

**OPTIMAL LOAD SHEDDING  
FOR  
POWER SYSTEM SECURITY  
USING  
PATTERN RECOGNITION TECHNIQUE**

Thesis

Submitted in partial fulfillment of the  
requirements for the degree of  
DOCTOR OF PHILOSOPHY

By

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CERTIFICATE

This is to certify that the thesis entitled  
'OPTIMAL LOAD SHEDDING FOR POWER SYSTEM SECURITY  
USING PATTERN RECOGNITION TECHNIQUE', submitted by  
Shri A.K.Sinha for the award of the Ph. D. degree  
of the Institute, embodies original work done by  
him under my supervision.

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## CHAPTER 1

### INTRODUCTION

#### 1.1 INTRODUCTION TO THE PROBLEM

The main objective in control of power system is to minimize the cost of generated power while maintaining its quality (frequency and voltage) at acceptable level and satisfying the system security constraints.

Under normal operating conditions - system servicing the load demand with voltage and frequency within acceptable limits and all equipments working within their operating limits - the major concern of the power system operator is economic dispatch whereby the total system generated power is allocated among the participating generators in such a way that the system operating cost is minimum. Power system operators will like to run the system in this mode indefinitely. Unfortunately, certain uncontrollable events such as unplanned outages of generator or transmission equipments may force the system into emergency operating conditions, where some lines become overloaded and/or some bus voltages are beyond acceptable limits.

Though at the planning stage the effects of probable contingencies on the system are analysed and some redundancy is built into the system to make it withstand these contingencies, it is not possible to foresee at the planning stage all the operating conditions into which a system may find itself during its operation, also due to economic and other reasons it is possible to build only a limited amount of redundancy into a



system. All these produce a severe burden on the power system operators trying to maintain continuous supply to the consumers at acceptable level of quality under all circumstances.

In the past when systems were small in size and operating in isolation it was possible for the operators to rely on their experience and certain tools - distribution factors, worst case load flow results etc - provided by the system planners, to take appropriate control actions in the event of unplanned outage conditions. With the rapid interconnections of the power systems the system response to disturbances have become very complex. Also, these interconnections along with their many advantages have created certain problems such as blackouts caused by cascade tripping initiated by the outage of some element. All these factors along with the need for a reliable power source have led to a new concept of on-line monitoring and control of power systems - security control systems<sup>(1)</sup>. The concept of security control is all encompassing in that it not only takes control decisions during normal operating conditions but also during emergencies, and since its introduction has been a major field of research in power systems<sup>(2-9)</sup>.

The security control scheme consists of continuously monitoring the power system operating state and the effects of probable outage contingencies on it, and if any dangerous operating conditions are detected preventive control actions - preferably optimal according to some criterion - are computed and initiated to overcome the danger on the system. The simplest security

monitoring concept is to determine the effects of probable transmission and generator outage contingencies on the steady state performance of the system at its current operating condition. Although this concept ignores the dynamic transition of the system from its operating state to its post outage steady state, in the absence of any practical transient security assessment scheme<sup>(8)</sup> it is generally accepted as an useful assessment of security<sup>(53)</sup>. Though the application of security control concept appears quite straight forward, the major problem in its implementation has been the lack of accurate and fast methods for security monitoring and control.

This thesis examines this problem critically and brings to bear upon it the latest techniques from widely differing fields like pattern recognition and linear programming, to develop an accurate and fast algorithm, for the security constrained economic dispatch problem which is suitable for on-line application.

## 1.2 OUTLINE OF THE THESIS

In this thesis the large and complex security constrained economic dispatch problem has been decomposed into three smaller manageable problems: (i) economic dispatch, (ii) security assessment and (iii) security control. Each of these problem is taken up individually and using the latest state of the art techniques very fast and accurate algorithms are developed for each of them separately. These algorithms are then properly co-ordinated to obtain the solution to the overall problem.



The Chapter 2 of the thesis deals with the economic dispatch problem. On the basis of detailed review of the existing economic dispatch methodologies it was found that the classical coordination equation method using the transmission loss penalty factors is the most widely used method in practice. The main criticism of this penalty factor method is due to the use of B coefficients for calculation of the transmission loss penalty factors. Although the B coefficients are dependent on system operating conditions, in practice B coefficients are considered constant. This leads to error in calculation of penalty factors.

In this thesis an accurate and fast method for penalty factor calculation is presented. The method uses the system voltages and LU factors of  $[B']$  matrix (same as that used in fast decoupled load flow (FDLF)). The economic dispatch method proposed uses the coordination equations, penalty factor calculations and FDLF in an iterative mode. The method is accurate, fast and has storage requirements similar to that of FDLF. The method is well suited for on-line applications. The effectiveness of the method is demonstrated through several examples.

The power system security assessment problem is formulated in Chapter 3. On undertaking detailed survey of the existing literature on this subject it is found that the existing methods for security assessment are though well suited for planning studies, they are not suited for on-line applications. This is mainly because these methods either require large computertime

(especially when a large number of contingencies are to be considered), or do not provide accurate results. This made it necessary to explore alternative methods. For this purpose pattern recognition method was investigated and fully exploited.

The pattern recognition process with main emphasis on pattern classification process is discussed in Chapter 4. The pattern classification process is primarily a two stage process. The first stage called the training process involves generating a pattern classifier (also called decision function, decision surface, separating surface etc.) through a training process. Both statistical and deterministic methods used for training of the classifier are discussed. The second stage of the process consists of using the trained classifier obtained through the training process for classifying the unknown patterns.

In power system security assessment one is mainly concerned about the security status of the system operating condition (secure/insecure). This problem is formulated as a two class security classification problem. For the training set (used to generate the security classifier) a large set of patterns (power system operating conditions) of known classification (secure/insecure) are generated considering the entire operating spectrum of the system and using EDLF and contingency analysis. The variables characterising the power system operating conditions constitute the pattern vector. In a power system the network topology keeps changing from time to time. This has been modelled by considering logical variables for the planned outages in the system.



As the number of variables characterising a power system operating condition is generally large a feature selection method based on Fisher linear discriminant measure is used to select a small number of effective variables (features) to characterise the pattern vectors.

For the security classifier a linear decision function is generated using an iterative training procedure involving the pattern vectors in the test set. The training procedure involves solution of linear inequalities. The problem is formulated as an optimisation problem which is solved using Fletcher-Reeves conjugate gradient method with periodic restarting. The method does not require storage of any matrices making it possible to solve large pattern classification problem on small computers. The particular choice of the objective function which is strictly convex for the inconsistent case (when patterns are linearly inseparable) leads to an optimal security classifier in all cases.

The security classifier concept is further extended to obtain individual security classifiers for each contingency. These single disturbance classifiers are then used to identify the contingencies which force the power system operating condition into insecure state.

A new security control index (SCI) is proposed. The SCI is based on the severity of the operating conditions (found from the value of the security function) and the probability of the occurrence of contingency. The computation of SCI requi-

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res only one multiplication operation when the probability of occurrence of contingency is known. The SCI is computed for each contingency individually and is very helpful in making decision about initiating security control actions.

In case when the value of SCI is more than the predetermined threshold value of SCI, indicating severe breach in security, preventive security control should be initiated to keep the system out of the danger. This problem is discussed in Chapter 5. On the basis of detailed review of the literature on this subject it is found that the linear programming (LP) methods, especially the dual revised simplex methods are well suited for on-line applications.

A method of real and reactive power rescheduling for security control is presented. The real and reactive power rescheduling problems are decomposed into two subproblems each of which is solved using dual revised simplex method with reduced basis formulation and coordinated by FDLF to obtain the solution to the full problem. Two modifications in the algorithm are proposed to incorporate piece-wise linear cost functions in the objective without increasing the storage requirement. This enhances the accuracy of the model as well as avoids large shifts in only a few control variables. In the reactive power problem all control variables including transformer tap changes are represented as reactive power injection changes. This results in much simpler formulation. The use of a load bus as voltage reference to avoid the illconditioning of the  $[B'']$  results in



a formulation for reactive power rescheduling problem which is exactly similar to its counterpart real power problem.

In certain contingency conditions it is not possible to relieve line overloads by rescheduling the real power sources only. In such cases load shedding has to be resorted to. The cost for load shedding can not be calculated only in terms of revenue lost. The consumer resentment due to load shedding should also be taken into account. For this purpose the concept of resentment cost due to load shedding is introduced. As the consumer resentment rises rapidly with the increase in load shedding, the resentment cost is modeled in this thesis by a quadratic function and is given a very high value in order to use load shedding only as a last resort. By incorporating higher resentment costs for certain buses a priority scheme for load shedding is developed. As only real power load can not be dropped at any bus a proportional amount of reactive load is also dropped to keep the power factor at the bus constant.

The preventive security control in many cases may be very costly, e.g., involving preventive load shedding etc. under these circumstances it may be preferable not to use preventive control at all but to prepare a contingency plan which is to be initiated immediately if the contingency really occurs.

The economic dispatch, security assessment and security control methods are effectively coordinated to obtain an accurate and fast solution to the overall problem of security constrained economic dispatch. The effective use of single disturbance classifiers and SCIs greatly reduce the computation time as security



control actions are to be computed only for the conditions which cause breach in security. The effectiveness of the method is demonstrated through several test examples.

The Chapter 6 presents the general conclusions of thesis about the effectiveness of the proposed method.

## CHAPTER 2

ECONOMIC LOAD DISPATCH2.1 INTRODUCTION

A given load demand in a power system with several generators can be met in an infinite number of ways. The purpose of economic dispatch is to allocate the total system load among different generators, in such a way that the total system operating cost is minimum. The system load demand keeps changing with time, but for the purpose of economic dispatch the system load demand is sampled at regular intervals (5-10 minutes) and the load is assumed to be constant during the sampling interval. For economic load dispatch all the generating units that are to take load are assumed to be committed and no-load running costs are incurred irrespective of whether they are assigned to take any load or not.

Initially the 'base load method' and 'best point loading' methods were used for load dispatch<sup>(14,16)</sup>. Later it was realised that these methods do not lead to most economic system operating condition and it was recognised that the "equal incremental cost method"<sup>(13-16)</sup> led to the most economic operating condition. With the development of inter-connected power systems and interconnections between operating companies for the purpose of economic interchange of power, it became necessary to consider not only the generator operating (production) costs but also the transmission line losses in the system. The inclusion of transmission line losses for the

economic operation led to the incremental transmission loss formula<sup>(14,16)</sup> which when incorporated with incremental production cost led to the classical "coordination equation method"<sup>(14-19)</sup> for economic load dispatch. This method because of its simplicity and robustness is very popular and is in wide use with minor variations for real-time economic load dispatch<sup>(16)</sup>.

The methods for economic load dispatch discussed above consider the allocation of real power only. A breakthrough in mathematical formulation of the economic dispatch problem was achieved by Carpentier in early '60's<sup>(20)</sup>. He treated the entire network in an exact manner and formulated an optimal load flow problem including both real and reactive power injections in the system. This resulted in a nonlinear optimisation problem formulation. Since then a great deal of interest has been generated in formulation of the optimal load dispatch as a nonlinear optimisation problem with equality and inequality constraints and has been subjected to much study through the present **(26-40)**. Because of large problem size and convergence difficulties the implementation of optimal load dispatch in real-time still remains a challenge.

In this chapter an economic load dispatch method for real-time application is proposed. The method is based on incremental transmission loss calculations and the coordination equation method. The penalty factors for transmission loss is calculated using the load-flow data which gives more accurate results than the classical B-coefficient method.



## 2.2 GENERATOR OPERATING COST

In economic load dispatch problem generator operating cost characteristics is the most important factor. The major component of generator operating cost for fuel-fired units is the cost of fuel input/h, while the cost of maintenance, water etc. contribute only to a small extent<sup>(11)</sup>. In case of hydro units the fuel cost has no meaning but its generation is constrained by water discharge rate which is used for their scheduling. In this thesis only fuel fired units are considered.

The input-output characteristics of a fuel-fired unit can be expressed in terms of million calories/h, or directly in terms of Rs/h versus output in megawatts. Since it is difficult to express the costs of labour, maintenance, supplies etc. as a function of output it is usual to assume these costs as a fixed percentage of the fuel cost at that output. The generator operating cost characteristic for a typical fuel fired unit is shown in Fig. 2.1. By fitting a suitable degree polynomial, an analytical expression for the generator operating cost characteristics can be written as

$$C_j(P_j) \text{ Rs/h at output } P_j.$$

Where  $j$  stands for the unit number. It generally suffices to fit a second-order polynomial to  $C_j$ <sup>(14)</sup>, i.e.,

$$C_j(P_j) = a_j P_j^2 + b_j P_j + d_j \text{ Rs/h} \quad (2.1)$$

Where  $a$ ,  $b$  and  $d$  are constants.

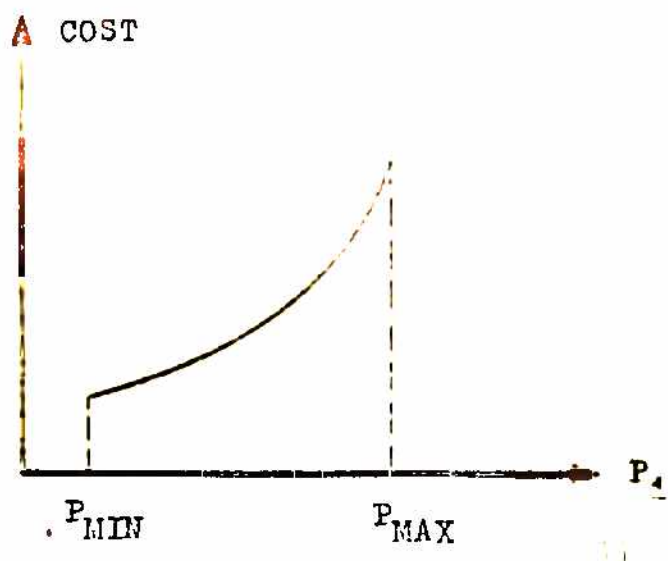


FIG.2.1 TYPICAL GENERATOR COST CURVE

2.3 ECONOMIC DISPATCH METHODS

In this section a review of economic dispatch methods are presented.

2.3.1 Merit Order Scheduling (13,16)

(a) Base Load Method : In this method the most efficient unit is called for supplying the load at light loads. As the load increases the power is supplied by this unit until its generating capacity is reached. Then for further increase in load the next most efficient unit starts supplying load and a third unit will not be called upon to supply load until the maximum capacity of second unit is reached and so on.

(b) Best Point Loading: Here the units are successively loaded to their lowest heat rate (maximum efficiency) point, beginning with the most efficient unit and working down to the least efficient unit.

2.3.2 Incremental Cost Method(13,14,16)

It was realised as early as 1930's that merit order scheduling does not lead to most economic operating condition, and it was recognised that incremental cost method yielded the most economic results. The idea of incremental cost loading is that the next increment of load is to be taken by the unit whose incremental cost is minimum. The net effect of this is to equalise the incremental costs of all the units. That is for most economic loading:

$$\frac{dC_1}{dP_1} = \dots \frac{dC_j}{dP_j} = \dots \frac{dC_n}{dP_n} = \lambda \text{ (a constant)} \quad (2.2)$$

Where  $\frac{dC_j}{dP_j}$  is the incremental production cost of unit  $j$ .  
 When the generator operating cost characteristics is represented by a second-order polynomial as in equation (2.1), then the incremental production cost of the  $j$ th unit is given by

$$\frac{dC_j}{dP_j} = a_j P_j + b_j \quad (2.3)$$

### 2.3.3 The Coordination Equation Method (14, 16)

In the equal incremental cost method the effect of transmission losses are neglected. A modern electric utility serves a vast area of relatively low load density. The transmission losses may vary from 5-15% of the total load and therefore it is essential to account for transmission losses in any economic load dispatch formulation. It is evident that when transmission losses are included, the equal incremental cost method is no longer applicable. But, the change in transmission losses with the variation in generation scheduling can be taken care by coordinating the generator production cost with transmission loss as follow:

Let,

$C_j$  = Production cost of generator  $j$  in Rs/h.

and  $C_t =$  Total production cost in Rs/h

then,  $C_t = \sum_{j=1}^n C_j(P_j)$ , for a  $n$  generating plant system.

The problem is



$$\text{Minimise } C_t = \sum_{j=1}^n C_j(P_j) \quad (2.4)$$

subject to

$$g = P_D + P_L - \sum_{j=1}^n P_j = 0 \quad (2.5)$$

and

$$h_j^m = P_j - P_j^m \geq 0 \quad (2.6)$$

$$h_j^M = P_j^M - P_j \geq 0$$

Where,

$P_j$  = Generation of plant  $j$

$P_L$  = Total transmission loss

$P_D$  = total system load demand

$P_j^m$  = Lower limit of generation at plant  $j$

$P_j^M$  = Upper limit of generation at plant  $j$ .

The Lagrangian function for this system can be formed as

$$L = C_t + \lambda g - \sum_{j=1}^n \mu_j^m h_j^m - \sum_{j=1}^n \mu_j^M h_j^M \quad (2.7)$$

Where,  $\lambda$ ,  $\mu_j^m$  and  $\mu_j^M$  are the dual variables associated with equations (2.5) and (2.6).  $\mu_j^m$  and  $\mu_j^M$  is(are) zero when the corresponding constraint(s) is(are) not effective (i.e., when  $P_j$  is not on a limit). The negative sign in equation (2.7) for terms with  $\mu_j$  are used so that in case of limit violations (i.e.,  $h_j < 0$ ) the contribution of these terms towards Lagrangian function  $L$  is positive. The necessary condition for minimum operating cost is given by

$$\frac{\partial L}{\partial P_j} = 0 = \frac{\partial C_t}{P_j} + \frac{\partial \mathcal{E}}{\partial P_j} - \mu_j^m \frac{h_j^m}{P_j} - \mu_j^M \frac{h_j^M}{P_j} \quad (2.8)$$

or

$$\frac{dC_j}{dP_j} - \left(1 - \frac{P_L}{P_j}\right) - \mu_j^m + \mu_j^M = 0 \quad (2.9)$$

When no unit is operating at limit both  $\mu_j^m$  and  $\mu_j^M$  are zero and equation (2.9) becomes

$$\frac{dC_j}{dP_j} + \lambda \frac{\partial P_L}{\partial P_j} = \lambda \quad (2.10)$$

the classical coordination equation for economic load dispatch.

Instead of using the coordination equation it is sometimes more convenient to use the penalty factor form of the equation (2.10) as given below:

$$\frac{dC_j}{dP_j} PF_j = \lambda \quad (2.11)$$

Where,  $PF_j$  = Penalty factor of plant  $j$  to account for the transmission loss, and

$$PF_j = \frac{1}{1 - \frac{\partial P_L}{\partial P_j}} \quad (2.12)$$

Since the transmission losses are usually small the penalty factor is close to unity. (For positive incremental transmission loss the penalty factor is greater than unity making that plant less attractive. For negative incremental transmission losses the penalty factor is less than unity which has the effect of making that plant incrementally more attractive.)

In case of generation capacity limitation on any plant  $k$ , the following holds

$$\mu_k^m = \frac{dC_k}{dP_k} - \lambda \left(1 - \frac{\partial P_L}{\partial P_k}\right) = \frac{dC_k}{dP_k} - \lambda / PF_k$$

or

$$\mu_k^M = -\frac{dC_k}{dP_k} + \lambda \left(1 - \frac{\partial P_L}{\partial P_k}\right) = \lambda / PF_k - \frac{dC_k}{dP_k} \quad (2.13)$$

Where  $\mu_k^m$  or  $\mu_k^M$  is the incremental loss due to capacity limitations and incremental cost of delivered power  $\lambda$  is given by

$$\lambda = \frac{dC_j}{dP_j} PF_j \text{ for plant } j \text{ not on limit.}$$

### 2.3.3.1 Incremental transmission losses

In order to calculate  $\lambda$  and  $\mu_k$ 's it is necessary to know the incremental transmission losses with respect the power at the generations (equations (2.10) and (2.13)) for the total system load demand. Some methods for calculating the incremental transmission loss are found in literature<sup>(14, 17, 18, 19)</sup>. In industry the method of B coefficients is widely used for calculation of incremental transmission loss<sup>(16)</sup>. In this method the transmission loss in the system is given by

$$P_L = \sum_{i=1}^n \sum_{j=1}^n P_i B_{ij} P_j ; \quad i, j = 1, 2, \dots, n \quad (2.14)$$

The incremental transmission loss of plant  $j$  thus becomes

$$\frac{\partial P_L}{\partial P_j} = \sum_{i=1}^n 2 B_{ij} P_i \quad (2.15)$$



The B matrix (matrix of coefficients  $B_{ij}$ ) represents a set of coefficients through which an approximation to the exact transmission loss is obtained. The B matrix is a square, symmetric real matrix. The  $B_{ij}$  coefficients in B matrix remain constant under following assumptions:

- (i) All loads are assumed to follow a certain form in their variation with total load demand change. Any nonconforming load is treated as negative generation.
- (ii) All bus voltages and voltage phase angles are assumed constants.
- (iii) All source (generators, tie-lines and non-conforming loads) reactive powers are assumed to remain a certain percentage of the source real power as they change.

Several sets of B matrices are often used to better reflect the actual system voltages, angles and transmission configuration. The popularity of loss formula is because of its simplicity in its use for either on-line or off-line calculations. The deviation of B coefficients are given in reference (14).

Some other methods<sup>(18,23)</sup> for calculation of transmission loss use  $Z_{BUS}$  matrix and system state (i.e., real and reactive powers at all the buses and voltage magnitude and angles at all the buses). This makes these methods more accurate but less suitable for real-time application because of large computation time.

### 2.3.3.2 Economic dispatch

Whatever may be the method used for calculation of transmission loss and incremental transmission loss, the solution procedure for economic dispatch is usually similar. When scheduling for a specified total generation, the  $\lambda$  s are computed iteratively using

$$\lambda^1 = \lambda^{i-1} + (P_T^S - P_T^{i-1}) \left( \frac{\lambda^{i-1} - \lambda^{i-2}}{P_T^{i-1} - P_T^{i-2}} \right) \quad (2.16)$$

Where,  $i$  = iteration being conducted

$i-1$  = iteration just concluded

$i-2$  = iteration concluded just before  $i-1$ .

$P_T$  = total system generation

$$= \sum_{i=1}^m P_{\text{Gen } i}; \quad m = \text{no. of generating buses}$$

$P_T^S$  = specified total system generation.

The algorithm for iterative solution of (2.16) is as follows:

Step 1 : Select  $\lambda^{i-1}$  and  $\lambda^{i-2}$  arbitrarily,  $\lambda > 0$ .

Step 2 : Using equation (2.10) compute  $P_{\text{Gen } j}^{i-1}$  and  $P_{\text{Gen } j}^{i-2}$ ;  $j = 1, \dots, m$ . If for any  $j$ ,  $P_{\text{Gen } j}$  violates its operating limit set it at the limit it is violating.

Step 3 : Compute  $P_T^{i-1}$  and  $P_T^{i-2}$ , using  $P_T = \sum_{j=1}^m P_{\text{Gen } j}$ .

Step 4 : If  $(P_T^S - P_T^{i-1}) < \text{tolerance}$ , set  $\lambda = \lambda^{i-1}$  go to step 7. Otherwise continue.

Step 5 : Calculate  $\lambda^1$  using equation (2.16).

Step 6 : Set  $i = i + 1$ , go to step 2.

Step 7 : Print  $\lambda$ ,  $P_{\text{Gen } j}$ ,  $P_T$ , return.

The algorithm for calculating  $\lambda$ s using equation (2.16) was first suggested by Kirchmayer<sup>(14)</sup>. The algorithm converges very fast for monotonically increasing cost functions like the quadratic cost functions usually used for economic dispatch purpose.

#### 2.3.4 Optimal Load Flow (20,26-40)

Work started in late 1950's to improve upon the loss formula approach<sup>(16)</sup>. Around the same time digital computer methods for load flow made its appearance. A load flow is characterized by the voltage magnitude and angles at the buses of the network under study, real and reactive power injections at the buses. The object of load flow study is to find out the voltage magnitude and angles at all the buses in the power system using the real and reactive power injections at the generator and load buses and the transmission network admittances.

An optimal load flow is a load flow in which fuel costs or some other quantity is optimised with the ordinary load flow constraint around all the buses and additional constraints such as voltage limits etc. also recognized. When fuel costs are minimised, the optimal load flow serves as economic dispatch and determines the real and reactive power output of the generators and that of other reactive sources and sets transformer taps to the optimal position.

The general problem of static optimisation of the operating costs of a power system subject to the equality



constraints imposed by the load flow equations as well as the inequality constraints imposed by the equipment ratings and voltage limits was first formulated by J. Carpentier in 1962<sup>(20)</sup>. The resulting optimisation problem was a nonlinear programming problem and sufficiency conditions of the theorem of Kuhn and Tucker were applied to derive the optimisation equations which must be satisfied at the optimum.

Kuhn and Tucker conditions for power flow optimisations:

Let,  $[x]$  be the vector of dependent (unknown) variables which consists of  $V$  and  $\theta$  on  $(P, Q)$  buses, and  $\theta$  on  $(P, V)$  buses; fixed parameters  $P, Q$  on  $(P, Q)$  buses and  $\theta$  on slack bus be denoted by vector  $[p]$ ; the vector of control variables  $[u]$  consist of parameters as voltage magnitude on generator buses, generator real powers  $P$ , and transformer tap ratios.

The optimisation problem for load flow with inequality constraints for control parameters can be stated as

$$\begin{array}{l} \text{Minimise } f(x, u) \\ [u] \end{array} \quad (2.17)$$

subject to equality constraints

$$g(x, u, p) = 0 \quad (2.18)$$

the load flow equations, and the inequality constraints

$$[u] - [u^{\max}] \leq 0 \quad (2.19)$$

$$[u^{\min}] - [u] \leq 0 \quad (2.20)$$



Assuming convexity of the functions (2.17) to (2.20) the Kuhn and Tucker theorem gives the necessary condition as

$$[\nabla L] = 0 \text{ (gradient w.r.t. } u, x, \lambda) \quad (2.21)$$

and, the exclusion equations

$$[\mu^{\max}]^t ([u] - [u^{\max}]) = 0$$

$$[\mu^{\min}]^t ([u^{\min}] - [u]) = 0 \quad (2.22)$$

$$[\mu^{\max}] \geq 0; [\mu^{\min}] \geq 0$$

Where,  $L$  is the Lagrangian function accounting for both the equality and inequality constraints; the augmented function is

$$L = f(x, u) + [\lambda]^t [g(x, u, p)] + [\mu^{\max}]^t ([u] - [u^{\max}]) + [\mu^{\min}]^t ([u^{\min}] - [u]) \quad (2.23)$$

Where,  $[\lambda]$  is the dual variable associated with the equality constraint and  $[\mu^{\max}]$  and  $[\mu^{\min}]$  are the dual variables associated with the upper and lower limits of the inequality constraints. If  $u_i$  reaches a limit, it will be either  $u_i^{\max}$  or  $u_i^{\min}$  and not both (otherwise it will be included in equality constraints); therefore either inequality constraint (2.19) or (2.20) is active, i.e., either  $\mu^{\max}$  or  $\mu^{\min}$  exists but never both. The equation (2.21) becomes

$$\begin{bmatrix} \frac{\partial L}{\partial x} \end{bmatrix} = \begin{bmatrix} -\frac{f}{x} \end{bmatrix} + \begin{bmatrix} -\frac{g}{x} \end{bmatrix}^t \begin{bmatrix} \lambda \end{bmatrix} = 0 \quad (2.24)$$

$$\begin{bmatrix} \frac{\partial L}{\partial u} \end{bmatrix} = \begin{bmatrix} -\frac{f}{u} \end{bmatrix} + \begin{bmatrix} -\frac{g}{u} \end{bmatrix}^t \begin{bmatrix} \cdot \end{bmatrix} + \begin{bmatrix} \mu \end{bmatrix} = 0 \quad (2.25)$$

Where

$$\mu_1 = \mu_1^{\max}, \text{ if } \mu_1 > 0$$

$$\mu_1 = -\mu_1^{\min}, \text{ if } \mu_1 < 0$$

$$\begin{bmatrix} \frac{\partial L}{\partial \lambda} \end{bmatrix} = \begin{bmatrix} g(x, u, p) \end{bmatrix} \quad (2.26)$$

at the optimum  $\begin{bmatrix} \mu \end{bmatrix}$  must fulfil the exclusion equations (2.22) which say that

$$\mu_1 = 0 \quad \text{if } u_1^{\min} \leq u_1 \leq u_1^{\max}$$

$$\mu_1 = \mu_1^{\max} \geq 0 \quad \text{if } u_1 = u_1^{\max}$$

$$\mu_1 = -\mu_1^{\min} \leq 0 \quad \text{if } u_1 = u_1^{\min}$$

Carpentier used Gauss-Siedel iterative method for the solution of nonlinear optimisation equations (2.24) to (2.26). The convergence behaviour of which proved to be very difficult and erratic. At present, the gradient methods for optimisation of power system operating conditions is the most widely used method. Some other nonlinear programming methods such as Powell and Fletcher-Powell<sup>(29)</sup> method, Fiacco and McCormick or SUMT<sup>(28)</sup> methods have also been used. But when number of variables increases to above hundred

(which is a general case for moderate size power systems) then these methods develop convergence problems<sup>(30)</sup>.

Among the gradient methods of optimisation, Dommel and Tinney's<sup>(27)</sup> formulation of the power system operating condition optimisation is the most simple, elegant and widely used. Dommel and Tinney in reference (27) extended the Newton's load flow method to an optimal load flow in the following manner:

The problem is

$$\begin{aligned} &\text{Minimise } f(x, u) && (2.27) \\ &\quad [u] \end{aligned}$$

subject to the equality constraints of the load flow equations

$$g(x, u, p) = 0 \quad (2.28)$$

The augmented Lagrangian function is given by

$$L = f(x, u) + [\lambda]^t [g(x, u, p)] \quad (2.29)$$

The set of necessary conditions for the minimum are

$$\left[ \frac{\partial L}{\partial x} \right] = \left[ \frac{\partial f}{\partial x} \right] + \left[ \frac{\partial g}{\partial x} \right]^t [\lambda] = 0 \quad (2.30)$$

$$\left[ \frac{\partial L}{\partial u} \right] = \left[ \frac{\partial f}{\partial u} \right] + \left[ \frac{\partial g}{\partial u} \right] [\lambda] = 0$$

$$\left[ \frac{\partial L}{\partial \lambda} \right] = [g(x, u, p)] = 0 \quad (2.32)$$

Equation (2.32) represents the load flow solution, the control parameters 'u' need only be in the feasible region



but not necessarily optimal. Equation (2.30) contains the transpose of the Jacobian matrix of the load flow which can be solved for  $\lambda$  as

$$\lambda = - \left[ \frac{\partial g}{\partial x} \right]^t^{-1} \left[ \frac{\partial f}{\partial x} \right] \quad (2.33)$$

$\left[ \frac{\partial L}{\partial u} \right]$  in equation (2.31) represents the gradient vector<sup>(27)</sup>

$$\left[ \nabla f \right] = \left[ \frac{\partial f}{\partial u} \right] + \left[ \frac{\partial g}{\partial u} \right]^t \left[ \lambda \right] \quad (2.34)$$

The computational process proceeds as follows: a set of feasible control parameters 'u' is assumed and a load flow by means of Newton's method is obtained. At this solution point, a repeat solution is carried out for  $\lambda$  in (2.33). The gradient  $\left[ \nabla f \right]$  is next calculated using equation (2.34) from which a correction in 'u' is obtained as

$$\left[ \Delta u \right] = - C \left[ \nabla f \right] \quad (2.35)$$

The above method is straight forward with the exception of the selection of factor 'C'; too small a value causes slow convergence, whereas too large a value causes oscillations. It can be seen that the adjustments in 'u' are in outer loop whereas, the inner loop consists of the load flow. The limits on 'u' are enforced according to the Kuhn and Tucker conditions. Inequality constraints on dependent variables (functional constraints) such as voltage limits at (P,Q) buses are handled by penalty function approach wherein, any violation of functional constraints penalises



the objective function.

Although the optimal load flow results in an exact economic dispatch problem formulation, its real-time application still remains a challenge. The main reason for this is the large computation time taken for the solution of nonlinear optimisation problem for any realistic size system, also the convergence of these methods are not guaranteed (i.e., the solution may diverge from the optimum). Because of these difficulties in the computational procedures, the nonlinear optimisation methods have not become attractive for practical applications in the power utilities<sup>(16)</sup>.

The coordination equation method using the penalty factors are still by and large the most popular method for economic dispatch. As reported in reference<sup>(16, 17)</sup> the rigorous methods using exact network models used for economic dispatch do not offer significant savings as compared to those obtained by classical coordination equation methods. As already discussed in section (2.3.3) the main disadvantage of using coordination equation method with the penalty factors lies in using B coefficients for calculation of penalty factors. These disadvantages are mainly because of large storage needed for storing many sets of [B] matrices as well as many assumptions used in calculating the B coefficients are no longer valid in real time applications. To overcome these disadvantages of [B] matrix usage some other methods for calculating penalty factors more accurately and requiring less storage have been suggested<sup>(17,19,21,22)</sup>.

## 2.4 METHODS FOR CALCULATING PENALTY FACTORS

- (1) Stevenson's Method<sup>(22)</sup> : This method for calculating accurate penalty factors using system operating condition data are based on the fact that incremental transmission loss for any generating plant  $m$  can be expressed as

$$\frac{\partial P_L}{\partial P_m} = \sum_{i=1}^N \frac{\partial P_L}{\partial \theta_i} \cdot \frac{\partial \theta_i}{\partial P_m} \quad (2.36)$$

Where  $\theta_i$  is the voltage angle at bus  $i$  in a  $N$ -bus system.

If the voltage magnitude is assumed to be constant at the value obtained from the load flow solution then we can write

$$\frac{\partial P_L}{\partial \theta_i} = 2 \sum_{j=1}^N |V_i| |V_j| G_{ij} \sin(\theta_j - \theta_i) \quad (2.37)$$

Where,  $G_{ij}$  is the real part of  $Y_{ij}$ , the  $ij$  element of the system bus admittance matrix  $Y_{BUS}$ . The details for the derivation of equation (2.37) is given in Appendix A2.1.

The terms for  $\frac{\partial \theta_i}{\partial P_m}$  are calculated using a sensitivity approach involving several load flow solutions.  $\frac{\partial \theta_i}{\partial P_m}$  is determined by finding variations in voltage phase angles ( $\theta_i$ ) w.r.t. discrete change in generated power  $P_m$ , all other generations remaining constant. An increment in  $P_m (= \Delta P_m)$  with all other generations remaining constant requires a correspond-

ing change in total received load in order to satisfy the law of conservation of energy. The algorithm for computation of  $\frac{\partial P_L}{\partial P_m}$  coefficients is as follows:

Step 1 : Select a generating bus (say  $m$ ) as the slack bus.

Step 2 : Obtain load flow solution for the base case loading condition. Store bus voltage angles  $[\theta^0] = (\theta_1^0, \dots, \theta_N^0)$ , and slack bus real power  $P_m^0$ .

Step 3 : Increase the load at all buses by a small amount (say 2%). Obtain load flow solution for the new loading condition. Store new bus voltage angles  $\theta' = (\theta_1', \dots, \theta_N')$ , and slack bus real power  $P_m'$ .

Step 4 : Compute  $\Delta \theta_i = \theta_i' - \theta_i^0$ ,  $i = 1, 2, \dots, N$ .

$$\text{Also } \Delta P_m = P_m' - P_m^0$$

Step 5 : Compute  $A_{im} = \frac{\Delta \theta_i}{\Delta P_m}$  (2.38)

Step 6 : If all generator buses over, stop. Otherwise make next generator bus a slack bus. Go to Step 2.

In reference (22) it is claimed that the coefficients  $A_{im}$  (equation (2.38)) are essentially constant and need to be evaluated only once and stored permanently. The calculation of  $\frac{\partial P_L}{\partial P_m}$  for any loading condition is carried out using the stored  $A_{im}$  coefficients and  $\frac{\partial P_L}{\partial \theta_i}$  is calculated for that opera-



ting condition using equation (2.37). On extensive testing on different systems, it is found that  $A_{im}$  factors remain constant for large variations on load levels only if the system bus voltages remain constant. In cases where system bus voltage magnitudes varied considerably on changes in load, it was found the  $A_{im}$  coefficients no longer remained constant. A variation as large as 10.4% in  $A_{im}$  coefficient is found in some cases on variation of load from full load to half load.

The advantages of the method over B coefficient method are much simpler computation for  $A_{im}$  coefficients using sparse  $Y_{BUS}$  matrix and fast load flow methods, a more accurate penalty factor calculation as system operating condition including bus voltages (via equation 2.37) are used. The disadvantages are large storage needed for storing  $A_{im}$  factors as well as in cases where bus voltage variations are large the  $A_{im}$  factors no longer remain constant and so some accuracy is lost.

- (11) Happ's Method<sup>(17)</sup>: This method uses the Jacobian matrix of the converged Newton-Raphson load flow for calculating the penalty factors. The methods in references (19, 21, 22) are similar. The incremental transmission loss for any generating plant  $m$  may be written as

$$\frac{\partial P_L}{\partial P_m} = \sum_{i=1}^N \frac{\partial P_L}{\partial V_i} \cdot \frac{\partial V_i}{\partial P_m} \quad (2.39)$$

Where  $V_i = |V_i| \angle \theta_i$ , the complex voltage at bus  $i$  in the  $N$ -bus system. Similarly we may write

$$\frac{\partial P_L}{\partial Q_m} = \sum_{i=1}^N \frac{P_L}{V_i} \cdot \frac{V_i}{Q_m} \quad (2.40)$$

Where  $Q_m$  is the reactive power generation at plant m.

Combining equations (2.39) and (2.40) in a single matrix equation for all the N-buses gives

$$\begin{aligned} \left[ \frac{\partial P_L}{\partial P} \mid \frac{\partial P_L}{\partial Q} \right] &= \left[ \frac{\partial P_L}{\partial \theta} \mid \frac{\partial P_L}{\partial |V|} \right] \begin{bmatrix} \frac{\partial \theta}{\partial P} & \frac{\partial \theta}{\partial Q} \\ \hline \frac{\partial |V|}{\partial P} & \frac{\partial |V|}{\partial Q} \end{bmatrix} \\ &= \left[ \frac{\partial P_L}{\partial \theta} \mid \frac{\partial P_L}{\partial |V|} \right] \begin{bmatrix} D \end{bmatrix} \end{aligned} \quad (2.42)$$

post multiplying both sides by  $[D]^{-1}$  gives

$$\left[ \frac{\partial P_L}{\partial \theta} \mid \frac{\partial P_L}{\partial |V|} \right] = \left[ \frac{\partial P_L}{\partial P} \mid \frac{\partial P_L}{\partial Q} \right] \begin{bmatrix} D^{-1} \end{bmatrix} \quad (2.43)$$

Where  $[D^{-1}]$  in terms of its submatrices may be represented as

$$\begin{bmatrix} D^{-1} \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial |V|} \\ \hline \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial |V|} \end{bmatrix} \quad (2.44)$$

Now transposing (2.43), we get

$$\begin{bmatrix} \frac{\partial P_L}{\partial \theta} \\ \hline \frac{\partial P_L}{\partial |V|} \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial |V|} \\ \hline \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial |V|} \end{bmatrix}^t \begin{bmatrix} \frac{\partial P_L}{\partial P} \\ \hline \frac{\partial P_L}{\partial Q} \end{bmatrix} \quad (2.45)$$

Where the  $\begin{bmatrix} \frac{\partial P}{\partial \theta} & \frac{\partial P}{\partial |V|} \\ \hline \frac{\partial Q}{\partial \theta} & \frac{\partial Q}{\partial |V|} \end{bmatrix}^t$  matrix in equation (2.45) is

the transpose of the Jacobian matrix used in N-R load flow

method augmented for slack and PV buses.

The  $(\partial P_L / \partial P_i)$  terms for the generation buses are computed by solving for equation (2.45) using the LU factors of the Jacobian matrix. This method provides quite accurate, simple and fast method for calculating the penalty factors. The use of full Jacobian matrix may pose some limitations in terms of storage and in systems where fast decoupled methods of load flows are used it may pose coordination problem in the sense that full Jacobian matrix has to be calculated after the load flow solution is available.

## 2.5 PROPOSED METHOD

The economic dispatch method proposed here is based the "coordination equation method and penalty factors are used to account for the transmission losses in the system.

### Penalty factor calculations

The method for penalty factor calculation proposed here are based on certain simplifying modification of the Happ's method and is presented below.

If it is assumed that the transmission losses are mainly due to real power flow (i.e., contribution of reactive power flows towards transmission losses are negligible) then the three fourth of the Jacobian matrix in (2.45) can be conveniently dropped giving



$$\left[ \frac{\partial P_L}{\partial \theta} \right] = \left[ \frac{\partial P}{\partial \theta} \right]^t \left[ \frac{\partial P_L}{\partial P} \right] \quad (2.46)$$

While calculating the elements of matrix  $\left[ \frac{\partial P}{\partial \theta} \right]$  if it is assumed that the voltages are constant at 1 pu. and if the elements of transmission system which predominantly effect the reactive power flow (e.g. line shunt reactances and off nominal transformer taps etc.) are neglected then the  $\left[ \frac{\partial P}{\partial \theta} \right]$  matrix reduces to a matrix  $[B^+]$  a matrix exactly similar to  $[B']$  matrix used in fast decoupled load flow but augmented by row and column for the slack bus. The new equation can now be written as

$$\left[ \frac{\partial P_L}{\partial \theta} \right] = [B^+]^t \left[ \frac{\partial P_L}{\partial P} \right] \quad (2.47)$$

The presence of all buses including the slack bus may lead to illconditioning of the  $[B^+]$  matrix. To overcome this difficulty (with little loss in accuracy as borne out by the numerical tests)  $\frac{\partial P_L}{\partial P_{\text{slack}}}$  is assumed to be zero and the row and column corresponding to the slack bus are deleted from  $[B^+]$  making it identical with the  $[B']$  matrix. The final equation may now be written as

$$\left[ \frac{\partial P_L}{\partial \theta} \right] = [B']^t \left[ \frac{\partial P_L}{\partial P} \right] \quad (2.48)$$

The  $\frac{\partial P_L}{\partial P_m}$  for generators at each generating bus (except that at the slack bus, whose coefficient is assumed zero) is computed by solving the equation (2.48). The penalty factors are calculated using

$$PF_m = 1 / (1 - P_L / P_m) \quad (2.49)$$

### Economic dispatch

In the proposed method it is assumed that,

- (i) The cost curve for the generators connected at a bus is represented by a single quadratic cost function and is known, and
- (ii) The loads at all the buses are accurately known.

