Alternating Current Electrical Engineering

BY

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PREFACE TO THE SIXTH EDITION

An attempt has been made in the present volume to produce a text-book which covers in a general way the main ground included in the title without going into too great detail in any one particular branch.

The author has endeavoured to combine theory with practice, since a certain amount of the former is necessary to understand properly the application to practical work. The success with which the earlier editions have met has led the author to expand the original work, and include in it many of the more important of the latest developments, but the general plan of adhering to principles, rather than the describing of particular types of machinery and apparatus, has been maintained.

The ground covered enables the work to be used as a text-book in connection with courses preparing for National Certificates and Diplomas in Electrical Engineering, both "Ordinary" and "Higher," as well as to serve as an aid to students in the study of the principles of alternating currents generally. Students preparing for the Degree of B.Sc.(Eng.), and similar examinations, will also find it of help, whilst engineers whose regular student days are over may use it to refresh their memories when called upon to solve problems which are out of their normal course of work.

The necessity for keeping the volume within due bounds has enforced a careful pruning of all the sections that do not represent the latest practice, but it is hoped that it is thoroughly up-to-date.

The notation used is, in the main, that recommended by the International Electrotechnical Commission.

The great majority of the illustrations have been specially prepared for this book, and the author takes this opportunity of thanking The British Thomson-Houston Co., Ltd., The Cambridge Instrument Co., Ltd., Messrs. Everett Edgcumbe and Co., Ltd., Metropolitan-Vickers Electrical Co., Ltd., Messrs. H. Tinsley and Co., the *Electrician*, and the Institution of Electrical Engineers for their kindness in supplying the blocks from which the remaining illustrations have been taken.

Philip Kemp.

The Polytechnic, Regent Street,
London, W.1.
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CHAPTER I

GENERAL CONSIDERATIONS OF ALTERNATING E.M.F. AND CURRENT

Production of Electromotive Force.—When an electrical conductor cuts a magnetic field there is an E.M.F. induced in the former, the magnitude of which is proportional to the rate of cutting lines of flux. Since both the paths of the electric current and the magnetic flux are closed loops, this is equivalent to saying that when the number of linkages is changed an E.M.F. is induced in the electric circuit, a linkage meaning one line of flux linked with one turn. In a number of problems it will be more convenient to speak of a change of linkages rather than the cutting of a magnetic flux, since the latter may be linked with more than one turn. Thus

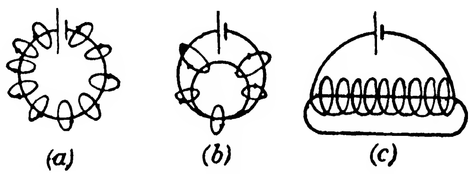


FIG. 1.—Linkages.

ten linkages may be produced by ten lines of flux linked with one turn as in Fig. 1 (*a*), or by five lines of flux linked with two turns as in Fig. 1 (*b*), or by one line of flux linked with ten turns as in Fig. 1 (*c*), or by any such combination.

The idea of linkages certainly gives a truer idea than that of flux cut by a conductor, since, in the latter case, one has to be careful to remember the return circuit both of the electric current and the magnetic flux. Notwithstanding this disadvantage, it will still be found expedient to consider a large number of problems from the point of view of cutting flux, but the student should make himself familiar with both conceptions. In the case where an electrical conductor not forming part of a complete circuit moves across a magnetic field, it will have an E.M.F. induced in it, but no current will flow and no work will be done on the conductor, which will consequently exert no reaction on the magnetic field. If the circuit be closed, current will flow which will endeavour to set up its own magnetic field, producing a reaction upon the main field. This effect is seen in the distortion of the magnetic flux in direct current dynamos and motors, and is particularly important in alternating current work.

Law of a Simple Alternating E.M.F.—When a conductor rotates with a constant angular velocity in a uniform magnetic field, as indicated in Fig. 2, it cuts the flux at varying rates depending upon the instantaneous direction of motion. The induced E.M.F. is therefore of a varying character as well. When the conductor is passing the centre of the pole, it is moving momentarily at right angles to the field and is cutting the flux at its maximum rate; its maximum E.M.F. therefore occurs at this point. When the conductor has advanced 90° from this point, it is, for a moment, moving parallel to the field, and has no E.M.F. induced in it. To obtain the value of the induced E.M.F. at any point throughout the complete revolution, it is necessary to determine that component of the velocity which is at right angles to the magnetic field. A knowledge of the

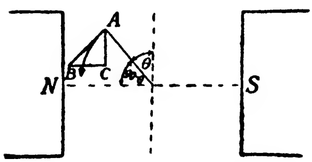


FIG. 2.—Generation of Alternating E.M.F.

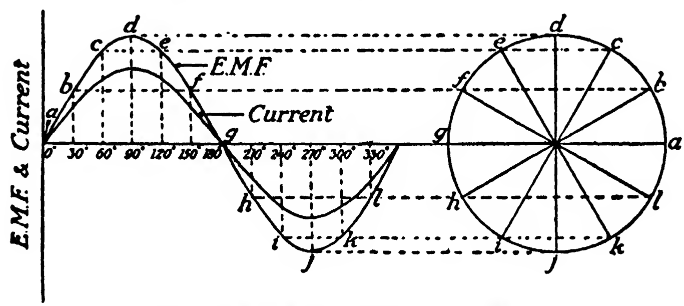


FIG. 3.—Graphical Construction of Sine Wave.

direction in which the magnetic field is being cut is also necessary, as this determines the direction of the induced E.M.F. If the conductor is rotating with a constant peripheral velocity of v cm. per second, represented in Fig. 2 by AB, then AC represents the useful component and is equal to $v \sin \theta$, θ being the angle moved through taking the vertical line as the starting point. Let l equal the length of the conductor in cm. and B the density of the magnetic flux in lines per square cm., then the E.M.F. induced in the conductor at any instant is equal to $Blv \sin \theta \times 10^{-8}$ volts. The only quantity which varies throughout the revolution is $\sin \theta$, and therefore the induced E.M.F. obeys a simple sine law as represented in Fig. 3. This diagram also shows a simple graphical method of constructing a sine wave. A circle is constructed as shown. and radiating lines are drawn every 30°. From the points *a*, *b*, *c*, *d*, etc., horizontal lines are drawn until they intersect the ordinates erected at 0°, 30°, 60°, 90°, etc. The intersections give points on the required curve.

Law of a Simple Alternating Current.—If the ends of the above active conductor are connected to the ends of a simple resistance, a current will flow the value of which at any instant is equal to the resultant E.M.F. divided by the resistance. If the resistance is constant and no other E.M.F. acts upon the circuit, the current will be exactly proportional to the E.M.F. and will follow the same law.

Alternating Magnetic Flux.—Whenever a current flows in a circuit, it always sets up a magnetic flux linking with the electric circuit. The magnitude of this flux is proportional to the strength of the current, assuming no iron to be present, for when the current is zero the flux which it sets up is zero, and when the current reaches its maximum value the flux does likewise. Thus an alternating flux is produced by an alternating current, both following the same law, and the number of linkages of lines with turns is continually changing. But a line of flux is imagined to grow from point size by gradually swelling, and thus getting larger. It certainly does not grow after the fashion of a broken thread, gradually encircling the conductor by moving end on, for a line of flux is always a closed loop. Bearing this fact in mind, it is easy to realize that during the production of a line of flux it must cut the conductor which produces it at some time during its generation. The result of this action is to generate an E.M.F. in the conductor. This can be summarized by saying that when the current changes the flux also changes, and thus the conductor is cut by its own flux, resulting in a self-induced E.M.F. proportional to the change of linkages per second. The direction of this self-induced E.M.F. is such as to oppose the change. This effect can be seen in a D.C. circuit, particularly in the shunt field circuit of a dynamo or motor where, when the E.M.F. is first applied, the induced voltage opposes the rise of the current, thus retarding its growth; similarly, when the circuit is opened, the induced voltage tries to keep the current flowing and produces a vicious spark at the opening contacts of the switch.

If the magnetic circuit consists of non-magnetic materials, the flux which is set up is proportional to the current and obeys the same law, whether this be a simple sine law or not; and if the resistance of the circuit is constant, the resultant E.M.F. producing the current also obeys the same law. But the resultant E.M.F. is obtained from a combination of the applied voltage and the self-induced back voltage due to the conductor cutting its own magnetic flux. There is, however, no guarantee that this latter voltage obeys the same law as the current, and consequently it cannot be said that the applied voltage follows the same law as the current. The presence of iron in the magnetic circuit exerts a further disturbing influence. As a matter of fact, there is very often a considerable difference in practice in the laws followed by the current and the applied voltage. It should, however, be borne in mind that the resultant E.M.F. acting upon the circuit, the current which it produces when the resistance is constant and the magnetic flux which is set up when no iron is present, all obey the same law, whatever it may be.

Frequency.—For a particular circuit, the instantaneous value of the current, or the voltage, may be plotted against time as a base. This curve may be an ideal sine wave, or it may take some

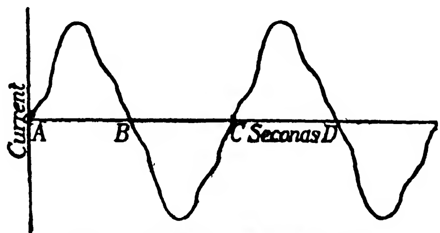


FIG. 4.—Non-sinusoidal Wave Form.

such form as is indicated in Fig. 4. It is said to be periodic, *i.e.* it keeps on repeating itself over and over again. The whole series of events included from a given point to the next similar point in the same direction is called a *period* or a *cycle.* For example, the series of events which occurs between *A* and *C* or between *B* and *D* is a complete cycle, whilst the time taken to accomplish this is called the *periodic time.* The number of complete cycles per second is called the *frequency.* The frequency of both current and voltage is, of course, the same. The wave form, as it is called, may vary considerably from the ideal sine wave, and in old-fashioned alternating current machinery it often was very far indeed from the desired shape, but in modern machinery, owing to the advance in methods of design, a good approximation to the ideal sine wave is usually achieved, although it may not be exactly correct in a mathematical sense. With the ordinary forms of alternating current generators both halves of the wave must be identical, *i.e.* any two points situated 180° apart must have the same value.

Non-sinusoidal Wave Form.—In order that the wave form shall be sinusoidal, *i.e.* follow a sine law, it is necessary that the conductor should cut the magnetic flux at a rate proportional to sin θ, the speed being constant. Now consider the elementary alternating current generator shown in Fig. 5, where there is one turn mounted on an iron core in the field of an electro-magnet excited by means of a direct current. The magnetic flux in this case may be considered to be radial across the air-gap, and as the conductor rotates with constant speed it cuts the flux at a constant rate whilst under the influence of the pole. Throughout the remainder of the revolution the conductors are not in the magnetic field at all, thus generating no voltage during these periods. A rectangular wave form of E.M.F. is obtained as shown in Fig. 6. In between this and the original sine wave obtained from the arrangement shown in Fig. 2

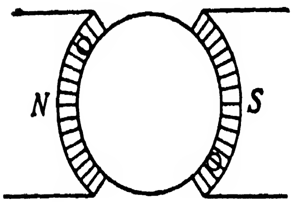


FIG. 5.—Conductors Rotating in Uniform Field.

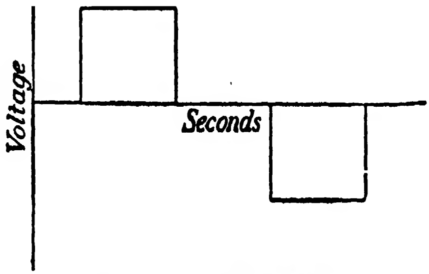


FIG. 6.—Rectangular Wave Form.

there are an infinite number of possibilities, and it is from these that the wave forms obtained in practice are built up.

In the majority of problems it will be necessary to make the calculations on the assumption that the quantities concerned follow a sine law, since the mathematics would otherwise be complicated to an undesired extent. Nevertheless, it must not be forgotten that the actual wave forms dealt with nearly always depart, to a more or less extent, from the ideal wave form.

EXAMPLES.

(1) By the aid of a graphical construction draw a sine wave having a maximum value of 100 units, and determine its amplitude at 20°, 135°, and 320° from the commencement.

(2) A conductor 10 cms. long revolves in a magnetic flux the density of which is 10,000 lines per cm.^{2}. The peripheral speed is 10 metres per second. What is the maximum value of the induced E.M.F.?

(3) Why is the wave form of current and voltage not always the same?

CHAPTER II

MAXIMUM, R.M.S. AND AVERAGE VALUES

Variation of an Alternating Current.—In dealing with alternating currents, it is at once necessary to fix upon a unit of current which must obviously have some differences from the ampere as used in direct current work. The current is varying at a very rapid rate, and although its value at any one instant can be compared with the direct current ampere, yet it is not self-evident what is the resultant effect. Take, for example, a circuit the frequency of which is 50 cycles per second, the current obeying a sine law and rising up to an instantaneous maximum of 10 amperes. The current rises from zero to its maximum value in the time taken to execute one-quarter of a cycle, that is in $\frac{1}{200}$th second. Starting from an instant when the current is zero, the value of the current is 5, 7·07 and 8·66 amperes at times equal to $\frac{1}{600}$th, $\frac{1}{400}$th and $\frac{1}{300}$th second respectively. Moreover, after $\frac{1}{100}$th second has elapsed, the current has reversed and is following the same procedure in the opposite direction for $\frac{1}{100}$th second. If, then, a moving coil ammeter is placed in circuit, with the object of measuring the current, it will endeavour to indicate all these different values of the current from instant to instant, including those where the current is flowing in the opposite direction. But, owing to the mechanical inertia of the moving system of the instrument, it cannot follow out all the rapid variations of the current, with the result that the pointer merely remains in the mean of the positions that it would like to take up. Since, however, there is as much current flowing in one direction as in the other, it follows that this mean position is the zero of the scale, independent of the actual value of the current. Thus an ordinary moving coil ammeter is useless for the purpose of measuring an alternating current, nor is it desired, at this present juncture, that the various instantaneous values should be recorded.

Maximum Value.—Since it would be extremely inconvenient to have to designate the value of an alternating current by stating its various values from instant to instant, apart from the practical difficulty of carrying this out, it is necessary to choose one representative value to specify the strength of the current. The first that suggests itself for this purpose is the maximum value, but there are obvious objections to this immediately currents of different wave forms are considered. Take, for example, the two currents of different wave form indicated in Fig. 7. They have the same maximum value, but there is obviously more current flowing in the first case than in the second, so that it is not desirable to represent these two currents by the same number. Therefore, the maximum value of the current is not a suitable measure of it unless the wave form is known, and this latter condition puts it out of court so far as practical purposes are concerned.

Average Value.—The next representative value that suggests itself is the average or mean value. Since both the positive and negative [1] halves of the wave are equal, the average value taken over a complete cycle is zero, as was previously seen when discussing the action of the moving coil ammeter. This will not do, but it would be possible to consider the average value taken over a half-

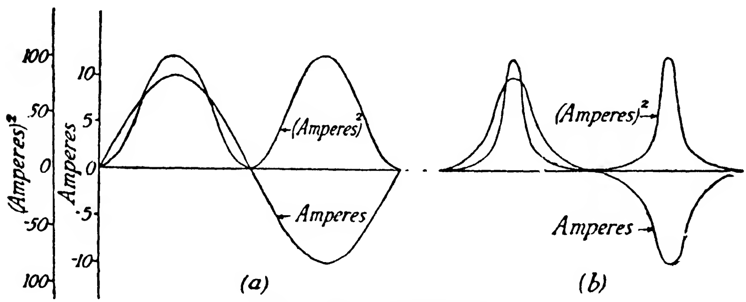


FIG. 7.—Values of (Current)².

period, since this would have a quite definite numerical meaning. Following out this idea, suppose that the current represented in Fig. 7 (*b*) is passed through a simple resistance, the instantaneous value of the power being represented by i^2r, where i is the instantaneous value of the current and r is the resistance. This power is varying at a very rapid rate, since, in a circuit the frequency of which is 50, the power rises from zero to a maximum value in $\frac{1}{200}$th second. In the great majority of cases, it is the average value of the power over a given time that is desired, and consequently the average value of i^2r over a complete cycle must be determined. Since r is a constant, the power at any instant is proportional to i^2. The curve of i^2 is shown in Fig. 7 (*b*), having been obtained by squaring all the ordinates of the current curve. The average power is equal to (average value of i^2) $\times$ r, and on measuring the average value of i^2 from the curve this is found to be 50, whilst the average value of

[1] The term "negative" is used merely to indicate the fact that the current is flowing in the opposite direction, and has no absolute meaning.

i is found to be 5·30. Thus if r be 1 ohm, the true value of the average power is 50 watts, whilst the value of the expression

$$(\text{average value of current})^2 \times r = 5{\cdot}30^2 \times 1 = 28{\cdot}1.$$[1]

From this it follows that if the average value of the current, taken over a half-period, is to be the representative value chosen, then the power in watts must be represented by an expression of the type

$$\text{watts} = k \times I^2r,$$

where k is some constant. Moreover, if different wave forms be considered, it will be found that the value of k changes, so that again it is necessary to know the wave form in order to determine the power in the circuit, rendering the average value of the current also impracticable as a representative value.

Root-Mean-Square Value.—From the above it is seen that the value of the current desired is such that the power expended in a simple resistance shall be given by the expression I^2r independent of wave form. This means that the average value of the squares of the various instantaneous values of the current is to be multiplied by the resistance.

In D.C. work the watts are given by the expression $I_d{}^2r$, where I_d is the direct current.

In A.C. work, the average watts are to be given by an expression

$$\text{average value of } (i^2) \times r,$$

where i is the instantaneous current. If r be given the same value in each case, then in order that the power shall be the same in both cases it follows that

$$I_d{}^2 = \text{average value of } (i^2).$$

It is now seen that the representative value of the alternating current, which has the same effect as a direct current in producing power in a circuit, is given by the expression

$$\sqrt{(\text{average value of } i^2)},$$

and when this expression has a certain value it means that the current has the same power effect, or heating effect, as a direct current of the same value, whilst the watts are given by the expression I^2r without the introduction of any constant, I being the

$$\sqrt{(\text{average value of } i^2)}.$$

The foregoing argument has made no assumptions as to wave form and, indeed, is true for all periodic wave forms, so that it is

[1] The student is warned to distinguish carefully between the (average value of current)^2 and the average value of (current)^2.

not necessary to know the various instantaneous values in order to determine the resultant value of any current.

This value of the current is known as the "Root-Mean-Square" or "R.M.S." value, the name expressing its meaning. It is also sometimes known as the effective value and as the virtual value, although the first name is by far the most commonly used.

Current Indicated by Ammeters.—At first sight the above chosen representative value appears somewhat complex, but it is only an appearance. This is seen when the action of various ammeters is considered (see pages 144 *et seq.*). Take, for example, the hot wire ammeter; the heating at any instant is proportional to the square of the current, and it is to the average value of this quantity that the indications of the instrument are proportional. When such an ammeter is connected in an alternating current circuit it would register an amount proportional to the mean square of the current and the square root of the deflection would be proportional to the R.M.S. current. Consequently the hot wire ammeter indicates that value of the current which it is desired to know. In the actual instrument the scale is not exactly a square scale, being somewhat modified by the construction and control, but this does not affect the fact that it is the mean square of the current to which the deflection is proportional.

Similarly, when considering a moving iron ammeter, it is again seen that the torque at any moment is proportional to the square of the instantaneous current. The current induces a certain pole strength in the moving iron which is proportional to the current provided the iron is not saturated, whilst the solenoid itself can be replaced by an equivalent magnet having a pole strength proportional to the current. The force of attraction or repulsion is proportional to the product of these pole strengths and, consequently, to the square of the current. Modifying factors are again introduced in practice, due to the construction and method of control, but the main fact still remains, viz., that the instrument pointer takes up a mean position indicating the average value of the square of the current. Thus a moving iron ammeter indicates the R.M.S. current.

A dynamometer type instrument also gives indications which are proportional to the products of the currents in the moving coil and in the fixed coil and registers accordingly the R.M.S. current.

Thus it is seen that, instead of the R.M.S. value of the current being a somewhat obscure quantity, it is actually the value recorded by the ordinary types of ammeters.

Representative Value of Voltage.—The question of settling a unit of E.M.F. can now be dealt with in the same way as for current. Imagine an alternating voltage applied to a simple resistance. The instantaneous value of the power is $\frac{e^2}{r}$, where e is the instantaneous value of the voltage and r is the resistance. The average value of the power is the average value of $\frac{e^2}{r}$. Suppose that a direct pressure of E_d be applied to a similar resistance of r ohms. In this case the power is $\frac{E_d^2}{r}$, and it is desired that the power in both cases shall be the same. From this it follows that

$$\frac{E_d^2}{r} = \frac{\text{average value of } e^2}{r}$$

or that $$E_d = \sqrt{(\text{average value of } e^2)}.$$

Thus it is seen that the R.M.S. value of the voltage ($= E$) is also the representative value which enables the expression

$$\text{average watts} = \frac{(\text{R.M.S. volts})^2}{\text{Resistance}} = \frac{E^2}{r}$$

to remain true without the addition of any multiplying constant. No assumption with regard to wave form has been made, and consequently the above expression holds good for any periodic wave form.

Voltage Indicated by Voltmeters.—With regard to the indications given by various voltmeters, it should be remembered that in all electromagnetic instruments it is the current which produces the deflection, and since the instruments previously discussed indicate the R.M.S. current, they will also indicate the R.M.S. voltage.

Graphical Method of Determining R.M.S. Value.—Let the wave be represented by AYB in Fig. 8. The R.M.S. value can be determined

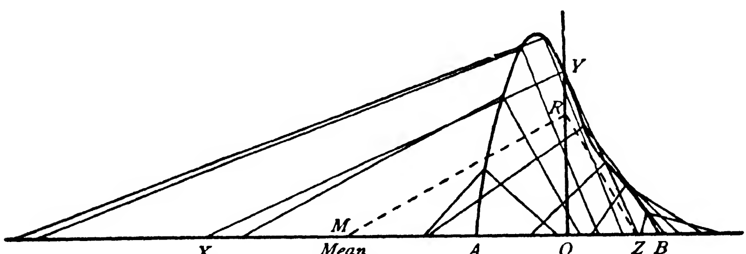


FIG. 8.—Graphical Determination of R.M.S. Value. (" World Power," July, 1929.)

by the following construction. A vertical is erected at O, midway between the two zero points, intersecting the graph of the wave at Y. The base line AB is next divided into a number of equal divisions, and in the centre of each division a mid-ordinate is erected. These mid-ordinates can conveniently be taken at $\theta = 10°$, $30°$, $50°$, etc., up to $170°$. There are thus four on each side of the original $90°$ ordinate. It is not necessary actually to draw these ordinates, it being sufficient to mark their points of intersection on the graph. A point Z is now chosen on the base line situated m units to the right of O, where m is any arbitrary constant of convenient value. A set square is now laid on the graph in such a manner that the point of the right angle coincides with the point Y on the graph, one side of the set square being adjusted to lie along the line YZ. The point of intersection of the other side of the set square and the base line is marked at X.

The two triangles ZOY and YOX are similar, so that $\frac{OZ}{OY} = \frac{OY}{OX}$ and, therefore

$$OY^2 = OZ \times OX = m \times OX.$$

Therefore $$OX = \frac{OY^2}{m}.$$

This procedure is repeated for all the other chosen ordinates, thus obtaining a number of points of intersection on the base line. Since the same number of measurements are taken from the right and from the left of O, the mean position of these points gives the distance OM which is equal to $\frac{\text{mean square value}}{m}$. The set square is again placed on the graph, and adjusted so that one side passes through Z and the other through M. It is now moved about until the point at the right angle lies in the central ordinate OY. This point is represented by R in the diagram.

As before, $$OR^2 = OZ \times OM$$
$$= m \times \frac{\text{mean square value}}{m}$$
$$= \text{mean square value},$$

and $$OR = \sqrt{\text{mean square value}}.$$

The R.M.S. value is thus obtained by simply scaling OR, and is independent of the value of m. The value chosen for m does, however, affect the accuracy and the convenience. It should be so chosen that the angle XZY is from 45° to 60°.

Ohm's Law for Alternating Current Circuit.—Ohm's Law holds good for an alternating current circuit where the R.M.S. values of the current and voltage are employed.[1] It obviously holds good when the maximum values are considered and also in the case of the average values, because if the circuit consists of a simple resistance the current is directly proportional to the voltage at

[1] In order to avoid confusion it should be noted that the usual way of expressing Ohm's Law as applied to alternating currents is to state that $I = \frac{E}{Z}$, where Z is the impedance (see page 21), but $I = \frac{E}{r}$ is strictly correct if E be the resultant voltage acting on the circuit.

any instant and the two wave forms are similar. The ratio of the average value to the maximum is the same, therefore, for both current and voltage, and since Ohm's Law holds good for maximum values it also holds good for average values when the current and voltage wave forms are similar.

Relationship between Maximum, Average, R.M.S. and Instantaneous Values.—In the succeeding pages, whenever the term "current" or "voltage" is used without any further qualification, it is the R.M.S. value which is meant, other values being defined specifically when they are used.

It is desirable to know the mathematical relationship that exists between the maximum, average, R.M.S. and instantaneous values of the current and voltage; but it is only necessary to consider one of them, since what applies to the current also applies to the voltage.

In the case of a current obeying a sine law, the instantaneous value of the current is given by the expression

$$i = I_m \sin \theta,$$

where i is the instantaneous value of the current, I_m the maximum value and θ the angle passed through from the commencement when the current was zero and beginning to rise in the positive direction.

The average value,[1] taken over half a period, is

$$\begin{aligned} I_{av} &= I_m \times \text{average value of } \sin \theta \\ &= I_m \times \frac{2}{\pi} \\ &= 0{\cdot}637\ I_m. \end{aligned}$$

The value $\frac{2}{\pi} = 0{\cdot}637$ can be verified by drawing a sine wave and measuring the average height graphically.

The R.M.S. value of a sine wave can be determined by drawing the curve of $\sin^2 \theta$ and measuring the average height graphically. The square root of this quantity is the R.M.S. value and is equal to

[1] The average height of a sine wave is given by the area divided by the base and is equal to

$$\begin{aligned} &\frac{1}{\pi}\int_0^\pi \sin \theta\, d\theta \\ &= \frac{1}{\pi}\Big[-\cos \theta\Big]_0^\pi \\ &= \frac{1}{\pi}\Big[(-\cos \pi) - (-\cos 0)\Big] \\ &= \frac{1}{\pi}\Big[(+1) - (-1)\Big] \\ &= \frac{2}{\pi}. \end{aligned}$$

$$\frac{1}{\sqrt{2}} \times \text{maximum value}$$

$$= 0{\cdot}707 \times \text{maximum value.}$$[1]

Form Factor.—It has been shown that

$$I_{av} = \frac{2}{\pi} I_m \text{ or } I_m = \frac{\pi}{2} I_{av}$$

and
$$I = \frac{1}{\sqrt{2}} I_m \text{ or } I_m = \sqrt{2}I.$$

Therefore
$$\sqrt{2}I = \frac{\pi}{2} I_{av}$$

and
$$\frac{I}{I_{av}} = \frac{\pi}{2\sqrt{2}} = 1{\cdot}11.$$

The ratio $\frac{\text{R.M.S. value}}{\text{average value}}$ is called the *Form Factor* and is equal to 1·11 in the case of a sine wave. The student will find it a useful exercise to draw some particular wave form and determine its form factor. As an example, this will be done for the wave shown in Fig. 9. In order to find the average value, divide the base into, say, twelve sections and take the ordinate in the middle of each section. A fruitful source of error is to take the ordinates at 0, 1, 2, etc., dividing by 13. Another source of error is to take the ordinates at 1, 2, 3, etc., whereas they should be taken at 0·5, 1·5, 2·5, etc. The form factor can now be worked out in the form of a table as follows :—

[1] The R.M.S. value can be determined in a simple manner by means of the calculus, thus :—

Average value of $\sin^2\theta = \frac{1}{2\pi}\int_0^{2\pi} \sin^2\theta \, d\theta$

and R.M.S. value
$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi} \sin^2\theta \, d\theta}$$
$$= \sqrt{\frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right) d\theta}$$
$$= \sqrt{\frac{1}{2\pi}\left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_0^{2\pi}}$$
$$= \sqrt{\frac{1}{2\pi}\left[\frac{2\pi}{2} - 0 - 0 + 0\right]}$$
$$= \frac{1}{\sqrt{2}} = 0{\cdot}707.$$

No. of Ordinate.	Value of Ordinate.	Square of Ordinate.
1	1	1
2	2	4
3	2	4
4	2	4
5	2	4
6	1	1
Sum total	10	18
Average	1·67	3
R.M.S................	—	$\sqrt{3} = 1{\cdot}73$

$$\text{Form factor} = \frac{\text{R.M.S.}}{\text{average}} = \frac{1{\cdot}73}{1{\cdot}67} = 1{\cdot}04.$$

If twice as many mid-ordinates are taken, the values of the first and second are 0·5 and 1·5 respectively. The sum total of the squares of the ordinates is now 37, and the R.M.S. value works out to $\sqrt{\frac{37}{12}} = 1{\cdot}76$. The form factor now becomes 1·06 (to 3 figures). The mathematically correct value is 1·058 (to 4 figures). This shows the importance of taking as large a number of ordinates as possible.

As a general rule, the wave may be assumed to be flatter than a sine wave if the form factor is less than 1·11 and more peaked if this ratio is greater than 1·11.

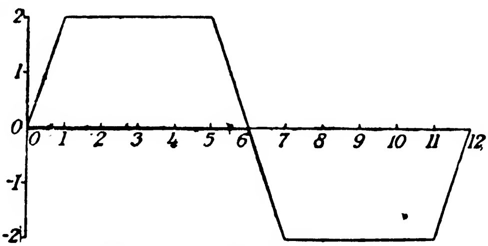


Fig. 9.—Example of Wave Form.

Examples.

(1) Determine the R.M.S. value of a semicircular wave, the maximum value of which is 100 units.

(2) Calculate the form factor of a triangular wave.

(3) The following values refer to a certain current wave. Draw the wave and calculate its R.M.S. and average values.

θ	0°	20°	40°	60°	80°	100°	120°	140°	160°	180°
i	0	17·1	17·1	30·1	69·5	115·6	132·7	103·7	47·2	0

CHAPTER III

INDUCTANCE, REACTANCE AND IMPEDANCE

Back E.M.F. set up due to Alternating Current.—When a varying current flows in a circuit it sets up a varying flux which is in phase with the current, *i.e.* it goes through similar phases at the same instant, such as, for example, the maximum value. The form of the circuit can be so arranged that the flux is negligible in strength although it can never be eliminated entirely. If a portion of a circuit be wound in the form of a coil, then the flux set up by each turn, due to the passage of the current, can be made to link with all the other turns and thus set up a considerable number of linkages, the effect being increased considerably if an iron core is placed in the coil. Such a coil is known as a choking coil, and may be either air-cored or iron-cored.

The alternating current causes the flux to alternate with it, and consequently the linkages vary from instant to instant. This sets up an E.M.F. in the coil the direction of which is such as to oppose the change, the effect being somewhat akin to inertia. If the flux set up is proportional to the current, then the back E.M.F. which is generated is proportional to the rate at which the current is changing and is numerically equal to

$$-\frac{\text{linkages set up per ampere} \times \text{change of amperes per second}}{10^8} \text{ volts.}$$

Inductance.—The number of linkages set up per ampere is a definite quantity for a given coil, and if the core be made of a non-magnetic material it is strictly a constant, but if an iron core be introduced this quantity varies to some extent, depending upon the saturation of the iron. For example, if the current be increased to ten times its former value, the total linkages may only be increased seven times, and thus the linkages per ampere will only be 0·7 times their former value. In the majority of elementary calculations, however, this quantity is assumed to be a constant for a given coil, and is known as the *inductance* or the *coefficient of self-induction*, the former term being the one most used at the present time.

On the practical system of units, the unit of inductance is the *Henry*, which is the inductance of a coil setting up 10^8 linkages per ampere. If, therefore, the current varies at the rate of one ampere per second in such a coil, there will be an E.M.F. of one volt set up. The inductance of any other coil, measured in henries, is

$$L = \frac{\text{number of linkages set up per ampere}}{10^8}.$$

As an example, the flux will be determined in a choking coil having 100 turns and an inductance of 0·2 henry, when a current of 5 amperes is passing.

$$\begin{aligned}\text{Linkages per ampere} &= L \times 10^8 \\ &= 0{\cdot}2 \times 10^8 \\ \text{Total linkages} &= 5 \times 0{\cdot}2 \times 10^8 \\ \text{Total flux} &= \frac{5 \times 0{\cdot}2 \times 10^8}{100} \\ &= 10^6 \text{ lines or 1 megaline.}\end{aligned}$$

Rate of Change of Current.—The next thing to determine is the

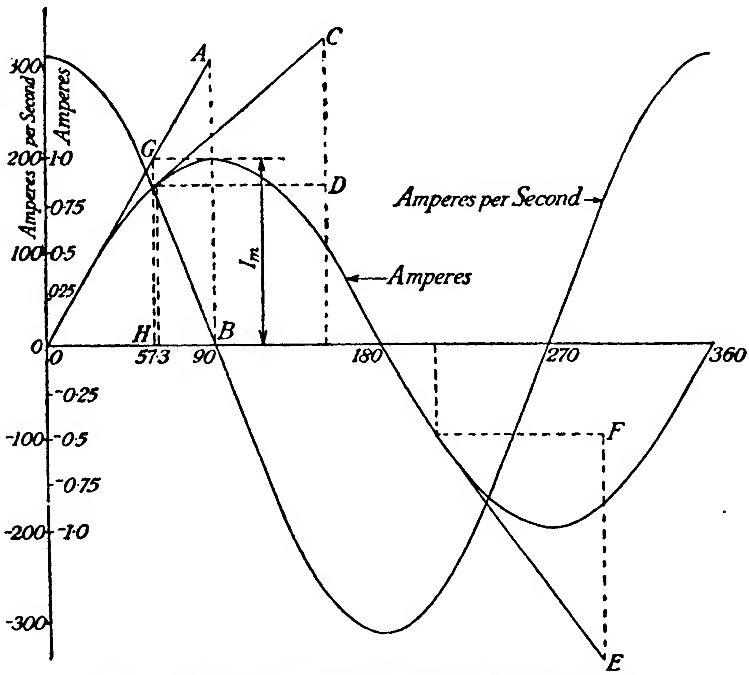


FIG. 10.—Relation between Current and Rate of Change of Current.

rate of change of the current in the circuit. This depends upon three things, the maximum value of the current (I_m), the wave form and the frequency (f). Fig. 10 represents the current curve where the wave form is sinusoidal. The rate of change of the current at any instant is measured by the slope of the curve at that particular point. At 0°, this is equal to AB amperes per 90°. Scaling this off the diagram, it is found to be $1{\cdot}57 \times I_m$ amperes per 90°, or $6{\cdot}28 \times I_m$ amperes per cycle, or $314 \times I_m$ amperes per second, assuming a frequency of 50.

At 60°, the slope of the curve is CD amperes per 90°. Scaling this off the diagram, it is found to be equal to $0{\cdot}785 \times I_m$ amperes per 90°, or $157 \times I_m$ amperes per second. Repeating this again at 210°, the slope is found to be $4 \times 50 \times EF \times I_m$ (= 272) amperes per second, but this time the current is changing in the reverse direction, so that this statement means that the current is decreasing at the rate of 272 amperes per second. Plotting a number of values obtained in this manner, the curve of change of current in amperes per second is obtained. On examination, this curve is seen to be a sine curve also, but is 90° ahead of the current curve. In reality it is a cosine curve. This establishes the fact that if the current varies according to a sine law, the rate of change of current also obeys a sine law. The maximum value of the rate of change curve occurs when the actual value of the current is zero. If a horizontal line is drawn through the vertex of the current curve, it is seen that the maximum slope is GH amperes in OH degrees. On scaling off the diagram, OH will be found to be one radian or 57·3° approximately. Therefore, the maximum value of the rate of change of current is I_m amperes per radian, or $2\pi I_m$ amperes per cycle, since there are 2π radians in 360°. Measured in amperes per second, this is equal to $2\pi f I_m$, where f is the frequency.[1]

To prove that the maximum rate of change of the current is I_m amperes per radian, consider the line OG which is a tangent to the sine curve. For very small angles, the sine of an angle and its radian measure have the same value, so that the tangent OG also represents angle plotted against angle. Taking the maximum height of the sine curve as unity and measuring horizontally in radian measure, it is seen that the tangent OG rises to a height of

[1] This can be determined by the calculus as follows :—

$$i = I_m \sin\theta$$

$$\frac{di}{dt} = \frac{d(I_m \sin\theta)}{dt}$$

$$= I_m \times \frac{d(\sin\theta)}{d\theta} \times \frac{d\theta}{dt}$$

$$= I_m \cos\theta \times 2\pi f.$$

The maximum value of $\frac{di}{dt} = 2\pi f I_m$.

one unit when the base line OH is one radian. Therefore the maximum rate of change of a sine curve is one unit per radian, and if the maximum height be made equal to I_m amperes, then the maximum rate of change is equal to I_m amperes per radian, or $2\pi I_m$ amperes per cycle, or $2\pi f I_m$ amperes per second.

The horizontal scale in Fig. 10 is measured in degrees, corresponding to the angle moved through by a conductor rotating in a magnetic field. Let the angular velocity of this conductor be ω radians per second. Then at the end of t seconds, the conductor will have swept out an angle $\theta = \omega t$ radians. But in one second the total angle moved through is $2\pi f$ radians, where f is the frequency, so that $\omega = 2\pi f$.

Reactance.—It has been shown that a voltage is set up in an alternating current circuit equal to

$$-\frac{\text{linkages set up per ampere}}{10^8} \times \text{(change of amperes per second) volts.}$$

This minus sign denotes the fact that it is a back voltage. The maximum value of this voltage is given by the expression

$$E'_m = -L \times 2\pi f I_m \text{ volts}$$

and the value at any other instant by

$$\begin{aligned} e' &= -2\pi f L I_m \cos \omega t \text{ volts} \\ &= -2\pi f L I_m \sin (\omega t + 90^\circ) \text{ volts.} \end{aligned}$$

This back voltage has to be overcome by the application of an equal and opposite forward voltage having a value

$$e = 2\pi f L I_m \sin (\omega t + 90^\circ) \text{ volts.}$$

This voltage is seen to vary according to a sine law also, but it *is not in phase with the current.* When ωt is 0°, the current is zero; but this voltage has its maximum value at this instant, since $\sin (\omega t + 90^\circ)$ is equal to unity. The voltage is said to *lead* the current by 90°, or the current is said to *lag* behind the voltage by 90°. The R.M.S. value of the applied voltage is obviously equal to $E = 2\pi f L I$, where I is the R.M.S. value of the current.

When the circuit contains nothing but resistance, the voltage is given by the expression

$$E = R \times I.$$

If the circuit has no resistance, but is linked with a magnetic flux, the applied voltage is given by the expression

$$E = 2\pi f L \times I.$$

Thus the quantity $2\pi f L$ can be considered as being in some respects analogous to resistance, inasmuch as the current has to be multiplied by it in order to obtain the voltage. Strictly speaking, it is more in the nature of an E.M.F. than a resistance, but owing to the similarity of the equations $E = RI$ and $E = 2\pi fLI$, it is usually considered as a kind of resistance.

The quantity L is called the *Inductance* and is measured in henries, whilst the quantity $2\pi fL$ is called the *Reactance*[1] and is measured in apparent ohms. They are not true ohms, because they do not result in the production of heat in the circuit, but have the appearance of being ohms because they are the ratio of the volts to the amperes.

Very often the inductance is given in milli-henries, *i.e.* thousandths of a henry, in order to avoid dealing with very small numbers. For example, if $L = 10$ milli-henries and $f = 50$ cycles per second, the reactance is equal to

$$X = 2\pi \times 50 \times \tfrac{10}{1000}$$
$$= 3{\cdot}14 \text{ apparent ohms.}$$

Circuit containing Resistance and Reactance.—The previous example has not taken into consideration the effect of resistance, but it must be borne in mind that it is theoretically impossible to have a circuit without resistance and also without reactance, since wherever there is a current of electricity there must always be a magnetic flux set up linking with the circuit. The reactance may be negligible in a large number of cases, but in very few circuits is the effect of resistance negligible in comparison with the reactance. It is therefore necessary to study the circuit when both are present in appreciable proportions.

Consider a circuit consisting of an ordinary choking coil having a resistance R and an inductance L. In order to force an alternating current of I amperes through this circuit, a voltage must be applied in order to overcome the resistance (RI volts), and, in addition, a further voltage ($2\pi fLI$ volts) must be applied in order to overcome the reactance. The former voltage is in phase with the current, whilst the latter leads it by 90°. At any instant represented by the point P (Fig. 11) there will be required a voltage PQ to overcome the resistance and a voltage PR to overcome the reactance. The total voltage required at this instant is given by $PS = PQ + PR$, the current being PT. Repeating this for all other points throughout the cycle, a curve representing the resultant applied voltage is obtained as shown. The effect is just as if the reactance were concentrated at one part of the circuit and the resistance at another, the two being placed in series. The resultant voltage also follows a sine law, since it is the sum of two sine waves. The maximum value is less than the sum of the maximum values of the two com-

[1] When a circuit possesses capacitance as well as inductance, another term must be added to the expression for the reactance as shown on page 55.

ponents, but is greater than either of them taken separately. Another point to notice is that the resultant voltage is neither exactly in phase with the current, nor does it lead by 90°, being somewhere in between these two limiting values, the angle of lead of the voltage becoming greater the more the reactance predominates over the resistance, and approaching more and more nearly to zero

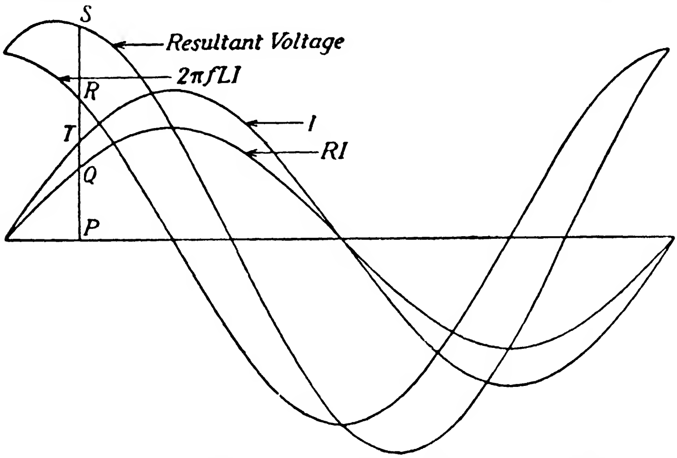


FIG. 11.—Current and Voltage Curves in a Circuit containing Resistance and Inductance.

as the resistance is increased. If the resistance and reactance were equal, the current would lag behind the voltage by 45°.

Phase Difference between Current and Voltage.—The exact angle of phase difference between the current and voltage can be determined in general terms by the application of a little trigonometry. The total voltage required to overcome the effects of resistance and reactance combined is equal to

$$RI_m \sin \omega t + XI_m \cos \omega t$$

where $$X = 2\pi fL.$$

Multiplying and dividing by $\sqrt{R^2 + X^2}$ we get

Voltage required

$$= (\sqrt{R^2 + X^2})\left(\frac{R}{\sqrt{R^2 + X^2}} I_m \sin \omega t + \frac{X}{\sqrt{R^2 + X^2}} I_m \cos \omega t\right).$$

Let $$\frac{X}{R} = \tan \alpha.$$

Then $$\cos \alpha = \frac{R}{\sqrt{R^2 + X^2}} \text{ and } \sin \alpha = \frac{X}{\sqrt{R^2 + X^2}}.$$

Thus the voltage required

$$= (\sqrt{R^2 + X^2})\,(I_m \sin \omega t \cos \alpha + I_m \cos \omega t \sin \alpha)$$

$$= (\sqrt{R^2 + X^2}) \times I_m \sin (\omega t + \alpha).$$

Obviously the curve representing this quantity is a sine curve, not in phase with the current, but leading it by an angle α,

where $$\alpha = \tan^{-1}\frac{X}{R}.$$

Thus α can be calculated for every value of X and R and will always lie between the limits of 0° and 90°.

Instead of saying that the voltage leads the current, it is more usual to say that the current lags behind the voltage, the two statements being synonymous.

Impedance.—The expression representing the current in a simple series circuit is

$$i = I_m \sin \omega t.$$

If nothing but resistance is present the voltage is

$$e = RI_m \sin \omega t.$$

If reactance only is present the voltage is

$$e = XI_m \sin (\omega t + 90°).$$

If both resistance and reactance are present the voltage is

$$e = (\sqrt{R^2 + X^2})\, I_m \sin (\omega t + \alpha).$$

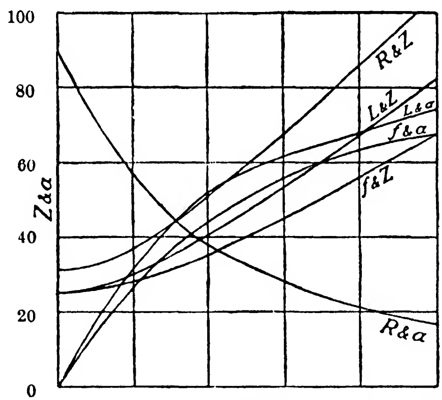


FIG 12.—Effect of Resistance, Frequency and Inductance upon Impedance.

The quantity $\sqrt{R^2 + X^2}$ is called the *Impedance* and obviously depends on both the resistance and reactance of the circuit, although it must be borne in mind that it is not the arithmetic sum of the two. The impedance is the ratio of volts to amperes and will be denoted by the symbol Z.

$$\frac{E}{I} = Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + (2\pi f L)^2}.$$

Fig. 12 shows the way in which the impedance varies with the resistance, frequency and inductance for a particular circuit, and also how the angle of lag depends upon these quantities.

Iron Cored Choking Coils.—The addition of an iron core to a choking coil results in a great increase in its inductance, but, unfortunately, this does not now remain constant, but depends to some extent upon the value of the current flowing. At first, as the current is gradually increased, the flux is proportional to the current and the linkages per ampere remain constant. When the iron becomes saturated, however, the flux increases at a slower and slower rate compared with the current, and the *linkages per ampere* become less. Thus the inductance actually is reduced when the iron gets saturated. This is unfortunate, since it means that the choking coil has a reduced value when on full load at the very time when, presumably, it is desired to give its full value. The

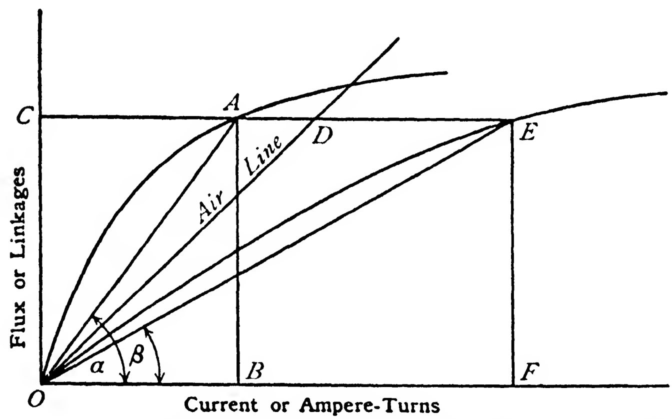


FIG. 13.—Inductance of Iron Cored Choking Coils with and without an Air-gap.

actual amount of the change depends upon the shape of the *B–H* curve of the iron, as illustrated in Fig. 13. The inductance is proportional to the rate of change of linkages (or flux) with current, and its average value is therefore proportional to $\tan \alpha = \frac{AB}{OB}$. (The angle α is not the geometrical angle, since the scales of *AB* and *OB* are different.) As the current is increased, the point *A* moves up the curve, and $\tan \alpha$ decreases. Thus the change in the slope of the line *OA* is an indication of the amount of change in the average inductance. More accurately, the inductance at any point is proportional to the slope of the curve at that point. Thus the inductance is high at the commencement, afterwards dropping to a very low value. As soon as the current drops, the inductance rises again.

This change in inductance is greatly minimized by the introduction of an air-gap in the magnetic circuit. In such a case a certain number of ampere-turns are required to force a given flux through the iron part of the circuit, and an additional number of ampere-turns are required to force this flux across the air-gap. The latter ampere-turns are strictly proportional to the flux, since the permeability of air is constant. It is possible, therefore, to draw a line such as OD, called the air line, to represent the ampere-turns or magnetizing current required to force the flux across the air-gap. The total ampere-turns for a given flux now consist of two components, (1) those required for the iron part of the circuit, $OB = CA$, and (2) those required for the air-gap, CD. The sum of these is CE, thus giving a point, E, on the required curve. This procedure can be repeated for different values of the flux, so enabling the complete curve OE to be constructed. The inductance of the choking coil with an iron core and an air-gap is now given by tan $\beta = \frac{EF}{OF}$, and it is seen that the change in the angle β for different positions of E is much less than in the previous case when no air-gap was included. In fact, β may almost be regarded as constant. Stability has been obtained at the sacrifice of the actual magnitude of the inductance, since tan β is smaller than tan α, but the choking coil is very much smaller than would have been the case had an air core only been used.

Mutual Inductance.—Two circuits are frequently placed so that when a current is passed through one of them the flux set up by it becomes linked with the other. This alternating flux induces a voltage in the second circuit in the same way that a back E.M.F. is induced in an ordinary inductance. In the case of the two circuits, a voltage is induced in either one when the current changes in the other and the circuits are said to possess *Mutual Inductance* (M). This quantity is comparable with inductance, sometimes called *Self-Inductance* (L) to distinguish it, and is measured in the same units.

Two coils are said to possess a mutual inductance of one henry if 10^8 linkages are set up in one coil due to a current of one ampere in the other. If the two coils have the same number of turns, it can be proved that the same current in either coil will set up the same number of linkages with the other coil. If the two coils have different numbers of turns, let these be represented by T_1 and T_2 respectively. When one ampere is passed through the first, T_1 ampere-turns are set up. Assuming the flux to be proportional to the ampere-turns, and that all the flux links with both coils, this flux can be represented by $\Phi_1 = kT_1$, which links with the second circuit and gives rise to $\Phi_1 T_2 = kT_1 T_2$ linkages. If the original current of one ampere had been passed through the second circuit, it would have set up T_2 ampere-turns. The flux corresponding to this traverses the same path as before, and is $\Phi_2 = kT_2$. The linkages produced by this flux in conjunction with the first circuit are $\Phi_2 T_1 = kT_1T_2$, the same as before. The flux set up per ampere is different, but the linkages are the same. The same is true if only a portion of the total induced flux links with both coils. In considering the mutual inductance of a pair of coils, it is therefore immaterial which coil is supplied with current and which has the voltage induced in it, or, in other words, which is the primary and which the secondary.

In an ordinary inductance the induced voltage is 90° out of phase with the current, so that in a pure reactance the applied voltage leads the current by 90°, but in a circuit which possesses mutual inductance with respect to a second circuit, the voltage is 90° out of phase with the *current in the second circuit.* In a general case, the latter may have any phase whatsoever with respect to the current in the first, so that the voltage set up by mutual inductance may have any phase, and in certain cases it may therefore tend to neutralize the effects of self-inductance.

When two coils possess mutual inductance, there is also a certain proportion of flux which links with one coil and not with the other, so that the two circuits each possess self-inductance as well.

Relation between Self- and Mutual Inductance.—If two coils, each possessing appreciable self-inductance, are placed near to one another, all or a portion of the flux set up by one coil may link with the turns of the other coil. This is the flux associated with mutual inductance. The remainder of the flux, linking only with the coil that produces it, gives rise to self-inductance only. These fluxes are represented diagrammatically in Fig. 14.

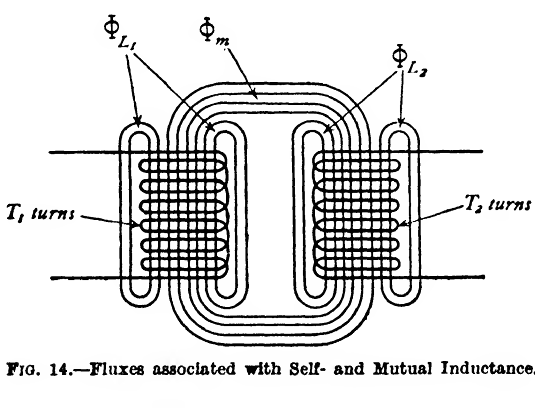


FIG. 14.—Fluxes associated with Self- and Mutual Inductance.

Consider first the case where the whole of the flux links with all the turns of both coils. If no current is flowing in the second coil (assumed to be on open-circuit), the current necessary to set up the flux, Φ_m, is I_1 amperes flowing through the first coil. This coil therefore possesses (by definition) a self-inductance of $L_1 = \frac{\Phi_m T_1}{I_1 \times 10^8}$ henries.

Similarly, if the first coil be open-circuited and a current I_2 passed through the second coil of such a magnitude as to set up the same flux Φ_m, then $L_2 = \frac{\Phi_m T_2}{I_2 \times 10^8}$ henries.

Again by definition, the mutual inductance, M, is equal to $M = \frac{\Phi_m T_2}{I_1 \times 10^8}$ henries and also $M = \frac{\Phi_m T_1}{I_2 \times 10^8}$ henries.

Multiplying these two equations together, we get

$$M^2 = \frac{\Phi_m T_2}{I_1 \times 10^8} \times \frac{\Phi_m T_1}{I_2 \times 10^8}$$
$$= \frac{\Phi_m T_1}{I_1 \times 10^8} \times \frac{\Phi_m T_2}{I_2 \times 10^8}$$
$$= L_1 L_2,$$

and
$$M = \sqrt{L_1 L_2}.$$

Consider next the more general case where only a part of the flux set up by one coil links with the turns of the other coil.

The flux set up by $I_1 T_1$ ampere-turns in the first coil is (see Fig. 14) $\Phi_m + \Phi_{L_1}$. Here, Φ_m is the flux mutually linking both coils, whilst Φ_{L_1} is that portion of the flux which only links with the turns of the winding producing it. This flux may be called the *primary leakage flux.*

In the same way the flux $\Phi_m + \Phi_{L_2}$ is set up by $I_2 T_2$ ampere-turns in the second coil, Φ_m being the same mutual flux as before, and Φ_{L_2} being the *secondary leakage flux.*

With the second coil on open-circuit, the first coil possesses a self-inductance of

$$L_1 = \frac{\Phi_m T_1}{I_1 \times 10^8} + \frac{\Phi_{L_1} T_1}{I_1 \times 10^8}$$
$$= \frac{\Phi_m T_1}{I_1 \times 10^8} + L_{s_1},$$

where L_{s_1} is the primary *leakage self-inductance.*

Similarly, the second coil possesses a self-inductance of

$$L_2 = \frac{\Phi_m T_2}{I_2 \times 10^8} + \frac{\Phi_{L_2} T_2}{I_2 \times 10^8}$$
$$= \frac{\Phi_m T_2}{I_2 \times 10^8} + L_{s_2},$$

where L_{s_2} is the secondary *leakage self-inductance.*

The mutual inductance, M, can be written

$$M = \frac{\Phi_m T_2}{I_1 \times 10^8} \text{ or } \frac{\Phi_m T_1}{I_2 \times 10^8}.$$

Therefore
$$M^2 = \frac{\Phi_m T_2}{I_1 \times 10^8} \times \frac{\Phi_m T_1}{I_2 \times 10^8}$$
$$= \frac{\Phi_m T_1}{I_1 \times 10^8} \times \frac{\Phi_m T_2}{I_2 \times 10^8}$$
$$= (L_1 - L_{s_1}) \times (L_2 - L_{s_2})$$
and
$$M = \sqrt{(L_1 - L_{s_1}) \times (L_2 - L_{s_2})}.$$

The above expression can be written
$$M = k\sqrt{L_1 L_2},$$
where k is a constant called the *coupling factor* and whose value lies between the limits of unity and zero.

Skin Effect.—When an alternating current passes through a conductor it does not distribute itself uniformly throughout the cross-section, but tends to concentrate itself in those portions of the conductor which are situated nearest the surface. This is called the *skin effect*, and in certain cases may be of very appreciable magnitude. In order to see the reason for this effect, consider the case of a solid conductor of circular cross-section, and imagine that it is replaced by a large number of small conductors in parallel, the total cross-sectional area being unchanged. This bundle of small conductors must be bunched together so that they occupy the same space as the original conductor, each one carrying a small fraction of the total current. The flux set up by a surface element of current will link with the whole conductor, but some of the flux set up by an internal element of current will not extend to the surface of the conductor. Thus the surface portions of the conductor will be linked with less flux than the more central portions and will have less inductance per unit of cross-sectional area. This naturally leads to an unequal distribution of the current, the density of which gradually diminishes as the distance from the surface of the conductor increases. The current tends to avoid the central portions of the conductor. However, the watts lost in a particular conductor for a given current are always greater for a non-uniform than for a uniform distribution of current. To demonstrate this, take the case of a conductor having a resistance of R ohms and carrying a current of I amperes and suppose that this conductor be divided up into n equal parallel filaments. The resistance of one of these filaments is nR ohms, and for a uniform distribution of the current the total watts lost would be
$$n \times \left(\frac{I}{n}\right)^2 nR = I^2R \text{ watts.}$$
Next, consider the case of a non-uniform distribution where each one of half the filaments carries a current of $\left(\frac{I}{n} + I'\right)$ amperes,

whilst each of the other half carries a current of $\left(\frac{I}{n} - I'\right)$ amperes. The total current is I amperes the same as before, but the total watts dissipated in the non-uniform case would be

$$\frac{n}{2} \times \left(\frac{I}{n} + I'\right)^2 nR + \frac{n}{2}\left(\frac{I}{n} - I'\right)^2 nR$$

$$= \frac{n^2R}{2}\left(2\frac{I^2}{n^2} + 2I'^2\right)$$

$$= I^2R + n^2I'^2R \text{ watts.}$$

Thus the watts lost in a conductor due to ohmic resistance depend upon the distribution of the current and are a minimum when the distribution is uniform. If no disturbing factors entered into the case the current would naturally distribute itself uniformly, but when this condition is destroyed an increased power is necessary to drive the current through a given conductor. But if the watts are still considered as being equal to I^2R, then R must be given a higher value, due to the lack of constancy of the current density. The effect is dependent on the frequency, the effective resistance increasing as the frequency goes up. It is very marked in the case of iron and steel conductors, because a greater proportion of the flux set up actually confines itself to the conductor due to its magnetic properties. This is the reason why the voltage drop in steel rails is so much larger with alternating than with direct currents. The effect is also very noticeable in radio circuits, the high frequency resistance, as it is termed, often being many times its low frequency value. Indeed, at radio frequencies, the resistance of a conductor is defined as equal to the watts dissipated in it divided by the square of the current.

Analogy of Inductance and Inertia.—A self-induced E.M.F. always opposes any change in the current which produces it. It is therefore impossible for an instantaneous change to take place in the strength of the current, all such changes being necessarily of a gradual character. It takes a definite time for the current to rise after the application of an E.M.F., and a definite time for it to die away after the E.M.F. has ceased to act. The analogy of the inductance of a circuit to the mechanical inertia of a body is suggested. A choking coil may be likened to a flywheel. Neglecting resistance on the one hand and friction on the other, it is seen that a certain amount of energy is required to start the current or the rotation as the case might be. When the full current is flowing or when full speed is attained, no further supply of energy is required to maintain the conditions. If a small amount of resistance or friction be present, a certain amount of power must be supplied continually to overcome these losses. When the flywheel stops, it gives out the whole of its kinetic energy. Similarly, when the current in the choking coil dies down to zero, it gives out all the energy which had previously been supplied to it when the current was started.

Examples.

(1) Determine the instantaneous value of an E.M.F. of 100 volts, after 0·001 second has elapsed. The voltage starts from zero and the frequency is 50.

(2) The current in a circuit obeys the law $i = 100 \sin \omega t$, the frequency being 25 cycles per second. How long will it take for the current to rise to 50 amperes?

(3) An A.C. reaches a maximum value of 100 amperes, the frequency being 50. Determine the rate of change of the current in amperes per second when (*a*) $t = 0{\cdot}00166$ sec., (*b*) $t = 0{\cdot}01$ sec., and (*c*) $t = 0{\cdot}015$ sec.

(4) A choking coil has a resistance of 2 ohms and an inductance of 0·015 henry. What is the p.d. across its ends when a current of 10 amperes passes through the coil at a frequency of 50?

(5) A circuit possesses a resistance of 20 ohms and an inductance of 0·1 henry. Determine the angle of lag of the current (*a*) when the frequency is 50 and (*b*) when it is 25 cycles per second.

CHAPTER IV

VECTOR REPRESENTATION

Vector Representation.—Imagine a point P moving with a circular motion around a point O called the origin, the distance OP

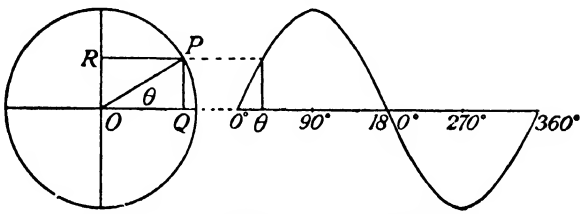


FIG. 15.—Vector Representation of Sine Wave.

being constant. Let $PQ = RO$ be the vertical height of the point P after an angle θ has been swept out. Then

$$\frac{PQ}{OP} = \sin \theta,$$

and since OP is constant, PQ is proportional to $\sin \theta$ and

$$PQ = OP \sin \theta.$$

Thus if OP is made equal to the maximum value of some quantity which is varying according to a sine law, PQ will represent its instantaneous value.

In order to represent a sinusoidal quantity, it is only necessary to draw OP to a scale equal to the maximum value. Then as OP rotates round O as centre, the vertical height of the point P above the horizontal axis will trace out all the instantaneous values throughout the cycle for one complete revolution of OP. Thus in the case of an alternating voltage obeying the law

$$e = E \sin \theta,$$

E is made equal to OP, θ is the angle made by OP with the horizontal, and e is the instantaneous voltage equal to PQ. The line OP is called a *vector*, and voltage is called a vector quantity. By means of this method of representation all that is necessary to specify a voltage fully is a line representing to scale the maximum value. There is no necessity even to draw in the vertical and horizontal axes unless it be desired. Now imagine the vector to rotate in a counter-clockwise direction at such a speed that it makes one complete revolution for every cycle. The vertical projection of *OP* at any instant of time will represent the value of the voltage at the same instant, provided the zero position of *OP* corresponds with the zero value of the voltage. If the whole diagram be imagined to rotate in the reverse direction with a speed of one revolution per cycle, or, if it be preferred, if the observer be imagined to rotate forward with the same speed, the line *OP* will appear to be fixed in position, whilst the axes will appear to be travelling in a clockwise direction at the rate of one revolution per cycle. Although *OP* is now apparently fixed in space, its actual position may be anywhere depending upon the relative times when *OP* and the axis pass the same point. The position of *OP* at any instant with respect to its zero position is called its *phase.*

Any quantity which varies according to a sine law can be represented in this way and is called a vector quantity, but there are only four of these with which the student need concern himself at present, viz., alternating voltage, current, M.M.F., and magnetic flux.

Vector Diagram.—When the various quantities in a circuit are represented after this manner in one diagram, the latter is called a *vector diagram*, or, by some writers, a *clock diagram.* Since these are frequently drawn without the axes being put in, it is necessary to indicate which is the moving end and which the origin. For this purpose it is the convention to draw an arrow head on the moving end, which is always made to rotate in a counter-clockwise direction. In order to differentiate further between voltage, current, ampere-

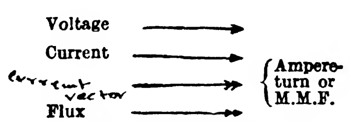


FIG. 16.—Convention of Arrow Heads.

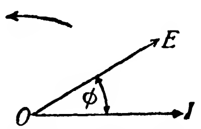


FIG. 17.—Simple Vector Diagram.

turn and flux vectors, different types of arrow heads will be used throughout this book as indicated in Fig. 16.

Fig. 17 shows an example of a simple vector diagram where the current and voltage are represented simultaneously, the current lagging behind the voltage by an angle ϕ which remains constant. *OI* is drawn to a scale of amperes and *OE* to a scale of volts, using R.M.S. values.

Vector Sum of Two Quantities.—Suppose there are, in a particular instance, two alternating voltages acting in series, one leading the other by a fixed angle α [see Fig. 18 (*a*)]. When the second vector has moved through $\theta°$ from the start, the first will have moved through $(\theta + \alpha)°$. At this moment the instantaneous value of E_1 is the vertical projection ON, that of E_2 being the vertical projection OP. The resultant instantaneous value is $ON + OP = OR$. Similarly, the resultant horizontal projection is $OM + OQ = OS$. The vector giving OR and OS as its vertical and horizontal projections at this instant is OE_r, which is obviously the diagonal of the parallelogram the sides of which consist of OE_1 and OE_2. The corresponding sine curves are drawn to the right of Fig. 18 (*a*) and serve to emphasise the relationship existing between the vector representation and the graphical representation. A vector diagram

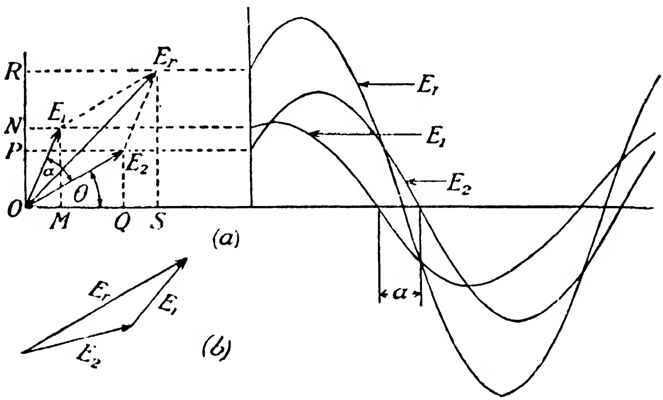


FIG. 18.—Vector Sum of Two Voltages.

of this kind is frequently drawn as shown in Fig. 18 (*b*) for the sake of convenience, just as in drawing a diagram for the parallelogram of forces. Thus voltages, and, of course, currents and fluxes, can be added vectorially just the same as forces or other vector quantities.

Resolution of Vectors.—Vectors can be resolved along any two axes in a similar way to which they are compounded, the majority of cases where this is done requiring a resolution into two components mutually at right angles. Take, for example, the case shown in Fig. 19 where (*a*) the current and (*b*) the voltage are resolved into two components, one in phase with the voltage and current respectively and the other 90° out of phase. The component in phase is called the active component, whilst that out of phase by 90° is called the reactive component there being, as will be shown later, no net transference of power associated with it. All the power in the circuit must be associated, therefore, with the other component, which is called the active component. Thus both currents and voltages can be split up into active and reactive currents and voltages. Take, for example, the case shown in Fig. 19 (*a*). The total current is split up into an active current of magnitude $I \cos \phi$ and a reactive current of magnitude $I \sin \phi$, the angle ϕ being fixed.

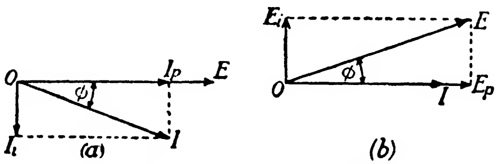


FIG. 19.—Resolution of Vectors.

Since the scales to which the vectors are drawn are arbitrarily chosen, the lengths of the vectors may be made to represent the R.M.S. values of the current and voltage, instead of the maximum values; it merely means a different scale.

Vector Diagrams of Simple Series Circuit.—In the case of a circuit consisting of a simple resistance, the vector diagram would be as shown in Fig. 20 (*a*), the current being in phase with the voltage, whilst in the case of a simple reactance the vector diagram would be as shown in Fig. 20 (*b*), the current lagging 90° behind the voltage. The actual inclination of the lines does not matter; it is the angle between the various components which is definite. It must be borne in mind that the voltage here represented is only the voltage overcoming resistance or reactance, as the case may be, and does not in any way refer to the total E.M.F. acting in the circuit.

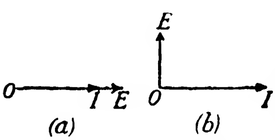


FIG. 20.—Vector Diagrams of Simple Series Circuit.

In the case of a circuit containing resistance and reactance in series, the voltage over the combination must be sufficient to overcome both effects, *i.e.* the vectorial sum of RI and $2\pi fLI$ as shown in Fig. 21. From this diagram it is obvious that

$$\begin{aligned} E_r &= \sqrt{(RI)^2 + (2\pi fLI)^2} \\ &= I\sqrt{R^2 + (2\pi fL)^2} \\ &= IZ, \end{aligned}$$

where Z is the impedance and is equal to $\sqrt{R^2 + (2\pi fL)^2}$.

Impedance Diagram.—In Fig. 21 we have a right-angled triangle the sides of which are equal to RI, XI and ZI, X being the reactance and equal to $2\pi fL$. The value of I is the same in all three expressions, since it is a series circuit, and thus another triangle can be constructed, similar to the first, by dividing each side by the current

I. This new triangle is shown in Fig. 22 and is called the impedance diagram. Since it is similar in shape to the vector diagram of voltages and currents, the angle ϕ also represents the angle of lag in the circuit. Thus $\frac{R}{Z} = \cos\phi$ and $\frac{X}{R} = \tan\phi$. The angle of lag of the current behind the voltage is therefore given by $\tan^{-1}\frac{X}{R}$.

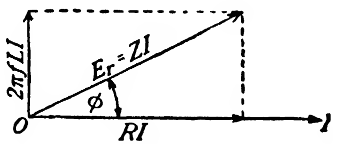


FIG. 21.—Vector Diagram of Circuit containing Resistance and Reactance in Series.

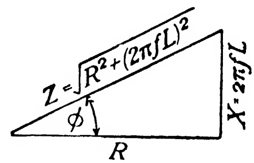


FIG. 22.—Impedance Diagram.

Two Impedances in Series.—In the case where there are two impedances in series, each consisting of resistance and reactance in different proportions, the vector diagram takes the form shown in Fig. 23, the resultant voltage being given by Z_rI. If every line in the diagram be divided by the current, an impedance diagram is obtained, similar in every geometrical respect to the vector diagram. In order to determine the value of two impedances in series, it is necessary to know the relative amounts of resistance and reactance in each. The resultant impedance can also be calculated by means of the various trigonometrical relations, instead of using the graphical method adopted above.

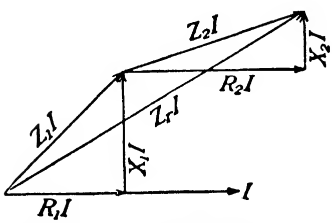


FIG. 23.—Vector Diagram of Two Impedances in Series.

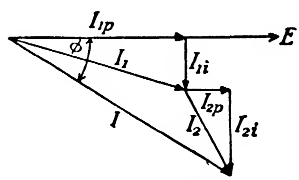


FIG. 24.—Vector Diagram of Two Impedances in Parallel.

Two Impedances in Parallel.—In calculating the resultant value of two impedances in parallel, a vector diagram of currents is drawn with reference to the applied voltage. In Fig. 24 each current is shown split up into its active, and reactive components, I_{1p} and I_{2p} being in phase with the voltage, whilst I_{1i} and I_{2i} are lagging by 90°. The resultant current is given by I, the total active current by $I_{1p} + I_{2p}$, and the total reactive current by $I_{1i} + I_{2i}$. The angle of lag of the total current is given by

$$\phi = \tan^{-1}\left(\frac{I_{1i} + I_{2i}}{I_{1p} + I_{2p}}\right).$$

The resultant impedance is given by $\frac{E}{I}$, where I is the resultant current.

Admittance.—The reciprocal of impedance is called admittance and hence

$$\text{Current} = \text{Admittance} \times \text{Voltage},$$

or

$$I = YE,$$

where Y is the admittance in mhos.

When dealing with a number of circuits in parallel, it is usually best to work from the point of view of the various admittances, adding them together to get the total admittance. This addition must be performed vectorially. Having determined the total admittance of the circuit, the resultant impedance is found by taking the reciprocal.

Summarizing the above, the combined impedance is the vector sum of the component impedances in series circuits, whilst the combined admittance is the vector sum of the component admittances in parallel circuits.

Conductance and Susceptance.—Just as impedance can be resolved into two components at right angles to one another, so can the admittance be split up in the same way. The component in phase with the E.M.F. is called the *conductance*, G, and corresponds to the resistance, whilst the reactive component is called the *susceptance*, B, and corresponds to the reactance. The unit in which these new quantities are measured is the *mho*, and they can be represented graphically in a triangle as shown in Fig. 25, which should be compared with Fig. 22. From this it is seen that

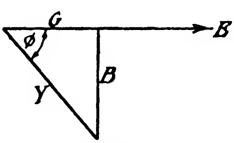


FIG. 25.—Admittance Triangle.

$$(\text{Admittance})^2 = (\text{Conductance})^2 + (\text{Susceptance})^2$$

or

$$Y^2 = G^2 + B^2.$$

Also

$$G = Y \cos\phi = \frac{1}{Z} \times \frac{R}{Z} = \frac{R}{Z^2} = \frac{R}{R^2 + X^2}.$$

and

$$B = Y \sin\phi = \frac{1}{Z} \times \frac{X}{Z} = \frac{X}{Z^2} = \frac{X}{R^2 + X^2}.$$

Numerical Example.—An example, containing both impedances in series and in parallel, is illustrated in Fig. 26. The impedance of the $R = 20$, $X = 30$ branch is 36·0 apparent ohms, and its admittance is consequently $\frac{1}{36 \cdot 0} = 0 \cdot 0278$, whilst $\tan \alpha = \frac{30}{20}$ and $\alpha = 56 \cdot 3°$. The impedance of the $R = 5$, $X = 60$ branch is 60·2 apparent ohms and its admittance is

$$\frac{1}{60 \cdot 2} = 0 \cdot 0166.$$

$\text{Tan } \beta = \frac{60}{5}$ and $\beta = 85 \cdot 2°$. The conductance, G, is

$$0 \cdot 0278 \cos 56 \cdot 3° + 0 \cdot 0166 \cos 85 \cdot 2°$$
$$= 0 \cdot 0167.$$

The susceptance, B, is

$$0 \cdot 0278 \sin 56 \cdot 3° + 0 \cdot 0166 \sin 85 \cdot 2°$$
$$= 0 \cdot 0397.$$

The resultant admittance is therefore

$$\sqrt{0 \cdot 0167^2 + 0 \cdot 0397^2}$$
$$= 0 \cdot 0430,$$

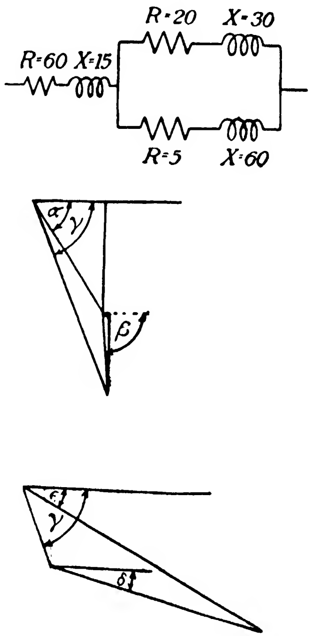


FIG. 26.—Admittance and Impedance Diagrams.

corresponding to an impedance of $\frac{1}{0 \cdot 0430} = 23 \cdot 3$ apparent ohms, whilst the angle γ is

$$\tan^{-1} \frac{0 \cdot 0397}{0 \cdot 0167}$$
$$= 67 \cdot 2°.$$

Next consider the impedance diagram, adding the resultant impedance already obtained to that in series with it. A line is drawn at 67·2° below the horizontal to represent the impedance of 23·3 apparent ohms. The impedance line for the $R = 60$, $X = 15$ branch is inclined at an angle δ to the horizontal such that $\tan \delta = \frac{15}{60}$; thus $\delta = 14 \cdot 0°$. The resultant horizontal component in the impedance diagram, or the equivalent resistance, is

$$23 \cdot 3 \cos 67 \cdot 2° + 60$$
$$= 69 \cdot 0.$$

The resultant vertical component, or the equivalent reactance, is

$$23{\cdot}3 \sin 67{\cdot}2^\circ + 15$$
$$= 36{\cdot}5.$$

The resultant impedance is therefore

$$\sqrt{69{\cdot}0^2 + 36{\cdot}5^2}$$
$$= 78{\cdot}1 \text{ apparent ohms.}$$

The angle ϵ is

$$\tan^{-1}\frac{36{\cdot}5}{69{\cdot}0}$$
$$= 27{\cdot}9^\circ.$$

These results could, of course, have been obtained by scaling off the diagram, but unless great care is taken as to accuracy it is preferable to work out the figures by calculation.

In most cases it is easier to work out problems by means of vector diagrams rather than by the more tedious method of drawing the various sine curves. It must be remembered, however, that the vector diagram is based upon the assumption that the various quantities concerned obey the simple sine law.

Examples.

(1) Two E.M.F.'s each of 100 volts are 60° out of phase and act on a circuit possessing a resistance and reactance of 8 and 6 ohms respectively. Determine the magnitude of the current.

(2) Two impedances each of 100 ohms are connected in series. Their resistances are 10 and 90 ohms respectively. Determine the combined impedance.

(3) A choking coil having an impedance of 25 ohms and a resistance of 15 ohms is connected in parallel with a non-inductive resistance of 10 ohms. Determine the total admittance, conductance and susceptance.

(4) An admittance of 0·2 mho is connected in parallel with a pure reactance, the susceptance of which is 0·15 mho. The combined admittance is 0·314 mho. What is the magnitude of the resistance in the circuit?

(5) Determine the combined impedance of a resistance R and a reactance X connected in parallel.

CHAPTER V

POWER AND POWER FACTOR

Power in a Circuit.—The power developed in a circuit at a given instant of time is equal to the product of the instantaneous values of the current and voltage. If the current and voltage are obeying a simple sine law, it follows that the magnitude of the power developed varies from instant to instant. The simplest case to consider is that of a circuit containing nothing but resistance, the current being in phase with the voltage. The power curve (see Fig. 27) is obtained by multiplying the instantaneous values of current and voltage thoughout the cycle. It will be noticed that this power curve never falls below the zero line, although it touches it at the moments when the current and voltage are zero. Furthermore, the watt curve obeys a sine law with a displaced axis, the frequency being double that of the current or voltage. This can be shown mathematically as follows. Let the voltage and current at any instant be represented by $E_m \sin \omega t$ and $I_m \sin \omega t$ respectively. The expression for the watts is therefore

$$E_m I_m \sin^2 \omega t = E_m I_m \left(\tfrac{1}{2} - \tfrac{1}{2} \cos 2\omega t\right).$$

Thus the power consists of two components, viz., $\tfrac{1}{2}E_m I_m$ and $-\tfrac{1}{2}E_m I_m \cos 2\omega t$. The former component is independent of the particular instant of time, whilst the latter obviously obeys a sine law of double frequency. The watt curve is really a sine wave displaced from the original axis by an amount $\tfrac{1}{2}E_m I_m$. Since the average value of a sine or a cosine taken throughout a complete period is zero, the average value of the power is equal to $\tfrac{1}{2}E_m I_m$, E_m and I_m being the maximum values of the voltage and current respectively, and since the maximum values are equal to $\sqrt{2}$ times the R.M.S. values, the average power becomes

$$\tfrac{1}{2}\left(\sqrt{2}E \times \sqrt{2}I\right) = EI,$$

thus obtaining the same expression as with direct currents.

Power in a Reactive Circuit.—When a circuit contains both resistance and reactance, the current lags behind the voltage by an angle ϕ, the value of which depends upon the relative magnitudes of the resistance and reactance. Figs. 28, 29 and 30 illustrate these conditions, the angle ϕ having values of 30°, 60° and 90° respectively. The watt curves are obtained in the same way as before, viz., by multiplying the instantaneous values of current and voltage. Each of these curves cuts the zero line four times in every cycle, the points occurring when either the current or voltage is zero. Also, during those portions of the cycle where the current and applied voltage are acting in opposite directions the power is negative, which means that during these intervals the circuit actually is sending energy back to the source of supply.

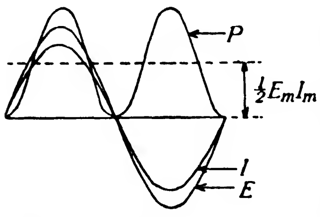


FIG. 27.—Power Curve, $\phi = 0°$.

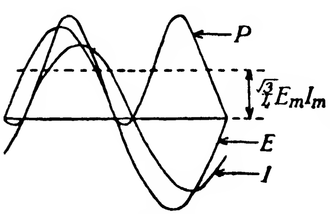


FIG. 28.—Power Curve, $\phi = 30°$.

The expression for the power is given by

$$E_m \sin \omega t \times I_m \sin (\omega t - \phi).$$

Expanding this expression it becomes

$$E_m I_m \sin \omega t \,[\sin \omega t \cos \phi - \cos \omega t \sin \phi]$$
$$= E_m I_m [\sin^2 \omega t \cos \phi - \sin \omega t \cos \omega t \sin \phi]$$
$$= E_m I_m [\tfrac{1}{2} \cos \phi - \tfrac{1}{2} \cos 2\omega t \cos \phi - \tfrac{1}{2} \sin 2\omega t \sin \phi]$$
$$= E_m I_m [\tfrac{1}{2} \cos \phi - \tfrac{1}{2} \cos (2\omega t - \phi)].$$

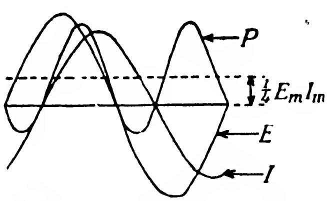


FIG. 29.—Power Curve, $\phi = 60°$.

Since the average value of a cosine, taken throughout a complete period, is zero, the average value of the above expression is

$$\tfrac{1}{2} E_m I_m \cos \phi,$$

or $$EI \cos \phi,$$

considering R.M.S. values.

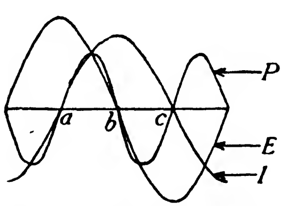


FIG. 30.—Power Curve, $\phi = 90°$.

Power Factor.—Since the value of $\cos \phi$ can never be greater than unity, it follows that the power developed in a circuit can never be greater than EI, although it may be less. Obviously the amount of power developed, when the current and voltage are fixed, depends upon the angle of phase difference between the current and voltage. The factor by which the volt-amperes must be multiplied in order to arrive at the watts is called the *power factor*, the value of which may be anything from unity to zero. Also, it is equal, numerically, to cos ϕ in the case where the simple sine law is obeyed, but where this law is not obeyed it is impossible to speak of the angle of lag with any definite meaning, since it may be different at various parts of the cycle. However, the ratio $\frac{\text{watts}}{\text{volt-amperes}}$ still is called the power factor, and an equivalent sine wave may be substituted for the actual wave, the angle of lag, ϕ, being made such that cos ϕ is equal to the power factor. The product of volts and amperes is termed the apparent power, which is measured in volt-amperes.

Power Curves for a Reactive Circuit.—Figs. 27–30 show the power curves for four circuits where the current lags behind the voltage by 0°, 30°, 60° and 90° respectively, the maximum value of current and voltage being the same in each case, and the sine law obeyed throughout. It will be noticed that as the angle of lag increases the proportion of the watt curve below the zero line increases, indicating that the circuit is returning to the source of supply a larger fraction of the energy supplied to it. It follows, therefore, that the net power absorbed by the circuit is less for the same volts and amperes. This is consistent with the fact that the power factors in the four cases are cos 0°, cos 30°, cos 60° and cos 90°, or numerically, 1·000, 0·866, 0·500 and zero. The particular case of Fig. 30 is worthy of note. The current here is a purely reactive one, lagging by 90°, the power factor is zero and the net amount of power supplied is zero. This does not mean that the value of the power at any instant is zero, but means that the circuit gives back to the source of supply as much energy as it receives, and therefore the net transference of power is zero. This is indicated in the watt curve by the portions below the zero line being equal to the portions above the zero line.

The watt curve is the same shape and size for all values of ϕ, but the amount by which the axis of the curve is displaced from the true zero line depends solely upon the value of ϕ. For example, when ϕ is zero and the power factor consequently unity, the watt curve is wholly on one side of the true zero line, just touching it twice every cycle. When ϕ is 60° and the power factor 0·5, the average height of the power curve is

$$\tfrac{1}{2}E_m I_m \cos 60° = \tfrac{1}{4}E_m I_m,$$

this being the amount by which the axis of the curve is displaced from the true zero line. The maximum height of the curve in this case is therefore

$$\tfrac{1}{4}E_m I_m + \tfrac{1}{2}E_m I_m = \tfrac{3}{4}E_m I_m,$$

and the curve falls below the true zero line by an amount $\tfrac{1}{4}E_m I_m$.

Power Curves for Non-sinusoidal Wave Form.—When the wave forms of the current and voltage are not sinusoidal the shape of the power curve reflects these irregularities, being obtained as before by the multiplication of the instantaneous values of current and voltage. The average height of this curve represents the average value of the power. When the voltage wave form contains irregularities the current wave form will usually differ in shape, as shown in Fig. 31, which illustrates a typical non-sinusoidal case.

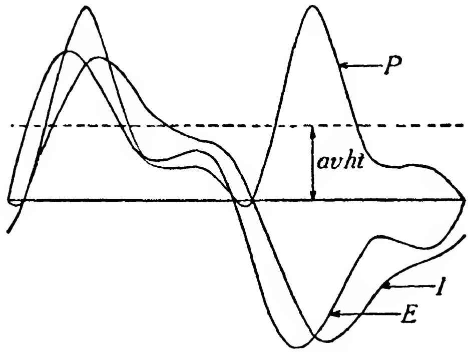


FIG. 31.—Power Curve for Non-sinusoidal Wave Form.

Measurements of Power by Means of a Wattmeter.—The simplest method of measuring power is by means of a wattmeter (see page 151). A wattmeter has two elements, one of which acts like, and is connected as, an ammeter, being called the *current coil*, whilst the other acts like, and is connected as, a voltmeter, being called the *voltage coil.* Fig. 32 illustrates the method of connecting a wattmeter so as to measure the power in a circuit.

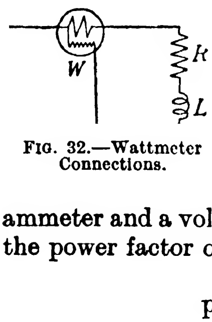


FIG. 32.—Wattmeter Connections.

The wattmeter, used in conjunction with an ammeter and a voltmeter, presents the simplest method of measuring the power factor of a circuit, for

$$\text{power factor} = \frac{\text{watts}}{\text{volts} \times \text{amperes}}.$$

Three Voltmeter Method of Measuring Power.—This is an instructive method of measuring the power in a circuit without the use of a wattmeter. Suppose that it is required to measure the power absorbed by a partially inductive resistance. A non-inductive resistance of somewhere about the same value is chosen, connected in series with the unknown inductive resistance, and an alternating E.M.F. applied to the circuit. The voltage drop over each component part and over the whole circuit must be measured, and also the current flowing. Fig. 33 shows the diagram of connections of the circuit and explains the symbols used. It also shows a vector diagram of the circuit. The power absorbed in the unknown inductive resistance is obviously

$$P_1 = AV_1 \cos\phi,$$

ϕ being the angle of lag in the inductive resistance.

From the geometry of the figure we have

$$\begin{aligned} V_3^2 &= V_2^2 + V_1^2 + 2V_2V_1\cos\phi \\ &= V_2^2 + V_1^2 + \frac{2V_2P_1}{A}. \end{aligned}$$

Therefore

$$\begin{aligned} P_1 &= \frac{A}{2V_2}(V_3^2 - V_2^2 - V_1^2) \\ &= \frac{1}{2R}(V_3^2 - V_2^2 - V_1^2). \end{aligned}$$

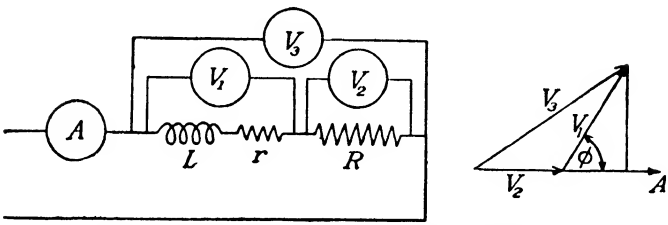


FIG. 33.—Three Voltmeter Method of Measuring Power.

If the value of the resistance R is known there is no necessity for the ammeter A, although usually it is desirable to have it in circuit to avoid overheating. The reason why the resistance R should be chosen so as to be of the same order of magnitude as the unknown resistance is to get the vectors V_1 and V_2 approximating to one another in magnitude and thereby getting the best experimental accuracy with given instruments.

With the addition of a wattmeter to measure directly the power consumed by the unknown inductive resistance, this forms a very valuable experiment for students to perform in the laboratory. By varying the value of the resistance R and adjusting the applied voltage so as to keep the current constant, a number of observations of the power absorbed by the unknown inductive resistance can be obtained. These should, of course, all agree with one another. The resistance of the unknown inductive resistance can be obtained by dividing the power absorbed by the square of the current. The value of the resistance obtained in this manner may be larger than the true ohmic resistance, since there may be an appreciable power expended due to iron loss consisting of hysteresis and eddy currents. The reactance can also be determined from the observations already made. Referring to the vector diagram in Fig. 33, it is seen that the voltage overcoming the reactance is given by

$$AX = \sqrt{V_1^2 - (Ar)^2},$$

where r is the equivalent resistance determined as shown above. The reactance can therefore be determined as follows :—

$$X = \frac{\sqrt{V_1^2 - (Ar)^2}}{A}$$

$$= \sqrt{\left(\frac{1}{A}\right)^2 - r^2}.$$

The power factor of the inductive circuit can be obtained without the use of the ammeter at all, and without knowing the value of the resistance R, for

$$\text{power factor} = \frac{P_1}{AV_1}$$

$$= \frac{A}{2V_2}(V_3^2 - V_2^2 - V_1^2) \times \frac{1}{AV_1}$$

$$= \frac{(V_3^2 - V_2^2 - V_1^2)}{2V_1V_2}.$$

Three Ammeter Method of Measuring Power.—The principle involved in this method of measuring power is very similar to that in the three voltmeter method, the difference being that, instead of having two component voltages and their vector sum, there are two component currents and their vector sum. For this purpose it is necessary to have two parallel circuits and to measure the currents in the two branches as well as the total current. Fig. 34

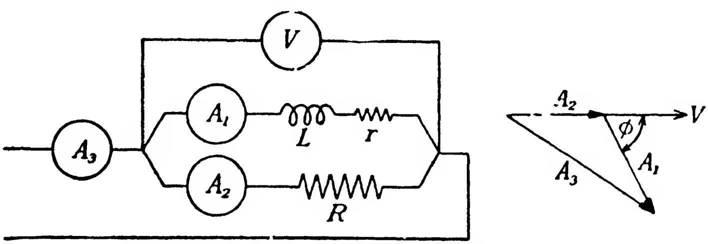


FIG. 34.—Three Ammeter Method of Measuring Power.

shows a diagram of the circuit and the necessary instruments, together with a vector diagram showing how the various quantities are related together. As before, L, r is an unknown inductive resistance, whilst R is a non-inductive resistance, which, if its value is known, renders the voltmeter V unnecessary; otherwise R must be determined from observations of V and A_2.

From the geometry of the vector diagram we get

$$A_3^2 = A_2^2 + A_1^2 + 2A_2A_1 \cos\phi,$$

ϕ being the angle of lag in the branch inductive circuit. The power in the inductive resistance is given by

$$P_1 = A_1 V \cos\phi.$$

Therefore
$$A_3^2 = A_2^2 + A_1^2 + \frac{2A_2P_1}{V},$$

and
$$P_1 = \frac{V}{2A_2}(A_3^2 - A_2^2 - A_1^2)$$
$$= \frac{R}{2}(A_3^2 - A_2^2 - A_1^2),$$

whilst the power factor of the inductive circuit is given by

$$\frac{P_1}{A_1V} = \frac{V}{2A_2}(A_3^2 - A_2^2 - A_1^2) \times \frac{1}{A_1V}$$
$$= \frac{(A_3^2 - A_2^2 - A_1^2)}{2A_1A_2}$$

and may be determined without the use of the voltmeter at all.

As in the previous case, it is desirable to have the magnitudes of A_1 and A_2 more or less of the same order. Knowing the watts, volts and amperes in the inductive resistance, it is a simple matter to determine the reactance and the equivalent resistance.

The student can check this reasoning experimentally by inserting a wattmeter in the circuit so as to measure the current through and the volts across the inductive resistance. By keeping the voltage constant and varying the value of R, a number of observations can be made, all of which should, of course, give the same value for the power absorbed in the partially inductive resistance.

A point of practical importance in carrying out either of these tests is that the instrument losses must be negligible, *i.e.* the voltmeter must take a negligible current and the ammeters must absorb a negligible voltage. A disadvantage of both methods is that the results depend upon the differences of the squares of the observed quantities and this tends to magnify the experimental errors.

Active and Reactive Components.—Very frequently it is desirable to consider the current as split up into two components, one in phase with the voltage and the other 90° out of phase, or, as it is sometimes termed, in *quadrature* with the voltage. In certain cases it is the voltage which is thus resolved, but the method of treatment is just the same.

Fig. 35 shows the vector diagram of a circuit where the current is lagging by an angle ϕ behind the voltage. The current is resolved into two components as indicated above. The magnitude of that component of the current in phase with the voltage is $I_p = I \cos \phi$, whilst the magnitude of the component in quadrature with the voltage is $I_i = I \sin \phi$. When the former component is multiplied by the voltage it gives the total power in the circuit and is called the active component, because all the watts in the circuit are associated with it. There is no net power associated with that component of the current which is in quadrature with the voltage, for the expression for the instantaneous value of the power, as far as this component is concerned, is,

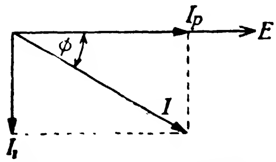


FIG. 35.—Active and Reactive Currents.

$$E_m \sin \omega t \times I_m \sin \phi \sin (\omega t - 90°)$$
$$= - \tfrac{1}{2}E_m I_m \sin 2\omega t \sin \phi.$$

Since the average value of $\sin 2\omega t$ is zero, there is no net power. This component of the current is therefore called the reactive, or wattless, component.

Energy Stored in the Magnetic Medium.—Whilst there is no continued expenditure of energy involved in the passage of a current through an inductance, yet there is a quite definite, though small, amount of energy expended in building up the field, the whole of which is returned to the circuit when the field is destroyed. This energy is stored up in the magnetic medium and can be calculated in the following way.

Consider a theoretical circuit having an inductance of L henries and absolutely no resistance, and that an E.M.F. is applied to it in such a manner that the current increases at a constant rate. Then

$$E = LI',$$

where E is the instantaneous value of the applied E.M.F. and I' is the rate of growth of the current measured in amperes per second (see page 16). Since both L and I' are assumed constant, the applied E.M.F. must also be constant. In other words, a constant voltage suddenly applied to a circuit containing only inductance would produce a uniformly increasing current, this going on indefinitely. Since the resistance cannot be eliminated in actual practice, the current tends towards a limiting maximum value $\frac{E}{R}$. Returning to the theoretical case with no resistance, suppose that the voltage be applied until the current reaches the value of I

amperes. The time necessary for this is $\frac{I}{I'}$ seconds and the average power expended during this interval of time is

$$E \times \tfrac{1}{2}I,$$

since E is constant throughout and $\tfrac{1}{2}I$ is the average current.

The total energy supplied is

$$\tfrac{1}{2}EI \times \frac{I}{I'}$$
$$= \tfrac{1}{2}LI'I \times \frac{I}{I'}$$
$$= \tfrac{1}{2}LI^2 \text{ joules.}$$[1]

The total energy stored up in the magnetic field is independent of the rate at which the field was produced, and so it does not matter whether it was produced at a uniform rate or not.

As an example, the energy stored up in the magnetic field of a circuit having an inductance of 0·2 henry, when a current of 5 amperes is flowing, is

$$\tfrac{1}{2} \times 0{\cdot}2 \times 5^2 = 2{\cdot}5 \text{ joules.}$$

The whole of this energy is restored to the circuit when the magnetic field is destroyed.

If a sinusoidal E.M.F. be applied to a pure reactance having an inductance of L henries the resulting current will lag by 90°. The magnetic field is completely built up in the interval between the times when the current is zero and at its maximum value of I_m amperes. During this interval the voltage is decreasing from the maximum value of E_m volts to zero. The expression for the power, taking the time when the current is zero as a starting point, is

$$E_m \sin(\omega t + 90°) \times I_m \sin \omega t$$
$$= 2\pi f L I_m^2 \sin \omega t \sin(\omega t + 90°)$$
$$= \pi f L I_m^2 \sin 2\omega t.$$

[1] This can be determined quite simply by the aid of the calculus, as follows :—

$$e = L\frac{di}{dt}$$
$$ei = p = Li\frac{di}{dt}.$$
$$\int p\,dt = \int Li\,di.$$

Integrating between the limits 0 and I :—

$$\text{Energy} = \tfrac{1}{2}LI^2.$$

The maximum value of the instantaneous power is, therefore,

$$\pi f L I_m^2 \text{ watts,}$$

and hence the average value of the power during the interval taken is

$$\frac{2}{\pi} \times \pi f L I_m^2 \text{ watts (see page 12)}$$
$$= 2 f L I_m^2 \text{ watts.}$$

Since the duration of this interval of time is $\frac{1}{4f}$ seconds, the total energy supplied in building up the field is therefore

$$2 f L I_m^2 \times \frac{1}{4f}$$
$$= \tfrac{1}{2} L I_m^2 \text{ joules.}$$

EXAMPLES.

(1) A coil having an inductance of 0·05 henry and a resistance of 0·5 ohm has a pressure of 100 volts at 50 cycles applied to it. Calculate the power absorbed and the power factor.

(2) An iron cored choking coil has a non-inductive resistance of 3 ohms connected in series with it, the voltage drops being 66 and 50 volts respectively. The applied pressure is 104 volts. What is the power absorbed by the choking coil and what is its power factor?

(3) Two circuits are connected in parallel across A.C. mains at 100 volts. One circuit takes 5 amperes at 0·6 p.f. lagging, whilst the other takes 10 amperes at 0·8 p.f. lagging. Determine the current in the main circuit and the power factor of the arrangement.

(4) A choking coil has a resistance of 5 ohms and a reactance of 12 ohms, and has an E.M.F. of 78 volts (R.M.S.) applied to it at 40 cycles. What is the energy stored in the electromagnetic field at the instant that the applied voltage is 88 volts and increasing in magnitude?

CHAPTER VI

CAPACITANCE AND CONDENSERS

Condensers.—Two conducting bodies separated by a dielectric are said to possess *capacitance* and the combination is called a *condenser*. There are usually a number of plates of conducting material separated by thin sheets of dielectric, alternate plates being connected together electrically so as to form two groups. In this case the areas of the two sets of plates are equal. If a difference of potential be applied to the two conducting bodies, no current actually flows *through* the condenser, unless the insulation be broken down, but the conducting plates become charged. This means that a certain definite quantity of electricity is stored up on the plates, one of which is positively and the other negatively charged, these charges being equal. The amount of charge which a condenser takes in given conditions depends upon its dimensions and the material of the dielectric, being proportional to the area of the conducting plates and inversely proportional to the thickness of the separating dielectric. If a number of condensers be taken, having equal dimensions but with various materials as the dielectric, it will be found that they take different charges for the same potential difference. This is due to a property of the dielectric called its *permittivity*,[1] corresponding in some ways to specific resistance in the case of a conductor. The permittivity of air is taken as unity.

Unit of Capacitance.—The charge which a given condenser stores up is proportional to the charging voltage. Since the charge consists of a quantity of electricity, it is measured in coulombs on the practical system of units. Thus we have

$$Q = CE,$$

where Q is the charge measured in coulombs, E is the charging voltage, and C is some constant. If a particular condenser takes a charge of one coulomb when a potential difference of one volt is applied, the value of the constant C is also unity. Such a condenser is said to possess unit capacitance, and the name of the unit is the *farad*. Thus the capacitance of any condenser is given by the expression

$$C = \frac{Q}{E},$$

[1] Older names for the same property are *dielectric constant* and *specific inductive capacity*.

case of the charge flowing into or out of the condenser as the case may be.

The effect of an alternating E.M.F. on a condenser will now be discussed, a sinusoidal wave form being assumed.

Let the applied E.M.F. be represented by

$$E_m \sin \omega t.$$

As shown on page 16, the maximum rate of change of voltage is E_m volts per radian or $2\pi f E_m$ volts per second, and is positive when the actual value of the voltage is zero and increasing. But since

$$Q = CE,$$

$$\text{Current} = \text{rate of change of } Q$$
$$= C \times \text{rate of change of } E.$$

Therefore

$$I_m = C \times \text{maximum rate of change of } E$$
$$= C \times 2\pi f E_m.$$

Considering R.M.S. values

$$I = 2\pi f C E.$$ [1]

On examining the curves in Fig. 36, which represents the conditions in a capacitive circuit, it is seen that the charging current leads

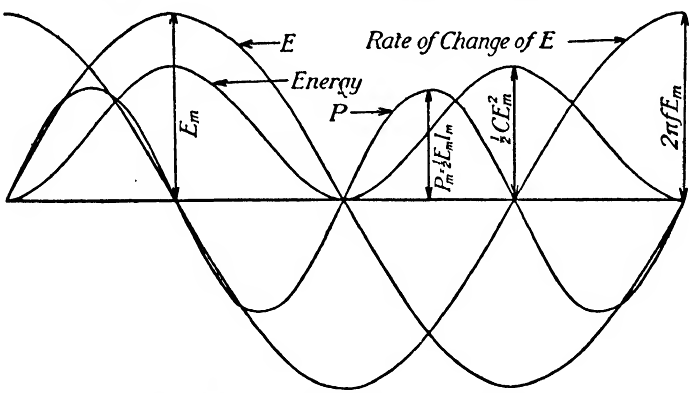


FIG. 36.—Power and Energy Curves for a Condenser Circuit.

[1] The current can be calculated by means of the calculus, thus :—

$$I = \frac{dQ}{dt}$$
$$= C\frac{de}{dt}$$
$$= C\frac{d(E_m \sin \omega t)}{dt}$$
$$= \omega C E_m \cos \omega t$$
$$= 2\pi f C E_m \sin (\omega t + 90°).$$

This shows that the current leads the voltage by 90°.

the voltage by 90°, being exactly opposite in this respect to an inductance. The reason for this is that, in an inductance, the applied voltage is equal and opposite to the back voltage set up, this being proportional to the rate of change of the current, whilst, in a condenser, the current is proportional to the rate of change of voltage.

Physical Meaning of Charging Current.—At the commencement of the cycle, when the value of the voltage is zero, the rate at which the voltage is increasing is a maximum, and consequently the rate at which the charge is increasing is a maximum. The current flowing into the positive plate is, therefore, a maximum at this instant. During the progress of the next quarter of a cycle the rate at which the voltage is increasing continually diminishes and the current flowing into the positive plate continually decreases until, at the moment when the voltage is no longer increasing, the current falls to zero. All this time current has been flowing into the positive plate of the condenser which contains its maximum charge at the moment when the current ceases to flow in. Thus, at the moment of maximum voltage the charge on the plates is a maximum, but the current is zero. As the voltage begins to decrease, the charge also begins to decrease, and this results in an apparent current flowing the other way through the condenser. Really it is the charge flowing out of the plates. The voltage continues to decrease at a greater and greater rate and, consequently, the current continues to increase. This goes on until the voltage has fallen to zero and the condenser is completely discharged. The voltage now begins to rise in the other direction and the complete chain of events is repeated in the next half-cycle. In a complete cycle, the condenser is twice charged and discharged, once in each direction.

Power and Power Factor of Capacitive Circuit.—It has been shown that the current leads the voltage by 90°, and if the voltage is represented by the expression

$$e = E_m \sin \omega t$$

the current is represented by the expression

$$i = I_m \sin (\omega t + 90°).$$

The instantaneous value of the power is represented, therefore, by the expression

$$\begin{aligned} ei &= E_m I_m \sin \omega t \sin (\omega t + 90°) \\ &= \tfrac{1}{2} E_m I_m \sin 2\omega t. \end{aligned}$$

Since the average value of $\sin 2\omega t$, taken throughout a complete cycle, is zero, it follows that the average power supplied to the condenser is zero. The power factor of a circuit containing only capacitance is, therefore, also zero. This can be realized when it is remembered that no heat is produced and no work done. As a matter of fact, in commercial condensers, there is a slight energy loss in the dielectric, but the mechanism of this action is somewhat complicated and lies outside the scope of this book. There is also a minute I^2R loss, due to the fact that the current has to flow over the plates which have a definite ohmic resistance. The result of these two actions is to raise the power factor from zero to a value which usually lies between 0·003 and 0·01. For ordinary purposes, the power factor can be taken as zero.

As in the case of an inductance, although the average power supplied is zero, the power at a given instant may have quite a definite value. The power curve of a condenser is shown in Fig. 36. It is seen that the condenser is receiving charge up to the moment when the voltage attains a maximum and that it is being supplied with energy during this interval. During the next quarter of a cycle, the voltage falls to zero and the condenser is discharged. The power curve shows that during this interval the condenser delivers back to the line the same amount of energy that it previously received, and this chain of events is continually repeated. The fourth curve in Fig. 36 shows the amount of energy stored up in the condenser at different moments throughout the cycle.

