ON SOME INVESTIGATIONS IN OPTIMISATION METHODS IN THE AREA OF WATER DISTRIBUTION SYSTEM AND QUALITY MANAGEMENT

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C E R T I F I C A T E

This is to certify that the thesis entitled "ON SOME INVESTIGATIONS IN OPTIMISATION METHODS IN THE AREA OF WATER DISTRIBUTION SYSTEM AND QUALITY MANAGEMENT" and submitted by **RAM KARAN SINGH** ID. No. **93PHXF013** for award of **Ph.D.** Degree of the Institute, embodies original work done by him under my supervision.

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ABSTRACT

The study covers two major aspects of water resources management, namely, the optimization of water distribution system both under deterministic and under uncertain conditions, qualitative management of water resources for multiobjective water resources system planning. In water distribution system, often, large linear programming problems are encountered. It has been shown here how such a large problem can be solved more effectively by means of decomposition principle. The prboblem is that of determining minimum cost of branching, network for supplying water to several demand points to meet the discharge and head requirement. The cost of the system is taken as function of length of alternative pipe sizes used and friction losses produced by the length of several pipe to be used in the system as well as magnitude of head loss required for equilibrium. The algorithm developed is also applicable for looped network layout. In order to consider the varying water demand of consumers in network optimization and design fuzzy linear programming (FLP) technique is applied with objective to minimize the cost of the system and results are compared with linear programming problem and it has been seen that there is net reduction in cost of 9.54 percent of total cost which

indicates the FLP is more rational than LPP which is most commonly used in water distribution network optimization, in network analysis and basic equation formulation, linear graph theory, primarily an area of research and development in Electrical network analysis, has been used. The analysis has many distinct advantages over the existing methods commonly used in analysis. For qualitative management of water resources case study is taken on Indian rivers. Water quality index is obtained using Delphi method on the scale of 100 is 24.6162 indicates water can be used for domestic purposes only after conventional treatment. Factor score curves are developed for sixteen water quality parameters and thirty two sampling stations for twenty rivers covering major part of the country. The rating for the extend of pollution is also done for all twenty rivers. The analysis is also done to check the suitability of water for drinking and irrigation purposes based on the standards of World Health Organisation (WHO), Bureau of Indian Standards and U.S. Salinity chart. Finally linear goal programming model is developed for its application to Rivers stretch for multiobjective water resources system planning.

CONTENTS

LIST OF FIGURES

LIST OF SYMBOLS

a_{ji}	-	Factor loading of variable j on factor i
BOD	-	Biochemical Oxygen Demand
C_{hw}	-	Hazen William's Coefficient
C	-	Rating
C_j	-	Cost per meter length of pipe diameter j
D	-	Diameter of pipe
DO	-	Dissolved oxygen
h_f	-	Head loss
H_k'	-	Maximum allowable head loss in fundamental circuit k
λ_i	-	Eigen value of the ith Factor
I_{TM}	-	Trace metal sub index
I_{tox}	-	Subindex of toxic metals
I_{LCZ}	-	Subindex of lithium, copper and zinc
I_H	-	Subindex for hardness
I_d	-	Subindex of water suitability for dirnking
I_{cr}	-	Subindex for contact recreation
I_{fish}	-	Index of mercury in fish
I_{amb}	-	Ambient index
I_{turb}	-	Index for turbidity
L	-	Length of Pipe
l	-	Length of Pipe in line
l_i	-	Length of pipe in line j
l_{ij}	-	Length of pipe in line i, having diameter j

l_{ijk}	-	Length of pipe in line i, having diameter j and belonging to fundamental circuit k
m	-	Total number of lines in network
n	-	Total number pipe diameter available
q	-	Rate of discharge
S_f	-	Loss of head due to friction per meter length of pipe
S_{ijk}	-	Friction head loss per meter length of pipe in line i, having diameter j and belonging to fundamental circuit k
W	-	Weight
WQI1	-	Unweighted arithmatic index
WQI2	-	Unweighted solway index
WQI3	-	Unweighted gemetric index
WQI4	-	Weighted arithmatic index
WQI5	-	Weighted solway index
y_j	-	Standardized value of the known variable j
β	-	Degree of truth.

ABBREVIATION USED

ASCE	-	American Society of Civil Engineers
AWWA	-	American Water Works Association
CPCB	-	Central Pollution Control Board
Co	-	Company
Div.	-	Division
EC	-	Electrical Conductivity
Engg.	-	Engineering
Fico	-	Fical Coliform
FLPP	-	Fuzzy Linear Programming Problem
Hyd.	-	Hydraulics
GP	-	Goal Programming
IWWA	-	Indian Water Works Association
IPHE	-	Institution of Public Health Engineer
IS	-	Indian Standard
IEI	-	Institution of Engineers India
Instn	-	Institutions
IAEM	-	Indian Associations of Environmental Management
LPP	-	Linear Programming Problem
LGPP	-	Linear Goal Programming Problem
MPN	-	Most Probable Number
NEERI	-	National Environmental Engineering Research Institute

No.	-	Number
Proc.	-	Proceedings
PP	-	Page
SAR	-	Sodium Absorption Ratio
Soc	-	Society
Turb	-	Turbidity
Trans	-	Transactions
Temp	-	Temperature
Vol.	-	Volume
WHO	-	World Health Organisation
WQI	-	Water Quality Index

CHAPTER - 1

INTRODUCTION

1.1 Introduction to Water Distribution System and Quality Management:

Water is likely to be one of the most critical resource issues of the developing world in the early of the 21st century. A balanced and sustainable approach to water development is mandatory if the adverse impacts of the impending water crisis is to be avoided. Water has been basically ignored in the international agenda in recent years.

The United Nations Conference on Environment and Development (UNCED) held at Rio De Janerio in June 1992. Issues like climate change, deforestation, biodiversity and ozone depletion took the centre stage at Rio: water was at best a 'bit' player, largely confied to the wings.

The United Nations Water Conference, held at Mardel plata in 1977, recommended that the decade (1981-1990) should be designated as the 'International Drinking Water Supply and Sanitation Decade'. It has been unanimously endorsed by the thirtieth World Health Assembly. The

proposed objectives have been highelighted as:
"A concentrated effort will be required by countries and the international community to ensure reliable drinking water supply and provide basic sanitary facilities to all urban and rural communities. Specific target should be setup by each country taking into consideration its sanitary, social and economic conditions."

A survey done by the World Health Organisation (WHO) in developing countries in 1975 indicated that about 77 percent of urban population and only about 22 percent or rural population had adequate water supply (World Health Statistics Report, 1976). It is also estimated that each year about 50 million people are affected by unsafe drinking water supply resulting in the poor health of the people which in turn is a stupendous loss of man-power.

The demand of safe water is increasing day by day. In every sector, the basic need is water for its development. If adequate safe water is not supplied, the economy of the country is badly affected, especially, in the case of developing countries.

Present conditions and future prospects vary enormously from country to country and from region to region. The greatest number of people needing better survices (i.e.

those who are now supplied from public outlets but who lack house connections).In case of India, there have been some outstanding examples of positive action, notably, the efforts to promote urban water supplies under the 5-year plans of the Government of India.

As per recommendations of United Nations Conference of Human settlement, 1976, Government of India has fixed by the target of 100 percent coverage for water supply from the present 30% for rural and in the case of urban, 82%. The estimated sum of Rupees 42 billion ($1 = Rs. 14.60) will be spent during the International Water Supply and Sanitations Decade to achieve the goal.

1.2 Object of Present Investigations:

The expenditure involved in investment for conventional water supply system is so large that alternative and cheaper means of providing potable water has to be devised. The main objective of these schems are to provide treated water to the consumers and it is widely believed that the most costly parts of water supply systems are the facilities for water distribution. The distribution system includes pipe networks, the pumping units and storage tanks and reservoirs. Due to population growth, the continuous increase in water consumption makes the system dynamic. In developing countries, the facilities to serve the various needs of consumers are limited and the dimension of work ahead to attain the target is formidable. It stands to reason, therefore, that the system needs develop a comprehensive, unified methodology for the total water distribution design process.

The distribution system accounts for about 75% of the cost of water supply schemes. The optimal design of water distribution system has quite recently received a great deal of attention by a number of researchers but much works still remain to be done. The present investigation is, therefore, aimed to study the system critically and to analyse the system in a simpler but in a more comprehensive way than the conventional analysis commonly applied. The study relates to the network analysis, optimization of the network, under deterministic and under various uncertain economic, hydraulic and population growth condtions.

The concept of optimization is also to be extended for water quality management of river stretch for multiobjective water resources system planning.

CHAPTER - 2

LITERATURE REVIEW

2.1 Optimization Models for Water Distribution System and Quality Management:

The pipe distribution network of a water supply scheme may comprise a major part of the capital cost of such scheme. In addition to the initial capital cost, there are continuous operating costs for pump operation, maintenance and repairs. Since the distribution system alone costs about 70% or more of the entire cost of the distribution system, it has become increasingly important to have a distribution system that gives the least cost in terms of investment and operating cost.

While in the preliminary analysis, the cost of the water distribution scheme may be based on the analysis and appraisal of an arbitrary assumed design, there my exist numerous arrangements of pipe sizes, within the network, which will satisfy the field requirements. Each arrangement results in a different total cost of the project. The economic appraisal of several different designs is laborious and there is every likelihood that the best solution obtained may be farther from the global optimum.

Various researchers have proposed the use of mathematical programming techniques in identifying optimal solution to water supply design problems. Such contributions have focussed mainly on either a branch system or a loop system. The optimisation techniques for solving pipe networks can be categorized as follows:

(i) Equivalent diameter concept

(ii) Linear programming

(iii) Non linear programming

(iv) Dynamic programming

Although, 'check design' using pipe network analysis has been in common use, the pursuit of direct optimal design is not a new idea. Tong et al. (1961) have introduced the concept of equivalent length of pipe which, as claimed, would enable a water network to be designed with minimum length of pipe from known pressure head surface profile. The method is based on the concept that the total amount of pipe in a loop is least, if, for a given condition of head, inflow and outflow, and the geometric pattern of the network, the sum total of the equivalent lengths of pipe of single size and with the same coefficient of roughness, is minimum in a loop. Briefly, the method suggested by Tong et al. (1961) consists of finding, by an iterative procedure the equivalent length of 200 mm diameter pipe making the algebraic sum of the equivalent lengths of pipe in each and every loop in a network equal to zero, $\Sigma L_e=0$. This concept of $\Sigma L_e = 0$, and thereby, $\Sigma k = 0$ (resistance or head loss coefficient) used for arriving at the minimum amount of pipe and is not arrived at mathematically, but is based on observation and experience.

Further work on this line was carried out by Raman and Raman (1961), Deb and Sarkar (1971), Jeppson (1982), Featherstone and El-Jumaily (1985) and many others. Raman and Raman (1961), while agreeing with the equivalent pipe concept, demonstrated mathematically that the algebraic sum of L_e/Q must be zero instead of ΣL_e. A correction factor for flow $Q = \Sigma L_e / \Sigma^{1.85}(L_e/Q)$, applied to initially assumed flows in the pipe is then derived. The assumed flows are subsequently corrected by applying the correction factor and the process is repeated until the equivalent pipe lengths are balanced. However, the requirement of known pressure surface in this method leads to some limitation. Deb and Sarkar (1971), however, disagreed with the concept and pointed out that the equivalent length is inversely proportional to the diameter raised to the power 4.87 and that by minimising the equivalent length of the pipe, the cost of network is increased. They developed an 'equivalent diameter' method for network optimisation using an assumed pressure surface profile. Cost functions for pipelines including initial power consumption and maintenance costs are used. By imposing an assumed pressure surface profile over the network, the pipes are replaced by equivalent pipes of 100m length and equivalent diameter, D_e = 1.106 x $Q^{0.381}/H^{0.206}$ (using Hazen-Williams formula in which Q is the pipe discharge in litres/min. and H is the head loss in meters of water). The total cost Y of the pipes in a loop is expressed in the form $Y = KM \Sigma Q_i^{0.381m}/H_i^{0.206m}$ in which M and m are constants derived from cost functions. In each closed loop, the discharges in all pipes are expressed in terms of Q_1, the discharge in the first loop. The initial assumed pipe flows are corrected using the Hardy-Cross method, which results in the corrected flows and the equivalent pipe diameters. The equivalent pipe diameter are then corrected into actual pipe diameters using the Hazen-Williams formula, incorporating the actual pipe lengths. However, this method as in the case of the equivalent length method, has two major drawbacks; firstly it lacks mathematical justification for cost equivalent pipes and second, a hydraulic pressure surface over the network must be artificially created (Swamee and Khanna, 1974).

Jeppson (1982) described a method for reducing groups of parallel pipes to single equivalent pipes in order to reduce computer costs associated with the analysis of large piping networks. It is pointed out that the time required for solving any real system by reducing parallel pipes to the equivalent one is one quarter of the time required by other methods. The potential saving is modestly more if the analysis utilizes the Hazen-Williams (or Manning's) formula instead of Darcy_Weisbach, since the hydraulic characteristics of the equivalent pipe are not required to be updated for each iteration of the network solution. The potential savings are likely to be much larger if the analysis is to determine the performance of the network hourly in a day (24 time steps) in which demand variability causes rising or falling water surface in the tanks and the pumps are turned on or off depending upon conditions encountered in the network.

In practice, in a network, pipes smaller than 100 mm or sometimes even 150 mm are either ignored (Fair et al. 1971) or grouped together and replaced by equivalent pipe (Jeppson, 1982; Walski, 1983). This process is termed 'skeletonisation' of the network. However, if many pipes are removed during skeletonization, the resultant model may poorly represent the distribution network. Eggener and Polkowski (1976) have studied the impact of skeletonisation on the accuracy in the distribution network analysis.

The method suggested by Featherstone and El-Jemaily (1985) is the design procedure based on either the head balance or quantity balance methods of analysis of flows and pressure distributions in the networks of assumed pipe sizes. These initial diameters, individually, are successively and systematically adjusted until the global cost of the network is at a minimum. After each adjustment, the network is reanalysed. The mathematical concept of optimisation is based on the whole system, thus, resulting in global minimum, as opposed to other methods in which the optimisation is based on a typical elemental loop.

A number of techniques are available for solving nonlinear/dynamic problems once they have been expressed mathematically. The network model generally uses one of these mathematical techniques to arrive at an optimal solution.

The first significant linear programming optimisation model was developed by Shamir (1968). the decision variables are the pipe diameters. The objective function considers a single loading condition and related to the energy loss are the flows through all the pipes. The steady state hydraulic solution is obtained by the Newton-Raphson method with the Jacobian of the solution used to compute the components of the gradient. The linear programming (LP) technique truly yields the global optimum solution for branching distribution networks only. For a known nodal demands, the discharge in all the links of the network can be uniquely computed (a link is a segment of the network that has constant flow and no branches). Each link can consist of one or more pipes connected in series (a pipe is a segment of the link that has constant diameter). Each link can consist of one or more pipes connected in series (a pipe is a segment of the link that has constant diameter). As the resistance and cost of the pipe are linear functions of its length, the different pipe lengths constituting a link are taken as the decision variable in the formulation of the LP problem. In looped network, however, the link discharges cannot be computed apriori and therefore the LP technique cannot be directly applied.

Pitchai (1966) has formulated the general problem of estimating the optimal diameters of a distribution network for arbitrary supply and demand inputs. The objective function including capital, energy and penalty cost terms for violating constraints is non-convex and so are the constraints. Both the cost function and constraints are expressed in nonlinear integer programming is employed. Since at that time no algorithm was available for direct computation of solutions to nonlinear integer optimization problem, the author adopted random sampling of diameters, starting at various feasible points and improving the value of the objective function towards local minima.

Jacoby (1968) has proposed a nonlinear programming technique of gradient variables for looping networks. The final design is obtained by selecting the size of commercial pipe closest to the theoretical diameter. This rounding may cause the selected design to be infeasible. The method is complex and is applied on a network consisting of two loops of seven pipes (with one pipe in common), five consumption nodes and one pump supply node. The combined cost of the pump and the pipelines is aimed to be minimised subject to the physical laws of fluid flows and demand boundary conditions. Hardy-Cross method is applied to eliminate this unfeasibility. Because the objective function has many local optima, the technique does not assure that the global optima will be found. It is pointed out that care should be exercised to avoid local minima.

Karmeli et al. (1968) have suggested a methodology for the design of branching network in steady state flow conditions. A linear programming model is formulated by choosing the pipe length as the decision variables. Like previous researchers, the model only considers the initial cost in the objective function.

Gupta (1969) has adopted a similar approach of Karmeli et al. (1968) in dealing with a simple branched network. The decision variables are taken as the lengths of pipe segments with a specified diameter, as both pipe resistance and pipe cost are linear functions of its length. The constraints on heads at nodes are considered as linear. However, the approach is not applicable on looped networks.

Kohlhass et al. (1971) have used separable programming technique to determine not only the optimal diameters but also the pumps and reservoirs for a looped system with all heads known. If the flows are decision variables, the constraints become linear for given heads. Parameters can be computed directly from the Hazen-William equation with heads and flows fixed. The nonlinear objective function constrained the cost of pipes, pumps and reservoirs. Both Lai and Schaake (1969) and Kohlhass and Mattern (1971) treated the case in which the head distribution in the network is fixed.

Swamee, Kumar and Khanna (1973) have handled the problem of minimizing the cost of a single source tree distribution system. Using dynamic programming, the authors have developed closed form solution with an objective function covering pipe, pump and elevated reservoir capital and maintenance cost plus pumping energy cost.

Lam (1973) developed a discrete gradient optimisation technique for a water distribution system consisting only of a single source, pipes and demands. the pipe diameters are treated as discrete variables. This technique avoids the rounding off continuous diameter variable to the nearest commercially available size.

Watanatada (1973) has developed an optimisation technique for multiple source networks and applied it to real networks of moderate size. The problem of hydraulic network optimisation is a constrained nonlinear optimising problem since the design vectors, viz. diameters, heads and discharges are not permitted to vary freely. In solving the constrained nonlinear optimisation problems, two approaches are generally followed, (i) minimise the functions explicitly and keep the variables within the feasible region where the constraints are not violated or (ii) transform the constrained problem into an unconstrained one which can be solved by available unconstrained minimization algorithm. Watanatada (1973) has followed the second approach, wherein, the inequality constraints are eliminated using the method employed by Box (1966) while the equality constraints are handled using the "gradient Balancing" technique suggested by Haarhoff and Buys (1970). However, it is mentioned, the method of solution employed is not suitable to a large network problem as the computing time required to solve a problem increases sharply with the number or variables. This is caused primarily because of the inefficiency in function minimisation procedure of Fletcher and Powell (1961) used in the analysis. this method is applied to a network with two loops with no reference to more complex system. The method also determines the theoretical and not the commercially available diameters.

Cembrowicz and Harrington (1973) have transformed the non-convex capital cost function of a hydraulic network to subsets to nonlinear convex functions by a decomposition principle using graph theory. The variables are the flows and the head losses and the constraints are linear expression of the head losses. The global minimum cost solution follows using a standard nonlinear programming algorithm. Since the inputs at each node enter the formulation explicitly, pumps, pressure controls can be incorporated, the pressure potentials are controlled by the constraint set. In an illustrative example, it is shown that a network with four loops and 13 branches has 2944 minimal feasible solutions. However, the number of feasible solutions can be reduced by introducing a reference node which is equivalent to omitting a node equation and by eliminating the whole families of inferior solutions that show recirculation, the number of feasible solutions are reduced from 2944 to 141.

Shamir (1974) has extended his earlier work by developing a methodology for handling both the optimal design and the operation of a water distribution system under one or several loading conditions. Optimisation is obtained by a combination of the generalised reduced gradient (GRG) and penalty methods. The objective function includes initial cost of the design and the cost of the operation. The author claims that the physical measures of the performance and penalties for violating constraints may be incorporated into the objective function but offers little guidance on the penalty defining measures.

In fact, the methods of design of looped system can be separated into two categories: (i) methods which require the use of network solver i.e. at each iteration of the optimisation, first solves for the heads and the flows in the network, then use this solution in some procedure to modify the design. Such an approach is followed by Jocoby (1968), Watanatada (1973), Shamir (1974) (ii) the methods which do not use a conventional network solver but may assume the head distribution in the network as fixed. Lai and Schaake (1969) and Kolhaas and Mattern (1971) have followed this approach. However, in all these studies, the flow solution is incorporated in the network optimisation procedure by making certain assumptions about the hydraulic solution of the network.

Deb (1976) has considered a distribution network with the decision variable as the size of the pipes, the pressure surface over the network, the height and the location of the elevated service reservoir and the capacity of the pumping station. The objective function encompases the initial cost of pipes, pumps and elevated storage reservoir, operation costs and maintenance costs. A gradient like technique is used to perform the optimisation. The author attempts to develop a comprehensive optimal design of the water distribution system.

Alperovits and Shamir (1977) employed a method called the linear programming gradient (LPG) method in optimizing a distribution system including pipes, pumps, valves and reservoirs. The decision variables include design parameters i.e. the pipe diameters, the pump capacities and the reservoir elevations and the operational parameters. The objective function includes overall capital costs. The LPG method deals with looped networks and decomposes the optimisation problem into a hierarchy of two levels.

Firstly, the distribution of flows in the network is assumed to be known. Once the flows are assumed, constraints equations are easily formulated and using LP, a set of optimal segments such that the network is hydraulically balanced and Q in all links is obtained. The next stage is to develop a method for systematically changing Q with the aim of improving cost. The method of changing Q is based on computing the gradient of the total cost with respect to the changes in the flow distribution. The gradient is used to change the flows so that the optimum is approached. Alperovits and Shamir (1977) have stated that the initial flow distribution for each loading condition is arbitrary. However, a poor choice of initial flow distribution for a large problem may lead to excessive computer time wastage in arriving at a feasible (balanced) solution. The modified pipe flows are then used and the process is repeated until it converges to a locally optimum solution.

Quindry et al. (1979,81) have shown that Alperovits and Shamir (1977) have not included the interaction of the loop constraints with the other loop, source and nodal head constraints in their gradient expression. Interaction occurs when another flow constraint has at least one link in common with the loop whose gradient is being computed. Quindry et al. (1979) has introduced additional term to account for such interaction and when applied to the problem solved by Alperovits and Shamir (1977) it yielded 8% reduction in total cost.

Bhave (1978) has developed a manual iterative approach for minimizing the cost of a single source distribution system. The heads at the demand nodes are treated as independent variables and iteratively changed until convergence to an optimal solution occurs. diameters are continuous rather than discrete variables. Later on, the author (1982,88) has developed a method for the optimal design of multisource, looped, gravity fed water distribution systems subjected to single loading pattern. The method is based on two steps (i) Conversion of the looped distribution system to the branching one using the classical transportation problem principle and (ii) Optimization of this branching distribution systems. Both these techniques are based on LP formulation. The solution of the transportation problem gives the design paths to all the demand nodes gives the various distribution which leads to the design distribution graph. The network cost minimization model using LP techniques is then formulated.

Chiplunkar and Khanna (1983) have reported a rational procedure for the optimization of the branched rural water supply networks using Lagrangian multiplier techniques which appends the equality constraints to the objective function (cost function) through Lagrangian multiplier. The minimization of Lagrangian function amounts to the minimization of the objective function as the constraint is satisfied. The solution of these equations results in a minimum cost design if the Lagranigian multipliers are positive (Rao, 1978).

The method is applied to branched water supply networks. This method is, therefore, more suitable to the rural water supply schemes which are substantially different from the urban installations. The rural water supply schemes are characterised by dead-end distribution where the flow quantities flowing in various sections are known. The distributions are single branched or multiple branched system depending on the population distribution. For a single branch dead end distribution, Lagrangian multiplier technique is quite suitable. For a multibranch system, the number of nonlinear equations increases warranting a standard nonlinear algebraic equation solutions. This procedure is simplified by considering each branch to be initially independent (Thereby requiring one Lagrangian multiplier) while estimating the multiplication factor for that branch and later combining all branches by choosing the maximum multiplication factor for common sections. This is an approximation to the exact design where all branches are considered simultaneously (thereby, requiring as many Lagrangina multiplier as the branches). It is reported that using the approximation does not alter the final design in significant manner.

The use of linear programming (LP) or dynamic programming (DP) to solve water quality management problems has been successfully embloged by many reachers, including Locuks et al. (1967), ReVelle et al. (1968), and Liemban and Lynn (1966). The issue of planning for capacity expansion was addressed by deLucia et al. (1978), who also used a mathematical programming based model. An additional concern that has received more attention recently in the issue of the probabilistic nature of water-quality phenomena. Although not initially addressed in the planning context, the randomness inherent in the water quality process by Loucks and Lynn (1966). A comprehensive optimization model based on the modeling framework of Fuziwara et al. (1986) was presented by Ellis (1987), to incorporate the waste allocation process.

2.2. Summary:

The linear programming (LP) techniques truly yield the global optimum solution for branching distribution network only. As the resistance and cost of the pipe are linear function of its length, the different pipe lengths constituting a link are taken as the decision variable and the LP problem is formulated. However, for a looped network, the link discharges cannot be computed apriori and thus the problem is not directly amenable to LP analysis. Also in any case, the nonlinear objective function or the nonlinear constraints are approximated as linear functions. In such cases, the optimal solution may not be the global minimum. The use of dynamic programming in network optimisation has been limited to tree shaped network only. Much work still remains to be done on its application on the design of multiple looped network. Generally optimisation is based on minimisation of cost, however, most of the cost of hydraulic parameters are uncertain or difficult to quantify. The uncertainty is the estimation of the hydraulic and economic parameters and its effects on the optimal design of the network system has not received the attention it deserves.

At the same time goal programming solution and the optimum cost solution for stream standards also generate information such as trade of function between treatment costs and achieving stream quality for variouses and the impact of designed users quality levels on regional budget goals, which can be used as a basis for finalising a management plant needs much attention for further research.

2.3 Identification of Thrust Areas for Further Research:

Review of the literature has revealed the following areas in the optimal design of hydraulic network systems and water quality management which needs further research effort.

1. There is a need to develop an optimisation of water supply system in its entirely based on more exact relationship for head loss.

2. A new approach for optimal design of both small and large distribution networks for water supply, considering the pipe diameter as discrete function.

3. A method for determining optimal diameter of pipe under uncertain conditions of water supply.

4. Multiobjective analysis for regional water quality management.

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CHAPTER - 3

DECOMPOSITION PRINCIPLE BASED OPTIMIZATION OF WATER DISTRIBUTION SYSTEM.

3.1 Introduction:

A water distribution system transports water from sources of supply to various demand points. A system consists of sources of water supply, pumping stations and demands for water, all connected by pipe lines. In a city of moderate size, there may be number of supply centres and thousands of demand points. For the sake of simplicity, a small system will be used as an example. Figure 3.3.1 shows the system where there are one supply centre, three demand points, and one pumping station.

A linear programming formulation for an optimal design of a water supply system is presented by Gupta [1] the costs of pipes in a given pipe network satisfying customers' demand for water is minimised. Work on optimal design and operation of water distribution systems upto 1973 has been reviewed by Shamir [2]. Subsequent works in this area are those by Watanatada [1973], Hamberg [1974], Rasmusen [1976], Bhave [1978], Bhave [1983], and Martin [1988]. We have shown here how a large linear programming problem for a water distribution system can be solved by means of decomposition principle. To the best of our knowledge solving linear programming problem for a water distribution system presented in this paper is first to incorporate the optimisation of water distribution system by decomposition principle.

3.2 A Brief Account of Decomposition Principle:

The decomposition principle is a systematic procedure for solving large-scale linear programs that contain constraints of special structure. The constraints are devided into two sets: general constraints and constraints with special structure.

The strategy of the decomposition principle is to operate on two separate linear programs: one over the set of general constraints and one over the set of special constraints. The linear program over the general constraints is called the master problem, and the linear program over the special constraints is called the subproblem. The information is passed back and forth between the two linear programs until a point is reached where the solution to the original proglem is achieved.

Let us consider the following linear program,

Minimize cx
s.t. $Ax = b$
$x \in S$

Where S is a polyhedral set representing constraints of special structure, A is a mxn matrix, and b is an m column vector. Assume that S is bounded. Let x_1, x_2,....x_t be extreme points of S. Then any point x∈S can be expressed as

$$x = \sum_{j=1}^{t} \lambda_j x_j ,$$

$$\sum_{j=1}^{t} \lambda_j = 1,$$

$$\lambda_j \geq 0, \; j = 1,2,\ldots\ldots,t$$

Hence the given optimization problem is transformed into the following problem called master problem in the variablesλ_1,...,λ_t

$$\text{Minimize} \quad \sum_{j=1}^{t} (cx_j)\lambda_j \qquad \ldots (3.2.0)$$

$$\text{s.t} \quad \sum_{j=1}^{t} (Ax_j)\lambda_j = b \qquad \ldots (3.2.1)$$

$$\text{Minimize} \quad \sum_{j=1}^{t} \lambda_j = 1 \qquad \ldots (3.2.2)$$

$$\lambda_j \geq 0, \; j=1,2,\ldots.,t \qquad \ldots (3.2.3)$$

We follow the following steps to solve the above problem.

Initialization Step:

Find an initial basic feasible solution of the system defined by equations (3.2.1), (3.2.2) and (3.2.3). Let the basis be B, and the dual variables corresponding to equation (3.2.1) and (3.2.2) be w & α respectively. Then $(w,\alpha)= \hat{c}_B \times B^{-1}$, where $\hat{C}_B$ is the cost of the basic variables with $\hat{c}_j = cx_j$ for each basic variables λ_j. The basis inverse B^{-1}, the dual variables (w,α), the values of the basic variables

$$\bar{b} = B^{-1}\begin{pmatrix} b \\ 1 \end{pmatrix}$$

and the objective function $\hat{c}_B \bar{b}$ are displayed by the following master array.

BASIS INVERSE	RHS
(W,α)	$\hat{c}_B \bar{b}$
B^{-1}	$\bar{b}$

Main Step:

1. Solve the following subproblem:

$$\begin{aligned} &\text{Maximize} \quad (wA-c)\,x + \alpha \\ &\text{s.t.} \qquad x \in S \end{aligned}$$

Let x_k be an optimal basic feasible solution with objective value of

$$z_k - \hat{c}_k = \underset{1 \le j \le t}{\text{Maximum}}\ (w,\alpha)\begin{bmatrix} Ax_j \\ 1 \end{bmatrix} - cx_j$$

$$= \underset{1 \le j \le t}{\text{Maximum}}\ wAx_j + \alpha - cx_j\,;$$

If $z_k - \hat{c}_k = 0$, stop; the basic feasible solution of the last master step is an optimal solution of the overall problem. Otherwise go to step 2.

2. Let $y_k = B^{-1} \begin{bmatrix} Ax_j \\ 1 \end{bmatrix}$

Adjoine the updated column $\begin{pmatrix} z_k - \hat{c}_k \\ y_k \end{pmatrix}$ to the master array. Pivot at y_{rk}, where the index r is determined by

$$\frac{\bar{b}_r}{y_{rk}} = \underset{1 \le i \le m+1}{\text{minimum}} \left\{ \frac{\bar{b}_i}{y_{ik}} : \quad y_{ik} > 0 \right\}$$

This updates the dual variables, the basis inverse and the RHS. After pivoting, the column corresponding to λ_k is deleted and step 1 is repeated.

Now we will discuss and analyse a water distribution system through decomposition principle.

3.3 Problem formulation and Solution:

Take S_1 as a source of water supply, and D_1, D_2, D_3 be three demand points. D_1 requires a net head to 10.0m or more with discharge rate of 0.10 m^3 per second. D_2 requires a net head of 12m or more with discharge rate of 0.09 m^3 per second. D_3 requires a net head of 15 m or more with discharge rate of 0.08 m^3 per second. Pump P_1 generates a head of 70 m. General layout of the network is shown in

Figure 3.3.1. Source S_1 is big enough to meet water demands of D_1, D_2 and D_3. The different pipe size available are 10 cm, 15 cm, 25 cm and 40 cm in diameter. Costs of these pipes per m length is Rs. 3, Rs. 4, Rs. 6 and Rs. 8 respectively. Neglect minor losses in the fittings.

The problem is to design an optimum pipe network using decomposition principle.

<u>First Step</u>: Detail of the problem.

Referring to Figure 3.3.1 the following can be concluded about the network: there are two nodes n_1 and n_2 there are three open loops, loop l_1, l_2 and l_3; there are five lines, lines 1,2,3,4 and 5.

Let use number the lines as shown in Figure 3.3.1.

<u>Second step</u>: Find discharge rates througout the networks.

Discharge rates in lines 2, 4 and 5 are known. Applying the continuity equation discharge in line 1 = 0.08 + 0.09 + 0.1 = 0.27 cubic m per second and line 3 = 0.27 - 0.08 = 0.19 cubic m per second.

<u>Third Step</u>: Values of the coefficient can be computed for each pipe diameter by making use of Hazen-Williams formula. Table 3.3.1 gives various values of S_1, obtained from

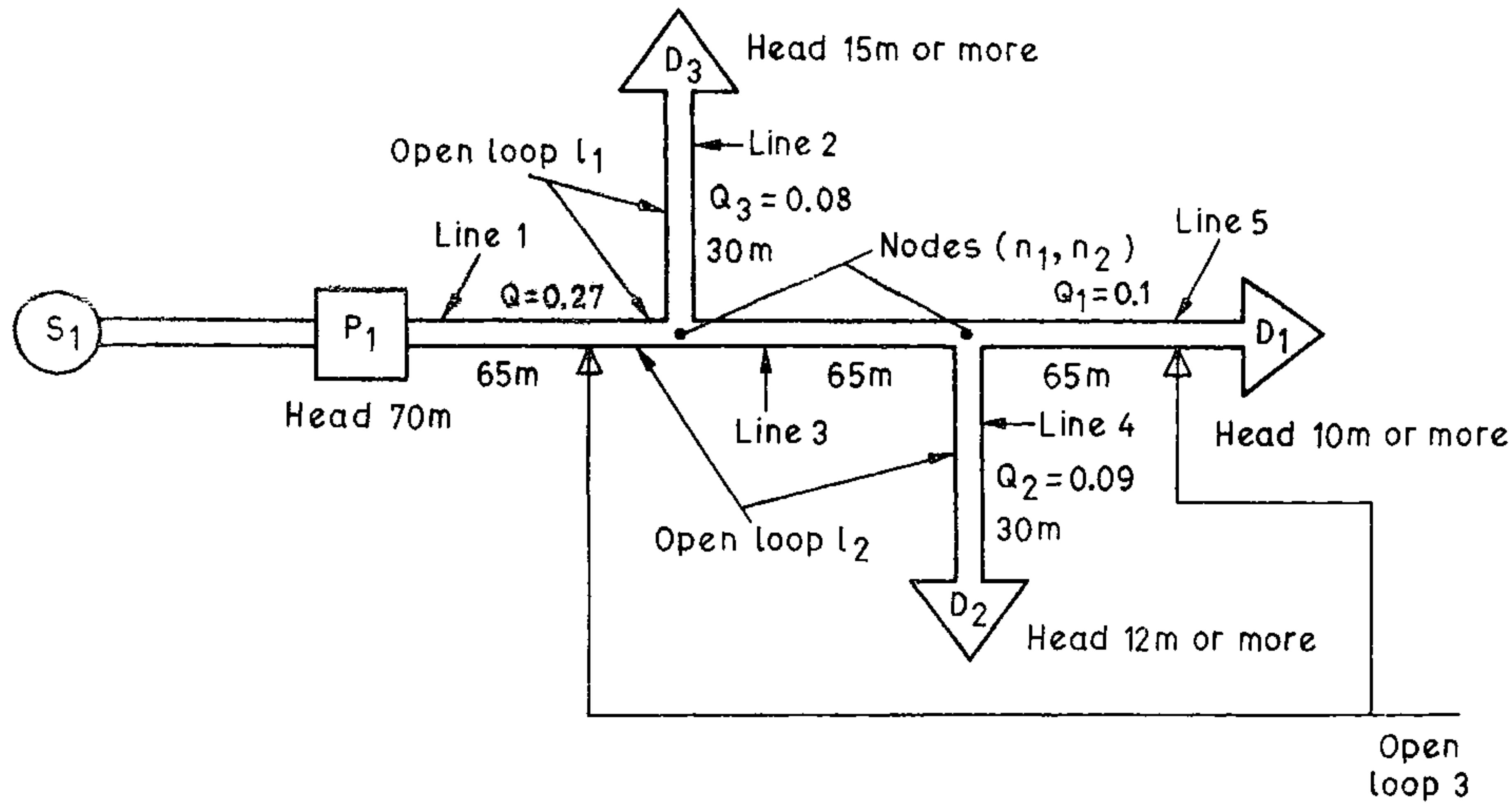

Fig. 3.3.1 Detail of Network layout

equation 3.3.1.

Hazen William' formula used is given as

$$S_f = \frac{h_f}{L} = \frac{10.68\ q^{1.852}}{C_{hw}^{1.852}\ D^{4.87}} \qquad \text{.... (3.3.1)}$$

where

h_f = Head Loss (m)

q = Rate of flow (m^3/s)

C_{hw} = Hazen William's coefficient (130 in our case)

L = Length of pipe (m)

D = Diameter of pipe (m)

S_f = Coefficient of friction using above equation values of S_f is given in following Table 3.3.1.

Table 3.3.1
Values of S_f

Pipe diameter in (cm)	Coefficient S_f				
	Line 1	Line 2	Line 3	Line 4	Line 5
15	1.18	0.124	0.617	0.155	0.188
20	0.291	0.031	0.152	0.038	0.046
30	0.04	0.0043	0.021	0.0053	0.0064
40	0.00996	0.001	0.003	0.0013	0.0016

<u>Fourth Step</u>: Calculate maximum allowable friction losses in each open loop.

Maximum allowable friction head loss in open loop 1 = 70-15 = 55.

Maximum allowable friction head loss in open loop l_2 = 70–12 = 58.

Maximum allowable friction head loss in open loop l_3 = 70–10 = 60.

<u>Fifth Step</u>: Set up restriction. L_{ij} will be the length of pipe where first subscripts denote for line and second subscripts denote for diameter, i = 1 to 5, j = 1 to 4.

Since there are three open loops and five lines. Therefore, the total number of restrictions will be eight.

1 – Loop l_1 (consists of line 1 and 2)

$[1.18\ L_{11} + 0.29\ L_{12} + 0.04\ L_{13} + 0.01\ L_{14} + 0.12\ L_{21} + 0.03\ L_{22} + 0.004\ L_{23} + 0.001\ L_{24} + 0.0\ L_{31} + 0.0\ L_{32} + 0.0\ L_{33} + 0.0\ L_{34} + 0.0\ L_{41} + 0.0\ L_{42} + 0.0\ L_{43} + 0.0\ L_{44} + 0.0\ L_{51} + 0.0\ L_{52} + 0.0\ L_{53} + 0.0\ L_{54}] \leq 55$

2. Loop l_2 (consists of lines 1,3 and 4)

$[1.18\ L_{11} + 0.29\ L_{12} + 0.04\ L_{13} + 0.01\ L_{14} + 0.0\ L_{21} + 0.0\ L_{22} + 0.0\ L_{23} + 0.0\ L_{24} + 0.62\ L_{31} + 0.15\ L_{32} + 0.02\ L_{33} + 0.003\ L_{34} + 0.16\ L_{41} + 0.04\ L_{42} + 0.005\ L_{43} + 0.001\ L_{44} + 0.0\ L_{51} + 0.0\ L_{52} + 0.0\ L_{53} + 0.0\ L_{54}] \leq 58$

3. Loop l_3 (consists of lines 1, 3 and 5)

$[1.18\ L_{11} + 0.29\ L_{12} + 0.04\ L_{13} + 0.01\ L_{14} + 0.0\ L_{21} + 0.0\ L_{22} + 0.0\ L_{23} + 0.0\ L_{24} + 0.62\ L_{31} + 0.15\ L_{32} + 0.02\ L_{33} + 0.003\ L_{34} + 0.0\ L_{41} + 0.0\ L_{42} + 0.0\ L_{43} + 0.0\ L_{44} + 0.19\ L_{51} + 0.05\ L_{52} + 0.01\ L_{53} + 0.002\ L_{54}] \le 60$

4. Line l_1

$L_{11} + L_{12} + L_{13} + L_{14} = 65$

5. Line l_2

$L_{21} + L_{22} + L_{23} + L_{24} = 30$

6. Line l_3

$L_{31} + L_{32} + L_{33} + L_{34} = 65$

7. Line l_4

$L_{41} + L_{42} + L_{43} + L_{44} = 30$

8. Line l_5

$L_{51} + L_{52} + L_{53} + L_{54} = 65$

9. $L_{ij} \ge 0$, $i = 1$ to 5, $j = 1$ to 4

<u>Sixth Steps</u>: Set up objective function using equation.