The first assumption is not binding but is used mainly to reduce storage and some simplifications in computation. The second assumption is generally true, but if telemetered measurements are not very accurate a state estimation method<sup>(116)</sup> can be used to obtain accurate data.

The economic dispatch for the given total generation is obtained by calculating the incremental cost of delivered power ( $\lambda$ ) using the iterative scheme used for equation (2.16) and the coordination equation (2.11). In order to satisfy the total received load the economic dispatch for total generation must satisfy the power balance equation

$$P_G = P_D + P_L \quad (2.50)$$

Where,

$P_G$  = Total real power generation in the system.

$P_D$  = Total real power load in the system.

$P_L$  = Total transmission loss in the system.

As initially the transmission loss  $P_L$  is not known accurately so the economic dispatch for the estimated total generation, in general, may not satisfy the compatibility

condition of equation (2.50) and a new dispatch must be found to satisfy the condition. The change in generation dispatch will change the transmission loss in the system and so the penalty factors will also change.

The problem can be solved iteratively by calculating the transmission loss and the new penalty factors using a load flow solution in the iteration loop with the iterations carried till the condition of equation (2.50) is satisfied. The complete scheme for economic dispatch is shown in Fig. 2.2.

Blocks A and B in the flow diagram represent the input of data and initialisation. The initial total generation is estimated 10% more than the total system load demand to account for the transmission losses.

The block C represents the economic dispatch function wherein the  $\lambda$ s and economic generation schedule are computed. The internal loop 1 represents the iterative loop for computing  $\lambda$ .

In block D fast decoupled load flow solution with the generations as obtained in block C, is obtained to find the new system operating state.

Block E and F are merely for convergence check. If the change in generation at any bus from its value in the previous iteration is more than the tolerance value then the process proceeds to block G, otherwise the present values for generations constitute the economic dispatch for the



INPUT DATA: DEMAND, GENERATION  
COST CURVE COEFFICIENTS AND  
OPERATING LIMITS

A

INITIALIZE  
(1) TOTAL SYSTEM GENERATION  
 $P^S = 1.1 \times \text{TOTAL SYSTEM LOAD}$   
(11) ALL GENERATIONS  $P_{GEN} = 0$  ✓  
(111) ALL PENALTY FACTORS = 1.0

B

ECONOMIC DISPATCH FOR  $P_T^i$  WITH  
PENALTY FACTORS  $PF_j^i$  USING  
EQUATION (2.16) AND ALGORITHM IN  
SECTION 2.3.3.2

C

$K = K + 1$

FAST DECOUPLED LOAD FLOW

D

COMPUTE  
(1)  $\Delta P_{GEN} = P_{GEN}^i - P_{GEN}^{i-1}$  FOR ALL GENERATORS  
(11)  $P_T^i = 2 \times P_{GEN}^i$

E

FOR ALL GENERATORS  
 $\Delta P_{GEN} < \text{TOLERANCE}$

YES

PRINT  $\lambda$ ,  $P_T^i$   
AND  $P_{GEN j, j=1, \dots, m}$

STOP

NO

COMPUTE NEW PENALTY FACTORS  $PF_j^K$   
USING EQUATION (2.47) AND (2.12)

G

FIG. 2.2 ALGORITHM FOR ECONOMIC DISPATCH

given received load.

In block G new penalty factors are calculated using the system state found in block D and the process returns to block C for a new economic dispatch schedule with the new penalty factors.

The algorithm is very fast as both the fast decoupled load flow and the penalty factor calculations involve constant matrices the LU factors of which are computed and stored once for all at the beginning of the process.

## 2.6 RESULTS AND DISCUSSIONS

The economic dispatch method developed in this chapter was extensively tested on (i) 5 bus/7 line/2 generator, (ii) 8 bus/14 line/4 generator and (iii) 14 bus/20 line/6 generator systems. The details of the study systems are given in Appendix 1. The numerical results obtained are summarised and presented below:

Tables 2.1 to 2.3 present the  $A_{im}$  coefficients for the study systems. From these tables it can be seen that  $A_{im}$  coefficients remain essentially constant for the voltage control buses but there is some variation in the values of these coefficients when the loading conditions are changed. This can be seen from Table 2.4 where it can be seen that the maximum variation in  $A_{im}$  coefficient for the 14 bus system is more than 10% when the system loading condition is changed from full load to half load.

The proposed method for economic dispatch is compared with the Stevenson's method ( $A_{1m}$  method, referred to as method 1 in tables) and Happ's method (method 2 in tables). In order to make the comparisons meaningful the economic dispatch for the given received load is carried out using the scheme in Fig.2.2 in all the methods. The only change being in block G for the calculation of the incremental transmission losses and the penalty factors using the different methods. For the convergence check the tolerances used in both block C and D is 0.05MW and MVAR and that used in block G is 0.1MW. The tolerance used in block G is kept higher than those used in block C and D mainly to avoid conflict in tolerances and ending up in an infinite loop.

Table 2.5 shows the penalty factors calculated using the three different methods. From this table it is seen that the penalty factors calculated by the three methods are quite different. Table 2.6 shows the results for economic dispatch by the three methods for the full load condition. From this table it is seen that though the generation dispatch by the three methods are different the production cost for the economic dispatch by the three methods are almost same. In order to check the effectiveness of the proposed method an economic dispatch was carried out using the scheme in Fig. 2.2 but keeping all the penalty factors at 1.0. It is seen that there is



considerable increase in transmission loss as well as the production cost as compare to the proposed method. The results are presented in the last column of Table 2.6. Table 2.7 presents the details of the number of iterations required for convergence. It is seen that the number of iterations required for convergence for the outer loop (loop 3) is independent of the system size for the sample systems studied.

In order to find the effect of choice of slack bus on the economic dispatch by the proposed method, the economic dispatch by the proposed method is carried out with different choice of slack buses. The results are presented in Table 2.8 and 2.9. Table 2.8 shows that the penalty factors change with the choice in slack bus, but the results in Table 2.9 show that the generation dispatch as well as the production cost is independent of the choice of slack bus.

TABLE 2.1

 $A_{im}$  Coefficients - 5 Bus System

(a) Full load condition

Bus No \ Generator Bus No	4	5
1	-0.03294	-0.07077
2	-0.04031	-0.07080
3	-0.04067	-0.06839
4	0.00	-0.04193
5	-0.00977	0.0

(b) Half load condition

Bus No \ Generator Bus No	4	5
1	-0.03267	-0.07042
2	-0.03994	-0.07040
3	-0.04020	-0.06790
4	0.00	-0.04171
5	-0.00975	0.00

TABLE 2.2

A<sub>1m</sub> Coefficients - 8 Bus System

(a) Full load condition

Bus No \ Generator Bus No	5	6	7	8
1	-0.00566	-0.00577	-0.00497	-0.00854
2	-0.00973	-0.01021	-0.00614	-0.00970
3	-0.00472	-0.00913	-0.00427	-0.00500
4	0.00414	-0.00893	-0.00697	-0.00697
5	0.00	-0.00831	-0.00517	-0.00542
6	-0.00603	0.00	-0.00380	-0.00808
7	-0.00604	-0.00697	0.00	-0.00710
8	-0.00424	-0.00921	-0.00506	0.0

(b) Half load condition

Bus No \ Generator Bus No	5	6	7	8
1	-0.00562	-0.00573	-0.00493	-0.00847
2	-0.00954	-0.01001	-0.00596	-0.00949
3	-0.00470	-0.00908	-0.00424	-0.00496
4	-0.00410	-0.00884	-0.00689	-0.00689
5	0.00	-0.00826	-0.00516	-0.00540
6	-0.00599	0.00	-0.00379	-0.00801
7	-0.00601	-0.00693	0.00	-0.00703
8	-0.00423	-0.00916	-0.00596	0.00



TABLE 2.3

A<sub>1m</sub> Coefficients - 14 Bus System

(a) Full load condition

Bus No. \ Generator Bus No	3	4	7	8	13	14
1	-0.09652	-0.02353	-0.10685	-0.27828	-0.15463	-0.18361
2	-0.08681	-0.01879	-0.10277	-0.27421	-0.15230	-0.18102
3	0.00	-0.00982	-0.09537	-0.26681	-0.14913	-0.17679
4	-0.10206	0.00	-0.08714	-0.25858	-0.14501	-0.17184
5	-0.11062	-0.01830	-0.09978	-0.27122	-0.14233	-0.17547
6	-0.18185	-0.09081	-0.13264	-0.30408	-0.06900	-0.15510
7	-0.14366	-0.04566	0.00	-0.17144	-0.13776	-0.14551
8	-0.14366	-0.04566	0.00	0.00	-0.13776	-0.14551
9	-0.16561	-0.06976	-0.06058	-0.23202	-0.13395	-0.13167
10	-0.17702	-0.08246	-0.08140	-0.25284	-0.13285	-0.14298
11	-0.18792	-0.09559	-0.11540	-0.28684	-0.11180	-0.15653
12	-0.20499	-0.11507	-0.15317	-0.32461	-0.05082	-0.15753
13	-0.19567	-0.10495	-0.14004	-0.31148	0.00	-0.13253
14	-0.17914	-0.08561	-0.09657	-0.26801	-0.07502	0.00

## (b) Half load condition

Bus No \ Generator Bus No	3	4	7	8	13	14
1	-0.09729	-0.02334	-0.10682	-0.27663	-0.15660	-0.20277
2	-0.08766	-0.01885	-0.10296	-0.27242	-0.15435	-0.19997
3	0.00	-0.1002	-0.09529	-0.26655	-0.14917	-0.17626
4	-0.10213	0.00	-0.08725	-0.25870	-0.14560	-0.17124
5	-0.11129	-0.01833	-0.09975	-0.26921	-0.14398	-0.19319
6	-0.18048	-0.08551	-0.12728	-0.29670	-0.06881	-0.16561
7	-0.14355	-0.04558	0.00	-0.17125	-0.13751	-0.14571
8	-0.14355	-0.04558	0.00	0.00	-0.13751	-0.14571
9	-0.16686	-0.06904	-0.05993	-0.22938	-0.13425	-0.13262
10	-0.17821	-0.08016	-0.08034	-0.25000	-0.13246	-0.15532
11	-0.18726	-0.09083	-0.11147	-0.28093	-0.10987	-0.16822
12	-0.20206	-0.10713	-0.14534	-0.31480	-0.04837	-0.16531
13	-0.19563	-0.10479	-0.13986	-0.31152	0.00	-0.13206
14	-0.17899	-0.08536	-0.09627	-0.26753	-0.07498	0.00

TABLE 2.4

Variation in  $A_{im}$  coefficients for load change from full load to half load

	$A_{im}$ for which change is maximum	% maximum change in $A_{im}$
5 Bus system	$A_{3,4}$	1.15%
8 Bus system	$A_{2,8}$	2.16%
14 Bus system	$A_{1,14}$	10.4%

TABLE 2.5

Penalty Factors

(a) 5 Bus system

Generator Bus No	Method 1	Method 2	Proposed Method*
4	1.02925	1.05215	0.98742
5	1.04025	1.06378	1.0

\*Slack bus = Bus No. 5



TABLE 2.5 (Contd.)

## (b) 8 Bus system

Generator Bus No	Method 1	Method 2	Proposed Method*
5	1.00792	1.01346	0.99748
6	1.00694	1.01270	0.99637
7	1.00564	1.01227	0.99520
8	1.00999	1.01538	1.00

\*Slack bus = Bus No. 8

## (c) 14 Bus system

Generator Bus No	Method 1	Method 2	Proposed Method*
3	1.05545	1.11305	0.89585
4	1.02465	1.10530	0.871574
7	1.03169	1.08023	0.88385
8	1.03169	1.03914	0.88385
13	1.03754	1.14742	0.89697
14	1.14244	1.20922	1.0

\*Slack bus = Bus No. 14

TABLE 2.6

Comparison of Economic Dispatch Schemes

(a) 5 Bus system

	Method 1	Method 2	Proposed Method	Incremental Production Cost Method
P <sub>4</sub> (MW)	89.937	89.962	90.076	89.385
P <sub>5</sub> (MW)	78.780	78.754	78.64	79.639
Total MW Generation(MW)	168.717	168.716	168.716	169.024
Transmission Loss (MW)	3.717	3.716	3.716	4.024
Production Cost(Rs/h)	20833.36	20833.13	20832.40	20888.38

(b) 8 Bus system

	Method 1	Method 2	Proposed Method	Incremental Production Cost Method
P <sub>5</sub> (MW)	350.00*	350.00*	350.00*	350.00*
P <sub>6</sub> (MW)	350.00*	350.00*	350.00*	350.00*
P <sub>7</sub> (MW)	347.139	346.836	347.26	346.205
P <sub>8</sub> (MW)	532.805	533.108	532.68	534.02
Total Generation (MW)	1579.94	1579.94	1579.94	1580.22
Transmission Loss(MW)	9.94	9.94	9.94	10.22
Production Cost(Rs/h)	481238.33	481232.88	481240.20	481390.90

\*Generations on limits.

TABLE 2.6 (Contd)

(c) 14 Bus system

	Method 1	Method 2	Proposed Method	Incremental Production Cost Method
P <sub>3</sub> (MW)	99.992	101.944	100.774	98.834
P <sub>4</sub> (MW)	75.00*	75.00*	75.00*	75.00*
P <sub>7</sub> (MW)	90.942	93.475	91.870	86.256
P <sub>8</sub> (MW)	50.00*	50.00*	50.00*	50.00*
P <sub>13</sub> (MW)	102.861	96.910	103.00	98.834
P <sub>14</sub> (MW)	97.592	99.167	95.488	108.960
Total Generation(MW)	516.386	516.416	516.232	517.89
Transmission Loss(MW)	16.386	16.416	16.232	17.89
Production Cost (Rs/h)	63859.40	63856.80	63860.70	63951.68

\* Generators on limits.

TABLE 2.7

Details of Convergence Characteristics

	Number of iterations for load flow (loop 1)		Number of iterations for (loop 2)*		Number of iterations for outer loop (loop 3)*
	Maximum	Average	Maximum	Average	
5 Bus system	5	3.5	5	4.03	3
8 Bus system	5	3.5	4	4.0	3
4 Bus system	9	5.3	5	4.03	3

\* Loops 1, 2 and 3 correspond to those in Fig. 2.2.



Effect of Slack Bus on Penalty Factors

(a) 5 Bus system

Generator Bus No	Slack Bus No	
	4	5
4	1.00	0.98742
5	1.01200	1.0

(b) 8 Bus system

Generator Bus No	Slack Bus No			
	5	6	7	8
5	1.00	1.00104	1.00225	0.99748
6	0.99889	1.00	1.00118	0.99637
7	0.99769	0.99878	1.0	0.99520
8	1.00247	1.00353	1.00476	1.0

(c) 14 Bus system

Generator Bus No	Slack Bus No					
	3	4	7	8	13	14
3	1.00	1.03314	1.02188	1.01898	1.00590	0.89585
4	0.96672	1.00	0.98927	0.98663	0.97446	0.87157
7	0.97529	1.00924	1.00	0.99752	0.98430	0.88385
8	0.97529	1.00924	1.00	1.00	0.98430	0.88385
13	0.98684	1.02128	1.01073	1.00801	1.00	0.89697
14	1.09932	1.14372	1.12883	1.12649	1.11323	1.00

TABLE 2.9

Effects of Slack Bus on Generation Dispatch

(a) 5 Bus system

Slack Bus No.	No. of Iterations (loop 3)	MW Dispatch			Production Cost (Rs/h)
		P <sub>4</sub> (MW)	P <sub>5</sub> (MW)	Total (MW)	
4	3	90.025	78.689	168.714	20832.53
5	3	90.075	78.639	168.714	20832.40

(b) 8 Bus system

Slack Bus No	No. of iterations (loop 3)	MW Dispatch					Production Cost (Rs/h)
		P <sub>5</sub> (MW)	P <sub>6</sub> (MW)	P <sub>7</sub> (MW)	P <sub>8</sub> (MW)	Total (MW)	
5	3	350.00*	350.00*	347.246	532.698	1579.944	481240.50
6	3	350.00*	350.00*	347.243	532.701	1579.944	481240.69
7	3	350.00*	350.00*	347.243	532.701	1579.944	481240.20
8	3	350.00*	350.00*	347.256	532.688	1579.944	481240.20

\* Generations on limit.

TABLE 2.9 (Contd)

(c) 14 Bus system

Slack Bus No	No. of iterations (loop 3)	MW Dispatch							Production Cost (Rs/h)
		P3	P4	P7	P8	P13	P14	Total (MW)	
3	3	102.513	75.00*	91.712	50.00*	102.301	94.538	516.064	63872.70
4	2	101.252	75.00*	92.037	50.00*	103.196	94.692	516.178	63833.09
7	3	101.214	75.00*	91.771	50.00*	103.059	95.062	516.105	63866.42
8	3	101.278	75.00*	91.775	50.00*	103.100	94.999	516.142	63875.04
13	3	101.531	75.00*	92.034	50.00*	102.523	95.075	516.163	63878.48
14	3	100.774	75.00*	91.870	50.00*	103.000	95.488	516.132	63860.67

\* Generations on limit.



## CONCLUSIONS

An economic dispatch method involving transmission loss penalty factors is presented in this chapter. The penalty factor calculation does not involve any precalculated coefficients like  $B_{mn}$  coefficients or  $A_{im}$  coefficients.

The use of previously calculated and factored  $[B']$ , fast decoupled load flow matrix requires no extra storage and the computation of penalty factors are very fast. The method coordinates very well with the fast decoupled load flow algorithm. The economic dispatch method was extensively tested on sample systems. The most significant result was that the production cost for economic dispatch schedule by the proposed method was very close to that of the more rigorous method used in reference (17). This indicates a high accuracy for the proposed method for economic dispatch purpose. As the method has not been tested on practical systems, because of the limitations of the computing facilities available (IBM 1130 system with 16 k words core memory), so any conclusive claim for accuracy can not be made as yet, but as indicated by the results of the sample systems tested, a high accuracy of results can be expected.

CHAPTER 3

REVIEW OF SECURITY ASSESSMENT METHODS

3.1 INTRODUCTION

Power system security means immediate future reliability of the system. Unplanned outages of generators, transmission equipment or faults may cause disruption of service in a part of the system, or a complete blackout in the system through cascade tripping of circuit breakers due to overloading of equipments in the other parts of the system. During planning and design stage some redundancy is built into the system to meet these contingencies. But, due to economic and other considerations only a limited amount of security can be built into the system. Also, some unforeseen events can occur during the system operation which may lead to dangerous operating conditions. Thus an on-line security monitoring of the system is essential for reliable operation of the system.

Power system security assessment is the determination of the security status (secure/insecure) of the power system operating condition based on a given next contingency list. The next contingency list contains the list of contingencies which may occur in immediate future and usually includes: loss of generation, loss of a transmission line or a transformer, sudden change in load demand, faults in any part of the system, etc.. For any contingency condition the power system security status can be determined for both dynamic instability and steady state emergency conditions. The trend in power system



security analysis is to have separate analysis for dynamic (or transient) and steady state security<sup>(4)</sup>. The work presented here deals with only steady state security of power system. In the sequel the term "security" will be used for steady state security.

In the last decade a great deal of interest has been generated on steady state security analysis of power systems. Several approaches to computer simulated security assessment methods are available<sup>(46-63)</sup>. Basically these approaches start with the knowledge of the present state (base case) of the system obtained through power flow solution or state estimation methods, the system is then tested for various contingencies by solving for the changes in the system operating conditions applying the contingencies in the list one by one. The post contingency operating condition for each contingency is then checked against the system operating constraints. Essentially these methods use sensitivity analysis, simplified power flows or distribution factor methods for finding the post contingency operating conditions.

In this chapter basic introduction to the power system security assessment problem is given and a short review of the existing methods is presented.

### 3.2 THE PRINCIPAL OPERATING STATES OF A POWER SYSTEM

Power system security is the capability of the system to withstand contingencies (i.e., unplanned events) without violating system load constraints and operating limit cons-



straints. The power system operating condition is never static as the system load demand is always changing but in the course of operation a power system is considered to transit from one operating state to another with small time steps. For the purpose of security analysis these operating conditions can be characterized in terms of four operating states - normal, alert, emergency and restorative<sup>(1,4)</sup>. These operating states of the system can be defined with the help of three sets of constraints. These three sets of constraints are:

- (1) Load constraints;  $G(x, u) = 0$
- (2) Operating constraints;  $H(x, u) = 0$
- (3) Security constraints;  $S(x, u) = 0$

The load constraint simply recognises the physical equations which satisfy the requirement that load demand will be met by the system. The operating constraints stipulate the objective of using system components within permissible limits and to meet the quality standard of the system. Overloaded components, bus voltages outside the tolerance, frequency deviations are violations of operating constraints. For the system to be secure it should be able to withstand contingencies without violating the load and operating constraints. That is, to be secure it has to satisfy certain security constraints. Accessibility and location of generating reserves, limits on transmission line capacities are examples of security constraints.

With the help of these three constraint set the four operating states of the system can be defined as:

(1) Normal state

All three sets of constraints satisfied. That is, when operating in this state the system is meeting all the load demand at proper frequency without overloading of any transmission equipment and generator, as well as it is able to withstand contingencies.

(2) Alert state

Load and operating constraints satisfied, security constraints not satisfied. That is, when operating in this state the system is meeting the load demand at proper frequency without overloading of any transmission equipment or generator but is not able to withstand contingencies. The operating objective in this state is to take proper corrective action so as to bring the system to the normal operating state.

(3) Emergency state

Operating and security constraints not satisfied, load constraints not necessarily satisfied. A power system operating in this state is not able to meet all the load demand, and, or the quality of service in terms of voltage and frequency is out of tolerance limits. The operating objective in this state is to stop the emergency from spreading and to obtain generation load balance quickly by means of reallocating generation and, or by shedding some load.

#### (4) Restorative state

Operating constraints are satisfied, load and security constraints are not satisfied. This state follows emergency state. In this the deterioration of system conditions have been halted, but the operating conditions are far from normal. That is, all the consumer loads are not satisfied or, the quality of service is not normal. The operating objective in this state is to restore all the loads and to return to the normal operating state.

The objective of power system operator is to keep the system in normal operating state as long as possible and should the system go into alert or emergency state, corrective control actions should be taken to bring the system back to the normal state.

The philosophy underlying the security assessment of a power system may be characterized as follows: The system operating condition is considered secure if the current state of the system can withstand the contingencies in the contingency list without violating the load and operating constraints. Thus the security criterion for the system is dependent on the contingency list. The larger is the contingency list considered, more stringent is the security criterion for the given system.

### 3.3 CONTINGENCY LIST

The following contingencies are usually considered for security analysis<sup>(63)</sup>:



- (1) Outage of a transmission line
- (2) Outage of a transformer
- (3) Outage of shunt capacitor or reactor
- (4) Loss of a generation
- (5) Sudden loss of a load
- (6) Sudden change in flow in a tie-line.

The outages of type (1) — (3) can be simulated by changes in the network admittance parameters and can be classified as network outage contingencies. The outages of type (4) — (6) can be simulated by changes in bus power injections and so, can be classified as power outage contingencies.

In case of network outage contingencies the practice in security analysis is to assume the network configuration and the bus power injections, in the contingency state, to remain same as in the pre-contingency state (base case) except for the changes in network admittances to account for the simulated outages. In case of power outages the network configuration and admittances remain same as that of the pre-contingency state. But the bus power injections for the other generators are rescheduled, according to some generation distribution law, to take into account the lost generation. The generation distribution law in its simplest form can be distribution of the lost generation equally among all other generators taking generators physical constraints into consideration (i.e., generation of any generator

should not exceed its maximum generation capacity), or generation distribution can be done in the inverse proportion to the penalty factors of the plants. An optimal load flow or economic despatch can be carried out for redistribution of generation but, it will be very time consuming especially when a large number of outage cases are to be simulated, and so is generally not preferred.

The preparation of a list of contingencies to be considered is a major sub-problem in large power system. The methods generally used are heuristic and operator's experience with his system is usually the only guide in preparation of such a list. Of late some contingency selection and ranking methods have been proposed<sup>(41-45)</sup>. These methods usually define a real power-flow index of the type<sup>(41)</sup>

$$J = \sum_{l=1}^{NL} \frac{1}{2n} \left( \frac{P_l}{P_l^{\max}} \right)^{2n} \quad (3.1)$$

Where,  $P_l$  = real power flow in line l

$P_l^{\max}$  = Maximum power transmission capacity of line l

NL = Number of lines in system

n = non-negative real index and usually, n = 1.

For each line outage index J is calculated using dc load flow method and contingencies are ranked in ascending order of J.

Though, the method is fast the accuracy of the results are not adequate, also it provides solution only for real power

overload constraints. As there is very little correlation between those contingencies which produce line overloads and those which produce unacceptable voltage levels<sup>(46)</sup>, a voltage reactive power index is needed to indicate the contingencies which produce unacceptable voltage levels or reactive power overloads in the transmission lines.

Similar to the real power-flow index a voltage reactive power flow index is defined as

$$J_{VQ} = \sum_{\beta} w_V \frac{|V_1 - V_1^{lim}|}{V_1^{lim}} + \sum_{\gamma} w_Q \frac{|Q_1 - Q_1^{lim}|}{Q_1^{lim}} \quad (3.2)$$

Where,

$V_1$  = Voltage magnitude at bus 1.

$V_1^{lim}$  = Upper or lower voltage limit (whichever is effective) at bus 1.

$w_V$  = Voltage weighing factor.

$Q_1$  = Reactive power injection at bus 1.

$Q_1^{lim}$  = Reactive power injection limit at bus 1.

$\beta$  = Set of buses at which voltage goes beyond limit.

$\gamma$  = Set of buses at which reactive power injection goes beyond limit.

$J_{VQ}$  is evaluated using the fast d.c. load flow type technique<sup>(46)</sup> and the contingencies are ranked in order of severity as indicated by  $J_{VQ}$ .

Though these contingency selection methods are quite fast their accuracy is suspect also the large number of contingencies to be evaluated still takes quite some time.



But these methods can be of help to the operators in choosing the small number of severe contingencies to be evaluated by more accurate methods.

### 3.4 SECURITY ASSESSMENT - STATEMENT OF THE PROBLEM

Basically security assessment answers the question "What if a contingency out of a set of probable contingencies (contingency list) takes place?" The possibilities are that either the system will ride through the disturbance and settle down to a normal state, or it will find itself in an emergency state. In the later case the system is insecure and security assessment function must inform the operator as to which contingency is causing insecurity and the nature and severity of the anticipated emergency.

The steady state operating condition of the power system is characterized by a set of nonlinear algebraic equations:

$$\begin{aligned} P_p - |V_p| \sum_{q=1}^N ((G_{pq} \cos \theta_{pq} + B_{pq} \sin \theta_{pq}) |V_q|) &= 0 \\ Q_p - |V_p| \sum_{q=1}^N ((G_{pq} \sin \theta_{pq} - B_{pq} \cos \theta_{pq}) |V_q|) &= 0 \end{aligned} \quad (3.3)$$

$$p = 1, 2, \dots, N.$$

Where,

$N$  = Number of buses in the system

$|V_q| \angle \theta_q$  = Voltage magnitude and angle at bus  $q$ .

$G_{pq}, jB_{pq}$  = Real and reactive parts of the  $pq$  element of  $Y_{BUS}$  matrix respectively.

$$\theta_{pq} = \theta_p - \theta_q$$

Equations (3.3) are power flow equations and represent the steady state equilibrium condition of the power network. For the purpose of making the statement of security assessment problem clearer, here we define three sets of vectors;

$$(1) \quad S = (P_1, P_2, \dots, P_N, Q_1, Q_2, \dots, Q_N)$$

a vector having  $2N$  dimension and representing the real and reactive power injections at different buses in the system.

$$(2) \quad C = (G_{11}, G_{12}, \dots, G_{1N}, G_{21}, \dots, G_{2N}, \dots,$$

$$G_{NN}, B_{11}, B_{12}, \dots, B_{1N}, B_{21}, \dots, B_{2N}, \dots, B_{NN})$$

a vector of  $2N^2$  dimension, representing the network admittance parameters.

$$(3) \quad X = (V_1, V_2, \dots, V_N, \theta_1, \theta_2, \dots, \theta_N)$$

a vector of dimension  $2N$ , representing the state variables of the system for the given operating condition.

The three vectors ( $X, S, C$ ) completely define the operating condition of the system. Any network outage will bring a change in network parameter vector  $C$  only and similarly any power outage will bring a change in power injection vector  $S$  only. If a power system operating condi-

tion (base case) is defined by  $(X^0, S^0, C^0)$ , then the post contingency states of the system can be given by

$(X^i, S^0, C^i)$ ,  $i = 1, 2, \dots, n$ , for network outages and  
 $(X^i, S^i, C^0)$ ,  $i = (n + 1), \dots, (n + p)$ , for power outages in the system.

Thus the security assessment problem can be stated as; given the base case operating condition  $(X^0, S^0, C^0)$

and contingency definitions

$C^i$ ;  $i = 1, 2, \dots, n$ , the network outages  
 and  $S^i$ ;  $i = (n + 1), \dots, (n + p)$ , the power outages determine  $X^i$ , the post contingency system operating state and identify if any dangerous operating conditions (line overloads, voltage beyond tolerance limit etc.) appear in the event of the contingency.

### 3.5 POWER SYSTEM SECURITY ASSESSMENT - A SHORT REVIEW

Various methods for power system security assessment have been proposed<sup>(46-65)</sup>, of which a few are briefly reviewed here mainly to highlight the different trends and approaches in security assessment methodology. The various methods for power system security assessment can be broadly classified into the following three types of approaches:

- (1) Probability methods;
- (2) Simulation methods;
- (3) Pattern recognition methods.



The probability method for security assessment was first proposed by A.D.Patton<sup>(64)</sup>. All probability methods<sup>(5, 64, 65)</sup> for security assessment define an index of security which reflects the probable future performance of the systems for the contingencies under consideration (contingency list). These methods assess the security of the system over a time period in near future considering the probability of outages for the given system operating configuration.

In these methods a security function, which expresses the risk of system operation as a function of future time, is defined. A number of different formulations for security functions are possible. A simple function which defines the system risk is given in reference (64) as:

$$S(t) = \sum_1 P_1(t) \cdot Q_1(t) \quad (3.4)$$

Where the summation in theory is over all possible system states, but in practice over states obtainable from initial state by some maximum number of contingencies, i.e., contingencies in the contingency list.

$P_1(t)$  = the probability of the system being in state 1 at time t into the future.

$Q_1(t)$  = the probability of violating the load and/or operating constraints at the time t into the future.

The security index  $S(t)$  is compared with the maximum allowable value of the index ( $S^m$ ). Where  $S^m$  defines the maximum allowable risk of inadequate system performance which can be

accepted. If  $S(t) > S^m$ , the security of the system has been breached and the operating condition must be modified to improve the system security.

If the load is considered deterministic  $Q_1(t)$  will either be zero or one. Several formulations of security functions are given in (5, 64, 65). Which can take care of dynamical operation of the system also into the consideration. The probabilistic methods of security assessment have been applied for generation system security assessments but have not gained much attention for the composite generation-transmission systems as the computational effort becomes quite complex. Further, its scope is limited to planning or off-line studies only because of the large computational time involved for computing the system state probability  $P_i(t)$ . For on-line applications where the system operating conditions are accurately known through telemetered data or state estimation process, the simulation methods appear to be more promising and have been gaining much attention in the last decade.

So far simulation methods for security assessment have received much attention of the power system engineers (41-63). The simulation methods for security assessment can be further classified into two classes; one which uses pre-calculated distribution factors to simulate the outage effects and the other which uses repeat solution of power flows to simulate the outage effects.

In the distribution factor methods the changes in power flows caused by the power and network outages are computed using pre-calculated distribution factors, and superimposed on the base case values<sup>(24, 46-48)</sup>. Figure 3.1 illustrates the effect of a line and/or generator outage on another line. The resultant post-fault complex power flow  $S_{rs}$  in line 'rs' due to the outage of a generator 'g' and/or line 'pq' can be written as

$$S_{rs} = S_{rs}^o + G_{g,rs} S_g^o \quad (3.5)$$

for generator 'g' outage, and

$$S_{rs} = S_{rs}^o + G_{pq,rs} S_{pq}^o \quad (3.6)$$

for line 'pq' outage, and for combination of line and generator outage

$$S_{rs} = S_{rs}^o + G_{g,rs} S_g^o + G_{pq,rs} S_{pq}^o \quad (3.7)$$

Where the superscript 'o' indicates the pre-outage condition and  $G_{g,rs}$  and  $G_{pq,rs}$  are the distribution factors for power flow in line 'rs' due to outage of generator 'g' and line 'pq' respectively. The distribution factors can be derived using the elements of line impedance and Z-matrix of the system<sup>(48)</sup>. Once the distribution factors are calculated and stored, the computation of power flows in different lines in the system for outages of lines or generators becomes very fast and can be implemented for on-line application in security assessment. The main drawback of the method is that



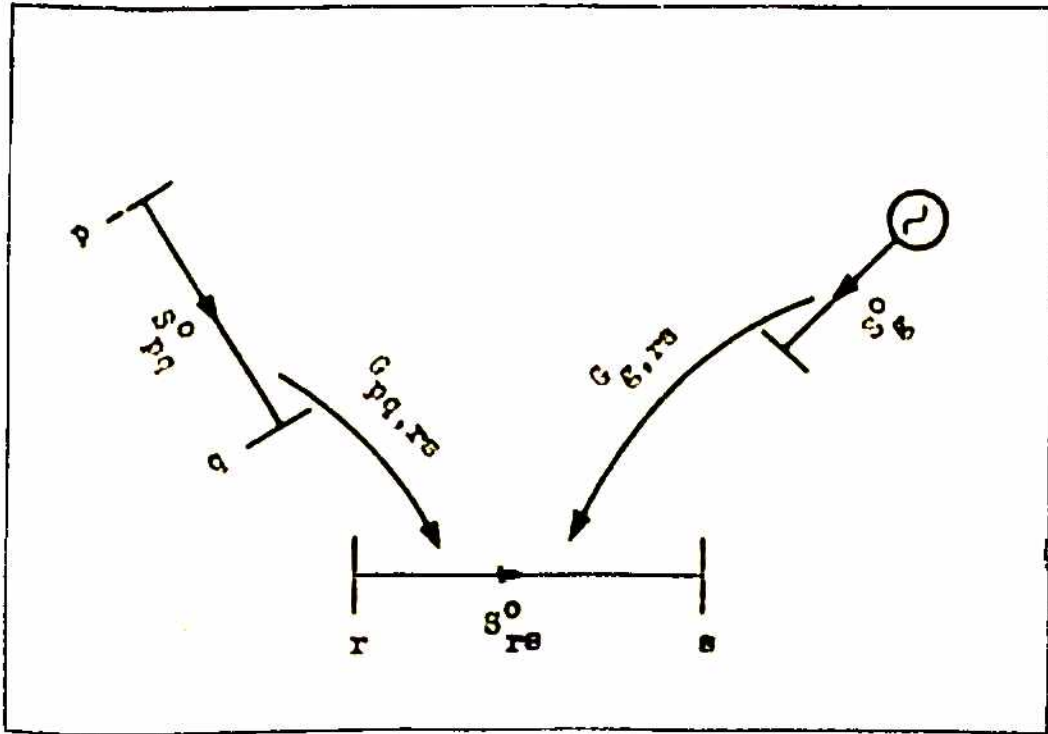


FIG.3.1

only power flow information due to outage conditions are obtained and no information about the nodal voltages are available. Also since the distribution factors are calculated using only the system impedance matrix elements (i.e., assuming  $V = 1.0$  pu throughout) the accuracy of the results are suspect. If system operating conditions are also taken into account then the distribution factors have to be updated to better represent the system conditions. This will increase the computational burden by a great extent as a large set of distribution factors have to be computed and stored.

The outage contingencies can be formulated as nonlinear power flow problem and solved using the Newton-Raphson method. The accuracy of results obtained will be very good but the computing time needed to process a large number of contingencies will be excessive. Noting that the accuracy of solution for outage simulations is not as demanding as that in case of power-flow solutions, simplified power-flow methods using d.c. power flow, sensitivity methods or linearized power flow methods are commonly used for contingency simulation and security assessment.

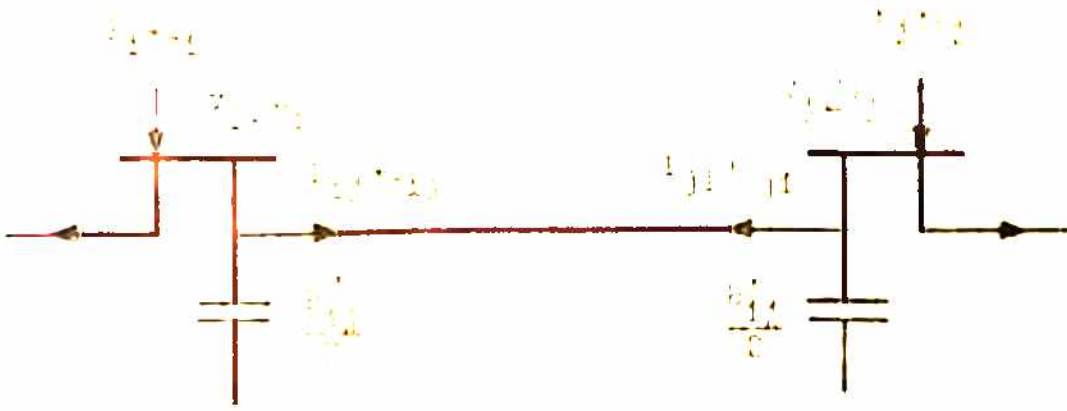
D.C. power flow methods give very fast solution for contingency analysis but the accuracy of the results are not adequate, also only real power flow solution is available and no voltage information can be obtained. Some Z-matrix methods for contingency evaluation have been proposed<sup>(49-51)</sup>. The methods are quite fast for contingency simulation but their main shortcoming is that storage requirements for large

non-sparse Z-matrix is excessive. Some of the methods overcome this shortcoming by using Z-matrix axis discarding technique and considering only a small number of limiting lines for overload monitoring<sup>(50,51)</sup>.

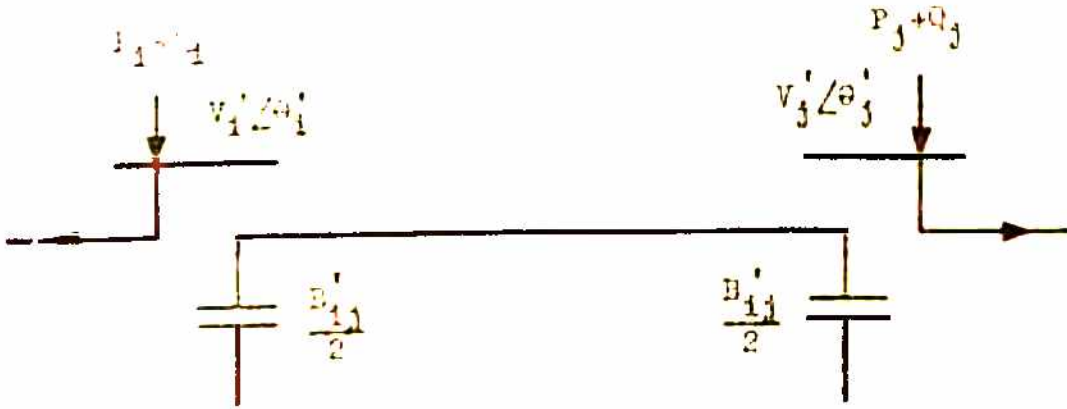
By and large the Y-matrix methods for contingency power flows are more popular because of the reduced storage requirement of the sparse Y-matrix. Since 1970's the trend in security assessment methods is towards use of constant matrix methods for contingency analysis. These methods linearise the power system model about the base case operating point and use either sensitivity techniques or linear iterative power flow methods.

Sachdev and Ibrahim<sup>(54)</sup> have proposed a fast contingency evaluation method using the base case sensitivity matrix. Transmission line or transformer outage is mathematically modelled by its pre-contingency configuration by adding changes in power injections at the end buses of the outage branches. The method can be explained with reference to Fig. 3.2. The basic and final state of the system buses  $i$  and  $j$  (for a outage simulation of line  $i-j$ ) are shown in Fig. 3.2(a) and (b). The voltages at these two buses will not change from that in the final system state, if the line  $i-j$  is reconnected and power equal to that flowing into the line is injected into these buses. Thus the <sup>structure of</sup> Jacobian matrix representing the system after the line outage remain unchanged from that in the basic state except that its elements are slightly different due to the change of magnitude and phase

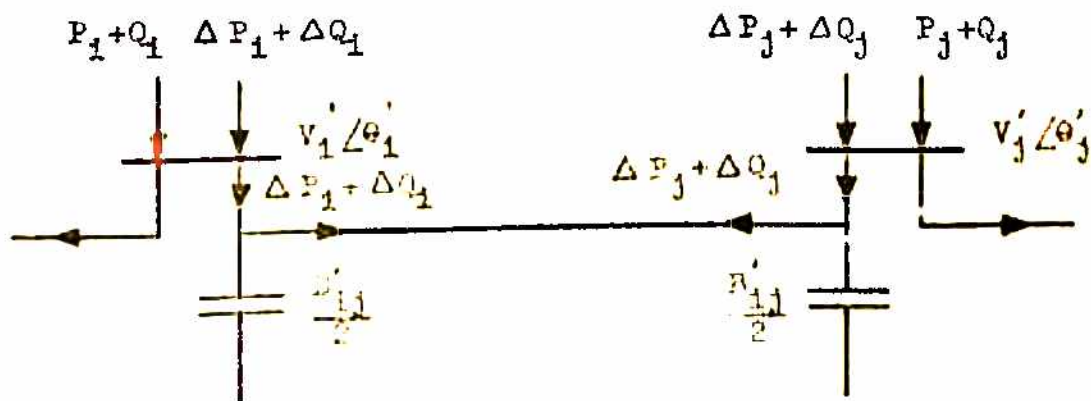




(a) BASIC SYSTEM STATE



(b) FINAL SYSTEM STATE



(c) SIMULATED FINAL SYSTEM STATE

FIG 3.2

angles of the voltages. Thus the retriangularization of the Jacobian matrix  $[J]$  to account for the network changes due to outage is avoided. The modification of the power injection into the end buses  $i$  and  $j$ , which would simulate the outage effect, is computed using the sensitivity matrix  $S (= [J]^{-1})$  of the basic state in an iterative mode using the respective nonlinear power flow equations. The main-drawback of the method is low accuracy because with the use of linear sensitivity of the basic system the method tries to satisfy the conditions in the outaged branch more readily than in the rest of the system. To overcome this shortcoming a power injection modification index is proposed but this requires off-line studies. The method in reference (61) tries to overcome the above shortcoming by trying to meet the scheduled demand at the end buses irrespective of the flow in the outage element.

