (iv)

To observe overall behavior of performance for all control factor combinations, reliability as a performance measure is computed by running the simulation 200 times separately. For computation of reliability, tolerance range around the target point is selected by trial and error basis because it was observed that wider the tolerance range higher the reliability and tighter the tolerance poorer the reliability for all factor combinations. Subsequently based on tolerance ranges chosen for experiment, reliability is computed. This tolerance range is assumed same for all the cases. The rational behind keeping this same is to make comparison between the cases easy. The change of performance measure i.e. reliability, against combination number is displayed in Figs. 3.11, 3.12, 3.13 and 3.14 for case (i), (ii), (iii) and (iv) respectively.

For case (i), it is observed that maximum reliability is 0.07 at combination number 49. From above figure it is observed that better performance is found between 16^{th} and 33^{rd} and 48^{th} and 64^{th} combination numbers. In this case, there are six combinations which showed poor performance. The value of reliability, is found to be zero in these combinations, indicating that the end-effector has not reached the specified region even once in 200 simulations.

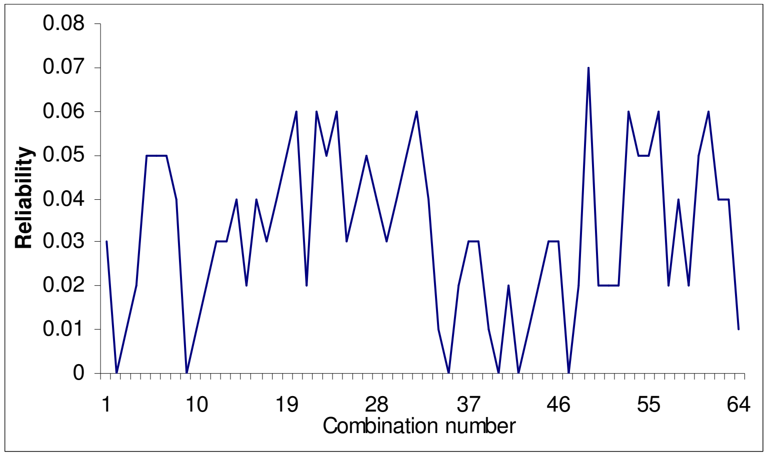

Fig 3.11 Reliability for case (i)

For case (ii), in Fig. 3.12, it is observed that maximum reliability is 0.07 at combination number 22. Better performance is found between combination number 47th and 58th. There are sixteen combinations for which reliability is zero. This indicates that the end-effector has not reached the region specified even once. The reason for showing poor performance in all the above cases are the tolerance value chosen around the target point.

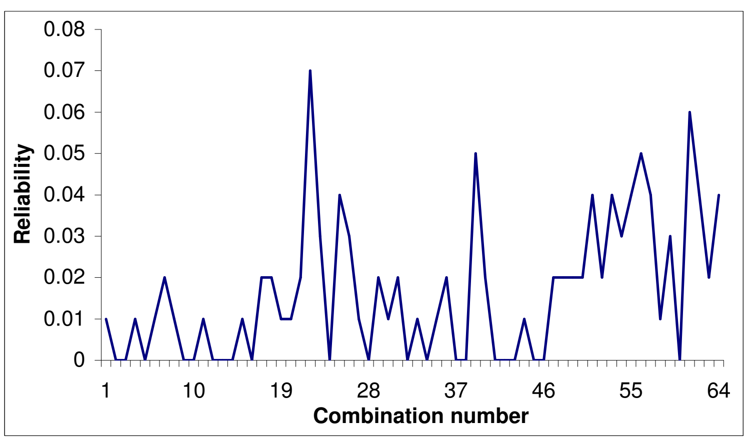

Fig 3.12 Reliability for case (ii)

Similarly for case (iii), computed performance measure reliability is plotted against the combination number. In Fig. 3.13, it is observed that maximum reliability is 0.25 at combination number 25, 51 and 61. Poor performances are found for more than forty combinations.

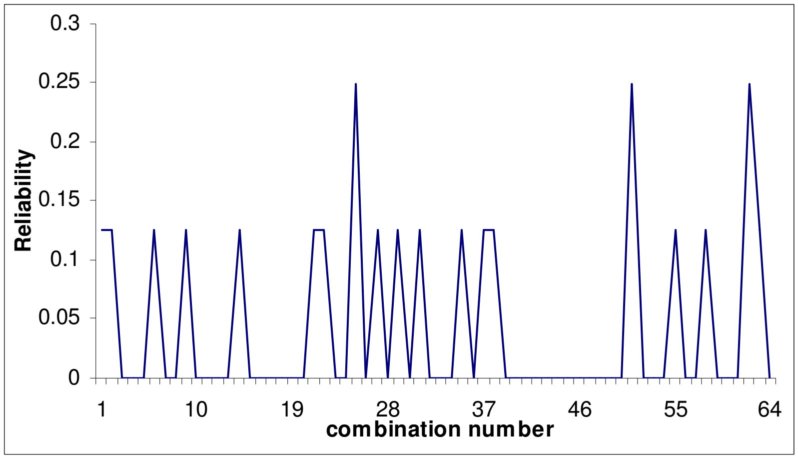

Fig 3.13 Reliability for case (iii)

For case (iv) in Fig. 3.14, it is observed that maximum reliability is 0.16 at combination number 61. Poor performances are found for more than twenty one combinations, indicating that the end-effector has not reached the specified region even once in 200 simulations.

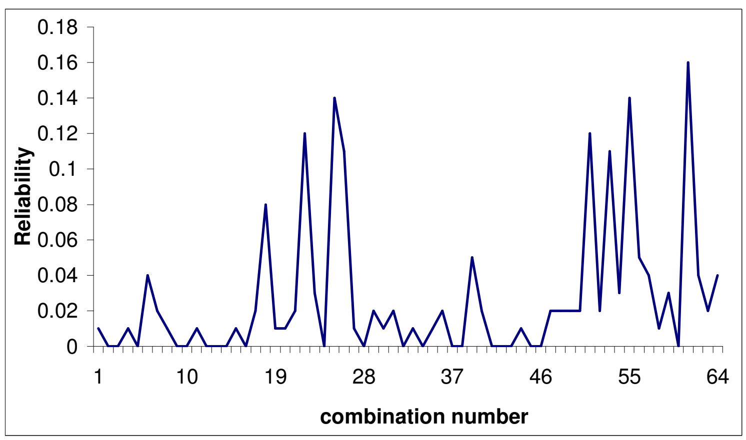

Fig 3.14 Reliability for case (iv)

Referring to above graphs combination number which delivers optimal performance measure or poor performance can easily be observed. In addition, these graphs help in understanding and capturing the relationship between the performance measures. The detailed discussions and analysis are provided in next section.

3.7 DISCUSSION AND ANALYSIS OF PERFORMANCE

After comparing the values of performance measure, optimal combinations which delivers optimal performance are identified and tabulated in Tables 3.9, 3.10, 3.11 and 3.12 for cases (i), (ii), (iii) and (iv) respectively. For case (i) the results of ANOVA are presented in Table 3.5 which clearly indicates that all control factors are statistically significant. All control factors have significant role to play in performance measure apart from the interactions. Therefore, none of the control factors go out of contention for parameter design. Subsequently mean positional error, SN ratio and reliability are used to find the suitable control factor combinations for which performance is optimal.

Table 3.9 Optimum Parameters for Different Performance Measure - case (i)

Control factor	**Target Point** $P(0.4, 0.3)$		
	SN ratio	**Reliability**	**Mean Positional Error in (m)**
Value (Combination No.)	32.7652 (44)	0.07 (49)	0.00735 (29)
l_1 (m)	0.50	0.50	0.40
l_2 (m)	0.30	0.40	0.40
m_1 (kg)	8	7	8
m_2 (kg)	5	5	6
τ_1 (Nm)	–800	–500	–500
τ_2 (Nm)	–105	–105	–100

Simulation results in terms of mean positional error, SN ratio and reliability for case (i) have been given in Figs. 3.3, 3.7 and 3.11, respectively for 64 parameter combinations. The optimum parameter combination using above performance measure are shown in Table 3.9. The basis for selection of optimum parameter combination is the combination number which has maximum value of SN ratio, reliability and minimum mean positional error.

Table 3.10 Optimum Parameters for Different Performance Measure - case (ii)

Control Factor	Target Point $P(0.5, 0.4)$		
	SN ratio	**Reliability**	**Mean Positional Error in (m)**
Value (Combination No.)	70.501272 (12)	0.07 (22)	0.009648 (56)
l_1 (m)	0.40	0.40	0.50
l_2 (m)	0.30	0.40	0.40
m_1 (kg)	8	7	7
m_2 (kg)	5	6	6
τ_1 (Nm)	–800	–500	–800
τ_2 (Nm)	–105	–105	–105

For case (ii) the results of ANOVA are presented in Table 3.6 and it is observed that control factor m_1 is statistically insignificant. The simulated performance measures are shown in Figs. 3.4, 3.8 and 3.12 for 64 parameter combinations. The optimum parameter combinations are presented in Table 3.10. Similarly, for case (iii) ANOVA of simulated results are presented in Table 3.7. In this case once again, control factor m_1 is observed to be statistically insignificant. The performance measures are shown in Figs. 3.5, 3.9, 3.13, and optimum parameter combinations are given in Table 3.11. The performance measures of case (iv) are shown in Figs. 3.6, 3.10, 3.14 for 64 parameter combinations. The optimum parameter combinations are presented in Table 3.12. Results of ANOVA are presented in Table 3.8 and conclude that control factor l_1 is statistically insignificant.

Table 3.11 Optimum Parameters for Different Performance Measure - case (iii)

Control factor	Target point $P(0.4, 0.4)$		
	SN ratio	**Reliability**	**Mean Positional Error in (m)**
Value (Combination No.)	42.57 (2)	0.25 (25)	0.00593 (34)
l_1 (m)	0.40	0.40	0.50
l_2 (m)	0.30	0.40	0.30
m_1 (kg)	7	8	7
m_2 (kg)	5	5	5
τ_1 (Nm)	–500	–500	–500
τ_2 (Nm)	–105	–100	–105

Table 3.12 Optimum Parameters for Different Performance Measure - case (iv)

Control factor	Target point $P(0.3, 0.3)$		
	SN ratio	**Reliability**	**Mean Positional Error in (m)**
Value (Combination No.)	47.65 (55)	0.16 (61)	0.0088 (31)
l_1 (m)	0.50	0.50	0.40
l_2 (m)	0.40	0.40	0.40
m_1 (kg)	7	8	8
m_2 (kg)	6	6	6
τ_1 (Nm)	–800	–500	–800
τ_2 (Nm)	–100	–100	–100

It has been observed that statistically significant factors are different for different target points in workspace. This indicates that different factors have different contribution to performance variation as target position changes or in other word, performance is dependent on the target point and for each target point different control factors are significant. But there is no evidence to show a correlation between them. It is important to note that statistically significant factors are major contributor to performance variation of manipulator, even though there is mathematical relationship between the parameters and these relations are utilized in simulating the performance. In addition to this optimum combination of control factors required to perform task are different for different cases. This indicate that one set of parameters of manipulator for one task will behave differently for other type of task.

Finally the optimum factor combinations obtained using mean positional error, SN ratio and reliability do not agree in the all the cases considered. The trends of performance measures observed to be different in different cases and peaks observed for different combination numbers are equally comparable in both the figures. Possible reason for disagreement in optimal solution can be due to the transformation of positional error into Taguchi's SN ratio which may not be same as the untransformed result obtained from reliability computation. It is observed that most of the combinations for case (i), (ii), (iii) and (iv) show poor simulation performance, possible reason may be attributed to the assumed value of retarding torque. Moreover the torque supplied at joint may not be able to satisfy the requirement. Other reason may be the inherent round off error present in search solution computation.

3.8 LIMITATIONS OF HEURISTICS BASED SEARCH SIMULATION METHOD

Though the proposed method has been found to be useful in simulating and obtaining optimal parameter combinations, but it had certain limitations. While analyzing the limitations of present method, an attempt has been made to develop generalized simulation method for the optimal manipulator parameter design. The limitations of present method are listed below.

1. Limitations in using start point of the task carried out by manipulator,
2. Limitations in using time required to reach start to destination point by manipulator,
3. Limitations in incorporating end conditions at the start as well as destination point,
4. Limitations in explicit use of noise factors like friction, joint clearances, environmental condition and manufacturing tolerances,
5. Limited scope to handle higher degree of freedom manipulator because of the complexity in kinematic and dynamic models.
6. Accuracy in results is restricted due to termination conditions and time required to simulate performance and compute performance measure.

3.9 EPILOGUE

The work presented in this chapter gives an insight to the use of simulation method for modeling and optimizing the performance of robot manipulators. It illustrates search-heuristic based method to simulate various performance measure of manipulator and use of DOE technique to determine statistically significant parameters. While simulating, performance of manipulator no constraint i.e. angular velocity and accelerations have been applied. The approach discussed is an initial attempt in determining significant factors responsible for performance variations and selection of the optimum factors rather than spending effort in controlling performance of manipulator. The novelty of this exploration can be summarized as given below:

(a) Strategies to incorporate effect of noise for simulation of real life performance.

(b) Use of both kinematic and dynamic models of manipulator for simulation of the performances i.e. mean positional error, SN ratio and reliability, to incorporate dynamic effects of parameters on performance of manipulators.

(c) Use of DOE technique to analyze the performance of manipulator.

(d) Use of ANOVA technique to analyze the statistical significance of kinematic and dynamic parameters that contribute most to the observed performance variations.

(e) Use of performance measure i.e. reliability, to investigate overall performance of factorial combinations.

Present work explores the performance variations problem of a manipulator in the perspective of robot designer and its manufacturer for its possible solution.

CHAPTER-4

SCREENING OF FACTORS INFLUENCING THE PERFORMANCE OF MANIPULATOR

4.1 INTRODUCTION

The attempt to analyze the effect of different design, process and noise parameters on performance of manipulator is novel in the sense that the designer can focus on those parameter that has significant influence on performance variations rather than focusing on several unwanted parameters. Without proper knowledge on the impact of parameter values on the performance, robotic system designer often decides parameter values by intuition fulfilling some kinematic performance criteria like manipulability index, condition number etc. [Craig 1989].

The intention behind taking up the problem of screening of parameters of the robot manipulator is to develop the knowledge regarding the impact of parameters on performance. In past, many researchers had attempted statistical analysis of robot performance by changing several process parameters. However, use of simulations model and design of experiments technique to study influence of design, process and noise parameters are rare. In this chapter the screening of parameters by using the Fractional factorial DOE approach is carried out to identify the important parameter. The chapter discusses how this technique is applied to robot manipulator design problem.

The parameters responsible for performance variations were explored in the previous chapter using search based heuristics method. In this chapter these are investigated once again using another novel method. In this method limitations of the earlier method discussed in earlier chapter are removed. The parameters relating to the manipulator designs were classified as the control factors and noise factors in previous chapter. To analyze the statistical significance of effects of these factors, DOE technique has been used. Effects of these factors are investigated using parameters at different level values. As the number of levels to be taken into consideration increase for each control factor and noise factor, the number of combinations becomes large, and unmanageable. Therefore, running experiments with all the combinations are cumbersome, uneconomical and time consuming. As the focus of this chapter is to identify the parameters responsible for performance variations, parameter combinations are generated using “Fractional factorial design of experiment approach”. Where Fractional factorial designs are used for screening of parameters in an experiment. Screening of parameters experiment is usually performed in the early stages of a design project when it is likely that many of the parameters initially considered have little or no effect on the performance. In this chapter many factors i.e. control and noise factors, are considered with the purpose of identifying those factors that have statistically significant effects on performance variations.

It has been observed in the previous chapter that simulation of real life performance of a manipulator is quite complex due to nature of kinematic and dynamic model. To avoid above difficulty, a novel method has been proposed which is different from the method discussed in earlier chapter. Since there are no closed form equations to simulate the real life performance of manipulator, a new method has been proposed to simulate the performance. The performances of the experiments are simulated using proposed approach. Obtained performances are subsequently used to study the statistical significance of parameters. As manipulator is expected to perform various tasks following different trajectories in the workspace, the statistical significance of the control and noise factors of manipulator are investigated. For investigating how tasks and trajectories affect the performance variations of manipulator, different tasks following different trajectories, have been used. The performance measure i.e. positional error defined in previous chapter is used for the analysis of parameters responsible for the performance variations in reaching the destination point.

This chapter is organized in seven sections. In section 4.2, steps for application of Fractional factorial DOE technique to screen the parameters and the importance of this type of analysis are discussed. The application of Fractional factorial DOE technique to a 2-DOF RR planar manipulator has been discussed in section 4.3. The methodology used to simulate the performance of the manipulator is discussed in section 4.4. The assumed manipulator data for simulation and analysis of results of experiment are presented in section 4.5 and 4.6 respectively. In section 4.7, parametric sensitivity of manipulator performance at destination is also investigated to compliment the statistical analysis carried out in this chapter. Effects of particular class of parameters on performance of manipulator have been carried out and results have been presented.

4.2 FRACTIONAL FACTORIAL (2^{k-p}) DOE TECHNIQUE

For parameter screening experiment using DOE technique, already identified control and noise factors in Chapter 3 needed to be considered at several levels. Since with increase in number of factors and number of levels increase the number of experimental runs, to keep number of experiment into a manageable level, factors at two levels are considered. Therefore, experiment with k factors will have 2^k combinations. As the number of factors k in a 2^k factorial increases the number of combinations required for a complete replicate of the design rapidly outgrows the resources of most experiments. To avoid this difficulty, a particular type of experimental design is used where not all parameter combinations are required for experimentation. However, for conducting such experiments it is assumed that high order factor interactions have negligible impact on performance. With this assumption, the main factor and low-order factor interaction effects are obtained by running only a fraction of the complete factorial experiment.

The steps utilized for conducting experiments remains almost same as discussed in Chapter 3. Using same steps the parameters that have significant impact on the performance variations of a manipulator are studied. In place of repeating the steps followed, the modifications in adopted procedures are discussed in detail. The methodology described below is used for Fractional factorial DOE technique:

Step 1. Statement of the problem

The statement of problem and objective of the experiment are clearly spelt out.

Step 2. Choice of factors and levels

In this step, the factors to be varied in the experiment, the number of levels and the ranges over which these factors will be varied are decided.

Step 3. Selection of the response variable

The experimenter selects the response variable that really provides useful information about the product or process under investigation.

Step 4. Choice of suitable fractional factorial experimental design, generators and resolution

A 2^k factorial design containing 2^{k-p} combinations is called a $1/2^p$ fraction of the 2^k design. For creating 2^{k-p} combinations it require proper selection of p independent generators. The defining relations for the design consist of the p generators initially chosen and their $2^p - p - 1$ generalized interactions [Montgomery 2001]. The alias structures are found by multiplying each effect column by the defining relation. For choosing the generators, proper care should be taken so that effects of potential parameters are not aliased with each other. To avoid a factor getting aliased with other factors, suitable design resolution is selected. A design is of resolution R, if no p-factor performance is aliased with another effect containing less than $R - p$ factors. Design resolutions are always specified by Roman numeral subscripts. For conducting experiments 2^{k-p} Fractional factorial design of resolution V is selected. In this design resolution main effects are clear of two-factor interactions and two factor interactions are not aliased with each other. The smallest word in the defining relation of such a design must have five letters. This design allows the estimation of all the main effects and two factor interactions.

Step 5. Perform the Experiment

It is vital to carefully monitor the process or simulation to ensure that everything is done as per plan. In this step, experiment is conducted and performance measure is obtained.

Step 6. Analysis of Performances

Statistical methods are used to analyze the data so that results and conclusions are made about the factors in the experiment.

4.3 APPLICATION OF FRACTIONAL FACTORIAL DOE TECHNIQUE FOR PARAMETER SCREENING

Fractional factorial DOE technique has been satisfactorily applied in several engineering applications effectively. However, application to manipulator design problem has not been attempted so far. To invesigate the effect of parameters on performance of manipulator a 2-DOF RR planar manipulator explained in previous chapter is considered. The step-by-step application of Fractional factorial DOE technique discussed above to a robot manipulator parameter-screening problem is presented here.

4.3.1 Statement of the Problem

The objective is to identify the manipulator parameters responsible for performance variations. It is desired that the manipulator must reach the commanded destination in the workspace accurately following a particular trajectory. For the 2-DOF RR planar manipulator, the task is always specified by the start and target point in the workspace of the manipulator. The start point is denoted by $P(x_i, y_i)$ and the destination point by $P(x_f, y_f)$, where workspace is *xy*-plane. The kinematic and dynamic models of the manipulator are used to simulate the performance in reaching the destination point and performance along a trajectory of manipulator. The method used is discussed in next section.

4.3.1.1 Kinematic and Dynamic Models of 2-DOF RR Planar Manipulator

Conducting experiments to ascertain the statistical significance of parameters are very costly and uneconomical. Along with this difficulty, the measurements of performances are still costlier and difficult to measure. Developed approach provides information regarding the performances of a manipulator and works like a simulator. For easy reference considered 2-DOF RR planar manipulator is shown in Fig. 3.1. The kinematic model in terms of D-H (Denavit-Hartenberg) notation for the homogenous transformation matrix [Mittal 2003] is given by

$$ {}^{0}T_2 = \begin{bmatrix} C_{12} & -S_{12} & 0 & l_1C_1 + l_2C_{12} \\ S_{12} & C_{12} & 0 & l_1S_1 + l_2S_{12} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.1} $$

where $C_i = \cos\theta_i, S_i = \sin\theta_i,\ C_{ij} = \cos(\theta_i + \theta_j),\ S_{ij} = \sin(\theta_i + \theta_j)$ with *i, j* = 1, 2 for the two links, link 1 and link 2, respectively. The coordinates of end-effector position $P(x, y)$ are obtained from last column of equation (4.1) or from equations (3.1) and (3.2), respectively.

The links are assumed to be rigid thin rods and gravity loading is considered. Based on Lagrange-Euler formulation and considering that manipulator is moving freely in its workspace. The dynamic behavior of joint *i* of the manipulator with contributions of viscous friction is given by [Mittal 2003]

$$ \tau_i = \sum_j M_{ij}\ddot{q}_j + \sum_j \sum_k h_{ijk}\dot{q}_j\dot{q}_k + B_i\dot{q} + G_i \tag{4.2} $$

where, M_{ij} the symmetric inertia matrix, h_{ijk} the centrifugal and coriolis force coefficients, B_i the viscous friction coefficient at the joint, G_i the gravity force vector, $\tau(t)$ the joint torque vector and $q(t), \dot{q}(t), \ddot{q}(t)$ are the joint position, velocity, acceleration vectors, respectively. The dynamic model used here has few exceptions as compared to the dynamic model considered in earlier chapter. For simulating performance of manipulator, the dynamic model is used in vector or matrix representation as compared to algebraic representation. The frictions at the joints are considered as compared to no friction in previous chapter. Therefore, torque equations for the two joints in generalized form are

$$\tau_1 = M_{11}\ddot{\theta}_1 + M_{12}\ddot{\theta}_2 + H_1 + B_1\dot{\theta}_1 + G_1 \tag{4.3}$$

$$\tau_2 = M_{21}\ddot{\theta}_1 + M_{22}\ddot{\theta}_2 + H_2 + B_2\dot{\theta}_2 + G_2 \tag{4.4}$$

where, $M_{11} = \left(\frac{1}{3}m_1 l_1^2 + m_2 l_1^2 + \frac{1}{3}m_2 l_2^2 + m_2 l_1 l_2 C_2\right)$

$$M_{12} = \left(\frac{1}{3}m_2 l_2^2 + \frac{1}{2}m_2 l_1 l_2 C_2\right) = M_{21}$$

$$M_{22} = \left(\frac{m_2}{3} l_2^2\right)$$

$$H_1 = -m_2 l_1 l_2 S_2 \dot{\theta}_1 \dot{\theta}_2 - \frac{m_2}{2} l_1 l_2 S_2 \dot{\theta}_2^2$$

$$H_2 = \frac{m_2}{2} l_1 l_2 S_2 \dot{\theta}_1^2$$

$$G_1 = \left(\frac{m_1}{2} + m_2\right) g l_1 C_1 + \frac{m_2}{2} g l_2 C_{12}$$

$$G_2 = \frac{m_2}{2} g l_2 C_{12}$$

where, θ_1, θ_2, $\dot{\theta}_1$, $\dot{\theta}_2$, $\ddot{\theta}_1$, $\ddot{\theta}_2$ are angular displacements, velocities, accelerations of link one and two, respectively; m_1 and m_2 are masses of link 1 and link 2, respectively and g is the acceleration due to gravity.

To determine the torque required at the joints of the manipulator following approach is used. Based on the trajectory chosen to perform the task, torque required at joints are determined. It is known that torque required to follow two trajectories are different. Thus by choosing different trajectories, influence of torque profile on performance variations of manipulator has also been investigated. In this chapter, effects of two trajectories i.e. cubic and quintic are studied to investigate the influence of time law and end conditions on performance variations of manipulator. The two trajectories used for the exploration are discussed below.

Trajectory I – Cubic Path

For the manipulator to follow a cubic trajectory, the initial (*i*) and final (*f*) boundary conditions are: $q_i = (\theta_{1i}; \theta_{2i})$; $q_f = (\theta_{1f}; \theta_{2f})$; $\dot{q}_i = (0; 0)$ and $\dot{q}_f = (0; 0)$ where q_i, q_f are joint coordinates and $\dot{q}_i$, $\dot{q}_f$ are joint velocities, respectively and time to reach the destination is T_g. The time law of cubic trajectory is given [Mittal 2003] by:

$$q(t) = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \end{bmatrix} = \begin{bmatrix} a_0 + a_1 t + a_2 t^2 + a_3 t^3 \\ b_0 + b_1 t + b_2 t^2 + b_3 t^3 \end{bmatrix} \tag{4.5}$$

where, a_0, a_1, a_2, a_3, b_0, b_1 b_2, b_3 are constants. Applying boundary conditions to equation (4.5), the constants are given as:

$$a_0 = \theta_{1i}\,;\; a_1 = \dot{\theta}_{1i};\; a_2 = \frac{3(\theta_{1f} - \theta_{1i})}{T_g^2};\; a_3 = -\frac{2(\theta_{1f} - \theta_{1i})}{T_g^3};$$

$$b_0 = \theta_{2i}\,;\; b_1 = \dot{\theta}_{2i};\; b_2 = \frac{3(\theta_{2f} - \theta_{2i})}{T_g^2};\; b_3 = -\frac{2(\theta_{2f} - \theta_{2i})}{T_g^3};$$

Trajectory II - Quintic Path

For the manipulator to follow a quintic trajectory, the initial (*i*) and final (*f*) boundary conditions are:

$q_i = (\theta_{1i}; \theta_{2i})$; $q_f = (\theta_{1f}; \theta_{2f})$; $\dot{q}_i = (0; 0)$; and $\dot{q}_f = (0; 0)$ $\ddot{q}_i = (0; 0)$ and $\ddot{q}_f = (0; 0)$

where q_i, q_f are joint coordinates, $\dot{q}_i$, $\dot{q}_f$ are joint velocities, $\ddot{q}_i$, $\ddot{q}_f$ are joint accelerations respectively and time to reach the destination is T_g. The time law of quintic trajectory is given [Craig 1989] by:

$$q(t) = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \end{bmatrix} = \begin{bmatrix} a_0 + a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5 \\ b_0 + b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5 \end{bmatrix} \tag{4.6}$$

where a_0, a_1, a_2, a_3, a_4, a_5, b_0, b_1, b_2, b_3, b_4, b_5 are constants. Applying boundary conditions to equation (4.6), the constants are given as:

$$a_0 = \theta_{1i}\,;\; a_1 = \dot{\theta}_{1i};\; a_2 = \ddot{\theta}_{1i};\; a_3 = \frac{10(\theta_{1f} - \theta_{1i})}{T_g^3};\; a_4 = -\frac{15(\theta_{1f} - \theta_{1i})}{T_g^4};\; a_5 = \frac{6(\theta_{1f} - \theta_{1i})}{T_g^5};$$

$$b_0 = \theta_{2i} \; ; \; b_1 = \dot{\theta}_{2i} ; b_2 = \ddot{\theta}_{2i} ; \; b_3 = \frac{10(\theta_{2f} - \theta_{2i})}{T_g^3} ; \; b_4 = -\frac{15(\theta_{2f} - \theta_{2i})}{T_g^4} ; \; b_5 = \frac{6(\theta_{2f} - \theta_{2i})}{T_g^5} ;$$

Using equation (4.5) for cubic time law and equation (4.6) for quintic time law in dynamic model of manipulator $\tau(t)$ is computed using process of inverse dynamics.

4.3.2 Identification of Factors and Levels

Referring to the kinematic model equations (3.1) and (3.2) and dynamic model of 2-DOF RR planar manipulator equations (4.3) and (4.4), the parameters which directly affect the working of manipulator are identified. The parameters are classified as control factors and noise factors.

4.3.2.1 The Control Factors

As discussed in chapter 3, control factors are identified from the kinematic and dynamic model of manipulator. Unlike to discussion in section 3.4.2 of Chapter 3, in this chapter first four i.e. link lengths l_1 and l_2, link masses m_1 and m_2 become control factors. For these parameters, suitable values can be chosen to conduct experiment.

4.3.2.2 The Noise Factors

The noise factors were identified in chapter 3, and enlisted in section 3.4.2. The noise variables are known for their random natures but it is assumed that these parameters are controllable for the experimentation.

4.3.3 Performance Measures for Manipulator - The Response Variable

To identify the contribution of individual parameters on performance variations the performance measure "positional error" i.e. ε_i defined in equation (3.17) and mean positional error i.e. $\bar{\varepsilon}$ in equation (3.18) are used. This performance measure provides the information regarding the deviation in performance at the destination while moving along a particular trajectory.

4.3.4 Design the Experiment – Choice of Experimental Design

For economic reasons, Fractional factorial designs, are commonly used. This design consists of a subset or fraction of full factorial designs. Optimal fractions are chosen according to the resolution and minimum aberration criteria. Aliasing of factorial effects is a consequence of using fractional factorial designs.

4.3.5 Performing and Analyzing the Experiment

For conduct of experiment, combined array approach [Montegomery 2001] has been used. This design is called combined array design because it contains control and noise parameters. In this approach the inner array for control parameters and outer array for noise parameters proposed by Taguchi [Park 1998] has been avoided. The main consideration in adopting this approach is to investigate the statistical significance of control and noise parameters in place of getting a design, which is insensitive to noise parameters.

To investigate the effect of parametric variation on performance of manipulator, experiments are conducted with the help of design matrix. Each combination is run for finite number of replications to capture the effect of noise. The developed methodology and data utilized to simulate the performance is provided in sections 4.4 and 4.5.

From each replication positional error is obtained as outcome of the experiment and thereafter the performance measure mean positional error are computed for each combination. Statistical analyses of performance of experiment are carried out using analysis of variance (ANOVA) technique; subsequently F-test is used to obtain statistically significant parameters and interacting parameters. For current study, statistically significant parameters and interacting parameters are determined using ANOVA, and its results are discussed in section 4.7.

4.4 APPROACH TO SIMULATE THE PERFORMANCE OF MANIPULATOR INCORPORATING EFFECT OF NOISE

The concept of uncertainty plays an important role in the investigation of manipulator performances. In manipulator's performance analysis, unknown variations in material distribution, uncertainty in manufacturing and assembly and uncertainty in boundary condition leads to variation in performance. As it is known that the dynamic model of a manipulator is highly coupled and non-linear, simulation of real time performance incorporating of effect of noise become difficult. It is commonly observed that robots manufactured with same design specifications have different performance variability. The main reason for this is the design tolerances prescribed for the manufacturing and assembling operations and other noise factors discussed in Chapter 3.

Given the randomness of robot parameters, models for their probability distributions are needed. For the sake of conciseness, the variations of the parameter are assumed to obey Gaussian distributions with nonzero mean and nonidentical standard deviations. However, assumption of Gaussian distributions for tolerances on dimensions of any manufacturing processes reflects the actual distribution found in practice. Indeed the justification for representing many complicated phenomenon by Gaussian distribution functions lies in central limit theorem [Montgomery 2001]: If $x_1, x_2,\ x_3 \,..........x_n$ are independent random variables with mean μ_i and variance σ_i^2, for parameter x_i and if, $y = x_1 + x_2 + x_3 \,.......... + x_n$, then the distribution

$$\frac{y - \sum_{i=1}^{n} \mu_i}{\sqrt{\sum_{i=1}^{n} \sigma_i^2}} \tag{4.7}$$

approaches the $N(0, 1)$ distribution as n approaches infinity i.e. if y is the sum of n independent random variables having nonidentical density functions, then y tends to have a Gaussian density function as n approaches infinity. Hence the central limit theorem not only justifies approximate Gaussian distribution assumption but also simplifies the probability computations [Rao 1988]. Therefore, the variations in length and mass of the links are assumed to follow the Gaussian distribution and standard deviations are dependent on the specified tolerance for link lengths and link masses.

Robot performing commanded task can be modeled as a stochastic process which is driven by actuators and when the time interval between sample values of supplied torque are small, the process becomes highly correlated over time. As torque being supplied to manipulator joint is in time order and highly correlated, the supplied torque by the actuators are treated as random vectors. These vectors are assumed to follow Gaussian stochastic process with Markov properties. The stochastic process with Markov properties has been used successfully to model Brownian motion process, diffusion processes, control systems and vibrations of structures [Ariaratnam 1988]. It is widely used in the area of Engineering, Communication theory and Management Sciences also. Therefore, supplied torque at manipulator joint is assumed to follow Gaussian stochastic process with Markov properties. Where the future values of a stochastic process depend only on immediate past or present but not all past events. Thus, a stochastic process with Markov properties is represented mathematically as:

The family of random variables $\{X_{t_n}\}$ indexed by the continuous real variable t and $t_1 < t_2 < .... < t_{n+1} \quad \{n = 1,2......\}$ represents points in time,

$$P(X_{t_{n+1}} = u_{n+1} \big| X_{t_n} = u_{n,}\ X_{t_{n-1}} = u_{n-1,} .... X_{t_1} = u_1) = P(X_{t_{n+1}} = u_{n+1} \big| X_{t_n} = u_n) \tag{4.8}$$

where, X_{t_n} denote the state of the process at time t_n and u_n, u_{n-1}, u_{n-2},....... are the values of the process parameters. $P(X_{t_{n+1}} = u_{n+1} \big| X_{t_n} = u_n)$ is called transition probability for $(u(t)_{n+1} = u_{n+1})$, given that $(u(t_1) = u_1,........... u(t_n) = u_n)$. The dynamics of a system can be written in state space representation as

$$du(t) = f(t, u(t))dt \tag{4.9}$$

Equation (4.9) shows that the change in the system parameter $u(t)$ is a function of the value of $u(t)$ at time t and does not depend on the values of u at previous times. It is assumed that the length and mass of link one and two of manipulator are random variables following Gaussian distribution, and the supplied torque at the joints by the actuators are random vectors following Gaussian stochastic process with Markov properties. The strategy adopted to simulate the joint torque vector is discussed below.

4.4.1 Simulation of Joint Torque

As the fluctuation in supplied joint torque vector $\tau(t)$ is assumed to follow Gaussian stochastic process with Markov properties, it is simulated as a time series. The time series is a sequence of observation taken sequentially over time. These observations in a time series are regarded as a sample realization from an infinite population of such time series that could have been generated by the stochastic process. The stochastic model, which can be used for simulation of supplied joint torque vector, is the auto-regressive process model. In this model, the current values of the process are expressed as a finite, linear aggregate of previous values of the process and a shock a_t. A generalized autoregressive process [Box 1994] of order p is represented as:

$$\tilde{\tau}_t = \phi_1 \tilde{\tau}_{t-1} + \phi_2 \tilde{\tau}_{t-2} + \phi_3 \tilde{\tau}_{t-3} + ... + \phi_p \tilde{\tau}_{t-p} + a_t \tag{4.10}$$

where, the values of the process at equally spaced time $t, t-1, t-2, ...$ is denoted by $\tau_t, \tau_{t-1}, \tau_{t-2}, ...$ .Also the $\tilde{\tau}_t, \tilde{\tau}_{t-1}, \tilde{\tau}_{t-2}, ...$ are the deviation from mean μ ; e.g. $\tilde{\tau}_t = \tau_t - \mu$ and the symbol $\phi_1, \phi_2, \phi_3, ...\phi_p$ are the finite set of weight parameters. For simulation of random joint torque vector first order auto-regressive (Markov) process is assumed and equation (4.10) become

$$\tilde{\tau}_t = \phi_1 \tilde{\tau}_{t-1} + a_t \tag{4.11}$$

where, ϕ_1 is unknown constant and a_t is normally and independently distributed with mean zero and standard deviation σ_a. The observations $\tilde{\tau}_{t_n}$ from such a model have zero mean and standard deviation $\sigma_a / \sqrt{(1-\phi_1^2)}$ and the observations that are k periods apart (τ_t and τ_{t-k}) have correlation coefficient ϕ_1^k. The relationship between the standard deviation of random shocks σ_{a_i} and the process σ_{τ_i} is given by

$$\sigma_{a_i} = \sigma_{\tau_i} \sqrt{(1-\phi_1^2)} \tag{4.12}$$

To satisfy condition of stationary process, value of ϕ_1 is chosen between $[-1, +1]$, where stationary process has constant mean and standard deviation over time *t.* The noise parameters used in simulation process are denoted by

(i) Error due to manufacturing and assembly tolerances are denoted by standard deviations of Gaussian distribution i.e. $\sigma_{l_1}, \sigma_{l_2}, \sigma_{m_1}$ and σ_{m_2},

(ii) Fluctuations in supplied joint torque are denoted by standard deviations of Gaussian stochastic process i.e. σ_{τ_1} and σ_{τ_2},

(iii) Viscous friction at the manipulator joints are denoted by constants B_1 and B_2,

(iv) Joint clearances are denoted by standard deviations of Gaussian distribution i.e. σ_{θ_1} and σ_{θ_2}.

The methodology adopted to simulate the performance of manipulator is discussed next.

4.4.2 Computation of Manipulator Performance

The dynamic model of a manipulator is highly coupled and non-linear therefore, present approach emphasizes a novel way of finding out the joint accelerations, velocities and positions because of difficulties associated with analytical method. Using process of inverse dynamics, the input vectors $\tau(t)$ required for the manipulator joints are obtained. For computation of torque vector nominal values of manipulator geometric parameters, Cartesian coordinates of start and destination point, and type of trajectory to reach destination are used. Then torque vector required at the individual joints of manipulator are simulated using method discussed in sub-section 4.5.2. Using process of forward dynamics, the actual output vectors available at joints i.e. $q(t), \dot{q}(t), \ddot{q}(t)$ are computed. The output vectors at the joint are obtained by integrating the dynamic model numerically. For each time step (sampling time) the effect of noise for each control factor, in the form of individual random error, are generated and incorporated in respective parameters. The noise parameters like manufacturing and assembly tolerances and fluctuation in supplied torque are incorporated into control parameters. Where as the noise parameters like viscous friction and effect of joint clearances and play, on performance are used individually in integration process. Subsequently joint coordinates $q(t)$ are transformed, using the kinematic equations, to obtain the Cartesian coordinates of the point reached at each time step and destination reached finally. After computing these performances of manipulator positional error at the destination while moving along a trajectory are obtained.

4.5 SIMULATION

To simulate the performance a computer programme is developed using the approach discussed in section 4.4. For the computer programme, MATLAB software and its commands are used. The numerical values used to simulate the real life performance of 2-DOF RR planar manipulator the level values for geometric parameters i.e. link lengths and link masses, are assumed. The noises of geometric parameters are assumed to follow Gaussian distribution, with mean zero and standard deviation σ, and the noise in torque vector is assumed as AR stochastic process with mean zero and standard deviation σ_{τ_i}.

4.5.1 Control and Noise Parameters

The combined effect of control and noise parameters is studied. The manipulator control parameters and noise parameters have been chosen at two level values. Each of fourteen parameters is assigned a capital letters A, B, C, D, E, F, G, H, J, K, L, M, N, and O. The assumed values of control parameters and noise parameters are provided in Table 4.1.

Table 4.1 Level Values of Control and Noise Parameters

Sl. No.		Factor	Symbol used for ANOVA	Low Level	High Level
1	Control Factors	l_1 (m)	A	0.40	0.50
2		l_2 (m)	B	0.30	0.40
3		m_1 (kg)	C	7	8
4		m_2 (kg)	D	5	6
5	Noise Factors	σ_{τ_1} (Nm)	E	0.05	0.10
6		σ_{τ_2} (Nm)	F	0.05	0.10
7		σ_{θ_1} (degree)	G	0.05	0.1
8		σ_{θ_2} (degree)	H	0.05	0.1
9		B_1 (Ns)	J	3.5	4
10		B_2 (Ns)	K	2	2.5
11		σ_{l_1} (m)	L	5×10^{-5}	1×10^{-4}
12		σ_{l_2} (m)	M	5×10^{-5}	1×10^{-4}
13		σ_{m_1} (kg)	N	2.5×10^{-3}	5×10^{-3}
14		σ_{m_2} (kg)	O	2.5×10^{-3}	5×10^{-3}

4.5.2 Generation of Design Matrix

The numbers of control and noise factors considered are 14 and each factor has two levels, then complete one replication of this design amounts to 16384 simulations. Another critical decision for any experimental design is the choice of the number of replicates to run. If numbers of replicates assumed are eight, then this experiment will require 131072 simulations. Conducting and managing so many simulations and data is quite difficult. As the focus of investigation is to screen the parameters and interactions, which have strongest influence on performance variations, experiments are conducted using Fractional factorial design of experiment approach. To have manageable experiments 2^{14-6} Fractional factorial design of resolution V is selected. In a resolution V design some factors are aliased with four factor interactions and some two-factor interactions are aliased with three factor interactions. It can be inferred that any resolution V experimental design, the individual factors are strongly clear and two factor interactions are clear. Therefore, experimental design of resolution V is selected to capture the variations contributed by the individual factors, which are not aliased with other factors or two-factor interactions.

Since 2^{14} factorial design has 14 main factors and ${}^{14}C_2$ two factor interactions, to estimate the effect of these, $\frac{1}{64}$ fraction or a 2^{14-6} design is considered. To find the new columns, of Fractional factorial design six design generators are used. The factor generator used for the Fractional factorial design matrix are given below.

To generate this design matrix combinations, a 2^8 factorial design cotaining factors A, B, C, ..……, H are written and then rest six columns for J, K…, O are added. In this design the column J is found using J = ABCDE, and K = ABCFG, L = ABDEFG, M = ABDFH, N = ADEGH, O = ACEFGH respectively. These ABCDE, ABCFG,……, ACEFGH are called design generators for design matrix. The few combinations of design matrix is given in Table 4.2. The strength of the selected design plan is that any two factor interaction is strongly clear, weakness in individual factors are aliased with four factor interactions.

Design matrix indicating all 256 combinations, used for the experimentation is provided in Appendix B1. The fractional factorial design chosen has 256 combinations, when each combination is run for 8 replications, leads to 2048 simulations.

Table 4.2 Design Matrix for 2^{14-6} Fractional factorial Design

Sl. No.	A (m)	B (m)	C (kg)	D (kg)	E (Nm)	F (Nm)	G (deg)	H (deg)	J (Ns)	K (Ns)	L ($\times10^{-4}$ m)	M ($\times10^{-4}$ m)	N (kg)	O (kg)
1	0.4	0.3	7	5	0.05	0.05	0.05	0.05	3.5	2	1	0.5	0.0025	0.005
2	0.5	0.3	7	5	0.05	0.05	0.05	0.05	4	2.5	0.5	1	0.005	0.0025
3	0.4	0.4	7	5	0.05	0.05	0.05	0.05	4	2.5	0.5	1	0.0025	0.005
4	0.5	0.4	7	5	0.05	0.05	0.05	0.05	3.5	2	1	0.5	0.005	0.0025
5	0.4	0.3	8	5	0.05	0.05	0.05	0.05	4	2.5	1	0.5	0.0025	0.0025
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
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.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
251	0.4	0.4	7	6	0.1	0.1	0.1	0.1	4	2.5	0.5	0.5	0.0025	0.005
252	0.5	0.4	7	6	0.1	0.1	0.1	0.1	3.5	2	1	1	0.005	0.0025
253	0.4	0.3	8	6	0.1	0.1	0.1	0.1	4	2.5	1	1	0.0025	0.0025
254	0.5	0.3	8	6	0.1	0.1	0.1	0.1	3.5	2	0.5	0.5	0.005	0.005
255	0.4	0.4	8	6	0.1	0.1	0.1	0.1	3.5	2	0.5	0.5	0.0025	0.0025
256	0.5	0.4	8	6	0.1	0.1	0.1	0.1	4	2.5	1	1	0.005	0.005

The chosen fractional factorial design will provide statistical significance of two factor interactions, which are not aliased with each other. Simulations are run for the task specified in Table 4.3, following cubic and quintic trajectories.

Assumed Process Parameters

(i) For simulation of performance of manipulator, different tasks are assumed. The task specifications are provided in Table 4.3.

Table 4.3 Manipulator Task Specifications

Case	**Coordinates of Start point** (x_i m, y_i m)	**Coordinates of Destination point** (x_f m, y_f m)	**Time to travel (sec)**
(i)	(0.65, 0)	(0.4, 0.3)	2
(ii)	(0.65, 0.05)	(0.4, 0.3)	2
(iii)	(0.65, 0.1)	(0.4, 0.3)	2
(iv)	(0.65, 0.05)	(–0.4, 0.3)	2
(v)	(0.65, 0.1)	(–0.4, 0.3)	2
(vi)	(0.4, 0.3)	(0.65, 0)	2

(ii) Weight parameter for first order autoregressive process $\phi_1 = 0.8$.

(iii) Time step for numerical integration, 0.001s.

4.6 ANALYSIS OF PERFORMANCE AND DISCUSSION

Using above numerical data, and discussed procedure, manipulator performance is simulated. Subsequently performance is analyzed using half normal plotting and ANOVA. Description regarding these methods and results are discussed next.

(i) Analysis of Performance using Half Normal Plot

For analysis of statistically significant parameters help of half normal plot proposed by Daniel [Daniel 1959] has been used. This analysis provides a simple way to examine the outcome of experiments i.e responses. As per Daniel, the responses that are negligible are normally distributed with zero mean and variance σ^2, and these tend to fall along a straight line on this plot, where as statistically significant effects will have nonzero means and will not lie along the straight line. Thus, the preliminary model will be specified to contain those effects that are apparently nonzero based on the normal probability plot. In this plot absolute value of the response against their cumulative normal probabilities are plotted. The straight line on the half normal plot always passes through the origin and it should also pass close to 50^{th} percentile data value. For plotting, help of freely downloadable software Design Expert Version 6 [Design Expert 6 1999] has been taken.

The half normal plot of all the tasks with positional error (ε_i) as performance is presented. In this plot X-axis represent the positional error (ε_i) or effect, and Y-axis represent cumulative normal probabilities. These plots shows the statistically significant control and noise parameters contribute most to performance variations. The half-normal plot for cases (i), (ii), (iii), (iv), (v) and (vi) are provided in Figs. 4.1- 4.6 respectively.

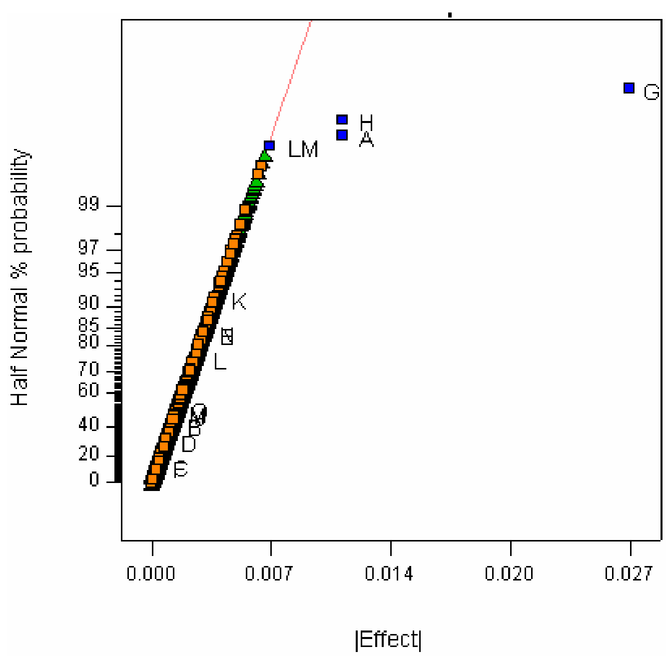


Fig. 4.1 Half Normal Plot of Performance for Cubic Trajectory - case (i)

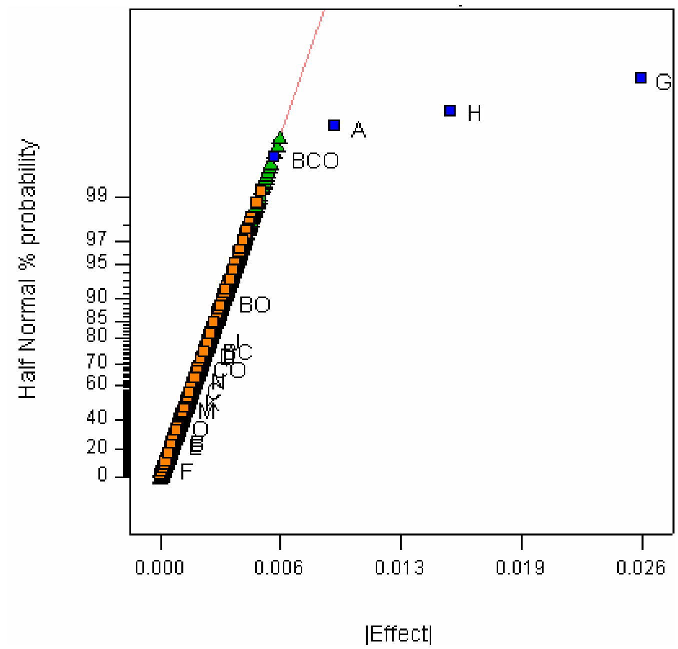


Fig. 4.2 Half Normal Plot of Performance for Cubic Trajectory - case (ii)

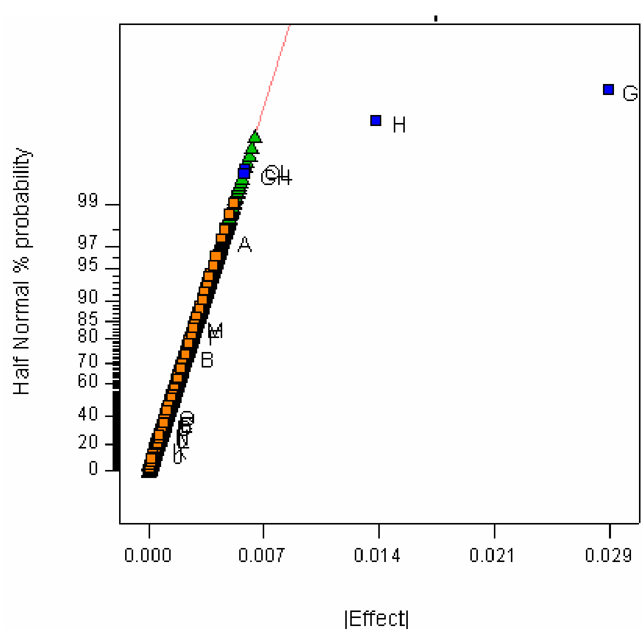


Fig. 4.3 Half Normal Plot of Performance for Cubic Trajectory - case (iii)

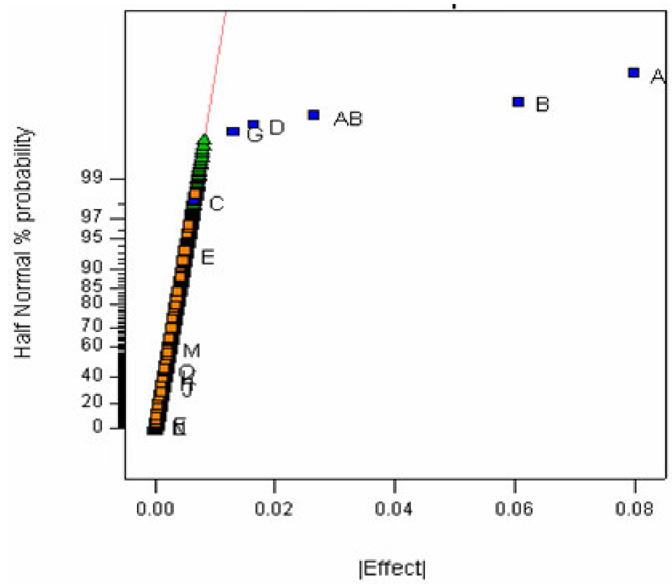


Fig. 4.4 Half Normal Plot of Performance for Cubic Trajectory - case (iv)

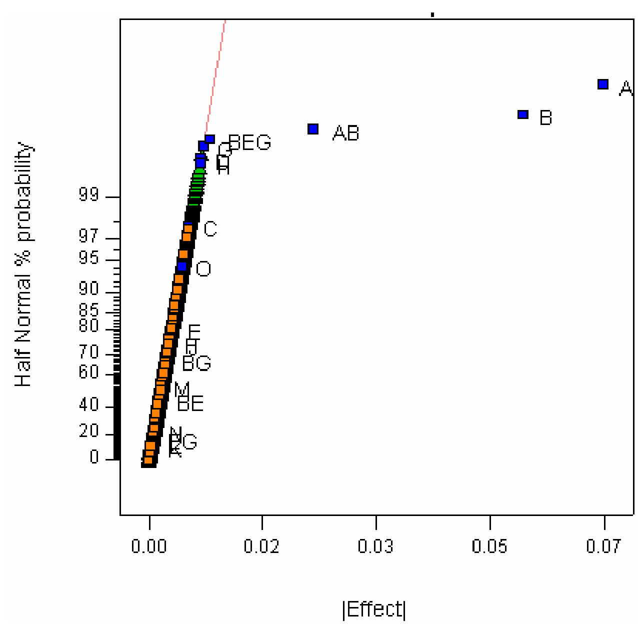


Fig. 4.5 Half Normal Plot of Performance for Cubic Trajectory - case (v)

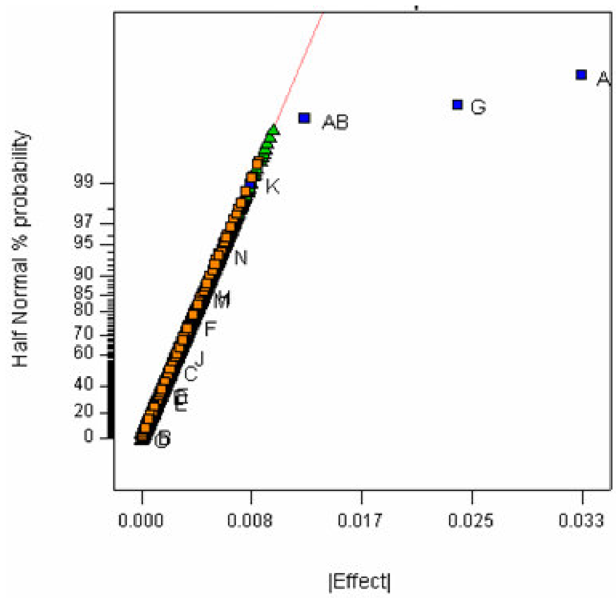


Fig. 4.6 Half Normal Plot of Performance for Cubic Trajectory - case (vi)

From Fig. 4.1 for case (i) it is observed that factors A, G and H do not lie on straight line. Hence these factors, become statistically significant. For the case (ii) from Fig 4.2 following cubic trajectory, factors A, G and H, are observed to be statistically significant. It is observed in Fig. 4.3 for case (iii) following cubic trajectory factors G, H, are statistically significant. From Fig. 4.4, for case (iv) following cubic trajectory factors A, B, D, G and factor interaction AB are statistically significant. It has been observed that in Fig. 4.5 for case (v) factors A, B, O and factor interactions AB, DEG are statistically significant. From Fig. 4.6 for case (vi) factor A, G and factor interaction AB are observed to be statistically significant. Similarly, the half normal plot for the manipulator performing task using quintic trajectory are provided in Figs. 4.7 - 4.12, which displays the factors responsible for the variations of performance i.e. positional error. In these graphs, the parameters which do not lie on straight line indicate they have statistically significant effect on performance variations.