$F\ min = (3L_{11} + 4L_{12} + 6L_{13} + 8L_{14})+(3L_{21} + 4L_{22} + 6L_{23}+ 8L_{24})+(3L_{31} + 4L_{32} + 6L_{33} + 8L_{34})+(3L_{41} + 4L_{42} + 6L_{43} + 8L_{44})+ (3L_{51} + 4L_{52} + 6L_{53} + 8L_{54})$.

<u>Seventh Step</u>: Mathematical solution by decomposition principle [5,6,7].

Initialization Step:

The first three constraints are handled as $A\,x \le b$ and constraints the remaining as S. The problem is transformed into $\lambda_1, \lambda_2 \ldots\ldots \lambda_t$ as follows ($x_1, \ldots\ldots, x_t$ are the extreme points of S)

$$\text{Minimize} \sum_{j=1}^{t} (c\,x_j)\,\lambda_j \quad \text{that} \sum_{j=1}^{t} (A\,x_j)\,\lambda_j + I_s = b$$

$$\sum_{j=1}^{t} \lambda_j = 1$$

$$\lambda_j \ge 0, \quad j = 1, 2, \ldots\ldots 3$$

Where

$$A = \begin{bmatrix} 1.18 & .29 & .04 & .01 & .12 & .03 & .004 & .001 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1.18 & .29 & .04 & .01 & 0 & 0 & 0 & 0 & .62 & .15 & .02 & .003 & .16 & .04 & .005 & .001 & 0 & 0 & 0 & 0 \\ 1.18 & .29 & .04 & .01 & 0 & 0 & 0 & 0 & .62 & .15 & .02 & .003 & 0 & 0 & 0 & 0 & .19 & .05 & .01 & .00 \end{bmatrix}$$

$b = [55\ 58\ 60]^T$, $C = [3,4,6,8,3,4,6,8.3,4.6.8,3,4,6,8,3,4,6,8,3,4,6,8]$

$x = [L_{11}, L_{12}, L_{13}, L_{14}, \ldots\ldots\ldots\ldots\ldots\ldots\ldots, L_{51}, L_{52}, L_{53}, L_{54}]^T$

I is the indentity matrix of order 3x3 and $S \ge 0$ is the slack vector.

Note That

$x_1 = (0,0,0,65,0,0,0,30,0,0,0,65,0,0,0,30,0,0,0,65)^T \in S$ and $Ax_1 \le b$ is satisfied. Therefore initial basis consists of corresponding x_1 and slack variables s_1, s_2, s_3. Now

$$Ax_1 = \begin{bmatrix} 0.68 \\ 0.845 \\ 0.975 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 & 0.68 \\ 0 & 1 & 0 & 0.845 \\ 0 & 0 & 1 & 0.975 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B^{-1} \begin{bmatrix} 1 & 0 & 0 & -0.68 \\ 0 & 1 & 0 & -0.845 \\ 0 & 0 & 1 & -0.975 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$cx_1 = 1520$, $b - Ax_1 = [54.32 \quad 57.155 \quad 59.025]^T$

The Initial Table 3.3.2 is given below:

Table 3.3.2

	Basis inverse				RHS	λ_2
z	0	0	0	1520	1520	755
s_1	1	0	0	−0.68	54.32	79.62
s_2	0	1	0	−0.845	57.155	116.155
s_3	0	0	1	−0.975	59.025	(128.375)
λ_1	0	0	0	1	1	1

Let the dual variables corresponding to (1) and (2) are $w = (w_1, w_2, w_3)$ and α.

Iteration 1

<u>Subproblem:</u> Solve the following subproblem.

Maximize $(wA-c)x + \alpha$ subject to $x \in S$.

Here $w = (0,0,0)$ and $\alpha = 1520$ therefore subproblem is as follows:

$$\text{Maximize } -3L_{11} - 4L_{12} - 6L_{13} - 8L_{14} \ldots\ldots\ldots\ldots - 3L_{51} - 4L_{52} - 6L_{53} - 8L_{54} + 1520$$

subject to $x \in S$

This problem is seperable in the vectors $(L_{11}, L_{12}, L_{13}, L_{14}), \ldots\ldots\ldots\ldots, (L_{51}, L_{52}, L_{53}, L_{54})$

Here the problem decomposes into five problems in $(L_{11}, L_{12}, L_{13}, L_{14})$, ..., $(L_{51}, L_{52}, L_{53}, L_{54})$ respectively. The optimal solutions of these problems by simplex method are $(65,0,0,0)^T$, $(30,0,0,0)^T$, $(65,0,0,0)^T$, $(30,0,0,0)^T$, $(65,0,0,0)^T$ with objective values $-195, -90, -195, -90, -195$ respectively.

Hence the optimal solution of the subproblem is

$x_2 = (65,0,0,0,30,0,0,0,65,0,0,0,30,0,0,0,65,0,0,0)^T$ with objective $z_2 - \hat{C}_2 = -195-90-195-90-195+1520 = 755$. Since $z_2 - \hat{C}_2 = 755 > 0$, therefore λ_2 corresponding to x_2 enters the basis.

<u>Master Problem:</u> $z_2 - \hat{C}_2 = 755$, $Ax_2 = [80.3 \; 117.0 \; 129.35]^T$

Then

$$\begin{bmatrix} Ax_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 80.3 \\ 117.0 \\ 129.35 \\ 1 \end{bmatrix}$$

$$y_2 = B^{-1} \begin{bmatrix} Ax_2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0.68 \\ 0 & 1 & 0 & 0.845 \\ 0 & 0 & 1 & 0.975 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 80.3 \\ 117.0 \\ 129.35 \\ 1 \end{bmatrix} = \begin{bmatrix} 79.62 \\ 116.155 \\ 128.375 \\ 128.375 \end{bmatrix}$$

Inserting the column $[z_2 - \hat{C}_2 / y_2]$ into the Table 3.3.2 and pivoting we have the Table 3.3.3.

Table 3.3.3

	Basis inverse				RHS	λ_3
z	0	0	−5.881	1525.663	1172.7	389.903
s_1	1	0	−0.62	−0.083	17.695	−3.022
s_2	0	1	−0.905	0.026	3.724	−8.200
λ_2	0	0	−0.008	−0.008	0.460	0.320
λ_1	0	0	−0.008	1.008	0.54	(0.680)

Subproblem:

Iteration 2 Solve the following subproblem

Max $(wA-c)x + \alpha$ such that $x \in S$

Here $w = (0,0,-5.881)$ and $\alpha = 1525.663$

$wA-c = (-9.940,-5.705,-6.235,-8.059,-3,-4,-6,-8,-6.646,-4.882,-6.118,-8.018,-3,-4,-6,-8,-4.117,-4.294,-6.059,-8.012)$

The subproblems is Max. $-9.94L_{11}-5.705L_{12}-6.235L_{13}$ $8.059L_{14}$..........$-4.117L_{51}-4.294L_{52}-6.059L_{53}-8.012L_{54}$ $+ 1525.663$

such that $x \in S$

This problem decomposes into five problems involving $(L_{11}), L_{12}, L_{13}, L_{14})$, ... and $(L_{51}), L_{52}, L_{53}, L_{54})$. The optimal solution by simplex method are $(0.65,0,0)^T$, $(30,0,0,0)^T$, $(0.65,0,0)^T$, $(30,0,0,0)^T$, $(65,0,0,0)^T$ with objective values $-370.825, -90, -317.33, -90$, -267.605 respectively.

Hence the optimal solution of the subproblem is x_3 = $(0,65,0,0,30,0,0,0,0,65,0,0,30,0,0,0,65,0,0,0)^T$ with objective $z_3-\hat{c}_3$ = -370.825-90-317.33-90-267.605 + 1525.663 =389.903.

Since $z_3-\hat{c}_3 > 0$, therefore λ_3 corresponding to x_3 will enter the basis.

<u>Master Problem</u> $z_3-\hat{c}_3$ = 389.903

$$AX_3 = \begin{bmatrix} 22.45 \\ 28.60 \\ 40.95 \end{bmatrix}. \quad y_3 = B^{-1}\begin{bmatrix} A*3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -0.62 & -0.83 \\ 0 & 1 & 0 & -0.905 & -0.026 \\ 0 & 0 & 0 & -0.008 & -0.008 \\ 0 & 0 & 0 & 1 & 0.008 \end{bmatrix} \begin{bmatrix} 22.45 \\ 28.60 \\ 40.90 \\ 1 \end{bmatrix} = \begin{bmatrix} -3.022 \\ -8.200 \\ 0.320 \\ 0.680 \end{bmatrix}$$

Now inserting the column $[z_3-\hat{C}_3/y_3]$ into the Table 3.3.3 and pivoting, we get the Table 3.3.4.

Table 3.3.4

	Basis inverse				RHS	λ_4
z	0	0	-1.202	947.827	863.117	32.899
s_1	1	0	-0.656	4.396	20.94	-20.058
s_2	0	1	-1.003	12.178	10.235	- 0.387
λ_2	0	0	0.012	-0.482	0.206	0.376
λ_3	0	0	-0.012	1.482	0.794	0.624

Subproblem:

Iteration 3

Solve the following subproblem

Maximize $(wA-c)x + \alpha$ such that $x \in S$.

Here w = 0,0,-1.202) and α = 947.827

$wA-c$ = (-4.418,-4.349,-6.068, 8.012,-3,-4,-6,-8,-

3.745,-4.180,-6.024,-8.004,-3,-4,-6,-8,-3.228,-4.060,-6.012,-8.002)

Therefore the subproblem is as follows:

Max. $\quad -4.418L_{11}-4.349L_{12}-6.048L_{13} \quad 8.012L_{14}$.............

$\quad -3.228L_{51}-4.06L_{52}-6.012L_{53}-8.002L_{54} + 947.827$

such that $X\in S$

This problem decomposes into five problems involving $(L_{11},L_{12},L_{13},L_{14})$,............. and $(L_{51},L_{52},L_{53},L_{54})$. The optimal solution by simplex method are $(0,65,0,0)^T,(30,0,0,0)^T,(65,0,0,0)^T,(30,0,0,0)^T,(65,0,0,0)^T$ with objective values -282.685,-90,-242.425,-90,-209.82 respectively.

Hence the optimal solution of the subproblem is $x_4 = (0,65,0,0,30,0,0,0,65,0,0,0,30,0,0,0,65,0,0,0)^T$ with objective $z_4-\hat{c}_4 = -282.685-90-242.425-90-209.82 + 947.827 = 32.897$.

<u>Master Problem</u>:

$$z_4-\hat{c}_4 = 32.897$$

$$Ax_4 = [22.45 \quad 59.15 \quad 71.50]^T$$

$$y_4 = B^{-1}\begin{bmatrix} Ax_4 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.656 & 4.394 \\ 0 & 1 & -1.003 & 12.178 \\ 0 & 0 & 0.012 & -0.482 \\ 0 & 0 & -0.012 & 1.482 \end{bmatrix} \begin{bmatrix} 22.45 \\ 59.15 \\ 71.50 \\ 1 \end{bmatrix} = \begin{bmatrix} -20.058 \\ -0.387 \\ 0.376 \\ 0.624 \end{bmatrix}$$

Now inserting the column $[z_4-\hat{c}_4/y_4]$ into the Table 3.3.4 and pivoting, we get the Table 3.3.5.

Table 3.3.5

	Basis inverse				RHS
z	0	0	-2.255	990.001	845.089
s_1	1	0	-0.014	-21.318	31.086
s_2	0	1	-0.0991	11.682	10.447
λ_4	0	0	0.032	- 1.282	0.548
λ_3	0	0	-0.032	2.282	0.452

λ_5
2.701
0.559
-0.299
0.028
(0.972)

Iteration 4

Subproblem:

Solve the following subproblem

Maximize $(wA-c)x + \alpha$ such that $x \in S$.

Here $w = (0,0,-2.255)$ and $\alpha = 990.001$ Therefore

$wA-c$ = (-5.661,-4.654,-6.09,-8.023,-3,-4,-6,-8,-4.398, -4.398,-6.045,-8.007,-3,-4,-6,-8,-3.428,-4.113,-6.023,-8.005)

Therefore the subproblem is as follows:

Max. $-5.661L_{11}-4.654L_{12}-6.09L_{13}$ $-8.023L_{14}$ .... $-8.005L_{54}$ $+ 990.001$

such that $x \in S$

This problem decomposes into five problems involving $(L_{11},L_{12},L_{13},L_{14})$,.........and $(L_{51}),L_{52},L_{53},L_{54})$.

Their optimal solution by simplex method are $(0,65,0,0)^T$, $(30,0,0,0)^T$, $(0,65,0,0)^T$,$(30,0,0,0)^T$,$(65,0,0,0)^T$ with objective values -302.51-90-281.97-90-222.82 respectively.

Hence the optimal solution of the subproblem is x_5 = (0,65,0,0,30,0,0,0,0,65,0,0,30,0,0,0,65,0,0,0)^T with objective $z_5 - \hat{c}_5$ = -302.51-90-281.97-90-222.82 + 990.001 = 2.701

∴ $z_5 - \hat{c}_5 > 0$, λ_5 corresponding x_5 enters the basis

<u>Master Problem:</u>

$$z_5 - \hat{c}_5 = 2.701$$

$$Ax_5 = [22.45 \quad 28.60 \quad 40.95]^T$$

$$y_5 = B^{-1}\begin{bmatrix} Ax_5 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -0.014 & 21.318 \\ 0 & 1 & -0.991 & 11.682 \\ 0 & 0 & 0.032 & -1.282 \\ 0 & 0 & -0.032 & 2.282 \end{bmatrix} \begin{bmatrix} 22.45 \\ 28.60 \\ 40.65 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.559 \\ -2.299 \\ 0.028 \\ 0.972 \end{bmatrix}$$

Now inserting the column $[z_5 - \hat{c}_5 / y_5]$ into the Table 3.3.5 and pivoting, we get the table 3.3.6.

Table 3.3.6

	Basis inverse			RHS	λ_6
z	0	0	−2.166	983.659	665.225
s_1	1	0	0.004	− 22.631	30.826
s_2	0	1	−1.001	12.384	10.586
λ_4	0	0	0.033	− 1.348	0.535
λ_5	0	0	−0.033	2.348	0.465

Iteration 5

Subproblem:

Solve the following subproblem

Maximize $(wA- c)x + \alpha$ such that $x \in S$.

Here $w = (0,0,-2.166)$ and $\alpha = 983.659$ Therefore

$wA-c = (-5.556,-4.628,-6.087,-8.022,-3,-4,-6,-8,-4.343,-4.325,-6.043,-8.006,-3,-4,-6,-8,-3.412,-4.108,-6.022,-8.004)$

Therefore the subproblem is as follows:

Max. $-5.556L_{11}..........-8.005L_{54} + 983.659$

such that $x \in S$

This problem decomposes into five problems involving $(L_{11},L_{12},L_{13},L_{14})$,.........and $(L_{51},L_{52},L_{53},L_{54})$.

There optimal solution by simplex method are $(0,65,0,0)^T$, $(30,0,0,0)^T$, $(65,0,0,0)^T$,$(30,0,0,0)^T$,$(65,0,0,0)^T$ with objective values -300.82,-90,-281.125,-90,-221.78 respectively.

$x_6 = (0,65,0,0,30,0,0,0,65,0,0,30,0,0,0,65,0,0,0)^T$ with objective $z_6-\hat{c}_6$ = -300.82,-90,-281.125,-90,-221.78 + 983.659 = -0.066 $\leq$ 0 which is the terminal criterion. Therefore optimal solution is reached. The optimal solution x^* from Table 3.3.6 is given by

$x^* = \lambda_4 x_4 + \lambda_5 x_5 = 0.535(0,65,0,0,30,0,0,0,65,0,0,0,30,0,0,0,65,0,0,0)^T + 0.465(0,65,0,0,30,0,0,0,0,65,0,0,30,0,0,0,65,0,0,0)^T = (0,65,0,0,30,0,0,0,35,30,0,0,30,0,0,0,65,0,0,0)^T$ with objective value = 665.225.

The above results are given in Table 3.3.7 and analysed.

Table 3.3.7

Values of Length			
L_{11}	0	L_{33}	0
L_{12}	65	L_{34}	0
L_{13}	0	L_{41}	30
L_{14}	0	L_{42}	0
L_{21}	30	L_{43}	0
L_{22}	0	L_{44}	0
L_{23}	0	L_{51}	65
L_{24}	0	L_{52}	0
L_{31}	35	L_{53}	0
L_{32}	30	L_{54}	0

with objective = 665.225

3.4 Analysis of Results:

Line 1 consits of 15 cm diameter pipe 65 meter long. Line 2 consists of a 10 cm diameter pipe 30 meter long. Line 3 consists of a 10 cm. diameter pipe 35 meter long and 15 cm. diameter pipe 30 meter long. Line 4 consists of 10 cm. diameter pipe 30 meter long. Line 5 consists of 10 cm diameter pipe 65 meter long.

Remark: The work in this Chapter is published in the Journal of Indian Water Works Association [8] and has won the Kancharedii Award of 1995.

3.5 References:

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CHAPTER - 4

PIPE NETWORK ANALYSIS AND OPTIMIZATION USING FUZZY LINEAR GOAL PROGRAMMING

4.1 Introduction :

Urban water distribution systems are interconnected networks of closed conduits, valves of various types, pumps and reservoirs etc. One can arrive at suitable combination of pipe sizes leading to the least cost of the entire water distribution system satisfying the head requirement and discharge at demand points by using appropriate mathematical modelling and computer programming technique.

Earlier works on optimization of water distribution systems such as Karmeli et al. (1968) [10], Gupta (1969) [5], and Gupta et al. (1972) deal with optimal design of branched water distribution system with fixed consumptions rate at demand points. Later, Kally (1972) [7] extended the method to looped water distribution systems with same constraints and objective function. Kohlhass and Mattern (1971) used separable programming to optimize the looped water distribution system for which all heads were known in advance. Deb and Sorkar (1971) [4] used a special formulation, called equivalent diameter, to optimize the parameters of a looped system for the known pressure profiles and heads at inlet. Bhave (1978) [2] has developed an iterative procedure for minimizing the cost of a single source distribution system. Chiplunkar and Khanna (1983) [3] have reported procedure for the optimization of the branched rural water supply networks using Lagrangian multiplier techniques which appends the equality constraints to the objective function. Kessler et al. (1989) [9] has given a theoretical analysis of the Linear Programming Gradient method for optimal design of water distribution system. The method was first proposed by Shamir (1977) [1] and has received much attention in last 10 years. Nathan et al. (1994) [14] has developed mathematical model for finding out the optimal head using Separable Programming Technique.

As such no technique takes care of the variation in hourly, daily and seasonal demand of Urban Water Distribution System for the optimal design purpose. In the present work effort has been made to overcome this problem by fuzzyfying [16] the discharge at the demands points in order to get more rational results for the parameters (to be optimized) as compare to the Classical Linear Programming which does not take in account the variation of demand but most commonly used in optimization of the water distribution system. The present methodology is applicable to both looped and branched network system.

The complete methodology is described and supported by an example for which the solution was obtained using the computer program given in appendix A4.1 and A4.2.

4.2 Basic Terms of Linear Graph Theory Applied to Water Distribution System:

4.2.1 Linear Graph:

A linear graph is a graphical representation of a set of inter connected line segments. These line segments are called the branches of the linear graph. A linear graph in which all the branches have a definite orientation is called an oriented linear graph. The end point of a branch is called a node. Thus every branch has two nodes. Branch n is incident to node j if j is an end point of branch n the linear graph shown in Fig. 4.2.1a has 4 nodes (d,e,f,g)and 8 branches (1,2,.....8). Branches 1,2,3 and 6 are incident to node d. Branches 1,7 and 4 are incident to node e. Similarly one can write the incident branches of other nodes.

4.2.2 Path of Linear Graph:

A route traced out through a linear graph which goes through no node more than once is called a path. In Figure

4.2.1a branches 1,6 and 4 constitute a path, i.e. branch 2 joining nodesd and t is a subgraph of Fig. 4.2.1a graph. If for a given Graph there exists a path between every pair of nodes, then the graph is connected graph. Figure 4.2.1a represents a connected grap.

4.2.3 Circuit of Linear Graph:

A circuit is a subgraph of a given connected graph which is connected and has precisely two branches of the subgraph incident to each node of the subgraph. In figure 4.2.1a branches 1,6 and 7 and branches 4,7 and 8 forms circuit.

4.2.4 Tree of a Linear Graph:

A tree is a subgraph of a given connected graph which is (i) connected (ii) contains all the nodes of a given graph, and (iii) contains no circuits. In linear graph of Figure 4.2.1a branches 6,7 and 8 from a tree.

The tree branches are indicated inhached lines in Figure 4.2.1b other possible trees are (1,3,5),(1,4,7), (1,6,8), (2,3,7), (2,6,7), (4,3,5) and (4,6,8).

4.2.5 Chords of a Linear Graph:

Those branches of a connected graph which are not included in a selected tree are called chords. For the present case of tree (6,7,8) the chord branches are 1,2,3,4 and 5. A special class of tree called the Lagrangian Tree arises when all the tree branches of the linear Graph incident to one node. Figure 4.2.1b shows a Lagrangian Tree.

4.2.6 Fundamental Circuit of Linear Graph:

Once a tree has been selected for a connected graph, a special class of a circuits called the fundamental circuit can be defined. A tree does not contain any circuit. When one chord is added to it, exactly one circuit is formed called the fundamental circuit. Figure 4.2.1b whose the associated fundamental circuit for the choice of tree (6,7,8) if chord 1 is added then the formed fundamental circuit in (1,6,7).

4.2.7 Modelling of Physical System using Linear Graph Theory:

4.2.7.1 Across and through variable:

In modelling of physical systems using linear graph theory, we associate with each branch of the linear graph a pair of variables of the system under consideration, called the across variable and through variable. conventionally, the across variables are represented by x, and through variable is represented by y.

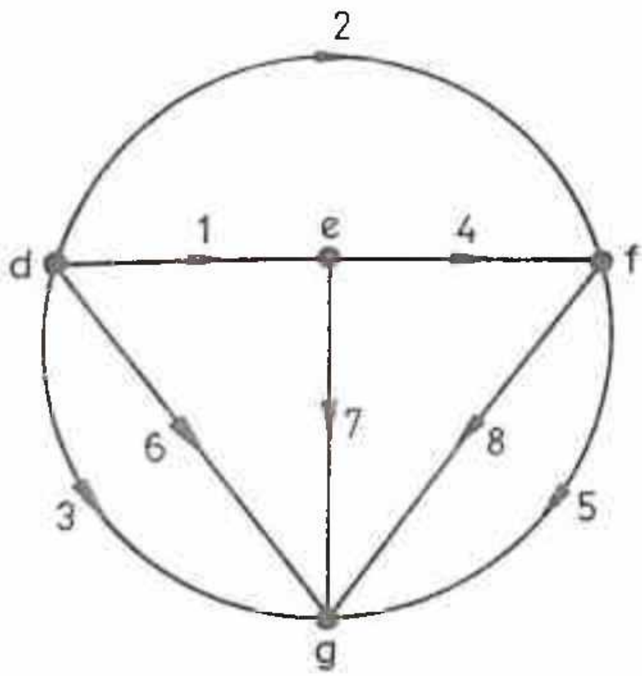

Fig.4.2.1(a) A Typical Linear Graph

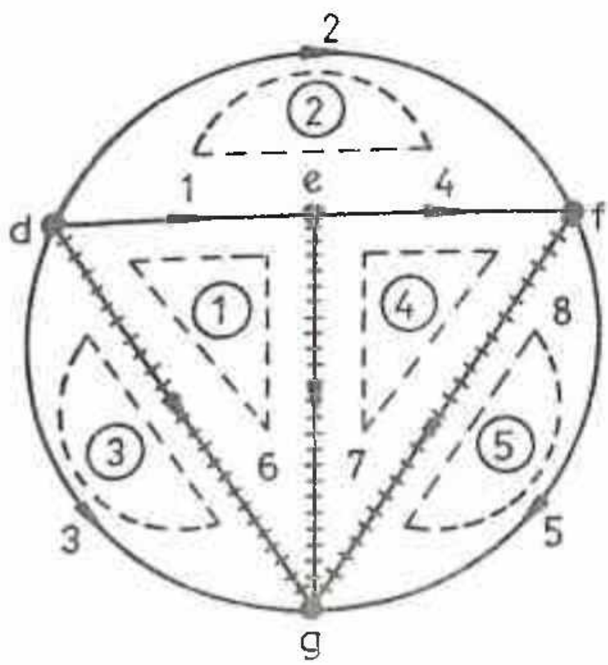

Fig.4.2.1(b) A possible tree (shown in hatched lines) and associated fundamental circuits indicated by encircled numbers around respective chords.

For an oriented linear graph, the algebraic sum of the across variables around each fundamental circuit is zero. Intaking the algebraic sum around the fundamental circuit, if the orientation of a branch constituting the fundamental circuit is coincident with that of the defining chord, then the across variable for that branch is taken as positive, it is taken as negative otherwise.

The fundamental circuit equations for the oriented linear graph shown in Figure 4.2.1b are

$x_1 + x_7 - x_6 = 0$; defined by chord 1

$x_2 + x_8 - x_6 = 0$; defined by chord 2

$x_3 - x_6 = 0$; defined by chord 3

$x_4 + x_8 - x_7 = 0$, defined by chord 4

$x_5 - x_8 = 0$; defined by chord 5 ..... (4.2.7.1.1)

Arranging above equation in matrix form

$$\text{Chords}\;\begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \end{array} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} = 0 \quad ..(4.2.7.1.2)$$

Rearranging the elements of the above matrix gives

$$
\begin{array}{c c}
 & \begin{array}{ccccc|ccc} \multicolumn{5}{c|}{\text{Chords}} & \multicolumn{3}{c}{\text{Tree branches}} \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array} \\
\text{Chords} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \left[\begin{array}{ccccc|ccc}
1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & -1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -1 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -1
\end{array}\right]
\end{array}
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ \hline x_6 \\ x_7 \\ x_8 \end{bmatrix} = 0 \qquad (4.2.7.1.3)
$$

Thus, the fundamental circuit matrix can be divided into two submatrices, one corresponding to the tree branches and other to the chord branches. It can be observed that the submatrix corresponding to the chord branches is a unit matrix I. The submatrix corresponding to the tree branches is symbolically represented by $\overline{S}$. Similarly, the column submatrix corresponding to chord branches is represented by x_c and the submatrix corresponding to tree branches by x_t. Thus we can write the equation (4.2.7.1.3) using the above notation as

$$
\left[I \mid \overline{S} \right] \begin{bmatrix} x_c \\ \hline x_t \end{bmatrix} = 0 \qquad \ldots \qquad (4.2.7.1.4)
$$

Expanding the above equation

$$
I x_c + \overline{S} x_t = 0
$$

$$
x_c = -\overline{S} x_t \qquad \ldots \qquad (4.2.7.1.5)
$$

Thus the across variables associated with the chords can be written in terms of the associated variables with the tree branches.

This is very important relation and would be made use subsequently. It is instructive to note that $\overline{S}$ matrix could be written directly from the topography of the branches of the Figure 4.2.1b, as follows

$$\overline{S} = \text{Chords} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \overset{\begin{matrix}\text{Tree branches} \\ 6 \quad 7 \quad 8\end{matrix}}{\begin{bmatrix} -1 & 1 & 0 \\ -1 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{bmatrix}} \quad \cdot\cdot \quad (4.2.7.1.6)$$

The tree branches are arranged column wise in ascending order and the chord branches are arranging row wise in ascending order.

Entries in a row corresponding to a particular chord are made as follows:

(1) Enter 1's corresponding to the tree branches in the circuit oriented in the same direction as the chord.

(2) Enter -1's corresponding to the tree branches in the circuit oriented in the direction opposite to chord.

(3) Enter 0's corresponding to the tree branches not included in the circuit of the chord.

In a parallel way, the relation between the through variables associated with the tree branches y_t can be expressed as a function of through variables associated with the chord branches y_c. The relation is given by

$$y_t = -\overline{F}\, y_c \qquad .. \qquad (4.2.7.1.7)$$

where $\overline{F}$ matrix can be written down from the fundamental cut set equations. A simpler way of getting the F matrix is to make use of the relation between F and S matrices, given by

$$\overline{F} = -\overline{S}^T \qquad .. \qquad (4.2.7.1.8)$$

Details of the above relation can be found in Nagrath & Gopal (1982)[12].

4.3 Terms and Definitions:

4.3.1 Water Distribution System:

A water distribution system transport water from source of supply to various locations where there is varying water demand for residential, commercial and industrial uses. A system consists of sources of water supply, pumping stations and demands for water, all connected by pipe lines. In a city of moderate size, there may be a number of supply centres and thousands of demand points. For the sake of simplicity, a small system will be used as an example. Figure 4.3.1 shows a typical system where there are two

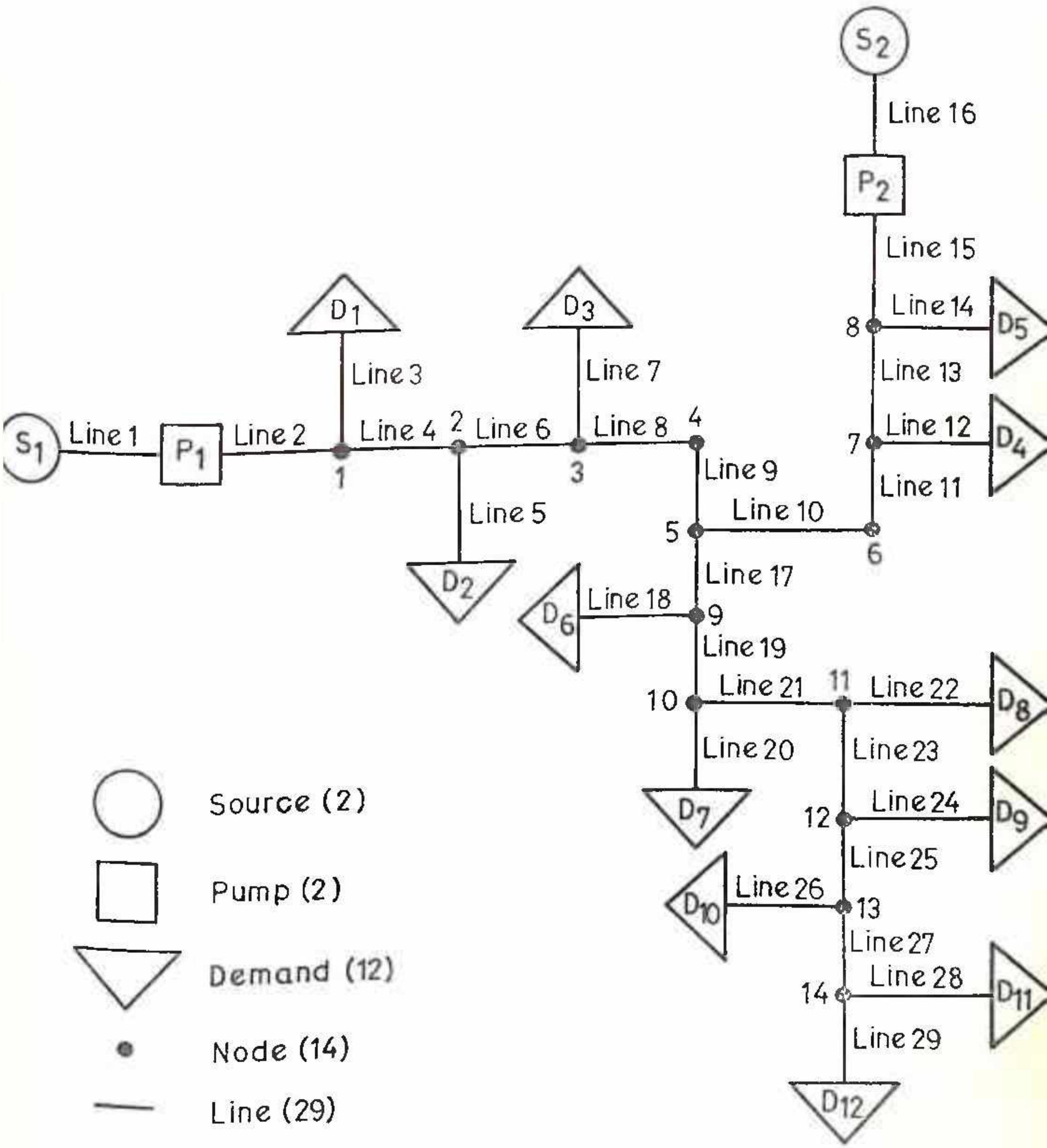

Fig. 4.3.1 Typical Water Distribution System

supply centres, 12 demand points, two pumping stations, 29 lines. (Parts of the loop between two nodes, or between a node and a demand centre, or between a node and a supply point, will be referred to as line).

4.3.2 Problem Definition:

The problem is to formulate a mathematical model of the whole pipe line system that will choose the right combination of various pipe sizes to minimize the cost of lines and at the same time satisfy the customer demand for water usage and supply pressure.

In designing even a single pipe line, millions of combinations of various pipe lengths and diameter can be used to satisfy flow and pressure requirements.

The problem is to find the optimum, least cost combination needed to build the system. In this case, a system design is required to minimize the cost of pipes subject to meeting the demand requirements.

4.3.3 Losses in Pipe flow:

For water distribution system analyses the most commonly used of the empirical formulas is the Hazen - Williams equation Most water system Engineers have a very good feel for the meaning of the Hazen - Williams C..

factor, while pipe roughness remains a mystery to many practicing engineers. For this reason, the application of the Hazen - Williams equation is used in current study. It is most commonly written as

$$S = \frac{hf}{L} = \frac{10.68\ q^{1.852}}{C_{hw}^{1.852}\ D^{4.87}}$$

where

hf = Head loss (m)

q = Rate of flow (m^3/sec)

C_{hw} = Hazen William's coefficient

L = Length of pipe in (m)

D = Diameter of pipe in (m)

S = Coefficient of friction

Virtually any practicing engineer in the water field knows that a C_{hw} of 130 is indicative of very smooth new pipe; old pipe in good condition has a C_{hw} in the range 100 to 120; and pipe with tuberculation or heavy scaling have C_{hw} factors of 80 to as low as 40.

4.4 Computer Program used for the solution of the system equation:

4.4.1 Using Simplex:

Simplex in a program that takes as input a maximization or minimization problem in form of variables and constraints and outputs the solution if possible. Simplex is the executable version obtained from the c source, simplex 1.c.

Simplex takes its input from the keyboard and sends its output to the screen. We recommended that the input for larger problems be redirected to the key board using standard key notation, <.

The usage then is

Simplex

Simplex <input file> output file

4.4.2 Order of Inputs

The order of input is as follows:

(1) 0 or 1 (0 for minimization, 1 for maximization)

(2) The number of variables in the objective function.

(3) The value of the constants in the objective function. eg. if the problem is maximization $3x_1 + 4x_2$, the inputs would be

```
1
2
-3  -4
```

(4) The total number of constraints

(5) The coefficient of constraint 1

(6) The nature of relations

1 for <=, 2 for =, 3 for >=

(7) The RHS of constraint 1

Input step (5), (6) and (7) are repeated for the remaining constraints.

4.5 LPP Formulation of the Problem:

4.5.0 Problem Definition:

Consider the network layout shown in Fig. 4.5.1 in which S_1 as a source of water supply, P_1 is the pumping station, D_1, D_2, D_3, D_4 and D_5 are demand points forming the closed loop network. Other details concerning to demand points are given in following Table 4.5.1. Pump P_1 generates net head of 100m. Source S_1 is big enough to meet water demands of D_1, D_2, D_3, D_4, & D_5. The different pipe sizes available are 20 cm, 25 cm, 32 cm, and 40 cm in diameter. Costs of these pipes per m length are Rs. 40, Rs. 60, Rs. 80 and Rs. 105 respectively. Neglect minor losses in the fittings.

Table 4.5.1

S.No.	Demand points	Head required (m)	Varying Disharge rate (m^3/sec)
1.	D_1	10	<0.06, 0.1, 0.14>
2.	D_2	15	<0.04, 0.17, 0.25>
3.	D_3	15	<0.04, 0.17, 0.25>
4.	D_4	12	<0, 0.09, 0.13>
5.	D_5	12	<0.05,0.09,0.13>

The value of S for different pipe diameter and length

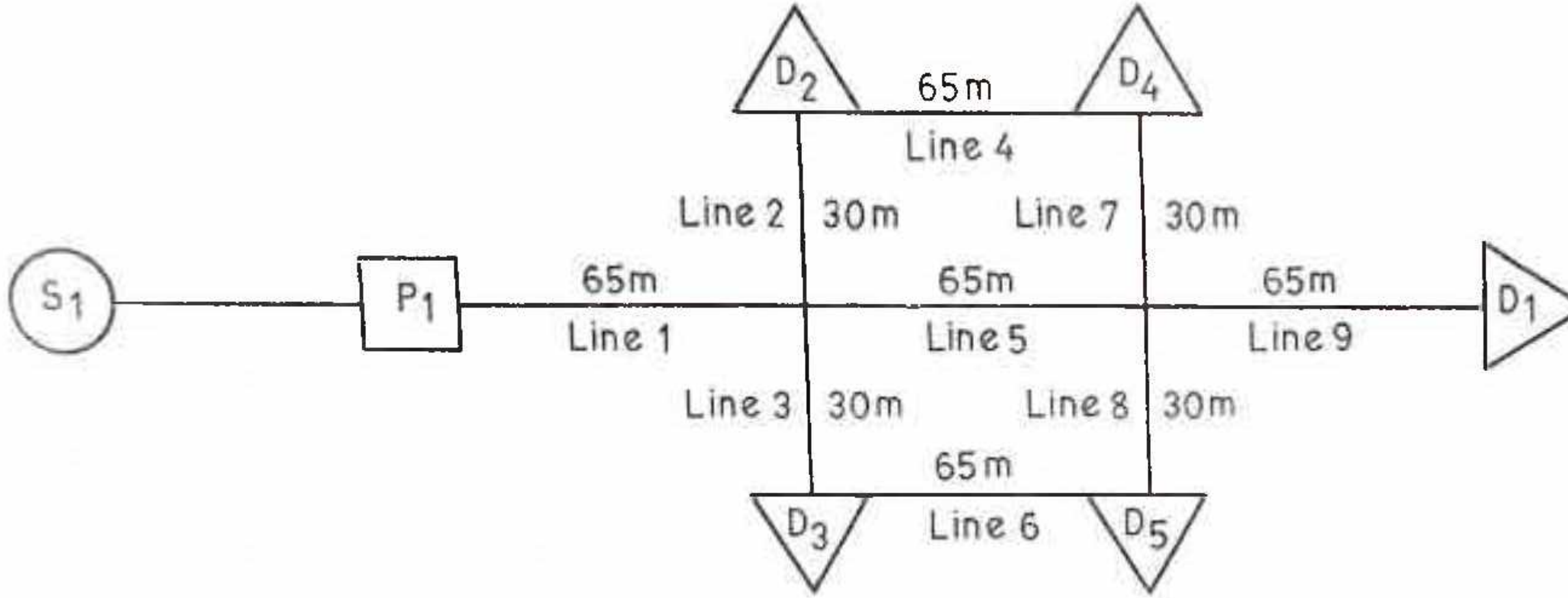

Fig.4.5.1 Detail of Looped Network Layout

had been calculated by Hazen-Williams formula and is given in following Table 4.5.2.