The method proposed by Pai and Khan<sup>(57)</sup> uses a fictitious state model concept and matrix inversion lemma of Shermon and Morisson to compute the sensitivity matrix for the contingency case from the base case triangular factors of the Jacobian matrix. The post contingency state of the system is obtained using this sensitivity matrix in an iterative mode.

A linear iterative power flow technique for contingency evaluation has been proposed by Peterson et al.<sup>(53)</sup>. The algorithm uses decoupled formulation for real and reactive power. The two Jacobian sub-matrices for the decoupled

formulation are evaluated at the no-load point and the triangular factors of these submatrices are stored. The effects of network outage simulations are obtained without modifying the triangular factors using compensation method<sup>(52)</sup>. The nonlinearity and coupling effects are introduced using an iterative scheme. The method is quite fast but the results obtained are only approximate solutions.

Stott and Alsac<sup>(55)</sup> extended their decoupled load flow algorithm to include the outage calculations. The method uses the matrix inversion lemma of Sherman and Morrison to simulate the network outage changes in the Jacobian without modifying the triangular factors of the Jacobian. The method is fast and does not have large storage requirement as only triangular factors of the sparse symmetric decoupled submatrices need to be stored. The method requires two to three B-V iterations to achieve the required accuracy. The details of the method are described in Appendix A3.1.

Recently Zaborszky et al.<sup>(66)</sup> have proposed a concept of "concentric relaxation" for contingency evaluation. The method makes use of the sparsely connected topological structure of the system and utilises the diagonal dispersion nature of the power system. The basic idea is that the effects of disturbance get attenuated as they travel away from the location of the disturbance. A concentric relaxation method is used by treating the system or a piece of it ("a float")



as electrically rigid and gradually relaxing the results tier by tier (a tier consists of all the buses directly connected to the buses of the previous tier) to account for the actual flexibility of the transmission system. The term "concentric relaxation" refers to relaxing the conditions of the network to fit the changes injected by the contingency. The tier by tier updating of the variables are done using a Gauss-Siedel type algorithm. The speed of solution is derived from the exploitation of structure of the system as the changes beyond third tier are negligible<sup>(66)</sup>. Though fast contingency simulation methods have been developed, the large number of contingencies to be simulated require considerable computational effort and time making them unattractive for on-line applications. Thus some better approach to security assessment which is accurate as well as very fast and reliable needs to be developed.

The application <sup>^</sup>pattern recognition to security assessment appears promising for on-line applications. These use a large set of training data and an off-line training process to design security classifiers. The security classifiers are used for classifying any power system operating state into secure or insecure state. Once the classifier is available the classification process requires negligible computational effort and time and can be used for on-line applications. The details of the pattern recognition process is presented in the next chapter.

## CHAPTER 4

SECURITY ASSESSMENT THROUGH PATTERN RECOGNITION TECHNIQUE4.1 INTRODUCTION

Pattern, in a general sense can be defined as description of an object or a situation. Recognition of patterns is a basic attribute of human as well as other living beings. It is thought that the decision making ability; for example, the next move in a game of chess is based on the present pattern on the board. The goal of pattern recognition systems is to clarify these complicated decision-making process and automate these functions using computers. The pattern recognition problem offers unique challenge because recognition is something everybody performs but the process underlying it is not very well understood, apparently because most everyday perceptual processes are carried below the conscious level.

The lack of complete theory of perception has not deterred people from trying more modest tasks. Many of these involve pattern classification - the assignment of a physical object or event to one of the several prespecified categories. Extensive study of classification problems has led to some abstract mathematical models that provide the theoretical basis for classification. Though these models are general, still for specific problems one has to come to grips with special characteristics of the problem at hand.



In this chapter a brief overview of the pattern recognition problem is presented. Then the power system security assessment is formulated as a pattern recognition problem. Some existing methods in literature are briefly reviewed. In the end a pattern classification algorithm based on solution of linear inequalities is proposed for power system security assessment.

#### 4.2 PATTERN RECOGNITION - BASIC APPROACH(67-72)

Pattern recognition in general encompasses a large variety of problems. Many of these can be characterised as classification of an object or an event into different pre-specified categories. This is usually termed as "pattern classification" problem. In this thesis we will confine ourselves to this category of problem only.

In general, pattern classification system design involves several subproblems. The first one is the sensing problem. This concerns with the representation of the input data which can be measured from the object or event to be recognised. Each measured quantity describes a characteristic of the pattern. Usually these measurements are arranged in the form of a measurement or pattern vector (a  $n$ -dimensional vector for  $n$  measurements used to characterise the pattern). The pattern vector contains all the measured information available about the patterns. When the measurements yield information in the form of real numbers the pattern vector can be considered as a point in  $n$ -dimensional Euclidean space.



In most of the pattern recognition problems the dimension of pattern vector is quite large. This makes the design of the recognition system quite complex and costly. So, the second problem concerns the reduction of the dimensionality of the problem. This can be achieved by extraction of characteristic features or attributes from the received input data. This is generally referred to as preprocessing or characterisation problem. The features characterising attributes common to all patterns belonging to the same class are called intraset features and the features which represent difference between patterns of different classes are called interset features. The elements of intraset features which are common to all pattern classes under consideration contain no discriminatory information for pattern classification and so can be dropped conveniently. If a complete set of discriminatory features can be determined for each pattern class then the pattern classification problem will become a simple feature matching problem, in which any new pattern will be classified by matching its features with the features for different pattern classes. Unfortunately, in all practical problems it is very difficult, if not impossible, to get a complete set of discriminatory features. But still some of the discriminatory features may be found which can be of use for classification problem. ( Feature extraction is a very important aspect in automatic pattern recognition process. )

The third important aspect in pattern recognition

process is the determination of a decision rule or a discriminant function which is able to classify the patterns into different classes. After the measurements for the patterns have been made and expressed in terms of pattern vector, each pattern can be viewed as a point in the Euclidean space. If the patterns are to be classified into  $M$  distinct classes, then the classification problem can be seen as dividing the Euclidean space into  $M$  distinct regions, one for each class. Thus the classification process is generation of decision boundaries which separate the  $M$  pattern classes based on the measurement data. Let  $w_1, w_2, \dots, w_M$  be designated as  $M$  possible pattern classes, and let  $X = (x_1, x_2, \dots, x_n)^t$  be the feature measurement vector, where  $x_i$  represents the  $i$ th feature measurement. Then we can define a decision or discriminant function associated with class  $w_i$  as

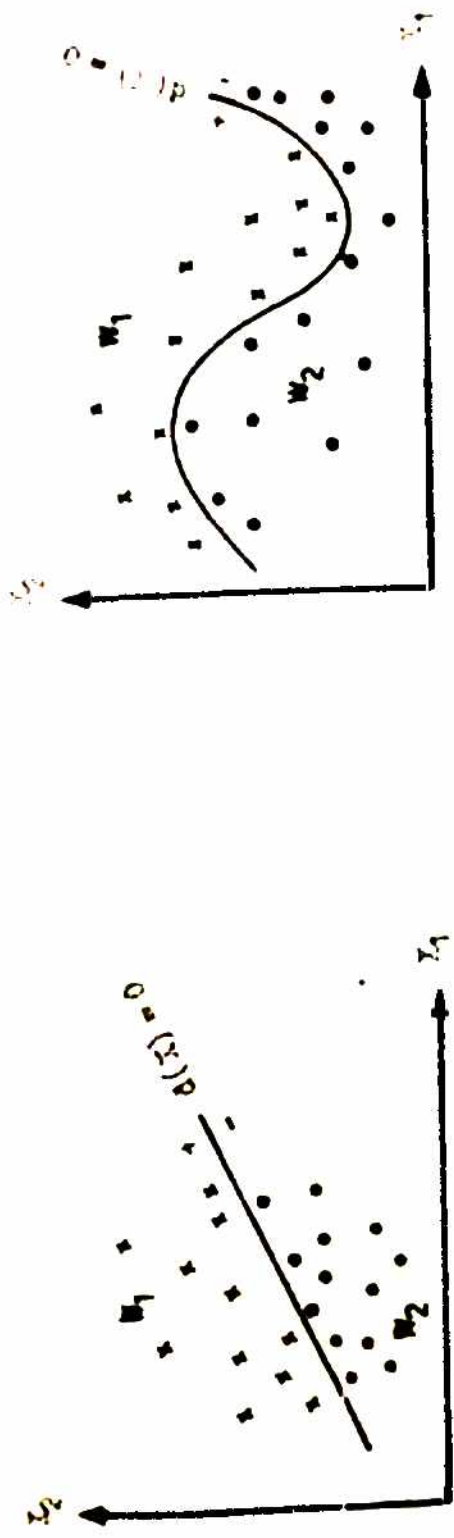
$$d_i(X) \geq d_j(X), \quad i \neq j \quad \text{and} \quad i, j = 1, 2, \dots, M \quad (4.1)$$

if the pattern vector  $X$  belongs to class  $w_i$ . That is if the decision function,  $d_i(X)$  has the largest value for a pattern  $X$ , then  $X \in w_i$ .

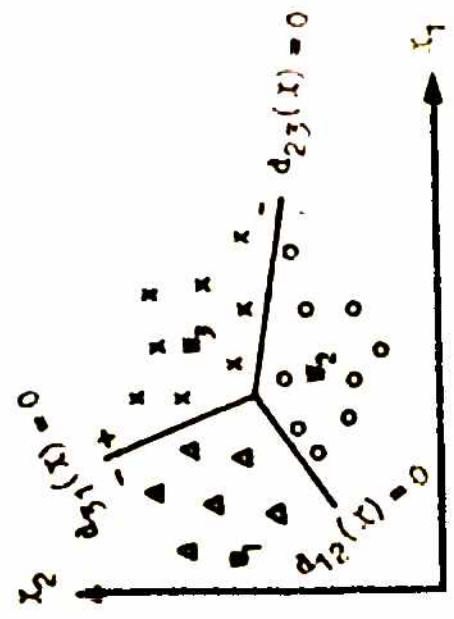
Thus in Euclidean feature space, the boundary of partition (the decision boundary) between regions associated with class  $w_1$  and  $w_j$ , respectively, is given by

$$d_{1j}(X) = d_1(X) - d_j(X) = 0 \quad (4.2)$$

The geometrical interpretation of the above concept is illustrated in Fig. (4.1) for a two-dimensional pattern space



(a) TWO CLASS LINEAR DECISION FUNCTION (b) TWO CLASS LINEAR DECISION FUNCTION

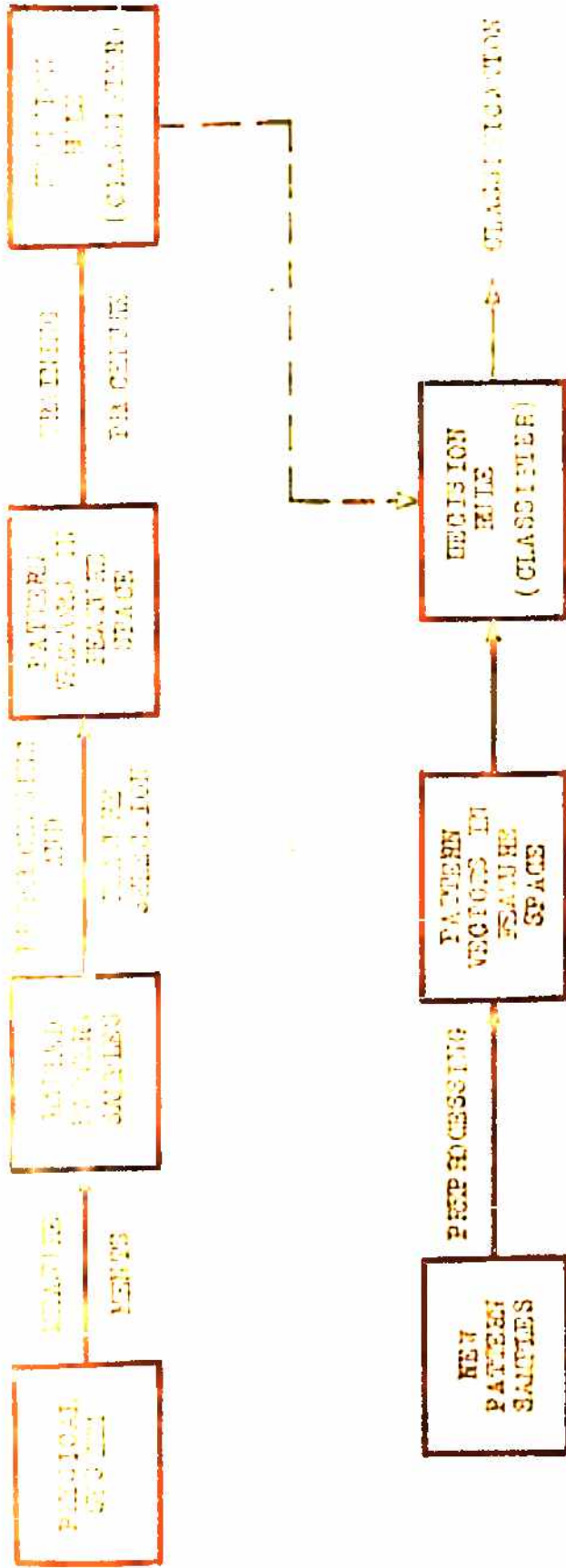


(c) THREE CLASS LINEAR DECISION FUNCTION

FIG. 4.1. WEIGHTED FUNCTIONS



STAGE I : TRAINING PROCESS (OFF-LINE)



STAGE II : CLASSIFICATION PROCESS (ON-LINE)

FIG.4.2 PATTERN RECOGNITION PROCESS

model. Many different forms satisfying equation (4.2) can be selected for  $d_1(X)$ . The common among these are Linear Discriminant Functions, Piecewise Linear Discriminant Functions, Polynomial Discriminant Functions etc.

After the classifier has been designed it is used for classifying unknown pattern samples. Thus the pattern recognition process is a two stage process. The first stage consists of designing or generating the classifier using the measurement data of the known pattern samples and the second stage is using this classifier for classifying unknown samples. The block diagram for the two stage pattern recognition process is shown in Fig. 4.2 .

#### 4.3 CHARACTERISATION (PREPROCESSING) PROBLEM<sup>(69,73-78)</sup>

As seen from the above discussion preprocessing is used for extracting those attributes from the measurements of the pattern samples which contain discriminatory information with regard to the pattern classes. The feature extraction process is a very important aspect in pattern recognition scheme as it strongly affects the design and performance of the classifier. Based on the difference in the basic approach to the problem, the feature extraction process can be divided into two different groups:

- ✓(1) Feature extraction;
- ✓(2) Feature Selection.

### 4.3.1 Feature Extraction

The feature extraction process can be considered as mapping of the original measurements into a usually much smaller and more effective set of features. This tantamounts to finding a transformation for the data from the n-dimensional measurement space to a m-dimensional feature space, where  $m \ll n$ . The main objective of this transformation is to reduce the dimensionality of the data and to retain or enhance the discriminatory information for the pattern samples. Both linear and nonlinear transformations<sup>(76)</sup> can be used for this purpose. But since nonlinear transformations are in general quite complex and time consuming if the data set is large, Usually, linear transformation methods are preferred<sup>(69,73-75)</sup>.

#### Karhunen Loe've Expansion<sup>(69,73-74)</sup>

The most important and widely used linear transformation method for feature extraction is based on Karhunen-Loe've (K-L) expansion. The method makes use of the orthonormal transformation of the data from measurement space  $R_n^n$  to the feature space  $R_t^m$  in such a way that the mean-square error in transformation is minimum. Let the  $N$  pattern samples in  $n$ -dimensional space are represented by pattern vectors as

$$\begin{aligned} P_1 &= (p_{11}, p_{12}, \dots, p_{1n})^t \\ P_2 &= (p_{21}, p_{22}, \dots, p_{2n})^t \\ &\vdots \\ P_N &= (p_{N1}, p_{N2}, \dots, p_{Nn})^t \end{aligned} \quad (4.3)$$



The problem is to find similarly represented orthonormal vectors in  $m$ -dimensional feature space:

$$\begin{aligned} F_1 &= (f_{11}, f_{12}, \dots, f_{1n})^t \\ &\vdots \\ F_m &= (f_{m1}, f_{m2}, \dots, f_{mn})^t \end{aligned} \quad (4.4)$$

such that

$$\begin{aligned} Q(F_1, F_2, \dots, F_m) &= \frac{1}{N} \sum_{i=1}^N \left\| P_i - \sum_{j=1}^m a_j^{(1)} F_j \right\|^2 \\ &= \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^n \left( p_{it} - \sum_{j=1}^m a_j^{(1)} f_{jt} \right)^2 \end{aligned} \quad (4.5)$$

is minimised. Where,

$$a_j^{(1)} = (P_i \cdot F_j), \text{ the dot product of } P_i \text{ and } F_j. \quad (4.6)$$

Equation (4.5) provides a relationship for finding a set of vectors which yields the least error averaged over all sample points. The solution to this problem is the discrete K-L expansion<sup>(74)</sup>. A simple method for solution is as follows:

- (1) Construct the  $n \times n$  covariance matrix  $[C]$  for the measurement data, where,

$$C_{j k} = \frac{1}{N} \sum_{i=1}^n P_{ij} P_{ik} \quad (4.7)$$

- (2) Solve the eigenvalue problem

$$[C] [F_1] = \sigma_1^2 [F_1] ; 1 = 1, 2, \dots, n$$

(4.8)

Where  $[F_1]$  is the 1<sup>st</sup> eigenvector and  $\lambda_1$  the corresponding eigenvalue. The normalised eigenvectors  $[F_1]$  form the orthonormal vectors.

- 3) Choose  $m$  eigenvectors  $[F_1, i = 1, \dots, m]$  corresponding to the  $m$  largest eigenvalues obtained in step 2, to construct the transformation matrix.

$$[T] = \begin{bmatrix} F_1^t \\ \vdots \\ F_m^t \end{bmatrix} \quad (4.9)$$

The features  $(x_{i1}, \dots, x_{im})$  for  $i$ th pattern sample are obtained using

$$\begin{bmatrix} x_{i1} \\ \vdots \\ x_{im} \end{bmatrix} = [T] \begin{bmatrix} p_{i1} \\ \vdots \\ p_{in} \end{bmatrix} \quad (4.10)$$

K-L expansion method provides the best orthonormal transformation as it minimises the mean-square distance between the samples in  $n$ -dimensional measurement space and their projections in the  $m$ -dimensional feature space.

The main drawback to K-L expansion method for feature extraction is that it involves the difficult problem of computing eigenvalues and eigenvectors of a large matrix. Also, the information compression property of the K-L expansion is not always associated with the compression

of discriminatory information<sup>(77)</sup>.

#### 4.3.2 Feature Selection<sup>(69,77)</sup>

The feature selection process is radically different in approach from that of feature extraction in the sense that, feature selection process finds the smallest subset of measurements for the pattern, from the original measurements, which will yield optimum classification. This suggests a search technique to be used for this purpose. A simple algorithm for this purpose can be as follows:

Step 1) For each positive integer  $m < n$ , find the subset  $S_m^*$  of dimension  $m$  for which classification performance  $f(S_m)$  is maximum.

Step 2) Find the smallest value of  $m$  for which

$$f(S_m) > f^* \quad (4.11)$$

Where  $f^*$  is the minimum desired performance.

The main problem in implementing the above algorithm is that, even for moderate values of  $m$  and  $n$  the number of subsets to be searched is extremely large. For example, for  $n = 20$  and  $m = 10$ , the total number of subsets will be  $\binom{20}{10} = 184,756$ . This makes the implementation of the above scheme almost impossible<sup>(77)</sup>.

Another approach to feature selection can be as follows:

Step 1) For each measurement  $x_i$ ,  $i = 1, \dots, n$ , characterising the pattern vector, find out the classification per-



formance measure  $f(x_i)$  on an individual basis.

Step 2) Rank the measurements according to the values of  $f(x_i)$ ,  $i = 1, \dots, n$ .

Step 3) Select the top ranking measurements as features.

A simple performance measure which can be used in above algorithm can be based on the interclass distance between the means for the measurement. The main shortcoming of the method is that each measurement is considered independent and no interaction between features is taken into account. In the extreme case this may lead to the choice of k highly correlated measurements as features and so the classification performance using these m features will be same as that using a single feature. A feature selection procedure which avoids this drawback is proposed in Section 4.7.

#### 4.4 PATTERN CLASSIFICATION (69-72, 78, 79)

Once the set of features has been selected and the data concerning the patterns and their features are available, the next problem is the determination of a decision function or a "classifier" of these features based on the given data. For a two class problem (extension to multi-class problem will be presented later) the classification problem can be stated as,

$$\text{Find, } d(X) \tag{4.12}$$

such that

$$d(X) = \begin{cases} \geq 0 & X \in w_1 \\ < 0 & X \in w_2 \end{cases} \quad (4.13)$$

Usually an iterative process is used for finding  $d(X)$  as given by (4.13) using the given data. This iterative process of finding  $d(X)$  is termed as "training" or "learning" process. When data for the patterns with known classification is available the training process is termed as supervised learning or "training with a teacher". When no prior knowledge about the classifications of the pattern vectors are available the training process becomes an "unsupervised learning" process. The approach used in unsupervised learning are usually heuristic and mostly some distance criterion like cluster analysis or nearest neighbour rules are used for the training of the classifier. Fortunately, in many practical classification problems the apriori knowledge of the classification for the pattern vectors, used in the training process is known and we will limit our discussion to the supervised learning methods only.

There are two main approaches used for supervised learning process:

- (1) statistical methods;
- (2) deterministic methods.

#### 4.4.1 Statistical Methods (68-72)

Bayes decision theory is fundamental statistical approach to the pattern classification problem. This approach is based on the assumption that the classification problem is

posed in probabilistic terms, and that all relevant probability values are known.

Let,  $X$  be a pattern vector, and we want to decide whether  $X$  belongs to  $w_1$  or  $w_2$ . A decision rule based simply on probabilities may be written as

$$P(w_1|X) \geq P(w_2|X) \rightarrow X \begin{matrix} \in w_1 \\ \in w_2 \end{matrix} \quad (4.14)$$

The a posteriori probabilities  $P(w_j|X)$  can be determined using the a priori probabilities  $P(w_j)$  and the conditional probability density functions  $p(X|w_j)$  using Bayes theorem

$$P(w_j|X) = \frac{p(X|w_j)P(w_j)}{P(X)} ; j = 1, 2 \quad (4.15)$$

Where,

$$P(X) = \sum_{j=1}^2 p(X|w_j)P(w_j) \quad (4.16)$$

Substituting (4.15) in (4.14) and dropping  $P(X)$  as it is common on both sides of the inequality, we get the decision rule as

$$p(X|w_1)P(w_1) \geq p(X|w_2)P(w_2) \rightarrow X \begin{matrix} \in w_1 \\ \in w_2 \end{matrix} \quad (4.17)$$

Decision rule given by (4.17) is the Bayes decision rule for minimum error in classification. It is an optimum decision rule as use of this rule will lead to minimum classification errors. Because of this important property this rule is very frequently used in pattern classification and forms a basis of comparison for all other statistical classification algorithms. The implementation of (4.17)



requires a priori knowledge of  $P(w_1)$  and  $p(X|w_1)$  for  $i = 1, 2$ . In case when these are not known a priori, these can be estimated from the data available. When a set of pattern samples with known classification (also called training set) is available then estimation of  $P(w_1)$  is quite simple, but the a priori class conditional pdf  $p(X|w_1)$  can be estimated only if the statistical distribution of the pattern samples in each class is either known or assumed.

In case when a training set of pattern samples with known classification is available then the pattern classifier can be designed using a training procedure. Using (4.15) and (4.17) the decision rule can be modified as follows:

The Bayes decision rule in (4.17) is equivalent to implementation of the decision function

$$d_1(X) = p(X|w_1)P(w_1), \quad i = 1, 2 \quad (4.18)$$

Where a pattern is assigned to class  $w_1$  if  $d_1(X) > d_j(X)$ ,  $i \neq j$ . An expression that is equivalent to (4.18) but does not require explicit knowledge of  $p(X|w_1)$  and  $P(w_1)$  can be obtained using (4.15) and (4.18) as

$$d_1(X) = p(w_1|X)P(X), \quad i = 1, 2 \quad (4.19)$$

However, since  $P(X)$  does not depend on  $i$  it can be dropped, yielding, a decision function

$$d_1(X) = p(w_1|X), \quad i = 1, 2 \quad (4.20)$$

The decision boundary for this two class case is given as

$d_1(X) - d_2(X) = 0$ . Thus the equivalent decision boundary function can be derived as

$$\begin{aligned} d(X) &= d_1(X) - d_2(X) \\ &= p(w_1|X) - p(w_2|X) \\ &= p(w_1|X) - [1 - p(w_1|X)] \end{aligned}$$

$$\text{OR} \quad d(X) = 2p(w_1|X) - 1 \quad (4.21)$$

This leads to the decision rule

$$p(w_1|X) \geq \frac{1}{2} \rightarrow \begin{matrix} X \in w_1 \\ X \in w_2 \end{matrix} \quad (4.22)$$

Implementation of decision rule (4.20 - 4.22) requires the knowledge of pdf  $p(w_1|X)$ . Unfortunately, the pdf  $p(w_1|X)$  is not directly available and the only information available is the class membership of each pattern vector in the training set. Using the information about the class membership and feature measurements of the pattern vector for all the patterns in the training set the pdf  $p(w_1|X)$  is estimated using some iterative learning (or training) algorithm involving stochastic approximation methods. Several algorithms are available in literature<sup>(69-72, 78,79)</sup> on this problem. The advantage of stochastic methods are that in the limit they always converge to the optimum Bayes classifier. The main drawback of these methods is the very slow rate of convergence<sup>(68)</sup> which makes the implementation of these methods difficult.

#### 4.4.2 Deterministic Methods (68,69,72,78)

From the above discussion it is clear that the statistical methods either require prior knowledge about the statistical distribution of the pattern samples for each class or the learning algorithms used for stochastic approximations are very slow. Both these shortcomings are overcome in deterministic methods. Basically, the idea is to find  $d(X)$  that works well at least on the given samples of known classification. Restating the two-class classification problem in (4.13), we have the decision rule:

$$d(X) = \begin{cases} \geq 0 & X \in w_1 \\ < 0 & X \in w_2 \end{cases} \quad (4.13)$$

Let after feature selection the pattern vectors in the training set are arranged into two sets

$$X(N1) = \left[ \begin{matrix} x_1^{w_1} & \dots & x_{N1}^{w_1} \end{matrix} \right]^t \text{ patterns in class } w_1 \quad (4.23)$$

$$X(N2) = \left[ \begin{matrix} x_1^{w_2} & \dots & x_{N2}^{w_2} \end{matrix} \right]^t \text{ patterns in class } w_2 \quad (4.24)$$

Where  $x_j^{w_1} = (x_{j1}^{w_1}, x_{j2}^{w_1}, \dots, x_{jm}^{w_1})$  is a row vector consisting of  $m$  features denoting  $j$ th pattern vector in class  $w_1$  and two sets of  $N1$  and  $N2$  ( $N1 + N2 = N$ ) patterns for class  $w_1$  and  $w_2$  respectively.

Let us consider a linear decision function (see Appendix 4.1)



$$d(X) = \underline{a}^t X \quad (4.25)$$

such that a pattern  $X_1$  is classified correctly if

$$d(X) = \underline{a}^t X_1 > 0 \quad \text{and} \quad X_1 \in w_1 \quad (4.26)$$

or  $d(X) = \underline{a}^t X_1 < 0 \quad \text{and} \quad X_1 \in w_2$

In the later case, we see that  $X_1$  is correctly classified if  $\underline{a}^t (-X_1) > 0$ . This suggests replacement of all patterns in class  $w_2$  by its negatives. With this normalisation we can look for a parameter vector  $\underline{a}$  such that

$$\underline{a}^t X_1 > 0 ; \quad i = 1, 2, \dots, N \quad (4.27)$$

or  $AW > 0 \quad (4.28)$

Where,

$$A = \begin{bmatrix} \begin{matrix} w_1 \\ X_{11} \dots\dots\dots X_{1m} \end{matrix} & \begin{matrix} w_1 \\ 1 \end{matrix} \\ \vdots \\ \begin{matrix} X_{N11} \dots\dots\dots X_{N1m} \end{matrix} & \begin{matrix} 1 \end{matrix} \\ \hline \begin{matrix} w_2 \\ -X_{11} \dots\dots\dots -X_{1m} \end{matrix} & \begin{matrix} w_2 \\ -1 \end{matrix} \\ \vdots \\ \begin{matrix} w_2 \\ X_{N21} \dots\dots\dots -X_{N2m} \end{matrix} & \begin{matrix} w_2 \\ -1 \end{matrix} \end{bmatrix} \quad \begin{matrix} (N_1 + N_2)(m + 1) \\ \text{matrix} \end{matrix} \quad (4.29)$$

and  $W = (a_1 \dots\dots\dots a_m \ a_{m+1})^t \quad (4.30)$

In (4.29) the pattern vectors have been augmented by ones and minus ones in the last column to indicate the known classification of the patterns.

Thus the problem of finding the decision function  $d(X)$  is nothing but finding a solution to the linear inequalities given in (4.27) or (4.28). If there is a single vector  $W^*$  which satisfies all the inequalities in (4.28) then the system of inequalities is said to be a consistent set (classes are separable) and  $W^*$  is a solution. Otherwise the system of inequalities is inconsistent (classes are inseparable) and solution  $W^*$  is a vector which satisfies the maximum number of inequalities in (4.28).

Many algorithms are available for solution of the linear inequalities<sup>(69,78,109,110)</sup>. A very simple algorithm for solution of (4.28) based on reward-punishment concept and commonly known as perceptron algorithm is as follows:

Let  $W(1)$  be the initial weight vector, which may be arbitrarily chosen. Then, at  $k$ th training step;

If  $W^t(k)X(k) \leq 0$ , replace  $W(k)$  by

$$W(k + 1) = W(k) + CX(k) \quad (4.31)$$

Otherwise leave  $W(k)$  unchanged, i.e.

$$W(k + 1) = W(k) \quad (4.32)$$

Where,  $X(k)$  is the pattern vector under consideration at the  $k$ th iteration step and  $C$  is the correction increment, a constant with condition  $C > 0$ .

A more general procedure for solving the linear inequalities is to transform the problem into an optimisation problem the solution of which also guarantees a solution to

(4.28). For this purpose we write (4.28) as

$$AW \geq B \quad (4.33)$$

Where  $B = (b_1, b_2, \dots, b_N)^t$  with  $b_i > 0, i = 1, 2, \dots, N$ .

The solution of linear inequalities in (4.33) can be formulated as a linear programming problem as follows:

Find a vector  $W = (a_1, a_2, \dots, a_{m+1})^t$  that minimises the linear objective function

$$Z = \alpha^t W \quad (4.34)$$

subject to

$$AW \geq B \quad (4.35)$$

Where  $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_{m+1})^t$  is the cost vector. For pattern recognition problems linear programming methods are rarely used because much simpler algorithms (mainly using gradient techniques) are available. One of the most important properties of the gradient vector of a function  $f$  is that the negative of the gradient points the direction of the maximum rate of decrease of  $f$ . On this basis iterative schemes are devised for finding the minimum of a function so chosen that it achieves its minimum whenever  $AW \geq 0$ . For example let us consider the criterion function

$$J(W) = (|AW| - AW) \quad (4.36)$$

It is evident that the minimum of this function is  $J(W) = 0$  and that this minimum results whenever  $AW \geq 0$ , excluding the



trivial case of  $W = 0$ . Using the gradient descent procedure for minimising (4.36) we are immediately led to

$$W(k+1) = W(k) - c \left[ \frac{\partial J(W)}{\partial W} \right]_{W=W(k)} \quad (4.37)$$

Where  $W(k)$  represents the value of  $W$  at  $k$ th step and  $c > 0$ . By choosing different criterion functions different algorithms may be created. A very useful algorithm based on gradient descent is the Least-Mean Square Error (LMSE) algorithm<sup>(80)</sup>. This algorithm has the advantage that in addition to being convergent for separable classes, it indicates in the course of operation that the classes under consideration are not separable, if it is the case. In this formulation the linear inequality (4.28) is expressed as:

$$AW = B \quad (4.38)$$

where  $B = (b_1, b_2, \dots, b_{m+1})^t$  with  $b_i > 0$  for  $i = 1, \dots, m+1$ . And consider the criterion function

$$J(W, B) = \frac{1}{2} \|AW - B\|^2 \quad (4.39)$$

Where  $\| \cdot \|$  is the norm of vector ( $\cdot$ ). The function  $J(W, B)$  achieves its minimum whenever (4.38) is satisfied. Since the function  $J(W, B)$  depends on both variables  $W$  and  $B$  a gradient descent procedure involving both variables will improve the convergence of the algorithm. The gradients associated with the problem are

$$\frac{\partial J(W, B)}{\partial W} = A^t (AW - B) \quad (4.40)$$

and 
$$\frac{\partial J(W, B)}{\partial B} = - (AW - B) \tag{4.41}$$

Since  $W$  is not constrained in anyway, we can set  $\partial J / \partial W = 0$  and obtain

$$W = (A^t A)^{-1} A^t B = A^{\#} B \tag{4.42}$$

Where  $A^{\#}$  is called the generalised inverse of  $A$ . Since all components of  $B$  are positive, the variation in  $B$  is achieved by letting,

$$B(k + 1) = B(k) + \delta B(k) \tag{4.43}$$

Where

$$B(k) = c \left[ (AW(k) - B(k)) + | AW(k) - B(k) | \right] \tag{4.44}$$

In equation (4.44)  $k$  denotes the iteration count and  $c$  is the positive correction increment. From (4.42) and (4.43) we obtain

$$\begin{aligned} W(k + 1) &= A^{\#} B(k + 1) \\ &= A^{\#} [B(k) + \delta B(k)] \\ &= A^{\#} B(k) + A^{\#} \delta B(k) \\ &= W(k) + A^{\#} \delta B(k) \end{aligned} \tag{4.45}$$

by letting

$$e(k) = AW(k) - B(k) \tag{4.46}$$

We have the following algorithm

$$W(1) = A^{\#} B(1), \quad b_1(1) > 0, \quad i = 1, \dots, m + 1,$$

$$e(k) = AW(k) - B(k)$$

$$W(k+1) = W(k) + c A^{\#} \left[ e(k) + |e(k)| \right]$$

$$B(k+1) = B(k) + c \left[ e(k) + |e(k)| \right] \quad (4.47)$$

When the inequalities  $AW \geq 0$  has a solution the algorithm converges for  $0 < c \leq 1$ . Furthermore, if all components of  $e(k)$  cease to be positive (but are not all zero) at any iteration step, it indicates that the classes are not linearly separable. The algorithm shows very good convergence property for linearly separable class but except for indicating linear inseparability it does not provide any solution for linearly inseparable case. Another shortcoming of the algorithm is that it requires the computation of generalised inverse of a large matrix. Many other different algorithms can be obtained using gradient descent technique by simply choosing different criterion functions. In fact, our ability to create new algorithms is limited only by our ability to find new meaningful criteria<sup>(78)</sup>.

#### The generalisation problem

One of the basic problems with the deterministic methods is the question of generalisation, i.e., assessment of the "goodness" of the decision function  $d(X)$ . In statistical methods the generalisation problem can be viewed as finding the probability  $P(w_1 | X)$  itself, as the statistical



distribution of the patterns is either known or is estimated. In deterministic case the generalisation problem is usually tackled by testing the effectiveness of the classifier on a set of pattern samples with known classification (test set), but not used in designing the classifier. To make this process of generalisation effective the number of pattern samples used in test should be large enough.

#### 4.4.3 Extension to Multiclass Problem

Any multiclass problem can always be effectively reduced into a collection of two class problem. The manner in which reduction is carried out depends on the separability that exists in the multiclass problem. The separability in multiclass problem can be of three main types:

Type 1 : Each class may be separable from all the rest by a single decision surface. Then we may take the decision according to

$$d_i(X) \geq 0 \rightarrow X \begin{cases} \in w_i \\ \in \text{otherwise} \end{cases} \quad (4.48)$$

This reduces the multiclass problem to  $M-1$  two class problems.

Type 2 : Each class may be separable from each other class. In this case we have  $M(M-1)/2$  two class problems and as many decision functions such that

$$d_{ij} \geq 0 \rightarrow X \begin{cases} \in w_i \\ \in w_j \end{cases} \quad (4.49)$$

The pattern X is classified as in class  $w_1$  only if

$$d_{1j}(X) > 0 \text{ for all } j \neq 1 \quad (4.50)$$

Type 3 : There exists  $M d_1(X)$  such that X belongs to  $w_1$  only if

$$d_1(X) > d_j(X) \text{ for all } j \neq 1 \quad (4.51)$$

This is nothing but a special case of Type 2 where we have

$$d_{1j}(X) = d_1(X) - d_j(X) \text{ for all } j \neq 1 \quad (4.52)$$

The way in which a multiclass problem is reduced to two class problems depends on the individual problem.

#### 4.5 POWER SYSTEM SECURITY ASSESSMENT - A PATTERN RECOGNITION PROBLEM

The operating state of a power system is characterised by the variables such as power injections, load, voltage magnitudes and angles at all the buses and power flows in transmission lines, transformers, etc. These variables can be obtained from the measurements through state estimation or power-flow solutions. An operating state of the power system may be considered as a pattern with the variables characterising the state of the system constituting the pattern vector  $X(x_1, \dots, x_n)$ .

In power system security assessment one is confronted with the question, "Is the present operating state of the system a secure state?". The answer to this question can be

provided by a classifier which is able to classify the power system operating states (patterns) into secure and insecure classes. Thus, power system security assessment may be treated as a two class pattern classification problem. In fact, one can go one step further and design classifiers which are able to classify the operating state into secure and insecure class for each contingency. These single disturbance security classifiers will point out if a particular disturbance will result in insecurity or not.

### Training Set

The design of these security classifiers requires a training set of pattern samples with known classification. Ideally the training set should consist of patterns representing all possible operating conditions of the power system. This is not possible as it will lead to infinitely large number of pattern samples, and instead of it, a large number of pattern samples representing the whole spectrum of the power system operating range is considered. Using the contingency simulation procedure (see Appendix A3.1) the security status (secure/insecure) for all the patterns, constituting the training set, is determined and the patterns are labeled secure(S) and insecure(I).

Once the training set of patterns with known classification (S or I) are available, security classifiers are designed using some learning algorithm.



#### 4.6 REVIEW OF LITERATURE ON POWER SYSTEM SECURITY ASSESSMENT BY PATTERN RECOGNITION METHOD

Pattern recognition method has not been extensively used for power system security assessment. To the best of my knowledge only a few papers<sup>(81-86)</sup> are available in the literature on this topic. One of the reasons for this may be the lack of familiarity of power system engineers with pattern recognition methodology. Here we present a short critical review of the different methodologies existing in the literature surveyed.

In reference (81) a pattern recognition method for power system transient stability studies was presented by Dewey and Tuel. They proposed a linear programming method for the design of the classifier. The main drawback of this approach is that unless the pattern classes are linearly separable the method will not give any solution, since in power system security assessment case linear separability of pattern classes can not be guaranteed this method may not work at all.

Pang, El-Abiad and others<sup>(82,83)</sup> proposed a pattern recognition method for both steady state and transient security assessment of power systems. Till now this work remains as the most important work on this topic. They proposed a feature selection method with feature ranking based on a distance criterion, this enabled them to reduce the number of features in each pattern. The classifier design was based on solution of linear inequalities by an

optimal search method<sup>(78)</sup>. Though the method works well for consistent case there is no proof that it will converge to optimal solution in inconsistent case also. Also the method is quite slow<sup>(109)</sup> increasing the computation burden. Their main contribution is for proposing single disturbance classifiers which will be able to pinpoint - "What disturbance will cause the system to go into insecure region?". Though design of single disturbance classifiers increases the computation burden but they are very helpful for on-line classification once the classifiers are available. The main drawback of the method is that it considers a constant topology network for power system which is not realistic. And so, for major network changes the classifiers have to be updated. Poncelet<sup>(84)</sup> proposed a method very similar to that of Pang et al. but considered the effect of network changes by including them as logical variables in the pattern vector (i.e., 1 for a line or branch in service and zero where it is out of service). This does increase the number of variables in the pattern vector. The limitations of the method are same as that of Pang's method.

A pattern recognition method for transient stability studies was proposed by Saito et al.<sup>(85)</sup>. The main contribution of this paper is an algorithm for selection of pattern samples to be used in the training process. The algorithm helps in choosing pattern samples near the boundary and so it is hoped that a good classifier can be obtained using these patterns. The drawback of the method is that no



criterion for choosing initial samples for this iterative process is provided, and if the separating surface obtained with initial samples are not closer to the final separating surface then the process may not converge at all. Also a perceptron method is used for classifier design; this method has the drawback that it does not converge (it merely keeps oscillating) in case the pattern samples are not linearly separable.

