

## Chapter 5

# Application of ACO to Train Scheduling Problem

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### 5.1 Introduction

The railway traffic system forms a backbone transport system of a nation. The rail transport system started with the invention of steam engine in 1785. Initially, it was mostly used for commodity freight transportation and later it was extended for passenger transportation. In the year 1825, for the first time steam-locomotive hauled a passenger train and people were transported in open coal wagons. The Railway transportation in the mid-eighteen century were quite different than today. There was no timetable at all; the railway operators could run their trains, whenever the trajectory was free; they literally fought for the right of using the tracks. There was virtually no safety system and collision was avoided only by the low speed of the trains.

The pioneers of passenger railways ( Maroti, 2006 ) would be quite astonished to see what their dreams have evolved into. Railways are now part of our everyday life. Trains operate according to carefully set-up timetables, safety has highest priority and comfortable carriages make long journeys easily bearable. For over a century, the development level of a country was directly measured by the density of its railway network. Till today, passenger and freight railway transportation play an important role in the economy of many countries.

For many years, railway transportation did not have to face much competition in public passenger and freight transportation. In the past two decades, the railways have lost a large part of their market share to automobiles and in last decade air traffic took over many middle and long distance train travelers. These developments have forced the railway operators to raise their service level in order to attract more customers and to cut their costs by working more efficiently. Rail operators can reach these goals by improving their planning process.

Railway companies have nearly inexhaustible sources of planning problems. Most of them are dealt with manually and lack automation and optimization. Railway applications have attracted the attention of mathematical research. Many problems are combinatorial in nature and suitable for Operations Research methods. In the last decade, computer-aided tools turned out to improve the railway planning process significantly and has contributed a lot to these successful applications. It shall be exciting to see, to what extent railway planning can be automated and optimized in the coming years and decades.

## **5.2 Railway Transportation as an Optimization Problem**

The Railway transportation industry (Cordeau et al., 1998) is rich in terms of problems that can be modeled and solved using mathematical optimization techniques. However, the related literature has experienced a slow growth and most contributions deal with simplified models or smaller problem instances failing to incorporate the characteristics of real life applications. This situation was surprising, given the considerable potential savings and performance improvements that may be realized through better resource utilization. In next subsection, we discuss the steps involved in the planning process and provide a brief description on the optimization problems that arise in each of these steps.

### **5.2.1 Railway Planning as an Optimization Problem**

Railway transport planning is a very complex task, which needs to be carried out keeping various stakeholder's interest in mind. The complexity of railway transport planning ( Ghoseiri et al., 2004 and Lindner, 2000 ) process can be divided into several steps. Figure 5.1 shows this decomposition.

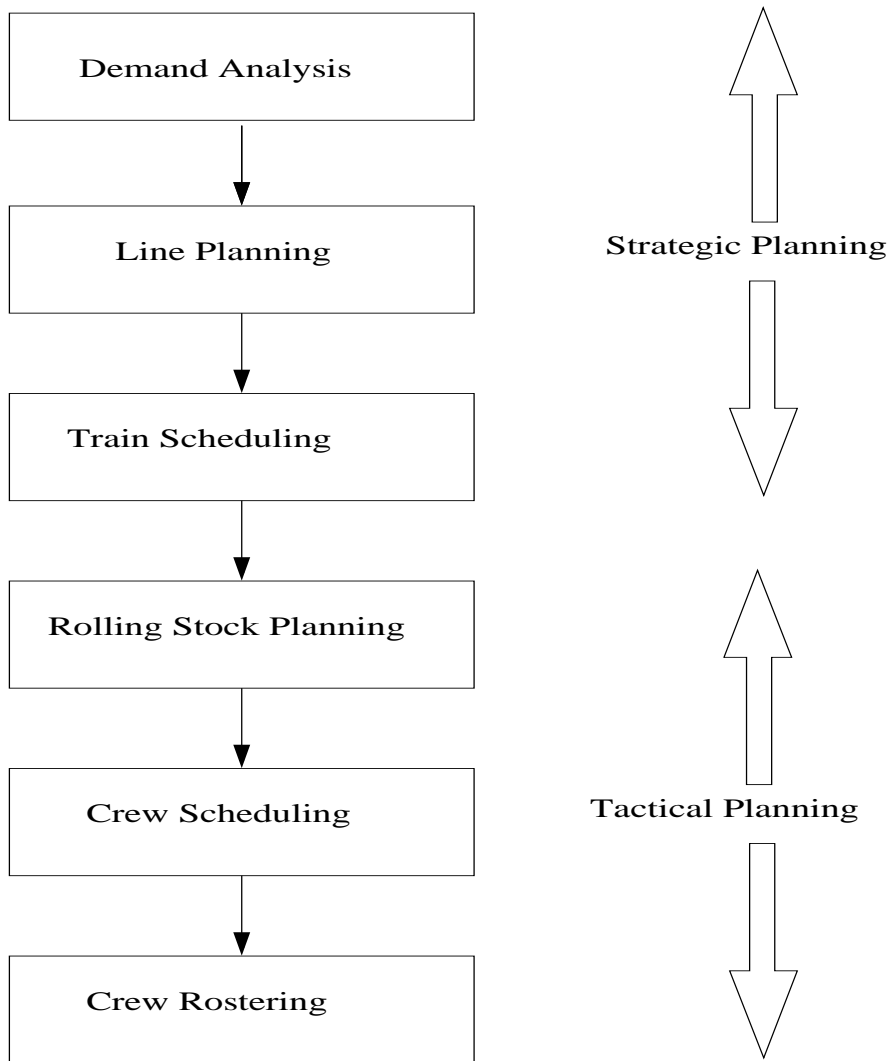


Figure 5.1: The hierarchical planning process in public rail transport.

The following is a brief description on the hierarchical planning process.

1. **Demand Analysis** - In order to plan railroad services ( Bussieck et al., 1997 ), one needs to have the information on the number of the passengers traveling from each location to any other location in the network along certain routes. Such information can be obtained by traffic counts and spot tests of passenger interrogations on some stations of the network. This method suffers from inherent inconsistencies like traffic counts that are not collected simultaneously. The optimization problem in demand analysis involves in deriving a reliable estimates for the origin-destination demands. Cascetta and Nguyen (1987) suggested an estimation method based on least squares and maximum likelihood. Sherali et al. (1994) proposed a linear programming approach for the Origin-Destination (OD) -matrix estimation problem considering all paths between an OD-pair.
2. **Line Planning** - Lines are the fundamentals of periodically scheduled railway transport systems. A line is a route in the railroad network connecting two terminal stations. The frequency of a line is the number of trains that serve this route in a fixed time interval. The line optimization problem consists in choosing a set of operating lines and its frequencies to serve the passenger's demand and to optimize some of the given objectives. Claessens et al.(1995) worked on minimum cost line plans. Bussieck et al. (1997) suggested a model for improving the comfort of the travelers considering the line plan design.
3. **Train Scheduling** - A train schedule consists of arrival and departure times for trains at each station. Each line  $l$  in the network is associated with the events i.e., arrival and departure of trains with a certain frequency of basic period  $T$  as described in a line plan. The consecutive events are subject to several constraints, since trains have to share the resources of the network. Hence, it is necessary to allocate enough time to enter and leave a line and to provide a safety distance between subsequent trains on the same track or platform. The usual objective for the evaluation of train schedules is to

minimize the traveling time. More recently, the problem of finding a periodic time table has received a lot of attention in the literature. Serafini and Ukovich (1989) proposed the Periodic Event Scheduling Problem (PESP) model to handle the periodic time table problem.

4. **Rolling Stock Planning** - The rail stock consists of locomotives and wagons. These stocks are combined to have a train. The length of the train depends on the number of the wagons. The coupling, decoupling operations are carried out at the station terminals depending on the operational requirements. The rolling stock planning process consists of acquisition of new rolling stock, assignment of different lines to the network, etc. The main objective of the process is to find rolling stock schedules with low operational costs and high service quality. Alfieri et al. (2006) focused on the determination of appropriate numbers of train units of different types together with their efficient circulation on a single line. Abbink et al. (2004) proposed a model to find an optimal allocation of train types and subtypes during peak hours time.
5. **Crew Scheduling** - Crew Scheduling is a part of *Crew Planning*. The Crew Planning is a typical problem arising in large transit system like railways. The crew scheduling can be further decomposed into *Crew Scheduling* and *Crew Rostering*. Crew Scheduling involves assignment of crew for various *train services* (movements of passenger or freight trains between stations) that need to be performed on a day to day basis. The crew assignment proceeds as follows: Each train service is first split into a sequence of *trips* also called as train journey segments. Each trip is characterized by departure station, departure time, arrival station, arrival time and the journey time. After arriving at the various trips, crew scheduling involves assignment of crew located at various depots to each of these trips ensuring all the operational constraints are met at the minimal cost. Although, the cost may depend on several factors, the main objective is to minimize the global number of crews needed to perform all the train services. Kroon and Fischetti

(2000) proposed a model based on set covering with additional constraints for scheduling train drivers and conductors for the Dutch railway operator NS Reizigers. Abbink et al. (2004) proposed a model for duty allocation to drivers and conductors based on planning support system called TURNI. TURNI is a set covering model that solves the problem by a combination of dynamic column generation, Lagrangian relaxation, and heuristic search methods.

6. **Crew Rostering** - The Crew Rostering Problem (CRP) aims at determining an optimal sequencing of a given set of trips into rosters satisfying operational constraints that are derived from union contract and company regulations. The main objective of the CRP is to evenly distribute the workload among the crews and to use the minimum number of crews while constructing the roster for the given period. Caprara et al. (1998) developed a heuristic based on a mixed integer programming formulation to determine a roster with a minimum number of weeks such that each duty is done once every day. Hartog et al. (2009) proposed a method for solving the cyclic crew rostering problem.

The planning process follows the top-down approach. The top-down approach has the advantages like problem decomposition, in which problem can be broken down to manageable size and can be individually solved. In addition, decomposition supports various planning interval that arise from other classical subdivision consisting of *Strategic*, *Tactical* and *Operational* procedures ( Assad, 1980 ). In strategic planning level, some decisions are made about infrastructure investments. These decisions are long-term decision and require greater costs. These decisions are greatly affected by political considerations. The infrastructure of the network develops in this phase. The tactical planning level is in fact the resource allocation phase. Most of line planning details and train schedule planning are done in this phase. Operational planning is just the day-by-day decisions. The decisions regarding unexpected events like breakdowns, special trains or short-term changes in the infrastructure caused by construction sites, certain parts of the schedule,

Table 5.1: Planning levels in train transport.

Planning stages	Time horizon	Objective
Tactical level	1 - 5 years	Resource allocation
Strategic level	5 - 15 years	Resource acquisition
Operational level	24 hours - 1 year	Daily planning

rolling stock, or crew assignment patterns will be taken. This work focuses on time table generation and to arrive at the schedule order for the train that has a minimal waiting time for the entire scheduled trains. Table 5.1 shows the time horizon and objective of each planning stage.

The train timetable generation is a tedious and time consuming task. Traditionally, timetable is generated manually by trial and error method based on experience and information. The advent of computer aided tools have helped the planner to come up with the effective timetable ( Caprara et al., 2007 ,Kroon et al., 2009 ) and to assess the effectiveness in terms of robustness in routing ( Zwaneveld et al., 2001 ), revenue profitability etc. The aim of the train scheduling problem is to come up with the ideal timetable that satisfies several objectives. The objectives can be; maximizing the number of passengers, minimizing the number of conflicts, waiting time of the passengers, revenue maximization and so on. Hence, scheduling is a multi-objective optimization problem. In next subsection, we will discuss about the set of constraints related to the train scheduling problem.

### 5.2.2 Constraints in Train Scheduling Problem

The idealistic timetable (Tormos, 2008) needs to satisfy the following constraints and these can be grouped into three categories.

1. **User Requirements :**

- a. *Interval for the Initial Departure* - Users expect that each train should leave the starting station within specified time bound.
- b. *Interval for the Arrival time* - Users expect that each train should arrive at the final station within specified time bound.



- c. *Maximum Delay* - Users expect that each train should complete its journey within the specified upper bound journey time.

## 2. Traffic Constraints :

- a. *Journey time* - The time needed to travel from one station to another station.
- b. *Crossing* - Two trains traveling in opposite directions cannot occupy same track at the same time.
- c. *Commercial Stop* - Each train is expected to stop in station for  $C$  units of time.
- d. *Overtaking on the track section* - Overtaking must be avoided between any two trains going in the same direction on any double-track sections of their journeys.
- e. *Delay for unexpected stop* - When a train stops in a station to avoid conflicts with other trains (overtaking/crossing) and no commercial stop is planned in that station, then a time delay needs to be incorporated in the journey time of train to reflect the delay in the arrival time of next station.
- f. *Reception Time* - The difference between the arrival times of any two trains in the same station.
- g. *Expedition Time* - The difference between the departure and arrival times of any two trains in the same station.
- h. *Simultaneous Departure* - The difference between departure times of two trains leaving from the same station moving in opposite directions.

## 3. Infrastructure Constraints :

- a. *Finite Capacity of Stations* - A train can arrive at station, if at least one track is available for stoppage.
- b. *Closing Time* - The station will be closed during closing time for maintenance operations. The closing time imposes constraints over regular

operations. During closing time, trains can pass but cannot stop in the station or it may not be allowed neither pass nor stop.

- c. *Headway Time* - If two trains are traveling in the same direction, then  $\Delta$  time difference need to be maintained in terms of departure of trains.

It is possible to deduce many more constraints to reflect the realistic railway schedules. The constraints described above are generic constraints that need to be satisfied by any idealistic timetable.

## 5.3 Literature Review

In this section, we will review the literature related to railway scheduling and orienteering problem, which will be used as a benchmark program for train scheduling problem.

### 5.3.1 Railway Scheduling

The train scheduling problem involves determining the arrival and departure at each station, ensuring no collisions occur between the trains and all the operational constraints are satisfied. The train timetable design is carried for a larger fragment of railway network, a longer time horizon and larger number of trains. The physical railroad network is shared by a large number of trains; it is indeed necessary to synchronize the use of the available resources. Also, the simultaneous scheduling of freight and passenger trains has an important impact on the quality and level of service provided to the public.

The earlier models for train scheduling considered only the set of stations connected by a single line. Nemhauser (1969) developed the timetable for passenger trains on a line of stations. The problem of finding a periodic train timetable that minimizes the passenger waiting time in station has received a lot of attention in the literature. Some of the optimization models are proposed for that purpose by Cedar (1991), Nachtigall (1996), Nachtigall and Voget (1996) and Odijk (1996). The strategy of choosing a set of operating lines and their frequencies to serve

demand and maximize the number of travelers on direct connections was studied by Bussieck et al. (1996). Zwaneveld et al. (1996) and Kroon et al. (1997) by developing models and algorithms for routing trains through railway stations. Nachtigall and Voget (1997) proposed a model for choosing the track segments to be upgraded so as to reduce train running times and to minimize total passenger waiting time.

As already mentioned, the train scheduling problem belongs to a category of NP-hard problems and it is complex for both modeling and solving. The huge search space to explore, when solving real-world instances of scheduling problems makes heuristic techniques a suitable approach to obtain a feasible time table. Heuristic techniques provide the result in reasonable amount of time. These techniques use the domain knowledge of the problem to arrive at the solution. Cai and Goh (1994) used the greedy heuristic approach to resolve the conflicts. A set of rules were devised to determine the best way to resolve the conflicts. The results obtained using heuristics often deviate more from the optimal solution. Kraay et al. (1991) proposed a heuristics based on Local Heuristics Search (LSH) that tries to resolve the conflicts by looking into previous schedules. The LSH technique replaces the existing solution with the better solution by searching in the neighborhood region. The train conflicts that happens at sidings were shifted by one position and solution is accepted, if it results in minimum conflict delay. Higgins et al. (1997) proposed a methodology that combines LSH and tabu search to shift more than one conflicts at a time with the intention to have a reduced total conflict delay. Kwan and Mistry (2003) used the co-evolutionary approach to generate the automatic timetable. Tormas et al. (2008) used genetic algorithms to solve the train timetable problem. The chromosomes were encoded using the activity list representation and genes are represented with a sequence of (train, section) pair. The genes are organized to satisfy the feasibility constraint that ensures no two trains occupy the same track at the same time. The problem involves generating a population of chromosomes that satisfy the feasibility constraints and selecting the best chromosome that results in minimum conflict delay.

### 5.3.2 Orienteering Problem (OP)

The name Orienteering Problem (Vansteenwegen et al., 2011) originates from the sport, game of orienteering. In this game, individual competitors start at a specified control point, try to visit as many checkpoints as possible and return to the control point within a given time frame. Each checkpoint has a certain score and the objective is to maximize the total collected score. The OP is a combination of vertex selection and determining the shortest Hamiltonian path between the selected vertices. As a consequence, the OP can be seen as a combination of Knapsack Problem and the TSP. The OP's goal is to maximize the total score collected, while the TSP tries to minimize the travel time or distance. Furthermore, not all vertices have to be visited in OP and determining the shortest path between the selected vertices will be helpful to visit as many vertices as possible in the available time. An OP can be stated as follows: given a set of  $n$  nodes and scores for each node, the goal is to find the subset of nodes starting from vertex 1 to vertex  $n$  that maximizes the total score within the time  $T_{max}$ . The edges connecting the vertex are associated with time  $t$  and once a vertex is covered, it should not be considered for further inclusion in journey. It should be noted that, all the vertices may not be covered due to timing constraints  $T_{max}$ . Hence, problem of finding the multiple paths from source to destination vertex with the timing constraints is an NP hard problem. The OP reduces to generalized TSP, if  $T_{max}$  is relaxed or sufficiently large with the possibility to cover all the vertices.