Energy of Charge.—In order to determine the energy associated with the charge, assume that a uniformly increasing voltage is applied at the rate of E' volts per second for a period of t seconds. The total voltage applied, E, is equal to $E't$. If the capacitance of the condenser be C farads, then the charge on the plates is equal to CE coulombs. Since the rate of change of voltage is constant the current is constant and equal to

$$I = \frac{Q}{t} = \frac{CE}{t} \text{ amperes.}$$

The average power supplied is

$$\tfrac{1}{2}E \times I$$
$$= \tfrac{1}{2}E \times \frac{CE}{t} \text{ watts.}$$

The total energy supplied is

$$\tfrac{1}{2}E \times \frac{CE}{t} \times t$$
$$= \tfrac{1}{2}CE^2 \text{ joules.}[1]$$

[1] Taking the case when a sinusoidal E.M.F. is applied, this can be proved by means of the calculus as follows :—

Instantaneous power $= ei = \tfrac{1}{2}E_m I_m \sin 2\omega t$

where $\omega = 2\pi f$ and $t =$ time in seconds.

In Fig. 36 the height of the energy curve at the end of a quarter of a cycle is equal to $\frac{1}{2}CE_m^2$ joules. It is immaterial whether the voltage is applied at a uniform rate or not, the energy associated with a given charge being dependent only on the magnitude of the applied voltage and independent of the rate at which it is applied.

When the applied voltage obeys a sine law the instantaneous value of the power is given by

$$\begin{aligned} ei &= \tfrac{1}{2}E_m I_m \sin 2\omega t \\ &= \tfrac{1}{2}E_m \times \omega C E_m \sin 2\omega t \\ &= \tfrac{1}{2}\omega C E_m^2 \sin 2\omega t \text{ watts.} \end{aligned}$$

The maximum value of the power is, therefore,

$$\tfrac{1}{2}\omega C E_m^2 \text{ watts}$$

and the average value, taken over a quarter-cycle, is

$$\frac{2}{\pi} \times \tfrac{1}{2}\omega C E_m^2 = \frac{2}{\pi} \times \pi f C E_m^2 \text{ watts.}$$

The total energy supplied in a quarter of a cycle is

$$\begin{aligned} &\frac{2}{\pi} \times \pi f C E_m^2 \times \frac{1}{4f} \\ &= \tfrac{1}{2}CE_m^2 \text{ joules.} \end{aligned}$$

Dependence of Current on Voltage, Capacitance and Frequency.—If any two of these quantities be kept constant and the third one varied, the current will be directly proportional to the quantity which is varied. Fig. 37 shows the relations which exist in a particular circuit, the data being given in the diagram.

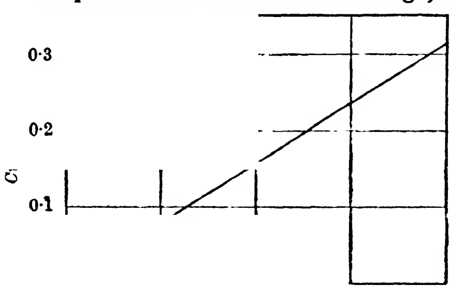


FIG. 37.—Dependence of Current on Voltage, Capacitance and Frequency.

Energy supplied during a quarter of a cycle

$$\begin{aligned} &= \int^{\frac{\pi}{2\omega}} ei dt = \int_0^{\frac{\pi}{2\omega}} \tfrac{1}{2}E_m I_m \sin 2\omega t\, dt \\ &= \left[-\frac{1}{4\omega} E_m I_m \cos 2\omega t \right]_0^{\frac{\pi}{2\omega}} \\ &= \frac{E_m I_m}{2\omega}. \end{aligned}$$

But $\qquad I_m = 2\pi f C E_m = \omega C E_m.$

Therefore energy $\qquad = \tfrac{1}{2}CE_m^2.$

The capacitance of a number of condensers in parallel is the sum of their individual capacitances, but condensers in series, or in cascade, as it is sometimes called, obey a law of the following type :—

$$\frac{1}{\text{resultant capacitance}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

Impedance of a Capacitive Circuit.—The relation between current and voltage in a capacitive circuit is given by

$$I = 2\pi fCE,$$

and since the impedance of a circuit is given by the ratio of volts to amperes, it follows that

$$Z = \frac{E}{I} = \frac{1}{2\pi fC} = \frac{1}{\omega C}.$$

The formula is similar to the corresponding one for a purely inductive circuit with the important exception that $2\pi fC$ comes in the *denominator* whilst $2\pi fL$ comes in the *numerator* of the respective expressions. The quantity $\frac{1}{2\pi fC}$ is called the *capacitive reactance*, to distinguish it from the quantity $2\pi fL$ which should now be called *inductive reactance.*[1]

This relationship enables us to calculate the capacitance of the circuit if the voltage, current and frequency are known; for accurate measurements, however, this method is not to be commended, as the presence of a condenser in the circuit has the effect of magnifying any distortion of the current wave form, and the factor 2π is only correct in the case of a sine wave.

Circuit containing Resistance and Capacitance.—In order to force a current through a circuit containing both resistance and capacitance in series, a voltage must be applied capable of overcoming both the ohmic resistance and the impedance of the condenser. The magnitude of the former component is IR volts and is in phase with the current, whilst the magnitude of the latter component is $\frac{1}{2\pi fC} \times I$ and lags behind the current by 90°. The resultant voltage which has to be applied is the vector sum of these two components and lags behind the current by some angle less than 90°.

Fig. 38 shows graphs of these various quantities and also a

[1] In practice the word *inductive* is often dropped except when it is specially desired to avoid confusion.

vector diagram of the circuit. It is seen that the resultant voltage E is equal to

$$\sqrt{E_R^2 + E_C^2}$$
$$= \sqrt{(RI)^2 + \left(\frac{1}{2\pi fC} \times I\right)^2}$$
$$= I\sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}.$$

The impedance Z is equal to

$$\frac{E}{I} = \sqrt{R^2 + \left(\frac{1}{2\pi fC}\right)^2}.$$

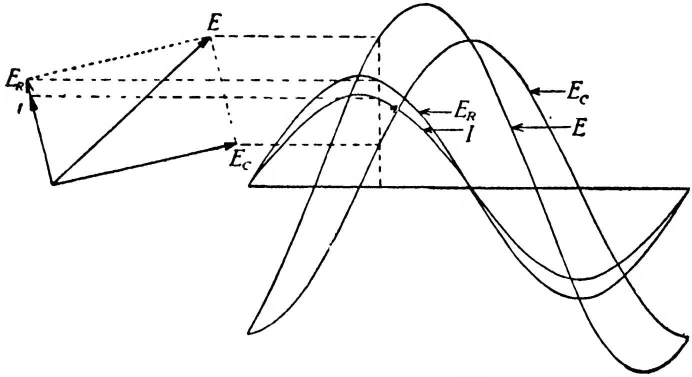


FIG. 38.—Voltage Curves for Circuit Containing Resistance and Capacitance.

As an example, take the case of a circuit consisting of a resistance of 40 ohms in series with an adjustable condenser, the voltage being 100 and the frequency 50. When the value of the condenser is 60 μF the current is

$$I = \frac{100}{\sqrt{40^2 + \left(\frac{10^6}{2\pi \times 50 \times 60}\right)^2}}$$
$$= 1{\cdot}507 \text{ amperes.}$$

The voltage across the resistance is

$$E_R = 40 \times 1{\cdot}507$$
$$= 60{\cdot}3 \text{ volts.}$$

The voltage across the condenser is

$$E_C = \frac{10^6}{2\pi \times 50 \times 60} \times 1{\cdot}507$$
$$= 79{\cdot}9 \text{ volts.}$$

The values of the current and impedance for various values of the capacitance are shown graphically in Fig. 39.

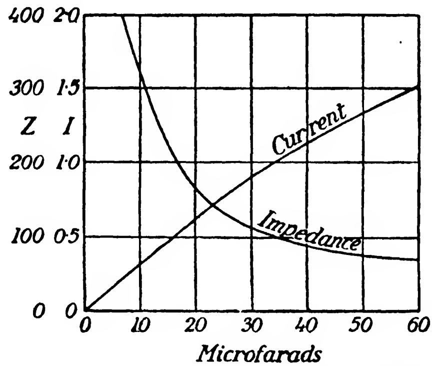


FIG. 39.—Variation of Current and Impedance with Capacitance.

Circuit containing Resistance, Inductance and Capacitance.—When a circuit contains all these three quantities, peculiar conditions are set up due to the various phase relationships. In the case of an inductance the current lags by 90° behind the voltage, whilst in the case of a capacitance the current leads by 90°. If these two are in series the current must have the same phase throughout,

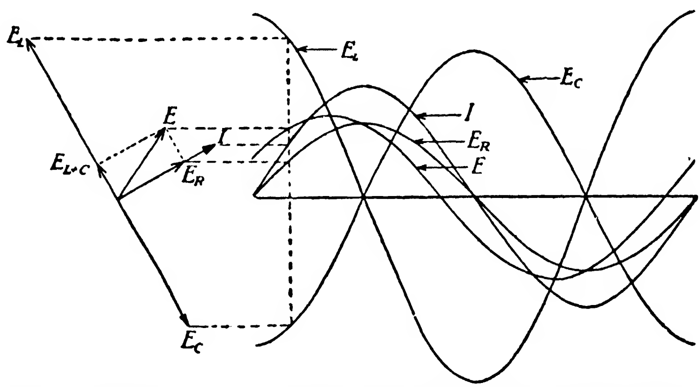


FIG. 40.—Voltage Curves for Circuit containing Resistance, Inductance and Capacitance.

and consequently the voltage must have a phase difference of 180° over these two portions of the circuit. This means that the two voltages are diametrically opposite and the combined voltage over the two is their arithmetical difference. It is quite possible for each of these component voltages to be considerably in excess of the impressed E.M.F. Fig. 40 shows the vector diagram and also the various sine curves for a simple series circuit containing resistance, inductance and capacitance. The resultant voltage may lead or lag behind the current, this depending on whether the inductance or the capacitance predominates. The magnitude of the voltage absorbed by the inductance plus the capacitance is given by

$$E_L - E_C = 2\pi fLI - \frac{1}{2\pi fC} \times I$$
$$= I\left(2\pi fL - \frac{1}{2\pi fC}\right).$$

Combining this with the voltage absorbed by the resistance, we get, for the total applied voltage,

$$E = \sqrt{I^2R^2 + I^2\left(2\pi fL - \frac{1}{2\pi fC}\right)^2}$$
$$= I \times \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}.$$

The impedance of the circuit is therefore given by

$$Z = \frac{E}{I} = \sqrt{R^2 + \left(2\pi fL - \frac{1}{2\pi fC}\right)^2}.$$

If $\frac{1}{2\pi fC}$ is greater than $2\pi fL$, the quantity inside the bracket is negative and this signifies a leading current. However, the square of this quantity is always positive, so that the impedance can never be less than R, no matter what may be the values of $2\pi fL$ and $\frac{1}{2\pi fC}$.

Resonance.—When a circuit contains both inductance and capacitance it is said to possess *resonance.* The effects may be likened to those produced on a pendulum in which considerable vibrations can be set up by the successive application of very small blows, providing these blows are timed correctly. The frequency of the blows corresponds to the frequency of the applied voltage. In the case of the pendulum, if the blows are applied at a different rate the resulting swing will be considerably less, even though the impulses may be of greater magnitude. Similarly, in the electrical case, if the frequency be changed, the current will be considerably reduced. When the inductance and capacitance are in series, resonance is shown by the increased current taken and by the rise in voltage over the choking coil and the condenser when the frequency is adjusted to the correct value. In this case the circuit is said to possess *voltage resonance.*

It is obvious from the expression for the impedance that its minimum value occurs when

$$2\pi fL - \frac{1}{2\pi fC} = 0.$$

Of course, a decrease in the resistance always lowers the impedance, and *vice versa*. The resonance is only affected by the inductive and capacitive portions of the circuit. With a given amount of inductance and capacitance there is one particular value of the frequency which will give maximum resonance. This is obtained when

$$2\pi fL - \frac{1}{2\pi fC} = 0.$$

Then

$$2\pi fL = \frac{1}{2\pi fC},$$

$$(2\pi f)^2 = \frac{1}{LC}$$

and

$$f = \frac{1}{2\pi\sqrt{LC}}.$$

Resonance of a lesser amount will occur at other frequencies.

If a circuit contains a fixed amount of inductance and the frequency is kept constant, then maximum resonance can be obtained by varying the capacitance in the circuit. The necessary conditions are obtained when

$$C = \frac{1}{(2\pi f)^2 L}.$$

As an example, take the case of a series circuit containing a capacitance of 50 μF, an inductance of 0·2 henry and a resistance of 5 ohms. The frequency for maximum resonance is

$$f = \frac{1}{2\pi\sqrt{0{\cdot}2 \times 50 \times 10^{-6}}}$$
$$= 50{\cdot}33 \text{ cycles per second.}$$

At this frequency the impedance is exactly 5 ohms, so that if the voltage were 100 the current would be 20 amperes.

The voltage across the condenser is given by

$$E_C = \frac{I}{2\pi fC}$$
$$= \frac{20 \times 10^6}{2\pi \times 50{\cdot}33 \times 50}$$
$$= 1268 \text{ volts.}$$

The voltage across the inductance is given by

$$\begin{aligned} E_L &= 2\pi f L I \\ &= 2\pi \times 50{\cdot}33 \times 0{\cdot}2 \times 20 \\ &= 1268 \text{ volts.} \end{aligned}$$

If the frequency falls to 49, then the impedance would be

$$Z = \sqrt{5^2 + \left(2\pi \times 49 \times 0{\cdot}2 - \frac{10^6}{2\pi \times 49 \times 50}\right)^2}$$

$= 6{\cdot}02$ apparent ohms.

The current would be

$$I = \frac{E}{Z} = \frac{100}{6{\cdot}02} = 16{\cdot}6 \text{ amperes}$$

and would lead the voltage since $\frac{1}{2\pi f C}$ is now greater than $2\pi f L$.

The voltage now across the condenser is

$$\begin{aligned} E_C &= \frac{16{\cdot}6 \times 10^6}{2\pi \times 49 \times 50} \\ &= 1078 \text{ volts,} \end{aligned}$$

and the voltage across the inductance is

$$\begin{aligned} E_L &= 2\pi \times 49 \times 0{\cdot}2 \times 16{\cdot}6 \\ &= 1022 \text{ volts.} \end{aligned}$$

The voltage across the two combined is

$$\begin{aligned} &1078 - 1022 \\ &= 56 \text{ volts.} \end{aligned}$$

The voltage across the resistance is $16{\cdot}6 \times 5 = 83$ volts.

If the frequency were raised to 51 the impedance would be

$$Z = \sqrt{5^2 + \left(2\pi \times 51 \times 0{\cdot}2 - \frac{10^6}{2\pi \times 51 \times 50}\right)^2}$$

$= 5{\cdot}28$ apparent ohms.

The current would be $\frac{100}{5{\cdot}28} = 18{\cdot}94$ amperes.

A variation of the frequency in either direction brings about a reduction in the current. Fig. 41 shows how the inductive and the capacitive reactances vary with the frequency. The impedance is the difference of these two reactances (neglecting resistance). As the frequency is raised, the impedance falls, becomes zero, and then rises again. The point of resonance occurs when the impedance is zero. At lower frequencies the resultant reactance is capacitive in nature and the current leads, whilst at higher frequencies the resultant reactance is inductive in character and the current lags. At resonance, the voltage across each individual part of the circuit may be very greatly in excess of the total voltage applied.

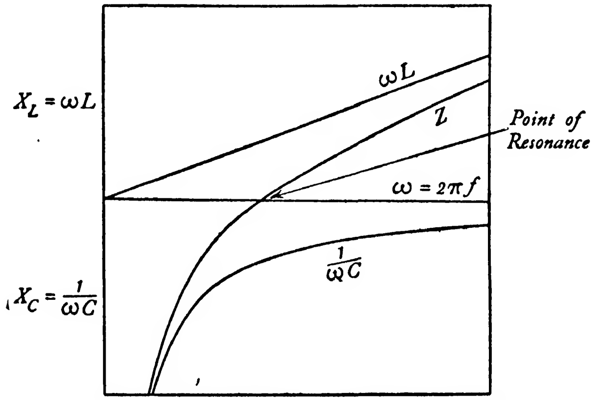


FIG. 41.—Effect of Frequency in Resonating Circuit.

In actual practice, it is impossible to avoid a certain amount of resistance, and so the impedance never really falls to zero. The ratio of the voltage across either the inductance or the capacitance to the total voltage applied to the circuit is called the *step-up* ratio of the circuit.

Current Resonance.—The above example serves as an illustration of voltage resonance in a series circuit. When, however, a condenser is placed in parallel with a choking coil, with a fixed voltage across each, a local circulating current is set up producing what is known as *current resonance.* Fig. 42 illustrates such a circuit. A

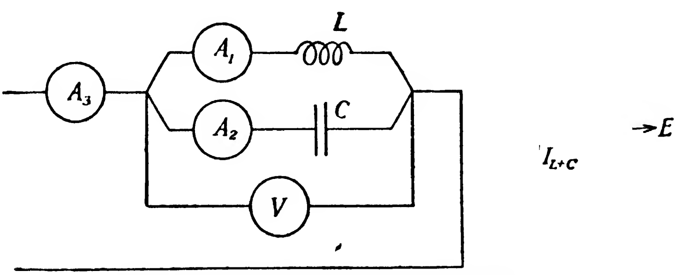


FIG. 42.—Circuit possessing Current Resonance.

voltage, E, applied to the circuit produces, through the inductance, L, a current

$$I_L = \frac{E}{2\pi f L},$$

lagging 90° behind the voltage, and, through the condenser, C, a current

$$I_C = 2\pi fCE,$$

leading the voltage by 90°. The resultant current is the numerical difference of these two components and is given by

$$\begin{aligned} I &= I_L - I_C \\ &= E\left(\frac{1}{2\pi fL} - 2\pi fC\right). \end{aligned}$$

If perfect resonance takes place in an ideal resistanceless circuit, then $I = 0$, and the circuit behaves as a perfect insulator.

Therefore $$\frac{1}{2\pi fL} - 2\pi fC = 0$$

and $$f = \frac{1}{2\pi\sqrt{LC}}$$

as before.

If only partial resonance takes place, *i.e.* if the above equation is not fulfilled, then the current supplied from the source is the difference of the currents in the two branches. Assuming that the current in the inductive branch is the larger, then the difference between this and the main current is

$$I_L - I = I_C.$$

The conditions correspond to a circulating current equal to that in the weaker branch, together with a current from the supply to make up the current in the stronger branch. This external current leads or lags by 90°; the former if the current in the condenser predominates, and the latter if the current in the choking coil predominates.

In practice there is usually an appreciable amount of resistance in the inductive circuit, and an example will be worked out in order to illustrate the procedure in this case. The circuit in the example on series resonance is chosen for this purpose, the condenser being in one branch and the inductance and resistance in series in the other branch. At a frequency of 50·33 the current through the condenser is

$$\begin{aligned} I_C &= 2\pi \times 50{\cdot}33 \times 50 \times 10^{-6} \times 100 \\ &= 1{\cdot}581 \text{ amperes.} \end{aligned}$$

The current through the inductive circuit is

$$\begin{aligned} I_{L+R} &= \frac{100}{\sqrt{5^2 + (2\pi \times 50{\cdot}33 \times 0{\cdot}2)^2}} \\ &= 1{\cdot}576 \text{ amperes.} \end{aligned}$$

inductance is zero, and hence the magnetic field is zero and has no energy stored up in it. A quarter of a cycle later the field has reached its maximum value and has its maximum amount of energy stored up in it. But now the voltage across the condenser has dropped to zero and it is discharged. If perfect resonance is taking place, these two maximum amounts of energy are equal, for

$$2\pi fL = \frac{1}{2\pi fC}.$$

Multiplying each side by I_m^2,

$$\begin{aligned}2\pi fLI_m^2 &= \frac{I_m^2}{2\pi fC}\\ &= \frac{(2\pi fCE_m)^2}{2\pi fC}\\ &= 2\pi fCE_m^2.\end{aligned}$$

Therefore $$\tfrac{1}{2}LI_m^2 = \tfrac{1}{2}CE_m^2.$$

At intermediate points in the cycle there is energy present in both magnetic and electrostatic form, but the total quantity is always the same. For instance, when the voltage has the instantaneous value of

$$e = E_m \sin \omega t,$$

the energy stored up in the condenser is

$$\tfrac{1}{2}Ce^2 = \tfrac{1}{2}CE_m^2 \sin^2 \omega t.$$

The current in the inductance is

$$i = I_m \sin (\omega t - 90°),$$

and the energy stored up in the inductance is

$$\begin{aligned}\tfrac{1}{2}Li^2 &= \tfrac{1}{2}LI_m^2 \sin^2 (\omega t - 90°)\\ &= \tfrac{1}{2}LI_m^2 \cos^2 \omega t.\end{aligned}$$

But since $$\tfrac{1}{2}LI_m^2 = \tfrac{1}{2}CE_m^2$$

$$\tfrac{1}{2}Li^2 = \tfrac{1}{2}CE_m^2 \cos^2 \omega t,$$

and the total energy stored up in both condenser and inductance is, therefore,

$$\begin{aligned}&\tfrac{1}{2}CE_m^2 \sin^2 \omega t + \tfrac{1}{2}CE_m^2 \cos^2 \omega t\\ &= \tfrac{1}{2}CE_m^2 (\sin^2 \omega t + \cos^2 \omega t)\\ &= \tfrac{1}{2}CE_m^2.\end{aligned}$$

After the currents have been once started there is no further supply of power necessary. The condenser in discharging liberates sufficient energy to build up the magnetic field, and this in turn, on its collapse, provides the necessary energy to charge the condenser. If resistance is present in the local circuit, energy is being late the current, the voltage drop over the condenser and the maximum charge in the condenser in micro-coulombs.

(2) A circuit consisting of a resistance of 5 ohms, an inductance of 0·05 henry and a capacitance of 200 μF in series has 100 volts applied to it at a frequency of 50. Calculate the impedance, the current and the voltage across the condenser. Repeat for a frequency of 60.

(3) A circuit contains an inductance of 1·5 henries; what value of condenser will be necessary to bring the current in phase with the volts on a 50 cycle supply?

(4) An inductance of 0·407 henry and a capacitance of 100 μF are connected in series and the frequency adjusted for perfect resonance. The R.M.S. voltage across the capacitance is 600 volts. What is the maximum amount of energy stored in the electromagnetic field of the inductance?

But the angle ACD is also equal to ϕ, CD being a vertical line dropped from C on to AB. Thus

$$AC \times \cos \phi = CD$$

and the power becomes proportional to

$$AB \times CD.$$

Since AB is constant, the power absorbed by the circuit is given by the vertical line CD to a suitable scale of watts. This scale having been determined experimentally for one value of the resistance, the power absorbed for all other values of the resistance can be scaled off from the diagram. This power becomes a maximum

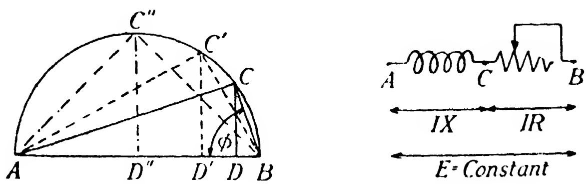


FIG. 45.—Circuit with Constant Reactance.

when the point D is situated at D'' midway along AB, the height $C''D''$ being equal to the radius of the circle. Moreover, at this point the angle of lag is 45° and the power factor is, therefore, $\frac{1}{\sqrt{2}}$ or 0·707. If the current and the voltage be measured under these conditions the power is given by

$$EI \times 0{\cdot}707,$$

and this enables the scale of watts to be settled. The power factor is given by $\frac{CB}{AB}$, and since AB is constant, CB is proportional to the power factor. The scale can be ascertained from the fact that at C'' the angle of lag is 45°, and consequently $C''B$ is 0·707. The total impedance is equal to $\frac{E}{I}$, and hence is proportional to $\frac{1}{AC}$.

If a wattmeter be included in the circuit, and the value of the resistance gradually reduced, the power will first be seen to increase, then attain a maximum value, and finally decrease again, even though the current is increasing throughout. The reason for this is that as the resistance is decreased the current goes up, but the power factor is reduced since the ratio of resistance to reactance is getting less. At first, the current increases at a greater rate than the power factor decreases, the power increasing, whilst beyond a certain point this is reversed and the power begins to decrease again. At the commencement, the current is small, but the power factor is high, whilst later on the current is large, but the power

Again $$AC = AE \cos\alpha + EB$$

$$= EB \times \frac{\cos\alpha}{\sin\alpha - \cos\alpha} + EB$$

$$= EB\left(\frac{\cos\alpha}{\sin\alpha - \cos\alpha} + 1\right)$$

$$= EB \times \frac{\sin\alpha}{\sin\alpha - \cos\alpha}$$

$$= EB \times k,$$

where $$k = \frac{\sin\alpha}{\sin\alpha - \cos\alpha} = \frac{AC}{EB}.$$

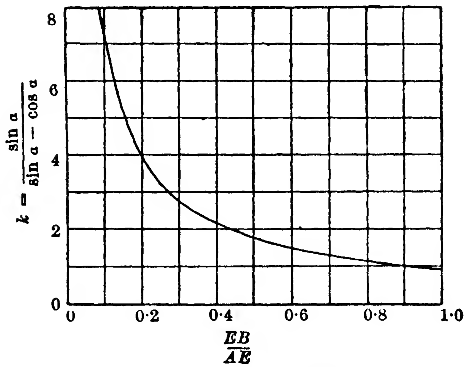


FIG. 47.—Values of $k = \frac{\sin\alpha}{\sin\alpha - \cos\alpha}$.

The quantity $k = \left(\frac{\sin\alpha}{\sin\alpha - \cos\alpha}\right)$ can be determined from the voltage ratio $\frac{EB}{AE}$, and this relationship is shown in Fig. 47. The reactance of the choking coil, therefore, is equal to the external resistance multiplied by the constant k which depends upon the ratio of the voltages over the resistance and choking coil. The observations must, of course, be taken at the point where the wattmeter reading is a maximum.

Circuit containing Capacitance and Resistance.—The circle diagram in this case is similar to the previous ones, with the exception that the voltage over the resistance leads the applied voltage and consequently is drawn from the left end of the semicircle as shown in Fig. 48. Otherwise, this case can be treated in the same way as a pure inductance, since the energy losses in the condenser will be negligible.

Two Impedances in Series.—If a constant impedance be connected in series with a variable impedance, the latter being varied by means of its resistance only, the example resolves itself into a case of two constant reactances, a constant resistance and a variable resistance in series. The circle diagram for this circuit is illustrated in Fig. 49. AB, on which the semicircle ACB is erected, represents

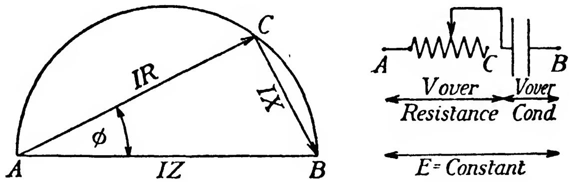


FIG. 48.—Circuit containing Capacitance and Resistance.

the constant applied voltage. This base AB is divided at D so that $\frac{AD}{DB} = \frac{X_1}{X_2}$, where X_1 and X_2 are the reactances of the two impedances, and a semicircle AED is erected on the base AD. Now any line drawn from A, such as AC, will be divided at E so that $\frac{AE}{EC} = \frac{X_1}{X_2}$.

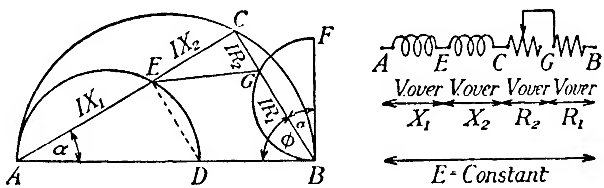


FIG. 49.—Two Impedances in Series.

This can be seen from a consideration of the similar triangles AED and ACB. A vertical line BF is next erected at the point B, its length being such that $\frac{BF}{AD} = \frac{R_1}{X_1}$, where R_1 is the constant resistance of the first impedance. A semicircle BGF is erected on BF intersecting the line CB at G. Since the angle $GBF = \alpha$, therefore

$$GB = BF \cos \alpha$$

and

$$AE = AD \cos \alpha.$$

Therefore

$$\frac{GB}{AE} = \frac{BF}{AD} = \frac{R_1}{X_1}.$$

Since it is a series circuit and AE represents the voltage across X_1, GB will represent the voltage across R_1, and consequently CG will represent the voltage across the variable resistance R_2. The voltage across the variable impedance is given by EG and the voltage across the constant impedance by the vector sum of AE and GB.

Circle Diagram for Two Parallel Circuits.—If a circuit be built up of two parallel branches, each consisting of a resistance and a reactance, but connected so that the resistance of one branch is opposite to the reactance of the other, then a complete circle diagram can be drawn as shown in Fig. 50.

The voltage across the two points C and D is given by the vector

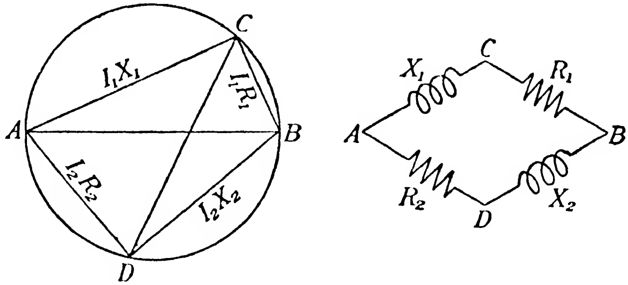


FIG. 50.—Circle Diagram for Two Parallel Circuits.

CD, its maximum value being equal to the applied voltage and occurring whenever C and D are at opposite ends of a diameter. The conditions necessary to make the voltage CD a maximum are that

$$I_1R_1 = I_2R_2$$

and
$$I_1X_1 = I_2X_2$$

or
$$\frac{R_1}{R_2} = \frac{X_1}{X_2}.$$

The phase difference between the voltage CD and the applied voltage can be adjusted to any value by regulating the amounts of resistance and reactance in the circuit. In order to obtain a phase difference of 90°, the resistance of each branch must be made equal to the corresponding reactance.

A similar circle diagram can be constructed for a circuit in which the choking coils have been replaced by condensers, the only difference being that the reactive voltage is drawn to the right in the top half of the diagram, and to the left in the bottom half, since the current leads the voltage in a condenser.

A further case of parallel circuits which may be considered is that wherein one branch consists of a choking coil and a resistance, whilst the other consists of a condenser and a resistance. The circle diagram for such a circuit is illustrated in Fig. 51. A point to be noted is that this time the two resistances are opposite to one another, instead of being opposite to the choking coils. In this case, the conditions necessary to make the voltage CD a maximum are that

$$I_1R_1 = I_2X_2$$

and
$$I_1X_1 = I_2R_2$$

or
$$\frac{R_1}{X_1} = \frac{X_2}{R_2}.$$

If, in addition, $R_1 = X_1$ and, consequently, $R_2 = X_2$, the figure *ACBD* becomes a square and the voltage across *CD* is 90° out of phase with that across *AB*.

Again, if X_1 and X_2 are made equal to $\sqrt{3} \times R_1$ and $\sqrt{3} \times R_2$ respectively, the angles *CAB* and *DAB* will each be 30°, the triangle *ACD* will become equilateral, and the three voltages *AC*, *CD* and *DA* will all be 120° out of phase with each other.

If, however, additional circuits are connected to these various points and current taken off, the relative phases of the voltages are affected and the above relations no longer hold good.

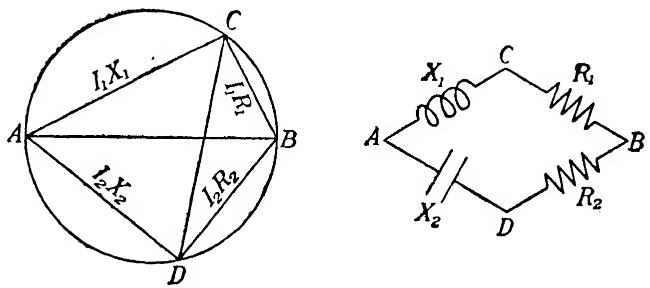


FIG. 51.—Circle Diagram for Two other Parallel Circuits.

Inversion.—In drawing the impedance diagram the line repre senting inductive reactance is drawn upwards, so that the impedance appears to be ahead in phase with respect to the resistance. The applied voltage is equal to *IZ* and assumes the phase indicated by the impedance line. In the corresponding admittance diagram the line representing the susceptance is drawn downwards, so that the admittance appears to lag behind the conductance. The current is equal to *EY* and assumes the phase indicated by the admittance line. Let these two diagrams be drawn together on the same base line as shown in Fig. 52, where OZ_1 is intended to represent an impedance to a certain scale and OY_1 its admittance, the scale of the latter being so chosen that the actual lengths of OZ_1 and OY_1 are the same. The angle above and below the horizontal is the same in each case. Another impedance of different magnitude and with a different phase angle is represented by OZ_2, its admittance being OY_2. The angles are again equal, but this time, since OZ_2 is larger than before, OY_2, being the reciprocal, is smaller than before. The points Y_1, Y_2, etc., are called the inverse points to Z_1, Z_2, etc., and the product $OZ_1 \times OY_1 = OZ_2 \times OY_2$ is called the constant of inversion.

In the case of a circuit containing a variable impedance, the ends of the impedance and admittance lines each trace out a locus, these two being the inverse of each other. If now the admittance lines be drawn on the wrong side of the zero line, they will appear at Y_1', Y_2', etc. Assuming these two loci to be known, as shown in Fig. 52, any line drawn from the origin to the impedance locus will also cut the admittance locus (the line must be produced if necessary) and the intercept will give the value of the admittance directly.

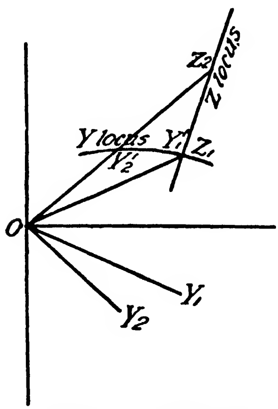


Fig. 52.—Inverse Points.

Simple Inversion Diagram.—Consider the case of a circuit consisting of a known constant resistance and a variable reactance in series. The locus of the impedance is a vertical straight line as shown in Fig. 53, the horizontal line OR representing the constant resistance. It can be shown geometrically that the inverse of such a straight line is any circle passing through O, its centre lying in OR or OR produced. The diameter of the circle can be chosen at will, this merely affecting the admittance scale and altering the constant of inversion. The inverse of the point R is at Y, thus fixing the admittance scale, for $OY = \frac{1}{OR}$ and OR is known. For any value of the

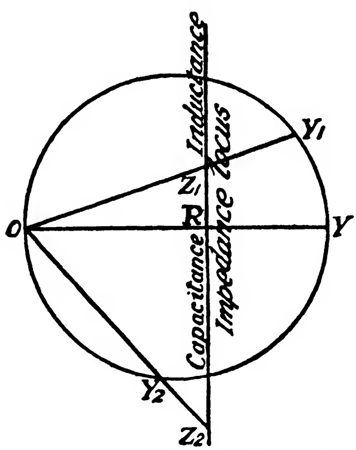


Fig. 53.—Simple Inversion Diagram.

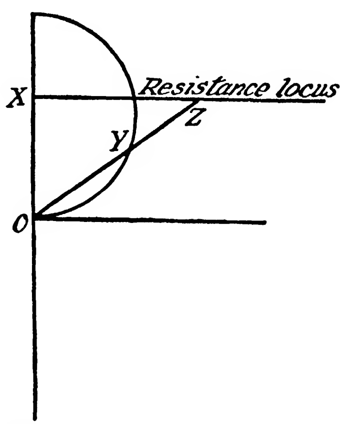


Fig. 54.—Inversion Diagram for Constant Reactance.

reactance such as RZ_1 or RZ_2, giving an impedance OZ_1 or OZ_2, the corresponding admittance is OY_1 or OY_2. Inductive reactances are drawn upwards and capacitive reactances downwards, but it must be remembered that the admittances come on the wrong side of the line. This can be rectified if desired by re-drawing them in their correct position.

Inversion Diagram for Constant Reactance.—The circuit is here supposed to consist of a constant reactance and a variable resistance in series. The constant reactance is represented by OX in Fig. 54 and the resistance locus by a horizontal line to the right commencing at X. The impedance for any resistance is given by a line drawn from O to this locus, the value of the resistance being set off from X. As before, the inverse of the impedance is given by a circle passing through O, but this time its centre lies in OX (or OX produced). The left-hand half of the circle is here omitted, for this would correspond to negative resistances. The admittance corresponding to any impedance OZ is given by OY.

Inversion Diagram for Two Parallel Circuits.—The case will be considered of a constant reactance and variable resistance in series, and a constant impedance in parallel with the combination. The variable impedance is first inverted to obtain its admittance, after which the constant admittance is added (see page 35). The combination is then re-inverted to obtain the resultant impedance.

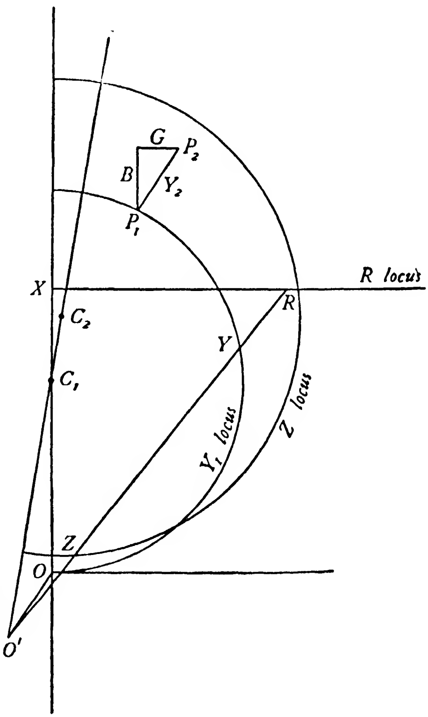


FIG. 55.—Inversion Diagram for Two Parallel Circuits.

First of all the impedance of the variable circuit is inverted in the manner already described, thus obtaining the Y_1 locus shown in Fig. 55. To this must be added the constant admittance Y_2 consisting of a conductance G and a susceptance B. To effect this every point P_1 must be raised through a vertical distance B, and moved to the right by a horizontal distance G, thus obtaining the point P_2. The semicircle may now be re-drawn with every point displaced by a distance P_1P_2, but it is simpler to move the origin back to O' by an equivalent amount. Lines drawn from O' to the Y_1 locus now represent the combined admittance of both branch circuits. Thus for any value of the resistance XR, the resultant admittance is $O'Y$.

The next step is to re-invert this admittance to obtain the final impedance. It can be shown geometrically that the inverse of points lying in a circle lie in another circle, the centres and the origin lying in a straight line. Again, when one circle is the inverse of another circle, a tangent to either circle passing through the origin is also a tangent to the other circle. C_2 is chosen as the centre of the final impedance circle, the scale being different from the original impedance scale, since the size of this new circle has been chosen arbitrarily. The resultant impedance corresponding to a value of the variable resistance equal to XR is given by $O'Z$, to the new scale, the inclination of this line to the horizontal also giving the correct phase angle. It should be emphasized that the " near " point on the Z locus is the inverse of the " far " point on the Y locus and *vice versa*.

Inversion Diagram for Series-parallel Circuits.—In order to obtain the resultant impedance or admittance of a complex circuit with sections in series and in parallel, the admittance of two parallel branches is first obtained, after which this is inverted. The resulting impedance is then added to the portion in series with it. This inversion and re-inversion can be carried out as many times as is necessary.

Examples.

(1) A choking coil and a non-inductive resistance are connected in series across 100 volts, the voltage drops being 65 and 55 volts respectively. Determine the voltage across the choking coil for maximum power.

(2) An inductive reactance of 5 ohms and a capacitive reactance of 8 ohms each have an adjustable resistance in series with them, and the two circuits are connected in parallel across 200 volts. Draw the circle diagram and determine the p.d. between the terminals of the inductance and capacitance which are not connected together, when each resistance is adjusted to 10 ohms.

(3) A choking coil having a resistance and reactance of 3·85 and 9·25 ohms respectively is connected in parallel with its duplicate, but in series with the latter is an adjustable resistance. Draw the inversion diagram and determine the combined impedance when the adjustable resistance is set to 20 ohms.

(4) Draw the inversion diagram for a condenser and resistance in series, the combination being in parallel with an inductance and resistance in series.

(5) An air-cored choking coil has a reactance of 10 ohms and a resistance of 2 ohms. It is connected in series with an adjustable resistance which can be varied between zero and 50 ohms. Draw the inversion diagram of the circuit and plot a curve showing the relation between current and angle of lag. Assume a supply of 100 volts.

CHAPTER VIII

IRON LOSSES

Hysteresis.—The flux density which results, in a specimen of iron, from the application of a given magnetic force depends upon the previous magnetic history of the specimen. The B—H curve lies higher when taken with descending than with ascending values of H, and if the magnetic force be reversed the flux density appears to lag behind the magnetic force. In fact, if the magnetic force be carried through some cycle, finally returning to the starting point, the curve connecting the flux density with the magnetic force will describe a figure which may or may not be a closed loop. If the specimen be taken through the same cycle of events a number of times it will eventually come into what is termed a *cyclical* state, and the complete B—H curve will then be a closed figure known as the *Hysteresis* loop. It is usually determined by magnetizing the iron to equal extents in either direction so that the hysteresis loop is symmetrical about the origin of the curve. This really amounts to an alternating flux with the same maximum in either direction.

In changing the state of magnetization of the iron, some of the molecular magnets are displaced relatively to one another, and this results in a kind of molecular friction, heat being engendered. Hence energy must be supplied to the iron to provide for this wastage. It has been shown that a certain definite quantity of energy must be supplied in building up the magnetic field, but in the case of materials exhibiting hysteresis the whole of this energy is not given back to the source of supply, for when the magnetizing current has dropped to zero some magnetic flux still remains, retaining a certain amount of energy. Hence, during the complete cycle there is a net amount of energy supplied to the specimen, this energy being converted into heat, and it can be shown that the amount thus lost is proportional to the area of the hysteresis loop.

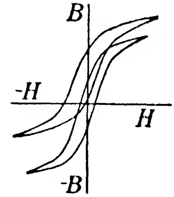


FIG. 56.—Hysteresis Loops.

In order to get the iron into a cyclical state so as to obtain a normal hysteresis loop, it is necessary to take the specimen through the complete cycle about fifty times, always taking care to travel round the loop in the same direction. Fig. 56 shows a typical example of a hysteresis loop when the specimen has been carried through a symmetrical and an unsymmetrical cycle. The hysteresis loss increases appreciably when the cycle is changed from a symmetrical to an unsymmetrical one, indicated by an increase in the area of the loop, although the maximum vertical height remains unaltered.

Steinmetz's Law.—Steinmetz enunciated the law, based on experimental observation, that the area of the hysteresis loop is proportional to the 1·6th power of the maximum flux density, for moderately low flux densities, although this index has a larger value for higher flux densities. This figure is known as the *Steinmetz index.* For very accurate work, a slight correction has to be made, since the hysteresis loop has no appreciable area if the maximum flux density is not carried beyond about 100 lines per sq. cm., and the corrected expression takes the form $(B - 100)^x$, the index x being very slightly modified. The value of this correction becomes apparent when measurements at low flux densities are made, although for most practical purposes it may be neglected. This variation is sometimes allowed for by using different values of the index for different maximum values of the flux density, this rising to as high a value as 4 in extreme cases (see pages 83 and 236).

The hysteresis loss per cycle is proportional to the volume of the iron or steel used, but it has been found that over very wide ranges the loss of energy per cycle is unaffected by change of frequency, from which it follows that the power loss in watts is directly proportional to the frequency. The final expression for the power wasted in hysteresis is

$$P_h = h \times v \times f \times B^x \times 10^{-7} \text{ watts,}$$

where h is a constant known as the *hysteretic constant* and usually has a value ranging from 0·0005 to 0·001 in the case of soft annealed plates, and where x has a value which ranges from 1·6 to about 4. The volume v is measured in c.c.

In determining the area of the hysteresis loop, a first approximation can be made by replacing the loop with a rectangle of equal width and height to the actual loop, as shown in Fig. 57. This is allowable, since the horizontal width of the loop is approximately constant throughout. The area of this rectangle is $2Bw$, and is proportional to B^x, from which it follows that w is proportional to B^{x-1}. This is useful when it is desired to predetermine a particular loop, the loop at another flux density being known. For example, if the maximum value of B be increased by 10 per cent. and x has a value of 1·7, the horizontal width of the loop is increased in the ratio $1{\cdot}1^{0{\cdot}7} \doteqdot 1{\cdot}07$, or by 7 per cent.

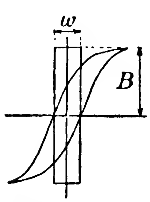


FIG. 57.—Approximate Area of Hysteresis Loop.

The hysteresis loss measured in watts per c.c. is shown graphically in Fig. 58, the bounding lines of the shaded portion representing good and bad specimens of soft annealed iron plates. In order to determine the loss at other frequencies, the values obtained from the curve must be changed in the direct ratio of the frequencies.

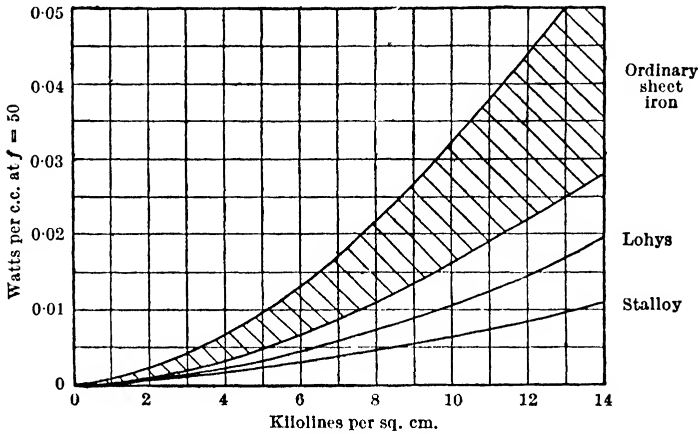


FIG. 58.—Hysteresis Loss in Iron.

Rotating Hysteresis.—When the induced flux in the iron is of a rotating instead of an alternating character, the curves of hysteresis are modified to a very considerable extent as illustrated in Fig. 59.

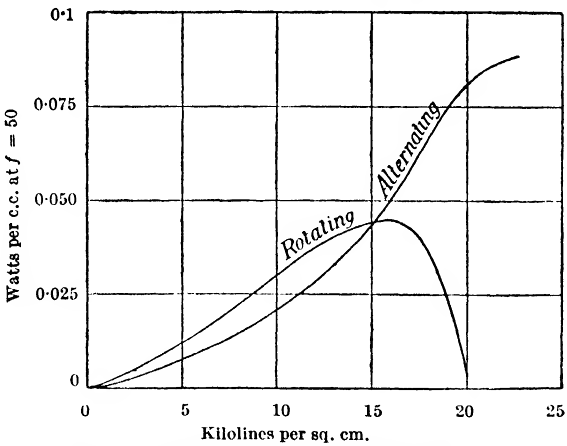


FIG. 59.—Alternating and Rotating Hysteresis.

A striking peculiarity about rotating hysteresis is that as the flux density is gradually raised the hysteresis suddenly disappears in the neighbourhood of $B = 20{,}000$. In certain cases it may therefore be desirable actually to increase the flux density in order to bring about a reduction of the hysteresis loss, although the exciting ampere-turns may be considerably increased thereby.

Ageing of Iron.—It has been determined experimentally that the hysteresis loss in annealed iron stampings increases with lapse of time, this increase being as much as 50 per cent. in some cases of very old iron. The temperature at which the iron is worked is a very important factor, since this ageing, as it is called, only becomes of such a magnitude if the working temperature is allowed to exceed about 85° C. Consequently, in modern apparatus efforts are made to avoid higher temperatures than this. A suggested comparison of different brands of iron with respect to their ageing properties is the percentage increase of hysteresis loss caused by keeping the sample at a temperature of 100° C. for a period of 600 hours. With ordinary iron stampings, the ageing, measured on this basis, should be below 15 per cent. It can be reduced by suddenly cooling the plates from a red heat, whilst unannealed iron plates show scarcely any signs of ageing at all.

A special silicon-iron alloy containing about 3½ per cent. of silicon, known as *stalloy*, has a very much lower hysteresis loss than pure iron, and is largely used for certain purposes.

It has been observed that mechanical pressure also increases the hysteresis loss to an appreciable extent, so that it is desirable that excessive pressures should not be used in the building up of iron cores, although a certain amount is absolutely indispensable from the constructional point of view.

Eddy Currents.—When an alternating magnetic flux follows an iron path, small but definite E.M.F.'s are set up in the iron itself, due to the fact that it is an electrical conductor being cut by lines of force. These E.M.F.'s act upon closed electrical circuits in the iron, and set up local circulating currents known as *Foucault* or *Eddy Currents* and give rise to a loss of power, since these currents flow through paths of definite resistance. The energy thus dissipated reappears as heat in the same way as the hysteresis loss, but must be supplied from an external source in the first place.

The lamination of the iron should be carried out in such a plane that the thickness of the plate corresponds to the length of the conductor which is having an E.M.F. induced in it, this being proportional to the thickness of the laminations. But the resistance of the mean path of the eddy currents is inversely proportional to the thickness of the plates as well, so that the watts lost per lamination, $\left(\frac{E^2}{R}\right)$, are proportional to the cube of the thickness. However, the total number of laminations in a given volume of iron is inversely proportional to the thickness, so that finally the watts per c.c. are proportional to the square of the thickness. They are obviously proportional also to the total volume of iron used.

The induced E.M.F. is proportional to the maximum value of a sine wave is 1·11. The total R.M.S. voltage induced in the whole winding is

$$E = 4{\cdot}44 \times \Phi f T \times 10^{-8} \text{ volts},$$

and, neglecting IR drop, this is equal to the applied pressure. If the wave form is not sinusoidal the R.M.S. voltage is equal to

$$E = 4k\Phi f T \times 10^{-8} \text{ volts},$$

where k is the form factor. Knowing the number of turns, frequency and voltage, the flux can be calculated as follows :—

$$\Phi = \frac{E \times 10^8}{4{\cdot}44 f T} \text{ lines.}$$

In cases where the IR drop is not negligible, it must be subtracted vectorially from the applied voltage.

By applying different voltages different fluxes are obtained, and the corresponding densities can be determined by dividing by the cross section of the iron. The values obtained really constitute the tips of successive hysteresis loops. If the B—H curve were a straight line, the magnetizing ampere-turns would be proportional to the flux density at any instant and the wave form of the magnetizing current would be similar to that of the flux and voltage, but this is not so in the case of iron and steel, and hence the current and voltage wave forms are not similar. The actual determination of the current wave form will be dealt with in Chapter IX.

In order to make accurate determinations of the B—H curve, the maximum values of the current and voltage should be obtained, since these correspond to the maximum value of the flux density, this being rendered necessary by the change of wave form.

Measurement of Iron Loss.—In order to measure the iron loss in a specimen, all that is necessary is to measure the input to such a ring or square as is described above. This is done most conveniently by means of a wattmeter (see page 40). The magnetizing component of the current does not cause any loss of power, since it is in quadrature with the voltage, and an allowance can be made for the I^2R and instrument losses. Since the power factor will be very low, care should be taken to obtain a wattmeter which will read accurately on a low power factor. It is also desirable that the E.M.F. wave form should be as nearly as possible a sine wave.

The great disadvantage of the ring-shaped specimen is the fact that it must be hand wound, but on the other hand any shape which allows a former wound coil to be slipped into position must contain one or more magnetic joints, and these exert a considerable effect upon the magnetic reluctance. If, however, it is the core loss which it is desired to measure, this will not be of much moment, since, although the magnetizing current will be increased, the watts lost will remain the same. For this reason, therefore, straight specimens built up into the form of a square are largely used.

Epstein's Iron Testing Apparatus.—In this apparatus, the sample laminations are built up into four cores, which are arranged in the form of a square, as shown in Fig. 60. The dimensions of the laminations are 500 mm. by 30 mm., whilst the cores are built up to a thickness of about 25 mm., having a mass of 2·5 kilogrammes

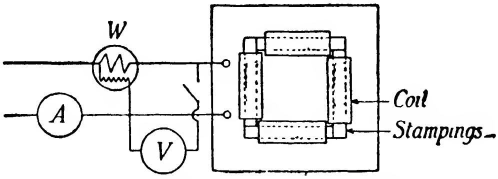


FIG. 60.—Epstein Iron Testing Apparatus.

each. The individual laminations are insulated from each other by thin sheets of paper, whilst at each corner, where a butt joint is formed, a thin layer of paper or press-spahn is placed so as to avoid any additional eddy current loss which might be caused by the short-circuiting of adjacent stampings. The four cores are held in position by means of wooden clamps, one of which is placed at each corner.

Each core carries a winding which consists of 150 turns in series, the total cross section of the wire being 14 mm.^{2}. This is wound upon a press-spahn tube having a bore of 38 mm. and a length of 435 mm. These four coils, connected in series, have a total resistance of about 0·18 ohm.

In making a measurement of the total iron loss, the wattmeter reading is observed, whilst the flux density is calculated after the manner shown on page 79. Since the total power absorbed is very small, the voltmeter should be disconnected whilst taking the wattmeter reading and the other instrument losses allowed for if necessary, as well as the copper loss of the magnetic square itself.

The loss is usually stated in watts per kilogramme or per lb. at a stated flux density and frequency.

Sometimes a rectangular core with a removable fourth limb is built up so that different samples of iron can be tested in succession. The losses in the other three limbs must be measured and an allowance made in measurements on different samples.

Separation of Hysteresis and Eddy Current Loss.—In order to separate the total iron loss into the hysteresis and eddy current components, a series of observations is made at constant flux density, but with varying frequency. To maintain constant flux density, all that is necessary is to keep the ratio $\frac{\text{volts}}{\text{frequency}}$ constant, for

$$\frac{E}{f} = 4{\cdot}44 \times 10^{-8}\Phi T.$$

Since T is constant for a given winding, the flux Φ is solely dependent upon the ratio $\frac{E}{f}$. The voltage must therefore be varied proportionally to the frequency.

In these conditions the only variable quantity in the expression for the iron loss is the frequency, and the hysteresis and eddy current losses can be represented by the expressions $P_h = af$ and $P_e = bf^2$ respectively, a and b being constants. The total iron loss is therefore equal to

$$P = P_h + P_e = af + bf^2$$

and
$$\frac{P}{f} = a + bf.$$

If the quantity $\frac{P}{f}$ be plotted against frequency, the resulting curve should be a straight line [see Fig. 61 (a)], the value at zero frequency giving the constant a and the slope of the curve determining b. The hysteresis loss at any frequency is given by af, whilst the remaining power is due to eddy currents. Let a horizontal line be

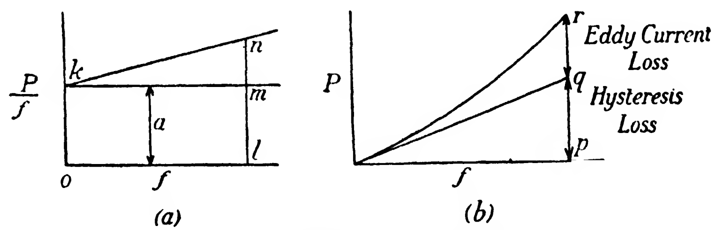


FIG. 61.—Separation of Iron Losses.

drawn through k [Fig. 61 (a)] and a vertical line lmn at any chosen frequency. The area $olmk = af$ represents the hysteresis loss, whilst the area kmn is proportional to $[km]^2 = f^2$ and represents the eddy current loss.

Another way of arriving at this result is to plot the total watts lost against the frequency, drawing a tangent to the curve at the origin. At any frequency f [see Fig. 61 (b)] the height pq represents the hysteresis loss, since this height is proportional to the frequency. The remaining height, qr, represents the eddy current loss. The reason why the tangent at the origin is chosen is that at very low frequencies the eddy current loss is negligible, since it is proportional to the square of the frequency. It is important to get some experimental observations at low frequencies in order to determine the position of the tangent with accuracy.

Steinmetz Index.—The hysteresis loss in a specimen of iron or steel is proportional to B^x, where x is known as the Steinmetz index. At moderately low flux densities, this has a value of 1·6, but for higher flux densities it rises considerably, and may reach a value as high as 3 or 4. The numerical value of the Steinmetz index can be calculated in a particular case from experimental observations in the following manner.

It was shown on p. 80 that the flux is given by

$$E = 4k\Phi fT \times 10^{-8} \text{ volts},$$

so that the flux density is equal to

$$B = \frac{E \times 10^8}{4kfTa},$$

where a is the cross-sectional area of the iron specimen. The total iron loss is

$$\begin{aligned} P &= \text{Hysteresis loss} + \text{Eddy current loss} \\ &= k_h f B^x + k_e f^2 B^2. \end{aligned}$$

Substituting for B, this becomes

$$\begin{aligned} P &= k_h f \frac{E^x \times 10^{8x}}{4^x k^x f^x T^x a^x} + k_e f^2 \frac{E^2 \times 10^{16}}{4^2 k^2 f^2 T^2 a^2} \\ &= \frac{k_h E^x \times 10^{8x}}{4^x k^x f^{x-1} T^x a^x} + \frac{k_e E^2 \times 10^{16}}{4^2 k^2 T^2 a^2}. \end{aligned}$$

If a series of experimental observations be taken at constant voltage but with varying frequency (the wave form being assumed unchanged), then the total iron loss can be expressed as

$$P = \frac{m}{f^{x-1}} + n.$$

The coefficient n can be calculated from the test for the separation of iron loss (see p. 81).

The hysteresis loss is now equal to

$$P - n = \frac{m}{f^{x-1}} = mf^{1-x}.$$

Therefore $\log (P - n) = \log m + (1 - x) \log. f$. On plotting $\log (P - n)$ against $\log f$, the slope of the curve, if a straight line, gives $(1 - x)$, which is negative since x is greater than unity. The index, x, can now be calculated. The graph is found to be not a straight line, and this indicates that (1) x is not a constant, (2) the subtracted eddy current loss is not constant, or (3) the coefficient k is not constant. If the wave form changes, as it probably does, k is not constant, and this further affects the magnitude of the eddy current loss. On the other hand it certainly appears as if x does alter, reaching a value of 3 or even 4 in extreme cases.

Effect of Iron Loss on Impedance.—In the case of a choking coil having an iron core, the total power losses occurring in it consist of a copper loss due to its ohmic resistance and an iron loss due to hysteresis and eddy currents. Such a choking coil could be replaced by an equivalent one having no iron loss at all, but an increased copper loss to make the total the same in the two cases. This results in an apparent increase in the ohmic resistance, and the choking coil can be considered to have an equivalent resistance higher than its true ohmic resistance so as to take into account the whole of the losses occurring in the choking coil. Thus iron loss has the effect of increasing the impedance of a choking coil.

Circuit Equivalent to an Actual Choking Coil.—When a voltage is applied to a choking coil, part of it is absorbed in overcoming the ohmic resistance, whilst the remainder, considered vectorially, is used for overcoming the back E.M.F. set up, due to the change of magnetic flux. In other words, the circuit may be considered as a pure resistance connected in series with a resistanceless choking coil. But, due to the iron loss in the latter, the current does not lag behind its volts by exactly 90°, since there must be a power

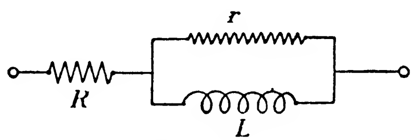


FIG. 62.—Circuit Equivalent to Choking Coil.

component. The current may therefore be resolved into two components, one in phase and the other in quadrature with the voltage acting on this part of the circuit, which may, consequently, be considered as a pure reactance, L, in parallel with a pure resistance, r. The value of r is such that $\frac{E_L^2}{r}$ is the total iron loss, where E_L is the voltage remaining after the IR drop has been subtracted vectorially. Fig. 62 shows the final equivalent circuit. This can be further simplified into a single resistance in series with a reactance.

Flux Distribution.—In most cases in practice the flux is more or less non-uniformly distributed. This is due to the fact that the flux endeavours to choose the shortest path and hence it tends to crowd together in certain parts of the magnetic circuit. This effect is counterbalanced by the repelling action of the lines on one another resulting in a tendency to cover as large a cross section as possible. In the case of a ring or a rectangular specimen, the former effect tends to make the lines crowd about the inside edge, whilst the latter effect forces some of them towards the outside into longer paths. The flux densities over different paths are inversely proportional to the relative reluctances of these paths, and hence the density gradually increases as the inner edge is approached.

The actual iron losses as determined by experiment are frequently in excess of the calculated losses, this being accounted for by the non-uniform distribution of the flux, which always results in an increase in the losses.

In a number of cases of machines of different types, the flux crosses an air-gap and then has to traverse a path which consists of a number of iron teeth in parallel with a number of slots. In a case like this, the major portion of the flux passes down the teeth, but some of the lines must pass down the slots since they have a definite reluctance. The actual distribution of the flux is somewhat complicated, but lends itself to mathematical treatment.

Calculation of Magnetizing Ampere-turns.—The simplest case is that of a ring specimen without any magnetic joints. Knowing the magnetization curve of the material, the maximum value of H can be determined for a given maximum value of B, and the total maximum ampere-turns required are given by the formula

$$H_m l = \frac{4\pi}{10} \times I_m T.$$

Knowing the number of turns in the winding, the maximum and R.M.S. values of the current can be obtained.

Let the total maximum flux be $\Phi = B_m a$, where a is the cross sectional area. Then

$$H_m = \frac{\Phi}{a\mu},$$

$$I_m T = \frac{10\Phi l}{4\pi a\mu},$$

and

$$I_i = \frac{10}{4\pi\sqrt{2}} \times \frac{\Phi l}{a\mu T},$$

assuming sinusoidal wave forms.

This is a purely reactive current lagging by 90°, but there is also an active component due to the iron losses. This can be determined by dividing the watts lost by the voltage. The resultant current is the vectorial sum of the two.

The effect of magnetic joints is quite appreciable, and is equivalent to adding a small air-gap, thus resulting in a considerable increase in the magnetizing current.

Where composite magnetic circuits are in question, the problem is more complicated and is dealt with by the method of determining the ampere-turns necessary to force the flux through each part of the magnetic circuit in succession. The sum total of these ampere-turns enables the magnetizing current to be determined.

Examples.

(1) An iron cored choking coil takes 5 amperes at a power factor of 0·6 when supplied at 100 volts. When the iron core is removed

and the voltage reduced to 15 volts the current rises to 6 amperes at a power factor of 0·9. Determine the iron loss in the core.

(2) An air cored coil of 200 turns has a resistance of 1·5 ohms, and passes a current of 30 amperes when supplied at 100 volts, 50 cycles. What is the value of the flux and the inductance of the coil?

(3) The hysteresis and eddy current losses in a certain specimen of iron are each equal to 1 kW at 50 cycles and $B = 10{,}000$ lines per sq. cm. Determine the hysteresis and eddy current losses separately when B is raised to 12,000 lines per sq. cm. The Steinmetz index is to be taken as 1·66.

(4) The iron loss in a choking coil is 280 watts when the applied voltage is 100 and $f = 50$, and is 110 watts when the applied voltage is 50 and $f = 25$. Determine the hysteresis and eddy current losses separately at 80 volts, 40 cycles.

CHAPTER IX

WAVE FORM

Non-sinusoidal Wave Form.—All wave forms actually obtained in practice differ more or less from the standard sine wave, and in cases where the difference is considerable, it is necessary to take account of it. Whether sinusoidal or not, the wave form is periodic in its character, each cycle being similar to the preceding one. It can be demonstrated mathematically that any periodic curve can be split up into a number of pure sine waves of different frequencies and amplitudes superimposed on one another. One of these component curves, called the *fundamental*, has the same frequency as the resultant complex curve. The other components have frequencies which are exact multiples of the fundamental frequency.

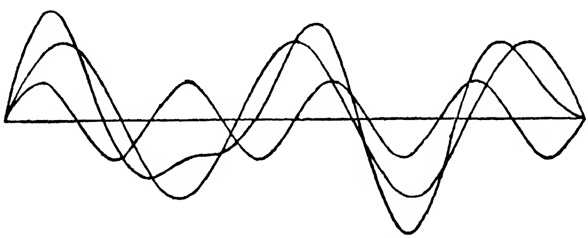


FIG. 63.—Effect of Fractional Harmonic.

If the frequency of one of these other components bore a fractional ratio to that of the fundamental, it is easy to see that the second cycle would not be a repetition of the first as illustrated in Fig. 63, where the ratio of the frequencies is chosen as 1·6.

These various components are known as *harmonics*, and are distinguished by means of a number. For example, the third harmonic is that component sine wave which has a frequency three times that of the fundamental. The maximum values of these various harmonics may be anything; but usually they get smaller the higher the frequency, although special circumstances may result in a particular one being well developed. The relative phase of the different harmonics may also be anything; it is not necessary for them to pass through the zero values at the same instant.

The three things which specify the harmonic are (1) the frequency,

(2) the amplitude or maximum value of the component, and (3) the relative phase. Thus the instantaneous value of a complex wave may be expressed as

$$e = E_1 \sin(\omega t + \alpha_1) + E_2 \sin(2\omega t + \alpha_2) + E_3 \sin(3\omega t + \alpha_3) + \ldots,$$

where E_1, E_2, E_3, etc., are the maximum values of the fundamental and the various harmonics respectively, ωt is the angle, measured at fundamental frequency, moved through since the commencement, and α_1, α_2, α_3, etc., are angles representing the phase of the various quantities at the instant from which the effects are measured. For example, at the instant of commencement the instantaneous value of the fundamental may have been half the maximum value.

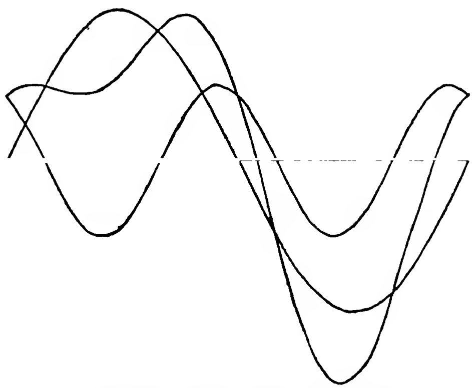


FIG. 64.—Effect of Second Harmonic.

Then $\sin(\omega t + \alpha_1) = 0{\cdot}5$, and since ωt is zero, by hypothesis α_1 must be 30°. Again, at the end of a given time imagine that the fundamental has advanced by 40°. Then the instantaneous value of the fundamental is $E_1 \sin(40° + 30°) = 0{\cdot}94E_1$ or 0·94 its maximum value.

Even Harmonics.—An even harmonic is one the frequency of which is an even number of times the fundamental frequency. One of the effects of an even harmonic is to make the two halves of the wave dissimilar, and this is not possible with the ordinary types of A.C. generators having a constant speed, for whatever occurs under one pole is repeated under the next which is of opposite polarity. Thus the two halves of the wave must be similar. Fig. 64 represents a wave with a 50 per cent. second harmonic passing through the zero 60° later than the fundamental, and it is seen that not only are the + and − portions dissimilar, but that one occupies a larger proportion of the base line than the other. All the even harmonics, therefore, are absent in the wave forms obtained from the ordinary types of generators.

Odd Harmonics.—The presence of odd harmonics does not render the two halves of the wave dissimilar, for when the fundamental has advanced through half a period the odd harmonics have advanced through a number of complete periods plus half a period, and hence their instantaneous values are in the same direction relative to the

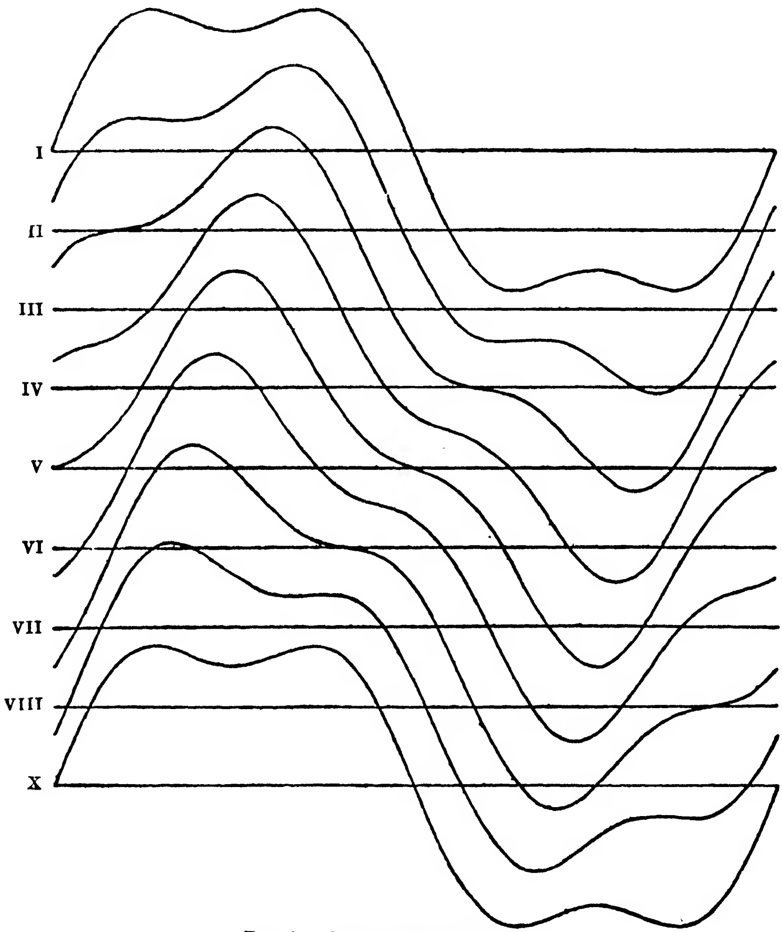


FIG. 65.—Effect of Third Harmonic.

instantaneous value of the fundamental. There is therefore no inherent reason why odd harmonics should not be present in E.M.F. and current waves, and they are, indeed, found there.

Phase of Harmonics.—The various harmonics may go through their zero values at any moment relative to the zero of the fundamental, and this question of the relative phase alters the resulting character of the wave to a very large extent. Fig. 65 shows a series of curves illustrating the effect of a 25 per cent. third harmonic, *i.e.* a third harmonic the maximum value of which is 25 per cent. that

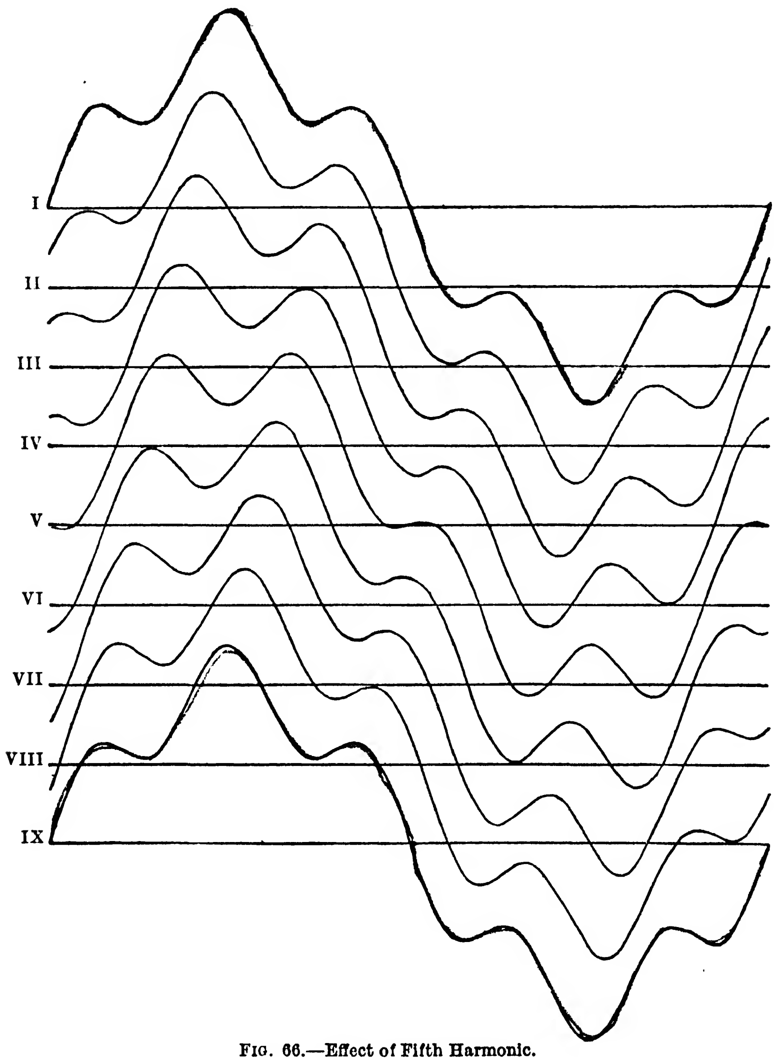


FIG. 66.—Effect of Fifth Harmonic.

of the maximum value of the fundamental. The different curves result from the application of different values to the angle α in the expression

$$e = E_1 \sin \omega t + E_3 \sin (3\omega t + \alpha).$$

There is no need to consider an angle in the first term corresponding to α, for this would simply mean drawing the curve from a different starting point. Fig. 66 shows a similar series of curves for a 25 per cent. fifth harmonic.

Curve V in Fig. 65 and curve I in Fig. 66 are termed *peaked* waves, whilst curve I in Fig. 65 and curve V in Fig. 66 are examples of what are termed *dimpled* waves. These two terms are merely indicative of the general character of the wave, there being no rigid demarcation between the two.

R.M.S. Value of a Complex Wave.—Taking an E.M.F. wave as an example, let the instantaneous value be represented by

$$e = E_1 \sin \omega t + E_3 \sin (3\omega t + \alpha) + E_5 \sin (5\omega t + \beta) + \dots$$

Then the instantaneous value of e^2 is given by

$$e^2 = E_1^2 \sin^2 \omega t + E_3^2 \sin^2 (3\omega t + \alpha) + E_5^2 \sin^2 (5\omega t + \beta) + \dots$$

+ (a number of terms containing the products of two sines).

It can be shown mathematically that the average product of two sines of different frequencies is zero.[1] Consequently the average value of e^2 is equal to the average value of

$$E_1^2 \sin^2 \omega t + E_3^2 \sin^2 (3\omega t + \alpha) + E_5^2 \sin^2 (5\omega t + \beta) + \dots$$

But the average value of $\sin^2 \omega t$, or $\sin^2 (3\omega t + \alpha)$, etc., is $\frac{1}{2}$.[2]

Therefore the average value of e^2 is equal to

$$\frac{E_1^2}{2} + \frac{E_3^2}{2} + \frac{E_5^2}{2} + \dots$$

[1] Average value of product of two sines

$$= \frac{1}{2\pi}\int_0^{2\pi} \sin a\theta \sin b\theta d\theta$$
$$= \frac{1}{4\pi}\int_0^{2\pi} \left\{\cos (a-b)\theta - \cos (a+b)\theta\right\} d\theta$$
$$= \frac{1}{4\pi}\left[\frac{1}{a-b}\sin (a-b)\theta - \frac{1}{a+b}\sin (a+b)\theta\right]_0^{2\pi}$$
$$= \frac{1}{4\pi} \times 0 = 0.$$

[2] This follows from the following reasoning :—

$$\text{Average value of } \sin^2\theta = \frac{1}{2\pi}\int_0^{2\pi} \sin^2\theta \, d\theta$$
$$= \frac{1}{2\pi}\int_0^{2\pi}\left(\frac{1}{2} - \frac{1}{2}\cos 2\theta\right) d\theta$$
$$= \frac{1}{2\pi}\left[\frac{\theta}{2} - \frac{1}{4}\sin 2\theta\right]_0^{2\pi}$$
$$= \frac{1}{2\pi}\left(\frac{2\pi}{2} - \frac{1}{4}\sin 4\pi - 0 + \frac{1}{4}\sin 0\right)$$
$$= \frac{1}{2}.$$

and the R.M.S. value is

$$E = \sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{2}}.$$

In a similar manner, the R.M.S. value of a complex current wave is found to be

$$I = \sqrt{\frac{I_1^2 + I_3^2 + I_5^2 + \ldots}{2}}.$$

Effect of Magnetic Saturation on Current Wave Form.—In the case of a choking coil having an iron core, the current wave form is distorted when magnetic saturation occurs, this being due to the fact that the *B—H* curve is not a straight line. The simplest case to take is that of a choking coil having no ohmic resistance at all. The applied volts must therefore be balanced at every instant by the induced back E.M.F. If the wave form of the applied voltage is sinusoidal, this must be counterbalanced by an induced back E.M.F. of the same magnitude. The latter is proportional to the rate of change of the flux, so that the flux setting up this E.M.F. must be sinusoidal as well, although it is displaced by 90° in phase. The next step, therefore, is to determine the current wave form necessary to produce a sine wave of flux. If the flux were proportional to the current, no distortion would occur, but this is not so. As the *B—H* curve bends over, each additional ampere of magnetizing current produces a smaller and smaller amount of flux, and consequently each increment of flux requires a larger and larger increase in the current. The actual wave form can be determined by means of the following graphical construction.

Let the left-hand diagram in Fig. 67 represent the *B—H* curve of the iron used, or, preferably, let it represent the relation between the total flux and the magnetizing current, the total flux being obtained by multiplying the density by the cross sectional area and the magnetizing current from the formula

$$H = \frac{4\pi}{10} \times \text{ampere-turns per cm.}$$

The sinusoidal flux wave is drawn to the right. In order to obtain the current required at any instant to produce a given flux, such as *ab*, a projection is drawn on to the left-hand diagram, the necessary current being given by *oc*. The point *d* is then obtained by making *ad* equal to *oc*, and *d* is then a point on the required current wave. This is repeated until a sufficient number of points is obtained enabling the current wave to be drawn in, a characteristic example of which is shown in the figure. The outstanding feature of the curve is the fact that it is more peaked than the sine wave and usually contains a prominent third harmonic. It is a purely reactive current, since it passes through the zero at the same instant as the flux which lags behind the applied voltage by 90°, and, moreover, both halves of the wave are equal. The resulting power curve is a double frequency one, but is not sinusoidal in character, although the average value is zero. Thus it is seen that magnetic saturation in the iron causes a distortion in the current wave, but does not result in any loss of power.

The above is only an approximate representation of what happens, for it is seen that there is a third harmonic in the current wave, whilst the E.M.F. wave is assumed to be sinusoidal. It is impossible for an E.M.F. of one frequency to set up directly a current of another frequency, so that since the third harmonic is undoubtedly present in the current wave, there must be a third harmonic E.M.F.

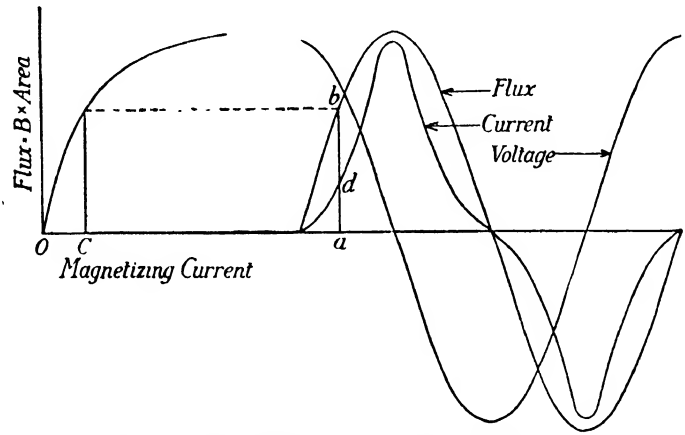


FIG. 67.—Effect of Magnetic Saturation on Wave Form.

operating on the circuit. By hypothesis this does not come from the source of supply, so that it must be induced in the winding.

The sinusoidal applied voltage induces a sine wave of magnetizing current. This does not reach the maximum value desired, so that the resulting flux wave is flat-topped, containing a third harmonic. This causes a third harmonic E.M.F. to be induced, which is the source of the third harmonic current. The whole of this harmonic E.M.F. is dropped in the winding itself, and so it does not appear at the terminals. The harmonic magnetizing current thus set up supplies in large measure the deficiency in the flux wave. There are also the higher harmonics to be considered, but these are here neglected. In an actual case where resistance is present and where the supplying circuit possesses impedance of its own, the flux wave never is restored absolutely to its ideal shape.

Effect of Hysteresis on Current Wave Form.—In the preceding section the same B—H curve was taken for ascending and descending values, with the result that the current wave was symmetrical about its maximum value and was purely reactive. When the effect of hysteresis is considered, the value of the magnetizing current required to produce a given flux is less when the current is decreasing than when it is increasing, and since the flux dies away at the same rate that it is built up, it follows that the current falls away at a greater rate, resulting, in general, in the downward part of the current wave being steeper than the upward portion. The current wave thus loses its symmetry, with the result that more energy is supplied to the circuit in building up the field than is returned on its destruction. Consequently, an active component has been introduced into the current wave, which was to be expected, since hysteresis results in a loss of power.

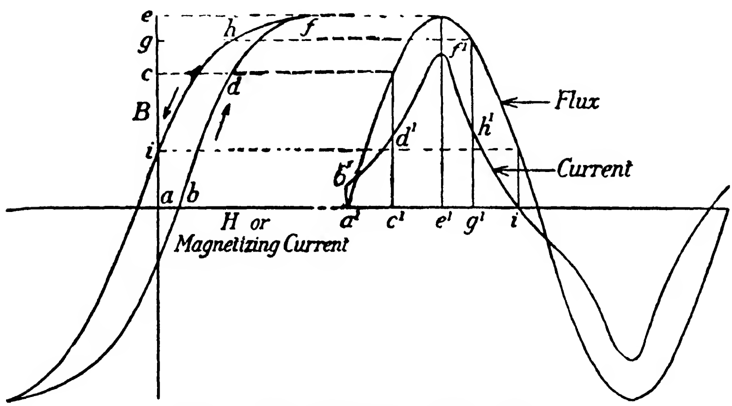


FIG. 68.—Effect of Hysteresis on Wave Form.

In order actually to determine the current wave, the graphical construction explained in the last paragraph is again employed, using a hysteresis loop instead of a simple *B*—*H* curve (see Fig. 68). The maximum value of *B* in the hysteresis loop is made equal to the maximum value of the flux density in the iron. Care must be exercised in order that the correct portion of the hysteresis loop shall be used. When the flux is zero and about to take a positive value, the necessary magnetizing current is given by *ab*, which length is transferred to *a′b′* in the right-hand diagram. When the flux has a value *ac* in the positive direction, the corresponding magnetizing current is *cd*, which is reproduced at *c′d′*. Similarly, *ef* is reproduced at *e′f′*. After the maximum value has been passed the flux follows the upper curve in the hysteresis loop, and when it has decreased to a value *ag* the magnetizing current is *gh*, from which *g′h′* is plotted. When the flux has dropped to *ai* the magnetizing current has dropped to zero, after which it commences to rise in the opposite direction before the flux has changed sign. It is thus seen that the current wave leads the flux wave to a certain extent and hence is less than 90° behind the voltage wave. On this account, therefore, the average value of the power assumes a positive value, representing the hysteresis loss.

If the magnetizing current be defined as that current which sets up the flux *and* provides the energy which is dissipated on account of hysteresis loss (and the second cannot exist without the first), then this magnetizing current can be split up into an active and a reactive component. With a sine wave of applied voltage, the former is necessarily sinusoidal also, since the average power developed by any harmonic current is zero (see p. 100). The reactive component is complex in form, since it is the difference between the total magnetizing current (complex) and the active component (sinusoidal). This reactive component may show considerable irregularities.

Only the fundamental component of the magnetizing current is obtained directly from the source of supply. The harmonic components are due to harmonic E.M.F.'s set up by the iron in the manner already explained (see p. 93).

Effect of Harmonics on Inductive Reactance.—When the wave form of the applied E.M.F. is not sinusoidal, it can be split up into a number of component sinusoidal E.M.F.'s of different frequencies, the fundamental frequency being the same as that of the complex wave. These different E.M.F.'s can be considered as acting independently of each other, each producing its own current. But the reactance of a given choking coil is proportional to the frequency of supply, and if the frequency is raised the reactance increases. Consequently, the reactance, as far as the third harmonic is concerned, is three times what it is to the fundamental, and the amperes produced per volt of the third harmonic will only be one-third of the amperes produced per volt of the fundamental. With the higher harmonics this effect is accentuated still more. In other words, the circuit offers a higher impedance to the harmonics than it does to the fundamental, with the result that the corresponding harmonics in the current wave will be diminished in magnitude and the current wave will be a closer approximation to the ideal than the voltage wave. Thus reactance has the effect of damping out the harmonics in the current wave, and the higher the harmonic the greater is the damping effect.

Consider a circuit having an inductance L and an applied voltage obeying the law

$$e = E_1 \sin \omega t + E_3 \sin (3\omega t + \alpha) + E_5 \sin (5\omega t + \beta) + \dots$$

The R.M.S. value of the voltage is

$$E = \sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \dots}{2}}$$

and the fundamental E.M.F. produces a maximum current of $\frac{E_1}{2\pi f L}$ amperes, whilst the third and fifth harmonics produce maximum currents of $\frac{E_3}{2\pi \times 3f \times L}$ and $\frac{E_5}{2\pi \times 5f \times L}$ amperes respectively, and so on. The R.M.S. value of all these currents combined is

$$I = \sqrt{\frac{\left(\frac{E_1}{2\pi f L}\right)^2 + \left(\frac{E_3}{6\pi f L}\right)^2 + \left(\frac{E_5}{10\pi f L}\right)^2 + \ldots}{2}}$$

$$= \frac{1}{2\pi f L} \times \sqrt{\frac{E_1^2}{2} + \frac{E_3^2}{18} + \frac{E_5^2}{50} + \ldots}$$

The reactance, considered with respect to the complex wave, is given by

$$X' = \frac{E}{I} = \frac{\sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{2}}}{\frac{1}{2\pi f L} \times \sqrt{\frac{E_1^2}{2} + \frac{E_3^2}{18} + \frac{E_5^2}{50} + \ldots}}$$

$$= 2\pi f L \times \sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{E_1^2 + \frac{E_3^2}{9} + \frac{E_5^2}{25} + \ldots}}$$

Thus another effect of harmonics in the E.M.F. wave form is to increase the reactance in the ratio

$$\sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{E_1^2 + \frac{E_3^2}{9} + \frac{E_5^2}{25} + \ldots}}$$

The relative phases of the various harmonics do not affect the value of this ratio, which is only dependent upon the relative magnitudes of the various harmonics.

As an example, consider the case where there is a 20 per cent. third harmonic, a 10 per cent. fifth harmonic and a 5 per cent. seventh harmonic in the E.M.F. wave form. This voltage is applied to a circuit having an inductance L. The reactance corresponding to this on a sine wave would be $2\pi f L$, but on the complex wave in question it is

$$2\pi f L \times \sqrt{\frac{1^2 + 0{\cdot}2^2 + 0{\cdot}1^2 + 0{\cdot}05^2}{1^2 + \frac{0{\cdot}2^2}{9} + \frac{0{\cdot}1^2}{25} + \frac{0{\cdot}05^2}{49}}}$$

$$= 2\pi f L \times 1{\cdot}023.$$

Thus the reactance is 2·3 per cent. higher on this particular wave form than on a pure sine wave. Strictly speaking, the factor 2π

is only correct on the sine wave hypothesis, and for other wave forms a slightly different constant should be used.

If resistance is also present in the circuit the conditions are somewhat modified. The current produced by each component of the E.M.F. is calculated from a knowledge of the impedance corresponding to the particular frequency, whilst the angle of lag for each component is calculated from the formula

$$\omega t = \tan^{-1}\frac{X}{R}.$$

This angle will be different for the various harmonics on account of the change in X. The sum of these component currents gives the resultant from which the wave may be plotted.

Effect of Harmonics on Capacitive Reactance.—In the case of a circuit containing nothing but condensers, the impedance is inversely proportional to the frequency, and treating a complex wave in the same way as in the previous paragraph, it is seen that each harmonic produces a proportionally greater current than the fundamental. Thus the third harmonic produces three times the current per volt that the fundamental does, and so on. Consequently, the circuit appears to possess a reduced impedance when the E.M.F. wave contains harmonics.

If the circuit contains a capacitance of C farads the fundamental E.M.F. produces a maximum current of $2\pi fCE_1$ amperes where E_1 is the maximum value of the fundamental E.M.F. The third harmonic produces a current of $2\pi \times (3f) \times CE_3$ amperes, and so on. The R.M.S. value of the resultant current is

$$I = \sqrt{\frac{(2\pi fCE_1)^2 + (6\pi fCE_3)^2 + (10\pi fCE_5)^2 + \ldots}{2}}$$

$$= 2\pi fC \times \sqrt{\frac{E_1^2 + 9E_3^2 + 25E_5^2 + \ldots}{2}}.$$

The impedance is therefore given by

$$\frac{E}{I} = \frac{\sqrt{\dfrac{E_1^2 + E_3^2 + E_5^2 + \ldots}{2}}}{2\pi fC \times \sqrt{\dfrac{E_1^2 + 9E_3^2 + 25E_5^2 + \ldots}{2}}}$$

$$= \frac{1}{2\pi fC} \times \sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots\ldots}{E_1^2 + 9E_3^2 + 25E_5^2 + \ldots\ldots}}.$$

The impedance is therefore reduced in the ratio

$$\sqrt{\frac{E_1^2 + E_3^2 + E_5^2 + \ldots}{E_1^2 + 9E_3^2 + 25E_5^2 + \ldots}}.$$

monics. The effect of resonance is to produce a very large current per volt and to render the impedance very small over a certain limited range of frequency. Thus the current due to a particular harmonic in the E.M.F. wave may be greatly magnified when it appears in the current wave, resulting in an extraordinary distortion. In fact, in some extreme cases it appears at first sight as if the frequency of the complex wave were that of the resonating harmonic, but this is not so.

The conditions which arise in such a circuit are very well illustrated by means of a concrete example. Consider the case of an E.M.F. wave represented by

$$e = 100 \sin \omega t + 10 \sin (3\omega t + 180^\circ).$$

The circuit consists of a resistance of 2 ohms, an inductance of 0·02 henry, and a capacitance of 55 μF, all connected in series, the frequency being 50 cycles per second.

The maximum value of the fundamental current is

$$\frac{100}{\sqrt{2^2 + \left(2\pi \times 50 \times 0{\cdot}02 - \dfrac{10^6}{2\pi \times 50 \times 55}\right)^2}}$$
$$= 1{\cdot}94 \text{ amperes.}$$

The maximum value of the third harmonic current is

$$\frac{10}{\sqrt{2^2 + \left(2\pi \times 150 \times 0{\cdot}02 - \dfrac{10^6}{2\pi \times 150 \times 55}\right)^2}}$$
$$= 4{\cdot}88 \text{ amperes.}$$

The angle of lead of the fundamental current over the fundamental voltage is

$$\tan^{-1} \frac{\left(2\pi \times 50 \times 0{\cdot}02 - \dfrac{10^6}{2\pi \times 50 \times 55}\right)}{2} = 88^\circ.$$

The angle of lead of the harmonic current over the harmonic voltage is

$$\tan^{-1} \frac{\left(2\pi \times 150 \times 0{\cdot}02 - \dfrac{10^6}{2\pi \times 150 \times 55}\right)}{2} = 13^\circ.$$

Strictly speaking, these angles are negative angles of lag.

The expression for the current is therefore

$$1{\cdot}94 \sin (\omega t + 88^\circ) + 4{\cdot}88 \sin (3\omega t + 193^\circ).$$

Both the voltage and the current waves are represented in Fig. 70, which shows the effect of resonance with the third harmonic.

When a length of alternating current mains is switched into circuit, a capacitance or charging current flows even on open circuit, The impedance with respect to the various components is

Fundamental $\sqrt{10^2 + (2\pi \times 50 \times 0{\cdot}02)^2} = 11{\cdot}8$ apparent ohms.
3rd Harmonic $\sqrt{10^2 + (2\pi \times 150 \times 0{\cdot}02)^2} = 21{\cdot}3$,, ,,
5th ,, $\sqrt{10^2 + (2\pi \times 250 \times 0{\cdot}02)^2} = 33{\cdot}0$,, ,,
7th ,, $\sqrt{10^2 + (2\pi \times 350 \times 0{\cdot}02)^2} = 45{\cdot}0$,, ,,

The angles of lag of the currents behind their respective voltages are

Fundamental $\tan^{-1}\dfrac{2\pi \times 50 \times 0{\cdot}02}{10} = 32{\cdot}1°$

3rd Harmonic $\tan^{-1}\dfrac{2\pi \times 150 \times 0{\cdot}02}{10} = 62{\cdot}0°$

5th ,, $\tan^{-1}\dfrac{2\pi \times 250 \times 0{\cdot}02}{10} = 72{\cdot}3°$

7th ,, $\tan^{-1}\dfrac{2\pi \times 350 \times 0{\cdot}02}{10} = 77{\cdot}2°$

The various power factors are

Fundamental $\cos 32{\cdot}1° = 0{\cdot}85$
3rd Harmonic $\cos 62{\cdot}0° = 0{\cdot}47$
5th ,, $\cos 72{\cdot}3° = 0{\cdot}30$
7th ,, $\cos 77{\cdot}2° = 0{\cdot}22$.

The expression for the current is

$$i = \frac{100}{11{\cdot}8}\sin(\omega t - 32°) + \frac{20}{21{\cdot}3}\sin(3\omega t + 60° - 62°)$$
$$+ \frac{10}{33{\cdot}0}\sin(5\omega t + 150° - 72°) + \frac{5}{45{\cdot}0}\sin(7\omega t + 300° - 77°)$$
$$= 8{\cdot}48\sin(\omega t + 328°) + 0{\cdot}94\sin(3\omega t + 358°)$$
$$+ 0{\cdot}30\sin(5\omega t + 78°) + 0{\cdot}11\sin(7\omega t + 223°).$$

The R.M.S. value of the current is

$$\sqrt{\frac{8{\cdot}48^2 + 0{\cdot}94^2 + 0{\cdot}30^2 + 0{\cdot}11^2}{2}}$$
$$= 6{\cdot}02 \text{ amperes.}$$

The R.M.S. value of the voltage is

$$\sqrt{\frac{100^2 + 20^2 + 10^2 + 5^2}{2}}$$
$$= 72{\cdot}5 \text{ volts.}$$

The total power is

$$\frac{100}{\sqrt{2}} \times \frac{8{\cdot}48}{\sqrt{2}} \times 0{\cdot}85 + \frac{20}{\sqrt{2}} \times \frac{0{\cdot}94}{\sqrt{2}} \times 0{\cdot}47$$
$$+ \frac{10}{\sqrt{2}} \times \frac{0{\cdot}30}{\sqrt{2}} \times 0{\cdot}30 + \frac{5}{\sqrt{2}} \times \frac{0{\cdot}11}{\sqrt{2}} \times 0{\cdot}22$$
$$= 360 + 4{\cdot}4 + 0{\cdot}45 + 0{\cdot}06$$
$$= 365 \text{ watts say.}$$

The resultant power factor is given by

$$\frac{\text{Total Average Power}}{\text{R.M.S. Voltage} \times \text{R.M.S. Current}}$$
$$= \frac{365}{72{\cdot}5 \times 6{\cdot}02}$$
$$= 0{\cdot}84.$$

P.D. and Current Waves in the Case of an Arc.—In the case of an arc supplied from an A.C. source, with a steadying resistance in series, a peculiar distortion is brought about in both the current wave and that of the p.d. across the arc terminals. To commence with, the supply voltage must rise to a certain value before the arc is struck, and thus the current wave is flat during this interval. When the arc is struck the current increases rapidly, and this current, flowing through the series steadying resistance, causes a

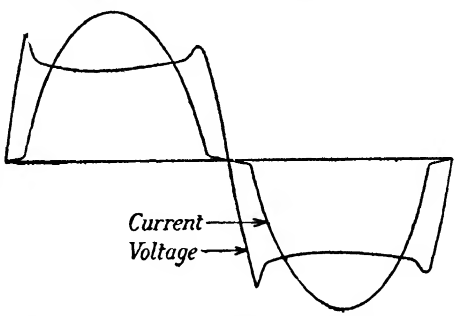


FIG. 71.—Current and Voltage Waves for an Arc.

considerable drop in voltage, which has the effect of suddenly lowering the p.d. across the arc itself. This sudden peak is shown in Fig. 71, which represents a typical case. As the current rises the voltage drop in the series resistance increases so that the wave form of the p.d. across the arc shows a broad hollow, which continues until the arc suddenly goes out. The current then drops to zero, whilst another smaller peak is obtained in the p.d. wave. The arc is then built up in the opposite direction and the operations are repeated.

This striking distortion of the current wave gives rise to a peculiar state of affairs with regard to the power consumed. The power curve can be constructed in the usual way from the current and voltage waves, but when its average value is measured the result is very much lower than the product of the R.M.S. current and the R.M.S. volts. Therefore, notwithstanding the fact that the current is in phase with the voltage, as far as this can be said of two waves so widely dissimilar, the power factor of the arc is considerably less than unity and may fall as low as 0·6 in certain cases where hard carbons are used.

The measurement of the amperes, volts and watts consumed by an A.C. arc forms an instructive experiment, a power factor of less than unity being obtained in spite of the absence of inductance and capacitance.

The horned appearance of the p.d. wave is very much accentuated if the arc is struck in coal gas, the tip of the initial peak representing a voltage many times as great as is obtained throughout the remainder of the time.

Other Causes of Wave Distortion.—Ordinary resistances are often the cause of a slight distortion of the current wave form on account of their variation of temperature throughout the cycle. In the case of an ordinary metal filament glow lamp, the temperature of the filament varies to a considerable extent throughout the cycle, and since it cannot get rid of its heat instantaneously it follows that the resistance when the voltage is decreasing is on the whole higher than when the voltage is increasing. Consequently, the current in the second quarter of the cycle is slightly less than it is in the first quarter. This results in a small distortion of the wave form, which is thrown slightly forward. The angle of lead obtained in this manner is of the order of 2° or 3°, resulting in a drop in the power factor of approximately 0·1 per cent. In all ordinary measurements this is, of course, negligible.

Another prolific source of wave distortion is the variation of the inductance of certain machines in different positions, certain cases of which will be dealt with later.

Equivalent Sine Wave.—In the case of a distorted wave form the power factor is no longer equal to cos ϕ, for this is based on the assumption of a sinusoidal wave form. Indeed, ϕ will probably vary throughout the cycle. In some cases it may be simpler to treat problems by substituting an equivalent sine wave for the actual one in question. The R.M.S. value of this equivalent sine wave is the same as that of the actual wave, and consequently the maximum value of the sine wave is $\sqrt{2}$ times the R.M.S. value of the actual wave. The phase of the equivalent sine wave can be determined from the power factor. The angle of lag, ϕ', is chosen so that cos ϕ' is equal to the actual power factor. In the case of an A.C. arc, the equivalent current would be taken to be in phase with the voltage, its magnitude being reduced so as to get the right power.

A warning is issued against a too promiscuous use of the equivalent sine wave, as it is only an approximation, and often only a very rough one at that.

Effect of Wave Form on Insulation Testing.—When insulation is subjected to an alternating electrical stress it is the maximum voltage which causes the greatest effect. The application of a peaky wave form is a severer test than if a flat wave form is used, assuming the two R.M.S. values to be equal. On the other hand, the breakdown of the insulation is not solely due to the maximum voltage, but depends to some extent upon the rapidity of its growth in the near neighbourhood of the maximum point.

Harmonic Analysis.—Commencing from any point, the wave may be represented by

$$e = E_1 \sin(\theta + \alpha_1) + E_3 \sin(3\theta + \alpha_3) + E_5 \sin(5\theta + \alpha_5) + \ldots$$

where $\omega t = \theta.$

This can be expanded as follows :—

$$\begin{aligned} e = {} & E_1 \sin\theta \cos\alpha_1 + E_1 \cos\theta \sin\alpha_1 \\ & + E_3 \sin 3\theta \cos\alpha_3 + E_3 \cos 3\theta \sin\alpha_3 \\ & + E_5 \sin 5\theta \cos\alpha_5 + E_5 \cos 5\theta \sin\alpha_5 \\ & + \ldots\ldots \end{aligned}$$

Since α_1, α_3, α_5, etc., are constants, this can be written

$$\begin{aligned} e = {} & A_1 \sin\theta + A_3 \sin 3\theta + A_5 \sin 5\theta + \ldots \\ & + B_1 \cos\theta + B_3 \cos 3\theta + B_5 \cos 5\theta + \ldots, \end{aligned}$$

where $A_1 = E_1 \cos\alpha_1$

and $B_1 = E_1 \sin\alpha_1$, etc.

Again $A_1^2 + B_1^2 = E_1^2 (\cos^2\alpha_1 + \sin^2\alpha_1) = E_1^2$

and $E_1 = \sqrt{A_1^2 + B_1^2}.$

Also $\dfrac{B_1}{A_1} = \dfrac{E_1 \sin\alpha_1}{E_1 \cos\alpha_1} = \tan\alpha_1.$

Summarizing

$$\begin{aligned} E_1 &= \sqrt{A_1^2 + B_1^2} \\ E_3 &= \sqrt{A_3^2 + B_3^2} \\ E_5 &= \sqrt{A_5^2 + B_5^2}, \text{ etc.,} \end{aligned}$$

and

$$\begin{aligned} \alpha_1 &= \tan^{-1}\frac{B_1}{A_1} \\ \alpha_3 &= \tan^{-1}\frac{B_3}{A} \\ \alpha_5 &= \tan^{-1}\frac{B_5}{A_5}, \text{ etc.} \end{aligned}$$

Most of the methods of analyzing periodic wave forms at present in use are rather tedious to carry out and necessitate a good deal of time being spent on the evaluation of the various constants. The method here outlined is an effort on the part of the author to expedite calculations of this kind and to provide a ready method, by means of a simple series of equations, for the analysis of periodic wave forms.

The method consists in taking a series of pairs of points on the wave to be analyzed, equidistantly spaced on either side of the point chosen as the zero. Let one pair of these ordinates be represented by y_θ and $y_{-\theta}$ respectively. Then

$$\begin{aligned} y_\theta + y_{-\theta} &= E_1 \sin(\theta + \alpha_1) + E_3 \sin(3\theta + \alpha_3) + \ldots \\ &\quad + E_1 \sin(-\theta + \alpha_1) + E_3 \sin(-3\theta + \alpha_3) + \ldots \\ &= 2E_1 \sin\alpha_1 \cos\theta + 2E_3 \sin\alpha_3 \cos 3\theta + \ldots \end{aligned}$$

The algebraic sum of each pair of readings is equal to a series of terms of the type $2E \sin\alpha \cos\theta$, since they are the sum of two sines. Here, θ is equal to some known angle, determined by the position of the points chosen, and E and α are constants depending upon the composition of the wave. In a similar manner, the algebraic differences of the same pairs of points give rise to a series of terms of the type $2E \cos\alpha \sin\theta$, the symbols having the same meaning as before. When a sufficient number of such equations is obtained, they can be solved in terms of the quantities $E \sin\alpha$ and $E \cos\alpha$.

In solving up to, say, the seventeenth harmonic there are eighteen unknown quantities to be determined, namely, the amplitudes and phase angles of the fundamental and the eight harmonics. Thus eighteen simultaneous equations are required, the necessary data being obtained from eighteen chosen ordinates. In order to minimize errors due to the incorrect drawing of the curve, those ordinates are chosen at equal distances apart, viz. at 5°, 15°, 25°, . . . 165° and 175° after the point which it is desired to regard as zero. The values of these various ordinates will be represented by y_5, y_{15}, y_{25}, . . . y_{165} and y_{175}.

Let the wave be represented by the expression :—

$$y_\theta = E_1 \sin(\theta + \alpha_1) + E_3 \sin(3\theta + \alpha_3) + \ldots + E_{17} \sin(17\theta + \alpha_{17}).$$

Then
$$\begin{aligned} y_{85} &= E_1 \sin(85° + \alpha_1) + E_3 \sin(255° + \alpha_3) \\ &\quad + E_5 \sin(425° + \alpha_5) + \ldots + E_{17} \sin(1445° + \alpha_{17}) \end{aligned}$$

and
$$\begin{aligned} y_{-85} &= E_1 \sin(-85° + \alpha_1) + E_3 \sin(-255° + \alpha_3) \\ &\quad + E_5 \sin(-425° + \alpha_5) + \ldots + E_{17} \sin(-1445° + \alpha_{17}) \\ &= -y_{95}. \end{aligned}$$

$$\begin{aligned} y_{85} + y_{95} &= y_{85} - y_{-85} \\ &= 2E_1 \cos\alpha_1 \sin 85° + 2E_3 \cos\alpha_3 \sin 255° \\ &\quad + 2E_5 \cos\alpha_5 \sin 425° + \ldots + 2E_{17} \cos\alpha_{17} \sin 1445°. \end{aligned}$$

Similarly,
$$\begin{aligned} y_{75} + y_{105} &= y_{75} - y_{-75} \\ &= 2E_1 \cos\alpha_1 \sin 75° + 2E_3 \cos\alpha_3 \sin 225° \\ &\quad + 2E_5 \cos\alpha_5 \sin 375° + \ldots + 2E_{17} \cos\alpha_{17} \sin 1275°. \end{aligned}$$

Other readings are taken every 10° until y_5 and y_{175} are reached. Then the resulting nine simultaneous equations are solved, thus obtaining $E_1 \cos \alpha_1, E_3 \cos \alpha_3$, etc., in terms of $(y_{85} + y_{95}), (y_{75} + y_{105}), \ldots (y_5 + y_{175})$.

A second series of calculations must now be made as follows :—

$$\begin{aligned} y_{85} - y_{95} &= y_{85} + y_{-85} \\ &= 2E_1 \sin \alpha_1 \cos 85° + 2E_3 \sin \alpha_3 \cos 255° \\ &\quad + 2E_5 \sin \alpha_5 \cos 425° + \ldots + 2E_{17} \sin \alpha_{17} \cos 1445°. \\ y_{75} - y_{105} &= y_{75} + y_{-75} \\ &= 2E_1 \sin \alpha_1 \cos 75° + 2E_3 \sin \alpha_3 \cos 225° \\ &\quad + 2E_5 \sin \alpha_5 \cos 375° + \ldots + 2E_{17} \sin \alpha_{17} \cos 1275°. \end{aligned}$$

In this way another nine simultaneous equations are obtained and $E_1 \sin \alpha_1$, $E_3 \sin \alpha_3$, etc., are evaluated in terms of $(y_{85} - y_{95})$, $(y_{75} - y_{105}), \ldots (y_5 - y_{175})$.

