STUDY OF EFFECTIVE INTERACTIONS IN NUCLEI N=28 AND Z=21 TO 28

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CERTIFICATE

This is to certify that the thesis entitled " Study of Effective Interactions in Nuclei N = 28 and Z = 21 to 28 ", submitted by Mr. G. Ramachandra Rao, I.D.No. ________ for award of Ph.D. degree of the Institute, embodies original work done by him under my supervision.

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C O N T E N T S

INTRODUCTION

There have been many theoretical and experimental investigations on the $1f_{7/2}$ shell nuclei. The experimental work on these nuclei[1-17] and the theoretical investigations[18-30] shed much light on the structure and properties of these nuclei. Shell model approach is well suited for these nuclei because there are three closed shell nuclei viz. ^{40}Ca, ^{48}Ca and ^{56}Ni in this area of the nuclear chart. The description of a system of interacting particles in terms of a system of independent particles moving in an average central field is called Shell Model and systems of a few particles in addition to a closed shell core can very well be described in this model by introducing a residual interaction between the extra core particles. Of the three closed shell nuclei the nucleus of ^{48}Ca is known to be a better closed shell core. Seth et.al,[31] concluded from $^{40,48}Ca$ $(^{3}He,d)^{41,49}$ Sc reactions that the stripping to $1d_{3/2}^{-1}$ states in ^{48}Ca is 8 times smaller than it is in ^{40}Ca. Experimental data on ^{48}Ca $(d,p)^{49}Ca$ and ^{48}Ca $(p,d)^{47}Ca$ reactions also indicate negligible amound of the core excitations in ^{48}Ca[32-34]. Therefore, the N = 28 isotones from ^{49}Sc to ^{56}Ni are under better control in truncated shell model calculations than the other nuclei in this region. In the case of the nucleus of ^{52}Cr, which is in the middle of the shell, it is believed that deformations set in. Lips et.al. attribute the small discrepancies in their calculatins[25] for ^{52}Cr to the possible deformations. However the calculations based on quadrupole defo-

rmations for even isotopes of Titanium[35] indicate that it may not be the case for 50Ti. Recent calculations[36] showed many neighbouring nuclei but not ^{52}Cr possibly due to closed neutron shell, could be regarded as a mixture of prolate and oblate in their shapes and of rotational and vibrational motions in their energy levels. In the present work ^{52}Cr is considered under shell model with effective interactions.

The calculations using pure $(1f_{7/2})^n$ configurationsfor the protons above ^{48}Ca core have repreduced the ground state energies satisfactorily[18-20]. Such pure configuration calculations do not predict enough energy levels with any choice of residual interaction since the space is small. In such calculations the excited levels are not well reproduced. The M1 transition operator is an odd tensor single-body operator. Matrix elements of odd tensor operators vanish between any two states of j^n configuration with different seniorities and also the matrix elements, diagonal in seniority, are independent of the number of particles since pairs coupled to J = 0 do not contribute. This means that within a j^n configuration of identical nucleons no M1 transitions occur in the usual longwave approximation for the accompanying electromagnetic radiation. Even the E2 transitions are forbidden between the states of the same seniority for ^{52}Cr nucleus since this nucleus is in the middle of the shell and E2 transition operator is an even rank tensor[37]. Experimentally the E2 transitions from the $6^+(\nu = 2)$ state to the $4^+(\nu = 2)$ state is found to be about half of the transition to the $4^+(\nu = 4)$ state[12,13]. If the

$J^{\pi} = 1^{+}$ states were pure states there would be no E2 transition from the $6^{+}(\nu = 2)$ state to one of the $J^{\pi} = 4^{+}$ states. M1 transitions are actually observed in ^{51}V and ^{53}Mn[11]. A streightforward evidence against pure configuration description comes from the stripping reactions[2] which show $l_p = 1$ transitions to some lowlying states. indicating clearly that the transfered nucleon, with a non-zero strength, is in a p-state. The ground state of ^{51}V is, essentially a $\nu = 1$ $J^{\pi} = 7/2^{-}$ state. Addition of a proton in $f_{7/2}$ orbit will give a state with $\nu = \nu \pm 1$ only. Seniority is always a non negative integer. Thus the state $\nu = 2$ $J^{\pi} = 4^{+}$ should be the only state to be exbited in these transfer reactions if there were no mixing of seniorities. In fact that both the $J^{\pi} = 4^{+}$ (with $\nu = 2$ and 4) are excited indicate that the two 4^{+} states contain $\nu = 2$ components and seniority mixing. Any two body interaction in a j^{n} configuration with $j = 7/2$ is diagonal in the seniority scheme. This implies that if the states were pure then the states with different seniority do not mix in a j^{n} configuration for $j = 7/2$, as long as the interactions are of one and two-body type[37]. These observations provide an excellent ground in favour of configuration mixing and seniority mixing.

In doing mixed configuration calculations for these nuclei, the valance protons are to be excited from the $1f_{7/2}$ subshell to $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ levels because core excitations are negligible. Shell model calculations in which more than one particle is raised, are very large. Earlier calculations[24-26] in which

only one proton is raised to $2p_{3/2}$ and $1f_{5/2}$ levels have given good results, and the pure configuration components were found to be dominant for the lowest states of each angular momentum, in most cases. The single particle energies of $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ protons above ^{48}Ca core are obtained as weighted averages, weighted by the spectroscopic factors, over a set of states obtained in single proton transfer reactions. They are obtained to be 9.62, 6.08, 4.93, 3.58 MeV respectively[8-10] and the lowlying levels below say 4 MeV of excitation are not expected to contain much contribution from configurations higher than those considered. It has also been found that small variations of the single particle energies do not have much effect on the energy levels[24,25]. In the calculation on N = 28 isotones the energy of the $1f_{7/2}$ proton was found to be 9.72 MeV nad that a small variation in energy levels due to variation of energy of $2p_{3/2}$ proton could very well be simulated by slight changes in the interaction between $1f_{7/2}$ and $2p_{3/2}$ protons. These mixed configuration calculations could very well predict both the ground state energies as well as the excited state energies. These calculations could also account for the spectroscopic factors and electromagnetic transition rates with an effective charge derived by I. Talmi. However, the M1 rates and mixing ratios show discrepancies.

There is one theoretical objection to the mixed configuration studies mentioned above. With any newtron excess core, the states in which only protons are excited to higher orbits do not have good isospin[38,39] and they can not represent true

nucleus unlike the other nuclei[40,47]. It may be noted that the calculations reported by Nomura et. al.[44] and Cochavi et.al.[45] take the deformations in to consideration. The calculations with a fixed and simple effective charge e_p = 1.6e reproduced B(E2) transition rates[24,25] and quadrupole moments satisfactorily[29].

Nuclear theory attempts to derive observed properties from interactions. This is a many body problem. Empirical evidence suggests[40] that to a good approximation, each nucleon in a nucleus moves independently in an average potential well due to all other nucleons. This independent particle model provides a basis of states in an infinite dimensional Hilbert space in which the residual interaction, the remaining part of the interaction after taking single particle potential, is to be diagonalized to get energy states and other properties. In shell model calculations such as those described above only a truncated space is chosen. The residual interaction between particles of independent particle model gets renormalized due to such truncation. Thus the effective interaction depends upon the choice of truncation. One chooses the trunctation suitable to the chosen states of a nucleus or nuclei and their properties to be obtained. It means that the interaction to be used in shell model calculations gets modified due to Pauli principle and other many body effects. This renormalized or effective interaction is to be diagonalized in the model space of finite size to get energies which would be the same as those obtained by diagonalizing the actual residual interaction in the original complete Hilbert space.

The nucleon-nucleon interaction in non-relativistic approximation may be represented by a potential. Such a version of NN interaction could explain scattering data up to 300 MeV while the energies of nucleons in a nucleus in collision are of the order of 160 MeV only. This potential picture for NN interaction is quite satisfactorily used in shell model studies for lowlying states. The matrix elements of such an interaction between shell model states are primary input data in nuclear structure calculations.

Assuming only two-body interaction, the many body problem requires solution of $(\Sigma T_i + \Sigma V_{ij})\Psi = E\Psi$, where Ti is keinetic energy operator, E is the total energy, eigen value, and Ψ is the many body wavefunction. Introducing auxiliary one-body potential, also called shell model potential Ui, we get

$$[H_0 + H_1]\Psi = [(\Sigma T_i + \Sigma U_i) + (\Sigma V_{ij} - \Sigma U_i)]\Psi = E\Psi \quad \ldots\ldots (1)$$

The condition on Ui is simply that the Schrodinger equation $(T_i + U_i)\phi = \epsilon\phi$ should have simple and analytic solutions, and that H1 should be fairly weak compared to H0 so that it may be treated as a perturbation. If $\phi_1, \phi_2, \phi_3 \ldots$ are solutions of one particle Hamiltonian, subsets of them define configuration denoted by $[n]$. The many body functions are Slater determinants $\Phi_{[n]}$. We write $\Psi = \Sigma a_n \Phi_{[n]}$ and determine $a_{[n]}$ by diagonalising the full Hamiltonian (note that H_0 is already diognal). This is a problem of infinite dimensional space. The single particle potential obtained self consistantly in Hartree-Fock theory is usually non-local and non-spherical. Harmonic oscellator appro-

ximation is quite satisfactory. However the single particle energies obtained in this manner do not contain interaction of extracore particles with the core and therefore one usually takes experimental energies in standard shell model calculations with inert core.

After chosing the configuration space to be used, the problem is to choose or derive a suitable interaction for diagonalization in the truncated space to get correct energies and properties. To achieve this consider

$$H_1\Psi = (H - H_0)\Psi \;\Rightarrow (E - H_0)\sum a_{[n]}\Phi_{[n]} = \sum a_{[n]}(E - E_{[n]})\Phi_{[n]}$$

$$\therefore a_{[n]}(E - E_{[n]}) = \langle \Phi_{[n]} \mid H_1\Psi\rangle$$

$$= \sum \langle \Phi_{[n]} \mid H_1 \Phi_{[m]}\rangle a_{[m]} \qquad \ldots\ldots \quad (2)$$

Now if we write $|\Psi^d\rangle = P^d|\Psi\rangle$, projection on the truncated space we have $\Psi^d = \sum_{[n]\in d} a_{[n]}\Phi_{[n]}$. The interaction for the model space will have by requirement these functions as eigenfunctions corresponding to truceenergies.

we need $H^d\Psi^d = \epsilon\Psi^d$, $\Psi^d = P\Psi$ and $H^d = H_0 + H_1^d$

and we get $\langle \Phi_n \mid H_1\Psi\rangle = a_n(E - E_n) = \langle \Phi_n \mid H_1^d\Psi^d\rangle$ .... (3)

Now we write $H_1\Psi = H_1\Psi^d + H_1 Q\Psi$, $Q = 1 - P$

$$Q\Psi = \sum_{n\notin d} a_n|\Phi_n\rangle = \sum' |\Phi_n\rangle \cdot \frac{\langle \Phi_n \mid H_1\Psi\rangle}{(E - \epsilon_n)} = \frac{Q}{E - H_0} H_1\Psi$$

and therefore $\langle \Phi_n \mid H_1^d\Psi^d\rangle = \langle \Phi_n \mid H_1\Psi\rangle = \langle \Phi_n \mid \{H_1\Psi^d + H_1\frac{Q}{E-H_0}H_1\Psi\}\rangle$

$$\text{or} \quad H_1^d = H_1\left[1 + \frac{Q}{E - H_0}H_1^d\right]$$

$$\ldots\ldots \quad (4)$$

we can also write,

$$\langle \phi_n | H_1^d \psi^d \rangle = \langle \phi_n | H_1 \psi^d \rangle + \langle \phi_n | H_1 \left\{ \sum_{\ell \notin d} \frac{|\phi_\ell\rangle\langle\phi_\ell|}{E - H_0} \right\} H_1 \psi \rangle$$

$$= \langle \phi_n | H_1 \psi^d \rangle + \sum_{\ell \notin d} \langle \phi_n | H_1 | \phi_\ell \rangle \frac{1}{(E - H_0)} \langle \phi_\ell | H_1 \psi \rangle$$

Now since the right hand side contains ψ we can again use (3) and write

$$\langle \phi_n | H_1^d \psi^d \rangle = \langle \phi_n | H_1 \psi^d \rangle + \sum_{\ell \notin d} \langle \phi_n | H_1 | \phi_\ell \rangle \frac{1}{E - H_0} \langle \phi_\ell | H_1^d \psi^d \rangle$$

or $\langle \phi_n | H_1^d | \phi_m \rangle = \langle \phi_n | H_1 | \phi_m \rangle + \sum_{\ell} \langle \phi_n | H_1 | \phi_\ell \rangle (1/E - H_0) \langle \phi_\ell | H_1^d | \phi_m \rangle \ldots\ldots$ (5)

This equation together with

$$(\epsilon - \epsilon_j) a_j = \sum_{\ell \in d} \langle \phi_j | H_1^d | \phi_\ell \rangle a_\ell$$

form a set of coupled equations to be solved [49-51].

Since we know that the Hamiltonian is scalar, J and T are good quantum numbers and similarly parity is also a good quantum number, these equations may be thought to be good in a JTP sub-space. The price paid in reducing the problem of diagonalization to a finite dimensional problem is that the effective interaction becomes state dependent, and the problems of self consistancy also arise. A further reduction is achieved in standard shell model calculations by core separation in which core particles are assumed to be in the lowest configuration and valance particles are allowed to occupy a certain manageable number of valance orbits. In this case the total energies are expressed with respect to the energy of the closed shell core. The handling of infinite sum on the right hand side of equation (5) is done by using cluster expansion techniques[52-55].

The expression for effective interaction is obtained in the form of an infinite series, eq. (5). The series must converge because the residual interaction in shell model calculations is weak. The energy dependence, core separation, choice of Q, choice of single particle potential, choice of single particle wavefunctions for model space levels as well as outside the model space, self-consistancy problems, and hard core in NN interaction are the many problems in evaluating the effective interactions. Different techniques are used in evaluating them. Rewritting the infinite series so that partial summations are possible is one of them. Reaction matrix of Bruckner, expansion of H_1^{eff} as a series in 1-body, 2-body, 3-body .... etc. terms, core polarization diagrams of different orders are of this kind. Other techniques often used are expanding out a certain part of the energy denominator and writting the interaction as a sum of two terms. Reference spectrum method[56], and separation method[57,58] to evaluate the reaction matrix and folded diagram expansion of effective interaction (energy independent series) are of this nature.

In all the calculations only first few terms of the effective xx interaction series are evaluated to study the relative importance of the methods, of different partial summations of the series, and approximations. There is no clearcut way to decide which method leads to fast convergence of the series except that we can get some cluse regarding useful way to get the effective interaction. Nuclear matter, closed shell nuclei and open shell nuclei are separately taken to study different aspects. Effective interactions obtained for closed shell nuclei are often

different for open shell nuclei. Detailed account is given in review articles by Barrett et.al.[59] and Kuo[60].

Apart from the microscopic theories, there is no unique way to obtain an effective interaction for use in standard shell model calculations. By-passing all the difficulties, standard shell model studies assume a suitable effective interaction and a model space for the valance nucleons. Effects of truncation and methods usually employed in SM calculations to obtain effective interaction are studied by Barrett et.al.[61] for the mass A A = 18-20 region.

There are several ways of determinign the effective interaction. The two-body matrix elements of effective residual interaction may be treated as adjustible parameters without any explicit reference to the kind of interaction[62]. Such an empirical interaction is expected to contain the effects of all the neglected configurations. The renormalization due to truncation depends upon the neglected configurations and the interaction determined in this way need not resemble the free nucleon interaction. Empirical interactions have been determined for different regions. Determination of them involve large SM calculations. The amount of work and number of parameters increase rapidly with increase in the size of the model space. In addition to empirical effective two-body interaction calculations[24-26] there have been several calculations using i) experimental two nucleon energies of nearest two particle systems, ii) phenomenological interactions that resemble free nucleon interaction[63], iii) zero range surface interactions such as surface delta interaction[64-66], and Pairing plus

different for open shell nuclei. Detailed account is given in review articles by Barrett et.al.[59] and Kuo[60].

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surface tensor interaction[67], iv) the interaction that depends upon classically defined angle between the angular momentum vactors of the two interacting particles [68-71] and v) the realistic interactions obtained in reaction matrix formalism[72]. There is yet another method in which the matrix elements of interactions in the realative coordinates are treated as free adjustible parameters[46,51,63].

Experimental energies of two particle (or equivalent) systems may be used to deduce the matrix elements of V_{pn}. For example assuming ^{42}Sc as $(1f_{7/2})^2$ configuration the effective interaction matrix elements are obtaind as follows.

^{42}Sc is asystem of a proton and aneutron above ^{40}Ca core. The total energy contains energy of ^{40}Ca in ground state, rest mass energies of proton and neutron, interactions of proton and and neutron with the core, and finally the pn interaction. This is expressed by

$$E(^{42}Sc\ JT) = E(^{40}Ca\ gnd) + (p \times {}^{40}Ca\ gnd) + (n \times {}^{40}Ca\ gnd) + p + n + V_{JT}(pn)$$

similarly $$E(^{41}Ca) = E(^{40}Ca\ gnd) + (n \times {}^{40}Ca\ gnd) + n$$

$$E(^{41}Sc) = E(^{40}Ca\ gnd) + (p \times {}^{40}Ca\ gnd) + p$$

$$\therefore\ E(^{42}Sc\ JT) - E(^{41}Ca) - E(^{41}Sc) + E(^{40}Ca\ gnd) = V_{JT}(pn)$$

Expressed interms of total binding energies, this becomes

$$BE(^{42}Sc\ JT) + BE(^{40}Ca\ gnd) - BE(^{41}Sc\ 7/2) - BE(^{41}Ca\ 7/2) = V_{JT}(pn)$$

This procedure may be used to obtain the particle-hole matrix elements of interaction also. Particle-hole conjugation is obtained by S.P. Pandya[73] and Goldstein and Talmi[74] who obtained energy levels of ^{38}Cl from the experimental spectrum of ^{40}K.

Schiffer[68] plotted the relative matrix elements defined by $R_T(J,J') = e_T(J,J')\left[\Sigma_{T=1} / \Sigma_{T=0}\right]$ against classical angle $\theta_{12} = \cos^{-1}\left[\frac{J(J+1) - J'(J'+1) - 2}{2\sqrt{J(J+1)J'(J'+1)}}\right]$ and found that they lie on a smooth curve of inverted bell shape. This suggests that there is a possibility of a universal effective interaction.

In the case of phenomenological interaction with central and non central parts the V_{eff} is assumed to have an explicit form, with short range radialddependence, exchange terms and resembling the NN interaction. One writes

$$V_{eff} = V_0 \left[W + B P_\sigma + H P_\tau + M P_\sigma P_\tau \right] f(r)$$

where V_0, W, B, H, M are parameters and f(r) is radial shape such as Gausian, $\exp(-r^2/\mu^2)$, with range parameter μ, P_τ and P_σ are isospin and spin exchange operators. The radial shape be chosen to be different for different parts. The parameters are varied to get best fit to experimental energies. Usually the number of parameters in this case is less than the empirical interaction method.

One may choose a delta function radial dependence. The

delta interaction is one of a successful interactions. The matrix elements of Schiffers interaction resemble those of delta function interaction. The surface delta interaction of Moszkowski is successful probably because in a sense it is a density dependent interaction. It does not act where the density of particles is constant but acts where the density varies, i.e. at the surface of the nucleus. More recent delta type interactions of Vautherin, Brink and Skyrme[75] and Moszkowski[76] are successful in obtaining ground state properties of doubly closed shell nuclei. Sharp and Zamic[77] however point out that these interactions are not as successful for open shell nuclei. They show that these interactions do not have enough pairing force to push the ground state, 0^+ , of two particle systems sufficiently down. The interaction between valance nucleons via virtual core excitation is a long range force. Addition of long range interaction such as in pairing plus quadrupole interaction is known to improve the overall agreement with experiment indicating the importance of long range parts in effective interaction. Some of these interactions are described in the following chapters and the results of calculations using them are presented.

The purpose of the present study is to investigate the usefulness of different effective interaction models currently in vogue for the nuclei of ^{49}Sc, ^{50}Ti, ^{51}V, ^{52}Cr, ^{53}Mn, ^{54}Fe and ^{55}Co. In view of the recent experimental results and aforesaid considerations the calculations are first done

by fixing the single particle energies. The energy of $1f_{7/2}$ proton is later determined to give better binding energies of the nuclei ^{49}Sc, ^{50}Ti, and ^{51}V in the first three chapters. The single particle wavefunctions are chosen to be harmonic oscellator functions because they are good approximations to more realistic Woods-Saxon wavefunctions[78]. The model space used contains $1f_{7/2}^{n}$ and $1f_{7/2}^{n-1}\ 2p_{3/2}$ configurations and in some cases excitations to $1f_{7/2}^{n-1}\ 1f_{5/2}$ are also considered. The residual interaction is assumed to be of two-body kind only. In a recent calculation by Kirson and Eisenstein[79] in which three-body interactions are considered in addition to the usual two-body interaction, it was found that the three-body interaction are quite small for the $N = 28$ isotones. Therefore, in the present investigation the three-body interactions are neglected and the two-body interactions are determined to fit the energy levels of ^{50}Ti and ^{51}V, spectroscopic factors for ^{51}V ^{51}V (d, ^{3}He) ^{50}Ti reactions and the B(E2) transition rates in ^{51}V. The interactions determined in this way and the $2p_{3/2}\ 1f_{7/2}$ interaction determined by Horie et. al.[80] are used to simulate isospin corrections and to obtain explicit dependence of the interaction on the number of valance particles. The resulting interaction is used to determine the energy levels of the nuclei from ^{52}Cr to ^{55}Co. These calculations are presented in 5 chapters. Chapter I contains the calculation with surface interactions that contains zero range and long range

parts and Chapter II contains the calculations with empirical interactions in relative states. Phenomenological interaction that acts when the centre of mass of the interacting particles is at the effective nuclear surface is treated in Chapter III. This interaction contains short range parts with Gausian radial dependence. Similar calculations including $1f_{7/2}^{n-1}\ 1f_{5/2}$ configuration in addition are presented in Chapter IV. This chapter also contains calculations for the three two particle systems, ^{42}Ca, ^{50}Ti and ^{58}Ni where the two identical nucleons occupy all Pauli allowed states in f-p shell. It is customary to determine effective interaction from two-particle systems because there is a possibility that the interaction parameters may depend on the number of particles. In a recent calculation with surface delta interaction[85] the strength is found to decrease linearly with the number of particles. However in the present work this n-dependence is assumed to come about only due to isospin considerations. Also excitation energies of lowest states of two particle systems ^{50}Ti, ^{42}Ca and ^{58}Ni are similar. It is felt that, though the spectra of ^{42}Ca and ^{58}Ni are known to show core excitation features they are neglected in the present calculation hoping complete f-p shell calculations might be able to reproduce the spectra. This is an assumption in the present work like in earlier works [46,81]. Similarly the nucleus ^{54}Fe is known to exhibit some deformation features usually obtained by core excitations. These are also neglected in the present calculation like in

earlier calculations [24,25,26,85] The heavier isotones of ^{48}Ca, ^{52}Cr through ^{55}Co are treated in Chapter V with some of the interactions determined for ^{50}Ti and ^{51}V. In this chapter isospin is also considered. This chapter is followed by overall concluding remarks. Additional relevant material is given in the following Appendices A through F.

CHAPTER - I

SURFACE INTERACTION WITH ZERO RANGE PARTS

Nucleus regarded as a system of interacting fermions contains too many particles for exact treatment. It suffers from over abundance of degrees of freedom. Therefore, Nuclear shell model considers a limited number of active nucleons interacting in a space spanned by a limited number of orbitals. The single particle wave functions and energies are input parameters of such a model. The single particle potential obtained in a self consistant derivation such as Hartree - Fock theory is non local and therefore shell model assumes simple locall approximation to it. Harmonic oscellator wavefunctions are chosen because it is a satisfactory approximation and because of mathematical convenience. The residual interaction may be parameterized. This residual interaction may be chosen to contain long range parts and short range parts. One argument infavor of ~~short~~ zero range interaction is that the range of nuclear force is smaller than the size of the nucleus. The short range interaction such as delta force is then a valid approximation and it contains only the strength as a parameter.

One of the basic assumptions of shell model is that closed shell core does not contribute to the properties of the nucleus. When the core term is removed one considers

a diagonal effective single body term that includes renormalizations due to core separation rather in an adhock way. In the absence of residual interaction, the particles move freely in common single particle potential. This is the situation at the centre of the nucleus where the density is fairly uniform, while the particles at the surface where the density falls off to zero experience the residual interaction. Noszkowski et al[64] considered simple delta type interaction at the surface and assumed that the radial integrals are the same at the nuclear surface. Shell model calculations treating the effective interaction matrix elements as free parameters exhibit a common feature that they are repulsive on the average. In view of this observation pairing plus surface tensor interaction has been introduced[67]. These surface interactions have been successfully employed in several calculations.

The delta force may be written as

$$\delta(\bar{r}_1 - \bar{r}_2) = \frac{1}{r_1 r_2} \delta(r_1 - r_2)\, \delta(\Omega_{12})$$

The matrix elements of this interaction between two particle wave functions in LS coupling scheme are

$$\langle n_1 l_1 n_2 l_2 LM \mid \delta(\bar{r}_1 - \bar{r}_2) \mid n_3 l_3 n_4 l_4 LM \rangle$$

$$= \left[\int R_{n_1 l_1}(r_1) R_{n_3 l_3}(r_1) \frac{dr_1}{r_1^2} \right] \times \left[\frac{1}{2L+1} (-1)^{l_1 - l_2 + l_3 - l_4} \langle L \| Y_{l_1}(\Omega_1) \| l_2 \rangle \langle L \| Y_{l_3} \| l_4 \rangle \right]$$

These matrix elements may be used to get the matrix elements for jj-coupling wavefunctions. Since the reduced matrix elements in the above expression vanish when $l_1 + l_2 + L$ (or $l_3 + l_4 + L$) is odd and since the antisymmetry of the wave functions imply $l_1 + l_2 + L + S$ (or $l_3 + l_4 + L + S$) is even, we get that the interaction acts only in states with zero total spin. The transformation from LS coupling to jj-coupling contains only one term with S = 0. Finally we get[82]

$$\langle j_1 j_2 J M | \delta(\bar{r}_1 - \bar{r}_2) | j_3 j_4 J M \rangle$$

$$= (\text{Radial Integral}) \times \frac{1}{4\pi} (-1)^{l_1+l_3} h_J(j_1 j_2)\, h_J(j_3 j_4)$$

where $h_J(j_1 j_2) = [2j_2+1]^{1/2} \langle j_2\, J\, -\tfrac{1}{2}\, 0 \,|\, j_1\, -\tfrac{1}{2} \rangle$ if l_1+l_2+J is even

$= 0$ if l_1+l_2+J is odd.

we can write $\delta(\bar{r}_1 - \bar{r}_2) \times \delta(r_1 - R) = \dfrac{\delta(r_1-R)\,\delta(r_2-R)}{R^2}\, \delta(\Omega_{12})$

for the surface delta interaction. The radial integral for this interaction will be simple $\frac{1}{R^2}\, \pi\, (R_{n_i l_i}(R))$

The matrix elements of tensor force are more complicated. They can be obtained by tensor expansion of the operator. The tensor interaction is written as

$$S_{12}\, V(r_{12}) = \left[\frac{(\sigma_1 \cdot \bar{r}_{12})(\sigma_2 \cdot \bar{r}_{12})}{r_{12}^2} - \frac{1}{3} (\sigma_1 \cdot \sigma_2) \right] V(r_{12})$$

which can be expanded as

$$\Big\{ r_1^2 \Big[(s_1 \cdot \hat{r}_1)(s_2 \cdot \hat{r}_1) - \tfrac{1}{3}(s_1 \cdot s_2) \Big] + r_2^2 \Big[(s_1 \cdot \hat{r}_2)(s_2 \cdot \hat{r}_2) - \tfrac{1}{3}(s_1 \cdot s_2) \Big]$$

$$- r_1 r_2 \Big[(s_1 \cdot \hat{r}_2)(s_2 \cdot \hat{r}_1) + (s_1 \cdot \hat{r}_1)(s_2 \cdot \hat{r}_2) - \tfrac{2}{3}(s_1 \cdot s_2)(\hat{r}_1 \cdot \hat{r}_2) \Big] \Big\}$$

$$* V(r_{12}) / r_{12}^2$$

We introduce $c_1 = c_1^{(1)} = \hat{r}_1 = \bar{r}_1 / r_1$ and $c_{1\mu}^{(k)} = \sqrt{\frac{4\pi}{2k+1}}\, Y_\mu^k(\theta_1 \phi_1)$

so that $(s_1 \cdot \hat{r}_1)(s_2 \cdot \hat{r}_1) - \frac{1}{3}(s_1 \cdot s_2)$

$$= \sum_{\rho} (-1)^{-\rho+\rho'} s_{1-\rho} s_{2\rho'} \sqrt{\tfrac{2}{3}} \langle 1 -\rho\, 1\rho' \mid 112\, -\rho+\rho' \rangle\, c^2_{1\,\rho-\rho'}$$

and a similar expression for $(s_1 \cdot \hat{r}_2)(s_2 \cdot \hat{r}_2) - \frac{1}{3}(s_1 \cdot s_2)$.

The third term with $r_1 r_2$ becomes after simplification

$$(s_1 \cdot \hat{r}_1)(s_2 \cdot \hat{r}_2) + (s_1 \cdot \hat{r}_2)(s_2 \cdot \hat{r}_1) - \tfrac{2}{3}(s_1 \cdot s_2)(\hat{r}_1 \cdot \hat{r}_2)$$

$$= \sum_{\rho\rho'} \sum_{\gamma\gamma'} (-1)^{\rho-\rho'} s_{1+\rho} s_{2\rho'} \; 5 \begin{pmatrix} 1 & 1 & 2 \\ \gamma & -\gamma' & -\gamma+\gamma' \end{pmatrix} \begin{pmatrix} 1 & 1 & 2 \\ -\rho & +\rho' & \rho-\rho' \end{pmatrix}$$

$$* \Big\{ c_{1\gamma}\, c_{2\,-\gamma'} + c_{2\gamma}\, c_{1\,-\gamma'} \Big\}$$

If we expand $V(r_{12})/r_{12}^2$ we get

$$V(r_{12})/r_{12}^2 = \sum_k f_k(r_1 r_2) \sum_m (-1)^m c^k_{1m}\, c^k_{2\,-m}$$

The case with $V(r_{12}) = \delta(r_1 - R)\, \delta(r_2 - R)\, r_{12}^2 / R^2$ is much simpler with only one term having $k = 0$.

Consider $c^{k_1}_{m_1}\, c^{k_2}_{m_2} = \sum (-1)^{m} \frac{(2\ell+1)}{\sqrt{(2k_1+1)(2k_2+1)}}\, c^{\ell}_{m}$

$$c^{\ell}(k_1 - m_1\; k_2 - m_2)$$

which defines $c^{\ell}(k_1 - m_1\; k_2 - m_2) = [k_1 k_2]^{1/2} \begin{pmatrix} k_1 & k_2 & \ell \\ -m_1 & -m_2 & m \end{pmatrix} \begin{pmatrix} k_1 & k_2 & \ell \\ 0 & 0 & 0 \end{pmatrix}$

so that we get

$$\Big\{ r_1^2 \Big[(s_1 \cdot \hat{r}_1)(s_2 \cdot \hat{r}_1) - \tfrac{1}{3}(s_1 \cdot s_2) \Big] + r_2^2 \Big[(s_1 \cdot \hat{r}_2)(s_2 \cdot \hat{r}_2) - \tfrac{1}{3}(s_1 \cdot s_2) \Big] \Big\} * V(r_{12})/r_{12}^2 =$$

$$\sum_{ll'k\mu m} (-1)^{l+l'+m} f_k(r_1 r_2) \delta_{l-l'\mu} \sqrt{\frac{5}{15}} \frac{(-\mu+1)}{\sqrt{2k+1}} C^{\mu}(l\,-l+l'\;k\;m) \times$$

$$\langle l\,-l\;l' \mid l\,l\,2\,-l+l' \rangle \left\{ r_1^{k} C^{(k)}_{l'\,-l+l-m} C^{(k)}_{2m} + r_2^{k} C^{(k)}_{l\,-l'-m} C^{(k)}_{1m} \right\}$$

Similarly the term containing $r_1 r_2$ may be expanded. Using the definitions

$$W^{(k_3)}_{\mu_3} = \left[U^{(k_1)} \times V^{(k_2)} \right]^{(k_3)}_{\mu_3} = \sum_{\mu_1 \mu_2} \langle k_1 \mu_1\, k_2 \mu_2 \mid k_1 k_2 k_3 \mu_3 \rangle U^{(k_1)}_{\mu_1} V^{(k_2)}_{\mu_2}$$

and $$\left(U^{(k)} \cdot V^{(k)} \right) = (-1)^k \sqrt{(2k+1)} \left[U^{(k)} \times V^{(k)} \right]^{(0)}$$

the terms can be regrouped and finally we get[83]

$$S_{12} V(r_{12}) = \sum_k (-1)^{1+\frac{k+k}{2}} f_k(r_1 r_2) \sqrt{\frac{2}{15}} (2k+1) \left(\frac{1}{2} C_{2lk} \right)^{1/2} \times$$

$$\left\{ r_1^{k} \left(\left[S_1 \times S_2 \right]^{(2)} \cdot \left[C^{(k)}_1 \times C^{(k)}_2 \right]^{(2)} \right) + r_2^{k} \left(\left[S_1 \times S_2 \right]^{(2)} \cdot \left[C^{(k)}_2 \times C^{(k)}_1 \right]^{(2)} \right) \right\}$$

$$- \sum (-1)^{k+\frac{l-t}{2}} r_1 r_2 f_k(r_1 r_2) (2l+1)(2t+1) \left\{ \frac{1}{4} \left(C_{1lk}\, C_{1tk} \right) \right\}^{1/2} \times$$

$$W(1\,l\,t\,;\,2k) \left(\left[S_1 \times S_2 \right]^{(2)} \cdot \left\{ \left[C^{l}_1 \times C^{t}_2 \right]^{(2)} + \left[C^{l}_2 \times C^{t}_1 \right]^{(2)} \right\} \right)$$

This expression is particularly suited for LS coupling wave functions. However for use with jj-coupling wavefunctions a change of coupling transformation may be used for wave functions or the tensor products of the above experession. In the case of surface tensor interaction we take

$$V(r_{12}) = \delta(r_1 - R)\, \delta(r_2 - R)\, r_{12}^2 / R^2$$

so that the radial function becomes exactly the same as the one for surface delta interaction. Complete expression for jj-coupling matrix elements are given by D. Banerjee et al[84] which is used in the present work.