LaCarna and Johnson<sup>(86)</sup> propose a method of optimal rescheduling for power system preventive security using a pattern recognition method for security assessment of the system. But, they assume that either the classifier is available or it can be obtained using a method similar to that proposed by Pang et al., and thus they do not provide anything new on this topic.

#### 4.7 PROPOSED METHOD

For any security assessment method to work properly, the measurements characterizing the operating state of the system should be accurate. In general, the measurements available in practical systems may not be very accurate. The inaccuracies in measurements may be due to noise present in measuring instruments or communication channels. Sometimes grave inaccuracies can be present due to malfunctioning of the instrument etc. The noise present in the measurement can be effectively filtered using state estimation techniques. In the sequel we assume that the noise in measurements have been filtered using state estimation technique



and quite accurate measurements of the power system operating state are available.

#### 4.7.1 Pattern Vector

The measurements for the variables characterizing the power system operating state constitute the pattern vector. The variables generally used for characterizing the power system operating state and so the pattern vector are real and reactive power bus injections, the real and reactive power line flows, total real and reactive power generations and loads, spinning reserves, bus voltages etc.

#### Network topology changes

Due to planned outages of transmission equipments for maintenance or other purposes the network topology of the system changes from time to time. The network topology changes are incorporated in the pattern vector by the use of logical variables one or zero. Where one is used when the particular transmission equipment is up and zero is used when the transmission equipment is down (i.e., out of service).

#### 4.7.2 Feature Selection

In general the number of variables (measurements) characterising a power system operating state (a pattern in this case) is quite large. This makes the security classifier design complicated and requires large computational resources. Also, all the variables characterizing the system operating state may not contain useful information

for the purpose of classification. Thus, there is a need for use of some feature extraction or selection procedure to reduce the number of variables to be used for classifier design.

In power system problem it will be simpler and better to use the feature selection method. The advantage is that the number of variables to be obtained accurately will be reduced. The feature selection is achieved by ranking the variables according to the discriminant information contained in them and selecting a small number of high ranking variables as features. The discriminant information contained in any variable 'q' will be large if the difference between the means  $|\bar{q}_S - \bar{q}_I|$  is large. Unfortunately, this measure is sensitive to scaling and to obtain a good discriminant measure the difference between the means should be large with respect to some measure of standard deviation for each class.

The discriminant measure proposed here is the Fisher linear discriminant<sup>(68)</sup> and for any variable 'q' is given as

$$M_q = \frac{|\bar{q}_S - \bar{q}_I|}{\left[ (NS - 1)\sigma_{q_S}^2 + (NI - 1)\sigma_{q_I}^2 \right]} \quad (4.53)$$

Where,

S  $\longrightarrow$  Secure class

I  $\longrightarrow$  Insecure class

NS = Number of patterns in secure class

$NI$  = Number of patterns in insecure class

$\bar{q}_S$  =  $\sum_{i \in S} (q_i/NS)$  = Estimated mean value for variable 'q' for patterns in class S.

$\sigma_{q_S}$  = Estimated standard deviation for variable 'q' for patterns in class S.

and  $\bar{q}_I$  and  $\sigma_{q_I}$  are the estimated mean and standard deviation respectively for variable 'q' in class I.

Using the values of  $M$  for all variables, the variables are ranked in descending order. For features, first the top ranking variable (variable with highest  $M$ ) is considered and all variable highly correlated to it (correlation coefficient  $\geq 0.9$ ) are dropped from the variable set. Next the remaining highest variable is chosen as second feature and the process is continued till all the variables have been considered or the required number of features are obtained. The details for calculation of standard deviations and correlation coefficients is presented in Appendix 4.4.

The advantages of the method are:

- (1) The discriminant measure  $M$  in equation (4.53) is insensitive to any scaling so no data normalisation is required.
- (2) Computational effort required for calculating  $M$  for each variable is small.
- (3) Since all those variables which are highly correlated to any selected feature and so not containing any additional discriminant information are dropped,



only those variables which contain distinct information regarding the class separation are selected as features.

#### 4.7.3 Pattern Classifier (Security Classifier) Design

Once the feature selection process is over the pattern vectors in the training set can be arranged in two sets:

$$X^S = (x_1^S, x_2^S, \dots, x_{NS}^S)^t \text{ patterns of secure class} \quad (4.54)$$

$$X^I = (x_1^I, x_2^I, \dots, x_{NI}^I)^t \text{ patterns of insecure class} \quad (4.55)$$

Where,

$$x_1^S = (x_{11}^S, \dots, x_{1m}^S)$$

is a row vector consisting of  $m$  variables (features) denoting  $i$ th pattern in secure class.

Now the problem of security classifier design can be stated as:

Find a decision function (security classifier)  $d(X)$  such that

$$\begin{aligned} d(x_1) &\geq 0; \rightarrow x_1 \in S \\ d(x_1) &< 0; \rightarrow x_1 \in I \end{aligned} \quad (4.56)$$

Consider a linear decision function

$$d(X) = a^t X; \text{ where } a^t = (a_1, a_2, \dots, a_m) \quad (4.57)$$

the problem in (4.56) is equivalent to find parameter vector  $W$ , such that

$$AW > 0 \quad (4.58)$$

Where

$$A = \begin{bmatrix} x_{11}^S & \dots & x_{1m}^S & 1 \\ x_{21}^S & \dots & x_{2m}^S & 1 \\ \vdots & & \vdots & \\ x_{NS1}^S & \dots & x_{NSm}^S & 1 \\ \hline x_{11}^I & \dots & x_{1m}^I & -1 \\ \vdots & & \vdots & \\ x_{NI1}^I & \dots & x_{NI m}^I & -1 \end{bmatrix} \quad \begin{array}{l} (NS + NI)(m+1) \\ \text{matrix} \end{array} \quad (4.59)$$

$$\text{and } W = (a_1 \dots a_m a_{m+1})^t \quad (4.60)$$

The problem (4.58) is further equivalent to,

Find  $W$  such that

$$AW \geq e > 0 \quad (4.61)$$

Where  $e$  is a prespecified  $(m+1)$  vector; without loss of generality,  $e = (1, 1, \dots, 1)^t$  in the sequel.

Now (4.61) can be stated as an optimisation problem,

Find  $W$  such that

$$f(W) = \left\| (AW - e) - |AW - e| \right\|^2 \quad (4.62)$$

is minimum.

Properties of  $f(W)$ : for the consistent case  $f(W)$  is a convex function and  $f(W^*) = 0$  if  $W^*$  is a solution, for inconsistent case  $f(W)$  is strictly convex and, therefore, its minimum is global and unique (details of proof are given in Appendix 4.2).

The Fletcher-Reeves conjugate gradient algorithm<sup>(107-109)</sup> with periodic restarting along the gradient direction is used to minimise the function  $f(W)$  as defined in (4.62). Let the negative gradient direction of  $f(W)$  at any point  $W$  be denoted by

$$g(W) = -\frac{1}{4} \nabla_W f(W) \quad (4.63a)$$

$$= A^t \left[ |AW - e| - (AW - e) \right] \quad (4.63b)$$

$$= A^t p \quad (4.63c)$$

Where

$$p = \left[ |AW - e| - (AW - e) \right] \quad (4.64)$$

The algorithm is as follows:

Let  $k$  denote the iteration index,  $r$  the restarting period (normally  $r = m$  or  $(m + 1)$ ),  $\gamma$  the number of inequalities (4.58) satisfied by  $W$ , and  $W^*$  denote the solution vector.

Step 1 : Set  $k = 0$ ,

Select  $W^0$  arbitrarily

Compute  $AW^0$  and  $\gamma^0$

if  $0 < \gamma < N/2$ , set  $W^0 = -W^0$ ,  $AW^0 = -AW^0$  and

$$\gamma^0 = N - \gamma^0$$



Step 2 : If  $\gamma^k = N$ , set  $w^* = w^k$ , stop

if  $\gamma^k = 0$ , set  $w^* = -w^k$ , stop

Otherwise continue

Step 3 : Compute  $g^k$ ; if  $g^k = 0$ , stop

otherwise compute  $\lambda^k \triangleq 1/\|g^k\|^2$  and continue.

Step 4 : Compute  $\theta^k$ ;  $\theta^k = 0$  if  $k$  is an integral multiple of  $\gamma$ , otherwise  $\theta^k = 1$ .

Set  $D^k = \theta^k D^{k-1} + \lambda^k g^k$

Step 5 : Find  $\mu^k$  such that  $F(\mu) = D(w^k + \mu D^k)$  is minimised with respect to  $\mu$ .

Step 6 : Set  $w^{k+1} = w^k + \mu^k D^k$

$$Aw^{k+1} = Aw^k + \mu^k AD^k$$

Compute  $\gamma^{k+1}$

Step 7 : Set  $k = k + 1$ , go to step 2.

The algorithm for linear minimisation w.r.t.  $\mu$  in step 5 is given in Appendix 4.3.

The advantages of the algorithm are:

- 1) The algorithm gives optimum solution in both consistent (linearly separable) and inconsistent (linearly inseparable) cases.
- 2) In the inconsistent case, which may generally be present in power system security assessment application, the algorithm converges to the 'best' solution i.e., a

solution in which maximum number of inequalities are satisfied, because in this case the objective function is strictly convex and so its minimum is global and unique.

- 3) Computation of generalised inverse of large matrices, as required by the least mean square error algorithm<sup>(80)</sup>, is no longer required. This reduces the computation burden by a great extent. Where the LMSE algorithm merely points out the inconsistency of the linear inequalities, if it is the case, this method provides an optimal solution for this case also.
- 4) The method is particularly well suited for large problems, which will be a general case in power system security assessment problems, as it does not require storage of any matrices.

#### 4.7.4 The Algorithm

The algorithm for the proposed security assessment scheme is as follows:

- Step 1 : For different power system operating states, covering the whole spectrum of the system operating range, obtain the pattern vectors with known classifications by using contingency evaluation method.
- Step 2 : Arrange the pattern vectors into two sets,  $S_S$  for patterns in secure class and  $S_I$  for patterns in insecure class.

- Step 3 : Randomly select large number of patterns from  $S_G$  and  $S_I$  to form the training set ( $S_{TR}$ ). The rest of the patterns form the test set ( $S_{TS}$ ).
- Step 4 : Using the pattern vectors in  $S_{TR}$  compute the discriminant measure  $M_i$  for each variable ( $x_i$ ) in the pattern vector using the equation (4.53).
- Step 5 : Starting with variable with largest value of  $M$ , rank the variables ( $x_i$ ) in descending order according to the values of  $M_i$ . Set  $k = 1$ .
- Step 6 : Compute the correlation coefficient  $C_{ij}$  for all variables ( $x_j$ ) ranked  $k + 1, \dots, n$ , w.r.t. the variable  $x_i$  ranked  $k$ .
- Step 7 : Set  $M_j = 0$ ; if  $C_{ij} \geq 0.9$ . Otherwise continue.
- Step 8 : Set  $k = k + 1$ ; if  $k < n$ . Otherwise go to step 10.
- Step 9 : If  $M_k = 0$ , go to step 8. Otherwise go to step 6.
- Step 10: Choose a small subset of  $m$  variables with highest  $M_s$  as the features.
- Step 11: Using the  $m$  variables chosen as features in step 10 form the matrix  $A$  (defined in equation (4.59)) with patterns in the training set ( $S_{TR}$ ).
- Step 12: Using the training algorithm described in section 4.7.2 obtain the optimal security classifier  $W^*$ .
- Step 12: If the classification efficiency (% correct classification) with  $W^*$  is not adequate ( $< 90\%$ ), set  $m = m + 1$ , go to step 10. Otherwise stop.



For checking the effectiveness of the security classifier obtained in step 12 it is used on the test set patterns as follows:

Step 14 : Using the features obtained in step 10 form the vector  $X_1 = (x_{11}, \dots, x_{1m}, 1)$  for the pattern vector  $i$  in the test set.

Step 15 : Compute the security function value

$$D(W) = X_1^t W^*$$

If  $D(W) \geq 0$  and the  $i$ th pattern is from secure class, the classification is correct. Otherwise it is incorrect.

If  $D(W) < 0$  and the  $i$ th pattern is from insecure class, the classification is correct. Otherwise it is incorrect.

The % of the correct classifications by  $W^*$  on test set patterns provide the generalized efficiency of classification.

#### 4.7.5 Single Disturbance Classifiers

The security classifier  $W^*$  obtained by the above algorithm is able to classify any power system operating state into secure and insecure state by computing the value of security function  $d(W) = X^t W^*$  which requires only  $m$  multiplication operations and so requires very little computation time. In fact it is possible to go one step further and design one security classifiers ( $W_1^*$ ) for each contingency. These are called single disturbance security classifier and aid in identifying the

contingencies which will force the given power system operating state into insecure state. As will be seen in Chapter 5, these single disturbance classifiers are of great help in security control applications.

#### 4.7.6 Security Control Index

Many different types of global security indices have been proposed (5,65). These indices provide a measure of the general health of the system for the given operating condition. These global security indices can be used as a measure for initiating security control actions. The main drawback of the methods using security indices is the large computational time involved in computing these indices, also these indices do not provide any information about what particular contingency causes insecurity and if so how severe is its effect.)

Here a security control index is proposed which overcomes the disadvantages of the global security indices proposed by Patton and others. The security control index (SCI) is computed for each contingency and provides a measure of severity caused by the particular contingency concerned. The magnitude of  $d(W)$  for any operating condition (pattern) indicates the distance of the pattern from the security classifier surface. For insecure operating condition  $d(W)$  is negative and  $|d(W)|$  gives a measure of severity of the operating condition from the secure operating condition threshold. Thus, if for any particular contingency  $i$ , the system becomes insecure ( $d(W_i) < 0$ ) but  $|d(W_i)|$  is small, then it may not



be necessary to take control actions if the probability of occurrence of that contingency is also small. Based on this argument the security control index for any contingency  $i$  is defined as

$$SCI_i = -d(W_i)P_i \quad (4.65)$$

Where,  $d(W_i)$  is the value of security function for  $i$ th contingency and  $P_i$  is the probability of occurrence of the contingency  $i$  (failure rate for the equipment causing  $i$ th contingency).

If  $SCI_i > \overline{SCI}_i$  ( $\overline{SCI}$  is the predetermined threshold value of  $SCI_i$  and depends on the system as well as the degree of reliability and quality of service required) then security control actions need to be initiated as severity level for the contingency is greater than the tolerance acceptable, otherwise security control actions can be bypassed as the severity caused by the contingency is lower than the acceptable tolerance. The advantage of the proposed SCI is the ease in its computation (it requires only one multiplication operation in case where equipment failure rate is available) and its effectiveness in security control applications.

### RESULTS AND DISCUSSIONS

The power system steady state security assessment method using pattern recognition technique as proposed in section 4.7 is programmed and tested using IBM1130 system on three test systems: (i) 5 Bus/7 line/2 generator system, (ii) 8 Bus/



14 line/4 generator system (iii) 14 Bus/20 line/6 generator system. The details of the system parameters and loading data is given in Appendix 1. Details of the pattern generation and test results are as follows:

(a) Generation of Patterns for Training Set and Test Set:

For generation of patterns for training and test set, the system operating conditions over the complete operating spectrum (minimum system load (0.8 base load) to the maximum system load (1.25 times system peak load) is considered. Different loading conditions of the system is considered by assuming the loads to be conforming (i.e., loads at all the buses remain a fixed percentage of total system load) and total system load demand changing in discrete steps of 2-5%. To account for the non-conformity of loads, different system load levels with random loadings at individual load buses were also considered. For each of these loading conditions different unit commitments were considered with the constraint that system spinning reserve is always greater than the generating capacity of the largest generator in operation. This constraint made sure that even for the outage of largest operating generator a feasible steady state operating condition is obtained. For the given loading conditions and unit commitments, economic load dispatch (described in Chapter 2) was conducted to obtain the voltages at the buses and power flows in the lines for each case (pattern). The system operating condition data formed the pattern vectors. Outage studies for all line

outage and generator outages (considering one outage at a time) is carried out using fast decoupled load flow to find out the security status (secure/insecure) for each pattern. These patterns with known classification formed the complete pattern set, out of which some secure class patterns and some insecure class patterns were randomly selected to form the training set, whereas the rest of the patterns formed the test set.

For the 14-bus system two more network topologies with

- (i) planned outage of one circuit of transmission line between bus 1 and 2 (simulated by doubling the impedance of the transmission line), and
- (ii) planned outage of transmission line between bus 4 and 9, are considered by including 100 more patterns (54 for case (i) and 46 for case (ii)) with known classifications to account for the network topology changes.

The variables considered to constitute the pattern vectors representing the power system operating states include

- (i)  $V, \theta$ ; the voltage magnitude and angle at each bus,
- (ii)  $P_1, Q_1$ ; real and reactive power load at each bus,
- (iii)  $P_g, Q_g$ ; real and reactive power generation at generating buses
- (iv)  $P_f, Q_f$ ; real and reactive power flow in each transmission line,



- (v)  $P_{SR}, Q_{SR}$  ; real and reactive power spinning reserves in the system.

To account for the network topology changes (in the 14 bus system) logical variables for each topology change are also included. The details for the pattern vectors for each system is shown in Table 4.1.

For each system the variables in the pattern vector are ranked according to the discriminant measure  $M$  and highly correlated variables (as discussed in section 4.7.2) were discarded. The remaining variables formed the features to be used. Table 4.2 shows the uncorrelated variables selected for use as features along with their values of  $M$ .

For the training process to design the classifier the top ranking  $m$  features are used. Table 4.3 shows the classifier efficiency with respect to value of  $m$  (number of features selected). The column for the K-L method in the Table 4.3 shows the classifier efficiency when the features are extracted using the Karhunen-Loeve' expansion method. It can be seen from the table that the proposed feature selection method is much superior to the K-L expansion method as far as the classification efficiency is concerned. This is mainly because the proposed feature selection method is based on discriminant information w.r.t. class separability whereas the K-L method is more concerned about the fidelity of the data. It is also seen from the Table 4.3 that the number of features required for quite high classification efficiency ( $>95\%$ ) is usually quite small. This again proves



the effectiveness of the feature selection method. The exact number of features to be selected depends on the system under consideration and can be found by trial and error only. This is not a great disadvantage as the whole process has to be done off-line. The results of the classification efficiency on the test set with classifier designed by training process is shown in Table 4.4.

For obtaining single disturbance classifiers for the 14 bus system, 10 line outages and one generator outage contingencies for fixed network topology case and 12 line outages and 2 generator outage for network with topology changes were considered, the others outages led to very few cases of insecurity. The same thirteen features which are used for security assessment are used for single disturbance classifiers also. In fact for each single disturbance classifier design a separate set of features can be selected using the feature selection scheme but as the same set of features gave quite good results, the feature selection was not carried out for each single disturbance classifier. The results for single disturbance classifiers on training set as well as on test set are shown in Table 4.5.

For calculating the security control index for the base case as well as particular contingencies, the probability of occurrence of any particular contingency was arbitrarily chosen as  $q_1 = 1 \times 10^{-3}$  (this corresponds to an outage rate of 0.365 days/year). The probability of occurrence of any contingency

when the system is operating normally is found using the relationship

$$q_0 = 1 - \prod_{i=1}^n (1 - q_i)$$

where  $q_i$  is the probability of occurrence of  $i$ th contingency and  $n$  is the number of postulated contingencies considered. For the 14 bus system, twelve contingencies are considered giving the value of  $q_0$  (probability of occurrence of a contingency) equals 0.01489.

For the patterns in insecure class in the training and test set for base case (o) and individual contingencies (i) the security control index value  $SCI_i$ ,  $i = 0, 1, \dots, n$ , were computed and compared with the contingency evaluation results of the patterns. The highest value of  $SCI_i$  for each case (base case and individual contingencies) for which the branch overflow violations were limited to 5% and/or bus voltage violations not more than 0.02 pu, was found and set as the threshold value of  $SCI_i$  ( $\overline{SCI}_i$ ) for that case. The values of  $\overline{SCI}_i$ s are presented in Table 4.6.

Details of Pattern Samples

	5-BUS SYSTEM	8-BUS SYSTEM	14-BUS SYSTEM
Maximum load demand in system ( $P_D^M$ )MW	200	2,000	500
Minimum load demand in system ( $P_D^m$ )MW	50	400	150
Number of load levels considered:			
(i) Conforming loads ( $L_c$ )	31	17	36
(ii) Non-conforming loads ( $L_n$ ) (randomly selected)	40	20	19
(iii) Total ( $L = L_c + L_n$ )	71	37	55
Number of unit commitments considered for each load level ( $U_c$ )	2	4	6
Number of operating conditions with intact network ( $N_1 = L \times U_c$ )	142	148	330
Number of topology changes considered (excluding the intact network topology)	-	-	2
Number of operating conditions with topology changed	-	-	100
Total number of operating conditions (patterns) considered	142	148	430
Outage studies			
(i) No. of network outages considered	7	14	19
(ii) No. of power outages considered	2	4	6
(iii) Total number of outages considered	9	18	25



Number of patterns in secure class	62	61	169
Number of patterns in insecure class	80	87	261
Training set (patterns randomly selected):			
(i) Number of patterns in secure class	50	50	100
(ii) Number of patterns in insecure class	50	50	100
(iii) Total number of patterns in training set	100	100	200
Test set:			
(i) Number of patterns in secure class	12	11	69
(ii) Number of patterns in insecure class	30	37	161
(iii) Total number of patterns in test set	42	70	230
Number of variables in pattern vector	42	48	112*

\*includes two logical variables used for the two network topology changes considered.

TABLE 4.2

Results of Feature Selection(a) 5 Bus system

Sl No.	Variable*	Discriminant measure M
1	$\theta_3$	264.5
2	$P_5^1$	211.3
3	$P_1^1$	158.7
4	$V_2$	143.5
5	$Q_4$	108.2
6	$\theta_4$	94.6
7	$P_{L1}$	78.2
8	$P_5^1$	32.1
9	$\theta_1$	11.5
10	$P_{G4}$	3.01
11	$P_7^1$	1.03

(b) 8 Bus system

Sl No.	Variable*	Discriminant measure M
1	$P_3^1$	256.30
2	$V_1$	198.72
3	$\theta_4$	173.96
4	$Q_{G7}$	139.59
5	$V_5$	85.45
6	$P_{12}^1$	59.28
7	$P_7^1$	45.91
8	$P_9^1$	41.76
9	$\theta_6$	19.40
10	$Q_{G8}$	7.53
11	$V_4$	7.51
12	$\theta_2$	4.88
13	$P_{G8}$	1.661

(c) 14 Bus System

(i) Fixed Topology

Sl. No.	Variable*	Discriminant Measure M	Sl. No.	Variable*	Discriminant Measure M
1	$V_5$	187.69	16	$P_{10}^1$	5.88
2	$P_{12}^1$	55.17	17	$P_{11}^1$	5.72
3	$P_4^1$	41.84	18	$P_{G4}$	5.37
4	$V_2$	40.85	19	$P_8^1$	4.91
5	$P_{13}^1$	39.00	20	$Q_{G8}$	4.72
6	$\theta_8$	34.67	21	$P_{G13}$	3.41
7	$V_{10}$	31.52	22	$P_{G3}$	3.03
8	$Q_{G4}$	27.10	23	$P_{G7}$	2.19
9	$P_2^1$	13.54	24	$P_{20}^1$	1.52
10	$\theta_7$	10.89	25	$Q_{G13}$	1.52
11	$P_9^1$	10.31	26	$P_{L5}$	1.43
12	$P_5^1$	8.54	27	$\theta_4$	0.85
13	$P_3^1$	8.33	28	$Q_{G7}$	0.73
14	$P_{14}^1$	6.36	29	$P_{17}^1$	0.34
15	$P_{G8}$	6.36	30	$Q_{G14}$	0.06



## (ii) Including Topology Changes

Sl No.	Variable*	Discriminant Measure M	Sl No.	Variable*	Discriminant Measure M
1	$V_5$	138.32	16	$V_{10}$	8.42
2	$P_{12}^1$	111.12	17	$P_{10}^1$	6.37
3	$L_1$	98.64	18	$P_{L11}$	6.04
4	$P_4^1$	73.21	19	$P_{G4}$	6.00
5	$P_{13}^1$	54.17	20	$P_8^1$	5.08
6	$\theta_9$	45.62	21	$Q_{G8}$	4.87
7	$L_2$	41.38	22	$P_{L12}$	4.53
8	$\theta_7$	41.02	23	$P_{G3}$	4.21
9	$Q_{G4}$	37.87	24	$P_{G7}$	4.13
10	$P_2^1$	31.08	25	$P_{20}^1$	3.98
11	$P_9^1$	29.22	26	$Q_{G13}$	3.47
12	$P_5^1$	29.21	27	$P_{L5}$	2.10
13	$P_3^1$	27.36	28	$\theta_4$	1.73
14	$P_{14}^1$	16.24	29	$Q_{G3}$	1.08
15	$P_{G8}$	13.10	30	$P_{17}^1$	0.07
			31	$P_{L6}$	0.03

\*  $P_i^1$  = real power flow in line i,

$P_{G1}, Q_{G1}$  = real and reactive power generation at bus 1

$P_{Li}$  = real power load at bus i

$V_1, \theta_1$  = Voltage magnitude and angle at bus 1

$L_1, L_2$  = Logical variables for planned outage class (i) and (ii) respectively.

TABLE 4.3

## (a) 5 Bus system

Number of features used	Classifier efficiency using feature selection by	
	Proposed method	K-L expansion method
5	72.0	70.0
7	87.0	72.0
8	98.0	78.0
11	98.0	83.0
20	**	87.0
40*	96.0	96.0

## (b) 8 Bus system

Number of features used	Classifier efficiency using feature selection by	
	Proposed method	K-L expansion method
5	65	68
8	88	72
10	97	71
11	97	78
13	97	74
30	**	90
70*	96	96

\* All variables used as features

\*\* Number of features used exceeds the number of incorrelated features selected by the proposed method.

(c) 14 Bus system

(i) Fixed topology

Number of features used	Classification efficiency using feature selection by	
	Proposed method	K-L expansion method
5	67.0	65.0
8	78.5	73.0
11	91.5	86.5
13	98.5	87.5
15	98.5	87.0
21	98.0	91.0
30	98.0	91.5

(ii) With topology changes

Number of features used	Classification efficiency using feature selection by	
	Proposed method	K-L expansion method
5	71.5	63.5
8	79.0	68.5
11	90.5	79.5
13	97.5	90.5
15	97.5	91.5
21	97.5	91.5
30	97.5	91.5



Results for Classification Efficiency of the Classifier Designed by the Proposed Method on Test Set.

## (a) 5 Bus system

No. of features used	No. of secure samples misclassified as insecure	No. of Insecure samples Misclassified as secure	Total No. of Misclassification	Classification efficiency (%)
8	1/12	1/30	2/42	95.2%

## (b) 8 Bus system

Number of features used	No. of secure samples misclassified as insecure	No. of Insecure samples Misclassified as secure	Total No. of Misclassification	Classification efficiency (%)
10	2/11	0/37	2	95.8%

## (c) 14 Bus system

	Number of features used	No. of secure samples misclassified as insecure	No. of Insecure samples misclassified as secure	Total No. of misclassification	Classification efficiency (%)
(i)*	13	2/36	2/94	4	96.9%
(ii)*	13	6/69	4/161	10	95.6

\* (i) Fixed topology  
(ii) With topology change

Single Disturbance Classifiers

14 Bus system

(a) Fixed topology

Single disturbance classifier	Training set				Test set			% Correct classification
	No. of secure patterns misclassified as insecure	No. of insecure patterns misclassified as secure	Total misclassified	% Correct classification	No. of secure patterns misclassified as insecure	No. of insecure patterns misclassified as secure	Total misclassified	
Line 2-3 Out	2/143	1/57	3/200	98.5	1/88	2/42	3/130	97.7%
Line 3-4 Out	1/157	0/43	1/200	99.5	0/102	1/28	1/130	99.2%
Line 4-5 Out	1/132	1/68	2/200	99.0	3/76	0/54	3/130	97.7%
Line 6-12 Out	0/113	1/87	1/200	99.5	2/73	2/57	4/130	96.9%
Line 7-9 Out	0/142	0/58	0/200	100.0	2/58	3/72	5/130	96.15%
Line 9-10 Out	2/108	0/142	2/200	99.0	3/77	4/53	7/130	94.6%
Line 9-14 Out	0/122	1/78	1/200	99.5	3/68	0/62	3/130	97.7%
Line 10-11 Out	0/152	0/48	0/200	100.0	0/83	0/47	0/130	100.0%
Line 12-13 Out	0/136	1/64	1/200	99.5	1/98	0/32	1/130	99.2%
Line 13-14 Out	1/119	2/81	3/200	98.5	1/47	3/83	4/130	96.9%
Gen. 3 Out	1/163	0/37	1/200	99.5	2/102	0/28	2/130	98.5%



(b) With topology changes

Single disturbance classifier	Training set				Test set			
	No. of secure patterns misclassified as insecure	No. of insecure patterns misclassified as secure	Total misclassification	% Correct classification	No. of secure patterns classified as insecure	No. of insecure patterns misclassified as secure	Total misclassification	% Correct classification
Line 1-5 Out	3/132	2/78	4/200	98.0	2/162	3/68	5/230	97.8
Line 2-3 Out	2/146	1/54	3/200	98.5	1/159	2/79	3/230	98.7
Line 3-4 Out	0/163	2/37	2/200	99.0	3/154	5/76	8/230	96.5
Line 4-5 Out	3/152	2/48	5/200	97.5	6/147	3/83	9/230	96.1
Line 4-7 Out	1/162	0/38	1/200	99.5	3/164	1/66	4/230	98.3
Line 6-12 Out	1/133	0/67	1/200	99.5	2/128	2/102	4/230	98.3
Line 7-9 Out	0/136	1/64	1/200	99.5	5/154	1/76	5/230	97.8
Line 9-10 Out	2/115	2/85	4/200	98.0	8/104	3/126	11/230	95.2
Line 9-14 Out	4/125	1/176	5/200	97.5	3/122	1/108	4/230	98.3
Line 10-11 Out	1/153	1/47	2/200	99.0	4/163	3/67	7/230	97.0
Line 12-13 Out	3/133	1/67	4/200	98.0	11/201	2/29	13/230	94.3
Line 13-14 Out	1/128	2/78	3/200	98.5	6/157	2/73	8/230	96.5
Gen 3 Out	3/148	4/52	7/200	96.5	10/191	4/39	14/230	93.9
Gen 13 Out	1/165	1/35	2/200	99.0	4/171	1/59	5/230	97.8



Table 4.6

Threshold values of Security Control Index

Outage Contingency	$\overline{SCI}$
Base Case	0.1883
Line 1-5 Out	$2.78 \times 10^{-3}$
Line 2-3 Out	$4.27 \times 10^{-3}$
Line 3-4 Out	$0.88 \times 10^{-3}$
Line 4-5 Out	$0.92 \times 10^{-3}$
Line 4-7 Out	$7.32 \times 10^{-3}$
Line 6-12 Out	$4.78 \times 10^{-3}$
Line 7-9 Out	$21.56 \times 10^{-3}$
Line 9-10 Out	$11.37 \times 10^{-3}$
Line 9-14 Out	$0.96 \times 10^{-3}$
Line 10-11 Out	$3.71 \times 10^{-3}$
Line 12-13 Out	$0.69 \times 10^{-3}$
Line 13-14 Out	$5.10 \times 10^{-3}$
Gen 3 out	$2.36 \times 10^{-3}$
Gen 13 Out	$3.47 \times 10^{-3}$

## Conclusions

In this chapter a pattern recognition method for power system security assessment is proposed. The pattern recognition process used is a two stage process. The first stage is an off-line 'learning' or 'training' process where the security classifiers are obtained using a large set of training set samples. The bulk of the simulation effort required goes in generation of the training samples. In practical systems the simulation effort required for generation of training set samples will be reduced considerably as a large number of training samples can be obtained directly from the system planning records.

Network topology changes have been incorporated by including logical variables in the pattern vectors. In order to make the training process simpler and economical the number of variables in the pattern vector is drastically reduced using a feature selection algorithm. The algorithm is very effective as can be seen from the numerical results.

The training process for the security classifier design is formulated as a solution of linear inequalities which is solved by transforming it into an optimisation problem. The choice of the quadratic objective function which is strictly convex when patterns are not linearly separable ensures that an optimal classifier is obtained in all cases. The Fletcher-Reeves conjugate gradient algorithm is used as it does not require inversion and storage of any matrices. This

makes it possible to solve the large pattern recognition problem even on small computers. Single disturbance classifiers for individual contingencies are designed. These aid in identifying the contingencies which will force the system into emergency state. The very high classification efficiency obtained for both training and test set samples for all classifiers designed indicate the effectiveness of the method.

The second stage of the pattern recognition process is the use of the security classifiers in classifying unknown operating states. This process requires only  $m$  multiplication operations (where  $m$  is the number of features used) and so requires negligible computation time and is well suited for on-line applications. The high classification efficiency of the classifiers designed on the test set samples indicates the high reliability of the classifiers in classifying unknown power system operating states.

A security control index (SCI) is proposed. Separate index is computed for each contingency and provides a measure of the severity caused in case the contingency occurs. The computation of SCI is very simple and requires only one multiplication operation. The security control index can be very effectively used for on line security control applications.



## CHAPTER 5

SECURITY CONTROL5.1 INTRODUCTION

The aim of the electric utility service is to continuously fulfil the system load demand of the system at acceptable level of voltage and frequency. The unscheduled outages produce very severe burden on operators trying to fulfil this function. System security assessment function provides information about the security status (insecure/secure) of the system operating condition in the face of uncertain future (unschedule outages). In case where an unforeseen contingent event indeed causes a transition into emergency state, we would like to find out what control actions can be taken to bring back the system to secure state. Although the design and planning of the system is done in an exhaustive manner, the complexity and variety of conditions into which a system can find itself requires that these decisions be made on-line.

In situations where the number of contingent events and number of possible states are small, the operator of a system can rely on his past experience. In large and complex system this may not be possible as well as any wrong decision may lead to catastrophic consequences. Thus, accurate means for determination of security control action are very important. In the last decade this problem has generated much interest among the power system researchers

and large number of papers are available in the literature<sup>(87-103)</sup>. In this chapter we present a short review of the existing methodologies and algorithms presently available in the literature. Then, we present a method of security control <sup>based on</sup> linearized power system model using dual-simplex method of linear programming (LP) with a reduced basis formulation to minimise the storage requirements. In the end some test results are presented to show the effectiveness of the method.

## 5.2 SECURITY CONTROL PROBLEM

The security control problem consists of: (i) looking if any danger to the system (abnormal operating conditions, e.g., over loading of lines, bus voltages beyond limits etc.) exists; and if so, (ii) finding actions to avoid this danger by well defined and accurate means; and then, (iii) performing this action manually or automatically.

Depending on the operating conditions and strategies used, security control may be defined at two levels;

- (i) emergency control,
- (ii) preventive control.

At both the control levels security control may be applied for steady state or transients depending on whether the danger exists due to static or transient conditions. As already stated in this thesis we confine ourselves to steady state conditions.



Emergency control function determines and executes a corrective action to overcome the danger existing in the intact system. Thus, the problem of emergency control is to reschedule the controllable quantities in the system in such a way that the new steady state operating condition is devoid of any danger. The rescheduling of controllable quantities is usually done in an optimal manner generally according to an economic objective. In overconstrained cases where danger can not be overcome by rescheduling process, some loads have to be shed using some optimal criterion which minimizes the amount of load shed or using some priority routine for loads to be shed.

Preventive control function determines and executes a corrective action to overcome the danger which would appear in the system in case of a contingency (unscheduled outage). The main aim of preventive control is to avoid probable cascade tripping and complete failure of the system on occurrence of the contingent event. Preventive control function is similar to emergency control but is much broader in scope as it covers probable events which may occur in near future and force the system into emergency state that is, it also includes security assessment function. Also, in case of preventive control if the danger due to probable events cannot be overcome by rescheduling the controllable quantities and preventive load shedding has to be resorted to, then under these conditions it may be preferable not to use preventive control at all but to



build a "contingency plan", i.e., to prepare a set of actions to be taken if the contingency really occurs. This approach we feel is more prudent as it avoids, (i) unnecessary changes in system operating condition from the optimal schedule as well as (ii) unnecessary load shedding in extreme cases, for a probable condition (failure) which may not appear at all.

### 5.3 REVIEW OF SECURITY CONTROL METHODS

The problem of security control, for both emergency control and preventive control, is the location of the limiting transmission lines and bus voltages, and to determine the generation and voltage schedule as well as location and amount of load shedding (if any) that is necessary to overcome the line overloads and abnormal bus voltages. A wide spectrum of fast computational approaches have been developed for security control applications. Though most of the approaches have been developed mainly for emergency control but with the incorporation of a fast security assessment and contingency evaluation algorithms they can as well be applied for preventive control also.

The most general approach to determining optimal security control action is to include the security constraints (line flow limits and voltage limits for both intact and outage cases) in the constraint set of the optimal power flow (OPF) programme. In reference (20) the security constraint was already included in the form of constraint on the amount of spread in voltage angles for transmission lines.

In reference (87) the OPF is formulated with security constraints for power flow in transmission lines and bus voltage constraints for both intact and outage conditions. The exact system model and non-linear cost curves for generators were considered making it a non-linear optimisation problem with equality and inequality constraints (including security constraints). The method of generalized reduced gradient (GRG) was used to solve this problem. The method is quite general but suffers from large computation time and storage requirements. The method of reference (88) is similar in approach but tries to reduce the problem size by considering only those functional inequality constraints, which are violated, to be solved by reduced gradient algorithm, which is used successively along with the load flow algorithm.

Stott and Alsac<sup>(89)</sup> decouple the real and reactive power problems and use the reduced gradient algorithm of Dommel and Tinney<sup>(27)</sup> for the solution of decoupled OPF. Security constraints were handled by penalty function to further reduce the storage requirements. The method suffers from the same disadvantage of choosing proper acceleration factor as that of reference (27). The solution of security constrained OPF with exact nonlinear formulation is quite difficult because of large number of constraints involved and the methods suffer from long computation times, a lack of reliability in convergence characteristics and large storage requirements. This has prompted many researchers to simplify the problem by either using linearized models or



reduce the exact problem.

In reference (90) the emergency control problem with line power flow constraint is formulated as a real power optimisation problem only, neglecting reactive power flows as well as constant bus voltage magnitudes are assumed. The problem is further simplified by using linearized P- $\theta$  decoupled load flow equations to impose line flow limits. This reduces the problem to a formulation in which cost function is nonlinear and the constraints are linear. A gradient projection technique is used to solve this particularly structured problem. The algorithm is much faster than the general OPF methods but only real power solutions are obtained in this case.

Of late linear programming (LP) techniques are becoming popular for security control applications. The L.P. algorithm has the advantage of assured convergence thus making them more dependable. The major difficulty in application of L.P. approach in its classical formulation to the security control problem is the prohibitively large storage requirement and/or large computation time for realistic size power systems. Also the system model has to be linearized as well as the objective function (usually the cost function) has to be linear, thereby reducing the accuracy of the results. In the past decade a number of specialized LP algorithms have been developed which make use of the special structure of the problem to reduce both the storage requirements as well as the computation time. Some of the



methods can use even piecewise linear cost curves making them quite accurate.

Wells in reference (91) presents a modified simplex method for solution of transmission line real power overload alleviation problem, which is formulated as a dual L.P. problem. The method uses system linearisation using d.c. load flow matrix and piecewise linear cost curves for the real power injection sources. The optimisation objective used is the system operating cost. In reference (92) Shen and Laughton present a similar formulation (dual L.P. solved by revised simplex method) for real power security constrained optimisation. The system linearisation is done about the initial optimal operating point and uses piecewise linear cost curves. To reduce storage requirements and computation time only those constraints which are effective are included in the formulation explicitly other inequality constraints are handled implicitly. References (93-95) present similar formulations except for minor modifications in computation strategies and/or objective functions used.

In reference (96) a successive LP approach with primal simplex formulation and successive linearisation of the system model about the operating points is used. The method is applied for rescheduling of real power for both emergency and preventive control. The drawbacks of the method are large storage requirement for storing all constraint equations and large computation time as basis matrix has large number of rows (as many as constraint equations).

Also initial feasible solution has to be found (phase I of primal simplex method).

Wollenberg and Stadlin<sup>(97)</sup> present a security control algorithm for real time application. The functional inequality constraints for line flows for both intact and outage cases are constructed using a sensitivity relation and are used to penalise the objective function. The solution process is based on Dantzig-Wolfe decomposition technique. Since functional inequality constraints are implicitly taken care of, the storage requirement is reduced but because of poor convergence characteristics of Dantzig-Wolfe decomposition technique<sup>(105)</sup>, the solution time is more. The problem is initially formulated for real power control only but method for extending it for reactive power and voltage control is also presented.

In reference (98) Ejebe et.al. present a load curtailment algorithm for evaluating the ability of the power transmission system to withstand generation and transmission contingencies. An upper bounding LP is used to determine the optimum phase shifter settings, generation dispatch and minimum load curtailment required to suppress the overloads. An iterative constraint search (ICS) technique is used to form a reduced set of only relevant line flow constraints to be included in the LP optimization process. This reduces the storage requirement and computation time considerably. The objective used is to minimise load shedding and penalise generation shifts from nominal schedule. The main drawback



of the method is that only real power case is considered and reactive power and voltage effects are completely neglected making this algorithm inadequate for real time emergency control in cases where voltage profile and scheduling of var sources are important. This drawback was overcome in the algorithm of reference (99) where both real and reactive power injections are used as control variables. A distinguishing feature of this algorithm is the consideration of sensitivity of loads to voltage magnitude. It enables this algorithm to select voltage reduction as an alternative to load shedding or real power allocation. The storage requirement is reduced by storing the basis matrix in factored form by which the inverse need not be computed explicitly and a reordering scheme is used to preserve the sparsity during updating process. The algorithm can handle piecewise linear objective functions but this increases the number of variables (as one variable is added for each linear segment of the objective function) thus offsetting to some extent the advantage gained by sparsity oriented technique, in terms of storage and computation time. Also since it uses a primal LP formulation so the initial basic feasible solution (phase I of LP simplex method) has to be found.