## 5.4 A Model for Train Scheduling Problem

In this section, we will discuss the generally used terminology, set of referred notations and the input requirements of the model. The model is described by objective functions and the constraints that are supposed to be satisfied.

### 5.4.1 Definitions

1. *Sidings* - It is an unexpected stop that occurs on partially double track section for crossing or passing of trains.
2. *Conflict Delay* - The amount of time spent by train due to sidings on the track.
3. *Minimum Headway* - The minimum duration of time separating two trains on a single track.
4. *Train Conflict* - It occurs under two circumstances on single track line:
  - a. When two trains approach each other.
  - b. When a fast train catches up the slow train.
5. *Resolving a Conflict* - If two trains are involved in conflict, then one of the trains must be forced for sidings so that other train can cross or pass it.
6. *Line Time* - The time taken by the train to cover the line.
7. *Dwell Time* - The waiting time of a train in a station.
8. *Station* - A place where passengers will board or get down from the train.

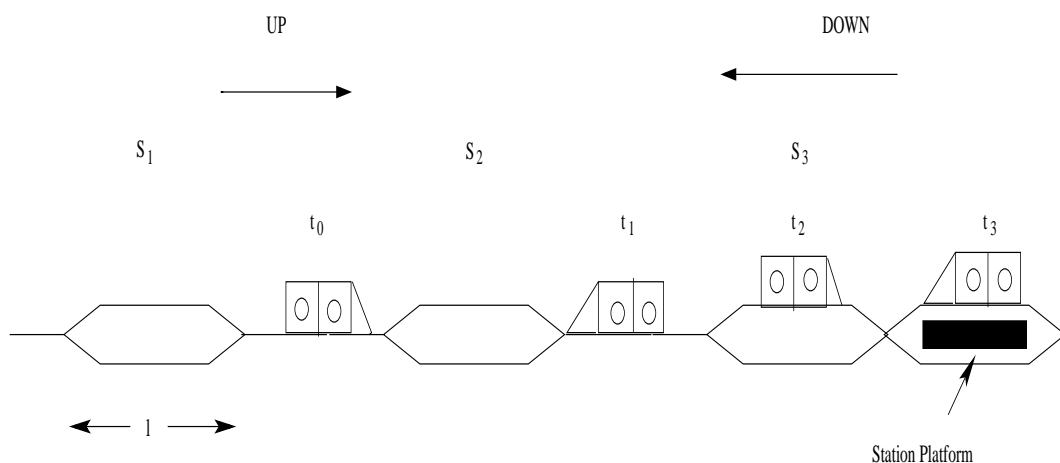


Figure 5.2: Train diagram.

Figure 5.2 shows the single track which is divided into lines  $l$ . The partial double lines  $\{S_1, S_2, S_3\}$  repeat at every alternate line. It can be observed that, one of the siding is present next to station. The train  $t_0$  can conflict with  $t_1$  and conflict can be resolved by siding one of the trains, so that crossing can take place. The UP trains move from left to right direction and the DOWN trains move from right to left direction.

### 5.4.2 Notations

The following notations were used to represent the model:

- $T$  : Set of trains  $\{t_1, t_2, \dots, t_n\}$ .
- $T_{up} \subset T$  : Set of  $n/2$  trains moving in up directions.
- $T_{down} \subset T$  : Set of  $n/2$  trains moving in down directions.
- $T = T_{up} \cup T_{down}$  and  $T_{up} \cap T_{down} = \emptyset$ .
- $L$  : Set of lines  $\{l_1, l_2, \dots, l_m\}$ . Set of lines connecting a pair of stations.
- $S$  : Set of stations  $\{s_1, s_2, \dots, s_j\}$   
 $j \ll m$  and  $S \subset L$ .
- $pml$  : Set of partial multiple lines. It can be obtained by

$$pml = L \text{ mod } k$$

$k$  controls the re-occurrence of partial multiple lines.

- $\Phi$  : Time spent by the train to cover the line.
- $\Delta$  : Specifies the minimum headway time.

### 5.4.3 Variables

- $A_i^{t_j}$  : Represents the arrival of train  $t_j$  on line  $i$ .
- $D_i^{t_j}$  : Represents the departure of train  $t_j$  from line  $i$ .
- $\gamma_i^{t_j}$  : Commercial stoppage or Dwell time of train  $t_j$  on line  $i$ .

### 5.4.4 Objective function

The objective function is to minimize the total conflict delay that arises due to unavoidable sidings.

$$Min_{cd} = \sum_{i \in T} \sum_{k \in DL} A_k^{t_i} - D_k^{t_i} \quad (5.1)$$

subject to **constraints**:

- **Line time constraints** specify the minimum amount of time, the train need to cover the line.

$$D_i - A_i = \Phi$$

- **Headway constraints** specify the minimum time difference need to be maintained between the departure of a train and the arrival of another train in the same direction on the same line.

$$A_i^{t_m} - D_i^{t_n} = \Delta$$

- **Train dispatch constraints**

– Train scheduled in *up* direction will use the lines in the following order:

$$\{ l_i, l_{i+1}, l_{i+2}, \dots, l_k \}$$

– Train scheduled in *down* direction will use the lines in the following order:

$$\{ l_k, l_{k-1}, l_{k-2}, \dots, l_i \}$$

where  $i \geq 0$ ,  $k \leq n$  and  $i < k$ .

- **Stop time constraints** specify the minimum time, the train needs to stop on a station line.

$$A_i^{tm} - D_i^{tm} = \Upsilon_i^{tm}$$

### 5.4.5 Assumption

The model is designed, keeping Indian Railways (IR) in mind. However, model's robustness lies in its ability to expand beyond the scope of application. The following assumptions are made with regard to the model and an attempt is made to keep the assumptions as realistic as possible by considering the operational activities of IR:

- All the stations are identified with respect to line numbers  $l_i$  and will have partial double lines. These partial double lines are in addition to  $DL$ .
- All the trains will travel with the same speed, will have same length, dwell time and have equal weightage, in the sense there will be only one type of train.
- The train  $t_i$  that starts its journey in *up* direction from source to destination station will come back again to source station in its *down* journey, but with the different train identifier say  $t_j$ .
- All the trains will run on a daily basis.
- All the lines are of equal length. The length of line will be more than the length of the train, which ensures crossing can be done without any problem.
- A train cannot be rescheduled in order to adjust the crossings with respect to unscheduled train.
- At any time, line is occupied by only one train (to ensure operational safety).
- The time measurement is expressed in minutes. The range of time spans from 1 - 1440 (24 X 60 ). We refer each minute as one time unit.



### 5.4.6 Input

The following input parameters are considered for the model:

- $Org_{t_i}$  = Origin station for train  $t_i \in T$  expressed in terms of line number.
- $Dst_{t_i}$  = Destination station for train  $t_i \in T$  expressed in terms of line number.
- $n$  = Number of trains.
- $L$  = Number of lines.
- $S$  = Number of stations.
- $\Delta$  = Headway time, expressed in terms of time unit.
- $\Phi$  = line time, expressed in terms of time unit.
- $\gamma$  = Commercial waiting time in station, expressed in terms of time unit.

## 5.5 ACO framework for Train Scheduling Problem

The train scheduling problem can be transformed into OP problem, since it generalizes the TSP. The train scheduling problem is a minimization problem with an objective to minimize the conflicting time whereas OP is a maximization problem with an objective to maximize the collected scores. A complete graph  $G=(V, E)$  called Train Schedule Order Graph (TSOG) is defined, where  $V$  is the set of vertices  $\{v_1, v_2, \dots, v_n\}$  and  $E$  is the set of edges  $\{e_1, e_2, \dots, e_m\}$ .  $V$  corresponds to set of trains and  $e_i \in E$  connecting  $t_i$  with  $t_j$  depicts the schedule order (i.e.,  $t_i$  followed by  $t_j$  or vice versa). Each edge is undirected, weighted, symmetric and associated with pheromone trail that stores the goodness of selecting the edge in previous schedules. The ants will use TSOG representation to move from one vertex to another vertex in order to construct the train schedule order.

Figure 5.3 shows the TSOG representation consisting of 5 trains. If ant has selected  $t_0$  as the starting train, then the possible schedule selection order can be  $t_0 \rightarrow t_2 \rightarrow t_4 \rightarrow t_3 \rightarrow t_1$ .

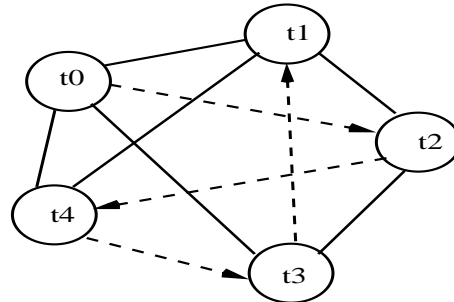


Figure 5.3: Train Schedule Diagram.

The generic pseudocode for Ant Based Train Scheduling Problem is given by Algorithm 13:

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**Algorithm 13** Ant Based Train Scheduling Problem Algorithm