There is a difficulty with the delta interaction since the matrix elements vanish unless $\ell_1 + \ell_2 + L$ and $\ell_3 + \ell_4 + L$ are both even. Similarly the tensor interaction acts only in spin triplet states. Since the matrix elements of the tensor interaction vanish between $1f_{7/2}^{\,2}$ and $1f_{7/2}\,2p_{3/2}$ wavefunctions it can not mix the two configurations. Additional interaction of the type

$$\left(V_0 \frac{1-\sigma_1\cdot\sigma_2}{4} + V_1 \frac{3+\sigma_1\cdot\sigma_2}{4}\right) \delta(r_1 - R)\,\delta(r_2 - R)/R^2$$

may be used to compensate for the deficiency. The matrix elements of $(\sigma_1\cdot\sigma_2)$ may be easily obtained either by evaluating first in LS coupling and transforming to jj-coupling scheme or may be directly evaluated using the expressions

$$\langle j_1 j_2\, JM \,|\, (T^{(k)}_{(1)}\cdot U^{(k)}_{(2)}) \,|\, j_3 j_4\, J'M' \rangle = (\text{Radial integral}) * \delta_{JJ'}\delta_{MM'}$$

$$* (-1)^{j_2 + j_3 + J} \begin{Bmatrix} j_1 & j_2 & J \\ j_4 & j_3 & k \end{Bmatrix} (\ell_1 j_1 \| T^{(k)}_{(1)} \| \ell_3 j_3)(\ell_2 j_2 \| U^{(k)}_{(2)} \| \ell_4 j_4)$$

$$\text{and } (j_1 j_2 J \| T^{(k)}_{(1)} \| j_1' j_2' J') = (-1)^{j_1 + j_2 + J' + k} [J J']^{1/2} *$$

$$\begin{Bmatrix} j_1 & j_1' & k \\ J' & J & j_2 \end{Bmatrix} (j_1 \| T^{(k)}_{(1)} \| j_1')\, \delta_{j_2 j_2'}$$

The radial integral for this interaction is also the same as the one for surface delta interaction. The matrix elements of SDI, STI and $(\sigma_1\cdot\sigma_2)$ at the surface are evaluated and are given apart from radial integrals in Appendix B. Finally the interaction used is

$$V = \left[V_0 \frac{1 + P_{12}}{4} + V_1 \frac{3 + P_{12}}{4} + V_{SDI}\, \delta(r_{12}) + V_{STI}\, S_{12}\, r_{12}^2 + V_P\, P \right] \delta(r_1 - R)\, \delta(r_2 - R)/R^2$$

The Pairing interaction P is of the usual kind given by

$$\langle j_1 j_2 JM | P | j_3 j_4 J' M' \rangle = [j_1 j_3]^{1/2}\, \delta_{j_1 j_3}\, \delta_{j_2 j_4}\, \delta_{J0}$$

The effective Hamiltonian for the valance nucleons contains simple diagonal one body part and a two body part. The matrix elements of the one body part are simply $\Sigma \epsilon_j n_j$ where n_j is the number of particles in the orbit j and $\Sigma n_j = n$ the total number of particles is valance orbits. The single particle energy of the $2p_{3/2}$ proton for the isotones of ^{48}Ca may be taken to be 3.5 Mev with respect to that of $1f_{7/2}$ proton. The binding energy of $1f_{7/2}$ proton taken to be positive is 9.62 MeV[10] while it is obtained at 9.72 MeV in earliar calculations[24,25]. The energy of $1f_{7/2}$ proton is usually varied to fit the binding energies while the energies of $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ protons may be fixed with respect to that if $1f_{7/2}$ proton.

The choice of the model space wave functions is explained in the introduction. Since isotopic spin is maximum for identical particle configurations it is dropped. The basis functions of the space are $|j^n J\rangle$, $|j^{n-1}(J_1) j' J\rangle$ The Hamiltonian is scalar in space spin variables and therefore diagonal in J. The Hamiltonian matrix for each allowed

value of J is set up. For example in the case of J = 3/2 of ^{51}V the space contains $|j^3\ 3/2\rangle$, $|j^2(J1)j'\ 3/2\rangle$ J1 = 0, 2 functions where j = $1f_{7/2}$ and j' = $2p_{3/2}$. The matrix elements of one and two body operators between n-body wave functions are written in terms of matrix elements between one and two body wavefunctions using s tandard method. These are given in Appendix D. The matrix elements of Hamiltonian contain the parameters of the interaction, the strengths, and the single particle energies. The eigen values depend upon the parameters and they are to be compared with the experimental total energies minus the ground state energy of ^{48}Ca nucleus. The values of the parameters are to be varied to reproduce the chosen energy levels satisfactorily. There are four lowest levels of ^{50}Ti and five lowest levels of ^{51}V well determined experimentally. The parameters are varied to minimise the root mean square deviation for these energy levels. The procedure followed in fitting is given in Appendix F. The wavefunctions are used to calculate the spectroscopic factors for the pick up reactions $^{51}V(\ d\ ,\ ^3He\ )\ ^{50}Ti$, $^{50}Ti(\ d,\ ^3He)\ ^{49}Sc$, $^{50}Ti\ (\ ^3He\ d)\ ^{51}V$ and $^{49}Sc\ (\ ^3He\ d)\ ^{50}Ti$ and the electric quadrupole transition rates in ^{51}V and ^{50}Ti. These properties are explained in Appendix E. The spectroscopic factors for single particle transfers are usually chosen because these reactions single out and excite simple degrees of freedom associated with the single particle states,

existance of which is the back bone of shell model, leaving all other degrees of freedom unaffected. Spectroscopic factors are squares of overlap integrals between initial and final states and the electromagnetic transition rates are squares of matrix elements of transition operators. These quantities are sensitive to the choice of model space and form a test of the wavefunctions.

In view of the results of earlier calculations the energy of $1f_{7/2}$ proton is first fixed at 9.72 MeV in the present calculation. This value affects essentially the total energies of ground states while the excitation energies depend upon the difference between the unperturbed energies of configurations and the residual interaction. The energy of $1f_{7/2}$ proton is later determined to give best binding energies. The radial integrals are assumed to be same at the nuclear s urface and are absorbed in the strength parameters. The results are given under the caption calculation I, in the tables IA to IG, in which the results are compared with the calculations using surface delta interaction[85] and surface tensor interaction[86] and experiment. The energy levels are taken from Nuclear Level Scheme A=45 through A = 257[87], E2 transition rates are taken from Afonin et al[1] and B.A. Brown et al[88]. The spectroscopic factors for (d, ^{3}He) and (^{3}He,d) reactions are taken from Cuec et al[4], Newman et al[5] and O'Brian et al[7]. The strengths of different interactions are presented in table IA and the jj-coupling matrix elements of interaction are given in table IB. The ground state energies and the excitation energi

	X	V_o	V_1	V_{SDI}	V_{STI}	V_P
Cal. I	-	1.410	-1.350	0.850	0.097	0.055
Cal. II	-	0.804	-1.349	1.286	-	-
Cal. III	2.5	0.1400	-0.1240	0.0826	0.0132	0.0020
Cal. IV	2.5	0.0427	-0.1127	0.1230	-	-

Table I-B Matrix elements of the interaction in MeV in jj-coupling scheme between antisymmetric wave functions. Positive matrix elements represent attraction and negetive matrix elements represent repulsion.

J	Cal.I	Cal.II	Cal.III	Cal.IV	Empirical
0	2.343	2.295	2.331	2.309	2.290
2	0.503	0.323	0.631	0.240	0.465
4	-0.121	-0.276	-0.055	-0.320	-0.420
6	-0.890	-0.901	-0.904	-0.860	-0.815
2	-0.558	-0.845	0.680	1.013	0.655
4	-0.218	-0.329	0.265	0.395	0.400
2	1.112	1.123	1.623	1.334	2.675
3	-0.131	-0.426	-0.084	-0.648	-0.875
4	-0.442	-0.453	-0.529	-0.549	-0.100
5	-1.400	-1.349	-1.840	-1.584	-2.200

where $j = 1f_{7/2}$ and $j' = 2p_{3/2}$. The empirical matrix elements are those determined by K. Lips et.al.

Table I-C Energy levels of 50Ti, 51V and 49Sc in MeV. The binding energies are taken to be positive. The results of SDI and PSTI are taken from R.Sanyaman et.al. and D.Banerjee et.al., while experimental energies are taken from Nuclear Level Schemes A = 45 through A = 257 ... 1973, and the ground state binding energies are taken from Binding Energy Tables 1964.[41] Only the lowest states for each angular momentum are fitted in the search . The binding energies of ground states are fitted as explained in the text.

Nucleus	J	Cal.I	Cal.II	Cal.III	Cal.IV	SDI	PSTI	Expr.
^{49}Sc (g.s.)	7/2	9.670	9.724	9.648	9.718			9.62
^{50}Ti(g.s.)	0	21.683	21.743	21.628	21.746			21.79
	2	1.736	1.729	1.528	1.700	2.09		1.554
		4.835	4.915	4.381	4.845	5.35		4.323
	4	2.451	2.542	2.368	2.588	2.69		2.677
		6.298	6.277	6.367	6.399	6.07		4.804
	6	3.233	3.196	3.235	3.170	2.91		3.201
^{51}V(g.s.)	7/2	29.938	29.881	29.958	29.865			29.85
	3/2	0.746	0.814	0.528	0.835	1.36	0.51	0.929
		2.977	2.882	3.195	3.083	2.96	3.19	2.409
		4.216	4.409	4.149	4.704	4.67	4.11	3.215
	5/2	0.697	0.585	0.445	0.518	1.03	0.63	0.320
		4.188	4.419	3.951	4.621	4.13	4.00	3.082
		5.031	5.167	5.183	5.871	4.91	4.46	
	7/2	4.452	4.408	4.256	4.457	4.48	4.09	
		5.437	5.476	5.785	5.902	4.95	4.94	
	9/2	1.647	1.764	1.558	1.814	1.83	1.52	1.813
		4.072	3.976	3.538	3.861	4.10	4.27	
		5.834	6.053	6.141	6.524			
	11/2	1.922	1.886	1.856	1.866	1.72	1.63	1.609
		5.494	5.583	5.481	5.771			
		6.559	6.502	7.155	6.937			
	15/2	3.072	2.981	3.143	2.915	2.24	2.84	2.699
		7.834	7.544	8.704	7.934			
R.M.S.								

Table I-D Spectroscopic factors for single proton transfer reactions. Experimental values contain square of the Clebsch-Gordon Coefficient for isospin coupling and are taken from Ref. .

Stripping reactions:

Final	State	Transfer	Cal.I	Cal.II	Cal.III	Cal.IV	Expr.
^{50}Ti	0	7/2	2.0	2.0	2.0	2.0	
	2	7/2	1.933	1.850	1.878	1.764	
		3/2	0.034	0.075	0.061	0.118	
	4	7/2	1.994	1.984	1.992	1.978	
		3/2	0.003	0.008	0.004	0.011	
	6	7/2	2.0	2.0	2.0	2.0	
	2_2	7/2	0.067	0.150	0.122	0.236	
		3/2	0.966	0.925	0.939	0.882	
^{51}V	7/2	7/2	0.748	0.745	0.747	0.743	0.75
	3/2	3/2	0.001	0.002	0.001	0.004	0.012
	$(3/2)_2$	3/2	0.975	0.963	0.920	0.941	0.45

Pickup reactions:

Final	State	Transfer	Cal.I	Cal.II	Cal.III	Cal.IV	Expr.
^{49}Sc	7/2	7/2	2.0	2.0	2.0	2.0	1.93
^{50}Ti	0	7/2	0.748	0.745	0.747	0.743	0.74
	2	7/2	0.391	0.359	0.373	0.332	0.37
		3/2	0.002	0.004	0.003	0.005	
	4	7/2	0.744	0.736	0.742	0.732	0.75
	6	7/2	1.080	1.076	1.079	1.074	1.14
	2_2	7/2	0.010	0.019	0.036	0.077	
		3/2	0.001	0.001	0.001	0.001	
	4_2	7/2	0.002	0.005	0.004	0.009	

of lowest states, which are taken for the least squares fitting and other states obtained are given in table I-C. The spectroscopic factors and B(E2) rates are given in table I-D and I-E. The order of levels in ^{50}Ti and ^{51}V are quite good even through they are off by about 0.20 MeV. The B(E2) rates show maximum discrepancy but the spectroscopic factors are only a little smaller than the experimental results. The spectroscopic factors calculated for $^{51}V(d, ^{3}He)^{50}Ti$ reaction involve the ground state wave function of ^{51}V which contains small contributions from different excited configurations and essentially only one component of the wave functions of ^{50}Ti. Also the reaction $^{50}Ti(^{3}He,d)^{51}V$ involves the ground state of ^{50}Ti which is a pure $(f_{7/2})^2$ $J = 0$ state the present model. The B(E2) rates calculated involve different states of ^{51}V all of which contain different amounts of contributions from excited configurations. Signs of the components of wavefunctions $|j^{n-1}(J_1)j'J\rangle$ depend upon the signs of the matrix elements of interaction $\langle j^2|V|jj'\rangle_J$ which are responsible for mixing the wavefunctions. The energies depend upon the numerical values of these matrix elements but insensitive to their signs. Compared to the matrix elements empirically determined, these are of opposite sign. A change in the sign of these matrix elements therefore changes the B(E2) rates in ^{51}V quite drastically while calculated spectroscopic factors are affected less leaving the energies unaltered. This discrepancy with the sign of the non diagonal matrix elements of surface delta interaction is essentially

Table I-E The B(E2) values in units of e^2. 10^{-50}. cm^4. The effective charge used is 1.6 e. The calculated values are compared with the experimental values taken from Ref. and the calculations of D. Banerjee et.al., and R. Saayaman et.al.

Nucleus ^{50}Ti.

Transition	Cal.I	Cal.II	Cal.III	Cal.IV	DB	RS	Expr.
2 — 8	0.426	0.369	0.603	0.616		0.92	0.66
4 — 2	0.443	0.399	0.592	0.611		0.82	0.60
6 — 4	0.207	0.191	0.272	0.292		0.36	0.34

Nucleus ^{51}V.

Transition	Cal.I	Cal.II	Cal.III	Cal.IV	DB	RS	Expr.
7/2 —3/2	0.167	0.146	0.285	0.292	0.31		0.27
5/2	0.675	0.529	1.142	1.147	0.74		0.92
9/2	0.221	0.189	0.343	0.379	0.11		0.22
11/2	0.814	0.777	0.900	0.892	0.34		0.90
e_{eff}	1.76	1.85	1.49	1.49			
3/2 —7/2	0.334	0.292	0.569	0.583		0.63	0.72
5/2	0.899	0.705	1.522	1.528		1.95	1.54
9/2	0.177	0.151	0.274	0.303		0.40	0.27
11/2	0.542	0.518	0.600	0.595		0.89	0.78

due to the assumption that the radial integrals are equal at the surface. On the other hand further iteration leads to reduction in the strengths of pairing and tensor parts of the interaction. This situation can be understood from the fact that the delta interaction has some pairing paroperty and the required property that the interaction should be repulsive on the average for the $|JJ'T\rangle$ states can be obtained from the $(\sigma_1 \cdot \sigma_2)$ part of the interaction. These matrix elements are given in Appendix B. It is not possible to improve the situation without allowing some of the strengths to be negative which is quite unphysical. The least squares fitting calculations are done by removing the pairing and tensor parts of the interaction. This results in an increase in the delta interaction strength and a reduction in the central interaction in S = 0 states. The results of this calculation II are given in tables. There is slight improvement in the energies in the present calculation while the spectroscopic factors and B(E2) rates decrease further away from the experimental results. The B(E2) rates are calculated with an effective charge $e_{eff} = 1.6e$[29] and a change in this value would not help much.

Next possible candidate for modification is the assumption that radial integrals are equal at the nuclear surface. As suggested already a change in the sign of the mixing matrix elements of the interaction can be obtained by relaxing this condition. A closer examination of the radial integrals reveals that it does. At least for fp shell orbits the radial integrals are not equal at the surface. The radial integrals

for a zero range interaction that acts throughout the nucleus and these for the one that acts at the effective surface are

$$\int R_{n_1 l_1}(r_1) R_{n_2 l_2}(r_2) R_{n_3 l_3}(r_1) R_{n_4 l_4}(r_2) \, dr_1 \, dr_2$$

and

$$\int R_{n_1 l_1}(r_1) R_{n_2 l_2}(r_2) R_{n_3 l_3}(r_1) R_{n_4 l_4}(r_2) \frac{(r_1 - r_2)}{R^2} \, dr_1 \, dr_2$$

Apart from a common factor the latter integrals become, for fp shell harmonic oscellator functions

$$F1 = F(1f, 1f, 1f, 1f) = 0.2857\, x^4$$

$$F2 = F(1f, 1f, 1f, 2p) = 0.5345(2.5 - x^2)\, x^2$$

and

$$F3 = F(1f, 2p, 1f, 2p) = (2.5 - x^2)^2$$

where $x = R/\beta$, $\beta = (\hbar/m\omega)^{\frac{1}{2}}$ and R is the effective radius of the nucleus. The radius of ^{48}Ca nucleus is about 4.5 fm. With value of derived by I. Talmi[29] we get that x is around 2.4. This makes F2 negative and change the sign of the matrix elements that bring about the configuration mixing. The values of F1, F2 and F3 for x = 2.4 are 9.478, -10.03 and 10.62 respectively and for x = 2.5 they are 11.160, -12.527 and 14.063 respectively.

Calculations are repeated by treating the effective radius as a free parameter. In calculation III all the five components of interaction are included and the results are given in tables. The Hamiltonian matrices and wavefunctions are given in tables I-F and I-G. The B(E2) rates improved very much in this calculation. The value of the parameter x is obtained at 2.5, close to the required value. The exci-

tation energy of the lowest 2^+ state of ^{50}Ti is obtained at 1.53 MeV with only a limited effect on the other lowest states of ^{50}Ti. However the energies of lowest levels of ^{51}V are badly affected in the present calculation. Further variation of parameters could only reduce pairing and tensor parts without improving the results very much. Calculations are repeated fourth time with only central and delta parts of the interaction. In this calculation IV again the value of x is obtained at 2.5. The B(E2) rates improved a little. The energy levels and spectroscopic factors are at least as good as in the other calculations. Except in the calculation III, the energy levels of ^{50}Ti and ^{51}V are quite similar. In calculation IV the excitation energies are off by a maximum of 0.15 MeV in ^{50}Ti. The lowest $3/2^-$ and $5/2^-$ states in ^{51}V are much better in this calculation. The levels $(9/2^-)_1$ and $(11/2^-)_1$ in ^{51}V are reversed in all these calculations. This can be understood by comparing the matrix elements of the present interaction with those determined empirically. Striking differences are that the non diagonal matrix element $\langle j^2|V|jj'\rangle_2$ is much larger in the present calculation which is essentially due to SDI. In the calculation III in which this is better, the 2^+ state of ^{50}Ti and 5/2 state of ^{51}V are closer to the experiment but the 4^+ state of ^{50}Ti and 3/2 state of ^{51}V are spoiled very much. The same interaction is responsible for larger repulsion in the state jj' $J = 4$ which requires more mixing to pull $(4^+)_1$ state fairly up. The resulting $(2^+)_1$

and $(4^+)_1$ states are off by 0.15 MeV from the experimental levels. Compared to the calculations in which all the 10 matrix elements as free parameters, the present calculation with only a few parameters provides a good description of those two nuclei. The results in tables I-C and I-D are compared with those of PSTI and of SDI, both are surface interactions, and experimental results. Banerjee et. al. did not calculate the energy levels of ^{50}Ti. The present interaction is certainly much better then pure SDI or PSTI evident from the $(3/2^-)_1$ and $(5/2^-)_1$ states of ^{51}V. The $(9/2)_1$ and $(11/2)_1$ are reported in reversed order in SDI and PSTI. The energy levels of ^{51}V are a little more separated in the present calculation compared to SDI and PSTI calculations. Saayaman et. al. did not calculate the spectroscopic factors. The spectroscopic factors reported by Banerjee et. al. and the results of the present calculation are equally comparable to the experiment. Banerjee et. al. reported B(E2) rates for absorption and Saayaman et. al. reported for emission. Therefore B(E2) rates are calculated for both transitions to make the comparison easier. These values are obtained in the present work much closer to the experiment than those of SDI , PSTI calculations. This may be due to the number of parameters. But in general the results are comparable to those of empirical interaction[25] calculations. The $(11/2)_1$ and $(9/2)_1$ are reversed in all the four calculations presented here like in PSTI calculations. In empirical ineraction

Table I-F Hamiltonian matrices. Since the matrices are symmetric and some of the components do not mix, due to angular momentum coupling rules, only the relevant components are given. A constant n* 5.02, where n is the number of particles, should be added to all the diagonal elements, which is done after diagonalization.

CALCULATION I:

50Ti

J = 2	9.903	-0.558	7.012			
J = 4	9.279	-0.218	5.458			

51V

J = 1.5	14.139	-0.000	0.684	12.022	0.195	10.985
J = 2.5	14.124	0.550	0.623	10.275	0.455	10.770
J = 3.5	15.012	-0.255	-0.133	9.604	-0.031	10.578
J = 4.5	13.360	-0.028	0.231	9.301	0.419	10.871
J = 5.5	13.045	0.098	-0.451	8.486	-0.136	9.578
J = 7.5	11.954	0.098	7.197			

CALCULATION II:

50Ti

J = 2	9.723	-0.845	7.023			
J = 4	9.124	-0.329	5.447			

51V

J = 1.5	13.656	-0.000	1.036	11.877	0.295	10.788
J = 2.5	13.755	0.832	0.942	9.991	0.689	10.309
J = 3.5	14.772	-0.316	-0.201	9.364	-0.047	10.406
J = 4.5	12.992	-0.042	0.350	8.972	0.635	10.668
J = 5.5	12.786	0.148	-0.681	8.349	-0.207	9.321
J = 7.5	11.822	0.149	7.270			

CALCULATION III:

50Ti

J = 2	10.031	0.680	7.523			
J = 4	9.345	0.265	5.381			

51V

J = 1.5	14.376	0.000	-0.833	11.845	0.266	11.249
J = 2.5	14.356	-0.669	-0.758	10.273	0.622	11.134
J = 3.5	15.091	0.310	0.162	9.347	-0.043	10.863
J = 4.5	13.517	0.034	-0.282	9.106	0.573	11.482
J = 5.5	13.169	-0.119	0.549	7.979	-0.136	9.701
J = 7.5	11.968	-0.120	6.412			

CALCULATION IV:

50Ti

J = 2	9.640	1.013	7.234			
J = 4	9.080	0.395	5.351			

51V

J = 1.5	13.501	0.001	-1.241	11.645	0.396	10.661
J = 2.5	13.635	-0.997	-1.129	9.674	0.927	10.111
J = 3.5	14.761	0.462	0.241	8.949	-0.064	10.360
J = 4.5	12.913	0.050	-0.420	8.584	0.854	10.793
J = 5.5	12.756	-0.178	0.817	7.932	-0.278	9.167
J = 7.5	11.888	-0.178	6.882			

Table I-G Wavefunctions of all states presented in Table I-C.

CALCULATION I:

50Ti	Energy					
J=0	0.0	1.0	-			
J=2	1.736	0.983	0.183			
	4.835	-0.183	0.983			
J=4	2.451	0.998	0.057			
	6.298	-0.057	0.998			
J=6	3.233	1.0	-			
51V	Energy					
J= 1.5	0.746	0.979	0.018	0.204		
	2.977	-0.049	0.988	0.149		
	4.216	-0.199	-0.156	0.968		
J= 2.5	0.697	0.970	-	0.153	0.188	
	4.188	-0.239	-	0.458	0.856	
	5.031	0.044	-	-0.876	0.480	
J= 3.5	0.0	0.999	-	-0.047	-0.030	
	4.452	-0.026	-	0.039	-0.999	
	5.437	0.048	-	0.998	0.038	
J= 4.5	1.647	0.996	-	-	0.003	0.092
	4.072	-0.090	-	-	0.246	0.965
	5.834	-0.020	-	-	-0.969	0.245
J= 5.5	1.922	0.992	-	-	0.025	-0.128
	5.494	-0.130	-	-	0.116	-0.985
	6.559	-0.010	-	-	0.993	0.118
J= 7.5	3.072	0.999	-	-	-	0.021
	7.834	-0.021	-	-	-	0.999

CALCULATION II:

50Ti	Energy					
J=0	0.0	1.0	-			
2	1.729	0.961	0.276			
	4.915	-0.276	0.961			
4	2.542	0.996	0.089			
	6.277	-0.089	0.996			
6	3.196	1.0	-			
51V	Energy					
J=1.5	0.814	0.950	+0.098	0.311		
	2.882	-0.098	0.982	0.165		
	4.409	-0.298	-0.187	0.936		
J=2.5	0.585	0.937		0.227	0.266	
	4.419	-0.349		0.565	0.748	
	5.167	0.020		-0.793	0.609	
J=3.5	0.0	0.997		-0.070	-0.045	
	4.408	-0.041		0.061	-0.997	
	5.476	0.073		0.996	0.057	
J=4.5	1.764	0.989			0.013	0.149
	3.976	-0.145			0.323	0.935
	6.053	-0.036			-0.946	0.321
J=5.5	1.886	0.981			0.040	-0.188
	5.583	-0.192			0.194	-0.962
	6.502	-0.002			0.980	0.198
J=7.5	2.981	0.999				0.033
	7.544	-0.033				0.999

CALCULATION III:

50Ti	Energy					
J=0	0.0	1.0	-			
2	1.528	0.969	0.246			
	4.381	-0.246	0.969			
4	2.368	0.998	0.066			
	6.367	-00.066	0.998			
6	3.235	1.000	-			
51V	Energy					
J=1.5	0.528	0.970	-0.024	-0.244		
	3.195	0.091	0.959	0.268		
	4.149	0.228	0.282	0.932		
J=2.5	0.445	0.955		-0.179	-0.236	
	3.951	0.290		0.393	0.873	
	5.183	-0.063		-0.902	0.427	
J=3.5	0.000	0.998		0.053	0.038	
	4.256	0.036		0.035	-0.999	
	5.785	-0.055		0.998	0.033	
J=4.5	1.558	0.991			-0.010	-0.137
	3.538	0.136			0.226	0.965
	6.141	0.021			-0.974	0.225
J=5.5	1.856	0.988			-0.028	0.154
	5.481	0.156			0.099	-0.983
	7.155	0.012			0.995	0.103
J=7.5	3.143	0.999				-0.022
	8.704	0.022				0.999

CALCULATION IV:

50Ti	Energy					
J=0	0.0	1.0	-			
J=2	1.700	0.939	0.343			
	4.845	-0.343	0.939			
J=4	2.588	0.995	0.104			
	6.399	-0.104	0.995			
J=6	3.170	1.0	-			

51V	Energy					
J=1.5	0.835	0.932	-0.060	-0.356		
	3.083	0.139	0.970	0.199		
	4.704	0.334	-0.235	0.913		
J=2.5	0.518	0.917	-	-0.259	-0.305	
	4.621	0.399		0.548	0.735	
	5.871	-0.023		-0.795	0.606	
J=3.5	0.0	0.996		0.078	0.053	
	4.457	0.0481		0.061	-0.997	
	5.902	-0.081		0.995	0.056	
J=4.5	1.814	0.980			-0.027	-0.197
	3.861	0.195			0.329	0.924
	6.524	0.040			-0.944	0.328
J=5.5	1.866	0.976			-0.047	0.215
	5.771	0.220			0.204	-0.954
	6.937	0.000			0.978	0.210
J=7.5	2.915	0.999				-0.036
	7.934	0.036				0.999

calculations the space chosen is the same as the one in the present calculations but the interaction has more freedom, having ten parameters. The calculations of SDI and PSTI are with in much larger spaces. The SDI calculations are done in complete fp shell and the PSTI calculations are done with all Pauli allowed configurations of the form $(1f_{7/2})^3$, $(1f_{7/2})^{n1}\ (2p_{3/2})^{n2}$ $n_1 + n_2 = 3$; $(1f_{7/2})^2\ (1f_{5/2})^1$ and $(1f_{7/2})^1\ (1f_{5/2})^2$. This implies that these two levels need more parameters in the interaction.

The lowest levels of both ^{50}Ti and ^{51}V are more close to the experiment than in the earlier calculations with surface interactions. The B(E2) rates are calculated with $e_{eff} = 1.6e$ as in the earlier calculations. More suitable effective charge is also calculated in all the calculations. This is even less than 1.6 for the calculations with effective radius and more than 1.6e for the others. The strength of the SDI part in the ppresent calculation is 0.35 in calculation I and 1.29 in calculation II, where as it is 0.55 for ^{50}Ti and 0.52 for ^{51}V in SDI calculations of Saayaman et al. The wavefunctions in present calculations are similar in general to those of earlier calculations.

The one useful result in the present calculations is that the relaxation of the assumption regarding the radial integrals improves the situation very much. The value of x obtained at 2.5 is very close to its experimental counter part.

It is seen that at least for the surface delta interaction this modification is necessary to make the mixing matrix elements to have correct sign compared to those of empirical interaction. It is also seen that this sign is is important to improve the properties of nuclear states, perticularly the B(E2) rates . Satis factory B(E2) rates obtained by Saayaman et al w ith SDI could be the result of the chddce of the model space. It a ppears that instead of assuming a well defined effective radiis where the inte raction ants, the interaction may be allowed to act in an extended region near the surface possibly would improve the results without drastic changes in the model.

CHAPTER - II

EMPIRICAL INTERACTION IN RELATIVE COORDINATES

Characterisation of effective interaction is an important feature in shell model. One popular method is to dispense with explicit potential picture and to treat the interaction matrix elements that enter the shell model calculations as free parameters[62]. The effective interaction matrix elements of this kind have been determined for many nuclear regions and they give a remarkally good fit to experimental energy levels. Such an empirical interaction is supposed to contain core effects. The advantage in this method is that it provides information about the usefulness of the model space of wavefunctions. To be more specific it tells whether a chosen model space can give raise to the observed levels and dynamic properties at all with any effective interaction. This can be understood because having chosen a model space, the interaction that can be determined suitable for the space can utmost have as much freedom as the two body interaction matrix elements needed. It may be noted that Cohen et al[89] emphasised that the effective interaction determined this way can not gaurentee that the wavefunctions obtained in such a calculation would give raise to the dynamic properties satisfactorily. If the model space is not apropriately chosen then the effective interaction and wavefunctions in such a

determination may bear very little similarity to the physically correct effective interactions and wavefunctions. On the other-hand renormalisation of the residual interaction due to neglected configurations may not be a simple local interaction and it may not be possible to approximate it to a perticular potential. Thus the two body matrix elements of the effective interaction which give raise to the observed structure and dynamical properties of the nuclear states are more important than a perticular form of the effective interaction. Such matrix elements of effective interaction form a guideline to understand the usefulness of methods and approximations used in obtaining the effective interaction from the free nucleon interaction in microscopic theories. For example, the importance of including G_{3p1h} in reaction matrix is understood from the observation the matrix elements so obtained resemble those determined empirically. In the case of identical nucleons in $1f_{7/2}$ orbit there are four matrix elements of the interaction. If one of the particles is raised to $2p_{3/2}$ orbit only then there will be ten matrix elements and if the raised particle is allowed to be in one of $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$ orbits the number raises to 30. These are $\langle j^2 J|V|j^2 J\rangle$ (four), $\langle j^2 J|V|jj'J\rangle$ (six) and $\langle jj'J|V|jj''J\rangle$ (twenty) where $j=1f_{7/2}$ and $j',j'' = 2p_{3/2}$, $1f_{5/2}$ or $2p_{1/2}$. This number increases further if more particles are raised or if T = 0 matrix elements are needed. It becomes more and more difficult to

treat these matrix elements as free parameters in shell model calculations with large model space. The usual phenomenological potentials, which are simple and local depend upon only the relative coordinates of the interacting particles. All the details of the interaction are contained in the matrix elements of the form.

$$I^{ST}_{n\ell n'\ell'} = \langle n\ell SjT|V|n'\ell' SjT\rangle$$

where $(n\ell)$ designate the wavefunction in relative coordinaes. The transformation from this system of coordinates to the jj-coupling system involves only geometry while all the physical nature of the interaction is contained in these matrix elements. In calculations with realistic potentials the radial integrals in relative coordinates are derived first and the reaction matrix elements are further improved to include core excitation effects[72]. The matrix elements of the effective interaction determined in relative coordinates for use in shell model calculations may be directly compared to those obtained from realistic potentials. Empirical determination of these matrix elements in relative and centre of mass system is of great value. The number of such matrix elements does not increase with the size of the model space if the valance particles remain the same fp shell. This is a real advantage since there are only 16 T = 0 and 16 T = 1 relative states for fp shell nuclei where as there are 30 T = 1 states in

jj - coupling system.

The two body matrix elements of the interaction in jj - coupling scheme can be expressed in terms of those in relative and centre-of-mass system. It can be radily shown that the Hamiltonian for two particles in common harmonic oscellator potential will remain to have the same form in relative and centre-of-mass system and that the wavefunctions are products of harmonic oscellator functions in relative and centre-of-mass coordinates. We define $\bar{r} = (\bar{r}_1 - \bar{r}_2)/\sqrt{2}$ and $\bar{R} = (\bar{r}_1 + \bar{r}_2)/\sqrt{2}$, since the nucleons have practically the same mass, so that $\bar{p} = (\bar{p}_1 - \bar{p}_2)/\sqrt{2}$ and $\bar{P} = (\bar{p}_1 + \bar{p}_2)/\sqrt{2}$. The Hamiltonian is given by $H = (p_1^2 + p_2^2 + r_1^2 + r_2^2)/2 = (p^2 + P^2 + r^2 + R^2)/2 = H'$ and the angular momentum is given by $\bar{L} = (\bar{\ell}_1 + \bar{\ell}_2) = (\bar{r}_1 x \bar{p}_1 + \bar{r}_2 x \bar{p}_2)$ $= (\bar{r} \times \bar{p} + \bar{R} \times \bar{P}) = \bar{\ell} + \bar{\mathfrak{L}} = \bar{L}'$. The Hamiltonian in the new system can be seen to be exactly in the same form as the one in the old system. Therefore the solutions in the new system are also products of harmonic oscellator functions in r and R. The transformation requires Brody-Moshinsky brackets[90]. The matrix elements between antisymmetric two particle wave-functions in jj-coupling are expressed in terms of the matrix elements in relative and centre-of-mass system as follows

$$
{}_A\langle n_1 \ell_1 j_1 n_2 \ell_2 j_2 JT | V | n_3 \ell_3 j_3 n_4 \ell_4 j_4 JT \rangle_A
$$

$$
= \left(\left[1+\delta_{n_1 n_2} \delta_{\ell_1 \ell_2} \right] \left[1+\delta_{n_3 n_4} \delta_{\ell_3 \ell_4} \right] \right)^{-1/2} \ *
$$

$$\Sigma \langle n \ell NL\Lambda | n_1 \ell_1 n_2 \ell_2 \Lambda \rangle \langle n' \ell N'L'\Lambda' | n_3 \ell_3 n_4 \ell_4 \Lambda' \rangle$$

$$\delta_{NN'}\, \delta_{LL'}\, (-1)^{\Lambda+\Lambda'} \left[1-(-1)^{\ell+S+T}\right] /2 * \Sigma [\Lambda j \Lambda' j]^{1/2} (-1)^{L+\ell+S+j} \cdot$$

$$\begin{Bmatrix} L & \ell & \Lambda \\ S & J & j \end{Bmatrix} \begin{Bmatrix} L & \ell' & \Lambda' \\ S & J & j \end{Bmatrix} \langle n\ell S(j)JT|V|n'\ell' S(j)JT\rangle$$

where the symbols have their usual meaning. The 6-J symbols and the 9-J symbols enter this exoression because of recoupling of angular momenta. The quantities $\langle n \ell NL\Lambda | n_1 \ell_1 n_2 \ell_2 \Lambda \rangle$ enter because of transformation from LS coupling wavefunctions to the wavefunctions in the relative and centre-of-mass coordinate system (RCM). These are known as Brody-Moshinsky brackets are simply brackets. This expression is a result of successive application of transformations, first from jj-coupling to LS-coupling, then to the RCM system and finally a change of coupling transformation to couple the realtive orbital angular momentum to the total spin of the two particles. The factor $[1 - (-1)^{\ell+S+T}]$ in the expression ensures anti-symmetrization[91], since the effect of a transposition (1,2) on the ket in the RCM system, $|n\ell S(j)NL,JT\rangle$ is to multiply the relative coordinate part by $(-1)^{\ell}$, spin part by $(-1)^{1+S}$ and isospin part by $(-1)^{1+T}$, the exchange integral is obtained

by multiplying the direct integral by the factor equal to $(-1)^{\ell+S+T}$. The quantum numbers n, ℓ designate relative states and N,L designate the wavefunction in centre-of-mass coordinates. Due to the energy consideration and since the frequency of oscellator remains the same for the new wavefunctions, the brackets vanish if $2n_1 + \ell_1 + 2n_2 + \ell_2 \neq 2n + \ell + 2N + L$. If only central interactions are assumed then for a single major shell such as fp shell, the matrix elements are diagonal in n, ℓ and independent of NL. The matrix elements are

$$I_{n\ell ST} = \langle n\ell S(j)JT|V|n\ell S(j)JT\rangle$$
$$= \int [R_{n\ell}(r)]^2 * V(r)dr$$

and contain all the information about the interaction. There are only 16 T = 1 matrix elements for the fp-shell orbits. These are the $I_{n\ell}$'s such that $2n+\ell \leq 6$. It is a tremendous simplification to treat $I_{n\ell}$'s as free parameters. Since the jj-coupling matrix elements are simply linear combinations of $I_{n\ell}$'s with the coefficients depending on goemetry only, the coefficients can be evaluated once for all. Some of these coefficients are tabulated in Appendix C. Simple generalizations such as inclusion of non-central parts or dependence of interaction on centre-of-mass may be done.