7. Pai and Parajothi in reference (100) proposed a dual LP formulation for security control problem. They use a decoupled load flow formulation for system model with linearization about the initial operating point using



sensitivity matrix. The contingency constraint relations are also obtained using sensitivity matrices. To increase the accuracy of the LP formulation it is coordinated with decoupled load flow after each LP optimisation. The main drawback of the method is the use of single segment linear objective function. This may cause many of the control variables to be forced to their minimum or maximum limits. Also the computation time is greatly increased due to the computation of sensitivity matrices at each successive iteration point.

In reference (101) Stott and Hobson present a dual LP method for real power security control which was extended to reactive power voltage control in (102) by Hobson. The main feature of this algorithm is the use of product form of inverse for the basis matrix updating process to reduce the storage requirement as well as the computation time. This method has the advantage of reduced storage and computation time only if the number of iterations are small, as the E-strings for the product form of inverse for basis matrix goes on increasing with each updating of the basis matrix.

In reference (103) Stott and Marinho present a dual LP method for security control where the problem formulation is similar to that of reference (101). The algorithm is not a standard LP algorithm but it combines the dual revised simplex method with separable programming method to handle piecewise linear generator cost curves. The

linearized system model is obtained using d.c. load flow matrix which is triangularised and stored once for all reducing the computing time. The programme exploits the problem structure to reduce the storage as well as computation time. Multisegment piecewise linear cost curves can be handled without increasing the number of variables in the basis thus reducing the computation time by a large extent as well as increasing the accuracy of the LP optimisation. Till now this method appears to be the most suitable method for real-time application. The only drawback is that the method is applied only to real power control neglecting the reactive power and voltage constraints which can be very important in many practical cases. The method proposed here while being somewhat similar to that of reference (103) extends this method to reactive power rescheduling also and the accuracy of the method is greatly enhanced by coordinating it with fast decoupled load flow.

#### 5.4 PROPOSED METHOD

For any security control algorithm to be effective for real-time application, it should be highly reliable, fast and accurate. In general security constrained optimal power flows using nonlinear programming techniques provide accurate solutions but suffer from large computation times as well as unreliable convergence characteristics, which is very much problem dependent. The linear programming (LP) methods for security control are reliable and computationally



quite fast when programmed properly, but suffer from the drawback of inadequate accuracy of solution. The linearization of the system network model about the operating point is generally adequate for security control applications<sup>(101)</sup>, but the main factor attributing inadequate solution accuracy in LP methods is the brute force linearization of the cost functional in the objective function by a single segment linear function.

The proposed method for security control is based on a LP formulation in which the security control problem for real and reactive powers are decomposed using a decoupled formulation. The system model is linearized using fast decoupled power flow matrix and the objective function consists of multisegment piece-wise linear cost functionals. The accuracy of the solution is greatly enhanced by coordinating the LP solution with fast decoupled load flow. The algorithm used is not a standard LP formulation but is a specially programmed algorithm which combines many advantageous aspects of dual revised simplex formulation, separable programming, relaxation methods and iterative constraint search algorithms and fully exploits the particular problem structure to reduce the computer storage requirements as well as computational time.

#### 5.4.1 Statement of the Problem

Given an insecure operating condition it is desired to eliminate the insecurity by finding adjustments ( $\Delta u$ ) of the control devices - generators, transformer taps, phase



shifters, load shedding etc. in an optimal manner. In other words

$$\text{Min } F(\Delta U) = \sum_{i=1}^n C_i(\Delta u_i) \quad (5.1)$$

$C_i$  = cost coefficient associated with control variable  $i$ ;  
 $n$  = number of control variables.

subject to

$$(1) \quad u_i \text{ min} \leq u_i^0 + \Delta u_i \leq u_i \text{ max} \quad (5.2)$$

double sided limits on control devices.

(ii) Constraints on line flows and bus voltages for the intact network (emergency constraints)

$$h_{\text{min}}^0 \leq (h^0(x^0, u^0, p^0) + \Delta h^0) \leq h_{\text{max}}^0 \quad (5.3)$$

and (iii) Security constraints (preventive) imposed on line flow and bus voltages due to postulated contingencies

$$h_{\text{min}}^k \leq (h^k(x^k, u^0, p^k) + \Delta h^k) \leq h_{\text{max}}^k$$

$k = 1, 2, \dots, n_c$ , the number of postulated contingencies.

#### 5.4.2 Problem Formulation

The above security control problem is formulated as a linear programming problem whose objective is to minimise the objective function in equation (5.1) subject to the linear constraints

$$[A] \Delta U \geq b$$

and  $\Delta u_i \max \geq \Delta u_i \geq \Delta u_i \min, i = 1, 2, \dots, n,$

where  $[A]$  is a  $m \times n$  matrix containing the sensitivity of the line flows and/or voltage magnitudes with respect to  $\Delta u$ .

$b$  is the  $m$ -vector of line flows and/or voltage magnitude limits.  $\Delta u = (\Delta u_1, \Delta u_2, \dots, \Delta u_n)^t$  is the  $n$ -vector of adjustments in real and/or reactive power injections and is referred to as "generations" in the sequel.

$\Delta u_{\max}$  and  $\Delta u_{\min}$  are the upper and lower limits on adjustments  $\Delta u$ .

The linearized full real and reactive power rescheduling problem is decoupled into two separate linearized problems as

- (i) real power rescheduling problem;
- (ii) reactive power rescheduling problem;

The proof for the convergence of the decoupled problem to the solution of the overall problem is provided in reference (104). The main advantages of this decoupling are

- (i) The storage requirement and the computation time for the decoupled problem is much less than the overall problem.
- (ii) Different optimising objectives can be used for the two decoupled problems.
- (iii) In systems where voltage and reactive power flow constraints are not very critical, only real power problem can be solved to further reduce the computation time.

5.4.3 Linearised Real Power Optimal Rescheduling Problem

The real power rescheduling problem finds the adjustments in real powers of the controllable "generations" in such a way that all real power branch flow overloads present in the initial operating state are completely alleviated. The real power rescheduling problem is formulated as an incremental model in which the system model is linearized about the initial operating point obtained from the state estimator or load flow solution. The "goodness" of the solution is dictated to the mathematical model via ~~and~~ objective function which is to be optimised.

5.4.3.1 Control variables

In real power rescheduling problem the adjustments in active power are used to alleviate the branch flow overloads. The active power adjustments can be applied only at those locations where controllable active power sources are available and these constitute the active power control variables. The active power control includes bus generation changes, phase-shifter control and load shedding. All these control actions can be represented as changes in bus injections, i.e., equivalent generation changes. Thus all buses where equivalent generation changes can be effected can be termed as "generation control buses" and real power variable at these buses as controllable "generations" and these constitute the control variables represented as

$$[\Delta P_c] = [\Delta P_1, \dots, \Delta P_n]^t \tag{5.6}$$



for  $n$ -controllable "generations"  $[\Delta P_c]$  is the control variable vector.

#### 5.4.3.2 Objective function

For real power rescheduling problem many different kinds of optimising objectives can be used. Some of these are as follows:

Objective 1 : Minimum cost of operation. This objective is the standard objective used in economic dispatch problem and is represented as:

$$F(\Delta P_c) = \sum_i C_i \Delta P_i, \quad i = 1, 2, \dots, n \quad (5.7)$$

Where,  $\Delta P_i$  = real power adjustment of  $i$ th controllable "generation".  $C_i$  = cost coefficient of the  $i$ th controllable "generation".  $n$  = number of controllable "generations" in the system.

The desire behind this objective is to keep the system operating cost minimum.

Objective 2 : Minimum weighted deviation from the initial operating point. The main desire behind this objective is to minimise departure from the initial (probably) economic operating point. This objective can be represented as:

$$F(\Delta P_c) = \sum \alpha_i |\Delta P_i| \quad ; \quad i = 1, \dots, n. \quad (5.8)$$

The  $i$ th component of the objective (5.8) can be depicted as in Fig. 5.1(a). The advantage of this objective is the ease with which it can be implemented in the LP formulation (100-101). The main disadvantage of this objective is that the LP process will force many of the controllable generations, rescheduled to their maximum or minimum operating limits.

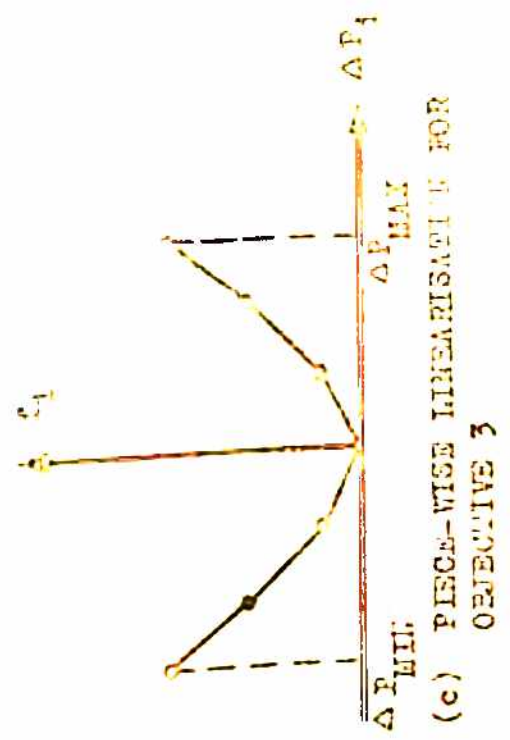


FIG. 5.1 OBJECTIVE FUNCTIONS

Objective 3 : Minimum weighted least square deviation from the initial operating point, i.e.,

$$F(\Delta P_i) = \sum \alpha_i (\Delta P_i)^2 ; i = 1, \dots, n \quad (5.9)$$

This objective is depicted in Fig. 5.1(b). The desire behind this objective is to heavily penalise large deviations from the initial operating point. This objective overcomes the shortcoming of the objective 2 by discouraging large changes in small number of units to relieve overloads. The disadvantage in this objective is that the objective being nonlinear it has to be linearised using piecewise linear segments (Fig. 5.1(c)) to make it suitable for implementation in the LP formulation. Another disadvantage is that minimum deviation from initial operating point does not ensure most economic operating point.

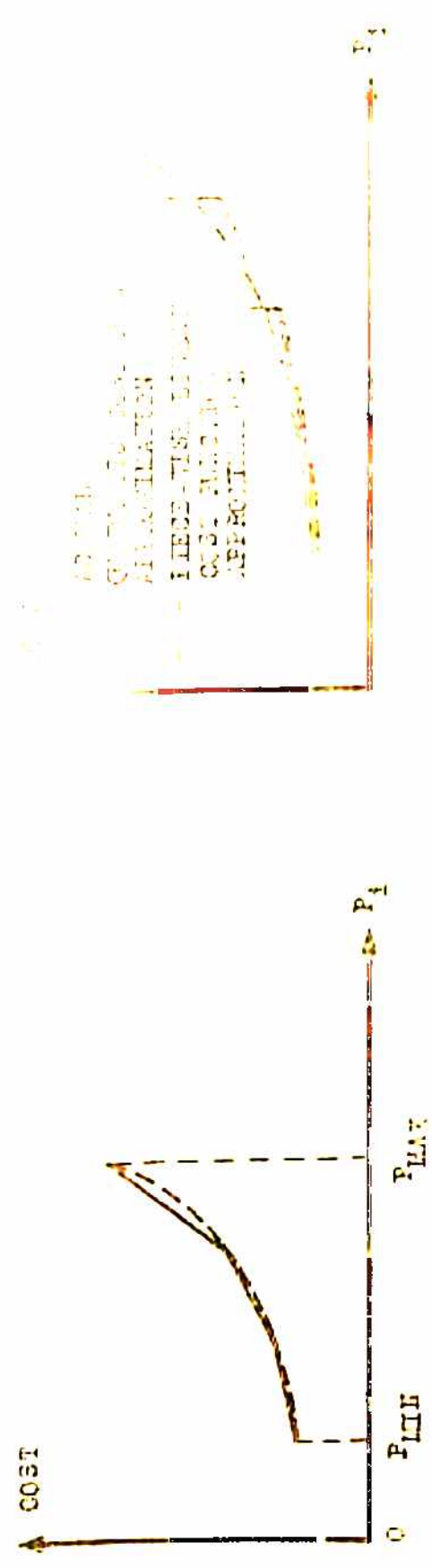
Objective 4 : Minimum number of controllable "generations" rescheduled. This objective can be implemented by a pseudo-LP approach in which those units are chosen for rescheduling which on moving to its operating limit will cause maximum reduction in overloads. The greatest disadvantage of this objective is its complete disregard for the cost of operation as well as unnecessary large changes in controllable "generations" have to be made.

From above it can be seen that in real power application, where the changes in real power injections have direct relation to the system operating cost, the most suitable objective to be used is the objective 1, the minimum cost of operation, i.e.,





(a) TYPICAL GENERATION COST CURVE



(c) PIECE-WISE LINEARISATION OF COST CURVE



(d) COST CURVE FOR MULTI-VALVE TURBO-GENERATOR

FIG.5.2

$$F(\Delta P_c) = \sum C_1 \Delta P_1 \tag{5.10}$$

In general the unit generation cost is represented by a second order nonlinear cost function as shown in Fig. 5.2(a). Since the LP formulation requires the objective function to be linear, this cost function has to be approximated by linear functions as shown in Fig. 5.2(b) and (c). In Fig. 5.2(b) the nonlinear cost function is crudely linearized by a single segment linear function. The disadvantages of this crude linearization is gross approximation of the cost function as well as the LP process may force many of the controllable "generations" to their operating limits. The multisegment piecewise linearised approximation of the cost function is much more accurate, the accuracy depending on the number of linear segments used for the approximation. The larger the number of piecewise linear segments used the better will be the approximation but larger will be the storage requirements. For multivalve units the traditional second order convex cost curves themselves are gross approximations Fig.5.2(d), chosen mainly for computational convenience<sup>(103)</sup>. In case of multivalve units the piecewise linear cost curves can be more accurate than the conventional cost curves used and may be used to promote valve point loading to provide more economic operation than the conventional representation.

### 5.4.3.3 Load shedding

As mentioned earlier load shedding may be used as a control function to alleviate the branch flow overloads in the real power rescheduling problem. Load shedding can be incorporated in the security control algorithm easily by considering the load shed at <sup>a</sup> bus as equivalent generation at that bus. Thus the load shed by an amount  $\Delta P_i$  at the bus  $i$  can be considered as equivalent generation of amount  $\Delta P_i$  at the bus  $i$ . Where the equivalent generation at bus  $i$  has a minimum operating limit at zero (no load shed) and maximum operating limit at  $P_{Ti}$  (all load shed).

Load shedding results not only in reduced income to the utility but <sup>also</sup> causes resentment among the consumers. The consumer resentment increases rapidly with the increase in frequency, magnitude and duration of load shedding. This resentment in the extreme cases may result in consumers opting for captive power plants or other sources for their energy requirements resulting in permanent loss of customers. Thus the cost of load shedding can not be considered only in terms of the revenue lost due to load shed but must also take the consumer "resentment cost" due to load shed into account.

As the consumer resentment increases rapidly with the increase in frequency and amount of load shed a second order cost functional is chosen to represent the cost for load shedding at each load bus. In order to reduce the resentment of the consumers and its associated disadvantages the load shedding must be used only as a last resort when other control options are not very effective. This is incorporated in the programme by keeping



the incremental cost for the cost functional much higher (5 to 10 times) than that for other control options like generation changes or phase shifter tap changes. A multi-segment piecewise linearisation of the second order cost functional for load shedding (similar to that for generator costs) is used to incorporate it in the LP model. In case of large amount of load shedding the piecewise linear segments for the cost functional ensure that the total load shed is spread over many buses in the system. By using different cost functionals for load shedding buses a priority scheme (high priority loads having higher cost associated with load shedding) is incorporated.

It is not possible to shed only real power load at any bus, an equivalent amount of reactive load is also shed at that bus. Thereby maintaining the power factor of the loads at the buses constant even after load shedding.

#### 5.4.3.4 Linearized system network model

The power system network model is basically nonlinear. To adopt it for use in LP formulation, this network model is linearized about the initial operating point as an incremental real power model, as shown below:

$$\begin{bmatrix} \Delta P \end{bmatrix} = \begin{bmatrix} H \end{bmatrix} \begin{bmatrix} \Delta \theta \end{bmatrix} \quad (5.11)$$

where,

$\begin{bmatrix} \Delta P \end{bmatrix}$   $\rightarrow$  (N - 1)-vector of incremental changes in bus real power injections, whose nonzero elements are net/at 'controllable generation' buses  $\Delta P_i$ s

$[\Delta \theta]$  — (N - 1)-vector of changes in bus voltage angles

$[H]$  — (N - 1) x (N - 1) matrix for a N bus system.

The 'exact' linearisation of the network model about the initial point makes  $[H]$  the non-symmetric  $[\partial P / \partial \theta]$  submatrix of the Newton load flow Jacobian matrix. By incorporating some approximations, like considering all bus voltages to be one pu and eliminating the effects of those elements which primarily control the reactive power (e.g., line charging capacitances, transformer tap positions etc.) the  $[H]$  matrix can be represented by the fast decoupled load flow matrix. The main advantage of choosing  $[H]$  as the fast decoupled load flow matrix is that  $[H]$  becomes a constant matrix which can be triangularised and stored once for all at the beginning of the process. This will save considerable computation time. Because of these reasons the fast decoupled matrix has been chosen as the  $[H]$  matrix for the linearized network model in the proposed method.

#### 5.4.3.5 Basic LP problem

The problem is to minimise the objective function

$$F(\Delta P) = \sum C_i \Delta P_i = [C]^t [\Delta P_0] \quad (5.12)$$

(Where  $C_i$  is the incremental cost for controllable "generation" 1 and  $\Delta P_i$  is the change in controllable 'generation' 1. For the present we will assume a single incremental cost for each controllable 'generation', the extension to multisegment cost curve will be discussed later.)

Subject to

- (1) Power balance equation

$$\beta_1 \Delta P_1 + \beta_2 \Delta P_2 + \dots + \beta_n \Delta P_n = 0 \quad (5.13)$$

Where  $\beta$ s are the transmission loss penalty factors and are obtained as explained in Chapter 2. and

- (2) a large set of double sided inequality constraints for upper and lower limits on controllable 'generation' changes and transmission line power flows. The above set can be written concisely as

$$L_j \min \leq [A_j] [\Delta P_c] \leq L_j \max, \quad j = 1, 2, \dots, m \quad (5.14)$$

In case of limit constraints on  $i$ th controllable generation changes the row vector  $[A_j]$  is null except for 1 in position  $i$ . For the limit constraint on branch power flow changes the vector  $[A_j]$  is non-sparse and contains the distribution factors relating the change in branch flow with a change in generation to the slack bus. For limit checking on branch flows instead of using equation (5.14) each time, it is much faster and easier to use the voltage angle limits ( $|\theta_{1j}^{\max}| = |\theta_1 - \theta_j^{\max}|$ ).

The above LP problem stated by (5.12) - (5.14) can be represented in standard LP format as

$$\text{Minimise } [c]^t [\Delta P_c]$$



$$\text{Subject to } [A][\Delta P_c] \geq [b] \quad (5.15)$$

$$[\Delta P_c] \text{ unrestricted in sign}$$

by breaking each double sided constraint into two lower bound constraint equations as

$$[I][\Delta P_c] \geq [\Delta P_c]_{\min}$$

$$- [I][\Delta P_c] \geq [\Delta P_c]_{\max} \quad (5.16)$$

Where  $[I]$  is identify matrix. This greatly increases the number of constraint equations and thus the storage requirement for simplex method. The storage requirement can be reduced by using an upper bounding algorithm<sup>(98,99)</sup> and handling the "generation" inequality constraints implicitly.

Another approach to solving (5.15) can be by using the dual problem of (5.15).

Intimately associated with each LP problem of (5.15) is another problem called its dual given as

$$\text{Maximise } [b]^t [\mu]$$

$$\text{Subject to } [A]^t [\mu] = [c]$$

$$[\mu] \geq 0 \quad (5.17)$$

Where  $[\mu]$  is a m-vector of dual variables. It is a well

known fact in LP theory that the optimal cost (if it exists) is the same for both the problems<sup>(105,106)</sup>. Moreover, if  $[B]$  is a basic feasible solution to the dual problem, then the corresponding primal variables  $[\Delta P_B]$  is given by

$$[\Delta P_B] = [B_d^{-1}] [b_B] \quad (5.18)$$

where  $[B_d]$  is a  $n \times n$  basis matrix of the dual problem, and  $[b_B]$  is the dual cost associated with  $[\omega_d]$ . Therefore, if the optimal solution to the dual problem is known the optimal solution to the primal problem can be found immediately using (5.18). Thus it is possible to solve the optimisation problem (5.15) using the dual formulation instead of the primal. The main advantages of the dual formulation over primal are:

- (1) In cases where the number of constraint equations,  $m \gg n$ , the number of control variables (as in the case of power system security control problem), the dual formulation offers considerable saving in computer storage and computation. This is because the dual formulation requires a  $n \times n$  basis matrix whereas the primal formulation requires  $m \times m$  basis matrix.
- (2) In many cases the initial dual feasible solution is apparent without any computation moreover it tends to be 'closer' to the optimal solution than the initial feasible solution given by phase I of the primal approach<sup>(105)</sup>. Intuitively, the simplest dual feasible initial solution

can be a primal infeasible solution which incurs least cost but violates some of the constraints. In the case of security control problem, the dual feasible starting point can be economic dispatch operating point which may be violating some security constraints.

One of the short-coming of dual formulation is that the decision variables are no longer the physical variables of interest. But this shortcoming can be overcome by using dual simplex method which solves the dual problem by working explicitly with the primal variables and only implicitly with the dual variables. The proposed security control algorithm is a heuristic LP approach based on dual simplex method.

#### 5.4.3.6 Reduced basic matrix

The solution to any LP problem lies at the vertices of the constraint set, i.e.,  $n - 1$  constraint equations must be binding (on the limit). The process thus restricts its attention to power system operating states at such vertices, moving from one to another iteration by iteration until the solution point is reached. Each iteration consisting of a pivot in which a previously binding constraint is relaxed (made free) to enforce a violated constraint at its limit by including it as an equality in the basis. At any stage in this process the power system operating state is given by the "basis matrix" equation

$$[L] = [B] [\Delta P_0] \quad (5.19)$$



Where  $[B]$  is the  $n \times n$  basis matrix

$[L]$  is the  $n$ -vector of limit constraints.

The first row of basis matrix consists of the terms corresponding to power balance equation (5.13) and the rest  $n - 1$  rows consist of the constraint coefficient row vectors  $[\Delta_j]$  from (5.14) for the relevant binding constraints. In the basis matrix equation (5.19) if  $k - 1$  branch flow constraints are binding (i.e., enforced as equality) and are numbered 2 to  $k$ , then the basis matrix equation (5.19) can be written as

$$\begin{array}{l}
 \text{Power balance term (zero) and } \\
 \text{k-1 branch flow limit violations} \rightarrow \\
 \\
 \text{Controllable generation limits} \rightarrow
 \end{array}
 \begin{bmatrix} L_f \\ L_g \end{bmatrix} = \begin{bmatrix} B_f & B_1 \\ 0 & \begin{matrix} 1 & & \\ & \ddots & \\ & & 1 \end{matrix} \end{bmatrix} \begin{bmatrix} \Delta P_f \\ \Delta P_1 \end{bmatrix}
 \begin{array}{l}
 \leftarrow \text{k-free controllable generations} \\
 \\
 \leftarrow \text{n-k binding controllable generations}
 \end{array}
 \tag{5.20}$$

As seen from equation (5.20) for  $k - 1$  binding branch flow constraints there have to be exactly  $k$  free controllable "generations" with no equality constraints in the basis. The binding controllable "generation" rows in lower part of (5.20) are trivial and can be handled implicitly; thus requiring only the storage of  $k$ -non sparse rows of basis matrix which forms the "reduced basis" equation:

$$[L_f] = [B_f][\Delta P_f] + [B_1][\Delta P_1] = [B_r][\Delta P_c] \tag{5.21}$$

Free controllable "generation" changes  $[\Delta P_f]$ , whenever required can be obtained from:

$$[\Delta P_f] = [B_f]^{-1} ( [L_f] - [B_1] [\Delta P_1] ) \quad (5.22)$$

Since  $k$  in general, will be very small (2 to 6 say)<sup>(103)</sup> the computational effort required for evaluation of factors of  $[B_f]^{-1}$  or even its explicit inverse will be quite small.

#### 5.4.3.7 Basis exchange process

The optimisation process in dual simplex method is carried out step by step. At each step a violated constraint from (5.14) is enforced by introducing it as an equality in the basis matrix equation (5.20). This necessitates an existing binding controllable "generation" or branch flow equality constraint to be freed (relaxed). The basis exchange process is a move to new operating state. In general, the choice among the currently violated constraints to be enforced is arbitrary. But the choice of currently most violated constraint to be enforced is generally preferred as the enforcement of this constraint in general, will lead to satisfying of many other violated constraints present in the previous step. Of course, the choice of the binding constraint to be freed is strictly governed by the "eligibility test" and "ratio test" to make sure that the enforcement of the violated constraint will lead to minimum increase in the objective function (i.e., the binding constraint to be freed is selected optimally).

The eligibility test ensures that a binding constraint does not violate its constraint limit immediately on being freed. The eligibility test depends mainly on the sensitivity  $S_i$  of the  $i$ th binding branch flow or "generation" constraint with respect to the violated constraint to be enforced by entering it in the basis. The sensitivity  $S_i$  is given by

$$S_i = \phi_e / \phi_i \quad (5.23)$$

Where  $\phi_e$  is the amount by which the violated branch or "generation" constraint will be changed by entering it in the basis and  $\phi_i$  is the amount by which the existing binding constraint  $i$  ("generation" or branch flow) will change when freed.

Thus if the entering violated constraint  $e$  is violating its upper (lower) limit, then the binding constraint  $i$  in the basis is eligible to be freed only if (i) it is at its upper (lower) limit and  $S_i$  is positive, or (ii) it is at lower (upper) limit and  $S_i$  is negative.

The row vector  $[S]$  of the sensitivities of the binding constraints w.r.t. the violated constraint  $e$ , entering the basis, is computed using the relation

$$[S] = [A_e] [B]^{-1} \quad (5.24)$$

Where  $[A_e]$  is the row  $A_j$  in equation (5.14) for the violated constraint entering the basis. Using the reduced basis structure (Structure of  $[B]^{-1}$  will be same as that of  $[B]$ ) and



ordering  $[A_e]$  as  $[A_f : A_1]$  the vector of sensitivity can be obtained from

$$[S_f] = [A_f] [B_f]^{-1} \quad (5.25)$$

$$[S_1] = [A_1] - [S_f] [B_1]$$

Among the eligible binding constraints the choice for the constraint to be freed is decided on the basis of "ratio test" which involves the incremental dual cost  $\mu_1$  of each binding constraint. The ratio test computes the ratio  $\left| \frac{\mu_1}{S_1} \right|$  and chooses the constraint  $i$  with minimum  $\left| \frac{\mu_1}{S_1} \right|$  as the binding constraint to be freed. The row vector of incremental dual cost  $[\mu]$  is computed using

$$[\mu] = [C] [B]^{-1} \quad (5.26)$$

Where  $[C]$  is the row vector of incremental cost for the controllable "generations". Partitioning  $[\mu]$  and  $[C]$  leads to

$$[\mu_f] = [C_f] [B_f]^{-1} \quad (5.27)$$

and

$$[\mu_1] = [C_1] - [\mu_f] [B_1]$$

#### 5.4.3.8 Incorporation of multisegment piecewise linear cost curves

The multisegment piecewise linear cost curves can be included in linear programming by using the separable programming method. The main disadvantage of this approach is that each linear cost segment has to be represented by a control variable. This considerably increases the number of LP variables for multisegment representation of cost curves and so the order of the basis matrix. Thereby, greatly increasing the storage requirement and making this approach unsuitable for real time applications.

The proposed method for incorporating the multisegment cost curves uses only one LP variable  $\Delta P_i$  for each controllable "generation"  $i$  and does not increase the storage requirement except the storage needed for storing the slopes of the segments of the cost curves.

The method is based on the fact that at any stage in the LP process a controllable "generation"  $i$  can be on only one particular segment (segment  $p$ ) of its cost curve. And so the incremental cost associated with that segment ( $p$ ) will be the only cost coefficient  $C_i$  associated with controllable "generation"  $i$ . It is this current value of  $C_i$  which is used in objective function (5.12) and equation (5.27) for calculation of  $\mu_i$ . Now if during the LP process the controllable "generation"  $i$  moves, if it is free (or wants to move if it is a binding "generation" eligible to be freed), from its currently designated segment ( $p$ ) to some other segment ( $q$ )

then the cost coefficient  $C_1$  associated with this generation will change to the incremental cost of the newly designated segment (q) and will replace the old cost coefficient in (5.12) and (5.27). Thus we see that if any controllable generation moves from its currently designated segment then the objective function (5.12) also changes. This makes it necessary to introduce the following modifications in the above discussed LP scheme:

- (1) Modification in eligibility test : As discussed above, the eligibility test determines whether a particular binding "generation" is eligible to be freed or not. In case of multisegment cost curves, a binding "generation" on the limit of its cost segment is always eligible to be freed, except when it is at the extremities of the cost curve (where it may not be eligible). Let us assume that the binding generation is at the upper limit of its currently designated segment (say point B of segment AB in Fig. 5.3) and from eligibility test it is found that on being freed it will move to the right (i.e., towards BC) and so will violate its currently designated segment. This can be taken care of by designating the "generation" i to be on the lower limit of the next segment to the right (i.e., on the same point B but as a lower limit of segment BC), and using incremental cost of the segment BC as the cost coefficient  $C_1$  in equation (5.27). For the situation in which the generation i is at lower limit of its current cost segment and wants to





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move to the left, it can be designated as on the upper limit of the next segment to the left and the incremental cost of the new segment has to be used in equation (5.27). This modification forms an integral part of the eligibility test.

(ii) Modification for violation of cost segment by free generators: After the changes in free controllable "generations"  $[\Delta P_f]$  are obtained using equation (5.22), each of these free controllable "generations" are checked for the violation of their designated cost segment and the generator with maximum violation of its designated segment is chosen (say  $\Delta P_j$ ). Let  $\Delta P_j$  be now operating at point F having violated the upper limit of its designated segment BC in Fig. 5.3. Now according to LP theory a free variable should not be enforced by relaxing a binding variable until it is the cheapest thing to do. So enforcing the free controllable "generation" j at point C (the upper limit of its designated segment) may not preserve optimality as the problem may still be optimal with  $\Delta P_j$  at F (still a free variable) or at the upper limit of an intermediate segment (i.e., as a binding variable at D or E). The optimal operating point for  $\Delta P_j$  is found as follows:

With  $\Delta P_j$  assumed to be operating at F calculate  $[\mu]$  vector using (5.27). Calculate the ratio  $R_j = \left| \frac{\mu_j}{S_j} \right|$  and  $R_{\min} = \min \left| \frac{\mu_i}{S_i} \right|$ , i for all eligible binding "generations". If  $R_j \leq R_{\min}$ ,  $\Delta P_j$  remains a free variable operating at point F. Otherwise go to the next applicable segment (DE in this case)

and repeat the above process till  $R_j \leq R_{\min}$  or segment BC is reached, then  $\Delta P_j$  is fixed at the upper limit of that segment and becomes a binding generator and the binding generator with minimum ratio is made free in its place.

In case the new operating point found for  $\Delta P_j$  be at  $P' > P_{j \max}$  and  $R_j$  for the extreme segment  $EG \leq R_{\min}$  for eligible binding "generation", the  $\Delta P_j$  is fixed as a binding "generation" operating at point G and the binding "generation" with minimum ratio  $R_{\min}$  is freed. Otherwise the above process is repeated for the applicable segments. Again using equation (5.22) the free "generations" are calculated and again the above process is repeated till no free "generations" violate their designated cost segment.

With these two modifications the multisegment cost curves can be incorporated in the LP process.

#### 5.4.3.9 The algorithm (P) :

The algorithm for the real power rescheduling process is as follows:

Step 1 : Using the economic dispatch program in chapter 2 obtain economic schedule and conduct fast decoupled load flow for this operating condition. Store the LU factors of  $[B']$  matrix as  $[H]$  matrix. Compute limit on controllable "generations".

Step 2 : Compute and store the slopes and limits for the segments of cost curves for all controllable "generations". Initialise the book keeping arrays and record the



controllable "generations" **position on its cost** curve.

Step 3: Search for the "generation" which lies on the segment with least incremental cost. Without loss of generating let this be the  $n$ th generation. Initially the rest  $n-1$  generation are binding (with  $\Delta P_1 = 0$ ). The  $n$ th generation is free (even though  $\Delta P_n = 0$ )

Step 4: Set the first row of basis matrix as the coefficients of  $[\Delta P_c]$  in the power balance equation (5.13) where, the penalty factors  $\beta_s$  for the generators are calculated as explained in Chapter 2 and  $\beta_s$  for non-generator variables is equal to 1. Factorise the basis matrix into  $[B_f]$  and  $[B_b]$  associated with free and binding "generations" respectively.

Step 5: Compute the flow in every line using voltage angles

$$\theta_{rS} = |\theta_r - \theta_s| \quad (5.28)$$

and store the indices of all lines with flow more than 75% of its rated, in an array  $[ICS]$ .

Step 6: Check for line overloading. If no violations found go to step 20.

Step 7: Search for most over loaded line. Let the branch connecting node  $r$  and  $s$  is most overloaded. Set a  $(N-1)$  vector

$$D^t = (0 \dots 0 \underset{\substack{\uparrow \\ \text{position} \\ r}}{+} \text{Hrs } 0 \dots 0 \underset{\substack{\uparrow \\ \text{position} \\ s}}{-} \text{Hrs } 0 \dots 0) \quad (5.29)$$

Where  $H_{rs}$  is the  $rs$  element of  $[H]$  matrix, the upper sign is used when the flow in line is from  $r$  to  $s$  and the lower sign is used in the opposite case. Now solve for  $(n - 1)$  vector  $[G]$  using equation (5.30)

$$[H][G] = [D] \quad (5.30)$$

In solving for  $[G]$  using the LU factors of  $[H]$ , the sparsity of  $[D]$  can be exploited. The vector  $[G]$  contains the sensitivity of line flow w.r.t. the controllable "generations"  $[\Delta P_o]$  except for the generation at slack bus, whose sensitivity to the line flow is always zero.

Step 8 : Set up a  $n$ -vector  $[A]$  by extracting the elements corresponding to the controllable "generations" from the vector  $[G]$ . Factorise  $[A]$  into  $[A_f]$  and  $[A_b]$  corresponding to the free and binding "generations".

Step 9 : Using equation (5.25) compute  $[S_f]$  and  $[S_b]$  and check for the eligibility of binding line and generations. The 1st row in the basis matrix consisting the power balance equation (5.13) is never eligible to be freed. A binding line  $j$  in the basis is eligible to be freed only if  $S_j$  is positive. A binding "generation" at the upper (lower) limit of the cost curve is eligible to be freed only if  $S_i$  is positive (negative). A binding "generation"  $i$  at the corner of two cost segments is always eligible to be freed. If  $S_i > 0$  and the binding "generation"  $i$  is at the lower limit of its designated cost segment then it is designated to be operating at the upper limit of the cost segment to the left (i.e. at the same point) and its cost segment pointer is updated. If

$S_i < 0$  and the binding "generation" is operating at the upper limit of its designated cost segment then it is designated to be operating at the lower limit of the cost segment to the right and its cost segment pointer is updated.

Step 10 : If no binding line or generation is eligible to be freed then problem is infeasible go to step 19. Otherwise continue.

Step 11 : Set up a  $n$ -vector  $[C]$  of the incremental costs associated with the segments on which the controllable "generations" are operating. Factorise  $[C]$  as  $[C_f]$  and  $[C_b]$ . Using equation (5.27) compute  $[\mu_f]$  and  $[\mu_b]$ .

Step 12 : Perform the ratio test by computing

$$R_i = \left| \frac{\mu_i}{S_i} \right| ; \quad i \text{ eligible binding lines and "generations"}. \quad (5.31)$$

The eligible binding constraint or "generation", with smallest  $R_i$  is freed. If a binding line is freed its row in the reduced basis  $[B_r]$  is removed; if a binding "generation" is freed, its status pointer is updated to show its new status.

Step 13 : The row  $[A]$  found in step 8 for line constraint being enforced is inserted in basis  $[B_r]$  and contents of  $[B_r]$  are factored into  $[B_f]$  and  $[B_b]$  for free and binding generations respectively. The amount of change in line flow  $L_j$  to back off the overloaded line  $j$  (which is being enforced) to its capacity is computed as below:



$$L_j = - \left[ (\theta_r - \theta_s) \mp \theta_{rs}^{max} \right] H_{rs} \quad (5.32)$$

Where  $\theta_{rs}^{max}$  is real power flow limit of line between nodes r and s in terms of voltage angle separation. The upper sign is used when flow is from r to s and lower sign is used when flow is from s to r. The value of  $L_j$  is stored in  $L_f$  in same row in which row  $[A]$  is stored in  $[B_r]$ .

Step 14 : The changes in the values of free "generations" is computed using equation (5.22).

Step 15 : Check for violations of their designated cost segments by the free "generations". If no free "generation" violates its cost segment go to step 18. Let the generation j with maximum amount of violation moves from its designated segment BC to point F on segment EG (Fig. 5.3) as calculated in step 14. Now the "generation" j should be allowed to move from BC towards F uptill it is the cheapest thing to do, i.e.,  $\Delta P_j$  can move to a new segment on the cost curve from its original segment BC till:

$$|R_j^0 + \alpha_j \Delta C_j| \leq \min_1 |R_1^0 + \alpha_1 \Delta C_j| \quad (5.32)$$

Where  $\Delta C_j$  is the value of change in incremental cost for generation j on moving to the new segment.  $R_1^0$  is the ratio  $\left| \frac{\mu_1}{S_1} \right|$  for all eligible binding "generations" at the beginning of step 14. If no binding generation is eligible to be freed, the problem is infeasible. Go to step 19.

$(\alpha_1 \Delta C_j)$  is the amount of change in  $\mu_1$  due to change in  $\mu_j$  caused by change in  $C_j$ . For details and calculation of  $\alpha_1$

see Appendix 5.1.

Step 16 : If  $\Delta P_j$  can move to segment EG then "generation"  $j$  remains free with its new operating point at point F. Its cost segment pointer is updated. Go to step 17. If  $\Delta P_j$  moves upto some other segment (BC, CD or DE) it becomes a binding "generation" with its operating point at the upper limit of the applicable segment. Its cost segment pointer and status indicator are updated. The binding "generation" with minimum  $R_1$  as found in (5.32) is made free and its status indicator updated. Update  $[B_f]$  and  $[B_1]$  matrices.

Step 17 : Go to step 14.

Step 18 : Set up a  $(N - 1)$  vector  $[X]$  containing elements from vector  $[\Delta P_c]$  for buses with controllable "generations" and 0 for buses where controllable "generations" are not present. Solve for the changes in voltage phase angle using equation

$$[H] [\Delta \theta] = [X] \quad (5.33)$$

Check for the constraint violations for lines in array  $[ICS]$ . If any violation detected go to step 7. Otherwise go to step 20.

Step 19 : Print problem infeasible. Either accept the results with some constraints still violated and go to step 20, or relax constraints on line flows by substituting their steady state flow ratings by emergency ratings and go to step 5.

Step 20 : Print results.

This ends the LP algorithm for real power rescheduling.

#### 5.4.4 Linearised Reactive Power Optimal Rescheduling Problem

The solution to the optimal reactive power rescheduling problem is the adjustments (changes) in reactive power control sources to alleviate the system bus voltage violations and line reactive power flows limit violations. The problem is formulated as a linearised incremental model and the objective function is optimised using the LP method exactly similar to that discussed for the real power rescheduling problem.

##### 5.4.4.1 Control variables

The reactive power and/or voltage control in a power system can be obtained by controlling reactive power of generators, switchable capacitors/inductors etc. and can be represented as equivalent reactive power bus injections, transformer taps control the bus voltages as well as reactive power flows and can be considered as equivalent reactive power injections at the bus on the tap side of the transformer (see Appendix 5.2b). Thus all reactive power control devices can be represented as equivalent reactive power bus injections. The control variable in this case is

$$[\Delta Q_c] = [\Delta Q_1, \dots, \Delta Q_m] \quad (5.34)$$

Where  $m$  is the number of equivalent reactive power bus injection sources, called "reactive generations" in the sequel.

##### 5.4.4.2 Objective function

Similar to the objective function used for real power rescheduling the objective function for reactive power resch-



eduling is also formulated as

$$F(\Delta Q) = \sum_{i=1}^m C_i^1 \Delta Q_i \quad (5.35)$$

Where  $C_i^1$  is the pseudo-cost associated with the equivalent "reactive generation"  $i$ .

Unlike real power case where actual cost is associated with generation, in the reactive power case there is no real cost associated with reactive generation and so a minimum cost objective (objective 1, sec 5.4.3.2) can not be used. Since the reactive power flow does cause real power losses in the transmission system, an objective minimising the power loss in the transmission system can be used in the reactive power case. But in our case since we have assumed complete decoupling of real and reactive power (with its added advantages as discussed before) it is not possible to use this objective. The disadvantage due to this is not great as, in general, the real power losses because of reactive power flow in the transmission system is usually small.

The other objectives which can be used for reactive power rescheduling problem are (i) weighted minimum deviation from initial operating point, (ii) weighted least square deviation from initial operating point and (iii) minimum number of control variables rescheduled. The merits and demerits of all these objectives have already been discussed in section 5.4.3.2. The objective (iii) is mainly a sensitivity based objective and may lead to large changes in a few equivalent

reactive generations making them unnecessarily operate at their limits. The objective (i) is well suited for LP formulation <sup>but</sup> it suffers from the same disadvantage as that of objective (iii). Among the above three objectives the objective (ii) appears to be the most suited as it spreads the control over a number of variables with small changes from their initial operating point.

