A STUDY OF THE OPERATION OF RIHAND RESERVOIR

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CERTIFICATE

This is to certify that the thesis entitled "A STUDY OF THE OPERATION OF RIHAND RESERVOIR" and submitted by G.N. Yoganarasimhan, ID No. 69/Ph.D/4 for award of Ph.D. Degree of the Institute, embodies original work done by him under my supervision.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Figure

ABSTRACT

The long term on line optimal control and management of stored water in hydroelectric projects with particular reference to Rihand reservoir in Uttar Pradesh has been studied. The problem is studied through Digital Computer Simulation and Mathematical programming techniques, and different operational strategies have been examined.

A simulation program, using a detailed model of an isolated hydroelectric station with storage considering spill and evaporation, subject to continuity and operational constraints has been developed. The Markovian model developed by Thomas and Fiering is used to generate monthly stream flows by a separate program, and the output of this is used as an input to the simulation program. The simulation program is used in various experiments to find the best starting point for annual operation of reservoir, firm energy capability, energy generation capability with carry over storage.

A problem of two reservoirs on separate streams is analyzed analytically to arrive at the operational strategy to maximize energy generation.

The first operational strategy is based on the premise that the hydroplant has its own load and has to satisfy its contractual obligations. Its long term operational plan is to maximize its energy generation, smoothing its surplus or deficiency with reference to the load. The problem is formulated as a nonlinear programming problem and is solved by a gradient method. The gradient calculation is simplified taking advantage of the special structure of the problem. The unconstrained minimization algorithm of Fletcher and Reeves is modified to take into account the bounds on the variables and is implemented on digital computer.

The second strategy is the long term integrated operation of the hydroplant with thermal stations, the hydrostation replacing the costly peak requirements of the system load in the load duration curve. The objective here is explicitly economic, that is to minimize the cost of energy production. The problem is solved by Forward dynamic programming in discrete time.

The uncertainties in the stream flow is proposed to be taken care of through stream flow forecasting. A simple method of forecasting stream flow from operational point of view is explored. The method essentially consists of identifying the deterministic and the stochastic components of the monthly stream flow data. The deterministic components are treated through time series analysis and the stochastic component is modelled through first order Markov chain. To take care of the uncertainties in the stream flow forecasts, adaptive method of correction in the operation plan is suggested.

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CHAPTER 1

INTRODUCTION

1.1 THE PROBLEM OF OPERATION OF RESERVOIRS

The problem of water resources management has two important aspects (i) planning (ii) operation of water resource system. The concepts of system analysis has a direct bearing on both these aspects of water resources management. Planning for a unified development of a river basin consists of the collection of data base followed by a series of decisions - whether, where and when to build each dam and other connected works. Operation of a water resource system is concerned with decisions that are necessary to best accomplish the objectives of an existing system.

While the operation of an existing water resource may be considered disjointly from the planning function, the planning for the expansion of an existing system definitely must encompass the hypothesized future operation of the system. From the point of view of this thesis, operation is concerned with the optimization of an existing system.

An operating policy is a set of rules guiding the determination of releases from each reservoir or quantification of the alternatives available to the managers of storage reservoirs. It must be established at the beginning of a season when the stream flow is still unknown.

The derivation of discharge or operating policy of reservoirs can be viewed as a high dimensional control problem; over the period of study, one seeks to optimize a performance criterion subject to constraints inherent in the system. The performance criterion can be formulated in terms of deterministic variables or expected values of stochastic variables when uncertainty is present.

The output of a reservoir which is most difficult to handle is the power generation. This is because energy production is a function of the amount of water released and the head on the turbine when it is released. Further the management of the system require a priori to make commitments for the supply of electric energy and water over a given time span. Because of uncertainty of hydrology in terms of inflow of water to the reservoirs the problem may be approached in one of the following directions:

(i) Characterize the random nature of the water inflow by specific probability distribution and then analyze the problem by using some stochastic models or

(ii) Develop a model that uses deterministic hydrology making use of short term forecasts based on the pattern of historical data.

The model is completely defined under a deterministic form. The optimal controls to be determined are the discharges from the reservoirs or schedules of hydropower generation. The management objectives could be to satisfy its contractual obligations with reference to power supply. The energy generation in any year is determined by the inflow into the reservoir in that year. The management can enter into an agreement with a neighbouring utility for exchange of its deficit or excess energy. This exchange of energy may be at a constant rate from economic point of view. Alternatively the management objective could be the overall economy in energy generation with a co-ordinated operation of hydro and thermal stations. Under this type of operation the peak load is allocated to hydrostations and the balance to thermal stations. The extent of peak load allocated to hydrostations is determined by inflow into the reservoir and the generation capacity of the hydrostations.

1.2 REVIEW OF LITERATURE

1.2.1 Optimal Operation of Reservoirs

The problem of reservoir operation dates back to the pioneering work of Ripple (1883). His work was the basis for the hydrologic design of most of the reservoirs now in existence, and is only being challenged of late by newer developments. The Ripple method is based on specifying the hydrologic objective, namely that of providing a reservoir to meet a specified water demand. Essentially it is a graphical comparison of the cumulative quantity of water involved in meeting a water demand, with the cumulative supply of water from some stream for which the records are available. The maximum departure between the two lines indicates the cumulative deficiency between supply and demand and hence the volume of storage required to meet that deficiency. The reservoir operating rule implied by the Ripple analysis is to meet the specified demand at all times. The most fundamental criticism of this kind of design and operation is that it is based on historical records of stream flow. As with all deterministic methods of analysis, the particular set of stream flow figures used is just one sample from a large population and hence conclusions based on that sample will include a sampling error of unknown magnitude. Also the objective is not economic, there is no comparison of costs and benefits when using this method. It could be modified to reflect the value of water supply available with different sizes and costs of reservoirs.

In the 1950's mathematicians Gani (1957), Kendall (1957), Moran (1956) and others approached the problem using statistical theory. These and others derived probability distribution of reservoir storage levels and releases for given release rules by applying Queuing theory. These studies have improved the understanding of the interaction between the factors involved. Moran (1959), Fiering (1961), Thomas and Fiering (1962) and Hufschmidt and Fiering (1966), introduced the use of Montecarlo simulations to extend Queuing theory to more complex systems for the purpose of searching optimal reservoir designs and operating policies. It is well recognized, however that neither Queuing theory nor simulation has an internal optimization structure. These procedures are extremely limited in flexibility with respect to operating rules. Due to limitations, this approach cannot be used when there is an economic objective. Also multipurpose use of reservoir poses operating problems outside the range of this method.

Most work that has been reported in the last decade has been devoted to find the best reservoir operating rules, being one of the major application of concepts of system analysis to the problem of water resources management. Linear programming applications have followed the early work of Manne (1960), Masse (1946). In this approach a price per unit volume of water released from storage is assigned for each of the time periods, and the objective is defined to maximize the revenue subject to continuity conditions. Additional constraints regarding minimum release in every time period and specified maximum and minimum levels of storage at various intervals can be introduced. The return function can be restated so as to approximate a decreasing unit price by means of piecewise linearization.

In continuity equations, the evaporation is to be approximated by defining it as a fraction of end of time storage. This formulation can be extended to multi-reservoirs. The formulation is deterministic in nature in that the design is based on historical stream flow record even though it is only one sample of a much larger population. The form of the return function does not suit the case for hydroelectric power generation where the return is a function of both the quantity of water used and the head on the turbines. As the storage is affected by the releases of water, this makes the objective function non-linear, being a function of two variables, volume and storage level or head, not just volume. Suggestions have been made by Roefs (1968) about dealing with this problem by guessing a set of storage values and using them as constraints in solving the problem, which then becomes linear again. Using the solution so obtained one then checks to see what storage values are implied and whether it is consistent with the selected one. Hopefully, a series of adjustments could then be made to bring the chosen storage levels close to those implied by the solution. There is no certainty that this procedure will converge on a solution.

Thomas and Watermeyer (1962) were the first to propose a stochastic linear programming solution to the reservoir operation problem. This formulation involves coarse discretization of the state variables if the problem is to be computationally of suitable size, and this severely limits its usefulness. A newer formulation of this problem has been proposed by Loucks (1966, 1967) being a development of a suggestion by Manne (1960). In this formulation, the joint probability of events are calculated, given constraints on these based on the transition probabilities of stream flows in one time period to the next. Also the value to be associated with these joint situations is calculated from the parameters of the situation described, with the objective being to maximize the return obtained from the operation of the system. As the objective function is made up of products of the joint probabilities (which are the variables), and the value placed on these situations, for example; the return from the release of fixed quantity of water with a specified storage and at a given time period, the problem is a linear program regardless of the form of return function from which the values are derived.

The number of constraints in the problem is the product of the number of ranges of inflow, ranges of storage, and time intervals. For a medium size problem this can get to over one thousand and the number of variables will be as many times this number as there are possible releases.

The solution to the problem is a set of joint probabilities $X_{d,s,q,t}$ of releasing amount d, while at storage s, with an inflow q, occurring at time t of various combinations of time, inflow, release and storage occurring together. From these, the optimal policy can be derived by noting that the optimal policy d will be for conditions of inflow q and storage s and time t when

$$\text{Probability} \quad P(d/sq)_t = X_{d,s,q,t} / \sum_d X_{d,s,q,t} = 1$$

The policy of releases d is specified in terms of storage s and inflow q in the same time period t as that for which the release is to be made. For actual operation basing on current inflow is unrealistic so that some means of forecasting is needed to provide inflow figures one period in advance.

This method of solution can be implemented with considerable success and although it requires the use of a large computer to solve the linear program, it does recognize the stochastic nature of the stream flow, which is regarded as serially correlated with a lag of one. By expressing the problem as one of products of values and probabilities, the whole difficulty of a nonlinear return function is dealt with but unfortunately it appears infeasible at this time to use this method for a multi-reservoir operating problem as the number of variables and constraints make it too large for current computing capabilities. The randomized decision rule to which this method introduces which is referred to as mixed strategy, may be quite logical and superior to any deterministic rule called a pure strategy in game theory. The randomized rule is introduced only to aid in development of a linear programming model whose solution allows computation of the optimal decision function. It turns out here that this optimal decision rule will always be deterministic.

The approach taken by Revelle et al. (1969) is one of first stochastic modelling techniques that can be extended to a multireservoir problem. Their approach was to formulate the problem as one of chance-constrained linear programming in which releases were assumed to be proportional to reservoir contents. Revelle and Kirby (1970) showed its application under various design criteria, and Joers et al. (1971) and Nayak and Arora (1971) showed its applications to multireservoir operation and design problems. Its application to real problems is limited, however, because of assumption of linear decision rule. Difficulties will also be encountered in establishing probability levels for constraints. The advantage of this method is that it gives a quick solution to complex problem and allows the analysis of its behaviour under different sets of conditions easily and inexpensively.

Chance constrained programming, although accounts for the randomness of natural inflows, does not define the magnitude by which the system fails. This may be a serious problem because a small failure may be relatively unimportant, whereas a large failure may have long term effects. This difficulty can be overcome with a simulation study of operating rules selected by the optimization models performed for instance with the historical data. Another difficulty arises when chance constrained programming is used, which results in total natural inflows that are significantly smaller than those observed even in dry years. The chance constrained programming assumes that the inflow are serially and spatially uncorrelated. In large reservoirs this may be serious drawback. One possible solution of this difficulty is the determination of a critical year whose characteristics are yet to be defined would be related to the level of risk or developing programming techniques to account for correlated inflows. The solution by this method do not represent the absolute optimum because of its inability to model every aspect of the physical system,it has the strong advantage of rapidly providing insights into the structure.

Little (1955) introduced Dynamic programming approach to the problem. The work of Hall and Buras (1961), Hall and Howell (1963) have followed with further application of Dynamic programming. Hall et al. (1968) used dynamic programming formulation in their operation study of a dam in California. The requirements to use this technique are that the historical stream flows must be known, but beyond that it is more flexible. No restriction is imposed on the kind of return function so that hydroelectric power returns cause no trouble. Allowance can be made for minimum withdrawals from reservoir and physical features of the reservoir with regard to its upper and lower limits etc. In Halls study he exploited this flexibility to the full and was able to reflect to a very large extent the realities of actual reservoir.

This approach, using a deterministic dynamic programming formulation is one which when applied to a multireservoir case, becomes so large as to be unmanageable. Heidari et al. (1971) applied a procedure called discrete differential dyhamic programming to a system of four reservoirs and four control variables. Restrictions in that paper included the assumption of deterministic inflows and one to one correspondence between state and control variables.

A few papers have addressed the multiple-state-multiple decision variable problem. Meier and Brighter (1967) introduced a branch compression technique for decomposing parallel reservoir system, but their approach did not address temporal allocation over seasons. Parikh (1966) put forth the idea of spatial decomposition by applying dynamic programming to subsystems under an initial set of prices. Then releases over time from all subsystems were allocated over space by linear programming from which dual values were used to adjust initial prices assigned to subsystems.

In LP analysis of Parikh, the outputs of the several reservoir systems are combined 'optimally' by a Master for contracting under the sales arrangements, on and off peak water and electrical energy for any sequence of outputs from individual reservoir subsystems. The solution of the dual of this problem gives a set of shadow prices that impute the value of an increase in availability of any one of the four commodities in each and every time period.

The shadow prices are used as a fictitious price structure for the operations of the individual reservoir systems. These prices do not have the same numerical value as contract schedule, but rather will be different for each month of an N months planning period. However, they have the effect of adjusting the reservoir operation to modify the monthly availability of firm power and water, thus relaxing the constraints that limited the linear programming optimum. By this device all the hydrologic and system constraints are properly accounted for by the dynamic programming analysis and reflected in the optimized availability of an on-and-off peak water and electrical energy used as constraints on the LP analysis. This cycle of iteration continues until no further refinement in output availabilities can be introduced by the new shadow price set, or until a new shadow price set is exactly same as the last set.

In this formulation time continuity is maintained within the sub-problem. Nothing explicit is said about space continuity. What is implied is that the reservoir in question are in parallel that is, the releases of each reservoir does not flow into any other reservoir. Roefs (1968) has presented a more general formulation in which there is a reservoir in series and multicommodity production is the motivation. It has been observed by him that if the reservoirs in series had significant energy production capability the above formulation would not be adequate.

Bodin (1970) argued for the extension of Parikh's work took to decomposition over time, but substantial computational difficulties were encountered when applied to a system of three reservoirs. All these analysis assumed a deterministic hydrology. Trott and Yeh (1973) have determined the multiple reservoir operation policy by decomposing the original multiple state variable dynamic programming by Bellman's method of successive approximation into a series of subproblems of one state variable in such a manner that the sequence of optimization over the subproblems converge to the solution of the original problem.

A combination of Dynamic programming and multivariable search technique method has been demonstrated by Erickson et al. (1969) in the analysis of one reservoir system. Chin-Shu-Lin and Tedrow (1973) used this technique to establish lake regulation rules in a multilake system.

The Dynamic programming is the most promising tool available to determine optimum reservoir operating rules. Deterministic Dynamic programming gives the optimum policy, so determined is actually hind sight, namely what was best that could have been done given perfect knowledge of all stream flows. For assessment of trial design when the stochastic nature of river flows is recognized and for real time operation when the inflow is uncertain two possible extensions have been advocated. The first has been called "Montecarlo Dynamic Programming" and has been explored by Young (1967). Basically his method is to generate, for the river in question, a number of series of synthetic annual stream flow sequences using the Montecarlo technique. For each of these he uses a dynamic programming formulation with a forward computation procedure. The optimum policies obtained for each of the synthetic stream flow sequences are then used in a regression analysis in an attempt to determine the causal factor influencing the optimal policy. This method has a considerable strength although there may be difficulties with it if a return function is used which has discontinuities. The computational efficiency of this method is high and more work is required to demonstrate the validity of the results obtained as being a good approximation of the true optimal policy.

The alternative to this Montecarlo approach is to formulate the problem as a pure stochastic dynamic programming. This formulation provides a policy which is capable of being used both for design studies and for actual operation. In it the state of the system is described by two variables, the flow of the preceding month and the quantity of water in storage. Use of this policy, which maximizes the expected value of the return will always be inferior to that which depends on full knowledge of hydrologic events. All that this policy requires is that the future hydrology should have the same statistical properties as the past.

Kunuyoshi Takeuchi et al. (1974) has formulated the monthly operation problem as a convex piecewise linear programming problem in which the objective function contains, in addition to immediate losses, the expected value of economic efficiency losses over all future months, and unknown function of end of month state variables. That function can be estimated however, by solving a stochastic dynamic programming problem in which the LP problem is nested.

Attempts have been made to overcome the computational difficulties in multireservoir operation studies (as reported by Roefs (1968)). One approach is based on aggregating all reservoir storage and inflow so that one storage and inflow represent all reservoirs in a multireservoir system. This model is then run over time finding an optimal aggregated system policy. After this allocation over time is completed, an expost facto allocation of water over space is performed by maintaining equal expectations of spill at all storage facilities. This method produces over estimates of the values of the recursive function, principally because the spill from disaggregated actual storage will always be greater than the spill from fictional system reservoir. The extent to which this distorts operating policies based on the overestimated recursive function is not clear.

Nonlinear programming has been used to determine the optimal operation of reservoirs by Lee and Waziruddin (1970). The problem considered is a three reservoir system in series. The objective function is quadratic in discharge release for irrigation, and a penalty function for deviation from the desired storage at any time. The problem is solved by conjugate gradient and gradient projection methods.

1.2.2 Operational Hydrology

The review here is concerned only with prediction or generation of monthly stream flows, with similar historical data only.

Approaches currently used in operational hydrology may be summarized in the following terms. The method uses random numbers of one or several variables which are distributed as independent normal, log normal, gamma or according to other theoretical distributions or as an empirical distribution. The stochastic dependence process or cyclic movement is process superimposed on the sequence of these independent variables. The deterministic element may be any of a variety of time functions and may be a linear composition of several such functions. One kind of deterministic element is a periodic function such as a sine wave.

The use of Markov model to generate monthly stream flows is reported by Thomas and Fiering (1962). This model takes into consideration the variation of population parameter with seasons or months and is written as a recursion relation:

$$q_{i+1,j} = \bar{q}_{i+1} + b_{i+1}(q_{i,j} - \bar{q}_i) + t_i s_{i+1}(1 - r_{i+1}^2)^{1/2} \quad - 1.1$$

where $q_{i+1,j}$ and $q_{i,j}$ represent the non historic monthly flow during year j for the month i+1 and month i respectively; $\bar{q}_{i+1}$ and $\bar{q}_i$ represent the average monthly flow of the historic stream flow record for month i+1 and month i, respectively, b_{i+1} is the regression coefficient for estimating flow during month i+1 from the flow during the month i; the value s_{i+1} is the standard deviation of the historic stream flow record for the month i+1; r_{i+1} is the correlation between flow for month i+1 and month i and t_i is a random deviate from a normal distribution with zero mean and unit variance.

Yagil (1963) derived a proof which asserts that a recursion equation of the above form, Equation 1.1 does preserve the mean, the variance and correlation between successive flows of the historic monthly stream flow sequence under the assumption that all monthly flows are normally distributed about their respective means. This is a useful proof; it assures that the non-historic stream flows will be statistically indistinguishable from the historic stream flow record although, by virtue of the addition of the random component, they are still chronologically different. Yagil's proof is of sufficient generality to permit the use of transformed variables.

Harms and Campbell (1967) extended the Thomas-Fiering model to preserve (i) normal distribution of annual flows (ii) log normal distribution of monthly flows and (iii) correlation between annual flows (iv) correlation between monthly flows. Making use of the wide accepted assumption in hydrologic literature that annual flows are very nearly normally distributed, and observations of occasional negative monthly stream flow obtained in synthesized stream flows by Thomas-Fiering model, and actual better fit of log transformed monthly flows to the normal distribution they proposed an alternative. The basis for the proposed stream flow model is stated as follows:

(i) Annual stream flow represents a regressive process that may be represented by a 1st order regressive model. In addition, annual flows are normally distributed. The basic recursion equation given by Thomas and Fiering may therefore be used to generate non-historic annual flow.

(ii) Imbedded in the annual stream flow sequence is a monthly stream flow sequence which may also be represented by a regressive process. Monthly flows are assumed to be log normally distributed. By using logarithms of monthly flows the basic recursive equation of Thomas and Fiering may be used to generate logarithm of non-historic monthly flows.

(iii) By the above development the non-historic annual sequence is independent of the non-historic monthly sequence. To render the weighted average of monthly flows equal to the annual flow a proportional adjustment of the monthly flows will need to be made.

In this it is noted that a distinct difference occurs only in the autocorrelation for the first month of water year. A property of the scaling factor used to make the monthly flows tally with the annual flows is such that a discontinuity exists in this scaling factor between the twelve months of one water year and the first month of subsequent year.

Thomas Fiering (1963) show that to treat non-normal distribution with their equation it is sufficient to alter the distribution of the random additive component and thus maintain higher moments of observed data. If the monthly flow values are derived from a parent gamma-distribution, it is necessary and sufficient that the standard random deviates be distributed like gamma, with skewness dependent upon, but not equal to, the skewness of the observed values.

To consider skewness they replaced the random component t_{i+1} by T_{i+1} which is defined as

$$T_{i+1} = \frac{2}{G_T}\left(1 + \frac{G_T\, t_{i+1}}{6} - \frac{G_T^2}{36}\right)^3 - \frac{2}{G_T}$$

where the skewness of T, denoted by G_T, is related to the estimate of the skewness of original data denoted by G_x by

$$G_T = \frac{1 - \rho_x^3(1)}{(1 - \rho_x^2(1))^{3/2}} \; G_x$$

If t_{i+1} is assumed to be normally distributed with zero mean and unit variance, then T_{i+1} is approximately distributed as gamma, with zero mean, unit variance and skewness G_T.

The basic assumption in the above regression models is that the time period to time period dependency is adequately expressed by a homoscedastic Markovian model. In the case of monthly stream flows there can be a physical argument against this assumption. For the stream flow contributed only by rainfall,the net rainfall during the month contributes to runoff of that month or it is retained in below ground or surface storage. If one postulates that there is some limit to natural basin storage then the assumption of homoscedacity is not strong. The dependency might very well be Markovian but not estimable by regression analysis.

Mandelbrot and Wallis (1968) have suggested Gaussian distribution with 'Memory' for annual stream flow values.

The first forecasting model was proposed by Carlson et al. (1970) for annual stream flow based on linear random models of Box and Jenkins (1968, 1970).

1.3 OBJECTIVE OF THIS THESIS

This thesis considers the problem related with long term management of hydropower reservoirs and quantification of the decision alternatives available, for the use of managers of such reservoirs. Simulation is a class of technique that involves setting up a model for real situation and then performing experiments on the model. Here it is the study of the system behaviour against the target output. The two alternative operating rule models provide a realistic means of planning operation in an existing system. In the first alternative the economic criteria is implicit whereas it is explicit in the second one.

Chapter two is concerned with estimation of firm power of Rihand hydroelectric station through simulation. Chapter three and four investigate the alternative strategies of planning for operation of reservoir. The model formulation is kept as general as possible, but the numerical computation is confined to a single reservoir problem, namely Rihand reservoir. Chapter five explores a stream flow forecasting technique.

In chapter two, a considerably realistic model for a hydropower station is presented. The simulation program is used to estimate the firm power capability of the Rihand reservoir, the starting point for annual operation, and storage carry over possibilities. The model of Thomas and Fiering as proposed originally, and modified, for monthly stream flow generation are examined for the operational hydrology. Strategy to assist simulation program of a two reservoir case on different streams is investigated analytically.

Chapter three deals with a nonlinear programming formulation of on line control of a multireservoir operation for power generation in a deterministic environment. Efficient organization of the computation is presented, and the problem of single reservoir is solved numerically.

Chapter four considers the problem of integrated operation of the reservoir with a thermal station of infinite capacity. Discrete Forward Dynamic programme is used to solve the problem. The computational procedure is discussed in detail.

Both the strategies presented take care of the stochastic nature of inflow to the reservoir in terms of the forecasts. Chapter five explores a simple method of stream flow forecasting. The method is essentially a decomposition technique. The attempt is to identify the underlying pattern in the monthly stream flow sequence, thus decomposing it into two major multiplying components. The deterministic component is dealt through time series analysis techniques and the stochastic component is modelled through first order Markov chain.