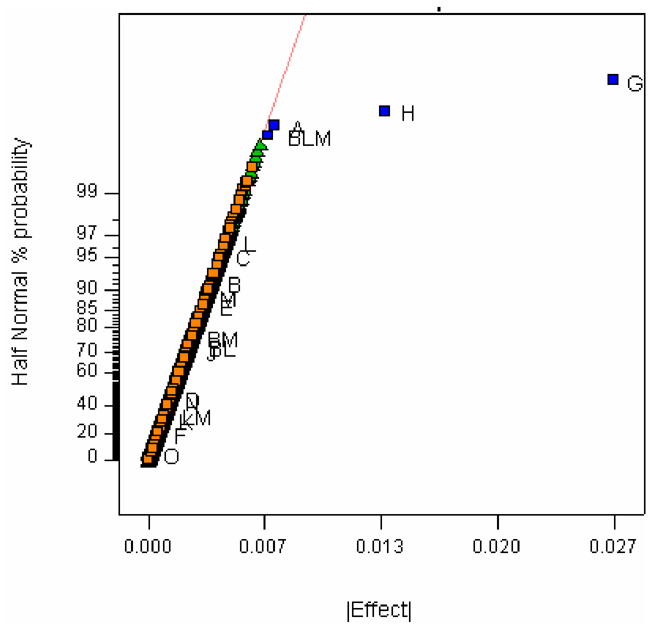


Fig. 4.7 Half Normal Plot of Performance for Quintic Trajectory - case (i)

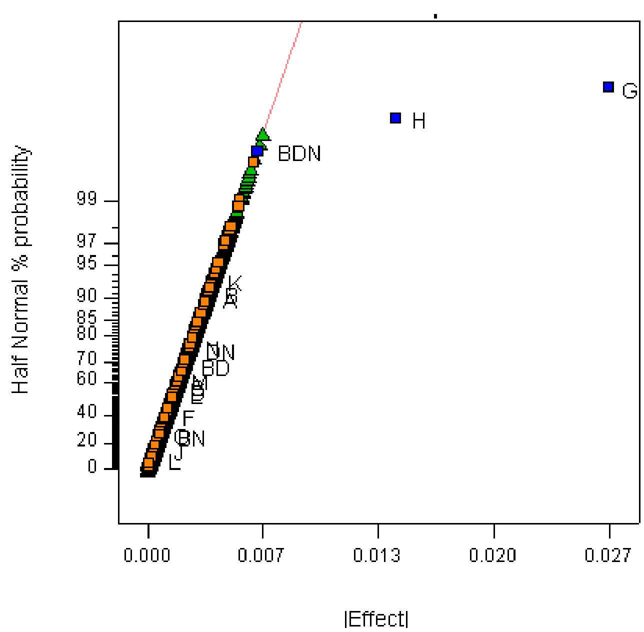


Fig. 4.8 Half Normal Plot of Performance for Quintic Trajectory - case (ii)

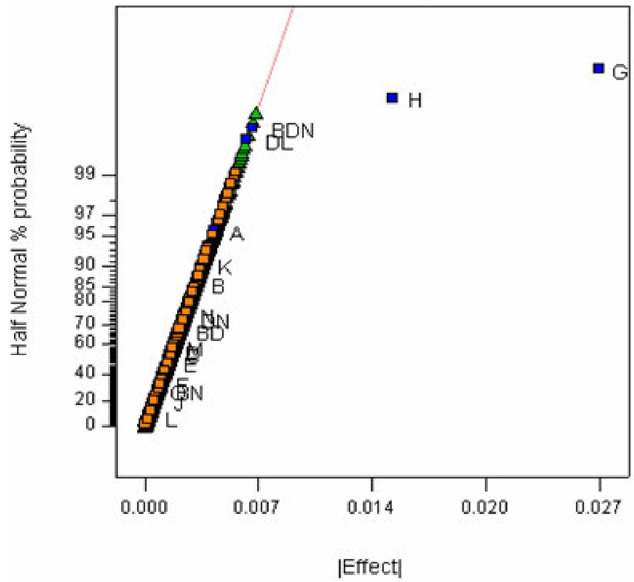


Fig. 4.9 Half Normal Plot of Performance for Quintic Trajectory - case (iii)

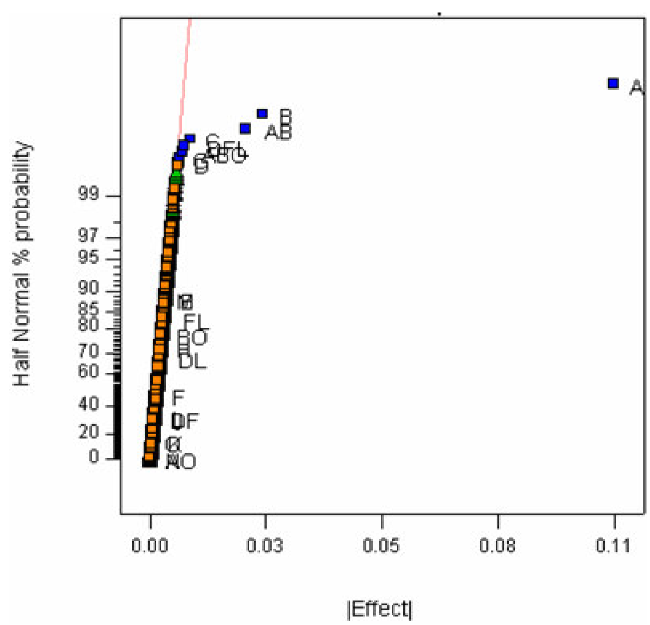


Fig. 4.10 Half Normal Plot of Performance for Quintic Trajectory - case (iv)

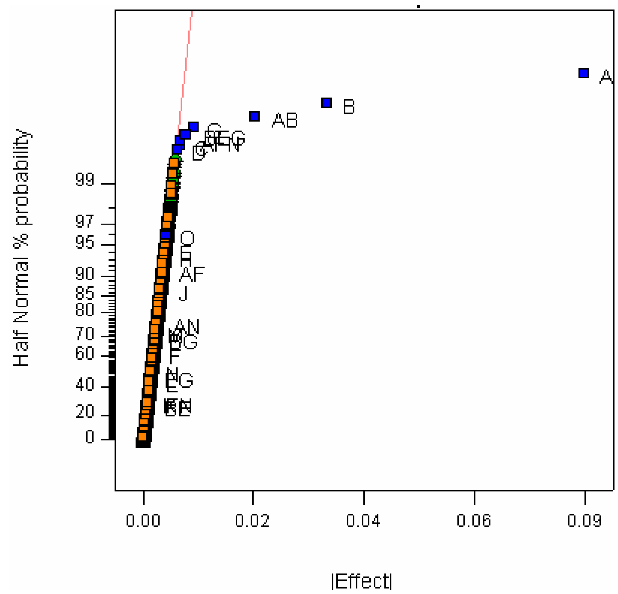


Fig. 4.11 Half Normal Plot of Performance for Quintic Trajectory - case (v)

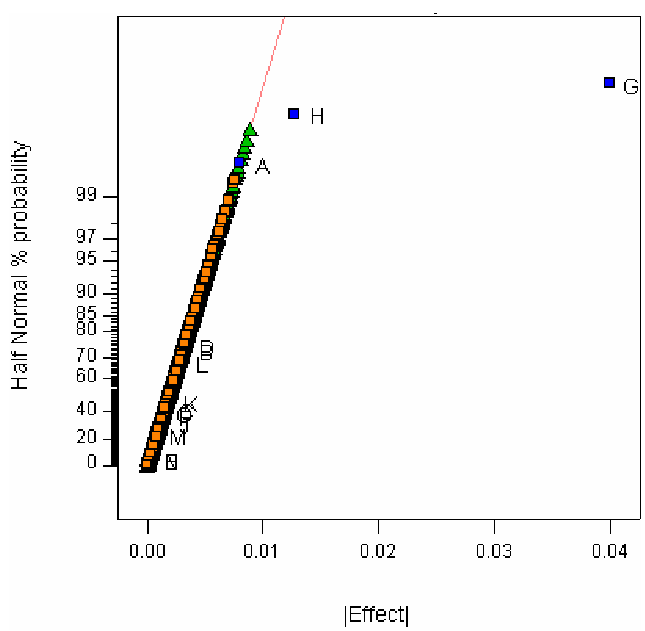


Fig. 4.12 Half Normal Plot of Performance for Quintic Trajectory - case (vi)

Half normal plots of the performance obtained from the experiments are presented for the tasks following quintic trajectory. It is observed that from Fig. 4.7 for case (i) factors A, G, H and factor interaction BLM are statistically significant. Similarly from Fig. 4.8 for case (ii) factors G and H are observed to be statistically significant. It is observed from Fig. 4.9 for task (iii) following quintic trajectory factors G and H are statistically significant. It has been observed that from Fig. 4.10 for case (iv) following quintic trajectory factors A, B, C, and factor interaction AB are statistically significant. From Fig. 4.11 for case (v), factors A, B, C, D and factor interaction AB, BEG are statistically significant. Similarly, From Fig. 4.12 for case (vi) following quintic trajectory factors G, H are statistically significant. It can be noted here that above figures indicate the factors that have strongest influence on performance variation, do not lie on straight line. However, to know exactly what is the contribution of each factor on performance variations, ANOVA technique is used for all the considered cases.

(ii) Analysis of Performance using ANOVA technique

The performance utilized for analysis of experiment using ANOVA is positional error (ε_i). To investigate the impact of a particular trajectory on performance, same set of task specifications are used for the manipulator following cubic and quintic trajectories. Results of ANOVA for each case is provided in individual table. For analysis using ANOVA, corresponding half normal plot is referred. With the help of this plot statistically significant factors and factor interactions which do not lie on line are selected along with remaining individual factors. The main motive behind this selection is to asses the impact of all the considered control factors and noise factors on performance variations.

To summarize the results of ANOVA clearly and in a compact form, statistically insignificant parameters are not indicated. In addition to above analysis, for mean positional error analysis is also carried out. The purpose of analyzing these performance measures are to obtain the factors contributing to mean performance variations. The results of ANOVA for significant parameters are presented in tabular form.

Statistical analysis of the data obtained from the experiment is analysed using ANOVA. This analysis is carried out for positional error as the performance while carrying out a particular task following cubic and quintic trajectories. The summary of results of ANOVA for all the discussed tasks are provided in tabular form. The ANOVA of positional error as performance are provided in Tables 4.4-4.9. In ANOVA tables, a factor can be identified whether it is statistically significant or not. A statistically significant factors is determined by comparing observed F_o statistic value obtained in the ANOVA table against tabulated F statistic (F) value. During analysis, the level of significance is kept at 0.05. The values of F tabulated is 4.0, i.e. $F_{0.05,\nu_1,\nu_2} = F_{0.05,1,2032} \approx F_{0.05,1,\infty} = 3.84$ [Montegomery 2001].

Table 4.4 ANOVA of Performance for Cubic Trajectory - case (i)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.059983	1	0.059983	26.61713	Significant
B	0.000673	1	0.000673	0.298494	–
C	6.02×10^{-5}	1	6.02×10^{-5}	0.026735	–
D	0.000339	1	0.000339	0.150332	–
E	0.004347	1	0.004347	1.928909	–
F	5×10^{-5}	1	5×10^{-5}	0.022176	–
G	0.378029	1	0.378029	167.7483	Significant
H	0.060371	1	0.060371	26.78938	Significant
J	0.000895	1	0.000895	0.397174	–
K	0.00668	1	0.00668	2.964344	–
L	0.003065	1	0.003065	1.360079	–
M	0.001004	1	0.001004	0.445679	–
N	0.004469	1	0.004469	1.983	–
O	0.001094	1	0.001094	0.485314	–
LM	0.023017	1	0.023017	10.21385	Significant
Residual	4.579207	2032	0.002254		
Corrected Total	5.123283	2047			

Table 4.5 ANOVA of Performance for Cubic Trajectory - case (ii)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.044885	1	0.044885	21.15018	Significant
B	0.000241	1	0.000241	0.11347	–
C	0.001439	1	0.001439	0.677846	–
D	0.002775	1	0.002775	1.307762	–
E	0.0002	1	0.0002	0.094436	–
F	1.16×10^{-5}	1	1.16×10^{-5}	0.005466	
G	0.339426	1	0.339426	159.9412	Significant
H	0.123057	1	0.123057	57.986	Significant
J	0.003437	1	0.003437	1.619729	–
K	0.001077	1	0.001077	0.507311	–
L	0.002811	1	0.002811	1.324687	–
M	0.000864	1	0.000864	0.407044	–
N	0.001741	1	0.001741	0.820444	–
O	0.000489	1	0.000489	0.230507	–
BC	0.002999	1	0.002999	1.413276	–
BO	0.005604	1	0.005604	2.640584	–
CO	0.002206	1	0.002206	1.039724	–
BCO	0.019091	1	0.019091	8.996027	Significant
Residual	4.30593	2029	0.002122		
Corrected Total	4.858285	2047			

Table 4.6 ANOVA of Performance for Cubic Trajectory - case (iii)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.01042	1	0.01042	4.994808	Significant
B	0.002554	1	0.002554	1.224226	–
C	0.000458	1	0.000458	0.21948	–
D	0.000421	1	0.000421	0.201968	–
E	0.000461	1	0.000461	0.220796	–
F	0.003622	1	0.003622	1.73609	–
G	0.41676	1	0.41676	199.772	Significant
H	0.102138	1	0.102138	48.95951	Significant
J	5.63×10^{-5}	1	5.63×10^{-5}	0.026998	–
K	9.6×10^{-5}	1	9.6×10^{-5}	0.046035	–
L	0.000227	1	0.000227	0.108894	–
M	0.004051	1	0.004051	1.941592	–
N	0.000254	1	0.000254	0.121579	–
O	0.000598	1	0.000598	0.286538	–
CL	0.018467	1	0.018467	8.852267	Significant
GH	0.018069	1	0.018069	8.661114	Significant
Residual	4.237027	2031	0.002086		
Corrected Total	4.815679	2047			

Table 4.7 ANOVA of Performance for Cubic Trajectory - case (iv)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	3.054097	1	3.054097	898.0368	Significant
B	1.751592	1	1.751592	515.0439	Significant
C	0.019943	1	0.019943	5.86409	Significant
D	0.131779	1	0.131779	38.74869	Significant
E	0.01149	1	0.01149	3.378425	–
F	1.42×10^{-5}	1	1.42×10^{-5}	0.004166	–
G	0.082211	1	0.082211	24.17369	Significant
H	0.000843	1	0.000843	0.248016	–
J	0.000645	1	0.000645	0.189777	–
K	0.001084	1	0.001084	0.318795	–
L	4.65×10^{-6}	1	4.65×10^{-6}	0.001367	–
M	0.002512	1	0.002512	0.738743	–
N	4.42×10^{-6}	1	4.42×10^{-6}	0.001299	–
O	0.001315	1	0.001315	0.386732	–
AB	0.337693	1	0.337693	99.29641	Significant
Residual	6.910547	2032	0.003401		
Corrected Total	12.30578	2047			

Table 4.8 ANOVA of Performance for Cubic Trajectory - case (v)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	2.439403	1	2.439403	691.7229	Significant
B	1.657409	1	1.657409	469.9789	Significant
C	0.019233	1	0.019233	5.453792	Significant
D	0.032664	1	0.032664	9.262155	Significant
E	0.006038	1	0.006038	1.712062	–
F	0.004813	1	0.004813	1.364775	–
G	0.035936	1	0.035936	10.18997	Significant
H	0.032316	1	0.032316	9.163493	Significant
J	0.00472	1	0.00472	1.338377	–
K	6.18×10^{-5}	1	6.18×10^{-5}	0.017528	–
L	9.12×10^{-5}	1	9.12×10^{-5}	0.025869	–
M	0.001869	1	0.001869	0.529929	–
N	0.000292	1	0.000292	0.082855	–
O	0.013107	1	0.013107	3.716786	–
AB	0.321191	1	0.321191	91.07767	Significant
BE	0.001223	1	0.001223	0.346814	–
BG	0.003509	1	0.003509	0.994904	–
EG	0.000149	1	0.000149	0.042124	–
BEG	0.045155	1	0.045155	12.80422	Significant
Residual	7.151864	2028	0.003527		
Corrected Total	11.77104	2047			

Table 4.9 ANOVA of Performance for Cubic Trajectory - case (vi)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.566893	1	0.566893	110.6564	Significant
B	3.61×10^{-6}	1	3.61×10^{-6}	0.000705	–
C	0.002365	1	0.002365	0.461634	–
D	0.000979	1	0.000979	0.191021	–
E	0.001032	1	0.001032	0.201508	–
F	0.006613	1	0.006613	1.290787	–
G	0.293333	1	0.293333	57.25809	Significant
H	0.010831	1	0.010831	2.114288	–
J	0.003522	1	0.003522	0.687552	–
K	0.034392	1	0.034392	6.713259	Significant
L	0.000725	1	0.000725	0.141613	–
M	0.010324	1	0.010324	2.015169	–
N	0.017779	1	0.017779	3.47052	–
O	1.07×10^{-7}	1	1.07×10^{-7}	2.09×10^{-5}	–
AB	0.077978	1	0.077978	15.2211	Significant
BD	0.00335	1	0.00335	0.653827	–
BN	0.001567	1	0.001567	0.305807	–
CH	0.036228	1	0.036228	7.071686	Significant
DN	0.005483	1	0.005483	1.070345	–
BDN	0.039517	1	0.039517	7.713659	Significant
Residual	10.38432	2027	0.005123		
Corrected Total	11.49723	2047			

From the Table 4.4 for case (i) it is observed that factors A, G, H and factor interaction LM are statistically significant. And rest of the factors are insignificant because their computed F_o statistic values are less as compared to tabulated F statistic value. It is important to mention here that in ANOVA table, tabulated F statistic value used is equal to 3.84, though degrees of freedom for the tabulated F statistic are different for different cases i.e., $F_{0.05,\nu_1,\nu_2} = F_{0.05,1,2029} \approx F_{0.05,1,\infty} = 3.84$.

Therefore, throughout the analysis same F value is used for comparison. From Table 4.5 for case (ii) following cubic trajectories performance analysis is carried out. It is observed that factors A, G, H and factor interaction BCO are statistically significant. And rest of the factors are insignificant because their F_o statistic values are less as compared to tabulated F statistic value. Similarly for the case (iii), from Table 4.6 factors A, G, H and factor interactions CL and GH are observed to be statistically significant. And rest of the factors are insignificant. From Table 4.7, for the case (iv) it is observed that factors A, B, C, D, G, and factor interaction AB are statistically significant.

It is observed from Table 4.8 that for the case (v) following cubic trajectory, factors A, B, C, D, G, H and factor interactions AB, BEG are statistically significant. From Table 4.9, it is observed that for case (vi) following cubic trajectory, factors A, G, K and factor interactions AB, CH and BDN are statistically significant. Similarly the analysis using ANOVA technique for the manipulator performing task following quintic trajectory are provided in Tables 4.10-4.15.

Similar procedure is adopted as discussed in the above case. Comparison of computed F_o statistic value is compared with tabulated F statistic values to identify statistically significant factors.

Table 4.10 ANOVA of Performance for Quintic Trajectory - case (i)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.026478	1	0.026478	11.75528	Significant
B	0.006593	1	0.006593	2.927079	
C	0.008715	1	0.008715	3.869159	Significant
D	0.000815	1	0.000815	0.361666	–
E	0.004967	1	0.004967	2.205382	–
F	0.000141	1	0.000141	0.062618	–
G	0.363438	1	0.363438	161.3541	Significant
H	0.093932	1	0.093932	41.70272	Significant
J	0.002496	1	0.002496	1.108161	–
K	0.000327	1	0.000327	0.144966	–
L	0.009892	1	0.009892	4.391542	Significant
M	0.005605	1	0.005605	2.488253	–
N	0.00072	1	0.00072	0.319765	–
O	6.28×10^{-6}	1	6.28×10^{-6}	0.002788	–
BL	0.002674	1	0.002674	1.187145	–
BM	0.00315	1	0.00315	1.398475	–
LM	0.000408	1	0.000408	0.181241	–
BLM	0.023922	1	0.023922	10.62048	Significant
Residual	4.570173	2029	0.002252		
Corrected Total	5.124452	2047			

Table 4.11 ANOVA of Performance for Quintic Trajectory - case (ii)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.006012	1	0.006012	2.69184	–
B	0.006365	1	0.006365	2.849962	–
C	0.001413	1	0.001413	0.632667	–
D	0.001289	1	0.001289	0.577334	–
E	0.0012	1	0.0012	0.537127	–
F	0.00061	1	0.00061	0.273152	–
G	0.366809	1	0.366809	164.2393	Significant
H	0.106979	1	0.106979	47.90006	Significant
J	7.76×10^{-5}	1	7.76×10^{-5}	0.034768	–
K	0.007197	1	0.007197	3.222459	–
L	2.25×10^{-5}	1	2.25×10^{-5}	0.010067	–
M	0.001654	1	0.001654	0.740397	–
N	0.003103	1	0.003103	1.389551	–
O	0.000257	1	0.000257	0.115285	–
BD	0.002228	1	0.002228	0.997729	–
BN	0.000233	1	0.000233	0.104138	–
DN	0.002958	1	0.002958	1.324461	–
BDN	0.020888	1	0.020888	9.35274	Significant
Residual	4.531532	2029	0.002233		
Corrected Total	5.060828	2047			

Table 4.12 ANOVA of Performance for Quintic Trajectory - case (iii)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.008744	1	0.008744	3.991303	Significant
B	0.004659	1	0.004659	2.126677	–
C	0.001223	1	0.001223	0.558434	–
D	0.001281	1	0.001281	0.5848	–
E	0.000918	1	0.000918	0.418853	–
F	0.000436	1	0.000436	0.199041	–
G	0.37498	1	0.37498	171.1659	Significant
H	0.111268	1	0.111268	50.79024	Significant
J	0.000146	1	0.000146	0.066469	–
K	0.005946	1	0.005946	2.71416	–
L	1.74×10^{-5}	1	1.74×10^{-5}	0.007954	–
M	0.001476	1	0.001476	0.673671	–
N	0.002892	1	0.002892	1.319966	–
O	0.000289	1	0.000289	0.132117	–
BD	0.002082	1	0.002082	0.950154	–
BN	0.000304	1	0.000304	0.138716	–
DL	0.018891	1	0.018891	8.623002	Significant
DN	0.002665	1	0.002665	1.21655	–
BDN	0.020822	1	0.020822	9.504352	Significant
Residual	4.442823	2028	0.002191		
Corrected Total	5.001862	2047			

Table 4.13 ANOVA of Performance for Quintic Trajectory - case (iv)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	5.962904	1	5.962904	2389.953	Significant
B	0.355509	1	0.355509	142.4892	Significant
C	0.046508	1	0.046508	18.64048	Significant
D	0.023415	1	0.023415	9.384907	Significant
E	0.006025	1	0.006025	2.414771	–
F	0.000995	1	0.000995	0.398849	–
G	0.024138	1	0.024138	9.674527	Significant
H	0.002988	1	0.002988	1.197597	–
J	0.000382	1	0.000382	0.152978	–
K	8.05×10^{-5}	1	8.05×10^{-5}	0.032247	–
L	0.000436	1	0.000436	0.17473	–
M	0.006082	1	0.006082	2.437635	–
N	1.76×10^{-7}	1	1.76×10^{-7}	7.06×10^{-5}	–
O	7.25×10^{-5}	1	7.25×10^{-5}	0.029039	–
AB	0.257749	1	0.257749	103.3067	Significant
AO	5.31×10^{-7}	1	5.31×10^{-7}	0.000213	–
BO	0.003684	1	0.003684	1.476615	–
DF	0.000396	1	0.000396	0.158807	–
DL	0.002458	1	0.002458	0.98507	–
FL	0.004622	1	0.004622	1.852673	–
ABO	0.03097	1	0.03097	12.41293	Significant
DFL	0.034078	1	0.034078	13.65855	Significant
Residual	5.052351	2025	0.002495		
Corrected Total	11.81584	2047			

Table 4.14 ANOVA of Performance for Quintic Trajectory - case (v)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	3.776665	1	3.776665	1523.78	Significant
B	0.653784	1	0.653784	263.7837	Significant
C	0.025992	1	0.025992	10.48691	Significant
D	0.022442	1	0.022442	9.054772	Significant
E	0.008974	1	0.008974	3.620937	–
F	0.001858	1	0.001858	0.74979	–
G	0.049656	1	0.049656	20.03465	Significant
H	0.008376	1	0.008376	3.379579	–
J	0.005558	1	0.005558	2.242665	–
K	0.000326	1	0.000326	0.131677	–
L	0.000847	1	0.000847	0.341624	–
M	0.002844	1	0.002844	1.147481	–
N	0.001143	1	0.001143	0.461019	–
O	0.01034	1	0.01034	4.171979	Significant
AB	0.240784	1	0.240784	97.14965	Significant
AF	0.007013	1	0.007013	2.829646	–
AN	0.003345	1	0.003345	1.349569	–
BE	0.000284	1	0.000284	0.114682	–
BG	0.002557	1	0.002557	1.031582	–
EG	0.000953	1	0.000953	0.384454	–
FN	0.000351	1	0.000351	0.141814	–
AFN	0.0267	1	0.0267	10.77256	Significant
BEG	0.03509	1	0.03509	14.15788	Significant
Residual	5.016451	2024	0.002478		
Corrected Total	9.902333	2047			

Table 4.15 ANOVA of Performance for Quintic Trajectory - case (vi)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.029581	1	0.029581	8.32181	Significant
B	0.004334	1	0.004334	1.219195	–
C	0.001006	1	0.001006	0.283038	–
D	0.004687	1	0.004687	1.318664	–
E	1.22×10^{-5}	1	1.22×10^{-5}	0.003424	–
F	0.000798	1	0.000798	0.224418	–
G	0.736214	1	0.736214	207.1114	Significant
H	0.07496	1	0.07496	21.08766	Significant
J	0.00057	1	0.00057	0.16044	–
K	0.001361	1	0.001361	0.382951	–
L	0.003442	1	0.003442	0.968255	–
M	0.000328	1	0.000328	0.092368	–
N	1.36×10^{-5}	1	1.36×10^{-5}	0.003824	–
O	0.000903	1	0.000903	0.254084	–
Residual	7.226655	2033	0.003555		
Corrected Total	8.084866	2047			

It has been observed that from Table 4.10 for case (i) following quintic trajectory, factors A, C, G, H, L and factor interaction BLM are statistically significant. From Table 4.11 for case (ii) it is observed that, factors G, H, and factor interaction BDN are statistically significant. It is observed that from Table 4.12 for case (iii) following quintic trajectory, factors A, G, H, and factor interactions DL and BDM are statistically significant. Similar to all the previous cases discussed, for the case (iv) it is observed from Table 4.13 that, factors A, B, C, D, G and factor interactions AB and ABO, DFL are statistically significant. From Table 4.14 it is observed that for case (v) following quintic trajectory factors A, B, C, D, G O and factor interactions AB and AFN, BEG are statistically significant. It has been observed from Table 4.15 that for case (vi) following quintic trajectory factors A, G and H, are statistically significant. From all these tables following inferences can be drawn. Number of factors influencing the performance variation are more when the manipulator perform task (iv) and (v) as compared to task (i), (ii), (iii) and (vi). To summarize the results obtained from above analysis, factors responsible for performance variations are provided in tabular form. Table 4.16 presents the parameters responsible for performance variations for all six cases following cubic and quintic trajectories.

Table 4.16 Summary of Positional Error Analysis Using ANOVA

Case	Significant Factors using ANOVA for task following Cubic trajectory	Significant Factors using ANOVA for task following Quintic trajectory
(i)	A, G, H, LM	A, C, G, H, L, BLM
(ii)	A, G, H, BCO	G, H, BDN
(iii)	A, G, H, CL, GH	A, G, H, DL, BDN
(iv)	A, B, C, D, G, AB	A, B, C, D, G, AB, ABO, DFL
(v)	A, B, C, D, G, H, AB, BEG	A, B, C, D, G, O, AB, AFN, BEG
(vi)	A, G, K, AB, AB, CH, BDN	A, G, H

To add strength to the analysis performed, ANOVA of mean positional error of all the considered cases are carried out. The figures of half normal plot and ANOVA table, for mean positional error of each task are not provided to prevent the duplication of results. In most of the cases, parameters found to be statistically significant are same as compared to the analysis done using positional error. To represent the outcome of ANOVA analysis, results are put in a tabular form to observe the common parameters responsible for performance variations. Therefore, the control and noise parameters and interacting parameters responsible for performance variations are provided in Table 4.17.

Table 4.17 Summary of Mean Positional Error Analysis Using ANOVA

Case	Significant Factors using ANOVA for task following Cubic trajectory	Significant Factors using ANOVA for task following Quintic trajectory
(i)	A, G, H	A, G, H, L, BLM
(ii)	A, G, H, LM, BCO	G, H, DL, BDN, DGJ
(iii)	A, G, H, BH, CL, GH, AJN	G, H, DL, FO, BDN, BJM, FGO
(iv)	A, B, C, D, E, G, AB, FGH, FGO	A, B, C, D, G, AB
(v)	A, B, C, D, G, H, O, AB, DM, BEG,	A, B, C, D, G, AB, BEG
(vi)	A, G, K, AB, DL, BDN	A, G, H

The objective of this chapter is to screen the parameters responsible for performance variations, from ANOVA results a consolidated table is prepared. It is always desired that the manipulator designed should perform all task specified with lesser performance variations while following different trajectories. To obtain robust design of manipulator, parameters should be selected which has prominient influence on performance variations. Therefore, to indicate the parameters responsible for performance variations in all the considered cases, stars (*) are put against the individual parameters. The results are summarized in Table 4.18

Table 4.18 Summary of Analysis Using ANOVA

Case	Traje -ctory	A (l_1)	B (l_2)	C (m_1)	D (m_2)	E (σ_{τ_1})	F (σ_{τ_2})	G (σ_{θ_1})	H (σ_{θ_2})	J (B_1)	K (B_2)	L (σ_{l_1})	M (σ_{l_2})	N (σ_{m_1})	O (σ_{m_2})
(i)	**Cubic**	*						*	*						
	Quintic	*		*				*	*			*			
(ii)	**Cubic**	*						*	*						
	Quintic							*	*						
(iii)	**Cubic**	*						*	*						
	Quintic	*						*	*						
(iv)	**Cubic**	*	*	*	*			*							
	Quintic	*	*	*	*			*							
(v)	**Cubic**	*	*	*	*			*	*						
	Quintic	*	*	*	*			*							*
(vi)	**Cubic**	*						*			*				
	Quintic	*						*	*						

From analysis, it is observed that statistically significant factors are different for different tasks in workspace indicating that each parameters have different contribution to performance variations as start and target position of task change. It has also been observed that the for the same task, parameters responsible for performance variations are different while following cubic and quintic trajectories. As a manipulator is supposed to perform different task in the workspace using different trajectories, for screening the parameters responsible for performance variations, union of all the cases i.e. tasks and trajectories are taken. In all cases, if the parameters are significant only once then those parameters are treated as unimportant. Before eliminating these parameters, half normal plots are referred once again to ascertain, the contribution to performance variations.

To summarize whole analysis the parameter are categorized into three types. These classifications are given below.

(i) The parameters not statistically significant even once in all the considered cases are E, F, J, M and N. Therefore, further investigation on these parameters should not be pursued.

(ii) The parameters statistically significant only once are K, L and O. It is decided that the parameters which are significant only once should be treated as insignificant parameters. Further invetsigations should not be attempted.

(iii)The parameters A, B, C, D, G, and H are statistically significant in more than one cases. Therefore parameters A, B, C, D, G and H are selected for further analysis.

The results of the analysis are provided in tabular form subsequently used to obtain the optimal parameter which will deliver the optimal performance. For computations of optimal parameters respose surface methodology is adopted. The approach used for the optimal design is discussed in Chapter 5.

4.7 PARAMETRIC SENSITIVITY OF MANIPULATOR

To provide insight to proposed investigation, the influence of individual parameters on performance variations of manipulator is studied. Exploration of individual parameter sensitivity will lead to large number of computation and figures. To keep number of computations and figures low, two dimensionless numbers are proposed. One of them is link length ratio $\beta = l_2/l_1$ and other one is link mass ratio $\alpha = m_2/m_1$.

While investigating parameter sensitivity, except considered factor all other factors have been kept at a particular value. To conduct the sensitivity study, sum of two link lengths are assumed to be constant. This imply comparison can be carried out for a family of robots which has same workspace. But by not assuming link length sum constant, the problem become difficult to draw valid conclusion. Moreover by increasing one link length and keeping other link length fixed may lead to situation where it may not be able perform the desired task. This sensivity study is nothing but conducting experiments by changing these ratio values one at a time or changing both values simultaneously in steps.

To investigate the performance of manipulator, sum of link lengths and masses are kept constant i.e. $l_1 + l_2 = 0.90$ m, and $m_1 + m_2 = 14$ kg. The parameters β and α are assumed to vary from 0.35 to 1.0 in step of 0.05. The noise factor tolerances are expressed interms of standard deviation of gaussian distribution.

Assumed noise factor values for link length one and two are i.e. $\sigma_{l_1}, \sigma_{l_2} = 0.0006$ m, link masses variation $\sigma_{m_1}, \sigma_{m_2} = 0.03$ kg, joint torques variation $\sigma_{\tau_1}, \sigma_{\tau_2} = 0.3$ Nm and joint clearance variation $\sigma_{\theta_1}, \sigma_{\theta_2} = 0.1$ degree. To conduct experiments, methodology discussed in section 4.4 has been used for simulating the performance. For comparison of performances, same task have been considered following cubic and quintic trajectories. To have a thorough understanding of the impact of parameter change on the performance, three independent studies are carried out.

i) By changing β value in steps and keeping all other parameters constant,

ii) By changing α value in steps and keeping all other parameters constant,

iii) By changing both β and α values simultaneously and keeping rest of the parameters constant.

For comparison, mean positional error has been used as performance measure. To get overall behaviour of manipulator, simulations are run for thousand times for the above discussed tasks in workspace.

4.7.1 By changing Link Length ratio (β)

For sensitivity of link length ratio investigation, dimensionless parameter has been incremented from 0.35 to 1.0 in a step of 0.05 and the performance observed are mean positional error. To explore the effect of time law of trajectory on β, simulations are carried out for manipulator following cubic and quintic trajectory. The results of simulations are shown in Figs. 4.13 and 4.14 respectively.

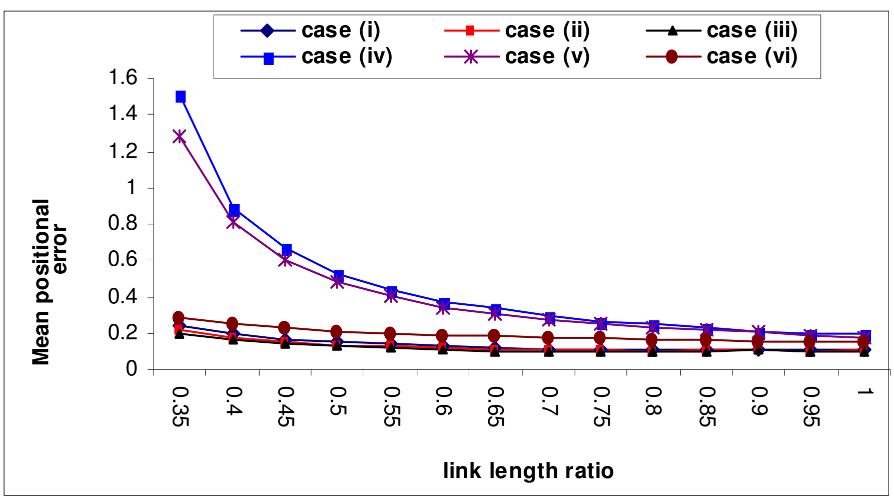


Fig 4.13 Link Length Ratio Sensitivity for Cubic Trajectory

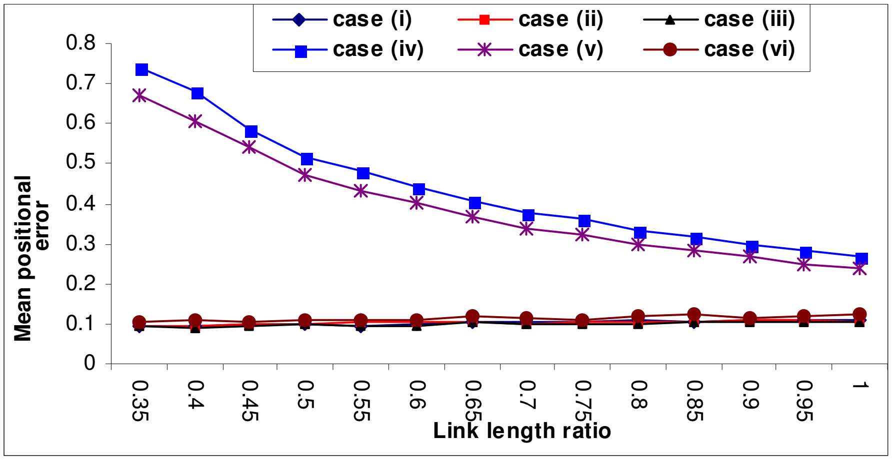


Fig 4.14 Link Length Ratio Sensitivity for Quintic Trajectory

From Fig. 4.13, change in performance measure i.e. mean positional error ($\times 10^{-2}$ m) is represented for different cases. For cases (iv) and (v) performance is observed to be poor for the β value 0.35. It is observed that in most of the cases with increase in β value, performance variations reduce, though it is not that prominient in cases (i), (ii), (iii) and (vi) respectively. From Fig. 4.13 it is observed that poorest performance is 1.6×10^{-2} m at β value 0.35. From Fig.4.14 it is observed that poorest performance is 0.75×10^{-2} m at β value 0.35. Similar to previous case it is observed that for case (iv) and (v) performance is poor for the β value 0.35 and it improves as β value increase. It is observed that performance improvement takes place with increase in β value. But it is not that prominient in cases (i), (ii), (iii) and (vi) respectively. Comparing both the graphs, it can be clearly infered that the trend of improvement in performance is similar. Poor performance is observed in case of tasks following cubic trajectory.

4.7.2 By changing Link Mass ratio (α)

For sensitivity of link mass ratio, dimensionless parameter has been incremented from 0.35 to 1.0 in a step of 0.05 and the performance observed is mean positional error. To study to effect of time law of trajectory on α, simulations are carried out for manipulator following cubic and quintic trajectory. The results of simulations have been presented in Figs. 4.15 and 4.16 respectively.

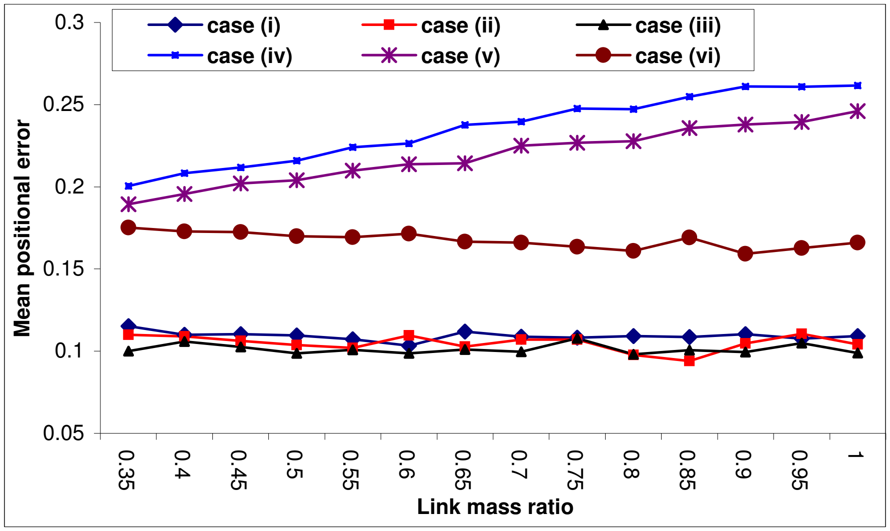


Fig 4.15 Link Mass Ratio Sensitivity for Cubic Trajectory

From Fig. 4.15 change in performance measure is represented for change in α values. It is observed that for cases (i) (ii) (iii) and (v) there are almost no change in performance measure with increase in α value. For case (iv) and (v) poor performance is observed with increase in α value. Performance variations remain at lowest value for α equal to 0.35. It is observed that poorest performance is 0.25×10^{-2} m at α value equal to 1.0.

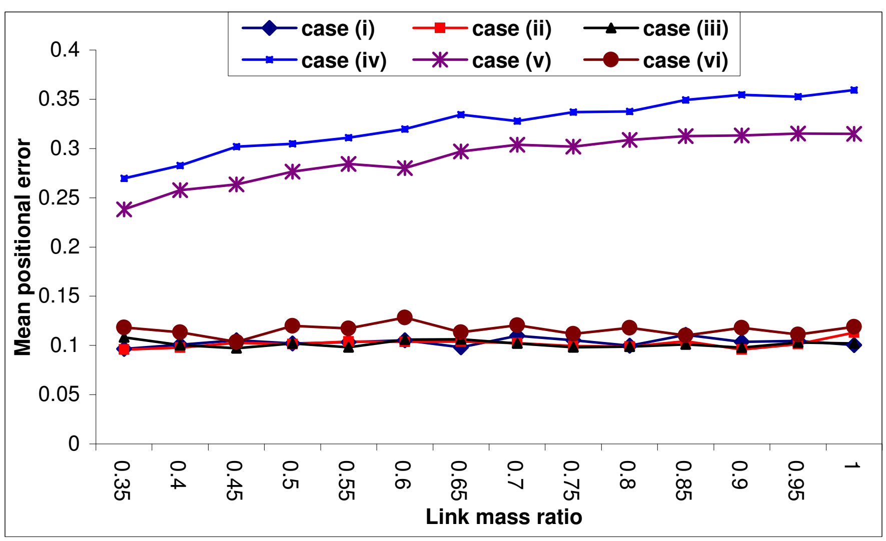


Fig 4.16 Link Mass Ratio Sensitivity for Quintic Trajectory

Similar to above, Fig. 4.16 represent the change in performance measure when manipulator performs tasks following quintic trajectory. It is observed that for cases (i) (ii) (iii) and (v) there are very less change in performance measure with increase in α value. For cases (iv) and (v) performance are observed to be poor with increase in α value. Performance variations remain at lowest value for α value 0.35. It is also observed that poorest performance is 0.35×10^{-2} m for α value 1.0.

The important inference drawn from this investigation is that, change in α value has less influence on the performance variations. Range of change in performance measure is between 0.1×10^{-2} m to 0.25×10^{-2} m for tasks following cubic trajectory and between 0.1×10^{-2} m to 0.35×10^{-2} m for tasks following quintic trajectory.

4.7.3 By changing β and α simultaneously

The impact of simultaneous change of link length and link masse ratios, have been investigated. In this dimensionless parameter β and α have been incremented from 0.35 to 1.0 in a step of 0.05 and the performance observed are mean positional error. To examine the effect of time law of trajectory on simultaneous change, simulations are carried out for manipulator following cubic and quintic trajectories. The results of simulations are presented with the help of contour plot. In this plot the performance measure is plotted in z-axis to understand the effect of parameters change (link mass ratio β and link length ratio α). The results has been presented in Figs. 4.17-4.22 and 4.23-4.28 respectively.

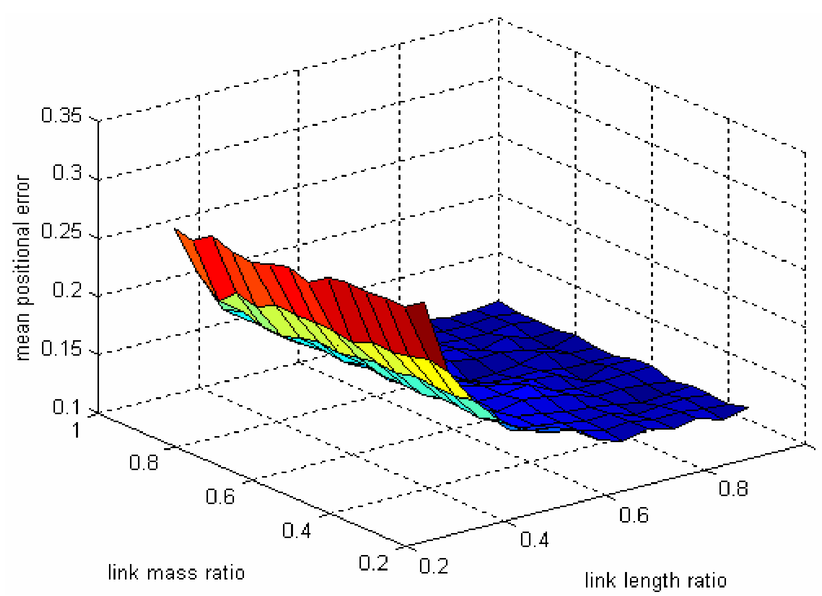


Fig 4.17 Contour Plotting of Performance Measure for Cubic Trajectory - case (i)

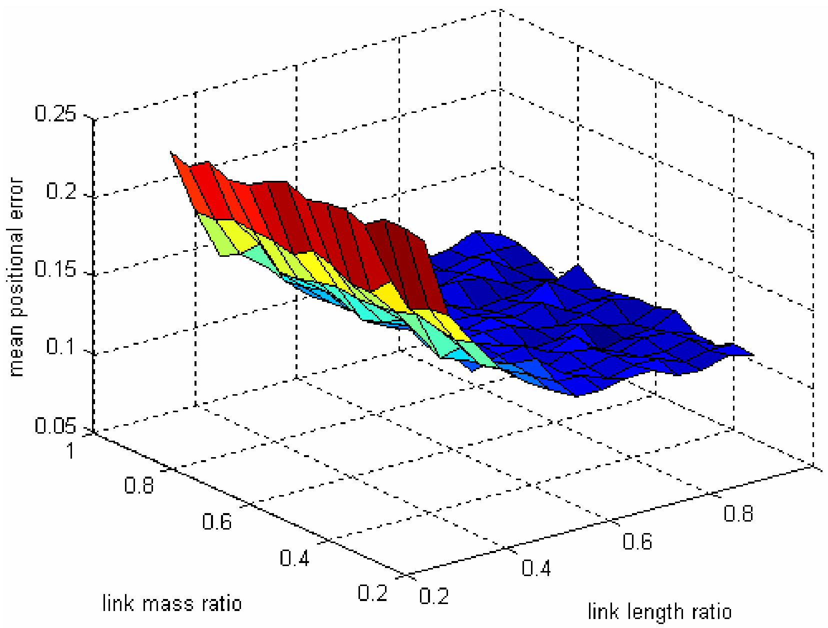


Fig 4.18 Contour Plotting of Performance Measure for Cubic Trajectory - case (ii)

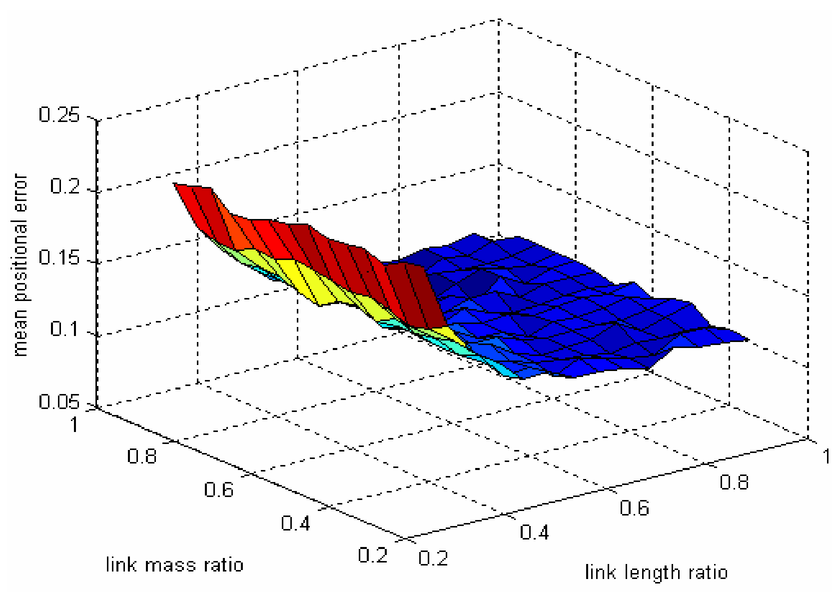


Fig 4.19 Contour Plotting of Performance Measure for Cubic Trajectory - case (iii)

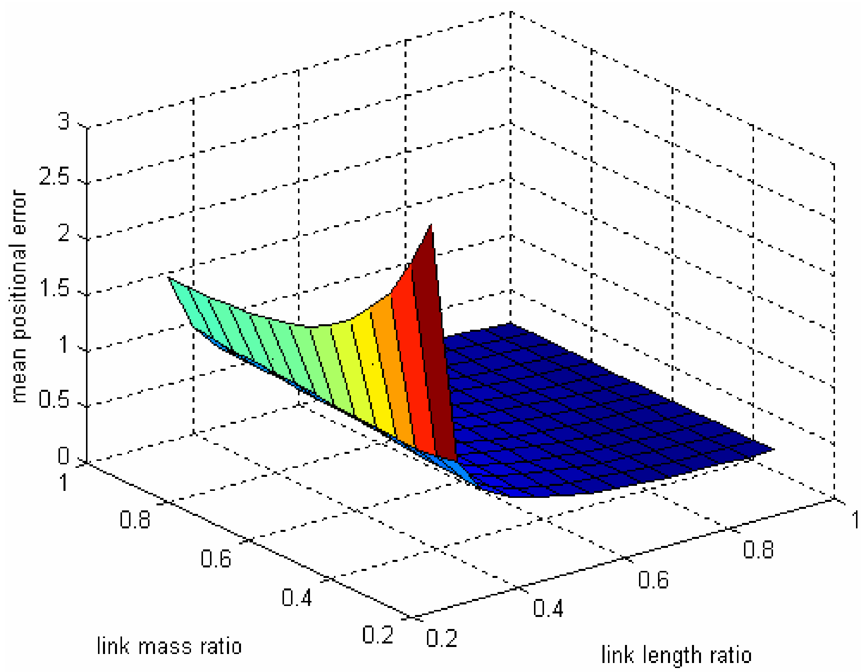


Fig 4.20 Contour Plotting of Performance Measure for Cubic Trajectory - case (iv)

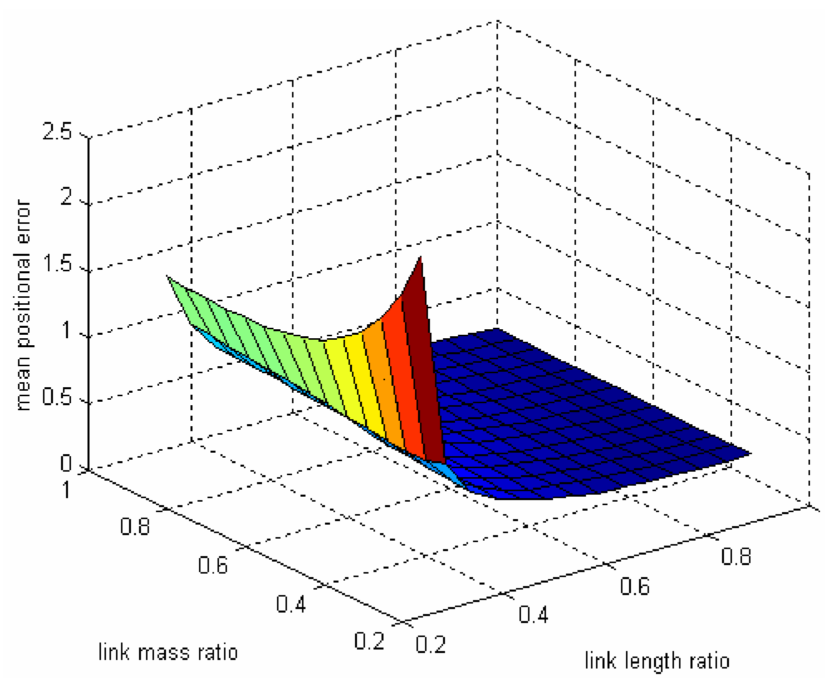


Fig 4.21 Contour Plotting of Performance Measure for Cubic Trajectory - case (v)

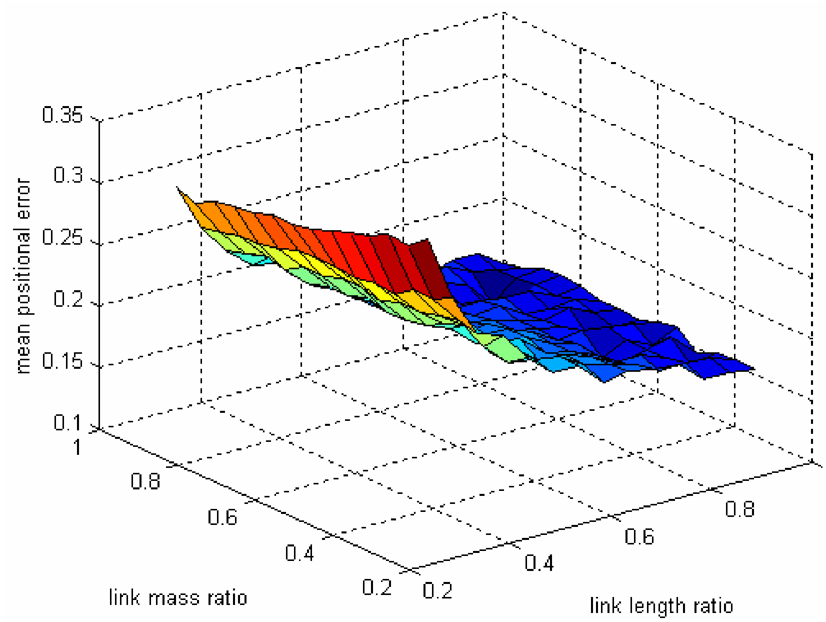


Fig 4.22 Contour Plotting of Performance Measure for Cubic Trajectory - case (vi)

From Fig. 4.17 for case (i) following cubic trajectory, it is observed that performance measure improves as the value of link length ratio β increase. Important thing observed in this graphs is the effect of change in link mass ratio. Performance measure does not change with change in link mass ratio α which is the case in earlier investigation. From figure it is observed that poorest performance is 0.25×10^{-2} m at β value 0.35 and improved performance is observed in case of link length ratios of 0.8 to 1.0. In Fig. 4.18, for case (ii) following cubic trajectory, it is observed that performance measure improves as the value of link length ratio β increase. Similar to previous case, performance measure do not change with change in link mass ratio α. Which is the case in earlier study. From graph it is observed that poorest performance is 0.22×10^{-2} m for β value 0.35 and improved performance is observed in case of link length ratios 0.8 to 1.0.

It is observed from Fig. 4.19, that for case (iii) mean positional error reduces as the value of link length ratio β increase. Similar to previous case performance measure do not change with change in link mass ratio α. The poorest performance is 0.2×10^{-2} m at β value 0.35 and improved performance is observed in case of link length ratios 0.8 to 1.0. From Fig. 4.20 for case (iv) it is observed that performance measure improve as the value of link length ratio β increase. Similar to case (iii) performance measure do not change much link mass ratio α.

But there is a remarkable change in performance measure with change in α value at $\beta = 0.35$. The poorest performance is 2.5×10^{-2} m at β value 0.35 and α value 0.35 and improved performance is observed in case of link length ratios β = 0.8 to 1.0 and α value between 0.35 to 1.0.

It has been observed that from Fig. 4.21 for case (v), mean positional error reduces as the value of link length ratio β increase. Similar to previous case (iv) performance measure do not change much link mass ratio α. But there is significant change in performance with change in α value at β =0.35. The poorest performance is 2×10^{-2} m at β value 0.35 and α value 0.35 and improved performance is observed in case of link length ratio 0.8 to 1.0 and α value between 0.35 to 1.0.

From Fig. 4.22 for the task (vi), similar feature is observed as discussed in case (i) (ii) and (iii). Performance measure reduces as the value of link length ratio β increase. The poorest performance is 0.28×10^{-2} m at β value 0.35 and α value 0.35, and lowest performance measure is observed in case of link length ratio β = 0.8 to 1.0 and α value 0.5.