Table 4.5.2
Coefficient S x 10^{-3}

Pipe diameter (cm)	Line 1	Line 2	Line 3	Line 4	Line 5	Line 6	Line 7	Line 8	Line 9
20	1440.72	252.65	252.65	603.30	75.26	75.26	75.26	75.26	66.33
25	485.99	85.22	85.22	203.51	25.93	25.93	25.93	25.93	29.12
32	146.05	25.61	25.61	61.16	7.63	7.63	7.63	7.63	8.75
40	49.27	8.64	8.64	20.63	2.57	2.57	2.57	2.57	2.95

4.5.1 Linear Programming Formulation:

To design, a linear programming model, the following are necessary

(1) An objective function

(2) A set of restrictions

It has been also assumed that the following information has been specified and must be met by design (1) Pump Head at source (2) Minimum head required at each demand point (3) Minimum flow requirement at each demand (4) length of each line.

The LPP formulation of the problem became a combination of "n" available pipe diameters in the designing of a network of "m" lines, so that the total cost of pipe can be minimized in such a way that the discharge requirement of each of the "p" customers were satisfied simultaneously.

The LP model for such a system is written as follows:

4.5.1.1 Objective function:

A loop may have one or more lines, and a line may consists of one or more pipe sections with different diameter. Different diameter pipes cost different amounts per meter length of pipe, that is, the meter cost is higher for larger diameters. The problem is to choose the minimum cost combination of pipe diameters and lengths head line the objective function can be represented by

$$F_{min} = \sum_{j} \sum_{i} C_j l_{ij}$$

4.5.1.2 Constraints for fundamental circuit:

By knowing the pump head at each source of supply and the head required at each demand point, the total allowable friction head loss in each and every line can be compared. Minor losses because of fittings will be neglected. Loss in any fundamental circuit can be computed as

$$H = H_p - H_d$$

where

H = Maximum allowable friction loss in a fundamental circuit

H_p = Pump head

H_d = Minimum head required at each demand point thus can represented mathematically as follows:

$$\sum_{j}^{n} \sum_{i}^{m} S_{ijk}\, l_{ijk} \leq H_k$$

for each k fundamental circuit (k = 1,2,.....,p)

where S_{ijk} = 0 for all lines not belonging to the k^{th} fundamental circuit.

where

n = Total number of various pipe diameter available in network.

m = Total number of lines in the system

k = Total number of fundamental circuit in the linear graph drawn for the system

S_{ijk} = Friction head loss per meter length of pipe in line i, diameter j and belonging to fundamental circuit k.

l_{ijk} = length of pipe section in line i, having diameter j and belonging to fundamental circuit k.

C_j = Cost per meter length of pipe of diameter j.

H_k = Maximum allowable friction head loss in fundamental circuit k and

l_i = length of line i.

4.5.1.3 Constraint for pipe length in each line:

The length of each line should be as given below mathematically represented as:

$$\sum_{j} l_{ij} = l_i,\ i = 1,2,.......,m$$

l_{ij} = length of pipe in line i having diameter j.

Let i denote line i = 1,2,......,m

j denote diameter, $j = 1,2,\ldots\ldots,n$.

k denote fundamental circuit $= 1,2,\ldots\ldots,p$.

Above two set of restrictions can be summarized as

Set 1: Frictional loss of head should not exceed a certain value in each loop.

Set 2: Lengths of each line in each loop should be as given.

The problem thus reduces to choosing, with consideration of the variety of pipe costs, the right combination of pipe diameters and lengths which meet the above set of restrictions and have the minimum cost. Such a design will be an optimum design for simultaneously meeting all minimum demands within the given layout of the system and for the pipe variety available.

Since the system has p demand centres (forming p restrictions) and m lines (forming m additional restrictions), there are a total of (p + m) restrictions in the formulation. These are n diameters of pipe and m lines and therefore mn unknowns to find.

Each unknown, l, represents the length in m of a particular diameter pipe within a particular line. Each unknown has in own coefficient of frictional per m loss, S. All pipe lengths of the same diameter share the same coefficient of per m cost, C. In general, there are more unknown, mn, than there are conditions, p + m, so that an infinite number of solution may be feasible. The LP formulation allows choosing among these according to some criterion, in this case, minimum cost.

4.5.2 Objective Function of the Problem:

Objective function of the problem in which cost is to be minimized is given as follows:

$$F_{min} = (45L_{11}+60L_{12}+80L_{13}+105L_{14})+(45L_{21}+60L_{22}+80L_{23}+105L_{24})$$
$$+ (45L_{31}+60L_{32}+80L_{33}+105L_{34})+(45L_{41}+60L_{42}+80L_{43}+105L_{44}) +$$
$$(45L_{51}+60L_{52}+80L_{53}+105L_{54}) +(45L_{61}+60L_{62}+80L_{63}+105L_{64}) +$$
$$(45L_{71}+60L_{72}+80L_{73}+105L_{74}) +(45L_{81}+60L_{82}+80L_{83}+105L_{84}) +$$
$$(45L_{91}+60L_{92}+80L_{93}+105L_{94})$$

4.5.3 Constrains for Fundamental circuits of the Problem:

Constrains for 7 fundamental circuits which are obtained from the following linear graph Figure 4.5.3.1 drawn for the actual problem in given as follows:

1. Constraint for fundamental circuits 1 (consists of line 1 and 2):

$$H_1 + H_2 \leq P_1 - D_2$$

$$[1440.72 \times 10^{-3}L_{11}+485.99\times10^{-3}L_{12} + 146.05\times10^{-3}L_{13} + 49.27\times 10^{-3}L_{14} + 252.65\times 10^{-3}L_{21} + 85.22\times 10^{-3}L_{22} + 25.61\times 10^{-3}L_{23} + 8.64\times 10^{-3}L_{24}] \leq 85$$

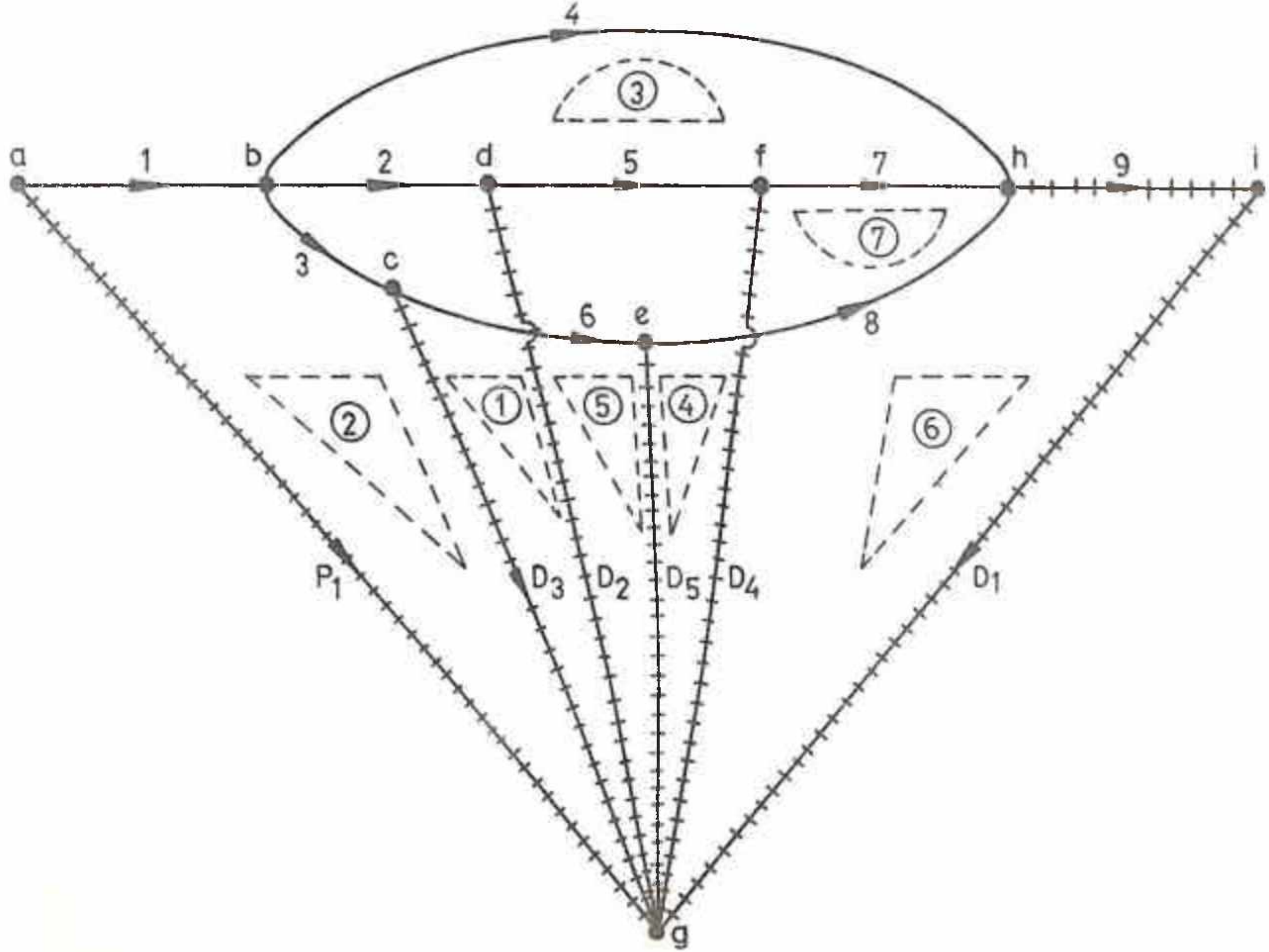

Fig.4.5.3.1 A possible tree (shown in hatched lines) and associated fundamental circuits indicated by encircled numbers around respective chords.

2. Constraint for fundamental circuit 2(consists of line 1 and 3):

$$H_1 + H_3 \le P_1 - D_3$$

$[1440.72 \times 10^{-3}L_{11} + 485.99\times10^{-3}L_{12} + 146.05\times10^{-3}L_{13} + 49.27\times 10^{-3}L_{14} + 252.65\times 10^{-3}L_{31} + 85.22\times 10^{-3}L_{32} + 25.61\times 10^{-3}L_{33} + 8.64\times 10^{-3}L_{34}] \le 85.$

3. Constraint for fundamental circuit 3 (consists of line 1,4, and 9).

$$H_1 + H_4 + H_9 \le P_1 - D_1$$

$[1440.72 \times 10^{-3}L_{11} + 485.99\times10^{-3}L_{12} + 146.05\times10^{-3}L_{13} + 49.27\times 10^{-3}L_{14} + 603.30\times10^{-3}L_{41} + 203.51\times10^{-3}L_{42} + 61.16\times 10^{-3}L_{43} + 20.63\times10^{-3}L_{44} + 86.33\times10^{-3}L_{91} + 29.12\times10^{-3}L_{92} + 8.75\times10^{-3}L_{93} + 2.95\times10^{-3}L_{94}] \le 90.$

4. Constraint for fundamental circuit 4 (consists of line 5):

$$H_5 \le D_2 - D_4$$

$[75.2\times10^{-3}L_{51} + 25.93\times10^{-3}L_{52} + 7.63\times10^{-3}L_{53} + 2.57\times10^{-3}L_{54}] \le 3.0.$

5. Constraint for fundamental circuit 5 (consists of line 6):

$[75.26\times10^{-3}L_{61} + 25.93\times10^{-3}L_{62} + 7.63\times10^{-3}L_{63} + 2.57\times10^{-3}L_{64}] \le 3.0.$

6. Constraint for fundamental circuit 6 (consists of line 7 and 9):

$$H_7 + H_9 \leq D_4 - D_1$$

$$[75.26x10^{-3}L_{71} + 25.93x10^{-3}L_{72} + 7.63x10^{-3}L_{73} + 2.57x10^{-3}L_{74} + 86.33x10^{-3}L_{91} + 29.12x10^{-3}L_{92} + 8.75x10^{-3}L_{93} + 2.95x10^{-3}L_{94}] \leq 2.0.$$

7. Constraint for fundamental circuit 7 (consists of line 8 and 9):

$$H_8 + H_9 \leq D_5 - D_1$$

$$[75.26x10^{-3}L_{81} + 25.93x10^{-3}L_{82} + 7.63x10^{-3}L_{83} + 2.57x10^{-3}L_{84} + 86.33x10^{-3}L_{91} + 29.12x10^{-3}L_{92} + 8.75x10^{-3}L_{93} + 2.95x10^{-3}L_{94}] \leq 2.0.$$

4.5.4 Constraints for line of the Problem:

Constraintx written to fulfill the length requirements of pipe in line is given as follows:

1. Line l_1:

   $L_{11} + L_{12} + L_{13} + L_{14} = 65$

2. Line l_2:

   $L_{21} + L_{22} + L_{23} + L_{24} = 30$

3. Line l_3:

   $L_{31} + L_{32} + L_{33} + L_{34} = 30$

4. Line l_4:

$L_{41} + L_{42} + L_{43} + L_{44} = 65$

5. Line l_5:

$L_{51} + L_{52} + L_{53} + L_{44} = 65$

6. Line l_6:

$L_{61} + L_{62} + L_{63} + L_{64} = 65$

7. Line l_7:

$L_{71} + L_{72} + L_{73} + L_{74} = 30$

8. Line l_8:

$L_{81} + L_{82} + L_{83} + L_{84} = 30$

9. Line l_9:

$L_{91} + L_{92} + L_{93} + L_{94} = 65$

10. $l_{ij} > 0$, $i = 1$ to 9 and $j = 1$ to 4.

4.6 F.L.P Formulation:

4.6.1 Fuzzy Linear program (FLP) :

The FLP belongs to certain type of decision problems. Such type of a decision problem requires that the aspiration level (β) concerning accomplishment of individual goals (objectives, constraints) to be satisfied by the problem under consideration.

In FLP problem it is usually assumed that aspiration level concerning the goal Zo and contraints $b_1, \ldots, b_m$ are not ordinary numbers but fuzzy numbers $Z_{o}, b_{1}, \ldots,$

b_{-m}, characterized by the triangular membership function of the following form :

$$\mu_{-z}(Y) = \langle \underline{Z}_o, Z_o, \overline{Z}_o \rangle; \quad \mu_{bi-}(\beta i) = \langle \underline{b}_i, b_i, \overline{b}_i \rangle; \quad (2.6.1)$$

$i = 1....m$ objectives

The criteria of satisfying the objective and constraints in form of the membership functions of a fuzzy goal is given by

$$\mu_o\left(\sum_{j=1}^{n} c_j x_j\right) \qquad (4.6.1.1)$$

and fuzzy constraints $\mu_i\left(\sum_{j=1}^{n} a_{i,j}\, x_j\right)$ can be determined in the following way

$$\mu_o(x) = \mu_o\left(\sum_{j=1}^{n} c_j x_j\right) = \begin{cases} 0 & \text{for } \sum_{j=1}^{n} c_j\, x_j < \underline{Z}_o \\ \sum_{j=1}^{n} c_j x_j - \underline{Z}_o / Z - \underline{Z}_o & \text{for } \underline{Z}_o \le \sum_{j=1}^{n} c_j x_j \le \overline{Z}_o \\ 1 & \text{for } \sum_{j=1}^{n} c_j x_j > 0 \end{cases}$$

(4.6.1.2)

And per analogy :

$$\mu_i(x) = \mu_i\left(\sum_{j=1}^{n} a_{i,j}\, x_j\right) \qquad (4.6.1.3)$$

For such a vector $x = (x_1, \ldots\ldots, x_n)$ for which the degree of satisfaction due to the simultaneous satisfaction of the constraints and accomplishment of the goal attains the highest possible value:

$$\sum_{j=1}^{n} a_{i,j}\, x_j - \beta(\bar{b}_i - b_i) \le \bar{b}_i ;\quad i = 1, \ldots\ldots, m$$

For such a system Fuzzy Linear Programming model can be written as follows

Objective Function :

$$Z_{min} = \sum_{j}^{n} \sum_{i}^{m} C_j\, l_{ij}$$

where:

C_j = Cost per meter length of pipe of diameter j

l_{ij} = Length of pipe in line i having diameter j

I Constraints for Fundamental Circuit :

Linear Graph Theory, Nagrath et al.(1982), is used to write the constraints for Headloss in a closed loop (fundamental circuit).

$$\sum_{j}^{n} \sum_{i}^{m} S_{ijk}\, L_{ijk} \le H_k \quad \text{for each fundamental circuit}$$

$$k = 1, 2, \ldots\ldots, p$$

Where S_{ijk} = 0 for all lines not belonging to the k^{th} fundamental circuit. Introducing the term S_{ijk} from Hazen-Williams formulae by fuzzyfying the discharge above formulae reduces to:

$$\sum_{j}^{n} \sum_{i}^{m} [10.68(q_{min}^{1.852} + \beta(q_{max}^{1.852} - q_{min}^{1.852}))/C_w^{1.852}\, D^{4.87}]\, L_{ijk} \le H_k$$

$$\bar{X} = \arg\max_{x \in X} \mu_D (x) \qquad (4.6.1.4)$$

Where $\mu_D (x)$ is a member function of fuzzy decision :

$$\mu_{D\sim}(x) = \min \{\mu_0(x), \mu_1(x) \ldots\ldots \mu_n(x)\} \qquad (4.6.1.5)$$

Problem (6) can be written in following equivalent form:

$$\bar{X} = \arg\max_{x \in X} \bar{\beta} \qquad (4.6.1.6)$$

Such that

$$\mu(x) \geq \bar{\beta}, \quad i = 1, \ldots\ldots, m$$

The solution of the problem is obtained by determining the highest value of β (which measures the degree of satisfaction related to the solution of the problem) for which the set of satisfying decisions :

$$X_\beta = \{ x : \mu_i(x) \geq \bar{\beta}, \; i = 0, 1 \ldots, m\} \qquad (4.6.1.7)$$

is nonempty

From the Zimmermann's formulation, problem (4.6.1.7) can be written as the Classical Linear Programming Problem :

$$\bar{X} = \arg\max_{x \in X} \beta \qquad (4.6.1.8)$$

$$\sum_{j=1}^{n} C_j X_j - \beta(Z_0 - \underline{Z}) \geq \underline{Z}$$

where: n = Total number of pipe diameter available,

m = Total number of lines in the network,

k = Number of fundamental circuits

S_{ijk} = Friction head loss per meter length of pipe in line i, having diameter j and belonging to fundamental circuit k,

l_{ijk} = length of pipe section in line i, having diameter j, and belonging to fundamental circuit k,

H_k = Maximum allowable friction headloss in fundamental circuit.

II Constraints for Line :

The generalised form of constraints relation for line i having j different diameter of pipe can be written as:

$$\Sigma l_{ij} = l_i, \quad (i = 1,2,......,m)$$

where:

l_{ij} = Length of pipe in line i having diameter j

l_i = Length of pipe in line i.

4.7. Fuzzy Liner Programming formulation of the Problem:

For the problem discussed in section 4.5 F.L.P. formulation of the problem is gives as follows:

4.7.1 Objective function:

Objective problem is to minimize the cost which is given as follows:

$$F_{min} = (45L_{11}+60L_{12}+80L_{13}+105L_{14}) + (45L_{21}+60L_{22}+80L_{23}+105L_{24}) + (45L_{31}+60L_{32}+80L_{33}+105L_{34}) + (45L_{41}+60L_{42}+80L_{43}+105L_{44}) + (45L_{51}+60L_{52}+80L_{53}+105L_{54}) + (45L_{61}+60L_{62}+80L_{63}+105L_{64}) + (45L_{71}+60L_{72}+80L_{73}+105L_{74}) + (45L_{81}+60L_{82}+80L_{83}+105L_{84}) + (45L_{91}+60L_{92}+80L_{93}+105L_{94})$$

4.7.2 Constraints for Fundamental circuits:

1. **Constraint for fundamental circuit 1 (consists of** line 1 and 2)

$$[\{234.25\times10^{-3}+ \beta(1206.5 \times 10^{-3})\}L_{11} +$$
$$\{79\times10^{-3}+ \beta(406.97 \times 10^{-3})\}L_{12} +$$
$$\{23.75\times10^{-3}+ \beta(122.31 \times 10^{-3})\}L_{13} +$$
$$\{8.01\times10^{-3}+ \beta(41.26 \times 10^{-3})\}L_{14} +$$
$$\{8.48\times10^{-3}+ \beta(244.20 \times 10^{-3})\}L_{21} +$$
$$\{2.86\times10^{-3}+ \beta(82.36 \times 10^{-3})\}L_{22} +$$
$$\{0.86\times10^{-3}+ \beta(24.75 \times 10^{-3})\}L_{23} +$$
$$\{0.29\times10^{-3}+ \beta(8.34 \times 10^{-3})\}L_{24}] \leq 85$$

2. Constraint for fundamental circuit 2 (consists of line 1 and 3)

$$[(234.25\times10^{-3}+ \beta(1206.5 \times 10^{-3}))L_{11} +$$
$$(79\times10^{-3}+ \beta(406.97 \times 10^{-3}))L_{12} +$$
$$(23.75\times10^{-3}+ \beta(122.31 \times 10^{-3}))L_{13} +$$
$$(8.01\times10^{-3}+ \beta(41.26 \times 10^{-3}))L_{14} +$$
$$(8.48\times10^{-3}+ \beta(244.20 \times 10^{-3}))L_{31} +$$
$$(2.86\times10^{-3}+ \beta(82.36 \times 10^{-3}))L_{32} +$$
$$(0.86\times10^{-3}+ \beta(24.75 \times 10^{-3}))L_{33} +$$
$$(0.29\times10^{-3}+ \beta(8.34 \times 10^{-3}))L_{34}] \leq 85$$

3. Constraint for fundamental circuit 3 (consists of line 1, 4 and 9)

$$[(234.25\times10^{-3}+ \beta(1206.5 \times 10^{-3}))L_{11} +$$
$$(879\times10^{-3}+ \beta(406.97 \times 10^{-3}))L_{12} +$$
$$(0.86\times10^{-3}+ \beta(24.75 \times 10^{-3}))L_{13} +$$
$$(8.01\times10^{-3}+ \beta(41.26 \times 10^{-3}))L_{14} +$$
$$(17.98\times10^{-3}+ \beta(585.30 \times 10^{-3}))L_{41} +$$
$$(6.06\times10^{-3}+ \beta(197.45 \times 10^{-3}))L_{42} +$$
$$(1.82\times10^{-3}+ \beta(59.34 \times 10^{-3}))L_{43} +$$
$$(0.61\times10^{-3}+ \beta(20.01 \times 10^{-3}))L_{44} +$$
$$(17.98\times10^{-3}+ \beta(68.30 \times 10^{-3}))L_{91} +$$
$$(6.06\times10^{-3}+ \beta(23.06 \times 10^{-3}))L_{92} +$$
$$(1.82\times10^{-3}+ \beta(6.93 \times 10^{-3}))L_{93} +$$
$$(0.62\times10^{-3}+ \beta(0.33 \times 10^{-3}))L_{94}] \leq 90$$

4. Constraint for fundamental circuit 4 (consists of line 5):

$[\beta(75.26x10^{-3})L_{51} + \beta(25.93x10^{-3})L_{52} + \beta(7.63x10^{-3})L_{53} + \beta(2.57x10^{-3})L_{54}] \leq 3$

5. Constraint for fundamental circuit 5 (consists of line 6):

$[\beta(75.26x10^{-3})L_{61} + \beta(25.93x10^{-3})L_{62} + \beta(7.63x10^{-3})L_{63} + \beta(2.57x10^{-3})L_{64}] \leq 3.$

6. Constraint for fundamental circuit 6 (consists of line 7 and 9):

$[\beta(75.26x10^{-3})L_{71} + \beta(25.93x10^{-3})L_{72} + \beta(7.63x10^{-3})L_{73} + \beta(2.57x10^{-3})L_{74} + \{17.98x10^{-3}+ \beta(68.30 \times 10^{-3})\}L_{91} + \{6.06x10^{-3} + \beta(23.06x10^{-3})\}L_{92} + \{1.82x10^{-3} + \beta(6.93x10^{-3})\}L_{93} + \{0.62x10^{-3}+ \beta(2.33 \times 10^{-3})\}L_{94}] \leq 2.0$

7. Constraint for fundamental circuit 7 (consists of line 8 and 9):

$\{12.82x10^{-3}+ \beta(62.4 \times 10^{-3})\}L_{81} +$

$\{4.32x10^{-3}+ \beta(21.06 \times 10^{-3})\}L_{82} +$

$\{1.3x10^{-3}+ \beta(6.33 \times 10^{-3})\}L_{83} +$

$\{0.43x10^{-3}+ \beta(2.13 \times 10^{-3})\}L_{84} +$

$\{17.98x10^{-3}+ \beta(68.30 \times 10^{-3})\}L_{91} +$

$\{6.06x10^{-3}+ \beta(23.06 \times 10^{-3})\}L_{92} +$

$\{1.82x10^{-3}+ \beta(6.93 \times 10^{-3})\}L_{93} +$

$\{0.62x10^{-3}+ \beta(2.33 \times 10^{-3})\}L_{94}] \leq 2.0$

4.7.3 Constraints for line

Constraints written for pipe length is given as follows:

1. Line l_1:

   $L_{11} + L_{12} + L_{13} + L_{14} = 65$

2. Line l_2:

   $L_{21} + L_{22} + L_{23} + L_{24} = 30$

3. Line l_3:

   $L_{31} + L_{32} + L_{33} + L_{34} = 30$

4. Line l_4:

   $L_{41} + L_{42} + L_{43} + L_{44} = 65$

5. Line l_5:

   $L_{51} + L_{52} + L_{53} + L_{44} = 65$

6. Line l_6:

   $L_{61} + L_{62} + L_{63} + L_{64} = 65$

7. Line l_7:

   $L_{71} + L_{72} + L_{73} + L_{74} = 30$

8. Line l_8:

   $L_{81} + L_{82} + L_{83} + L_{84} = 30$

9. Line l_9:

   $L_{91} + L_{92} + L_{93} + L_{94} = 65$

10. $l_{ij} > 0$, $i = 1$ to 9 and $j = 1$ to 4.

4.8 Results:

The optimum length of the intabulated as follows in Table No. 4.8 using L.P.P and F.L.P.

Table 4.8
Values of optimum lengths (m)

LPP		FLP	
L_{11} = 18.88	L_{61} = 27.50	L_{11} = 51.80	L_{61} = 56.67
L_{12} = 46.12	L_{62} = 37.50	L_{12} = 13.20	L_{62} = 08.33
L_{21} = 30.00	L_{71} = 0.00	L_{21} = 30.00	L_{71} = 03.33
L_{31} = 30.00	L_{72} = 30.00	L_{31} = 30.00	L_{72} = 26.67
L_{41} = 65.00	L_{82} = 30.00	L_{41} = 65.00	L_{81} = 02.50
L_{51} = 27.50	L_{92} = 31.67	L_{51} = 65.67	L_{82} = 27.50
L_{52} = 37.50	L_{93} = 33.33	L_{52} = 08.33	L_{92} = 65.00
Optimum Cost = Rs. 24,383.40		Optimum Cost = Rs. 22,260.45	

Yielded 9.54% reduction in total cost by F.L.P.

4.8.1 Interpretation of Results:

4.8.2 Interpretation of LP Results:

Line 1 consists of 20 cm diameter pipe 18,88 m long and 25 cm diameter pipe 46.12 m long. Line 2 consists of 20 cm diameter pipe 30 m long. Line 3 consists of 20 cm diameter pipe 30 m long. Line 4 consists of 20 cm diameter pipe 65 m long. Line 5 consists of 20 cm diameter pipe 27.5 m long and 25 cm diameter 37.5 m long. Line 6 consists of 20 cm diameter pipe 27.5 m long and 25 cm diameter pipe 37.5 m long. Line 7 consists of 20 cm diameter pipe 0 m long and 25 cm diameter pipe 30 m long. Line 8 consists of 25 cm diameter pipe 30 m long. Line 9 consists of 25 cm diameter pipe 31.67m long and 32 cm diameter pipe 33.33 m long with the overall optimum cost of Rs. 24,383.40.

4.8.3 Interpretation of FLP Results:

Line 1 consists of 20 cm diameter pipe 51.80 m long and 25 cm diameter pipe 13.20 m long. Line 2 consists of 20 cm diameter pipe 30 m long. Line 3 consists of 20 cm diameter pipe 30 m long. Line 4 consists of 20 cm diameter pipe 65 m long. Line 5 consists of 20 cm diameter pipe 56.67 m long and 25 cm diameter pipe 8.33 m long. Line 6 consists of 20 cm 56.67 m long and 25 cm diameter pipe 8.33 m long. Line 7 consists of 20 cm diameter pipe 3.33 m long and 25 cm diameter pipe 26.67 m long. Line 8 consists of 20 cm diameter pipe 2.5 m long and 25 cm diameter pipe 27.5 m long. Line 9 consists of 25 cm diameter pipe 65 m long with the overall optimum cost of Rs. 22,260.45. Result obtained in table 3.7 also indicated that it yielded reduction in total cost by 9.54%.

4.9 References:

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CHAPTER - 5

WATER QUALITY INDEX OF SOME INDIAN RIVERS USING MODIFIED DELPHI METHOD

5.1 Introduction

Considerable effort has been expended in the last few decades to develop comprehensive mathematical models for management of water quality in river basins. Water quality models have proved to be powerful tools in water resources management as they can incorporate the complexity of the relevant processes in the water body into a utilitarian form for management consideration. Through the use of models both diagnostic and predictive capability is provided, diagnostic in the sense of permitting the identification and isolation of specific factors affecting the water quality and predictive in permitting evaluation of future effects of proposed changes in the water body, its input or its uses.

5.2 Approach to a Water Quality Index

The water quality index (WQI) was patterned in part on Gross National Product (GNP) and Consumer Price Index (CPI). Economic indicators are quantitative rather than qualitative and only numerical data were used in WQI. This requirement has the consequence of eliminating factors which are presently unquantified such as smell and aspects of aesthetics.

Data had to be rational and comprehensive. However, if a general pattern of reasonably uniform information was assumed, it was assumed to be universal in scope. For clarity, Data was divided into two broad areas as shown in Fig. 5.2.1. One dealing with what is discharged into water, and the other with what is in the water environment as well as certain secondary effects of water quality.

Industrial and Municipal effluent Index: There are three major use-categories of water considered in the formulation of WQI.

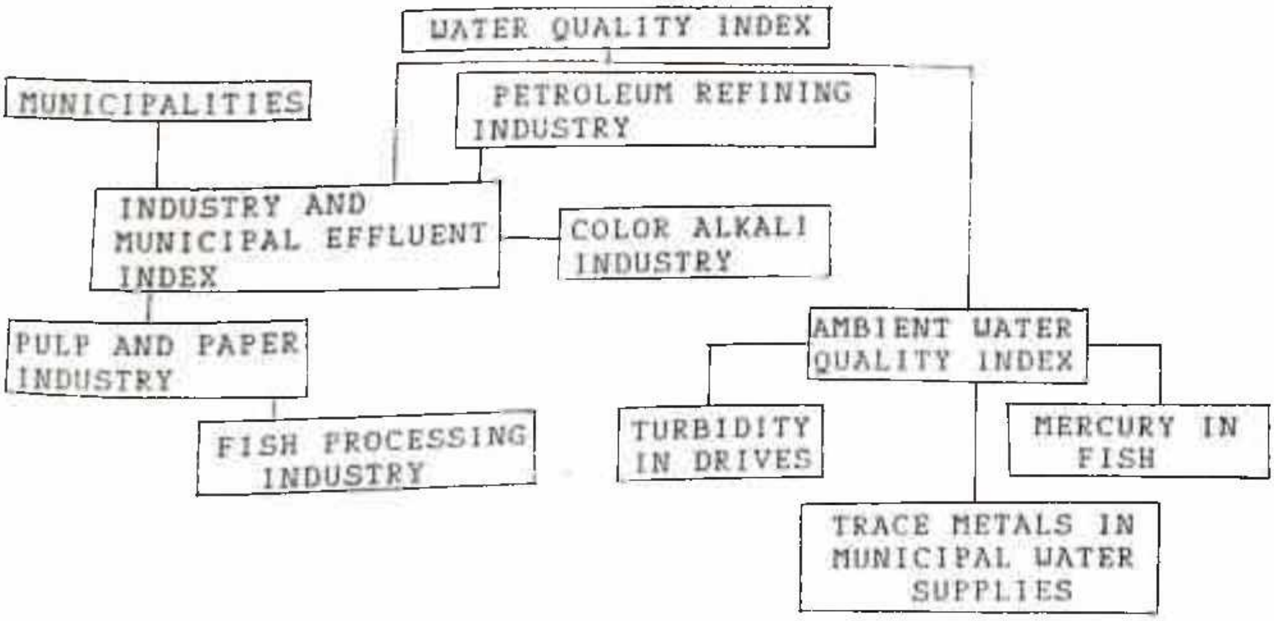


Fig 5.2.1: A Schematic Diagram of WQI

namely (a) drinking (b) fish and aquatic life and (c) recreation. Table 5.2.1 shows the objectives for concentration of various water constituents in ambient waters for each of three major user categories.

Table 5.2.1
Water Use Criteria and Weighting Factors, Mg^{-1} of Water[4]

Pollutant condition	Potable water	Fish & aquatic life	Aesthetics and Recreation	Most Stringent criterion	Weighting factor
DO	4	6	5	3	0.33
Suspended Solid	-	25	-	25	0.04
Ammonia	0.5	1	-	0.5	2
Phosphorous	-	0.015	-	0.015	66.7
Phenols	0.001	-	-	0.001	1000
Cyande	0.2	-	-	0.2	5
Mercury	0.005	-	-	0.005	200

Because quantities of water discharged are much higher in urban areas where industrial activity is the greatest, two types of indices was formulated in a comprehensive manner.

1. The normalised unit load index which shows the relative levels of pollution abatement for various sources. It is defined as follows:

$$I_{nui} = \frac{E_{ui}}{E_{uia}}$$

where:

E_{uia} = average unit load for a particular source

E_{ui} = unit load for a particular source defined as

$$E_{ui} = \frac{E_1}{X_1}$$

where:

X_1 = weight of product manufactured for industrial sources, or the population served by the municipal water systems for municipalities.

E_1 = equivalent effluent load from an industrial or municipal source and is then defined as,

E_1 = $W_1P_1 + W_2P_2 + \ldots\ldots$

where: W_1 = weighting factors for the first constituents

P_1 = actual weight discharged of the first constituent per year

2. The fatal load index conveys the idea of the actual equivalent loads being discharged to the water environment

$$I_1 = \frac{E_1}{E_{1t}}$$

where:

E_1 = as defined previously

E_{1t} = total equivalent load for all of certain locality for a particular source.

Inclusions of new types of data could bias any water quality index. These could be (i) a new waste constituent or one which was not considered important previously (ii) the change of weighting factors or (iii) the addition of a new industrial source for which previous data was not available.

5.2.1 Ambient water quality Index:

This index deals with what is in the water environment rather than what is discharged into it. This also deals with (a) trace metal contamination of waters supply (b) the suitability of rivers in terms of turbidity for drinking water supplies and contact recreation and (c) the mercury contamination of fish landed commercially.

The data was divided into three groups

(1) Cadmium and chromium (Chemicals which should be absent)

(2) Lithium, Copper and Zinc (chemicals for which definite water supply objectives are set) and

(3) Water hardness which can change the toxic effect of the above chemicals.

The subindices for the five metals and the water hardness of municipal water supplies were then combined as follows:

$$I_{TM} = \sqrt{\frac{(I_{tox})^2 + (I_{LCZ})^2 + (I_H)^2}{3}}$$

where:

I_{TM} = trace metal sub index

I_{tox} = subindex for toxic metals

I_{LCZ} = subindex for lithium, copper and zinc

I_H = subindex for hardness

Two aspects of water were considered for turbidity suitability index namely (1) suitability of water for drinking and (b) suitability for contact reaction.

Since it was judged that the drinking water and contact recreation suitability subindices would have approximately equal value in conducting an overall index for turbidity, they were given equal weight in the combination of two indices, then

$$I_{turb} = \sqrt{\frac{(I_d)^2 + (I_{cr})^2}{2}}$$

I_d = sub index for water suitability for drinking

I_{cr} = sub index for contact recreation.

The index of mercury in fish was given as

$$I_{fish} = \frac{W_1'C_1 + W_2'C_2 + \ldots}{0.5\ (W_1' + W_2' + \ldots)}$$

where:

W_1', W_2' = the landed weights for each particular species of fish

C_1, C_2 = the respective concentrations of mercury in each particular species in ppm.

Each of the three components of the ambient water quality index was given equal weights. Then

$$I_{amb} = \sqrt{\frac{(I_{TM})^2 + (I_{turb})^2 + (I_{fish})^2}{3}}$$

where:

I_{amb} = ambient index

I_{TM} = average index for trace metals
I_{turb} = Index for turbidity

I_{fish} = average index for mercury in fish.

5.2.2 Combined water quality Index:

Because the effluent and ambient water quality indices were judged to have an approximately equal weight in an overall water quality index, the equation adopted to integrating those subindices was as follows

$$I_{water} = \frac{(I_{it})^2 + (I_{amb})^2}{2}$$

5.3 Index Formulation

A water quality index may be defined as a rating reflecting the composite influence on overall quality of a number of individual quality characteristics.

5.3.1 Formulation of WQI:

One of the earlier attempts in formulating a water quality index was made by Horton [1]. He defined a WQI based on chemical and physical measurements with parameters. Sewage treatment, DO, pH, Coliform density, specific conductance, Carbon chloroform extract, alkalinity and chloride were selected; rating scales were assigned; and each parameter was weighted according to its relative significance in overall stream quality.

According to him, water quality index is

$$QI = \frac{\sum_{i=1}^{n} C_i W_i}{\sum_{i=1}^{n} W_i} \times M_1 M_2$$

where:

QI = water quality index

C = rating

W = weight

M_1 = temperature Value (1/2 or 1 depending on whether there is temperature pollution or not respectively)

M_2 = Obvious pollution Value (1/2 or 1 depending on whether there is obvious pollution or not)

The lesser the value, the higher the pollution.

Horton [1] considered 3 steps in formulation of WQI. They are as follows:

1. Selection of quality characteristics on which the index is to be based
2. Establishment of a rating scale for each characteristic
3. Weighting of several characteristics

The quality characteristics which are considered significant in the development of a WQI are given in Table 5.3.1.1.

Suggested rating scales for different characteristics and suggested weighting factors are also given in table 5.3.1.1.