The objective of weighted least square deviation from initial operating point is a non-linear objective Fig. (5.1(b)) which is linearized using piecewise linear segments as shown Fig. (5.1(c)). By using different weights for different control variables a priority scheme may be worked out, e.g., transformer tap setting changes can be awarded less weight (pseudo cost) than switchable reactances or generator reactive power changes, in order to give it priority over other controls etc.

#### 5.4.3 Linearised Network Model

The power system network model for the Q-V relationship is linearized using the fast decoupled load flow equation

$$\left[ \frac{\Delta Q}{|V|} \right] = \left[ B'' \right] \left[ \Delta |V| \right] \quad (5.36)$$

By assuming  $|V|$  on the left hand side of equation (5.36) to be equal to 1 pu, the above relation reduces to

$$\left[ \Delta Q \right] = \left[ B'' \right] \left[ \Delta |V| \right] \quad (5.37)$$

The fast decoupled load flow matrix  $\left[ B'' \right]$  in equation (5.37) does not contain the rows and columns of PV buses and slack

bus. In reactive power rescheduling problem the reactive power injections at the PV buses and slack bus is used to control the reactive power flow and voltages at different buses. Therefore, the Q-V relationship for these buses also have to be included in equation (5.37). This is done by augmenting the  $[B'']$  matrix with the elements for rows and columns corresponding to these buses and the Q-V relationship for all the buses can be written as

$$[\Delta Q] = [H'] [\Delta V] \quad (5.38)$$

Where  $[H']$  matrix is the  $[B'']$  matrix augmented for the PV and slack bus.

The augmentation of  $[B'']$  to contain all the buses (5.38) may cause the  $[H']$  matrix to become ill-conditioned or even singular in extreme cases, unless strong suceptive ties to ground are present. To avoid this illconditioning of the  $[H']$  matrix, a load bus with no interruptible reactive load is considered as a reactive power reference bus (say r) and the equation (5.38) can be factorised as

$$\begin{bmatrix} \Delta Q_r \\ \Delta Q'' \end{bmatrix} = \begin{bmatrix} A & D \\ D^t & H \end{bmatrix} \begin{bmatrix} \Delta V_r \\ \Delta V'' \end{bmatrix} \quad (5.39)$$

Equation (5.39) can be rearranged as



$$\begin{bmatrix} \Delta V_r \\ \Delta Q'' \end{bmatrix} = \begin{bmatrix} A^{-1} & -E^t \\ B & H'' \end{bmatrix} \begin{bmatrix} \Delta Q_r \\ \Delta V'' \end{bmatrix} \quad (5.40)$$

Since  $\Delta V_r$  is unknown and  $\Delta Q_r$  is zero (no interruptible reactive load at reference bus) so the first row and column in equation (5.40) can be removed as there is no equation corresponding to the reference bus. This leads to, the Q-V relation for the rest (N - 1) buses of the system as

$$[\Delta Q''] \quad [H''] \quad [\Delta V''] \quad (5.41)$$

Appendix 5.3 gives details for calculating the elements of the matrix  $[H'']$ .

The nonsparse constraint on  $\Delta Q_r$  (which is zero) is obtained in terms of the reactive controllable injections is obtained as

$$0 = \xi_1 \Delta Q_1 + \xi_2 \Delta Q_2 \dots + \xi_n \Delta Q_n \quad (5.42)$$

Equation (5.42) forms the reactive power balance equation similar to the real power balance equation. The coefficients  $\xi_i$  are calculated as shown in Appendix 5.3

#### 5.4.4 Linear Programming Formulation

The LP formulation for the reactive power rescheduling problem may be stated as:

Minimise the objective function

$$F(\Delta Q) = \sum_{i=1}^m C_i \Delta Q_i = [C] [\Delta Q_c] \quad (5.43)$$

Subject to the reactive power balance equation

$$\xi_1 \Delta Q_1 + \dots + \xi_m \Delta Q_m = 0 \quad (5.44)$$

Where  $\xi_s$  are the reference bus reactive power constraint coefficients w.r.t. the control variables  $[\Delta Q_c]$ .

And a large number of double sided constraints of upper and lower limits on "generations", bus voltage limits and reactive power flows in transmission lines or transformers. All these constraints when transformed in terms of linear constraints on equivalent reactive generations can be concisely expressed as

$$L_j \min \leq [A_j] [\Delta Q_c] \leq L_j \max \quad (5.45)$$

The reactive power rescheduling problem as formulated above is exactly similar to its real power counterpart and the solution procedure for this problem closely follows that of the real power.

In equation (5.45) the rows  $[A_j]$  contain the sensitivities of the constraint  $j$  w.r.t. the control variables  $[AQ_c]$ . In the case of bus voltages and branch reactive power flow constraints (see Appendix 5.2a), it is much

easier to represent these constraints as linear function of bus voltages as

$$L_1 \min \leq [G_1] [\Delta V^0] \leq L_1 \max \quad (5.46)$$

Where  $L_1 \min$  and  $L_1 \max$  are the  $i$ th constraint's upper and lower limits. The sparse row vector  $[G_1]$  consists of zeros except for 1 in  $i$ th position. For reactive power flow constraint on  $i$ th branch connecting buses  $j$  and  $k$  the row vector  $[G_1]$  consists of only two non-zero elements  $G_{1j}$  and  $G_{1k}$  (see Appendix A5.2a) in the  $j$ th and  $k$ th position. The transformation of equation (5.46) in terms of equivalent reactive power injections is carried out using equation (5.41) as

$$L_1 \min \leq [G_1] [H^0]^{-1} [\Delta Q] \leq L_1 \max$$

The row  $A_1$  containing the sensitivities of constraint 1 w.r.t. control variables  $[\Delta Q_c]$  is obtained by extracting elements corresponding to the control variables from vector  $[D]$  which is obtained by solving

$$[G_1]^t = [H^0] [D] \quad (5.47)$$

The non sparse row  $[A_1]$  is obtained only when required. The limit checking for bus voltage are carried out using

$$L_1 \min \leq V_1 \leq L_1 \max \quad (5.48)$$

Where,

$$L_1 \max = V_1 \max - V_1^0$$



$$L_{i \min} = V_{i \min} - V_i^0$$

The branch reactive power flow limit checking is carried out using

$$L_{jk \min} \leq Q_{jk} \leq L_{jk \max} \quad (5.49)$$

Where,

$$L_{jk \max} = (F_{jk}^2 - P_{jk}^0)^{\frac{1}{2}} - Q_{jk}^0$$

and

$$L_{jk \min} = - (L_{jk \max} + 2Q_{jk}^0)$$

The superscript 0 indicates the initial values and  $F_{jk}$  is the MVA flow limit of branch connecting bus j and k.

#### 5.4.5 The Algorithm (Q)

The algorithm for reactive power rescheduling is exactly similar to that of real power rescheduling algorithm barring small differences in some details. The steps for the algorithm are as follows:

Step 1 : Using the initial power flow solution as the nominal schedule compute the limits on control variables. Using a load bus a reactive power reference bus compute,  $(N - 1) \times (N - 1)$   $[H^Q]$  matrix. Factorise and store the LU factors of  $[H^Q]$ .

Step 2 : Same as step 2 of algorithm (P).

Step 3 : Same as step 3 of algorithm (P) except replace  $(\Delta P_c)$  by  $(\Delta Q_c)$ .

Step 4 : Set up the first row of basis matrix as coefficient of  $[\Delta Q_c]$  in the reactive power balance equation (5.42).

Factorise the basis matrix into  $[B_f]$  and  $[B_b]$  associated with free and binding variables.

Step 5 : Compute the rated reactive power flow in all branches using

$$Q_{jk} \text{ rated} = (F_{jk}^2 - P_{jk}^2)^{\frac{1}{2}} \quad (5.50)$$

Store the indices of all branches with reactive power flow more than 60% of its rated in the array  $[ICS]$ .

Step 6 : Check for bus voltage limit and branch reactive power flow limit violations. If no violations go to step 22.

Step 7 : If any bus voltage limit violation go to (a), otherwise go to (b).

(a) Search for the most violated bus voltage. Let it be voltage at bus  $j$ . Set up the  $(N - 1)$  row vector,

$$[G_j] = (0 \quad 0 \quad \dots \quad 1 \quad 0 \quad 0 \quad 0) \quad (5.51)$$

↑  
jth position

and solve for  $(N - 1)$  vector  $[D]$  using

$$[G_j]^t = [H'] [D] \quad (5.52)$$

Go to step (8)

(b) Search for the branch with maximum reactive power flow violation. Let this be branch  $i$  connecting bus  $j$  and  $k$ .

Set up a  $(N - 1)$  vector

$$[G_i] = (0 \ 0 \ \dots \ G_{ij} \ \dots \ G_{ik} \ 0 \ 0) \quad (5.53)$$

$\begin{array}{ccc} \uparrow & & \uparrow \\ \text{jth} & & \text{kth} \\ \text{position} & & \text{position} \end{array}$

Where  $G_{ij}$  and  $G_{ik}$  are obtained as explained in Appendix

5.2a. Solve for the  $(N - 1)$  vector  $[D]$  using

$$[G_i]^t = [H'] [D] \quad (5.54)$$

Step 8 : Set up a  $m$ -vector  $[A]$  by extracting from  $[D]$  the elements corresponding to the controllable reactive power injection buses. If a transformer is present between buses  $p$  and  $q$  with tap at bus  $p$  then the element corresponding to the  $q$ th bus is multiplied by  $-\alpha$  (see Appendix 5.2b).

Factorise  $[A]$  into  $[A_f]$  and  $[A_b]$  corresponding to the free and binding variables.

Step 9 : Using equation (5.25) compute  $[S_f]$  and  $[S_b]$ , the sensitivities between the incoming constraint and the free and binding control variables, and check for the eligibility of binding constraints and control variables for becoming free. A binding constraint or variable  $j$  is eligible to be free only if (a) both the incoming constraint and the binding constraint or variable are either at their upper and lower limits and  $S_j$  is positive or (b) both are at their opposite limits and  $S_j$  is negative. A binding control variable at the corner of two cost segments is always eligible to be freed. But if the incoming constraint is at the upper limit and the binding control variable



$j$  is at the upper limit of its cost segment with  $S_j$  positive, then the cost segment pointer for variable  $j$  is change to that of segment to the right with the variable  $j$  working at the lower limit of the new cost segment.

Similar logic holds for all other possible cases.

Step 10 : If no binding constraint or control variable is eligible to be freed, then the problem is infeasible, go to step 19. Otherwise continue.

Step 11 : Same as step 11 in algorithm (P).

$$R_i = \left| \frac{\mu_i}{S_i} \right| ; i \in \text{eligible binding constraints} \\ \text{and control variables} \quad (5.55)$$

The eligible binding constraint or control variable with smallest  $R_i$  is freed. If a binding constraint (bus voltage or branch reactive power flow) is freed its row in the reduced basis  $[B_r]$  is removed; if a binding control variable is freed its status pointer is updated to show its new status.

Step 13 : The row  $[A]$  computed in step 8 for the violated constraint being enforced is inserted in the basis  $[B_r]$  and the contents of  $[B_r]$  are factored into  $[B_f]$  and  $[B_b]$  corresponding to free and binding variables respectively. The amount of change  $L_j$  needed to back off the violated incoming constraint to its limit is computed and stored in  $L_2$  in the same row in which  $[A]$  is stored in  $[B_r]$ .

Step 14, 15, 16 and 17 : Same as that in algorithm (P) except that replace  $[\Delta P]$  with  $[\Delta Q]$ .

Step 18 : Set up a  $(N - 1)$  vector  $[X]$  containing elements from  $[\Delta Q_c]$  for buses with equivalent reactive injections and zeros for buses where equivalent reactive injections are not available. Solve for changes in bus voltages using

$$[H'''] [\Delta V'''] = [X] \quad (5.56)$$

Compute the reference bus voltage using

$$\Delta V_r = - [E]^t [\Delta V'''] \quad (5.57)$$

for details of computation of  $[E]$  see Appendix 5.3, go to step 20.

Step 19 : Print problem infeasible. Either except the results with some constraints still violated and go to step 22, or relax the constraints and go to step 5.

Step 20 : If the reference bus voltage violates the bus voltage limit, compute  $(N - 1)$  vector  $D$  by solving

$$- [E]^t = [H'''] [D] \quad (5.58)$$

go to step 8.

Step 21 : Check for constraint violations for bus voltages and branch reactive power flows for the branches in array  $[ICS]$ . If any violations detected go to step 7. Otherwise continue.

Step 22 : Print results.

This ends the algorithm for reactive power rescheduling.

## 5.5 OVERALL SCHEME FOR SECURITY CONTROL

The decoupled real and reactive power rescheduling scheme is coordinated with the fast decoupled power flow to check for the accuracy of the LP rescheduling scheme. In case of any violations detected after the power flow solution, the process reenters the LP rescheduling scheme. The process is repeated till no violations are detected after the power flow solution.

For preventive control application the whole scheme is coordinated with the pattern recognition security assessment scheme. The overall scheme uses either preventive security control or contingency plan mode for preventive security control. The flow chart for the overall scheme is given in Fig. 5.4.



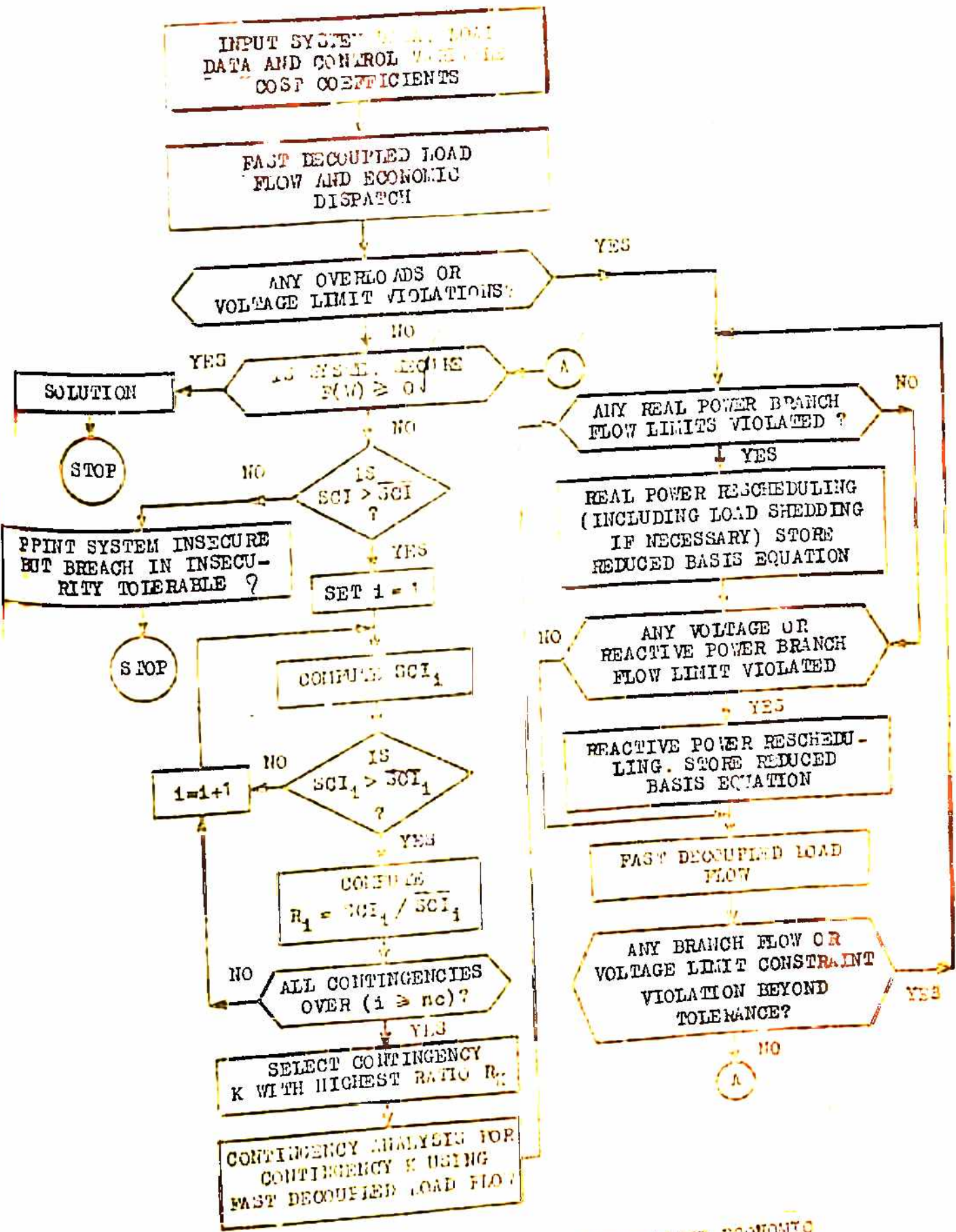


FIG.5.4 (a) FLOW CHART FOR SECURITY CONSTRAINED ECONOMIC DISPATCH USING PREVENTIVE SECURITY CONTROL.

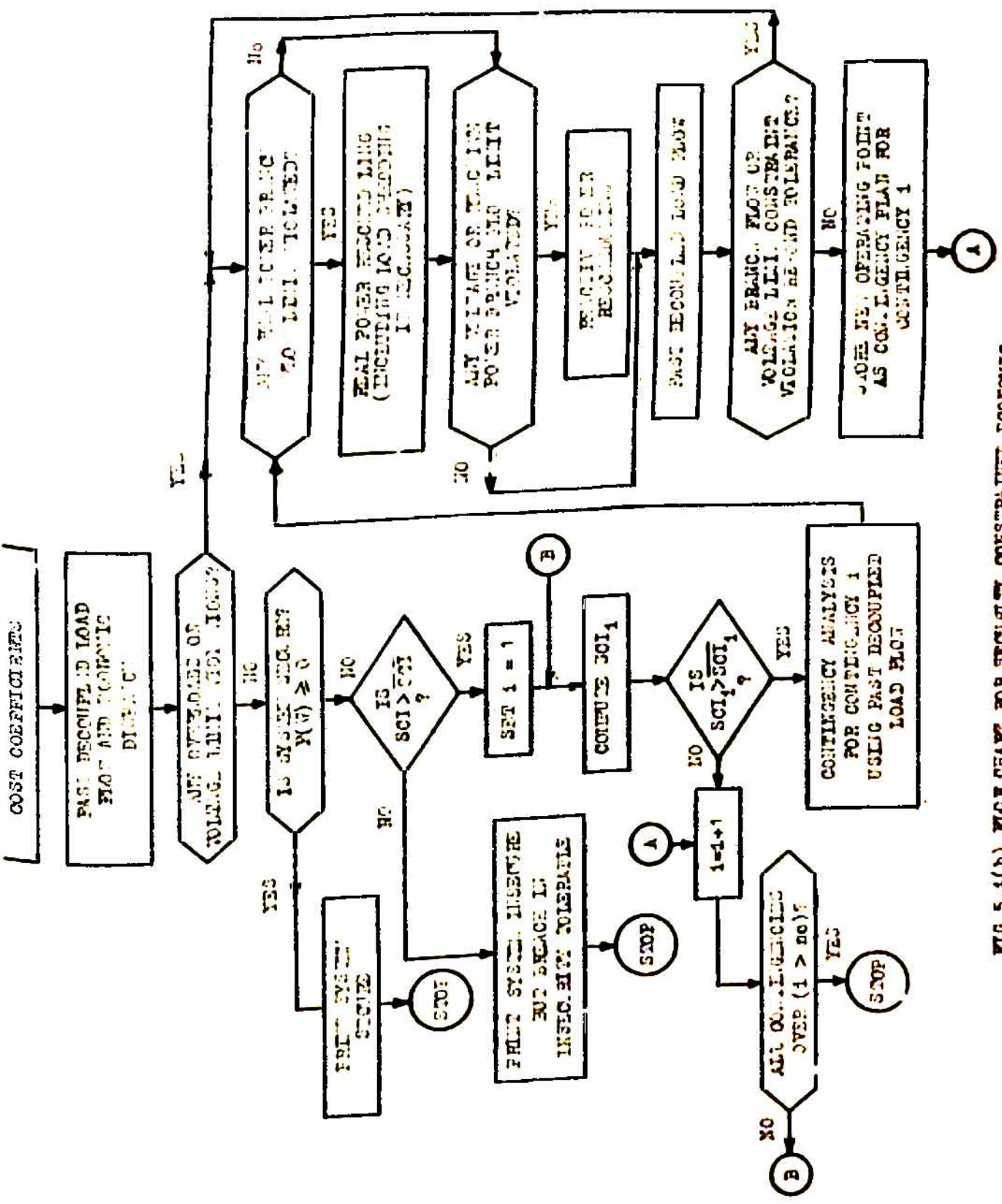


FIG. 5.4(b) FLOW CHART FOR SECURITY CONSTRAINED ECONOMIC DISPATCH USING CONTINGENCY PLAN METHOD.



## 5.6 RESULTS AND DISCUSSIONS

The security control method presented above was programmed and extensively tested on the three (5 Bus, 8 Bus and 14 Bus) systems. Some of the results are presented below to show the effectiveness of the method.

Different loading conditions were used for testing the algorithm. For each loading condition economic dispatch was carried out using the method proposed in Chapter 2. The resultant operating condition details are presented in Table 5.1 - 5.3.

### Security Assessment

For the given operating conditions security assessment was carried out using the security classifiers developed in Chapter 4. The results of security assessment along with the values of security function  $(F(W))$  are given in Table 5.4. For the 14 bus system the single disturbance classifiers developed in Chapter 4 were used to identify the contingencies which will cause the system to go into emergency operating state. The results of security assessment with single disturbance classifiers along with the security function values  $(F_i(W))$  for the postulated contingencies are shown in Table 5.5.

### Security Control Index

For the insecure operating state security control indices are calculated for the base case and individual contingencies. The values of security control indices  $(SCI_1)$  and its threshold



values ( $\overline{SCI}_i$ ) from Table 4.6 along with the ratio of  $SCI_i / \overline{SCI}_i$  are presented in Table 5.6.

### Security Control

In cases where SCI value for the base case is greater than  $\overline{SCI}$ , security control actions are computed using the algorithm presented in section (5.5). Both preventive security control and contingency plan algorithms are tested.

### Cost Functions

In case of real power rescheduling problem the cost functions used for real power generations at generator buses are identical with those used for economic dispatch. For load shedding at load buses the loads at different buses are classified as high priority loads and low priority loads. In order to make load shedding as last resort the incremental cost associated with the equivalent generations at load buses are kept high, (higher than the largest value for generators for low priority loads and twice that for high priority loads). All the quadratic cost functions associated with the controllable generations are modelled as piece-wise linear cost functions using 6 equal segments. In case of reactive power rescheduling problem there is no real cost associated with the control variables and the pseudo-costs associated with each variable merely provides the priorities for different controls. In this case the highest priority for control was accorded to transformers (lowest incremental cost) the next priority was

accorded to static capacitors (incremental cost twice that of transformers), and the generator reactive power changes were accorded the lowest priority (incremental cost twice that of static capacitors).

### Preventive Control

In preventive control scheme the contingency causing greatest severity (largest  $SCI/\overline{SCI}$  ratio) is taken up first for finding optimal control actions to satisfy the constraint violations due to this contingency. The reason for choosing this strategy is based on the heuristic reasoning (confirmed by the tests) that the control actions to satisfy the most severe insecurity condition will, in general, satisfy many of the less severe insecure conditions caused by other contingency conditions. This leads to less number of contingency evaluations (Table 5.8). The final operating condition and the changes in control variables for preventive security of the system is presented in Table (5.7) for both multisegment and single segment cost function representations. From the table it can be seen that the multisegment representation avoids large shifts in control variables by distributing the control action over larger number of variables and thus keeping the final operating state nearer to the base case (economic) state. In Table (5.9) the number of segments shifted for real power and reactive power rescheduling.

### Contingency Plan Method

For the contingency plan scheme the control actions for each individual contingency for which  $SCI > \overline{SCI}$  was computed. Some of the results are presented in Table (5.10) for the 14 Bus system. From the results it is seen that in contingency plan method the shifts in control variables due to rescheduling is much less than that for complete preventive control and thus the final operating state is more economical. Also the control actions need to be taken only after the contingency has really occurred.

The exact CPU time for the proposed security control method can not be given as IBM 1130 computer does not provide this information. But the computer time required including the output time for 14 Bus system preventive control and contingency plan method are given in Table (5.11).



TABLE 5.1

## 5 Bus System : System Operating Conditions

(a) Case I

Bus No.	Load		Generation		Operating Limits	
	Real Power (MW)	Reactive Power (MVAR)	Real Power (MW)	Reactive Power (MVAR)	$P_{max}$ (MW) $P_{min}$	$Q_{max}$ (MVAR) $Q_{min}$
1	40.0*	5.0	-	-	-	-
2	45.0**	15.0	-	-	-	-
3	60.0*	10.0	-	-	-	-
4	20.0	5.0	98.54	-32.53	100.00 0.00	50.00 -50.00
5	0.0	0.0	70.08	48.27	100.00 0.00	50.00 -50.00

Operating cost (Rs/h) = 20880.46

(b) Case II

Bus No	Load		Generation		Operating Limits	
	Real Power (MW)	Reactive power (MVAR)	Real Power (MW)	Reactive Power (MVAR)	$P_{max}$ (MW) $P_{min}$	$Q_{max}$ (MVAR) $Q_{min}$
1	28.0*	3.83	-	-	-	-
2	25.0**	7.50	-	-	-	-
3	40.0*	5.0	-	-	-	-
4	0.0	0.0	47.81	-21.73	100.00 0.00	50.00 -50.00
5	0.0	0.0	46.90	17.86	100.00 0.00	50.00 -50.00

Operating cost (Rs/h) = 10810.02

TABLE 5.2

## 8 Bus System : System Operating Conditions

(a) Case I

Bus No.	Load		Generation		Operating Limits	
	Real power (MW)	Reactive power (MVAR)	Real power (MW)	Reactive Power (MVAR)	$P^{\max}$ (MW) $P^{\min}$	$Q^{\max}$ (MVAR) $Q^{\min}$
1	240.0*	57.5	-	-	-	-
2	266.0*	61.0	-	-	-	-
3	244.0**	60.0	-	-	-	-
4	260.0**	60.0	-	-	-	-
5	40.0	0.0	191.55	135.0	350.00 50.00	175.00 -175.00
6	30.0	0.0	281.81	64.4	350.00 50.00	175.00 -175.00
7	0.0	0.0	207.03	-180.5	400.00 0.00	200.00 -200.00
8	20.0	0.0	326.70	194.6	600.00 50.00	300.00 -300.00

Operating cost (Rs/h) = 253417.66.

(b) Case II

Bus No.	Load		Generation		Operating Limits	
	Real power (MW)	Reactive power (MVAR)	Real power (MW)	Reactive power (MVAR)	$P^{\max}$ (MW) $P^{\min}$	$Q^{\max}$ (MVAR) $Q^{\min}$
1	340.00*	45.0	-	-	-	-
2	366.00*	60.0	-	-	-	-
3	444.00**	55.0	-	-	-	-
4	260.00**	60.0	-	-	-	-
5	40.0	0.0	350.0 <sup>+</sup>	135.0	350.00 50.00	175.00 -175.00
6	30.0	0.0	350.0 <sup>+</sup>	-2.71	350.00 50.00	175.00 -175.00
7	40.0	0.0	347.25	0.00	400.00 0.00	200.00 -200.00
8	50.0	0.0	532.69	79.90	600.00 50.00	300.00 -300.00

Operating cost (Rs/h) = 481240.44.



TABLE 5.3

## System Operating Conditions for 14 Bus System

(a) Case I

Bus No.	Load		Generation		Operating Limits	
	Real power (MW)	Reactive power (MVAR)	Real power (MW)	Reactive power (MVAR)	$P^{\max}$ (MW) $P^{\min}$	$Q^{\max}$ (MVAR) $Q^{\min}$
1	40.0*	7.5	-	-	-	-
2	35.0*	6.0	-	-	-	-
3	25.0	5.0	100.77	3.80	150.00 0.00	75.00 -75.00
4	20.0	2.5	75.0+	32.01	75.00 0.00	40.00 -40.00
5	65.0**	12.5	-	-	-	-
6	35.0*	6.0	-	-	-	-
7	25.0	2.5	91.87	26.30	150.00 0.00	75.00 -75.00
8	0.0	0.0	50.0*	2.12	50.00 0.00	25.00 -25.00
9	50.0*	10.0	-	-	-	-
10	40.0*	7.5	-	-	-	40.00 0.00
11	35.0*	6.0	-	-	-	40.00 0.00
12	65.0**	12.5	-	-	-	40.00 0.00
13	55.0	5.0	103.00	32.84	150.00 0.00	75.00 -75.00
14	50.0	5.0	95.49	-1.77	200.00 0.00	100.00 -100.00

Operating cost (Rs/h) = 63860.70

(b) Case II

Bus No.	Load		Generation		Operating Limits	
	Real power (MW)	Reactive power (MVAR)	Real power (MW)	Reactive power (MVAR)	$P_{\min}$ $P_{\max}$ (MW)	$Q_{\min}$ $Q_{\max}$ (MVAR)
1.	32.0*	6.0	-	-	-	-
2.	28.0*	5.0	-	-	-	-
3.	20.0	4.0	74.49	1.48	150.00 0.00	75.00 -75.00
4.	16.0	2.0	72.90	5.91	75.00 0.00	40.00 -40.00
5.	52.0**	4.0	-	-	-	-
6.	23.0*	2.4	-	-	-	-
7.	20.0	2.0	73.70	5.58	150.00 0.00	75.00 -75.00
8.	0.0	0.0	32.92	0.92	50.00 0.00	25.00 -25.00
9.	40.0**	8.0	-	-	-	-
10.	32.0*	3.0	-	-	-	40.00 0.00
11.	28.0*	2.4	-	-	-	-
12.	52.0*	10.0	-	-	-	40.00 0.00
13.	28.0	4.0	83.87	25.29	150.00 0.00	75.00 -75.00
14.	24.0	4.0	71.41	-2.08	200.00 0.00	100.00 -100.00

Operating cost (Rs/h) = 47660.641

(c) Case III

Bus No.	Load		Generation		Operating Limits	
	Real power (MW)	Reactive power (MVAR)	Real power (MW)	Reactive power (MVAR)	$p^{\max}$ (MW) $p^{\min}$	$Q^{\max}$ (MVAR) $Q^{\min}$
1.	52.0*	12.25	-	-	-	-
2.	81.2*	20.25	-	-	-	-
3.	0.0	0.0	75.2	8.77	150.00 0.00	75.00 -75.00
4.	20.0	0.0	75.0 <sup>+</sup>	40.00	75.00 0.00	40.00 -40.00
5.	53.8**	12.15	-	-	-	-
6.	73.6*	16.05	-	-	-	-
7.	15.0	2.5	84.2	6.67	150.00 0.00	75.00 -75.00
8.	0.0	0.0	45.2	1.73	50.00 0.00	25.00 -25.00
9.	40.0**	5.0	-	-	-	-
10.	21.3*	4.0	-	-	-	40.00 0.00
11.	21.3*	5.0	-	-	-	-
12.	22.3**	5.40	-	-	-	40.00 0.00
13.	15.0	2.50	87.8	33.66	150.00 0.00	75.00 -75.00
14.	30.0	5.0	93.6	-15.74	200.00 0.00	100.00 -100.00

Operating cost (Rs/h) = 54931.55

- \* Low priority load  
 \*\* High priority load  
 + Variable at high limit



TABLE 5.4

Security Assessment for the Operating Conditions (base case)

Operating Conditions		Security Assessment
		F(W), (Secure/Insecure)
5 bus system	Case I	-25.65, (Insecure)
	Case II	4.37, (Secure)
8 bus system	Case I	-5.21, (Insecure)
	Case II	1.68, (Secure)
14 bus system	Case I	-521.81, (Insecure)
	Case II	-76.32, (Insecure)
	Case III	-108.44, (Insecure)

TABLE 5.5

Security assessment for individual contingencies in  
14 bus system:

Contingency	Security Assessment $P_1$ (W)		
	Case I	Case II	Case III
branch 1-5 out	23.21	41.37	47.31
2-3 out	-6.21	11.25	7.94
3-4 out	1.37	6.32	4.69
4-5 out	-3.61	4.78	-11.27
4-7 out	-38.97	7.92	-26.13
6-12 out	15.13	26.87	19.27
7-9 out	-127.63	-89.34	-59.93
9-10 out	-231.74	-49.86	27.16
9-14 out	1.79	4.71	-1.62
10-11 out	-8.32	12.34	16.43
12-13 out	-2.45	-1.27	5.91
13-14 out	-8.21	6.53	6.94

TABLE 5.6

Security control indices for insecure operating conditions

(a) 5 bus system:

	Security control index (SCI)	Threshold value of SCI ( $\overline{\text{SCI}}$ )	SCI/ $\overline{\text{SCI}}$
Base case	0.179	$16.42 \times 10^{-3}$	10.90

(b) 8 bus system:

	Security control index (SCI)	Threshold value of SCI ( $\overline{\text{SCI}}$ )	SCI/ $\overline{\text{SCI}}$
Base case	0.0725	$9.32 \times 10^{-3}$	7.775



(c) 14 bus system:

Case I

Contingency	SCI	SCI	SCI/ SCI
base case	7.258	0.1893	38.545
branch 1-5 out	-	$2.78 \times 10^{-3}$	-
branch 2-3 out	$6.21 \times 10^{-3}$	$4.27 \times 10^{-3}$	2.234
branch 2-3 out	-	$0.88 \times 10^{-3}$	-
branch 4-5 out	$3.61 \times 10^{-3}$	$0.92 \times 10^{-3}$	3.924
branch 4-7 out	$38.97 \times 10^{-3}$	$7.32 \times 10^{-3}$	5.324
branch 6-12 out	-	$4.78 \times 10^{-3}$	-
branch 7-9 out	$121.63 \times 10^{-3}$	$21.56 \times 10^{-3}$	5.919
branch 9-10 out	$231.74 \times 10^{-3}$	$11.37 \times 10^{-3}$	20.382
branch 9-14 out	-	$0.96 \times 10^{-3}$	-
branch 10-11 out	$8.32 \times 10^{-3}$	$3.71 \times 10^{-3}$	2.242
branch 12-13 out	$2.45 \times 10^{-3}$	$0.69 \times 10^{-3}$	3.551
branch 13-14 out	$8.21 \times 10^{-3}$	$5.10 \times 10^{-3}$	1.609

Case II

Contingency	SCI	$\overline{\text{SCI}}$	SCI/ $\overline{\text{SCI}}$
base case	1.062	0.1883	5.637
branch 1-5 out	-	$2.78 \times 10^{-3}$	-
branch 2-3 out	-	$2.27 \times 10^{-3}$	-
branch 3-4 out	-	$0.38 \times 10^{-3}$	-
branch 4-5 out	-	$0.92 \times 10^{-3}$	-
branch 4-7 out	-	$7.32 \times 10^{-3}$	-
branch 6-12 out	-	$4.78 \times 10^{-3}$	-
branch 7-9 out	$89.34 \times 10^{-3}$	$21.56 \times 10^{-3}$	4.144
branch 9-10 out	$49.86 \times 10^{-3}$	$11.37 \times 10^{-3}$	4.385
branch 9-14 out	-	$0.96 \times 10^{-3}$	-
branch 10-11 out	-	$3.71 \times 10^{-3}$	-
branch 12-13 out	$1.27 \times 10^{-3}$	$0.69 \times 10^{-3}$	1.840
branch 13-14 out	-	$5.10 \times 10^{-3}$	-

Case III

Contingency	SCI	$\overline{SCI}$	SCI/ $\overline{SCI}$
Base case	1.508	0.1883	8.010
branch 1-5 out	-	$2.78 \times 10^{-3}$	-
branch 2-3 out	-	$4.27 \times 10^{-3}$	-
branch 3-4 out	-	$0.88 \times 10^{-3}$	-
branch 4-5 out	$11.27 \times 10^{-3}$	$0.92 \times 10^{-3}$	12.25
branch 4-7 out	$26.13 \times 10^{-3}$	$7.32 \times 10^{-3}$	3.569
branch 6-12 out	-	$4.78 \times 10^{-3}$	-
branch 7-9 out	$59.93 \times 10^{-3}$	$21.56 \times 10^{-3}$	2.780
branch 9-10 out	-	$11.37 \times 10^{-3}$	-
branch 9-14 out	$1.62 \times 10^{-3}$	$0.96 \times 10^{-3}$	6.687
branch 10-11 out	-	$3.71 \times 10^{-3}$	-
branch 12-13 out	-	$0.69 \times 10^{-3}$	-
branch 13-14 out	-	$5.10 \times 10^{-3}$	-



TABLE 5.7

## Changes in Control Variables for Preventive Security Control

(a) 5 Bus system (Case I)

Bus No.	Changes in control variables				Secure operating point			
	Using multisegment cost functions		Using single segment linear cost functions		Using multisegment cost functions		Using single segment linear cost functions	
	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)
1	-10.17**	-1.36	-20.24	-2.60	-29.83**	-3.64	-19.76**	-2.50
2	-	0.0	0.0	0.0	-45.00	-10.00	-45.00	-10.00
3	-10.00**	-1.67	0.0	0.0	-50.00**	-8.35	-60.00	-10.00
4	1.46*	0.0	1.46	0.0	100.00*	-32.53	100.00	-32.53
5	-23.35	-12.10	-23.31	-12.13	46.73	36.21	46.74	36.18
Operating cost (Rs/h)					23293.94		23518.80	

(b) 8 Bus system (Case II)

Bus No.	Change in control variables			
	Using multisegment cost functions		Using single segment linear cost functions	
	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)
1	0.0	0.0	0.0	0.0
2	-70.63**	-11.58	-70.63**	-11.58
3	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0
6	-19.11	0.0	-19.11	0.0
7	0.0	0.0	0.0	0.0
8	-51.87	-12.63	-51.87	-12.63
Operating cost (Rs/h)				

### Secure operating point

Using multisegment cost functions      Using single segment linear cost functions

P (MW)	Q(MVAR)	P(MW)	Q(MVAR)
-340.00	-45.00	-340.00	-45.00
-295.37	-48.42	-295.37	-48.42
-444.00	-55.00	-444.00	-55.00
-260.00	-60.00	-260.00	-60.00
350.00	135.00	350.00	135.00
330.89	-2.71	330.89	2.71
347.25	0.00	347.25	0.00
480.82	67.27	480.82	67.27
496884.28		496884.28	



## (c) 14 Bus system (Case I)

Bus No.	Changes in control variables				Secure operating point			
	Using multisegment cost functions		Using single segment linear cost functions		Using multisegment cost functions		Using single segment linear cost functions	
	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)
1	0.0	0.0	0.0	0.0	-40.0	-7.50	-40.0	-7.50
2	0.0	0.0	0.0	0.0	-35.0	-6.00	-35.0	-6.00
3	-0.77	0.0	0.0	0.0	100.00	-3.80	100.77	-3.80
4	0.0	0.0	0.0	0.0	75.00**	32.01	75.00*	32.01
5	0.0	$t_5 = +3.1\%$	0.0	$t_5 = +3.1\%$	-65.00	$t_5 = +3.1\%$	-65.00	$t = 3.1\%$
6	0.0	0.0	0.0	0.0	-35.00	-6.00	-35.00	-6.00
7	-24.75	0.0	0.0	0.0	67.12	26.30	91.87	26.30
8	-16.67	0.0	-41.43	0.0	33.33	2.12	9.57	2.12
9	0.0	0.0	0.0	0.0	-50.00	-10.00	-50.00	-10.00
10	-6.63**	36.3	0.0	40.0	-33.37**	28.80	-40.00	32.5
11	-5.83**	-1.0	-12.26	0.0	-29.17**	-5.00	-22.74	-3.93
12	0.0	37.8	0.0	37.3	-65.00	25.3	-65.00	24.8
13	22.00	-18.61	21.84	-21.60	125.0	14.23	124.84	11.24
14	8.13	-21.64	8.93	-20.3	103.62	-23.41	104.42	-22.07
Operating cost (\$/h)					66017.08**		68257.83**	

(c) 11. 14 Bus system (Case II)

Bus No.	Change in control variables				Secure operating point				
	Using multisegment cost functions		Using single segment linear cost functions		Using multisegment cost functions		Using single segment linear cost functions		
	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)	
1	-	-	-	-	-32.00	-6.00	-32.00	-6.00	
2	-	-	-	-	-28.00	-5.00	-28.00	-5.00	
3	-	-	-	-	74.49	1.48	74.49	1.48	
4	-9.38	-	-9.38	-	63.52	5.91	63.52	5.91	
5	-	-	-	-	-52.00	-4.00	-52.00	-4.00	
6	-	-	-	-	-28.00	-2.40	-28.00	-2.40	
7	-8.82	-	-8.82	-	64.88	5.58	64.88	5.58	
8	-	-	-	-	32.92	0.92	32.92	0.92	
9	-	-	-	-	-40.00	-8.0	-40.00	-8.0	
10	-	17.13	-	17.13	-32.00	14.13	-32.00	14.13	
11	-	-	-	-	-28.00	-2.40	-28.00	-2.40	
12	-	18.01	-	18.01	-52.00	8.01	-52.00	-8.01	
13	16.13	-	16.93	-	100.00	25.29	100.00	25.29	
14	2.79	-11.26	2.00	-11.26	74.20	-13.34	73.41	-13.34	
				Operating cost (Rs/h)				48105.38	
								48065.53	



## (c) iii 14 Bus system (Case III)

Bus No.	Changes in control variables				Secure operating point	
	Using multisegment cost functions		Using single segment linear cost functions		Using multisegment cost functions	Using single segment linear cost functions
	P (MW)	Q (MVAR)	F (MW)	Q (MVAR)	P (MW)	Q (MVAR)
1	-	-	-	-	-52.00	-12.25
2	-	-	-	-	-31.20	-20.25
3	12.40	-	12.40	-	87.60	8.77
4	-	-	-	-	75.00*	40.00
5	-	$t_5 = 4.1\%$	-	$t_5 = 4.1\%$	-53.60	9.13
6	-	-	-	-	-73.60	-16.05
7	-25.01	-	-	-	59.19	6.67
8	-11.87	-	26.89	-	33.33	1.73
9	-	-	-	-	-40.0	-5.0
10	-	-	-	-	-21.3	-4.0
11	-	-	-	-	-21.3	-5.0
12	-	9.41	-	9.41	-22.3	4.01
13	12.20	-	25.92	-	100.00	33.66
14	13.81	-7.32	-	-7.32	107.41	-23.06
Operating cost (\$/h)					55536.11	58287.37

\* Variable at limit

\*\* Load shed

\*\*\* Operating cost includes reserment cost of load shed.