Then
$$E_1 = \sqrt{(E_1 \sin \alpha_1)^2 + (E_1 \cos \alpha_1)^2},$$
$$\alpha_1 = \tan^{-1} \left[\frac{E_1 \sin \alpha_1}{E_1 \cos \alpha_1}\right],$$
and similarly for the various harmonics.

Care must be taken to observe the signs of $E \sin \alpha$ and $E \cos \alpha$, as these enable the quadrant in which α is situated to be determined.

The initial labour in solving the above equations is considerable, but this having been accomplished once and for all, the problem resolves itself into simply multiplying the various chosen ordinates by certain known constants, and the analysis becomes much less tedious an operation than it is by many of the other methods in common use.

In order further to facilitate the actual calculations, a schedule has been drawn up as shown in Table I. This table is self-explanatory; in fact it is not even necessary to be familiar with the underlying principles in order to work out an example.

If so desired, the fundamental or any particular harmonic may be determined by itself without carrying out the analysis any further.

A modified set of constants, for use in connection with the approximate analysis of a wave up to the fifth harmonic, may be of value in some cases. For this purpose six readings from the curve are necessary, and they may be taken conveniently at 15°, 45°, 75°, 105°, 135° and 165° after the zero.

It will be found convenient for quickness in working to arrange these approximate calculations also in the form of a schedule, an example of which is shown in Table II.

If no harmonics higher than the fifth are present, the constants enumerated in this table will give correct results, but, of course, the more pronounced the higher harmonics are, the larger the error will be in the determination of the equation to the wave by means of this second set of constants.

TABLE II.

						$E_1 \sin \alpha_1$ terms.			$E_3 \sin \alpha_3$ terms.			$E_5 \sin \alpha_5$ terms.		
Differences.	(1)		(2)		(3) = (1) − (2) Difference.	(4) Constants.	(5) + or −	(6) = (3) × (4)	(7) Constants.	(8) + or −	(9) = (3) × (7)	(10) Constants.	(11) + or −	(12) = (3) × (10)
	y_{75} =		y_{105} =			$+0{\cdot}086_3$			−1			$+0{\cdot}322_0$		
	y_{45} =		y_{135} =			$+0{\cdot}235_7$			−1			$-0{\cdot}235_7$		
	y_{15} =		y_{165} =			$+0{\cdot}322_0$			+1			$+0{\cdot}086_3$		
						sum of +terms			sum of +terms			sum of +terms		
						sum of −terms			sum of −terms			sum of −terms		
									sum total					
					A	sum total $= E_1 \sin \alpha_1$			($+0{\cdot}235_7 \times$ sum total) $= E_3 \sin \alpha_3$			sum total $= E_5 \sin \alpha_5$		
						$E_1 \cos \alpha_1$ terms.			$E_3 \cos \alpha_3$ terms.			$E_5 \cos \alpha_5$ terms.		
Sums.	(1)		(2)		(13) = (1) + (2) Sum.	(14) Constants.	(15) + or −	(16) = (13) × (14)	(17) Constants.	(18) + or −	(19) = (13) × (17)	(20) Constants.	(21) + or −	(22) = (13) × (20)
	y_{75} =		y_{105} =			$+0{\cdot}322_0$			−1			$+0{\cdot}086_3$		
	y_{45} =		y_{135} =			$+0{\cdot}235_7$			+1			$-0{\cdot}235_7$		
	y_{15} =		y_{165} =			$+0{\cdot}086_3$			+1			$+0{\cdot}322_0$		
						sum of +terms			sum of +terms			sum of +terms		
						sum of −terms			sum of −terms			sum of −terms		
									sum total					
					B	sum total $= E_1 \cos \alpha_1$			($+0{\cdot}235_7 \times$ sum total) $= E_3 \cos \alpha_3$			sum total $= E_5 \cos \alpha_5$		
					A^2	$(E_1 \sin \alpha_1)^2$			$(E_3 \sin \alpha_3)^2$			$(E_5 \sin \alpha_5)^2$		
					B^2	$(E_1 \cos \alpha_1)^2$			$(E_3 \cos \alpha_3)^2$			$(E_5 \cos \alpha_5)^2$		
					A^2+B^2	E_1^2			E_3^2			E_5^2		
					$\sqrt{A^2+B^2}$	E_1			E_3			E_5		
					$\frac{A}{B}$	$\frac{E_1 \sin \alpha_1}{E_1 \cos \alpha_1}$			$\frac{E_3 \sin \alpha_3}{E_3 \cos \alpha_3}$			$\frac{E_5 \sin \alpha_5}{E_5 \cos \alpha_5}$		
					$\tan^{-1}\left(\frac{A}{B}\right)$	α_1			α_3			α_5		
					y =		sin (θ + °) +			sin (3θ + °) +			sin (5θ + °)	

[*To follow* TABLE I.

Experimental Determination of Wave Form.—A number of different types of instruments have been devised for the purpose of determining wave forms experimentally, and some of these are described in Chapter XIII. One method which does not involve the use of any special type of measuring instrument, except an electrostatic voltmeter, will be described now.

Joubert's Contact Method.—The special piece of apparatus used consists of an arrangement whereby momentary contact is made once per cycle to an electrostatic voltmeter (see page 148), which consequently gives a deflection which is an indication of the instantaneous value rather than the R.M.S. value. By shifting the point of contact, the instantaneous values can be determined at other points of the wave, thus enabling it to be plotted.

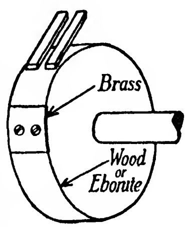


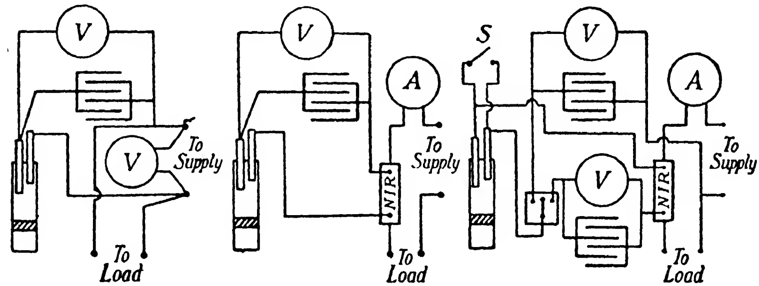
FIG. 72.—Joubert Contact.

One form of the Joubert contact consists of an insulated disc mounted on a metal hub, the whole being directly driven by means of a shaft extension of the machine supplying the power, or alternatively by a special synchronous motor (see Chapter XXIII). At one portion of the rim a little strip of brass is let in so that two copper brushes placed side by side and insulated from each other are connected together every time the strip of brass comes under them. One of these brushes is capable of being given a certain amount of lead by means of an adjustment, this regulating the duration of the interval of time over which contact is made. This affects the steadiness and magnitude of the voltmeter reading, and the best position is obtained by trial.

The two brushes are supported from a movable arm the position of which is indicated by means of a pointer moving over a scale. Assuming that the machine supplying the circuit has four poles, a displacement of 10° of the brushes corresponds to a phase displacement of 20°. If the brushes are moved round in the direction of rotation the instant of contact is deferred and the point obtained comes later in the wave, whilst if the brushes are moved in the opposite direction the reverse is the case.

In making a determination of the voltage wave the circuit is connected according to Fig. 73 (*a*), the electrostatic voltmeter measuring directly the instantaneous pressure across the mains, whilst the connections shown in Fig. 73 (*b*) are used when it is desired to make a determination of the current wave. Here the instantaneous voltage drop across a known non-inductive resistance is obtained, enabling the instantaneous current to be calculated. The other instruments indicated in the diagram serve to measure the R.M.S. values of the quantities concerned. An alternative method of doing this is to connect a short-circuiting switch across the two brushes and use one voltmeter throughout. A condenser is also placed in parallel with the voltmeter in order to maintain the deflection during the period when no electrical connection is made at the contact. The magnitude of the condenser should be determined by trial, but the best value will in most cases be found to be somewhere under a microfarad. By means of a change-over switch it can be arranged to take readings on the voltage and current waves together as indicated in Fig. 73 (*c*).

For the purpose of current wave form determinations, it is better to have a low reading electrostatic voltmeter, or else to set up the voltage by the addition of a constant voltage. This latter must be very constant, as otherwise the accuracy of the measurement is



(*a*) For Volt Wave. (*b*) For Current Wave. (*c*) For Volt and Current Waves.

FIG. 73.—Connections for Joubert Contact.

vitiated. A convenient piece of apparatus for this purpose is a battery of 100 cadmium cells arranged so that they can be switched in ten at a time.

EXAMPLES.

(1) Determine the R.M.S. value of an E.M.F. wave, the fundamental of which rises to 100 volts, and which contains a 20 per cent. third and a 10 per cent. fifth harmonic.

(2) An E.M.F. obeying the law

$$e = 100 \sin \omega t - 25 \sin 3\omega t$$

is applied to (*a*) an inductance of 0·1 henry, and (*b*) a capacitance of 100 microfarads, the frequency being 50. Determine the law of the current in each case, and plot the three waves on a common base in their proper phase positions.

(3) An E.M.F. containing a 15 per cent. third and a 2 per cent. seventh harmonic is applied to (*a*) an inductance of 0·1 henry, and (*b*) a capacitance of 100 microfarads, the frequency being 50. Determine the inductive and the capacitive reactances.

(4) Explain how the current wave becomes deformed in an iron-cored choking coil.

(5) Show how a flat-topped E.M.F. wave gives rise to a peaked current wave when applied to the terminals of a condenser.

(6) An E.M.F. represented by

$$e = 100 \sin \omega t + 20 \sin 3\omega t - 10 \sin 5\omega t$$

is applied to a series circuit consisting of a resistance of 5 ohms and inductive and capacitive reactances of 5 and 120 ohms respectively at fundamental frequency. Plot the E.M.F. and current waves in their correct positions. Assume the inductive reactance to be short-circuited and plot the new current wave.

(7) The following figures relate to the magnetization of an iron cored choking coil :—

Lines per sq. cm.	Exciting Current in Amperes.	Lines per sq. cm.	Exciting Current in Amperes.
0	2·0	18,000	2·6
4,000	2·6	16,000	1·1
8,000	3·2	12,000	− 0·1
12,000	3·9	8,000	− 0·8
16,000	5·1	4,000	− 1·4
18,000	6·5	0	− 2·0
20,000	10·0		

Determine the shape of the current wave when a sine wave of E.M.F. is applied, the maximum value of which corresponds to a flux density of 20,000 lines per sq. cm. Neglect I^2R effects.

(8) Develop, from first principles, an expression for the reactance of a condenser when the applied voltage is non-sinusoidal.

An E.M.F. containing a 15 per cent. third harmonic and a 10 per cent. fifth harmonic is applied to the plates of a 10 microfarad condenser. Calculate its reactance. The fundamental frequency is 50 cycles per second.

(9) An E.M.F. containing a 15 per cent. third harmonic and a 10 per cent. fifth harmonic has an R.M.S. value of 100 volts, and a frequency of 50 cycles per second. It is applied to the terminals of a 100-microfarad condenser. Calculate the R.M.S. value of the current.

(10) An E.M.F. wave contains a third harmonic of such a phase as to cause the resultant wave to be symmetrically flat-topped. This E.M.F. is applied to (*a*) an air-cored inductance of negligible resistance, (*b*) a non-inductive resistance, and (*c*) a bank of condensers. Sketch the three current waves in relation to the E.M.F. wave, giving reasons for the shapes of these current waves.

If the third harmonic in the E.M.F. wave has an amplitude equal to 30 per cent. of its fundamental, what are the amplitudes of the third harmonics in the three current waves?

CHAPTER X

POLYPHASE CURRENTS

Production of Two-Phase Currents.—The simple elementary alternating current generator consists of a single turn rotating with uniform speed in a bipolar magnetic field. Such a turn has a sinusoidal E.M.F. induced in it if certain conditions are fulfilled, and a *single-*

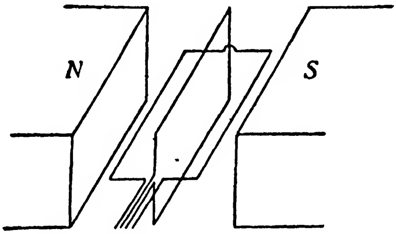


FIG. 74.—Simple Two-Phase A.C. Generator.

phase current is the result if the circuit is closed. Now imagine a second turn rigidly fixed to the first, the planes of the two coils being at right angles (see Fig. 74). This second turn will also produce a sinusoidal E.M.F. of the same magnitude as the first, the only difference being that when one coil has its maximum E.M.F. induced, the E.M.F. in the second is zero. In other words, there is a phase difference of 90° between the two E.M.F.'s induced in the two coils. The two E.M.F.'s are represented graphically in Fig. 75. Each coil may be connected across a non-inductive resistance producing a current, and these two currents will also have a phase difference of 90°, since they are in phase with their respective E.M.F.'s. Such a circuit is termed a *two-phase* circuit. If reactance is included in equal amounts in each circuit so that the magnitudes and phases of the two currents are equal, the two currents will still be 90° out of phase with each other. Such a combined circuit is

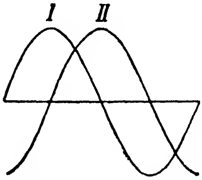


FIG. 75.—Two-Phase E.M.F.'s.

the current flowing *outwards* is represented by the vector OI_R, which is minus the current flowing inwards, since a reversal of the current is equivalent to changing its phase by 180°. The phase difference between the current in the third wire and either of the other two is 135°. In one case it is a lead and in the other a lag.

Power in a Two-Phase Circuit.—The power developed in a two-phase circuit is the sum of the powers developed in each phase

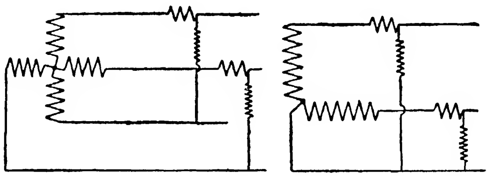


FIG. 80.—Measurement of Power in a Two-Phase Circuit.

separately and can be determined by two wattmeters connected so that each measures the power in one phase, as shown in Fig. 80. Sometimes these two wattmeters are combined in one instrument, both tending to deflect the pointer in the same direction, so that the resultant deflection is proportional to the total power. In measuring the power in this manner, it is not necessary to have the circuits balanced.

If the circuits are balanced, the power factor of each phase will be the same; but if the circuits are unbalanced, the resultant power factor will be given by

$$\frac{\text{Total Watts}}{\text{Volt-amperes of phase I} + \text{Volt-amperes of phase II}}.$$

Production of Three-Phase Currents.—If, instead of placing two coils at right angles on the elementary alternating current generator,

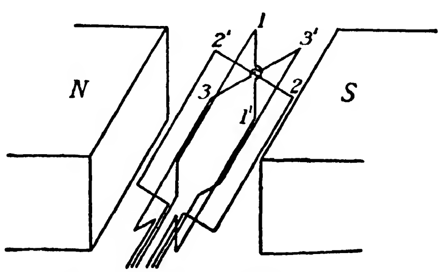


FIG. 81.—Simple Three-Phase A.C. Generator.

three coils had been placed on it mutually inclined at 120° to each other, as in Fig. 81, then the R.M.S. voltage induced in all three would be the same, but there would be a phase difference of 120°

between each pair of coils. Such a combination is termed a *three-phase* system, and the three E.M.F.'s are graphically represented in Fig. 82 both in curve form and in a vector diagram.

A three-phase circuit may be represented diagrammatically in

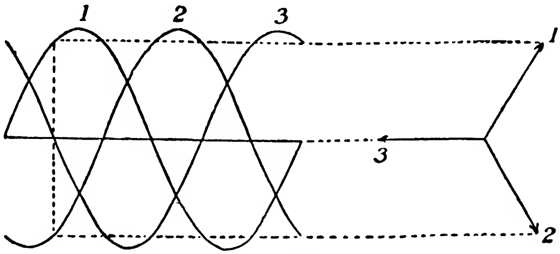


FIG. 82.—Three-Phase E.M.F.'s.

the way shown in Fig. 83, where the three coils of the simple A.C. generator are independently connected to three equal resistances. According to this method of connection, six wires are required for the transmission of the power, but all the return leads can be combined in one without upsetting the electrical conditions, since they are only joined on one pole. Thus the six wires can be reduced

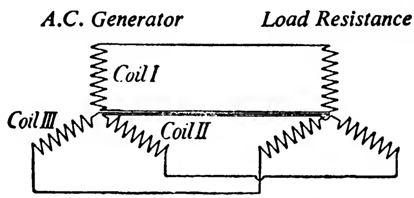


FIG. 83.—Three-Phase Circuits.

to four. The current returning through this fourth wire is the vector sum of all the currents flowing *outwards* in the other three wires. Assuming that the system is balanced, these three currents will be equal and will have a phase displacement of 120° with each other. Thus the current returning by the fourth wire can be represented by the expression

$$i = I \sin \omega t + I \sin (\omega t - 120°) + I \sin (\omega t - 240°).$$

This can be expanded with the following result :—

$$i = I [\sin \omega t + \sin \omega t \cos 120° - \cos \omega t \sin 120° + \sin \omega t \cos 240° - \cos \omega t \sin 240°$$

$$= I [\sin \omega t - \frac{1}{2} \sin \omega t - \frac{\sqrt{3}}{2} \cos \omega t - \frac{1}{2} \sin \omega t + \frac{\sqrt{3}}{2} \cos \omega t]$$

$$= 0.$$

Thus at every instant the current returning by way of the fourth wire is zero, and consequently this wire can be dispensed with. In this way, the important result is arrived at that only three wires are necessary to transmit power by means of a three-phase system. In certain cases in practice, however, where unbalanced circuits are dealt with, four conductors are used.

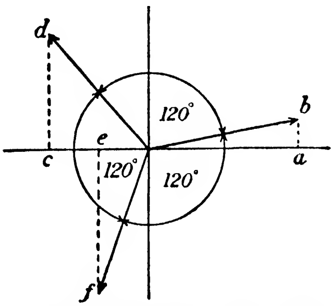


FIG. 84.—Vector Diagram of Three-Phase E.M.F.'s.

The three line wires may also be regarded from the point of view that each in turn serves as the return wire for the other two, for it is seen that the instantaneous current flowing outwards in any one is always equal and opposite to the vector sum of the instantaneous currents flowing outwards in the other two. This can also be seen from the vector diagram shown in Fig. 84, where, considering the vertical components, *ab* + *cd* is equal to *ef*, and, considering the horizontal components, *oa* is equal to *oc* + *oe*.

Star System of Connection.—In developing the circuit shown in Fig. 83, the first operation was to join all the inner ends together

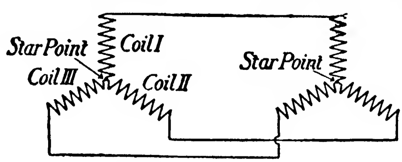


FIG. 85.—Three-Phase Star Connections.

into what is known as a *star point* and then to omit the fourth wire. The resulting system of connections, shown in Fig. 85, is known as a *star-connected* circuit, or sometimes as a Y-connected circuit.

An important relationship to be established is that existing between the volts per coil, or the phase volts, and the volts between any two line wires, or the line volts. Referring to the vector diagram (see Fig. 86), it is seen that the voltage between lines is equal to the vector *difference* of the two phase voltages concerned, for the vectors represent the E.M.F.'s acting away from the star point. Reversing E_2, therefore, and adding the voltage thus obtained to E_1, the vector E_L is obtained, and it is seen that there is a phase difference of 30° between E_1 and E_L. By dropping a perpendicular from E_1 on to OE_L it is seen that $\frac{E_L}{2}$ is equal to $E_1 \cos 30° = \frac{\sqrt{3}}{2}E_1$, and therefore E_L is equal to $\sqrt{3}E_1$. The important fact is therefore established that the line voltage is $\sqrt{3}$ times the volts per coil or per phase, and that there is a difference of phase of 30° in each case.

If the load at the receiving end of the line consists of three non-inductive resistances arranged in star, then the current through each resistance will be in phase with the voltage across it, and the same will hold good at the generator end of the line. But since there is a phase difference of 30° between the coil volts and the line volts, it follows that in a non-inductive load there is a phase

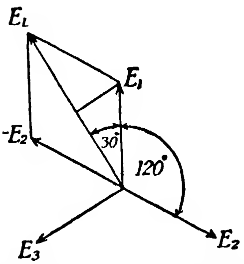


FIG. 86.—Line and Phase Volts for Star Connection.

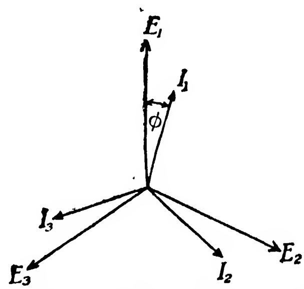


FIG. 87.—Vector Diagram showing Currents in Star-connected System.

difference of 30° between the line current and the line voltage. The voltage across lines I and II will lead the current in I by 30°, whilst the voltage across lines I and III will lag behind the current in I by 30°, as can be seen from Fig. 86. If the load circuit contains reactance as well as resistance, the current will lag behind the coil voltage by an angle ϕ, as shown in the vector diagram in Fig. 87. The angle of phase difference between current and line voltages will be $30° + \phi$ and $30° - \phi$.

Delta (or Mesh) System of Connection.—An alternative method of connecting the three coils of the simple alternating current generator is to connect the rear end of I to the front end of II, the rear end of II to the front end of III, and the rear end of III to the front end of I, thus making a closed local circuit. The three line wires are connected to the three joining points (see Fig. 88). Such a system is called a *mesh-connected* or *delta-* (from the Greek letter Δ) *connected* system. At first sight it appears as if there is a short circuit formed by the three coils, since they are all connected in series with one another and the circuit is closed. But if the three E.M.F.'s are added together at any instant it will be found that they always add up to zero. In other words, the sum of the E.M.F.'s of any two coils is always equal and opposite to the E.M.F. of the third. Here the line voltage is obviously equal to the phase voltage, but the current in each line wire is the resultant of the currents in two coils. Again, it will be seen that the current flowing out of line *A* is the vector difference of the currents in coils I and III. In order to determine what this is, reverse I_3 and add it to I_1, when the resulting

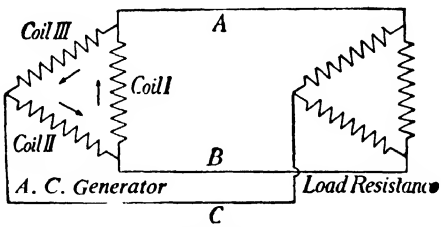


FIG. 88.—Three-Phase Delta or Mesh Connections.

line current will be found to be equal to $\sqrt{3}I_1$, in the same way that the line voltage was found to be equal to $\sqrt{3}$ times the coil voltage for a star-connected system. If three non-inductive resistances are connected up in the same manner at the receiving end, they will each take a current which is in phase with the voltage across their respective terminals. If reactance is present in equal amounts in the three circuits the current will lag behind the voltage by an angle ϕ, and combining each pair of phase currents the three line currents are obtained, the resulting vector diagram being shown in Fig. 89.

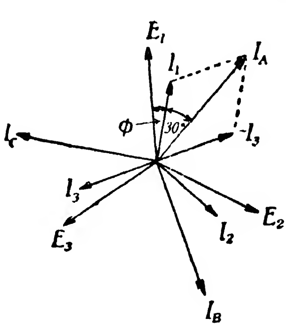


FIG. 89.—Vector Diagram of Three-Phase Delta- or Mesh-connected System.

The two systems of connection described above are interchangeable, and it is possible to have a star-connected generator with a delta-connected load and *vice versa*.

Power in a Star-connected Circuit.—The power given out by the simple alternating current generator when star-connected is the sum of the powers given out by the three coils. Assuming that both resistance and reactance are present so as to make the current lag by an angle ϕ behind the coil voltage, it is seen that the power output of each coil is $E_P I_P \cos \phi$, where E_P is the phase voltage and I_P is the phase current, which is also equal to I_L, the line current. But $E_P = \frac{1}{\sqrt{3}}E_L$ where E_L is the line voltage, and therefore

$$P = 3 \times \frac{1}{\sqrt{3}}E_L I_L \cos \phi$$
$$= \sqrt{3}E_L I_L \cos \phi.$$

It is to be remembered that the power factor is the cosine of the angle of phase difference between the coil voltage and the current, not the line voltage and the current.

Power in a Delta-connected System.—Here again the total power is the sum of the powers in the three separate phases and is therefore equal to

$$3E_P I_P \cos \phi.$$

But the line current is equal to $\sqrt{3}I_P$, and consequently the total power can be rewritten

$$3E_L \frac{I_L}{\sqrt{3}} \cos \phi$$
$$= \sqrt{3}E_L I_L \cos \phi,$$

the same as before.

Thus the power in a three-phase system is the same for both star and delta if the line voltage, line current, and power factor are the same. In the one case the line voltage is $\sqrt{3}$ times the phase voltage, whilst in the other the line current is $\sqrt{3}$ times the phase current.

Measurement of Power in a Three-Phase System.—The first obvious method of measuring the power in a three-phase system is to use three wattmeters, so that each measures the power developed or absorbed by one phase. The connections for doing this in the case of a star system are shown in Fig. 90, and at first sight it appears as if it is necessary to bring out a fourth wire from the star point. But the three ends of the volt coils of the wattmeters form a star point in themselves, and, considering them as a very small load, it is seen that no current flows through the fourth wire, so that it may be omitted. In other words, the potential of the star point formed by the wattmeter volt coils is the same as that at the generator or load ends.

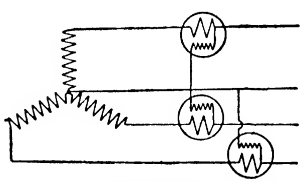


FIG. 90.—Power Measurement in Star System.

If the system were balanced one wattmeter would be sufficient if the star point were available, the reading being multiplied by 3. If a lead is brought out from the generator star point the connections would be those shown in Fig. 91 (*a*). On forming an auxiliary star point by means of three high resistances the connections shown in Fig. 91 (*b*) could be adopted. This obviates the necessity of bringing a fourth wire out from the generator. But since the resistance of the volt coil is comparable with each of these fine wire resistances, the value of the resistance in parallel with the volt coil must be chosen higher than the other two, so that when placed in parallel with the volt coil the combined resistance is equal to each of the other two. A further simplification can be made as shown in Fig. 91 (*c*). Here the volt coil is itself used as one of the three resistances, the values of the other two each being made equal to it.

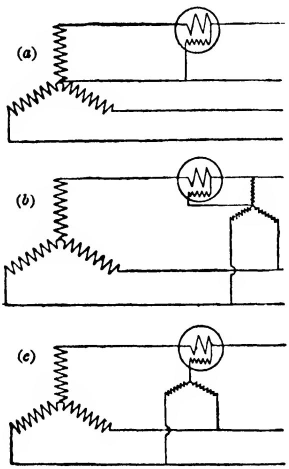


FIG. 91.—Wattmeter Connections for Balanced Three-Phase System.

Wattmeters are sometimes provided with these two extra resistances so that they can measure directly the power in a balanced three-phase circuit.

In the case of a delta-connected system three wattmeters could be used as in the previous case, but this would necessitate opening the three branches of the delta for the purpose of introducing the three current coils as shown in Fig. 92, and this would be very inconvenient, apart from the fact that it could only be done at either the generator or the load end of the line. But since the power in a delta system is the same as that in a star system, there is no need to resort to this arrangement, and the other more convenient methods can be adopted.

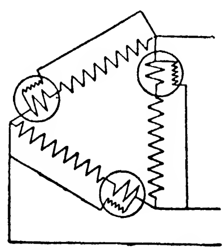


FIG. 92.—Power Measurement in a Delta System.

Two-Wattmeter Method of Measuring Power.—By far the most commonly used method of measuring the power in a three-phase system is that known as the *two-wattmeter method.* Each wattmeter has its current coil in a different lead, whilst the two volt coils are connected at one end to their respective current coils, the other ends being connected to the third lead, which has no current coil in it, as shown in Fig. 93. It will now be shown that the sum of the two wattmeter readings gives the total power, and that this measurement is independent of balance and wave form if the wattmeters are themselves accurate.

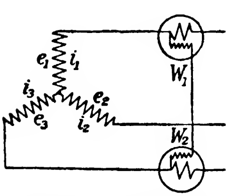


FIG. 93.—Two-Wattmeter Method of Measuring Power.

Let the instantaneous volts measured from the star point to the three line wires be e_1, e_2 and e_3 respectively, and let the currents flowing outwards in the three arms be i_1, i_2 and i_3 respectively. Then wattmeter W_1 measures $(e_1 - e_2)\, i_1$ and wattmeter W_2 measures $(e_3 - e_2)\, i_3$. The sum of the two readings is therefore

$$(e_1 - e_2)\, i_1 + (e_3 - e_2)\, i_3$$
$$= e_1 i_1 - e_2 (i_1 + i_3) + e_3 i_3.$$

But
$$i_2 = -(i_1 + i_3).$$

Therefore the sum of the two readings is

$$e_1 i_1 + e_2 i_2 + e_3 i_3,$$

which is the total power at that instant, and the wattmeters will indicate the average value of this quantity, in which no assumption as to balance or wave form is made.

The same reasoning holds good for a delta connection, for it does not matter, as far as the wattmeters are concerned, whether the power is generated in a delta or star system.

Vector Diagram for Two-Wattmeter Method.—It is interesting to observe what each wattmeter is really doing in this measurement, and this can be seen by a reference to the vector diagram in Fig. 94. Assuming again a balanced circuit and sinusoidal wave forms, it is seen that wattmeter W_1 measures the product of I_1, E_{L_1} and the cosine of the angle of phase difference between them. If E_{L_1} had

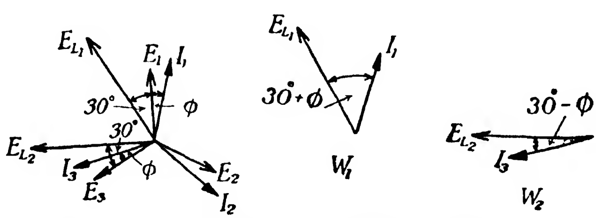


FIG. 94.—Vector Diagram for Two-Wattmeter Method.

been drawn in exactly the opposite direction, it would have been more than 90° out of phase with I_1 and would correspond to a negative power and a backward reading of the wattmeter. Similarly wattmeter W_2 measures the product of I_3, E_{L_2} and the cosine of the angle of phase difference between them. The quantities which the wattmeters record are shown separately in Fig. 94, where it is seen that

$$W_1 = EI \cos (30° + \phi)$$

and
$$W_2 = EI \cos (30° - \phi).$$

Therefore

$$\begin{aligned} W_2 + W_1 &= EI\,\{\cos(30^\circ - \phi) + \cos(30^\circ + \phi)\} \\ &= EI \times 2\cos 30^\circ \cos\phi \\ &= \sqrt{3}EI\cos\phi, \end{aligned}$$

which is again equal to the total power.

When ϕ attains a value of over 60°, $\cos(30^\circ + \phi)$ becomes negative and wattmeter W_1 commences to read backwards. In order to make the measurement, therefore, either the current coil or the volt coil must be reversed, and the forward reading thus obtained must be *subtracted* from that of the other wattmeter in order to obtain the total power. In the same way, when the angle of lead becomes greater than 60° the wattmeter W_2 commences to read backwards and the same procedure must be adopted.

Measurement of Power Factor from Wattmeter Readings.—On a sine wave hypothesis the measurement of the power factor devolves into a determination of $\cos\phi$, and this can be obtained from the two wattmeter readings mentioned above, for

$$\begin{aligned} W_2 - W_1 &= EI\,\{\cos(30^\circ - \phi) - \cos(30^\circ + \phi)\} \\ &= EI \times 2\sin 30^\circ \sin\phi \\ &= EI\sin\phi. \end{aligned}$$

Therefore

$$\frac{W_2 - W_1}{W_2 + W_1} = \frac{EI\sin\phi}{\sqrt{3}EI\cos\phi} = \frac{\sin\phi}{\sqrt{3}\cos\phi},$$

$$\sqrt{3}\,\frac{W_2 - W_1}{W_2 + W_1} = \tan\phi$$

$$3\left(\frac{W_2 - W_1}{W_2 + W_1}\right)^2 = \tan^2\phi$$

$$1 + 3\left(\frac{W_2 - W_1}{W_2 + W_1}\right)^2 = 1 + \tan^2\phi = \sec^2\phi$$

$$\frac{1}{1 + 3\left(\frac{W_2 - W_1}{W_2 + W_1}\right)^2} = \frac{1}{\sec^2\phi} = \cos^2\phi,$$

and

$$\cos\phi = \frac{1}{\sqrt{1 + 3\left(\frac{W_2 - W_1}{W_2 + W_1}\right)^2}}.$$

Letting r equal $\frac{W_1}{W_2}$ and dividing top and bottom of the part in the round brackets by W_2 we get

$$\cos\phi = \frac{1}{\sqrt{1 + 3\left(\frac{1 - r}{1 + r}\right)^2}}.$$

It is convenient to divide by the larger reading so that $\frac{W_1}{W_2}$ is always less than unity.

Another way of stating this relationship is obtained by multiplying top and bottom of the above equation by $\sqrt{(1+r)^3}$. Then

$$\cos\phi = \frac{\sqrt{(1+r)^3}}{\sqrt{\left\{1+3\left(\frac{1-r}{1+r}\right)^2\right\}\times(1+r)^3}}$$

$$= \sqrt{\frac{(1+r)^3}{(1+r)^3+3(1-r)^2\times(1+r)}}$$

$$= \sqrt{\frac{(1+r)^3}{4(1+r^3)}}$$

$$= \frac{1}{2}\sqrt{\frac{(1+r)^3}{1+r^3}}.$$

When the angle of lag is 60° the wattmeter W_1 is measuring

$$EI\cos(30° + 60°) = 0,$$

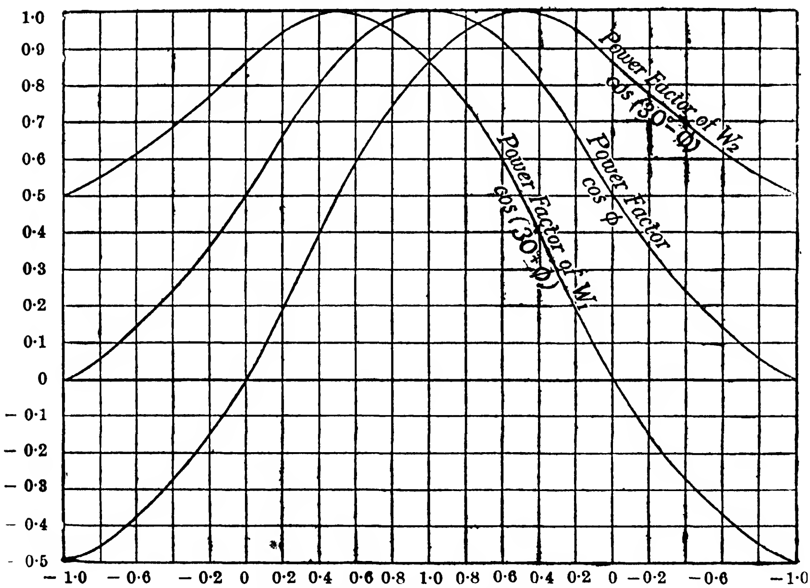


FIG. 95.—Power Factor from Wattmeter Readings.

and thus, when the power factor falls to 0·5 = cos 60°, the indication of the first wattmeter is zero. This result is also obtained by putting r equal to zero in the last equation. If the power factor is less than 0·5, it follows, from the same equation, that r must have a negative value, indicating that one wattmeter is reading backwards.

Again, when the angle of lead is 60° the wattmeter W_2 measures

$$EI \cos\{30° - (-60°)\} = 0,$$

and for power factors of less than 0·5 with the current leading, the second wattmeter gives a negative indication.

Fig. 95 shows graphically the relation between the power factor and the ratio of the two wattmeter readings, this ratio always having a fractional value. The power factors under which the two wattmeters themselves are working are also shown, it being seen that each of them reverses in sign over a portion of the range.

Reactive Volt-amperes.—The total volt-amperes of a single-phase circuit are given by EI and the power by $EI \cos \phi$. The reactive component of the volt-amperes is given by $EI \sin \phi$, and this quantity is usually referred to as the VAr (volt-amperes reactive). The same applies to three-phase circuits, where the kilovar ($kVAr$) is now commonly used. The most convenient way to obtain the $kVAr$ in a three-phase system is from the difference of the two wattmeter readings when these are connected so as to measure the total power.

It was shown on p. 119 that, assuming sine waves and balanced loads, the quantities recorded by the two wattmeters are $EI \cos (30° + \phi)$ and $EI \cos (30° - \phi)$ respectively. The difference of these two readings is

$$W_2 - W_1 = EI \cos (30° - \phi) - EI \cos (30° + \phi)$$
$$= EI \sin \phi,$$

and $\sqrt{3}$ times the difference of the two wattmeter readings gives the total $kVAr$ in the circuit. This method is, however, not accurate for unbalanced loads.

Three-Phase Load.—The conditions necessary for a three-phase load to be balanced are that the resistances of the three arms must be all equal, the inductive reactances must be all equal and the capacitive reactances must be all equal. It is not sufficient that the impedances should be equal, as this result might be attained with different proportions of resistance and reactance.

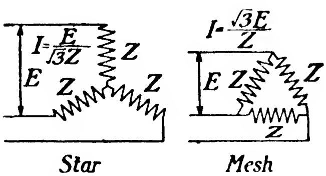


FIG. 96.—Impedances in Star and Delta.

The same impedances arranged in delta produce a larger current for a given E.M.F. than when arranged in star, for consider the general case where the impedance of each branch is Z and the line volts are E (see Fig. 96).

When arranged in star, the voltage across each branch is $\dfrac{E}{\sqrt{3}}$

and the current is $\frac{E}{\sqrt{3}Z}$, but when they are arranged in delta the current per branch is $\frac{E}{Z}$, so that the line current is $\sqrt{3}\frac{E}{Z}$, or three times the previous value. Thus the equivalent impedance of the system is reduced to one-third of its original value by changing over from star to delta.

Unbalanced Three-Phase Circuit.—An unbalanced system is produced when the impedances in the different branches are unequal. The three currents may be of different magnitude and may also lag behind their respective E.M.F.'s by different amounts, but the vector sum of all the currents flowing outwards must add up to zero if there are only three wires. In order that this condition may be brought about, the voltage is usually distributed in an unsymmetrical manner over the three branches, the general effect being to reduce the voltage on the heavily loaded side. One result of this is that the potential of the unsymmetrical star point is different from what it would be if the system were balanced.

The average power factor is a term which has rather a dubious meaning, but it may be defined as the

$$\frac{\text{Total power}}{\text{Sum of the volt-amperes for all phases}}.$$

This will not necessarily be the same as the average of the three individual power factors.

The line voltages may also be unbalanced, a difference in magnitude giving rise to an alteration in phase angle. The vector sum of the three line voltages is always zero, so that these can be represented in the form of a triangle ABC as shown in Fig. 97. Now suppose three circuits to be connected in delta between these three points. The current flowing between lines A and B lags by an angle ϕ and is represented by AB'. If the magnitudes of the other two currents are known, the current triangle $AB'C'$ can be drawn, for the vector sum of the three currents is zero, if only three line wires are employed. The phase angles of the other two currents are thus fixed.

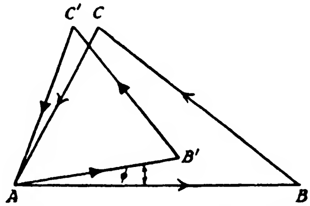


FIG. 97.—Unbalanced Three-Phase System. Load in Delta.

If the load circuit is connected in star and the three currents are known, they can again be drawn in the form of a closed triangle (see Fig. 98) as shown at (*b*), the triangle ABC in (*a*) representing the line voltages as before. The three currents can now be re-drawn in their correct relative phase in the form of a star as shown at (*c*).

If the power factors of the three legs of the star are known, then the phases of the three star voltages can be determined, these voltages being represented by E_1, E_2, E_3, leading their respective currents by ϕ_1, ϕ_2 and ϕ_3. (It should be noted that their magnitudes are not known unless the values of the three impedances are known.) If a tracing of these three voltage vectors be made and placed over the voltage triangle at (*a*), then the three lines of the star must pass

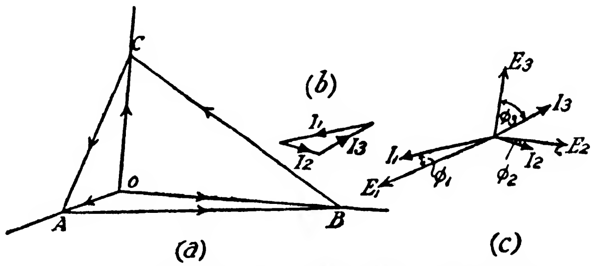


FIG. 98.—Unbalanced Three-Phase System. Load in Star.

through the three angles of the triangle. When the tracing is adjusted to bring this about, the star point can be marked by pricking through the star point of the tracing. The lengths of the lines OA, OB, OC now give the magnitudes and phases of the three star voltages.

Symmetrical Components.—The instantaneous sum of the line currents is zero in a three-phase three-wire system, and the vectors of these currents form a closed triangle, whether the load is balanced or not. If the star points of the generator and load be connected, however, the system becomes converted into a four-wire system. If the generator is earthed at its star point, then an earth fault on the system converts it into a four-wire one, the fault current to earth being the equivalent of a current in a fourth (neutral) conductor. In such a case the vector sum of the three line currents is no longer zero, and the phase voltages, which also formed a closed triangle in a three-wire transmission, become independent.

Any system of three-phase currents, whether balanced or not, and irrespective of whether there is, or is not, any current in a fourth conductor, can be resolved into three component systems, each of which is balanced and symmetrical. The first is a symmetrical three-phase system of currents having normal, or positive, sequence. By this is meant that the currents in lines A, B and C rise to a positive maximum in that order. The second symmetrical group of components has a phase sequence opposite to normal, and these are called the negative sequence components. The third group consists of three line currents all equal and in phase, these being called the zero phase sequence components. It can be proved that any system of three-phase currents can be resolved into components of the type mentioned above, the magnitudes and phases of the various components depending upon the magnitudes and phases of the actual currents flowing.

Fig. 99 shows the vectors of these three components. The vectors a_1, b_1 and c_1 form the positive sequence components, these

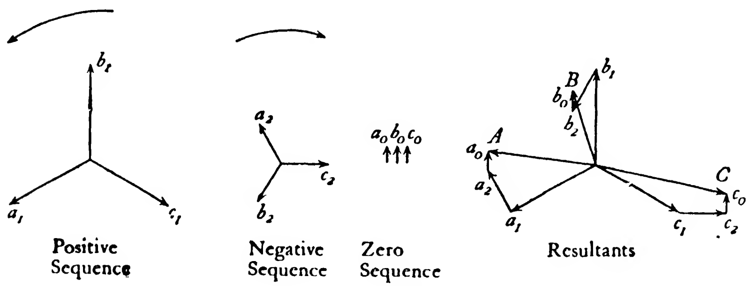


FIG. 99.—Symmetrical Components.

all being equal in magnitude and 120° out of phase with one another. The phase sequence is positive, since a_1 leads b_1. The vectors a_2, b_2, c_2 form the negative sequence system. This is also symmetrical, but the phase rotation is reversed. The vectors a_0, b_0, c_0 constitute the zero sequence system, these components all being equal and in phase. The resultant current in line A is given by the vector A, which is the sum of a_1, a_2 and a_0. Similarly, B is the sum of b_1, b_2, and b_0, and C is the sum of c_1, c_2 and c_0. Any degree of dissymmetry can be dealt with in this manner.

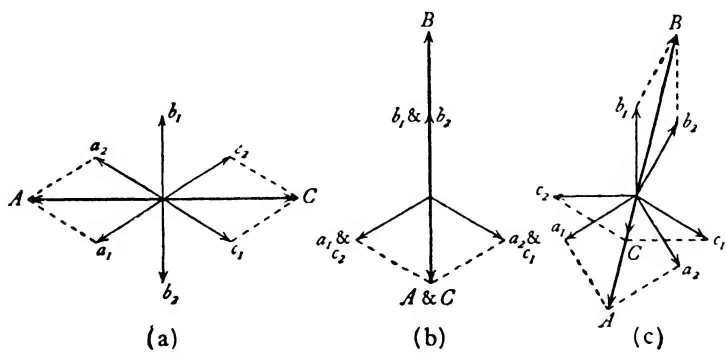
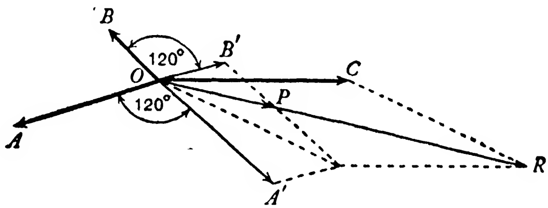


FIG. 100.—Symmetrical Components of Single-Phase Loads.

Symmetrical Components of Single-phase Loads.—When a single-phase load is taken from a three-phase system, the symmetrical components of current consist of positive and negative sequence components which are equal in magnitude. This can be shown by adding these components together with any relative phase angle whatsoever, the zero sequence components being absent. Three examples are shown in Fig. 100 of unsymmetrical loading. At (*a*) the line currents in lines *A* and *C* are equal in magnitude but 180° out of phase. The resultant current in line *B* is zero. This represents a single-phase load connected between lines *A* and *C*. At (*b*) the currents in lines *A* and *C* are equal and in phase, but are 180° out of phase with the current in line *B*, which is equal in magnitude to the sum of the currents in lines *A* and *C*. At (*c*) the current in line *B* is again equal to the sum of the currents in lines *A* and *C*, but the two latter currents are not themselves equal.

Resolution of Unsymmetrical Three-phase Systems.—In the general case, the current in the fourth conductor is the vector sum of the currents in the other three line conductors. If the vectors



OR = 3 × Positive Symmetrical Component
OP = Positive Symmetrical Component

FIG. 101.—Resolution of Unsymmetrical Three-Phase System.

of the three line currents be added together, they will form an unclosed triangle. The resultant current consists solely of three zero sequence components, and each zero sequence component is one-third of this resultant, since they are all equal and in phase. These currents can now be subtracted from the three line currents, leaving currents which consist only of positive and negative sequence components. The vectors *OA* and *OB* are now each rotated through 120° *towards OC* (*see* Fig. 101), the three vectors *OA′*, *OB′* and *OC* being added together to give the resultant *OR*. This, when divided by three, gives the positive symmetrical component of *OC*, as *OP*. The difference between *OC* and *OP*, *i.e. PC*, gives the negative symmetrical component of *OC*. The positive symmetrical components of *OA* and *OB* are obtained by drawing lines at 120° to *OP* and equal to it in length. The negative symmetrical components of *OA* and *OB* are obtained by drawing lines at 120° to *PC* and equal to it in length.

Retarding the phase of *OB* by 120° is equivalent to advancing its negative symmetrical component by the same angle. Similarly, advancing the phase of *OA* by 120° is equivalent to retarding its negative symmetrical component by the same angle. In the resultant OR, therefore, the negative symmetrical components of OA', OB' and OC are now all exactly 120° out of phase with one another, and their sum must equal zero. In the same way the positive symmetrical components of OA', OB', and OC are all in phase, so that the positive symmetrical component of OC is one-third their sum.

If the vectors OA and OB had each been turned *away* from OC, a similar construction would have given the negative symmetrical component, but this is unnecessary, since it is given directly by subtracting OP from OC.

Elimination of Third Harmonic.—All the harmonics which are divisible by 3 are absent in a balanced three-phase system, providing

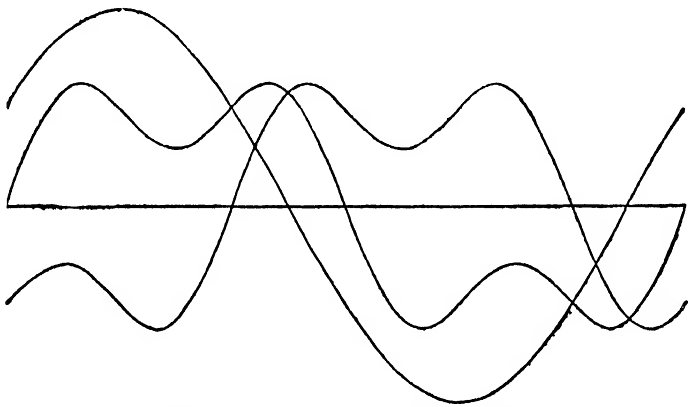


FIG. 102.—Disappearance of Third Harmonic in Three-Phase System.

no fourth conductor is used. The voltage between any two lines is the vector difference of two phase voltages acting away from the star point, and these voltages differ in phase by a third of a cycle. But if a third harmonic is present in the E.M.F. wave, there will be a phase difference of one complete cycle between these harmonic voltages in the two phases, and hence they will always be equal and opposite. This can be demonstrated graphically by subtracting two equal voltages containing third harmonics, as in Fig. 102, the phase difference being 120°. The resulting curve shows no third harmonic.

It can also be seen by adding together the voltages as follows :—

$$
\begin{aligned}
e_1 &= E_1 \sin \omega t + E_3 \sin (3\omega t \times \alpha) \\
e_2 &= E_1 \sin (\omega t - 120°) + E_3 \sin \{3 (\omega t - 120°) + \alpha\} \\
&= E_1 \sin (\omega t - 120°) + E_3 \sin (3\omega t + \alpha - 360°) \\
&= E_1 \sin (\omega t - 120°) + E_3 \sin (3\omega t + \alpha)
\end{aligned}
$$

$$e_1 - e_2 = E_1 \{\sin \omega t - \sin (\omega t - 120°)\} + E_3 \{\sin (3\omega t + \alpha) - \sin (3\omega t + \alpha)\}$$
$$= E_1 \{\sin \omega t - \sin (\omega t - 120°)\}$$
$$= 2E_1 \cos (\omega t - 60°) \sin 60°$$
$$= \sqrt{3}E_1 \cos (\omega t - 60°)$$
$$= \sqrt{3}E_1 \sin (\omega t + 30°).$$

The third harmonic has obviously disappeared.

In a similar manner, the ninth, fifteenth, etc., harmonics disappear, so that the only ones possible in a balanced three-phase system are the fifth, seventh, eleventh, thirteenth, etc.

This is not necessarily the case when a fourth wire is employed to join the star points, for although no third harmonic can exist in the voltage between lines, yet it can exist between any line wire

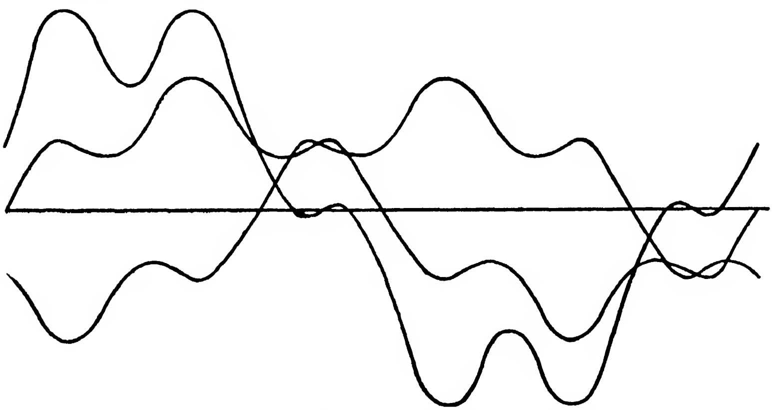


FIG. 103.—Reversal of Fifth Harmonic in Three-Phase System.

and the fourth conductor, and its presence may produce a harmonic current flowing round the circuit, consisting of the line wire and the fourth conductor together with the apparatus joining them.

Reversal of Fifth Harmonic.—When a fifth harmonic occurs in the phase voltage between the line and the star point, it appears in the line voltage with its phase reversed, *i.e.* if the harmonic is a peaking one in the phase voltage it becomes a dimpling or a flat-topping one in the line voltage, and *vice versa.* This is shown graphically in Fig. 103, where two equal voltages containing fifth harmonics are shown with a phase difference of 120°. The resultant line voltage is the difference of the two, and the peaked phase voltages are shown to produce a dimpled line voltage.

The effect can also be demonstrated by adding the voltages together as follows :—

$$e_1 = E_1 \sin \omega t + E_5 \sin (5\omega t + \alpha)$$
$$e_2 = E_1 \sin (\omega t - 120°) + E_5 \sin \{5 (\omega t - 120°) + \alpha\}$$

$$= E_1 \sin(\omega t - 120°) + E_5 \sin(5\omega t - 600° + \alpha)$$
$$= E_1 \sin(\omega t - 120°) + E_5 \sin(5\omega t + 120° + \alpha).$$
$$e_1 - e_2 = E_1 \{\sin \omega t - \sin(\omega t - 120°)\}$$
$$+ E_5 \{\sin(5\omega t + \alpha) - \sin(5\omega t + 120° + \alpha)\}$$
$$= 2\,E_1 \cos(\omega t - 60°) \sin 60°$$
$$+ 2\,E_5 \cos(5\omega t + \alpha + 60°) \sin(-60°)$$
$$= \sqrt{3}\,E_1 \cos(\omega t - 60°) - \sqrt{3}E_5 \cos(5\omega t + \alpha + 60°)$$
$$= \sqrt{3}\,E_1 \sin(\omega t + 30°) - \sqrt{3}E_5 \sin(5\omega t + \alpha + 150°)$$
$$= \sqrt{3}\,E_1 \sin(\omega t + 30°) + \sqrt{3}E_5 \sin(5\omega t + \alpha - 30°).$$

If $\alpha = 0$, then the fundamental and the harmonic reach a positive maximum value simultaneously in the phase voltage, but in the line voltage the fundamental reaches a positive maximum when $\omega t = 60°$. The value of the fifth harmonic is now

$$\sqrt{3}E_5 \sin(5 \times 60° - 30°) = \sqrt{3}E_5 \sin 270°,$$

which shows it to be at its maximum value in the reverse direction. The relative magnitude of the harmonic is, however, unchanged, but its phase is reversed in the line voltage.

In the same way it can be shown that the seventh harmonic is reversed in the line voltage. The ninth harmonic is eliminated, and the eleventh and thirteenth appear unchanged.

Oscillating Neutral.—If three equal resistances are connected in star on a three-phase system, the star point being insulated, the potential of the latter automatically assumes the same value as that of the star point of the generator supplying the system. In practice, this star point is usually earthed. There is, therefore, no p.d. between the insulated star point of the three resistances and earth.

Now suppose that the three resistances are replaced by three choking coils, the voltage being sufficient to cause magnetic saturation in their cores. Assume also that the line voltages are sinusoidal. It was shown on page 92 that in these circumstances the magnetizing current requires to have a third harmonic, and to produce this a third harmonic voltage is necessary. This does not exist in the voltage between line and line, but such a voltage is set up between line and star point in each phase of the load. The fundamental phase voltages acting away from the star point are mutually 120° out of phase with each other, and so these three harmonic voltages are $3 \times 120° = 360°$ out of phase with each other. In other words, they are all in phase, and as they are all equal, they cancel out between lines, leaving the line voltage undistorted.

The voltage between line and star point at the generator consists solely of a fundamental component, but the voltage between the lines and the star point of the saturated choking coils now consists of an equal fundamental component together with a third harmonic component. There is, therefore, a third harmonic voltage existing between the insulated star point and that of the generator. If the latter is earthed, the potential of the insulated star point oscillates about the neutral potential at third harmonic frequency. This insulated star point is known as an *oscillating neutral.*

Six-Phase Currents.—Six-phase currents can be obtained in the same way as two- and three-phase currents, by having six coils spaced 60° apart on the simple alternating current generator. But coil No. 4 is exactly the same as coil No. 1, with the exception that it is reversed, and thus six separate phases can be obtained from a three-phase source provided they are kept insulated. By means of transformers (see Chapters XIV–XVI) it is possible to produce six connected phases from a three-phase supply, and in most cases in practice where six-phase currents are employed they are obtained from a three-phase source. They are used in connection with certain types of apparatus (*e.g.* rotary converters, see Chapter XXV) the operation of which is improved by the use of a large number of phases.

The power in a six-phase system is six times the power in each phase, the latter being the product of the line current, the voltage between line and star point and the power factor. But as this voltage is the same as that between adjacent phases, as is seen on reference to the vector diagram, the total power is

$$P = 6EI \cos \phi,$$

where E is the voltage between adjacent phases, I is the line current and $\cos \phi$ is the power factor of each phase.

The total power may also be obtained by considering the system as a double three-phase system. Thus

$$P = 2 \times \sqrt{3}E'I \cos \phi,$$

where E' is the voltage between alternate phases. This voltage is $\sqrt{3}$ times the voltage between line and star point, and, consequently, between adjacent phases. Thus

$$E' = \sqrt{3}E$$

and

$$P = 2 \times \sqrt{3} \times \sqrt{3}E \times I \cos \phi = 6EI \cos \phi$$

as before.

Greater numbers of phases are also occasionally met with in practice, but here again their application is limited to certain particular types of apparatus.

Examples.

(1) Show how it is possible to transmit three-phase currents with only three wires.

(2) In a two-phase three-wire system the two phases are unequally loaded. The current in the leading phase is 47 amperes in phase with its own voltage. The current in the lagging phase is 39 amperes and lags by 22·6° behind its own voltage. What is the current in the common return?

(3) Three non-inductive resistances each of twenty ohms are connected in star across a three-phase system the line voltage of which is 480 volts. Three other non-inductive resistances are connected in delta so as to take the same line current. What are the values of these other resistances and what is the current flowing though each of them?

(4) Two wattmeters measure the total power in a three-phase circuit, and are correctly connected. One reads 4800 watts, whilst the other reads backwards. On reversing the latter it is found to read 400 watts. What is the total power and the power factor?

(5) The voltage between each pair of lines in a three-phase system is 500 volts, and three non-inductive resistances are connected in star across these lines, the currents being 100, 80 and 60 amperes respectively. What is the p.d. across each resistance and what is the voltage between their star point and the ideal star point with a balanced load?

(6) Show that in a three-phase, star-connected system, if a fifth harmonic is present in the voltage between line and neutral, it appears in the line voltage with its phase reversed.

CHAPTER XI

ROTATING FIELDS

Production of a Rotating Field.—A rotating magnetic field is one in which the flux rotates round a fixed axis and can be produced by merely rotating a magnet or a coil of wire in which a D.C. is flowing.

A rotating field can also be produced by a system of stationary coils supplied with polyphase currents. Consider Fig. 104, which

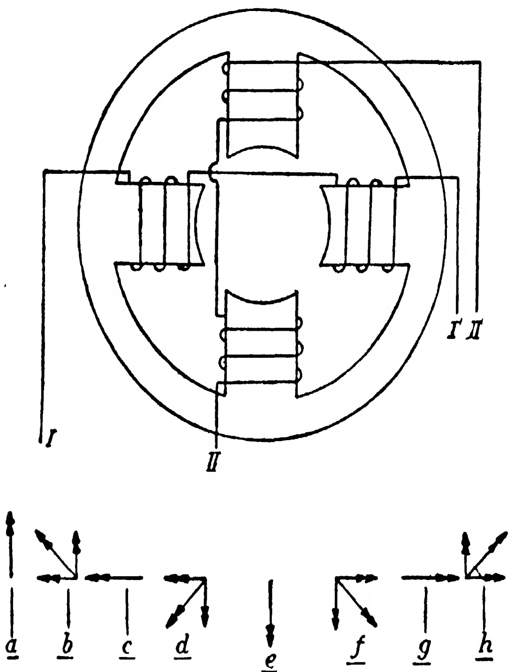


FIG. 104.—Production of Rotating Field.

represents a simple two-pole (not a four-pole) two-phase system, the two horizontal projections being supplied from one phase and the two vertical projections from the other. The combined effect of these two phases is to produce a resultant flux the axis of which changes from instant to instant, and it will be seen that this resultant flux gradually rotates whilst maintaining a constant magnitude. At the instant when the current in the first phase is zero that in the other is a maximum. The combined M.M.F. is therefore vertical, since there is no horizontal component, and the resultant flux is also vertical, as shown at (*a*). In order to determine the axis of the resultant flux, each phase will be imagined to produce its own flux, the two component fluxes then being added together vectorially. This is not theoretically accurate, but it is a convenient method of dealing with the problem and leads to accurate results. As the current in the first phase grows that in the second phase dies down. The vertical flux is now not so great, but there is a horizontal flux to be added to it vectorially, the result being a magnetic flux having an axis lying along an inclined line, as shown at (*b*). After a time, the current in phase I reaches a maximum, whilst that in phase II has died down to zero. The resultant flux is now horizontal, as shown at (*c*). The axis of the resultant flux at succeeding instants is shown at (*d*), (*e*), (*f*), etc., from which it is seen that the magnetic flux gradually revolves, completing one revolution in the time taken to pass through one cycle in a bipolar case. In a multipolar case the magnetic flux swings past a pair of poles for every cycle, and consequently the speed of revolution is $\frac{1}{n}$th that in the bipolar case, where n is the number of pairs of poles.

Care must be taken in determining the number of poles, which is the same as the number of poles *per phase*.

Two-Phase Rotating Field.—In order to determine the magnitude of the resultant field, a sinusoidal current wave form will be assumed and also a sinusoidal flux distribution. This latter assumption will be found to be unjustified later on, but it enables the resultant flux to be calculated with greater ease.

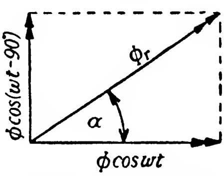


FIG. 105.—Combination of Fluxes (Two-Phase).

Let Φ represent the maximum flux produced by each phase separately. Considering an instant when the current in the first phase has advanced through an angle ωt from its maximum value, the flux due to this phase is represented by $\Phi \cos \omega t$, and lies along a horizontal axis. At the same instant the flux due to the other phase, which is lagging behind, is represented by $\Phi \cos (\omega t - 90°)$ and lies along a vertical axis. The resultant flux, Φ_r (see Fig. 105), is equal to

$$\begin{aligned}\Phi_r &= \sqrt{\{\Phi \cos \omega t\}^2 + \{\Phi \cos (\omega t - 90°)\}^2} \\ &= \Phi\sqrt{\cos^2 \omega t + \cos^2 (\omega t - 90°)} \\ &= \Phi\sqrt{\cos^2 \omega t + \sin^2 \omega t} \\ &= \Phi.\end{aligned}$$

It is thus seen that the resultant flux due to the two phases is equal to the maximum flux produced by each phase separately and is constant in magnitude and independent of the angle ωt.

The angle through which the axis of the resultant flux has rotated, corresponding to an advance in phase of the current of ωt, can also be calculated. This angle is represented by α in Fig. 105 and

$$\tan \alpha = \frac{\Phi \cos (\omega t - 90^\circ)}{\Phi \cos \omega t}$$

$$= \frac{\sin \omega t}{\cos \omega t} = \tan \omega t.$$

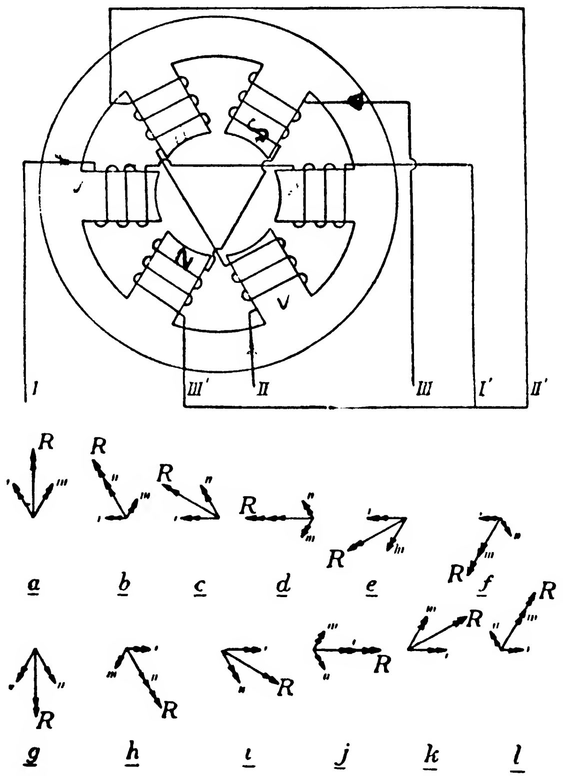


FIG. 106.—Three-Phase Rotating Field.

Therefore α is equal to ωt and an advance of ωt in phase corresponds to a rotation of α° of the magnetic flux. The flux is thus seen to rotate uniformly, having a constant magnitude all the time.

5) **Three-Phase Rotating Field.**—A simple bipolar arrangement for three phases would be provided with six projections (see Fig. 106)

instead of four, as in the two-phase case. The coils belonging to each phase are wound on to two opposite projections, care being taken to obtain a space displacement of 120° instead of 60°. This is done by connecting the three front ends, I, II, III, to the supply, the three rear ends, I′, II′, III′, being joined together to form a star point. A delta connection can also be adopted if desired.

Commencing with the current in phase I at zero, the current in phase II (lagging by 120°) is $-\frac{\sqrt{3}}{2}$ times its forward maximum value and the current in phase III (lagging by 240°) is $+\frac{\sqrt{3}}{2}$ times its forward maximum value. The flux vector diagram representing these conditions is shown at (a) in Fig. 106. After an interval of 30°, the current in phase I has risen to half its maximum value, the component flux due to this phase doing the same. The flux due to the second phase has now reached its maximum, whilst that due to phase III has died down to half its maximum value, as shown at (*b*). The flux due to phase III continues to die away, falling to zero 30° later, when the resultant flux is due to phases I and II only, as shown at (*c*). The values of the component fluxes at further successive intervals of 30° are shown at (*d*), (*e*), (*f*), etc., from which it is seen that the resultant magnetic field makes one complete revolution per cycle, as in the two-phase case. The magnitude of the resultant flux is also seen to be constant for all the instants illustrated, and it will now be shown that it is constant throughout.

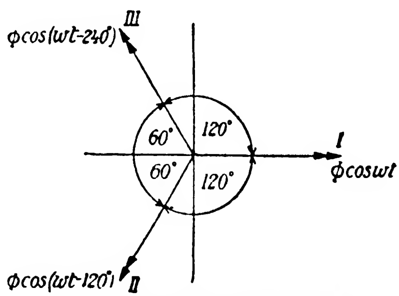


FIG. 107.—Combination of Fluxes (Three-Phase).

Using the same notation as before, the values of the component fluxes at any instant are $\Phi \cos \omega t$, $\Phi \cos (\omega t - 120°)$, and $\Phi \cos (\omega t - 240°)$. These fluxes act along axes which are mutually inclined to each other at 120°, so that in order to obtain an expression for the resultant flux all the instantaneous horizontal components and all the instantaneous vertical components will be determined and the two resultants added together vectorially.

The resultant horizontal component (see Fig. 107) is

$$\Phi \cos \omega t - \Phi \cos (\omega t - 120°) \cos 60° - \Phi \cos (\omega t - 240°) \cos 60°$$

$$= \Phi \left[\cos \omega t + \tfrac{1}{4} \cos \omega t - \frac{\sqrt{3}}{4} \sin \omega t + \tfrac{1}{4} \cos \omega t + \frac{\sqrt{3}}{4} \sin \omega t \right]$$

$$= \Phi \times \tfrac{3}{2} \cos \omega t.$$

The resultant vertical component is

$$0 - \Phi \cos(\omega t - 120^\circ)\sin 60^\circ + \Phi\cos(\omega t - 240^\circ)\sin 60^\circ$$

$$= \Phi\left[\frac{\sqrt{3}}{4}\cos\omega t - \tfrac{3}{4}\sin\omega t - \frac{\sqrt{3}}{4}\cos\omega t - \tfrac{3}{4}\sin\omega t\right]$$

$$= \Phi \times (-\tfrac{3}{2}\sin\omega t).$$

The resultant field is, therefore,

$$\Phi_r = \sqrt{\Phi^2 \times (\tfrac{3}{2}\cos\omega t)^2 + \Phi^2 \times (-\tfrac{3}{2}\sin\omega t)^2}$$

$$= \tfrac{3}{2}\Phi \times \sqrt{\cos^2\omega t + \sin^2\omega t}$$

$$= \tfrac{3}{2}\Phi.$$

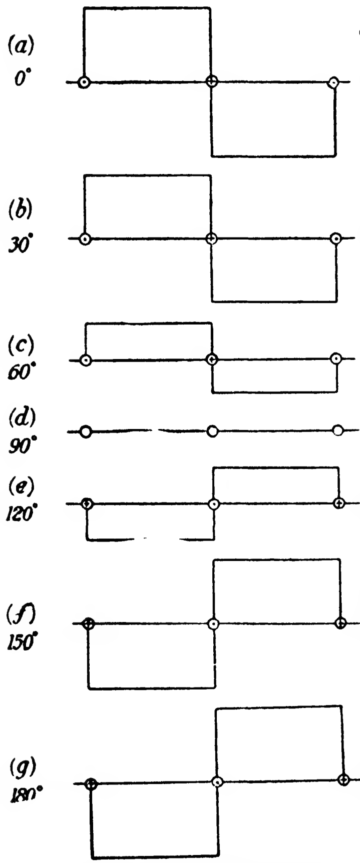


FIG. 108.—Flux Distribution with Concentrated Winding.

The resultant flux is again constant and independent of ωt as in the two-phase case, and is $\frac{3}{2}$ times the maximum flux produced by each phase separately. It will be shown later that this constant has a value of 2 when the non-sinusoidal flux distribution is taken into account. The flux also rotates with uniform speed, making one complete revolution per cycle in a bipolar case.

Flux Distribution with Concentrated Winding. It is usual to arrange the magnetizing winding in slots in preference to putting it on salient pole pieces, a uniform air-gap being thus obtained when a cylindrical rotor is placed in the field system. In a bipolar arrangement each coil lies in two slots and spans an arc of 180°. That portion of the air-gap lying between the two slots on one side forms one pole, whilst the other portion of the air-gap lying between the two slots on the other side forms the other pole. There is thus one coil per pair of poles. The M.M.F. is proportional to the ampere-turns, and has the same value over the whole arc of 180°. Over the remaining 180° it is equal in magnitude, but opposite in sign. As the reluctance is approximately uniform all round the air-gap, the flux density is constant all over the pole, and is represented by Fig. 108. When the magnetizing current dies down, the flux density diminishes proportionally, successive intervals of 30° being shown. After 90° the flux reverses and grows sinusoidally with time until it reaches a maximum, as shown in Fig. 108 (*g*). The action is then repeated.

For two phases four slots would be necessary, and for three phases six slots, these numbers being multiplied by the number of pairs of poles if a multipolar field is required.

Such a wave of magnetic flux is called a stationary wave, the zero points or *nodes* remaining fixed in space. At any particular instant of time the flux is distributed around the air-gap according to some law (in Fig. 108 the law is that of a rectangular wave), and this is called the space distribution. At any particular point in the air-gap the flux varies with time according to some law (in Fig. 108 a sinusoidal time variation is assumed), and this is called the time variation. Thus the strength of the flux varies both in time and space, the two laws being independent of one another. The time law depends upon the law obeyed by the magnetizing current, and the space law upon the distribution of the magnetizing turns around the air-gap.

Flux Distribution with Distributed Winding.—It is not usual, however, to concentrate the whole of the winding for one pole into a single slot, the general practice being to distribute it over a number of slots. This modifies the shape of the flux wave to a certain extent, since the coils in the different slots embrace slightly different arcs. Consider a case where there are five slots per pole for one phase [see Fig. 109 (*a*)]. The flux set up by each of the five coils is confined to the space lying between the two conductors of that coil. Since the current in each coil is the same, there are five times the ampere-turns per coil acting on the space between the inner conductors. Between these and the next adjacent conductors there are only four times the ampere-turns per coil, since the inner conductors are inoperative on their outside. Finally, in the space between the outer conductors and the next ones inside there is only that due to the single outside coil. The curve of flux distribution for this pole is shown in curve (*a*), Fig. 109 (*a*). Next considering the flux over the adjacent pole pitch, the same considerations lead to a curve of flux distribution as shown in curve (*b*), and combining these two flux curves together the resultant flux distribution is obtained as shown in curve (*c*). Owing to the fringing of the lines, it is sufficiently near to consider the sloping zigzag lines as straight lines, in which case the curve of flux distribution takes the form shown in Fig. 110. The horizontal distance between *a* and *b* is equal to the pole pitch, whilst that between *c* and *d* is equal to half the pole pitch in a two-phase case and one-third the pole pitch in a three-phase case. The space between *d* and *e* is filled up by the remaining phases, each of which produces a component flux wave flux density being 1·41 times that due to one phase alone. This further reacts on the wave form of the current and distorts it to a certain extent by introducing harmonics.

Calling the height of the flat-topped wave unity, the average flux density, which is given by $\frac{\text{area}}{\text{base}}$, is 0·75 when the wave is flat-topped and 0·71 when the wave is peaked (triangular), having a maximum value of 1·41. The mean value is $\frac{0{\cdot}75 + 0{\cdot}71}{2} = 0{\cdot}73$,

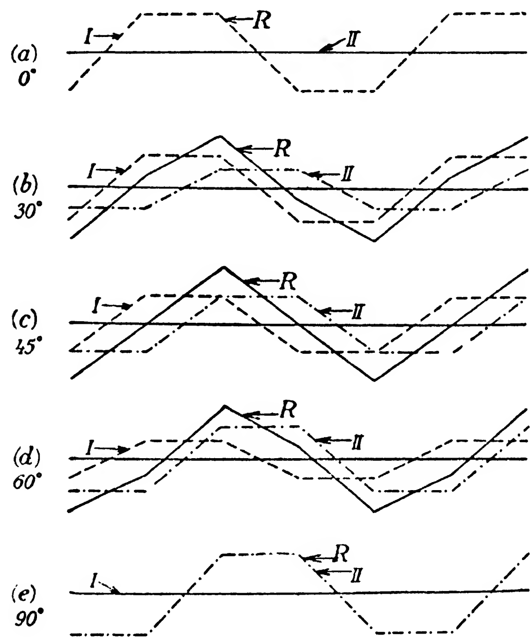


FIG. 112.—Flux Distribution with Two-Phase Winding.

and the ratio of maximum to average flux density is, therefore, $\frac{1{\cdot}4}{0{\cdot}73} = 1{\cdot}94$.

Resultant Flux Distribution due to Three Phases.—Each of the phases will now utilize one-third of the available winding space, so that the coils cover an arc of 60° instead of 90° in the two-phase case. The flux distribution is shown in Fig. 113. At (*a*) the current in phase I is a maximum, and hence the currents in phases II and III are equal to half their maximum values, but are in the reverse direction. Advancing by 120° to the right of the flux wave due to phase I, the flux wave due to phase II is plotted. This is in the reverse direction and has only half the amplitude, since a sine and the horizontal component $\Phi \sin \omega t$. The former are in phase with one another, but the latter are in opposite directions, owing to the different directions of rotation. The horizontal components thus cancel each other for all values of ωt. The resultant vertical component is $2\Phi \cos \omega t$, and the two rotating fluxes can be replaced by a single alternating flux having a maximum value equal to twice the magnitude of each component rotating flux. The axis of this resultant alternating flux lies along the line where the two rotating fluxes cross.

The converse of the above is that any single phase alternating flux can be resolved into two equal rotating fluxes, rotating in opposite directions, each of them having a magnitude equal to half the maximum value of the alternating flux.

Reversal of Rotation.—In order to reverse the direction of rotation, the connections of any two phases are interchanged. If this is done in a three-phase case, it is seen that instead of advancing in the order 1, 2, 3, the flux advances in the order 3, 2, 1.

Examples.

(1) Prove that, on the assumption of sine waves throughout, the rotating flux due to a three-phase system of magnetizing currents is constant in magnitude.

(2) Trace out the variations in M.M.F. due to a belt of conductors representing one phase of a three-phase magnetizing system.

(3) Calculate the maximum value of the M.M.F. due to a three-phase winding at a time corresponding to $\omega t = 10°$ for the first phase, in terms of the maximum M.M.F. due to any one phase acting separately.

(4) Each phase of a three-phase winding occupies one-third of a pole pitch, and is assumed to give rise to a flux wave of trapezoidal shape. Draw the resultant flux wave at the instant when (*a*) it is most peaked and (*b*) it is most flat-topped. Assuming the area of the graph to be proportional to the total flux, draw an equivalent sine wave of flux.

a strict square law, but is usually a little contracted at the top end as well as being very contracted at the beginning.

In the case of ammeters, the whole current cannot be led into and out of the moving system by means of the controlling springs, and to get over this difficulty the connections shown in Fig. 114 are sometimes used. The two fixed coils, C, C, are placed in series with one another and a resistance, R, across the terminals, T_1, T_2. The moving coil, $M.C.$, is connected in series with the resistance, r, also across the terminals, T_1, T_2. The ratio of resistance to reactance in each branch circuit must be the same, so that the phase relationships are not disturbed.

They are affected by stray magnetic fields, but to counteract this they can be effectively screened and, if necessary, they can be wound astatically.

Most instruments of this type are provided with a spring control and an air damping device.

The spring control produces a restoring torque proportional to the angle of deflection, and is similar to that used on moving coil instruments, whilst the various air damping devices adopted are similar to those used on moving iron instruments.

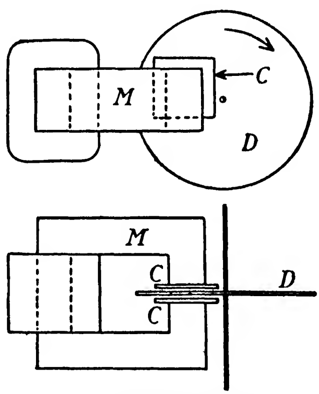


FIG. 115.—Principle of Induction Instrument.

The Kelvin Balance and the Siemens Dynamometer are examples of instruments of this type which are suitable for use on direct as well as alternating currents.

High-class dynamometer instruments are only affected to a very small degree by changes of wave form and frequency, and sometimes not at all.

Induction Ammeters and Voltmeters.—The main class of induction instruments are designed upon the *shielded pole* principle and were originally due to Ferraris.

A specially shaped spring-controlled aluminium disc, D (see Fig. 115), is arranged to rotate between the poles of an electro-magnet, M, energized by the current to be measured. Two copper plates, C, C, partially shield the poles, so that part of the flux goes straight across the aluminium disc, and part goes through the copper plates on the way. Due to the fact that the flux is alternating, eddy currents are set up in both copper plates and the aluminium disc, and since these currents are flowing in the same direction at any given instant of time, they attract each other. A clockwise rotation of the movable disc is therefore set up, this being opposed by a spiral spring. By suitably shaping the aluminium disc, a deflection of

300° can be obtained, the long scale being one of the distinctive features of the instrument. The deflecting torque at any instant is proportional to the square of the instantaneous current, and consequently the instruments indicate R.M.S. values, but owing to the action of the eddy currents they are very sensitive to changes of frequency unless compensated. The indications are practically independent of wave form, but temperature errors are considerable unless compensation is again provided.

A variation of this type of induction ammeter or voltmeter has a laminated iron core energized by a coil as before, but instead of the two copper plates, a portion of the iron is cut away and a short-circuited copper ring is placed on one of the projections as shown in Fig. 116. A spring-controlled pivoted disc is provided as before. Two

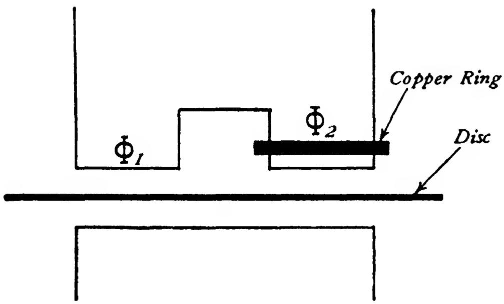


FIG. 116.—Shielded Pole.

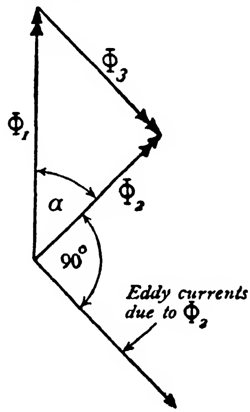


FIG. 117.—Fluxes in Shielded Pole Arrangement.

fluxes, Φ_1 and Φ_2, are set up in the unshielded and shielded portions of the air-gap. These two fluxes differ not only in position, but also in phase, due to the induced current in the copper ring. Each flux may be assumed to induce eddy currents in the disc. The flux, Φ_2, induces eddy currents in the disc which are proportional to the magnitude of Φ_2 and to the frequency, f. These eddy currents react on the flux, Φ_1, and a torque is set up proportional to the *average product* of Φ_1 and the eddy currents due to Φ_2 (see Fig. 116). This torque is therefore proportional to $f\Phi_1 \Phi_2 \cos (90° + \alpha)$, where $(90° + \alpha)$ is the phase angle between Φ_1 and the eddy currents due to Φ_2 (see Fig. 117). But $\cos (90° + \alpha)$ is numerically equal to $\sin \alpha$, so that the torque is proportional to $f\Phi_1 \Phi_2 \sin \alpha$, and since both Φ_1 and Φ_2 may be assumed to be proportional to the current, I, the average torque is proportional to $fI^2 \sin \alpha$. The same result is obtained if the flux, Φ_2, is considered in conjunction with the eddy currents due to Φ_1. The instrument therefore fulfils the general condition that the torque shall be proportional to the square of the current. It is also obvious that these instruments are very sensitive to change in frequency.

Currents are induced in the ring, these reducing the value of the flux in the affected region, and also modifying its phase. The E.M.F. in the ring is proportional to the rate of change of the flux, Φ_2, and lags behind it by 90°. Assuming the ring to be non-inductive, its current also lags by 90° behind the flux, Φ_2, and it may be said to set up a hypothetical demagnetizing flux, Φ_3, shown in Fig. 117. The resultant flux, Φ_2, in the shielded region is therefore the

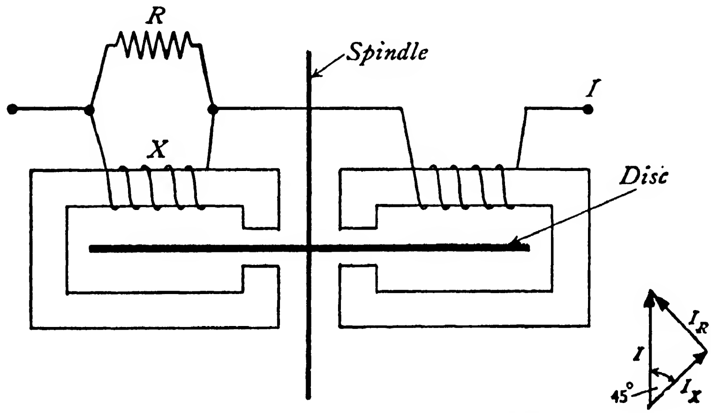


FIG. 118.—Connections for Induction Ammeter.

vector sum of Φ_1 and Φ_3, and $\Phi_2 = \Phi_1 \cos \alpha$. Therefore, the torque is proportional to

$$\begin{aligned} & f\Phi_1\,\Phi_2 \sin \alpha \\ = & f\Phi_1^2 \sin \alpha \cos \alpha \\ = & \tfrac{1}{2} f\Phi_1^2 \sin 2\alpha, \end{aligned}$$

and its maximum value occurs when $\sin 2\alpha =$ unity, or $\alpha = 45°$.

The connections of a modern type of induction ammeter are shown in Fig. 118, the phase difference of 45° being obtained by shunting one of the magnetizing coils with a non-inductive resistance, the current through X lagging behind the total current I.

Hot Wire Ammeters and Voltmeters.—Since the rate of production of heat is proportional to the square of the current, these instruments tend to obey, with slight modifications, the square law common to all alternating current ammeters and voltmeters, and consequently indicate R.M.S. values. In fact the A.C. ampere is defined as being that alternating current which produces the same heating effect as one ampere of direct current. Obviously, wave form, frequency and stray magnetic fields have absolutely no effect on the indications, but the instruments suffer from other inherent disadvantages which render them not so accurate as some of the other types described.

Thermo-junction Ammeters and Voltmeters.—Another type of thermal instrument is the moving coil millivoltmeter supplied from a thermo-junction heated by the alternating current to be measured. In the Paul instrument of this type, the thermo-junction and heater are contained in a glass bulb about 25 mm. diameter exhausted to a high degree of vacuum. Both the heater and the thermo-junction, which consists of an iron-eureka couple, are supported on platinum leading-in wires and are lightly soldered together. Only a proportional part of the current is used to heat the thermo-junction, *J* (see Fig. 119), the remainder being carried by the shunt, *S*. A small resistance, *R*, is used for purposes of adjustment. These "thermal converters," as they are called, can be used in conjunction with shunts on ammeters as well as on voltmeters.

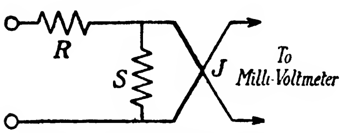


FIG. 119.—Connections of Thermal Converter.

In the Duddell thermo-ammeter the heater consists of a sheet of platinized mica, the platinum being scraped away in places so as to form a kind of grid. In this way high resistances are readily obtained in a very small space. The thermo-junction lies just above the heater, and is part of the moving system of the instrument, so that no current passes through the controlling spring.

These instruments are particularly suitable for high frequency work, since they are absolutely independent of frequency. They are also suitable for the measurement of small alternating currents.

Electrostatic Voltmeters.—These instruments can only be used as voltmeters, since they act like condensers and only take a small capacitance current. They can be divided into two main types: (1) where a pair of plates or quadrants are charged to different potentials, whilst a movable vane to which a pointer is attached is connected to one or other of the fixed vanes or is charged to some intermediate potential; and (2) where the moving vanes are connected to one terminal and the fixed vanes are connected to the other, the moving system being attracted bodily to the fixed system, causing a rotation round its axis.

One of the best known electrostatic instruments is the Kelvin multicellular voltmeter, so named because a number of cells act together on a common spindle. The working parts are represented diagrammatically in Fig. 120, where a number of movable vanes, *M.V.*, are threaded on to a spindle carrying the pointer, *P*, and are interleaved between a number of fixed vanes, *F.V.*

The moving system is suspended by means of a fine wire, *S*, from the underneath side of a coach spring, *C*, to prevent injury due to accidental vibration. The zero is adjusted by means of a torsion head, *T.H.*, and tangent screw, *T.S.*, attached to the suspension. The underneath side of the moving system ends in a vertical perforated paddle, *P*, moving in a glass vessel containing oil, this serving as the damping device. To avoid injury during transit, the moving system is clamped against a collar.

These instruments are only used on low tension circuits; for high tension, a different pattern is used. In this case the vanes are vertical and the moving system is supported on knife edges or by means of ligaments.

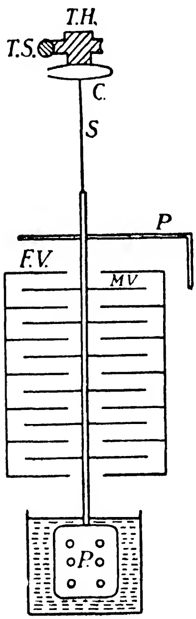


FIG. 120.—Multicellular Electrostatic Voltmeter.

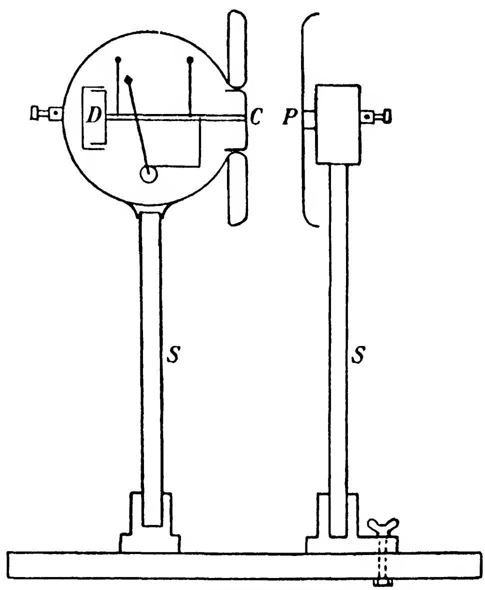


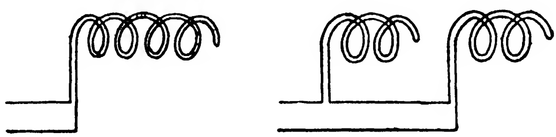
FIG. 121.—Abraham Electrostatic Voltmeter.

The Abraham electrostatic voltmeter is suitable for measuring pressures up to 200,000 volts. The principle of operation depends upon the electrostatic attraction between a metal plate and a metal cup and the instrument is shown diagrammatically in Fig. 121. The metal cup, *C*, is attached to an arm which is suspended by means of ligaments. To this arm is fastened a silk cord which passes round a pulley mounted on a spindle. When the voltmeter is charged the cup is attracted towards the plate and the motion is transmitted by the cord to the pulley and thence to the pointer. Damping is provided by means of the air dash pot, *D*. The metal plate, *P*, can be moved to or from the metal cup, and in this way

The readings on the milliammeter are found to be less for increasing values of A.C. volts. This arrangement does not suffer from the limitations of the previous one.

Wattmeters.—Wattmeters consist of two essential elements, viz. a voltage and a current coil, these two parts being connected in the circuit as a voltmeter and an ammeter respectively. The simplest type of wattmeter to understand is the dynamometer type, where the current passes through the fixed current coils and the voltage is applied to the moving volt coil, producing a current proportional to the voltage.

The torque which is produced at any moment is proportional to the product of the instantaneous values of voltage and current, and, assuming that the current lags behind the voltage by an angle ϕ, the average value of this product is $EI \cos \phi$. In the ideal wattmeter, the volt coil must have negligible inductance and capacitance compared with its resistance, so that the current flowing through it shall be in phase with the voltage across it. In actual instruments the phase error is less than 1°.



(a) (b)

FIG. 124.—Non-inductive Winding in Sections.

As the power factor decreases the error due to this phase angle increases at a greater and greater rate, so that an instrument which is approximately accurate on high power factors is subject to an appreciable error on low power factors. This has led to the development of a number of wattmeters having no iron in their construction, the object being to reduce the inductance of the voltage circuit. High voltage wattmeters are easier to deal with in this respect, since a large non-inductive resistance can be placed in series with the moving coil. These series resistances are wound back on themselves, as in Fig. 124, so as to make them non-inductive; but this introduces a capacitance effect, since there are conductors at different potentials lying close to one another and separated by a dielectric.

To reduce this capacitance effect and still have a non-inductive winding the coil is divided into sections as in Fig. 124 (b). This reduces the average p.d. between adjacent conductors, the total capacitance being inversely proportional to the number of sections.

This capacitance, if not too great, tends to neutralize the effect of inductance, and therefore is beneficial up to a certain amount. Eddy currents also tend to produce errors, and to minimize these the case and constructional details are made of some insulating material rather than metal, whilst the current coil is carefully stranded when the current is large.

An inherent error in wattmeter measurements lies in the fact that the instrument always includes in its reading the power absorbed by either the current coil or the volt coil. In Fig. 125 (*a*) the power lost due to the voltage drop across the current coil is included in the power measured, whilst in Fig. 125 (*b*) the power lost due to the current in the volt coil is included. On a constant voltage

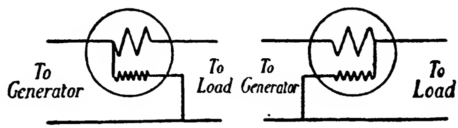


FIG. 125.—Wattmeter Connections.

circuit the latter will provide a constant error, whilst in the former case the error will be different for each value of the current. It does not always follow, however, that Fig. 125 (*b*) is the better method of connection, for when very small currents are being measured the watts lost in the current coil will be smaller than those lost in the volt coil, and in such cases the connections shown in Fig. 125 (*a*) are preferable.

In some wattmeters a fine wire compensating coil is placed inside the main current coil and in opposition to it, so that a negative torque is produced reducing the deflection by the amount of power lost in the volt coil. The turns on this compensating winding are made equal to those on the main current coil. This unfortunately increases the inductance of the voltage circuit, which is so undesirable.

Dynamometer Type Wattmeters.—Dynamometer type wattmeters form one of the commonest classes met with in practice, the moving coil being used as the voltage element and the fixed coil or coils as the current element. The moving coil, *V.C.*, is attached to a spindle, *S*, in the plane of the fixed current coils, *C.C.*, as shown in Fig. 126. When the current is sufficiently large, the fixed coil is wound with copper strip. Variations of wave form and frequency have very little effect in good dynamometer wattmeters.

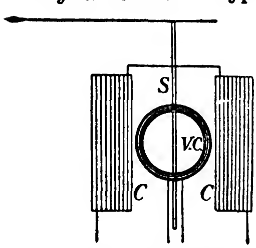


FIG. 126.—Dynamometer Wattmeter.

Induction Wattmeters.—The wattmeter contains a volt coil, *V.C.*, and a current coil, *C.C.*, as before, in addition to a copper or aluminium disc, *D* (see Fig. 127), whilst damping is obtained by means of a permanent magnet producing eddy currents in the disc. The coils are wound on to soft iron cores between the pole pieces of which the disc rotates. The voltage circuit is made very highly inductive, so that the current and flux produced by it lag behind the applied voltage by practically 90°. The eddy currents induced in the disc are further proportional to the rate of change of the flux and are again 90° out of phase. The induced current in the disc is therefore in phase opposition with the line voltage, whilst the flux produced by the current coil is in phase with the main current, since the coil is wound in series with the line. The torque which is developed, therefore, between the disc and the current coil is proportional to

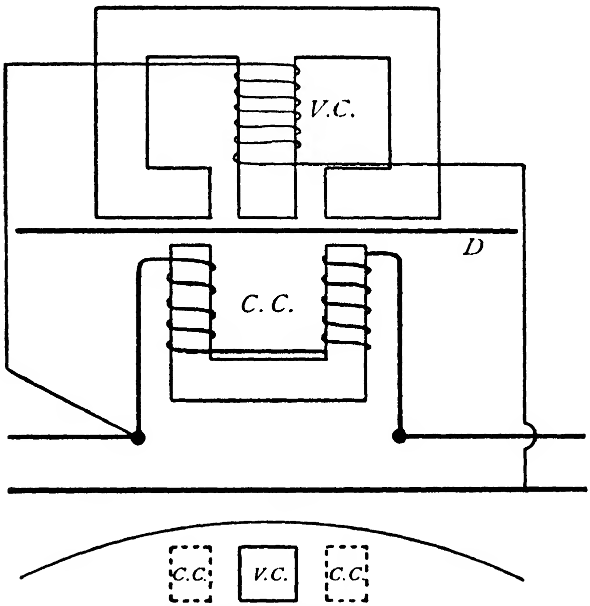


FIG. 127.—Induction Wattmeter. Single-Phase.

their instantaneous product and the deflection is proportional to the true power.

In practice, however, the flux produced by the volt coil does not lag by exactly 90° behind the voltage on account of the resistance, nor is the flux produced by the current coil exactly in phase with the current, since there are power losses produced by hysteresis and eddy currents. Various compensating devices are used to get over this difficulty, and usually consist of auxiliary windings arranged so that, when in conjunction with the main coil, a flux of the desired phase is produced.

Frequency has an effect on the accuracy of the instrument, the wattmeter reading decreasing as the frequency is varied on either side of the normal. Change of voltage and wave form also affect the accuracy to a slight extent. The accuracy is not, as a general rule, as high as in dynamometer wattmeters.

Electrostatic Wattmeters.—These instruments have a moving vane and a system of fixed vanes as in the case of electrostatic voltmeters, but instead of the fixed vanes being connected to points at the same potential, opposite pairs are connected to the two ends of a non-inductive shunt, the potential difference between them being proportional to the current (see Fig. 128). The moving vane is connected to the other line wire, so that the full voltage exists between the moving and fixed vanes, and it can be shown that the deflection is proportional to the true watts.

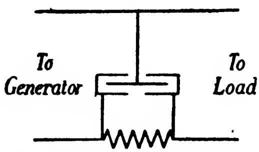


FIG. 128.—Electrostatic Wattmeter Connections.

The sphere of usefulness of these instruments lies, at present, in measurements involving very high voltages and low currents.

Two-Phase Wattmeters.—In the case of a two-phase circuit the total power is the sum of the powers in the two phases, and if two wattmeters have their moving systems attached to the same pointer the resulting deflection will depend upon the total power in the circuit. Instruments are made on this principle so as to avoid having to use two separate wattmeters, there being eight terminals

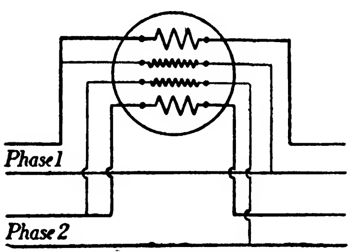


FIG. 129.—Two-Phase Wattmeter Connections.

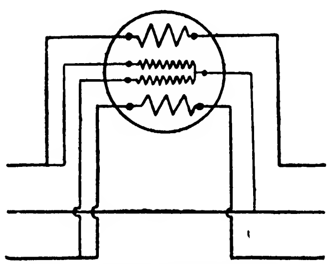


FIG. 130.—Three-Phase Wattmeter Connections.

in such a case, as shown in Fig. 129. Such wattmeters may be used for balanced or unbalanced circuits. Occasionally in connecting up such instruments the two parts are put in opposition, and the wattmeter will then give a small deflection which is due to the difference of the powers supplied by the two phases.

Three-Phase Wattmeters.—In the case of three-phase measurements it has been shown that only two wattmeters are necessary, and these can be mounted on the same spindle in the same way as in the two phase case. Thus the total power can be obtained from a single observation on one instrument which is supplied with seven terminals and connected as shown in Fig. 130. The number of terminals in some instruments is reduced to five by

0·02 ampere, and since the resistance of each is 100 ohms, additional

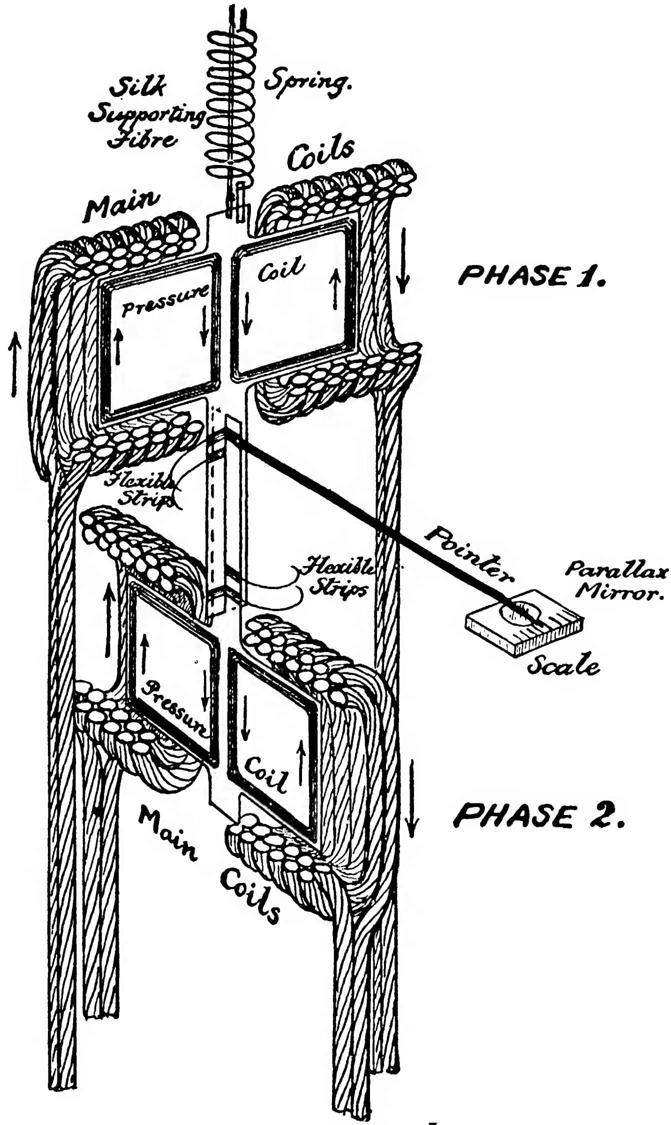


FIG. 131.—Arrangement of Coils in Drysdale Double Standard Wattmeter.

resistance has to be added externally to make it suitable for use on commercial voltages.

The reading always includes the loss in either the voltage coil or the current coil, but in most cases it is best to make it include the former, as this can be easily calculated and deducted from the reading. The swinging coil should always be joined to the nearest point possible of the main current coils, in order that electrostatic forces between the two may be minimized and to obviate any danger of sparking between them.

Summation Wattmeters.—It is sometimes required to measure the sum of the powers in a number of circuits, *e.g.* in the several bus bar sections of a power station, there being no point at which the total power can be measured directly. By means of suitably connected instrument transformers (see page 206) a resultant current is passed through the wattmeter, thus enabling it to indicate the total power.

Supply Meters.—A large number of A.C. watt-hour meters, or *integrating wattmeters*, as they are sometimes called, have been developed, but only one typical case will be discussed.

Induction Motor Meters.—These meters depend for their movement upon the interaction of a magnetic field and a metal disc placed in the field. They consist of four main parts: (1) the driving combination, consisting of a laminated iron core suitably wound with volt and current coils; (2) the moving system, or rotor; (3) an eddy current brake; and (4) the registering train.

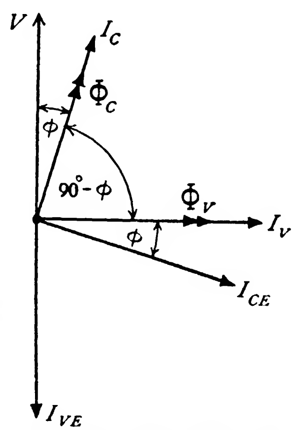


FIG. 132.—Vector Diagram of Induction Motor Meter.

The instruments do not give an indication by means of a pointer, but record the total number of revolutions made, this being an indication of the B.T. units consumed. Fig. 127 represents diagrammatically a meter of this type. The rotor consists of a flat disc of copper or aluminium with a vertical spindle running in a jewelled footstep bearing, friction being reduced to a minimum by making the rotor as light as possible. The brake disc rotates between the poles of a permanent magnet which has been aged artificially to ensure constancy. The registering train consists of a system of gear wheels and indicators simply measuring the total number of revolutions.

The volt coil is made as inductive as possible, so that the flux lags by practically 90° behind the volts, whilst the current coil is made of low resistance and should be non-inductive, so that the phase of the current is not disturbed and will be determined by the circuit. In this way, two fluxes are produced differing in phase by $(90^\circ - \phi)$, where ϕ is the angle of lag.

The auxiliary shunt winding tends to produce a flux, Sh_2, which when combined with Sh_1 produces the resultant shunt flux, Sh, lagging by exactly 90° behind the series flux, Se. The final adjustment is made by varying the non-inductive resistance in the auxiliary shunt circuit.

In the second method of compensation, an auxiliary series coil is wound in opposition to the main series coil, a certain amount of extra inductance being included in the circuit. The flux vector diagram is now represented by Fig. 134, where Sh is the shunt flux, Se_1 the main series flux and Se_2 the auxiliary series flux. Combining the two latter, the resultant series flux, Se, is obtained, this being in exact quadrature with the shunt flux, Sh.

Polyphase Supply Meters.—These are constructed for two- and three-phase circuits in the same way as wattmeters, viz. two separate instruments operate on the same spindle. Instead of adding the deflections, the speeds are added so that the total revolutions of the disc are proportional to the sum of the B.T. units measured by each portion of the instrument. The connections are the same as have been already shown in Figs. 129 and 130 in connection with wattmeters.

Reactive kVA-hour Meters.—These meters measure the reactive component only of the total kVA-hours. This result is attained by using a kW-hour meter, and displacing the phase of the current in the volt coils by 90°. They are usually designed for three-phase working.

If two single-phase kW-hour meters are connected so as to measure the total energy (see two-wattmeter method, page 154), and if the voltage or current coil of one meter be reversed, the total registration is a measure of $(W_2 - W_1)$, where W_2 and W_1 are the readings of the two meters. This is equal to $EI \sin \phi$ (see page 120), so that if this quantity be multiplied by $\sqrt{3}$, the total reactive kVA-hours are obtained. When two such meters are combined in this way to form a single three-phase meter, it can therefore be calibrated directly in kVA-hours.

EXAMPLES.

(1) Why does wave form affect the calibration of a moving iron ammeter or voltmeter?

(2) Explain the method of altering the range of (*a*) a dynamometer voltmeter and (*b*) an Abraham electrostatic voltmeter.

(3) Describe the theory of operation of one form of induction type wattmeter.

(4) When one element of a three-phase watt-hour meter is reversed, the speed of the disc is reduced to one-half in the negative direction. With this element restored to its original connection, and the other element reversed, the speed is reduced to one-half in the positive direction. What is the power factor?

balanced so that the line of suspension passes through the centre of gravity of the coil. This coil is suspended in the field of a permanent magnet and may have a periodic time up to as high as 1500 complete vibrations per second, although much lower figures are usual. The coil is held in tension by two phosphor bronze strips and the natural period of vibration is altered by adjusting this tension.

In the case of the moving coil instrument, the coil is vibrating in a strong field, with the result that relatively large E.M.F.'s are induced in it, which tend to damp out its movement very rapidly. The moving coil, therefore, comes quickly to rest, so long as there is a closed external circuit, whereas the moving iron galvanometer continues to vibrate until the very small mechanical friction absorbs the energy stored up in the system.