Table 5.3.1.1
Quality characteristics and rating scales for development of WQI

Sewage Treatment (% pop.served)	Rating	pH	Rating	Sp.cond (μ moles)	Rating
95-100	100	6-8	100	0-750	100
70-95	80	5-6;8-9	80	750-1500	80
70-80	60	4-5;9-10	40	1500-2500	40
60-70	40	<4;>10	0	>2500	0
50-60	20				
>50	0				

DO (%saturation)	Rating	Coliforms (MPN/100ml)	Rating	CCE (1x10[illegible] mg/l)	Rating
>70	100	<1000	100	0-100	100
50-70	80	1000-5000	80	100-200	80
30-50	60	5000-10,000	60	200-300	60
10-30	30	10,000-20,000	30	300-400	30
<10	0	>20,000	0	>400	0

Alkalinity (mg/l)	Rating	Chloride (mg/l)	Rating
20-100	100	0-100	100
5-20;100-200	80	100-175	80
0-5;>200	40	175-250	40
Acid	0	>250	0

WEIGHTING			
Sewage Treatment	4	Alkalinity	1
DO	4	Chloride	1
pH	4	CCE	1
Coliforms	2	Sp.Cond	1

5.3.2 Delphi Method:

The Delphi method developed by Rand corporation [2] has also been used for development of WQI. Horton's approach uses a subjective ratings whereas in Delphi's method an opinion poll is conducted. Brown et al. have used the approach to develop a weighted mean index of form [2]

$$WQI = \sum_{i=1}^{n} W_i q_i$$

where:

WQI = water quality index, a number between 0 to 100

Q_i = quality of i^{th} parameter, a number between 0 to 100

W_i = unit weight of i^{th} parameter, a number between 0 and 1

$\sum_{i=1}^{n} W_i = 1$, n = number of parameters

There are many modifications of the Delphi method. One such modified approach was used to develop a WQI for Indian rivers and is described here in detail.

The following steps were carried out

Relative importance of parameters were determined [4]. (Table 5.3.2.1) of which DO and fecal coliforms have been rated as the most important parameters. The score rating of each parameter were developed.

Table 5.3.2.1
Relative importance ratings and weights for six parameters

Parameters	Mean of relative Importance rating	Temporary weights	Final weights
DO	2.55	1	0.25
Fecal coliform	2.58	0.99	0.25
BOD(5)	3.60	0.71	0.17
pH	4.05	0.63	0.15
Turbidity	6.65	0.38	0.09
Temperature	7.02	0.36	0.09
		4.07	1.00

They are shown in figures 5.3.2.1, 5.3.2.2, 5.3.2.3, 5.3.2.4, 5.3.2.5 & 5.3.2.6).

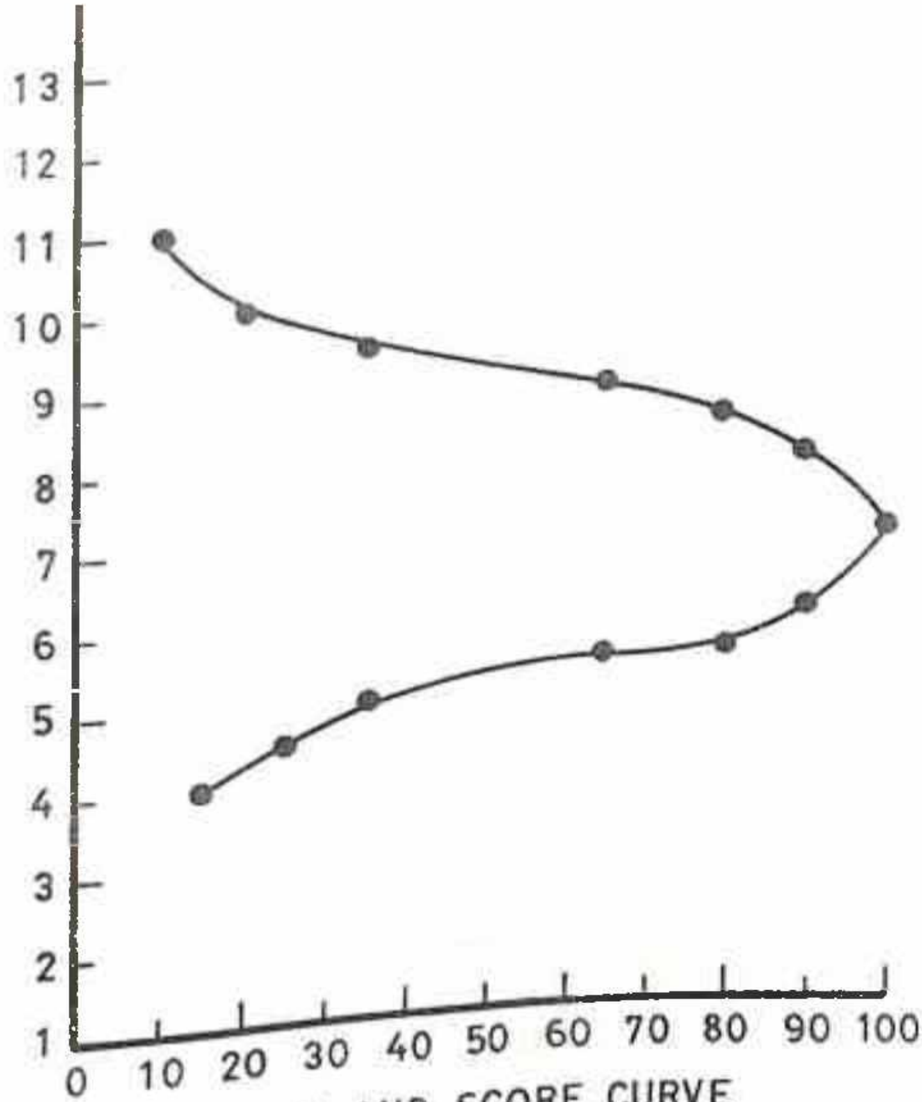
5.4 Calculation and results of WQI:

Using table 5.3.2.1 and figures 5.3.2.1, 5.3.2.2, 5.3.2.3, 5.3.2.4, 5.3.2.5 & 5.3.2.6 the WQI has been calculated for some of the rivers in India. Calculations of WQI of each river are shown in table 5.4.1.

Water quality index of each river by different methods of calculations are shown in table 5.4.2. It shows that the solway weighted method is the best one.

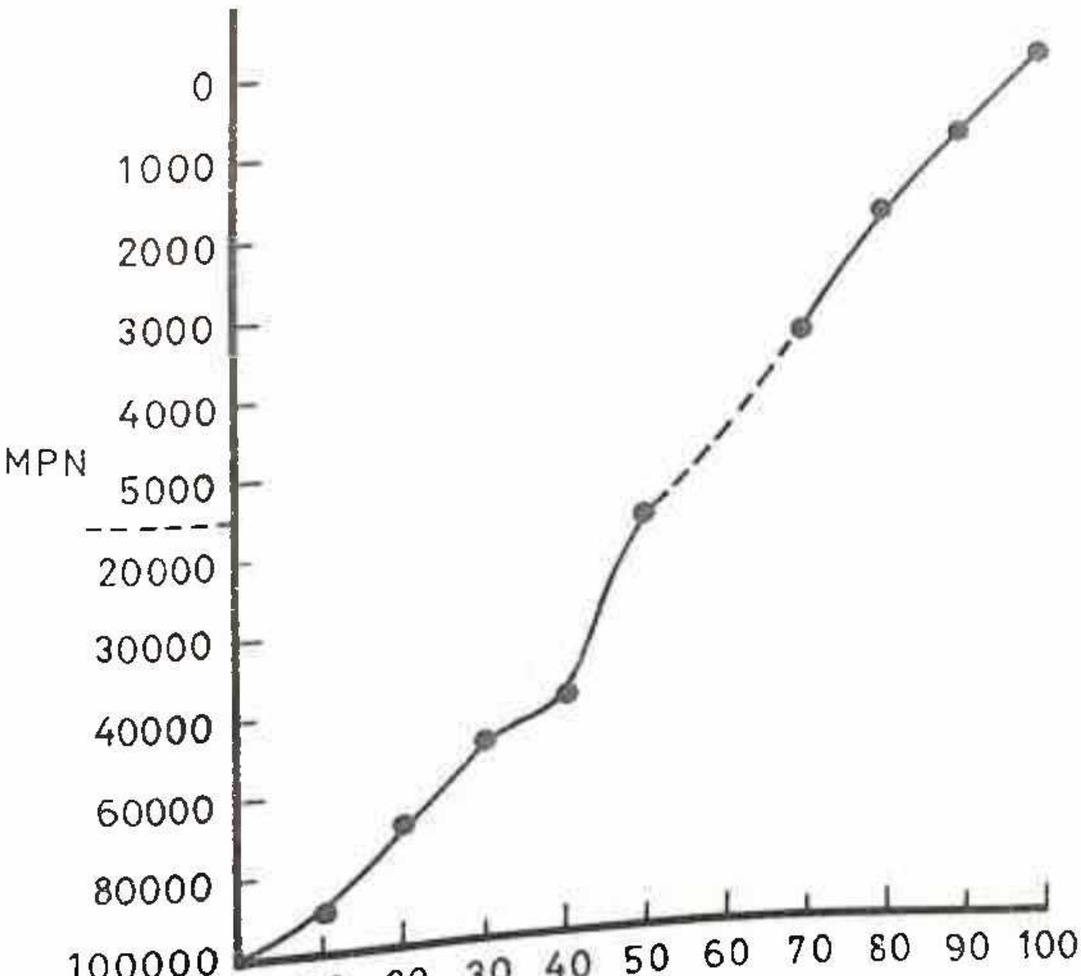
The following is the value of WQI of India

$$WQI = \frac{1}{A_R} \sum_{i=1}^{10} A_i I_{wi} = \frac{1}{23} \sum_{i=1}^{10} I_{wi} = 24.6162$$



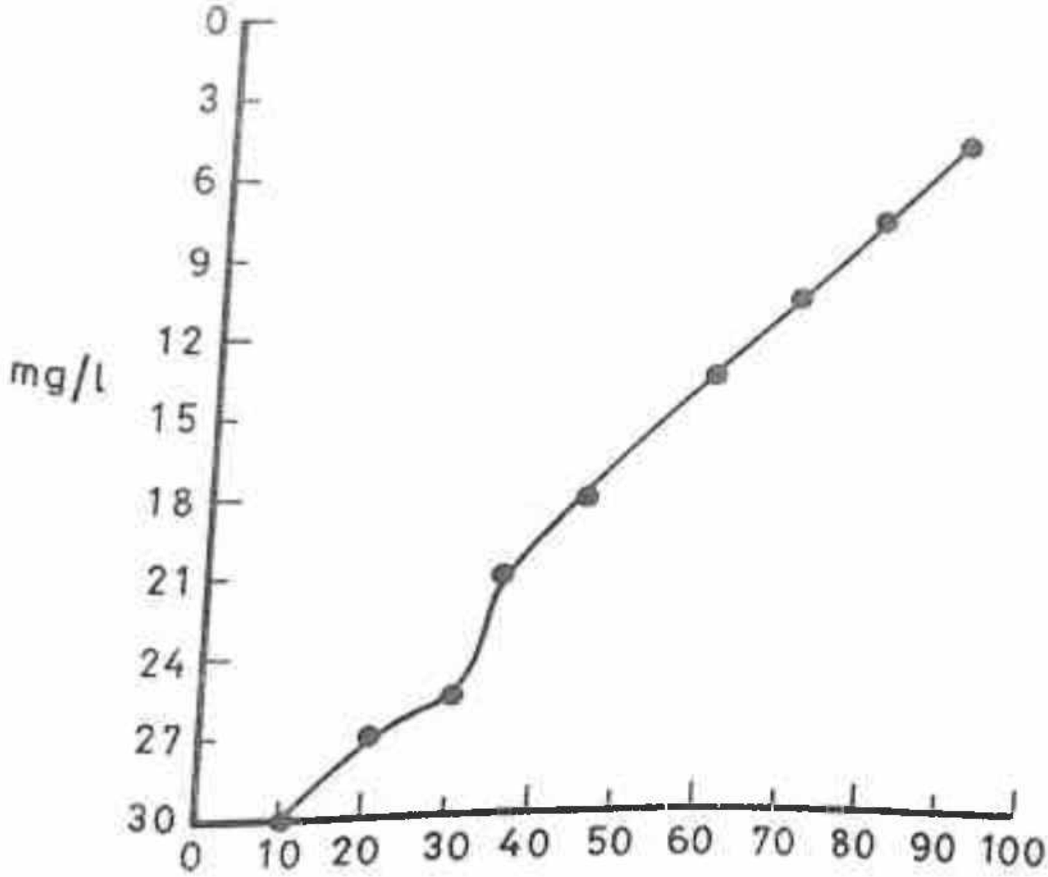
pH AND SCORE CURVE

Fig.5.3.2.1



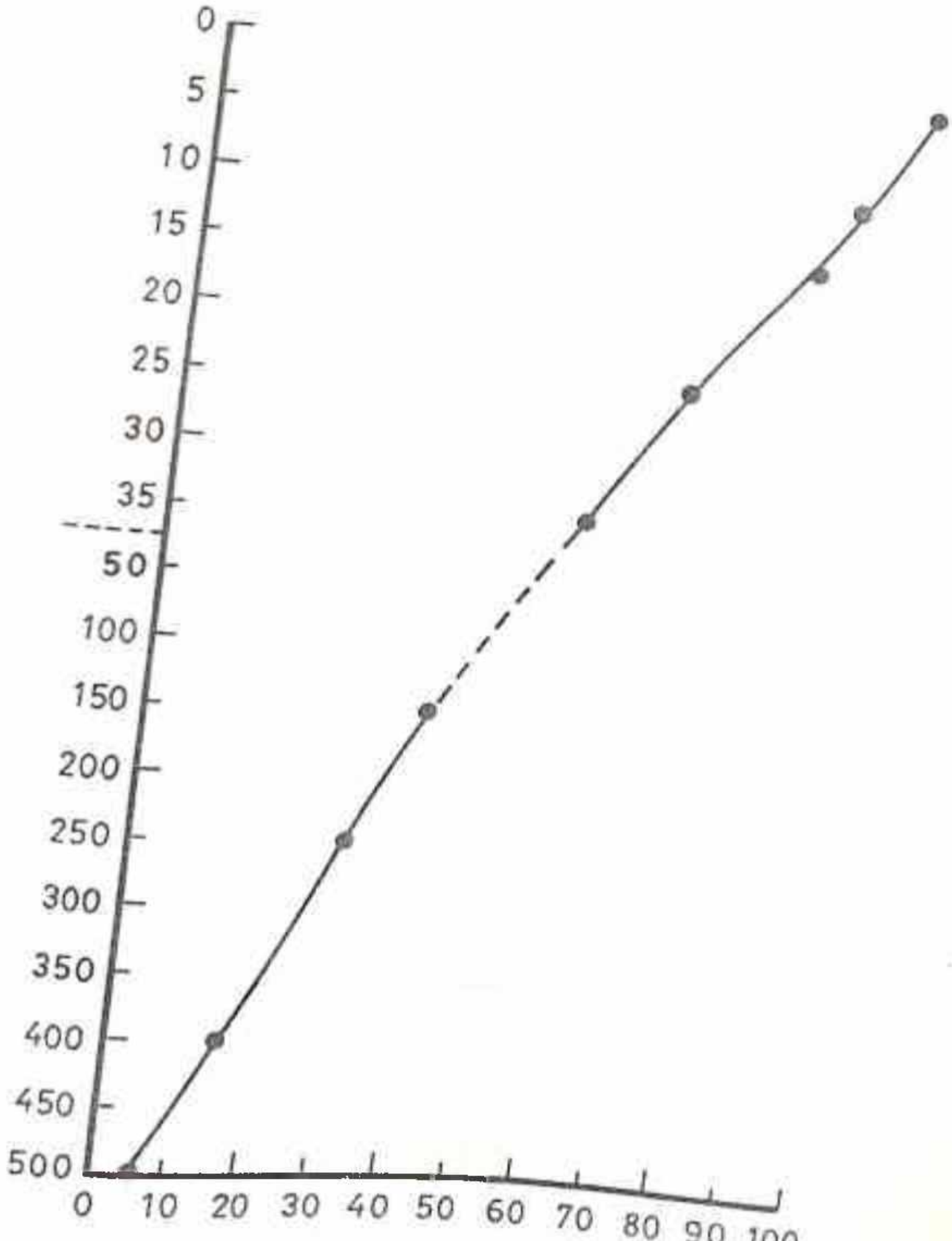
FECAL COLI AND SCORE CURVE

Fig.5.3.2.2



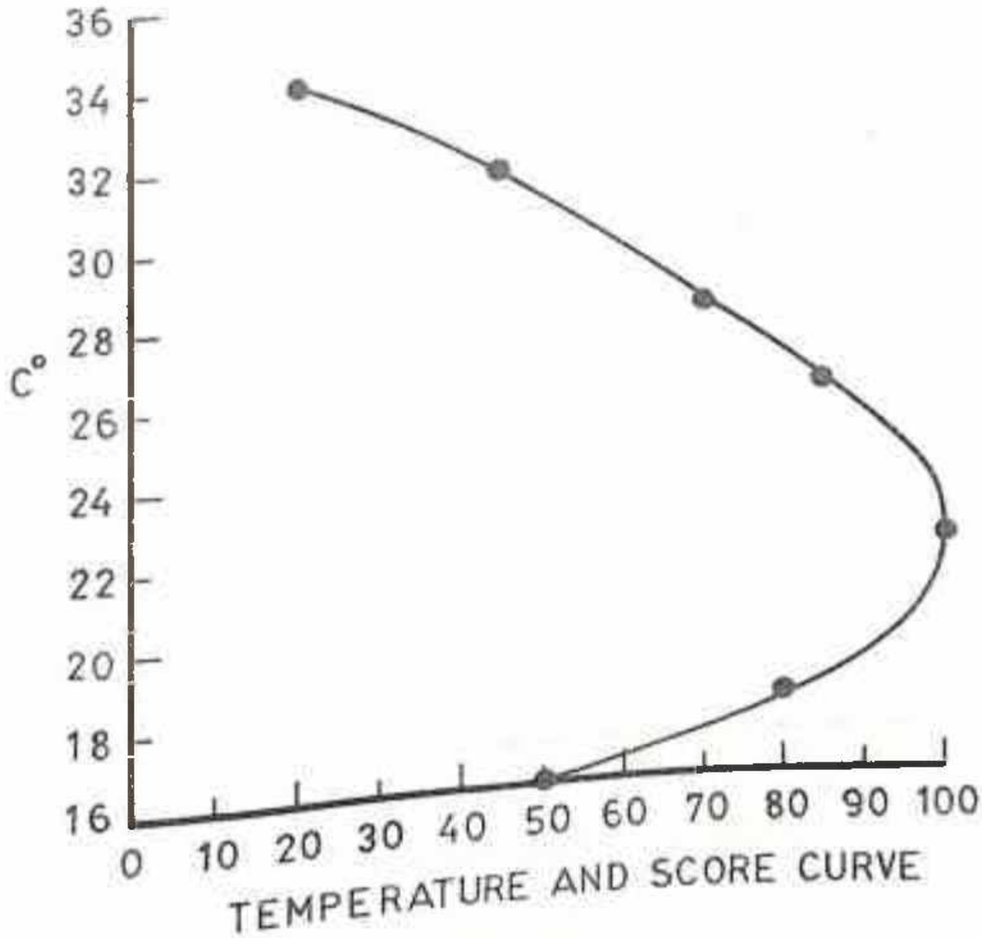
BOD_5 AND SCORE CURVE

Fig.5.3.2.3



TURBIDITY AND SCORE CURVE

Fig.5.3.2.4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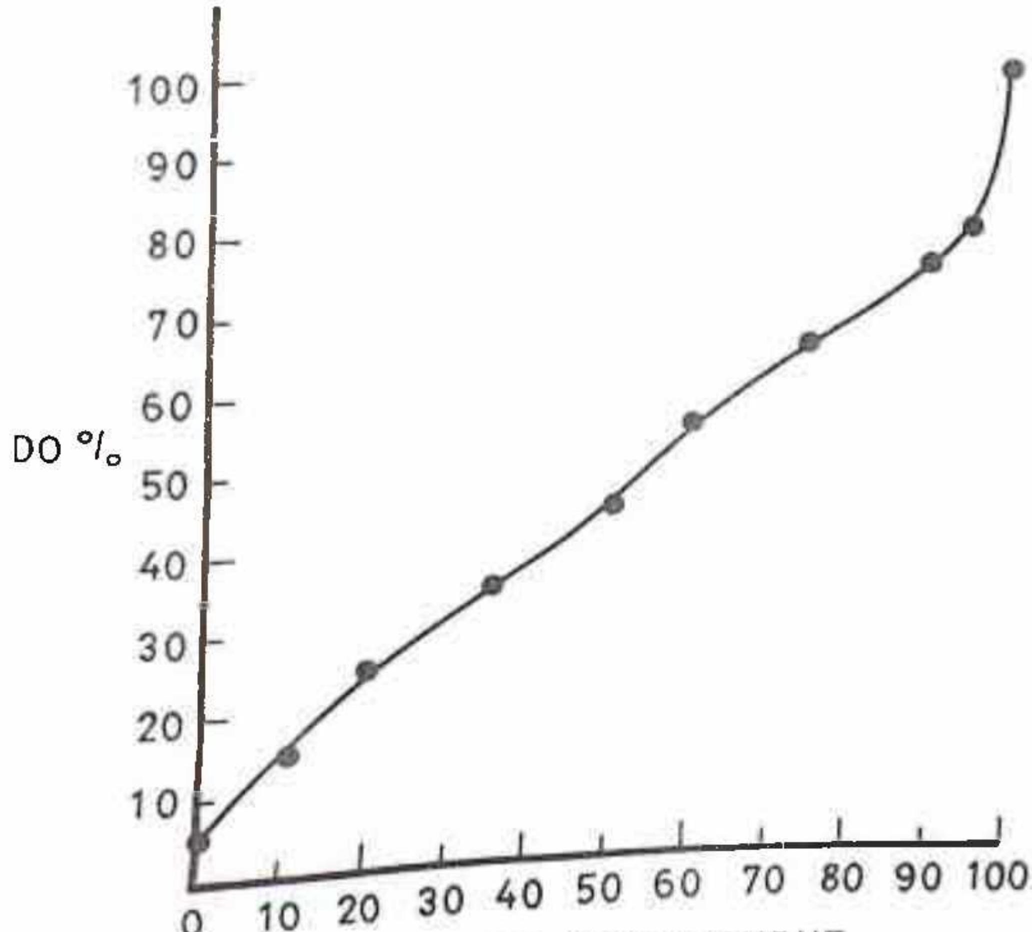
TEMPERATURE AND SCORE CURVE

Fig. 5.3.2.5



DO AND SCORE CURVE

Fig. 5.3.2.6

Application in India: WQI of different rivers by different calculations are shown in table 5.4.3. It shows that the solway weighted method is the best.

WQI1 = unweighted arithmatic index

$$= \frac{1}{n} \sum_{i=1}^{n} q_i$$

WQI2 = unweighted Solway index

$$= \frac{1}{100} \left(\frac{1}{n} \sum_{i=1}^{n} q_i \right)^2$$

WQI3 = unweighted geometric index

$$= \left(\prod_{i=1}^{n} q_i \right)^{1/n}$$

WQI4 = weighted arithmetic index

$$= \sum_{i=1}^{n} q_i w_i$$

WQI5 = weighted solway method

$$= \frac{1}{100} \left(\frac{1}{n} \sum_{i=1}^{n} q_i w_i \right)^2$$

Table 5.4.1
Calculation and Results of WQI of Some Indian Rivers

Parameters	Case - 1			Case - 2	
	w_i	q_i	$q_i w_i$	q_i	$q_i w_i$
DO	0.25	5.1	1.275	0	0
Fico	0.25	91	22.75	0	0
BOD	0.17	90	15.3	0	0
pH	0.15	80	12	86	12.9
Turb	0.09	68	6.12	48	4.32
Temp	0.09	57	5.13	20	1.8
$\Sigma q_i w_i$	62.575			19.02	
$\frac{1(\Sigma q_i w_i)^2}{100}$	39.156			3.617	

Table 5.4.1 (Continue)

Parameters	Case - 3		Case - 4	
	q_i	$q_i w_i$	q_i	$q_i w_i$
DO	4.2	1.05	3.9	0.975
Fico	0.0	0.00	0.0	0.0
BOD	90.0	15.3	90	15.3
pH	67.0	10.0	67	10.0
Turb	66.0	5.94	66	5.96
Temp	40.0	3.6	40	3.6
$\Sigma q_i w_i$	35.089		35.815	
$\frac{1(\Sigma q_i w_i)^2}{100}$	12.88		12.8271	

Table 5.4.1 (Continue)

Parameters	Case - 5		Case - 6	
	q_i	$q_i w_i$	q_i	$q_i w_i$
DO	3.8	0.95	2.5	0.625
Fico	48	12.0	99	24.75
BOD	90	15.3	90	15.3
pH	75	11.25	80	12
Turb	31	2.79	22	1.98
Temp	40	3.6	20	1.8
$\Sigma q_i w_i$	48.95		64.55	
$\frac{1(\Sigma q_i w_i)^2}{100}$	21.05		31.871	

Table 5.4.1 (Continue)

Parameters	Case - 7		Case - 8	
	q_i	$q_i w_i$	q_i	$q_i w_i$
DO	4.1	1.025	1.5	0.375
Fico	95	23.70	-	-
BOD	78	13.26	65	11.05
pH	86	12.9	80	12
Turb	12	1.08	60	5.4
Temp	20	1.8	65	5.85
$\Sigma q_i w_i$	53.816		34.675	
$\frac{1(\Sigma q_i w_i)^2}{100}$	28.961		12.02	

Table 5.4.1 (Continue)

Parameters	Case - 9		Case - 10	
	q_i	$q_i w_i$	q_i	$q_i w_i$
DO	1.5	0.375	3.5	0.875
Fico	–	–	100	25
BOD	78	13.26	90	15.3
pH	75	11.25	80	12
Turb	55	4.95	65	5.85
Temp	68	6.12	50	4.5
$\Sigma q_i w_i$	35.955		63.525	
$\frac{1(\Sigma q_i w_i)^2}{100}$	12.92		40.35	

Table 5.4.1 (Continue)

Parameters	Case - 11		Case - 12	
	q_i	$q_i w_i$	q_i	$q_i w_i$
DO	2.5	0.625	1.4	0.350
Fico	100	25	96	24.0
BOD	90	15.3	90	15.3
pH	80	12	100	5.22
Turb	25	2.25	58	3.33
Temp	50	4.5	37	
$\Sigma q_i w_i$	59.675		63.200	
$\frac{1(\Sigma q_i w_i)^2}{100}$	35.611		39.940	

Table 5.4.1 (Continue)

Parameters	Case - 13		Case - 14	
	q_i	q_iw_i	q_i	q_iw_i
DO	1.7	0.425	1.9	0.475
Fico	75	18.75	89	22.25
BOD	90	15.3	90	15.3
pH	97	14.55	97	14.55
Turb	87	7.83	72	6.48
Temp	32	2.88	32	2.88
Σq_iw_i	59.735		61.935	
$\frac{1(\Sigma q_iw_i)^2}{100}$	35.682		38.359	

Table 5.4.1 (Continue)

Parameters	Case - 15		Case - 16	
	q_i	q_iw_i	q_i	q_iw_i
DO	1.4	0.350	2.0	0.50
Fico	88	22.25	48	12.0
BOD	90	15.3	35	5.95
pH	92	13.8	94	14.1
Turb	59	5.31	17	1.58
Temp	40	3.6	20	1.8
Σq_iw_i	60.610		35.930	
$\frac{1(\Sigma q_iw_i)^2}{100}$	36.735		12.900	

Table 5.4.1 (Continue)

Parameters	Case - 17		Case - 18	
	q_i	$q_i w_i$	q_i	$q_i w_i$
DO	3.8	0.95	3.8	0.950
Fico	48	12.0	0	0
BOD	90	15.3	90	15.3
pH	86	12.9	75	11.25
Turb	12	1.08	59	5.31
Temp	32	2.88	40	3.6
$\Sigma q_i w_i$	45.110		36.410	
$\frac{1(\Sigma q_i w_i)^2}{100}$	20.349		13.250	

Table 5.4.1 (Continue)

Parameters	Case - 19		Case - 20	
	q_i	$q_i w_i$	q_i	$q_i w_i$
DO	1.8	0.450	1.6	0.40
Fico	-	-	82	20.5
BOD	71	12.07	90	15.3
pH	75	11.25	80	12.0
Turb	49	4.41	56	5.04
Temp	40	3.6	50	4.5
$\Sigma q_i w_i$	31.780		57.740	
$\frac{1(\Sigma q_i w_i)^2}{100}$	10.090		33.333	

Table 5.4.1 (Continue)

Parameters	Case -21		Case -22		Case -23	
	q_i	$q_i w_i$	q_i	$q_i w_i$	q_i	$q_i w_i$
DO	5.6	1.4	1.9	0.475	1.8	0.45
Fico	0	0	82.0	20.5	82	20.5
BOD	89	15.13	90.0	15.3	90	15.3
pH	81	12	80.0	12	80	12.0
Turb	45	4.05	59.0	5.31	44	3.96
Temp	50	4.5	68	6.12	57	5.13
$\Sigma q_i w_i$	25.200		59.705		57.340	
$\frac{1(\Sigma q_i w_i)^2}{100}$	6.3504		35.6468		32.878	

Table 5.4.2
Water quality index of Indian rivers by different methods of calculation

Case	WQI 1	WQI 2	WQI 3	WQI 4	WQI 5
1	65.18	42.48	48.46	62.575	39.156
2	25.667	6.584	0	19.02	3.617
3	44.03	19.829	0	35.89	12.88
4	44.23	19.56	0	35.815	12.8271
5	47.96	28.00	33.9332	45.89	21.05
6	52.25	27.80	30.366	54.455	31.871
7	49.18	24.18	29.25	53.816	28.961
8	45.25	20.47	17.668	34.670	120.2
9	46.25	21.39	17.893	35.955	12.92
10	64.75	41.92	44.896	63.525	40.35
11	57.91	33.53	36.1990	59.675	35.611
12	63.93	40.61	37.071	63.20	39.94
13	63.78	40.67	38.18	59.835	35.682
14	68.65	40.51	38.779	61.935	38.359
15	61.93	38.10	36.60	60.61	36.735
16	36.00	12.96	21.801	35.93	12.90
17	45.30	20.52	28.55	45.11	20.349
18	44.63	19.91	0	36.41	13.25
19	19.466	15.57	16.30	31.78	10.09
20	59.93	35.91	37.188	57.74	33.33
21	45.10	20.34	0	25.2	3.35
22	63.48	40.22	40.63	59.705	35.64
23	59.13	34.96	37.235	57.34	82.878

Table 5.4.3
Desired and Existing Water Quality Levels

River	Desired water quality level	Existing Water quality level	Critical parameters
Case 1	A	Below C	Coliforms,BOD
Case 2	E	-	-
Case 3	C	Below C	Coliforms
Case 4	C	Below C	Coliforms
Case 5	A	Below C	Coliforms,BOD
Case 6	A	C	Coliforms,BOD
Case 7	A	C	Coliforms,BOD
Case 8	C	Below C	BOD,Coliforms
Case 9	B	C	Coliforms,BOD
Case 10	C	C	
Case 11	C	C	-
Case 12	E	-	Coliforms
Case 13	B	C	Coliforms,pH
Case 14	B	C	Coliforms,BOD,DO
Case 15	B	C	Coliforms,BOD
Case 16	C	Below C	Coliforms
Case 17	C	Below C	-
Case 18	C	-	BOD,Coliforms
Case 19	C	Below C	-
Case 20	C	C	-
Case 21	C	C	BOD
Case 22	C	C	-
Case 23	C	C	-

Detail of the Cases discussed previously

Case 1	Sabarmati, Dharoidam, Gujarat
Case 2	Sabarmati,Ahmedabad, Gujarat
Case 3	Mahi, Sevali, Gujarat
Case 4	Mahi, Vassd, Gujarat
Case 5	Narmada, Garudeshwar, Gujarat
Case 6	Tapi, NepaNagar, MP
Case 7	Tapi, Burhanpur, MP
Case 8	Godavari, Ashti, Maharastra
Case 9	Godavari, Dhalegaon, Maharastra
Case 10	Godavari, Mancherial, A.P.
Case 11	Godavari, Polavaram, A.P.
Case 12	Periyar, Alwaye, Kerala
Case 13	Periyar, Kalady, Kerala
Case 14	Charliyar, Koolimadu, Kerala
Case 15	Charliyar, Kallapally Kerala
Case 16	Subarnarekha, Ranchi, Bihar
Case 17	Subarnarekha, Jamshedpur, Bihar
Case 18	Krishna, Vijayawada, A.P.
Case 19	Bhima, Takali, Maharashtra
Case 20	Tungabhadra, Ullanuru, Karnataka
Case 21	Penner, Nellore, A.P.
Case 22	Cauveri, KRS Dam, Karnataka
Case 23	Cauveri, Sathyalam, Karnataka

	Beneficial Uses	Classification
1.	Drinking water and domestic supplies without treatment, but with disinfection	A
2.	River bathing, swimming and water contact sports	B
3.	Source of raw-water for municipal supplies consumed only after conventional water treatment	C
4.	Propagation of wild life, animal husbandry and fisheries	D
5.	Agriculture, industrial cooling and washing, hydro-power generation and controlled waste disposal	E

The primary water quality objectives for each of the water use listed above have been defined in the Central Board Publication "Scheme for Zoning and Classification of Indian Rivers, Estuaries and Costal Waters (Parn One: Sweet Water)". These quality criteria are given in Table 5.4.4.

Table 5.4.4
Primary Water Quality Criteria

Designated Best Use	Class of Water	Criteria
Drinking water source without Conventional treatment bur after disinfection	A	1. Total Coliforms organisms MPN/100ml shall be 50 or less. 2. pH between 6.5 and 8.5 3. Dissolved Oxygen 6mg/l or more 4. Biochemical Oxygen Demand 5 days 20°C 2mg/l or less
Outdoor bathing (organised)	B	1. Total Coliforms organisms MPN/100ml shall be 500 or less. 2. pH between 6.5 and 8.5 3. Dissolved Oxygen 5mg/l or more 4. Biochemical Oxygen Demand 5 days 20°C 3mg/l or less
Drinking Water source with conventional treatment followed by disinfection	C	1. Total Coliforms organisms MPN/100ml shall be 5000 or less. 2. pH between 6 and 9 3. Dissolved Oxygen 4mg/l or more 4. Biochemical Oxygen Demand 5 days 20°C 3mg/l or less
Propagation of Wild Life, Fisherims	D	1. pH between 6.5 and 8.5 2. Dissolved Oxygen 4mg/l or more 3. Free Ammonia (as N) 1.2 mg/l or less
Irrigation, Industrial cooling, Controlled Waste Disposal	E	1. pH between 6.0 and 5.89 2. Dissolved Oxygen 4mg/l or more 3. Sodium Absorption Ratio Max 26 4. Boron, Max 2mg/l

5.5 Result And Discussion:

The WQI of Indian rivers = 24.6162 on a scale of 100. This shows that water is suitable for drinking but only after conventional water treatment. This coincides with the results obtained by the central board for the prevention and control of water pollution as presented in table 5.4.4.

Remark: The work of this chapter is published as per given in the reference [5].

5.6 References:

1. Horton, R.K. "An Index-number System for rating water quality", J. Water Pollution Control Federation, 3, 37:300-305, (1965).

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4. Bindu, N. Lohani, Environmental Quality Management, South Asian Puslishers, New Delhi, 63-66 (1984).

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CHAPTER - 6

ASSESSMENT OF WATER QUALITLY OF SOME INDIAN RIVERS BASE ON FACTOR ANALYSIS

6.1 Introduction

Developing countries are undergoing a transition period from a largely agrarian economy to intensive industrial activity. People and establishments congregate at certain areas and their activities produce external effects, whether beneficial or adverse on the environment that support them. One of the most vital components of the physical environment is water. Because of a growing global awareness in the maintenance of a "Clean World", public and private agencies have come to realize the importance of surface water to a nations economy. Knowledge of water quality thus plays a significant role in the development of water quality control and management strategies.

An index is a number, usually dimensionless, which expresses the relative magnitude of some complex phenomenon of condition. A Water Quality Index is also a "Communication tool for transfer of water quality". It provides an instrument of tool that appropriately consolidates and presents as a single number, the values of multiple parameters selected to enter into the index formulation. The composite influence of significant physical, chemical and biological parameters is reflected in the index.

Some possible uses of Water Quality Index are for resource allocation, location ranking, standards enforcement, trend analysis, public information and scientific research. Horton [4] first proposed the concept of indices to represent gradation in water quality. The importance of such an evaluation tool was ultimately realized, and several authors have developed their own rating schemes [2,3,5,7].

The objective of study involved in this chapter is to find out the extent of pollution in Indian rivers based on the analysis of water quality parameters. The analysis is carriedout with the help of factor analysis the brief description is given as follows:

6.2 Methodology Used-Factor Analysis:

The method proposed to solve the water quality index is based on factor analysis [8]. Factor analysis was first originated to aid in the explanation of psychological data but has been widely applied to other fields. Its best feature lies in the parsimonius reduction of data [6]. Factor analysis seeks to express a large number of variables in terms of a smaller, more manageable number of factors based on linear relationships between the original variables. These derived components or factors may be regarded as underlying influences of dimensions that, on further measurement can be substituted for the more numerous variables [4].

Factor analysis provides a mathematical model which can be used to describe certain areas of nature. A series of test scores or other measures are intercorrelated to determine the number of dimensions the test space occupies, and to identify these dimensions in terms of traits or other general concepts. The interpretations are done by observing which tests fall on a given dimension and inferring what these tests have in common that is absent from tests not falling on the dimension. Tests correlate to the extent that they measure common traits. By observing and analyzing the pattern of intercorrelations, the operation of one or more underlying traits or other sources of common variance is inferred.

It is thus by establishing the basic sources of variance in a field of investigation and determining the nature of each measure in terms of basic categories that we can know what types of variations the tests (or other variables) are measuring, the interrelationships of these measures, and what needs to be done to modify or improve them when they are not entirely satisfactory for our purposes.

It has been pointed out that a proper order for research programs might be, first to use a set of priori measures in a field of investigation and factor analyze them to determine the basic traits or other sources of variance operating: second, to study these factors, one at a time, by the techniques of analysis of variance to determine how they are affected by different experimental conditions or how they vary among groups that differ with respect to age, sex, education, or other pertinent background variables; and lastly, to study them experimentally in the laboratory for specific groups under carefully controlled conditions. While this order need not be invariable it does help to point up the proper function of each type of approach. In factor analysis the "individual differences" represented by a large number of measures, that are given to a single population, usually at one time under a standard set of conditions, are studied to detect possible common sources of variation or variance. In analysis of variance a single measure is administered over a series of occasions and conditions to determine the significance of group differences. The individual differences by this latter approach are often used as an estimate of "error variance" against which to evaluate the group differences.

It is quite understandable that some psychologists have used the results of factor analysis in the area of congnitive tests as support or disproof of theories of intelligence. The results of factor analysis can serve only as indirect evidence for this purpose, however, since factor analysis attempts to account statistically for differences in traits among individuals rather than for the mental organization within any one individual. The fact that the observed differences can be accounted for by weighted sums of measures of the reference variables obtained from a factor analysis should not be construed to mean that any one individual's behavior is a resultant of the additive combination of these hypothetical traits. They merely serve to account mathematically for observed or predicted individual differences. Moreover, as methods of measurement and prediction become more refined and exact, factorial methods based on multiplicative or more complicated relationships for combining variances undoubtedly will be developed. While nonlinear relationships may be more appealing logically, they have yet to prove their usefulness at our present stage of knowledge.

The interpretations of the results of factor analysis, as is true of all scientific interpretations, are tentative.

Just as the theory of relativity has replaced Newtonian physics as an interpretation of observed facts, so may present theories based on factorial results be superseded by other interpretations, if they more adequately account for the data. Factors are not eternal veritics; they merely serve to represent the fundamental underlying sources of variation operating in a given set of scores or other data observed under a specified set of conditions.