Calculations are done in two steps assuming pure central interaction by treating the matrix elements of the interaction in relative coordinates, $I_{n\ell}$'s as free parameters. Pure central

interaction is assumed to make the interaction simple with a small number of parameters. In Calculation - I only interaction in relative s-and p - states is considered while all other interactions are taken to be zero. Interaction in relative d-states is also included in Calculation - II. Interaction in higher angular momentum states are neglected because for lowlying states they may not contribute much. In both the calculations the method of calculation is the same as described in chapter I, i.e. construction of Hamiltonians in terms of two body matrix elements of the interaction in jj-coupling, which are in turn expressed as linear combinations of interaction matrix elements in relative states whose parameters are varied to fit experimental data of ^{50}Ti and ^{51}V nuclei. The energy of $1f_{7/2}$ proton, $\epsilon_{1f_{7/2}}$ is later determined to improve the binding energies of the ground states of these nuclei. By treating the I_{nl} 's as freeparameters the excitation energies of lowlying states of ^{50}Ti and ^{51}V are fitted and spectroscopic factors for single proton transfer reactions and the reduced electric quadrupole transition rates in ^{51}V are calculated.

The results of these calculations show striking similarities. The matrix elements of the interaction in both calculations have the property that the interaction in $|f_{7/2}\ p_{3/2}^{J}\rangle$ states is repulsive on the average. The

Table II-A Empirical interaction matrix elements, I_{nl}, in relative states, in MeV. Suffixes n and l are radial and orbital angular momentum quantum numbers of the relative states in harmonic oscellator representation. The interaction is assumed to be purely central and acts in relative s-, p-, and d- states only.

n	l	Cal. I	Cal. II
0	0	26.012	24.836
1	0	34.825	46.700
2	0	- 7.239	-28.700
3	0	-41.154	-37.605
0	1	- 8.611	- 8.030
1	1	-12.483	-11.132
2	1	37.698	62.193
0	2	-	- 7.500
1	2	-	- 2.540
2	2	-	-16.020

Table II-B Matrix elements of interaction in jj-coupling representation between antisymmetric states. Positive sign indicates attraction and negative sign indicates repulsion.

J	Cal. I	Cal.II	Lips et.al.
0	2.2910	2.3163	2.290
2	0.5425	0.5018	0.465
4	-0.3916	-0.4269	-0.420
6	-0.8032	-0.8365	-0.815
2	0.5711	0.6142	0.655
4	0.1980	0.3867	0.400
2	2.6978	2.6717	2.675
3	-0.8303	-0.8715	-0.875
4	0.0578	-0.1299	-0.100
5	-2.3946	-2.2012	-2.200

WHERE $j = 1f_{7/2}$ and $j' = 2p_{3/2}$. The last column contains empirical matrix elements of Lips et.al. who treated the jj-coupling matrix elements as free parameters.

value of $\Sigma (2J+1) \langle jj'J|V|jj'J\rangle / \Sigma(2J+1)$ is about -0.57 in both the calculations. This quantity for the empirical interaction calculations of Lips et al is -0.558 . The non-diagonal elements in the 10-parameter calculation are slightly larger indicating more configuration mixing, than in the other calculation. The parameters of the interaction and the jj-coupling matrix elements are given in Tables II-A and II-B respectively.

The groundstate binding energies and excitation energies of ^{50}Ti and ^{51}V are given in Table II-C. The calculated values in both cases are similar to each other and are close to the experimental values. In both cases the excitation energies of the lowest $2^+, 4^+$ and 6^+ states in ^{50}Ti are very close to the experiment. The (2_1^+) and (4_1^+) states are within 20 KeV white the 6^+ states is within 100 KeV from the experiment. The excited 2^+ states are however below the experiment by about 1 MeV and the excited 4^+ states are above the experiment by about the same amount. The lowest 3/2 state in ^{51}V is about 150 KeV above the experimental result and the 5/2 state is about 50 KeV above the experimental result in calculation I while they are much improved in calculation II, in which they are brought within 15 KeV and 40 KeV respectively from the experimental results. The lowest 9/2 and 11/2 states in calculation I are reversed compared to the experiment, while in the calculation II they are brought in correct order. This

Table II-C Energy levels in MeV. The binding energies are taken to be positive. Results of Lips et.al. (Model A) are given for comparison.

Nucleus	J	Cal. I	Cal. II	Lips(A)	Expr.
^{49}Sc (g.s.)	7/2	9.705	9.712	9.72	9.62
^{50}Ti (g.s.)	0	21.700	21.741	21.730	21.79
	2	1.539	1.574	1.551	1.554
		3.303	3.385	3.389	4.323
	4	2.670	2.697	2.660	2.677
		5.746	5.992		4.804
	6	3.094	3.153	3.105	3.201
^{51}V (g.s.)	7/2	29.910	29.883	29.902	29.85
	3/2	1.096	0.942	0.908	0.929
		2.855	2.696	2.730	2.409
		3.308	3.725	3.736	3.215
	5/2	0.371	0.357	0.349	0.320
		3.088	3.144	3.186	3.082
		6.362	6.339		
	7/2	3.066	3.285	3.237	
		6.017	5.930		
	9/2	1.722	1.876	1.829	1.813
		2.189	2.137	2.125	
		7.350	7.281		
	11/2	1.770	1.773	1.747	1.609
		4.725	5.002		
		7.930	7.735		
	15/2	2.923	2.971	2.912	2.699
		9.154	8.869		

is essentially because of the kind of configuration mixing in calculation I due to the non-diagonal elements. These two states are obtained in the correct order only in the 10-parameter calculation but not even in the seven parameter calculation indicating that the effective interaction for these two states or at least one of them may be more complicated than the effective interaction for the other states. In calculations with surface interactions presented in chapter I also these states are reversed. It could also be due to the small space chosen in these calculations or the kind of interactions employed. The energies in calculation II are much closer to the experimental results. The results in calculation I are also fairly good. The ground state binding energies which depend linearly on the single particle energy of $1f_{7/2}$ proton are obtained within 50 KeV for the ^{51}V and within 100 KeV for the ^{50}Ti nuclei. The value of $\epsilon_{1f_{7/2}}$ is also very close to the value obtained in earlier calculations. On the whole the energies obtained in these calculations are very close to the experiment like in the calculations of Lips et al except the $(11/2)_1$ and $(9/2)_1$ states obtained in the present calculations. The higher excited states of each angular momentum obtained in the present calculations are about as good as the results of Lips et al. It may be possible to improve the results with a larger space of wavefunction.

The wavefunctions obtained in both calculation are also

quite similar. These are given in Table II-C. The lowest states of ^{50}Ti and ^{51}V in calculation I have slightly large pure configuration components than in the calculation II as a result of smaller configuration mixing, but the lowest 9/2 state obtained however differs much. All these states contain above 90 / of the pure configuration component except the lowest 9/2 state, which contains about equal amounts of contribution from the pure $|j^3J>$ and $|j^2(6)j'J>$ states. These are 0.66 in calculation I and 0.62 in calculation II and the $|j^2(6)j'J>$ contribution is slightly more in both the calculations. The $(9/2)_2$ state has more pure configuration component than $(9/2)_1$ state like in the empirical interaction calculations of Lips et al. The spectroscopic factors in both calculations are fairly close to the experimental values. The B(E2) rates in ^{51}V and ^{50}Ti are also close to the experiment. The spectroscopic factors are given in Table II-D and the B(E2) rates are given in Table II-E. The spectroscopic factors for the stripping reaction to ^{51}V are much better in the present calculation compared to the calculations of chapter I, and are close to the results of Lips et al. This is quite striking in the case of transfer of a 3/2 proton to the second 3/2 state of ^{51}V. The B(E2) rates are calculated with an effective charge equal to 1.6 e and are close to the experiment in both ^{50}Ti and ^{51}V. However an effective charge equal to 1.5 e may be a more suitable choice as indicated by the present calculation

Table II-D Spectroscopic factors for single proton transfer reactions. The results of the present work are compared with experiment (see Table I-D) and those of Lips et.al. (Model A).

Stripping reactions

Final State		Transfer	Cal.I	Cal.II	Lips(A)	Expr.
^{50}Ti	0 0	7/2	2.	2.	2.	
	2	7/2	1.762	1.734	1.702	
		3/2	0.119	0.133	0.149	
	4	7/2	1.956	1.972	1.970	
		3/2	0.022	0.014	0.015	
	6	7/2	2.	2.	2.	
	2_2	7/2	0.238	0.266	0.298	
		3/2	0.881	0.867	0.851	
^{51}V	7/2	7/2	0.748	0.747	0.746	0.75
	3/2	3/2	0.003	0.001	0.001	0.012
	$3/2_2$	3/2	0.356	0.641	0.662	0.45

Pick up reactions

Final State		Transfer	Cal.I	Cal.II	Lips(A)	Expr.
^{49}Sc	7/2	7/2	2.	2.	2.	1.92
^{50}Ti	0	7/2	0.748	0.747	0.746	0.74
	2	7/2	0.343	0.352	0.361	0.37
		3/2	0.001	0.004	0.002	
	4	7/2	0.746	0.721	0.735	0.75
	6	7/2	1.080	1.078	1.073	1.14
	2_2	7/2	0.069	0.062	0.071	
		3/2	0.001	0.001	0.001	
	4_2	7/2	0.003	0.016		

Table II-E The B(E2) values in units of $e^2 10^{-50}$ cm^4. The effective charge used in 1.6 e while more suitable effective charge is also given. The calculated values are compared with experiment and those of Lips et.al. (Model A).

Nucleus $^{50}_{Ti}$

Transition	Cal.I	Cal.II	Expr.
2 —— 0	0.616	0.618	0.66
4 —— 2	0.593	0.616	0.60
6 —— 4	0.271	0.299	0.34

Nucleus $^{51}_{V}$

Transition	Cal.I	Cal.II	Lips et.al. (A)	Expr.
7/2 —— 3/2	0.288	0.292	0.220	0.27
7/2 —— 5/2	1.154	1.137	0.996	0.92
7/2 —— 9/2	0.697	0.732	0.323	0.22
7/2 —— 11/2	0.924	0.854	0.952	0.90
e_{eff}	1.482	1.507		
3/2 —— 7/2	0.576	0.583		0.72
5/2 —— 7/2	1.538	1.516		1.54
9/2 —— 7/2	0.557	0.586		0.27
11/2 —— 7/2	0.616	0.570		0.78

Table II-E The B(E2) values in units of $e^2 10^{-50}$ cm^4. The effective charge used in 1.6 e while more suitable effective charge is also given. The calculated values are compared with experiment and those of Lips et.al. (Model A).

Nucleus ^{50}Ti

Transition	Cal.I	Cal.II	Expr.
2 —— 0	0.616	0.618	0.66
4 —— 2	0.598	0.616	0.60
6 —— 4	0.271	0.299	0.34

Nucleus ^{51}V

Transition	Cal.I	Cal.II	Lips et.al. (A)	Expr.
7/2 —— 3/2	0.288	0.292	0.220	0.27
7/2 —— 5/2	1.154	1.137	0.996	0.92
7/2 —— 9/2	0.697	0.732	0.323	0.22
7/2 —— 11/2	0.924	0.854	0.952	0.90
e_{eff}	1.482	1.507		
3/2 —— 7/2	0.576	0.583		0.72
5/2 —— 7/2	1.538	1.516		1.54
9/2 —— 7/2	0.557	0.586		0.27
11/2 —— 7/2	0.616	0.570		0.78

Table II-F Hamiltonian matrices as in Chapt. I.

CALCULATION I

$^{50}_{Ti}$

J = 2	9.943	0.571	8.598			
= 4	9.008	0.198	5.958			

$^{51}_{V}$

J = 1.5	13.526	-0.028	-0.664	11.757	0.246	12.147
= 2.5	14.232	-0.561	-0.630	9.817	1.711	10.817
= 3.5	14.881	0.242	0.156	9.048	-0.662	11.676
= 4.5	12.909	0.011	-0.249	8.112	1.657	12.405
= 5.5	13.055	-0.085	0.445	7.271	-0.950	9.637
= 7.5	11.971	-0.090	5.743			

CALCULATION II

$^{50}_{Ti}$

J = 2	9.902	0.614	8.572
= 4	8.973	0.387	5.770

$^{51}_{V}$

J = 1.5	13.416	0.170	-0.968	11.784	0.467	11.976
= 2.5	14.118	-0.612	-0.723	9.921	1.744	10.660
= 3.5	14.820	0.392	0.024	9.015	-0.432	11.489
= 4.5	12.803	0.137	-0.182	8.152	1.654	12.291
= 5.5	12.948	-0.200	0.591	7.339	-0.774	9.742
= 7.5	11.870	-0.175	5.983			

Table II-G Wavefunctions as in [illegible]

CALCULATION I

^{50}Ti	Energy					
J = 0	0.0	1.0	0.0			
J = 2	1.539	0.939	0.345			
	3.303	-0.345	0.939			
J = 4	2.670	0.998	0.065			
	5.746	-0.065	0.998			
J = 6	3.094	1.0	0.0			

^{51}V	Energy					
J = 1.5	1.096	0.923	-0.058	-0.379		
=	2.855	0.337	0.597	0.728		
	3.308	0.184	-0.800	0.571		
J = 2.5	0.371	0.945		-0.205	-0.255	
	3.088	0.328		0.561	0.760	
	6.362	-0.013		-0.802	0.597	
J = 3.5	0.0	0.999		0.037	0.041	
	3.066	0.031		0.234	-0.972	
	6.017	-0.045		0.972	0.232	
J = 4.5	1.722	0.660			-0.233	-0.714
	2.189	0.751			0.226	0.621
		0.017			-0.946	0.323
J = 5.5	1.770	0.988			-0.039	0.149
		0.154			0.304	-0.940
		-0.009			0.952	0.306
J = 7.5	2.923	0.999				-0.014
		0.014				0.999

CALCULATION II

^{50}Ti	Energy					
J = 0	0.0	1.0	-			
= 2	1.574	0.931	0.364			
	3.385	-0.364	0.931			
= 4	2.697	0.993	0.118			
	5.992	-0.118	0.993			
= 6	3.153	1.0	-			

^{51}V	Energy					
J = 1.5	0.942	0.891	-0.029	-0.454		
	2.696	0.292	0.801	0.523		
	3.725	0.349	-0.598	0.722		
= 2.5	0.357	0.931		-0.232	-0.282	
	3.144	0.365		0.585	0.725	
	6.339	-0.004		-0.778	0.629	
=3.5	0.0	0.998		0.067	-0.002	
	3.285	-0.013		0.165	-0.986	
	5.930	-0.066		0.984	0.166	
= 4.5	1.876	0.619			-0.239	-0.749
	2.137	0.785			0.232	0.575
		0.036			-0.943	0.331
=.5.5	1.773	0.980			-0.060	0.188
		0.197			0.275	-0.941
		0.004			0.960	0.281
= 7.5	2.971	0.999				-0.297
		0.297				0.999

Like in the calculations III and IV of chapter I, the B(E2) rate involving the 3/2 state is much better than the result of Lips et al while that involving 9/2 state is worse. The reduced matrix element for the electric quadrupole transition, B(E2), for the transition 7/2 → 5/2 is about 20 % larger than the experiment (and 15 % larger when calculated with are effective charge 1.5 e) and the one for the transition 7/2 → 11/2 is less in the second calculation by about 11 % than the experiment (about 12 % less when calculated with effective charge 1.5 e). The largest discrepancy is with the B(E2) rate for the transition 7/2 to 9/2 which is about three times the experimental result. It should be noted that the $(9/2)_1$ and $(11/2)_1$ states are the two states which are not very close to the experiment and that these two states are reversed in calculation I. The reduced transition rates are squares of matrix elements of one body operators. As a result the choice of the model space affects the calculated values very much. However it appears from the present results and earlier results [25] that the model space including configurations in which only one proton is raised to $2p_{3/2}$ orbit may be just enough to describe most of the lowlying states of ^{50}Ti and ^{51}V satisfactorily.

The results show that the interaction is mostly in relative s- and p- states. As pointed out earlier the proce-

dure outlined can be extended to include interaction in other relative states, non central parts such as tensor interaction, or dependence on the centre-of-mass coordinates. However this involves a large number of parameters and may be employed for a limited number of states such as the (9/2) and (11/2) states of ^{51}V. It appears that these states require special attention.

Finally it may be noted that some of the parameters, $I_{n\ell}$'s, the radial integrals in relative states, are negetive indicating that the interaction may be repulsive in those states. It is impossible to get the jj-coupling matrix elements of the interaction havin' the same features as those of empirical interaction Lips et al, for example the repulsion on the average, without making some of the parameters negetive. More specifically, the $\langle f_{7/2}\ p_{3/2}\ J = 5\ |V|\ f_{7/2}\ p_{3/2}\ J = 5\rangle$ matrix element can be made negetive only if some of the $I_{n\ell}$'s $\ell = 1$ are negetive. In this connection it should be born in mind that the interaction and the model space chosen are very simple.

CHAPTER - III

SHORT RANGE INTERACTION AT THE SURFACE

Empirical interaction in relative states and surface interaction with zero range components have been presented in the preceeding chapters. It has been pointed out that the parameters of the interaction are a few and that the calculations are much easier, they do not increase with the size of the model space, when confined to the fp shell, if the matrix elements of the interaction in relative states are treated as parameters. The empirical interaction in relative states is chosen because the usual interactions depend upon the relative distance of the two interacting particles. It has also been pointed out that generalisations to the simple kind that has been considered are possible. Some kind of generalisation may be needed to get correct excitation energies for the lowest 9/2 and 11/2 states of ^{51}V. Inclusion of non central parts and dependence on the centre of mass are the possibilities one can think of, but such generalisations make the number of parameters increase drastically. In the case of surface interaction with zero-range components, presented in chapter I, it has been found that the assumption of pure surface delta interaction is a little too narrow since it acts only in a limited number of states. It has been found that the lowest 2^+ state of ^{50}Ti

is obtained at about 150 KeV too high and that the lowest 9/2 and 11/2 states are reversed. The zero range of the interaction is an approximation to the small range of the interaction because of the observation that this range is smaller than the size of the nucleus. It has also been pointed out that the usual assumption on the radial integrals should be relaxed inorder to get better wavefunctions. The range of the interaction may have significant effect on the energy levels and wavefunctions. It is known that the effective interaction is important at the nuclear surface and that the density dependence may be important. The usual phenomenological interactions with central and non central parts with short range radial shape and exchange terms have been used. The number of parameters in such calculations do not seem to be adquate to get good results. These considerations make it a natural modification to make the residual interaction act when the centre of mass of the two interacting particles is in a small region near the surface. The dependence on relative distance may be chosen to have short range like Gaussian type with central, spin-orbit and tensor parts in usual manner. The region in which the density falls off to zero from maximum, surface region, is small and since a short range, non-zero range, is taking, the delta type dependence on the centre of mass coordinate can be assumed. Tensor interaction is known to be important from the considerations of the deuteron. Since

the nucleons have spin 1/2 and therefore can not possess moments of any kind higher that of dipoles, the non central interaction that can be thought of is of the kind dipole-dipole interaction. This is the tensor interaction, obtained by taking a scalar product of a second degree tensor of spins, $[\sigma_1 \times \sigma_2]^{(2)}$ and a second degree tensor constructed from space coordinates. We have therefore $V_2(12) = \left[[\sigma_1 \times \sigma_2]^{(2)} \cdot [\hat{r} \times \hat{r}]^{(2)}\right]^{(0)}$ This may also be written in the usual fashion $S_{12} = 3(\sigma_1 \cdot \hat{r})(\sigma_2 \cdot \hat{r}) - (\sigma_1 \cdot \sigma_2)$ The simple scalar force $V_0(12) = f_0(r) + (\sigma_1 \cdot \sigma_2)\ f(r)$ may also be written as $V_0 f_0(r)\left[(1 - \sigma_1 \sigma_2)/4\right] + V_1 f_1(r)\left[(3 + \sigma_1 \sigma_2)/4\right]$ separating the spin singlet and spin triplet parts. The central force is known to be dominant in nuclear force. The vector force $V_1(12) = \left([\sigma_1 \times \sigma_2]^{(1)} \cdot \bar{r}\right) * f_1(r)$ is not useful since we want parity conserving interaction and the expectation value of this interaction in any configuration vanishes. The vector interaction that conserves parity is the spin-orbit interaction. Unlike the other interactions so far discussed this interaction depends upon derivatives. This momentum dependent interaction is written as $V_1(12) = \left((\sigma_1 + \sigma_2) \cdot \bar{\ell}\right)$ where $\bar{\ell} = (\bar{r}_1 - \bar{r}_2) \times (\bar{p}_1 - \bar{p}_2)/2$ is the relative angular momentum operator. This is the simplest interaction that depends upon velocities. Such interactions are known to be important from scattering experiments. The interaction considered in the present work is

$$V = \left[V_0 f_0(r) \frac{1 - \sigma_1 \sigma_2}{4} + V_1 f_1(r) \frac{3 + \sigma_1 \sigma_2}{4} + V_T f_T(r) S_{12} + V_{ls} f_{ls}(r) \bar{\ell} \cdot \bar{s}\right] \delta(R - R_{cm})$$

In this interaction V_o and V_1 are the strengths of the central part in spin singlet and spin triplet states, V_T and $V_{\ell S}$ are those of the tensor and spin orbit parts. The factor $\delta(R-R_{ef})$ assures that the interaction acts at the effective nuclear surface, spherical, with effective radius equal to R_{ef}. These quantities $V_o, V_1, V_T, V_{\ell S}$ and R_{ef} are parameters of the interaction. The radial dependence may be chosen to be Gaussian such as $f(r) = \exp(-\beta r^2)$ where $r=(r_1 - r_2)/\sqrt{2}$ and $R = (r_1+r_2)/\sqrt{2}$. Due to the dependence on the centre-of-mass, the matrix elements of the interaction in jj-coupling when expanded interms of the matrix elements in RCM system will contain terms with $(N'L') \neq (NL)$. The use harmonic oscellator wavefunctions. This makes the transformation to the RCM system simple. The wave function in the new system are products of oscellator functions in relative coordinates and in centre-of-mass coordinates with the same frequency. The transformation from jj-coupling wave function to those in relative and centre-of-mass system is

$$|j_1 j_2 JM\rangle = \sum [j_1 j_2 \Lambda S]^{1/2} \begin{Bmatrix} \ell_1 & 1/2 & j_1 \\ \ell_2 & 1/2 & j_2 \\ \Lambda & S & J \end{Bmatrix} \sum \langle n\ell NL\Lambda | n_1\ell_1 n_2\ell_2 \Lambda\rangle * |n\ell NL(\Lambda)SJM\rangle$$

Since we have spin-orbit interaction and tensor interaction we couple $\bar{\ell}$ and $\bar{S}$ and write

$$|n\ell NL(\Lambda)SJM\rangle = \sum [\Lambda j]^{1/2} W(jLS\Lambda; J\ell) |n(Sj)NLJM\rangle$$

The matrix elements of the interaction in the new system are $\langle n\ell S(j)NLJM|V|n'\ell'S(j)N'L'J\rangle$. Due to the factor

$\delta(R-R_{ef})$ these matrix elements will have a factor $\langle NL||\delta(R-R_{ef})||N'L'\rangle$ where (N,L) characterise oscellator function in centre-of-mass coordinates. We have

$$\langle NL||\delta(R-R_{ef})||N'L'\rangle = \delta_{LL'}\int R_{NL}(R)\,\delta(R-R_{ef})\,R_{N'L'}(R)dR$$

The oscellator functions $R_{NL}(R)$ are given by

$$R_{NL}(R) = \left\{ \frac{2^{L-N+2}\,(2\nu)^{L+3/2}\,(2L+2N+1)!!}{\sqrt{\pi}\,[(2L+1)!!]^2\,N!} \right\}^{1/2} \times$$

$$R^{L+1}\,e^{-\nu R^2} \sum (-1)^K\, {}_NC_K\, \frac{(2L+1)!!}{(2L+2K+1)!!}\,(2\nu R^2)^K$$

where $\nu = M\omega/2\hbar$. With these radial functions the matrix elements of $\delta(R-R_{ef})$ are given by

$$(2y_0)^{2L+2}\cdot 2^{-N-N'}\left[\frac{(2L+2N+1)!\,(2L+2N'+1)!\,N!N'!}{\pi\,(N+L)!\,(N'+L)!}\right]^{1/2} \times e^{-y_0^2} \times$$

$$\left[\left\{\sum \frac{(-1)^K(2y_0)^{2K}(L+K)!}{(N-K)!\,K!\,(2L+2K+1)!}\right\}\left\{\sum \frac{(-1)^{K'}(2y_0)^{2K'}(L+K')!}{(N'-K')!\,K'!\,(2L+2K'+1)!}\right\}\right]$$

where $y_0 = \sqrt{2\nu}\,R_{ef} = \sqrt{2\nu}\left(\left|\frac{r_1+r_2}{\sqrt{2}}\right|\right)_{ef}$

The matrix elements $\langle n\ell Sj||V||n'\ell Sj\rangle$ for different parts of the interaction are well known[90,91]. Some useful information is given in Appendix-C.

The parameters in the calculation are the parameters of the interaction and the single particle energies. The

single particle energies are first fixed and the $\epsilon_{1f_{7/2}}$ is determined later as explained in earlier chapters. The parameters of the interaction are the effective radius and strengths and ranges of various parts of the interaction. The ranges are usually chosen to be equal for all the parts. However the tensor range is known to be larger than that of central part and the spin orbit range is smaller. Since the harmonic oscellator functions are chosen for the single particle wavefunctions, they are proportional to $e^{-\nu r^2}$ where $r = |r_1 - r_2|/\sqrt{2}$ and for Gaussian radial shape of the interaction we write $f(r) = e^{-\beta r^2}$. In an earlier calculation for the Nickel isotopes the values of $\xi = (\beta/\nu)$ chosen are 2, 4, 1 respectively for central, spin orbit and tensor parts. The value of y_o corresponding to the experimental radius of ^{48}Ca nucleus (about 4.8 fm) and the oscellator constant obtained by I.Talmi[29] is about 3.4 . It is very difficult to vary y_o and ξ continuously like the other parameters. Therefore these two parameters may be fixed while the other parameters, viz. the strengths may be varied to obtain a best fit to the lowying excited energy states of ^{50}Ti and ^{51}V nuclei. Calculation method is the same as given in earlier chapters.

Calculations are done in two steps. The matrix elements in jj-coupling representation of different parts of the interaction are calculated for various choices of y_o and ξ. These matrix elements are used to evaluate the

Hamiltonian matrices for all the different angular momentum values of ^{50}Ti and ^{51}V. If H_o, H_1, H_T and H_S are the Hamiltonians for any given J, obtained from the central (spin singlet and triplet) parts, tensor and spin-orbit parts of the two body interaction, then the total Hamiltonian matrix is simply given by $H = H^{SP} + V_o H_o + V_1 H_1 + V_T H_T + V_{\ell S} H_{\ell S}$ where H^{SP} is the diagonal single particle term. The parameters of this effective Hammiltonian are the strengths V_o, V_1, V_T and $V_{\ell S}$ which are varied in search program for the best fit. As pointed out earlier an attempt is made to mix the two kinds of the interaction used in earlier chapters. Empirical interaction is assumed to be central and only in relative s-states, in addition to the short range interaction, to make the interaction as simple as possible. This kind of mixing has been done by Lawson et al[63] but their interaction is not a surface interaction. They introduced these parameters in order to improve the results obtained with out them. This makes the number of parameters equal to eight, four strengths and four additional interaction parameters. Calculations are done for different combinations of y_o and $\frac{1}{2}$.

The results are given in the following Tables III-A through III-G. A total of six calculations are done. Calculation I is with the value of y_o equal to 3.2 and with equal ranges while Calculation II is with $y_o = 3.2$ and unequal ranges.

Table III-A Parameters of finite range surface interaction. V_0, V_1, V_{LS}, V_T are the strengths of central, spin singlet and triplet parts, spin-orbit and tensor parts. I_{nl} are parameters of additional interaction in relatives states. All of the parameters are in MeV. y_0 is the effective ratius parameter while F is the range parameter. F=1 indicates that the ranges of all the parts are equal while F=2 indicates unequal ranges, with Gaussian radial dependence.

		Cal.I	Cal.II	Cal.III	Cal.IV	Cal.V	Cal.VI
y	y_o	3.2	3.2	3.4	3.4	3.6	3.6
	F	1.	2.	1.	2.	1.	2.
	V_o	140.511	214.737	268.79	433 .15	523.51	7768.22
	V_1	-53.915	-126.29	-91.0515	-230 .71	-174.30	-437.62
	V_{LS}	10.573	279.31	27.686	621.53	85.238	1467.6
	V_T	15.024	6.695	17.947	1.926	164.10	65.753
	I_{oo}	-40.326	-31.763	-85.164	-73.769	-171.95	-136.61
	I_{1o}	-60.549	-60.744	-153.07	-152.30	-351.30	-318.76
	I_{2o}	13.861	108.67	330.93	526.76	1416.4	1642.5
	I_{3o}	1702.35	948.95	1560.00	72.00	1702.2	360.00

Table III-B Matrix elements of interaction in JJ coupling representation between antisymmetric states positive sign indicates attraction and negative sign repulsion. In the following $j = 1f_{7/2}$ and $j' = 2p_{3/2}$. The last column contains the matrix elements from the calculations Col. III Chapter I, for comparison.

J	Cal.I	Cal.II	Cal.III	Cal.IV	Cal.V	Cal.VI	Cal.III of Chapt. I
0	2.340	2.295	2.303	2.293	2.331	2.313	2.331
2	0.672	0.347	0.656	0.565	0.663	[illegible]	0.621
4	-0.376	-0.379	-0.320	-0.343	-0.219	-0.304	-0.[illegible]
6	-1.032	-0.762	-0.930	-0.[illegible]39	-0.764	-0.[illegible]92	-[illegible].[illegible]4
2	0.595	0.733	0.565	0.650	0.745	0.[illegible]69	0.[illegible]3[illegible]
4	0.054	0.560	0.218	0.[illegible]83	0.[illegible]14	0.[illegible]91	0.[illegible]65
2	2.143	2.045	2.[illegible]53	2.[illegible]31	2.[illegible]16	2.[illegible]32	1.[illegible]2[illegible]
3	-1.112	-1.[illegible]33	-1.[illegible]3[illegible]	-1.[illegible]91	-1.[illegible]2[illegible]	-1.[illegible]6[illegible]	-[illegible].[illegible]6[illegible]
4	0.114	0.20[illegible]	0.[illegible]65	-[illegible].[illegible]2[illegible]	[illegible].106	-0.118	0.519
5	-1.970	-1.981	-2.020	-2.133	-2.119	-2.159	-1.640

Table III-C Energy levels in MeV. The results of Cal. III, Chapter I are given for comparison. Experimental results are as in earlier chapters.

Nucleus	J	Cal.I	Cal.II	Cal.III	Cal.IV	Cal.V	Cal.VI	Cal.III of Chapt. I	Exper.
^{49}Sc(g.s.)	7/2	9.727	9.710	9.710	9.719	9.574	9.683	9.648	9.62
^{50}Ti(g.s.)	0	21.793	21.715	21.723	21.731	21.679	21.678	21.628	21.79
	2	1.505	1.687	1.466	1.477	1.568	1.590	1.528	1.554
		3.860	4.011	3.151	3.421	3.448	3.359	4.381	4.323
	4	2.714	2.620	2.614	2.586	2.705	2.574	2.368	2.677
		5.728	5.572	5.750	5.857	5.960	5.974	6.367	6.804
	6	3.361	3.077	3.233	3.212	3.285	3.105	3.235	3.201
^{51}V(g.s.)	7/2	29.848	29.900	29.895	29.890	29.924	29.925	29.958	29.85
	3/2	0.946	0.937	0.835	0.673	0.813	0.856	0.528	0.929
		2.750	3.012	2.578	2.668	2.746	2.766	3.195	2.409
		3.072	3.549	3.357	3.609	3.756	3.863	4.149	3.215
	5/2	0.245	0.526	0.244	0.249	0.349	0.454	0.445	0.320
		3.251	3.725	2.981	3.097	3.219	3.169	3.951	3.082
		6.384	6.626	6.592	6.479	7.067	6.605	5.183	
	7/2	3.166	3.118	3.040	3.080	3.065	3.271	4.256	
		5.545	5.940	5.661	5.810	6.168	6.069	5.785	
	9/2	1.883	1.900	1.785	1.835	1.739	1.836	1.558	1.914
		2.841	2.761	2.389	2.161	2.201	2.116	3.538	
		7.278	7.304	7.411	7.356	7.835	7.423	6.141	
	11/2	1.896	1.781	1.809	1.771	1.879	1.782	1.856	1.609
		4.991	4.827	5.012	5.153	5.195	5.131	5.481	
		7.439	7.478	7.538	7.611	7.936	7.739	7.155	
	15/2	3.293	2.830	3.141	3.091	3.088	2.915	3.143	3.099
		8.506	8.343	8.610	8.789	9.030	8.880	8.704	

Similarly the calculations III and IV are with $y_o = 3.4$ and calculations V and VI are with $y_o = 3.6$.

The results show that the calculations with unequal ranges has a tendency to lower 4_1^+ and raise 2_1^+ states in ^{50}Ti as compared to the calculations with equal ranges. Similarly the $(3/2)_1$ is lowered (except when $y_o = 3.6$) and $(5/2)_1$ is raised. The order of the levels $(9/2)_1$ and $(11/2)_1$ is brought to correct position by lowering $(11/2)_1$ and raising $(9/2)_1$. Changes in the excitation energies of the lowest states with change in the effective radius parameter may be seen from the following table.

State		Equal range			Expr.	Unequal range		
		3.2	3.4	3.6		3.2	3.4	3.6
^{50}Ti	2	1.505	1.466	1.568	1.554	1.687	1.477	1.590
	4	2.714	2.614	2.705	2.677	2.620	2.586	2.574
	6	3.361	3.233	3.285	3.201	3.077	3.212	3.105
^{51}V	1.5	0.946	0.835	0.813	0.929	0.937	0.673	0.856
	2.5	0.245	0.244	0.349	0.320	0.240	0.249	0.454
	4.5	1.833	1.785	1.739	1.813	1.900	1.835	1.836
	5.5	1.896	1.809	1.879	1.609	1.781	1.774	1.782

The table shows that while 2^+ and 4^+ states are lower at $y_o = 3.4$ (a kind of minima) the 6^+ state is higher at $y_o = 3.4$. This trend is seen both for equal range as well as unequal range calculations. For the 4^+ state the equal range seems to be favourable while for the 2^+ state the unequal range is favourable. The lowest states of ^{51}V do not show this simple behaviour but unequal ranges with y_o somewhere between 3.4 and 3.6 may be favourable, perticularly to get the 9/2 and 11/2 states close to the experiment. Another important result in the present calculations is that the 3/2 and 5/2 levels in ^{51}V are much compressed compared to the results of calculations of chapter I and are comparable to the experiment. In ^{50}Ti the 2^+ levels are much closer to each other than in experiment, while the 2_2^+ levels obtained in chapter I are quite comparable to the experiment. The 4_2^+ levels obtained in present calculations is much better than that of chapter I, though still it is off by about 1 MeV compared to experiment.

