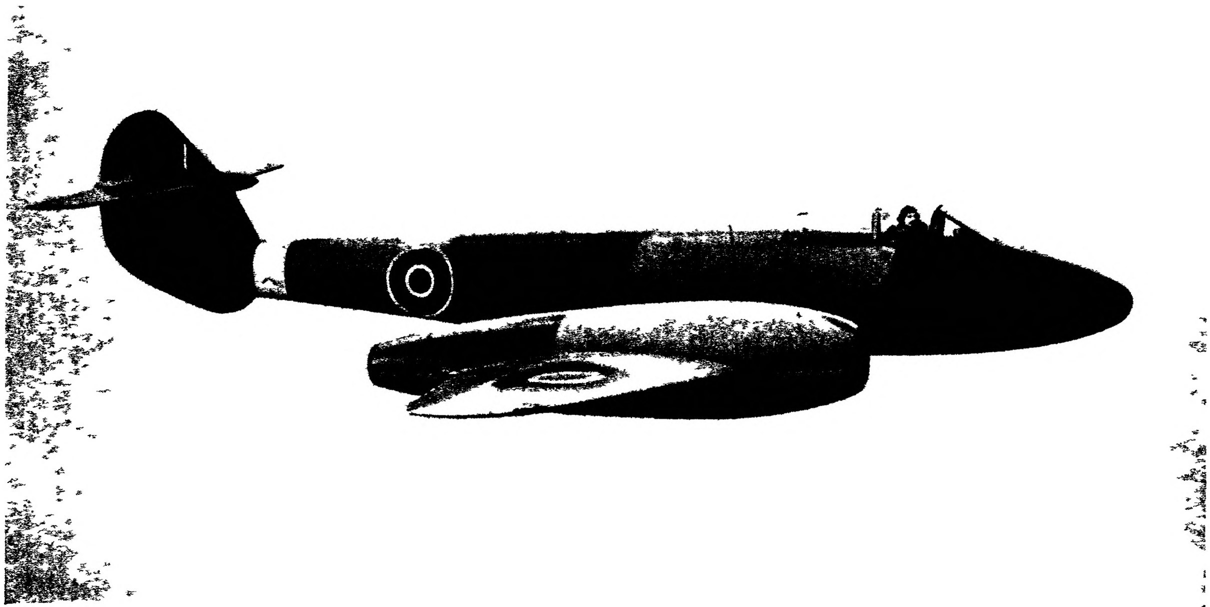
ELEMENTARY THEORY OF
GAS TURBINES AND
JET PROPULSION



THE GLOSTER METEOR MK IV JET PROPELLED FIGHTER, POWERED BY TWO ROLLS ROYCE WHITTLE GAS TURBINES MAXIMUM SPEED 606 M P H

ELEMENTARY THEORY OF GAS TURBINES AND JET PROPULSION

BY

J. G. KEENAN

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PREFACE

THIS book has been written to meet what I believe to be a genuine need for information on the elementary theory of the gas turbine. The prime mover itself, although not new in principle, has only been brought to a state of practical operation within the last few years. Owing to the impetus given to its development in recent years, it now bids fair to occupy a position on a par with the steam turbine and the reciprocating internal-combustion engine in the not-so-distant future. Since the engine is still in its infancy, gas-turbine practice is not yet sufficiently standardized to render possible the compilation of a comprehensive text-book on the subject. Any such attempt to lay down rigid laws would, indeed, be premature at the present stage of development. This book is, therefore, an endeavour to explain the basic principles of the gas turbine in a simple manner without, however, making it completely non-mathematical. For this reason the important conception of entropy has been omitted and the theory based entirely on pressure-volume considerations.

Much of the text has previously been published, in one form or another. The chapters on nozzles, impulse turbines, and reaction turbines have largely been derived from the mass of information available on steam turbines. The theory of the centrifugal compressor is well known in connexion with aero-engine superchargers and industrial blowers. General gas-turbine theory and test results have been discussed by Dr. Adolf Meyer and others before the Institution of Mechanical Engineers.

Several combinations of turbine and compressor types are possible and at this juncture it is difficult to indicate which may predominate in the future. Different applications may, in fact, demand different combinations. For this reason an attempt has been made to present the theory of each turbine type and compressor type in as impartial a manner as possible. Constructional details, too, may vary widely and those given should be regarded as possibilities rather than accepted practice. The latter term, indeed, as yet can hardly be applied to the gas turbine.

In view of the novelty of the subject and the scarcity of published experimental data, it is, perhaps, too much to hope that, in spite of careful checking, the text is entirely free from error. I would be glad to receive intimation of any such errors and suggestions for improvement. I humbly present this book to the technical public in the hope that, despite its many imperfections, it will be of some use to those to whom the gas turbine is still little more than a new and exciting name.

In conclusion, I would like to express my gratitude to Miss Esther Wharry for her work in the preparation of the manuscript and to F./Lt. W. H. Bond, R.A.F., and F./Lt. P. R. Heaton, B.Sc., R.A.F.V.R., for their patient and careful checking of text and drawings. Acknowledgements for the use of figures, diagrams, etc., are made below.

J. G. K.

ILLUSTRATIONS

THE frontispiece and Figs. 184, 185 A, 189, 190, and 193 appear as separate plates facing the title and pages 246, 247, 252, 253, and 256 respectively. Thanks are due to the following for permission to reproduce them.

ACKNOWLEDGEMENTS

The Gloster Aircraft Co. for the frontispiece; U.S. Air Forces (Official Photo) for Fig. 184; The Bristol Aeroplane Co., Ltd., for Fig. 185 A; and Messrs Brown Boveri, Ltd., for Figs. 189, 190, and 193.

Thanks are also due to the following authors, editors, and publishers for permission to reproduce figures in the text. H. Holzwarth, *The Gas Turbine* (Charles Griffin & Co., Ltd.), for Figs. 8 and 21; Flight Publishing Co., Ltd., for Figs. 32 and 182; *The Journal of the Royal Aeronautical Society*, vol. 48 (Sir Roy Fedden, 'Aircraft Power Plant—Past and Future'), for Figs. 169, 181, 185, and 186; *The Proceedings of the Institution of Mechanical Engineers*, 1928 ('Reports of the Steam Nozzles Research Committee'), for Fig. 103; vol. 141 (A. Meyer, 'The Combustion Gas Turbine—Its History, Development and Prospects'), for Figs. 59, 60, 163, 166, and 194; and vol. 150 (A. Meyer, 'First Gas Turbine Locomotive'), for Figs. 82, 148, 187, 188, and 191.

CONTENTS

I

INTRODUCTION TO THE GAS TURBINE

It is only within recent years that the gas turbine has attained a state of development whereby it has become a practical prime mover; yet more than a century and a half has passed since the first patent for a gas turbine was taken out. From time to time during these years some of the world's best scientific brains have been applied to the subject and considerable sums of money spent on experiment, with little or no success. The idea of obtaining continuous rotary motion directly from a combustible mixture, without the interposition of a steam boiler or reciprocating mechanisms, was an intriguing one, but one that defied practical realization, while its competitors, the reciprocating steam engine, the steam turbine, and the reciprocating internal-combustion engine advanced rapidly, each advance making the problem of the gas turbine more difficult since, to be a commercial success, the new prime mover would have to show some worth-while advantage over existing engines.

The two major factors which militated against the realization of the gas turbine were the lack of an efficient rotary air compressor and suitable materials, capable of running under high temperature and stress conditions, for the turbine blading. The development of the aeroplane, however, opened up new fields of aerodynamic knowledge which have now permitted the construction of rotary compressors of fairly high efficiency, and recent metallurgical research has yielded metals capable of withstanding the high temperatures and stresses to which the turbine blades are subjected. But a great deal of work still remains to be done before the gas turbine outstrips its competitors. Indeed, it cannot yet be said to threaten the displacement of the steam turbine or the reciprocating internal-combustion engine, except for certain highly specialized purposes for which the latter types of prime mover are unsuitable. Perhaps the most outstanding application of the gas turbine is in the jet-propelled aircraft, the comparatively rapid development of which has been forced by the need for higher, and still higher, speeds, although considerable development has been carried out on gas turbines for rail traction and on stationary plants for electricity generation.

The term 'gas turbine' is a wide one, and is used, in general, to distinguish from the steam turbine those turbines which use the hot products of combustion as their working fluid. Gas turbines can be divided into two distinct classes:

1. 'Combustion' turbines and
2. 'Explosion' turbines.

These two basic types are briefly described below.

The combustion turbine

The basic components of a combustion turbine are a compressor *A* (Fig. 1), a combustion chamber *C*, a turbine *D*, and

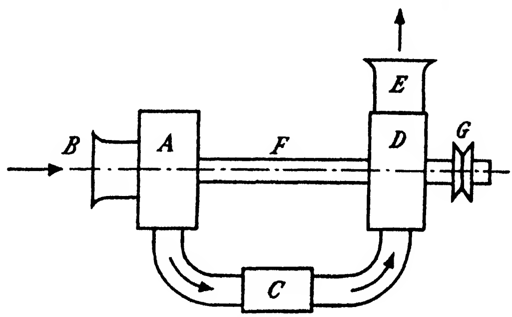


FIG. 1

a shaft *F* upon which the rotating elements of both compressor and turbine are mounted. Air is drawn into the compressor through the intake *B* and is delivered to the combustion chamber *C*, into which fuel is introduced and burned smoothly and continuously. The hot products of combustion pass into the turbine casing, where they do useful work in rotating the turbine rotor, and then pass to atmosphere through the exhaust *E*. The turbine drives the compressor, and the power for external use, such as driving a generator, is taken off at *G*.

The important feature of this type of machine is that the combustion is continuous and takes place at the delivery pressure of the compressor. There is no explosion, as in the cylinders of a reciprocating internal-combustion engine, and there is no rise in pressure in the combustion chamber.

Practically all modern gas turbines are of this type, the full name of which is the 'constant-pressure continuous-combustion turbine'. For ease of reference this may be abbreviated to 'combustion turbine'.

The explosion turbine

The essentials of an explosion turbine are very similar to those of a combustion turbine, namely, a compressor, one or more combustion chambers, and a turbine. Generally, several combustion chambers are employed and these are fitted with valves. The compressor, which is considerably smaller than that required for an equivalent combustion turbine, charges each chamber in turn with air, the charging sequence being controlled by the inlet valve gear. As each chamber is charged, its inlet valve is closed and a quantity of fuel is introduced and ignited. The resulting explosion causes a rapid rise in pressure, whereupon the outlet valve is opened, permitting the products of combustion to issue in a high-speed stream which is directed upon the blades of the turbine wheel. As the pressure in the chamber drops, the velocity of the jet decreases, and when the pressure reaches atmospheric the outlet valve is closed, the inlet valve opened, and the chamber is recharged with air. A single explosion chamber is illustrated diagrammatically in Fig. 2. Several of these chambers are arranged in a circle, concentric with the turbine shaft, and the valve gear is so designed that they are discharged consecutively, thus subjecting the wheel to a series of impulses, which propagate its rotation.

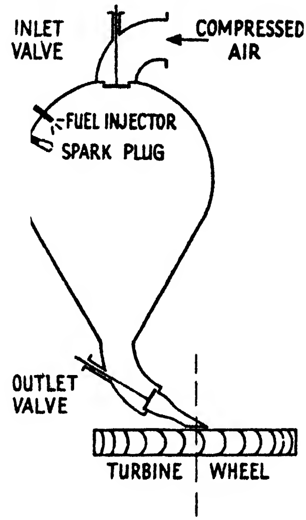


FIG. 2

In the early days of gas-turbine research, the problems of the combustion turbine presented so many difficulties that considerable attention was devoted to the explosion turbine which, despite its greater mechanical complication, offered better chances of success. A number of explosion turbines were built and run, but none of them attained their calculated thermal efficiencies and consequently were never produced in quantity. To-day attention has veered from the explosion to the combustion turbine which, due to advances in aerodynamical and metallurgical knowledge, is at last a practical prime mover and is in quantity production, for jet-propelled aircraft, in several countries. It has been successfully applied to at least one railway locomotive, the trials of which were extremely promising, and it would seem that production in this field will follow during the coming years.

The history of the gas turbine

It will, perhaps, be of some assistance to an understanding of the subject if we consider briefly the history of the gas turbine, since a knowledge of the various obstacles encountered in the past contributes to an appreciation of the relative importance of the major factors in successful gas turbine design.

The fact that heat, when applied to gases, causes them to expand, and that they may, thereby, be caused to act upon and to rotate a wheel for the production of useful power, is by no means new. In fact a number of propositions for the construction of crude turbines were put forward long before the invention of the piston-cylinder-crank combination. The very first steam-engine, Hero's of Alexandria, was a species of turbine, crude it is true, but nevertheless it did utilize the reaction principle which is so important in modern turbines.

The smokejack

Probably the first known gas turbine was the medieval 'Smokejack', the origin of which cannot be definitely traced, but has been attributed to Leonardo da Vinci. This apparatus is described in Bishop Wilkins's book *Mathematical Magick,* which was published in 1648. The form of this apparatus is illustrated in Fig. 3, taken from an engraving in the edition of 1680. Bishop Wilkins says:

> But there is a better invention to this purpose mentioned in Cardan (Cardan *De Variet. Rerum,* l. 12, C. 58), whereby a spit may be turned (without the help of weights) by the motion of the air that ascends the Chimney; and it may be useful for the roasting of many or great joynts: for as the fire must be increased according to the quantity of meat, so the force of the instrument will be augmented proportionably to the fire. In which contrivance there are these conveniences above the Jacks of ordinary use:
>
> 1. It makes little or no noise in the motion.
> 2. It needs no winding up, but will constantly move of itself, while there is any fire to rarifie the air.
> 3. It is much cheaper than the other instruments that are commonly used to this purpose. There being required with it only a pair of

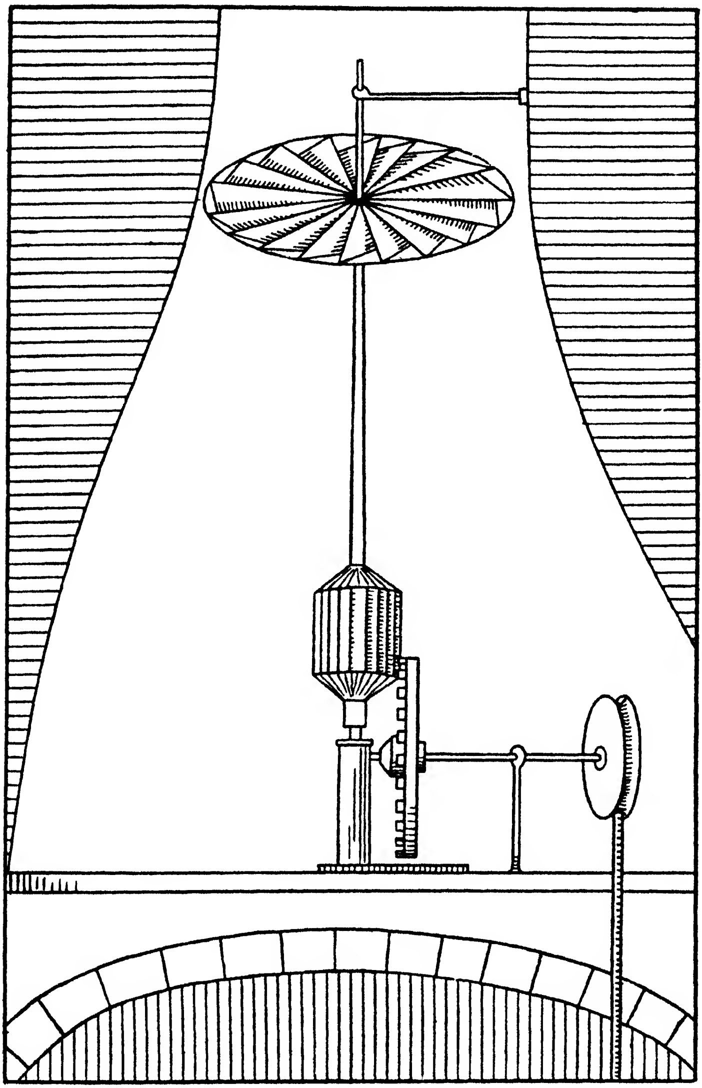


FIG. 3

sails, which must be placed in that part of the Chimney where it begins to be straightened, and one wheel, to the axis of which the spit line must be fastened.

These sails will always move both day and night, if there is but any fire under them, and sometimes though there be none. For if the air without be much colder than that within the room, then must this which is more warm and rarified, naturally ascend through the Chimney, to give place unto the more condensed and heavy, which does usually blow in at every chink and cranny, as experience shews.

Barber

There appears to have been no further effort to design a gas turbine for some 140 years, when we find that an Englishman named Barber in 1791 took out a patent for a gas turbine. His patent, British Patent No. 1833, anticipated in a remarkable way many of the developments in modern gas-turbine theory. The patent covers the distillation of the gas from wood, coal, or oil, its combustion, with the correct amount of air, in a combustion chamber, and the discharge of the final fluid through a nozzle on to the blades of a turbine wheel. Provision was also made for the injection of water into the combustion chamber to reduce the temperature of the gases acting upon the turbine wheel. When it is remembered that Murdock did not begin his experimental investigations into the manufacture of coal gas until 1792, one year after Barber's patent, and only made his results public in 1797, Barber's ideas seem all the more remarkable.

Although surprisingly complete in its theoretical conception, Barber's gas turbine involved constructional features, the problems of which were far from being solved in his day. Even to-day some of these problems, such as the production of suitable metals for the turbine blading, are a perpetual headache to the designer.

Tournaire

In 1853 the Frenchman, M. Tournaire, Ingénieur des Mines, gave a very remarkable paper before the Académie des Sciences, in which he described a scheme for a gas turbine put forward by M. Burdin. He said:

> Elastic fluids acquire enormous velocities, even under the influence of comparatively low pressures. In order to utilise these pressures advantageously upon simple wheels analogous to hydraulic turbines, it would be necessary to permit a rotative motion of extraordinary rapidity, and to use extremely minute orifices, even for a large expenditure of fluid. These difficulties may be avoided by causing the steam or gas to lose its pressure, either in a gradual and continuous manner, or by successive fractions, making it react several times upon the blades of conveniently arranged turbines.

We must attribute the origin of the researches which we have made upon this subject to the communications which M. Burdin, Ingénieur en Chef des Mines, and Membre Correspondant de l'Institut, has had the courtesy to make to us, and which go back to the close of 1847. M. Burdin, who was then engaged upon a machine operated by hot air, desired to discharge the compressed and heated fluid upon a series of turbines fixed upon the same axis. Each one of these wheels was placed in a closed chamber, the air to be delivered through injector nozzles and discharged at a very low velocity. The author proposed to compress the cold air by means of a series of blowers arranged in a similar manner. The idea of employing a number of successive turbines in order to utilise the tension of the fluid a number of times seemed to us a simple and fertile one; we perceived in it the means of applying the principle of reaction to steam and air engines.

However, despite his understanding of the basic principles and his appreciation of the possibilities, Tournaire appears to have made no attempt to initiate experimental work on the gas turbine, and it was left to later workers to put the ideas into practice.

Stolze

Notwithstanding the fact that a great deal of thought had evidently been expended upon the gas turbine, and several patents taken out, no one seems to have done anything practical towards actually getting a machine running, and proving or disproving the several theories expounded upon the subject until, in 1872, a Dr. Stolze of Charlottenburg, Berlin, applied for a patent licence from the Prussian Government for what he called a 'fire-turbine'. The design of Stolze's hot-air turbine followed very closely the principles and lay-out described by Tournaire in his paper before the Académie des Sciences in 1853, and probably owed much to this source. It worked on a constant-pressure cycle and was fitted with a multi-row axial-flow compressor. A sectional view of the turbine is shown in Fig. 4.

Air was drawn into the compressor at A, and passed through the blade-rings. The moving blades were fixed to the rotor B and the fixed blade-rings were attached to the casing C. No details of the pressure-ratio developed are available, but in view of the poor state of aerodynamic knowledge at the time, it is unlikely that it was more than about one and one-half atmospheres. The air left the compressor and entered the heating chamber D, which was annular, where it was heated by the burning of fuel in the duct which surrounded *D*. The heated air passed into the blade-rings of the turbine, the rotor of which was on the same shaft as the rotor of the compressor. The exhaust from the turbine passed down the duct *E*, in which was situated a butterfly-valve *F*, so that control of the power output would seem to have been by regulation of the back-pressure at the exhaust side of the turbine, thus altering the pressure drop through the blade-rings.

From the evidence available it would not appear that Stolze's turbine gave any useful results. If it worked at all, it certainly gave very little shaft power output. The useful shaft energy was, therefore, practically zero, and the overall efficiency almost nil. However, the fact that such a machine was actually built and incorporated most of the basic essentials of the modern combustion turbine, marked a definite step forward in the development of this most desirable prime mover, despite its failure as an immediately useful engine.

Perhaps the greatest difficulty in the construction of a successful combustion turbine at that time was the lack of an efficient rotary compressor. The aeroplane had still to be developed, and there was little aerodynamic knowledge available upon which to draw for the design of the aerofoil-section compressor blades. The shape of the blade-sections, and the pitch-setting of the rotor blades were, therefore, largely a matter of trial and error. Consequently the turbulence losses were large, leading to a low compressor efficiency. The result of this was, of course, that even for small pressure-ratios, so much of the power output of the turbine wheel was used up in driving the compressor that the net useful work was negligible. The design of the combustion chamber also required a certain amount of knowledge of aerodynamic theory and the nature of combustion, which in Stolze's day was, at best, rudimentary.

The turbine itself appears to be of the reaction type, but since the work of Parsons on the reaction turbine was not to be made public for another decade, it is doubtful whether or not Stolze's rotor attained anything like the efficiency possible with modern turbines. Another problem was the provision of suitable metals for the turbine blading and the heating chamber. The wheels of combustion turbines run at considerably higher temperatures than those of steam turbines, and the heat-resisting metals developed for the steam turbine were yet to come.

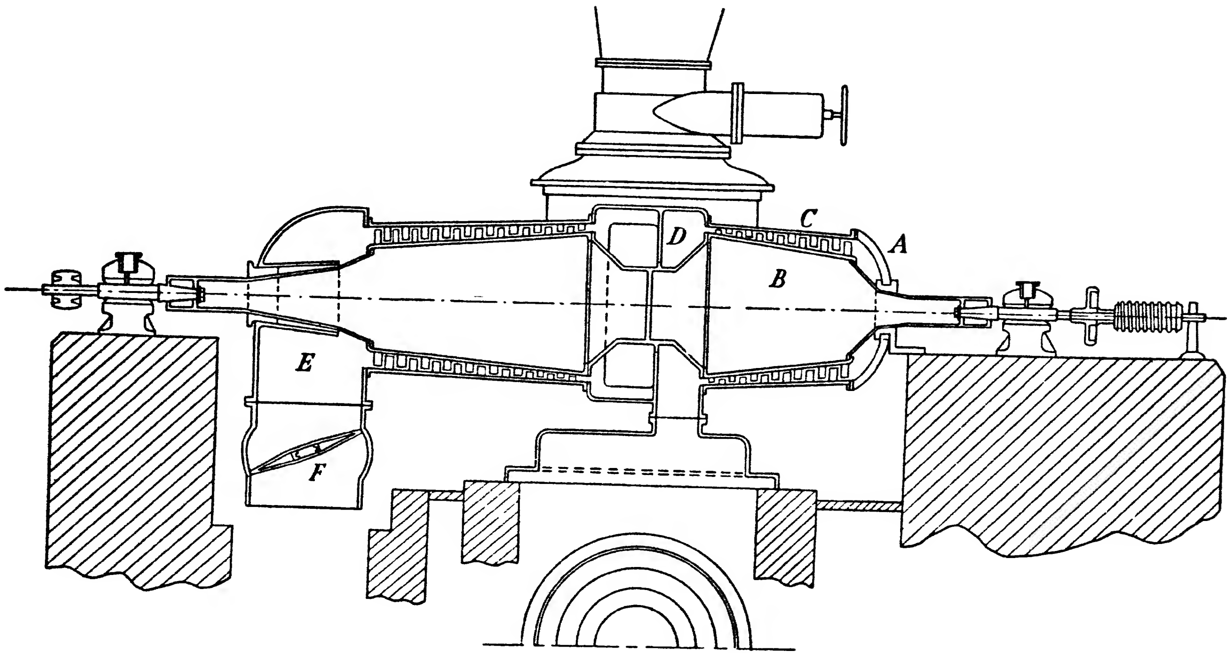


Fig. 4

Parsons

The next reference to the combustion turbine is to be found in the original patent of 1884 of the Hon. C. A. Parsons, the father of the Parsons reaction steam turbine. In his patent specification he says:

> Motors, according to my invention, are applicable to a variety of purposes, and if such an apparatus be driven, it becomes a pump, and can be used for actuating a fluid column or producing pressure in a fluid. Such a fluid pressure-producer can be combined with a multiple motor, according to my invention, to obtain motive power from fuel or combustible gases of any kind. For this purpose I employ the pressure-producer to force air or combustible gases into a furnace into which there may or may not be introduced other fuel (liquid or solid). From the furnace the products of combustion can be led in a heated state to the multiple motor which they actuate. Conveniently, the pressure-producer and multiple motor can be mounted on the same shaft, the former to be driven by the latter; but I do not confine myself to this arrangement of parts.

It is clear that Parsons foresaw the development of the axial-flow compressor, designed something on the lines of his axial-flow reaction turbine, compressing air which was to be passed through a combustion chamber, and then over the blades of a reaction turbine. Apart from this statement of the possibilities, however, he seems to have done little towards the construction of a combustion turbine, although he did, later, develop an axial-flow compressor. The rapid development of the centrifugal compressor pushed the axial-flow type into the background, and after Parsons's early development work, it fell largely into abeyance.

Karavodine

During the first few years of the twentieth century there took place a great development of new ideas. All the work of the latter half of the previous century culminated in a spate of mechanical invention which produced a whole series of new machines during these few years, and this acceleration of development brought the gas turbine a certain share of attention. Although, in a short space of time, more work and initiative were given to the subject than in the whole of the previous century, it is to be regretted that the results of experiments were not carried farther, despite their initial promise. It is true, however, that early results indicated that an enormous amount of expensive experimental work would be required to make the gas turbine a commercially practicable proposition, and in the meantime the reciprocating internal-combustion engine was making giant strides forward, a fact which perhaps obscured the issue.

In 1906 a Monsieur Karavodine built, in Paris, a small

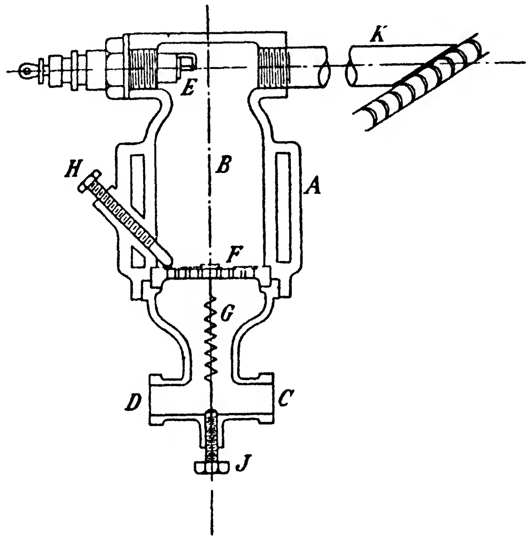


FIG. 5

explosion-turbine developing about 2 h.p., which ran consistently and successfully for some time. A sectioned view of one of the combustion chambers of this machine, together with one of the nozzles and a portion of the turbine wheel, is shown in Fig. 5. The turbine was fitted with four explosion chambers, each equipped with a separate nozzle. The wheel itself was of the De Laval type, 5·9 inches in diameter, and was mounted on a flexible shaft.

Referring to the drawing, the combustion chamber *B* was made of cast-iron, and its inlet end was water-jacketed at *A*. This had the effect of keeping the inlet end fairly cool. The fuel, either combustible gas or a vaporized hydrocarbon such as petrol, was fed in via the entry connexion *C* The air necessary for combustion was drawn in through a similar connexion *D*, and mixing of the fuel and air to form a combustible mixture took place before entry into the combustion chamber proper.

Both the inlet for the fuel and that for the air were fitted with throttles, by means of which the mixture strength could be manually regulated. There was a plate valve *F*, made of sheet steel, covering the inlet ports, and held on its seat by a light spring *G*, the tension of which could be adjusted by means of the screw *J*. The lift of the plate valve was determined by the adjustment of the set-screw *H*. The ignition of the explosive mixture was effected by the sparking-plug *E*. The discharge nozzle *K* was screwed into a fitting at the end of the combustion chamber.

In order to start the machine, the air valve was closed, and a blast of air was blown through the fuel inlet *C*, carrying with it sufficient fuel to form an explosive mixture. The mixture, on entering the combustion chamber, was ignited by the sparking-plug, and the products of combustion were ejected through the nozzle on to the turbine wheel. Immediately the first explosion took place, the air port *D* was opened and the further intake of mixture became automatic, the operation being as follows: When explosion takes place there is a pressure rise which is not immediately dissipated due to the inertia of the mass of gas in the nozzle. The gas then accelerates and passes down the nozzle, the pressure in the chamber falling at the same time, until it finally reaches atmospheric pressure, but, since gas is still moving down the nozzle, a slight negative pressure is induced in the combustion chamber. While this is going on, the water-jacket around the inlet end of the chamber cools the walls, and when the pressure has reached atmospheric level, this cooling effect causes a sufficient contraction of the gas in the combustion chamber to set up a substantial negative pressure. The period during which this negative pressure is dominant is sufficiently short for the inertia of the mass of gas in the nozzle to prevent an inflow from the wheel-case. The negative pressure causes the plate-valve to lift, and a fresh volume of explosive mixture is drawn into the chamber. This process was found to be completely automatic, and would repeat itself indefinitely. After a time the outlet end of the combustion chamber became so hot that the sparking-plug could be switched off and ignition of the mixture took place by virtue of the heat of the walls at the outlet end.

The complete cycle took 0·026 seconds, which means that there were 38 explosions per second. The maximum pressure attained in the combustion-chamber was 19·4 lb. per sq. in. absolute, and the minimum pressure was 12·6 lb. per sq. in. absolute, which corresponds to a maximum of about 4½ lb. per sq. in. gauge, and a suction of about 2 lb. per sq. in. gauge. The mean effective pressure worked out at 16·25 lb. per sq. in. absolute.

The volume of each combustion chamber of Karavodine's turbine was approximately 14 cu. in. and each nozzle was 9 feet long and of 0·63 inch bore. The rotational speed of the De Laval wheel was 10,000 r.p.m., which gave a tip-speed of 258 ft. per sec. The shaft output was 1·6 b.h.p., and the power required to overcome bearing friction and windage was approximately 0·5 h.p. Thus the total wheel output was 2·1 h.p. When running on petrol, the fuel consumption was 10·3 lb. per hour, which represented a specific consumption of 4·9 lb. per h.p. per hour. Thus, although this turbine actually worked, its overall thermal efficiency was far too low to compare seriously with contemporary reciprocating internal-combustion engines.

Armengaud and Lemale

While Karavodine was doing his initial work, a great deal of important experimental work on the combustion turbine was being done by the Frenchmen Armengaud and Lemale. In 1905 they had constructed a small machine which was running successfully at the plant of the Société des Turbo-moteurs, in Paris, and from which much useful information was gained.

This machine was made from a converted De Laval steam turbine of 25 b.h.p. which was supplied with compressed air from a separate high-speed compressor, via a combustion chamber. The lay-out of the original combustion chamber is shown in Fig. 6. This was made of cast-iron, lined with carborundum with an elastic backing of asbestos to allow for differential expansion. The nozzle was fitted with a water-jacket to keep the temperature within reasonable limits, and cooling of the body of the combustion chamber was effected by embedding a spiral water-tube in the chamber walls, the steam generated in this spiral tube being introduced into the combustion chamber just before expansion through the nozzle. The temperature of combustion was about 1,800° C., and the effect of the introduction of steam from the spiral tube was to reduce this to about 400° C. at the nozzle.

The problem of keeping the turbine wheel sufficiently cool was overcome by installing an economizer, in the form of a flash boiler, in the exhaust duct. Low-pressure steam was generated

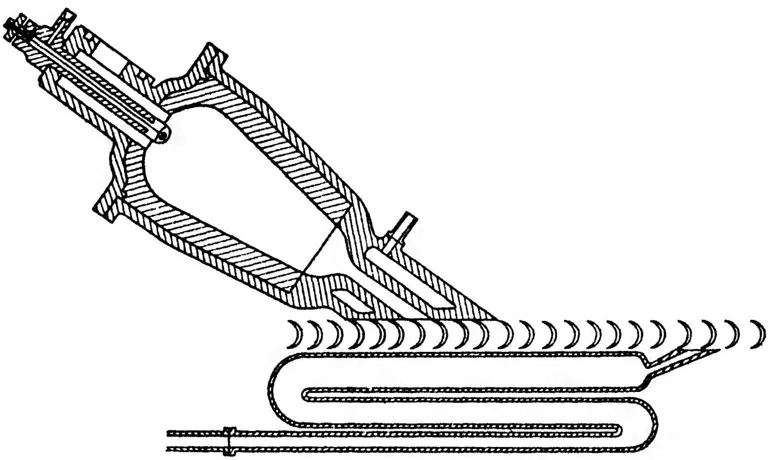


FIG. 6

in the boiler, and passed over the blades of the wheel immediately after the gas from the combustion chamber, so that the blades passed first through the jet of hot gases, and then through the jet of much cooler steam, the average temperature of the blading thus being kept at a reasonably low value. Liquid fuel was sprayed into the chamber through an atomizer, and ignition was carried out electrically.

The promising results obtained from this little 25 h.p. turbine led to the construction of a larger machine, which was intended to give a shaft output of about 400 h.p. The turbine itself was of the Curtis type and had a rotational speed of 4,000 r.p.m. The wheel was water-cooled by means of passages drilled radially from the hub, and continued into the blades themselves. The difference in temperature between the blades and the hub caused a difference in the density of the water, which, in conjunction with the centrifugal force of rotation, was sufficient to set up an adequate circulation.

For this turbine, an improved type of combustion chamber was designed, and a sectioned view of this is shown in Fig. 7. It will be seen that the actual combustion space, instead of being pear-shaped as in the early experimental machine, was made divergent in an attempt to obtain a more efficient constant-pressure combustion. The cooling of the body of the combustion chamber by means of a coil of water-tubing was retained in this improved design, as also was the water-cooling of the nozzle and the steam-cooling of the turbine wheel-blades.

The air compressor used in this machine was a centrifugal

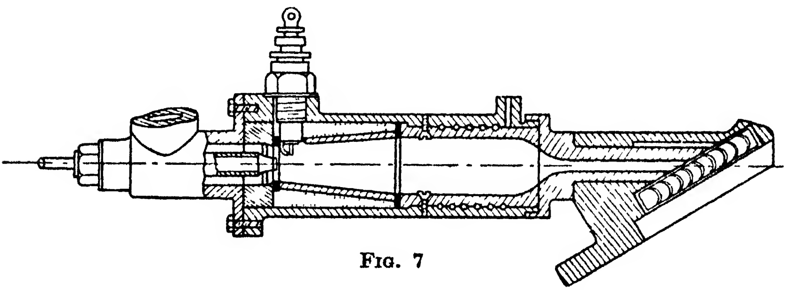


FIG. 7

blower of large dimensions, designed by Professor Rateau, and constructed by Brown, Boveri & Co., of Baden, Switzerland. This is the earliest mention of the name of that very enterprising firm in connexion with the combustion turbine, and was probably the beginning of their long association with this type of prime mover. The compressor was water-cooled between stages in the usual manner for a turbo-blower, and gave an overall efficiency of about 65 per cent., an excellent figure for those early days. The delivery of the compressor was approximately 28 cu. ft. per sec. at a final pressure of about 70 lb. per sq. in. gauge.

The compressor was direct-coupled to the turbine through a flexible coupling. Speed regulation was effected by a throttle valve in the air intake pipe for small speed variations and by a change in the fuel injection for large variations. This was accomplished through a Hartung governor. The fuel pump and water pump were driven off the turbine shaft. The bulk of the plant, for the moderate output of 400 b.h.p., was considerable. The fuel consumption worked out at 2·65 lb. of petrol per b.h.p. hour,[1] which even at that time was far too high for industrial purposes.

[1] Suplee, *The Gas Turbine*.

Holzwarth

During this first decade of the century, while Karavodine, Armengaud, and Lemale were experimenting in France, a great deal of research on the explosion turbine was being carried out by Holzwarth, in Germany, in conjunction with Messrs. Körting. This culminated in 1908 in the construction of a small vertical explosion turbine, the results of tests on which were considered sufficiently promising to warrant the construction of a second and larger machine, designed for an output of 1,000 b.h.p. This second machine, intended for electricity generation at a steel plant, was completed and tested in 1911.

The sectioned drawing, Fig. 8, shows the lay-out of the turbine. The combustion chambers are set in a ring, coaxial with the turbine shaft, and form the base of the machine. The turbine shaft itself is hung from the main bearing at the top of the machine, and carries the turbine rotor at its lower end. This main shaft also carries the generator rotor, the generator being installed above the turbine assembly, and the drives for the valve gear are provided by the worm between the turbine and the generator. The turbine exhaust ring leads out to a single connexion immediately above the turbine wheel. Each combustion chamber is provided with an air valve, a gas valve, and a nozzle valve which controls the emission to the turbine blades. Compressed air is supplied by a rotary compressor and combustible gas by a separate rotary compressor, both compressors being driven by an external electric motor.

With the turbine running, the sequence of operations in each combustion chamber is as follows. When ignition of the mixture takes place, there is a very rapid pressure rise. The nozzle valve is opened, allowing the products of combustion to issue in a high-speed stream upon the blades of the turbine wheel, which is forced to rotate. As the exhausting of the chamber proceeds the velocity of the jet diminishes, in consequence of the fall in combustion chamber pressure. The flow ceases when the pressure in the chamber becomes the same as that in the wheel-casing. The air valve is then opened and a blast of air sweeps through the chamber, scavenging it of residual exhaust gases. The blast is of short duration and the nozzle valve is immediately closed, the air valve remaining open until the chamber is fully charged with air. At the same time the gas valve is opened and a charge of combustible gas forced into the

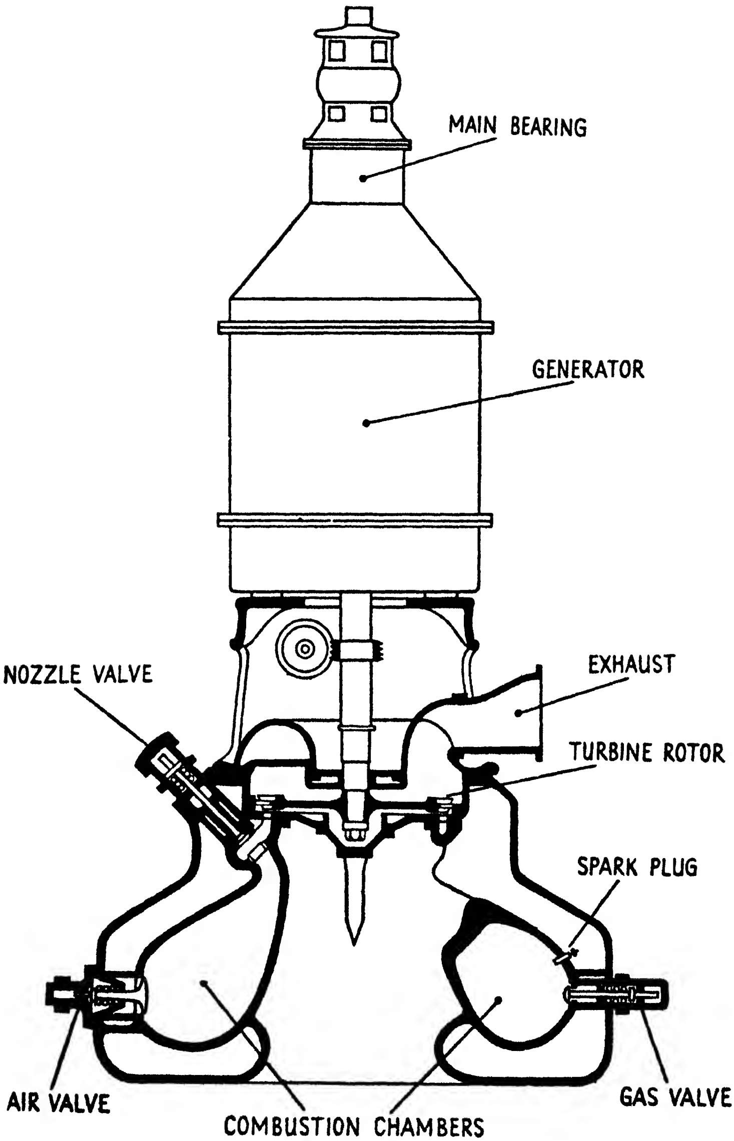

MAIN BEARING
GENERATOR
NOZZLE VALVE
EXHAUST
TURBINE ROTOR
SPARK PLUG
AIR VALVE
GAS VALVE
COMBUSTION CHAMBERS

Fig. 8

chamber. This may be blast furnace gas or coal gas, or if a liquid fuel is used the gas valve may be replaced by an injector valve. When the air and gas valves are closed, the explosive mixture is ignited electrically, by means of the sparking-plugs, and there is a rapid rise in pressure. The whole sequence of operations is then repeated. The valve-gear timing is such that the chambers are charged and fired in sequence, the frequency of the explosions being high, so that the turbine wheel is subjected to a rapid and continuous series of impulses.

In this particular plant, in order to obtain as high an overall thermal efficiency as possible, the hot exhaust gases were used to heat a steam boiler, the steam being employed to provide the power for the air and gas compressors, thus obviating the necessity of taking the power supply for the auxiliaries from the main generator.

This turbine was intended to deliver 1,000 b.h.p. at 3,000 r.p.m., but during its trials it gave only about 200 b.h.p., corresponding to an overall thermal efficiency of about 4 or 5 per cent., and consequently it was never put into continuous operation.

For the following twenty years very little was done with either the combustion turbine or the explosion turbine, but in 1933 Messrs. Brown Boveri, of Switzerland, built a Holzwarth type explosion turbine incorporating some very considerable improvements of their own. The performance of this machine was satisfactory and it was installed at a German steel plant where it ran continuously for a number of years. But by this time advances in combustion-turbine design were such that attention veered from the explosion turbine, and shows little indication of returning.

Meyer

Within recent years the results of aerodynamical and metallurgical research have made possible the construction of highly efficient air compressors and have produced metals capable of withstanding the combination of high temperature and stress to which the blades of a combustion turbine are subjected. The application of these to the combustion turbine has, at last, made that prime mover a commercial proposition. Nevertheless, it should be borne in mind that the constant-pressure continuous-combustion gas turbine is still in its infancy, and much remains to be done before it can compete, over a wide field of application, with the reciprocating petrol and diesel engines and with the steam turbine.

Much of the work involved in bringing the combustion turbine to its present stage of development was carried out by Dr. Meyer and Messrs. Brown Boveri, of Switzerland, who have produced

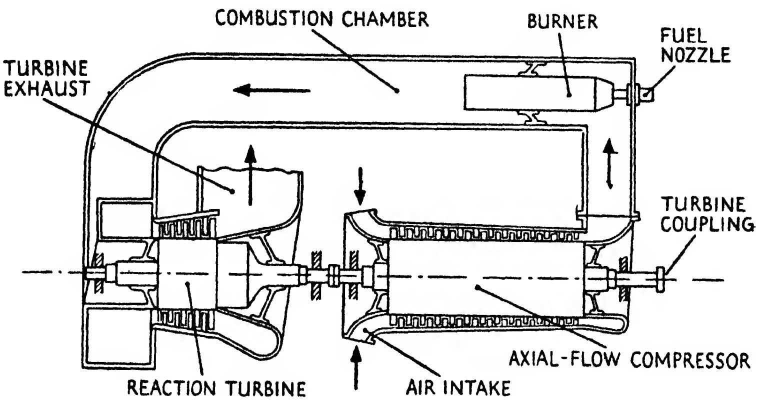


Fig. 9

a number of machines which have been put into continuous operation with a considerable degree of success. The majority of these were stationary plants, but in 1941 a combustion turbine-electric railway locomotive was constructed, the trials of which were extremely promising and would seem to pave the way for further construction.

A typical Brown Boveri lay-out is illustrated diagrammatically in Fig. 9. Air is drawn into the multi-stage axial-flow compressor and delivered, under pressure, through a waste-heat economizer to the combustion chamber. Fuel is introduced and combustion is smooth and continuous, the hot gaseous products being fed to the multi-stage reaction turbine, and so to atmosphere. The turbine and compressor rotors are mounted on the same shaft, the former driving the latter, and the generator is driven through gearing. Overall thermal efficiencies of 18 per cent. and over are attainable with machines of this type.

Whittle

In one highly specialized field the combustion turbine has undergone intensive development, and been brought to a quantity-production stage, namely, as the power unit for jet-propelled aircraft. Basically, jet propulsion consists of drawing into the aircraft a continuous stream of air, accelerating it, and finally ejecting it with considerably increased velocity, the change in momentum of the air stream handled by the aircraft providing the forward thrust. The propeller of a normal aircraft is replaced by a compressor, and the combustion turbine has proved the most suitable engine for dealing with the very large quantities of air involved.

The successful application of the constant-pressure combustion gas turbine to aircraft jet propulsion is in large measure due to the pioneer work of Air-Commodore Sir Frank Whittle, whose first engine ran in 1937. The research on, and construction of, the Whittle engine was carried out by Power Jets, Ltd., a company formed for the purpose.

These Power Jets engines consisted of a single-stage centrifugal compressor, with a double-sided impeller, driven by a single-stage turbine mounted on the same shaft. The air delivered by the compressor was fed into a number of separate combustion chambers in which fuel was injected and burned. The hot gases were then passed through the turbine blading where part of the available pressure drop was utilized to provide the necessary shaft power to drive the compressor. The remainder of the expansion, down to atmospheric pressure, was carried out in a convergent nozzle at the extremity of the turbine exhaust duct, the gases leaving the engine in the form of a high-velocity jet. It was one of these engines which powered the first British jet-propelled aircraft, built by the Gloster Aircraft Co., on its successful initial flight in 1941.

Towards the end of the 1939–45 war production of jet engines based on the Whittle formula was initiated by the aero-engine firms of Rolls-Royce and De Havilland. From the 'Welland', which was substantially similar to the Whittle engines, were developed the more powerful Rolls-Royce 'Derwent' and 'Nene' engines, the latter giving a static thrust of 5,000 lb. The De Havilland 'Goblin' and 'Ghost' afford a variation of the Whittle formula, in that the compressors have single-sided impellers. During the same period a jet engine having an axial-flow compressor and a single annular combustion chamber was constructed by the firm of Metropolitan-Vickers.

II

THE PHYSICS OF GASES

In considering the production of mechanical energy by means of a gas turbine, or any other form of heat engine for that matter, it is necessary to know something of the behaviour of gases when they are subjected to various conditions of pressure and temperature. The function of a heat engine is to convert the heat energy of combustion into a form of mechanical energy which can be employed to do useful work. Since the direct conversion of heat energy to mechanical energy is not a practical process, it is usual to interpose a gaseous substance whose response to the application of heat is manifested by a change in its condition which can be utilized to actuate the mechanical parts of the engine. Gases, under varying conditions of pressure and temperature, are obedient to certain physical laws, and a knowledge of at least the principal of these laws is essential to any form of study of the gas turbine.

The first law of thermodynamics

Broadly speaking, this may be stated as follows: Heat and mechanical energy are mutually convertible, and the 'rate of exchange' is constant and can be measured.

The rate of exchange is known as 'Joule's equivalent' and is usually denoted by the letter J. The value of J is 1,400 ft. lb. per Centigrade heat unit, or 778 ft. lb. per British Thermal Unit. That is to say, 1,400 ft. lb. of mechanical work would be required to raise the temperature of 1 lb. of water through one degree Centigrade, or 778 ft. lb. to raise the temperature of 1 lb. of water through one degree Fahrenheit.

For example, if two rotating surfaces are in contact, and are not lubricated, the heat generated by the friction between them will soon raise their temperature by a measurable amount. Thus mechanical energy is converted into heat, but although this process is quite simple, it is far more difficult to convert heat to mechanical energy. The latter, indeed, is seldom accomplished without considerable loss, as the comparatively poor efficiency of the majority of heat engines amply demonstrates.

The second law of thermodynamics

This may be stated as: Heat cannot be conveyed from one body to another which is at a higher temperature, without the expenditure of energy supplied from an external source.

In other words, heat energy always runs down a temperature gradient, like water down a hill, and will not climb up the gradient unless forced to do so by the application of energy from an external source.

The nature of a gas

A gas may be considered as a collection of molecules, each one of which behaves as though it were a perfectly elastic body. If a quantity of a gas is contained in a closed vessel the molecules, travelling in all directions at very high speeds, set up a continuous bombardment on the walls of the vessel, and it is the force of this bombardment which gives rise to the pressure of the gas on the walls of the vessel. Since the molecules may be regarded as perfectly elastic bodies, there is no loss of energy when they rebound from the walls and from each other, so that their movement is always maintained at the same level, provided no outside influences are brought to bear, and their motion is not slowed up due to the collisions which occur. Hence, if the vessel is completely gas-tight and is maintained at the same temperature, the gas pressure will remain at the same value indefinitely.

A gas offers no resistance to gradual change of shape (i.e. it has no resistance to shear stress) and automatically takes up the shape of any vessel in which it is contained, completely filling the vessel. The volume of a given weight of gas is dependent upon the pressure and, conversely, the pressure is dependent upon the volume. Thus, if a given weight of gas is introduced into an evacuated, closed vessel of a certain volume, the pressure will be at the value determined by the volume, at that temperature. If now we introduce the same weight of gas into a larger vessel, at the same temperature, the gas will expand until it completely fills the vessel, and the pressure will be at a lower value, dependent upon the volume of the vessel. For this reason it is usual to express the dimensional particulars of a gas as being those at certain standard conditions. Since the volume of a gas varies with both temperature and pressure, its standard volume is taken as the volume of 1 lb. of the gas at 0° C., or 32° F., and the standard sea-level pressure of 14·7 lb. per sq. in. These standard conditions are known as 'Normal Temperature and Pressure', or simply N.T.P., and the volume of 1 lb. of a gas at N.T.P. is expressed in cubic feet. 1 lb. of air, for instance, has a volume of 12·39 cu. ft. at N.T.P.

Boyle's law

If a given mass of gas is compressed at constant temperature, its absolute pressure is inversely proportional to its volume.

If P is the pressure and v the volume of the gas, then

$$P \text{ is proportional to } \frac{1}{v}$$

or

$$Pv = \text{constant.}$$

It should be noted that P is the absolute pressure, and not the pressure above atmospheric level. The latter is referred to as 'gauge' pressure.

Charles's law and absolute zero

If the pressure of a given mass of gas is kept constant, and its temperature raised by one degree, its volume will increase by a certain amount. If the temperature is now raised by a further one degree, the volume increases by a precisely similar amount. Thus, if the volume of a mass of gas is v_0 at 0° C., α is the increase in volume per cubic foot of gas per degree rise, and t is the increase in temperature, the increase in total volume is

$$v_0 \alpha t,$$

and the new volume after a temperature increase of t degrees is

$$\begin{aligned} v_1 &= v_0 + v_0 \alpha t \\ &= v_0(1+\alpha t). \end{aligned}$$

The quantity α is called the 'coefficient of volumetric expansion', and has the same value for all gases. The value of α is $\frac{1}{273}$ of the volume at 0° C., or $\frac{1}{492}$ of the volume at 32° F., measured on the Centigrade and Fahrenheit scales respectively.

In the same way, if the volume of a given mass of gas is kept constant, and its temperature is increased by t degrees, there is an increase in the pressure of the gas, the new pressure being given by the expression

$$P_1 = P_0(1+\alpha t),$$

where α has the same value as before.

Since the change in volume of a gas per degree of temperature change is $\frac{1}{273}$ of its volume at 0° C., or $\frac{1}{492}$ of its volume at 32° F., it is clear that the volume will approximate to zero at $-273°$ C. or $-460°$ F. Similarly the pressure will approximate to zero at $-273°$ C. or $-460°$ F. The temperature $-273°$ C., or $-460°$ F., is called 'absolute zero', and if this temperature is used as the starting-point of a temperature scale, 0° C. will be 273° absolute and 32° F. will be 492° absolute, the two absolute scales being measured according to the Centigrade and Fahrenheit scales respectively. For instance, if t is a temperature on the Centigrade scale, the corresponding temperature T on the absolute scale will be $t+273°$ C. absolute.

From the above, we can say that at constant pressure the volume of a gas is proportional to its absolute temperature, and at constant volume the pressure is proportional to the absolute temperature. Therefore v is proportional to T and P is proportional to T, at constant pressure and constant volume respectively, or

$$\frac{v}{T} = \text{constant, at constant pressure}$$

and

$$\frac{P}{T} = \text{constant, at constant volume.}$$

This is Charles's law.

Combination of Boyle's and Charles's laws

In most practical cases when a quantity of gas undergoes changes of pressure, volume, or temperature, the changes are seldom such that either Boyle's or Charles's law alone is obeyed, but generally both laws are operative at the same time. Boyle's law states that $Pv =$ constant when $T =$ constant, and Charles's law states that $P/T =$ constant when $v =$ constant.

If the initial condition of a given mass of gas is P_1, v_1, and T_1, and the final condition is P_3, v_3, and T_3, after changes of pressure, volume, and temperature have taken place, we can conveniently consider the changes as taking place in two stages.

Firstly, let us assume the volume remains constant while the change in pressure takes place. This will result in the temperature having some value T_2, which is intermediate between T_1 and T_3. Since $P/T =$ constant, we have

$$\frac{P_1}{T_1} = \frac{P_3}{T_2} \quad \text{or} \quad \frac{P_1}{P_3} = \frac{T_1}{T_2}.$$

Secondly, the first change having been made, let us consider the change of volume taking place with the pressure remaining constant at the value P_3. Then

$$\frac{v_1}{T_2} = \frac{v_3}{T_3} \quad \text{or} \quad \frac{v_1}{v_3} = \frac{T_2}{T_3}.$$

The combination of the two laws can now be effected by multiplying these two equations, as follows:

$$\frac{P_1 v_1}{P_3 v_3} = \frac{T_1}{T_3} \quad \text{or} \quad \frac{P_1 v_1}{T_1} = \frac{P_3 v_3}{T_3}.$$

Putting this in another way, we can say that

$$\frac{Pv}{T} = \text{constant.}$$

Let R be the constant. Then,

$$Pv = RT.$$

The constant R can be evaluated, and its value is 96 when the Centigrade scale is used, and 53·18 when the Fahrenheit scale is employed.

Specific heat at constant volume

Let us consider a weight of W lb. of a gas in a completely closed vessel, Fig. 10. If heat is applied to the vessel the temperature of the gas will be increased but, owing to the constraint of the retaining walls, the volume will remain constant. All the heat is accounted for by an increase in the internal energy of the gas, which results in an increase in pressure. If C_v is the 'specific heat at constant volume' of the gas, T_1 is the initial temperature, and T_2 the final temperature, then the quantity of heat absorbed by the gas is

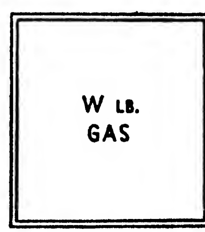


FIG. 10

$$H = WC_v(T_2 - T_1).$$

If the temperature is measured on the Centigrade scale, H is in C.H.U., and if measured on the Fahrenheit scale, H is in B.Th.U. For 1 lb. of gas

$$H = C_v(T_2 - T_1).$$

Specific heat at constant pressure

If a quantity of a gas is heated at constant pressure, the value of the specific heat differs from that which obtains when heating is carried out at constant volume. Suppose a quantity of gas is contained in a cylinder fitted with a gas-tight frictionless piston, Fig. 11, the initial position of the piston being indicated by the broken lines. If now heat is applied to the gas, an expansion takes place which forces the piston upwards, the gas pressure remains constant, and the temperature of the gas is increased. The heat imparted to the gas does two things:

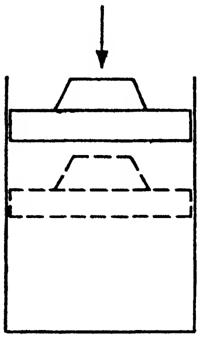


FIG. 11

1. It increases the internal energy of the gas, as indicated by the temperature rise.
2. It does external work in moving the piston against the constant downward force.

The increase in internal energy per 1 lb. of gas is given by

$$C_v(T_2-T_1),$$

where T_1 is the initial and T_2 the final temperature. The external work done in moving the piston is

$$P(v_2-v_1).$$

If C_p is the 'specific heat at constant pressure' of the gas, then

$$C_p(T_2-T_1) = C_v(T_2-T_1)+P(v_2-v_1)/J$$
$$= C_v(T_2-T_1)+R(T_2-T_1)/J.$$

Therefore $$C_p = C_v+R/J.$$

It is clear from this that the specific heat at constant pressure is greater than the specific heat at constant volume due to the external work done.

The ratio of the two specific heats is denoted by γ, so that

$$\gamma = \frac{C_p}{C_v}.$$

From this it will be seen that a low value of γ indicates that the gas to which it applies has a relatively large capacity for storing internal energy. Hence γ may be stated as

$$\frac{\text{energy absorbed internally} + \text{energy used to do external work}}{\text{energy absorbed internally}}.$$

In general, the more complex the molecular structure of a gas, the greater the amount of energy absorbed internally, and the lower the value of γ. Thus, monatomic gases, such as argon and helium, have a value of γ approximately equal to 1·67, diatomic gases, such as oxygen and hydrogen (and also air, which is chiefly a mixture of oxygen and nitrogen) have a value of γ approximately equal to 1·40, and triatomic gases, such as carbon dioxide and steam have a value of γ approximately equal to 1·33.

Work done during expansion

In the general case of the expansion of a gas there is a change in pressure, volume, and temperature, and the expansion is according to the law Pv^n = constant. Let us, for instance, consider the introduction of a quantity of gas, the pressure of which is above that of the atmosphere, into a cylinder, Fig. 12, fitted with a piston. The initial position of the piston is at A, but due to the expansion of the gas it is forced, against the resistance of the atmosphere on its outer face, to B, and hence, by virtue of the expansion, a certain amount of external work is done. The work done during the expansion can be calculated on the basis of the law of expansion for the gas.

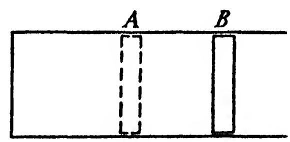


FIG. 12

Let the initial condition of the gas be P_1, v_1, and T_1, the final condition be P_2, v_2, and T_2, and the law of expansion be $Pv^n = k$. If the pressure and volume at a number of points during the expansion are plotted, a curve such as that shown in Fig. 13 is obtained, A being the initial condition, and B the final condition of the gas. At any point during the expansion, let the pressure be P and the volume v. At this point, during a very small increase of volume δv, the pressure may be considered constant at the value P, so that the small amount of work done during this very small expansion is approximately equal to $P\,\delta v$, and is represented by the area of the shaded strip in the diagram. It follows that the total work done during the expansion is represented by the entire area under the curve, between v_1 and v_2, and the entire area is equal to the sum of all such

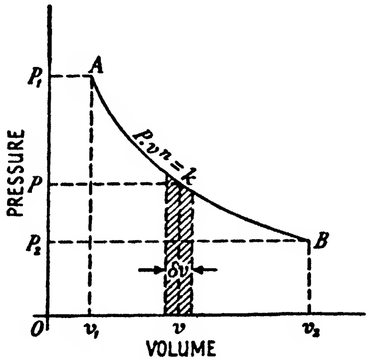


FIG. 13

elemental strips as that shown. Therefore,

$$\text{work done} = \int_{v_1}^{v_2} P\,dv.$$

But
$$P = \frac{k}{v^n}.$$

Hence,

$$\text{work done} = \int_{v_1}^{v_2} \frac{k}{v^n}\,dv = k\left[\frac{1}{1-n}v^{1-n}\right]_{v_1}^{v_2}$$

$$= \frac{k}{1-n}(v_2^{1-n}-v_1^{1-n}) = \frac{P_1 v_1^n}{1-n}(v_2^{1-n}-v_1^{1-n}),$$

and since $P_1 v_1^n = P_2 v_2^n$, the work done becomes

$$\frac{P_1 v_1 - P_2 v_2}{n-1}.$$

Since $Pv = RT$, the work done may also be written as

$$\frac{R(T_1-T_2)}{n-1}.$$

These expressions for the total work done during an expansion may be written in several forms, as follows:

$$\text{Work done} = \frac{P_1}{1-n}(v_1^n v_2^{1-n} - v_1) = \frac{P_1 v_1}{1-n}(v_1^{n-1}v_2^{1-n}-1)$$

$$= \frac{P_1 v_1}{1-n}\left\{\left(\frac{v_1}{v_2}\right)^{n-1}-1\right\} = \frac{P_1 v_1}{n-1}\left\{1-\left(\frac{v_1}{v_2}\right)^{n-1}\right\}$$

Also, since $$P_1 v_1^n = P_2 v_2^n,$$

$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^n.$$

Substituting this value of the pressure ratio for the volume ratio in the expression for work done, we see that

$$\text{work done} = \frac{P_1 v_1}{n-1}\left\{1-\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}$$

Provided the pressures are expressed in pounds per square foot and the volumes in cubic feet, the work done is given in foot-pounds per 1 lb. of gas.