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Initialize the parameters relevant to ACO and train scheduling application
while termination condition is not met do
  for  $i = 1, 2, \dots, n$  do
    for  $j = 1, 2, \dots, m$  do
      ScheduleTrain
    end for
    UpdatePheromone
  end for
end while

```

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The train scheduling problem is encoded with the TSOG  $G = (V, A)$ , a completely connected graph whose nodes  $V$  represent the trains and arcs  $A$  represent the schedule order between the train. Finding a solution means constructing a feasible walk in  $G$ . The pseudo-code essentially consists of two procedure namely *ScheduleTrain* and *UpdatePheromone*. The *ScheduleTrain* procedure constructs the train schedule order by traversing TSOG. The *UpdatePheromones* procedure modifies the pheromone on TSOG arcs. In subsequent sections, the solution construction procedure followed by the ants will be described.

### 5.5.1 Tour Construction

The solution construction proceeds as follows: The edges in TSOG will be initialized with pheromone trail of quantity  $\tau_0 = \frac{1}{n \cdot \eta_0}$ , where  $n$  is the number of trains and  $\eta_0$  is the least time difference between two scheduled trains. Initially, each ant will be randomly assigned the starting train. The selection of next train scheduled by ant  $k$  will be based on probabilistic function of next nearest schedule time of the train and the amount of pheromone trail present on the connecting path. The probabilistic function is given by the equation:

$$p_{t_i t_j}^k = \frac{[\tau_{t_i t_j}]^\alpha \cdot [\eta_{t_i t_j}]^\beta}{\sum_{k \notin \text{tabulist}} [\tau_{t_i t_j}]^\alpha \cdot [\eta_{t_i t_j}]^\beta} \quad (5.2)$$

$\tau_{t_i t_j}$  is the heuristic information, that specifies the amount of pheromone present on the path between  $t_i$  and  $t_j$  and  $\eta_{t_i t_j}$  is the visibility factor, computed as  $\frac{1}{D_{Org_{t_i}}^{t_i} - D_{Org_{t_j}}^{t_j}}$ . The two parameters  $\alpha$  and  $\beta$  control the importance of previous experience and the visibility factor. A *tabu list* is maintained for each ant  $k$  in order to keep track of selected trains and to ensure that no train is selected more than once in a given iteration. After the selection, train will be scheduled ensuring all the constraints are satisfied. If there are  $n$  trains, then each ant will make  $n - 1$  trains selection to come up with a single train schedule order.

### 5.5.2 Pheromone Updation

The train during its journey experiences the conflict delays in the form of crossing and passing which affects its total journey time. The individual ants construct the individual train schedule order and the selection experience can be expressed as *Total Conflict Time* (TCT). The TCT time is defined as sum of all the conflict delays experienced by all the trains in a single schedule order. The TCT will be used for trial reinforcement along the selection path of the trains. The trial reinforcement is done according to the following equation:

$$\tau_{t_i t_j} = \rho \cdot \tau_{t_i t_j} + \Delta \tau_{t_i t_j} \quad (5.3)$$

where  $\rho \in [0,1]$  is pheromone coefficient such that  $(1-\rho)$  represents the evaporation rate.

$$\Delta\tau_{t_i t_j} = \sum_{k=1}^m \Delta\tau_{t_i t_j}^k$$

where  $\Delta\tau_{t_i t_j}^k$  is the amount of pheromone trail laid by the ant  $k$  on the edge  $(t_i, t_j)$  and it is given by equation.

$$\Delta\tau_{ij}^k = \begin{cases} Q/L_k & \text{if the } k^{\text{th}} \text{ ant travels on } \textit{edge}(t_i, t_j) \\ 0 & \text{otherwise} \end{cases} \quad (5.4)$$

where  $Q$  is a constant and  $L_k$  is the total conflict time of the  $k^{\text{th}}$  ant.

## 5.6 Experimental Study

The proposed model is extensively simulated to identify the schedule that has minimal conflict delay. The model was analyzed with basic ant variants available in the literature as well as proposed algorithms which are listed in Table 5.2.

Table 5.2: ACO variants used to analyze the train scheduling model.

Class of Algorithms	Algorithms
<b>Basic Ant Variants</b>	AS ACS EA RA MMAS+GB MMAS+GB+PTS
<b>Punished Ants System</b>	PEAS PRAS
<b>Performance Linked Influential Elitist Ant System</b>	PLIEASMR PLIEASM PLIEASMed PLIRASMR PLIRASM PLIRASMed
<b>Punished Performance Linked Influential Elitist Ant System</b>	PPLIEASMR PPLIEASM PPLIEASMed PPLIRASMR PPLIRASM PPLIRASMed
<b>Cluster Integrated ACO variants</b>	GS-ACO k-Means ACO k-Medians ACO DBSCAN ACO

### 5.6.1 Parameter Settings and Input to the Algorithm

The parameters relevant to ACO were set as follows:

- $\alpha, \beta$  was varied from 1 to 5.
- $\rho$  was varied from 0.7 to 1.0 with incremental value of 0.5.
- Number of ants  $m$  was set to 20.
- Number of punished ants were varied from 5 to 15.

The parameters relevant to train scheduling problem were set as follows:

- Number of trains  $n = 20$ .
- Number of lines  $L = 200$ .
- Number of stations  $S = 15$ .
- Headway time  $\Delta = 6$ .
- line time  $\Phi = 3$ .
- Commercial waiting time  $\mathcal{T} = 2$ .
- $k$  that controls the repetition of double line was set to 8.

The input to the algorithm were set as follows:

- The line number  $l = \{ 0, 17, 38, 47, 56, 72, 80, 95, 117, 126, 149, 163, 179, 190, 199 \}$ , where  $l_i \subseteq L$  and each  $l_i$  represents the line number associated with the station.
- The train origin station and destination station are expressed in line numbers. The train details are expressed in tuple  $TP = \langle Org_{t_i}, Dst_{t_i}, Dept_{Org_{t_i}} \rangle$ , where  $Org_{t_i}, Dst_{t_i}$  represents the origin and destination station of train  $t_i$  and  $Dept_{Org_{t_i}}$  represents the departure time of train  $t_i$  from the origin station. The train details considered for the experimental purpose is  $\langle \{38, 117, 330\}, \{95, 190, 680\}, \{56, 149, 740\}, \{126, 199, 1120\}, \{17, 179, 445\}$ ,

{72, 190, 870}, {80, 199, 1300}, {38, 126, 540}, {0, 199, 1430}, {56, 163, 990}, {117, 38, 1202}, {190, 95, 475}, {149, 56, 670}, {199, 126, 1400}, {179, 17, 800}, {190, 72, 100}, {199, 80, 1350}, {126, 38, 340}, {199, 0, 550}, {163, 56, 1190} >.