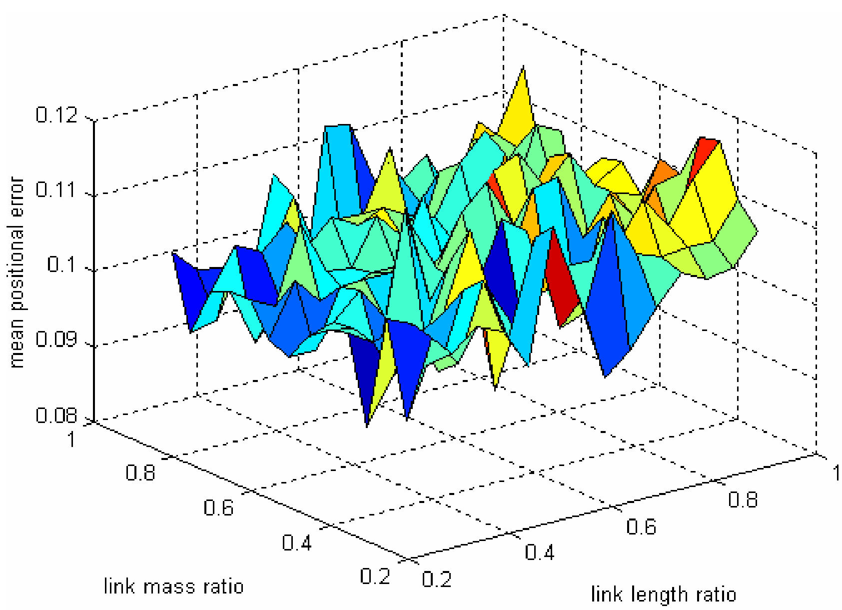


Fig 4.23 Contour Plotting of Performance Measure for Quintic Trajectory - case (i)

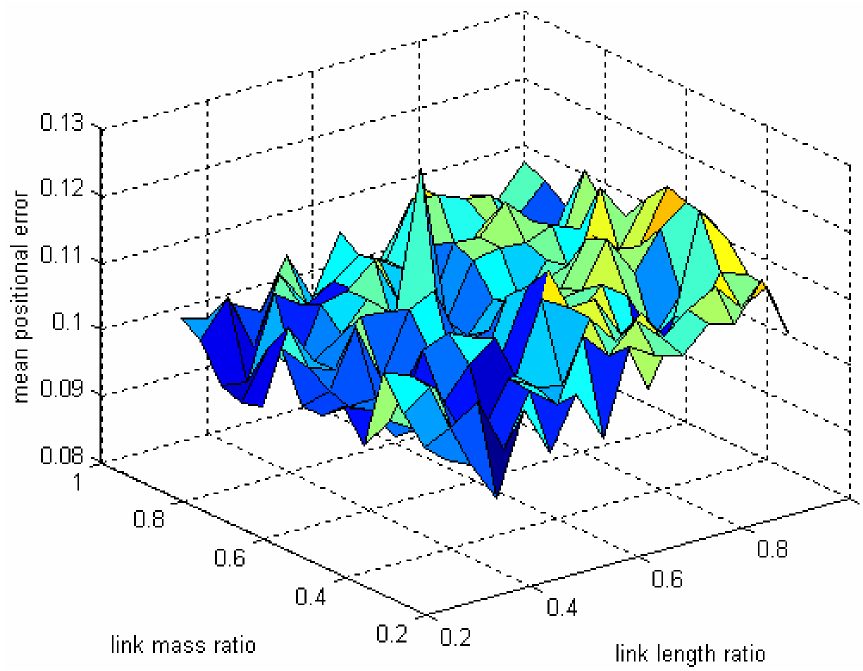


Fig 4.24 Contour Plotting of Performance Measure for Quintic Trajectory - case (ii)

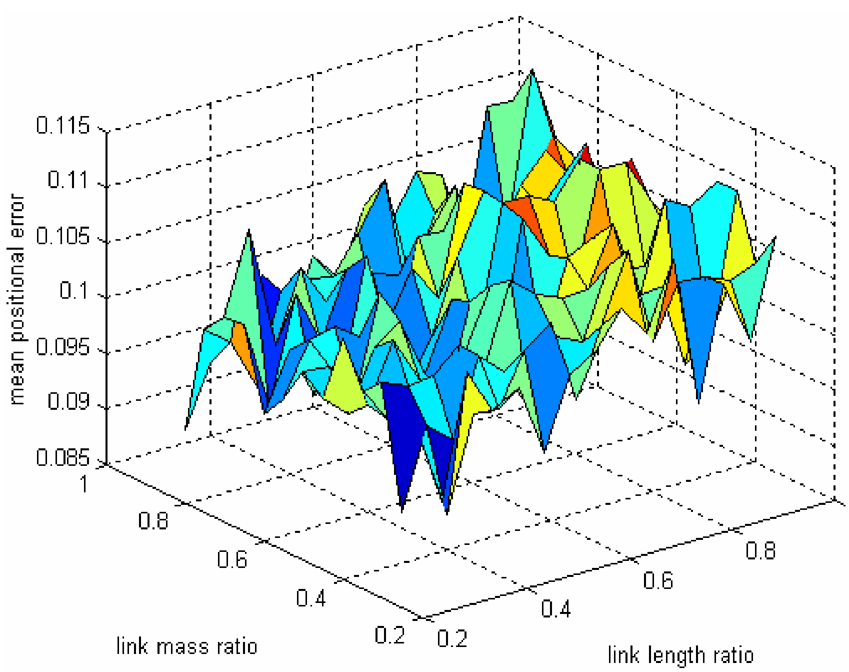


Fig 4.25 Contour Plotting of Performance Measure for Quintic Trajectory - case (iii)

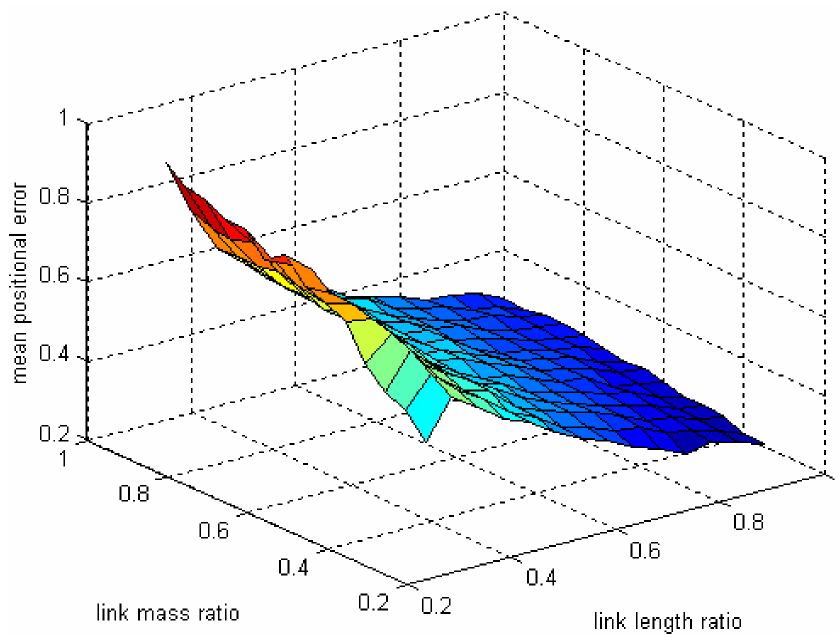


Fig 4.26 Contour Plotting of Performance Measure for Quintic Trajectory - case (iv)

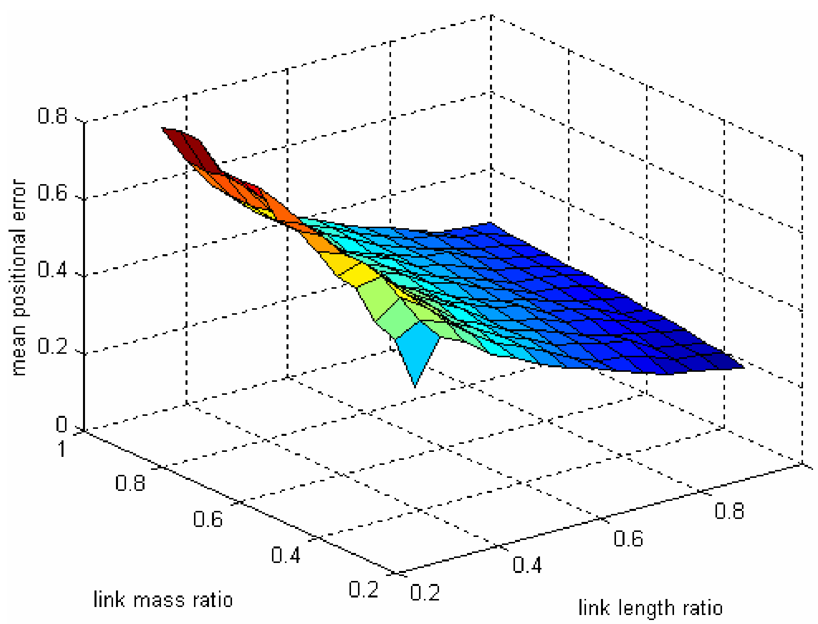


Fig 4.27 Contour Plotting of Performance Measure for Quintic Trajectory - case (v)

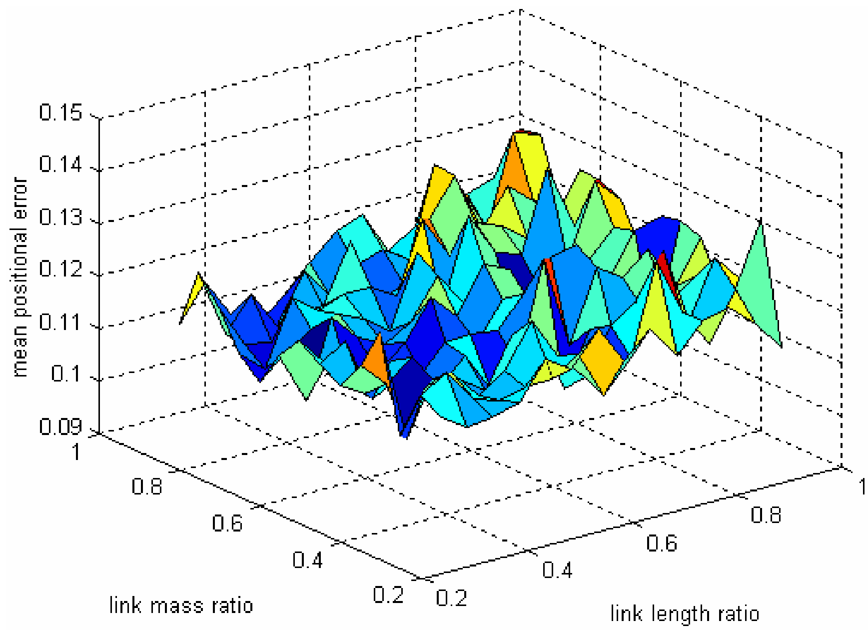


Fig 4.28 Contour Plotting of Performance Measure for Quintic Trajectory - case (vi)

From Fig. 4.23, for case (i) following quintic trajectory, performance measure observed (mean positional error) is quite intriguing. The performance obtained from the simulation does not provide clear indication, for optimum link mass ratio and link length ratio i.e. which will provide best performance measure. The range of performance varies between 0.08×10^{-2} m to 0.12×10^{-2} m for all parameter combinations. This indicates none of the combination is bad. There are quite a number of optimal combination which deliver better performance.

Similar to previous case, it is observed that from Fig. 4.24 for case (ii) the change in performance measure, with change in parameter values are difficult to understand. The range of performance varies between 0.09×10^{-2} m to 0.11×10^{-2} m . This indicates all the combinations are equally good. From Fig. 4.25 for case (iii) following quintic trajectory, it is observed that the range of performance is between 0.09×10^{-2} m to 0.11×10^{-2} m for all parameter combinations. It is difficult to say which combination will deliver very less performance variations. All the combinations considered are equally good.

It has been observed that from Fig. 4.26 for task (iv) following quintic trajectory, range of performance varies between 0.2×10^{-2} m to 0.8×10^{-2} m. The best performance is observed for the combination having link length ratio β = 0.8 to 1.0 and α value 0.35 to 1.0. Change in link mass ratio has very less influence in change in performance. This feature is similar to the trend observed in case of task performed following cubic trajectories.

From Fig. 4.27 for case (v) following quintic trajectory, feature observed is similar to case (iv). It is observed that performance measure vary between 0.2×10^{-2} m to 0.8×10^{-2} m. The best performance is observed for the parameter combination having link length ratio β = 0.8 to 1.0 and α value 0.35 to 1.0 like in previous case. Change in link mass ratio has very less influence in change in performance. In Fig. 4.28 similar feature is observed as discussed in case (i) (ii) and (iii). Unlike previous cases (iv) and (v), it is difficult to say which combination will deliver very less performance variations. For the case (vi), range of performance vary between 0.1×10^{-2} m to 0.13×10^{-2} m. All the considered combinations considered are equally good or equally bad.

4.7.4 Results and Discussion

It has been observed that in Figs 4.13 and 4.14 link length ratio change has significant influence on performance in some cases and insignificant in some cases. For cases (iv) and (v) performance measure (mean positional error) decrease till β is equal to 0.75. And remain stable with increase in β value. Trends of mean positional error are same for cases (i),(ii),(iii) and (v). Similar trend is also observed in case of manipulator following cubic and quintic trajectory

It can be seen that in Figs 4.15 and 4.16 link mass ratio change has insignificant influence on performance measure in some cases. In cases (iv) and (v) performance measure, increase with increase of α. Trends of mean positional error are same for cases (i),(ii),(iii) and (vi). Similar trend is observed in case of manipulator following cubic and quintic trajectories. The important thing to be observed in these figures are the contribution of link mass ratio and link length ratio on performance variations. Performance variations contribution has been less by link mass ratio change as compared to link length ratio change.

It can be observed that simultaneous change of link length and link mass ratios have significant effect on performance variations of manipulator. For β equal to 08 to 1.0 and α value between 0.35 to 1, have lowest performance variations. The trend of change is observed to be similar in Figs. 4.17-4.22 and Figs. 4.26-4.27 respectively. For considered cases performance variations have similar trend following cubic trajectory. The Figs. 4.23-4.25

indicate that link length and link mass ratios change have insignificant effect on performance variations. Therefore in these cases, drawing conclusion would be difficult.

4.8 EPILOGUE

This chapter presents a probabilistic approach to simulate the real life performance of manipulator. For simulating performance of manipulator different performance measures are defined. The methods adopted to simulate the performance for tasks following cubic and quintic path are discussed. Since numbers of control and noise factors considered are fourteen, the Fractional factorial design approach has been adopted to create the design matrix. Taking these fraction factorial combinations of design matrix performance of manipulator are simulated using the proposed probabilistic approach. Half normal probability plotting and ANOVA tables are used to determine statistically significant factors. This chapter discusses and explains an approach to screening the parameters responsible for performance variations. The focus of this investigation is the subsequent use of screened parameters for robust design. Uniqueness of above investigations are given below.

(a) Development of approach to incorporate effect of noise for simulation of real life performance.

(b) Use of both kinematic and dynamic models of manipulator for simulating the performances. Subsequently use of performance measure like positional error, mean positional error for analysis.

(c) Application of combined array approach and Fractional factorial design technique 2^{14-6} of resolution V for generating combination of control and noise factors as prototypes and simulation of performance measures.

(d) Use of ANOVA technique to analyze the statistical significance of control and noise parameters that contribute most to the observed performance variations.

(e) Exploration of impact of parameter changes on performance measure by parametric sensitivity study.

Present chapter emphasizes the performance variation problem of manipulator in robot manufacturer and designer's perspective and identifies the parameter responsible for performance variations.

CHAPTER-5

OPTIMAL MANIPULATOR PARAMETER SELECTION USING RESPONSE SURFACE METHODOLOGY

5.1 INTRODUCTION

This chapter deals with the selection of optimal parameters for optimal performance using response surface approach. In Chapter 3, search based heuristics method along with design of experiments approach is applied to obtain statistically significant manipulator parameters. However, to get rid of the limitations in this study, another investigation is initiated in Chapter 4. The effect of control and noise parameters on performance are explored and factors influencing the most to the performance variations are identified. From selected fourteen control and noise parameters, only six control and noise parameters are identified to be statistically significant in previous chapter.

Taking these factors and factor bounds into consideration, optimal manipulator design factor which is insensitive to noise factors and delivers optimal performance has been obtained. As the focus is to optimize the performance at destination while following a particular trajectory, the significant parameters selected from previous chapters are used to develop the design matrix. Using the probabilistic approach discussed in Chapter 4, desired performances are simulated taking parameter combinations of design matrix. The response surface approach is applied to develop an empirical model which connects simulated performance of manipulator with the design and noise factors of manipulator. Subsequently using the empirical model, response equation for mean performance and variance in performances has been computed. These empirical relations are finally used to obtain the optimal control parameter combination, which deliver minimum mean performance and performance variation less than the specified limit.

Response Surface Methodology (RSM) is conventionally applied to problems where true functional relationship between performance and the independent parameters is unknown. As the mathematical relations relating manipulator performances and the control and noise factors are not available, therefore, application of RSM to this problem becomes pertinent. RSM has been used to develop the relationship between independent control and noise factors with the performances. To optimize parameters that deliver optimum performance, an effort is made to apply robust design concepts to address this problem.

The rest of this chapter is organized in seven sections. In section 5.2, robust design principles using RSM techniques have been discussed. In section 5.3 steps utilized for application of RSM to relate the factors and the performances are discussed. The application of RSM technique to a 2-DOF RR planar manipulator has been discussed in section 5.4. The methodology used to optimize the performances of the manipulator is discussed in section 5.5. The assumed manipulator data for simulation optimization and analysis of results of experiment are presented in section 5.6.

5.2 ROBUST DESIGN PRINCIPLES

Theoretically robust design principles can be applied at any stage of the product development processes. However, it is ideal to apply it in the conceptual stages. Early application of these techniques in the design process would allow grater flexibilities to make changes as well as larger amount of cost savings. The aim of robust design principle is to find the optimal setting of the control factors such that the variability in the performance (response) due to noise factors are minimized with mean performance achieving a specified target. Many methods for handling uncertainty or developing a robust design have been proposed. The best-known approach for robust design is that proposed by Taguchi [Park 1998]. However there have been many drawbacks in analytical techniques employed by him. Some criticisms include lack of flexibility in modeling design factors and the lack of economy in experimental design plan. Taguchi advocated highly fractionated designs that do not enable one to analyze control factor interactions. In response to these limitations, the use of response surface methodology has been chosen as a viable alternative for solving the robust design problem. To avoid inefficiency of cross array design of experiment approach proposed by Taguchi, a single experimental design for both control and noise factors was proposed by Welch et al. [Welch 1990]. The design is known as a combined array in which the experiments are run at various combinations of control and noise parameters. The response obtained form these experiments are linked to the factors by the response equations.

5.2.1 Robust Design through Response Surface Methodology

RSM is an important tool used in product design. This method is applied in many industrial settings where several variables influence the desired performance or outcome (e.g. performance of manipulator at destination). RSM consists of techniques from mathematical optimization and statistics. RSM is used for the parameter design of products or processes that may be sensitive to uncontrollable or noise factors. By developing a model containing both the noise and the controllable factors, a combination of control factor setting can be determined such that the response will be "robust" to changes in the noise factors.

This methodology was developed by Box and his coworkers in late 50s and 60s [Box 1987]. Traditionally, RSM application did not consider the existence of noise factors. It is after the introduction of Taguchi's philosophy that RSM was extended to address robust design issues with noise factors included. Usually, the relationship between the dependent variables and the independent variables is too complex, or unknown. RSM provides a procedure, which solves this problem. It is assumed that the designer is concerned with a system involving some response variable Y that depends on the input variables x_i i.e. $x_1, x_2, ..., x_n$. It is assumed that each x_i is continuous and controllable. The functional relationship between the response and *n* inputs is written as:

$$Y = f(x_1, x_2 ....x_n) + \in \tag{5.1}$$

where, $\in$ represent the error observed in response Y. A mechanistic model for such a relationship does not necessarily exist. Thus, the first step in RSM is to find a suitable approximation for function $f(.)$ using a low-order polynomial in some region of the independent variables (input variables). If the approximated function has linear variables, a first-order polynomial can be used and written in terms of the design variables.

$$Y = a_0 + \sum_{i=1}^{n} a_i x_i + \in \tag{5.2}$$

where, $a_0, a_1, \ldots a_n$ are regression coefficients or constants.

Otherwise a second-order polynomial is used to develop the relationship. The second order model is given as:

$$Y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} b_i x_i^2 + \sum_{i<j}^{n} \sum_{j=1}^{n} c_{ij} x_i x_j + \in \tag{5.3}$$

where, b_i's and c_{ij}'s are regression coefficients of quadratic and interacting terms respectively.

In most of RSM applications second order model is used. The justification behind the selection is the ability of second order models to include the nonlinear behavior of the system. But the experimental designs for fitting a second order response surface require at least three levels of each factor so that the coefficients in the model can be estimated. When there are k factors it will lead to 3^k factorial combinations.

It is important for the second order model to provide good predictions through out the region of interest. One way is to define "good" that the model has a reasonably consistent and stable variance of the predicted response at points of interest $\mathbf{x}$. Box and Hunter suggested that a second order response surface design should be rotatable. This means that variance of predicted response is same at all points $\mathbf{x}$ that are at the same distance from the design center. That is the variance of predicted response is constant on spheres. A design with this property will leave the variance of $\hat{y}$ unchanged when the design is rotated about the center $(0,0,......,0)$, hence the name rotatable design. Rotatability is a reasonable basis for the selection of a response surface design. Because the purpose of RSM is optimization and location of the optimum is unknown prior to running of the experiment. Hence, the orientation of the design is an important factor with regard to the response surface and will affect the collection of data and the fitting of the response surface.

There are two experimental designs in the class of the 3^k factorial designs that can be used for fitting a second order model to response surfaces. These designs are: Central Composite Design (CCD) and Box-Behnken Design (BBD) [Montgomery 2001]. Both the designs are a fraction of the 3^k factorial design and rotatable. This property is important for the experimental design of a three factorial 3^k design. For the experimental design, the levels of each factor are assumed to be equally spaced. A least-square approach is used to estimate the coefficients of the polynomials. The response surface analysis then proceeds in terms of the fitted response surface. The eventual goal of RSM is to determine optimal factor levels in the system that will deliver optimal performance.

As discussed in the previous section, to get robust design control factor and noise factors needed are to be considered and then model should be developed with these factors. To represent the relationship in terms of control and noise factors following notations are used. Where the control and noise variables are represented in vector form as x and z, respectively. The set of control variables x is assumed to vary within region of interest R_x, and set of noise variables z is varied within region of interest R_z. By means of proper linear transformation on x and z, the R_x and R_z are defined by

$$R_x = \{x : -1 \le x_i \le +1,\ i = 1,2,....,l\} \text{ and } R_z = \{z : -1 \le z_i \le +1,\ i = 1,2,.....,m\}$$

For instance the linear transformation.

$x_{i,trans} = (x_{i,org} - a_i)/b_i$ where $x_{i,trans}$ and $x_{i,orig}$ are the transformed and original values of x_i respectively. and a_i , b_i are obtained using following relations,

$$a_i = \{\max(x_i) + \min(x_i)\}/2 \quad b_i = \{\max(x_i) - \min(x_i)\}/2$$

Assuming that a set of control variables is denoted by $x' = (x_1, x_2......x_l)$ and set of noise variables by $z' = (z_1, z_2......z_m)$.

The functional relationship in equation (5.2) is well represented by the first order polynomial regression model,

$$y(x,z) = \beta_0 + x'\beta + z'\gamma + z'\Delta x + \in \tag{5.4}$$

where, β_0 is a unknown coefficient representing the intercept of the plane and β, γ are unknown vectors of coefficients for the control and noise factors to be estimated from data, of size $(l \times 1)$ and $(m \times 1)$ respectively. Similarly Δ is matrix of size $(l \times m)$ representing coefficient of interaction terms. The, x' is vector of control factors of size $(1 \times l)$ and l represent the number of control factors. Similarly z' is the vector of noise factors of size $(1 \times m)$ and m represent the number of noise factors. The error $\in$ is random error associated with y.

The response equation for second order model discussed in equation (5.3) is expressed explicitly in the following form,

$$y(x,z) = \beta_0 + x'\beta + x'Bx + z'Rz + z'\gamma + z'\Delta x + \in \tag{5.5}$$

where B is a matrix of size $(l \times l)$ whose diagonals are the coefficients for the pure quadratic effects of the control factors and whose off-diagonals are one-half of the interaction effects of the control factors, R is a $(m \times m)$ matrix and Δ is a $(l \times m)$ matrix of the control factors and noise factor interaction effects.

The empirically fitted second order model of equation (5.5) by least squares method is written as [Park 1998].

$$\hat{y}(x,z) = b_0 + x'b + x'\hat{B}x + z'\hat{R}z + z'r + z'\hat{\Delta}x \tag{5.6}$$

where, $\hat{y}(x,z)$, b, $\hat{B}$, $\hat{R}$, r and $\hat{\Delta}$ are predicted or estimated values. Then to obtain the estimated mean response $\hat{m}(x)$ at x is the above equation (5.6) is averaged over the noise variables z, where z is uniformly distributed over R_z:

$$\hat{m}(x) = k \int_{R_z} \hat{y}(x,z)dz \tag{5.7}$$

Integrating and simplifying the equation (5.7) will give $\hat{m}(x)$ as:

$$\hat{m}(x) = b_0 + x'b + x'\hat{B}x + \frac{1}{3}\mathrm{tr}\hat{R} \tag{5.8}$$

where $k = \dfrac{1}{\int_{R_z} dz}$ and $\mathrm{tr}\hat{R}$ is the trace of matrix $\hat{R}$.

Similarly, the mean square variation about the mean response $\hat{m}(x)$ i.e., variance $\hat{v}(x)$ is estimated as

$$\hat{v}(x) = k \int_{R_z} [\hat{y}(x,z) - \hat{m}(x)]^2 dz \tag{5.9}$$

Substituting $\hat{m}(x)$ in equation (5.9) will result,

$$\hat{v}(x) = k \int_{R_z} [\hat{y}(x,z) - (b_0 + x'b + x'\hat{B}x + \frac{1}{3}\mathrm{tr}\hat{R})]^2 dz$$

Simplifying the above equation will give the variance of response as

$$\hat{v}(x) = k \int_{R_z} [z'\hat{R}z + z'r + z'\hat{\Delta}x + \frac{1}{3}\mathrm{tr}\hat{R})]^2 dz$$

and finally,

$$\hat{v}(x) = (r + \hat{\Delta}x)'(r + \hat{\Delta}x) + A \tag{5.10}$$

where $A = \frac{1}{45}\left[4\sum_{j=1}^{r_Z} r_{jj}^2 + 5\sum_{j=1}^{r_Z-1}\sum_{k=j+1}^{r_Z} r_{jk}^2\right]$ and r_{jk} is the j^{th} row and k^{th} column element of the matrix $\hat{R}$. The detailed explanation is also available in Box [Box 1992].

The models for the mean and variance are derived in terms of the response variables of interest, which has following characteristics:

(1) The mean and variance models involve only the control variables, meaning that potential set of the control variables can be chosen properly to achieve a target value of the mean and minimize the variability transmitted by the noise variable.

(2) The variance model involves only the control factors. It also involves the interaction regression coefficients between the control and noise parameters. This is how the noise parameter influences the response.

(3) The variance model is a quadratic function of the control parameters.

(4) The variance model is simply the square of the slope of the fitted response model in the direction of the noise parameters.

5.3 STEPS TO OPTIMAL PARAMETER DESIGN USING RSM

As most of the mechanical systems show nonlinear behavior, to understand the relationship between the performance of manipulator and the parameters a second order model has been adopted. The key steps for obtaining the optimal parameter design through use of RSM have been outlined below. It can be observed that the steps provided are similar to design of experiment technique except few exceptions. The exceptions are discussed here.

Step 1. Identify performances to be optimized

This method used is same as discussed in previous chapter.

Step 2. Identify the control and noise factors and their feasible ranges

As the focus of the investigation is to obtain a robust design, control parameters and noise parameters and their level values are identified.

Step 3. Construct experimental design

Design of experiments is carried out to understand the influence of the control factors, noise factors and their interaction on the response.

In experiments where the effects of noise are not considered, a single design array can be used to investigate the relationship between the response and the factors. However, in this exploration, both the control and noise factors are studied together using combined array approach. The justification behind the use of combined array approach is the requirement of lesser number of experimental run. To generate combined array which will be useful for the RSM, the help of CCD approach is taken [Montgomery 2001].

5.3.1 Central Composite Design

It was felt that the quadratic effects would be important for the analysis and thus a central composite Design (CCD) has been used to generate the design points. It is also known that CCD allows efficient estimation of the first and second-order coefficients. The resulting designs are usually very efficient in terms of the number of required runs, and they are rotatable or nearly rotatable. Where, rotatability is important for the second order model to provide good predictions throughout the region of interest.

The CCD has following features;

(i) A complete 2^k or fractional 2^{k-p} first order factorial design, where k is the number of factors and p is the p^{th} fraction.

(ii) One or more center points

(iii) Two axial points on the axis of each design variable at a distance, α from design center.

The factorial portion is used to estimate the linear and two factor interaction terms, the axial points (denoted by $\pm\alpha$) contribute to the estimation of both the linear and the quadratic terms, and the center points will give information about curvature and also contribute to estimation of the quadratic terms in the model. Using the standard ±1 scaling on the design variables, the axial points are chosen based on the region of interest and region of operability. The axial or spherical distance is $\alpha = \sqrt{k}$ where k represents the number of variables in the model. The axial distance does not have to be *k* and can be chosen based on other criteria.

Step 4. Conduct the experiments

The experiment is run using CCD combined array i.e. design matrix and subsequently the response of the experiments are used to develop empirical model.

Step 5. Construct the RSM models

Depending on the order of the model chosen, response surfaces are developed. Developed model gives empirical relationship between the considered parameters and the response.

Step 6. Develop and evaluate the mean and the variance equation using the response equations

Using procedure discussed in section 5.2, mean and variance of response equation in parametric form are developed for further use.

Step 7. Formulate the optimization problem taking mean and variance equations and perform a constrained optimization

Using mean and variance of response equation and optimization problem is formulated. These steps are explained in this section. With regard to the formulation of constrained optimization for performance, however, some explanation is needed. There are many ways of formulating the optimal parameter design problem. The following optimization measures proposed by Taguchi are used depending upon the type of quality characteristics.

(a) **Nominal-is-best case**: $\min_{x \in R_X} [\hat{m}(x) - \tau]^2$ subjected to $\hat{v}(x) \leq c$, where τ is the target of response required to be achieved, c is some upper bound on the variance response.

(b) **Larger-the-better case**: $\max_{x \in R_X} \hat{m}(x)$ subjected to $\hat{v}(x) \leq c$,

(c) **Smaller-the-better case**: $\min_{x \in R_X} \hat{m}(x)$ subjected to $\hat{v}(x) \leq c$,

Hence, it can be seen that the formulation adopted for a particular problem would vary depending on the problem at hand as well as the relative importance the designer places on these two statistics i.e. mean and variance.

Step 8. Obtain the optimal designs

This is the last step used in RSM. The optimization problem is developed using parametric model equations of mean and variance and subsequently solutions of the problem is obtained. In RSM the problem formulated is of nonlinear type; therefore, nonlinear optimization technique is used to solve the problem.

5.4 ROBUST DESIGN OF MANIPULATOR USING RSM

In Chapter 4, it was observed that the performance of manipulator at the destination while traveling along a trajectory is important. In this chapter parameter design of manipulator for the improved performance at the target has been attempted. For determination of robust parameters of manipulator RSM approach has been adopted. The optimal parameter levels values are selected for different tasks following different trajectories.

It has been observed that statistically significant parameters which contribute to the performance variation at the destination point are only six from the fourteen selected parameters. It is decided to conduct the experiment taking parameters, which are statistically significant during analysis of positional error as the performance. In this investigation, more emphasis is given to minimizing the response characteristics and reducing performance variations to certain limit. Normally the steps 1-3, are grouped under the general heading of experimental design, while steps 4 and 5 fall within the domain of model fitting.

5.4.1 Experimental Design for RSM

Factor information

In Chapter 4, fourteen parameters control and noise factors were identified to be important. The design information of the factors is provided in Table 4.1. The factors considered in that investigation were continuous in nature. Factors A to D were control factors while factors from E to O were noise factors. After performing screening experiment statistically significant control and noise parameters were identified.

In case of 2-DOF RR planar manipulator, the noise parameters i.e. variations of link lengths, variations in supplied joint torque, friction at two joints, and uncertainty in link mass distribution have been found to be statistically insignificant. For performing experiment using these parameters, which are not statistically significant, are kept at high level value. These factors have been allowed to vary as per explained procedure in section 4.4, during experiment.

5.4.2 Design Array using Central Composite Design

The focus of this investigation has been to optimize the performance of manipulator using RSM approach. Central Composite Design has been used to generate design matrix. Since the region of interest and operability is the same, the value of α is set to unity to obtain the face centered cubical design (FCC). For the control and noise factors, it is altered between -1 and $+1$ when the other control factors are at their axial settings. This design has (2^6) 64 factorial points, 12 axial points and 10 center points. Therefore, total number of combinations for which experiment to be carried out for the manipulator design problem are 86.

5.4.3 Conducting Experiments

The design matrix generated using CCD method has been utilized to simulate the performances. While simulating the performance it has been ensured that the noise parameters which are not statistically significant do not change their level values but vary as per discussion in Chapter 4. The procedure discussed is used for simulating the performance. In this method effect of noise is simulated using probabilistic method. Assumption for the probability distributions kept same as discussed earlier. Subsequently numerical integration based method is used to simulate the performance at the target points. For simulation of performance probabilistic method discussed has been used. To analyze the performance of manipulator, performance i.e. positional error is used. The second order response surface model for the specified performance is built. After this a set of control parameters are searched for by which optimization of performance is achieved.

5.4.4 Model Fitting

The functional relationship between the performance and the factors was obtained using regression analysis. A second order polynomial has been fitted for the response using the regression approach. The regression approach is widely used for the model building. To judge the adequacy of the fitted model help of coefficient of determination (R^2) is taken. In order to judge the significance of the model, the coefficient of determination R^2 measure is widely used. This measure ranges between 0 and 1. A value of 0.8 implies that 80% of the variations in the response is explained by the model. One drawback of this measure is that as the number of variables in the model increase, the R^2 measure will increase, until it reaches the theoretical maximum of 1.

5.5 SIMULATION

Developed computer programme in MATLAB, for Chapter 4 is used once again to simulate the performance of the manipulator. The numerical level values chosen for simulation of performance have been provided in Table 5.1. The parameters are selected at three level values i.e. low, medium, and high. The low and high values are same as discussed in Chapter 4. Medium level is found by taking average of the two levels.

Table 5.1 Level Values of Statistically Significant Control and Noise Parameters

Control and Noise Factors	Symbol used for ANOVA	Low Level Value	Medium Level Value	High Level Value
l_1 (m)	A	0.40	0.45	0.50
l_2 (m)	B	0.30	0.35	0.40
m_1 (kg)	C	7	7.5	8
m_2 (kg)	D	5	5.5	6
σ_{θ_1} (degree)	G	0.05	0.75	0.1
σ_{θ_2} (degree)	H	0.05	0.75	0.1

During simulation except parameters shown in Table 5.1 and remaining noise factors are fixed at a particular value i.e., no change of level value. The assumed values of noise parameters for the simulation are provided in Table 5.2.

Table 5.2 Values of Statistically Insignificant Noise Parameters

Noise Parameters	Values
σ_{τ_1} (Nm)	0.10
σ_{τ_2} (Nm)	0.10
B_1 (Ns)	4
B_2 (Ns)	2.5
σ_{l_1} (m)	1×10^{-4}
σ_{l_2} (m)	1×10^{-4}
σ_{m_1} (kg)	5×10^{-3}
σ_{m_2} (kg)	5×10^{-3}

The tasks considered remain same as discussed in Chapter 4 and provided once again for easy reference in Table 5.3.

Table 5.3 Manipulator Task Specifications

Case	Coordinates of Start point (x_i m, y_i m)	Coordinates of Destination point (x_f m, y_f m)	Time to travel (sec)
(i)	(0.65, 0)	(0.4, 0.30)	2
(ii)	(0.65, 0.05)	(0.4, 0.30)	2
(iii)	(0.65, 0.10)	(0.4, 0.30)	2
(iv)	(0.65, 0.05)	(–0.4, 0.30)	2
(v)	(0.65, 0.10)	(–0.4, 0.30)	2
(vi)	(0.40, 0.30)	(0.65, 0)	2

Assumed Process Parameters

These parameters are used while simulating the performance of manipulator. The values chosen for the process parameters are kept same as discussed in Chapter 4. For easy reference it is provided below:

(i) Weight parameter for first order autoregressive process $\phi_1 = 0.8$.

(ii) Time step for numerical integration, 0.001 s.

Design Matrix using Control and Noise Parameters

In this chapter focus is to obtain the robust deign: design which is insensitive to noise factors. The manipulator control parameters and noise parameters have been chosen at three level values. Six parameters are indicated by capital letters A, B, C, D, E, and F for easy reference. For design matrix CCD design is used and 86 design combinations are generated using this approach. Few combinations of CCD matrix are shown in Table 5.4. The design matrix showing all 86 combinations are provided in Appendix C1.

Table 5.4 Central Composite Design Matrix for RSM

Combination number	l_1 (A) (m)	l_2 (B) (m)	m_1 (C) (kg)	m_2 (D) (kg)	σ_{θ_1} (E) (deg)	σ_{θ_2} (F) (deg)
1	0.4	0.3	7	5	0.05	0.05
2	0.5	0.3	7	5	0.05	0.05
3	0.4	0.4	7	5	0.05	0.05
4	0.5	0.4	7	5	0.05	0.05
76	0.45	0.35	7.5	5.5	0.075	0.1
77	0.45	0.35	7.5	5.5	0.075	0.075
78	0.45	0.35	7.5	5.5	0.075	0.075
79	0.45	0.35	7.5	5.5	0.075	0.075
80	0.45	0.35	7.5	5.5	0.075	0.075
81	0.45	0.35	7.5	5.5	0.075	0.075
82	0.45	0.35	7.5	5.5	0.075	0.075
83	0.45	0.35	7.5	5.5	0.075	0.075
84	0.45	0.35	7.5	5.5	0.075	0.075
85	0.45	0.35	7.5	5.5	0.075	0.075
86	0.45	0.35	7.5	5.5	0.075	0.075

5.6 OPTIMIZATION AND DISCUSSION

For the task specified in Table 5.3 and combinations provided in Table 5.4, simulations are run following cubic and quintic trajectories. The performances obtained from simulation have been used to fit a response surface model. In this investigation response surface of second order has been chosen and for fitting this model help of Design Expert software has been taken.

The performance utilized for analysis of experiment using ANOVA is positional error (ε_i). To obtain optimal parameters of manipulator, mean and variance of response equations are developed and subsequently these parametric equations are used for optimization. To optimize, available *fmincon* subroutine in MATLAB is used

Performance of Manipulator Following Cubic and Quintic trajectory

Statistical analysis of the response obtained from the experiment has been analysed using ANOVA. This analysis is carried out for positional error as the performance, while manipulator performing a task following either cubic or quintic trajectories. The summary of results of ANOVA for all the discussed tasks are provided in tabular form. The ANOVA of performance for all the cases following cubic trajectory are provided in Tables 5.5-5.10. During analysis the level of significance is taken as 0.05 to identify statistically significant parameters. Statistically significant parameters are those for which F_o statistic is greater than tabulated F statistic.

For assumed, level of significance, F tabulated is equal to 4.0, i.e. $F_{0.05,\nu_1,\nu_2} = F_{0.05,1,58} \approx 4.00$, [Montegomery 2001]. The observed statistic F_o values have been provided in respective tables. The statistically significant parameter are indicated in the tables.

Table 5.5 ANOVA of Performance for Cubic Trajectory - case (i)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.28023479	1	0.28023479	6.796715469	Significant
B	0.044384773	1	0.044384773	1.076492576	-
C	0.00223738	1	0.00223738	0.054264614	-
D	0.066320152	1	0.066320152	1.608505506	-
E	1.46985314	1	1.46985314	35.64929807	Significant
F	0.291292774	1	0.291292774	7.064911881	Significant
A^2	0.00215896	1	0.00215896	0.052362658	-
B^2	0.035678565	1	0.035678565	0.865335304	-
C^2	0.223857868	1	0.223857868	5.429369538	Significant
D^2	0.00277868	1	0.00277868	0.067393129	-
E^2	0.006331736	1	0.006331736	0.15356768	-
F^2	0.059447248	1	0.059447248	1.441812514	-
AB	0.025761046	1	0.025761046	0.624799304	-
AC	0.000567437	1	0.000567437	0.013762424	-
AD	0.082317528	1	0.082317528	1.996500202	-
AE	0.003170462	1	0.003170462	0.076895263	-
AF	0.006614928	1	0.006614928	0.160436132	-
BC	0.014926364	1	0.014926364	0.362018764	-
BD	0.027521496	1	0.027521496	0.667496627	-
BE	0.101118824	1	0.101118824	2.45250019	-
BF	0.061824375	1	0.061824375	1.49946652	-
CD	0.002851073	1	0.002851073	0.069148916	-
CE	5.15303×10^{-6}	1	5.15303×10^{-6}	0.00012498	-
CF	0.021979788	1	0.021979788	0.53309	-
DE	0.179636911	1	0.179636911	4.356850085	Significant
DF	0.007310279	1	0.007310279	0.17730091	-
EF	1.88604×10^{-6}	1	1.88604×10^{-6}	4.57434×10^{-5}	-
Residual	2.391393007	58	0.041230914		
Corrected Total	5.378020412	85			

Table 5.6 ANOVA of Performance for Cubic Trajectory - case (ii)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.432608	1	0.432608	11.89585	Significant
B	2.67×10^{-5}	1	2.67×10^{-5}	0.000733	-
C	0.002083	1	0.002083	0.057281	-
D	0.01258	1	0.01258	0.345922	-
E	1.808514	1	1.808514	49.73045	Significant
F	0.474973	1	0.474973	13.0608	Significant
A^2	0.011367	1	0.011367	0.312578	-
B^2	0.034441	1	0.034441	0.947046	-
C^2	0.077664	1	0.077664	2.135613	-
D^2	0.059783	1	0.059783	1.64392	-
E^2	0.001332	1	0.001332	0.036632	-
F^2	0.084537	1	0.084537	2.324593	-
AB	0.072166	1	0.072166	1.984406	-
AC	0.004187	1	0.004187	0.11514	-
AD	0.000736	1	0.000736	0.020233	-
AE	0.023197	1	0.023197	0.637865	-
AF	0.007033	1	0.007033	0.193406	-
BC	0.10631	1	0.10631	2.923304	-
BD	0.049679	1	0.049679	1.366067	-
BE	0.019449	1	0.019449	0.534803	-
BF	0.01934	1	0.01934	0.531807	-
CD	0.000421	1	0.000421	0.011589	-
CE	0.0967	1	0.0967	2.659056	-
CF	0.033892	1	0.033892	0.93197	-
DE	0.082814	1	0.082814	2.277229	-
DF	0.018188	1	0.018188	0.500138	-
EF	0.071416	1	0.071416	1.963789	-
Residual	2.109247	58	0.036366		
Corrected Total	5.69771	85			

Table 5.7 ANOVA of Performance for Cubic Trajectory - case (iii)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	8.17×10^{-5}	1	8.17×10^{-5}	0.424863	-
B	3.79×10^{-6}	1	3.79×10^{-6}	0.019728	-
C	0.000708	1	0.000708	3.684603	-
D	0.000248	1	0.000248	1.289699	-
E	0.012015	1	0.012015	62.51191	Significant
F	0.002978	1	0.002978	15.49104	Significant
A^2	9.23×10^{-5}	1	9.23×10^{-5}	0.480283	-
B^2	5.16×10^{-5}	1	5.16×10^{-5}	0.268378	-
C^2	1.66×10^{-5}	1	1.66×10^{-5}	0.086564	-
D^2	0.000238	1	0.000238	1.240238	-
E^2	0.000148	1	0.000148	0.772045	-
F^2	2.35×10^{-5}	1	2.35×10^{-5}	0.01223	-
AB	0.000777	1	0.000777	4.044232	Significant
AC	4.78×10^{-7}	1	4.78×10^{-7}	0.002489	-
AD	3.95×10^{-5}	1	3.95×10^{-5}	0.205682	-
AE	5.35×10^{-5}	1	5.35×10^{-5}	0.278151	-
AF	0.000163	1	0.000163	0.847661	-
BC	0.000294	1	0.000294	1.52796	-
BD	4.13×10^{-5}	1	4.13×10^{-5}	0.214709	-
BE	3.7×10^{-5}	1	3.7×10^{-5}	0.192679	-
BF	2.38×10^{-5}	1	2.38×10^{-5}	0.123587	-
CD	3.97×10^{-5}	1	3.97×10^{-5}	0.206288	-
CE	4.95×10^{-7}	1	4.95×10^{-7}	0.002574	-
CF	0.000259	1	0.000259	1.346628	-
DE	6.26×10^{-5}	1	6.26×10^{-5}	0.325591	-
DF	5.36×10^{-5}	1	5.36×10^{-5}	0.278989	-
EF	0.000201	1	0.000201	1.047896	-
Residual	0.011148	58	0.000192		
Corrected Total	0.029744	85			

Table 5.8 ANOVA of Performance for Cubic Trajectory - case (iv)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.081821209	1	0.081821209	339.9088934	Significant
B	0.066501775	1	0.066501775	276.2675485	Significant
C	0.000270459	1	0.000270459	1.12356479	-
D	0.001549082	1	0.001549082	6.43533394	Significant
E	0.000663716	1	0.000663716	2.75726625	-
F	0.000352562	1	0.000352562	1.464642293	-
A^2	6.93816×10^{-5}	1	6.93816×10^{-5}	0.288231189	-
B^2	1.46057×10^{-7}	1	1.46057×10^{-7}	0.000606762	-
C^2	8.19093×10^{-5}	1	8.19093×10^{-5}	0.340274841	-
D^2	0.000343134	1	0.000343134	1.425477935	-
E^2	1.1954×10^{-5}	1	1.1954×10^{-5}	0.049660318	-
F^2	0.000398205	1	0.000398205	1.654259381	-
AB	0.012338405	1	0.012338405	51.25729331	Significant
AC	6.82792×10^{-5}	1	6.82792×10^{-5}	0.283651627	-
AD	0.000404669	1	0.000404669	1.681109799	-
AE	0.000136893	1	0.000136893	0.568692678	-
AF	0.000316702	1	0.000316702	1.315671687	-
BC	0.000549897	1	0.000549897	2.284429174	-
BD	4.76324×10^{-5}	1	4.76324×10^{-5}	0.197878837	-
BE	0.000174263	1	0.000174263	0.723939163	-
BF	0.001127692	1	0.001127692	4.684757514	Significant
CD	8.30172×10^{-5}	1	8.30172×10^{-5}	0.344877194	-
CE	0.000226807	1	0.000226807	0.942223186	-
CF	3.07169×10^{-6}	1	3.07169×10^{-6}	0.012760704	-
DE	0.000722125	1	0.000722125	2.999913529	-
DF	0.001296747	1	0.001296747	5.387061353	Significant
EF	0.00056622	1	0.00056622	2.352240594	-
Residual	0.013961477	58	0.000240715		
Corrected Total	0.183971574	85			

Table 5.9 ANOVA of Performance for Cubic Trajectory - case (v)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.090186	1	0.090186	579.2061	Significant
B	0.046283	1	0.046283	297.2465	Significant
C	0.000267	1	0.000267	1.714127	-
D	0.004688	1	0.004688	30.10623	Significant
E	0.001782	1	0.001782	11.44144	Significant
F	0.001815	1	0.001815	11.6541	Significant
A^2	0.000107	1	0.000107	0.687437	-
B^2	1.55×10^{-8}	1	1.55×10^{-8}	9.94×10^{-5}	-
C^2	2.79×10^{-5}	1	2.79×10^{-5}	0.179149	-
D^2	0.00021	1	0.00021	1.347576	-
E^2	0.000554	1	0.000554	3.561173	-
F^2	5.4×10^{-6}	1	5.4×10^{-6}	0.03465	-
AB	0.008959	1	0.008959	57.54005	Significant
AC	0.000545	1	0.000545	3.499666	-
AD	6.48×10^{-6}	1	6.48×10^{-6}	0.041647	-
AE	0.001324	1	0.001324	8.505045	Significant
AF	0.00021	1	0.00021	1.348951	-
BC	0.00019	1	0.00019	1.219886	-
BD	8.17×10^{-5}	1	8.17×10^{-5}	0.524555	-
BE	8.44×10^{-5}	1	8.44×10^{-5}	0.542083	-
BF	0.000666	1	0.000666	4.278045	Significant
CD	0.000237	1	0.000237	1.519223	-
CE	3.12×10^{-5}	1	3.12×10^{-5}	0.200507	-
CF	2.28×10^{-5}	1	2.28×10^{-5}	0.146434	-
DE	0.000473	1	0.000473	3.038876	-
DF	0.000527	1	0.000527	3.382968	-
EF	0.00053	1	0.00053	3.401196	-
Residual	0.009031	58	0.000156		
Corrected Total	0.168802	85			

Table 5.10 ANOVA of Performance for Cubic Trajectory - case (vi)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.018136896	1	0.018136896	26.15700368	Significant
B	0.000793666	1	0.000793666	1.144623444	-
C	0.002543281	1	0.002543281	3.667916078	-
D	5.85754×10^{-5}	1	5.85754×10^{-5}	0.084477418	-
E	0.007219869	1	0.007219869	10.41248365	Significant
F	0.001153722	1	0.001153722	1.663895936	-
A^2	5.47808×10^{-5}	1	5.47808×10^{-5}	0.079004723	-
B^2	0.000311275	1	0.000311275	0.448920255	-
C^2	0.000195993	1	0.000195993	0.282660877	-
D^2	0.000213567	1	0.000213567	0.308006242	-
E^2	0.000191002	1	0.000191002	0.275463205	-
F^2	0.000816603	1	0.000816603	1.17770426	-
AB	0.00252402	1	0.00252402	3.64013773	-
AC	0.000369135	1	0.000369135	0.532365315	-
AD	0.000203487	1	0.000203487	0.293468153	-
AE	0.000182922	1	0.000182922	0.263810185	-
AF	0.000837965	1	0.000837965	1.208511852	-
BC	0.000113793	1	0.000113793	0.164111934	-
BD	0.000104497	1	0.000104497	0.150705345	-
BE	4.5816×10^{-7}	1	4.5816×10^{-7}	0.000660757	-
BF	8.84423×10^{-5}	1	8.84423×10^{-5}	0.127551307	-
CD	5.01246×10^{-5}	1	5.01246×10^{-5}	0.072289666	-
CE	0.006001465	1	0.006001465	8.655304263	Significant
CF	0.00070947	1	0.00070947	1.023196331	-
DE	0.000365187	1	0.000365187	0.526672607	-
DF	9.08471×10^{-5}	1	9.08471×10^{-5}	0.131019564	-
EF	5.0355×10^{-5}	1	5.0355×10^{-5}	0.072621891	-
Residual	0.040216378	58	0.000693386		
Corrected Total	0.08397218	85			

From Table 5.5, for case (i) it is observed that factors A, E and F , quadratic effect of factor C i.e. C^2 and interactions DE are statistically significant. In Table 5.6, for case (ii) following cubic trajectory performance analysis is carried out. It is observed that factors A, E and F , are statistically significant. And single factors effects, quadratic factor effects and interacting factor effects are found statistically insignificant. From Table 5.7, for case (iii) it is observed that factors E and F and interacting factors AB are statistically significant. Similarly from Table 5.8, for case (iv) performance analysis is done. It is observed that factors A, B and D and interacting factors AB and BF are statistically significant. For this task, effect of other factors are observed to be statistically insignificant. Likewise in Table 5.9, for case (v) it is observed that factors A, B, D, E and F and interacting factors AB, AE and BE are statistically significant and effect of other factors are statistically insignificant. From Table 5.10, for case (vi) following cubic trajectory, it is observed that factors A and E and interacting factor CE are statistically significant.

Similarly the ANOVA table for the manipulator performing task following quintic trajectory are provided in Tables 5.11-5.16. During analysis the level of significance is assumed at 0.05 to identify statistically significant parameters. Statistically significant parameters are those for which F_o statistic is greater than tabulated F statistic. For assumed, level of significance, F tabulated is equal to 4.0 i.e. $F_{0.05,\nu_1,\nu_2} = F_{0.05,1,58} \approx 4.00$ [Montgomery 2001]. The observed statistic F_o values have been provided in respective tables. The statistically significant parameter are indicated in the tables.

Table 5.11 ANOVA of Performance for Quintic Trajectory – case (i)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	6.15×10^{-5}	1	6.14821×10^{-5}	0.280443275	-
B	0.000223	1	0.000222902	1.01673947	-
C	0.000604	1	0.000604401	2.756904604	-
D	0.000123	1	0.000123426	0.562995203	-
E	0.010408	1	0.010408246	47.47598924	Significant
F	0.00321	1	0.003209979	14.64194011	Significant
A^2	0.002012	1	0.002012244	9.17861287	Significant
B^2	0.000555	1	0.000555138	2.532195719	-
C^2	1.96×10^{-5}	1	1.95673×10^{-5}	0.089253972	-
D^2	3.14×10^{-7}	1	3.13503×10^{-7}	0.001430005	-
E^2	8.35×10^{-5}	1	8.3483×10^{-5}	0.380798055	-
F^2	0.000462	1	0.000462173	2.108147804	-
AB	1.04×10^{-5}	1	1.03885×10^{-5}	0.047386078	-
AC	0.000154	1	0.000153714	0.701146042	-
AD	8.3×10^{-5}	1	8.30172×10^{-5}	0.378672972	-
AE	0.000261	1	0.000260536	1.188403897	-
AF	1.68×10^{-5}	1	1.6768×10^{-5}	0.076485263	-
BC	0.001298	1	0.001298386	5.922435262	Significant
BD	0.000108	1	0.000107633	0.490954541	-
BE	0.000561	1	0.000561033	2.559083752	-
BF	5.38×10^{-5}	1	5.37894×10^{-5}	0.245353965	-
CD	0.000161	1	0.000160963	0.734214409	-
CE	0.000129	1	0.000128899	0.587958172	-
CF	8.97×10^{-6}	1	8.96628×10^{-6}	0.040898638	-
DE	0.00042	1	0.00042045	1.917832694	-
DF	0.001262	1	0.001261733	5.755244472	Significant
EF	3.89×10^{-9}	1	3.89064×10^{-9}	1.77467×10^{-5}	-
Residual	0.012715	58	0.000219232		
Corrected Total	0.034583	85			

Table 5.12 ANOVA of Performance for Quintic Trajectory – case (ii)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.033112	1	0.03311206	0.897823434	-
B	0.135901	1	0.135901267	3.684921477	-
C	0.001064	1	0.001064054	0.028851509	-
D	0.020438	1	0.020437679	0.554161439	-
E	2.074954	1	2.074953809	56.26174091	Significant
F	0.640384	1	0.640384302	17.36382541	Significant
A^2	0.008121	1	0.008121494	0.220211835	-
B^2	0.030813	1	0.030812909	0.835482651	-
C^2	0.064677	1	0.064676615	1.753686736	-
D^2	0.007669	1	0.007668866	0.207938955	-
E^2	0.036573	1	0.036573412	0.991676921	-
F^2	1.31×10^{-5}	1	1.31379×10^{-5}	0.000356231	-
AB	0.019508	1	0.019508004	0.528953583	-
AC	0.054177	1	0.05417663	1.46898283	-
AD	0.014024	1	0.014024255	0.380263408	-
AE	0.003553	1	0.003552896	0.096335688	-
AF	0.014912	1	0.014911519	0.404321293	-
BC	0.003286	1	0.00328601	0.089099171	-
BD	0.017288	1	0.017287855	0.46875492	-
BE	0.021717	1	0.021717315	0.588858382	-
BF	0.018754	1	0.018754318	0.508517613	-
CD	0.01095	1	0.010949982	0.296905437	-
CE	0.336736	1	0.336735511	9.130480878	Significant
CF	0.046258	1	0.046257781	1.254265659	-
DE	0.073798	1	0.073798362	2.001020123	-
DF	0.011161	1	0.011160772	0.302620925	-
EF	0.098473	1	0.098473166	2.670069917	-
Residual	2.139061	58	0.03688037		
Corrected Total	5.976475	85			

Table 5.13 ANOVA of Performance for Quintic Trajectory – case (iii)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.00016	1	0.00016	0.768577	-
B	0.000193	1	0.000193	0.92799	-
C	0.000257	1	0.000257	1.233651	-
D	0.000118	1	0.000118	0.568072	-
E	0.015809	1	0.015809	75.86714	Significant
F	0.002166	1	0.002166	10.39536	Significant
A^2	6.75×10^{-5}	1	6.75×10^{-5}	0.324157	-
B^2	0.000386	1	0.000386	1.85017	-
C^2	0.000911	1	0.000911	4.371622	Significant
D^2	2.07×10^{-5}	1	2.07×10^{-5}	0.099547	-
E^2	0.000151	1	0.000151	0.722519	-
F^2	1.81×10^{-5}	1	1.81×10^{-5}	0.086809	-
AB	4.31×10^{-7}	1	4.31×10^{-7}	0.002068	-
AC	7.29×10^{-7}	1	7.29×10^{-7}	0.0035	-
AD	4.12×10^{-6}	1	4.12×10^{-6}	0.019791	-
AE	0.000889	1	0.000889	4.265308	Significant
AF	0.000745	1	0.000745	3.577254	-
BC	0.000102	1	0.000102	0.48825	-
BD	1.97×10^{-6}	1	1.97×10^{-6}	0.009443	-
BE	0.00038	1	0.00038	1.823225	-
BF	0.000449	1	0.000449	2.15376	-
CD	3.83×10^{-5}	1	3.83×10^{-5}	0.183601	-
CE	1.33×10^{-5}	1	1.33×10^{-5}	0.063613	-
CF	0.000131	1	0.000131	0.62676	-
DE	5.31×10^{-6}	1	5.31×10^{-6}	0.025498	-
DF	0.000357	1	0.000357	1.711163	-
EF	0.000917	1	0.000917	4.400129	Significant
Residual	0.012086	58	0.000208		
Corrected Total	0.036153	85			

Table 5.14 ANOVA of Performance for Quintic Trajectory – case (iv)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.173937	1	0.173936943	684.8691003	Significant
B	0.016114	1	0.016114063	63.44841578	Significant
C	0.000157	1	0.000157448	0.619944114	-
D	0.000605	1	0.000605109	2.382592029	-
E	0.001084	1	0.001084413	4.269827097	Significant
F	1.51×10^{-6}	1	1.50547×10^{-6}	0.005927722	-
A^2	0.000119	1	0.000118609	0.46701621	-
B^2	0.002468	1	0.002468214	9.718484208	Significant
C^2	0.000138	1	0.000138168	0.544028678	-
D^2	0.000521	1	0.000521095	2.051787005	-
E^2	7.16×10^{-5}	1	7.1578×10^{-5}	0.281835309	-
F^2	3.07×10^{-5}	1	3.07204×10^{-5}	0.120960378	-
AB	0.004937	1	0.004936626	19.43774755	Significant
AC	0.000262	1	0.000262306	1.032819838	-
AD	0.000184	1	0.000184352	0.725877539	-
AE	9.11×10^{-6}	1	9.11059×10^{-6}	0.03587254	-
AF	0.000673	1	0.000672734	2.648862463	-
BC	0.000586	1	0.000585815	2.306622744	-
BD	0.000176	1	0.000176109	0.6934234	-
BE	2.55×10^{-5}	1	2.55467×10^{-5}	0.100589039	-
BF	0.000866	1	0.000865956	3.409662608	-
CD	0.000145	1	0.000145326	0.572214916	-
CE	5.62×10^{-5}	1	5.61956×10^{-5}	0.221267865	-
CF	0.000256	1	0.000255876	1.007500609	-
DE	0.000199	1	0.000198553	0.781792795	-
DF	0.000498	1	0.000498367	1.962296504	-
EF	0.000208	1	0.000208344	0.820345241	-
Residual	0.01473	58	0.000253971		
Corrected Total	0.219318	85			

Table 5.15 ANOVA of Performance for Quintic Trajectory – case (v)

Source	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.138	1	0.138	678.1193	Significant
B	0.018561	1	0.018561	91.20753	Significant
C	0.000774	1	0.000774	3.801765	-
D	0.003149	1	0.003149	15.47513	Significant
E	0.001601	1	0.001601	7.868047	Significant
F	0.001044	1	0.001044	5.129201	Significant
A^2	2.18×10^{-6}	1	2.18×10^{-6}	0.010714	-
B^2	0.000492	1	0.000492	2.417406	-
C^2	1.09×10^{-5}	1	1.09×10^{-5}	0.053719	-
D^2	0.000295	1	0.000295	1.451612	-
E^2	0.000928	1	0.000928	4.562133	Significant
F^2	2.63×10^{-5}	1	2.63×10^{-5}	0.12924	-
AB	0.005864	1	0.005864	28.81294	Significant
AC	2.4×10^{-5}	1	2.4×10^{-5}	0.117887	-
AD	4.48×10^{-6}	1	4.48×10^{-6}	0.021991	-
AE	0.000677	1	0.000677	3.325897	-
AF	0.000432	1	0.000432	2.122993	-
BC	1.56×10^{-5}	1	1.56×10^{-5}	0.076543	-
BD	0.000359	1	0.000359	1.763436	-
BE	0.000143	1	0.000143	0.704453	-
BF	0.001081	1	0.001081	5.310226	Significant
CD	4.27×10^{-5}	1	4.27×10^{-5}	0.209662	-
CE	1.07×10^{-8}	1	1.07×10^{-8}	5.26×10^{-5}	-
CF	0.000154	1	0.000154	0.754711	-
DE	0.000196	1	0.000196	0.961821	-
DF	0.000634	1	0.000634	3.115829	-
EF	0.000117	1	0.000117	0.576295	-
Residual	0.011803	58	0.000204		
Corrected Total	0.188122	85			

Table 5.16 ANOVA of Performance for Quintic Trajectory – case (vi)

Source	**Sum of Squares**	**Degrees of Freedom**	**Mean Square**	**F_o Value**	**Remark**
A	0.020076	1	0.020076	0.498822	-
B	0.240127	1	0.240127	5.966273	Significant
C	0.006853	1	0.006853	0.17027	-
D	0.013245	1	0.013245	0.329098	-
E	2.825707	1	2.825707	70.20845	Significant
F	0.113018	1	0.113018	2.808073	-
A^2	0.110069	1	0.110069	2.734812	-
B^2	0.038762	1	0.038762	0.963096	-
C^2	0.409232	1	0.409232	10.16792	Significant
D^2	0.006068	1	0.006068	0.150765	-
E^2	0.000577	1	0.000577	0.014333	-
F^2	0.00651	1	0.00651	0.161759	-
AB	1.15×10^{-7}	1	1.15×10^{-7}	2.85×10^{-6}	-
AC	0.028162	1	0.028162	0.699716	-
AD	0.125204	1	0.125204	3.110859	-
AE	0.045139	1	0.045139	1.121541	-
AF	0.037491	1	0.037491	0.931515	-
BC	0.00145	1	0.00145	0.036031	-
BD	0.057805	1	0.057805	1.436254	-
BE	0.025254	1	0.025254	0.627458	-
BF	3.49×10^{-5}	1	3.49×10^{-5}	0.000866	-
CD	0.007148	1	0.007148	0.177592	-
CE	0.029364	1	0.029364	0.729594	-
CF	0.105985	1	0.105985	2.633342	-
DE	0.066306	1	0.066306	1.647471	-
DF	0.027602	1	0.027602	0.685814	-
EF	0.071242	1	0.071242	1.770114	-
Residual	2.334348	58	0.040247		
Corrected Total	6.757714	85			

Similarly, analysis of performance is carried out for the task following quintic trajectory. In Table 5.11 for case (i) it is observed that factors E and F and quadratic effect of factor A i.e. A^2 and interacting factors BC and DE are statistically significant. From Table 5.12 for case (ii) following quintic trajectory it is observed that factors E and F and interacting factor CE are statistically significant. Similarly from Table 5.13 it has been observed that for case (iii) factors E and F and quadratic effect of factor C i.e. C^2 and interacting factors AE and EF are statistically significant.