Adiabatic expansion

Suppose a quantity of gas is contained in a cylinder which is jacketed with a material which is a perfect heat insulator, so that no heat can either enter or leave the gas, the cylinder being fitted with a gas-tight, frictionless piston. Should the gas expand, forcing the piston along the cylinder, both the pressure and temperature of the gas will drop and the volume will increase, but the total heat content cannot be increased, owing to the insulation of the cylinder. Therefore the external work done in moving the piston must be at the expense of the initial internal energy of the gas, as indicated by the temperature drop. This type of expansion is known as an 'adiabatic' expansion.

We have shown that

$$C_p = C_v + R/J.$$

Therefore $$\frac{C_p}{C_v} - 1 = \frac{R}{JC_v}$$

or $$\gamma - 1 = \frac{R}{JC_v},$$

and hence $$JC_v = \frac{R}{\gamma - 1}.$$

Now we have shown that in the general case of the expansion of a gas, according to the law $Pv^n = k$, the work done during the expansion is

$$\frac{R(T_1 - T_2)}{n-1}.$$

Considering an adiabatic expansion, the work done is at the expense of the internal energy, which decreases by an amount

$$JC_v(T_1-T_2),$$

or since

$$JC_v = \frac{R}{\gamma-1},$$

the internal energy is decreased by an amount

$$\frac{R(T_1-T_2)}{\gamma-1}.$$

The loss of internal energy must be equal to the external work done, so that

$$\frac{R(T_1-T_2)}{\gamma-1} = \frac{R(T_1-T_2)}{n-1}.$$

In other words, when an expansion is adiabatic, n is replaced by γ, and the law of expansion becomes

$$Pv^{\gamma} = k.$$

Hence the work done during an adiabatic expansion per 1 lb. of gas is

$$\frac{R(T_1-T_2)}{\gamma-1},$$

or, if we write this in terms of the pressures and volumes, the work done is

$$\frac{P_1v_1-P_2v_2}{\gamma-1} = \frac{P_1v_1}{\gamma-1}\left\{1-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\}.$$

If, on the other hand, heat is allowed to flow into the gas during expansion so that the temperature is maintained constant, the rate of inflow of heat being such that the amount of internal energy expended in doing external work is immediately replaced, the expansion is said to be 'isothermal'. In this case $n = 1$, and the law of expansion becomes

$$Pv = k.$$

In most practical heat engines the expansions and compressions approximate to the adiabatic, chiefly because the changes take place so rapidly that there is little time for leakage of heat either out of, or into, the gas.

Relations between pressure, volume, and temperature during expansion

In the general case of an expansion, we have

$$P_1 v_1^n = P_2 v_2^n = \text{constant} \tag{1}$$

and
$$\frac{Pv}{T} = R,$$

or
$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2},$$

or
$$\frac{P_2 v_2}{P_1 v_1} = \frac{T_2}{T_1}. \tag{2}$$

From (1) we get
$$\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^n. \tag{3}$$

Substituting (3) in (2) gives us

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1}. \tag{4}$$

Also, from (1) we get
$$\frac{v_2}{v_1} = \left(\frac{P_1}{P_2}\right)^{\frac{1}{n}}. \tag{5}$$

Substituting (5) in (2) gives us

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}. \tag{6}$$

Using equation (6), the work done per 1 lb. of gas during an expansion can be written as

$$\frac{P_1 v_1}{n-1}\left\{1 - \frac{T_2}{T_1}\right\}.$$

If pressure-volume curves are plotted for expansions with different values of n, they will be similar to those illustrated in Fig. 14. In general, the greater the value of n, the steeper the curve.

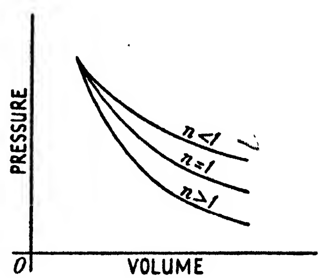


FIG. 14

Compression of a gas in a compressor

Let us consider what happens when a quantity of gas is compressed in the cylinder of a reciprocating compressor. The cycle of operations is shown in Fig. 15, and commences at D, at which point the crank is at top dead centre. The piston begins to move down the cylinder and the inlet valve opens, so that gas is sucked into the cylinder above the receding piston. When the crank reaches bottom dead centre, movement of the piston momentarily ceases, the inlet valve closes, and the piston moves upwards on the return stroke, compressing the gas until, when the pressure attains a predetermined value, the delivery valve opens and the gas is forced out of the cylinder by the moving piston. At top dead centre the delivery valve closes, the piston again starts to move downwards, and the inlet valve opens, admitting a fresh charge of gas, the whole sequence of operations being repeated.

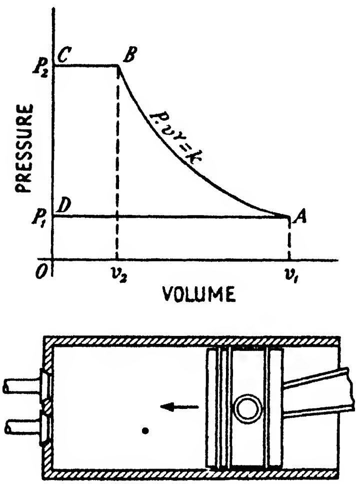


Fig. 15

Thus, there are three distinct actions during a single rotation of the crank; firstly the suction on the downward stroke, secondly the compression during the first part of the upward stroke, and thirdly the delivery during the remainder of the upward stroke.

The suction takes place at a constant pressure P_1, and a volume v_1 of gas is drawn into the cylinder. Hence, the amount of work done during suction is

$$P_1 v_1.$$

Ideally the compression is adiabatic and is according to the law $Pv^\gamma =$ constant.

During compression the volume of the gas is decreased from v_1 to v_2 and the pressure is increased from P_1 to P_2, the conditions being represented by the point B on the pressure-volume diagram, Fig. 15. As we have previously shown, the amount of work done during an adiabatic expansion or compression is given by the expression

$$\frac{P_2 v_2 - P_1 v_1}{\gamma - 1}$$

and the absolute temperature rises from T_1 at A to a value T_2 at B, where

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}.$$

At the point B the delivery valve opens and the gas is forced out of the cylinder at the constant pressure P_2, top dead centre being reached at C, at which point the cylinder is completely empty. The amount of work done during delivery is, therefore,

$$P_2 v_2.$$

Hence the total amount of work done on 1 lb. of gas during a single cycle is equal to the work of compression plus the work of delivery minus the work of suction. This is, of course, represented by the area $ABCD$ enclosed within the diagram. Therefore the total work done per 1 lb. of gas per cycle is

$$\frac{P_2 v_2 - P_1 v_1}{\gamma-1} + P_2 v_2 - P_1 v_1 = \frac{\gamma(P_2 v_2 - P_1 v_1)}{\gamma-1}.$$

If the pressures are in pounds per square foot and the volumes in cubic feet, the work done is in foot-pounds.

Since $Pv = RT$ the expression may also be written in the form

$$\frac{R\gamma(T_2 - T_1)}{\gamma-1}.$$

III

GAS-TURBINE CYCLES

THE majority of heat engines use a gas as the working fluid. The reason for this is that gases are almost perfectly elastic, and these elastic properties are most sensitive to the application of heat. By heating a quantity of gas it can be made to expand and do useful work during the expansion. Those heat engines which fall within what may be broadly termed the internal-combustion group use air as the working fluid, the burning of fuel in which causes an increase in the pressure or volume (or both together) of the fluid. The result of this is the setting up of forces on the moving parts of the engine which cause their motion. The fuel, of course, needs the oxygen of the air charge in order to accomplish the combustion, and after the chemical reaction has taken place the charge no longer consists of pure air. Nevertheless, the primary purpose of the combustion is to apply heat to the charge, whether it consists of pure air or not, so that the resulting elastic changes in the condition of the gaseous charge can be utilized to do useful work.

In order to obtain some theoretical basis on which engines of widely differing types, and using different kinds of fuel, can be readily compared, it is usual to assume that they use pure air as their working fluid. The chemical changes consequent upon the combustion of fuel are ignored, and only the effects on the air of the addition of a quantity of heat energy, which might be represented by the fuel, are taken into account.

The air cycle

When a heat engine takes in a charge of air, the charge may be compressed, causing its volume to decrease and its pressure to increase; then it may be subjected to the addition of heat energy by the combustion of a quantity of fuel, which may cause an increase in its pressure, its volume, its temperature, or, more often than not, all three together; it may then be allowed to expand in order to do useful work on the moving parts of the engine, suffering while it does so a reduction in its pressure and temperature and an increase in its volume; finally, its task accomplished, it is released to atmosphere where it returns to its original condition before it was taken in to the engine Once the charge of air has passed through the system, a fresh charge is taken in, and the whole process is repeated. This process is the cycle of operations for the engine and can be represented on a diagram by plotting the changes which take place in the pressure of the air against the corresponding changes in its volume, from the moment the air enters the system to the moment it is finally exhausted and returns to its original condition. This diagram is known as the 'air-cycle diagram', or the 'air-standard cycle' for the engine, and calculations based upon it assume that the weight of the air charge is 1 lb.

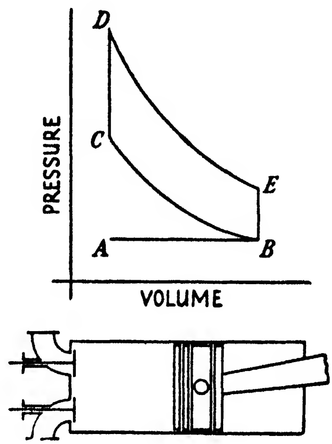


FIG. 16

For instance, let us consider the changes of pressure and volume which, ideally, take place in the cylinder of an ordinary four-stroke petrol engine during a single cycle of operations. With the piston at top dead centre, the inlet valve is opened. The conditions of pressure and volume are then indicated by the position of the point A, Fig. 16. The piston travels down the cylinder on the induction stroke, and air is drawn in at constant atmospheric pressure until the cylinder is full. This operation can be represented on the diagram by the line AB. The inlet valve is then closed and the piston comes up on the compression stroke. This causes the pressure of the air charge to increase and its volume to decrease, so that, when the piston reaches top dead centre the condition of the air can be represented by the point C on the diagram. The compression is, ideally, adiabatic and can be represented by the curve BC. Ignition of the fuel now takes place. The combustion is rapid and approximates to an explosion, the sudden addition of heat energy causing the pressure to jump to a value represented by the point D. Since the piston has had no time to move any appreciable distance down the cylinder, the volume of the charge remains approximately constant during the explosion. The increased pressure in the cylinder forces the piston downwards, the adiabatic expansion of the charge resulting in a drop in pressure until, when the piston reaches bottom dead centre, the pressure-volume conditions are as indicated by the point E, the expansion curve being DE. At E the exhaust valve is opened and the sudden release of the charge causes a very rapid drop in pressure, at almost constant volume, along the line EB. Finally, the piston comes up on the exhaust stroke and the remainder of the charge is ejected, at constant pressure, along the line BA, thus completing the cycle of operations. This then, is the ideal air-cycle diagram for the petrol engine, and diagrams for gas turbines can be obtained in the same way.

Work done

The area enclosed within the diagram $BCDE$ is of considerable importance. An area is the product of two linear dimensions and, broadly speaking, the area of the cycle diagram is proportional to its length parallel to the volume axis multiplied by its height parallel to the pressure axis. The length is in units of volume and the height is in units of pressure. If these are in cubic feet and pounds per square foot, respectively, then the area of the diagram is in

$$\frac{\text{lb.}}{\text{ft.}^2}\text{ft.}^3 = \text{ft. lb.}$$

Hence the area of the diagram is in foot-pounds, and the foot-pound is a unit of work.

Now the amount of work done on the gas by the piston during the compression stroke is represented by the area under the adiabatic BC (see Chap. II), and the amount of work done on the piston by the expanding gas is represented by the area under the adiabatic DE. The difference between the two areas is equal to the area of the diagram $BCDE$. It follows that the area $BCDE$ represents the amount of useful work obtained during the cycle, as a result of the heat energy released by the combustion of the fuel. Since the weight of the air charge is assumed to be 1 lb., the area of the diagram represents the ideal useful work done in foot-pounds per 1 lb. of air passed through the system.

Air standard efficiency

An expression for the thermal efficiency of an engine may be derived from the air-cycle diagram. The efficiency value so obtained, however, is considerably higher than that which would be found if tests were carried out on an actual engine, since it is assumed that the engine is working with pure air and no account is taken of the energy lost in the bearings, cooling water, lubricating oil, etc. Nevertheless, the 'air standard efficiency' as it is called, calculated from the air-cycle diagram, enables a good comparison of different engine types to be made, and forms the basis for performance estimates on a projected design.

Any useful work obtained from an engine must come from the heat energy put into the system by the combustion of the fuel. It follows that the temperature changes which occur in the air charge during the cycle are a measure of the corresponding energy changes.

Referring to Fig. 16, ignition takes place at point *C* and, due to the combustion, the pressure rapidly rises to a value indicated by point *D*. At the same time the temperature of the air charge increases and is much higher at *D* than at *C*. Hence the addition of heat energy to the charge during the cycle can be said to take place along the line *CD*. When the expansion of the charge has forced the piston down to bottom dead centre, the pressure-volume condition of the air is represented by point *E*, but since an adiabatic expansion results in a drop in temperature (see Chap. II), the temperature at *E* is lower than that at *D*. This indicates that the air charge contains less heat energy at *E* than it contained at *D*, the difference between the two values being the work done in pushing the piston along the cylinder. At *E* the exhaust valve is opened and the charge, which is still at a pressure and temperature considerably above atmospheric, is suddenly released, but is still hot when it leaves the cylinder, thus carrying away with it a quantity of heat energy which is eventually dissipated in the outside atmosphere. It can be said that the heat energy carried away in the exhaust is rejected by the engine along the line *EB*.

Thus we have a quantity of heat energy added to the air charge by the combustion of fuel, and a smaller quantity of heat energy rejected in the exhaust. It follows that the difference between the heat energy added and that rejected is, in theory, the useful work obtained during the cycle and is the same as the useful work represented by the area of the diagram

BCDE. The air standard efficiency, or theoretical cycle efficiency, is therefore

$$\eta_{\text{cycle}} = \frac{\text{useful work done}}{\text{heat energy added}}$$

$$= \frac{\text{heat energy added}-\text{heat energy rejected}}{\text{heat energy added}}.$$

The constant-pressure turbine cycle

A simple constant-pressure turbine, or combustion turbine, is illustrated diagrammatically in Fig. 17. Air is drawn through

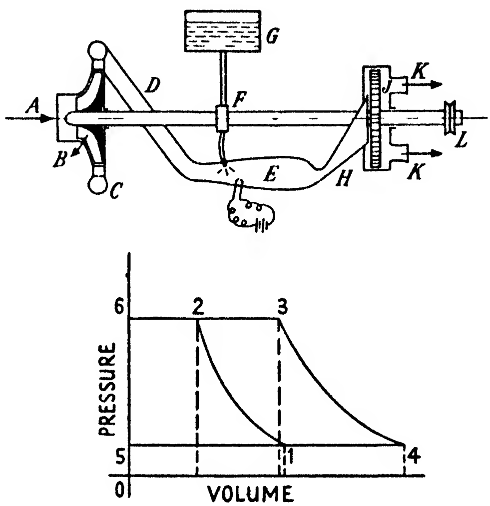


FIG. 17

the intake *A* into the centre of the impeller *B* of the centrifugal compressor, which is mounted on the same shaft as the turbine wheel *J*. On leaving the impeller it passes into the volute casing *C*, at increased pressure, and thence along the duct *D* into the combustion chamber *E*. Fuel is fed from the tank *G* into the combustion chamber by the pump *F*, where it is burnt. The combustion takes place at constant pressure and, due to the addition of heat energy from the fuel, the volume of the air is considerably increased and its temperature raised. The fluid is then expanded to atmospheric pressure through the turbine nozzle *H*, from which it emerges in the form of a high-speed jet, a considerable proportion of its heat energy being converted to velocity energy. The stream of gas impinges upon the blades of the turbine wheel *J*, causing it to rotate and so drive the compressor impeller *B*. Finally the hot gases, which are at atmospheric pressure in the turbine casing, leave the system through the outlets *K*. The power developed by the turbine wheel, over and above that required to drive the compressor, is taken off at *L* and is the useful work, or net work, of the machine.

Fig. 17 also illustrates the constant-pressure air cycle upon which the plant functions. It receives its name from the fact that the combustion takes place at a constant pressure, the sole effect of the addition of heat energy in the combustion chamber being to increase the volume of the air.

Let us consider the behaviour of 1 lb. of air in its passage through the system, plotting its pressure against its volume at a number of points between inlet and exhaust. This diagram will then correspond to the air-cycle diagram for a reciprocating engine. Air enters the compressor at atmospheric pressure at the point 1 on the cycle diagram and, ideally, is compressed adiabatically to the point 2, at which instant it may be assumed to enter the combustion chamber. Combustion takes place at constant pressure from 2 to 3, the air increasing in volume due to the addition of heat energy consequent upon the burning of the fuel. Expansion through the turbine nozzle takes place along the adiabatic 3,4 and finally, when the gas has been exhausted from the system, it returns to its original condition before intake along the constant-pressure line 4,1.

The work done on the air by the compressor is represented by the area 1,5,6,2 (see Chap. II). The total work, or gross work, of the cycle is represented by the entire area 4,5,6,3. This is the work done during the expansion through the turbine nozzle in giving to the air its high velocity and is the theoretical kinetic energy of the jet. The useful work, or net work, which may be taken off the turbine shaft at *L*, is the difference between the gross work obtained from the turbine wheel and the 'negative' work which is put into the compressor. Ideally, if there were no losses in the compressor or the turbine, and if there were no friction losses in the bearings, then the energy given to the air in the compressor would be just sufficient to enable the turbine to keep the compressor running. No fuel would need to be used in the system and no useful work would be produced, since both the gross work and the negative work would be equal to the area 1,5,6,2. The system would then be in a state of perpetual motion which, of course, is impossible, due to the inevitable losses in the mechanism. From this it is clear that the greater the losses in the compressor, the greater will be the amount of negative work required to keep it running, the negative work being obtained at the expense of the useful shaft work available at L.

For a given pressure-ratio the gross work of the cycle depends on the length of the constant-pressure line 6,3, since the longer this line, the greater the area of the diagram. The length of 6,3 is usually fixed by the maximum temperature at which the turbine can be allowed to run, in view of the fact that the temperature at point 3 is raised to a very high value by the addition of heat energy along 2,3, and hence this maximum permissible temperature tends to fix the amount of gross work obtainable from the system. It is, therefore, of the utmost importance that the negative work should be maintained at as low a value as possible, for a given pressure-ratio, in order that a maximum amount of useful work may be obtained.

Air standard efficiency of the combustion-turbine cycle

An expression for the efficiency of the air cycle can be simply derived in the following manner:

Let

T_1 = temperature at point 1 in degrees absolute.
T_2 = temperature at point 2 in degrees absolute.
T_3 = temperature at point 3 in degrees absolute.
T_4 = temperature at point 4 in degrees absolute.
C_p = specific heat of air at constant pressure.
γ = the ratio of the specific heats of air.
P_1 = absolute pressure at points 1 and 4.
P_2 = absolute pressure at points 2 and 3.
η_{cycle} = air standard efficiency.

With a weight of 1 lb. of air passing through the system we defined the air-cycle efficiency of an engine as

$$\eta_{\text{cycle}} = \frac{\text{heat energy added} - \text{heat energy rejected}}{\text{heat energy added}}$$

Now both the addition and rejection of heat energy take place at constant pressures, along the lines 2,3 and 4,1 respectively.

$$\text{heat added} = C_p(T_3 - T_2)$$

and

$$\text{heat rejected} = C_p(T_4 - T_1).$$

Therefore

$$\eta_{\text{cycle}} = \frac{C_p(T_3-T_2)-C_p(T_4-T_1)}{C_p(T_3-T_2)} = 1-\frac{T_4-T_1}{T_3-T_2}.$$

Since both compression and expansion are assumed to be adiabatic, we have

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{and} \quad T_4 = T_3\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}.$$

Therefore $$C_p(T_3-T_2) = C_p\left\{T_3-T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\}$$

and $$C_p(T_4-T_1) = C_p\left\{T_3\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}-T_1\right\}.$$

Therefore

$$\eta_{\text{cycle}} = \frac{C_p\left\{T_3-T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\}-C_p\left\{T_3\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}-T_1\right\}}{C_p\left\{T_3-T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\}}.$$

Dividing throughout by the denominator this reduces to

$$\eta_{\text{cycle}} = 1-\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

Hence we see that the efficiency of the air cycle is dependent solely upon the pressure-ratio of the system. The extension of the line 2,3 by the addition of more heat at constant pressure does not necessarily increase the air standard efficiency since there is, at the same time, a proportionate increase in the quantity of heat rejected from the system along the line 4,1. Therefore, the only way in which the air-cycle efficiency can be increased is by an increase in the pressure-ratio. In practice, however, the pressure-ratio cannot be increased indefinitely since compressor losses are apt to be large at high pressures and there are limits to the maximum temperatures at which the turbine can be run.

Fig. 18 shows a curve of air standard efficiency, derived from the above expression, plotted against pressure-ratio. Values of efficiency given by this curve cannot, of course, be attained by actual combustion turbines, since no account has been taken of the very considerable losses which occur in both compressor and turbine, and which necessitate a modification of the simple expression to render it of practical use. This is dealt with in a later chapter.

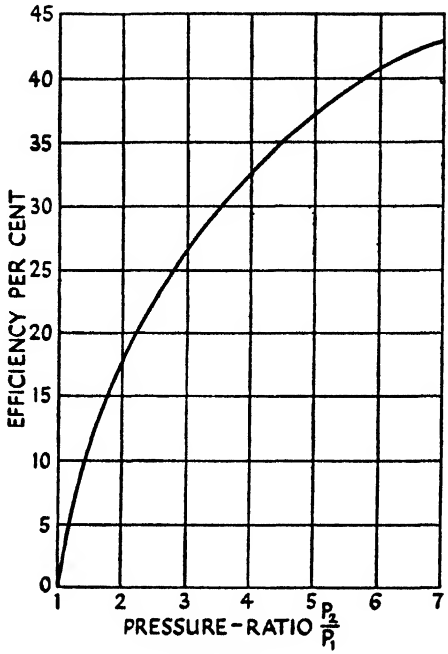


FIG. 18

The explosion-turbine cycle

There are, broadly speaking, two types of explosion turbine, one having no air compressor, and the other being fitted with a small compressor. The first of these is of academic, rather than practical, interest at the present time although, in a later chapter, the cycle upon which it functions will be encountered as a practical jet-propulsion cycle.

The essentials of the compressorless type are illustrated diagrammatically in Fig. 19. A mixture of combustible gas and air enters the combustion chamber B through the spring-loaded plate-valve A, and is ignited by an electric spark at F. Combustion takes place at approximately constant volume, and there is a rapid pressure rise which causes a discharge of gas through the long nozzle C on to the blades of the turbine wheel D. When the pressure in the chamber has dropped to that of the outside atmosphere, the cooling effect of the water-jacket *E* causes a contraction of the gases in the chamber, with a consequent small sub-atmospheric pressure drop. This lifts the plate-valve *A* off its seat and a fresh charge is drawn in. Meanwhile, the inertia of the mass of gas in the long nozzle *C* prevents any

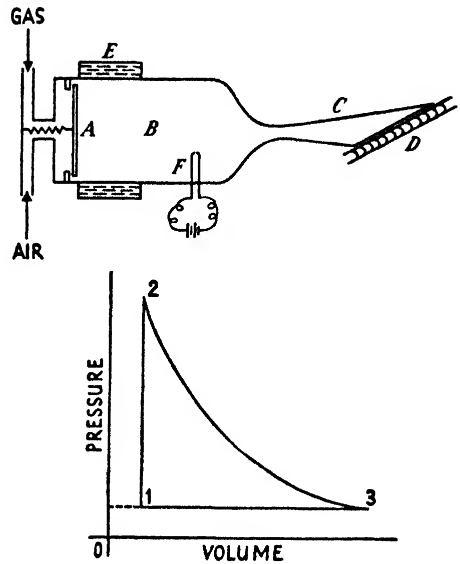


FIG. 19

serious back-flow from the wheel casing. Ignition again takes place, the pressure rise consequent upon the explosion closes the plate-valve and the whole cycle of operations is repeated.

The ideal air-cycle diagram is also shown in Fig. 19. With the contents of the chamber at atmospheric pressure, ignition takes place at the point 1 and combustion is, theoretically, at constant volume along the line 1,2. Expansion through the long nozzle is along the adiabatic 2,3 and all the power developed by the turbine wheel is taken out in the form of useful shaft work. The gas leaves the system at point 3 at atmospheric pressure, and returns to its original condition before intake along the constant-pressure line 3,1. The experimental Karavodine turbine of 1906, described in Chapter I, operated on this cycle.

The type of explosion turbine fitted with a compressor is illustrated diagrammatically in Fig. 20. Air is drawn into the compressor at *A* and passes through the vane-channels of the impeller *B*. From the volute casing *C* it passes along the duct *D* to the combustion chamber *E*, which it enters through the valve *F*. When the chamber is fully charged, the valve *F* is closed and a metered quantity of fuel is injected through the fuel injector *K*. The fuel is supplied from the tank *J* by the

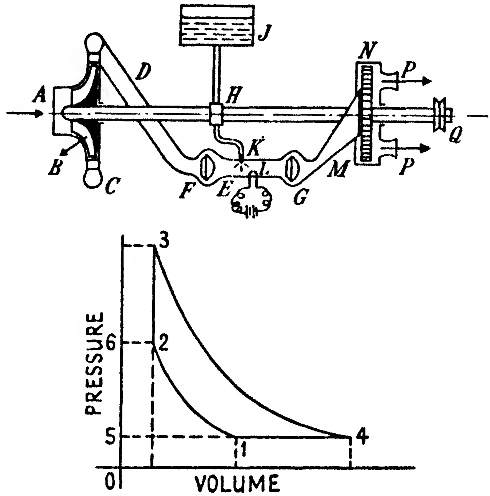


Fig. 20

pump *H*. The mixture is immediately ignited by the electric sparking-plug *L* and an explosion takes place, which causes a very rapid pressure rise. Valve *G* is then opened and the released gases leave the combustion chamber through the turbine nozzle *M*. The high-speed jet of gas leaving the nozzle is directed on to the blades of the turbine wheel *N*, which is forced to rotate. The turbine wheel is mounted on the same shaft as the compressor impeller and the useful shaft energy obtainable from the turbine wheel, over and above that required to drive the compressor, is taken off at *Q*. The exhaust gases leave the turbine casing at *P*.

Referring to the air-cycle diagram, Fig. 20, compression takes place, ideally, along the adiabatic 1,2. At the point 2 the air is assumed to enter the combustion chamber, fuel is injected and the mixture is ignited. The consequent pressure rise is very rapid and the combustion is at constant volume, along the line 2,3. At the point 3 the valve *G* opens and the gas is allowed to expand, along the adiabatic 3,4, to atmospheric pressure. The working fluid leaves the system at 4 and returns to its original condition before intake along the constant-pressure line 4,1.

Air standard efficiency of the explosion-turbine cycle

The air-cycle efficiency for an explosion turbine can be calculated in a manner similar to that used for the combustion turbine.

Dealing first with the cycle for the compressorless type, Fig. 19, and assuming that a weight of 1 lb. of air is passing through the system, we have, as before,

$$\eta_{\text{cycle}} = \frac{\text{heat energy added}-\text{heat energy rejected}}{\text{heat energy added}}.$$

Now the heat energy added is, in this case, at constant volume and is equal to

$$C_v(T_2-T_1),$$

where C_v is the specific heat of air at constant volume. The rejection of heat energy in the exhaust is at constant pressure, along the line 3,1, and is equal to

$$C_p(T_3-T_1).$$

Hence the air standard efficiency is

$$\eta_{\text{cycle}} = \frac{C_v(T_2-T_1)-C_p(T_3-T_1)}{C_v(T_2-T_1)} = 1-\gamma\frac{T_3-T_1}{T_2-T_1}.$$

Similarly, for the explosion turbine fitted with a compressor, Fig. 20, we have

$$\eta_{\text{cycle}} = \frac{\text{heat energy added}-\text{heat energy rejected}}{\text{heat energy added}}.$$

Referring to the air-cycle diagram, the addition of heat energy is at constant volume, along the line 2,3, and is equal to

$$C_v(T_3-T_2).$$

The rejection of heat energy is at constant pressure, along the line 4,1, and is equal to

$$C_p(T_4-T_1).$$

Therefore

$$\eta_{\text{cycle}} = \frac{C_v(T_3-T_2)-C_p(T_4-T_1)}{C_v(T_3-T_2)} = 1-\gamma\frac{T_4-T_1}{T_3-T_2}.$$

The compressor of an explosion turbine is smaller than that of an equivalent combustion turbine and the adiabatic compression is not so vital a feature of the cycle. Its delivery pressure is relatively low and its primary purpose is to ensure good scavenging and adequate charging of the combustion chambers. Consequently the negative work of an explosion turbine is, in theory, less than that of an equivalent combustion turbine.

The explosion turbine has a theoretical advantage over the combustion turbine in that, for a given maximum turbine-wheel temperature, the combustion-chamber temperature can be much higher. This is due to the fact that in an explosion turbine the addition of heat at constant volume causes an increase in pressure as well as in temperature and, consequently, the pressure drop and temperature drop in the turbine nozzles is greater than in an equivalent combustion turbine. The maximum permissible temperature at the point 3 in the cycle diagram, Fig. 20, may also be greater than the combustion-chamber temperature of the combustion turbine, by virtue of the cooling effect of the fresh charge after each explosion.

These theoretical advantages of the explosion-turbine cycle are, however, largely offset in practice by certain mechanical drawbacks inherent in the lay-out. In the first place, mechanical valves give bad breathing characteristics and, apart from the turbulence losses, there is an inevitable throttling effect. In addition, the design of valves large enough to deal with the very large volumes of air involved, and yet retain very rapid opening and closing characteristics, is a matter of considerable difficulty. Two types of nozzle valve used on the Holzwarth explosion turbine,[1] described in Chapter I, are illustrated in Fig. 21. A further serious loss of efficiency is provided by the intermittency of the explosions. When the peak pressure at the point 3 in the cycle diagram is reached and the nozzle valve opens, the discharge from the nozzle is at a very high velocity. As the pressure in the combustion chamber falls, however, so the nozzle velocity correspondingly falls. This variation in the discharge velocity causes a progressive alteration in the relative angle of the jet to the turbine blades, with a consequent increase in shock losses.

The explosion-turbine cycle has been described in some detail here, for the sole reason that it is a workable gas-turbine cycle and machines operating on it have been built and have run

[1] Holzwarth, *The Gas Turbine.*

although, in the majority of cases, their efficiency has been poor. Nevertheless, it should be emphasized that the explosion turbine has not proved popular, and that the current trend of development is concentrated on the combustion turbine. The essential

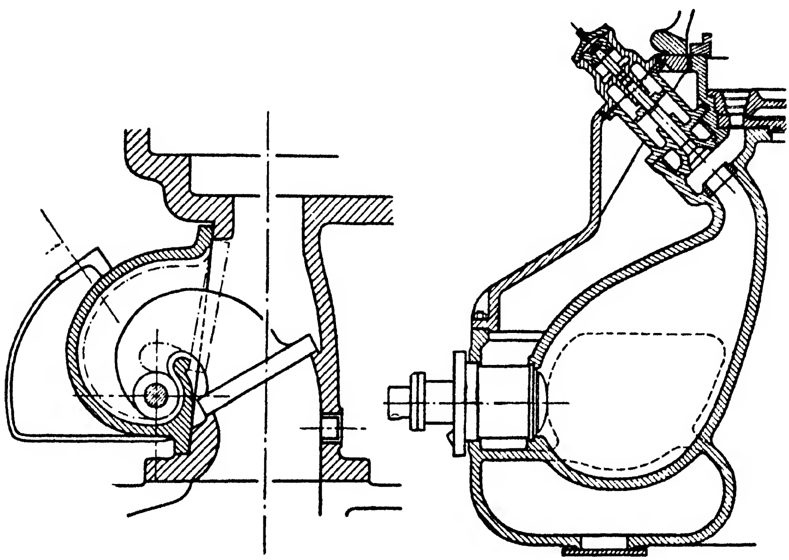


FIG. 21

advantages of a turbine motor over a reciprocating motor are simplicity, lightness in weight, and the provision of smooth and continuous rotary motion, combined with a comparable thermal efficiency. With the exception of continuous rotary motion, the explosion turbine possesses none of these, and even its rotary motion is not as smooth as that of the combustion turbine.

IV

THE CENTRIFUGAL COMPRESSOR

THE centrifugal compressor is a light and simple rotary compressor capable of handling large quantities of air. In view of its large capacity, in relation to its comparatively small dimensions, it has become popular for a variety of purposes where very high delivery pressures are not demanded, its extreme simplicity of construction being conducive to reliability and ease of maintenance.

The lay-out of a single-stage centrifugal compressor is illustrated in diagrammatic section in Fig. 22, together with a sectioned perspective view. The rotor, or impeller, *A* is mounted on the shaft *B* and enclosed within the casing *C*, the running clearance between impeller and casing being very small in order to prevent, as far as possible, leakage of compressed air. The impeller, itself, is a circular metal disk, on one surface of which is a number of radial vanes. In small compressors both disk and vanes may be in one piece, machined from a solid forged blank of steel or light alloy, but in large compressors the vanes may be fabricated from sheet metal and riveted to the disk. The impeller shown has an open face, but some types are fitted with an annular ring, or shroud, of sheet metal, which covers the open face, leaving only a circular hole for entry of the air. Such an impeller is said to be 'completely shrouded', and an example is illustrated in Fig. 23. Other types have vanes on both sides of a central disk, Fig. 24, and are thus able to handle a greater quantity of air without increase in diameter.

Referring to Fig. 22, around the impeller *A* is a ring of fixed guide vanes *E*, known as the 'diffuser ring', which are attached to the casing. These guide vanes direct the compressed air smoothly into the casing, the periphery of which is of spiral form and is sometimes known as a 'volute'. The spiral casing terminates in the delivery connexion *F*, the compressor shown having only one such connexion, although a number of connexions may, if necessary for a particular installation, be taken off around the periphery of the casing.

The functioning of the compressor is as follows: The impeller *A* is rotated at high speed in a clockwise direction and air is drawn axially through the entry *D* into the centre of the impeller, where it is trapped in the vane channels and carried around with the impeller. But due to the centrifugal force, consequent upon the high rotational speed, it is hurled outward

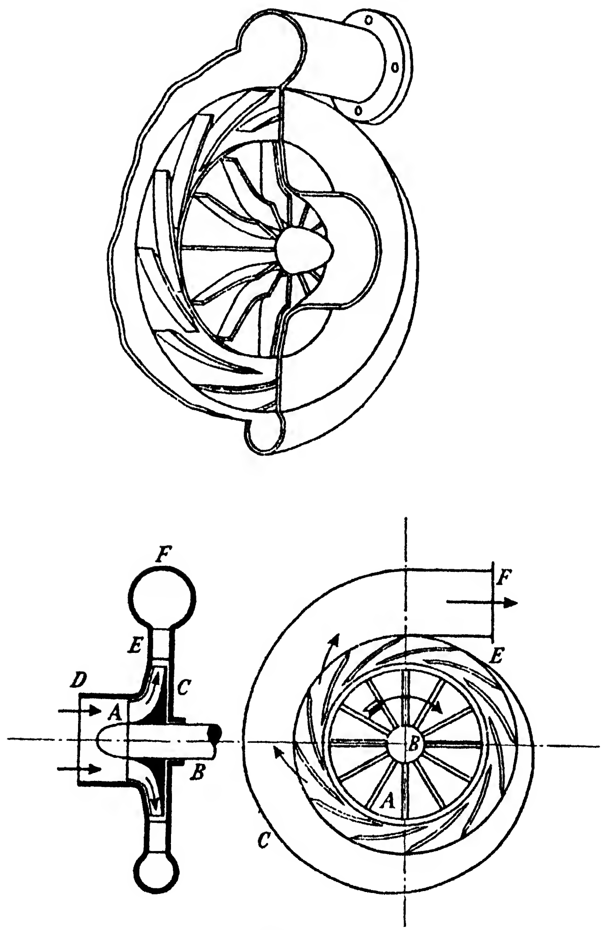


Fig. 22

to the periphery of the impeller and in so doing is subjected to an increase in both pressure and velocity. On leaving the vane channels it has acquired, in addition to its radial velocity, a tangential velocity, and the angle of the fixed diffuser vanes is such that their inner edges are in line with the direction of the resultant velocity of the air leaving the impeller, so that it is picked up smoothly and guided into the spiral casing. The high velocity of the air leaving the impeller represents a considerable

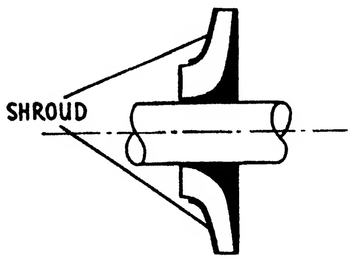


FIG. 23

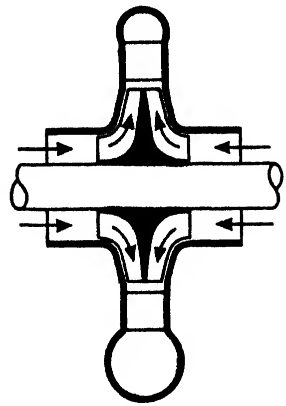


FIG. 24

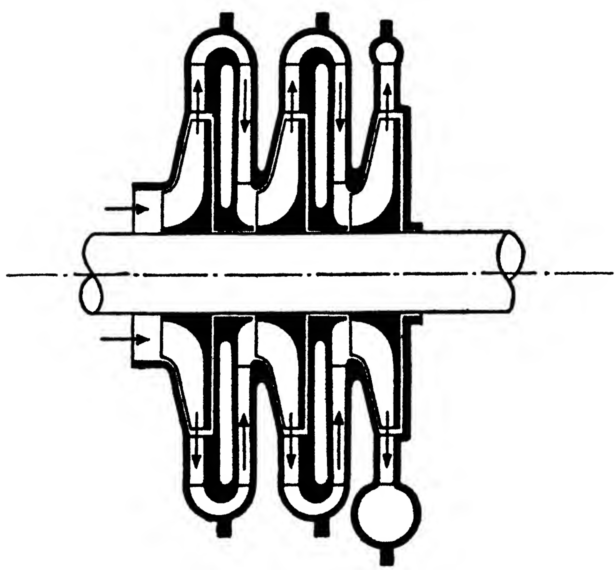


FIG. 25

proportion of the energy acquired in its passage through the impeller, and it is one of the functions of the diffuser to slow up the air by guiding it through channels of increasing area, in order to convert the velocity energy to pressure energy. Hence there is a pressure rise, not only in the impeller, but also in the diffuser. The pressure-ratio developed is largely dependent upon the tip speed of the impeller. The higher the tip speed the greater the pressure-ratio, and vice versa. Consequently, the rotational speeds of centrifugal compressors tend to be high.

A single-stage compressor, such as that in Fig. 22, will generate a maximum pressure-ratio of about 4 : 1. When greater pressure-ratios are required it is usual to mount a number of such units in series, thus forming a multi-stage compressor. A diagrammatic section through a multi-stage compressor is illustrated in Fig. 25, the path of the air through the impellers and diffusers being indicated by the arrows. This type of compressor, however, has not found great favour in combustion-turbine design, partly because of its considerable bulk when more than one stage is employed, and partly because its efficiency is slightly lower than that of the more popular axial-flow compressor.

The velocity diagram

Three types of impeller vane are possible, namely, the back-swept vane, the straight radial vane as in Fig. 22, and the

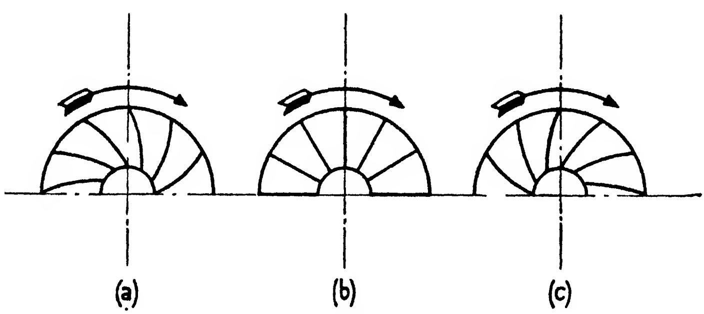


FIG. 26

forward-swept vane. The three types are drawn in Fig. 26, the directions of rotation being indicated by the arrows. Both the back-swept and straight-vaned types are widely used, but the forward-swept type is never employed, for reasons which will be discussed later in the chapter.

As regards the former types, it is evident that the straight radial vane is merely a particular case of the curved back-swept vane, in which the blade entry angle, near the hub, is equal to the blade outlet angle, at the periphery. Hence, any calculations for the back-swept type will also be applicable to the straight type, provided modifications allowing for the differences in vane angles are incorporated. For this reason, in what follows, the back-swept vane is taken as a general case, with indications

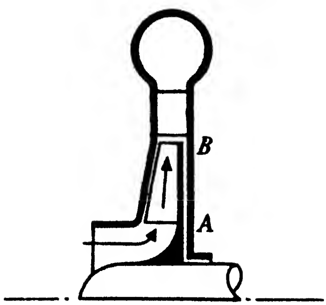


FIG. 27

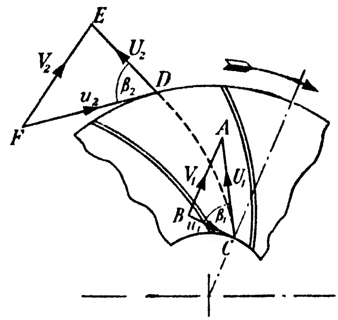


FIG. 28

as to what mathematical corrections are necessary to accommodate the straight radial vane.

One further assumption is made, namely, that the vanes extend only from the periphery of the entry duct to the periphery of the impeller, Fig. 27. It is assumed that in the space immediately adjacent to the hub the direction of the air flow, at first parallel to the axis of the impeller shaft, changes through a right angle, so that when the air enters the vane channels it is moving in an approximately radial direction with the same absolute velocity which it possessed immediately prior to its change in direction. Thus the calculations are concerned only with the changes in the condition of the air which take place within the vane channels, between points A and B, Fig. 27.

Two vanes of an impeller are drawn in Fig. 28. The path taken by the air during its journey through the channel may be considered to be the mean line drawn equidistant between the two curved vanes and shown dotted in the diagram. Air entering the channel is assumed to travel in a radial direction with an absolute velocity V_1, which may be represented, to any convenient scale, by the vector BA. But, due to the rotation of the impeller, the entry edges of the vanes have a tangential velocity u_1 relative to the air. This may be represented by the vector BC. The velocity of the air relative to the entry edges of the vanes is found by drawing CA to complete the triangle

ABC. The vector *CA* gives the velocity of the air relative to the impeller, to the scale to which the diagram is drawn. Let this relative velocity be U_1. Then *ABC* is the velocity diagram at entry to the vane channel, and the vane inlet angle β_1 is so designed as to bring the edges of the vanes in line with the direction of the relative velocity.

The velocity diagram at outlet from the channel is obtained in the same way. The vector *DE* is drawn, to scale, to represent

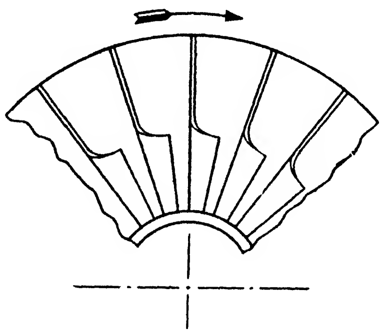


FIG. 29

the velocity U_2 of the air relative to the impeller. The vector *FD* represents the tangential velocity u_2 of the impeller rim and consequently *FE*, the third side of the triangle, represents, to scale, in magnitude and direction, the absolute velocity V_2 of the air at the impeller outlet. Hence *DEF* is the velocity diagram at outlet from the impeller. The angle β_2 is the vane outlet angle.

For purposes of calculation, the condition of the air at entry may be taken as that indicated by the velocity diagram *ABC* but, in order to minimize shock losses at entry, it is usual to curve the entry edges of the vanes in the direction of rotation, Fig. 29. These curved portions are sometimes referred to as the 'entry impeller', and may be manufactured as a separate unit designed to be mounted on the impeller shaft so that the curved vanes mate up with the main impeller vanes.

Work done during compression

Fig. 30 shows a curved-vane impeller, the direction of rotation of which is indicated by the arrow. Only two vanes are drawn, for the sake of clarity. The mean path of the air flowing between these two vanes is the mean line, shown dotted, drawn midway between the vanes, and we may consider all the conditions as those which appertain along this line. That this is not strictly true, and that a correction to this basic assumption must be applied, will be demonstrated later. Air flows axially into

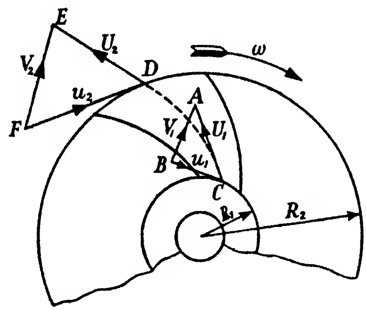


FIG. 30

the centre of the impeller and we shall assume that the vanes do not run right into the hub, but stop off at the periphery of the entry annulus. The entering air turns through a right angle and flows radially outwards over the face of the impeller, but where it first enters the vane channels its direction of flow is not truly radial. This is due to the fact that the air in immediate contact with the impeller disk is being whirled around, due to the skin-friction at the disk and, since air is a viscous fluid, this rotational effect is transmitted, with decreasing intensity, to the incoming air for some distance in front of the intake. Thus the air approaches the impeller with a corkscrew motion.

Let

V_1 = Absolute velocity of entry air in feet per second.
U_1 = Velocity of entry air relative to impeller in feet per second.
u_1 = Tangential velocity of blade edges at entry in feet per second.
V_2 = Absolute velocity of outlet air in feet per second.
U_2 = Velocity of outlet air relative to impeller in feet per second.
u_2 = Peripheral velocity of impeller in feet per second.
R_1 = Radius of entry annulus in feet.
R_2 = Radius of impeller in feet.
ω = Angular velocity of impeller in radians per second.

Let us consider 1 lb. weight of air passing through the impeller. Calculations on momentum involve the product of MASS and velocity. It should be borne in mind that MASS = WEIGHT/g. A change in the velocity of the air from V_1 to V_2 takes place.

$$\text{Momentum at } C = \frac{V_1}{g}.$$

$$\text{Momentum at } D = \frac{V_2}{g}.$$

Now let us resolve the absolute velocities V_1 and V_2 tangentially and radially. It is clear that the radial components, since their line of action passes through the centre, have no moment of momentum about the centre. Therefore,

$$\text{moment of momentum at } C = \frac{V_{w1}}{g} R_1$$

and

$$\text{moment of momentum at } D = \frac{V_{w2}}{g} R_2,$$

where V_{w1} and V_{w2} are the tangential components of the absolute velocities V_1 and V_2. V_{w1} and V_{w2} are referred to as the 'velocities of whirl' of the air.

The change in angular momentum which the air experiences is the angular momentum at D minus the angular momentum at C.

Hence

$$\text{change of angular momentum} = \frac{1}{g}(V_{w2} R_2 - V_{w1} R_1)$$

and this is also the change of angular momentum per second.

Newton's second law states that the rate of change of momentum of a body is proportional to the applied force. If this is applied to the air flow through the vane channel, we see that the rate of change of momentum of the air is a measure of the force exerted on the air by the impeller vanes. If all the quantities involved are expressed in their appropriate units, then we can say that the applied force is equal to the rate of change of momentum. Since, in this case, we are dealing with angular momentum, the rate of change of angular momentum is equal to the applied torque. Using the foot-pound-second system of units, we have:

$$\text{applied torque} = T = \frac{1}{g}(V_{w2} R_2 - V_{w1} R_1) \text{ lb. ft.}$$

and

$$\text{work done per second} = \omega T = \frac{\omega}{g}(V_{w2}\,R_2 - V_{w1}\,R_1) \text{ ft. lb.}$$

But $\qquad \omega R_1 = u_1 \quad \text{and} \quad \omega R_2 = u_2.$

Therefore

$$\text{work done per second per 1 lb. air} = \frac{1}{g}(u_2\,V_{w2} - u_1\,V_{w1}) \text{ ft. lb.}$$

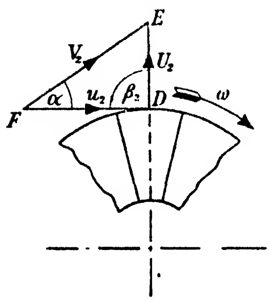


FIG. 31

If W lb. of air per sec. are passing through the impeller, then the total work done per second by the impeller is

$$\frac{W}{g}(u_2\,V_{w2} - u_1\,V_{w1}) \text{ ft. lb.}$$

The horse-power equivalent of the work done on the air by the impeller is

$$\frac{W}{550g}(u_2\,V_{w2} - u_1\,V_{w1}).$$

Now, in practice, $u_1\,V_{w1}$ is often small compared with $u_2\,V_{w2}$, in which case we may neglect it. Therefore the horse-power equivalent of the work done on the air by the impeller becomes

$$\frac{W u_2\,V_{w2}}{550g}.$$

Let us now briefly consider the straight-vaned impeller. The triangle of velocities is as shown in Fig. 31 and the angle $\beta_2 = 90°$. We have shown that the horse-power equivalent of the work done on the air by the impeller is

$$\frac{W u_2\,V_{w2}}{550g},$$

but in this case the velocity of whirl V_{w2}, which is equal to $V_2 \cos\alpha$, acts in the same straight line as the tip velocity u_2. From the geometry of the figure, it is clear that $V_{w2} = u_2$. Hence, for a straight-vaned impeller the horse-power equivalent of the work done on the air is

$$\frac{W u_2^2}{550g}.$$

A further expression for work done

The work done on. the air during compression can also be considered in terms of the temperature changes which take place. Ideally, the nature of the compression should be adiabatic, but due to the very considerable skin-friction effect of the high impeller speed and the rapid flow of air through the vane channels, a perfect adiabatic compression is unobtainable. Therefore, the outlet temperature is considerably higher than that which would be predicted by an assumption of pure adiabatic compression, but we can, nevertheless, still treat the conditions as obeying adiabatic theory, provided allowance is made for the fact that the compression index is greater than γ, which is approximately 1·4 for air.

Let

P_1 = Absolute pressure of air at inlet condition.
V_1 = Absolute velocity of air at inlet condition.
T_1 = Absolute temperature of air at inlet condition.
P_2 = Absolute pressure of air at outlet condition.
V_2 = Absolute velocity of air at outlet condition.
T_2 = Absolute temperature of air at outlet condition.
J = Mechanical equivalent of heat.
ρ_1 = Density of air at inlet condition.
ρ_2 = Density of air at outlet condition.
R = Gas constant.
C_v = Specific heat of air at constant volume = 0·169.

Let us consider 1 lb. of air in its passage through the compressor.

$$\text{Total energy at inlet} = \frac{P_1}{\rho_1}+\frac{V_1^2}{2g}+JC_vT_1.$$

$$\text{Total energy at outlet} = \frac{P_2}{\rho_2}+\frac{V_2^2}{2g}+JC_vT_2.$$

Then,

energy at inlet+work done on air = energy at outlet.

The heat loss by radiation, convection, etc. is usually neglected. Therefore

$$\frac{P_1}{\rho_1}+\frac{V_1^2}{2g}+JC_vT_1+\text{work done on air} = \frac{P_2}{\rho_2}+\frac{V_2^2}{2g}+JC_vT_2.$$

It is usual to design the compressor so that the inlet velocity is equal to the outlet velocity. Therefore $V_1 = V_2$, the outlet velocity being taken as the velocity of the air in the delivery pipe. Hence

$$\frac{P_1}{\rho_1}+JC_vT_1+\text{work done on air} = \frac{P_2}{\rho_2}+JC_vT_2.$$

The work done per pound of air per second is

$$\left[\frac{P_2}{\rho_2}+JC_v T_2\right]-\left[\frac{P_1}{\rho_1}+JC_v T_1\right]$$

Let v_1 be the volume of 1 lb. of air at the inlet pressure, and v_2 be the volume of 1 lb. of air at the outlet pressure. Then

$$\frac{1}{\rho_1}=v_1 \quad \text{and} \quad \frac{1}{\rho_2}=v_2.$$

Therefore the equation becomes

$$\text{work done} = (P_2 v_2+JC_v T_2)-(P_1 v_1+JC_v T_1).$$

From the gas laws we have

$$P_1 v_1 = RT_1 \quad \text{and} \quad P_2 v_2 = RT_2.$$

Hence,

$$\begin{aligned}\text{work done} &= (RT_2+JC_v T_2)-(RT_1+JC_v T_1)\\ &= T_2(R+JC_v)-T_1(R+JC_v).\end{aligned}$$

From the gas laws $\quad R+JC_v = JC_p.$

Therefore

$$\text{work done per pound of air per second} = JC_p(T_2-T_1)\text{ ft. lb.}$$

If the compressor deals with a weight of W lb. of air per second, then,

$$\text{work done} = WJC_p(T_2-T_1)\text{ ft. lb. per sec.}$$

and

$$\text{horse-power equivalent of work done} = \frac{WJC_p}{550}(T_2-T_1).$$

In the above theory T_2 is the true measured temperature of the outlet air in degrees absolute, and is higher than the theoretical adiabatic temperature. Since the conditions considered are those at the compressor entry and in the delivery pipe, the shape of the vanes, whether curved or straight, is not involved.

Stage pressure-ratio

An air compressor is always designed to deliver a given quantity of air in unit time at a given pressure, the calculated pressure-ratio being attained with the impeller running at its designed speed under the given conditions. The pressure-ratio developed by a centrifugal compressor can be calculated on a basis of the factors which we have already considered, in the following manner:

The horse-power equivalent of the work done on the air by the impeller is

$$\frac{W}{550g}(u_2 V_{w2} - u_1 V_{w1}).$$

If the ideal form of compression is assumed to be adiabatic, and if T_2' is the theoretical absolute delivery temperature, assuming adiabatic compression, then the horse-power equivalent of the work done on the air by the impeller is also equal to

$$\frac{WJC_p}{550}(T_2' - T_1).$$

Therefore

$$\frac{WJC_p}{550}(T_2' - T_1) = \frac{W}{550g}(u_2 V_{w2} - u_1 V_{w1})$$

or

$$JC_p(T_2' - T_1) = \frac{1}{g}(u_2 V_{w2} - u_1 V_{w1}).$$

Since the compression is adiabatic, then

$$T_2' = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}};$$

and we may say that

$$JC_p\left[T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - T_1\right] = \frac{1}{g}(u_2 V_{w2} - u_1 V_{w1})$$

or

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 = \frac{u_2 V_{w2} - u_1 V_{w1}}{gJC_p T_1}.$$

Therefore

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{u_2 V_{w2} - u_1 V_{w1}}{gJC_p T_1} + 1.$$

Hence the pressure-ratio developed in the stage is

$$\frac{P_2}{P_1} = \left[\frac{u_2 V_{w2} - u_1 V_{w1}}{gJC_p T_1} + 1\right]^{\frac{\gamma}{\gamma-1}}.$$

If we consider $u_1 V_{w1}$ to be small enough to be neglected, the expression becomes

$$\frac{P_2}{P_1} = \left[\frac{u_2 V_{w2}}{gJC_p T_1} + 1\right]^{\frac{\gamma}{\gamma-1}}.$$

Thus we see that the pressure-ratio depends upon the tip speed of the impeller, the velocity of whirl at the impeller outlet, and the inlet temperature in degrees absolute, the other quantities on the right-hand side of the equation being constants.

The foregoing theory is for the general case of a compressor fitted with a back-swept vane impeller, but if the impeller has

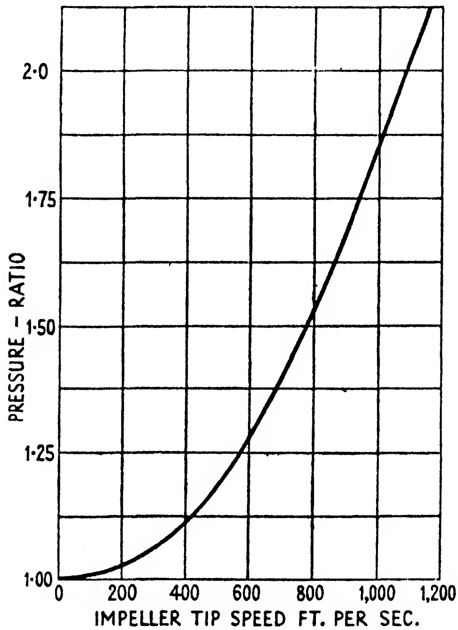


FIG. 32

straight radial vanes the expression for the pressure-ratio becomes

$$\frac{P_2}{P_1} = \left[\frac{u_2^2}{gJC_pT_1} + 1\right]^{\frac{\gamma}{\gamma-1}}.$$

If we assume that the inlet temperature T_1 is constant, then it is clear that the pressure-ratio depends only upon the impeller tip-speed u_2. Since it is not possible, in practice, to obtain a perfect adiabatic compression, the value of the compression index, in an uncooled compressor, is invariably greater than γ. If we let n denote the actual value of the compression index, the expression becomes

$$\frac{P_2}{P_1} = \left[\frac{u_2^2}{gJC_pT_1} + 1\right]^{\frac{n}{n-1}}.$$

Pressure-ratios in the region of 2 to 1 are attainable with impeller tip speeds of the order of 1,000 ft. per sec. Fig. 32

shows a curve,[1] illustrating the relationship between pressure-ratio and tip speed, derived as the result of tests on a large number of aero-engine super-chargers.

Adiabatic efficiency

Due to the inevitable losses which occur, the efficiency of compression is always less than unity. These losses are a consequence of the considerable friction between the fast moving air and the surfaces of the ducts and channels, and of the eddies and vortices which are formed. Very little energy is actually lost from the system, but a considerable portion of the energy imparted to the air by the rotating impeller goes, by virtue of the friction and turbulence losses, into heating the air to a temperature higher than that which would be attained if a perfect adiabatic compression were possible.

It follows from this that in order to compress a given quantity of air from its initial pressure to a higher pressure in a centrifugal compressor, the energy input to the compressor must be considerably greater than that which would be required if a perfect adiabatic compression were possible. The efficiency of compression is termed the 'adiabatic efficiency' and is denoted by the symbol η_{ad}. Then

$$\eta_{ad} = \frac{\text{work required for perfect adiabatic compression}}{\text{work actually required to compress the air}}.$$

If T'_2 is the delivery temperature which would be attained with a perfect adiabatic compression, and if T_2 is the actual delivery temperature, both temperatures being measured on the absolute scale, then

$$T'_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

and
$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}.$$

The value of n is, of course, greater than γ (1·4 for air) and is in the region of 1·6 for an uncooled compressor.

If the compressor deals with W lb. of air per sec., the work required if the compression were a perfect adiabatic would be

$$WJC_p(T'_2-T_1) \text{ ft. lb. per sec.},$$

[1] Null and Pfau, *Journal of the Society of German Engineers*, 20 Sept. 1941.

and the work which is actually required to effect the compression is

$$WJC_p(T_2-T_1) \text{ ft. lb. per sec.}$$

Hence, the adiabatic efficiency is

$$\eta_{ad} = \frac{WJC_p(T_2'-T_1)}{WJC_p(T_2-T_1)}$$

$$= \frac{T_2'-T_1}{T_2-T_1}.$$

A well-designed centrifugal compressor may attain an adiabatic efficiency of 75 to 80 per cent. when running under its optimum conditions, but the efficiency falls away rapidly on either side of the optimum, due to increases in the losses.

The mechanical efficiency η_m of the compressor is defined as follows:

$$\eta_m = \frac{\text{horse-power equivalent of actual work done on air}}{\text{shaft horse-power input to compressor}}$$

$$= \frac{\frac{WJC_p}{550}(T_2-T_1)}{\text{shaft horse-power input to compressor}}.$$

Since the construction of the centrifugal compressor is simple, and the bearings are few in number, the mechanical losses are low, and the mechanical efficiency is, in most cases, little less than unity.

The overall efficiency of the compressor is the product of the adiabatic efficiency and the mechanical efficiency. Therefore

$$\eta_{overall} = \eta_{ad}\,\eta_m.$$

A further expression for adiabatic efficiency

It is sometimes found convenient to express the adiabatic efficiency in terms of the pressure-ratio, and this can be done in the following manner:

Referring to the previous article, the work which would be required for a perfect adiabatic compression is given by the expression

$$WJC_p(T_2'-T_1)$$

and the work actually required to effect the compression, by the expression

$$WJC_p(T_2-T_1).$$

But
$$T_2' = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

and
$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}.$$

If we now substitute these values of the delivery temperatures in the expressions for work done, we see that the ideal adiabatic work may be written as

$$\text{adiabatic work} = JC_pT_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$$

and the actual work required may be written as

$$\text{actual work} = JC_p\,T_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right]$$

Since the adiabatic efficiency is the ratio

$$\frac{\text{adiabatic work}}{\text{actual work}},$$

it is clear that
$$\eta_{\text{ad}} = \frac{(P_2/P_1)^{(\gamma-1)/\gamma}-1}{(P_2/P_1)^{(n-1)/n}-1}.$$

If we denote the pressure-ratio P_2/P_1 by r, the expression becomes

$$\eta_{\text{ad}} = \frac{r^{(\gamma-1)/\gamma}-1}{r^{(n-1)/n}-1}.$$

It will be observed that, in order to determine the adiabatic efficiency from this expression, it is first necessary to know the value of n. This is difficult to obtain by theoretical means, and it is, therefore, usual, in the design of a given compressor, to have recourse to the results of tests on previous compressors of similar type and construction.