Vibration galvanometers are extremely sensitive to changes of frequency, this practically precluding their use as deflectional

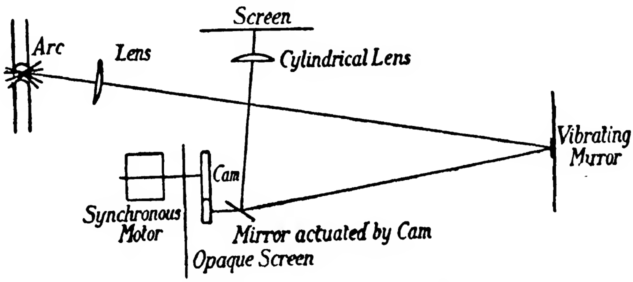


FIG. 136.—Optical Arrangement of Oscillograph.

instruments, although they are of great value in zero methods of test.

Oscillographs.—These instruments are employed for the purpose of obtaining the exact shape of a pressure or current wave form.

All oscillographs are in reality only galvanometers, the moving systems of which are capable of following the extremely rapid variations of the p.d. or current, enabling these to be reproduced on the screen. There are several types at present in use operating on the moving coil, electrostatic and cathode ray principles.

The galvanometer part gives a rapidly varying deflection which appears as a straight line. The beam of light is then given a movement in a direction at right angles to this, and the spot of light now traces out a curve. This second movement is so arranged that the spot of light is deflected over equal distances in equal times, so that the curve which is plotted automatically is the relation between deflection and time and is, consequently, the wave which it is desired to record. The optical arrangements are shown in Fig. 136, where the light from an arc or other powerful source is passed through a lens before impinging on the vibrating mirror of the galvanometer. The light is focused on to this mirror, and the reflected ray is made to vibrate at right angles to the plane of the paper. The ray of light then impinges on a plane mirror or a totally reflecting prism, which is made to vibrate in such a direction as to give a movement to the ray at right angles to the movement already impressed on it. The reflected ray then passes through a cylindrical lens before reaching the screen, where it has a movement in the plane of the paper proportional to the time and a movement at right angles to the plane of the paper proportional

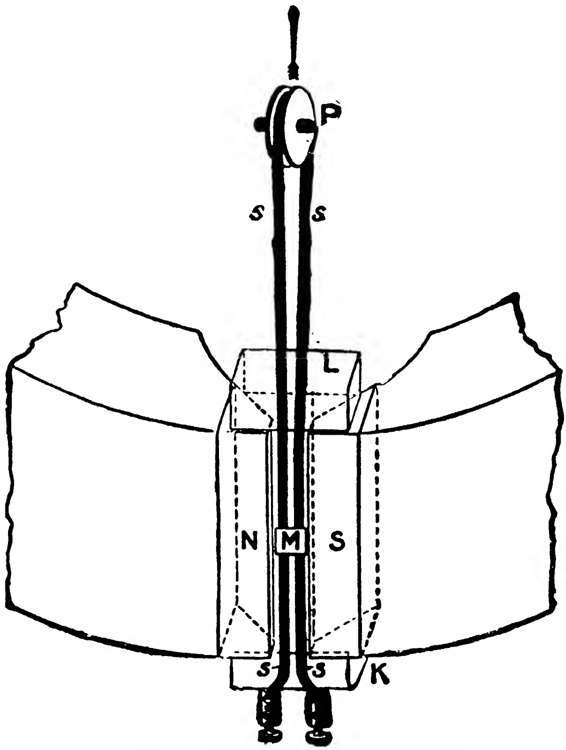


FIG. 137.—Duddell Moving Coil Oscillograph.

to the instantaneous value of the current or voltage. The second mirror is actuated by means of a cam driven from a four-pole synchronous motor (see Chapter XXIII), which is a motor rotating exactly once for every two periods of the alternating current. The cam is of peculiar design and is so arranged that during one and a half periods the mirror is turning with uniform angular velocity, whilst during the remaining half-period the mirror executes a quick return swing, an opaque screen being interposed to cut off the light and avoid confusing the diagram.

Duddell Moving Coil Oscillograph.—The first instruments of this type were due to M. Blondel, but the successful development has been brought about by the late Mr. Duddell. The oscillograph consists of a powerful permanent electromagnet, *NS* (see Fig. 137), with a narrow air-gap in which are stretched two parallel conductors, *ss*, formed of a strip of phosphor bronze bent round an ivory pulley, *P*. A suitable tension is kept on the strips by means of a spiral spring attached to the pulley. When a current passes through the strips, one is urged forward and the other back, so that the mirror, *M*, is deflected. Very often two such *vibrators*, as they are called, are placed side by side for the purpose of recording the current and voltage waves simultaneously. The guide piece, *L*, serves to limit the length of the vibrating portion, which is immersed in an oil-bath for the purpose of damping the movement and making it dead beat. There is also a third mirror for the purpose of recording the zero, the other mirrors being brought to zero by an adjustment of the guide piece, *L*. The clearance between the edge of the strip and the walls of the magnet face is very small, varying from 0·04 to 0·15 mm.

The inductance possessed by this type of oscillograph is quite negligible, so that it can be shunted as an ammeter, the safe working current being 0·1 ampere in some instances and 0·5 ampere in others.

Ho Electrostatic Oscillograph.—In the electrostatic type a double strip is employed as shown in Fig. 138, these strips oscillating in an electrostatic field instead of in an electromagnetic one. This electrostatic field is provided by applying the alternating pressure to the two metal plates, F_1 and F_2. In determining the voltage wave for low pressures, the supply is connected directly to these plates, but for high voltages, the especial field of this particular type of oscillograph, the working E.M.F. is reduced by means of the series condenser, *C*. The two vibrating strips, s_1 and s_2, are insulated from each other at *g* and *h*, being joined by a silk thread which passes over a pulley, *p*, and kept in tension by the spring, *q*. This tension can be adjusted at *n*. The terminals of the strips are at *j* and *e*, and these are connected to a battery, *B*, giving about 300 volts, being thus oppositely charged. In order to make the potentials of the strips definite in relation to the field plates, two condensers, C_1 and C_2, are inserted in series and the mid-point of the battery, *k*, is connected to *d*, the mid-point of the condensers.

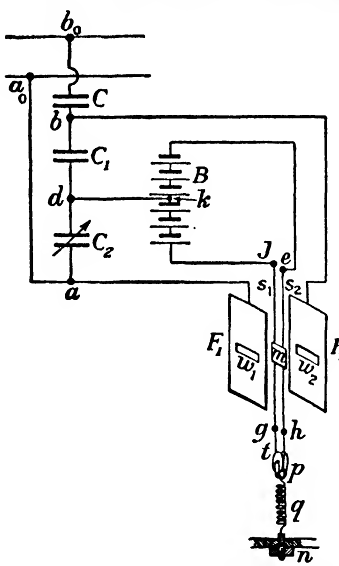


FIG. 138.—Ho Electrostatic Oscillograph

If the condensers are equal, and the strips occupy a position midway between the plates, they will vibrate equally when a voltage is applied. Since it is not possible to ensure that the strips are in the exact midway position, the condenser, C_2, is made variable in order to allow for this. A mirror, m, is attached to the strips, and small windows, w_1 and w_2, are cut in the plates, F_1 and F_2, through one of which the light from an arc can be passed and reflected.

Oscillograph Connections.—Fig. 139 indicates the method of connecting up a double strip oscillograph, the current strip, I, together with the resistance, R_2, measuring the p.d. across the known resistance, R_1. The resistance, R_2, is included so as to avoid overrunning the strip and to enable the deflections to correspond to some suitable number of amperes per division. The volt strip, E, is connected in series with a high resistance, R_3, across the mains. A most important point is to have one end of this strip directly

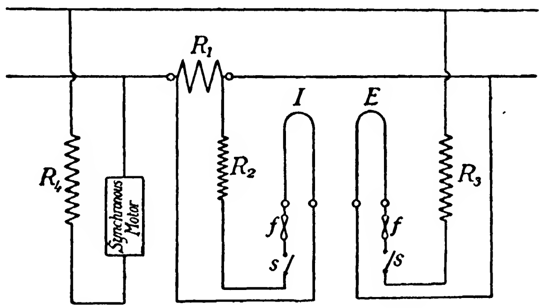


FIG. 139.—Oscillograph Connections.

connected to the same main as the current one, so as to avoid any large difference of potential between the two strips themselves. If this point is neglected, a short circuit between the two is most likely to occur owing to the small clearances employed. The value of R_3 can also be regulated so as to obtain a convenient volt scale on the curve. A fuse, f, and a switch, S, are further included in each circuit, whilst the synchronous motor is run, in series with the resistance, R_4, across the mains.

In some cases the wave is reflected on to a glass screen, where it can be observed by eye or traced by hand by placing a sheet of paper over the screen. Where a photographic record is desired, a falling plate camera is used, the plate falling directly across the path of the ray which is reflected from the oscillograph, the synchronous motor with the vibrating shutter being removed. If the plate falls from the height of a few feet the speed remains very nearly constant during the interval of time in which the record is made.

Apparatus is also made to enable a kinematograph record to be obtained.

Cathode Ray Oscillograph.—This instrument does not possess perhaps the accuracy of the types already described, but has a great advantage on the score of cost and simplicity in construction, and for many purposes is convenient and easy to manipulate. The oscillograph consists of a cathode ray tube of special design, a pencil of cathode rays producing a bright spot on a fluorescent screen. Such a beam of cathode rays can be deflected in any direction by applying an electric or magnetic field across the beam near its source. The beam is deflected in the same direction as the electric field and at right angles to the direction of the magnetic field. If two fields are applied so that their respective deflections are at right angles, then the end of the beam traces out a curve which shows the relation between the two fields on a system of rectangular co-ordinates. If the quantities concerned are periodic in their variation, the same curve will be traced repeatedly and appears as a stationary picture. Such an oscillograph differs from those previously described inasmuch as the quantities are not plotted against time as the variable.

FIG. 140.—Cathode Ray Oscillograph.

The oscillograph consists of a glass tube (see Fig. 140) about 30 cms. long. The cathode consists of a filament coated with active oxides, and emits the electrons for the rays when raised to a dull red heat. The anode is a small platinum tube set about a millimetre from the filament, which is bent into the form of a circle. Between the filament and the anode is a small circular screen with a hole in the centre just smaller than the circular filament.

A battery of 250–400 volts is connected between anode and cathode, and the electrons which are supplied by the hot filament are accelerated by the electric field thus produced. A small fraction of the electrons pass completely through the tubular anode and constitute the cathode rays. These rays pass between two pairs of deflecting plates set at right angles to each other, and fall upon the fluorescent screen at the large end of the tube, where they become visible.

To determine the shape of a particular wave, it is convenient to apply a sinusoidal E.M.F. to one pair of deflecting plates and the E.M.F. of unknown wave form to the other. The resulting diagram is of the form shown in Fig. 141, which also shows the construction for obtaining the wave form. When the sine wave has zero value, the magnitude of the unknown is *OO* and the value of the curve θ° later is given by the point *P*.

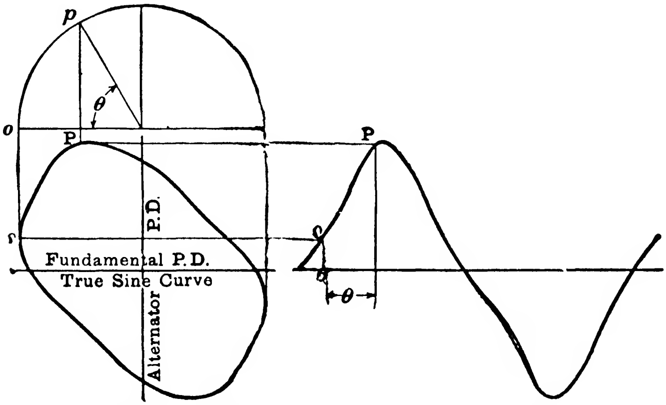


FIG. 141.—Cathode Ray Oscillogram and Wave Form.

Time Base.—An oscillogram of the type shown in Fig. 141 is inconvenient for practical purposes, since it demands additional labour to convert it into a wave form plotted against time as a base. The oscillographic record can, however, be converted directly into the required form by means of a *time base*. To do this, the two plates of the oscillograph not connected to the A.C. supply must have applied to them a voltage which increases in magnitude at a

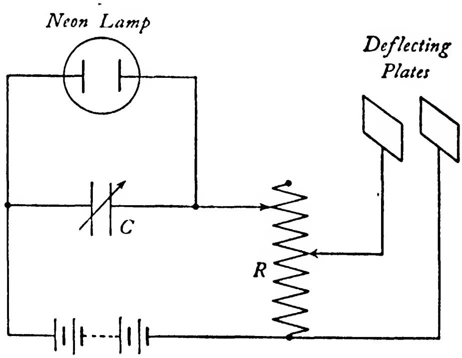


FIG. 142.—Circuit for Time Base.

constant rate. The spot on the screen is now displaced at a uniform rate in a direction perpendicular to that of the deflection due to the A.C. under test.

A suitable linear time base is obtained by the use of a neon lamp in parallel with a variable condenser, the connections being shown in Fig. 142. A neon lamp only passes current when the applied voltage reaches a certain value. It also ceases to pass current when the applied voltage falls below a certain value. (The "failing" voltage is lower than the "striking" voltage.) The action of the time base is as follows. The battery charges up the condenser, C, during which time the current through R is decreasing. Suddenly, the neon lamp "strikes", and additional current flows through R in such a way as to cause the p.d. across the deflecting plates of the cathode ray oscillograph to increase uniformly with time, this p.d. being proportional to the current through R. The battery voltage being constant, the increase in p.d. across R causes a decrease in the p.d. across the neon lamp. After a certain definite time, therefore, the neon lamp "fails" and the current through R suddenly decreases. The condenser now charges up again, the current through R again decreasing. This continues until the neon lamp again "strikes", when the whole action is repeated. The neon lamp blinks, and a voltage which rises gradually and then falls rapidly is applied to one pair of deflecting plates on the oscillograph. The time base thus obtained is approximately linear, whilst the frequency of the blink can be controlled by adjustment of the condenser, C.

Reactive Current Ammeters and Reactive Volt-Ampere Meters.—These reactive current instruments measure the reactive component of the current or volt-amperes only, viz. $I \sin \phi$ or $EI \sin \phi$, where ϕ is the angle of lag. Such instruments are useful on the switchboard for noting how near to unity power factor the plant is operating, but the true power factor indicators have largely supplanted them.

The principle of the iron-cored Sumpner instruments (see page 160) is applied in one form of reactive current ammeter. The supply voltage must be kept constant, and is used to excite the shunt electromagnet as before, whilst the moving coil is connected across a low non-inductive resistance placed in series with the mains like an ammeter shunt. If the angle of lag in the main circuit is ϕ, the instantaneous deflecting torque is proportional to

$$\begin{aligned} &\Phi \sin(\omega t - 90^\circ) \times I \sin(\omega t - \phi) \\ &= -\Phi \cos \omega t \times I (\sin \omega t \cos \phi - \cos \omega t \sin \phi) \\ &= -\Phi I (\tfrac{1}{2} \sin 2\omega t \cos \phi - \cos^2 \omega t \sin \phi). \end{aligned}$$

The average value of $\sin 2\omega t$ being zero and the average value of $\cos^2 \omega t$ being 0·5, the average value of the torque is proportional to

$$\tfrac{1}{2}\Phi I \sin \phi,$$

and if Φ is constant the deflection is proportional to $I \sin \phi$, which is the reactive component of the current. This assumes a constant maximum flux and a constant voltage. If the latter varies, the instrument measures $EI \sin \phi$, acting as a reactive volt-ampere meter.

Power Factor Indicators.—These instruments can be constructed on the dynamometer principle, the spring control being removed. There is a fixed coil, *CC* (see Fig. 143), connected in series with the line, and two moving coils, MC_R and MC_L, connected across the mains like voltage coils, the first, MC_R, having a non-inductive resistance and the second, MC_L, a choking coil connected in series with it. The magnitudes of the resistance and reactance, as well as the turns on the coils, are so adjusted that the ampere-turns of the two coils are exactly equal. The two moving coils are further rigidly attached at right angles to one another on the same spindle.

If the line current is in phase with the line voltage; the moving coil, MC_R, will endeavour to set itself vertically. The fixed coils, *CC*, produce a flux which is in phase with that produced by MC_R. The combination of these two fluxes, trying to take up as short a path as possible, tends to pull the coil into the vertical position.

When the main current lags behind the volts by 90°, the reverse is the case, and the moving system takes up the position with MC_L vertical. For intermediate angles of lag, the moving system takes

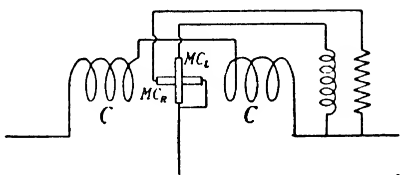


FIG. 143.—Power Factor Indicator.

up an intermediate position, the pointer (not shown on the diagram) moving over a scale. For leading currents, the moving system would deflect the other way, with the result that these instruments frequently have very long scales.

Instruments working on this principle can also be constructed for three-phase circuits, the moving system being wound with three coils star-connected on to the mains.

Power factor indicators are also constructed on the moving iron principle. In a three-phase instrument a pair of current coils are employed as in Fig. 143, together with three moving irons, excited by means of three volt coils, and rigidly fixed at 120° to each other on a spindle. The latter tends to set itself in a particular position, depending upon the relative phase of the voltage and the current, and so indicates the power factor.

Frequency Meters.—The first type to be described is based upon the fact that when a steel reed is brought near to the poles of an alternating current electromagnet it is set into resonant vibration at one particular frequency, this being independent of the voltage or wave form. In one frequency meter of this type there are a number of such reeds, each tuned to vibrate at a particular frequency, the reeds themselves being tipped with a white enamelled flag. These flags appear on the dial of the instrument, the frequency corresponding to each reed being marked on the scale opposite to it. These reeds are very sharply tuned, *i.e.* they only vibrate for the particular frequency which they are supposed to indicate, the indications dying away very rapidly indeed as the frequency is slowly changed.

In most power stations the frequency only varies over very narrow limits and a short range on the scale is all that is desired.

If a direct current equal to, or slightly greater than, the maximum value of the alternating current be passed through the winding at the same time, one half of the wave is neutralized whilst the other half is strengthened. The reed which happens to be vibrating is therefore attracted only once per cycle instead of twice, and is

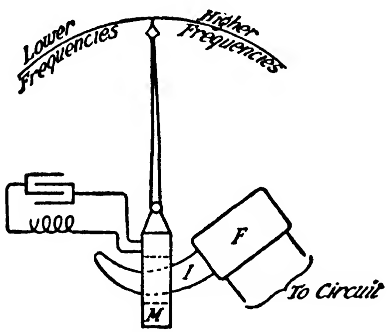


FIG. 144.—B.T.H. Frequency Meter.

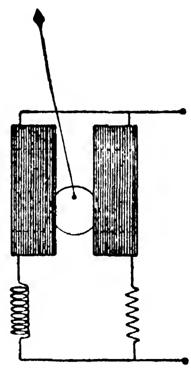


FIG. 145.—Coleman Frequency Meter.

thrown into resonance by double the frequency which originally made it indicate. This, therefore, provides a simple means of doubling the range of the instrument.

A frequency meter on a totally different principle is that represented diagrammatically in Fig. 144. This has a moving coil, M, which is connected across an inductance and a capacitance in series, adjusted for resonance at the mid-point of the scale. There is no spring or other form of mechanical control. A fixed coil, F, connected to the supply, sets up an alternating field, the strength of which is governed by an iron core, I, which passes through the moving coil. An E.M.F. is induced in the moving coil, and the secondary current leads or lags depending upon the frequency. If the frequency falls the inductive reactance falls, whilst that due to the capacitance rises. The current therefore leads. Alternatively, a rise in frequency makes the induced current lag. Lagging currents cause the coil to be repelled to a weaker part of the field, whilst leading currents bring about an attraction. For every frequency the moving system automatically re-tunes itself, the amount of movement depending upon the rate of change of the inductance of the coil as it moves over the core. The openness of the scale is controlled by the shape of the iron core.

The Coleman frequency meter is also of the deflectional type, and it depends upon the fact that the current through an inductive coil varies with the frequency. There are two coils (see Fig. 145), one of which is connected in series with a resistance and the other with an inductance. The combination is then connected across the mains like a voltmeter. A moving iron swings between the two coils and is subjected to pulls in each direction. For a given voltage the pull exerted by the coil in series with the resistance is constant, but the pull exerted by the other coil depends upon the frequency. An increase in the frequency causes a decrease in the pull and *vice versa.*

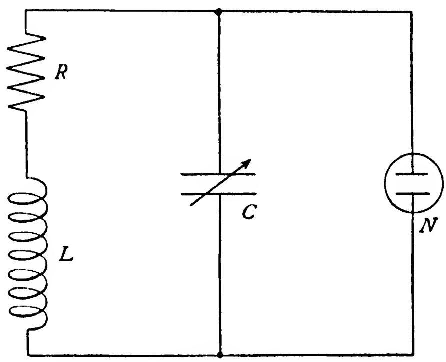


FIG. 146.—Circuit Diagram of Wavemeter.

As the relative values of the two currents vary, the iron takes up different positions, thus rotating the spindle and causing the pointer to indicate on the scale. A small change in voltage will affect the currents in both coils to the same extent and so will have no effect on the reading of the instrument.

When the instrument is required to indicate over a narrow frequency range, say 49 to 51 cycles per second, an inductance and capacitance are connected in series with each coil, one being tuned to resonate at 49 and the other at 51 cycles per second.

Wavemeter.—The wavemeter is an instrument which measures the frequency or wavelength of the currents in a nearby circuit operating at radio frequency. It consists essentially of an inductance in series with a variable condenser together with a detecting device. The condenser enables the circuit to be tuned to any given frequency in its range, a pointer attached to the moving vanes of the condenser giving a reading on a scale. The wavelength or frequency is then ascertained by reference to a calibration chart.

The internal connections of such a wavemeter are shown in

Fig. 146. An inductive coil, L, is connected across the terminals of a variable condenser, C. This coil necessarily possesses a certain amount of resistance, this being indiçated in the diagram by R.

The wavemeter is placed close to the circuit carrying the A.C. the radio frequency of which is to be measured. An alternating E.M.F. is induced in the coil, L, and the wavemeter circuit is then brought into resonance by adjustment of the variable condenser. The condition of resonance can be detected by means of a small lamp, or more accurately by means of a milliammeter, connected in series with the condenser. Alternatively, a valve voltmeter (*see* p. 150) may be connected in parallel with the condenser. A cheaper method, just as effective, is to connect a neon tube, N, across the condenser. When the current is a maximum, the p.d. across the condenser is also a maximum, and if this p.d. exceeds 140 volts, the neon tube becomes conducting and emits a bright red glow. This action ceases if the p.d. falls below 120 volts. Such an instrument is said to be voltage controlled.

To make a measurement, the variable condenser is adjusted until the neon tube gives a flash. The distance between the wavemeter and the exciting circuit is now altered until there is only just sufficient voltage to cause the neon tube to glow, the slightest adjustment of the condenser either way causing the glow to disappear. The indication on the dial is now observed, and the frequency or wavelength ascertained by reference to the calibration chart.

A.C. Potentiometers.—The A.C. potentiometer does not reach the degree of accuracy met with in the D.C. apparatus, for not only is there the difficulty in obtaining an accurate potential gradient, but there follows the difficulty of standardizing the E.M.F.'s upon the potentiometer without an A.C. standard cell. In addition, there is the need for measuring not only the magnitude of the alternating E.M.F., but also its phase relationship with regard to some datum.

Drysdale A.C. Potentiometer.—This potentiometer is of the polar type, *i.e.* it measures the magnitude of a voltage together with its phase angle. It employs a main circuit consisting of the usual slide wire, standard resistances and regulating rheostat as shown in Fig. 147. By means of two change-over switches the slide wire circuit may be connected to either the D.C. or the A.C. source at will. The potentiometer is standardized against the standard cell, $S.C.$, using the D.C. galvanometer, G, and the deflection of the milliammeter, $M.A.$, on D.C. is noted. The change-over switches are now put in the A.C. positions and the current through the milliammeter, $M.A.$, is adjusted to the previous reading obtained on D.C., by regulating the A.C. supply. The potential gradient on the slide wire is now the same as when standardized on D.C. To measure an unknown alternating p.d., the movable contacts on the slide wire and standard resistances are first adjusted until a minimum deflection is obtained on the vibration galvanometer, $V.G.$ The phase of the A.C. supply voltage is now altered by the phase-shifting transformer, *P.S.T.* (see page 208), by turning its rotor round, so as to get a new minimum deflection on *V.G.* Finally, small adjustments of the movable contacts and rotor position of the phase-shifting transformer are made until an exact balance is obtained.

The p.d. across the potentiometer has now been brought into phase with that to be measured, and the tapped portion of the slide wire circuit indicates the magnitude of this p.d. The position of the rotor of the phase-shifting transformer gives the phase of the p.d. under test.

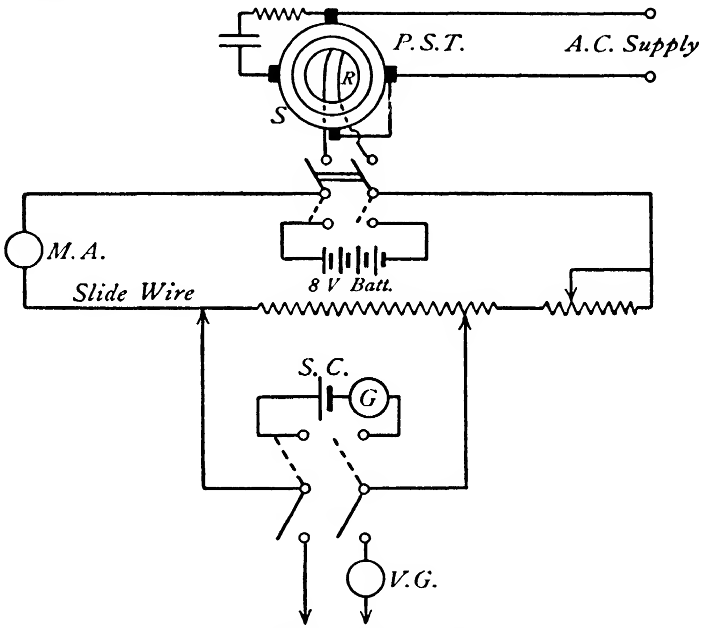


FIG. 147.—Drysdale A.C. Potentiometer.

The phase-shifting transformer consists of a two-phase stator, the phase-splitting being effected by including capacitance and resistance in the circuit of one phase winding.

Although one phase of the rotor only is required to supply the slide wire circuit, a two-phase winding is fitted, the second winding being connected to a compensating coil. This has resistance and inductance equal to that in the slide wire circuit, thus ensuring that the reaction of the rotor is constant for all positions.

Gall A.C. Potentiometer.—This potentiometer is of the co-ordinate type. Two separate potentiometers are employed as shown in Fig. 148. One, termed the " in-phase " potentiometer, is supplied with A.C. through a sensitive dynamometer. If D.C. be substituted this potentiometer can be calibrated against a standard cell. The calibration is maintained by observation of the dynamometer reading.

Reverting to the A.C. supply, the second potentiometer is supplied with an A.C. which is 90° out of phase with the first. In order to obtain this a quadrature transformer (see page 208) is employed. In the second potentiometer circuit the dynamometer is replaced by a mutual inductance in which the secondary current is 90° out of phase with the primary current. The second or "quadrature" potentiometer can now be calibrated by balancing the secondary E.M.F. of the mutual inductance against that of the

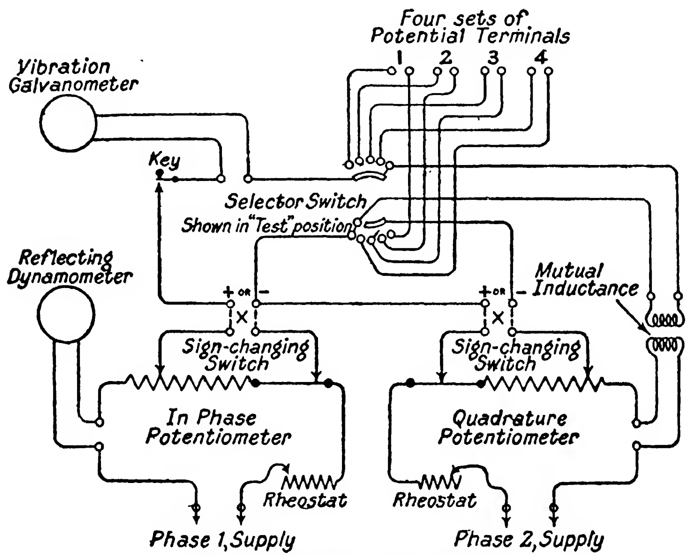


FIG. 148.—Gall A.C. Potentiometer.

in-phase potentiometer, these two E.M.F.'s being in phase. Adjustment of the potential gradient along the potentiometer wires is made by means of a series rheostat in both cases. The accuracy of this calibration is dependent upon that of the device employed for measuring the frequency.

The E.M.F.'s to be measured are applied to the potentiometer terminals of the instrument and may be determined both in phase and magnitude. If these voltages come in another quadrant, one or both of the reversing or sign-changing switches are thrown over.

EXAMPLES.

(1) Compare the various types of oscillograph in ordinary use, stating the particular field of usefulness of each type.

(2) Draw a diagram of connections for an oscillograph to measure current and voltage simultaneously, and explain how the oscillograms may be calibrated.

(3) Describe two types of frequency meter and show how the range of the instrument can be extended by the addition of auxiliary apparatus.

(4) What are the great points of difference between a D.C. and an A.C. potentiometer? Explain why the accuracy of the latter is less than that of the former.

CHAPTER XIV

TRANSFORMERS.—PRINCIPLES AND CONSTRUCTION

General Principle.—When a choking coil is supplied with an alternating voltage, a back E.M.F. is set up, due to the continual rate of change of flux. This E.M.F. is due to the turns of the winding linking with the flux, and if some other turns are placed side by side with the original winding but insulated from it electrically, the turns of the second winding will also have an E.M.F. set up in them, due to the same cause. Neglecting for the moment any losses that might occur, the back E.M.F. in the first or *primary* winding must equal the applied E.M.F. Each turn will provide its own proportion of the total voltage, and if there are T_1 turns on the primary, the back volts per turn will be $\frac{E_1}{T_1}$, where E_1 is the primary applied voltage. Each turn on the other or *secondary* winding will also have the same E.M.F. induced in it, viz. $\frac{E_1}{T_1}$ volts, and if there are T_2 total turns in series on the secondary, the total induced voltage will be $E_1 \times \frac{T_2}{T_1}$. The ratio of the voltages in the two windings is therefore seen to be the same as the ratio of the turns, and a simple means is thus provided of transforming from one voltage to another. Such a piece of apparatus is known as a *transformer*.

Flux in a Transformer.—Consider a rectangular core built up of laminations wound with a primary and secondary winding, shown on opposite limbs in Fig. 149 for the sake of clearness. If the applied voltage follows a sine law, the back voltage, the rate of change of flux and the flux itself must also each follow a sine law. Let the maximum value of the flux induced in the iron core be Φ lines. The total flux cut per cycle by each turn is therefore 4Φ, since the whole flux dies away in a quarter of a period, and the total flux cut per second by each turn is $4f\Phi$, where f is the frequency. The *average* E.M.F. induced in each turn is therefore $4f\Phi \times 10^{-8}$

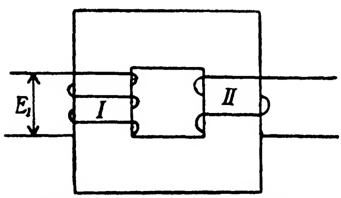


FIG. 149.—Simple Transformer.

and, since the form factor $\left(= \frac{\text{R.M.S. value}}{\text{average value}}\right)$ is 1·11 for a sine wave, the R.M.S. voltage induced per turn is

$$1{\cdot}11 \times 4f\Phi \times 10^{-8}.$$

The total induced voltage in the primary is

$$E'_1 = 4{\cdot}44 f\Phi T_1 \times 10^{-8} \text{ volts.}$$

Another way of arriving at this result is to consider the maximum rate of change of the flux, which is $2\pi f\Phi$ lines per second, since the flux is sinusoidal in character. The maximum E.M.F. induced per turn is therefore $2\pi f\Phi \times 10^{-8}$ volts, and the total R.M.S. voltage induced in the complete winding is

$$\frac{2\pi}{\sqrt{2}} f\Phi T_1 \times 10^{-8} \text{ volts}$$
$$= 4{\cdot}44 f\Phi T_1 \times 10^{-8} \text{ volts,}$$

the same as before.

This flux links with both the primary and the secondary windings, so that the total R.M.S. voltage induced in the secondary is

$$4{\cdot}44 f\Phi T_2 \times 10^{-8} \text{ volts.}$$

The frequency of the E.M.F. is obviously the same in both cases.

The maximum value of the flux in the ideal transformer can therefore be expressed as

$$\Phi = \frac{E'_1 \times 10^8}{4{\cdot}44 f T_1} = \frac{E_2 \times 10^8}{4{\cdot}44 f T_2}.$$

Effect of Secondary Current.—If the secondary winding be left on open circuit, the primary acts like an ordinary choking coil and takes a small current due to its high impedance, this being called the no-load current. This no-load current consists of a small active component and a relatively large reactive component. The former is necessary to account for the I^2R and iron losses, whilst the latter is due to the interlinking flux making the winding inductive.

Since there is an E.M.F. induced in the secondary winding, a current will flow if the terminals are connected to the ends of a non-inductive resistance. Assuming for the moment that this winding contains no resistance or reactance, the secondary current will be in phase with the secondary E.M.F. But this current tends to produce a flux in the opposite direction to that already existing in the core, and the momentary effect is to reduce the flux, thus causing a reduction in the primary reactance. This causes an increased current to flow in the primary until a state of equilibrium is attained, when the flux reaches its original value. The secondary ampere-turns must be counterbalanced by an equal and opposite number of ampere-turns in the primary, so that the total primary current is the vector sum of the no-load current and the additional current required to counterbalance the current in the secondary. Neglecting the effect of the no-load current, it is seen that the ampere-turns of both primary and secondary must be equal, and consequently the currents must be in the inverse ratio of the turns, or, since

$$I_1T_1 = I_2T_2,$$

therefore

$$\frac{I_1}{I_2} = \frac{T_2}{T_1}.$$

This can also be seen from considerations of the conservation of energy, for, since the power factor is unity and there are no losses,

$$E_1I_1 = E_2I_2$$

and

$$\frac{I_1}{I_2} = \frac{E_2}{E_1} = \frac{T_2}{T_1}.$$

Effect of Lagging Current in Secondary.—If the secondary winding be closed through a partially inductive resistance, the current will lag behind the secondary E.M.F. by some definite angle, and in order that the ampere-turns thus set up shall be counterbalanced at every instant, it is necessary that the corresponding primary ampere-turns shall have a phase exactly opposite to them. This is apparent when it is considered that for two sine waves to neutralize each other completely at every instant they must be exactly equal and opposite in phase. The effect of a lagging current in the secondary is, therefore, to cause a lagging current to flow in the primary. The actual angle of lag in the primary is usually greater than that in the secondary, since the no-load current itself has a very large angle of lag, thus tending to make the power factor of the primary a little less than that of the secondary.

In the case of a leading current being taken from the secondary, the primary current consists of a component leading the primary voltage by the same amount together with the no-load current lagging by a large angle. A certain amount of resonance is therefore set up in the primary, with the result that the power factor is slightly higher than that in the secondary.

Effect of Ohmic Resistance.—A transformer with its secondary on open circuit may be likened to a choking coil, the applied voltage being divided into two components, overcoming the resistance and reactance respectively. The same thing occurs when the transformer is giving out a secondary current. Part of the primary applied voltage is absorbed by the IR drop in the winding, the remaining component (obtained by vectorial subtraction) producing the flux which generates the voltage in the secondary. When the secondary is on open circuit, the primary current is very small and there is practically no difference between the applied voltage and the voltage producing the flux, particularly since the IR drop, small in itself, is almost in quadrature with the applied voltage as the current lags by nearly 90°. When a current is taken from the secondary, the primary current goes up and the IR drop is increased, so that it may no longer be negligible and must, consequently, be taken into account. But the flux is proportional to the voltage remaining after an allowance has been made for the IR drop (see page 80), and, consequently, falls very slightly as the load on the secondary goes up. Neglecting this slight variation, the flux remains constant for all values of the load on the transformer.

In a similar way, due to the ohmic resistance of the secondary winding, the secondary terminal voltage is rather less than the total induced secondary E.M.F., when it is delivering current, and hence the ratio of transformation does not remain strictly constant as the ratio of the turns, but varies somewhat, due to the resistances of the two windings and the currents flowing.

Magnetic Leakage.—The major portion of the magnetic flux produced by the primary current passes through the iron core, but since the air also has a definite permeability, although very much less than that of the iron used, a certain amount of flux traverses an air path as indicated in Fig. 150. These air paths are in parallel with the iron paths, but whereas the flux traversing the iron cuts the secondary winding, that following air paths serves no useful purpose and forms a *primary leakage flux,* in contradistinction to the *main flux* following an all-iron path and linking with both windings. In a similar way, the secondary current tries to set up a back flux opposing the existing main flux, but the primary automatically takes a larger current to provide sufficient ampere-turns to overcome this effect as far as the main flux is concerned. Notwithstanding this, a certain flux is set up by the secondary winding following leakage air paths for the most part, and this flux constitutes what is known as the *secondary leakage flux.*

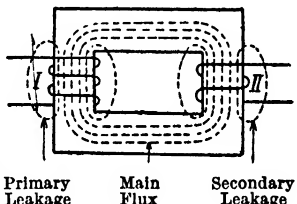


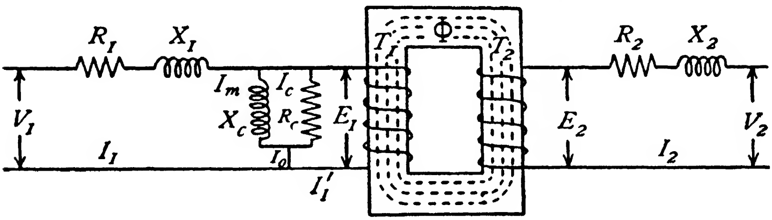
FIG. 150.—Main and Leakage Fluxes in Transformer.

The effect of the primary leakage flux is to add a certain amount of reactance to the primary winding, which serves no useful purpose and uses up a certain amount of the primary applied voltage in the same way that the resistance of the primary winding does.

The secondary leakage flux acts in much the same manner and has the effect of diverting more and more of the main flux into leakage paths as the current is increased, for inside the secondary winding the leakage flux is in exact opposition to the main flux. As the currents in the two windings increase, the difference in the ampere-turns of the two windings does not alter very much, the main flux actually decreasing slightly. The leakage fluxes are, however, practically proportional to the currents in the respective windings, and so the total leakage of the transformer increases as the load goes up until on very heavy overloads the majority of the flux has been diverted into leakage paths.

The effect of magnetic leakage upon the ratio of transformation is to reduce the secondary terminal voltage for a given primary applied voltage.

Equivalent Circuits.—A commercial transformer may be represented, for purposes of explanation, as consisting of an ideally perfect transformer having no losses or magnetizing current, together with various additions to allow for these effects. In Fig. 151 is shown such an ideal transformer having a resistance, R_1, and a reactance, X_1, in series with the primary winding, representing the resistance and reactance of the primary respectively. The reactance is that associated with the primary magnetic leakage flux,



$$n = \frac{T_1}{T_2}$$

FIG. 151.—Equivalent Transformer Circuits.

and is equal to $2\pi f L_1$, where L_1 is the primary inductance due to magnetic leakage. Another resistance, R_c, and a reactance, X_c, are shown connected in parallel with the primary. The resistance, R_c, is such that when connected to the supply voltage the power absorbed is equal to the core loss in the iron consisting of hysteresis and eddy currents, whilst the reactance, X_c, is such that it takes a purely reactive lagging current equal to the magnetizing current of the transformer. Further, a resistance, R_2, and a reactance, X_2, are shown in series with the secondary circuit, to account for the resistance and leakage reactance of the secondary winding respectively.

The currents flowing through X_c and R_c are I_m and I_c respectively, these representing the purely magnetizing component and the core loss component. The vector sum of these two currents is I_0, the no-load current of the transformer.

If desired, the resistance and reactance of the secondary can be combined with those of the primary to form one resistance and one reactance. This combination does not consist of simple addition, but necessitates taking into consideration the ratio of transformation. For example, in the case of a step-down transformer the impedance of the secondary is made less than that of the primary, because it deals with larger currents, and it must be multiplied by some constant in order to refer it to the primary side. The subject of equivalent resistance and equivalent reactance is further dealt with on page 217.

Vector Diagram.—In drawing the vector diagram of a transformer, it is convenient to start with the flux vector, Φ, as being the connecting link between primary and secondary, and for ease of illustration it is convenient to choose a transformer having a ratio of transformation of 1 : 1.

The induced secondary E.M.F., E_2, lags by 90° behind the flux (see Fig. 152), since it is proportional to minus the rate of change of the flux, or,

$$E_2 = -T_2 \frac{d\Phi}{dt} \times 10^{-8} \text{ volts.}$$ [1]

The graph showing the rate of change of a sine function is shown in Fig. 10, and the rate of change curve is there seen to lead the actual sine function by 90°, so that the graph of minus the rate of change must lag by 90°. (Fig. 10 deals with current and rate of change of current, but the same argument applies when flux is substituted for current.) In developing this vector diagram, constant reference should be made to the equivalent circuit diagram in Fig. 151, since the quantities in the vector diagram are all lettered in the same way as the corresponding quantities in the equivalent circuit diagram.

With the secondary on open circuit, the terminal voltage, V_2, is equal to the induced secondary E.M.F., E_2, but when the secondary is connected to a load circuit, the terminal voltage falls slightly. (In certain cases where the load gives rise to leading currents the voltage may rise instead of fall.) In Fig. 152 the usual case of a lagging secondary current is considered. This current is represented by I_2, its magnitude and phase depending upon the character of the load circuit. The current, I_2, flows through the secondary resistance, R_2, and causes a voltage drop equal to I_2R_2 in phase with the current, I_2, as shown in the vector diagram. A further voltage drop is occasioned by the passage of I_2 through the secondary reactance, X_2. This secondary reactance is caused by the secondary magnetic leakage flux. The voltage drop, I_2X_2, leads the secondary current by 90°. The combination of these two drops gives I_2Z_2, the impedance voltage drop in the secondary winding. When this quantity is subtracted vectorially

[1] Let the instantaneous value of the flux be $\Phi \sin \omega t$. Then the rate of change of flux is equal to $\omega\Phi \cos \omega t$ and

$$\begin{aligned} E_2 &= -T_2 \times \omega\Phi \cos \omega t \\ &= T_2\omega\Phi \sin(\omega t - 90°). \end{aligned}$$

i.e. E_2 lags by 90° behind Φ.

from the induced E.M.F., E_2, the secondary terminal voltage, V_2, is obtained. The power factor of the secondary load circuit is given by cos ϕ_2. It should be noted that ϕ_2 is the angle between the secondary current and the secondary *terminal* voltage, not the secondary induced E.M.F., E_2.

When the secondary is on open circuit, the primary current consists only of the no-load current, I_0, which is small compared with the full load current. The no-load current consists of two components, the magnetizing current, I_m, necessary for magnetizing

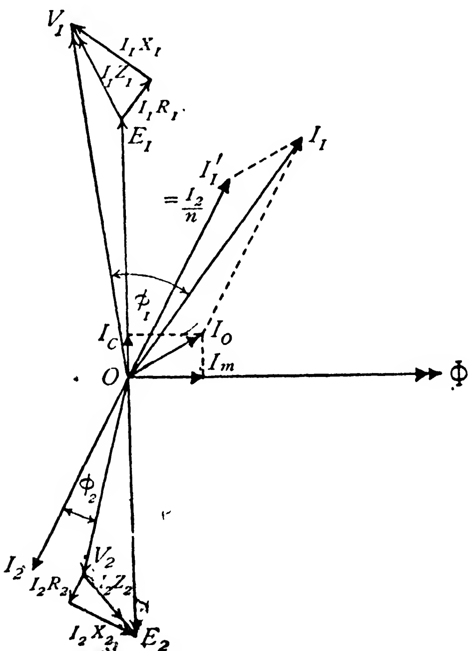


FIG. 152.—Transformer Vector Diagram.

the core, and the core loss current, I_c, which supplies the core loss consisting of hysteresis and eddy currents. The magnetizing current is in phase with the flux, Φ, and thus lags by 90° behind the E.M.F., E_1. The core loss component of the current is in phase with E_1, since it represents a power loss.

If the transformer has an air core, the no-load current will be in phase with the flux produced by it, neglecting the effect of eddy currents in adjacent metal. With an iron core, magnetized so that a definite hysteresis loop results, the current has a positive value before the flux commences to grow, and also the current dies down to zero whilst the flux has still a positive value. The current obviously leads the flux, when hysteresis is taken into account.

The phase difference is further accentuated by the effect of eddy currents. These oppose the change of the flux, and so cause it to lag. In other words, the component of the primary current producing the eddy currents leads the flux.

When the secondary delivers a current, I_2, a corresponding current, I'_1, flows in the primary, equal in magnitude (in a 1 : 1 transformer) and exactly opposite in phase. The ampere-turns of the two currents, I_2 and I'_1 thus cancel out, leaving the original magnetizing current to set up the flux as before. The total primary current is now I_1, which is the vector sum of I'_1 and I_0. Since, however, the majority of transformers have ratios which differ greatly from unity, there would be a great disparity in the lengths of the vectors representing I_2 and I'_1, if they were drawn to the same scale. Either one would be cumbrously large or the other impracticably small. To overcome this, different scales are chosen, the ratio of the scales being $n : 1$, where n is the turn ratio of the transformer. In other words, 1 cm. represents n times as many amperes on the secondary side as it does on the primary side. The actual lengths of the vectors I_2 and I'_1 now become exactly equal. The same device is adopted in the choice of voltage scales, but here 1 cm. on the primary side represents n times as many volts as 1 cm. on the secondary side.

The alternating flux sets up a back E.M.F. in the primary winding which necessitates the application of a voltage, E_1, in order to neutralize it. In addition, a voltage, I_1R_1, has to be supplied to provide for the resistance drop, this being in phase with the primary current, I_1. A further voltage, I_1X_1, has to be supplied to provide for the reactance drop occasioned by the primary leakage flux. This leads I_1 by 90°. The combination of the resistance and reactance drops gives the primary impedance voltage drop, I_1Z_1. The resultant applied voltage is therefore given by V_1, which is the vector sum of E_1 and I_1Z_1, where I_1Z_1 is the vector sum of I_1R_1 and I_1X_1. The primary power factor is cos ϕ_1, where ϕ_1 is the phase angle between V_1 and I_1.

The primary leakage flux may be taken as being in phase with the primary current, I_1, and since this flux links with the primary winding, in addition to the mutual flux Φ, the total flux linking with the primary winding is given by the vector sum of these two component fluxes. The phase of this total flux linking with the primary winding is slightly ahead of that of the flux Φ linking with both windings. In the same way the secondary leakage flux may be regarded as being in phase with the secondary current, I_2. This leakage flux, when combined with Φ, gives the total flux linking with the secondary winding, and this lags behind Φ in phase by a small amount. The total fluxes linking with each of the two windings are thus not quite equal, either in magnitude or phase.

The efficiency is given by $\frac{V_2 I_2 \cos \phi_2}{V_1 I_1 \cos \phi_1}$ without reference to the scales, the actual lengths of the lines only being required, since the change in current scale is exactly counterbalanced by the change in voltage scale in the opposite direction. The watt scale is the same for both primary and secondary. Similarly, the regulation is given by $\left(1 - \frac{V_2}{V_1}\right)$, for when V_1 is referred to the secondary side it assumes the same scale as V_2.

As the load increases, both the phase and magnitude of V_2 alter slightly, due to the increase in $I_2 Z_2$; so also do the phase and magnitude of V_1. But the conditions under which a transformer works in practice are that the primary applied voltage is constant, and not the flux, as has been assumed in the above diagram. Since the change is but a small one, the necessary correction can be made by choosing a new scale of voltage for each load, obtained by making V_1 represent the constant applied voltage in each case. This change does not affect the current vectors, but does affect all the voltage vectors and the flux vector proportionally. Thus the flux is seen to decrease slightly as the load goes up, and also to lag a little more than 90° behind the primary applied voltage.

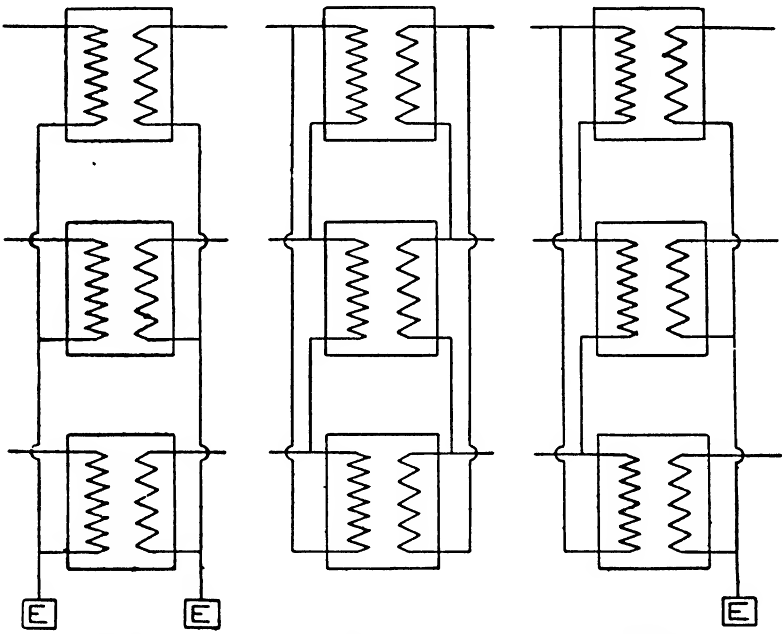
The primary and secondary terminal voltages are thus seen to be no longer in exact phase opposition, and the small difference between the actual phase angle between V_1 and V_2 and the ideal phase angle of 180° is called the phase displacement angle.

When the load is partially inductive, the primary power factor is always less than that of the secondary, except when it is less than that on no-load, which is not likely to occur in practice. One effect of the lag in the secondary circuit is to bring the two currents more nearly into phase opposition and to increase those components of $I_1 Z_1$ and $I_2 Z_2$ which are in phase with V_1 and V_2 respectively. This causes a reduction in the secondary terminal voltage as the secondary power factor is decreased, other things being equal, and increases the ratio of transformation. Another point of interest with respect to magnetic leakage is that neither the primary nor the secondary leakage flux is in phase with the main flux, since the leakage fluxes are in phase with the primary and secondary currents which produce them.

When the secondary current is a leading one, the power factor of the primary is higher than that of the secondary, due to the no-load magnetizing current neutralizing a portion of the capacitance current required to balance the secondary current. In fact, for one particular value of the secondary current, the primary power factor is unity, although the secondary current is a leading one. For lower values of the secondary current, the primary current lags, whilst that in the secondary leads. The ratio of transformation with a leading current is much more nearly constant for varying values of the load than is the case when the load is inductive.

The vector diagram for a three-phase transformer consists simply of three single-phase diagrams arranged mutually at 120° to each other. The line values of voltage and current on no-load and on load can easily be determined, for both star- and delta-connected systems. Usually, however, it will be found more desirable to deal with the problem as a single-phase one, except in the cases where unbalanced circuits are concerned.

Polyphase Transformers.—Two-phase currents can be transformed by means of two similar single-phase transformers, the secondaries being independent or interconnected according to choice.



Star to Star — Delta to Delta — Delta to Star

Ratio of Transformation = n. Ratio of Transformation · ation = $\frac{n}{\sqrt{3}}$.

(*a*) (*b*) (*c*)

FIG. 153.—Single-Phase Transformers on Three-Phase Circuits.

Similarly, three-phase currents can be transformed by means of three similar single-phase transformers, the secondaries either being independent or connected in star or delta. It is also possible to connect the primaries in delta and the secondaries in star, and *vice versa*, the ratio of transformation of the line voltages being, of course, affected by the connections. Fig. 153 (*a*), (*b*) and (*c*) illustrates some of the methods of connection.

If the three primary windings were wound on the same simple core which has been already described, a single-phase alternating flux would be produced and the three secondary windings would have equal voltages of the same phase induced in them, so that only a single phase supply could be obtained from their combination. But by adopting a three-limbed core of the type shown in Fig. 154, and winding each phase on to a separate limb, it is possible to build one transformer which will transform three-phase currents by itself. The phase of the flux bears the same relationship to that of the voltage in each of the three phases, and consequently the sum total of the flux either upwards or downwards is zero at every instant (see Chapter X). In other words, the flux in any limb is always the sum of the fluxes in the opposite direction in the other two limbs. By winding a secondary coil on to each limb, a three-phase supply can be obtained, for in each limb the secondary E.M.F. lags by 90° behind its respective flux, which lags by 90° behind the impressed voltage.

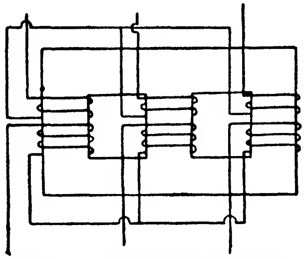


FIG. 154.—Three-Phase Transformer.

Single-Phase Core Construction.—The simple type of transformer in which a laminated iron core is surrounded by copper coils is called the *Core Type*. In the case of small transformers, the iron core is built up of two shapes of stampings, one set being ⊔-shaped and the other consisting of rectangular strips to close the iron circuit. The thickness of these stampings usually ranges from 0·35 to 0·45 mm. A bundle of stampings is clamped together by bolts or rivets, the latter being insulated from the core in order to reduce the eddy currents. Since it is difficult to make a good magnetic butt joint without introducing additional eddy currents, the plates are sometimes dovetailed into each other, alternate stampings being cut long and short for this purpose. For larger transformers the stamping of the ⊔-shaped pieces involves a considerable waste of material, and the practice is to build up such cores from four bundles of rectangular strips, the thickness of the bundles all being equal (see Fig. 155). These are bolted together between end cheeks at the top and bottom, the latter also being held tightly in position by means of tie rods.

Cross Section of Limb.—Since the length of the turn does not influence its flux-producing properties, it is advisable to reduce it to a minimum, both from the point of view of its resistance and also the amount of material used. With a fixed magnetic density, the cross section of the iron is fixed and the problem resolves itself into finding a figure which has the minimum periphery for a given cross-sectional area. The ideal figure is the circle, but this involves each stamping having a different width, which is impracticable. By having two, three or four sizes of stampings, however, a good approximation to the circle can be obtained, the coil itself being made circular, since it does not follow exactly the outline of the core.

A typical limb cross section with three sizes of stampings is shown in Fig. 156. Surrounding these stampings is an insulating cylinder, on the outside of which is wound the low voltage winding. On the outside of this, leaving an annular space, is another insulating cylinder, and on the outside of this is wound the high voltage winding. Transformers are usually oil-immersed, except in very small sizes, and so the various spaces are filled with oil which cir-

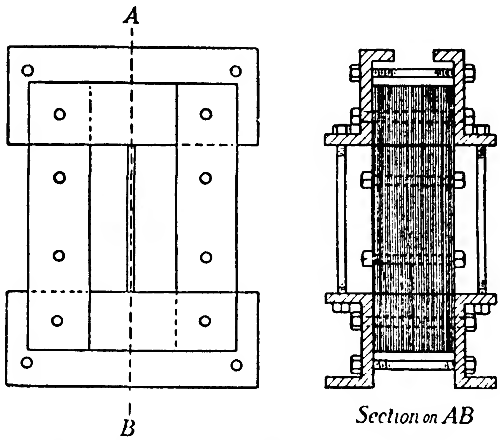


FIG. 155.—Construction of Core Type Transformer.

culates naturally due to convection currents set up by the heated copper and iron.

In certain cases, particularly in small sizes, a rectangular section of limb is adopted, the depth usually being of the order of 1·5 times the width. This shape is not recommended for large sizes, since one effect of a short circuit is to make the coil endeavour to assume a

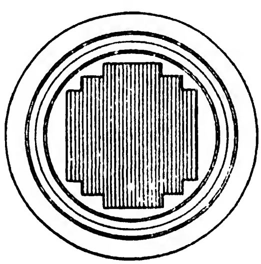


FIG. 156.—Limb Cross Section.

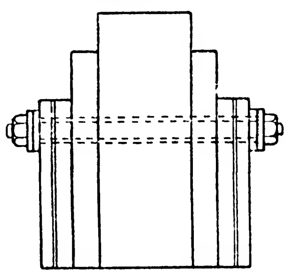


FIG. 157.—Magnetic Yoke Section.

circular shape. The opposite sides of a turn repel each other when carrying current, and the circle is the shape that encloses the largest area for a given length of turn.

Magnetic Yokes.—The upright limbs are connected at top and bottom by magnetic yokes, either interleaved or with butt joints. The section of the yoke is stepped in a somewhat similar manner to the limb, the number of shapes employed being the same as in the limb. The stampings in the centre have the greatest depth, those on either side being rather less deep. The arrangement is shown in Fig. 157. The depths of the various yoke stampings must bear the same ratio to one another that the widths of the limb stampings do. This is to prevent an interchange of flux between the different portions, which would otherwise set up excessive eddy currents.

Single-Phase Shell Construction.—The core type of transformer already discussed consists of an iron core around which copper coils are wound. The same effect is produced if an iron core is wound round the copper coil, this being the characteristic feature of the shell type transformer. Fundamentally there is no difference between the two types, since the principle involved in each is the linkage of flux with ampere-turns. The appearance of the two

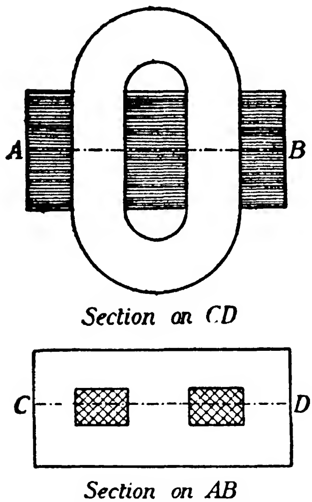


FIG. 158.—Shell Type Transformer.

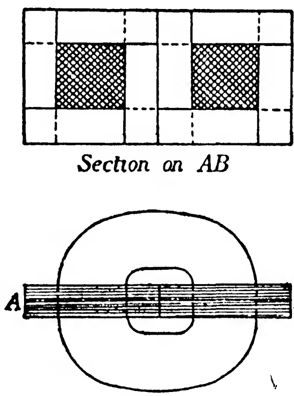


FIG. 159.—Shell Type Transformer.

types, however, differs considerably, since, in the shell type transformer, the coils are more or less embedded in the iron, which serves also as a mechanical protection.

A common example of the shell type transformer is the one having a three-limbed core shown in Fig. 158. The iron section consists of two E-shaped stampings facing each other so as to form two square holes, through which the coils, both primary and secondary, are threaded. By employing two stampings it is possible to adopt former wound coils, the stampings being cut alternately long and short in order to minimize the effect of the air-gap at the joint. There are only three magnetic joints in this form of construction, as opposed to four in the simple core type transformer built up of four rectangular strips, and since the flux is divided outside the central limb, the section of the iron in these parts need only be half that of the central limb.

For large transformers, the core is often built up from two simple rectangular cores placed side by side (see Fig. 159), the coils being mounted in an upright position.

Another example of shell construction is that shown in Fig. 160, where a circular coil, including both primary and secondary, is linked with bundles of stampings which are placed all the way round. Each bundle consists of an upright rectangular strip inside the coil, the magnetic circuit being completed by a ⊔-shaped piece. From the nature of the construction, the stampings leave air spaces between them at their outer edges, whilst they are crowded together in the interior. This ensures a large cooling surface both for the copper and the iron.

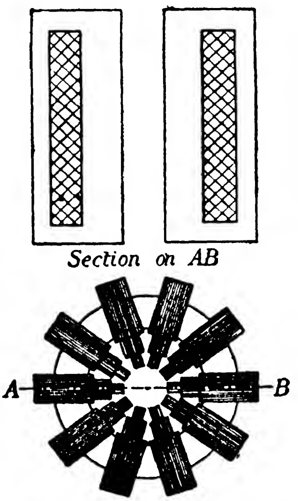


FIG. 160.—Shell Type Transformer with Circular Coil.

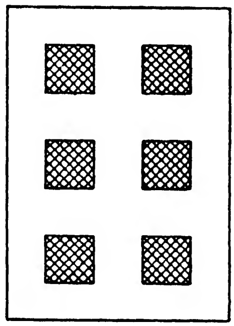


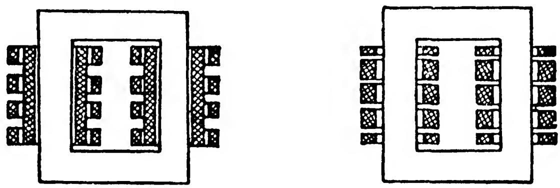
FIG. 161.—Three-Phase Shell Construction.

Shell type transformers usually provide a shorter magnetic path, and hence the magnetizing current is usually less than in the corresponding core type transformer. Also the amount of copper required is less, but, due to the embedding of the coils, the natural cooling is poorer. A point against the shell type transformer is the necessity for dismantling the laminations when the coils have to be withdrawn for repairs, this not being necessary in the core type transformer.

Three-Phase Core Construction.—Theoretically the simplest three-phase construction is to have three upright limbs set in a triangle and joined at the top and bottom by a number of plate stampings. In practice, however, the three limbs are arranged in a line, as in Fig. 154, this not introducing any serious asymmetry.

Five Limb Cores.—This construction is sometimes adopted for transformers of very large output. The three inner limbs are wound, the two outer limbs constituting return paths for the magnetic flux. In the three limb type, the flux from one limb always returns by way of the other two limbs. In very large transformers, however, the reluctance of the top and bottom yokes becomes appreciable, and the two extra limbs provide additional return paths for the flux.

Three-Phase Shell Construction.—A three-phase transformer of this type is constructed by mounting three single-phase shells on top of one another as shown in Fig. 161. The first phase, both primary and secondary, is wound in the top pair of holes, the second phase in the middle pair, and the third phase in the lowest pair. The full flux for each phase passes down the central iron path, but this flux divides, half passing to the right and half to the left, so that the iron section on the outside is made considerably less than it is in the middle. The section of iron immediately above and below the middle phase can also be reduced by reversing the whole of the connections of the middle phase, both primary and secondary. The horizontal flux in these regions is now equal to the flux per phase instead of being $\sqrt{3}$ times the flux per phase.



(a) (b)

FIG. 162.—Arrangement of Windings.

Types of Winding.—It is not the practice to wind one limb with the primary and the other with the secondary, although transformers are shown diagrammatically in this manner for the sake of clearness. On the contrary, a portion of each winding is placed on each wound limb, the L.T. (low tension) side being nearest the core. This only necessitates high grade insulation on the primary (H.T.) side and between primary and secondary, whereas if the H.T winding were nearest the core there would have to be high grade insulation on the primary itself, between primary and core and between primary and secondary. In addition, the H.T. winding is divided into sections as shown in Fig. 162 (*a*), in order to limit the voltage between adjacent turns.

Another method of winding is to subdivide both primary and secondary and sandwich the sections, as shown in Fig. 162 (*b*). This arrangement requires more insulation than the previous one.

Heavy current windings are sometimes wound in a single or double layer spiral, as represented by the inner winding in Fig. 162 (*a*). The H.T. windings of relatively small transformers are often wound with "cross-over" coils. These have a number of layers with several turns per layer. One end comes from the inside layer and the other from the outside layer. A number of such coils are connected in series, as represented by the outer winding in Fig. 162 (*a*). Another type of coil consists of a number of flat circular discs, each formed of a number of turns wound on top of one another radially outwards.

In very heavy current windings, each turn is split up into a number of separately insulated conductors, these being transposed from time to time, so as to equalize their impedance. Otherwise they would not share the current equally.

Effect of Winding Arrangement on Magnetic Leakage.—The distribution of flux shown in Fig. 150 is only intended to illustrate roughly how the leakage fluxes are set up, and is not intended as an

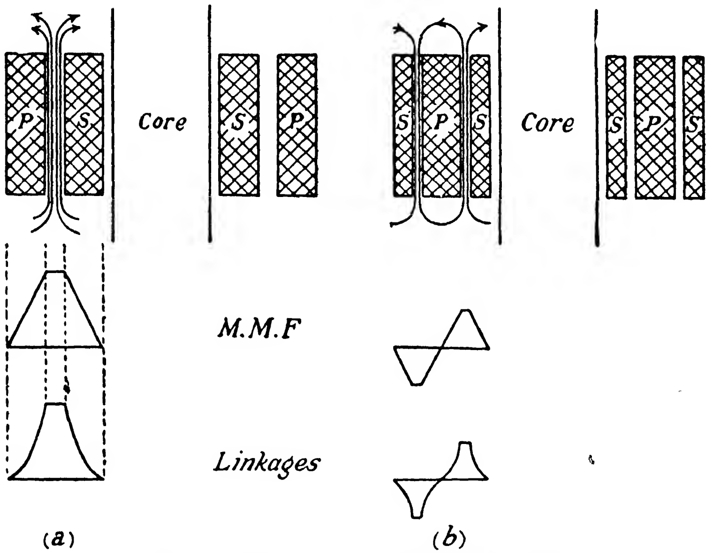


FIG. 163.—Magnetic Leakage with Concentric Coils.

accurate representation. The windings are not arranged with the primary on one limb and the secondary on the other, but are usually distributed as indicated in Fig. 162.

The distribution of leakage flux corresponding to the winding arrangement represented in Fig. 162 (*a*) is shown in Fig. 163 (*a*), where the secondary is placed next to the core and the primary has only one coil per limb, although the figure might be extended to show the effect of a subdivided primary. The M.M.F.'s of the two coils are supposed to be equal and opposite. The leakage flux passes between the coils and returns through the iron on the one side and through the air on the other. The space between the coils may be regarded as being somewhat similar to the interior of a solenoid, except that it is an annular space. The return path of the primary leakage flux has an infinite section so that its reluctance may be neglected.

Similarly the return path of the secondary leakage flux is in iron, and its reluctance may also be neglected. The M.M.F. over the space between the two coils is constant, but this falls away uniformly from the interior to the exterior of the coils. The curve of M.M.F. over the section is shown. The density of the leakage flux is proportional to the M.M.F., since the reluctance of the return paths is negligible. It is seen that some of the leakage flux actually passes through the windings, and these lines link with a proportionally smaller number of turns, depending upon the depth of the line in the winding. The linkage distribution over the cross section is therefore that shown, the curved portions of the graphs obeying a square law.

On occasion, when a very low leakage flux is desired, the arrangement shown in Fig. 163 (*b*) is adopted, half the secondary being wound next to the core and the other half outside the primary. The

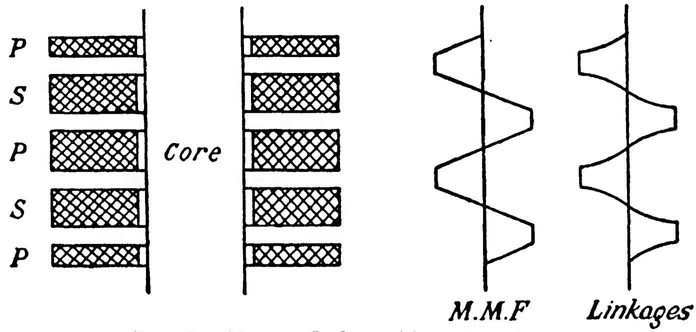


FIG. 164.—Magnetic Leakage with Sandwich Coils.

general distribution of the leakage flux is shown in the diagram, but its magnitude is much less than before. The curve of M.M.F. over the section is now partly positive and partly negative as shown. The leakage flux is a maximum in each space between the windings, but its direction is opposite in the two cases. The graph of the linkages is of the same general shape as in the previous case, except that it is in two halves, each the reverse of the other.

When the coils are sandwiched as shown in Fig. 162 (*b*) the distribution of M.M.F. and linkages is as shown in Fig. 164, the same general principles being applied. This case is also applicable to shell type transformers, where the windings in each window are arranged in layers alternately primary and secondary. In the simple shell case where there is no subdivision of the winding, a portion of the above M.M.F. curve can be picked out corresponding to one primary and one secondary coil.

Mechanical Stresses on Windings.—When adjacent primary and secondary coils carry currents, these are opposite in direction, and the two coils consequently exert a repelling force on each other. With normal currents, this effect is not of much account; but in the

Electrical Stresses on End Turns.—When the voltage is first applied to the primary of a transformer, the end turns have to be charged up to the line potential before any current can flow into the remainder of the winding, and since the two extreme turns are separated by a dielectric from the earth and from each other, they constitute a kind of condenser. These extreme turns take, consequently, a small but definite charging current, and then the next turns receive the voltage and are charged up in the same way. All this occurs very rapidly, but the effect is to concentrate the voltage on to the end turns at the moment of switching on. To avoid possible breakdowns of the insulation, the practice is to put extra insulation on the end turns to enable them to withstand the extra stress to which they are subjected at the moment of switching on.

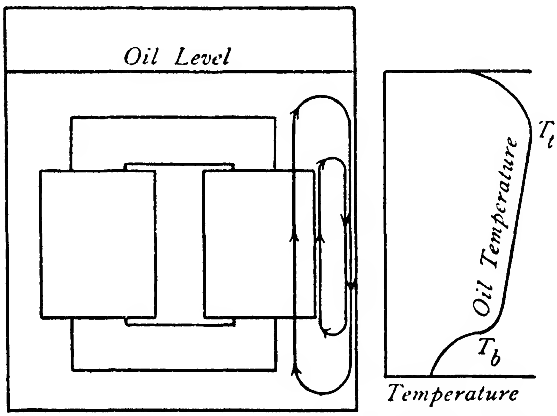


FIG. 167.—Convection Currents and Oil Temperatures in Transformer.

Artificial Cooling.—For small transformers natural cooling is sufficient, but for the larger sizes artificial means are employed to assist in the dissipation of the heat, since there is no rotating element to set up ventilation. Oil insulation is better than air in this respect, since the oil is a better conductor of heat. The final temperature rise depends upon the cooling facilities as well as upon the losses. In the case of oil-immersed transformers, the heat generated in the core and windings has to be transmitted to the tank, from the external surface of which it is dissipated into the atmosphere. When the oil is heated it becomes lighter and rises, cooler oil flowing in from the bottom of the tank to take its place. A circulation of the oil is thus set up. Fig. 167 shows the temperature of the oil at different points in a core type transformer. At the bottom of the tank the temperature is T_b ; the oil moves upwards and its temperature rises, finally reaching a maximum value T_t at the top. It then passes outwards to the tank and sinks downwards, the heat being gradually transferred to the tank wall. Another method of cooling consists of an air-blast, which is forced through the windings and ventilating ducts of the core by means of a small air-pump.

Transformer Tanks.—The tank containing the oil and the transformer may be plain or corrugated. Additional cooling surface may be obtained by means of a number of external tubes connected to the tank at their upper and lower ends, the former point being just below the oil level. The sides of the tank consist of boiler plate. Convection currents in the oil cause it to circulate through these tubes, which thus exert a great cooling effect.

A further development with very large transformers is to replace the numerous cooling tubes with several radiators. Each of these consists of a top and bottom header, connected by a number of cooling pipes, the whole being placed in a cold air blast. The oil headers are connected to the transformer tank, and the whole arrangement may be regarded as a special development of the tubular tank.

Conservators and Breathers.—Transformer oil should not be permitted to come into contact with the open atmosphere, since the absorption of even a minute amount of moisture causes a great deterioration in its insulating properties. To prevent this, many transformers are provided with *conservators* or *oil-expansion chambers*, especially those working at extra high voltages and those designed for outdoor operation. The conservator consists of a metal drum fixed above the level of the top of the tank, and connected by a pipe with the main tank, the joints being rendered air- and oil-tight. The main tank is then completely filled with oil, even when cold, the oil also partially filling the conservator. The surface of the oil exposed to the atmosphere is therefore greatly reduced. The convection currents in the main tank do not penetrate to the conservator, so that any sludge that is formed remains in the conservator. The oil expands as it heats up, so that ultimately the conservator is practically full of oil, the air being expelled. When the transformer cools down again, the oil level sinks, and air is drawn in. This action is known as *breathing*, the in-drawn air being passed through an auxiliary piece of apparatus, called a *breather*, for the purpose of abstracting the moisture.

A breather consists of a small vessel containing calcium chloride or other drying agent, through which the air is passed, the moisture being abstracted in its passage. This ensures that only dry air is in contact with the transformer oil.

Buchholz Relay.—This is a mechanical device which may be attached to an oil-immersed transformer, being fitted in the pipe connecting the transformer tank with the conservator tank. It consists of two (or sometimes only one) floats, as shown diagrammatically in Fig. 168. Two pairs of electrical contacts are also provided as shown, these being short-circuited in certain circumstances.

A breakdown in the insulation in a transformer is always accompanied by the generation of gas in the oil. A serious fault results in the rapid generation of this gas which, rushing through the pipe, pushes the lower float to the right. This causes the two lower contacts to be bridged, thus closing the trip circuit of the circuit breaker and cutting the transformer out of circuit. If a fault should develop slowly, gas is only generated slowly and this may be insufficient to move the lower float. This gas, however, accumulates gradually in the top of the relay chamber, so that there is a progressive lowering of the oil level in it. This causes the upper float to sink, so that it ultimately closes the second pair of contacts. This trips the circuit breaker as before, or it may be arranged to ring an alarm bell. An incipient fault can thus be detected, and the transformer switched out by hand before any major fault develops.

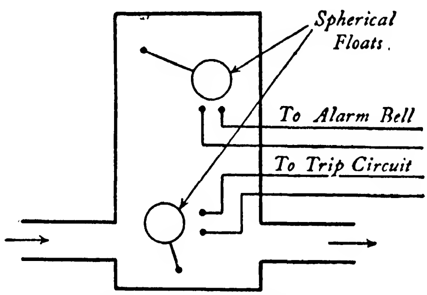


FIG. 168.—Buchholz Relay.

Water-Cooling.—In this type of cooling, water is passed through a system of pipes arranged in the upper portion of the tank, where the oil temperature is the highest.

Forced Cooling.—In this system of cooling, the hot oil is drawn off from the tank and passed through an external cooler of the surface condenser type, a motor-driven oil pump being employed to effect the circulation. The cooling tubes or radiators may now be omitted, plain transformer tanks being employed.

In certain large installations a mixture of radiator and forced cooling is adopted. Up to about half full load, the radiators are relied upon for the necessary cooling, but for the higher loads, the oil pump is brought into action, often automatically.

Transformer Oil.—Transformer oil is a mineral oil obtained by refining crude petroleum. It is an excellent insulator, its loss by evaporation is negligibly small, and its tendency to form a sludge is much less than with the resin oils which were once used. This sludge is due to the slow formation of solid hydro-carbons. When deposited on the windings and in the cooling ducts, it tends to produce overheating. Unfortunately, its dielectric strength is affected to an enormous extent by the presence of even a minute percentage of moisture, and great care must be taken in practice to keep the oil dry.

Terminal Bushings.—The terminals are brought through the tank by means of bushings, generally made of porcelain or paper and bakelite or other insulating varnish in the case of high voltage transformers. If a plain cylindrical tube is used, the potential gradient in the insulation is very high on the inside, this decreasing towards the outside. In order to reduce the maximum value of this potential gradient, the dielectric flux density must be reduced at the surface of the conductor. This can be effected either by increasing the diameter of the conductor or by increasing the thickness of the insulation.

The condenser type of bushing makes the outer layers of the insulation carry their proper share of the voltage in a different way. It consists of a number of concentric condensers of tinfoil and paper in series between the central conductor and the cover of the tank. The voltage drop across each of these component condensers is inversely proportional to their capacitance, and to make the potential gradient uniform, they all must have the same capacitance. Since the insulation thickness is made the same, the area of the various tinfoils must be the same. This area is equal to πDL, where D is the diameter and L the length parallel to the conductor. It follows, therefore, that D must be made inversely proportional to L, and the bushing takes on the shape shown in Fig. 169. The length of the portion projecting into the tank is less than that projecting outwards, since the oil is a better insulator than air. Unfortunately, however, the bushing is not economically designed for surface leakage, since the distance on the surface is not the same between different layers. The shape is, therefore, modified somewhat in practice. The bushing is also surrounded with a cylinder of fibre which is filled with compound to prevent the formation of corona.

An alternative to the above is the oil-filled bushing. In this type, the conductor is passed through a large hollow porcelain insulator which is filled with oil. When intended for very high voltages, a number of insulating cylinders are inserted, these being concentric with the conductor and graduated in length. These baffle tubes, as they are called, prevent particles of foreign matter from forming themselves into a continuous chain between the conductor and the wall of the insulator. A glass tube is mounted on the top of the bushing, this being filled with oil when the maximum operating temperature is reached. A small breather is attached to the outer end of this tube, to prevent the absorption of moisture, as in the case of the main transformer.

Polyphase Transformer Connections.—The connections of two-phase transformers or two single-phase transformers used on a two-phase supply are quite simple, each phase being connected up independently and the system linked or not according to desire. With three-phase transformers, however, a number of methods of connection are possible, these methods also being applicable when three single-phase transformers are used. The several methods employed are as follows:—

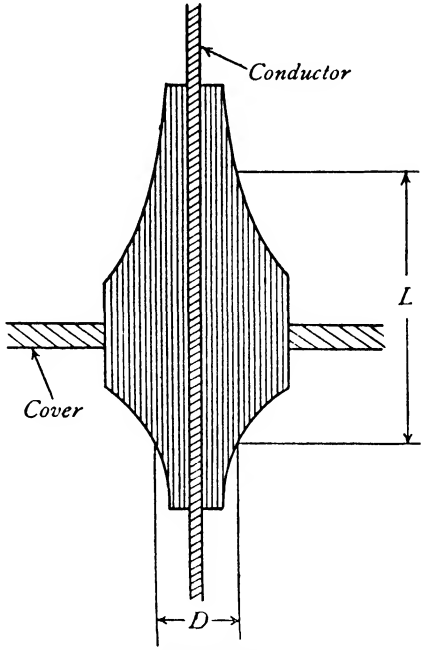


FIG. 169.—Condenser Type Bushing.

(*a*) *Star- or* $\curlywedge$*-Connection.*—Both primaries and secondaries are connected in star [see Fig. 170 (*a*)], but if one phase should fail it puts two phases out of action, which practically means a complete shut-down of the transformer.

(*b*) *Mesh- or* Δ*-Connection.*—Both primaries and secondaries are connected in delta [see Fig. 170 (*b*)], but if one phase winding should fail the other two would continue to supply a true three-phase current. In this respect the delta connection is preferable to the star. The new system is called the V-connection or open Δ.

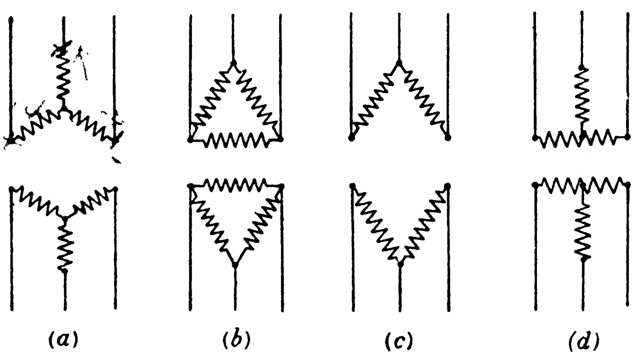


FIG. 170.—Three-Phase Transformer Connections.

It is also possible to have the primaries connected in delta and the secondaries in star, and *vice versa.*

(*c*) V- *or open* Δ-*Connection.*—Only two transformers are needed [see Fig. 170 (*c*)], but since the phase difference between the two secondaries is the same as that between the two primaries, the three-phase supply is maintained. The current in the common wire is the vector sum of the currents in the other two.

(*d*) T-*Connection.*—Only two transformers are needed in this system of connection [see Fig. 170 (*d*)], but the voltages which are applied to their terminals are slightly different, that transformer which is connected to the middle point of the other only being supplied with $\frac{\sqrt{3}}{2} = 0{\cdot}866$ times the voltage between line wires. The ratio of transformation of both transformers is the same, so that a true three-phase supply is obtained at the free terminals of the secondaries.

Interconnected Windings.—Three-phase transformers are sometimes used to supply three separate single-phase circuits, the neutral acting as the common return. They are also used in connection with three-wire rotary converters (see page 437), the out-of-balance direct current being brought to the star point of the transformer. In these cases the currents in the different phase windings are usually unequal. This affects the magnetizing current and may result in an appreciable increase in the iron loss owing to flux distortion, and to minimize this effect interconnected windings or zigzag connections are sometimes adopted. There are two secondaries to each phase connected as shown in Fig. 171. A secondary from one phase is connected in series with a secondary from the next phase, the latter being reversed so that the phase difference between the E.M.F.'s in the two windings is only 60° instead of 120°. Thus the winding *gh* is connected in series with *on* (not *no*), the resultant voltage being $\sqrt{3}$ times the voltage across each of the component windings. The phase of the resultant voltage is also shifted by 30°. The advantage of this system of connections lies in the fact that if the load on the secondary is unbalanced, the inequality is reduced in the primary circuit, for each secondary directly affects two of the primary windings instead of one in the ordinary method of connection.

Tap-Changing under Load.—The object and arrangement of tapping coils have already been explained (see page 193). If the transformer can be withdrawn from the circuit when any change is made, a system of links or switches is all that is necessary, but when the tap-changing has to take place whilst the transformer is under load, a rather more complicated arrangement has to be adopted. The circuit must not be opened, even momentarily, or dangerous sparking will ensue, whilst the temporary short-circuit of a section of the transformer winding is equally inadmissible.

The principal method of effecting the change employs a reactor provided with a mid-point tapping. The tappings are brought out to a number of contacts (see Fig. 348), and in the normal position, two fingers, connected to the two ends of the reactor, rest on the same contact. One half of the total current now flows through each half of the reactor, but in *opposite directions*, so that this becomes practically non-inductive, and therefore causes only a negligible voltage drop. In changing from one tapping to the next, two adjacent contacts are bridged momentarily, but the reactor prevents an excessive short-circuit current from flowing, since it is highly inductive to a current flowing straight through it A further

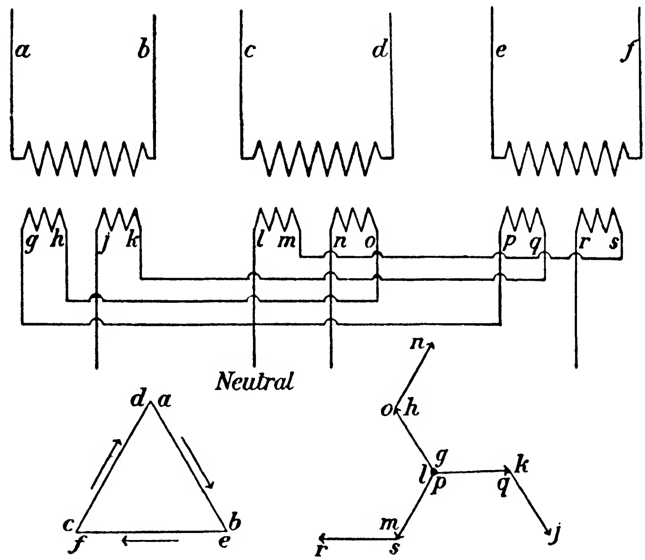


FIG. 171.—Interconnected Windings.

advantage of this method is that it gives an additional number of voltages, due to the intermediate positions, and so reduces the number of tapping coils required.

Tertiary Winding.—It has already been shown that third harmonic currents cannot flow in a balanced three-phase three-wire system (see page 127), and that when three similar saturated choking coils are connected in star, the potential of the insulated neutral point oscillates at third harmonic frequency about a mean value (see page 129). These conditions are obtained in a star-star three-phase core type transformer, which accordingly has an oscillating neutral. If either the primary or the secondary is connected in delta, third harmonic magnetizing currents are permitted to circulate, thus restoring the flux waves to their sinusoidal wave form, and stabilizing the potential of the neutral. If, however, both windings are connected in star, the neutral potential may oscillate to a very undesirable extent, and to prevent this a third winding, called a *tertiary winding,* is sometimes added. This consists of three small auxiliary windings connected in delta and insulated from the other two sets of windings. The resultant E.M.F. at fundamental frequency around this tertiary delta is zero, so that no fundamental current flows. On the other hand, the tertiary delta constitutes a short-circuit path for the third harmonic currents, and these currents provide the harmonic deficiency in the magnetizing currents flowing in the primary windings. The flux waves are thus restored to their normal shape, and the potential of the insulated neutral is stabilized.

If an earth fault should develop on one of the main lines, the E.M.F. in one of the legs of the tertiary delta will disappear. There

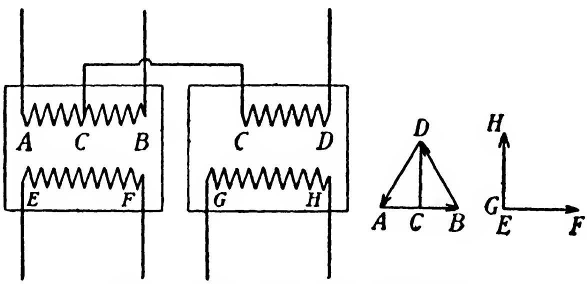


FIG. 172.—Scott System of Transformation.

is now an unbalanced resultant E.M.F. acting around the closed tertiary delta, and this may cause it to burn out.

Parallel Running.—In a number of the three-phase transformer connections there is a phase displacement of the secondary line voltage, as in the case of a delta-star connected transformer. If the primaries of this be paralleled with the primaries of a star-star transformer, then it is not possible to run the secondaries in parallel, for the line voltages in the two cases have a permanent phase displacement of 30°.

A delta-star group may be paralleled with a star-delta or with a star-interconnected star group, but not with a star-star or delta-delta group. These last two may be, however, paralleled with each other.

Scott System of Transformation.—The Scott system of transformation is a method whereby the supply may be changed from three-phase to two-phase, or *vice versa.* Two transformers have their primaries T-connected on to the three-phase supply, whilst their secondaries are connected independently and deliver a true two-phase supply (see Fig. 172). The primaries of these two transformers are wound for voltages in the ratio of $1 : \frac{\sqrt{3}}{2}$, since the primary of the second one only receives this fraction of the line voltage. As is seen from the vector diagram, CD is 90° out of phase with AB, and consequently GH is 90° out of phase with EF. If the ratio of transformation were the same in the two transformers, however, the voltage of the second phase would be too low, and so the voltage given by the secondary of the second transformer is increased relatively to that of the first one in the ratio $\frac{2}{\sqrt{3}}$. This equalizes the voltages, which are already 90° out of phase with each other.

The ampere-turns on both windings on the two-phase side are the same, whereas this is not apparent on the three-phase side, since the two windings carry the same current and have different numbers of turns. The current flowing from A to C, however, is 120° out of phase with that flowing from B to C, and is therefore 60° out of phase with the current flowing from C to B. On combining the ampere-turns of the windings AC and CB, the resultant is found to be equal to $\sqrt{3}$ times the ampere-turns due to AC or CB acting separately, and so is seen to be the same as the ampere-turns due to CD.

Although this system of connections gives rise to a balanced arrangement as far as the fundamental is concerned, it is not balanced when harmonics are considered.

Three- to Six-Phase Transformation.—For a number of purposes a six-phase supply is desired, *e.g.* rotary converters (see page 435), and a simple means of obtaining this is available by the use of a three-phase transformer or three single-phase transformers. On the secondary side, all six ends are brought out separately, instead of connecting them to form a star or delta system. If desired, the mid-points of the three secondary windings may be linked to form a star point, although this is not absolutely necessary. The ends of the first secondary give phases 1 and 4, the second giving phases 3 and 6, and the third 5 and 2. This system of connections is known as the *diametric* connection.

A six-phase supply can also be obtained by means of a double delta arrangement.

Three- to Single-Phase Transformation.—On occasion a single-phase supply is desired from a three-phase source. The easiest method is to use one phase only, leaving the other two phases unloaded, but this causes unbalance. A better way is to connect the three secondaries in series as shown in Fig. 173, although this also leads to unbalance.

Auto-Transformers.—In an ordinary transformer there is no electrical connection between primary and secondary, but since the volts per turn are the same in each case, there is no fundamental reason why the two windings should not lie side by side without any intervening insulation. Carrying this idea further, the same wire might be used for carrying the two currents. Since it is presumed that one winding has more turns that the other, this superimposition only exists in that part of the winding which is common to both primary and secondary. Fig. 174 illustrates in

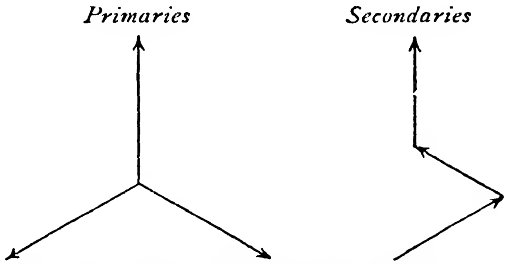


FIG. 173.—Three- to Single-Phase Transformation.

diagrammatic form the principle of the auto-transformer, the voltage being reduced in this instance, although the reverse is also possible. Since the primary and secondary currents act in opposition, the resultant current in BC is the vector sum of the two, this being very nearly the arithmetic difference. Thus suppose the primary voltage and current were 100 volts and 10 amperes respectively and the secondary voltage and current 10 volts and 100 amperes respectively, AC would carry 10 amperes whilst BC would carry $100 - 10 = 90$ amperes.

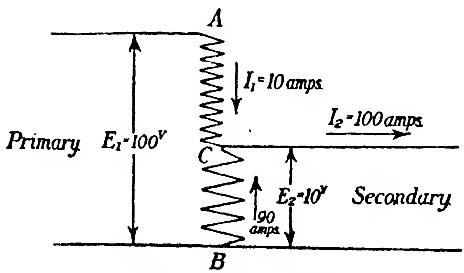


FIG. 174.—Auto-Transformer.

The auto-transformer is not merely a potential divider, although it does act in this capacity, but there is true transformer action, for the secondary current exceeds that of the primary when the ratio of transformation is greater than unity.

A great disadvantage of the auto-transformer is that there is a direct electrical connection between the primary and secondary. If the primary is supplied at H.T., there is a possibility of a dangerous voltage reaching the secondary circuit in the event of a fault occurring in the winding, and this usually precludes the use of an auto-transformer in such circumstances. Its usefulness is therefore limited to cases where the voltages and ratios of transformation are low, and for certain special purposes.

Three-phase auto-transformers are usually connected in star or in V as shown in Fig. 175.

One very useful application is as a starter for induction motors. A number of tappings are made on the winding so as to obtain a variable voltage on the secondary, these tappings being brought out to contacts after the manner of an ordinary D.C. motor starter. Such a piece of apparatus is termed a *compensator*, for it allows

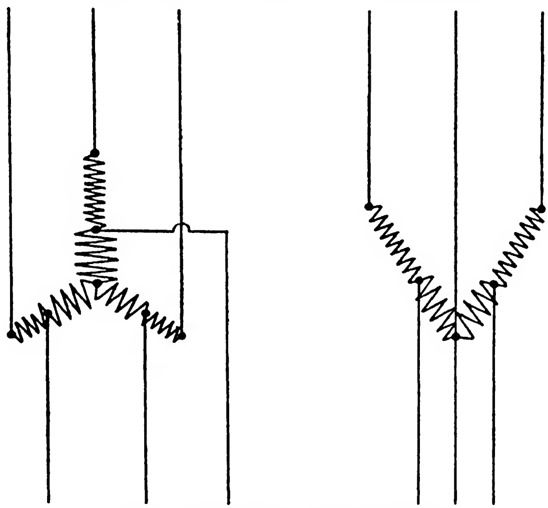


FIG. 175.—Three-Phase Auto-Transformers.

the motor to take an excess current without putting a heavy overload on the mains. In the majority of small compensators there is only one tapping corresponding to only one starting stop in addition to the full running position, the windings being connected in V.

Relative Sizes of Auto- and Double Wound Transformers.—An ordinary double wound transformer can be converted into an auto-transformer by connecting the primary and secondary windings in series, as shown in Fig. 176. It is essential that the two windings be connected in such a way that the total voltage is the sum (and not the difference) of the voltages across the two individual windings.

If the input be denoted by $E_1 I_1$ and the output by $E_2 I_2$ (losses being neglected), an ordinary double wound transformer would have to be designed for this output. The voltage across the first winding of the auto-transformer is, however, only $(E_1 - E_2)$ volts, and so this winding need only be designed to deal with an input of $(E_1 - E_2)I_1$. Similarly, since the load current is I_2, the current taken by the second winding is only $(I_2 - I_1)$ and this winding need only be designed to deal with an output of $E_2(I_2 - I_1)$. The quantity $(E_1 - E_2)I_1$ can be re-written

$$(nE_2 - E_2) \times \frac{I_2}{n}$$
$$= E_2 I_2 \left(\frac{n-1}{n}\right).$$

Also

$$E_2(I_2 - I_1) = E_2\left(I_2 - \frac{I_2}{n}\right)$$
$$= E_2 I_2 \left(\frac{n-1}{n}\right)$$

the same as before.

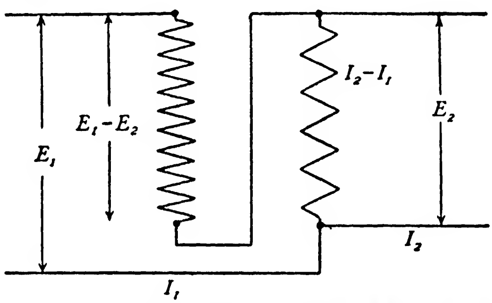


FIG. 176.—Double Wound Transformer connected as Auto-Transformer.

An auto-transformer having an output of $E_2 I_2$ may therefore be designed as if it were an ordinary transformer having an output of $E_2 I_2\left(\frac{n-1}{n}\right)$. An auto-transformer thus has a smaller core and smaller windings than an ordinary double wound transformer of the same output, the ratio being $\frac{n-1}{n}$. Thus an auto-transformer having an output of 100 kVA with a 4 to 1 ratio would be of the same size as an ordinary transformer of $100 \times \frac{4-1}{4} = 75$ kVA. The saving in material is thus considerable for small ratios of transformation, but is not of great importance where large ratios are concerned.

Transformers for Very High Voltages.—Groups of transformers giving single-phase voltages up to one million volts are required for testing high voltage apparatus. The currents obtained from such testing equipments are quite small, insulation being the chief problem. They usually operate with one terminal earthed.

There are practical difficulties in obtaining the total voltage from one transformer, so that three auto-transformers are connected in series to give 1,000,000 volts. Fig. 177 shows the arrangement which is called the *cascade* connection. The middle point of the winding of the first transformer is electrically connected to its tank, so that the voltage between winding and tank is limited to 166,000 volts. The whole of this transformer is then mounted on a base insulated from earth for 333,000 volts. Tappings from the high potential end of the first transformer act as the feed for the second

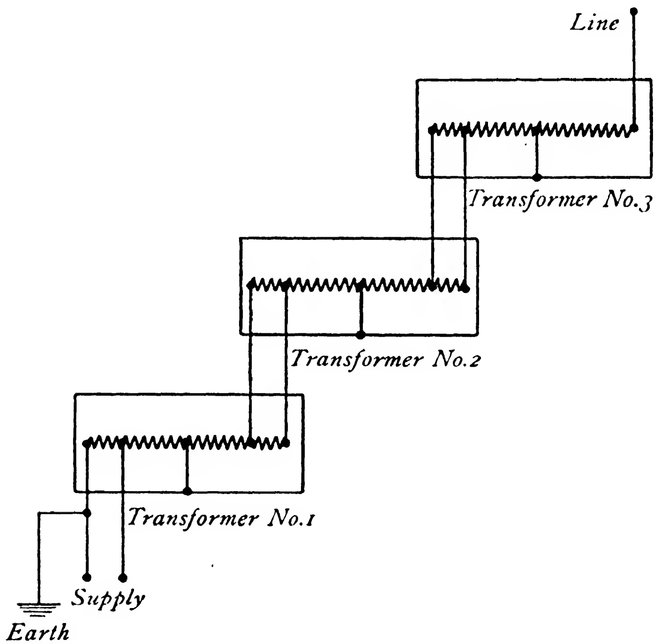


FIG. 177.—Cascade Connections for 1,000,000 Volts.

transformer, the base of this being insulated from earth for 666,000 volts. The process is again repeated, the base of the third transformer being insulated from earth for the full million volts.

Voltage and Current Transformers.—Small transformers are often used in conjunction with voltmeters for voltage measurements on H.T. lines. The primary is wound with many turns of fine wire, whilst the secondary is designed to give a low voltage which is applied to the terminals of the voltmeter [see Fig. 178 (*a*)], the scale being arranged in most cases to read the voltage on the primary side. Such a transformer is called a *voltage transformer* and works under a small constant load due to the voltmeter itself. The ratio of transformation must be determined with the instrument connected in position, for the voltage drop is such that a considerable difference would be made if the voltmeter were removed or if another one of different impedance were substituted.

In a similar way, small instrument transformers, called *current transformers*, are used in conjunction with ammeters. The connections of an ammeter with a current (or series) transformer are shown in Fig. 178 (*b*). In this case the primary current is determined by what current is flowing in the mains rather than by the impedance of the winding, and, with the secondary circuit closed, the secondary current is such that the primary and secondary ampere-turns balance each other, neglecting magnetic leakage. The currents are therefore approximately in the inverse ratio of the turns and the ammeter can be calibrated to read the primary current, provided that the same instrument is always used in conjunction with the same current transformer. This arrangement takes the place of the shunt in D.C. measurements and enables heavy currents to be measured without the trouble of designing instruments with heavy leads, since the instrument transformer, which is only small, can

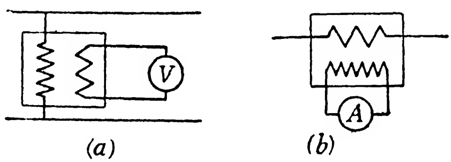


FIG. 178.—Voltage and Current Transformers.

be placed in the path of the main lead, whilst the small secondary leads can be carried to the instrument, which may then be situated wherever convenient.

A.C. wattmeters may also be used in conjunction with both a voltage and a current transformer.

Another application of voltage and current transformers is their use in the operation of protective relays.

The ratio of transformation should be constant within close limits. It is possible to calibrate the instrument with its own transformer, thereby avoiding the effect of ratio variation, but it is more usual to design the transformer so that the instrument may be calibrated independently of it. This necessitates low resistance of the windings, low flux densities and small magnetic leakage between the windings. The voltage drop between open circuit and the rated secondary load should not exceed half of one per cent. In power measurements the phase displacement between primary and secondary voltages is important and should be kept within 1° of the ideal value. This phase displacement is roughly proportional to the secondary current.

In the case of current transformers it is not essential that the voltage ratio should be constant nor that the primary and secondary currents should be exactly in phase, but constancy of the current ratio and phase displacement is important. For this condition to be fulfilled the shape of the vector diagram should remain unaltered throughout the working range. To reduce the magnetizing current and the iron loss, silicon steel or stalloy is used and a low flux density is employed. The primaries usually have from 600 to 1200 ampere-turns at full load. The rating of a current transformer is expressed in volt-amperes and usually lies between 15 and 50 volt-amperes at full load current.

Should it be necessary to remove the ammeter from the secondary of a current transformer, the secondary should first of all be short-circuited. The primary current would normally be unaffected by the removal of the ammeter, so that the resultant ampere-turns on the transformer core would be very greatly increased. The flux density would rise considerably, and the iron losses would be enormous. The voltage across the primary would rise and in all probability the transformer would be destroyed.

The ranges of instrument transformer secondaries have now been standardized, 110 volts and 5 amperes being adopted for voltage and current transformers respectively.

Quadrature Transformers.—A quadrature transformer is one in which the secondary current is 90° out of phase with the primary current instead of the usual 180°. They are used in making measurements with certain A.C. instruments, *e.g.* the Gall potentiometer, the connections being those of an ordinary current transformer. A peculiar construction is adopted, a long air-gap being included in the magnetic circuit to make it as leaky as possible. In fact a wholly non-magnetic core might be used, but this tends to increase the size and is not necessary. The secondary circuit has a high non-inductive resistance placed in series with it, so as to cause only a small secondary current to flow, this being in phase with the secondary E.M.F. Since the load on the transformer is low and the no-load ampere-turns are comparatively large, the primary current lags by practically 90° behind the voltage across its terminals. The secondary current, which is in phase with the secondary voltage, lags by practically 180° behind the primary voltage, and consequently by 90° behind the primary current.

Phase Shifting Transformers.—The phase shifting transformer is in reality a small induction motor (see page 323), with both stator and rotor wound two- or three-phase according to requirements. The stator winding acts as the primary and causes an E.M.F. to be induced in the rotor winding which is constant for any position relative to the stator when a constant E.M.F. is applied to the primary. The only variable condition is the phase angle between the applied and the induced voltages. This is varied by altering the relative positions of stator and rotor.

For single-phase systems a two-phase transformer is used, one set of windings being connected in series with a condenser, so as to obtain the phase difference necessary to set up the rotating field (see page 132).

These phase shifting transformers are very useful for testing wattmeters, power factor indicators, etc.

Boosting Transformers.—Sometimes a transformer is used for boosting up the supply pressure, in which case the primary is connected across the mains, whilst the low voltage secondary is placed in series with the mains as in Fig. 179. A number of tappings are taken from different points along the secondary to the contacts of a multi-way switch, the handle of which is connected to the line. The switch blade is made in two insulated parts connected by a low resistance or reactance after the manner of a battery switch, so that in moving from one contact to another the circuit is never opened, nor is one section of the secondary momentarily short-circuited. In this way, a variable boost is obtained depending upon the position of the switch handle. Looked at from another point of view, such a transformer may be regarded as a step-up auto-transformer. (See also "Induction Regulators," page 425.)

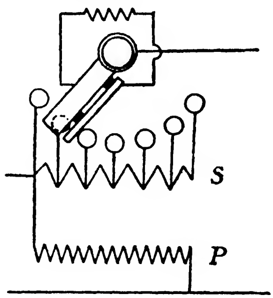


FIG. 179.—Boosting Transformer.

EXAMPLES.

(1) The following data refer to a single-phase transformer: Turn ratio, 19·5 : 1. $R_1 = 25$, $X_1 = 100$, $R_2 = 0{\cdot}06$, $X_2 = 0{\cdot}25$. No-load current = 1·25 amperes, and leads the flux by 30°. The secondary delivers 200 amperes at a terminal pressure of 500 volts and a power factor of 0·8 (lagging).

Determine by the aid of a vector diagram—

(1) the primary applied voltage;
(2) the primary current;
(3) the primary power factor;
(4) the efficiency;
(5) the regulation.

(2) Derive from first principles a formula for the induced E.M.F. of a transformer.

A single-phase transformer is supplied at 111 volts, 50 cycles, the number of primary turns being 250. Calculate the flux in the core.

(3) A three-phase transformer has its primary delta-connected and its secondary star-connected. The primary and secondary line volts are 6600 and 380 volts respectively. The flux is $2{\cdot}0 \times 10^6$

lines and the frequency is 50. Determine the number of turns on each primary and secondary, neglecting losses. How would the losses affect the result?

(4) Draw the equivalent circuits for a single-phase transformer, explaining the significance of each portion of the circuits. Construct a vector diagram, comparing it step by step with the equivalent circuits diagram.

(5) Write an account of the various methods of cooling power transformers.

(6) A group of transformers is Scott-connected for transforming from two- to three-phase. Each primary takes 20 amperes at 6000 volts, and the secondaries deliver power at 500 volts between lines. There are 1000 turns on each primary. How many turns are there on the secondaries, and what is the current in each secondary, neglecting losses?

(7) An auto-transformer has 200 volts applied to the full winding, the secondary circuit of which is supplied from 60 per cent. of the turns. When a secondary current of 50 amperes is flowing, determine the current in each part of the winding, neglecting the losses and no-load current. How would these affect the result?

(8) A three-phase transformer has its primaries connected in delta and its secondaries in star. It has an equivalent resistance of 1·0 per cent. and an equivalent reactance of 6·0 per cent. The primary applied voltage is 6600 volts. What must be the ratio of transformation in order that it shall deliver 480 volts on full load current at 0·8 power factor, lagging?

(9) What are "tap" coils on a transformer? Why are they employed, and where are they situated in the windings?

(10) A three-phase transformer has its primary windings connected in star and its secondary windings connected in delta. With the secondary windings disconnected from their external loads it is observed that a current is circulating through the closed delta. Explain the origin of this and show how it assists in stabilizing the potential of the primary star point which is insulated from earth.

CHAPTER XV

TRANSFORMERS.—PERFORMANCE AND TESTING

No-load Current.—The no-load current of a transformer consists of a small active component necessary on account of the iron loss in the core and a comparatively large reactive component which supplies the magnetizing ampere-turns. The total no-load current is the vector sum of these two components. The transformer may be supplied on either the high or low tension side, the two different no-load currents being in the inverse ratio of the number of turns on the two windings. Neglecting the small IR drop, the flux produced is proportional to the applied voltage, so that if the relation between the reactive component of the magnetizing current and voltage be plotted for various applied voltages, the resulting curve should have the same shape as the B—H curve for the iron. If the iron is unsaturated, the wave form of the magnetizing current is sinusoidal, assuming a sinusoidal applied E.M.F., but as soon as the iron is taken beyond the straight line part of the B—H curve, harmonics are introduced into the magnetizing current. With the flux densities now employed, these harmonics become very important. The third harmonic is the largest, and after that the fifth. In a three-phase core type transformer with insulated neutral, third harmonic currents cannot flow, and this further complicates matters. In practice, the no-load current frequently has a very distorted wave form.

If the applied voltage be kept constant and the frequency varied, it is seen from the general equation that a high frequency corresponds to a low no-load current, for

$$\Phi = \frac{E_1 \times 10^8}{4{\cdot}44 f T_1}$$

(see page 177). The flux is inversely proportional to the frequency, an increase in which means a reduction of the flux and consequently of the magnetizing current.