In all these calculations the I_{oo} and I_{10} are negetive and indicate that the central interaction with short range is a little large in these calculations. It may be noted that I_{30} does not contribute to the jj-coupling matrix elements except for the $\langle j^2 0|V|j^2 0\rangle$ and further due to the factor $e^{-y_o^2}$ term, the coefficient is quite small. Therefore the value of I_{30} obtained is very large in all the six calculations.

Unlike the calculations of Lawson et al, the strengths of V_o, V_1, V_T and V_{ls} obtained in the present calculations are all physically acceptable.

Spectroscopic factors for the stripping reactions from ^{50}Ti to ^{51}V seem to improve for the $(3/2)_2$ state as effective radius is increased but slightly spoiled as ranges are changed. On the otherhand the spectroscopic factors for the pickup reactions are only little changed and mostly a slight improvement is observed as y_o is increased or ranges made unequal. The B(E2) rates are calculated with effective charge equal to 1.6 e. They seem to improve and more closer to the experiment, except for the 9/2 → 7/2 transition, as the effective radius is increased or the ranges made unequal. In general the spectroscopic factors and B(E2) rates are fairly good. It may be that the values of the ranges intermediate between the two sets may improve the results further. It may be noted that the lowest 9/2 and 11/2 states are still not very satisfactory from the excitation energies or B(E2) rates.

In an effective interaction calculation with phenomenological interaction such as the kind considered in these calculations, the values of the parameters obtained do not have direct resemblance to the parameters of the nucleon-nucleon interaction in freespace. A straight forward physical interpretation can not be given. As an effective interaction

Table III-D Spectroscopic factors of one proton transfer reactions as in earlier Chapters.

Stripping reactions

Final state		Trans-fer	Cal.I	Cal.II	Cal.III	Cal.IV	Cal.V	Cal.VI	Expt.	Lips.
^{50}Ti	0	7/2	2.	2.	2.	2.	2.	2.		2.
	2	7/2	1.862	1.776	1.819	1.814	1.816	1.755		1.702
		3/2	0.069	0.112	0.091	0.128	0.192	0.133		0.149
	4	7/2	1.998	1.970	1.991	1.969	1.978	1.974		1.970
		3/2	0.001	0.115	0.005	0.015	0.011	0.013		0.015
	6	7/2	2.	2.	2.	2.	2.	2.		2.
	2_2	7/2	0.138	0.225	0.182	0.256	0.284	0.267		0.298
		3/2	0.931	0.888	0.909	0.872	0.808	0.867		0.851
^{51}V	7/2	7/2	0.746	0.746	0.748	0.746	0.747	0.747	0.75	0.746
	3/2	3/2	0.013	0.001	0.006	0.001	0.006	0.001	0.012	0.001
	$(3/2)_2$	3/2	0.337	0.977	0.393	0.587	0.562	0.591	0.45	0.662

Pick up reactions

^{49}Sc	7/2	7/2	2.	2.	2.	2.	2.	2.	1.93	2.
^{50}Ti	0	7/2	0.746	0.746	0.748	0.746	0.747	0.745	0.74	0.746
	2	7/2	0.353	0.353	0.359	0.353	0.310	0.352	0.37	0.344
		3/2	0.001	0.004	0.002	0.005	0.003			0.004
	4	7/2	0.748	0.724	0.744	0.717	0.733	0.721	0.75	0.733
	6	7/2	1.077	1.078	1.080	1.078	1.078	1.079	1.11	1.079
	2_2	7/2	0.048	0.059	0.052	0.060	0.107	0.061		0.071
	4_2	7/2	0.001	0.016	0.003	0.019	0.010	0.015		

Table III-E The B(E2) rates in units of $e^2 fm^4 = 26.4 e^2 b^2$. For effective charges $e_{eff} = 1.0$ e. Core polarization effects are not included.

Nucleous $^{50}_{Ti}$

Transition	Cal.I	Cal.II	Cal.III	Cal.IV	Cal.V	Cal.VI	Exper.
2 —— 0	0.606	0.616	0.612	0.617	0.617	0.614	0.66
4 —— 2	0.583	0.616	0.593	0.618	0.609	0.617	0.60
6 —— 4	0.252	0.302	0.273	0.303	0.293	0.297	0.34

Nucleus $^{51}_{V}$

7/2—— 3/2	0.253	0.336	0.275	0.298	0.279	0.290	0.37
5/2	1.156	1.143	1.151	1.135	1.118	1.138	0.92
9/2	0.450	0.457	0.548	0.609	0.707	0.622	0.22
11/2	0.958	0.871	0.906	0.844	0.891	0.853	0.90
e_{eff}	1.474	1.494	1.485	1.509	1.494	1.506	
3/2—— 7/2	0.506	0.671	0.550	0.597	0.557	0.581	0.72
5/2	1.541	1.524	1.534	1.514	1.531	1.518	1.51
9/2	0.360	0.365	0.438	0.487	0.506	0.498	0.37
11/2	0.639	0.581	0.604	0.562	0.584	0.569	0.78

is the renormalised one due to truncation of model space and due to neglect of many body parts of the effective interaction. The matrix elements of the effective interaction in jj-coupling are more important. The mixing matrix elements obtained in all the six calculations are positive automatically, probably due to the dependence on the centre-of-mass. A particular for of the effective interaction can be thought of as an intermediate step to understand the structure and properties of nuclear states, just like a useful model. It appears that the effective interaction could be simulated fairly well by an interaction which depends upon the centre-of-mass of the interacting particles. The calculations presented here clearly indicate that, for the shell model calculations, an effective interaction acting near the effective nuclear surface can reproduce the observed properties very well. The interaction chosen contains short range parts as well as empirical interaction in relative states. Though this later interaction acts only in s-states the excitation energies and the properties calculated are fairly close to the experimental results. In an earlier calculation with similar interaction that acts throughout the nucleus the values of some of the strengths obtained are unphysical[92]. Infact they obtained a repulsive spin orbit term where as in the present calculation both the tensor and spin orbit terms are attractive.

The values of the parameters given in table III-A show

that when the ranges are taken equal the central interaction in both spin singlet and triplet states are less strong compared to the case in which the ranges are taken unequal. The values of I_{0c} and I_{1c} decrease numerically and also the tensor part, while the spin orbit part increases numerically. The parameters change very much with a change in y_o. The values of the strengths of the first four parts as well as the I_n 's are numerically large essentially due to the factor $\exp(-y_o^2)$ comming from the centre-of-mass dependence. It appears that simple delta type dependence on the centre-of-mass coordinate may be replaced by a function which vanishes outside the region of the surface and takes either a constant or a Ganssian shape in the region for better results.

The ground state energies and the excitation energies show small variation in these calculations. The lowest states in ^{50}Ti are less sensitive to a change in ranges at $y_o = 3.4$. The lowest excited states in ^{51}V on the other hand seem to be more sensitive to the choice of y_o. The $(9/2)_1$ and $(11/2)_1$ states in ^{51}V are reversed in calculations with equal ranges while they are brought to right position in the other calculation. The spectroscopic factors and reduced E2 transition rates do not vary much. The results of these calculations together with those presented in chapter I indicate that the effective residual interaction should be made to depend upon the centre of mass of interacting particles in shell model calculations

and that it acts in a larger region near the surface rather than at a perticular effective radius. Finally the present calculations show more satisfactory trend of excitation energies than the surface interaction calculations of chapter I. Compared to those calculations the lowest excited states of ^{50}Ti and ^{51}V in the present calculations are fairly better. In the chapter I they are obtained a little too high, for example 2_1^+ and $(3/2)_1$ states. This satisfactory trend in the present calculation could be due to the large number of parameters compared to those of chapter I.

Table III-F Hamiltonian matrices as in earlier chapters.

CALCULATION I

$^{50}_{Ti}$

J =2	10.072	0.598	8.043
=4	9.024	0.084	6.014

$^{51}_{V}$

J =1.5	13.644	-0.171	-0.515	11.832	0.211	12.062
=2.5	14.250	-0.582	-0.628	9.913	1.691	10.257
=3.5	14.746	0.161	0.266	9.454	-0.690	11.395
=4.5	12.816	-0.078	-0.320	8.192	1.644	11.347
=5.5	12.828	-0.012	0.366	7.798	-0.968	9.354
=7.5	11.475	-0.038	6.263			

CALCULATION II

$^{50}_{Ti}$

J =2	9.747	0.733	7.945
=4	9.031	0.366	6.168

$^{51}_{V}$

J =1.5	13.452	0.036	-1.007	11.348	0.046	11.811
=2.5	13.899	-0.725	-0.837	9.544	1.630	10.290
=3.5	14.842	0.391	0.113	9.239	-0.841	11.470
=4.5	12.907	0.091	-0.267	8.232	1.610	11.506
=5.5	12.939	-0.175	0.640	7.897	-1.075	9.687
= 7.5	12.036	-0.165	6.532			

CALCULATION [illegible]

^{50}[illegible]

J =2	10.056	0.570	8.432
=4	9.074	0.213	5.966

^{51}V

J =1.5	13.753	-0.010	-0.685	11.826	0.413	12.242
=2.5	14.328	-0.561	-0.633	10.012	1.941	10.435
=3.5	14.848	0.253	0.143	9.363	-0.599	11.680
=4.5	12.977	0.022	-0.241	8.206	1.857	11.844
=5.5	12.988	-0.094	0.454	7.750	-0.957	9.495
=7.5	11.721	-0.096	6.255			

CALCULATION IV

^{50}Ti

J =2	9.968	0.650	8.521
=4	9.057	0.403	5.887

^{51}V

J =1.5	13.656	0.173	-1.015	11.779	0.431	12.117
=2.5	14.174	-0.647	-0.763	9.931	1.869	10.571
=3.5	14.804	0.410	0.030	9.165	-0.541	11.641
=4.5	12.925	0.141	-0.196	8.184	1.784	12.040
=5,5	12.933	-0.208	0.622	7.560	-0.892	9.580
=7.5	11.736	-0.182	6.051			

CALCULATION V

^{50}Ti

J =2	10.003	0.740	8.843
=4	9.190	0.342	6.008

^{51}V

J =1.5	13.993	0.062	-0.984	12.132	0.529	12.467
=2.5	14.425	-0.731	-0.339	10.068	2.159	10.779
=3.5	15.276	0.378	0.132	9.278	-0.589	12.121
=4.5	13.318	0.075	-0.281	8,314	2.050	12.501
=5.5	13.291	-0.163	0.632	7.775	-1.002	9.832
=7.5	12.210	-0.154	6.277			

CALCULATION VI

^{50}Ti

J =2	9.888	0.600	8.589
=4	9.096	0.381	5.782

^{51}V

J =1.5	13.697	0.170	-0.950	11.763	0.515	11.984
=2.5	14.158	-0.597	-0.707	9.885	1.860	10.658
=3.5	14.951	0.385	0.021	9.003	-0.442	11.634
=4.5	13.070	0.137	-0.177	8.219	1.761	12.265
=5.5	13.078	-0.197	0.580	7.504	-0.810	9.694
=7.5	12.056	-0.172	6.102			

Table III-G Wavefunctions as in earlier chapters.

CALCULATION I

^{50}Ti	Energy					
J =2	1.505	0.965	0.262			
	2.860	-0.263	0.965			
J =4	2.714	0.999	0.028			
	5.728	-0.028	0.999			

^{51}V	Energy					
J =1.5	0.946	0.950	-0.113	-0.291		
	2.750	0.301	0.581	0.757		
	3.072	0.064	-0.307	0.585		
J =2.5	0.2.5	0.955		-0.201	-0.220	
	3.251	0.298		0 0.689	0.709	
	6.384	-0.002		-0.742	0.670	
J =3.5	0.0	0.997		0.021	0.074	
	3.166	0.065		0.309	-0.949	
	5.545	-0.043		0.951	0.307	
J =4.5	1.853	0.939			-0.126	-0.320
	2.841	0.3			0.372	0.862
		0.010			-0.920	0.393
J =5.5	1.896	0.993			-0.024	0.116
		0.115			0.436	-0.893
		-0.029			0.900	0.436
J =7.5	3.293	0.999				-0.007
		0.007				0.999

CALCULATION [illegible]

^{50}Ti	Energy					
J = 2	1.687	0.942	0.[illegible]			
	4.011	-0.333	0.942			
J = 4	2.620	0.992	0.123			
	5.572	-0.123	0.992			

^{51}V	Energy					
J = 1.5	0.937	0.903	0.028	-0.428		
	3.012	0.039	0.999	0.146		
	3.549	0.427	-0.148	0.892		
J = 2.5	0.526	0.928		-0.238	-0.287	
	3.725	0.373		0.574	0.729	
	6.626	-0.009		-0.784	0.621	
J = 3.5	0.0	0.998		0.067	0.017	
	3.118	-0.006		0.317	-0.949	
	5.940	-0.069		0.946	0.316	
J = 4.5	1.900	0.966			-0.066	-0.249
	2.761	0.256			0.376	0.891
		0.035			-0.924	0.380
J = 5.5	1.731	0.975			-0.076	0.208
		0.220			0.422	-0.379
		-0.021			0.903	0.429
J = 7.5	2.830	0.999				-0.030
		0.030				0.999

CALCULATION III

^{50}Ti Energy

J = 2	1.466	0.954	0.301			
	3.451	-0.301	0.954			
J = 4	2.614	0.998	0.068			
	5.750	-0.068	0.998			

^{51}V Energy

J = 1.5	0.835	0.925	-0.074	-0.372		
	2.578	0.333	0.627	0.705		
	3.357	0.181	-0.770	0.605		
J = 2.5	0.244	0.945		-0.218	-0.244	
	2.981	0.327		0.631	0.703	
	6.592	0.001		-0.744	0.668	
J = 3.5	0.0	0.998		0.042	0.037	
	3.040	0.026		0.239	-0.971	
	5.661	-0.050		0.970	0.238	
J = 4.5	1.785	0.905			-0.148	-0.399
	2.339	0.425			0.359	0.831
		0.020			-0.921	0.388
J = 5.5	1.809	0.990			-0.042	0.138
		0.143			0.405	-0.903
		-0.017			0.913	0.407
J = 7.5	3.141	0.999				-0.018
		0.018				0.999

CALCULATION IV

^{50}Ti	Energy					
J = 2	1.477	0.934	0.358			
	3.421	-0.358	0.934			
J = 4	2.586	0.992	0.124			
	5.357	-0.124	0.992			

^{51}V	Energy					
J = 1.5	0.673	0.894	-0.016	-0.448		
	2.668	0.298	0.767	0.569		
	3.609	0.334	-0.642	0.690		
J = 2.5	0.249	0.925		-0.245	-0.290	
	3.097	0.380		0.596	0.707	
	6.479	-0.000		-0.765	0.645	
J = 3.5	0.0	0.997		0.072	-0.003	
	3.080	-0.013		0.202	-0.979	
	5.810	-0.070		0.977	0.203	
J = 4.5	1.835	0.893			-0.133	-0.430
	2.161	0.449			0.342	0.826
		0.037			-0.930	0.365
J = 5.5	1.774	0.979			-0.068	0.192
		0.204			0.348	-0.915
		-0.005			0.935	0.355
J = 7.5	3.091	0.999				-0.032
		0.032				0.999

CALCULATION V

^{50}Ti	Energy					
Jπ= 2	1.566	0.899	0.438			
	3.448	-0.438	0.899			
J = 4	2.705	0.994	0.106			
	5.960	-0.106	0.994			

^{51}V	Energy					
J = 1.5	0.813	0.888	-0.079	-0.453		
	2.746	0.352	0.750	0.561		
	3.756	0.295	-0.657	0.693		
J = 2.5	0.349	0.903		-0.279	-0.326	
	3.219	0.429		0.583	0.690	
	7.067	-0.003		-0.763	0.647	
J = 3.5	0.0	0.998		0.060	0.050	
	3.065	0.018		0.198	-0.980	
	6.168	-0.064		0.979	0.196	
J = 4.5	1.739	0.693			-0.254	-0.675
	2.201	0.720			0.283	0.633
		0.030			-0.925	0.379
J = 5.5	1.879	0.980			-0.062	0.190
		0.199			0.375	-0.905
		-0.015			0.925	0.380
J = 7.5	3.088	0.999				-0.026
		0.026				0.999

CALCULATION VI

^{50}Ti	Energy					
J = 2	1.590	0.931	0.364			
	3.359	-0.364	0.931			
J = 4	2.574	0.994	0.113			
	5.974	-0.113	0.994			

^{51}V	Energy					
J = 1.5	0.816	0.911	-0.024	-0.411		
	2.766	0.280	0.769	0.575		
	3.863	0.302	-0.639	0.707		
J = 2.5	0.454	0.931		-0.233	-0.282	
	3.169	0.366		0.586	0.723	
	6.665	-0.003		-0.776	0.631	
J = 3.5	0.0	0.998		0.065	-0.002	
	3.271	-0.013		0.160	-0.987	
	6.069	-0.063		0.985	0.160	
J = 4.5	1.836	0.878			-0.139	-0.458
	2.116	0.477			0.324	0.817
		0.034			-0.936	0.351
J = 5.5	1.782	0.963			-0.059	0.176
		0.186			0.307	-0.933
		0.001			0.950	0.313
J = 7.5	2.915	0.999				-0.029
		0.029				0.999

CHAPTER - IV

EXTENDED SPACE MODEL

The model space of configurations chosen in earlier chapters contains simple $2p_{3/2}$ admitures only. That is the space contains $(1f_{7/2})^n$ and $(1f_{7/2}^{n-1}\ 2p_{3/2})$ configurations. An extension of the space may be done from energy considerations and the properties of the states underconsideration. For example, the unperturbed energy of the $(1f_{7/2}^{n-2}\ 2p_{3/2}^{2})$ would be around 7 MeV above that of $(1f_{7/2}^{n})$ since the energy of $2p_{3/2}$ proton is 3.5 MeV above that of the $1f_{7/2}$ proton. The energy of $1f_{5/2}$ proton is 4.7 MeV above that of $1f_{7/2}$ proton and therefore addition of $1f_{7/2}^{n-1}\ 1f_{5/2}$ could be more useful than $(1f_{7/2}^{n-2}\ 2p_{3/2}^{2})$ for lowlying states of ^{50}Ti and ^{51}V. All higher configurations may be assumed to contribute negligible amount to such states. Another consideration for the choice of model space depends upon the properties. For example the reduced matrix element $(j||f^{(L)}||j')$ vanishes and does not contribute to the property under consideration unless $|j - j'| \leq L$. Only B(E2) rates are the properties considered in this work and therefore $p_{1/2}$ orbit will not contribute to this property. The diagonal reduced matrix elements of E2 operator between $1f_{7/2}$ $2p_{3/2}$ and $1f_{5/2}$ orbits are, apart from a common factor, 6.94, 4.50, 5.89 and are of the same order of magnitude. Inclusion of $1f_{5/2}$

could affect the B(E2) rates very much. Lips and Me Ellistrem considered the configuration $(1f_{7/2}^{n})(1f_{7/2}^{n-1}\ 2p_{3/2})$ and $(1f_{7/2}^{n-1}\ 1f_{5/2})$ and found that the results were satisfactory. They took the same interaction determined in the small space calculations and introduced surface delta interaction, with modification, for that part of the interaction in which $1f_{5/2}$ participates. The single proton energy of $1f_{5/2}$ proton determined by Erskine et al[10] was found to be satisfactory. Compared to their small space calculations the wave functions obtained in the larger space calculations are such that the $1f_{5/2}$ admixtures are built at the expense of pure configuration components and that the pureconfiguration components are still dominant for the lowest states of each angular momentum. For the $(5/2)_2$ state in ^{51}V and other odd isotones the $1f_{5/2}$ admixtures are large the discrepancies between the theory and experiment are largest. This can be understood from that the interaction remains same for the $2p_{3/2}$ admixtures and the interaction, MSDI, used for $1f_{5/2}$ admistures is quite small. To be more specific, the MSDI matrix elements $\langle f_{7/2}f_{5/2}|V|f_{7/2}f_{5/2}\rangle_J$ and $\langle f_{7/2}f_{5/2}|V|f_{7/2}p_{3/2}\rangle_J$ are all zero for odd values of J. The transition rates and the branching ratios improved for ^{51}V with the inclusion of $1f_{5/2}$ admixtures. In general a larger space can be expected to describe the system and the effective interaction better and it is useful to study how the interaction

gets modified with a change of the model space. The present chapter is devoted to the study of these changes in the effective interaction.

For the two nuclei ^{50}Ti and ^{51}V the space chosen contains $(1f_{7/2}^{n})$, $(1f_{7/2}^{n-1}\ 2p_{3/2})$ and $(1f_{7/2}^{n-1}\ 1f_{5/2})$ configurations. Higher configurations are assumed not to contribute for the lowlying states of these nuclei. The single particle energies $\epsilon_{2p3/2}$ and $\epsilon_{1f5/2}$ are fixed at 3.5 MeV and 4.7 MeV respectively above that of the $1f_{7/2}$ proton. Three model interactions are chosen for the present study. They are i) empirical interaction in relative s- and p- states (calculation I), ii) surface interaction with zero range and long range parts, without the effective radius parameter (calculation -II) and the surface interaction aththe effective radius parameter X = 2.5 (calculation III). Similar to the situation encountered in small space calculations (CHAPTER - I) the surface interaction in which all the five parts are included results in unphysical strengths. In particular the pairing part becomes negetive even though the binding energies and excitation energies are obtained well. This implies that the pairing nature of the delta interaction is quite enough, a conclusion drawn from the calculations of chapter I. The radial integrals in the calculation II are assumed to be equal at the surface and the pairing part is removed while for the calculation III, the

assumption is relaxed and the effective radius parameter is fixed at X = 2.5, a result taken from the calculations of chapter I. In calculation III the tensor part is also eliminated because of satisfactory results in chapter I so that comparison may be possible with these calculations. In the case of empirical interaction in relative states only s- and p- state interaction is considered because the results obtained these are quite satisfactory.

The results are presented in the following tables. The values of parameters obtained in least squares search are given in table IV-A and table IV-B contains the two particle matrix elements of the determined effective interaction between antisymmetric states in jj-coupling. The excitation energies of ^{50}Ti and ^{51}V and the binding energies of ground states, taken to be positive, with respect to that of the ^{48}Ca ground state are given in table IV-C. The spectroscopic factors and reduced electric quadrupole transition rates are given in table IV-D and IV-E while the Hamiltonian matrices and wavefunctions of ^{50}Ti and ^{51}V are respectively presented in table IV-F and IV-G.

The energy levels, presented in table IV-C, for the empirical interaction are quite satisfactory. The lowest states of ^{50}Ti are a little low compared to the experiment by about 50 KeV. These states are little more separated than experiment

and similar calculations of chapter II. The 6_1^+ state however is much better in the present calculations. The excited states of ^{51}V on the otherhand are quite well reproduced. The lowest states are much closer to experiment than those of (calculation I) chapter II. In perticular the $(9/2)_1$ is pushed up and $(11/2)_1$ pushed down and are brought in correct order and are quite close to the experiment. The $(3/2)_1$ and $(5/2)_1$ states are also much better in the present calculation. The binding energies of ground states are well reproduced in the present calculation. The single particle energy of $1f_{7/2}$ proton, ^{49}Sc state, is obtained at 9.73 MeV, a little more than the results of earlier calculations. The wavefunctions of lowest states of 2^+ of ^{50}Ti and 3/2 and 9/2 of ^{51}V in present calculations contain more pureconfiguration components compared to those obtained in chapter II. The situation is reversed in the case of lowest states of 7/2, 11/2 and 15/2 of ^{51}V and 4^+ of ^{50}Ti. In all those cases where pure configuration components are increased, the components of $|j^2(J1)j'J>$ decreased drastically for large J1 and the component of smaller J1 changed only little. In other cases the $p_{3/2}$ admixtures do not change much. These changes are more pronounced for the $(9/2)_1$ state whose pure configuration component changes from 0.66 to 0.954, a change brought at the expense of the component of $|1f_{7/2}^2(6)2p_{3/2}\ J = 9/2>$ essentially. The components $|1f_{7/2}^2(2)2p_{3/2}\ J = 5/2>$, $|1f_{7/2}^2(4)2p_{3/2}\ J = 9/2>$ and $|1f_{7/2}^2(6)2p_{3/2}\ J = 11/2>$ do not change much. The matrix

elements of the interaction in jj-coupling show reoulsion on the average for the $|\ f_{7/2}\ p_{3/2}\ J>$ states but the $<f^2_{7/2}|V|\ f_{7/2}\ p_{3/2}>_T$, J = 4 is obtained negetive. This is a situation very undesirable in the present calculations. This is reflected in the properties of the wavefunctions such as the spectroscopic factors and B(E2) rates. The spectroscopic factors for transfer of a $lf_{7/2}$ proton are spoiled compared to the results of chapter II. The B(E>) rates are calculated with an effective charge equal to 1.6 e. They may improve with larger effective charge. The two body matrix elements in jj-coupling are compared with those of reaction matrix calculation with 3p 1h inclusion, calculated from Hamada-Johnston potential by Kuo and Brown[72]. Except a few matrix elements the present interaction is comparable to the KB interaction. The experimental results given for comparison in tables IV-D and IV-E are same as those given in earlier chapters.

In the case of surface interaction calculations (calculations II and III) the excitation energies obtained are about the same kind as those of chapter I for ^{51}V and a little worse for ^{50}Ti. They tend more towards the results of R. Saayaman et al with surface delta interaction. Except the 2^+_1 state of ^{50}Ti and $(5/2)_1$ state of ^{51}V, the excitation energies in calculation II are a little better than the results

of calculations in chapter I without effective radius parameter. The excitation energies obtained in calculation III with the surface interaction with the effective radius parameter X=2.5, also show the same trend compared to similar calculations of chapter I. While the two $(9/2)_1$ and $(11/2)_1$ states of ^{51}V are in reverse order in calculation II, they are brought in correct order in calculation III. The levels of ^{50}Ti are more separated in the present calculations and are comparable to those of SDI calculations with complete fp shell configurations. The single particle energy of $1f_{7/2}$ proton is obtained at 9.68 MeV in cal. III better than that of other calculations but the binding energies of ground states of ^{50}Ti and ^{51}V are off by about 100 KeV. The pure configuration components of lowest states of ^{50}Ti as well as those of ^{51}V in present calculations are less than those of calculations of chapter I. In the present calculations with inclusion of $1f_{5/2}$ orbit, the higher angular mementum states are improved. With the inclusion of $1f_{5/2}$ excitations the $1f_{5/2}$ admixtures are built at the expense of pure configuration components for all the lowest states while $2p_{3/2}$ admixtures also increased in some cases. This is a result unlike the results of Lips et al. This is because the interaction in the present calculations is completely determined for the larger space while in empirical interaction calculations of Lips et al the interaction is determined separately for the $1f_{5/2}$ admixtures. The matrix elements

$\langle j^2|V|jj'\rangle_J$, J = 2 and 4 are not changed in their calculations while they are increased slightly in the present calculations compared to those of chapter I. The wavefunctions of $1f_{5/2}$ admixtures for higher states of ^{51}V are built at the expense of both $(1f_{7/2})^n$ and $(1f_{7/2}^{n-1}\ 2p_{3/2})$ components. In several cases the largest component among $|1f_{7/2}^{n-}\ (J1)2p_{3/2}J\rangle$ states shifted from one to the other with the inclusion of $1f_{5/2}$ admixtures. The B(E2) rates obtained in the present calculations are much better than their counter parts obtained in chapter I, perticularly those involving $(9/2)_1$ state. But, the B(E2) rates involving $(11/2)_1$ state are a little spoiled in the present calculation. The spectroscopic factors for transfer of a single $1f_{7/2}$ proton decreased with the inclusion of $1f_{5/2}$ admixtures. In most cases these are better reproduced in calculations of chapter I. The spectroscopic factors for stripping reactions involving transfer of a $2p_{3/2}$ or $1f_{5/2}$ proton are increased in the present model. They are much larger than the experiment for the $(3/2)_2$ and $(5/2)_2$ state of ^{51}V.

Compared to its counter part in chapter I, the tensor part of calculation II is stronger and the SDI part is weaker in the present calculations (calculation II). The central part in spin singlet states becomes repulsive in calculation II. In calculation III, however, the interaction in spin

singlet states as well as SDI are about halved. In both calculations the spin triplet interaction (central) charged little. This decreased a little where there is tensor part while it increased in the other slightly.

A closer examination of changes in excitation energies of ^{50}Ti and ^{51}V reveals that the lowest states tend towards the results of calculations with SDI, with the inclusion of $1f_{5/2}$ admixtures. The SDI calculations of R. Saayaman et al are done in complete fp shell considering all Pauli allowed states of the configurations $(fp)^n_\pi$ and are closer to experiment. The excited levels of each angular momentum are more separated than those of experiment. A more interesting result is that the matrix elements obtained in calculation III resemble those of Kuo and Brown obtained using reaction matrix formalism. The Kuo and Brown matrix elements of effective interaction include 3p 1h core excitations and the matrix elements obtained in calculation III with only 3 parameters for the surface interaction are very similar to them. In general the present interaction is much stronger specially those involving $2p_{3/2}$ or $1f_{5/2}$. Except $\langle j^2|V|j^2\rangle_J$ and $\langle jj'|V|jj'\rangle_J, J = 4$ which are very weak compared to others in KB interaction, the matrix elements of the present interaction and of KB interaction have same sign.

It appears that the excitation energies are not

Table IV-A Parameters of the effective interactions obtained in fitting. Calculation I is with effective interaction in relative s- and p- states, while calculation II and calculation III are surface interactions of the kind presented in Chapter I.

Calculation I: Empirical interaction

I_{oo}	I_{1o}	I_{2o}	I_{3o}	I_{o1}	I_{11}	I_{21}
4.8842	5.0542	18.1484	-77.350	-4.6367	-1.6147	6.5494

Surface interaction

	X	V_o	V_1	V_{SDI}	V_{STI}
Calculation II	-	-0.0847	-1.1627	0.6168	0.7280
Calculation III	2.5	0.02053	-0.12652	0.06733	-

Table IV-B Matrix elements of interaction in jj-coupling.

J	Cal. I	Cal. II	Cal. III	K.B.
0	2.3466	2.3365	2.5313	2.068
2	0.8069	0.3115	0.1410	0.755
4	-0.4139	-0.3265	-0.4578	0.036
6	-1.2723	-1.0036	-1.0023	-0.287
2	0.2271	-0.8633	1.1805	0.609
4	-0.3209	-0.3360	0.4595	0.356
2	0.4964	1.0549	1.3501	0.918
3	-0.5809	-0.5158	-0.8929	-0.086
4	-0.5136	-0.5267	-0.6635	0.083
5	-1.9405	-1.4216	-1.7791	-0.379
2	0.5655	0.5168	0.7605	0.127
4	0.7259	0.6587	1.0057	0.450
6	1.1425	0.6728	1.0033	0.705
1	-0.5217	-1.6481	-1.4119	-0.134
2	-0.3840	-1.3645	-1.0392	-0.121
3	-1.1314	-1.3245	-1.4119	-0.122
4	-0.4434	-0.7014	-0.3518	-0.132
5	-1.7388	-0.7421	-1.4119	-0.200
6	1.0599	0.6008	1.0456	0.852
2	0.0372	-0.1733	0.5783	0.1044
3	-0.2995	0.3603	0.0000	0.107
4	0.6176	0.0597	0.4844	0.182
5	0.0000	0.3813	0.0000	-0.031

where $j = 1f_{7/2}$, $j' = 2p_{3/2}$ and $j'' = 1f_{5/2}$.

Table IV-C Energy levels in MeV, as in earlier chapters.

Nucleus	J	Cal. I	Cal.II	Cal.III	Expr.
^{49}Sc	7/2	9.7314	9.7315	9.6841	9.62
^{50}Ti	(g.s.)0	21.8094	21.7995	21.8995	21.79
	2	1.474	1.732	1.759	1.554
		5.363	5.029	5.164	4.323
		7.485	8.442	8.419	
	4	2.638	2.551	2.703	2.677
		6.088	6.384	6.647	4.804
		7.885	7.827	7.918	
	6	3.158	3.200	3.197	3.201
		6.448	6.576	6.522	
^{51}V	(g.s.) 3.5	29.8373	29.8438	29.7770	29.85
	1.5	0.963	0.846	1.036	0.929
		3.469	3.892	3.108	2.409
		4.191	4.485	5.144	3.215
		6.792	7.202	7.791	
		7.701	8.151	8.349	
	2.5	0.311	0.606	0.624	0.320
		3.116	3.961	3.474	3.082
		4.698	4.674	5.210	
		5.421	5.494	6.208	
		6.074	6.491	6.602	
		7.491	7.889	8.161	
	3.5	4.255	4.526	4.626	
		4.832	5.561	5.437	
		5.793	5.863	6.430	
		7.831	7.755	8.285	
		8.039	9.206	8.960	
	4.5	1.766	1.711	1.841	1.813

	4.601	4.124	4.915	
	5.282	5.684	5.560	
	6.573	6.057	7.123	
	8.191	7.556	8.261	
	8.477	8.922	8.973	
5.5	1.699	1.825	1.705	1.609
	5.266	5,677	5.456	
	6.574	6.233	6.585	
	7.351	6.694	7.492	
	8.458	8.471	8.574	
7.5	3.037	2,802	2,574	2.699
	8.102	6.995	7.738	
	8.987	8.025	8.479	

Table IV-D Spectroscopic factors for single particle transfer reactions.

Stripping reactions

Final state	J	transfer	Cal.I	Cal.II	Cal.III	Expr.
^{50}Ti	0	7/2	2.000	2.000	2.000	
	2	7/2	1.976	1.835	1.649	
		3/2	0.003	0.076	0.156	
		5/2	0.009	0.007	0.020	
	4	7/2	1.953	1.954	1.872	
		3/2	0.004	0.007	0.019	
		5/2	0.020	0.016	0.045	
	6	7/2	1.720	1.917	1.797	
		5/2	0.140	0.041	0.101	
	2_2	7/2	0.006	0.154	0.331	
		3/2	0.997	0.923	0.830	
		5/2	0.001	0.001	0.005	
	4_2	7/2	0.001	0.011	0.076	
		3/2	0.837	0.988	0.885	
		5/2	0.163	0.006	0.077	
^{51}V	7/2	7/2	0.715	0.738	0.729	0.75
	3/2	3/2	0.032	0.002	0.006	0.012
	5/2	5/2	0.023	0.001	0.001	
	$(3/2)_2$	3/2	0.859	0.951	0.929	0.45
	$(5/2)_2$	5/2	0.551	0.615	0.713	0.24

Pickup reactions

Final state	J	transfer	Cal.I	Cal.II	Cal.III	Expr.
^{49}Sc	7/2	7/2	2.000	2.000	2.000	1.93
^{50}Ti	0	7/2	0.715	0.738	0.729	0.74
	2	7/2	0.398	0.356	0.301	0.37
		3/2	0.002	0.005	0.006	
		5/2	0.027	0.002	0.003	
	4	7/2	0.687	0.712	0.661	0.75
		3/2	0.006	0.002	0.003	
		5/2	0.008	0.003	0.006	
	6	7/2	0.764	0.999	0.888	1.24
		5/2	0.003	0.003	0.007	
	2_2	7/2	0.002	0.020	0.099	
		3/2	0.001	0.001	0.001	
		5/2	0.001	0.001	0.001	
	4_2	7/2	0.001	0.003	0.034	
		3/2	0.002	0.001	0.001	
		5/2	0.001	0.001	0.001	

Table IV-E B(E2) rates as in earlier chapters.