Factor analysis has many limitations and those who apply it should have considerable skill in experimental design and the-orizing to obtain meaningful results. Without such skill and insight the considerable effort involved will be wasted.

6.3 Data Analysis:

Data was obtained from CPCB, New Delhi as given in table 6.3.1 for 32 sampling stations located on twenty different rivers in India. The following distribution was based for selection of variables :(I) Parameters commonly included in water quality indices for general use of water; (II) availability of data as wide as range of sampling period as possible, and (III) by allowing compression with water quality indicators.

By using the Diagonal method of the factor analysis.

the correlation matrix from the data matrix has been obtained [6]. Using this correlation matrix, we find the factor loadings for the first factor. Similarly, other factor loadings are calculated. The factors along with their factor loadings, communality, eigen values and cumulative variance are listed in the table 6.3.2. The results of principal factor analysis with varimax rotation met the objective and were employed for index building Rotated factors in principal factor analysis always have 100% cumulative variance. The last column of table 6.3.4 only depicts the cumulative variance explained by a set of factors if higher factors are ignored.

Factors with eigen values higher than 1.00 were considered significant. Eigen values represent the sum of the sequences of the factor loadings across all variables on that factor. Column entries in the Table 6.3.2 denote factor loadings, which are measured of how much each variable contributes to the explanation of the extracted factors. Variables that load heavily on a factor are most representative of that factor. There is, however, no strict criteria on what constitutes a high loading. In order to interpret the factor loading matrix some degree of subjectivity must be used. The communality is a measure of the portion of a variable's total variance attributable to other variables in the set. The greater the communality, the greater the interrelationships within the set of variables. In this analysis variables having factor loadings higher than or equal to 0.500 are considered significant.

Factor loadings of factor scores are plotted against the factors for the sixteen variables in Figures 6.3.1(a), 6.3.1(b). These figures indicate, how much each variable or parameter loads on every factor. Suppose, if the factor loading is a high value for a particular factor, it implies that the variable loads heavily on that factor and so is that variable, most representative of that factor. And accordingly if the factor loading is a low value, it shows that the variable is less representative of that factor. The graphical representation is shown to give a more clearer idea of how a variable varies in loading on, from one factor to the next factor.

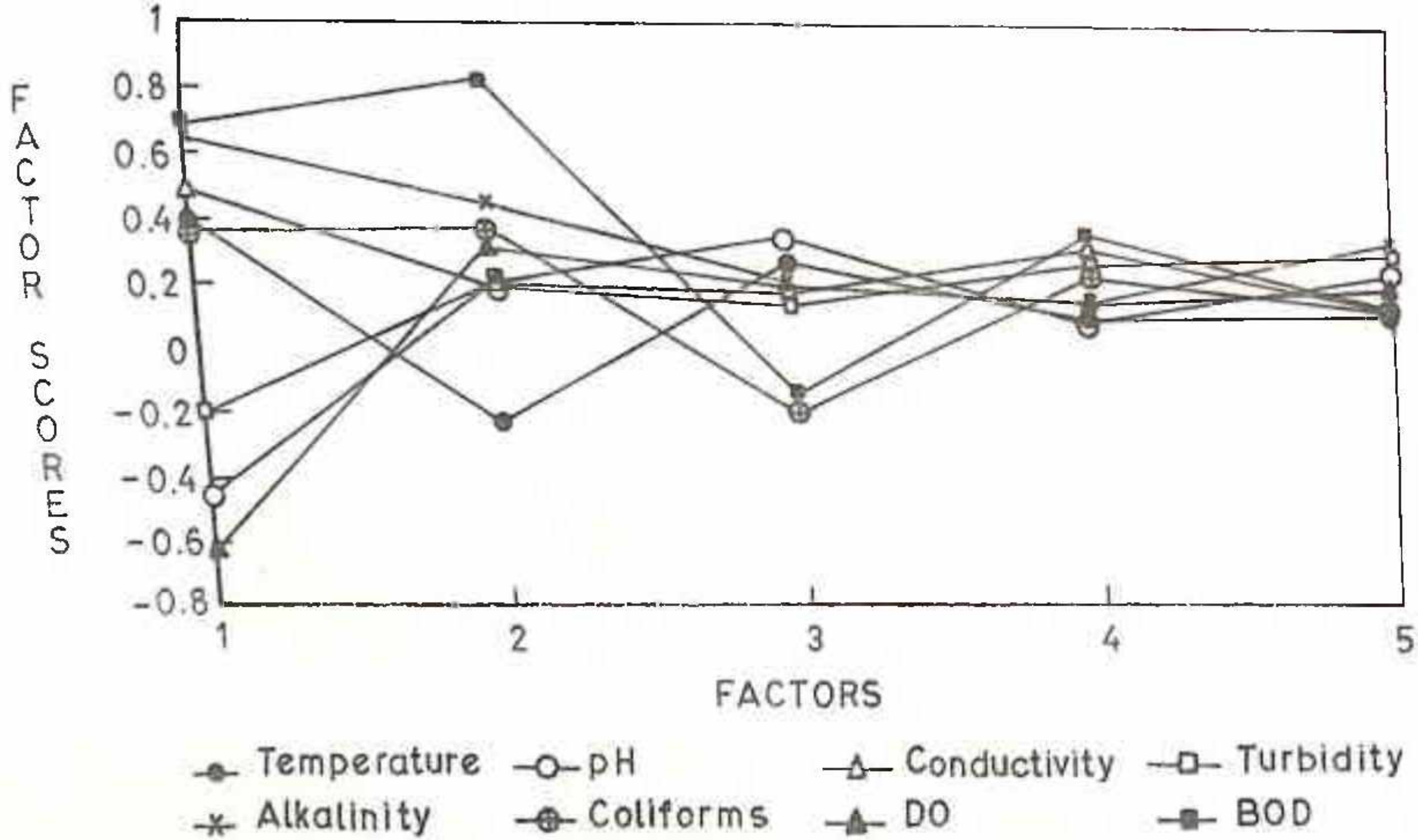

Fig.6.3.1(a) Factor Scores Curve

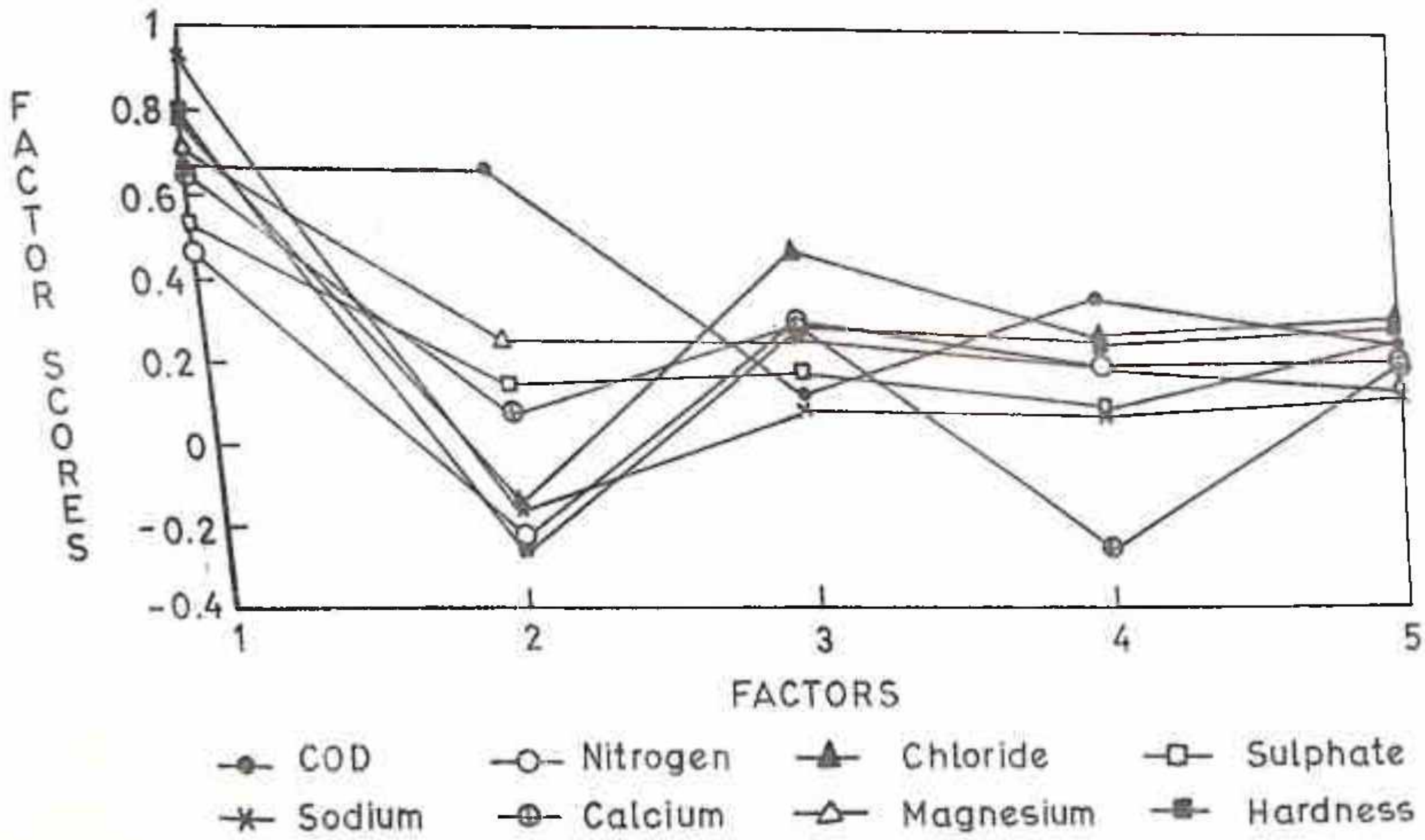

Fig.6.3.1(b) Factor Scores Curve

Table 6.3.1
PHYSICAL, CHEMICAL AND BIOLOGICAL PARAMETERS OF IMPORTANT RIVERS IN INDIA

River, Location, State	Temperature (°C)	pH	Conductivity (µmhos/cm)	Turbidity (JTU)	Alkalinity (mg/lt)
	1	2	3	4	5
1. Bhima, Takali, Maharashtra	27.9	7.8	696	54.7	142.2
2. Brahmaputra, Pandu, Assam	24.3	7.4	120	59	75
3. Cauvery, KRS Dam, Karnataka	26.2	7.9	188.5	17	78
4. Cauvery, Satyagalam, Bridge Karnataka	26.6	8.0	250.8	42.7	109.2
5. Godavari, Asthi, Maharashtra	24.6	7.7	293.7	35.7	120.8
6. Godavari, Dhalegaon, Maharashtra	24.9	7.9	500.8	58.9	163.4
7. Godavari, Mancherial, A.P.	28.3	8.0	538.9	27.2	187.3
8. Godavari, Polavaram, A.P.	29.0	7.9	218.6	105.9	100.1
9. Krishna, Vijayawada, A.P.	29.2	8.2	432.5	22.3	130.2
10. Pennar, Nellore, A.P.	28.8	8.1	862.1	42.7	189.4
11. Tungabhadra, Ullanuru, Karnataka	27.3	8.0	470.7	31.2	129.7
12. Periyar, Alwaye, Kerala	30.7	6.5	1220.4	30.7	72.8

Table 6.3.1 (Continue)

	1	2	3	4	5
13. Periyar, Kalady, Kerala	30.8	7.1	129.4	9.8	23.4
14. Chaliyar, Koolimadu, Kerala	30.0	7.1	135.5	14.0	38.8
15. Chaliyar, Kollapalli, Kerala	30.4	7.1	244.8	24	68.3
16. Sabarmati, Dharojdam, Gujarat	26.4	7.9	313.4	17.2	142
17. Sabarmati, Ahmedabad, Gujarat	29.3	7.8	1677.8	53.9	460.2
18. Mahi, Sevalia, Gujarat	27.3	8.2	281.8	25.7	127.8
19. Mahi, Vasad, Gujarat	27.5	8.2	347.1	22.9	193.9
20. Narmada, Gurudeshwar, Gujarat	26.9	8.1	271.8	146.7	166.1
21. Tapi, Nepanagar, M.P.	26.0	8.0	313.1	130.9	200.8
22. Tapi, Burhapnur, M.P.	26.7	8.0	499.4	275.2	240.1
23. Sutlej, Amritsar, Punjab	22.5	7.8	1980	56	75
24. Beas, Goindwal, Punjab	22.0	8.3	2190	57	86
25. Mahanadi, Sambalpur, Orissa	27.5	7.7	302.8	590.6	94
26. Subarnarekha, Ranchi, Bihar	25.3	7.2	296.2	239.3	72.2
27. Subarnarekha, Jamshedpur, Bihar	28.5	7.8	250	59	62.1

Table 6.3.1 (Continue)

	1	2	3	4	5
28. Musi, Hyderabad, A.P.	28.9	8.0	700	35	124
29. Ganges, Kannauj, U.P	24.9	7.7	300	123	167
30. Ganges, Allahabad, U.P.	26	8.3	370	130	166
31. Yamuna, Allahabad, U.P.	25.9	8.3	480	144	192
32. Yamuna, Karnal, Haryana	19.8	7.6	920	156	71

Table 6.3.1 (Continue)

River, Location, State	Coliforms (MPN/100ml)	D.O. (mg/lt)	B.O.D (mg/lt)	C.O.D (mg/lt)	Nitrogen (mg/lt)
	6	7	8	9	10
1. Bhima, Takali, Maharashtra	2322	6.3	5.4	34.2	1.220
2. Brahmaputra, Pandu, Assam	9296	7.5	0.8	5.6	0.02
3. Cauvery, KRS Dam, Karnataka	765	7.1	1.3	13.9	0.001
4. Cauvery, Satyagalam Bridge, Karnataka	701.0	7.0	1.5	16.6	0.001
5. Godavari, Asthi, Maharashtra	3176	5.9	5.8	42.9	0.250

Table 6.3.1 (Continue)

	6	7	8	9	10
6. Godavari, Dhalegaon, Maharashtra	1425	6.5	4.7	32.4	0.240
7. Godavari, Mancherial, A.P.	34	8.1	2.4	21.9	0.520
8. Godavari, Polavaram, A.P.	31.0	7.3	1.6	23.6	0.580
9. Krishna, Vijayawada, A.P.	155	8.2	2.2	23.6	0.78
10. Pennar, Nellore, A.P.	106.0	8.7	3.2	30.2	0.6
11. Tungabhadra, Ullanuru, Karnataka	1006	7.1	1.7	27.8	0.01
12. Periyar, Alwaye, Kerala	1342	5.8	1.4	36.4	4.23
13. Periyar, Kalady, Kerala	4038	7.6	1.2	26.8	0.89
14. Chaliyar, Koolimadu, Kerala	2695	7.2	1.2	25.9	0.64
15. Chaliyar, Kollapalli, Kerala	4147	6.1	2.4	26.2	2.5
16. Sabarmati, Dharojdam, Gujarat	6203	9.6	2.2	6.9	0.4
17. Sabarmati, Ahmedabad, Gujarat	17E6	1.0	73.3	193.6	1.18
18. Mahi, Sevalla, Gujarat	26E6	8.9	2.0	6.9	1.31
19. Mahi, Vasad, Gujarat	15E5	8.4	1.6	7.6	1.06
20. Narmada, Gurudeshwar, Gujarat	31E5	8.0	2.4	16.5	0.54

Table 6.3.1 (Continued)

	6	7	8	9	10
21. Tapi, Nepanagar, M.P.	1156	7.3	2.4	16.8	0.47
22. Tapi, Burhanpur, M.P.	1578	7.2	4.3	32.6	0.49
23. Sutlej, Amritsar, Punjab	2944	7.8	2.5	3.6	2.4
24. Beas, Goindwal, Punjab	3497	7.7	3.1	3.9	2.733
25. Mahanadi, Sambalpur, Orissa	9659	8.2	2.3	26	0.28
26. Subarnarekha, Ranchi, Bihar	15E3	4.7	7.2	41.9	2.05
27. Subarnarekha, Jamshedpur, Bihar	6312	7.2	2.5	7.6	1.89
28. Musi, Hyderabad, A.P.	3894	7.1	1.6	124	217
29. Ganges, Kannauj, U.P	12990	7.2	2.3	8.9	0.908
30. Ganges, Allahabad, U.P.	9719	8.3	2.1	16.6	1.749
31. Yamuna, Allahabad, U.P.	8492	8.2	1.3	1.656	11
32. Yamuna, Karnal, Haryana	1800	5.4	3.5	15.1	0.325

Table 6.3.1 (Continue)

River, Location, State	Chlorides (mg/lt)	Sulphates (mg/lt)	Sodium (mg/lt)	Calciam (mg/lt)	Magnesium (mg/lt)	Hardness (mg/lt)
	11	12	13	14	15	16
1. Bhima, Takali, Maharashtra	92.8	75.8	32.4	49.6	151.1	187.4
2. Brahmaputra, Pandu, Assam	6	120	2	45	20	66
3. Cauvery, KRS Dam, Karnataka	15.2	30.1	9.7	13.1	46.3	60.4
4. Cauvery, Satyagalam Bridge, Karnataka	19.2	29.5	13.6	14.8	62.9	84.2
5. Godavari, Asthi, Maharashtra	41.2	16.3	15.3	36.4	100.1	131.1
6. Godavari, Dhalegaon, Maharashtr	72.2	69.7	20.7	38.2	113.6	171.0
7. Godavari, Mancherial, A.P.	37	44.2	79	66.4	82.4	148.5
8. Godavari, Polavaram, A.P.	15.3	11.9	19.3	44.9	29.4	74.9
9. Krishna, Vijayawada, A.P.	45.3	34.7	47.6	49.6	44.1	101.4
10. Pennar, Nellore, A.P.	113.6	77.6	135.2	82.9	76.5	155.4
11. Tungabhadra, Ullanuru, Karnataka	49.6	32.1	40.7	40.1	71.4	104.4
12. Periyar, Alwaye, Kerala	866.3	71.8	367.9	108.1	102.3	409.9
13. Periyar, Kalady, Kerala	1147.9	9.2	68.6	21.9	42.3	64

Table 6.3.1 (Continue)

	11	12	13	14	15	16
14. Chaliyar, Koolimadu, Kerala	39.7	4.2	16.4	18.6	13.4	35.1
15. Chaliyar, Kollapalli, Kerala	307	25.9	135.3	69.8	82.6	161.3
16. Sabarmati, Dharojdam, Gujarat	39.6	13.4	32.5	75.7	54.1	128.6
17. Sabarmati, Ahmedabad, Gujarat	300.9	113.3	354.9	121.6	114.4	238.1
18. Mahi, Sevalia, Gujarat	28.4	7.9	34.6	54.2	66.1	119.5
19. Mahi, Vasad, Gujarat	47.7	10.7	53.4	60.2	90.1	151.2
20. Narmada, Gurudeshwar, Gujarat	27.3	12.6	35.4	67.1	95.6	171.8
21. Tapi, Nepanagar, M.P.	21.8	17.6	30.7	74.6	68	140.6
22. Tapi, Burhanpur, M.P.	48.5	37.4	39.5	110	96.5	208.8
23. Sutlej, Amritsar, Punjab	32	26	3.0	30.0	23.0	92.0
24. Beas, Goindwal, Punjab	39	49	2.0	46	29	97
25. Mahanadi, Sambalpur, Orissa	21	4	29	50	16	64
26. Subarnarekha, Ranchi, Bihar	17.3	19.8	11.8	51.4	40.5	89.5
27. Subarnarekha, Jamshedpur, Bihar	38.9	12.5	16.4	47.8	30.9	80.1
28. Musi, Hyderabad, A.P.	32	12	30	62	53	126

Table 6.3.1 (Continue)

	11	12	13	14	15	16
29. Ganges, Kannanj, U.P	8	16	2	88	61	151
30. Ganges, Allahabad, U.P.	60	13	6	92	70	153
31. Yamuna, Allahabad, U.P.	18	12	7	98	88	174
32. Yamuna, Karnal, Haryana	61	14	12	58	38	96

Table 6.3.2

ROTATED FACTOR LOADING MATRIX FROM FACTOR ANALYSIS

Parameters	Stations					Commu-nality
	Factors					h^2
	1	2	3	4	5	
1. Temperature	0.392	-0.224	0.262	0.090	0.103	0.291
2. pH	-0.456	0.216	0.338	0.088	0.227	0.428
3. Conductivity	0.489	0.189	0.174	0.313	0.139	0.422
4. Turbidity	-0.199	0.189	0.130	0.252	0.297	0.244
5. Alkalinity	0.644	0.446	0.201	0.134	0.324	0.777
6. Coliforms	0.366	0.370	-0.198	0.223	0.115	0.373
7. D.O.	-0.627	0.317	0.186	0.142	0.177	0.586
8. B.O.D.	0.687	0.830	-0.146	0.352	0.111	1.322
9. C.O.D.	0.671	0.665	0.123	0.362	0.254	1.103
10. Nitrogen	0.474	-0.231	0.302	0.196	0.209	0.451
11. Chloride	0.785	-0.147	0.475	0.262	0.308	1.027
12. Sulphates	0.535	0.148	0.180	0.090	0.258	0.415
13. Sodium	0.928	-0.175	0.081	0.070	0.126	0.919
14. Calcium	0.656	0.076	0.294	-0.256	0.198	0.627
15. Magnesium	0.714	0.251	0.255	0.191	0.134	0.692
16. Hardness	0.782	-0.267	0.286	0.248	0.283	0.907
Eigenvalue	6.047	2.004	0.970	0.804	0.754	-
Percentage of Variance	51.02	27.4	11.1	6.4	4.1	-
Cumulative % of variance	51.02	78.4	89.5	95.9	100	-

Table 6.3.3
Eigenvalues and cumulative percentage of variance of the five factors

Factors	Eigenvalues	Cumulative % of variance
1	6.047	51.02
2	2.004	78.4
3	0.970	89.5
4	0.804	95.9
5	0.754	100.0

6.4 Water Quality Index Development:

In factor analysis, once the loadings or weights for each variable are known, sampling index in which each variable is weighted proportionately in its involvement in the patterns may be constructed. The greater the involvement, the higher the weight. An index I(1), calculated from the first factor may be expressed as

$$I_1 = \sum_{j=1}^{n} \frac{a_{j1}\, y_j}{\lambda_1} \qquad (6.4.1)$$

where,

λ_1 = eigenvalue for the first factor;

y_j = standardized value of known variable j and

a_{j1} = factor loading of variable j on the first factor

An index calculated from the foregoing formulation is valid only if the first factor extracts a large proportion of total variance among variables. None of the first factors had a cumulative variance, exceeding 0.70 and this limits the construction of a single index. Thus separate indices were calculated for factors having eigenvalues greater than 1.00. The general form is

$$I_i = \sum_{j=1}^{n} \frac{a_{ji} \, y_j}{\lambda_i} \qquad (6.4.2)$$

where,

λ_i = eigenvalue of the ith factor;

y_j = standardized value of the variable j and

a_{ji} = factor loading of variable j on factor i

Table 6.3.4
Calculation of Water Quality Index of 32 Stations

S.N.	River,Station,State	$\Sigma a_{ji} y_j$	$(\Sigma a_{ji} y_j / \lambda i)$	Area WQI.1 as a (%)	Rating
1.	Bhima, Takali,Maharashtra	4.3527	0.7198	71.98	9
2.	Brahmaputra, Pandu,Assam	2.061	0.3408	34.08	26
3.	Cauvery, KRS Dam,Karnataka	1.2959	0.2142	21.42	31
4.	Cauvery, Satyagalam Bridge, Karnataka	1.9345	0.3199	31.99	27
5.	Godavari, Asthi,Maharashtra	4.584	0.7581	75.81	8
6.	Godavari, Dhalegaon,Maharashtra	3.9901	0.6598	65.98	12
7.	Godavari, Mancherial,A.P	3.9912	0.6601	66.01	11

Table 6.3.4 (Continue)

S.N.	River,Station,State	$\Sigma a_{ji} y_j$	$(\Sigma a_{ji} y_j / Ai)$	Area WQI.1 as a (%)	Rating
8.	Godavari, Polavaram, A.P.	1.412	0.2334	23.32	30
9.	Krishna, Vijayawada, A.P.	2.693	0.4454	44.54	23
10.	Pennar, Nellore, A.P.	5.241	0.8668	86.68	5
11.	Tungabhadra, Ullanuru, Karnataka	2.846	0.4706	47.06	22
12.	Periyar, Alwaye, Kerala	5.8624	0.9695	96.95	2
13.	Periyar, Kalady, Kerala	1.8725	0.3097	30.97	28
14.	Chaliyar, Koolimadu, Kerala	0.7314	0.1209	12.09	32**
15.	Chaliyar, Kollapalli, Kerala	4.8141	0.8086	80.86	7
16.	Sabarmati, Dharojdam, Gujarat	3.0392	0.5026	50.26	21
17.	Sabarmati, Ahmedabad, Gujarat	5.9638	0.9862	98.62	1*
18.	Mahi, Sevalia, Gujarat	2.4194	0.4001	40.01	24
19.	Mahi, Vasad, Gujarat	5.4257	0.8973	89.73	4
20.	Narmada, Gurudeshwar, Gujarat	3.4276	0.5668	56.68	18
21.	Tapi, Nepanagar, M.P.	4.9395	0.5678	56.78	17
22.	Tapi, Burhanpur, M.P.	5.4748	0.9053	90.53	3

Table 6.3.4 (Continue)

S.N.	River,Station,State	$\Sigma a_{ji} y_j$	$(\Sigma a_{ji} y_j / A_i)$	Area UQI.1 as a (%)	Rating
23.	Sutlej, Amritsar,Punjab	3.5359	0.5847	58.47	16
24.	Beas, Goindwal,Punjab	4.2941	0.7101	71.01	10
25.	Mahanadi, Sambalpur,Orissa	1.8288	0.3024	30.24	29
26.	Subarnarekha, Ranchi,Bihar	4.9345	0.8160	81.60	6
27.	Subarnarekha, Jamshedpur,Bihar	2.7204	0.3757	37.57	25
28.	Musi, Hyderabad,A.P.	3.3773	0.5585	55.85	19
29.	Ganges, Kannauj,U.P	3.9379	0.6512	65.12	13
30.	Ganges, Allahabad,U.P.	3.8667	0.6394	63.94	15
31.	Yamuna, Allahabad,U.P.	3.9196	0.6482	64.82	14
32.	Yamuna, Karnal,Haryana	3.1945	0.528;	52.83	20

* Most Polluted
** Least Polluted

The calculated indices are then transformed to values in the "area under the normal curve (Table 6.3.4). The body of the Table 6.3.4, may be regarded as degree of pollution values. the proportion of total area column is this value, normalized to a linear variable. Values thus range from 0-100, with zero representing good water quality and progressively becoming poorer as the value tends towards 100. With these values, the rating has been done for 20 rivers at the 32 surface stations (Table 6.3.4). The station which is ranked 1 is the most polluted, while the station ranked 32 is the least polluted among the 32 stations. In Table 6.3.4, only the WQI for the first factor has been calculated. Similarly, WQI can be calculated for other factors also. A bar graph, surface stations Vs. water Quality Index percentage, has been drawn (Fig. 6.3.2) which shows the comparision in the extent of pollution between the 32 surface stations of the 20 rivers.

6.5 Conclusions:

From the WQI-1 (Table 6.3.4), we can conclude that the river Sabermati at Ahmedabad in Gujarat is the most polluted. It is followed by the river Periyar at Alwaye in Kerala and so on as the ratings given. The least polluted river is the Chāliyar at Koolimadu in Kerala, preceded by the Cauvery at KRS Dam in Karnataka. All the other rivers are moderately polluted.

With the availability of statistical packages, factor

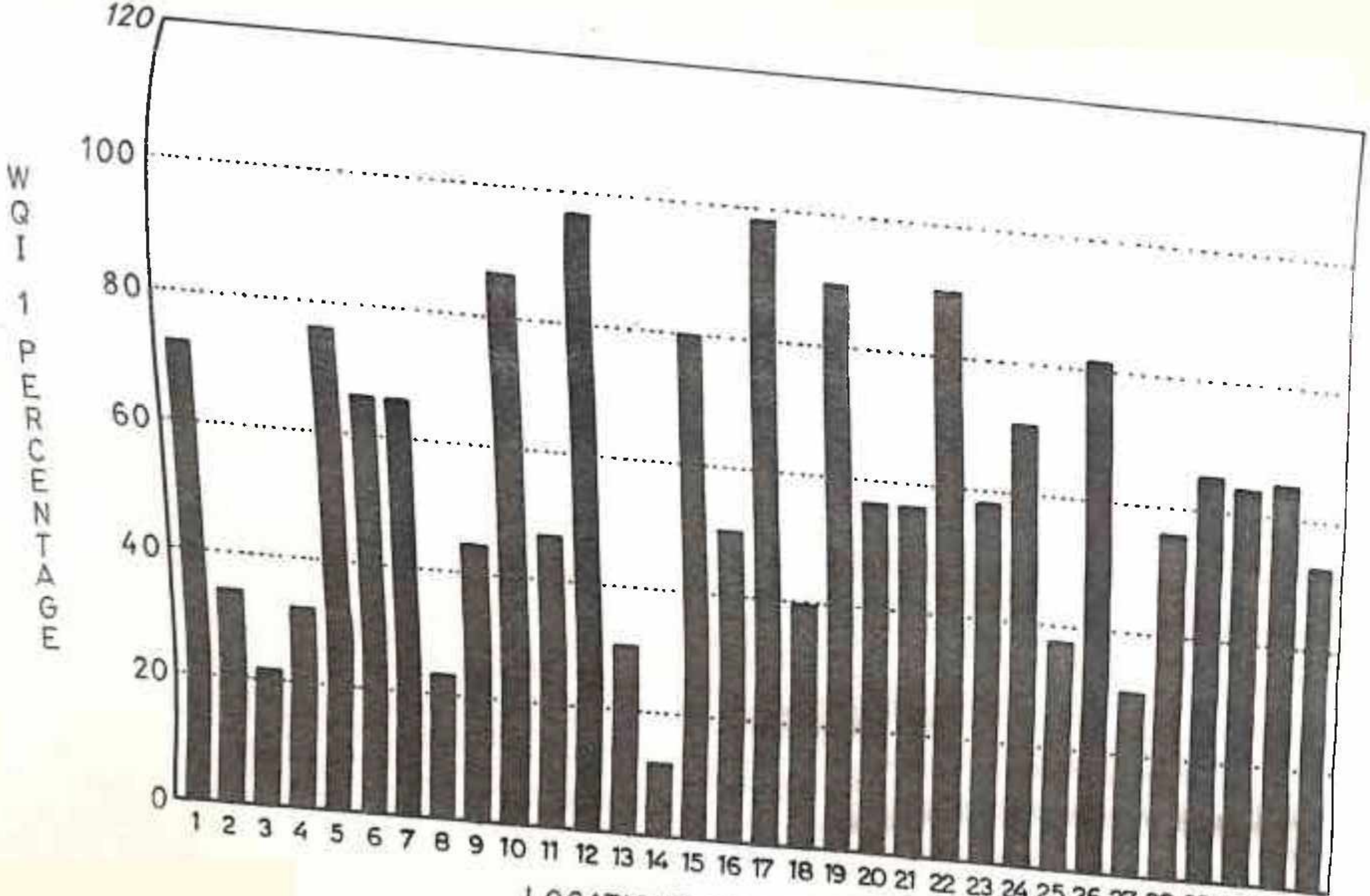

Fig.6.3.2 WQI OF VARIOUS STATIONS

analysis provides a quick, simple and analytical method of quality evaluation. The factor scores may be further utilized for the development of composite index for water quality assessment. Further different processes may be calculated. The development of index may be summarized as: data collection, test for normality, missing value estimation, cluster analysis, factor analysis, and transformation to normal curve index values.

Remark: Work of this Chapter is accepted for publication in an International Journal of "Pollution Research" under special supplement "Pollution and Biomonitoring of Indian Rivers."

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CHAPTER - 7

POTABILITY AND SUITABILITY OF WATER FOR IRRIGATION PURPOSE OF SOME INDIAN RIVERS

7.1 Introduction:

River water forms a major source of water supply for urban and rural people of India. It is now generally recognised that quality of river water is just as important as its quantity. The quality of public health depends to a great extent on the quality of water. Environmental degradations, one of the undesirable side effect of Industrialisation, "a necessary evil" essential to provide basic necessities of life. However, if our environment is degraded in the process, not only does the quality of life suffer, but also the existence of life is threatened. Pollution of water thus is an important aspect of environmental pollution. The quality of water is usually characterised in terms of certain water quality parameters, according to its physical, chemical and biological properties. The number of such parameters necessary to fully specify the quality of water however is quite large. In a developing country like India, it may be too expensive or even unfeasible to determine all of them due to lack of laboratory facilities. In this study we analyse various water quality data (mainly physico-chemical parameters) of various Indian rivers shown in Figure 7.1.1. It's suitability for drinking and irrigation purposes at various locations is then determined.

7.2 Water Quality Criteria:

In the water resources evaluation, the quality nowadays is of growing importance. The study of river water quality involves a description of the occurrence of various constituents and relation of these constituents in river water, transfer of water quality informations among individuals and groups require the use of standardised techniques and terminology to assure accurate understanding by all concerned. The approach is based on using quantitative and reproductible parameters that are as descriptive and ambiguous as possible. It increases the accuracy and ease of measuring water quality parameters and conveying the information to others.

7.2.1 Systems for Measuring Water Quality:

Water quality is dynamic, subject to major changes with or without human intervention. Also, many different parameters are used to describe and specify quality characteristics. Therefore, decisions on parameters should be made carefully. Analytical measurements of water quality may be divided as

Fig. 7.1.1 Detail of some Indian Rivers and sampling stations used in Analysis

1. Analysis based on measuring concentration of specific constituents of water.
2. Analysis based on direct and quantitative measurements of quality characteristics.
3. Measurements of specific constituents
4. Measurements of the effects of constituents as per their concentration in water.
5. Description of water characteristics in qualitative terms.

Analytical techniques used in water supply and wastewater disposal are based largely on standardised procedures, the most common one being published in standard methods for the examination of water and wastewater (water pollution control federation, 1980).

7.3 Standards for Water:

Standards are means by which regulatory agencies define their requirements for water courses and for each individual. The development of standards include careful attention to programme objective and quality criteria for the designated use, but they also consider several additional facts of the matter, including economic aspects, public need and desires, political reaction and practical attainability. The water quality should satisfy the requirements or standards set for the specific use, namely domestic, stock agricultural and Industrial purposes.[2,3,4]

7.3.1 Drinking water standards:

Degree of purity required for drinking purpose is highest in relation to other uses of water. Water for this purpose should be free from suspended impurities, dissolved harmful salts, and disease producing bacteria [2,3]. The drinking water standard laid by world health organisation (WHO) and Indian standards Institute are shown in Table 7.3.1.1.

Table 7.3.1.1

S.No. Constituent	W.H.O (1984)		ISI (1983)	
	A	B	A	B
1. Turbidity (JTU Units)	5	25	5	25
2. pH	7.0-8.5	6.5 -9.2	7.0-8.5	6.5 -9.2
3. Chloride as Cl^-	250	1000	250	1000
4. Magnesium Mg^{2+}	50	150	30	100
5. Sulphate SO_4^{2-}	200	400	150	upto 400 if Mg <30
6. Calcium Ca^{2+}	75	200	75	200
7. Total Coliform (MPN/100 ml)	-	-	0	50
8. Electrical conductivity in (micromho/cm)	-	-	1500	2500

A = Recommended maxm. Concentration (a)
(Mg/Lit:except where shown otherwise)

B = Maximum permissible concentration (b)
(Mg/Lit:except where shown otherwise)

(a) Constituents should not be present in excess of listed concentrations where other suitable supplies are available.

(b) Constituents in excess of concentrations listed shall constitute grounds for rejection of water supply.

7.3.2 Irrigation Water Standards:

The suitability of river water for irrigation is contigent on the effects of the mineral constituents of the water on both the plant and soil salts may harm plant growth physically by limiting the uptake of water through modification of osmotic processes or chemically by metabolic reactions such as those caused by toxic constituents. Effects of salts on soils causing changes in soil structure, permeability affects plant growth.

It is difficult to fix up specially the standards of water for irrigation purposes based on the chemical characterisation of water alone as the same quality of water represent difficulty to other factors like the nature of salts and their percentage present in the soil, drainage characteristics of soil and land to be irrigated, climatic conditions, crops to be grown and irrigation operations to be adopted.

Generally, suitability of water to be used for irrigation purposes is indicated by the total salt content and sodium content present in that water sodium concentration is important in classifying in irrigation water because sodium reacts with soil to reduce its permeability. Soils containing a large proportion of sodium with carbonate as the predominant anion are alkali soils and those with chloride or sulphate is saline soil. The salt content of irrigation water is usually expressed in any of the following ways.

- parts per million
- Milli equivalents per litre
- electrical conductivity, expressed in micro mho/cm

As a result in excessive quantity, these salts reduce the osmotic activity of plants, thus preventing the absorption of nutrients from the soil, in addition they may have indirect chemical effect on the metabolism of the plant and may reduce soil permeability.

Sodium content is also important in classifying an irrigation water because sodium reacts with soil to reduce its permeability. A high sodium concentration leads to development of an alkali soil. Further the high sodium saturation directly causes calcium deficiency. Irrigation water is judged by the concentration of sodium on the basis of relative proportion of cations of sodium or Sodium Adsorption Ratio (S.A.R.) given by following equation.

$$S.A.R. = \frac{Na^{+}}{\sqrt{\frac{Ca^{2+} + Mg^{2+}}{2}}} \qquad 7.3.2.1$$

where all the constituents are expressed in milli equivalents per litre (Meq/Litre).

The U.S Regional Salinity Laboratory Department of Agriculture has constructed a diagram Fig. 7.5.1 for the classification of water describing 16 fields with reference to sodium adsorption ratio given by eqn. 7.3.2.1 as an index for sodium hazard and electric conductivity as an index for salinity hazard. The quality of water falling in various fields is classified as given in following Table 7.3.2.1.