The hysteresis loss is proportional to fB^x, where x is greater than unity, so that an increase in frequency causes B^x to decrease more rapidly than f increases. The hysteresis loss therefore decreases as the frequency is raised. The eddy current loss is proportional to f^2B^2, and the increase in f^2 balances the decrease in B^2, the loss being independent of frequency. Thus the total losses are reduced, and the active component of the current as well as the magnetizing (reactive) component decreases with an increase in frequency which enables a smaller core to be used in a transformer for a given duty resulting in a lowering of the first cost.

No-Load Current Wave Form in Three-Phase Transformer.—If both primary and secondary windings of a three-phase core-type transformer are connected in star, the two star points being insulated, no third harmonic current can flow (*see* p. 127), so that the magnetizing current required to produce a sine wave of flux is not possible. The flux wave is flattened to a certain extent, a third harmonic being introduced into the flux wave. This induces third harmonic E.M.F.'s in both primary and secondary windings, all of which, in one circuit or the other, have a phase difference of $3 \times 120° = 360° = 0°$ with respect to one another. In other words, these third harmonic E.M.F.'s are all exactly in phase, when considered acting away from the star point. The resultant third harmonic E.M.F. between lines is thus zero in all cases, no third harmonic E.M.F. appearing. Between line and star point, however, there is a third harmonic E.M.F., and this causes the potential of the star point to fluctuate, if it is insulated from earth (see p. 129).

Assuming the star point of the generator supplying the transformer to be earthed, the voltage between line and earth can be assumed to be sinusoidal. The voltage between line and the transformer star point now consists of a fundamental, which is imagined to equal the generator phase voltage, together with a third harmonic voltage. The transformer star point thus has a voltage to earth which consists of this third harmonic component, and thus varies in potential, with respect to earth, at third harmonic frequency. Such a star point is called an *oscillating neutral*, and the voltage oscillation may be very considerable.

If the transformer primary windings are connected in delta, different conditions are set up. Assuming the supplying generator to induce a sinusoidal E.M.F., this can be supposed to produce a sine wave of magnetizing current in the first instance. As has already been shown, this results in a flat-topped wave of flux, which may be supposed to consist of a sine wave together with a third harmonic. E.M.F.'s of corresponding frequency are induced in the transformer windings. The induced E.M.F. of fundamental frequency neutralizes the applied E.M.F., whilst the third harmonic E.M.F.'s produce third harmonic currents which aid in the magnetization of the core. There are three of these third harmonic E.M.F.'s, each having a phase difference of $3 \times 120° = 360° = 0°$ with respect to one another. The three E.M.F.'s thus being in phase, the closed delta constitutes a short-circuit to them, and since the impedance is very low, a very considerable third harmonic current is set up by even a very small third harmonic E.M.F. These third harmonic magnetizing currents supply the deficiency in the magnetizing current derived from the supply and restore the flux wave almost to its ideal sine shape. Thus there are practically no third harmonic E.M.F.'s induced in the secondary windings, and the potential of the secondary star point is stabilised at earth potential.

If the transformer primary windings are connected in star, the secondary windings being connected in delta, similar conditions are set up. The fundamental magnetizing current is derived from the supply, this producing in the first instance a flux wave containing a third harmonic. Third harmonic E.M.F.'s are induced in the secondary windings and these, being connected in delta, again constitute a short-circuit to the resulting third harmonic currents. The harmonic magnetization is now provided by the secondary windings, and the flux wave restored almost (but not quite) to its ideal sine shape. The potential of the primary star point is now stabilised and the oscillating neutral is again avoided.

If both primary and secondary windings of the transformer are connected in delta, the harmonic magnetization is provided partly by each set of windings.

Tertiary windings are sometimes used when both primaries and

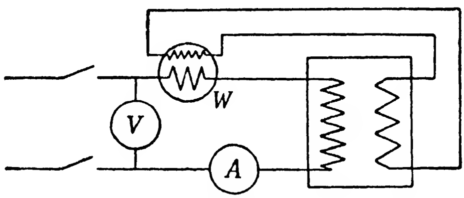


FIG. 180.—Connections for Measuring Iron Loss.

secondaries are connected in star. In this case the fundamental magnetization is provided by the primary windings, the harmonic magnetization being obtained from the circulating currents in the tertiary delta. The lower the impedance of these latter windings, the more complete is the correction of the flux wave, and the less the variation in the potentials of the two star points.

No-load Losses.—The no-load losses of a transformer consist of a very small I^2R loss and an iron loss due to hysteresis and eddy currents. The latter can be determined in the way indicated in Chapter VIII by simply measuring the power input to the primary by means of a wattmeter. The iron losses can be separated by the method involving a constant flux obtained by varying the applied voltage and the frequency at the same rate (see page 81). Ordinarily, the wattmeter reading includes the small I^2R loss, but if it is desired to take account of this, the connections shown in Fig. 180 can be adopted, the voltage coil of the wattmeter being connected across the secondary. The wattmeter reading multiplied by the ratio of transformation gives the iron loss only, for the voltage actually used for transformer action is only that which is left after the small IR drop has been subtracted vectorially. The voltage required is therefore the secondary voltage multiplied by the ratio of transformation.

Effect of Wave Form on Iron Loss.—When the applied E.M.F. wave form is not sinusoidal the flux is given by

$$\Phi = \frac{E_1 \times 10^8}{4kfT_1},$$

where k is the form factor. Thus a high form factor causes a reduction in the flux for a given R.M.S. value of applied E.M.F., and results in a decrease in the hysteresis loss, for the work done in carrying the iron through a complete magnetic cycle is independent of the rate at which this is done.

A correction may also be applied to allow for the effect of form factor on the eddy current component of the losses. This may be determined by taking three or four tests with different values of form factor, k, and plotting a curve connecting total losses and k^2. This curve should be linear and the total losses corresponding to $k = 1{\cdot}11$ may be read off the graph by extrapolating to $k = 1{\cdot}11^2 = 1{\cdot}232$. The change in wave form is most conveniently obtained by inserting a resistance or an air-cored inductance in series with the magnetizing circuit, this bringing about a change in wave form. The correction is negligible if the form factor of the induced E.M.F. departs less than 2 per cent. from the sinusoidal value of 1·11.

The total iron loss is therefore slightly reduced when a peaked E.M.F. wave form is employed, since in such cases the form factor is greater than that of a sine wave. Alternatively, a flat-topped wave causes a slight increase in the iron loss.

The disadvantage of a peaked wave is that it imposes a greater strain on the insulation.

Regulation of a Transformer.—By the regulation of a transformer is meant the drop in the secondary terminal voltage experienced when full-load current is taken from the transformer, and is usually expressed as a percentage of the open circuit secondary voltage. It is usual, also, to specify the power factor of the load, as this has a most important effect, the voltage drop being considerably greater for low power factors. At no-load the ratio of transformation is practically equal to the ratio of the number of turns on the two windings, but as the load comes on, the secondary terminal voltage drops, the applied primary voltage being assumed constant. The ratio of transformation, therefore, is no longer the exact ratio of the turns, but becomes slightly larger, due to the presence of resistance and reactance in both windings. Referring to the vector diagram in Fig. 152, it is seen that the drop in voltage on the secondary side is the numerical difference between E_2 and V_S, and that this is *not* the same as the voltage absorbed in overcoming the secondary impedance, which is I_2Z_2. The same thing occurs on the primary side, so that in general the voltage drop is less than the voltage absorbed by the impedance of the windings.

The regulation can be approximately determined experimentally by connecting the transformer as shown in Fig. 181. The primary voltage should be kept constant at its normal value. If the load is non-inductive, the wattmeter may be omitted, but otherwise it is necessary, in order to determine the power factor.

Since the drop is measured by the difference of two very similar voltages, great accuracy in the voltage measurements is necessary in order to ensure a moderate accuracy in the result. This dis-

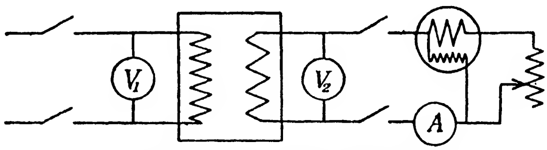


FIG. 181.—Regulation Test of a Transformer.

advantage is overcome in the back-to-back method, which will next be described.

Back-to-Back Regulation Test.—For this test two approximately similar transformers are required. The primaries are connected in parallel across the mains as shown in Fig. 182, whilst the secondaries are also connected in parallel, but with a low reading voltmeter inserted in one lead. It is important that the two secondaries should be in opposition and not aiding one another, as in the latter case the low reading voltmeter would be burnt out. In order to test this before the voltmeter is inserted, the voltage across the

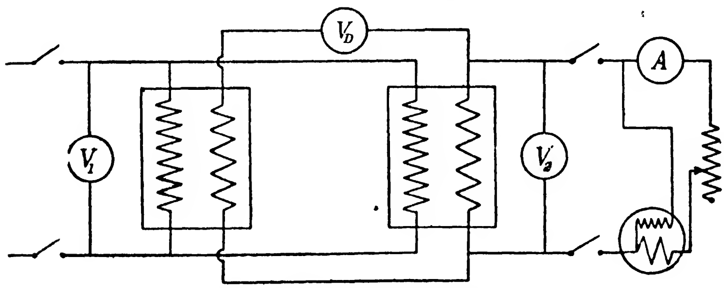


FIG. 182.—Back-to-Back Regulation Test.

free ends should be tested by means of another voltmeter or lamp capable of standing double the secondary voltage of either transformer. If a high voltage is observed, the connections to one transformer must be interchanged. Having arranged this satisfactorily, a load circuit is connected to one secondary as shown in the diagram. The other transformer will not contribute anything towards the load current, for it has the voltmeter V_D directly in series with it. If the transformers are similar, the voltmeter V_D should indicate no voltage at all when the load circuit has the switch open. If there is a slight voltage indicated, it means that the two transformers are not exactly similar. When a load is applied the terminal voltage of the loaded transformer will fall slightly, and V_D will give a small indication. The increase of the reading of V_D gives the voltage drop of the loaded transformer. This can be repeated for all loads up to full load and further. If a partially inductive load is used, a wattmeter is necessary in the secondary circuit, the same as before.

The above arrangement assumes that the two secondary voltages are in exact phase opposition, and this is not justified for accurate work. As shown in the vector diagram in Fig. 152, the secondary voltage is not exactly 180° out of phase with the primary voltage when on load, so that the voltmeter V_D is reading the difference between two voltages that are not in exact opposition. This is taken into account in the arrangement shown in Fig. 183. Across the secondary terminals of the unloaded transformer is connected a high resistance that can be continuously tapped. The total

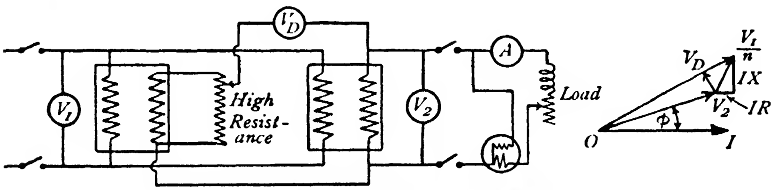


FIG. 183.—More accurate Back-to-Back Regulation Test.

secondary voltage of the unloaded transformer is given by $\frac{V_1}{n}$, as shown in the vector diagram in Fig. 183, where n is the ratio of transformation. The secondary terminal voltage of the loaded transformer is given by V_2. If the voltmeter V_D is connected to the extreme end of the tapped resistance, it measures the vectorial difference between $\frac{V_1}{n}$ and V_2. Moving the tapping point on the resistance is equivalent to sliding it along the vector $\frac{V_1}{n}$. The point of contact is adjusted until the voltmeter V_D reads a minimum, this being equal to $V_2 V_D$ in the vector diagram. Knowing $\frac{V_1}{n}$ and V_2, the drop is given by $\frac{V_1}{n} - \sqrt{V_2^2 - V_D^2}$.

Equality of Ratio.—When two transformers are to work in parallel it is not only necessary that the secondaries should give the same voltage on no-load, but also that they should give practically the same voltage on load. Otherwise they would not share this equally and circulating currents would be set up between them. The connections shown in Fig. 182 enable this point to be tested.

Both secondaries are now loaded independently, and the low reading voltmeter detects any difference in terminal voltage at the various loads. Alternatively the two loads may be so adjusted as to make this difference voltmeter read zero, the magnitudes of the two loads then indicating how the two transformers would share such a total load.

Equivalent Resistance, Reactance and Impedance.—For purposes of calculation, it is often convenient to consider the whole of the transformer impedance as concentrated in the secondary winding. The primary is now supposed to possess no impedance at all, whilst the secondary is supposed to possess a higher impedance than is actually the case. This hypothetical impedance is called the *equivalent impedance*, Z_{eq}, of the transformer referred to the secondary side, and it has a value such that it gives the same voltage drop at the secondary terminals as the real primary and secondary impedances. The resistances and reactances of the two windings can be treated in the same way. The equivalent resistance, R_{eq}, is such that it gives rise to a copper loss in the secondary equal to the sum of the two real copper losses; the equivalent reactance, X_{eq}, gives a reactive voltage drop at the secondary terminals equal to that due to the real primary and secondary reactances. These equivalent values are *not* equal to the numerical sum of the real primary and secondary values.

The equivalent impedance, etc., is usually expressed as a percentage, this being the percentage of the total voltage that is absorbed with full load current, *i.e.* $I_2 Z_{eq}$, etc.

The vector diagram employing the idea of equivalent impedance is shown in Fig. 183. When the secondary is delivering a load current, I, lagging by an angle ϕ behind the secondary terminal voltage, V_2, the applied primary voltage, referred to the secondary side being $\frac{V_1}{n}$, the equivalent resistance drop is given by IR_{eq} and the equivalent reactance drop by IX_{eq}, in phase and in quadrature with I respectively. If V_2 is made equal to 100 per cent., then IR_{eq} and IX_{eq} represent to scale the percentage resistance and reactance drops respectively.

It is seen from the vector diagram in Fig. 152 that, neglecting the no-load current, I_0, the resistance drops, I_1R_1 and I_2R_2 are in phase. The equivalent resistance drop, I_2R_{eq}, has the same phase, and is the resultant of both these individual drops. But $I_1R_1 = \frac{I_2}{n}R_1$, and when this is transferred to the secondary side of the transformer, this voltage becomes $\frac{1}{n} \times \frac{I_2}{n}R_1 = I_2 \times \frac{R_1}{n^2}$. The total resistance drop is therefore $I_2\left(\frac{R_1}{n^2} + R_2\right)$ and the ohmic value of the equivalent resistance is $R_{eq} = \left(\frac{R_1}{n^2} + R_2\right)$. Similarly, the ohmic value of the equivalent reactance is $X_{eq} = \left(\frac{X_1}{n^2} + X_2\right)$, and the ohmic value of the equivalent impedance is $Z_{eq} = \left(\frac{Z_1}{n^2} + Z_2\right)$.

The equivalent resistance and reactance of an auto-transformer, referred to the secondary side are

$$R_{eq} = \frac{R_1}{n^2} + R_2\left(\frac{n-1}{n}\right)^2$$

and

$$X_{eq} = \frac{X_1}{n^2} + X_2\left(\frac{n-1}{n}\right)^2$$

respectively, where R_1, R_2, X_1 and X_2 are the actual ohmic values in the two windings.

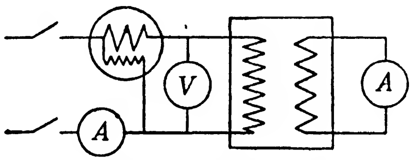


FIG. 184.—Short-Circuit Test.

Short-Circuit Test.—The object of this test is to enable the regulation of a transformer to be determined without actually putting it on load. The secondary is short-circuited through an ammeter capable of reading the full load current, whilst the applied primary voltage is measured by the voltmeter V (see Fig. 184). The applied voltage must be very low. Usually it is necessary to have a considerable resistance in series with the supply for the purpose of cutting down the voltage. The addition of the wattmeter enables the copper losses to be determined. If the secondary current be adjusted to its full load value, and then, without altering the applied voltage, the secondary ammeter be replaced by a low reading voltmeter, the latter will record the open-circuit voltage. Alternatively, the primary can be short-circuited, when it will be found that, in order to cause full load current to circulate, the same voltage must be applied to the secondary as was previously observed on the low reading voltmeter. The whole of this was absorbed previously by the impedance of the two windings, and the ratio of this voltage to the secondary current gives the equivalent impedance referred to the secondary side. But on open circuit $V_S = V_P \times \frac{T_2}{T_1}$, so that the equivalent impedance is equal to $\frac{V_P}{I_2} \times \frac{T_2}{T_1}$.

The total active component of the voltage drop, referred to the primary side, is $\frac{P}{I_1}$, where P is the total power registered by the wattmeter. When this quantity is referred to the secondary side it becomes $\frac{P}{I_1} \times \frac{T_2}{T_1} = \frac{P}{I_2}$, so that the equivalent resistance is $\frac{P}{I_2{}^2}$. The equivalent reactance referred to the secondary side is, therefore,

$$\sqrt{\left(\frac{V_P T_2}{I_2 T_1}\right)^2 - \left(\frac{P}{I_2{}^2}\right)^2}$$

$$= \frac{1}{I_2}\sqrt{\frac{V_P{}^2 \times T_2{}^2}{T_1{}^2} - \frac{P^2}{I_2{}^2}}.$$

The determination of the regulation, using these results, is explained in the next paragraph.

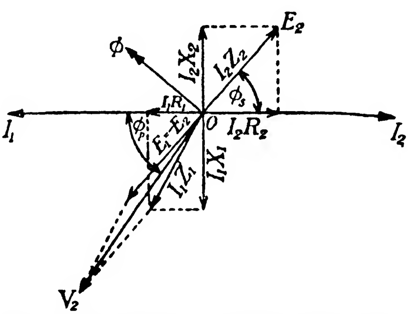


FIG. 185.—Vector Diagram for Short-Circuit Test.

Fig. 185 shows the vector diagram for this short-circuit test. Due to the low applied voltage, the flux is very small, so that the open-circuit current is negligible and the primary and secondary currents are practically in phase opposition. I_2 and I_1 represent the secondary and primary currents respectively. In order to generate I_2, a voltage E_2 is necessary, having for its active and reactive components I_2R_2 and I_2X_2 respectively. On the primary side there are the corresponding voltages I_1R_1 and I_1X_1, the total primary applied voltage consisting of the vector sum of E_1, I_1R_1 and I_1X_1. The small flux vector Φ is at right angles to E_1 and E_2.

Since the flux is very small, the iron losses are negligible, so that the wattmeter reading practically consists of the copper losses of both windings, and this forms a very convenient method of measuring them. If the resistances are measured with direct currents, the results obtained are usually too low, due to the fact that with alternating currents the current distribution is often unequal, resulting in an increased power loss.

Calculation of Regulation.—The equivalent circuit vector diagram is re-drawn in Fig. 186. CF is an arc of a circle having O as its centre, and CE is a vertical dropped on to OA produced. If OA represents 100 per cent., then the percentage regulation is equal to AF. This is approximately equal to $IR \cos \phi + IX \sin \phi$, where IR and IX are the equivalent resistance and reactance percentage drops respectively. This neglects the small distance EF, and taking this into account for greater accuracy, a further term must be added.

Considering the right-angled triangle OEC, $EC^2 = OC^2 - OE^2 = (OC + OE)(OC - OE)$, so that

$$EF = (OC - OE) = \frac{EC^2}{(OC + OE)}.$$

But $EC = IX \cos \phi - IR \sin \phi$, and OC and OE may each be taken as 100 per cent., so that

$$EF = \frac{(IX \cos \phi - IR \sin \phi)^2}{200}.$$

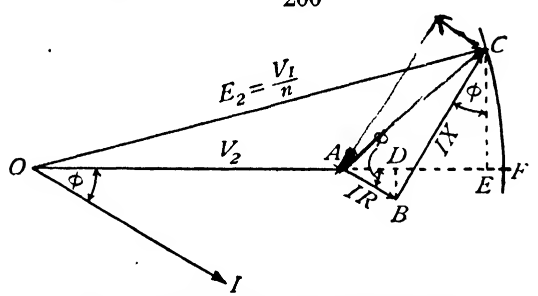


FIG. 186.—Equivalent Circuit Vector Diagram.

Adding this term to the original approximate expression, the percentage regulation becomes

$$IR \cos \phi + IX \sin \phi + \frac{(IX \cos \phi - IR \sin \phi)^2}{200}.$$

Kapp's Regulation Diagram.—By means of a graphical construction proposed by Kapp, the voltage drop for a given load can be determined for all power factors. For this purpose it is necessary to know the equivalent resistance and the equivalent reactance, both referred to the secondary side. In Fig. 187 AB and AD represent the active and reactive components respectively of the equivalent impedance voltage AC. Taking the secondary current line, CI, as the axis of reference, and assuming an angle of lag ϕ in the load circuit, the secondary terminal voltage lies along a line CV_2. The open-circuit voltage, which is kept constant, is given by AV_2, being the vector sum of AC and CV_2. The point V_2, therefore, always lies on an arc of a circle of radius AV_2 and centre A.

Another arc of a circle having the same radius is now drawn with C as centre, and if CV_2 be produced until it meets the other circle at E_2, then V_2E_2 represents the voltage drop at the load corresponding to the secondary current CI and the power factor $\cos\phi$. When V_2 is above CI it corresponds to a lagging current, and when it is below it corresponds to a leading current. All that is necessary, then, in order to determine the voltage drop at any power factor is to draw a line CE_2 making an angle ϕ with CI such that $\cos\phi$ is the power factor, when V_2E_2 can be read directly from the diagram.

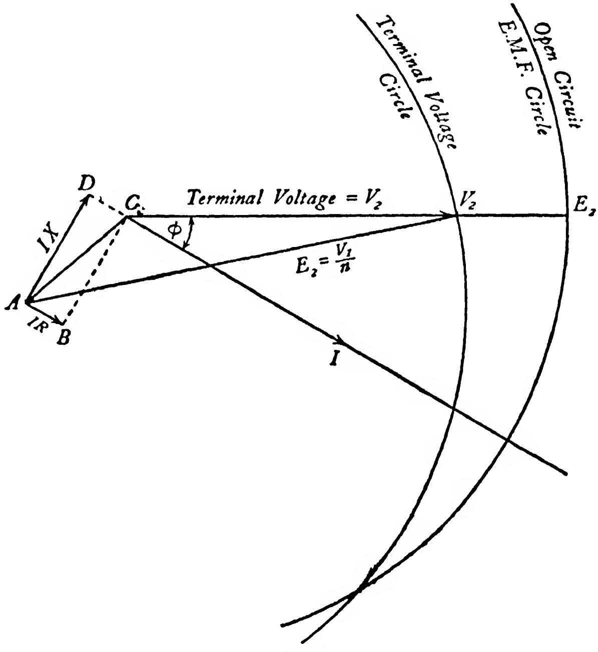


FIG. 187.—Kapp's Regulation Diagram.

It is seen that the more the current lags in the secondary circuit the greater is the voltage drop until AV_2 and AC are in phase. For leading currents the voltage drop decreases until it finally becomes zero. If the current is made to lead still further, the terminal voltage on load is actually greater than it is on open circuit.

Efficiency Test.—The simplest method of measuring the efficiency of a transformer consists in measuring the output and input by means of two wattmeters and is usually stated for a particular load which is given by an ammeter in the secondary main circuit. If the load is non-inductive, a voltmeter and ammeter are sufficient to measure the power, but in most cases it is desirable to check the power factor by means of wattmeter observations. In any case, a wattmeter must be included in the primary circuit, since there is a quite appreciable reactive component of the primary current. The efficiency is, of course, given by the ratio of the output to the input. This method is a wasteful one, as it necessitates the whole of the power used being dissipated in some form or other.

An indirect method of determining the efficiency consists in measuring both the iron loss and the copper loss separately. The power input at no-load is practically all iron loss, and this remains substantially constant for all loads. The copper loss is obtained from the short circuit test, as already explained. Adding these losses to the output at any load, the input is obtained, knowing which, the efficiency may be calculated.

The direct and indirect methods explained above are equally applicable in the case of three-phase transformers, the only difference being that three-phase wattmeters are necessary, or, if these are

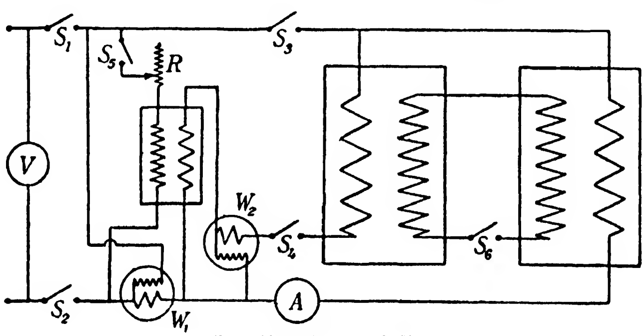


FIG. 188.—Sumpner's Test.

not available, single-phase instruments may be used for measuring the three-phase power, as explained in Chapter X.

Sumpner's Test.—This test is carried out on two similar transformers, and not only enables the efficiency to be determined, but, in addition, measures the copper and iron losses separately. The first transformer supplies the second, which is loaded back on to the mains after the fashion of the Hopkinson test with direct current machines. The net power taken from the mains is therefore only that required to make up the losses of the two transformers and the various instruments. The two transformers are connected back to back as described on page 215, the second one being worked the reverse way, so that the winding which is normally the secondary now acts as the primary. If there were no losses in the circuit, the acting secondary (real primary) of this transformer would deliver back to the mains a current equal to that taken by the first transformer. In order to enable this action to take place in an actual case, the primary of the first transformer must have its applied voltage boosted up a little, this being done by means of a small auxiliary transformer the secondary of which is connected in series with the primary of the transformer which requires the boost. The full diagram of connections necessary for this test is shown in Fig. 188.

If the primary of the auxiliary transformer be open-circuited the secondary acts merely like a high impedance in series with the primary of the main transformer. The magnetizing current of the latter is therefore supplied from the second transformer and the ammeter A registers the total magnetizing current of both. The wattmeter W_1 registers the total power supplied, and this represents the iron loss of the two transformers. If the wattmeter still givès an indication on opening S_3, this is due to instrument losses and should be deducted from the former reading. When S_5 is closed the auxiliary transformer supplies an additional voltage which can be regulated by means of the adjustable resistance R. The extra power supplied in this manner is measured by the wattmeter W_2, and since this causes a circulating current to be set up between the two transformers and loads them up, the extra power supplied must be on account of the copper losses. The reading of W_2 therefore indicates the total copper loss. This does not cause the reading of W_1 to go up by a like amount, for the power supplied by the auxiliary transformer is received in its primary, and this primary current does not go through the wattmeter W_1, which therefore continues to register the total iron loss. By varying the value of R, the load may be varied over a wide range, enabling the efficiency to be determined from no-load up to any desired overload. If the instrument losses are appreciable, the switches S_3 and S_6 may be opened, when the wattmeters will register their value. These losses may be taken as being proportional to the square of the current.

If the auxiliary transformer acts in opposition to the supply voltage, it reduces the terminal voltage on the test transformer, but this does not matter, as it simply means that, instead of supplying the second test transformer, it is supplied from it.

The output of each transformer, in volt-amperes, may be taken as the product of the current read on the ammeter A and the voltage read on the voltmeter V, whilst the total input may be obtained by adding the copper loss and the iron loss to the output. The individual efficiency of each transformer, at any power factor $\cos\phi$, is obtained as follows :—

Efficiency of each transformer

$$= \frac{VA \cos\phi}{VA \cos\phi + \frac{1}{2} \times \text{total losses}}.$$

Separation of Losses.—The total copper loss and the total iron loss may be determined separately or from the Sumpner efficiency test. The copper loss is divided between primary and secondary, and the watts wasted in each can be determined from a knowledge of their relative resistances and the current flowing in each, and will usually be somewhere about the same. The iron loss can be separated into hysteresis and eddy current loss by the method explained on page 81, where the flux is kept constant by varying the frequency at the same rate as the voltage. In this test the secondary is left entirely disconnected.

All-Day Efficiency.—Since some transformers, notably those used for lighting purposes, work for very considerable periods every day at loads much less than full load, it is advisable to take this into account when considering the suitability of a transformer for a particular duty. In this connection, an expression termed the *all-day efficiency* is introduced, this being defined as

$$\frac{\text{Kilowatt-hours output per 24 hours}}{\text{Kilowatt-hours output per 24 hours} + \text{Kilowatt-hours wasted per 24 hours}}.$$

In order to get a high value for this ratio, it is necessary to have the maximum efficiency in the neighbourhood of the load at which the transformer works for the major portion of the time. But the maximum efficiency occurs when the copper loss equals the iron loss,[1] so that in a case of a lighting transformer the efficiency is designed to reach a maximum at very much less than full load, being usually about half full load. This means that at full load the copper losses considerably exceed the iron losses, whilst in a transformer designed to have its maximum efficiency at full load these two are equal. In order to reduce the iron loss, the flux density must be reduced, and this can be brought about either by increasing the cross-sectional area of the core or by increasing the number of turns. The former method involves an increased length of mean turn, resulting in an increased copper loss, whilst the latter method also increases the resistance of the windings. There is a practical limit to this, since an increase in the copper losses makes

[1] This can be proved as follows :—

Let output $= EI\cos\phi$ and input $= EI\cos\phi + P_i + I^2R$, where P_i represents the constant iron loss and I^2R the copper loss.

$$\text{Efficiency} = \eta = \frac{EI\cos\phi}{EI\cos\phi + P_i + I^2R},$$

$$y = \frac{1}{\eta} = 1 + \frac{P_i}{EI\cos\phi} + \frac{I^2R}{EI\cos\phi}.$$

For $\eta =$ max. or $\frac{1}{\eta} =$ min., $\frac{dy}{dI} = 0$.

$$\therefore \quad \frac{dy}{dI} = 0 - \frac{P_i}{EI^2\cos\phi} + \frac{R}{E\cos\phi}$$

$$\frac{R}{E\cos\phi} = \frac{P_i}{EI^2\cos\phi},$$

$$R = \frac{P_i}{I^2} \text{ and } I^2R = P_i.$$

the regulation of the transformer worse, and after a certain point this becomes prohibitive.

As an example, consider the case of two 10 kW transformers, the first having a maximum efficiency of 97 per cent. occurring at full load and the second having the same maximum efficiency occurring at half full load. The full load losses of the first are 300 watts, divided equally between the iron and the copper. At one-quarter full load the copper loss is $\frac{150}{16} = 9{\cdot}4$ watts and the efficiency $\frac{2500}{2500 + 150 + 9{\cdot}4} \times 100 = 94{\cdot}0$ per cent., the no-load loss being 150 watts. The losses of the second transformer at half full load are $\frac{3}{100} \times 5000 = 150$ watts. The iron loss is 75 watts and the full load copper loss is $4 \times 75 = 300$ watts. The full load efficiency is therefore $\frac{10{,}000}{10{,}000 + 75 + 300} \times 100 = 96{\cdot}4$ per cent.

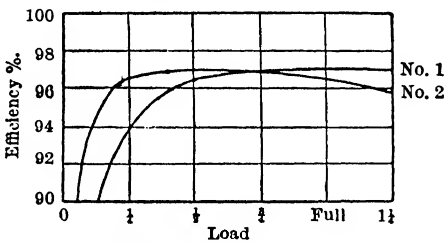


FIG. 189.—Efficiency Curves of Two Transformers.

The efficiency at one-quarter full load is

$$\frac{2500}{2500 + 75 + 18{\cdot}75} \times 100 = 96{\cdot}4 \text{ per cent.}$$

Now assume that each transformer is kept on full load for six hours, one-quarter full load for twelve hours, and on no-load for six hours every day. The total energy output during a day is $6 \times 10 + 12 \times 2{\cdot}5 = 90$ kWh. The total energy losses of the first transformer are $24 \times 0{\cdot}15 + 12 \times 0{\cdot}0094 + 6 \times 0{\cdot}15 = 4{\cdot}61$ kWh. The all-day efficiency is therefore $\frac{90}{90 + 4{\cdot}61} \times 100 = 95{\cdot}1$ per cent. The total energy losses of the second transformer are $24 \times 0{\cdot}075 + 12 \times 0{\cdot}01875 + 6 \times 0{\cdot}3 = 3{\cdot}825$ kWh. The all-day efficiency of this transformer is $\frac{90}{90 + 3{\cdot}825} \times 100 = 95{\cdot}9$ per cent., and is an improvement on the other. If a greater period of the time were spent on no-load, the improvement would be still more marked. Fig. 189 shows the efficiency curves of these two transformers.

Heat Tests.—When a transformer is run on a constant load its temperature gradually rises to a maximum value, which depends upon the total losses which have to be dissipated. If the heat were produced uniformly throughout the transformer and also dissipated uniformly from its surface, the temperature rise of all the parts would be equal. In this case, the rate of production of heat is constant, whilst the rate of dissipation of heat is proportional to the temperature rise at that moment. The instantaneous temperature rise, θ, is then given by the equation

$$\theta = \theta_m \left(1 - \epsilon^{\frac{-t}{T}}\right),$$

where θ_m is the maximum temperature rise and T is a constant called the heating time constant [1] (see page 574).

This latter quantity is the time taken to reach the final temperature if the initial rate of increase of temperature were maintained and is indicated in Fig. 190, which shows the general form of the temperature rise curve. The cooling curve is the same shape as the heating curve, except that it is inverted.

Unfortunately, the heating is not uniform in an actual case; the copper may have more watts to dissipate than the iron and may have less cooling facilities, or *vice versa*. The different parts of a transformer, therefore, may heat up unequally, and in any case the heating curve, in the majority of instances, does not exactly agree with the theoretical curve, the continued increase in temperature after the bend has been passed being slightly greater in an actual case than is inferred from the theoretical curve.

A peculiar point sometimes observed is that on switching off the load the copper continues to rise in temperature for a short

[1] This law can be worked out as follows :—

Let H and $\frac{1}{T}\theta$ be the rate of production and of dissipation of heat respectively.

Then $$\frac{d\theta}{dt} = H - \frac{\theta}{T}, \text{ and } \frac{d\theta}{H - \frac{\theta}{T}} = dt.$$

Integrating, we get $$\int \frac{d\theta}{H - \frac{\theta}{T}} = \int dt \text{ or } - T \log\left(H - \frac{\theta}{T}\right) = t + k.$$

When $t = 0$, $\theta = 0$ and $k = - T \log H$.

Therefore $$t = T \log H - T \log\left(H - \frac{\theta}{T}\right) = T \log \frac{H}{H - \frac{\theta}{T}}.$$

$$\epsilon^{\frac{t}{T}} = \frac{H}{H - \frac{\theta}{T}} \text{ and } \theta = TH\left(1 - \epsilon^{\frac{-t}{T}}\right).$$

But $\theta = \theta_m$ when $t = \infty$ and $\epsilon^{\frac{-t}{T}} = 0$, therefore $\theta_m = TH$ and $\theta = \theta_m \left(1 - \epsilon^{\frac{-t}{T}}\right)$.

time. This is due to the fact that the iron is hotter than the copper, and, when cooling commences, the iron gets rid of some of its heat to the less hot copper and temporarily raises its temperature.

To carry out such a heating test, the transformer must be run on full load until it reaches its final temperature, which is usually not for several hours. Various methods are adopted to reduce the wastage of energy.

In the equivalent short-circuit test, the secondary is short-circuited and a low voltage is applied to the primary, so as to cause full load current to circulate. The losses are now only those due to copper losses, and in order to allow for the iron losses, which must have been measured previously, the circulating current is increased until the copper losses are equal to the total losses for full load.

In the equivalent open-circuit test, the secondary is open-circuited and the applied voltage raised above the normal until

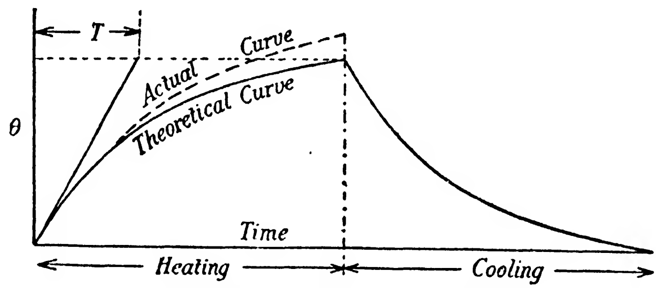


FIG. 190.—Heating and Cooling Curve.

the iron losses are equal to the total losses for full load. Instead of raising the applied voltage, the frequency may be lowered, thus increasing the flux and flux density. The open-circuit method is preferable when the iron losses normally exceed the copper losses, and *vice versa.*

The loading back method may be used, employing the connections for the Sumpner test, the circulating current being adjusted as already explained. Actual full load conditions are reproduced in this test, and on that ground it is to be preferred.

Three-phase transformers, or groups of three single-phase transformers intended for three-phase operation, may be connected in delta-delta and a suitable voltage applied. The secondary delta is opened and a single-phase E.M.F. is injected into the circuit at the same frequency. This sets up a circulating current which is reflected in the primary, the magnitude being controlled by the single-phase voltage. If necessary, the auxiliary voltage can be injected into the primary, the secondary delta being left closed and disconnected from any other circuit.

It is preferable to determine the temperature of the copper by the increase of resistance method, as this measures the average temperature of all the turns and not merely that of the surface.

Short Time Heat Tests.—In a number of cases it is undesirable to carry out a heat test for the length of time necessary to arrive at a constant temperature. But running the transformer for a shorter period than this enables the initial part of the temperature rise curve to be determined, and knowing the law which is followed the final temperature can be calculated. One method of estimating this, due to the late Prof. S. P. Thompson, is to determine graphically the point at which the slope of the curve has been reduced to half its initial value. θ_m is then twice the temperature rise at that point.[1] Unfortunately, this method is very often found to be too inaccurate even for practical purposes. One reason for this is that it depends upon the initial rate of rise of temperature, and this is the most irregular part of the curve in the majority of actual cases.

Short time tests are not as accurate as the direct method of measurement. Their chief advantage lies in the fact that they cause a great reduction in the amount of energy consumed and the time taken to complete the test.

Load Sharing.—Two transformers operating in parallel share the total load in the ratio of their rated outputs, if the percentage resistances and reactances are the same. (The same percentage values mean different ohmic values when the rated outputs are not the same.) If one transformer has a larger percentage impedance than the other, it delivers less than its proper share of the load, which is now divided in the inverse ratio of the percentage impedances, assuming the same rated output. Again, two transformers may have the same percentage impedance, one having a larger percentage resistance and the other a larger percentage reactance. In this case the power factor of the loads delivered by the two transformers differs, that with the larger percentage reactance delivering more than its proper share of the reactive component of the load.

EXAMPLES.

(1) Explain how the efficiency and regulation of a transformer can be calculated by the aid of data obtained from the open-circuit and short-circuit tests.

(2) What is meant by the equivalent resistance and equivalent reactance of a transformer?

A transformer with a 5 to 1 ratio has a primary resistance and

[1] $\frac{d\theta}{dt} = \frac{\theta_m}{T}\epsilon^{-\frac{t}{T}}$, and this is equal to $\frac{\theta_m}{2T}$.

$$\therefore \quad \epsilon^{-\frac{t}{T}} = \tfrac{1}{2} \text{ and } \theta = \theta_m \left(1 - \tfrac{1}{2}\right) = \tfrac{1}{2}\theta_m.$$

reactance of 0·3 and 1·5 ohms respectively, and a secondary resistance and reactance of 0·01 and 0·05 ohm respectively. Determine the percentage regulation when delivering 150 amperes at 600 volts and 0·8 power factor (lagging).

(3) The equivalent resistance drop and equivalent reactance drop in a single-phase transformer are 1 per cent. and 3 per cent. respectively. Determine, by the aid of a graphical construction, the regulation on full load at power factors of unity and 0·8 lagging.

(4) A 1000 kVA transformer has an iron loss of 8·8 kW and a copper loss of 9·8 kW at full load current. Determine its efficiency at 25 per cent. overload and 0·85 power factor. At what kVA load is the efficiency a maximum, assuming unity power factor?

(5) A 500 kVA transformer reaches an efficiency of 98·33 per cent. at 50 per cent. overload and unity power factor, and has an efficiency of 98·20 per cent. at half full load and unity power factor, maximum efficiency occurring in the neighbourhood of full load. Determine the efficiency, and the core loss and copper loss separately at full kVA load and a power factor of 0·8.

(6) A 100 kVA single-phase transformer takes 900 watts with open-circuited secondary when operated at its normal voltage of 500 volts and at its normal frequency of 50 cycles per second, the secondary terminal voltage being 100 volts. The short-circuit test shows that the input is 1600 watts for full load secondary current, the primary applied voltage being 25 volts. Calculate the percentage regulation with full load current at 0·8 power factor (lagging).

CHAPTER XVI

TRANSFORMERS.—PRINCIPLES OF DESIGN

Volts per Turn.—Let Φ and IT be the magnetic and electric loading respectively and let $a = \Phi/IT$. The output $=$ kVA $=$

$$4{\cdot}44 f\Phi \times IT \times 10^{-11}$$

$$= 4{\cdot}44 f\frac{\Phi^2}{a} \times 10^{-11},$$

so that the flux, Φ,

$$= \sqrt{\frac{\text{kVA} \times a \times 10^{11}}{4{\cdot}44f}}.$$

The volts per turn, V_t,

$$= 4{\cdot}44f\Phi \times 10^{-8}$$

$$= 4{\cdot}44f \times 10^{-8} \times \sqrt{\frac{\text{kVA} \times a \times 10^{11}}{4{\cdot}44f}}$$

$$= \sqrt{4{\cdot}44af \times \text{kVA} \times 10^{-5}}.$$

If a transformer be designed with a relatively large electric loading, then a will be small, and the volts per turn will be low. There will be a large number of turns and the copper loss of such a transformer would be large. On the other hand, since the flux is relatively low, the iron loss would be small, and the transformer would reach its maximum efficiency at a fraction of its full load. For the efficiency to be a maximum at full load, the copper and the iron losses must be equal. This means a larger flux and a smaller number of turns; in other words, the constant a is increased. Also the flux must increase as the frequency decreases (from the E.M.F. formula). In practice, the constant a is approximately inversely proportional to the frequency, or

$$af = \text{constant}.$$

The volts per turn may now be written

$$V_t = \sqrt{4{\cdot}44 \times \text{constant} \times \text{kVA} \times 10^{-5}}$$

$$= \text{constant} \times \sqrt{\text{kVA}}.$$

This constant has the following average values :—

Core Type Power Transformers . . .	0·6
Shell " " " . . .	1·2
Core Type Distributing "	0·5

The distributing type transformer has a smaller constant in order to give it a small core loss and a high all-day efficiency. The difference between the constants for the shell and the core types is due to the difference in construction.

Allowance for Voltage Drop.—A few extra turns must be provided on the secondary in order to allow for the voltage drop in the windings. The normal values for the regulation on unity power factor are about two per cent. at 100 kVA, falling to about one per cent. at 1000 kVA. Above this size, the regulation is usually round about one per cent., except where the transformer is required for a special purpose, such as operating a rotary converter.

The regulation on unity power factor is approximately the same as the percentage copper loss.

Having estimated the total voltage drop, the total secondary turns can be calculated from

$$T_2 = T_1 \times \frac{E_2 + \text{total drop in volts}}{E_1}.$$

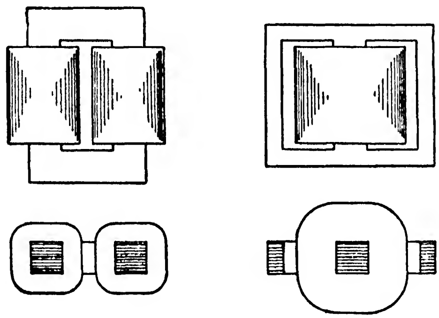
Comparison of Core and Shell.—Consider the core type transformer in Fig. 191. The section of the limb is shown square, although the depth is sometimes made greater than the width in order to improve the shape of the tank. More frequently the limb is made cruciform in section, or stepped as shown in Fig. 156, so as to improve the shape of the coil. Rectangular coils are bad not only on account of their lack of economy of copper, but also due to the fact that in the event of a short-circuit, each turn attempts to assume a circular shape, the resulting deforming forces often being of serious magnitude.

If the coil on the left-hand limb in Fig. 191 (*a*) be placed on the other limb, and the iron core be split into two equal parts arranged as shown in Fig. 191 (*b*), then a shell type transformer is obtained having the same magnetic and electric loading as before. This shell is squat in shape and the tank to contain it occupies considerable floor space. The proportions for shell type transformers are therefore modified considerably, the depth of the central iron limb usually being made about three times its width.

For the same rating, the flux is about double and the ampere-turns half the value in the corresponding core type. This makes the ratio a about four times its previous value, but as the volts per turn are proportional to the square root of this ratio, the constant given above is double for the shell type what it is for the core type.

Size of Conductor.—The best distribution of the copper losses is obtained when the current density in primary and secondary is the same, and this means that the cross sections of the two windings should be proportional to their respective currents. The usual current densities adopted vary considerably with the output of the transformer, and with other factors, such as the depth of the windings. Average figures are 200–300 amperes per sq. cm., the lower figure corresponding to the higher efficiency and the lower temperature rise.

Number and Arrangement of Coils.—The question of whether to adopt concentric or sandwich coils having been settled, the number of coils and the turns per coil can now be decided. The voltage per coil should not exceed 1500 volts, and the voltage between layers should not exceed 250 volts. When the pressure is over 3000 volts, the coils should always be wound in sections. The



(a) (b)

FIG. 191.—Core and Shell Type Transformers with same Copper and Iron.

end coils are always separate and should be reinforced for voltages exceeding 3500 volts.

The radial depth of each coil should not greatly exceed 2·5 cms. between oil ducts, except in the case of the coil nearest to the core, which for this purpose may be regarded as a cooling agent equal to an oil duct.

With double cotton covered conductors, 0·25 mm. should be added to the bare diameter. A strip of fuller-board or other insulating material such as varnished cloth about 0·35 mm. thick should be placed between layers. The number of layers in a coil can thus be settled, and then the number of turns per layer. Thus the total winding length is fixed.

As a check the copper space factors shown in Fig. 192 may be consulted.

Specific Electric Loading.—The approximate winding height of the limbs can be provisionally estimated by considering the specific electric loading, or ampere-turns per winding per cm. length of the wound portion of the limb. The following table gives average values, but these figures vary considerably with different manufacturers, and particularly with regard to the reactance of the transformer. A high reactance goes with high ampere-turns per cm.

kVA	0	500	1000	2000	3000	4000	5000
Ampere-turns per winding per cm.	400	600	700	850	930	960	980

Insulation of Windings.—An insulating cylinder is placed between the L.T. winding and the core, and between the L.T. and H.T. windings. There is also insulation at each end of the coils and sometimes between limbs.

The cylinders may be made of bakelite or they may consist of a number of layers of Manila paper cemented with bakelite varnish.

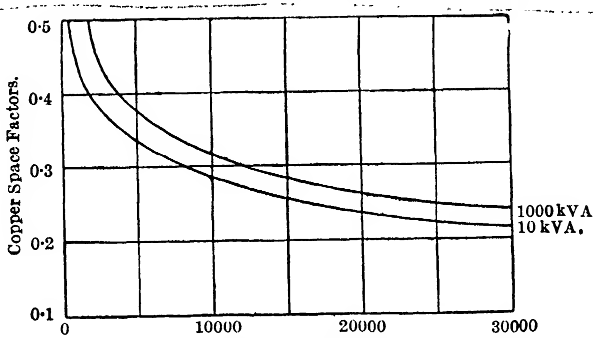


Fig. 192.—Copper Space Factors.

The end insulation commonly consists of a series of fuller-board washers and spacing blocks. These must be so arranged as to preserve continuity of the oil ducts.

The conductors themselves are usually insulated with d.c.c., sometimes augmented with tape (particularly end turns) or with strips of insulation between turns.

For insulation between layers a thin sheet of fuller-board or a few thicknesses of varnished cloth may be used.

The leads from the coils to the terminals are encased in bakelite tubes.

Flux.—The number of turns having been determined, the flux can be calculated directly from the E.M.F. formula.

Flux Density.—The flux density usually lies between the extreme limits of 11,000 and 14,000 lines per sq. cm. The higher value is economical in iron, but gives rise to a large iron loss and a considerable temperature rise in the iron. It also necessitates a large magnetizing current containing very considerable harmonics. The following table gives the maximum permissible flux densities in lines per sq. cm. The 50 cycle figures should be reduced by about 2000 for 75 cycles, and by a further 1000–1500 for 100 cycles.

kVA Output.	Flux Density.	
	25 cycles.	50 cycles.
0–5 (air cooled) ...	12,500	11,500
0–5 (oil cooled) ...	13,000	12,500
5–25...	13,500	13,000
25–150	13,800	13,500
150–500	14,200	13,800
Above 500	14,600	14,000

Section and Height of Limb.—Knowing the flux and assuming a suitable flux density, the approximate net cross section of the limb can be estimated. In view of the insulation between adjacent stampings the gross iron section is larger, and a stacking factor of 0·9 may be assumed where

$$\text{Stacking factor} = \frac{\text{net section of iron}}{\text{gross} \;\;,, \;\;,, \;\;,,}.$$

If necessary one or more oil ducts may be provided in the iron. The actual shape and section of the limb can now be settled, together with its length. The latter dimension is greater than the winding length, from one to two cms. being allowed at each end for additional insulation and for adjustable coil supports to allow for shrinkage of the winding.

It is usual to design the limb with several sizes of stampings as shown in Fig. 156. The relationship between the various dimensions can be worked out mathematically,[1] the following table giving

[1] As an example, the relation between a and b for Fig. 193 is worked out as follows :—

Diameter of circle $= d = (a^2 + b^2)^{\frac{1}{2}} =$ constant.

$\therefore \quad b = (d^2 - a^2)^{\frac{1}{2}}.$

Total area
$$= A = ab + 2\frac{a - b}{2}b$$
$$= 2ab - b^2$$
$$= 2a(d^2 - a^2)^{\frac{1}{2}} - d^2 + a^2.$$

For maximum area,
$$\frac{dA}{da} = 2a \times \tfrac{1}{2}(d^2 - a^2)^{-\frac{1}{2}} \times 2a + 2(d^2 - a^2)^{\frac{1}{2}} + 2a = 0.$$

Substituting b for $(d^2 - a^2)^{\frac{1}{2}}$,
$$-2\frac{a^2}{b} + 2b + 2a = 0.$$

Rearranging and multiplying by $-\frac{b}{2}$,
$$a^2 - ab - b^2 = 0.$$
$$\therefore \quad a = \frac{b \pm \sqrt{b^2 + 4b^2}}{2}$$
$$= \frac{b}{2} \pm \frac{\sqrt{5}}{2}b = 1{\cdot}62b,$$

since the $-$ sign is inadmissible.

Total area $= 3{\cdot}24b^2 - b^2 = 2{\cdot}24b^2.$

$b = 0{\cdot}67 \times \sqrt{\text{area}}$ and $a = 1{\cdot}09 \times \sqrt{\text{area}}.$

the values for two and three sizes of stampings (see Figs. 193 and 194).

	Two Sizes of Stampings.	Three Sizes of Stampings.
a	$1{\cdot}09 \times \sqrt{\text{Gross Area}}$	$1{\cdot}11 \times \sqrt{\text{Gross Area}}$
b	$0{\cdot}67 \times \sqrt{\text{Gross Area}}$	$0{\cdot}87 \times \sqrt{\text{Gross Area}}$
c	———	$0{\cdot}52 \times \sqrt{\text{Gross Area}}$
diameter	$1{\cdot}28 \times \sqrt{\text{Gross Area}}$	$1{\cdot}22 \times \sqrt{\text{Gross Area}}$

Dimensions of Magnetic Yokes.—The magnetic density in the yokes is made less than in the limbs in order to keep down the iron losses. The section of the limb is made as small as is economically possible in order to keep down the length of the mean turn of the winding, and thus keep the copper losses within limits. This consideration does not apply to the yokes joining the limbs, where an increased section actually reduces the iron losses in spite of the

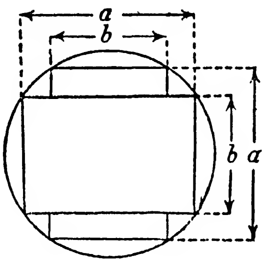


FIG. 193.—Cross Section of Core. Two Sizes of Stampings.

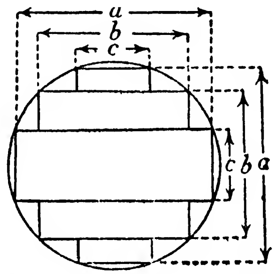


FIG. 194.—Cross Section of Core. Three Sizes of Stampings.

greater volume employed. The section of the yokes is usually made about 20 per cent. larger than that of the limbs, and has a shape as shown in Fig. 157, but the proportions are kept the same as far as possible, as otherwise additional eddy currents are set up.

Efficiency.—In order to estimate this, the copper and the iron losses must be determined.

The secondary copper loss for full load current is obtained by estimating the length of the mean turn, and thence the total length of the winding. Knowing the cross section of the conductor the resistance can be estimated and then the value of I^2R. When the cross section is large, however, the current is not distributed uniformly over it, due to skin effect and eddy currents in the conductors. On this account large conductors are laminated and arranged so that the various strips are interchanged at intervals. In such a case and with ordinary current densities, the total secondary copper loss is about 30 per cent. greater than is calculated on the assumption of uniform distribution of current. The percentage increase varies, of course, very considerably on either side of the figure quoted, in individual cases.

To estimate the primary copper loss, the primary current must be provisionally calculated. This is done by assuming a suitable efficiency (and power factor if necessary). The resistance is estimated, as in the previous case, by determining the length of the mean turn and the total length of the winding. The length of the mean turn will probably be slightly different in primary and secondary. The percentage increase due to lack of uniformity of current distribution will also probably not be the same as in the secondary winding, but in the absence of any further data, the same (or slightly less) percentage increase may be assumed. In both primary and secondary windings an allowance should be made for rise in temperature, the increase probably being of the order of 20 per cent.

The iron loss depends upon the quality of the iron, its volume, and the flux density. Taking first the main limbs, the weight is

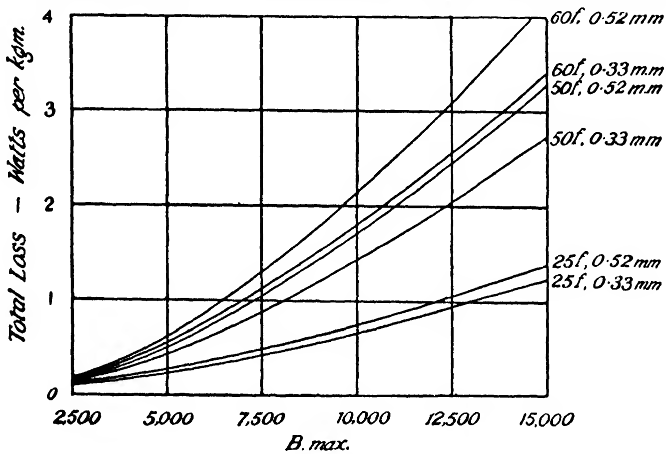


FIG. 195.—Iron Losses in Transformer Stampings.

estimated and the iron loss in watts per kgm. is read off the curves shown in Fig. 195, which refer to modern stalloy plates. The weight may be estimated from the volume, taking the density as 0·0078 kgm. per c.c. In estimating the weight, it should be remembered that only 0·9 of the gross section is net iron. This calculation is then repeated for the magnetic yokes, which are usually worked at a lower magnetic density.

The hysteresis loss is frequently quoted as being proportional to $B^{1·6}$, but whilst this is accurate for low flux densities, it is definitely not true for high flux densities. The loss can be expressed as proportional to B^x, where x gradually rises and may reach a value as high as 4 at $B = 14{,}500$. Round about $B = 13{,}500$, the hysteresis loss is proportional to B^2.

Having determined all the losses the efficiency at unity power factor can now be calculated. The resulting figure should be checked against average practice as represented in the following table :—

kVA	1	10	100	5000
Efficiency	93	96	98	98·5

The following table gives representative values for iron and copper losses for three-phase core type transformers with secondary voltages about 400 volts.

The corresponding losses for single-phase transformers may be estimated by considering the loss per limb. For example, a 1000 kVA single-phase transformer with two limbs has approximately the same loss per limb as a 1500 kVA three-phase transformer with three limbs. The latter has (see table) an iron loss of 1500 watts per limb and a copper loss of 4000 watts per limb. The loss for the 1000 kVA single-phase transformer may therefore be taken as $2 \times 1500 = 3000$ watts and $2 \times 4000 = 8000$ watts respectively.

kVA.	Iron loss in watts.			Copper loss at 15° C. in watts.
	3300 volts.	6600 volts.	11,000 volts.	
25	300	325	350	580
50	485	510	540	875
100	700	750	800	1,500
250	1,400	1,450	1,500	3,000
500	2,300	2,400	2,500	5,000
1,000	3,500	3,500	3,500	9,000
1,500	4,500	4,500	4,500	12,000

Temperature Rise.—The temperature rise in the windings, measured by increase in resistance, should not exceed 55° C., and in the oil, measured by thermometer, should not exceed 50° C. There may, however, be a temperature variation of anything up to 15° C. between oil at the top and bottom of the tank.

The rise can be calculated approximately by means of the formula,

$$\text{Temp. rise in } ^\circ\text{C.} \times k = \frac{\text{watts lost}}{\text{cooling surface in sq. cms.}}$$

The constant is thus equal to the watts dissipated per sq. cm. of cooling surface per degree rise in temperature. The following table gives the maximum permissible loss which can be dissipated per sq. cm. of cooling surface for a temperature rise of 50° C., which is the figure commonly specified.

Type.	Watts per sq. cm.		
	Core without ducts.	Core with ducts.	Coils.
Air insulated—Self cooled	0·035	0·025	0·035
„ „ —Air blast	0·16	0·12	0·155
Oil insulated—Self cooled	0·14	0·11	0·14
„ „ —Water „	0·15	0·115	0·16
„ „ —Forced oil „	0·16	0·12	0·18
„ „ —Air blast	0·15	0·115	0·15

When considering the temperature rise of any individual part of the transformer, the loss dissipated by that part and its own particular cooling surface only are considered. For example, when considering the coils, the copper cooling surface only is taken into account, and the copper loss only is supposed to be dissipated. Similarly, the iron is supposed to dissipate the iron loss only. On the other hand, as the whole of these losses are passed on to the oil, the whole of the losses have to be dissipated by the oil to the atmosphere.

With oil-immersed transformers, the quantity of oil required varies from about 2 gallons per kVA in the small sizes down to about 0·4 gallon per kVA in the large sizes, the specific gravity ranging from 0·86 to 0·9, the former figure being the more usual.

With ordinary plain or tubular tanks, the cooling constant varies between 0·06 and 0·03 watt per sq. cm. for a transformer temperature rise of 50° C. The total cooling surface in sq. cms. required by the tank is, therefore, $\frac{\text{total watts lost}}{\text{constant}}$. The necessary surface can be provided by the employment of cooling tubes or ribs where required. The cooling constant has the following average values :—

Plain tank	... 0·06	watt per sq. cm. for 50° C. rise.
Tank with one row of tubes ...	... 0·045	,, ,, ,, ,, ,, ,,
,, ,, two rows ,, ...	... 0·04	,, ,, ,, ,, ,, ,,
,, ,, three ,, ,, ...	... 0·038	,, ,, ,, ,, ,, ,,
,, ,, four ,, ,, ...	... 0·033	,, ,, ,, ,, ,, ,,

The top and bottom of the tank are not included in the cooling surface.

Predetermination of No-load Current.—The no-load current consists of an active and a reactive component, the former of which supplies the losses and the latter the magnetizing current necessary to force the flux round the magnetic circuit. The relative magnitudes of these two components are such that the resultant no-load current usually lags behind the voltage by an angle of about 45° to 60°. The active component is obtained by dividing the total losses in watts by the primary applied voltage in a single-phase case, whilst for a polyphase transformer each phase is supposed to take its proper share of the watts, and the current per phase is obtained by dividing this value by the volts acting across each phase, *e.g.* $\frac{1}{\sqrt{3}}$ times the line voltage in the case of a star-connected three-phase transformer.

In calculating the purely magnetizing current, the total reluctance of the magnetic circuit is required, and this usually includes a number of joints of somewhat indefinite reluctance. As an approximation the reluctance of these may be taken as being equal to 20 per cent. of the reluctance of the remainder of the iron circuit when the joints are interleaved. From the B—H curve of the iron the number of ampere-turns per cm. may be obtained, corresponding to the maximum flux density in the core. This value is multiplied by the total length of the magnetic path and an allowance made for the joints, giving the total maximum ampere-turns required. Dividing by the number of primary turns and by $\sqrt{2}$ to obtain the R.M.S. value, the magnetizing current is obtained. The formula to be used is, therefore,

$$\text{Magnetizing current} = \frac{\text{Ampere-turns per cm.} \times \text{length in cm.} \times 1{\cdot}2}{\text{primary turns} \times \sqrt{2}}.$$

In three-phase transformers the primary turns per phase are taken, giving as the result the magnetizing current per phase.

The total no-load current is then given by

$$\text{No-load current} = \sqrt{(\text{magnetizing current})^2 + (\text{iron loss current})^2},$$

and the angle of lag, ϕ, is equal to

$$\tan^{-1}\frac{\text{magnetizing current}}{\text{iron loss current}}.$$

An approximate formula for the magnetizing current of three-phase transformers is

Percentage magnetizing current

$$= \frac{2 \times \text{frequency}}{100} \times \frac{\text{constant} \times \text{wt. of iron in kgm.}}{10 \times \text{kVA}},$$

where the constant has the following values :—

Flux density in limbs	14,000	13,000	12,000	11,000	10,000
Constant	37	23	13	8·8	4

For single-phase transformers, the above figures should be reduced by 25 per cent.

Predetermination of Regulation.—The voltage drop at the terminals of the secondary winding when on full load is due to the combined effect of the resistance and the reactance of the two windings. The total drop due to the resistance, reduced to the secondary side, is

$$I_1R_1 \times \frac{T_2}{T_1} + I_2R_2 \text{ volts.}$$

The calculation of the total drop due to the internal reactance is given by

Percentage reactance

$$= \frac{0{\cdot}006 \times \text{Ampere-turns per limb (total)} \times l \times (A + B + C)}{\text{Total flux per limb in megalines} \times \text{Length of shorter winding in mms.}},$$

where l is the mean length of turn (average of primary and secondary) in metres, A is the depth of the secondary winding in mms., B is the depth of the primary winding in mms., and C is three times the distance between the windings in mms.

The impedance voltage triangle can now be calculated and the full load voltage drop obtained from Kapp's regulation diagram shown on page 221, or by calculation (see page 220).

The normal percentage reactances for 50 cycle, single- and three-phase transformers, from 6000 to 11,000 volts, are given in the following table :—

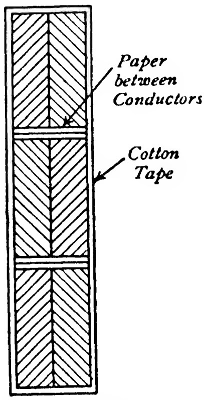


FIG. 196.—Secondary Conductor.

kVA.	Percentage Reactance.	
	Single-Phase.	Three-Phase.
10	2·5	2·0
50	3·3	2·8
100	3·5	3·25
500	4·5	3·8
1,000	5·0	5·0
5,000	6·5	6·0
10,000		6·0

Example of Design.—As an example, a design is worked out for a core type transformer for 300 kVA, three-phase, 50 cycle, 6600 volts (delta-connected with tappings at plus and minus 5 per cent. to compensate for supply variations) to 400 volts (star-connected with neutral brought out for 231 volts for lighting load). The temperature rise is not to exceed 50° C. by thermometer, and it is to be provided with a boiler plate tubular tank.

Secondary current (full load) $= \dfrac{300 \times 10^3}{\sqrt{3} \times 400} = 433$ A.

Primary „ („ „) $= \dfrac{300 \times 10^3}{\sqrt{3} \times 6600 \times 0{\cdot}982} = 27$ A.

„ „ (phase) $= \dfrac{27}{\sqrt{3}} = 15{\cdot}6$ A.

Volts per turn $= 0{\cdot}55 \times \sqrt{300} = 9{\cdot}5$.

Assume regulation at full load and unity power factor to be approximately 1·25 per cent.

L.T. volts on no-load $= 400 \times 1{\cdot}0125 = 405$ V.

Core Section :—

Flux density (assumed). $= 13800$.

Flux per limb. $= \dfrac{9{\cdot}5 \times 10^8}{4{\cdot}44 \times 50}$

$= 4{\cdot}28 \times 10^6$ lines.

Net cross section of iron required $= \frac{4{\cdot}28 \times 10^6}{13800}$

$= 310$ sq. cms.

Gross cross section of iron required $= \frac{310}{0{\cdot}9} = 344$ sq. cms.

Choose three sizes of stampings.

Width of largest stampings $= 1{\cdot}11 \times \sqrt{344}$

$= 20{\cdot}6$ cms.

„ „ intermediate „ $= 0{\cdot}87 \times \sqrt{344}$

$= 16{\cdot}1$ cms.

„ „ smallest „ $= 0{\cdot}52 \times \sqrt{344}$

$= 9{\cdot}7$ cms.

Diameter of circumscribing circle $= 1{\cdot}22 \times \sqrt{344}$

$= 22{\cdot}6$ cms.

Gross cross section $= 20{\cdot}6 \times 9{\cdot}7 + 16{\cdot}1\ (16{\cdot}1 - 9{\cdot}7) + 9{\cdot}7\ (20{\cdot}6 - 16{\cdot}1)$

$= 347$ sq. cms.

Secondary Winding:—

Secondary turns per phase $= \frac{405}{\sqrt{3} \times 9{\cdot}5} = 25.$

This will be placed nearest to the core and wound in two layers.

Secondary turns per layer $= 12\frac{1}{2}$.

Assumed current density $= 240$ A/sq. cm.

Cross section of secondary (tentative) $= \frac{433}{240} = 1{\cdot}8$ sq. cms.

Secondary conductor = six strips in parallel, each $0{\cdot}95 \times 0{\cdot}32$ sq. cm. (see Fig. 196).

Total copper cross section $= 6 \times 0{\cdot}95 \times 0{\cdot}32$

$= 1{\cdot}82$ sq. cms.

The conductor is insulated with tape to a thickness of 0·5 mm., giving overall dimensions (including paper between piles of conductors) of $[3 \times 0{\cdot}95 + 2 \times 0{\cdot}05 + 0{\cdot}1] \times [2 \times (0{\cdot}32 + 0{\cdot}05)] =$

$3{\cdot}05 \times 0{\cdot}74$ sq. cms.

Radial depth of winding $= 2 \times 0{\cdot}74 + 0{\cdot}05$

$= 1{\cdot}53$ cms.

For calculating the height of the winding, the first turn must be counted as two, on account of the spiral effect.

Height of secondary winding $= 3{\cdot}05 \times 13{\cdot}5 = 41{\cdot}2$ cms.

Clearance at ends :—

This is larger at the top than at the bottom due to the H.T. clamping ring.

Top clearance = 4 cms.

Bottom clearance = 2 cms.

This is made up of fuller-board, wound in with the end turns.

Inside diameter of winding	= 25 cms.
Outside ,, ,, ,,	$= 25 + 2 \times 1{\cdot}53$
	= 28·1 cms.

Primary Winding :—

Primary turns per phase (normal winding)	$= \dfrac{25 \times 6600 \times \sqrt{3}}{405}$
	= 705.
5 per cent. tappings	= 35 turns.
Turns per limb	= 740 (+ 5%)
	= 705 (normal)
	= 670 (– 5%).
Primary conductor	= No. 12, S.W.G.
	= 0·264 cm. diameter.
Cross section of conductor	= 0·0548 sq. cm.
Current density	$= \dfrac{15{\cdot}6}{0{\cdot}0548} = 285$ A/sq. cm.

On account of the better cooling, the current density is increased in the outer winding.

Ordinary coils per limb	= 8.
Conductor diameter (d.c.c.)	= 0·30 cm.
Turns per coil	= 83.
Turns per layer	= 12.
Layers	$= 6\frac{11}{12}$.
Insulation between layers	$= 2 \times 0{\cdot}125$ Empire cloth.
,, ,, coils (spacer)	= 0·5 cm.
,, ,, tapping coils (two spacers to allow taps to be brought out)	= 1·0 cm.
Volts between layers	$= 2 \times 12 \times 9{\cdot}5 = 228$.
,, ,, pairs of coils	$= 83 \times 2 \times 9{\cdot}5 = 1580$.
Reinforced coils per limb	= 2.
Conductor diameter (t.c.c.)	= 0·315 cm.
Turns per coil	= 38.
Turns per layer	= 7.
Layers	$= 5\frac{3}{7}$.
Insulation between layers	$= 4 \times 0{\cdot}125$ Empire cloth.
,, ,, coils (spacer)	= 0·5 cm.
Volts between layers	$= 2 \times 7 \times 9{\cdot}5 = 133$.
,, ,, pairs of coils	$= (83 + 38) \times 9{\cdot}5 = 1150$.
Ordinary coil length	$= 12 \times 0{\cdot}30 + 0{\cdot}15$ (tape)
	= 3·75 cms.
,, ,, depth	$= 7 \times 0{\cdot}30 + 0{\cdot}15 + 0{\cdot}15$ (tape)
	= 2·40 cms.
Reinforced coil length	$= 7 \times 0{\cdot}315 + 0{\cdot}15$ (tape)
	= 2·35 cms.
,, ,, depth	$= 6 \times 0{\cdot}315 + 0{\cdot}25 + 0{\cdot}15$ (tape)
	= 2·30 cms.

Total height of primary winding	$= 3{\cdot}75 \times 8 + 2{\cdot}35 \times 2 + (10 \text{ spacers} = 5{\cdot}0)$
	= 40 cms. (say).
Top clearance (for symmetry)	= 4·6 cms.
Bottom ,, ,, ,,	= 2·6 cms.
Inside diameter of winding	= 32 cms.
Outside ,, ,, ,,	$= 32 + 2 \times 2{\cdot}4$
	= 36·8 cms.

Yoke Section :—

Flux density (assumed)	$= 0{\cdot}8 \times 13800$
	= 11040.
Gross cross section of iron required	$= \frac{344}{0{\cdot}8} = 430$ sq.cms.
Height of largest stampings	$= \frac{20{\cdot}6}{0{\cdot}8} = 25{\cdot}75$ cms.
,, ,, intermediate ,,	$= \frac{16{\cdot}1}{0{\cdot}8} = 20{\cdot}1$ cms.
,, ,, smallest ,,	$= \frac{9{\cdot}7}{0{\cdot}8} = 12{\cdot}1$ cms.
Distance between limb centres	$= 36{\cdot}8 + 1{\cdot}2$ (clearance)
	= 38 cms.
Overall height of core	$= 47{\cdot}2 + 2 \times 25{\cdot}75 = 98{\cdot}7$ cms.
,, width ,, ,,	$= 2 \times 38 + 20{\cdot}6 = 96{\cdot}6$ cms.
Weight of core :— Limbs, 327 kgm.; Yokes, 528 kgm.	= 855 kgm.
Core loss :— Limbs, 870 watts; Yokes, 930 watts	= 1800 watts.
Magnetizing current :— Limbs	$= \frac{2 \times 50}{100} \times \frac{34 \times 327}{10 \times 300} = 3{\cdot}7$ per cent.
Yokes	$= \frac{2 \times 50}{100} \times \frac{8{\cdot}8 \times 528}{10 \times 300} = 1{\cdot}6$ per cent.
Total	= 5·3 per cent.
Secondary copper loss :— Length of mean turn	= 84 cm.
Weight	= 101 kgm.
Resistance per phase (hot)	$= \frac{1{\cdot}6}{10^6} \times \frac{84 \times 25}{1{\cdot}8} \times 1{\cdot}2$
	= 0·00224 ohm.
Copper loss (allowing 15 per cent. for eddy currents, and 5 per cent. for connections)	$= 3 \times 433^2 \times 0{\cdot}00224 \times 1{\cdot}2$
	= 1510 watts.

Primary copper loss :—

Length of mean turn	$= 108$ cms.
Weight	$= 124$ kgm.
Resistance per phase (hot)	$= \frac{1\cdot6}{10^6} \times \frac{108 \times 705}{0\cdot0548} \times 1\cdot2$
	$= 2\cdot67$ ohms.
Copper loss (no allowance for eddy currents but 5 per cent. for connections)	$= 3 \times 15\cdot6^2 \times 2\cdot67 \times 1\cdot05$
	$= 2240$ watts.
Total copper loss (hot)	$= 1510 + 2240$
	$= 3750$ watts.
Percentage Reactance drop (from formula on page 239)	$= 3\cdot77$ per cent.
Percentage Resistance drop	$= \frac{\text{Total copper loss in watts}}{\text{kVA} \times 10}$
	$= \frac{3750}{300 \times 10}$
	$= 1\cdot25$ per cent.
,, Impedance	$= \sqrt{1\cdot25^2 + 3\cdot77^2} = 4\cdot0$ per cent.
Regulation at full load and unity p.f. (from formula on page 220)	$= 1\cdot31$ per cent.
Efficiency, full load	$= \frac{300 \times 100}{300 + 3\cdot75 + 1\cdot8}$
	$= 98\cdot2$ per cent.

Examples.

(1) Draw sketches of the core of a three-phase core type transformer, and the shell of a single-phase shell type transformer, indicating the differences.

(2) Explain how the relative price of copper and iron may affect the economical design of a transformer.

(3) A certain 100 kVA single-phase transformer is designed for operating on 500 volts, 50 cycles, the secondary voltage being 100 volts. It has a maximum efficiency of 98 per cent. occurring at full load. It is desired to operate it on 440 volts, 40 cycles. What output may be expected from it for the same ultimate temperature rise? How would its behaviour be affected?

(4) What are the relative advantages and disadvantages of the rectangular and cruciform sections for the iron cores of transformers?

In a particular case the section of the limb is 100 sq. cms. A winding 8 cms. deep is to be placed on the core, allowing a minimum clearance of 2 cms. between core and winding. Calculate the lengths of the mean turn, using (*a*) a rectangular section and (*b*) two sizes of stampings.

(5) A 250 kVA single-phase transformer operating on 2200 to 550 volts at 50 cycles has a core of square section. The width of the iron is 25 cms. and the gross depth of the iron is also 25 cms., with two vent ducts each 1 cm. wide. The available winding length per limb is 75 cms., and the distance between limbs is 30 cms. Design suitable windings, indicating their arrangement.

(6) Determine the main dimensions of the core of a 50 kVA, 60-cycle, core type transformer to operate on 500–200 volts, together with the number of turns in each winding. Use a limb of square cross section and a flux density of 10,000 lines per sq. cm., the iron loss at this density being 0·015 watt per c.c. The efficiency is to be a maximum at full load.

(7) A 25 kVA, 50-cycle, single-phase core type transformer has a limb the net iron cross section of which is 130 sq. cms. The primary and secondary voltages are 500 and 100 volts respectively. Determine the number of turns on primary and secondary windings, and the cross sections of the conductors. Use a flux density of 11,000 lines per sq. cm. and a current density of 200 amperes per sq. cm.

(8) The limb of a core type transformer has three sizes of stampings, the widths of which are 6·0, 10·0, and 12·8 cms. respectively. The top yoke is to be designed with a flux density equal to 80 per cent. of that in the limbs. Make a dimensioned sketch of the magnetic part of the yoke section. What is the object of this reduced flux density?

(9) A 500 kVA, 6600-415 volt, single-phase, 50-cycle transformer has a limb with two sizes of stampings. The gross section of the main iron stampings is 25 cms. $\times$ 18 cms., and on each side of this is a smaller batch of stampings each 6 cms. $\times$ 15 cms.

Determine a suitable number of turns for each winding, and a suitable section for the top and bottom yokes.

Employ a flux density of 12,000 lines per sq. cm. for the limbs.

(10) A 250 kVA, single-phase, 50 cycle, core type transformer operating on 6600-500 volts is to be designed with approximately 6·5 volts per turn, and a flux density of approximately 11,800 lines per sq. cm.

Design a suitable core section and yoke section, using two sizes of stampings. The width of the smaller stampings should be approximately 0·615 time the width of the wider stampings.

CHAPTER XVII

ALTERNATORS.—PRINCIPLES AND CONSTRUCTION

Simple Alternator.—An alternator is an alternating current generator and in its elementary form consists of a coil of wire rotating between the poles of a magnet, as shown in Fig. 2. The two ends of the coil do not need to be connected to a commutator and are, instead, connected to a pair of slip rings, on each of which a brush presses for the purpose of collecting the current. In an ideal case the induced E.M.F. obeys a sine law of the type

$$e = E_m \sin \omega t.$$

An increase in the number of turns results in an increase in the induced voltage.

Frequency and Number of Poles.—The majority of actual alternators are multipolar machines, the number of poles employed being, in general, much greater than is the case in D.C. generators of the same size. Alternators are usually designed to generate at definite frequencies, and there is a rigid relationship between the speed, the number of poles, and the frequency. If there are p poles, the E.M.F. goes through $\frac{p}{2}$ cycles per revolution or $\frac{pn}{2 \times 60}$ cycles per second, where n equals the r.p.m. Thus

$$f = \frac{pn}{120}.$$

The r.p.m. at which alternators must run for various frequencies and numbers of poles are given in the following table :—

No. of Poles	2	4	6	8	10	12	16	20
Speed for f = 25 ...	1500	750	500	375	300	250	187·5	150
„ „ f = 40 ...	2400	1200	800	600	480	400	300	240
„ „ f = 50 ...	3000	1500	1000	750	600	500	375	300
„ „ f = 60 ...	3600	1800	1200	900	720	600	450	360
„ „ f = 100...	6000	3000	2000	1500	1200	1000	750	600

Polyphase Alternators.—A simple two-phase alternator may be obtained by adding a second coil similar to the first, but situated at right angles to it so that this angle is always maintained during rotation (see Fig. 74). In the case of multipolar machines, corresponding conductors in the two phases are always situated half a pole pitch distant from one another.

A simple three-phase alternator is shown in Fig. 81, corresponding conductors being always situated two-thirds of a pole pitch distant from each other, this being independent of the total number of poles.

Rotating Field and Rotating Armature.—From a theoretical point of view it is immaterial whether the armature rotates between the poles or whether the poles rotate inside a stationary armature. The rotating field type is now standard practice, except for very small experimental machines, the armature presenting a hollow cylindrical surface to the field system. The latter now consists of a yoke supported from the shaft, with the poles projecting radially outwards and facing the armature. The stationary element is called the *stator* and the rotating element the *rotor*.

There are several inherent advantages of the rotating field type. A higher peripheral speed may be employed, since it is easier to make a sound mechanical job of the poles and pole windings than of the armature winding with its end connections, since, owing to the centrifugal force, the end connections of the armature must be braced securely in position. Also, since the armature is on the outside, there is more room for its winding, whilst in the case of high voltage machines it is preferable to have the H.T. winding stationary. There is a further advantage in connection with the slip rings. A three-phase alternator with a rotating armature requires three slip rings, and a two-phase machine four, whilst only two are necessary in either case with a rotating field. With a rotating armature, also, the slip rings must be insulated for the full armature voltage.

Low Speed Alternators.—In the usual form of low and medium speed alternators, the magnet poles are bolted on to a yoke ring, which is supported by means of spider arms from the rotor hub. Since the majority of the weight of the rotor lies near its outer edge, its flywheel effect is so considerable that it avoids the necessity for any extra flywheel. This would otherwise be necessary, since a uniform angular velocity is desired for smooth running. In fact, this type of alternator is often referred to as the *flywheel type*.

Owing to the low speed, these alternators usually have a large number of poles, this tending to make the diameter much larger for a given output than is the case in a D.C. generator. The general tendency is for the machines to become large in diameter and small in length to a noteworthy extent. Even with the low speeds of revolution which are adopted, the peripheral speeds of the pole faces are kept up, due to the large diameters, the values of the peripheral speed ranging from 60 to 70 metres per second.

This type of alternator is usually directly driven from a reciprocating engine and frequently has only one bearing. The end of the shaft on the engine side then terminates in a forged coupling which is bolted directly on to the engine crankshaft.

High Speed Turbo-alternators.—The proportions of high speed alternators which are intended for direct coupling to steam turbines are much different from those of the low speed type. Owing to the high speeds of rotation which are employed, the diameters are made very much less, resulting in a corresponding increase in the axial length. Even then the peripheral velocities of the rotating element may be as high as 140 metres per second, and this necessitates a very sound mechanical rotor construction. Special types of rotating field systems are thus called into being, and these will be dealt with in detail later. The stator construction is not very different from that in low speed alternators except for the greatly changed ratio of diameter to length. The number of poles is invariably either two or four.

Since the speed is very much higher, the bulk of a turbo-generator is very much less than that of a flywheel type alternator of the same output, and consequently the total surface available for dissipating heat is very much less. In order, therefore, to prevent an undue temperature rise, artificial cooling must be employed, and this usually takes the form of an air-blast. The outer casing which guides the air past the heated surfaces serves the additional purpose of deadening the noise of the machine, which would otherwise be considerable, due to its high speed and the restricted air passages in its construction.

A separate class of alternators has been developed in conjunction with water turbines having a vertical shaft.

Inductor Alternators.—In these machines both armature and field windings are stationary, no rotating contacts being employed at all. The rotor consists of a heavy iron system with two rings of projecting pole pieces. These are magnetized by a central fixed exciting coil, so that all the poles of one ring have one polarity, whilst all the poles of the other ring have the opposite polarity. The E.M.F. in the armature coils is induced by the variation in the magnetic flux, due to the fact that the magnetic circuit is made alternately good and bad as the inductors move past the armature coils. The flux never cuts the armature coils in the reverse direction, simply varying between a maximum and a minimum value. This necessitates a large amount of flux, which, although it cannot be called leakage flux, is wholly waste as far as the generation of E.M.F. is concerned and results in a considerable increase in the iron weight of the machine. One characteristic of this class of alternator is the falling away of the voltage as the load comes on, which, although of advantage for parallel running, is undesirable as far as the voltage regulation is concerned.

Stator Construction.—The stator frame is merely an arrangement for holding the armature stampings and windings in position and does not fulfil any magnetic function. In large low speed alternators the size of the stator frame usually results in it being cast in two or three sections, these being bolted together as indicated in Fig. 197, which shows the general arrangement of rotor and stator, the frame of the latter being cut in two halves. In order to increase

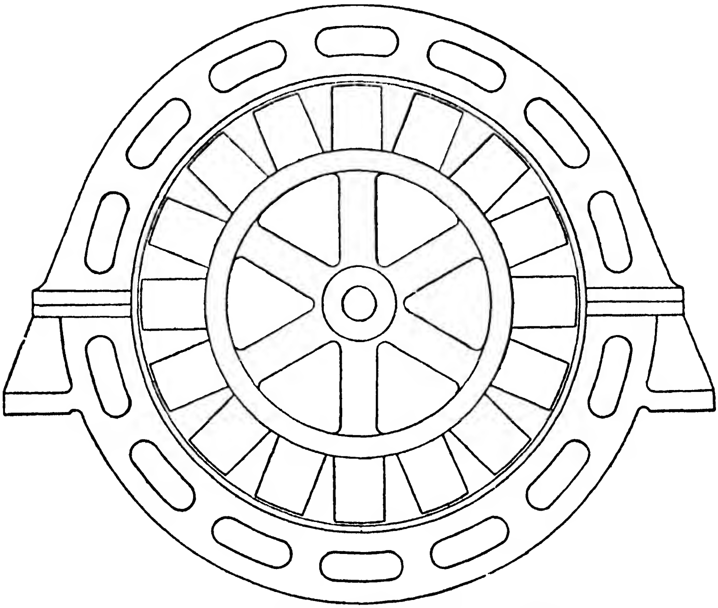


FIG. 197.—General Arrangement of Low Speed Alternator.

the ventilation a number of holes are cast in the frame, this being allowable since it does not interfere with the magnetic flux and does not unduly weaken the mechanical strength if done in moderation. Fig. 198 is one example of stator construction, the diagram showing

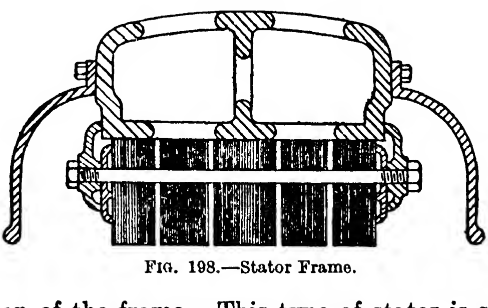


FIG. 198.—Stator Frame.

a cross section of the frame. This type of stator is suitable, with modifications, for medium and large size alternators. The stampings are held in position by being clamped between two end cheeks, a number of bolts passing right through from side to side. Radial ventilating ducts are provided in the stampings by the insertion of distance pieces, so that air can be thrown out from the air-gap by centrifugal force, escaping by the vent holes in the outer case.

The stators of turbo-alternators present very little real difference from those of low speed sets, although their external appearance is widely different. The great points of difference are the smaller diameter and the increased length. They frequently consist of a box type frame fabricated from mild steel plates welded together and supporting the laminated iron core. The compartments in the frame serve as passages for the ventilating air. The stator frames of low speed, as well as high speed, machines are now frequently fabricated from mild steel plates, this type of construction gradually displacing the cast type with many manufacturers.

Cast-iron end shields, built up in sections, are bolted on to both sides of the exterior of the stator frame for the purpose of providing a mechanical protection for the end connections, which frequently are at high potentials.

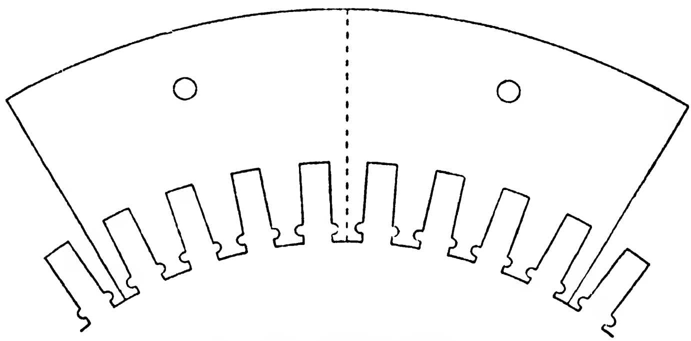


FIG. 199.—Stator Stampings.

Stator Core.—In the rotating field type of alternator the stator stampings consist of annular rings with the slots along the inside edge. For all except small machines these stampings have to be built up in sections (see Fig. 199), being bolted and keyed on to the frame at the back. In order to nullify the effect of the joint as far as possible, the sections are staggered so that the joints of adjacent stampings do not coincide. Radial ventilating ducts are provided at intervals by inserting spacers. These may consist of skeleton castings so arranged as not to impede the passage of the air, or they may be built up from two stampings thicker than the rest and separated by means of a number of distance pieces welded on. One type of construction for ventilating ducts employing channel iron spacers is shown in Fig. 200.

Slots.—Three distinct types of slots are used in A.C. machines, the first being the *open type* as used also in D.C. machines (see Fig. 201). These are frequently recessed near the mouth for the purpose of inserting a wooden or fibrous strip designed to keep the winding in place. Open slots enable former wound coils to be employed and are thus easier to wind than the other types. They also have the advantage of allowing the slots to be thoroughly

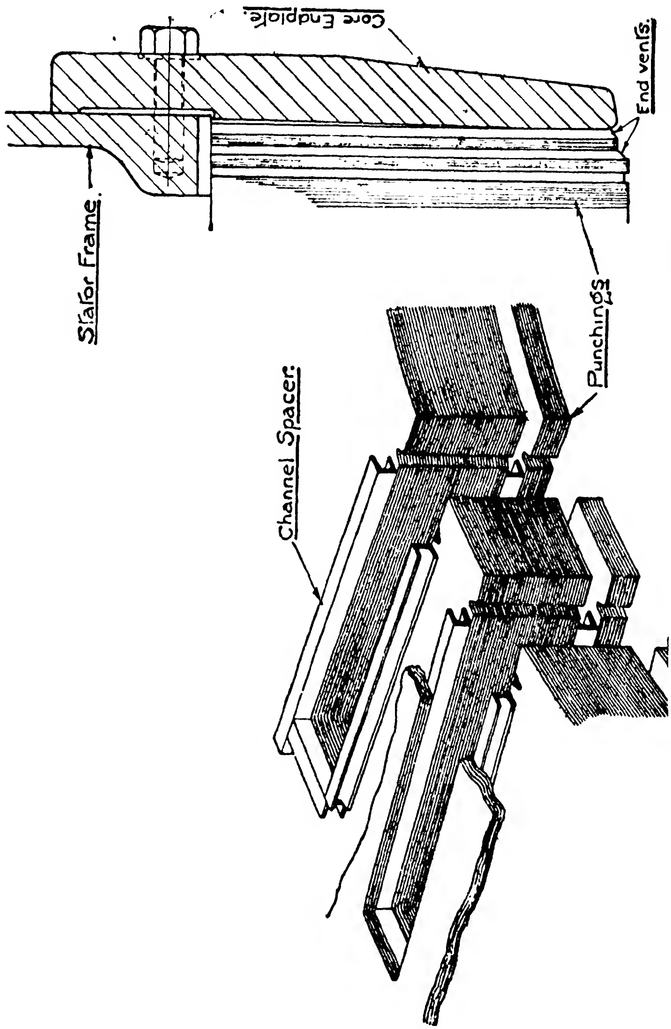


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FIG. 200.—Ventilating Duct Construction.

impregnated with insulating varnish before the coils are wound. Their disadvantage is that the available area for the air-gap is less than is the case when the other types are employed and that they cause slight oscillations of the flux from side to side as the teeth move past the pole face. This is due to the great difference in permeability of the tooth and slot. The lines of force cling on to the retreating tooth as long as possible, and then snap across the slot to the next tooth coming along. This tends to produce harmonics in the E.M.F. wave form and necessitates laminated pole shoes in order to reduce the eddy currents which would otherwise be generated to a considerable extent.

The use of *totally closed slots* or *tunnels* removes these disadvantages, since a uniform iron surface is presented to the pole face, but other disadvantages are introduced. Due to the fact that the winding is totally surrounded by iron, the inductance of the coils is materially increased, and this has the effect of causing an increased drop in the voltage as the load current is increased, thus making the regulation of the machine worse. To reduce this effect as much as possible, the bridge over the roof of the tunnel is made very thin in order to force up the flux density of the leakage lines in this neighbourhood and thus decrease the permeability. But if this bridge is made less than about 1 mm. the tunnels are apt to become broken open during the operation of stamping, this fact exercising

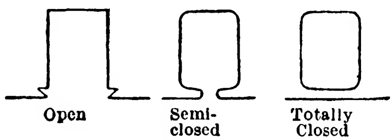

FIG. 201.—Types of Slots.

a practical limit to the thickness of the roof. Another disadvantage of the tunnel type of slot is that the coils must be hand wound by threading them through the holes, thus increasing the labour of winding and the danger of abrasion during the process.

The *semi-closed slot* is a compromise between the wide open slot and the tunnel. The danger of bursting the narrow bridge is avoided and an air-gap is inserted in the path of the leakage lines of force. Moreover, nearly the whole of the surface of the armature is useful for flux bearing purposes, which tends to eliminate ripples from the wave form. The actual winding is easier to carry out than when totally closed slots are employed, as the conductors can be passed through the mouth of the slot, although more labour is involved than when open slots with former wound coils are adopted.

High Voltage Generators.—For many years the limit of generator voltage was regarded as about 11,000 volts, but a number of generators have now been constructed for 33,000 volts. Two methods are employed for providing the necessary insulation on the stator windings. In the first, an improved quality and greater thickness of insulation is employed, whilst in the second a special concentric conductor is adopted. The stator slot is now circular in section, with a relatively small slot opening. There are three concentric conductors per slot, known as the bull, inner, and outer respectively. In connecting the conductors in the various slots to form a winding, all the outers of one phase are connected in series, one end forming the star point. These are then connected in series with the corresponding inner conductors, and finally with the bull conductors. There is thus never more than one-third of the phase voltage between any bull and inner in the same slot, between any inner and outer in the same slot, and between any outer and the core itself which is, of course, earthed.

A 33,000 volts generator costs more than a machine of the same output but for a lower voltage, but it has definite advantages. There

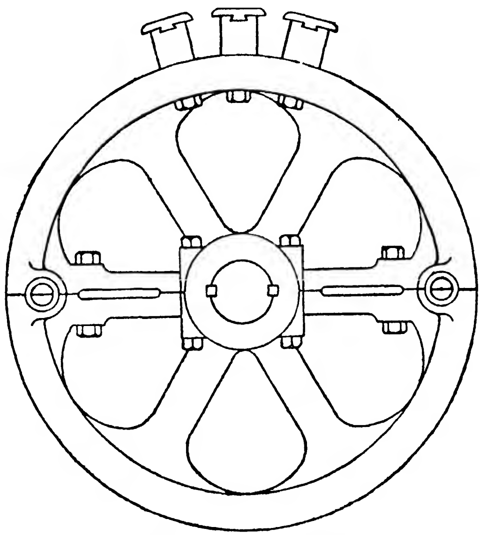

FIG. 202.—Split Rotor System.

are no transformers required for stepping up the voltage to 33,000 volts, transformer losses are eliminated, and there is an appreciable reduction in the cost of the heavy current cables required.

Rotors.—The yoke ring and spider form a flywheel upon which the poles are mounted. Up to a diameter of two or three metres the spider is generally cast in one piece, but for larger sizes it is usual to cast it in sections, which are bolted together. To increase the mechanical strength, wrought iron rings are shrunk on to the hub and over projections near the rim as shown in Fig. 202. The rim has to be of sufficient cross section to carry half the flux per pole, and also to give the necessary flywheel effect, the calculation of which involves a knowledge of the engine which is to drive the alternator.

The poles may be fixed to the rim in several ways. A method in common use consists in screwing them on from the under side of the rim by means of set screws as shown in Fig. 202. When laminated pole shoes are used with cast steel poles they may be attached by dovetailing them on to the yoke ring, lateral movement being prevented by means of keys. When the poles are laminated throughout they may be dovetailed on to the yoke ring in the same

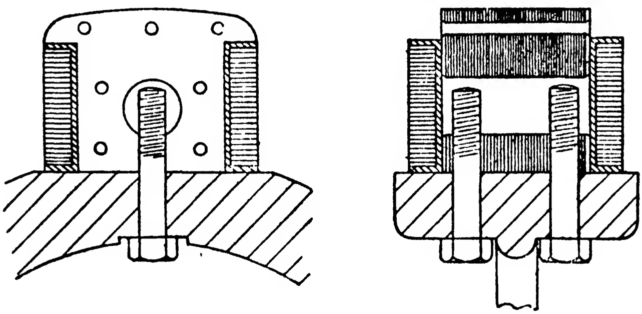

FIG. 203.—Laminated Pole Construction.

way. When ventilating ducts are employed in the poles, holes should be made through the yoke ring to meet them, and the ducts should further be in line with the stator air ducts so as to obtain a through passage for the air.

Sometimes laminated poles are attached to the yoke ring by means of set screws. In this case the laminations are pierced from side to side by a wrought iron bar so as to provide something solid for the set screws to grip (see Fig. 203).

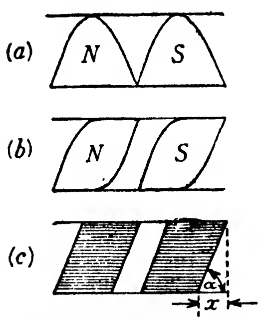

FIG. 204.—Shaped Pole Shoes to produce Sine Wave.

Shape of Pole Shoe.—If a rectangular pole shoe is employed with a uniform air-gap the resulting theoretical wave form obtained is a rectangular one. Actually, due to fringing, the two vertical sides of the rectangle become sloping ones, but notwithstanding this the wave form obtained in such a case is very far from being a sine wave. The E.M.F. induced in a particular conductor is proportional to its length in the magnetic field, the density of the magnetic field and the velocity of the conductor. The latter must be kept constant so that either the active length of the conductor or the density in the air-gap must be made to vary according to a sine law. The active length of the conductor is settled by the shape of the pole face. Fig. 204 (*a*) shows a development of the air-gap wherein the active length of the conductor is proportional to sin θ. Apart from the mechanical difficulties presented by such a construction, there would be a tremendous amount of magnetic leakage across the adjacent tips of neighbouring poles. By turning the second part of each pole face round, however, as shown in Fig. 204 (*b*), the disadvantage of the magnetic leakage is obviated, and by the approximation shown in Fig. 204 (*c*) the manufacturing difficulties are done away with.

The skewing of the pole shoes does not appear very noticeable in practice, as it is only necessary to do it to a small extent. Take the case of a 6-pole alternator with an air-gap diameter of 36 cms. With a ratio of pole arc to pole pitch of 0·65, the distance x in Fig. 204 (*c*) would be

$$\frac{\pi \times 36}{6}(1 - 0 \cdot 65) = 6 \cdot 6 \text{ cms.}$$

With an armature core length of 18 cms. this means that the angle α is given by

$$\tan \alpha = \frac{18}{6 \cdot 6} = 2 \cdot 73,$$

whence $$\alpha = 70^\circ.$$

Another method of obtaining a good approximation to a sine wave consists in having an air-gap of variable length, so that the magnetic density in the gap varies according to a sine law. The minimum air-gap occurs along the centre line of the pole and corresponds to the point of maximum flux density and the peak of the wave. If this length is represented by l, the length of the gap at any other point displaced by θ° electrical from the centre line is given by

$$x = \frac{l}{\cos \theta}.$$

Fig. 205 shows such a variable air-gap. When $\cos \theta$ gets small the air-gap becomes very long and the pole shoe is cut away altogether, the small amount of flux required being produced by fringing. Supposing a ratio of pole arc to pole pitch of 0·667 be employed, the length of the air-gap at the extreme end of the pole shoe is

$$\frac{l}{\cos(0 \cdot 667 \times 90^\circ)} = \frac{l}{\cos 60^\circ} = 2l.$$

It is rarely worth while carrying the pole shoe beyond this.

A simple approximation to this form of pole shoe can be obtained by chamfering the pole tips so that the air-gap length at the tips of the pole shoe is double the value in the centre.

Field Coils.—In addition to the usual coil winding for the poles, a type of winding frequently adopted consists in having a single layer of strip wound on edge, the strip being bent to the right shape before assembly. The adjacent layers are insulated by a layer of paper or other insulation, the outside surface being coated with varnish. This construction is very sound mechanically, as there is no tendency for adjacent conductors to roll over one another, due to centrifugal force. Another advantage of this type of coil is its superior heat dissipating properties.

Slip Rings.—The slip rings necessary for conveying the field current to the rotor are generally made of gun metal, brass or cast iron. The usual type of construction consists of a cast iron spider carried from the shaft upon which the slip rings are bolted, insulated bushes and washers being employed as in the brush spindles of D.C. machines. A common type of construction is shown in Fig. 206, there being three lugs cast on to each ring for bolting on to the spider. Another lug is also cast on for the purpose of attaching

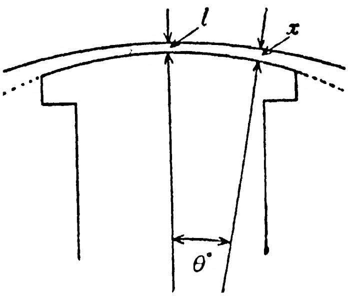

Fig. 205.—Variable Air-gap to produce Sine Wave.

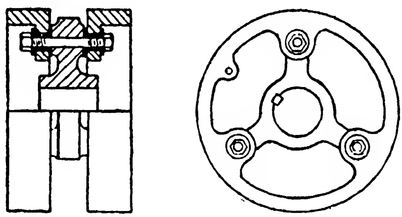

Fig. 206.—Slip Rings.

the lead carrying the current to the field coils. The dimensions of the slip rings are usually settled from mechanical considerations, as these demand larger sections than would be required on purely electrical grounds.

Carbon brushes are employed, current densities of from 5 to 7 amperes per square cm. being adopted with ordinary types of brushes.

Rotors for Turbo-alternators.—Owing to the high speed at which turbines run, the number of poles is either two or four, the diameter of the rotor being kept down in order to prevent the peripheral speed from exceeding the safe limit. This results in the appearance differing greatly from that of low-speed rotors and has led to the development of the *cylindrical* type of rotor, where the field windings are distributed in slots.

The cylindrical type of rotor consists of a core and shaft generally forged in one piece, although in some cases it is built of boiler plate. Radial slots are cut around the periphery, as shown in Fig. 207.

The slots are omitted or modified over certain regions, thus forming the poles. This results in a uniform air-gap being formed all the way round. The slots receive the pole winding, which consists of former wound coils of copper strip laid in flat and suitably insulated. A phosphor bronze or steel wedge is driven into the mouth of each slot so as to keep the winding in place. The end connections are also covered by means of phosphor bronze end shields, which prevent them from flying outwards under the action of centrifugal force. Small slots are frequently cut at the bottom of the main slots, so as to act as additional ventilating ducts, these being shown in Fig. 207, communicating with the radial ducts.

In certain cases where very large outputs are concerned, the rotor is constructed from three forgings. There is a central portion consisting of a cylinder with its centre removed, and a portion at each end which looks like a giant forged half coupling. The three portions are assembled by means of shrink links. Long slots with

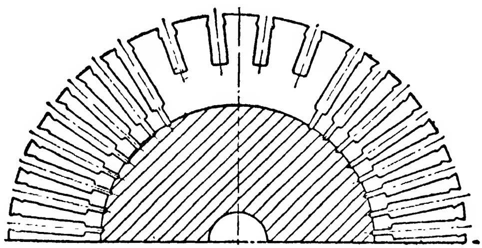

FIG. 207.—Cylindrical Type Rotor.

expanded ends are cut so as to join the central portion with each end portion, heated links being inserted in these slots which, on cooling, shrink and bind the two forgings securely together. These are called *three piece* rotors.

The cylindrical type of rotor has certain advantages over the salient pole type, inasmuch as a superior cooling surface is obtained, due to the distribution of the winding. Further, the mechanical balancing of the rotor is not so difficult, and, due to the smooth surface exposed by the cylindrical type, noiseless running is more likely to be attained.

Since the winding of the cylindrical type rotor is distributed over several slots, the flux density in the air-gap increases towards the centre of the pole, thus tending towards the generation of the ideal sine wave. For example, if the winding is distributed over three pairs of slots the theoretical wave form is of the type shown in curve (*a*), Fig. 208, whilst if the same winding were concentrated in the outermost slots the theoretical wave form would be given by curve (*b*), Fig. 208. The latter arrangement generates a higher R.M.S. value of the voltage, but does so at the expense of the wave form.

Ventilation.—Turbo-alternators are much smaller than low speed sets for a given output, so that the natural cooling surface is necessarily less. If the efficiency is of the same order, then approximately the same losses have to be dissipated. It follows, therefore, that an efficient ventilating system is essential and forced ventilation is now always adopted.

On the smaller sets cold air is drawn in by centrifugal fans mounted on the ends of the rotor, this air passing over the end-bells and traversing the air-gap. From here it passes outwards into the radial vent ducts which are provided at frequent intervals in the stator core.

In larger machines an excessive air pressure is required to force the air through the air-gap, so that additional air passages are required. Both radial and axial ducts are now provided, Fig. 209 representing a turbo-alternator of this type. Air enters the rotor through axial holes extending the full length of the field core. These holes feed ventilating ducts provided every few inches. From these ducts the air moves across the air-gap and escapes through radial

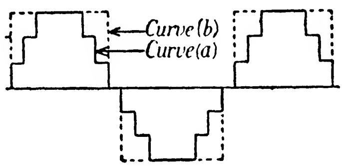

FIG. 208.—Theoretical Wave Form.

ducts in the stator core. Particular care must be taken to cool the rotor end connections, some of the air being diverted directly over these.

One of the troubles met with lies in the dust which accumulates in the ducts, clogging them so as to impair the efficacy of the whole ventilating system. To avoid this the closed circuit ventilating system has been developed. Instead of the hot air being allowed to escape, it is cooled by a series of cold water pipes, and used over and over again. This air is also dried by means of calcium chloride or some other drying agent, so that moisture is excluded from the machine as well as dust, a very important point, since moisture has a very deleterious effect upon the insulation. Arrangements are also made to throw the machine open to the atmosphere in case of need.

Hydrogen Cooling.—The substitution of hydrogen for air as a cooling medium in a closed ventilating system does not involve any fundamental change in construction. To avoid leakage of air into the cooling circuit, the hydrogen is maintained at a pressure slightly above that of the atmosphere. The advantages of hydrogen as a cooling medium are reduced windage and ventilating losses, due to the density of the gas, and an increased life of the insulation, due

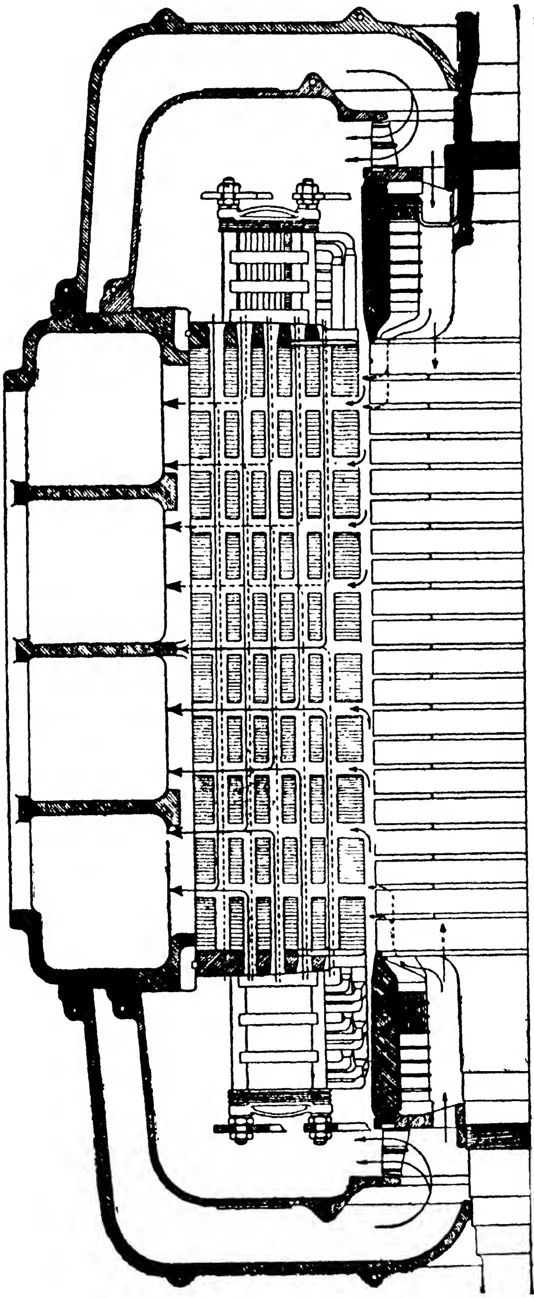

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FIG. 209.—Turbo-Alternator with Radial-axial Ventilation.

to the absence of oxygen and moisture. There is also less noise. The reduction in the losses may be such as to bring about an increase in efficiency up to half of one per cent. The purity of the hydrogen is maintained above 95 per cent., this being well above the explosive limit. It is also necessary to provide a special sealing device at the shaft to prevent the escape of hydrogen.