Eddy velocity

Due to the existence of a number of subsidiary factors, which all have their influence on the results attainable, the straightforward theorv we have considered up to now will not give us a completely accurate statement of the performance which we can expect from an impeller. Some of these factors are difficult, or impracticable, of estimation by calculation, and recourse to experimental data is necessary. One of the most important subsidiary factors, however, is the influence of the phenomenon known as 'eddy velocity', which is capable of simple mathematical treatment.

Fig. 33 shows two vanes of a back-swept vane type impeller, the direction of rotation of which is indicated by the arrow *A*.

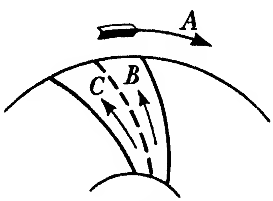


FIG. 33

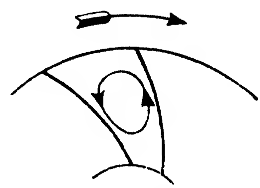


FIG. 34

The stream of air passing through the vane channel from inlet to outlet does not, as we have previously assumed, possess a uniform velocity which may be taken as the velocity along the mean line (shown dotted) drawn midway between the two vanes. On the contrary, it is found that the air at the leading edge of the channel, in the vicinity of the arrow *B*, has a higher velocity than the portion of the stream at the trailing edge of the channel, in the vicinity of the arrow *C*. This gives rise to the conception of a vortex formation in the vane channel, Fig. 34, the direction of rotation of the vortex being in the opposite sense to that of the impeller. It should not be inferred, however, that a particle of air entering the channel makes one complete circuit around the vortex, or eddy, centre before leaving the impeller, since the vortex is itself moving through the channel with the mean velocity of the stream, and such a particle would only traverse a very small portion of the hypothetical eddy path before being ejected from the impeller. It is important to realize that the vortex is simply a convenient fiction, the assumed existence of which renders the phenomenon capable of straightforward mathematical treatment, and has no existence in reality. It is, in fact, rather similar to the conception of 'circulation' around an aeroplane wing section.

The eddy velocity is the peripheral velocity of the vortex about its centre and is considerably influenced by the number of vanes on the impeller, because, the wider the vane channel, the bigger the hypothetical vortex, and hence the higher the velocity around the imaginary eddy centre. If the impeller had an infinite number of vanes, the channel width would be zero and consequently the eddy velocity would be zero, but since most impellers have a comparatively small number of vanes the eddy velocity is large enough to have a pronounced effect on the performance.

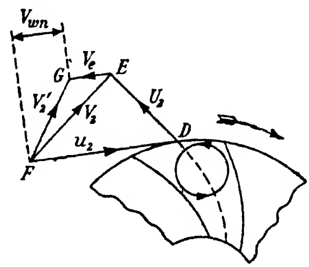


FIG. 35

Fig. 35 shows the velocity diagram drawn at the point where the mean line between the two vanes cuts the periphery of the impeller. The tip speed u_2, the relative velocity U_2, and the absolute velocity V_2 of the air leaving the impeller form the triangle of velocities *DEF* upon which our previous calculations were based. On this we must now impose the eddy velocity V_e, the direction of which is opposite to that of u_2. If *EG* is drawn, to scale, to represent the eddy velocity, then *FG* gives, to the same scale, the true value of the absolute velocity of the air, both in magnitude and direction. Let V_2' be the new value of the absolute velocity at outlet from the impeller. Clearly V_2' is smaller than V_2. Thus we see that the effect of the eddy velocity is to reduce the value of the absolute velocity of the air leaving the impeller, and to alter its direction.

Now the velocity of whirl at the impeller outlet V_{w2} is the projected length of V_2 on the line of action of u_2 and, due to the influence of the eddy velocity, this also is reduced. Let V_{wn} be the new value of the velocity of whirl. We showed that the work done on the air by the impeller was

$$\frac{W}{g} u_2 V_{w2} \text{ ft. lb. per sec.,}$$

but this now becomes

$$\frac{W}{g} u_2 V_{wn} \text{ ft. lb. per sec.}$$

Since V_{wn} is smaller than V_{w2} there is a reduction in the amount of work done per second on the air by the impeller, and the greater the eddy velocity, the greater the reduction. It follows from this that the smaller the number of vanes on the impeller, the smaller the amount of useful compression work that can be obtained from the impeller. It should be noted, however, that this is not a loss of work, but merely a limitation to the compression work that can be got out of the impeller, and a limitation to the pressure-ratio which can be obtained as per the equation

$$\frac{P_2}{P_1} = \left[\frac{u_2 V_{w2}}{gJC_p T_1} + 1\right]^{\frac{\gamma}{\gamma-1}}.$$

Thus it is a disadvantage to use a very small number of vanes, but, on the other hand, if a very large number of vanes is employed there is a considerable increase in surface area which increases the frictional resistance to air flow and lowers the adiabatic efficiency. In addition, the increased 'solidity' of the impeller renders necessary an increase in dimensions in order to accommodate a given mass flow of air. It is, therefore, necessary to effect a compromise in the number of vanes to be used, and twelve to twenty-four are probably the most common numbers found in practice.

Stodola's method for eddy velocity

A simple approximate method for the determination of eddy velocity was originated by Professor Stodola. While this does not give an exact result, it is an approximation sufficiently close for most practical purposes.

The peripheral distance p between the vane tips, Fig. 36, is known as the pitch of the vanes. Let a perpendicular CB be dropped from the tip of the leading vane to the trailing vane. This perpendicular is assumed to be the diameter of a circle which constitutes the path of the eddy motion (i.e. the eddy is assumed to be circular, and of outside diameter x) and its position in the vane channel is taken as that indicated by the diametral line CB. It is clear that this is not strictly correct, but it, nevertheless, forms a good working basis.

The next assumption is that the tip portions of the vanes are straight. Since their curvature is, in any case, relatively small near the tips, this is reasonable. It is also assumed that the pitch length AC is a straight line, which is justified by the fact that the distance p, measured along the periphery of the wheel, is little greater than the length of a straight line drawn from

A to C. The figure ABC is now an approximation to a triangle, in which the angle β_2 is known.

It is now necessary to determine the rotational speed of the vortex about its own centre. After one revolution of the

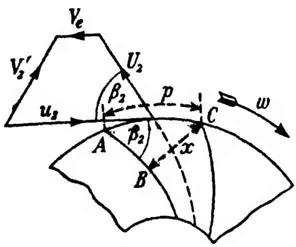


FIG. 36

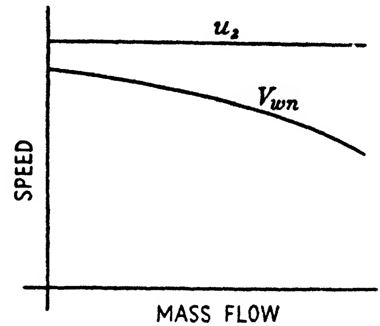


FIG. 37

impeller, the vane channel will return to its initial position, and the conditions within the channel will be the same as they were at the start of the rotation. Therefore, during a single revolution of the impeller, the eddy must make one revolution about its own centre. If the speed of rotation of the impeller is ω rad. per sec., then the rotational speed of the vortex about its own centre is also ω rad. per sec. Since the diameter of the vortex is x, the eddy velocity is

$$V_e = \frac{\omega x}{2}.$$

From the triangle ABC

$$x = p \sin \beta_2.$$

Therefore
$$V_e = \frac{\omega p \sin \beta_2}{2}.$$

If R_2 is the outside radius of the impeller, and if n is the number of vanes, then

$$p = \frac{2\pi R_2}{n},$$

and consequently

$$V_e = \frac{\omega}{2} \frac{2\pi R_2}{n} \sin \beta_2 = \frac{\pi u_2}{n} \sin \beta_2.$$

Kearton showed that, for a constant impeller tip speed u_2, the velocity of whirl V_{wn} decreases, Fig. 37, with increase in the mass flow of air through the impeller, the mass flow being controlled, for test purposes, by a valve placed in the delivery pipe.

The diffuser

The diffuser is the name given to the ring of fixed guide vanes B surrounding the impeller A, Fig. 38. In order to obtain a clear

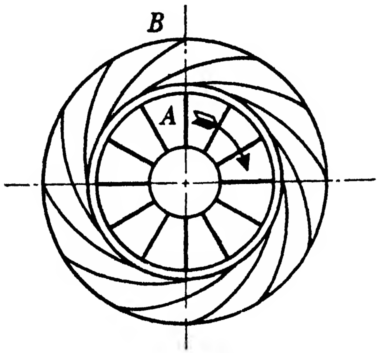


FIG. 38

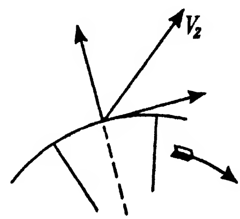


FIG. 39

conception of the behaviour of the air immediately it leaves the rim of the impeller, let us first consider what would happen if the diffuser were absent.

The air, on leaving the impeller, Fig. 39, has a certain absolute velocity V_2, which is its velocity relative to the compressor casing. This absolute velocity may be resolved into two components, one radial to, and the other tangential to, the rim of the wheel. If we consider the flow of 1 lb. of air, it follows from the laws of conservation of momentum that once this 1 lb. of air has left the impeller its angular momentum remains constant, since no forces are brought to bear which would cause a change. Now the angular momentum at any point is the mass of the air multiplied by the tangential component of the absolute velocity multiplied by the radial distance of the point from the centre of the impeller. The mass is constant and the momentum is constant. Therefore the tangential component of the absolute velocity must be inversely proportional to the radial distance from the centre. The radial component of the absolute velocity, at any radius, depends upon the circumferential area available for the air to flow through at that radius. If we assume, for the moment, that the density of the air remains constant after leaving the impeller, and that the axial width of the casing is constant, then the radial component of the absolute velocity, at any point, is also inversely proportional to the radial distance of the point from the centre of the impeller.

Therefore, since both components are inversely proportional to the radius, it follows that the absolute velocity, itself, is inversely proportional to the radius. In addition, the angle which the direction of the absolute velocity, at any point, makes with the tangent at that radius is constant. These are the conditions for a logarithmic spiral, and the path of the air leaving the impeller, under these conditions, is a logarithmic spiral, Fig. 40.

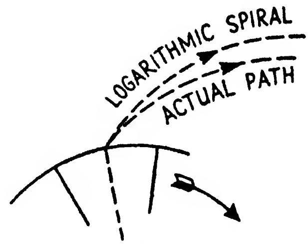


FIG. 40

The fact that the absolute velocity of our 1 lb. of air decreases more and more, the farther away from the impeller it gets, implies that its kinetic energy also decreases. This energy, however, is not lost, but goes into raising the pressure of the air. The absolute velocity of the air leaving the impeller is considerably greater than its velocity at the compressor inlet and, since it is usual to make the velocity in the delivery pipe the same as the velocity at the inlet, the amount of kinetic energy to be converted is by no means small.

Due to the increase in pressure, there is a reduction in the volume of the air, the compression being, ideally, adiabatic. This reduction in volume causes a progressive reduction in the value of the radial component of the absolute velocity, so that the radial component, at any point, is not actually inversely proportional to the radial distance of that point from the centre of the impeller. The tangential component, however, remains unaffected. The result of this is that the actual path of the air is not a true logarithmic spiral, but lies slightly inside the logarithmic spiral, Fig. 40. When diffuser vanes are fitted, they are so designed as to guide the air approximately along the logarithmic spiral, their curvature being that which is best suited to the natural path of the air leaving the impeller when the compressor is running under its optimum operating conditions. As the mass flow of air through the compressor is varied, the magnitude and direction of the absolute velocity of the air leaving the impeller vary, and consequently the shape of the diffuser vanes is not such as would afford guidance of the air along its natural path under all operating conditions.

One object of the diffuser is to provide a smooth canalization of the flow from the impeller, so as to reduce to a minimum the considerable amount of turbulence which would otherwise be set up in the volute casing. Another, and more important, object is that of compelling the air to flow through passages of increasing sectional area, thus reducing its velocity and causing a conversion of kinetic energy to pressure energy. But the action of a diffuser, in practice, is invariably accompanied by fairly high losses, which arise from the fact that, although it is easy to convert pressure energy to velocity energy with a high degree of efficiency, it is a far more difficult matter to carry out the reverse process efficiently. With the compressor running under its optimum conditions, the fixed vanes may be set at an angle at which the air is picked up smoothly, but over the length of the vanes the flow tends to break away from the surface, forming areas of turbulence which cause an increase in the air temperature, but which detract from the efficiency of the desired energy conversion. The frictional effect of the high-speed stream flowing through the diffuser channels is also an important contributory factor to the undesirable temperature rise.

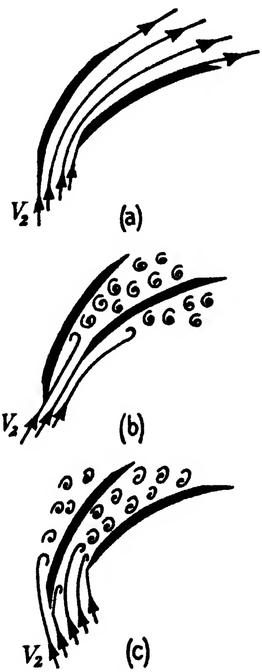


Fig. 41

As the operating conditions diverge from the optimum, the variation in the direction of the absolute velocity of the air leaving the impeller gives rise to a rapid increase in turbulence and causes a reduction in the efficiency of compression. This is illustrated diagrammatically in Fig. 41. In Fig. 41 (*a*) the absolute velocity V_2 of the air leaving the impeller is in the optimum direction and the flow through the diffuser channel is fairly smooth and approximately stream line. In Fig. 41 (*b*) and (*c*), however, the effect of variation in the direction of the absolute velocity is shown. The flow breaks away from the vanes, forming areas of violent turbulence which result in an undesirable temperature rise over and above the temperature rise which is normally consequent upon adiabatic compression.

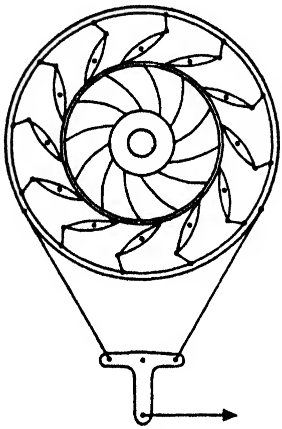


FIG. 42

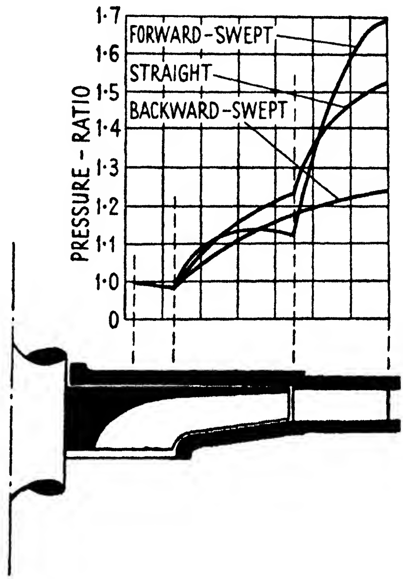


FIG. 43

The angle through which V_2 may be varied, without excessive loss of efficiency, is comparatively small, being generally only a few degrees on either side of the designed optimum direction. Some increase in the efficient operating range is afforded by the system of movable vanes developed by Messrs. Brown Boveri of Switzerland, and illustrated in Fig. 42, in which the angle of the vanes can be altered to suit varying conditions of flow from the impeller.

Typical curves of pressure distribution through compressors with back-swept, straight, and forward-swept vane impellers are shown in Fig. 43. The curves are derived on a theoretical basis, and take no account of the diffuser losses. It will be seen that the forward-swept type gives the highest pressure-ratio, the rise in the impeller being small and that in the diffuser being very large. With such a considerable absolute velocity of the air leaving the impeller, the diffuser losses, however, are extremely high and, due to the general inefficiency of the diffuser action, the overall efficiency of the compressor is low. The backward-swept type, although giving a lower pressure-ratio, has its greatest pressure rise in the impeller and, since the absolute velocity of the air entering the diffuser is much lower than in the forward-swept type, the compressor efficiency is relatively high. The straight-vaned impeller type falls roughly between the other two, with an almost equal pressure rise in impeller and diffuser.

Losses

In the foregoing articles mention has been made of most of the major losses which occur in the compressor during compression. Some of these vary considerably with the size and type of compressor. Bearing-friction losses, for instance, expressed as a percentage of the shaft horse-power input to the compressor, tend to decrease with increasing compressor capacity, but increase as the delivery pressure is raised because of the necessity for additional stages which cause an increase in the weight of the rotating elements and may call for the introduction of intermediate bearings. Again, disk-friction and leakage losses tend to decrease with increase in compressor size, the latter being also governed, to a considerable extent, by the delivery pressure. It should be borne in mind, however, that, apart from bearing-friction and leakage losses, the majority of the energy losses are not losses in the true sense of the word. That is, the 'lost' energy is not lost from the system, but goes into heating the air to a temperature higher than that which would be predicted by an assumption of perfect adiabatic compression, and it is, therefore, necessary to supply the compressor with a greater shaft horse-power input than would be required if the compression were perfectly adiabatic. It will be shown, in a later chapter, that the combustion turbine does actually obtain a small benefit from the additional heating due to the losses, and that the energy 'lost' in the compressor is not entirely wasted, but it should be emphasized that the benefit so obtained is small in comparison with the very great increase in plant efficiency which could be expected if a perfect adiabatic compression were possible. For these reasons, every effort is made to keep the compressor losses as low as possible.

The following table is very approximately representative of the nature of the energy distribution in a centrifugal compressor, each item being expressed as a percentage of the shaft horse-power input:

Bearing-friction	1·5 per cent.
Disk-friction	7·5 ,, ,,
Surface-friction	5·0 ,, ,,
Leakage losses	2·0 ,, ,,
Shock losses	6·0 ,, ,,
Useful adiabatic work	78·0 ,, ,,
Shaft input to compressor	100·0 ,, ,,

Intercooling

In multi-stage compressors some reduction in the actual amount of work required to carry out compression can be effected by the judicious cooling of the air between stages.

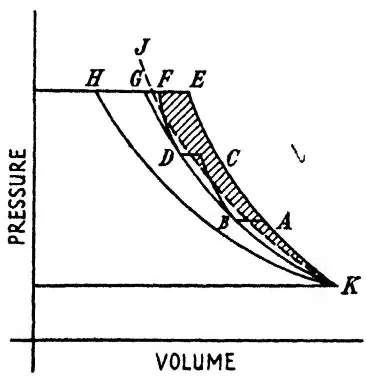
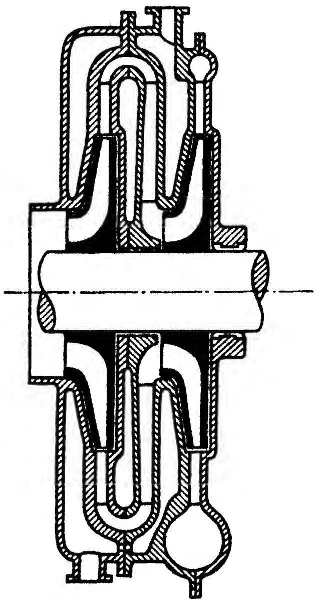


Fig. 44

In Fig. 44 three pressure-volume curves, KH, KG, and KE, respectively, are drawn. KH is an isothermal and is obtained from the expression $Pv =$ constant; KG is an adiabatic and is obtained from the expression $Pv^\gamma =$ constant; KE is the actual compression curve for an uncooled three-stage compressor, and is obtained from the expression $Pv^n =$ constant. Now suppose the air to be cooled between stages. Compression in the first stage takes place along the curve KA, and the air then enters the first intercooler where it is cooled (theoretically at constant pressure) along the line AB, with a consequent reduction in volume. At the point B it enters the second stage and is compressed along the curve BC. Then it enters the second inter-cooler and is cooled along the line CD. Compression in the third stage takes place along the curve DF and the compressed air enters the delivery pipe in the condition indicated by the point F. If a mean curve KJ, shown as a broken line in the diagram, is drawn through the stepped curve, the compression may be taken as conforming to this curve. The amount of work saved by intercooling is represented by the area between the mean curve *KJ* and the original curve *KE* for the uncooled compressor.

In theory the intercooling could be sufficient to continue the constant-pressure lines *AB* and *CD* to meet the isothermal curve *KH*, thus reducing the temperature of the air, between stages, to that at the compressor inlet. In practice, however, intercoolers have some serious disadvantages. To effect any considerable temperature reduction the intercoolers have to be large, and the air is forced to negotiate a tortuous path around a system of baffles and through banks of tubes carrying the flow of cooling water. This results in a pressure drop across the cooler which, to a great extent, offsets the advantages obtained by intercooling. Large coolers involve considerable bulk and expense, which in many cases may not be justified by the net benefit obtained from their use, and hence they have in the past found little favour amongst gas-turbine designers.



TWO-STAGE JACKET-COOLED COMPRESSOR

Fig. 45

A lesser benefit may be obtained by surrounding the compressor casing with a cooling jacket, through which a steady flow of water is maintained. This method gives a considerably more compact installation than can be obtained by the employment of intercoolers, but the amount of cooling which can be effected is very much smaller. A two-stage jacket-cooled compressor is shown in section in Fig. 45.

Compressor characteristics

Suppose a valve is located in the delivery pipe of a centrifugal compressor so that the delivery volume can be controlled by variation of the valve setting. If a number of readings of pressure and delivery volume are taken for a series of valve settings from the fully closed to the fully open position, and these readings of pressure are plotted against the corresponding readings of volume, or mass flow, the compressor running at a constant speed throughout the test, the curve so obtained is known as the characteristic curve for the compressor when running at that particular speed. If the speed is altered and another test carried out, a different curve is obtained. By this means a family of curves, indicating the characteristics of the compressor over the whole of its operating range, can be built up. It should be borne in mind that each curve is a constant-speed curve. One such curve is shown in Fig. 46. In practice it is not possible, for reasons explained below, to obtain the portion of the curve which lies to the left of the highest point, although it is possible to determine the pressure-ratio for zero delivery.

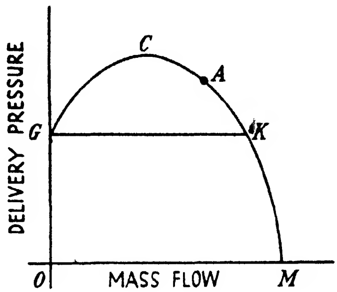


FIG. 46

It will be seen that the pressure rises from G, where the delivery is zero, to a maximum at C, and then drops to zero at M, where the delivery is very large, all the power being absorbed in circulating the air against the internal resistances. The efficiency is a maximum at the highest point of the curve, which corresponds to the maximum pressure-ratio, and the conditions in the neighbourhood of this point are the conditions under which the compressor is designed to operate. The efficiency is, of course, zero at the point of zero delivery and also at the point of zero pressure.

It is usual to design a centrifugal compressor to work at a point A on the characteristic curve, a little to the right of the highest point C. With the machine running at a constant speed, if the delivery is decreased by closing the valve in the delivery pipe the operating point A moves along the curve towards C. On the other hand, if the delivery is increased, the operating point moves along the curve away from C. Let us now imagine the slow closure of the valve so that the operating point moves to the left along the curve until it eventually coincides with C, the delivery pressure then being a maximum. If the valve is closed still further, so that the operating point moves to the left of C, then the pressure generated by the compressor drops, momentarily, below the pressure in the delivery system. This tends to cause a temporary backward flow of air from the delivery system into the compressor, and delivery momentarily

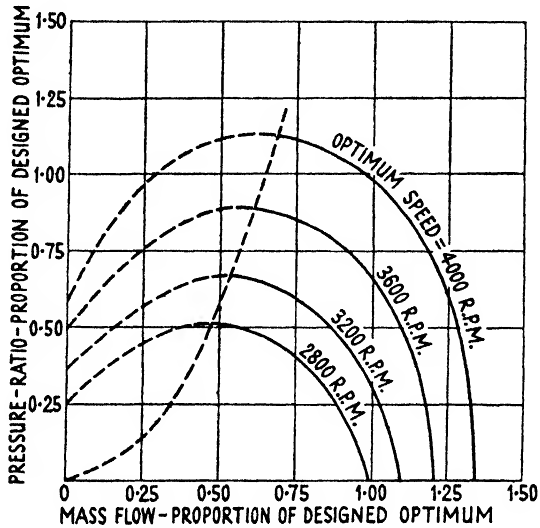


FIG. 47

ceases, the operating point moving to G. Since the demand for air still exists, the pressure in the delivery system rapidly falls until it reaches a value lower than that at G. The compressor once more begins to deliver, and the operating point jumps to K. At the point K, however, the delivery is greater than the demand, so that the operating point moves to the left along the curve until it once more coincides with the point C and delivery again ceases. The whole cycle of events is repeated with considerable rapidity, and represents an unstable flow condition. This phenomenon is known as 'surging', and explains why it is not possible to plot the portion CG of the curve. C is called the 'surge point'.

Four characteristic curves, each for a different compressor speed, are drawn in Fig. 47. Each curve has its surge point and, if the points are joined the curve so obtained is of parabolic form and passes through the origin. This curve is called the 'surge line', and a centrifugal compressor is always designed so that its operating point lies in the stable region to the right of the surge line. The position of the surge point is considerably influenced by the shape of the impeller vanes. With back-swept vanes the most efficient operating point is fairly far removed from the surge point, and lies well within the stable region. With straight radial vanes the best operating point is nearer the surge point but, since the characteristic curve is somewhat

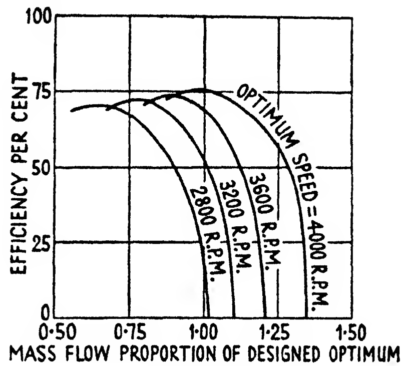


Fig. 48

flatter than that of the back-swept type, the compressor can be operated farther away from the surge point without undue loss of efficiency. With forward-swept vanes the best operating point lies within the surge region, and is a further reason why this type is never used.

If the compressor efficiency is determined for a series of settings of the valve in the delivery pipe, with the impeller running at a constant speed, and the efficiencies plotted against mass flow, the curve obtained is somewhat similar in form to the characteristic curve. As in the case of the characteristic curves, if the speed is altered different curves may be obtained. Four such curves are shown in Fig. 48. It will be seen that although, for each speed, the variation in efficiency with mass flow is considerable, the peak efficiency is roughly the same for each curve. It follows that if both the speed and the delivery are altered in the same ratio, the efficiency will remain approximately constant. In this case the pressure-ratio will vary as the square of the speed.

V

THE AXIAL-FLOW COMPRESSOR

A LONGITUDINAL section through an axial-flow compressor is illustrated diagrammatically in Fig. 49. Below this is a development of the fixed and moving blade-rings, and at the bottom of the figure is a curve of absolute pressure through the compressor, together with a curve of absolute air velocity in the fixed and moving rings.

Air is drawn into the system through the intake *A* in the direction of the arrows and enters the first row of moving blades B_1. These blades are attached to the periphery of the rotor *E*, mounted on the shaft *F* and, due to the rotation, have a mean tangential velocity in the direction shown. The blade inlet angle is so designed that the entry of the air is smooth and as shock-free as possible when the compressor is running under the designed optimum conditions, although, as will be shown later, it is not always possible to maintain the conditions such that the entry is shock-free over the whole operating range. In passing through the ring of moving blades the air is given a certain tangential velocity in the direction of rotation of the blades, so that it emerges from the ring with its original axial velocity parallel to the rotor axis plus a tangential velocity component. If these two velocities are added vectorially it will be seen that the resultant velocity, which is the absolute velocity of the air leaving the moving ring, is greater than the original absolute velocity of the air on entry to the ring, and that the direction of this absolute velocity is no longer parallel to the axis of the rotor, but is inclined to it at an angle. Thus, the absolute velocity of the air is increased due to its passage through the ring of moving blades.

On leaving the moving blade-ring the air enters the first row of fixed blades C_1, attached to the compressor casing *G*. The entry angle of these fixed blades is so designed that the air is picked up smoothly and without shock loss, which means that the inlet edges of the blades are in line with the direction of the air leaving the moving blade-ring. The function of these fixed blades is to guide the air back to its original direction parallel to the axis of the rotor, so that it enters the second moving blade-ring B_2 in the same direction in which it entered the first moving blade-ring B_1. In so doing, the tangential velocity component which the air acquired during its passage through the moving blade-ring is destroyed and the air leaves the fixed

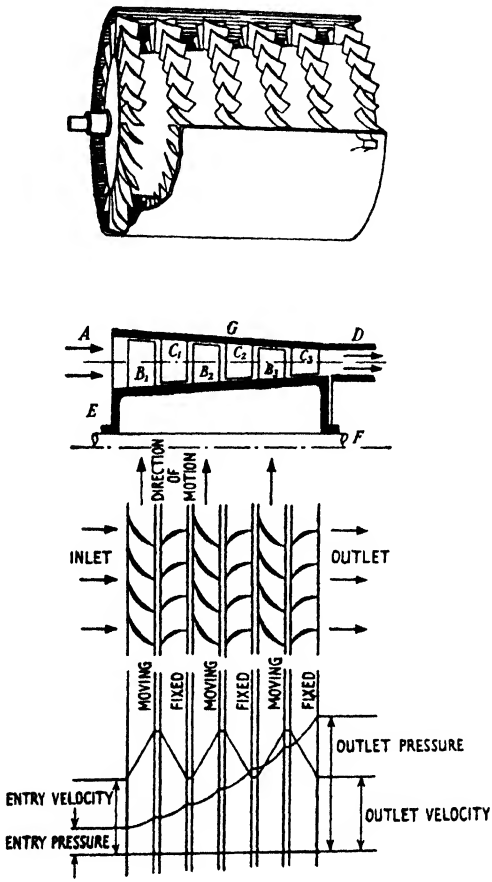


FIG. 49

blades in an axial direction with the same absolute velocity with which it entered the first ring of moving blades B_1. Thus, the absolute velocity of the air is decreased during its passage through the fixed blade-ring. The air then enters the second moving blade-ring B_2, passes to the second fixed blade-ring C_2, and the process is repeated until it finally reaches the compressor outlet D.

It will be observed that the changes to which the air is subjected are the same in each pair of blade-rings, consisting of one moving ring and one fixed ring. The combination of one moving and one fixed ring is called a 'stage' of the compressor, and the number of stages which a compressor possesses is governed by the required delivery pressure at the outlet D. The combination of one moving and one fixed blade is known as a 'blade pair'

We have stated that in the moving blade-ring the air velocity is increased, and that in the fixed ring this increase in velocity is destroyed and the absolute velocity restored to its original value. Consequently the air undergoes an increase in kinetic energy in the moving ring and a reduction in kinetic energy in the fixed ring. The velocity changes are brought out clearly by the velocity curve, Fig. 49. The energy which is thus imparted to the air and then apparently destroyed does not disappear, but is converted to pressure energy, so that as the air passes through the stage it experiences a pressure rise. There is a further increase of pressure in each subsequent stage, and if a pressure curve is drawn it will appear something like that shown in Fig. 49. The mean diameters of all the blade-rings illustrated in Fig. 49 are the same and both rotor and casing are progressively tapered from inlet to outlet to accommodate the reduced air volume due to the increasing pressure, on the assumption that the axial component of the flow velocity is constant over the whole length of the compressor.

The velocity diagram

Fig. 50 shows one blade in the first moving blade-ring. By virtue of the rotation of the rotor drum the blade CD is moving in the direction indicated with a velocity u ft. per sec. Let us imagine a particle of air approaching the blade-ring in a direction parallel to the axis of the compressor with an absolute velocity V_1 ft. per sec. along the line AC. When we first encounter the particle, it is at some position A in front of the blade-ring and at the same instant the blade is at position H, as shown dotted in the diagram. The air particle moves from A to C in a time t seconds, so that the distance AC is equal to $V_1\,t$ feet. In the same time the blade moves from H to C with a velocity u, and hence the distance HC is equal to ut feet. Now suppose some imaginary observer were situated on the blade at H as it moved towards C. If he were watching the approach of the air particle it would appear to him as though it were approaching the blade in a direction AH. Hence the velocity of the particle relative to the blade is given by the distance AH

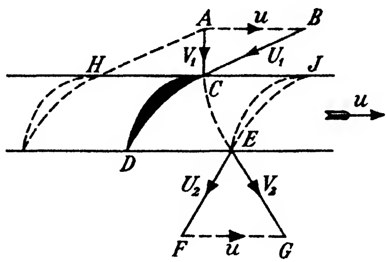


FIG. 50

divided by the time t which the particle takes to reach the blade-ring. Therefore the velocity of the air particle relative to the moving blade-ring is

$$U_1 = \frac{AH}{t}$$

and is in the direction AH. If BC is drawn parallel and equal to AH, and the triangle ABC completed by drawing the line AB, the sides of the triangle represent the distances moved by the air particle and the blade and the apparent distance travelled by the particle relative to the blade in a time t. These distances are proportional to the respective velocities of the particle and the blade and the velocity of the particle relative to the blade. Hence the triangle can be used as a velocity diagram.

The air particle then travels over the surface of the blade and finally leaves it to pass into the first ring of fixed blades, located immediately behind the moving blade-ring and not shown in the diagram. In the same time the blade itself moves from C to J, so that the air particle actually leaves at E. The true path of the particle is the dotted curve CE. The true path of the particle on leaving the blade is in the direction EG, and in an interval of time t it would cover the distance EG, with its new absolute velocity V_2, so that $EG = V_2 t$. Now its direction relative to the blade is along EF which is, of course, in line with the outlet edge of the blade and to an observer situated on the blade it would appear to depart with a relative velocity U_2. In the same interval of time t the blade itself would move a distance FG with velocity u, and clearly $FG = ut$. The apparent distance EF which the particle would move in time t gives it a velocity relative to the blade which is

$$U_2 = \frac{EF}{t}.$$

As in the case of the inlet conditions, the three sides of the triangle EFG represent the true distance travelled by the particle, the apparent distance travelled, and the distance

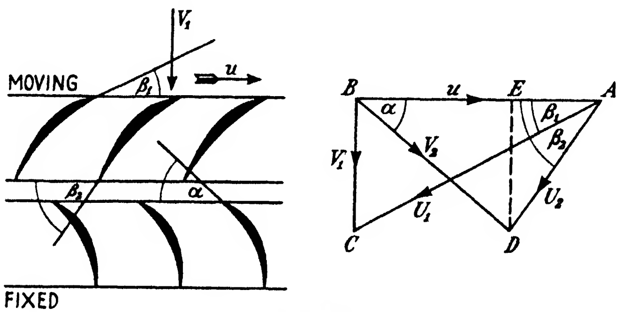


FIG. 51

travelled by the blade, in time t. The sides of the triangle are respectively proportional to the absolute velocity of the particle, its velocity relative to the blade, and the velocity of the blade itself. Hence the triangle can be used as a velocity diagram for the conditions at outlet from the blade-ring.

Fig. 51 shows one moving and one fixed blade-ring. Air enters the moving ring in an axial direction with an absolute velocity V_1 and the blades themselves are moving with a peripheral velocity u. The velocity diagram for the inlet and outlet conditions in the moving blade-ring can be conveniently combined, as shown in Fig. 51, to form a single diagram. Set off BA to represent the blade velocity u to any convenient scale, and then draw BC to represent V_1 to the same scale. Join A and C. Then AC represents, to the same scale, the velocity U_1 of the air relative to the moving blade-ring. Now ABC is a right angle and U_1 makes an angle β_1 with the plane of the blade-ring. In order that the entry of the air into the blade-ring shall be smooth and free from shock, the entry angle of the blades must be made equal to β_1 so that their leading edges are in line with the direction of the relative velocity.

The velocity diagram at outlet from the blade-ring may now be superimposed on the inlet diagram in the following manner: The blade velocity at outlet is, of course, the same as the blade velocity at inlet and therefore the vector BA can be used to represent the blade velocity u in the outlet triangle. The direction of the air velocity relative to the blades at outlet is determined by the blade outlet angle β_2 and the vector AD can be set off at an angle β_2 with BA, to represent the relative velocity U_2 to scale.

In the design of an axial-flow compressor it may be assumed that the axial rate of flow of the air is constant from intake to final delivery. That is to say, although the magnitude and direction of the absolute air velocity changes from point to point during the passage through the blade-rings, the axial component of the absolute velocity at every point is the same. Referring to the velocity diagram, Fig. 51, if we drop a perpendicular DE to cut AB at E, then the length of DE represents, to scale, the axial component of the absolute velocity V_2 at outlet from the moving ring. But if we assume that the axial component of the absolute velocity is constant throughout the compressor, we see that ED is equal and parallel to BC, and therefore the triangles ABC and ABD are of equal area.

It is not absolutely essential that the axial component of the absolute velocity should remain constant. It may, in fact, progressively increase or decrease through the compressor. But the rate of increase or decrease will be determined by the design of the compressor and the ratio of DE to BC will be known. The location of the point D can then be determined as above.

Join B and D to complete the outlet triangle. Since BA and AD represent to scale the velocities u and U_2, respectively, then BD represents, to the same scale, the resultant velocity V_2 of u and U_2. Clearly V_2 is the absolute, or true, velocity of the air at outlet from the moving blade-ring and its direction makes an angle α with the plane of the ring.

When the air is ejected from the ring of moving blades, it enters the fixed blade-ring. The entry of the air should be as shock-free as possible, and to ensure this the leading edges of the fixed blades should be in line with the direction of the absolute velocity of the air leaving the moving ring. Thus the entry angle of the fixed blades should be made equal to α. The outlet angle is, of course, 90°, so that the air leaves the fixed blade-ring in an axial direction.

The equation of continuity

This simple, but very important, equation will be referred to again and again in the study of the flow of compressible gases through the various component parts of the combustion turbine. Consider, for instance, the simple divergent duct of Fig. 52. A

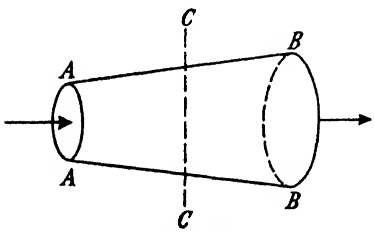


FIG. 52

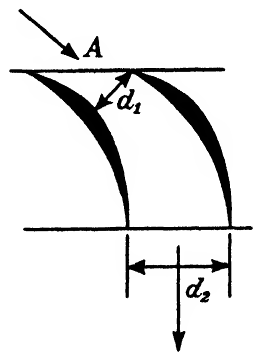


FIG. 53

stream of gas enters the section A–A, flows through the duct, and leaves at B–B. Now the gas must fill the duct at every point. That is, at any section such as C–C, the gas stream completely fills the section. Let a be the area of the section C–C, and let V be the velocity of the stream at that point. Then the volume of gas flowing through the section in unit time is equal to Va. If W is the weight of gas which flows past the section in unit time, and v is its specific volume, then the volume flowing through the section in unit time is also equal to Wv. Therefore

$$Va = Wv \quad \text{or} \quad V = \frac{Wv}{a}.$$

This is the equation of continuity, and for smooth flow through a duct, or channel, it must be satisfied at all points in the duct, or channel.

For instance, consider the blade-channel of a fixed blade-ring shown in Fig. 53. Air from the moving blade-ring enters the channel at an angle, as indicated by the arrow A, and the effective width of the channel is d_1. At outlet from the channel, however, the effective width is d_2, which is greater than d_1. The equation of continuity must be satisfied at both inlet and outlet. The mass flow is constant and the variation of a is known, so that in order to satisfy the equation there must be a variation in either V or v, or in both. In practice there is a suitable reduction in both velocity and specific volume and the equation is satisfied.

Work done per stage

In order to analyse the physical changes which take place in the air stream during its passage through the compressor, we shall investigate the occurrences in a single stage, and treat the whole compressor as a series of essentially similar stages. The amount of work done on the air in a given stage can be determined from a consideration of the momentum changes in the air stream during its passage through the stage, in the following manner:

The velocity diagram for the moving blade-ring is drawn in Fig. 54. The absolute velocity of the air at entry is V_1, in a direction parallel to the rotor axis, and the absolute velocity at outlet is V_2 in the direction BD, which is at an angle $90° - \alpha$ to the rotor axis. The magnitude of the absolute velocity is increased from V_1 to V_2 and its direction changed through an angle $90° - \alpha$. We have already stated that the axial component of the absolute velocity is the same at outlet as at inlet, so that ED and BC are equal and parallel. It follows that if C and D are joined, the line CD represents, to the same scale as the rest of the diagram, the additional velocity given to the air during its passage through the ring of moving blades, CD being equal in magnitude to the tangential component of the absolute velocity V_2. Since the axial component of the absolute velocity is assumed constant, it will be observed that the change in velocity is purely tangential. This tangential velocity is known as the 'velocity of whirl' and is denoted by V_w.

Let us investigate what happens to 1 lb. weight of air in its journey through the moving blade-ring from inlet to outlet. Since V_w is the change in velocity, and is measured in feet per second, then the change of momentum per second of the 1 lb. of air is

$$\frac{V_w}{g}.$$

The force exerted on the air by the blades is equal to the change in momentum per second (from Newton's laws) and is, therefore, equal to

$$\frac{V_w}{g}\ \text{lb.}$$

Now the blades, while exerting this tangential force on the air, are moving with a peripheral velocity u ft. per sec. Since work

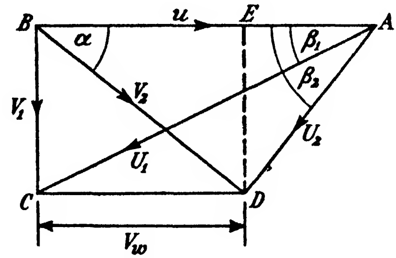


FIG. 54

done is the product of force and distance, the work done per second on the air by the blades is

$$\frac{uV_w}{g}\ \text{ft. lb. per lb. of air.}$$

If the total weight of air flowing through the blade-ring is W lb. per sec., then the total work done on the air by the blades is

$$\frac{WuV_w}{g}\ \text{ft. lb. per sec.,}$$

and the horse-power equivalent of the work done on the air is

$$\frac{WuV_w}{550g}\ \text{h.p.}$$

Referring to the velocity diagram, Fig. 54, the angle ABC is a right angle and, therefore,

$$\frac{V_1}{u} = \tan\beta_1.$$

Let
$$\frac{V_1}{u} = \rho.$$

Then
$$\rho = \tan\beta_1.$$

If we again refer to the velocity diagram we see that

$$V_w = u - U_2\cos\beta_2,$$

and since the axial component of the absolute velocity is assumed constant, then

$$ED = V_1 = U_2 \sin\beta_2.$$

From this
$$U_2 = \frac{V_1}{\sin\beta_2}.$$

If we substitute this value of U_2 in the equation for V_w, we get

$$V_w = u - \frac{V_1}{\sin\beta_2}\cos\beta_2$$
$$= u - V_1\cot\beta_2$$
$$= u\left\{1 - \frac{V_1\cot\beta_2}{u}\right\}.$$

Substituting ρ for V_1/u, we get

$$V_w = u(1-\rho\cot\beta_2).$$

If we now substitute this value of V_w in the expression for the work done per 1 lb. of air per second, we see that the work done is

$$\frac{u^2(1-\rho\cot\beta_2)}{g}\text{ ft. lb.}$$

Or since $\rho = \tan\beta_1$ the work done per 1 lb. of air per second may be stated as

$$\frac{u^2(1-\tan\beta_1\cot\beta_2)}{g}\text{ ft. lb.}$$

Now, ideally, once the blade form and blade angles have been decided upon, β_1 and β_2 are constants. This is not strictly true under all conditions of operation, and then other factors have to be taken into consideration, but for the present let us assume that they are constant, the direction of the velocity of the air relative to the blade-ring being always equal to the blade angles. From the final expression for the work done, it is clear that the blade velocity u is the only variable, and consequently the work done per second, and therefore the horse-power equivalent, is proportional to the square of the blade speed. That is, for any given compressor, the work done per second on the air is proportional to the square of the rotational speed. For the full flow of W lb. of air per second, the horse-power equivalent of the work done on the air per second may be written as

$$\frac{Wu^2(1-\tan\beta_1\cot\beta_2)}{550g}\text{ h.p.}$$

To quote an example, suppose the rotational speed of the compressor is doubled. Then, since the rate at which work is done on the air is proportional to the square of the rotational speed, four times the original work done per second will be done on the air, and four times the power will be required to drive the compressor.

Stage pressure-ratio

Air approaches the moving blade-ring, Fig. 55, in a direction parallel to the axis of the compressor, but its direction relative

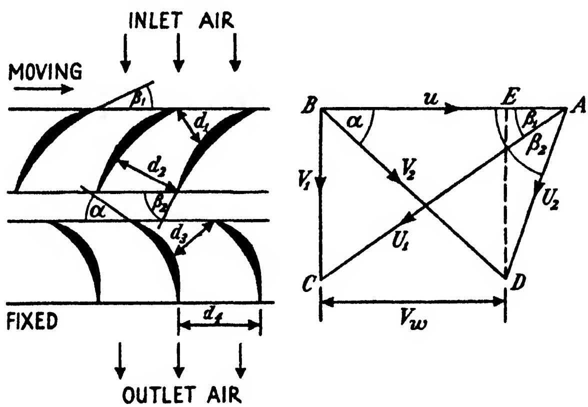


FIG. 55

to the blades themselves is given by the angle β_1, where $\tan\beta_1 = V_1/u$. The entry area available for the flow is proportional to the channel width d_1 perpendicular to the direction of the flow. The air passes through the channel from inlet to outlet and, due to the curvature of the blades, is turned through an angle, so that its direction relative to the ring at outlet is β_2. The angular change relative to the blade-ring is $\beta_2-\beta_1$. Now the area available for the flow at outlet is proportional to the channel width d_2 at outlet and d_2 is greater than d_1, so that the area perpendicular to the direction of the flow relative to the blade-ring at outlet is greater than that at inlet. Hence the channel is divergent.

The equation of continuity must be satisfied at all points between inlet and outlet, so that the whole section of the channel is completely filled with air at every point. It follows that either the relative velocity, the specific volume, or both these quantities must vary between inlet and outlet. In practice both the relative velocity and the specific volume decrease. This will be clear on reference to the velocity diagram, which shows that U_2 is substantially smaller than U_1. In addition to the reduction in relative velocity, however, there is a reduction in the specific volume of the air. Any reduction in specific volume implies an increase in pressure and there is, therefore, a pressure rise through the moving blade-ring. Putting this in another way, the air at entry possesses a kinetic energy equal to $U_1^2/2g$ ft. lb. per lb. relative to the moving blade-ring, but at outlet this is reduced to $U_2^2/2g$ ft. lb. per lb., due to the reduction in relative velocity from U_1 to U_2. The amount of energy released cannot simply vanish from the system, but is converted to pressure energy. Consequently there is a pressure rise through the blade-ring.

On leaving the moving ring the air enters a fixed ring. The inlet angle of the fixed blades is made equal to α so that they pick up the air stream smoothly. The function of the fixed blades is to guide the air back to its original axial direction of motion and to reduce the value of the absolute velocity to that which it possessed before entering the stage. Clearly the change in direction of the air stream through the fixed blade-ring is equal to $90°-\alpha$. As in the case of the moving blades, the channel width d_3 at entry is less than the width d_4 at outlet and, therefore, the sectional area available for the flow at outlet is greater than that at inlet. Due to the divergence of the channel from inlet to outlet, there is a reduction in the stream velocity such that the specific volume is also reduced, which, of course, implies a pressure rise.

Thus we see that there is a reduction in specific volume, with a rise in pressure, in both the moving and the fixed blade-rings. Air enters the stage axially with an absolute velocity V_1, and this is increased to V_2 in the moving ring. At the same time there is a reduction in relative velocity from U_1 to U_2, which results in a pressure rise. In the fixed ring the air stream has its axial direction restored and its absolute velocity reduced from V_2 to the original value V_1, again accompanied by an increase in pressure due to the conversion of kinetic energy. The air then passes to the second stage under the same velocity conditions with which it entered the first stage, but at a higher pressure.

Let

P_1 = absolute pressure at inlet to stage.
P_2 = absolute pressure at outlet from stage.
v_1 = specific volume at inlet to stage.
v_2 = specific volume at outlet from stage.
γ = adiabatic compression index.

If we assume that the compression is adiabatic, as it would be if there were no losses, then the compression work done per 1 lb. of air per second is equal to

$$\frac{\gamma}{\gamma-1}(P_2 v_2 - P_1 v_1) = \frac{\gamma}{\gamma-1} P_1 v_1 \left\{\frac{P_2 v_2}{P_1 v_1} - 1\right\}$$

and substituting for the volume ratio v_2/v_1, the expression for the work done per second can be written as

$$\frac{\gamma}{\gamma-1} P_1 v_1 \left\{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\}.$$

In the previous article we showed that the work done per 1 lb. of air per second in the stage was equal to

$$\frac{u^2(1-\rho \cot \beta_2)}{g}.$$

Now these quantities must be equal, so that

$$\frac{\gamma}{\gamma-1} P_1 v_1 \left\{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right\} = \frac{u^2(1-\rho \cot \beta_2)}{g}.$$

We can now isolate the pressure-ratio, as follows:

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 = \frac{(\gamma-1)u^2(1-\rho \cot \beta_2)}{g\gamma P_1 v_1},$$

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{(\gamma-1)u^2(1-\rho \cot \beta_2)}{g\gamma P_1 v_1} + 1,$$

and hence $$\frac{P_2}{P_1} = \left\{\frac{(\gamma-1)u^2(1-\rho \cot \beta_2)}{g\gamma P_1 v_1} + 1\right\}^{\frac{\gamma}{\gamma-1}}.$$

If we assume that the pressure and specific volume at inlet to the stage remain constant, we then have an expression for the stage pressure-ratio which involves only one variable, namely the mean blade speed u. In practice, due to the various unavoidable losses in the system, it is not possible to obtain a perfect adiabatic compression and consequently the compression index is at some value n, where n is greater than 1·4 and may be about 1·6. Rewriting the above expression, using n in place of γ, we get

$$\frac{P_2}{P_1} = \left\{\frac{(n-1)u^2(1-\rho\cot\beta_2)}{g\gamma P_1 v_1} + 1\right\}^{\frac{n}{n-1}}.$$

Fig. 56 shows two curves, derived from the above expression, of pressure-ratio against mean blade speed in feet per second, one for $n = \gamma = 1{\cdot}4$ and the other for a value of $n = 1{\cdot}6$. The

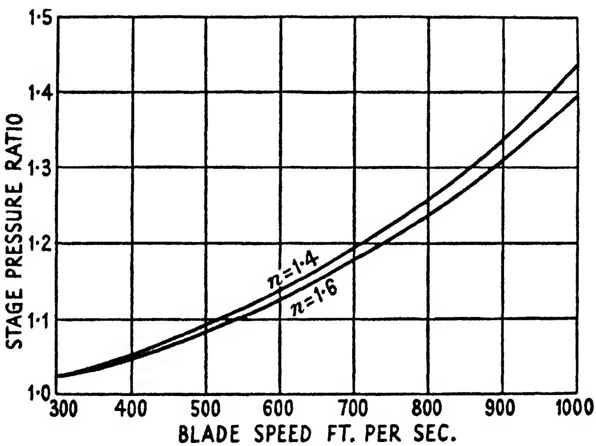


FIG. 56

inlet and outlet angles of the moving blades, β_1 and β_2, were taken as 30° and 40° respectively, and the inlet pressure was taken as 14·7 lb. per sq. in. Average pressure-ratios per stage are in the neighbourhood of 1·1 to 1·2, which is a considerably lower figure than can be obtained with the centrifugal compressor. On the other hand multi-stage axial-flow compressors are more compact and are, in general, less costly than multi-stage centrifugal compressors.

Work done in terms of the temperature changes

In the same way as for the centrifugal compressor, the work done on the air in the stage can be expressed in terms of the temperature changes consequent upon the compression.

Let

V_1 = absolute velocity at inlet to stage.
V_2 = absolute velocity at outlet from stage.
T_1 = absolute temperature at inlet to stage.
T_2 = absolute temperature at outlet from stage.
ρ_1 = air density at inlet to stage.
ρ_2 = air density at outlet from stage.
R = gas constant.
J = mechanical equivalent of heat.
C_v = specific heat of air at constant volume.
C_p = specific heat of air at constant pressure.

The other symbols used are the same as before.

The total energy possessed by 1 lb. of air at inlet is

$$\frac{P_1}{\rho_1}+\frac{V_1^2}{2g}+JC_vT_1,$$

and the total energy at outlet is

$$\frac{P_2}{\rho_2}+\frac{V_2^2}{2g}+JC_vT_2.$$

Now the total energy of the 1 lb. of air at outlet, assuming there are no losses, must be equal to the total energy at inlet plus the work done on the air during its passage through the stage. Therefore,

$$\frac{P_1}{\rho_1}+\frac{V_1^2}{2g}+JC_vT_1+\text{work done}=\frac{P_2}{\rho_2}+\frac{V_2^2}{2g}+JC_vT_2.$$

We are working on an assumption of a constant axial component of the absolute velocity through the stage, and we have stated that $V_1 = V_2$, bearing in mind that in this case V_2 is taken as the absolute air velocity at outlet from the stage, i.e. at outlet from the fixed blade-ring. This being the case, the above expression can be written as

$$\frac{P_1}{\rho_1}+JC_vT_1+\text{work done}=\frac{P_2}{\rho_2}+JC_vT_2,$$

and, therefore, the work done per 1 lb. of air per second is

$$\left\{\frac{P_2}{\rho_2}+JC_vT_2\right\}-\left\{\frac{P_1}{\rho_1}+JC_vT_1\right\}.$$

If v_1 is the specific volume of the air at inlet to, and v_2 the specific volume at outlet from, the stage, then

$$\frac{1}{\rho_1} = v_1 \quad \text{and} \quad \frac{1}{\rho_2} = v_2.$$

Hence the expression for work done becomes

$$(P_2 v_2 + JC_v T_2) - (P_1 v_1 + JC_v T_1).$$

Now from the gas laws,

$$P_1 v_1 = RT_1 \quad \text{and} \quad P_2 v_2 = RT_2,$$

and, therefore, the work done is

$$(RT_2 + JC_v T_2) - (RT_1 + JC_v T_1) = T_2(R + JC_v) - T_1(R + JC_v).$$

But from the gas laws,

$$R + JC_v = JC_p.$$

Substituting this in our expression for the work done, we see that the work done per 1 lb. of air per second is equal to

$$JC_p(T_2 - T_1).$$

If we are working in foot-pound-second units the work done is in foot-pounds per second. For the total mass flow of W lb. per sec., the work done is

$$WJC_p(T_2 - T_1) \text{ ft. lb. per sec.},$$

and the horse-power equivalent of the work done on the air is

$$\frac{WJC_p(T_2 - T_1)}{550}.$$

In the above, T_2 is the true outlet temperature in degrees absolute, whether the compression is a perfect adiabatic or not, provided, of course, no energy is lost from the system during the compression.

Adiabatic efficiency

The adiabatic efficiency of compression is the ratio of the amount of work which would be required to deliver the air at the stipulated pressure, assuming the compression to be a perfect adiabatic, to the amount of work actually required to deliver the air at the stipulated pressure. The adiabatic efficiency is denoted by the symbol η_{ad}.

In the previous article T_2 was taken as the actual absolute temperature of the air at outlet from the stage. Let T_2' be the theoretical absolute delivery temperature if the compression were a perfect adiabatic, i.e. $n = \gamma = 1{\cdot}4$. Then

$$T_2' = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \quad \text{and} \quad T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}.$$

Now the work done per 1 lb. of air per second can be written as

$$JC_p(T_2'-T_1)$$

for a perfect adiabatic compression. As shown in the previous article, the actual work done is

$$JC_p(T_2-T_1).$$

The adiabatic efficiency for the stage is equal to

$$\eta_{\text{ad}} = \frac{\text{ideal adiabatic work}}{\text{actual work done on air}}$$

$$= \frac{JC_p(T_2'-T_1)}{JC_p(T_2-T_1)}$$

$$= \frac{T_2'-T_1}{T_2-T_1}.$$

This is the same as the expression for the adiabatic efficiency of the centrifugal compressor. In practice a well-designed axial-flow compressor may attain an adiabatic efficiency of 85 per cent., when running under optimum conditions.

The mechanical efficiency of the compressor is

$$\eta_m = \frac{\text{horse-power equivalent of actual work done on air}}{\text{shaft horse-power input}}.$$

For the full flow of W lb. of air per second the horse-power equivalent of the work done on the air in the stage per second is

$$\frac{WJC_p(T_2-T_1)}{550}$$

and therefore the mechanical efficiency is

$$\eta_m = \frac{WJC_p(T_2-T_1)/550}{\text{shaft horse-power input to stage}}.$$

Due to the simple construction of the compressor and the fact that only two bearings, one at each end of the rotor, are employed the mechanical efficiency is very little less than unity.

The overall efficiency of the compressor is given by the product of the adiabatic and mechanical efficiencies, and is

$$\eta_{\text{overall}} = \eta_{\text{ad}}\,\eta_m.$$

A further expression for stage pressure-ratio

A further expression for the stage pressure-ratio, which is useful in some types of problem, can be obtained as follows:

We have shown that the work done on the air per 1 lb. of air per second is equal to

$$\frac{uV_w}{g} \text{ ft. lb.}$$

and is also equal to

$$JC_p(T_2'-T_1) \text{ ft. lb.}$$

assuming the compression is a perfect adiabatic. Equating these two expressions,

$$JC_p(T_2'-T_1) = \frac{uV_w}{g}.$$

Now
$$T_2' = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}.$$

Therefore
$$JC_p\left\{T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - T_1\right\} = \frac{uV_w}{g}.$$

We can now proceed to isolate the pressure-ratio, so that

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1 = \frac{uV_w}{gJC_p T_1}$$

and
$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \frac{uV_w}{gJC_p T_1} + 1.$$

Hence the pressure-ratio becomes

$$\frac{P_2}{P_1} = \left\{\frac{uV_w}{gJC_p T_1} + 1\right\}^{\frac{\gamma}{\gamma-1}}.$$

If we substitute for V_w, the expression becomes

$$\frac{P_2}{P_1} = \left\{\frac{u^2(1-\rho\cot\beta_2)}{gJC_p T_1} + 1\right\}^{\frac{\gamma}{\gamma-1}}$$

which gives us the pressure-ratio in terms of the mean blade speed u and the absolute temperature T_1 of the inlet air.

A further expression for adiabatic efficiency

We can obtain a further expression for the adiabatic efficiency of the stage, this time involving the pressure-ratio, in a manner similar to that employed in the theory of the centrifugal compressor.

We know that $$T_2' = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

and $$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}},$$

where n is greater than γ. Therefore

$$JC_p(T_2'-T_1) = JC_p T_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right\}$$

and $$JC_p(T_2-T_1) = JC_p T_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right\}.$$

We have already shown that the adiabatic efficiency is equal to

$$\eta_{\text{ad}} = \frac{JC_p(T_2'-T_1)}{JC_p(T_2-T_1)}$$

$$= \frac{JC_p T_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1\right\}}{JC_p T_1\left\{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1\right\}} = \frac{\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}-1}{\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}-1}.$$

If we let $r = P_2/P_1$, this can more conveniently be written as

$$\eta_{\text{ad}} = \frac{r^{(\gamma-1)/\gamma}-1}{r^{(n-1)/n}-1}.$$

Compressor characteristics

The characteristic curve of an axial-flow compressor is obtained when the delivery pressure is plotted against the mass flow, with the compressor running at a constant speed. The method of determination is similar to that employed for the centrifugal compressor.

A valve is fitted in the delivery pipe, or duct, and a series of readings of the delivery pressure is taken with the valve set at different positions, the compressor running at a constant speed. The inlet pressure to the compressor is kept at a constant value. Readings of the velocity of the delivery air are also taken. The resulting information enables us to plot a curve of delivery pressure against mass flow for the particular speed at which the compressor is running. If the rotational speed is now altered and another set of readings taken at the new constant speed, a fresh curve is obtained. In this manner a whole family of curves may be derived, each for a particular rotational speed, and these are the characteristic curves for the compressor.