TABLE 5.8

Number of contingency evaluations needed:

Operating condition	Method using ( $\overline{SCI} / \overline{SCI}$ ) ratio	Without using ( $\overline{SCI} / \overline{SCI}$ ) ratio
Case I	3	5
Case II	2	4
Case III	3	3

TABLE 5.9

Changes in number of segments of cost functions

Operating	Real power rescheduling	Reactive power rescheduling	Number of load flows in loop 6
5 Bus (Case I)	4	0	2
8 Bus (Case II)	4	0	2
14 Bus (Case I)	13	7	4
(case II)	8	4	3
(Case III)	11	3	3

## Changes in Control Variables for Contingency Plan Method

(a) Outage in branch 15 (7-9); (Case I) 14 Bus system

Bus No.	Changes in control variables				Secure operating point			
	Using multisegment cost functions		Using single segment linear cost functions		Using multisegment cost functions		Using single segment linear cost functions	
	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)
1	0.0	0.0	0.0	0.0	-40.00	-7.50	-40.00	-7.50
2	0.0	0.0	0.0	0.0	-35.00	-6.00	-35.00	-6.00
3	0.0	0.0	0.0	0.0	100.77	-3.80	100.77	-3.80
4	-0.0	0.0	-0.0	0.0	75.00	32.01	75.00	32.01
5	0.0	0.0	0.0	0.0	-65.00	-12.50	-65.00	-12.50
6	0.0	0.0	0.0	0.0	-35.00	-6.00	-35.00	-6.00
7	-24.09	0.0	0.0	0.0	67.78	26.30	91.87	26.30
8	-16.67	0.0	-41.36	0.0	33.33	2.12	8.64	2.12
9	0.0	0.0	0.0	0.0	-50.00	-10.00	-50.00	-10.00
10	0.0	0.0	0.0	0.0	-40.00	-7.50	-40.00	-7.50
11	0.0	0.0	0.0	0.0	-35.00	-6.00	-35.00	-6.00
12	0.0	36.80	0.0	36.80	-65.00	24.30	-65.00	24.30
13	22.00	-24.32	29.23	-24.32	125.00	8.52	124.23	8.52
14	20.26	0.0	12.68	0.0	115.75	-1.77	116.17	-1.77
Operating cost (\$/h)					65018.317		66767.34	



(b) Outage in branch 17(9-10); (Case II) 14 Bus system

Bus No.	Changes in control variables				Secure operating point			
	Using multisegment cost functions		Using single segment linear cost functions		Using multisegment cost functions		Using single segment linear cost functions	
	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)
1	0.0	0.0	0.0	0.0	-32.00	-6.00	-32.00	-6.00
2	0.0	0.0	0.0	0.0	-28.00	-5.00	-28.00	-5.00
3	0.0	0.0	0.0	0.0	74.49	1.48	74.49	1.48
4	0.0	0.0	0.0	0.0	72.90	5.91	72.90	5.91
5	0.0	0.0	0.0	0.0	-52.00	-4.00	-52.00	-4.00
6	0.0	0.0	0.0	0.0	-28.00	-2.40	-28.00	-2.40
7	-13.29	0.0	-13.29	0.0	60.41	5.58	60.41	5.58
8	0.0	0.0	0.0	0.0	32.92	0.92	32.92	0.92
9	0.0	0.0	0.0	0.0	-40.00	-8.00	-40.00	-8.00
10	0.0	17.13	0.0	17.13	-32.00	14.13	-32.00	14.13
11	0.0	0.0	0.0	0.0	-28.00	-2.40	-28.00	-2.40
12	0.0	18.01	0.0	18.01	-52.00	8.01	-52.00	8.01
13	16.13	0.0	16.64	0.0	100.00	25.29	100.51	25.29
14	0.28	-11.26	0.0	-11.26	71.69	-13.34	71.41	-13.34
Operating cost (\$/h)					48367.08		48421.052	



(c) Outage in branch 15 (7-9), (Case III) 14 Bus system:

Bus No.	Changes in control variables				Secure operating point				
	Using multisegment cost functions		Using single segment linear cost functions		Using multisegment cost functions		Using single segment linear cost functions		
	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	
1	0.0	0.0	0.0	0.0	-52.00	-12.25	-52.00	-12.25	
2	0.0	0.0	0.0	0.0	-81.20	-20.25	-81.20	-20.25	
3	24.8	0.0	33.14	0.0	100.00	8.77	100.34	8.77	
4	0.0	0.0	0.0	0.0	75.00	40.00	75.00	40.00	
5	0.0	0.0	0.0	0.0	-53.80	-12.15	-55.00	-12.15	
6	0.0	0.0	0.0	0.0	-73.60	-16.05	-75.60	-16.05	
7	-26.03	0.0	0.0	0.0	58.17	6.67	84.2	6.67	
8	-11.87	0.0	-37.91	0.0	33.33	1.73	7.25	1.73	
9	0.0	0.0	0.0	0.0	-40.00	-5.00	-40.00	-5.00	
10	0.0	0.0	0.0	0.0	-21.30	-4.00	-21.30	-4.00	
11	0.0	0.0	0.0	0.0	-21.30	-5.00	-21.30	-5.00	
12	0.0	0.0	0.0	0.0	-22.30	-5.40	-22.30	-5.40	
13	12.20	0.0	4.61	0.0	100.00	33.66	92.61	33.66	
14	0.0	0.0	0.0	0.0	93.60	-15.74	93.60	-15.74	
Operating cost (\$/h)					55340.08				57287.24

TABLE 5.11

Time taken for complete security control scheme:

	Preventive security	Contingency plan
Maximum time taken	8.33 min	11.28 min
Minimum time taken	1.48 min	1.48 min
Average time taken	4.76 min	5.34 min

## 5.7 CONCLUSION

In this chapter the security control problem is formulated. Different existing techniques are reviewed and it is shown that the LP method fulfils the desired security control function requirements better than the nonlinear programming methods. An LP method based on dual revised simplex algorithm is presented. The decoupling of real and reactive power problem is carried out to save the computer storage requirements as well as computation time. The storage requirements are further reduced by using the reduced basis formulation. The nonlinear cost functions are modelled by multisegment piece-wise linear cost functions for greater accuracy. A heuristic method for incorporating piece-wise linear cost functions without increasing the 'basis' matrix size is used.

Multisegment piece-wise linear representation of the cost function is more accurate and avoids large changes in control variables keeping the system nearer to the most economic (base case) operating state.

The use of pattern recognition method for security assessment provides very fast security assessment and single disturbance classifiers identify the particular contingencies for which the system operating condition will become insecure. This avoids unnecessary contingency evaluations for all the



contingencies under consideration. Also security assessment for the operating state after rescheduling is checked using the security control index. This makes the security control scheme very fast and suitable for on-line application.

CHAPTER 6CONCLUSIONS6.1 CONCLUSIONS

In this thesis a method for secure economic dispatch is presented. The model is based on decomposition of the large and difficult problem of security constrained economic dispatch into three subproblems namely: economic dispatch, security assessment and security control. Each of the subproblems are solved independently by simple and fast algorithms and are coordinated to obtain the solution for the complete problem.

A model for economic dispatch is presented where the effect of transmission losses are taken care by penalty factors. A method based on system admittances and voltages is used for calculating the penalty factors. The use of LU factors of the fast decoupled load flow matrix results in saving in storage as well as computation time. The economic dispatch is obtained using a model in which the coordination equations, penalty factors calculation and fast decoupled load flow are used in an iterative scheme.

The security assessment problem is modeled as a pattern recognition problem. The pattern recognition model used is deterministic and is a two stage process. The first stage involves generation of security classifiers using a large set of patterns with known classification and is an off-line process. The second stage involves using the security classifiers generated

in the first stage to classify patterns (power system operating state) of unknown classification. The second stage requires negligible computation effort and can be used for on-line applications.

The large amount of computational effort required during the training process does not constitute a serious drawback as it is to be done off-line. Also the bulk of effort required in generating the training set patterns will be mitigated when the method is applied to practical systems, as large number of training samples will be available from the system planning study records.

In a power system the network topology does not remain fixed all the time and this has been modelled by including the planned outages of the transmission equipment as logical variables in the pattern vector. It is not necessary to use all the variables in the pattern vector to obtain a good security classifier, rather the use of large number of variables (features) sometime leads to inefficient classifier design. A model based on Fisher linear discriminant is used to select only highly efficient features and thus reduce the number of features used for security classifier design. The method is simple, fast and does not involve the time consuming computation of eigenvalues and eigenvectors of a large matrix as is case of Karhunen-Loe've expansion method.



The problem of security classifier design is formulated as an optimisation problem involving solution of linear inequalities. The strict convexity of the objective function for the inconsistent case (a proof for this is provided) ensures that security classifiers with optimum classification efficiency is obtained even when the problems are not linearly separable. This is a significant advantage as pattern vectors for power system security assessment problem in general are linearly inseparable. The conjugate gradient method of Fletcher-Reeve's is used to minimise the objective function. This results in considerable saving of storage as no matrices need to be stored. The test results indicate that the security classifiers obtained have high classification efficiency with very little misclassification.

Single disturbance classifiers have been obtained for each member in the contingency list. These single disturbance classifiers are effectively used later in security control application to identify the contingencies which will lead to insecure operating conditions.

A security control index (SCI) is proposed. For security control applications the proposed SCI has distinct advantage over global security indices proposed by Patton and others. The computational effort required for calculating SCI is negligible. The SCIs calculated for different contingencies in the contingency list have been effectively used for identifying contingencies which will cause dangerous operating conditions and thus

needing initiation of suitable security control actions, from for those which there is no immediate danger and so do not require any preventive control action to be taken.

A model for security control problem is presented. The model is based on decomposition of the real and reactive power problems, the decoupled real and **reactive power problem** is solved independently.

The real power control **problem** **adjusts** the controllable generations in an optimal way to alleviate any branch overloads in the intact network or in contingencies. The problem is formulated as a dual simplex LP problem with the objective to **minimize** the total operating cost. The linearized model for power system network is obtained using fast decoupled load flow **matrix** and the **nonlinear generator** cost curves are modeled using piece-wise linear cost curves. Two heuristic modifications are presented which are **along with reduced basis** formulation to reduce the storage requirement and the **computation** time.

Load shedding is included as a control variable by considering the amount of load shed as equivalent generation. To avoid indiscriminate load shedding resentment costs are associated with load shedding. To **make** load shedding a last resort these costs for load shedding are always kept higher than the operating costs for the generators. The priorities of different loads are incorporated by according higher costs for high priority loads.



A reactive power security control model developed adjusts the transformer taps and reactive generations at different locations to overcome any violations of bus voltage limits and reactive power flows in the branches for intact network as well as contingency condition. The model considers all reactive power control variables including transformer taps as equivalent reactive generation at the bus-bars. This enables to arrive at a simple model for reactive power control which <sup>is</sup> solved by the dual LP method similar to real power problem. The objective function used is the weighted least square deviation from initial condition and is modeled by piecewise linear segments. The network linearisation for reactive power model is carried out using the  $[B'']$  matrix of the fast decoupled load flow which is augmented for the slack bus and the PV buses. To avoid illconditioning of this augmented matrix a load bus is considered a reference bus with its reactive power remaining constant. This leads to a reactive power balance equation similar to that in the real power case, the row and column corresponding to the reference bus are deleted and the elements of  $[B'']$  are suitably modified.

The decoupled real and reactive power control model are iterated with the fast decoupled load flow in the outer loop to obtain an accurate solution for the full ac security control problem. Two security control options are explored. The preventive security control option adjusts the control variables to keep the system always in a secure state. Whereas the contingency plan option finds the security control actions required for all contingent event, but does not initiate any control action rather



it stores the required actions for each contingent event as a contingency plan which is to be actuated if and when the particular contingency really occurs. The contingency plan method appears more prudent as it avoids unnecessary departure from the economic operating condition for contingent events which may not occur at all.

An algorithm for secure economic dispatch is developed by coordinating the algorithm models developed above. The whole scheme is reliable, accurate and fast making it suitable for real-time applications.

The algorithm has been tested on three sample systems and uses about 10.5 K words of memory for the programmes with two links and 12 local subroutines. It uses about 4K words for variables when used for 14 bus system. The core memory of the IBM 1130 computer available was limited to 16K words only. This prevented us from testing the algorithm on larger systems.

## 6.2 SCOPE FOR FURTHER WORK

It will be a worthwhile attempt to extend the pattern recognition method discussed in Chapter 4 to transient security assessment. A major hurdle in this is the excessively large computational effort required for generating the training set patterns, and some method should be found to overcome this problem. Two simple approaches towards this can be (i) use of some fast transient security analysis methods (recently some progress in this direction have been reported<sup>(117)</sup>), and (ii) developing some training process which does not require large

training set to generate the classifier with high confidence level.

Some further work also needs to be done to find a method which automatically updates the security classifiers as new generator or load buses are added into the system.

A logical extension of the security control method discussed in Chapter 5 will be to extend it to hydro-thermal systems. Environmental constraints may also be included.

## APPENDIX 1

### Details of the Test Systems

The three test systems used for testing the algorithms developed in this thesis are:

- (i) 5 Bus/7 Lines/2 Generator system.
- (ii) 8 Bus/14 Lines/4 Generator system. and
- (iii) 14 Bus/20 Lines (2 Transformers)/6 Generator system.

The topological configurations for the above three systems are shown in Fig. A1.1-A1.3.

The details of the system parameters (line impedances) *along with the branch power flow limits* in terms of MVA flow limit and maximum voltage angle separation are given in Tables A1.1(a), A1.2(a) and A1.3(a) for three systems. Two power flow limits-normal and emergency are given for both MVA flow limit and voltage phase angle separation. The emergency limits are 20% higher than the normal steady state limit as transmission equipments can be subjected to small over loads for short duration of time (say 30 minutes), and are used for limit checking during outage conditions.

Tables A1.1(b), A1.2(b) and A1.3(b) present the bus loading data for peak load condition along with the load flow results for this loading condition for the three systems.

The quadratic cost function ( $C_i = a_i P_i^2 + b_i P_i + d_i$ ) parameters for the generators and loads (for load shedding) are given in Tables A1.1(c), A1.2(c) and A1.3(c). The



voltage magnitude limits and cost coefficient parameters for reactive power changes is presented at the end.

TABLE A1.1

5 Bus system

(a) System parameters

Sl. No.	Bus designation p - q	Line Impedance			Line MVA flow limit		$ \theta_p - \theta_q _{\max}^*$	
		R (p.u.)	X (p.u.)	$Y_{sh}^{1/2}$ (p.u.)	Normal	Emergency	Normal	Emergency
1	1 - 2	0.08	0.24	0.025	60.0	72.0	7.5	9.0
2	1 - 4	0.04	0.12	0.015	75.0	90.0	5.0	6.0
3	2 - 3	0.01	0.03	0.010	100.0	120.0	1.5	1.8
4	2 - 4	0.06	0.18	0.020	50.0	60.0	5.0	6.0
5	3 - 4	0.06	0.18	0.020	50.0	60.0	5.0	6.0
6	3 - 5	0.08	0.24	0.025	50.0	60.0	5.0	6.0
7	4 - 5	0.02	0.06	0.030	100.00	120.0	3.0	3.6

\* Angles in degrees.

(b) Bus data:

Bus No.	Load		(Peak load) Generation		Voltage	
	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	Magnitude (p.u.)	Angle (°)
1	40.0	5.0	-	-	1.009	-3.538
2	45.0	15.0	-	-	1.004	-4.019
3	60.0	10.0	-	-	1.007	-3.937
4	20.0	5.0	98.55	-32.54	1.03	-0.677
5	0.0	0.0	70.08	48.27	1.06	0.0

(c) Cost function data:

Generation:

Bus No.	a	b	d
4	0.65	43.0	2000.0
5	0.83	35.0	1800.0

Load Shedding:

	a	b	d
Low priority load	1.0	250.0	0.0
High priority load	2.0	500.0	0.0

TABLE A1.2

8 Bus system:

(a) System parameters

Sl. No.	Bus designation	Line Impedance			Line MVA flow limit		$ \theta_p - \theta_q $ max	
		R (p.u.)	X (p.u.)	$Y_{sh}/2$ (p.u.)	Normal	Emergency	Normal	Emergency
1	1-5	0.160	2.100	0.000	450.0	540.0	5.0	6.0
2	1-6	0.100	1.500	0.000	500.0	600.0	4.0	4.8
3	1-7	0.210	3.110	0.000	350.0	420.0	5.0	6.0
4	2-6	1.100	8.100	0.080	150.0	180.0	5.0	6.0
5	2-7	0.320	3.000	0.000	300.0	360.0	5.0	6.0
6	2-8	1.000	7.000	0.050	150.0	180.0	5.0	6.0
7	3-5	1.000	7.000	0.050	150.0	180.0	5.0	6.0
8	3-7	0.200	1.300	0.000	400.0	480.0	3.0	3.6
9	3-8	0.210	1.000	0.000	500.0	600.0	3.0	3.6
10	4-5	0.210	1.000	0.000	500.0	600.0	3.0	3.6
11	4-6	0.200	1.300	0.000	400.0	480.0	3.0	3.6
12	4-8	0.200	1.000	0.000	500.0	600.0	3.0	3.6
13	5-8	0.200	1.000	0.000	500.0	600.0	3.0	3.6
14	6-7	0.350	2.000	0.000	450.0	540.0	5.0	6.0
		0.750	6.300	0.060	200.0	240.0	5.0	6.0
		0.800	6.500	0.030	200.0	240.0	5.0	6.0
		0.300	3.000	0.000	300.0	360.0	5.0	6.0
		0.250	2.300	0.000	400.0	480.0	5.0	6.0

\* Angles in degrees.



(b) Bus data:

(Peak load)

Bus No.	Load		Generation		Voltage	
	P(MW)	Q(MVAR)	P(MW)	Q(MVAR)	Magnitude (p.u.)	Angle (°)
1	340.0	45.0	-	-	1.028	-1.722
2	366.0	60.0	-	-	1.0178	-3.015
3	444.0	55.0	-	-	1.0310	-1.217
4	260.0	60.0	-	-	1.026	-2.104
5	40.0	0.0	350.0*	135.0	1.038	-0.596
6	30.0	0.0	350.0*	-2.72	1.030	-0.351
7	40.0	0.0	353.0	0.70	1.030	-0.579
8	50.0	0.0	527.0	79.90	1.040	0.000

\* Generation at upper limit.

(c) Cost function data:

(i) Generations

Bus No.	a	b	d
5	0.65	43.0	2000.0
6	0.65	43.0	2000.0
7	0.83	35.0	1800.0
8	0.51	50.0	2400.0

(ii) Load Shedding

	a	b	d
High priority load	2.0	1400.0	0.0
Low priority load	1.0	700.0	0.0



TABLE A1.3

14 Bus system :

(a) System parameters :

Sl. No.	Bus designation p-q	Line Impedance			Line MVA flow limit		$ \theta_p - \theta_q $ max	
		R (p.u.)	X (p.u.)	$Y_{sh}/2$ (p.u.)	Normal	Emergency	Normal	Emergency
1	1-2	0.01938	0.05917	0.02640	150.0	180.0	5.0	6.0
2	1-5	0.05403	0.22304	0.02460	100.0	120.0	10.0	12.0
3	2-3	0.04699	0.19797	0.02190	75.0	90.0	7.5	9.0
4	2-4	0.05811	0.17388	0.01870	75.0	90.0	7.5	9.0
5	2-5	0.05695	0.17388	0.01700	75.0	90.0	7.5	9.0
6	3-4	0.06701	0.17103	0.01730	75.0	90.0	7.5	9.0
7	4-5	0.01335	0.04211	0.00640	175.0	210.0	5.0	6.0
8	4-7	0.00000	0.20912	0.00000	75.0	90.0	7.5	9.0
9	4-9	0.00000	0.55618	0.00000	50.0	60.0	7.5	9.0
10	5-6	0.00000	0.25202	0.00000	50.0	60.0	5.0	6.0
11	6-11	0.09498	0.19890	0.00000	50.0	60.0	7.5	9.0
12	6-12	0.12291	0.25591	0.00000	75.0	90.0	10.0	12.0
13	6-13	0.06615	0.13027	0.00000	150.0	180.0	15.0	18.0
14	7-8	0.00000	0.17615	0.00000	75.0	90.0	7.5	9.0
15	7-9	0.00000	0.11001	0.00000	75.0	90.0	5.0	6.0
16	9-10	0.03181	0.08450	0.00000	100.0	120.0	5.0	6.0
17	9-14	0.12711	0.19207	0.00000	100.0	120.0	5.0	6.0
18	10-11	0.08205	0.19207	0.00000	75.0	90.0	10.0	12.0
19	12-13	0.22092	0.19988	0.00000	75.0	90.0	7.5	9.0
20	13-14	0.17093	0.23802	0.00000	75.0	90.0	15.0	18.0
					150.0	180.0	20.0	24.0

\* Angles in degrees.

## Transformer data:

Branch No.	Bus designation (tap side)	Tap position
10	5-6 (5)	0.0%
15	7-9 (9)	+2.0%

## Static capacitor data:

Bus No.	MVAR Limit
10	+40.0
12	+40.0

## (b) Bus data : (peak load)

Bus No.	Load		Generation		Voltage	
	P (MW)	Q (MVAR)	P (MW)	Q (MVAR)	Magnitude (p.u.)	Angle (°)
1	40.0	7.50	-	-	0.99522	-5.73087
2	35.0	6.00	-	-	1.00316	-4.67295
3	25.0	5.00	100.77	1.055	1.03000	0.40465
4	20.0	2.50	75.00*	29.68	1.02000	-2.62225
5	65.0	12.50	-	-	0.99941	-4.69864
6	35.0	6.00	-	-	0.97104	-7.84706
7	25.0	2.50	91.87	9.40	1.02000	1.39107
8	0.0	0.00	50.00*	2.12	1.02000	6.24725
9	50.0	10.0	-	-	1.02000	-3.58112
10	40.0	7.50	-	-	0.98670	-6.46057
11	35.0	6.00	-	-	0.95642	-9.07082
12	65.0	12.50	-	-	0.90712	-10.29386
13	35.0	5.0	103.00	35.20	1.02000	-4.81094
14	30.0	5.0	95.49	-7.46	1.05000	0.00000

\* Generation at upper limit.

(c) Cost function data:

(i) Generation

Bus No.	a	b	d
3	0.65	43.0	2000.0
4	0.65	43.0	2000.0
7	0.83	35.0	1800.0
8	1.05	45.0	0.0
13	0.65	43.0	2000.0
14	0.51	50.0	2400.0

(ii) Load Shedding

	a	b	d
High priority load	2.0	500.0	0.0
Low priority load	1.0	250.0	0.0



Voltage limits and cost coefficients for reactive power rescheduling:

The upper and lower voltage magnitude limits at all the buses are 1.10 p.u. and 0.95 p.u. respectively for all the three test system considered.

For reactive power generation changes the cost function is considered as  $C_i = \alpha_i (\Delta Q_i)^2$  with the limits on  $Q$  as

$$Q_i \max = Q_i \max^0 - Q_i^0$$

$$Q_i \min = Q_i^0 - Q_i \min^0$$

For all transformers  $\alpha_i = 1.0$

For all static capacitor banks  $\alpha_i = 2.0$

and For all generators  $\alpha_i = 4.0$

in all the three test systems considered.

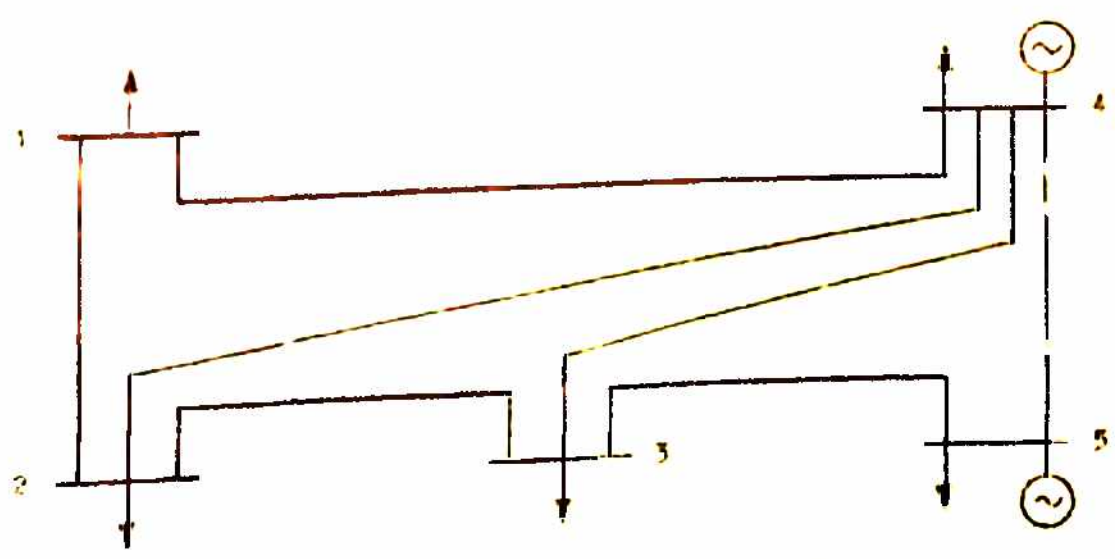


FIG.A1.1 5 BUS SYSTEM

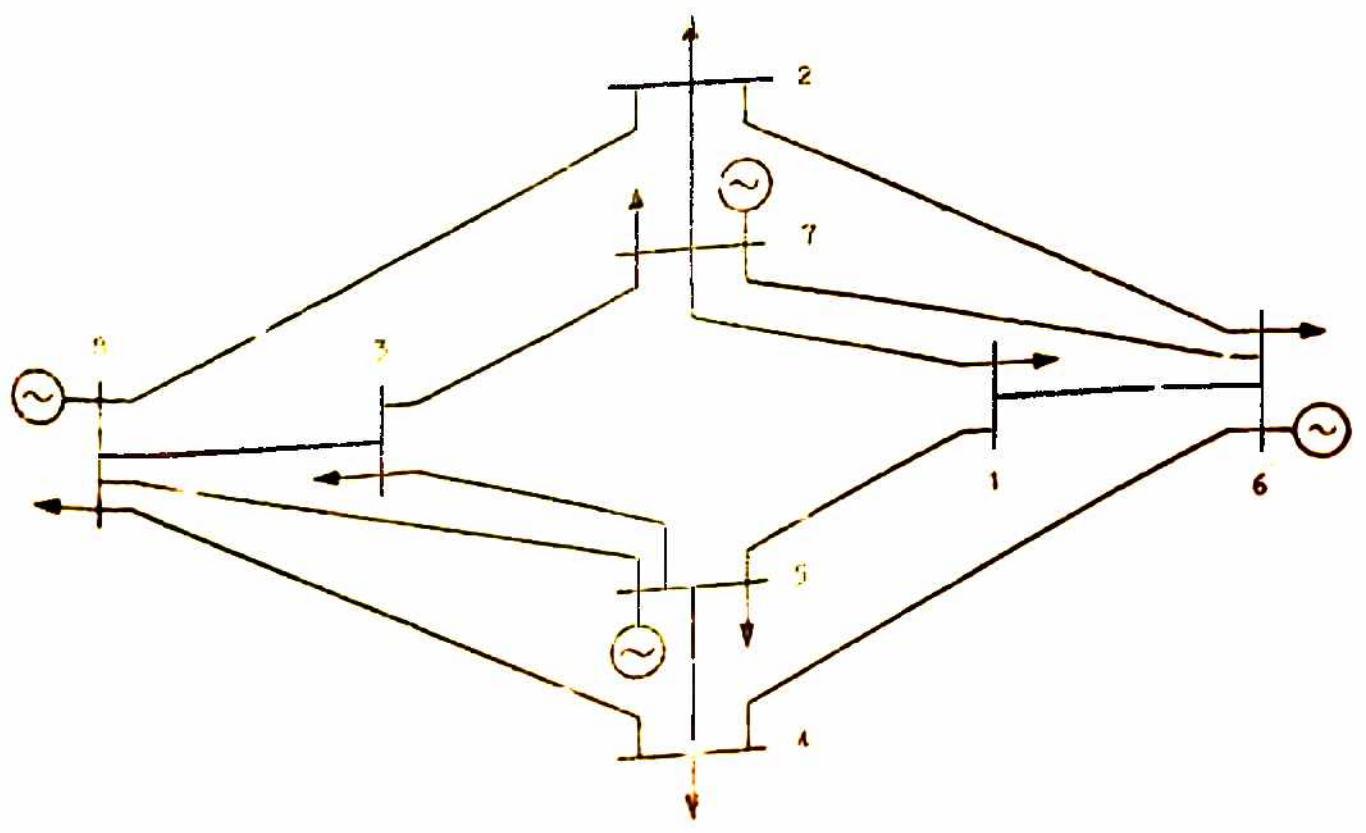


FIG.A1.2 9 BUS SYSTEM





APPENDIX 2.1DETERMINATION OF FACTORS  $\partial P_L / \partial \theta_j$  FROM LOAD FLOW DATA

For a N-bus system the real power injection at any bus p is given by

$$P_p = \text{Re} [V_p I_p^*] \quad (\text{A2.1.1})$$

$$= \text{Re} \left[ V_p \sum_{q=1}^N Y_{pq}^* V_q \right] \quad (\text{A2.1.2})$$

Where

$$\text{Re} [\cdot] = \text{real part of } [\cdot]$$

$$V_p = |V_p| \angle \theta_p, \text{ the phasor voltage at bus } p$$

$$Y_{pq} = G_{pq} + jB_{pq}, \text{ the } p\text{-}q \text{ element } Y_{\text{BUS}} \text{ matrix}$$

$$\theta_{pq} = \theta_p - \theta_q$$

On expanding the equation (A2.1.2) and simplifying, we get,

$$P_p = |V_p| \left[ \sum_{q=1}^N (G_{pq} \cos \theta_{pq} + B_{pq} \sin \theta_{pq}) |V_q| \right] \quad (\text{A2.1.3})$$

The total power loss in the system is equal to the algebraic sum of the real power injections (load powers being negative injections). Thus,

$$\text{Power loss } P_L = \sum_{p=1}^N P_p \quad (\text{A2.1.4})$$

Substituting the value of  $P_p$  from (A2.1.3) in (A2.1.4) and

simplifying, we get

$$P_L = \sum_{p=1}^N \sum_{q=1}^N |V_p| |V_q| G_{pq} \cos(\theta_p - \theta_q) \quad (\text{A2.1.5})$$

Therefore  $\partial P_L / \partial \theta_j$  will be given by

$$\frac{\partial P_L}{\partial \theta_j} = 2 \sum_{q=1}^N |V_j| |V_q| G_{jq} \sin(\theta_q - \theta_j) \quad (\text{A2.1.6})$$

APPENDIX 3.1FAST DECOUPLED LOAD FLOW AND CONTINGENCY ANALYSIS

The nonlinear load flow equations

$$F(X, U, P) = 0 \quad (A3.1.1)$$

is solved by Newton-Raphson (N-R) method through an iterative solution of linearized equations.

Let  $X_0$  be the assumed solution to equation (A3.1.1) and  $X_0 + \Delta X$  be the correct solution to equation (A3.1.1). Expanding the Taylor series about  $X_0$  and dropping second and higher order terms, we have the linear relationship

$$F(X_0 + \Delta X, U, P) = 0 \approx F(X_0, U, P) + \left[ \frac{\partial F(X_0, U, P)}{\partial X} \right] \cdot \Delta X$$

$$\text{or } \left[ \frac{\partial F(X_0, U, P)}{\partial X} \right] \Delta X = -F(X_0, U, P) \\ = \begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} \quad (A3.1.2)$$

Where  $\left[ \frac{\partial F(X_0, U, P)}{\partial X} \right]$  is the Jacobian matrix of partial derivatives of  $F$  w.r.t.  $X$ , evaluated at point  $X_0$ , and  $\Delta P$ ,  $\Delta Q$  are the residuals or the power mismatch, defined as

$$\Delta P_p = P_p^{\text{specified}} - |V_p| \sum_{q=1}^N ((G_{pq} \cos \theta_{pq} + B_{pq} \sin \theta_{pq}) |V_q|)$$

$$\text{and } \Delta Q_p = Q_p^{\text{specified}} - |V_p| \sum_{q=1}^N ((Q_{pq} \sin \theta_{pq} - B_{pq} \cos \theta_{pq}) |V_q|)$$



Where,

$\theta_p, V_p$  = voltage angle, magnitude at bus p.

$$\theta_{pq} = \theta_p - \theta_q$$

$$G_{pq} + jB_{pq} = (p, q) \text{ element of bus admittance matrix } \begin{bmatrix} G \\ + j \end{bmatrix} \begin{bmatrix} B \end{bmatrix}$$

$N$  = Number of buses with system.

By partitioning the Jacobian matrix, equation (A3.1.2) can be written as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix}_k = \begin{bmatrix} H & N \\ M & L \end{bmatrix}_k \begin{bmatrix} \Delta \theta \\ \frac{\Delta |V|}{|V|} \end{bmatrix}_k \quad (\text{A3.1.3})$$

Where  $\Delta \theta$  is the vector of incremental voltage angles at all buses except slack bus. The voltage increment  $\Delta |V|$  at (P, Q) buses is divided by  $|V|$  to bring symmetry in the elements of the Jacobian matrix. The submatrices H, N, M, L represent the negated partial derivatives <sup>of</sup>  $\begin{bmatrix} P \end{bmatrix}$  and  $\begin{bmatrix} Q \end{bmatrix}$  w.r.t.  $\theta$  and  $|V|$ .

Noting that for power transmission system steady state operating condition, there is weak coupling between (P -  $\theta$ ) and (Q -  $|V|$ ) problem, the N and M submatrices of the Jacobian can be dropped, giving

$$[\Delta P] = [H] [\Delta \theta] \quad (A3.1.4)$$

$$[\Delta Q] = [L] \left[ \frac{\Delta |V|}{|V|} \right] \quad (A3.1.5)$$

Where, for  $p \neq q$

$$H_{pq} = L_{pq} = |V_p| |V_q| (G_{pq} \sin \theta_{pq} - B_{pq} \cos \theta_{pq}) \quad (A3.1.6)$$

and for  $p = q$

$$H_{pp} = -B_{pp} |V_p|^2 - Q_p \quad (A3.1.7)$$

$$L_{pp} = -B_{pp} |V_p|^2 + Q_p \quad (A3.1.8)$$

Making some more justifiable simplifications, which are valid in practical systems, such as

$$\cos \theta_{pq} \approx 1$$

$$G_{pq} \sin \theta_{pq} \ll B_{pq} \quad (A3.1.9)$$

$$Q_p \ll B_{pp} |V_p|^2$$

The elements of H and L matrices become

for  $p \neq q$

$$H_{pq} = L_{pq} = -|V_p| |V_q| B_{pq} \quad (A3.1.10)$$

and for  $p = q$

$$H_{pp} = L_{pp} = -B_{pp} |V_p|^2 \quad (A3.1.11)$$

Thus equations (A3.1.4) and (A3.1.5) can be written as

$$\begin{bmatrix} \Delta P \end{bmatrix} = \begin{bmatrix} |V| & B' & |V| \end{bmatrix} \begin{bmatrix} \Delta \theta \end{bmatrix} \quad (\text{A3.1.12})$$

$$\begin{bmatrix} \Delta Q \end{bmatrix} = \begin{bmatrix} |V| & B'' & |V| \end{bmatrix} \begin{bmatrix} \Delta |V| / |V| \end{bmatrix} \quad (\text{A3.1.13})$$

The elements of matrices  $\begin{bmatrix} B' \end{bmatrix}$  and  $\begin{bmatrix} B'' \end{bmatrix}$  are elements of  $\begin{bmatrix} -B \end{bmatrix}$  matrix. The decoupling process and the final algorithmic forms are completed by:

- 1) Omitting from  $\begin{bmatrix} B' \end{bmatrix}$  the representation of those network elements that predominantly affect the MVAR flows, i.e., shunt reactances and off nominal in phase transformer taps.
- 2) Omitting from  $\begin{bmatrix} B'' \end{bmatrix}$  the angle shifting affects of phase-shifters.
- 3) Taking the left-hand  $|V|$  terms in (A3.1.12) and (A3.1.13) on the left hand side of the equations, and then in equation (A3.1.12) removing the influence of MVAR flows on calculation of  $\Delta \theta$  by setting all the right hand  $|V|$  terms to 1.0 p.u. Note that the  $|V|$  terms on the left hand side of (A3.1.12) and (A3.1.13) affect the behaviour of the defining functions and not the coupling.
- 4) Neglecting the series resistance in calculation of elements of  $\begin{bmatrix} B' \end{bmatrix}$  which then becomes the DC-approximation



load flow matrix. This is of minor importance but is found experimentally to give slightly improved results.

With above modifications the final fast decoupled load flow equations become

$$\left[ \frac{\Delta P}{|V|} \right] = \left[ B' \right] \left[ \Delta \theta \right] \quad (\text{A3.1.14})$$

$$\left[ \frac{\Delta Q}{|V|} \right] = \left[ B'' \right] \left[ \Delta |V| \right] \quad (\text{A3.1.15})$$

Both  $[B']$  and  $[B'']$  are real and sparse and have structures of  $[H]$  and  $[L]$  respectively. Since they contain only network admittances they are constant and need to be evaluated once only at the beginning. If phase shifters are not present both  $B'$  and  $B''$  are symmetrical and only lower or upper triangular matrices need to be stored.

#### APPLICATION TO OUTAGE STUDIES

Network outages like line or transformer outages are simulated by an adaption of the inverse matrix modification technique applied to  $[B']$  and  $[B'']$ . The experiments have shown<sup>(55)</sup> that it is only necessary to simulate the removal of series transmission elements from these matrices. Shunt capacitors and reactors, line charging capacitance and shunt branches of off-nominal-tap transformer equivalents can remain in  $[B'']$  without affecting convergence noticeably. All outages must of course be reflected correctly in calculation of  $\left[ \frac{\Delta P}{|V|} \right]$  and  $\left[ \frac{\Delta Q}{|V|} \right]$ .

Let either (A3.1.14) or (A3.1.15) be represented in the base case problem as the equation

$$[R] = [B_0] [E_0] \quad (\text{A3.1.16})$$

for which a solution

$$[E_0] = [B_0]^{-1} [R] \quad (\text{A3.1.17})$$

can be obtained using factors of  $[B_0]$ . In the most general case, the outage of a line (neglecting charging capacitance) or transformer can be reflected in  $[B_0]$  by modifying two elements in row  $k$  and two in row  $m$ . The new outage matrix is then

$$[B_1] = ([B_0] + b M^t M) \quad (\text{A3.1.18})$$

where

$b$  = Line or nominal transformer series admittance

$M$  = Row vector which is null except for  $M_k = a$ , and

$$M_m = -1.$$

$a$  = Off-nominal turns ratio referred to the bus corresponding to row  $m$ , for a transformer

= 1 for a line.

Depending on the type of connected buses, only one row,  $k$  or  $m$ , might be present in  $[B']$  or  $[B'']$ , in which case either  $M_k$  or  $M_m$  above is zero, as appropriate. If both connected buses are either PV or slack, then  $[B'']$  requires no modifications.

It can be shown using the Sherman-Morrison formula that

$$\left[ B_1 \right]^{-1} = \left[ B_0 \right]^{-1} - C X M \left[ B_0 \right]^{-1} \quad (A3.1.19)$$

Where,

$$C = (1/b + MX)^{-1} \quad (A3.1.20)$$

$$\text{and } X = \left[ B_0 \right]^{-1} M^t \quad (A3.1.21)$$

The solution vector  $E_1$  to the outage problem is

$$\left[ E_1 \right] = \left[ B_1 \right]^{-1} \left[ R \right] \quad (A3.1.22)$$

and from (A3.1.17), (A3.1.19) and (A3.1.21), we get

$$\left[ E_1 \right] = \left[ E_0 \right] - C X M \left[ E_0 \right] \quad (A3.1.23)$$

Thus for outage of a branch two non-sparse vectors  $X'$  and  $X''$  has to be calculated, each requiring one repeat solution using factors of  $\left[ B' \right]$  and  $\left[ B'' \right]$  respectively. After each solution of (A3.1.14),  $\left[ \Delta \theta \right]$  is corrected by an amount

$$- C' X' M' \left[ \Delta \theta \right] \quad (A3.1.24)$$

Similarly after each solution of (A3.1.15),  $\left[ \Delta |V| \right]$  is corrected by an amount

$$- C'' X'' M'' \left[ \Delta |V| \right] \quad (A3.1.25)$$

Usually (10 - 1 |V|) scheme is used and 2-3 iterations are required to obtain results with adequate accuracy.



APPENDIX 4.1LINEAR DECISION FUNCTIONS

Linear decision functions are the most popular form of decision functions in use in pattern recognition application. This is mainly due to the ease with which they can be implemented. In fact, any complex decision function can always be treated as if it were linear by virtue of a straight forward transformation as shown below:

Let us consider a generalised decision function

$$d(X) = a_1 f_1(X) + a_2 f_2(X) + \dots + a_k f_k(X) + a_{k+1} \quad (\text{A4.1.1})$$

$$= \sum_{i=1}^{k+1} a_i f_i(X) \quad (\text{A4.1.2})$$

Where,  $a_i$ s are the weights and  $f_i(X)$ ;  $i = 1, \dots, k$ , are real single valued functions of pattern  $X$ ,  $f_{k+1}(X) = 1$ , and  $(k + 1)$  is the number of terms used in the expression.