From Table 5.14 for case (iv) factors A, B and E and quadratic effect of factor B i.e. B^2 and interacting factor AB are statistically significant. It is observed from Table 5.15 for case (v), factors A, B, D, E and F and quadratic effect of factor E i.e. E^2 and interacting factor AB and BF are statistically significant. Lastly from Table 5.16 for case (vi) following quintic trajectory, factors B and E and quadratic effect of factor C i.e. C^2 are observed statistically significant. Rest of the factor effects are found to be statistically insignificant.

Subsequently the fitted second order response equations are provided in tabular form. The fitted emperical models are provided for the manipulator following cubic and quintic path to explore the impact of the time law on performance variations. While fitting the response equation using software, it gave informations regarding the requirement of any data transformations method and the suitable type of data transformation method.

As suggested by the software the desired data transformation method has been adopted for some cases. In two to three cases it has been observed that natural logarithm is the data transformation method required. One advantage in the use of this transformation method is the improvement in R^2 value. The responses are transformed and second order models are fitted. The response equations obtained for all six cases are provided in Tables 5.17(a)-(b) respectively. To present the finding conviniently two tables are used by which response equation of three case are accomodated in each table. The response equations for cases (i) to (iii) are provided in Table 5.17(a).

Table 5.17(a) Response Equation in terms of Coded Parameter for Positional Error at Destination - cases (i) to (iii)

For case (i) to (iii)		
Case	**Manipulator following Cubic trajectory**	**Manipulator following Quintic trajectory**
(i)	$\ln \varepsilon = -2.512546483$ $+0.065161198 \times A - 0.025932539 \times B$ $+0.005822344 \times C + 0.031699382 \times D +$ $0.149233046 \times E + 0.06643438 \times F$ $-0.03006576 \times A^2 + 0.122223174 \times B^2$ $-0.306151256 \times C^2 - 0.034109013 \times D^2$ $+ 0.051488621 \times E^2 + 0.157766845 \times F^2$ $-0.02006281 \times A \times B + 0.002977618 \times A \times C$ $-0.035863789 \times A \times D + 0.007038357 \times A \times E$ $-0.010166526 \times A \times F - 0.015271688 \times B \times C$ $-0.020737005 \times B \times D + 0.039748983 \times B \times E$ $+0.031080635 \times B \times F + 0.00667443 \times C \times D$ $+0.000283754 \times C \times E - 0.018531977 \times C \times F$ $+0.052979493 \times D \times E + 0.010687521 \times D \times F$ $-0.000171667 \times E \times F$	$\varepsilon = 0.079969599$ $-0.000965167 \times A + 0.001837742 \times B$ $+0.003026152 \times C$ $+0.001367515 \times D + 0.012557894 \times E$ $+0.006973955 \times F$ $-0.029026198 \times A^2 + 0.015245802 \times B^2$ $+0.002862302 \times C^2 + 0.000362302 \times D^2$ $-0.005912198 \times E^2 + 0.013910802 \times F^2$ $+0.000402891 \times A \times B + 0.001549766 \times A \times C$ $-0.001138922 \times A \times D - 0.002017641 \times A \times E$ $-0.000511859 \times A \times F + 0.004504141 \times B \times C$ $+0.001296828 \times B \times D - 0.002960766 \times B \times E$ $+0.000916766 \times B \times F + 0.001585891 \times C \times D$ $+0.001419172 \times C \times E - 0.000374297 \times C \times F$ $+0.002563109 \times D \times E - 0.004440109 \times D \times F$ $+7.79687E{-}06 \times E \times F$
(ii)	$\ln \varepsilon = -2.670878006$ $+0.080960915 \times A + 0.000635467 \times B$ $+0.005618039 \times C$ $-0.013805974 \times D + 0.165534658 \times E$ $+0.084832572 \times F$ $-0.068988817 \times A^2 - 0.120084018 \times B^2$ $+0.180327048 \times C^2 - 0.158212193 \times D^2$ $+0.023617419 \times E^2 + 0.188136501 \times F^2$ $+0.033579562 \times A \times B - 0.008088595 \times A \times C$ $-0.003390696 \times A \times D - 0.019038119 \times A \times E$ $-0.010483206 \times A \times F - 0.04075648 \times B \times C$ $-0.027860934 \times B \times D + 0.017432374 \times B \times E$ $+0.017383479 \times B \times F - 0.002566146 \times C \times D$ $-0.038870799 \times C \times E - 0.023012337 \times C \times F$ $-0.035971873 \times D \times E - 0.016857943 \times D \times F$ $-0.033404668 \times E \times F$	$\ln \varepsilon = -2.51846141$ $-0.022398613 \times A + 0.045377418 \times B$ $+0.004015225 \times C + 0.01759721 \times D + 0.177309599 \times E$ $+0.098502754 \times F$ $+0.058313328 \times A^2 - 0.113583753 \times B^2$ $+0.164559707 \times C^2 - 0.05666507 \times D^2$ $-0.123746408 \times E^2 + 0.002345381 \times F^2$ $-0.017458882 \times A \times B - 0.029094842 \times A \times C$ $+0.014803006 \times A \times D + 0.007450772 \times A \times E$ $-0.015264091 \times A \times F - 0.007165467 \times B \times C$ $-0.016435411 \times B \times D - 0.018420995 \times B \times E$ $+0.017118301 \times B \times F + 0.01308027 \times C \times D$ $+0.072536145 \times C \times E + 0.026884528 \times C \times F$ $-0.033957317 \times D \times E + 0.013205569 \times D \times F$ $-0.039225543 \times E \times F$
(iii)	$\varepsilon = 0.072863408$ $+0.001112348 \times A + 0.000239697 \times B -$ $0.003275758 \times C + 0.00193803 \times D + 0.013492667 \times$ $E + 0.006716712 \times F$ $+0.006217083 \times A^2 - 0.004647417 \times B^2$ $-0.002639417 \times C^2 + 0.009990583 \times D^2$ $-0.007882417 \times E^2 + 0.000992083 \times F^2$ $-0.003485109 \times A \times B - 8.64531E{-}05 \times A \times C$ $+0.000785953 \times A \times D - 0.000913984 \times A \times E$ $-0.001595547 \times A \times F - 0.002142172 \times B \times C$ $-0.000803016 \times B \times D - 0.000760703 \times B \times E$ $+0.000609234 \times B \times F - 0.000787109 \times C \times D$ $-8.79219E{-}05 \times C \times E - 0.002011047 \times C \times F$ $+0.000988859 \times D \times E + 0.000915359 \times D \times F$ $-0.001774016 \times E \times F$	$\varepsilon = 0.074069692$ $-0.001557727 \times A - 0.001711667 \times B + 0.00197353 \times$ $C + 0.001339212 \times D + 0.015476561 \times E$ $+0.005728848 \times F$ $-0.005317984 \times A^2 + 0.012705016 \times B^2$ $-0.019529484 \times C^2 + 0.002947016 \times D^2$ $+0.007939516 \times E^2 + 0.002752016 \times F^2$ $-8.20625E{-}05 \times A \times B + 0.00010675 \times A \times C$ $-0.000253844 \times A \times D - 0.003726531 \times A \times E$ $+0.00341275 \times A \times F - 0.001260813 \times B \times C$ $+0.000175344 \times B \times D - 0.002436406 \times B \times E$ $+0.002648063 \times B \times F + 0.000773156 \times C \times D$ $-0.000455094 \times C \times E + 0.0014285 \times C \times F$ $+0.000288125 \times D \times E - 0.002360344 \times D \times F$ $-0.003784969 \times E \times F$

The Table 5.17 (b) provides the response equation for the manipulator following the cubic and quintic trajectories.

Table 5.17(b) Response Equation in terms of Coded Parameter for Positional Error at Destination - cases (iv) to (vi)

For case (iv) to (vi)		
Case	**Manipulator following Cubic Trajectory**	**Manipulator following Quintic Trajectory**
(iv)	$\varepsilon = 0.230935654$ $+0.035209591 \times A - 0.031742758 \times B - 0.002024318 \times C$ $+0.004844682 \times D + 0.003171167 \times E + 0.002311242 \times F$ $+0.005389792 \times A^2 + 0.000247292 \times B^2$ $-0.005856208 \times C^2 - 0.011986208 \times D^2$ $-0.002237208 \times E^2 + 0.012912292 \times F^2$ $-0.013884797 \times A \times B + 0.001032891 \times A \times C$ $+0.002514547 \times A \times D - 0.001462516 \times A \times E$ $-0.002224516 \times A \times F - 0.002931234 \times B \times C$ $-0.000862703 \times B \times D + 0.001650109 \times B \times E$ $-0.004197641 \times B \times F + 0.001138922 \times C \times D$ $-0.001882516 \times C \times E - 0.000219078 \times C \times F$ $-0.003359047 \times D \times E - 0.004501297 \times D \times F$ $-0.002974422 \times E \times F$	$\varepsilon = 0.31610373$ $+0.051336227 \times A - 0.015625379 \times B - 0.00154453 \times C + 0.003027924 \times D + 0.004053455 \times E + 0.00015103 \times F$ $-0.00704706 \times A^2 - 0.03214706 \times B^2$ $+0.00760594 \times C^2 + 0.01477094 \times D^2$ $+0.00547444 \times E^2 + 0.00358644 \times F^2$ $-0.008782641 \times A \times B - 0.002024484 \times A \times C$ $-0.001697203 \times A \times D + 0.000377297 \times A \times E$ $+0.003242141 \times A \times F - 0.003025453 \times B \times C$ $+0.001658828 \times B \times D - 0.000631797 \times B \times E$ $-0.003678391 \times B \times F - 0.001506891 \times C \times D$ $+0.000937047 \times C \times E + 0.001999516 \times C \times F$ $-0.001761359 \times D \times E - 0.002790516 \times D \times F$ $-0.001804266 \times E \times F$
(v)	$\varepsilon = 0.216931807$ $+0.036965561 \times A - 0.026481288 \times B - 0.002010955 \times C$ $+0.008427697 \times D + 0.005195424 \times E + 0.005243485 \times F$ $-0.006694514 \times A^2 - 8.05136E{-}05 \times B^2$ $-0.003417514 \times C^2 - 0.009373014 \times D^2$ $+0.015236986 \times E^2 + 0.001502986 \times F^2$ $-0.011831719 \times A \times B - 0.002917938 \times A \times C$ $+0.000318312 \times A \times D - 0.004548844 \times A \times E$ $+0.001811594 \times A \times F + 0.00172275 \times B \times C$ $+0.001129688 \times B \times D - 0.001148406 \times B \times E$ $+0.003226156 \times B \times F + 0.001922531 \times C \times D$ $+0.000698437 \times C \times E - 0.000596875 \times C \times F$ $+0.002719063 \times D \times E - 0.002868875 \times D \times F$ $-0.002876594 \times E \times F$	$\varepsilon = 0.288046123$ $+0.045726424 \times A - 0.016769864 \times B - 0.003423788 \times C + 0.006907667 \times D + 0.00492547 \times E + 0.003976848 \times F$ $-0.000955446 \times A^2 - 0.014351946 \times B^2$ $-0.002139446 \times C^2 - 0.011121446 \times D^2$ $+0.019716054 \times E^2 - 0.003318446 \times F^2$ $-0.009571719 \times A \times B + 0.00061225 \times A \times C$ $-0.000264438 \times A \times D - 0.003252 \times A \times E$ $+0.002598188 \times A \times F + 0.000493344 \times B \times C$ $+0.002367969 \times B \times D - 0.001496656 \times B \times E$ $+0.004109156 \times B \times F + 0.0008165 \times C \times D$ $-1.29375E{-}05 \times C \times E - 0.001549125 \times C \times F$ $+0.001748813 \times D \times E - 0.003147625 \times D \times F$ $-0.001353688 \times E \times F$
(vi)	$\varepsilon = 0.146836455 +$ $0.016577136 \times A + 0.003467742 \times B + 0.006207621 \times C$ $- 0.000942076 \times D + 0.010459061 \times E - 0.004180985 \times F$ $- 0.00478921 \times A^2 - 0.01141621 \times B^2$ $+ 0.00905879 \times C^2 - 0.00945621 \times D^2$ $- 0.00894271 \times E^2 + 0.01849079 \; F^2$ $- 0.006279953 \times A \times B - 0.002401609 \times A \times C$ $-0.001783109 \times A \times D - 0.001690609 \times A \times E$ $-0.003618453 \times A \times F + 0.001333422 \times B \times C$ $+0.001277797 \times B \times D + 8.46094E{-}05 \times B \times E$ $-0.001175547 \times B \times F - 0.000884984 \times C \times D$ $+0.009683641 \times C \times E + 0.003329484 \times C \times F$ $-0.002388734 \times D \times E + 0.001191422 \times D \times F$ $-0.000887016 \times E \times F$	$\mathrm{Ln}\,\varepsilon = -2.407953421$ $+0.017440928 \times A + 0.060318206 \times B - .010189802 \times C + 0.014166405 \times D + 0.206914806 \times E + 0.041381008 \times F$ $+0.214675478 \times A^2 + 0.127395308 \times B^2$ $-0.413937421 \times C^2 - 0.050404465 \times D^2$ $-0.015541271 \times E^2 + 0.052209876 \times F^2$ $+ 4.23152E{-}05 \times A \times B - 0.020976831 \times A \times C$ $-0.044230209 \times A \times D - 0.02655745 \times A \times E$ $-0.024203259 \times A \times F - 0.004760119 \times B \times C$ $-0.030053461 \times B \times D - 0.019864202 \times B \times E$ $-0.00073811 \times B \times F + 0.010567933 \times C \times D$ $-0.021420004 \times C \times E + 0.040694199 \times C \times F$ $-0.032187541 \times D \times E + 0.020767388 \times D \times F$ $-0.033364106 \times E \times F$

To know whether the fitted model is able to predict the system behaviour or performance properly, R^2 value has been computed. The R^2 value for each case with different trajectories are provided in Table 5.18.

Table 5.18 R^2 Value of the Fitted Model

Case	Manipulator following Cubic trajectory	Manipulator following Quintic trajectory
(i)	0.56	0.59
(ii)	0.63	0.59
(iii)	0.63	0.67
(iv)	0.93	0.94
(v)	0.95	0.94
(vi)	0.52	0.66

To validate as to how the developed second order response equation predicts the performance of manipulator, a comparison between predicted values of performance and desired values of performance has been made. The positional error obtained from simulation and from the emperical model are plotted against the combination numbers. This comparison indicate the closeness between the desired and predicted values. Comparison of results are shown in Figs. 5.1-5.6 for manipulator performing task while following cubic trajectory.

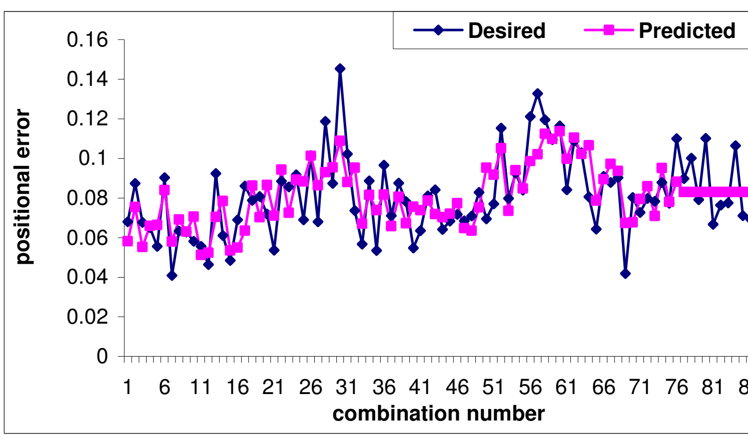


Fig. 5.1 Comparison of Desired and Predicted Performance for Cubic Trajectory - case (i)

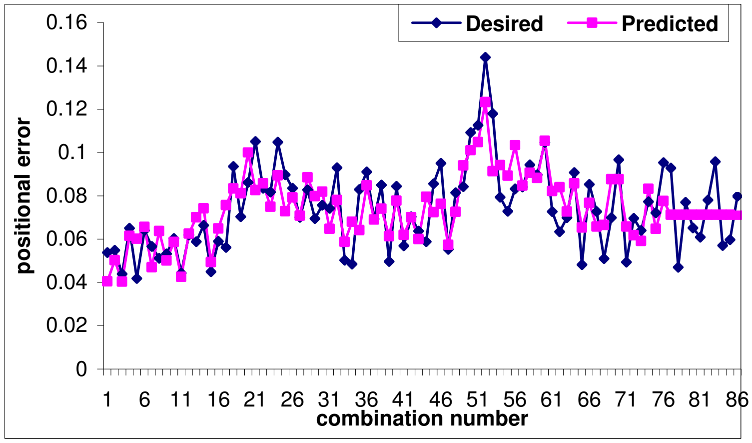


Fig. 5.2 Comparison of Desired and Predicted Performance for Cubic Trajectory - case (ii)

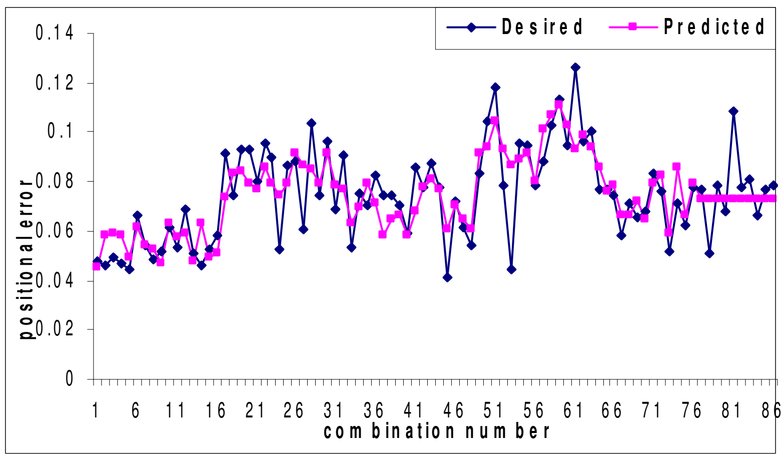


Fig. 5.3 Comparison of Desired and Predicted Performance for Cubic Trajectory - case (iii)

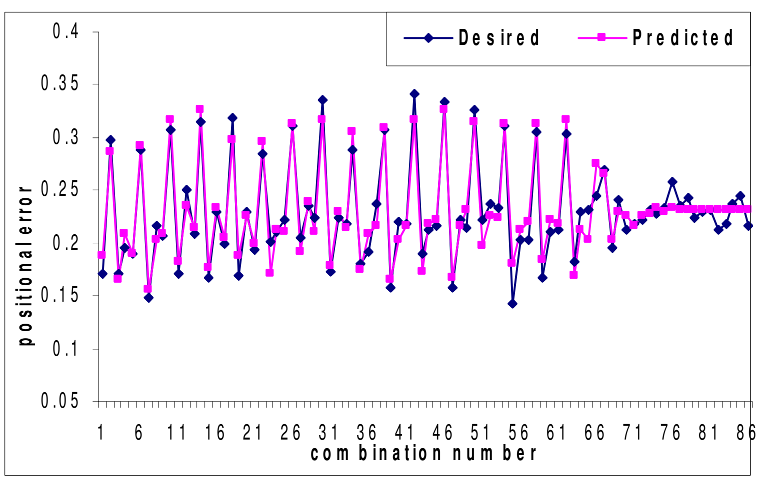


Fig. 5.4 Comparison of Desired and Predicted Performance for Cubic Trajectory - case (iv)

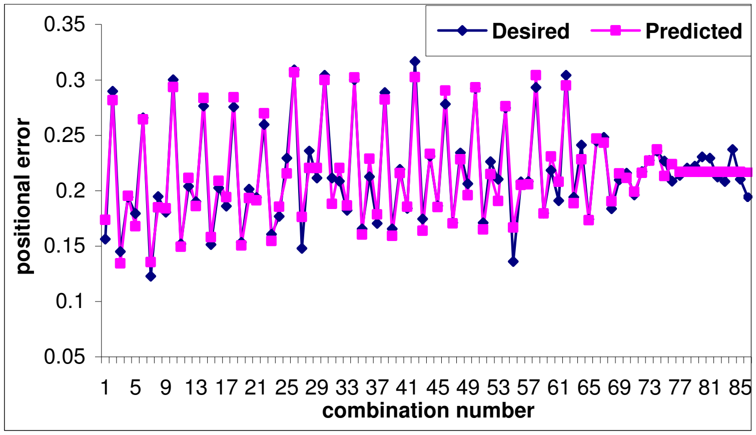


Fig. 5.5 Comparison of Desired and Predicted Performance for Cubic Trajectory - case (v)

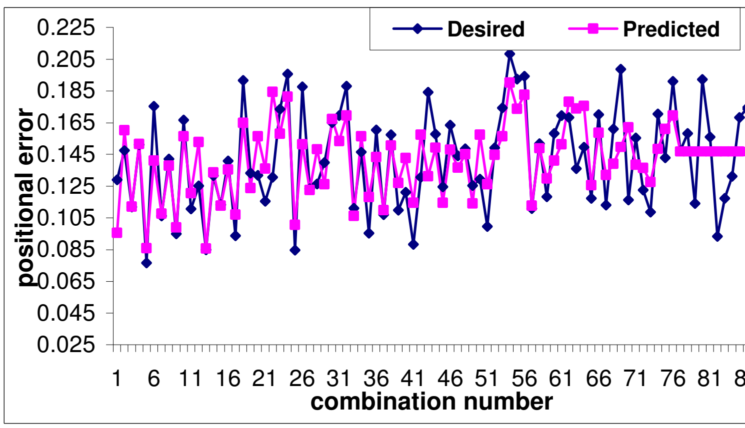


Fig. 5.6 Comparison of Desired and Predicted Performance for Cubic Trajectory - case (vi)

It is observed that from Fig. 5.1 for case (i) following cubic trajectory trend both the performances are close. For the considered 86 combinations performance i.e. positional error vary between 0.04×10^{-2} m to 0.14×10^{-2} m. The predicted value differs from the desired values in few cases i.e. at combination numbers 30, 52 and combinations number from 78 to 86.

From Fig. 5.2 for case (ii) trend of desired performance and predicted performances are found to match closely. Similar to earlier case, considered 86 combinations performance i.e. vary between 0.04×10^{-2} m to 0.14×10^{-2} m. The difference in predicted value differs from the desired values is observed for combination number from 78 to 86.

It is observed that from Fig. 5.3 for the case (iii) trend of desired performance and predicted performances are not that close. The performance for the considered combinations varies between 0.04×10^{-2} m and 0.12×10^{-2} m. Predicted performance of combination numbers 45, 54, 61 and combination number 78 to 86 are different as compared to observed performance. It is observed that from Fig. 5.4 for case (iv) predicted performance trend is matching closely with the desired performance. The range of performance is between 0.12×10^{-2} m and 0.35×10^{-2} m. Predicted performances of combination numbers 78 to 86 are different as compared to observed performance.

Similarly from Fig. 5.5 for case (v) following cubic trajectory, predicted performance trend is observed close to the desired performance. The range of performance for the considered combinations lies between 0.12×10^{-2} m to 0.32×10^{-2} m. It is found that from Fig. 5.6 for case (vi) predicted performance trend is close to the desired performance. The performance for the considered combinations vary between 0.085×10^{-2} m and 0.205×10^{-2} m. Predicted performance of combination number 48 and combination number from 78 to 86 are observed to be different as compared to observed performance. Likewise the comparison between the simulated performance and the predicted performance for the manipulator performance following quintic trajectory are shown in Figs. 5.7-5.12.

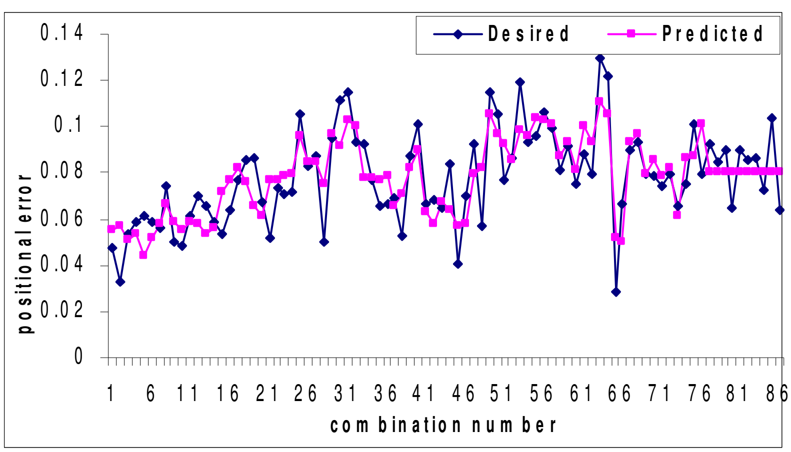


Fig. 5.7 Comparison of Desired and Predicted Performance for Quintic Trajectory - case (i)

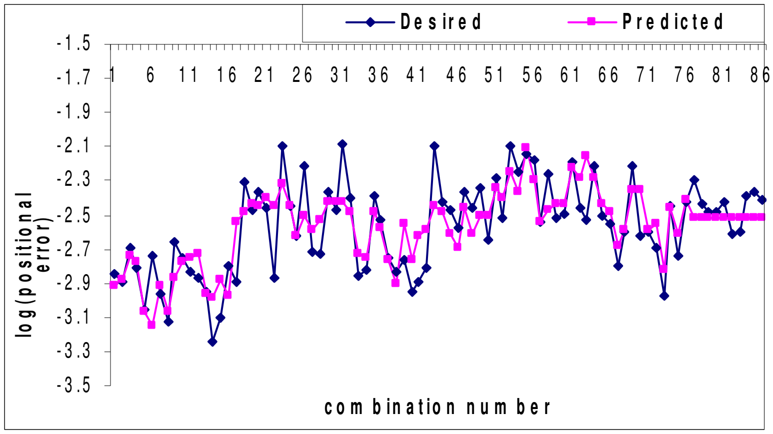


Fig. 5.8 Comparison of Desired and Predicted Performance for Quintic Trajectory - case (ii)

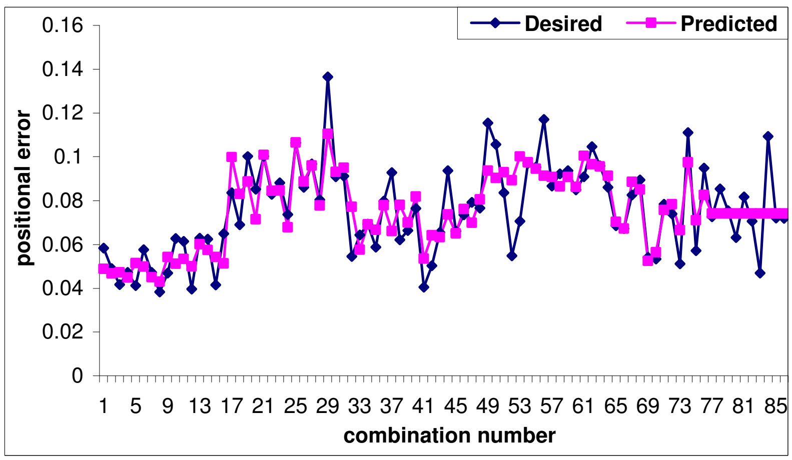


Fig. 5.9 Comparison of Desired and Predicted Performance for Quintic Trajectory - case (iii)

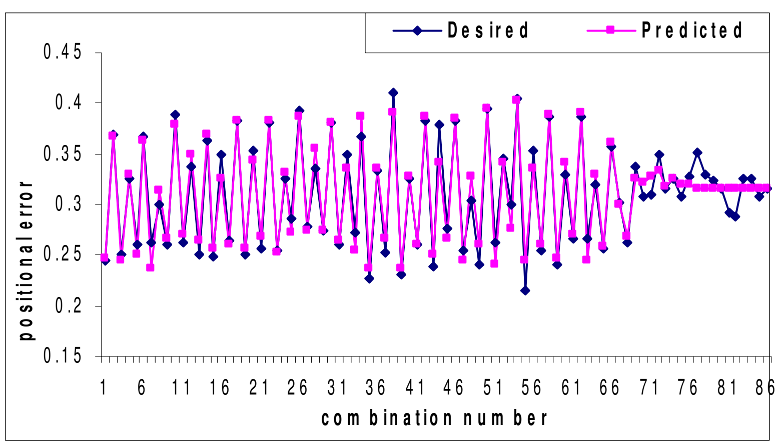


Fig. 5.10 Comparison of Desired and Predicted Performance for Quintic Trajectory - case (iv)

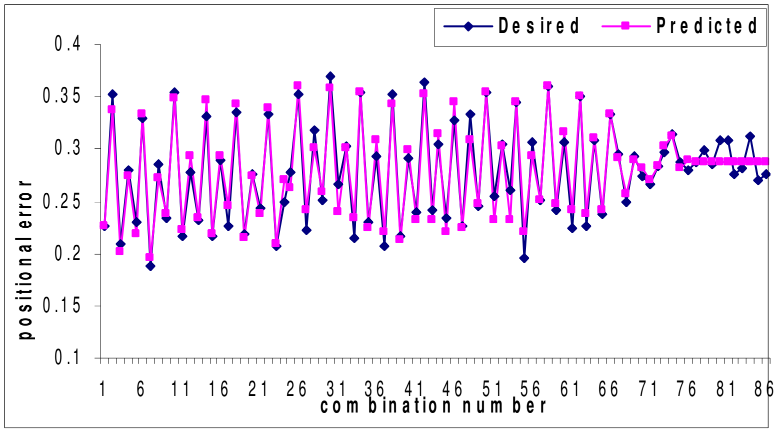


Fig. 5.11 Comparison of Desired and Predicted Performance for Quintic Trajectory - case (v)

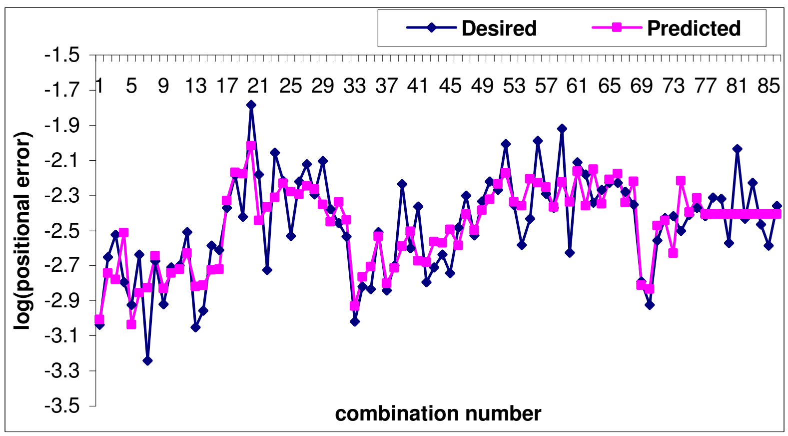


Fig. 5.12 Comparison of Desired and Predicted Performance for Quintic Trajectory - case (vi)

From Fig. 5.7 for case (i) following quintic trajectory predicted performance trend is found close to the desired performance. The ranges of performance for the considered combinations is between 0.02×10^{-2} m and 0.12×10^{-2} m. Predicted performance of combination numbers 63, 66 and combination numbers 78 to 86 are found to be different as compared to observed performance.

It has been observed from Fig. 5.8 for case (ii) predicted performance trend is not matching with the desired performance. The ranges of performance for the considered combinations lie between -3.3×10^{-2} m to -2.1×10^{-2} m. The negative value is due to the logarithmic data transformations method used. Predicted performance of combination numbers 14, 21, 42, and

66 are observed to be different as compared to observed performance. From Fig. 5.9, for the case (iii) the predicted performance trend is not close as compared to the desired performance. The range of performance is observed to vary between 0.04×10^{-2} m to 0.14×10^{-2} m. Predicted performance of combination numbers 29, 49, 56 and combination number from 78 to 86 are found to be different as compared to observed performance. Similarly it has been observed that from Fig. 5.10 for case (iv) predicted performance trend is close to the desired performance. The performance is observed to vary between 0.22×10^{-2} m and 0.4×10^{-2} m. Predicted performance of combination numbers from 78 to 86 are found to be different as compared to observed performance.

From Fig. 5.11 for case (v) predicted performance trend is found close to the desired performance. The performance is observed to vary between 0.18×10^{-2} m to 0.36×10^{-2} m. Predicted performance of combination numbers from 78 to 86 are found to be different as compared to observed performance.

From Fig. 5.12 for case (vi) predicted and desired performance trend is observed to be close. The ranges of performance are observed between -3.243×10^{-2} m and -1.734×10^{-2} m. Predicted performance of combination numbers from 78 to 86 are found to be different as compared to observed performance. In some combinations, the predicted and simulated performances are significantly different, because of the logarithmic transformation of the data.

Using the equations (5.8) and (5.10), discussed in section 5.2 mean of predicted value of positional error and variance of predicted value of positional error, in parametric forms are obtained. The equations of mean and variances of performances in terms of control parameters are provided in tabular form. The mean performance of manipulator for specified task are provided in Table 5.19. The variance in performances equations obtained form the obtained response equation are provided in Table 5.20. The exact evaluation of the expected means and variances are computationally intensive.

Table 5.19 Mean of Response Equation for Positional Error

Case	Manipulator following Cubic Trajectory	Manipulator following Quintic Trajectory
(i)	$\hat{m}(\mathrm{L}n(\varepsilon))$ = –2.512546483 +0.065161198× A – 0.025932539× B +0.005822344× C +0.031699382 × D–0.03006576× A^2 +0.122223174 × B^2 –0.306151256× C^2 –0.034109013× D^2 –0.02006281× A × B +0.002977618× A × C –0.035863789× A × D–0.015271688× B × C –0.020737005× B × D +0.00667443× C × D	$\hat{m}(\varepsilon)$ = 0.079969599 –0.000965167×A +0.001837742 × B +0.003026152× C +0.001367515×D –0.029026198× A^2 +0.015245802×B^2 +0.002862302× C^2+0.000362302×D^2 +0.000402891×A× B +0.001549766×A×C –0.001138922×A×D +0.004504141×B ×C +0.001296828×B× D +0.001585891×C×D
(ii)	$\hat{m}(\mathrm{L}n(\varepsilon))$ = –2.670878006 +0.080960915× A +0.000635467× B +0.005618039× C –0.013805974× D–0.068988817 × A^2 –0.120084018× B^2 +0.180327048× C^2 –0.158212193× D^2 +0.033579562× A × B –0.008088595× A × C –0.003390696×A × D–0.04075648 × B × C –0.027860934× B × D– 0.002566146× C × D	$\hat{m}(\mathrm{L}n(\varepsilon))$ = –2.51846141 –0.022398613× A +0.045377418× B +0.004015225× C +0.01759721× D +0.058313328× A^2 – 0.113583753×B^2 +0.164559707× C^2–0.05666507× D^2 –0.017458882 × A × B –0.029094842× A × C +0.014803006 × A × D –0.007165467 × B × C –0.016435411 × B × D +0.01308027 × C × D
(iii)	$\hat{m}(\varepsilon)$ = 0.072863408 +0.001112348× A +0.000239697 × B –0.003275758× C +0.00193803× D +0.006217083 × A^2 – 0.004647417 × B^2 –0.002639417× C^2 +0.009990583 × D^2 –0.003485109× A × B – 8.64531E–05× A × C +0.000785953× A × D– 0.002142172× B × C –0.000803016× B × D – 0.000787109 × C × D	$\hat{m}(\varepsilon)$ = 0.074069692 –0.001557727× A –0.001711667× B +0.00197353× C +0.001339212×D–0.005317984× A^2 +0.012705016× B^2 –0.019529484×C^2 +0.002947016×D^2 –8.20625E–05×A× B +0.00010675×A×C –0.000253844× A× D –0.001260813×B× C +0.000175344×B× D + 0.000773156×C× D
(iv)	$\hat{m}(\varepsilon)$ = 0.230935654 +0.035209591× A –0.031742758× B – 0.002024318 ×C +0.004844682 × D+0.005389792× A^2 +0.000247292× B^2 –0.005856208× C^2 –0.011986208 × D^2 –0.013884797× A × B +0.001032891× A × C +0.002514547× A × D–0.002931234× B × C –0.000862703× B × D +0.001138922× C × D	$\hat{m}(\varepsilon)$ = 0.31610373 +0.051336227× A –0.015625379× B – 0.00154453× C +0.003027924× D–0.00704706 × A^2 –0.03214706× B^2 +0.00760594×C^2 +0.01477094× D^2 –0.008782641× A × B –0.002024484×A × C –0.001697203× A × D–0.003025453× B × C +0.001658828× B × D–0.001506891× C × D
(v)	$\hat{m}(\varepsilon)$ = 0.216931807 +0.036965561× A –0.026481288× B –0.002010955 × C +0.008427697× D–0.006694514× A^2 – 0.000080513×B^2 –0.003417514× C^2 –0.009373014× D^2 –0.011831719× A × B –0.002917938× A × C +0.000318312× A × D +0.00172275 × B × C +0.001129688× B × D +0.001922531× C × D	$\hat{m}(\varepsilon)$ = 0.288046123 +0.045726424×A –0.016769864× B – 0.003423788 × C +0.006907667× D–0.000955446× A^2 – 0.014351946×B^2 –0.002139446× C^2 – 0.011121446× D^2 –0.009571719× A × B +0.00061225× A × C –0.000264438× A × D +0.000493344× B × C +0.002367969× B × D + 0.0008165× C × D
(vi)	$\hat{m}(\varepsilon)$ = 0.146836455 + 0.016577136×A + 0.003467742× B + 0.00620762× C – 0.000942076× D– 0.00478921× A^2 – 0.01141621× B^2 + 0.00905879 × C^2 – 0.00945621 × D^2 – 0.006279953 × A × B – 0.002401609×A × C –0.001783109× A × D + 0.00133342 × B × C +0.001277797× B × D –0.000884984× C × D	$\hat{m}(\mathrm{L}n(\varepsilon))$ = –2.407953421 +0.017440928×A+0.060318206× B – 0.010189802×C +0.014166405×D+0.214675478 × A^2 +0.127395308×B^2 –0.413937421× C^2 –0.050404465× D^2 + 4.23152E–05×A × B –0.020976831 × A × C –0.044230209×A × D –0.004760119× B × C –0.030053461×B × D +0.010567933 × C × D

Table 5.20 Variance of Response Equation for Positional Error

Case	Manipulator following Cubic Trajectory	Manipulator following Quintic Trajectory
(i)	$\hat{v}(\mathrm{Ln}(\varepsilon))$ = 1/3 ((0.149233 + 0.007038×A + 0.03975×B + 0.0002838×C+0.0523×D)^2 + (0.0664343 – 0.01016653×A+0.03108×B –0.018532×C+0.01069×D)^2) + 1/45(4(–0.0514886)^2+(0.1577668)^2) +5/45(–0.0000858)^2	$\hat{v}(\varepsilon)$ = 1/3((0.012557894 –0.002017641×A – 0.002960766×B + 0.001419172×C + 0.002563109×D)^2+(0.006973955 –0.000511859×A+ 0.000916766×B–0.000374297×C –0.004440109×D)^2) +1/45(4(–0.005912198)^2 + (0.013910802)^2)+5/45(0.000007)^2
(ii)	$\hat{v}(\mathrm{Ln}(\varepsilon))$ = 1/3 ((0.16553– 0.019038×A+0.017432×B – 0.038871×C–0.035971×D)^2 +(0.08483–0.010483×A+0.017383×B+0.0230123×C –0.016857×D)^2) +1/45(4(0.02362)^2+(0.18814)^2)+5/45(–0.01671)^2)	$\hat{v}(\mathrm{Ln}(\varepsilon))$ = 1/3((0.177309599– 0.018420995×B+0.072536145×C –0.033957317×D + 0.007450772×A)^2 + (0.098502754–0.015264091×A + 0.017118301 × B+0.026884528× C+0.013205569× D)^2) + 1/45(4×(–0.123746408)^2+(0.002345381)^2) + 5/45(–0.039225543)^2
(iii)	$\hat{v}(\varepsilon)$ = 1/3((0.013493–0.000913×A –0.0007607× B–0.000087×C +0.000989×D)^2 + (0.0067167–0.001596×A+0.0006092×B –0.0020114C+0.000915×D)^2) + 1/45(4(–0.007882)^2+(0.000992)^2)+5/45(– 0.000885)^2	$\hat{v}(\varepsilon)$ = 1/3((0.015476 – 0.003726×A – 0.002436×B – 0.000456×C + 0.0002881×D)^2 +(0.005729 + 0.003413×A + 0.002648×B + 0.001428×C –0.0023602×D)^2) + 1/45(4(0.00794)^2+(–0.00189)^2)+5/45(–0.00189)^2
(iv)	$\hat{v}(\varepsilon)$ = 1/3 ((0.003171–0.001463×A+0.0016501×B –0.0018825×C–0.003359×D)^2+ (0.002311 –0.002224×A–0.0041976×B–0.000219×C –0.0045013×D)^2) +1/45(4(–0.002237)^2+(0.012912)^2)+5/45(–0.001487)^2	$\hat{v}(\varepsilon)$ = 1/3((0.004053+0.0003772×A – 0.0006317×B+0.00094×C –0.001763×D)^2+(0.000151+0.003242×A –0.003678×B+0.002×C–0.0027905×D)^2) + 1/45(4(0.00547)^2+(0.003586)^2)+5/45(–0.000902)^2
(v)	$\hat{v}(\varepsilon)$ = 1/3((0.005195–0.004549×A –0.0011484×B+0.000698×C+0.002719×D)^2 + (0.0052434+0.0018116×A+0.0032261×B –.000567×C –0.002869×D)^2) + 1/45(4(0.015237)^2+(0.0018116)^2) +5/45(– 0.001438)^2	$\hat{v}(\varepsilon)$ = 1/3 ((0.004925–0.003252×A+0.002598×B – 0.000013×C+0.0017488×D)^2 +(0.003976–0.00362×A–0.004109×B–0.0015491×C – 0.0003147×D)^2) +1/45(4(0.0197161)^2+(–0.0033184)^2) +5/45(– 0.0006768)^2
(vi)	$\hat{v}(\varepsilon)$ = 1/3 ((0.01046 –0.00169×A+0.0000846×B+0.00968×C– 0.002389×D)^2 + (–0.04181–0.00362×A –0.001176×B+0.00333×C+0.001191×D)^2) + 1/45{4(–0.00894)^2+(0.01845)^2}+5/45(–0.000445)^2	$\hat{v}(\mathrm{Ln}(\varepsilon))$ = 1/3((0.20691–0.026557×A–0.019864×B –0.02142×C – 0.0321875×D)^2 + (0.041381–0.024203×A–0.0007381×B+0.040649×C –0.020767×D)^2)+ 1/45(4(–0.0155413)^2+ (0.0152209)^2)+5/45(–0.01668)^2

To evaluate maximum and minimum value of mean and variances of performance, equations provided in the Tables 5.9 and 5.10 are used. For computation, region of interest is taken as $R_x = \{(A, B, C, D \mid -1 \le x_i \le 1,\ i = 1,2,3,4\}$. To evaluate these equations a computer programme is developed. The obtained results are provided in tabular form. For different cases the values are provided in Tables 5.21 and 5.22 respectively. The Table 5.21 provides the mean and variances of performances of manipulator following cubic trajectory.

Table 5.21 Maximum and Minimum Values of Mean and Variance from Predicted Model

Case	**Manipulator following Cubic Trajectory**	
	Mean $\min_{x\in R_x} \hat{m}(x) \le \hat{m}(x) \le \max_{x\in R_x} \hat{m}(x)$	Variance $\min_{x\in R_x} \hat{v}(x) \le \hat{v}(x) \le \max_{x\in R_x} \hat{v}(x)$
(i)	$0.046904 \le \hat{m}(x) \le 0.0976106$	$0.000362 \le \hat{v}(x) \le 0.025826$
(ii)	$0.0417258 \le \hat{m}(x) \le 0.084078$	$0.0003175 \le \hat{v}(x) \le 0.0030198$
(iii)	$0.063164 \le \hat{m}(x) \le 0.094417$	$0.002991 \le \hat{v}(x) \le 0.00471$
(iv)	$0.162317 \le \hat{m}(x) \le 0.313646$	$0.00366 \le \hat{v}(x) \le 0.0087$
(v)	$0.13022 \le \hat{m}(x) \le 0.288669$	$0.1018 \le \hat{v}(x) \le 0.93$
(vi)	$0.09509 \le \hat{m}(x) \le 0.0171489$	$0.002814 \le \hat{v}(x) \le 0.000887$

Where as Table 5.22 provides the mean and variances of performance of manipulator following quintic trajectory.

Table 5.22 Maximum and Minimum Values of Mean and Variance from Predicted Model

Case	**Manipulator following Quintic Trajectory**	
	Mean $\min_{x\in R_x} \hat{m}(x) \le \hat{m}(x) \le \max_{x\in R_x} \hat{m}(x)$	Variance $\min_{x\in R_x} \hat{v}(x) \le \hat{v}(x) \le \max_{x\in R_x} \hat{v}(x)$
(i)	$0.047595 \le \hat{m}(x) \le 0.112058$	$0.0789 \le \hat{v}(x) \le 0.136$
(ii)	$0.0688121 \le \hat{m}(x) \le 0.10644$	$0.05187 \le \hat{v}(x) \le 0.035621$
(iii)	$0.03977 \le \hat{m}(x) \le 0.09259$	$0.000179 \le \hat{v}(x) \le 0.00061$
(iv)	$0.223981 \le \hat{m}(x) \le 0.2187$	$0.0007 \le \hat{v}(x) \le 0.00005$
(v)	$0.27013 \le \hat{m}(x) \le 0.192654$	$0.00036 \le \hat{v}(x) \le 0.00109$
(vi)	$0.0383091 \le \hat{m}(x) \le 0.13701$	$0.03218 \le \hat{v}(x) \le 0.004305$

It has been observed that for different tasks the response equations are different, therefore optimal solution for a particular task may not deliver the desired performance. The optimal solutions obtained from emperical models of different tasks and following different tarjectories have been provided in tabular form. The novelity of this approach is once the designer is able to model the performance of manipulator against the different design and noise parameters, it becomes easier to search for the optimal parameter which will deliver performance insensitive of noise parameters. Therefore expectation of the performance i.e. mean performance, is considered to be relatively important than variance. As such the objective of the experiment is to minimize the performance while achieving an acceptable amount of variability $\sigma^2_{\min}$ in performance. Hence the optimization problem is formulated as

$$\min E(\hat{y}) \qquad \text{such that } V(\hat{y}) = \sigma^2_{\min} \tag{5.11}$$

Since focus of the thesis is the design aspect of manipulator meant for different tasks following different trajectories, the optimal parameter combination which will deliver minimum performance with minimum variance is searched for. The corresponding models for each case have been used for optimization. To solve above constrained optimization problem *fmincon* routine available in MATLAB has been used. The optimal parameter values are provided in Tables 5.23 and 5.24 respectively. The results provided in Table 5.23 represent optimal control parameter values of manipulator performing task following cubic trajectory.

Table 5.23 Optimal Solutions from Fitted Response Model

Case	**Manipulator following Cubic Trajectory**					
	A	**B**	**C**	**D**	**Minimum value of positional Error**	**Minimum Value of Variance**
(i)	0.4	0.365	7	5	–2.60217*	0.001725*
(ii)	0.4	0.301	7.4	6	–2.63694*	0.003175*
(iii)	0.467	0.40	7.94	5.4	0.073017	0.000047
(iv)	0.403	0.40	8	6	0.162317	0.000005
(v)	0.40	0.392	8	5.02	0.15706	0.000027
(vi)	0.40	0.30	7.35	5.98	0.09618	0.000492

*** Logarithmic transformation**

Similarly Table 5.24 represents the optimal control parameter values of manipulator performing task following quintic trajectory. These tables provide corresponding minimum mean performance and variance at the optimum value of control factors.

Table 5.24 Optimal Solutions from Fitted Response Model

Case	Manipulator following Quintic Trajectory					
	A	**B**	**C**	**D**	**Minimum Value of Positional Error**	**Minimum Value of Variance**
(i)	0.5	0.3652	7.22	5.22	0.06155	0.000036
(ii)	0.4373	0.30	7.38	6	0.05155	0.00523
(iii)	0.4684	0.3332	7	5.89	0.06045	0.000062
(iv)	0.4	0.4	7.58	5.46	0.23361	0.00001
(v)	0.4	0.3911	8	5.04	0.19265	0.000037
(vi)	0.4727	0.3606	8	6	–2.3454*	0.004305*

*** Logarithmic transformation**

Using ANOVA techniques, the statistical significance of individual effects, quadratic effects of parameters and interaction effect of parameters have been investigated. It is observed that for the considered cases while following cubic trajectory, link length one, clearances present in joint one and two have significant impact on performance variations. Except two cases, quadratic effect of parameters are observed to be insignificant. This indicate that the parameters have small nonlinear effect on the performance variations.

In analysis of performance for the task following quintic trajectory it is observed that clearance in joint1 and joint 2 factors are statistically significant in five out of six cases. For cases (i), (iii), (iv), (v) and (vi) nonlinear effect of link length one, link mass one, link length two and clearance in joint 2 are observed to be statistically significant respectively. Apart from control factors and few two factor interactions effects are also observed to be statistically significant.

To represent the ability of the developed emperical model to predict the performance of manipulator help of R^2 is taken. It has been observed that R^2 values are close to 0.90 for cases (iii), (iv) and (v) following cubic and quintic trajectories, which is ideal for any emperical model. The R^2 values are found to be small in cases (i), (ii) and (vi) following cubic and quintic trajectories. This small value indicate that the emperical model is unable to fit the total variability present in responses. In addition to R^2 values of emperical model comparison between the desired and predicted values of performances are provided in figures. In most of cases the predicted values are closer to the desired value and trends are similar. Subsequently developed response equations for mean performance and variances in performance are used for optimization. For selection of the optimal parameter the problems are formulated as equation (5.8). It is desired that mean performance should be minimum and the variance in performance should take a predecided minimum value. Taking these conditions, optimization routine has been run and the results are provided in tables. It is observed that for different cases optimal parameters are different. Mostly manipulators perform repetitive tasks, therefore for robust design of manipulator above procedure can adopted to obtain the optimal parameter setting. This setting will provide the desired performance with minimum variability.

5.7 EPILOGUE

The present chapter discusses how optimal parameters of manipulator has been selected that will have minimum performance variations. A probabilistic method has been used to simulate the performance of the manipulator. Experimental design techniques are used to run experiments required for developing empirical models of the performance. The final model to predict the performance of manipulator has been obtained using regression analysis procedure. A response surface approach has been employed to minimize the performance variation while keeping its performance at minimum value. Solution to this optimization problem is expected to produce a reasonable improvement in performance of manipulator. Using the RSM approach, the design parameters of manipulator are allocated optimally by the approximated function. The suggestions for possible design improvement are obtained by referring to the results from the statistical analysis. This approach enables designers to have immediate feedback suggestions for design improvement. With the presented mathematical and statistical parameter design model, it is possible that a high quality and cost-effective manipulator design can be achieved during the early stages of design.

CHAPTER-6

MANIPULATOR PARAMETER TOLERANCE DESIGN USING CROSS ARRAY DESIGN OF EXPERIMENTS APPROACH

6.1 INTRODUCTION

The next step to parameter design is tolerance design. In tolerance design, suitable tolerances on control factors are selected. The chapter discusses how design of experiment approach can be used to select the optimal control parameter tolerances to reduce performance variations. In Chapter 5, optimal control parameter values were obtained to give very low variations. In this chapter, an attempt has been made to further reduce the performance variations. As discussed in Chapter 2 Taguchi's off-line quality control for product design method has three steps. These steps are system design, parameter design and tolerance design. In tolerance design, the quality of performance variation is reduced further as compared to whatever achieved in parameter design.

Tolerance design identifies the quality sensitive parameters and applies tolerances to these parameters to meet the required level of variation of the output. In tolerance design, a trade off is usually made between reduction in quality variation and increase in manufacturing cost. That is selective specification of higher-grade parts, materials or components to reduce tolerances in order of their cost effectiveness. For selection of optimal tolerances, use of design of experiments technique becomes pertinent. Two conflicting criteria exist for the optimization of tolerances in mechanical systems: manufacturing cost and performance variations. Tightening tolerances decreases the performance variations, but increase the manufacturing cost and vice versa. So as to achieve tighter tolerances, a lot of manufacturing effort is spent in terms of machine tool, process and skill.

Variations in both processes and materials, such as, setup errors, material property variations, environment, and so on, mean that parts cannot be produced to desired dimensions. The dimensional variations that occur, displace parts from their required dimension, location, and geometry, and negatively affect the assembly and performance of the product. The cost of production is exponentially proportional to the magnitude of the tolerance – smaller is the tolerance higher is the cost. If optimal tolerances are determined to ensure assembly and function, the assembly will be acceptable and cost will be minimal. The study of the aggregate behavior of a series of individual factor variations on the tolerances is referred as “tolerance analysis”. In tolerance analysis, the cumulative effect of individual dimensional variations on the overall assembly is studied; the resultant assembly and function of the product are verified and checked against the design requirements.

This chapter discusses offline simulation strategy by which real time performance is obtained without using a prototype and selection of optimum tolerance of robot parameters using cross array experimentation. By using cross array experimental strategy, the significant parameters and their interactions are identified and the performance is expressed as a function of tolerance. The results of the cross array experimentation are validated by Monte Carlo simulation and parametric tolerance sensitivity has been carried out to complement the statistical analysis. To illustrate the application 2-DOF RR planar manipulator has been used.

The rest of this chapter is organized in seven sections. In section 6.2, tolerance design of manipulator is discussed. Cross array, design of experiment approach for tolerance design is presented in section 6.3. The steps utilized for cross array design of experiment approach are discussed in section 6.4. The application of cross array design of experiment approach to tolerance design of 2-DOF RR planar manipulator is discussed in section 6.5. The results of parametric tolerance sensitivity on performance variations are presented in section 6.6.

6.2 TOLERANCE DESIGN OF MANIPULATOR

Industrial robots have poor accuracy and repeatability for various applications because of inaccuracies present in geometrical, kinematic and dynamic parameters. Few researchers have investigated the effect of different parameters on performance variations of robot but, exploration and analysis of effect of parameter tolerances and their interactions on performance variations are rare. This investigation is carried out without imposing any constraints such as maximum torque, maximum angular velocity and angular acceleration of links.

Conducting experiments on actual manipulator by varying tolerance of parameters and finding optimum values of tolerances for optimal performance is very tedious, time consuming and uneconomical though not impossible. In this chapter a simulation approach has been adopted to conduct the experiments. A cross array experimentation strategy is employed to investigate the effect of tolerances of control factors systematically that contain the manipulator's parameters at their different nominal values. To investigate the effect of noises due to kinematic and dynamic parameters tolerance on performance maximum positive and negative variations from nominal values are considered. After application of experimental design technique a performance measure signal to noise ratio (SN ratio) proposed by Taguchi [Park 1998] has been used. The response of the experiments has been analyzed using the analysis of variance (ANOVA) technique, and the manipulator parameters that contribute the most to the observed performance variations has been identified.