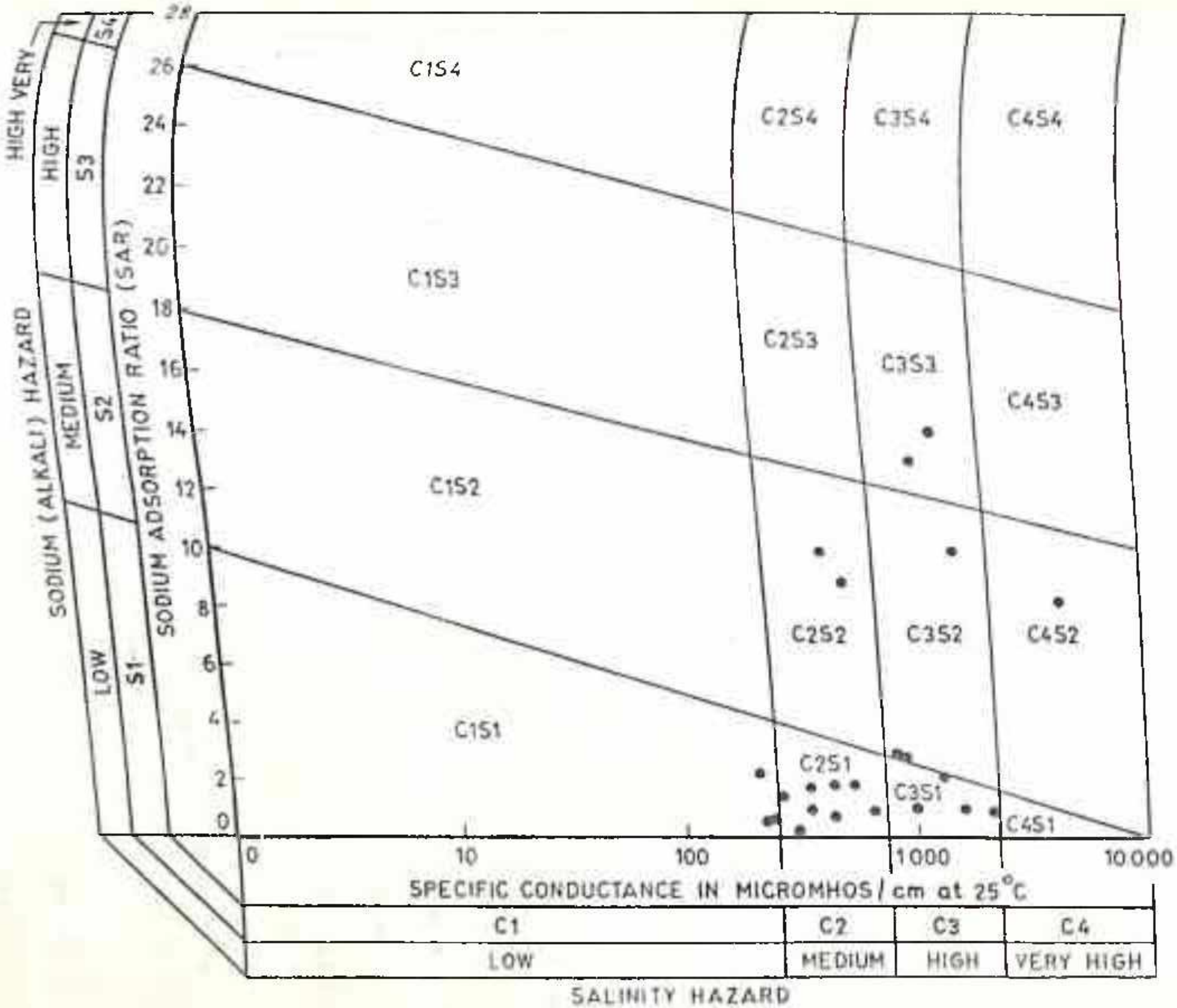

Fig.7.5.1 Diagram for classification of irrigation waters (after Richards) for some Indian Rivers.

Table 7.3.2.1

Zone of quality for irrigation purposes	Salinity fields
Good	C1S1,C2S1
Moderate	C1S2,C2S2, C3S1,C3S2
Bad	All other groups

The classification into various zones were based on the following table 7.3.2.2.

Table 7.3.2.2

S.No.	Salinity Hazard Electric conductivity (Micro mho/cm)	Alkali hazard S.A.R.	Water class
1.	<250	upto 10	Excellent
2.	250-750	10-18	Good
3.	750-2250	18-26	Permissible
4.	2250-4000	>26	Doubtful
5.	>4000		Unsuitable

7.4 Interpretation of Data to Check Potability of River Water

Comparing table 7.3.1.1,7.4.1 and 7.4.2 we find that based on turbidity all the rivers except Sabarmati at Dharoidam Gujarat, chaliyar at Koolimadu, Kerala tall beyond the turbidity limit mentioned. All the rivers fall within the range specified with reference to pH. In case of chlorides, except the rivers chaliyar at Kallapally, Kerala, Periyar at Alwaye and Kalady Kerala all other rivers fall within the range mentioned by the standards. For magnesium, rivers, Godavari at Dhalegaon, Maharashtra, Bhima at Takali Maharashtra Periyar at Alwaye, & Kalady, Penner at Nellore, Narmada at Gurudeshwar do not fall in the range specified. In case of sulphate most of the rivers fall below the lower limit range. As for calcium, periyar at Kalady and Alwaye do not fall in the range specified by the standards. The forms of coliform count, the water quality at Tapi and Periyar are found to be at desired level indicating status of "drinking with conventional treatment".

7.4.1 Electrical Conductivity:

The waters of the Cauveri, Subarnarekha, Chaliyar, Tapi, Narmada, Mahi, Sabarmati at Dharoi dam, Godavari at Ashti and Polavaram maintained a conductivity of less than 500 micro mho/cm. The electrical conductivity of water of Sabarmati at downstream of Ahmedabad remained about 1000 micromho/cm. The water of the periyar remained low in Electrical Conductivity except at Alwaye where it was more than 3000 micromho/cm, indicating many fold increase in the mineral constituent of water. The EC at the remaining places varied from as low as 200 to as high as 5000 micro-mho/cm indicating that the variation in mineral constituent of the water at these locations is quite significant.

7.4.2 Calcium-Magnesium Sodium:

As expected calcium, magnesium and sodium together form the major fraction of local cations. The concentration is in the order Ca^{2+}>Mg^{2+}> Na^{+}, till interfered by human this inequality holds good for Tapi at Nepanagar and low some extent for Tapi at Burhampur only.

At all the others locations due to human interference this inequality disturbed. In the Periyar and Chaliyar sodium was the dominating cation due to the saline intrusions. Out of the three cations analysed, calcium is the least dominating. The concentration of calcium was very low in Cauvery. Periyar at kalady and Chaliyar at Koolimadu. In the Mahi Narmada, Godavari, Subarnarekha, Bhima and Tungabhadra rivers the level of calcium was low.

Magnesium is the most dominating among the three cations in the Cauvery. Bhima, Narmada and Godavari in Maharashtra. The level of magnesium was very low in the Subarnarekha, the Chaliyar at Koolimadu, the Krishna at Vijayawada, Godavari at Polavaram and Periyar at Kalady.

The concentration of magnesium was very low in Periyar at Alwaye also, except where many fold increase in all the major concentrations of cations and anions was observed sodium is observed to be the most dominating cations in the Sabarmati, Periyar, Chaliyar, Pennar, Mahi, Krishna at Vijayawada, Tungbhadra at Uuanuru and Godavari at Mancherial. Like other ions, sodium concentration was also very high in the Savarmati as Ahmedabad followed by the Penner at Nellore. In the Chaliyar at Kallapally due to saline instruction sometimes the sodium concentration increased many fold but generally the concentration was little above 2 Meq/L. In Periyar at Alwaye the level of sodium concentration was sometimes more than 100 times than the level which generally prevailed in river.

7.4.3 pH:

In all the rivers pH value vary between 6 & 9 except in Subarharekha at Ranchi where it has gone below 6. The level of pH was on the lower side in the periyar at Alwaye where it ranged between 5.7 & 7.2 otherwise all the stations average value varied between 7.2 & 8.3.

7.4.4 Turbidity:

The water at both the stations on the Mahi and the Chaliyar. Bharoidam on Sabarmati, Ashti and Mancheial on

Godavari, Kalady on the periyar and KRS dam on the Cauvery is less burbid than at other locations. Ninety percent of time, turbidity is found to be less than 50 JTU at these monitoring stations.

7.4.5 Sulphate-Chloride:

Among all the anions sulphate contributes the least at most all places. The average value of SO_4^{2-} is found to be less than 1 meq/l or 48 mg/l at most of locations. Among all, sulphate is found to be highest at down stream of Ahmedabad on the Sabarmati with 2.3 meq/l or 112 mg/l as average value of sulphate exceeded 1 meq/l at Dhalegaon in the Godavari, Tapi on the Bhima, Nellore on the Periyar and Alwaye on the Periyar. Chloride is the most dominating anion in periyar, the Penner, the Bhima, the Godavari in Maharashtra, Subarnarekha at Ranchi and Sabarmati at Ahmedabad. Generally the level of chloride seens not to be very high in chaliyar and the Pariyar. At Kalady in the Periyar, the level of chloride is usually found to be very high out sometimes it reaches as low as 0.5 meq/l.

7.4.6 Total Coliform:

The rivers like Sabarmati, the Mahi, the Narmada, the Subarnarekha and the Chaliyar are critical in terms of coliforms indicating that the domestic sewage is the dominant source of pollution at these stations. Also the rivers like Bhima at Takali, the Periyar at Kalady and the Godavari at Ashti, the coliform count is formed to be very high.

Table 7.4.1

S.No.	River	Location	Ca^{2+} Meq/Lit	Mg^{2+} Meq/Lit	Na^{+} Meq/Lit	Electrical Conductivity Micromho/Cm	S.A.R. = $\frac{Na^{+}}{\sqrt{\frac{Ca^{2+}+Mg^{2+}}{2}}}$
1.	Godavari	Ashti Maharashtra	2.675	9.995	4.8285	520	1.918
2.	Godavari	Dhalegaon Maharashtra	4.661	14.735	3.443	660	1.106
3.	Godavari	Mancherial A.P.	3.892	11.645	24.950	563	8.895
4.	Godavari	Polavaram A.P.	2.145	2.613	3.377	205	2.025
5.	Godavari	KRS dam Karnataka	1.405	7.000	1.472	242.5	0.718
6.	Cauveri	Sathayalam Karnataka	1.4795	6.412	1.8097	350	0.911
7.	Sabarmati	Dharoda Gujarat	5.002	8.337	4.505	349.5	1.768
8.	Sabarmati	Ahmedabad Gujarat	5.197	11.837	37.735	1364.6	12.930
9.	Tapi	Nepanagar M.P.	4.454	9.275	3.303	261	1.261
10.	Tapi	Burhampur M.P.	7.335	9.883	5.289	438	1.802
11.	Periyar	Alvaye Kerala	12.60	102..88	63.04	4691.5	8.292
12.	Periyar	Kalady Kerala	11.57	65.93	61.763	477.2	9.92
13.	Mahi	Sevalia Gujarat	2.902	6.992	1.914	240.4	0.861
14.	Mahi	Vasad Gujarat	4.942	9.357	5.100	284.75	1.963
15.	Chaliyar	Koolimadu Kerala	[illegible]	4.854	1.916	[illegible]	2.799

Table 7.4.1 (Continue)

S.No.	River	Location	Ca^{2+} Meq/Lit	Mg^{2+} Meq/Lit	Na^{+} Meq/Lit	Electrical Conductivity Micromho/Cm	S.A.R. = $\frac{Na^{+}}{\sqrt{\frac{Ca^{2+}+Mg^{2+}}{2}}}$
16.	Chaliyar	Kailapally Kerala	21.876	53.578	60.95	1116.25	9.927
17.	Subarnalekha	Ranchi Kerala	3.044	3.51[illegible]	1.009	444.5	0.6572
18.	Subarnalekha	Jamshedpur Bihar	3.094	5.495	0.5481	402	0.264
19.	Krishna	Vijayawada A.P.	2.994	5.511	2.232	1005	1.082
20.	Bhima	Takali Maharashtra	5.02	22.448	3.654	2120	0.985
21.	Tunghabadra	Ullanuru Karnataka	2.919	10.633	2.414	1600	0.927
22.	Penner	Nellore A.P.	5.152	12.491	6.7425	870	2.270
23.	Narmada	Gurudeshwar Gujarat	5.229	13.984	2.196	243.6	0.708

Data Source: CBPCWP, Delhi (1986)

* All the concentrations were actually expressed in Mg/L which are converted to Meq/L by following facto

Ca^{2+} = 0.04990, Mg^{2+} = 0.08226, Na^{+} = 0.04350 [1].

Table 7.4.2

S.No.	River	Location	pH	Cl^- (Mg/l)	Mg^{2+} (Mg/l)	$SO4^-$ (Mg/l)	Ca^{2+} (Mg/l)	Turbidity (JTU)	Total Coliform (MNP/100 ml)
1.	Godavari	Ashti Maharashtra	7.4	119	121.5	110	53.6	376	6203
2.	Godavari	Dhalegaon Maharashtra	7.715	227.25	179.1	115	93.4	656.85	17E6
3.	Godavari	Mancherial A.P.	7.55	72.5	144	66	78	237.5	26E6
4.	Godavari	Polavaram A.P.	7.6	29	34.2	19.3	55	563	15E5
5.	Godavari	Krs dam Karnataka	8.05	43	85.1	57.2	28.5	855	31E5
6.	Cauveri	Sathayalam Karnataka	7.75	85	77.95	77.2	29.65	230.5	1156
7.	Sabarmati	Dharoidam Gujarat	7.8	49	101.35	106	100.25	21.5	1578
8.	Sabarmati	Ahmedabad Gujarat	7.9	395	143.9	106	104.15	73.5	3176
9.	Tapi	Nepanagar M.P.	7.8	37	112.75	53	89.25	1052	1425
10.	Tapi	Burhampur M.P.	8.2	81.5	120.15	68.25	147	4002	34
11.	Periyar	Alwaye Kerala	6.45	3221	1250.75	165	252.6	55.1	31
12.	Periyar	Kalady Kerala	6.85	3152	801.5	155	232.05	33	1342
13.	Mahi	Sevalia Gujarat	8.25	36	85	24.2	58.15	180	4038
14.	Mahi	Vasad Gujarat	7.9	172.5	113.75	72.7	83	100	2695
15.	Chaliyar	Koolimadu Kerala	7.45	592.25	83.33	14.35	107.2	25.7	4147
16.	Chaliyar	Kallapally Kerala	7.15	2483	650.6	230.5	438.4	36.2	15E3
17.	Subarnalekha	Ranchi Kerala	6.7	61.5	42.7	16	61	800	96590
18.	Subarnalekha	Jamshedpur Bihar	7.6	30.35	66.8	29.5	62	1901.5	155
19.	Krishna	Vijayawada A.P.	7.7	78.5	67	43	60	52.7	2322
20.	Bhima	Takali Maharashtra	7.51	228	272.9	425	100.6	490	1006
21.	Tunghabadra	Ullanare Karnataka	7.7495	85.5	129.4	50.5	58.5	121	106
22.	Penner	Nellore A.P.	8.0	114	151.85	87.8	103.25	175	765
23.	Narmada	Gurudeshwar Gujarat	7.9	54	170	53	104.8	1453	601

Data Source: CBPCWP, Delhi (196

7.5 Salinity Diagram and Its Interpretation:

In order to check the suitability of water of various rivers for irrigation purposes we have plotted the electrical conductivities on the X axis and S.A.R. on the Y axis as a diagram developed for the classification of irrigation quality of water (refer Table 7.3.2.1,7.3.2.2 and Figure 7.5.1). The table 7.5.1 gives the suitability of water for irrigation purpose.

Table 7.5.1

S.No.	River	Location	Position in Graph	Suitability for Irrigation
1	2	3	4	5
1.	Godavari	Ashti Maharashtra	C_2S_1	Good
2.	Godavari	Dhalegaon Maharashtra	C_2S_1	Good
3.	Godavari	Mancherial A.P.	C_2S_2	Moderate
4.	Godavari	Potavaram A.P.	C_1S_1	Good
5.	Godavari	KRS dam Karnataka	C_1S_1	Good
6.	Cauveri	Sathayalam Karnataka	C_2S_1	Good
7.	Sabarmati	Dharoidam Gujarat	C_2S_1	Good
8.	Sabarmati	Ahmedabad Gujarat	C_3S_3	Bad
9.	Tapi	Nepanagar M.P.	C_2S_1	Good
10.	Tapi	Burhampur M.P.	C_2S_1	Good
11.	Periyar	Alwaye Kerala	C_4S_2	Bad
12.	Periyar	Kalady Kerala	C_2S_2	Moderate

Table 7.5.1 (Continue)

1	2	3	4	5
13.	Mahi	Sevalia Gujarat	C_1S_1	Good
14.	Mahi	Vasad Gujarat	C_1S_1	Good
15.	Chaliyar	Koolimadu Kerala	C_1S_1	Good
16.	Chaliyar	Kallapally Kerala	C_3S_2	Moderate
17.	Subarnalekha	Ranchi Kerala	C_2S_1	Good
18.	Subarnalekha	Jamshedpur Bihar	C_2S_1	Good
19.	Krishna	Vijayawada A.P.	C_3S_1	Moderate
20.	Bhima	Takali Maharashtra	C_3S_1	Moderate
21.	Tunghabadra	Ullunuru Karnataka	C_3S_1	Moderate
22.	Penner	Nellore A.P.	C_2S_3	Bad
23.	Narmada	Gurudeshwar Gujarat	C_2S_1	Good

The plots on the Salinity diagram are mostly in the field of C_2S_1 & C_1S_1. Most of the river waters are thus in the category for irrigation purposes. These water can be used by plants which have moderate salt tolerance capacity on almost all soils with little danger of the development of harmful leaves of exchangeable sodium, however sodium sensitive crops may accumulate injurious concentrations of sodium. Rivers like Godavari (Mancherial), Periyar (Kalady), Chaliyar (Kallapally), Krishna (Vijayawada), Bhima (Takali) & Tungabadra (Ullanuru) are found to be moderately suitable for irrigation. However rivers Sabarmati (Ahmedabad), Periyar (Alwaye) and Penner (Nellore) are bad for irrigation pruposes.

7.6 CONCLUSIONS

The water of various rivers of India after careful analysis is found to be more directly suitable for irrigation rather than drinking purpose. Except for rivers Sabarmati (Ahmedabad). Periyar (Alwaye) and Penner (Nellore), all other rivers are good or moderate for irrigation purpose. As for drinking is concerned no river is suitable for direct consumption all the rivers are excessively turbid. Most of the rivers, because of excessive mineral constituents remain impotable. Thmse rivers require proper treatment before taken to use.

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HAPTER - 8

APPLICATION OF LINEAR GOAL PROGRAMMING WITH PARTITION ALGORITHM IN WATER QUALITY MANAGEMENT OF A RIVER STRETCH FOR MULTIOBJECTIVE WATER RESOURCES SYSTEM PLANNING

8.1 Introduction:

Decision makers are often found with the task of obtaining solutions to the problem that inherently involves noncommensurable and conflicting objectives. For example, the competing water uses between flood control and hydropower dictate conflicting operating policies: One should keep the reservoir as empty as possible for flood control purposes and as full as possible for hydropower and water supply. The problem of determining the optimal or best solution for a problem involving conflicting objectives is very complex because of following reasons:

1. Different criteria may be measured on different scales such as costs (Rupees), and the probability of violation of safe limits on water quality. In general, it is not possible to scale down all these units to a common scale of evaluation that may aid in reducing the vector minimization problem to the one of scalar minimization; and

2. Cause multiple objectives are involved in a vector minimization objective in a consistent manner comes a

tedious problem, and even more complicated when multiple decision makers are involved.

In developing countries where the government has full control over its water resource. The main objective of water resource planning is to carry out the social responsibilities to satisfy the customer requirements and not maximization of profit in monetary terms so that the governmental duties with regard to social responsibilities are satisfied. The profit motive is secondary which is mainly to make the particular water resource project at least self sustaining. In this approach different objectives are transformed into goals by using deviational variables and aspiration levels. All the constraints are also converted to subgoals as they may not be binding to the same extent. In such cases the system analysis is not basically to optimize a single objective function but to evolve a realistic best solution. Such flexibility is provided by the method called Goal programming.

Goal programming or multiobjective programming is the mathematical programming in which there are more than one objective functions and it seeks to minimize the deviations between the desired goals and the actual results according to the priorities assigned. In this approach different objectives are transformed into goals by using deviational variables and aspiration levels. All the constraints are also converted to subgoals as they may not be binding to the same extent. A complicated water resources system can be more exactly represented by such a model. Each goal and subgoals is assigned a priority level and attempt is made to satisfy all the goals by minimizing deviation from the goals. Acceptable solution is arrived at when all goals are satisfied or if some of them are violated, they are of lower order in priority.

The pioneering work by Cooper [1,2,4,9,10,11] has clearly pointed out that for a real life problem goal programming is a more appropriate approach for decision making than a single objective function approach. For a linear goal programming problem the solution procedure for GP resembles the simplex procedure [5]. For nonlinear problems the application is quite challenging.

Although goal programming has got wider application in various branches of Engineering, problems related mainly with planning and decision making, but its potential in the planning of water resources systems has not been explored much so far.

In this paper effort has been made to high light the important aspects of the "Linear Goal Programming with partition algorithm. The Methodology is also illustrated with the help of an example applied to water quality management of a River stretch.

8.2 A Brief Account of Linear Goal Programming Problem (LGPP):

One of the most promising techniques for multiple objective decision analysis is goal programming. Goal programming is a powerful tool which draws upon the highly developed and tested technique of linear programming, but provides a simultaneous solution to a complex system of competing objectives. Goal programming can handle decision problems having a single goal with multiple subgoals [9]. The technique was originally introduced by Charnes and Cooper [15,16,17] and further developed by Ijiri [18] and Lee [19].

Often goals set by management compete for scarce resources. Furthermore, these goals may be incommensurable. Thus there is a need to establish a hierarchy of importance among these conflicting goals so that low order goals are satisfied or have reached the point beyond which no further improvements are desirable. If the decision maker can provide an ordinal ranking of goals in terms of their contributions or importance to the organization and if all relationships of the model are linear, the problem can be solved by goal programming.

In goal programming, instead of attempting to maximize or minimize the objective criterion directly, as in linear programming, the deviations between goals and what can be achieved within the given set of constraints are minimized. In the simplex algorithm of linear programming such deviations are called slack variables. These variables take on a new significance in goal programming. The devotional variable is represented in two dimensions, both positive and negative deviations from each subgoal or goal. Then the objective function becomes the minimization of these deviations based on the relative importance or priority assigned to them.

The solution of any linear programming problem is based on the cardinal value such as profit or cost. The distinguishing characteristic of goal programming is that it allows for an ordinal solution. The decision maker may be unable to obtain information about the value or cost of a goal or a subgoal, but often can determine its upper or lower limits. Usually, the decision maker can determine the priority of the desired attainment of each goal or subgoal and rank the priorities in an ordinal sequence. Obviously, it is not possible to achieve every goal to the extent desired. Thus, with or without goal programming, the decision maker attaches certain priority to the achievement of a particular goal. The true value of goal programming, therefore, is its contribution to the solution of decision problems involving multiple and conflicting goals according to the decision maker's priority structure.

Goal programming has been applied to a wide range of planning, resource allocation, policy analysis, and functional management problems. The first application was made by Charnes et al. [8] for advertising media planning. However, the first real-world application was in the area of manpower planning by Charnes et al. [7]. Subsequently, goal programming was applied to aggregate production planning [9,13], transportation logistics [12], academic resource planning [10] hospital administration [9], marketing planning [14], capital budgeting [9], portfolio selection [11], municipal economic planning [9], and resource allocation for environmental protection [6].

8.3 Problem Formulation for a River Stretch:

Water quality management along the stretch of a river Godavari (Maharashtra, A.P.) reach which flows down by cities Ashti (Maharashtra), Dhalegaon (Maharashtra), and Mancherial (A.P.). Cities Ashti and Dhalegaon dispose waste at sites 1 and 2 in the river after some treatment. The amounts of wastes to be treated at these sites are 300 and 200 units respectively. The desired water quality indicator (oxygen indicator) in this case is 8 ppm and 7 ppm at sites 1 and 2 respectively. For each unit of waste removal at site 1 improves the quality index at site 2 by 0.3 ppm and at site 3 by 0.2 ppm. Similarly the unit of waste removal at site 2 improves the quality index at site 3 by 0.3 ppm. The percentage waste removal at the sites should be at least 40% but not be more than 95% as beyond this costs is expected to be prohibitive. The current concentration at site 1 and 2 are 4 ppm and 3 ppm respectively.

The cost of per unit waste removal is Rs. 30,000 at site 1 and Rs. 20,000 at site 2 and the total cost is to be within Rs. 45,000.

8.3.1 Goal Programming Formulation:

The equation when simplified result in the following goal programming problem:

The goals in the order of priorities P_1, P_2, P_3 and P_4 are respective as follows:

(a) The total cost of waste removal at sites 1 and 2 (Priority P_1) should not exceed Rs. 45,000.

(b) The water quality indicator (oxygen indicator) is desired not below the level of 8 ppm at site 1 (Priority P_1).

(c) The water quality indicator (oxygen indicator) is desired not below the level of 7 ppm at site 2 (Priority P_2).

(d) The minimum percentage waste removal at sties 1 and 2 should be at least 40% (Priority P_3).

(e) The maximum percentage waste removal at sites 1 and 2 should be at most 95% (Priority P_4).

Explanation:

Let x_1 and x_2 be the units of waste to be removed at sites 1 and 2 respectively.

(1) Since x_1 and x_2 are the units of waste to be removed at sites 1 and 2 respectively and per unit of waste removal cost is Rs. 30,000 at site 1 and Rs. 20,000 at site 2 and total cost should not exceed Rs. 45,000.

$$30x_1 + 20x_2 \leq 45 \; (P_1) \qquad (8.3.1.1)$$

(II) Since water quality indicator (oxygen indicator) deficit at sites 1 and 2 is 4 ppm and for each unit of waste removal at site 1 improves the quality index at site 2 by 0.3 ppm and at site 3 by 0.2 ppm.

$$x_1 \geqslant 4 \times 0.3 \quad (P_1)$$

i.e. $$x_1 \geqslant 1.2 \quad (P_1) \qquad (8.3.1.2)$$

$$x_1 + x_2 \geqslant 4 \times 0.3 + 4 \times 0.3 \quad (P_2)$$

i.e. $$x_1 + x_2 \geqslant 2 \quad (P_2) \qquad (8.3.1.3)$$

(III) Since minimum percentage of waste removal at sites 1 and 2 should be at least 40%,

$$x_1 \geqslant 0.40 \quad (P_3) \qquad (8.3.1.4)$$

$$x_2 \geqslant 0.40 \quad (P_3) \qquad (8.3.1.5)$$

(IV) Since maximum percentage of waste removal at sites 1 and 2 should be atmost 95%,

$$x_1 \leqslant 0.95 \quad (P_4) \qquad (8.3.1.6)$$

$$x_2 \leqslant 0.95 \quad (P_4) \qquad (8.3.1.7)$$

The equations when simplified results in the following goal programming problem:

$$\text{Min } P_1(s_1^+ + s_3^-) + P_2(S_2^-) + P_3(s_6^- + s_7^-) + P_4(s_4^+ + s_5^+)$$

S.t.

$$30x_1 + 20x_2 + s_1^- - s_1^+ = 45 \quad \ldots \qquad p_1(8,3.1.8)$$

$$x_1 - s_2^+ + s_2^- = 1.2 \quad \ldots \qquad p_1(8.3.1.9)$$

$x_1 + x_2 - s_2^+ + s_2^- = 2$ .... P_2(8.3.1.10)

$x_1 - s_6^+ + s_6^- = 0.40$ .... P_3(8.3.1.11)

$x_2 - s_7^+ + s_7^- = 0.40$ .... P_3(8.3.1.12)

$x_1 + s_4^- - s_4^+ = 0.95$ .... P_4(8.3.1.13)

$x_2 + s_5^- - s_5^+ = 0.95$ .... P_4(8.3.1.14)

where s_i's are surplus and slack variables. The problem is shown graphically in the Figure 8.3.1.1.

Solution by Partition Algorithm (See Appendix A8-1, A8-2)

First Subproblem:

Min $s_1^+ + s_3^-$

s.t $30X_1 + 20X_2 + s_1^- - s_1^+ = 45$

$X_1 - s_3^+ + s_3^- = 1.2$

B.V.	a_0	x_1	x_2	s_1^+	s_3^+	s_1^-	s_3^-	Sol
a_0		~~0~~ 1	0	-1	~~0~~ -1	0	~~1~~ 0	~~0~~ 1.2
s_1^-		30	20	-1	0	1	0	45
← s_3^-		[1]	0	0	-1	0	1	1.2
a_0		0	0	-1	0	0	-1	0
s_1^-		0	20	-1	30	1	-30	9
x_1		1	0	0	-1	0	1	1.2

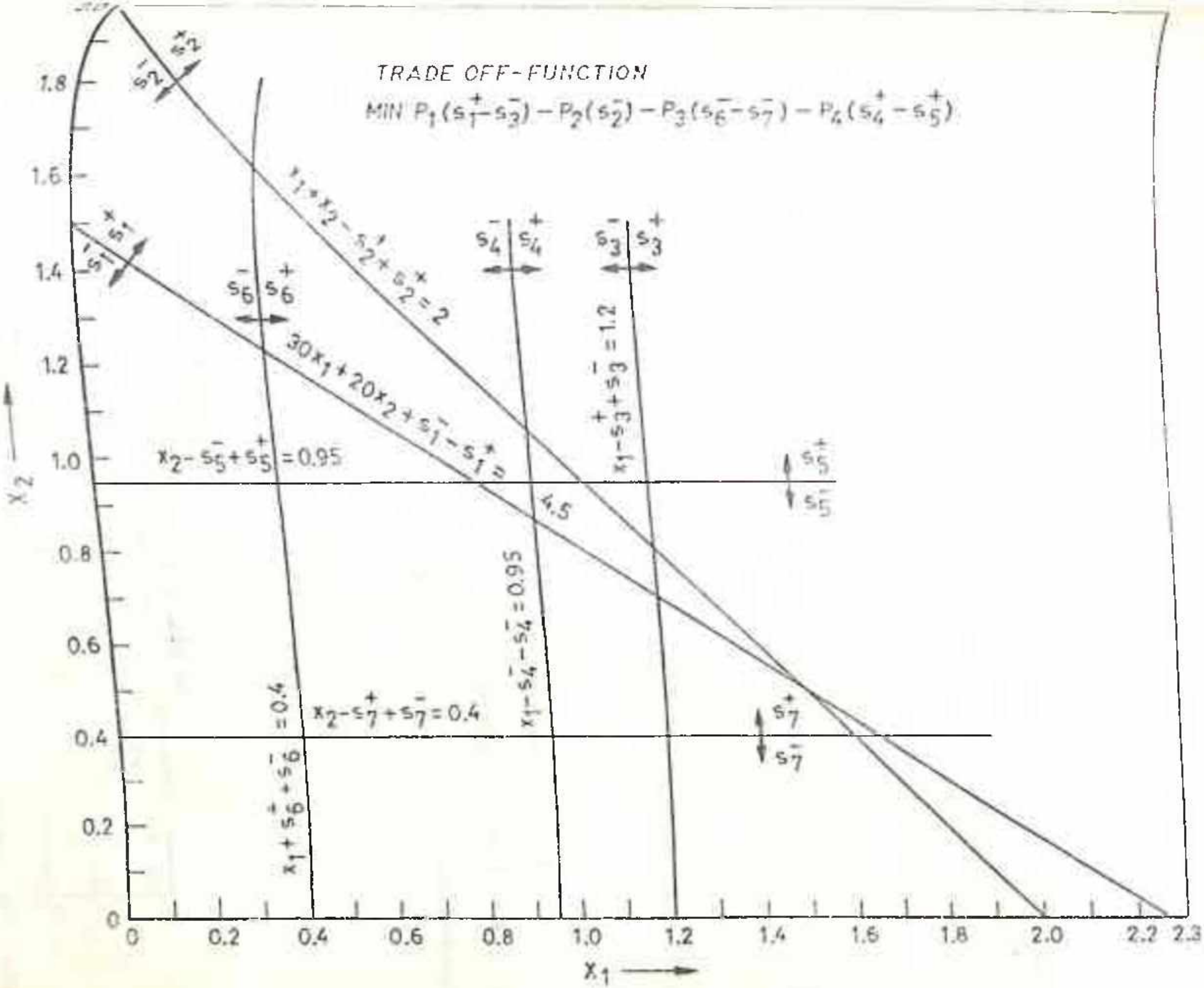

Fig. 8.3.1.1 GRAPHICAL PRESENTATION OF THE PROBLEM UNDER CASE STUDY

Optimal table Alternatives sol. exists because $z_2 - x_2 = 0$

Second Subproblem:

Drop variables s_1^+ & s_3^-

B.V.	s_0	x_1	x_2	s_3^+	s_1^-	s_2^+	s_2^-	Sol.
s_0		0	~~0~~ 1	~~0~~ 1	0	~~0~~ −1	~~1~~ 0	~~0~~ .8
← s_1^-		0	[20]	30	1	0	0	9
x_1		1	0	− 1	0	0	0	1.2
s_2^-		0	1	1	0	−1	1	.8
s_0		0	0	−1/2	−1/20	−1	0	7/20
x_2		0	1	3/2	1/20	0	0	9/20
x_1		1	0				0	1.2
s_2^-		0	0					7/20

Optimal Table no alternative Sol.

The optimal sol. is $x_1 = 1.2$, $x_2 = 0.45$, $s_2^- = .35$, $s_4^+ = 0.25$, $s_5^- = 0.5$, $s_6^+ = 0.8$, $s_7^+ = 0.05$, $s_1^- = s_1^+ = s_2^+ = s_3^+ = s_3^- = s_4^- = s_5^+ = s_6^- = s_7^- = 0$ and the achievement vector is

$a = (0, 0.35, 0, 0.25)$

8.4 Analysis of Result:

Solution of problem:

$x_1 = 1.20$	$s_2^+ = 0$	$s_4^+ = 0.25$	$s_6^+ = 0.80$
$x_2 = 0.45$	$s_2^- = 0.35$	$s_4^- = 0$	$s_6^- = 0$
$s_1^+ = 0$	$s_3^+ = 0$	$s_5^+ = 0$	$s_7^+ = 0.05$
$s_1^- = 0$	$s_3^- = 0$	$s_5^- = 0.50$	$s_7^- = 0$

It is observed that 1.2 units of waste removal at site

1 and 0.45 unit of waste removal at site 2 keeps the total cost of waste removal at sites 1 & 2 at the target level of Rs. 45,000.

8.5 Conclusions:

Goal programming is especially suitable when constraints are conflicting and wherever linear programming fails. The approach can also be extended to nonlinear problems. The application of goal programming to typical water resources system problem clearly demonstrates that water resources is one area where the method can be ideally adopted. In case of large scale complicated system there may be number of different priority structures which may be adopted. In our future work we plan to look into the implications of priority ranking and to select the most appropriate structure for a particular problem.

8.6 References:

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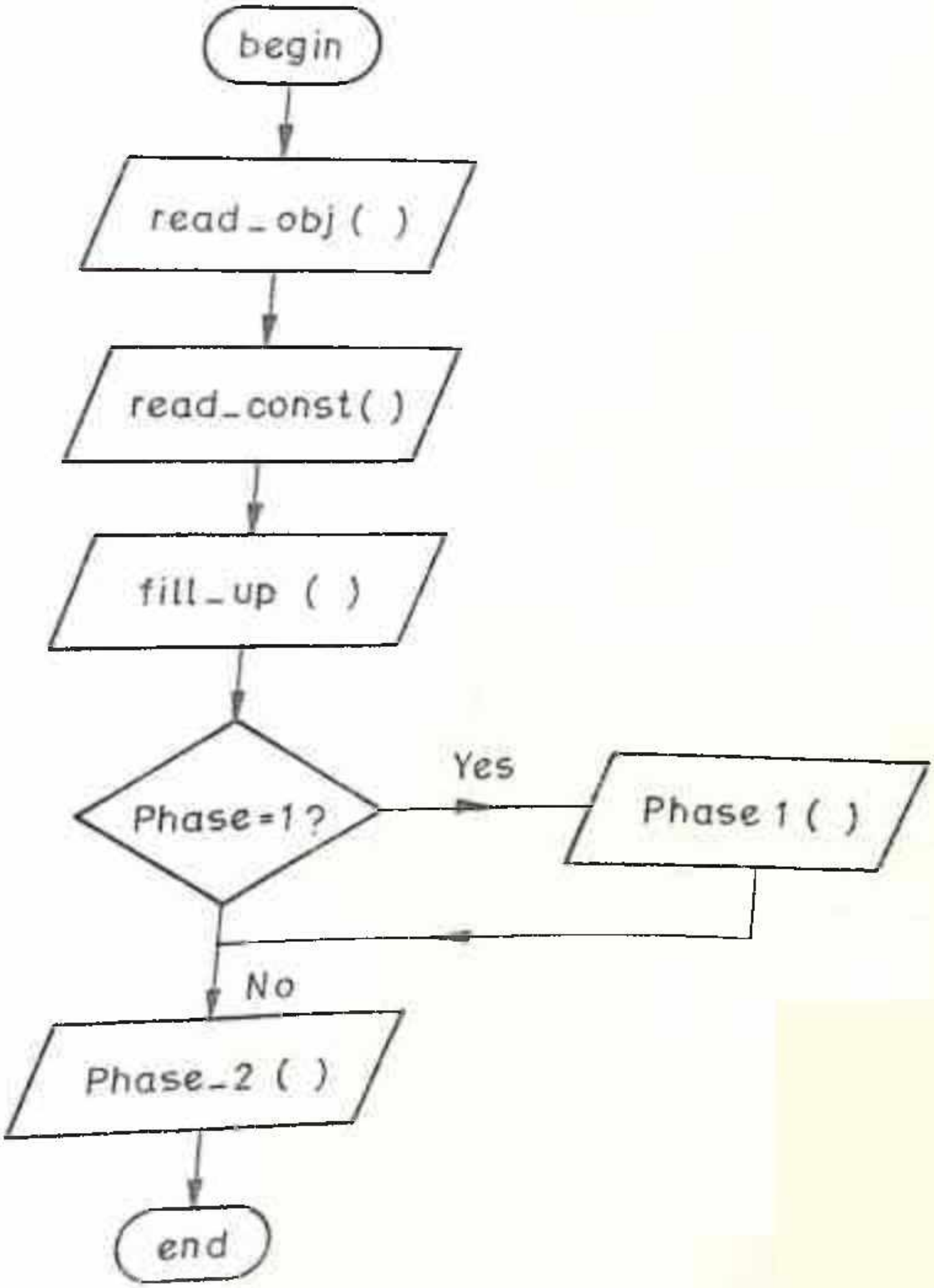
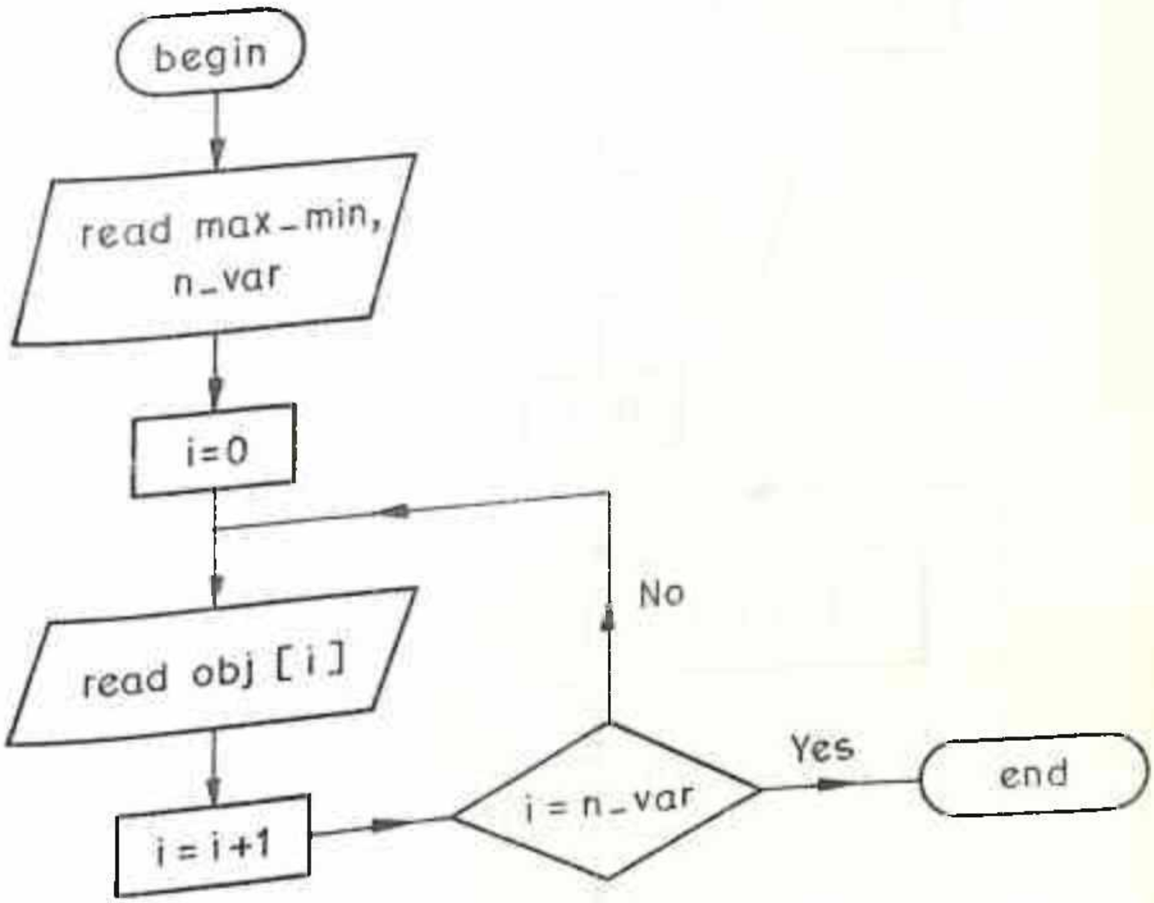
APPENDIX A 4.1