Now, let us define a vector  $X^*$  whose components are the functions  $f_i(X)$ , i.e.,

$$X^* = \begin{bmatrix} f_1(X) \\ f_2(X) \\ \vdots \\ f_{k+1}(X) \end{bmatrix} \quad (\text{A4.1.3})$$

With the help of (A4.1.3) expression (A4.1.2) can be written as

$$d(X) = \underline{a}^t X^* \quad (\text{A4.1.4})$$

Where

$$\underline{a} = (a_1 \dots a_{k+1})^t$$

Once evaluated, the functions  $f_i(X)$  are nothing but a set of real numerical values and  $X^*$  is simply a  $(k + 1)$ -dimensional vector. Therefore, (A4.1.4) represents a linear function with respect to a new vector  $X^*$ . By evaluating the functions  $f_i(X)$  for all  $X$ , the problem has been effectively transformed into a linear representation.

APPENDIX 4.2PROOF FOR STRICT CONVEXITY OF  $F(W)$  FOR INCONSISTENT SET OF LINEAR INEQUALITIES

Rewriting the problem as defined in (4.61), we have

Find  $W$ , such that

$$AW \geq e > 0 \quad (\text{A4.2.1})$$

Where  $A$ ,  $W$  and  $e$  are as defined in 4.59, 4.60 and 4.61 respectively. The set  $S \triangleq [W | AW \geq e]$  is a polyhedral convex set for every  $e > 0$ ; for  $e = 0$ ,  $S$  will be a convex cone,  $S'$  say. If (A4.2.1) is consistent, i.e., it has a solution, then  $S$  is nonempty; if not, then  $S$  is empty.

Now (A4.2.1) can be stated as an optimisation problem.

Find  $W$ , such that

$$F(W) = \left\| (AW - e) - |AW - e| \right\|^2 \quad (\text{A4.2.2})$$

is minimum.

Properties of  $F(W)$  : If (A4.2.1) is consistent, then  $S$  will be nonempty and  $F(W) = 0 \forall W \in S$  and  $> 0 \forall W \notin S$  (i.e.  $W \in \bar{S}$ ). If (A4.2.1) is inconsistent then  $S$  will be empty and  $F(W) > 0 \forall W \in \mathbb{R}^n$ .

Let us first take the inconsistent case: Since  $S$  is empty  $\bar{S}$  (complement of  $S$ ) consists of whole of  $\mathbb{R}^n$ . Now, let  $W, (W + h) \in \bar{S}$  and  $0 < \mu < 1$  be such that  $(W + \mu h) \in \bar{S}$ . Then for  $F(W)$  to be strictly convex function over  $\bar{S}$ , we must have

$$F(W + \mu h) < \mu F(W + h) + (1 - \mu) F(W), \quad 0 < \mu < 1 \quad (\text{A4.2.3})$$



Let us define  $(NS + NI = N)$ -vector

$$Y = (y_1, y_2, \dots, y_N)^t \triangleq (a_1^t W - e_1, \dots, a_N^t W - e_N)^t \quad (\text{A4.2.4})$$

$$\text{and } Z = (z_1, z_2, \dots, z_N)^t \triangleq (a_1^t h, \dots, a_N^t h)^t \quad (\text{A4.2.5})$$

$$\text{where } A^t = (a_1, a_2, \dots, a_N)$$

Then, we have

$$F(W) = \sum_{i=1}^N \left[ y_i - |y_i| \right]^2 \quad (\text{A4.2.6})$$

$$F(W+h) = \sum_{i=1}^N \left[ (y_i + z_i) - |y_i + z_i| \right]^2 \quad (\text{A4.2.7})$$

$$F(W + \mu h) = \sum_{i=1}^N \left[ (y_i + \mu z_i) - |y_i + \mu z_i| \right]^2 \quad (\text{A4.2.8})$$

Let us define two sets  $S_1 = (y_i | y_i < 0)$  and  $S_2^c(y_i | y_i > 0)$ . Clearly for inconsistent case  $S_1$  is non empty and  $S_2$  can be either nonempty or empty. If  $y_i \in S_1$ , the following four possibilities exist:

$$\begin{aligned} (1) \quad & \left. \begin{array}{l} y_i + z_i > 0 \\ y_i + \mu z_i < 0 \end{array} \right] \quad \left. \begin{array}{l} (2) \quad y_i + z_i > 0 \\ y_i + \mu z_i > 0 \end{array} \right] \\ (3) \quad & \left. \begin{array}{l} y_i + z_i < 0 \\ y_i + \mu z_i < 0 \end{array} \right] \quad \left. \begin{array}{l} (4) \quad y_i + z_i < 0 \\ y_i + \mu z_i > 0 \end{array} \right] \end{aligned}$$

Let us dispose each of above, one by one.

$$(1) \quad \left. \begin{array}{l} y_1 < 0 \\ y_1 + z_1 > 0 \\ y_1 + \mu z_1 < 0 \end{array} \right\} \Rightarrow \left| \frac{z_1}{y_1} \right| > 1 \quad \text{and} \quad \mu \left| \frac{z_1}{y_1} \right| < 1 \quad (\text{A4.2.9})$$

From above condition (A4.2.9) and using relations (A4.2.6-8) we get,

$$F_1(W) = (2y_1)^2$$

$$F_1(W+h) = 0$$

$$F_1(W+uh) = [2(y_1 + \mu_1 z_1)]^2 = 4y_1^2 (1 - \mu \left| \frac{z_1}{y_1} \right|)^2$$

Now substituting in (A4.2.3), we get

$$4y_1^2 (1 - \mu \left| \frac{z_1}{y_1} \right|)^2 < (1 - \mu) 4y_1^2$$

or 
$$(1 - \mu \left| \frac{z_1}{y_1} \right|) < \sqrt{1 - \mu}$$

True, this condition (A4.2.3) is satisfied.

$$(2) \quad \left. \begin{array}{l} y_1 < 0 \\ y_1 + z_1 > 0 \\ y_1 + \mu_1 z_1 > 0 \end{array} \right\} \Rightarrow \left| \frac{z_1}{y_1} \right| > 1 \quad \text{and} \quad \mu_1 > \left| \frac{y_1}{z_1} \right| \quad (\text{A4.2.10})$$

from above condition (A4.2.10) and using relations (A4.2.6-8),

we get

$$F_1(W) = (2y_1)^2$$

$$F_1(W+h) = 0$$

$$F_1(W + \mu h) = 0$$

Thus condition (A4.2.3) is satisfied.

$$(3) \quad y_1 < 0$$

$$y_1 + z_1 < 0$$

(A4.2.11)

$$y_1 + \mu_1 z_1 < 0$$

from above condition and using relations (A4.2.6-8), we get

$$F(W) = (2y_1)^2 = 4y_1^2$$

$$F(W + h) = [2(y_1 + z_1)]^2 = 4y_1^2 + 8y_1 z_1 + 4z_1^2$$

$$F(W + \mu h) = [2(y_1 + \mu z_1)]^2 = 4y_1^2 + 8\mu y_1 z_1 + 4\mu^2 z_1^2$$

Thus L.H.S. of (A4.2.3) is

$$F_1(W + \mu h) = 4y_1^2 + 8\mu y_1 z_1 + 4\mu^2 z_1^2$$

and R.H.S. of (A4.2.3) is

$$F_1(W + h) + (1 - \mu)F(W) = 4\mu_1 y_1^2 + 8\mu y_1 z_1 + 4\mu z_1^2 + 4(1 - \mu)y_1^2$$

Thus condition (A4.2.3) is satisfied as  $0 < \mu < 1$ .

$$(4) \quad y_1 < 0$$

$$y_1 + z_1 < 0$$

(A4.2.12)

$$y_1 + \mu z_1 > 0$$

as  $0 < \mu < 1$  the above condition is not possible.



If  $y_1 \in S_2$ , then again four possibilities exist.

$$(1) \quad y_1 > 0$$

$$y_1 + z_1 > 0$$

(A4.2.13)

$$y_1 + \mu z_1 > 0$$

This leads to  $F(W) = 0$ ;  $F(W + h) = 0$  and  $F(W + \mu h) = 0$ ,

Thus the contribution of this type of  $y_1$  and  $z_1$ 's is zero in (A4.2.3).

$$(2) \quad y_1 > 0$$

$$y_1 + z_1 > 0$$

(A4.2.14)

$$y_1 + \mu z_1 < 0$$

This condition is not possible as  $0 < \mu < 1$

$$(3) \quad \left. \begin{array}{l} y_1 > 0 \\ y_1 + z_1 < 0 \\ y_1 + \mu z_1 < 0 \end{array} \right\} \Rightarrow \left| \frac{z_1}{y_1} \right| > 1; \mu > \left| \frac{y_1}{z_1} \right| \quad (\text{A4.2.15})$$

Using above condition (A4.2.15) and relations (A4.2.6-8),

we get

$$F(W) = 0$$

$$F(W + h) = 2(y_1 + z_1)^2 = 4y_1^2 \left[ 1 - \left| \frac{z_1}{y_1} \right| \right]^2$$

$$F(W + \mu h) = 2(y_1 + \mu z_1)^2 = 4y_1^2 \left[ 1 - \mu \left| \frac{z_1}{y_1} \right| \right]^2$$

substituting is (A4.2.3)

$$4y_1^2 \left[ 1 - \mu \left| \frac{z_1}{y_1} \right| \right]^2 < \mu 4y_1^2 \left[ 1 - \left| \frac{z_1}{y_1} \right| \right]^2$$

$$1 - 2\mu \left| \frac{z_1}{y_1} \right| + \mu^2 \left| \frac{z_1}{y_1} \right|^2 < \mu - 2\mu \left| \frac{z_1}{y_1} \right| + \mu \left| \frac{z_1}{y_1} \right|^2$$

$$(1 - \mu) < \mu \left| \frac{z_1}{y_1} \right|^2 (1 - \mu)$$

True. Thus condition (A4.2.3) is satisfied.

$$(4) \quad y_1 > 0$$

$$y_1 + z_1 < 0 \quad (\text{A4.2.16})$$

$$y_1 + \mu_1 z_1 > 0$$

using above conditions and relations (A4.2.6-8), we get

$$F(W) = 0$$

$$F(W + h) = \left[ 2(y_1 + z_1) \right]^2$$

$$F(W + \mu h) = 0$$

Thus condition (A4.2.3) is satisfied.

In the inconsistent case, since  $S_1$  is nonempty and  $S_2$  may be empty or nonempty, thus as seen from above the condition of strict inequality of (A4.2.3) is always satisfied so the function  $F(W)$  is strictly convex function over  $S = \mathbb{R}^n$  for inconsistent case.

In the consistent case set  $S$  is nonempty and  $S_1$  may be empty or nonempty.  $S_1$  is empty if  $W$  is a solution. In which case  $F(W) = 0$ . If  $W$  is not a solution to (A4.2.1) then  $S_1$  is nonempty and strict inequality of (A4.2.3) holds. From above it is clear that for consistent case  $F(W)$  is a convex function,



APPENDIX 4.3

THE LINEAR MINIMISATION ALGORITHM

This appendix deals with the minimisation in step 5 of the algorithm in Sec (4.7.4). Let us drop the superscript  $k$  and denote

$$F(\mu) = F(W + \mu d)$$

and  $u^*$  correspond to the minimum of  $F(\mu)$ . Then  $\mu^*$  is a solution of  $F'(\mu) = dF(\mu)/d\mu = 0$ ; i.e.,

$$(A^t d) [A(W + \mu d) - e - |A(W + \mu d) - e|] = 0$$

An analytical solution of this equation is not possible unless we know the interval  $[\check{\mu}, \hat{\mu}]$  in which the sign of each component of  $A(W + \mu d)$  remains constant and which contains  $\mu^*$ . The algorithm given below exploits this particular problem structure.

$F(\mu)$  is a convex function in  $\mu$ .  $\mu^*$  is unique if  $F(\mu^*) > 0$ . Let us define  $N$ -vectors,

$$Y = (y_1, \dots, y_N)^t \triangleq (a_1^t W - e_1, \dots, a_N^t W - e_N)^t$$

and

$$Z = (z_1, \dots, z_N)^t \triangleq (a_1^t d, \dots, a_N^t d)^t$$

Then, 
$$F(\mu) = \sum_{i=1}^N F_i(\mu)$$

Where 
$$F_i(\mu) = [(y_i + \mu z_i) - |y_i + \mu z_i|]^2$$

Clearly  $F_i(\mu) \geq 0$ ,  $i = 1, \dots, N$ ;

$F_i(\mu) = 0$  if either

$$1) \quad \mu \geq \mu_1 (= -y_1/z_1) \text{ if } z_1 > 0$$

$$\text{or } 2) \quad \mu \leq \mu_1 (= -y_1/z_1) \text{ if } z_1 < 0.$$

Let us define two sets

$$S_1 = (\mu_1 | \mu_1 = -y_1/z_1, z_1 > 0)$$

$$\text{and } S_2 = (\mu_1 | \mu_1 = -y_1/z_1, z_1 < 0)$$

and let

$$\bar{\mu} = \max (\mu_1 | \mu_1 \in S_1)$$

$$\text{and } \underline{\mu} = \min (\mu_1 | \mu_1 \in S_2)$$

Clearly  $S_1$  can not be empty, for if it were then we would be at the solution.  $S_2$  may be empty in which case  $\underline{\mu}$  does not exist and any  $\mu^* \in (\bar{\mu}, \infty)$  corresponds to the minimum of  $F(\mu)$  and of  $F(W)$ . If  $S_2$  is not empty,  $\mu^*$  lies in the interval with  $\bar{\mu}$  and  $\underline{\mu}$  as extreme points. There exists three possibilities, we will consider each one by one.

1)  $\bar{\mu} < \underline{\mu}$ , in this case,  $\mu^* \in (\bar{\mu}, \underline{\mu})$  minimizes  $F(\mu)$ , see Fig. (A4.3.1). To avoid a solution at boundary point

$$\text{set } \mu^* = (\bar{\mu} + \underline{\mu})/2.$$

2)  $\bar{\mu} = \underline{\mu}$ , In this case  $\mu^* = \bar{\mu} = \underline{\mu}$  minimises  $F(\mu)$ .

3)  $\bar{\mu} > \underline{\mu}$ , In this case  $\mu^* \in (\bar{\mu}, \underline{\mu})$ .

To determine  $\mu^*$ , first the interval  $(\check{\mu}, \hat{\mu})$  is determined where  $(\check{\mu}, \hat{\mu}) \subseteq (\underline{\mu}, \bar{\mu})$  such that  $\check{\mu} \in S_1 \cup S_2$  and

and  $\hat{\mu} \in S_1 \cup S_2$  are adjacent in  $(\underline{\mu}, \bar{\mu}) \cap (S_1 \cup S_2)$  and satisfy  $F(\check{\mu}) < 0$  and  $F(\hat{\mu}) > 0$ , respectively. Then clearly  $\mu^* \in (\check{\mu}, \hat{\mu})$ , see Fig. (A4.3.2). For finding  $\check{\mu}$  and  $\hat{\mu}$ , the elements of  $\mu_1 \in (\underline{\mu}, \bar{\mu}) \cap (S_1 \cup S_2)$  are ordered in ascending order of magnitude and  $F(\mu)$  evaluated at successive values of  $\mu$  starting from  $\underline{\mu}$  until  $\hat{\mu}$  is found. (This task is simplified by setting  $\check{\mu} = 0$  initially and then evaluating  $F(\mu)$  for  $\mu_1 > 0$  only in the above ordered array because we know that  $d$  is a descent direction and therefore,  $\mu^* > 0$  always. In the interval  $(\check{\mu}, \hat{\mu})$ ,  $F(\mu) = \sum_{i \in I} 4(y_i + \mu z_i)^2$ , where for  $i \in I \subseteq (1, 2, \dots, N)$ , either (1)  $z_i > 0$  and  $\mu_1 > \check{\mu}$  or (2)  $z_i < 0$  and  $\mu_1 < \hat{\mu}$ . Now  $\mu^*$  solves  $F(\mu) = 0$ , i.e.,

$$\sum_{i \in I} (y_i + \mu z_i) z_i = 0$$

The algorithm is given below and is a subroutine which returns the value of  $\mu^*$  to the main algorithm in Sec(4.7.4).

Step 1 : Compute  $Z \triangleq Ad$

Step 2 : Form the sets  $S_1$  and  $S_2$ . Compute  $\bar{\mu}$ .

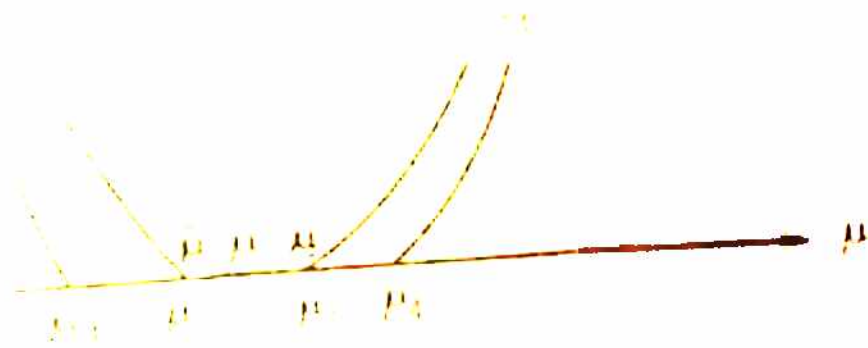
Step 3 : If  $S_2$  is empty, set  $\mu^* > u$  (arbitrary) and return  $\mu^*$ ; else continue.

Step 4 : Compute  $\underline{\mu}$  and  $(\bar{\mu} - \underline{\mu})$ .

Step 5 : If  $(\bar{\mu} - \underline{\mu}) < 0$ , set  $\mu^* = (\bar{\mu} + \underline{\mu})/2$ , return  $\mu^*$ ; else continue.

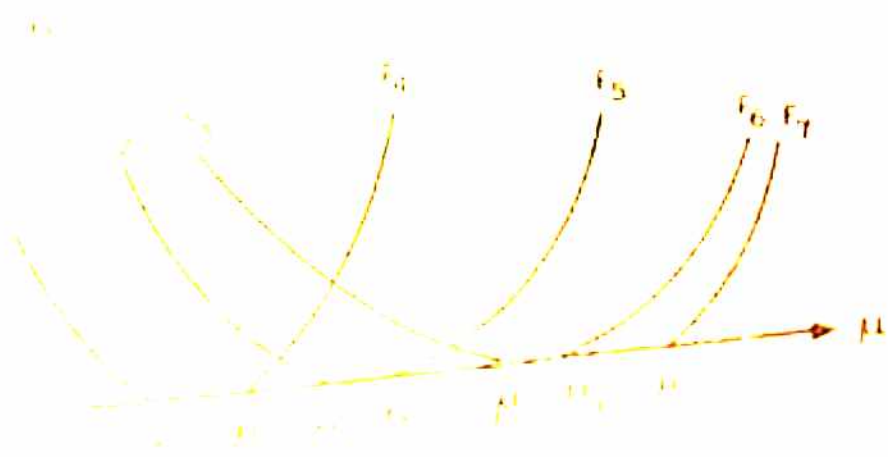
Step 6 : If  $(\bar{\mu} - \underline{\mu}) = 0$ ; set  $\mu^* = \bar{\mu}$ , return  $\mu^*$ ; else continue.





$$f(\mu) = \mu^2 + \mu_1^2 - (\mu_2 - \mu_1) \cdot \mu = (\mu_1 + \mu_2) \mu$$

$$f'(\mu) = 2\mu + \mu_1 - \mu_2 = 0 \Rightarrow \mu = \frac{\mu_2 - \mu_1}{2}$$



$$f(\mu) = \mu^2 + \mu_1^2 - (\mu_2 - \mu_1) \cdot \mu$$

$$f'(\mu) = 2\mu + \mu_1 - \mu_2 = 0 \Rightarrow \mu = \frac{\mu_2 - \mu_1}{2}$$

Step 7 : Set  $\check{u} = \max(0, \underline{u})$ .

Step 8 : Order the elements of the set  $(S_1 \cup S_2) \cup (\check{u}, \bar{u})$  in ascending order of magnitude. Let this set be  $S = (s_1, s_2, \dots, s_p)$ . Set  $i = 0$ .

Step 9 : Set  $i = i + 1$ ,  $\mu = s_i$ . Compute

$$F'(\mu) = \sum_{i=1}^N z_i [(y_i + \mu z_i) - |y_i + \mu z_i|].$$

If  $F(\mu) < 0$ , set  $\check{\mu} = \mu$ , go to step 9. If  $F(\mu) = 0$ , set  $u^* = \mu$ , return  $\mu^*$ . If  $F(\mu) > 0$ , set  $\hat{\mu} = \mu$ , go to step 10.

Step 10: Determine the set

$$I = \left[ i \mid \begin{array}{l} \text{either } z_i > 0 \text{ and } \mu_i > \check{u} \\ \text{or } z_i < 0 \text{ and } \mu_i < \hat{u} \end{array} \right]$$

Step 11: Compute  $\mu^* = - \sum_{i \in I} y_i z_i / \sum_{i \in I} z_i^2$ , return  $\mu^*$ .

This completes the linear minimisation algorithm.

APPENDIX 4.4.CALCULATION OF MEAN, VARIANCE AND CORRELATION COEFFICIENTMEAN

The mean, also called arithmetic mean is one of the most commonly used measure of central tendency and is defined as

$$\mu = \frac{\sum_{i=1}^N X_i}{N} \quad (\text{A4.4.1})$$

for a population. If we have a sample from the population then the mean is called sample mean and is given by

$$\bar{X} = \frac{\sum_{i=1}^n X_i}{n} \quad (\text{A4.4.2})$$

VARIANCE AND STANDARD DEVIATION

The variance and standard deviation are the most widely used and important measures of dispersion. The variance is the square of standard deviation and represents the average squared deviation from the mean value. Where the complete population is known the variance and standard deviation are calculated as follows:

$$\sigma^2 = \frac{\sum_{i=1}^N (X_i - \mu)^2}{N} \quad (\text{A4.4.3})$$

and

$$\sigma = \sqrt{\frac{\sum_{i=1}^N (X_i - \mu)^2}{N}} \quad (\text{A4.4.4})$$

If the data represents a sample from the population, rather than a complete population, then the sample variance ( $S^2$ )



and standard deviation (S) are calculated according to the following:

$$s^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1} \quad (\text{A4.4.5})$$

and

$$S = \sqrt{\frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n - 1}} \quad (\text{A.4.4.6})$$

The use of (n-1) provides the unbiased estimate of variance and standard deviation and is justified in the statistical *theory* in that the estimate is based on data from a sample. As the *sample size* becomes larger and closer to the total population size, the resulting S will become closer to  $\sigma$ .

### CORRELATION COEFFICIENT

The correlation coefficient  $r$  is a measure of the degree of relationship between two variables. It can range from -1 to +1 where both *extremes* represent perfect relationship or correlation. The +1 indicates a perfect direct, or positive linear correlation, and the -1 indicate a perfect inverse, or negative, linear correlation. The sample linear correlation coefficient, also called simple correlation is defined by

$$r = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2 \sum_{i=1}^n (Y_i - \bar{Y})^2}} \quad (\text{A4.4.7})$$

APPENDIX 5.1

Details of computation for ratio test modification for multisegment piecewise linear cost curve representation:

In case of multisegment piecewise linear representation of the cost function, a free generator violating its currently designated segment (as found by solving equation 5.22) can move to a new segment till it is the cheapest thing to do.

Whenever a free generation moves from its currently designated cost segment, the change in the incremental cost for the new segment causes a change in all  $\mu$ s and thus a change in ratio  $R_s$ . The change in  $R_s$  due to the change  $\Delta C_j$  in the incremental cost of the free generator  $j$  on moving to a new segment is found as follows:

From equation (5.26), we have

$$[\mu] = [C] [B]^{-1} \quad (\text{A5.1.1})$$

Using the reduced basis formulation and arranging the row vectors  $[\mu]$  and  $[C]$  into two sets  $[\mu_f], [C_f]$  for free generations and  $[\mu_1], [C_1]$  for binding generations, equation (A5.1.1.) can be written as

$$[\mu_f] = [C_f] [B_f]^{-1} \quad (\text{A5.1.2})$$

$$\text{and} \quad [\mu_1] = [C_1] - [u_f] [z_1] \quad (\text{A5.1.3})$$

A change in  $C_j (= C_j^0 + \Delta C_j)$ ,  $j$ th element of  $[C_f]$ , will change  $[\mu_f]$  and  $[\mu_1]$  as follows:

$$[\mu_f] = ([C_f^o] + [\Delta C]) [B_f]^{-1} \quad (\text{A5.1.4})$$

Where  $[\Delta C]$  is a very sparse row vector whose only nonzero element is  $\Delta C_j$  in  $j$ th column. From equation (A5.1.4) we get

$$\begin{aligned} [\mu_f] &= [C_f^o] [B_f]^{-1} + [\Delta C] [B_f]^{-1} \\ &= [\mu_f^o] + [\Delta C] [B_f]^{-1} \end{aligned} \quad (\text{A5.1.5})$$

and 
$$[\mu_1] = [C_1] - [\mu_f] [B_1] \quad (\text{A5.1.6})$$

from (A5.1.5) and (A5.1.6) we get

$$\begin{aligned} [\mu_1] &= [C_1] - ([\mu_f^o] + [\Delta C] [B_f]^{-1}) [B_1] \\ &= [C_1] - [\mu_f^o] [B_1] - [\Delta C] [B_f]^{-1} [B_1] \\ &= [\mu_1^o] - [\Delta C] [B_f]^{-1} [B_1] \end{aligned} \quad (\text{A5.1.7})$$

Superscript o indicates values at the beginning of the process.

Thus, the new ratio  $R_j$  for the  $j$ th generation is

$$R_j = \left| \frac{\mu_j}{S_j^o} \right| = \left| \frac{\mu_j^o + \Delta \mu_j}{S_j^o} \right|$$

or 
$$R_j = R_j^o + \Delta R_j \quad (\text{A5.1.8})$$

Where 
$$\Delta R_j = \Delta C_j I_{jj} S_j^o = \alpha_j \Delta C_j \quad (\text{A5.1.9})$$

$I_{jj}$  is the  $jj$  element of  $[B_f]^{-1}$  matrix.

and 
$$\alpha_j = I_{jj} S_j^o$$



The new ratio  $R_i$  for any eligible binding generation is given by

$$R_i = \left| \frac{\mu_i}{S_i^0} \right| = \left| \frac{\mu_i^0 + \Delta \mu_i}{S_i^0} \right|$$

or  $R_i = R_i^0 + \Delta R_i = R_i^0 + \alpha_i \Delta C_j$  (A5.1.10)

Where  $\alpha_i$  is the  $i$ th element of the  $j$ th row of  $[B_f]^{-1}$   
 $[B_1] / S_i^0$ .

As the explicit inverse of  $[B_f]$  is available the computation of  $\alpha_i$  involves only  $m$  multiplications ( $m$  = number of free generations) and one division.

APPENDIX 5.2

Linear incremental models for transmission lines and transformers:

5.2 (a) Transmission Lines:

The transmission line model is shown in Fig. A5.2.1. For this transmission line model the complex power flowing in the line is given by

$$S_{pq} = P_{pq} - jQ_{pq} = E_p^* [E_p - E_q] y_{pq} + E_p^* E_p (y_{sh}/2)$$

where  $E_p = V_p \angle \theta_p$ ;  $y_{pq} = G_{pq} - jB_{pq}$ ;  $(y_{sh}/2) = jB_1$

and  $(.)^*$  is complex conjugate of  $(.)$ .

$$\text{or } P_{pq} - jQ_{pq} = V_p^2 (G_{pq} - jB_{pq}) - V_p V_q (\cos \theta_{pq} - j \sin \theta_{pq})$$

$$(G_{pq} - jB_{pq}) + jV_p^2 B_1; \theta_{pq} = (\theta_p - \theta_q)$$

Separating real and imaginary parts; we get

$$P_{pq} = V_p^2 G_{pq} - V_p V_q G_{pq} \cos \theta_{pq} + V_p V_q B_{pq} \sin \theta_{pq} \quad (A5.2.1)$$

$$\text{and } Q_{pq} = V_p^2 B_{pq} - V_p V_q B_{pq} \cos \theta_{pq} - V_p V_q G_{pq} \sin \theta_{pq} - V_p^2 B_1 \quad (A5.2.2)$$

For practical systems generally,

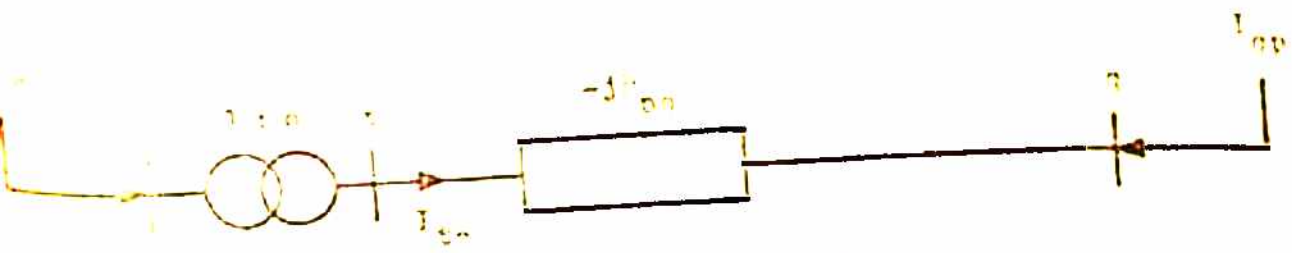
$$V_p \approx V_q \approx 1.0 \text{ pu}; G_{pq} \cos \theta_{pq} \approx G_{pq}; \sin \theta_{pq} \approx \theta_{pq}$$

$$G_{pq} \sin \theta_{pq} \ll B_{pq} \cos \theta_{pq}$$

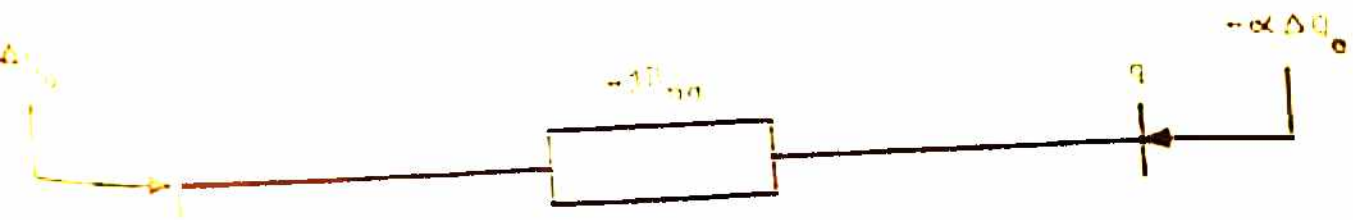
Therefore, the real power flow in line pq becomes



Fig. A5.2.1 APPROXIMATE EQUIVALENT CIRCUIT



(a) TRANSFORMER FORM.



(b) EQUIVALENT CIRCUIT FORM

Fig. A5.2.2



$$P_{pq} = B_{pq} (\theta_p - \theta_q) \quad (A5.2.3)$$

Thus the limit on real power flow in the transmission line can be written in terms of voltage angle spread  $(\theta_p - \theta_q)$  as,

$$P_{pq}^{lim} = |\theta_p - \theta_q|_{max} \quad (A5.2.4)$$

The reactive power flow in line p-q from equation (A5.2.2)

$$Q_{pq} \approx V_p^2 (B_{pq} - B_l) - V_p V_q B_{pq} \cos \theta_{pq}$$

For finding incremental changes in reactive power flow in line p-q due to incremental changes in bus voltage magnitudes, we apply small voltage increments at the buses p and q resulting an incremental change in reactive power flow from p to q i.e.,

$$Q_{pq}^0 + \Delta Q_{pq} = (V_p^0 + \Delta V_p)^2 (B_{pq} - B_l) - (V_p^0 + \Delta V_p)(V_q^0 + \Delta V_q) B_{pq} \cos \theta_{pq}.$$

rearranging and ignoring terms with the product of voltage increments we get

$$\Delta Q_{pq} = \Delta V_p (2V_p^0 (B_{pq} - B_l) - V_q^0 B_{pq} \cos \theta_{pq}) - \Delta V_q (V_p^0 B_{pq} \cos \theta_{pq})$$

$$\text{or } \Delta Q_{pq} = G_{ip} \Delta V_p + G_{iq} \Delta V_q \quad (A5.2.5)$$

$G_{ip}$  and  $G_{iq}$  for all lines are calculated at the beginning of the LP process and are stored to be used throughout the

reactive power LP process.

(b) Transformer

The model of a tap changing under load transformer is shown in Fig. A5.2.2(a). For this model of the transformer

we have

$$E_p / E_t = \frac{I_{tq}}{I_{pq}} = \frac{1}{a}$$

Therefore,

$$\begin{aligned} I_{pq} &= a I_{tq} = a(E_t - E_q) (-jB_{pq}) \\ &= a(aE_p - E_q) (-jB_{pq}) \end{aligned} \quad (\text{A5.2.6})$$

$$\text{and } I_{qp} = (E_q - E_t) (-jB_{pq}) = (E_q - aE_p) (-jB_{pq})$$

The complex power injected into the transformer at bus p is

$$\begin{aligned} S_{pq} &= P_{pq} - jQ_{pq} = E_p^* I_{pq} \\ &= E_p^* a(aE_p - E_q) (-jB_{pq}) \\ &= a^2 V_p^2 (-jB_{pq}) - a E_p^* E_q (-jB_{pq}) \\ &= -j a^2 V_p^2 B_{pq} - a V_p V_q (\cos \theta_{pq} - j \sin \theta_{pq}) \\ &\quad (-jB_{pq}) \end{aligned}$$

Separating real and reactive part, we get

$$Q_{pq} = a^2 V_p^2 B_{pq} - a V_p V_q B_{pq} \cos \theta_{pq} \quad (\text{A5.2.7})$$

Similarly the reactive power injected in the transformer at

at bus q is given by

$$Q_{qp} = V_q^2 B_{pq} - a V_p V_q B_{pq} \cos \theta_{pq} \quad (\text{A5.2.8})$$

As seen from (A5.2.7) the additional reactive power injection at bus p due to the change in tap ratio from nominal 1:1 to (1:a) is

$$\Delta Q_{pq}^o = (a_o^2 - 1) V_p^2 B_{pq} - (a_o - 1) V_p V_q B_{pq} \cos \theta_{pq} = Q_p^o \quad (\text{A5.2.9})$$

Similarly the additional reactive power injection at bus q is

$$\Delta Q_{qp}^o = -(a_o - 1) V_p V_q B_{pq} \cos \theta_{pq} = Q_q^o \quad (\text{A5.2.10})$$

To find out the change in reactive power injection at the buses p and q due to changes in tap position, an incremental analysis is carried out as follows:

$$(Q_p^o + \Delta Q_p) = [(a + \Delta a)^2 - 1] V_p^2 B_{pq} - [(a_o + \Delta a) - 1] V_p V_q B_{pq} \cos \theta_{pq}$$

ignoring  $(\Delta a)^2$  terms we get

$$\Delta Q_p = \Delta a [2V_p^2 B_{pq} - V_p V_q B_{pq} \cos \theta_{pq}] = \Delta Q_e \quad (\text{A5.2.11})$$

Similarly the change in reactive power injection at bus q due to change in tap setting to  $(a + \Delta a)$  is

$$\Delta Q_q = -\Delta a [V_p V_q B_{pq} \cos \theta_{pq}] \quad (\text{A5.2.12})$$



As only one injected power  $\Delta Q_e$  is required in the IP process as the control variable,  $\Delta Q_q$  is represented in terms of  $\Delta Q_e$  as

$$\Delta Q_q = -\alpha \Delta Q_e \quad (\text{A5.2.13})$$

Where

$$\alpha = V_q \cos \theta_{pq} / (2V_p - V_q \cos \theta_{pq}) \quad (\text{A5.2.14})$$

The incremental transformer model using equations (A5.2.12) and (A5.2.13) is shown in Fig. 5.2.2(b).

Details of computation of elements of  $[H'']$ ;  $[E]^t$  and  $[f]$ :

The reactive power network model including all PV and slack bus is

$$[\Delta Q] = [H'] [\Delta V] \quad (A5.3.1)$$

The inclusion of all buses will, in general, make  $[H']$  singular unless ground susceptive ties are strong. To avoid this, singularity of  $[H']$  a load bus with no reactive power control ( $\Delta Q = 0$ ) is chosen as a reference bus. Writing the equation (A5.3.1) in expanded form and choosing rth bus as reference we have

$$\Delta Q_1 = H'_{11} \Delta V_1 + \dots + H'_{1r} \Delta V_r + \dots + H'_{1N} \Delta V_N \quad (A5.3.2)$$

$$\vdots$$

$$\Delta Q_r = H'_{r1} \Delta V_1 + \dots + H'_{rr} \Delta V_r + \dots + H'_{rN} \Delta V_N$$

$$\Delta Q_N = H'_{N1} \Delta V_1 + \dots + H'_{Nr} \Delta V_r + \dots + H'_{NN} \Delta V_N$$

rearranging the set of equations in (A5.3.2), we get

$$\Delta V_r = \frac{\Delta Q_r}{H'_{rr}} - \frac{H'_{r1}}{H'_{rr}} \Delta V_1 - \dots - \frac{H'_{rN}}{H'_{rr}} \Delta V_N \quad (A5.3.3)$$

and

$$\Delta Q_1 = H'_{11} \Delta V_1 + \dots + H'_{1r} \left( \frac{\Delta Q_r}{H'_{rr}} - \frac{H'_{r1}}{H'_{rr}} \Delta V_1 - \dots - \frac{H'_{rN}}{H'_{rr}} \Delta V_N \right) + \dots + H'_{1N} \Delta V_N \quad (A5.3.4)$$

or

$$\begin{aligned}
\Delta Q_1 &= \frac{H'_{1r}}{H'_{rr}} \Delta Q_r + \left( H'_{11} - \frac{H'_{1r} H'_{r1}}{H'_{rr}} \right) \Delta V_1 + \dots \\
&+ \left( H'_{1i} - \frac{H'_{1r} H'_{ri}}{H'_{rr}} \right) \Delta V_i + \dots \\
&+ \left( H'_{1N} - \frac{H'_{1r} H'_{rN}}{H'_{rr}} \right) \Delta V_N \\
&= H''_{rr} \Delta Q_r + H''_{11} \Delta V_1 + \dots + H''_{ii} \Delta V_i + \dots + H''_{iN} \Delta V_N
\end{aligned}
\tag{A5.3.5}$$

Writing the set of new network incremental model with bus  $r$  as the reactive power reference bus, we get

$$\begin{array}{|c|} \hline \Delta V_r \\ \hline \Delta Q'' \\ \hline \end{array} = \begin{array}{|c|c|} \hline A & -E^t \\ \hline E & H'' \\ \hline \end{array} \begin{array}{|c|} \hline \Delta Q_r \\ \hline \Delta V'' \\ \hline \end{array}
\tag{A5.3.6}$$

where  $\Delta Q''$  is  $(N-1)$  vector of changes in reactive power bus injections at all buses except the reference bus.

$\Delta V''$  is  $(N-1)$  vector of changes in bus voltage magnitudes,  $E_i = (H'_{ri} / H'_{rr})$  th  $i$ th element of  $(N-1)$  vector  $E$ .

$H''_{ij} = (H'_{ij} - \frac{H'_{ir} H'_{ri}}{H'_{rr}})$ , the  $ij$  element of matrix  $H''$ .

$\Delta V_r$  = change in voltage of reference bus

$\Delta Q_r = 0$ , the change in reactive power injection at reference bus.

By choosing a load bus with least number of buses directly



connected to it the sparsity of  $[H'']$  will be similar to that of  $[H']$  matrix.

As  $\Delta Q_r$  is zero and thus known, the first row and column of the matrix in equation (A5.3.6) can be deleted, giving the network model

$$\begin{bmatrix} \Delta V'' \end{bmatrix} = \begin{bmatrix} H'' \end{bmatrix} \begin{bmatrix} \Delta Q'' \end{bmatrix} \quad (\text{A5.3.7})$$

The matrix  $H''$  is no longer singular as the sum of the elements in its rows will no longer add up to zero.

Method for calculating  $\xi_1 = \frac{\partial Q_r}{\partial Q_1}$

We have

$$\Delta Q_r = \sum_{\substack{i=1 \\ i \neq r}}^N \frac{\partial Q_r}{\partial Q_i} \Delta Q_i$$

or

$$\begin{aligned} 0 &= \frac{\partial Q_r}{\partial Q_1} \Delta Q_1 + \dots + \frac{\partial Q_r}{\partial Q_N} \Delta Q_N \\ &= \xi_1 \Delta Q_1 + \dots + \xi_N \Delta Q_N \end{aligned} \quad (\text{A5.3.8})$$

also,

$$\begin{aligned} \frac{\partial Q_r}{\partial V_1} &= \frac{\partial Q_r}{\partial Q_1} \frac{\partial Q_1}{\partial V_1} + \dots + \frac{\partial Q_r}{\partial Q_N} \cdot \frac{\partial Q_N}{\partial V_1} \\ &\vdots \\ \frac{\partial Q_r}{\partial V_N} &= \frac{\partial Q_r}{\partial Q_1} \cdot \frac{\partial Q_1}{\partial V_N} + \dots + \frac{\partial Q_r}{\partial Q_N} \cdot \frac{\partial Q_N}{\partial V_N} \end{aligned} \quad (\text{A5.3.9})$$

Writing the set of  $(N-1)$  equations (no equation for  $\frac{\partial Q_r}{\partial V_r}$ ) in matrix form, we get

$$\left[ \frac{\partial Q_r}{\partial V^n} \right] = \left[ H^n \right] \left[ \frac{\partial Q_r}{\partial Q^n} \right] \quad (\text{A5.3.10})$$

Where  $\partial Q_r / \partial V_1$  is the  $ri$  element of the  $B^n$  matrix of fast decoupled load flow.

The elements  $\frac{\partial Q_r}{\partial Q_1} = \xi_{1r}$  are obtained by solving (A4.3.10).

The matrix  $[H^n]$  is factored and stored at the beginning of the process.

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