6.3 CROSS ARRAY DESIGN OF EXPERIMENTS APPROACH FOR TOLERANCE DESIGN

Design of experiment (DOE) technique has been used traditionally to design an experiment, analyze data and optimize processes or products. In this chapter, effect of control factor tolerances on performance of manipulator has been investigated. To obtain optimal tolerance of control factors that is insensitive to noise due to control factor variations, cross array design of experiments approach is utilized. Using this approach effect of systematic variations of control parameters on performance is studied. The procedure of cross array experimental design approach is similar to factorial design of experiment approach discussed in Chapter 3. The steps used in this study for selection of optimum tolerance of manipulator parameters, are discussed below.

6.3.1 Identification of Control and Noise Factors

The control factors and noise factors were identified in Chapter 3 and their effects have been studied extensively in Chapter 4. To study the effect of control factor tolerance on performance variations, the noise factors, which are considered in this investigation, are enumerated below.

(a) Manufacturing tolerances, errors in manufacture and assembly leading to geometric and inertial parameter variations in link length and mass.

(b) Variation in supplied joint torques at joints.

To incorporate these noise factors in experiments a novel approach has been proposed and is explained in next section.

6.3.2 Design the Experiment

For the tolerance design, the factors studied are the tolerance on kinematic and dynamic parameters, which are under control of the designer. The random deviation from nominal value of each kinematic and dynamic parameter within a given tolerance can be treated as an independent variable. This deviation is treated as an error due to noise. To capture the effect of this type of noise the experiments should be replicated large number of times as it is done in Monte Carlo simulation procedure. This approach is very computationally intensive. Therefore, cross array experimental strategy is adopted in which a limited set of discrete values of noise are considered to capture the effect of noise. But in Monte Carlo simulation effect of noise is studied randomly over the distribution space.

For designing experiment, the factors and their levels need to be determined. In experimental design, two levels (low and high) are recommended, to reduce the number of experiments. Sometimes three levels (low, medium and high) are also used, when nonlinear effects need to be investigated. In this case, it is assumed that there are no nonlinear effects and hence, only two levels are considered.

In the crossed array, experimental design approach control factor array and noise factor array are used to study the effect of noise factors on control factors. The control factor array is used to determine the significance of control factors, and to select the levels of significant factors to optimize the performance measure, while noise factor array is used to introduce the effect of noise into the experiment in a systematic manner. Thus, the results of crossed array experiment would be more “robust” against the noise of control factors. To incorporate the effect of noises, noises are taken as the deviations from a nominal value, corresponding to the specified tolerance value. These represent “worst case” tolerance deviations and satisfy the 3-sigma limits of Gaussian variability. Each noise factor array is a noise combination that is treated as repetitive data in the control factor array. For each experimental run in the control factor array consisting of specific tolerance for each kinematic and dynamic parameter, the noise factor array provides noise for each kinematic and dynamic parameter. Thus, if the noise factor array has O rows for each run of the control factor array with I rows, the size of the experiment becomes $I \times O$.

6.3.3 Performance Measure – The SN ratio

The performance measure used for the study and analysis of manipulator are positional error and SN ratio. These performance measures are already defined in Chapter 3. As per, Clausing [Clausing 1989] SN ratio is a good performance measure and results found using SN ratio will be the "Best Set" of kinematic and dynamic parameter tolerances to get minimal performance variations.

6.4 STEPS TO TOLERANCE DESIGN

The key steps to obtain the optimal control parameter tolerances through use of cross array design of experiments approach is outlined below. These are similar to design of experiment technique except few exceptions.

Step 1. Identify the number of control factors, (manipulator kinematic and dynamic parameters)

Step 2. Select the nominal values and levels of control factors (tolerance level values of manipulator kinematic and dynamic parameters)

Step 3. Design the control factor array.

Step 4. Design the noise factor array.

Step 5. For each combination of control factor tolerances, simulate the response i.e. position vector (x_f, y_f) reached, for cross array experiment.

Step 6. Compute the positional error ε_i

Step 7. Calculate the SN ratio.

Step 8. Test statistical significance.

Step 9. Find the optimum set of parameter tolerances.

The methodology adopted to simulate the manipulator performance is described next.

6.4.1 Simulation of Performance for Tolerance Design of Manipulator

To simulate the performance of manipulator probabilistic approach discussed in Chapter 4 is used. The control parameters of manipulator identified in Chapter 3, i.e. link lengths and link masses, are considered to be independent random variables following Gaussian distribution, with assumed mean value μ and standard deviation σ, and the torque vector is assumed AR stochastic process.

To determine the torque vector required at the joints of the manipulator without effect of noise, following approach is used. Based on the trajectory chosen to perform the task, torque vector required at joints are determined. Therefore, influence of torque profile on performance variations of manipulator are also investigated. In this chapter two trajectories i.e. cubic and quintic are studied to investigate the influence of time law and end conditions on performance variations of manipulator.

For the tolerance design, tolerance limits are specified by taking $\pm C$ multiples of standard deviation σ around the mean size μ. It is customary to take ± 3 i.e. six sigma, spread for control parameters, by which the upper and lower tolerance limits become $\mu + 3\sigma$ and $\mu - 3\sigma$, respectively.

6.5 SIMULATION OF PERFORMANCE FOR 2-DOF RR PLANAR MANIPULATOR

In Chapter 4 and 5 manipulator performances are simulated based on the design factor and noise factor changes. In this chapter effect of control factor tolerances on performance variations is investigated. The control parameters for the 2-DOF RR planar manipulator are same as already identified. To implement above discussed approach computer programme is developed using MATLAB commands. Subsequently numerical values are assumed to simulate the performance.

Design Parameters

To investigate the effect of manipulator tolerances, previously identified six control factors have been chosen at two level values. Thus, the level values of tolerance are set either at Loose (*L*) i.e. original value or Tight (*T*) i.e. half-the-original value. The assumed values of nominal parameter and parameter tolerances, which are at two levels for six control factors are given in Table 6.1.

Table 6.1 Robot Parameter Nominal Values and Tolerance Level Values

Parameter	Symbols Used for ANOVA	Nominal Parameter Values	Tight Tolerance (*T*)	Loose Tolerance (*L*)
l_1 (m)	A	0.45	$\pm 15\times10^{-5}$	$\pm 3\times10^{-4}$
l_2 (m)	B	0.30	$\pm 15\times10^{-5}$	$\pm 3\times10^{-4}$
m_1 (kg)	C	6	$\pm 75\times10^{-4}$	$\pm 15\times10^{-3}$
m_2 (kg)	D	4.5	$\pm 75\times10^{-4}$	$\pm 15\times10^{-3}$
τ_1 (Nm)	E	variable	$\pm 7.5\times10^{-2}$	$\pm 15\times10^{-2}$
τ_2 (Nm)	F	variable	$\pm 7.5\times10^{-2}$	$\pm 15\times10^{-2}$

Process Parameters

The numerical values assumed for the process parameters are same as used in Chapter 4. To maintain consistency of study, Cartesian coordinate of the task indicating the start point, destination point and time to reach the destination are kept same as provided in Table 4.3. Weight parameter for first order autoregressive process for torque simulation is taken as $\phi_1 = 0.8$. The time step used for numerical integration is assumed as 0.001s.

6.5.1 Control and Noise Factor Arrays

The six control factors at two level values give 2^6=64 possible combinations of parameter tolerances of the control factor array. All 64 combinations have been considered to avoid any alias structure that may be present in experimentations, if lesser number combinations are used. Few combinations of control factor array are shown in Table 6.2. For all the 64 combinations of control factor array Appendix D1 can be referred. The combinations of control factor array are generated in such a way that increase in combination number indicate increasing order of tightness of parameter tolerances.

Table 6.2 Control Factor Array in terms of Tolerances for Tolerance Design

Combination number	$3\sigma_{l_1}$ (m) ($\times10^{-2}$)	$3\sigma_{l_2}$ (m) ($\times10^{-2}$)	$3\sigma_{m_1}$ (kg)	$3\sigma_{m_2}$ (kg)	$3\sigma_{\tau_1}$ (Nm) ($\times10^{-2}$)	$3\sigma_{\tau_2}$ (Nm) ($\times10^{-2}$)
1	0.03	0.03	0.015	0.015	15	15
2	0.03	0.03	0.015	0.015	15	7.5
3	0.03	0.03	0.015	0.015	7.5	15
4	0.03	0.03	0.015	0.015	7.5	7.5
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
61	0.015	0.015	0.0075	0.0075	15	15
62	0.015	0.015	0.0075	0.0075	15	7.5
63	0.015	0.015	0.0075	0.0075	7.5	15
64	0.015	0.015	0.0075	0.0075	7.5	7.5

For this study six control factors are studied, therefore to incorporate the effect of noise due to control factors systematically, help of orthogonal array proposed by Taguchi has been taken. There are many orthogonal arrays proposed by Taguchi. For noise factor array L_8 OA is chosen, which has 7 columns and 8 rows. The reason behind L_8 OA, selection is the number of control factors for which effect of noise required to be studied. The selected OA is presented in Table 6.3, with 1's for low level and 2's for high level to account for the random variations about the nominal values. In the noise factor array only first six columns of Table 6.3 are used for noise on parameters: length and mass of the two links, and torque at joint one and two respectively.

Table 6.3 Noise Factor Array (L_8 Taguchi's Orthogonal Array)

Experiment Number	Column Number						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

Using Taguchi's L_8 array the noise factor array for cross array experimentation has been developed. For example, for combination number 64 of Table 6.2, i.e. Tight (T) tolerances, and based on values in Table 6.3, the noise factor array will be as shown in Table 6.4. Similarly, for each tolerance combination in control factor array, noise factor array is developed to simulate the outcome of the experiment.

Table 6.4 Noise Factor Array for Combination Number 64

Experiment Number	**Column Number**					
	1	2	3	4	5	6
1	$l_1 - T$	$l_2 - T$	$m_1 - T$	$m_2 - T$	$\tau_1 - T$	$\tau_2 - T$
2	$l_1 - T$	$l_2 - T$	$m_1 - T$	$m_2 + T$	$\tau_1 + T$	$\tau_2 + T$
3	$l_1 - T$	$l_2 + T$	$m_1 + T$	$m_2 - T$	$\tau_1 - T$	$\tau_2 + T$
4	$l_1 - T$	$l_2 + T$	$m_1 + T$	$m_2 + T$	$\tau_1 + T$	$\tau_2 - T$
5	$l_1 + T$	$l_2 - T$	$m_1 + T$	$m_2 - T$	$\tau_1 + T$	$\tau_2 - T$
6	$l_1 + T$	$l_2 - T$	$m_1 + T$	$m_1 + T$	$\tau_1 - T$	$\tau_2 + T$
7	$l_1 + T$	$l_2 + T$	$m_1 - T$	$m_2 - T$	$\tau_1 + T$	$\tau_2 + T$
8	$l_1 + T$	$l_2 + T$	$m_1 - T$	$m_2 + T$	$\tau_1 - T$	$\tau_2 - T$

6.5.2 Cross Array Experimentation

The torque vector required at joints of manipulator is computed using nominal values of control factors and assumed process parameters, by inverse dynamics process. For a given tolerance of torque vector in control factor array and the desired direction of deviation in noise factor array, the torque vector actually available at joint has been modeled. For a given combination of control factor array and noise factor array torque vector are simulated and used in dynamic model equation of manipulator provided in Chapter 4 i.e. equations (4.3) and (4.4). Subsequently these equations are integrated numerically for time duration of 2 seconds with a time step of 0.001 seconds. The simulation is conducted, to get outcome of experiment i.e. joint coordinates. The obtained joint coordinates are used in kinematic model equation of manipulator to get the Cartesian coordinates of the point actually reached by end-effector. The positional error is computed using equation (3.17) and the simulation for same tolerance combination is run for 8 times and SN ratio is computed using equation (3.20). For all 64 combinations of tolerances shown in Table 6.2, simulations are carried out for a task following cubic trajectory and SN ratios are obtained.

(i) Analysis of Performance using SN ratio

To investigate influence of different task on performance, simulations are carried out and results are plotted in Figs. 6.1 to 6.6. Important features of these graphs are described below.

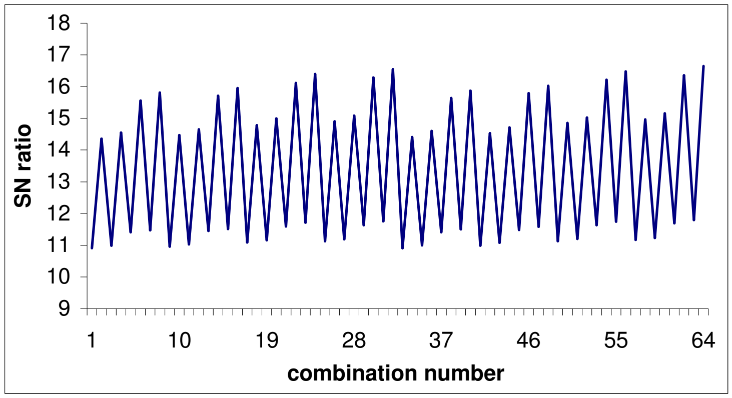


Fig. 6.1 SN ratio from Cross Array Experiments for Cubic Trajectory - case (i)

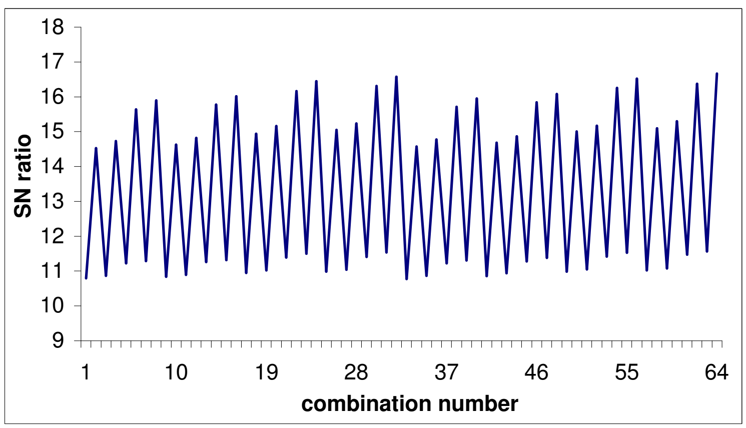


Fig. 6.2 SN ratio from Cross Array Experiments for Cubic Trajectory - case (ii)

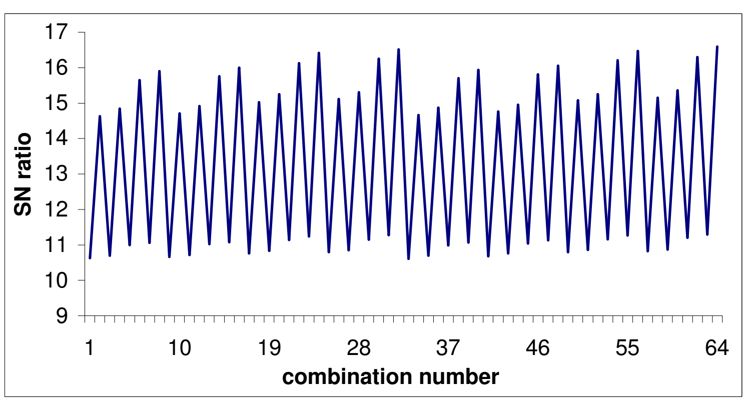


Fig. 6.3 SN ratio from Cross Array Experiments for Cubic Trajectory - case (iii)

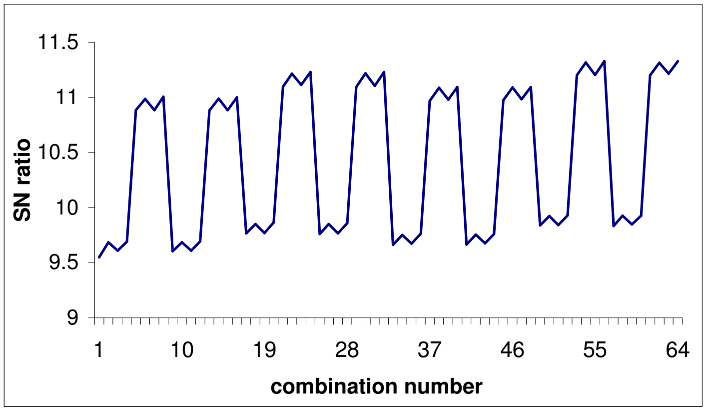


Fig. 6.4 SN ratio from Cross Array Experiments for Cubic Trajectory - case (iv)

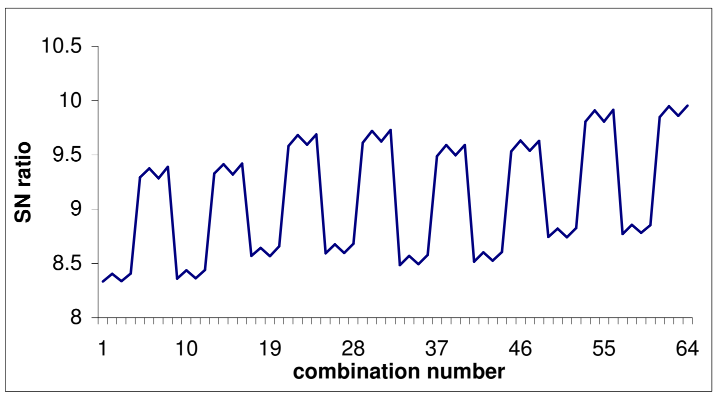


Fig. 6.5 SN ratio from Cross Array Experiments for Cubic Trajectory - case (v)

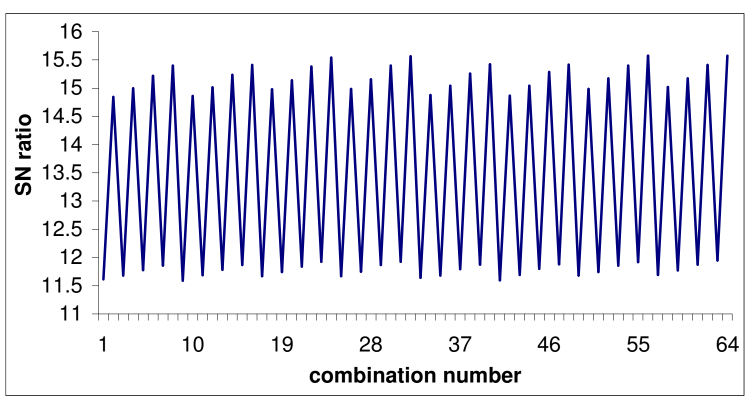


Fig. 6.6 SN ratio from Cross Array Experiments for Cubic Trajectory - case (vi)

From Fig. 6.1, for case (i) it is observed that the SN ratio values fluctuate alternately and maximum SN ratio value is observed to be 16.66 dB at combination number 64 and minimum SN ratio is 10.79 dB at combination number 1. In Fig. 6.2 it is observed that for case (ii) following cubic trajectory, similar to above case SN ratio values fluctuate alternately. And maximum SN ratio value is observed to be 16.66 dB at combination number 64 and minimum SN ratio is 10.79 dB at combination number 1. It can be seen from Fig. 6.3 that case (iii) has similar trend and feature as compared to cases (i) and (ii). Maximum SN ratio value is observed to be 16.56 dB at combination number 64 and minimum SN ratio is 10.62 dB at combination number 1. From Fig. 6.4, for case (iv) trend is observed to be different as compared to previous three cases. But SN ratio fluctuates alternately. Maximum SN ratio value is observed to be 11.32 dB at combination number 52, 54, 64 and minimum SN ratio is 9.55 dB at combination number 1 and 10. Referring to Fig. 6.5 for case (v) it is observed trend is similar to case (iv). And maximum SN ratio value is observed to be 9.95 dB at combination number 64 and minimum SN ratio is 8.33 dB at combination number 1. In Fig. 6.6 for case (vi) SN ratio values observed to fluctuate alternately. The maximum SN ratio value is observed to be 15.56 dB at combination number 64 and minimum SN ratio is 11.61 dB at combination number 1. One important feature of all these figures is the availability of number of equally competing solutions.

Likewise for different tasks following quintic trajectory, SN ratios obtained from simulation are represented with the help of Figs. 6.7-6.12. Important features of all these graphs are discussed below.

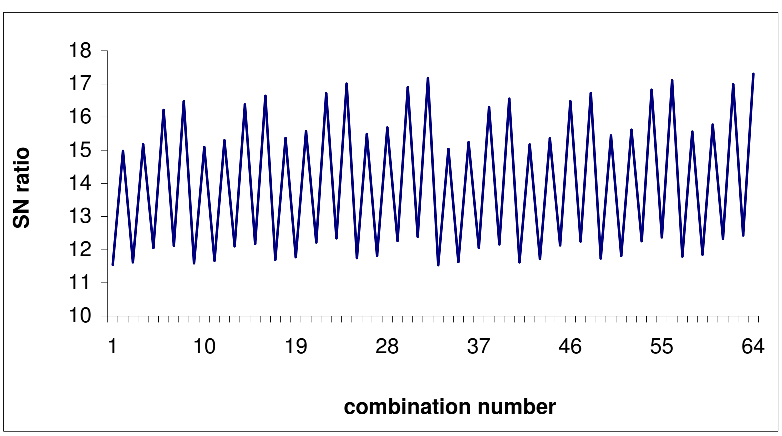


Fig. 6.7 SN ratio from Cross Array Experiments for Quintic Trajectory - case (i)

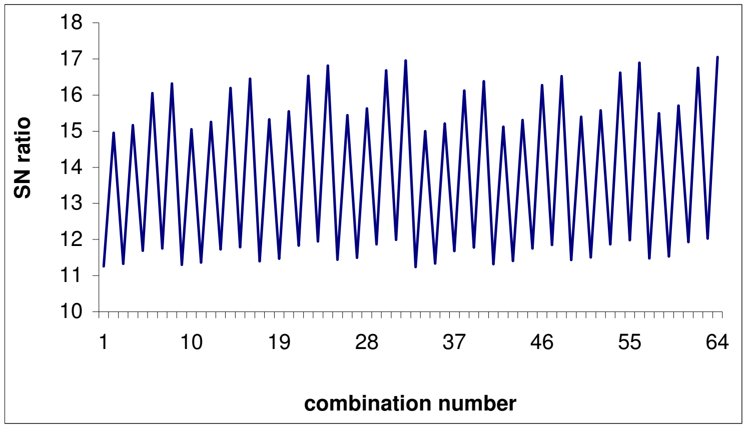


Fig. 6.8 SN ratio from Cross Array Experiments for Quintic Trajectory - case (ii)

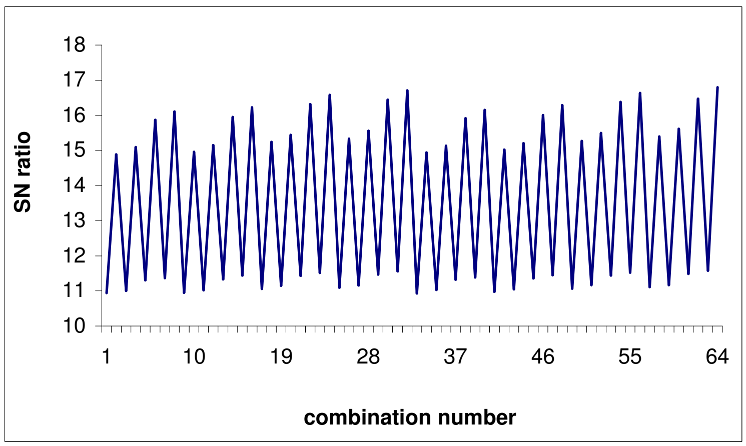


Fig. 6.9 SN ratio from Cross Array Experiments for Quintic Trajectory - case (iii)

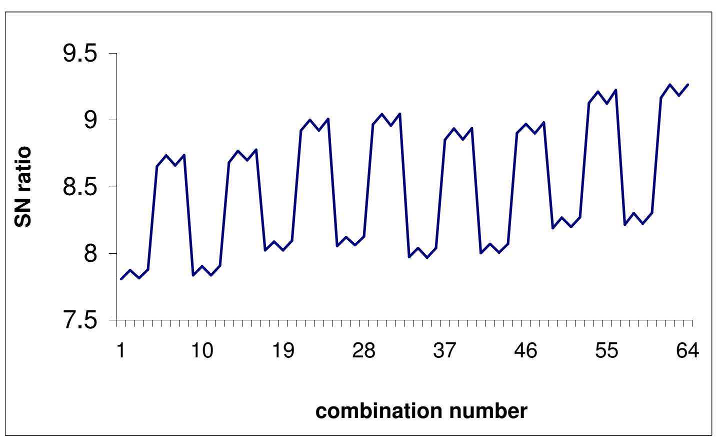


Fig. 6.10 SN ratio from Cross Array Experiments for Quintic Trajectory - case (iv)

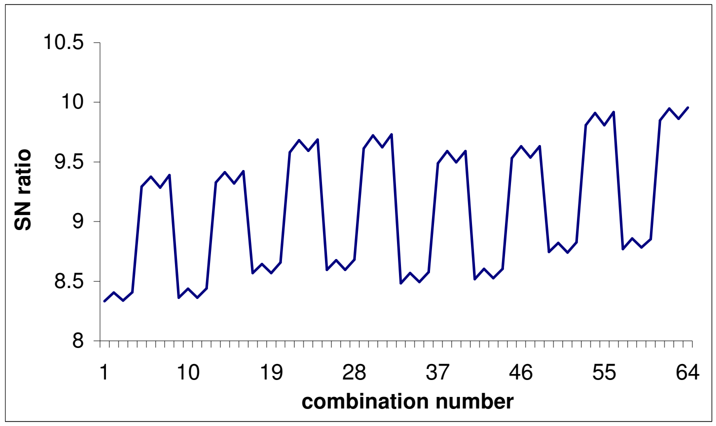


Fig. 6.11 SN ratio from Cross Array Experiments for Quintic Trajectory - case (v)

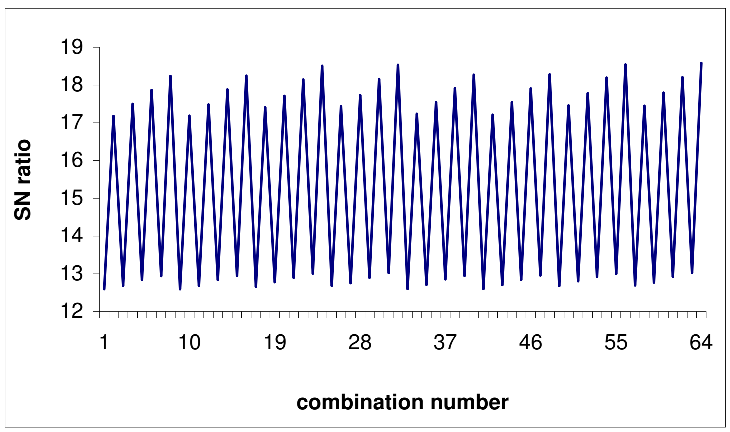


Fig. 6.12 SN ratio from Cross Array Experiments for Quintic Trajectory - case (vi)

From Fig 6.7, for case (i) following quintic trajectory, SN ratio value is observed to fluctuate as combination number increase. Maximum and minimum SN ratio is found to be 17.70 dB and 11.54 dB respectively. Similar to case (i) from Fig. 6.8 for case (ii) SN ratio value is observed to fluctuate alternately as combination number increase. Maximum SN ratio is found to be 17.05 dB at combination number 64 and minimum SN ratio value is 11.25 dB at combination number 1.

In Fig. 6.9 for case (iii) it is observed that trend is similar as compared to previous two cases. SN ratio value is found to fluctuate alternately and range of variation is between 16.79 dB and 10.93 dB. From Fig. 6.10 for the case (iv) it is seen that the trend is different as compared earlier three cases (i), (ii) and (iii). The SN ratio is observed to fluctuate alternately. And maximum and minimum SN ratio value is 9.26 dB at combination number 64 and 7.80

dB at combination number 1 respectively. In Fig. 6.11 for case (v) it is observed that the trend is similar to case (iv). The SN ratio is observed to fluctuate between 9.95 dB and 8.33 dB. From Fig. 6.12 it is found that SN ratio fluctuate alternately as combination number increase. The range of SN ratio is found to lie between 18.58 dB and 12.59 dB.

(ii) Analysis of Performance using Half Normal Plot

To represent the statistically significant parameters and parameter interactions graphically help of half normal plot is taken. Description regarding the half normal plot is provided in Chapter 4. For detailed explanation section 4.5 can be referred. Taking simulation results i.e positional error obtained from cross array experimental strategy for the manipulators performing the tasks following cubic trajectory the half normal plots are plotted. The plots indicating significant parameters are shown in Figs. 6.13-6.18.

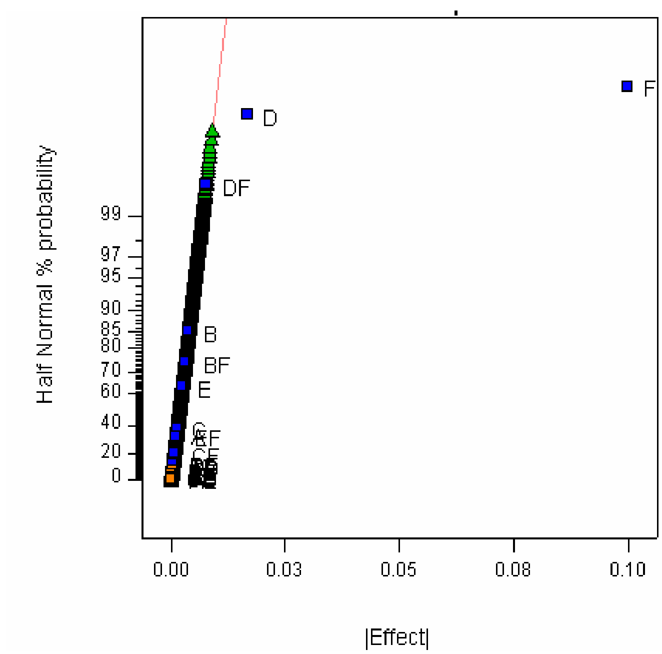


Fig. 6.13 Half Normal Plot of Performance for Cubic Trajectory - case (i)

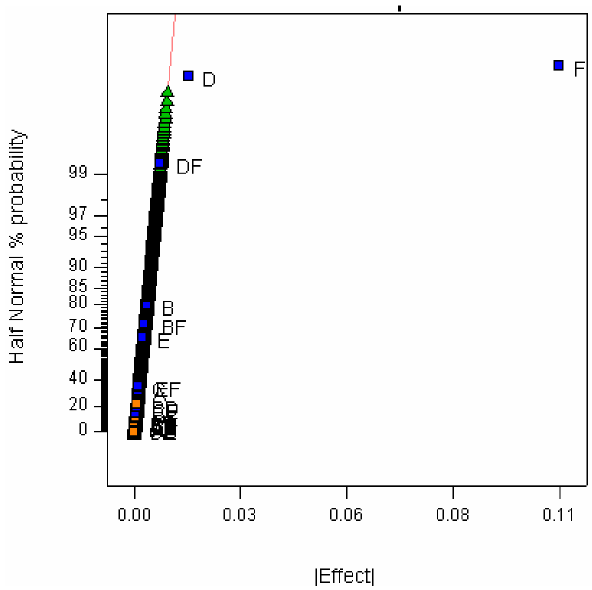


Fig. 6.14 Half Normal Plot of Performance for Cubic Trajectory - case (ii)

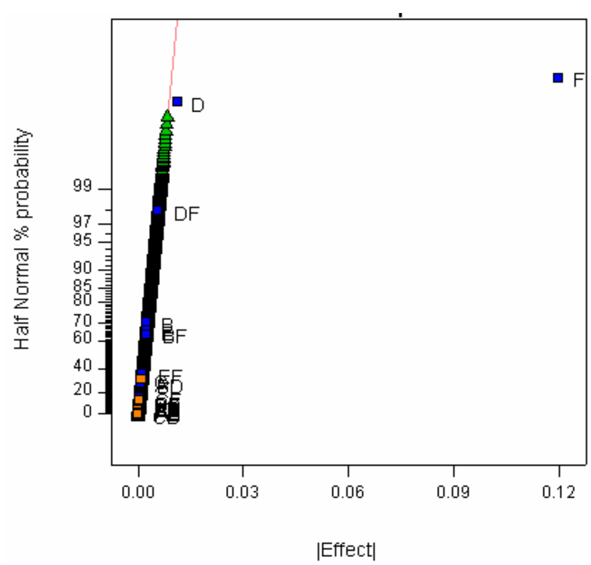


Fig. 6.15 Half Normal Plot of Performance for Cubic Trajectory - case (iii)

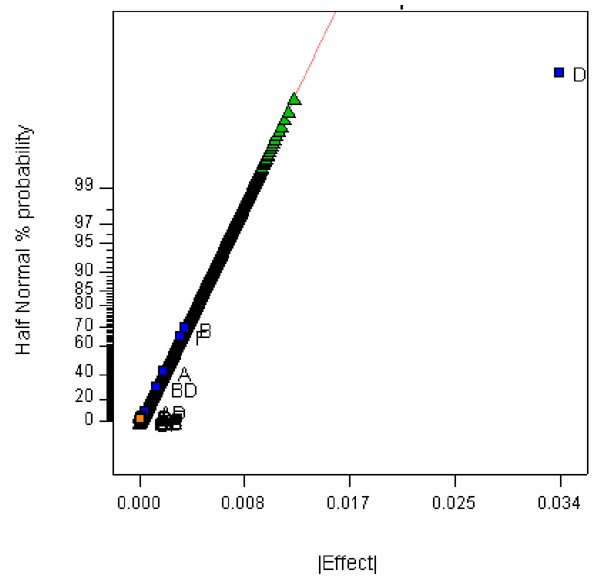


Fig. 6.16 Half Normal Plot of Performance for Cubic Trajectory - case (iv)

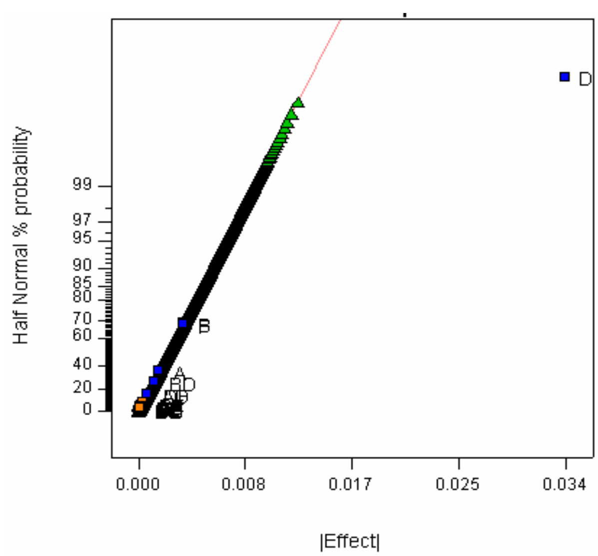


Fig. 6.17 Half Normal Plot of Performance for Cubic Trajectory - case (v)

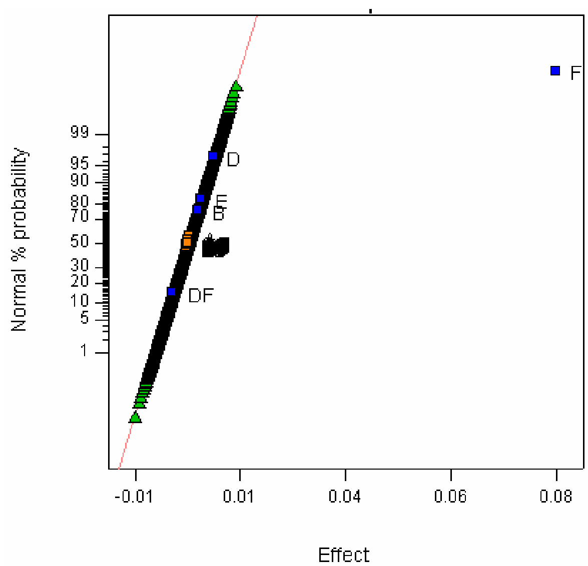


Fig. 6.18 Half Normal Plot of Performance for Cubic Trajectory - case (vi)

In this half normal plot it can be observed that the factors which have significant effect on performance variation lie away from the straight line. From Fig. 6.13 for case (i) it is observed that factors D and F are significant. Similarly in Fig. 6.14, for case (ii) factors D and F are observed to be statistically significant.

From Fig. 6.15 it is found that results for case (iii) is same as discussed in cases (i) and (ii) respectively. It is observed that factors D and F are statistically significant. In Fig. 6.16 for case (iv) factor D is found to affect the performance variations. Similarly it is observed that factor D is statistically significant, from Fig. 6.17 for case (v). In Fig. 6.18 for case (vi) factor F is found statistically significant.

As discussed in previous case half normal plots for the manipulators performing the task following quintic trajectory are provided in Figs. 6.19-6.24.

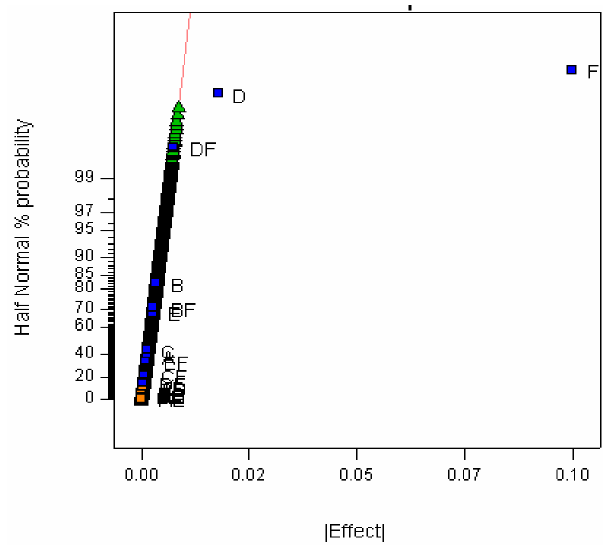


Fig. 6.19 Half Normal Plot of Performance for Quintic Trajectory - case (i)

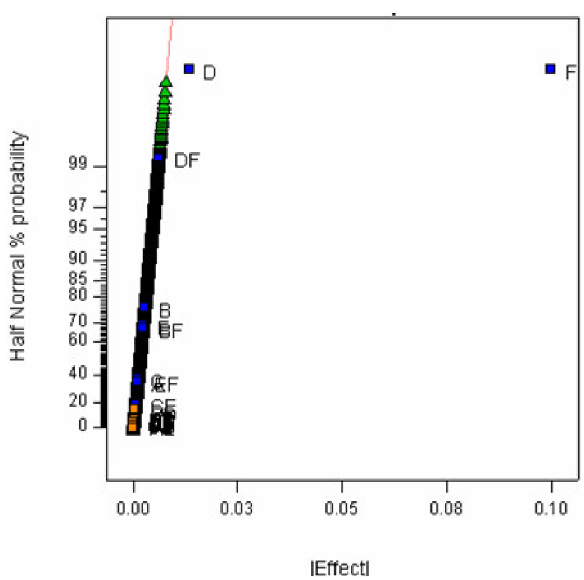


Fig. 6.20 Half Normal Plot of Performance for Quintic Trajectory - case (ii)

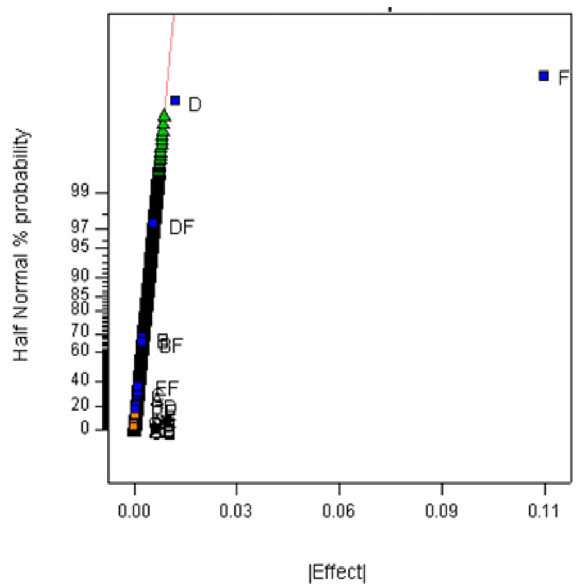


Fig. 6.21 Half Normal Plot of Performance for Quintic Trajectory - case (iii)

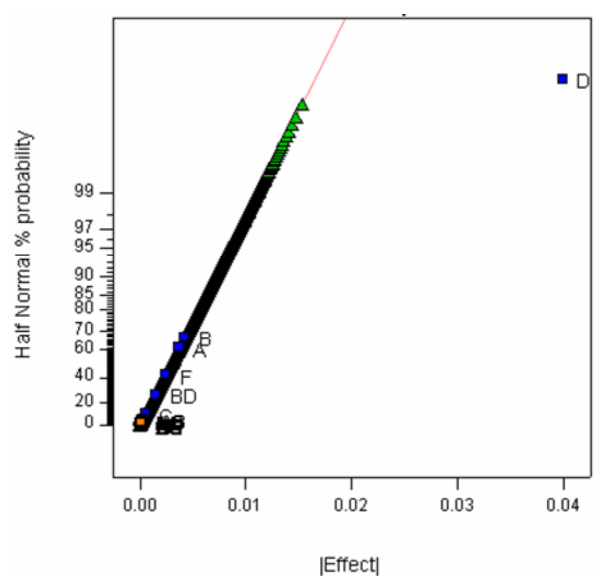


Fig. 6.22 Half Normal Plot of Performance for Quintic Trajectory - case (iv)

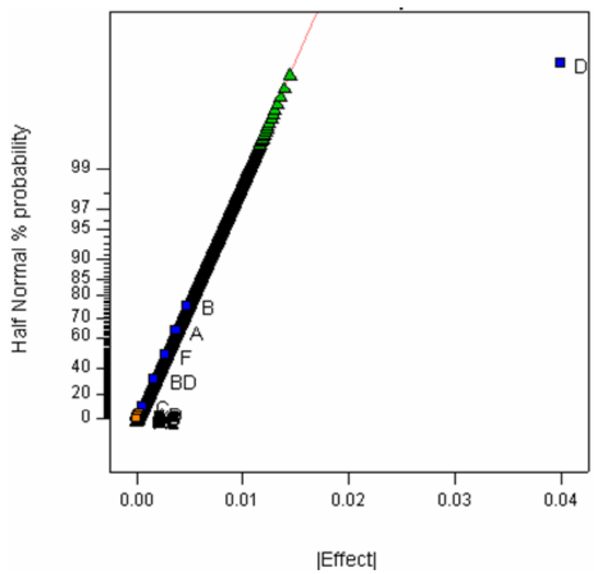


Fig. 6.23 Half Normal Plot of Performance for Quintic Trajectory - case (v)

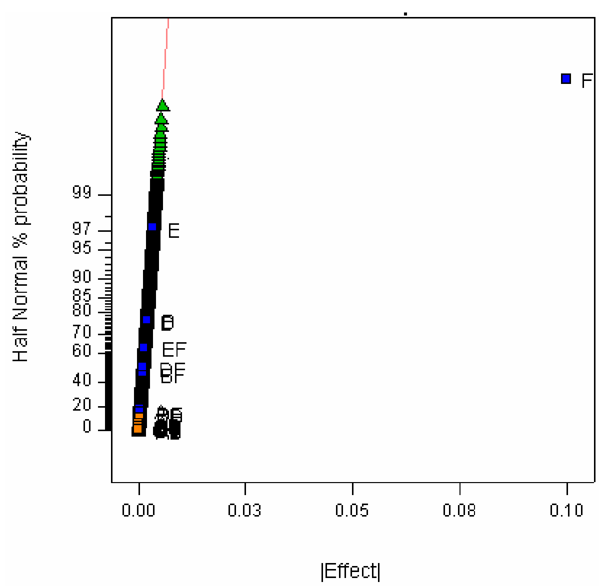


Fig. 6.24 Half Normal Plot of Performance for Quintic Trajectory - case (vi)

From Fig. 6.19 for case (i) following quintic trajectory it is observed that factors D and F are statistically significant. It is observed in Fig. 6.20 for case (ii) factors D and F are statistically significant. It is found that in Fig. 6.21 for case (iii), factors D and F are statistically significant. This result is similar to cases (i) and (ii).

It is observed that factor D is statistically significant from Figs. 6.22 and 6.23 for cases (iv), and (v) respectively. From Fig. 6.24 for case (vi), it is observed that factor F is statistically significant.

(iii) Analysis of Performance using ANOVA technique

Statistical analysis of the performance obtained from the cross array design of experiment strategy is analysed using ANOVA. For analysis, positional error (ε_i) is taken as the performance while manipulator performs a particular task following cubic and quintic trajectories. ANOVA technique provides the quantitative measure of degree of statistical significance.

The results of ANOVA are presented in tabular form for cubic trajectory and quintic trajectory in Tables 6.5-6.10 and 6.11-6.16 respectively. As each tolerance combination is run for 8 times, it gave rise to 512 numbers of simulations. Therefore, available degrees of freedom for ANOVA is 511.

The summary of results of ANOVA for all the discussed tasks are provided in tabular form and provided in Tables 6.5-6.10. In ANOVA tables a factor is considered to be statistically significant by comparing observed F statistic value (F_o) obtained in the ANOVA table against tabulated F statistic (F) value. During analysis the level of significance is kept at 0.05, for all cases. The values of F tabulated is 3.84 i.e. $F_{0.05,\nu_1,\nu_2} = F_{0.05,1,511} \approx F_{0.05,1,\infty} = 3.84$ [Montgomery 2001].

Table 6.5 ANOVA of Performance for Cubic Trajectory - case (i)

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.001412	1	0.001412	0.191553328	-
B	0.015563	1	0.015563	2.111163328	-
C	0.001889	1	0.001889	0.256181392	-
D	0.30113	1	0.30113	40.84855221	Significant
E	0.006142	1	0.006142	0.833174473	-
F	10.71367	1	10.71367	1453.319954	Significant
AB	$9.27 \cdot 10^{-6}$	1	$9.27 \cdot 10^{-6}$	0.001257605	-
AC	$1.78 \cdot 10^{-5}$	1	$1.78 \cdot 10^{-5}$	0.002410471	-
AD	0.000105	1	0.000105	0.014279311	-
AE	$1.05 \cdot 10^{-6}$	1	$1.05 \cdot 10^{-6}$	0.000141919	-
AF	$2.77 \cdot 10^{-5}$	1	$2.77 \cdot 10^{-5}$	0.003751221	-
BC	$5.82 \cdot 10^{-5}$	1	$5.82 \cdot 10^{-5}$	0.007892332	-
BD	0.000144	1	0.000144	0.019545756	-
BE	$1.85 \cdot 10^{-5}$	1	$1.85 \cdot 10^{-5}$	0.002508589	-
BF	0.009758	1	0.009758	1.323706328	-
CD	$3.96 \cdot 10^{-6}$	1	$3.96 \cdot 10^{-6}$	0.0005365	-
CE	$2.24 \cdot 10^{-5}$	1	$2.24 \cdot 10^{-5}$	0.003045021	-
CF	0.000469	1	0.000469	0.063636299	-
DE	0.000168	1	0.000168	0.022828026	-
DF	0.061859	1	0.061859	8.391217612	Significant
EF	0.001308	1	0.001308	0.177482132	-
Residual	3.61228	490	0.007372		-
Corrected Total	14.72605	511			-

Table 6.6 ANOVA of Performance for Cubic Trajectory - case (ii)

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.001124	1	0.001124	0.16771	-
B	0.01083	1	0.01083	1.615454	-
C	0.001358	1	0.001358	0.202597	-
D	0.206251	1	0.206251	30.7662	Significant
E	0.005991	1	0.005991	0.893641	-
F	12.76136	1	12.76136	1903.592	Significant
AB	$1.26 \cdot 10^{-5}$	1	$1.26 \cdot 10^{-5}$	0.001875	-
AC	$3.08 \cdot 10^{-5}$	1	$3.08 \cdot 10^{-5}$	0.004596	-
AD	$7.88 \cdot 10^{-5}$	1	$7.88 \cdot 10^{-5}$	0.011753	-
AE	$3.39 \cdot 10^{-6}$	1	$3.39 \cdot 10^{-6}$	0.000505	-
AF	$2.44 \cdot 10^{-5}$	1	$2.44 \cdot 10^{-5}$	0.003638	-
BC	$6.54 \cdot 10^{-5}$	1	$6.54 \cdot 10^{-5}$	0.009757	-
BD	0.000452	1	0.000452	0.067419	-
BE	$5.28 \cdot 10^{-5}$	1	$5.28 \cdot 10^{-5}$	0.007883	-
BF	0.007844	1	0.007844	1.170055	-
CD	$3.17 \cdot 10^{-6}$	1	$3.17 \cdot 10^{-6}$	0.000473	-
CE	$2.68 \cdot 10^{-5}$	1	$2.68 \cdot 10^{-5}$	0.003998	-
CF	0.000345	1	0.000345	0.051457	-
DE	0.000165	1	0.000165	0.024671	-
DF	0.049214	1	0.049214	7.341113	Significant
EF	0.001391	1	0.001391	0.207509	
Residual	3.28496	490	0.006704		
Corrected Total	16.33158	511			

Table 6.7 ANOVA of Performance for Cubic Trajectory - case (iii)

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.00085	1	0.00085	0.137452	-
B	0.006897	1	0.006897	1.115602	-
C	0.000939	1	0.000939	0.15196	-
D	0.131602	1	0.131602	21.28675	Significant
E	0.005705	1	0.005705	0.922727	-
F	14.9238	1	14.9238	2413.941	Significant
AB	$1.06{\cdot}10^{-5}$	1	$1.06{\cdot}10^{-5}$	0.00172	-
AC	$4.27{\cdot}10^{-5}$	1	$4.27{\cdot}10^{-5}$	0.006901	-
AD	$5.99{\cdot}10^{-5}$	1	$5.99{\cdot}10^{-5}$	0.009684	-
AE	$6.25{\cdot}10^{-6}$	1	$6.25{\cdot}10^{-6}$	0.00101	-
AF	$2.9{\cdot}10^{-5}$	1	$2.9{\cdot}10^{-5}$	0.004688	-
BC	$9.72{\cdot}10^{-5}$	1	$9.72{\cdot}10^{-5}$	0.015717	-
BD	0.000732	1	0.000732	0.118364	-
BE	0.000109	1	0.000109	0.017582	-
BF	0.005208	1	0.005208	0.842391	-
CD	$1.07{\cdot}10^{-8}$	1	$1.07{\cdot}10^{-8}$	$1.74{\cdot}10^{-6}$	-
CE	$4.15{\cdot}10^{-5}$	1	$4.15{\cdot}10^{-5}$	0.006714	-
CF	0.000217	1	0.000217	0.035055	-
DE	$8.39{\cdot}10^{-5}$	1	$8.39{\cdot}10^{-5}$	0.013577	-
DF	0.033802	1	0.033802	5.46749	Significant
EF	0.001319	1	0.001319	0.213373	
Residual	3.02918	490	0.006182		
Corrected Total	18.14073	511			

Table 6.8 ANOVA of Performance for Cubic Trajectory - case (iv)

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.003888	1	0.003888	0.314443463	-
B	0.01331	1	0.01331	1.076354423	-
C	$4.77{\cdot}10^{-6}$	1	$4.77{\cdot}10^{-6}$	0.000385371	-
D	1.16888	1	1.16888	94.52254516	Significant
E	$4.1{\cdot}10^{-5}$	1	$4.1{\cdot}10^{-5}$	0.003319468	-
F	0.01114	1	0.01114	0.900811477	-
AB	$1.26{\cdot}10^{-6}$	1	$1.26{\cdot}10^{-6}$	0.000101815	-
AC	$2.06{\cdot}10^{-6}$	1	$2.06{\cdot}10^{-6}$	0.000166706	-
AD	0.000167	1	0.000167	0.013538809	-
AE	$9.52{\cdot}10^{-6}$	1	$9.52{\cdot}10^{-6}$	0.000769892	-
AF	$1.66{\cdot}10^{-5}$	1	$1.66{\cdot}10^{-5}$	0.001343828	-
BC	$1.26{\cdot}10^{-5}$	1	$1.26{\cdot}10^{-5}$	0.001017059	-
BD	0.001802	1	0.001802	0.145752192	-
BE	$8.38{\cdot}10^{-6}$	1	$8.38{\cdot}10^{-6}$	0.000677849	-
BF	$1.06{\cdot}10^{-7}$	1	$1.06{\cdot}10^{-7}$	$8.53408{\cdot}10^{-6}$	-
CD	$1.97{\cdot}10^{-6}$	1	$1.97{\cdot}10^{-6}$	0.000158996	-
CE	$7.67{\cdot}10^{-6}$	1	$7.67{\cdot}10^{-6}$	0.000620203	-
CF	$7.14{\cdot}10^{-6}$	1	$7.14{\cdot}10^{-6}$	0.000577423	-
DE	$1.5{\cdot}10^{-5}$	1	$1.5{\cdot}10^{-5}$	0.001210935	-
DF	$6.54{\cdot}10^{-5}$	1	$6.54{\cdot}10^{-5}$	0.005287471	-
EF	$1.66{\cdot}10^{-6}$	1	$1.66{\cdot}10^{-6}$	0.000133954	-
Residual	6.059394	490	0.012366		
Corrected Total	7.258774	511			

Table 6.9 ANOVA of Performance for Cubic Trajectory - case (v)

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.002767	1	0.002767	0.21626	-
B	0.013173	1	0.013173	1.029472	-
C	$3.65 \cdot 10^{-5}$	1	$3.65 \cdot 10^{-5}$	0.00285	-
D	1.182802	1	1.182802	92.43601	Significant
E	$3.53 \cdot 10^{-6}$	1	$3.53 \cdot 10^{-6}$	0.000276	-
F	0.012939	1	0.012939	1.011216	-
AB	$1.68 \cdot 10^{-5}$	1	$1.68 \cdot 10^{-5}$	0.001312	-
AC	$4.98 \cdot 10^{-5}$	1	$4.98 \cdot 10^{-5}$	0.003896	-
AD	0.000438	1	0.000438	0.034233	-
AE	$1.94 \cdot 10^{-5}$	1	$1.94 \cdot 10^{-5}$	0.001517	-
AF	$2.76 \cdot 10^{-5}$	1	$2.76 \cdot 10^{-5}$	0.002153	-
BC	$4.04 \cdot 10^{-5}$	1	$4.04 \cdot 10^{-5}$	0.003156	-
BD	0.0015	1	0.0015	0.117221	-
BE	$1.45 \cdot 10^{-5}$	1	$1.45 \cdot 10^{-5}$	0.001135	-
BF	$9.87 \cdot 10^{-5}$	1	$9.87 \cdot 10^{-5}$	0.007713	-
CD	$3.49 \cdot 10^{-5}$	1	$3.49 \cdot 10^{-5}$	0.002729	-
CE	$4.35 \cdot 10^{-5}$	1	$4.35 \cdot 10^{-5}$	0.003401	-
CF	$1.72 \cdot 10^{-5}$	1	$1.72 \cdot 10^{-5}$	0.001345	-
DE	$2.36 \cdot 10^{-6}$	1	$2.36 \cdot 10^{-6}$	0.000184	-
DF	0.000471	1	0.000471	0.036838	-
EF	$2.42 \cdot 10^{-5}$	1	$2.42 \cdot 10^{-5}$	0.001893	-
Residual	6.27004	490	0.012796		
Corrected Total	7.48456	511			

Table 6.10 ANOVA of Performance for Cubic Trajectory - case (vi)

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F Value	Remark
A	0.000206	1	0.000206	0.019722	-
B	0.005061	1	0.005061	0.484454	-
C	$1.7 \cdot 10^{-5}$	1	$1.7 \cdot 10^{-5}$	0.001625	-
D	0.035788	1	0.035788	3.425746	-
E	0.009336	1	0.009336	0.893645	-
F	7.038495	1	7.038495	673.7442	Significant
AB	$5.82 \cdot 10^{-6}$	1	$5.82 \cdot 10^{-6}$	0.000558	-
AC	$9.89 \cdot 10^{-7}$	1	$9.89 \cdot 10^{-7}$	$9.46 \cdot 10^{-5}$	-
AD	$3.41 \cdot 10^{-7}$	1	$3.41 \cdot 10^{-7}$	$3.27 \cdot 10^{-5}$	-
AE	$1.53 \cdot 10^{-5}$	1	$1.53 \cdot 10^{-5}$	0.00146	-
AF	$7.42 \cdot 10^{-5}$	1	$7.42 \cdot 10^{-5}$	0.007105	-
BC	$2.22 \cdot 10^{-5}$	1	$2.22 \cdot 10^{-5}$	0.002128	-
BD	$9.06 \cdot 10^{-5}$	1	$9.06 \cdot 10^{-5}$	0.008676	-
BE	$1.71 \cdot 10^{-6}$	1	$1.71 \cdot 10^{-6}$	0.000164	-
BF	$6.13 \cdot 10^{-5}$	1	$6.13 \cdot 10^{-5}$	0.005872	-
CD	$6.51 \cdot 10^{-6}$	1	$6.51 \cdot 10^{-6}$	0.000623	-
CE	$3.53 \cdot 10^{-6}$	1	$3.53 \cdot 10^{-6}$	0.000338	-
CF	$6.27 \cdot 10^{-6}$	1	$6.27 \cdot 10^{-6}$	0.0006	-
DE	$1.75 \cdot 10^{-5}$	1	$1.75 \cdot 10^{-5}$	0.001675	-
DF	0.011619	1	0.011619	1.112225	-
EF	$4.87 \cdot 10^{-5}$	1	$4.87 \cdot 10^{-5}$	0.00466	-
Residual	5.11903	490	0.010447		
Corrected Total	12.2199	511			

From Table 6.5 for case (i), it is observed that factors D, F and factor tolerance interaction DF are statistically significant. And rest of the factors are insignificant because their F statistic values are less as compared to tabulated F statistic value. Similarly from Table 6.6, for case (ii) factors D and F and factor interactions DF are found to be statistically significant. In Table 6.7 for case (iii), it is observed that factors D and F and factor interaction DF are statistically significant. And rest of the factors and two factor interactions are statistically insignificant. From Table 6.8 for case (iv), it is observed that factor D is statistically significant. And rest of the factors and two factor interactions are statistically insignificant. In Table 6.9 for case (v) following cubic trajectory, factor D is statistically significant. From Table 6.10 for case (vi) it is observed that factor F is statistically significant and rest of the factors and two factor interactions are statistically insignificant.