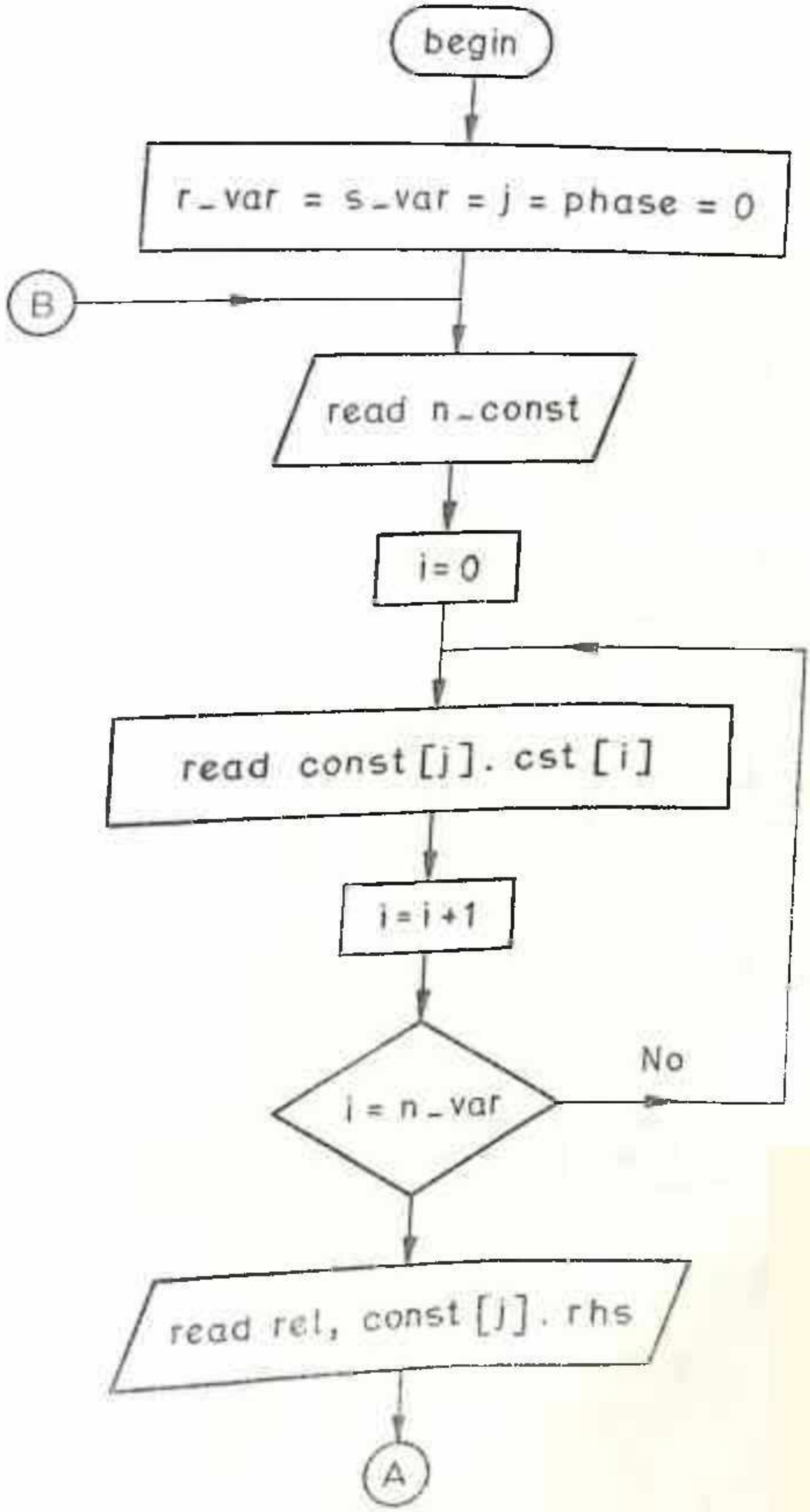
ALGORITHM FOR MAIN PROGRAM

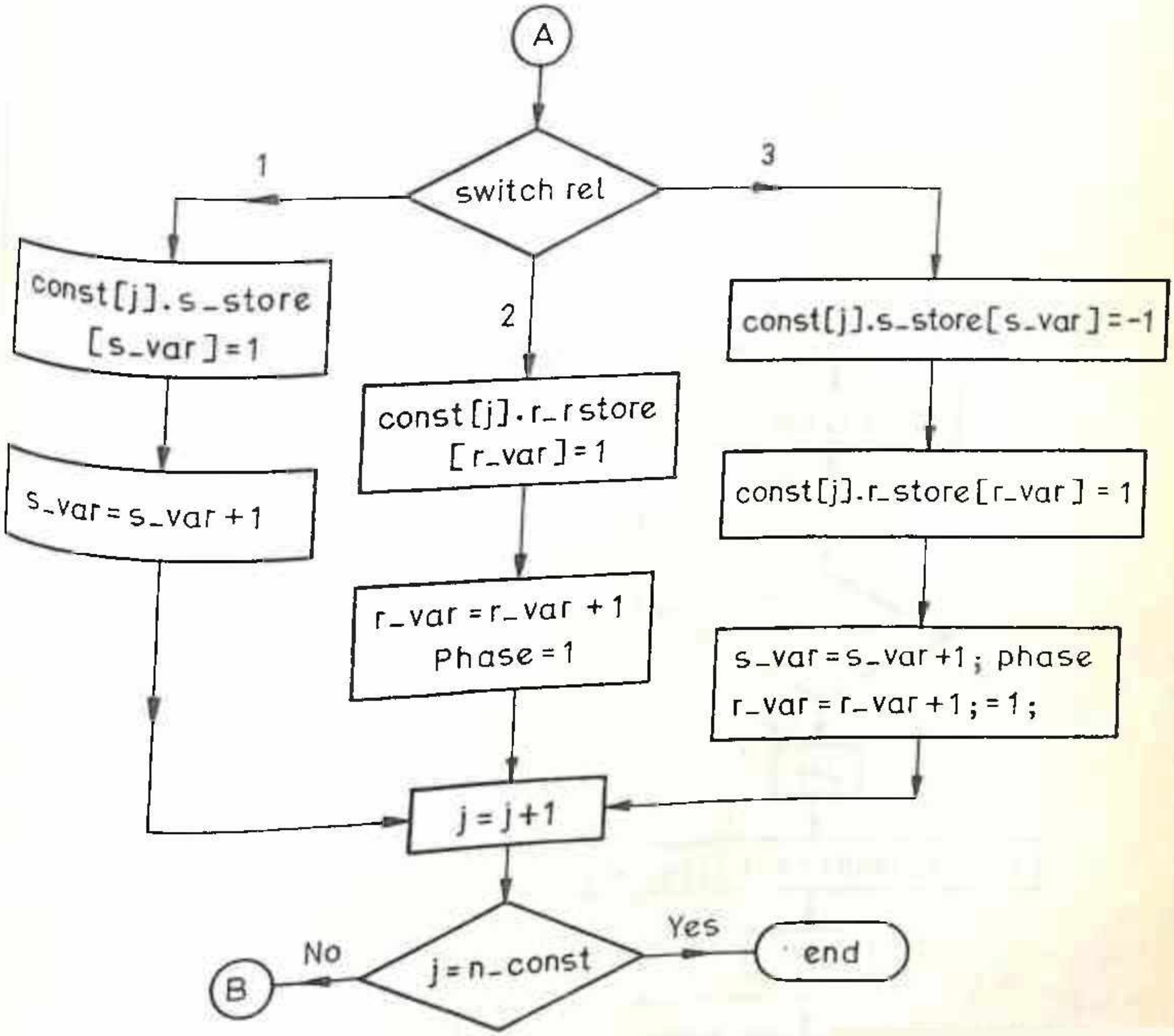


FLOW CHART FOR read_obj( )

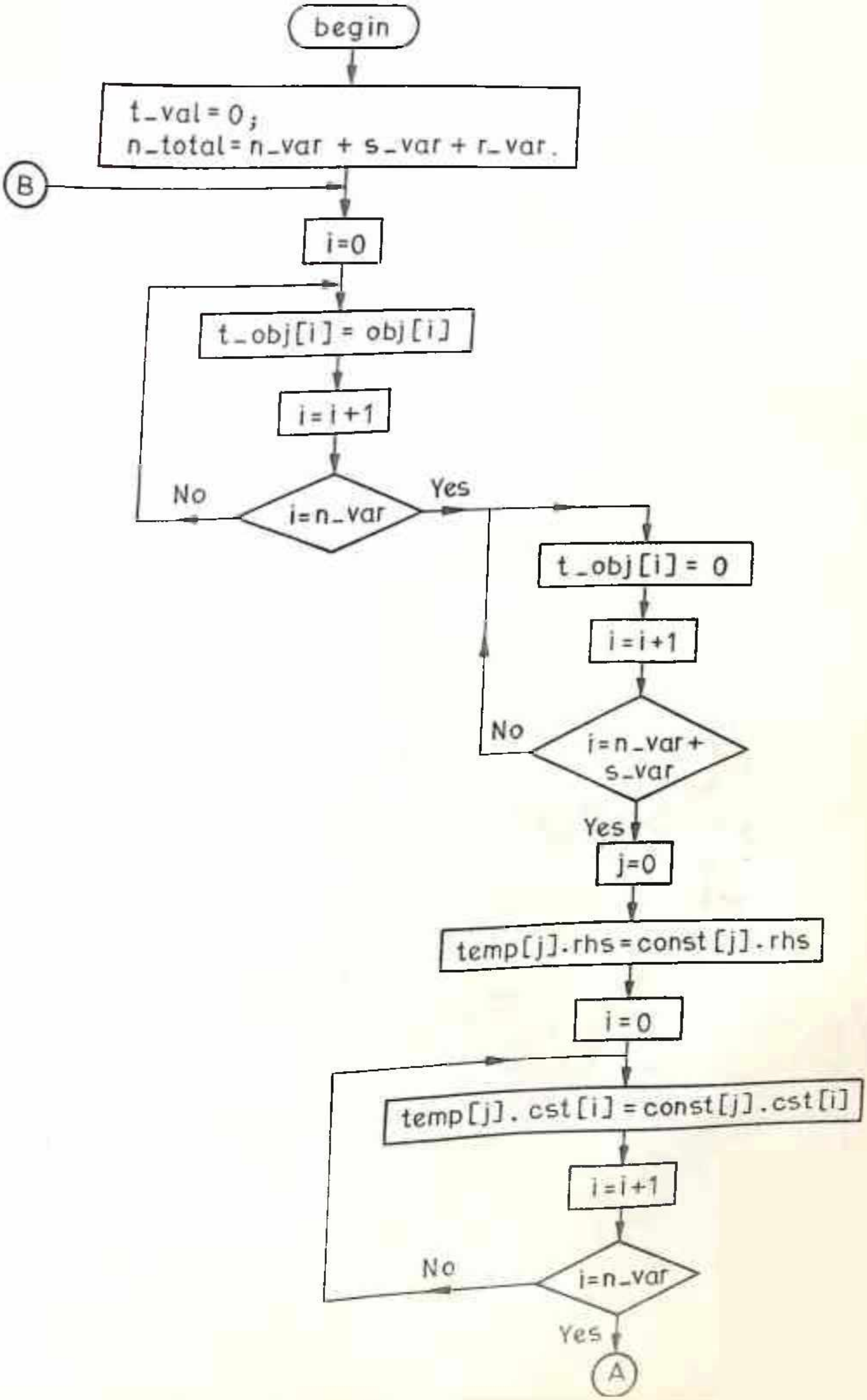


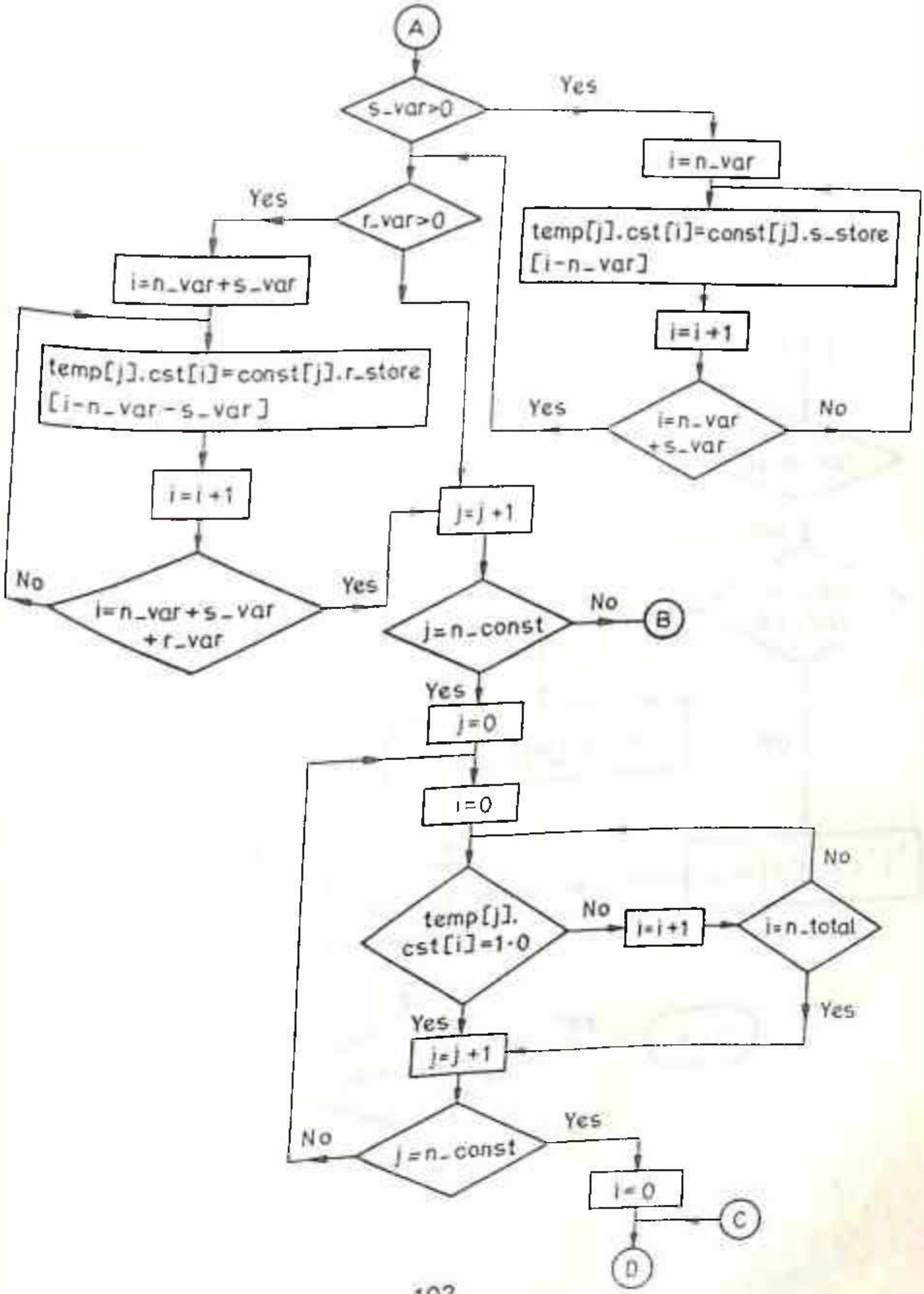
FLOW CHART FOR read_const()



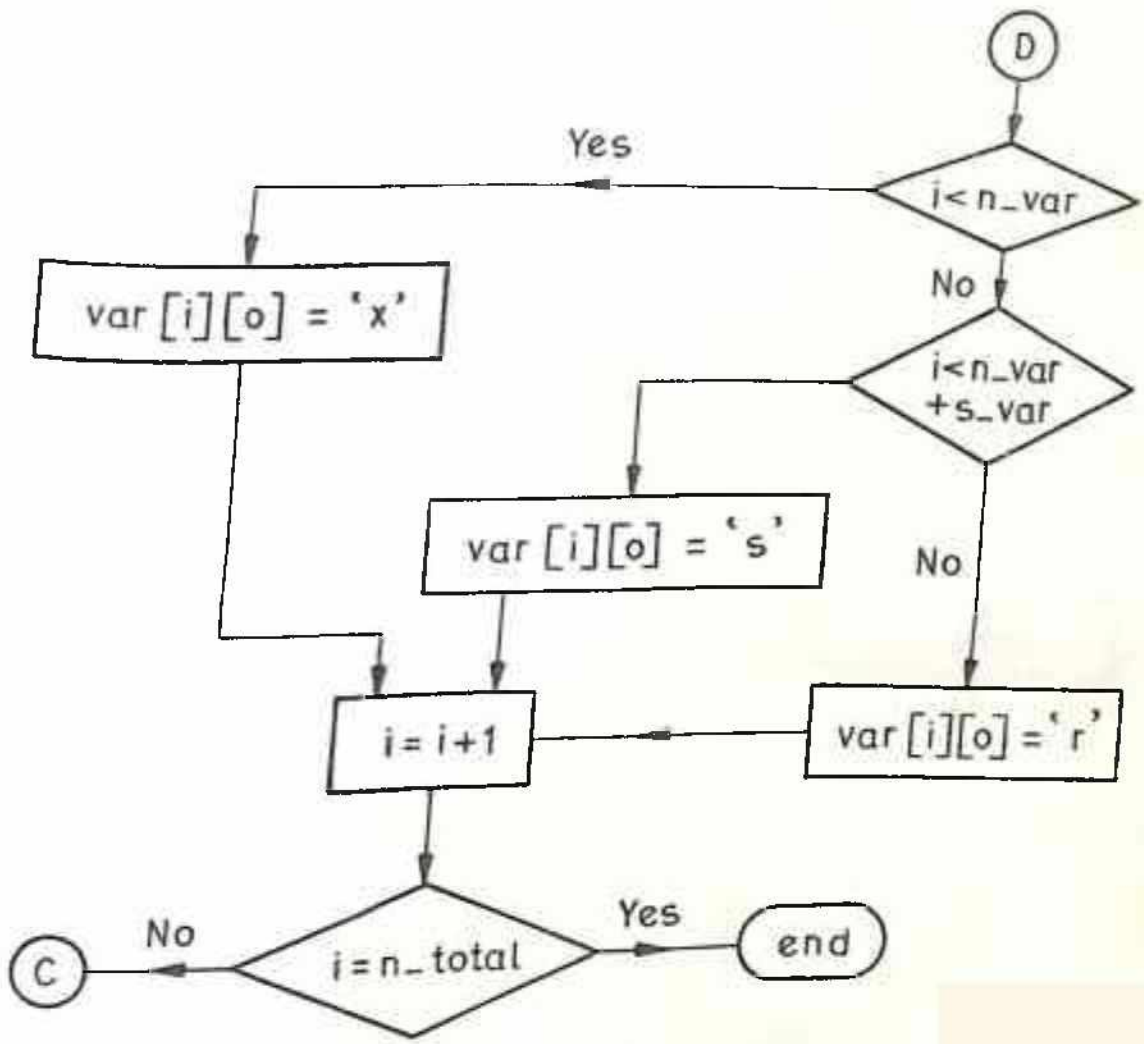

A
switch rel
1
2
3
const[j].s_store [s_var]=1
s_var=s_var+1
const[j].r_rstore [r_var]=1
r_var=r_var+1
Phase=1
const[j].s_store[s_var]=-1
const[j].r_store[r_var] = 1
s_var=s_var+1; phase
r_var=r_var+1; =1;
j=j+1
j=n_const
No
Yes
B
end

FLOW CHART FOR fill-up ( )

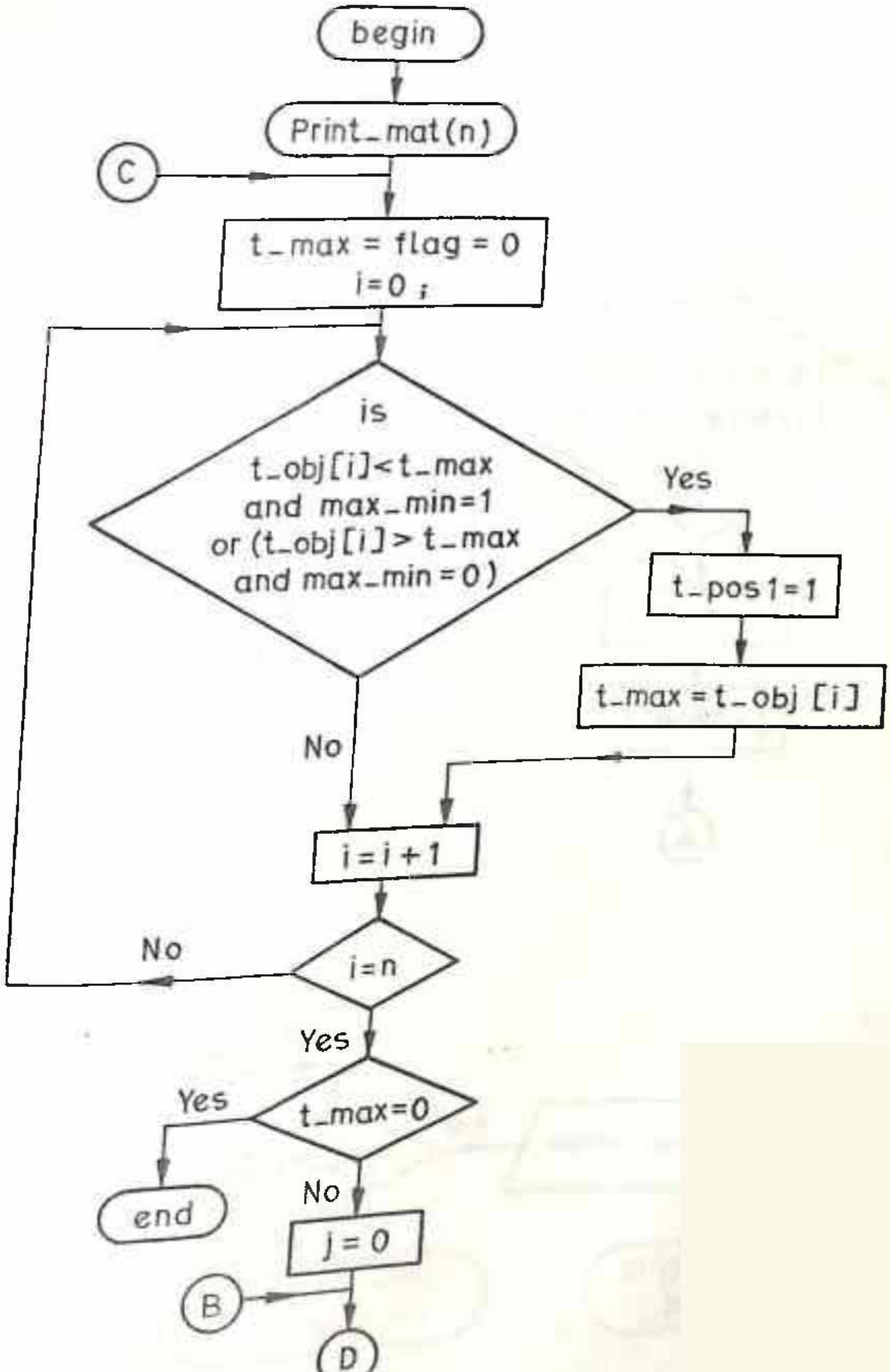


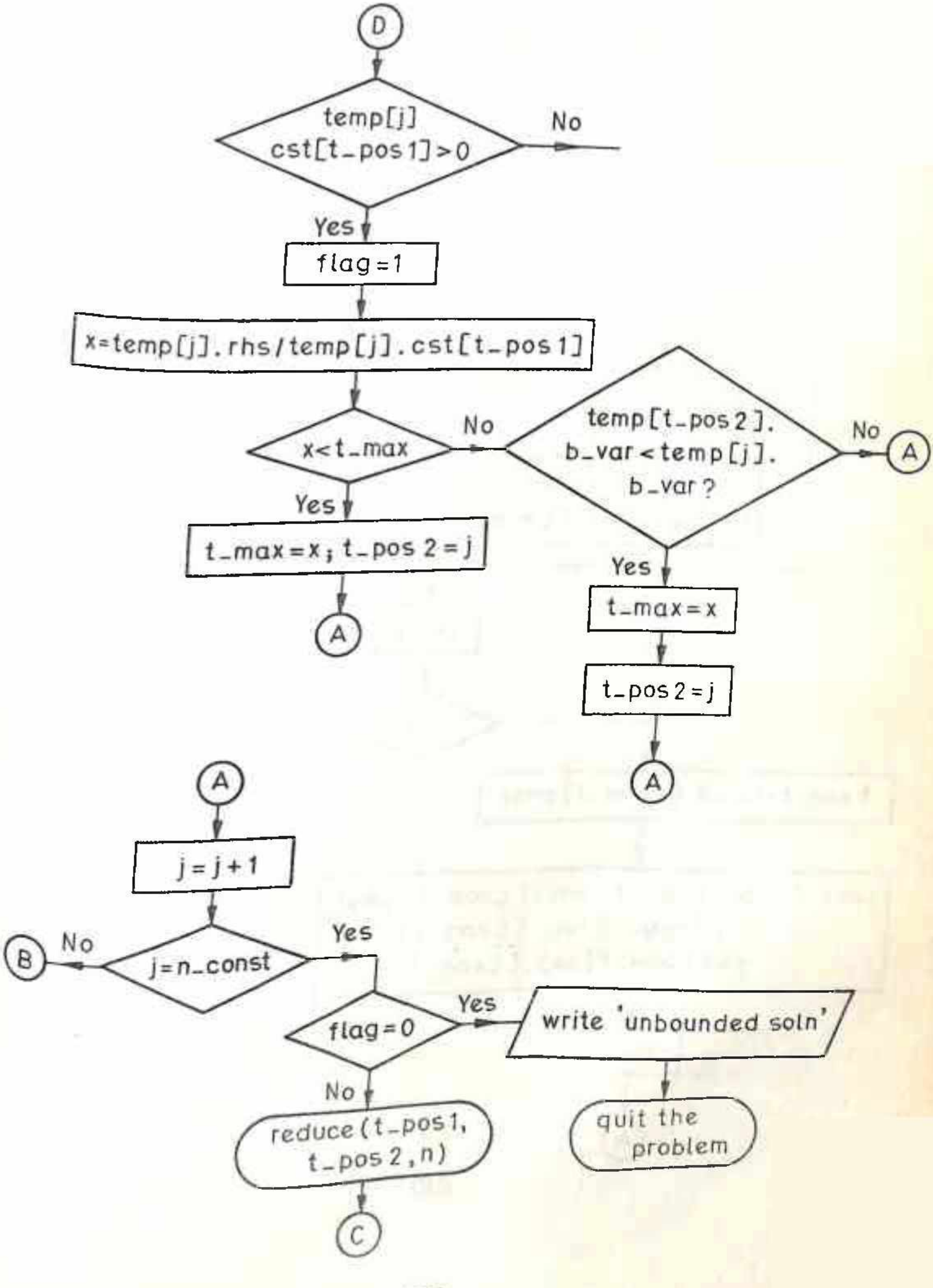


A
s_var>0
Yes
i=n_var
temp[j].cst[i]=const[j].s_store
[i-n_var]
i=i+1
i=n_var
+s_var
Yes
No
r_var>0
Yes
i=n_var+s_var
temp[j].cst[i]=const[j].r_store
[i-n_var-s_var]
i=i+1
i=n_var+s_var
+r_var
No
Yes
j=j+1
j=n_const
No
B
Yes
j=0
i=0
temp[j].
cst[i]=1·0
No
i=i+1
i=n_total
No
Yes
Yes
j=j+1
j=n_const
No
Yes
i=0
C
D

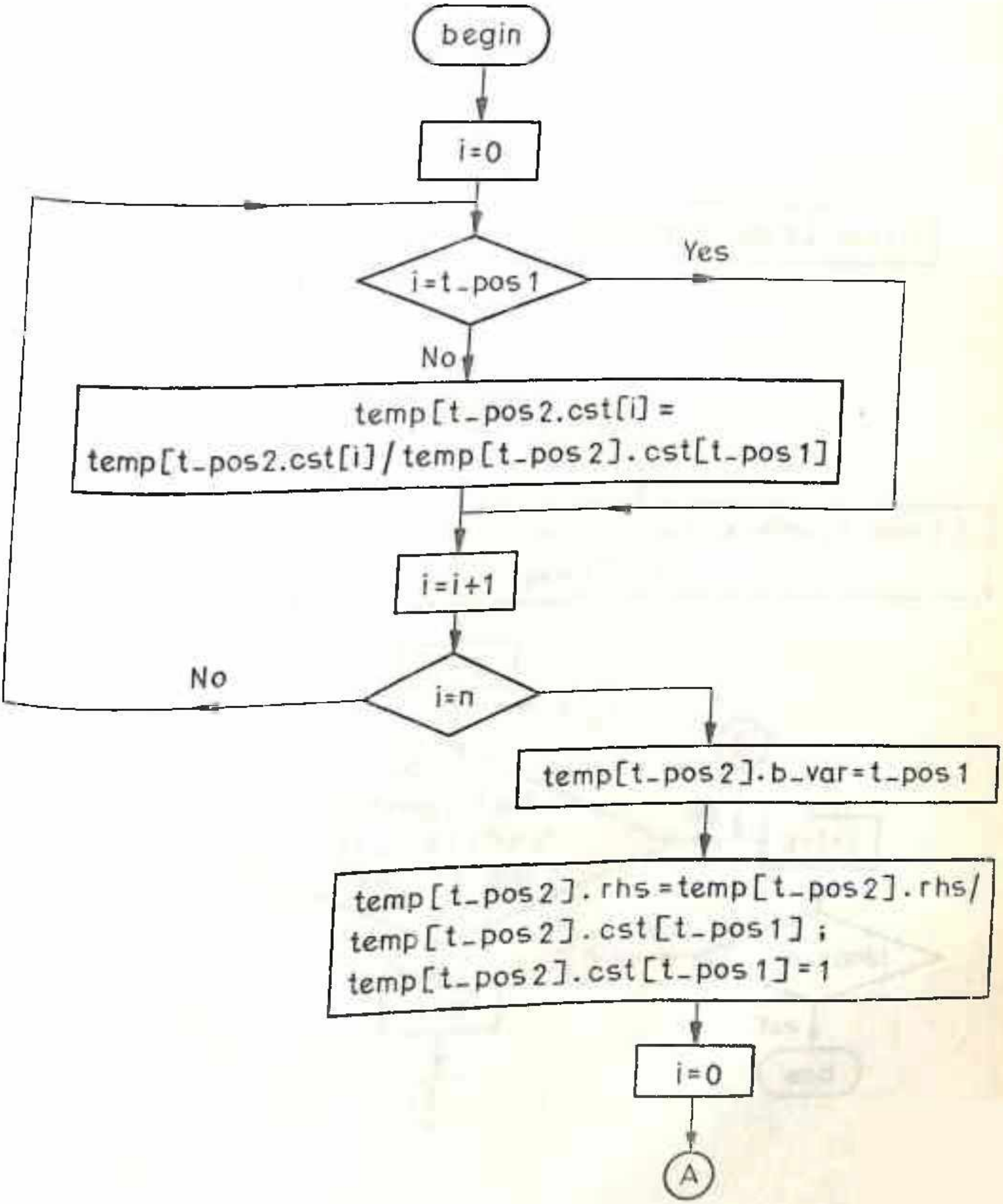

D
Yes
i< n_var
No
var [i][o] = 'x'
i<n_var
+s_var
var [i][o] = 's'
No
i = i+1
var [i][o] = 'r'
No
Yes
C
i = n_total
end

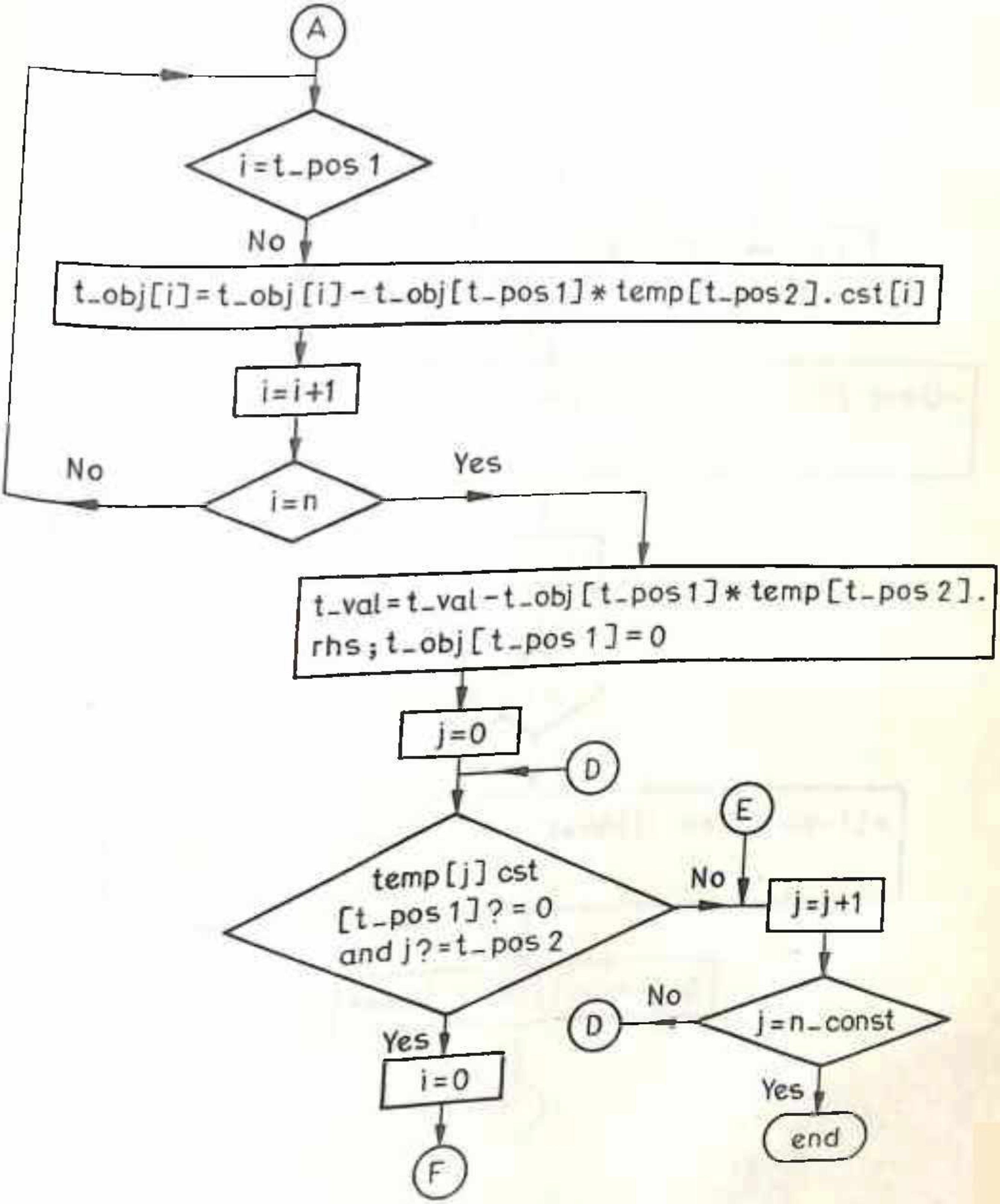
FLOW CHART FOR phase_2 (intn)

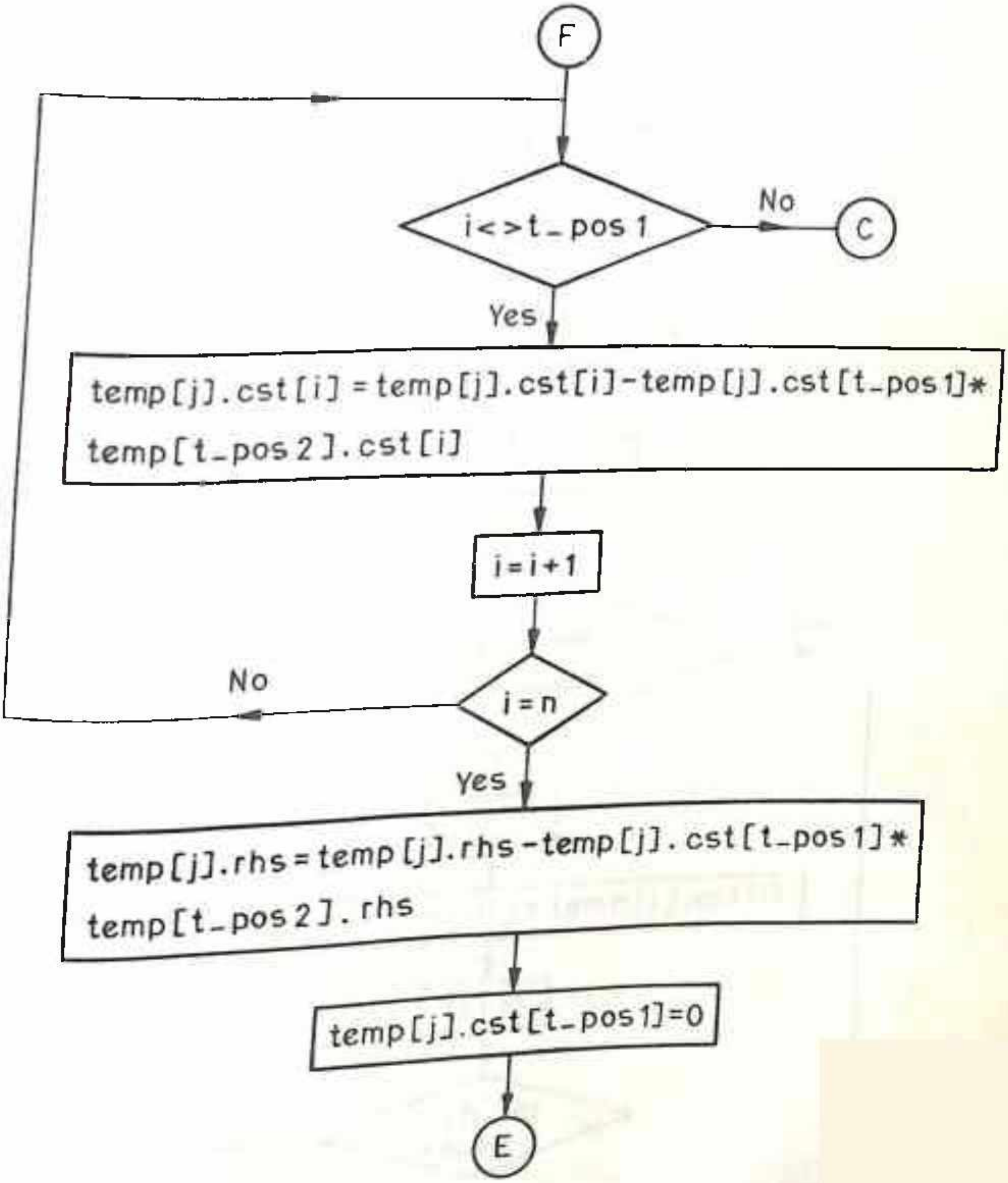



D
temp[j]
cst[t_pos 1] > 0
No
Yes
flag = 1
x=temp[j].rhs/temp[j].cst[t_pos 1]
x < t_max
No
temp[t_pos 2].
b_var < temp[j].
b_var ?
No
A
Yes
t_max = x ; t_pos 2 = j
A
Yes
t_max = x
t_pos 2 = j
A
A
j = j + 1
B
No
j = n_const
Yes
flag = 0
Yes
write 'unbounded soln'
No
reduce (t_pos1, t_pos 2, n)
quit the problem
C