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CHAPTER II

SIMULATION PROGRAM, ESTIMATION OF FIRM POWER CAPABILITY OF RIHAND RESERVOIR

2.1 STATEMENT OF THE PROBLEM AND THE SIMULATION MODEL

2.1.1 Introduction

In evaluating the economic benefits of a hydropower station, it is necessary to distinguish between "dependable" or firm power, which is the continuous output available throughout every year, and "secondary power" which is the power available intermittently or for only portion of the year because of large stream flow requirements for irrigation, pollution control, navigation and so on.

In any reservoir problem, there are three non-structural variables. They are (i) controlled variable i.e., the release in any period which is under the control of the decision maker, (2) partly controlled variable i.e., the storage at the beginning or end of any period which is influenced by the decision maker through their relationships with controlled variables, which is also influenced by the uncontrolled variable in any period, and (3) the uncontrolled variable in any period i.e., inflow in any period which is determined outside the system under consideration and may be forecast by the decision maker but cannot be influenced by him.

The structural variable in an already existing reservoir is the live storage. This is constrained on one side by the minimum requirement of the turbines installed and on the other by the full reservoir level. The power generation capacity is a function of the discharge and the effective head at the time of discharge.

2.1.2 A typical model for reservoir type hydroelectric scheme

A detailed model for a storage type hydroelectric station is shown in Fig. 2.1. The model presented needs modification under the following conditions:

(i) If the reservoirs are located in series and are hydraulically connected, the tail water level is affected by the down stream reservoir level.

(ii) If the forebay is located away from reservoir and the two are connected by a limited capacity water conductor such as canal, or tunnel, the tail water level is a function of power discharge and possibly of the discharge in the stream to which the power station is discharging water. The effective head is computed based on forebay level which is in turn equal to reservoir level minus the head loss in the water conductor.

(iii) There is always a restriction on full gate discharge affecting the power conversion factor.

2.1.3 Factors to be considered and the simplifying assumption in the analysis

(i) The amount of usable reservoir storage is a function of reservoir elevation.

(ii) Respective tail water elevation varies somewhat with the total river flow in the tail water area.

(iii) The net head is the difference between the forebay and tail water levels less the losses through penstocks, turbines, and draft tube.

(iv) Plant efficiency is a function of net head on the turbine even though it is assumed that the units can be added or shut down to maintain essentially best unit efficiency for the total water available and the operating head corresponding to the forebay elevation at any time.

(v) The energy output at any plant is a function of the net head, the conversion efficiency, and the total water passing through the plant, the net head being the function of the initial water storage, inflow, and outflow from the reservoir.

(vi) The forebay elevation is an independent variable and cannot be reduced too rapidly because of water discharge limits, shore erosion control, and recreational aspects.

(vii) At the start of a critical storage use season, the reservoir is full, so the head is at its maximum. As a critical season continues with less and less water remaining in the reservoir, there is a continued drop in the net head, resulting in less energy from each unit of natural flow or storage use than when it is nearly full.

(viii) Evaporation losses from the surface of reservoir can be a significant portion of the total inflow and consequently evaporation losses must be included in the model.

The number of variables mentioned may be reduced by eliminating those which will produce relatively minor changes in the final results. They are;

(i) An average tail water elevation is selected, thereby eliminating the complication of change of this variable with total discharge.

(ii) Natural inflow to the reservoir is assumed to occur at an average, uniformly throughout the month. This may introduce some error if natural flow is very high, concentrated to any part of the month. However considerable simplification in data requirements and computation results and hence this appears to be justified.

(iii) No variation in efficiency of conversion from hydro to electric energy is assumed, a proper averaged efficiency is used. The time honoured principle of always operating each unit in its best efficiency range may not result in the optimum co-ordination operation with say thermal system except when river flow permits greater generation. Thus the assumed average efficiency is justified and it is conservative.

(iv) Energy capability is computed in discrete time. The stream flow period is considered to consist of discrete time intervals. The interval considered in the study is a calendar month.

(v) The evaporation loss from reservoir surface is an empirical quantity dependent on climatic conditions such as weather, geographical location, and time of the year. The monthly evaporation coefficients available are used with average monthly reservoir area to estimate the monthly evaporation.

2.2 SALIENT FEATURES OF RIHAND HYDROELECTRIC SCHEME

Mainly designed for power generation, the Rihand project consists of a main dam across the Rihand river, a tributary of Sone. The straight gravity concrete dam has a catchment area of 5148 sq. miles. The gross storage capacity of the reservoir is 8.6 m.a.f. out of which 7.28 m.a.f. are the live storage for power generation. The average rainfall over the catchment area is 56.3 inches per year and the impounded water is expected to generate a minimum of 919,800,000 kwh per year. The other features are as under;

Estimated average annual run off	= 5.138 MAF
Full reservoir elevation	= 880.00 ft
Dead storage elevation	= 775.00 ft

Average tail water elevation	=	632.00 ft
Power station capacity	=	300.00 MW
Turbines	=	6x70,000 HP rated at 225' head
Generators	=	6x55,000 KVA 90% power factor

The tabulated functions specified in the model vide Fig. 2.1 are presented in Tables 2.1, 2.2 and 2.3.

TABLE 2.1 Relation between reservoir elevation, gross volume and area

Elevation (ft)	Volume (MAF)	Area (1000 acres)
772	1.23	29.25
777	1.39	31.93
782	1.56	34.70
787	1.74	37.42
792	1.94	40.44
797	2.15	43.53
802	2.37	46.67
807	2.62	50.14
812	2.88	53.25
817	3.16	57.56
822	3.45	61.28
827	3.76	66.00
832	4.09	70.71
837	4.40	75.50
842	4.80	80.00
847	5.22	84.45
852	5.68	89.00
857	6.17	93.60
862	6.65	98.12
867	7.16	102.92
872	7.69	107.87
877	8.25	112.85
882	8.83	117.75

TABLE 2.2 Tail water Discharge vs Elevation (only in the power discharge range)

Discharge cusecs	Elevation ft
2000	628.50
3000	629.40
4000	630.00
5000	631.00
10000	633.50

TABLE 2.3 Monthly evaporation coefficients

Month	Evaporation in inches
January	2.60
February	3.93
March	5.21
April	6.98
May	8.08
June	9.38
July	9.42
August	8.64
September	7.66
October	3.73
November	1.71
December	1.50

Monthly river flow at the dam site are available from 1945 onwards. However the data available beyond May 1959 shows negative inflows into the reservoir in some of the months. Therefore for this study discharge data available from June, 1945 to June, 1959 are only considered. The inflows during this period are shown in Table 2.4. Though the discharge data used in the study is only for a period of 14 years, it appears that these figures are truly representative of the basin covering a full range of possible discharges from a very high annual discharge to a very low annual discharge.

The tabulated plant characteristics can be represented by fitting curves to portions of tabulated data. These include the following characteristics.

1. Tail water elevation vs. discharge

The polynomial of order 1 that fits is

Power of X	Coefficient
0	628.3793
1	0.00043

The polynomial of order 2 that fits is

Power of X	Coefficient
0	627.78623
1	0.00057
2	-0.000004

Here X is the discharge in cusecs and tail water elevation is in feet.

TABLE 2.4 Monthly inflow volumes to reservoir in M.A.F.

Year	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	March	April	May
1945	0.804	1.145	1.089	1.095	0.338	0.113	0.032	0.023	0.014	0.01	0.008	0.005
1946	0.506	1.749	1.843	0.732	0.409	0.075	0.018	0.01	0.015	0.008	0.005	0.003
1947	0.008	0.594	1.231	1.600	0.296	0.022	0.013	0.07	0.069	0.022	0.01	0.007
1948	0.322	0.956	1.350	1.476	0.335	0.235	0.079	0.06	0.065	0.018	0.008	0.007
1949	0.110	0.730	2.700	0.843	0.367	0.261	0.099	0.057	0.049	0.06	0.02	0.01
1950	0.112	1.605	4.465	1.033	0.371	0.238	0.215	0.203	0.161	0.176	0.249	0.135
1951	0.286	0.731	2.644	1.487	0.368	0.199	0.087	0.05	0.053	0.027	0.012	0.011
1952	0.389	1.234	1.929	1.175	0.177	0.074	0.056	0.055	0.031	0.016	0.01	0.007
1953	0.032	1.260	0.889	0.882	0.202	0.064	0.041	0.029	0.031	0.014	0.006	0.005
1954	0.069	0.333	0.776	0.503	0.071	0.185	0.011	0.461	0.021	0.006	0.005	0.00
1955	0.106	0.910	0.795	0.581	0.205	0.041	0.02	0.012	0.012	0.006	0.005	0.042
1956	0.553	1.128	2.270	1.178	0.458	0.203	0.073	0.084	0.054	0.033	0.017	0.006
1957	0.087	1.651	0.929	0.461	0.047	0.019	0.013	0.011	0.066	0.05	0.01	0.008
1958	0.058	1.626	1.316	1.129	0.693	0.112	0.054	0.088	0.059	0.018	0.01	0.005
1959	0.056	0.939	2.208	1.210	0.761	0.113	0.061	0.052	0.023	0.02	0.014	0.008

2. Storage vs. Reservoir area

The polynomial of order 6 that fits is

Power of X	Coefficient
0	-10.79214
1	54.95260
2	-27.36920
3	9.13704
4	-1.59039
5	0.13598
6	0.00452

The polynomial order 7 that fits is

Power of X	Coefficient
0	-8.60453
1	50.28798
2	-23.46643
3	7.46583
4	-1.19146
5	0.08243
6	-0.00075
7	-0.00010

X here is the volume of reservoir in million acre ft and the reservoir area is in 1000 acres.

3. Storage vs. Reservoir level

The polynomial of order 7 that fits is

Power of X	Coefficient
0	723.67788
1	45.63248
2	-3.47898
3	-2.37959
4	1.02233
5	-0.17694
6	0.01454
7	-0.00046

The polynomial of order 8 that fits is

Power of X	Coefficient
0	750.60952
1	-20.36454
2	62.11733
3	-37.05222
4	11.74361
5	-2.17564
6	0.23531
7	-0.01375
8	0.00033

Here X is the storage in million acre ft and reservoir level is in feet.

In the initial stages of study these polynomial functions were used in the simulation program. But in the final results the tabulated functions were directly used with linear interpolations between the tabulated values. When programs are taxing on computer storage space, polynomial functions will be helpful.

2.3 SIMULATION PROGRAM AND ITS MODIFICATIONS

The flow chart of the computer program used to operate the reservoir is shown in Fig. 2.4. The computer output for a target firm power of 105 MW is shown in Table 2.5 and the variations of reservoir levels is exhibited in Fig. 2.5.

TABLE 2.5 Working Table of Rihand Reservoir

Study period - Oct. 1945 to Oct. 1959

Power generated = 105 MW

Year	Month	Inflow	Evpn.	Outflow	Res. Vol.	Res. Level	Spill
1945	9	0.000	0.000	0.000	8.598	880.000	0.000
	10	0.338	0.033	0.366	8.535	879.465	0.000
	11	0.113	0.016	0.356	8.276	877.228	0.000
	12	0.032	0.013	0.372	7.922	874.074	0.000
1946	1	0.023	0.023	0.377	7.544	870.628	0.000
	2	0.014	0.034	0.345	7.178	867.175	0.000
	3	0.010	0.043	0.389	6.755	863.036	0.000
	4	0.008	0.056	0.383	6.323	858.601	0.000
	5	0.005	0.062	0.404	5.861	853.854	0.000
	6	0.506	0.071	0.395	5.901	854.260	0.000
	7	1.749	0.076	0.396	7.177	867.167	0.000
	8	1.843	0.078	0.375	8.566	879.728	0.000
	9	0.732	0.073	0.354	8.598	880.000	0.272
	10	0.439	0.034	0.365	8.598	880.000	0.039
	11	0.075	0.016	0.355	8.300	877.439	0.000
	12	0.018	0.013	0.371	7.933	874.170	0.000
1947	1	0.010	0.023	0.377	7.542	870.607	0.000
	2	0.015	0.034	0.345	7.177	867.163	0.000
	3	0.008	0.043	0.389	6.752	863.003	0.000
	4	0.005	0.056	0.383	6.317	858.534	0.000
	5	0.003	0.062	0.404	5.853	853.768	0.000
	6	0.008	0.069	0.400	5.392	848.870	0.000

(Contd.)

Table 2.5 (Contd.)

Year	Month	Inflow	Evpn.	Outflow	Res. Vol.	Res. Level	Spill
	7	0.594	0.068	0.417	5.501	850.054	0.000
	8	1.231	0.065	0.408	6.258	857.917	0.000
	9	1.600	0.063	0.378	7.415	869.409	0.000
	10	0.296	0.030	0.382	7.297	868.299	0.000
	11	0.022	0.014	0.374	6.930	864.754	0.000
	12	0.013	0.012	0.392	6.538	860.840	0.000
1948	1	0.070	0.020	0.399	6.188	857.193	0.000
	2	0.069	0.030	0.379	5.847	853.713	0.000
	3	0.022	0.038	0.413	5.418	849.153	0.000
	4	0.010	0.048	0.408	4.970	844.027	0.000
	5	0.007	0.053	0.434	4.489	837.693	0.000
	6	0.322	0.058	0.429	4.324	835.346	0.000
	7	0.956	0.060	0.439	4.780	841.733	0.000
	8	1.350	0.060	0.422	5.647	851.649	0.000
	9	1.476	0.059	0.390	6.673	862.233	0.000
	10	0.335	0.028	0.394	6.585	861.326	0.000
	11	0.235	0.013	0.384	6.422	859.630	0.000
	12	0.079	0.011	0.401	6.088	856.166	0.000
1949	1	0.060	0.019	0.407	5.720	852.414	0.000
	2	0.065	0.028	0.374	5.382	848.762	0.000
	3	0.018	0.036	0.423	4.940	843.672	0.000
	4	0.008	0.045	0.420	4.482	837.584	0.000
	5	0.007	0.048	0.448	3.991	830.506	0.000
	6	0.110	0.051	0.448	3.600	824.428	0.000
	7	0.730	0.051	0.467	3.812	827.790	0.000
	8	2.700	0.057	0.432	6.022	855.493	0.000
	9	0.843	0.060	0.389	6.416	859.564	0.000
	10	0.367	0.028	0.399	6.356	858.937	0.000
	11	0.261	0.013	0.387	6.215	857.474	0.000
	12	0.099	0.011	0.405	5.897	854.223	0.000
1950	1	0.057	0.019	0.411	5.523	850.303	0.000
	2	0.049	0.028	0.378	5.166	846.360	0.000
	3	0.060	0.035	0.427	4.762	841.484	0.000
	4	0.020	0.044	0.425	4.313	835.186	0.000
	5	0.010	0.047	0.454	3.821	827.929	0.000
	6	0.112	0.050	0.455	3.428	821.624	0.000
	7	1.605	0.053	0.458	4.520	838.123	0.000
	8	4.465	0.068	0.400	8.517	879.301	0.000
	9	1.033	0.073	0.354	8.598	880.000	0.524
	10	0.371	0.033	0.365	8.569	879.751	0.000
	11	0.238	0.016	0.355	8.435	878.601	0.000
	12	0.215	0.014	0.368	8.267	877.153	0.000

(Contd.)

Table 2.5 (contd.)

Year	Month	Inflow	Evpn.	Outflow	Res. Vol.	Res. Level	Spill
1951	1	0.203	0.024	0.371	8.075	875.439	0.000
	2	0.161	0.036	0.337	7.862	873.538	0.000
	3	0.176	0.047	0.377	7.614	871.284	0.000
	4	0.249	0.061	0.368	7.433	869.577	0.000
	5	0.135	0.070	0.384	7.114	866.549	0.000
	6	0.286	0.079	0.375	6.945	864.892	0.000
	7	0.731	0.080	0.387	7.208	867.458	0.000
	8	2.644	0.078	0.375	8.598	880.000	0.800
	9	1.487	0.073	0.353	8.598	880.000	1.059
	10	0.368	0.033	0.365	8.566	879.725	0.000
	11	0.199	0.016	0.355	8.393	878.237	0.000
	12	0.087	0.014	0.370	8.096	875.626	0.000
1952	1	0.050	0.023	0.374	7.747	872.515	0.000
	2	0.053	0.034	0.355	7.410	869.365	0.000
	3	0.027	0.044	0.385	7.007	865.506	0.000
	4	0.012	0.057	0.379	6.582	861.297	0.000
	5	0.011	0.064	0.399	6.129	856.589	0.000
	6	0.389	0.072	0.391	6.054	855.824	0.000
	7	1.234	0.075	0.398	6.815	863.618	0.000
	8	1.929	0.076	0.380	8.287	877.320	0.000
	9	1.175	0.073	0.355	8.598	880.000	0.435
	10	0.177	0.033	0.367	8.374	878.070	0.000
	11	0.074	0.016	0.358	8.073	875.424	0.000
	12	0.056	0.013	0.374	7.741	872.455	0.000
1953	1	0.055	0.023	0.379	7.393	869.199	0.000
	2	0.031	0.033	0.347	7.042	865.848	0.000
	3	0.016	0.043	0.391	6.623	861.728	0.000
	4	0.010	0.055	0.385	6.192	857.233	0.000
	5	0.007	0.061	0.406	5.730	852.519	0.000
	6	0.032	0.068	0.402	5.292	847.786	0.000
	7	1.260	0.069	0.412	6.070	855.980	0.000
	8	0.889	0.068	0.401	6.489	866.333	0.000
	9	0.882	0.062	0.380	6.928	864.728	0.000
	10	0.202	0.029	0.391	6.709	862.583	0.000
	11	0.064	0.013	0.383	6.376	859.148	0.000
	12	0.041	0.011	0.402	6.002	855.294	0.000
1954	1	0.029	0.019	0.409	5.602	851.157	0.000
	2	0.031	0.028	0.377	5.227	847.086	0.000
	3	0.014	0.035	0.426	4.779	841.713	0.000
	4	0.006	0.044	0.424	4.315	835.224	0.000
	5	0.005	0.047	0.454	3.819	827.894	0.000
	6	0.069	0.049	0.456	3.382	820.830	0.000

(Contd.)

Table 2.5 (Contd.)

Year	Month	Inflow	Evpn.	Outflow	Res. Vol.	Res. Level	Spill
	7	0.333	0.046	0.484	3.184	817.416	0.000
	8	0.776	0.042	0.483	3.433	821.720	0.000
	9	0.503	0.038	0.462	3.435	821.745	0.000
	10	0.071	0.017	0.487	3.001	814.169	0.000
	11	0.185	0.007	0.489	2.689	808.333	0.000
	12	0.011	0.005	0.530	2.163	797.308	0.000
1955	1	0.461	0.009	0.552	2.062	794.924	0.000
	2	0.021	0.012	0.524	1.547	781.622	0.000
	3	0.006	0.014	0.000	1.538	781.359	0.000
	4	0.005	0.019	0.000	1.523	780.920	0.000
	5	0.000	0.022	0.000	1.500	780.249	0.000
	6	0.106	0.026	0.000	1.579	782.544	0.000
	7	0.910	0.029	0.587	1.873	790.329	0.000
	8	0.795	0.029	0.564	2.074	795.205	0.000
	9	0.581	0.027	0.537	2.091	795.604	0.000
	10	0.205	0.011	0.570	1.714	786.281	0.000
	11	0.041	0.005	0.000	1.749	787.245	0.000
	12	0.020	0.004	0.000	1.765	787.627	0.000
1956	1	0.012	0.008	0.000	1.768	787.722	0.000
	2	0.012	0.012	0.000	1.768	787.712	0.000
	3	0.006	0.016	0.000	1.758	787.452	0.000
	4	0.005	0.021	0.000	1.741	787.031	0.000
	5	0.042	0.025	0.000	1.757	787.449	0.000
	6	0.553	0.029	0.566	1.715	786.311	0.000
	7	1.128	0.032	0.564	2.246	799.202	0.000
	8	2.270	0.041	0.496	3.979	830.328	0.000
	9	1.178	0.047	0.431	4.678	840.318	0.000
	10	0.458	0.023	0.435	4.678	840.312	0.000
	11	0.203	0.010	0.424	4.445	837.083	0.000
	12	0.073	0.009	0.448	4.061	831.572	0.000
1957	1	0.084	0.014	0.461	3.669	825.544	0.000
	2	0.054	0.020	0.430	3.272	818.948	0.000
	3	0.033	0.024	0.496	2.785	810.175	0.000
	4	0.017	0.028	0.507	2.266	799.643	0.000
	5	0.006	0.027	0.565	1.679	785.326	0.000
	6	0.087	0.028	0.000	1.737	786.941	0.000
	7	1.651	0.035	0.543	2.809	810.652	0.000
	8	0.929	0.039	0.497	3.201	817.713	0.000
	9	0.461	0.036	0.473	3.151	816.854	0.000
	10	0.047	0.015	0.502	2.680	808.164	0.000
	11	0.019	0.006	0.513	2.179	797.667	0.000
	12	0.013	0.004	0.571	1.615	783.546	0.000

(Contd.)

Table 2.5 (Contd.)

Year	Month	Inflow	Evpn.	Outflow	Res. Vol.	Res. Level	Spill
1958	1	0.011	0.007	0.000	1.618	783.638	0.000
	2	0.066	0.011	0.000	1.673	785.144	0.000
	3	0.050	0.015	0.000	1.707	786.090	0.000
	4	0.010	0.021	0.000	1.695	785.773	0.000
	5	0.008	0.024	0.000	1.679	785.310	0.000
	6	0.058	0.028	0.000	1.708	786.124	0.000
	7	1.626	0.034	0.546	2.752	809.554	0.000
	8	1.316	0.041	0.491	3.536	823.388	0.000
	9	1.129	0.042	0.447	4.175	833.215	0.000
	10	0.693	0.021	0.447	4.399	836.418	0.000
	11	0.112	0.010	0.434	4.066	831.646	0.000
	12	0.054	0.008	0.461	3.650	825.235	0.000
1959	1	0.088	0.013	0.477	3.247	818.511	0.000
	2	0.059	0.018	0.447	2.840	811.240	0.000
	3	0.018	0.021	0.521	2.315	800.771	0.000
	4	0.010	0.024	0.541	1.760	787.510	0.000
	5	0.005	0.025	0.000	1.740	787.002	0.000
	6	0.056	0.029	0.000	1.766	787.667	0.000
	7	0.939	0.031	0.567	2.106	795.965	0.000
	8	2.208	0.039	0.505	3.770	827.155	0.000
	9	1.210	0.045	0.437	4.497	837.792	0.000

The simulation program was modified to carry out the following experiments:

Experiment 1. To study the effect of starting point of operation

This experiment needed least modification in the program. Only the change in the input data was sufficient. The various starting points of operation tried were beginning of August, September, October and November. This study has shown that, for the general pattern of inflow contained in the historical flow of 14 years used in this study,the starting point of October results in least loss of water in the form of spill.

Experiment 2. To estimate the firm power capacity of the hydroplant with end condition of the reservoir to lie anywhere in the range of maximum and minimum reservoir conditions

The modifications required for this study is shown in the flow diagram vide Fig. 2.6. This program is run with monthly stream flow generated using Thomas Fiering model. It is well established that this stream flow synthesis procedure as proposed originally retains the statistical properties such as mean, variance and serial correlation. As such no attempt was made to verify these. However, as noted by many, this algorithm did produce negative flows in some of the months which was arbitrarily set to zero. The modified model of Thomas and Fiering with random component conforming to Gamma distribution is found not suitable in this case as it distorted the pattern of monthly distribution of flow in any year. The procedure adopted for estimating the firm power is a trial and error one and to keep the computer time limited, the estimation is limited to a precision of 1 MW. It is found that the firm power capacity of this reservoir is 85 MW.

Experiment 3. To examine whether there is any specific advantage with reference to power generation, of operating the reservoir with carry over storage varying from zero to 14 years

For this study the historical stream flows are used. The modifications of the main program required for this study is shown in the flow diagram vide Fig. 2.7. The result of this study is presented in the matrix form in Table 2.6. The first row shows the power generation capability with annual operation. The second row shows power generation capability with operation over a period of two years and so on. No sanctity can be attached to these numerical values as they do change with the inflow distribution in various months in a year and its distribution over the years of study. However the average generation capability exhibit the relative advantage of operation with various periods of operation. The last column of Table 2.6 shows the average generation capability with durations of operation. The results show that it is better to operate this reservoir on year by year basis to maximize energy production. This fact is used in the operation planning of this reservoir through mathematical programming in chapters three and four. In estimating the generation capacity with various durations of carryover, it is specified that the reservoir should attain its maximum water condition at the end of the set period of operation. The constraints on the reserve to be within the maximum and minimum conditions at any intermediate stage is imposed as usual.