Similar to above, performance obtained from experiments while following quintic trajectory are analysed using ANOVA technique. The results of ANOVA are summarized in Tables 6.11 to 6.16.

Table 6.11 ANOVA of Performance for Quintic Trajectory - case (i)

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.00152	1	0.00152	0.258236	-
B	0.010847	1	0.010847	1.843052	-
C	0.001936	1	0.001936	0.328977	-
D	0.296437	1	0.296437	50.36817	Significant
E	0.006084	1	0.006084	1.03382	-
F	9.280937	1	9.280937	1576.94	Significant
AB	$9.23\cdot10^{-6}$	1	$9.23\cdot10^{-6}$	0.001569	-
AC	$1.73\cdot10^{-5}$	1	$1.73\cdot10^{-5}$	0.002942	-
AD	0.000113	1	0.000113	0.019175	-
AE	$7.72\cdot10^{-7}$	1	$7.72\cdot10^{-7}$	0.000131	-
AF	$2.34\cdot10^{-5}$	1	$2.34\cdot10^{-5}$	0.003984	-
BC	$4.34\cdot10^{-5}$	1	$4.34\cdot10^{-5}$	0.007371	-
BD	$9.4\cdot10^{-5}$	1	$9.4\cdot10^{-5}$	0.015979	-
BE	$1.8\cdot10^{-5}$	1	$1.8\cdot10^{-5}$	0.003062	-
BF	0.006714	1	0.006714	1.140861	-
CD	$1.08\cdot10^{-5}$	1	$1.08\cdot10^{-5}$	0.001835	-
CE	$1.72\cdot10^{-5}$	1	$1.72\cdot10^{-5}$	0.002917	-
CF	0.00047	1	0.00047	0.079833	-
DE	0.000205	1	0.000205	0.034829	-
DF	0.052088	1	0.052088	8.850447	Significant
EF	0.001151	1	0.001151	0.195571	
Residual	2.88365	490	0.005885		
Corrected Total	12.54239	511			

Table 6.12 ANOVA of Performance for Quintic Trajectory - case (ii)

Sources of Variation	**Sum of Squares**	**Degrees of Freedom**	**Mean Square**	**F_o Value**	**Remark**
A	0.001205	1	0.001205	0.203592	-
B	0.008371	1	0.008371	1.414031	-
C	0.001405	1	0.001405	0.237381	-
D	0.204703	1	0.204703	34.57914	Significant
E	0.00606	1	0.00606	1.023749	-
F	11.20947	1	11.20947	1893.542	Significant
AB	$1.54 \cdot 10^{-5}$	1	$1.54 \cdot 10^{-5}$	0.0026	-
AC	$2.58 \cdot 10^{-5}$	1	$2.58 \cdot 10^{-5}$	0.004363	-
AD	$7.49 \cdot 10^{-5}$	1	$7.49 \cdot 10^{-5}$	0.012659	-
AE	$1.87 \cdot 10^{-6}$	1	$1.87 \cdot 10^{-6}$	0.000316	-
AF	$2.33 \cdot 10^{-5}$	1	$2.33 \cdot 10^{-5}$	0.00394	-
BC	$5.18 \cdot 10^{-5}$	1	$5.18 \cdot 10^{-5}$	0.008749	-
BD	0.000146	1	0.000146	0.024728	-
BE	$3.45 \cdot 10^{-5}$	1	$3.45 \cdot 10^{-5}$	0.005826	-
BF	0.005768	1	0.005768	0.974308	-
CD	$7.64 \cdot 10^{-6}$	1	$7.64 \cdot 10^{-6}$	0.00129	-
CE	$1.9 \cdot 10^{-5}$	1	$1.9 \cdot 10^{-5}$	0.003213	-
CF	0.000324	1	0.000324	0.054799	-
DE	0.000198	1	0.000198	0.033492	-
DF	0.042237	1	0.042237	7.134745	Significant
EF	0.001224	1	0.001224	0.206699	
Residual	2.9008	490	0.00592		
Corrected Total	14.38217	511			

Table 6.13 ANOVA of Performance for Quintic Trajectory - case (iii)

Sources of Variation	**Sum of Squares**	**Degrees of Freedom**	**Mean Square**	**F_o Value**	**Remark**
A	0.000837	1	0.000837	0.139573	-
B	0.00585	1	0.00585	0.975816	-
C	0.000973	1	0.000973	0.162342	-
D	0.140463	1	0.140463	23.42842	Significant
E	0.005918	1	0.005918	0.987137	-
F	13.40498	1	13.40498	2235.866	Significant
AB	$6.51 \cdot 10^{-7}$	1	$6.51 \cdot 10^{-7}$	0.000109	-
AC	$4.55 \cdot 10^{-6}$	1	$4.55 \cdot 10^{-6}$	0.000759	-
AD	$5.99 \cdot 10^{-7}$	1	$5.99 \cdot 10^{-7}$	$9.99 \cdot 10^{-5}$	-
AE	$6.26 \cdot 10^{-7}$	1	$6.26 \cdot 10^{-7}$	0.000104	-
AF	$7.1 \cdot 10^{-5}$	1	$7.1 \cdot 10^{-5}$	0.011834	-
BC	$1.07 \cdot 10^{-6}$	1	$1.07 \cdot 10^{-6}$	0.000178	-
BD	0.000423	1	0.000423	0.070489	-
BE	$4.22 \cdot 10^{-5}$	1	$4.22 \cdot 10^{-5}$	0.007043	-
BF	0.005351	1	0.005351	0.892546	-
CD	$5.28 \cdot 10^{-5}$	1	$5.28 \cdot 10^{-5}$	0.008804	-
CE	$1.16 \cdot 10^{-7}$	1	$1.16 \cdot 10^{-7}$	$1.93 \cdot 10^{-5}$	-
CF	0.000281	1	0.000281	0.046822	-
DE	0.00011	1	0.00011	0.018306	-
DF	0.030203	1	0.030203	5.03771	Significant
EF	0.001339	1	0.001339	0.223363	
Residual	2.93755	490	0.005995		
Corrected Total	16.53507	511			

Table 6.14 ANOVA of Performance for Quintic Trajectory - case (iv)

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.01138	1	0.01138	0.752858	-
B	0.014547	1	0.014547	0.962367	-
C	0.000229	1	0.000229	0.01517	-
D	1.318729	1	1.318729	87.23886	Significant
E	$2.75 \cdot 10^{-5}$	1	$2.75 \cdot 10^{-5}$	0.001821	-
F	0.004786	1	0.004786	0.316625	-
AB	$5.68 \cdot 10^{-5}$	1	$5.68 \cdot 10^{-5}$	0.003756	-
AC	$4.44 \cdot 10^{-8}$	1	$4.44 \cdot 10^{-8}$	$2.94 \cdot 10^{-6}$	-
AD	$2.33 \cdot 10^{-5}$	1	$2.33 \cdot 10^{-5}$	0.00154	-
AE	$1.68 \cdot 10^{-6}$	1	$1.68 \cdot 10^{-6}$	0.000111	-
AF	$1.16 \cdot 10^{-6}$	1	$1.16 \cdot 10^{-6}$	$7.68 \cdot 10^{-5}$	-
BC	$1.21 \cdot 10^{-7}$	1	$1.21 \cdot 10^{-7}$	$8.01 \cdot 10^{-6}$	-
BD	0.001759	1	0.001759	0.116346	-
BE	$1.15 \cdot 10^{-7}$	1	$1.15 \cdot 10^{-7}$	$7.62 \cdot 10^{-6}$	-
BF	$2.3 \cdot 10^{-6}$	1	$2.3 \cdot 10^{-6}$	0.000152	-
CD	$9.26 \cdot 10^{-7}$	1	$9.26 \cdot 10^{-7}$	$6.12 \cdot 10^{-5}$	-
CE	$1.65 \cdot 10^{-6}$	1	$1.65 \cdot 10^{-6}$	0.000109	-
CF	$1 \cdot 10^{-6}$	1	$1 \cdot 10^{-6}$	$6.62 \cdot 10^{-5}$	-
DE	$1.12 \cdot 10^{-6}$	1	$1.12 \cdot 10^{-6}$	$7.43 \cdot 10^{-5}$	-
DF	$3.09 \cdot 10^{-5}$	1	$3.09 \cdot 10^{-5}$	0.002044	-
EF	$8.76 \cdot 10^{-7}$	1	$8.76 \cdot 10^{-7}$	$5.8 \cdot 10^{-5}$	-
Residual	7.40684	490	0.015116		
Corrected Total	8.75843	511			

Table 6.15 ANOVA of Performance for Quintic Trajectory - case (v)

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.012465	1	0.012465	0.844481	-
B	0.020552	1	0.020552	1.392344	-
C	0.000282	1	0.000282	0.019112	-
D	1.444124	1	1.444124	97.83657	Significant
E	$2.68 \cdot 10^{-5}$	1	$2.68 \cdot 10^{-5}$	0.001816	-
F	0.006484	1	0.006484	0.439297	-
AB	$2.74 \cdot 10^{-5}$	1	$2.74 \cdot 10^{-5}$	0.001858	-
AC	$1.98 \cdot 10^{-7}$	1	$1.98 \cdot 10^{-7}$	$1.34 \cdot 10^{-5}$	-
AD	$6.08 \cdot 10^{-5}$	1	$6.08 \cdot 10^{-5}$	0.004119	-
AE	$5.51 \cdot 10^{-8}$	1	$5.51 \cdot 10^{-8}$	$3.73 \cdot 10^{-6}$	-
AF	$1.34 \cdot 10^{-5}$	1	$1.34 \cdot 10^{-5}$	0.000908	-
BC	$1.43 \cdot 10^{-5}$	1	$1.43 \cdot 10^{-5}$	0.000969	-
BD	0.002535	1	0.002535	0.171728	-
BE	$2.09 \cdot 10^{-7}$	1	$2.09 \cdot 10^{-7}$	$1.41 \cdot 10^{-5}$	-
BF	$3.38 \cdot 10^{-6}$	1	$3.38 \cdot 10^{-6}$	0.000229	-
CD	$1.98 \cdot 10^{-6}$	1	$1.98 \cdot 10^{-6}$	0.000134	-
CE	$3.06 \cdot 10^{-7}$	1	$3.06 \cdot 10^{-7}$	$2.07 \cdot 10^{-5}$	-
CF	$1.48 \cdot 10^{-6}$	1	$1.48 \cdot 10^{-6}$	0.000101	-
DE	$1.88 \cdot 10^{-6}$	1	$1.88 \cdot 10^{-6}$	0.000128	-
DF	$8 \cdot 10^{-6}$	1	$8 \cdot 10^{-6}$	0.000542	-
EF	$1.99 \cdot 10^{-6}$	1	$1.99 \cdot 10^{-6}$	0.000135	-
Residual	7.23289	490	0.014761		
Corrected Total	8.71949	511			

Table 6.16 ANOVA of Performance for Quintic Trajectory - case (vi)

Sources of Variation	Sum of Squares	Degrees of Freedom	Mean Square	F_o Value	Remark
A	0.000132	1	0.000132	0.048502	-
B	0.004007	1	0.004007	1.472162	-
C	$3.41 \cdot 10^{-7}$	1	$3.41 \cdot 10^{-7}$	0.000125	-
D	0.003941	1	0.003941	1.448138	-
E	0.013385	1	0.013385	4.918022	Significant
F	11.12709	1	11.12709	4088.359	Significant
AB	$9.22 \cdot 10^{-9}$	1	$9.22 \cdot 10^{-9}$	$3.39 \cdot 10^{-6}$	-
AC	$1.69 \cdot 10^{-6}$	1	$1.69 \cdot 10^{-6}$	0.000622	-
AD	$9.43 \cdot 10^{-6}$	1	$9.43 \cdot 10^{-6}$	0.003465	-
AE	$8.05 \cdot 10^{-7}$	1	$8.05 \cdot 10^{-7}$	0.000296	-
AF	$2.2 \cdot 10^{-5}$	1	$2.2 \cdot 10^{-5}$	0.008096	-
BC	$5.8 \cdot 10^{-6}$	1	$5.8 \cdot 10^{-6}$	0.002131	-
BD	$8.95 \cdot 10^{-5}$	1	$8.95 \cdot 10^{-5}$	0.03289	-
BE	$5.72 \cdot 10^{-6}$	1	$5.72 \cdot 10^{-6}$	0.0021	-
BF	0.001077	1	0.001077	0.395707	-
CD	$8.97 \cdot 10^{-6}$	1	$8.97 \cdot 10^{-6}$	0.003294	-
CE	$1.98 \cdot 10^{-7}$	1	$1.98 \cdot 10^{-7}$	$7.28 \cdot 10^{-5}$	-
CF	$2.19 \cdot 10^{-6}$	1	$2.19 \cdot 10^{-6}$	0.000803	-
DE	$9.58 \cdot 10^{-5}$	1	$9.58 \cdot 10^{-5}$	0.035195	-
DF	0.00127	1	0.00127	0.466786	-
EF	0.00222	1	0.00222	0.815845	-
Residual	1.33378	490	0.002722		
Corrected Total	12.4911	511			

From Table 6.11, it is observed that factors D and F are and factor interactions DF are statistically significant. And rest of the factors and factor interactions are insignificant because their computed F statistic values are less as compared to tabulated F statistic value. In Table 6.12 for the case (ii) it is observed that factors D and F and factor interaction DF are statistically significant and rest of the factors are found to be insignificant. From Table 6.13 for the case (iii) it is observed that factors D and F and factor interaction DF are statistically significant. Similarly analysis is carried out for the case (iv) and observed that factor D is statistically significant in Table 6.14. It is observed from Table 6.15 for case (v) factor D is statistically significant. From Table 6.16 for the case (vi) factors E and F are found to be statistically significant.

6.5.3 Monte Carlo Simulation for Validation

For validating the results obtained in cross array experimentation strategy, the Monte Carlo simulation approach is used. In this approach simulations are run without using noise factor array. Depending on the tolerance combination of the control factors i.e. tolerances of kinematic and dynamic parameters, values are generated randomly as per the probability density functions assumption, for thousand runs. Using equation (3.20) Performance measure SN ratio is computed after getting the performance after simulations, for all 64 parameter tolerance combinations following cubic and quintic trajectories. The performance variations of manipulator in terms of SN ratio are shown in Figs.6.25 - 6.30 and Figs. 6.31 - 6.36 respectively, for both the trajectories.

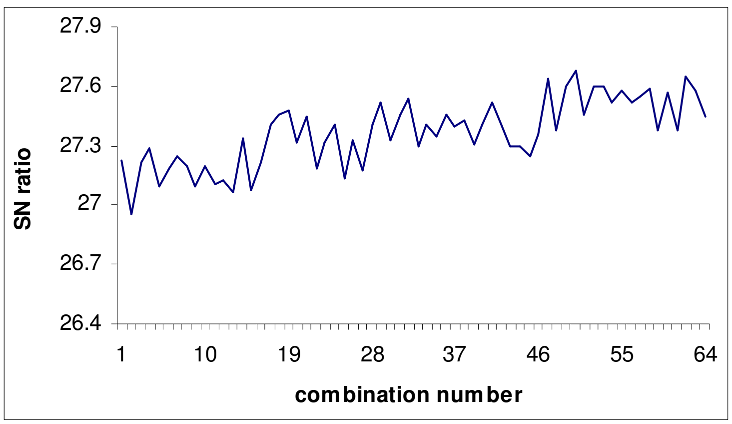


Fig. 6.25 SN ratio from Monte Carlo Simulation for Cubic Trajectory - case (i)

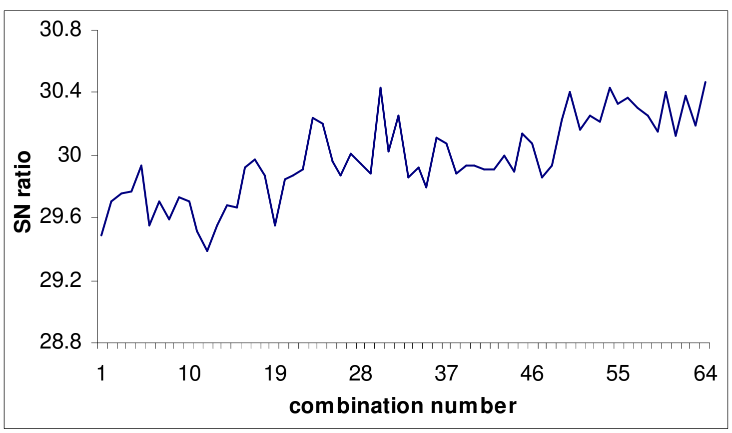


Fig. 6.26 SN ratio from Monte Carlo Simulation for Cubic Trajectory - case (ii)

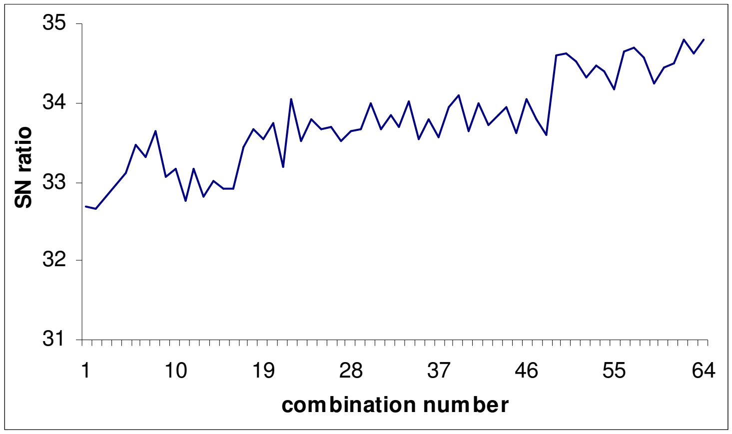


Fig. 6.27 SN ratio from Monte Carlo Simulation for Cubic Trajectory - case (iii)

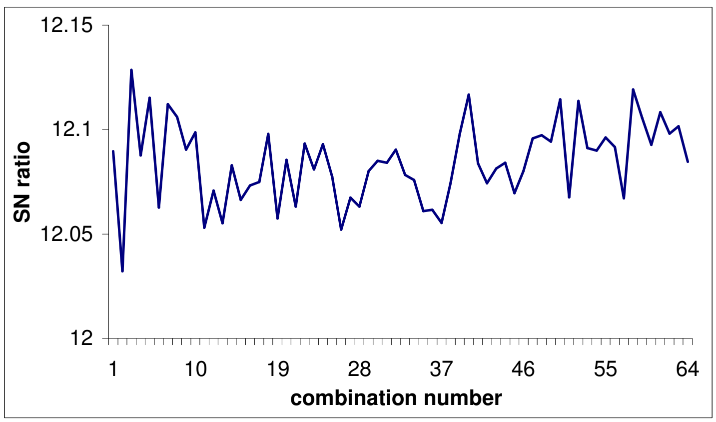


Fig. 6.28 SN ratio from Monte Carlo Simulation for Cubic Trajectory - case (iv)

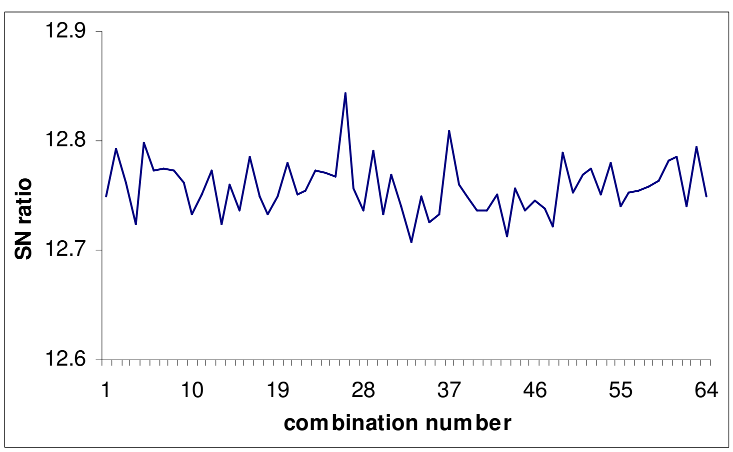


Fig. 6.29 SN ratio from Monte Carlo Simulation for Cubic Trajectory - case (v)

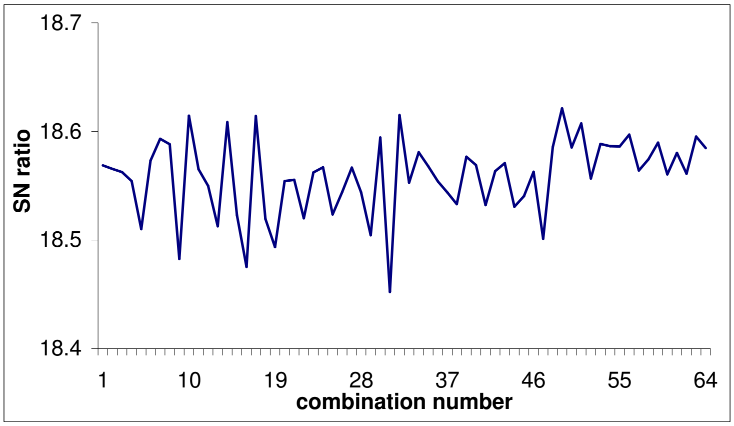


Fig. 6.30 SN ratio from Monte Carlo Simulation for Cubic Trajectory - case (vi)

It is observed from Fig. 6.25 for case (i), Fig. 6.26 for case (ii) and Fig. 6.27 for case (iii) following cubic trajectories, the SN ratio ranges between 27 dB to 27.6 dB, 29.5 dB to 30.5 dB and 32.5 dB to 34 dB respectively. Similar to earlier cases, from Figs. 6.28 and 6.29 for cases (iv) and (v) respectively SN ratio ranges between 12.04 dB to 12.2 dB and 12.7 dB to 12.85 dB respectively. In Fig. 6.30 for case (vi) SN ratio is observed to vary between 18.45 dB to 18.62 dB. Similarly, to validate the results obtained from cross array experimentation strategy, for the tasks following quintic trajectory, Monte Carlo simulations are carried out. The SN ratio values obtained corresponding to all 64 combinations for different tasks are plotted in Figs 6.31-6.36. These graphs show the trend of change with increase of combination number and performance at a combination number.

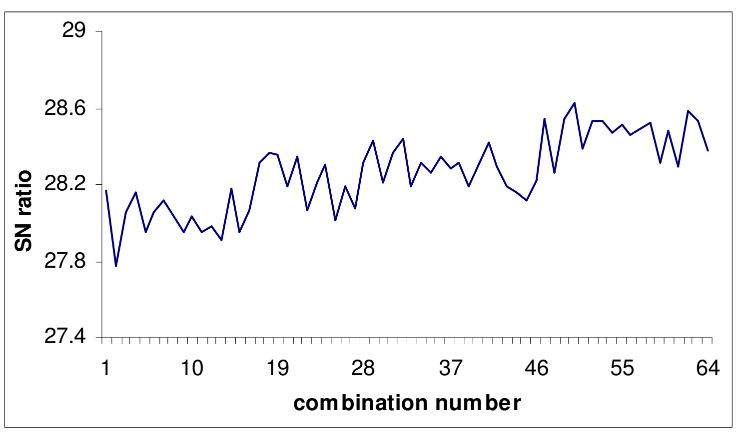


Fig. 6.31 SN ratio using Monte Carlo Simulation for Quintic Trajectory - case (i)

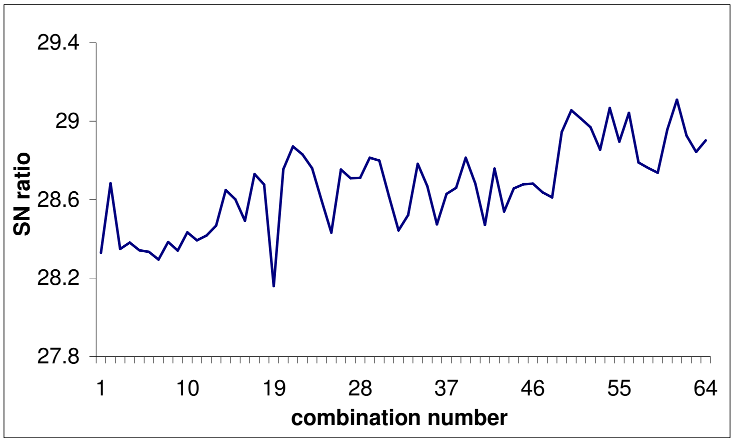


Fig. 6.32 SN ratio using Monte Carlo Simulation for Quintic Trajectory - case (ii)

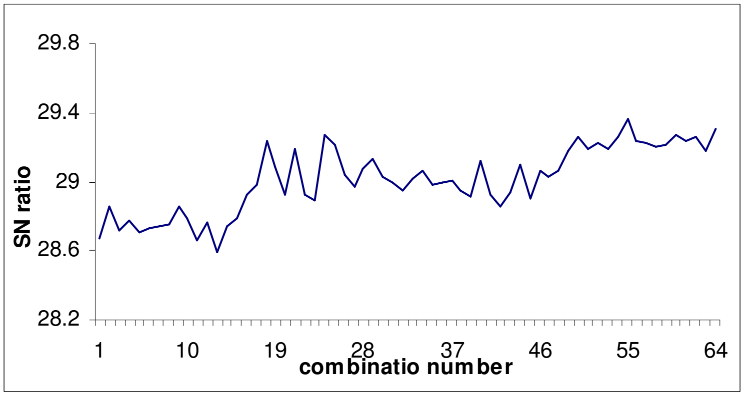


Fig. 6.33 SN ratio using Monte Carlo Simulation for Quintic Trajectory - case (iii)

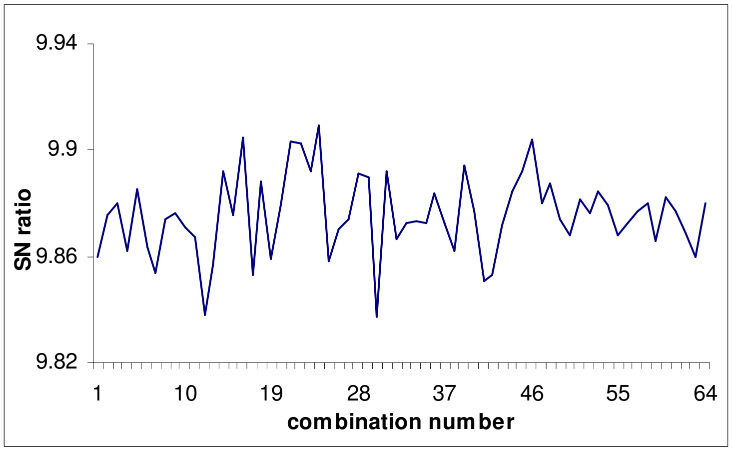


Fig. 6.34 SN ratio using Monte Carlo Simulation for Quintic Trajectory - case (iv)

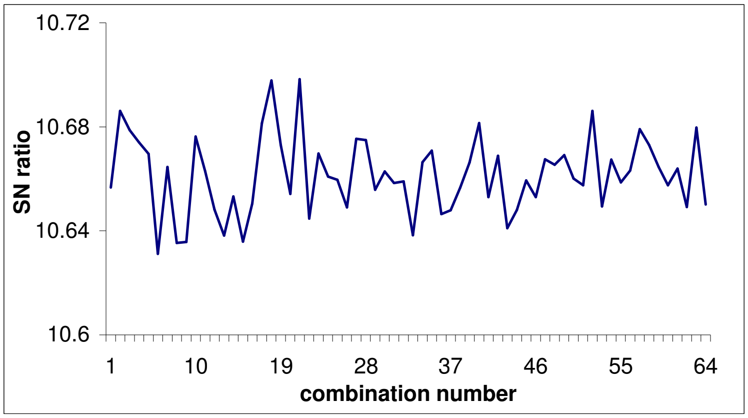


Fig. 6.35 SN ratio using Monte Carlo Simulation for Quintic Trajectory - case (v)

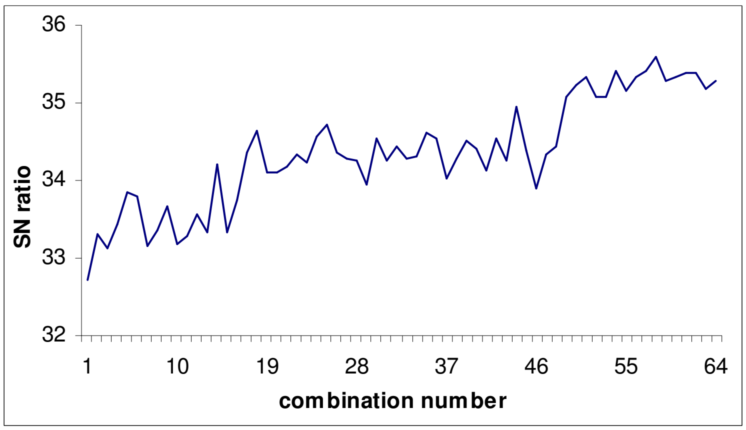


Fig. 6.36 SN ratio using Monte Carlo Simulation for Quintic Trajectory - case (vi)

From Fig. 6.31 for case (i), Fig. 6.32 for case (ii) and Fig. 6.33 for case (iii) the SN ratio values are observed to lie between 27.8 dB to 28.6 dB, 28 dB to 29 dB, 28.6 dB to 29.3 dB respectively. Similarly, in Figs. 6.34 and 6.35, it is observed that SN ratio values range between 9.84 to 9.9 dB and 10.64 to 10.68 dB for the cases (iv) and (v) respectively. From Fig. 6.36 for case (vi) the SN ratio values is observed to lie between32.72 dB to 35.2 dB.

An important feature of all these graphs is that there is improvement in performance as combination number increase. Above all performances observed, for cases (iv) and (v) following cubic as well as quintic trajectories are poor as compared to the cases (i), (ii), (iii) and (vi). The reason being the SN ratios obtained in these cases are less as compared to rest four cases.

6.5.4 Discussion

As it is known that, the use of tighter tolerance combinations for kinematic and dynamic parameter lead to very less performance variations but then manufacturing cost becomes prohibitively high. The strategy should be to tighten the tolerances of statistically significant parameters and widen tolerances of insignificant parameters. Once the values have been determined, the tolerances can be further fine-tuned to make the end effector performance better and reduce overall cost. The simulation results of different experiments are analyzed below.

It important to observe that in almost all the cases the SN ratio values are close to each other. There are nearly 16 combinations that are almost same or close and more importantly all the 64 combinations can be categorized into four classes of performance. This indicates that the designer has multiple solutions available at hand and based on manufacturing considerations, designer can choose to tighten or loosen the parameter tolerance, rather than tightening all the parameter tolerances. Similar results are obtained for the tasks following quintic trajectory. Another feature which comes out from the analysis of cross array experimentation strategy is the similarities in the performance of cases (i), (ii), (iii) and (vi).The performances in cases (iv) and (v) are similar in both the trajectories.

From analysis of experiment results it can be observed that irrespective of trajectory followed for cases (i), (ii) and (iii) computed *F* values are greater than tabulated values for $m_2, \tau_2, \tau_2 m_2$ parameters and their interactions respectively. Similarly results are observed for the cases (iv) and (v) irrespective of trajectory followed, factor m_2 is statistically significant. Only exception is in case (vi) where factor τ_2 is statistically significant when following cubic trajectory and factors τ_1 and τ_2 are significant when following quintic trajectory. This indicates that the motion following either cubic or quintic time law, has same impact on the performance variation, though the quintic time law is less jerky as compared to the cubic time law. The computed *F* values for (F_o) $l_1, l_2,\ m_1$ and τ_1 are found to be less as compared to tabulated *F* values and hence tolerance on parameters i.e. $l_1, l_2,\ m_1$ and τ_1 are insignificant and can be widened in both the cases without affecting manipulator performance to get overall reduction in cost of manipulator. These indications from ANOVA are amply clear in half normal plot. In half normal plot (Figs. 6.13-6.18 and 6.19-6.24) statistically significant parameters are clearly indicated. So results in plot complements the results provided in ANOVA.

It is observed that there is an increase in SN ratio as combination number increases. For combination number 50 onwards performance measures are always best among all the combinations considered. It can also be observed that the ranges within which SN ratios vary are small. This low variation is indicative of a situation where only one or two parameters are responsible for performance variations. This validation procedure complements the conclusions drawn from ANOVA and half normal plots. The SN ratios obtained from cross array experimentation approach, are smaller and the reason for this is the worst-case strategy adopted to reduce the amount of computations. The numbers of evaluations in former case are 64×8 as compared to 64×1000 evaluations in latter case. However, considering competing optimal solutions, the optimal tolerance combinations numbers are observed to be same in both the cross array experimentation and Monte Carlo simulations. Summary of ANOVA and optimal SN ratio for cross array experimentation and Monte Carlo simulation are provided in Table 6.17.

Table 6.17 Summary of ANOVA and SN ratio Results

Trajectory	Case	Statistically Significant Factors from ANOVA	Optimal SN ratio by Cross Array Experimentation (dB)	Optimal SN ratio by Monte Carlo Simulation (dB)
Cubic	(i)	D, F and DF	16.54	27.6
	(ii)	D, F and DF	16.57	30.5
	(iii)	D, F and DF	16.58	34
	(iv)	D	11.31	12.2
	(v)	D	9.9	12.85
	(vi)	F	15.56	18.6
Quintic	(i)	D, F and DF	17.18	28.6
	(ii)	D, F and DF	17.05	29
	(iii)	D, F and DF	16.79	29.3
	(iv)	D	9.26	9.9
	(v)	D	9.95	10.68
	(vi)	E and F	18.58	35.2

Referring to the results in the Table 6.17, important conclusions which can be drawn are that irrespective of the trajectory followed same factors and factor interactions are statistically significant but from task to task, factors responsible for performance variations are different. Assuming that the manipulator is intended to perform all the tasks, then designer can tighten the tolerances of factors D, E and F to achieve still lesser performance variations.

6.6 PARAMETRIC TOLERANCE SENSITIVITY

The statistical results obtained from simulation may be questioned for intentional incorporation of noise in control factors therefore; another study is conducted to corroborate the results. In this investigation, the contribution of individual parameter tolerances on performance variation of manipulator has been determined. The simulations are run independently for different tolerances of individual kinematic and dynamic parameters keeping all other parameters fixed at a particular value. It is similar to running experiment for one factor at a time. The tolerance set values assumed for parametric tolerance sensitivity are given in Table 6.18. The ten values of tolerance for a parameter are obtained as a multiple of tight tolerance of the parameter in Table 6.1.

The performance measure chosen for sensitivity study is SN ratio. For simulating the performance procedure discussed in Chapter 4 was used with the exception that parameter tolerance combinations at different levels are replaced by tolerance sets for each kinematic or dynamic parameter. The performance variations contributed by each set of parameter tolerance is captured by running the simulation for thousand times, for manipulator following cubic and quintic trajectories. The influence of different parameter tolerance sets on manipulator performance following cubic trajectory are shown in Figs. 6.37-6.42. Similarly, the influence of different parameter tolerances on manipulator performance following quintic trajectory are shown in Figs. 6.43-6.48.

Table 6.18 Tolerance Sets for Parameter Tolerance Sensitivity

Parameter	Tolerance set									
	1	2	3	4	5	6	7	8	9	10
l_1 (m)	$\pm 7.5\times10^{-5}$	$\pm 15\times10^{-5}$	$\pm 22.5\times10^{-5}$	$\pm 3\times10^{-4}$	3.75×10^{-4}	4.5×10^{-4}	5.25×10^{-4}	$\pm 6\times10^{-4}$	$\pm 6.75\times10^{-4}$	$\pm 7.5\times10^{-4}$
l_2 (m)	$\pm 7.5\times10^{-5}$	$\pm 15\times10^{-5}$	$\pm 22.5\times10^{-5}$	$\pm 3\times10^{-4}$	3.75×10^{-4}	4.5×10^{-4}	5.25×10^{-4}	$\pm 6\times10^{-4}$	$\pm 6.75\times10^{-4}$	$\pm 7.5\times10^{-4}$
m_1 (kg)	$\pm 3.75\times10^{-3}$	$\pm 7.5\times10^{-3}$	$\pm 11.25\times10^{-3}$	$\pm 15\times10^{-3}$	$\pm 18.75\times10^{-3}$	$\pm 22.5\times10^{-3}$	$\pm 26.25\times10^{-3}$	$\pm 30\times10^{-3}$	$\pm 33.75\times10^{-3}$	$\pm 37.5\times10^{-3}$
m_2 (kg)	$\pm 3.75\times10^{-3}$	$\pm 7.5\times10^{-3}$	$\pm 11.25\times10^{-3}$	$\pm 15\times10^{-3}$	$\pm 18.75\times10^{-3}$	$\pm 22.5\times10^{-3}$	$\pm 26.25\times10^{-3}$	$\pm 30\times10^{-3}$	$\pm 33.75\times10^{-3}$	$\pm 37.5\times10^{-3}$
τ_1 (Nm)	$\pm 3.75\times10^{-2}$	$\pm 7.5\times10^{-2}$	$\pm 11.25\times10^{-2}$	$\pm 15\times10^{-2}$	$\pm 18.75\times10^{-2}$	$\pm 22.5\times10^{-2}$	$\pm 26.25\times10^{-2}$	$\pm 30\times10^{-2}$	$\pm 33.75\times10^{-2}$	$\pm 37.5\times10^{-2}$
τ_2 (Nm)	$\pm 3.75\times10^{-2}$	$\pm 7.5\times10^{-2}$	$\pm 11.25\times10^{-2}$	$\pm 15\times10^{-2}$	$\pm 18.75\times10^{-2}$	$\pm 22.5\times10^{-2}$	$\pm 26.25\times10^{-2}$	$\pm 30\times10^{-2}$	$\pm 33.75\times10^{-2}$	$\pm 37.5\times10^{-2}$

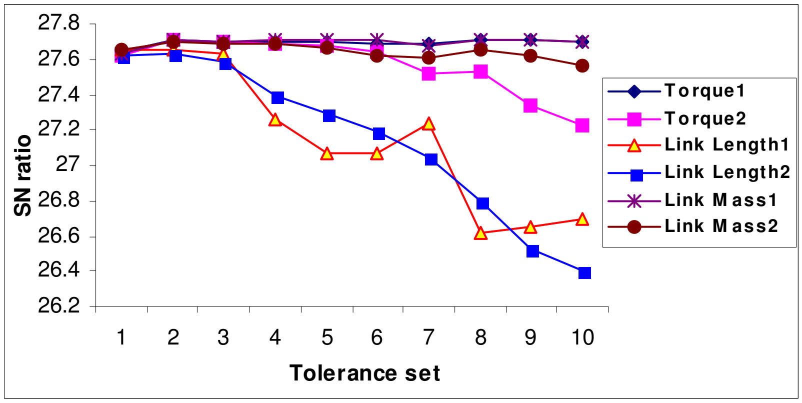


Fig. 6.37 Manipulator Parameter Tolerance Sensitivity for Cubic Trajectory - case (i)

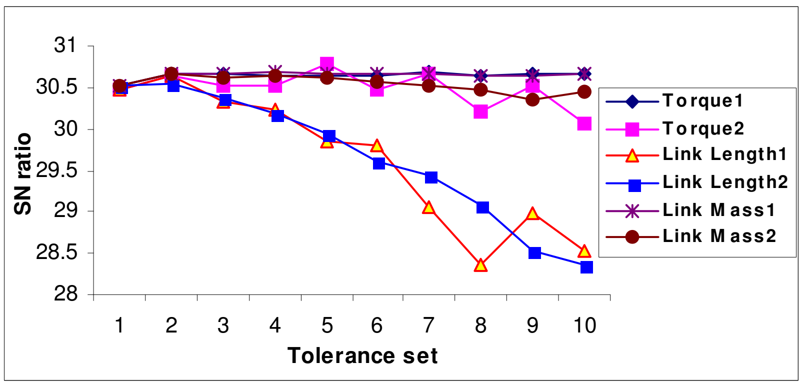


Fig. 6.38 Manipulator Parameter Tolerance Sensitivity for Cubic Trajectory - case (ii)

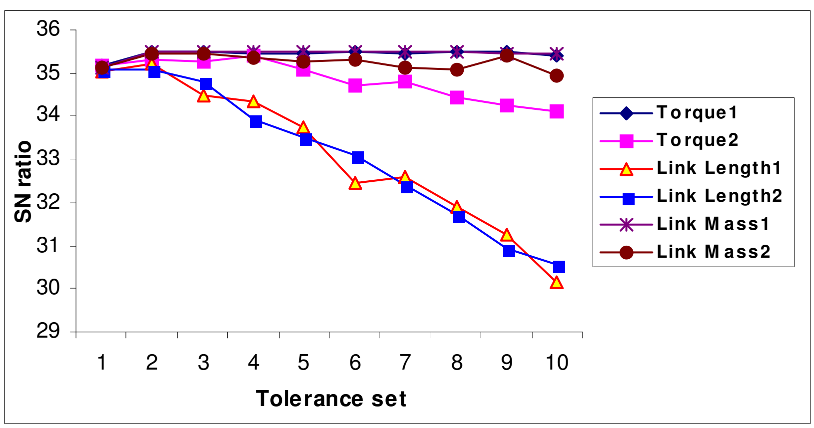


Fig. 6.39 Manipulator Parameter Tolerance Sensitivity for Cubic Trajectory - case (iii)

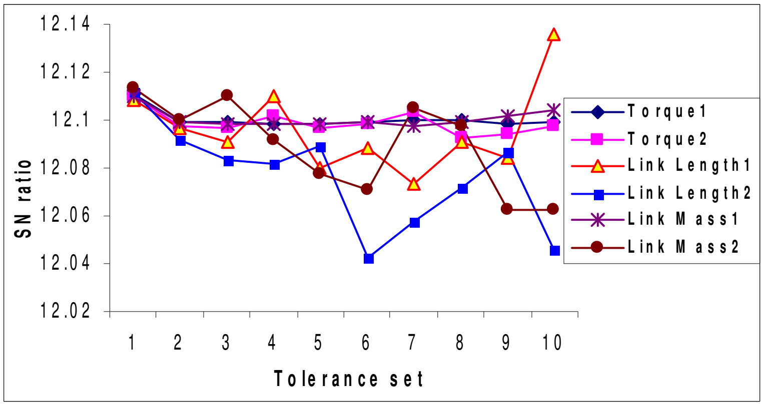


Fig. 6.40 Manipulator Parameter Tolerance Sensitivity for Cubic Trajectory - case (iv)

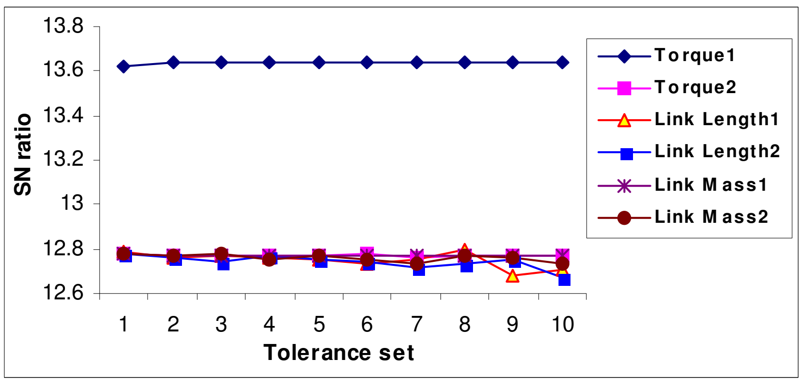


Fig. 6.41 Manipulator Parameter Tolerance Sensitivity for Cubic Trajectory - case (v)

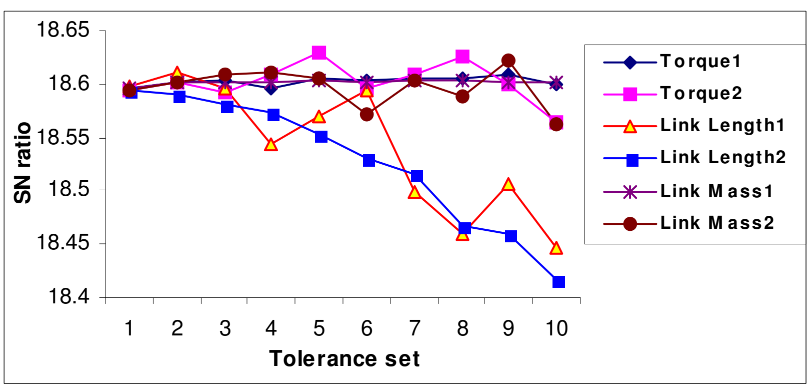


Fig. 6.42 Manipulator Parameter Tolerance Sensitivity for Cubic Trajectory - case (vi)

From Fig. 6.37 for case (i), it is observed that performance variations increase as parameter tolerances increase, resulting in decrease in SN ratio. The maximum SN ratio value is 27.6 dB and minimum value is 26.4 dB. The variations in performance is pronounced when there is an increase in tolerance of link lengths. But increase in tolerance of joint torque has less impact on performance variations. Except the increase of three factors tolerance value, rest of the three factors has negligible impact on performance variations. Similarly, from Fig. 6.38 for case (ii), it is observed that maximum SN ratio value is 30.5 dB and minimum value is 28.3 dB. As discussed the variations in performance is observed to be more for increase in tolerance value of link lengths. Increase in tolerance of torque at joint has moderate impact on performance variations. As shown in Fig. 6.39 for cases (iii), maximum SN ratio value is 35.05 dB and minimum value is 30 dB. The trends observed are similar to cases (i) and (ii). From Fig. 6.40 for case (iv), the maximum SN ratio value is found to be 12.10 dB and minimum value is 12.02 dB. The trends observed are totally different in comparison with cases (i), (ii) and (iii). Referring to graphs, it can be seen, all factor tolerances contribute significantly to performance variations. This is highlighted by low SN ratio values. But interestingly the range of SN ratio variations are less with increase in tolerance values for most of the factors. As shown in Fig. 6.41 for case (v), the maximum SN ratio value is observed to be 13.6 dB and minimum value is 12.08 dB. The trends observed are totally different in comparison with all the above cases. Increase of factor tolerance has very less influence on performance variations. From Fig. 6. 42 for case (vi), the maximum SN ratio value is observed to be 18.6 dB and minimum value is 18.41 dB. The trends observed are similar to cases (i), (ii) and (iii). There is deterioration of performance variation with increase of link length tolerances. Similar to above, the impact of individual tolerances on performance variations is investigated while manipulator performs a task following quintic trajectory.

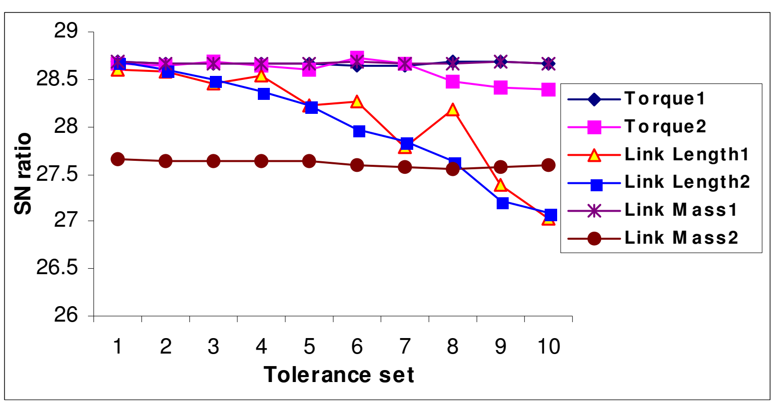


Fig. 6.43 Manipulator Parameter Tolerance Sensitivity for Quintic Trajectory - case (i)

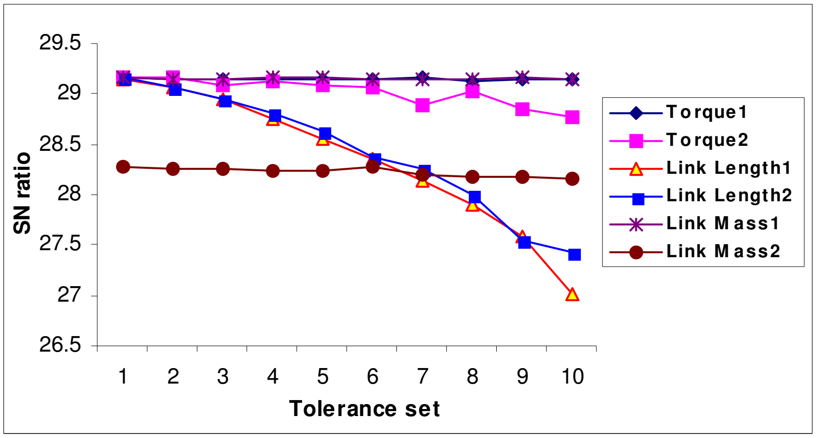


Fig. 6.44 Manipulator Parameter Tolerance Sensitivity for Quintic Trajectory - case (ii)

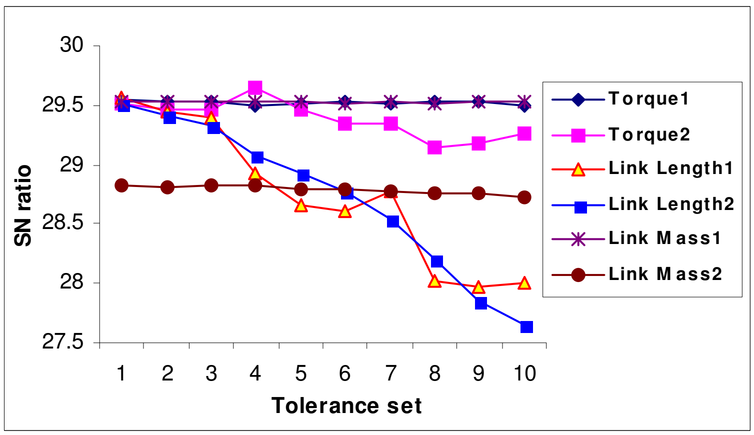


Fig. 6.45 Manipulator Parameter Tolerance Sensitivity for Quintic Trajectory - case (iii)

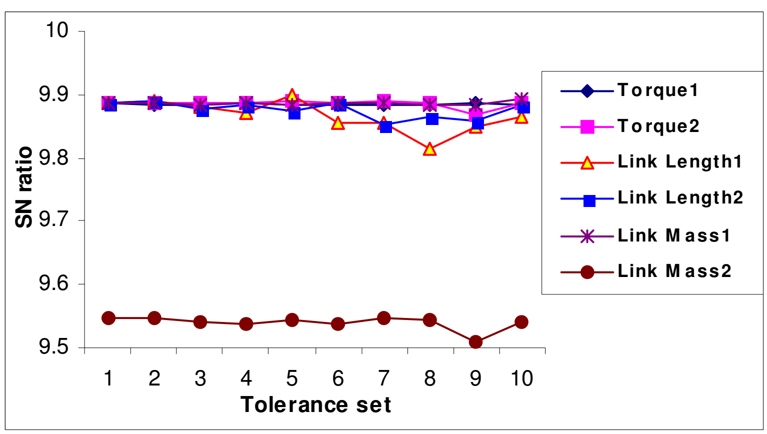


Fig. 6.46 Manipulator Parameter Tolerance Sensitivity for Quintic Trajectory - case (iv)

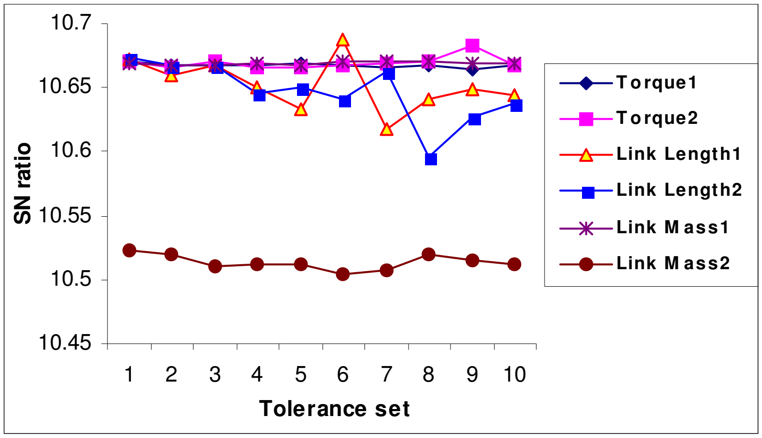


Fig. 6.47 Manipulator Parameter Tolerance Sensitivity for Quintic Trajectory - case (v)

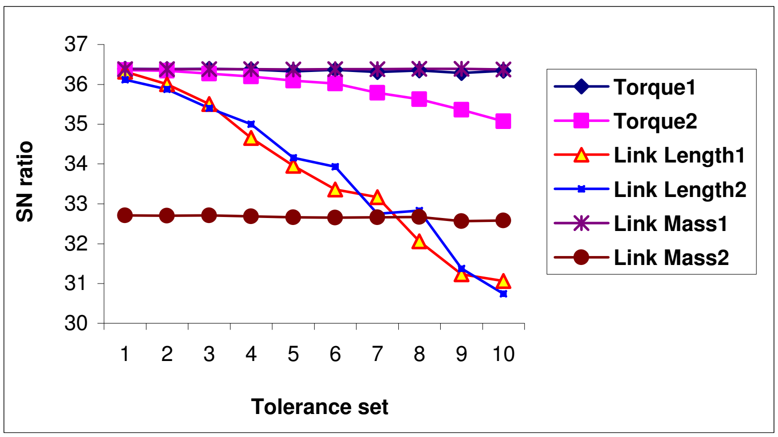


Fig. 6.48 Manipulator Parameter Tolerance Sensitivity for Quintic Trajectory - case (vi)

As shown in Fig. 6.43 for case (i), the maximum SN ratio value is observed to be 28.6 dB and minimum value is 26.8 dB. Link length one and two show increase in performance variations with increase in tolerance values and link mass two show poorest performance amongst the factors considered. As seen from Fig. 6.44 for case (ii), the maximum SN ratio value is observed to be 29.2 dB and minimum value is 27 dB. It is seen from Fig. 6.45 for case (iii), the maximum SN ratio value is 29.5 dB and minimum value is 27.6 dB. This case show similar trend as shown in cases (i) and (ii). In Fig. 6.46 for case (iv), the maximum and minimum SN ratio value is observed to be 9.9 dB and 9.5 dB respectively. All the parameter tolerance value has high influence on performance variations. Amongst the parameters considered link mass two has significant influence. From Fig. 6.47 for case (v), the maximum SN ratio value is observed to be 10.675 dB and minimum value is 10.52 dB.

Link mass two has maximum influence on performance variations. In Fig. 6.48 for case (vi), the maximum and minimum SN ratio value is found to be 36.3 dB and 30.5 dB respectively. This case show similar trend in comparison with cases (i), (ii) and (iii).

6.6.1 Discussion

For the task following both cubic and quintic trajectories, link length tolerances have the maximum influence on performance variations (Figs. 6.37-6.39 and 6.43-6.45). Similar observation can be made for case (vi) (Figs. 6.42 and 6.48). It is observed that SN ratio is not significantly sensitive to increase in tolerances on link mass and torque individually. It is important to observe that in cases (iv) and (v) (Figs. 6.40-6.41 and 6.46-6.48) none of the factor tolerance increase leads to deterioration of performance.

Therefore, from these graphs, it can be inferred that individual control factor tolerances do not contribute significantly to performance variations. This analysis points out to the possibilities of wrong conclusions drawn form one factor tolerance at a time experimentation analysis. The tolerances on link lengths contribute the most to the performance variation because of the direct sensitivity of geometric length parameter on the positional error. The values of SN ratios are less for cubic trajectory compared to quintic trajectory because the cubic trajectory is quite jerky compared to quintic trajectory. The reason for jerky motion is due to sudden acceleration and deceleration in case of cubic trajectory as compared to quintic trajectory.

6.7 EPILOGUE

This chapter discusses the methodology used to select tolerance of manipulator kinematic and dynamic parameters that give minimum performance variation. To investigate the dynamic behavior physical model error such as link lengths, link masses and joint torques are examined. To simulate the real life performance of manipulator discussed, probabilistic approach in Chapter 4 has been used to account for the effects of manipulator design parameters and process parameter uncertainties. The present work uses statistical technique to find optimum combination of tolerances for manipulator performing task using different trajectories. To validate the results obtained from the experimental technique, performance measure SN ratio is utilized. It can be noted that the proposed experimental technique is computational in nature. This technique is useful for robot system engineering before initial construction of manipulator begins. The advantages of proposed method are that it does not involve any capital investment in equipment and in process. In addition, above experimental design technique requires fewer computations as compared to Monte Carlo simulation. One important conclusion drawn is that the statistically significant factors and interacting factors remain same irrespective of type of trajectory time law. The approach will assist robotic system designers in making decisions regarding the tolerances of parameters before a costly manufacture of manipulator is undertaken.

CHAPTER-7

OPTIMAL MANIPULATOR PARAMETER SELECTION USING HYBRID DIFFERENTIAL EVOLUTION TECHNIQUE

7.1 INTRODUCTION

The optimal design of robotic manipulator parameters to satisfy the desired performance requirement to perform a task is complex. This chapter describes the optimization method developed for selecting optimal parameters of manipulator to perform a task with minimal performance variability under the real world uncertainties. This optimization problem is solved by making the search space finite through discretization. It is known that the order of the optimization problem is compounded by each parameter. Even for relatively low dimensional problems searches are difficult to complete in a reasonable amount of time if an all-inclusive search is attempted. Developed approach attempts to provide optimal solution with fewer computations.