## 5.6.2 Result Analysis of Algorithms for Fixed Commercial Waiting Time

The experimental simulations were carried out in two parts. In first part, commercial waiting time for all the stations was fixed as specified in the parameter settings and in second part it was varied. In reality, commercial waiting time depends on various factors like the importance of station, operational activities like loading of parcel items, coupling and decoupling of locomotives etc to name a few. In the second part comparative analysis will be done for both *Fixed Commercial Waiting Time* and *Variable Commercial Waiting time*. The assessment was performed for basic as well as newly proposed ant variants.

### Basic Ant Variants

Table 5.3 shows the comparative analysis for some of the important ant algorithm variants available in the literature. The model assessment was done with respect to TCT by considering the fact that best schedule order paths will have smaller TCT value. The smaller TCT value indicates the smaller waiting time for the scheduled trains. Table 5.3 reports the TCT and the observed parameter values. The experiment was carried out for partial double lines (pml=2) and partial triple lines (pml=3). For the partial double lines, best TCT was obtained for RA and for partial triple lines, ACS provides the best result. The MMAS+IB and MMAS+IB+PTS variants suffer from search stagnation and this may be due to ‘limiting the pheromone strength’ mechanism of algorithm. It can be observed that, limiting mechanism overcomes the search stagnation for ACO algorithms which use TSP as a benchmark problem, but for an OP problem, it leads to search stagnation. The results obtained when pml=3 are better than the results

when  $pml=2$ , demonstrating the fact that better availability of lines for sidings results in lesser waiting time on stations/ partial multiple lines. The comparison of best results for  $pml=2$  and  $pml=3$  parameter settings reveals that waiting time for  $pml=3$  has decreased by 8.56%. Another interesting observation can be made with respect to ACO parameters for the obtained TCT values. The observed parameter values for  $\alpha$  is predominantly high for both  $pml=2$  and  $pml=3$ . Similarly, lower  $\beta$  values are observed for  $pml=3$ .

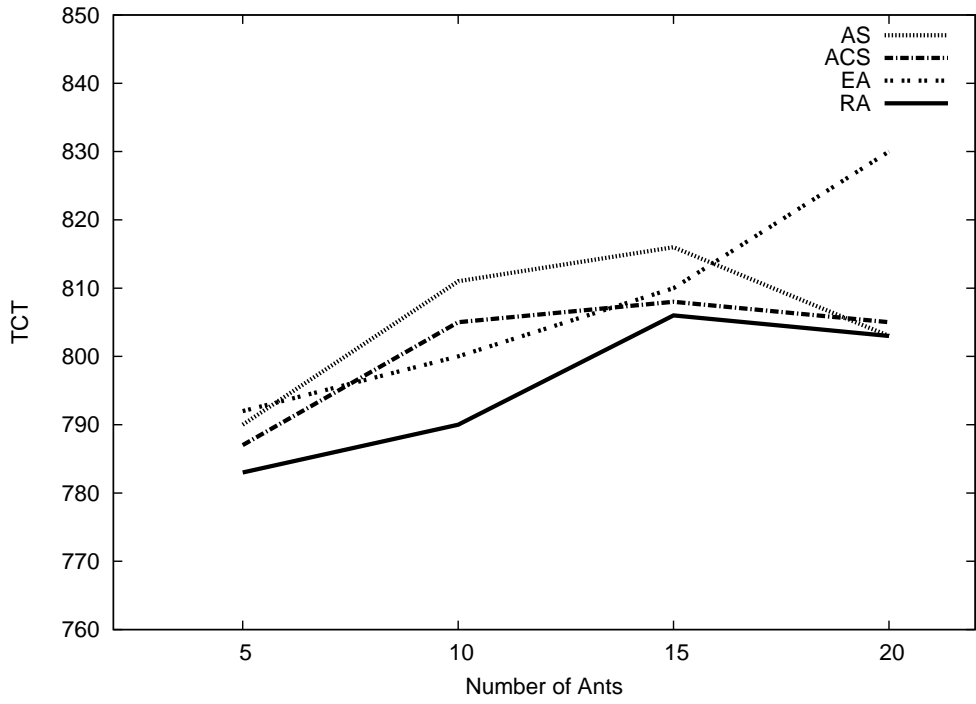
Table 5.3: Performance of basic ant variants for fixed commercial waiting time.

Algorithms	pml = 2		pml = 3	
	TCT	Parameter details	TCT	Parameter details
AS	790	$\alpha=3 \beta=3 \rho=0.75$	757	$\alpha=4 \beta=3 \rho=0.75$
ACS	787	$\alpha=3 \beta=4 \rho=0.85$	<b>716</b>	$\alpha=4 \beta=1 \rho=0.85$
EA	792	$\alpha=4 \beta=1 \rho=0.75$	751	$\alpha=5 \beta=1 \rho=0.7$
RA	<b>783</b>	$\alpha=4 \beta=2 \rho=0.85$	722	$\alpha=3 \beta=1 \rho=0.9$
MMAS+IB	1023	$\alpha=1 \beta=4 \rho=0.70$	1025	$\alpha=2 \beta=1 \rho=0.8$
MMAS+IB+PTS	1023	$\alpha=1 \beta=4 \rho=0.85$	1025	$\alpha=4 \beta=3 \rho=0.9$

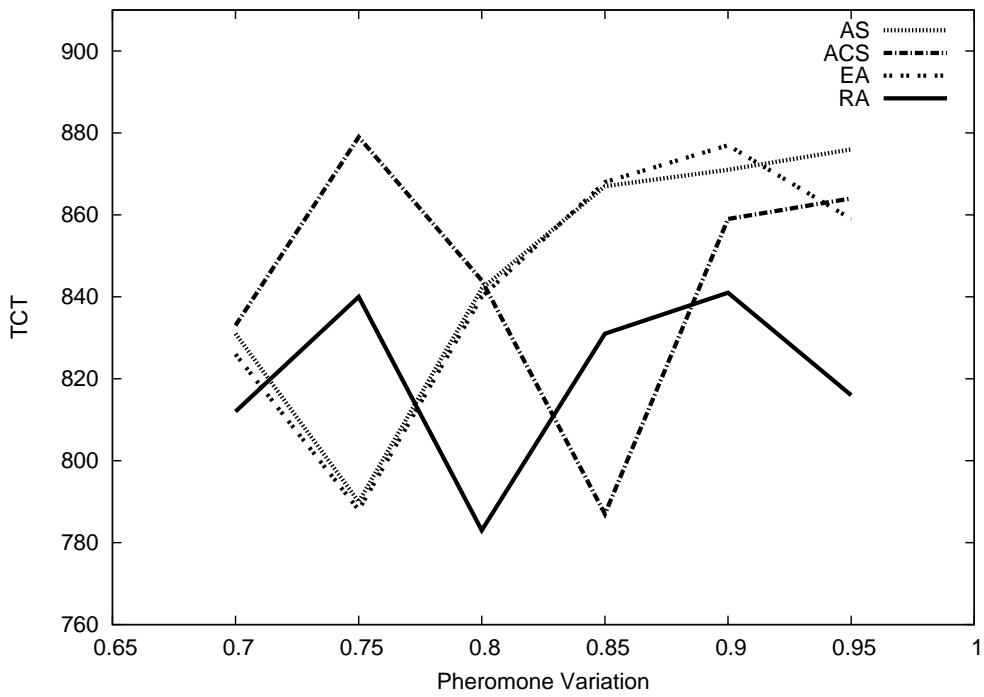
Figure 5.4 provides the comparative analysis of basic ant variants for  $pml=2$ . It can be seen from Figure 5.4(a) that all the variants have lesser waiting time for smaller ant population of size 5. RA variant provides consistently good result in terms of lesser waiting time to the variation in number of ants. Similarly, EA exhibits large variation for the observed waiting times. It is quite interesting to see that AS, ACS and RA have nearby waiting time and show a converging trend for ant population of size 20. Figure 5.4(b) shows the performance graph for variation in pheromone trial. The RA variant exhibits lesser variation in the observed waiting time. All the variants have least waiting time for  $\rho$  in the range of 0.75-0.85.

Figure 5.5 provides the comparative analysis of basic ant variants for  $pml=3$ . Figure 5.5(a) shows that ACS and RA have lesser waiting time than AS and EA for varying number of ants. Infact, ACS and RA provide least waiting time for  $n=15$ . The ACS variant has consistently outperformed and AS variant showed





(a) Variation in number of Ants



(b) Variation in Pheromone trial

Figure 5.4: Performance comparison of basic ant variants for partial double lines (fixed commercial waiting time).

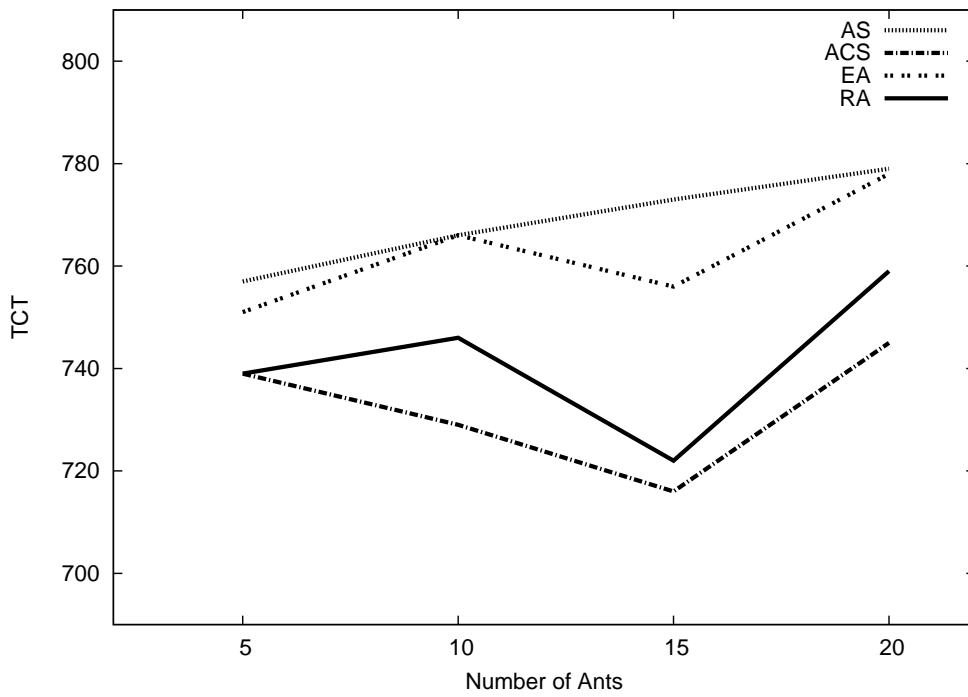
worst performance in terms of waiting time across the variation in number of ants. Similarly, 5.5(b) depicts the algorithms performance for variation in trial strength. The graph looks quite interesting as relatively lesser deviation in waiting time has been observed for lower pheromone strength and it increases after  $\rho=0.8$  across all the variants. RA variant exhibits comparatively smaller variations and AS shows larger variation in waiting time for varying trial strength.

### **Punished and Performance Linked Ant Variants**

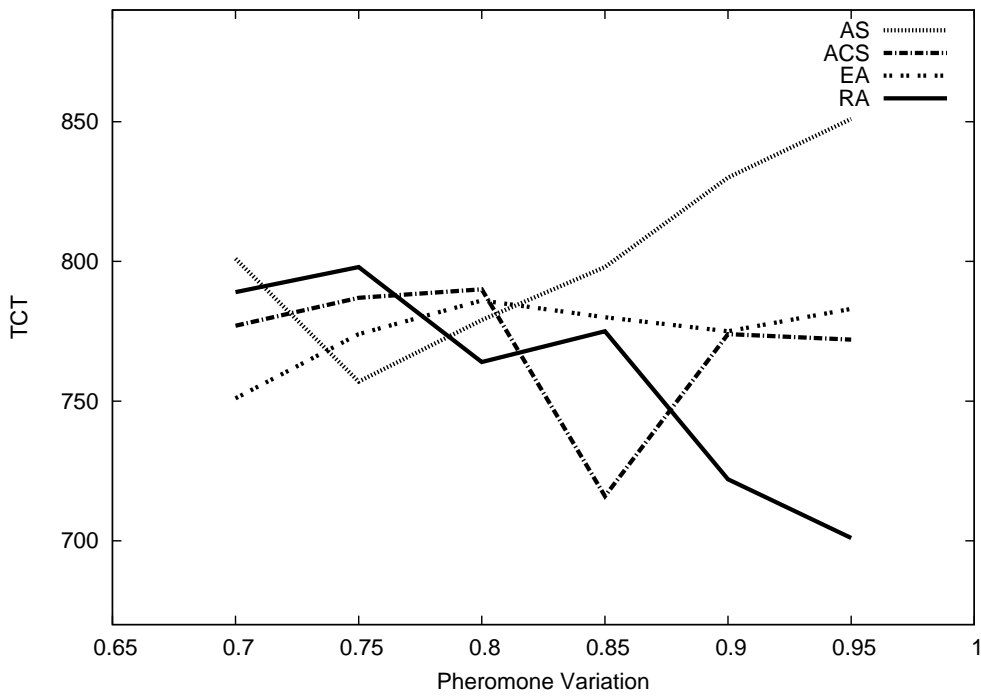
The Chapter 2 discusses the integration of punishment mechanism and influential ant selection mechanism in ACO variants. The punishment mechanism is characterized by punishing the non-elite ants, in which specified amount of pheromone trails will be taken out of the paths traveled by the non-elite ants and the number of non-elite ants will be specified as a part of parameter settings. The chapter 2 also discusses the dynamic ant selection mechanism called Influential Ants (IA) selection mechanism that identifies the elite and non-elite ants based on the IA specification.

The IA selection mechanism was extended to EA and RA ant variants resulting in PLIEAS and PLIRAS class of algorithms. The statistical tools like mean, median and mid-range were integrated with these class of algorithms that resulted in several variants. In PLIEAS, additional reinforcement will be done for the elite paths proportionate to the quality of solution found by the ants and in case of PPLIEAS, additional pheromone enforcement will be done for elite paths and non-elite paths will be punished by additional pheromone evaporation proportional to the quality of solution.

Inorder to demonstrate the superiority of proposed mechanisms; PEAS, PLIEAS and PPLIEAS were applied to train scheduling problem. The TCT is used as a measure to assess the performance of train scheduling problem and it will form the basis for pheromone updation. In case of PEAS, PLIEAS and PPLIEAS, TCT's reported by elite ants will be a basis for additional reinforcement on the schedule order paths and additional pheromone evaporation will be done for non-elite