Fig. 57 illustrates four typical curves for an axial-flow compressor, plotted for four different speeds. When the valve in the

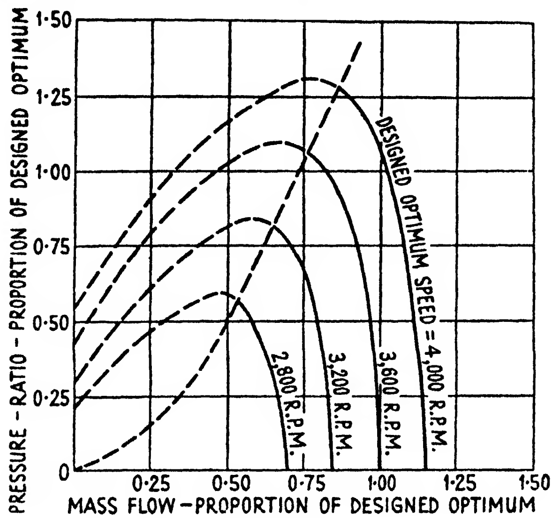


Fig. 57

delivery pipe is completely closed, the mass flow is zero, but nevertheless a certain pressure is generated against the face of the valve. As the valve is slowly opened the delivery increases, and the pressure rises until it reaches a maximum value, after which it begins to fall off with further opening of the valve. When the valve is wide open and there is unrestricted delivery straight to atmosphere, the pressure falls to zero, all the energy put into the compressor being used to circulate the very large volume of air against the considerable internal resistances which obtain when the compressor is running under conditions so far removed from the optimum. The efficiency is a maximum a little to the right of the highest point of the curve. The efficiency is zero at the point of zero delivery and also at the point of zero pressure.

Due to the fact that the compressor surges when operated at points to the left of the peaks of the curves, it is not possible to obtain readings for these portions, and the approximate positions of the portions of the curves which lie within the surge range are shown as broken lines. Characteristic curves for a centrifugal compressor are shown in Fig. 58, and it will be seen that these are very much flatter than the characteristic curves

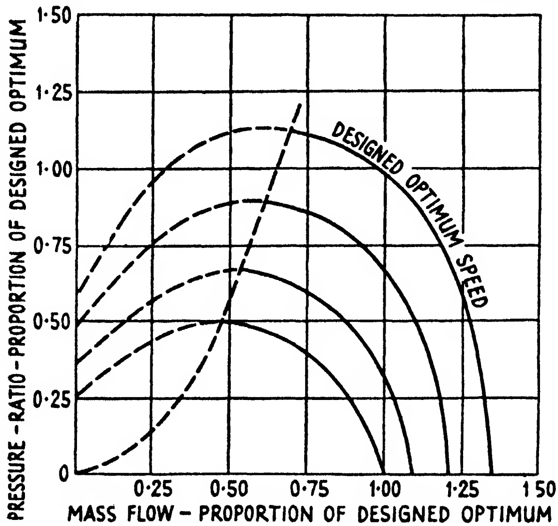


Fig. 58

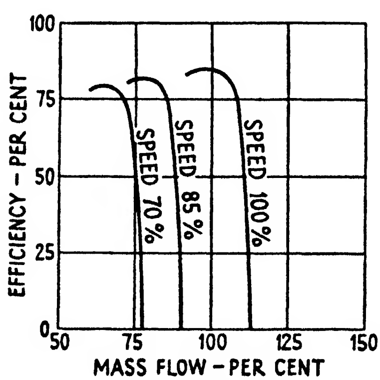


Fig. 59

for the axial-flow compressor. Due to the steepness of its characteristic curves, the range of efficient operation of the axial-flow type is rather limited, but on the other hand its maximum efficiency, when running under optimum conditions, is somewhat greater than that of the centrifugal compressor.

If, for each speed, the overall efficiency of the compressor is plotted against the mass flow, a series of curves such as those shown in Fig. 59[1] can be obtained. The curves fall away very steeply from the point of maximum efficiency, and it is clear that should the operating conditions differ by more than a comparatively small amount from the optimum, the drop in efficiency can be considerable. Some of the reasons for this

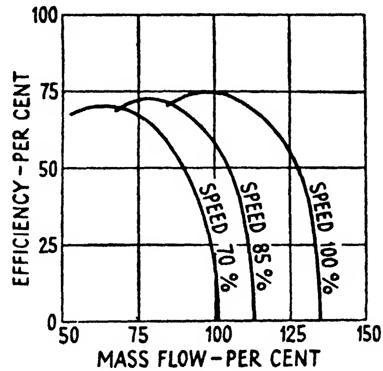


FIG. 60

sharp efficiency drop are discussed below. Some efficiency curves, plotted on the same basis, for a centrifugal compressor are shown in Fig. 60,[1] and it will be observed that they are very much flatter than those for the axial-flow compressor, although the maximum efficiency is somewhat lower.

Losses

Up to now, for the sake of clarity we have shown the blades, both moving and fixed, as being of simple curved section with tapered leading and trailing edges. Blade sections are, however, not quite as simple and straightforward as might have been inferred from previous diagrams. The actual sections used are rather similar to that illustrated in Fig. 61. The similarity of this section to the wing section of an aircraft will be immediately apparent, and indeed the behaviour of the blades of an axial-flow compressor is in many respects akin to that of certain aircraft sections. For this reason the study of blade design is closely allied to the theory of aerodynamics, and a detailed analysis of blade theory is outside the scope of this

[1] Meyer, 'The Combustion Gas Turbine—Its History, Development, and Prospects', *Proc. I. Mech. E.*, 1939.

work. Some of the salient features of blade behaviour are, however, described below.

Let us consider the blade section shown in Fig. 62 (*a*). The blade is attached to the rotor at such an angle that when the compressor is running with maximum efficiency at its optimum operating conditions the direction of the relative velocity U_1

FIG. 61

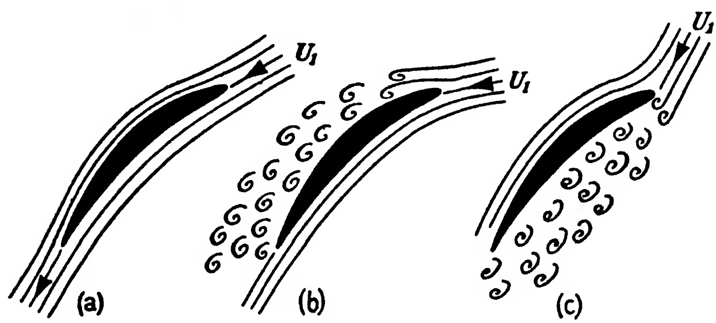


FIG. 62

at entry to the moving blade-ring is such that the stream is smoothly picked up by the blade and flows over the surfaces of the blade with, ideally, a perfect stream-line motion. The flow, in practice, is never of a perfect stream-line nature, but with good design a close approach to the ideal may be obtained.

With the compressor running at a constant speed, let us see what happens when the mass flow is suddenly decreased. Since the quantity of air passing through the compressor per second is now less, it follows that the axial velocity of flow V_1 is reduced and, since the blade speed u remains the same, the angle which the relative velocity U_1 makes with the plane of the blade-ring is decreased. If the variation in the direction of the relative velocity is small, there is little difference beyond a small reduction in the efficiency of the blade, but if the divergence from the optimum angle is large, the blade stalls and the reduction in efficiency is considerable. Fig. 62 (*b*) shows what happens when blade-stalling takes place. The air-flow tends to break away from the convex surface of the blade, and an area of turbulence is formed, the powerful vortices of which are a very considerable source of energy loss. This energy is not, of course, lost from the system, but the useful compression work is decreased and the adiabatic efficiency reduced.

Similarly, should the mass flow be increased beyond the optimum value, the axial flow velocity is increased, with a consequent increase in the angle which the relative velocity makes with the plane of the blade-ring, and stalling may take place in the opposite direction, as illustrated in Fig. 62 (*c*).

It follows that if good efficiency is required over more than a small range of operation, it is necessary to vary the compressor speed in conjunction with the variation in mass flow, in

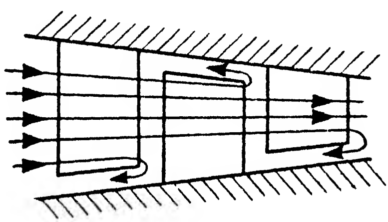


FIG. 63

order to maintain the direction of the relative velocity of the air-stream as nearly constant as possible.

A further source of energy loss is the leakage of air through the clearance spaces between the rotor blades and the casing and between the fixed blades and the rotor shell. The clearances are made as small as possible, but inevitably there is a certain amount of leakage, Fig. 63, since the pressure at outlet from a blade-ring is higher than the pressure at inlet to the ring, and there is a tendency for flow to take place down the pressure gradient in the reverse direction to the main flow within the blade channels.

The clearance spaces give rise to yet another type of loss. Fig. 64 shows three rotor blades moving in the direction of the arrow. The pressure on the concave side of the blade is slightly greater than that on the convex side, and consequently there is a tendency for air to flow over the tip of the blade from one surface to the other. This gives rise to a tip vortex, the magnitude of which is dependent upon the clearance. The vortex serves no useful purpose and is a source of energy loss. Again, it should be borne in mind that the energy is not lost from the

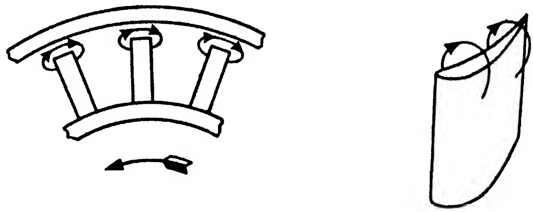


FIG. 64

system, but gives rise to an undesirable temperature increment and a reduction in the useful compression work.

Surging

The phenomenon of surging occurs in the axial-flow compressor in a manner basically similar to that in the centrifugal compressor. Fig. 65 shows a single curve of delivery pressure plotted against mass flow, with the compressor running at a constant speed.

Suppose the compressor is running at a point A on the curve, corresponding to a delivery pressure P_A and a mass flow Q_A. Let us imagine a valve installed in the delivery system, and let this valve be slowly closed so that the mass flow is progressively reduced. The operating point moves along the curve until it reaches the peak C, at which point the pressure is P_C and the mass flow Q_C. If the valve is closed still further, the operating point moves to the left of C and the pressure at the compressor outlet immediately drops below P_C. This drop in pressure does not instantaneously affect the pressure in the delivery system, which is then momentarily greater than the pressure at the compressor outlet. Consequently there is a tendency for the flow to reverse its direction. Delivery from the compressor immediately ceases, and the operating point moves to B. The air remaining in the delivery system is rapidly dissipated, causing the pressure in the delivery system to drop below P_B. When this happens, the operating point jumps to K and delivery recommences. The operating point then moves up the curve to A and continues to C, when delivery again momentarily ceases, and the whole process is repeated.

It follows that to the left of the point C the delivery is unstable, and the compressor surges. If a number of curves for different speeds are plotted on the same diagram, Fig. 66, their individual surge points may be joined by a curve which passes through the origin. This is the surge line, and the area to the

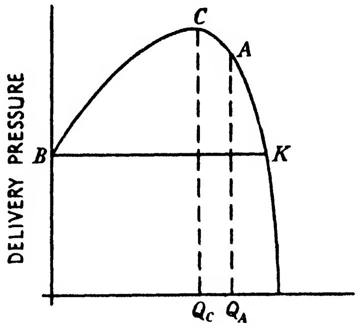


FIG. 65

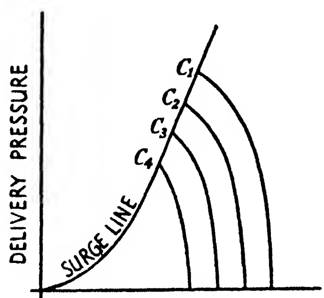


FIG. 66

left of the surge line represents conditions of unstable delivery. Consequently, a compressor should always be so designed that its operating point lies to the right of the surge line. It is for this reason that it is not possible to plot the portion of the characteristic curve which, in theory, is in the unstable region to the left of the surge line.

Combined axial and centrifugal compressor

In assessing the relative merits of the axial-flow and centrifugal compressors, the following factors stand out:

(*a*) The maximum efficiency of the centrifugal type is less than that of the axial-flow type.

(*b*) The characteristic curve of the centrifugal compressor is flatter than that of the axial-flow compressor.

(*c*) The stage pressure-ratio of the centrifugal type is about 2, whereas that of the axial-flow type is only 1·1 to 1·2.

(*d*) The axial-flow compressor is considerably more efficient and is probably more compact than the equivalent multi-stage centrifugal compressor, assuming the required pressure ratio is greater than can be supplied by a single-stage centrifugal compressor. On the other hand, if the required pressure-ratio is sufficiently low to be generated in a single stage, as a result of (*c*) above, the centrifugal type is considerably the lighter and more compact.

In weighing up the various conflicting factors for any given installation, it is almost inevitable that some compromise be made in the choice of compressor. It is possible, however, to obtain some of the advantages of both types by employing a compressor which combines a number of axial stages with a final centrifugal stage. Such a lay-out is shown in Fig. 67.

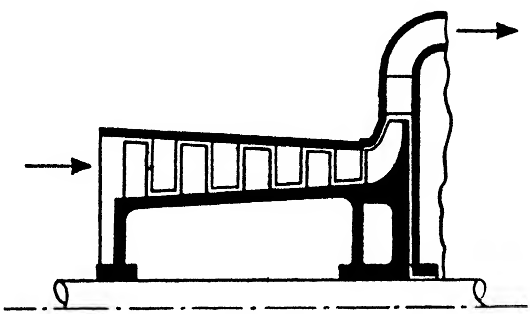


FIG. 67

The efficiency of such a compressor is clearly less than that of a purely axial-flow type, but is, on the other hand, greater than that of a purely centrifugal compressor. The characteristic curve of the combination cannot be as flat as that of the purely centrifugal type, due to the presence of the axial stages, but is, in a well-designed machine, unlikely to be as sharply peaked as the characteristic curve of the purely axial-flow compressor. For the pressure-ratios of 3:1 or 4:1 employed in current combustion-turbine practice, this type of machine is both light and compact. It represents possibly the best compromise between the conflicting factors, especially where economy of space is of primary importance as is the case in, for example, aircraft installations. Ideally, it is possible that the purely axial-flow compressor, with blades of variable angle on both rotor and casing, will become the predominating type in the future, but such a machine has yet to be developed.

VI

COMBUSTION CHAMBERS

THE careful design of the combustion chambers is a matter of the utmost importance, from an efficiency point of view, in the combustion turbine. The quantity of heat to be transferred to the moving air-stream is very large and must, of necessity, take place during a very short period of time. The transfer must therefore be smooth and as uniform as possible over the whole length and cross-section of the chamber, and the mixing must be good. Attention to detail in the design of the compressor and the turbine wheel can be largely offset by a badly designed combustion chamber, with a consequent serious reduction in the overall efficiency of the turbine. It should be constantly borne in mind that the machine is a constant-pressure continuous-combustion turbine, and this definition should at all times be closely adhered to, and each component carefully examined to ensure that the basic principles inherent in the definition are not contravened.

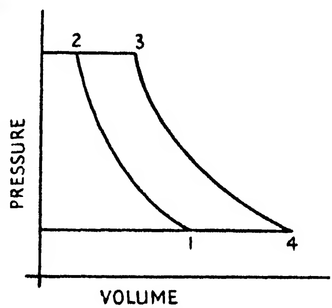


FIG. 68

The constant-pressure turbine cycle is illustrated in Fig. 68. When compression has taken place along the adiabatic 1,2 in the compressor, the working fluid enters the combustion chamber and heat energy is added to it along the line 2,3, causing an increase in specific volume at constant pressure. It is necessary to know just what happens when the fuel is burned in the stream and what effect the combustion has on the pressure, the volume, the density, and the velocity of the air-stream. If we know how the stream reacts to the addition of large quantities of heat energy under different conditions, then it will be possible to design a combustion chamber which will function efficiently under the particular conditions existing in the combustion turbine.

The straight duct

Let us consider the straight parallel-sided duct of Fig. 69. Air from the compressor enters the duct *A* at *B*, and passes from

C to the turbine nozzles. A burner D is inserted at the centre of the duct and is supplied with a steady flow of fuel to provide a continuous combustion.

At a first glance it might be assumed that the addition of a quantity of heat energy at D would cause the air to expand and move along the duct to the turbine nozzles with a considerably

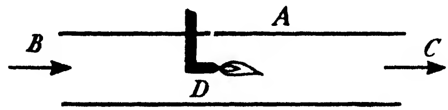


FIG. 69

increased velocity, but on closer examination it will be seen that this assumption is inaccurate and contravenes the basic physical laws. Newton's Second Law states that the change in the momentum of a body is proportional to the applied force, and his Third Law states that action and reaction are equal and opposite. It is clear that if the velocity of the stream is greater at C than at B, then between these two points it must have experienced an acceleration, and consequently an increase in momentum. To produce this increase in momentum there must be some force acting on the stream in the region of the burner D. Hence, in order to accelerate the stream, the pressure at D must be greater than the pressure at B, which contravenes our initial stipulation of a constant-pressure cycle.

Again, it is a design assumption that the pressure at B shall be the same as the pressure at C, and must therefore be lower than the pressure at D. Since action and reaction are equal and opposite, the pressure at D will act uniformly in both directions along the duct. This back-pressure, acting towards the point B, will cause a reduction in the mass flow through the compressor, which will then have to deliver against an increased head represented by the pressure at D. In short, a combustion chamber of this type would be extremely inefficient and would afford little useful increase in the total energy of the stream.

Effect of a sudden increase of section

Let us now examine a type of chamber in which the section of the duct is increased in order to provide a combustion space. Such a chamber is shown diagrammatically in Fig. 70. Air from the compressor enters at B and flows into the combustion chamber A, the cross-sectional area of which is greater than that of the duct B. The burner D is situated inside the chamber A, near the junction of the two ducts.

Let us first investigate what happens when a steady stream of air flows through the ducting with no combustion taking place.

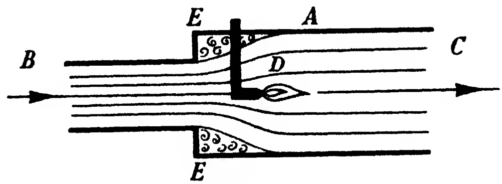


FIG. 70

The main flow will approximately follow the stream lines in the diagram, broadening out on reaching the junction and finally becoming parallel again before entering the turbine nozzles at C. If no fuel is being supplied, then, from the equation of continuity, the stream velocity in the chamber A will be less than the velocity in the duct B, and the ratio of the velocities will be inversely proportional to the ratio of the cross-sectional areas. If a_1 and a_2 are the cross-sectional areas of the duct B and the chamber A respectively, and if V_1 and V_2 are the corresponding stream velocities, then

$$a_1 V_1 = a_2 V_2$$

and

$$\frac{V_1}{V_2} = \frac{a_2}{a_1}.$$

If we now assume that the flow is of an incompressible nature, then clearly the reduction in momentum consequent upon the decrease in stream velocity will give rise to an increase in the static pressure in the chamber A. The change which takes place between B and C can be approximately represented by Bernoulli's theorem, which for a unit mass flow is

$$\frac{P_1}{\rho} + \frac{V_1^2}{2g} = \frac{P_2}{\rho} + \frac{V_2^2}{2g},$$

where P_1 and P_2 are the static pressures in duct B and chamber A, respectively, and ρ is the density of the air, which, on the assumption of incompressibility, is the same throughout.

It will be seen from the drawing, however, that at the points E, near the junction of the ducts, there is a very considerable amount of turbulence due to the breakaway of the stream from the walls of the duct. This turbulence represents a loss of energy, and is a potent source of inefficiency. The amount of energy lost in this way can be calculated and is sufficiently large to cause a reduction in the overall turbine efficiency.

Now let us suppose a steady flow of fuel is supplied to the burner D, and that combustion takes place smoothly and continuously. When the air reaches the burner it will be heated

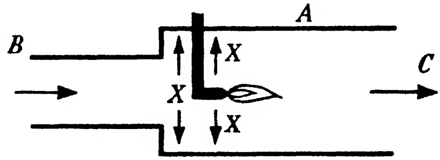


Fig. 71

and will expand, but since the section of the chamber A is greater than that of the duct B, the expansion will take place laterally, as shown by the arrows X in Fig. 71. Now the increase in static pressure in the chamber A was due to the reduction in the momentum of the stream, which in turn was caused by the decreased velocity in chamber A. The decrease in velocity was due to the fact that, in order to fill the whole cross-section of chamber A, the air had to expand laterally and contract longitudinally, its density remaining unaltered. But if we now supply sufficient heat energy to create the necessary lateral expansion by reducing the density of the air, then the velocity of the stream in chamber A will be the same as the velocity in duct B, since there is now no need for a longitudinal contraction in order to provide the lateral expansion. Under these conditions the static pressure in A will be the same as the static pressure in B, but the air density in A is less than that in B.

It is clear, therefore, that this type of combustion chamber fulfils the requirements of constant pressure and continuous combustion in a more or less efficient manner. The considerable turbulence at the sharp junction is, however, a by no means negligible source of energy loss, and detracts from the efficiency of the system. It should be noted that with the combustion of fuel, the flow is no longer of an incompressible nature.

Stating it in general terms, we may say that in order successfully to impart energy to the air passing through the combustion chamber, the sectional area of the chamber should be greater than that of the compressor delivery duct, and at maximum heat input there should be no change in the velocity of the stream. Under the same condition there should be no increase in the static pressure of the air passing through the chamber. In point of fact it is possible to increase the velocity of the stream in the combustion chamber by using a form of local heat engine, but this will be discussed later. For the present it is necessary to assume that, when fuel is burned in the air-stream, no change in velocity takes place.

The divergent duct

Let us now consider the nature of the flow through the divergent duct shown in Fig. 72. At the section *A–A*, the air

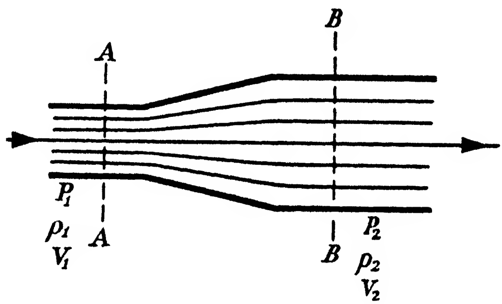


FIG. 72

will be at a pressure P_1 and will have a velocity V_1 and a density ρ_1. At the section *B–B* the pressure will be P_2 and the velocity and density will be V_2 and ρ_2 respectively. If the velocity is sub-sonic and we can assume an incompressible flow, then Bernoulli's equation states the nature of the flow which is

$$\frac{P_1}{\rho_1}+\frac{V_1^2}{2g}=\frac{P_2}{\rho_2}+\frac{V_2^2}{2g}.$$

Since the cross-sectional area of the duct is greater at *B–B* than at *A–A*, and the flow is incompressible, then the velocity at *B–B* will be less than the velocity at *A–A*. At the same time, from Bernoulli's equation, the static pressure at *B–B* will be greater than the static pressure at *A–A*.

But if now we burn fuel at a point near the throat of the duct, Fig. 73, we can cause a modification of Bernoulli's flow condition which can be applied to the design of a suitable combustion chamber. If the quantity of heat added to the air at this point is adequate for the divergence of the duct, there will be no change in the velocity of the air as it passes from the section *A–A* to the section *B–B*.

As the fuel burns during the passage of the air through the divergent part of the duct, the air expands laterally, in the direction of the arrows *X*, as it does when no heat is added. But

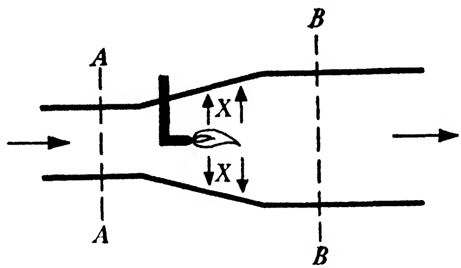


FIG. 73

in the case where no fuel is burnt in the duct, the lateral expansion takes place at the expense of forward velocity, so that there is actually a contraction in a longitudinal direction. This contraction implies a deceleration, which in turn implies, due to the reduction in velocity, that a force be set up. This force is manifested in the increased pressure in the larger duct section at *B–B*.

But when fuel is burnt in the divergent portion of the duct none of the original energy of the air need be used up in the lateral expansion, and the air may continue in its passage through the duct with undiminished velocity. The energy for the lateral expansion is, of course, supplied by the burning of the fuel, and since there is no alteration in the stream velocity in a longitudinal direction, there will be no alteration in the static pressure. This follows from the fact that there has been no change in the momentum of the stream. It should be noted that the flow is not now of an incompressible nature, because there is a reduction in the density of the gas. The reduction in density does not imply a reduction in static pressure, since the temperature is now considerably increased, and it should be borne in mind that pressure is a function of temperature as well as density.

Hence we see that this type of combustion chamber works on essentially the same principles as the chamber of Fig. 71, but due to the elimination of the sharp junction of the compressor delivery duct and the chamber itself, the violent turbulence is eliminated and the efficiency consequently improved. By this we mean that the resistance to flow is reduced to as low a value as possible in an endeavour to obtain a smooth flow. On the other hand, it must be borne in mind that the actual combustion

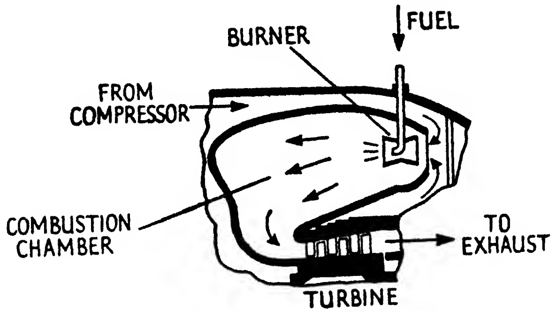


Fig. 74

takes place in a comparatively small region around the burner, or burners, the greater portion of the stream being heated by mixing of the hot products of combustion. The velocity of the stream is fairly high, and it is therefore of some importance to ensure that the mixing is good and that the whole cross-section of the stream is uniformly heated before entering the turbine nozzles. A certain measure of turbulence is necessary in order to achieve this aim, and successful designs must be a compromise between the minimum amount of turbulence required for adequate mixing and the maximum that can be permitted without an excessive loss of efficiency.

Examples of this type of combustion chamber are the Lemale chamber of Fig. 88, described later in this chapter, and the combustion chamber of the proposed Milo aircraft turbine, part of which is shown in Fig. 74. The divergent duct need not necessarily be of truncated conical form, for this assumes a uniform heating of the stream through the whole length of the divergent portion, whereas the actual effect of the heat transfer will be dependent upon the nature and location of the fuel nozzle, or nozzles, the rate of flow of the stream, and the quality of the mixing.

In aircraft turbines employing an annular combustion chamber taking its air supply from the entire periphery of the diffuser of a centrifugal compressor, a distinction should be drawn between the actual and apparent divergencies of the duct. Such a turbine is shown diagrammatically in Fig. 75. Air from the impeller *A* enters the ring of diffuser vanes *B* which guide it along a path which is approximately a logarithmic spiral, so that when it enters the combustion chamber *C* its

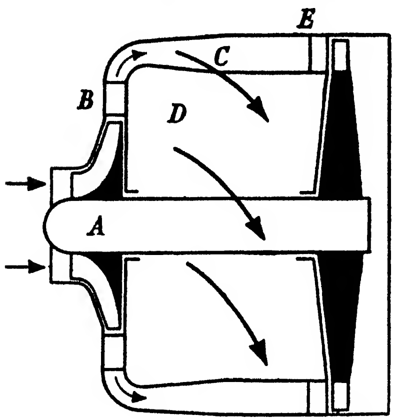


FIG. 75

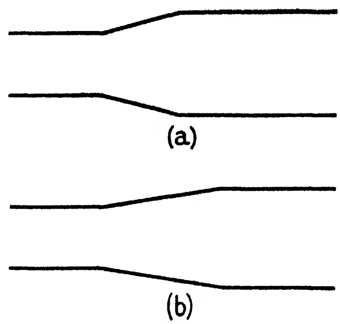


FIG. 76

path is not parallel to the axis of the turbine. It will, in fact, describe a helical path around the circular core *D* of the chamber, as shown by the arrows, before entering the nozzle ring *E*.

Thus the length of the actual path of the air through the chamber is greater than the distance, measured parallel to the axis, between the diffuser and the nozzle ring. Fig. 76 (*a*) is a section through the combustion chamber parallel to the axis of the turbine, whereas if a section along the actual path of the air is developed, Fig. 76 (*b*) will be obtained. Thus, to obtain the required divergent angle along the actual path of the air-stream the angle of Fig. 76 (*a*) must be greater than that of Fig. 76 (*b*). The net effect of the helical air-flow in such an installation is to render possible a shorter and more compact combustion chamber without sacrificing the length required for complete combustion and mixing, no small advantage from the point of view of aircraft installations where compactness is of primary importance.

The effect of combustion

We have described above how the addition of energy to the air in the combustion chamber results in an increase in the volume of the air. It is the ultimate aim to convert as much as possible of the total energy of the moving air stream into velocity energy. All this final velocity energy may be used to rotate the turbine wheel, or, alternatively, only a part may be used to rotate the turbine wheel, and the remainder ejected

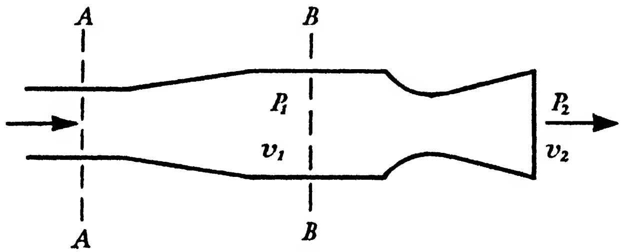


FIG. 77

through a suitable orifice in the rear of an aircraft to provide forward thrust. A nozzle is used to convert the energy of the air into velocity energy before it passes over the turbine wheel, and it remains to be seen how this conversion takes place in the nozzle.

Let us assume, for the present, that all the energy is used in driving the turbine, the useful work of the plant being taken out in the form of shaft horse-power. Fig. 77 gives a diagrammatic illustration of the combustion chamber and one of the turbine nozzles. Since the velocity of the stream in the duct at B is small compared with the final exit velocity from the nozzle, we will assume that it is zero.

Let:

P_1 = Absolute pressure in the combustion chamber.
v_1 = Volume of 1 lb. of gas in the combustion chamber.
P_2 = Atmospheric pressure.
v_2 = Volume of 1 lb. of gas after expansion through the nozzle.
V_n = Exit velocity of the gas from the nozzle.
n = Expansion index for the nozzle.

Then the nozzle velocity will be

$$V_n = \sqrt{\left[2g\left(\frac{n}{n-1}\right)P_1 v_1\left\{1-\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}}\right\}\right]}.$$

How this expression is derived is shown in the chapter on nozzle theory and design, but for the present let us take it for granted. It will be seen from this expression that the nozzle velocity depends on the ratio P_2/P_1, and the smaller the value of this fraction the higher will be the nozzle velocity. If the value of $P_1 v_1$ is increased by increasing P_1, then the velocity will benefit, not only by the increase in the value of $P_1 v_1$ but also by the reduction in the value of the fraction P_2/P_1. But the product $P_1 v_1$ may also be increased by increasing the value of v_1. The beneficial effect on the velocity is not so great in this case, because the value of P_2/P_1 is not reduced.

Hence, from the expression given above for the nozzle velocity, we can say that an increase in nozzle velocity may be obtained by heating the air in the combustion chamber so that it undergoes an increase in volume at constant pressure. It is this increase in volume which supplies the useful work energy of the constant pressure cycle.

Combustion-chamber proportions

We now wish to know what dimensions, or proportions, our divergent duct must assume in order that the air-stream may absorb the heat energy of the fuel under the conditions defined above.

Let:

a_1 = Cross-sectional area at A–A.
a_2 = Cross-sectional area at B–B.
P = Pressure in the system.
ρ_1 = Density at A–A.
ρ_2 = Density at B–B.
V = Constant flow velocity through the duct.
H = Heat energy added to the air in thermal units.
J = Mechanical equivalent of heat.

Then, equating the total energy of the stream per pound of air at A–A, to the total energy at B–B, we get

$$\frac{P}{\rho_1}+\frac{V^2}{2g}+JH = \frac{P}{\rho_2}+\frac{V^2}{2g}+JC_v(T_2-T_1). \qquad (1)$$

Now the quantity of heat, H, which must be added to the air-stream is determined by the amount of energy required to drive the turbine and will have been previously calculated. Similarly, the pressure P and the velocity V with which the air leaves the compressor casing will have been determined during the design of the compressor. The density of the compressed air ρ_1 before combustion is known, but the density ρ_2 after combustion is a

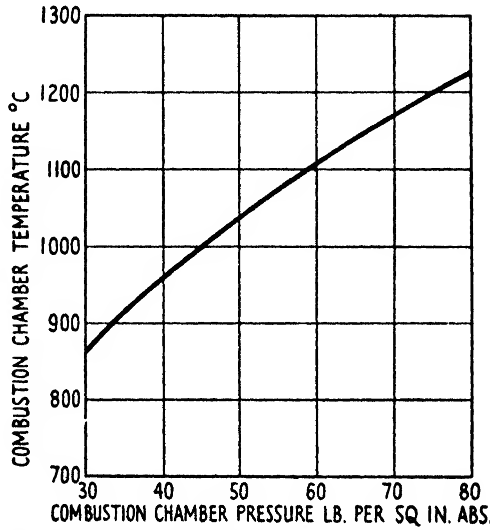


FIG. 78. Maximum combustion chamber temperature to maintain impulse wheel blades at 650° C. at sea-level.

variable factor depending upon the temperature to which the air is heated.

Equation (1) becomes

$$\frac{P}{\rho_1}+JH=\frac{P}{\rho_2}+JC_v(T_2-T_1).$$

Therefore
$$\frac{1}{\rho_2}-\frac{1}{\rho_1}=\frac{J}{P}\{H-C_v(T_2-T_1)\}.$$

Let the absolute temperature of the air leaving the compressor be T_1. The temperature to which the air is to be raised by the burning of the fuel is determined by the temperature at which the turbine blades will run and is calculated from the pressure drop through the turbine nozzles, as illustrated by the curve, Fig. 78. This temperature is therefore known and will be T_2° absolute. If v_1 is the volume of 1 lb. of air before combustion, and v_2 the volume of 1 lb. of air after combustion, then

$$v_2=\frac{v_1T_2}{T_1}.$$

If W lb. of air are passed through the duct per second, then

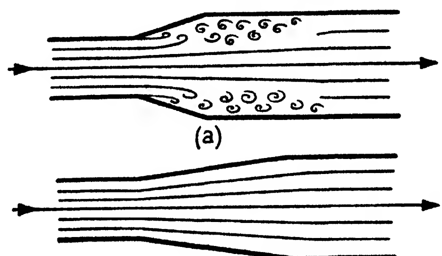
$$Wv_2 = a_2 V = \frac{Wv_1 T_2}{T_1}.$$

But $$Wv_1 = a_1 V.$$

Therefore $$a_2 V = a_1 V \frac{T_2}{T_1}$$

and $$a_2 = a_1 \frac{T_2}{T_1}.$$

If we assume that the burning of the fuel is uniform through the divergent duct, this expression will give the greatest cross-



(a)

(b)

FIG. 79

sectional area of the duct. It is one of the objects of combustion-chamber design to place the fuel injection nozzles so that the combustion in the divergent duct will be smooth and progressive.

As previously stated, the ideal divergent angle of the duct must be so chosen that combustion and expansion take place in the shortest practicable time, and yet without excessive turbulence. If the angle is too great the air flow will break away from the duct wall, forming a region of violent turbulence, as in Fig. 79 (*a*), whereas in a well-designed chamber it will approximate to the stream-line flow indicated in Fig. 79 (*b*). Again, emphasis must be placed upon the importance of a suitable compromise so that the turbulence is sufficient for good mixing. This is largely a matter of experiment.

The Brown Boveri burner

We have assumed up to this point that both the pressure and velocity of the air, during its passage through the combustion chamber, must remain constant, but we mentioned previously that there was a possible method of speeding-up the air in the duct, without affecting the pressure, by using a form of 'local heat engine'. A simple divergent-duct combustion chamber, such as that we have discussed, while being excellent in theory, will, in practice, have a number of disadvantages. In the first place, it is bulky and, from the point of view of aircraft installations in particular, it will entail a most undesirable increase in

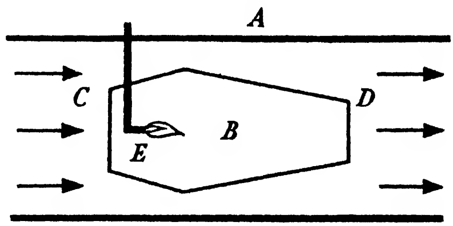


FIG. 80

duct diameter. Secondly, it is not conducive to good mixing. It is most desirable that the burning of the fuel should heat the whole cross-section of the stream as uniformly as possible, not only to avoid the undesirable turbulence which would otherwise be set up, but also to comply as nearly as possible with the conditions of constant pressure and constant velocity. To ensure very good mixing in such a large diameter duct, it might be necessary to provide a complicated system of fuel injectors distributed around the periphery and over the whole length of the divergency. Thirdly, the surface area is large, and the amount of heat lost by radiation is considerable, especially since, in aircraft practice, considerations of weight and accessibility for maintenance purposes prohibit any great amount of lagging. If the velocity of the stream can be increased, it is desirable to do so in order to reduce the dimensions of the duct and to prevent, as far as possible, the loss of heat energy by radiation.

A means by which the above objects are to a certain extent achieved has been originated by Messrs. Brown Boveri of Switzerland. This consists of the introduction of a 'local heat engine' into the combustion-chamber duct. This heat engine is similar to the 'ducted radiator' propulsion motor described in Chapter XII.

Fig. 80 shows the method of installation. The divergent-convergent shroud *B* is located inside the main duct *A* conveying the compressed air from the compressor to the turbine. A part of the main stream enters the shroud at C, and experiences a Bernoulli pressure rise. The main burner E is located inside the mouth of the shroud and combustion takes place at constant pressure. But it should be observed that this pressure is higher than the pressure in the main duct A. After combustion the gas

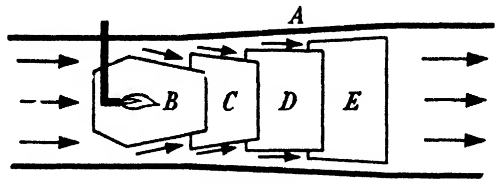


FIG. 81

leaves the shroud at D with a velocity which is greater than the entry velocity at C. Its pressure is then the same as the pressure in the main duct A. The fast-moving stream leaving the shroud mixes with the air in the main duct and the velocity of the whole mass is increased at the expense of the velocity of the jet from the shroud. As the mixing takes place, the hot gases from the shroud lose some of their heat energy to the air in the main duct, and the mean temperature of the whole mass is increased. Thus the entire mass of gas, in its passage from the compressor to the turbine nozzle, experiences an increase both in velocity and temperature.

To improve the mixing, the shrouding arrangement shown in Fig. 81 is superimposed on the simple duct B. This consists of the series of truncated conical ducts C, D, and E, of successively increasing diameter. Air from the main stream in the duct A enters, and mixes with, the high-speed stream from the shroud B through the annular slits, thus ensuring good mixing and a smooth increase in the mean velocity of the whole stream, with turbulence reduced to a minimum. A drawing of the Brown Boveri burner, as used in a combustion-turbine locomotive supplied to the Swiss Federal Railways, is shown in Fig. 82.[1]

This type of burner is more economical than the simple divergent-duct combustion chamber, described earlier in this chapter, in that the radiation losses are lower. The shroud system is surrounded by a stream of moving air which cools the

[1] Meyer, 'The First Gas Turbine Locomotive', *Proc. I. Mech. E.*, 1943.

burner shroud and at the same time increases its own temperature. Thus, over almost the whole length of the combustion chamber, the shrouds are surrounded by an insulating layer of air which considerably reduces the radiation loss from the walls of the main duct *A*.

If a stream of gaseous fluid is moving along a smooth duct

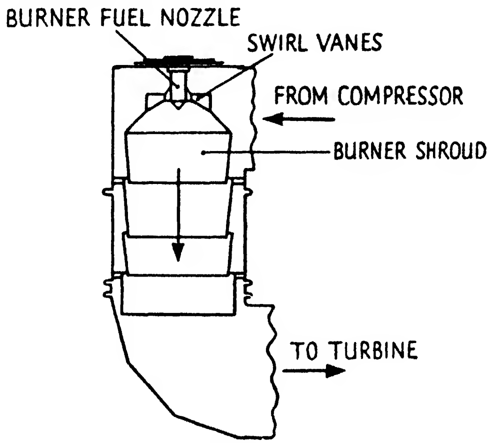


Fig. 82

and enters a portion of the duct which is of greater cross-sectional area, the divergence being sufficiently gradual to retain stream-line flow, then the fluid will experience an increase in static pressure and a decrease in velocity, according to Bernoulli's theorem. We have shown, however, that if the correct amount of heat energy is added to the gaseous fluid during its passage through the divergent portion of the duct there will be no change in the pressure or the velocity of the stream. But in the case of the burner shroud *B* it is both necessary to add heat energy to the stream and to obtain a pressure rise.

If no fuel were burned in the system, and if a given pressure rise only is required, then the duct *A* (Fig. 83) would be needed. On the other hand, if both pressure and velocity were to remain constant during the burning of the fuel, duct *B* (Fig. 83) would be required. If now both a pressure rise and a combustion of fuel are required, then a duct of greater transverse dimension at *D–D* than either *A* or *B* is necessary. This is shown at *C*. If the sectional area at *D–D* of duct *A* is greater than the entry area Q by an amount a, and if the sectional area at D–D of duct B is greater than Q by an amount b, then the sectional area at D–D of duct C must be greater than the entry area Q by an amount equal to $(a+b)$. The length of duct C, however, may have to be greater than that of duct A since, if the divergence

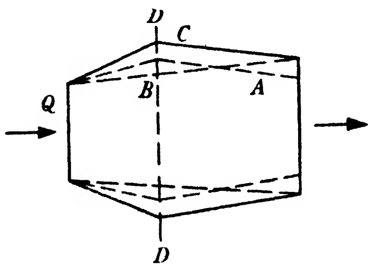


FIG. 83

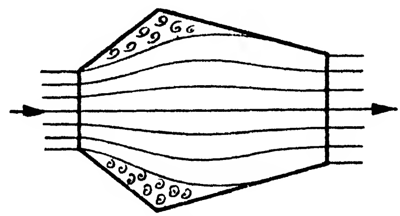


FIG. 84

is too rapid, the smooth flow will break away from the duct wall and set up an area of turbulence, as shown in Fig. 84, which will adversely affect the flow conditions.

Excessive turbulence within the ducting considerably increases the resistance to fluid flow, upsets the flow conditions, and increases the amount of work required to force the air through the system to the turbine nozzles. On the other hand, a turbulent flow gives a rapid and efficient mixing, with a consequent good distribution of heat energy over the whole cross-section of the duct. Here again it is necessary to achieve a compromise between the degree of turbulence which can be permitted and the quality of the mixing. A successful compromise may be attained as the result of experiment, and no definite ruling can be made at this stage. Too little is known of the actual process of combustion, and much research remains to be carried out on combustion-chamber form before design becomes as stereotyped as that of the cylinders of modern internal-combustion engines.

It should be emphasized that the use of a 'local heat engine' within the main duct in order to speed up the air flow does not imply that the main duct is parallel-sided. On the contrary, since the stream emerging from the burner shroud is at a considerably higher temperature than the main air flow, the subsequent mixing will cause an expansion which will necessitate a divergence of the main duct, the amount of divergence in any particular design being dependent on the shape of the burner shroud. The temperature of the gases leaving the burner shroud may be determined by the theory of Chapter III, which deals with this type of constant-pressure cycle.

Whether or not the 'local heat engine' type of shroud is used, it is invariably an advantage, from the point of view of good mixing, to employ a shroud around the burner, as illustrated in Fig. 85. The dimensions of the shroud should be such that the quantity of air entering the shroud is a little greater than that required for a chemically correct fuel-air mixture. This gives a rapid combustion, the products of which emerge from the shroud C and mix with the main body of the stream, raising its temperature and causing it to expand through the divergent portion of the main duct A.

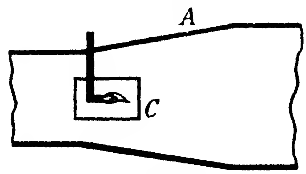


FIG. 85

Combustion-chamber construction

The problem of providing suitable materials for the combustion chambers is a difficult and complex one. Most metals suffer a reduction in tensile strength with considerable increase in temperature, a fact which, in the case of aircraft turbines, means a most undesirable increase in weight, although this consideration is of smaller importance in turbines intended for land vehicles. Temperatures up to 1,200° C. may have to be allowed for and, combined with this, the working fluid provides an oxidizing atmosphere, since the proportion of air to fuel is many times greater than that required for a chemically correct mixture strength. Hence the quantity of free oxygen present in the stream is considerable. The problem of hot corrosion is serious and tends to cause a rapid burning away of the material, assisted, in no small degree, by the scouring effect of the high-speed stream on the walls of the chamber.

For these reasons the first design trend was to line the combustion chamber with a heat-resistant material. A number of such refractory materials is available, but in general they are not homogeneous, and consequently their mechanical properties, and in particular their tensile strengths, are poor. This disadvantage can, however, be overcome by using a metal combustion-chamber shell, in order to provide the necessary strength, and lining the interior of the shell with the refractory material. This does not necessarily maintain the metal shell at a low temperature, but protects it from the violent corrosive effects of the hot gases. The metal itself may be one which

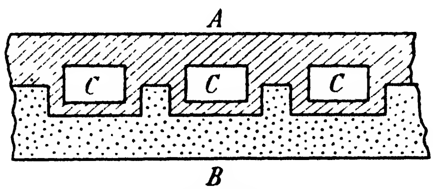


Fig. 86

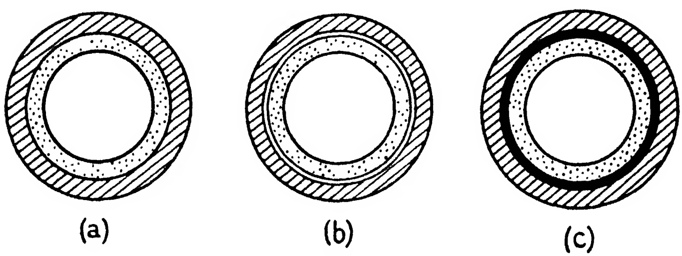


Fig. 87

maintains a considerable degree of strength at elevated temperatures, or it may be one of the more common, and consequently cheaper, metals, in which case a water-cooling system can be interposed between the heat-resistant lining and the combustion-chamber shell.

A portion of the wall of such a chamber is shown in Fig. 86. *A* is the metallic wall, and *B* the heat-resistant non-homogeneous material. Cooling passages *C* are provided in the outer shell, and are of sufficient size to keep the greater part of the outer shell thickness at a suitably low temperature so that the required degree of strength is maintained.

There is, however, another important factor to be taken into consideration in the design of any combustion chamber whose walls are composed of two concentric layers of different materials. This is the question of differential expansion, since, in general, the coefficients of expansion of the two materials differ considerably. When the turbine is at rest and the system is cold, the inner lining fits snugly into the metal shell, as at (*a*), Fig. 87. When the turbine is running and the combustion chamber gets hot, both the inner and the outer shell will expand.

But, in general, the coefficient of expansion of refractory materials is very low, whilst that of most metals is comparatively high. Hence, in spite of the fact that the lining is at a higher temperature than the outer shell, the metal will have a greater total expansion than the heat-resisting material. This would leave the lining unsupported, as at (*b*), Fig. 87, with the result that the internal gas pressure would probably burst it.

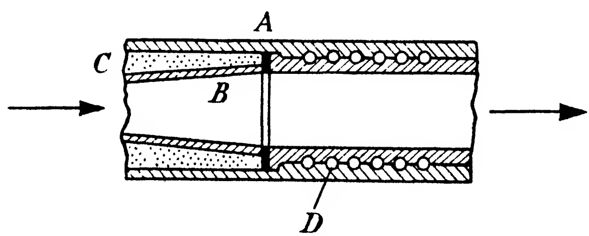


FIG. 88

The problem of differential expansion can be overcome in one of two ways. Firstly, the material of the outer shell can be so chosen that its coefficient of expansion gives it a total expansion, at the working temperatures, which is exactly matched to the total expansion of the lining. This, however, is a solution extremely difficult of achievement. Secondly, an elastic backing of some fairly heat-resistant material can be placed between the lining and the outer shell, as shown at (*c*), Fig. 87. If this is done, the total expansion of the shell can be greater than that of the lining, and yet the lining will still be supported by the metal shell, the stresses being transmitted through the elastic material. Nevertheless, the difference between the total expansions of lining and shell should be kept as small as possible.

An example of this type of construction is the Lemale combustion chamber illustrated in section in Fig. 88. The outer shell, *A*, was of cast-iron, and the heat-resistant lining was of carborundum. Between the lining and the shell was an elastic backing, *C*, of asbestos. A cooling system was provided between the shell and the casing, and this consisted of a coil of tubing, *D*, through which water was circulated in order to keep the shell at a reasonable temperature. The designed operating temperature of the chamber was about 1,800° C.

Chambers lined with refractory materials are, however, only of use when impulse wheels are employed. The expansion of the hot gases through the nozzles of an impulse turbine reduces their temperature considerably, and the blade temperature is very much lower than that of the combustion chamber. The magnitude of the temperature drop depends upon the pressure-ratio; the greater the pressure-ratio, the higher the combustion-chamber temperature for a given blade temperature. At the present time it is doubtful whether pressure-ratios are sufficiently high to justify a lined chamber, with the consequent increase in cost, but in view of the progress towards higher pressure-ratios, necessitated by the demand for increased plant efficiency, it is possible that their adoption will follow in the near future.

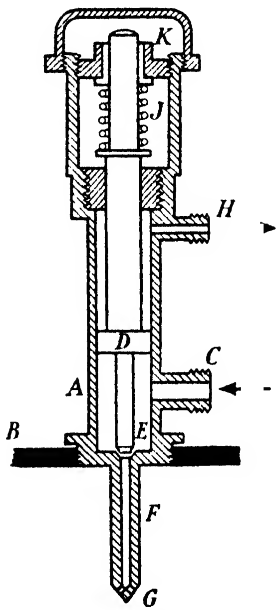


Fig. 89

If a reaction turbine is employed, a lined combustion chamber is unnecessary because the chamber temperature is the same as that of the blades of the first stage of the turbine. In this case the chamber is made of heat-resisting steel. Since the governing temperature is that of the highly stressed turbine blades, it follows that combustion-chamber temperatures are much lower when a reaction turbine is employed than when an impulse turbine is used.

In all cases it is necessary to provide for the expansion, both lateral and longitudinal, of the chamber. The expansion when running is not uniform throughout the chamber because the inlet end is continuously cooled by the flow of air from the compressor, while the outlet end runs at the full temperature of the hot gases. This differential expansion must be allowed for in the design of the expansion joints, which must be gas-tight under all running conditions.

Fuel-injection nozzles, or burners

The lay-out of a proposed fuel-injection nozzle, or burner, is shown in Fig. 89. Since it is possible to run a combustion turbine on practically any form of liquid fuel, it follows that the number of types, and variations of types, of fuel nozzles can be almost as multifarious as the types of fuel. For every turbine the fuel nozzles should be designed to suit the fuel on which the turbine is intended to run.

The lay-out shown is suitable for a comparatively light fuel. Fuel from the pump and main control valve enters the nozzle

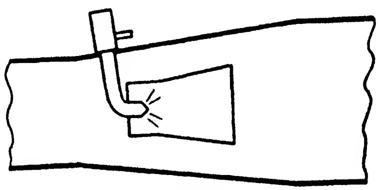


FIG. 90

body A, which is screwed into a seating in the combustion-chamber wall B, through the inlet connexion C. Once the fuel pressure has risen to a sufficiently high value on the underside of the piston D, the valve E is lifted off its conical seat and fuel enters the nozzle-head F, whence it is sprayed into the combustion chamber through the orifices G. While the turbine is running the valve E is held off its seat by the fuel pressure on the piston D, which causes compression of the spring J against the adjusting-plug K. When the fuel supply is cut off, or the pressure falls below the pre-set minimum value, the spring J automatically returns the valve to its seat. Any leakage of fuel past the piston is drained back to the tank through connexion H.

The fuel nozzle is shown with its body screwed into the combustion-chamber wall, but it need not necessarily occupy this position. In many installations, where compactness is required, the entire valve assembly may be located parallel to the axis of the combustion chamber and connected to the nozzle-head F by a short pipe. In other cases the nozzle-head itself may be elongated and curved, as in Fig. 90, in order to discharge the fuel along the axis of the combustion chamber.

VII

NOZZLES

When the air drawn into the turbine system has passed through the compressor and its pressure has been increased, and when its internal energy has been further increased by the burning of fuel in the combustion chamber, it is still not in a suitable state for acceptance by an impulse turbine wheel. In order that the energy of the working fluid may be harnessed by the turbine wheel and converted into mechanical energy, as much as possible must be turned into kinetic energy so that the gas is released from the combustion chamber in the form of a high-speed jet, the action of which on the turbine blades will force the wheel to rotate.

Clearly the gas must leave the combustion chamber through one, or more, apertures, or orifices, and in so doing will undergo a pressure drop and an increase in velocity. These apertures, or orifices, are the instruments used to effect the conversion into kinetic energy of the other forms of energy possessed by the gas before leaving the combustion chamber, and upon their good design will depend the proportion of the total energy of the gas which can be harnessed to do useful mechanical work. When an aperture, or orifice, is so designed as to convert the maximum possible proportion of the total energy of the gas in the combustion chamber into kinetic energy, it is termed a nozzle. For purposes of calculation the gas must be treated as a perfectly elastic fluid during its passage through a nozzle. It is of the utmost importance to be able to determine what velocity will be acquired under various stipulated conditions of expansion, and also what dimensions the profile of the nozzle must assume in order to accommodate the mass flow under these conditions.

The continuity equation

It is a basic stipulation in nozzle design that the flow must be continuous through the nozzle, and this condition can be set out in the form of a simple equation.

At any section across the nozzle, and at any instant, let W be the weight of gas in pounds per second passing the section, v the volume of 1 lb. of gas at the section in cubic feet, a the area of the section in square feet, and V the velocity of flow in feet per second. Then, at any instant, the volume of gas passing the section is

$$Wv \text{ cu. ft. per sec.}$$

and is also equal to

$$aV \text{ cu. ft. per sec.}$$

Therefore $$Wv = aV.$$

This is the equation of continuity. It should be satisfied at every section throughout the length of the nozzle.

Energy distribution

Let us consider the general case of the distribution of the energy of 1 lb. of gas in its passage through a nozzle. A convergent-divergent nozzle is shown in Fig. 91, gas being delivered

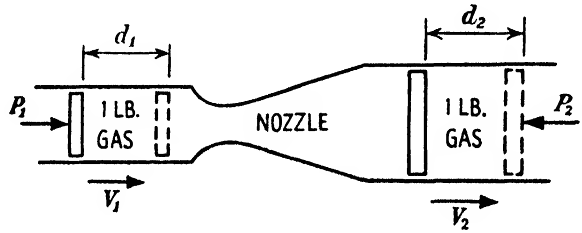


FIG. 91

to it at a pressure P_1 lb. per sq. ft. and velocity V_1 ft. per sec. and taken away from it after expansion at a pressure P_2 lb. per sq. ft. and velocity V_2 ft. per sec. In practice, of course, the jet leaving the nozzle would impinge upon the blades of the turbine wheel, but for the moment let us imagine the turbine wheel replaced by a cylindrical duct attached to the mouth of the nozzle. Let the sectional areas of the inlet and outlet pipes be a_1 and a_2 sq. ft. respectively, and let the volume of 1 lb. of gas at inlet be v_1 cu. ft. and at outlet be v_2 cu. ft.

Now let us imagine the inlet and outlet pipes each fitted with a smooth frictionless piston which moves with the same velocity as the gas. The distance moved by the piston in the inlet pipe during the flow of 1 lb. of gas through the system will be

$$d_1 = \frac{v_1}{a_1}.$$

The total force on the piston is $P_1 a_1$, and hence the work done by the gas in forcing into the system the gas which lies immediately in front of it will be

$$P_1 a_1 d_1 = P_1 v_1 \text{ ft. lb. per lb.}$$

In order that all quantities of energy may be expressed in the same units, we shall convert this to Centigrade Heat Units by dividing by J, the mechanical equivalent of heat. Then the work done on the inlet piston is

$$\frac{P_1 v_1}{J} \text{ C.H.U. per lb.}$$

Similarly the work done by the gas which is leaving the system in forcing out the gas which lies directly in front of it will be

$$\frac{P_2 v_2}{J} \text{ C.H.U. per lb.}$$

The kinetic energy of the gas at entry to the system is

$$\frac{V_1^2}{2gJ} \text{ C.H.U. per lb.}$$

and at outlet is

$$\frac{V_2^2}{2gJ} \text{ C.H.U. per lb.}$$

Let E_1 be the internal energy of 1 lb. of gas at inlet, and E_2 be the energy at outlet, both expressed as C.H.U. per lb.

If no energy is lost from, or gained by, the system during the passage of the gas from inlet to outlet, then, by the principle of conservation of energy, the total energy of 1 lb. of gas at its inlet condition must be the same as the total energy of 1 lb. of gas at its outlet condition. Hence

(Work done by gas at inlet in forcing 1 lb. of gas into system) + (internal energy at inlet) + (kinetic energy at inlet)
= (Work done by gas at outlet in expelling 1 lb. of gas from system) + (internal energy at outlet) + (kinetic energy at outlet).

Substituting in this equation the energy expressions derived above, we get

$$\frac{P_1 v_1}{J} + E_1 + \frac{V_1^2}{2gJ} = \frac{P_2 v_2}{J} + E_2 + \frac{V_2^2}{2gJ}.$$

This is known as the steady-flow equation, and is a general equation covering the energy distribution in a unit mass of gas during its expansion through a nozzle system.

The adiabatic equation

When a perfect gas (that is, a gas which precisely obeys the laws of Boyle and Charles and is completely elastic) is expanded through a nozzle the nature of the expansion is, ideally, adiabatic. Of course this assumption implies that there are no losses or extraneous influences, and that the expansion is taking place under exactly the conditions for which the nozzle was designed, whereas in practice small modifications are necessary in order to account for divergences from the ideal. But the theory is essentially based upon this assumption of an expansion which is adiabatic in form.

It can be shown that when a perfect gas expands adiabatically, the pressure P and the volume v are related by the expression

$$Pv^{\gamma} = k,$$

where γ is the ratio of the specific heats of the gas and k is a constant. If we now put this in logarithmic form we get

$$\log P + \gamma \log v = \log k,$$

which indicates a straight-line relationship. In other words, if corresponding values of pressures and volumes are substituted in this logarithmic equation and plotted, the curve obtained would be a straight line.

At very high temperatures it is possible to obtain a variation in the specific heats of a gas, which, in turn, causes a variation in the value of their ratio γ. This variation is due to a dissociation of a proportion of the molecules, the proportion, and consequently the variation, increasing with increase in temperature. In such a case the curve plotted from the logarithmic equation is no longer a straight line, but its deviation at normal working temperatures is not sufficiently great to affect the basic theory to any considerable extent, and γ will, therefore, be assumed constant throughout. The value of γ for air may be taken as 1·4, although the exact value to be used in any combustion-turbine design will depend on the fuel-air ratio and the type of fuel. The value is, however, not greatly different from that for air.

Velocity determination

The cycle upon which the constant-pressure turbine works is shown in Fig. 92. We showed in Chapter III that the useful work of the cycle is represented by the cross-hatched area

ABCD, but that the gross work, or total work, is represented by the complete area *EFCD*, the area *EFBA* being the negative work required to drive the compressor. It is clear that since the negative work has to be taken from the turbine wheel, then the gross work, represented by the complete area *EFCD*, must be manifested as kinetic energy in the jet from the turbine nozzles. Therefore area *EFCD* represents the kinetic energy of the jet, acquired in the nozzle.

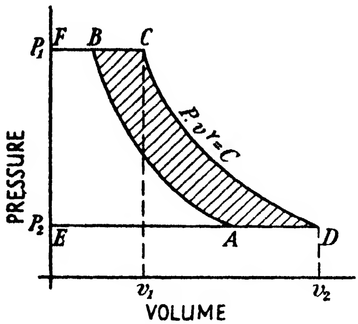


Fig. 92

Now *EFCD* is a Rankine area and, provided *CD* is an adiabatic, the work represented by this area can be shown to be

$$\frac{\gamma}{\gamma-1}(P_1 v_1 - P_2 v_2) \text{ ft. lb. per 1 lb. of gas.}$$

If the expansion index differs from γ and is at some value n, say, then this becomes

$$\frac{n}{n-1}(P_1 v_1 - P_2 v_2).$$

If V_1 is the velocity of the gas at entry to the nozzle and V_2 the velocity of the jet on leaving the nozzle, the increase in kinetic energy obtained during the passage through the nozzle is

$$\frac{V_2^2 - V_1^2}{2g} \text{ ft. lb. per 1 lb. of gas.}$$

Therefore $$\frac{V_2^2 - V_1^2}{2g} = \frac{\gamma}{\gamma-1}(P_1 v_1 - P_2 v_2).$$

In practice V_2 is very large and V_1 is, in comparison, small, so that V_1 may be neglected. The expression then becomes

$$\frac{V_2^2}{2g} = \frac{\gamma}{\gamma-1}(P_1 v_1 - P_2 v_2).$$

Hence $$V_2 = \sqrt{\left\{\frac{2g\gamma}{\gamma-1}(P_1 v_1 - P_2 v_2)\right\}} \text{ ft. per sec.}$$

This equation is usually written in the more convenient form of

$$V_2 = \sqrt{\left[\frac{2g\gamma}{\gamma-1} P_1 v_1 \left\{1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\}\right]} \text{ ft. per sec.}$$

by dividing the bracketed quantity by $P_1 v_1$ and substituting for the volume ratio v_2/v_1. It should be emphasized that P_1 is the pressure in the combustion chamber and P_2 the pressure after expansion in pounds per square foot, v_1 is the specific volume before and v_2 the specific volume after expansion, in cubic feet. When using the expression, care should be taken to see that all the quantities are in their appropriate dimensions.