When optimization of robot parameters is attempted using conventional optimization techniques, major hindrance comes in form of the way these techniques work. The disadvantages are the requirements of objective function in terms of decision variables and use of point-to-point local information to decide which point to explore next. These disadvantages lead to the premature convergence of optimization process at a local optimum. In addition, many of the conventional search techniques require specific knowledge of the problem to be analyzed, for example, gradient-based techniques require derivative information and a good initial guess. However, there are many efficient search methods available, but most of them are incompatible and inapplicable to robot manipulator design for optimal performance problem. The formulations of objective functions in terms of decision variables are difficult and attempted by few researchers. Even the kinematic and dynamic models of manipulator, used to simulate the real life performance are coupled and non-linear. To overcome above shortcomings, evolutionary based techniques are employed in place of conventional optimization methods. Though evolutionary techniques do not require formal objective functions in terms of decision variables, but it is known for its inefficiency to handle stochastic nature of performance. Therefore, to optimize the performance a novel approach is proposed. This chapter focuses on selection of parameter, namely the determination of optimum design such that the effects of noise factors on performance are less. To incorporate the effect of noise in performance a novel method was discussed in Chapter 4. The consequence of this simulation procedure is that the performances become random, therefore the performance measure to be optimized can not be called global optimum. For this reason, a novel approach is developed by which the optimization problem will have a global optimum and will not vary wildly based on number of simulation run.

Evolution based techniques are employed to overcome the shortcoming of conventional optimization methods. Genetic algorithm and their variants are some of the evolutionary techniques and have been extensively used for modular robot design, inverse and forward kinematics, calibration and optimum motion and path planning. Differential Evolution (DE) is a recently developed evolutionary approach proposed by Storn [Storn 1997] for minimizing nonlinear and non-differentiable continuous space functions. Later Storn with other researchers [Price 2005] presented a generalized algorithm on DE to optimize variety of problems. It has been successfully applied to the optimum design of digital filter and communication control, and many other diverse domains such as batch fermentation process and many more.

The proposed technique is a hybrid approach, which combines Differential Evolution (DE) optimization technique and orthogonal array (OA) of the Taguchi method. The OA helps in simulating and optimizing the performance incorporating effect of noise. The hybrid approach uses all genetic operators of differential evolution with one modification. The novelty in approach is the evaluation procedure used for the cost function. In this approach, evaluation of the entire population member is carried out with the help of hyper-rectangular design space created by OA. The improvement of parameter design in successive generations has been achieved by conducting OA experiments to reduce the mean positional error of a pool of candidate designs from one generation to the next. The proposed hybrid evolutionary optimization technique has been applied to optimal parameter deign of 2-DOF RR planar manipulator for determining optimum performance based on task specifications. The steps required to obtain the optimal parameters for optimal performance incorporating effect of noises are discussed and the results obtained are discussed. The objective of the developed parameter optimization problem is to minimize the mean positional error considering kinematic and dynamic models of manipulator for a specified task subject to reachability constraints.

The rest of this chapter is organized in five sections. In section 7.2, optimization approaches required in the chapter are discussed. Hybrid evolutionary technique for optimal parameter design of manipulator is presented in section 7.3. In section 7.4, optimal parameter design of 2-DOF RR planar manipulator using hybrid evolutionary technique are discussed. The design parameters assumed for the simulation and results of the simulations are presented in section 7.5.

7.2 OPTIMIZATION APPROACHES

The goal of optimization is to find the values of the variables of the product that delivers the best performance criterion. Typical real life problems have many solutions. Optimization is concerned with selecting the best among the entire set by efficient quantitative methods. To optimize design parameters for reduced performance variations, hybrid evolutionary optimization technique is used. The distinctive feature of technique is the way objective function (cost function) is evaluated. To develop optimization technique suitable for optimal parameter that delivers optimized performance, hybrid of DE with OA used in the Taguchi method are made. To explain the novelty of the proposed method, first important features and working principle of DE are explained and subsequently proposed hybrid approach is discussed.

7.2.1 Differential Evolution Technique

DE is a search and optimization technique, which works differently, compared to classical search and optimization methods. DE is increasingly applied to various search and optimization problems in the recent past. Unlike GA that uses binary coding for representing design parameters, DE uses real coding of floating point numbers. The advantage of DE is its simple structure, ease of use, speed and robustness [Storn 1995]. Storn gave the working principle of DE. The key parameters to be used for a problem are to be determined by trial and error.

A cost function, C is used to rate the individual vector according to their capability to minimize the objective function. The genetic operation of mutation in the DE uses the vector differentiation method (adding the weighted difference between two population vectors to the third vector) to generate a new vector. DE is a parallel search method that operates on *D* dimensional parameter vectors. The number of vectors is equal to user defined population size. The initial vector population is chosen randomly. The DE process starts from selecting a target vector. Then, it randomly selects two other vectors and generates a difference vector, which is multiplied with a user defined weighting factor F to obtain 'weighted difference vector'. The weighted difference vector and randomly chosen mutation vector are added to create a noisy vector, which is subjected to crossover process with the target vector in order to generate the trial vector. The cost function of trial vector is then compared with the cost function of original target vector. The vector having low cost function is allowed into the new population. The algorithm used for the optimization is discussed below.

7.2.2 Algorithm for Differential Evolution Technique

The key parameters for differential evolution technique are selected by trial and error. The key parameters of control are:

NP - the population size, CR - the crossover probability, F - the weight applied to random differential (scaling factor). The detailed DE algorithm used for D number of design variables, is given below.

Step 1. Initialize the value of D, NP, CR, F and Number of generation.

Step 2. Initialize all the vector population randomly in the given upper and lower limit.

Step 3. Evaluate the cost C of each vector.

Step 4. Perform mutation, crossover, selection and evaluation of the objective function for a specified number of generations.

(a) For each vector x_t (target vector), select three distinct vectors x_a, x_b, and x_c randomly from a current population other than vector x_t.

(b) Generate difference vector $x_d = (x_a - x_b)$

(c) Multiply weighted factor F to difference vector to obtain 'weighted difference vector' i.e. $\mathrm{F} \times x_d$

(d) Perform mutation by adding weighted difference vectors to the third vector x_c to get noisy vector $x_n = \mathrm{F} \times (x_a - x_b) + x_c$

(e) Perform crossover with probability CR for each target vector with noisy vector to create a trial vector

(f) Perform selection for each target vector, x_t by comparing its cost with that of the trial vector. For minimization problem, vector with lower cost is selected for next generation.

(g) Repeat the procedure by selecting the next target vector of the population.

Step 5. Check for termination criteria, if satisfied stop or otherwise repeat the same procedure with the new generation

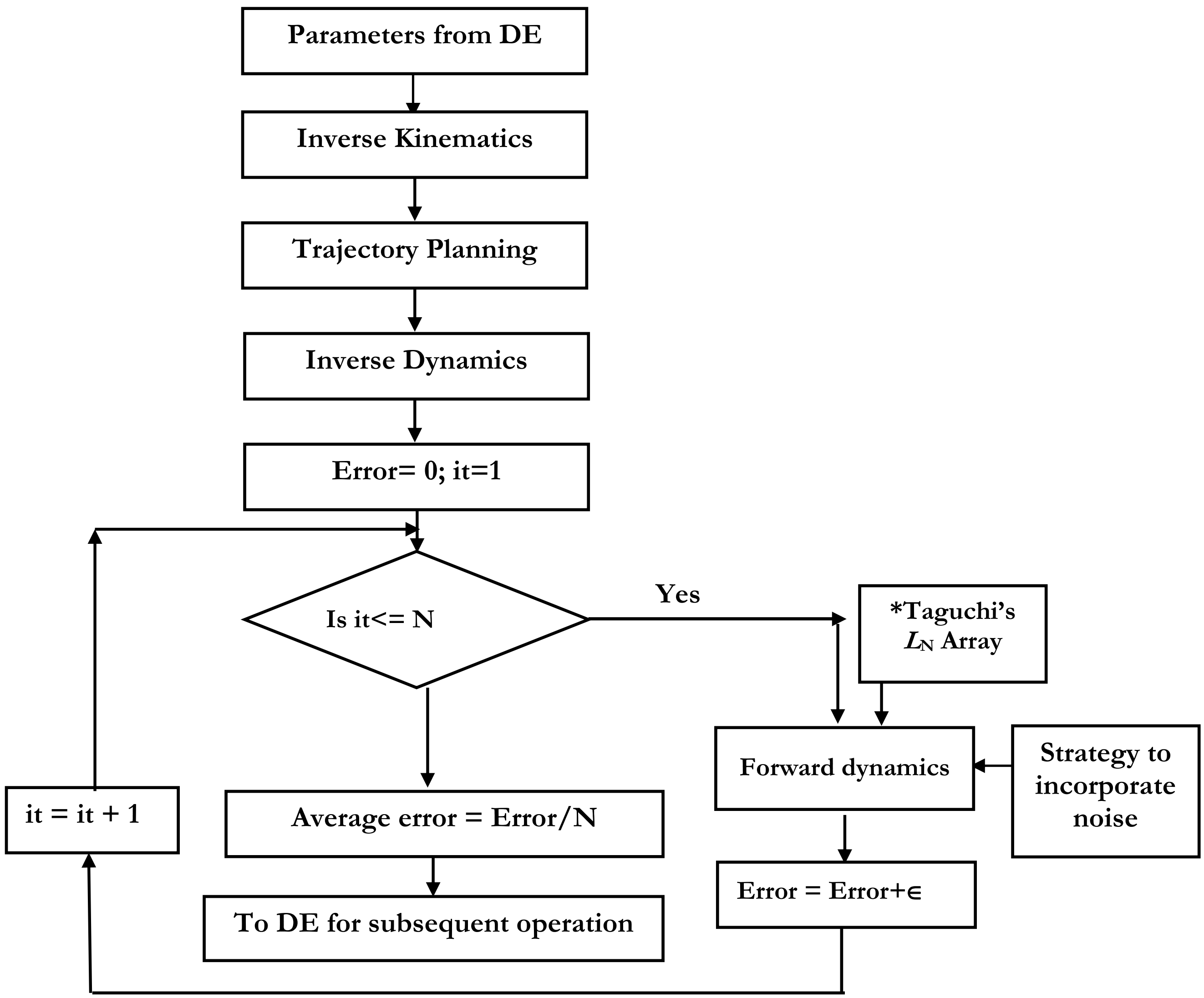
In DE technique, the function f to be optimized is called an objective function, or cost function C. A cost function, C is used to rate the individual vectors according to their capability to minimize/maximize the objective function f.

7.2.3 Hybrid Differential Evolution Approach

From the DE technique it is evident that, the cost function evaluation of members of population is carried out once indicating its suitability for deterministic optimization problems. Because this investigation focuses on reducing the performance variations of the manipulator due to noises while performing a task and the performances vary from one experiment to other. Therefore, to capture the variability of performance in optimization process a hybrid approach is proposed. In this hybrid approach, OA proposed by Taguchi for planned experimentation is coupled with the DE optimization technique which results in a hybrid DE approach. The proposed approach is described below and can be used to optimize the processes where the effect of noise is considered.

The performance variations are attributed to noises in design and process parameters, and to model these noises it is assumed that noise factors follow particular probability distribution function. Therefore, the cost function to be optimized will follow a combined probability distribution function and finding its optimum will be difficult. For this reason, in place of probability distribution of noise parameters only uncertainties ranges (levels) are considered for the design and process parameters (factors). To incorporate effect of noise, tolerances associated with the nominal dimensions of each component in the assembly are considered. For instance, any deviation in design parameters would result not in single-point designs but rather in hyper-rectangular design region where each manipulator performance change over a finite set of values. As opposed to single point designs where every population member results in a particular performance, each candidate design produces a range of performance.

To make it possible, systematic use of OA is planned. By use of hyper-rectangular space created by OA, variability in that domain is captured. This evaluation procedure in hybrid evolutionary technique provides ‘worst case’ scenario for determination of performance variations. Design points (corner point) in hyper rectangular region are evaluated for performance and subsequently obtained performances are transformed to provide cost function. Successively the designs evolve by comparing cost function for each members of population. The strategy proposed to handle effect of noise is discussed below and shown with the help of a flowchart in Fig 7.1.



*for incorporating effect of noise

Fig. 7.1 Flow Diagram for the Proposed Hybrid Evolutionary Approach

The systematic method to incorporate effect of noises in design and process parameters by use of OA is discussed below. An OA is a fractional factorial design which assures a balanced comparison of levels of any factor or interaction of factors. It is a matrix of numbers arranged in rows and columns where each row represents the level of the factors in each run, and each column represents a specific factor that can be changed from each run. The array is called orthogonal because all columns can be evaluated independently of one another. The advantage in use of OA is that it allows the evolutionary technique to sample only a fraction of the design space which is efficient as compared to other stochastic optimization methods, such as Monte Carlo simulation, which depend on large number of sample point to ensure high accuracy. In the proposed method, each population member creates a hyper-rectangular space by use of OA. Thereafter, corner point of the hyper-rectangular space is evaluated for the performance. Subsequently all the performances obtained are transformed to mean positional error. This becomes the cost function of the population member. The choice of the cost function for the evolutionary technique is an important aspect, which directs the algorithm towards the desired target response region. Furthermore, the cost function should not only be able to asses the relationship exists between the nominal design specifications, but it should also consider variability associated with the simulated performances. Among many candidate designs in such feasible regions, the evolutionary approach attempts to evolve selectively for designs which have minimum performance variations. Therefore, to have minimum variability in performance mean positional error is chosen as the cost function. Finally, the mean positional error of the candidate design is sent to DE for further processing.

There is no strong mathematical proof of convergence that the evolutionary technique will find the global optimum. In addition, it is not known yet, which is the best way to terminate the algorithm. In many cases, maximum number of generations are fixed in advance and the optimization process terminates when this criteria is meet. However, predetermination of maximum number of generations implies that the duration of the evolutionary search is fixed, irrespective of the search success.

7.3 OPTIMAL DESIGN OF MANIPULATOR PARAMETERS

Traditional methods of experimenting with various prototypes are often expensive and are not always successful in complex systems. The developed methodology is a viable alternative to the costly prototype testing because for optimal parameter design, help of kinematic and dynamic models of manipulator are utilized to simulation real life performance. Using these mathematical models and optimization technique optimal design parameters are selected to minimize performance variability.

In order to find optimum manipulator kinematic and dynamic parameters that give minimal performance variations for performing a task, specified as the required pose of the end-effector. Assuming $P(x_f, y_f, z_f)$ be the coordinate of the point which manipulator is required to reach by manipulator following a trajectory and $P_i(x_a, y_a, z_a)$ be the actual coordinate reached in i^{th} experiment. To express this performance deviation at the destination for each experiment ε_i defined in Chapter 3 is used. For computation of positional error (ε_i) equation (3.16) is used. To express the overall performance deviation for N experiments while performing a task the mean positional error $\bar{\varepsilon}$ is defined in Chapter 3 is taken as the objective function. To compute $\bar{\varepsilon}$ equation (3.18) is used. The mean positional error ($\bar{\varepsilon}$) from each design are to be minimized. Hence, the objective function is expressed mathematically as:

$$\text{Minimize } f(x) = \bar{\varepsilon} \tag{7.1}$$

The constraint on the optimization problem is to include physical constraints and the structural characteristics of each link. On this optimization problem the minimum and maximum values of the link parameters (link length and mass), and reachability of the manipulator constraints are imposed. The limits on the variables are specified as:

$$l_{il} \le l_i \le l_{iu} \tag{7.2}$$

$$m_{il} \le m_i \le m_{iu} \tag{7.3}$$

where, l_{il}, m_{il} and l_{iu}, m_{iu} are the lower bounds and the upper bounds of the length and mass of link i, respectively. The constraints on the joint limits or range of motion of the manipulator are imposed due to physical constraints as:

$$\theta_{i,\min} \le \theta_i \le \theta_{i,\max} \tag{7.4}$$

where θ_i is the joint variable for joint i and $\theta \in R^n$ is the real valued joint variable with n being the number of joints. Above all it is assumed that the required torque at the joints to perform the specified task is available without any restriction.

7.3.1 Procedure for Robot Parameter Optimization

The formulation of the optimization problem is already discussed, where, objective function of the problem, the design parameters and the constraints are identified. For the manipulator parameter optimization problem, the desired start and destination point in Cartesian space, time for the motion and type of trajectory and control parameter bound are the information required.

The design parameters are randomly generated in DE routine and values are checked for any constraint violation. If constraints are not violated then design parameters are used to evaluate the objective function (cost function). These evaluations are used in the optimization routine where new values for the design variables are generated. One function evaluation is completed when one set of design variables is analyzed. For evaluating the cost function discussed novel strategy in section 7.2.3, is used. With the help of orthogonal array, noises are incorporated in design parameter to generate several design combinations. Each design combination is equivalent to one corner point of the hyper-rectangular region created by OA.

These designs points are utilized to simulate the performance i.e. positional error at the target point. For simulating the performance steps followed are inverse kinematics – where joint coordinates at the start and destination point are computed, joint space trajectory generation – where depending on trajectory joint positions, joint velocities and joint accelerations are computed, inverse dynamics – where torque required at the joints are computed, incorporate effect of noise where control parameters with noise are computed, forward dynamics – where parameter with noise are used to compute the joint positions, joint velocities and joint accelerations with effect of noise, and lastly forward kinematics – where actual position reached by the manipulator is computed. Then this response is used for computation of positional error. By this process performance of one design combination i.e., one corner point of OA is evaluated. When all the corner point of OA is evaluated by the discussed process, taking all the responses of designs i.e. positional error, mean positional error is computed. Then the mean positional error obtained becomes the cost function for one population member. Subsequently all population member are evaluated to get cost function. And these cost functions provide value for comparison of design variables in optimization process. This process is repeated until criteria for number of generations are met.

The implementation procedure for the manipulator performance optimization is discussed in Fig. 7.2. This figure provides the steps used to get optimal manipulator parameters.

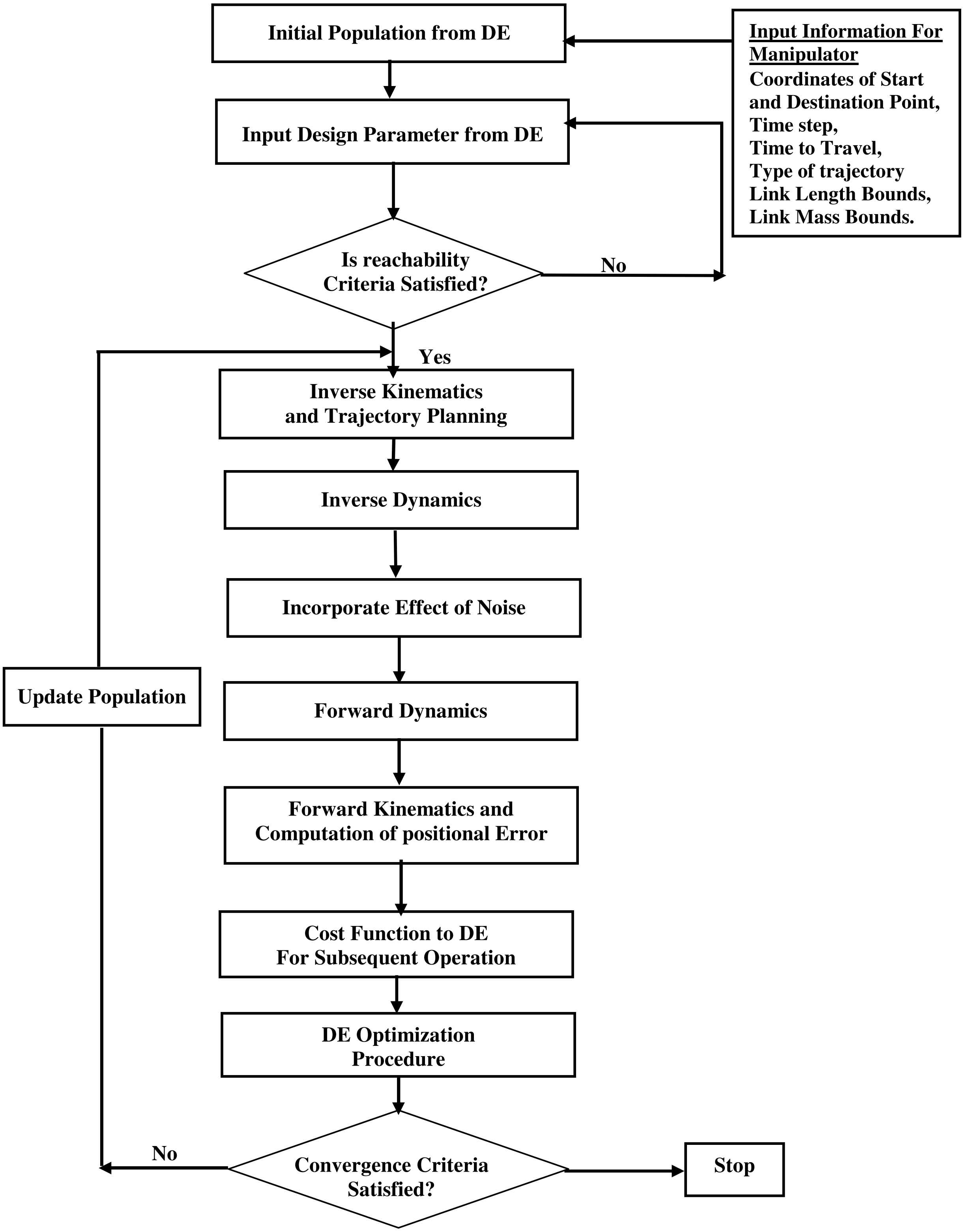


Fig 7.2 Flowchart for Robot Parameter Optimization

7.4 PARAMETER DESIGN OPTIMIZATION OF 2-DOF RR PLANAR MANIPULATOR

The applications of proposed hybrid approach for optimal parameter design of 2 DOF RR planar manipulator has been discussed. The manipulator models, kinematics, inverse kinematics and dynamic analyses for the manipulator considered are based on the DH parameters. The optimization problem for the 2-DOF RR planar manipulator has been formulated as given in equation (7.1). The positional error ε_i of the above manipulator is computed using equation (3.17). For mean positional error computation equation (3.18) is used. Subject to constraints discussed in equations (7.2) and (7.3). For the discussed problem no constraint is posed on the required torque to perform the task, thus no minimum and maximum values are assumed.

7.4.1 Check for Reachability of Manipulator

In addition to the constraints discussed while searching for the optimal design it must be ensured that the task to be performed lie within the range of workspace. Thus, it should satisfy the conditions given below:

$$\mathrm{r} \geq (l_1 - l_2) \tag{7.5}$$

$$\mathrm{r} \leq (l_1 + l_2) \tag{7.6}$$

where, $\mathrm{r} = \sqrt{(x^2 + y^2)}$. If the Cartesian coordinates for a manipulator satisfies the above conditions, then it is considered reachable and the design parameter vector generated by DE participates in optimization process.

7.4.2 Kinematic and Dynamic Models of 2-DOF RR Planar Manipulator

Considering the 2-DOF RR planar manipulator in Fig. 3.1, the kinematic model given in Chapter 3 in terms of D-H notation for the homogenous transformation matrix is used. By using kinematic model coordinates of end-effector position (x_i, y_i) are determined. While using forward and inverse kinematic model l_1 and l_2 are taken lengths of two links, and θ_1 and θ_2 are as joint coordinates respectively. Already available dynamic model of 2-DOF RR planar manipulator is used for computation of performance. For computation of performance same procedure as discussed in section 4.4 of Chapter 4, is used with exception that number of noise parameters considered.

7.4.3 Forward Dynamics

The closed form solutions are difficult to obtain, the torque obtained from inverse dynamics are integrated to compute the joint coordinates, velocities and accelerations with effect of noise. In this work Euler numerical integration method [Craig 1989] has been applied, for obtaining angular velocities and angular positions from angular accelerations. The torque equation for the manipulator at the start point $t = 0$ is given by

$$\tau_0 = M(q)\ddot{q}_0 + h(q_0, \dot{q}_0) + G(q_0) \tag{7.7}$$

From equation (7.7) joint acceleration can be computed as,

$$\ddot{q}_0 = M^{-1}(q_0)\left[\tau_0 - h(q_0, \dot{q}_0) - G(q_0)\right] \tag{7.8}$$

Therefore, to obtain future positions and velocities equation (7.8) is integrated forward in time by steps of size Δt using numerical integration technique. Iteratively the angular velocities and positions at the $(i+1)^{th}$ instance are obtained using

$$\dot{q}_{i+1} = \dot{q}_i + \ddot{q}_i \Delta t \tag{7.9}$$

$$q_{i+1} = q_i + \dot{q}_i \Delta t + \frac{1}{2}\ddot{q}_{i+1}(\Delta t)^2 \tag{7.10}$$

For each iteration, equation (7.8) is used to compute the angular acceleration and subsequently equations (7.9) and (7.10) are used to compute the position, and velocity of the manipulator, caused by supplied torque.

7.5 SIMULATION AND DISCUSSION

To implement above discussed hybrid evolutionary optimization approach computer programme are developed using MATLAB commands. The numerical values for different parameters are provided below. The cubic and quintic polynomials have been used as joint trajectories specified in Chapter 4. The task specification for deterministic cases and cases with effect of noise are provided in Tables 7.1(a) and (b) respectively. The task specified in Table 7.1(b) is same as the task considered in Chapter 4, 5 and 6. For easy reference, the task is provided once again.

Table 7.1(a) Manipulator Task Specifications for Deterministic case

Trajectory	Coordinates of Start point (x_i m, y_i m)	Coordinates of Destination point (x_f m, y_f m)	Time to travel (sec)
Cubic	(0.65, 0)	(0.4, 0.3)	2
Quintic	(0.65, 0)	(0.4, 0.3)	2

Table 7.1(b) Manipulator Task Specifications for case with Effect of Noise

Case	Coordinates of Start point (x_i m, y_i m)	Coordinates of Destination point (x_f m, y_f m)	Time to travel (sec)
(i)	(0.65, 0)	(0.4, 0.3)	2
(ii)	(0.65, 0.05)	(0.4, 0.3)	2
(iii)	(0.65, 0.1)	(0.4, 0.3)	2
(iv)	(0.65, 0.05)	(–0.4, 0.3)	2
(v)	(0.65, 0.1)	(–0.4, 0.3)	2
(vi)	(0.4, 0.3)	(0.65, 0)	2

To determine the influence of task specifications on optimal parameter design, number of cases has been considered. The constraints imposed during deterministic and cases with effect of noise, for the optimization process are given in Table 7.2.

Table 7.2 Parameter Constraints for Manipulator

Case	Link 1 length l_1 range (m)	Link 2 length l_2 range (m)	Link 1 mass m_1 range (kg)	Link 2 mass m_2 range (kg)
(i)	$0.45 \le l_1 \le 0.55$	$0.35 \le l_2 \le 0.45$	$6 \le m_1 \le 10$	$4 \le m_2 \le 8$
(ii)	$0.45 \le l_1 \le 0.55$	$0.35 \le l_2 \le 0.45$	$6 \le m_1 \le 10$	$4 \le m_2 \le 8$
(iii)	$0.45 \le l_1 \le 0.55$	$0.35 \le l_2 \le 0.45$	$6 \le m_1 \le 10$	$4 \le m_2 \le 8$
(iv)	$0.45 \le l_1 \le 0.55$	$0.35 \le l_2 \le 0.45$	$6 \le m_1 \le 10$	$4 \le m_2 \le 8$
(v)	$0.45 \le l_1 \le 0.55$	$0.35 \le l_2 \le 0.45$	$6 \le m_1 \le 10$	$4 \le m_2 \le 8$
(vi)	$0.45 \le l_1 \le 0.55$	$0.35 \le l_2 \le 0.45$	$6 \le m_1 \le 10$	$4 \le m_2 \le 8$

It can be seen that the constraints are kept same for both deterministic optimization and optimization with effect of noise. Even for different tasks, constraints are assumed to remain same. The optimization of the deterministic case, while following cubic and quintic trajectories are carried out using method discussed in sections 7.2. The evolution control parameters used for optimization are provided in Table 7.3.

Table 7.3 Evolution Control Parameters for DE Technique

Control Parameter	Value
Population size (NP)	40
Number of generations	50
Crossover probability (CR)	0.5
Weighting factor (F)	0.8

In deterministic optimization, the evaluation of cost function for each population member is carried out once because during simulation of performance, effect of noise is not considered. By this process the positional error obtained from simulation is nothing but the computational error. Therefore, by application of DE optimization technique to the deterministic problem the design parameter which minimizes computational error is obtained. The evolution control parameters used for the DE optimization approach is specified in Table 7.3. To arrive at these parameter values help of trial and error method is taken, by which optimal performance of DE is observed. The generation wise reduction in positional error i.e. computational error is provided in Figs. 7.3 and 7.4 respectively.

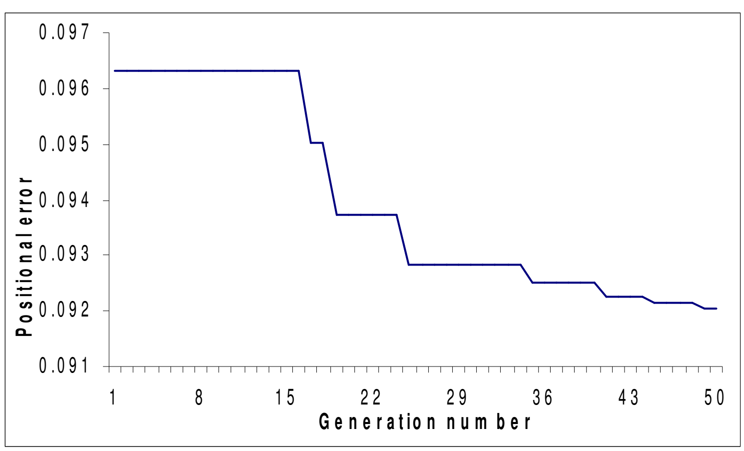


Fig. 7.3 Function History of Deterministic case for Cubic Trajectory

For the specified task following cubic trajectory, it is observed that the positional error reduces from 0.0965×10^{-2} m to 0.0921×10^{-2} m in 50 generations.

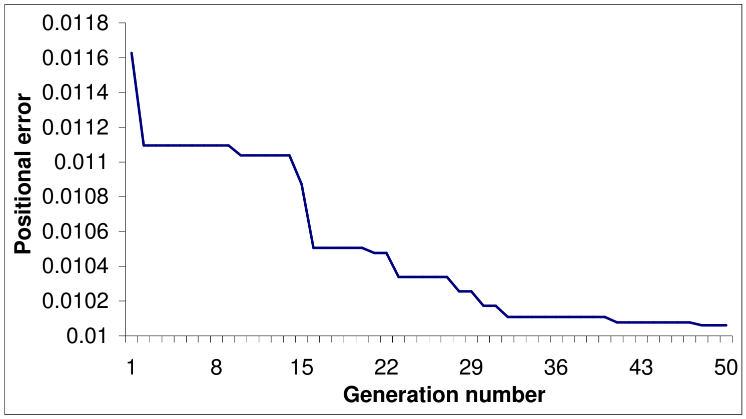


Fig. 7.4 Function History of Deterministic case for Quintic Trajectory

Similarly from Fig. 7.4 for the task following quintic trajectory, it is observed that the positional error reduces from 0.0116×10^{-2} m to 0.0101×10^{-2} m in 50 generations. The optimal parameter values for deterministic cases are given in Table 7.4.

Table 7.4 Optimal Parameters of Manipulator for Deterministic case

Trajectory	Link 1 length l_1 (m)	Link 2 length l_2 (m)	Link 1 mass m_1 (kg)	Link 2 mass m_2 (kg)	Function Value $\times10^{-2}$ (m)
Cubic	0.4502313	0.44993491	6.255106	7.985928	0.092029
Quintic	0.54994198	0.41548853	9.975794	4.014191	0.010061

It can be seen that the optimal parameter value obtained using DE technique provide different solution for different trajectories. It is also observed that the function value for the task following quintic trajectory is quite less as compared to task following cubic path. This indicates that the task following quintic trajectory have high chance to get closer to target point as compared to other case.

Subsequently, to obtain the optimal design parameters which delivers optimized real life performance, proposed hybrid technique is applied. For these cases, effect of noise is incorporated while simulating the performance, subsequently taken into consideration in optimization process. The tasks considered for this optimization process are provided in Table 7.1(b). The evolution control parameters for the hybrid DE optimization approach are provided in Table 7.3. It can be observed that parameters used in deterministic optimization are same as in this case. It is also observed that these parameters provide optimal solution with lesser functional evaluations. Therefore, same parameters are used for both the case.

As per discussion in Chapter 3 there are six control factors (lengths of the two links, its corresponding masses and fluctuations in joint torques). Correspondingly, for these control factors there will be six noise parameters. Since there are six noise parameters and two levels for each parameter (–3 sigma and +3 sigma), 2^6 combinations are possible and conducting all experiments are impractical and leads to increased computation time. To reduce the amount of computation and incorporate effect of noise systematically Taguchi's OA is selected. The rational behind this selection is the number of factors for which effect of noise needs to be incorporated. In the present chapter only effect of link length and link mass variation is incorporated in the optimization process. Therefore, for the proposed technique, L_8 OA [Park 1998] is selected. This L_8 array is same as discussed in Chapter 6. The array is provided once again for easy reference. For assumed tolerances of each design parameter and specific parameter values of each kinematic and dynamic parameter, the OA selected provides noise to each kinematic and dynamic parameter. The noises are deviations of the tolerance value from nominal parameter value. These represent 'worst case' tolerance deviations and satisfy the 3-sigma limits of normal Monte Carlo variability. Each row of OA is a noise combination that is treated as repetitive data for each member of population. To incorporate effect of noise 1's and 2's of L_8 array presented in Table 7.5 has been used. These numbers provide the direction in which population parameter vector deviate from the specified values.

Table 7.5 L_8 Orthogonal Array

Experiment No.	Column Number						
	1	2	3	4	5	6	7
1	1	1	1	1	1	1	1
2	1	1	1	2	2	2	2
3	1	2	2	1	1	2	2
4	1	2	2	2	2	1	1
5	2	1	2	1	2	1	2
6	2	1	2	2	1	2	1
7	2	2	1	1	2	2	1
8	2	2	1	2	1	1	2

The assumed noises of six control factors are specified in Table 7.6. These noises are utilized in hybrid DE to incorporate noise in simulation and optimization process.

Table 7.6 Noise for Design and Process Parameters

Parameters	Tolerances
Lengths of link l_1 and l_2 (m)	± 0.0003
Mass of link m_1 and m_2 (kg)	± 0.015
Torque at joint τ_1 and τ_2 (Nm)	± 0.05

The detailed use of OA in hybrid DE has been provided in Tables 7.7, 7.8 and 7.9 respectively. To explain the use of OA in hybrid DE approach, an example has been considered. In which all the eight possible combinations are generated for the set of four parameters l_1, l_2, m_1 and m_2. The assumed tolerances for link lengths l_1, l_2 and masses m_1, m_2 are provided in Table 7.7 and assumed parameter vector value sent by DE for optimization are specified in Table 7.8. The resulting eight combinations with the use of L_8 array are given in Table 7.9.

Table 7.7 Assumed Tolerances for Control Factors

Parameters	Tolerances
Link Lengths l_1, l_2 (m)	± 0.0003
Link Masses m_1, m_2 (kg)	± 0.015

Table 7.8 Assumed Population Member Values Sent by DE

l_1 (m)	l_2 (m)	m_1 (kg)	m_2 (kg)
0.425	0.375	6	4

Table 7.9 Combinations Created from the Population Member Values Using OA

Experiment No.	Column number Parameter			
	1 l_1 (m)	2 l_2 (m)	3 m_1 (kg)	4 m_2 (kg)
1	0.4247	0.3747	5.985	3.985
2	0.4247	0.3747	5.985	4.015
3	0.4247	0.3753	6.015	3.985
4	0.4247	0.3753	6.015	4.015
5	0.4253	0.3747	6.015	4.985
6	0.4253	0.3747	6.015	4.015
7	0.4253	0.3753	5.985	3.985
8	0.4253	0.3753	5.985	4.015

The experiment is conducted for each combination and the average of all the experiments is taken. This out come i.e. mean positional error, from experiments is sent for comparison in DE optimization process. It is important to mention, noise in supplied torque is modeled as Gaussian stochastic process with Markov property as discussed in Chapter 4. To incorporate the effect of noise similar approach discussed in Chapter 4 is used. Thus to incorporate effect of noise, assumed standard deviation (tolerance) of fluctuation in supplied torque is used in the simulation. And these parameters are not assigned to columns of L_8 orthogonal array.

The results of the optimization process utilizing the hybrid evolutionary technique are presented in Tables 7.10(a) and 7.10(b).

Table 7.10(a) Optimal Parameters for Cubic Trajectory with Effect of Noises

Case	Link 1 length l_1 (m)	Link 2 length l_2 (m)	Link 1 mass m_1 (kg)	Link 2 mass m_2 (kg)	Mean positional error $\bar{\varepsilon}$ $\times 10^{-2}$ (m)	Number of function Evaluations
(i)	0.45115423	0.44025207	6.585616	7.886889	0.130762	698
(ii)	0.4516324	0.44804167	7.843923	7.99053	0.120789	796
(iii)	0.45044698	0.35094645	6.191169	7.998867	0.231646	812
(iv)	0.45021022	0.35036626	6.257418	7.95036	0.236336	728
(v)	0.45713918	0.44867156	6.727582	7.969148	0.111177	764
(vi)	0.45083597	0.44943789	6.037671	7.714803	0.125251	735

Table 7.10(b) Optimal Parameters for Cubic Trajectory with Effect of Noises

Case	Link 1 length l_1 (m)	Link 2 length l_2 (m)	Link 1 mass m_1 (kg)	Link 2 mass m_2 (kg)	Mean positional error $\bar{\varepsilon}$ $\times 10^{-2}$ (m)	Number of function evaluations
(i)	0.53526522	0.44429454	9.728534	7.9974	0.120093	694
(ii)	0.53052684	0.44841063	9.185676	7.976033	0.101264	812
(iii)	0.52195604	0.44669770	9.498453	7.947742	0.097292	812
(iv)	0.45139675	0.44965546	9.898766	7.609616	0.252178	726
(v)	0.45054925	0.44976276	9.920583	7.85658	0.218845	754
(vi)	0.45159362	0.44882879	9.955927	7.763131	0.054702	755

In this table, the optimal values of the design parameters, the functional value, and number of function evaluations are presented. It is observed that the hybrid DE approach consistently obtains smaller function value with less function evaluations. The number of function evaluations is an indication of the computing effort required in reaching the optimum function value for the same number of generations. In case of DE after the trail vector is computed each member of the trail vector is subjected to bound check and only those members who pass the bound check will have cost function evaluation. Thus the function evaluations of the members which fail the bound check have been avoided and this results in reduction of the number of function evaluations.

The objective function history for tasks following cubic trajectory are shown in Figs. 7.5 to 7.10.

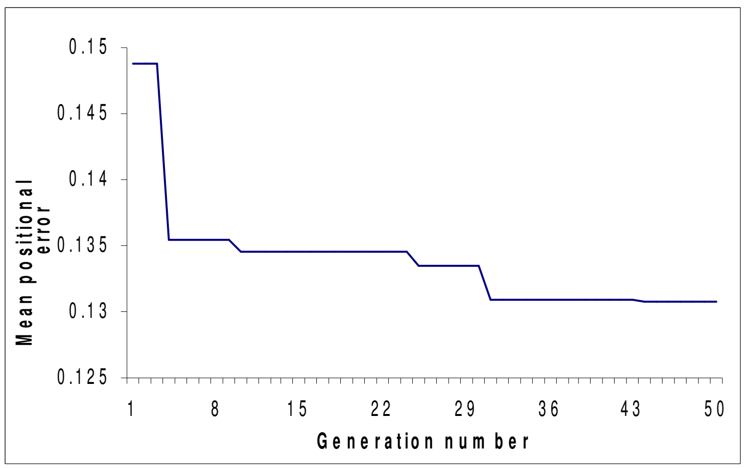


Fig. 7.5 Function History for Cubic Trajectory with effect of Noise - case (i)

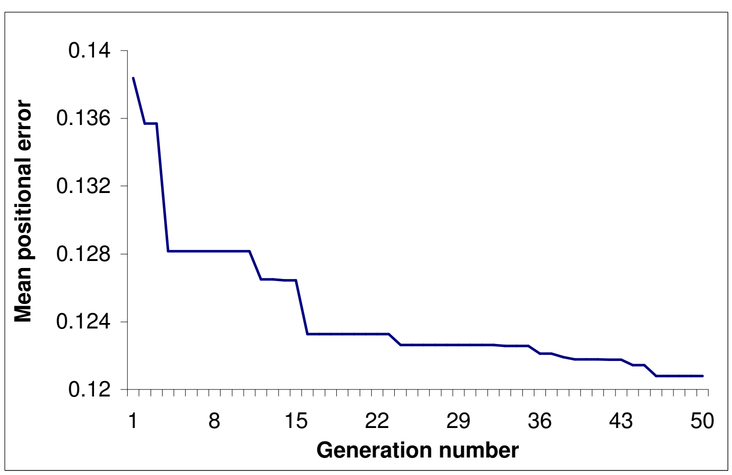


Fig. 7.6 Function History for Cubic Trajectory with effect of Noise - case (ii)

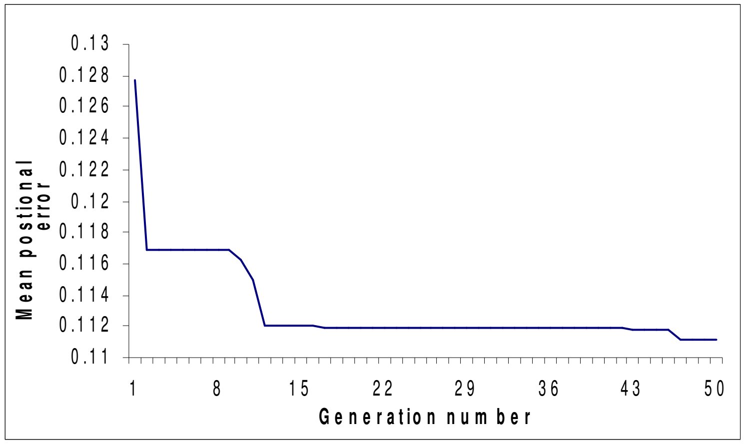


Fig. 7.7 Function History for Cubic Trajectory with effect of Noise - case (iii)

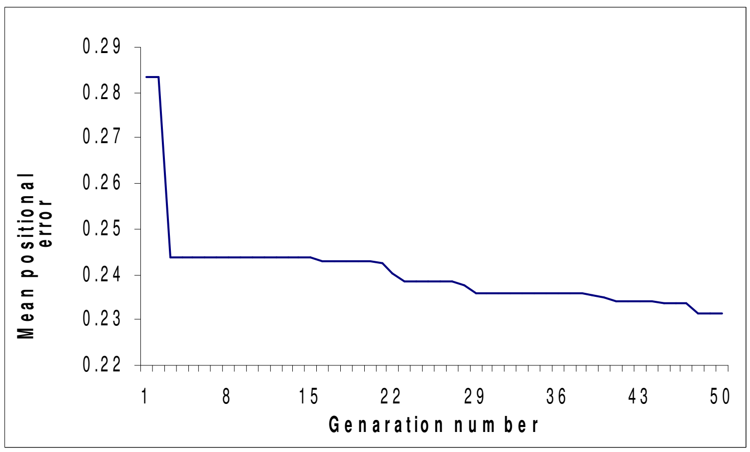


Fig. 7.8 Function History for Cubic Trajectory with effect of Noise - case (iv)

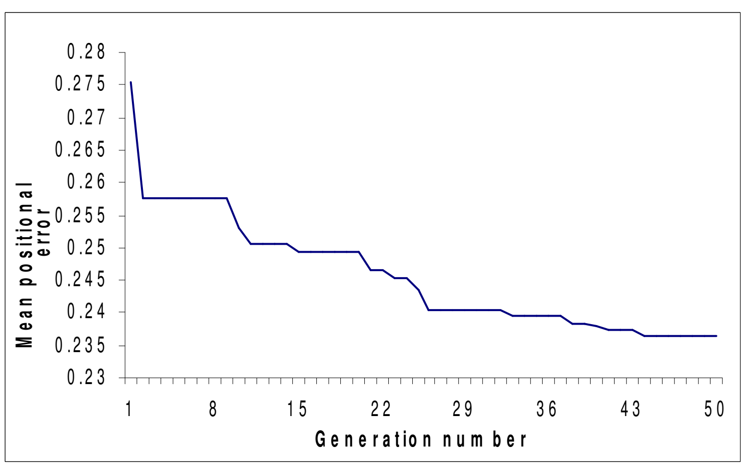


Fig. 7.9 Function History for Cubic Trajectory with effect of Noise - case (v)

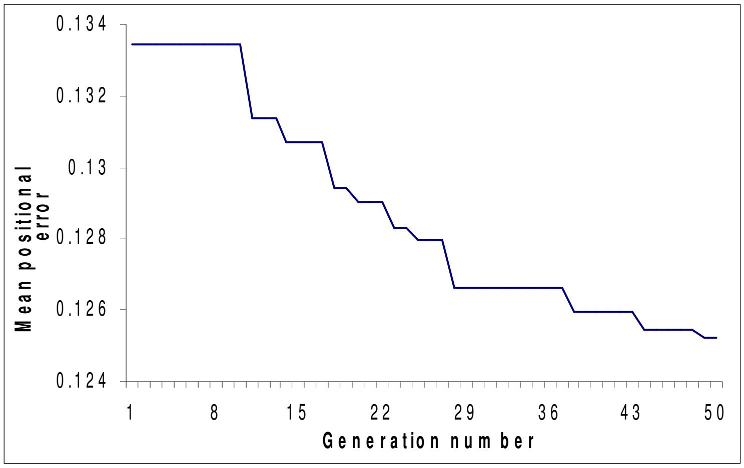


Fig. 7.10 Function History for Cubic Trajectory with effect of Noise - case (vi)

From Fig. 7.5, for case (i) following cubic trajectory cost function i.e. mean positional error $\bar{\varepsilon}$ reduces from 0.149×10^{-2} m to 0.131×10^{-2} m in 50 generations. It is also observed that mean positional error $\bar{\varepsilon}$ reduces from 0.14×10^{-2} m to 0.12×10^{-2} m, from Fig. 7.6 for case (ii). Similarly in Fig. 7.7 for case (iii) mean positional error reduces from 0.128×10^{-2} m to 0.112×10^{-2} m while from Fig. 7.8 for case (iv) objective function value is observed to reduce from 0.284×10^{-2} m to 0.236×10^{-2} m. In Fig. 7.9 for case (v) mean positional error is observed to reduce from 0.273×10^{-2} m to 0.238×10^{-2} m. In all these cases objective fumction value reduces monotonically. From Fig. 7.10 for case (vi) mean positional error $\bar{\varepsilon}$ reduces from 0.133×10^{-2} m to 0.125×10^{-2} m in 50 generations.

The objective function history for tasks following quintic trajectories are shown in Figs. 7.11 to 7.16. It is observed that the DE converges in a monotonic fashion in all the considered situations. From Fig. 7.11 for the case (i) following quintic trajectory cost function i.e. mean positional error $\bar{\varepsilon}$ is observed to reduce from 0.101×10^{-2} m to 0.125×10^{-2} m in 50 generations. Simililarly from Figs. 7.12 and 7.13 mean positional error $\bar{\varepsilon}$ is found to reduce from 0.1015×10^{-2} m to 0.0972×10^{-2} m and 0.1052×10^{-2} m to 0.1015×10^{-2} m in 50 generations for case (ii) and (iii), respectively.

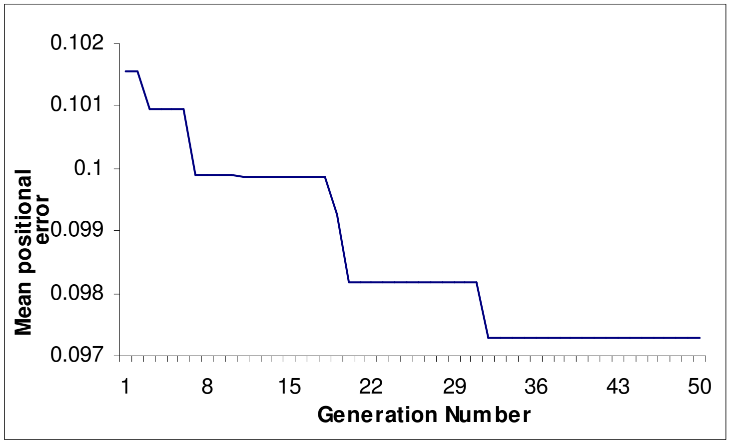


Fig. 7.11 Function History for Quintic Trajectory with effect of Noise - case (i)

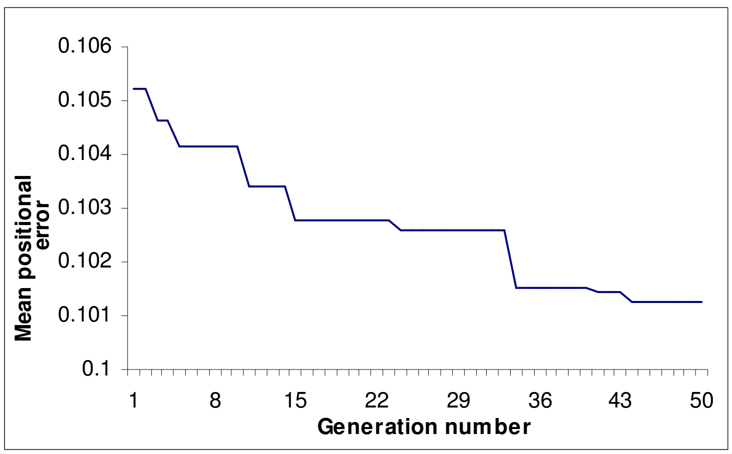


Fig. 7.12 Function History for Quintic Trajectory with effect of Noise - case (ii)

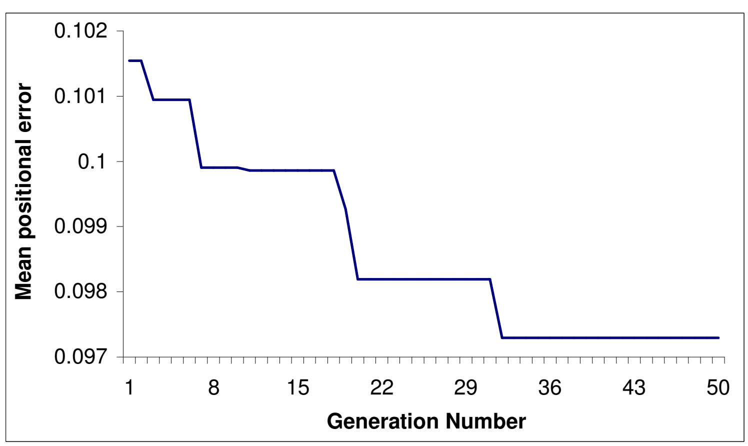


Fig. 7.13 Function History for Quintic Trajectory with effect of Noise - case (iii)

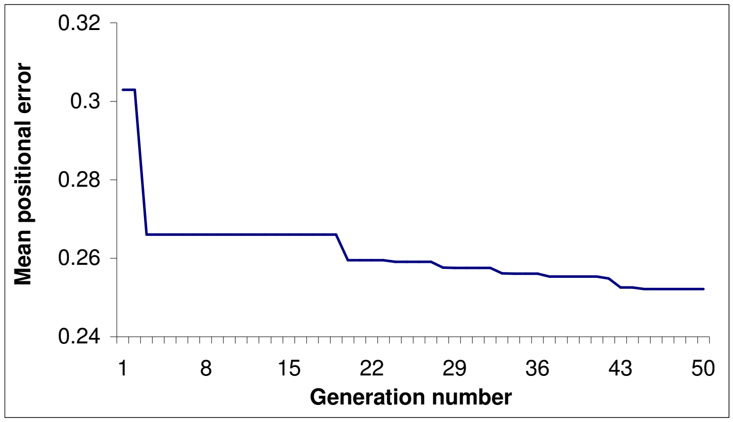


Fig. 7.14 Function History for Quintic Trajectory with effect of Noise - case (iv)

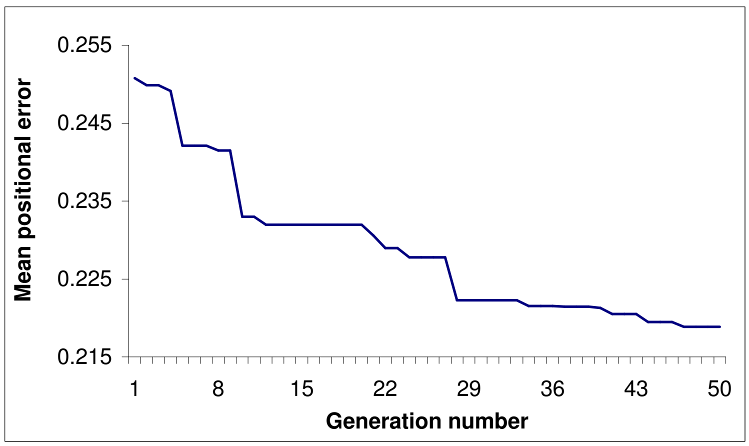


Fig. 7.15 Function History for Quintic Trajectory with effect of Noise - case (v)

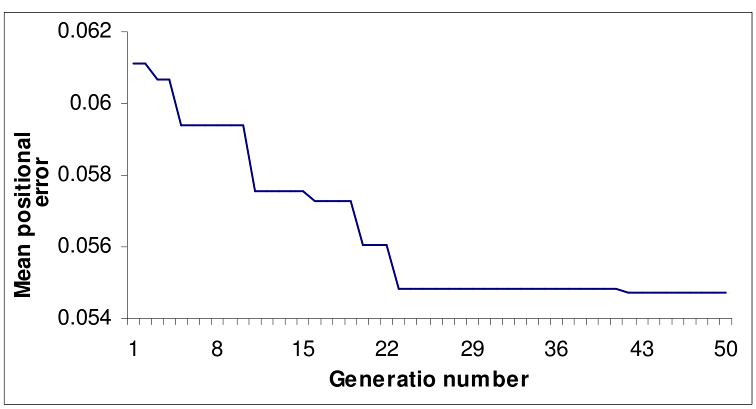


Fig. 7.16 Function History for Quintic Trajectory with effect of Noise - case (vi)

From Figs. 7.14 and 7.15 the manipulator performance is found to reduce from 0.305×10^{-2} m to 0.255×10^{-2} m and 0.250×10^{-2} m to 0.220×10^{-2} m for cases (iv) and (v) respectively in 50 generations. In Fig. 7.16 for case (vi) mean positional error is observed to reduce from 0.061×10^{-2} m to 0.055×10^{-2} m. In all of the above cases cost function is observed to reduce in a monotonic fashion. This behaviour illustrates the power of the differential evolution strategy. Focus of the present investigation is to reduce the performance variation by proper selection of design parameters. Therefore, optimal results obtained after 50^{th} generation is certainly going to deliver the best performance i.e. minimum mean positional error.

7.6 EPILOGUE

A hybrid approach has been proposed which couples DE optimization techniques with OA of the Taguchi method to incorporate real world uncertainties into optimal parameter design. The design optimization process considered here uses the kinematic and dynamic model of the manipulator. The objective has been to minimize mean positional error, required to perform the defined motion subject to constraints on link parameters (length, and mass). In this chapter, developed hybrid DE optimization approach, has been applied to obtain optimal parameter design of 2-DOF RR planar manipulator. The results of above approach are presented to show the convergence and number of functional evaluations. The results indicate that the hybrid DE reaches to a ‘steady-state’ objective function quickly with smaller number of generations requiring less function evaluations. The proposed hybrid approach best suits the purpose of parameter design considering effect of noise. The fast performance of hybrid DE indicates that this approach can be a viable optimization technique, for processes where development of objective function is difficult. The results show that the evolutionary optimization technique is reliable, despite the fact that it is relatively slow. However, this is not a serious drawback, since the selection of optimal design is an off-line procedure.

CHAPTER-8

CONCLUSIONS AND FUTURE PERSPECTIVES

8.1 INTRODUCTION

The robot performance problem has been dealt in this thesis, to get minimum manipulator performance variability by selecting control parameters and tolerances for optimal robust designs of manipulators. The solutions are developed for the corresponding problems formulated in section 1.3. Strengths and limitations of each proposed solution are summarized and recommendations for future research have been given in this chapter.

8.2 CONCLUSIONS

To meet increasing demands on improved robot performance, different design optimization techniques have been developed. For implementation of optimization techniques new techniques have been proposed and approaches have been developed. None of the considered design optimization techniques for performance improvement can be implemented easily as each required performance simulation while incorporating effects of noise. For simulation of performance for parameter and tolerance optimization, software has been developed and analysis of simulated results has been carried out using Design Expert software [Design Expert 1999]. Because of the dynamic model of manipulator is highly nonlinear and coupled, the robotic manipulators pose a significant challenge in achieving the objectives, namely, robust manipulator design for improved quality of performance.

Application of design of experiments (DOE) technique for optimal design of parameters and their tolerances is a powerful concept, which contributes potentially to the improved quality of performance and helps in understanding the effect of different parameters on performance variability. By the use of proposed approaches, optimal design parameters have been computed. In addition, the required manufacturing tolerances and supplied actuator torque tolerances that will give the optimal manipulator performance with focus on optimizing cost, have been obtained. The performance of manipulator depends on both the errors in kinematic and dynamic parameters and joint parameters. In this thesis the kinematic and dynamic parameters are treated as independent random variable and considering variations in joint variables, performance of the manipulator is simulated and subsequently effect of control and noise parameters are studied.

Present thesis explores the objective i.e. to study the performance of robot to get a better evaluation of the performance of robot manipulators. Results of this thesis can be readily implemented in industry since there is a continuing need to improve product quality through the employment of better techniques.

In Chapter 2, the chronological review of published literature is organized in two parts. The first part presents the review on approaches, techniques and design procedures to improve the performance of robot and the second part reviews the Taguchi method and taxonomic reviews on robust design techniques. This chapter highlights the development of methodologies and approaches, over the years to improve performance of both product and process design optimization. It has been observed that robust design is a multi-objective and non-deterministic problem. The objective is to optimize the mean and minimize the variability in the performance response that results from uncertainty represented through noise variables. Based on the study following conclusions can be drawn.