^{50}Ti

Transition	Cal.I	Cal.II	Cal.III	Expr.
2 — 0	0.5231	0.3542	0.5789	0.66
4 — 2	0.5250	0.4093	0.6056	0.60
6 — 4	0.2028	0.2062	0.3112	0.34

^{51}V

7/2 — 3/2	0.1150	0.1462	0.3000	0.27
5/2	0.8385	0.5321	1.1717	0.92
9/2	0.1785	0.1580	0.3216	0.22
11/2	0.6220	0.6618	0.6102	0.90
3/2 — 7/2	0.2300	0.2922	0.6003	0.72
5/2	1.1178	0.7095	1.5622	1.54
9/2	0.1400	0.1264	0.2573	0.27
11/2	0.4146	0.4411	0.4068	0.78

Table IV-F Hamiltonians and wavefunctions of ^{50}Ti.

Calculation I.

J = 2	H	10.2068	0.2171	0.5655	0.2171	6.3964	0.0372	0.5655	0.0372	4.3160
	X	0.9939	0.0566	0.0947	-0.0570	0.9984	0.0023	-0.0944	-0.0077	0.9955
J = 4	H	8.9861	-0.3209	00.7259	-0.3209	5.3865	0.6176	0.7259	0.6176	4.2566
	X	0.9882	-0.0620	0.1399	0.0003	0.9151	0.4033	-0.1530	-0.3985	0.9043
J = 6	H	8.1277	0.0000	1.1425	0.0000	1.0000	0.0000	1.1425	0.0000	5.7599
	X	0.9272	0.0000	0.3744	-0.3744	0.0000	0.9272			

Calculation II.

J = 2	H	9.7115	-0.8633	0.5168	-0.8633	6.9549	-0.1733	0.5168	-0.1733	3.3355
	X	0.9578	-0.2758	0.0814	0.2772	0.9608	-0.0069	-0.0763	0.0292	0.9967
J = 4	H	9.0735	-0.3360	0.6587	-0.3360	5.3733	0.0597	0.6587	0.0597	3.9986
	X	0.9885	-0.0852	0.1246	0.0755	0.9939	0.0806	-0.1307	-0.0702	0.9889
J = 6	H	8.3964	0.0000	0.6728	0.0000	1.0000	0.0000	0.6728	0.0000	5.3008
	X	0.9791	0.0000	0.2036	-0.2036	0.0000	0.9791			

Calculation III.

J = 2	H	9.5490	1.1805	0.7605	1.1805	7.2501	0.5783	0.7605	0.5783	3.6609
	X	0.9079	0.3947	0.1411	-0.4068	0.9108	0.0700	-0.1009	-0.1209	0.9875
J = 4	H	8.9422	0.4595	1.0057	0.4595	5.2366	0.4844	1.0057	0.4844	4.3482
	X	0.9674	0.1372	0.2130	-0.1945	0.9408	0.2777	-0.1622	-0.3101	0.9368
J = 6	H	8.3977	0.0000	1.0033	0.0000	1.0000	0.0000	1.0033	0.0000	5.7456
	X	0.9480	0.0000	0.3182	-0.3182	0.0000	0.9480			

Table IV-G Hamiltonians and wavefunctions of ^{51}V.

Calculation I. J = 1.5

j^3	$j^2(0)j'$	$j^2(2)j'$	$j^2(2)j''$	$j^2(4)j''$
13.6430	-0.4668	0.3264	0.2058	-0.3358
- 0.4668	11.2246	-0.1938	0.0811	-0.4991
0.3264	-0.1938	10.5912	-0.0302	0.4323
0.2058	0.0811	-0.0302	7.7604	0.3531
- 0.3358	-0.4991	0.4323	0.3531	7.3810
13.7806	**11.2737**	**10.5525**	**7.9509**	**7.0424**
0.9770	0.1955	-0.0449	0.0024	0.0723
- 0.1801	0.9268	0.3054	0.0449	0.1155
0.1068	-0.2747	0.9474	-0.0622	-0.1100
0.0288	0.0187	0.0062	0.8845	-0.4652
- 0.0284	-0.1645	0.0865	0.4601	0.8677

J = 2.5

j^3	$j^2(2)j'$	$j^2(4)j'$	$j^2(0)j''$	$j^2(2)j''$	$j^2(4)j''$
14.2506	-0.1954	-0.1372	0.2955	0.6504	0.2959
- 0.1954	9.5135	0.3771	-0.0662	-0.0020	0.1344
- 0.1372	0.3771	9.6212	0.4075	-0.2827	0.1933
0.2955	-0.0662	0.4075	10.8822	1.3608	0.6497
0.6504	-0.0020	0.2827	1.3608	9.4049	-0.4107
0.2959	0.1344	0.1933	0.6497	-0.4107	7.6759
14.4319	**11.6276**	**10.0455**	**9.3217**	**8.6693**	**7.2524**
0.9726	-0.2054	-0.0849	-0.0130	-0.0864	-0.0607
- 0.0415	0.0223	-0.5450	0.8129	-0.1867	-0.0709
- 0.0261	0.1408	-0.7571	-0.3954	0.4988	0.0328
0.1500	0.8615	-0.0210	-0.1380	-0.3570	-0.2973
0.1642	0.4329	0.3092	0.3822	0.6339	0.3769
0.0455	0.0889	-0.1626	-0.1323	-0.4316	0.8717

J = 3.5

j^3	$j^2(2)j'$	$j^2(4)j'$	$j^2(2)j''$	$j^2(4)j''$	$j^2(6)j''$
14.5075	−0.2065	0.3887	−0.6581	−0.5705	−0.5477
− 0.2065	9.2075	−0.4238	0.1118	−0.2180	−0.2860
0.3887	−0.4238	9.5869	−0.2820	0.4026	0.0859
− 0.6581	0.1118	−0.2820	10.3370	0.8758	−0.3242
− 0.5705	0.2180	0.4026	0.8758	7.1417	0.1005
− 0.5477	−0.2860	0.0859	−0.3242	0.1005	6.9279
14.7431	10.4879	9.9118	8.9502	6.9120	6.7035
0.9966	0.1859	0.0257	−0.0139	0.1030	0.0122
− 0.0392	0.1137	0.5313	0.8260	0.1427	0.0243
0.0780	−0.1973	−0.7876	0.5559	−0.1240	0.1017
− 0.1649	0.9313	−0.2019	0.0032	−0.0945	0.2364
− 0.0878	0.1772	−0.2278	0.0252	0.6549	−0.6925
− 0.0604	−0.1223	−0.0641	−0.0887	0.7183	0.6734

J = 4.5

j^3	$j^2(4)j'$	$j^2(6)j'$	$j^2(2)j''$	$j^2(4)j''$	$j^2(6)j''$
12.5987	−0.2834	−0.2882	−0.1711	−1.1157	0.5302
− 0.2834	8.1066	0.4063	0.1098	−0.2231	0.4528
− 0.2882	0.4063	9.4390	0.2558	0.1053	−0.0621
− 0.1711	0.1098	0.2558	9.2331	1.4909	0.6174
− 1.1157	−0.2231	0.1053	1.4909	7.6182	0.1051
0.5302	0.4528	−0.0621	0.6174	0.1051	6.5617
12.9768	10.1418	9.4609	8.1700	6.5515	6.2564
0.9539	0.2352	−0.0299	−0.0321	0.1633	−0.0788
− 0.0507	0.0672	−0.2692	0.9056	0.2173	−0.2307
− 0.1014	0.2822	−0.9100	−0.2807	−0.0083	0.0568
− 0.1359	0.8182	0.2428	0.0507	−0.4428	−0.2335
− 0.2351	0.3932	0.1968	−0.1675	0.8470	0.0766
0.0593	0.1911	0.0308	0.2637	−0.1119	0.9365

J = 5.5

j^3	$j^2(4)j'$	$j^2(6)j'$	$j^2(4)j''$	$j^2(6)j''$
12.5231	0.2161	-0.0875	1.0623	1.1745
0.2161	7.4379	-0.4329	0.3223	-0.4953
-0.0875	-0.4329	8.4957	-0.5697	0.3981
1.0623	0.3223	-0.5697	9.1322	-0.0419
1.1744	-0.4953	0.3981	-0.0419	6.7809
13.0436	9.4768	8.1693	7.3924	6.2847
0.9475	-0.1885	0.1688	0.0546	-0.1877
0.0398	0.2723	0.2315	00.8508	0.3832
-0.0403	-0.5988	-0.6765	0.4175	-0.0887
0.2647	0.6916	-0.6689	-0.0617	0.0221
0.1702	-0.2313	-0.1136	-0.3082	0.8998

J = 7.5

j^3	$j^2(6)j'$	$j^2(6)j''$
10.8687	0.1451	-1.9075
0.1451	5.8748	0.2660
-1.9075	0.2660	7.3591
11.7059	6.6409	5.7557
0.9158	0.3722	0.1510
0.0045	0.3665	-0.9304
-0.4016	0.8527	0.3340

Calculation II.

J = 1.5

j^3	$j^2(0)j'$	$j^2(2)j'$	$j^2(2)j''$	$j^2(4)j''$
13.5307	0.0000	1.0573	0.1888	-0.3025
0.0000	11.7669	0.3009	0.1970	0.3381
1.0573	0.3009	10.6312	0.4021	0.0550
0.1888	0.1970	0.4021	7.5983	0.1058
-0.3025	0.3381	0.0550	0.1058	6.6449
13.9037	11.8572	10.2647	-7.5478	6.5987
0.9461	-0.1029	-0.3032	-0.0045	0.0478
0.0435	0.9751	-0.2041	-0.0447	-0.0605
0.3152	0.1731	0.9247	-0.1245	-0.0118
0.0492	0.0587	0.1038	0.9868	-0.0974
-0.0343	0.0722	0.0234	0.0929	0.9922

J = 2.5

j^3	$j^2(2)j'$	$j^2(4)j'$	$j^2(0)j''$	$j^2(2)j''$	$j^2(4)j''$
13.6232	0.8501	0.9625	0.0000	0.3455	0.2389
0.8501	9.8272	0.7041	-0.1608	0.1489	0.1527
0.9625	0.7041	10.0979	0.2761	0.0120	0.2958
0.0000	0.1608	0.2761	10.5822	0.3517	-0.2219
0.3455	0.1489	0.0120	0.3517	8.2989	-0.2026
0.2389	0.1527	0.2958	0.2219	-0.2026	6.9511
14.1433	10.7878	10.0750	9.2555	8.2581	6.8607
[illegible]	-0.1674	0.3149	0.0165	-0.0519	-0.0271
0.2366	0.2284	-0.5417	-0.7692	-0.0883	-0.0363
0.2671	0.3434	-0.6373	0.6277	0.0639	-0.0810
-0.0226	-0.8856	-0.4349	0.0551	-0.1281	-0.0818
0.0587	-0.1315	-0.0644	-0.0952	0.9683	0.1687
0.0445	-0.0191	-0.0895	0.0450	-0.1772	0.9779

J = 3.5

j^3	$j^2(2)j'$	$j^2(4)j'$	$j^2(2)j''$	$j^2(4)j''$	$j^2(6)j''$
14.6502	-0.3940	-0.2058	-0.2358	-0.4033	-0.4952
-0.3940	9.2008	-0.0482	0.0608	0.1347	0.1937
-0.2058	-0.0482	10.2116	-0.0781	0.0670	0.2481
-0.2358	0.0608	-0.0781	8.8798	0.2224	-0.1211
-0.4033	0.1347	0.0670	0.2224	7.0458	0.0063
-0.4952	0.1937	0.2481	-0.1211	0.0063	5.5957
14.7493	10.2236	9.1879	8.8858	6.9945	5.5429
0.9922	-0.0434	0.0874	0.0351	0.0450	0.0523
-0.0738	0.0576	0.9816	-0.1514	-0.0488	-0.0488
-0.0474	-0.9937	0.0607	0.0610	-0.0203	-0.0506
-0.0409	0.0710	0.1402	0.9796	-0.1114	0.0388
-0.0549	-0.0081	0.0619	0.1017	0.9913	0.0107
-0.0560	-0.0481	0.0405	-0.0454	-0.0122	0.9953

J = 4.5

j^3	$j^2(4)j'$	$j^2(6)j'$	$j^2(2)j''$	$j^2(4)j''$	$j^2(6)j''$
12.8153	-0.0423	0.3578	0.0937	-0.8176	0.4972
-0.0423	8.7651	0.6488	-0.1574	0.3990	-0.1854
0.3578	0.6488	10.4143	-0.1733	-0.2765	0.2267
0.0937	-0.1574	-0.1733	8.9654	0.4271	0.2158
-0.8176	0.3990	-0.2765	0.4271	7.5779	-0.0027
0.4972	-0.1854	0.2267	0.2158	-0.0027	5.9043
13.0387	10.6249	9.0657	8.6922	7.1930	5.8276
0.9733	-0.1477	-0.0128	-0.1036	0.1218	-0.0713
-0.0040	0.3313	-0.0743	-0.8921	-0.2879	0.0773
0.1540	0.9211	-0.1249	0.3129	0.1047	-0.0592
0.0038	-0.1399	-0.9511	0.1023	-0.2484	-0.0593
-0.1536	-0.0202	-0.2625	-0.2820	0.9086	-0.0443
0.0730	0.0093	-0.0713	0.0745	0.0633	0.9899

J = 5.5

j^3	$j^2(4)j'$	$j^2(6)j'$	$j^2(4)j''$	$j^2(6)j''$
12.5854	0.1512	-0.6966	0.5314	0.9902
0.1512	8.1508	-0.2110	0.1933	0.2383
-0.6966	-0.2110	9.0600	0.1644	0.1427
0.5314	0.1933	0.1644	8.5459	-0.0698
0.9902	0.2383	0.1427	-0.0698	6.5050
12.9240	9.0728	8.5162	8.0554	6.2788
0.9674	-0.1119	-0.1520	-0.0151	-0.1679
0.0498	0.0985	0.4747	-0.8633	-0.1307
-0.1670	-0.9202	-0.2443	-0.2328	-0.1068
0.1110	-0.3539	0.8293	0.4087	0.0880
0.1462	-0.0755	-0.0647	-0.1822	0.9672

J = 7.5

j^3	$j^2(6)j'$	$j^2(6)j''$
11.5512	0.1520	-1.2734
0.1520	7.0217	0.4416
-0.2734	0.4416	7.8530
11.9472	7.7544	6.7243
0.9550	0.2488	0.1617
0.0029	0.5373	-0.8434
-0.2967	0.8059	0.5124

Calculation III.

J = 1.5

j^3	$j^2(0)j'$	$j^2(2)j'$	$j^2(2)j''$	$j^2(4)j''$
13.1115	-0.0000	-1.4457	0.2720	-0.4790
-0.0000	11.5663	0.4461	-0.2694	-0.4624
-1.4457	0.4461	10.3806	-0.2892	-0.0752
0.2720	-0.2694	-0.2892	6.9193	0.2675
-0.4790	-0.4624	-0.0752	0.2675	6.7171
13.7888	1.7168	9.6803	7.0333	6.4755
0.9105	0.1548	0.3737	0.0227	0.0831
-0.0763	0.9643	-0.2301	0.0953	0.0484
-0.3997	0.1741	0.8963	0.0817	0.0023
0.0539	-0.0620	-0.0395	0.8440	-0.5285
-0.0504	-0.1099	-0.0508	0.5209	0.8435

J = 2.5

j^3	$j^2(2)j'$	$j^2(4)j'$	$j^2(0)j''$	$j^2(2)j''$	$j^2(4)j''$
13.2879	-1.1624	-1.3162	0.0002	0.5276	0.3387
-1.1624	9.2666	1.0822	0.2200	0.1377	-0.2089
-1.3162	1.0822	9.6785	0.3775	-0.0164	-0.0022
0.0002	0.2200	0.3775	10.9138	0.8505	0.6459
0.5276	0.1377	-0.0164	0.8505	8.3682	-0.5897
0.3387	-0.2089	-0.0022	0.6459	-0.5897	7.1484
14.2009	11.3503	9.6144	8.6172	8.2229	6.6577
0.8953	0.1236	-0.4085	-0.0774	-0.0686	-0.0751
-0.2850	0.1369	-0.5595	0.4096	-0.6463	0.0389
-0.3317	0.1961	-0.6521	-0.4683	0.4547	-0.0164
-0.0315	0.9189	0.2879	-0.1006	-0.1177	-0.2187
0.0662	0.2681	-0.0693	0.6681	0.5369	0.4292
0.0431	0.1067	0.0839	-0.3878	-0.2623	0.8721

J = 3.5

j^3	$j^2(2)j'$	$j^2(4)j'$	$j^2(2)j''$	$j^2(4)j''$	$j^2(6)j''$
14.6281	0.5388	0.2815	-0.3472	-0.6159	-0.7383
0.5388	8.5178	-0.0925	0.2582	-0.1843	-0.2649
0.2815	-0.0925	10.6614	0.1069	-0.4744	-0.3392
-0.3472	0.2582	0.1069	9.2418	0.5379	-0.3132
-0.6159	-0.1843	-0.4744	-0.5379	6.7554	0.0468
-0.7383	-0.2649	-0.3392	-0.3132	0.0468	6.0056
14.8247	10.1990	8.3874	8.3952	6.5393	5.8645
0.9862	-0.0286	-0.0392	0.1197	0.0732	0.0732
0.0869	-0.0312	-0.3231	-0.9338	0.1018	0.0691
0.0535	0.9520	0.2380	-0.1157	0.1171	0.0840
-0.0591	0.2336	-0.9040	0.2883	-0.1747	0.1057
-0.0785	0.1783	-0.1084	0.1224	0.9633	-0.0952
-0.0855	-0.0855	0.0922	0.0476	0.1097	0.9811

J = 4.5

j^3	$j^2(4)j'$	$j^2(6)j'$	$j^2(2)j''$	$j^2(4)j''$	$j^2(6)j''$
12.5076	0.0583	-0.4893	0.1626	-1.2246	0.7657
0.0583	8.0786	1.0000	0.2152	0.0929	0.2534
-0.4893	1.0000	10.4855	0.2370	-0.3781	0.3286
0.1626	0.2152	0.2370	8.7534	1.0732	0.6094
-1.2246	0.0929	0.3781	1.0732	7.3369	-0.0360
0.7657	0.2534	0.3286	0.6094	-0.0360	6.1150
12.9840	10.9101	9.2652	7.7023	6.5633	5.8522
0.9433	0.2292	-0.0312	0.0645	0.2145	-0.0796
-0.0346	0.3434	0.0279	-0.9350	-0.0475	-0.0610
-0.2225	0.8645	0.2912	0.3371	-0.0278	-0.0627
-0.0216	0.2317	-0.8586	0.0566	-0.3834	-0.2419
-0.2247	0.0900	-0.3971	0.0638	0.8698	0.1520
0.0925	0.1428	-0.1365	-0.0281	-0.2174	0.9510

$J = 5.5$

j^3	$j^2(4)j'$	$j^2(6)j'$	$j^2(4)j''$	$j^2(6)j''$
12.3677	-0.2067	0.9525	0.7762	1.4855
-0.2067	7.4526	-0.3391	0.3158	-0.3259
0.9525	-0.3391	8.7277	-0.2248	0.6582
0.7762	0.3158	-0.2248	8.9852	-0.2710
1.4855	-0.3259	0.6582	-0.2710	6.7779
13.1198	9.3678	8.2399	7.3326	8.2511
-0.9300	0.0731	0.2798	0.0548	-0.2201
-0.0538	0.2479	0.0425	0.9551	0.1470
0.2344	-0.5096	-0.7916	0.2014	-0.1351
0.1421	0.7954	-0.5413	-0.1941	0.1282
0.2389	-0.2020	0.0181	-0.0806	0.9462

$J = 7.5$

j^3	$j^2(6)j'$	$j^2(6)j''$
11.4666	-0.2078	-1.9141
-0.2078	6.3743	0.2086
-1.9141	0.2086	7.9088
12.3111	7.0870	6.3455
0.9154	0.4021	0.0172
-0.0461	0.1472	-0.9880
-0.3998	0.9037	0.1533

drastically affected by increase in the size of the model space. The discrepancies in the calculated energies are about the same when compared to smaller space calculations. In the case of empirical interaction in relative states it is not possible to get both binding energies and the property that jj-coupling matrix elements of the interaction be repulsive on the average without making some of the $I_{n\ell}$'s negetive. This is because of the coefficients of transformation for the matrix elements $\langle f_{7/2}\, p_{3/2}|V|f_{7/2}\, p_{3/2}\rangle_J$, J= 2 and 4 for all s-,p- and d- states. This has been pointed out earlier in connection with small space calculations. The results could not charge much inspite of inclusion of $f_{5/2}$ admixtures. In the case of surface interaction also it is more or less the same. An attempt to determine the strengths of surface interaction to fit the empirical matrix elements of Lips et al results in one of SDI, STI and Pairing being repulsive. The surface interaction determined however reproduces the energy levels and properties very well with only a few parameters. In summary the large space calculations improve the energy levels of ^{51}V satisfactorily. The B(E2) rates and S-values are much affected by inclusion of $1f_{7/2}$ admixtures for the surface interaction with effective radius. The results tend towards those of SDI calculations within complete fp shell model space.

So far simple models have been considered for the

nuclei ^{50}Ti and ^{51}V. Different interactions have been used for these nuclei with the model space containing only the lowest configuration and those obtained by raising one particle to $2p_{3/2}$ and $1f_{5/2}$ orbits. However the orbits $1f_{7/2}$ $2p_{3/2}$ $1f_{5/2}$ and $2p_{1/2}$ belong to the same degenerate single oscellator shell. Their single particle energies differ because of their interactions with the core and possibly other renormalizations. Therefore for any n-particle system one would like to consider all the Pauli allowed states in fp shell, $(1f_{7/2}\ 2p_{3/2}\ 1f_{5/2}\ 2p_{1/2})^n$. This is a very big problem for $n \geq 3$ in fp shell and calculation is not possible with the available computers. There are 30 two particle states of the kind $|jj'J>$ where $j,j' = 1f_{7/2}, 2p_{3/2}, 1f_{5/2}$ and $2p_{1/2}$ and they are

$\|1f_{7/2}\ 1f_{7/2}\ J>$	J = 0 2 4 6
$\|1f_{7/2}\ 2p_{3/2}\ J>$	= 2 3 4 5
$\|1f_{7/2}\ 1f_{5/2}\ J>$	= 1 2 3 4 5 6
$\|1f_{7/2}\ 2p_{1/2}\ J>$	= 3 4
$\|2p_{3/2}\ 2p_{3/2}\ J>$	= 0 2
$\|2p_{3/2}\ 1f_{5/2}\ J>$	= 1 2 3 4
$\|2p_{3/2}\ 2p_{1/2}\ J>$	= 1 2
$\|1f_{5/2}\ 1f_{5/2}\ J>$	= 0 2 4
$\|1f_{5/2}\ 2p_{1/2}\ J>$	= 2 3
$\|2p_{1/2}\ 2p_{1/2}\ J>$	= 0
Total	30

Therefore the dimensions N_J of these spaces are N_J = 4,3,8,5,6,2,2 respectively for the angular momenta J = 0,1,2,3,4,5 and 6. The total number of two body matrix elements required are $\Sigma N_J(N_J+1)/2 = 94$. The sizes of the spaces for allowed angular momenta of n-particle systems increase rapidly even for maximum isospin states with n. Moreover to determine the two body part of the effective interaction one considers two particle systems because the 3-body and n-body n > 3 parts of the effective interaction do not operate for two particle systems. There are three closed shell nuclei in fp shell region and many two particle or equivalent nuclei. Shell model calculations for nuclei like ^{50}Ti is possible because of the dimensionality mentioned above. To determine effective interaction for ^{50}Ti there are not many levels to be used for fitting. Inclusion of ^{42}Ca and ^{58}Ni for which T = 1 matrix elements of the interaction are needed provides a few more states for use in fitting, with the assumption that the nuclear shell model effective interaction is charge independent. The lowlying spectra of these nuclei are not too far different from that of ^{50}Ti. In the case of ^{58}Ni the dimensionality of the spaces will be, since $1f_{7/2}$ neutron shell is closed, N_J = 3,2,5,2,2 for J = 0,1,2,3 and 4 respectively. Therefore calculations can be done. Putting three nearly two particle systems together for the determination of the effective interaction is compalible with the

belief that a universal effective may exist. In what follows results of calculations in which effective interactions are determined to fit the lowlying state of ^{50}Ti, ^{42}Ca and ^{58}Ni nuclei.

Experimental and theoretical investigations on ^{50}Ti have already been cited. The ground state of ^{48}Ca is a good closed shell core and therefore ^{50}Ti can safely be described as a two proton system above inert ^{48}Ca core. The experimental energies are taken from Nuclear Level Schemes...[87] and the single particle energies are taken from the work of Erskine et al[10] for this nucleus. In a recent experiment, two neutron transfer, J.G. Pronko et al[93] it was found that the states of ^{50}Ti near 4 MeV excitation contain considerable amount of neutron excitation from the $f_{7/2}$ neutron shell. The lowest states of this nucleus are however simple two proton states above ^{48}Ca. These states of ^{50}Ti are considered for fitting.

There are many experimental and theoretical investigations on ^{42}Ca. This is a two neutron system above ^{40}Ca core which contains equal number of neutrons and protons filling up to and including 2s 1d shell. The energy levels of this nucleus are given by PM Endt and Van der Leun[94]. The spectrum of this nucleus contains many lowlying 0^+ and 2^+ states which are thought of as due to possible core excitations and deformations. A sequence of levels 0^+ 2^+ 4^+ and 6^+ would

be obtained in shell model $(f_{7/2})^2$ configuration. The configurations $f_{7/2}\ p_{3/2}$, $f_{7/2}\ p_{1/2}$ and $f_{7/2}\ f_{5/2}$ contribute to the spectrum at higher energy. Pairing vibrational states start at about 6 MeV excitation. In a reaction like $^{41}Ca(dp)^{42}Ca$ the $(f_{7/2})^2$ states and $(f_{7/2}\ p_{3/2}\ p_{1/2}\ f_{5/2})^2$ states are excited. CW Towsley et al[95] find that, since E2 operator does not connect f and d orbits, in order to get E2 properties one has to resort to large space and instead coexistance model gives useful results. In this model the states are assumed to be mixtures of $(fp)^2$ shell model states and some complex states, presumably deformed. The authors calculated wavefunctions from the E2 matrix elements. They obtained the complex states as follows.

0_1^+	0.0 MeV	0.52
0_2^+	1.836	-0.85
2_1^+	1.523	-0.76
2_2^+	2.423	0.65
4_1^+	2.751	0.44
4_2^+	3.250	-0.90
6_1^+	3.191	0.45
6_2^+	5.790	-0.89

They also showed that 0_2^+, 2_1^+, 4_2^+ and 6_2^+ belong to a rotational band. On the otherhand the 2_2^+ also contains sufficienlly the complex state contribution. MC Grory et al[96,97], D. Banerjee et al[81] consider simple shell model wavefunctions. The energy

levels and spectroscopic factors obtained by D. Banerjee et al quite satisfactory. In the following the nucleus of ^{42}Ca is considered as a system of two neutrons in fp shell above ^{40}Ca inert core and the single particle energies are taken as $\epsilon_{1f_{7/2}} = 8.36$ MeV, $\epsilon_{2p_{3/2}} = 6.29$ MeV, $\epsilon_{1f_{5/2}} = 2.86$ MeV and $\epsilon_{2p_{1/2}} = 4.23$ MeV respectively as done by D. Banerjee et al and the lowest states are considered for fitting.

In the case of ^{58}Ni nucleus the experimental energies are taken from Nuclear Level Schemes ...[87]. There are several theoretical calculations on this nucleus. The single particle energies are taken as $\epsilon_{2p_{3/2}} = 10.257$ MeV $\epsilon_{1f_{7/2}} = 9.477$ MeV and $\epsilon_{2p_{1/2}} = 9.177$ MeV like every one does [46]. The binding energy of $2p_{3/2}$ neutron is taken from the Binding Energy tables, 1964. The nucleus of ^{58}Ni is considered from shell model by several authors though the ^{56}Ni is well known to be a deformed and not simple doubly closed core. Poor results of E2 rates are usually attributed to this aspect. S.P. Pandya and B.P. Singh[46] consider this nucleus to consist of two neutrons in fp shell and state dependence of effective charge to reproduce E2 transition rates. They consider that the neglect of core excitations results in such a complex effective charges. In the present calculations however simple $(fp)^2$ configurations assumed for the two neutrons above the ^{56}Ni core in order to obtain an effective interaction suitable for all the three nuclei ^{58}Ni ^{50}Ti and ^{42}Ca. No attempt is made to calculate

the E2 rates.

Calculations are done with two interactions, one with empirical interaction in relative s- and p- states and the other with surface interaction of the kind taken in chapter I. The single particle energies used are given in table IV-H, the parameters of interactions obtained are given in table IV-I, while the energy levels and wavefunctions are given in tables IV-J and IV-K respectively. For comparison experimental results[27,94] and theoretical results[85,31,46] are also given in table IV-J.

In the present calculations all the $I_{n\ell}$'s of empirical interaction are obtained positive. The repubsion on the average property does not seem to hold good for this complete shell calculation. Compared to the calculations with smaller space, $2p_{3/2}$ admixtures and $1f_{5/2}$ admixtures the parameters in the present calculation are quite different. They are small and are all positive. In a similar calculation for ^{58}Ni alone and taking all the thirteen excited states for least squares fitting, S.P. Pandya and B.P. Singh obtained a negetive value for I_{21}. The interaction determined by them pushed the 2_1^+ 4_1^+ and 6_1^+ of ^{50}Ti and ^{42}Ca for below the experiment though ^{58}Ni levels are well reproduced. In the present calculation the lowest 2^+ 4^+ 6^+ states of ^{50}Ti are with in 10 KeV from the experiment and those of ^{42}Ca are with in 50 KeV for 4^+ and 6^+

while the 2^+ is about 70 KeV above the experiment. It has already been mensioned that the 2^+ states of ^{42}Ca contain more of complex states than the 4^+ and 6^+ states. The worst disagreement is with the 2_1^+ state of ^{58}Ni which is obtained at 0.95 MeV while the experimental value is at 1.454 MeV. The lowest 2^+ and 3^+ states of ^{58}Ni are pushed down by about 0.5 MeV while the lowest 1^+ is by about 0.7 MeV. The lowest 4^+ is obtained at about 0.2 MeV. The states obtained in this calculation are in general pushed down and compressed in the case of the ^{58}Ni nucleus, while the states of ^{50}Ti and ^{42}Ca are more spread than the experiment.

According to OLE Hansen,[98] the 0^+ near 6 MeV contains $p^2_{3/2}$ $p^2_{1/2}$, $f^2_{5/2}$ and the other $(fp)^2$ states lie above the multiparticle multihole and deformed states around 3.2 MeV in ^{42}Ca. MC Grory et al[96] conclude from their calculations that the 0^+ state of ^{42}Ca at 1.84 MeV and the 2^+ state at 2.42 MeV are not simple shell modela states. There are several 2^+ and 4^+ states below 6 MeV. The present results, a 0^+ state at 6.14 MeV and 2^+ states at 4.59 MeV and 6.23 MeV in the empirical interaction calculation are in agreement with their observations. According to JG Pronko et al[93] the 0^+ level at 3.87 MeV in ^{50}Ti and 2^+ state at 4.31 MeV are composed of one and two neutron excitations from the $f_{7/2}$ orbit. A new 0^+ state at 7.19 MeV excitation has been recently found[99]. Therefore the states obtained in the present calculation, a 0^+ at 8.53 MeV and a 2^+

at 5.96 MeV could be compared with the experimental states at 7.19 MeV and 5.70 MeV respectively and indicate that these are simple 2 proton states of the kind considered.

The surface interaction calculations also lead to more or less same kind of spectra for the nuclei ^{50}Ti, ^{42}Ca and ^{58}Ni. In the case of ^{58}Ni the lower states of even angular momentum are more close to the experimental states than in the other calculation, though, the odd angular momentum states are a little more excited than the experiment. In the case of ^{50}Ti and ^{42}Ca the results can be compared with those of R. Saayaman et al and D. Banerjee et al. The states obtained in the present calculation are quite similar to those of R. Saayaman et al. In fact the 2_1^+ state obtained in the present calculation is more closer to the experiment. In the case of ^{42}Ca also the present results are comparable to those of PSTI calculations of D. Banerjee et al. The lowest 2^+ 4^+ and 6^+ states obtained in the present calculations are a little better than those of PSTI calculations, probably because the number of parameters in the present calculation is five against two of the PSTI, which is a part of the interaction considered here. The binding energy of the ground state also is quite close to the experiment and their result. The PSTI part of the present interaction is very small compared to the other part of the interaction. According to D. Banerjee et al the two 2^+ states

of ^{42}Ca at 1.52 and 2.42 MeV contain deformed components and their unperturbed shell model state should be around 1.99 MeV. The present result is 1.74 MeV a little closer to their result. The scond 2^+ obtained in these calculations, like in PSTI calculations is at 3.87 MeV representing the two 2^+ states at 3.39 and 3.65 MeV. In these calculations it appears that it is possible to get an interaction equally suitable for all the two particle systems near the closed shell nuclei ^{40}Ca, and ^{56}Ni. More information about the structure of the levels of these nuclei would be very much helpful to get a reasonably good interaction for the fp shell.

Table IV-II Single particle energies used in the calculations for the three two-particle systems, in MeV.

	$1f_{7/2}$	$2p_{3/2}$	$1f_{5/2}$	$2p_{1/2}$	
^{50}Ti	6.04	2.50	1.35	0.00	(3.58)
^{42}Ca	5.50	3.43	0.00	1.37	(2.864)
^{58}Ni	-	1.08	0.30	0.00	(9.177)

Table IV-I Parameters of interactions determined.

Calculation I - Empirical interaction in relative s- and p- states.

I_{oo}	I_{1o}	I_{2o}	I_{3o}	I_{o1}	I_{11}	I_{21}
5.5769	1.4007	8.4995	1.0951	0.0356	4.1919	3.2325

Calculation II - Surface interaction containing SDI and STI

V_0	V_1	V_{SDI}	V_{STI}	V_p
0.8667	-1.0009	0.4437	0.0188	0.0011

Table IV-J Energy levels of the two-particle systems in MeV. Experimental values and theoretical values are given for comparison.

Nucleus	J	Empirical interaction Cal.I	Surface interaction Cal.II	Irvin	D.B.	B.P.S.	Expr.
^{50}Ti	(g.s.)0	22.80	22.02	-	-	-	21.79
		8.530	8.447	-	-	-	3.87
	2	1.560	1.842	2.09	-	-	1.56
		5.957	5.016	5.35	-	-	4.31
		6.2649	8.192	-	-	-	5.70
	4	2.670	2.538	2.69	-	-	2.68
		6.418	6.130	6.07	-	-	4.80
		7.728	7.618	-	-	-	-
	6	3.187	3.144	3.20	-	-	3.20
		6.847	6.509	-	-	-	-
^{42}Ca	(g.s.)0	20.30	19.612	-	19.65	-	19.835
		6.136	5.871	-	5.37	-	1.84
		-	-	-	-	-	5.85
	2	1.588	1.742	-	1.92	-	1.52
		4.588	3.873	-	3.83	-	2.42
		6.228	6.712	-	6.00	-	3.39
		-	-	-	-	-	3.65
		-	-	-	-	-	4.75
		-	-	-	-	-	4.86
		-	-	-	-	-	5.20
		-	-	-	-	-	6.27

Contd.

	4	2.695	2.643	-	2.50	-	2.75
		5.030	4.812	-	4.56	-	4.45
		-	-	-	5.82	-	5.01
		-	-	-	-	-	6.10
	6	3.235	3.306	-	3.36	-	3.19
^{58}Ni	(g.s.)0	22.982	22.865	-	-	-	-
		1.574	3.323	-	-	2.675	2.943
		3.465	5.069	-	-	3.950	3.531
	1	2.226	4.143	-	-	2.898	2.902
		2.599	4.445	-	-	3.278	3.593
	2	0.951	1.293	-	-	1.489	1.454
		2.113	3.284	-	-	2.692	2.775
		2.629	3.864	-	-	2.922	3.038
		3.032	4.090	-	-	3.355	3.263
		3.960	4.618	-	-	3.821	3.898
	3	3.031	3.569	-	-	2.967	3.420
		3.962	4.983	-	-	3.975	3.774
	4	2.276	2.778	-	-	2.291	2.459
		3.918	4.405	-	-	3.665	3.620

Table IV-K Wavefunctions of states obtained in the calculations.