It will be observed that the quantity under the root sign may be considered as the product of two variables and a constant. $2g\gamma/(\gamma-1)$ is clearly a constant, but the value of $P_1 v_1$ may be altered by varying either P_1, v_1, or both together. Similarly the value of

$$1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

may be altered by a variation in the ratio P_2/P_1.

It is clear, therefore, that if we require an increase in the nozzle velocity we have two methods of obtaining it. If we increase the combustion-chamber pressure P_1, then the product $P_1 v_1$ will be increased, and also, due to the decrease in the value of the ratio P_2/P_1, the quantity

$$1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

will be increased, assuming P_2 remains unaltered. In consequence, the nozzle velocity V_2 will be increased.

But if now we allow the combustion-chamber pressure P_1 to remain constant, as is done in the constant-pressure turbine, we may obtain an increase in nozzle velocity by increasing the value of the specific volume v_1. This produces an increase in the value of the product $P_1 v_1$, but the quantity

$$1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

remains unaltered.

Determination of nozzle area

Since the expansion is smooth and continuous over the whole length of a nozzle, it follows that the sectional area of the nozzle, at any point in its length, must be such as exactly to suit the flow conditions at that point. This object may be achieved provided the equation of continuity is satisfied.

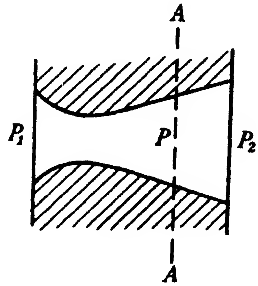


FIG. 93

Fig. 93 is a diagrammatic representation of any nozzle, and we wish to know the flow conditions at any section A–A of the nozzle. Let the sectional area at this position be a sq. ft., the pressure be P lb. per sq. ft., and the mass flow be W lb. of gas per sec. Similarly, let the gas velocity at this section be V ft. per sec. and the specific volume be v cu. ft. Then if the equation of continuity is to be satisfied,

$$\frac{W}{a} = \frac{V}{v}.$$

If v_1 is the specific volume at the pressure P_1 in the combustion chamber, then

$$v = v_1\left(\frac{P_1}{P}\right)^{\frac{1}{\gamma}}$$

and hence
$$\frac{W}{a} = \frac{V}{v_1}\left(\frac{P}{P_1}\right)^{\frac{1}{\gamma}}.$$

But the velocity V is given by the expression

$$V = \sqrt{\left[\frac{2g\gamma}{\gamma-1}P_1 v_1\left\{1-\left(\frac{P}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\}\right]}.$$

Therefore

$$\frac{W}{a} = \frac{1}{v_1}\left(\frac{P}{P_1}\right)^{\frac{1}{\gamma}}\sqrt{\left[\frac{2g\gamma}{\gamma-1}P_1 v_1\left\{1-\left(\frac{P}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right\}\right]}$$

$$= \sqrt{\left[\frac{2g\gamma}{\gamma-1}\frac{P_1}{v_1}\left\{\left(\frac{P}{P_1}\right)^{\frac{2}{\gamma}}-\left(\frac{P}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right\}\right]}.$$

Now W/a is the mass rate of flow per unit area, and P_1 and v_1 are, to all intents and purposes, constants. The value of P, however, will vary for different positions in the nozzle. Hence it is clear that the mass rate of flow per unit area is a function of the ratio of P to P_1.

Critical pressure

Associated with the flow of gas through a nozzle is a peculiar phenomenon which occurs under certain pressure conditions. These conditions have been given the name Critical Pressure, but should not be confused with the true critical pressure of a gas. The choice of term is perhaps a little unfortunate, and it should be borne in mind that the critical pressures referred to here are peculiar to nozzle systems.

Fig. 94 shows two chambers A and B, separated by a diaphragm containing a convergent nozzle C. Gas can be fed into chamber A through the large diameter inlet pipe and leaves chamber B through the outlet pipe, which is fitted with a valve D. The only connexion between the two chambers is the nozzle C, and the passage of gas from chamber A to chamber B must, therefore, take place through this nozzle.

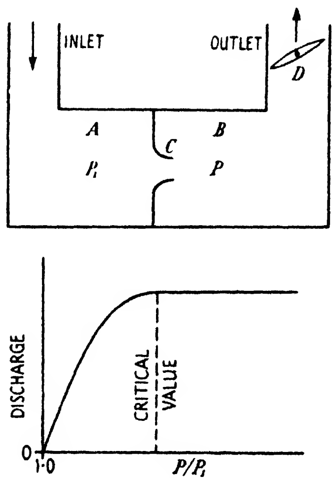


Fig. 94

Let us assume that the pressure in chamber A is kept at a constant value P_1 throughout, and that the pressure P in chamber B can be varied at will by opening or closing the valve D. With the pressure in chamber A kept constant, let us plot the mass flow through the nozzle at different values of pressure in chamber B, as in Fig. 94.

Starting with the valve D completely closed, it is clear that the pressure P will be equal to P_1 and, consequently, there will be no flow through the nozzle. As the valve is gradually opened, the pressure P progressively drops and the discharge increases, as shown by the rising curve in Fig. 94. But when the pressure in chamber B reaches a certain value the curve flattens out and, although the pressure P is reduced still further, the mass flow through the nozzle does not increase but remains constant. The value of the pressure P at which the mass flow ceases to increase is known as the critical pressure, and is slightly more than half the value of the steady pressure P_1. Corresponding to this phenomenon of critical pressure is, of course, a critical velocity. The critical velocity is the value of the velocity of the gas emerging from the nozzle at the instant when the mass flow ceases to increase with increase of the pressure difference between the chambers A and B.

The explanation of the strange behaviour of the gases passing through the nozzle is comparatively simple, and is as follows: a curve of nozzle velocity V for a convergent nozzle, plotted against the pressure-ratio P/P_1, is shown in Fig. 95, and on the same diagram is drawn a curve of the local acoustic velocity S at the appropriate pressures and densities of the gas. Suppose the pressure P is equal to the pressure P_1 and there is no flow through the nozzle. Then any sudden disturbance which took place in chamber B would be transmitted through the nozzle into chamber A with the speed of sound S in the gas at that pressure and density. Now suppose the pressure P is reduced to, say, $0{\cdot}75P_1$, and V is the consequent flow velocity through the nozzle. A sudden disturbance in chamber B would then be transmitted up stream into chamber A with a velocity $S-V$. Since V is less than S (Fig. 95), the quantity $S-V$ is positive, so that if a reduction of pressure takes place in chamber B its effect is transmitted back through the stream with a velocity $S-V$ and the mass flow through the nozzle is suitably augmented. When, however, the pressure in chamber B is reduced to the critical value, the two curves intersect and the quantity $S-V$ becomes zero. If any further reduction in pressure in chamber B takes place, its effect can no longer be transmitted back through the stream emerging from the nozzle outlet and, consequently, there is no further increase in the mass flow.

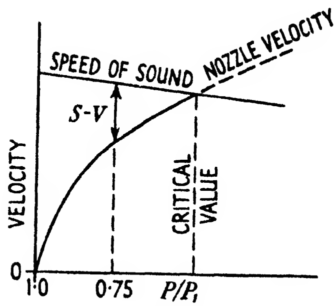


FIG. 95

Mathematical proof of the existence of a critical pressure

The existence of a critical pressure in nozzle flow can be demonstrated mathematically without any great difficulty. We have previously shown that the mass rate of flow through a nozzle, per unit sectional area, is equal to

$$\frac{W}{a} = \sqrt{\left[\frac{2g\gamma}{\gamma-1}\frac{P_1}{v_1}\left\{\left(\frac{P}{P_1}\right)^{\frac{2}{\gamma}} - \left(\frac{P}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right\}\right]}.$$

It will be seen that all the quantities under the root sign are constants, with the exception of P, which is a variable. Consequently the ratio P/P_1 is a variable, and therefore the mass rate of flow per unit area is a function of the variable P/P_1. For the sake of simplicity let us write r for P/P_1. Hence

$$\frac{W}{a} = \sqrt{\left[\frac{2g\gamma}{\gamma-1}\frac{P_1}{v_1}\left\{r^{\frac{2}{\gamma}} - r^{\frac{\gamma+1}{\gamma}}\right\}\right]}.$$

Since the only variable on the right-hand side of the equation is the quantity $r^{2/\gamma} - r^{(\gamma+1)/\gamma}$, then it is clear that the mass rate of flow per unit sectional area will be a maximum when $r^{2/\gamma} - r^{(\gamma+1)/\gamma}$ is a maximum. The value of r which makes this expression a maximum may be obtained by differentiating the expression with respect to r and equating to zero. In other words, the mass rate of flow per unit sectional area, W/a, is a maximum when

$$\frac{d}{dr}\left(r^{\frac{2}{\gamma}} - r^{\frac{\gamma+1}{\gamma}}\right) = 0,$$

that is, when

$$r = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}.$$

This value of r is the critical pressure-ratio and depends on the value of γ. Should the expansion index differ slightly from γ, which is the ratio of the specific heats of the gas, we may write this expression for the critical pressure-ratio as

$$\frac{P}{P_1} = \left(\frac{2}{n+1}\right)^{\frac{n}{n-1}},$$

where n is the actual expansion index.

Nozzle form

We have as yet said little about the actual form of a nozzle. This will depend upon the rate of pressure drop along the axis of the nozzle. For instance, if we decide that the rate of pressure drop shall be, say, 20 lb. per sq. in. per inch length of nozzle, then we shall get a certain nozzle shape. On the other hand, if we first choose the nozzle shape from the point of view of other considerations, the rate of pressure drop along the axis will be governed by the chosen shape. Let us first see what happens if we design a nozzle so that the rate of pressure drop along the axis is uniform.

Suppose the pressure in the combustion chamber is somewhere about 100 lb. per sq. in. absolute and we design on the assumption of an equal pressure drop per inch length of the

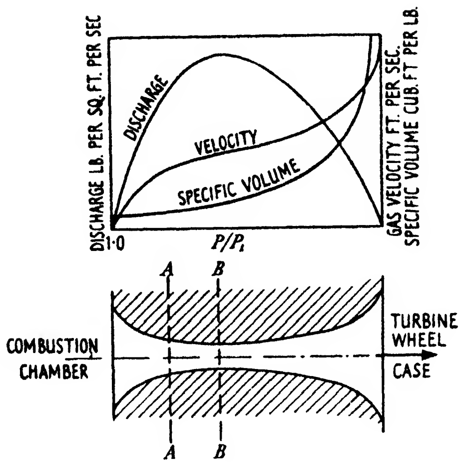


Fig. 96

nozzle. Then we shall find that our nozzle has a form something like that shown in Fig. 96. If the combustion-chamber pressure remains at a constant value, and the back-pressure (i.e. the pressure in the turbine wheel case) is fairly high, say about 80 lb. per sq. in., then instead of using the long nozzle illustrated we should need only a short one which terminated in the region of the section *A–A*.

If the pressure in the turbine wheel case were now reduced to the critical pressure, the required nozzle would terminate at the narrowest section *B–B*. If the pressure in the turbine wheel case were reduced to atmospheric pressure, then we should obtain the full form shown. It follows that if the pressure in the wheel case is greater than the critical pressure, a convergent nozzle must be used, whereas if it is less than the critical value, a convergent-divergent nozzle is called for. The pressure at the narrowest portion of the nozzle, which is called the throat, is the critical pressure, and the throat area is calculated on this assumption. Fig. 96 also indicates the form of the velocity, discharge per unit cross-sectional area, and specific volume curves for such a nozzle.

Actual nozzles, however, tend to differ from the form shown in Fig. 96. A typical nozzle is shown in Fig. 97. Here the convergent portion may be designed according to the method just

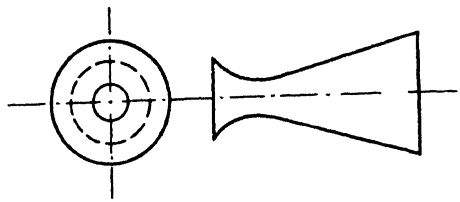


FIG. 97

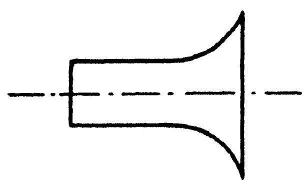


FIG. 98

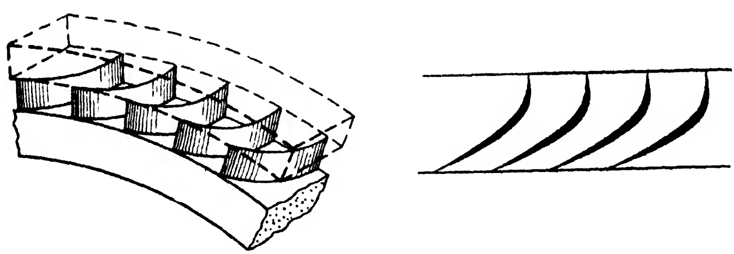


FIG. 99

described, but the divergent portion has a straight taper. Consequently the pressure drop per unit length of nozzle is not uniform between the throat and the nozzle outlet. Some convergent nozzles may have a short parallel outlet as in Fig. 98. The rectangular section nozzle illustrated in Fig. 99 may also be used. This consists of a series of vanes shaped so as to provide the required nozzle form and to deliver the gas stream at an angle suitable to the turbine blades.

In order that the gas stream shall impinge upon the turbine blades at the required angle, the axis of the nozzle is inclined to the axis of the turbine, as shown in Fig. 100, and the shape of the mouth of the nozzle is unsymmetrical in a plane at right angles to its axis. This has an effect on the jet which will be discussed later.

When designing a nozzle it is most important that the throat area and the outlet area should be correct, for on these depends

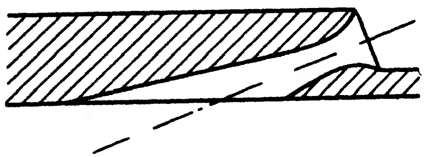


FIG. 100

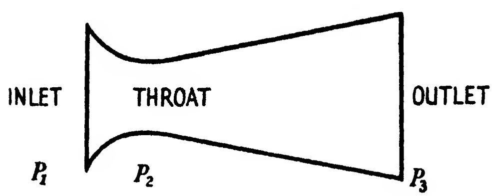


FIG. 101

the efficient expansion of the gas. Suppose expansion takes place adiabatically through the nozzle shown in Fig. 101, from a pressure P_1 to a pressure P_3. Let P_2 be the pressure at the throat, this being, of course, the critical pressure. Using the. same symbols as before, we showed that

$$\frac{W}{a}=\sqrt{\left[\frac{2g\gamma}{\gamma-1}\frac{P_1}{v_1}\left\{\left(\frac{P}{P_1}\right)^{\frac{2}{\gamma}}-\left(\frac{P}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right\}\right]},$$

where W is the mass flow in pounds per square foot of sectional area per second, a is the sectional area at any point on the nozzle axis, and P is the pressure at that point. Then it is clear that the sectional area of the throat will be

$$a_2=\frac{W}{\sqrt{\left[\frac{2g\gamma}{\gamma-1}\frac{P_1}{v_1}\left\{\left(\frac{P_2}{P_1}\right)^{\frac{2}{\gamma}}-\left(\frac{P_2}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right\}\right]}},$$

where W is the required mass flow in pounds per second. Using foot-pound-second units, the area a_2 will, of course, be in square feet. Similarly the outlet area of the nozzle will be

$$a_3 = \frac{W}{\sqrt{\left[\frac{2g\gamma}{\gamma-1}\frac{P_1}{v_1}\left\{\left(\frac{P_3}{P_1}\right)^{\frac{2}{\gamma}}-\left(\frac{P_3}{P_1}\right)^{\frac{\gamma+1}{\gamma}}\right\}\right]}}.$$

Nozzle friction

Up to now we have based all our theory on the assumption that the expansion of the gas from a higher to a lower pressure takes place through a perfectly smooth nozzle. This enabled us,

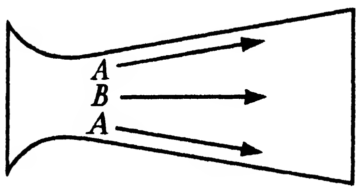


FIG. 102

without undue difficulty, to arrive at certain important mathematical conclusions. In practice, however, it is not possible to obtain a perfectly smooth nozzle. However fine the finish, the walls of the nozzle will inevitably have a small amount of roughness. After a nozzle has been in use for some time, the corrosive effect of the hot gases, combined with the scouring action of the high-speed stream, tends to cause pitting of the walls, thus giving rise to considerable roughness.

If the walls of even the smoothest nozzle are examined under a microscope, it will be seen that they contain innumerable tiny projections. These minute projections act as a brake on the layers of gas in immediate contact with the walls and decrease their velocity. Consequently the centre of the stream, indicated by the arrow B in Fig. 102, is moving faster than the boundaries, indicated by the arrows A. Due to this relative motion between parts of the stream, there is naturally a certain amount of fluid friction which will decrease the quantity of useful energy obtainable from the jet. Similarly, there is friction between the stream and the nozzle walls, and the greater the roughness of the walls, the greater will be the loss of useful energy. The effect of nozzle friction is to reduce the efflux velocity and also the mass flow below the values predicted on an assumption of perfect adiabatic expansion. In a well-designed nozzle the reduction is, however, comparatively small.

If V is the calculated efflux velocity, on an assumption of perfect adiabatic expansion, and if V' is the actual velocity when nozzle friction is taken into account, then the velocity coefficient for the nozzle is

$$\eta_v = \frac{V'}{V}.$$

Fig. 103 shows some curves derived as the result of tests carried out on nozzles using steam as the working fluid.[1] The

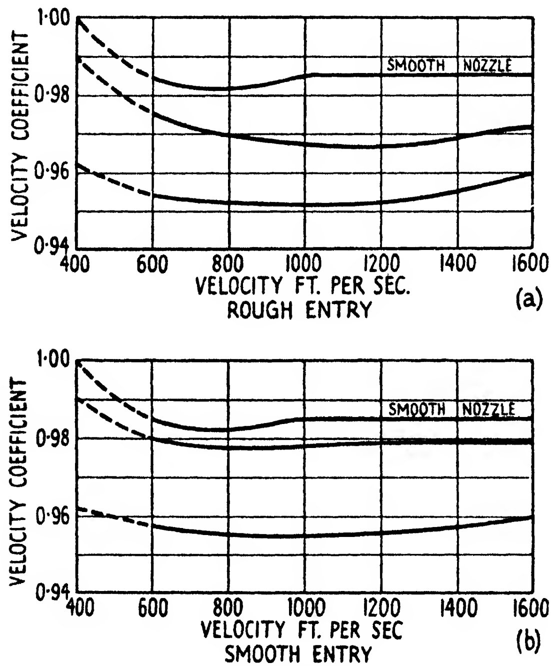


Fig. 103

nozzle of Fig. 103 (*a*) has a rough entry curve, and that of Fig. 103 (*b*) a smooth entry curve, the portion between the throat and the outlet being given various degrees of roughness. As will be seen from the curves, the velocity coefficient for a smooth nozzle is between 0·98 and 0·99, so that the reduction in useful energy, due to nozzle friction, is small.

[1] Report of the Steam Nozzles Research Committee, *Proc. I. Mech. Eng.*, 1928.

The effect of back-pressure variation

When designing a nozzle it is usual to cater for a fixed inlet pressure and a fixed back-pressure, which gives a fixed pressure drop through the nozzle. The back-pressure is, of course, the

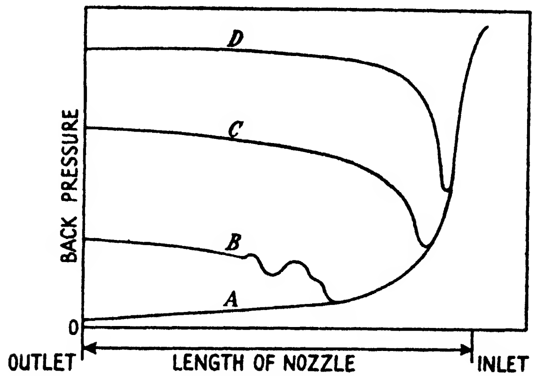


FIG. 104

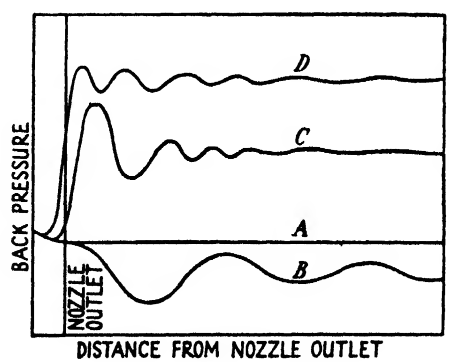


FIG. 105

absolute pressure in the turbine wheel case. Should a nozzle be called upon to operate under pressure conditions other than those for which it was designed, the drop in efficiency can be quite considerable. A good deal of information on the effects of a variation in back-pressure was obtained by Professor Stodola in the course of experiments on steam nozzles. Some typical curves[1] are set out in Figs. 104 and 105.

Fig. 104 shows the nature of the pressure distribution within the nozzle as the back-pressure is gradually increased. Curve *A*

[1] After Stodola.

gives the pressure distribution along the axis of the nozzle when operating under the designed pressure conditions. Curves B, C, and D represent increased values of back-pressure. The pressure follows the correct expansion line along curve A for a certain distance, and then suddenly rises to a higher value towards the nozzle outlet. The explanation given for this is that the gas completely fills the nozzle section for a certain distance and expands at a rate controlled by the sectional area of the nozzle. At some point, however, the flow conditions break down and the stream enters a portion of the nozzle in which the gas velocity is lower, with a consequent impact which gives rise to a sudden increase in pressure. The loss of useful energy due to the impact has been estimated, and can be quite considerable.

Fig. 105 shows the nature of the pressure distribution in the jet after it has left the nozzle outlet. Curve A is drawn for the nozzle operating against the designed back-pressure and indicates a smooth and constant pressure distribution along the jet. Curve B indicates what happens when the back-pressure is reduced below the designed value. The pressure distribution is no longer smooth, but fluctuates violently. Similarly, if the back-pressure is at some value greater than the designed value, as indicated by curves C and D, the fluctuations in pressure are violent and there is considerable impact at the nozzle outlet. The fluctuations are damped out after the jet has travelled a few inches, but the effect on the turbine blades is adverse, since the clearance between the nozzle outlet and the first moving blade ring is small. The loss of useful energy due to impact at the nozzle outlet can be by no means negligible.

Jet deflexion

Since the nozzle must be inclined to the plane of the turbine wheel, it follows that the nozzle outlet must be oblique, as shown in Fig. 106, which illustrates a convergent type. If the back-pressure in the turbine wheel case is not less than the critical pressure, then the jet will leave the nozzle at the designed geometrical angle α. But if the back-pressure is less than the critical pressure, there will be a certain amount of expansion in the wedge-shaped space ABC and the jet will be deflected, so that it issues at an angle α' which is greater than the designed nozzle angle α.

A similar argument can be applied to the convergent-divergent type of nozzle. A convergent-divergent nozzle is illustrated in Fig. 107. Strictly speaking, the working portion terminates at the section AB, the extension BC being parallel to the nozzle axis. Provided the nozzle operates at the designed pressure values, the jet will issue at the correct nozzle angle α.

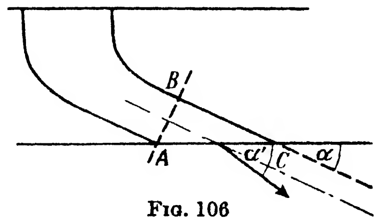


Fig. 106

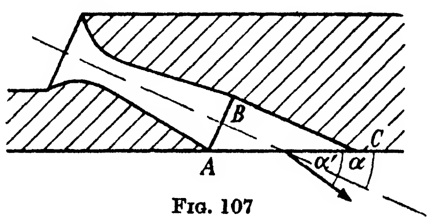


Fig. 107

If, however, the back-pressure is at some value lower than the designed value, then there will be a certain amount of expansion in the wedge-shaped portion ABC and the jet will be deflected so that it issues from the nozzle outlet at an angle α' which is greater than α.

If the nozzle is made divergent for its entire length, as in Fig. 108, the section AB still being the correct outlet area, the divergence of BC will be too great and the flow will be adversely affected. If, on the other hand, the projected section CD is made equal to the correct outlet area, the nozzle still being divergent for its whole length, a certain amount of expansion will take place in the wedge-shaped portion ABC, and there will be deflexion of the jet even when the nozzle is operating under the designed pressure conditions.

In the case of rectangular section nozzles having thick vanes, such as that illustrated in Fig. 109, a few degrees of deflexion in the opposite direction may be produced. The reason given for this is that the space in the acute angle between the jet and the edge of the vane at A sets up a slight suction which deflects the jet so that the issuing angle α' is less than the nozzle angle α. It follows that very thin vanes are more efficient than thick

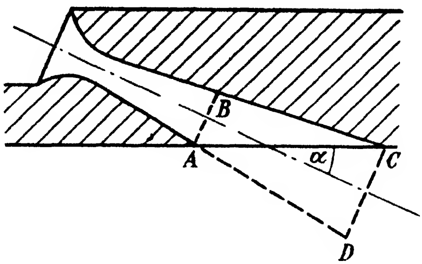


FIG. 108

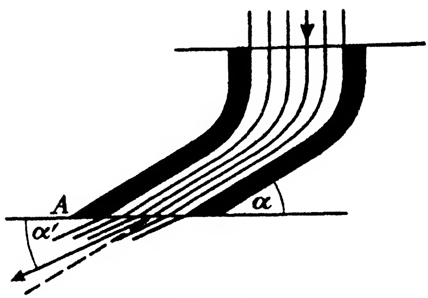


FIG. 109

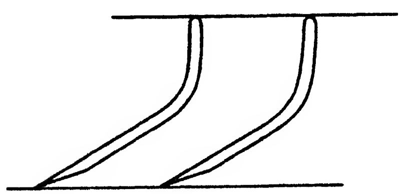


FIG. 110

ones, but for constructional reasons thick vanes are often necessary. In order to mitigate to a certain extent the tendency to flow instability of thick vanes, the outlet edges can be chamfered, as in Fig. 110.

VIII

IMPULSE TURBINES

A SIMPLE single-row impulse turbine is shown diagrammatically in Fig. 111. Only the half of the turbine above the horizontal axis of the rotor shaft is drawn. The lower portion of the diagram illustrates a development of the nozzle and blading, together with curves of pressure and absolute gas velocity through the system.

Gas enters the nozzle *B* from the combustion chamber *A*, and is expanded in the nozzle from the combustion chamber pressure down to atmospheric pressure. As a result of this expansion, the gas velocity is very considerably increased and the gas leaves the nozzle as a high-speed jet in the direction indicated by the arrow. The jet then impinges upon the blades *C* of the turbine wheel *D*, which, due to the rotation of the wheel, are moving relative to the nozzle in the direction shown. In its passage over the curved surfaces of the blades the direction of the gas stream is changed so that it emerges roughly parallel to the axis of the wheel. The velocity of the gas leaving the wheel is very much lower than the nozzle velocity and the consequent change in momentum, as will be discussed in detail later, generates a force upon the wheel blades which acts in their direction of motion and perpetuates the rotation of the wheel.

Referring to the curves, the pressure in the combustion chamber is constant up to the nozzle entry. As the gas expands through the nozzle, it undergoes a rapid, but smooth, drop in pressure, emerging at atmospheric pressure, at which value it remains constant during its passage through the blade-ring and into the exhaust system. At the same time the gas velocity, initially low in the combustion chamber, increases rapidly in the nozzle and may reach a value between 1,000 and 3,000 ft. per sec. There is a reduction in absolute velocity in the blade-ring, due to the gas stream giving up a large proportion of its kinetic energy to the turbine wheel, and the gas leaves the blade-ring with a certain residual velocity which is known as the 'leaving velocity', or 'carry-over velocity'. It is naturally desirable that this leaving velocity should be as low as possible in order to minimize the consequent loss of kinetic energy.

We see, therefore, that the essential components of a turbine of this type are a nozzle, to generate a high gas velocity, and a ring of suitably shaped blades attached to a wheel, or rotor, to convert the kinetic energy of the high-speed jet into a form which can be utilized to do mechanical work. The turbine of Fig. 111 is known as a single-stage single-row impulse turbine.

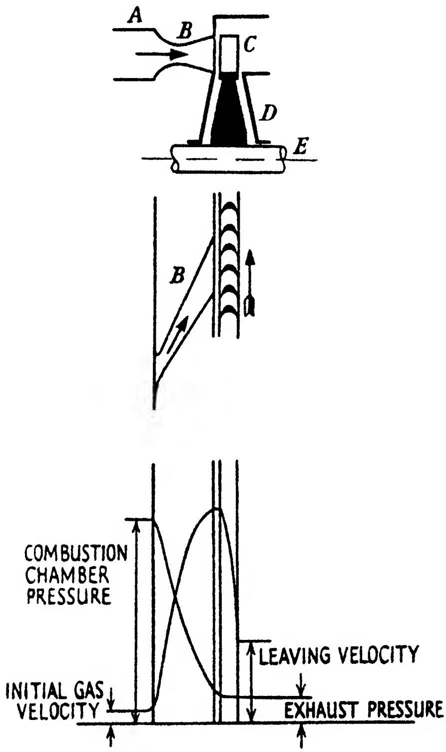


Fig. 111

All such turbines which rely on the use of a high-speed jet, with no pressure drop taking place in the blade-ring itself are known as impulse turbines, to distinguish them from the type of turbine in which the pressure drop takes place during the passage of the gas through the blade-rings. The latter are known as 'reaction' or 'impulse-reaction' turbines, and will be fully described later.

The single-stage turbine usually has a very high blade speed which may amount to as much as 1,200 or 1,500 ft. per sec. In many cases this is undesirable, and the pressure drop may then be divided into a number of stages, so that instead of the complete expansion taking place in the single nozzle, or nozzle-ring, of Fig. 111, only a partial expansion takes place, the gas then being passed to another nozzle for further expansion, and so on for any suitable number of stages. Each unit, consisting of a

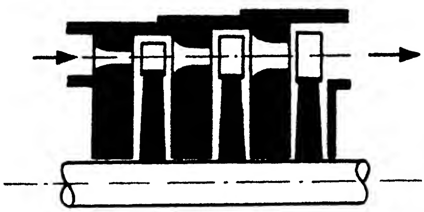


FIG. 112

nozzle and a blade-ring, is called a stage, and such a turbine is said to be 'pressure-compounded'. A pressure-compounded impulse turbine is shown diagrammatically in Fig. 112. When the gas passes through the nozzle, as well as the drop in pressure there is a considerable drop in temperature. It is clear, therefore, that for equal pressure drops the blade temperature of a single-stage turbine is less than the blade temperature of the first stage of a multi-stage turbine, due to the fact that the pressure drop in the first set of nozzles is not complete. For this reason the pressure-compounded arrangement is not ideal for the combustion turbine, since, if a high blade temperature is to be tolerated, it is better to use a reaction type of turbine, although an impulse wheel may still be used as the first stage in combination with a reaction turbine.

A single-stage turbine may have more than one row of blades on the wheel. Such a lay-out is illustrated diagrammatically in Fig. 113. In this case the rotor carries two rows of blades *C* and *D*, and between them is situated a row of fixed blades *E*, attached to the casing. The function of these fixed blades is to pick up the stream of gas leaving the first moving blade-ring *C* and redirect it at a suitable angle on to the second moving blade-ring *D*. The number of moving blade-rings is, of course, not restricted to two, but if more than two are used, the total friction loss in the blading may become quite large. The rotor of Fig. 113 is said to be a 'two-row wheel'.

In a single-row single-stage impulse turbine the entire conversion of kinetic energy to useful mechanical energy is carried out in the single blade-ring, and this results in a high blade speed. If, however, more than one row of moving blades is employed, the energy conversion can be divided amongst them

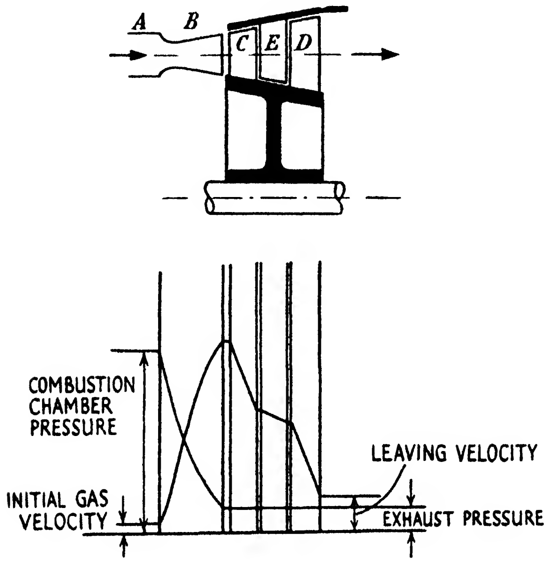


Fig. 113

so that the amount of energy dealt with by each moving blade-ring is considerably less than in the case of the single-row wheel. This results in a lower blade speed, which, in many applications, is most desirable. The friction losses in the blading are higher for a multi-row wheel than for a single-row type, but on the other hand the leaving loss is considerably reduced, due to the lower residual velocity. The nature of the velocity drop through the blading is indicated by the curves of Fig. 113. This type of turbine is said to be 'velocity-compounded'. Multi-row wheels can be used in a pressure-compounded turbine in place of the single-row wheels shown in Fig. 112, in which case the machine is called a 'pressure-velocity-compounded' turbine.

It will be observed that all the foregoing types of impulse turbine, with the exception of the unsuitable pressure-compounded type, are fitted with convergent-divergent nozzles. In the chapter on nozzles we showed that if a nozzle of this type operates under pressure conditions which differ more than slightly from the designed values, the drop in nozzle efficiency is considerable. It follows that turbines of this type, in the form described, should run at a more or less constant load and constant speed if good efficiency is to be maintained. The method of output control for load variation in steam turbines employing convergent-divergent nozzles is normally by blanking off one or more of the nozzles by means of valves, so that the pressure conditions across the remaining nozzles are not appreciably affected, but owing to the very high nozzle inlet temperatures employed in the combustion turbine, this method of control presents some difficulty.

The velocity diagram

Since the functioning of a turbine depends entirely upon the behaviour of the gas during its period of contact with the blades,

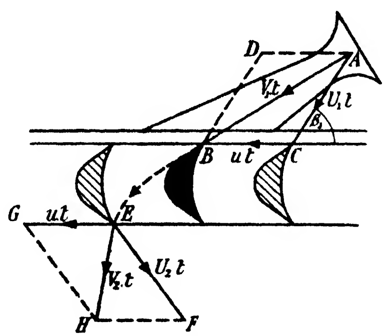


FIG. 114

it is very necessary that we should be able to determine the variations in the velocity of the gas from the moment it leaves the nozzle outlet until it eventually passes into the exhaust system. The velocity conditions are usually set out graphically in the form of a velocity diagram, giving a clear picture of the changes which take place.

Fig. 114 shows a nozzle, whose axis is the line AB, directing a stream of gas on to the moving blades of an impulse wheel. For clarity only one blade is drawn, and the entire discharge from the nozzle is assumed to be along the axis AB. Let us consider a particle of gas at some point A in the nozzle. This particle is travelling with the main stream in the direction AB, and will eventually emerge from the nozzle at the point B. At the instant the gas particle is at A, the blade is at some position C (shown hatched). Let t be the very short period of time for the particle to move from A to B, and let V_1 be its average velocity during this period of time. Then $AB = V_1 t$. Now the blade, which is attached to the rotor, is moving in the direction CB with a velocity u, and must cover the distance CB in the same interval of time t. Therefore $CB = ut$. Now suppose some imaginary observer were situated on the blade as it moved from C to B. Then he would see the gas particle approach him along an apparent path AC. The particle must, of course, cover the apparent distance AC in the same length of time t as it takes to cover the actual distance AB. Its apparent velocity along AC is U_1, where

$$U_1 = \frac{AC}{t}.$$

Hence U_1 is the velocity of the particle relative to the moving blade. We can now draw a parallelogram $ACBD$, such that AB is the diagonal. Then clearly $V_1 t$ is the vector sum of $U_1 t$ and ut, and ABC can be used as the triangle of velocities at the inlet side of the blade.

The gas particle now travels over the surface of the blade until it finally leaves the blade-ring and passes into the exhaust system. During the time the particle takes to pass through the blade-ring the blade is, of course, still moving with its velocity u, so that when the particle leaves the blade-ring, the blade will be at some position E. So the actual path of the particle is the curve BE. Clearly the particle will leave the blade-ring in a direction, relative to the blade, which is determined by the blade angle and will be along the line EF. If there were no friction effects in the blade channels, the relative velocity of the particle and the blade would be the same at exit as at entry, but, owing to the inevitable friction, the relative velocity at exit will be at some value U_2 which is less than U_1. During the short interval of time t, the particle will have moved a distance $EF = U_2 t$ relative to the blade on exit from the blade-ring. During the same period of time the blade itself will have advanced a distance $EG = ut$. If V_2 is the true velocity of the particle on leaving the blade-ring, then the actual distance it will have travelled in time t is EH, where $EH = V_2 t$ and is the vector sum of $U_2 t$ and ut as given by the diagonal of the parallelogram $EFHG$ in the diagram. The triangle EHG can be used as the triangle of velocities at outlet from the blade-ring.

It is usual to incorporate both the inlet triangle and the outlet triangle in a single diagram. This makes for ease of working, and is accomplished as follows:

Let:

V_1 = absolute velocity of gas at nozzle outlet.
U_1 = velocity of gas relative to blades at inlet.
V_2 = absolute velocity of gas at outlet from blades.
U_2 = velocity of gas relative to blades at outlet.
u = peripheral velocity of blades.
α = nozzle angle (assuming there is no jet deflexion).
β_1 = inlet angle of blades.
β_2 = outlet angle of blades.
γ = true direction of gas at outlet from blade-ring.

To any convenient scale set off AB (Fig. 115) to represent the peripheral velocity u of the blades. At the angle α to AB draw AC to represent the nozzle velocity V_1 in magnitude and direction. Draw BC to join C and B. Then BC represents, in magnitude and direction, the velocity U_1 of the gas relative to the blades, and ABC is the triangle of velocities at inlet. Now the magnitude of U_2 will have been estimated, and its direction determined by the choice of blade outlet angle β_2. Therefore set off BD at an angle β_2 to AB to represent U_2. Draw AD. Then AD represents to scale the true velocity V_2, in magnitude and direction, of the gas at outlet from the blade-ring. $ABCD$ is the complete velocity diagram.

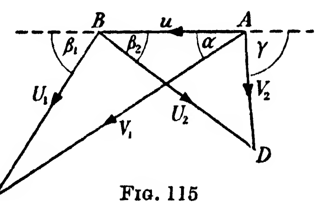


FIG. 115

Forces acting on the blades

In order to estimate the power obtainable from a turbine wheel we must first find the tangential force induced at the rim of the wheel by the action of the gas jet on the blades. The function of the blades is to change the direction of motion of the jet in as smooth a manner as possible. In changing its direction the jet experiences a change of momentum in the original direction of motion. Now Newton's second law states that when a body experiences a change of momentum, the rate of change of momentum is proportional to the force which produces the change. Consequently, since the blades cause a change in the momentum of the jet, they experience a force which is proportional to the rate of change of momentum.

For instance, consider the blade of Fig. 116. The blade is held stationary and a jet of gas, whose absolute velocity is V, is directed smoothly upon it. Assuming that there is no friction effect, and that the inlet and outlet angles of the blade are equal, the jet will leave the vane with the same velocity V. The velocity at inlet and outlet may be resolved into two components, V_t and V_a. The component V_a is not affected from inlet to outlet, but the component V_t experiences a reversal in direction. There is, therefore, a change in velocity in the direction AB which amounts to $2V_t$. If the weight of gas passing over the blade is W lb. per sec. and if the velocities are expressed in feet per second, then the change of momentum per second is

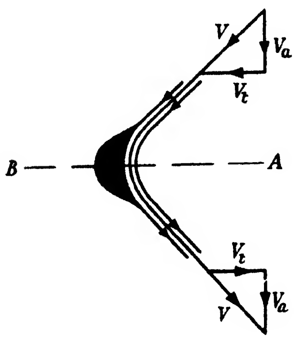


FIG. 116

$$\frac{2WV_t}{g} \text{ lb.}$$

Since force is proportional to rate of change of momentum, then the force exerted on the blade in the direction AB by the jet is

$$F = \frac{2WV_t}{g} \text{ lb.}$$

Now let us consider the velocity diagram of Fig. 117. As before, ABC is the inlet diagram and BDA is the outlet diagram. The first thing we wish to know is the change in the absolute velocity of the gas which takes place during its passage through the blade-ring. The absolute velocity at inlet V_1 is represented by the vector AC, and the absolute velocity at outlet V_2 is represented by the vector AD, so that if we find the vectorial difference of AC and AD we shall know the change which has taken place in the absolute velocity of the gas. If we draw the line CD, then CD will be equal to the vectorial difference of AC and AD, and represents to scale the change in the absolute velocity.

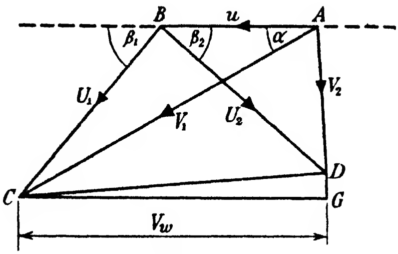


FIG. 117

If the weight of gas discharged over the blades per second is W lb., then the change of momentum per second is

$$\frac{W.CD}{g} \text{ lb.}$$

and consequently the thrust on the blades is

$$F = \frac{W.CD}{g} \text{ lb.}$$

But it will be observed that this total thrust is not parallel to the direction of motion of the blades. It can, therefore, be

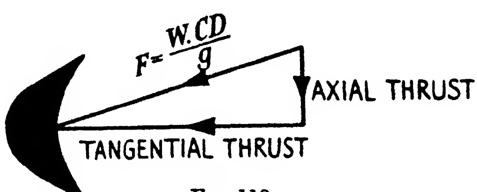


FIG. 118

resolved into two components, Fig. 118, one parallel to the direction of motion of the blades, and one parallel to the axis of the wheel. The former is the useful tangential thrust which causes the rotation of the wheel, while the latter is merely an axial thrust which has to be taken up by a thrust bearing in order to prevent the shaft and rotor moving bodily. Referring to Fig. 117, it is clear that CD also can be resolved into two components. The tangential component CG is known as the 'velocity of whirl' and is denoted by V_w. Hence,

$$\text{tangential thrust} = \frac{W}{g} CG = \frac{W}{g} V_w.$$

We are now in a position to calculate the power obtainable from the turbine wheel. The velocity of the blades is u ft. per sec., and therefore

$$\text{work done per second} = \frac{WV_w u}{g} \text{ ft. lb.}$$

$$= \frac{WV_w u}{550g} \text{ h.p.}$$

This is known as the 'rim horse-power', since certain losses are involved before energy can be taken from the shaft to do useful work.

Blade efficiency

The energy of the jet is its kinetic energy, which is

$$\frac{WV_1^2}{2g} \text{ ft. lb. per sec.}$$

The quantity of useful energy harnessed by the blades, as given above, is

$$\frac{WV_w u}{g} \text{ ft. lb. per sec.}$$

Hence the efficiency of the blades will be given by the ratio of the useful energy developed by the blades to the energy of the jet.

Blade efficiency $$\eta_b = \frac{WV_w u}{g} \frac{2g}{WV_1^2}$$

$$= \frac{2uV_w}{V_1^2}.$$

One of the most important factors in turbine-wheel design is the ratio of the blade speed to the jet speed. This is known as the 'blade-speed ratio' and is denoted by the Greek letter ρ, where

$$\rho = \frac{u}{V_1}.$$

The wheel attains its maximum efficiency when running at the correct blade-speed ratio appropriate to the particular type of impulse turbine, whether simple or velocity-compounded. On either side of the correct ratio the efficiency falls away fairly rapidly.

Let us first consider the case of a simple single-stage single-row turbine as illustrated in Fig. 111. Fig. 119 gives the

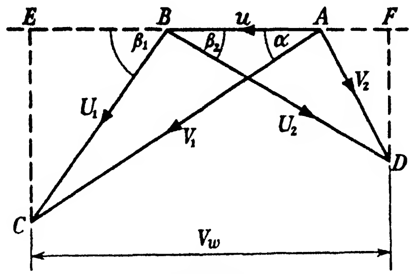


FIG. 119

velocity diagram. Due to the friction effects in the blade channels U_2 is less than U_1, and the ratio of U_2 to U_1 is denoted by k. Therefore

$$\frac{U_2}{U_1} = k.$$

In all cases k is less than unity. We have shown above that the blade efficiency

$$\eta_b = \frac{2uV_w}{V_1^2}.$$

Referring to the velocity diagram:

$$\begin{aligned} V_w &= BE+BF \\ &= U_1\cos\beta_1+U_2\cos\beta_2 \\ &= U_1\cos\beta_1\left(1+\frac{U_2\cos\beta_2}{U_1\cos\beta_1}\right). \end{aligned}$$

Now the difference between β_1 and β_2 is always small and in many cases the angles may be equal. We shall assume that $\beta_1 = \beta_2$.

Therefore

$$\begin{aligned} V_w &= U_1\cos\beta_1\left(1+\frac{U_2}{U_1}\right) \\ &= U_1\cos\beta_1(1+k). \end{aligned}$$

Referring to the diagram,

$$U_1\cos\beta_1 = V_1\cos\alpha-u.$$

Therefore $$V_w = (1+k)(V_1\cos\alpha-u)$$

and $$2uV_w = 2u(1+k)(V_1\cos\alpha-u).$$

We now introduce into this equation the blade-speed ratio ρ.

Since $u = \rho V_1$, then

$$2uV_w = 2V_1^2(1+k)(\rho\cos\alpha-\rho^2).$$

Since the blade efficiency is given by the expression

$$\eta_b = \frac{2uV_w}{V_1^2},$$

we can say that

$$\eta_b = \frac{2uV_w}{V_1^2} = 2(1+k)(\rho\cos\alpha-\rho^2).$$

This equation gives us a relationship between the blade efficiency η_b and the blade-speed ratio ρ, so that we can now draw a curve

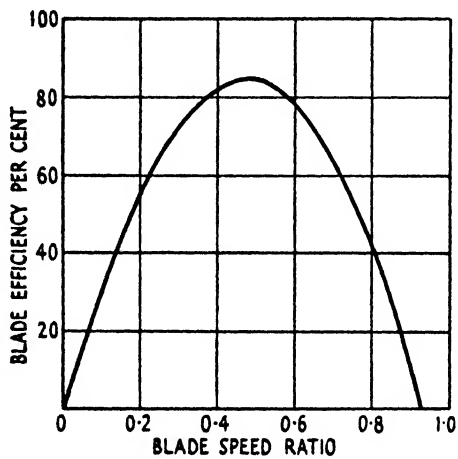


FIG. 120

showing the theoretical variation of η_b with ρ. Such a curve for a single-stage single-row impulse turbine is shown in Fig. 120. It will be observed that the maximum value of blade efficiency is attained at a blade-speed ratio of approximately 0·5, and on either side of this value the efficiency falls off sharply. The value of k is assumed to be 0·9.

The maximum value of the blade efficiency can be determined theoretically by differentiating η_b with respect to ρ and equating to zero. Thus

$$\frac{d\eta_b}{d\rho} = 2(1+k)(\cos\alpha-2\rho).$$

If the right-hand side of this expression is equated to zero, we obtain an optimum value of ρ, which is

$$\rho_{\text{optimum}} = \frac{\cos\alpha}{2}.$$

If we now substitute this value of ρ in our original equation, we get

$$\eta_{b\,\text{maximum}} = \tfrac{1}{2}(1+k)\cos^2\alpha,$$

which gives us the theoretical maximum value of η_b.

Gross stage efficiency

A 'stage' in an impulse turbine may be defined as a nozzle or nozzle ring, in which there takes place a pressure drop, plus a moving blade-ring, or group of moving blade-rings together with their associated fixed blade-rings, through which there is no pressure drop. Hence a nozzle system is an essential part of a stage and, when assessing the efficiency of such a stage, it is necessary to take into account the efficiency of the nozzle as well as the efficiency of the blading. We have already obtained an expression for the blade efficiency η_b, and in the chapter on nozzles we discussed the nozzle efficiency η_n. The product of the blade efficiency and the nozzle efficiency gives the theoretical overall efficiency for the stage, which is known as the 'gross stage efficiency' and is denoted by η_s. Therefore

$$\eta_s = \eta_b\,\eta_n.$$

This value of the gross stage efficiency does not, of course, take into account such factors as bearing losses, leakage losses, etc., but is merely the combined efficiency of the nozzle and blading.

Provided the pressure conditions remain constant, the nozzle efficiency will be constant, and hence the gross stage efficiency will attain its maximum value at the same value of blade-speed ratio at which η_b is a maximum. It follows that if a curve is plotted of gross stage efficiency against blade-speed ratio, it will be of substantially the same form as the curve of Fig. 120. The value of the maximum blade efficiency, and consequently the value of the maximum gross stage efficiency, is dependent upon the nozzle angle α. The smaller the nozzle angle, the higher the theoretical gross stage efficiency, since by reducing the nozzle angle the axial component of the nozzle velocity is reduced and the leaving loss is diminished. In practice other factors tend to limit the minimum angle which can be employed and it is doubtful whether α would ever be less than about 12°. In jet-propulsion installations where it may be necessary that the gas stream leaving the turbine blades should still possess a considerable amount of kinetic energy, the nozzle angle may be very much greater than this minimum.

Velocity-compounding

We discussed briefly the velocity-compounded type of impulse turbine wheel at the beginning of this chapter. Whereas with a single-row impulse wheel the entire energy conversion takes place in a single blade-ring, in a multi-row wheel an attempt is made to divide the energy conversion amongst two or more rows of moving blades, thus reducing both the blade speed and the leaving loss. This arrangement, however, is not without its disadvantages since the increased area of the blading results in an increase in the friction and other losses.

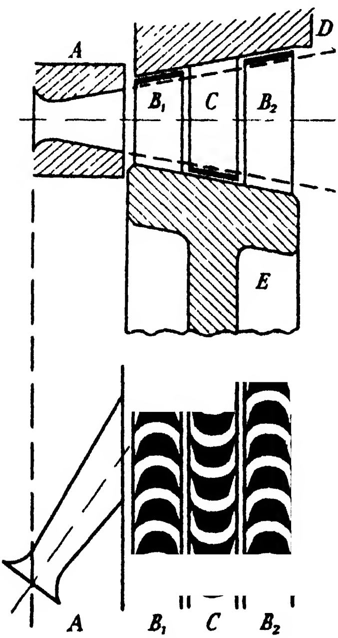


FIG. 121

Fig. 121 is a diagrammatic section and developed plan of a two-row lay-out. The convergent-divergent nozzle A discharges the jet on to the first row of moving blades B_1 attached to the wheel E. From the first moving row the gas passes into the fixed blade-ring C, attached to the turbine casing D, where it experiences a change in direction, and from which it is discharged on to the second moving blade-ring B_2. The passage of the gas through both moving and fixed blade-rings takes place at constant pressure, but its velocity is reduced from inlet to outlet. As the velocity is reduced, the sectional area of the passages through which the gas flows must, of course, be increased so that the equation of continuity may be satisfied. In addition, a certain amount of reheating is caused by the friction effect in the blade channels which results in an increase in specific volume and, consequently, demands a further increase in the sectional area available for the flow. The required increase in sectional area is partly provided by progressively

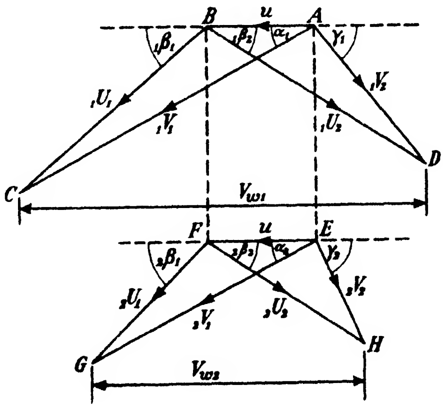


FIG. 122

increasing the blade height from the first to the last blade-ring, and partly by increasing the blade outlet angles.

The method of setting out the velocity diagrams for the two-row wheel is shown in Fig. 122. The following notation is employed.

$_1V_1$ = absolute velocity of gas at inlet to first moving row.
$_1U_1$ = relative velocity of gas at inlet to first moving row.
$_1V_2$ = absolute velocity of gas at outlet from first moving row.
$_1U_2$ = relative velocity of gas at outlet from first moving row.
$_2V_1$ = absolute velocity of gas at inlet to second moving row.
$_2U_1$ = relative velocity of gas at inlet to second moving row.
$_2V_2$ = absolute velocity of gas at outlet from second moving row.
$_2U_2$ = relative velocity of gas at outlet from second moving row.

As in the case of the single-row wheel, the inlet and outlet angles of the first moving blade-ring are $_1\beta_1$ and $_1\beta_2$ respectively. To ensure smooth and shock-free entry, the inlet angle of the fixed blades must be made equal to γ_1 since this is the true direction of the gas stream leaving the first moving blade-ring. The direction of the gas leaving the fixed blade-ring is given by α_2 and its velocity is ${}_2V_1$. Since there is friction in the fixed blade-channels, then ${}_2V_1$ is less than ${}_1V_2$.

The rim horse-power is calculated in the same way as for the single-row wheel. If W is the weight of gas in pounds discharged per second from the nozzle system, and if V_{w1} is the velocity of whirl in the first moving row in feet per second, then the work done per second in the first moving row is

$$\frac{W}{g}uV_{w1} \text{ ft. lb.},$$

where u is the peripheral velocity of the blades in feet per second. Similarly, the work done per second in the second moving row is

$$\frac{W}{g}uV_{w2} \text{ ft. lb.},$$

where V_{w2} is the velocity of whirl in the second moving row. Therefore the total work done is

$$\frac{Wu}{g}(V_{w1}+V_{w2}) \text{ ft. lb. per sec.}$$

This expression can be applied to a wheel with more than two moving blade-rings, so we can make it more general by writing it in the following form:

$$\text{work done} = \frac{Wu}{g}\sum V_w \text{ ft. lb. per sec.}$$

Therefore the power developed is

$$\frac{Wu}{550g}\sum V_w \text{ rim h.p.}$$

Blade efficiency in a two-row wheel

The blade efficiency can be determined in the same way as for the single-row wheel. Referring to the velocity diagram, Fig. 122, the kinetic energy of the jet at the nozzle outlet is

$$\frac{W_1V_1^2}{2g} \text{ ft. lb. per sec.}$$

Therefore the blade efficiency is

$$\eta_b = \frac{Wu \sum V_w}{g} \frac{2g}{W_1 V_1^2}$$
$$= \frac{2u \sum V_w}{{}_1V_1^2}.$$

If η_n is the nozzle efficiency, then the gross stage efficiency is

$$\eta_s = \eta_b \eta_n.$$

As in the case of the single-row wheel there is a certain blade-speed ratio at which the blade efficiency, and consequently the gross stage efficiency, is a maximum. Let ρ be the blade-speed ratio, where

$$\rho = \frac{u}{{}_1V_1}.$$

If α is the nozzle angle, then it can be shown analytically that the maximum gross stage efficiency is attained when

$$\rho = \frac{\cos\alpha}{4}.$$

This value of blade-speed ratio (for frictionless flow) is just one-half that for a single-row wheel. In a similar manner it can be shown that the maximum gross stage efficiency of a three-row wheel is attained when

$$\rho = \frac{\cos\alpha}{6}.$$

Fig. 123 shows two curves (*a*) and (*b*), for a single-row and a two-row impulse wheel, respectively. Gross stage efficiency is plotted against blade-speed ratio, the pressure conditions through the nozzle remaining constant. The curves are only approximate, but are sufficiently accurate to illustrate the nature of the variation of efficiency with blade-speed ratio. It will be seen that while the optimum value of ρ for the two-row wheel is about half that for the single-row wheel, the maximum gross stage efficiency is somewhat lower. This is largely due to the losses in the blading, but in some installations it may be that the lower efficiency will be outweighed by the advantage of a lower blade speed. Like the single-row impulse wheel, it is possible to use the two-row velocity-compounded wheel as the first stage of a reaction turbine, in which case its disadvantages might be outweighed by the fact that a comparatively large heat drop could be adopted in the first stage.

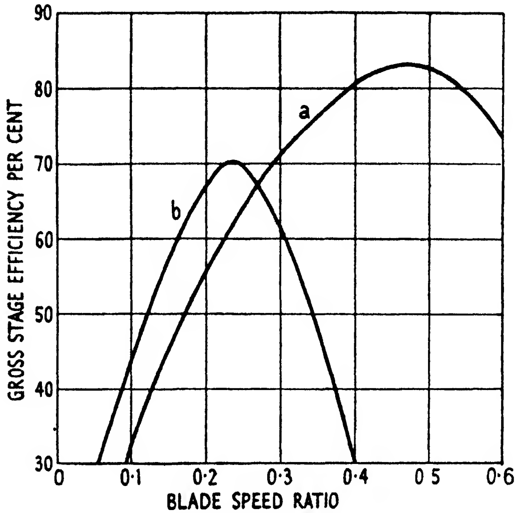


FIG. 123

Blade sections

The shape of the blades is most important from an efficiency point of view, since it is very necessary that the gas stream should be guided through the blade channels in as smooth and shock-free a manner as possible. Fig. 124 is a section through the simplest type of impulse turbine blade. This is the so-called plate blade, made from sheet metal and rolled to the correct curvature, the curve being a circular arc. In order to present a sharp edge to the gas stream at inlet and to prevent violent eddies at outlet the blade is chamfered, the chamfered edges being set at the inlet and outlet angles β_1 and β_2. This type of blade has the advantage of being cheap and simple to manufacture and is comparatively light, so that the centrifugal stresses imposed upon the rotor

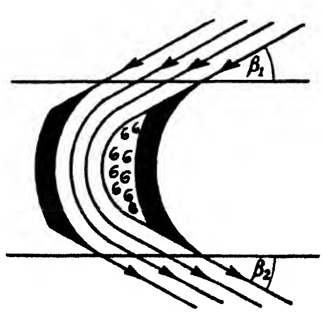


FIG. 124

are fairly low. It has, on the other hand, one serious disadvantage. It will be seen from Fig. 124 that the main stream does not follow the convex surface of the blade, but breaks away, thus forming a comparatively large eddy space in which powerful vortices are generated. This area of turbulence results

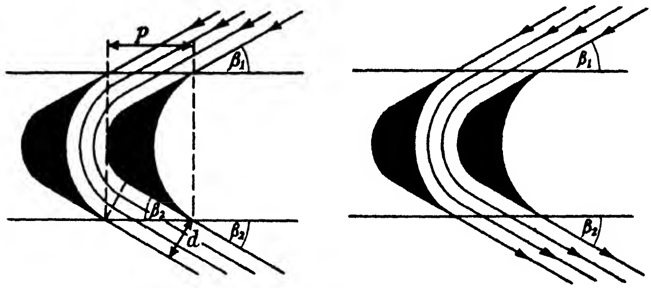


FIG. 125

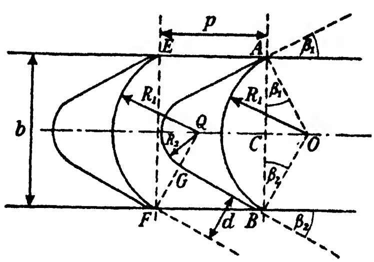


FIG. 126

in a by no means negligible loss of energy and detracts from the efficiency of the blading.

This eddy space can be eliminated by the adoption of the type of blade known as a profile blade, illustrated in Fig. 125. In this type of blade the section is increased so that it extends into what would otherwise be the eddy space and takes up the whole of it. By this means a very much smoother flow is obtained and the efficiency considerably improved. On the other hand, the cost of manufacture is increased and, owing to the greater weight of such sections, the edge-loading of the rotor is somewhat greater than with plate blades. The improvement in efficiency, however, would in most cases probably outweigh the increased cost.

The geometry of the profile blade is as follows. Referring to Fig. 126, if the centres of curvature of the convex and concave sides of adjacent blades are made coincident, then the channel width can be kept constant, i.e. Q, the centre of curvature of the convex side of blade AB, is coincident with the centre of curvature of the concave side of blade EF. The centre of curvature of blade AB is determined by setting off the line AO at the angle β_1 to AB, and BO at the angle β_2. The point of intersection O of AO and BO is the centre of curvature.

If the blade is assumed to be equiangular, i.e. $\beta_1 = \beta_2$, it is clear that

$$b = AC+BC = 2R_1 \cos\beta_1,$$

which determines the radius of curvature of the concave side as

$$R_1 = \frac{b}{2\cos\beta_1}.$$

The radius of curvature R_2 of the convex side is dependent upon the blade pitch p and is governed by the necessity for keeping the channel width constant.