(a) Variation in number of Ants



(b) Variation in Phormone trial

Figure 5.5: Performance comparison of basic ant variants for partial triple lines (fixed commercial waiting time).

schedule order paths reported by non-elite ants for PPLIEAS. For the experimentation purpose, number of non-elite ants *nea* were varied from 5 to 15 and in case of PLIEAS and PPLIEAS, elite and non-elite ants selection will be done as per algorithmic specification.

Table 5.4 shows the comparative results for PEAS, PLIEAS and PPLIEAS. In Punished ant system, PEAS provides the best TCT value and its performance is at par with basic ant variants (see Table 5.3  $pml=2$ ). Similarly, in PLIEAS and PPLIEAS class, PLIRASM and PPLIRASM provide the best TCT value with decrease in waiting time of 1.66% and 0.70% respectively, when compared with the basic ant variants. Another interesting observation is that punishment mechanism hasn't improved the performance of IA selection mechanism and shows an overall increase in waiting time of 0.90%. In general, it can be observed that punished and performance linked ant variants provide improved TCT values than the basic ant variants.

Figure 5.6 reveals the performance of the ants for Punished Ant Systems. Figure 5.6(a) reveals that both the PEAS and PRAS variants provide the best waiting time for smaller number of non-elite ants population *nea* size of 5. It can be observed that, waiting time increased with the increase in the number of punished ants indicating the negative effect of larger punished ants. Figure 5.6(b) shows the comparative analysis of algorithms for different trial strength. The figure reveals that PEAS variant provides better waiting time for higher trail strength than the lower trial strength and least waiting time is observed for  $\rho=0.85$ . Similarly, for PRAS variant, trains experience more conflicts leading to longer waiting time for  $\rho$  setting in the range of  $0.8 < \rho < 0.9$  and provides better TCT values for  $\rho=0.95$ .

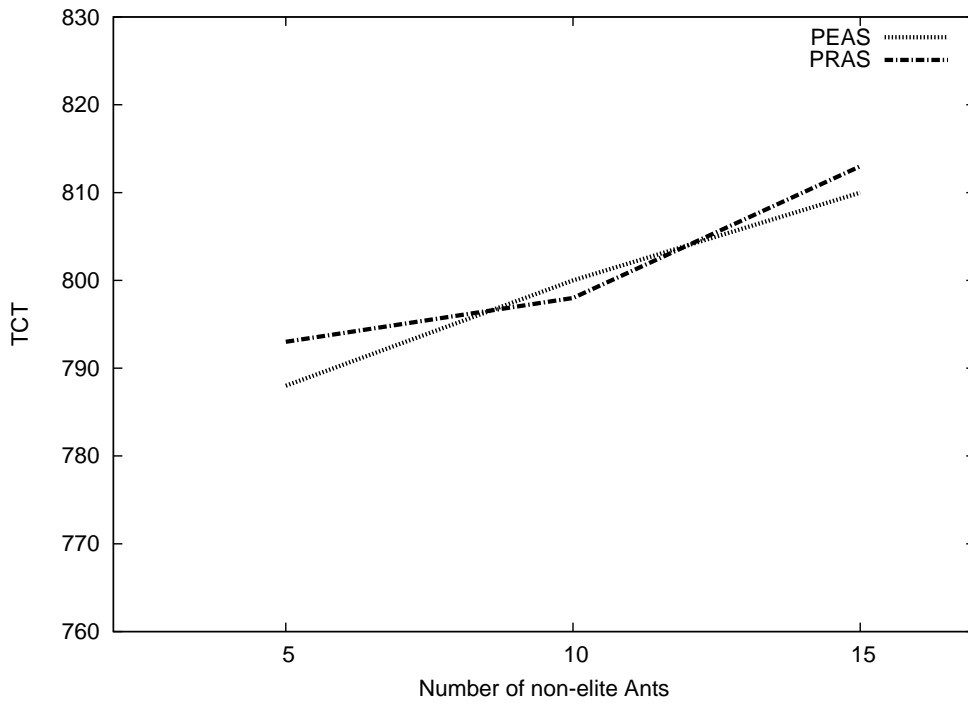
Table 5.4: Performance of punished and performance linked ACO variants on Train Scheduling Problem for fixed commercial waiting time.

Class	pml = 2		
	Algorithms	TCT	Parameter details
Punished Ant System	PEAS	<b>788</b>	$\alpha=4 \beta=3 \rho=0.85$
	PRAS	792	$\alpha=1 \beta=3 \rho=0.95$
Performance Linked Influential Elite Ant Systems	PLIEASMR	793	$\alpha=3 \beta=1 \rho=.75$
	PLIEASM	794	$\alpha=4 \beta=1 \rho=0.75$
	PLIEASMed	783	$\alpha=5 \beta=1 \rho=0.70$
	PLIRASMR	776	$\alpha=5 \beta=1 \rho=0.8$
	PLIRASM	<b>770</b>	$\alpha=3 \beta=1 \rho=0.7$
	PLIRASMed	776	$\alpha=5 \beta=1 \rho=0.8$
Punished Performance Linked Influential Elite Ant Systems	PPLIEASMR	800	$\alpha=5 \beta=3 \rho=0.75$
	PPLIEASM	792	$\alpha=4 \beta=1 \rho=0.95$
	PPLIEASMed	800	$\alpha=5 \beta=3 \rho=0.75$
	PPLIRASMR	805	$\alpha=5 \beta=5 \rho=0.85$
	PPLIRASM	<b>777</b>	$\alpha=5 \beta=1 \rho=0.85$
	PPLIRASMed	793	$\alpha=5 \beta=1 \rho=0.75$

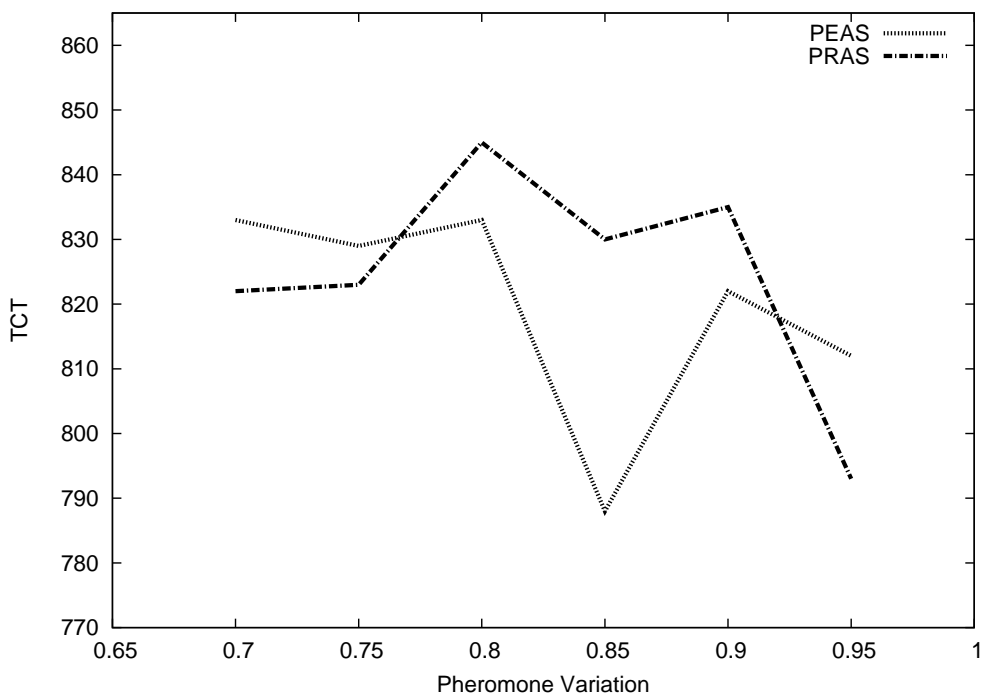
Figure 5.7 shows the comparative performance of PLIEAS for train scheduling problem. One of the interesting observations that can be made from Figure 5.7(a) is that, PLIEASM, PLIEASMR and PLIEASMed have similar waiting times for  $n=15$ . The PLIEASM, PLIEASMR and PLIEASMed provide least waiting time, when the number of ants settings  $n$  is 20, 10, 20 respectively. Figure 5.7(b) shows the performance of PLIEAS algorithms for varying pheromone trail. The variants exhibit larger deviation in the observed waiting time for lower pheromone trail compared to the higher pheromone trail and provide least waiting time for  $\rho$  in the range of 0.7-0.75 for all the variants.

Figure 5.8 shows the performance of PPLIEAS variants, when applied to train scheduling problem. Figure 5.8(a) shows that PPLIEASM has least sensitiveness in waiting time variation compared to the other variants for varying ants population. The PPLIEASMR variant has comparatively longer waiting time than the other variants. The PPLIEASM, PPLIEASMR and PPLIEASMed provide minimal waiting time, when number of ants  $n$  was set to 20, 10, 15 respectively. Figure 5.8(b) shows that PPLIEASMR is sensitive to the varying pheromone trail. The waiting time increased rapidly with the increase in the pheromone strength. The PPLIEASM variant is comparatively insensitive to the varying trail strength compared to its other peer variants. The PPLIEASM, PPLIEASMR and PPLIEASMed provide optimal waiting time, when trail strength  $\rho$  was 0.95, 0.75 and 0.85 respectively.

Figure 5.9 reveals the comparative analysis of PLIRAS variants. It can be observed from Figure 5.9(a) that waiting time of PLIRASMed is comparatively longer than the PLIRASMR and PLIRASM for varying number of ants. It is interesting to see that PLIRASMR has a better waiting time for smaller ant population and PLIRASM for larger ant population. The PLIRASM, PLIRASMR and PLIRASMed provide least waiting time for ants population size of 20, 10 and 15 respectively. Figure 5.9(b) shows that conflict among the trains increases rapidly for PLIRASM variant compared to other variants with the increase in trail strength. In general, it can be concluded that higher pheromone persistence factor

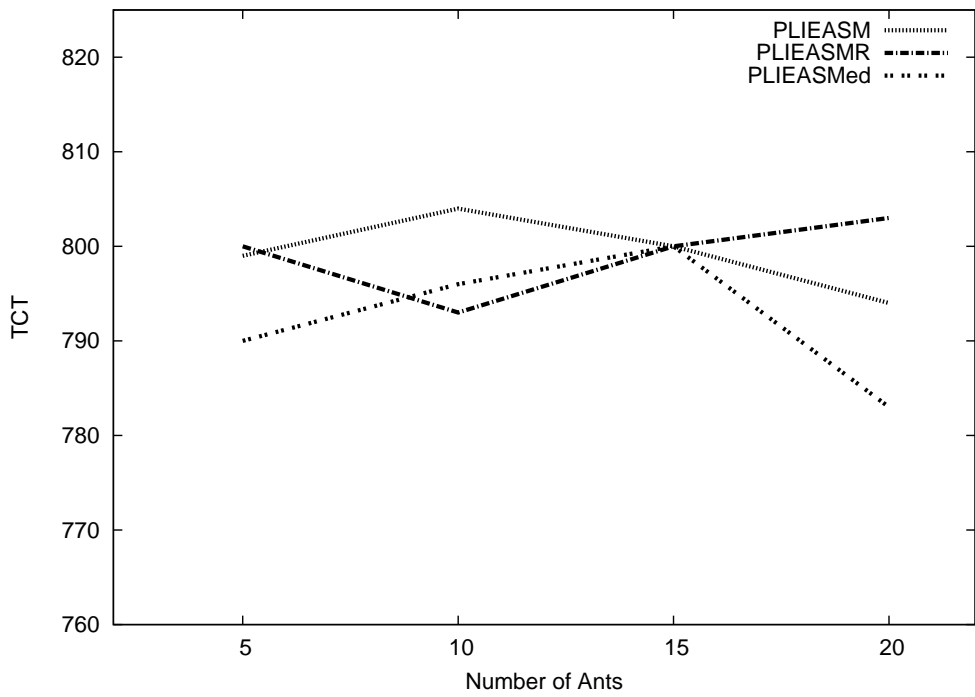


(a) Variation in number of Ants

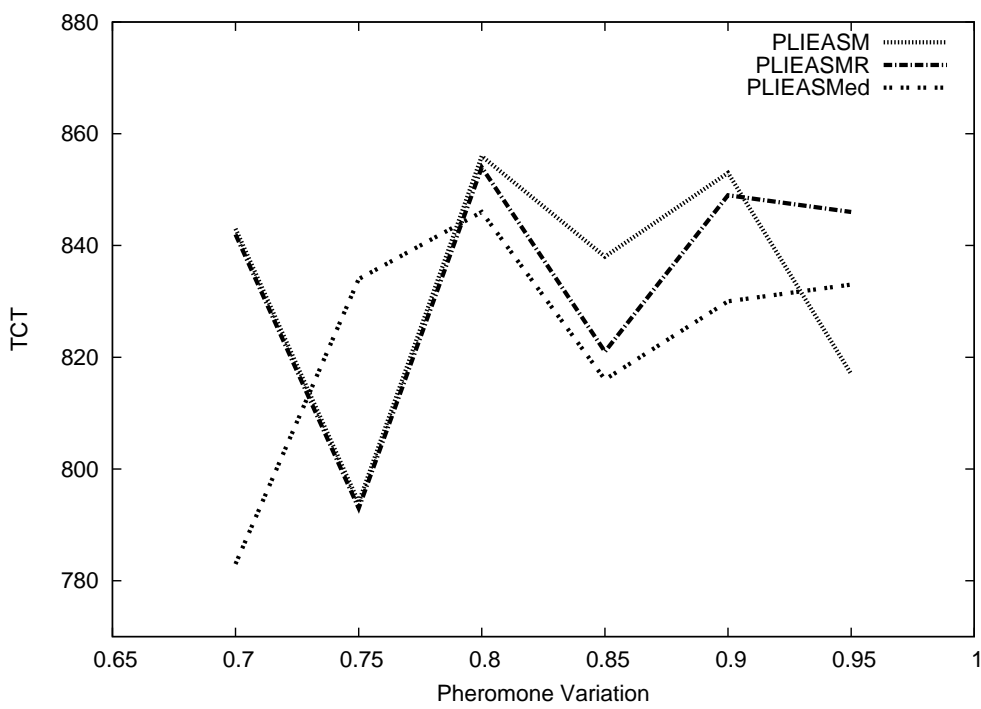


(b) Variation in Pheromone trial

Figure 5.6: Performance comparison of Punished Ant Systems for partial double lines (fixed commercial waiting time).



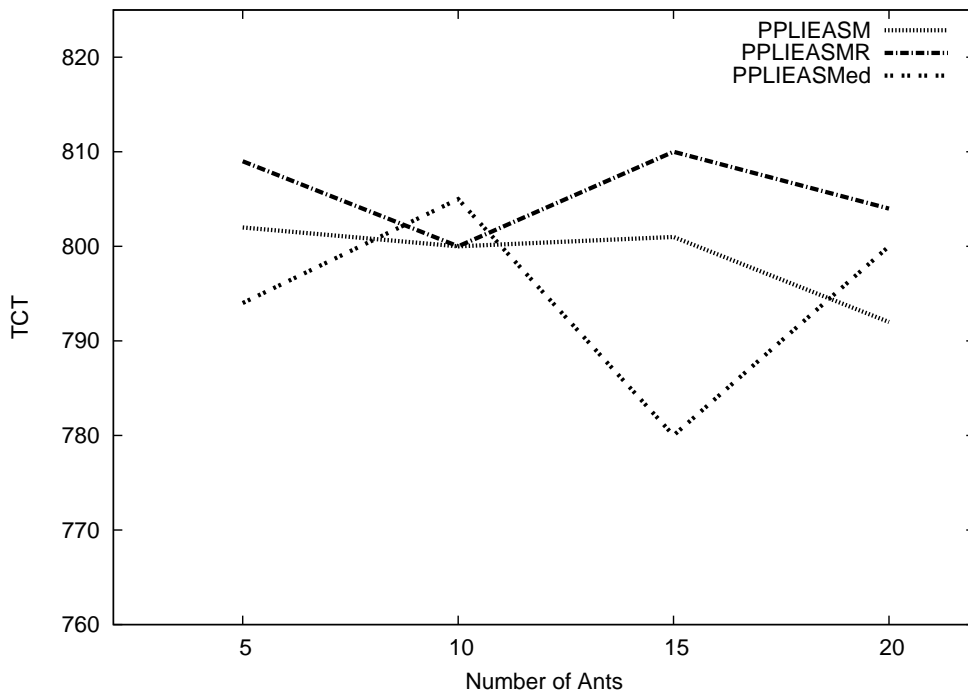
(a) Variation in number of Ants



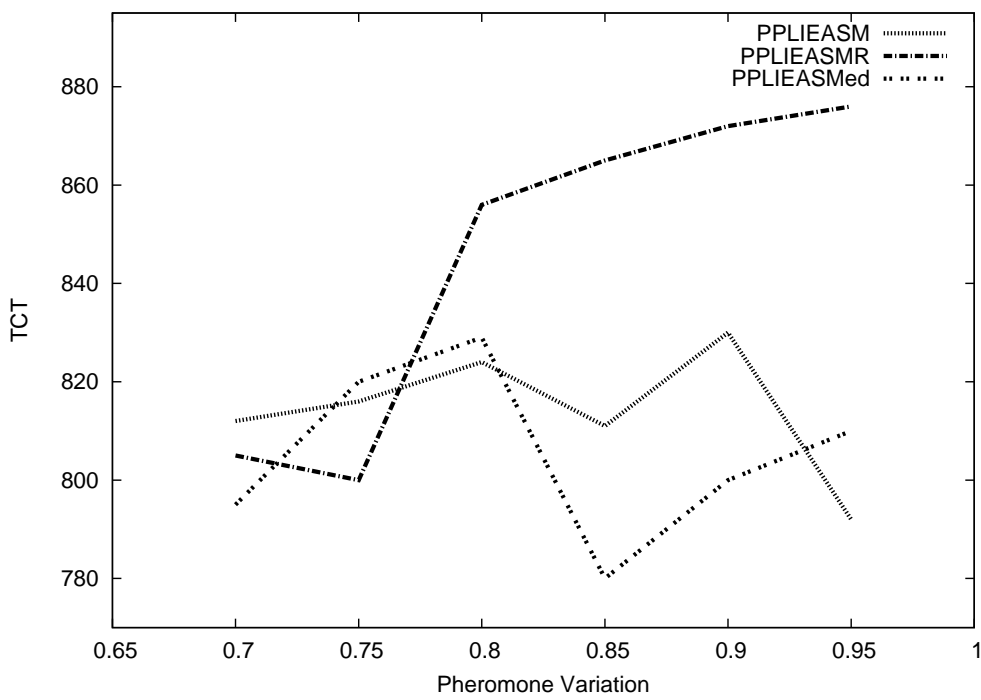
(b) Variation in Pheromone trial

Figure 5.7: Performance comparison of PLIEAS variants for partial double lines (fixed commercial waiting time).





(a) Variation in number of Ants



(b) Variation in Pheromone trial

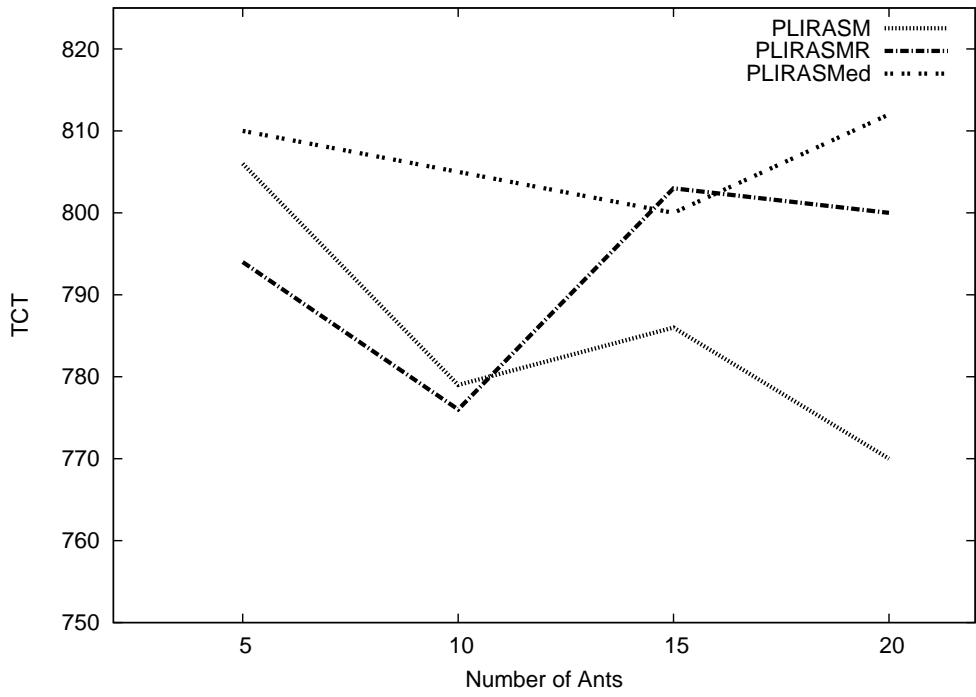
Figure 5.8: Performance comparison of PPLIEAS variants for partial double lines (fixed commercial waiting time).

increases the waiting time of trains for all the variants.

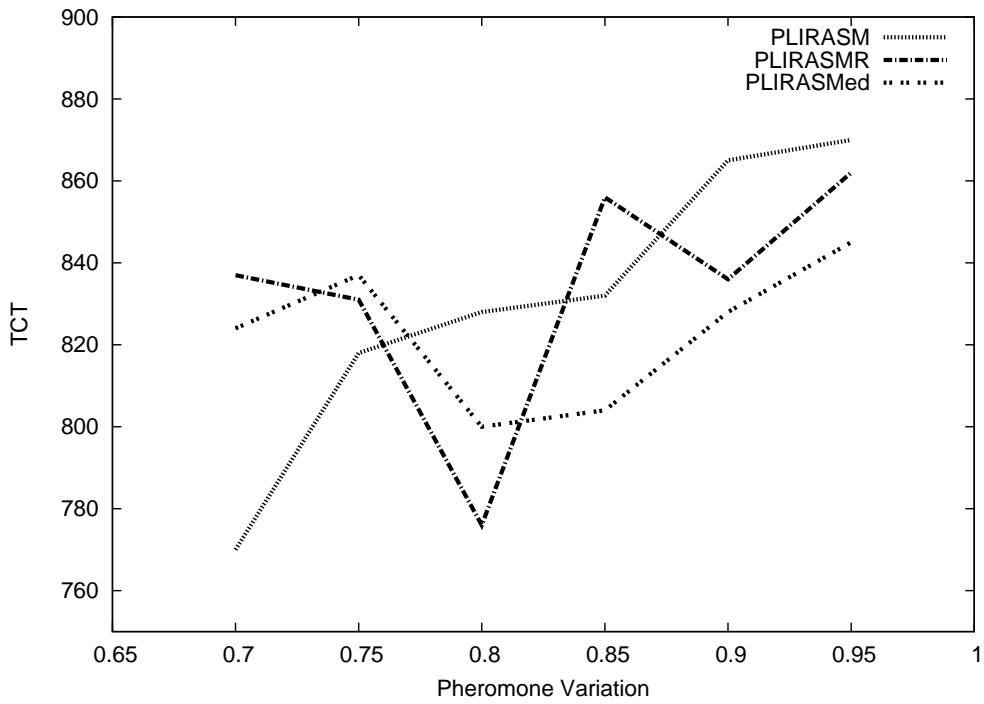