2.4 STRATEGY FOR STORAGE RELEASES FOR POWER GENERATION IN THE CASE OF TWO RESERVOIRS ON DIFFERENT STREAMS

Hydropower reservoirs filled during monsoon, have to be depleted in the non-monsoon period to meet the demand for power. Not much of flexibility exists if each reservoir is operated independent of the other and has its own load. If two or more reservoirs are operated as a system to meet the combined load, lot of flexibility in storage releases of

TABLE 2.6 Firm power capability with years of carry-over

Carry-over year	Firm power capability (MW)														Av.Power
0	110	76	93	107	168	145	114	68	33	60	112	78	87	106	96.93
1	76	85	99	134	145	114	68	33	47	85	78	82	99		88.08
2	85	91	118	137	114	68	33	47	67	78	82	92			84.33
3	91	106	123	114	68	33	47	67	70	82	92				81.18
4	106	110	114	68	33	47	67	70	73	89					77.70
5	110	111	68	33	47	67	70	72	78						72.89
6	111	68	33	47	67	70	72	76							68.00
7	68	33	47	67	70	72	76								61.88
8	33	47	67	70	72	76									60.83
9	47	67	70	72	76										66.40
10	67	70	72	76											71.25
11	70	72	76												72.67
12	72	76													74.00
13	76														76.00

these reservoirs exists. For a given storage at various reservoirs, load, and inflow, thecretically there can be infinite number of release combinations possible. In simulation programs, these infinite number of alternatives can be reduced to a few preference storage release policies for a defined objective. These alternatives can be derived under simplified assumptions and further, the release policies can be refined via simulation runs. This in effect reduces the time and effort on simulation and extraction of the results to satisfy the objective.

The problem considered here is a two reservoir case on different streams. The effect of depletion of storage is the loss of energy which otherwise would have been generated by the inflow to the reservoir. Therefore the objective is to minimize this loss of energy.

Let the release policy for a reservoir be represented in the form of a linear function of time;

$$d_t = a + bt \qquad (2.1)$$

where d_t = release rate of stored water at time t,

a = release rate of water at time $t = 0$

b = rate of change of release rate.

Let S = total volume of storage withdrawal

T = total duration of storage withdrawal

Therefore $$\int_{t=0}^{t=T} (a+bt)\, dt = S$$

$$aT + \frac{bT^2}{2} = S$$

$$\text{or} \quad a = \frac{S}{T}\left(1 - \frac{bT^2}{2S}\right)$$

define a quantity $\alpha = \frac{bT^2}{2S}$. This is a measure of the rate of change of storage release. With this substitution we can now write equation 2.1 as

$$d_t = \frac{S}{T}\left[1 - \alpha + 2\alpha t/T\right] \tag{2.2}$$

Total volume of water released upto any time t

$$= \int_0^t d_t . dt = \frac{St}{T}\left[1 - \alpha + \alpha t/T\right]$$

If ΔH is the loss of head due to release of stored water of volume S, the loss of head upto time t can be written as

$$\Delta H . \frac{t}{T}\left[1 - \alpha + \alpha t/T\right]$$

with the assumption that loss of head is proportional to the storage withdrawal. If F is the average rate of inflow, the loss of energy upto time t can be written as

$$\Delta H \frac{Ft}{T}\left[1 - \alpha + \alpha t/T\right]$$

Here it is assumed that product of discharge and head is a measure of energy.

Total loss of energy in the period 0 to T is

$$= \int_0^T \Delta H . F . \frac{t}{T}\left[1 - \alpha + \frac{\alpha t}{T}\right] dt$$

$$= \Delta H . F . T \left(\frac{1}{2} - \frac{1}{6}\alpha\right) \tag{2.3}$$

In the case of two reservoirs called 1 and 2 the expression 2.3 may be written with all quantities subscripted with respect to the reservoir it represents. Thus,

The total loss of energy $EL = \Delta H_1 F_1 T_1 (\frac{1}{2} - \frac{1}{6}\alpha_1) + \Delta H_2 F_2 T_2 (\frac{1}{2} - \frac{1}{6}\alpha_2)$

The particular cases of the general release policies considered are

POLICY	PARAMETER VALUES	LOSS OF ENERGY
A. Uniform release of stored water, storage 1 is used for 1st half of the period and storage 2 is used for the 2nd half of the period	$\alpha_1=0$, $\alpha_2 = 0$, $T_1=T_2$ from $t=0$ to $t=T/2$; $T_2=T/2$ from $t=T/2$ to $t=T$	$\Delta H_1 F_1 \frac{T}{4} + \Delta H_2 F_2 \frac{3T}{4}$
B. Uniform release of stored water from both storages over the total duration	$\alpha_1=0$, $\alpha_2=0$, $T_1=T$, $T_2=T$	$\Delta H_1 F_1 \frac{T}{2} + \Delta H_2 F_2 \frac{T}{2}$
C. Deferred release, both storage release is uniform and is in the later half of the total duration	$\alpha_1=0$, $\alpha_2=0$, $T_1=T/2$, $T_2=T/2$, both from $t=T/2$ to T	$\Delta H_1 F_1 \frac{T}{4} + \Delta H_2 F_2 \frac{T}{4}$
D. Rate of release from both storages increasing with time	$\alpha_1=+1$, $\alpha_2=+1$, $T_1=T$, $T_2=T$	$\Delta H_1 F_1 \frac{T}{3} + \Delta H_2 F_2 \frac{T}{3}$
E. Rate of release increasing from storage one and decreasing from storage two with respect to time	$\alpha_1=+1$, $\alpha_2=-1$, $T_1=T$, $T_2=T$	$\Delta H_1 F_1 \frac{T}{3} + \Delta H_2 F_2 \frac{2T}{3}$

POLICY	PARAMETER VALUES	LOSS OF ENERGY
F. Rate of release decreasing with time from storage one and increasing from storage two	$\alpha_1=-1$, $\alpha_2=+1$, $T_1=T$ $T_2=T$	$\Delta H_1 F_1 \frac{2T}{3} + \Delta H_2 F_2 \frac{T}{3}$
G. Rate of release decreasing with time from both the storages	$\alpha_1=-1$, $\alpha_2=-1$, $T_1=T$ $T_2=T$	$\Delta H_1 F_1 \frac{2T}{3} + \Delta H_2 F_2 \frac{2T}{3}$

The relative weights of these energy losses can be expressed by dividing the energy loss terms by $12\Delta H_2 F_2 T$ and writing $K = \Delta H_2 F_2 / \Delta H_1 F_1$.

Thus we get

Release policy	Relative weights of energy loss
A	3+9K
B	6+6K
C	3+3K
D	4+4K
E	4+3K
F	8+4K
G	8+8K

These release policies and the relative loss of energy are shown in Fig. 2.8 and the preference order for storage release is also indicated.

The analysis can be extended to multireservoir systems. The preference solution can be used to determine the storage release policies to minimize the energy loss due to storage depletion. Deviations from the linear decision policies can be tried via simulation and final release policies can be arrived at in advance with inflow forecasts.

2.5 CONCLUSIONS

The preliminary studies reported here have shown that:

(i) The continuous power generation capability of Rihand reservoir is 85 MW.

(ii) It is preferable to operate this reservoir on annual basis with a view to maximize average power production over the years.

(iii) The starting point of operation in any year should be beginning of October when the reservoir level should be at its maximum.

(iv) Flexibility exists if this reservoir is operated with other hydropower station, or thermal station or both. The latter two aspects are dealt in subsequent chapters.

(v) Preference storage release policies to minimize the energy loss due to storage depletion can be worked out when two or more reservoirs are operated as a system. This is to be further refined by Simulation program. The preference solutions obtained by total storage available for release, the corresponding loss of head and the average stream flow anticipated saves considerable time and effort in the simulation of the system operation.

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CHAPTER III

HYDROPOWER SCHEDULING UNDER AN INDEPENDENT OPERATIONAL STRATEGY

3.1 OPERATION STRATEGY AND THE MATHEMATICAL MODEL

3.1.1 Introduction

In chapter two, the firm power capability of a hydropower reservoir is estimated through simulation. The model can be extended to find firm power capability of a group of reservoirs. Computations using this model consume quite a lot of computer time. Further the result obtained is only helpful in economic evaluation of the project before inception. In actual practice the power generation must conform to the demand. The energy capability of a hydropower station does vary from year to year because of the variation in the inflow to reservoir. In this chapter an operation strategy is examined wherein a group of hydropower stations, administered by a separate agency, has to take care of its load. To ensure proper supply of power to the consumers, the hydro utility has to schedule its generation in advance over a planning horizon say a twelve months period. The strategy for such planning is to maximize the average energy generation and keep the generation level in any month at a constant surplus or deficit level relative to the fluctuating demand.

Depending on whether there is surplus or deficit, the management may plan to sell out to or purchase from, the neighbouring utility at a constant rate. Thus in this method of operation, the system resource, that is, the stored water releases are scheduled monthly in a manner so as to produce a uniform power excess or deficit for a twelve month period. The uniform power deficit so established represents a demand, which under stipulations of the firm power sales contract, the hydro utility will purchase from the other utility. The human judgement in such situations to choose proper storage release sequence does not guarantee optimum results. This indicates the needs for construction of a faithful mathematical model for the hydro system.

3.1.2 Mathematical Model

The two objectives in the problem to be solved are (1) To maximize the average energy production, (2) To minimize the sum of the deviations from the demand. To state the problem in mathematical form, the following notations are used.

- K - Time interval index (month)
- $S_j(K)$ - Storage in reservoir j at the start of interval K
- $Q_j(K)$ - Water released for power generation from reservoir j during interval K
- $IN_j(K)$ - Inflow from independent catchment to reservoir j during interval K

n - Number of storage reservoirs in the system

m - Total number of hydro schemes in the system

P(K) - Power generation during interval K

L(K) - System load during the interval K

t(K) - Power generation duration in the interval K

The system equation for storage reservoirs:

$$S_j(K+1) = S_j(K) + IN_j(K) + \sum_{i=1}^{n} \delta_{ij} Q_i(K) - Q_j(K) \qquad (3.1)$$

$$j = 1,n$$

$$\text{where } \delta_{ij} = \begin{cases} 1 & \text{if reservoir } i \text{ is directly upstream from reservoir } j \\ 0 & \text{otherwise} \end{cases}$$

Similarly we can write for run off the river schemes;

$$IN_j(K) + \sum_{i=1}^{n} \delta_{ij} Q_i(K) - Q_j(K) = 0 \qquad (3.2)$$

$$j = n,m$$

δ_{ij} is defined similarly.

Equations (3.1) and (3.2) are water conservation equations which can be written as

$$A\bar{Q} + B\bar{S} + \bar{C} = 0 \qquad (3.3)$$

where A is a (mk × mk) square matrix and B is matrix rectangular having mk rows and nk columns. $\bar{Q}$ is a vector of elements Q_{jk}, $\bar{S}$ is the vector of elements S_{jk} and $\bar{C}$ is a vector, with elements consisting of initial storage and inflows. A and B matrices are sparse, and the distribution of their non zero elements depends on the topology of the system and its ordering. The total power generated in the interval K over a period of one year is;

$$P(K) = \sum_{j=1}^{n} p_j(K) + \sum_{j=n}^{m} p_j(K) \qquad (3.4)$$

$$K = 1,2,\ldots,12$$

where $p_j(K)$ represents the average power produced by jth hydrostation in the Kth interval.

For a storage reservoir:

$$p_j(K) = p_j(K)\ (Q_j(K),\ S_j(K),\ S_j(K-1),\ S_i(K),\ S_i(K-1)) \qquad (3.5)$$

where ith reservoir is below jth one. If there is no coupling between the two reservoirs, the expression can be written as;

$$p_j(K) = p_j(K)\ (Q_j(K),\ S_j(K),\ S_j(K-1)) \qquad (3.6)$$

$$j = 1,n$$

For a run off the river scheme

$$p_i(K) = p_i(K)\ (Q_i(K)) \qquad (3.7)$$

$$i = n,m$$

Thus the system annual generation capability can be defined as

$$\underline{P} = \frac{1}{T} \sum_{K=1}^{12} P(K).t(K) \qquad (3.8)$$

where $t(K)$ is the generation duration in the Kth interval and $T = \sum_{k=1}^{12} t(K)$.

The first objective is to maximize the average power production, which can be stated as minimize the function

$$\frac{1}{T}\sum_{k=1}^{12} (L(K) - P(K)).t(K) \tag{3.9}$$

The second objective can be taken as a constraint and can be incorporated into the objective function in the form of deviation squared multiplied by a penalty term. Thus the whole objective function for a twelve year period can be written as,

$$\sum_{k=1}^{12} \frac{1}{T} (L(K) - P(K)) \; t(K) + W \sum_{K=1}^{12} ((L(K) - P(K)) - \frac{1}{T} \sum_{K=1}^{12} (L(K) - P(K)) \; t(K))^2 \tag{3.10}$$

which is to be minimized, where W is a penalty term.

For an energy deficient system wherein the demand is always more than generation capability the objective function can simply be written as

$$\sum_{k=1}^{12} (L(K) - P(K))^2 \tag{3.11}$$

which is to be minimized.

This is a much simpler objective function and easy to work with. Even the surplus system wherein the energy generation capability is more than the system demand, the system can be converted to a deficient system by artificially augmenting the load by a constant known quantity so as to satisfy the condition that the augmented load is much more than the energy capability of the generating system.

The system constraints can be discharge limitations of turbines which is a function of storage, and the storage contents of the reservoir. The latter can be represented as

$$S_j^{min} \leq S_j(K) \leq \hat{S}_j(K) \leq S_j^{max} \qquad (3.12)$$

where S_j^{min} - Minimum storage at the jth reservoir

S_j^{max} - Maximum storage at the jth reservoir

$\hat{S}_j(K)$ - Specified storage level at jth reservoir at the beginning of K interval

Other constraints on releases and storages depending on special situation of the problem can also be imposed, along with the continuity constraints.

3.2 APPLICATION TO THE SPECIFIC PROBLEM

The problem considered here is to schedule water releases over a period of one year, from Rihand hydropower station such that there is a uniform power deficit over the planning period. The problem is a deterministic discrete one in as much as the loads and stream flow forecasts are known in discrete form. The objective is simply to minimize the function

$$\sum_{k=1}^{12} (L(K) - P(K))^2 \tag{3.13}$$

Subject to

$$S^{min} \leq S(K) \leq S^{max} \tag{3.14}$$

and the continuity constraints.

There are no other special requirements with respect to storage or releases from the reservoir. The problem is treated as a deficient system, if necessary by augmenting, and will be solved.

The problem can be solved by any of the unconstrained methods if the bounds on the reservoir storage is taken care of. The method used here is the conjugate gradient method of Fletcher and Reeve's.

3.3 COMPUTATIONAL PROCEDURE

The method requires gradient to be computed. Since the objective function is in terms of P(K), which is determined by Q(K) and S(K), and Q(K) is related to S(K) through the system equation, the variables of the problem basically are a set of 12 storage values at the end of 12 months, the initial one to start with being specified. Since

$$A\bar{Q} + B\bar{S} + \bar{C} = 0$$

$$\bar{Q} = -A^{-1} (B\bar{S} + \bar{C}) \tag{3.15}$$

and gradient

$$\nabla F(S) = \frac{\partial F}{\partial S} + \left(\frac{\partial Q}{\partial S}\right)^T \frac{\partial F}{\partial Q} \qquad (3.16)$$

where

$$\left(\frac{\partial Q}{\partial S}\right)^T = -B^T(A^T)^{-1}$$

$\partial F/\partial S$ and $\partial F/\partial Q$ are obtained by chain rule of differentiation.

$$F = \sum_{k=1}^{12} (L(K) - P(K))^2$$

$$\frac{\partial F}{\partial S(I)} = 2 \sum_{k=1}^{12} (L(K) - P(K))\left(- \frac{\partial P(K)}{\partial h(I)} \cdot \frac{\partial h(I)}{\partial S(I)}\right), \; I=1,\ldots,12$$

where h is the effective head corresponding to storage S and $S(I)$ represents variable S at time I.

$$\frac{\partial F}{\partial S(I)} = -2 \frac{\partial h(I)}{\partial S(I)} \left[\frac{\partial P(I)}{\partial h(I)} (L(I) - P(I)) + \frac{\partial P(I+1)}{\partial h(I)} (L(I+1)-P(I+1)) \right]$$

since $P(I) = C\, Q(I)(h(I)+h(I-1))/2$

where C is a energy conversion constant, Q and h represents the discharge and effective head respectively.

$$\frac{\partial F}{\partial S(I)} = -C \frac{\partial h(I)}{\partial S(I)} \left[Q(I)(L(I)-P(I))+Q(I+1)(L(I+1)-P(I+1)) \right]$$

$$I = 1,2,\ldots,12 \qquad (3.17)$$

Similarly

$$\frac{\partial F}{\partial Q(I)} = -C \left[(h(I)+h(I-1))(L(I)-P(I)) \right] \qquad (3.18)$$

$$I = 1,2,\ldots,12$$

The matrix A in this case is an Identity matrix and matrix B is lower triangular matrix with elements $B(i,j) = 1$ if i = j and equal to -1 if i = j-1 and equal to zero otherwise. Therefore the gradient

$$\overline{\nabla F}_S = \frac{\partial F}{\partial S} + \begin{bmatrix} -\frac{\partial F}{\partial Q(1)} + \frac{\partial F}{\partial Q(2)} \\ -\frac{\partial F}{\partial Q(2)} + \frac{\partial F}{\partial Q(3)} \\ -\frac{\partial F}{\partial Q(3)} + \frac{\partial F}{\partial Q(4)} \\ \vdots \\ -\frac{\partial F}{\partial Q(11)} + \frac{\partial F}{\partial Q(12)} \\ -\frac{\partial F}{\partial Q(12)} \end{bmatrix} \qquad (3.19)$$

The method now used is to find the unconstrained minimum of multivariable nonlinear function of the form

$$F(S(1),\ S(2)\ \ldots.\ S(12))$$

The basic procedure is described by Fletcher and Reeves (1964). The algorithm proceeds as follows:

1. A starting point is selected.

2. The direction of steepest descent is determined by determining the following direction vector components at the starting point

$$M_n(I) = \left\{ \frac{-\partial F/\partial S(I)}{\left[\sum_{k=1}^{12} (\partial F/\partial S(I))^2 \right]^{1/2}} \right\}_n, \qquad (3.20)$$

$$I = 1, 2, \ldots, 12$$

where n = 0 for the starting point.

3. A one dimensional search is then conducted along the direction of steepest descent using the relation

$$S(I)_{(new)} = S(I)_{(old)} + dM(I), \qquad I = 1,2,\ldots,12 \tag{3.21}$$

where d is the distance moved in the direction $\bar{M}$. When a minimum is obtained along the direction of steepest descent, a new "conjugate direction" search direction is evaluated at the new point with the normalized components.

$$M_n(I) = \frac{-(\partial F/\partial S(I))_n + \beta_{n-1} \cdot M_n(I-1)}{\left[\sum_{K=1}^{12} (-(\partial F/\partial S(K))_n + \beta_{n-1} M_n(K))^2\right]^{1/2}} \tag{3.21}$$

$$I = 1,2,\ldots,12$$

where $\beta_{n-1} = \dfrac{\sum_{K=1}^{12} \left[(\partial F/\partial S(X))_n\right]^2}{\sum_{K=1}^{12} \left[(\partial F/\partial S(X))_{n-1}\right]^2}$

4. A one dimensional search is then conducted in this direction. When a minimum is found, an overall convergence check is made. If convergence is achieved, the procedure terminates. If convergence is not achieved, new "conjugate direction" vector components are evaluated as per step (3) at the minimum point from the current one dimensional search. This process is continued until convergence is achieved or 12+1 directions have been searched. If a cycle of 12+1 directions have

been completed, a new cycle is started consisting of a steepest descent direction (step 2) and 12 conjugate directions (step 3).

To take care of bounds on $\bar{S}$ the following modifications are necessary. The gradients at step 2 and step 3 are projected on to the bounds on $\bar{S}$ if they are at a very small distance from the bounds. Thus

$$\begin{aligned} M_n(I) &= 0 \quad \text{if } (S^{max} - S_n(I)) \leq \epsilon \text{ and } M_n(I) > 0 \\ &= 0 \quad \text{if } (S_n(I) - S^{min}) \leq \epsilon \text{ and } M_n(I) < 0 \\ &= M_n(I) \text{ otherwise} \end{aligned} \tag{3.22}$$

and in the one dimensional search step the maximum step length at any stage is limited to

$$d^{max} = \underset{I}{\text{Min}} \left\{ \underset{M_n(I)>0}{\text{Min}} \frac{S^{max} - S_n(I)}{M_n(I)}, \underset{M_n(I)<0}{\text{Min}} \frac{S^{min} - S_n(I)}{M_n(I)} \right\} \tag{3.23}$$

The one dimensional search procedure used here is the Davidon's linear search technique. Davidon's (1959) has suggested that the linear minimizations which constitute a necessary part of the multivariate minimization technique should make use of values of the gradient in addition to values of the objective function, and has specifically recommended that cubic interpolation using function values and gradients at two points be adopted.

Davidon's linear search technique is as follows. Given a point $\bar{x}_i$, and the direction $\bar{d}_i = -H_i \bar{g}_i$, the function and gradient are evaluated at $\bar{x}_i$ and $\bar{x}_i+\lambda_1\bar{d}_i$, to give values f_o, $\bar{g}_o$, f_1 and $\bar{g}_1$ respectively. A suitable choice of λ_1 is given by

$$\lambda_1 = \min \left\{ 2, \frac{-2(f_o - f^*)}{\bar{g}_o \bar{d}_i} \right\}$$

where f^* is an estimate of the minimum of the function provided by the user. Here in the problem considered λ_1 is further limited by the equation (3.23).

If $f(\bar{x}_i+\lambda_1 \bar{d}_i) \geq f(x_i)$, and/or $\bar{d}_i' \bar{g}(\bar{x}_i+\lambda_i\bar{d}_i) \geq 0$, then the minimum has been straddled. A cubic is then fitted through these two points, and the minimum of the cubic $\bar{x}_i+\lambda^*\bar{d}_i$, used as an estimate of the minimum along the line, where

$$\frac{\lambda^*}{\lambda_1} = 1 - \frac{(\bar{g}_1' \bar{d}_i + w - z)}{(\bar{g}_1' \bar{d}_i - \bar{g}_o' \bar{d}_i + 2w)}$$

in which $$w = \left[z^2 - (\bar{g}_o' \bar{d}_i).(\bar{g}_1' \bar{d}_i) \right]^{1/2}$$

and $$z = \frac{3}{\lambda_1}(f_o - f_1) + \bar{g}_o' \bar{d}_i + \bar{g}_1' \bar{d}_i.$$

This particular representation for the root of the cubic was chosen by Davidon as being the most accurate for automatic computation.

If a minimum has not been straddled, then a new step is estimated along $\bar{d}_i$ starting from $\bar{x}_i + \lambda_1 \bar{d}_i$ instead of $\bar{x}_i$.

3.4 DATA USED, COMPUTATIONAL ASPECTS

The reservoir characteristics used in the study are given in Chapter 2. Evaporation is not explicitly taken into consideration. The time unit considered is the standard month of 31.5 days. The evaporation can be taken care of by adjusting the inflow forecasts for evaporation. The partial derivative $\partial h/\partial S$ required in the computation of gradient is computed at discrete points and is assumed to be constant in between the two successive points. The inflow, the load data used in the sample program and the results of the computer program are summarized in Table 3.1 and the variations of reservoir contents are exhibited in Fig. 3.2. The solution of the same problem via simulation program presented in Chapter two is furnished in Table 3.2. The optimizing program has invariably reduced the objective function as compared to that obtained by simulation program, and has shown an increase in the energy capability of the system. Further the optimizing program has taken much less of computer time compared to that taken by the simulation program. The number of function evaluation in the optimizing program is generally of the order of 10 to 30.

TABLE 3.1 Table of power generation schedule (from optimization program)

Month	Load MW	Inflow M.A.F.	Power generation MW	End of month Res. content M.A.F.	Reservoir level	Short fall* in power MW
October	100	0.075	82.2	8.390	878.201	17.8
November	110	0.013	92.2	8.082	875.486	17.8
December	120	0.010	102.2	7.733	872.363	17.8
January	120	0.015	102.2	7.384	869.091	17.8
February	100	0.003	82.2	7.090	866.293	17.8
March	110	0.005	92.2	6.758	863.037	17.8
April	120	0.003	102.2	6.3[illegible]1	859.181	17.8
May	110	0.008	92.2	6.041	855.666	17.8
June	100	0.594	82.5	6.324	858.580	17.3
July	90	1.231	73.2	7.286	868.163	16.8
August	100	1.600	84.1	8.590	879.908	15.9
September	100	0.296	84.6	8.598	880.000	15.4

*The maximum short fall 17.8 MW is to made up by purchasing it from other utility on contract basis.