Calculation - I Empirical interaction in relative states.

Nucleus ^{50}Ti

J	Energy	77	33	55	11
0_1	0.00	0.995	0.069	0.048	0.049
0_2	8.53	-0.087	0.801	0.589	0.067

		77	73	75	33	35	31	55	51
2_1	1.56	0.978	0.033	-0.187	0.023	0.004	0.065	-0.040	0.012
2_2	5.96	0.075	0.803	0.562	0.141	-0.017	0.084	0.068	0.031
2_3	6.27	0.169	-0.585	0.789	0.075	0.017	0.014	0.032	0.007

		77	73	75	71	35	55
4_1	2.67	0.996	0.025	-0.072	0.045	0.033	-0.003
4_2	6.42	-0.018	0.927	0.272	0.228	0.117	0.042
4_3	7.73	0.072	-0.299	0.948	0.058	0.049	0.037

		77	75
6_1	3.19	0.986	0.168
6_2	6.85	-0.168	0.986

Nucleus ^{42}Ca

		77	33	55	11
0_1	0.0	0.992	0.102	0.043	0.068
0_2	6.14	-0.112	0.981	0.140	0.077

Contd.

		77	73	75	33	35	31	55	51
2_1	1.59	0.980	0.055	-0.156	0.043	0.005	0.100	-0.031	0.015
2_2	4.59	-0.057	0.977	0.083	0.153	-0.018	0.097	0.035	0.024
2_3	6.23	0.036	-0.189	0.489	0.829	0.033	0.165	0.079	0.038

		77	73	75	71	35	55
4_1	2.70	0.994	0.057	-0.060	0.069	0.039	-0.002
4_2	5.03	-0.061	0.952	0.102	0.265	0.094	0.025

		77	75
6_1	3.24	0.991	0.138

Nucleus $^{58}_{Ni}$

		33	55	11
0_1	0.00	0.646	0.753	0.126
0_2	1.57	-0.750	0.657	-0.081
0_3	3.47	-0.144	-0.042	0.989

		35	31
1_1	2.23	1.00	0.00
1_2	2.60	0.00	1.00

		33	35	31	55	51
2_1	0.95	0.855	0.129	0.387	0.300	0.115
2_2	2.11	-0.497	0.258	0.777	0.247	0.146
2_3	2.63	-0.004	-0.892	0.321	0.097	-0.305
2_4	3.03	-0.142	0.013	-0.361	0.913	-0.126
2_5	3.96	-0.049	-0.349	-0.114	0.680	0.926

Contd.

		35	51
3_1	3.03	0.978	0.214
3_2	3.96	-0.214	0.978

		35	55
4_1	2.28	0.994	0.114
4_2	3.92	-0.114	0.994

Calculation - II Surface interaction

Nucleus $^{50}_{Ti}$

J	Energy	77	33	55	11
0_1	0.00	0.951	0.182	0.233	0.088
0_2	8.45	-0.250	0.919	0.221	0.210

		77	73	75	33	35	31	55	51
2_1	1.84	0.934	-0.268	0.132	0.064	-0.058	0.074	0.134	-0.088
2_2	5.02	0.314	0.915	-0.053	-0.094	0.122	-0.093	-0.041	0.168
2_3	8.19	-0.128	0.121	0.975	0.072	-0.007	0.053	0.096	-0.014

		77	73	75	71	35	55
4_1	2.54	0.963	-0.099	0.210	-0.083	-0.066	0.086
4_2	6.13	0.178	0.876	-0.184	0.365	0.179	-0.045
4_3	7.62	-0.194	0.287	0.926	-0.075	-0.076	0.105

		77	75
6_1	3.14	0.960	0.280
6_2	6.51	-0.280	0.960

Contd.

Nucleus ^{42}Ca

		77	33	55	11
0_1	0.00	0.932	0.271	0.207	0.126
0_2	5.87	-0.312	0.922	0.062	0.222

		77	73	75	33	35	31	55	51
2_1	1.74	0.852	-0.444	0.116	0.118	-0.078	0.125	0.117	-0.121
2_2	3.87	0.496	0.838	0.001	-0.112	0.093	-0.107	-0.000	0.137
2_3	6.71	-0.081	0.225	0.042	0.849	-0.011	0.467	0.024	-0.018

		77	73	75	71	35	55
4_1	2.64	0.952	-0.170	0.184	-0.134	-0.082	0.075
4_2	4.81	0.235	0.873	-0.048	0.399	0.147	-0.016

		77	75
6_1	3.31	0.974	0.228

Nucleus ^{58}Ni

		33	55	11
0_1	0.00	0.830	0.429	0.356
0_2	3.32	-0.458	0.889	-0.001
0_3	5.07	-0.317	-0.163	0.935

		35	31
1_1	4.14	0.998	-0.056
1_2	4.45	0.056	0.998

		33	35	31	55	51
2_1	1.29	0.686	-0.170	0.637	0.191	-0.240
2_2	3.28	0.422	0.633	0.026	-0.376	0.528
2_3	3.86	-0.590	0.292	0.744	0.038	0.113
2_4	4.09	0.056	0.463	-0.186	0.864	0.025
2_5	4.62	0.008	-0.520	0.074	0.271	0.806

		35	51
3_1	3.57	0.957	-0.290
3_2	4.98	0.290	0.957

		35	55
4_1	2.78	0.989	-0.151
4_2	4.41	0.151	0.989

CHAPTER V.

ISOTOPIC SPIN AND HEAVIER ISOTONES

Shell model calculations with inert core allow three kinds of choices. The configuration space for valance nucleons is chosen from considerations of energies of low lying states and experimental results of transfer reactions for such states. Suitability of such a choice is checked by comparing other properties such as transition rates with the calculated properties. The form of the effective residual interaction chosen determines such configuration mixing and energy levels. A study of these choices explains some of the features of the residual interaction. A third choice in shell model calculations is the set of nuclei whose energy levels and properties are to be explained. This chapter is devoted to the study of the isotones of ^{48}Ca from ^{50}Ti through ^{56}Ni with some of the interactions determined in earlier chapters.

Different interactions are determined in the preceeding chapters to fit lowlying levels of ^{50}Ti and ^{51}V for different choices of model space. The isotopic spin has been neglected in these calculations since such corrections are small if the number of valance particles is small. The calculations are to be done with correct wavefunctions which have a definite isospin. Exact treatment of isospin involves more complications and neglect of isospin introduces changes in the effective residual interaction. This effect can be

seen to depend upon the number of valance particles.

Consider the isospin of a neutron to be given by $t = t_z = \frac{1}{2}$ and that of a proton to be $t = \frac{1}{2}$, $t_z = -\frac{1}{2}$. The isospin of the ground state of ^{40}Ca is $T = T_z = 0$. The isospin of a state with n neutrons in $1f_{7/2}$ orbit ($n \leq 8$) above the ^{40}Ca is simply $T = T_z = n/2$. Even if some of the neutrons are raised to a higher orbit such as $2p_{3/2}$ the isospin remains to be the same. The situation is different if the doubly closed shell core has excess neutrons, such as ^{48}Ca. The isospin of a state of n protons in $1f_{7/2}$ orbit about ^{48}Ca core is given by $T = T_z = (2j+1-n)/2$, $j = 7/2$. If a proton is raised to $2p_{3/2}$ orbit then the state does not have a definite isospin but is a mixture of $T = T_< = (2j+1-n)/2 = T_z$ and $T = T_> = T_< + 1$, $T_z = T_<$. These are represented diagramatically as follows.

^{40}Ca ground state		$T = T_z = 0$
$40+n_{Ca}$	$(\nu\, 1f_{7/2})^n$ 	$T = T_z = n/2$
Isotones of ^{48}Ca	$(\pi\, 1f_{7/2})^n$ 	$T = T_z = (2j+1-n)/2$
	$(\pi\, 1f_{7/2}^{n-1})(\pi\, 2p_{3/2})^1$ 	$T_z = (2j+1-n)/2$ a mixture of $T = T_<$ & $T_>$ $T_< = (2j+1-n)/2$

This situation will be the same if the excited proton is in any one of the higher orbits and can be obtained by coupling wave functions of (n - 1) protons in ($1f_{7/2}$) orbit with that of a proton in a higher orbit. The case in which more protons are raised also can be obtained in the same way. Therefore, strictly speaking, these two configurations should not be mixed essentially because of neutron excess core. Only that component of the excited configuration which has the correct isospin $T = T_<$ should be mixed with the lowest configuration wave function.

Consider the case of ^{51}V which has $n = 3$ protons above ^{48}Ca ground state. The case of higher isotones differ only in n, the number of protons, and therefore in the numerical values of the Clebsch Gordon coefficients for isospin coupling. The angular momentum has a definite value in the following.

$$: (T, T_z) = (T_<, T_z)\ T_< = \frac{2j+1-n}{2} = T_z = \alpha/2$$

$$: \frac{1}{(\alpha+2)^{\frac{1}{2}}} \left[(T_<, T_z) \sqrt{\alpha+1} + (T_>, T_z) \right]$$

$$T_> = T_< + 1$$

$$: 1/ (\alpha + 2)^{\frac{1}{2}} \left[- (T_<, T_z) + \sqrt{\alpha + 1}\, (T_>, T_z) \right]$$

From these two equations we can get ($T_<$, T_z) and ($T_>$, T_z).

$$(T_<, T_z) = \sqrt{\frac{\alpha+1}{\alpha+2}}$$  $$- \frac{1}{\sqrt{\alpha+2}}$$ 

$(T_>, T_z)$ = $\frac{1}{\sqrt{\alpha+2}}$  + $\sqrt{\frac{\alpha+1}{\alpha+2}}$ 

It is the former one to be mixed with  . And inorder to do this we will need neutron particle hole interaction, that of a proton with a neutron particle and a neutron hole. Instead, it is possible to derive a modification in the effective interaction due to neglect of isospin in the wavefunctions asffollows.

Consider T_+  =  $= (T_>, T_>)$

and T_-  $= \sqrt{\alpha+2}\ (T_>, T_z)$ $T_z = T_> - 1 = T_<$

$= ($  $+\sqrt{\alpha+1}$  $)$

This can be used to eleminate  from $(T_<, T_<)$. We get finally $\Psi_{T_<} = (T_<, T_<) = \sqrt{\frac{\alpha+2}{\alpha+1}}\left(\right.$  $-\frac{T_-}{\alpha+2}$  $\left.\right)$

In this way we are able to eliminate neutron hole from the wavefunction. Now the nuclear Hamiltonean is to be diagonalized between the wavefunctions $(j^n) \equiv$  and $\Psi_{T_<}$.

The total wavefunction can be written as

$$\Psi = a \mid j^n \; J > + \sum'_{J1} b_{J1} \left[\Psi^J_{T_<}(J1) \right]$$

where $J1$ is the angular momentum of the $(n-1)$ protons in the $1f_{7/2}$ orbit in $\Psi_{T_<}$.

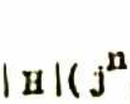
The matrix elements of the nuclear Hamiltonean to be evaluated are

i) $\langle$  $|\ H\ |$ $\rangle$ $= \langle (j^n)\ |\ H\ |\ (j^n)\rangle$

ii) $\langle$  $|\ H\ |\ \Psi_{T_<}\rangle$ $= \langle (j^n)\ |\ H\ |\ \Psi_{T_<}\rangle$

iii) $\langle \Psi_{T_<}\ |\ H\ |\ \Psi_{T_<}\rangle$

While the matrix elements of the first kind remain as before, we have

$$\langle \Psi_{T_<}\ |H|(j^n)\rangle = \langle \sqrt{\frac{\alpha+2}{\alpha+1}}\left(\text{  } - \frac{T_-}{\alpha+2} \text{} \right)|H\ |\ (j^n)\rangle$$

Since the nuclear Hamiltonean is independent of charge we get $\langle (j^n)\ |\ H|\ T_-$ $\rangle = 0$

Thus the consideration of isospin has modified the matrix element $\langle (j^n)\ |\ H\ |$ $\rangle$ to $\sqrt{\frac{\alpha+2}{\alpha+1}}\langle (j^n)\ |\ H\ |$ $\rangle$

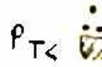
These matrix elements are linear combinations of the two-body matrix elements of the interaction, $\langle j^2\ J\ |\ v\ |\ j\ j'\ J\rangle$ and therefore the new factor $\sqrt{\frac{\alpha+2}{\alpha+1}}$ could be absorbed in the interaction matrix elements.

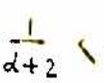
Finally the matrix elements $\langle \Psi_{T_<}\ |\ H\ |\ \Psi_{T_<}\rangle$ might be evaluated by re-writing the wave-function $\Psi_{T_<}$ as follows.

$$\Psi_{T_<} = \sqrt{\frac{\alpha+2}{\alpha+1}}\left(\text{} - \frac{T_-}{\alpha+2} \text{} \right)$$

$$= \sqrt{\frac{\alpha+2}{\alpha+1}}\left(1 - \frac{T_- T_+}{\alpha+2} \right) \text{}$$

The operator $P_{T<} = (1 - T_- T_+ /(\alpha +2))$ thus projects out $\psi_{T<}$ from  . Therefore

$\langle \psi_{T<} | H | \psi_{T<} \rangle = (\alpha+2/\alpha+1) \langle P_{T<}$  $| H | P_{T<}$  $\rangle$

$= \frac{\alpha+2}{\alpha+1} \{ \langle$  $| H |$  $\rangle - \frac{1}{\alpha+2} \langle$  $| H | T_- T_+$  $\rangle \}$

Expansion of these matrix elements in terms of the two body matrix elements of the interaction can be done and we can see that the isospin consideration is equivalent to replacing $\langle j\ j'J\ |v|\ jj'\ J \rangle$ by

$$\frac{\alpha+2}{\alpha+1} \langle j_\pi j'_\pi\ J | v | j_\pi j'_\pi\ J \rangle - \frac{1}{\alpha+1} \langle j_\pi j'_\nu\ J | v | j_\pi j'_\nu\ J \rangle$$

In a calculation for a nucleus if the isospin is neglected the results obtained are to be interpreted to contain the isospin effects in the interaction matrix elements. Due to the factor $(\alpha+2/\ \alpha+1)$ we see that the isospin corrections depend upon the number of valance protons. On the otherhand in a calculation for several isotones together the above expressions may be used with the wavefunctions free of isospin. The proton-neutron interaction is another input parameter in such calculations.

Since the proton-proton interaction has been determined in earlier chapters, it is used together with the required proton-neutron interaction taken from H. Horie and K. Ogawa to obtain energy levels of the isotones of ^{48}Ca. They calculated the proton neutron interaction for the $N = 29$

isotones with ^{48}Ca inert core, by limiting the protons to $1f_{7/2}$ orbit and allowing the last neutron in any of the $^{2p}_{3/2}$, $^{1f}_{5/2}$, $^{2p}_{1/2}$ orbits and took the experimental energy levels of the $N = 28$ isotones for the proton proton interaction. The pn interaction must contain noncentral forces because the spin orbit splitting between $p_{1/2}$ and $p_{3/2}$ neutrons changes as we go from ^{48}Ca to ^{56}Ni. Inorder to obtain this interaction a least squares fitting is resorted to. The pn interaction thus obtained contains all central and non-central interactions and is supposed to give raise to the changes mentioned earlier. Additional neutron, from their calculations, seems to break down seniority scheme indicated by large contribution from the higher s_p components in the ground states obtained in their calculations. According to these calculations the neutron excitations from $1f_{7/2}$ orbit are important above 2 Mev excitation in the $N = 29$ isotones. It should be noted that the neutron in these calculations is allowed in $p_{3/2}$, $f_{5/2}$ and $p_{1/2}$ orbits and therefore the pn interaction determined contains effects of neglecting other configurations where as in the present calculations only pn interaction between $f_{7/2}$ proton and $p_{3/2}$ neutron are needed, which naturally does not contain the effects of neglecting $f_{5/2}$ and $p_{1/2}$ neutron excitations. However it is not expected to change the interaction very much and therefore the pn interaction of Horie and Ogawa is used in the present calculations.

Present calculations are done with three of the interactions determined in earlier chapters which reproduce energy levels of ^{50}Ti and ^{51}V very well. These are calculation IV of Chapter I with surface interaction containing zero range parts, the empirical interaction in relative s- , p- , d- states (Chapter II),calculation VI of Chapter III with short range interaction at the surface and the phenomenological interaction that acts at the surface with $y_o = 3.2$ and $F_1 = 1.0$, without spin-orbit and tensor parts , (Chapter III). In the case of the last interaction the parameters are further varied to get better excitation energies for ^{50}Ti nucleus. These parameters are given below and the corresponding TBMES in jj-coupling follow.

Parameters of the phenomenological interaction

V_o	V_1	I_{oo}	I_{10}	I_{20}	I_{30}
144.6	-53.30	32.70	70.89	59.89	1461.3

$\langle j^2 \mid V \mid j^2 \rangle_J$	J = 0	2.3057
	2	0.5307
	4	-0.4823
	6	-0.8438
$\langle j^2 \mid V \mid jj' \rangle_J$	2	0.6019
	4	0.3101
$\langle jj' \mid V \mid jj' \rangle_J$	2	2.5539
	3	-1.2186
	4	0.6425
	5	-1.7585

Calculations are done for the four interactions with and without isospin considerations. The results are presented in Tables V-A and V-B. Table V-A contains the energy levels of the nuclei from ^{50}Ti through ^{56}Ni. The experimental values and results of the theoretical calculations by R. Saayaman et. al. and K Lips et.al. are also given for comparison. The wavefunctions obtained are given in Table V-B where the strengths of largest component and total pure configuration components, of both the seniorities in the case of ^{52}Cr $J = 2$ and 4, are given.

The binding energies of all the nuclei calculated are quite satisfactory owing particularly to the fixed single $E_{1f_{7/2}}$ proton energy equal to 9.72 MeV. The effect of including the isotopic spin in all nuclei is to seperate the levels more compared to the calculations without isotpic spin considerations. This is essentially due to the change brought by the proton neutron interaction in the matrix elements $\langle f_{7/2}\, p_{3/2} | V | f_{7/2}\, p_{3/2} \rangle_J$. This proton neutron interaction is fairly attractive. This consideration of isospin improves the 2^+ states in ^{50}Ti where the 2^+_1 is obtained high otherwise and the changes in 4^+ states is small. For ^{51}V the $(3/2)_1$ state is further pushed down and the $(5/2)_1$ state is improved by the same effect in all the calculations. The order of $(9/2)_1$ and $(11/2)_1$ states is spoiled since the $(9/2)_1$ is pushed down more than the $(11/2)_1$. In general the changes obtained in the levels of

Table V-A Energy levels of the nuclei ^{50}Ti through ^{56}Ni in MeV. The ground state binding energies taken to be positive. The results are compared with those of SDI calculations of R. Saayaman et.al. and empirical interaction calculations of K. Lips etal and experiment. The binding energies are taken from 1964 Binding Energy Tables and the excitation energies are taken from Nuclear Level Schemes A = 45 through A = 257.

Nucleus	J	Cal. IV of Chapt I		Cal. II of Chapt II		Cal. VI of Chapt. III				EXPR.	SDI	Lips et al (Model A)
		Ispin		Ispin		Ispin		Ispin				
^{50}Ti	(g.s.)0	21.749	21.749	21.756	21.756	21.757	21.757	21.746	21.746	21.79		21.730
	2	1.700	1.644	1.574	1.501	1.594	1.521	1.561	1.499	1.56	2.09	1.551
		4.845	4.823	3.385	3.189	3.363	3.164	3.466	3.275	4.31	5.35	3.389
	4	2,588	2.588	2.697	2.691	2.578	2.572	2.748	2.741	2.68	2.69	2.660
		6.399	6.502	5.992		5.978		5.203	5.138	4.80		
	6	3.170	3.170	3.153	3.153	3.109	3.109	3.150	3.150	3.20	2.91	3.105
^{51}V	(g.s.) 7/2	29.870	29.873	29.906	29.908	30.039	30.041	29.857	29.859	29.85		29.902
	3/2	0.835	0.801	0.942	0.887	0.859	0.807	0.884	0.786	0.93	1.36	0.908
		3.083	3.507	2.696	2.977	2.767	3.026	2.379	2.613	2,41	2.96	2.730
		4.704	4.941	3.725	3.923	3.864	4.080	2.770	2.845	3.22	4.67	3.736
	5/2	0.518	0.463	0.358	0.305	0.457	0.405	0.260	0.201	0.32	1.03	0.349
		4.621	4.858	3.144	3.186	3.172	3.210	2.816	2.832	3.08	4.13	3.186
		5.871									4.91	
	7/2	4.457	4.600	3.285	3.195	3.274	3.180	2.312	2.047		4.48	3.237
		5.902		5.931							4.95	
	9/2	1.814	1.804	1.876	1.646	1.839	1.695	5.293	5.799	1.81	1.83	1.829
								1.804	1.553			

		3.801	3.827	2.137	2.087	2.119	1.983	2.181	2.151		4.10	2.125
	11/2	1.866	1.850	1.773	1.758	1.785	1.772	1.743	1.723	1.61	1.72	1.747
		5.771		5.002	5.103	5.134	5.243	4.247	4.236			
	15/2	2.915	2.918	2.971	2.973	2.918	2.920	2.978	2.979	2.70	2.24	2.912
^{52}Cr(g.s.)	0	40.201	40.201	40.338	40.342	40.607	40.611	40.237	40.241	40.35		40.325
		5.856		4.249	4.143	4.198	4.084	2.505	2.049	2.65		2.56
										4.74		
	2	1.606	1.580	1.492	1.457	1.541	1.509	1.367	1.295	1.43	1.83	1.457
		2.930	2.921	2.961	3.098	3.025	3.037	2.516	2.578	2.97	3.14	2.951
		4.429	4.822	3.238	3.222	3.116	3.220	2.894	2.799	3.16	4.45	3.188
				5.263	5.483	5.325	5.544	4.265	4.309	3.77		
										4.71		
	4	2.449	2.442	2.514	2.489	2.516	2.451	2.389	2.348	2.37	2.32	2.470
		2.760	2.732	2.705	2.700	2.673	2.657	2.708	2.622	2.77	2.82	2.672
		5.392	5.746	3.639	3.544	3.692	3.592	2.966	2.807	3.41	5.02	3.589
		5.989		5.302	5.830	5.402	5.925	4.092	4.395			
								5.444	5.726			
	5	3.536	3.522	3.690	3.679	3.597	3.587	3.625	3.596	3.62	3.32	3.624
				5.975				5.164	5.285		5.60	
	6	3.089	3.085	3.132	3.134	3.091	3.093	3.113	3.113	3.11	2.52	3.079
				4.374	4.200	4.495	4.334	4.056	3.847			
	8	5.034	5.034	5.145	5.150	5.046	5.050	5.151	5.155	3.90	4.75	

.458	46.758	46.769	46.91		46.950
.209	0.755	0.926	1.29	1.29	1.265
.134	1.432	1.981	2.41	2.46	2.122
.468	1.875	2.403	2.88	3.85	3.184
			2.91		
424	0.128	0.094	0.38	0.94	0.301
975	1.361	1.505	2.27	3.41	2.299
031	1.996	2.237		3.70	3.358
	4.982	5.884		3.96	
	5.532				
302	0.877	0.711	2.57	3.68	1.978
531	1.295	1.365		3.62	
350	4.389	3.701			
		5.546			
654	0.952	0.866	1.62	1.55	1.576
423	1.850	1.931			2.094
214	3.025	3.307			
758	1.637	1.663	1.44	1.44	1.693
187	3.098	3.585			
	5.026	5.909			
155	2.991	3.000	2.69	1.97	2.941
509	3.942	4.111			

538	55.475	55.479	55.76		55.770
557	1.196	1.377	2.56	4.55	2.834
			4.29		
683	1.062	1.220	1.41	1.57	1.433
881	1.7296	2.163	2.86	3.84	2.671
750	2.602	3.092	3.16		
	5.776		4.58		
412	1.944	1.882	2.54	1.99	2.635
485	2.469	2.610	3.30	4.20	3.520
	3.006	4.152	3.84		
	4.335	5.288	4.03		
			4.27		
975	2.835	2.866	2.95	2.44	3.021
261	3.290	3.335			
					60.816
391	60.428	60.368	60.82		
627	0.244	1.428		3.64	2.010
	3.512				
					68.000
471	67.403	67.403	68.01		
			4.95		

^{53}Mn(g.s.)	7/2	46.727	46.727	46.923	46.927	47.453	47.458	46.758	46.769	46.91		46.950
	3/2	1.028	1.052	1.291	1.327	1.191	1.209	0.755	0.926	1.29	1.29	1.265
		2.708	3.951	2.083	2.933	2.284	3.134	1.432	1.981	2.41	2.46	2.122
		4.228	5.132	3.189	4.123	3.506	4.468	1.875	2.403	2.88	3.85	3.184
										2.91		
	5/2	0.537	0.514	0.313	0.294	0.440	0.424	0.128	0.094	0.38	0.94	0.301
		3.811	4.680	2.288	2.764	2.506	2.975	1.361	1.505	2.27	3.41	2.299
		4.732	5.580	3.371	3.809	3.576	4.031	1.996	2.237		3.70	3.358
		5.551		5.600		5.862		4.982	5.884		3.96	
								5.532				
	7/2	3.757	4.558	2.004	2.215	2.096	2.302	0.877	0.711	2.57	3.68	1.978
		4.614	5.230	0.030	3.368	3.202	3.531	1.295	1.365		3.82	
		5.142		4.214	5.071	4.477	5.350	4.389	3.701			
				5.756					5.546			
	9/2	1.668	1.668	1.623	1.643	1.659	1.654	0.952	0.866	1.62	1.55	1.576
		3.639	4.263	2.128	2.286	2.229	2.423	1.850	1.931			2.094
		5.636	6.	4.413	5.020	4.608	5.214	3.025	3.307			
	11/2	1.679	1.689	1.732	1.744	1.749	1.758	1.637	1.663	1.44	1.44	1.693
		5.100		4.183	4.961	4.413	5.187	3.098	3.585			
								5.026	5.909			
	15/2	2.946	2.946	3.005	3.009	2.951	2.955	2.991	3.000	2.69	1.97	2.941
				4.989	5.416	5.171	5.609	3.942	4.111			

^{54}Fe(g.s.)	0	55.353 4.659	55.353 6.	55.730 2.872	55.733 3.594	56.535 2.874	56.538 3.557	55.475 1.196	55.479 1.377	55.76 2.56 4.29	4.55	55.770 2.834
	2	1.734 4.035 5.857	1.751 5.488	1.597 2.656 4.361	1.634 3.589 5.464	1.661 2.922 4.652	1.683 3.881 5.750	1.062 1.7296 2.602 5.776	1.220 2.163 3.092	1.41 2.26 3.16 4.58	1.57 3.84	1.433 2.671
	4	2.151 5.061 5.773	2.174	2.492 3.572 4.763 5.862	2.505 4.189	2.404 3.845 5.032	2.412 4.485	1.944 2.469 3.006 4.335	1.882 2.610 4.152 5.288	2.54 3.30 3.84 4.03 4.27	1.99 4.20	2.635 3.520
	6	2.770	2.802	2.983 4.478	3.006 4.887	2.955 4.806	2.975 5.261	2.835 3.290	2.866 3.335	2.95	2.44	3.021
^{55}Co(g.s.)	7/2	60.274 3.486 5.489	60.253	60.702 2.024 5.435	60.691 4.200	61.899 2.264 5.784	61.891 4.427	60.428 0.244 3.512	60.368 1.428	60.82	3.64	60.816 2.010
^{56}Ni(g.s.)	0	67.203	67.203	67.863	67.863	69.471	69.471	67.403	67.403	68.01 4.95		68.000

other nuclei are also same. The high spin states in general are much less affected by the isospin considerations. The wavefunctions are less affected in all the states. obtained leaving the order of levels and the strengths except for the $(9/2)_1$ and $(11/2)_1$ states of ^{51}V which changes the order and those of ^{53}Mn.

The energy levels obtained are quite satisfactory. The lowest states obtained for each angular momentum in ^{50}Ti ^{51}V are much better than the results of SD I calculations, except the $(9/2)_1$ and $(11/2)_1$ states in ^{51}V, (4^+_1) states of ^{52}Cr, $(1.5)_1$, $(5.5)_1$ states of ^{53}Mn and 2^+ states of ^{54}Fe. The higher states however are not much better compared the SD I results. The number of low-lying levels is quite large in the nuclei ^{52}Cr, ^{53}Mn and ^{54}Fe. The calculated levels are comparable to those of SD I calculations in many cases. In general the results with the surface interactions determined in earlier chapters are much better than those calculated with short range interaction components at the surface without the tensor and spin orbit parts. It is to be noted thatthe SD I calculations take complete shell and all the nuclei are considered for fitting whereas the interactions used here are determined for the ^{50}Ti and ^{51}V nuclei only. The results of Lips et.al are however much better. The wavefunctions of SD I calculations anyway support the simple configurations considered. In the empirical interaction calculations of Lips et.al

the interaction is a 10 parameter interaction. As mentioned earlier the interactions chosen in the present calculation have less number of parameters and have less freedom. Even in the case of the empirical interaction in relative states the interaction is assumed to be purely central and is also limited than that of empirical interaction in jj coupling states. Inspite of these aspects of the interaction, the results obtained indicate that the surface interaction with explicit reference to the effective radius can give satisfactory results.

The case of ^{53}Mr and ^{54}Fe is different. The density of levels at low energies is high in these nuclei. The spectra of these nuclei are known to exhibit core excitation features particularly the one and two neutron excitations. Even though the g.s. of ^{48}Ca. (core) is known to have only 4% excitation components the neutron excitations are important in these nuclei because of the number of protons and because they are near ^{56}Ni nucleus. In the coriolis coupling model calculations of Scholz and Malik deformations appear to be important in these nuclei. But in the shell model calculations for the ^{51}V and ^{52}Cr and the SD I calculations of R. Saayaman et.al. do not support the deformations at least for these two nuclei. In the case of ^{54}Fe, however, two and neutron excitations seem to be important.

The lowest states in many cases contain large pure configuration component. In ^{51}V, $(9/2)_1$ state the pure configuration is only 38% in relative state empirical inter-

action calculations like in the calculations of Lips et. al. The same case is with the $(9/2)_1$ state of ^{53}Mr. In the case of ^{52}Cr the two 2^+ states and the two 4^+ states contain admixtures from the two seniorities. The existence of $l_p = 3$ transition strength in stripping reactions and branching of E2 transition from 6_1^+ state to the two 4^+ states indicates that these states are not pure states. The seniority mixing comes through the other excited configurations. The lowest 2^+ state is known to contain large $\nu = 2$ component and the lowest 4^+ state contains large $\nu = 4$ component. The ratio of the $\nu = 2$ components in the two states is 0.63. In the present calculation the 2_1^+ state contains large $\nu = 2$ component in all the calculations. The second 2^+ state obtained in empirical interaction calculations and the short range surface interaction without tensor and spin orbit part it is the third 2^+ state which has large $\nu = 4$ component. The second 2^+ in these cases contain large $| j^3 (3.5) j' 2\rangle$ component. The 4^+ states obtained in these calculations also are different. The empirical interaction in relative states and the short range interaction at the surface (without tensor and spin orbit part) lead to large $\nu = 4$ component in 4_1^+ state but not others. The ratio of the $\nu = 2$ component in these calculations are, in the order,

$$\frac{77}{20}, \quad \frac{74}{24}, \quad \frac{18}{76}, \quad \frac{16}{78}, \quad \frac{72}{24}, \quad \frac{66}{30}, \quad \frac{7}{74}, \quad \frac{7}{31}$$

In the calculations hewhere it is less than 1° , it is much less than the ratio of experimental C^2S . In the SD I calculations of R Saayaman, et.al the seniority components are not given whilein the empirical interaction calculation of Lips et.al. the ratio of the spectroscopic factors for stripping reactions is (0.259/0.989). In the present calculation isospin consideration has decreasing effect on this ratio.

It should be noted that in the present calculations the interaction contains fewer number of parameters and acts at the nuclear surface and contains explicit dependence on the effective ratio. This interaction determined to fit the lowlying states of ^{50}Ti and ^{51}V in only a small configuration space works well for the higher isotones of ^{48}Ca except for the states for which neutron excitations are known to be important.

Table V-B Wavefunctions of states presented in Table V-A. The strengths of dominating components and total pure configuration component are given. The numbers given are percentage contributions. The component $| j^2 (J1)\ j'\ J >$ where $j = 1f_{7/2}$, $j' = 2p_{3/2}$ is simply represented by J1. The symbol s stands for the seniority.