Figure 5.10 shows the comparative analysis of PPLIRAS variant. Figure 5.10(a) reveals that PPLIRASM provides the smallest waiting time for most of the varying size of ants population. The PPLIRASM and PPLIRASMR variants deliver best waiting time for smaller ant population size  $n$  of 5 and 10 respectively. Similarly, PPLIRASMed provides least waiting time for larger population of size 20. Similarly, Figure 5.10(b) reveals that, algorithms exhibit most of the time a similar trend in the variation of waiting time for the varying pheromone trail strength. The PPLIRASM, PPLIRASMR and PPLIRASMed provide least waiting time for trail strength  $\rho$  of 0.85, 0.85 and 0.75 respectively.

### **Cluster Integrated Ant Variants**

The Chapter 3 discusses the mechanism to integrate clustering philosophy and the ACO algorithms. The basic purpose of integration is to group the nearby tour performance and to update the paths in each group with same quantity of pheromone trail. The cluster approach can be applied to train scheduling problem by grouping nearby TCTs and updating the scheduled order paths within the group with the same amount of pheromone trail. The updation can be done in two possible ways : primary updation and secondary updation. In primary updation, clustering is done immediately after ants report the TCT values and then pheromone is reinforced as mentioned above. In secondary updation, scheduled order paths will be updated twice. Firstly, scheduled order paths will be updated with pheromone trials proportionate to the solution quality found by the ants and then followed by additional reinforcement using cluster based updation strategy. Table 5.5 shows the comparative results for cluster approach to train scheduling problem. The results obtained using cluster approach are comparatively inferior to basic ant variants, PEAS, PLIEAS and PPLIEAS and upon comparison of best results, one can notice the increased waiting time of 2.73%, 2.11%, 4.34% and 3.47% respectively.

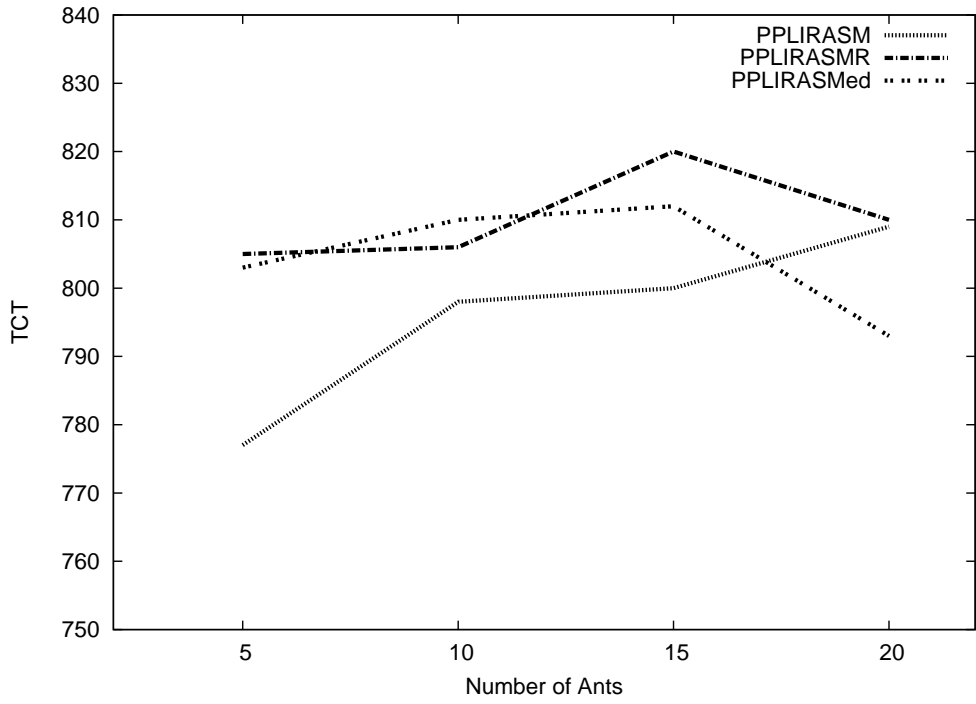


(a) Variation in number of Ants

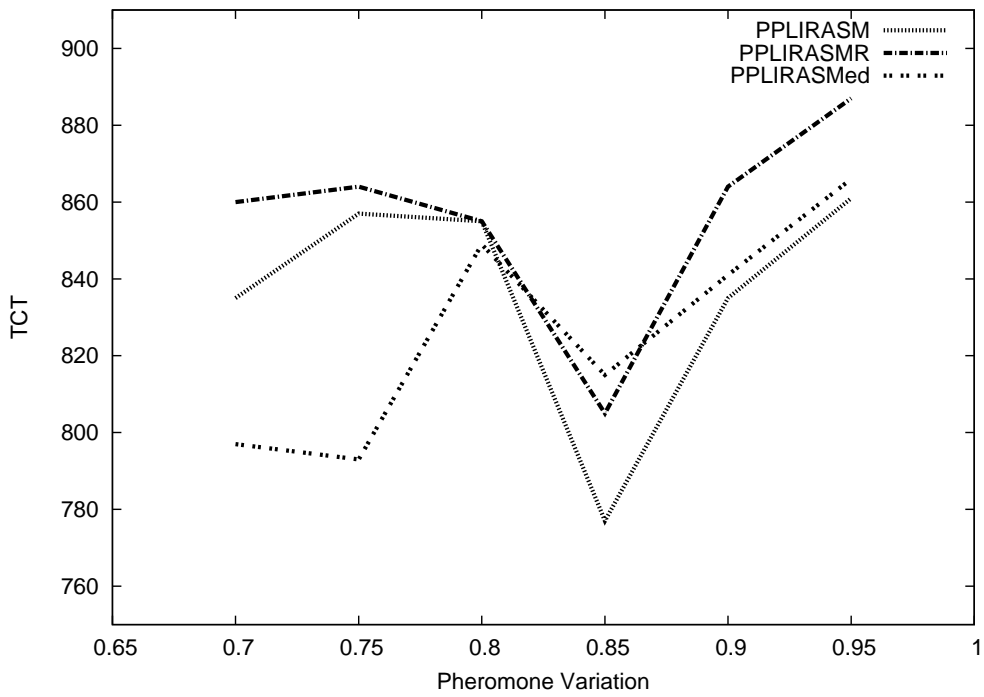


(b) Variation in Phormone trial

Figure 5.9: Performance comparison of PLIRAS variants for partial double lines (fixed commercial waiting time).



(a) Variation in number of Ants



(b) Variation in Pheromone trial

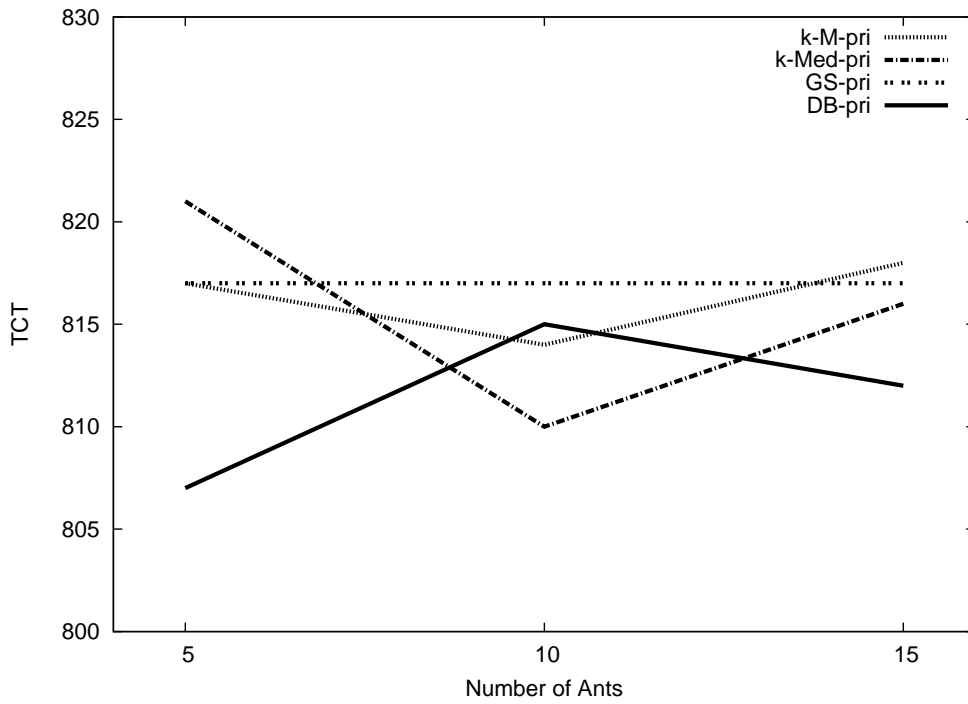
Figure 5.10: Performance comparison of PPLIRAS variants for partial double lines (fixed commercial waiting time).

Table 5.5: Performance of cluster integrated ACO variants on Train Scheduling Problem.

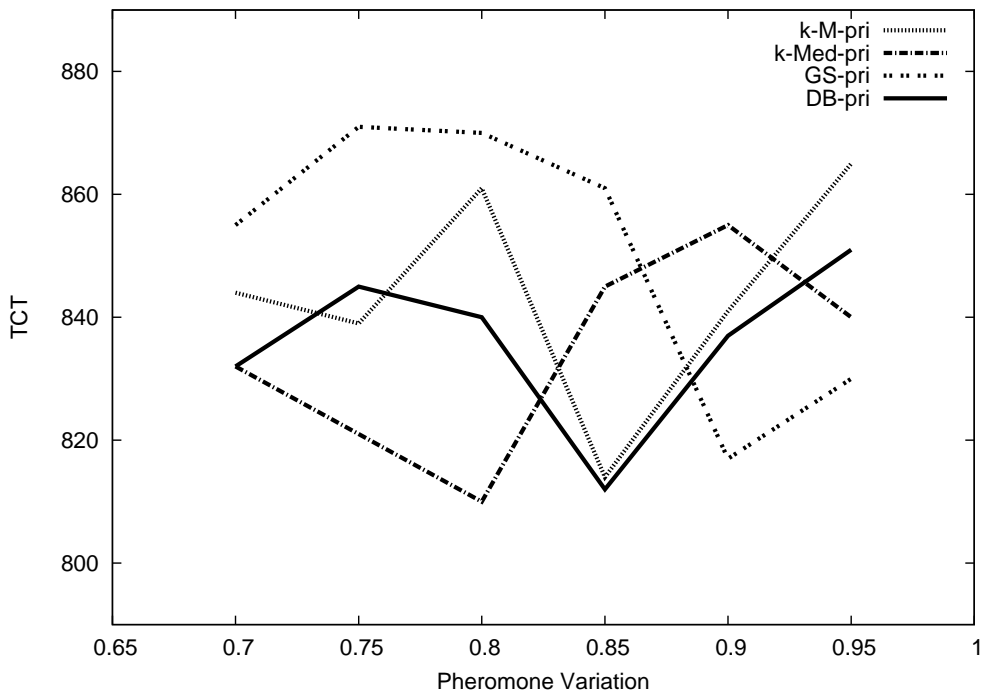
Class	pml = 2		
	Algorithms	TCT	Parameter details
GS-ACO	GS-pri	<b>817</b>	$\alpha=4 \beta=4 \rho=0.90$
	GS-sec	829	$\alpha=3 \beta=1 \rho=0.7$
k-Means ACO	k-M-pri	814	$\alpha=5 \beta=1 \rho=0.85$
	k-M-sec	<b>798</b>	$\alpha=3 \beta=1 \rho=0.70$
k-Medians ACO	k-Med-pri	<b>810</b>	$\alpha=4 \beta=3 \rho=0.80$
	k-Med-sec	817	$\alpha=3 \beta=5 \rho=0.90$
DBSCAN-ACO	DBC-pri	<b>805</b>	$\alpha=3 \beta=4 \rho=0.75$
	DBC-sec	810	$\alpha=3 \beta=5 \rho=0.80$

Figure 5.11 shows the behavior of the cluster integrated primary updation ACO variant applied to train scheduling problem. In the Figure 5.11(a) an interesting fact about the GS-primary (GS-pri) updation variant of GS-ACO class was observed. The variant gave consistently same waiting time for varying ant's population. However, DBC-primary (DBC-pri) updation provided relatively shorter waiting time compared to other variants and least waiting time among all the variants for ant's population of size 5. Figure 5.11(b) displays the behavior of algorithm for different trial strength. It can be observed that GS-pri exhibit larger volatility in observed waiting times compared to other variants. The least waiting time for all the variants is observed in the range of  $0.8 \leq \rho \leq 0.9$ .

Figure 5.12 shows the behavior of secondary updation in cluster integrated ACO variants applied to train scheduling problem. It can be observed from Figure 5.12(a) that GS-secondary (GS-sec) updation exhibits a similar trend as that of GS-pri primary updation and has a longer waiting time compared to other variants for the varying ants population. The k-Median secondary (k-Med-sec) and DBC-secondary (DBC-sec) show the same trend in waiting time variation for the varying ants population. However, DBC-sec provides the smallest waiting time among all the variants and best waiting time is obtained for population of size 10. Figure 5.12(b) shows the behavior of algorithms for varying trial strength. The waiting times of k-Med-sec and DBC-sec are lesser than k-M-sec and GS-sec variants.



(a) Variation in number of Ants



(b) Variation in Pheromone trial

Figure 5.11: Performance comparison of primary updation in cluster integrated ant variants for partial double lines (fixed commercial waiting time).

However, k-M-sec provides least waiting time compared to other variants and all the variants provide least waiting time in the pheromone range of  $0.7 \leq \rho \leq 0.8$ .

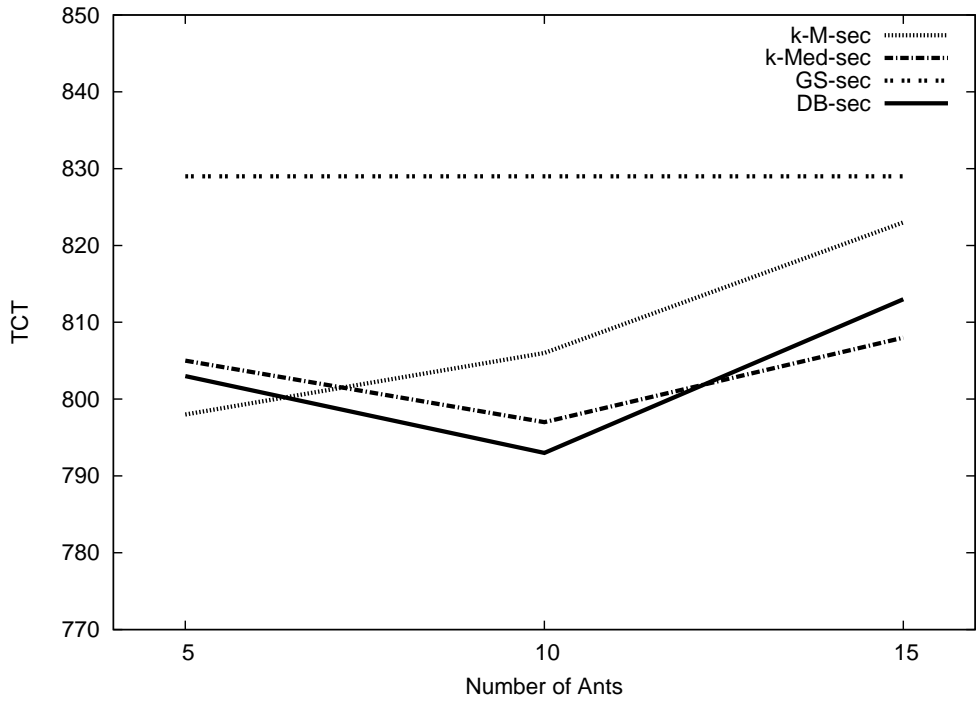
### 5.6.3 Result Analysis of Algorithms for Variable Commercial Waiting Time

In Variable Commercial Waiting Time, trains dwell time duration varies for each of the stations they travel as a part of the journey. The variable commercial waiting time can be expressed as a tuple  $VT = \langle s_i, \gamma_i \rangle$ , where  $s_i$  is the station and  $\gamma_i$  is the commercial waiting time associated with the line  $i$ . The input considered for this purpose is  $\langle \{0, 3\}, \{17, 4\}, \{38, 8\}, \{47, 2\}, \{56, 6\}, \{72, 2\}, \{80, 5\}, \{95, 4\}, \{117, 2\}, \{126, 4\}, \{149, 3\}, \{163, 5\}, \{179, 10\}, \{190, 3\}, \{199, 2\} \rangle$ .

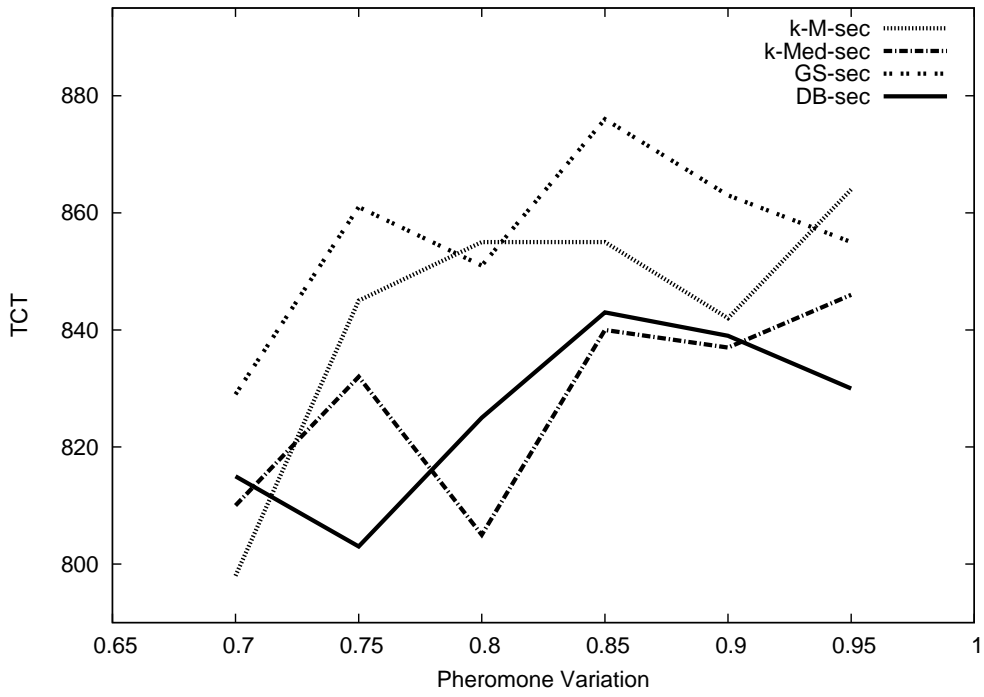
#### Basic Ant Variants

Table 5.6 shows the comparative results for the variable commercial time. The algorithm simulation was carried for pml=2 and pml=3 settings. It can be observed from table that RA provides best TCT values for both pml=2 and pml=3 parameter settings. The MMAS and MMAS+IB+PTS variants report search stagnation due to ‘limiting of pheromone’ mechanism. The results obtained for pml=3 are better than the results when pml=2, once again demonstrating the fact that better availability of lines for sidings leads to lesser waiting time for trains with decrease in waiting time of 2.55% for RA variant. However, AS variant reports sharp decrease in waiting time of 3.65%.