There has been relatively little or not much work available to handle effects of noise and since, noise is rarely considered in design optimization of robotic manipulator. Therefore, methods to minimize the effect of uncertainties on manipulator design and performance are of paramount concern to researchers and practitioners.

In Chapter 3 the first attempt was to establish the use of simulation methods for modeling and optimizing the performance of robot manipulators. The P-diagram was used to identify the key factors of a manipulator that influence the performance and these are classified as control factors and noise factors. A heuristic based search method to simulate various performance measures of manipulator and use of DOE technique to determine statistically significant parameters was carried out. In the chapter the strategies needed to incorporate effect of noise in both kinematic and dynamic models of manipulator for real life performance were developed. Subsequently the design matrix of DOE technique was used to simulate the performances. Finally the simulated performances of manipulator were analyzed using ANOVA technique for statistical significance of kinematic and dynamic parameters.

To understand the overall behavior of manipulator performance at the target point, four cases were used to simulate the performance. The simulated performances were analyzed case-by-case basis. To summarize the findings, it was observed that four control factors are statistically significant in all the considered cases. Control factor m_1 was observed to be statistically insignificant for only two out of four cases. Similarly, control factor l_1 was observed to be statistically insignificant in one out of four cases. Subsequently for all cases mean positional error, SN ratio and reliability were used to find the suitable control factor combinations for which performance were optimal. It was observed that statistically significant factors were different for different target points in workspace. This indicates that different factors have different contribution to performance variations as target point changes or in other words, performance is dependent on the target point and for each target point different control factors are significant. In addition to this, optimum combination of control factors required to perform task are different for different cases. This indicates that one set of parameters of manipulator for one task will behave differently for other type of task. Finally the optimum factor combinations obtained using mean positional error, SN ratio and reliability do not agree in all the cases considered. Possible reason for disagreement in optimal solution could be due to the transformation of positional error into the Taguchi's SN ratio, which may not be same as the untransformed result obtained from reliability computation.

In Chapter 4 probabilistic approach to simulate the performance of manipulator has been presented. The methods adopted to simulate the performance for tasks following cubic and quintic path were discussed. Because of large number of control and noise factors considered, the fraction factorial design approach was adopted to create the design matrix. Taking factorial combination of design matrix, performance of manipulator has been simulated. Half normal probability plotting and ANOVA tables are used to determine statistically significant parameters. The approach to screening the parameters responsible for performance variations and subsequent use of these parameters for robust design was developed. In addition to statistical analysis, to complement the investigation, parametric sensitivity studies were also carried out. Effect of change of dimension-less parameters β i.e. link length ratio, α i.e. link mass ratio and simultaneous change of β and α on performance were analyzed. From the analysis in the chapter it was concluded that as the task change, statistically significant factors also change. For the same task, parameters responsible for performance variations were different while following cubic and quintic trajectories. Lastly, the most important outcome was the segregation of identified fourteen control and noise parameters into three categories. The first category consist of those parameters, which were not statistically significant even once in all the considered cases. Therefore, further study should not be pursued. The second category belongs to those parameters which were statistically significant only once in all the considered cases. To reduce the number of parameters for further study, it was decided that these parameters should be treated as insignificant parameters and should be switched to first category. Then third category consist of those parameters which were statistically significant more than once in all the considered cases. Therefore, further study was pursued taking all these parameters into consideration. It was observed that only six out of fouteen parameters fall into third category. Investigations on effect of dimensionless parameters on performance variations indicated that, contributions of parameters vary depending on tasks and the trajectories. It was observed that link length ratio increase, reduces performance variations i.e. specifically for β value between 0.75 and 1 stabilizes the performance measure. But trend of variations was observed to be same for manipulator following cubic and quintic trajectories. Link mass ratio change was observed to have insignificant influence on performance measure in most cases. To conclude the important finding of this study, the performance variations contribution has been observed to be less by link mass ratio change as compared to link length ratio change. By simultaneous change of link length and link mass ratios, performance variations of manipulator were observed to be significantly different for the tasks and the trajectories.To generalize, for β values between 0.8 to 1.0 and α values between 0.35 to 1, performance measure was found to be lowest. Whereas in few cases following quintic trajectory, drawing similar type conclusions were difficult.

The robust design approach to obtain optimal design parameters of manipulator, which will give minimum performance variations, was developed and described in Chapter 5. Experimental design techniques were used to run experiments to simulate the performance. Taking simulated performance, a model to predict the performance of manipulator was obtained using response surface approach. Taking these response equations, parametric models of means of performance and variances of performance were developed. Thereafter an optimization problem was formulated using these models. The objective of the formulated problem was to minimize the mean performance variation while keeping variance of performance at a decided value. This constrained optimization problem was solved for optimal value using *fmincon* routine available in MATLAB.

The simulated performances were analyzed using ANOVA techniques and it was observed that for the tasks following cubic trajectory, link length 1, clearances present in joint one and two, were statistically significant. Except two cases, quadratic effect of parameters were observed to be insignificant. This indicates that the parameters have small nonlinear effect on the performance variations. For the task following quintic trajectory it was observed that clearance in joint1 and joint 2 factors were statistically significant in five out of six cases. Similarly for five out of six cases, nonlinear effect of link length1, mass 1, length 2, clearance in joint2 and link mass 1 were observed to be statistically significant respectively. To know the ability of developed emperical model to predict the manipulator performance, help of R^2 was taken. It was observed that R^2 values were close to 0.90 for three out of six cases, and were small for rest of the cases following cubic and quintic trajectories. In addition to R^2 values, comparison between the desired and predicted values of performances were shown with the help of figures, to observe the closeness of trend. In most of the cases, the predicted values were closer to the desired values and trends were similar. Subsequently developed response equations for mean performance and variances in performance were used for optimization. After solving the formulated optimization problem, it was observed that for different cases optimal parameters were different.

In Chapter 6, the methodology to select optimal tolerance on manipulator kinematic and dynamic parameters i.e. tolerances on link dimensions, masses and joint torques that results in minimum performance variations and individual effect of these tolerances were investigated. In Chapter 4, for screening experiment, these factors were identified as noise factors, and effect of these factors on performance variations were investigated. This chapter attempts to further reduce the performance variations that have been achieved by robust design principle i.e. optimal solutions of Chapter 5. To obtain optimal tolerances of these parameters, cross array design of experiment approach was utilized. Uniqueness of this DOE approach is the ability to investigate the effect of noise due to kinematic and dynamic parameters. Subsequently using probabilistic approach, the performance of manipulator were simulated and analyzed to obtain the parameters that contribute the most to the observed performance variations. The results of this study may raise objections for intentional incorporation of effect of noise in control factors; therefore, another study was conducted to investigate the influence of individual parameter tolerance on performance variations.

One important conclusion drawn was that the statistically significant factors were independent of trajectory time law. But, from task to task, factors responsible for performance variations were different. Out of six only two factor tolerances i.e. tolerances of m_2 and τ_2, can be tightened to further reduce the performance variations. To validate the results obtained from the experimental design technique, Monte Carlo simulation approach was used and for comparison, performance measure SN ratio was utilized. The advantages of proposed method are that it does not involve any capital investment in equipment. This technique requires fewer computations as compared to Monte Carlo simulation. From parameter tolerance sensitivity study, it was observed that for the majority of the cases following both cubic and quintic trajectories, link length tolerances had significant influence on performance variations. It was observed that change in SN ratio values was not significantly sensitive to increase in tolerances on link mass and torque individually. In few cases, none of the factor tolerance increase leads to deterioration in performance. This analysis points out to the possibilities of wrong conclusions, from one factor tolerance at a time experimentation analysis.

In Chapter 7, a hybrid approach has been proposed for optimal parameter design, which couples Differential Evolution optimization techniques with Orthogonal Array of the Taguchi method to incorporate real world uncertainties. For the design optimization process considered, the objective was to minimize mean positional error at destination point. The optimization problem was subjected to constraints on reachability and link parameters i.e. link lengths and link masses. The results obtained using hybrid approach has been presented for the convergence and number of functional evaluations. The results indicate that the hybrid DE reaches to a 'steady-state' objective function quickly requiring less function evaluations. The fast performance of hybrid DE indicates that it can be a viable optimization approach for processes for which development of objective function is difficult. Proposed hybrid DE technique can be used for the selection of optimal design, which is an off-line procedure.

To consolidate the achievements of this thesis, it was observed that, out of fourteen parameters only six parameters were responsible for performance variations of manipulator at the target point. In these six factors, four were control factors and two were noise factors. By applying robust design technique, suitable control factors were chosen to make the design insensitive to these two noise factors. To further reduce the performance variations at the destinations, tolerance design of control factors were carried out. It was observed that by tightening only two control factor tolerances, performance variations can be lowered. By doing so the cost of production or manufacturing would be less. In the entire thesis statistically significant parameters were observed to be different for different tasks. Optimal parameter values were different for different tasks, indicating that an optimal setting for one task would perform differently when task changes. Except Chapter 6, it was observed that statistically significant parameters were different for different type of trajectory followed to perform the task. To determine the optimal parameters of manipulator for minimum performance variations, an evolutionary optimization technique was used and it was observed that for different tasks the optimal solutions were different and even different for the trajectories followed.

The results of the simulation experiment with robust design optimization techniques are important. The methodology for improving a robot performance also promises to be useful for robot manufacturers and users. A robot manufacturer can use this method to examine the influence of design parameters on robot performance.

8.3 FUTURE PERSPECTIVES

For testing the developed techniques a 2-DOF RR planar manipulator has been used throughout, as is the standard practice in the published literature. A more rigorous testing would require application of the developed techniques to manipulators with higher degrees of freedom and other industrial manipulators like SCARA, PUMA, STANFORD etc.

An important subject for further research should be to consider the effect of other parameters i.e. control and noise parameters on performance of the manipulator at different speeds, process constraints, and trajectory time law.

Immediate extension of the present work would be to develop a setup of 2-DOF RR planar manipulator to experimentally verify the results obtained in the present work. The exact probability distributions of various errors can be determined experimentally or analytically and utilized later to get accurate performance. The method utilized here will require very little modification because assumption is valid for many physical phenomena. This investigation is a comprehensive exercise.

Considering the work presented in the thesis, extensive research can be carried out in area of manipulator design and development of sophisticated robust design techniques to handle complex industrial manipulator design problems. For example development of a method/technique, that will optimize parameters and its tolerances simultaneously, for optimal performance.

The links of the manipulator were assumed to have uniform cross section and fixed inertia. An attempt to investigate the effect of link cross section on performance of manipulator should be made. Along with this, effects of noise due to changes in moment of inertia of links on performance of manipulator also need to be investigated. Results of this simulation can further be experimentally verified to confirm the findings.

While simulating performance of manipulator except fluctuation of torque, other factors influencing actuator dynamics have not been considered. Effects of parameters of transmission systems and transmission element on performance of manipulator can also be investigated. The dynamic model of manipulator including the effects of actuation system parameters and friction can also be studied further.

Computer programs for the developed techniques are to be converted into manipulator performance simulation package to simulate the performance of manipulator considering effect of noise. The package should be able to provide the required information on optimal parameters, significant parameters and optimal tolerances required for the manipulator for a given task.

Attempt can also be made to look for optimal parameters of manipulator, which will not only optimize performance but also optimize others like energy usage, stiffness and weight. Even explorations can further be extended to optimize the discussed performance measure of manipulator with kinematic and dynamic performance indices.

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APPENDIX - A

Table A1 Design Matrix for Factorial Design

Combination Number	l_1 (m)	l_2 (m)	m_1 (kg)	m_2 (kg)	τ_1 (Nm)	τ_2 (Nm)
1	0.40	0.30	7	5	–500	–100
2	0.40	0.30	7	5	–500	–105
3	0.40	0.30	7	5	–800	–100
4	0.40	0.30	7	5	–800	–105
5	0.40	0.30	7	6	–500	–100
6	0.40	0.30	7	6	–500	–105
7	0.40	0.30	7	6	–800	–100
8	0.40	0.30	7	6	–800	–105
9	0.40	0.30	8	5	–500	–100
10	0.40	0.30	8	5	–500	–105
11	0.40	0.30	8	5	–800	–100
12	0.40	0.30	8	5	–800	–105
13	0.40	0.30	8	6	–500	–100
14	0.40	0.30	8	6	–500	–105
15	0.40	0.30	8	6	–800	–100
16	0.40	0.30	8	6	–800	–105
17	0.40	0.40	7	5	–500	–100
18	0.40	0.40	7	5	–500	–105
19	0.40	0.40	7	5	–800	–100
20	0.40	0.40	7	5	–800	–105
21	0.40	0.40	7	6	–500	–100
22	0.40	0.40	7	6	–500	–105
23	0.40	0.40	7	6	–800	–100
24	0.40	0.40	7	6	–800	–105
25	0.40	0.40	8	5	–500	–100
26	0.40	0.40	8	5	–500	–105
27	0.40	0.40	8	5	–800	–100
28	0.40	0.40	8	5	–800	–105
29	0.40	0.40	8	6	–500	–100
30	0.40	0.40	8	6	–500	–105
31	0.40	0.40	8	6	–800	–100
32	0.40	0.40	8	6	–800	–105
33	0.50	0.30	7	5	–500	–100
34	0.50	0.30	7	5	–500	–105
35	0.50	0.30	7	5	–800	–100
36	0.50	0.30	7	5	–800	–105
37	0.50	0.30	7	6	–500	–100
38	0.50	0.30	7	6	–500	–105
39	0.50	0.30	7	6	–800	–100
40	0.50	0.30	7	6	–800	–105
41	0.50	0.30	8	5	–500	–100
42	0.50	0.30	8	5	–500	–105
43	0.50	0.30	8	5	–800	–100

(Table A1 contd…)

Combination Number	l_1 (m)	l_2 (m)	m_1 (kg)	m_2 (kg)	τ_1 (Nm)	τ_2 (Nm)
44	0.50	0.30	8	5	–800	–105
45	0.50	0.30	8	6	–500	–100
46	0.50	0.30	8	6	–500	–105
47	0.50	0.30	8	6	–800	–100
48	0.50	0.30	8	6	–800	–105
49	0.50	0.40	7	5	–500	–100
50	0.50	0.40	7	5	–500	–105
51	0.50	0.40	7	5	–800	–100
52	0.50	0.40	7	5	–800	–105
53	0.50	0.40	7	6	–500	–100
54	0.50	0.40	7	6	–500	–105
55	0.50	0.40	7	6	–800	–100
56	0.50	0.40	7	6	–800	–105
57	0.50	0.40	8	5	–500	–100
58	0.50	0.40	8	5	–500	–105
59	0.50	0.40	8	5	–800	–100
60	0.50	0.40	8	5	–800	–105
61	0.50	0.40	8	6	–500	–100
62	0.50	0.40	8	6	–500	–105
63	0.50	0.40	8	6	–800	–100
64	0.50	0.40	8	6	–800	–105

APPENDIX - B

Table B1 Design Matrix for 2^{14-6} Fractional factorial Design

Sl No.	A (m)	B (m)	C (kg)	D (kg)	E (Nm)	F (Nm)	G (deg)	H (deg)	J (Ns)	K (Ns)	L ($\times10^{-4}$ m)	M ($\times10^{-4}$ m)	N (kg)	O (kg)
1	0.4	0.3	7	5	0.05	0.05	0.05	0.05	3.5	2	1	0.5	0.0025	0.005
2	0.5	0.3	7	5	0.05	0.05	0.05	0.05	4	2.5	0.5	1	0.005	0.0025
3	0.4	0.4	7	5	0.05	0.05	0.05	0.05	4	2.5	0.5	1	0.0025	0.005
4	0.5	0.4	7	5	0.05	0.05	0.05	0.05	3.5	2	1	0.5	0.005	0.0025
5	0.4	0.3	8	5	0.05	0.05	0.05	0.05	4	2.5	1	0.5	0.0025	0.0025
6	0.5	0.3	8	5	0.05	0.05	0.05	0.05	3.5	2	0.5	1	0.005	0.005
7	0.4	0.4	8	5	0.05	0.05	0.05	0.05	3.5	2	0.5	1	0.0025	0.0025
8	0.5	0.4	8	5	0.05	0.05	0.05	0.05	4	2.5	1	0.5	0.005	0.005
9	0.4	0.3	7	6	0.05	0.05	0.05	0.05	4	2	0.5	1	0.005	0.005
10	0.5	0.3	7	6	0.05	0.05	0.05	0.05	3.5	2.5	1	0.5	0.0025	0.0025
11	0.4	0.4	7	6	0.05	0.05	0.05	0.05	3.5	2.5	1	0.5	0.005	0.005
12	0.5	0.4	7	6	0.05	0.05	0.05	0.05	4	2	0.5	1	0.0025	0.0025
13	0.4	0.3	8	6	0.05	0.05	0.05	0.05	3.5	2.5	0.5	1	0.005	0.0025
14	0.5	0.3	8	6	0.05	0.05	0.05	0.05	4	2	1	0.5	0.0025	0.005
15	0.4	0.4	8	6	0.05	0.05	0.05	0.05	4	2	1	0.5	0.005	0.0025
16	0.5	0.4	8	6	0.05	0.05	0.05	0.05	3.5	2.5	0.5	1	0.0025	0.005
17	0.4	0.3	7	5	0.1	0.05	0.05	0.05	4	2	0.5	0.5	0.005	0.0025
18	0.5	0.3	7	5	0.1	0.05	0.05	0.05	3.5	2.5	1	1	0.0025	0.005
19	0.4	0.4	7	5	0.1	0.05	0.05	0.05	3.5	2.5	1	1	0.005	0.0025
20	0.5	0.4	7	5	0.1	0.05	0.05	0.05	4	2	0.5	0.5	0.0025	0.005
21	0.4	0.3	8	5	0.1	0.05	0.05	0.05	3.5	2.5	0.5	0.5	0.005	0.005
22	0.5	0.3	8	5	0.1	0.05	0.05	0.05	4	2	1	1	0.0025	0.0025
23	0.4	0.4	8	5	0.1	0.05	0.05	0.05	4	2	1	1	0.005	0.005
24	0.5	0.4	8	5	0.1	0.05	0.05	0.05	3.5	2.5	0.5	0.5	0.0025	0.0025
25	0.4	0.3	7	6	0.1	0.05	0.05	0.05	3.5	2	1	1	0.0025	0.0025
26	0.5	0.3	7	6	0.1	0.05	0.05	0.05	4	2.5	0.5	0.5	0.005	0.005
27	0.4	0.4	7	6	0.1	0.05	0.05	0.05	4	2.5	0.5	0.5	0.0025	0.0025
28	0.5	0.4	7	6	0.1	0.05	0.05	0.05	3.5	2	1	1	0.005	0.005
29	0.4	0.3	8	6	0.1	0.05	0.05	0.05	4	2.5	1	1	0.0025	0.005
30	0.5	0.3	8	6	0.1	0.05	0.05	0.05	3.5	2	0.5	0.5	0.005	0.0025
31	0.4	0.4	8	6	0.1	0.05	0.05	0.05	3.5	2	0.5	0.5	0.0025	0.005
32	0.5	0.4	8	6	0.1	0.05	0.05	0.05	4	2.5	1	1	0.005	0.0025
33	0.4	0.3	7	5	0.05	0.1	0.05	0.05	3.5	2.5	0.5	1	0.0025	0.0025
34	0.5	0.3	7	5	0.05	0.1	0.05	0.05	4	2	1	0.5	0.005	0.005
35	0.4	0.4	7	5	0.05	0.1	0.05	0.05	4	2	1	0.5	0.0025	0.0025
36	0.5	0.4	7	5	0.05	0.1	0.05	0.05	3.5	2.5	0.5	1	0.005	0.005
37	0.4	0.3	8	5	0.05	0.1	0.05	0.05	4	2	0.5	1	0.0025	0.005
38	0.5	0.3	8	5	0.05	0.1	0.05	0.05	3.5	2.5	1	0.5	0.005	0.0025
39	0.4	0.4	8	5	0.05	0.1	0.05	0.05	3.5	2.5	1	0.5	0.0025	0.005
40	0.5	0.4	8	5	0.05	0.1	0.05	0.05	4	2	0.5	1	0.005	0.0025
41	0.4	0.3	7	6	0.05	0.1	0.05	0.05	4	2.5	1	0.5	0.005	0.0025
42	0.5	0.3	7	6	0.05	0.1	0.05	0.05	3.5	2	0.5	1	0.0025	0.005
43	0.4	0.4	7	6	0.05	0.1	0.05	0.05	3.5	2	0.5	1	0.005	0.0025
44	0.5	0.4	7	6	0.05	0.1	0.05	0.05	4	2.5	1	0.5	0.0025	0.005
45	0.4	0.3	8	6	0.05	0.1	0.05	0.05	3.5	2	1	0.5	0.005	0.005
46	0.5	0.3	8	6	0.05	0.1	0.05	0.05	4	2.5	0.5	1	0.0025	0.0025
47	0.4	0.4	8	6	0.05	0.1	0.05	0.05	4	2.5	0.5	1	0.005	0.005
48	0.5	0.4	8	6	0.05	0.1	0.05	0.05	3.5	2	1	0.5	0.0025	0.0025
49	0.4	0.3	7	5	0.1	0.1	0.05	0.05	4	2.5	1	1	0.005	0.005
50	0.5	0.3	7	5	0.1	0.1	0.05	0.05	3.5	2	0.5	0.5	0.0025	0.0025
51	0.4	0.4	7	5	0.1	0.1	0.05	0.05	3.5	2	0.5	0.5	0.005	0.005
52	0.5	0.4	7	5	0.1	0.1	0.05	0.05	4	2.5	1	1	0.0025	0.0025
53	0.4	0.3	8	5	0.1	0.1	0.05	0.05	3.5	2	1	1	0.005	0.0025
54	0.5	0.3	8	5	0.1	0.1	0.05	0.05	4	2.5	0.5	0.5	0.0025	0.005

(Table B1 contd…)

Sl No.	A (m)	B (m)	C (kg)	D (kg)	E (Nm)	F (Nm)	G (deg)	H (deg)	J (Ns)	K (Ns)	L ($\times10^{-4}$ m)	M ($\times10^{-4}$ m)	N (kg)	O (kg)
55	0.4	0.4	8	5	0.1	0.1	0.05	0.05	4	2.5	0.5	0.5	0.005	0.0025
56	0.5	0.4	8	5	0.1	0.1	0.05	0.05	3.5	2	1	1	0.0025	0.005
57	0.4	0.3	7	6	0.1	0.1	0.05	0.05	3.5	2.5	0.5	0.5	0.0025	0.005
58	0.5	0.3	7	6	0.1	0.1	0.05	0.05	4	2	1	1	0.005	0.0025
59	0.4	0.4	7	6	0.1	0.1	0.05	0.05	4	2	1	1	0.0025	0.005
60	0.5	0.4	7	6	0.1	0.1	0.05	0.05	3.5	2.5	0.5	0.5	0.005	0.0025
61	0.4	0.3	8	6	0.1	0.1	0.05	0.05	4	2	0.5	0.5	0.0025	0.0025
62	0.5	0.3	8	6	0.1	0.1	0.05	0.05	3.5	2.5	1	1	0.005	0.005
63	0.4	0.4	8	6	0.1	0.1	0.05	0.05	3.5	2.5	1	1	0.0025	0.0025
64	0.5	0.4	8	6	0.1	0.1	0.05	0.05	4	2	0.5	0.5	0.005	0.005
65	0.4	0.3	7	5	0.05	0.05	0.1	0.05	3.5	2.5	0.5	0.5	0.005	0.0025
66	0.5	0.3	7	5	0.05	0.05	0.1	0.05	4	2	1	1	0.0025	0.005
67	0.4	0.4	7	5	0.05	0.05	0.1	0.05	4	2	1	1	0.005	0.0025
68	0.5	0.4	7	5	0.05	0.05	0.1	0.05	3.5	2.5	0.5	0.5	0.0025	0.005
69	0.4	0.3	8	5	0.05	0.05	0.1	0.05	4	2	0.5	0.5	0.005	0.005
70	0.5	0.3	8	5	0.05	0.05	0.1	0.05	3.5	2.5	1	1	0.0025	0.0025
71	0.4	0.4	8	5	0.05	0.05	0.1	0.05	3.5	2.5	1	1	0.005	0.005
72	0.5	0.4	8	5	0.05	0.05	0.1	0.05	4	2	0.5	0.5	0.0025	0.0025
73	0.4	0.3	7	6	0.05	0.05	0.1	0.05	4	2.5	1	1	0.0025	0.0025
74	0.5	0.3	7	6	0.05	0.05	0.1	0.05	3.5	2	0.5	0.5	0.005	0.005
75	0.4	0.4	7	6	0.05	0.05	0.1	0.05	3.5	2	0.5	0.5	0.0025	0.0025
76	0.5	0.4	7	6	0.05	0.05	0.1	0.05	4	2.5	1	1	0.005	0.005
77	0.4	0.3	8	6	0.05	0.05	0.1	0.05	3.5	2	1	1	0.0025	0.005
78	0.5	0.3	8	6	0.05	0.05	0.1	0.05	4	2.5	0.5	0.5	0.005	0.0025
79	0.4	0.4	8	6	0.05	0.05	0.1	0.05	4	2.5	0.5	0.5	0.0025	0.005
80	0.5	0.4	8	6	0.05	0.05	0.1	0.05	3.5	2	1	1	0.005	0.0025
81	0.4	0.3	7	5	0.1	0.05	0.1	0.05	4	2.5	1	0.5	0.0025	0.005
82	0.5	0.3	7	5	0.1	0.05	0.1	0.05	3.5	2	0.5	1	0.005	0.0025
83	0.4	0.4	7	5	0.1	0.05	0.1	0.05	3.5	2	0.5	1	0.0025	0.005
84	0.5	0.4	7	5	0.1	0.05	0.1	0.05	4	2.5	1	0.5	0.005	0.0025
85	0.4	0.3	8	5	0.1	0.05	0.1	0.05	3.5	2	1	0.5	0.0025	0.0025
86	0.5	0.3	8	5	0.1	0.05	0.1	0.05	4	2.5	0.5	1	0.005	0.005
87	0.4	0.4	8	5	0.1	0.05	0.1	0.05	4	2.5	0.5	1	0.0025	0.0025
88	0.5	0.4	8	5	0.1	0.05	0.1	0.05	3.5	2	1	0.5	0.005	0.005
89	0.4	0.3	7	6	0.1	0.05	0.1	0.05	3.5	2.5	0.5	1	0.005	0.005
90	0.5	0.3	7	6	0.1	0.05	0.1	0.05	4	2	1	0.5	0.0025	0.0025
91	0.4	0.4	7	6	0.1	0.05	0.1	0.05	4	2	1	0.5	0.005	0.005
92	0.5	0.4	7	6	0.1	0.05	0.1	0.05	3.5	2.5	0.5	1	0.0025	0.0025
93	0.4	0.3	8	6	0.1	0.05	0.1	0.05	4	2	0.5	1	0.005	0.0025
94	0.5	0.3	8	6	0.1	0.05	0.1	0.05	3.5	2.5	1	0.5	0.0025	0.005
95	0.4	0.4	8	6	0.1	0.05	0.1	0.05	3.5	2.5	1	0.5	0.005	0.0025
96	0.5	0.4	8	6	0.1	0.05	0.1	0.05	4	2	0.5	1	0.0025	0.005
97	0.4	0.3	7	5	0.05	0.1	0.1	0.05	3.5	2	1	1	0.005	0.005
98	0.5	0.3	7	5	0.05	0.1	0.1	0.05	4	2.5	0.5	0.5	0.0025	0.0025
99	0.4	0.4	7	5	0.05	0.1	0.1	0.05	4	2.5	0.5	0.5	0.005	0.005
100	0.5	0.4	7	5	0.05	0.1	0.1	0.05	3.5	2	1	1	0.0025	0.0025
101	0.4	0.3	8	5	0.05	0.1	0.1	0.05	4	2.5	1	1	0.005	0.0025
102	0.5	0.3	8	5	0.05	0.1	0.1	0.05	3.5	2	0.5	0.5	0.0025	0.005
103	0.4	0.4	8	5	0.05	0.1	0.1	0.05	3.5	2	0.5	0.5	0.005	0.0025
104	0.5	0.4	8	5	0.05	0.1	0.1	0.05	4	2.5	1	1	0.0025	0.005
105	0.4	0.3	7	6	0.05	0.1	0.1	0.05	4	2	0.5	0.5	0.0025	0.005
106	0.5	0.3	7	6	0.05	0.1	0.1	0.05	3.5	2.5	1	1	0.005	0.0025
107	0.4	0.4	7	6	0.05	0.1	0.1	0.05	3.5	2.5	1	1	0.0025	0.005
108	0.5	0.4	7	6	0.05	0.1	0.1	0.05	4	2	0.5	0.5	0.005	0.0025
109	0.4	0.3	8	6	0.05	0.1	0.1	0.05	3.5	2.5	0.5	0.5	0.0025	0.0025
110	0.5	0.3	8	6	0.05	0.1	0.1	0.05	4	2	1	1	0.005	0.005
111	0.4	0.4	8	6	0.05	0.1	0.1	0.05	4	2	1	1	0.0025	0.0025
112	0.5	0.4	8	6	0.05	0.1	0.1	0.05	3.5	2.5	0.5	0.5	0.005	0.005
113	0.4	0.3	7	5	0.1	0.1	0.1	0.05	4	2	0.5	1	0.0025	0.0025

(Table B1 contd…)

Sl No.	A (m)	B (m)	C (kg)	D (kg)	E (Nm)	F (Nm)	G (deg)	H (deg)	J (Ns)	K (Ns)	L ($\times 10^{-4}$ m)	M ($\times 10^{-4}$ m)	N (kg)	O (kg)
114	0.5	0.3	7	5	0.1	0.1	0.1	0.05	3.5	2.5	1	0.5	0.005	0.005
115	0.4	0.4	7	5	0.1	0.1	0.1	0.05	3.5	2.5	1	0.5	0.0025	0.0025
116	0.5	0.4	7	5	0.1	0.1	0.1	0.05	4	2	0.5	1	0.005	0.005
117	0.4	0.3	8	5	0.1	0.1	0.1	0.05	3.5	2.5	0.5	1	0.0025	0.005
118	0.5	0.3	8	5	0.1	0.1	0.1	0.05	4	2	1	0.5	0.005	0.0025
119	0.4	0.4	8	5	0.1	0.1	0.1	0.05	4	2	1	0.5	0.0025	0.005
120	0.5	0.4	8	5	0.1	0.1	0.1	0.05	3.5	2.5	0.5	1	0.005	0.0025
121	0.4	0.3	7	6	0.1	0.1	0.1	0.05	3.5	2	1	0.5	0.005	0.0025
122	0.5	0.3	7	6	0.1	0.1	0.1	0.05	4	2.5	0.5	1	0.0025	0.005
123	0.4	0.4	7	6	0.1	0.1	0.1	0.05	4	2.5	0.5	1	0.005	0.0025
124	0.5	0.4	7	6	0.1	0.1	0.1	0.05	3.5	2	1	0.5	0.0025	0.005
125	0.4	0.3	8	6	0.1	0.1	0.1	0.05	4	2.5	1	0.5	0.005	0.005
126	0.5	0.3	8	6	0.1	0.1	0.1	0.05	3.5	2	0.5	1	0.0025	0.0025
127	0.4	0.4	8	6	0.1	0.1	0.1	0.05	3.5	2	0.5	1	0.005	0.005
128	0.5	0.4	8	6	0.1	0.1	0.1	0.05	4	2.5	1	0.5	0.0025	0.0025
129	0.4	0.3	7	5	0.05	0.05	0.05	0.1	3.5	2	1	1	0.005	0.0025
130	0.5	0.3	7	5	0.05	0.05	0.05	0.1	4	2.5	0.5	0.5	0.0025	0.005
131	0.4	0.4	7	5	0.05	0.05	0.05	0.1	4	2.5	0.5	0.5	0.005	0.0025
132	0.5	0.4	7	5	0.05	0.05	0.05	0.1	3.5	2	1	1	0.0025	0.005
133	0.4	0.3	8	5	0.05	0.05	0.05	0.1	4	2.5	1	1	0.005	0.005
134	0.5	0.3	8	5	0.05	0.05	0.05	0.1	3.5	2	0.5	0.5	0.0025	0.0025
135	0.4	0.4	8	5	0.05	0.05	0.05	0.1	3.5	2	0.5	0.5	0.005	0.005
136	0.5	0.4	8	5	0.05	0.05	0.05	0.1	4	2.5	1	1	0.0025	0.0025
137	0.4	0.3	7	6	0.05	0.05	0.05	0.1	4	2	0.5	0.5	0.0025	0.0025
138	0.5	0.3	7	6	0.05	0.05	0.05	0.1	3.5	2.5	1	1	0.005	0.005
139	0.4	0.4	7	6	0.05	0.05	0.05	0.1	3.5	2.5	1	1	0.0025	0.0025
140	0.5	0.4	7	6	0.05	0.05	0.05	0.1	4	2	0.5	0.5	0.005	0.005
141	0.4	0.3	8	6	0.05	0.05	0.05	0.1	3.5	2.5	0.5	0.5	0.0025	0.005
142	0.5	0.3	8	6	0.05	0.05	0.05	0.1	4	2	1	1	0.005	0.0025
143	0.4	0.4	8	6	0.05	0.05	0.05	0.1	4	2	1	1	0.0025	0.005
144	0.5	0.4	8	6	0.05	0.05	0.05	0.1	3.5	2.5	0.5	0.5	0.005	0.0025
145	0.4	0.3	7	5	0.1	0.05	0.05	0.1	4	2	0.5	1	0.0025	0.005
146	0.5	0.3	7	5	0.1	0.05	0.05	0.1	3.5	2.5	1	0.5	0.005	0.0025
147	0.4	0.4	7	5	0.1	0.05	0.05	0.1	3.5	2.5	1	0.5	0.0025	0.005
148	0.5	0.4	7	5	0.1	0.05	0.05	0.1	4	2	0.5	1	0.005	0.0025
149	0.4	0.3	8	5	0.1	0.05	0.05	0.1	3.5	2.5	0.5	1	0.0025	0.0025
150	0.5	0.3	8	5	0.1	0.05	0.05	0.1	4	2	1	0.5	0.005	0.005
151	0.4	0.4	8	5	0.1	0.05	0.05	0.1	4	2	1	0.5	0.0025	0.0025
152	0.5	0.4	8	5	0.1	0.05	0.05	0.1	3.5	2.5	0.5	1	0.005	0.005
153	0.4	0.3	7	6	0.1	0.05	0.05	0.1	3.5	2	1	0.5	0.005	0.005
154	0.5	0.3	7	6	0.1	0.05	0.05	0.1	4	2.5	0.5	1	0.0025	0.0025
155	0.4	0.4	7	6	0.1	0.05	0.05	0.1	4	2.5	0.5	1	0.005	0.005
156	0.5	0.4	7	6	0.1	0.05	0.05	0.1	3.5	2	1	0.5	0.0025	0.0025
157	0.4	0.3	8	6	0.1	0.05	0.05	0.1	4	2.5	1	0.5	0.005	0.0025
158	0.5	0.3	8	6	0.1	0.05	0.05	0.1	3.5	2	0.5	1	0.0025	0.005
159	0.4	0.4	8	6	0.1	0.05	0.05	0.1	3.5	2	0.5	1	0.005	0.0025
160	0.5	0.4	8	6	0.1	0.05	0.05	0.1	4	2.5	1	0.5	0.0025	0.005
161	0.4	0.3	7	5	0.05	0.1	0.05	0.1	3.5	2.5	0.5	0.5	0.005	0.005
162	0.5	0.3	7	5	0.05	0.1	0.05	0.1	4	2	1	1	0.0025	0.0025
163	0.4	0.4	7	5	0.05	0.1	0.05	0.1	4	2	1	1	0.005	0.005
164	0.5	0.4	7	5	0.05	0.1	0.05	0.1	3.5	2.5	0.5	0.5	0.0025	0.0025
165	0.4	0.3	8	5	0.05	0.1	0.05	0.1	4	2	0.5	0.5	0.005	0.0025
166	0.5	0.3	8	5	0.05	0.1	0.05	0.1	3.5	2.5	1	1	0.0025	0.005
167	0.4	0.4	8	5	0.05	0.1	0.05	0.1	3.5	2.5	1	1	0.005	0.0025
168	0.5	0.4	8	5	0.05	0.1	0.05	0.1	4	2	0.5	0.5	0.0025	0.005
169	0.4	0.3	7	6	0.05	0.1	0.05	0.1	4	2.5	1	1	0.0025	0.005
170	0.5	0.3	7	6	0.05	0.1	0.05	0.1	3.5	2	0.5	0.5	0.005	0.0025
171	0.4	0.4	7	6	0.05	0.1	0.05	0.1	3.5	2	0.5	0.5	0.0025	0.005
172	0.5	0.4	7	6	0.05	0.1	0.05	0.1	4	2.5	1	1	0.005	0.0025

(Table B1 contd…)

Sl No.	A (m)	B (m)	C (kg)	D (kg)	E (Nm)	F (Nm)	G (deg)	H (deg)	J (Ns)	K (Ns)	L ($\times10^{-4}$ m)	M ($\times10^{-4}$ m)	N (kg)	O (kg)
173	0.4	0.3	8	6	0.05	0.1	0.05	0.1	3.5	2	1	1	0.0025	0.0025
174	0.5	0.3	8	6	0.05	0.1	0.05	0.1	4	2.5	0.5	0.5	0.005	0.005
175	0.4	0.4	8	6	0.05	0.1	0.05	0.1	4	2.5	0.5	0.5	0.0025	0.0025
176	0.5	0.4	8	6	0.05	0.1	0.05	0.1	3.5	2	1	1	0.005	0.005
177	0.4	0.3	7	5	0.1	0.1	0.05	0.1	4	2.5	1	0.5	0.0025	0.0025
178	0.5	0.3	7	5	0.1	0.1	0.05	0.1	3.5	2	0.5	1	0.005	0.005
179	0.4	0.4	7	5	0.1	0.1	0.05	0.1	3.5	2	0.5	1	0.0025	0.0025
180	0.5	0.4	7	5	0.1	0.1	0.05	0.1	4	2.5	1	0.5	0.005	0.005
181	0.4	0.3	8	5	0.1	0.1	0.05	0.1	3.5	2	1	0.5	0.0025	0.005
182	0.5	0.3	8	5	0.1	0.1	0.05	0.1	4	2.5	0.5	1	0.005	0.0025
183	0.4	0.4	8	5	0.1	0.1	0.05	0.1	4	2.5	0.5	1	0.0025	0.005
184	0.5	0.4	8	5	0.1	0.1	0.05	0.1	3.5	2	1	0.5	0.005	0.0025
185	0.4	0.3	7	6	0.1	0.1	0.05	0.1	3.5	2.5	0.5	1	0.005	0.0025
186	0.5	0.3	7	6	0.1	0.1	0.05	0.1	4	2	1	0.5	0.0025	0.005
187	0.4	0.4	7	6	0.1	0.1	0.05	0.1	4	2	1	0.5	0.005	0.0025
188	0.5	0.4	7	6	0.1	0.1	0.05	0.1	3.5	2.5	0.5	1	0.0025	0.005
189	0.4	0.3	8	6	0.1	0.1	0.05	0.1	4	2	0.5	1	0.005	0.005
190	0.5	0.3	8	6	0.1	0.1	0.05	0.1	3.5	2.5	1	0.5	0.0025	0.0025
191	0.4	0.4	8	6	0.1	0.1	0.05	0.1	3.5	2.5	1	0.5	0.005	0.005
192	0.5	0.4	8	6	0.1	0.1	0.05	0.1	4	2	0.5	1	0.0025	0.0025
193	0.4	0.3	7	5	0.05	0.05	0.1	0.1	3.5	2.5	0.5	1	0.0025	0.005
194	0.5	0.3	7	5	0.05	0.05	0.1	0.1	4	2	1	0.5	0.005	0.0025
195	0.4	0.4	7	5	0.05	0.05	0.1	0.1	4	2	1	0.5	0.0025	0.005
196	0.5	0.4	7	5	0.05	0.05	0.1	0.1	3.5	2.5	0.5	1	0.005	0.0025
197	0.4	0.3	8	5	0.05	0.05	0.1	0.1	4	2	0.5	1	0.0025	0.0025
198	0.5	0.3	8	5	0.05	0.05	0.1	0.1	3.5	2.5	1	0.5	0.005	0.005
199	0.4	0.4	8	5	0.05	0.05	0.1	0.1	3.5	2.5	1	0.5	0.0025	0.0025
200	0.5	0.4	8	5	0.05	0.05	0.1	0.1	4	2	0.5	1	0.005	0.005
201	0.4	0.3	7	6	0.05	0.05	0.1	0.1	4	2.5	1	0.5	0.005	0.005
202	0.5	0.3	7	6	0.05	0.05	0.1	0.1	3.5	2	0.5	1	0.0025	0.0025
203	0.4	0.4	7	6	0.05	0.05	0.1	0.1	3.5	2	0.5	1	0.005	0.005
204	0.5	0.4	7	6	0.05	0.05	0.1	0.1	4	2.5	1	0.5	0.0025	0.0025
205	0.4	0.3	8	6	0.05	0.05	0.1	0.1	3.5	2	1	0.5	0.005	0.0025
206	0.5	0.3	8	6	0.05	0.05	0.1	0.1	4	2.5	0.5	1	0.0025	0.005
207	0.4	0.4	8	6	0.05	0.05	0.1	0.1	4	2.5	0.5	1	0.005	0.0025
208	0.5	0.4	8	6	0.05	0.05	0.1	0.1	3.5	2	1	0.5	0.0025	0.005
209	0.4	0.3	7	5	0.1	0.05	0.1	0.1	4	2.5	1	1	0.005	0.0025
210	0.5	0.3	7	5	0.1	0.05	0.1	0.1	3.5	2	0.5	0.5	0.0025	0.005
211	0.4	0.4	7	5	0.1	0.05	0.1	0.1	3.5	2	0.5	0.5	0.005	0.0025
212	0.5	0.4	7	5	0.1	0.05	0.1	0.1	4	2.5	1	1	0.0025	0.005
213	0.4	0.3	8	5	0.1	0.05	0.1	0.1	3.5	2	1	1	0.005	0.005
214	0.5	0.3	8	5	0.1	0.05	0.1	0.1	4	2.5	0.5	0.5	0.0025	0.0025
215	0.4	0.4	8	5	0.1	0.05	0.1	0.1	4	2.5	0.5	0.5	0.005	0.005
216	0.5	0.4	8	5	0.1	0.05	0.1	0.1	3.5	2	1	1	0.0025	0.0025
217	0.4	0.3	7	6	0.1	0.05	0.1	0.1	3.5	2.5	0.5	0.5	0.0025	0.0025
218	0.5	0.3	7	6	0.1	0.05	0.1	0.1	4	2	1	1	0.005	0.005
219	0.4	0.4	7	6	0.1	0.05	0.1	0.1	4	2	1	1	0.0025	0.0025
220	0.5	0.4	7	6	0.1	0.05	0.1	0.1	3.5	2.5	0.5	0.5	0.005	0.005
221	0.4	0.3	8	6	0.1	0.05	0.1	0.1	4	2	0.5	0.5	0.0025	0.005
222	0.5	0.3	8	6	0.1	0.05	0.1	0.1	3.5	2.5	1	1	0.005	0.0025
223	0.4	0.4	8	6	0.1	0.05	0.1	0.1	3.5	2.5	1	1	0.0025	0.005
224	0.5	0.4	8	6	0.1	0.05	0.1	0.1	4	2	0.5	0.5	0.005	0.0025
225	0.4	0.3	7	5	0.05	0.1	0.1	0.1	3.5	2	1	0.5	0.0025	0.0025
226	0.5	0.3	7	5	0.05	0.1	0.1	0.1	4	2.5	0.5	1	0.005	0.005
227	0.4	0.4	7	5	0.05	0.1	0.1	0.1	4	2.5	0.5	1	0.0025	0.0025
228	0.5	0.4	7	5	0.05	0.1	0.1	0.1	3.5	2	1	0.5	0.005	0.005
229	0.4	0.3	8	5	0.05	0.1	0.1	0.1	4	2.5	1	0.5	0.0025	0.005
230	0.5	0.3	8	5	0.05	0.1	0.1	0.1	3.5	2	0.5	1	0.005	0.0025
231	0.4	0.4	8	5	0.05	0.1	0.1	0.1	3.5	2	0.5	1	0.0025	0.005

(Table B1 contd…)

Sl No.	A (m)	B (m)	C (kg)	D (kg)	E (Nm)	F (Nm)	G (deg)	H (deg)	J (Ns)	K (Ns)	L ($\times10^{-4}$ m)	M ($\times10^{-4}$ m)	N (kg)	O (kg)
232	0.5	0.4	8	5	0.05	0.1	0.1	0.1	4	2.5	1	0.5	0.005	0.0025
233	0.4	0.3	7	6	0.05	0.1	0.1	0.1	4	2	0.5	1	0.005	0.0025
234	0.5	0.3	7	6	0.05	0.1	0.1	0.1	3.5	2.5	1	0.5	0.0025	0.005
235	0.4	0.4	7	6	0.05	0.1	0.1	0.1	3.5	2.5	1	0.5	0.005	0.0025
236	0.5	0.4	7	6	0.05	0.1	0.1	0.1	4	2	0.5	1	0.0025	0.005
237	0.4	0.3	8	6	0.05	0.1	0.1	0.1	3.5	2.5	0.5	1	0.005	0.005
238	0.5	0.3	8	6	0.05	0.1	0.1	0.1	4	2	1	0.5	0.0025	0.0025
239	0.4	0.4	8	6	0.05	0.1	0.1	0.1	4	2	1	0.5	0.005	0.005
240	0.5	0.4	8	6	0.05	0.1	0.1	0.1	3.5	2.5	0.5	1	0.0025	0.0025
241	0.4	0.3	7	5	0.1	0.1	0.1	0.1	4	2	0.5	0.5	0.005	0.005
242	0.5	0.3	7	5	0.1	0.1	0.1	0.1	3.5	2.5	1	1	0.0025	0.0025
243	0.4	0.4	7	5	0.1	0.1	0.1	0.1	3.5	2.5	1	1	0.005	0.005
244	0.5	0.4	7	5	0.1	0.1	0.1	0.1	4	2	0.5	0.5	0.0025	0.0025
245	0.4	0.3	8	5	0.1	0.1	0.1	0.1	3.5	2.5	0.5	0.5	0.005	0.0025
246	0.5	0.3	8	5	0.1	0.1	0.1	0.1	4	2	1	1	0.0025	0.005
247	0.4	0.4	8	5	0.1	0.1	0.1	0.1	4	2	1	1	0.005	0.0025
248	0.5	0.4	8	5	0.1	0.1	0.1	0.1	3.5	2.5	0.5	0.5	0.0025	0.005
249	0.4	0.3	7	6	0.1	0.1	0.1	0.1	3.5	2	1	1	0.0025	0.005
250	0.5	0.3	7	6	0.1	0.1	0.1	0.1	4	2.5	0.5	0.5	0.005	0.0025
251	0.4	0.4	7	6	0.1	0.1	0.1	0.1	4	2.5	0.5	0.5	0.0025	0.005
252	0.5	0.4	7	6	0.1	0.1	0.1	0.1	3.5	2	1	1	0.005	0.0025
253	0.4	0.3	8	6	0.1	0.1	0.1	0.1	4	2.5	1	1	0.0025	0.0025
254	0.5	0.3	8	6	0.1	0.1	0.1	0.1	3.5	2	0.5	0.5	0.005	0.005
255	0.4	0.4	8	6	0.1	0.1	0.1	0.1	3.5	2	0.5	0.5	0.0025	0.0025
256	0.5	0.4	8	6	0.1	0.1	0.1	0.1	4	2.5	1	1	0.005	0.005

APPENDIX - C

Table C1 Central Composite Design Matrix for RSM

Combination number	l_1 (A) (m)	l_2 (B) (m)	m_1 (C) (kg)	m_2 (D) (kg)	σ_{θ_1} (E) (deg)	σ_{θ_2} (F) (deg)
1	0.4	0.3	7	5	0.05	0.05
2	0.5	0.3	7	5	0.05	0.05
3	0.4	0.4	7	5	0.05	0.05
4	0.5	0.4	7	5	0.05	0.05
5	0.4	0.3	8	5	0.05	0.05
6	0.5	0.3	8	5	0.05	0.05
7	0.4	0.4	8	5	0.05	0.05
8	0.5	0.4	8	5	0.05	0.05
9	0.4	0.3	7	6	0.05	0.05
10	0.5	0.3	7	6	0.05	0.05
11	0.4	0.4	7	6	0.05	0.05
12	0.5	0.4	7	6	0.05	0.05
13	0.4	0.3	8	6	0.05	0.05
14	0.5	0.3	8	6	0.05	0.05
15	0.4	0.4	8	6	0.05	0.05
16	0.5	0.4	8	6	0.05	0.05
17	0.4	0.3	7	5	0.1	0.05
18	0.5	0.3	7	5	0.1	0.05
19	0.4	0.4	7	5	0.1	0.05
20	0.5	0.4	7	5	0.1	0.05
21	0.4	0.3	8	5	0.1	0.05
22	0.5	0.3	8	5	0.1	0.05
23	0.4	0.4	8	5	0.1	0.05
24	0.5	0.4	8	5	0.1	0.05
25	0.4	0.3	7	6	0.1	0.05
26	0.5	0.3	7	6	0.1	0.05
27	0.4	0.4	7	6	0.1	0.05
28	0.5	0.4	7	6	0.1	0.05
29	0.4	0.3	8	6	0.1	0.05
30	0.5	0.3	8	6	0.1	0.05
31	0.4	0.4	8	6	0.1	0.05
32	0.5	0.4	8	6	0.1	0.05
33	0.4	0.3	7	5	0.05	0.1
34	0.5	0.3	7	5	0.05	0.1
35	0.4	0.4	7	5	0.05	0.1
36	0.5	0.4	7	5	0.05	0.1
37	0.4	0.3	8	5	0.05	0.1
38	0.5	0.3	8	5	0.05	0.1
39	0.4	0.4	8	5	0.05	0.1
40	0.5	0.4	8	5	0.05	0.1
41	0.4	0.3	7	6	0.05	0.1
42	0.5	0.3	7	6	0.05	0.1
43	0.4	0.4	7	6	0.05	0.1

(Table C1 contd…)

Combination number	l_1 (A) (m)	l_2 (B) (m)	m_1 (C) (kg)	m_2 (D) (kg)	σ_{θ_1} (E) (deg)	σ_{θ_2} (F) (deg)
44	0.5	0.4	7	6	0.05	0.1
45	0.4	0.3	8	6	0.05	0.1
46	0.5	0.3	8	6	0.05	0.1
47	0.4	0.4	8	6	0.05	0.1
48	0.5	0.4	8	6	0.05	0.1
49	0.4	0.3	7	5	0.1	0.1
50	0.5	0.3	7	5	0.1	0.1
51	0.4	0.4	7	5	0.1	0.1
52	0.5	0.4	7	5	0.1	0.1
53	0.4	0.3	8	5	0.1	0.1
54	0.5	0.3	8	5	0.1	0.1
55	0.4	0.4	8	5	0.1	0.1
56	0.5	0.4	8	5	0.1	0.1
57	0.4	0.3	7	6	0.1	0.1
58	0.5	0.3	7	6	0.1	0.1
59	0.4	0.4	7	6	0.1	0.1
60	0.5	0.4	7	6	0.1	0.1
61	0.4	0.3	8	6	0.1	0.1
62	0.5	0.3	8	6	0.1	0.1
63	0.4	0.4	8	6	0.1	0.1
64	0.5	0.4	8	6	0.1	0.1
65	0.4	0.35	7.5	5.5	0.075	0.075
66	0.5	0.35	7.5	5.5	0.075	0.075
67	0.45	0.3	7.5	5.5	0.075	0.075
68	0.45	0.4	7.5	5.5	0.075	0.075
69	0.45	0.35	7	5.5	0.075	0.075
70	0.45	0.35	8	5.5	0.075	0.075
71	0.45	0.35	7.5	5	0.075	0.075
72	0.45	0.35	7.5	6	0.075	0.075
73	0.45	0.35	7.5	5.5	0.05	0.075
74	0.45	0.35	7.5	5.5	0.1	0.075
75	0.45	0.35	7.5	5.5	0.075	0.05
76	0.45	0.35	7.5	5.5	0.075	0.1
77	0.45	0.35	7.5	5.5	0.075	0.075
78	0.45	0.35	7.5	5.5	0.075	0.075
79	0.45	0.35	7.5	5.5	0.075	0.075
80	0.45	0.35	7.5	5.5	0.075	0.075
81	0.45	0.35	7.5	5.5	0.075	0.075
82	0.45	0.35	7.5	5.5	0.075	0.075
83	0.45	0.35	7.5	5.5	0.075	0.075
84	0.45	0.35	7.5	5.5	0.075	0.075
85	0.45	0.35	7.5	5.5	0.075	0.075
86	0.45	0.35	7.5	5.5	0.075	0.075

APPENDIX - D

Table-D1 Control Factor Array in terms of Tolerances of Control Factors

Combination number	$3\sigma_{l_1}$ (m) ($\times10^{-2}$)	$3\sigma_{l_2}$ (m) ($\times10^{-2}$)	$3\sigma_{m_1}$ (kg)	$3\sigma_{m_2}$ (kg)	$3\sigma_{\tau_1}$ (Nm) ($\times10^{-2}$)	$3\sigma_{\tau_2}$ (Nm) ($\times10^{-2}$)
1	0.03	0.03	0.015	0.015	15	15
2	0.03	0.03	0.015	0.015	15	7.5
3	0.03	0.03	0.015	0.015	7.5	15
4	0.03	0.03	0.015	0.015	7.5	7.5
5	0.03	0.03	0.015	0.0075	15	15
6	0.03	0.03	0.015	0.0075	15	7.5
7	0.03	0.03	0.015	0.0075	7.5	15
8	0.03	0.03	0.015	0.0075	7.5	7.5
9	0.03	0.03	0.0075	0.015	15	15
10	0.03	0.03	0.0075	0.015	15	7.5
11	0.03	0.03	0.0075	0.015	7.5	15
12	0.03	0.03	0.0075	0.015	7.5	7.5
13	0.03	0.03	0.0075	0.0075	15	15
14	0.03	0.03	0.0075	0.0075	15	7.5
15	0.03	0.03	0.0075	0.0075	7.5	15
16	0.03	0.03	0.0075	0.0075	7.5	7.5
17	0.03	0.015	0.015	0.015	15	15
18	0.03	0.015	0.015	0.015	15	7.5
19	0.03	0.015	0.015	0.015	7.5	15
20	0.03	0.015	0.015	0.015	7.5	7.5
21	0.03	0.015	0.015	0.0075	15	15
22	0.03	0.015	0.015	0.0075	15	7.5
23	0.03	0.015	0.015	0.0075	7.5	15
24	0.03	0.015	0.015	0.0075	7.5	7.5
25	0.03	0.015	0.0075	0.015	15	15
26	0.03	0.015	0.0075	0.015	15	7.5
27*	0.03	0.015	0.0075	0.015	7.5	15
28	0.03	0.015	0.0075	0.015	7.5	7.5
29	0.03	0.015	0.0075	0.0075	15	15
30	0.03	0.015	0.0075	0.0075	15	7.5
31	0.03	0.015	0.0075	0.0075	7.5	15

(Table D1 contd…)

Combination number	$3\sigma_{l_1}$ (m) ($\times10^{-2}$)	$3\sigma_{l_2}$ (m) ($\times10^{-2}$)	$3\sigma_{m_1}$ (kg)	$3\sigma_{m_2}$ (kg)	$3\sigma_{\tau_1}$ (Nm) ($\times10^{-2}$)	$3\sigma_{\tau_2}$ (Nm) ($\times10^{-2}$)
32	0.03	0.015	0.0075	0.0075	7.5	7.5
33	0.015	0.03	0.015	0.015	15	15
34	0.015	0.03	0.015	0.015	15	7.5
35	0.015	0.03	0.015	0.015	7.5	15
36	0.015	0.03	0.015	0.015	7.5	7.5
37	0.015	0.03	0.015	0.0075	15	15
38	0.015	0.03	0.015	0.0075	15	7.5
39	0.015	0.03	0.015	0.0075	7.5	15
40	0.015	0.03	0.015	0.0075	7.5	7.5
41	0.015	0.03	0.0075	0.015	15	15
42	0.015	0.03	0.0075	0.015	15	7.5
43	0.015	0.03	0.0075	0.015	7.5	15
44	0.015	0.03	0.0075	0.015	7.5	7.5
45	0.015	0.03	0.0075	0.0075	15	15
46	0.015	0.03	0.0075	0.0075	15	7.5
47	0.015	0.03	0.0075	0.0075	7.5	15
48	0.015	0.03	0.0075	0.0075	7.5	7.5
49	0.015	0.015	0.015	0.015	15	15
50	0.015	0.015	0.015	0.015	15	7.5
51	0.015	0.015	0.015	0.015	7.5	15
52	0.015	0.015	0.015	0.015	7.5	7.5
53	0.015	0.015	0.015	0.0075	15	15
54	0.015	0.015	0.015	0.0075	15	7.5
55	0.015	0.015	0.015	0.0075	7.5	15
56	0.015	0.015	0.015	0.0075	7.5	7.5
57	0.015	0.015	0.0075	0.015	15	15
58	0.015	0.015	0.0075	0.015	15	7.5
59	0.015	0.015	0.0075	0.015	7.5	15
60	0.015	0.015	0.0075	0.015	7.5	7.5
61	0.015	0.015	0.0075	0.0075	15	15
62	0.015	0.015	0.0075	0.0075	15	7.5
63	0.015	0.015	0.0075	0.0075	7.5	15
64	0.015	0.015	0.0075	0.0075	7.5	7.5