TABLE 3.2 Table of power generation schedule (from simulation program)

Month	Load MW	Inflow M.A.F.	Power generation MW	End of month Res. content M.A.F.	Reservoir level	Short fall in power MW
October	100	0.075	79.85	8.389	878.201	21.15
November	110	0.013	89.85	8.080	875.485	21.15
December	120	0.010	99.85	7.728	872.340	21.15
January	120	0.015	99.85	7.376	869.038	21.15
February	100	0.003	79.85	7.081	866.232	21.15
March	110	0.005	89.85	6.747	862.957	21.15
April	120	0.003	99.85	6.368	859.063	21.15
May	110	0.008	89.85	6.026	855.531	21.15
June	100	0.594	79.85	6.308	858.443	21.15
July	90	1.231	69.85	7.274	868.080	21.15
August	100	1.600	79.85	8.584	879.887	21.15
September	100	0.296	79.85	8.598	880.000	21.15

Since the direction of search is computed making use of tabulated values of reservoir characteristics, the program terminates the computation when the gradient is very small or when the function value evaluated in two successive iterations differ only by a small quantity. It is necessary that the initial values chosen must be feasible, satisfying both the continuity constraints and the upper and lower bounds. Otherwise, it is possible the program terminates prematurely giving negative outflows which is absurd. The method of optimization used is quite effective and even larger problems can easily be solved on a small computer like IBM 1130. Different starting points were used as multimodality of the objective function was suspected. From different starting points, the solution always converged to the same point for a given set of data.

3.5 CONCLUSIONS

1. A mathematical model is presented for solution of independent operational strategy of hydropower reservoir.
2. The problem is solved by modified Fletcher and Reeve's algorithm for one reservoir case namely Rihand reservoir.
3. The solution of the problem provides the Manager of the reservoir a quantified rational basis for operation of the reservoir.
4. For the reservoir under consideration, which has to supply its production to bulk consumers on contract,

the operational strategy appears quite realistic. This type of operation ensures continuous supply to the consumer under this utility.

5. The problem tagged with uncertainty of stream flow forecasts is to be tackled on either fixed horizon policy wherein the energy generation is to be changed adaptively as suggested in Appendix A1, or on moving horizon policy wherein the problem is solved every month with 12 months planning horizon. These suggestions need further investigation.

6. The formulation can be used to find the firm power of a multihydroelectric project avoiding the conventional trial and error method.

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CHAPTER IV

CO-ORDINATED OPERATION OF HYDRO AND THERMAL POWER STATIONS

4.1 STATEMENT OF THE PROBLEM AND MATHEMATICAL MODEL

4.1.1 Introduction

With the formation of regional grids, the co-ordinated operation of various electricity generation utilities have become a reality. This is necessary with the increased demand for power which usually results in decreased load factor and fluctuations in demand. By sharing the peak load of the combined load, the hydro-stations increase the load factor of thermal stations, thereby increasing the efficiency of thermal stations and decreasing the high-cost thermal power generation. The co-ordinated operation is based on explicit economic criteria of minimizing the cost of thermal power generation.

The problem considered here is the co-ordinated operation of a single hydropower station namely Rihand hydropower station with a thermal station of infinite capacity. The combined system load is to be met by hydro and thermal generation minimizing the out of pocket cost of thermal generation. Given the system load defined by duration curve, the stream flow forecast, the initial and final conditions of reservoir, the characteristics of the reservoir, the problem is to schedule hydrogeneration for a planning horizon over a period of one year. Such an operation enables the hydrostation to sell off its full energy potential, and the thermal station ensures uninterrupted energy supply despite low water year.

4.1.2 Representation of the Model

The model of the system includes (a) thermal plant represented by its cost characteristic, (b) Hydroplant represented by its characteristics depending on both water heads and discharges, (c) bounds on operating conditions of the hydrostation.

In order to describe the model mathematically, the following variables are introduced.

- K - Period index $K = 1,2,...,12$ months
- $V(K)$ - Volume of reservoir at the beginning of period K
- $D(K)$ - Discharge from reservoir for power generation during period K
- $S(K)$ - Spill from the reservoir during period K
- $E(K)$ - Evaporation from the reservoir during period K
- $\bar{V}$ - Upper bound on variable V
- $\underline{V}$ - Lower bound on variable V
- $EH(K)$ - Effective head for power generation during period K
- $Y(K)$ - Energy generated by hydropower station during period K
- $I(K)$ - Inflow to the reservoir during period K
- $h(K)$ - Number of hours in period K

τ - Normalized duration obtained by dividing any duration in a month by the total number of hours in month

$L(\tau)$ - Ordinate of the load duration curve (normalized) for the duration τ in any month

$L'(\tau)$ - Ordinate of the load duration curve (normalized) after replacement by hydrogeneration for the duration τ in any month

$C(P)$ - Hourly generation cost for power generation P

$GC(K)$ - Hydropower generating capacity during period K

MGC - Upper bound on hydrogenerating capacity.

The reservoir dynamics is described by

$$V(K+1) = V(K) + I(K) - D(K) - E(K) - \delta_K S(K) \tag{4.1}$$

$$\delta_K = \begin{cases} 1 & \text{if } V(K) + I(K) - D(K) - E(K) > \bar{V} \\ 0 & \text{otherwise} \end{cases}$$

Here E(K) is a function of the reservoir area which is changing continuously in a month. The computation of E(K) is based on the average reservoir level during the month under consideration.

The initial reservoir volume is known, and the final reservoir volume is specified, i.e.,

$$V(1) = V(t_o) \tag{4.2}$$

$$V(13) = V(t_f) \tag{4.3}$$

The hydropower generation restriction is described by the equation

$$0 \leq GC(K) \leq MGL \tag{4.4}$$

The storage content of the reservoir has an upper and lower bound

$$\underline{V} < V(K) < \overline{V} \tag{4.5}$$

The hydro energy generated in any given period

$$Y(K) = e.EH(K).D(K).CF \tag{4.6}$$

where e is the efficiency of conversion of potential energy to electrical energy and CF is the conversion factor.

The hydro replaced energy cost is the cost of generating system energy by thermal station less the cost of generating hydro displaced system energy by thermal station. The demand $L_K(T)$ must be satisfied by thermal and hydrogeneration. The hourly cost of generation of thermal power increases monotonically with generation level. Therefore to get maximum savings, the hydro-energy must share the system peak loads and the balance by thermal energy. Since the hydropower generation has an upper bound governed by the total installed capacity, it is not possible to replace all the expensive thermal energy. The possibility therefore is that hydrostation should replace the high cost thermal generation in the feasible region only. These aspects are shown in Figures 4.1 and 4.2.

The hydro-energy cost HYDROCOST which is a function of the hydro-energy generated in any duration K is expressed mathematically as

$$\text{HYDROCOST } (Y(K)) = -h_K \int_0^1 L_K(\tau)\, C(\tau).d\tau + h_K \int_0^1 L'_K(\tau).C(\tau).d\tau \qquad (4.7)$$

L'_K is a function of $Y(K)$, L_K and GC(K).

The objective function is therefore defined as

$$\sum_{K=1}^{12} \text{HYDROCOST } (Y(K)) \qquad (4.8)$$

which is to be minimized.

4.2 SOLUTION PROCEDURE

The above dynamic optimization problem is solved by dynamic programming. The procedure is based on the Bellman's principle of optimality which states that whatever the initial state and control are, the remaining controls in the sequence must be an optimal control sequence with regard to the state resulting from the first control. The usual computational procedures work backward in time. Here it is advantageous to work with the forward algorithm as the reservoir dynamics equation does not map the condition of the reservoir described by its storage content at time K+1 into the condition at K due to the presence of spill. Further it is desired to have an optimal trajectory from a given initial condition of the reservoir. Using forward dynamic programming solution it is easy to examine trajectories reaching many terminal states.

If I(X,t) represents the minimum cost corresponding to state X at time t, the definition in the discrete case is

$$I(X,K) = \underset{u(0),\ldots,u(K-1)}{\text{Min}} \left\{ \sum_{j=0}^{K-1} l\left[X(j),u(j),j\right] \right\} \quad (4.9)$$

where $$g\left[X(K-1),u(K-1),K-1\right] = X \quad (4.10)$$

where u represents the control and K the period and l the cost function and g the system dynamics.

Define a quantity $h\left[X,u(K-1),K\right]$ by

$$g\left[h\left[X,u(K-1),K\right],\ u(K-1),(K-1)\right] = X \quad (4.11)$$

then, the iterative equation can be written as

$$I(X,K) = \underset{u(K-1)}{\text{Min}} \left\{ l\left[h\left[X,\ u(K-1),K\right],\ u(K-1),K-1\right] + I\left[h\left[X,\ u(K-1),\ K\right],\ K-1\right] \right\} \quad (4.12)$$

This iterative procedure determines I(X,K) in terms of I(X,K-1); thus, the calculations do indeed go forward in time rather than backward. The initial condition to start the iteration is I(X,0) = 0 i.e., no cost is incurred before the system begins to operate.

The computation procedure is exactly analogous to that of backward dynamic programming; at a given quantized X and K try all possible quantized controls u(K-1); find the corresponding previous state X(K-1); evaluate the quantity inside the brackets in equation (4.12) using interpolation if necessary; and pick the optimal control and minimum cost.

4.3 DATA AND THE PROGRAM

The load duration curve assumed for various months in the planning horizon of 12 months is shown in Fig. 4.2. The reservoir characteristics such as volume-area-elevation relationship and evaporation data are used in the tabulated form given in Chapter two. The other parameters used in the sample problem solved are:

$\overline{V}$ = 8.598 million acre ft

$\underline{V}$ = 1.294 million acre ft

V(1) = 8.598 m.a.f.

V(13) = 8.598 m.a.f.

GC(1) = GC(2) = ... = GC(12) = 250 MW

No. of days in the month = 30.5 days

Efficiency of conversion = 0.85

The fuel cost $C(P) = a_1P + a_2P^2$

where C is cost in Rs/hour for generation of P megawatts, a_1 = 3.75 and a_2 = 0.075. This equation fairly represents the cost of thermal power generation in the country. The duration curve is approximated by 3 point representation and the values of a,b,c as defined in Fig.4.2 are assumed to be same in all the months. This is only for simplicity sake. The program is prepared to solve the problem for any values of a, b and c.

The Computer Program:

The algorithm to solve the problem of minimizing hydrocost by forward dynamic programming algorithm is programmed on IBM 1130 and constitute a main line program and the two subroutines. The subroutine called F takes in co-ordinate points a,b,c of the load duration curve, maximum demand and the upper bound on hydrogeneration from the main program. It computes the total area under the duration curve, i.e., the total system energy demand, and for a set megawatt hydropower generation computes the cost of thermal power for balance generation and the hydropower energy and returns it to the main program. The subroutine STATE, takes in the initial conditions of the reservoir, reservoir characteristics, evaporation constants and inflow into the reservoir and computes the final conditions of reservoir for a set energy generation. The conditions of the reservoir returned to the main line are the reservoir volume and its corresponding elevation and area. The evaporation in any month is calculated on the basis of average reservoir area during the month which is computed as the average of initial and final area of reservoir in any month. The final area is computed here by iterative process.

The main line programme is the basic dynamic programming algorithm, the computational procedure of which is already discussed earlier. All input and outputs are through the main program.

4.4 RESULTS OF THE SAMPLE PROBLEM

The results obtained for the sample problem is given in Table 4.1 and monthly generation schedule for hydropower and the corresponding reservoir contents are shown in Fig. 4.3. The program prepared is quite general and can take any load duration curve presented in three point representation, and the tabulated data of the reservoir characteristics and inflow forecasts. As the demand on the core memory was high, the problem was solved first for coarse discretized grid and was finally reduced to 1 MW level gradually only in the region of optimal solution.

4.5 CONCLUSIONS

1. The problem of long term scheduling of hydropower generation in co-ordination with thermal generation for an existing reservoir is solved via dynamic programming forward algorithm with deterministic formulation. The computer program prepared for IBM 1130, can take normalized monthly load duration curve represented by three co-ordinate points and forecasted inflow and gives the hydropower generation schedule for a planning horizon of 12 months.

2. The results obtained from the program serve as an operating guide to the reservoir manager, the data which he could not otherwise easily obtain.

TABLE 4.1 Results of the sample problem

Month	Hydropower schedule Mwhx 1000	System energy demand Mwhx 1000	Thermal generation cost Rupees	Av. cost Rs/Mwh	End of month Res. volume	End of month Res. elevation
October	54.9	217.404	33,16,105	15.25	8.37	878.11
November	77.592	241.560	33,70,320	14.43	8.01	874.85
December	97.356	265.716	35,50,280	13.37	7.54	870.66
January	114.924	289.878	38,49,364	13.27	6.98	865.27
February	114.924	289.878	38,49,364	13.27	6.39	859.32
March	97.356	265.716	35,50,200	13.37	5.86	853.91
April	98.488	265.716	35,23,000	13.26	5.31	848.02
May	77.592	241.560	33,70,320	14.43	5.35	848.45
June	114.924	289.878	38,49,364	13.27	6.44	859.62
July	54.9	217.404	33,16,105	15.25	7.93	874.16
August	35.136	193.248	31,62,554	16.36	8.40	878.37
September	35.136	193.248	31,62,554	16.36	8.59	880.00

3. The economic data obtained through the program can form the basis for actual operation, pricing of energy, and for entering into agreement for co-ordinated operation of the reservoir with thermal station.

4. The uncertainty factor involved in the stream flow forecast can be taken care of through moving horizon policy wherein the problem is solved every month with new stream flow forecast for next 12 months and a planning horizon of 12 months and the hydropower generation is adjusted accordingly, or a fixed horizon policy wherein the problem is solved once in a year and the changes necessary in different months of the year are established via adaptive control discussed in Appendix A1.

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CHAPTER V

STREAM FLOW FORECASTING

5.1 INTRODUCTION

The deterministic procedures discussed in Chapters two, three and four, takes care of uncertainties in the stream flow through forecasting. The main objective of the work presented here is the development of a simple model to forecast stream flows on a short term basis say a year ahead. Such models are needed for forecasting stream flows one or several months ahead based on the available flow information prior to a given month. These forecasts form the basis for determining the policies for optimal operation of reservoirs determined by deterministic optimization techniques. The models reviewed in Chapter one are not very useful in stream flow forecasting for operation planning of existing reservoirs. These models aim at maintaining the statistical properties of the historical sequence, but their sequence of occurrence is no concern. It is in this context forecasting and synthesis is differentiated. The second element that is always present in forecasting situations is uncertainty and the third element is the reliance of a forecast on information that is contained in historical data.

The forecasting of situations that are extremely stable over time is a different proposition from the forecasting of situations that are in a state of flux. In the latter case what is needed is a method that can be adapted continually to the most recent results and the latest information.

5.1.1 Characteristics of the Monthly Stream Flow Data

The distinguishing feature of the monthly river flow sequence is not stationary, the subsequence of flows in any particular month is weakly stationary. However the means and standard deviations of flows vary from month to month. If we denote the flow in the ith month by X(I), then the equation for X(I), may consists of (i) immediate lagged value like X(I-1), X(I-2), (ii) lagged values like X(I-12), X(I-24), (iii) deterministic sinusoidal trend terms with suitable harmonics, (iv) the disturbance with its lagged values. The appropriate number of type (i) to (iii) can be chosen by trial and error until the model yields satisfactory forecasts. The attempts to obtain stochastic difference equation to represent the available stream flow information at Rihand dam site was not quite successful. Further the power of the statistical tests designed to test for the presence of various terms in the model such as the sinusoidal terms is rather weak. This points to the need for a simple approach used in this work.

The problem of stream flow modelling is approached via time series analysis. This is because the information given by the observations of a few rain gauges on several square miles is very small for obtaining the rainfall run-off relation. The net rainfall over a water shed, which is not directly observable, is strongly causally related to the flow. The estimates of the net rainfall contribution derived from the observation of a net work of rain gauges in a water shed by techniques such as isohyetal and Theissen polygon methods are not reliable, primarily because of the low density of rain gauges and the non-random spatial distribution of the rain gauges. Finally, the model which involves such input variables is not useful for generating 'synthetic' trace of stream flow by simulation since there is no reliable model available for simulating the rainfall process over large water sheds.

The monthly stream flow data exhibit seasonal pattern because of its dependence on the weather. In the semi-arid regions, the streams which are not fed by snow, discharges are at low rate in mid summer and at a high rate by July-August i.e. during monsoons. Therefore, the monthly stream flow may be considered as a composite figure of trend, seasonal and cyclical components. These components may be considered additive, in which case, as is shown in the appendix, it leads to models of state space representation. This opens up a vast field of state space mathematics to the field of Hydrology. Alternatively as is done here it may be considered as factors of a multiplicative process.

5.2 FORECASTING MODEL

The forecasting procedure adopted here is based on seasonal and seasonally adjusted components of the monthly stream flow figures. The seasonal factor relates to the annual fluctuation in the basic underlying pattern. This component is one that repeats every twelve month. The trend-cyclical factor simply amounts to the long run linear projection or a sinusoidal behaviour over some long period of time. Depending on the actual data and the variable being forecast, the decision maker may have no reason to believe a cyclical pattern exists and thus basically the data may be factored to seasonal factor and trend factor. The residual factor may be considered random component and has to be analyzed separately.

So the mathematical form used to represent this decomposition is

$$S = T \times C \times U$$

where T is the trend-cyclic factor

C is the seasonal index

U is the randomness factor

In applying the decomposition technique the seasonal factor present is first sorted out. The basic data used is the monthly stream flow data for a period of 15 years.

Ratio to moving average method is used for separating monthly stream flow data into seasonal and seasonally adjusted components. A linear trend curve is fitted to the seasonally adjusted component and the residual is modelled by Morkov process. The essential structure of forecasting scheme proposed is shown in Fig. 5.1.

5.2.1 Separation of Seasonal Component

The seasonal component is comprised of one cycle per year frequency and few of its low frequency harmonics. Removing the seasonal component from the data requires band-pass filtering those frequency components. A linear-band pass filter is easy to analyze and can be implemented on a computer in the form of a weighted, moving average smoothing of the flow data. On the other hand a nonlinear operation can be used effectively in splitting the monthly flowdata into a seasonal component and a seasonally adjusted component. Most linear schemes incorporate a moving average for seasonal adjustment of data, but this results in a smooth seasonally-adjusted component and a noisy seasonal component. To make this component noise free it is proposed to use ratios of original data and an appropriate moving average as a starting point.

The ratio to moving average method obtains an estimate of the trend by using simple moving average which combines 180 successive months.

The original data is then divided by the moving averages, which gives a series of seasonal-noise ratios. The extreme values of seasonal noise ratios which might bias the results, are identified and are eliminated for computation of seasonal indices. An estimate of seasonal adjustment factors designated as seasonal indices are then obtained by averaging the seasonal noise ratios month by month, and assuming that the noise factor will be cancelled out in the process. Finally the flow data is seasonally adjusted by dividing by these seasonal indices.

The seasonal adjustment procedure can be expressed mathematically as follows:

Let $X(t)$ be the flow at month t

$$\text{then } Y(t) = \frac{1}{12} \sum_{i=-5}^{6} X(t+i) \qquad (5.1)$$

$$\text{and } X_1(t) = \frac{X(t)}{Y(t)} = \text{seasonal noise ratios} \qquad (5.2)$$

The identification of randomness factor is straight forward. Starting with the moving average, it is called that since it is a twelve month average, the seasonal fluctuations have been eliminated. Therefore what is represented by the moving average is

$$\text{Moving average} = \text{trend} \times \text{random component}$$

$$\text{Therefore random component} = \frac{\text{moving average}}{\text{trend}}$$

5.2.2 Treatment of the Seasonally Adjusted Component

Time polynomial regression technique is used to fit the moving averages, which has shown that a simple linear fit is quite sufficient. The moving average values are divided by the trend component to obtain component designated as the randomness factor. The randomness factor for a month is highly correlated with the previous month and successive month factors and its serial correlation is also very high. The static models such as regression models failed to satisfactorily forecast this particular factor, and a glance at these factor clearly shows that it is evolutionary in character and hence points to the use of dynamic models such as Markov chains.

5.2.3 Forecasting Random Component with the aid of Homogeneous Markov Chains

The state of the residual factor during the next month depend on the factor in the current month, and does not depend (or hardly depends) on how the system has achieved it. Hence the random process of variation of any such factor can be regarded as Markovian.

The random component show some persistence in its values indicating a considerable lag (memory). Therefore it is justified to assume that the laws, constraints, and relationships characteristic for the past and present will retain their validity also for some time in future. Therefore the Markov process of variation of the random factor of the seasonal and trend adjusted monthly flow values can be regarded as homogeneous over a time T+L, where T is the initial period and L the predicted period.

The process of evaluation of these factors denoted by U(t) is fixed at discrete instants of time. Thus we have a time series $u(t_l)$, $l = 1,\ldots,n$ which can be regarded as a single realization with discrete time and continuous states.

The process of variation of random component of monthly flow has a finite state space. It is convenient to transform this space from continuous to discrete. For this purpose the scale of values of the indicators $u(t_l)$, $l = 1,\ldots,n$ will be partitioned into a finite number K of intervals S_k. The value of K will be selected on the basis of initial information: the length of time series, the range of values of $u(t_l)$, and the required prediction accuracy. The limits of intervals are

$$\begin{aligned} a_1 &= \underline{u} \\ a_2 &= \underline{u} + h \\ a_3 &= \underline{u} + 2h = a_2 + h \\ &- - - - - - - \\ a_{K+1} &= \bar{u} \end{aligned} \tag{5.3}$$

where $\underline{u} = \min u(t_l)$ and $\bar{u} = \max u(t_l)$ in the period T

Here $h = \dfrac{\bar{u} - \underline{u}}{K}$

thus K discrete states of the investigated process:

$S_1 : (a_1, a_2)$,

$S_2 : (a_2, a_3)$,

$\vdots$

$S_k : (a_k, a_{k+1})$

The process will be in state S_i if its value at t_m lies in the interval $S_i : u(t_m) = i$ for $u(t_m) \in S_i$. Here it is assumed that the available statistics are sufficiently complete that we can consider as infinitely small the probability of transition of the process $u(t)$, for $t > T$, into state that does not belong to the set S_i, $i = 1, 2, \ldots, K$. In this case the variation of $u(t)$ can be regarded as a homogeneous finite Markov chain that is discrete in space and time. Such a chain is specified by a probability matrix of one step transitions p_{ij} $(i, j = 1, \ldots, k)$:

$$P = p_{ij} = \begin{array}{c|cccc|} & S_1 & S_2 & \cdots & S_k \\ \hline S_1 & p_{11} & p_{12} & \cdots & p_{1k} \\ S_2 & p_{21} & p_{22} & \cdots & p_{2k} \\ --- & - & - & - & - \\ S_k & p_{k1} & p_{k2} & \cdots & p_{kk} \end{array}$$

Here $p_{ij} = n_{ij}/n_i$, where n_{ij} is the number of transitions from state S_i to state S_j during one step, and n_i is the number of occurrences in S_i during the initial period T. This is a stochastic matrix, i.e.,

$$0 \leq p_{ij} \leq 1, \qquad \sum p_{ij} = 1.$$

For the transition probabilities from state S_i to S_j during N steps

$$p_{ij}(N) = \sum_{r=1}^{k} p_{ir}(m)\, p_{rj}(N-m), \qquad 1 \leq m \leq N$$

The transition probabilities of the process during 2,3,....,N constant steps are

$$\begin{aligned} P(2) &= P(1)\,P(1) = P^2, \\ P(3) &= P(1)\,P(2) = P^3, \\ &\text{- - - - - - - - - -} \\ P(N) &= P(m)\,P(N-m) = P^N \end{aligned} \qquad (5.5)$$

Hence by knowing the elements of the matrix P^N, and the initial state of the process $u(t)$, it is possible to find its transition probabilities during the desired number of steps, and the various numerical characteristics of the investigated indicator. For example, the mean of $u(t_1)$ during N steps is

$$\tilde{u}(t_1) = \sum_{j=1}^{k} p_{ij}(N)\, \bar{a}_j \qquad (5.6)$$

where $\bar{a}_j = \frac{a_{j+1} - a_j}{2} \quad (j = \overline{1,k-1})$

If the time series under investigation has a clear trend to increase, then for $t > T$ the values of $u(t_1)$, $(t_{n+1}, t_{n+2}, \ldots)$ will go outside the limits $\underline{u}$, $\overline{u}$ obtained on the basis of the original statistics $(1 = 1,\ldots,n)$ i.e., the state space of the process $u(t)$ will no longer be finite. Such processes cannot be predicted by the above algorithm. In such cases the time series considered will be of rate of increase.