Figure 5.13 shows the comparative graph of basic ant variants for pml=2. It can be observed from 5.13(a) that AS and RA have better waiting times for higher ants population and ACS, EA for lower ants population. Similarly, AS variant has longer waiting time for most of the varying ants population compared to other variants. Figure 5.13(b) shows the graph for varying pheromone trial strength. The EA variant exhibits least variation in waiting times compared to



(a) Variation in number of Ants



(b) Variation in Phormone trial

Figure 5.12: Performance comparison of secondary updation in cluster integrated ant variants for partial double lines (fixed commercial waiting time).

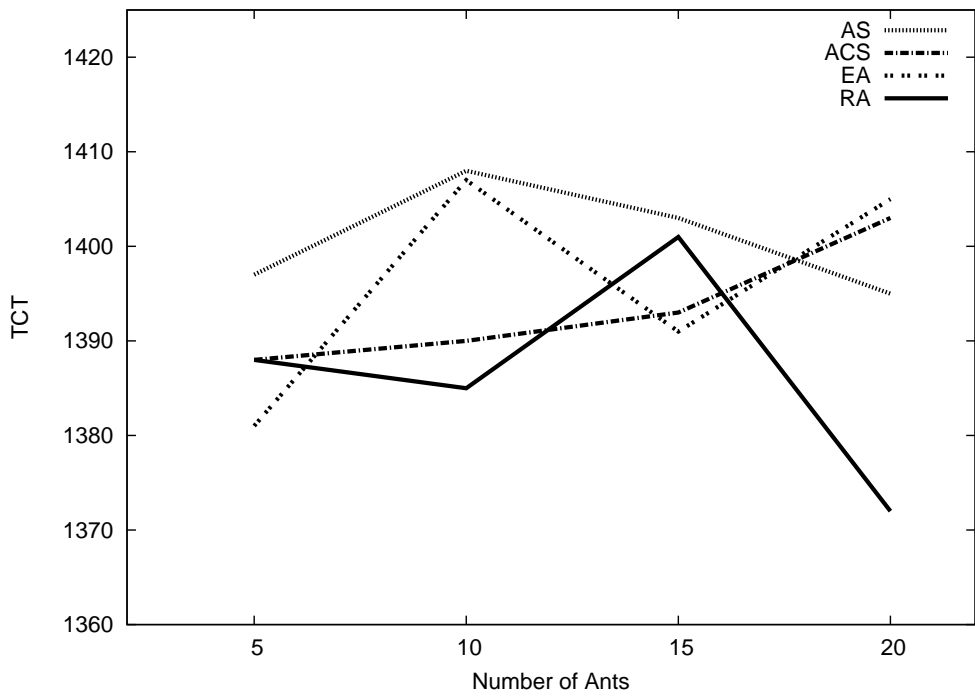


Table 5.6: Performance comparison of basic ant variants on Train Scheduling Problem with varying commercial waiting time.

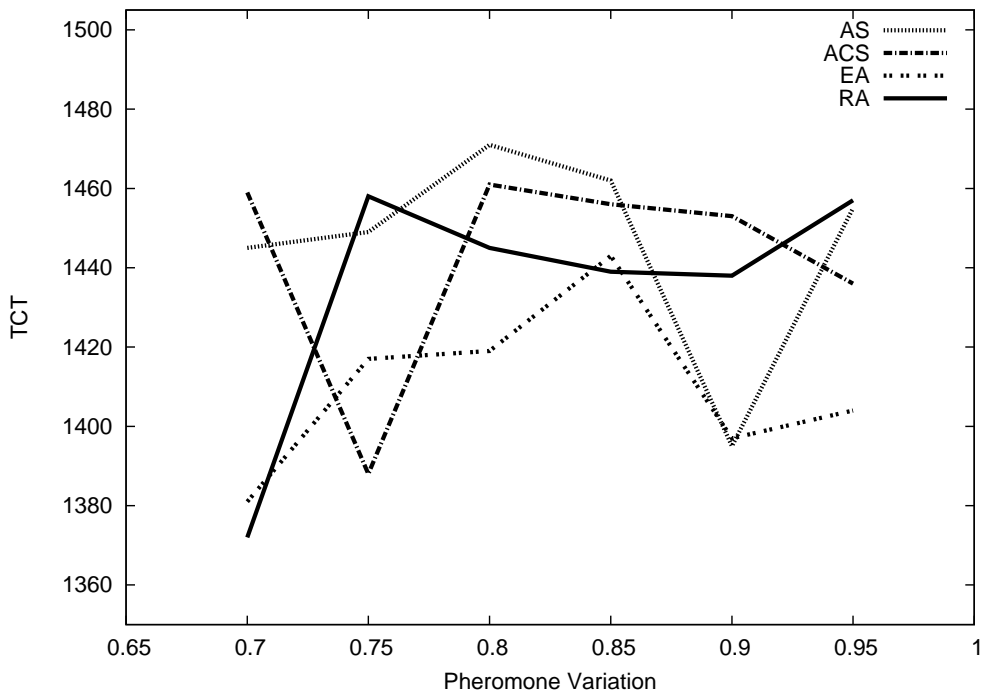
Algorithms	pml = 2		pml = 3	
	TCT	Parameter details	TCT	Parameter details
AS	1395	$\alpha=3 \beta=1 \rho=0.9$	1344	$\alpha=5 \beta=1 \rho=0.85$
ACS	1388	$\alpha=2 \beta=2 \rho=0.75$	1342	$\alpha=4 \beta=4 \rho=0.95$
EA	1381	$\alpha=5 \beta=1 \rho=0.7$	1339	$\alpha=5 \beta=1 \rho=0.9$
RA	<b>1372</b>	$\alpha=5 \beta=2 \rho=0.7$	<b>1337</b>	$\alpha=3 \beta=1 \rho=0.8$
MMAS+IB	1594	$\alpha=1 \beta=4 \rho=0.70$	1557	$\alpha=2 \beta=1 \rho=0.8$
MMAS+IB+PTS	1594	$\alpha=1 \beta=4 \rho=0.85$	1557	$\alpha=4 \beta=3 \rho=0.9$

other variants. The graph depicts that ACS, EA and RA provide least waiting time for lower trial strength  $\rho$  and in the range of 0.7-0.75. Similarly, RA provides the least waiting time for  $\rho=0.9$ .

Figure 5.14 shows the comparative graph of basic ant variants for pml=3. It can be observed from Figure 5.14(a) that except EA, all the other variants provide the shorter waiting times for smaller ant population and that too in the range of 5 to 10. The trend analysis reveals that for AS, ACS and RA, increase in ants population leads to increase in waiting time. Similar interesting observations can be made from Figure 5.14(b). The graph reveals a larger deviation in observed TCT values for all the ant variants and provides a least waiting time for wider values of  $\rho$  in the range of 0.8-0.95.

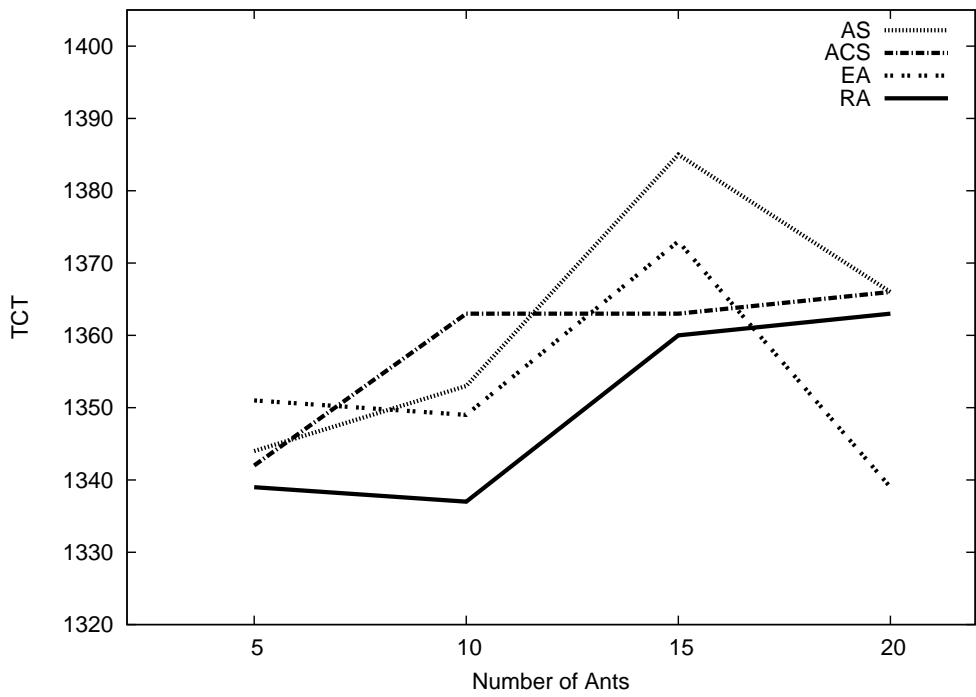


(a) Variation in number of Ants

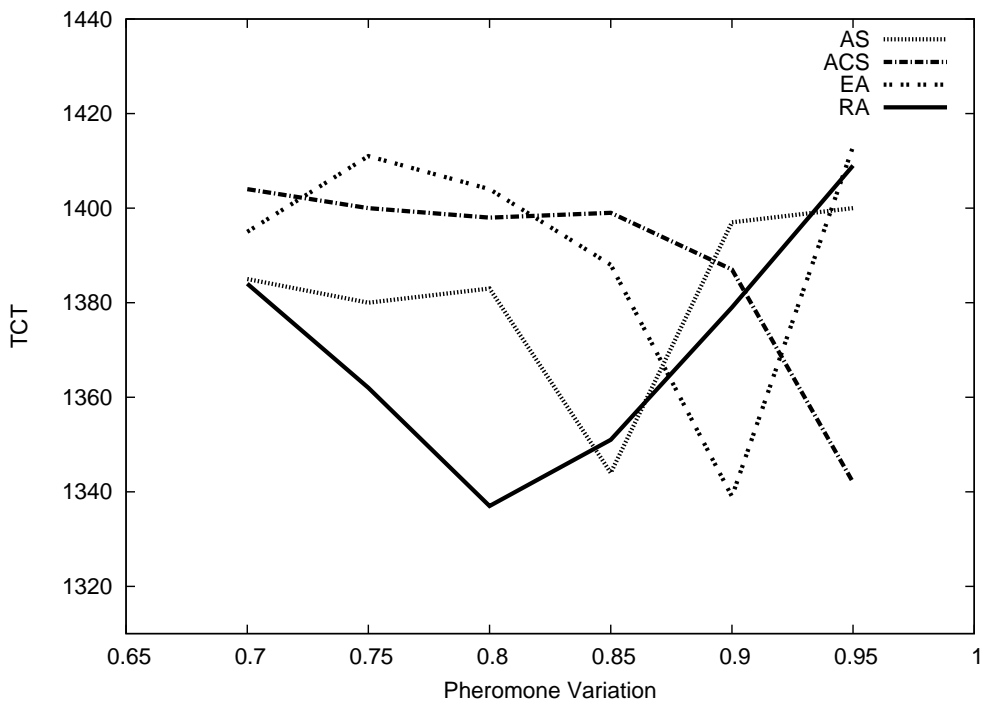


(b) Variation in Phomone trial

Figure 5.13: Performance comparison of basic ant variants for partial double lines (variable commercial waiting time).



(a) Variation in number of Ants



(b) Variation in Phormone trial

Figure 5.14: Performance comparison of basic ant variants for partial triple lines (variable commercial waiting time).

Table 5.7: Comprehensive performance comparison of new ACO variants on Train Scheduling Problem.

Class	pml = 2		
	Algorithms	TCT	Parameter details
Punished Ant System	PEAS	1372	$\alpha=4 \beta=4 \rho=0.9$
	PRAS	<b>1366</b>	$\alpha=5 \beta=1 \rho=0.8$
Performance Linked Elitist Ant System	PLIEASMR	1376	$\alpha=5 \beta=1 \rho=0.75$
	PLIEASM	<b>1371</b>	$\alpha=5 \beta=1 \rho=0.95$
	PLIEASMed	1393	$\alpha=4 \beta=1 \rho=0.95$
	PLIRASMR	1376	$\alpha=4 \beta=3 \rho=0.7$
	PLIRASM	1372	$\alpha=5 \beta=4 \rho=0.7$
	PLIRASMed	1376	$\alpha=4 \beta=3 \rho=0.7$
Punished Performance Linked Elitist Ant Systems	PPLIEASMR	1361	$\alpha=4 \beta=2 \rho=0.7$
	PPLIEASM	1366	$\alpha=4 \beta=3 \rho=0.85$
	PPLIEASMed	1361	$\alpha=4 \beta=2 \rho=0.7$
	PPLIRASMR	1363	$\alpha=2 \beta=1 \rho=0.75$
	PPLIRASM	<b>1360</b>	$\alpha=3 \beta=4 \rho=0.90$
	PPLIRASMed	1368	$\alpha=5 \beta=1 \rho=0.85$
GS-ACO	GS-pri	1409	$\alpha=5 \beta=1 \rho=0.7$
	GS-sec	<b>1393</b>	$\alpha=3 \beta=1 \rho=0.9$
k-Means ACO	k-M-pri	<b>1378</b>	$\alpha=4 \beta=2 \rho=0.7$
	k-M-sec	1403	$\alpha=5 \beta=1 \rho=0.95$
k-Medians ACO	k-Med-pri	1374	$\alpha=3 \beta=5 \rho=0.85$
	k-Med-sec	<b>1366</b>	$\alpha=4 \beta=4 \rho=0.9$
DBSCAN ACO	DBC-pri	<b>1346</b>	$\alpha=5 \beta=3 \rho=0.8$
	DBC-sec	1355	$\alpha=4 \beta=5 \rho=0.75$

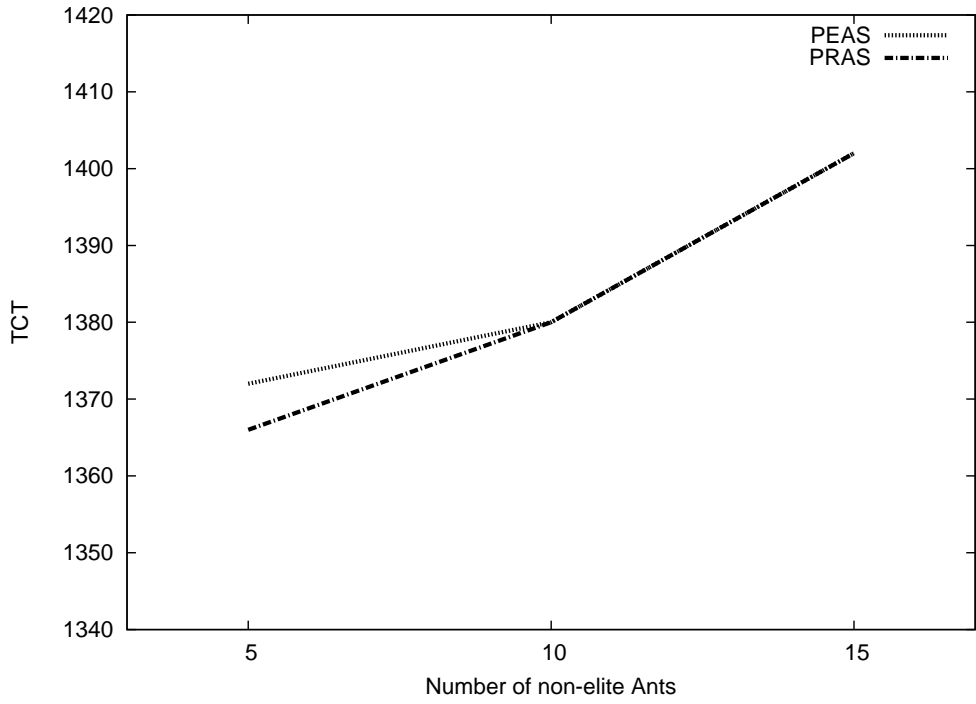
## Punished and Performance Linked Ant Variants

Table 5.7 shows the comparative analysis of various algorithms discussed in the Chapter 2 and Chapter 3 applied to train scheduling problem for variable commercial waiting time. The performance of PRAS in punished ant system class is slightly better than that of basic variants (see Table 5.6  $pml=2$ ) with decrease in waiting time of 0.43%. Similarly, PLIEASM and PPLIRASM in PLIEAS and PPLIEAS class show an improvement with decreased waiting time of 0.07% and 0.87% respectively. However, among the proposed approaches PPLIRASM shows an improvement of 0.43% over PRAS and PLIEASM in terms of best results.

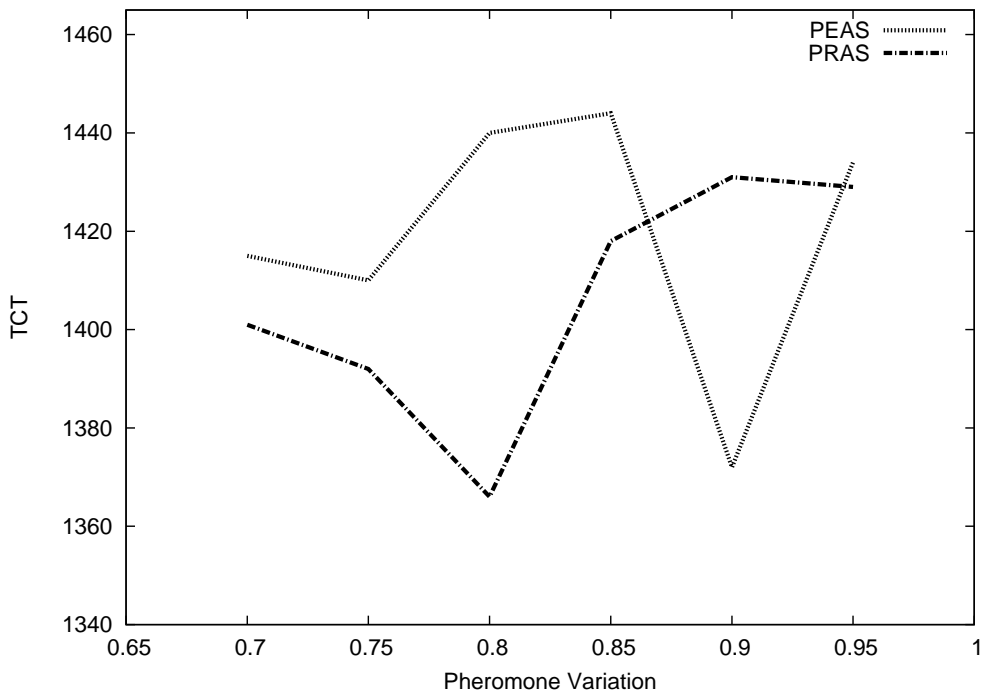
Figure 5.15 displays the performance of punished ant systems for the variable commercial waiting time. Figure 5.15(a) reveals that observed TCT values of PRAS is slightly better than PEAS variant. The PRAS variant has the best TCT value for non-elite ant's population  $nea$  of size 5. However, same waiting time is observed for the higher number of non-elite ants population. Similarly, Figure 5.15(b) reveals that PRAS has comparatively shorter waiting time than PEAS for varying trial strength. The PEAS variant has the least waiting time for trial strength  $\rho=0.9$  and PRAS for  $\rho=0.8$ .

Figure 5.16 displays the performance of PLIEAS for the variable commercial waiting time. Figure 5.16(a) shows that PLIEASMed variant provides sub-optimal TCT values compared to other variants for varying ant's diversity. Figure 5.16(b) reveals some interesting patterns for the varying trial strength. In general, all the variants show increase in waiting time for the higher pheromone persistence factor and shows sharp decrease in waiting time for  $\rho=0.95$ .

Figure 5.17 shows the behavior of PPLIEAS for the varying parameters. Figure 5.17(a) reveals that all the variants have nearby waiting times for smaller and higher number of ant's population  $n$  of size 5 and 20 respectively. However, shorter waiting times were observed for ants population size in the range of 5 to 10. Similarly, PPLIEASMR reacts sharply with an increase in waiting time, as the number of ants in the system increases. The higher concentration of pheromone trial has a bad effect on PPLIEASMR variant and waiting time increases with