Synchronous condensers (see p. 526) employing hydrogen cooling have also been used, the construction being here simplified since there is now no necessity for a shaft seal.

Armature Windings.—The ordinary type of winding used in D.C. machines is also suitable for alternator armatures, but is not usually

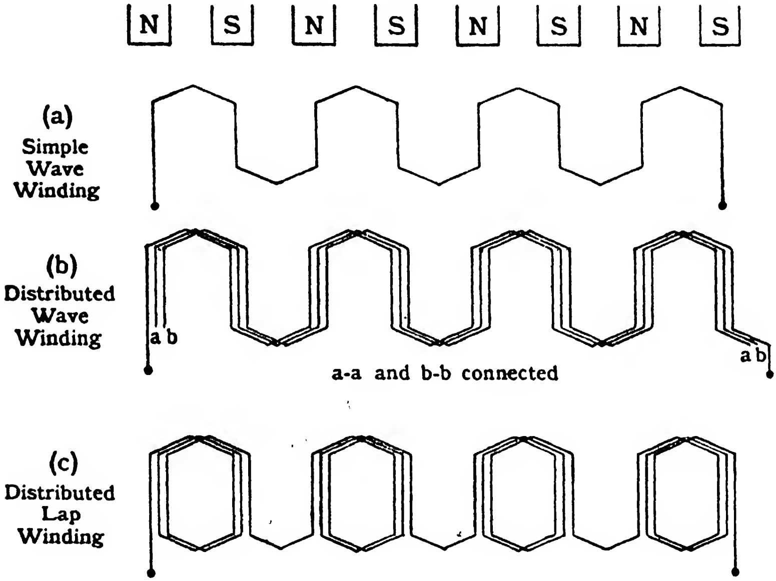


FIG. 210.—Single-Phase Bar Windings.

adopted. The ordinary D.C. winding is called a *closed circuit* winding, because it closes on itself, whilst the windings used on alternators are called *open circuit* windings, since there is no closed circuit in the armature itself.

The simplest example is the wave winding illustrated in Fig. 210 (*a*), where the winding is shown in a development. The E.M.F.'s in all the conductors aid one another, there being eight conductors in all (in an 8-pole alternator). This only utilizes one slot per pole, and if the winding is to be distributed in more slots, it takes the form shown in Fig. 210 (*b*), there being one unsymmetrical end connection for every time the winding goes right round the armature.

An alternative arrangement showing a distributed lap winding is represented in Fig. 210 (*c*). The winding now consists of a number of lap coils, each batch of coils being connected to the next batch by means of special end connections called *jumpers*. The winding is of the two layer type with diamond-shaped coils. The coils may span a pole pitch, but short-pitched (or *chorded*) coils are used wherever possible. In some cases, the coils span more than a pole pitch. It is not usual to fill up all the slots in a single-phase alternator, since the last conductors add but little to the total voltage, as the phase of the conductors in the various slots differs considerably. For example, if there are six slots per pole, the phase difference between conductors in adjacent slots is 30°, and the vector diagram in Fig. 211 shows the various voltages obtained by using 1, 2, 3, 4, 5 and 6 conductors. The following table shows the numerical values of the voltage obtained in the different cases, and as each conductor adds to the impedance of the winding, it is usual in such cases to limit the number of slots used to about two-thirds of the total.

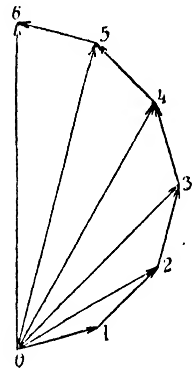


FIG. 211.—Vector Diagram showing Phase Difference of Conductors in Different Slots.

TABLE SHOWING EFFECT OF DISTRIBUTING WINDING.

No. of Slots used.	Voltage.	Voltage added by last Conductor.
1	1·00	1·00
2	1·93	0·93
3	2·73	0·80
4	3·34	0·61
5	3·73	0·39
6	3·86	0·13

Concentric Coil Windings.—The concentric coil type of winding can be derived from the distributed lap winding shown in Fig. 210 (*c*). Since all the conductors in one batch of coils are connected in series, a re-arrangement of the individual connections is permissible. This has been done in Fig. 212 (*a*), from which the concentric coil winding in Fig. 212 (*b*) is evolved. The final form of the winding is shown in Fig. 212 (*c*), the connections between the several turns in a coil being omitted for the sake of clarity.

A two-phase winding of this type is obtained by providing a second winding similar to the first, but displaced by half a pole pitch as shown in Fig. 213. The end connections of the two windings have to be of different shape, in order to enable them to clear one another. These end connections are bent back to save space and to enable them to be clamped firmly in position. The end connections of one phase project further from the stator core than do those of the other phase, thus forming what are called two plane or two range end connections as shown in Fig. 214. (See also Fig. 224.)

In the three-phase concentric coil winding, there are three separate windings displaced by 60° or one-third of a pole pitch from

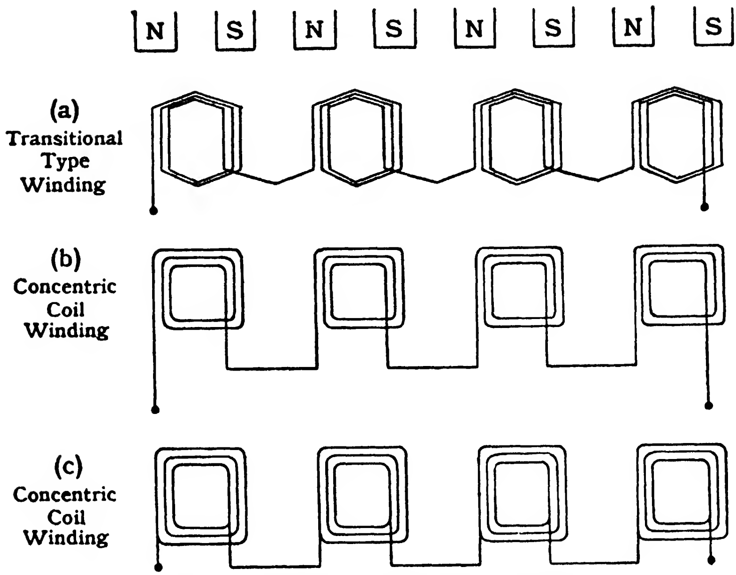


FIG. 212.—Single-Phase Concentric Coil Windings.

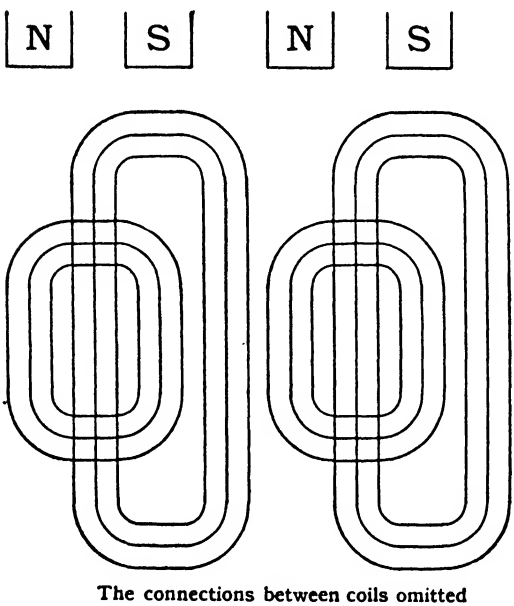
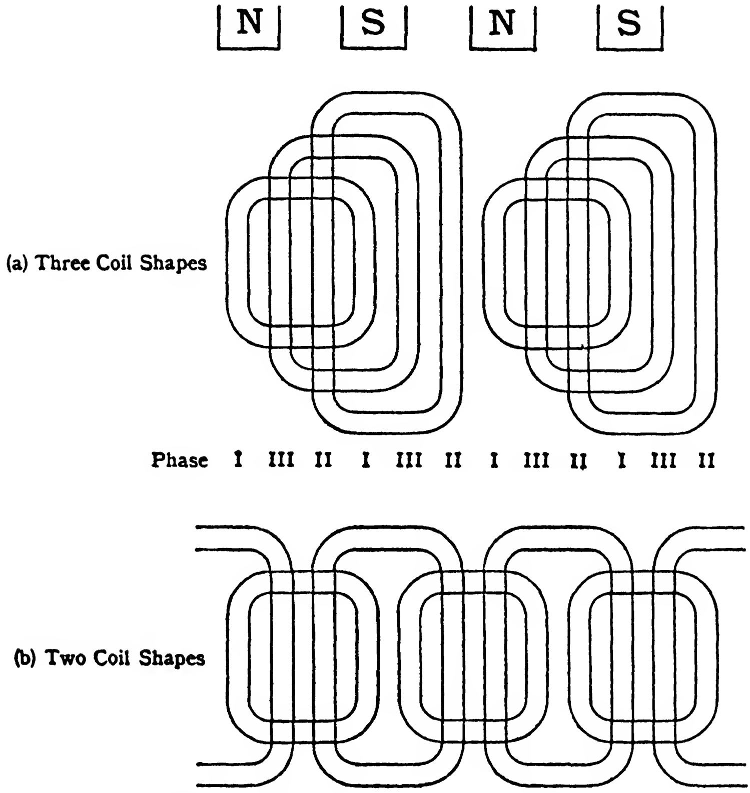
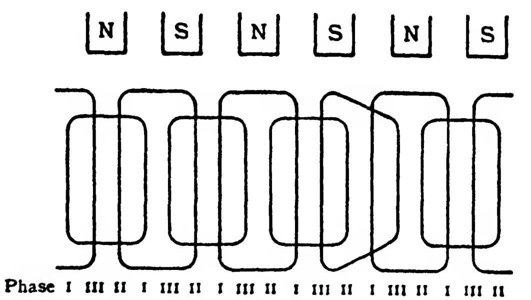


FIG. 213.—Two-Phase Concentric Coil Winding.



The connections between coils omitted

FIG. 215.—Three-Phase Concentric Coil Windings.



The connections between coils omitted

FIG. 216.—Six-Pole Three-Phase Concentric Coil Winding.

the winding is reduced; and, finally, it leads to an improvement in the wave form of the generated E.M.F.

Fig. 217 shows a three-phase diamond winding with full pitch coils, there being two slots per pole per phase. Only a portion of the winding is shown, extending over two pole pitches, but beyond this the winding repeats itself. The phase displacement between

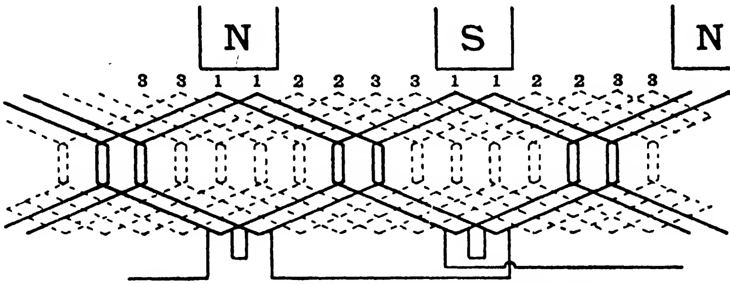


FIG. 217.—Three-Phase Diamond Winding. Full Pitch Coils.

adjacent slots is $\frac{180°}{2 \times 3} = 30°$, and the coils span six slot pitches or 180°.

Fig. 218 shows the same winding arranged with chorded coils, the only difference being that the coils span five slot pitches or 150°.

Fig. 219 shows a similar type of winding, also with chorded coils, but this time with a fractional number of slots per pole per phase. The winding represents a four-pole, three-phase winding with

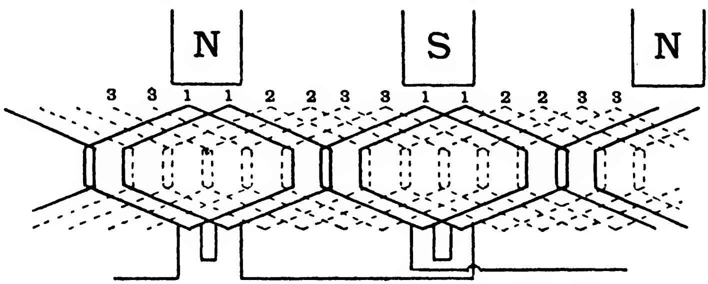


FIG. 218.—Three-Phase Diamond Winding. Chorded Coils.

18 slots, *i.e.* 1·5 slots per pole per phase. Since there are 9 slots in a double pole pitch, the phase displacement between adjacent slots is $\frac{360°}{9} = 40°$, and the coil span is four pitches or 160°.

When the winding has a fractional number of slots per pole, only certain numbers of slots can be used to give a balanced winding. Considering the fraction $\frac{\text{slots}}{\text{poles}}$, reduced to its lowest terms, the numerator must be a multiple of the number of phases, from which it follows that the denominator must not be. For example, 108 slots with 8 poles is permissible in a three-phase machine, since $\frac{108}{8} = \frac{27}{2}$, and 27 is a multiple of 3. Again, $4\frac{4}{9}$ slots per pole per phase cannot be used on a three-phase machine, because $\frac{40}{3}$ has a denominator equal to the number of phases.

Involute Windings.—In this type of winding, the end connections are arranged in a double layer as in the diamond coil winding. They are, however, bent back to an angle of 90°, and mounted in a plane at right angles to the axis of the stator. They are usually formed separately on this account, as is also done in many cases with con-

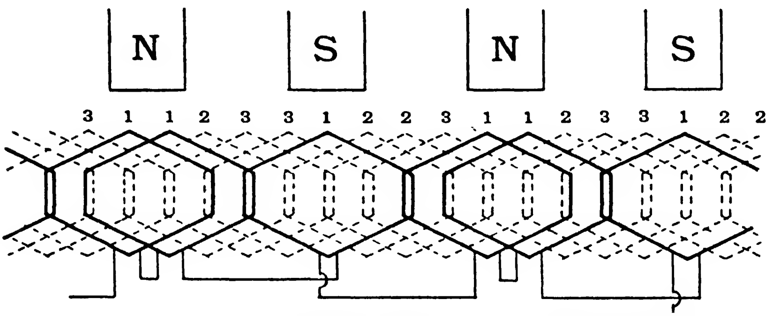


FIG. 210.—Three-Phase Diamond Winding. Chorded Coils. Fractional Number of Slots per Pole.

centric or diamond coils. The involute type of winding combines to some extent the advantages of both of the other two types described.

Armature Conductors.—In small machines the armature (stator) conductors are made of single wires or strips, but in the larger sizes they are built up by connecting several strips in parallel. Apart from convenience in manufacture, this minimizes the eddy current loss in the bars, due to different portions of the cross-section lying in magnetic fluxes of different density. These component strips are lightly insulated from each other, and the complete conductor is so constructed that each parallel element occupies different positions in succeeding slots. This is effected by the arrangement of the end connections. In other cases, the parallel elements are transposed in the slot itself.

Breadth Factor.—In the majority of instances the armature winding is not concentrated in one slot per pole per phase. As the result of this, the conductors in neighbouring slots have E.M.F.'s induced in them which differ slightly from one another in phase. Since the total E.M.F. induced in the coil is due to the E.M.F.'s in the component conductors connected in series, this difference in phase causes a loss of voltage, since the individual E.M.F.'s must be added vectorially instead of arithmetically. For example, in a three-phase alternator having three slots per pole per phase there are nine slots distributed over one pole pitch. The phase difference between the E.M.F.'s induced in conductors in adjacent slots is therefore 20°. The vector diagram for this case is shown in Fig. 220, the resultant voltage being 2·88 instead of three times the voltage per slot. Distributing the winding in three slots thus causes a 4 per cent. loss in voltage, but is nevertheless preferable to having it concentrated in one slot of three times the size, as this would tend towards the production of irregularities in the wave form. For this and other reasons, the single slot winding is not used.

The ratio of the voltage actually obtained to the voltage which would be obtained if the winding were all concentrated in one slot is called the *Breadth Factor*, and in the case shown in Fig. 220 its value is $\frac{2 \cdot 88}{3} = 0 \cdot 96$. The value of the breadth factor is unity if

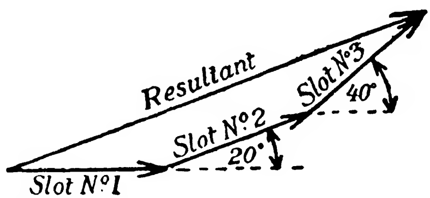


FIG. 220.—Effect of distributing Winding in Three Slots.

the winding is concentrated into one slot per pole per phase, but when it is distributed its value is always less than unity, decreasing as the number of slots is increased.

If the winding were distributed in an infinite number of slots extending over a complete pole pitch, then the diagram shown in Fig. 220 would become a semicircle with the resultant E.M.F. as the diameter, for a circle may be regarded as a polygon with an infinite number of sides. The breadth factor in this case is equal to $\frac{2}{\pi} = 0 \cdot 637$.

In the general case, let the phase angle between the E.M.F.'s induced in adjacent slots be α, and let one phase of the winding consist of conductors arranged in n consecutive slots. There are, therefore, n voltages to be added vectorially. The voltage per slot is represented by AB, BC, CD, etc., in Fig. 221. From O, draw a line OP perpendicular to AB. Now $\widehat{OBP} = \widehat{OBC}$, and $\alpha = 180° - \widehat{OBP} - \widehat{OBC} = 180° - 2 \,.\, \widehat{OBP}$. But $\widehat{OBP} = 90° - \widehat{BOP}$, so that $\alpha = 180° - 2\,(90° - \widehat{BOP}) = 2 \,.\, \widehat{BOP} = \widehat{AOB}$. Also, $PB =$

$OB \cdot \sin \widehat{BOP} = OB \sin \frac{\alpha}{2}$, and $AB = 2 \times OB \sin \frac{\alpha}{2}$. The total voltage induced by conductors in n adjacent slots is, therefore, $n \times 2 \times OB \sin \frac{\alpha}{2}$, when added arithmetically. Similarly the resultant voltage induced in the conductors in the same slots is $AD = 2 \times OB \sin \frac{n\alpha}{2}$. The breadth factor is given by

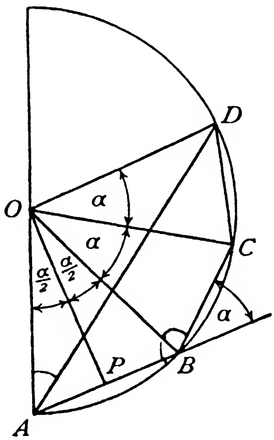


FIG. 221.—Breadth Factor. General Case.

$$k_2 = \frac{AD}{n \times AB}$$

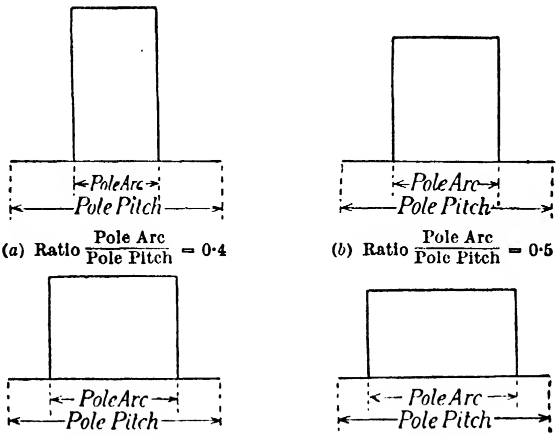
$$= \frac{2 \times OB \times \sin \frac{n\alpha}{2}}{n \times 2 \times OB \times \sin \frac{\alpha}{2}} = \frac{\sin \frac{n\alpha}{2}}{n \sin \frac{\alpha}{2}}.$$

According to present practice, the number of slots per pole per phase is usually limited to five in the case of flywheel type alternators and eight in the case of turbo-alternators. The following table gives the values of the breadth factor in a number of cases of single-, two- and three-phase machines.

TABLE OF BREADTH FACTORS.

Slots per Pole per Phase.	Single-Phase, Half Slots Wound.	Single-Phase, Two-thirds Slots Wound.	Two-Phase, All Slots Wound.	Three-Phase, All Slots Wound.
1	—	—	1·000	1·000
2	1·000	—	0·924	0·966
3	—	0·866	0·910	0·960
4	0·924	—	0·906	0·958
5	—	—	0·904	0·957
6	0·910	0·836	0·903	0·956
8	—	—	—	0·956

Effect of Ratio $\frac{\text{Pole Arc}}{\text{Pole Pitch}}$.—In order to study the effect of this ratio on the induced voltage, the case of an alternator will be discussed where the pole pitch and the flux per pole are fixed. As a first approximation, a rectangular pole shoe and a uniform air-gap will be assumed, the resulting wave form being, of course, rectangular. A variation in the ratio $\frac{\text{pole arc}}{\text{pole pitch}}$ is now obtained by choosing different lengths of pole arc, getting different magnetic densities at the pole face. The area of the resulting flux curve is constant, since this is proportional to the product of the magnetic density and the pole arc, and thus represents the total flux per pole, which is assumed constant. With a constant speed of rotation, the E.M.F. wave form is similar to the curve of flux distribution, and



(*a*) Ratio $\frac{\text{Pole Arc}}{\text{Pole Pitch}} = 0{\cdot}4$ (*b*) Ratio $\frac{\text{Pole Arc}}{\text{Pole Pitch}} = 0{\cdot}5$

(*c*) Ratio $\frac{\text{Pole Arc}}{\text{Pole Pitch}} = 0{\cdot}6$ (*d*) Ratio $\frac{\text{Pole Arc}}{\text{Pole Pitch}} = 0{\cdot}7$

FIG. 222.—Effect of Ratio $\frac{\text{Pole Arc}}{\text{Pole Pitch}}$ on Wave Form.

thus the area of the E.M.F. wave form is also constant. Working out the form factors and relative R.M.S. values of the E.M.F.'s for the various cases in Fig. 222, these are found to be different and they are tabulated in the following table.

Ratio $\frac{\text{Pole Arc}}{\text{Pole Pitch}}$.	Form Factor.	Relative R.M.S. Voltage.	Relative Max. Voltage.
0·4	1·58	0·632	1·000
0·5	1·41	0·566	0·800
0·6	1·29	0·516	0·667
0·7	1·19	0·478	0·571

According to the above table, the ratio which gives a form factor closest to that of a sine wave is approximately 0·7, but if the pole face is shaped it is found that the best value is somewhat lower than this.

Calculation of E.M.F.—Each conductor on the armature cuts Φp lines of flux in every revolution of the armature, Φ being the number of lines of flux per pole and p the number of poles. Since each turn consists of two active conductors together with the inactive end connections, the lines cut per turn per second are

$2\Phi pn$, where n represents the revolutions per second. But the frequency f is equal to $\frac{pn}{2}$, so that the *average* E.M.F. induced per turn is

$$2\Phi pn \times 10^{-8} \text{ volts}$$
$$= 4\Phi f \times 10^{-8} \text{ volts.}$$

With a sinusoidal wave form the form factor $\left(= \frac{\text{R.M.S. value}}{\text{average value}}\right)$ is 1·11, and therefore the R.M.S. value of the induced E.M.F. per turn is

$$4{\cdot}44\Phi f \times 10^{-8} \text{ volts.}$$

For other shapes of E.M.F. wave the R.M.S. voltage per turn is given by

$$4k_1\Phi f \times 10^{-8} \text{ volts,}$$

where k_1 is the form factor.

If there are T turns in series per phase the voltage per winding becomes

$$E = 4k_1\Phi fT \times 10^{-8} \text{ volts.}$$

It has been shown, however, that distributing the winding in several slots per pole results in a slight loss of voltage, and so the breadth factor has to be taken into account, when

$$E = 4k_1k_2\Phi fT \times 10^{-8} \text{ volts,}$$

k_2 being the breadth factor.

When a winding is adopted in which the turns do not span a complete pole pitch there is a further slight loss in voltage. The E.M.F. due to the two conductors in a turn is now rather less than twice the E.M.F. per conductor. Suppose the two conductors span only 170° instead of the full 180°. The two conductor E.M.F.'s are now 10° out of phase with each other and the resultant E.M.F. of the two is $2 \times \cos 5° = 1{\cdot}992$ times the conductor E.M.F. In general if the coil spans $180° - \Psi$, the reduction factor (or pitch factor) is $\cos \frac{\Psi}{2} = k_3$, so that

$$E = 4k_1k_2k_3\Phi fT \times 10^{-8} \text{ volts.}$$

There is a further slight loss of voltage if the coils which are connected in series are not situated exactly a pole pitch apart.

The voltage obtained in this way is the open-circuit voltage per winding, and, in the case of a three-phase star-connected alternator, the line voltage is obtained by multiplying by $\sqrt{3}$. To find the full load terminal voltage, the drop caused by the armature impedance and the effect of armature reaction must be taken into account, and this will be discussed later (see page 281).

Methods of obtaining a Sine Wave.—The ideal method of obtaining a sine wave of induced E.M.F. is to cause the conductors to cut a sinusoidally distributed flux at a constant rate. In practice, however, this is not possible, and all practical methods are only more or less close approximations to the ideal.

The effects of skewed pole shoes and air-gaps of variable length have already been discussed (see page 254), as well as the effect on the distribution of the field flux of distributing the field winding in a number of slots in the case of the cylindrical type rotor of a turbo-alternator (see page 257).

The shape of the wave can also be improved by various means on the stator. The stator slots may be skewed, this producing the same effect as skewing the pole shoes; the conductors enter the field more or less gradually. The armature winding is now always distributed in a number of slots per pole per phase. The resultant induced E.M.F. is the sum of the separate E.M.F.'s induced in the different conductors, taking into consideration their various phase angles. This results in a slight loss of voltage as far as the fundamental of the wave is concerned, but it causes the harmonics to cancel each other to a certain extent. The effect can be seen at once by drawing a number of trapezoidal waves, slightly displaced with respect to each other, and determining their resultant graphically. The resultant wave is a closer approximation to a sine wave. A somewhat similar effect is produced by the use of the chord winding, where the two sides of a coil span rather less than a complete pole pitch. The phases of the E.M.F.'s in the two conductors are not exactly the same and so tend to smooth out the harmonics. Again, the same effect can be produced by the use of a fractional number of slots per pole per phase or by the use of several empty slots. The value of the ratio of pole arc to pole pitch is also of importance, this effect being discussed on page 268.

Higher harmonics can be minimized by not having an exact number of teeth per pole arc. If this were not done the number of teeth under the pole shoe would be one more in certain positions of the rotor than in others. This would cause a fluctuation in the reluctance of the magnetic circuit and also cause small E.M.F.'s of high harmonic frequency to be induced. These particular higher harmonics are called *tooth ripples*. Another method of reducing these tooth ripples is to use an air-gap which is relatively long so as to increase the reluctance. The relative effect of the teeth is thus reduced.

The connection of the stator windings for a three-phase supply in itself eliminates harmonics of three and multiples of three times the fundamental frequency, provided the neutral be not earthed. This, however, is rarely the case.

All the above methods are employed (although not simultaneously) for the production of a close approximation to a sine wave of induced E.M.F., and modern machinery does closely approximate to the ideal in this respect.

Clamps for End Connections.—The end connections of the armature coils consist of a number of conductors carrying currents flowing

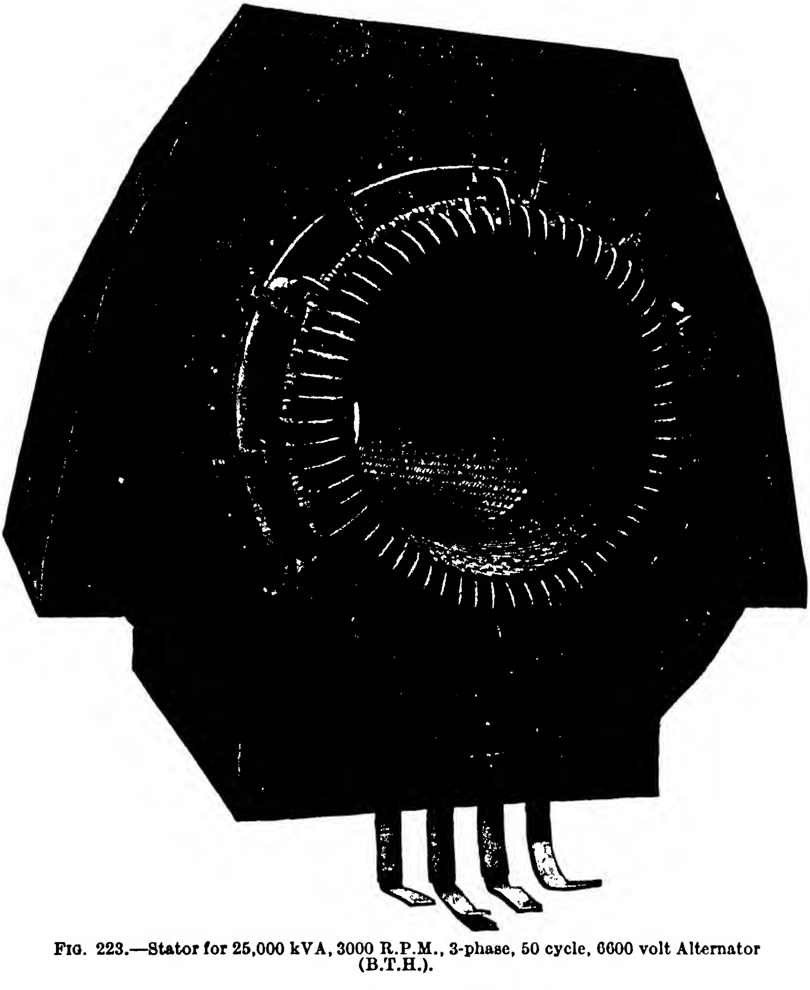


FIG. 223.—Stator for 25,000 kVA, 3000 R.P.M., 3-phase, 50 cycle, 6600 volt Alternator (B.T.H.).

in the same direction, and this results in considerable mechanical stresses being set up on occasion, particularly in the case of turbo-alternators. In ordinary low-speed alternators, no special precautions need be taken to deal with these stresses, but with turbo-alternators it is different. This is due to the longer pole pitches and to the greater number of ampere-turns per pole employed. Experience has shown that very heavy clamping devices are necessary in these cases to prevent the coils being torn from their fastenings under the influence of an accidental short-circuit. Figs. 223 and 224 show typical examples of the heavily clamped end connections of a turbo-alternator.

Damping Grids.—When a polyphase alternator delivers a current a rotating magnetic field is set up by the combined action of the

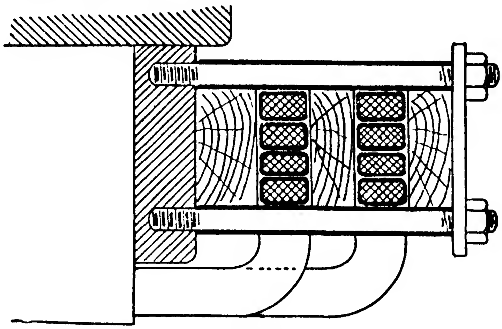


FIG. 224.—Clamping Arrangements for Stator End Connections.

armature currents, and if the speed of rotation of the rotor is not exactly uniform the poles will sometimes gain on this flux as it rotates and sometimes lag behind. The pole face can therefore be imagined to be situated in a magnetic field which oscillates from side to side, and any closed circuit in the neighbourhood will have a current induced in it. The generation of such a current tends to damp out the motion which produces it and thus tends

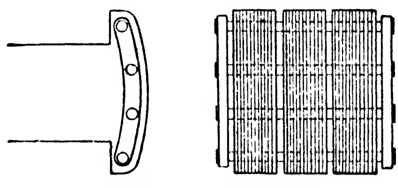


FIG. 225.—Damping Grid.

towards smoother running. In order to effect this, special windings are sometimes wound in slots or holes in the pole shoe. These windings are known as *damping grids* or *amortisseurs*, and consist of a number of heavy copper rods, one in each hole, riveted at the ends to a common bar so as to form a short-circuited grid, as shown in Fig. 225.

In cylindrical type rotors the damping grids take the form of a squirrel cage (see page 328). Surrounding the end connections is a copper ring with a number of fingers projecting into the slots. The ring and fingers are cut from a single large sheet of copper. Separate strips placed in the top of the slots between the slot wedges and the winding complete the squirrel cage.

In some cases a stationary squirrel cage damper is mounted on the stator, opposite to the rotor end connections, the object being to reduce the magnetic leakage from the latter.

Excitation and Exciters.—The ordinary types of alternator require to be excited by means of D.C. from some external source, and for this purpose it is usual to instal special D.C. generators called exciters in connection with large alternators. One system frequently adopted consists in having an exciter for every alternator, directly driven from the alternator shaft. An alternative method is to have one exciter of sufficient capacity to deal with the excitation of all the alternators in question, the exciter being steam-driven. In this case the exciter can also be made to supply additional power for the purpose of lighting the generating station and for other auxiliary services. Sometimes also there will be a pair of D.C. bus bars convenient, in which case the necessary exciting current can be obtained from the mains.

When each alternator is fitted with its own exciter, the regulation of the exciting current can be effected by means of a resistance in the exciter field, which deals with a comparatively small current, but when a common exciter serves a number of alternators the field regulation must be effected by resistances in the exciter main armature circuit. The field regulating resistances must therefore be capable of carrying larger currents, thus resulting in an increase in size.

In modern large turbo-alternators the exciting current required for full load current at the lowest rated power factor may be several times the exciting current required to give normal voltage on open-circuit. The terminal voltage of the exciter in the latter case is, therefore, only a relatively small percentage of its full voltage. This means that it is working on the straight line portion of its voltage-excitation characteristic, and a certain amount of voltage instability may be encountered. To obviate this, some alternators are provided with what is called a *service exciter*, the function of which is to supply the excitation of the main exciter. The latter now consists of a separately excited D.C. generator.

When an alternator delivers a leading current, its field is strengthened (see page 282) on account of the armature reaction, and in certain cases this strengthening effect is sufficient to cause a very considerable E.M.F. to be generated, even if the D.C. field current be interrupted. The leading current in the stator conductors sets up a rotating field of its own just as in the case of an induction motor (see Chapter XX). Not only may the E.M.F. be maintained on interrupting the D.C. field current, but an unexcited alternator may build up its voltage if switched on to a capacitance of sufficient magnitude. The residual magnetism induces a small voltage which produces a small capacitance current. This strengthens the field and the action goes on until the E.M.F. is completely built up. The value of the E.M.F. which can be obtained in this way depends upon the capacitance of the circuit to which the machine is connected. If the capacitance is large enough, very dangerous potentials may be reached, exceeding those normally obtained. This effect has a direct practical application in the switching on of unexcited alternators to long open-circuited feeders possessing considerable capacitance. This practice should always be avoided where possible.

Tirrill Regulator.—The Tirrill regulator is an automatic device employed for regulating the voltage of an alternator under varying

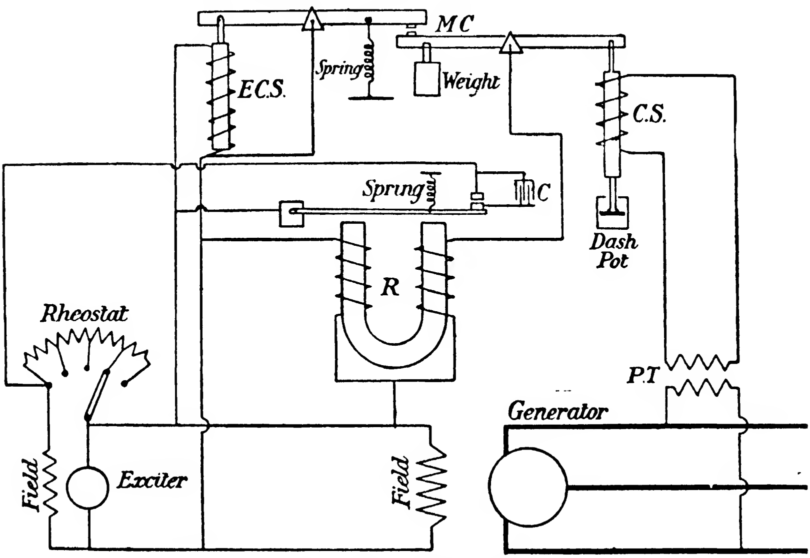


FIG. 226.—Tirrill Regulator.

loads, and is shown diagrammatically in Fig. 226. There is a main control solenoid, *CS*, connected across the alternator terminals, either directly or through a voltage transformer, *PT*, and an exciter control solenoid, *ECS*, connected across the exciter armature. A differentially wound relay, *R*, with a U-shaped core operates two contacts which open and close a short-circuit across the exciter field rheostat, these contacts being protected by a condenser, *C*, which is shunted across them. One of the differential relay windings is connected across the exciter terminals and the other is connected across the same points but in series with the main contacts, *MC*. The latter are mounted one on each of the control levers. The pull on the exciter control solenoid is balanced by springs and that exerted on the main control solenoid is damped by a dash pot. The pull on the main control solenoid is independent of the position of the core and is balanced by a weight.

If the generator voltage falls the plunger on the main control solenoid tends to sink and the main control lever to rotate in a clockwise direction, and *vice versa*. The lever is in equilibrium at one voltage only. When the voltage falls this lever closes the main contacts. This brings the second relay winding into operation, which neutralizes the other one and demagnetizes the relay. The spring then pulls off the keeper and closes the relay contacts, thus short-circuiting the exciter field rheostat. The exciter voltage then rises until the generator voltage is sufficient to restore equilibrium in the main control lever. The exciter voltage continues to increase by a small amount and this causes the exciter control solenoid to rotate its control lever so that the main contacts are opened. This now removes the short circuit across the exciter field rheostat. The only moving parts of the mechanism consist of very light contacts which, when the regulator is in circuit, are continuously vibrating, their maximum travel being only $\frac{1}{32}$ inch.

The time lag is practically non-existent and these regulators are most successful in maintaining a practically steady pressure.

EXAMPLES.

(1) Compare critically the rotating field and the rotating armature types of alternator, and explain the reasons which have led to the adoption of the former as the standard type.

(2) A three-phase low speed alternator has three slots per pole per phase. Make a sketch of the development of a suitable coil winding with end connections in two planes.

(3) Calculate the breadth factor of a three-phase, 4-pole, turbo-alternator having a total of 72 stator slots.

If the stator of this machine were re-wound for two-phase working, using the same rotor and the same conductors per slot, would the output be the same, greater or less than the original output? Give reasons and calculate the percentage difference.

(4) Explain what steps are taken to ensure that an alternator shall generate an E.M.F., the wave form of which shall be a close approximation to a sine wave.

(5) Write an account of the methods employed for ventilating large turbo-alternators.

(6) A three-phase star-connected alternator is required to give 6700 volts between lines on open-circuit at 50 cycles and 600 r.p.m. There are eight conductors per slot and three slots per pole per phase. Determine the useful flux per pole, assuming a sine wave.

(7) A three-phase, star-connected, 50 cycle, 4-pole alternator has 60 slots on the stator, with 2 conductors per slot. The armature winding is of the two layer type. Short pitch coils are used, a conductor in slot No. 1 being connected to a conductor in slot No. 13. Determine the useful flux per pole required to induce a line voltage of 6700 volts. Assume a sine wave of induced E.M.F.

CHAPTER XVIII

ALTERNATORS.—PERFORMANCE AND TESTING

Magnetization Curve.—The graphical relationship which exists between the exciting current and the terminal voltage of the armature is called the magnetization curve, and can be obtained experimentally by taking various values of the exciting current and observing the corresponding armature voltages, the machine being on open-circuit. Usually it is necessary to insert a considerable resistance in series with the field winding in addition to the ordinary shunt regulator, in order to bring down the exciting current so as to obtain points on the lower portion of the curve. The resulting graph follows the general shape of the *B*—*H* curves. Another magnetization curve can be obtained by making the alternator give out full load current at unity power factor and obtaining the relation between the exciting current and the terminal voltage as before. If the necessary inductances are available, the test can be repeated with a power factor of less than unity. Fig. 227 shows the nature of the curves which are obtained in the test.

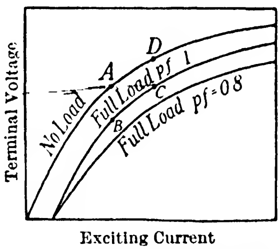


FIG. 227.—Magnetization Curves.

Load Characteristic.—The load characteristic of an alternator is obtained by determining the relationship between the terminal voltage and the load current, the exciting current and speed being kept constant.

As the armature current is increased the terminal volts drop, due to a number of causes. The resistance and reactance of the armature winding absorb some of the volts which are generated, whilst the armature reaction generally results in an actual decrease of the volts generated, owing to its weakening effect on the magnetic field. The magnitude of the latter effect depends to a very large extent upon the angle of lag or lead of the armature current and results in a strengthening of the magnetic field when a leading current is delivered. In fact, if the angle of lead be sufficient, the increase in the generated volts is such that, taking into consideration the phase of the volts absorbed by the armature impedance, the terminal voltage on load is actually higher than the no-load E.M.F. Fig. 228 shows a typical series of load characteristics obtained at different power factors, and clearly shows the effect of lagging and leading currents.

When the alternator is a polyphase machine, care must be taken to obtain a balanced load, both as regards power factor and current.

When performing the test at any particular power factor, it will be found convenient to insert a power factor indicator, as otherwise great difficulty will be experienced in maintaining it constant. An alternative method is to take a series of readings with a constant current but varying power factors, obtained by altering the relative amounts of resistance and reactance in the circuit. A number of such curves must be taken, each one corresponding to a different current (see Fig. 229). To obtain the load characteristic for a particular power factor, draw a vertical line through the value chosen and read off the voltages corresponding to the various currents from the curve. In this way, one point is

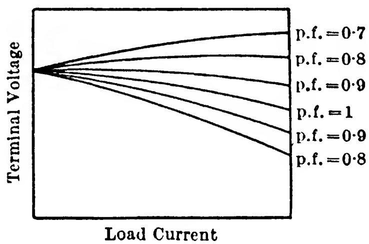


FIG. 228.—
Load Characteristics.

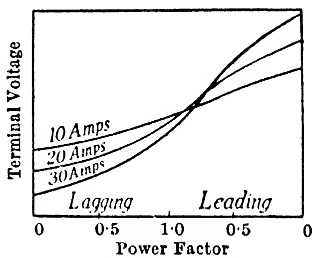


FIG. 229.—Effect of Power Factor on Terminal Voltage.

obtained from each of the curves in Fig. 229, but this can be repeated for as many power factors as desired.

Regulation.—When an alternator is subjected to a varying load, the voltage at the terminals of the armature varies to a certain extent, and the amount of this variation determines the *regulation* of the machine. The numerical value of the regulation is defined as the percentage rise in voltage when full load is switched off, the excitation being adjusted initially to give normal voltage. This gives a lower figure than the percentage fall in voltage when full load is switched on, and is favoured, therefore, by the manufacturers. The reason for this difference can be seen by referring to Fig. 227. Assume the machine to be on open-circuit and giving a voltage represented by the point *A* on the no-load curve. On switching on full load at unity power factor the voltage drops to that represented by the point *B* and the drop in voltage is given by the vertical distance between *A* and *B*. Now let the excitation be increased until the original voltage is reached, full load being maintained all the time. In this way point *C* is reached. On throwing off the load, the voltage rises to a value given by point *D*, and the vertical distance between *C* and *D* measures the rise in voltage. Owing to the greater saturation due to the larger exciting current, this rise is less than the fall obtained when the load is switched on. If this be repeated with a load having a power factor of less than unity, the current being a lagging one, the percentage fall or rise of the voltage would be correspondingly greater.

Close regulation is not asked for in modern alternators, as the protection of the machine is considered more important than its inherent regulation. It is now usual to design an alternator with a considerable amount of internal reactance, as this limits the short-circuit current, a most important point when dealing with modern high power units. The voltage is then maintained approximately

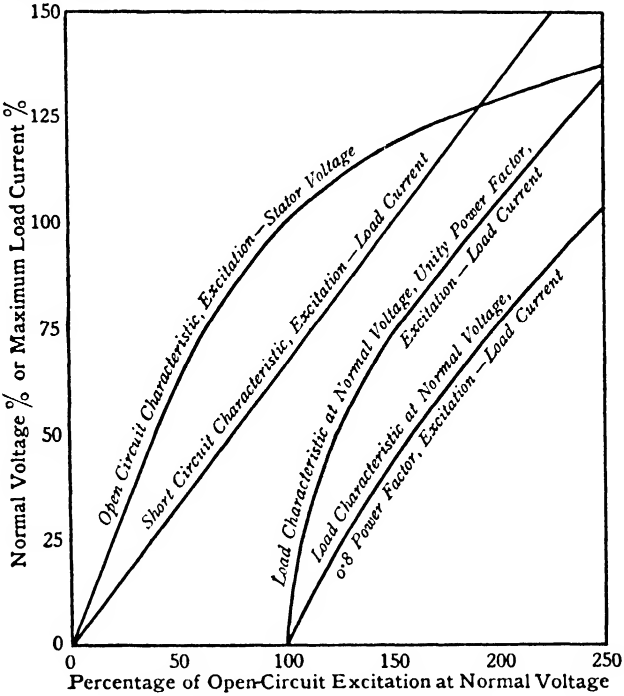


FIG. 230.—Turbo-Alternator Characteristics.

constant by the aid of automatic voltage regulators, of which the Tirrill regulator is an example (see page 275).

If it is not possible or desirable to measure the regulation by direct experiment, it can be obtained indirectly by other means which do not necessitate the alternator being fully loaded. For this purpose it is necessary to perform two tests, viz. the open-circuit test to obtain the magnetization curve (already described) and a short-circuit test to obtain the impedance of the alternator armature. The latter test will now be considered.

Short-Circuit Test.—In this test the armature is directly short-circuited through an ammeter, whilst additional resistances must be inserted in the field circuit to limit the exciting current so as not to burn out the armature. The relation between the armature current on short-circuit and the exciting current is then obtained over as large a range as possible, the speed being kept constant. As will be explained later small variations of the speed do not affect the armature current to any appreciable extent, so that there is no necessity to have close speed regulation. The resulting curve should be a straight line, as shown in Fig. 230. Under short-circuit conditions the armature current is practically 90° out of phase with the voltage, and the armature M.M.F. has a demagnetizing action on the field (see Figs. 235 and 236). The resultant ampere-turns inducing the armature E.M.F. are therefore the numerical difference between the field and the armature ampere-turns. This induced E.M.F. is equal to $A'B'$ in Fig. 238, and is also equal to IZ where I is the armature current and Z the impedance of the armature winding.

If the resistance of the winding be measured by means of a direct current, the reactance can be calculated and the impedance triangle determined.

In the majority of cases the resistance is small compared with the reactance, so that if the speed varies, the frequency and the reactance ($= 2\pi f L$) vary accordingly. Thus the impedance is practically proportional to the speed, and since the induced voltage is also proportional to the speed, the short-circuit current remains practically constant over a wide range.

As an example, consider an armature having a resistance of 0·1 ohm and a reactance of 0·8 apparent ohm at a frequency of 50. The impedance is $Z = \sqrt{0{\cdot}1^2 + 0{\cdot}8^2} = 0{\cdot}806$ apparent ohm, and if the induced voltage is 100 the short-circuit current is $\frac{100}{0{\cdot}806} = 124$ amperes. On halving the speed, the frequency drops to 25 and the impedance becomes $Z_1 = \sqrt{0{\cdot}1^2 + 0{\cdot}4^2} = 0{\cdot}412$ apparent ohm. The voltage is now 50 and the short-circuit current becomes $\frac{50}{0{\cdot}412} = 121$ amperes, a drop of only 3 per cent. for a speed reduction of 50 per cent. Fig. 231 shows a typical example of the variation of the short-circuit current with the speed.

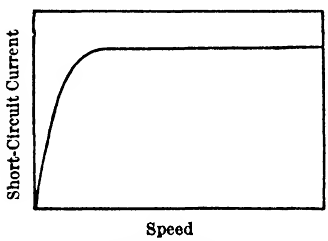


FIG. 231.—Effect of Speed on Short-Circuit Current.

Polyphase alternators behave in the same way as single-phase ones in this respect, all the terminals being short-circuited through the necessary ammeters.

The ratio of the field current required for normal volts on open-circuit to that required to circulate full load current on short-circuit is called the *short-circuit ratio*. Its value usually lies between 0·6 and 1·4. The steady short-circuit current with normal open-circuit excitation therefore also lies between 0·6 and 1·4 times full load current.

A low short-circuit ratio is obtained with a short air-gap length and results in poor regulation. Such a machine requires a relatively large change in field excitation for a given change of load.

Armature Reaction.—When the armature is supplying current there are two distinct sources of M.M.F. acting upon it, viz. that due to the main field system and that due to the armature current itself. The former is approximately constant, but the latter is not so in a single-phase case, since it alternates with time and rotates in space, assuming a rotating armature. Fig. 232 represents successive instants of time in a single-phase alternator having one conductor per pole, the current being supposed to be in phase with the E.M.F. Assuming a sinusoidal space distribution of the main flux and also that the curve of armature M.M.F. follows a sine law with respect to time, the maximum induced E.M.F. occurs when the conductors are passing the centre line of the poles. The armature current is also a maximum at this instant, and the curve of armature M.M.F. now has its maximum height. Points above the line are taken to represent M.M.F.'s aiding the main N pole. Successive instants are shown at intervals of 30° electrical, the armature M.M.F. dying down and reversing synchronously with the current. Now consider a point situated on the centre line of the N pole. There is the constant main M.M.F. acting, but in addition to this there is an alternating M.M.F. due to the armature alternately weakening and strengthening it. The average flux is unaffected, but it pulsates from instant to instant, now less than and now greater than its mean value. Next consider a point situated at the right-hand edge of the N pole (the trailing pole tip). This is subjected to similar fluctuations of M.M.F., but the strengthening is much more than the weakening. The opposite is the case for a point situated at the leading pole tip of the N pole where there is more weakening than strengthening. The same thing occurs under the S pole, with the result that there is no resultant strengthening or weakening, but

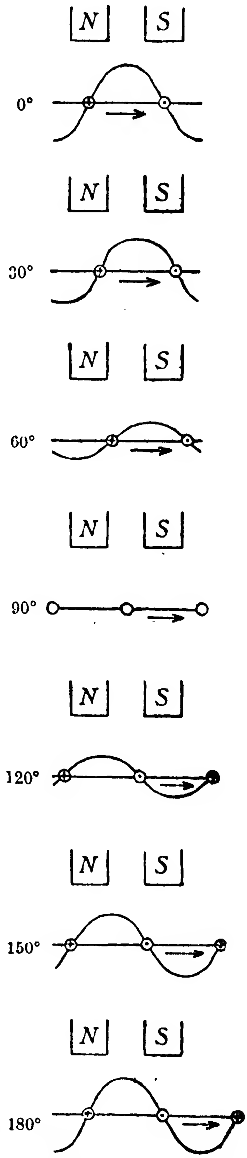


FIG. 232.—Armature Reaction Current in Phase.

only distortion. What one side of a pole gains the other side loses, so that it corresponds to a pulsating swing of the flux from side to side of the poles.

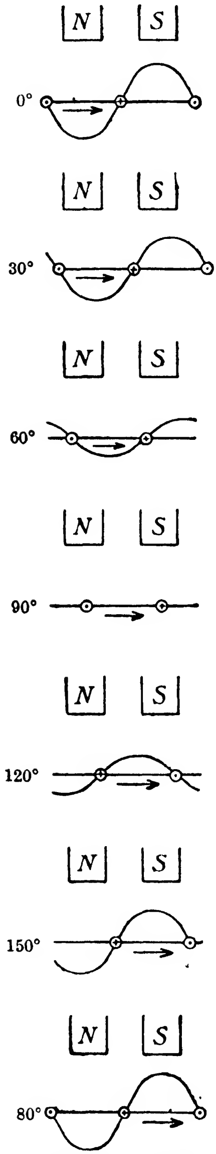


FIG. 233.—Armature Reaction. Current lagging 90°.

Next consider the same case when the current lags behind the induced E.M.F. by 90°. These conditions are represented in Fig. 233 and can be traced out in the same way as in the previous example. Taking a point lying under the centre line of a pole, it is found that there is always a weakening effect, although variable in magnitude. The trailing pole tip is subjected to an action which is mainly weakening, although a slight strengthening occurs about 120° from the start. The leading pole tip is also subjected to a general weakening effect with a slight strengthening about 60° from the start. The net result is a perpetual weakening with a slight periodical swaying of the resultant flux from side to side.

When the current lags by an angle less than 90° there is both weakening and distortion, the weakening getting greater and the distortion less as the angle of lag increases.

When a leading current is considered, the current arrives at its maximum value before the conductor gets to the centre line of the pole, and the weakening effect is turned into a strengthening one, again accompanied by distortion, the strengthening increasing and the distortion decreasing as the angle of lead is increased.

It is thus seen that when an alternator is delivering a lagging current the magnetic flux is decreased in magnitude so that the induced voltage is actually less than on no-load, whilst the phase relationship of the current causes an increased loss of the voltage which is generated, thus accounting for the increased rate at which the voltage falls away when the machine is supplying an inductive load.

Conversely, when an alternator is supplying a leading current the flux is increased, thus causing an increase in the voltage generated, whilst the phase relationship existing between the current and voltage reduces the drop in voltage and may even make it a rise if the current leads by a sufficient angle. The load characteristic of the alternator, therefore, rises with increase of load under these conditions.

The effects of armature reaction can be further studied by reference to Fig. 234. At (*a*) are shown the no-load conditions, the ampere-turns of the field winding acting along the centre line of the pole. The conductor which happens to be in this particular

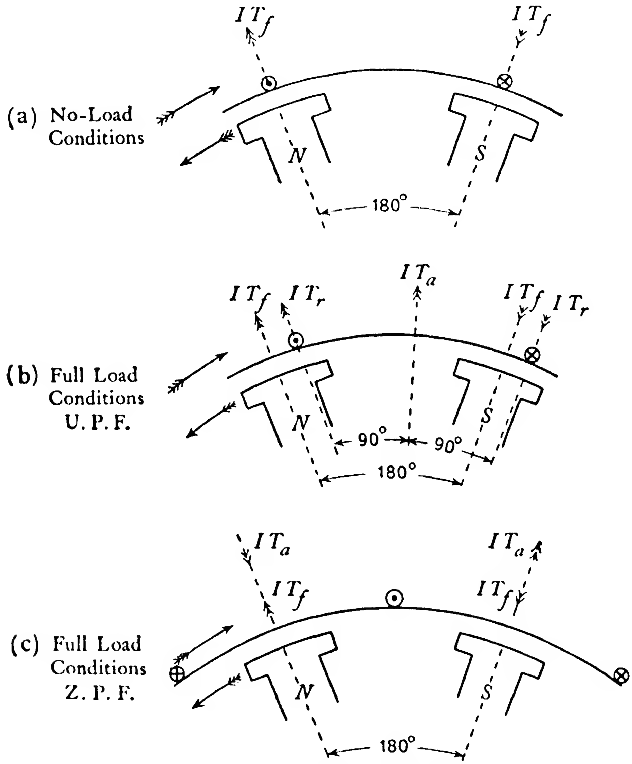


FIG. 234.—Distribution of M.M.F.'s.

position has its maximum E.M.F. induced in it at this instant. Since the armature current is zero at no-load, the armature exerts no reaction in these circumstances. When the armature delivers a load current, the flux distribution becomes modified after the manner indicated at (*b*), which represents the conditions for a unity power factor load. The resultant flux is, on the whole, displaced along the air-gap, towards the trailing pole tip, as can be seen from a detailed examination of Fig. 232. Let the magnetic axis of the resultant M.M.F. now lie along the line IT_r. The armature conductors which are having maximum E.M.F. induced in them are also lying on this line. The magnetic axis of the turn which is inducing its maximum E.M.F. is therefore along the line IT_a. Other turns to right and to left of this one will have magnetic axes to the right and the left of the line IT_a, but conditions of symmetry show that the line IT_a represents the centre line of the whole armature M.M.F. as well as the centre line of the single turn indicated in the diagram. The centre line of the field M.M.F. lies along the centre line of the pole as before. It is now seen that the centre line of the resultant M.M.F. lies between the centre lines of the field and the armature M.M.F.'s (considered in the same direction), as is to be expected. The axis of the resultant M.M.F. is nearer to that of the field M.M.F. than to that of the armature M.M.F., because in this instance the field ampere-turns are greater than the armature ampere-turns.

The conditions for an armature current lagging by 90° are shown at (*c*). There is now no distortion of the flux, but only weakening. The position of the armature conductors when inducing their maximum E.M.F. is along the centre line of the resultant M.M.F., which is now also the same as the centre line of the field M.M.F., since there is no distortion. Maximum current does not occur, however, until the moving system has rotated through a further 90°, or half a pole pitch. Conductors in this position are indicated in Fig. 234 (*c*), from which it is seen that the centre line of the armature M.M.F. lies along the same line as the centre line of the field M.M.F., but *the two are in opposition*. The armature reaction is now a pure weakening effect, there being no lateral distortion in either direction.

For currents lagging by less than 90°, there is a combination of the lateral distortion and the direct weakening, whilst for leading armature currents, the distortion is in the other direction, and the weakening effect is converted into a strengthening one.

Armature Reaction in Three-Phase Alternator.—In the case of a polyphase alternator with a stationary armature, the armature currents set up a rotating M.M.F. (see Chapter XI) which, if the load be a balanced one, is constant in magnitude and rotates synchronously, assuming sine waves. As shown on page 141, the actual wave of M.M.F. is alternately more peaked and more flat-topped than a true sine wave. This rotating M.M.F. thus remains fixed with respect to that due to the main field system. If the armature be rotating, the M.M.F. due to it rotates at synchronous speed with respect to it, but in the reverse direction, so that it is stationary in space, and again reacts in a constant manner on the main field, which is now stationary. In either case distortion is produced when the power factor is unity, this being combined with a weakening effect when the current is lagging and a strengthening effect when the current is leading.

The resultant M.M.F. is due to the combination of the field M.M.F. and the armature M.M.F., the latter in its turn being the

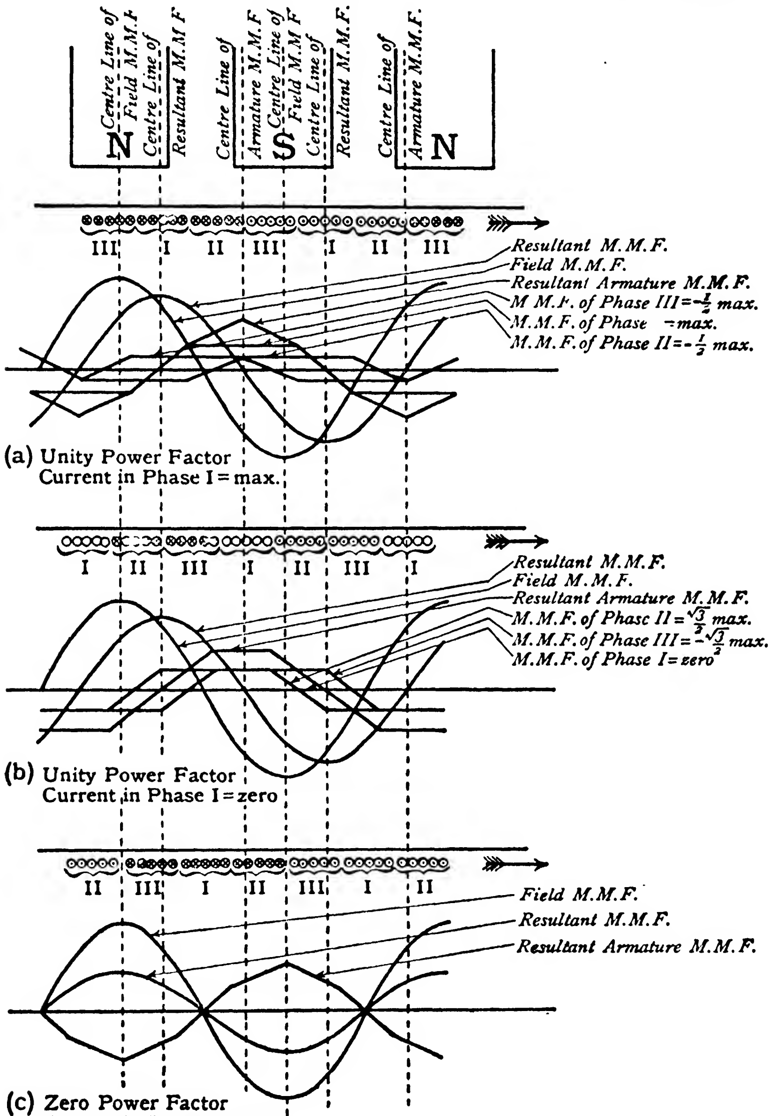


FIG. 235.—Armature Reaction in Three-Phase Alternator.

resultant effect due to the three phases. All these components have a phase displacement in space with respect to one another, *i.e.* the centre lines of their respective ampere-turns occur at different points in the air-gap. The centre line of the resultant M.M.F. differs from that of the field M.M.F. when the armature is delivering current, so that the position of the armature for maximum E.M.F. is altered. This changes the time at which the maximum E.M.F. occurs, or, in other words, alters its phase.

Fig. 235 (*a*) represents the conditions (in a rotating armature alternator) when the armature winding is delivering current to an external circuit at unity power factor. The centre line of the M.M.F. due to the field winding is taken as being the centre line of the pole, but the centre line of the resultant M.M.F. occurs further to the right. The diagram is drawn for the instant when the current in Phase I is a maximum. The M.M.F. space waves are drawn for all three phases separately, together with their resultant. The positions of these component M.M.F. waves are determined by the positions of the corresponding conductors, the centre of the Phase I conductors coinciding with the centre line of the resultant M.M.F. The resultant space wave of M.M.F. due to the armature is obtained by adding together the waves due to the three phases separately. The instantaneous values of the currents in Phases II and III are each equal to minus half their maximum value. (This diagram should be compared with Fig. 113.) The resultant armature M.M.F. wave is slightly more peaked than a true sine wave.

Next consider the conditions when the armature has moved half a pole pitch to the right [see Fig. 235 (*b*)]. The current in Phase I is now zero, the currents in Phases II and III being $\frac{+\sqrt{3}}{2}$ and $\frac{-\sqrt{3}}{2}$ times their maximum value respectively. The resultant armature M.M.F. wave is now slightly more flat-topped than a sine wave, but *its centre line is the same as before.* The armature M.M.F. has a constant angle of displacement with respect to the field M.M.F. at all instants during the cycle. The mean wave form of the armature M.M.F. is assumed to be a sine wave, and when combined with the M.M.F. due to the field winding, gives rise to a resultant wave of M.M.F. which is exactly 90° out of space phase with the armature M.M.F.

Fig. 235 (*c*) represents the conditions when the current in Phase I is a maximum, the power factor of the external circuit being zero. The current in Phase I now reaches a maximum 90° after the instant when the centre line of the Phase I conductors is passing the centre line of the resultant M.M.F. wave. The resultant armature M.M.F. is now wholly demagnetizing, there being no distortion as in the previous case. The resultant M.M.F. is the numerical difference of the field M.M.F. and the armature M.M.F. There is, consequently, no displacement in phase, and the E.M.F. when delivering a current at zero power factor is in phase with the E.M.F. on no-load. This is not the case when the power factor has other values; in such cases there is an alteration in the phase of the induced E.M.F.

The effect can be further studied by means of Fig. 236. At (*a*) the armature M.M.F. is represented as acting at 90° (half a pole pitch) to the resultant M.M.F., the latter being the vector sum of the M.M.F.'s due to field and armature. On no-load the phase of

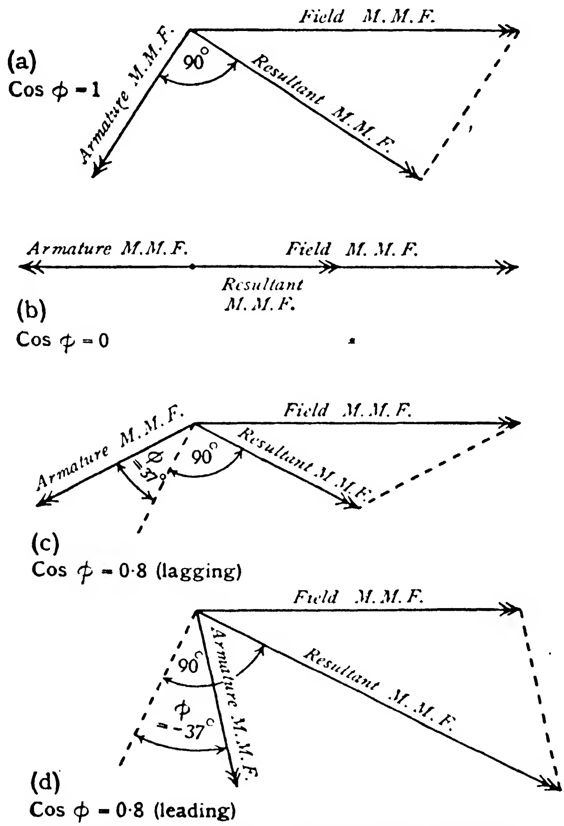


FIG. 236.—M.M.F. Diagrams for Three-Phase Alternator.

the induced E.M.F. is determined by the phase of the field M.M.F., since this now coincides with the resultant M.M.F. When delivering load, the phase of the induced E.M.F. clearly lags behind its phase on no-load. At (*b*) the armature is delivering a current at zero power factor (lagging). The armature M.M.F. now acts 180° instead of 90° behind the resultant M.M.F. There is obviously no phase displacement, and no distortion, but only weakening of the flux. At (*c*) the conditions are represented for a load power factor of 0·8 (lagging), and at (*d*) for a load power factor of 0·8 (leading).

At (*c*) there is both phase displacement (and flux distortion) together with a reduction in the resultant M.M.F., whilst at (*d*) there is a phase displacement (and flux distortion) together with an increase in the resultant M.M.F.

The diagrams in Fig. 236 should be compared with those in Fig. 234. At (*a*) in Fig. 236, the field M.M.F. is shown leading the resultant M.M.F. by a certain angle, and the resultant M.M.F. is shown leading the armature M.M.F. by 90°. At (*b*) in Fig. 234, the same sequence is observed.

Again at (*b*) in Fig. 236 the conditions are seen to be the same as at (*c*) in Fig. 234.

Lagging currents thus cause a diminution in the induced E.M.F., whilst leading currents bring about an increase. Lagging currents tend to demagnetize, and leading currents to magnetize, the alternator.

Armature Reactance.—When a current flows through the armature winding, local fluxes are set up at various places. Flux set up in this manner, linking with the armature winding but not with the field winding, is called *leakage flux*, and is the cause of the armature reactance. Broadly, it may be divided into (*a*) slot leakage and (*b*) end connection leakage. In the latter case, the leakage flux has a distinct independent existence, but in the case of slot leakage the flux has no separate physical existence, being merged in the main flux, which becomes, in consequence, distorted locally.

Armature reactance results in a voltage drop in the winding, in the same way as does its resistance, this voltage drop usually being of the order of 10–15 per cent.

Synchronous Impedance.—The method of determining the impedance of the armature described on page 280 assumes that the voltage generated when short-circuited is the same as when on open-circuit, and this assumption is not justified, for in addition to the armature possessing resistance and reactance there is a true armature reaction as well. Armature reaction is due to the magnetizing action of the armature upon the main field and is an M.M.F. effect. Armature reactance is due to an E.M.F. being induced in the armature conductors, on account of the current carried by it. The first effect, by weakening the field, reduces the voltage actually generated; the second uses up some of the generated volts before the terminals are reached. Really, the main induced E.M.F. and the E.M.F. of self-induction do not combine as they do not exist separately, but the M.M.F.'s which produce the respective fluxes exist and produce a resultant flux. It is convenient, however, to regard the two E.M.F.'s as separate from a mathematical standpoint.

When the current in the armature is a lagging one the effect of armature reaction is to weaken the main field besides distorting it, whilst when the armature current leads the weakening action becomes a strengthening one. Again, when the armature current lags armature reactance sets up an E.M.F. which is in partial opposition to the induced E.M.F., causing a drop in voltage, whilst when the armature current leads the reactance introduces an E.M.F. having a component which increases the induced E.M.F. From this point of view the armature reaction and reactance produce similar effects and may be combined in what is called the *synchronous reactance.* This is not a true reactance, but can be considered as such for a variety of purposes. When combined with the resistance in the usual way, a quantity called the *synchronous impedance* is obtained, which thus takes into consideration the armature reaction and the armature reactance in addition to its resistance.

The synchronous reactance is usually not constant, but pulsates periodically on account of the varying reluctance of the armature magnetic circuit as the rotor changes position. This reluctance also varies between maximum and minimum limits synchronously.

Predetermination of Regulation by Behn-Eschenburg's E.M.F. Method.—In this method the open-circuit magnetization curve and the short-circuit characteristic are necessary, so that the synchronous impedance may be obtained as explained above. The impedance triangle is then determined and the regulation can be obtained by drawing a vector diagram as shown in Fig. 237. In the first case the full load current is assumed, the power factor of the external circuit being unity. By multiplying each side of the impedance triangle by the current, a voltage triangle is obtained representing the voltages absorbed by the armature resistance and synchronous reactance. Taking the phase of the current as a datum line, this voltage loss triangle can be erected in position. The total generated voltage consists of the terminal voltage together with the impedance voltage added at its proper phase angle, so that the tip of the induced voltage vector OE lies along a horizontal line drawn through V_z. The magnitude of OE is obtained from the open-circuit magnetization curve, and the vector is drawn by taking O as centre and with OE as radius, striking an arc so as to cut the horizontal line through V_z. A line VE is then drawn equal and parallel to OV_z, when OV represents the terminal voltage.

When a lagging current is dealt with [see Fig. 237 (*b*)], the phase of OV is taken as the datum line and OI is drawn lagging behind it at the correct angle ϕ and the same construction repeated, remembering that the voltage overcoming the resistance of the armature is in phase with the current.

Fig 237 (*c*) shows the vector diagram for a leading current.

These vector diagrams can be repeated for other values of the load current by making the size of the impedance voltage triangle proportional to the current.

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In this way, points on the load characteristic can be calculated for all values of the load current and for any power factor.

In practice this method gives values for the voltage drop which are too high and has been named, in consequence, the *pessimistic method*.

Armature Ampere-turns.—The armature ampere-turns are approximately equal to the field ampere-turns necessary to force full load current through the armature on short circuit. This neglects that portion of the field ampere-turns which actually sets up the flux required to generate the small E.M.F. necessary to

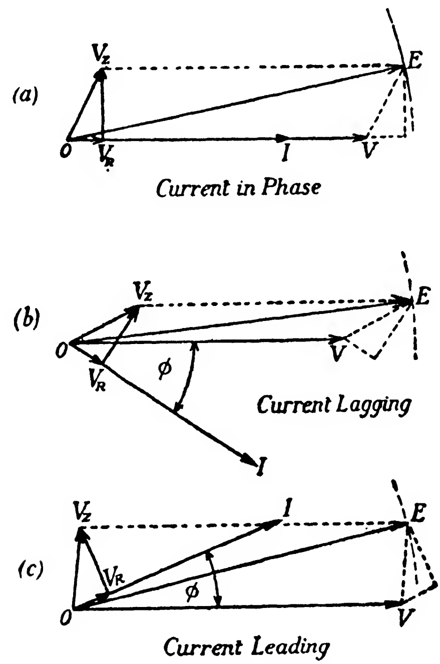


FIG. 237.—Behn-Eschenburg's E.M.F. Diagram.

produce the short-circuit current. The true value of the armature ampere-turns is less than is given by this method.

It was shown on page 141 that for a uniform air-gap machine, the resultant maximum effect of three phases varies from twice to $\sqrt{3}$ times the maximum effect due to any one phase acting by itself. If there be N total conductors on the armature, each carrying a current of I amperes, then the R.M.S. ampere-turns per phase may be taken as $\frac{IN}{6}$, the maximum value being $\frac{\sqrt{2}}{6}IN$. The resultant maximum ampere-turns of the whole armature winding, therefore, vary between $\frac{2\sqrt{2}}{6}IN = 0{\cdot}471IN$ and $\frac{\sqrt{3}\times\sqrt{2}}{6}IN = 0{\cdot}408IN$. The average of these two values is $0{\cdot}44IN$, so that the resultant maximum ampere-turns per pole are $\frac{0{\cdot}44IN}{p}$. This neglects the effect of distribution, but where cylindrical type rotors are employed this is largely neutralized by the corresponding distribution of the field winding.

When an alternator is short-circuited, the armature ampere-turns are wholly demagnetizing, and for a salient pole machine the total demagnetizing effect is less than for a machine with a cylindrical rotor. The constant 0·44 is now modified to a certain extent, the maximum armature ampere-turns per pole becoming approximately $\frac{0{\cdot}35\,IN}{p}$. The value of the constant depends to a certain extent upon the ratio of pole arc to pole pitch.

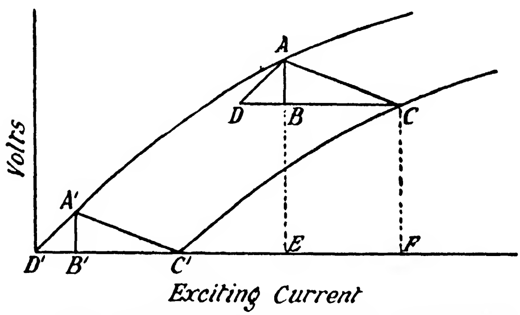


FIG. 238.—Separation of Armature Reaction and Reactance.

Separation of Armature Reaction and Reactance.—The method of calculating the regulation outlined above introduces errors because the armature reaction and reactance are treated in the same way. If these two effects, the true armature reaction (an M.M.F. effect) and the leakage reactance (an E.M.F. effect) be separated, then a very close approximation to the truth is obtained. This can be done by the aid of the magnetization curves on open-circuit and on full load at zero power factor. It is seen from Fig. 237 that if the current be made to lag by 90°, then the difference between the E.M.F. on open-circuit and on load is practically equal to the voltage drop due to the reactance of the winding. The small IR drop is at right angles to the terminal voltage and is negligible. In other words, to obtain the terminal volts, the IX drop (X is here the true leakage reactance and not the synchronous reactance) is subtracted arithmetically from the open-circuit voltage.

Considering the point A (see Fig. 238) on the open-circuit curve, the terminal voltage is AE and the exciting current $D'E$, the alternator being on no-load. On the application of full load current at zero power factor, there is a drop in terminal volts, AB, equal to IX, but in addition to this, the armature reaction is now acting, and this necessitates an increase in the exciting current in order to maintain the same effective flux. This increase in the exciting current is represented by EF, so that the point C on the full load curve corresponds to the point A on the open-circuit curve. The increase, EF, when multiplied by the total number of field turns, is equal to the total armature ampere-turns as calculated above. For the same armature current, however, the triangle ABC is of constant shape and size, so that the lower curve may be said to be a repetition of the upper one, only dropped by a constant distance AB and moved to the right by a constant distance BC. It follows, therefore, that the triangle $A'B'C'$ is the same as ABC and, moreover, since the curves are always practically straight lines near the origin, it may also be said that the two triangles $D'A'C'$ and DAC are equal. In order to determine a point such as A on the open-circuit curve corresponding to the point C on the full load curve, therefore, a horizontal line DC is drawn equal to the known length $D'C'$, and then from the point D, a line DA is drawn parallel to $D'A'$. This line cuts the open-circuit curve at A, after which the lengths AB and BC can be readily determined. In order to obtain a reasonable degree of accuracy it is desirable that the line DA should be inclined to the open-circuit characteristic as much as possible.

On dividing the voltage represented by AB by the corresponding armature current, the leakage reactance of the armature is obtained, whilst the exciting current EF may be considered as that required to overcome the effects of armature reaction at this current. This exciting current can, if so desired, be converted into ampere-turns.

Zero Power Factor Characteristic.—In actual practice it is difficult to obtain the complete zero power factor characteristic on experimental grounds. Large oil-immersed reactors, built like transformers but without any secondaries, are employed for this purpose. If one point be determined, together with the short-circuit point, then the graph may be obtained by means of the following construction.

When short-circuited, the current lags by an angle approximating to 90°, due to the large ratio of X to R, so that for this purpose the point C' (Fig. 238) may be considered as being at zero power factor. Let C be the one determined point on the zero power factor characteristic. The points A' and A are at present unknown, but by trial and error two parallel lines $A'C'$ and AC can be obtained equal in length. There is only one slope at which this occurs. Having found this slope other points may be determined by drawing further lines from the open-circuit curve, equal in length to AC and inclined at the same angle, until the whole curve is constructed.

Potier's Regulation Diagram.—The complete regulation diagram can now be drawn by a method originally suggested by Potier, combining both the E.M.F. and M.M.F. diagrams (see Fig. 239). The terminal voltage is represented by OV and the load current by OI lagging by an angle ϕ. To OV is added the resistance drop IR, parallel to OI, and the leakage reactance drop IX, at right angles to this. The vector sum of these voltages, OE, is the induced E.M.F. This is proportional to minus the rate of change of the effective flux and therefore lags behind it by 90°. The flux (and also the exciting current) may thus be represented by a vector 90° ahead of the induced E.M.F. The magnitude of the exciting current, I_v, is obtained from the open-circuit magnetization curve, and is that necessary to induce the voltage OE. To this must be added the vector I_a, drawn parallel to OI, this representing the exciting current, which when multiplied by the field turns per pole balances the

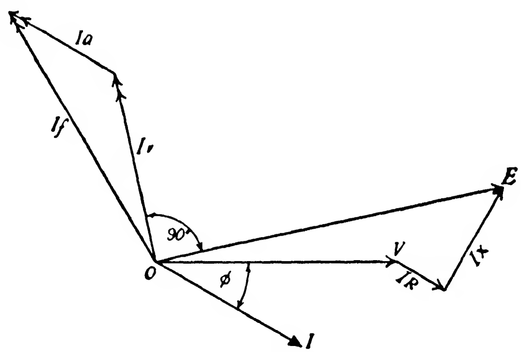


FIG. 239.—Potier's Regulation Diagram.

armature reaction ampere-turns per pole. It is in direct phase opposition to OI since it is required to neutralize it. But since I_a is drawn as a current, and not as ampere-turns, it can be obtained directly from Fig. 238, where it is represented by the length BC or EF. Alternatively it can be calculated from $0{\cdot}44IN/p$ as shown on page 290. The total field current required is given by I_f, the vector sum of I_v and I_a. The induced E.M.F. corresponding to this field current can now be determined from the open-circuit characteristic, from which the regulation can be calculated immediately.

The diagram can be repeated for any power factor and also for any load current by suitably altering the value of ϕ and by adjusting the lengths of all vectors proportional to the load current.

It is important to note that the right hand (E.M.F.) portion of this diagram is a time diagram; whereas the left hand (M.M.F.) portion is a space diagram.

The actual construction of the diagram may be conveniently carried out in the following manner. The open-circuit and zero power factor characteristics are required, these being plotted to percentage scales as shown in Fig. 240. By means of the construction shown in Fig. 238, the armature reactance and armature reaction are separated. The former is expressed as a percentage of normal line volts and the latter in terms of alternator exciting current. These are represented by IX and AT_a respectively. In the right hand portion of the diagram, OV is then drawn vertically to represent normal volts, OI being full load current at any specified power factor. The IR drop is then added to OV at the correct phase angle, this drop being $\sqrt{3}$ times the IR drop per phase, expressed as a percentage. The IX drop is next added, this being obtained from the first portion of the diagram. The induced E.M.F.,

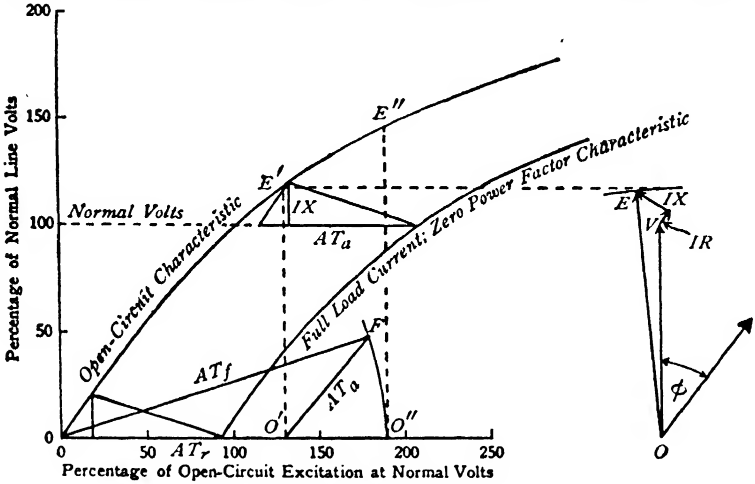


FIG. 240.—Construction for Regulation.

OE, is thus obtained, the value being marked on the open-circuit characteristic at $O'E'$. The resultant M.M.F. required to induce this voltage is given by OO' in terms of exciting current. To this must be added the exciting current necessary to overcome the effects of armature reaction, viz. AT_a, this vector being drawn parallel to OI. The total field current, OF, is thus obtained, and when the load is thrown off, this induces an E.M.F., $O''E''$. The regulation can now be calculated from the rise in volts above normal.

Cyclical Variation in Exciting Current.—It has been shown that the flux linked with each pole fluctuates slightly from instant to instant, and this induces an alternating E.M.F. in the exciting coil which produces a small alternating current superimposed upon the main direct exciting current. The frequency of this current is double that of the armature current, as can be seen from Fig. 232, for the flux of any particular pole goes through its whole cycle of events whilst the armature coils are advancing through one pole pitch, during which time the armature current goes through half a cycle. If a third harmonic be present in the main current wave, a fourth harmonic is produced in the field circuit. In general, only even harmonics are generated in the field windings, and their magnitude depends upon the shape of the armature current wave form and upon the angle of lag. The presence of these fluctuations can easily be shown by allowing an alternator to come to rest with its load still connected. Just before it reaches standstill the frequency is very low, and the field ammeter will indicate the beats in the exciting current. Under actual working conditions, their presence can be detected by means of an oscillograph or by a Joubert's contact, the instantaneous voltage drop over a known resistance placed in series with the field being measured.

Synchronizing.—The operation of paralleling two alternators is known as *synchronizing*, and certain conditions must be fulfilled before this can be effected. The incoming machine must have its voltage and frequency adjusted to that of the bus bars, and, in

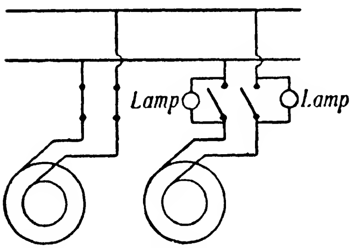


FIG. 241.—Synchronizing with Lamps dark.

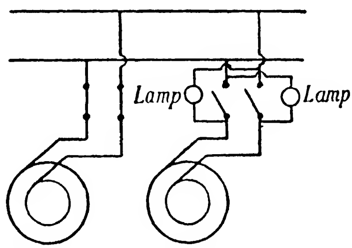


FIG. 242.—Synchronizing with Lamps bright.

addition, the phase of the two voltages must be the same for correct synchronizing. The instruments or apparatus for determining when these conditions are fulfilled are called *synchroscopes*.

The simplest method of synchronizing is by means of two lamps connected across the ends of the double pole paralleling switch, as shown in Fig. 241. If the conditions for synchronizing are fulfilled there is no voltage across the lamps and the switch may be closed. It is very rarely that the lamps are completely black for any length of time, and the speed of the incoming machine must be adjusted as closely as possible so that the lamps light up and die down at a very slow rate. The alternator may then be switched in at the middle of the period of darkness, which must be judged by the speed at which the light is varying. When the incoming machine is 180° out of phase, there is double the voltage of one machine acting across the two lamps in series, so that each lamp must be capable of standing the full terminal voltage of one alternator.

An alternative system of connections is shown in Fig. 242, where the lamp connections are crossed. For synchronizing there must be no volts across the switch, and as there are full volts between the two poles of the switch, it follows that the lamps will be lit with their maximum brilliance at the correct moment for synchronizing. This method has an advantage over the previous one inasmuch as the lamps are much more sensitive to changes of voltage at their maximum candle power than when they are quite black, and a sharper definition of the exact point of synchronism is thus obtained. Both methods are, however, employed.

When the voltage of the alternator is such that lamps cannot be used directly, small voltage transformers may be installed, the synchronizing lamps being connected across the secondaries. The primaries may be connected straight across the switch or they may be cross-connected as with low tension alternators.

A pair of special synchronizing bus bars are often provided on switchboards where there are a number of alternators running in parallel. There is only one set of synchronizing gear, and this is connected between these auxiliary bus bars and the main bus bars. The incoming machine is plugged on to the synchronizing bus bars, one voltmeter indicating the main bus bar voltage, and a second that of the incoming machine. The one set of synchronizing gear can thus be used for any of the alternators as desired.

For polyphase machines it is only necessary to synchronize one phase, as all the other phases will then be in synchronism as well. When connecting up an alternator in the first place, however, it is necessary that it should be *phased out* correctly, *i.e.* the phases must be connected in their correct order of 1, 2, 3 and not 1, 3, 2. In the latter case it is, of course, impossible ever to synchronize the machines. Any two leads of the new alternator must then be interchanged to reverse the sequence.

A commonly adopted method of synchronizing three-phase alternators, introduced by Siemens and Halske, consists in having three lamps connected as shown in Fig. 243 (*a*). The order in which the lamps light up shows whether the incoming machine is running too fast or too slow. The lamps are not connected symmetrically, as they would then light up and die down simultaneously, but L_1 is connected between A and D, L_2 is connected between B and F (not B and E), and L_3 is connected between C and E (not C and F). Fig. 243 (*b*) shows the corresponding vector diagram. If the frequencies of the two machines are slightly different, the two stars will have a relative movement with respect to each other. Assuming that the incoming machine DEF is rotating a little fast, the star DEF will have a slow rotation in a counter-clockwise direction with respect to the star ABC. The voltage AD on lamp L_1 will then be increasing from zero, the voltage CE on lamp L_3 will be increasing and near its maximum, and the voltage BF on lamp L_2 will be decreasing, having passed through its maximum. The lamps will then light up one after the other in the order 2, 3, 1, 2, 3, 1, etc. Next, suppose the machine DEF is running a trifle slow.

The star DEF in the vector diagram will now rotate slowly in a clockwise direction with respect to the star ABC. The voltage AD on lamp L_1 will be decreasing, having passed through its maximum some time previously. The voltage CE on lamp L_3 will be decreasing, having just passed through its maximum, whilst the voltage BF on lamp L_2 will be increasing up to its maximum value. The lamps will therefore light up in the order 1, 3, 2, 1, 3, 2, etc., which is the reverse order to that in the previous case. It is common practice to mount the three lamps at the angles of a triangle, and the apparent direction of rotation of the light indicates whether the machine is running too fast or too slow. The actual synchronizing is done when the lamp L_1 is in the middle of its dark stage.

Synchroscopes.—Synchronizing by means of lamps is not very exact, as a considerable amount of judgment is called for in the operator, and in large machines even a small angle of phase displacement causes a certain amount of shock to the machines. On this account a number of more complicated *synchroscopes* have been devised, of which two types will be described.

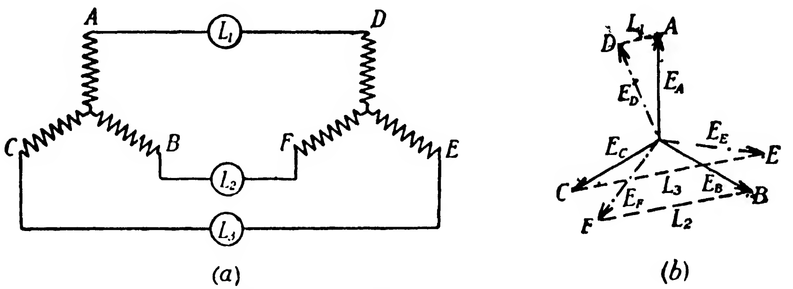


FIG. 243.—Siemens and Halske Synchronizer.

If an ordinary dynamometer wattmeter has both its fixed and moving windings made of fine wire and each is supplied with an alternating voltage, the deflection will depend upon the phase difference of these two voltages. When they are in quadrature the deflection will be zero, and if one advances in phase with respect to the other it will cause a continued deflection of the pointer until they are in phase, after which the pointer will slowly retrace its path. It will thus appear to oscillate from side to side, assuming a central zero. If a condenser is placed in series with one coil, the deflection will be zero when the two voltages are in phase, and this is what is required, and to compensate for the resistance in the condenser circuit a little reactance is placed in the other to establish exact quadrature. The switch is closed when the pointer is stationary and on the zero. In order to indicate whether the incoming machine is running too fast or too slow, an ordinary synchronizing lamp is arranged to illuminate the dial of the instrument. This lamp is bright when the pointer is swinging one way and dark on the return swing, which is consequently not seen. When viewed from a little distance the pointer has the appearance of rotating in a constant direction. In this way "too fast" is indicated by an apparent rotation in one direction and "too slow" by an apparent rotation in the other, since the lamp now lights up the opposite swing of the pointer.

A type of rotary synchroscope often employed is very similar in construction to the power factor indicator shown in Fig. 143. In reality it consists of a small motor, the field of which is provided by the bus bar volts, whilst the rotor currents are supplied by the incoming machine. The rotor is wound with two coils at right angles, the currents in which differ in phase by approximately 90°. This is obtained by connecting a resistance in series with one and a reactance in series with the other, as shown in Fig. 244. Only three slip rings are required, since a common return is used for

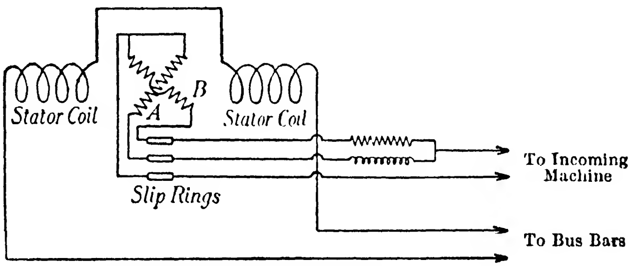


FIG. 244.—Rotary Synchroscope.

both coils. A pointer is attached to the rotor and serves to indicate the correct time for synchronizing. The E.M.F.'s of the bus bars and of the incoming machine will now be in phase, so that the currents in the stator and in coil *A* on the rotor will be in phase since they both lag by 90° behind their respective E.M.F.'s Coil *A* will set itself so that the path of the resultant flux is as short as possible. There will be no torque acting on coil *B*, since its current is in quadrature with the stator current. The pointer now rests on the zero of the scale. If the two alternators are running at the same speed but with a constant phase difference, the stator current will produce a torque in both the rotor coils, so that the rotor will take up a new position of equilibrium, the deflection of the pointer indicating the difference in phase of the bus bar volts and those of the incoming machine. If one machine is gaining on the other, the angle of phase difference is continually increasing, and this results in a continually increasing deflection. In other words, the pointer rotates with a speed proportional to the difference in the two frequencies. If the fast machine is now made to go slower than its companion the direction of rotation is reversed, and the correct time for synchronizing is when the pointer is stationary and vertical.

The actual instrument is made with four poles, and a double-ended pointer is used, either end being considered, since a gain of a complete cycle is now indicated by half a revolution.

For use in engine rooms where the synchroscope is viewed from a distance, a signalling arrangement is provided whereby a red or a green light is shown, depending upon whether the speed is too high or too low. This is obtained by a toothed disc on the rotor spindle, which engages with one of two pawls according to the direction of rotation. These operate a vertical arm which falls over to one side or the other and thereby interposes a red or a green glass in front of the lamp.

A diagram showing how the alternators and synchroscope are connected to the synchronizing bus bars is shown in Fig. 245.

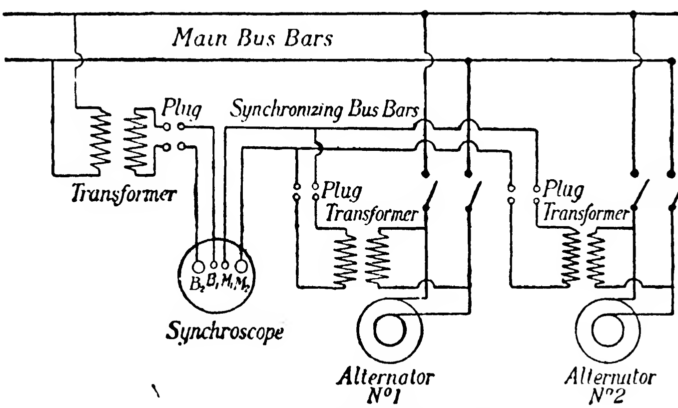


FIG. 245.—Synchroscope Connections.

Earthed Neutral.—Three-phase star-connected generators are frequently earthed at their star points in order to prevent the volts to earth rising to any undue value. With a balanced load and sinusoidal wave forms, this earth circuit will not carry any current, but this is not true with an unbalanced load. It was shown on page 127 that the third harmonic, and all the other harmonics which are a multiple of 3, neutralize one another in a three-phase star winding because they are exactly in phase opposition in two adjacent phases. But considering the local circuit formed by joining together a generator and a load star point, it is seen that a third harmonic current can flow. The ninth and fifteenth harmonics act in a similar way, but in practice the triple frequency currents are the only ones which need be considered. To limit this current, reactances may be inserted in the earth lead as shown in Fig. 246. (See also "Petersen Coil," page 557.) These choking coils provide a triple impedance to the third harmonic currents, the value of which automatically drops to one-third when dealing with an ordinary out-of-balance current.

If the earthing is done by means of resistances, it is the usual practice to earth one generator only.

Efficiency Test.—The most convenient method of testing an alternator for efficiency is by means of a motor the efficiency of which is known for all the required loads. The alternator can then be loaded under various conditions and the output and input measured, the latter being equal to the output of the motor. The efficiency can thus be obtained directly.

Measurement of Losses.—If it is desired to measure the various losses separately, a motor having a capacity of about one-tenth that of the alternator will be found to give the best results. The alternator can then be run unexcited at its correct speed and the watts output of the motor determined from its input. This gives the friction and windage loss. The alternator is now normally excited and the increase in the watts output gives the normal iron loss at no-load. In the same way, the iron loss at different excitations can be obtained.

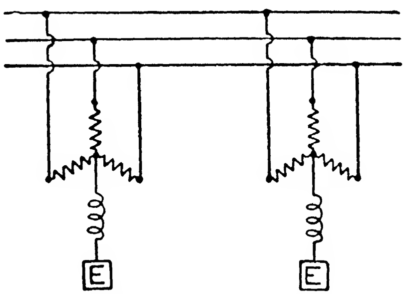


FIG. 246.—Earthed Neutrals.

In order to determine the armature copper losses, it is not sufficient to measure the resistance on D.C., as, owing to the skin effect and eddy currents, this gives values which are too low. A better way is to short-circuit the armature and excite the fields sufficiently to cause full load current to flow through the armature. The friction and the iron loss for this excitation must be determined by the auxiliary motor as described above, and then the increase in the power to drive the alternator when the armature is short circuited.

To measure the excitation loss, all that is necessary is to know the exciting voltage and exciting current, these being measured on a D.C. circuit.

The efficiency for any output can now be calculated by adding all the losses to the output to get the input. The power factor of the load should be specified, as this affects the armature copper loss. Strictly speaking, the other losses are also slightly affected, but not to such an extent as to make it worth while allowing for it.

Hopkinson Test.—This method of determining the efficiency is convenient when two similar alternators are available for the purpose. One acts as a generator and drives the other as a motor, the balance of the power being supplied mechanically by means of a third machine the efficiency of which at various loads must be known. In order to make a circulating current flow between the two alternators, their excitations are made different, and, in addition, the two alternators are rigidly coupled, so that they are out of phase to a certain extent. An angle of phase displacement of about 25° gives good results.

The output of the alternator can be measured by wattmeters in the usual way, and the total losses supplied by the third machine can be divided equally between the two test machines, and in this way the individual efficiency of each alternator can be calculated.

Retardation Test.—The retardation method of testing, which is particularly applicable in the case of flywheel type alternators, consists in measuring the rate at which an alternator slows down under different conditions when the driving power is removed. The rate at which the speed is decreasing is a measure of the rate at which kinetic energy is given out, this being used up to overcome the losses at that particular instant. The instantaneous power (in watts) given out by the rotating system when slowing down is

$0{\cdot}011 \times 10^{-7} \times Mn \times$ instantaneous rate of decrease of speed in (r.p.m.) per second,

where n is the speed in revolutions per minute and M is the moment of inertia in C.G.S. units.[1] The test, therefore, demands a knowledge of the moment of inertia of the rotor and the various instantaneous values of the speed and the slope of the speed-time curve. In order to carry out the test, the alternator is run up to speed, preferably by means of a belt drive. The belt is then thrown off and the alternator allowed to come to rest with nothing but friction and windage to cause it to slow down. If the alternator is motor-driven through a direct coupling, the power is switched off the motor and the combined set allowed to come to rest. The losses occasioned by the motor are then calculated by carrying out a similar test on the motor alone and deducted from the total to get the alternator losses.

[1] The kinetic energy E at any angular velocity ω is $\frac{1}{2}M\omega^2$. Then

$$P = \frac{dE}{dt} = \frac{d(\frac{1}{2}M\omega^2)}{dt} = M\omega\frac{d\omega}{dt} = M\frac{2\pi n}{60} \times \frac{2\pi}{60}\frac{dn}{dt}$$

$$= \frac{\pi^2}{900}Mn\frac{dn}{dt} = 0{\cdot}011\, Mn\frac{dn}{dt}\text{ C.G.S. units}$$

$$= 0{\cdot}011 \times 10^{-7} Mn\frac{dn}{dt}\text{ watts.}$$

Whilst the alternator is coming to rest, speed measurements are taken from instant to instant, and these are plotted as shown in Fig. 247. The test is then repeated with the alternator fields excited, the machines now coming to rest in a shorter time since there is the added drag due to the iron losses.

At any particular speed, the slope of the speed-time curve can be determined by drawing a tangent to the curve at that speed and measuring the drop in speed corresponding to one second, *i.e.* $\frac{AB}{BC}$ to the correct scales (Fig. 247). It now remains to determine the moment of inertia. This can be done by ordinary mechanical methods, but this is a cumbersome task and can be avoided in the following way. The total power to drive the alternator light at one particular speed is measured by noting the input, and the moment of inertia is determined from the equation

$P = 0{\cdot}011 \times 10^{-7} \times Mn \times$ instantaneous rate of decrease of speed,

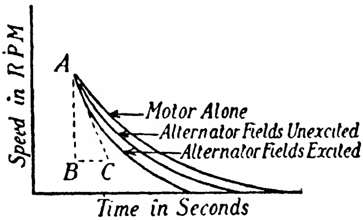


FIG. 247.—Retardation Curves.

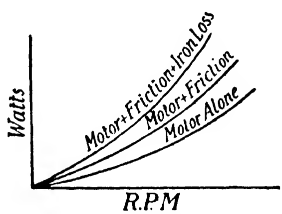


FIG. 248.—Separation of Losses by Retardation Test.

P being the watts required to drive the set at n revolutions per minute, the slope being obtained from the retardation curve. This equation can now be re-written

$$P = kn \times \text{instantaneous rate of decrease of speed,}$$

where the constant k can be evaluated as shown.

The losses can now be calculated at various speeds, and curves can be drawn, as shown in Fig. 248, from which the friction and the iron loss at any speed can be determined separately by means of the vertical distance between the various curves.

Heating Tests.—Before an alternator is put into actual operation it is desirable to test its temperature rise when working under full load. Since the temperature rise does not attain a maximum value until many hours have elapsed, this test involves a considerable expenditure of energy which, apart from the cost, it is difficult to dissipate. The Hopkinson test avoids the latter difficulty, but it is seldom that two similar alternators are available for test. It is therefore desirable to set up artificial conditions which will imitate the real heating effects, and one method of doing this is by reversing half the poles on the rotor, which method will now be described briefly.

Reversed Poles Test.—The field windings are divided into two halves, one of which is reversed so that the E.M.F.'s induced in the corresponding halves of the armature winding are in opposition. On short-circuiting the armature, therefore, no current flows since the E.M.F.'s are balanced. But by increasing the excitation in one half of the field winding or by decreasing the excitation in the other half the equilibrium of the armature E.M.F.'s is destroyed and a current flows the magnitude of which can be regulated by adjusting the excitations. Full load current can be obtained in this way without the expenditure of the full amount of power, since one half of the machine is acting as a generator whilst the other half is acting as a motor. It is, in fact, a kind of Hopkinson test on the two halves of a single machine. The grave disadvantage of this test is the large unbalanced mechanical stresses which are set up and which cause considerable vibration. This can be mitigated to a certain extent by reversing alternate pairs of poles instead of reversing one half completely. Thus in a 12-pole alternator poles Nos. 3, 4, 7, 8, 11 and 12 would be reversed. It is desirable that the alternator should have an even number of pairs of poles, as otherwise there would be one pair left over and these two poles would have to be cut out.

Examples.

(1) What is the meaning of the term "synchronous reactance" as applied to an alternator? How can the armature reaction be separated from the leakage reactance in an alternator?

(2) Draw a typical open-circuit magnetization curve for a turbo-alternator, and explain how the zero power factor characteristic can be obtained, if the only loading apparatus available consists of a large oil-immersed iron-cored choking coil of fixed inductance. Show how the effects of armature reaction and armature reactance can be separated, using the two graphs already mentioned. In what units are armature reaction and armature reactance measured in practice?

(3) A single-phase alternator has a resistance voltage drop of 3 per cent. and a leakage reactance voltage drop of 20 per cent. on full load. The armature reaction ampere-turns are 40 per cent. of the resultant field ampere-turns. Determine the regulation at full load current and (*a*) unity power factor and (*b*) a power factor of 0·8 lagging.

Magnetization curve :—

Volts	4000	5000	6000	7000	8000	8500
Amperes	23	32	48	72	102	124

Normal voltage = 6600 volts.

(4) A 1000 kVA, 11,000 volt, three-phase star-connected alternator has an effective resistance of 2 ohms per phase. The characteristics on open circuit and with full load current at zero power factor are :—

Field Current, amps. ...	40	50	110	140	180
O.C. Terminal volts ...	5,800	7,000	12,500	13,750	15,000
Terminal volts at *f.l.c.* and *z.p.f.*	0	1,500	8,500	10,500	12,400

Determine the percentage regulation for full load current at a lagging power factor of 0·8.

(5) Write an account of the uses of the open-circuit and short-circuit characteristics of an alternator.

(6) Explain why armature reaction exerts different distorting and weakening effects at different power factors.

(7) Describe fully one method of determining the efficiency of a large low speed alternator.

CHAPTER XIX

ALTERNATORS.—PRINCIPLES OF DESIGN

Speed and Number of Poles.—When designing an alternator the output, voltage and frequency must be specified, the speed depending to a large extent upon the output and the type of prime mover adopted. In general, the larger the output the slower the speed, since the diameter goes up and the peripheral velocity must be kept within limits. For turbo-alternators, the available speeds are very limited owing to the fact that either 2 or 4 poles are invariably employed. In any case, only certain definite speeds are available, as they must fulfil the equation

$$\text{number of poles} = \frac{120f}{\text{r.p.m.}}.$$

The table on page 246 shows these speeds for various numbers of poles and frequencies.

Output Coefficient.—The E.M.F. is given by

$$E = 4k_1k_2k_3f\Phi T \times 10^{-8}$$

and putting $k_1 = 1{\cdot}11$, $k_2 = 0{\cdot}96$, and $k_3 = 1{\cdot}00$,

$$E = 4{\cdot}26f\Phi T \times 10^{-8}.$$

This voltage in a three-phase case is that between line and neutral when star-connected.

The flux, Φ, is equal to $B_g \times \psi\tau \times L$, where B_g is the average flux density in the air-gap, τ is the pole pitch, ψ the ratio of pole arc to pole pitch, and L is the gross length of the core.

The total magnetic loading is, therefore, $p\Phi = pB_g\psi\tau L$.

Similarly the total electric loading is $m \times 2T \times I$ where m is the number of phases. The whole current, I, is here assumed to flow through each conductor. The ampere-conductors per cm. of periphery are $q = \dfrac{2mIT}{\pi D}$, where D is the air-gap diameter.

The output in watts $= mEI$

$$= m \times 4{\cdot}26 \times \frac{pn}{120} \times B_g\psi\tau L \times T \times 10^{-8} \times \frac{\pi q D}{2mT}$$

$$= 0{\cdot}0558\, npB_g\psi\tau qDL \times 10^{-8}.$$

Moreover, the pole pitch τ is equal to $\frac{\pi D}{p}$, so that

$$mEI = 0{\cdot}175\, nB_g\psi qD^2L \times 10^{-8}.$$

The output is thus proportional to the speed, n, the ratio of pole arc to pole pitch, ψ, the magnetic flux density in the air-gap, B_g, and the specific electric loading, q. Values for all these quantities are made as large as is economically practicable, the limiting figures depending upon the properties of the materials used. In any commercial design, therefore, the above equation can be written

$$\text{kVA} = knD^2L$$

where k is a constant known as the *output coefficient* and is equal to $0{\cdot}175\ B_g\psi q \times 10^{-11}$. This represents $\frac{4}{\pi}$ times the output per unit volume at one revolution per minute, and practice has shown that this constant varies with the size of the machine.