5.3 DATA ANALYSIS AND THE RESULTS

The monthly stream flow data presented in Table 2.4 is converted to monthly average discharge figures and is presented in Table 5.1. Table 5.2 shows the ratios of actual stream flow figures to the moving average. The final row in the Table gives the adjusted seasonal index. Table 5.3 gives the table of randomness factor. The seasonal factors and the randomness factor are presented in Fig. 5.2 and Fig. 5.3 respectively. It is observed that the randomness factor is very stable in various months in any year excepting for the monsoon months of July, August and September. The circle diagram shows the evolutionary process in its generation. The first order Markov transition matrix is given in Table 5.4. The range of randomness factor is 0.4 to 1.54 and is divided into 10 states. The transition probabilities of the process during 2,3,....,12 steps are shown in Table 5.5 to 5.15. The regression constants for the trend components are found to be $a = 7617.16$ and $b = -9.73$ where the trend equation is of the form $T = a+bt$.

TABLE 5.1 Monthly average inflow to reservoir (cusecs)

Year	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	March	April	May
1945	13518	19243	18303	18396	5679	1898	538	379	240	105	128	92
1946	8493	29392	30977	12306	6876	1259	300	168	253	127	92	54
1947	131	9990	20681	26885	4973	374	226	1179	1161	368	171	111
1948	5417	16074	22683	24806	5636	3948	1331	1011	1093	298	138	120
1949	1852	12263	45385	14172	6164	4382	1668	959	832	993	332	175
1950	1880	26966	75037	17367	6241	3995	3609	3406	2704	2958	4182	2273
1951	4804	12237	44431	24982	6187	3338	1463	840	883	455	193	176
1952	6532	20737	32418	19753	2974	1250	946	930	520	271	167	113
1953	540	21181	14938	14823	3395	1076	684	495	516	231	106	92
1954	1156	5593	13034	8461	1201	311	183	774	345	107	88	91
1955	1784	15239	13362	9762	3443	680	341	209	207	101	77	701
1956	9296	18956	38155	19789	7689	3412	1234	1419	902	557	283	97
1957	1465	27751	15604	7744	797	321	225	191	1099	837	176	126
1958	972	27326	22109	18972	11650	1881	913	1481	1004	297	162	77
1959	945	15776	37113	20331	12801	1905	1017	879	388	329	227	130

TABLE 5.2 Seasonal Index Factors

	June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	March	April	May
							8.2011	6.1668	3.4264	2.0544	1.7012	1.2067
	112.2527	389.2503	411.2082	163.3341	91.3026	16.7111	3.9868	2.4606	4.3363	2.8948	1.6344	0.9798
	2.4366	187.4677	382.0499	489.8126	90.2704	6.7625	4.0757	19.7788	17.9349	5.5466	2.6465	1.6880
	79.4795	232.7256	329.0902	360.1989	81.8941	57.3851	19.3346	15.3454	17.4482	3.6414	1.8982	1.6269
	25.1825	166.1990	615.4524	192.7529	83.1611	59.0055	22.4465	12.9013	9.5975	8.9133	2.9046	1.5260
	16.5263	233.8394	639.4023	146.0465	51.7620	32.2802	28.7469	26.6204	23.3677	32.7787	43.3013	23.5515
	50.0607	130.4687	482.7532	275.9772	69.9629	39.2088	17.5503	9.8938	9.6034	5.5480	2.4782	2.3397
	89.4110	285.5385	445.9111	272.8471	41.1525	17.3076	13.0939	13.8143	7.6836	5.1152	3.4171	2.2957
	10.9828	433.4908	307.9952	305.6603	70.0347	22.2264	14.1341	10.1008	14.3662	6.7286	3.6516	3.3822
	43.4809	213.9155	494.0797	322.4699	45.9595	11.9081	7.0074	29.0183	9.9085	3.0597	2.4393	2.3719
	46.6123	398.3065	352.4244	258.2710	91.1027	17.9977	8.8792	4.6686	4.3479	1.4794	0.9918	8.7446
	112.7619	227.8799	453.1876	233.4270	90.2868	39.9773	14.5485	18.1115	10.5347	8.3198	4.9831	1.8806
	30.2032	582.6369	334.7744	165.5489	16.9340	6.8207	4.7719	4.1048	23.7998	16.2217	2.8710	1.8000
	13.6326	380.1966	303.0672	260.3462	160.3708	25.9644	12.6165	20.4582	16.0085	3.9483	2.1086	0.9830
	12. 2199	203.7631	482.5039	266.0849	167.4847	24.8934						
*	44.1704	279.3811	424.2859	256.4099	80.6483	26.0567	12.2217	13.4971	12.0596	5.9992	2.7278	2.4871
**	45.6956	289.0285	438.9371	265.2641	83.4333	26.9565	12.6437	13.9631	12.4761	6.2063	2.8220	2.5730

*Medial average

**Seasonal index

TABLE 5.3 Table of Randomness Component

June	July	Aug.	Sep.	Oct.	Nov.	Dec.	Jan.	Feb.	March	April	May
0.8673	0.8129	0.9263	1.0679	1.0017	1.0163	1.0105	1.0092	1.0082	1.0096	1.0105	1.0114
1.0123	0.9197	0.7028	0.5879	0.7529	0.7325	0.7234	0.7235	0.7360	0.7472	0.7510	0.7529
0.7545	0.8158	0.8864	0.9105	0.8879	0.8967	0.9389	0.9529	0.9522	0.9527	0.9532	0.9541
0.9555	0.9154	0.8725	1.1375	1.0153	1.0229	1.0293	1.0347	1.0355	1.0359	1.0434	1.0471
1.0492	1.0510	1.2259	1.5779	1.6179	1.6211	1.6187	1.6440	1.6754	1.7001	1.7259	1.7743
1.8019	1.8395	1.6659	1.3006	1.3940	1.3953	1.3894	1.3654	1.3363	1.3161	1.2876	1.2409
1.2172	1.2400	1.3448	1.2000	1.1377	1.0999	1.0759	1.0711	1.0737	1.0706	1.0701	1.0713
1.0721	0.9993	1.0063	0.7904	0.7302	0.7365	0.7354	0.7332	0.7288	0.7299	0.7304	0.7307
0.7316	0.7404	0.5446	0.5213	0.4414	0.4142	0.4051	0.3993	0.4035	0.4019	0.4009	0.4013
0.4019	0.4105	0.5359	0.5409	0.5585	0.5883	0.5940	0.5969	0.5905	0.5896	0.5904	0.5912
0.6000	0.6992	0.7484	1.0748	1.2082	1.2658	1.3039	1.3177	1.3357	1.3470	1.3551	1.3600
1.3540	1.2518	1.3711	1.0718	0.9122	0.8213	0.7810	0.7687	0.7534	0.7572	0.7622	0.7620
0.7636	0.7581	0.7535	0.8433	0.9979	1.1478	1.1710	1.1823	1.2019	1.2025	1.1970	1.1987
1.2000	1.2015	1.0433	1.2534	1.2743	1.2925	1.2949	1.2985	1.2922	1.2856	1.2882	1.2913

TABLE 5.4 Stochastic Matrix

0.9166	0.0834	0	0	0	0	0	0	0	0
0	0.75	0.25	0	0	0	0	0	0	0
0.027	0.081	0.811	0.054	0.054	0	0	0	0	0
0	0	0.1250	0.75	0.0625	0	0.0625	0	0	0
0	0	0.0286	0.0571	0.8	0.1143	0	0	0	0
0	0	0	0	0.105	0.685	0.21	0	0	0
0	0	0	0	0.0833	0.125	0.7917	0	0	0
0	0	0	0	0	0	0	0	1.0	0
0	0	0	0	0	0	0.143	0	0.714	0.143
0	0	0	0	0	0	0	0	0.2	0.8

TABLE 5.5 Transition probability in second step

0.8401	0.1389	0.0208	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
0.0067	0.5760	0.3902	0.0135	0.0135	0.0000	0.0000	0.0000	0.0000	0.0000
0.0466	0.0865	0.6795	0.0873	0.0903	0.0061	0.0033	0.0000	0.0000	0.0000
0.0033	0.0067	0.1969	0.5728	0.1088	0.0149	0.0963	0.0000	0.0000	0.0000
0.0007	0.0015	0.0532	0.0900	0.6571	0.1697	0.0275	0.0000	0.0000	0.0000
0.0000	0.0000	0.0030	0.0059	0.1734	0.5074	0.3101	0.0000	0.0000	0.0000
0.0000	0.0000	0.0023	0.0047	0.1457	0.1941	0.6530	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1430	0.0000	0.7140	0.1430
0.0000	0.0000	0.0000	0.0000	0.0119	0.0178	0.2153	0.0000	0.5383	0.2165
0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0285	0.0000	0.3028	0.6685

TABLE 5.6 Transition probability in third step

0.7706	0.1754	0.0516	0.0011	0.0011	0.0000	0.0000	0.0000	0.0000	0.0000
0.0167	0.4536	0.4625	0.0319	0.0327	0.0015	0.0008	0.0000	0.0000	0.0000
0.0611	0.1054	0.5662	0.1073	0.1153	0.0149	0.0094	0.0000	0.0000	0.0000
0.0084	0.0159	0.2360	0.4464	0.1430	0.0347	0.1152	0.0000	0.0000	0.0000
0.0021	0.0040	0.0735	0.1079	0.5543	0.1948	0.0631	0.0000	0.0000	0.0000
0.0000	0.0001	0.0081	0.0145	0.2183	0.4062	0.3524	0.0000	0.0000	0.0000
0.0000	0.0001	0.0066	0.0120	0.1917	0.2312	0.5580	0.0000	0.0000	0.0000
0.0000	0.0000	0.0000	0.0000	0.0119	0.0178	0.2153	0.0000	0.5383	0.2165
0.0000	0.0000	0.0003	0.0006	0.0293	0.0405	0.2512	0.0000	0.4277	0.2501
0.0000	0.0000	0.0000	0.0000	0.0023	0.0035	0.0659	0.0000	0.3499	0.5781

TABLE 5.7 Transition probability in fourth step

0.7077	0.1986	0.0859	0.0036	0.0037	0.0001	0.0000	0.0000	0.0000	0.0000
0.0278	0.3666	0.4934	0.0508	0.0533	0.0049	0.0029	0.0000	0.0000	0.0000
0.0718	0.1158	0.5185	0.1187	0.1330	0.0246	0.0173	0.0000	0.0000	0.0000
0.0140	0.0254	0.2553	0.3557	0.1683	0.0545	0.1264	0.0000	0.0000	0.0000
0.0039	0.0072	0.0900	0.1165	0.4798	0.2046	0.0976	0.0000	0.0000	0.0000
0.0002	0.0005	0.0147	0.0232	0.2480	0.3472	0.3652	0.0000	0.0000	0.0000
0.0002	0.0004	0.0124	0.0203	0.2253	0.2500	0.4911	0.0000	0.0000	0.0000
0.0000	0.0000	0.0003	0.0006	0.0293	0.0405	0.2512	0.0000	0.4277	0.2501
0.0000	0.0000	0.0012	0.0022	0.0487	0.0625	0.2685	0.0000	0.3554	0.2613
0.0000	0.0000	0.0000	0.0001	0.0077	0.0109	0.1029	0.0000	0.3654	0.5125

TABLE 5.8 Transition probability in fifth step

0.6510	0.2126	0.1199	0.0076	0.0078	0.0005	0.0003	0.0000	0.0000	0.0000
0.0388	0.3039	0.4997	0.0676	0.0732	0.0098	0.0065	0.0000	0.0000	0.0000
0.0798	0.1206	0.4681	0.1246	0.1458	0.0342	0.0263	0.0000	0.0000	0.0000
0.0198	0.0340	0.2627	0.2902	0.1869	0.0724	0.1337	0.0000	0.0000	0.0000
0.0060	0.0106	0.1031	0.1196	0.4256	0.2072	0.1275	0.0000	0.0000	0.0000
0.0006	0.0012	0.0221	0.0328	0.2676	0.3118	0.3635	0.0000	0.0000	0.0000
0.0005	0.0010	0.0191	0.0287	0.2493	0.2584	0.4426	0.0000	0.0000	0.0000
0.0000	0.0000	0.0012	0.0022	0.0487	0.0625	0.2685	0.0000	0.3554	0.2613
0.0000	0.0000	0.0026	0.0044	0.0681	0.0819	0.2767	0.0000	0.3060	0.2598
0.0000	0.0000	0.0002	0.0005	0.0159	0.0212	0.1361	0.0000	0.3634	0.4623

TABLE 5.9 Transition probability in sixth step

0.6000	0.2202	0.1515	0.0126	0.0133	0.0013	0.0008	0.0000	0.0000	0.0000
0.0490	0.2581	0.4918	0.0620	0.0914	0.0159	0.0115	0.0000	0.0000	0.0000
0.0858	0.1226	0.4296	0.1271	0.1555	0.0434	0.0358	0.0000	0.0000	0.0000
0.0252	0.0413	0.2632	0.2425	0.2006	0.0276	0.1392	0.0000	0.0000	0.0000
0.0083	0.0140	0.1134	0.1196	0.3859	0.2065	0.1519	0.0000	0.0000	0.0000
0.0012	0.0021	0.0300	0.0411	0.2803	0.2896	0.3554	0.0000	0.0000	0.0000
0.0010	0.0018	0.0265	0.0368	0.2663	0.2608	0.4064	0.0000	0.0000	0.0000
0.0000	0.0000	0.0026	0.0044	0.0681	0.0519	0.2767	0.0000	0.3060	0.2598
0.0001	0.0002	0.0046	0.0074	0.0865	0.0985	0.2803	0.0000	0.2704	0.2516
0.0000	0.0000	0.0007	0.0013	0.0263	0.0334	0.1642	0.0000	0.3519	0.4218

TABLE 5.10 Transition probability in seventh step

0.5540	0.2234	0.1799	0.0184	0.0198	0.0025	0.0017	0.0000	0.0000	0.0000
0.0582	0.2242	0.4762	0.0933	0.1074	0.0228	0.0175	0.0000	0.0000	0.0000
0.0902	0.1223	0.3994	0.1274	0.1631	0.0519	0.0454	0.0000	0.0000	0.0000
0.0302	0.0473	0.2598	0.2075	0.2107	0.1004	0.1438	0.0000	0.0000	0.0000
0.0107	0.0173	0.1214	0.1178	0.3567	0.2046	0.1711	0.0000	0.0000	0.0000
0.0019	0.0033	0.0380	0.0484	0.2885	0.2749	0.3447	0.0000	0.0000	0.0000
0.0016	0.0029	0.0342	0.0442	0.2780	0.2599	0.3789	0.0000	0.0000	0.0000
0.0001	0.0002	0.0046	0.0074	0.0865	0.0985	0.2803	0.0000	0.2704	0.2516
0.0002	0.0004	0.0072	0.0107	0.1036	0.1124	0.2817	0.0000	0.2434	0.2400
0.0000	0.0000	0.0015	0.0025	0.0384	0.0464	0.1874	0.0000	0.3356	0.3878

TABLE 5.11 Transition probability in eighth step

0.5127	0.2234	0.2046	0.0246	0.0271	0.0042	0.0030	0.0000	0.0000	0.0000
0.0662	0.1987	0.4570	0.1018	0.1213	0.0301	0.0245	0.0000	0.0000	0.0000
0.0935	0.1208	0.3751	0.1264	0.1692	0.0599	0.0548	0.0000	0.0000	0.0000
0.0347	0.0520	0.2545	0.1817	0.2180	0.1108	0.1479	0.0000	0.0000	0.0000
0.0130	0.0204	0.1278	0.1153	0.3350	0.2023	0.1858	0.0000	0.0000	0.0000
0.0027	0.0047	0.0460	0.0548	0.2934	0.2643	0.3337	0.0000	0.0000	0.0000
0.0024	0.0041	0.0419	0.0509	0.2859	0.2572	0.3573	0.0000	0.0000	0.0000
0.0002	0.0004	0.0072	0.0107	0.1036	0.1124	0.2817	0.0000	0.2434	0.2400
0.0004	0.0007	0.0102	0.0143	0.1192	0.1240	0.2821	0.0000	0.2218	0.2268
0.0000	0.0001	0.0026	0.0041	0.0514	0.0596	0.2063	0.0000	0.3172	0.3582

TABLE 5.12 Transition probability in ninth step

0.4754	0.2214	0.2257	0.0311	0.0350	0.0063	0.0048	0.0000	0.0000	0.0000
0.0730	0.1792	0.4365	0.1079	0.1333	0.0375	0.0321	0.0000	0.0000	0.0000
0.0958	0.1186	0.3550	0.1247	0.1744	0.0672	0.0639	0.0000	0.0000	0.0000
0.0387	0.0556	0.2484	0.1625	0.2235	0.1193	0.1517	0.0000	0.0000	0.0000
0.0154	0.0233	0.1327	0.1125	0.3188	0.2001	0.1968	0.0000	0.0000	0.0000
0.0037	0.0062	0.0537	0.0603	0.2962	0.2563	0.3231	0.0000	0.0000	0.0000
0.0033	0.0056	0.0496	0.0567	0.2909	0.2535	0.3401	0.0000	0.0000	0.0000
0.0004	0.0007	0.0102	0.0143	0.1192	0.1240	0.2821	0.0000	0.2218	0.2268
0.0006	0.0011	0.0137	0.0181	0.1334	0.1339	0.2820	0.0000	0.2037	0.2131
0.0001	0.0002	0.0042	0.0062	0.0650	0.0725	0.2214	0.0000	0.2981	0.3319

TABLE 5.13 Transition probability in tenth step

0.4419	0.2179	0.2433	0.0375	0.0432	0.0089	0.0071	0.0000	0.0000	0.0000
0.0787	0.1641	0.4162	0.1121	0.1436	0.0449	0.0400	0.0000	0.0000	0.0000
0.0974	0.1161	0.3382	0.1227	0.1789	0.0740	0.0725	0.0000	0.0000	0.0000
0.0421	0.0584	0.2421	0.1480	0.2275	0.1262	0.1553	0.0000	0.0000	0.0000
0.0177	0.0259	0.1367	0.1097	0.3067	0.1981	0.2049	0.0000	0.0000	0.0000
0.0049	0.0079	0.0611	0.0651	0.2975	0.2498	0.3134	0.0000	0.0000	0.0000
0.0044	0.0071	0.0570	0.0618	0.2939	0.2494	0.3260	0.0000	0.0000	0.0000
0.0006	0.0011	0.0137	0.0181	0.1334	0.1339	0.2820	0.0000	0.2037	0.2131
0.0009	0.0016	0.0175	0.0219	0.1461	0.1422	0.2817	0.0000	0.1681	0.1996
0.0002	0.0004	0.0061	0.0086	0.0787	0.847	0.2336	0.0000	0.2792	0.3082

TABLE 5.14 Transition probability in eleventh step

0.4116	0.2134	0.2577	0.0437	0.0516	0.0119	0.0098	0.0000	0.0000	0.0000
0.0834	0.1521	0.3967	0.1148	0.1524	0.0522	0.0481	0.0000	0.0000	0.0000
0.0984	0.1135	0.3237	0.1205	0.1828	0.0802	0.0806	0.0000	0.0000	0.0000
0.0452	0.0604	0.2359	0.1371	0.2305	0.1319	0.1587	0.0000	0.0000	0.0000
0.0199	0.0283	0.1398	0.1072	0.2975	0.1964	0.2107	0.0000	0.0000	0.0000
0.0061	0.0096	0.0682	0.0691	0.2977	0.2443	0.3047	0.0000	0.0000	0.0000
0.0056	0.0088	0.0642	0.0662	0.2954	0.2452	0.3143	0.0000	0.0000	0.0000
0.0009	0.0016	0.0175	0.0219	0.1461	0.1422	0.2817	0.0000	0.1881	0.1996
0.0013	0.0022	0.0215	0.0257	0.1576	0.1493	0.2811	0.0000	0.1742	0.1866
0.0003	0.0006	0.0083	0.0112	0.0922	0.0962	0.2432	0.0000	0.2610	0.2865

TABLE 5.15 Transition probability in twelfth step

0.3842	0.2083	0.2693	0.0496	0.0600	0.0153	0.0130	0.0000	0.0000	0.0000
0.0871	0.1424	0.3784	0.1162	0.1600	0.0592	0.0562	0.0000	0.0000	0.0000
0.0989	0.1108	0.3112	0.1163	0.1854	0.0859	0.0882	0.0000	0.0000	0.0000
0.0478	0.0618	0.2302	0.1287	0.2328	0.1365	0.1619	0.0000	0.0000	0.0000
0.0220	0.0304	0.1424	0.1049	0.2904	0.1948	0.2147	0.0000	0.0000	0.0000
0.0075	0.0114	0.0749	0.0725	0.2972	0.2394	0.2965	0.0000	0.0000	0.0000
0.0068	0.0105	0.0710	0.0700	0.2959	0.2410	0.3045	0.0000	0.0000	0.0000
0.0013	0.0022	0.0215	0.0257	0.1576	0.1493	0.2811	0.0000	0.1742	0.1866
0.0018	0.0029	0.0257	0.0294	0.1679	0.1554	0.2805	0.0000	0.1617	0.1742
0.0005	0.0009	0.0110	0.0141	0.1052	0.1068	0.2508	0.0000	0.2436	0.2665

5.4 PREPARING A FORECAST BASED ON THE DECOMPOSITION METHOD AND ITS EVALUATION

Once the three components are determined, the relationship for forecasting is simply

$$S = T \times C \times U$$

Starting with the time period to be forecast, the seasonal factor for that period can be identified from the adjusted seasonal index, the trend factor can be determined by putting the time period in the trend equation and the randomness factor can be estimated through the Markov chain from the recent pattern in this factor. The forecast is simply S = seasonal × trend × randomness.

To determine the level of accuracy of this forecasting technique twelve years of data is used in determining the seasonal, trend, and the randomness component and the last two years data is used to test the accuracy of the forecasts based on these factors. This has been done in Table 5.16. The mean and standard deviation is also shown against each month of forecast. Since the slope of the trend line is small the forecast is based on a constant trend factor = 7617.16. On the annual basis the percentage error in the 1st year of forecast is -5.66 per cent and in the second year it is +2.65 per cent. This appears to be quite satisfactory. There is no other available method to check the accuracy of these forecasted figures.

TABLE 5.16 Forecast obtained by the decomposition method

Year	Month	Actual value cusecs	Mean	S.D.	Forecast cusecs
1958	June	972	3918	3845	2662
	July	27326	18587	6864	18490
	Aug.	22109	29614	16193	28257
	Sept.	18972	17236	5771	17359
	Oct.	11650	5713	3210	5626
	Nov.	1881	2001	1397	1850
	Dec.	913	978	846	890
1959	Jan.	1481	954	770	980
	Feb.	1004	809	605	822
	March	297	539	692	425
	April	162	434	1003	215
	May	77	294	549	135
	June	945	3918	3845	4203
	July	15776	15587	6864	14640
	Aug.	37113	29614	16193	39791
	Sept.	20331	17236	5771	24781
	Oct.	12801	5713	3210	7396
	Nov.	1905	2001	1397	2364
	Dec.	1017	978	846	905
1960	Jan.	879	954	770	1183
	Feb.	388	809	605	1005
	March	329	539	692	425
	April	227	434	1003	215
	May	130	294	549	131

Period	Forecasted av. annual flow	Actual annual av. inflow
1958 June to 1959 June	6825	7235
1959 June to 1960 June	7736	7575

5.5 CONCLUSIONS

1. A simple model based on time sequence of historical flows is presented and is applied to the monthly inflow figures available at the Rihand dam site. The forecast based on the model suggested appears to be quite satisfactory on the annual basis. Against individual months, there are some large discrepancies. A look at the inflow figures against these months in other years show a situation peculiar to the month under test only. As such these deviations may be considered as abnormal.

2. There is no other method available to test the accuracy of the forecasts. There is a need to develop such methods.

3. The appendix A2 suggests an alternative way of forecasting procedure, the mathematical sophistication of which has grown sufficiently in other branches of engineering.