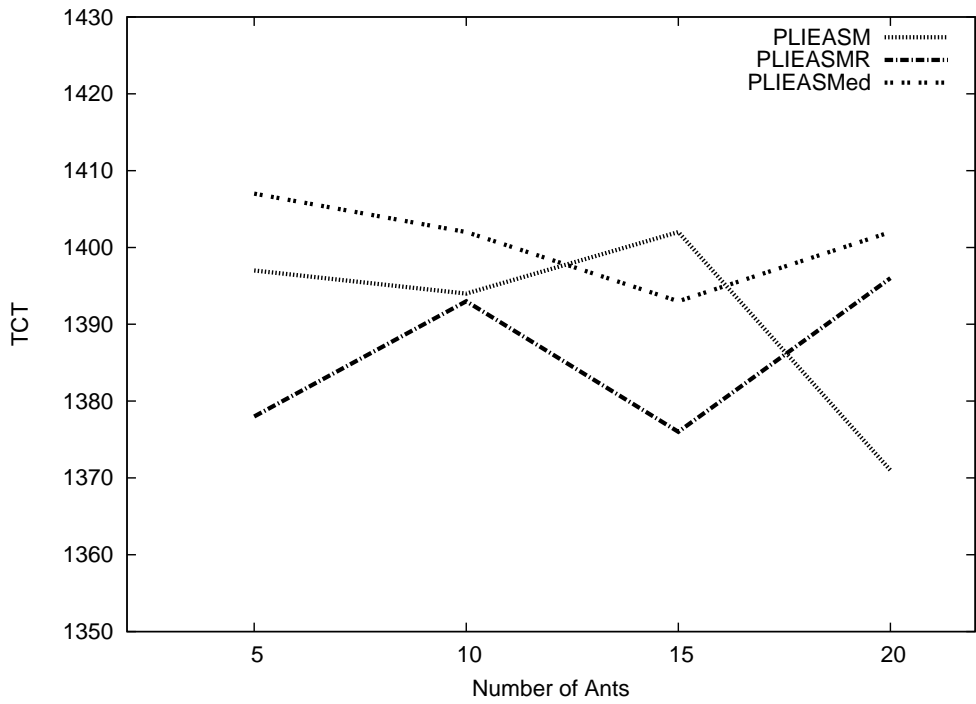


(a) Variation in number of Ants

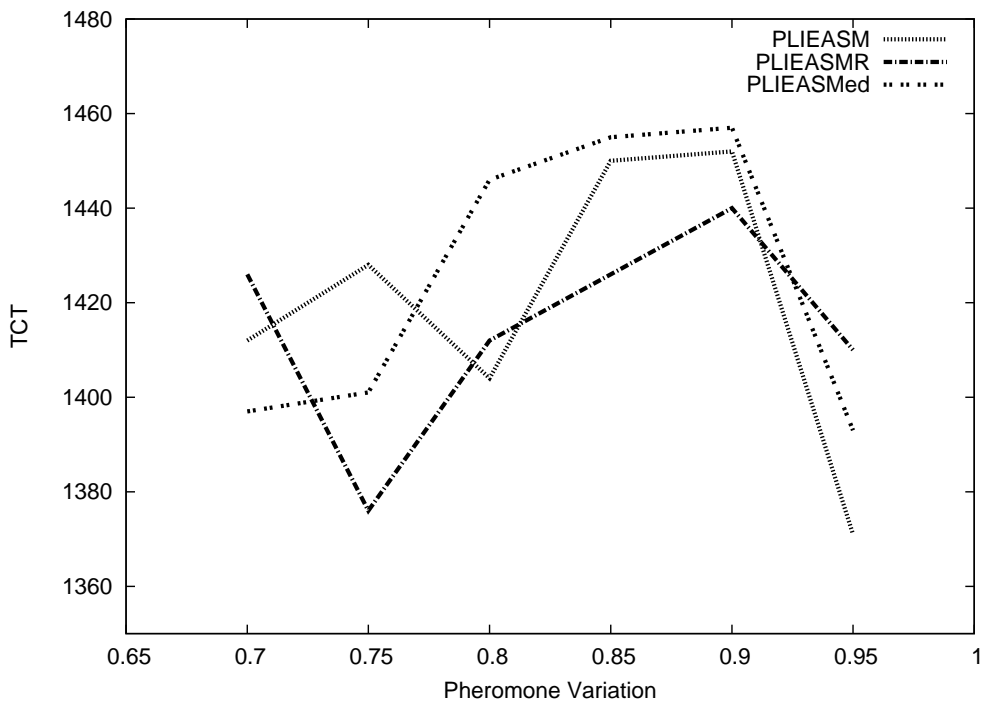


(b) Variation in Phomone trial

Figure 5.15: Performance comparison of Punished Ant Systems for partial double lines (variable commercial waiting time).



(a) Variation in number of Ants



(b) Variation in Phormone trial

Figure 5.16: Performance comparison of PLIEAS variants for partial double lines (variable commercial waiting time).

the increase in the pheromone persistence factor as seen from Figure 5.17(b). The shorter waiting time observed for varying  $\rho$  and for all the variants it is in the range of  $0.7 \leq \rho \leq 0.85$ .

Figure 5.18 depicts the behavior of PLIRAS class for train scheduling problem. Figure 5.18(a) reveals an interesting behavior of PLIRASMR variant, which reacts sharply to the varying ants population. However, PLIRASM and PLIRASMed variants report lesser volatility in waiting time for varying ants population. The trend analysis of Figure 5.18(b) reveals an increase in waiting time with the increase in pheromone persistence factor for PLIRASM and PLIRASMed variants and all the variants provide shorter waiting time for trial settings  $\rho$  in the range of 0.7-0.75.

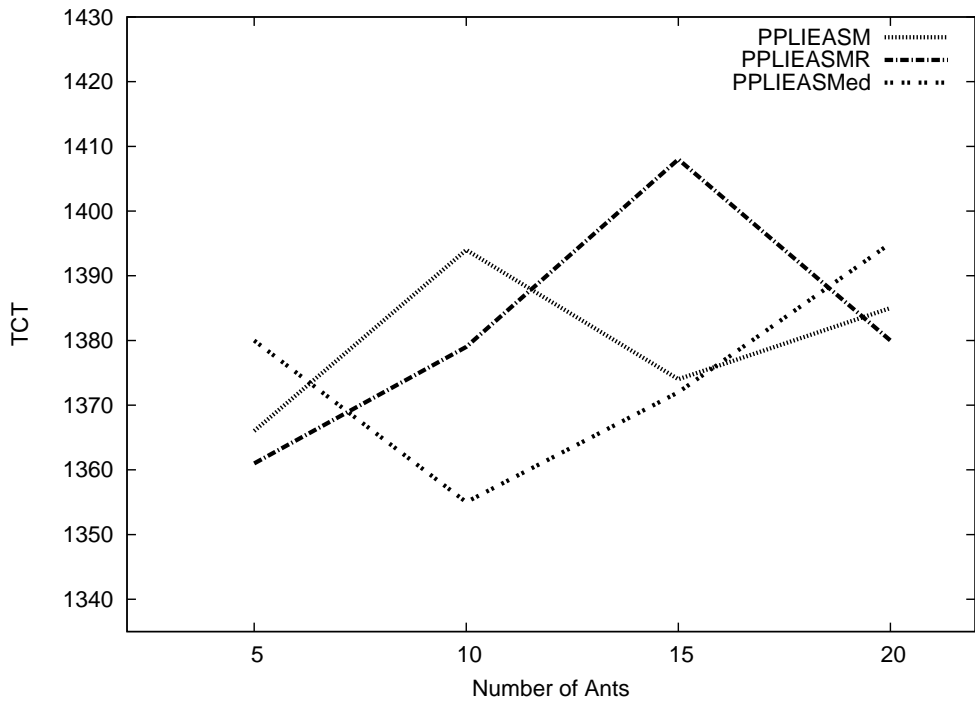
Figure 5.19 shows the performance of PPLIRAS class for parameter settings pertaining to ACO algorithm. Figure 5.19(a) reveals that PPLIRASM variant has consistently delivered larger TCT values compared to other variants for varying ants population. Similarly, Figure 5.19(b) shows larger fluctuation in waiting times for all the variants indicating their sensitiveness to the pheromone trial strength.

### **Cluster Integrated Ant Variants**

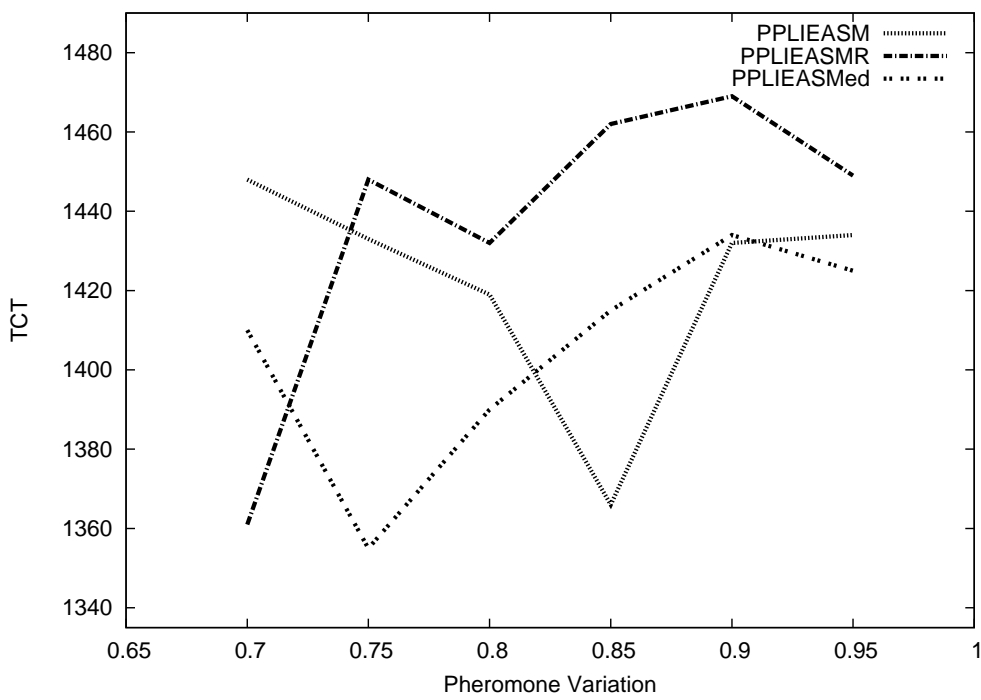
Table 5.7 reports the comprehensive analysis of cluster integrated ant approaches to variable commercial waiting time. The DBSCAN variant with primary updation provides the shortest waiting time compared to all the other cluster integrated variants with decrease in waiting time of 1.89%, when compared with basic ant variants. Similar conclusion can be drawn upon the comparison with the waiting times of PLIEAS and PPLIEAS class of algorithms. The decrease in waiting time was to the tune of 1.82% and 1.02% for PLIEAS and PPLIEAS classes respectively.

Figure 5.20 reveals the behavior of cluster integrated primary updation for variable commercial waiting time. Figure 5.20(a) shows interesting patterns of nearby waiting times for smaller and larger ants population and larger difference in the mid size ants population. It is interesting to see that performance of GS-primary (GS-pri) provides consistently same waiting time for the variation in



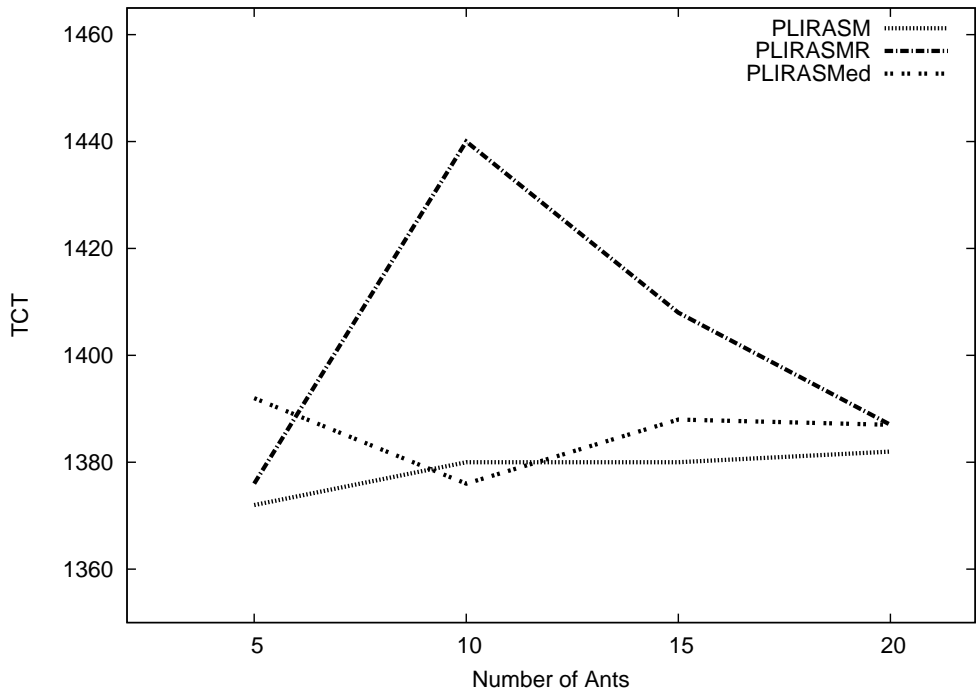


(a) Variation in number of Ants

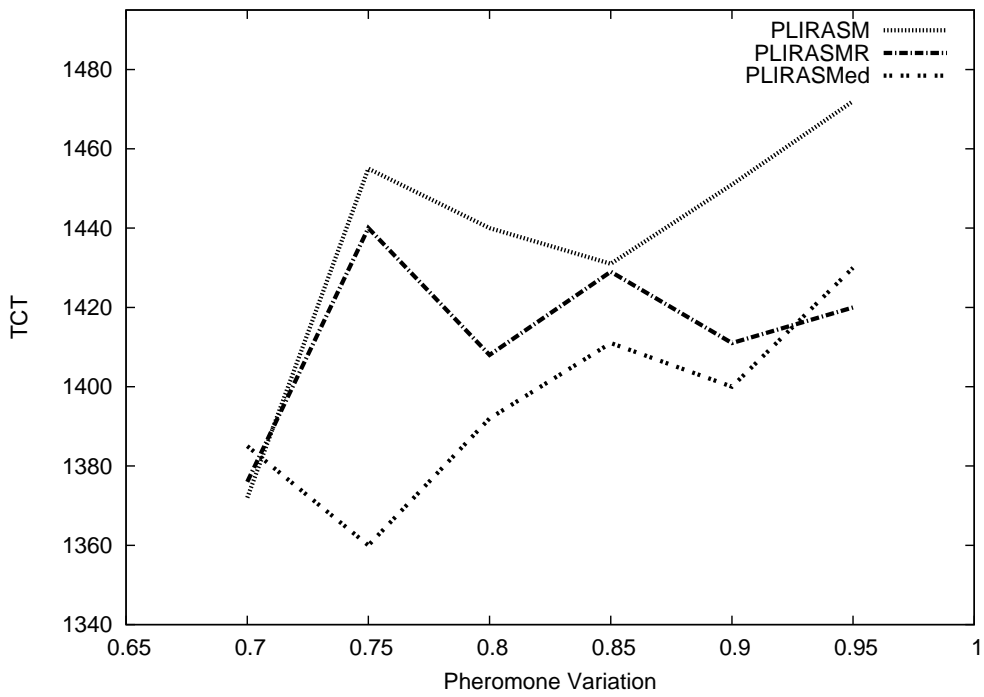


(b) Variation in Phomone trial

Figure 5.17: Performance comparison of PPLIEAS variants for partial double lines (variable commercial waiting time).

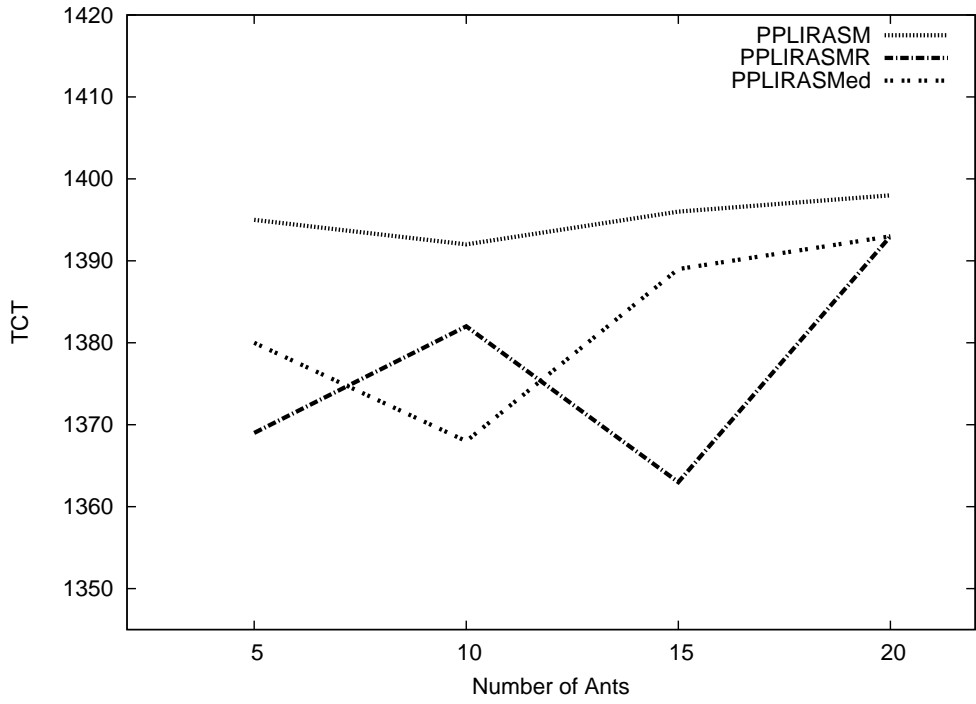


(a) Variation in number of Ants

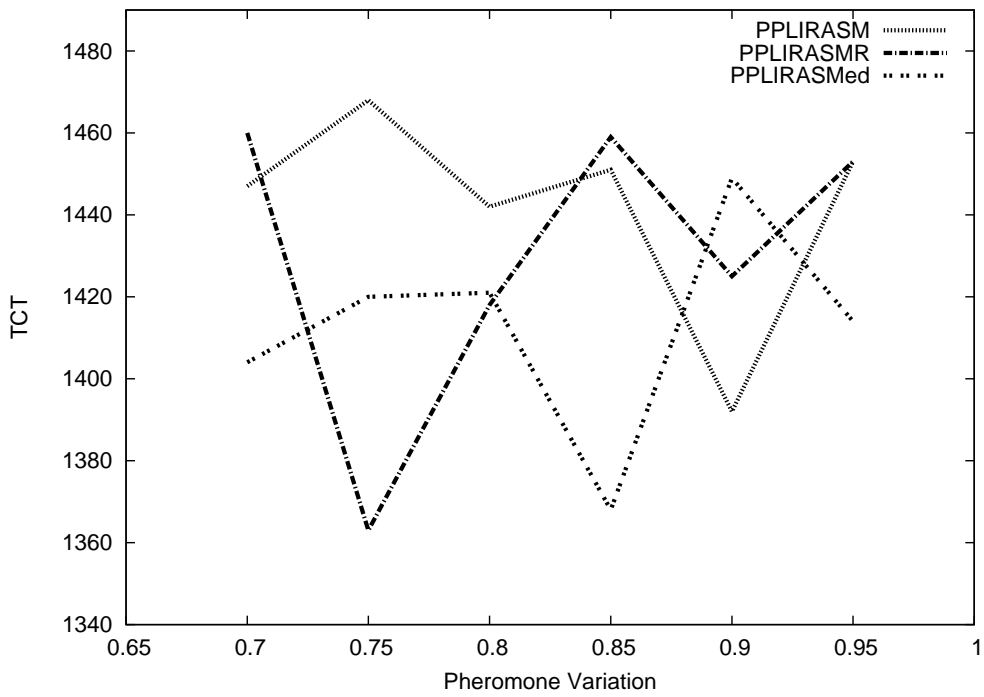


(b) Variation in Phomone trial

Figure 5.18: Performance comparison of PLIRAS variants for partial double lines (variable commercial waiting time).



(a) Variation in number of Ants



(b) Variation in Pheromone trial

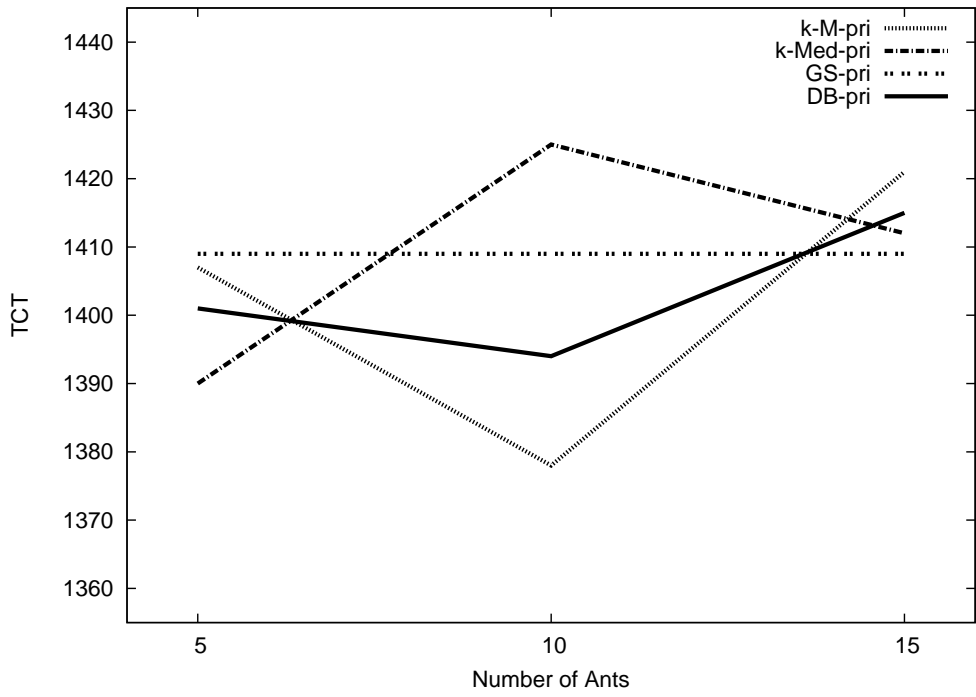
Figure 5.19: Performance comparison of PPLIRAS variants for partial double lines (variable commercial waiting time).

population size. Similarly, Figure 5.20(b) displays that all the variants provide shorter waiting time for lower pheromone strength  $\rho$  in the range of 0.7-0.8 and shows an increasing waiting time trend for the increased pheromone presence.

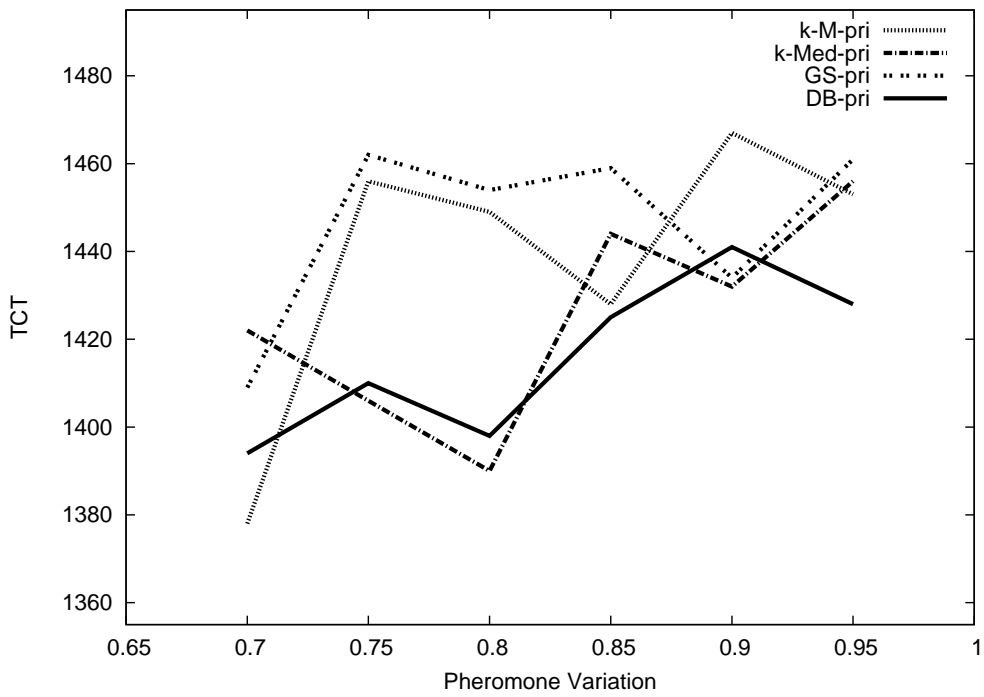
Figure 5.21 shows the nature of cluster integrated secondary updated ant variants for variable commercial waiting time. Figure 5.21(a) reveals that waiting time of GS-secondary (GS-sec) and DB-secondary (DBC-sec) are superior to k-Means-secondary (k-M-sec) and k-Medians-secondary (k-Med-sec) for the varying ants population. Similarly, 5.21(b) shows that k-Med-sec and DBC-sec have shorter waiting times than k-M-sec and k-Med-sec for lower trial strength  $\rho$  which is in the range of 0.7-0.85. The k-Med-sec and DBC-sec provide the least waiting time for  $\rho$  settings 0.75 and 0.8 respectively. Similarly, k-M-sec and k-Med-sec provide least waiting time for higher  $\rho$  values of 0.9 and 0.95 respectively.

## 5.7 Concluding Remarks

Railway train timetable problem is an important and challenging task in optimizing the performance and profit of the railway department. This chapter highlighted the potential of using ACO techniques for applications to train schedule planning. The problem is to arrive at train schedule order for a fixed timetable that has minimum waiting time on sidings. The proposed model is flexible to incorporate new rules (constraints) and policies (objectives) without any change to the existing model and also make some realistic assumptions that suits to the Indian Railways Systems. The ACO framework has been successful in providing the optimal result in reasonable interval of time. The new concepts described in previous chapters have been successful in intensifying the search in promising region to arrive at the optimal solution. Hence railway department should consider using this model with the suitable modifications. The model given in this thesis could be a basis for future research on traffic related problems.

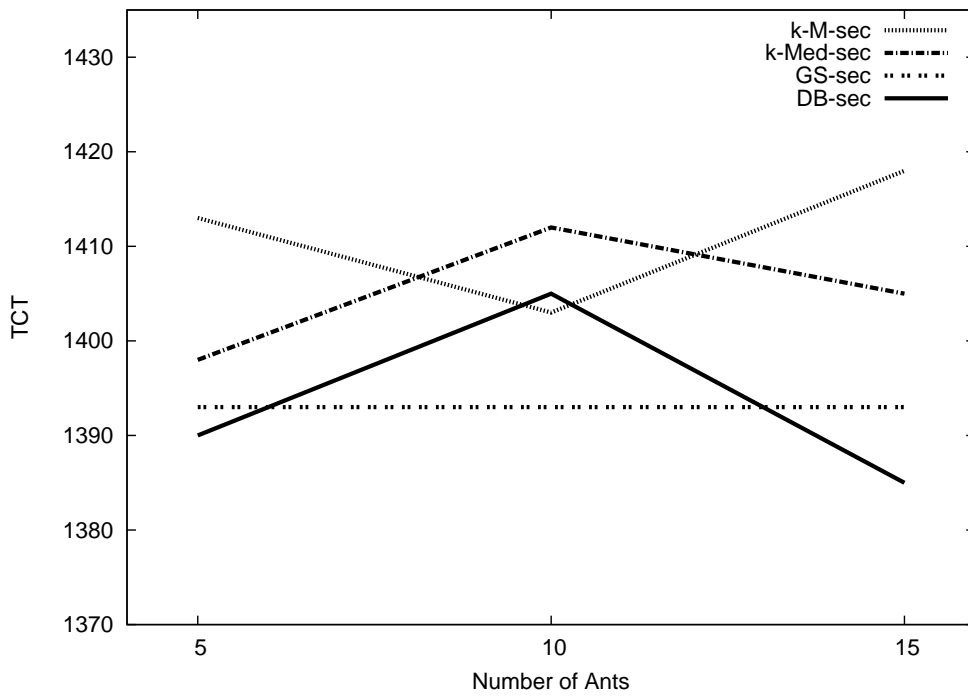


(a) Variation in number of Ants

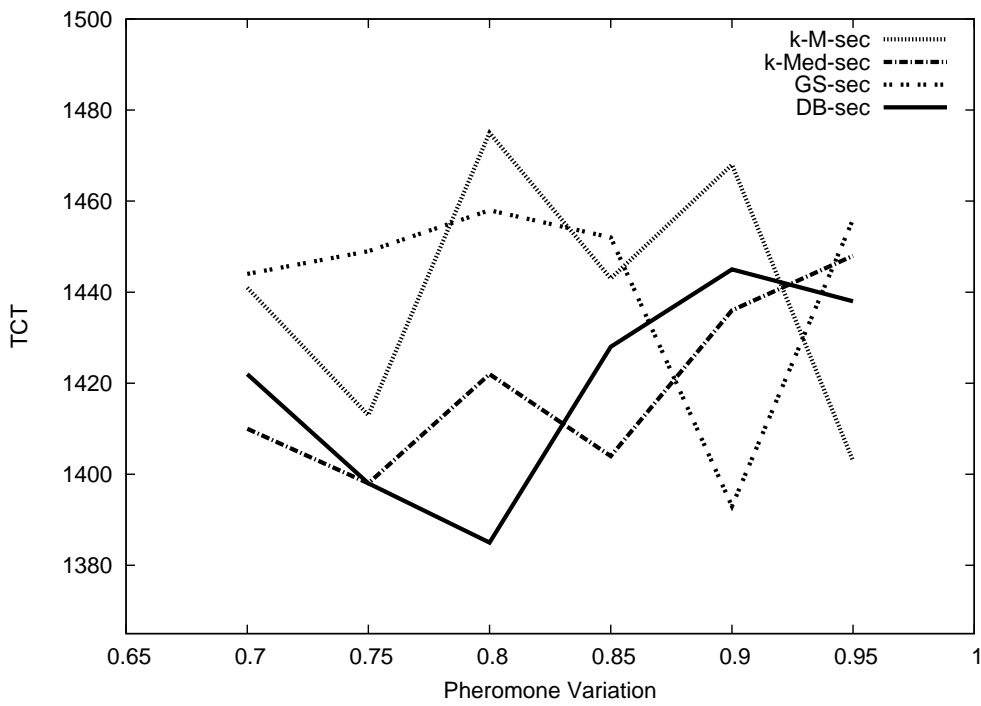


(b) Variation in Phormone trial

Figure 5.20: Performance comparison of primary updation in cluster integrated ant variants for partial double lines (variable commercial waiting time).



(a) Variation in number of Ants



(b) Variation in Phomone trial

Figure 5.21: Performance comparison of secondary updation in cluster integrated ant variants for partial double lines (variable commercial waiting time).