Blade heights and angles

It is not possible to obtain a smooth gas entry into the blade

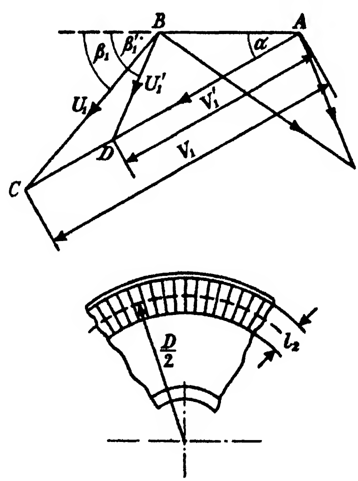


FIG. 127

channels at all speed and load conditions. Consequently the entry angle β_1 (Fig. 127) should be so designed as to give smooth entry when the turbine is running under cruising conditions, since this would give maximum economy. Should the pressure conditions through the nozzle be altered, there may be a change in the nozzle velocity from V_1 to some value V_1'. The nozzle angle α, however, remains constant, and consequently the shape of the velocity diagram is altered, as illustrated in Fig. 127. The direction of the relative velocity U_1 is changed from β_1 to β_1', and the flow into the blade channels is no longer smooth and shock-free. This, of course, involves a loss in blade efficiency.

The blade height is dependent upon the outlet angle β_2 and the outlet area required to accommodate the flow. Let:

p = mean circumferential pitch of the blades.
D = mean diameter of the blade-ring.
l_2 = height of blades on the outlet side.
n = number of blades in blade-ring.
d = effective width of blade channel at outlet.
v_2 = specific volume of gas at outlet.
W = mass flow of gas.

Referring to the diagram, it is clear that

$$d = p \sin \beta_2.$$

The effective cross-sectional area of each blade-channel at outlet is

$$l_2 p \sin \beta_2$$

and the total area available for the flow over the whole circumference of the wheel is

$$n l_2 p \sin \beta_2.$$

Now the area required by the flow is

$$\frac{W v_2}{U_2}$$

and if the equation of continuity is to be satisfied then the area required must be equal to the area available. Therefore

$$n l_2 p \sin \beta_2 = \frac{W v_2}{U_2}$$

and hence
$$\sin \beta_2 = \frac{W v_2}{n l_2 p U_2}$$

or
$$l_2 = \frac{W v_2}{n p U_2 \sin \beta_2}.$$

If the value of l_2 has been decided upon, then β_2 can be determined from the above equation, and conversely. If we assume that the blade outlet angle is equal to the inlet angle, the blade height at outlet is entirely dependent upon β_2. The blade heights in a velocity-compounded turbine can be determined by successive applications of the foregoing theory.

When the jet leaves the nozzle it diverges, as shown diagrammatically in Fig. 128. For this reason it is advisable to make

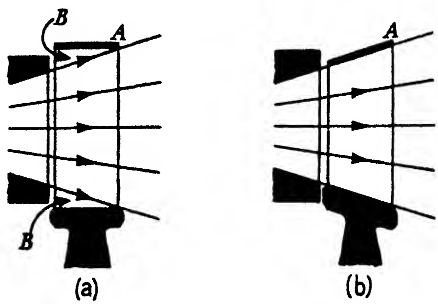


FIG. 128

the blade height slightly greater than the radial dimension of the nozzle outlet. If a shrouding strip A is employed, as is common in steam-turbine practice, then an area of turbulence may be located at B (Fig. 128 *a*). This may be avoided by the type of construction illustrated in Fig. 128 *b*.

Some recent research carried out in Russia[1] tends to indicate that the use of shrouding strips is inadvisable in combustion turbines. The reason given for this is that the cooling effect of the high-speed rotation on unshrouded blades is somewhat diminished by using a shrouding strip and that, therefore, the wheel must be run at a slightly lower temperature. It is suggested that the leakage loss consequent upon the absence of a shrouding strip can be compensated for by tilting the nozzle axis towards the centre of the wheel at a small angle of up to 10°. While this may be true for turbines fitted with a very small number of nozzles, it is doubtful whether or not it would hold for turbines having a complete nozzle ring, or nozzles extending over a considerable arc.

Losses in impulse wheels

We have already discussed nozzle losses, blade-friction losses, and the leaving loss due to the residual velocity of the gas

[1] Shevyakov, *Sovietskoye, Kotloturbostroynie*, No. 1, Jan. 1940.

passing into the exhaust system. One further source of loss should be mentioned, namely the disk-friction loss.

Any gas is slightly viscous, and no metal surface is completely smooth. Consequently, if a surface is moved through a gas, there is a certain amount of friction between the surface and the portion of the gas in immediate contact with it. Fig. 129

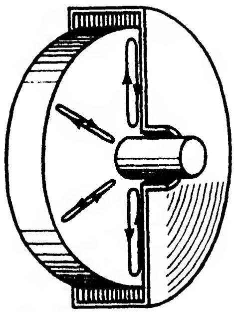


Fig. 129

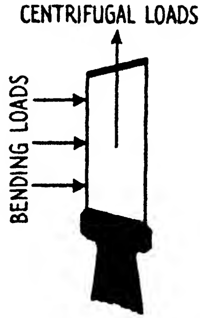


Fig. 130

shows a circular disk mounted on a shaft and enclosed in a casing, part of which is, for the sake of clarity, cut away. If the disk is rotated, the friction between the surface of the disk and the layers of gas in immediate contact with it will cause these layers of gas to be dragged around. As the speed of rotation is increased, the centrifugal action will force a flow of gas over the disk surface to the periphery, the return flow towards the centre being along the casing walls. Thus a circulation is set up, as indicated by the arrow-marked paths in the diagram. A very similar action takes place in the case of turbine disks, and there is a loss of energy due to the vortex effect in the wheel casing.

Construction

The centrifugal stresses on a turbine blade are high, but in addition to these the bending stresses consequent upon the impact of the gas stream on the blade must be taken into account, and the blade must be able successfully to withstand the stresses at high temperatures. At high blade speeds the centrifugal force on a blade may in fact amount to several thousand times the weight of the blade.

Two typical methods of blade attachment, as used in steam turbines, are illustrated in Fig. 131, and there is no reason to. believe why attachments of this nature should not prove popular in combustion-turbine practice. In 131 (*a*) the blades are fitted into individual slots in the wheel rim, and the tops of the blades

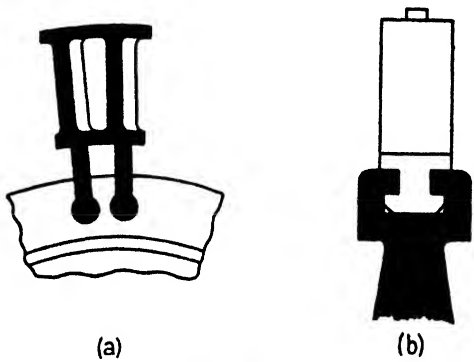


FIG. 131

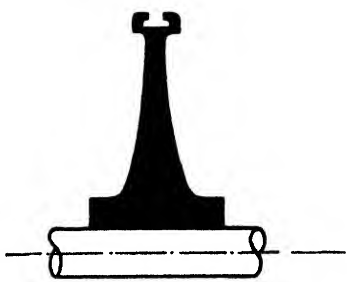


FIG. 132

butt together to form the continuous shrouding strip. The type of blade shown in Fig. 131 (*b*) has a T-section tang which fits into a continuous slot cut in the wheel rim and a separate shrouding strip is employed.

If a flat disk of constant thickness is employed as the turbine rotor, the stresses caused by the rotation will vary from centre to periphery. If a tapered section, such as that shown in Fig. 132, is used, a more uniform stress distribution can be obtained with consequent beneficial effect. Fig. 133 illustrates a section which may be described as a close approach to the ideal rotor section. This is the so-called 'disk of constant stress', and is so designed as to give an almost completely uniform distribution of stress from centre to rim. Unfortunately it suffers from the disadvantage of being a solid disk with no central hole, and the turbine shaft must, therefore, be split. The good balancing of turbine rotors is, of course, most important since at the high rotational speeds employed the consequences of vibration can be serious.

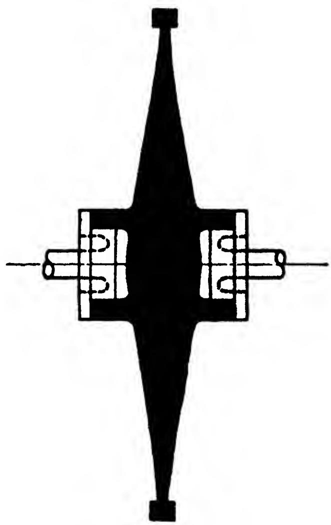


Fig. 133

IX

REACTION TURBINES

FIG. 134 is a diagrammatic representation of a turbine of the so-called 'reaction' type. The upper portion of the drawing shows a half-section through the rotor and casing, while below it is a development of the fixed and moving blade-rings, together with curves of pressure and velocity variation between inlet and exhaust. Gas from the combustion chamber A enters the first row of fixed blades B_1 and experiences a drop in pressure and an increase in velocity. The blades are designed to have an outlet angle which is considerably smaller than the inlet angle, so that the area at outlet is less than that at inlet. The blade channels are, therefore, convergent and approximate to convergent nozzles. On leaving the fixed blade-ring the gas enters the first moving blade-ring C_1 and gives up some of its kinetic energy. But again the outlet angle of the blades is smaller than the inlet angle, so that there is a nozzle effect in the moving blade-ring, which is accompanied by a drop in gas pressure. The gas then enters the second row of fixed blades B_2, and the process is repeated until the exhaust pressure is reached and the gas leaves the system at D. The fall of pressure between inlet and exhaust, and the variation in gas velocity through the fixed and moving blade-rings, are illustrated by the curves, Fig. 134.

The moving blade-rings are attached to a drum, or rotor, E which is keyed to the turbine shaft F, while the fixed blades are attached to the turbine casing intermediate between the rows of moving blades. The clearance between the tips of the moving blades and the casing and between the tips of the fixed blades and the rotor must, of necessity, be very small in order to prevent leakage of gas through the clearance space. Each 'pair' of blade-rings, that is, one fixed ring and its associated moving ring, comprises a 'stage'. The total number of stages in such a turbine may be considerable, the pressure drop in each stage being small as compared with the pressure drop in a single stage of an impulse turbine working between the same limits of inlet and exhaust pressure. It will be observed from the diagram, Fig. 134, that although the mean diameter of all the blade-rings, both fixed and moving, is the same, the blade height increases progressively between inlet and exhaust in order to accommodate the continuous expansion.

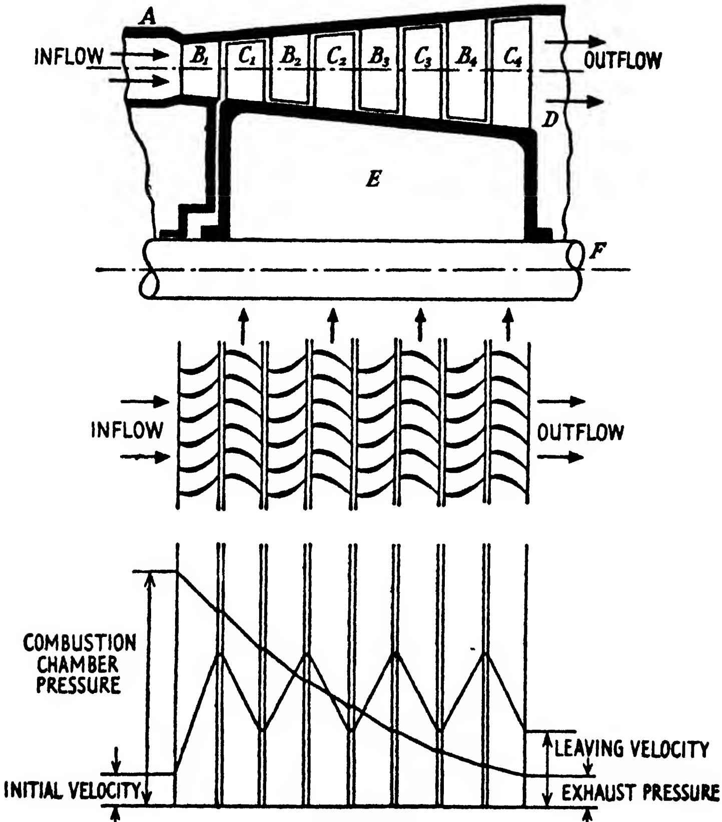


FIG. 134

When discussing the blade channel area required for the gas flow in an impulse turbine we derived the following expression for the blade outlet angle β_2:

$$\sin \beta_2 = \frac{Wv_2}{nl_2pU_2},$$

where

W = rate of flow, pounds per second.
v_2 = specific volume of gas at blade outlet.
n = number of blades in blade-ring.
p = mean circumferential pitch of blades.
l_2 = height of blades on the outlet side.
U_2 = velocity of gas relative to blade at outlet.

Now in the impulse turbine the gas pressure through the moving blades is constant. That is, the gas pressure at the blade inlet is the same as the gas pressure at the blade outlet. Consequently the blade outlet angle is made equal, or very nearly equal, to the blade inlet angle, and the expression for the blade outlet angle β_2 was derived on this basis. Suppose the pressure drop in the fixed nozzles is comparatively small, and that when the gas enters the first moving blade-ring its pressure is still considerably above that of the turbine exhaust (as is actually the case in a reaction turbine). If we now make the outlet angle β_2 very much smaller than the inlet angle β_1, Fig. 135, the above equation can still be applied provided the conditions at the blade outlet are taken into account. In the impulse turbine the equation provided a relationship between $\sin\beta_2$ and l_2, the specific volume v_2 and the relative velocity U_2 being assumed roughly constant through the blade-ring.

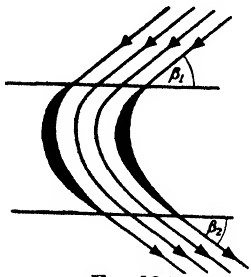


FIG. 135

If, however, β_2 is made very much smaller than β_1, then the area available for the flow at outlet becomes considerably smaller than the area at inlet. Assuming the blade height constant, this means that the moving blade channel will function in a manner similar to that of a convergent nozzle, and in order to satisfy the equation (which is nothing more nor less than the equation of continuity in another form) the flow velocity relative to the blades at outlet must be greater than the velocity relative to the blades at inlet. In the same way as in a nozzle, the increase in velocity is accompanied by a pressure drop through the moving blade channel, and an increase in the specific volume v_2. Since the gas stream undergoes an increase in velocity during its passage through the moving blade-ring, it follows that it also experiences an increase in momentum. This increase in momentum generates a force, or thrust, on the blades which is called a 'reaction' force, and gives the turbine its name. We see, therefore, that in the reaction turbine a useful thrust is obtained by a suitable expansion of the gas in the moving blade-rings themselves, quite apart from the expansion which takes place in the fixed blade-rings.

The velocity diagram

Suppose there were no reaction in the moving blade channels. Then we might obtain a velocity diagram something like that of Fig. 136 (*a*). The velocity of the gas U_2 relative to the blade at outlet would be represented by the vector BD, and the

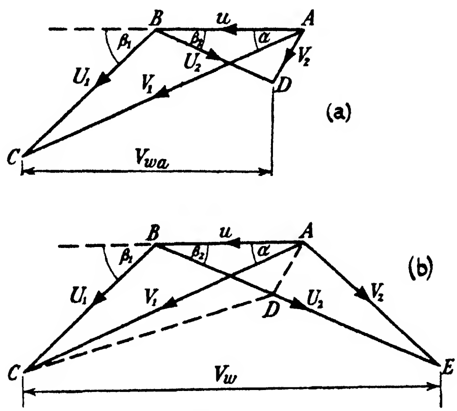


FIG. 136

impulsive thrust on the blades would be proportional to the velocity of whirl V_{wa}. Now let us see what happens when there is a reaction effect. The velocity at outlet is increased, so that it may now be represented by the vector BE, Fig. 136 (*b*), and the useful thrust on the blades is proportional to the velocity of whirl V_w. We see, therefore, that the useful thrust on the blades is derived from two sources

(*a*) the impulse due to the change in momentum of the gas stream leaving the fixed blade-ring, due to the deflexion caused by the curvature of the moving blades, and

(*b*) the reaction generated by the change in momentum of the gas stream due to expansion in the moving blade-ring itself.

If a force vector diagram were drawn, it would be similar to that shown in Fig. 137. Referring to the velocity vector diagram of Fig. 136 (*b*), the impulsive force is equal to $(W/g)DC$, and the reaction force is equal to $(W/g)ED$, where W is the rate of flow through the blade-ring. The lines of action of these two forces are in different directions, but they may be combined by means of the force diagram to give a single resultant force, as shown, the tangential component of which is the useful thrust on the moving blade-ring.

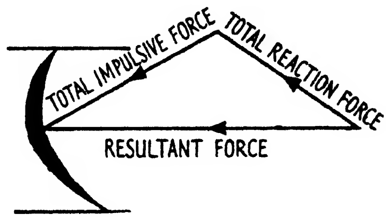


FIG. 137

The degree of reaction

Fig. 138 illustrates one fixed row and one moving row of blades, together with the velocity diagram for the moving row. Ideally the sections of each 'blade pair' would be the same for every stage, i.e. the sections of all the moving rows would be the

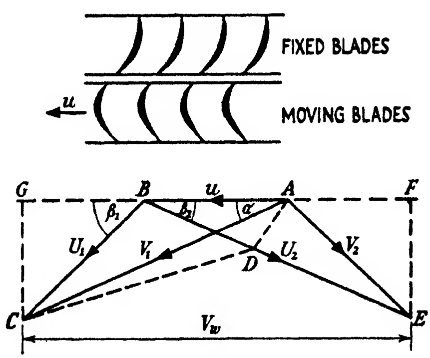


FIG. 138

same, but the sections of the fixed rows need not necessarily be the same as the sections of the moving rows. It follows, therefore, that the velocity diagrams would be the same for all the moving blade-rows, so that if the properties of any one pair of blade-rings, or stage, are determined, as far as the velocity diagram is concerned the results may be assumed applicable to all the stages. As regards all the fixed blade-rows, except the first, the entry velocity must clearly be equal to V_2, for V_2 is the absolute velocity with which the gas leaves the preceding moving blade-row. In the same way, the gas velocity at exit from the fixed blade-row is V_1.

The kinetic energy per pound of the gas stream entering the fixed blade-row, neglecting any entry losses, is

$$\frac{V_2^2}{2g}.$$

While passing through the fixed blade-row the velocity is increased, and the kinetic energy at outlet is

$$\frac{V_1^2}{2g}.$$

Consequently the gain of kinetic energy in the fixed blade-row is, ideally,

$$\frac{V_1^2-V_2^2}{2g}.$$

Referring to the velocity diagram, we see that

$$V_2^2 = U_2^2+u^2-2uU_2\cos\beta_2.$$

Now let

$$\rho_2 = \frac{u}{U_2}.$$

Then

$$V_2^2 = U_2^2(1+\rho_2^2-2\rho_2\cos\beta_2).$$

The gain of kinetic energy in the fixed blades may then be written as

$$\frac{V_1^2-U_2^2(1+\rho_2^2-2\rho_2\cos\beta_2)}{2g}.$$

Since the blade-ring can be considered as a ring of nozzles, we can let η_n represent the efficiency of the blades when considered as nozzles. In this case, although the above expression states the gain in kinetic energy, the total energy actually given up by the gas in producing this gain in kinetic energy would be

$$\frac{V_1^2-U_2^2(1+\rho_2^2-2\rho_2\cos\beta_2)}{2g\eta_n}.$$

If we now consider the moving blade-row, the velocity of the stream relative to the blades at entry is U_1 and at outlet is U_2. Due to the expansion in the moving blade channels, U_2 is greater than U_1. So, neglecting losses, we can say that the gain in kinetic energy in the moving blade-row is

$$\frac{U_2^2-U_1^2}{2g}.$$

From the velocity diagram it is clear that

$$U_1^2 = V_1^2 + u^2 - 2uV_1 \cos\alpha.$$

Now let

$$\rho_1 = \frac{u}{V_1}.$$

Then

$$U_1^2 = V_1^2(1+\rho_1^2 - 2\rho_1 \cos\alpha).$$

The gain of kinetic energy in the moving blades may then be written as

$$\frac{U_2^2 - V_1^2(1+\rho_1^2-2\rho_1 \cos\alpha)}{2g}.$$

Again, the moving blades can be considered as nozzles, and if we let η_n represent their efficiency when considered as nozzles, the total energy given up by the gas in producing the increase in kinetic energy may be written as

$$\frac{U_2^2 - V_1^2(1+\rho_1^2-2\rho_1 \cos\alpha)}{2g\eta_n}.$$

We have now derived expressions for the energy used up in increasing the kinetic energy of the stream in one fixed and one moving blade-row. Let E_f be the energy used up in the fixed blade-row and E_m be the energy used up in the moving blade-row. Then

$$\frac{E_m}{E_f + E_m} = R,$$

where R is known as the 'degree of reaction'. The degree of reaction is, therefore, the ratio of the energy needed to produce the required velocity increase in the moving blade-row to the energy needed to produce the velocity increases in both fixed and moving blade-rows.

Half-degree reaction

For ease of construction the blade-sections in both fixed and moving blade-rows may be made the same. Provided the mean diameter of the blade-rings is constant and the blade heights are increased progressively, as in Fig. 134, to accommodate the expansion, this involves a special case. The gas velocity at outlet from each row of blades, whether fixed or moving, is the same. Referring to Fig. 139, $U_2 = V_1$ and $\beta_2 = \alpha$ and, therefore, the triangles ABC and ABE are congruent.

As previously explained, the gas stream enters each row of fixed blades, except the first, with an absolute velocity V_2, which is also the velocity relative to the blades since the blade-row is stationary. Since the triangles are congruent, then $V_2 = U_1$. The velocity with which the gas leaves the

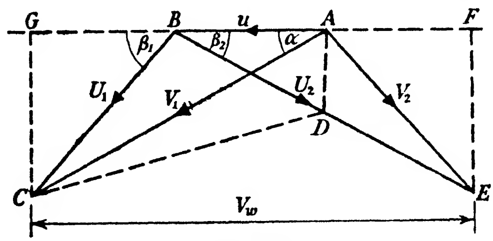


FIG. 139

fixed blade-row is V_1, and the gain in kinetic energy in the fixed blades is

$$\frac{V_1^2 - U_1^2}{2g}.$$

If we now consider the moving blades, the gas velocity relative to the blades at entry is U_1 and at outlet is U_2. Therefore the gain in kinetic energy in the moving blades is, ideally,

$$\frac{U_2^2 - U_1^2}{2g}.$$

But $U_2 = V_1$. Therefore the gain in kinetic energy is

$$\frac{V_1^2 - U_1^2}{2g}.$$

In other words, the gain of kinetic energy due to expansion in the moving blade-ring is the same as the gain of kinetic energy due to expansion in the fixed blade-ring. This, of course, neglects all losses, but since the blade-sections are similar it can reasonably be assumed that the losses in both fixed and moving blade-rows are the same. If E_f is the energy used up in producing the increase in kinetic energy in the fixed blade-ring and E_m is the energy used up in producing the gain in kinetic energy in the moving blade-ring, then, as before, we have the degree of reaction equal to

$$\frac{E_m}{E_f + E_m} = R.$$

But we have shown that in this case $E_f = E_m$. It is clear, therefore, that $R = 0{\cdot}5$, and the turbine is said to have 'half-degree reaction'.

Gross stage efficiency

The gross stage efficiency has the same significance in the reaction turbine as in the impulse turbine, but the 'stage' is, of course, a pair of blade-rings comprised of one fixed and one moving row. We first need to calculate the useful work done on the blades by the gas stream. Referring to Fig. 138, we see that the velocity of whirl is equal to

$$V_w = BG + FB$$
$$= V_1 \cos\alpha - u + U_2 \cos\beta_2.$$

Since we are working in ft.-lb.-second units the velocity of whirl is in feet per second. The work done per pound of gas is equal to

$$\frac{u}{g}(V_1 \cos\alpha - u + U_2 \cos\beta_2) \text{ ft. lb. per sec.,}$$

and if we have a rate of flow of W lb. of gas per second the work done on the moving blade-ring is

$$\frac{Wu}{g}(V_1 \cos\alpha - u + U_2 \cos\beta_2) \text{ ft. lb. per sec.}$$

The rim horse-power is

$$\frac{Wu}{550g}(V_1 \cos\alpha - u + U_2 \cos\beta_2).$$

If $\rho_1 = u/V_1$ and $\rho_2 = u/U_2$, then the expression for the work done per pound of gas per second can be written in the form

$$\frac{V_1^2(\rho_1 \cos\alpha - \rho_1^2) + \rho_2 U_2^2 \cos\beta_2}{g}.$$

The gross stage efficiency η_s is the ratio of the useful work done on the moving blade-ring to the total energy given up by the gas stream in the stage (i.e. in the fixed and the moving blade-ring). Therefore

$$\eta_s = \frac{V_1^2(\rho_1 \cos\alpha - \rho_1^2) + \rho_2 U_2^2 \cos\beta_2}{g(E_f + E_m)}.$$

The above is a general expression for any reaction turbine blade form, but the special case of the half-degree reaction turbine is of considerable importance and must be considered separately. Again, the velocity of whirl is equal to

$$V_w = BG+FB$$
$$= V_1 \cos\alpha - u + U_2 \cos\beta_2.$$

But $U_2 = V_1$ and $\beta_2 = \alpha$. Therefore

$$V_w = V_1 \cos\alpha - u + V_1 \cos\alpha$$
$$= 2V_1 \cos\alpha - u.$$

The useful work done on the moving blades per pound of gas per second is

$$\frac{uV_w}{g} = \frac{2uV_1 \cos\alpha - u^2}{g} \text{ ft. lb.}$$

If the full flow of gas is W lb. per sec., then the total useful work done on the blade-row per second is

$$\frac{W(2uV_1 \cos\alpha - u^2)}{g} \text{ ft. lb.}$$

and the rim horse-power is

$$\frac{W(2uV_1 \cos\alpha - u^2)}{550g}.$$

Since $U_2 = V_1$ it is clear that when a turbine has half-degree reaction $\rho_2 = \rho_1$, and therefore the expression for the work done per pound of gas per second may be written in the form

$$\frac{V_1^2}{g}(2\rho \cos\alpha - \rho^2) \text{ ft. lb.,}$$

where $\rho = u/V_1$. Referring to the last article, we showed that for a half-degree reaction turbine the increase in kinetic energy of the gas stream due to expansion in each row of blades, whether fixed or moving, was

$$\frac{V_1^2 - U_1^2}{2g} \text{ ft. lb. per lb. of gas per sec.}$$

Hence, for a complete stage, consisting of one fixed and one moving row, the total increase in kinetic energy is

$$\frac{V_1^2 - U_1^2}{g} \text{ ft. lb. per lb. of gas per sec.}$$

If we let η_n represent the efficiency of the blade-rows when considered as nozzles, this being assumed the same for both fixed and moving blade-rows in a half-degree reaction turbine, then the total energy used up in producing the increase in kinetic energy in the stage is

$$\frac{V_1^2-U_1^2}{g\eta_n} \text{ ft. lb. per lb. of gas per sec.}$$

Referring to the diagram, Fig. 139, it is clear that

$$U_1^2 = V_1^2+u^2-2uV_1\cos\alpha,$$

and writing
$$\rho = \frac{u}{V_1}$$

we can say that

$$U_1^2 = V_1^2(1+\rho^2-2\rho\cos\alpha).$$

Therefore the total energy used up per pound of gas per second may be written as

$$\frac{V_1^2-V_1^2(1+\rho^2-2\rho\cos\alpha)}{g\eta_n} = \frac{V_1^2(\rho^2-2\rho\cos\alpha)}{g\eta_n} \text{ ft. lb.}$$

The gross stage efficiency η_s is the ratio of the useful work obtained at the wheel rim to the total energy used up in the stage. The gross stage efficiency of a half-degree reaction turbine is, therefore, given by the expression

$$\eta_s = \frac{\eta_n(2\rho\cos\alpha-\rho^2)}{\rho^2-2\rho\cos\alpha}.$$

From this it is clear that the gross stage efficiency may be numerically equal to the efficiency of the blades considered as nozzles, but in practice it must inevitably be less than this due to the various losses which we have not taken into account, and which would increase the value of the denominator in the above expression.

The quantity $\rho = u/V_1$ may be taken as the blade-speed ratio for this type of turbine. The gas velocities are very much lower than in an equivalent impulse turbine working between the same limits of inlet and exhaust pressure, and the blade-speed ratios in the neighbourhood of the peak efficiency are much higher, the blade velocity being not very much lower than the gas velocity. Fig. 140 shows a typical curve of gross stage efficiency plotted against blade-speed ratio. On the same diagram is drawn a curve for a comparable single-stage single-row impulse turbine, and it will be observed that while the efficiency of the impulse turbine increases rapidly as the optimum blade-speed ratio is approached and falls off rapidly

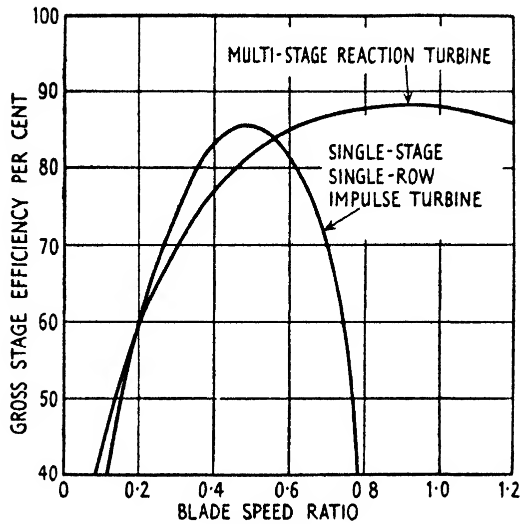


FIG. 140

at ratios greater than the optimum, the curve for the reaction turbine is relatively flat, thus giving a much improved operating range with only a small drop in efficiency on either side of the optimum blade-speed ratio. Taking into account the normal losses, it can be shown that the gross stage efficiency attains, ideally, its maximum value when $2\rho\cos\alpha - \rho^2$ is a maximum, and this is the case when $\rho = \cos\alpha$.

Blade leakage losses

There is invariably a certain amount of energy lost due to the clearance space between the rotor blades and the casing and the clearance between the fixed blades and the rotor drum. The loss is comparatively small when the rotor blades are shrouded, but when unshrouded blades are used there is a leakage of gas through the clearance spaces which has to be taken into account. This leakage gas does no useful work on the turbine blades and, therefore, the energy which it initially possessed is written off as going to waste. If l is the mean radial blade height in any given stage, and if c is the radial clearance between the blade-tips and the casing, let W be the mass flow of gas through the blade-ring and W_1 the gas which actually does useful work on the blades. Then the amount of gas lost per second due to

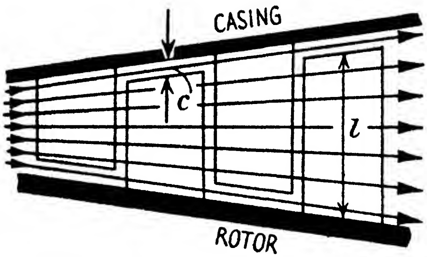


FIG. 141

leakage through the clearance space is $W-W_1$. The determination of the leakage loss by purely theoretical means is a matter of considerable difficulty. The following empirical formula has been derived from experiments on steam turbines:

$$W_1 = W\left(1-\frac{3c}{l}\right).$$

Taking the leakage loss into account in the half-degree reaction turbine, the expression for the useful work done per second in any given stage becomes

$$\frac{W_1 V_1^2(2\rho\cos\alpha-\rho^2)}{g}\ \text{ft. lb.}$$

Substituting from the leakage-loss formula, we get

$$W\left(1-\frac{3c}{l}\right)\frac{V_1^2(2\rho\cos\alpha-\rho^2)}{g}\ \text{ft. lb.}$$

as the actual useful work done per second by the gas on the moving blades of the stage.

Blade form

The form of the blade-sections of reaction turbines is illustrated in Fig. 142. As previously explained, the blade height is increased progressively from inlet to exhaust in order to accommodate the expansion. Let v be the specific volume of the gas in any stage, D the mean diameter of the blade-ring, l the blade height, p the mean circumferential pitch of the blades, and β_2 the blade outlet angle. If d is the effective mean width of each blade-channel at outlet, then

$$d = p \sin \beta_2$$

and the outlet area of each blade-channel is equal to

$$pl \sin \beta_2.$$

The number of blades in the ring is equal to

$$\frac{\pi D}{p}$$

and, therefore, the total area available for the flow is

$$\frac{\pi D}{p} pl \sin \beta_2 = \pi Dl \sin \beta_2.$$

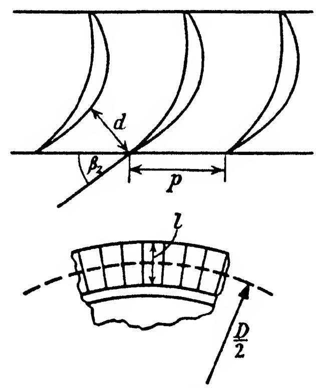


FIG. 142

If W_1 is the actual mass flow of gas through the blade-ring after deducting the gas lost by leakage through the clearance space, then the area required by the gas is

$$\frac{W_1 v}{V_1},$$

where V_1 is the gas velocity relative to the blades at outlet. This, of course, is for the half-degree reaction turbine, where $U_2 = V_1$. The area available for the flow should be made equal to the area required if we intend to cater for steady and progressive expansion between inlet and exhaust. Therefore,

$$\pi Dl \sin \beta_2 = \frac{W_1 v}{V_1}$$

and

$$l = \frac{W_1 v}{\pi V_1 D \sin \beta_2}.$$

From this the blade height l may be determined, provided the mean diameter D of the blade-ring is already fixed. The two quantities are dependent upon each other, but the value of D is influenced by other considerations and may, in general, be determined within fairly close limits before l is finally calculated.

The use of an impulse stage

One of the chief disadvantages of the reaction turbine is that the first moving blade-row runs at a temperature very little below that of the combustion chamber. In the single-stage impulse turbine the full expansion takes place in the fixed nozzles, with a consequently large temperature drop, and the wheel blading runs at what is virtually the exhaust temperature.

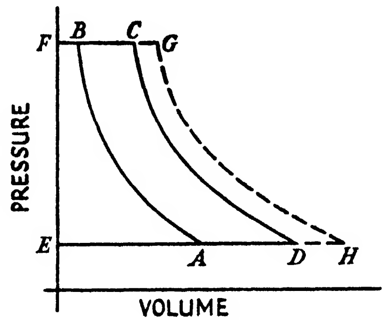


FIG. 143

Now the maximum temperature at which the blades can be run at the present time is in the neighbourhood of 700° C. If an impulse wheel is used, the combustion-chamber temperature can be several hundred degrees above this value, whereas if a reaction turbine is employed the combustion-chamber temperature cannot be very much greater than the permissible blade temperature on account of the comparatively small pressure drop, and hence the small temperature drop, in the first row of fixed blades. On the other hand, the characteristics of the reaction-turbine efficiency curve (Fig. 140) render it more desirable from the point of view of practical operation than the rather sharply peaked curve of the impulse turbine. In addition, the blade speeds of impulse wheels tend to be rather higher than those of reaction turbines, and the resulting higher stresses impose a slightly lower temperature limit on impulse blading than would be the case if equivalent turbines of both types could be run at the same blade speeds.

That it is an advantage to run the blades at a high temperature can be made clear from the constant-pressure cycle diagram, Fig. 143. Suppose we have a turbine the air-cycle diagram for which is given by *DEFC*. Then the useful work of the cycle is given by the area *ABCD*, and the negative work by the area *AEFB*. Suppose now the combustion-chamber temperature is increased so that the diagram is enlarged to *GH*. Clearly the blade temperature at *H* will be increased, but the useful work per pound of working fluid is now represented by the area *ABGH*, which is greater than *ABCD*. If reference is made to the expression for air-cycle efficiency, it will be remembered

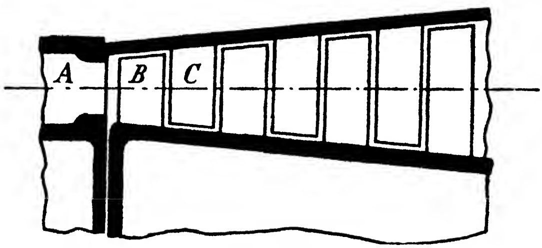


Fig. 144

that its value is dependent only upon the pressure-ratio, and that an increase in the useful-work area does not increase the efficiency. But this takes no account of the considerable losses in the compressor, so that if we can obtain a greater amount of useful work from each pound of working fluid by increasing the temperatures, then we can get the same total output from the machine by compressing a smaller quantity of air. This will mean decreased losses in the compressor, leading to higher overall efficiency.

A compromise may be achieved between the conflicting factors by using an impulse wheel as the first stage of what is essentially a reaction turbine. The layout is illustrated diagrammatically in Fig. 144. Gas from the combustion chamber enters the ring of convergent nozzles *A* and undergoes a relatively large pressure and temperature drop. After passing through the row of impulse blades *B* it enters the first row of fixed blades *C* of the reaction turbine proper, and thence continues its expansion through the reaction blade-rows in the usual manner. A velocity-compounded impulse wheel may, of course, be used in place of the single row shown in the diagram.

By utilizing this arrangement it is possible to employ higher combustion-chamber temperatures than would be permissible with a reaction turbine alone, but not as high as those obtainable with an impulse turbine alone, and yet keep the blade temperatures within reasonable bounds. The 'peaky' efficiency curve of the impulse stage naturally has an adverse effect on the very flat curve of the reaction turbine, but provided convergent nozzles are used and provided the nozzle back-pressure does not fall below the critical value, the net effect of the combination, in many types of installation, may prove advantageous.

Construction

There are several possible methods of attaching the blades to the rotor drum. One method is illustrated in Fig. 145. The blade root is fairly large and the serrations machined on it mate up with corresponding serrations in a slot machined in the rotor drum, the fixation being completed by the tangential insertion of a serrated packing-piece. In general, the method of blade attachment employed is dependent upon the conditions of temperature and stress pertaining to the particular machine, and the method of attachment may in fact be different at the inlet end of the rotor from the exhaust end, where the temperatures are lower and the blades longer.

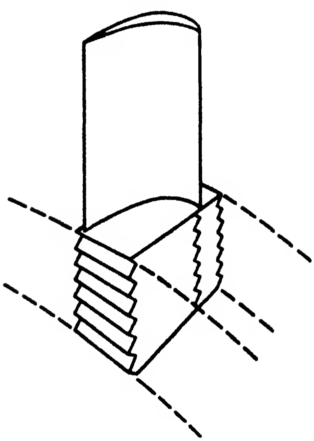


FIG. 145

Blade materials give rise to a certain amount of difficulty, since they are required to stand up to considerable stresses at very high temperatures and at the same time remain corrosion-free and retain their mechanical properties over long periods of operation. For high-temperature work in steam turbines the nickel-copper alloys, such as Monel metal and K-Monel have been extensively developed, but while possessing good corrosion-resisting properties, the limiting temperatures at which they may be continuously run are rather too low for combustion turbines. The high nickel-chrome alloy known as Inconel, which contains 80 per cent. nickel and up to 14 per cent. chromium, possesses good mechanical properties combined with corrosion resistance at fairly high temperatures and may be suitable for the last stages of a reaction turbine provided the temperatures do not exceed about 500° C. The best practical results up to date, however, have been attained by the use of austenitic steels, and consistently good service has been obtained by Messrs. Brown Boveri of Switzerland with an

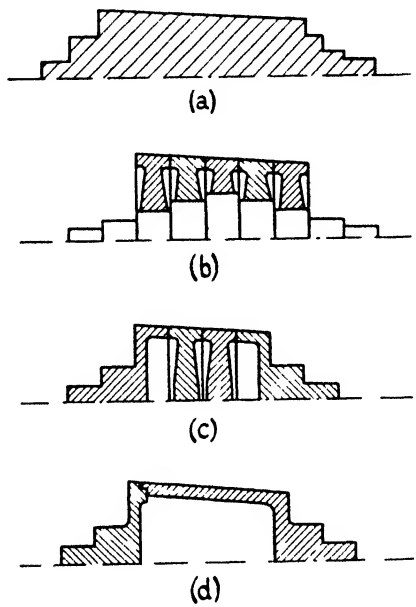


FIG. 146

austenitic steel of the following composition: chromium, 18 per cent.; nickel, 8 per cent.; silicon, 1·2 per cent.; magnesium, 0·6 per cent.; tungsten, 1·0 per cent.; iron, 71 per cent. Blades of this material have operated successfully over long periods at temperatures in the neighbourhood of 600° C. while subjected to a continuous stress of 5 tons per sq. in., which may be regarded as a normal working stress for this type of machine.

A number of different types of rotor drum are possible, varying from the solid machined forging of Fig. 146 (*a*) to the hollow forged drum of Fig. 146 (*d*). Fig. 146 (*b*) shows a rotor made up of a number of disks attached to a shaft, a form of construction which has certain disadvantages, while in Fig. 146 (*c*) the drum consists of a number of disks welded together, with no central shaft. In all cases the effects of expansion at the high operating temperatures must be taken into account, not only as regards the rotor as a whole, but including the differential expansion between various parts of the drum which are running at different temperatures. Although the rotational speeds of reaction rotors are generally lower than those of equivalent impulse wheels, it is nevertheless of considerable importance that they should be well balanced, since the vibrational consequences of out-of-balance forces may be serious.

X

HEAT-EXCHANGERS

THE exhaust gases from the turbine casing carry away with them a considerable quantity of heat energy, which goes to waste, and consequently, where possible, it is most desirable that there should be some form of waste-heat economization. By waste-heat economization is meant the utilization of a portion of the heat energy contained in the exhaust gases to do useful work, thereby increasing the net useful output of the plant and decreasing the quantity of fuel consumed per shaft horse-power. In other words, waste-heat economization results in an increase in the overall efficiency of the plant.

In combustion-turbine practice the hot exhaust gases may be used to pre-heat the compressed air when it leaves the compressor and before it enters the combustion chamber. The compressed air is passed through a number of tubes, or ducts, which may be of various sizes and sections, situated in the exhaust system, and is heated at constant pressure to some temperature which is intermediate between the compressor delivery temperature and the combustion-chamber temperature. The system of tubes, or ducts, in which the pre-heating takes place is known as a heat-exchanger.

The effect of a heat-exchanger can be seen on reference to the air-cycle diagram, Fig. 147. The air enters the compressor at atmospheric temperature at point 1 and is compressed adiabatically along the curve 1,2. At 2 it leaves the compressor and, due to the compression, its temperature is considerably higher than the temperature at 1. If there were no heat-exchanger the compressed air would then enter the combustion chamber and heating would take place at constant pressure along the line 2,3, the maximum permissible temperature being attained at point 3. The amount of heat energy imparted to the air stream, and consequently the quantity of fuel burned, in the combustion chamber is proportional to the length of the line 2,3. If now a heat-exchanger is installed between the compressor and the combustion chamber, the air delivered by the compressor can be considered as entering the heat-exchanger at point 2, and is heated at constant pressure along the line 2,3 until it reaches some point 5 at which it leaves the heat-exchanger and enters the combustion chamber where the remainder of the heating, from 5 to 3, necessary to bring it to the required maximum temperature, is carried out. Clearly, the amount of heat energy imparted to the air stream in the heat-exchanger is proportional to the length of the line 2,5 and the quantity of fuel which must be burned is reduced from 2,3 to 5,3.

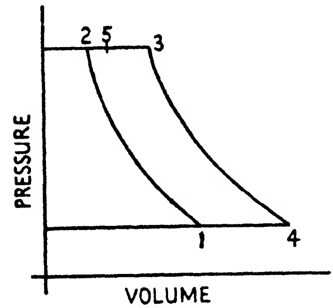


FIG. 147

The amount of pre-heating which can be carried out is proportional to the difference in temperature between the exhaust gases and the air leaving the compressor, i.e. to the difference between the temperatures at the points 4 and 2 in the air-cycle diagram. The greater the temperature difference the greater the amount of waste heat which can be economized.

Heat-exchanger layout

Heat-exchangers may be of many different shapes and sizes, depending on the type of combustion turbine, the pressure-ratios employed, the exhaust-gas temperature, the space available for the installation, and the limitations imposed by the capital cost of the plant.

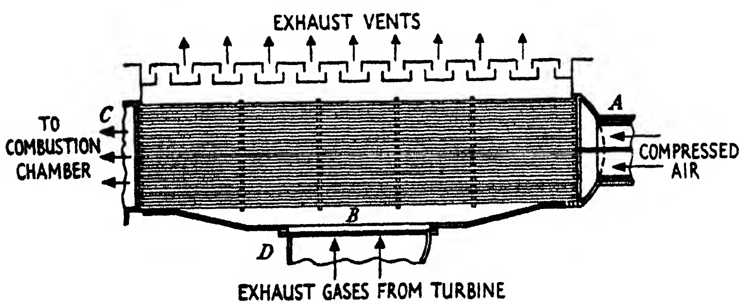


FIG. 148

A heat-exchanger installed in a 2,200 h.p. combustion-turbine locomotive built by Messrs. Brown Boveri for the Swiss Federal Railways is illustrated in Fig. 148.[1] The unit is mounted immediately above the combustion turbine and the hot exhaust

[1] Meyer, 'The First Gas Turbine Locomotive', *Proc. I. Mech. E.*, 1943.

gases pass from the turbine outlet D into the broad chimney duct, their velocity being considerably reduced on account of the large increase in duct sectional area available for the flow, and finally to atmosphere through vents in the roof of the locomotive. The compressed air delivered by the compressor is taken up the duct A to the heat-exchanger B, which is located in the exhaust duct, and consists of a large number of comparatively small diameter tubes designed to present as great a surface area as possible to the hot exhaust gases. Here the compressed air is heated at constant pressure, experiencing an increase in volume, and finally passes from the heat-exchanger to the combustion-chamber C at an increased temperature. The exhaust gases are, of course, cooled by contact with the tubes, since the temperature of the compressed air entering the heat-exchanger may be in the region of 200° C. lower than that of the exhaust gases.

Heat transfer by conduction

The compressed air, in its passage through the heat-exchanger, does not, of course, receive heat directly from the exhaust gases, but only by transmission through the walls of the metal tubes. The outside surfaces of the tubes are heated by the exhaust gases and the inside surfaces are cooled by contact with the compressed air, which thus picks up heat from the metal. Consequently there is a temperature gradient across the thickness of the tube walls, so that heat flows from the hot outside surface to the relatively cool inside surface.

The process by which heat is transferred through a solid body is in some ways similar to that by which electricity is conducted along a body, in that both processes appear to depend upon the presence of 'free' electrons, that is, of certain electrons which are associated with the molecules of the material as a whole, but not with any particular molecule. It follows that materials which are good conductors of electricity are, in general, also good conductors of heat, and materials which are good electrical insulators, due to the absence of free electrons associated with the molecules, are also good heat insulators. Obviously, it is desirable that the material of which heat-exchanger tubes are made should have good properties of heat conduction so that the maximum amount of heat is transferred from the exhaust gases to the compressed air.

Let us consider the heat flow through the rectangular block of material drawn in Fig. 149. If the two opposite faces of this block of uniform material are maintained at different temperatures, heat will flow from the hotter to the colder face. The

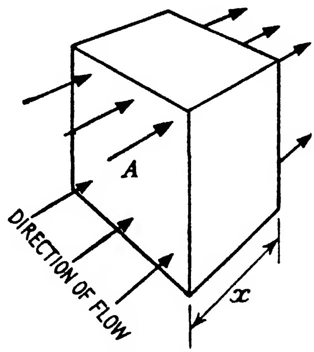


FIG. 149

direction of flow is indicated by the arrows in the diagram. The amount of heat transferred is proportional—

1. To the area A of the block measured at right angles to the direction of flow. The greater the area, the greater the amount of heat transferred, and vice versa.
2. To $1/x$, where x is the perpendicular distance between the two faces. The greater the thickness of the material, the smaller the heat transfer, since more energy is required to complete the motion, between the faces, of the free electrons.
3. To the time t during which the flow occurs.
4. To the temperature difference T between the two faces. The greater the temperature difference, the steeper the temperature gradient and the greater the heat flow.
5. To the 'conductivity' K of the material.

The conductivity K is defined as the quantity of heat conducted through a cube of the material of unit face when a temperature difference of 1° is maintained between the opposite faces. The value of K is the same whether the quantity of heat is measured in C.H.U. and the temperature in °C., or the quantity of heat in B.Th.U. and the temperature in °F.

If H is the quantity of heat which flows through the rectangular block of material in time t, T_1 is the temperature of the hotter face and T_2 the temperature of the other face, then

$$H = \frac{KA(T_1 - T_2)t}{x}.$$

The dimensions must, of course, be in the same units, that is, K in B.Th.U. or C.H.U. for a cube of 1 ft. side per °C. or per °F. per second, A in square feet, t in seconds, and x in feet. The values of K for some common materials are given below:

Material	K
Aluminium	0·024
Copper	0·048
Bronze	0·0048
Cast iron	0·0077
Asbestos	0·000032

Thin-walled tubes

Heat-exchanger tubes can, in general, be considered as thin-walled tubes, and the simple formula of the previous article can be applied to them. A different mathematical treatment is

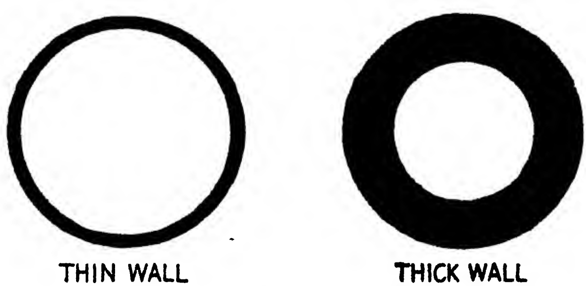


FIG. 150

required for thick-walled tubes since the difference between the inside and outside surface areas is considerable, and recourse must be made to the calculus. In the thin-walled tube, however, the difference between the surface areas is small, and may be neglected. The conditions can best be illustrated by means of an example.

A heat-exchanger tube 3 inches outside diameter and 0·1 inch thick transmits 15,000 C.H.U. per sq. ft. of surface area per minute. If $K = 0{\cdot}01$ C.H.U. for a cube of 1 ft. side per °C.

per second, calculate the temperature difference between the inner and outer surfaces of the tube.

$$H = \frac{KA(T_1 - T_2)t}{x}.$$

Now, inserting the appropriate values, and bearing in mind that all quantities must be in foot-second units, we have

$$15{,}000 = \frac{0{\cdot}01(T_1 - T_2)60}{0{\cdot}1/12}.$$

Therefore $$T_1 - T_2 = \frac{0{\cdot}1 \times 15{,}000}{60 \times 0{\cdot}01 \times 12} = 208^\circ\,\text{C}.$$

Hence the temperature difference between the outer and inner surfaces of the tube is 208° C.

This is the temperature difference across the thickness of the material between the two faces, and is not necessarily the temperature difference between the hot exhaust stream and the air flow inside the tube. The factors which affect the latter, and the nature of their influence, are discussed below.

Heat transfer by convection

When there is an exchange of heat between the hot exhaust gases from the turbine and the relatively cool flow of compressed air in the tubes of the heat-exchanger, the process is not merely the simple one of transmitting the heat through the metal walls of the tubes. As the exhaust gases pass through the heat-exchanger, the layers of gas which come into immediate contact with the surfaces of the tubes are cooled, while the gases which do not come into close contact with them remain hot. In consequence there is a flow of heat from the hotter to the cooler regions by virtue of the temperature gradient set up, and the outside surfaces of the tubes are actually at a lower temperature than the mean temperature of the gases immediately before their entry into the heat-exchanger. Inside the tubes a similar condition exists. The layers of compressed air adjacent to the tube walls become heated, while the air at the centres of the tubes remains relatively cool. Again a temperature gradient is set up and there is a flow of heat through the air from the hotter regions near the tube walls to the cooler regions at the centres. Thus the total drop from the turbine outlet temperature to the temperature of the compressed air is the result of three temperature gradients:

1. The temperature gradient in the exhaust gases.
2. The temperature gradient through the tube walls.
3. The temperature gradient in the stream of compressed air within the tubes.

The process by which heat is transmitted through a gas is rather different from that by which it is transmitted through a solid body. Since a quantity of gas consists of a very large number of molecules which move in all directions with high velocities, the first important factor to be taken into consideration is the natural diffusion of the molecules through the body of the gas. If the gas is contained in a closed vessel which is heated, molecules which strike the walls of the vessel pick up some of the heat and rebound with increased velocity, on account of their increased energy. In their passage through the gas these molecules collide with other molecules to which they impart some of their new-found energy, and so the process goes on until the entire quantity of gas attains the temperature of the containing vessel.

The second important factor in the transmission of heat through a gas is the mass movement of the gas itself. The movement may be due to outside influences, as is the case in the heat-exchanger tubes of a combustion turbine, where the turbulence of the flow causes an intermixing of the gas which tends to distribute the heat over the whole cross-section of the stream, or it may be due to changes in the density of portions of the gas. If the bottom of a closed vessel containing a quantity of gas is heated, the density of that portion of the gas in immediate contact with the bottom of the vessel is reduced and there is an upward flow of warm gas and a downward flow of denser cool gas, so that a circulation is set up in the vessel which causes an intermixing of the gases and a transmission, or distribution, of heat from one part of the gas to another. When the mass movement is due to outside influences the heat transmission through the gas is largely dependent upon the velocity of flow, and when due to differential heating is dependent upon the gas density.

The name 'convection' is given to the above processes whereby heat is transmitted through a gas. Convection is said to be 'natural' when the movement is due to differences of density of the gas, caused by variations of temperature, and 'forced' when the motion is due to outside influences compared with which the natural convection forces are negligible. The heating of the compressed air in the tubes, or ducts, of the heat-exchanger of a combustion turbine falls into the latter category, although the cooling of the exhaust gases tends towards the former.

For the heat transfer from a hot surface to a gas, per unit area in unit time, Reynolds gave the following expression:

$$H = \alpha T + \beta\rho v T,$$

where H is the quantity of heat, ρ the gas density, T the temperature difference between the gas and the surface, v the velocity of the gas flow, and α and β are constants determined by the conditions. If a is the cross-sectional area of the stream and W the mass flow, then since $W = av\rho$ this can also be written as

$$H = \left(\alpha + \beta\frac{W}{a}\right)T.$$

In addition to the above, it is clear that two other factors must be given consideration: firstly the 'hydraulic mean depth' of the air stream in the tubes, which is the ratio

$$\frac{\text{cross-sectional area of stream}}{\text{length of contact perimeter with tube walls}},$$

since this determines the proportion of molecules in contact with the tube walls at any one time; and secondly the nature of the flow, whether stream line or turbulent. The greater the turbulence, the better the mixing, and the greater the heat transmission.

The experimental consideration of conductivity is extremely complicated. The degree of turbulence is most difficult to assess and the heat transfer depends on many factors such as the density, viscosity, and specific heat of the gas, and the size, temperature, and shape of the heating surfaces.

Conditions in a heat-exchanger

We have shown that the heat flow from the hot exhaust gases through the tube walls of the heat-exchanger to the compressed air is dependent not only upon the thickness and material of the tube walls, but also upon the conditions which exist in the regions of the exhaust gases adjacent to the outer surfaces of the tubes and in the layers of compressed air adjacent to the inner surfaces of the tubes. In each case the conditions tend to impede the heat flow, and hence they may be described as resistances to the flow. The principal factors affecting the flow of heat from the exhaust gases to the compressed air may be set out as follows:

Beyond a short distance on either side of the tube walls the flow may be either stream line or turbulent and the heat transfer

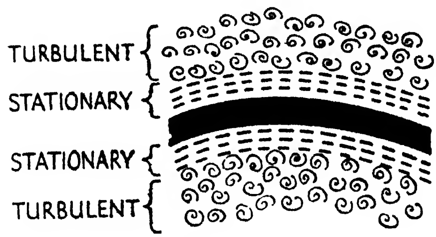


FIG. 151

depends both on natural diffusion and on the motion of the gases themselves. The greater the turbulence, the better the mixing of the hotter and cooler portions of the gas and the greater the heat transfer. As regards the compressed air within the tubes, the greater the velocity the greater the turbulence, and this effect is sufficiently great to be the dominating factor, the effect of natural diffusion being small in comparison. The velocity of the exhaust gases, on the other hand, is relatively low and, although the flow is far from being perfectly stream line, natural diffusion plays no small part in the heat transmission to the tube walls.

Immediately on each side of the tube walls is formed a thin, stationary boundary layer of gas, air on the one side and exhaust gases on the other, Fig. 151. The remainder of the gas inside and outside the tube is in a more or less turbulent state, but there is no sharply defined dividing line, the two conditions merging into each other. The formation of this boundary layer is due to the microscopic surface irregularities of the tube, which prevent motion of the very thin layers of gas in immediate contact with the surfaces, despite the motion of the main stream. The heat transmission through the boundary layer is due to natural diffusion alone. Thus, the stationary boundary layer acts as a form of heat insulator on both the inside and outside surfaces of the tubes. Fig. 152 illustrates an exaggerated section through a portion of a tube wall. If temperatures are plotted from a point in the exhaust stream outside the boundary layer to a point in the compressed-air stream, also outside the boundary layer, a curve of similar form to that shown in the diagram may be obtained. At some point in the turbulent exhaust flow the temperature is that of the turbine outlet T_g,

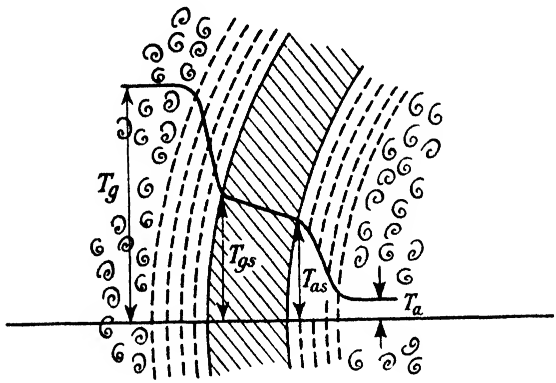


FIG. 152

but through the boundary layer there is a sharp drop to the temperature T_{gs} of the outer surface of the tube wall. Through the thickness of the material the temperature drop is comparatively small, but the boundary layer on the inside of the tube gives rise to a further sharp drop from the temperature T_{as} of the inner surface to the temperature T_a of the main body of the air stream. By far the greater portion of the temperature drop, and consequently the resistance to the heat flow, is due to the two stationary layers of gas, the resistance caused by the tube wall itself being relatively small.

It follows from the above that it is most desirable that the boundary layers should be as thin as possible, and this can be achieved on the inside of the tube by increasing the velocity of the air flow. An increase in flow velocity is not, however, without its disadvantages, despite the fact that the heat transmission is improved, since an increase in velocity results in an increase in the viscous resistance to fluid flow with a consequent increased pressure drop across the heat-exchanger. In the design of a heat-exchanger it is, therefore, necessary to arrive at a compromise between the conflicting factors. As regards the velocity of the exhaust stream, there is little that can be done towards increasing it, since exhaust is normally at atmospheric pressure and the flow through the heat-exchanger is in a vertical direction, by virtue of the sub-atmospheric density of the exhaust gases due to their high temperature. It is not possible to eliminate completely the stationary gas films, and their effect may be gauged from the fact that the resistance of the material of the tube walls may be only 1 or 2 per cent. of the total resistance to the heat flow.

Types of heat-exchangers

Heat-exchangers are generally classified according to the relative direction of flow of the fluids between which the

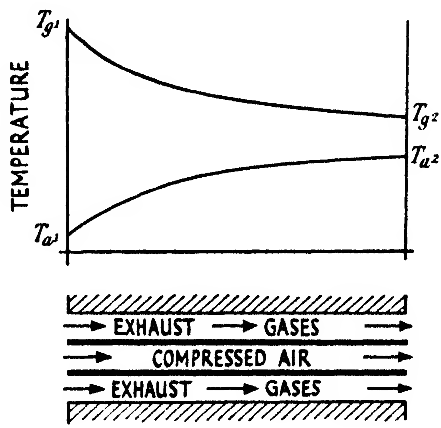


FIG. 153

exchange of heat takes place, and fall into the following three classes when both fluids are gases:

1. Parallel flow, where the fluids flow in the same direction over the separating wall.
2. Counter flow, where the fluids flow in opposite directions over the separating wall.
3. Mixed flow, where the flow is neither parallel nor counter, but one of the fluids takes an irregular direction with respect to the other.

The effect of parallel-flow heat exchange is illustrated diagrammatically in Fig. 153. The hot exhaust gases flow over the outer surface of the tube in the same direction as the cooler compressed air inside the tube. Exhaust-gas and compressed-air temperatures are plotted between inlet and outlet to the heat-exchanger. The inlet temperature of the exhaust gases is T_{g1}, but during their passage through the exchanger they are cooled by contact with the tubes and their temperature falls to T_{g2} at outlet. The compressed air, on the other hand, is heated from a

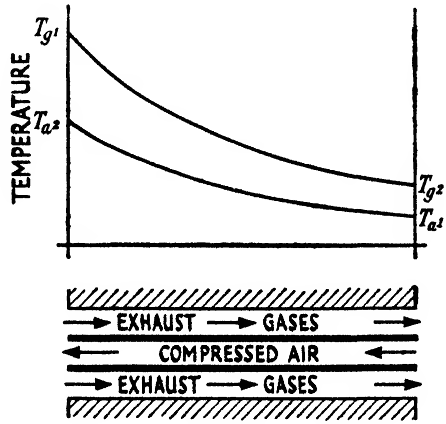


FIG. 154

temperature T_{a1} at inlet to a temperature T_{a2} at outlet, and the approximate forms of the curves are as shown.