4. The uncertainty factor attached with the forecasts demands some mid-course corrections on the preliminary plans of power generation based on forecasts. The appendix A1 suggests one adaptive control concept for such a situation.

Conclusions 3 and 4 mentioned above appear to be very promising and a detailed investigation on these lines on specific cases may lead to fruitful research.

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CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK

A case oriented research reported in this thesis is directed towards optimal utilization of stored water in a hydroelectric project through system analysis concepts and operations research techniques.

The simulation of the reservoir implemented on a digital computer using the synthetic traces of monthly stream flows based on the Thomas-Fiering model has shown that the firm power capability of Rihand hydroelectric project is 85 MW. Reliability of this computed figure is limited because the length of the historical stream flow data used in stream flow generation model is only of 14 years. The modified Thomas-Fiering model, in which the random component is generated corresponding to gamma distribution is found to produce significant distortion in the monthly distribution of flow compared to the historical data. The approximations outlined in Chapter 2, in computing the firm power capacity is conservative and is justified on the basis of the computer time and human effort needed in data processing. As there is no specific advantage with carry over storage, it is concluded that this reservoir should be operated on year to year basis, commencing from 1st week of October when the reservoir should be at its maximum capacity.

The strategy worked out analytically for two parallel reservoirs and similar analysis for reservoirs in series will be of use in multireservoir operations through simulation. It reduces the number of alternatives to be tried in simulation runs.

Two different strategies for planning annual operation of Rihand reservoirs are proposed. Both the strategies have economic objective, in one it is implicit whereas in the co-ordinated operation it is explicit. The strategy to be adopted depends on the organizational set-up. There is no common ground to compare these two strategies. Both aim at ensuring the assured supply of power to the consumers despite low water years. The computer programs can be used to work off line control policies through extensive experimentation and regression techniques.

The nonlinear programming formulation is preferable to hit and trial method for the type of operation proposed in Chapter 3 from computation point of view. The special structure of the computational problem in the nonlinear programming subroutine has been exploited to full advantage in as much as it avoids matrix manipulations which are time consuming, especially so when applied to multireservoir case.

The co-ordinated operation solved by dynamic programming minimizes the unit cost of power generation in the combined hydrothermal system.

In stream flow forecasting, it is essential to separate seasonal effects from monthly stream flow data before fitting a trend curve. This conclusion reached through time consuming experimentation may be helpful to other investigators. While the results of decomposition forecasting procedure appear good, they should be taken as primarily illustrative of the technique at this stage in as much as the universality of the method is not established despite its ability to produce good forecasts in the test case presented. The so-called randomness factor is unstable during the monsoon months from June to October and is slowly varying upto the commencement of next monsoon. This is an advantage because this factor is more or less determined at the starting point of operation of reservoir.

Suggestions for Future Work

The application of the nonlinear programming technique to multilake problem for single purpose is direct. For multiuse case, additional constraints are to be imposed which can be taken care of through penalty function techniques.

The application of dynamic programming to multiuse problem of a single reservoir is direct, but multireservoir case can be tackled through concept of equivalent single thermal station and an equivalent hydrostation. The apportionment of the hydropower target so arrived at may be attempted through nonlinear programming technique similar to that discussed in Chapter two.

The state space approach for stream flow forecasting, in the author's opinion, opens up a vast field in the area of stochastic hydrology and further investigations in this direction will be fruitful. The errors in forecasting and its implications need further investigations.

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APPENDIX A1

Adaptive Control Concept:

Suppose any one of the strategies of planning energy generation is used to establish power production plan over interval of time say a year. This plan gives the energy generation and the energy balance in the reservoir for each period in the interval. The plan is a preliminary one which at time zero establishes targets for power production and energy balance in the reservoir based on stream flow forecast. Since the actual inflow into the reservoir may differ from the forecast, it may be necessary to adjust the power generation plan from their originally planned levels.

Let E_t^* = Energy production planned for period t

E_t = Energy production finally adjusted for time t

X_t^* = Planned energy level in the reservoir at the end of time period t

X_t = Actual energy level at the end of period t

T = Number of periods in the planning horizon

τ = Lead time i.e. the number of periods between a decision to change the energy generation plan and the change becomes effective

I_t^* = Energy inflow forecast

I_t = Actual energy inflow

Define a correction factor $\Delta_t = E_t - E_t^*$ (A1-1)

$$X_t = X_{t-1} + I_t - E_t$$
$$X_t^* = X_{t-1}^* + I_t^* - E_t^* \qquad \text{(A1-2)}$$

A simple control rule to schedule energy generation E_{t+T+1} equal to originally planned rate E_{t+T+1}^* plus a correction equal to a fraction α of the energy level discrepancy projected at the end of period $t+T$ i.e.

$$E_{t+T+1} = E_{t+T+1}^* + \alpha \left[X_t^* - X_t - \sum_{j=1}^{T} (E_{t+j} - E_{t+j}^*) \right]$$
$$= E_{t+T+1}^* + \alpha \left[X_t^* - X_t - \sum_{j=1}^{T} \Delta_{t+j} \right] \qquad \text{(A1-3)}$$
$$0 \leq \alpha \leq 1$$

To avoid continually making minor changes in the plan an alternative plan could be used

$$E_{t+T+1} = \begin{cases} E_{t+T+1}^* & \text{if } |\Delta_{t+T+1}| \leq L \\ E_{t+T+1}^* + \Delta_{t+T+1} & \text{if } |\Delta_{t+T+1}| > L \end{cases} \qquad \text{(A1-4)}$$

where L is some selected minimum change level and

$$\Delta_{t+T+1} = \alpha \left[X_t^* - X_t - \sum_{j=1}^{T} \Delta_{t+j} \right] \qquad \text{(A1-5)}$$

When control rules as established like this, it may be possible to express the difference between the actual and planned energy level at the end of period $t+T$ as a linear combination of forecast errors for periods $1, 2, \ldots, t+T$. These aspects need further investigation.

APPENDIX A2

Component Models - Additive Form:

It is shown here that the component models in additive form may be analyzed within the state space framework found in the literature of control engineering.

The system of equations

$$Z(t) = M\, Z(t-1) + \Gamma\, W(t) \qquad (A2-1)$$

$$Y(t) = H\, Z(t) + B\, V(t) \qquad (A2-2)$$

where $Y(t)$ is a vector of outputs, $Z(t)$ is a $(q \times 1)$ vector of unobservables, referred to as a state vector, $V(t)$ and $W(t)$ are uncorrelated Gaussian process with covariance matrices R and Q respectively and

$$E(Z'(t).W(t)) = 0, \quad E(W(t).V'(t)) = 0$$

is the state space representation.

To illustrate the conversion of a component model to this form consider the decomposition of a series $Y(t)$ into trend $T(t)$, seasonal $S(t)$ and irregular $I(t)$, where

$$Y(t) = T(t) + S(t) + I(t)$$

$$T(t) = \beta_1\, T(t-1) + \beta_2\, T(t-2) + e_1(t) \qquad (A2-3)$$

$$S(t) = \beta_3\, S(t-4) + e_2(t) \qquad (A2-4)$$

$$I(t) = e_3(t) \qquad (A2-5)$$

and $E(e_j(t)) = 0$, $E(e_i(t), e_j(t)) = \sigma_i^2 \quad (i=j)$

$E(e_i(t).e_j(t)) = 0 \quad (i \neq j)$.

Equation A2-5 can be written as

$$\begin{bmatrix} T(t) \\ T(t-1) \\ S(t) \\ S(t-1) \\ S(t-2) \\ S(t-3) \end{bmatrix} = \begin{bmatrix} \beta_1 & \beta_2 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_3 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} T(t-1) \\ T(t-2) \\ S(t-1) \\ S(t-2) \\ S(t-3) \\ S(t-4) \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} e_1(t) \\ e_2(t) \end{bmatrix} \qquad \text{(A2-6)}$$

and

$$Y(t) = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T(t) \\ T(t-1) \\ S(t) \\ S(t-1) \\ S(t-2) \\ S(t-3) \end{bmatrix} + e_3(t) \qquad \text{(A2-7)}$$

Obviously equations A2-6, A2-7, have the form of A2-1, A2-2 clearly and can be encompassed within the structure.

Such a formulation has the advantage that a large volume of filtering and estimation theory may then be brought to bear upon such models and has the secondary benefits of greater mathematical lucidity, flexibility, in exposition of the feature extraction problem. By relating such models to the format that has wide spread use in other fields, it is possible to exploit any future computational and theoretical advances therein.

APPENDIX A3

Testing for the Markov Property:

A statistical test for the Markov property forms an important component of the experimenter's list of tools. The test distinguishes between the two alternative hypotheses that either the successive events are independent of each other (the null hypothesis) or the events are not independent. If not independent, they could form a first-order Markov chain. The test statistic λ is

$$\lambda = \prod_{i,j}^{m} (p_j/p_{ij})^{n_{ij}}$$

where $-2 \log e^{\lambda}$ is distributed asymptotically as χ^2 with $(m-1)^2$ degrees of freedom (Anderson and Goodmann, 1957). The expression is equivalent to the more convenient computational equation

$$-2 \log_e \lambda = 2 \sum_{i,j}^{m} n_{ij} \log_e (p_{ij}/p_j)$$

where p_{ij} = probability in cell i,j of the transition probability matrix

p_j = marginal probabilities for the ith row

$$(= \sum_{i=1}^{m} n_{ij} / \sum_{i,j}^{m} n_{ij})$$

n_{ij} = transition frequency total in cell i,j of the original tally matrix of observed transitions

m = total number of states

$(m-1)^2$ = number of degrees of freedom.

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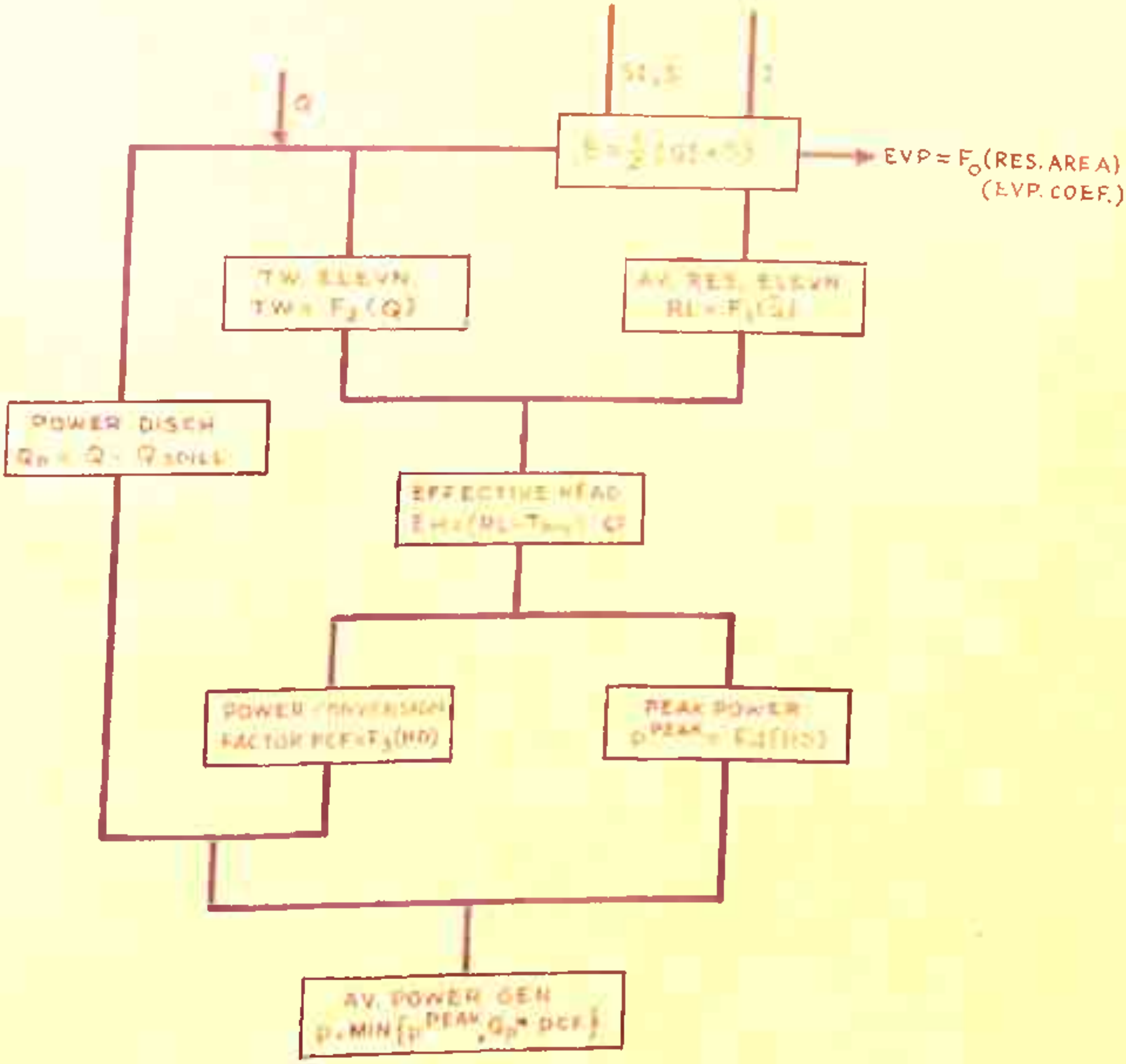
List of Computer programs prepared for this study:

1. Simulation program for a single hydroelectric scheme
2. Estimation of firm power of a hydroelectric storage
3. Study of carryover storage in power production
4. Stream flow generation by Thomas-Fiering model
5. Stream flow generation by modified Thomas-Fiering model
6. Modified Fletcher-Reeves Optimization Routine
7. Dynamic programming routine for co-ordinated operation of a hydrostation with a thermal station
8. Seasonal trend and random component analysis
9. First order Markov chain model
10. Stream flow forecasting routine
11. Least square curve fitting by orthogonal polynomials
12. Spectral analysis subroutine



Y = F(x) IMPLIES Y IS A FUNCTION OF x

SI = INITIAL STORAGE CONTENT FOR THE PARTICULAR TIME INTERVAL

S = END OF THE PERIOD STORAGE $SI_{i,j} = S_{i,j-1}$

Fig. 2.1: Model of high head Hydropower Station.

Fig. 2·2 Index Map of Rihand Hydel Project

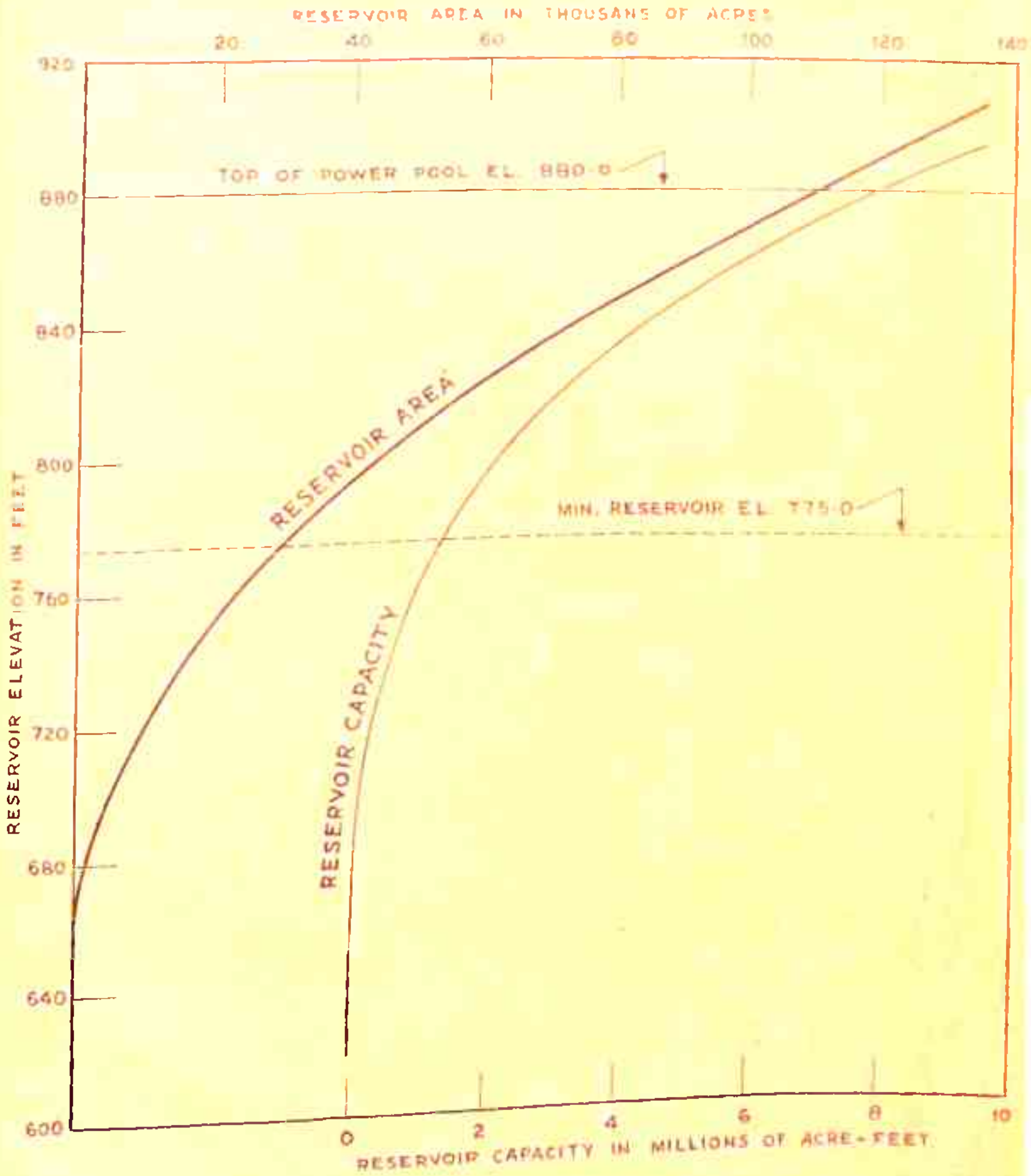


Fig. 2 3 Rihand Dam Area & Capacity Curves.

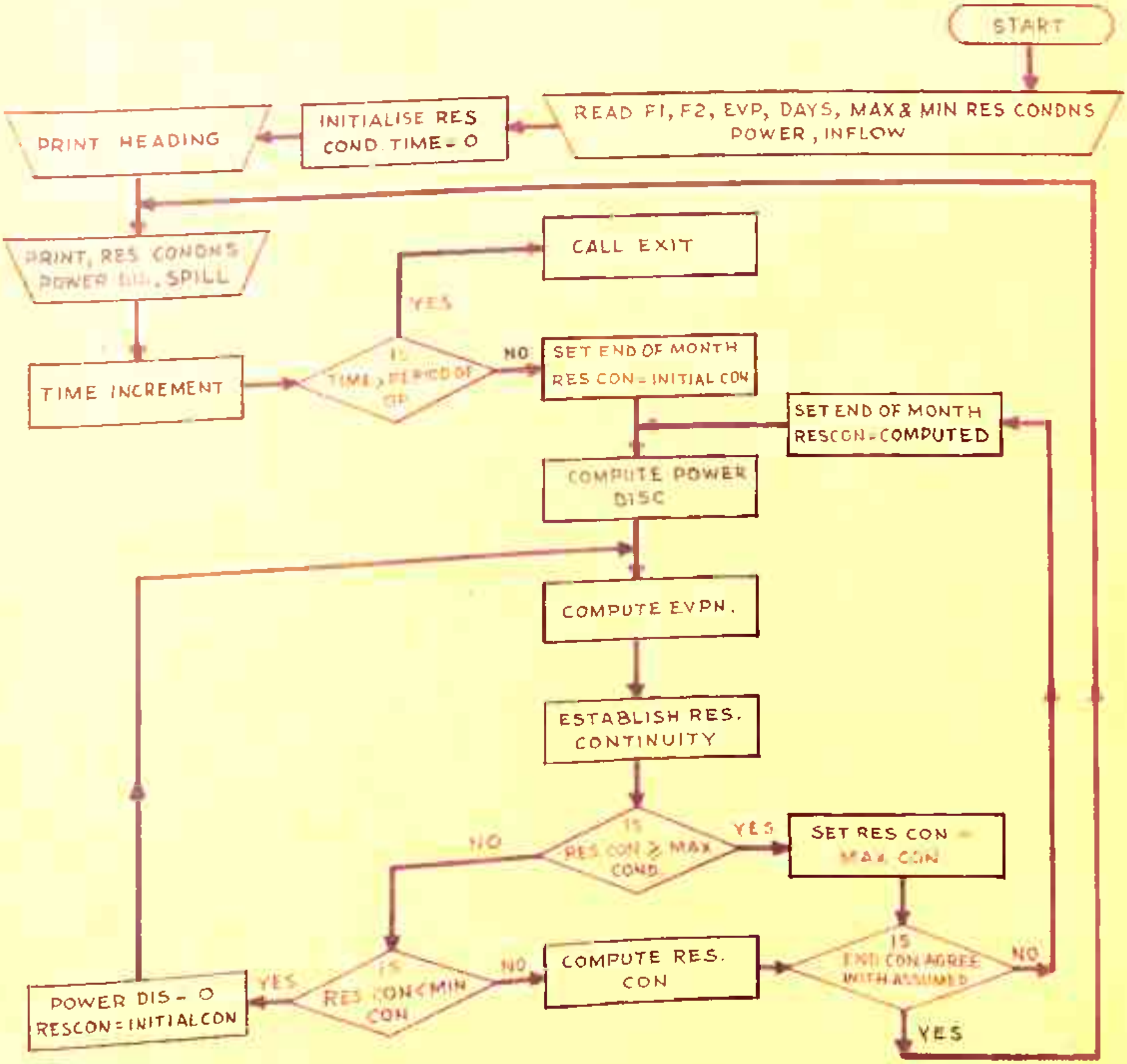


Fig. 2·4 : Flow Diagram for Reservoir Simulation.