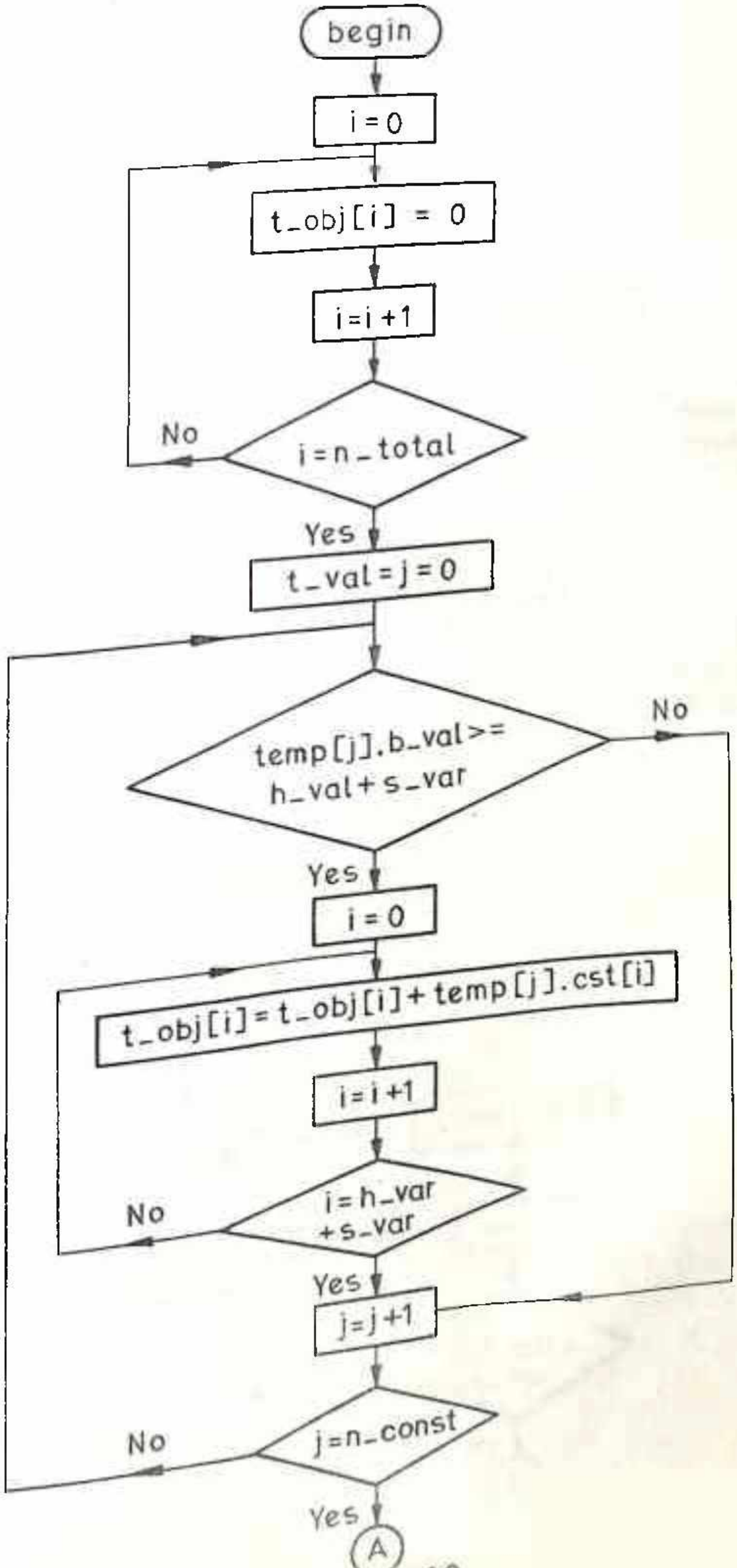
FLOW CHART FOR reduce (t_pos1 t_pos 2 n)

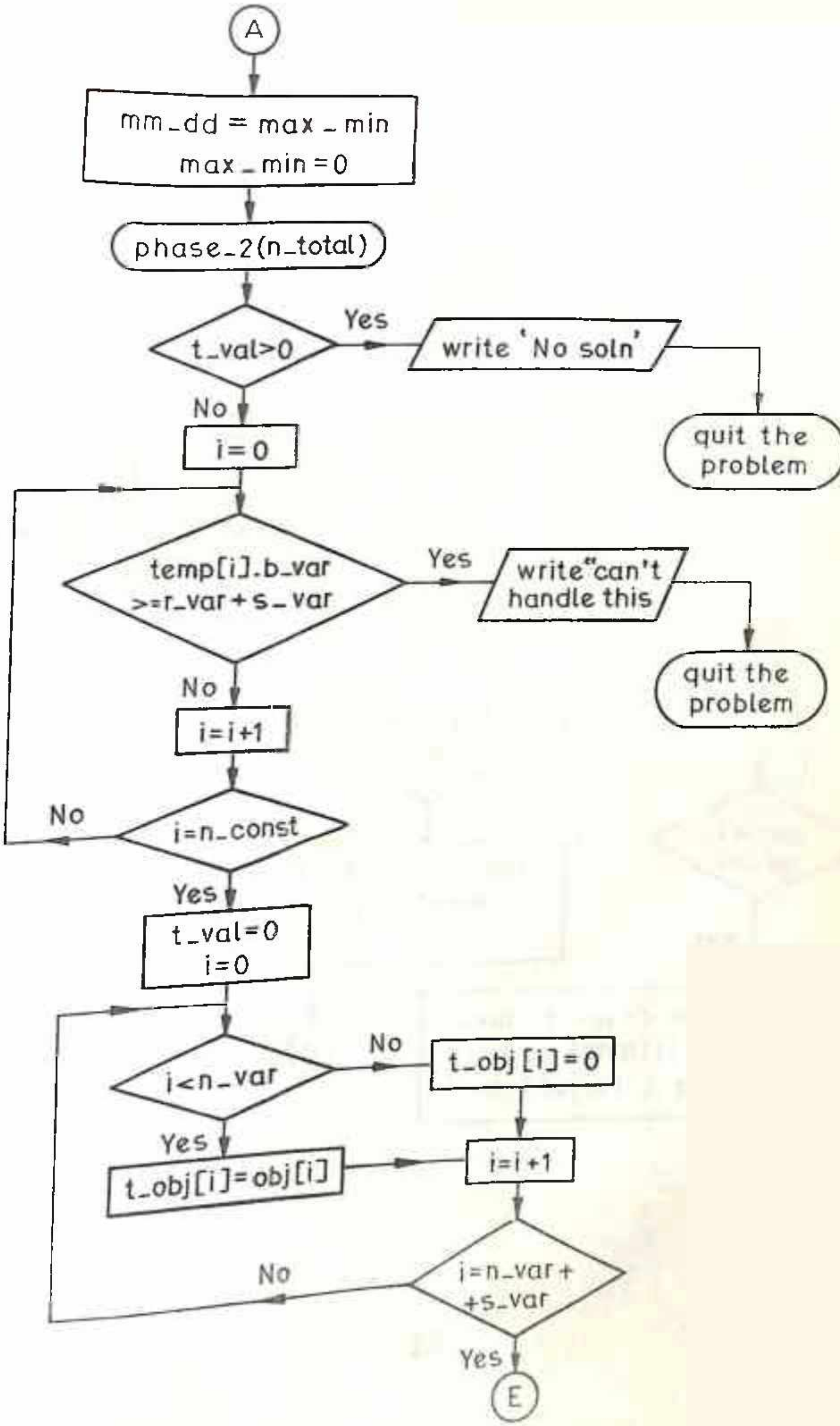


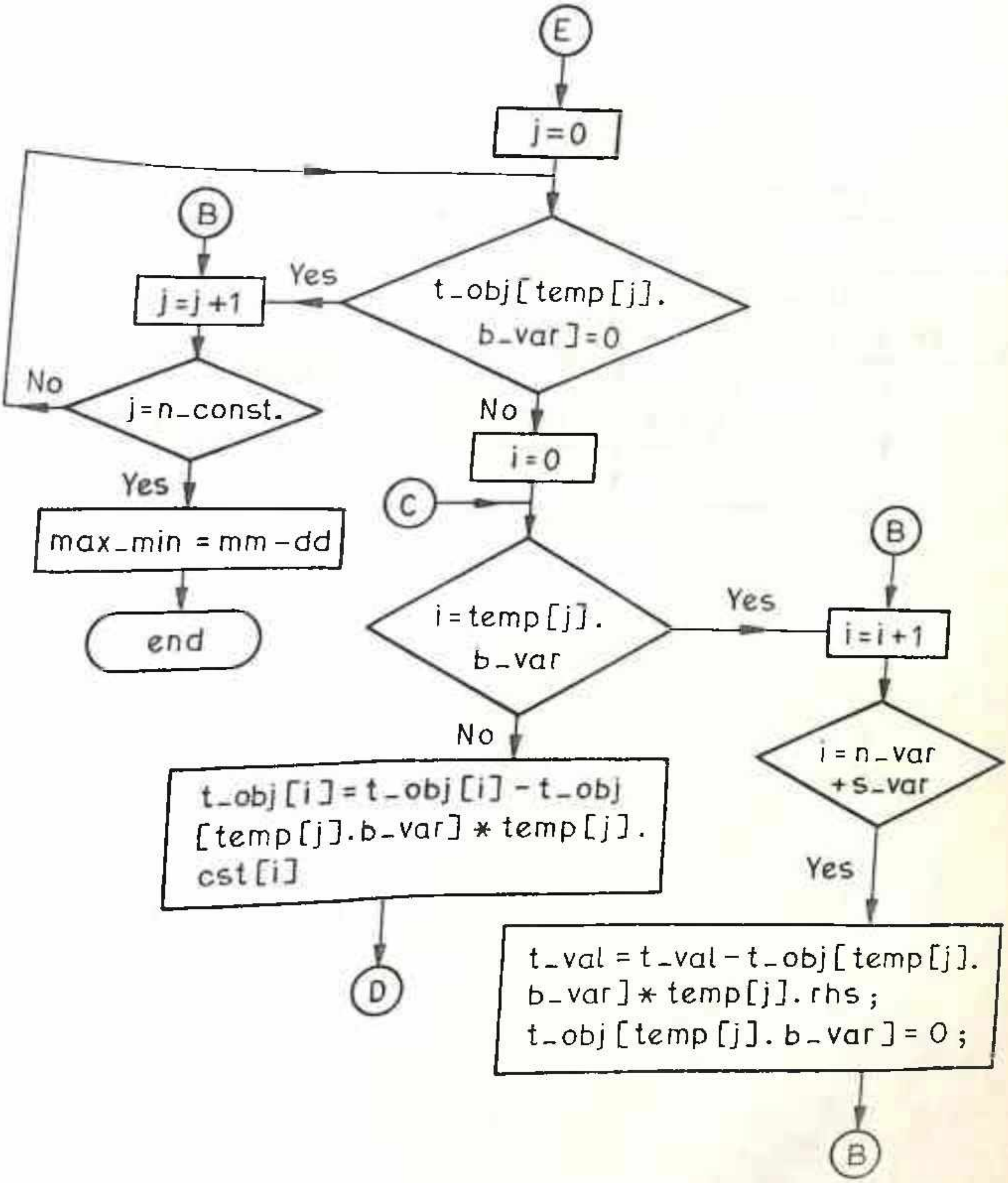

A
i=t_pos 1
No
t_obj[i]=t_obj[i]-t_obj[t_pos 1] * temp[t_pos 2]. cst[i]
i=i+1
No
Yes
i=n
t_val=t_val-t_obj[t_pos 1] * temp[t_pos 2].
rhs; t_obj[t_pos 1]=0
j=0
D
E
temp[j] cst
[t_pos 1] ?=0
and j?=t_pos 2
No
j=j+1
No
D
j=n_const
Yes
i=0
Yes
end
F


F
i<>t_pos 1
No
C
Yes
temp[j].cst[i] = temp[j].cst[i]-temp[j].cst[t_pos1]*
temp[t_pos 2].cst[i]
i=i+1
No
i=n
Yes
temp[j].rhs=temp[j].rhs-temp[j].cst[t_pos1]*
temp[t_pos 2].rhs
temp[j].cst[t_pos1]=0
E

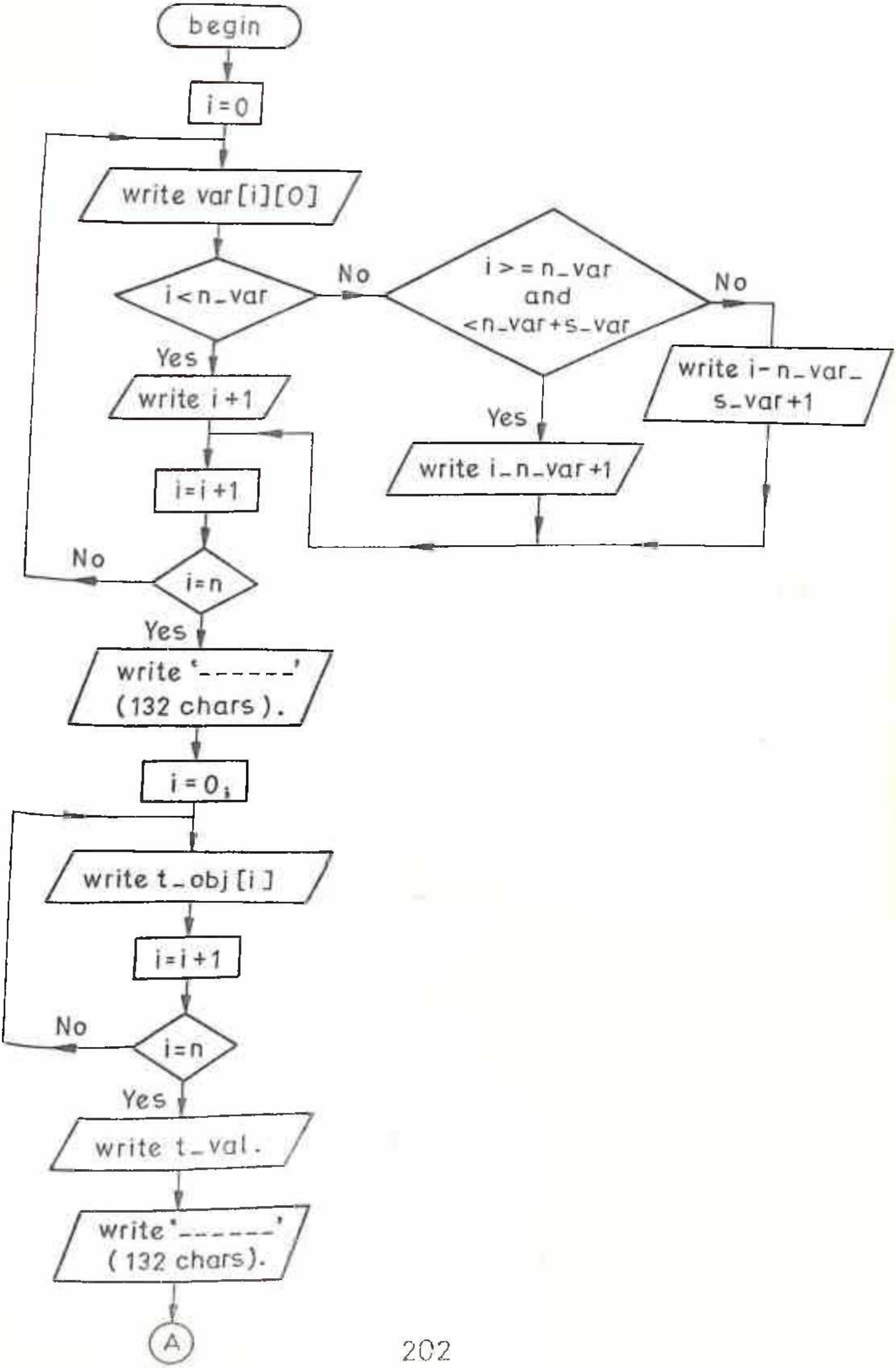
FLOW CHART FOR phase_1( )

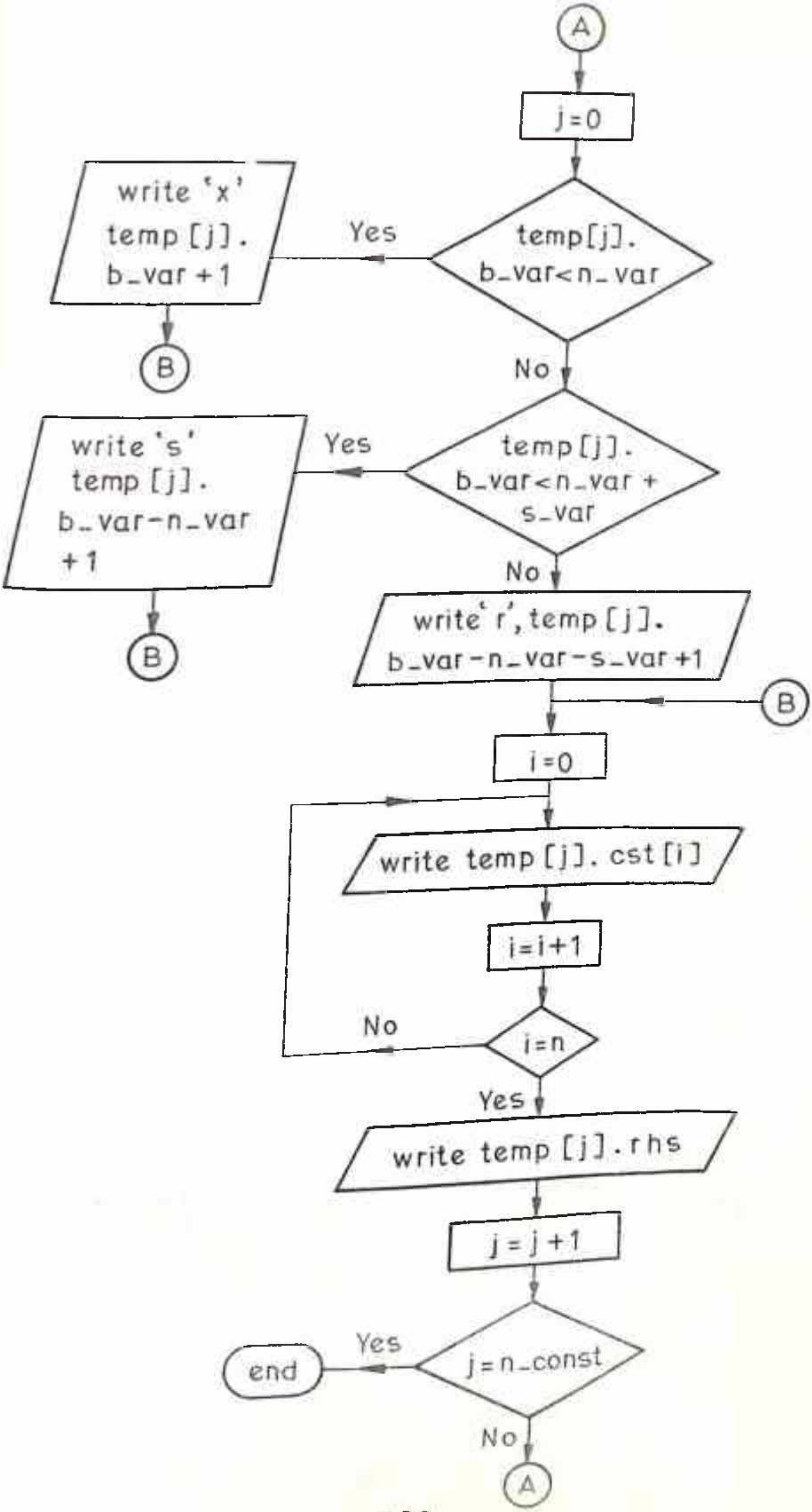



A
mm_dd = max_min
max_min = 0
phase_2(n_total)
t_val>0
Yes
write 'No soln'
quit the problem
No
i=0
temp[i].b_var >=r_var+s_var
Yes
write "can't handle this
quit the problem
No
i=i+1
i=n_const
No
Yes
t_val=0
i=0
i<n_var
No
t_obj[i]=0
Yes
t_obj[i]=obj[i]
i=i+1
i=n_var+ +s_var
No
Yes
E


E
j=0
B
Yes
j=j+1
t_obj[temp[j].
b_var]=0
No
j=n_const.
No
Yes
i=0
C
max_min = mm-dd
end
B
Yes
i=temp[j].
b_var
i=i+1
No
i=n_var
+s_var
t_obj[i]=t_obj[i]-t_obj
[temp[j].b_var] * temp[j].
cst[i]
Yes
D
t_val = t_val-t_obj[temp[j].
b_var] * temp[j].rhs ;
t_obj[temp[j].b_var] = 0 ;
B

FLOW CHART FOR Print_mat (intn)





A
j=0
temp[j].
b_var<n_var
Yes
write 'x'
temp [j].
b_var +1
B
No
temp [j].
b_var<n_var +
s_var
Yes
write 's'
temp [j].
b_var-n_var
+1
B
No
write 'r', temp [j].
b_var-n_var-s_var+1
B
i=0
write temp [j]. cst [i]
i=i+1
i=n
No
Yes
write temp [j].rhs
j=j+1
j=n_const
Yes
end
No
A

APPENDIX A4.2 LISTING OF PROGRAM USED

```
# include <stdio.h>
# include <ctype.h>

struct x1{
                int b_var;
                float cst[50];
                float rhs;
                float s_store[50];
                float r_store[50];
                } const_anc[50];

struct {
        float  cst[50], rhs;
        int b_var;
        } temp[50];

float obj[50], t_obj[50], t_val;
int n_var, n_const, s_var, r_var, n_total, phase, max_min;
char var[50][4];

read_obj()
{
int i;
printf(" Enter the nature of the problem \n");
printf(" 1 for max, 0 for min : ");
scanf("%d", &max_min);
printf(" Enter the no of variables in obj function : ");
scanf("%d", &n_var);
for ( i = 0 ; i < n_var ; i++)
{
        printf(" Enter objective : %d", i+1);
        scanf("%f",&obj[i]);
        printf("%.2f",obj[i]);
}
printf(" read_obj() over ... !\n");
}

read_const()
{
        int i,j, rel ;
        r_var = s_var = j = phase = 0 ;
        printf("Enter the number of constraints : ");
        printf(" If const. is 3x1+ 4x2 > 2 enter 3 4 \n");
        scanf("%d", &n_const);
        for ( j = 0; j < n_const; j++)
        {
                printf(" enter constraint %d ... ", j+1);
```

```
                for ( i = 0 ; i < n_var ; i++)
                        scanf("%f", &const_anc[j].cst[i]);
                printf("\nEnter the relation <:1 = :2>:3\n");
                scanf("%d", &rel);
        if(rel==1)      const_anc[j].s_store[s_var++] = 1;
        else if (rel ==3) {
                const_anc[j].s_store[s_var++] = (-1);
                const_anc[j].r_store[r_var++] = 1;
                phase = 1 ;
                     }
                else if (rel == 2) {
                     const_anc[j].r_store[r_var++] = 1;
                     phase = 1 ;
                     }
                printf("\n Enter the rhs value ... ");
                scanf("%f", &const_anc[j].rhs);
        }
        printf("\n No of constraints : %d\n", n_const);
        printf(" ******** SIMPLEX TABLE ***** \n");
        printf(" The objective is : ");
        for ( i= 0 ; i < n_var ; i++)
                printf("%1.2f ", obj[i]);
        printf("\n\n The Constraints : ");
        for ( j = 0; j < n_const; j++) {
                for (i = 0 ; i < n_var ; i++)
                printf(" %1.2f ", const_anc[j].cst[i]);
        if(s_var>0)
                for ( i = 0 ; i < s_var ; i++)
                printf(" %1.2f ", const_anc[j].s_store[i]);
        if(r_var>0)
                for ( i = 0 ; i < r_var ; i++)
                printf(" %1.2f ", const_anc[j].r_store[i]);
        printf("\n");
       printf(" read_const() over ... !\n");
}       }

fill_up()
{
        int i, j;
        printf(" Entered the function fill_up ! \n");
        t_val = 0;
        n_total = s_var + r_var + n_var ;
        for ( i = 0; i < n_var ; i++)
                t_obj[i] = obj[i];
        for ( i = n_var; i < n_var+s_var ; i++)
                t_obj[i] = 0 ;
        for ( j = 0; j < n_const ; j++)
        {
                for ( i = 0; i < n_var ; i++)
```

```
                temp[j].cst[i] = const_anc[j].cst[i];
                if(s_var> 0)
                for ( i = n_var; i < n_var+s_var ; i++)
          temp[j].cst[i] = const_anc[j].s_store[i-n_var];
          if(r_var> 0)
          for ( i = n_var+s_var; i < n_var+s_var+r_var; i++)
          temp[j].cst[i]=const_anc[j].r_store[i-n_var-s_var];
          temp[j].rhs = const_anc[j].rhs ;
          }
          for ( j = 0; j < n_const ; j++)
          for (i = n_var ; i < n_total ; i++)
          if(temp[j].cst[i] == 1.00){ temp[j].b_var = i ;
                      break ; }
         for ( i = 0; i < n_var ; i++)
               var[i][0] = 'x' ;
         for ( i = n_var; i < n_var+s_var ; i++)
               var[i][0] = 's' ;
        if(r_var > 0)
               for ( i = n_var+s_var; i < n_total; i++)
                            var[i][0] = 'r' ;
}

phase_2(int n)
{  /* okay */
      int t_pos2, t_pos1, flag , i , j ;
      float t_max, x;
printf(" Entered second phase of computations .... !\n");
print_mat(n);
while(1)
      {    /* okay */
              t_max = flag = 0 ;
              for ( i = 0 ; i< n; i++ )
              if( ((t_obj[i] < t_max) && (max_min ==1))
              || ( (max_min ==0) && (t_obj[i] > t_max)))
                    {  /* okay */
                          t_pos1 = i;
                          t_max = t_obj[i] ;
                    } /* finished */
             if(t_max==0) break ;
             else {  /* okay */
    printf(" The pos of entering var is %d\n", t_pos1);
                   t_max = 100 ;
                   flag = 0;
                   for (j= 0 ; j < n_const ; j++)
                         if(temp[j].cst[t_pos1] > 0)
                         { /* okay */
                         flag = 1;
   if((x=temp[j].rhs/temp[j].cst[t_pos1]) < t_max)
                        { /* okay */
                        t_max =x ; t_pos2 = j ;
```

```
                                        } /* finished */
else if(x==t_max && temp[t_pos2].b_var<temp[j].b_var)
                              {  /* okay */
                                        t_max =x ; t_pos2 = j ;
                                        }  /* finished */
                    } /* finished */
                    if(!flag)
                    { /* okay */
                    printf(" UNbounded soln !\n"); exit(1);
                    } /* finished */
                    else {    /* okay */
          printf(" Posn of leaving var : %d\n", t_pos2);
          reduce(t_pos1,t_pos2, n);
                              }  /* finished */

          } /* finished */
          print_mat(n);
          } /* finished */
          return ;
}

reduce(int t_pos1, int t_pos2, int n)
{
          int i, j;
          for (i =0 ; i < n ; i++)
                    if(i!=t_pos1)
temp[t_pos2].cst[i]=temp[t_pos2].cst[i]/temp[t_pos2].cst[t_pos1];
          temp[t_pos2].rhs /= temp[t_pos2].cst[t_pos1];
          temp[t_pos2].cst[t_pos1] = 1;
          temp[t_pos2].b_var = t_pos1 ;
          for (i =0 ; i < n ; i++)
                    if(i!=t_pos1)
          t_obj[i] -= t_obj[t_pos1]*temp[t_pos2].cst[i];
          t_val -= t_obj[t_pos1]*temp[t_pos2].rhs;
          t_obj[t_pos1] = 0 ;
          for (j =0 ; j < n_const ; j++)
                    if((temp[j].cst[t_pos1] !=0) && (j!=t_pos2))
                    {         for (i =0 ; i < n ; i++)
                                        if(i!=t_pos1)
temp[j].cst[i] -= temp[j].cst[t_pos1]*temp[t_pos2].cst[i];
temp[j].rhs -= temp[j].cst[t_pos1]*temp[t_pos2].rhs;
temp[j].cst[t_pos1] = 0;
                    }
}

phase_1()
{
          float t_min, x;
          int i, j , flag, t_pos1, t_pos2, mm_old;
          for ( i= 0 ; i< n_total ; i++)
```

```
                t_obj[i] = 0;
        t_val = 0;
        for ( j= 0 ; j< n_const ; j++)
                if(temp[j].b_var >= (n_var+s_var)) {
                        for ( i =0; i < n_var+s_var ; i++)
                t_obj[i] += temp[j].cst[i];
                t_val += temp[j].rhs ;
                }
        mm_old = max_min ;
        max_min = 0 ;
        phase_2(n_total);
        if(t_val > 0)
        {
                printf(" End of phase 1 ... ! \n");
                printf(" Soln set is empty ... !\n");
                exit(1);
        }
        else
        for (i= 0 ; i<n_const ; i++)
        if (temp[i].b_var >= (n_var +s_var)){
        printf(" This is a very rare prob .... !\n");
                                exit(1);
                        }
        t_val = 0;
        for ( i = 0 ; i < n_var ; i++)
                t_obj[i] = obj[i] ;
        for ( i = n_var ; i < (n_var+s_var) ; i++)
                t_obj[i] = 0;
        for ( j= 0 ; j< n_const ; j++)
        if (t_obj[temp[j].b_var])
                {
                for ( i = 0; i< (n_var +s_var) ;i++)
        if(i!=temp[j].b_var)
        t_obj[i] -= t_obj[temp[j].b_var]*temp[j].cst[i];
        t_val -= t_obj[temp[j].b_var]*temp[j].rhs;
        t_obj[temp[j].b_var] = 0;
                }
        printf(" End of phase 1 ... !\n");
        max_min = mm_old ;
}

print_mat(int n)
{
        int i, j;
        printf(" ***** SIMPLEX TABLE **** \n");
        printf("        ");
        for ( i= 0; i < n ; i++)
        {
                printf("%c", var[i][0]);
        if(i < n_var)
```

```
                printf("%d      ||", i+1);
        else if((i>=n_var) && (i < (n_var+s_var)))
                printf("%d      ||", i-n_var+1);
                else
                printf("%d      ||", i-n_var-s_var+1);
        }
        printf("\n");
        for (i = 0; i < 132,printf("-") ; i++) ;
        printf("\n");
        printf("        ");
        for ( i= 0; i < n ; i++)
                printf("%-6.2f|", t_obj[i]);
        printf("%-6.2f\n", t_val);
        for (i = 0; i < 132,printf("-") ; i++) ;
        for ( j = 0; j < n_const; j++)
        {
        if(temp[j].b_var < n_var)
        {
        printf("x"); printf("%d ||", temp[j].b_var+1);
        }
 else if((temp[j].b_var>n_var)&&(temp[j].b_var<(n_var+s_var)))
 {
 printf("s"); printf("%d ||", temp[j].b_var-n_var+1);
        }
        else
        {
printf("r");printf("%d ||",temp[j].b_var-n_var-s_var+1);
                }
        for ( i= 0; i < n ; i++)
                printf("%-6.2f|", temp[j].cst[i]);
        printf("%-6.2f\n", temp[j].rhs);
        }
}

main()
{
        read_obj();
        read_const();
        fill_up();
        printf(" Exited fillup ... !\n");
        if(phase==1) phase_1();
        phase_2(n_var+s_var);
        printf(" End of computation .... !\n");
}
```

APPENDIX A8-1:

Partition Algorithm for Linear Goal Programming Problem (LGPP):

Goal programming problems can be solved efficiently by the partition algorithm. The algorithm is based on the principle that goals with higher priorities must be optimized before lower order goals are even considered. With this, the problem becomes solving a series of linear programming sub-problems each using the previous optimal solution as the starting solution. The methodology is explained below

Step 1:

Bring the LGPP into the following standard form

$$\text{Minimise } x_0 = \sum_{i=1}^{m} p_i \left(w_i^+ s_i^+ + w_i^- s_i^-\right)$$

$$\text{subject to } \sum_{j=1}^{n} a_{ij} x_j + s_i^- - s_i^+ = b_i \quad \text{for all } i = 1 \text{ to } m$$

$$x_j, s_i^-, s_i^+ \geq 0 \quad \begin{array}{l}\text{for all } i = 1 \text{ to } m \\ \text{for all } j = 1 \text{ to } n\end{array}$$

where p_i is priority attached to the ith constraint and w_i^+ and w_i^- are the weights attached to s_i^+ & s_i^- respectively. $w_i^+ = 1$ or 0 according as s_i^+ is to be included or not to be included in the trade-off function. Similarly w_i^- is associated with s_i^-.

In the standard form of LGPP defined above, the main features are following two:

(a) Each constraint contains both the surplus and slack variables.

(b) The surplus and slack variables s_i^+ and s_i^- of the ith constraint are not present in jth $(j = i)$ constraint for all $j = i$, that is the surplus and slack variables of a constraint are present only in that constraint and not in any other constraint.

Step 2:

Solve the first sub-linear programming problem, which consists of the part of objective function involving first priority p_1 and the constraint (or constraints) which are given priority p_1. This sub-problem is solved by using simplex method because the slack variables give the identity matrix (starting basic feasible solution). Solve this and get optimal solution. There arise one of two cases:

(a) There exists an alternative optimal solution, (b) the optimal solution is unique. In case of (a) go to step (3) and in case of (b) go to step 4.

Step 3:

If alternative optimal solution exists then it is possible to negotiate with the constraint (or constraints) of next highest priority P_2. For this we follow the following logical steps.

(a) In the optimal table of step 2, no relative cost $z_j - c_j$ will be positive, because it is an optimal table of a minimization problem. If, in the optimal table, the relative cost corresponding to a slack or surplus variable is negative then this variable is dropped. This is because, these slack or surplus variables are not involved in any of the remaining constraints and they can not give solution better than given in optimal table of step 2, if entered at later stage.

(b) Delete the objective function row.

(c) Add the constraint (or constraints) with priority P_2, by sensitivity analysis. This will not create any problem because both s^- and s^+ are present. While adjusting the columns corresponding to basic variables to proper format, one of these will give the required identity column.

(d) Add the part of the objective function corresponding to priority P_2 by using sensitivity analysis (that is make the relative costs of basic variables to be zero).

(e) Use simplex method to get optimal table of the above second sub-problem.

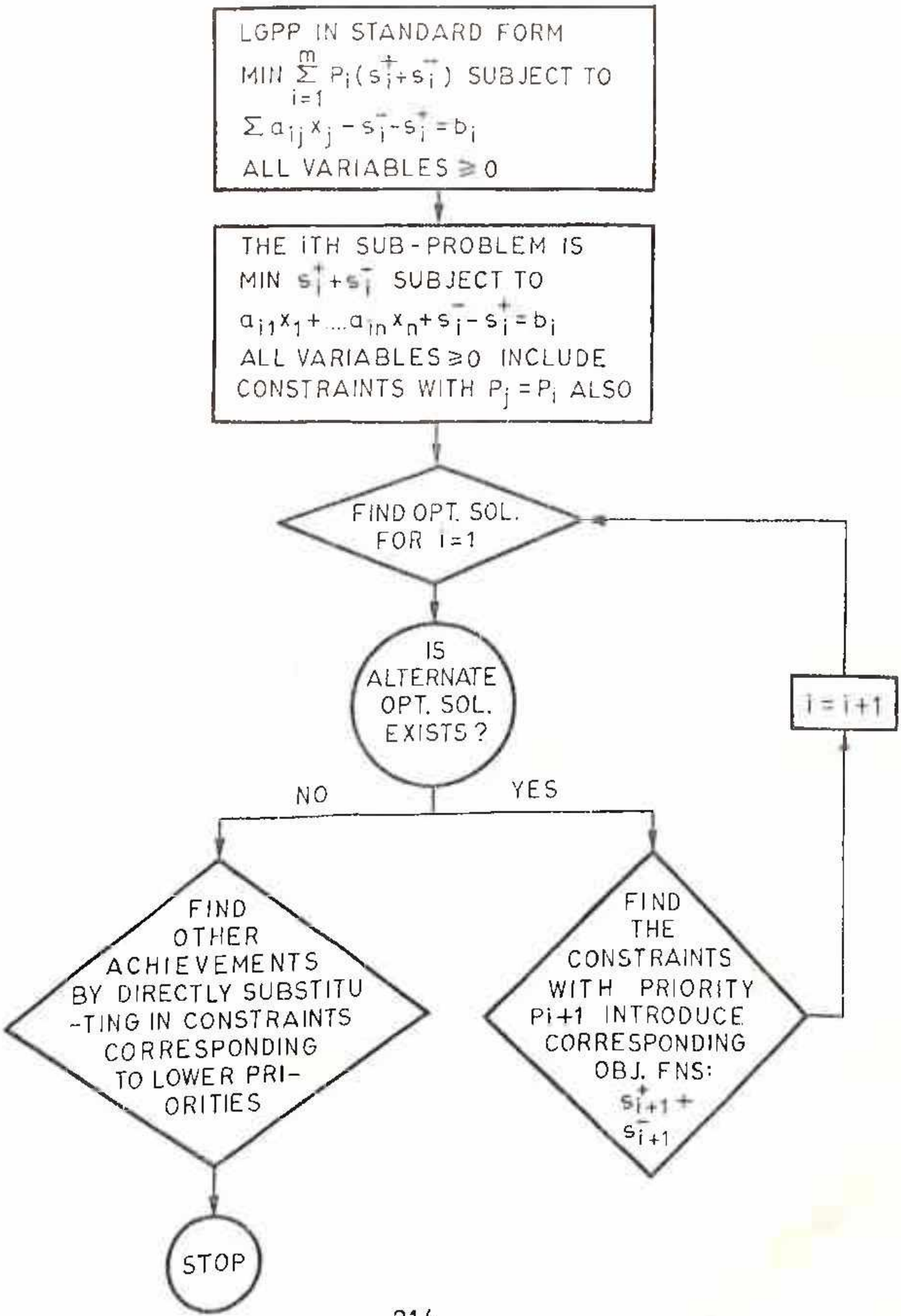
(f) Go to Step 2 with the above problem as first sub-problem.

Step 4:

In case there exists no alternative optimal solution, the optimal table of step 2 gives the optimal solution for the given LGPP with respect to all of the priorities. This is because P_1 is the priority of highest order and this subproblem has one and only one optimal solution. So there is no scope to accommodate the constraints with subsequent lower priorities. The values of other decision variables are obtained by substituting the values of the variables given by the optimal table of step 2 in the goal constraints of the lower priorities. This completes the algorithm.

APPENDIX A8-2

FLOW DIAGRAM OF PARTITION ALGORITHM



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