The nature of the temperature changes in a counter-flow cooler is shown in Fig. 154. The exhaust gases and compressed

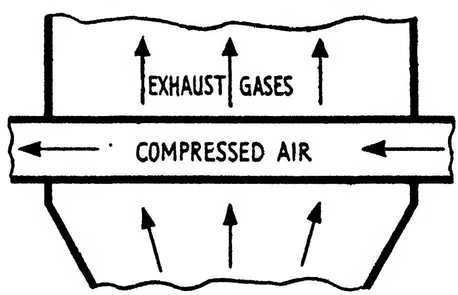


FIG. 155

air flow in opposite directions, the temperature of the exhaust gases falling from T_{g1} at inlet to T_{g2} at outlet, and the temperature of the compressed air increasing from T_{a1} to T_{a2}. The approximate forms of the temperature curves are as shown. For given flow conditions and given air and exhaust gas inlet temperatures the counter-flow type of cooler has an advantage over the parallel-flow type in that it may be made slightly shorter and more compact for the same heat transfer.

The temperature conditions in a mixed-flow heat-exchanger, such as that shown diagrammatically in Fig. 155, vary with the nature of the flow, and the forms of the temperature curves differ for individual designs. The locomotive heat-exchanger of Fig. 148 falls into this category.

Advantages and disadvantages of heat-exchangers

The advantage of using a heat-exchanger in a combustion-turbine installation largely depends on the difference between

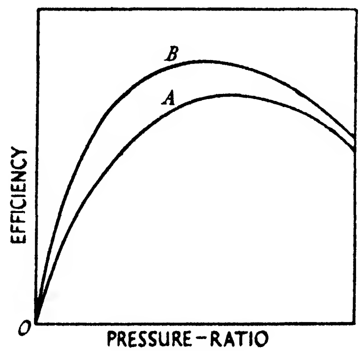


FIG. 156

the exhaust temperature and the compressor delivery temperature, since this temperature difference governs the magnitude of the heat transfer. The greater the temperature difference the greater the degree of waste-heat economization which can be effected and the greater the percentage increase in the over-all efficiency of the plant consequent upon the employment of the heat-exchanger. The effect of waste-heat economization on the combustion-turbine efficiency curve is illustrated in Fig. 156. Curve A is for a plant without an exchanger, whereas curve B indicates the form of the curve obtained when a heat-exchanger is installed. The maximum over-all efficiency of the plant is increased, and at the same time the peak of the curve is slightly displaced to the left.

The greater the surface area of the heat-exchanger tubes, the greater the increase in over-all efficiency which might be expected, but on the other hand, the greater the wetted area of the tubes presented to the flow of compressed air, the greater the frictional resistance to the air flow, and consequently the greater the pressure drop across the exchanger. The increased pressure loss must be offset against the increased waste-heat economization obtained by increase in heat-exchanger surface area, and the ultimate size of the exchanger is determined by the net over-all gain obtainable.

Heat-exchangers are bulky in comparison with the compactness of the combustion turbine and their use is prohibited by lack of space in many types of installation. In addition, large heat-exchangers considerably increase the capital cost of the plant, and this must be weighed against the saving in fuel cost over the life of the turbine, taking into consideration the amount of running expected from the plant.

XI

COMBUSTION-TURBINE PERFORMANCE

THE elements of a simple combustion turbine are illustrated in Fig. 157. The compressor rotor *A* and the turbine rotor *D* are mounted on the same shaft. Air is drawn into the compressor through the intake *F* and, after passing through the rows of fixed and moving blade-rings, is discharged, under pressure, into the duct *B*, which conveys it to the combustion chamber *C*. Here fuel is burnt and the temperature and volume of the stream considerably increased. The gases then pass into the turbine *D* and are expanded through the blade-rings, the liberated energy being absorbed in causing the rotation of the turbine rotor *D*. On leaving the turbine the gases pass out of the system at *E*. A large part of the power generated by the turbine is absorbed by the compressor, and the remainder may be taken off as useful shaft work at the coupling *G*.

The combustion turbine works on the constant-pressure cycle of Fig. 158, as explained in Chapter III. This, of course, is the ideal cycle, or air cycle, and involves the assumption that there are no losses of any kind from the moment the air enters the system to the moment when it is finally discharged back to atmosphere. In practice it is impossible to build a machine in which there are no losses, and we have indeed shown in previous chapters that the losses in both compressor and turbine can be by no means negligible. Nevertheless, the air cycle must be used as the basis of any estimation of the actual efficiency and output of a machine of this type, and corrections must be applied to the expression for the air standard efficiency in order to allow for the various losses.

Referring to Fig. 158, compression is along the adiabatic 1,2, combustion is at constant pressure along 2,3, and expansion takes place along the adiabatic 3,4. At 4 it is assumed that the gases leave the system, and eventually cool, at constant pressure along the line 4,1, back to their original condition before entry into the system. The negative work of compression is represented by the area 1,2,6,5, and the gross work, or the total work which could, ideally, be obtained from the turbine wheel if there were no losses anywhere in the system, is represented by the area 5,6,3,4. The area 1,2,3,4, which is the difference between 5,6,3,4 and 1,2,6,5, represents the useful work which can be taken from the turbine shaft. In other words, the useful work, or net work, is equal to the gross work of the turbine

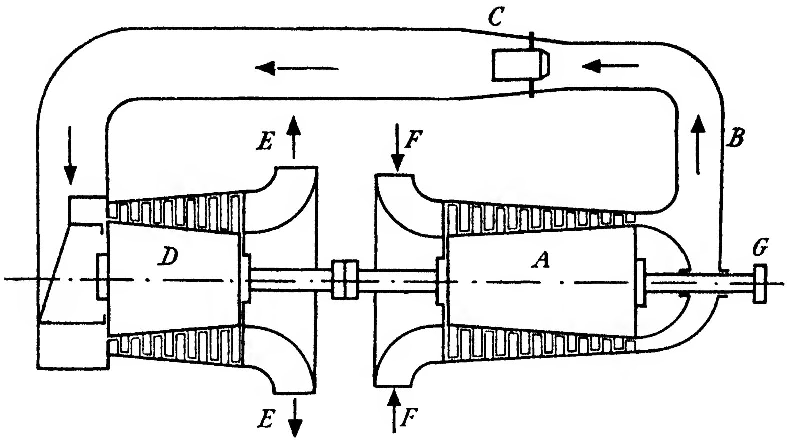


FIG. 157

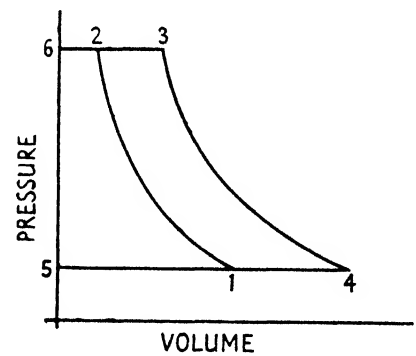


FIG. 158

rotor minus the negative work absorbed by the compressor. In Chapter III we derived an expression for the air standard efficiency of the constant-pressure cycle, which is

$$\eta_{\text{cycle}} = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}},$$

where P_1 is the intake pressure, invariably the atmospheric pressure, P_2 is the delivery pressure of the compressor, and γ is the ratio of the specific heats of air.

Correction for losses

In an actual combustion turbine it is necessary to take account of the various losses which occur in the system. The chief of these are the compressor losses and the turbine losses, the latter including the nozzle losses if an impulse wheel is used. Clearly it is necessary to multiply the air standard efficiency η_{cycle} by some factor which takes account of the fact that neither the compressor nor the turbine is 100 per cent. efficient.

The effect of the losses in the compressor is to increase the amount of negative work absorbed by the compressor, so that the actual amount of work put into the compressor, in order that it may deliver the required weight of air at the required pressure, must be greater than the compression work represented by the area 1,2,6,5 in the air-cycle diagram, Fig. 158. Let y be the ideal work of compression as given by the air cycle, and let η_c be the over-all efficiency of the compressor. Then the actual amount of work which must be put into the compressor shaft is

$$\frac{y}{\eta_c}.$$

The effect of the losses in the turbine is to decrease the amount of the gross work obtainable from the rotor. If the turbine were 100 per cent. efficient, the amount of work which could be taken from it would be represented by the area 5,6,3,4 in the air-cycle diagram, but, in view of the inevitable losses, the actual work obtainable is less than this value. Let z be the gross work, as given by the air cycle, and let η_t be the over-all efficiency of the turbine. Then the actual amount of work which can be obtained from the turbine shaft is

$$\eta_t z.$$

Now the useful work which can be delivered by the plant, as a whole, is the difference between the gross work and the negative work of compression. The useful work is represented on the air-cycle diagram by the area 1,2,3,4, but, since we acknowledge the fact that none of the components of the machine is 100 per cent. efficient, it is clear that the actual useful work which may be obtained from the system is less than that given by the air-cycle diagram. If x is the ideal useful work, as given by the air cycle, then

$$x = z - y.$$

Now let x' be the actual amount of useful work obtainable when the compressor and turbine losses are taken into account. Then

$$x' = \eta_t z - \frac{y}{\eta_c}.$$

The ratio of the actual useful work x' to the ideal useful work x is x'/x and clearly the air standard efficiency η_{cycle} must be multiplied by this ratio in order to obtain the actual efficiency η_{actual} of the combustion turbine. Therefore,

$$\eta_{\text{actual}} = \frac{x'}{x}\eta_{\text{cycle}}.$$

But

$$\frac{x'}{x} = \frac{\eta_t z - y/\eta_c}{z-y}$$

$$= \frac{\eta_t \frac{z}{y} - \frac{1}{\eta_c}}{\frac{z}{y} - 1}.$$

Let

$$K = \frac{z}{y} = \frac{\text{air-cycle gross work}}{\text{air-cycle negative work}}.$$

Then

$$\frac{x'}{x} = \frac{\eta_t K - 1/\eta_c}{K-1}$$

and

$$\eta_{\text{actual}} = \frac{\eta_t K - 1/\eta_c}{K-1}\eta_{\text{cycle}}$$

$$= \frac{\eta_t K - 1/\eta_c}{K-1}\left\{1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right\}.$$

This expression gives us the actual over-all efficiency of the combustion turbine when the necessary allowances are made for the losses in the compressor and turbine.

As an example, let us determine the actual efficiency of a combustion turbine which has a pressure-ratio of 4, a turbine efficiency of 0·85, and a compressor efficiency of 0·80. The maximum permissible temperature T_3 is 600° C., and the air temperature T_1 at inlet to the compressor is 15° C., the pressure being 14·7 lb. per sq. in. absolute. The value of γ, both for compression and expansion, may be taken as 1·4.

Let us consider the passage of a weight of one pound of air through the system. The first step is to determine K, the ratio of air-cycle gross work to air-cycle negative work.

$$\text{Air-cycle negative work} = \frac{\gamma(P_2 v_2 - P_1 v_1)}{\gamma - 1}.$$

Now
$$P_1 v_1^{\gamma} = P_2 v_2^{\gamma},$$
$$14{\cdot}7(12{\cdot}4)^{1{\cdot}4} = 58{\cdot}8 v_2^{1{\cdot}4}.$$

Therefore
$$v_2 = 4{\cdot}61 \text{ cu. ft.}$$

and the air-cycle negative work is equal to

$$144\,.\,3{\cdot}5(58{\cdot}8\,.\,4{\cdot}61 - 14{\cdot}7\,.\,12{\cdot}4) = 44{,}800 \text{ ft. lb. per 1 lb. air.}$$

$$\text{Air-cycle gross work} = \frac{\gamma(P_3 v_3 - P_4 v_4)}{\gamma - 1}.$$

Before we can evaluate this expression we must first determine v_3 and v_4. The absolute temperature T_1 at inlet to the compressor is $273 + 15 = 288°$ abs., and the absolute value of the maximum temperature T_3 is $273 + 600 = 873°$ abs.

$$T_2 = T_1 \left(\frac{P_2}{P_1}\right)^{\frac{\gamma - 1}{\gamma}} = 288\,.\,(4)^{\frac{1{\cdot}4 - 1}{1{\cdot}4}} = 430° \text{ abs.}$$

Now
$$\frac{v_3}{v_2} = \frac{873}{430} = 2{\cdot}04.$$

Therefore
$$v_3 = 4{\cdot}61\,.\,2{\cdot}04 = 9{\cdot}4 \text{ cu. ft.}$$

To determine v_4:
$$P_3 v_3^{\gamma} = P_4 v_4^{\gamma},$$
$$58{\cdot}8.(9{\cdot}4)^{1{\cdot}4} = 14{\cdot}7 v_4^{1{\cdot}4}.$$

Therefore
$$v_4 = 25{\cdot}2 \text{ cu. ft.}$$

and the air-cycle gross work is equal to

$$144\,.\,3{\cdot}5(58{\cdot}8\,.\,9{\cdot}4 - 14{\cdot}7\,.\,25{\cdot}2) = 91{,}800 \text{ ft. lb. per 1 lb. air.}$$

Hence
$$K = \frac{91{,}800}{44{,}800} = 2{\cdot}05,$$

$$\eta_{\text{actual}} = \frac{0{\cdot}85\,.\,2{\cdot}05 - 1/0{\cdot}80}{2{\cdot}05 - 1}\left\{1 - \left(\frac{14{\cdot}7}{58{\cdot}8}\right)^{\frac{1{\cdot}4 - 1}{1{\cdot}4}}\right\}$$
$$= 0{\cdot}466\,.\,0{\cdot}33$$
$$= 0{\cdot}154 = 15{\cdot}4 \text{ per cent.}$$

The effect of pressure-ratio on efficiency

Using the formula

$$\eta_{\text{actual}} = \frac{\eta_t K - 1/\eta_c}{K-1}\left\{1-\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right\}$$

a series of curves of actual combustion-turbine efficiency against pressure-ratio of compression has been derived, and these curves

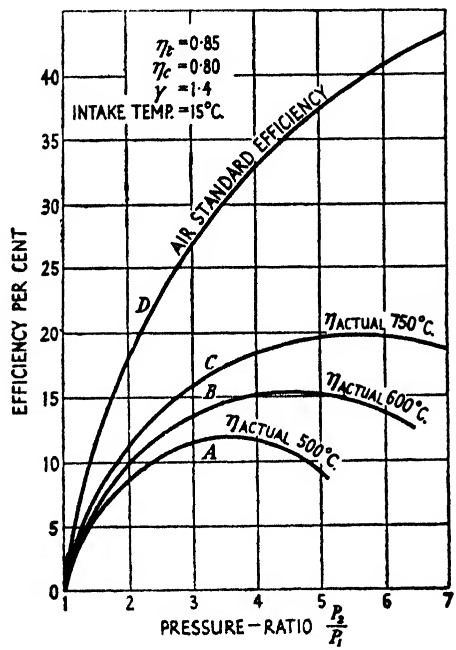


FIG. 159

are shown plotted in Fig. 159. In calculating the curves it was assumed that

Compressor efficiency $\eta_c = 0{\cdot}80$.
Turbine efficiency $\eta_t = 0{\cdot}85$.
Ratio of specific heats $\gamma = 1{\cdot}4$ = constant.
Intake pressure = 14·7 lb. per sq. in. at 15° C.

Curve *A* was obtained for a maximum permissible temperature T_3 of 500° C., curve *B* for 600° C., and curve *C* for 750° C. For the purpose of comparison a curve of air standard efficiency η_{cycle} is shown on the same diagram. This is curve *D*.

It will be observed that the efficiency increases at a decreasing rate from zero at a pressure-ratio of 1 (no compression) to a maximum value, and then begins to decrease with a further increase of pressure-ratio. Thus for each value of maximum temperature T_3 there is an optimum pressure-ratio at which the actual efficiency of the plant attains its maximum value. The

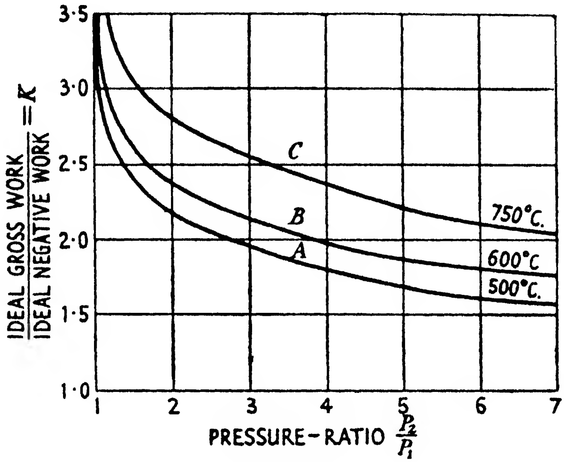


FIG. 160

efficiency increases as the maximum permissible temperature is raised, and at the same time the peak of the curve is moved to the right. Thus, with a value of $T_3 = 500°$ C., a maximum efficiency of 11·8 per cent. is attained at a pressure-ratio of 3·5, while with a value of $T_3 = 750°$ C. a maximum efficiency of 19·6 per cent. is attained at a pressure-ratio of 5·6. At these two pressure-ratios the values of the air-standard efficiency are 30 per cent. and 39·4 per cent. respectively, a comparison which emphasizes the importance of high compressor and turbine efficiencies.

Fig. 160 shows three curves *A*, *B*, and *C*, for the same respective temperatures of 500° C., 600° C., and 750° C., of the ratio of gross work to negative compression work in the ideal air cycle, plotted against pressure-ratio.

Corrections to the formula

The above formula for the actual over-all efficiency, however, while affording a good working basis, is not completely accurate, in that it takes no account of the heating effect of the compressor losses, nor of the variation in γ, the ratio of the specific heats, at elevated temperatures. Both these factors contribute towards a slight increase in the actual over-all efficiency of the plant.

In the chapters dealing with compressors we showed that, although the adiabatic efficiency of compression is invariably less than unity, there is, neglecting losses due to radiation from the compressor casing, no loss of energy from the system. The apparent loss of energy (i.e. the difference between the amount of work put into the compressor shaft and the adiabatic work of compression), due to the effects of turbulence, skin-friction, etc., in the blade-channels, actually goes into heating the air to a temperature which is higher than that which would result from a perfect adiabatic compression. Now the amount of energy which has to be added to the air stream in the combustion chamber is proportional to the difference between the maximum permissible temperature T_3 and the compressor delivery temperature T_2. If T_2 is increased, then the difference between T_3 and T_2 is decreased, and less fuel need be burned in order to produce the same maximum temperature T_3. The formula for η_{actual} was based upon the assumption of a perfect adiabatic compression with the energy loss, due to the fact that the compressor efficiency is less than unity, being considered as lost from the system, whereas if the heating effect due to the losses is taken into account, the actual over-all efficiency of the combustion turbine is slightly higher than the value given by the formula.

When the temperature of a quantity of air is raised, the specific heat ratio γ does not remain constant at its sea-level atmospheric temperature value. For small increases in temperature, the variation of γ is negligible, but when the temperature of the air is raised to a very high value, as it is in the combustion turbine, the specific heat ratio tends to decrease. This is due to the dissociation of some of the gas molecules, an effect which increases with increasing temperature, and which causes a small increase in the value of C_v, the specific heat at constant volume. This, in turn, results in a decrease in the value of γ on the high-temperature side of the system and raises the actual over-all efficiency to a value which is slightly higher than that predicted by the formula.

The three curves, *A*, *B*, and *C*, of actual efficiency against pressure-ratio, for temperatures of 500° C., 600° C., and 750° C. respectively, shown in Fig. 159 are redrawn in Fig. 161. The broken curves *D*, *E*, and *F*, for the same respective temperatures, illustrate the slight increase in efficiency, over and above

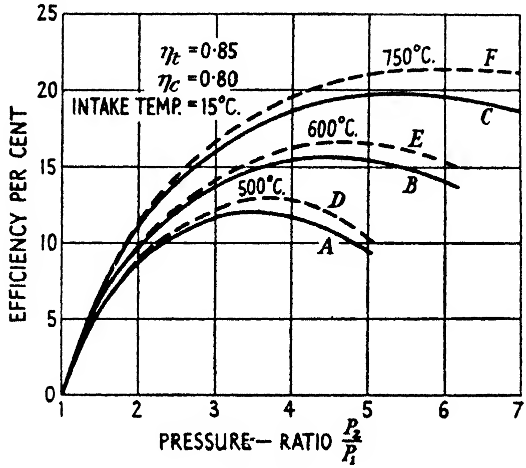


FIG. 161

the value given by the formula, when allowance is made for the heating effect of the compressor losses and the variation in specific heat. It will be observed that not only is there a small gain in efficiency, but the peaks of the curves are slightly displaced to the right, so that the optimum pressure-ratio for each value of maximum permissible temperature is a little higher than is the case when no account is taken of the above effects. Thus, with a maximum temperature of 500° C., curve *A* for instance gives a maximum efficiency of 11·8 per cent. at a pressure-ratio of 3·5, while curve *D* gives a maximum efficiency of 12·6 per cent. at a pressure-ratio of 3·7.

The effect of turbine and compressor efficiency

The formula for actual over-all efficiency can be rewritten in the form

$$\eta_{\text{actual}} = \frac{\eta_t \eta_c K - 1}{\eta_c (K-1)} \left\{ 1 - \left(\frac{P_1}{P_2} \right)^{\frac{\gamma-1}{\gamma}} \right\}.$$

The product $\eta_t \eta_c$ of turbine efficiency and compressor efficiency is a most important quantity. It will be seen, from the expression, that for a given value of the product $\eta_t \eta_c$, the actual over-all efficiency is inversely proportional to η_c. Thus a greater gain in combustion-turbine efficiency can be obtained from an increase in turbine efficiency than from an equal percentage increase in compressor efficiency. The difference, however, is not large and it is most essential that both the turbine efficiency

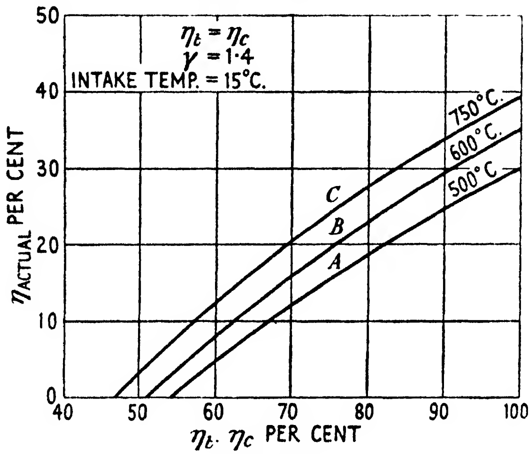


Fig. 162

and the compressor efficiency should be as high as possible. Clearly η_{actual} becomes zero when the quantity $\eta_t \eta_c K$ is equal to unity. The value of K depends upon the pressure-ratio and the maximum permissible temperature T_3, and hence, for given values of pressure-ratio and maximum temperature, the actual over-all efficiency depends upon the efficiency product.

Fig. 162 shows three curves of actual over-all efficiency η_{actual} plotted against efficiency product $\eta_t \eta_c$. The value of γ is taken as 1·4, the intake pressure is assumed to be 14·7 lb. per sq. in. at 15° C., and the turbine efficiency is taken as being equal to the compressor efficiency. Curve A is obtained for a maximum permissible temperature T_3 of 500° C., curve B for 600° C., and curve C for 750° C. In each case the pressure-ratio employed is the optimum value for the particular maximum temperature, as given by the curves of Fig. 159.

In the case of curve A, with a maximum temperature of 500° C., the efficiency becomes zero when the value of $\eta_t \eta_c$ falls below 0·54. At this particular value of efficiency product the turbine delivers just enough power to drive the compressor, and although, theoretically, the machine should be able to turn itself over, no useful work can be taken from the system. This value of $\eta_t \eta_c$ corresponds to individual turbine and compressor efficiencies of 73·5 per cent., a fairly high figure. Even with a maximum temperature as high as 750° C. the actual over-all efficiency is zero at an efficiency product value of 0·47, which corresponds to individual turbine and compressor efficiencies of 68·5 per cent. Thus, the importance of high turbine and compressor efficiencies is manifest, and it was for this reason that the early experimental combustion turbines were unsuccessful, since the poor state of aerodynamic knowledge in those days prevented the construction of sufficiently efficient turbines and compressors. At the present time, a value of efficiency product $\eta_t \eta_c$ of 73 to 75 per cent. is attainable, and maximum temperatures of 600° C. to 700° C. are permissible. Referring to the curves of Fig. 162, an actual over-all efficiency in the region of 20 per cent. may be obtained at the turbine coupling, although this takes no account of various small losses, such as the power required to drive the oil pumps, etc.

A slightly higher maximum over-all efficiency is attainable if the turbine is of the impulse type than if a reaction turbine is used. This is clear from the cycle diagram, Fig. 158. If a reaction turbine is used the temperature of the gases impinging on the turbine blades is T_3, whereas with an impulse turbine the blade temperature is T_4. This means that, for a fixed blade temperature, the maximum temperature in the system can be higher when an impulse turbine is used than when a reaction turbine is used, and consequently the actual over-all combustion-turbine efficiency is greater with the former than with the latter. On the other hand, the efficiency curve of an impulse wheel is sharply peaked, whereas that of a reaction turbine is comparatively flat, and the greater maximum efficiency has to be paid for by some reduction in the efficient operating range of the machine. A compromise between the conflicting factors may, however, be achieved by using an impulse wheel as the first stage of what is essentially a reaction turbine.

The effect of a heat-exchanger

The efficiency of a combustion turbine can be improved by the installation of a heat-exchanger in the exhaust duct, the purpose of the heat-exchanger being to economize some of the heat energy which would otherwise go to waste in the exhaust gases. The compressed air, on leaving the compressor, is led into the channels of the heat-exchanger, which are heated by the exhaust gases, and its temperature is raised before it is passed to the combustion chamber. Since the amount of fuel

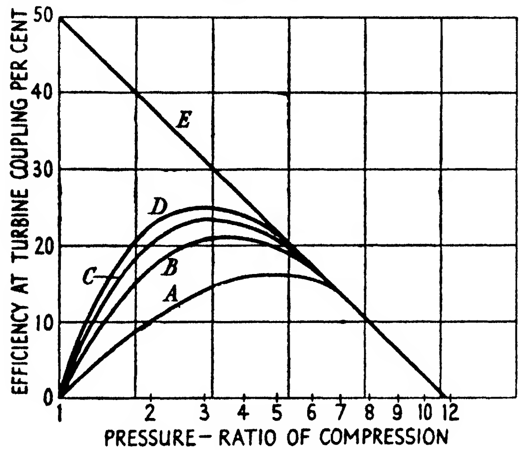


FIG. 163

which has to be burned in the combustion chamber in order to bring the temperature of the air stream to its desired maximum value T_3 is proportional to the difference between T_3 and the air temperature at entry to the combustion chamber, any economization of the exhaust heat which can be utilized to raise the temperature of the air before combustion takes place, without, of course, increasing the negative work of the system, results in a reduction in the amount of fuel which need be supplied, with a resulting increase in the actual over-all efficiency of the combustion turbine.

Ideally, the compressed air receives heat energy in the exchanger at constant pressure, and the amount of heat received is, broadly speaking, proportional to the surface area of the heat-exchanger, and also to the temperature difference between the compressed air and the exhaust gases, i.e. to the difference between T_4 and T_2. Fig. 163 illustrates five curves of actual over-all combustion-turbine efficiency plotted against pressure-ratio for plants equipped with heat-exchangers of various sizes, the maximum permissible temperature T_3 being

555° C., the air-inlet temperature T_1 20° C., the turbine efficiency 86 per cent., and the compressor efficiency 83 per cent.[1] The curves are plotted on a logarithmic base, since the curve for a plant assumed to be fitted with a heat-exchanger of infinite surface area then becomes a straight line. Curve *A* is for a simple combustion turbine with no heat-exchanger, and it will be observed that the maximum efficiency is about 16·5 per cent., but when a heat-exchanger of 5,000 sq. ft. surface area is employed (curve *B*), the actual over-all efficiency is increased to 21 per cent., an improvement of 27 per cent. on the efficiency with no heat-exchanger. Curve *C* is obtained with an exchanger of 15,000 sq. ft. surface area, curve *D* with an exchanger of 30,000 sq. ft. surface area, and curve *E* is for a plant assumed to have a heat-exchanger of infinite surface area. It should be noted that, in addition to raising the efficiency of a combustion turbine, the use of a heat-exchanger has the effect of shifting the peak of the efficiency–pressure-ratio curve to the left, so that the optimum pressure-ratio for a given maximum permissible temperature is lowered when the plant is equipped for economization of the exhaust heat.

Since the efficacy of a heat-exchanger depends upon the difference between the exhaust temperature T_4 and the compressor delivery temperature T_2, the greater the pressure-ratio, for a fixed maximum temperature T_3, the smaller the benefits obtained from the use of an exchanger. This must be so, because the greater the pressure-ratio, the higher becomes the compressor delivery temperature T_2 and the smaller becomes the difference between T_4 and T_2. The efficiency gain obtained by the use of a heat-exchanger is also dependent upon the surface area, but the size of the exchanger cannot be increased indefinitely since the pressure drop across the ducting, due to skin-friction and turbulence, attains a magnitude sufficiently great to detract considerably from the potential benefits of the economization in very large heat-exchangers. Capital cost, too, is a factor of some importance, for a large heat-exchanger adds considerably to the cost of the plant and, if the degree of economization so obtained is small, the extra complication and financial outlay entailed may not be worth while when balanced against the small saving in the running cost of the combustion

[1] Meyer, 'The Combustion Gas Turbine—Its History, Development and Prospects', *Proc. I. Mech. E.*, 1939.

turbine. In many types of installation, where compactness is the keynote, as for instance in aircraft installations, it is often impracticable to employ a heat-exchanger on account of its bulk and weight.

Reheating

A further increase in combustion-turbine efficiency can be obtained by reheating the gases at some point, or points, in

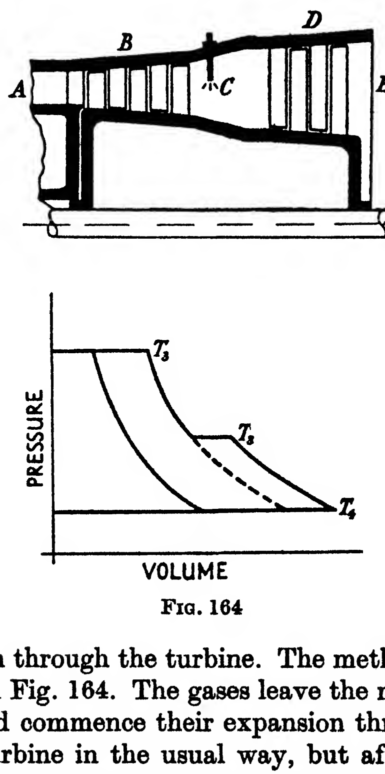


FIG. 164

their expansion through the turbine. The method of doing this is illustrated in Fig. 164. The gases leave the main combustion chamber *A* and commence their expansion through the blade-rings of the turbine in the usual way, but after a number of stages *B*, they enter a second combustion chamber *C*, where a further quantity of fuel is burned, raising the temperature of the gases back to the original value T_3. This second combustion is possible because the quantity of fuel burned in chamber *A* is far from sufficient to absorb all the oxygen of the air stream, and the gases, even on leaving the exhaust, still contain a considerable quantity of free oxygen. The gases, on leaving chamber *C*, complete their expansion through the remaining stages D of the turbine, and leave the system at the exhaust E, to pass to the heat-exchanger, if fitted, or otherwise to atmosphere.

The effect of reheating on the cycle diagram, Fig. 164, is to increase the useful work area without, at the same time, increasing the negative work of compression, and it is possible, by this means, to obtain an increase in the actual over-all efficiency of the plant, although the gain is not large. Ideally, a large number of reheatings during the expansion process should result in a considerable efficiency increase, but in practice the losses (chiefly pressure drop) in the various combustion chambers would prohibit the use of more than one or two reheatings. Naturally, reheating between stages adds to the bulk and weight of the plant, and the question of compactness, in many types of installation, would often make its employment difficult if not impossible.

To obtain the maximum benefit from a reheating process, it is necessary to employ a heat-exchanger. Referring to Fig. 164, it is clear that the exhaust temperature T_4 is considerably increased by the introduction of inter-stage reheating, and therefore the difference between T_4 and the compressor delivery temperature T_2 is increased, so that the compressed air may be raised to a higher temperature before it enters the main combustion chamber A than is the case when there is no reheating between stages. This, of course, gives rise to a further small increase in the actual over-all efficiency of the combustion turbine.

The twin-shaft arrangement

Some increase in actual over-all efficiency at partial loads, although not at full load, may be obtained by using two turbine rotors mounted on separate shafts, one to deliver the useful power, and the other driving the compressor only. The two turbines have separate controls, and by this means the compressor can be run at an efficient speed over most of the operating range of the plant, independently of the demand on the rotor delivering the useful power.

The layout is shown diagrammatically in Fig. 165. The compressor A delivers into a branched duct, part of the air being taken through the combustion chamber B to the turbine C, which is direct-coupled to the compressor, and the entire shaft output of which is absorbed by the compressor. The remainder of the air from the compressor *A* is delivered, through the combustion chamber *D*, to the turbine *E*, which is geared to the generator *F*, and provides the useful work

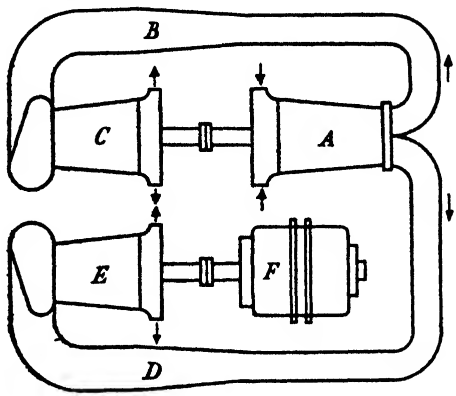


Fig. 165

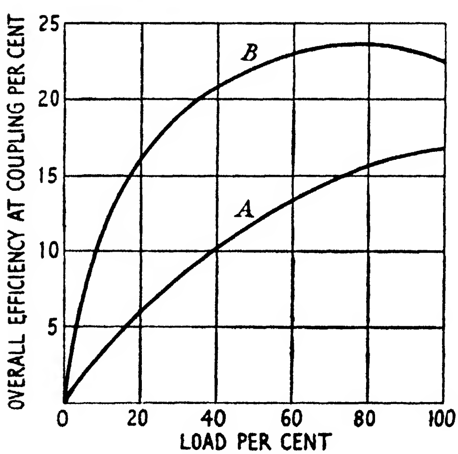


Fig. 166

output of the plant. Thus, whatever the demand on turbine *E*, the compressor set, being mounted on an independent shaft, can be run at a speed at which its efficiency remains at a fairly high value, whereas with the normal single-shaft arrangement the running conditions of the entire plant are dependent upon the load.

Fig. 166 illustrates two curves of actual over-all efficiency for two combustion turbines, each of 2,680 b.h.p. useful shaft output, with a maximum permissible temperature T_3 of 555° C.[1] The turbine of curve A has a single rotor and no heat-exchanger, whereas the turbine of curve B has two rotors, mounted on separate shafts, and is fitted with a heat-exchanger of 7,500 sq. ft. surface area. The efficiency values are plotted against percentage load, and the shapes of the curves indicate the considerable benefits obtainable at partial loads by the use of a twin-shaft arrangement.

Example

A combustion turbine has a useful shaft output of 3,000 b.h.p. at the turbine coupling, a turbine efficiency of 85 per cent., a compressor efficiency of 83 per cent., and the maximum permissible temperature is 600° C. The pressure and temperature of the air at inlet to the compressor are 14·7 lb. per sq. in. and 15° C. respectively, and the specific-heat ratio γ is 1·4. The plant has a single-shaft arrangement and there is no heat-exchanger. It is required to find the actual over-all efficiency of the combustion turbine, the weight of air consumed per second, the shaft horse-power input to the compressor, the ratio of the turbine-rotor output to the compressor-shaft input, and the power developed per pound of air handled by the plant.

$$\eta_t \eta_c = 0{\cdot}83\,.\,0{\cdot}85 = 0{\cdot}706.$$

From the curves of Fig. 162, the actual over-all combustion-turbine efficiency is

$$\eta_{\text{actual}} = 0{\cdot}16 = 16 \text{ per cent.}$$

From the curves of Fig. 159, the optimum pressure-ratio is 4·5. Therefore

$$T_2 = T_1\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = 288(4{\cdot}5)^{\frac{1\cdot4-1}{1\cdot4}} = 443° \text{ abs.}$$

The maximum temperature $T_3 = 273+600 = 873°$ abs. Therefore the temperature rise in the combustion chamber is

$$873-443 = 430° \text{ C.}$$

Heat required to raise 1 lb. air through 430° C.

$$= 430C_p = 430\,.\,0{\cdot}238 = 102{\cdot}3 \text{ C.H.U.}$$

[1] Meyer, loc. cit.

Useful output = 3,000 b.h.p., and $\eta_{\text{actual}} = 0{\cdot}16$. Hence the horse-power equivalent of the energy content of the fuel which must be burned is

$$\frac{3{,}000}{0{\cdot}16} = 18{,}750.$$

The energy content of the quantity of fuel burned per second is

$$18{,}750\,.\,550 \text{ ft. lb.} = \frac{18{,}750\,.\,550}{1400} = 7{,}380 \text{ C.H.U. per sec.}$$

Therefore,

$$\text{weight of air required} = \frac{7{,}380}{102{\cdot}3} = 72 \text{ lb. per sec.}$$

Now $$v_2^{1\cdot4} = \frac{14{\cdot}7(12{\cdot}4)^{1\cdot4}}{66{\cdot}2} = 7{\cdot}53$$

and $$v_2 = 4{\cdot}24 \text{ cu. ft.}$$

Adiabatic work done in compression

$$= \frac{W\gamma(P_2 v_2 - P_1 v_1)}{\gamma - 1}$$

$$= \frac{144\,.\,72\,.\,1{\cdot}4(66{\cdot}2\,.\,4{\cdot}24 - 14{\cdot}7\,.\,12{\cdot}4)}{1{\cdot}4 - 1}$$

$$= 3{,}550{,}000 \text{ ft. lb. per sec.}$$

But compressor efficiency $\eta_c = 0{\cdot}83$. Therefore shaft input to compressor

$$= \frac{3{,}550{,}000}{0{\cdot}83} = 4{,}280{,}000 \text{ ft. lb. per sec.}$$

$$= 7{,}780 \text{ h.p.}$$

Hence total output of turbine rotor

$$= 3{,}000 + 7{,}780 = 10{,}780 \text{ h.p.,}$$

and the ratio of turbine-rotor output to compressor-shaft input

$$= \frac{10{,}780}{7{,}780} = 1{\cdot}383.$$

The actual power which must be developed by the turbine rotor is 3·593 times the useful shaft power which may be taken from the plant to do external work, by far the greater portion of the rotor output being absorbed by the compressor.

The useful power developed per pound of air handled by the plant is

$$\frac{3{,}000}{72} = 41{\cdot}6 \text{ h.p.}$$

Of course, this example takes no account of the variations in specific heat at the high-temperature end of the system, nor of the heating effect of the compressor losses, but the results obtained are typical of this class of machine.

XII

AIRCRAFT JET PROPULSION

At the very high speeds attainable by some current types of aircraft the airscrew is considerably less efficient than it is at lower speeds, say, below 450 m.p.h. This is due to the fact that at high aircraft speeds the speed of the tips of the airscrew blades (which is the resultant of the forward speed of the aircraft and the tangential velocity of the blade-tips in the plane of the airscrew) approaches, and may even exceed, the speed of sound in air. When this occurs, the stream line flow over the blade sections breaks down and a 'shock wave' is formed. This is really a sound wave, and gives rise to a very considerable drop in the efficiency of the airscrew. Jet propulsion has been devised as a means of eliminating the airscrew for high-speed aircraft, thus enabling the attainment of aircraft speeds in excess of the limits imposed by the formation of airscrew shock waves. The adoption of jet propulsion does not, however, imply that aircraft speeds can be increased indefinitely since, as the aircraft itself approaches the speed of sound, the formation of shock waves may destroy the stream line flow over the wings and so increase the drag of the aircraft many fold.

A jet-propelled aircraft, Fig. 167, contains one or more ducts, whose axes are parallel to the longitudinal axis of the aircraft. As the aircraft moves forward, a stream of air passes through the duct, entering at the nose and leaving at the tail. Situated in the duct is an apparatus, usually a combustion turbine, the purpose of which is to increase the speed of the air stream. Thus the speed of the stream, relative to the aircraft, is greater at outlet than at entry. Since the speed of the stream is increased, its momentum is also increased, and in order to accomplish this it is necessary to apply a force to the air in a rearward direction. It is the function of the apparatus situated within the duct to apply this force. It follows from the fact that action and reaction are equal and opposite, that if the apparatus applies a force to the air an equal and opposite force is exerted on the apparatus. It is this reaction which provides the forward thrust on the aircraft. Thus, a jet-propulsion motor is purely and simply a piece of apparatus capable of accelerating a stream of air, and the forward thrust of the aircraft is obtained from the change in momentum of the mass of air handled by the plant.

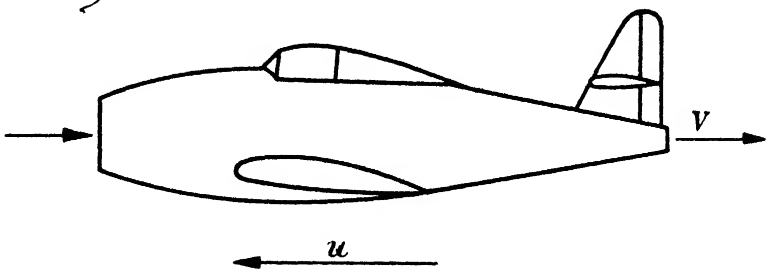


FIG. 167

Jet efficiency

Let us consider the effects of the change of momentum of the air stream passing through the jet-propulsion system of an aircraft, Fig. 167, which is flying straight and level. The velocity of the aircraft is u ft. per sec., and therefore the velocity of the air, relative to the aircraft, as it enters at the nose is also u ft. per sec., but in the reverse direction. During its passage through the system, the velocity of the air is increased and it leaves the aircraft in a rearward direction with a velocity V ft. per sec. relative to the aircraft. The relative velocity V of the jet is, of course, greater than the relative velocity u with which the air enters the system. If g is the gravitational acceleration in feet per second per second, then the energy, relative to the aircraft, of one pound of air in the jet is its kinetic energy, which is

$$\frac{V^2}{2g} \text{ ft. lb.}$$

Similarly, the kinetic energy, relative to the aircraft, of one pound of air entering at the nose is

$$\frac{u^2}{2g} \text{ ft. lb.}$$

If friction and other losses in the ducting are neglected, the amount of energy which must be added to the stream per pound of air, in order to accelerate the fluid to its ejection velocity, is

$$\frac{V^2}{2g} - \frac{u^2}{2g} \text{ ft. lb.}$$

Now the absolute velocity of the jet as it leaves the aircraft, relative to the surrounding atmosphere, is $V-u$ ft. per sec. and the increase in the velocity of the stream, due to its passage through the system, is $V-u$ ft. per sec. Therefore, the change in the momentum of one pound of air is

$$\frac{V-u}{g}.$$

If W pounds of air passes through the system per second, the change of momentum of the stream per second is

$$\frac{W(V-u)}{g}.$$

From Newton's laws of motion we know that the rate of change of momentum of a body is proportional to the applied force. Since we have an expression for the rate of change of momentum of the air stream, and since all the quantities involved are in the same basic units, the foot-pound-second system, it follows that the rearward force exerted on the air by the jet-propulsion apparatus is equal to

$$F = \frac{W(V-u)}{g} \text{ lb.}$$

This is the force which causes the acceleration of the air as it passes through the system and, due to the fact that action and reaction are equal and opposite, a force of equal magnitude, but of opposite direction, is exerted on the aircraft. This reaction, the magnitude of which is equal to F, is the forward thrust on the aircraft.

Work done is the product of force and distance. The aircraft moves forward a distance u feet during each second, under the action of the force F, and the work done on the aircraft per second is, therefore,

$$Fu = \frac{Wu(V-u)}{g} \text{ ft. lb.}$$

The jet efficiency is defined as the ratio of the actual work done on the aircraft in moving it forward against the external resistances, to the energy required, in the same length of time, to effect the necessary change in the momentum of the air stream passing through the aircraft.

The work done on the aircraft is

$$\frac{Wu(V-u)}{g} \text{ ft. lb. per sec.}$$

and the energy required to change the momentum of the air stream is

$$W\left(\frac{V^2}{2g}-\frac{u^2}{2g}\right) \text{ ft. lb. per sec.}$$

Therefore,

$$\text{jet efficiency} = \frac{2u(V-u)}{V^2-u^2}$$

$$= \frac{2u(V-u)}{(V+u)(V-u)}$$

$$= \frac{2u}{V+u}.$$

It should be borne in mind that this expression only gives the efficiency of the jet as a means of propulsion, and takes no account of the thermal efficiency of the jet-propulsion motor which is used to accelerate the air stream.

The expression for jet efficiency can be made as near unity as is desired, by making V and u approximate to equality. If $V = u$, the jet efficiency will be exactly unity, but it will be observed, from the expression for the forward thrust F, that if this is the case the thrust will be reduced to zero. On the other hand, when u is zero the thrust will be a maximum but, since the aircraft is stationary, no work is being done on it and, consequently, the efficiency will be zero. This, of course, assumes a constant jet velocity V relative to the aircraft. When $V = 2u$ the jet efficiency is 0·66.

Fig. 168 shows a curve of jet efficiency plotted against the ratio $(V-u)/u$. Now $V-u$ is the absolute velocity of the jet relative to the surrounding atmosphere, sometimes known as the 'slip' velocity, and u is the forward speed of the aircraft. Therefore,

$$\frac{V-u}{u} = \frac{\text{slip velocity}}{\text{aircraft speed}}.$$

When V and u are equal, the ratio is zero and the efficiency is 100 per cent. The greater the difference between V and u, the greater the value of the ratio and the lower the jet efficiency.

Curves illustrating a comparison of the propulsive efficiencies of airscrew, jet propulsion, and rocket, over a range of flight speeds, for a particular type of aircraft flying at 20,000 ft., are shown in Fig. 169.[1] The curve for the airscrew takes account of

[1] Fedden, 'Aircraft Power Plant—Past and Future', *Journal Roy. Aero. Soc.*, 1944.

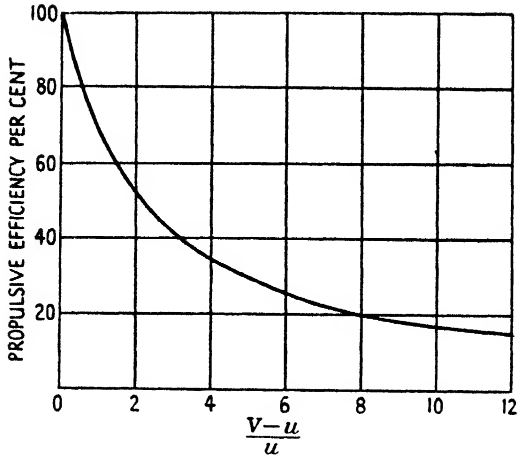


FIG. 168

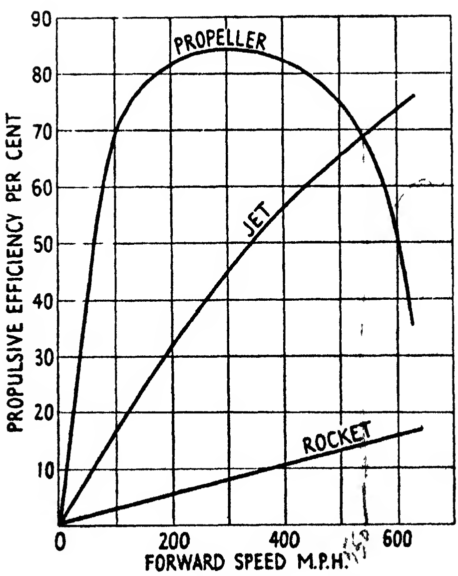


FIG. 169

the tip losses at high speeds. It will be seen that the efficiency of the jet does not equal that of the airscrew until a speed of about 550 m.p.h. is attained. If, however, the height at which the aircraft is flying is increased to 30,000 feet, the efficiencies become equal at about 450 m.p.h.

The impulse-duct jet motor

This is a very simple type of jet-propulsion motor and works on an explosion cycle. A diagrammatic section through a unit of this type is illustrated in Fig. 170 (*c*). Due to the forward motion of the aircraft, air enters the combustion chamber *B* through the battery of spring-leaf shutters *A*, the shutters being

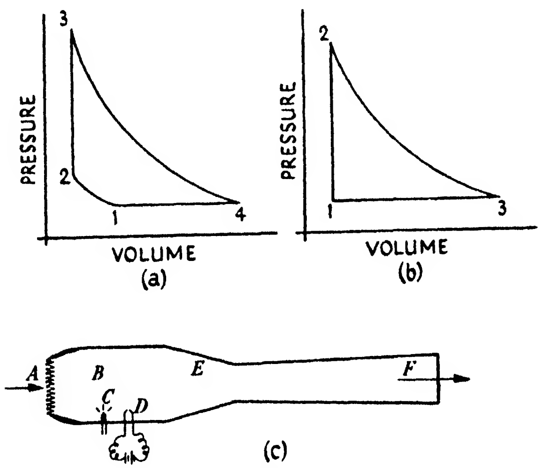


FIG. 170

forced open by the dynamic pressure of the air stream. The entrance to the combustion chamber is of divergent section and, ideally, there is a small pressure rise as the velocity of the air decreases. A metered quantity of fuel is then injected into the chamber through the fuel injector *C*, and the mixture ignited by the electric spark *D*. The increase in pressure consequent upon the explosion closes the spring shutters *A* and the pressure rapidly builds up, causing an acceleration of the gas along the tail nozzle *E*, from whence it emerges at *F* as a jet. As the gas passes along the nozzle, the pressure in the combustion chamber drops to atmospheric, when the shutters *A* again open and a fresh charge of air enters the chamber.

The ideal cycle upon which the system operates is shown in Fig. 170 (*a*). Air enters the combustion chamber at the point 1 and the small pressure rise due to the reduction in velocity is, theoretically, along the curve 1,2. In practice, however, the pressure rise is probably neutralized by the small, but proportionately important, throttling effect of the shutters. For this reason it is probable that the ideal cycle approximates more closely to Fig. 170 (*b*). Considering this latter cycle, entry of air and ignition may be assumed to take place at 1 and the pressure rise consequent upon the explosion is along the constant-volume line 1,2. Expansion through the tail nozzle is along the adiabatic 2,3 and, after leaving the system, the gas returns to its original condition before entry into the system along the constant-pressure line 3,1.

It will be observed that this cycle is similar to the cycle of the early experimental Karavodine explosion turbine described in Chapter III. In an actual motor the cycle diagram is not, of course, as sharply defined as that drawn in Fig. 170 (*b*), but is rounded, the peak pressure being lower than the ideal, due to the fact that while the explosion is actually taking place the gas is already beginning to accelerate along the tail nozzle.

Referring to the cycle diagram, Fig. 170 (*b*), the useful work of the system, manifested as kinetic energy in the jet, is represented by the area 1,2,3 of the diagram and the air standard efficiency is given by the expression

$$\eta_{\text{cycle}} = 1-\gamma\frac{T_3-T_1}{T_2-T_1}.$$

The derivation of this expression is set out in Chapter III. Since the discharge from this type of jet motor is intermittent and since the jet velocity varies with the combustion-chamber pressure, the propulsive efficiency of the jet is not constant. If η_{jet} is taken as the mean propulsive efficiency of the jet, then the over-all efficiency of the system as an aircraft motor is

$$\eta_{\text{aircraft}} = \eta_{\text{cycle}} \times \eta_{\text{jet}}.$$

The impulse-duct jet motor has undergone considerable development and has been employed as the power unit of pilotless aircraft used as flying bombs. A German flying bomb is illustrated in the next chapter. Pressure for the fuel injection is supplied by air bottles and current for the ignition spark by electric cells. This type of power unit is cheap and simple to manufacture, but has little commercial value as a general aircraft power plant on account of its very poor over-all efficiency, which is of the order of 5 per cent. at speeds in the region of 400 m.p.h., and the consequent limitation of aircraft range.

The constant-pressure duct jet motor

A diagrammatic illustration of this type of jet motor is shown in Fig. 171. It consists simply of a duct which, from nose to tail, is first divergent and then convergent. Due to the forward motion of the aircraft, air enters the duct at A and its velocity decreases as it approaches the section of maximum area. The decrease in the kinetic energy of the stream, consequent upon the reduction in velocity, causes a small pressure rise. This is represented by the curve 1,2 in the ideal-cycle diagram, Fig. 171. At D a steady spray of fuel is introduced into the air stream and is originally ignited by the electric spark E, after which burning is continuous. Combustion then takes place at constant pressure, the volume of the air increasing and its temperature rising. This is represented in the cycle diagram by the line 2,3.

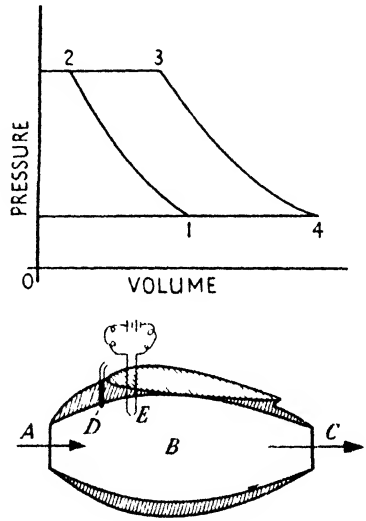


Fig. 171

As the stream leaves the duct through the convergent tail portion, the pressure falls to atmospheric and the velocity increases, the fluid emerging from the orifice C as a jet. The expansion is represented by the curve 3,4. Finally, when it has the left the system, the gas returns to its original condition before entry along the constant-pressure line 4,1, thus completing the cycle.

Due to the addition of heat energy to the stream, consequent upon the combustion of the fuel, the velocity of the jet at C is greater than the entry velocity at A. Thus there is an increase in the momentum of the stream as it passes through the duct, resulting in a reaction which supplies a forward thrust. The useful work of the cycle is represented by the area 1,2,3,4 in the diagram, and is manifested as kinetic energy in the jet.

It will be observed that this cycle is similar to that of the constant-pressure continuous-combustion turbine described in Chapter III, and the air-standard efficiency is, therefore,

$$\eta_{\text{cycle}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}},$$

where P_2/P_1 is the pressure-ratio of the duct. The theoretical over-all efficiency of the system is the product of the propulsive efficiency of the jet and the air-standard efficiency. Hence

$$\eta_{\text{aircraft}} = \eta_{\text{cycle}} \times \eta_{\text{jet}}.$$

The efficiency of the cycle is entirely dependent upon the pressure-ratio attainable and, since this is small, the over-all

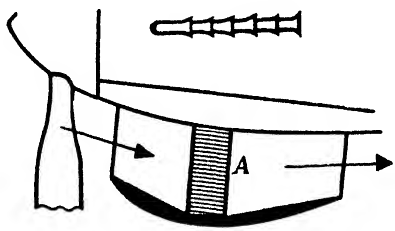


FIG. 172

efficiency is even lower than that of the impulse-duct jet motor. For this reason, this type of motor, considered purely and simply as a jet-propulsion motor, is of academic, rather than practical, interest. The principle of the system has, however, an important application in the ducted radiators employed in liquid-cooled reciprocating aero-engine installations, Fig. 172. In this case, heat energy is added to the air stream, not by the combustion of fuel in the duct, but by the hot coolant passing through the radiator elements A, which take the place of the fuel burners in the jet-propulsion system. Thus the cooling air leaves the radiator duct at a higher velocity than that with which it enters, the change in the momentum of the stream providing a small thrust, which largely offsets the aerodynamic drag of the radiator installation.

The combustion-turbine jet motor

This is the generally accepted type of jet-propulsion motor for man-carrying aircraft, and incorporates a combustion turbine. The layout is illustrated diagrammatically in Fig. 173, the combustion turbine employing a centrifugal compressor and a single-row impulse turbine wheel. Installations are not, of course, restricted to the use of these components and may use axial-flow compressors and reaction turbines, or any of the combinations which can be used in ordinary combustion turbines. The salient feature of the combustion turbine, used as a jet-propulsion motor, is that none of the useful work of the

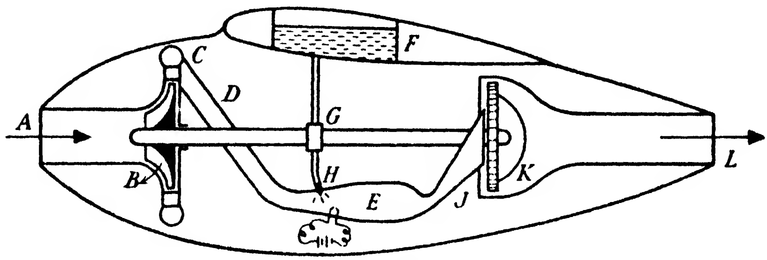


FIG. 173

system is taken out as shaft energy, but is utilized as kinetic energy in the jet.

Referring to Fig. 173, air enters the compressor at A and, after passing through the vane-channels of the impeller, is delivered through the duct D to the combustion chamber E. Fuel is fed from the tank F, by the pump G, to the burner nozzle H, through which it is sprayed into the combustion chamber and burned. The gas is then expanded through the turbine nozzle J, from which it emerges at high velocity, and is passed over the blades of the turbine wheel K. Finally, the gas leaves the turbine casing through the tail duct L, from which it emerges as a jet. The turbine rotor K is mounted on the same shaft as the compressor rotor B, and provides the power required to drive the compressor. The velocity of the jet leaving the tail duct L is greater than the velocity of the air stream entering the system at A, and the change in the momentum of the mass flow of air handled by the plant provides the forward thrust of the aircraft. It should be noted that the compressor receives a small benefit from the fact that, by virtue of the forward motion of the aircraft, a small additional pressure rise, or 'ram' effect, is obtainable.

The manner in which the useful work of the system is utilized is as follows: The turbine nozzle, Fig. 174, discharges a jet of gas, the direction of which is at an angle to the plane of the turbine wheel. The absolute velocity V of the jet has two components, one parallel to the plane of the wheel and the other parallel to the axis of the wheel. The component V_1 parallel to the plane of the wheel is the useful component, as far as the generation of shaft energy is concerned, and it is the aim of the designer completely to encompass the destruction of this

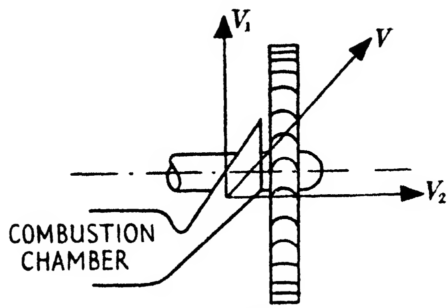


FIG. 174

velocity component in propagating the rotation of the turbine wheel. The axial component V_2, however, cannot be utilized for the provision of shaft energy, since its direction is perpendicular to the plane of rotation of the wheel. If all the useful work of the system is to be taken out in the form of shaft energy, as is the case in, for example, a combustion-turbine railway locomotive, the angle between the axis of the nozzle and the plane of the turbine wheel is made as small as possible, in order to reduce to a minimum the magnitude of the useless axial component V_2.

When the combustion turbine is used as a jet-propulsion motor, on the other hand, it is only necessary for the turbine wheel to generate sufficient shaft power to drive the compressor. The axial component is no longer useless, for this is the velocity, relative to the aircraft, of the propulsion jet and upon the magnitude of the kinetic energy contained in the jet depends the propulsive thrust of the aircraft. For this reason, the angle between the axis of the turbine nozzle and the plane of the wheel is increased to such a value that the magnitude of the velocity component V_1 is decreased to the point where the turbine wheel provides just sufficient power to drive the compressor. The consequent increase in the magnitude of V_2 results in the ejection of the gaseous fluid from the tail of the power plant at a velocity considerably greater than that with which it entered the system at the compressor intake.

It will be observed that the acceleration of the stream of air passing through the system takes place in the turbine nozzle. It follows that the point of application of the forward thrust on the aircraft is at the turbine nozzle, since it is here that the change of momentum takes place.

A second method of obtaining the thrust is to divide the expansion of the hot products of combustion into two stages. In this case the pressure drop in the turbine nozzles is not complete and the gases in the turbine casing are at a pressure greater than atmospheric. The remainder of the expansion takes place in a nozzle located in the tail duct of the aircraft and the point of application of the forward thrust on the aircraft is this second nozzle. The pressure drop in the turbine nozzles is sufficient to supply the power input to the compressor, and the angles between the axes of the nozzles and the plane of the wheel are made as small as practicable, so that as much as possible of the kinetic energy of the jets is converted into mechanical energy by the turbine wheel. Since the expansion in the turbine nozzles is incomplete, this method results in a lower velocity of the gases relative to the turbine blades, as compared with the first method, and the frictional losses in the blade channels are, therefore, lower.