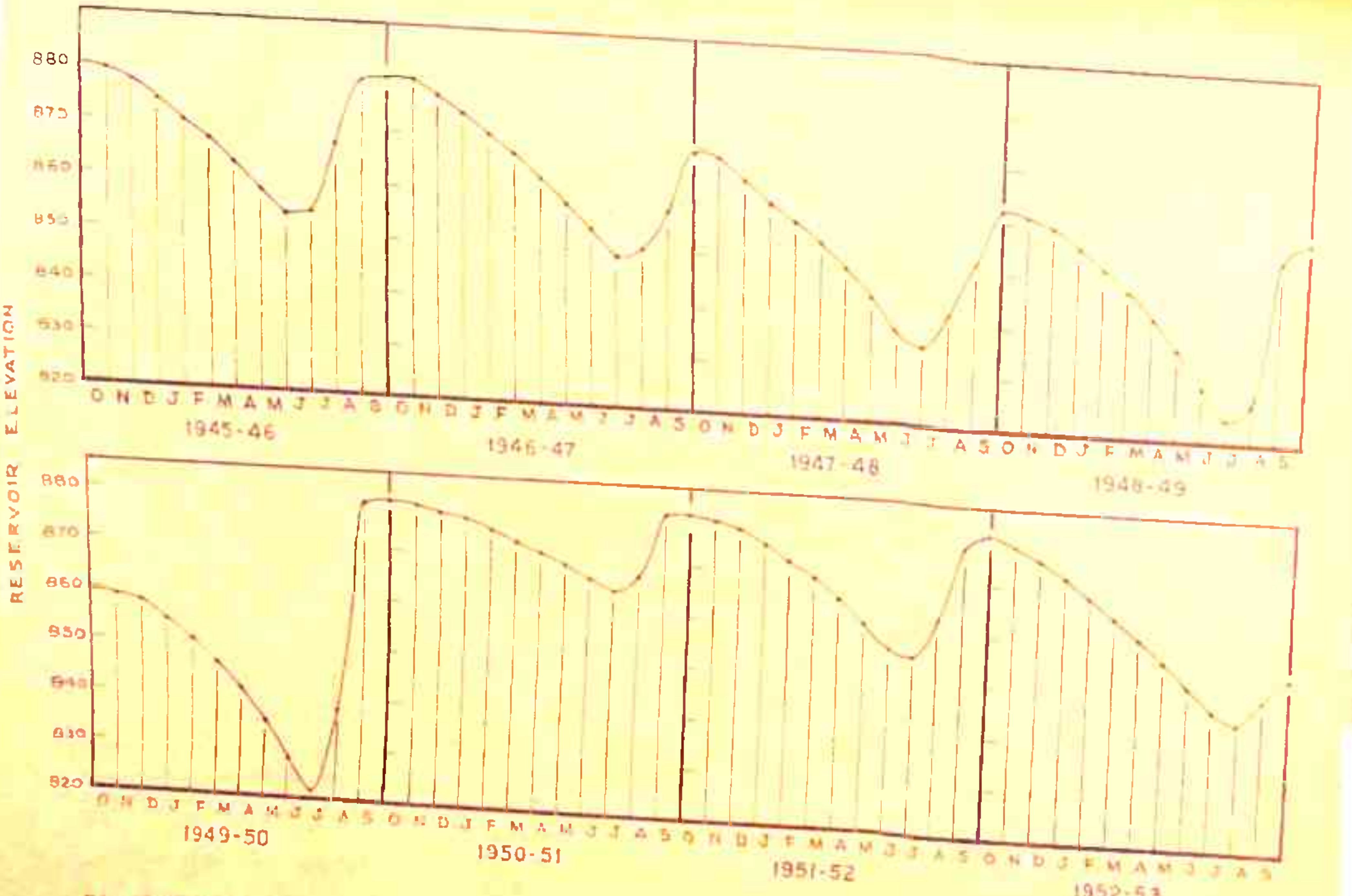
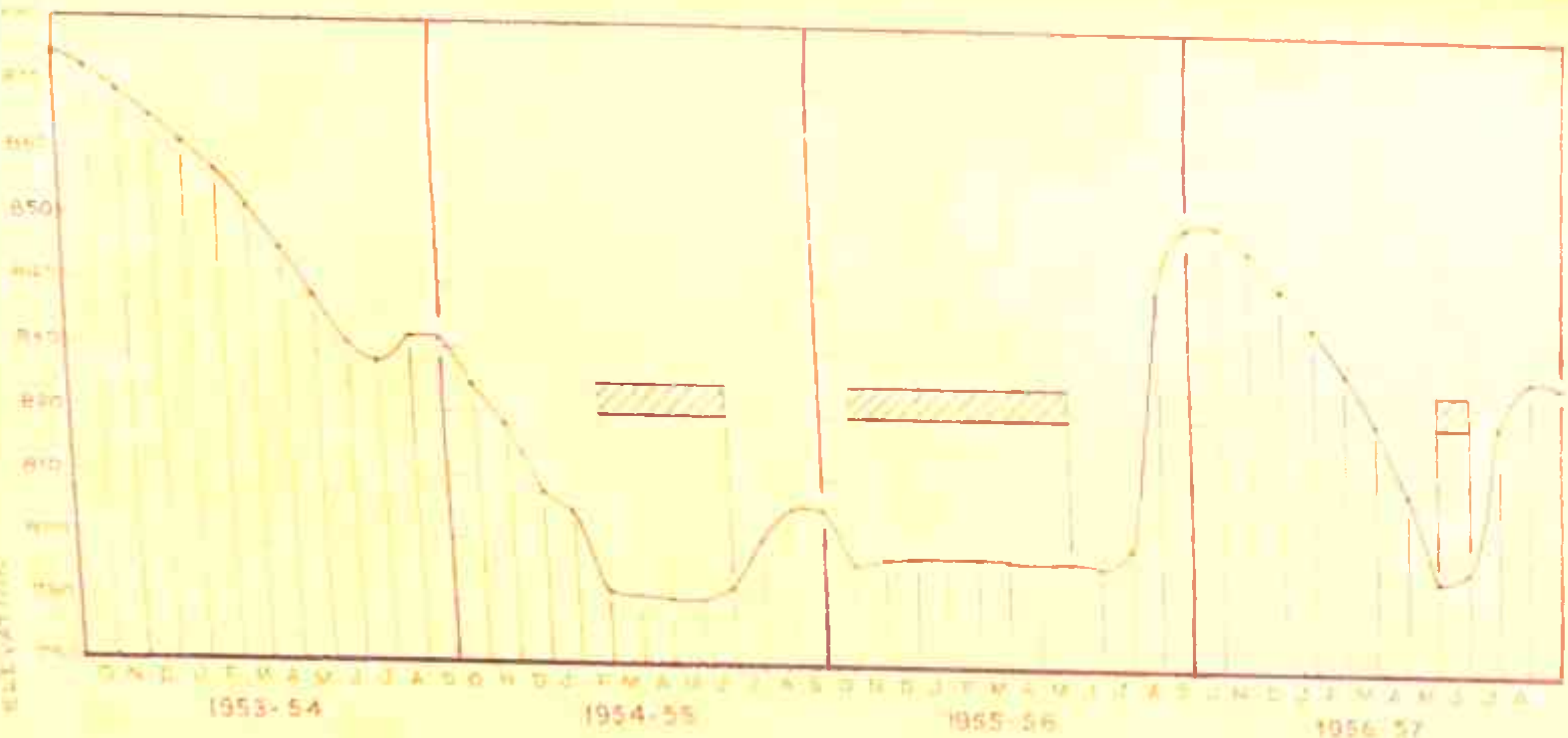
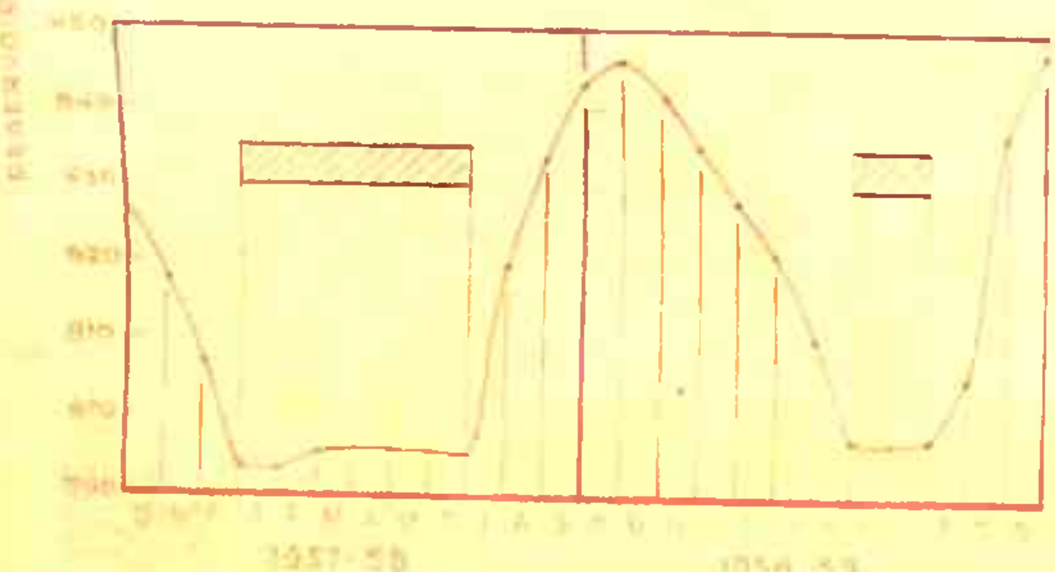


Fig. 2·5A: Water Level Variations - for Target output of 105 MW





INDICATES PERIOD OF ZERO POWER GENERATION

Fig. 2.58: Waterlevel Variations for Target output of 105 MW

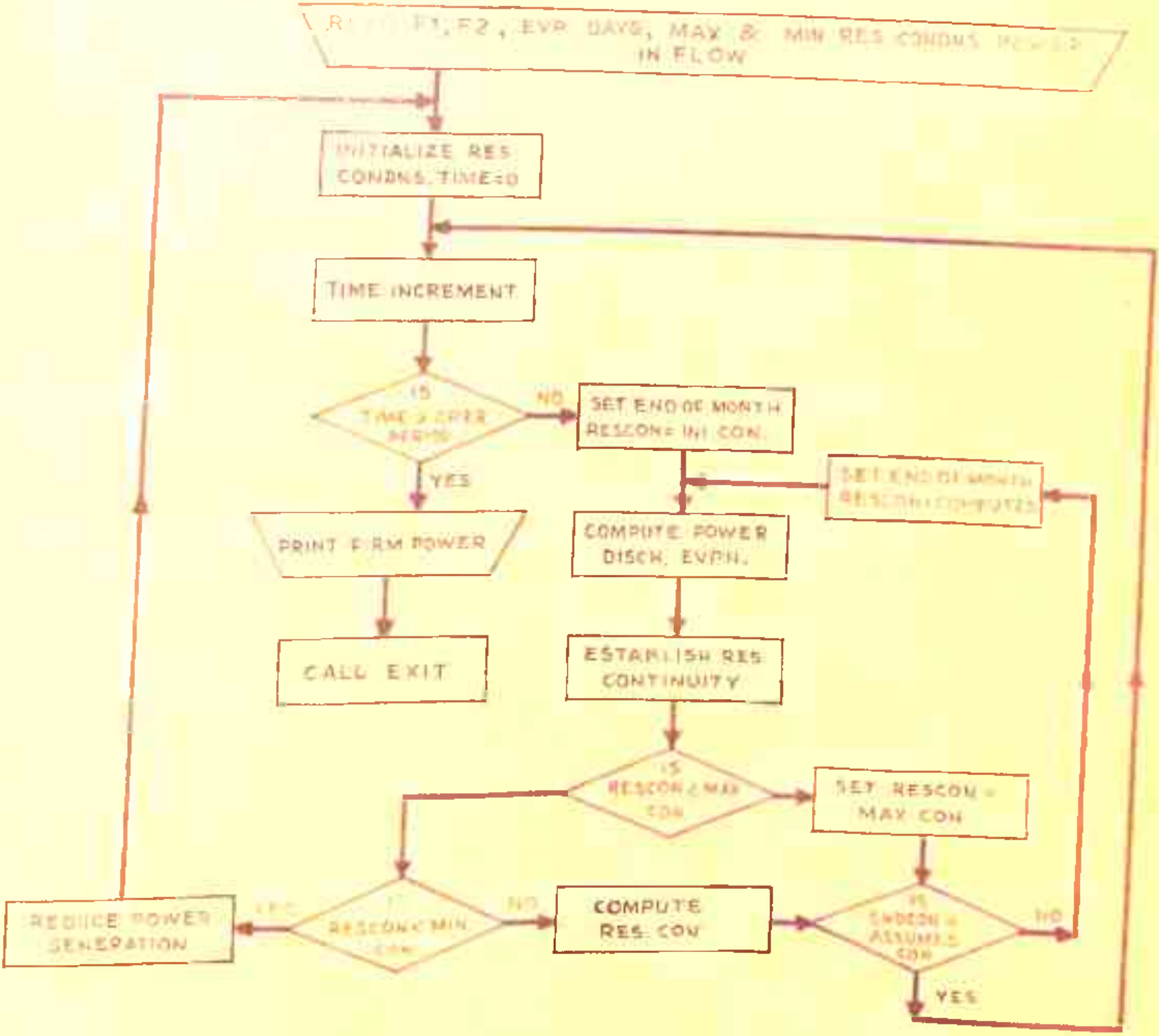


Fig. 2·6· Flow Diagram for Firm Power Estimation

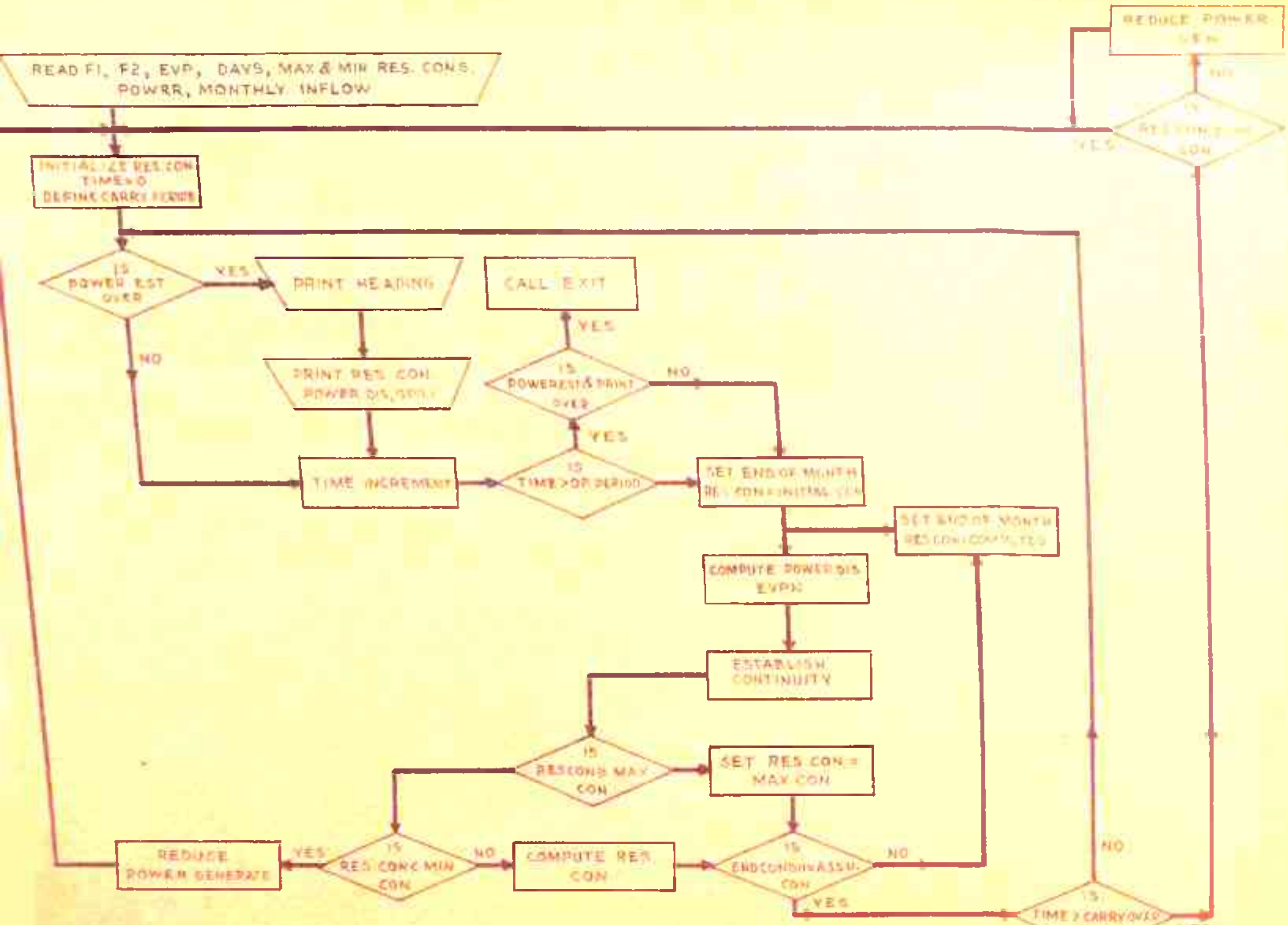
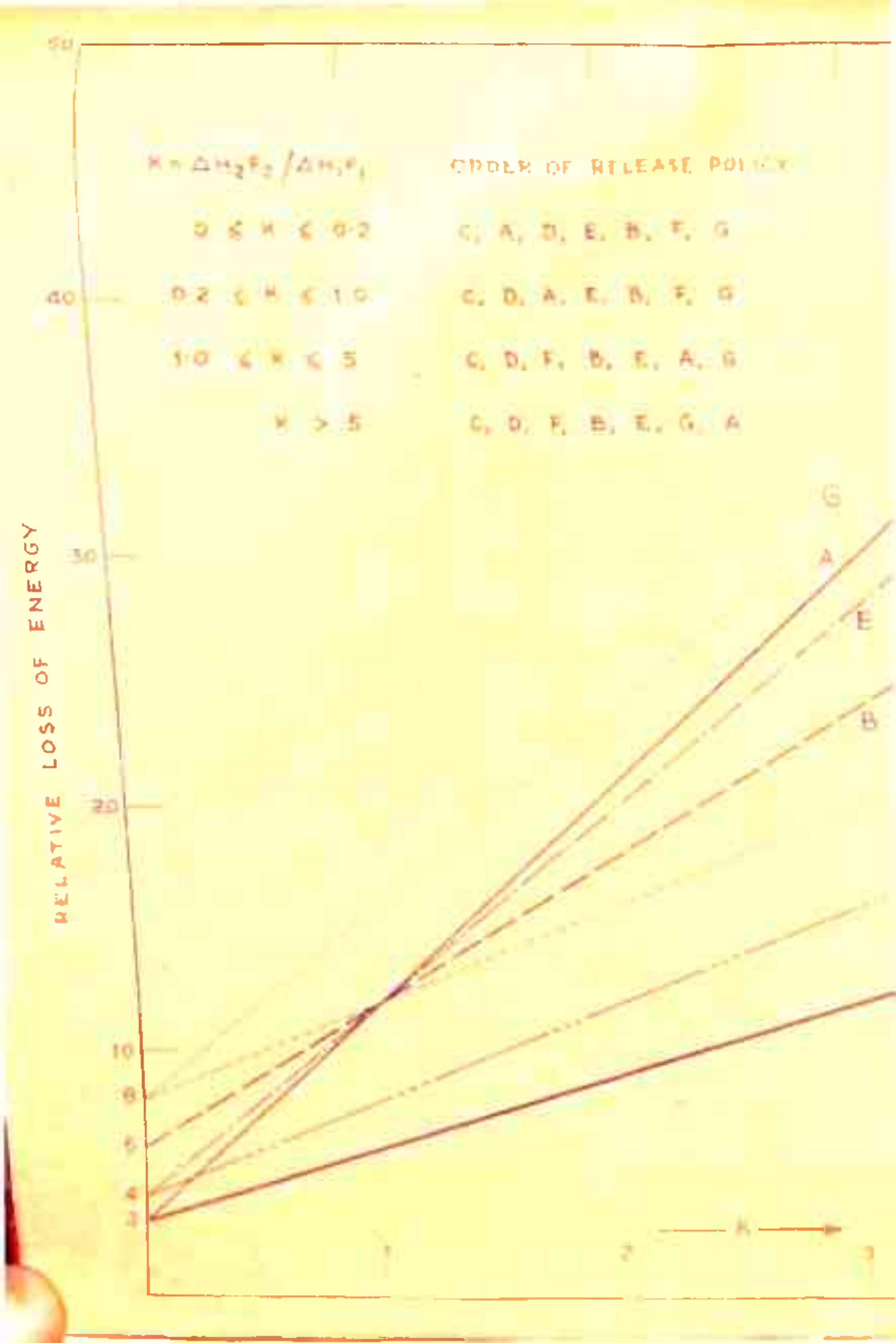


Fig 2.7: Flow Diagram for carry over Storage Operation



RELATIVE LOSS OF ENERGY
50
40
30
20
10
8
6
4
3
$K = \Delta H_2 F_2 / \Delta H_1 F_1$
ORDER OF RELEASE POLICY
0 ≤ K ≤ 0·2 C, A, D, E, B, F, G
0·2 ≤ K ≤ 1·0 C, D, A, E, B, F, G
1·0 ≤ K ≤ 5 C, D, F, B, E, A, G
K > 5 C, D, F, B, E, G, A
G
A
E
B
1
2
3
K

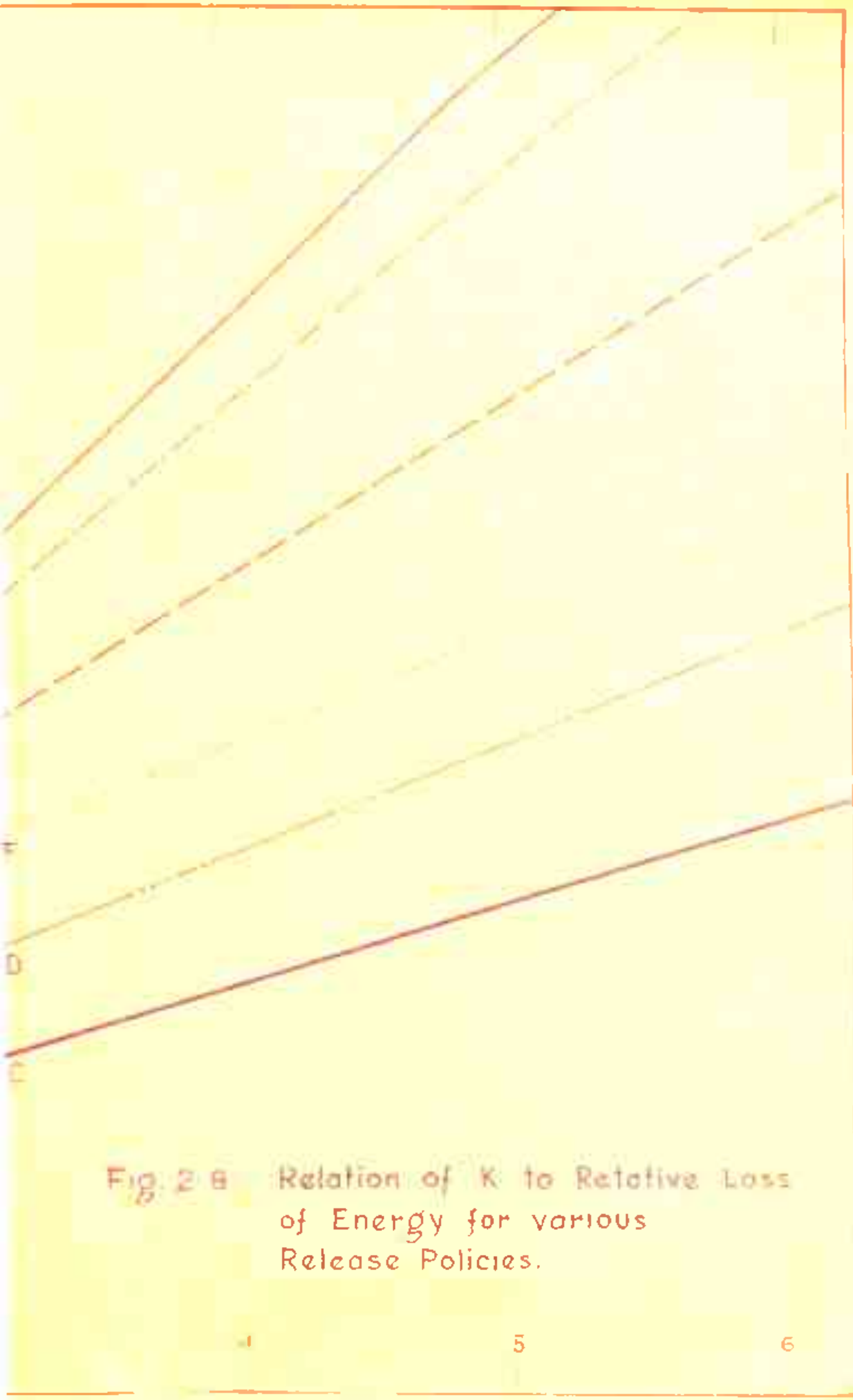


Fig. 2.8 Relation of K to Relative Loss of Energy for various Release Policies.