Average values of the output coefficient for polyphase alternators are given in the following table, but it must be emphasized that rigid adherence to these figures is not essential.

Output Coefficients and Ampere-conductors per cm.

$\frac{\text{kVA}}{\text{r.p.m.}}$	0·2	0·5	1·0	2·0	5·0	10·0	20·0	40·0
Output coefficient $\times 10^6$ $= k \times 10^6$	2·2	2·5	2·8	3·0	3·4	3·7	4·0	4·5
Ampere-conductors per cm. $= q$	275	310	340	370	420	460	500	550

Specific Electric Loading.—The ampere-conductors per cm. of periphery, or specific electric loading ($= q$), fixes the armature reaction, and largely determines the temperature rise of the winding. A high value for q results in a high leakage reactance and a large armature reaction, with a high value for the regulation. If close voltage regulation is desired, a low value for q must be adopted. When used in conjunction with some form of automatic voltage regulator, close regulation is not desirable, since parallel operation is not then so satisfactory. As a consequence the regulation is usually about 20–25 per cent. on unity power factor.

The above table gives average values for q for 50 cycle alternators up to about 6600 volts. For higher voltages, slightly lower figures should be adopted on account of the additional slot space occupied by insulation. For lower frequencies, the value of q may be slightly increased.

The high values for q are largely due to the increased depth permissible in the stator slots. This is offset to some extent by the choice of a somewhat lower current density on account of the great cross sectional area of the conductors in large sizes.

Air-gap Diameter.—When the value of D^2L is known there are a number of various values of D and L which can be inserted. For example, a 2000 kVA 50-cycle polyphase alternator running at 500 r.p.m. has an output coefficient $3{\cdot}12 \times 10^{-6}$ (cm. measure). The value of D^2L is, therefore,

$$D^2L = \frac{2000}{500} \times \frac{10^6}{3{\cdot}12}$$
$$= 1{\cdot}28 \times 10^6.$$

An upper limit for D is reached when the peripheral speed exceeds, say, 70 metres per second, and a lower limit is reached when

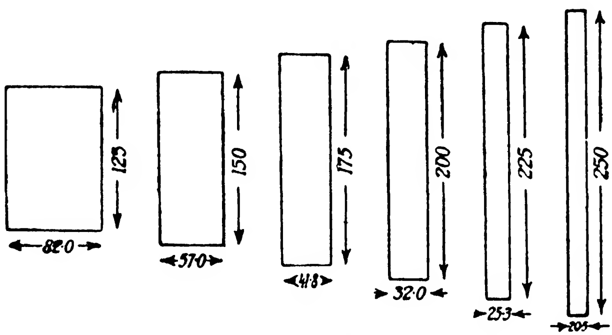


FIG. 249.—Shapes of Rotor.

the poles become too crowded together. The maximum permissible diameter therefore becomes $D = \dfrac{70 \times 60 \times 100}{\pi \times 500} = 268$ cms.

The following values of D and L are possible :—

D	125	150	175	200	225	250
L	82·0	57·0	41·8	32·0	25·3	20·5

The diagrams in Fig. 249 show the shapes of the section of the rotating element in the different cases.

Flux and Flux Density.—The total flux is directly proportional to the average density in the air-gap, and hence to the tooth density. High voltage machines require more slot insulation, thus limiting the size of the tooth, and this necessitates a reduction in the air-gap flux density. Too high a flux density in the air-gap leads to excessive densities in the iron and to high stator tooth losses. Too low a flux density, on the other hand, leads to an uneconomical use of the material. The flux densities are, in general, lower than in D.C. machines, because the latter are usually designed to operate at much lower than 50 cycles per second.

The permissible flux density increases with the output. An increased output is obtained by an increase in both flux and ampere-conductors, so that the flux does not increase so rapidly as the output. Assuming a constant efficiency, the iron losses are proportional to the output. With a constant flux density, however, the iron losses would not increase in proportion to the output, so that the maximum permissible flux density is increased.

The total flux is proportional to DL, so that for a given value of D^2L, the total flux is increased when D is reduced, since $DL = \frac{D^2L}{D}$. Turbo-alternators with a long core length have, therefore, a relatively heavy flux, and this is counterbalanced to some extent by reducing the maximum flux density.

The following table gives typical values of the maximum air-gap flux density.

Output in kVA.	Air-gap Flux Densities in Lines per sq. cm. (maximum value.)	
	Low and Medium Speed Alternators.	Turbo-Alternators.
0–500	5500–7000	5500–6500
500–1000	7000–8000	6500–7500
1000–2000	8000–8500	7500–8000
2000 and upwards	8500–9000	8000–8500

The above figures are for 50-cycle machines and may be increased by 10–15 per cent. for $f = 25$. Again, in the case of high voltage alternators, the densities should be reduced by about 5–10 per cent.

The diameter can be settled by assigning a value to the ratio $\frac{\tau}{L} = \frac{\text{pole pitch}}{\text{core length}}$. For economy of copper there is a best value for this ratio, and the value may be taken from the curve shown in Fig. 250.

Let $\frac{\tau}{L} = a$ (from the curve). Then $\frac{\pi D}{ap} = L$, $D^2L = \frac{\pi D^3}{ap}$, and $D = \sqrt[3]{\frac{apD^2L}{\pi}}$.

If D is increased, the flux is decreased, since L decreases more rapidly than D increases. The total electric loading is simultaneously increased, and the armature reactance and reaction are increased. This results in a relatively lower short-circuit current, with consequent added protection for the machine. Too small a value for D is not therefore desirable.

Air-gap Diameter of Turbo-Alternators.—Considerations of peripheral speed now determine the air-gap diameter in all except small machines. Peripheral speeds of 100–140 metres per second are adopted, but these figures necessitate a very strong mechanical construction. With 4-pole, 50-cycle machines the diameter is therefore limited to 125 cms. for 100 metres per second, and 175 cms. for 140 metres per second, the latter only being employed on very large sets. For 2-pole, 50-cycle turbo-alternators, the air-gap diameter is limited to 85 cms. in the largest sizes built, and to lesser values for smaller sets.

As a consequence of the limit to the air-gap diameter, the core length becomes very long in turbo-alternators of large output, thus necessitating special cooling arrangements.

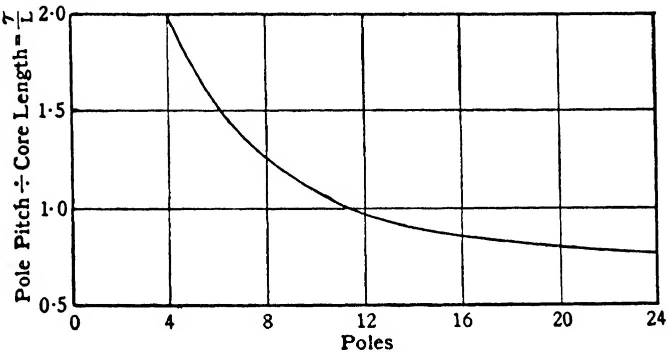


FIG. 250.—Pole Pitch and Core Length.

For 4-poles, the best value for $\frac{\tau}{L}$ is about 2, and adopting this value, $\frac{\pi D}{4L} = 2$, and D is approximately equal to $2{\cdot}5L$. The value of D^2L then becomes equal to $0{\cdot}4D^3$, and $D = 1{\cdot}36 \times \sqrt[3]{D^2L}$. If this value of D causes a peripheral speed higher than the maximum permissible, then D is taken at the maximum permissible value.

The output at which the maximum diameter is reached can be determined as follows :—

$$D^2L = 0{\cdot}4 \times 175^3 = 2{\cdot}14 \times 10^6.$$

$$\text{Output} = 3{\cdot}6 \times 10^{-6} \times 2{\cdot}14 \times 10^6 \times 1500 = 11{,}500 \text{ kVA}.$$

The output coefficient, $3{\cdot}6 \times 10^{-6}$, corresponds to the figure in the table on page 306 for $\frac{\text{kVA}}{\text{r.p.m.}} = \frac{11.500}{1500} = 7{\cdot}7$.

In the case of bipolar turbo-alternators, a suitable value for $\frac{\tau}{L} = 2{\cdot}7$, when $\frac{\pi D}{2L} = 2{\cdot}7$ and D is approximately $1{\cdot}72L$. The value of D^2L now becomes equal to $D^2 \times \frac{D}{1{\cdot}72} = 0{\cdot}58D^3$, and $D = 1{\cdot}2 \times \sqrt[3]{D^2L}$. From considerations of peripheral speed, however, D should not exceed 85 cms. for 50-cycle sets.

The output at which the maximum diameter is reached is determined as follows :—

$$D^2L = 0{\cdot}58 \times 85^3 = 0{\cdot}355 \times 10^6.$$

$$\text{Output} = 2{\cdot}8 \times 10^{-6} \times 0{\cdot}355 \times 10^6 \times 3000 = 3000 \text{ kVA}.$$

The output coefficient, $2{\cdot}8 \times 10^{-6}$, corresponds to the figure in the table on page 306 for $\frac{\text{kVA}}{\text{r.p.m.}} = \frac{3000}{3000} = 1$.

Air-gap Diameter of Hydroelectric Alternators.—There is a wide range of speed with these machines, and very often they are designed with a vertical shaft. The peripheral speed of a waterwheel on full load is much less than the velocity of the water, but on light load they are approximately equal. Suppose full load is suddenly thrown off and the governor does not act quickly enough. There may be a rise in speed of 50–100 per cent. Specifications commonly allow for 75 per cent. overspeed. The air-gap diameter is now limited by the maximum peripheral speed at the overspeed, with the result that the diameter is reduced and the core length increased in comparison with the values for alternators where this overspeed is not encountered. The core length is increased to such an extent in some cases that artificial ventilation in the middle becomes necessary.

Length of Stator Core.—The preliminary value for the length of the stator core is definitely fixed when once the values of the output coefficient and the air-gap diameter have been decided upon, since

$$L = \frac{\text{kVA}}{nkD^2} \text{ or } \frac{D^2L}{D^2}.$$

Ventilating Ducts.—The usual practice with respect to flywheel type alternators is to have a number of radial ventilating ducts in the stator about 1 to 1·5 cms. in width and spaced about 8 cms. apart. In the case of turbo-alternators an increased ventilation is necessary, since the overall dimensions are much smaller for a given output. This means that each cubic cm. of material must dissipate more heat, since the efficiency and the total losses are approximately the same in each case. The distance between the ventilating ducts is therefore reduced to about 5 or 6 cms.

Number of Slots.—These vary from three to five per pole per phase in the ordinary rotating field alternators. The larger the number of slots the nearer does the wave form approach the ideal sine wave, but the copper space factor of the slot goes down at the same time, particularly with high voltage machines. In the latter case, therefore, the tendency is to use three and four slots per pole per phase, whilst with low voltage machines four and five are frequently employed. With turbo-alternators, the pole pitch is usually considerably greater, and this leads to larger numbers such as five to eight slots per pole per phase. The total number of slots is then obtained by multiplying by the number of poles and the number of phases. Fractional numbers of slots per pole per phase are largely employed with diamond coil windings.

Armature Winding.—The number of armature conductors is settled by E.M.F. considerations, but this number must be divisible by the total number of slots. The induced voltage per winding is first calculated, this being equal to the line voltage in a single- and two-phase case and $\frac{1}{\sqrt{3}}$ times the line voltage in the case of a three-phase star-connected alternator. The total number of turns in series per phase is then given by the equation

$$
\begin{aligned}
E &= 4k_1k_2k_3\Phi fT \times 10^{-8} \text{ volts (see page 270)} \\
&= 4{\cdot}26\Phi fT \times 10^{-8} \text{ volts}
\end{aligned}
$$

as a first approximation. The approximate value of the useful flux per pole can be obtained from a knowledge of the dimensions of the pole shoe and by assuming a suitable flux density taken from the table on page 308. The number of turns required can then be calculated, and the nearest possible number is adopted, taking into consideration the number of slots. The flux is then adjusted to suit the number of conductors. The winding itself may be either of the concentric or diamond coil type. The total number of conductors decreases as the kVA increases, and with very large outputs indeed it is a common practice to employ one conductor per slot with several windings in parallel. In such cases, the resultant E.M.F.'s of the various parallel circuits must have no phase difference.

Size of Armature Conductors.—The section of the conductor to be employed is obtained from a knowledge of what is a suitable current density to adopt. This varies with the total current carried by the conductor, and values are shown in the curve in Fig. 251. Solid wire of circular cross section should be used up to sections of about 0·25 cm.^{2}, but for larger sizes the conductor becomes difficult to bend and two or more conductors are connected in parallel. For the larger sizes of conductor, rectangular strip is often employed, as this brings about a higher copper space factor in the slot.

On account of the eddy current loss in large bars, the conductors are twisted or transposed. In the former case there are two series-connected sections, each being the image of the other. In transposed conductors, each parallel element occupies every position in the slot in turn.

At the bottom of the slot where the magnetic leakage is negligible, thick bars are employed, bars of reduced thickness being used nearer the mouth of the slot. Bars of equal thickness are then grouped and twisted, each group being connected to a solid end-connector. For very heavy currents, several end-connectors are connected in parallel.

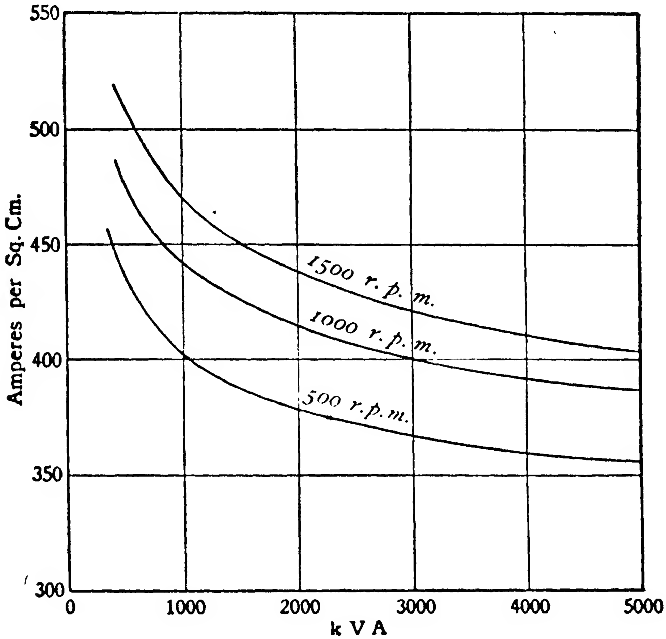


FIG. 251.—Approximate Current Densities.

Size of Slots.—When the number of conductors per slot and the cross-sectional area of the conductors are settled, the space that they will occupy, when covered with the necessary insulation, can be drawn out and the slot dimensions fixed. Since the width of slot plus tooth is fixed, the dimensions of the slot determine the dimensions of the tooth, and this should not be too narrow. A suitable maximum magnetic density in the teeth is 19,000 lines per sq. cm.

The slot width depends upon the slot pitch. The tooth width may be made equal to the slot width for a slot pitch of 2 cms., and equal to twice the slot width for a slot pitch of 8 cms. The maximum slot width is approximately 2·5 cms.

Sub-slots are also used on occasion for purposes of ventilation.

Armature Losses.—In order to predetermine the heating of the armature, it is necessary to obtain an estimate of the losses incurred by it. These will consist of copper and iron losses.

The copper loss can be predetermined from a knowledge of the number and dimensions of the conductors. The mean length of a turn must be estimated, and the number of turns in series per phase being known, the total length of conductor per phase can be obtained. Then, knowing the cross-sectional area, the resistance

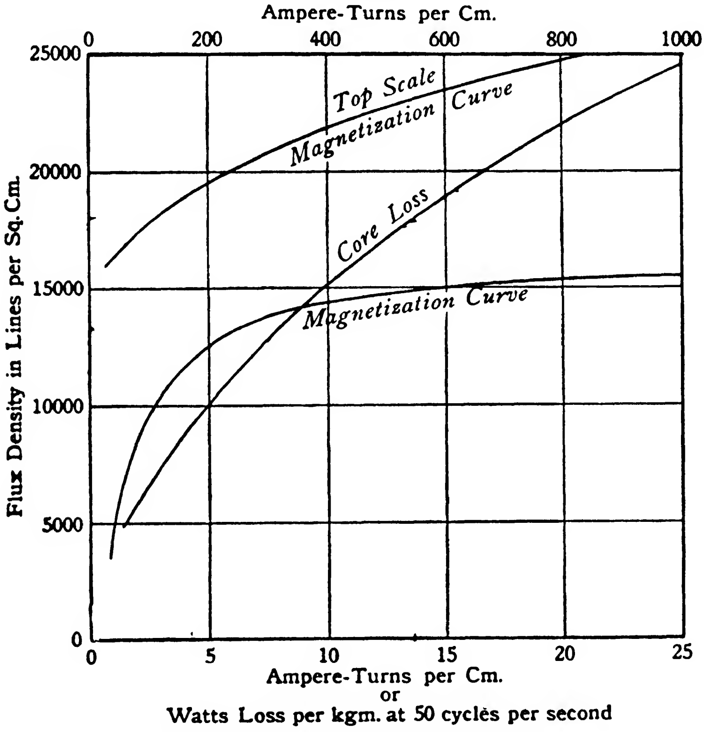


FIG. 252.—Core Losses and Magnetization Curve.

per phase can be calculated, taking care to allow for the rise of temperature. At a temperature of about 50° C. the specific resistance of copper is approximately 2 microhms per cm. cube. From the current per phase and the number of phases the total watts lost due to ohmic resistance can be calculated. The figures obtained in this way will be frequently too low, as the current is not always uniformly distributed over the conductors, due to eddy currents, and this always results in an increase in the watts lost in a conductor.

The increase may be as high as 60 per cent., an average value being about 30 per cent.

The iron loss, comprising the hysteresis and eddy current losses, is dependent upon the flux density in the core, and may be estimated approximately from Fig. 252, which refers to sheet steel for armature stampings of 0·35 mm. thickness.

Cooling Surface.—In determining the cooling surface of the armature, the inner and outer cylindrical surfaces of the stampings may be considered together with the two ends of the iron core and one side of each ventilating duct.

Estimated Temperature Rise.—The estimated temperature rise of the armature depends upon the watts which have to be dissipated per square cm. of the cooling surface. With the usual ventilating ducts, a temperature rise of approximately 40° C. will be obtained when the watts per square cm. are 0·12 to 0·15, the cooling surface being estimated in the above manner. Here also the temperature rise may be taken as being proportional to the watts per square cm. of cooling surface, but these preliminary calculations are by no means accurate and are subject to a number of disturbing conditions.

Flux per Pole.—The no-load useful flux per pole can be calculated from the E.M.F. formula, since the number of conductors is now settled. The form factor, k_1, the breadth factor, k_2, and the coil span factor, k_3, can also be estimated fairly accurately, and the useful flux per pole on no-load is given by

$$\Phi = \frac{E \times 10^8}{4k_1k_2k_3fT},$$

f being the frequency, T the turns in series per phase and E the no-load induced E.M.F. per phase. For the same terminal voltage the induced E.M.F. on full load must be greater owing to the loss of voltage occasioned by the impedance of the armature. The resistance may be calculated from a knowledge of the dimensions of the winding and an estimate may also be made for the reactance, but the calculations are rather involved. As a rough approximation, the latter may be taken as 10–15 per cent. of the no-load voltage. The full load useful flux per pole is thus obtained, to which must be added the waste leakage flux, which will ordinarily be of the order of 10 per cent. to 35 per cent. of the useful flux. Average values for the leakage factors are as follows :—

kVA. Output ...	2·5	5	10	25	50	100	200	300	500	1000	2000	10,000
Leakage Factor = λ	1·4	1·35	1·30	1·28	1·25	1·22	1·20	1·18	1·15	1·12	1·10	1·08

Air-gap.—The radial length of the air-gap in modern low speed alternators is fairly constant for a given size of machine, increasing as the bore of the stator goes up. Fig. 253 shows the approximate values of the air-gap length in the case of salient pole alternators.

Considerably longer air-gaps are employed in the case of turbo-alternators. The factor which determines to a great extent the length of the air-gap in these cases is the regulation which it is desired to obtain. In order to limit the value of the short-circuit current a comparatively large amount of synchronous reactance is deliberately provided for in the design of the machine. From this it follows that the armature must have a very considerable number of ampere-turns, and this in its turn necessitates very strong field coils. The ampere-turns of the latter are chiefly used up in overcoming the reluctance of the air-gap, the length of which is adjusted to suit individual conditions. The above considerations sometimes lead to the use of air-gaps of 5 cms. or more in turbo-alternators.

Other advantages of a long air-gap are, that it tends to

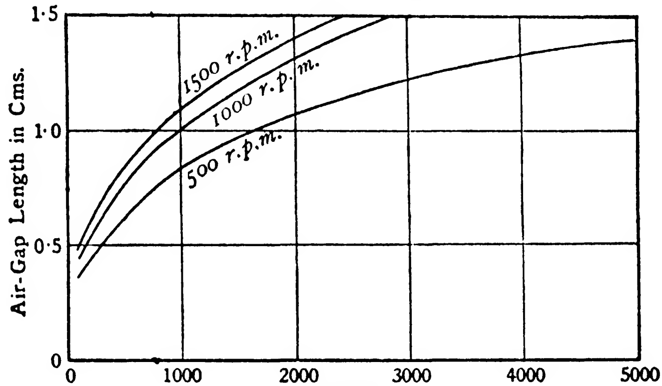


FIG. 253.—Approximate Air-gap Lengths.

keep down the iron loss in the teeth, it assists ventilation and tends to obviate noisy running, and it keeps down the magnitude of the unbalanced magnetic pull should the rotor become decentralized.

Ampere-turns per Pole.—Before the ampere-turns per pole can be calculated, it is necessary to know the flux densities in the various parts of the magnetic circuit, and also the lengths of the lines of force in the different materials. The various magnetic sections are settled by choosing suitable flux densities, and for this purpose reference may be made to the following table :—

	Lines per Sq. Cm.
Armature cores	10,000—12,000
Armature teeth	15,000—17,000
Magnet poles	14,000—17,000
Magnet yokes	10,000—12,000
Rotor teeth (cyl. type) ...	17,000—20,000
Rotor core („ „) ...	10,000—12,000

The flux in the armature core and teeth is alternating and thus sets up an iron loss. At higher frequencies, therefore, a lower density is adopted, this accounting for the considerable range shown in the table, where the higher figures correspond to a frequency of 25 and the lower figures to a frequency of 50.

The magnetic sections being chosen and the various flux densities calculated, the corresponding values of H must be obtained from a B—H curve of that particular material. From the formula

$$H = \frac{4\pi}{10} \times \text{Ampere-turns per cm.}$$

the ampere-turns required for the various parts can be obtained. Adding all these ampere-turns together, the total no-load ampere-turns required per pole can be evaluated. It is usual to consider the poles and yoke as carrying all the field leakage flux in addition to the useful flux, whilst the armature only carries the useful flux.

The above calculation should now be repeated for full load at the minimum specified power factor.

Field Winding.—A tentative length of winding space and an approximate depth of the winding must first be fixed. The length of the mean turn can be estimated from the known dimensions. Then

$$\begin{aligned}\text{Volts per Coil} &= \text{Exciting Current} \times \text{Resistance of Coil (Hot)} \\ &= \text{Exciting Current} \times \text{Turns} \times \text{Resistance of Mean Turn (Hot)} \\ &= \text{Ampere-turns} \times \text{Resistance of Mean Turn (Hot)}\end{aligned}$$

and $$\text{Resistance of Mean Turn (Hot)} = \frac{\text{Volts per Coil}}{\text{Ampere-turns}}.$$

From this the resistance per metre can be obtained, and a workshop rule to get the cold resistance is to multiply by $\frac{6}{7}$. The nearest wire to this in the wire tables is chosen and its insulated diameter noted. Allowing 5 per cent. for loss of winding space due to imperfect winding, the number of turns per layer and the number of layers can be calculated. The total number of turns per pole and the resistance of the winding are next evaluated, giving the current taken by the coil. The ampere-turns thus obtained should agree substantially with the number aimed at in the first instance. For a temperature rise of about 40° C. the watts per square cm. of cooling surface should be about 0·15 to 0·25 for rotating fields. Further, the watts wasted in excitation should range from about 4 per cent. for small alternators to about 0·4 per cent. for very large alternators.

Estimated Efficiency.—All the losses having been estimated, the probable efficiency can now be evaluated. In order to compete with rival machines, similar efficiencies must be obtained, and the following table shows average values.

Full Load Efficiency at Unity Power Factor.

kVA.	100	500	1000	10,000	100,000
Efficiency per cent. ...	92	94	95	97	98

Estimated Open-Circuit Magnetization Curve.—The ampere-turns for one value of the voltage and flux have already been worked out, and the corresponding exciting current can also be evaluated, since the field turns per pole are known. This gives one point on the curve, and a number of other points can be obtained in a similar manner. A value of the voltage is chosen and the corresponding flux calculated. The ampere-turns required to drive this flux through the various parts of the magnetic circuit are worked out, from which the exciting current is evaluated as above.

Example of Design.—As an example of a design, the main dimensions will be worked out for a three-phase rotating field alternator having an output of 500 kVA, 2200 volts, 50 cycles per second, and 375 r.p.m.

The number of poles is $\dfrac{120 \times 50}{375} = 16.$

The alternator being star-connected, the volts per winding will be $\dfrac{2200}{\sqrt{3}} = 1270$ volts.

The full load current will be $\dfrac{500{,}000}{\sqrt{3} \times 2200} = 132$ amperes.

An output coefficient of $2{\cdot}85 \times 10^{-6}$ will be chosen, and the value of D^2L becomes

$$D^2L = \frac{\text{kVA}}{\text{r.p.m.}} \times \frac{1}{k}$$

$$= \frac{500}{375} \times \frac{10^6}{2{\cdot}75} = 470{,}000.$$

Choosing a ratio of $\dfrac{\text{pole pitch}}{\text{core length}} = 0{\cdot}85$ (see Fig. 250),

$$\frac{\pi D}{16L} = 0{\cdot}85, \text{ and } L = 0{\cdot}231D.$$

Therefore $$D^2L = 0{\cdot}231D^3,$$

and $$D = \sqrt[3]{\frac{470{,}000}{0{\cdot}231}} = 127 \text{ cms.}$$

$$L = \frac{470{,}000}{127^2} = 29{\cdot}3 \text{ cms.}$$

An air-gap diameter of 125 cms. and a gross core length of 31 cms. will be adopted.

There will be three ventilating ducts, each 1 cm. wide, leaving four batches of stampings each 7 cms. in width.

The pole arc is $\frac{\pi \times 125}{16} \times 0{\cdot}65 = 16$ cms.

With three slots per pole per phase the total slots number

$3 \times 16 \times 3 = 144$, having a slot pitch of $\frac{\pi \times 125}{144} = 2{\cdot}72$ cms.

Assuming $q = 350$ (see page 306), the total ampere-conductors are $\pi \times 125 \times 350 = 137{,}000$, so that the total number of conductors should be approximately $\frac{137{,}000}{132} = 1040$. With 144 slots, 8 conductors per slot will be chosen.

The no-load useful flux per pole is

$$\Phi = \frac{2200}{\sqrt{3}} \times \frac{10^8}{4{\cdot}26 \times 50 \times 48 \times 4} = 3{\cdot}1 \times 10^6,$$

and the approximate air-gap flux density is

$$\frac{3{\cdot}1 \times 10^6}{16 \times 31 \times 0{\cdot}9} = 7000.$$

To determine the size of conductor, reference is made to Fig. 251. A current density of 430 amperes per sq. cm. will be assumed, giving a conductor having an approximate cross-sectional area of $\frac{132}{430} = 0{\cdot}307$ sq. cm. Using strip 0·40 cm. wide and 0·75 cm. deep, and allowing 0·25 cm. per side for the slot lining, a slot width of 1·35 cms. is required with two conductors side by side. The active slot depth required is $2 \times 0{\cdot}25 + 4 \times 0{\cdot}75 = 3{\cdot}5$ cms. The slot dimensions chosen are 1·35 cms. wide and 4·0 cms. deep. The minimum thickness of tooth is $2{\cdot}72 - 1{\cdot}35 = 1{\cdot}37$ cms.

The average flux per tooth (no-load) is

$$\frac{3{\cdot}1 \times 10^6}{9} = 0{\cdot}344 \times 10^6 \text{ lines.}$$

The maximum flux per tooth (no-load) is

$$\frac{\pi}{2} \times 0{\cdot}344 \times 10^6 = 0{\cdot}54 \times 10^6 \text{ lines.}$$

The minimum cross-sectional area of a tooth is

$$1{\cdot}37 \times (31 - 3) \times 0{\cdot}9 = 34{\cdot}5 \text{ sq. cms.}$$

and the maximum flux density (no-load) is $\frac{0{\cdot}54 \times 10^6}{34{\cdot}5} = 15{,}700$.

To get the density in the iron behind the teeth, the flux per magnetic circuit in the armature is

$$\frac{3{\cdot}1 \times 10^6}{2} = 1{\cdot}55 \times 10^6 \text{ lines.}$$

The diameter at the tooth roots is $125 + 2 \times 4{\cdot}0 = 133$ cms. If a depth of 7 cms. of iron behind the teeth be chosen, the flux density behind the teeth becomes

$$\frac{1{\cdot}55 \times 10^6}{7 \times 28 \times 0{\cdot}9} = 8800,$$

which is satisfactory.

The external diameter of the stator core is $133 + 2 \times 7{\cdot}0 = 147$ cms.

To calculate the iron loss refer to Fig. 252. Then

Watts per kgm. $= 1{\cdot}4$ with $0{\cdot}52$ mm. stampings.

Weight of iron in core	$= \frac{\pi}{4}(147^2 - 133^2) \times 28 \times 0{\cdot}9 \times 0{\cdot}0078$
	$= 625$ kgm.
Weight of iron in teeth	$= 144 \times 4{\cdot}0 \times 1{\cdot}37 \times 28 \times 0{\cdot}9 \times 0{\cdot}0078$
	$= 156$ kgm.
Watts per kgm. in core	$= 4{\cdot}0$
Watts lost in core	$= 4{\cdot}0 \times 625 = 2500$
Watts per kgm. in teeth	$= 10{\cdot}6$
Watts lost in teeth	$= 10{\cdot}6 \times 156 = 1650$
Total no-load iron loss	$= 2500 + 1650 = 4150$ watts.

The copper loss is calculated as follows :—

Pole pitch $= \frac{\pi \times 125}{16}$	$= 24{\cdot}6$ cms.
Length of end connection assumed	$= 30$ cms.
Embedded length of conductor	$= 31$ cms.
Length of turn $= 2 \times 31 + 2 \times 30$	$= 122$ cms.
Length per circuit $= 122 \times 192$	$= 23{,}400$ cms.
Cross-sectional area	$= 0{\cdot}30$ sq. cm.
Resistance per winding (hot)	$= \frac{2 \times 10^{-6} \times 23{,}400}{0{\cdot}30}$
	$= 0{\cdot}2$ ohm, say.
Total armature I^2R loss	$= 3 \times 132^2 \times 0{\cdot}2.$
	$= 10{,}400$ watts.
Total armature loss	$= 10{,}400 + 4150 = 14{,}550$ watts.

The calculations for the field system are as follows :—

No-load useful flux per pole	$= 3{\cdot}1 \times 10^6$.
Leakage coefficient λ (from page 314)	$= 1{\cdot}20$.
Total no-load flux per pole	$= 1{\cdot}20 \times 3{\cdot}1 \times 10^6$
	$= 3{\cdot}72 \times 10^6$ lines.
Resistance drop per phase at full load	$= 132 \times 0{\cdot}2$
	$= 26$ volts.
Synchronous Reactance per phase at full load	$= 0{\cdot}20 \times 1270$ (say)
	$= 254$ volts.

The E.M.F. required to be generated on full load at a power factor of 0·8 is obtained from the vector diagram shown in Fig. 237 and is 1440 volts.

The full load flux per pole is, therefore,

$$3{\cdot}72 \times 10^6 \times \frac{1440}{1270}$$

$$= 4{\cdot}21 \times 10^6 \text{ total lines,}$$

or $$3{\cdot}51 \times 10^6 \text{ useful lines.}$$

A length of air-gap (see Fig. 253) of 0·5 cm. will be chosen. The approximate air-gap flux density on full load is

$$\frac{3{\cdot}51 \times 10^6}{16{\cdot}0 \times \frac{31 + 28}{2} \times 0{\cdot}9} = 8250.$$

The field ampere-turns per pole required for the air-gap are

$$\frac{10}{4\pi} \times 8250 \times 0{\cdot}5 = 3290.$$

Maximum density in teeth

$$= \frac{\pi}{2} \times \frac{3{\cdot}51 \times 10^6}{9 \times 34{\cdot}5} = 17{,}800.$$

Ampere-turns per cm. from Fig. 252 = 13.
Ampere-turns for teeth = 13 × 4·0 = 52.
Density in armature core

$$= \frac{3{\cdot}51 \times 10^6}{2 \times 7{\cdot}0 \times 28 \times 0{\cdot}9} = 10{,}000.$$

Ampere-turns per cm. from Fig. 252 = 2·5.
Length of path = 20 cms. (say).
Ampere-turns for armature core = 2·5 × 20 = 50.
Section of pole = 15 × 20 with semicircular ends.

$$= \frac{\pi}{4} \times 15^2 + 15 \times 5$$

$$= 252 \text{ sq. cms.}$$

Density in pole $= \frac{4{\cdot}21 \times 10^6}{252} = 16{,}700.$

Ampere-turns per cm. = 60.
Assume a length of 8 cms.
Ampere-turns for pole = 60 × 8 = 480.
Assume section of yoke = 175 sq. cms.

Density in yoke $= \frac{4{\cdot}21 \times 10^6}{175 \times 2} = 12{,}000.$

Ampere-turns per cm. = 10.

Length of path $= \dfrac{\pi \times 100 \text{ (say)}}{16 \times 2} = 10$ cms. (say).

Ampere-turns for yoke $= 10 \times 10 = 100$.

Total ampere-turns per pole $= 3290 + 52 + 50 + 480 + 100$
$= 3972$.

Assume a D.C. supply of 480 volts.

Volts per coil $= \dfrac{480}{16} = 30$ volts.

Resistance of mean turn (hot) $= \dfrac{30}{3972} = 0{\cdot}0076$ ohm.

„ „ „ (cold) $= \dfrac{6}{7} \times 0{\cdot}0076$
$= 0{\cdot}0065$ ohm.

Estimated length of mean turn with 3 cm. depth of winding
$= \pi \times 18 + 2 \times 5 = 76{\cdot}5$ cms.
Ohm per metre $= 0{\cdot}0085$.

The nearest wire to this is 1/·064, having a resistance of 0·0083 ohm per metre and a diameter of 0·163 cm. (bare) and 0·195 cm. (d.c.c.).

Turns per layer $= \dfrac{8}{0{\cdot}195} \times 0{\cdot}95$
$= 39$.

Number of layers $= \dfrac{3}{0{\cdot}195} \times 0{\cdot}95$
$= 15$.

Turns per coil $= 39 \times 15 = 585$.

Length of coil $= 585 \times 0{\cdot}765$
$= 450$ metres.

Resistance of coil (hot) $= 450 \times 0{\cdot}0083 \times \dfrac{7}{6}$
$= 4{\cdot}4$ ohms.

Current $= \dfrac{30}{4{\cdot}4} = 6{\cdot}8$ amperes.

Ampere-turns $= 6{\cdot}8 \times 585$
$= 3980$.

Watts per coil $= 30 \times 6{\cdot}8 = 204$.

Total watts lost in excitation $= 16 \times 204 = 3270$ watts.

Allowing about 5000 watts for the frictional loss, and 50 per cent. of 4150 watts, say 2000 watts, for additional iron loss on load, the full load efficiency is, approximately

$$\frac{500{,}000}{500{,}000 + 10{,}400 + 4150 + 2000 + 3270 + 5000} \times 100$$
$$= 95 \text{ per cent.}$$

The question of the E.M.F. wave form has not been gone into, but by suitably skewing the pole shoe this could be made approximately sinusoidal.

Examples.

(1) A three-phase 50-cycle alternator is to run at 375 r.p.m. The value of D^2L in cm. units is $1{\cdot}2 \times 10^6$. It is desired to make the stator coils as close an approximation as possible to the circular form. Determine suitable values for D and L.

(2) Describe the method of predetermining the open-circuit magnetization curve of an alternator from the design data.

(3) A three-phase star-connected turbo-alternator requires a flux of $32{\cdot}5 \times 10^6$ lines per pole to generate a line pressure of 7200 volts at 50 cycles. There are four poles. Determine a suitable number of total stator slots and conductors per slot.

(4) Write an account of the closed air ventilation system of a modern high power turbo-alternator.

(5) Make a sketch of a two-pole rotor for a turbo-alternator. The flux per pole is to be 20 megalines, and the maximum ampere-turns per pole 10,000. The speed is 3000 r.p.m. Indicate the approximate dimensions.

(6) Describe how the regulation of a turbo-alternator is affected by the length of the air-gap. Explain how the length of the air-gap is determined in designing such a machine to a given specification.

(7) A 20,000-kVA, 1500 r.p.m., 6600 volt, 50 cycle turbo-alternator is to be designed with an output coefficient of $2{\cdot}0 \times 10^{-6}$ (cm. measure). Determine a suitable air-gap diameter, core length, number of stator slots and total stator conductors. The maximum air-gap flux density is to be approximately 7500 lines per sq. cm. on open-circuit.

(8) A 200-kVA, three-phase, 50-cycle, 2200-volt, 375 r.p.m. alternator is to be designed with a specific electric loading of approximately 300 ampere-conductors per cm. periphery. Design a suitable stator winding, giving the total number of slots and the conductors per slot.

The air-gap diameter is 125 cms.

CHAPTER XX

INDUCTION MOTORS.—PRINCIPLES AND CONSTRUCTION

General Construction.—An induction motor consists of two main parts, a stator and a rotor. The stator consists of a number of stampings slotted so as to receive the windings, these stampings being held between two end plates, which are carried from the outer carcase of the motor. This outer carcase serves merely as a mechanical protection for the stampings and does not carry any flux, as is the case in the outer yoke of a D.C. motor. The stator winding is called the primary, and is connected to the supply, producing the rotating field on which the induction motor depends for its action.

The rotor consists of another set of stampings mounted on a spider, which is carried by the shaft. These stampings are also slotted and receive the rotor winding, which is called the secondary, since it has currents induced in it, as in the case of the transformer. Since there are no salient poles, a uniform air-gap is obtained all the way round, this being, in general, smaller than in the case of a D.C. motor of similar size.

In some induction motors the rotor windings are short-circuited on themselves, having no external connections, whilst in others the rotor conductors are brought out to slip rings (usually three) and then joined through a resistance, which is used for starting up.

Production of Rotation.—The primary (stator) winding is supplied with a polyphase current in order to produce the rotating field, and this field, issuing from the stator stampings, crosses the air-gap and enters the rotor. It there cuts the rotor conductors, which form the secondary winding, and induces an E.M.F. in them. The rotor circuit being closed, this E.M.F. sets up a current in the rotor, which consequently absorbs a certain amount of power, this reappearing as heat. The winding then tends to place itself in such a position that it generates a minimum amount of electrical power, and in order to do this it follows the rotating field and commences to rotate. The induced E.M.F. is thereupon reduced, since this is proportional to the rate at which the rotor conductors are cut by the rotating field, and this in turn causes a drop in the rotor current. The power wasted in the rotor circuit is thus reduced, when rotation is set up.

Another way of looking at the problem is to consider the interaction of the fluxes set up by both the stator and the rotor windings. Fig. 254 (*a*) represents a portion of the air-gap showing the stator flux by itself rotating in a counter-clockwise direction. These lines of force are cutting the rotor conductors from right to left, and this is equivalent to a movement of the rotor conductors from left to right. E.M.F.'s are induced in these rotor conductors which are short-circuited, producing currents flowing away from the observer. A hypothetical secondary flux is thus set up, consisting of clockwise lines of force surrounding the rotor conductors, as shown in Fig. 254 (*b*). Combining the stator and rotor fluxes,

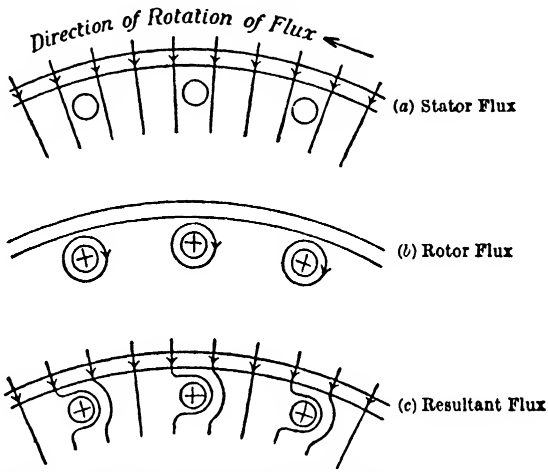


FIG. 254.—Distortion of Flux by Rotor Current.

we get the resultant flux actually existing. This is shown in Fig. 254 (*c*), and as the field tends to straighten itself out the rotor conductors are urged in a counter-clockwise direction and follow the field in its rotation.

Slip.—The rotating field revolves with the speed of synchronism, and if the rotor conductors were to follow at exactly the same speed there would be no relative movement of the field and the rotor. In this case there would be no E.M.F. induced in the rotor and the rotor current would drop to zero. The distortion of the field causing the motoring action now disappears, and the rotor immediately commences to slow down. As soon as it does this, however, the rotor conductors begin to be cut by the rotating field at a rate depending upon the difference in the speeds of rotation of the flux and the rotor. The slower the speed of the rotor the greater is the rotor E.M.F. and current, and, consequently, the greater is the motoring force. Actually, the speed of the rotor adjusts itself, so that the magnitude of the rotor current is just sufficient to exert the necessary torque to overcome the mechanical resistance to motion of the rotor. If it is running too fast, the rotor current is too small to maintain the rotation, and the speed falls. If it is running too slow, the rotor current is greater than necessary, and produces an acceleration which speeds the rotor up.

The difference between the speed of the rotating field and the actual speed of the rotor is called the *slip* of the motor, and may be expressed in revolutions per minute, but a much more common way is to express it as a percentage of the synchronous speed. With modern induction motors, the slip generally lies between zero and about 5 per cent.

On account of the fact that it must run at a speed rather less than that of synchronism, the induction motor is sometimes termed an *asynchronous motor.*

If f represents the frequency of supply and n the revolutions per second of the rotor, then the frequency of slip is $(f - n)$ in a two-pole motor and $(f - pn)$ in a multipolar motor, where p is the number of pairs of poles. Expressing this as a fraction of the supply frequency, it becomes $\frac{f - pn}{f}$ or $\frac{f - pn}{f} \times 100$ per cent.

Rotor Current.—When the rotating field is set up by switching on the stator current, the motor acts like a polyphase transformer with a short-circuited secondary. A current is produced in the rotor winding, which is usually wound for three phases, and this develops the torque necessary to set the rotor in motion. The magnitude of this current depends upon the induced E.M.F. per phase and also upon the impedance per phase. The induced E.M.F. is proportional to the rate of cutting lines of force, and this in turn is proportional to the difference in speed of the rotating field and the rotor, *i.e.* the slip. The resistance of the rotor winding is constant, but the reactance also depends upon the slip, since the frequency of the rotor currents is the same as the frequency of slip. If the slip is doubled the induced E.M.F. is also doubled, but the impedance is not doubled, although it is very materially increased. As a result, the rotor current is increased, but it does not increase proportionally to the slip. Another effect of the reactance in the rotor circuit is to cause the rotor current to lag behind the induced rotor E.M.F. by a certain angle, this having the twofold effect of reducing the power factor and the torque developed per ampere.

The rotor induced E.M.F. per phase is equal to the rotor open-circuit E.M.F. multiplied by the fractional slip, or

$$E_r = sE_2.$$

The rotor current is

$$I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{R_2^2 + s^2X_2^2}},$$

where $X_2 = 2\pi f L_2$ is the reactance per phase at standstill.

This current lags behind the rotor E.M.F. by an angle

$$\phi_r = \tan^{-1}\frac{sX_2}{R_2} = \cos^{-1}\frac{R_2}{\sqrt{R_2^2 + s^2X_2^2}}.$$

In a given motor operating at a fixed line voltage and frequency, the magnitude and phase of the current depend solely upon the slip, s.

Stator Construction.—The construction of the stator of a modern induction motor follows the general lines of that of the stator of a

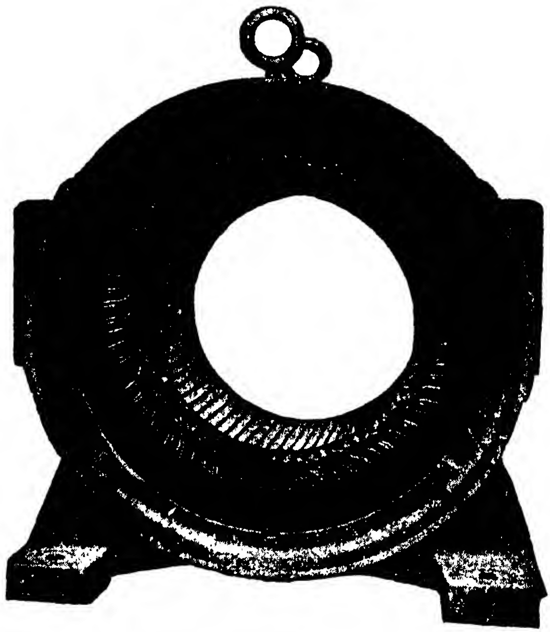


FIG. 255.—Induction Motor Stator. (B.T.H.)

rotating field alternator of a similar size (see Fig. 198). It consists in the main of a ring of stampings held between end cheeks and supported from the outer casting, which serves only as a mechanical support and fulfils no magnetic function. This outer casting is perforated with a number of large holes for ventilation and supports the two end shields, which also carry the bearings. Fabricated stator frames are also commonly used. The stator is wound in the same way as the armature of an alternator, the slots being either of the open or semi-closed type. Radial or axial ventilating ducts are provided in the stator core, these being so arranged, in conjunction with the rotor ventilating ducts, as to permit a passage of the air right through the machine.

A typical induction motor stator is shown in Fig. 255.

Stator Winding.—The stator can be wound either with concentric or diamond coil windings, exactly as in the case of alternators. With the concentric winding a semi-closed slot is employed, whilst with the diamond coil winding an open slot must be used, to enable the coils to be placed in position. The winding also is distributed over a number of slots per pole so as to utilize the surface of the stator to a greater extent. When concentric coil windings are employed, it is usual to have one coil per pair of poles for each phase, so that a four-pole three-phase motor has six coils in all and not twelve. The arrangement of the coils in this case is indicated in Fig. 256, which shows three shapes of coils. These have an actual space displacement of 30° (or 60° electrical), but the connections to the middle coil are reversed in order to get the correct phase angle. This arrangement enables the winding to

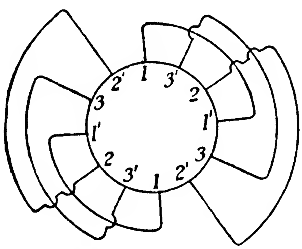


FIG. 256.—Arrangement of Coils in Four-Pole Three-Phase Stator.

be split up into two portions, so that in case of a burn-out in one coil it is not necessary to strip the whole stator winding.

The practice of winding one coil per pair of poles is not adopted in the case of two-pole motors, as it would mean that a very large bunch of end connections would have to lie on top of one another. By using two coils per pair of poles, no extra ampere-turns are required, and half the end connections are carried round one side of the stator, whilst the other half are carried round the other side.

Construction of Wound Rotors.—The usual type of wound rotor consists of a ring of sheet iron stampings mounted directly on the shaft in the smaller sizes and built on to a cast iron spider in the larger sizes. The rotor winding is carried in slots arranged along the outer periphery. Ventilating holes are punched in the plates to provide an entrance for the air, which is thrown outwards radially by means of the ventilating ducts. The three-phase winding is connected in star, and the three ends are brought out to three slip rings. In small motors, these usually lie on the outside of the bearing, the shaft being made hollow to allow the three conductors to pass through the bearing. The current is collected from these slip rings by means of carbon brushes from which it is led to a resistance which is used for starting purposes. When the motor is running, these slip rings are short-circuited by means of a collar which is pushed along the shaft and connects all three slip rings together on the inside. The brushes are provided in a large number of cases with a device for lifting them from the slip rings when the motor has been started up, thus reducing the wear and the frictional loss.

Rotor Winding.—The simplest form of rotor winding is a coil winding of the same type as that employed on the stator. There is an exact number of slots per pole per phase, with one coil for each phase per pair of poles.

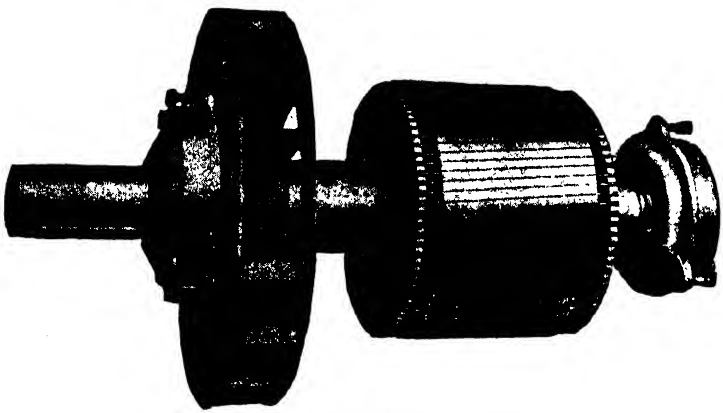


FIG. 257.—Squirrel Cage Rotor. (B.T.H.)

For heavier currents a diamond coil, or two layer, winding is adopted of the same type as is used for the stator and for the stators of alternators (see p. 263). Fractional numbers of slots per pole per phase are frequently used.

Squirrel Cage Rotors.—A simple and effective form of rotor is that known as the *squirrel cage,* which consists of a laminated core as in the wound rotor with a single conductor lying in each slot. These conductors consist of heavy copper bars lightly insulated from the core and short-circuited at each end by means of a pair of stout end rings. Each conductor is riveted or screwed on to both end rings. In another type of construction, the ends of the bars are fitted into slotted end rings, the joints being silver soldered. The slots should be skewed (see p. 271), as this is found to assist the motor in starting and in quiet running. The whole rotor winding thus consists of a permanently short-circuited system having as many phases as conductors.

Fig. 257 shows a typical squirrel cage rotor with a fan and bearing housings, the bearings being of the roller type at the driving end, and of the ball type at the other end.

The great advantage of such a rotor lies in the soundness and simplicity of the mechanical design. There are no moving contacts at all, and the construction of the whole squirrel cage rotor is exceedingly simple. Its great disadvantages lie in the small starting torque and large starting current which are characteristic of the type.

The starting torque can be improved by the employment of a double squirrel cage, one having a considerably higher resistance than the other. (It is shown on page 333 that an increase in the resistance of the rotor circuit increases the starting torque.) The high resistance winding is placed nearest to the air-gap.

In another form of squirrel cage rotor the winding is made of aluminium and is cast in position, both slot conductors and end rings, after the rotor core has been assembled.

Yet another form of short-circuited rotor consists of a solid cylinder of steel without any slots or conductors at all. The steel itself takes the place of the conductors, the motor really operating by virtue of the eddy currents set up in the rotor core. A variant of this consists in the cutting of narrow longitudinal slots in the rotor core, not for the reception of conductors, but merely to keep the rotor eddy currents in the best paths for the production of torque.

Torque.—The torque developed by the rotor is proportional to the instantaneous product of the rotor current and the strength of the magnetic field cutting the rotor. If the whole motor, stator and rotor together, be imagined to rotate in the *opposite* direction to that of the rotating field and at the same speed, the field appears to stand stationary in space and has an approximately sinusoidal distribution over the air-gap. The rotor appears to have a slow rotation in the opposite direction to that in which it is really rotating, the speed of this slow rotation being that of the slip. The induced rotor E.M.F. has a sinusoidal wave form, the maximum value occurring when it passes the centre line of the pole where the maximum flux density occurs. Owing to the rotor reactance, however, the maximum current does not occur until a little later, so that the field and the current are not quite in phase. The average force on a conductor, and the mean torque developed by it, are proportional to the mean product of current and flux, and, the conductors being distributed uniformly round the air-gap, the total torque developed at any instant is given by mean torque per conductor × number of conductors. The instantaneous torque is therefore proportional to

$$\Phi I_r \cos \phi_r,$$

where Φ is the useful air-gap flux per pole, I_r is the rotor current, and ϕ_r is its angle of lag. Since the conductors are distributed uniformly, and sinusoidal conditions are assumed, the total torque developed is constant from instant to instant.

The torque can therefore be expressed as

$$\begin{aligned} T &= k\Phi I_r \cos \phi_r \\ &= k\Phi \times \frac{sE_2}{\sqrt{R_2^2 + s^2X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + s^2X_2^2}} \\ &= \frac{k\Phi E_2 s R_2}{R_2^2 + s^2X_2^2}. \end{aligned}$$

The maximum value of the torque is obtained when $R_2 = sX_2$,[1] when the maximum torque becomes

$$T_m = \frac{k\Phi E_2 s R_2}{R_2^2 + R_2^2} = \frac{k\Phi E_2 s}{2R_2}, \text{ or } \frac{k\Phi E_2 s}{2sX_2} = \frac{k\Phi E_2}{2X_2}.$$

Reactance in the rotor circuit is thus seen to be harmful, inasmuch as it reduces the torque which is developed at any speed, including the maximum torque.

Relation between Slip and Torque.—If the resistance and inductance of a given rotor be kept constant, the magnitude of the torque depends solely upon the slip, providing the magnitude of the rotating field is also constant. For low values of the slip, the reactance is negligible compared with the resistance, and the expression $\frac{k\Phi E_2 s R_2}{R_2^2 + s^2X_2^2}$ becomes approximately $\frac{k\Phi E_2 s R_2}{R_2^2}$, so that the torque is practically proportional to the slip. For large values of the slip, the reactance is large compared with the resistance, so that the expression for the torque becomes approximately $\frac{k\Phi E_2 s R_2}{s^2X_2^2}$ or $\frac{k\Phi E_2 R_2}{sX_2^2}$. The torque is now approximately inversely proportional to the slip. As the slip increases from zero, therefore, the torque at first increases, approximately according to a straight line law, then reaches a maximum, when $R_2 = sX_2$. It then decreases approximately according to the law of a rectangular hyperbola.

[1] The maximum torque can be obtained by differentiation with respect to s and equating to zero. A simpler method is to find the minimum value of $y = \frac{1}{T}$, again by differentiation with respect to s and equating to zero.

$$y = \frac{R_2^2 + s^2X_2^2}{k\Phi E_2 s R_2} = \frac{R_2}{k\Phi E_2 s} + \frac{sX_2^2}{k\Phi E_2 R_2}.$$

$$\frac{dy}{ds} = -\frac{R_2}{k\Phi E_2 s^2} + \frac{X_2^2}{k\Phi E_2 R_2} = 0,$$

$$\frac{R_2}{k\Phi E_2 s^2} = \frac{X_2^2}{k\Phi E_2 R_2},$$

$$\frac{R_2}{s^2} = \frac{X_2^2}{R.} \text{ or } R_2 = sX_2.$$

As an example, consider the case of a motor operating on a frequency of 50, where $T_m = 1$, $k\Phi E_2 = 50$, and $R = \frac{5}{4}$.

Since $T_m = \frac{k\Phi E_2}{2X_2}$, $X_2 = \frac{k\Phi E_2}{2T_m} = 25$.

The expression $\frac{k\Phi E_2 s R_2}{R_2^2 + s^2 X_2^2}$ can now be evaluated for various values of s, and this has been done in the second line of the following table. Next let the resistance be increased to four times its original value, viz., $R = 5$. The new values of the torque are shown in the third line in the table, from which it is seen that the torque now attains a maximum value for four times the slip in the previous case.

s	0	0·02	0·04	0·05	0·10	0·20	0·30	0·40	0·60	0·80	1·00
Torque for $R = \frac{5}{4}$	0	0·69	0·98	1·00	0·80	0·47	0·32	0·25	0·17	0·12	0·10
Torque for $R = 5$	0	0·20	0·38	0·47	0·80	1·00	0·92	0·80	0·60	0·47	0·38

The above figures are plotted in Fig. 258, which shows the general shape of the torque-slip curves.

The condition for maximum torque is that $R_2 = sX_2$, so that the slip for maximum torque is equal to $s = \frac{R_2}{X_2}$. If R_2 is low compared with X_2, the maximum torque occurs with a small slip, and *vice versa*. For a given rotor, the inductance is fixed, but if it were possible to reduce it, the maximum torque would be increased, and would occur at an increased slip, assuming a constant resistance. A reduction of both resistance and inductance in the rotor would, therefore, increase the maximum torque and keep down the slip.

Relation between Rotor Resistance and Torque.—It was shown on page 330 that the torque developed in the rotor was proportional to an expression which included, amongst other terms, the rotor resistance. Of course the resistance of the rotor itself is a fixed quantity, but by inserting an external resistance in series with each phase the total amount can be varied. The maximum value of the torque is independent of the resistance, but the slip at which this maximum torque is obtained is proportional to the total resistance per phase. A means of speed regulation is thus obtained by varying the rotor resistance, since the maximum torque can be obtained at any speed from standstill up to that of synchronism simply by varying the rotor resistance. When no resistance is inserted at all, the rotor runs at full speed, and to increase this speed, *i.e.*

decrease the normal slip, it would be necessary to decrease the rotor resistance, which is only possible by re-designing the winding.

If the load on the shaft necessitates a certain torque being developed, an increase in the rotor resistance causes a corresponding increase in the slip and thus brings the speed down. To illustrate the relationship which exists between the rotor resistance and the slip for a given torque, three other curves have been drawn in Fig. 258, corresponding to $R = 10$, $R = 25$, and $R = 50$. All these curves obey the same law, their only difference being that they attain their maximum values at different points. In fact the last curve for $R = 50$ never does reach its maximum by the time standstill is reached, but it would attain the same height as

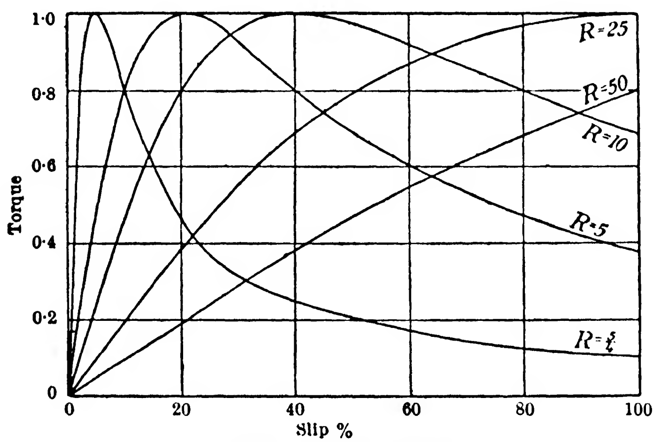


FIG. 258.—Torque-slip Curves.

the others at the hypothetical slip of 200 per cent., which, of course, is impossible in ordinary circumstances.

Another series of curves can be derived from those shown in Fig. 258, giving the relation between the rotor resistance and the slip for a given torque. These are obtained by drawing a horizontal line through, say, 0·5 × maximum torque and noting the corresponding values of the rotor resistance and the slip. These curves are drawn in Fig. 259 for values of the torque equal to 0·25, 0·50, 0·75, and 1·00 times the maximum.

Speed Regulation.—In ordinary circumstances, an induction motor is practically a constant speed machine, since the slip is very small even at full load. In this respect it is comparable with the D.C. shunt motor, the speed of which also drops very slightly as the load comes on. (The introduction of the rotor resistance, however, causes a reduction in speed, enabling all values from standstill up to very nearly synchronous speed to be obtained.) It is not possible to obtain an increase in speed in this way, since the motor must run at a speed less than that of synchronism. The only way in which this can be done is by increasing the frequency, which is not practicable, or by changing the number of poles.

The insertion of the additional resistance is brought about by bringing the three ends of the three-phase rotor to the slip rings, from which external connections are made to the three-phase variable resistance.

In the case of squirrel cage rotors, it is not possible to insert external resistance in circuit, and so no speed regulation is possible other than the small natural drop caused by the increase of load.

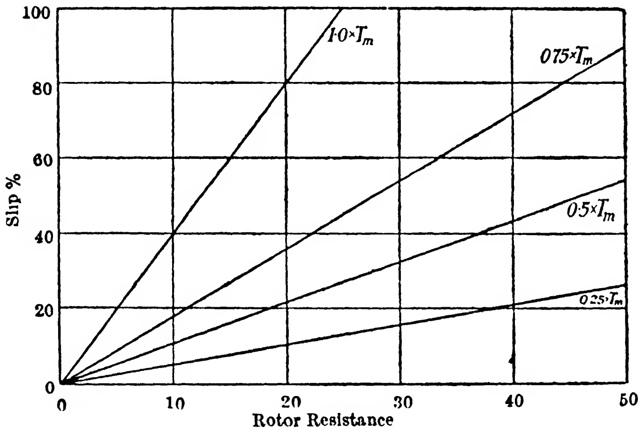


FIG. 259.—Relation between Slip and Rotor Resistance.

Starting Torque.—At the moment of switching on, the frequency of slip is the same as the frequency of supply, and thus the starting torque is obtained by substituting $s = 1$ in the expression for the torque, which now becomes

$$T_s = \frac{k\Phi E_2 R_2}{R_2^2 + X_2^2}.$$

The only practicable way to vary this is again to vary the resistance, and there is one particular value of R which will give the maximum starting torque. This is obtained when $R_2 = X_2$. Not only should the rotor resistance and reactance be equal, but they should both be small if a large starting torque is desired, since the maximum torque obtainable is inversely proportional to the inductance. Both a higher and a lower rotor resistance result in a decreased starting torque, since if a higher resistance is employed the increased impedance brings the rotor current down, whilst if a lower resistance is employed the reduction in the power factor more than counterbalances the increase in the current. The case is, in fact, similar to that discussed on page 66, where it was shown that the maximum power in a circuit occurs when the angle of lag is 45° and the resistance equals the reactance. In the present problem it is desired to get a rotor current which has a maximum component in phase with the field.

Considering the same example taken on page 331, the values of the starting torque have been worked out for various resistances,

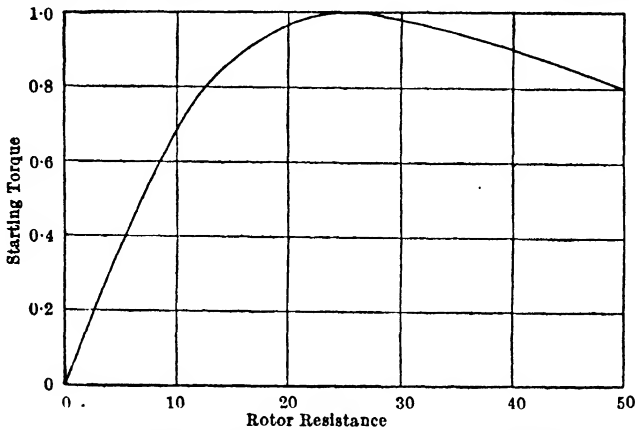


FIG. 260.—Relation between Starting Torque and Rotor Resistance.

and these values are tabulated in the following table and plotted in Fig. 260.

R	1	5	10	20	25	30	40	50
T_s	0·08	0·38	0·69	0·97	1·00	0·98	0·90	0·80

Starting Resistances.—The three-phase resistances used for starting up induction motors consist of three separate variable

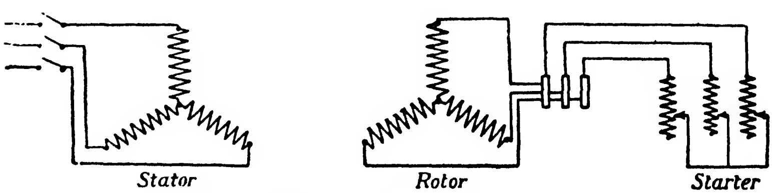


FIG. 261.—Diagram of Connections of Induction Motor.

resistances joined together by means of a three-armed handle which forms a star point. Three movable contacts are thus formed, rigidly attached to one another so as to make them move in unison, and these move over three rows of contacts. The arrangement of the various circuits is shown in diagrammatic form in Fig. 261, whilst the internal connections of a starting resistance are shown in Fig. 262. The three terminals make connection with the ends of the three resistances and are joined to the three slip rings. The rotation of the starting handle is limited to approximately one-third of a revolution by means of stops, the position shown being that when the supply is first switched on, all the resistance being in. To start the motor up, the main switch is closed and the starting handle slowly moved round in a clockwise direction through approximately 120°. The resistance is then all cut out, only that of the leads and switch contacts remaining. As, however, these are usually comparable with the resistance of the rotor itself, it is necessary to short-circuit the rotor at the slip rings themselves as well. This is done by means of a metal collar, which is forced along the shaft under the slip rings, touching them all, thereby eliminating the resistance of the brush contacts on the slip rings, which is quite appreciable. The brushes are then raised to reduce the wear and the frictional loss.

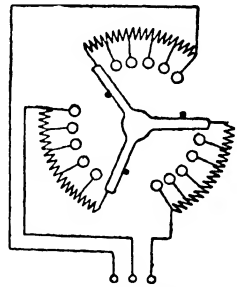


FIG. 262.—Internal Connections of Starting Resistance.

In the eddy current starter there are a few turns wound on each of three solid iron cylinders and connected in the rotor circuit. When first switched in the frequency is that of supply and a considerable eddy current loss is set up. This is the equivalent of a certain amount of resistance. As the motor speeds up, the frequency of slip falls and the equivalent resistance decreases automatically, until finally it is very low when the starter is short-circuited as usual.

Another type of starter is the liquid resistance. This consists of a tank containing the liquid into which three blades, *B* (see Fig. 263), are dipped gradually by means of the handle, *H*, until they engage in three short-circuited contacts, *C*, at the bottom. The short-circuiting device at the slip rings is then employed as before.

Auto-transformer Starters.—The use of the starting resistance is not possible in the case of squirrel cage rotors, and so other means must be adopted to limit the starting current. This is usually done by means of an auto-transformer starter, which consists, as its name implies, of an auto-transformer of which the primary is connected to the line, whilst the secondary, giving a reduced voltage, is connected to the stator of the induction motor. The effect of the reduced voltage on the motor is to reduce, proportionally, the strength of the rotating field. This in turn reduces the E.M.F. generated in the rotor circuit, and hence the rotor current also. Since the torque developed is proportional to the product of the rotor current and the strength of the rotating field, it is seen that it is proportional to the square of the voltage applied to the stator. A reduction in the applied voltage at starting, therefore, causes a

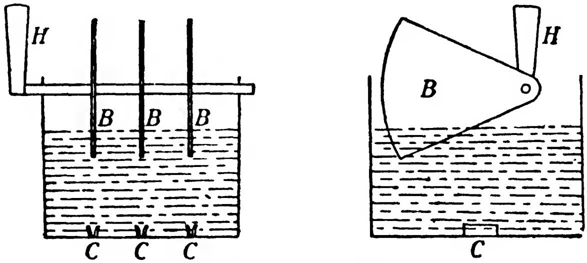


FIG. 263.—Liquid Starter.

considerable drop in the starting torque, but is necessary in view of the reduction in the starting current which it also brings about. By the action of the auto-transformer, however, the current drawn from the line is less than that supplied to the motor, so that an excess current can be taken by the motor without an overload being put upon the mains. For example, consider a motor which would

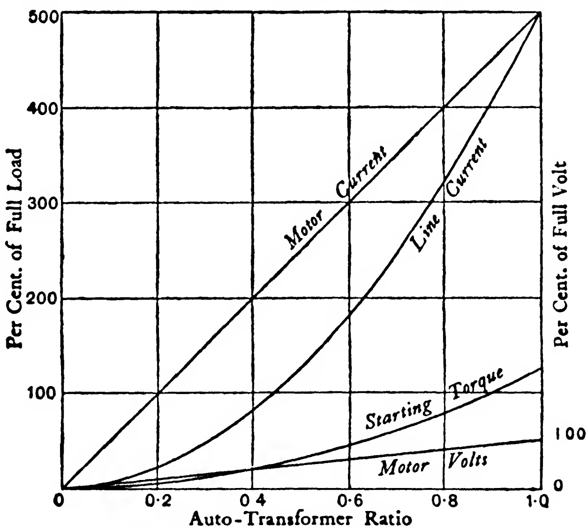


FIG. 264.—Effect of Auto-transformer.

take five times full load current if suddenly thrown straight on to the mains, developing 1·25 times full load torque in so doing. If the auto-transformer had a ratio of 2 : 1 the motor current would be reduced to 2·5 times full load current and the line current to 1·25 times full load current. The torque would go down as the square of the voltage ratio, and would thus be only $\frac{1 \cdot 25}{4} = 0 \cdot 31$ times full load torque. This is a very inferior starting torque, but it is practicable, whereas without the auto-transformer the motor would trip its circuit breaker. To obtain full load starting torque, the motor voltage must be reduced to $\frac{1}{\sqrt{1 \cdot 25}} = 0 \cdot 9$ (approximately) times the line voltage. The motor current is now $5 \times 0 \cdot 9 = 4 \cdot 5$ times full load current and the line current $4 \cdot 5 \times 0 \cdot 9 = 4$ times full load current, approximately. The values of the starting torque available, together with the corresponding values of the voltage applied to the stator and the motor and the line currents, expressed as percentages of the normal full load values, for various tappings on the auto-transformer, are shown in Fig. 264. It is thus seen that the squirrel cage induction motor is not very suitable for starting up under full load torque conditions in view of the heavy current taken, slip ring motors being much preferable in such cases.

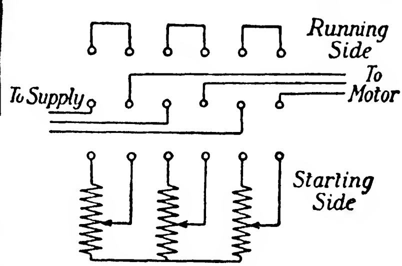


FIG. 265.—Auto-transformer Switch Connections

The auto-transformer may have a number of tappings on it, but it is usual, particularly in the smaller motors, to use one tapping only, this being the one giving the best all-round starting properties. The auto-transformer is then provided with a change-over switch, one position being for starting and the other for running, as shown in Fig. 265. To start up the motor the switch is closed on the "starting" side, and when a certain speed is attained it is thrown over quickly to the "running" side.

Star-delta Connections.—Another method of reducing the voltage on an induction motor at starting is to connect the phases in star for starting and in delta (or mesh) for running. In this way the volts per circuit are reduced to $\frac{1}{\sqrt{3}}$ times the line volts at starting, whilst when running each circuit receives the full line voltage. The motor operates as if an auto-transformer were used having a voltage ratio of $\sqrt{3} : 1$, the motor receiving 57·7 per cent. of the full voltage at starting. Since the torque is proportional to the square of the applied voltage, this arrangement results in a reduction in the starting torque to one-third of its value when switched on direct, whilst the current per phase winding is reduced in the ratio of $\sqrt{3} : 1$. Since the line current is $\sqrt{3}$ times the current per phase winding when delta connected, the line current at starting is also one-third of what it would be if switched on direct. The necessary operations may be performed by means of a change-over switch or by a small drum type controller, the connections of which are shown in Fig. 266.

The disadvantages of this method of starting are, (1) only one starting position is obtained, so that the method is not suitable for very large motors; (2) an induction motor runs better with star than with delta connections, since any lack of electrical balance in the latter case causes circulating currents to flow round the stator winding; and (3) the motor starts up with a low torque.

With two-phase motors the equivalent of star-delta starting is the series-parallel arrangement. Each phase is divided into two

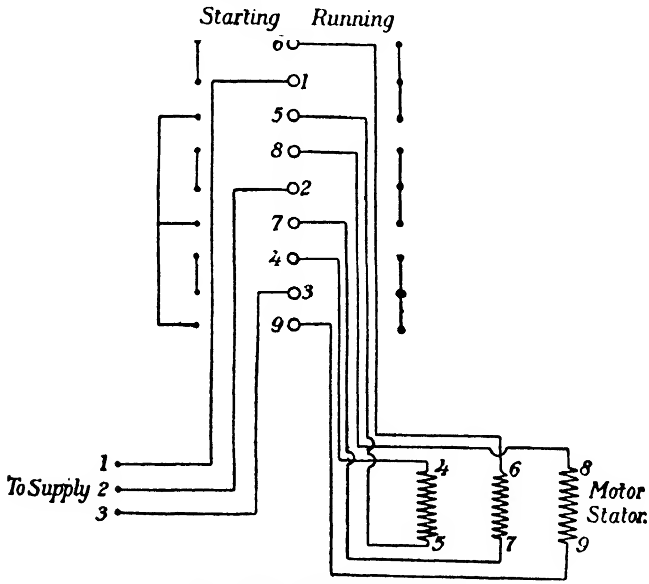


FIG. 266.—Star-delta Controller.

halves which are connected in series for starting and in parallel for running, this doing the same as an auto-transformer with a 2 : 1 ratio.

Induction motors with solid rotors (see p. 329) may be started up in the same way as ordinary squirrel cage motors.

Three-phase induction motors up to a few H.P. may be switched directly on to the mains.

Reversal of Rotation.—In order to reverse a two-phase motor, all that is necessary is to reverse the connections of one of the stator phases, this causing the rotating field to rotate in the opposite direction. In the case of a three-phase motor, any two of the stator leads must be interchanged, this having the same effect.

Crawling Speed.—When considering the rotating field of a three-phase induction motor it was found that the M.M.F. was not constant throughout the cycle (see page 141). The wave form changes from one extreme shape to the other in 30°, so that a cycle of disturbance is gone through in 60°. This corresponds to a sixth harmonic in the flux wave, and gives rise to a seventh harmonic in the rotor currents. The torque due to these is of such a nature as though the motor had seven times its actual number of poles, synchronous speed due to the harmonic being one-seventh of its fundamental synchronous speed. The shape of this torque-speed curve for the harmonic is the same as that due to the main fundamental current, although much smaller in magnitude. Just below one-seventh speed the harmonic torque aids the fundamental torque, but just above this speed the motor is running at hyper-synchronous speed from the harmonic point of view and the harmonic torque becomes negative. If the harmonic is pronounced, the resultant torque may fall below that necessary to overcome the load. When starting up, therefore, such a motor would not accelerate beyond one-seventh synchronous speed, this being called the *crawling speed.*

Output of Rotor.—Of the power supplied to the stator, a portion is wasted in stator copper loss and another portion in stator iron loss. The remainder is supplied to the rotor. The effect of the copper loss is to absorb a portion of the applied voltage, the remainder, obtained by vectorial subtraction, being utilized for the production of the rotating field. This corresponds to that component of the primary voltage of a transformer which remains after the IR drop has been deducted. The power associated with this voltage is transferred to the secondary through the medium of the magnetic flux. The same thing applies to the induction motor. The stator current, also, may be divided into two components, one of which supplies the no-load magnetizing current and the constant iron loss current. The other component provides the ampere-turns necessary to balance the ampere-turns set up by the rotor current. The power transferred to the rotor consists, therefore, of the product of this component of the current and the voltage producing the flux, multiplied by the cosine of the angle of phase difference between the two. Let I_s, E_s and $\cos\phi$ represent these quantities respectively, line values being considered. The power supplied to the rotor in a three-phase case is therefore $\sqrt{3}E_sI_s\cos\phi$. If n is the ratio of the turns per phase on the stator to the turns per phase on the rotor, the voltage induced in the rotor at standstill is $\frac{E_s}{n}$. When the rotor is rotating with a frequency of slip sf, the induced voltage becomes $\frac{sE_s}{n}$.

The rotor current is nI_s and the angle of lag is the same as in the stator, as was explained when discussing the transformer. The power wasted in heating the rotor is therefore

$$\sqrt{3}\frac{sE_s}{n} \times nI_s \times \cos\phi$$

$$= \sqrt{3}E_sI_s\cos\phi \times s.$$

The power output of the rotor is obtained by subtracting the losses from the input and is

$$\sqrt{3}E_sI_s \cos\phi - \sqrt{3}E_sI_s \cos\phi \times s$$
$$= \sqrt{3}E_sI_s \cos\phi \times (1 - s).$$

The efficiency of the rotor is thus

$$\frac{\sqrt{3}E_sI_s \cos\phi \times (1 - s)}{\sqrt{3}E_sI_s \cos\phi}$$
$$= 1 - s.$$
$$= \frac{\text{actual speed}}{\text{synchronous speed}}.$$

Similarly, the ratio of loss to input may be expressed by

$$s = \frac{\text{speed of slip}}{\text{synchronous speed}}.$$

Summarizing, we get

$$\frac{\text{Output}}{\text{Input}} = \frac{\text{actual speed}}{\text{synchronous speed}}.$$
$$\frac{\text{Loss}}{\text{Input}} = \frac{\text{speed of slip}}{\text{synchronous speed}}.$$
$$\frac{\text{Loss}}{\text{Output}} = \frac{\text{speed of slip}}{\text{actual speed}}.$$

It is thus seen that for a high efficiency the slip must be low. In fact, the efficiency is reduced by a percentage equal to the percentage slip.

In the above argument, the power necessary to supply the friction loss is included in the output of the rotor.

Equivalent Circuit of Induction Motor.—The rotor current is given by $I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{R_2^2 + s^2X_2^2}}$ and this demands a corresponding current in the stator equal to

$$\frac{I_r}{n} = \frac{sE_2}{n\sqrt{R_2^2 + s^2X_2^2}}.$$

The stator voltage required to produce this current is $E_1 = nE_2$, so that the rotor impedance, viewed from the stator, becomes

$$\frac{E_1}{\frac{1}{n}I_r} = nE_2 \times \frac{n\sqrt{R_2^2 + s^2X_2^2}}{sE_2}$$
$$= \frac{n^2}{s}\sqrt{R_2^2 + s^2X_2^2} = \sqrt{\left(\frac{n^2R_2}{s}\right)^2 + (n^2X_2)^2}.$$

As s decreases, the apparent resistance $\frac{n^2R_2}{s}$ increases, becoming infinite when $s = 0$. In the equivalent circuit, therefore, the rotor resistance may be regarded as consisting of two parts in series, n^2R_2 which remains constant, and $n^2R_2\left(\frac{1}{s} - 1\right)$ which varies between zero and infinity as s changes from 1 to 0. On the other hand, the reactance of the rotor appears to be constant when viewed from the stator. The reason for this is that when the actual reactance drops owing to the fall in the rotor slip frequency, the rotor voltage falls in the same proportion, thus giving rise to the *appearance* of constancy in the rotor reactance.

The right-hand portion of the equivalent circuit diagram in Fig. 267 thus represents the rotor circuit referred to the stator. Comparing it now with the transformer equivalent circuit diagram,

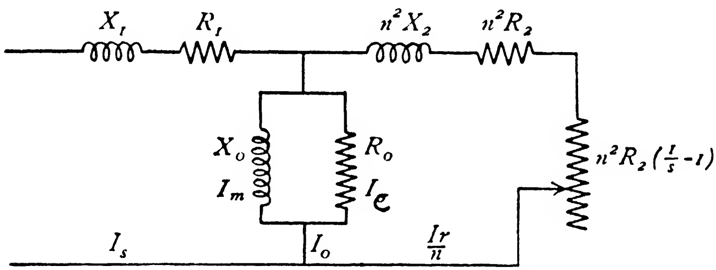


FIG. 267.—Equivalent Circuit of Induction Motor.

it is seen that the full stator current, I_s, is obtained by adding the no-load current, I_0, to the load component, $\frac{I_r}{n}$. This no-load current, I_0, itself consists of two components, I_m and I_c, the former being a magnetizing current represented as flowing through the reactance X_0, and the latter an active component necessary on account of stator iron and friction losses, represented as flowing through the resistance R_0.

Finally, there is a small voltage drop in the stator circuit due to its resistance and leakage reactance, this being allowed for by the inclusion of R_1 and X_1 in the equivalent circuit. The applied voltage is thus slightly larger than the voltage actually used for setting up the useful rotating flux in the air-gap.

Magnetic Leakage.—Magnetic leakage occurs in both the stator and rotor. The stator leakage flux consists of a number of lines of force set up by the stator current, but not linking with the rotor winding, whilst the rotor leakage flux consists of another number of lines of force set up by the rotor current, but not linking with the stator winding.

The stator leakage causes the stator winding to possess inductance, and this absorbs a certain portion of the applied voltage, although there is no direct loss of power associated with it. This is equivalent to reducing the voltage producing the useful rotating field cutting the rotor. This in turn reduces the induced E.M.F. in the rotor, and hence the rotor current. Since the torque developed depends upon both the strength of the rotating field and the rotor current, the presence of stator leakage reduces the torque.

The effect of rotor leakage is to give inductance to the rotor winding. Owing to the increased impedance of the circuit, the rotor current is decreased, thereby decreasing the torque, whilst it also causes the rotor current to lag behind the rotor induced E.M.F. This was shown on page 330 to result in a reduction of the torque as well, so that both the rotor and the stator leakage fluxes cause the output of the motor to be reduced and the power factor made worse.

When considering the performance of the induction motor, it is usual to imagine the whole of the leakage flux as being supplied from the stator winding, the rotor being non-inductive. This is permissible, since that portion of the induced rotor E.M.F. used up in overcoming the rotor reactance is supplied from the stator winding by transformer action. In fact, the whole of the rotor effects come from the stator, since the latter receives the external electrical supply.

Tooth Locking.—If there were the same number of teeth on stator and rotor, the reluctance of the magnetic circuit would be considerably less when the teeth were opposite to each other than when the teeth on one element were opposite to the slots on the other. The magnetic attraction between the stator and rotor teeth tends to make the rotor fall into the position of minimum reluctance and, particularly if the voltage is low, this may have a serious effect on the starting of the motor. The teeth on stator and rotor appear to be locked magnetically, and this effect exerts a reverse torque when the rotor teeth are moving away from those on the stator. Another name for this action is *cogging*, from the analogy of cog wheels.

The effect is present, though in a reduced degree, whenever the number of teeth on stator and rotor have a common factor. For this reason, squirrel cage rotors are designed with a prime number, or twice a prime number, of slots.

Number of Stator Conductors.—The number of stator conductors per phase required to produce the rotating field can be calculated in the same way as is done with transformers. The formula is (see page 177)

$$E = 4{\cdot}44 f \Phi T \times 10^{-8}.$$

E is now the E.M.F. per phase, Φ the flux per pole, and T the turns per phase. Substituting conductors for turns, it becomes

$$E = 2{\cdot}22 f \Phi N \times 10^{-8}$$

and
$$N = \frac{E \times 10^8}{2{\cdot}22 f \Phi}.$$

The constant 2·22 only holds good when all the turns link with all the flux, and this is only obtained when the winding is concentrated in one slot. In a practical case, the winding is distributed over a number of slots, and this introduces a constant called the *Breadth Factor*.

Breadth Factor.—In the case of alternators, the effect of distributing the winding is to cause a slight reduction in the induced E.M.F.; in the case of induction motors, the effect of employing several slots per pole per phase is to reduce slightly the number of linkages set up. In order to generate the same flux, therefore, slightly more turns are necessary, which means that the constant 2·22 is slightly reduced. This reduction factor is called the *Breadth Factor*, and has a value of approximately 0·95 for commercial three-phase motors and 0·9 for two-phase motors. When fractional pitches are employed, a further constant, called the pitch factor, or coil span factor, k_3, is introduced (see page 270).

The number of stator conductors per phase becomes, therefore,

$$N = \frac{E \times 10^8}{0{\cdot}95 \times k_3 \times 2{\cdot}22 f \Phi}$$

$$= \frac{E \times 10^8}{2{\cdot}1 k_3 f \Phi} \text{ for three-phase motors}$$

and
$$N = \frac{E \times 10^8}{0{\cdot}9 \times k_3 \times 2{\cdot}22 f \Phi}$$

$$= \frac{E \times 10^8}{2 k_3 f \Phi} \text{ for two-phase motors.}$$

Number of Rotor Conductors.—The number of rotor conductors bears no fixed ratio to those on the stator, since if a small number of turns is employed the cross section of the conductors can be made correspondingly large. The impedance of the short-circuited winding is thus proportional to the square of the number of turns, assuming a constant copper space factor, whilst the induced E.M.F. is directly proportional to the number of turns. The current is therefore inversely proportional to the number of turns, so that the total ampere-turns on the rotor are approximately constant, depending only on the available slot space.

Three-Phase Rotor for Two-Phase Motor.—It is not at all necessary for the rotor to be wound for the same number of phases as the stator, since the function of the stator winding is to set up a rotating field which can be obtained from any polyphase source. For example, a squirrel cage rotor is one which has as many phases as conductors, since the current in each bar has a slightly different phase from that in its neighbours.

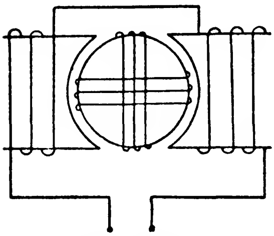


FIG. 268.—Single-Phase Induction Motor.

Since three-phase rotors only require three slip rings and three terminals, instead of four for two-phase rotors, and have a slightly higher breadth factor, they are always used for two-phase motors. Another very important advantage possessed by the three-phase rotor, from the manufacturer's point of view, lies in the fact that one rotor can be utilized for both two- and three-phase machines, and since many more three-phase motors are made than two-phase, it is convenient to make all the rotors three-phase.

Single-Phase Induction Motor.—A single-phase winding is only capable of producing an alternating magnetic field, and is not capable of producing a rotating field unaided. A single-phase motor constructed on the rotating field principle must have some auxiliary help, therefore, to enable a rotating field to be set up. It is found that if a polyphase induction motor has all its phases broken except one, the motor will continue to run, although it will not start up again after having been shut down. The reason for this is that when running the induced currents in the rotor set up a true rotating field of their own which enables the rotation to be maintained, but the effect disappears when the motor comes to rest. This can be shown by considering Fig. 268, which indicates a two-pole stator wound with a single-phase winding and a rotor on which two short-circuited windings are placed at right angles. First, consider the action of the vertical rotor winding, which acts like the secondary of a transformer and has a voltage induced in it proportional to the rate of change of the stator flux. This voltage is, therefore, in quadrature (*i.e.* 90° out of phase) with the flux. The horizontal coil has no induced voltage due to transformer action, since the turns do not link with the stator flux, but owing to the rotation of the rotor these turns cut the stator flux and have a voltage induced in them. This voltage is in phase with the stator flux. The two sets of turns produce two fluxes having a space displacement of 90° and differing in phase by 90°. The combination of these two sets up the desired rotating field. It is not, of course, necessary to confine the rotor winding to two simple coils at right angles, since any polyphase winding will produce the same result. In fact, a squirrel cage rotor is quite satisfactory. Since the whole of the magnetization must be supplied from the one phase, the magnetizing current is larger than in a similar polyphase motor, and the behaviour of the machine under load is very inferior as regards efficiency, power factor and overload capacity.

Torque-slip Characteristic of Single-Phase Induction Motor.— The behaviour of a single-phase induction motor can be represented approximately by assuming the alternating flux set up by the stator winding to be resolved into two hypothetical rotating fluxes, each constant and of half the amplitude of the maximum value of the alternating flux, rotating in opposite directions at synchronous speed (see page 142).

The torque-slip curves are drawn for each of these component fluxes, as shown in Fig. 269. Consider first the flux rotating in the same direction as the rotor. This sets up a positive torque, just as in the case of a three-phase motor. The curve is continued for slips up to 200 per cent., at which value the rotor is rotating

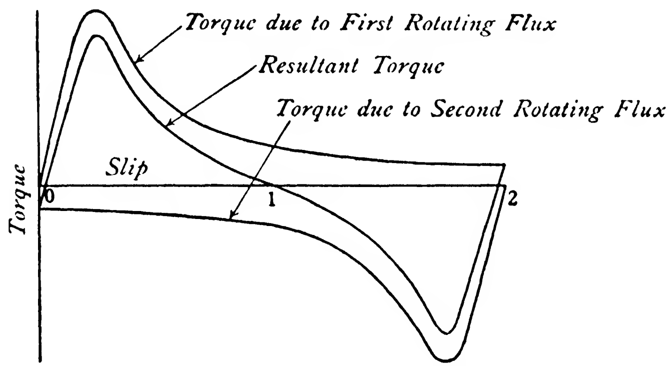


FIG. 269.—Torque-slip Curve of Single-Phase Induction Motor.

at 100 per cent. of synchronous speed in the *reverse* direction. Points on this part of the curve can be obtained by inserting values between 1 and 2 for s in the torque formula on page 330. This torque has the same sense as for values of s that are less than unity, but an induction motor will not run in these conditions, since a reversed torque is required to produce a reversed speed.

The second component flux is rotating in the opposite direction to the first. When the rotor is rotating synchronously with respect to the first flux, it is running with 200 per cent. slip with respect to the second flux, and moreover the torque set up by this flux is opposed to that set up by the first flux. The torque-slip curve for the second flux is, therefore, drawn as a negative torque below the zero line in Fig. 269. Again, 200 per cent. slip with respect to the first flux corresponds to synchronous speed with respect to the second flux, and the reverse torque due to the latter reaches its maximum value at some value of the slip rather less than 200 per cent. on the slip scale in the diagram. The resultant torque due to the combined action of the two component rotating fluxes is the algebraic sum of the two component torques, and is shown in the diagram. This curve of resultant torque passes through zero at a slip of 100 per cent., thus showing that there is no starting torque developed. The maximum torque is also less than would be expected in a three-phase induction motor. A further point is that the torque falls to zero again at rather less than synchronous speed, the torque for the latter speed being actually negative. As a result, the normal slip of a single-phase induction motor under load conditions is rather greater than in the corresponding three-phase motor.

The effect of adding rotor resistance is rather different in the single-phase induction motor from what it was in the three-phase one. In the latter case, the value of the maximum torque was

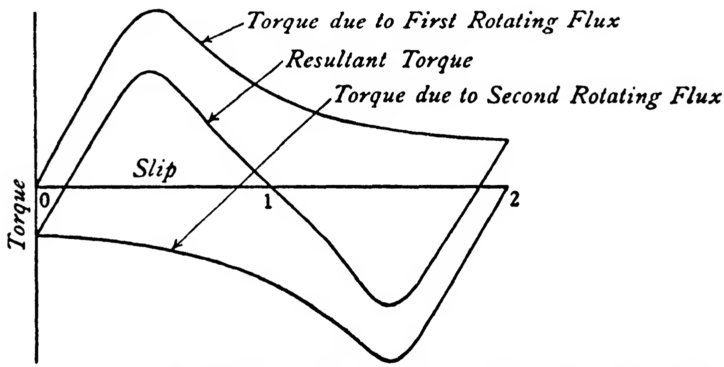


FIG. 270.—Torque-slip Curve of Single-Phase Induction Motor with Added Rotor Resistance.

not affected by the addition of rotor resistance, the only change being in the value of the slip at which maximum torque occurred. In the single-phase machine, the addition of rotor resistance results in an actual decrease in the value of the maximum torque developed, in addition to altering the slip at which it occurs. This is made apparent by reference to Fig. 270, which shows torque-slip curves for a higher rotor resistance, the maximum values of the component torques occurring at larger slips than in the previous diagram. The maximum value of the resultant torque occurs at a larger slip, but it is obviously reduced in magnitude as well.

This method of treatment is only approximate, as it assumes that the single-phase induction motor is equal to two mechanically coupled polyphase motors, of opposite phase rotation, and connected in *parallel*. More accurately, the single-phase induction motor is equivalent to two such polyphase motors connected in *series*, since the stator currents for the two component fluxes must be the same.

Starting of Single-Phase Induction Motors.—In order to enable the motor to start up, it is necessary to produce an initial rotating field, and this is usually done by means of a "*split phase.*" A second winding is placed on the stator after the manner of the second phase of a two-phase motor. This winding need not be as strong as the main winding, and is made of smaller wire, since it is only in use during the starting period. The main and the starting windings are connected in parallel, and both supplied from the same single-phase source, but to obtain the difference in phase an additional reactance is placed in the starting winding. The currents in the two branches now set up a rotating field, not uniform in magnitude, but sufficient to cause the motor to rotate. When a fair speed is attained, the starting winding is cut out. In some cases the two windings are connected in series, the phase displacement being obtained by shunting one with a resistance and the other with an inductance. This arrangement has the advantage that it limits the starting current. Instead of adding inductance to the starting winding, a condenser may be placed in series with it, whilst in some motors resistance is actually added, the idea being that since the two windings already possess a fair amount of inductance in themselves, the necessary phase displacement can be obtained by bringing the current in one branch more into phase with the voltage.

Single-Phase Capacitor Motor.—The ordinary single-phase induction motor obtains the necessary phase displacement of the current in its auxiliary winding by the addition of inductive reactance in series with this winding. In the capacitor motor, a condenser is substituted for the choking coil, so that the current in the auxiliary winding leads the current in the main winding, instead of lagging behind it. When the motor has attained its full speed, the auxiliary winding is cut out of circuit in the ordinary motor, since it reduces the power factor. With the capacitor motor, on the other hand, the auxiliary winding is left in circuit, since it takes a leading and not a lagging current. The main and the auxiliary windings thus combine to produce torque when running as well as during starting, and the motor approximates in performance to a two-phase induction motor.

A larger capacitance is used during the starting period than when running, and condensers of the electrolytic type may be used for the extra capacitance required during the short starting period.

It is claimed for this motor that it gives a better starting torque than the ordinary single-phase induction motor, and that it runs more quietly with a better efficiency and power factor. It can be operated with either a squirrel cage or wound rotor.

When delivering load both windings assist in providing the output, but on no-load the main winding may be found to be returning power to the mains. The action of the condenser in series with the auxiliary winding results in a certain amount of resonance being set up, so that the voltage across the auxiliary winding may be considerably in excess of the supply voltage The amount by which this voltage across the auxiliary winding is resonated up depends upon the capacitance of the condenser and the inductive reactance of the winding. The voltage across the condenser exceeds that across the winding. The vector diagram for no-load conditions is shown in Fig. 271, where E represents the supply voltage. The resonant conditions of the auxiliary circuit cause voltages E_a and E_c to be set up across the auxiliary winding and the condenser respectively, the phase angle between these two component voltages approaching 180°. The current in this circuit is I_a and leads the condenser voltage E_c by 90°. The component of the current I_a in phase with the supply voltage may be greater than that required to supply the losses of the motor, and so a certain amount of power must be returned to the line, which is performed by the main winding. This winding has a certain impedance at no-load, this determining the magnitude of the current flowing through it, and since the component of this current in *phase opposition* to E is fixed, the phase angle of the current in the main winding is thus determined. This current is represented by I_m in the vector diagram. The vector sum of I_m and I_a gives I the total current taken by the motor, the power factor being given by cos ϕ. The phase angle between E and I_m is greater than 90°, indicating that I_m has a reverse power component.

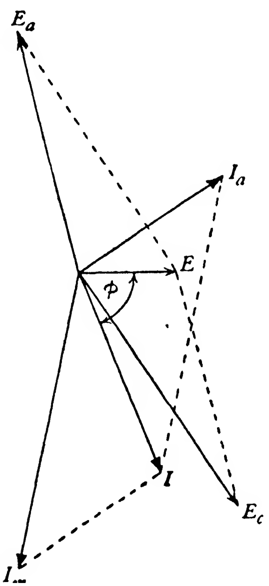


FIG. 271.—Vector Diagram of Capacitor Motor on No-Load.

Pole Changing Motors.—Two economical speeds can be obtained, one double the other, by arranging the stator windings so that the number of poles can be changed at will in some simple ratio such as 8 to 4. The span of the coils may be either equal to, greater than, or less than, the pole pitch and, provided the coil span is not too small, there is not a very great loss in efficiency. More copper is, of course, required with fractional pitch windings. For example, a winding which has a full pitch for eight poles only has a 50 per cent. pitch for four poles. This is about the smallest fractional pitch that is advisable. By suitably rearranging the connections to the various coils such a winding may be made to set up either an 8- or a 4-pole field. Again, in the case of a winding arranged to give full pitch coils for a 6-pole field, the same winding would, when re-connected, give two-thirds pitch coils in a 4-pole field, and four-thirds pitch coils in an 8-pole field.

Suppose a series of full pitch coils is arranged to set up eight poles [see Fig. 272 (*a*)]. By reversing alternate coils, consequent poles are set up giving a 4-pole arrangement [Fig. 272 (*b*)]. The four coils are connected in a closed net as shown in Fig. 273 (*a*) and (*b*). When fed at the terminals T_8 and T_8' an 8-pole arrangement is obtained, but when fed at T_4 and T_4' it is seen that the currents in coils *B* and *D* are reversed whilst the currents in coils *A* and *C* are

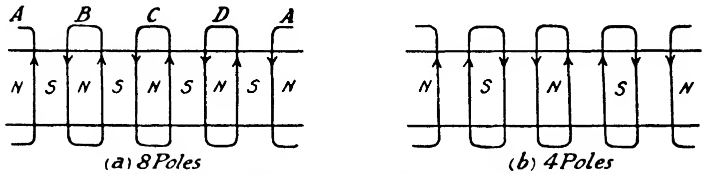


FIG. 272.—Arrangement for Eight and Four Poles.

unchanged. The winding now sets up four poles. In a three-phase motor three such groups of windings are necessary, connected in star. Either T_8' or T_4' forms the star point, the other terminals being connected to the line. The smaller number of poles, however, corresponds to a higher speed and if a greater output is desired, then a greater current must flow. This can be effected by connecting the coils in series for the low 8-pole speed and in series-parallel for the high 4-pole speed as shown in Fig. 274.

In the 4-pole case, shown in Fig. 274 (*b*), the voltage per coil is doubled, so that each coil takes double the current flowing in

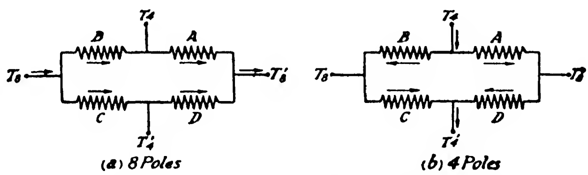


FIG. 273.—Coil Connections for Eight and Four Poles.

the 8-pole arrangement, shown in Fig. 274 (*a*). Since there are two coils in parallel, the total current taken is four times, so that the motor develops four times the power on the same voltage. Since the speed is doubled, it follows that the torque is also doubled. Such an increase in torque is not desired, however, in a number of cases, and the following arrangement may be adopted to get over this disadvantage. The coils are connected in delta for the low speed, and in parallel-star for the high speed as shown in Fig. 275. It is obvious that in the former case the currents in the two coils in one leg of the delta flow in the same direction, the terminals being those marked T_8. When the terminals marked T_4 are used, the currents in the same two coils must flow in opposite directions, so that the number of poles is halved, as shown in Fig. 272. For this connection the three T_8 terminals are short-circuited to form

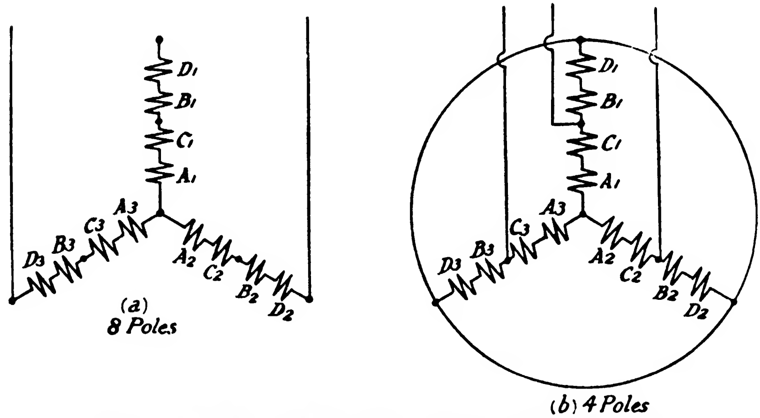


FIG. 274.—Series and Series-Parallel Connections for Three-Phase Motor.

a star point, n. The voltage per coil with the delta arrangement is $\frac{E}{2}$, where E is the line voltage. With the parallel-star arrangement the voltage per coil is $\frac{E}{\sqrt{3}}$, so that each coil now takes $\frac{2}{\sqrt{3}}$

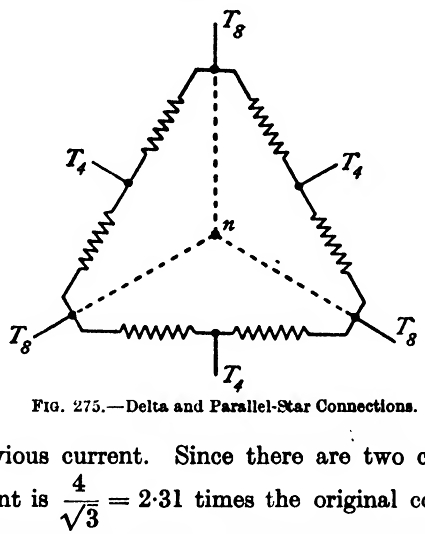


FIG. 275.—Delta and Parallel-Star Connections.

times its previous current. Since there are two coils in parallel, the line current is $\frac{4}{\sqrt{3}} = 2{\cdot}31$ times the original coil current, or $\frac{4}{3}$ times the original line current. Since the speed is doubled, the torque is $\frac{2}{3}$ times its previous value. The variation in torque is therefore seen to be much smaller than in the series and series-parallel connection shown in Fig. 274.

The rotor may be either of the squirrel cage or of the wound type. In the latter case the use of fractional pitch windings enables the rotor to operate in a rotating field with either number of poles.

Cascade Arrangement.—A system of connections sometimes adopted with polyphase motors for obtaining two speeds is that known as the *cascade arrangement*. This employs two motors mechanically coupled together, the stator of the first being connected to the supply. The rotor of the first is connected to the stator of the second, and the rotor of the second is connected to a starting resistance in the usual way. Usually the stator and rotor of the second machine are interchanged, thus doing away with the necessity for slip rings, since the connecting conductors can now be carried from one machine to the other through a hollow shaft. The diagram of connections is shown in Fig. 276.

The frequency of the current supplied to the second stator is the frequency of slip of the first motor, and the frequency of slip

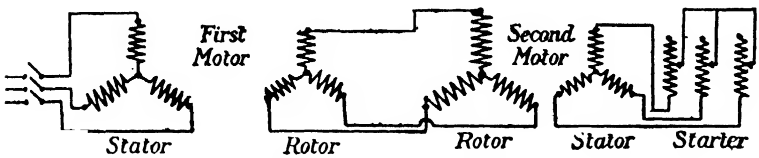


FIG. 276.—Cascade Connections.

of the second motor is the usual small value of a few per cent. Neglecting the second slip and assuming the same number of poles for both motors, it is seen that the synchronous speed of the second motor is the same as the speed of slip of the first. If there are p pairs of poles, the speed of the first motor is

$$\frac{f - sf}{f} \times \frac{f}{p} = \frac{f - sf}{p}. \text{ revs. per sec.}$$

Neglecting the second slip, the speed of the second motor is $\frac{sf}{p}$.

Equating these two, we get

$$\frac{f - sf}{p} = \frac{sf}{p},$$
$$f - sf = sf,$$
$$sf = \tfrac{1}{2}f.$$

The synchronous speed of the set connected in cascade is thus half the synchronous speed of either when connected in the ordinary way.

If the number of pairs of poles on the two motors be different, let them be represented by p_1 and p_2 respectively. The speed of the first motor is then $\frac{f - sf}{p_1}$, and the speed of the second $\frac{sf}{p_2}$.

But
$$\frac{f - sf}{p_1} = \frac{sf}{p_2},$$

and
$$fp_2 - sfp_2 = sfp_1,$$
$$sf(p_1 + p_2) = fp_2,$$
$$sf = f \times \frac{p_2}{p_1 + p_2} \cdot \text{ revs. per sec.}$$

The speed is therefore

$$\frac{sf}{p_2} = \frac{f}{p_1 + p_2} \cdot \text{ revs. per sec.}$$

In other words, the synchronous speed of the set is the same as that of a single motor having as many poles as both component machines added together.

If the two motors are not mechanically coupled together, the arrangement is unworkable, since the moment any load is put upon one motor it stops and acts just like a transformer, whilst the other motor runs up to its normal speed.

Sometimes a small auxiliary motor is cascaded with a large main motor, the former having only two poles and the latter a large number. For example, a small 2-pole motor may be cascaded with a large 20-pole motor, on a 50-cycle circuit. The synchronous speed of the latter by itself is 300 r.p.m., but when the auxiliary motor is introduced into the circuit, this synchronous speed drops to 273 r.p.m. (22-pole speed). By arranging the auxiliary motor to be driven against its own torque, a further speed of 333 r.p.m. can be obtained, corresponding to 18 poles.

Hunt Cascade Motor.—The two machines in a cascade set can be combined into one machine by the use of certain windings as in the Hunt motor. Suppose that a single machine is provided with two stator and two rotor windings, one of each being wound for each number of poles. As an example, an 8-pole stator winding induces E.M.F.'s in an 8-pole rotor winding, and these force currents through a second rotor winding wound for four poles. This now acts as the primary of the second set of windings and induces E.M.F.'s in the second secondary wound on the stator also for four poles. This last winding is connected to the usual starting resistances, which are short-circuited when running.

Instead of two distinct windings on both stator and rotor, a single winding is placed on each in the Hunt motor. Each phase is provided with a series of coils connected somewhat similarly to those shown in Fig. 273. The terminals T_4 and T_4' are now connected together. This makes no difference to the supply, for these are equipotential points with reference to the supply currents obtained through T_8 and T_8', these setting up an 8-pole field. The rotor is designed to set up, in addition, a 4-pole rotating field, and due to this the stator sets up an E.M.F. between the points T_4 and T_4', which, being short circuited, allows a secondary current to circulate. This current does not interfere with the supply, for, to it, the points T_8 and T_8' are also equipotential. The rotor winding also is designed to operate in rotating fields with two numbers of poles.