The efficiency of the combustion-turbine jet-propulsion system

Clearly the over-all efficiency of a system of this type is dependent, primarily, upon the efficiency of the combustion turbine. Combustion turbines used in land vehicles and installations, and used for the generation of shaft power, when equipped with all the aids to high efficiency, such as waste-heat economizers and reheat stages, may attain an over-all thermal efficiency of the order of 20 per cent.

In aircraft practice, however, questions of power-plant weight and compactness of installation are of paramount importance and, in general, preclude the employment of these aids to efficiency. In consequence, the efficiency figures obtainable in aircraft plants are comparable with those attained by basic combustion turbines in land installations, neglecting the reduction in efficiency due to the fact that the propulsive efficiency of the jet is less than unity.

On the other hand, aircraft turbines have certain advantages over combustion turbines for land installations. The compressor intake faces forward and the compressor obtains a small, but by no means negligible, benefit from the so-called 'ram' effect due to the forward motion of the aircraft. This results in an increase in the delivery pressure and the cycle efficiency is increased. In a land installation, where it is desired that all the useful work of the cycle should be taken out of the system in the form of shaft work, a certain amount of kinetic energy inevitably goes to waste in the exhaust, by virtue of the axial velocity component of the discharge from the turbine nozzles. Since, in a jet-propulsion system, the useful work is taken out in the form of kinetic energy in the aircraft jet, the energy represented by this axial component no longer goes to waste, but forms a definite part of the useful work of the system.

The 'slip' loss of the jet, however, is large and unavoidable. For a given jet velocity relative to the aircraft, the greater the speed of the aircraft, the higher the propulsive efficiency of the jet. In practice it is not possible to attain aircraft speeds sufficiently high, compared with the jet velocity, to enable the propulsive efficiency to approach unity. Nor is it possible, for a given aircraft speed, so to reduce the jet velocity as closely to approach this ideal since, to obtain the same value of thrust, it would be necessary to increase the mass of air handled by the plant at the reduced flow velocity in order to obtain the same change of momentum. An airscrew generates its thrust by giving a small increment of velocity to the very large mass of air which passes through the airscrew disk. A jet-propelled aircraft, on the other hand, gives a much larger increment of velocity to a smaller mass of air, in order to obtain a comparable total change of momentum and, hence, a comparable forward thrust. It follows that, over the speed range in which the airscrew maintains its high efficiency, the propulsive efficiency of the jet is inevitably lower on account of its greater 'slip' loss.

If the over-all efficiency of a jet-propelled aircraft is taken as the product of the thermal efficiency of the turbine and the propulsive efficiency of the jet, then, for example, an aircraft fitted with a combustion turbine of 18 per cent. thermal efficiency and flying at a speed at which the propulsive efficiency of the jet is 66 per cent., would have an over-all efficiency of

12 per cent. So low an efficiency value implies a high fuel consumption and a consequent limitation in range.

The density of the atmosphere decreases with increase in altitude and the resistance of the air to the forward motion of the aircraft at great altitude is considerably less than at sea-level. For a given power output, it is, therefore, possible for an aircraft to attain a much higher forward speed at great altitude than at sea-level. The effect of this is to increase the propulsive efficiency of a jet-propelled aircraft, and to indicate that such aircraft are essentially high-altitude machines, their sea-level efficiency, at least at the present time, being poor.

The effect of altitude on the combustion turbine

Atmospheric pressure decreases with increase in altitude and so also does the density of the atmosphere. The pressure at 20,000 feet, for example, is a little less than one-half of that at sea-level. In addition, the temperature of the atmosphere decreases with altitude at a rate of approximately 2° C. for every 1,000 feet. Curves of relative density (taking the sea-level density as being equal to unity at a temperature of 15° C.) and temperature are shown in Fig. 175. When a combustion turbine is taken up into the air, these changes in the condition of the atmosphere have a very considerable effect on its performance.

In order to study the nature of the effect, let us consider a combustion turbine designed to give an output of 3,000 useful shaft horse-power at sea-level and investigate its performance at different altitudes. It is more convenient to consider the useful work of the plant taken out as shaft energy than as kinetic energy in the jet of a jet-propelled aircraft, because in the latter case it is usual to speak of the output in thrust horse-power (i.e. the work done in unit time by the thrust force in moving the aircraft forward), and the thrust horse-power varies with the speed of the aircraft.

If the efficiency η_t of the turbine and the efficiency η_c of the compressor are both equal to 0·85, if the pressure-ratio is 4, the maximum permissible temperature 650° C., and the value of γ is taken as 1·4, the actual over-all efficiency of the combustion turbine is given by the formula

$$\eta_{\text{actual}} = \eta_t \frac{K-1/\eta_c}{K-1}\left\{1-\left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}\right\},$$

where K is the ratio of gross work to negative work in the ideal cycle, P_1 is the pressure at the compressor inlet, and P_2 is the maximum pressure in the system.

(Now, provided the rotational speed of the set is maintained constant, the pressure-ratio of the compressor remains constant)

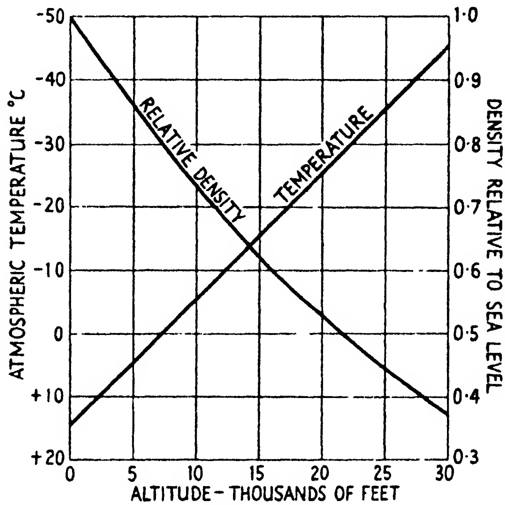


FIG. 175

regardless of changes in altitude and, consequently, in atmospheric density.)(The actual inlet and delivery pressures, however, both decrease with increase in altitude.) The values of turbine and compressor efficiency are assumed constant. The value of K does not remain constant, but increases with increase in altitude, for the following reason: Due to the decrease in atmospheric temperature with increase in altitude, the negative work of the ideal cycle, per pound of air handled, decreases but, since the maximum temperature in the system can be maintained at a constant value (650° C. in this case), the gross work of the ideal cycle remains constant. Hence the value of K increases with increase in altitude. A curve of K plotted against altitude, for this particular plant, is shown in Fig. 176, the value increasing from 2·155 at sea-level to 2·710 at 30,000 feet.

(The result)of this increase in K is an increase in the actual over-all efficiency of the combustion turbine.) This is illustrated by the curve of Fig. 177, which shows the variation in efficiency with altitude. At sea-level the efficiency is 18·75 per cent., while

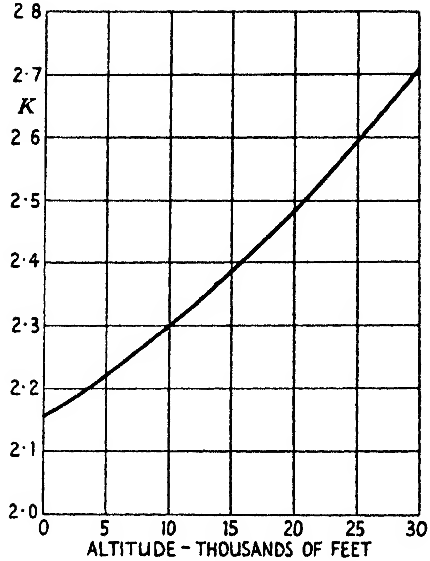


Fig. 176

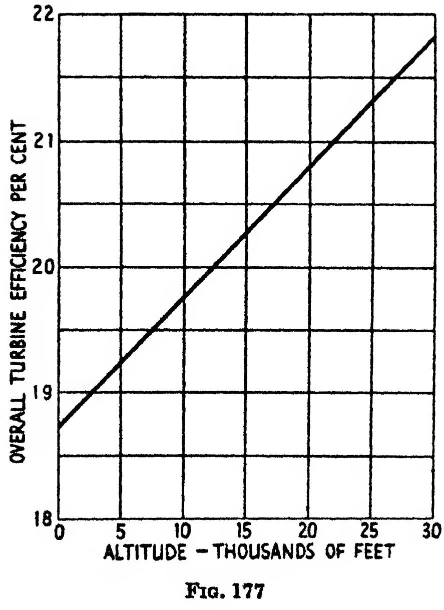


Fig. 177

at 30,000 feet it is 21·80 per cent., an improvement of 16·30 per cent. on the sea-level value. (Thus, one of the most important effects of an increase in altitude is a considerable increase in the efficiency of the plant.)

(Now the weight of free air passed through the system varies in proportion to the atmospheric density and, since the density

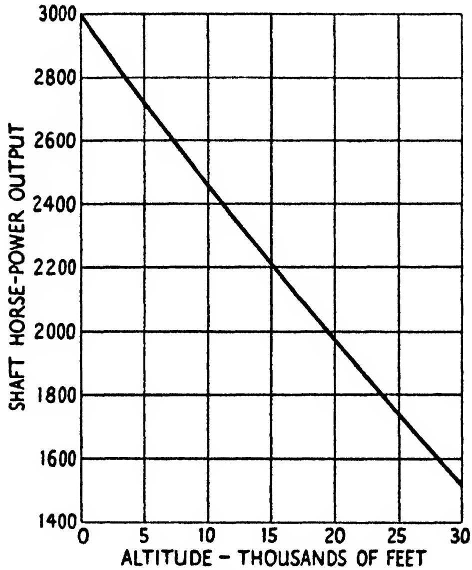


FIG. 178

decreases with increase in altitude, the power output of the combustion turbine falls off, in consequence of the smaller weight of air consumed. But, due to the fact that the efficiency of the plant increases with altitude, the power developed per one pound of air consumed also increases and, therefore, the power output decreases at a rather smaller rate than the atmospheric density.) At sea-level the turbine consumes 56·1 pounds of air per sec. for a useful output of 3,000 shaft h.p., the power developed per pound of air being 53·5 h.p. At 30,000 feet the output falls to 1,520 shaft h.p. with a consumption of 21·0 pounds of air per sec., and the power developed per pound of air is 72·3 h.p. The variation with altitude of the useful shaft output and of the useful shaft output per pound of air are shown in Figs. 178 and 179.

Since the power output falls off and the efficiency increases, the total fuel consumption decreases with increase in altitude, Assuming the turbine is run on paraffin having a calorific value of 10,850 C.H.U. per 1 lb., Fig. 180 illustrates the decrease in

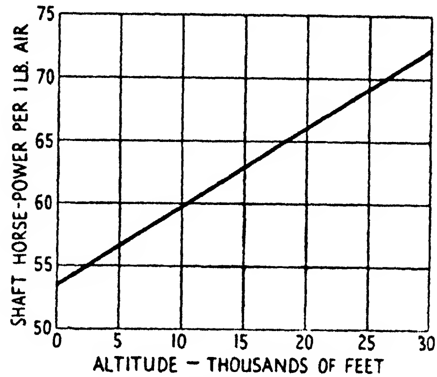


Fig. 179

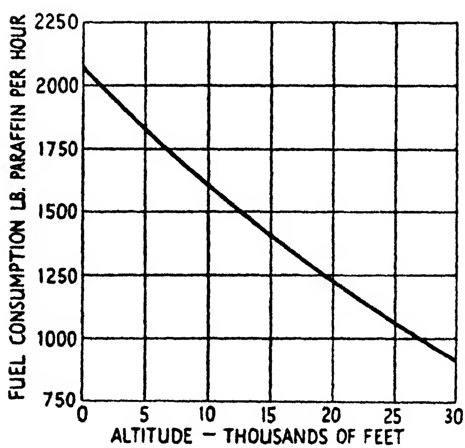


Fig. 180

fuel consumption with increase in altitude. Thus, at sea-level the consumption is 2,085 lb. per hour, corresponding to a specific consumption of 0·7 lb. per shaft h.p. per hour, while at 30,000 feet this is reduced to a total consumption of 910 lb. per hour, corresponding to 0·6 lb. per shaft h.p. per hour.

Summing up these results, we see that, although the power output of the combustion turbine decreases with altitude, due to the decrease in atmospheric density, the efficiency of the turbine increases in consequence of the reduction in the temperature of the atmosphere and that, therefore, the power output falls off at a lower rate than the air density.

If an aircraft in horizontal flight travels at a constant speed, its aerodynamic drag decreases with decreasing atmospheric

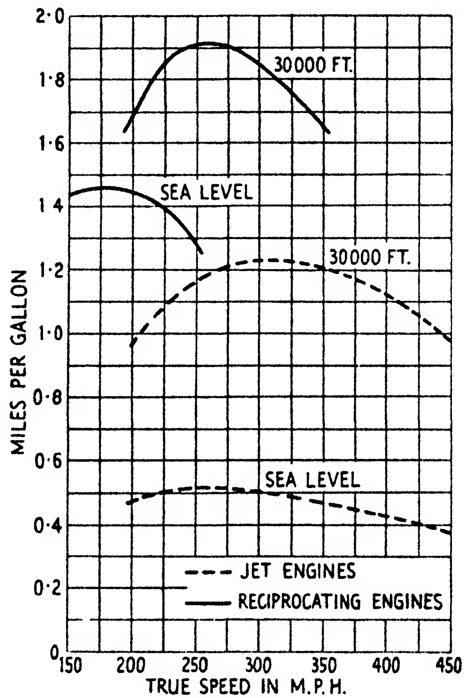


FIG. 181

density, and less power is required to maintain that speed at, say, 30,000 feet than at sea-level. The decrease in drag is not, however, in direct proportion to the decrease in density which occurs with altitude, for, provided the forward speed remains constant, the wing assumes a greater angle of incidence as the atmospheric density decreases in order to maintain the same total lift force on the aircraft, and this leads to an increase in induced drag. Thus the drag falls off at a lower rate than the atmospheric density. The decrease in drag varies considerably with the type of aircraft, the aspect-ratio of the wings, and other factors.

If we can assume, for a particular type of aircraft flying at a constant horizontal speed, that the rate of decrease with altitude of the power required to overcome the aerodynamic drag is the same as the rate of decrease of the power output of the combustion turbine (which, it will be remembered, also falls off at a lower rate than the air density), then the fuel consumption-altitude curve for the aircraft will be of the same form as that of Fig. 180. Hence, for the same performance, the fuel consumption is reduced to less than half by increasing the flying height from sea-level to 30,000 feet. If, however, the rate of decrease with altitude of the aerodynamic drag is such that an increase in speed is possible at altitude, a further benefit is obtained by virtue of the increase in the propulsive efficiency of the jet.

Clearly, then, it is an advantage to fly high, but, even so, the fuel consumptions of jet-propelled aircraft are, at the present time, nearly double those of equivalent airscrew-reciprocating-engine installations and a great deal still remains to be done by way of improving turbine efficiencies. Curves illustrating the estimated comparison of the fuel consumptions of a particular type of jet-propelled aircraft and an equivalent airscrew aircraft are shown in Fig. 181, the true speed in miles per hour being plotted against the air miles flown per gallon of fuel consumed at sea-level and at 30,000 feet.[1]

The combustion turbine and the airscrew

It is possible to use a combustion turbine to drive an airscrew, in exactly the same way as a reciprocating engine is used. In this case the useful work of the aircraft turbine is taken out in the form of shaft energy instead of kinetic energy in a jet. The performance is then comparable with that of equivalent turbines in land installations and the propulsive efficiency is enhanced, at least at speeds below about 450 m.p.h., by the replacement of the jet by an airscrew. The gases on discharge from the plant still, however, contain a considerable amount of kinetic energy and, if they are discharged in the form of a jet, the division of power between airscrew and jet has been estimated as 70 to 80 per cent. airscrew and 30 to 20 per cent. jet, depending on the type and function of the particular aircraft.[1]

Such turbine installations have an advantage over equivalent

[1] Fedden, loc. cit.

reciprocating engines in that their weight is little more than half that of the latter, but against this must be weighed their higher fuel consumption of about 0·6 lb. per b.h.p. per hour as compared with the figure of 0·4 lb. per b.h.p. per hour obtainable with reciprocating engines. Herein lies their primary disadvantage as commercial aircraft motors, until such time as the progress of research brings about a reduction in the specific fuel consumption to a value comparable with that of the reciprocating petrol motor.

Nevertheless, there are certain compensations in the use of a combustion turbine in combination with an airscrew. Perhaps the chief of these is that it is possible to run a turbine on practically any liquid fuel and therefore low volatility 'safety' fuels, such as paraffin or diesel oil, may be used, thus virtually eliminating the fire hazard prevalent with petrol as a fuel. A further advantage is that the construction of a turbine is considerably less complicated than that of a high-performance reciprocating aero-engine and the aircraft designer is, to a certain extent, in a position to have his power plant designed specifically for his projected aircraft, with consequent maximum performance, whereas this is rarely possible with reciprocating engines.

Some power-plant layouts are discussed in the next chapter.

XIII

GAS-TURBINE INSTALLATIONS

The impulse-duct jet motor

To date, the only practical application of this type of jet motor is as the power unit of small, pilotless aircraft used as flying bombs. A German flying bomb of the type F.Z.G. 76 is illustrated in Fig. 182.

The aircraft is of all-metal construction. The wing is a cantilever structure with a single tubular spar running right through the fuselage. The ribs are sheet-steel pressings. The fuselage is also made of sheet steel and is constructed in three sections which are bolted together. The forward section is the war-head and contains approximately 1 ton of explosive, together with the fusing gear. In the nose of the war-head is located the master compass which polices the directional gyroscopes. The forward portion of the centre section is the fuel tank, the capacity of which is about 135 gallons, while the rear portion houses the two wire-wound spherical air bottles which supply the air required for actuating the gyro-controlled servo-motors and for delivering the fuel to the jet motor. The tail section carries the tail-plane, elevators, fin, and rudder and contains the gyroscopic control gear, the fuel-metering mechanism, and the batteries which supply the electrical services.

The impulse-duct motor is of sheet-steel construction and is mounted above the rear portion of the fuselage. The mounting consists of a yoke at the forward end of the fuselage tail-section and a single-point attachment at the top of the fin. At the forward end of the duct is a grid containing 126 spring-leaf flap valves which control the admission of air. Attached to the grid are nine fuel jets, the supply to which is metered by the veeder counter in the fuselage. The sparking-plug is screwed into the top of the duct just aft of the valve grid.

The aircraft is rocket-launched along a rail track. When sufficient speed has been attained to enable the air stream to force open the spring-leaf valves, ignition takes place and the impulse-duct motor begins to function with an explosive frequency of about 40 per second. The fuel consumption of this particular type is slightly less than 1 gallon of petrol per air mile, which gives it a range of about 150 miles in still air at a

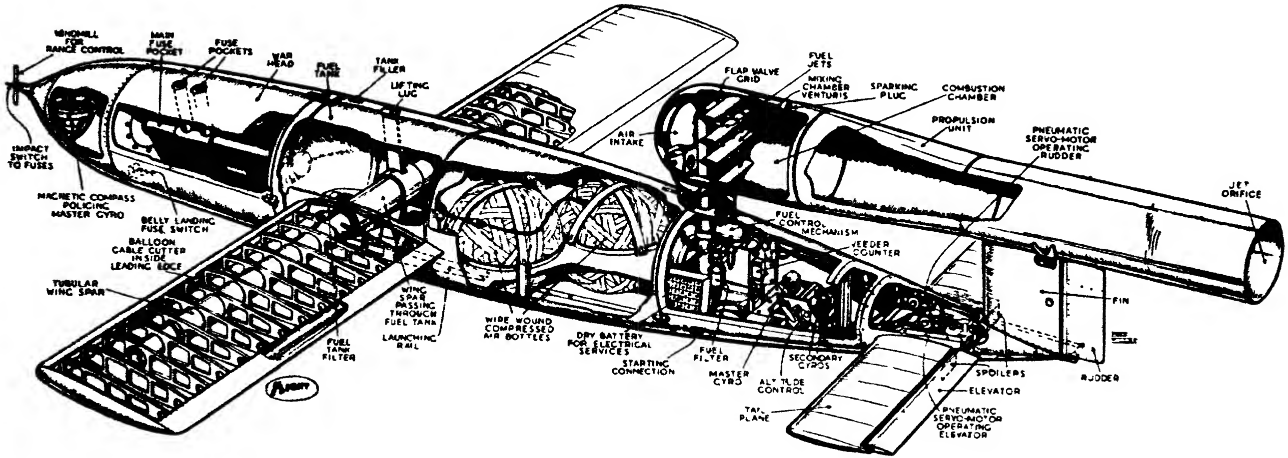

WINDMILL FOR RANGE CONTROL
MAIN FUSE POCKET
FUSE POCKETS
WAR HEAD
FUEL TANK
TANK FILLER
LIFTING LUG
IMPACT SWITCH TO FUSES
MAGNETIC COMPASS POLICING MASTER GYRO
BELLY LANDING FUSE SWITCH
BALLOON CABLE CUTTER INSIDE LEADING EDGE
TUBULAR WING SPAR
WING SPAR PASSING THROUGH FUEL TANK
FUEL TANK FILTER
LAUNCHING RAIL
WIRE WOUND COMPRESSED AIR BOTTLES
DRY BATTERY FOR ELECTRICAL SERVICES
STARTING CONNECTION
FUEL FILTER
MASTER GYRO
SECONDARY GYROS
AIR INTAKE
FLAP VALVE GRID
FUEL JETS
MIXING CHAMBER VENTURIS
SPARKING PLUG
COMBUSTION CHAMBER
PROPULSION UNIT
PNEUMATIC SERVO-MOTOR OPERATING RUDDER
JET ORIFICE
FUEL CONTROL MECHANISM
FIN
SPOILERS
RUDDER
ELEVATOR
TAIL PLANE
PNEUMATIC SERVO-MOTOR OPERATING ELEVATOR

Fig. 182

speed in the neighbourhood of 400 m.p.h. Under these conditions the over-all thermal efficiency is of the order of 5 per cent., the thrust about 600 lb., and the thrust horse-power about 580.

The combustion-turbine jet motor

A typical layout of a combustion-turbine jet motor suitable for a wing installation is illustrated in section in Fig. 183. This turbine is fitted with an 8-stage axial-flow compressor and a single-stage single-row impulse turbine. Other combinations of turbine and compressor types are, of course, possible.

Despite their sharply peaked efficiency curves, the axial-flow compressor and the impulse wheel are well suited to aircraft practice, since the greater portion of an aircraft's flying time is usually spent at, or near, the designed optimum cruising speed and the combustion turbine can, therefore, be operated under conditions of maximum efficiency for long periods. The choice of turbine and compressor types is also influenced by considerations of weight and compactness. The impulse turbine is considerably lighter and more compact than an equivalent reaction turbine and the axial-flow compressor is sometimes more compact and lighter than an equivalent multi-stage centrifugal type. The combination of these factors makes the choice of axial-flow compressor and impulse wheel a good one.

The compressor rotor and turbine wheel are mounted on the same hollow steel shaft which is carried in four bearings. The bearing housings and brackets are of light alloy, the latter being bolted to the compressor casing which is itself of light alloy. The front bearing bracket carries the auxiliary-drive gear-box which provides drives for the electrical generator, tachometer, oil pump, fuel pump, vacuum pump, etc. The leads and piping from these are housed in the four hollow streamline-section arms of the bracket. The petrol starter-motor, together with its epicyclic gearing, is mounted on the forward face of the gear-box. The compressor rotor is a built-up light alloy drum the end-plates of which are attached to the shaft. Due allowance is made throughout the design, in order to permit of expansion with increase in temperature. The combustion chambers, which are circular, are made of heat-resisting steel and joints are provided which allow for differential expansion of the chambers without leakage of gas. The nozzle ring and turbine blades are machined from heat-resisting steel. The wheel itself

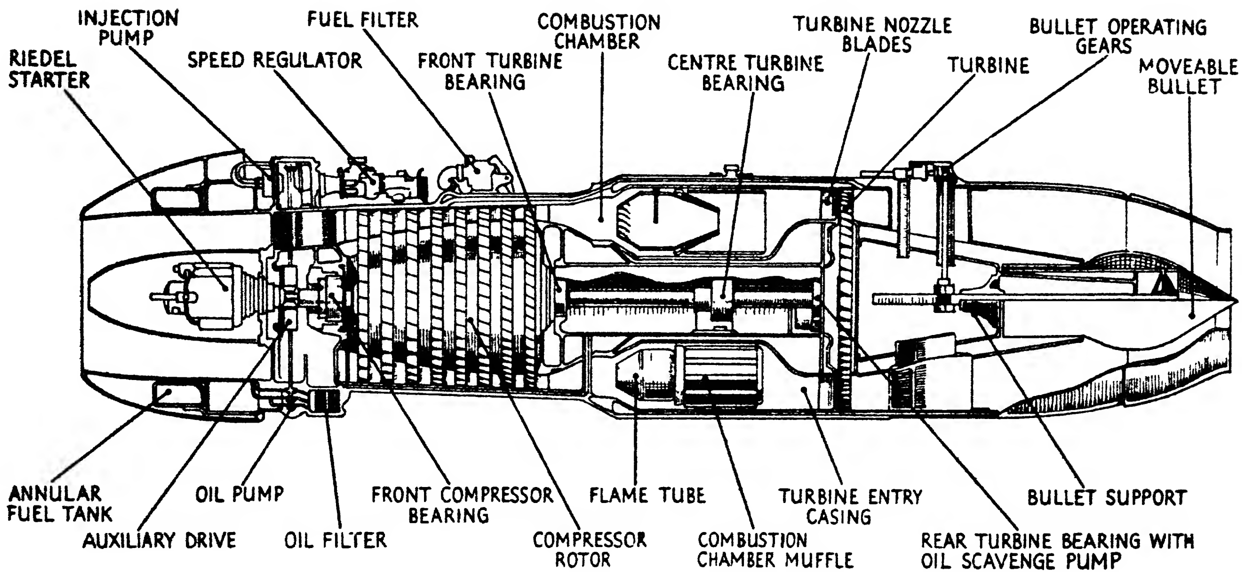
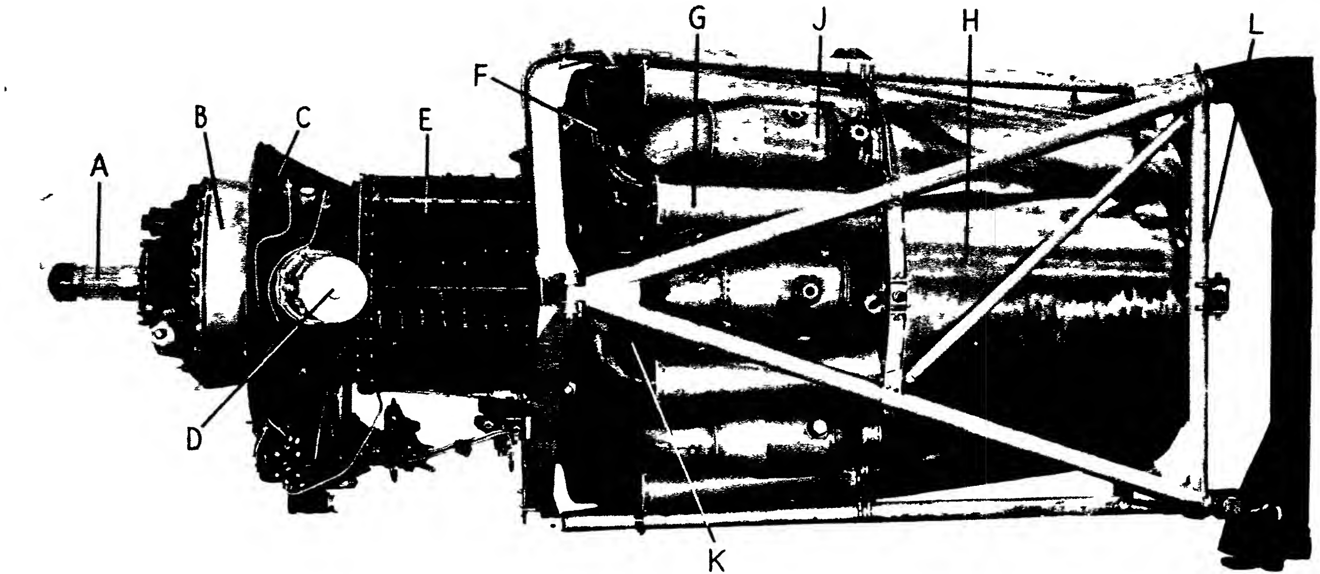


Fig. 183. Junkers Jumo 004 jet-propulsion motor with axial-flow compressor and impulse turbine.

FIG. 184. THE BELL 'AIRACOMET' TWIN-TURBINE JET-PROPELLED FIGHTER. 49 FT. SPAN. ALL-UP WEIGHT 5 TONS. POWERED BY TWO WHITTLE-TYPE G.E.C. GAS TURBINES



A—Airscrew shaft
B—Epicyclic reduction gear
C—Air intake
D—Electric starter motor

E—Axial flow compressor stages
F—Centrifugal compressor stage
G—Delivery pipe to heat exchanger
H—Heat exchanger

J—Combustion chamber
K—Turbine housing
L—Exhaust orifice

Fig 185a THE BRISTOL THESEUS MK I

Incorporating combined axial centrifugal compressor, heat exchanger, reverse gas flow and airscrew driven by separate turbine wheel is a 'disk of constant stress' and is overhung from the rear bearing. A number of shrouded burners is disposed in a circle within the combustion chambers.

To date, combustion-turbine jet motors have only been fitted to high-speed fighter aircraft, since the propulsive efficiency of the jet is poor at the lower speeds of transport and bomber aircraft and in the latter case the fuel consumption would be very much greater than that of equivalent reciprocating petrol motors. Twin turbine units slung beneath the wings form the most popular layout, thus making the fuselage space available for the installation of armament, cameras, radio equipment, etc. In designs of this type it is usual to set the tail-plane fairly high so as to be clear of the high-temperature jets. True air speeds in excess of 500 m.p.h. are attained by such aircraft.

The turbine-airscrew combination

A layout for a combustion turbine driving an airscrew is illustrated in Fig. 185 B.[1] The turbine is completely enclosed within the wing and drives contra-rotating pusher airscrews through an epicyclic reduction gear and an extension shaft. An axial-flow compressor and a multi-stage reaction turbine are employed.

Air enters the compressor through the leading-edge of the wing and the exhaust gases leave through branched ducts which discharge above the trailing edge. A certain amount of 'jet thrust' is obtained from the gas discharge and the jets are arranged so as to be clear of the airscrew disk. This is necessary because the velocity of the jets may be considerably higher than the velocity of flight, while gas temperatures may be up to 500° C., and the action of the airscrew blades entering and leaving the zones of high-velocity and low-density air might set up serious vibration in the blades and interfere with the airflow over them. The radii of the bends in the discharge ducts should be as large as possible in order to minimize losses.

At the present time the specific fuel consumption of such an installation would be in the region of 0·6 lb. per b.h.p. per hour, which does not compare favourably with the figure of about 0·40 lb. per b.h.p. per hour attainable with reciprocating petrol engines. When, however, the fact that the weight of such an installation is little more than half that of an equivalent

[1] Fedden, loc. cit.

reciprocating motor is taken into account, it may be that the turbine would show certain over-all advantages on short and medium distance flights.

Recent information indicates that the specific weight of an airscrew-turbine installation varies with the pressure-ratio and,

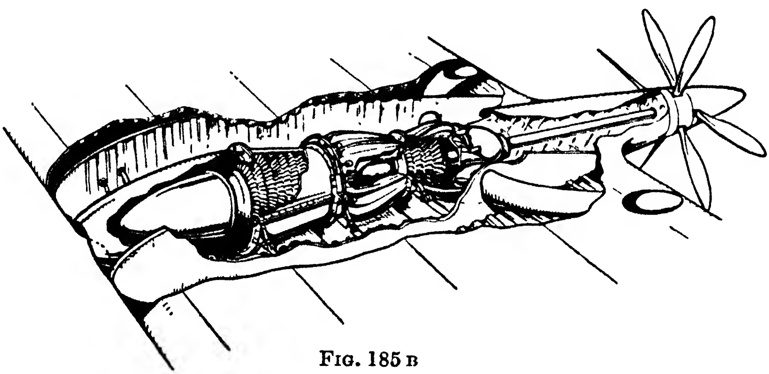


FIG. 185 B

in theory, quite high pressure-ratios could be attained before the specific weight approaches that of the reciprocating petrol engine. The use of pressure-ratios of the order of 8:1 or 12:1 would reduce the specific consumption to a value comparable with that of the reciprocating engine, but such pressure-ratios are not yet a practical possibility.

The mechanical design of the turbine and compressor is relatively simple and the provision of a smooth torque, together with a general absence of vibration, reduces the difficulties of high-power airscrew design and makes for lightness of installation fittings. The major factors likely to give rise to mechanical trouble are thermal distortion and compressor-blade vibration.

One of the major factors likely to influence the adoption of the combustion turbine for airscrew-driven aircraft is undoubtedly the fact that low-volatility 'safety' fuels can be used, thus eliminating the fire hazard.

Fig. 186 illustrates a projected civil aircraft of 200,000 lb. all-up weight employing four combustion turbines totalling 20,000 h.p. Such an aircraft fitted with pressure cabins should be capable of cruising at 300 m.p.h. at 25,000 to 30,000 feet over ranges of 2,000 to 3,000 miles.[1]

[1] Fedden, loc. cit.

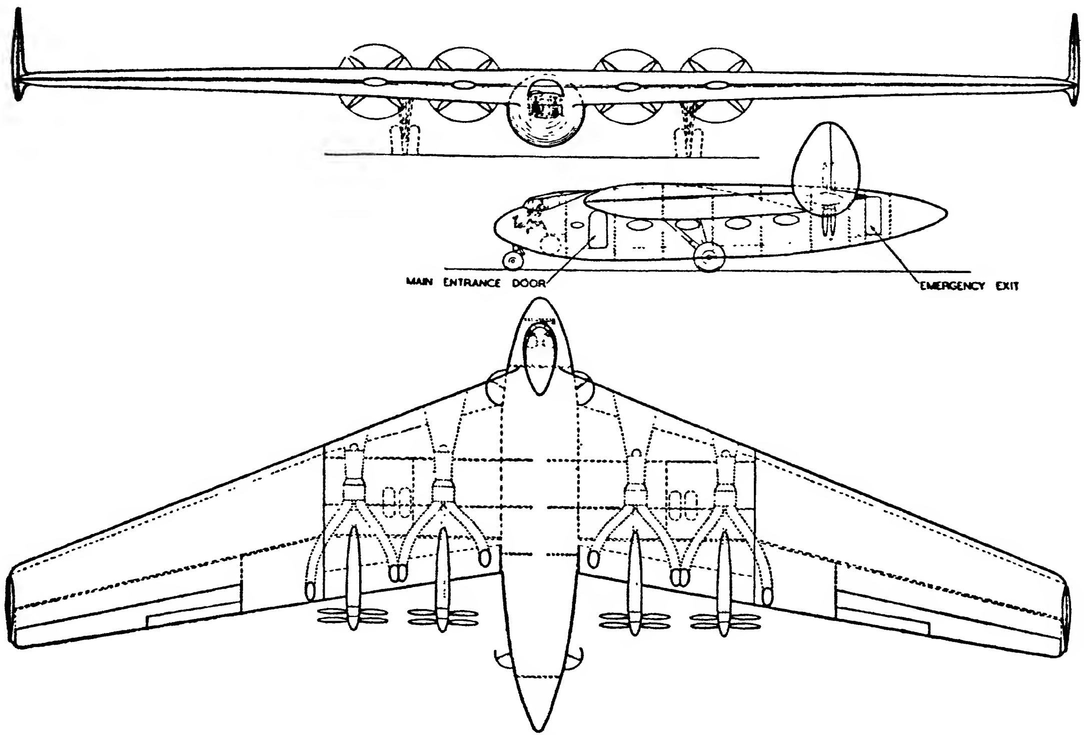


Fig. 186. Tailless aircraft of 200,000 lb. all-up weight fitted with four combustion turbines driving contra-rotating pusher airscrews.

Railway locomotives

The application of the combustion turbine to the railway locomotive may perhaps prove its most fruitful field of development. To date, only one such locomotive has been built, but the results of its trials were so promising that further construction is projected. The locomotive in question was designed and built by Messrs. Brown Boveri & Co., Ltd., Baden, Switzerland, for the Swiss Federal Railways and its trials were conducted in 1941.[1]

The transmission is electric, the single 2,200 h.p. combustion turbine being geared to a generator which supplies current to the motors on the driving axles. The continuous output of 2,200 h.p. is delivered at 5,200 r.p.m., the gearing reducing this to 812 r.p.m. at the generator. The tractive effort at the wheel rim is 29,000 lb. at starting and 11,000 lb. for continuous cruising at 45 m.p.h. The maximum speed is 70 m.p.h. and the effective weight with fuel tanks full is 93·5 tons. A sectioned view of the power unit is shown in Fig. 187.

A small auxiliary Diesel-driven generator is used to start the turbine. This spins the main power unit to a speed at which enough air is delivered by the compressor to permit the lighting of the burner. The time lapse between the starting by means of the auxiliary Diesel set and the lighting of the burner is 4 minutes. The Diesel set is then disconnected from the turbine and after a further 4 minutes the turbine attains its normal light-load speed. During running, control of the power output of the combustion turbine is by variation of the fuel supply. Since the transmission system is electric, the normal compressed-air brakes can be supplemented by power braking.

The actual output of the turbine rotor is 8,000 h.p., but of this total just under 6,000 h.p. is absorbed by the compressor. Thus, only a little more than one-quarter of the rotor output is available as useful work. The gas temperature at the turbine inlet varies from 455° C. to 610° C.

The operating characteristics of the combustion turbine are indicated by the curves of Fig. 188. It will be observed that the maximum over-all thermal efficiency is slightly less than 18 per cent. and that this value is attained at about three-quarters

[1] All diagrams, photographs, and figures concerning this locomotive are taken from 'The First Gas Turbine Locomotive' by Dr. Adolf Meyer, *Proc. I. Mech. E.*, 1943.

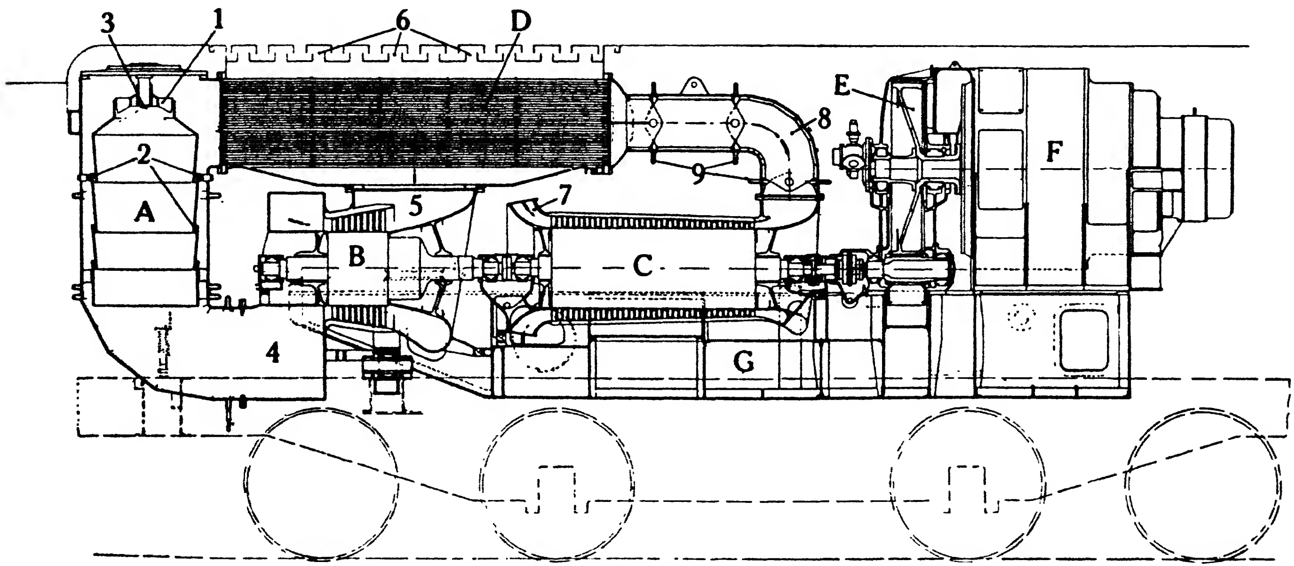


FIG. 187. Section through gas-turbine unit of the locomotive.

A Combustion chamber.
1 Swirl vanes.
2 Slits.
3 Injection nozzle.
4 Mixing space.

B Gas turbine.
5 Exhaust gas inlet to air heater *D*.
6 Exhaust gas from air heater *D* to atmosphere.

C Compressor.
7 Air inlet.
8 Air outlet pipe.
9 Expansion joints.

D Air heater.
E Gear.
F Generator set.
G Bedplate of unit.

full load. The following specific consumptions were obtained on test, burning fuel oil:

Half load	0·9302 lb. per h.p. per hour.
Three-quarters load	0·7971 lb. per h.p. per hour.
Full load	0·8797 lb. per h.p. per hour.

When, however, it is possible to take full advantage of the considerable amount of gas-turbine research carried out during

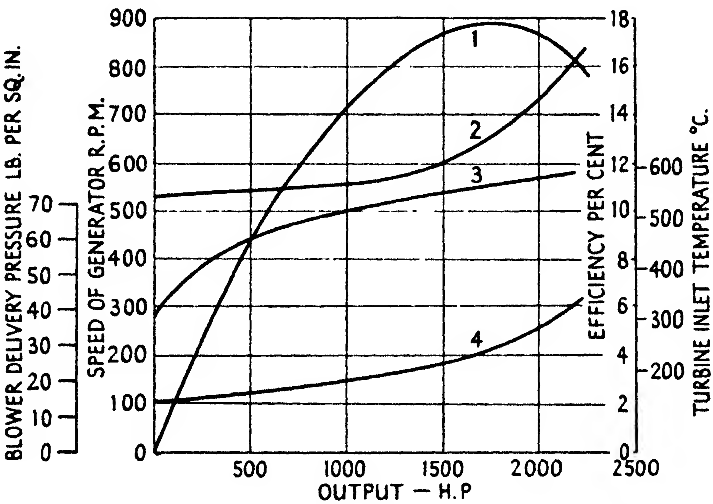


FIG. 188. Operating characteristics of a gas-turbine plant for traction purposes.

Curve 1. Percentage efficiency in relation to load.
2. Speed of generator in relation to load.
3. Turbine inlet temperature in relation to load.
4. Blower delivery pressure in relation to load.

the 1939–45 war it is, perhaps, reasonable to suppose that future locomotives may have specific consumptions of little more than 0·6 lb. per h.p. per hour.

The effects of changes in atmospheric temperature are illustrated by the curves of Fig. 191. The lower the temperature of the inlet air, the greater the maximum output, the higher the efficiency, and the lower the specific fuel consumption.

It is inevitable that the introduction of this new type of locomotive should involve a comparison with existing types and the table opposite sets out some of the economic factors affecting the operation of steam, Diesel-electric, and combustion turbine-

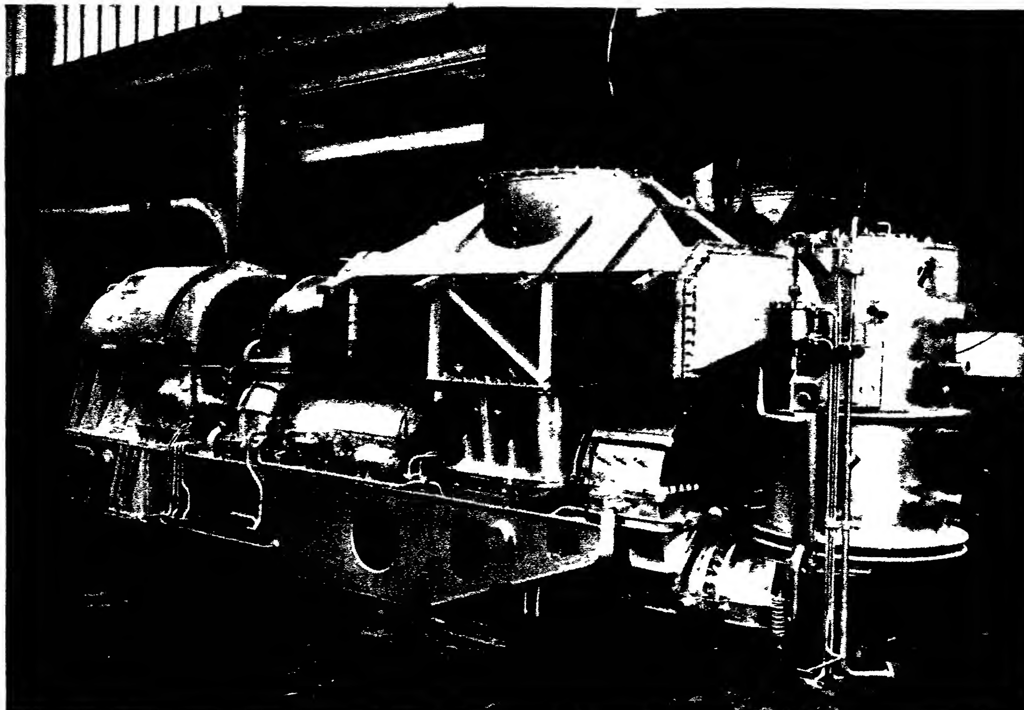


FIG. 189. 2,200 H.P. POWER PLANT FOR GAS TURBINE LOCOMOTIVE

FIG. 190. 2,200 H.P. GAS TURBINE LOCOMOTIVE

electric locomotives, the prices being based on pre-war U.S.A. figures. The efficiency figures are average operating values and not maxima.

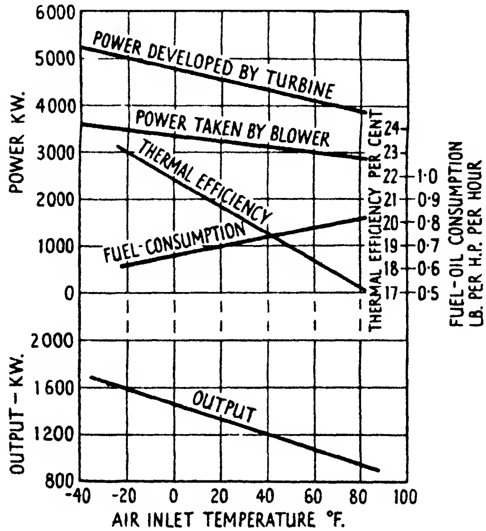


Fig. 191. Gas-turbine output and fuel consumption in relation to air-intake temperature in °F.

	Steam	Diesel-electric	Gas turbine
Initial cost per horse-power	£7	£17.10s.	£13
Efficiency at drawbar, per cent.	6–8	26–28	15–16
Mileage per year	180,000	250,000	Greater than 250,000
Filling and refuelling time	Greatest	Least	Small
High scheduled speed	Lowest	Higher	Higher
Track wear	Large	Less	Least
Power braking	None	Full power	Full power
Life in years	30	15–20	30
Maintenance	Lower	High	Least
Fuel costs (steam = 100 per cent.)	100	50–75	50–75
Lubrication costs as percentage of fuel costs	10	20–30	Less than 1
Water costs as percentage of fuel costs	10	Small	Nil
Starting effort	Least	Larger	Larger

From these figures it will be seen that the combustion turbine-electric locomotive, although still in its infancy, compares

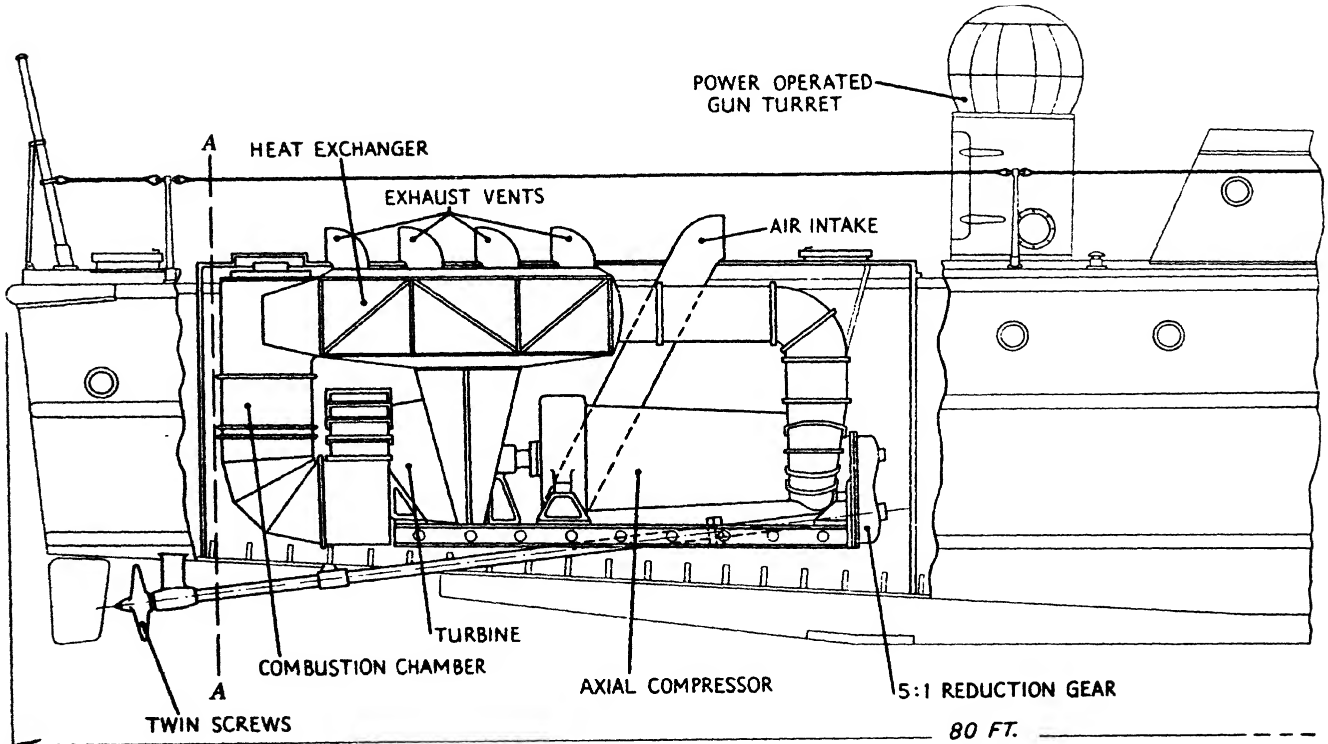

POWER OPERATED
GUN TURRET
A
HEAT EXCHANGER
EXHAUST VENTS
AIR INTAKE
TURBINE
COMBUSTION CHAMBER
AXIAL COMPRESSOR
5:1 REDUCTION GEAR
A
TWIN SCREWS
80 FT.

Fig. 192 A

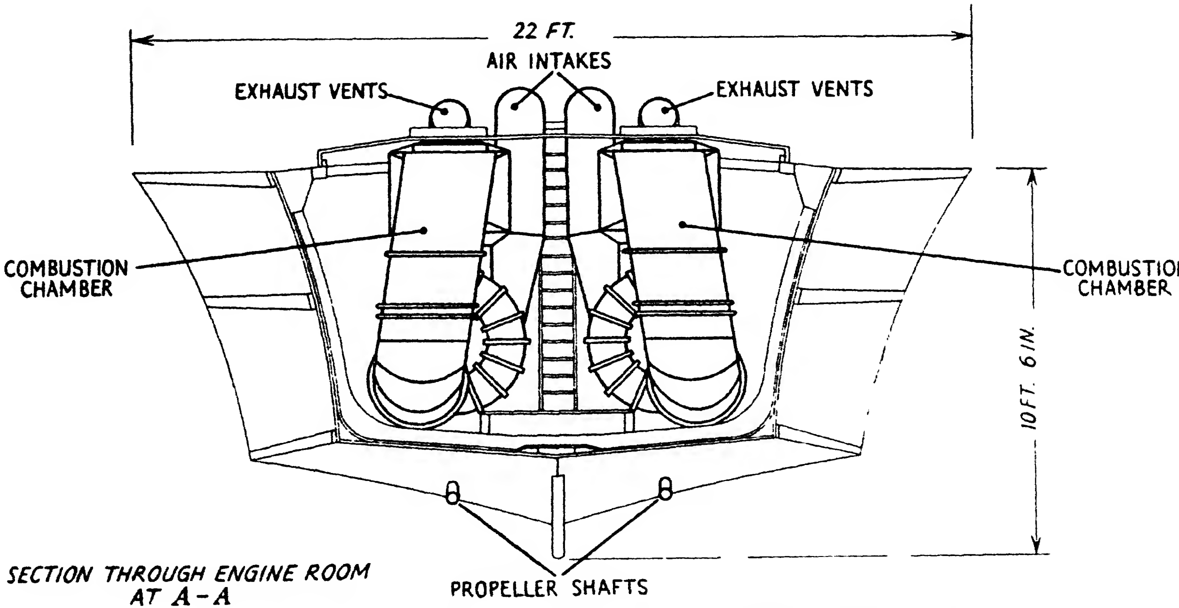


FIG. 192 B. Eighty-foot high-speed launch: two gas turbines of 2,500 b.h.p. each.

favourably with the Diesel-electric locomotive and in some respects possesses decided advantages. In all respects, except that of capital cost, it is superior to the traditional steam locomotive. Since the combustion-turbine locomotive will run on a large variety of liquid fuels, including coal derivatives, it is perhaps not unjustified to conjecture that in due course it may entirely replace the steam locomotive.

Marine installations

Figs. 192 A and 192 B show a projected 80-foot high-speed launch fitted with twin combustion turbines of 2,500 b.h.p. each.

In the larger types of craft the combustion turbine offers certain advantages over petrol-engine installations of the same power output. Engine vibrations are eliminated and the great reduction in noise would be an important factor in motor torpedo-boats and motor gun-boats. But perhaps the greatest advantage in the case of wooden-hulled fighting craft is the ability of the combustion turbine to use low-volatility 'safety' fuels, thus considerably reducing the fire hazard.

On the other hand, the fuel consumption at the present time would be about 40 per cent. greater than that of an equivalent petrol-engine installation, due to the lower efficiency of the combustion turbine. For craft of less than 70 feet length and of less than 3,000 h.p. it is doubtful whether a combustion-turbine installation would result in a shorter engine-room, but for craft of over 100 feet length and of more than 5,000 h.p. the combustion turbine offers decided advantages over the petrol engine in most respects other than that of minimum fuel consumption. The economy in space is largely due to the fact that in large, high-powered petrol-engined craft it is necessary to use a number of engines, since the output of the largest units at present available is of the order of only 1,000 h.p.

Combustion-turbine installations for large ocean-going vessels have been suggested, but at the present time the combustion turbine, while effecting a considerable economy in engine-room space due to the absence of boilers and condensers, cannot compete with the steam turbine in fuel economy over voyages of several thousand miles. It would appear that the introduction of the combustion turbine into marine practice on any large scale will have to wait until the progress of research results in turbines of higher efficiency.

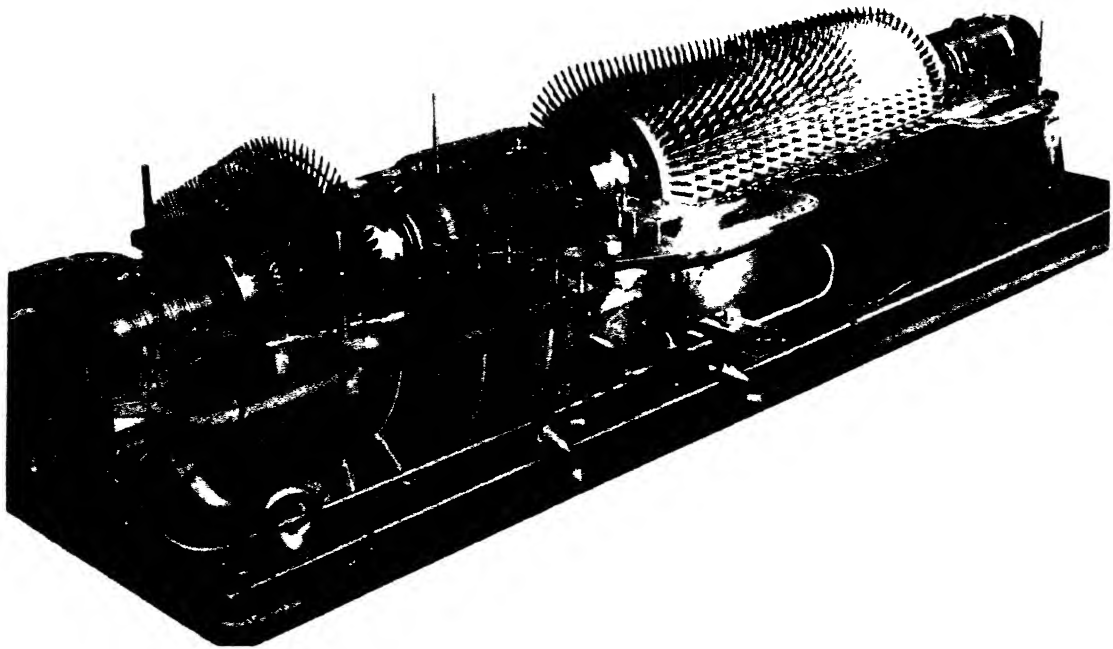


Fig. 193. BROWN-BOVERI GAS TURBINE INSTALLED AT SUN OIL COMPANY'S PLANT, MARCUS HOOK, PHILADELPHIA, U.S.A.

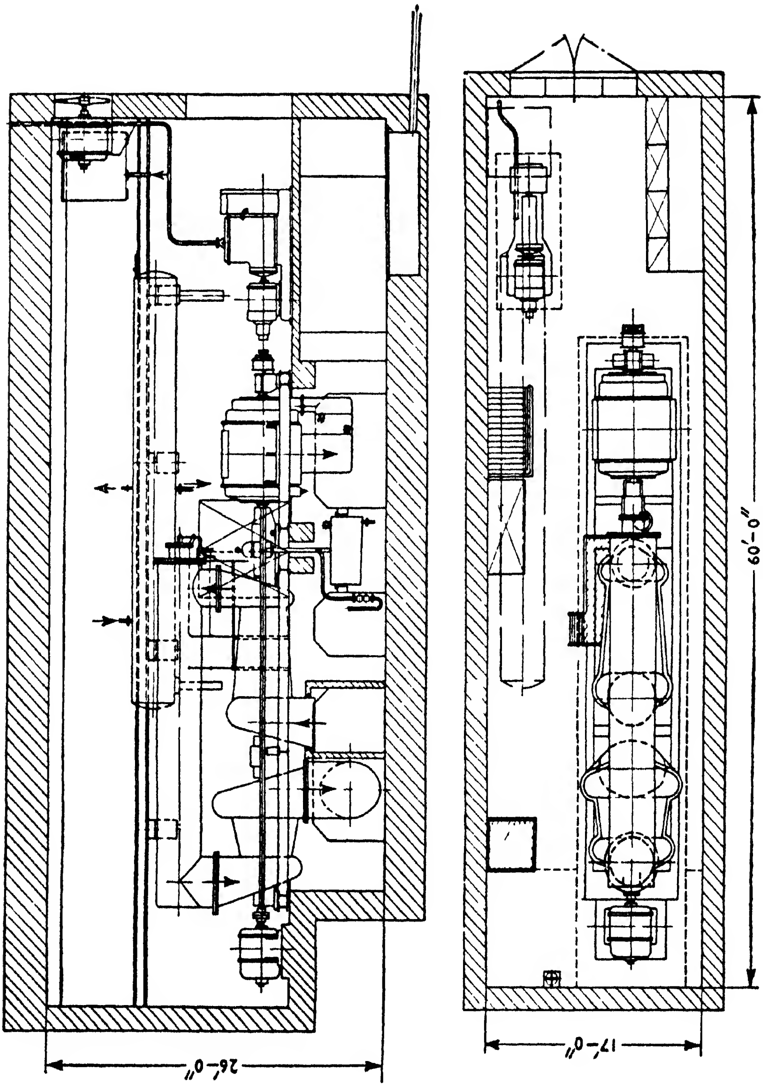


Fig 194

Stationary land installations

The combustion turbine is not yet efficient enough to compete on base-loads with the modern steam-turbine generating station fitted with condensers, feed-water heaters, etc.

Use has, however, been found for it as a stand-by generating plant to deal with peak or emergency loads. Fig. 194 illustrates the layout of a 4,000 kW. gas-turbine emergency-load generating set installed in a bomb-proof shelter at the city of Neuchâtel.[1] For such emergency operation the combustion turbine has the advantage of low capital cost. The plant is not fitted with a heat-exchanger since its running periods are too short for economy in fuel costs to be shown on the capital cost of the exchanger.

A combustion turbine built by Messrs. Brown Boveri & Co., Ltd., for the Sun Oil Company, Marcus Hook, Philadelphia, is illustrated in Fig. 193.[1] The purpose of the turbine is to supply the large quantities of compressed air required for the Houdry Oil Refinery Process. The photograph shows well the blade-rings of the axial-flow compressor and the turbine blading.

[1] Meyer, 'The Combustion Gas Turbine—Its History, Development and Prospects', *Proc. I. Mech. E.*, 1939.

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