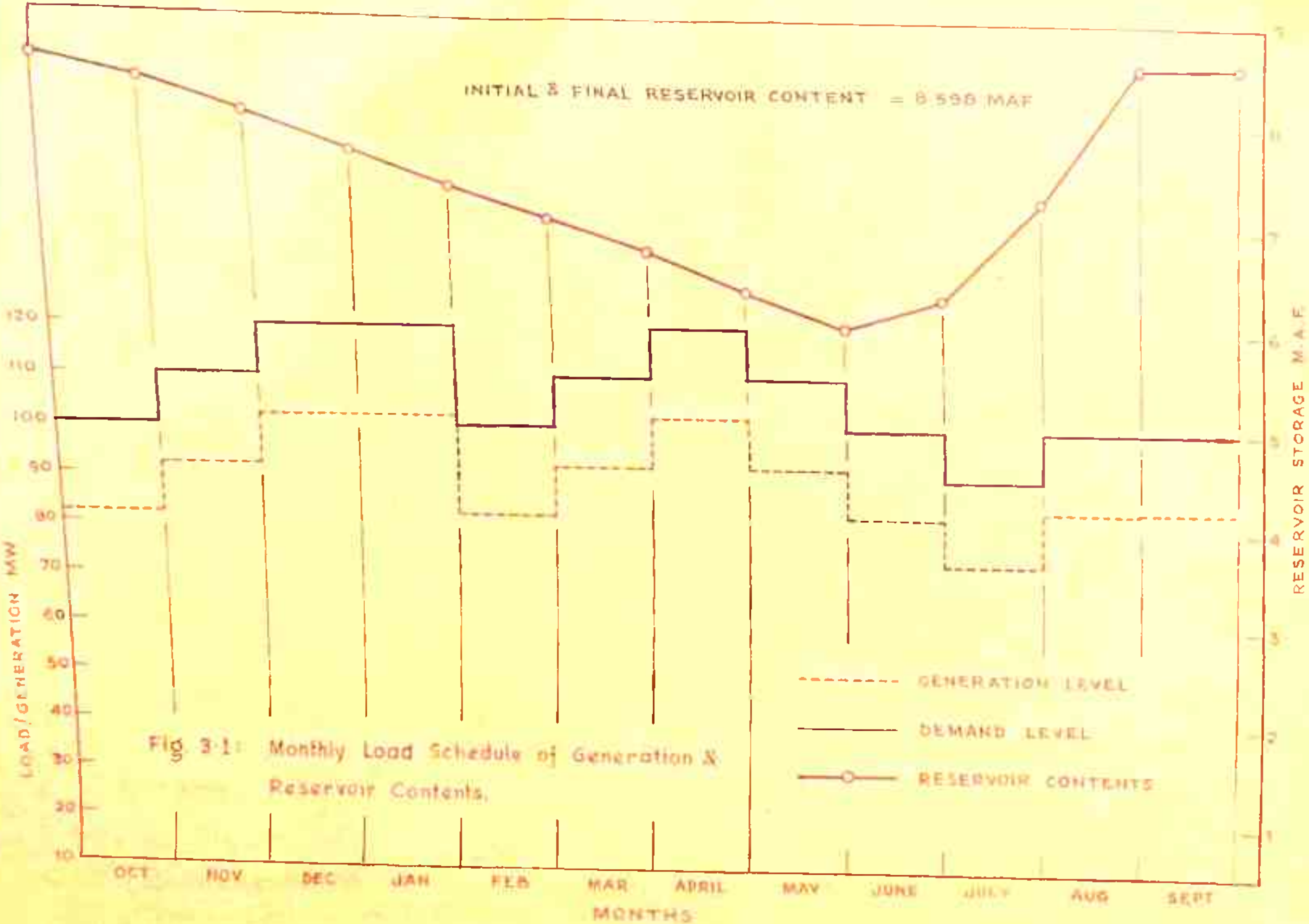


Fig 3·1: Monthly Load Schedule of Generation & Reservoir Contents.

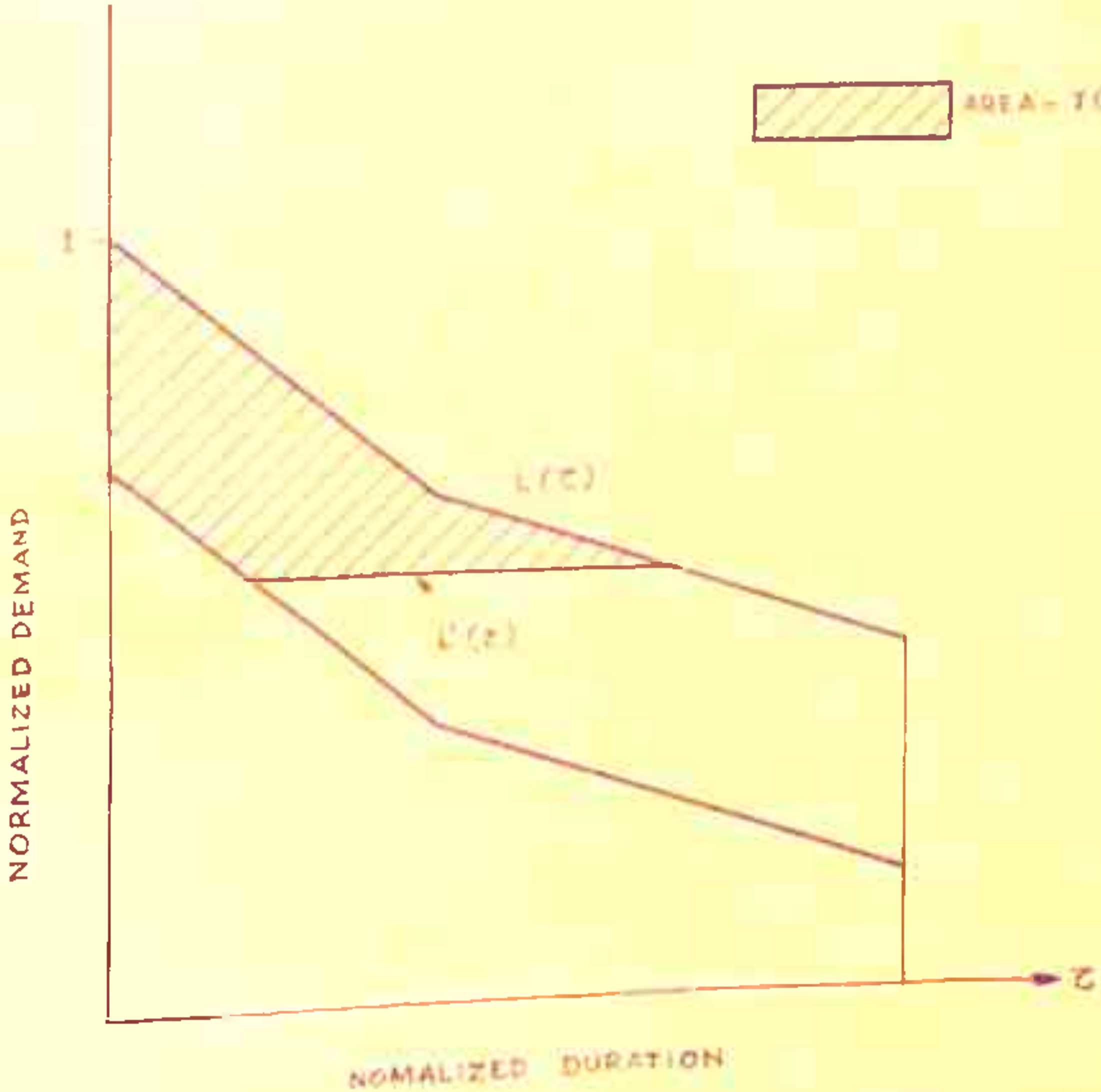
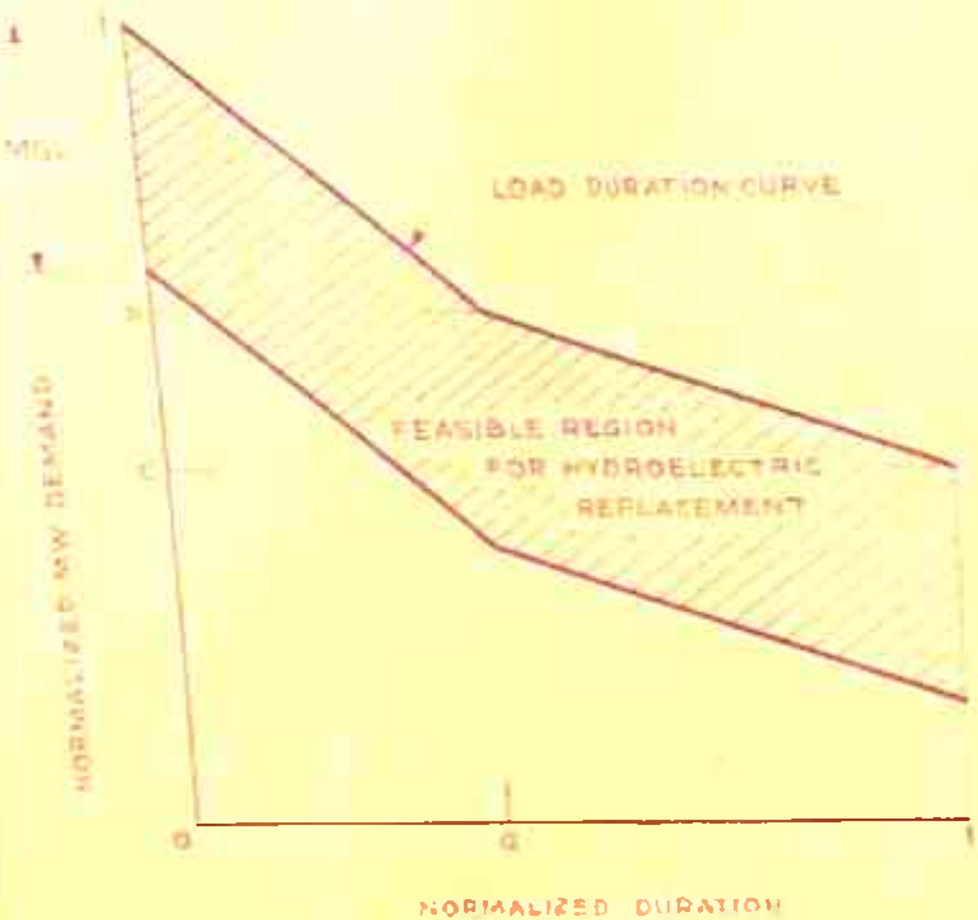


Fig 4.1: Hydro Energy Generation within the Feasible Region.



DATA ASSUMED IN THE SAMPLE PROBLEM

MONTH	a	b	c	MAX. DEMAND M.W.	HYDRO GENERATION
OCT	0.4	0.65	0.45	450	250
NOV	0.4	0.65	0.45	500	250
DEC	0.4	0.65	0.45	550	250
JAN	0.4	0.65	0.45	600	250
FEB	0.4	0.65	0.45	600	250
MAR	0.4	0.65	0.45	550	250
APR	0.4	0.65	0.45	550	250
MAY	0.4	0.65	0.45	500	250
JUN	0.4	0.65	0.45	600	250
JUL	0.4	0.65	0.45	450	250
AUG	0.4	0.65	0.45	400	250
SEP	0.4	0.65	0.45	400	250

MGL = NORMALIZED [illegible]

Fig. 4-2 Definition sketch for duration curve and data for sample problem.

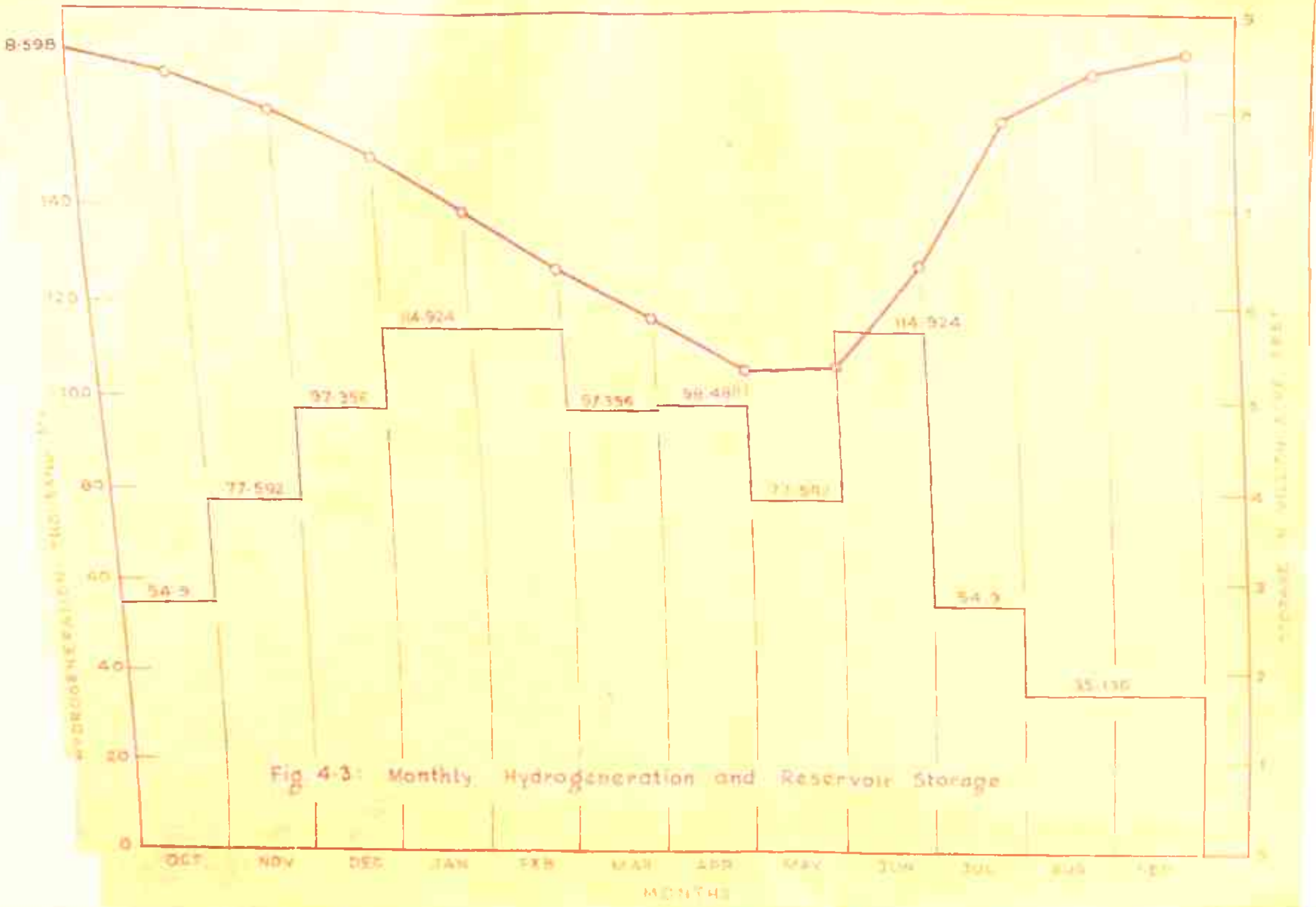


Fig. 4·3: Monthly Hydrogeneration and Reservoir Storage

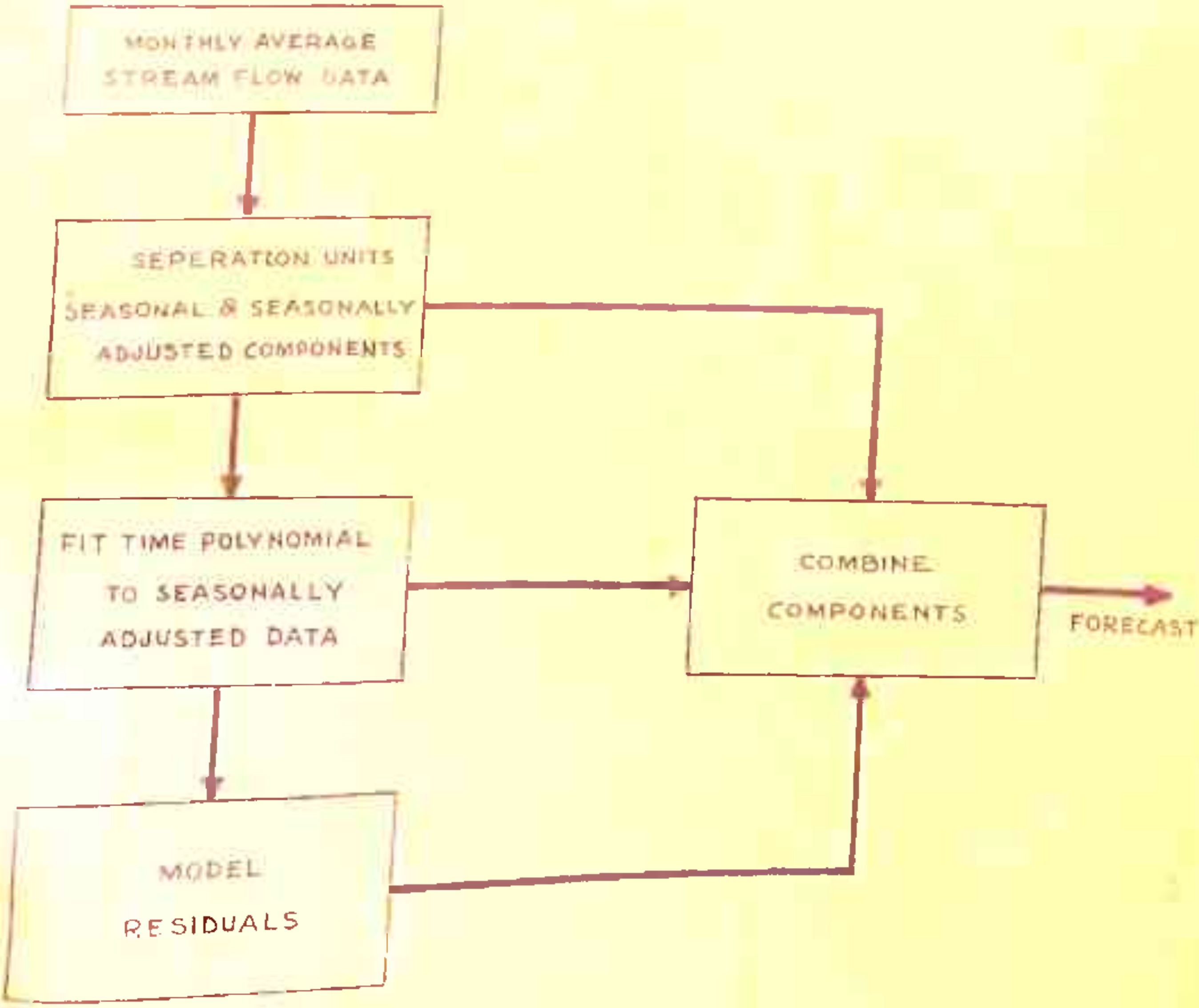


Fig. 5·1 : Structure of Forecasting Scheme

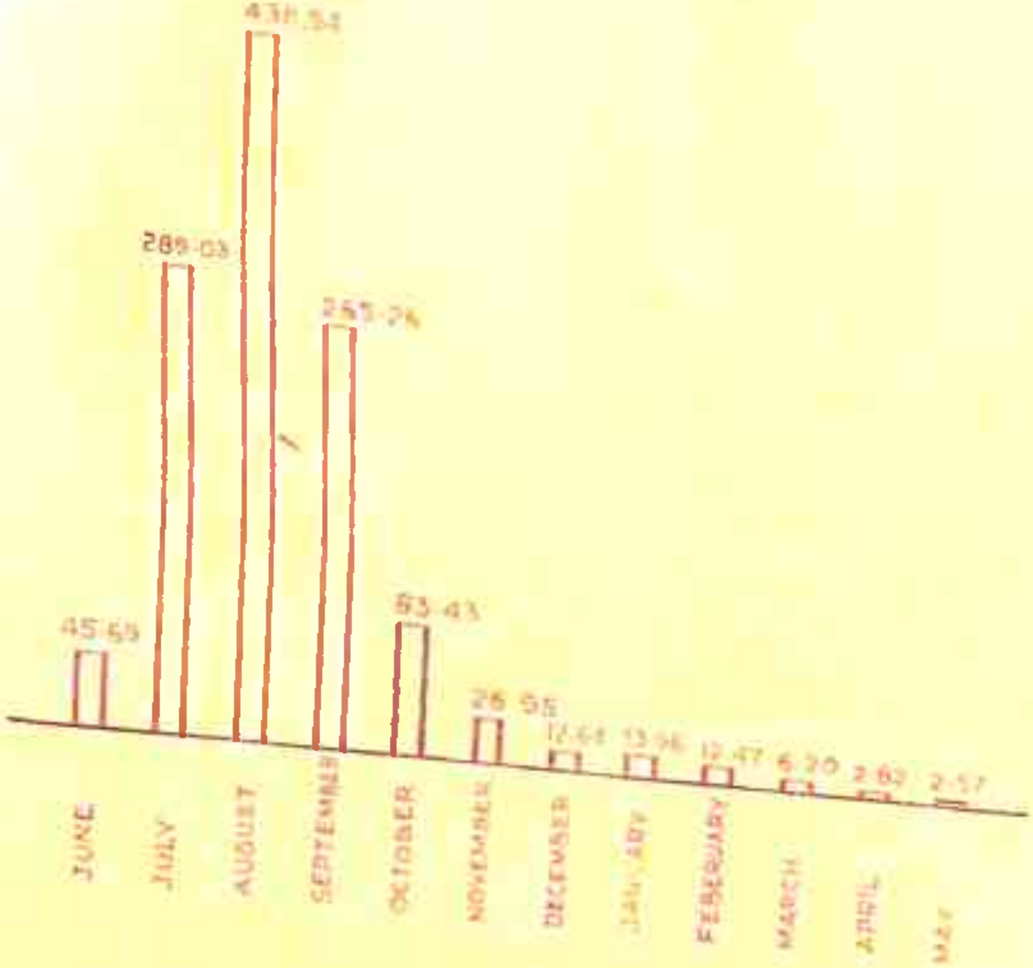


FIG. 5·2. SEASONAL FACTORS OF MONTHLY INFLOWS

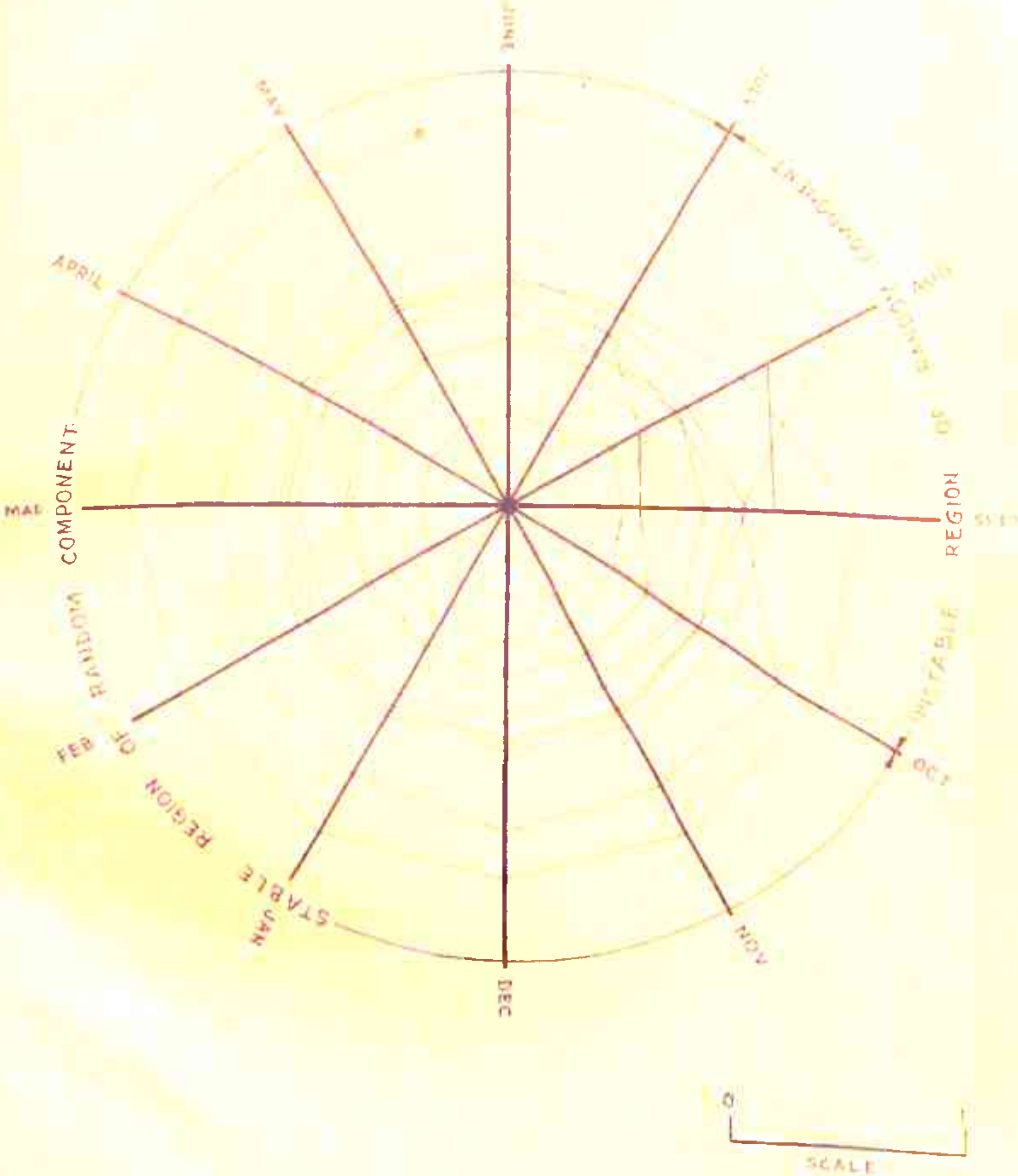


Fig. 5·3: Evolution of the Random Component