Nucleus	J	Cal. IV of Chapt. I				Cal. II ofCChapt. II				Cal. VI of Chapt. III							
		Ispin				Ispin				Ispin							
		Domin.	pure	Domin.	pure	Domin.	pure	Domin.	pure	Domin.	pure	Domin.	pure	Domin.	pure	Domin.	pure
$^{50}_{Ti}$	2		88		87		87		82		87		81		89		85
		88	12	87	13	87	13	82	18	86	13	81	19	89	11	85	15
	4		99		99		99		99		99		99		98		98
		99	1	99	1	99	1	98	2	99	1	99	1	98	2	98	2
$^{51}_{V}$	1.5		87		87		79		78		83		82		66		63
		J1=0	94 2	J1=0	94 2	J1=0	64 9	J1=0	49 12	J1=0	59 8	J1=2	45 2	J1=0	76 9	J1=2	45 26
		2	83 11	2	83 11	2	52 12	0	51 10	2	50 9	0	56 7	2	50 25	0	71 12
	2.5		84		83		87		85		87		85		85		83
		4	54 16	4	59 16	4	53 13	4	55 15	4	52 13	4	54 15	4	46 15	4	47 17
		2	63														
	3.5		99		99		99		99		99		99		99		99
		4	99	4	99	4	97	4	97	4	97	4	97	4	88	4	88
		2	99 1			2	97							2	88	2	87
	4.5		96		96	6	56 38	6	80 11		77	6	65 27	6	65 23	6	77 9
		6	85 4	6	86 4		62		89	6	67 23		73		77		91

	5.5	97	96	95	94	95	95	96	96
		6 87 4	6 87 4	6 73 5	6 73 6	6 91 5		6 89 4	6 88 4
	7.5	100	100	100	100	100	100	100	100
^{52}Cr	0	100	100	100	95	100	99	100	99
				1.5 99	1.5 99	1.5 99	1.5 99	1.5 99	1.5 99
	2	s=2 89 89	s=2 89 90	s=2 85 85	s=2 84 84	s=2 87 87	s=2 86 87	s=2 73 74	s=2 70 70
		J 1=4 88 88	4 89 90	J 1= 3.5 75 11	J 1=3.5 74 13	4 53 54	4 85 85	J 1=3.5 67 20	J 1=3.5 68 23
		J 1=3.5 87 6	J 1=3.5 89 8	s=4 80 83	s=4 79 83	J 1=3.5 56 43	J 1=3.5 81 13	s=4 41 47	s=4 40 47
				J 1=2.5 48 22	J 1=2.5 50 18	2.5 46 13	2.5 47 13	4 57 57	4 58 58
	4	s=2 77 93	s=2 74 93	s=4 70 88	s=4 72 88	s=2 72 93	s=2 66 92	4 76 83	4 73 80
		4 70 90	4 67 91	2 76 96	2 78 96	4 68 98	4 64 94	2 74 84	J 1=4.5 31 36
		J 1=5.5 47	J 1=5.5 61	J 1=5.5 63 4	J 1=5.5 63 5	J 1=5.5 61 3	J 1=5,5 61 4	J 1=5.5 51 16	s=2 67 68
		3.5 68 1		3.5 90 1	3.5 91 1	3.5 87 2	3.5 88 1	3.5 97 2	J 1=3.5 98 2
								4.5 38 15	4.5 41 14
	5	91	92	91	91	93	93	80	80
				5.5 70 8				5.5 59 19	5.5 60 20
	6	98	98	99	99	99	99	99	98
				7.5 88	7.5 89	7.5 86	7.5 88	7.5 80	7.5 81
	8	100	100	100	1100	100	100	190	100
^{53}Mn	1.5	89	93	86	93	92	95	J 1=2 s1=2 72 26	J 1=2 s1=2 57 43
		0 90 3	0 88 1	0 48 9	2 s1=2 70 5	2 s1=2 50 4	2 s1=2 72 3	J 1=0 85 14	54
		2 s1=2 83 7	2 s1=2 83 5	0 51 5	J 1=0 76 2	J 1=0 54 3	J 1=0 75 1	60	J 1=0 97 2

^{53}Mn 2.5 87 88 88 89 89 90 77 78

J 1=4 45 2 J 1=4 48 1 J 1=4 50 1 J 1=4 55 1 J 1=4 46 2 J 1=4 51 1 J 1=4 57 1 J 1=4 59

s1=2 s1=2 s1=4 s1=4 s1=4 s1=4 s1=4 s1=4

J 1=2 83 7 J 1=2 83 5 J 1=0 51 5 J 1=0 76 2 J 1=4 39 5 J 1=4 44 6 J 1=4 37 18 J 1=4 46 18

s1=2 s1=2 s1=2 s1=2 s1=2 s1=2

J 1=2 41 3 J 1=2 41 4 J 1=2 60 5 J 1=2 60 4

s1=2 s1=4 s1=4 s1=4

J 1=2 54

s1=2

3.5 98 99 98 98 98 98 98 98 97 97 95

J 1=4 J 1=4 50 J 1=5 47 J 1=5 60 1 J 1=5 46 1 J 1=5 58 1 J 1=5 55 2 J 1=5 65 4

s1=2 62 s1=2

J 1=5 53 J 1=5 43 J 1=4 62 J 1=4 76 J 1=4 60 J 1=4 73 J 1=4 72 J 1=4 87

s1=2 s1=2 s1=2 s1=2 s1=2 s1=2

J 1=4 48 1 J 1=4 66 J 1=4 72 J 1=4 66 J 1=4 72 J 1=4 67 J 1=4 70

s1=4 s1=4 s1=4 s1=4 s1=4 s1=4 s1=4

J 1=2 82 1 J 1=2 87 1

s1=2 s1=2

4.5 94 94 95 J 1=6 51 43 55 74 78 J 1=6 80 5 J 1=6 79 6

J 1=6 86 J 1=6 89 51 J 1=6 48 41 J 1=6 62 22 J 1=6 68 18 71 76

5 72 2 5 58 5 5 61 4 5 59 4 5 62 3 5 43 24 5 48 17

5.5 91 93 94 96 95 96 90 92

6 87 3 6 86 3 6 88 2 6 83 2 6 86 2 6 72 5 6 75 3

5 56 6 5 5

7.5 99 99 99 99 99 99 96 96

8 98 1 8 99 1 8 98 1 8 99 1 8 96 4 8 96 4

^{54}Fe J=0 100 100 99 99 99 99 98

J 1=1.5 99 1 J 1=1.5 99 1 J 1=1.5 99 1 J 1=1.5 99 1 J 1=1.5 98 2 J 1=1.5 98 98

2 92 94 85 92 90 94 3.5 39 2

J 1=3.5 88 2 J 1=3.5 93 2 J 1=3.5 75 9 J 1=3.5 86 5 J 1=3.5 78 6 J 1=3.5 86 3 54

2.5 58 4 2.5 63 5 2.5 66 3 2.5 59 4 2.5 61 2 45 J 1=3.5 56 34

4 90 93 91 93 92 94 2.5 43 25 2.5 56 11

3.5 73 1 5.5 71 1 5.5 76 2 5.5 66 1 5.5 72 1 5.5 45 13 5.5 41 18

5.5 63 3.5 87 1 3.5 83 1 63 70

4.5 71 8 3.5 93 4 3.5 99

4.5 43 19 4.5 45 12

6 92 95 95 97 96 97 74

7.5 91 1 7.5 92 1 7.5 90 1 7.5 91 1 7.5 65 18 76

7.5 65 19

^{55}Co 3.5 97 99 98 99 98 99 73

J 1=4 82 J 1=4 83 1 J 1=4 89 1 J 1=4 83 1 J 1=4 57 25 J 1=4 87 97

2 80 3 2 83 1 2 83 1 2 81 3 2

CONCLUSIONS

Standard shell model studies start with an effective Hamiltonian. In the present work the diagonal one body part is chosen from experiment while the interaction part is chosen in one of several ways. The single particle wave-functions are chosen usually to be harmonic oscillator wave-functions out of convenience. This harmonic oscillator basis is found to be satisfactory in many cases. With the choice of the model space and effective residual interaction shell model studies attempt to explain the connection between observed properties such as energy levels and electromagnetic transition rates.

There are two basic methods of choosing the effective residual interaction in standard shell model calculations; phenomenological interaction with variable parameters to be determined in a least squares fit and empirical interaction method in which two body matrix elements are treated as parameters. In the present work both methods are employed. In such studies effects of choice of model space on interaction and even odd effects, if any, when several nuclei are chosen such as isotones or isotopes are of some importance. Properties of the model interaction play a basic role. It is well known that the residual interaction acts at the nuclear surface.

In the present work short range and zero range surface interactions and empirical interaction in relative states are studied. Surface delta interaction and surface tensor inter-

action have been widely used in shell model calculations with the assumption that the radial integrals are equal at the surface. It is shown in Chapter I that correct radial integrals affect the B(E2) rates in ^{51}V. Also that since for delta interaction one has $(EJ + 2/EJ) = (J+1)^2/(J+2)^2$ for J^2 configuration (large j) the excitation energies would be proportional to $0(0^+)$, $1.0(2^+)$, $1.15(4^+)$ and $1.20(6^+)$.[105] This means only too strong a pairing force effect with the inclusion of STI, which does not have non diagonal matrix elements, configuration mixing do not seem to be adequate. An additional central interaction at the surface and correct radial integrals leads to very satisfactory effective radius. However the $(2^+)_1$ state in ^{50}Ti and, as a consequence, the $(3/2)_1$ and $(5/2)_1$ states in ^{51}V are still a little too high. This seems to be essentially due to zero range of the interaction. A possible improvement, to make the particles interact in a finite region near the surface, is suggested noting that the nuclear surface is not very sharp.

The calculations in the second chapter use empirical interaction in relative states. It is explained that this method has an edge over the usual empirical interaction in jj coupling states. From the results obtained it appears that the interaction in relative s- and p.states may be sufficient to describe the nuclei of ^{50}Ti and ^{51}V. Non-central interactions in relative states are not considered simply becasue it makes the calculations immensely large with probably slight improvement in results.

The question of short range of the effective interaction at the surface is taken in Chapter III. Calculations with two choices each of the relative ranges and effective radius are presented. It appears that equal range for the central, spin-orbit and tensor parts has the tendency to reverse the $(9/2)_1$ and $(11/2)_1$ states in ^{51}V. The results suggest that the centre of mass of interacting particles fixed at the experimental radius can reproduce the energy levels of both ^{50}Ti and ^{51}V satisfactorily with a small fonfiguration space as chosen.

Effects of the choice of the size of configuration space are studied in Chapter IV. The excitation energies in ^{50}Ti do not seem to improve much but those of ^{51}V change for the better with increase in the size of the model space. In particular the $(9/2)_1$ and $(11/2)_1$ states in ^{51}V are obtained in correct order with both empirical interaction and surface interaction with zero range parts with correct radial integrals with the effective radius parameter at 2.5. The $(3/2)_1$ and $(5/2)_1$ states are still not very satisfactory with the zero range interaction. Calculations for two particle systems in fp shell by taking all Pauli allowed states into consideration are also done in this chapter. It appears that a single interaction may very well account for the excitations energies of ^{50}Ti, ^{42}Ca and ^{58}Ni nuclei. Core excitation effects are usually considered to be important in low-lying levels of ^{42}Ca and ^{58}Ni but all the three

nuclei are equally well described in the present calculations. The pairing strength of surface interaction obtained is still very small. The empirical interaction parameters are all obtained to be positive though there is no strong evidence that they should be. The interactions seem to be different for the ^{58}Ni nucleus owing to, probably, different shell closure.

In the last chapter the higher isotones of ^{48}Ca are treated. In this chapter need for using correct isospin wavefunctions is explained and isospin corrections are derived and used. The energy levels are not fitted in these calculations but interactions determined for ^{50}Ti and ^{51}V are used. In the case of short range surface interaction slight variation of strengths is seen to describe even isotones very well but the odd isotones are spoiled. The results are compared to calculations with SDI and empirical interaction and are found to be quite reasonable.

The spectroscopic factors for transfer reactions and reduced electric quadrupole transition rates presented here are quite satisfactory. The present work assertains the usefulness of the small model space for the isotones of ^{48}Ca and proves that much simpler interactions than the empirical interaction in jj-coupling states can provide a good description of these nuclei. An attempt is made to study the effects of effective interactions in shell model calculations, effects of choice of model space and choice of set of nuclei in the present work.

It should be noted that in a recent work[100] it has been pointed out that all the usually employed effective i interactions give similar fits to the experimental data. By studying the two-particle transition density function in momentum space determined by truncation they find that the usually employed interactions lead only to long range correlations and that the short range correlations are to be incorporated into the nuclear wavefunctions. Only then useful interactions which exhibit possible symmetry schemes might be found.

The aim of the present work is to study the usefulness of some of the interactions presently in vogue and to study the effects of choice of model space. Surface interactions of two kinds, empirical central interaction in relative states are chosen. Their relative merits and difficulties are discussed. It is found that theyprovide similar spectra. The changes in the values of the parameters with the choice of the model space are not very significant. This is because in all calculations ^{48}Ca is assumed and the valance protons are confined to fp shell. Probably much larger spaces with core excitations and excitations to higher configurations may lead to significant changes. In the present calculations the interactions determined for the nuclei ^{50}Ti and ^{51}V are found to be satisfactory for higher isotones of ^{48}Ca and except for the ^{58}Ni with ^{56}Ni core, which is quite different from the ^{48}Ca core,

the nuclei ^{42}Ca and ^{50}Ti , which are not of much complicated spectrum in structure can be equally well described by the same effective interactions. It may be noted that these findings are qualitatively in agreement with the work of Knupfer et al.

Shell model calculations are bulky. Owing to non-availability of several quantities, all the required q uantities are recalculated in the present work. They include coefficients of pfractional parentage, Brody Moshinsky brackets , E2 transition matrices, quantities in Racah Algebra. Calculations would be impossible without an electronic computor. All the programs needed for every calculation are written and checked by reproducing earlier work published in Standard journals before using for the present calculations. The following Appendices contain relavent material to make the presentation complete.

COEFFICIENTS OF FRACTIONAL PARENTAGE

Wavefunctions of several nucleons in jj-coupling, completely antisymmetrized with respect to the particle index, can be built starting from antisymmetric wavefunctions of two nucleons by successively coupling wavefunctions of a single nucleon, antisymmetrization and normalization. The properties of these functions under rotations and change of coupling transformation make it possible to write antisymmetric wavefunctions of j^n configurations as linear combinations of wavefunctions which are antisymmetric in $(n-1)$ particles as follows:

$$\psi(j^n \alpha J T) = \sum [j^{n-1}(\alpha_1 J_1 T_1)\, j\, J T | \} j^n \alpha J T]\, \psi(j^{n-1}(\alpha_1 J_1 T_1)\, j\, J T) \qquad A1$$

where α stands for all other quantum numbers such as seniority that may be needed to identify the wavefunctions uniquely except for the phase factor which is arbitrary. The wavefunctions used in the text are wavefunctions for identical nucleons for which the isotopic spin takes the highest value and may be neglected. Seniority of a wavefunction is the number of unpaired particles in the sense that it can be obtained by successively coupling wavefunctions of two particles coupled to zero angular momentum from the wavefunction $\psi(j^n J)$, where $\psi(j^\nu J)$ can not be obtained from $\psi(j^\lambda J)$ $\lambda < \nu$. For $J = 7/2$ the wavefunctions are uniquely defined by J except when $n = 4$. In this case there are two orthogonal wavefunctions for each of

ν = 2 and 4 and are distinguished by the seniority ν = 2 & 4.

The co-efficients in equation A1 are called co-efficients of fractional parentage (CFP). They satisfy the orthonormality relations,

A2

These CFP's are calculated using the relation

A3

where are quantum numbers of the principal parent. Different principal parents may give raise to different functions. These wavefunctions with all allowed values of (α, J_1) form a complete basis of functions of n particles which are antisymmetric in coordinates of (n - 1) particles. Thus any wavefunction which is antisymmetric in all the n particle coordinates can be expressed in the form A1. The transformation can be considered as a projection operation on to the subspace of completely antisymmetric functions. This transformation cannot be inverted.

To evaluate matrix elements of two body operators it is necessary to separate n-particles into n - 2 and 2. The wavefunctions are conveniently written as

A4

The double CFP's appearing in this equation are obtained from

A5

In general one can define multiple CFP's as follows

A6

The CFP's given in Nuclear Shell Theory by A. de Shalit and I. Talmi contain some errors and therefore the need to calculate them. The CFP's used in the present work are calculated with an arbitrary phase factor and are given below.[109,110] The double CFP's needed are calculated using these CFP's and equation A5.

We write for convenience

A7

where J, J1, J2 and J' each represents both the index and the angular momentum as the case may be. In the following tables angular momenta are chosen in increasing order except for four particles due to seniority. In this case they are chosen in the order of increasing seniority as $\nu = 0$, $J = 0$, $\nu = 2$, $J = 2, 4, 6$ and $\nu = 4$, $J = 2, 4, 5, 8$. The CFP $(J, J1)$, and CFP $(J, J2, J')$ are tabulated as follows

OFF'S CALCULAT[illegible]

			-	-	[illegible]
-	[illegible]	[illegible]	[illegible]	-0.5000	-0.6009
[illegible]	[illegible]	0.5000	[illegible]	-	[illegible]
-	[illegible]	-0.8058	-	0.4767	-0.8790
[illegible]	- .7147	-			

		1.000	-	-	-
-	-	0.5774	-0.2782	+0.5000	-
0.2535	-0.5244	0.5774	0.5202	-0.3138	0.3892
-0.3619	-0.1231	0.5774	-0.2672	-0.4264	-0.5072
-	0.2334	-	0.7549	0.3015	-
0.5000	0.1581	-	0.1284	-0.6435	-0.3203
0.1759	0.6582	-	-0.5539	-0.4594	0.4699
0.3373	-0.3337	-	0.3740	-0.4835	0.7914
-	-				

		-	-	-	-0.3727
	-	-	-	-0.5000	-
[illegible]	-	-	-0.6009	-	[illegible]
-	-	-	0.3693	-0.3863	-
-	-	0.2330	-0.5133	-	-
-	-	0.0401	-	0.2667	-
-	-0.3355	-	-0.2330	0.2886	[illegible]
-	-0.5134	0.6321	-0.2330	-0.4365	0.2000
[illegible]	-0.2833	-			
-	-0.4886				
[illegible]					

-	-	-	0.2886	-	-
-0.1946	-0.3178	-	-0.1946	-0.4065	0.2164
0.2886	-0.3177	0.2164	0.5446	-	-
-	-	-	0.3350	0.2332	-
-	0.2383	-0.6623	-0.3784	-	-
-0.3784	0.2318	-	-	-	-
-	-0.3504	-0.3433	0.3848	-	-0.3433
0.1966	0.1004	-	0.3848	0.1005	0.5349
-	-	-	-	-	-
[illegible]	[illegible]	-	0.3256	-	0.5563
-	[illegible]	-0.5565	-	-	-
-	-	-	-	-	-0.2817
-	-	0.3302	0.3668	-	-0.2816
-	-				
-	[illegible]				
[illegible]					

$i = 3$ $\quad \overline{DM}\ (0.33,\ 0.33)$

-	[illegible]	[illegible]	-	0.5606	-0.2560
[illegible]	-	-	0.4284	0.1347	-0.3141
-	[illegible]	[illegible]	-	0.3162	-0.4082
[illegible]	-	-	-	-	-
[illegible]	[illegible]	-	0.2725	0.4774	0.1089
[illegible]	[illegible]	[illegible]	0.2007	-0.2450	-0.3971
-	[illegible]	-	0.5147	-	-
[illegible]	[illegible]	[illegible]	0.2140	0.3485	-0.7298
[illegible]	0.5000	-			
[illegible]	[illegible]				
[illegible]			-	-	-0.2507
	$\overline{DM}\ (0.33,\ 0.33)$	-	-	-0.2804	-
[illegible]	[illegible]	-			
[illegible]	-				
-0.1793					

-	-	-0.4548	-0.4072	-	-
-0.3704	0.1432	-	-	-	0.2928
-	0.2048	-.1464	-	[illegible]	-0.3754
0.3310	-	-	-0.2473	-0.0779	0.1812
-	[illegible]	-.1[illegible]	0.0335	-	-
[illegible]	-.[illegible]	-	-	-	[illegible]
-	-0.10[illegible]	0.198[illegible]	-	-	-0.2141
-0.067[illegible]	0.156[illegible]	0.3163	-0.2357	-0.316[illegible]	-0.3[illegible]1
-	-0.113[illegible]	0.2849	-0.1749	-	-0.2041
0.1719	0.2807	-	-	-0.2132	0.3931
-	-	0.2876	0.2953	-	0.1967
-0.1053	0.0200	-	-0.1016	0.2548	-0.1573
0.1581	0.1486	0.1268	-0.4416	-	-0.2913
-0.0538	-0.2926	-	-	0.4920	0.0766
-	-	-0.2184	0.0839	-	-
-0.2095	0.3785	-	0.1667	-0.1404	-0.2292
-	0.2659	0.0491	0.2671	0.1581	-0.0317
-0.3688	0.1745	-	-0.3299	0.1855	0.4219
-	-	-	0.1464	-	-
-	-0.1966	-	-	0.1508	-0.2780
-	-	-0.3890	-0.0606	-	-0.2857
0.1606	0.3654	0.1581	-0.2082	0.3055	-0.5314

N = 6 CFP (J, J1)

-	-	1.0000	-	-	-
0.2928	-0.6058	0.3332	-0.3212	0.5774	-
-0.4180	-0.1422	0.3332	0.6006	-0.3624	0.4496
-	0.2753	0.3332	-0.3085	-0.4924	-0.6897

CFP (J, J2, J')

0.3162	-	-	-	-	-0.4082
-	-	-	-	-0.5477	-
-	-	-	-0.6583	-	-
-	-	-	-	-	-
-	-	-	-	-	-
-	-	-	0.1059	-	-
0.1826	0.1704	-0.2448	-	-	-0.2447
0.0272	-0.1985	-	-	-0.1985	-0.3242
-	-0.2119	-0.1476	-	-	0.2975
0.2915	-0.3268	-	-	0.3056	0.2726
-	-	-	0.3286	-	-
0.1053	-	-	-0.1823	0.0202	-0.1480
0.1826	0.0203	0.1934	-0.3090	-	0.1479
-0.3091	0.1644	-	-0.1099	0.3122	0.1784
-	0.2172	-0.1244	-0.0635	-	-0.2277
-	-0.3890	-	-	-0.3306	-0.3189
-	-	-	0.1053	-	-
-0.1231	-0.2010	-	-0.1231	-0.2571	0.1368
0.1826	-0.2010	0.1369	0.3445	-	-
0.1484	-0.0908	-	-0.2026	-0.0529	-0.2816
-	-0.1690	0.3287	-	-	0.2038
-0.2654	0.4729				

[illegible] [illegible]

0.1[illegible]9	-0.4227	-0.5669	-0.6815

CFP (J, [illegible], J')

-	-0.1237	0.2370	-	-	-0.2560
-0.0[illegible]06	0.1877	0.1890	-0.1408	-0.1888	-0.2271
-	-0.1357	0.3406	-0.2102	-	-0.2440
0.2055	0.3556	-	-	-0.2549	0.4700

APPENDIX - B

MATRIX ELEMENTS FOR SURFACE INTERACTIONS

The surface interactions used in Chapter I are surface delta interaction and surface tensor interactions. The matrix elements are calculated using the methods given by I. Talmi and A de. Shalit.[37] The matrix elements for the surface delta interaction are given by S.A. Moszkowski et. al. and R. Arvieu and those for tensor interaction are given by I. Talmi and D. Banerjee et. al.

Surface delta interaction : $-4\pi\, G\, \delta(\Omega_{12})$

$$\langle j_1 j_2 J | V | j_3 j_4 J \rangle = -G\,(-1)^{l_1+l_3} h_J(j_1 j_2)\, h_J(j_3 j_4) \qquad \text{B1}$$

where $h_J = (2j_1+1)^{1/2} \langle j_2 -\tfrac{1}{2}\; J\,0 \,|\, j_1 -\tfrac{1}{2}\rangle$ if $l_1 + l_2 + J$ is even

$= 0$ otherwise B2

This can be easily derived by writing

$$\delta(\bar{r}_1 - \bar{r}_2) = \frac{1}{r_1 r_2}\, \delta(r_1 - r_2)\, \delta(\cos\theta_1 - \cos\theta_2)\, \delta(\varphi_1 - \varphi_2) \qquad \text{B3}$$

and noting $Y_{l_1 m_1}(\Omega_1)\, Y_{l_2 m_2}(\Omega_2) = \sum_L (-1)^{l-m} \begin{pmatrix} l & l_1 & l_2 \\ -m & m_1 & m_2 \end{pmatrix} (l \| Y_{l_1} \| l_2)\, Y_{lm}(\Omega_1)$

B4

$$\psi(l_1 j_1 l_2 j_2 J M) = \sum_{SL} [S L j_1 j_2]^{1/2} \begin{Bmatrix} s_1 & l_1 & j_1 \\ s_2 & l_2 & j_2 \\ S & L & J \end{Bmatrix} \psi(l_1 l_2 S L J M) \qquad \text{B5}$$

where $(l \| Y_{l_1} \| l_2) = (-1)^l [l l_1 l_2]^{1/2}\, 1/\sqrt{4\pi} \begin{pmatrix} l & l_1 & l_2 \\ 0 & 0 & 0 \end{pmatrix}$ B6

Spin-Spin interaction at the surface:

B7

where

We have

B8

and

B9

The pairing interaction in jj-coupling is defined by

B10

The tensor interaction S12 may be evaluated by the tensor expansion method explained in Appendix - D. This has been done by I. Talmi. D. Banerjee et. al. has given the formula for computing the matrix elements of surface tensor interaction as given below,

B11

X1

X3

B12

where

The matrix elements of these interactions for the complete fp shell except for the radial integrals are given in the following table. The radial integrals for the fp shell are, but for a common factor,

$$F1 = R(1f, 1f, 1f, 1f) = 0.2857\ X^4$$

$$F2 = R(1f, 1f, 1f, 2p) = 0.5345\ (2.5 - X^2 2)\ X^2$$

$$F3 = R(1f, 2p, 1f, 2p) = (2.5 - X^2)^2$$

where $X = R/\ ,\ = (h / Mw)^{\frac{1}{2}}$

In the following table the orbit numbers 1, 2, 3, 4 stand for $1f_{7/2}$, $2p_{3/2}$, $1f_{5/2}$ and $2p_{1/2}$.

Complete fp shall matrix elements of some interactions

j_a	j_b	j_c	j_d	J	1	$\sigma_1 \cdot \sigma_2$	SDI	STI	q_{12}
1	1	1	1	0	1.0	-1.2851	4.0000	0.5714	8.0000
				2	1.0	-1.0408	0.9524	0.4626	-
				4	1.0	-0.4694	0.4675	0.2086	-
				6	1.0	0.4286	0.2331	-0.1905	-
1	1	1	2	2	-	-	-1.3997	-	-
				4	-	-	-0.5448	-	-
1	1	1	3	2	-	-0.9998	0.4666	-0.0555	-
				4	-	-1.5489	0.4928	-0.0861	-
				6	-	-1.3997	0.5710	-0.0778	-
1	1	1	4	4	-	-	-0.6447	-	-
1	1	2	2	0	-	-	2.8284	-	5.6568
				2	-	-	0.6172	-	-
1	1	2	3	2	-	-	-0.5714	-	-
				4	-	-	-0.7310	-	-
1	1	2	4	2	-	-	0.8729	-	-
1	1	3	3	0	-	-1.8795	3.4641	-0.6598	6.9282
				2	-	-1.7317	0.9081	-0.5772	-
				4	-	-1.1487	0.3655	-0.3829	-
1	1	3	4	2	-	-	-1.0690	-	-
1	1	4	4	0	-	-	2.0000	-	4.0000
1	2	1	2	2	1.0	-1.2857	2.0571	0.4571	-
				3	1.0	-0.7143	-	0.2540	-
				4	1.0	0.0476	0.6349	-0.0169	-
				5	1.0	1.0000	-	-0.3556	-
1	2	1	3	2	-	-	-0.6857	0.3429	-
				3	-	-	-	0.4949	-
				4	-	-	-0.5743	0.5686	-
				5	-	-	-	0.5237	-

1	2	1	4	3	-	-0.9897	-	-0.0550	-
				4	-	-1.1269	0.7513	-0.0626	-
1	2	2	2	2	-	-	-0.9071	-	-
1	2	2	3	2	-	-0.9331	0.8399	0.0311	-
				3	-	0.1278	-	- 0.0426	-
				4	-	-0.1278	0.8518	0.0426	-
1	2	2	4	2	-	-	-1.2829	-	-
1	2	3	3	2	-	-	-1.1877	0.1979	-
				4	-	-	-0.4259	0.7667	-
1	2	3	4	2	-	-1.7457	1.5712	-0.5819	-
				3	-	-1.1429	-	-0.3810	-
1	3	1	3	1	1.0	1.0000	-	-0.6667	-
				2	1.0	0.5102	0.2236	-0.6522	-
				3	1.0	1.0000	-	-0.2222	-
				4	1.0	-0.6227	0.5195	-0.4109	-
				5	1.0	1.0000	-	0.5773	-
				6	1.0	-2.4286	1.3986	-0.0317	-
1	3	1	4	3	-	-	-	-	-
				4	-	-	-0.6795	-	-
1	3	2	2	2	-	-	0.3024	-	-
1	3	2	3	1	-	-	-	-0.9562	-
				2	-	-	-0.2799	0.8398	-
				3	-	-	-	-0.6639	-
				4	-	-	-0.7705	0.4238	-
1	3	2	4	1	-	-	-	-	-
				2	-	-	0.4276	-	-
1	3	3	3	2	-	-0.8484	0.3959	-0.3111	-
				4	-	-1.2108	0.3853	-0.4440	-
1	3	3	4	2	-	-	-0.5257	-	-
				3	-	-	-	-	-

1	4	1	4	2	1.0	0.4286	-	-0.6667	-
				4	1.0	-0.3333	0.8889	0.5185	-
1	4	2	3	3	-	0.7377	-	0.2459	-
				4	-	-1.5119	1.0079	-0.5040	-
1	4	3	3	4	-	-	-0.5040	-	-
1	4	3	4	3	-	-0.6598	-	-0.7698	-
2	2	2	2	0	1.0	-1.6667	2.0000	0.4444	4.0000
				2	1.0	-0.3333	0.4000	0.0889	-
2	2	2	3	2	-	-	-0.3703	0.8641	-
2	2	2	4	2	-	-1.8856	0.5657	0.0629	-
2	2	3	2	0	-	-	2.4495	-	4.8990
				2	---	-	0.5237	-	-
2	2	3	4	2	-	-	-0.6928	0.4619	-
2	2	[illegible]	4	0	-	-1.8856	1.4142	-0.6285	2.8284
2	3	2	3	1	1.0	1.0000	-	-0.6667	-
				2	1.0	0.6191	0.3429	-0.4127	-
				3	1.0	0.0476	-	-0.0317	-
				4	1.0	-0.7143	1.1429	0.4762	-
2	3	2	4	1	-	-	-	-0.8944	-
				2	-	-	-0.5237	0.6110	-
2	3	3	[illegible]	2	-	-	-0.4849	0.4849	-
				[illegible]	-	-0.5-	-0.5714	0.5714	-
[illegible]				3	-	-0.7127	0.6414	-0.2613	-
				4	-	0.3518	-	0.3133	-
[illegible]				1	1.0	1.0000	-	-0.6667	-
				3	1.0	-1.6667	0.8000	4.4444	-
[illegible]				[illegible]	-	-	0.7407	-	-
				[illegible]	-	-	-0.9796	0.3266	-
[illegible]				[illegible]	-	-			
2	[illegible]	[illegible]	4						

APPENDIX - C

BRODY-MOSHINSKY BRACKETS AND INTERACTION MATRIX ELEMENTS

The interaction between two particles depends upon the distance between them. To evaluate matrix elements of such an interaction between jj-coupling wavefunctions they are to be written interms of the interaction matrix elements in the relative-centre of mass coordinate system (RCM). If the common central potential is harmonic oscillator kind then the transformation is simple. The harmonic oscillator potential has the property

C1

where r = It turns out that the wavefunctions in the RCM system are also products of harmonic oscillator functions of relative coordinates and centre of mass coordinates with the condition $2n_1 + l_1 + 2n_2 + l_2 = 2N + L + 2n + l$. The transformation from jj-coupling wavefunctions to those of RCM system are carried out by repeated use of the change of coupling transformations.

C2

C3

C4

and

C5

The transformation from LS-coupling wavefunctions to the RCM system is known as Moshinsky transformation and is written as

C6

where $n_1, l_1, n_2, l_2, L/ n, l, N, \Lambda, L \rangle$ are known as Brody-Moshinsky brackets or simply brackets. They vanish unless $2n_1 + l_1 + 2n_2 + l_2$ is equal to $2n + l + 2N + \Lambda$. Complete derivations of these brackets and needed recurrence relations are given by T. Moshinsky. An alternative approach by writting the wavefunctions in RCM system interms of is given by L. Baranger and K. Davies[11] which does not require the recurrence relations. The brackets are evaluated by using *

C7

C8

C9

C10

C11

C12

The brackets used in the calculations are presented in the text are calculated using the equations C7 to C12. They are given in the following table.

Oscillator Shell

$n_1 = 0$, $l_1 =$

$n_2 = 0$, $l_2 =$

n	l	N	L	
0	0	3	0	0.22361
0	2	1	2	0.31622
1	0	2	0	-0.59161
1	2	0	1	-0.31622
2	1	0	1	0.0
0	1	2	1	0.0
0	3	0	3	0.0
1	1	1	1	0.0
2	0	1	0	0.59161
3	0	0	0	-0.22361

= 1

0	1	2	1	0.44721
0	3	0	3	0.20000
1	2	0	2	0.0
0	2	1	2	0.0
1	1	1	1	-0.74833
2	1	0	1	0.44721

= 2

0	0	2		0.14142
0	1	2	1	0.0
0	2	1	2	0.52372
0	3	0	3	0.0
0	4	0	2	-0.04140
1	1	0	3	0.0
1	3	0	2	-0.52372
2	0	0	1	0.0
2	2	0	0	-0.14142
0	1	1	3	0.0
0	2	0	4	0.04140
0	2	2	0	-0.28983
0	3	1	1	0.0
1	0	1	2	-0.34641
1	2	1	1	0.0
1	2	1	0	0.34641
2	0	0	2	0.28983
2	2	0	0	-0.14142

= 3

0	3	0	4	0.0
0	3	2	0	0.40000

0	0	0	1	-0.51962	0	4	0	2	0.0
0	1	0	3	-0.51962	1	2	0	2	0.0
1	0	0	1	0.33730					

$\Lambda = 4$

0	0	1	4	0.19365	0	1	0	5	0.0
0	1	1	3	0.0	0	2	0	4	0.53452
0	2	1	2	-0.19021	0	3	0	3	0.0
0	3	1	1	0.0	0	4	0	2	-0.53452
0	4	1	0	0.37031	0	5	0	1	0.0
1	0	0	4	-0.37031	1	1	0	3	0.0
1	2	0	2	0.19021	1	3	0	1	0.0
1	4	0	0	-0.19365					

$\Lambda = 5$

0	1	0	5	0.61237	0	2	0	4	0.0
0	3	0	3	-0.50000	0	4	0	2	0.0
0	5	0	1	0.61237					

$\Lambda = 6$

0	0	0	6	0.55902	0	1	0	5	0.0
0	2	0	4	-0.43301	0	3	0	3	0.0
0	4	0	2	0.43301	0	5	0	1	0.0
0	6	0	0	-0.55902					

Oscillator Shell

$n_1 = 1$, $l_1 = 1$

$n_2 = 0$, $l_2 = 3$

$\Lambda = 2$

n	l	N	L	
0	0	2	2	0.25981
0	1	2	1	-0.27386
0	2	1	2	-0.24054
0	3	0	3	0.0
0	4	0	2	-0.20284
1	1	0	3	-0.25000
1	2	0	2	0.24054
1	3	0	1	-0.33541
1	4	0	1	0.27386

n	l	N	L	
0	1	1	3	0.33541
0	2	0	4	0.20284
0	2	2	0	0.05916
0	3	1	1	0.25000
1	0	1	2	-0.28284
1	1	1	1	0.0
1	2	1	0	0.28284
2	0	0	2	-0.05916
2	2	0	0	-0.25981

$\Lambda = 3$

0	1	1	3	0.33541
0	2	1	2	-0.29881
0	3	1	1	0.05000
1	1	0	3	0.05000
1	3	0	1	0.33541

0	2	0	4	0.40089
0	3	0	3	-0.51962
0	4	0	2	0.40089
1	2	0	2	-0.29881

$\Lambda = 4$

0	0	1	4	0.37081
0	1	1	3	-0.18634
0	2	1	2	-0.20702
0	3	1	1	0.25000
0	4	1	0	0.06455
1	0	0	4	-0.06455
1	2	0	2	0.20702
1	4	0	0	-0.37081

0	1	0	5	0.39087
0	2	0	4	-0.25588
0	3	0	3	0.0
0	4	0	2	0.25588
0	5	0	1	-0.39087
1	1	0	3	-0.25000
1	3	0	1	0.18634

The transformation from jj-coupling to [illegible] system [illegible] given in Chapters II and III. The transformation to relative states may be written as

C13

C14

if V does not depend upon [illegible] coordinates, i.e. [illegible] then we can write

C15

C16

(a) For pure central interaction : V(r)

C17

(b) For spin-spin interaction:

C18

(c) For spin-orbit interaction:

C19

(d) For tensor interaction:

C20

C21

where

One then uses

C22

We can also calculate the matrix elements of this interaction noting the property that they vanish in spin singlet states. We have

C23

C24

where

and L is relative orbital angular momentum and S is the total spin of the two particles. The integration can be carried out to get the matrix elements of S12. They are given by Roy and Nigam as follows:

L	J + 1	J	J - 1
J + 1	$-\frac{2(J+2)}{2J+1}$	0	$6\sqrt{\frac{J(J+1)}{2J+1}}$
J	0	2	0
J - 1	$6\sqrt{\frac{J(J+1)}{2J+1}}$	0	$\frac{-2(J-1)}{2J+1}$

C25

[illegible] is evaluated

[illegible]

C26

where [illegible]

The Talmi integral for Gaussian radial dependence $f(r)=\exp($ [illegible] are

[illegible]

C27

Talmi integrals for other usual radiat dependence are given by [illegible]shinsky.

We can also write

[illegible]

C28

where $I(nl) = I(nl)(l's'j)$ of equation C16 when the interaction is assumed to be purely central and diagonal in nl. This is done in Chapter II by assuming $I_{n\ell}$ as parameters without explicit reference to the shape of the interaction. The required $F(nl)$ are calculated and tabulated below. The allowed combinations of (nl) are only 16 for the fp shell and are

(00) , (10), (20) , (30) ; (01) , (11) , (21) ;
(02) , (12), (22) ; (03) , (13) ; (04) , (14) ;
(05) ; and (06)

	(0,0)	(1,0)	(2,0)	(3,0)	(0,1)	(1,1)	(2,1)	(3,1)	(0,2)	(1,2)
BIAS:			1F7/2			1F7/1			1F5/1	
0	-0.02857	0.28000	0.00000	-0.02857	0.08571	0.24000	-0.05714	0.07429	-0.05714	0.00000
2	0.01020	0.00122	0.04835	0.0	0.09184	0.00734	-0.07824	0.18247	-0.00000	0.00000
4	0.01577	0.05011	0.0	0.0	0.10258	0.14163	0.0	0.17439	-0.15000	0.00000
6	0.00464	0.0	0.0	0.0	0.00155	0.0	0.0	0.00747	0.0	0.00000
BIAS:			1F7/1			1F1/2			1F1/1	
2	0.02000	0.07482	-0.01800	0.0	0.01000	-0.00001	0.0	-0.17001	-0.00247	0.00000
4	0.03000	0.01001	0.0	0.0	0.12121	-0.00002	0.0	-0.00005	0.02710	0.0
BIAS:			1F7/2			2F3/2			1F5/2	
2	0.07714	0.00143	0.00400	0.0	0.15857	0.04700	0.05714	0.11714	0.10700	0.07714
3	0.0	0.0	0.0	0.0	0.14200	0.05429	0.00133	0.21429	0.07000	0.0
4	0.00547	0.00100	0.0	0.0	0.17357	0.00052	0.0	0.05133	0.10041	0.0
5	0.0	0.0	0.0	0.0	0.57400	0.12000	0.0	0.0	0.0	0.0
BIAS:			1F7/2			1F7/3			1F7/1	
0	-0.00000	-0.13000	0.02000	0.0	-0.05000	-0.13700	-0.05000	0.05000	0.00000	0.00000
1	0.01472	0.03004	0.0	0.0	-0.05042	-0.13770	0.0	0.12504	0.01021	0.0
6	0.10000	0.0	0.0	0.0	-0.18182	0.0	0.0	0.00561	0.0	0.0
BIAS:			1F7/2			1F5/2			1F7/2	
1	0.0	0.0	0.0	0.0	0.20000	0.56000	0.25000	0.0	0.0	0.0
2	0.00245	0.01460	0.01029	0.0	0.15918	0.39673	0.11020	0.04403	0.01525	0.00245
3	0.0	0.0	0.0	0.0	0.14000	0.26000	0.0	0.0	0.0	0.0
4	0.01531	0.05672	0.0	0.0	0.15775	0.10103	0.0	0.13265	0.01003	0.0
5	0.0	0.0	0.0	0.0	0.37500	0.0	0.0	0.0	0.0	0.0
6	0.26785	0.0	0.0	0.0	0.05357	0.0	0.0	0.10071	0.0	0.0
BIAS:			1F7/2			2F3/2			1F7/2	
2	0.01374	0.03005	-0.00641	0.0	0.02291	-0.00458	0.0	-0.03040	-0.01047	0.01374
3	0.0	0.0	0.0	0.0	0.00943	-0.01389	0.0	0.0	0.0	0.0
4	0.03105	0.01055	0.0	0.0	0.00497	0.01000	0.0	-0.04320	0.01000	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

APPENDIX - B

MATRIX ELEMENTS OF OPERATORS

The calculations in Nuclear Shell model include diagonalization of Hamiltonian and evaluation of properties such as B(E2) rates. The Hamiltonian contains diagonal one body part and two body interaction. These calculations in shell require evaluation of matrix elements of operators between n body wavefunctions. The wavefunctions of identical particles used in the present work are of the kind $|j^n \alpha J\rangle$ i.e. $|j^{n-1}(\alpha_1 J_1) j' J\rangle$ and span the model space, where J & J1 are angular momenta and α and α_1 are additional quantum numbers such as seniority of completely antisymmetric wavefunctions of n and (n - 1) particle states. These basis states $|j^n \alpha J\rangle$ i.e. $|j^{n-1}(\alpha, J_1) j' J\rangle$ are constructed by coupling single particle wavefunctions to antisymmetric and normalized (n - 1) particle wavefunctions, antisymmetrization and normalization. Matrix elements of scalar two body interaction and tensor E2 transition operator between such states are given below, without derivation, for the purpose of record.

(a) Scalar two body interaction

$$\langle j^n \alpha J | \sum_{i<k} \vartheta_{ik} | j^n \alpha' J\rangle =$$

$$= n \cdot \frac{n-1}{2} \cdot \sum_{\alpha_2 J_2 J'} [j^{n-2}(\alpha_2 J_2) j^2(J') J \} j^n \alpha J] \, [j^{n-2}(\alpha_2 J_2) j^2(J') J \} j^n \alpha' J] \langle j^2 J' | \vartheta | j^2 J'\rangle \quad \text{B1}$$

$$\langle j^n \alpha J \| \sum_i \vartheta_{ik} \| j^{n-1}(\alpha_1 J_1)\, j' J \rangle =$$

$$(n-1)\sqrt{\frac{n}{2}} \sum' [j^{n-2}(\alpha_1 J_1)\, j^2(J^2)\, J \} j^n \alpha J][j^{n-2}(\alpha_1 J_1)\, j J_1 \} j^{n-1} \alpha_1 J_1] *$$

$$(-1)^{j'+J+J_2+1} [(2J_1+1)(2J'+1)]^{1/2} \begin{Bmatrix} J_2 & j & J_1 \\ j' & J & J' \end{Bmatrix} \langle j^2 J' | \vartheta | j^2 J' \rangle \qquad \text{D2}$$

$$\langle j^{n-1}(\alpha_1 J_1)\, j' J \| \sum \vartheta_{ik} \| j^{n-1}(\alpha_1' J_1')\, j'' J \rangle =$$

$$(n-1) \sum \mathrm{CFP}(J_1, J_2) \cdot \mathrm{CFP}(J_1' J_2) * (-1)^{j'+j''+1} [J_1 J_1']^{1/2} \cdot$$

$$\sum [J'] \begin{Bmatrix} J_2 & j & J_1 \\ j' & J & J' \end{Bmatrix} \begin{Bmatrix} J_2 & j & J_1' \\ j'' & J & J' \end{Bmatrix} \langle j j' J' | \vartheta | j j'' J' \rangle$$

$$+ \delta_{j'j''} \delta_{\alpha_1 \alpha_1'} \delta_{J_1 J_1'} \langle j^{n-1} \alpha_1 J_1 | \sum_i \vartheta_{ik} | j^{n-1} \alpha_1' J_1' \rangle \qquad \text{D3}$$

(b) Tensor one body operators

The Wigner - Eckart theorem states

$$\langle JM | T_k^K | J'M' \rangle = (-1)^{J-M} \begin{pmatrix} J & K & J' \\ -M & k & M' \end{pmatrix} (J \| T^{(K)} \| J') \qquad \text{D4}$$

The reduced matrix element on the right hand side is the required quantity independent of geometry and includes all the specific physical information contained in the tensor $T^{(K)}$.

If $T_k^K = \sum_i f_k^{K}(i)$ is a single body tensor operator of degree K the reduced matrix elements between n-body wave-functions are expressed interms of those between one body functions.

$$\langle j^n \alpha J \| \sum_i f_k^K(i) \| j^n \alpha' J' \rangle = n \langle j \| f^K \| j \rangle [J J']^{1/2} *$$

$$\sum \mathrm{CFP}(J, J_1)\, \mathrm{CFP}(J', J_1) (-1)^{j+J_1+J+K} \begin{Bmatrix} j & J & J_1 \\ J' & j & K \end{Bmatrix} \qquad \text{D5}$$

Similarly

$$\left(j^{n}\alpha_1 J_1 \| \sum_i f_i^{(n)} \| j^{n-1}(\alpha_2 J_2) j' J'\right) = \sqrt{n[J,J']}\,(j\|f^{n}\|j') \times$$

$$\text{coef}\,P(J,J_1)(-1)^{J_1+j'+J+K}\begin{Bmatrix} j & j' & j \\ K & J & J_1 \end{Bmatrix} \qquad \text{D6}$$

and

$$\left(j^{n-1}(\alpha_1 J_1) j' J \| \sum_i f_i^{(n)} \| j^{n}(\alpha_2 J_2) j'' J\right) =$$

$$(-1)^{J_1+j'+J'+K}\,\delta_{j'j''}\,[J J']^{1/2}\begin{Bmatrix} J_1 & J_2 & K \\ J' & J & j' \end{Bmatrix} \times$$

$$\left(j^{n-1}\alpha_1 J_1 \| \sum_i f_i^{(K)} \| j^{n-1}\alpha_2 J_2\right) \cdot$$

$$+\ (-1)^{J_1+j''+J+K}\,\delta_{\alpha_1\alpha_2}\,\delta_{J_1J_2}\,[J J']^{1/2}\begin{Bmatrix} j' & j'' & K \\ J' & J & J_1 \end{Bmatrix}(j'\|f^{(K)}\|j'') \qquad \text{D7}$$

In the preceeding Appendix evaluation of matrix elements are given. Here some more sophisticated techniques are given for completeness which are useful perticularly when the form of the interaction is not known. A very well known procedure is to expand the simple local two body interaction in terms of Legendre Polynomials of $\cos(\omega_{12})$ where ω_{12} is angle between the two radius vectors.

$$v(|\bar{r}_1-\bar{r}_2|) = \sum_K' v_K(r_1,r_2)\,P_K(\cos\omega_{12}) \qquad \text{D8}$$

where $$v_K(r_1,r_2) = \frac{2K+1}{2}\int v(|\bar{r}_1-\bar{r}_2|)\,P_K(\cos\omega_{12})\,d(\cos\omega_{12}) \qquad \text{D9}$$

Using the addition theorem of spherical harmonics we can also write

$$v(|\bar{r}_1-\bar{r}_2|) = \sum_K' v_K(r_1,r_2)\,C_k^{K*}(\Omega_1)\,C_k^{K}(\Omega_2) \qquad \text{D10}$$

where $C^{k}_{k}(\Omega) = \sqrt{4\pi/(2k+1)}\, Y_{kk}(\Omega)$ D11

The matrix elements may be evaluated very easily and the radial integrals may be treated as parameters.

In more general case when the interaction depends upon spin coordinates also we can write

$$V_{12} = \sum \vartheta_{ss',kk',\lambda}(r_1, r_2)\left(T^{(s,k)\lambda}(1) \cdot \bar{T}^{(s'k')\lambda}(2)\right) \quad \text{D12}$$

where $T^{(s,k)\lambda}$ is an irreducible tensor of degree r built out of an irreducible tensor of degree s of spin corrdinates and an irreducible tensor of degree k of space coordinates,

$$T^{(s,k)\lambda} = \left[\Sigma^{(s)} \times U^{(k)}\right]^{(\lambda)} \quad \text{D13}$$

the tensors $\Sigma^{(s)}$ are $\Sigma^{(0)} = 1$ and $\Sigma^{(1)} = \vec{\sigma}$ while the tensor $U^{(k)}$ which depends upon the space coordinates only must be proportional to $C^{(k)}$.

The matrix elements of such an interaction can be written in jj-coupling representation as

$$\langle j_1 j_2 JM | V_{12} | j_1' j_2' JM \rangle = \sum f_\lambda F^\lambda \quad \text{D14}$$

where $f_\lambda = (-1)^{j_1' + j_2 + J}(j_1 \| T^{(s)}_{(k)} \| j_1')(j_2 \| T^{(s)}_{(k)} \| j_2') *$

$$\begin{Bmatrix} j_1 & j_2 & J \\ j_1' & j_2' & \lambda \end{Bmatrix} \delta_{JJ'} \delta_{MM'} \quad \text{D15}$$

and $F^\lambda = \int R_{n_1 l_1}(r_1) R_{n_2 l_2}(r_2) \vartheta^{(r_1, r_2)}_{ss'kk'\lambda} R_{n_3 l_3}(r_1) R_{n_4 l_4}(r_2)\, dr_1\, dr_2$ D16

[illegible] are [illegible] for convenience [illegible]. The summation over r extends over the allowed values $|j_1 - j_1'| \leq k \leq (j_1 + j_1')$, $|j_2 - j_2'| \leq k \leq (j_2 + j_2')$, and for each value of r, s and s' take values 0 and 1 and t and t' take values $\pm \frac{1}{2}$, r only. The reduced matrix element in the above expression are

$$\left(j_1 \| T^{(s,k)_r} \| j_1'\right) = [k\, j_1\, j_1']^{1/2} \begin{Bmatrix} l_1 & s_1 & j_1 \\ l_1' & s_1' & j_1' \\ \delta & k & r \end{Bmatrix} *$$

$$\left(l_1 \| \Sigma_1^{(s)} \| l_1'\right) \left(s_1 \| U^{(k)} \| s_1'\right) \qquad b17$$

Note that, due to the properties of 9-J symbols, the reduced matrix elements vanish unless $(l_1 + l_1' + j_1 + j_1' + s + k + r)$ is even further restricting the number of terms to be computed.

The [illegible] depend essentially on geometry and not on the shape of the interaction and therefore they can be evaluated separately. The quantities F^r however depend upon the shape of the interaction for different states as well as for different cross terms. One can impose some restrictions on them and treat them as parameters to fit the experimental data.

Some times it may be convenient to couple spin parts and space parts separately, perticularly when the LS-coupling wavefunctions are used or when the interaction is assumed to contain scalar forces only. This can be done by applying change of coupling transformation. In such case we may write

$$V_{12}(\lambda_1 \lambda_2 l) = \sum v_{ss'kk'l}^{(\lambda_1 \lambda_2)} \left(T^{(ss')k} * T^{(kk')k} \right) \qquad \text{D18}$$

For example the scalar forces are

$$V_c(12) = f_c(r) + (\sigma_1 \cdot \sigma_2) f(r) \qquad \text{D19}$$

the vector forces are

$$V_l(12) = \left([\sigma_1 \times \sigma_2]^{(1)} \cdot \hat{r} \right) f_l(r) \qquad \text{D20}$$

and the tensor forces are

$$V_t(12) = \left([\sigma_1 \times \sigma_2]^{(2)} \cdot [\hat{r} \times \hat{r}]^{(2)} \right) \qquad \text{D21}$$

The change of coupling transformation needed is

$$\left[\left[\Sigma_{(1)}^{(s)} \times U_{(1)}^{(k)} \right]^{(\lambda)} \times \left[\Sigma_{(2)}^{(s')} \times U_{(2)}^{(k')} \right]^{(\lambda')} \right]^{I}$$

$$= \sum \left[\lambda \lambda' t t' \right]^{1/2} \begin{Bmatrix} s & k & \lambda \\ s' & k' & \lambda' \\ t & t' & I \end{Bmatrix} * \qquad \text{D22}$$

$$\left[\left[\Sigma_{(1)}^{(s)} \times \Sigma_{(2)}^{(s')} \right]^{(t)} \times \left[U_{(1)}^{(k)} \times U_{(2)}^{(k')} \right]^{(t')} \right]^{I}$$

Consider the δ force as an example for the tensor expansion.

$$\delta(\vec{\lambda}_1 - \vec{\lambda}_2) = \sum_k \frac{2k+1}{4\pi} \frac{\delta(\lambda_1 - \lambda_2)}{\lambda_1 \lambda_2} P_k(\cos \omega_{12}) \qquad \text{D23}$$

$$\text{so that} \quad v_k(\lambda_1 \lambda_2) = \frac{2k+1}{4\pi} \frac{\delta(\lambda_1 - \lambda_2)}{\lambda_1 \lambda_2} \qquad \text{D24}$$

$$\text{and} \quad F^k = \frac{2k+1}{4\pi} \int \frac{1}{r^2} R_{n_1 l_1} R_{n_2 l_2} R_{n_3 l_3} R_{n_4 l_4} \, dr \qquad \text{D25}$$

$$= \frac{2k+1}{4\pi} F_0$$

and therefore the δ force involves only one parameter which is usually absorbed in the strength.

The most general form of scalar, charge independent local two dbody interaction which does not depend upon velocities can be written as

$$V_{12} = V_c(r) + P_\sigma V_\sigma(r) + P_\tau V_\tau(r) + P_\sigma P_\tau V_{\sigma\tau}(r)$$
$$+ \left([\sigma_1 \times \sigma_2]^{(1)} \cdot \hat{r}\right) V_1(r) + P_\tau \left([\sigma_1 \times \sigma_2]^{(1)} \cdot \hat{r}\right) V_{\tau_1}(r)$$
$$+ \left([\sigma_1 \times \sigma_2]^{(1)} \cdot [\hat{r} \times \hat{r}]^{(1)}\right) V_2(r) + P_\tau \left([\sigma_1 \times \sigma_2]^{(1)} \cdot [\hat{r} \times \hat{r}]^{(2)}\right) V_{\tau_2}(r) \qquad \text{D26}$$

from which Wigner, Bartlet, Heisenberg and Majorana forces can be obtained as special cases. Charge dependence may be introduced by a factor $a + b(\tau_1 \cdot \tau_2)$.

APPENDIX - E

ELECTRIC QUDRUPOLE TRANSITIONS AND SPECTROSCOPIC FACTORS

Multipole mixing ratio is the ratio of reduced matrix elements of different multipoles contributing to a gamma transition between well defined nuclear states, is a valuable source of information about nuclear states. The multipole transition probabilities predicted in a nuclear model provide useful information about the validity of the model. The transition probability given by perturbation theory for the quantum mechanical treatment o. radiating systems. The transition probability per unit time is

$$T(L) = \frac{8\pi}{\hbar c}\,\frac{e^2}{c}\,\frac{(L+1)}{L\left[(2L+1)!!\right]^2}\,k^{2L+1}\,B(L) \qquad E1$$

where B(L) is the reduced transition rate and L, the degree of multipole radiation, is the total (orbital + intrinsic) angular momentum of accompanying radiation. This formula is valid in the long wave approximation, the wave length of the radiation is long compared to the size of the radiating system : $1/k = c/\omega \gg R$. Details of derivation may be found in standard books on quantum mechanics or quantum field theory, or Blatt and Weiskopf, Roy and Nigam, and Rose and Brink.

The reduced transition rate for electric multipole transition is given by

$$B(el, L) = \sum_{M M_f} |\langle J_f M_f | \tfrac{1}{2} \int \rho(\vec{r}) r^L Y_{LM}(\Omega) d^3r | J_i M_i \rangle|^2$$

E2

For a system of point charges we get

$$B(el, L) = \sum_{M M_f} |\langle J_f M_f | \tfrac{1}{2} \sum_i e_i r_i^L Y_{LM}(\Omega_i) | J_i M_i \rangle|^2$$

$$= \frac{1}{2J_i + 1} \left(J_f \| \tfrac{1}{2} \sum_i e_i r_i^L Y_L(\Omega_i) \| J_i \right)^2$$

E3

The wavefunctions of nuclear states in the mixed configuration shell model may be written as $\sum a_p \psi_{pJ} = \psi_J$ we get [114, 115]

$$B(el, L) = \frac{1}{2J_i + 1} \left[\sum a_p^i a_q^f \left(\psi_{q J_f} \| \tfrac{1}{2} \sum_i f^L(i) \| \psi_{p J_i} \right) \right]^2$$

E4

where $f_M^L(i) = e_i r_i^L Y_{LM}(\Omega_i)$

E5

The reduced matrix elements between n-particle wave functions are evaluated using the formulae given in Apprendix - D. The single particle matrix elements are

$$\left(n_f l_f j_f \| r^L Y_L \| n_i l_i j_i \right)$$

$$= \langle r^L \rangle (-1)^{j_f + \frac{1}{2}} \frac{[L j_i j_f l_i l_f]^{\frac{1}{2}}}{4\pi} \cdot \frac{1}{2} \left[1 + (-1)^{l_i + l_f + L} \right] \times \begin{pmatrix} l_i & l_f & L \\ 0 & 0 & 0 \end{pmatrix} \begin{Bmatrix} j_i & j_f & L \\ l_f & l_i & \frac{1}{2} \end{Bmatrix}$$

E6

where $\langle r^L \rangle = \int R_{n_f l_f} r^L R_{n_i l_i} dr$

E7

and $[abc \ldots] = (2a + 1)(2b + 1) \ldots\ldots$

In the case of electric quadrupole transition L = 2

and we have

$$\langle n l \mid r^2 \mid n l \rangle = (2n + l + 3/2)(1/2\nu)$$

$$\langle n L \mid r^2 \mid n+1\ l-2 \rangle = -2\sqrt{(n+1)(n+l+1/2)}(1/2\nu)$$

The value of ν has been obtained by I. Talmi. [25] Accordingly

$$(\hbar/2\nu) = 10.44$$

the B_{ij} matrices calculated between basis states for the transitions from ground state ($J = 7/2$) of 3 particles (51V) to the states $J_f = 3/2$, $5/2$, $9/2$ and $11/2$ are given in the tables.

The single particle transfer reactions are simple direct reactions in which single particle states are excited and therefore they provide valuable information about single particle structure of nuclear states. They are very much useful for shell model studies. For such reactions the differential cross section and the reduced width contain a factor which is a measure of the probability that the nucleons in the initial nuclens will find themselves in an arrangement corresponding to the final state of the resulting nuclens. [12] This factor is called spectroscopic factor and depends upon the wavefunctions of nuclear states involved and therefore provides a useful basis for comparing experiment and predictions from a nuclear model.

The measured differential cross-sections for pick up and stripping reactions are respectively $\frac{d\sigma}{d\Omega} = K C^2 S$ and

$\frac{d\sigma}{d\omega} = \left(\frac{C_k}{[J]}\right) N C^2 S$, where N is normalisation factor, C is Clebsch-Gordon coefficient for isospin coupling and S is the spectroscopic factor. The spectroscopic factor S is given by, for pure shell model states,

$$S = A\,[\,I(j_k)\,]^2 \qquad \text{E9}$$

where A is the number of nucleons in the target or final nucleus which ever is larger and j_k is the angular momentum of the transfered nucleon in a single nucleon transfer. The overlap integral, I, is given by

$$I(j_k) = \langle A\,J \mid (A-1)\,J_1 ;\, j_k\, J \rangle \qquad \text{E10}$$

Calculation of these overlap integrals depends upon the model wavefunctions. The wavefunctions in the mixed configuration shell model are

$$\psi(A\,J) = a \mid j^A \alpha\, J \rangle + \sum_1 b_{\alpha_1 J_1} \mid j^{A-1} (\alpha_1 J_1)\, j\, J \rangle \qquad \text{E11}$$

$$\psi(A-1, J_1') = a' \mid j^{A-1} \alpha_1' J_1' \rangle + \sum_1 b'_{\alpha_1' J_1'} \mid j^{A-2} (\alpha_1' J_1')\, j'\, J \rangle \qquad \text{E12}$$

Now the overlap integral is to be calculated after coupling a single nucleon wavefunction to the (A - 1) particle function. We can now write for convenience the A-particle wavefunctions as

$$\psi(A\,J) = a \sum_1 CFP(J, J1) \mid j^{A-1}(\alpha_1 J_1)\, j\, J \rangle + \sum_1 b_{\alpha_1 J_1} \sum_1 CFP(J_1 J_2) \mid j^{A-2}(\alpha_1 J_2)\, j\,(J1)\, j'\, J \rangle \qquad \text{E13}$$

$$\psi(A-1\ J1';\ j_k\ J) = a' \mid j^{A-1}(\alpha_1' J_1')\, j_k\, J \rangle + \sum b'_{\alpha_2' J_2'} \mid j^{A-2}(\alpha_1 J_2)\, J_k'(J_1')\, j_k\, J \rangle \qquad \text{E14}$$

where j_k is the angular momentum of the transfered nucleon. Now it can be seen that the overlap integral vanishes unless $j_k = j$ or j' it can be seen that the overlap integral vanishes unless $j_k = j$ or j'

the single particle is in j orbit, $J_T = \frac{1}{2}$ and

$$|j^{n-1}(\alpha_1 J_1)\, j\,(J_2)\, j'\, J\rangle = \sum (-1)^{J_1+j+j'-J_2} [J_1\ J_2]^{\frac{1}{2}} \times$$

$$\begin{Bmatrix} J_1 & j & J_2 \\ J & j' & J_1 \end{Bmatrix} |j^{n+1}(\alpha_1 J_1)\, j'(J_2)\, j\, J\rangle$$

and we get the overlap integral immediately

E15

$$I(j) = a\, a'\, \mathbf{C}FP(J J_1')\sqrt{A} + \sum_i b_{\alpha_1 J_1} b'_{\alpha_1 J_2} CFP(J_1 J_2) \times$$

$$(-1)^{J_1+j+j'+J_1'} [J_1\ J_1']^{\frac{1}{2}} \begin{Bmatrix} J_2 & j & J_1 \\ J & j' & J_1' \end{Bmatrix} \sqrt{(A-1)}$$

E16

when the transfered nucleon is in j' orbit, the overlap integral is simple and we have

$$I(j') = a'\, b_{\alpha_1' J_1'}$$

E17

The spectroscopic factors presented in the text do not contain the isospin coupling co-efficient $C = \langle T_c\, M_{T_c}, \frac{1}{2}, M_T - M_{T_c} | T\, M_T \rangle$ as it is customary to give C^2S in experimental results. The wavefunctions of configurations (j^n) are eigen functions of the operator T^2 and are said to have fixed isospin while those of $(j^{n-1}\ j')$ are mixtures of eigen functions of T^2 with different values of T. The Clebsch-Gordon coefficients for the transfers between pure configurations $(j^n) \rightleftharpoons (j^{n-1} * j)$ are simply equal to one. The problem comes only when we are dealing with mixed configurations and for j' transfers. The wavefunctions used in the text are not pure isospin functions and the impurities are small. Also the wavefunctions of lowest states are found to be dominated by pure (j^n) configurations. On the otherhand the experimental results contain large errors so that the experimental values C^2S may safely be compared with the calculated values of S.

E2 TRANSITION MATRIX

N = 2

$J_i = 2$ $J_f = 0$

-4.90990	-2.12131	0.85041
0.0	0.0	0.0
0.0	0.0	0.0

$J_i = 4$ $J_f = 2$

-6.57985	-0.82992	0.73564
-2.13211	-4.99434	1.58917
-1.51955	-1.27774	-5.94618

$J_i = 6$ $J_f = 4$

-5.33762	0.0	0.30816
-5.92451	0.0	2.28034
-1.90961	0.0	-3.85887

N = 3

$J_i = 3.5$ $J_f = 1.5$

-4.06573	0.0	-0.09718	2.79157	0.0
-5.19615	0.0	-6.21059	0.0	0.0
2.53561	0.0	-1.16023	-4.97390	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0

$J_f = 2.5$

8.40939	0.0	-0.80403	-1.39980	0.0
0.0	0.0	0.0	0.0	0.0
-2.71068	0.0	-7.41427	-3.54488	0.0
-2.84282	0.0	0.24270	2.78842	0.0
0.0	0.0	0.0	0.0	0.0

$J_f = 4.5$

4.46057	0.0	-2.13257	6.68218	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
-2.01529	0.0	3.21063	2.00391	0.0
-5.32552	0.0	0.0	1.66213	0.0

$J_f = 5.5$

-8.01714	0.0	-6.70761	3.03605	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.40451	0.0	-7.59775	-3.81630	0.0
3.26120	0.0	0.0	-4.74994	0.0

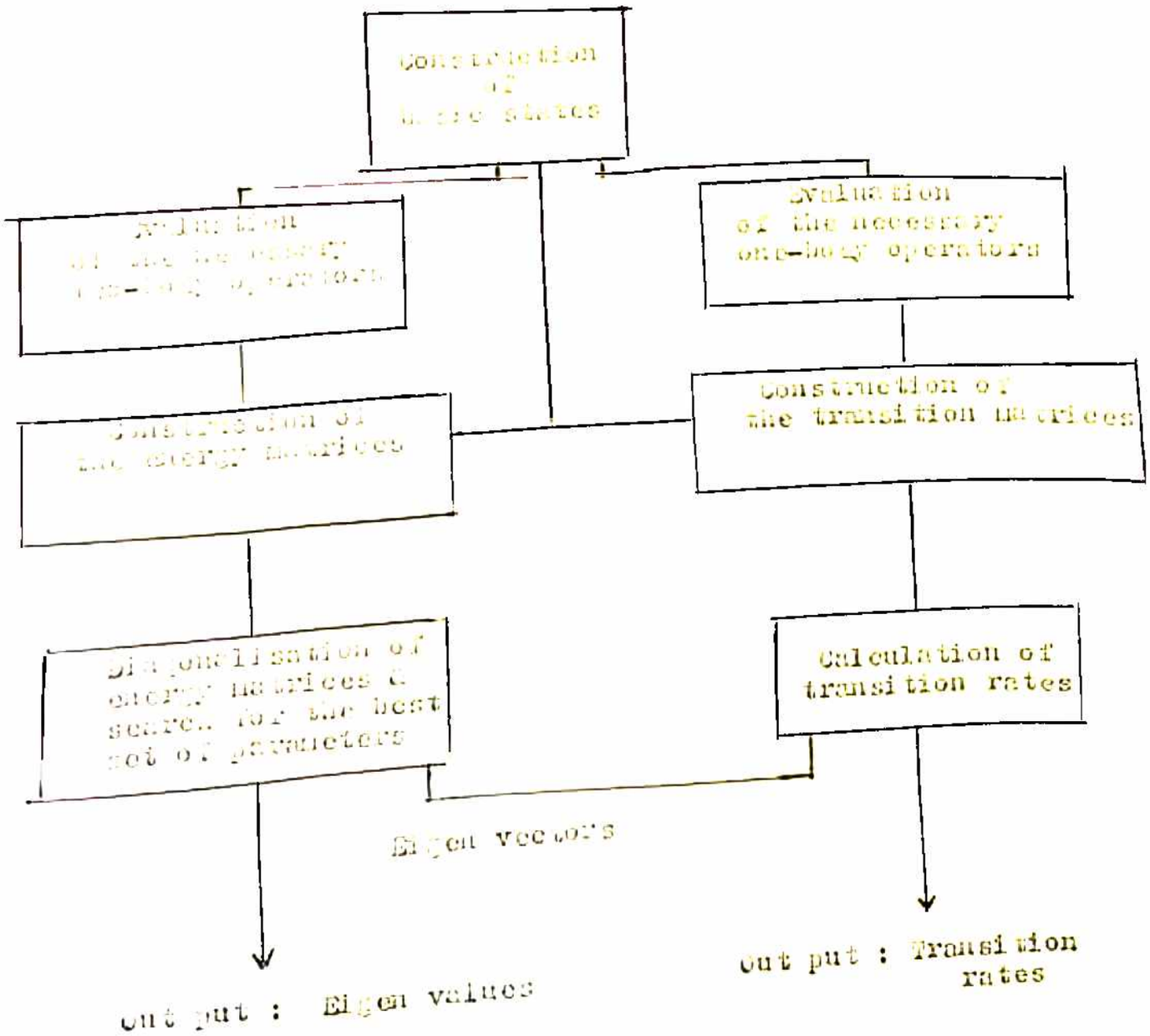
APPENDIX - F

AUTOMATED CALCULATIONS

The major problem in nuclear shell model calculations is the large number of allowed states. Even in a calculation with small severely restricted model space the amount of calculation is quite large. It includes calculation of matrix elements of required operators between all the basis states and construction and diagonalization of energy matrices for all angular momentum states of interest. In the following a brief description of the method employed in the present calculation is given.

Basically the shell model computations can be represented by the block diagram given in the following page.

In view of the limitations of the computer capabilities each block of the diagram is separately executed. The basis states are constructed by using coefficients of fractional parentage already evaluated for $j = 7/2$, since the model space in the present calculation does not contain general fp shell states $(1f7/2 ; 2p3/2 ; 1f5/2 ; 2p1/2)^n$ but only those of simple kind in which one particle raised to 2p3/2 or 1f5/2 orbit together with $(1f7/2)^n$. The evaluation of one-and two-body operators between basis states is performed as explained in other Appendices and are used in constructing the Hamiltonian. Since the parameters in the calculations are mostly the strengths of interaction components, corresponding interaction matrices are evaluated and



Flow chart for shell model calculations.

stored. The total interaction in the Hamiltonian is simply a linear combination of them. Using the single particle constribution, the Hamiltonian is diagonalised and the eigen values are compared to the required experimental results. The effective radius parameter and any other parameter of the interaction other than the strength are dealt with separately by repeating the complete process of function minimization for each set of values of such parameters.

There are several function minimization techniques. In the calculations presented in the text the Oak Ridge and Oxford method which depends upon first order Taylor expansion of theoretical quantities is employed.[113]

Let $\bar{P}\ (p_1 \ldots\ldots p_n)$ be the values of n parameters, and $f_\lambda(\bar{P})$, $\lambda = 1, 2, \ldots\ldots n$ are the n quantities calculated and to be compared with the experimental quantities $f_\lambda^{Exp.}$

Then the function to be minimised is

$$F(\bar{P}) = \sum \left(f_\lambda(\bar{P}) - f_\lambda^{Exp.} \right)^2 \omega_\lambda \qquad F1$$

where ω_λ is a suitable weight function, and is taken as $(\gamma_{\Delta f_\lambda^{Exp.}})^2$.

Now Taylor series expansions give

$$f_\lambda(\bar{P}) = f_\lambda(\bar{P}_0) + \sum_i \left(\partial f_\lambda / \partial p_i\right)_{P_0} \delta p_i \qquad F2$$

$$\left(\partial F / \partial p_i\right)_{P_0} = 2 \sum_i \left(f_\lambda(\bar{P}_0) - f_\lambda^{Exp.} \right) \omega_\lambda \left(\partial f_\lambda / \partial p_i \right)_{P_0} \qquad F3$$

$$\left(\partial^2 F / \partial p_i \partial p_j \right)_{P_0} = 2 \sum_i \omega_\lambda \left(\partial f_\lambda / \partial p_i \right)_{P_0} \left(\partial f_\lambda / \partial p_j \right)_{P_0} \qquad F4$$

A second order Taylor expansion and approximation to a quadratic surface $\bar{F}(\bar{P})$ we get , for small changes in the

parameters

$$\bar{F}(\bar{P}) = F(\bar{P}_0) + \sum_i (\partial F/\partial p_i)_{P_0} \delta p_i + \sum \left(\frac{\partial^2 F}{\partial p_i \partial p_j}\right)_{P_0} \delta p_i \, \delta p_j \qquad F5$$

where $\bar{P} = \bar{P}_0 + (\overline{\delta P})$

If $F(P)$ is minimum at P_0 we get,

$$\left[(\partial F/\partial p_i)_{P_0} + \sum_i (\partial^2 F / \partial p_i \partial p_j)_{P_0} \delta p_j \right] = 0 \qquad F6$$

$$\text{i.e.} \quad 2 \sum_i \left(f_\lambda(\bar{P}_0) - f_\lambda^{ex} \right) \omega_\lambda \left(\partial f_\lambda / \partial p_i \right)_{P_0} \qquad F7$$

$$+ 2 \sum_i \omega_\lambda \left(\partial f_\lambda / \partial p_i \right)_{P_0} \left(\partial f_\lambda / \partial p_j \right)_{P_0} \delta p_j = 0 \qquad F8$$

Now after performing the summation over we can rewrite these equations as a matrix equation

$$[A][B] = [C] \qquad F8$$

where $C_i = \sum_i (f_\lambda^{ex} - f_\lambda$ [illegible] $\qquad F9$

[illegible] $\qquad F10$

and B_i [illegible] $\qquad F11$

This equation may be easily solved for the best $(\overline{\delta P})$. Here lies the advantage of approximating the $F(\bar{P})$ to a quadratic surface and approximations used in Taylor expansions for $f_\lambda(P)$.

Since approximate quadratic χ surface is considered in the neighbourhood it is worthwhile to take a smaller $(\overline{\delta P})_0$, than given by $[B]$. Consider max $(B_i / P_i) = \phi_i$. The differential coefficients $($ [illegible] $/\partial p_i)$ are obtained as follows in actual computation.

[illegible] $\qquad F12$

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