Induction Generator.—If the rotor of an induction motor is driven mechanically from some external source at a speed higher than that of synchronism, the machine acts as a generator and the stator supplies a current to the mains. Unfortunately, such a machine is not capable of acting independently, but must run in parallel with another A.C. generator of the synchronous type, as otherwise there will be no rotating field set up and no generator action takes place. In other words, the other generator must supply the A.C. magnetizing current for the induction generator. The effect of raising the speed is to increase the output of the machine, the frequency remaining constant. In fact, the frequency is determined by the other generator and is independent of the speed. For this reason, the machine is sometimes called an *asynchronous generator*, since it does not run at synchronous speed.

Machines of this kind have not a wide application in practice, but the principle is sometimes employed with induction motors where a regenerative control is used, as in lift motors on the lightly-loaded journey and railway motors when descending a steep gradient.

Examples

(1) Draw a developed diagram of a single-layer winding suitable for a small 3-phase, 4-pole induction motor. The winding is to be accommodated in 36 slots. Show clearly all connections.

(2) A 6-pole, 50-cycle, three-phase induction motor develops a maximum torque of 100 lbs.-ft. at 880 r.p.m., the resistance per phase of the rotor being 0·6 ohm. Calculate the torque when operating with 5 per cent. slip.

(3) A squirrel cage motor takes 5 times full load current and develops 2·2 times full load torque when switched directly on to the mains. What torque will it develop and what will be the initial value of the current drawn from the supply when started up through an auto-transformer with a 60 per cent. tapping?

(4) Compare critically the various methods adopted for starting three-phase induction motors.

(5) Compare methods of obtaining speed regulation of three-phase induction motors by means of (1) rotor resistance, (2) cascade system, and (3) pole-changing. Give examples where each system may be employed with advantage.

(6) Show how multiple speeds are obtained by the cascade control of two induction motors.

A particular group consists of a 6-pole and a 10-pole motor. What speeds are possible with this combination neglecting slip? The frequency is 50.

(7) A 4-pole and a 6-pole three-phase induction motor are connected in cascade, the 6-pole machine being inverted and having its stator windings connected to a starting resistance. The frequency of supply is 50. Neglecting the slip of the second motor, determine the frequency in the intermediate circuit, and the actual speed of rotation of the rotating field in the second motor. Prove any formulæ used.

(8) A 4- and a 6-pole induction motor are connected in cascade on a 50-cycle supply. When running separately the former motor is rated at 30 B.H.P. and the latter at 20 B.H.P. What speeds can be obtained without the use of rheostatic control, and what B.H.P. can be obtained at each speed? What is the supply frequency to the second machine when cascaded? Neither machine is to be overloaded.

(9) Evolve an expression for the ratio of the rotor copper loss in a three-phase induction motor to the rotor mechanical output.

Why is speed regulation by the insertion of rotor resistance an uneconomical method?

(10) A three-phase squirrel cage induction motor is wound with four groups of coils per phase. Show by the aid of diagrams how these can be connected in series-star for use as an 8-pole motor, and in parallel-star for use as a 4-pole motor. If such a motor is operated through a transformer, show that a different ratio of transformation is desirable in the two cases.

(11) Describe one type of single-phase capacitor motor and show, by the aid of a diagram, how starting is effected. What advantages does such a motor possess over an ordinary single-phase induction motor?

CHAPTER XXI

INDUCTION MOTORS.—PERFORMANCE AND TESTING

No-load Conditions.—If the rotor winding be open-circuited by raising the brushes on the slip rings, no current flows in the rotor and no torque is developed. In these circumstances, the rotor remains stationary, and the stator winding takes only that current necessary to maintain the rotating field and to provide for the stator iron loss. The former component is a purely magnetizing current and lags behind the E.M.F. by 90° whilst the latter is an active component, relatively small in magnitude. On short-circuiting the rotor, the motor runs up to full speed, the only torque which is developed being that necessary to overcome the friction and the rotor iron loss. A very small rotor current is sufficient to produce this torque, so that the slip is extremely small and the motor runs at very nearly synchronous speed. Owing to the extremely low frequency in the rotor circuit, the rotor iron loss is quite negligible. This small rotor current causes a corresponding current to flow in the stator winding, and this component, combined vectorially with the current obtained in the stator with the rotor on open circuit, forms the no-load current of the motor.

The power factor is considerably less than 0·5 in an ordinary case, so that when measuring the power of a three-phase motor running light by means of the two-wattmeter method, one wattmeter reads negatively (see page 119), and the total power is obtained by subtracting the two wattmeter readings.

Effect of Voltage.—If the applied voltage be gradually lowered, the magnitude of the rotating field is reduced proportionally. The rotor current required to produce the no-load torque is, therefore, slightly increased, since the torque is proportional to the product of the field strength and the rotor current. This does not produce any appreciable effect on the no-load stator current, however, until very low voltages are reached, and in the meantime the magnetizing and the stator iron loss currents are falling with the voltage, so that the resultant stator current first of all falls as the voltage is reduced, but after a time rises again.

The power absorbed falls away as the voltage is reduced, but not so fast as the reactive magnetizing current, so that the power factor rises. In fact, when a motor is started up by means of an auto-transformer it happens quite frequently that the second wattmeter reads positive for a time, going behind the zero as the voltage is raised.

The slip remains practically constant over a wide range of voltage, but commences to increase at a considerable rate when very low voltages are reached.

Effect of Frequency.—The relation between the applied voltage, the flux per pole and the frequency is given by the formula

$$E = 2{\cdot}1 f \Phi T \times 10^{-8}.$$

With a constant applied voltage, therefore, the flux is inversely proportional to the frequency, so that reducing the frequency below its normal value causes an increased flux, which increases the magnetic saturation in the iron and thereby reduces the permeability. The magnetizing current is thus increased both on account of the increased flux and the decreased permeability. This makes the power factor poorer, and results in an increased copper and iron loss.

If the motor be designed for a low frequency in the first place, the flux density is prevented from reaching such high limits by employing a larger number of turns on the stator. With a given number and size of slots this results in a smaller size of wire being used, and this only permits a smaller current to flow. Both the output and the speed of the motor are thus reduced more or less proportionally.

For example, consider an 8-pole, 50-cycle motor having a synchronous speed of 750 r.p.m. If the same carcase be employed for a 25-cycle motor with the same number of poles, the synchronous speed would be 375 r.p.m., and there would be twice as many conductors per slot half the size of those in the first case. Carrying half the current, therefore, the motor would do half the output. If the number of poles be reduced to four, the synchronous speed would be 750 r.p.m., as in the first case. The pole pitch is thus doubled and the original cross section could be used, since there will be twice as many slots per pole. The same current per conductor can now be allowed as in the 50-cycle motor, so that the output is the same for the same synchronous speed.

The above 4-pole 25-cycle motor is superior to the 8-pole 50-cycle motor, since the pole pitch is longer, and in all probability less leakage will take place. The magnetic leakage plays a most important part in determining the performance of the motor and affects not only the power factor, but also the overload capacity of the motor.

In order to get a reasonable length of pole pitch, few poles are required, and this leads to high speeds unless low frequencies are adopted. In general, motors designed for low frequencies operate with better power factors than those intended for higher frequencies.

Load Test.—In order to carry out a load test on a three-phase induction motor, the motor is connected up as shown in Fig. 277, the instruments required being a voltmeter, an ammeter, and a three-phase wattmeter. Instead of the latter, two single-phase wattmeters may be used (see Fig. 93). A power factor indicator is unnecessary, as this can be determined by the ratio of the two

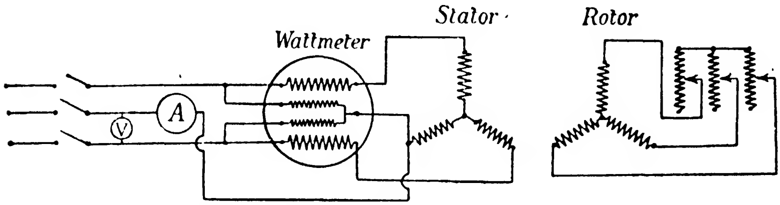


FIG. 277.—Electrical Connections for Load Test.

wattmeter readings. For low loads, the power factor will be probably below 0·5, so that one wattmeter will show a negative indication. The volt coil of this instrument must be reversed in order to get its reading. The rotor is connected up to a three-phase starting resistance if a slip ring rotor, whilst an auto-transformer should be used in the stator circuit if the rotor is of the squirrel cage type. Sometimes it is desired to measure the current in a slip ring rotor, and this is done by inserting an ammeter in one of the slip ring connections, but this is not very satisfactory, since the currents usually become unbalanced to an appreciable extent, owing to the fact that the resistance of the ammeter is quite comparable with the resistance of the rotor, which is, of course, very low. No extra apparatus, therefore, should be included in the rotor circuit other than that absolutely necessary.

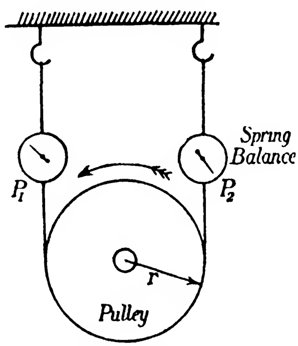


FIG. 278.—Arrangement for Band Brake.

The load can be taken up by another machine run as a generator (D.C. or A.C.) coupled up to the induction motor either directly or by belt. The generator is then loaded up on resistance or otherwise to obtain the required loads. The output of the generator is measured, and its efficiency at various loads must be known, so that the output of the motor can be determined. When a belt drive is employed, the loss in the belt must also be allowed for.

Another method of determining the output of the motor is to use some form of brake, such as the Prony brake or the eddy current brake.

A simple method of determining the output, when it is not kept on for more than a few minutes at a time, consists in loading up the motor by means of a simple band brake with a spring balance on each side to measure the pull on the tight and the slack sides of the belt. The arrangement is shown in Fig. 278, where P_1 and P_2 represent the two spring balances, the pulls being measured in lb. The net pull at the rim of the pulley is $(P_1 - P_2)$ lb. and the torque developed is $T = (P_1 - P_2) \times r$ lb.-ft., where r is the radius of the pulley in feet. The work done per revolution is

$$2\pi r(P_1 - P_2)$$

or $2\pi T$ foot-lb., and if n is the speed in r.p.m. the work done per minute is $2\pi nT$. The B.H.P. developed is $\frac{2\pi nT}{33,000}$, and this is equivalent to $\frac{2\pi nT}{33,000} \times 746 = 0{\cdot}142nT$ watts. To determine the

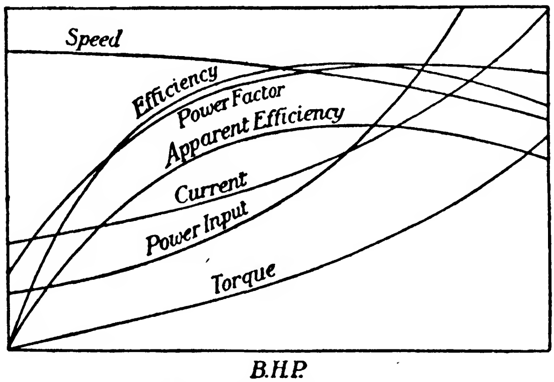


FIG. 279.—Performance Curves of Induction Motor.

output, therefore, n and T must be obtained, and this involves measurements of the speed and the pulls on the two spring balances and a knowledge of the radius of the pulley.

In carrying out the test, the tension on the brake is adjusted to a certain value, and the various measurements are taken. This is then repeated for a number of other tensions on the brake, so that a complete range of loads is obtained which should be plotted against B.H.P. as a base. The performance curves of a typical induction motor are shown in Fig. 279.

The disadvantage of this method of testing lies in the fact that if the load is considerable the band brake very soon gets extremely hot, so that the observations have to be taken quickly.

Power Factor.—The power factor when running light is very low, since the major portion of the stator current is reactive magnetizing current. As the load is increased, the no-load magnetizing current remains approximately constant, while the active component increases on account of the load. The current has now, however, another reactive component, due to the flux which is forced into the leakage paths, but this does not prevent the power factor from rising to a very high value. As the load is increased, the effect of this component becomes more marked, and the power factor begins to decrease again after having passed through a maximum value. The shape of the power factor curve is shown in Fig. 279.

Efficiency.—The mechanical output of the motor was shown on page 358 to be $0{\cdot}142nT$ watts, whilst the input is given by wattmeter readings and is equal to $W_3 = \sqrt{3}\ EI \cos\phi$ and $W_2 = 2EI \cos\phi$ in a three- and two-phase case respectively, E and I being the line values.

The efficiency is thus

$$\frac{0{\cdot}142nT}{W_3} \text{ or } \frac{0{\cdot}142nT}{\sqrt{3}EI\cos\phi} = \frac{0{\cdot}082nT}{EI\cos\phi} \text{ (3-phase)},$$

and

$$\frac{0{\cdot}142nT}{W_2} \text{ or } \frac{0{\cdot}142nT}{2EI\cos\phi} = \frac{0{\cdot}071nT}{EI\cos\phi} \text{ (2-phase)}.$$

The efficiency curve follows the same general law as in D.C. motors, commencing at zero for no-load and rising to a maximum, after which it commences to drop slightly, as shown in Fig. 279. The efficiency and power factor curves are very similar, their chief difference lying in the fact that the former starts from zero whilst the latter does not.

Apparent Efficiency.—If the effect of power factor be not taken into account, it appears as if the input were equal to $\sqrt{3}\ EI$, and this quantity is called the *apparent power*. The ratio of the output to the apparent power input is called the *apparent efficiency*, and is, of course, equal to the product of the true efficiency and the power factor. Values of the apparent efficiency are plotted in Fig. 279 along with the other performance curves.

Heat Run.—In order to carry out a heat run on an induction motor, it is necessary to load it up in such a manner that the load can be maintained for a number of hours. The band brake method previously described is not, therefore, applicable, and some other method must be adopted. The usual practice is to maintain full load for six hours and then to determine the temperature rises in various parts of the motor, usually by thermometer. In order to avoid wasting the energy involved in the test, which is considerable in the case of large motors, a D.C. generator is sometimes coupled to the motor and loaded back on to the D.C. supply. In this case, the power consumed is only that used up in overcoming the losses in the two machines.

Measurement of Slip.—There are three main methods of measuring slip, viz.

1. By measurement of synchronous and actual speeds.

2. By measurement of rotor frequency.
3. By stroboscopic method.

These will be dealt with in turn.

1. The actual speed of the motor is measured and also the speed of the driving alternator or any synchronous apparatus connected to the supply, the slip being obtained from the difference of the two readings. Expressed as a percentage, this is

$$\frac{\text{Synchronous speed} - \text{Actual speed}}{\text{Synchronous speed}} \times 100.$$

If the number of poles on the alternator is not the same as the number on the induction motor, the speed of the alternator must be multiplied by the ratio of the numbers of poles. This is not a very accurate method of determining the slip, albeit a convenient one, since it depends on the difference of two quantities very nearly equal, and a small error in determining either of the speeds leads to a very large error in the result.

2. Since the rotor frequency is very low, not often exceeding one or two cycles per second, it can be read on a moving coil ammeter by counting the oscillations of the pointer. The method adopted, therefore, is to connect a moving coil ammeter in series with one of the leads going from the slip rings to the starter and count the beats in a given time. If a central zero instrument be used, the number of complete to and fro swings must be counted, but in the ordinary type the reverse half of the wave will produce no effect, since the pointer will be pressing against the zero stop. The percentage slip is then given by

$$\frac{\text{Rotor frequency}}{\text{Stator frequency}} \times 100.$$

If the motor is of the squirrel cage type, this method must be modified, since there are no slip rings. There is usually a small portion of the magnetic field going right through the centre of the rotor and cutting the shaft, which has a small E.M.F. induced in it. This can be detected by pressing a lead against each end of the shaft, the other ends being connected to a sensitive moving coil millivoltmeter. The measurement is then made in the same way as with the wound rotor.

3. In the stroboscopic method, a disc, with alternate black and white sectors painted on it, is attached to the end of the motor shaft. This disc is illuminated by means of an arc or other lamp running from the same supply. If the disc is normally in a very poor light, an incandescent lamp is sufficient, this being preferable, since it is not so likely to unbalance the voltages of the system. Assuming the number of black sectors to be equal to the number of poles on the motor, one sector moves forward a pole pitch in half a cycle if there is no slip at all and the next black sector occupies the

original position of the first. But the disc is illuminated once every half-cycle, so that, viewed in the light of the arc, the disc appears to be stationary. But since a certain amount of slip occurs, the second black sector does not quite reach the initial position of the first in half a cycle, with the result that the disc appears to rotate slowly in a direction opposite to that of the motor. A complete apparent revolution of the disc thus corresponds to a slip of as many cycles as there are pairs of poles on the motor. If the apparent revolutions of the disc per minute are counted, the frequency of slip is obtained from

$$\frac{\text{apparent r.p.m.}}{60} \times \text{pairs of poles.}$$

The percentage slip is then calculated as before. For testing motors with different numbers of poles, a series of discs is required having different numbers of sectors painted on them.

Instead of using an arc or an incandescent lamp, an electrically driven tuning fork may be used (see Fig. 280). This must have a natural period of vibration equal to the frequency of supply, and is kept in action by means of a small electromagnet operated from accumulators through an ordinary make and break. When the circuit is made, the two prongs of the fork are attracted, thus breaking the circuit and allowing the prongs to fly back. At the top end of each prong of the fork is attached a metal flag with a slot cut in it. These slots are arranged to overlap in the normal position, so that when the fork is vibrating the observer can see through the flags twice per period. The tuning fork acts in the same way as the arc lamp, therefore, in allowing the observer to see the disc on the motor shaft twice per period. This arrangement is much more convenient than that of the arc lamp, but it suffers from the disadvantage of only operating on the one frequency.

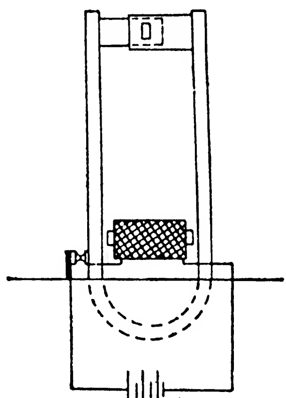


FIG. 280.—Tuning Fork for Slip Measurement.

In the slip stroboscope of Dr. Drysdale, a small synchronous motor drives a conical roller, and this in turn drives a small disc containing a number of slots. The speed of the disc is adjusted by sliding it up or down the roller until it is running at the same speed as the motor. The painted disc on the motor shaft now appears stationary and the slip is indicated by the position of the disc resting on the roller.

Measurement of Resistance.—The resistance of the stator windings can be measured conveniently by passing a D.C. through them, obtained from a few accumulators, and determining the current and voltage. If the windings are connected in star, the resistance per phase is half that observed, since there are two circuits in series, whilst if the circuits are connected in delta there are two windings in series connected in parallel with the third. If r be the resistance per circuit, then the combined resistance, as determined by the voltmeter and ammeter, is

$$R = \frac{1}{\frac{1}{r} + \frac{1}{2r}} = \frac{2r}{3}$$

and $$r = \tfrac{3}{2}R.$$

The same can be done for the rotor circuits if it is a slip ring motor, but care must be taken to measure the voltage drops across the rings directly and not across the brushes or terminals, since

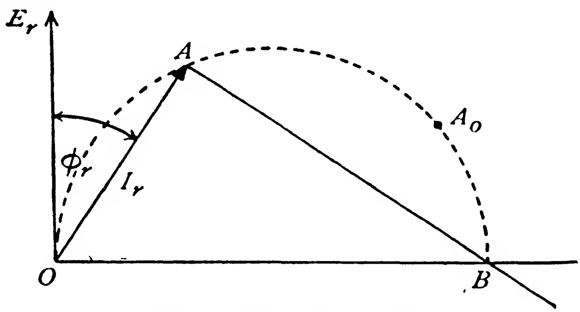


FIG. 281.—Rotor Circle Diagram.

the effect of the brush resistance is very appreciable, and this is cut out when the motor is running normally.

Simple Circle Diagram.—In order to simplify the development of the circle diagram, one phase only will be considered, this being legitimate since each of the phases acts in the same way. Certain assumptions are made in the first instance as follows : (1) the applied stator voltage is constant, (2) the voltages, currents, and fluxes are sinusoidal, (3) the flux is proportional to the magnetizing current, *i.e.* saturation is neglected, and (4) the stator resistance is negligible. A circle diagram for the rotor circuit will first be constructed, and then, by considering the motor as a special kind of transformer, a circle diagram for the stator will be developed, utilizing the transformer vector diagram for the purpose.

Rotor Circle Diagram.—In Fig. 281 let E_r be the induced rotor E.M.F. per phase (*i.e.* between slip ring and star point), and let I_r represent the rotor current to scale, lagging by an angle ϕ_r. Draw OB at right angles to OE_r and AB at right angles to OA, so as to intersect at B. Then if it can be proved that OB is constant for all values of I_r, it follows that the point A must always lie in a semicircle erected on OB as diameter, since the angle OAB is always a right angle by construction.

The angle OBA is equal to ϕ_r, so that

$$OB = \frac{OA}{\sin \phi_r} = \frac{I_r}{\frac{X_r}{Z_r}} = \frac{E_r}{Z_r} \times \frac{Z_r}{X_r} = \frac{E_r}{X_r}$$

$$= \frac{sE_2}{sX_2} = \frac{E_2}{X_2} = \text{constant.}$$

The locus of the rotor current vector is therefore a semicircle erected on OB as diameter drawn at right angles to OE_r. The

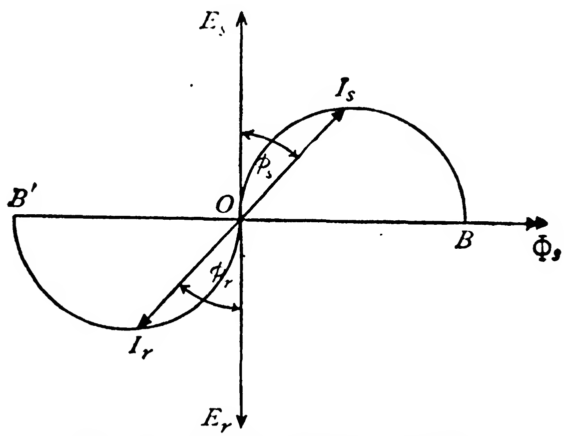


FIG. 282.—Derivation of Stator Circle Diagram.

length OB represents the hypothetical current which would flow in the rotor circuit at standstill, if no resistance were present.

At synchronous speed the rotor current is zero and A coincides with O, whilst at standstill the point A assumes some such position as A_0 and not B, since the rotor resistance prevents the rotor current from lagging by the full 90° behind the rotor E.M.F.

Stator Circle Diagram Neglecting Losses and Magnetizing Current.—In Fig. 282 let OE_s be the applied stator voltage per phase, and OI_s the stator current. The combination of the three-phase currents sets up a rotating M.M.F. and a rotating flux which can be represented by $O\Phi_s$ drawn at right angles to OE_s. This rotating flux causes counter E.M.F.'s to be induced in the stator windings so as to neutralize exactly the applied voltages, since losses are neglected. The rotating flux also cuts the rotor conductors and induces E.M.F.'s in them, giving rise to rotor currents. In their turn these rotor currents set up a rotating M.M.F. which rotates at slip frequency with respect to the rotor, and as the rotor is rotating at speed, this rotating rotor M.M.F. must rotate in space at synchronous frequency (= slip frequency + speed frequency). The rotating M.M.F.'s due to both stator and rotor therefore rotate in space at the same speed, and must exactly neutralize each other, since losses and magnetizing current are for the moment neglected. It is convenient to draw the rotor induced E.M.F. OE_r as shown in Fig. 282, despite the fact that it is operating at a different frequency from OE_s. The rotor current can now be represented by OI_r, exactly equal in length to OI_s and in exact phase opposition. In order to obtain this exact equality in length, different current scales are chosen for stator and rotor, 1 cm. representing n times as many amperes in the rotor as in the stator, where

$$n = \frac{\text{stator turns per phase}}{\text{rotor turns per phase}}.$$

(The same device was adopted in the transformer vector diagram.) Since the locus of I_r is a semicircle erected on OB', and OI_r is equal

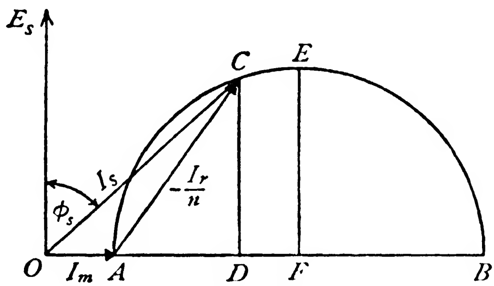


FIG. 283.—Simple Circle Diagram.

and opposite to OI_s, it follows that the locus of OI_s is a semicircle erected on OB as a diameter. (The full diagram is really a circle diagram and not a semicircle diagram, but the other half of the circle represents conditions where the machine is operating as a generator and not as a motor, and is not here considered.)

Allowance for Magnetizing Current.—In Fig. 282 the stator and rotor M.M.F.'s exactly neutralize each other, so that there is no resultant M.M.F. to set up the flux. Actually there must be a resultant M.M.F., and this is provided by an additional magnetizing current which flows through the stator windings. This stator magnetizing current lags behind the stator applied voltage by 90° and is represented in Fig. 283 by $OA = I_m$, which is constant in magnitude so long as Φ_s and E_s are constant. The semicircle is now moved bodily to the right by a distance OA. The stator current becomes $OC = I_s$ and consists of two components (1) $OA = I_m$ required to set up the rotating flux, and (2) $AC = -\frac{I_r}{n}$

required to neutralize the rotating M.M.F. due to the rotor current I_r in the corresponding rotor phase, exactly as in the case of the transformer. Alternatively, the resultant magnetizing current may be regarded as the vector sum of OC, the stator current, and CA, the rotor current (corrected for turn ratio).

As the load on the motor is increased, the point C moves round the semicircle away from A and towards B. The power supplied to the motor is proportional to $I_s \cos \phi$, and can thus be represented to scale by CD. It is seen that this line attains a maximum length when it coincides with EF, which makes it clear why an induction motor breaks down when a certain load is reached. The application of a greater load causes the current to increase, but this results in an actual decrease in the power supplied, with the result that the motor comes to rest. The maximum load which an induction motor can overcome without pulling up is called the *breakdown load.*

It is also seen that the power factor under which the motor operates varies with the load and attains a maximum value considerably before the breakdown load is reached. In fact, power factor usually attains a maximum value somewhere in the neighbourhood of full load in a commercial motor.

Dispersion Coefficient.—In the theoretical motor the performance of which is represented by the simple circle diagram in Fig. 283, the line OA represents the stator current when the motor is running light. This is a purely reactive magnetizing current, since the losses are neglected. As no torque is required, the motor runs with no slip (*i.e.* at synchronous speed), because in these circumstances there is no rotor E.M.F. generated, and hence no rotor current is produced. The flux set up by the stator is thus free to cut the rotor, because no back ampere-turns are set up in the rotor, with the result that the stator flux passes through the useful rotor paths and the waste leakage paths in parallel. Representing the reluctances of these paths by R_u and R_l respectively, the combined reluctance is given by $\dfrac{R_u \times R_l}{R_u + R_l}$ and the magnetizing current is represented by OA.

Next consider the same motor to be clamped so that the rotor cannot move. On applying the same E.M.F. as before to the stator, a reactive magnetizing current flows, but this time the motor acts like a transformer with its secondary short-circuited. An E.M.F. is induced in the rotor winding which produces a current owing to its being short-circuited. This current sets up a number of back ampere-turns which oppose the passage of the flux through the rotor. The stator now takes a current from the mains capable of setting up just sufficient ampere-turns to balance the rotor ampere-turns together with the ampere-turns necessary to magnetize the system. This really means that the whole of the stator flux has been deflected into the leakage paths. The magnitude of the flux has not been decreased, since this depends upon the applied voltage, which is maintained constant. It merely means that one of the parallel paths through which the flux passed has been cut off. The flux now being confined to the leakage paths, it follows that a larger magnetizing current is required, and this is given by *OB* (Fig. 283) since the current vector has passed completely round the semicircle.

The magnetizing currents in the two cases are proportional to the respective reluctances, since the flux is the same, and therefore

$$\frac{OA}{OB} = \frac{\dfrac{R_u \times R_l}{R_u + R_l}}{R_l} = \frac{R_u}{R_u + R_l} = \frac{1}{1 + \dfrac{R_l}{R_u}}.$$

But
$$\frac{R_l}{R_u} = \frac{\text{Useful flux}}{\text{Leakage flux}}.$$

Therefore
$$\frac{OA}{OB} = \frac{1}{1 + \dfrac{\text{Useful flux}}{\text{Leakage flux}}} = \frac{1}{\dfrac{\text{Leakage flux} + \text{Useful flux}}{\text{Leakage flux}}} = \frac{\text{Leakage flux}}{\text{Leakage flux} + \text{Useful flux}} = \frac{\text{Leakage flux}}{\text{Total flux}}.$$

This ratio, which is a most important one in induction motor design, is called the *Dispersion Coefficient* and is represented by σ, whilst its value largely settles the shape of the circle diagram.

Leakage Factor.—The leakage factor is defined as the ratio

$$\frac{\text{Total flux}}{\text{Useful flux}} = \lambda.$$

There is a direct relation between this ratio and the dispersion coefficient, for

$$\frac{1}{\lambda} = \frac{\text{Useful flux}}{\text{Total flux}}$$

$$1 - \frac{1}{\lambda} = \frac{\text{Leakage flux}}{\text{Total flux}} = \sigma.$$

Also
$$\lambda = \frac{1}{1 - \sigma}.$$

The magnitude of the dispersion coefficient thus settles the value of the leakage factor, and *vice versa.*

Maximum Power Factor.—The maximum power factor under which an induction motor can operate is directly connected also with the dispersion coefficient. The conditions for maximum power factor are shown in the simple circle diagram in Fig. 284, where OC, representing the stator current, is drawn tangential to the semicircle ACB. The angle of lag, ϕ, is now a minimum, and consequently the power factor is a maximum. But since the angle OCD is a right angle, the angle ϕ is equal to the angle ODC, and the power factor is given by $\frac{CD}{OD}$. But $CD = AD = \frac{1}{2}AB$. Therefore the maximum power factor is equal to

$$\frac{AB}{2 \times OD} = \frac{AB}{2 \times OA + 2 \times AD}$$

$$= \frac{AB}{2 \times OA + AB}$$

$$= \frac{AB}{OB + OA}$$

$$= \frac{OB - OA}{OB + OA}.$$

Dividing top and bottom by OB this becomes

$$\frac{\frac{OB}{OB} - \frac{OA}{OB}}{\frac{OB}{OB} + \frac{OA}{OB}}$$

$$= \frac{1 - \sigma}{1 + \sigma}.$$

A simple approximation to this formula is given by

$$\text{Maximum power factor} = (1 - 2\sigma),$$

which is near enough for the majority of purposes.

For example, a motor having a dispersion coefficient of 0·05 cannot work on a higher power factor than $(1 - 2 \times 0{\cdot}05) = 0{\cdot}90$.

Power Input.—The applied voltage being maintained constant, the power input is proportional to $I \cos \phi$. But ϕ is equal to the angle OCD (see Fig. 285), and I is represented by OC. Therefore $I \cos \phi$, and consequently the power input, is represented to scale by

$$OC \cos OCD = CD.$$

The power input for any current is thus represented to scale by a vertical line dropped from the point C on to OB. In this way, the maximum power input is represented by EF.

Overload Capacity.—The overload capacity may be defined as the maximum percentage overload that the motor will stand without breaking down and (neglecting all losses) is equal to

$$\frac{\text{Maximum power input} - \text{Full load input}}{\text{Full load input}} \times 100$$
$$= \frac{EF - CD}{CD} \times 100 \text{ (see Fig. 285),}$$

assuming the point C to represent the full load conditions. If the no-load current be expressed as k per cent. of the full load current, then

$$CD = OC \cos\phi$$
$$= \frac{100}{k} \times OA \times \cos\phi,$$

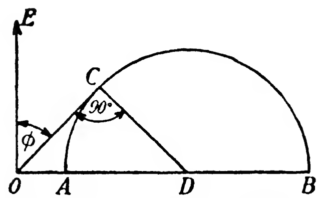


Fig. 284.—Construction for Maximum Power Factor.

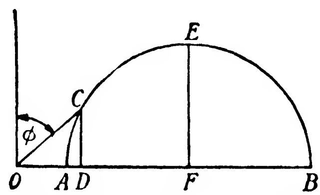


Fig. 285.—Construction for Power Input.

and the overload capacity becomes equal to

$$\left(\frac{k \times EF}{100 \times OA \times \cos\phi} - 1\right) \times 100$$
$$= \left(\frac{k \times AB}{200 \times OA \times \cos\phi} - 1\right) \times 100.$$

But
$$\frac{AB}{OA} = \frac{OB - OA}{OA} = \frac{OB}{OA} - 1$$
$$= \frac{1}{\sigma} - 1 = \frac{1 - \sigma}{\sigma}.$$

Therefore the overload capacity equals

$$\frac{k(1 - \sigma)}{2\sigma \cos\phi} - 100.$$

If the losses of the motor are taken into consideration, the overload capacity is less than is given by the above formula.

Effect of Losses.—The losses to be considered are the iron and friction loss and the stator and the rotor copper losses. The stator iron loss is practically constant for all loads, since it depends upon the applied E.M.F. The rotor iron loss is negligible, since the frequency is only that of the slip. The frictional loss is also practically constant, since for all working loads the speed is approximately constant. All these losses may therefore be regarded as a constant loss, and since the power input is represented by the height of vertical lines such as CD (Fig. 286), a horizontal line may be drawn through E, a little above AB, cutting off a portion of the vertical power lines. The height of this line is obtained from the no-load observation of the motor, OE representing the no-load current and the height of E above AB being determined by the no-load watts. Strictly speaking, the line through E should droop downwards slightly towards the right, since as the current increases the slip increases, thus decreasing the frictional watts slightly, whilst the increased stator IR drop causes a slight reduction in the flux, thus decreasing the iron loss. On the other hand, the rotor frequency increases and the rotor iron loss may cease to be negligible.

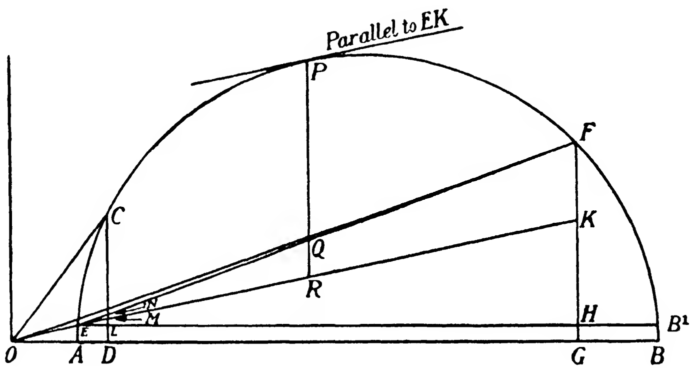


FIG. 286.—Circle Diagram for Commercial Motor.

Next consider the behaviour of the motor when the rotor is clamped. The whole of the power input is now wasted in losses, and the stator current vector takes up some position such as OF, the vertical height, FG, representing the total losses. Of these GH represents the constant loss, whilst HF represents the stator and rotor copper losses. But since the stator current is approximately proportional to the rotor current, the total copper losses may be taken as proportional to the (rotor current)^2, *i.e.* to EC^2 (not shown in the diagram for the sake of clearness). Now $\frac{EC}{EB'} = \cos CEB'$, and since EB' is constant, EC is proportional to $\cos CEB' = \frac{EL}{EC}$. (Note that the centre of the semicircle now lies in EB'.) EL is therefore proportional to EC^2, *i.e.* to the total copper losses. A line joining EF therefore cuts off portions of vertical power lines proportional to EL, *i.e.* to the total copper losses for the current in question. Furthermore, FH may be divided at K, so that HK represents the stator copper loss and KF the rotor copper loss. On joining EK, the copper losses for all currents are separated, the vertical distance between EH and EK representing the stator copper loss and the vertical distance between EK and EF representing the rotor copper loss.

Having accounted for all the losses, the remainder of the vertical through any point, C, represents the output. Thus CD represents the input, DL the constant loss, LM the stator copper loss, MN the rotor copper loss and NC the output.

The efficiency for any current or output can be determined directly from the circle diagram when the loss lines are drawn, it being given by $\frac{CN}{CD} \times 100$. The maximum output is obtained by drawing a tangent to the semicircle parallel to EF. The point of contact represents the point at which maximum output is obtained.

Torque Line.—It was shown on page 340 that

$$\frac{\text{Rotor input}}{\text{Rotor output}} = \frac{\text{Synchronous speed}}{\text{Actual speed}},$$

and the rotor output is equal to $2\pi nT$, n being the speed and T the torque. The rotor input is, therefore, equal to

$$2\pi nT \times \frac{\text{Synchronous speed}}{\text{Actual speed}}$$

$$= 2\pi T \times \text{Synchronous speed}.$$

For a constant frequency, the torque is therefore proportional to the rotor input, and this is given in Fig. 286 by the line CM, since CN is the output and NM the rotor copper loss. The scale, in lb.-ft., can be obtained from a knowledge of the rotor input, in watts, and the speed at any one point, the watts being converted into H.P. and then into foot-lb. per minute. The starting torque is given by FK, but by inserting resistance in the rotor circuit the point F is moved round the semicircle towards A, and the starting torque is thus increased up to a maximum value. The maximum torque is obtained by drawing another tangent to the semicircle, this time parallel to EK, the point of contact indicating the required maximum torque. The necessary starting resistance to obtain the maximum starting torque is obtained from the line PQR. QR represents the watts lost in the rotor and PQ those lost in the starting resistance. Since the currents are the same in both, the magnitude of the external resistance per phase is given by

$$\frac{PQ}{QR} \times \text{Rotor resistance per phase.}$$

Determination of Slip from Circle Diagram.—From the rotor circle diagram, and also from Fig. 44, it is seen that $\frac{OA}{AB} = \frac{I_r s X_2}{I_r R_2} = \frac{s X_2}{R_2}$, and therefore in Fig. 287, $\tan \alpha = \frac{EC}{CB'} = \frac{sX_2}{R_2}$

and
$$s = \frac{R_2}{X_2} \tan \alpha.$$

Considering the short-circuit point F, where $s = 1$, it follows that

$$s = 1 = \frac{R_2}{X_2} \times \frac{EF}{FB'}$$

and
$$\frac{R_2}{X_2} = \frac{FB'}{EF}.$$

Therefore
$$s = \frac{FB'}{EF} \tan \alpha.$$

The ratio $\frac{FB'}{EF}$ is a constant which can be scaled off the diagram, so that the slip is determined by the simple measurement of $\tan \alpha$.

The method can be elaborated by constructing a scale of slip on $O'V$, where O' is obtained by projecting $B'E$ backwards to meet OV. Suppose that it is desired to construct a scale for slips ranging from $s = 0$ to $s = 0{\cdot}1$, *i.e.* 10 per cent. slip. Then

$$s = 0{\cdot}1 = \frac{FB'}{EF} \tan \alpha \text{ and } \tan \alpha = 0{\cdot}1 \times \frac{EF}{FB'}.$$

Since EF and FB' are known, α can be determined. Then produce $B'C$ to cut $O'V$ at s, when $\frac{O'S}{O'B'} = \tan \alpha$, and $O'S = O'B' \tan \alpha = 0{\cdot}1 \times O'B' \times \frac{EF}{FB'}$. The length of $O'S$ can thus be evaluated, after which it can be divided into ten equal divisions each representing one per cent. slip.

If resistance is inserted in the rotor circuit, the short-circuit point F is moved to some such position as F', and the slip is increased in the ratio of $\frac{F'B'}{EF'} \div \frac{FB'}{EF} = \frac{F'B'}{FB'} \times \frac{EF}{EF'}$.

Experimental Determination of Circle Diagram.—In order to draw the circle diagram of a motor, it is necessary first to make an observation of the no-load current, watts and power factor at the correct voltage and frequency. Commencing with the main horizontal and vertical lines, the no-load current is set off to scale, the angle of lag being determined from the no-load power factor which is obtained from the voltmeter, ammeter and wattmeter readings. It is convenient to let the current vectors represent the line values, whilst the vertical power lines are made to represent the total power of all the phases. The point E on the circle diagram (Fig. 286) is thus determined, and a horizontal line is drawn through it to represent the constant iron and friction loss. It is now necessary to obtain another point or series of points on the semicircle, preferably far removed from E. For this purpose, the rotor may be clamped and the short-circuit current measured, thus obtaining the point F. The height of F above AB is determined either from the angle of lag of the current or from the watts supplied to the motor. Unfortunately, however, the current which would flow if the normal voltage were applied would be such as to burn out both stator and rotor. It is therefore necessary to reduce considerably the voltage applied to the stator to prevent this occurring. The stator current actually measured in this way must

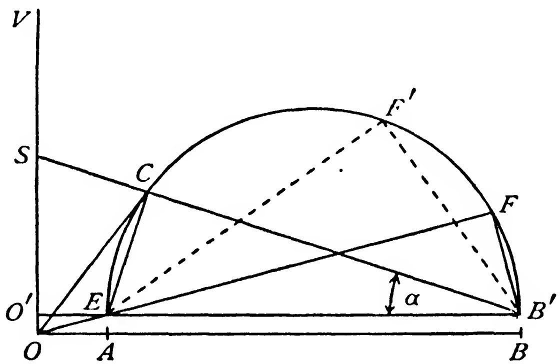


FIG. 287.—Construction for Slip.

then be increased in the direct ratio of the normal voltage to the actual voltage applied. This assumes that the stator current is proportional to the voltage, and this is not strictly true, on account of the different flux densities set up in the core. To minimize this error as much as possible, it is desirable to pass as large a current through the motor as is safe in order to approximate to the actual conditions. Even then the accuracy of the position of the short-circuit point is scarcely good enough to warrant the drawing of the semicircle without further data. In place of the short-circuit point a number of readings in the neighbourhood of full load may be employed, including some on considerable overload. The position of each point is marked off from a knowledge of the corresponding values of the current and watts. A semicircle is then drawn through the no-load point, the various load points and the short-circuit point if the latter has been obtained, the centre of the semicircle lying in EB'. The disadvantage of relying on the load points is that they are comparatively close to the no-load point, thus making it difficult to draw in the semicircle accurately, whilst the disadvantage of relying on the short-circuit point lies in its dubious accuracy. Obviously, the best effect is obtained by employing both methods, a mean semicircle being drawn through all the points. Points on the upper regions of the semicircle, including the unstable portion to the right of the maximum output point, can be obtained by clamping the rotor and inserting the starting resistance in the rotor circuit. This is equivalent to increasing the rotor resistance and the losses occurring in the rotor circuit. The vertical height, FK (Fig. 286), is thus increased, which means that the point F is moved round the semicircle towards the left. By putting the handle of the starting resistance in various positions, a number of observations may be made giving points in the unstable region. By still further increasing the resistance of the rotor circuit, points all round the semicircle can be obtained, and these additional points should agree with the load readings already obtained. Care must be taken to obtain these observations rapidly, as otherwise there is a danger of the starter or the motor being burnt out. The difference between these readings with the rotor clamped and those with the motor running and giving an output is that the power which goes to the external load in one case goes to heat up the starting resistance in the other. From the point of view of the circle diagram, it does not matter whether the watts are dissipated at the pulley or in the external resistance.

The lines representing the stator and rotor copper losses are next drawn in. For this purpose the stator and rotor resistances are measured, together with a corresponding pair of values for the currents in the two circuits. The watts lost in each are now calculated. If there is a different number of phases in the stator and rotor, this must be taken into account. A vertical line through F is now drawn and divided at K so that $\frac{HK}{HF} = \frac{\text{Stator copper loss}}{\text{Total copper loss}}$. If the rotor is of the squirrel cage type, the actual stator copper loss must be calculated for the current OF and HK measured off to the scale of watts already determined. If the short-circuit point, F, is not available, the losses for some known load must be measured. Points such as L, M and N corresponding to a current, OC, can thus be determined and the loss lines drawn through these points. Unfortunately, these points lie fairly close together, and a considerable error is liable to be introduced in drawing the diagram.

To obtain the torque scale, the slip is measured from the diagram for one particular output. This gives the speed from which the torque can be calculated, since the output is known.

Importance of σ.—It is seen that the relative shape of the circle diagram is largely dependent upon the value of σ. The no-load current is greatly affected, and this in turn affects the power factor of the motor. An increase in the value of σ results in a decrease in the maximum power factor, thus causing a slightly larger current to flow, which in turn causes a slight drop in the efficiency. An increase in σ also results in a decrease in the overload capacity of the motor. This is seen to be the case, since, for a constant applied voltage, the total stator flux *OB* (Fig. 286) may be considered constant, and an increase in σ will consequently result in an increase in *OA* and a decrease in *AB*. The diameter of the circle is thus reduced, resulting in a decrease in the maximum power output. If the full load is the same as before, this is equivalent to saying that the overload capacity is reduced. It is therefore desirable to aim at obtaining a low value for σ when designing an induction motor.

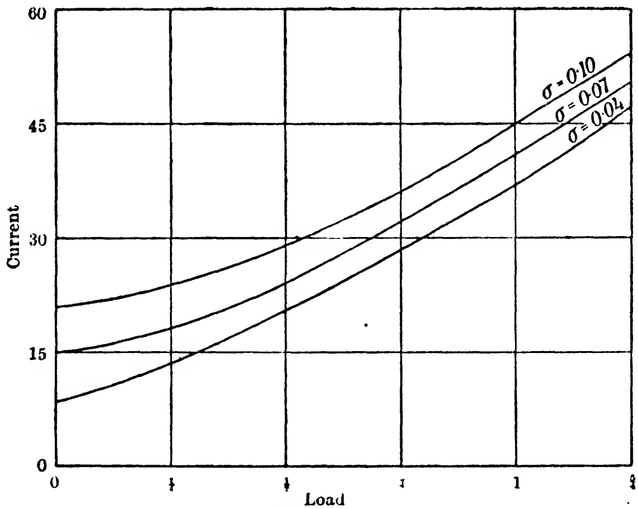


FIG. 288.—Effect of σ on Current.

Effect of Air-gap Length.—The length of the air-gap exerts a great influence on the magnitude of the leakage flux, inasmuch as the reluctance of the paths of the useful rotor flux is largely determined by it. The reluctance of the leakage paths is independent of this dimension, and so the value of σ depends to a very great extent upon the radial length of the air-gap. An increase in this direction, therefore, affects adversely both the power factor and the overload capacity of the motor to a very considerable degree, and to prevent this it should be reduced to the smallest possible limits consistent with good mechanical design. This is the reason why induction motors are designed with very much smaller air-gaps than D.C. motors of the same size, the lengths varying from 0·5 mm. for a rotor diameter of 12 cms. up to 3 mm. for a diameter of 1 metre.

Effect of σ on Characteristics.—In order to study the effect of

σ on the characteristics of motors of the same size and output, three cases have been chosen in which the values of σ have been taken to be 0·04, 0·07 and 0·10 respectively. From the three circle diagrams curves have been plotted in Figs. 288 and 289, showing the relationships between (*a*) the current and the output and (*b*) the power factor and the output for each case. The inferiority produced by the larger values of σ is clearly seen from these curves. The no-load currents for the three motors are widely different, whilst the breakdown loads are also affected. The following table gives

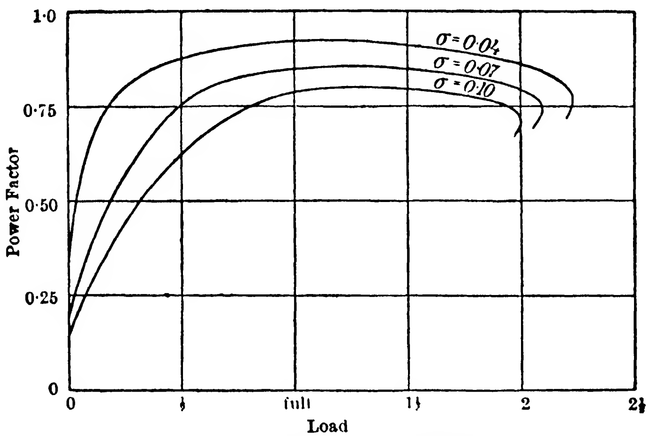


FIG. 289.—Effect of σ on Power Factor.

some comparative data for the three motors, obtained from their circle diagrams.

Motor No.	1	2	3
σ	0·04	0·07	0·10
No-load Current	8·5	15	21
Full Load Current...	37	41	45
Full Load Power Factor... ...	0·92	0·85	0·78
Overload Capacity	123%	112%	101%
Ratio $\frac{\text{No-load Current}}{\text{Full Load Current}}$	0·23	0·37	0·47

EXAMPLES

(1) A moving coil ammeter is connected in the rotor circuit of a 4-pole 50-cycle induction motor, and it is observed to give five swings every two seconds. What is the slip in r.p.m.?

(2) An 8-pole 50-cycle induction motor has an 8-sector stroboscopic disc mounted on its shaft. This disc appears to rotate at 25 r.p.m. What is the speed of the motor?

(3) A certain induction motor has a dispersion coefficient of 0·06. What is the maximum power factor under which the motor can operate, and what is its probable full load power factor?

(4) Prove from first principles that the performance of a three-phase induction motor can be represented by means of a circle diagram.

Derive an expression for the dispersion coefficient, and show how it is represented on the circle diagram.

(5) Draw the circle diagram for a 6-pole star-connected three-phase induction motor having the following data :—

Rated output 40 B.H.P. at 380 volts, 50 cycles.
No-load current 15 amperes.
Power absorbed at no-load 1500 watts.
Short-circuit (standstill) current 105 amperes at 130 volts and 0·23 power factor.
Resistance from terminal to terminal of stator winding (hot) 0·155 ohm.

Find the full load power factor, maximum output and maximum torque.

CHAPTER XXII

INDUCTION MOTORS.—PRINCIPLES OF DESIGN

Specification.—In order to design an induction motor, it is necessary to know the voltage, frequency and number of phases of the supply, together with the B.H.P. and speed required. With a given frequency, only certain speeds are possible, these being determined by the number of poles. The efficiency and power factor at which the motor is expected to work are specified also in a number of cases.

Number of Poles.—The relation between the frequency, number of poles, and synchronous speed is the same as for alternators, the formula being

$$p = \frac{120f}{n},$$

n being the synchronous speed in r.p.m. The actual speed is about 2 to 5 per cent. less than the synchronous speed, the difference depending on the slip.

Efficiency and Power Factor.—The average full load efficiencies and power factors are given in the following table :—

B.H.P.	1	5	10	50	100	200	500
Efficiency % ...	80	86	88	89	90	91	92
Power Factor ...	0·78	0·88	0·88	0·89	0·90	0·90	0·90

Motors having a large number of poles have a relatively large leakage flux owing to the reduced pole pitch. This increases the value of σ and reduces the power factor, so that for low speed motors with many poles, the power factor is lower than is given above. On the other hand, motors operating on a low frequency have a small number of poles for a given speed, so that 25-cycle motors may be expected to operate with slightly higher power factors than 50-cycle motors.

Output Coefficient.—The D^2L formula holds good with induction motors just as with alternators (see page 305), and is now expressed as

$$\frac{\text{B.H.P.}}{\text{r.p.m.}} = k \times D^2L,$$

D being the air-gap diameter, L the gross iron length of the core, and k the output coefficient.

The average values of the output coefficient for modern polyphase induction motors operating on a frequency of 50 cycles per second are shown in the following table. For 25-cycle motors these figures can be increased by about 15 per cent. The values of the output coefficient must not be taken as being rigidly fixed, since various manufacturers differ somewhat in this respect.

B.H.P.	10	50	100	500	1000
Output Coefficient $\times 10^6$ $= k \times 10^6$	1·2	2·5	3·3	4·4	4·4

Air-gap Diameter.—The air-gap diameter depends upon the B.H.P. and the speed in the same manner as the output coefficient. The diameter increases as the B.H.P. goes up, but since the bulk of the carcase is inversely proportional to the speed, it is also necessary to increase the diameter for the low speeds in order to make room for the larger number of poles. There is no simple way in which D^2L can be separated into its two components so as to give the best design. If the diameter is too large the overload capacity is small, whilst if the diameter is too small the magnetizing current is large and the power factor is poor. The air-gap diameters at present employed correspond to peripheral speeds up to 80 metres per second.

It is found that the ratio $\frac{\text{pole pitch}}{\text{core length}} = \frac{\tau}{L} = a$ usually lies between the limits of 0·5 and 1·6, and this enables an approximate value to be assigned to the air-gap diameter. The value of D^2L having been ascertained, this can be written $D^2 \times \frac{\pi D}{ap} = \frac{\pi}{ap} \times D^3$, from which

$$D = (0{\cdot}54 \text{ to } 0{\cdot}8) \times \sqrt[3]{p \times D^2L}.$$

A value for D can now be chosen by considering the following relationship which connects the flux per pole and the H.P., speed, and frequency, viz.,

$$\left.\begin{matrix}\text{Flux per pole}\\ \text{in megalines}\end{matrix}\right\} = \frac{k}{f} \times \sqrt{\text{H.P.} \times \text{synch. speed in r.p.m.}}$$

where k is equal to 0·35 for three-phase, and 0·37 for two-phase motors.

This formula can be derived in the following manner.

Let Total magnetic loading $= x \times$ Total electric loading.

or $$p\Phi = xIZ = 2xIT,$$
$$\Phi = 2xIT/p$$
and $$\Phi^2 = 2xIT\Phi/p.$$

Substituting $$p = \frac{120f}{n},$$

$$\Phi^2 = \frac{2xIT\Phi n}{120f}$$

and since $$E = 4{\cdot}44k_2k_3f\Phi T \times 10^{-8}$$

where k_2 is the breadth factor and k_3 is the pitch factor (see page 270),

$$\Phi^2 = \frac{2xEIn \times 10^8}{4{\cdot}44 \times 120f^2}$$

and $$\Phi = \frac{k'}{f}\sqrt{EIn}$$

$$= \frac{k}{f}\sqrt{\text{H.P.} \times n.}$$

On choosing a suitable flux density, the polar area can be calculated approximately, and an air-gap diameter selected to fit in with the above permissible range.

Core Length.—When the air-gap diameter has been settled the gross length of the core can be determined approximately, since the value of D^2L is known.

When a complete line of motors is to be designed, it is the usual practice to have two or three core lengths for each diameter. The most economical designs are not obtained in this way, considering each one separately on its own merits, but it leads to a reduction in the manufacturing costs, since it reduces the number of patterns required. If three motors have the same diameter and differ merely in the length of their cores a large number of their parts are similar. The middle machine is designed for the particular diameter in question, the short length one having a larger diameter and the long length one a smaller diameter than would be expected if they were designed separately.

Ventilating Ducts.—The usual practice with regard to the number and width of the radial ventilating ducts is similar to that which obtains in the case of alternators. They are spaced usually about 6 to 8 cms. apart and are about 1 cm. in width. Axial ventilation is also employed.

Specific Electric Loading.—The number of ampere-conductors per cm. periphery, or the specific electric loading, q, generally lies between certain limits. Average figures are given in the following table :—

B.H.P.	10	20	50	100	200	500	1000
q	180	230	300	350	400	440	440

Multiplying q by the periphery in cms. and dividing by the current per conductor enables the approximate number of conductors to be determined.

Number of Stator Slots.—With concentric windings, the number of slots must be a multiple of the phases times the poles, and it is usual to allow 3, 4, or 5 slots per pole per phase.

With double-layer windings, the number of conductors per slot must be an even integer. The choice of number of slots is not restricted as with concentric windings, and the selected number must be made to fit in with the total number of conductors desired. Fractional numbers of slots per pole per phase are frequently employed with windings of this type. The windings are practically always chorded, the pitch factor usually falling between the limits of 0·7 and unity. The slot pitch should not fall below 1·5 cms.

Number of Stator Conductors.—The final number of conductors is obtained by multiplying the number of slots by a suitable number of conductors per slot, choosing the latter value so that the total number of conductors is approximately the same as is given by consideration of the permissible electric loading.

Flux per Pole.—The useful flux per pole can now be calculated from the formulæ given on page 343, viz.

$$\Phi = \frac{E \times 10^8}{2{\cdot}1k_3fN} \text{ for three-phase motors}$$

or

$$\Phi = \frac{E \times 10^8}{2k_3fN} \text{ for two-phase motors.}$$

E is the voltage per phase and is $\frac{1}{\sqrt{3}}$ times the line voltage for three-phase star-connected motors, k_3 is the pitch factor or coil span factor, f is the frequency, and N is the number of stator conductors per phase. A small allowance may be made if required for the IR drop in the winding. The value of the flux obtained in this way should be checked against the value of the flux desired given in the flux formula on page 378.

Flux Densities.—Suitable values for the maximum flux densities in various parts of the magnetic circuit can be obtained from the following table. The flux distribution in the air-gap and in the teeth is not uniform, however (see page 141), and the maximum flux density is obtained by multiplying the average value by 1·57. This constant does not affect the flux in the iron at the back of the teeth in either stator or rotor, since this is due to the average effect of all the phases and averages itself out over the whole section.

The flux per pole being known and the dimensions of the various magnetic materials being determined, the flux densities can now be worked out and compared with the figures in the table on page 376.

The average air-gap flux density is equal to the total flux per pole divided by the area of the air-gap per pole, the maximum value of this density being 1·57 times the average value.

PART.	MAXIMUM FLUX DENSITY. Lines per sq. cm.
Air-gap	8,000
Stator Teeth	16,000
Stator Iron behind Teeth	10,000
Rotor Teeth	17,500
Rotor Iron behind Teeth	13,000

In the case of the air-gap flux density, the figure quoted above is based on the assumption that the area of the air-gap per pole is $\frac{\pi DL}{p}$, *i.e.* no allowance is made for the increase in air-gap reluctance due to the slots. The real flux density is, therefore, higher and may be calculated as follows. The reluctance of the air-gap is increased by the slot openings on both stator and rotor, and the reluctance assuming no slot openings must be multiplied by constants k_s and k_r to allow for this increase in reluctance. These two constants have the following values, where δ is obtained from Fig. 290.

$$k_s = \frac{\text{stator tooth pitch}}{\text{width of stator tooth} + \delta \times \text{air-gap length}}$$

and $$k_r = \frac{\text{rotor tooth pitch}}{\text{width of rotor tooth} + \delta \times \text{air-gap length}},$$

the measurements being taken at the air-gap surfaces. The real flux density is equal to $k_s k_r$ times the apparent density. In a number of cases the rotor slot opening is very small, and k_r can be taken as unity.

A check calculation should also be made relating to the probable flux density in the teeth. Assuming a slot width equal to the tooth width, the circumferential length of iron per pole is half the pole pitch. The axial length of iron is found by subtracting the total length of the vent spaces from the gross length of the core and then multiplying by 0·9 to allow for the laminations. The flux per pole is divided by this area and multiplied by 1·57 to obtain the maximum tooth flux density. This figure probably errs on the optimistic side.

Size of Stator Conductors.—Knowing the current, the size of the stator conductors can be determined by allowing a current density of from 250 to 400 amperes per square cm., the higher figures referring to the smaller sizes of conductors. For low currents wire will be most convenient to use, whilst for the higher currents former wound coils of bar or strip should be adopted, since anything above about 2 or 3 mm. diameter is difficult to wind. The use of strip has an advantage, since it brings about a higher space factor in the slot. A circular wire has a cross-sectional area of $\frac{\pi}{4}D^2$, but it occupies a space of D^2 in the slot, and, apart from any insulation at all, this brings the space factor down to 0·78.

The space factors ordinarily employed will vary from 0·25 to 0·4, high voltage machines having the lower values and bar windings being superior to wire windings.

Size of Stator Slots.—When the number of conductors per slot has been obtained, an approximate area of slot can be calculated. The width of tooth and slot should be made about the same, thus

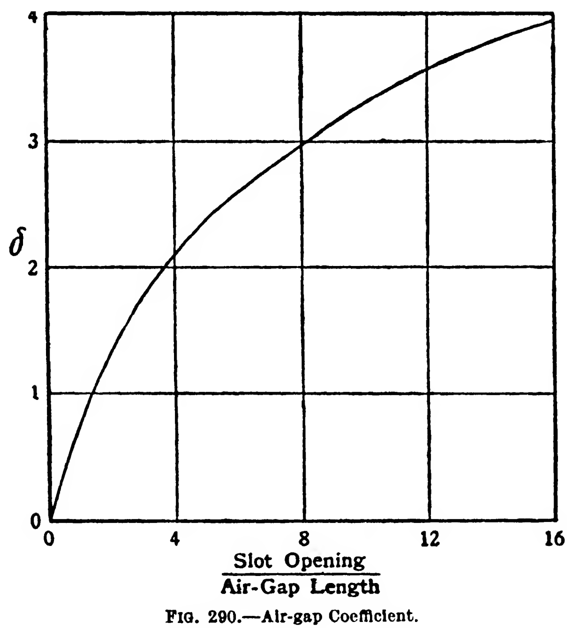


FIG. 290.—Air-gap Coefficient.

giving the depth, which should not exceed six times the width. The conductors should now be arranged in the slot, putting in the necessary insulation, and the final dimensions of the slot can be decided upon by adjusting it to fit the conductors.

Depth of Stator Iron.—The stator iron behind the teeth carries half the flux per pole, and by using the flux density given in the table on page 381 the required cross section of iron can be obtained. When this area is divided by the net iron length of the core, it gives the depth of iron required behind the stator teeth. This enables the outer diameter of the stator stampings to be calculated.

Stator Copper Loss.—The resistance of each winding can be calculated by estimating the length of wire and assuming a specific resistance of 2 microhms per cm. cube (hot). Knowing the current, the watts lost per phase and the total for all phases can now be determined.

Stator Iron Loss.—The combined hysteresis and eddy current loss may be obtained by reference to Fig. 252. The iron loss in the teeth and in the iron at the back of the teeth should be obtained separately.

Cooling Surface.—The cooling surface may be determined in just the same way as was done in the case of alternators. The inner and outer cylindrical surfaces are taken, together with the two ends of the core and one side of each ventilating duct.

Estimated Temperature Rise of Stator.—With normal designs a temperature rise of the stator of about 40° C. should be obtained when the watts to be dissipated are about 0·15 per sq. cm. As a first approximation, the temperature rise may be taken as being proportional to the watts dissipated per square cm., but this rule is by no means accurate, as the temperature rise in a particular motor depends to a very large extent upon its mechanical design.

Air-gap Length.—Very small air-gap lengths are adopted (see page 374); suitable values may be obtained from the following empirical rule :—

$$\text{Air-gap length in cm.} = 0{\cdot}32 - \frac{70}{D + 250}.$$

Number of Rotor Slots.—The same number of slots should not be used on both stator and rotor, since if this is done there are some positions of the rotor in which the magnetic reluctance is very much less than it is when the rotor is in intermediate positions. The rotor tends to jump into these positions of minimum reluctance and the motor does not start up as well as it otherwise would.

The number of rotor slots is usually between 115 per cent. and 130 per cent. or else between 70 per cent. and 85 per cent. of the number of stator slots. The number of rotor slots must also be a multiple of the poles × phases in the case of concentric windings.

In the case of squirrel cage rotors, it is usual to adopt a prime number, or twice a prime number, for the number of rotor slots.

Rotor Winding.—The actual number of rotor conductors does not much matter. If there is a large number of turns in series, the open-circuit voltage of the winding is considerable, whilst if fewer turns of larger cross section are employed, the current at the slip rings becomes increased. The rotor current can be estimated by taking the rotor ampere-turns as 0·85 times the stator ampere-turns on full load. It is therefore a simple matter to arrange a suitable winding.

Size of Rotor Slots.—The size of the rotor slots can be determined by assuming a similar current density to that in the stator winding and then finding the required slot space. Since the induced voltage per winding will probably be low, a fairly high space factor should be obtained. The dimensions of the slot can then be settled to suit the winding.

Depth of Rotor Iron.—The required depth of rotor iron behind the teeth can be determined in the same way as in the case of the stator, from which the internal diameter of the rotor stampings can be obtained. The question of whether to put in a spider or to key the stampings direct on to the shaft will be settled by this internal diameter of the stampings. If no spider is employed, axial ventilating ducts should be provided to supply the air inlet for the radial ventilating ducts from which the air is thrown out by centrifugal force.

Squirrel Cage Rotors.—The number of rotor slots should preferably be chosen a prime number, or twice a prime number, and as high as is practicable. The current per bar is

$$0{\cdot}85 \times \text{Stator current} \times \frac{\text{Stator conductors}}{\text{Rotor conductors}}.$$

With a density of 450—500 amperes per square cm., this settles the area of the rotor bar and consequently that of the rotor slot as well. The dimensions of the latter should be such as to leave sufficient tooth area to carry the magnetic flux.

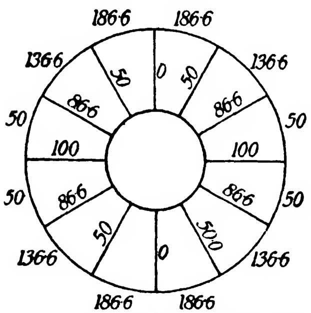


FIG. 291.—Currents in a Squirrel Cage Winding.

To calculate the current in the end ring, consider Fig. 291, which represents a two-pole squirrel cage winding, the two end rings being shown as two circles with the bars joining them. The figures against the bars represent the currents in them at a particular instant, 100 amperes being taken as the maximum. The currents in the other bars decrease according to a sine law. The current in the bar carrying 100 amperes flows into the end ring, 50 amperes in each direction. A little further on this 50 amperes is reinforced by 86·6 amperes from the next bar, making 136·6 amperes in the next section. The next bar adds another 50 amperes, making a total of 186·6 amperes, which is the maximum current carried by the end ring. This maximum current is thus seen to be the sum of the instantaneous values of the currents in half the bars per pole. Assuming a perfect sine law, the average current per bar is $\frac{2\sqrt{2}}{\pi}I = \frac{I}{1{\cdot}11} = 0{\cdot}9I$, and the maximum current in the end ring

becomes $$0{\cdot}9I \times \frac{N}{2p},$$

where p is the number of poles and N the number of bars. The R.M.S. current in the end ring is

$$\frac{1}{\sqrt{2}} \times \frac{2\sqrt{2}}{\pi} I \times \frac{N}{2p}$$
$$= \frac{IN}{\pi p}.$$

It is seen that not only does the current in any one section of the end ring vary sinusoidally with time, but that there is a sinusoidal space variation as well.

The section of the end ring is $\frac{IN}{\delta_r \pi p}$, where δ_r is the current density, a suitable value for this being 600 amperes per sq. cm.

Copper Loss in Squirrel Cage Rotors.—If A and L be the cross-sectional area and length, respectively, in cm. measure, the total copper loss in the bars is

$$N \times I^2 \times \rho\frac{L}{A} = IN\delta_b\rho L.$$

where δ_b is the current density in the bars, say 500 amperes per sq. cm.

The full load copper loss in the two end rings is

$$2 \times \left(\frac{IN}{\pi p}\right)^2 \times \rho\frac{\pi D\delta_r \pi p}{IN}$$
$$= IN \times 2\delta_r\rho\frac{D}{p},$$

D being the rotor diameter.

The total full load rotor copper loss is therefore

$$IN\left(\delta_b\rho L + 2\delta_r\rho\frac{D}{p}\right),$$

and this enables the full load slip to be determined.

Copper Loss in Wound Rotors.—In the case of wound rotors, the copper loss is obtained approximately by estimating the resistance of each phase from its dimensions and assuming that the rotor current

$$= 0{\cdot}85 \times \text{Stator current} \times \frac{\text{Stator conductors}}{\text{Rotor conductors}}.$$

Estimated Temperature Rise of Rotor.—The rotor has to dissipate only the I^2R loss in the winding, since the iron loss is negligible owing to the very low frequency, viz. that of slip. The frictional loss may be supposed to be dissipated in the bearings.

Estimating the cooling surface in the same way as for the stator, the temperature rise may be calculated by allowing 40° C. rise for every 0·3 watt per sq. cm. of cooling surface. It must be understood that this calculation is only very approximate.

Friction and Windage Loss.—This will vary from about 3 per cent. in a 5 B.H.P. motor down to about 1 per cent. in a 200 B.H.P. motor, whilst for the largest sizes of all it will drop to something of the order of $\frac{3}{4}$ per cent.

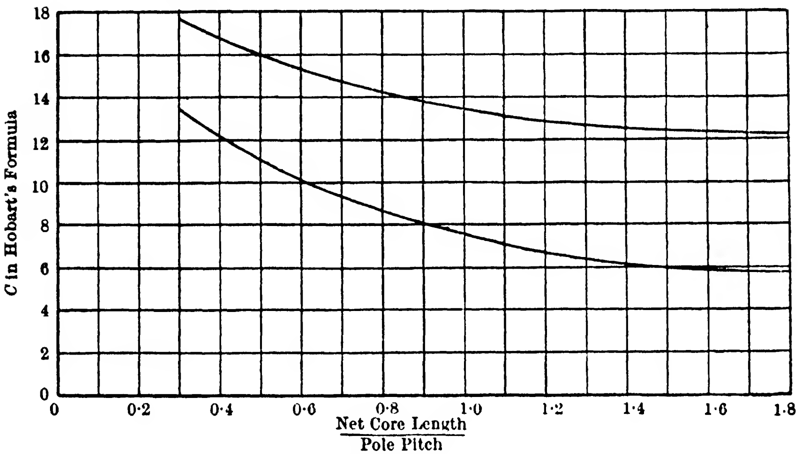


FIG. 292.—Values of *C* in Hobart's Formula.

No-load Current.—The ampere-turns per pole for the air-gap

$$= 0·8 \times l_g \times B,$$

where l_g is the length of the single air-gap and B is the (real) average air-gap density in lines per sq. cm. The total ampere-turns per pole are twice the ampere-turns per pole per phase for a three phase motor (see page 142).

The maximum ampere-turns per pole per phase therefore

$$= 0·8 \times l_g \times B \times \frac{1·57}{2},$$

and the R.M.S. ampere-turns per pole per phase

$$= 0·8 \times l_g \times B \times \frac{1·57}{2\sqrt{2}}$$

$$= 0·45 l_g \times B.$$

The magnetizing current per phase for the air-gap

$$= \frac{0·45 \times l_g \times B}{\text{turns per pole per phase}}.$$

The magnetizing current for the iron path is usually about 10 to 30 per cent. of the above, the value depending largely upon the width of the slot openings. The wider the slot opening the greater is the density in the teeth and the more the magnetizing current.

The active component of the no-load current

$$= \frac{\text{Power to drive motor light}}{\sqrt{3} \times \text{Line voltage}}.$$

The total no-load current

$$= \sqrt{(\text{Active component})^2 + (\text{Magnetizing current})^2}.$$

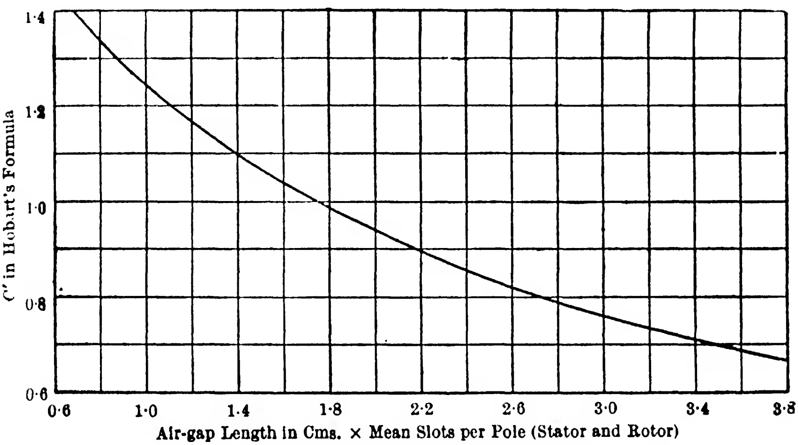


FIG. 293.—Values of C' in Hobart's Formula.

Estimation of σ.—The value of σ can be estimated fairly closely by a formula due to Hobart, viz.

$$\sigma = C \times C' \times \frac{\text{Air-gap length}}{\text{Pole pitch}}.$$

The values of C and C' can be read off the curves in Figs. 292 and 293. For squirrel cage motors a third constant, having an approximate value of 0·75, should be introduced.

Another formula for the estimation of σ is due to Behn-Eschenburg, and is

$$\sigma = \frac{3}{N^2} + \frac{l_g}{XN \times \tau} + \frac{6l_g}{\tau},$$

where N = average number of slots per pole for stator and rotor,
X = average width of slot opening,
l_g = air-gap length,
and τ = pole pitch.

Dimensions are in cms.

Predetermination of Circle Diagram.—The necessary data for predetermining the circle diagram have now been worked out. Referring to Fig. 286, the no-load magnetizing current enables the point A to be determined and $OB = \frac{OA}{\sigma}$. The circle can thus be drawn straight away. The active component of the no-load current fixes the vertical height of the line EH. The resistances of the stator and rotor windings being known, the point F can be obtained by trial, so that FH (to the same scale that HG represents the no-load power to drive) represents the total copper loss for the current OF. The accuracy of the current OF, called the short circuit current, cannot, however, be guaranteed. FH can now be divided at K, so that

$$\frac{FK}{KH} = \frac{\text{Rotor copper loss}}{\text{Stator copper loss}}.$$

The torque line is obtained by joining EK.

✓**Full Load Current, Power Factor and Efficiency.**—These values can now be obtained from the circle diagram in the usual way. The "apparent efficiency," which is the product of the true efficiency and the power factor, should be calculated also. If desired, the complete performance of the motor can be worked out from the circle diagram.

Example of Design.—As an example, a design will now be worked out for a 10 B.H.P., 3-phase, 50-cycle, 440-volt motor having a synchronous speed of 1000 r.p.m.

Number of poles = 6.

An efficiency of 0·85 and a power factor of 0·86 will be aimed at, these figures representing average practice. This gives the full load current as

$$\frac{10 \times 746}{\sqrt{3} \times 440 \times 0{\cdot}85 \times 0{\cdot}86}$$
$$= 13{\cdot}5 \text{ amperes.}$$

Choosing an output coefficient of $1{\cdot}1 \times 10^{-6}$, we get

$$D^2L = \frac{10}{1000 \times 1{\cdot}1 \times 10^{-6}} \text{ (see page 377)}$$
$$= 9100.$$

Assume an air-gap diameter $D = 30$ cms.

The core length $L = \frac{9100}{30^2} = 10$ cms.

There will be two ventilating ducts, each 8 mm. wide.

The flux per pole will now be worked out. The pole pitch $= \frac{\pi \times 30}{6} = 15{\cdot}7$ cms. and the net iron length of the core

$= 10 \times 0{\cdot}9 = 9$ cms. Semi-closed slots will be used in both stator and rotor, and the iron length per pole pitch $= 0{\cdot}8 \times 15{\cdot}7 = 12{\cdot}6$ cms. The air-gap area per pole

$$= 12{\cdot}6 \times 9 \times 1{\cdot}2 = 136 \text{ sq. cms.}$$

Assuming a maximum flux density of 8000 lines per sq. cm., the flux per pole is $\frac{8000}{1{\cdot}57} \times 136 = 0{\cdot}7 \times 10^6$ lines (approx.).

The required number of stator conductors per phase

$$= \frac{440 \times 10^8}{\sqrt{3} \times 2{\cdot}1 \times 50 \times 700{,}000} = 346.$$

Choosing three slots per pole per phase, this gives 54 total slots and 18 slots per phase. There will be, therefore, 20 conductors per slot, giving 180 turns per phase.

The ampere-conductors per cm. periphery

$$= \frac{13{\cdot}5 \times 360 \times 3}{30\pi} = 155,$$

which is satisfactory.

Using 1/·0930 wire for the stator conductors, this gives a current density of $\frac{13{\cdot}5}{0{\cdot}0068} = 310$ amperes per sq. cm. The diameter of this wire $= 0{\cdot}236$ cm. bare and $0{\cdot}264$ cm. with d.c.c. insulation. With a slot lining of 1 mm. of press-spahn the 20 conductors can be arranged in twelve layers, alternate layers being staggered, in a semi-closed slot 4 cms. × 1 cm. with an opening of 0·3 cm.

The flux carried by the stator iron behind the teeth

$$= \frac{700{,}000}{2} = 350{,}000 \text{ lines,}$$

and with a density of 10,000 lines per sq. cm. the required iron cross section

$$= \frac{350{,}000}{10{,}000} = 35 \text{ sq. cms.}$$

The net iron length

$$= 10 \times 0{\cdot}9 = 9 \text{ cms.}$$

Required depth of iron behind teeth

$$\frac{35}{9} = 3{\cdot}9, \text{ say 4 cms.}$$

Outer diameter of stator stampings

$$= 30 + 2(4 + 4) = 46 \text{ cms.}$$

Length per turn of winding

$$= 2 \times \text{pole pitch} + 2(\text{core length} + 2)$$
$$= 2 \times 15{\cdot}7 + 2 \times 15{\cdot}0 = 61{\cdot}5 \text{ cms. approximately.}$$

Resistance per winding (hot)

$$= 2{\cdot}0 \times 10^{-6} \times \frac{61{\cdot}5 \times 216}{0{\cdot}0068 \times 6{\cdot}45} = 0{\cdot}6 \text{ ohm.}$$

Total stator copper loss

$$= 3 \times 13{\cdot}5^2 \times 0{\cdot}6 = 330 \text{ watts.}$$

To get the stator iron loss, the volume of the stator iron is required. The average tooth width

$$= \frac{\pi(30 + 4)}{54} - 1 = 0{\cdot}98 \text{ cm.}$$

Volume of stator teeth

$$= 54 \times 0{\cdot}98 \times 4 \times 9 = 1900 \text{ c.c.}$$

Volume of iron behind teeth

$$= \frac{\pi}{4}(44^2 - 38^2) \times 9 = 3480 \text{ c.c.}$$

Density in teeth

$$= \frac{700{,}000 \times 1{\cdot}57}{9 \times 0{\cdot}98 \times 9} = 13{,}900.$$

Watts per kgm.

$$= 9 \text{ (see Fig. 252).}$$

Watts lost in teeth

$$= \frac{1900 \times 7{\cdot}8}{1000} \times 9 = 133 \text{ watts.}$$

Density in iron behind teeth

$$= \frac{350{,}000}{4 \times 9} = 9700.$$

Watts per kgm.

$$= 4{\cdot}5 \text{ (see Fig. 252).}$$

Watts lost in iron behind teeth

$$= \frac{3480 \times 7{\cdot}8}{1000} \times 4{\cdot}5 = 122 \text{ watts.}$$

Total stator iron loss

$$= 133 + 122 = 255 \text{ watts.}$$

Stator cooling surface

$$= \pi \times 46 \times 10 + \pi \times 30 \times 10 + 4 \times \frac{\pi}{4}(46^2 - 30^2) = 6200 \text{ sq. cms.}$$

Watts per sq. cm.

$$= \frac{330 + 255}{6200} = 0{\cdot}095.$$

The temperature rise of the stator will, therefore, be quite reasonable.

A wound rotor will next be designed.

Air-gap length

$$= 0{\cdot}32 - \frac{70}{30 + 250} = 0{\cdot}07, \text{ say } 0{\cdot}1 \text{ cm.}$$

Choosing four slots per pole per phase, this gives 72 slots in all. A bar winding will be adopted with four conductors per slot, giving 48 turns per phase. The rotor current is approximately

$$= 0{\cdot}85 \times 13{\cdot}5 \times \frac{216}{48} = 52 \text{ amperes.}$$

Choosing a strip with a section $0{\cdot}20 \times 0{\cdot}625 = 0{\cdot}125$ sq. cm., a current density of $\frac{52}{0{\cdot}125} = 416$ amperes per sq. cm. is obtained. This winding can be made to fit in semi-closed slots 0·55 cm. × 2·0 cms. with a slot opening of 0·25 cm.

As in the stator, a depth of iron of 4 cms. will be allowed behind the teeth, making the inner diameter of the rotor stampings

$$30 - 2(2 + 4) = 18 \text{ cms.}$$

The length of the mean turn of the rotor winding will be approximately the same as in the stator, and the rotor resistance per phase (hot)

$$= 2{\cdot}0 \times 10^{-6} \times \frac{61{\cdot}5 \times 48}{0{\cdot}125} = 0{\cdot}047 \text{ ohm.}$$

Rotor copper loss

$$= 3 \times 52^2 \times 0{\cdot}047 = 380 \text{ watts.}$$

Rotor cooling surface

$$= \pi \times 30 \times 10 + \pi \times 18 \times 10 + 4 \times \frac{\pi}{4} \times (30^2 - 18^2) = 3320 \text{ sq. cms.}$$

Watts per sq. cm.

$$= \frac{380}{3320} = 0{\cdot}115.$$

The temperature rise of the rotor will also be quite reasonable.

Taking the friction and windage loss at about 3 per cent. of the output, this becomes

$$0{\cdot}03 \times 7460 = 220 \text{ watts.}$$

Total losses

$$= 330 + 255 + 380 + 220 = 1185 \text{ watts.}$$

Magnetizing current per phase for the air-gap

$$= \frac{0\cdot45 \times 0\cdot1 \times 8000}{1\cdot57 \times 30} \text{ (see page 386)}$$
$$= 7\cdot7 \text{ amperes.}$$

Adding 10 per cent. for the iron path, this becomes

$$1\cdot1 \times 7\cdot7 = 8\cdot5 \text{ amperes.}$$

The active component of the no-load current

$$= \frac{255 + 220}{\sqrt{3} \times 440} = 0\cdot62 \text{ ampere.}$$

Actually this would be rather larger, since the no-load copper losses are neglected, but the difference is not noticeable in the final no-load current.

Total no-load current

$$= \sqrt{8\cdot5^2 + 0\cdot62^2} = 8\cdot5 \text{ amperes (approx.).}$$

This could be reduced by adopting a shorter air-gap.

The estimated value of σ, obtained from Figs. 292 and 293, is

$$\sigma = 10 \times 1\cdot27 \times \frac{0\cdot1}{15\cdot7} = 0\cdot081.$$

According to Behn-Eschenburg's formula

$$\sigma = \frac{3}{10\cdot5^2} + \frac{0\cdot1}{0\cdot275 \times 10\cdot5 \times 15\cdot7} + \frac{6 \times 0\cdot1}{15\cdot7}$$
$$= 0\cdot0272 + 0\cdot0022 + 0\cdot0382$$
$$= 0\cdot068.$$

A value of 0·075 will be assumed, taking both methods into consideration. This gives a maximum power factor of

$$\frac{1 - 0\cdot075}{1 + 0\cdot075} = 0\cdot86,$$

and a full load power factor of 0·85 is obtained from the circle diagram of the motor, for which the necessary data are now available.

The full load efficiency is

$$\frac{7460}{7460 + 1185} = 0\cdot86,$$

and the full load apparent efficiency is

$$0\cdot86 \times 0\cdot85 = 0\cdot73.$$

Full load current

$$= \frac{7460}{\sqrt{3} \times 440 \times 0{\cdot}73}$$
$$= 13{\cdot}4 \text{ amperes.}$$

Full load slip $= \dfrac{\text{Rotor } I^2R \text{ loss}}{\text{Rotor input}}$

$$= \frac{380}{7460 + 220 + 380}$$
$$= 0{\cdot}047, \text{ say 5 per cent.}$$

Full load speed $= 950$ r.p.m.

Alternative Squirrel Cage Rotor.—Choosing 43 slots, a slot pitch of $\frac{\pi \times 29{\cdot}8}{43} = 2{\cdot}18$ cms. is obtained. The current per bar is

$$0{\cdot}85 \times 13{\cdot}6 \times \frac{1080}{43} = 290 \text{ amperes.}$$

Using bars with a cross section of 0·8 cm. × 1·0 cm., a current density of 363 amperes per sq. cm. is obtained. These bars will go in slots 1 cm. × 1·3 cms. The maximum tooth width is 2·18 − 1·0 = 1·18 cms., and the minimum $2{\cdot}18 \times \frac{30 - 2 \times 1{\cdot}3}{30} - 1{\cdot}0 =$ 0·99 cm.

The R.M.S. current in the end ring is

$$\frac{290 \times 43}{\pi \times 6} \text{ (see page 385)}$$
$$= 660 \text{ amperes.}$$

The required cross section of the end ring, at 600 amperes per sq. cm.,

$$= \frac{660}{600} = 1{\cdot}1 \text{ sq. cms.}$$

An end ring with a cross section of 2·2 cms. × 0·5 cm. is suitable, since this gives the desired cross-sectional area.

The total full load rotor copper loss

$$= 290 \times 43 \left(363 \times 2 \times 10^{-6} \times 11{\cdot}5 + 2 \times 600 \times 2 \times 10^{-6} \times \frac{30}{6}\right)$$

(see page 385)

$= 250$ watts (approx.)

as compared with 380 watts for the wound rotor.

The full load slip would be about 3·2, say 3·5 per cent.

Examples

(1) The stator core of a three-phase induction motor has an internal diameter of 30 cms. and a length of 18 cms., including three ventilating ducts each one cm. wide. What output could be obtained from it if wound for four poles on 500 volts, 50 cycles? How many slots and conductors per slot would you employ?

(2) A certain design for a polyphase induction motor is found to give a poor power factor when the performance is calculated. Discuss the various modifications which could probably be made to the design with the object of improving its performance.

(3) There are 54 and 72 slots on the stator and rotor of an induction motor respectively. There are six poles and the air-gap diameter is 30 cms. The length of the air-gap is 1 mm. Semi-closed slots are employed, the opening being 3 mm. on the stator and 2·5 mm. on the rotor. Estimate the probable full load power factor.

(4) If the length of the air-gap in the motor described in the previous question be increased to 1·5 mm., what would be the effect upon its behaviour, particularly the power factor?

(5) What are the factors which determine the choice of the number of turns in the rotor winding of a three-phase slip ring induction motor?

CHAPTER XXIII

SYNCHRONOUS MOTORS

Alternator used as a Motor.—If two alternators are run in parallel and the driving force of one is suddenly cut off, the machine continues to run as a motor and takes the power necessary to drive it from the other machine, which is loaded to a certain extent on this account. The supply of D.C. to the field system must be maintained throughout. For an alternator to act as a motor it must, therefore, be supplied with A.C. for the armature and D.C. for the field system, and further, as will be seen later, the machine must be brought up to the speed of synchronism before the motoring action takes place.

In order to examine the action in detail, take the case of the elementary two-pole single phase synchronous motor shown in Fig. 294. This machine is supposed to be exactly the same as the corresponding alternator, and may have a stationary field and a rotating armature, or *vice versa*, the current being led into the rotating element by means of slip rings. Consider the conductor which arrives at *A* at the moment when the current is zero. The instantaneous value of the torque due to this conductor is also zero, since it is proportional to the product of the field strength and the armature current. The field strength is assumed to be constant throughout. A moment later the conductor has arrived at *B* and the armature has rotated through an angle θ. But the current has also advanced in phase by θ° and is supposed to be flowing away from the observer. This produces a torque in a clockwise direction and causes the armature to revolve. When the conductor reaches *C* the current has reached its maximum value, and by the time the conductor reaches *D* the current has died down to zero. Throughout the whole of this half-revolution, which has taken place whilst the current has advanced through half a cycle, the torque has been in the same direction. A little later the conductor arrives at *E*, but the current has now started to

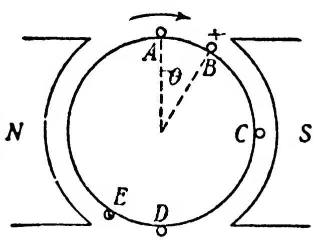


FIG. 294.—Action of Synchronous Motor.

grow in the *reverse* direction. However, since this reverse current is cutting the magnetic field in the reverse direction, the torque still tends to produce rotation in a clockwise direction. By the time the armature has completed one whole revolution the current has advanced through one whole period. This is the essential condition for the continuance of rotation; the armature must rotate synchronously with the current, and hence the machine is called a synchronous motor. The currents in the other armature conductors produce torques which aid one another, for during the first half-period of the current they are cutting the field in one direction, and during the second half-period, when the current reverses, they are cutting the field in the other direction.

The same principle operates in the case of two- and three-phase synchronous motors, these being much superior in their performance to the single-phase machines, their great advantage lying in the fact that they are much less likely to drop out of step when subjected to overloads.

Conditions for Running.—Such a motor as is described above is not self-starting, and it is necessary first to run up the machine to synchronous speed by some external means and then to synchronize it with the supply, the field having first been excited. In other words, the machine must be run up and synchronized exactly like an alternator, and the same conditions apply. The machine will then continue to run and take a motoring current from the supply as long as it keeps in step. By this is meant keeping in synchronism. If the armature does not rotate synchronously with the variations of the current, there come certain points in the revolution where a reverse torque is produced, and this tends to pull the motor up. The retardation of the armature causes it to fall still further out of step, with the result that the motor quickly comes to rest. Since the armature is still supplied with current, the circuit should be opened immediately, to prevent damage, since the motor at standstill has only the impedance of its armature to limit the current.

Speed.—The speed of a two-pole synchronous motor has already been shown to be one revolution per cycle, and this corresponds to a speed of $60f$ revolutions per minute, where f is the frequency. If the motor is a multipolar machine having p poles, the armature conductors will advance past one pole pair every cycle, or, in other words, the armature will rotate through $\frac{2}{p}$ths of a revolution. The revolutions per minute will then be equal to

$$\frac{2}{p} \times 60f = \frac{120f}{p}.$$

The speed of a polyphase synchronous motor is the same as that of the corresponding single-phase machine with the same number of poles.

The only way to vary the speed is to vary the frequency or the number of poles, and these are not practical methods. Synchronous motors are, therefore, essentially constant speed machines.

No-load Conditions.—When a synchronous motor is first synchronized with an alternator, the two machines momentarily act as generators in parallel across the same bus bars, their E.M.F.'s acting in the same direction. Considering the local circuit formed by the two armatures, these may be regarded as being in series with each other with their two E.M.F.'s always opposing, as shown in Fig. 295. The armature of the motor may, therefore, be considered as setting up a back E.M.F. equal to and opposite in phase to the applied voltage. The resultant voltage in this circuit is zero, and so no current is supplied to the motor armature. Since the latter receives no power in an electrical form from the supply, it immediately commences to slow down when the mechanical driving power is removed. But as soon as the motor armature falls behind the position where it should be if it maintained an absolutely synchronous motion, the back E.M.F. and the applied E.M.F. no longer neutralize each other, for, notwithstanding the fact that they are equal, no two voltages can completely neutralize one another unless they are in phase opposition. The applied and the back E.M.F. now produce a resultant voltage which causes a current to flow through the motor armature and supplies it with a certain amount of power. If this power is sufficient to maintain the rotation, the motor continues to rotate synchronously, but always lagging by a constant small angle. If the power supplied to the motor in this manner is not sufficient to overcome the losses at this speed, the armature is retarded and lags behind by a greater angle. The effect of this is to increase the resultant voltage and the armature current, and the power supplied to the motor is thereby increased. This action goes on until the motor lags by such an angle as to produce a resultant voltage which will cause the necessary amount of power to be transferred to the motor.

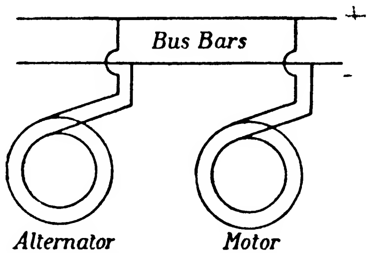


FIG. 295.—Alternator driving Synchronous Motor.

The action is similar in a way to that which takes place in a D.C. shunt motor where the back E.M.F. is exactly in opposition to the applied E.M.F., only in the A.C. case the back E.M.F. lags by rather more than 180° behind the applied E.M.F. instead of being in exact phase opposition.

Torque.—In a simple two-pole single-phase synchronous motor the axis of the armature field may be considered as lying along the line joining the two conductors which are connected to the slip rings, just as in a D.C. motor the armature field lies along the line joining the brushes. This field rotates with the armature, but also varies according to a sine law with time, and it reacts with the main field produced by the exciting current. Since the latter field is constant, the torque is proportional to that component of the armature field which is at right angles to the main field. The magnitude of the armature field after the armature has moved through $\theta°$ from the zero position may be represented by $\Phi \sin \theta$, where Φ is the maximum value. At the same instant, the vertical component of this (see Fig. 296) is $\sin \theta$ times its actual value, and thus the instantaneous torque is proportional to $\Phi \sin^2 \theta$, where $\theta = \omega t$. This expression shows that the torque is of a pulsating character, although it never reverses in direction.

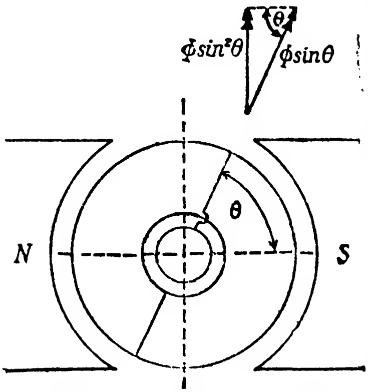


FIG. 296.—Hypothetical Armature Flux.

In the case of a three-phase synchronous motor, the armature field may be regarded as consisting of three components spaced 120° apart in space and differing by 120° in phase. The useful part of each component is obtained in the same way as before. The instantaneous magnitudes of the three hypothetical fluxes are $\Phi \sin \theta$, $\Phi \sin (\theta + 120°)$, and $\Phi \sin (\theta + 240°)$, as shown in Fig. 297. The vertical components are $\Phi \sin^2 \theta$, $\Phi \sin^2 (\theta + 120°)$, and $\Phi \sin^2 (\theta + 240°)$. The resultant vertical field is obtained by adding these together, and is equal to

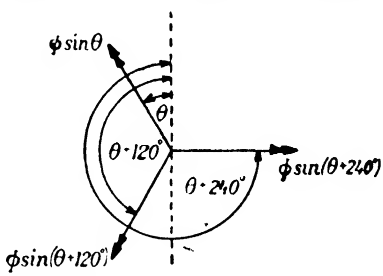


FIG. 297.—Hypothetical Armature Fluxes in Three-Phase Motor.

$$\Phi \sin^2 \theta + \Phi \sin^2 (\theta + 120°) + \Phi \sin^2 (\theta + 240°)$$

$$= \Phi \{\tfrac{1}{2} - \tfrac{1}{2}\cos 2\theta + \tfrac{1}{2} - \tfrac{1}{2}\cos 2(\theta + 120°) + \tfrac{1}{2} - \tfrac{1}{2}\cos 2(\theta + 240°)\}$$
$$= \Phi \{\tfrac{3}{2} - \tfrac{1}{2}\cos 2\theta - \tfrac{1}{2}\cos (2\theta + 240°) - \tfrac{1}{2}\cos (2\theta + 120°)\}$$
$$= \Phi \{\tfrac{3}{2} - \tfrac{1}{2}\cos 2\theta - \tfrac{1}{2}\cos 2\theta \cos 240° + \tfrac{1}{2}\sin 2\theta \sin 240° - \tfrac{1}{2}\cos 2\theta \cos 120° + \tfrac{1}{2}\sin 2\theta \sin 120°\}$$
$$= \Phi \left\{\tfrac{3}{2} - \tfrac{1}{2}\cos 2\theta + \tfrac{1}{4}\cos 2\theta - \frac{\sqrt{3}}{4}\sin 2\theta + \tfrac{1}{4}\cos 2\theta + \frac{\sqrt{3}}{4}\sin 2\theta\right\}$$
$$= \tfrac{3}{2}\Phi.$$

In other words, the magnitude of the torque is independent of the position of the armature, and the same statement holds true for a two-phase machine. The fact that the torque is constant in magnitude in a polyphase machine whilst it is of a rapidly pulsating character in a single-phase motor, dropping to zero twice per period, indicates the reason why a single-phase synchronous motor is so much more liable to drop out of step when a heavy overload or a sudden change of load occurs.

Effect of Load.—When a load is put upon the shaft of the motor, the first tendency is to retard the rotation, but as soon as the armature commences to drop behind, it causes the back E.M.F. to lag by a rather larger angle than before. It will be remembered that the angle of phase difference between the applied and the back voltage is rather more than 180°, the applied voltage leading. But an increase in the angle of lag of the back voltage causes an increase in the resultant voltage acting on the circuit, and this in turn causes an increased current to flow, with the result that the motor takes more power from the supply. The motor armature then drops behind until its position, relatively to that of the driving alternator, is such as to produce a resultant voltage capable of causing sufficient current to flow to deal with the load. The armature then continues to rotate synchronously with the driving alternator. If the motor should drop too far behind, the power which it takes from the supply is greater than is necessary, and so the armature is accelerated until it is in its correct relative position. Some motors are subject to this overshooting the mark, and when a motor is constantly retarded and accelerated in this manner the effect is called *hunting* or *phase-swinging*.

Approximate Vector Diagram.—The study of the vector diagram of the synchronous motor is the best means of understanding what is really going on inside the motor. For the sake of simplicity, the approximate vector diagram of a single-phase machine is first considered, but the same principles apply in the case of polyphase machines.

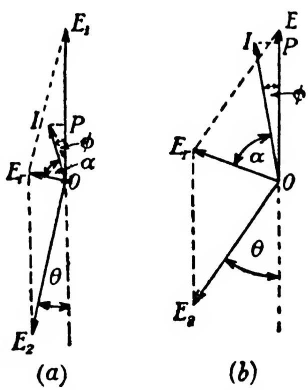


FIG. 298.—Vector Diagrams of Synchronous Motor.
(a) On No-load. (b) On Load.

In the approximate vector diagram shown in Fig. 298, armature reaction is entirely neglected as a separate effect, the armature reactance and armature reaction being combined together under the term synchronous reactance, as in the case of alternators (see page 288). The applied voltage is represented by OE_1 and the induced back E.M.F. by OE_2. These two voltages are taken as equal, representing the conditions which occur when the machine is perfectly synchronized, OE_2 lagging behind OE_1 by an angle $180° + \theta$. Combining these two voltages, the resultant voltage OE_r is obtained. This voltage leads the applied voltage by rather less than 90°, and it can be considered as acting on a circuit having a definite resistance, R, and synchronous reactance, X_s. Assuming these two quantities to remain constant, the angle of lag of the current behind the resultant will be constant. If the synchronous impedance of the armature be known, the current vector, OI, can be plotted, lagging behind OE_r by a fixed angle, α, equal to $\tan^{-1}\frac{X_s}{R}$, the magnitude of the current being given by $\frac{E_r}{\sqrt{R^2 + X_s^2}}$. The angle of lag or lead, ϕ, of this current with respect to the applied voltage determines the power factor under which the motor is operating, the current leading in the example shown in the figure. Since the applied voltage, E_1, is supposed to be constant, the power taken by the motor is represented to scale by OP, which is the projection of OI on OE_1. After subtracting the losses incurred by the motor, the output is obtained.

An increase of load results in an increase in the angle θ, as shown in Fig. 298 (b). This swings the vector OE_r round a little and increases its magnitude. The magnitude of the current is now also increased, since the synchronous impedance remains unaltered, and the power taken by the motor ($E_1 I \cos\phi$) goes up as well.

Ampere-turn Diagram.—In the previous paragraph a constant synchronous reactance was assumed, the effects of both armature reactance and armature reaction being included in it. The latter effect is, however, very important, and must be treated separately if accurate results are to be obtained. Let the armature be assumed, for the moment, to possess neither ohmic resistance nor leakage reactance. With this assumption, the induced back E.M.F. is exactly equal and opposite in phase to the applied voltage, and is wholly independent of the exciting current. The exciting current produces an M.M.F. which can be split up into two components, one producing the useful air-gap flux which generates the back E.M.F., and the other a neutralizing M.M.F. equal and opposite to that set up by the armature ampere-turns. The first component is in quadrature with the applied E.M.F., and the second is in phase opposition to the current. Their vector sum gives the total M.M.F. which must be supplied by the exciting current. The vectorial addition of the fluxes produced by these M.M.F.'s is, however, not strictly accurate, as it neglects the changes of permeability consequent upon the changes in flux density. If this point be neglected, the vector diagram might be drawn to a scale of ampere-turns instead of M.M.F.'s, since they are proportional to one another, or it may be drawn to a scale of exciting current even, since the turns on the field system are constant.

Fig. 299 (*a*) represents the ampere-turn vector diagram when the power factor of the motor is unity, the diagram being drawn to a scale of ampere-turns. E_1 represents the applied voltage and E_2 the back induced E.M.F., these two being exactly equal and opposite. The armature current, I, is in phase with E_1, since

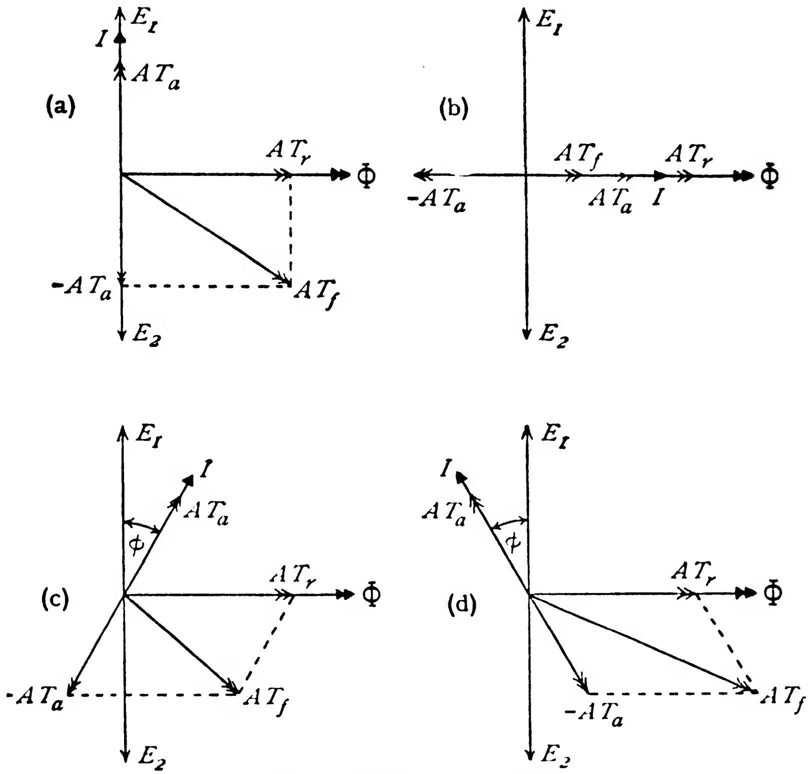


FIG. 299.—Ampere-turn Diagrams.

the power factor is unity, and its magnitude is determined by the load. The ampere-turn part of the diagram is a space diagram in which the ampere-turns due to the armature are in space quadrature with the resultant ampere-turns producing the useful flux which induces the armature E.M.F. In other words, the armature reaction consists solely of cross ampere-turns. Since the induced E.M.F. is proportional to minus the rate of change of the flux, the resultant M.M.F. producing this can be represented by a vector leading the induced E.M.F. by 90° and lagging behind the applied E.M.F. by 90° as in the case of the transformer (see page 182). Similarly the armature ampere-turns can be represented by a vector in phase with the armature current. The field current is now required to set up ampere-turns which will (*a*) neutralize the armature ampere-turns and (*b*) provide a resultant flux to induce the back E.M.F., E_2. These field ampere-turns are represented by the vector AT_f.

When the armature current lags by 90° behind the applied voltage, it constitutes a magnetizing current, just like the primary current of a transformer when it lags by 90° behind the applied voltage. Consequently less field current is required to set up the necessary flux. The field ampere-turns are reduced because the armature ampere-turns are doing part of their work.

Another way of viewing the problem is to consider the reduced field ampere-turns as consisting of two components (*a*) neutralizing the armature ampere-turns which are now in direct phase opposition, and (*b*) providing the resultant flux to induce the back E.M.F. as before. These conditions are represented in Fig. 299 (*b*).

The general intermediate case is shown in Fig. 299 (*c*), where the armature current lags by an angle ϕ, and where the armature ampere-turns consist partly of cross ampere-turns and partly of magnetizing ampere-turns.

The case of the leading current is shown in Fig. 299 (*d*), where the armature ampere-turns are now partly cross ampere-turns and partly demagnetizing ampere-turns. As a consequence of the latter, the field current now has to be increased, to maintain the armature in its leading phase.

If the armature current is negligibly small, the armature ampere-turns disappear and the field ampere-turns become equal to the resultant ampere-turns. These can be determined from the open-circuit magnetization curve when the machine is run as an alternator. In fact, it is sufficient merely to know the value of the field current, without troubling about the turns, the ampere-turn diagram then being drawn to a scale of exciting current. Similarly, if the resultant ampere-turns are made equal to zero, then the armature ampere-turns are equal to the resultant ampere-turns. These conditions are imitated by running the motor as an alternator on short-circuit. The magnitude of the armature ampere-turns can then also be determined in terms of exciting current, *i.e.* field amperes.

These ampere-turn diagrams should be compared with those in Fig. 236 relating to the alternator, remembering that the generated E.M.F. in the alternator corresponds to the back E.M.F. in the synchronous motor.

In actually drawing a diagram of this kind it is much simpler to draw it to a scale of exciting current rather than ampere-turns, as the unmber of field turns is not then required.

Complete Vector Diagram.—The complete vector diagram of the synchronous motor is a combination of the approximate vector diagram shown on page 399, and the ampere-turn diagram, and is illustrated in Fig. 300 (*a*) and (*b*). The applied voltage consists of (*a*) a component which neutralizes the induced E.M.F., E_2, and (*b*) a further component which provides for the impedance drop in the armature circuit. This drop is largely a reactance drop, but the reactance to be considered is only the true leakage reactance and not the synchronous reactance as in the approximate vector diagram. This is equivalent to saying that the resultant voltage acting on the armature circuit is the resultant of the applied

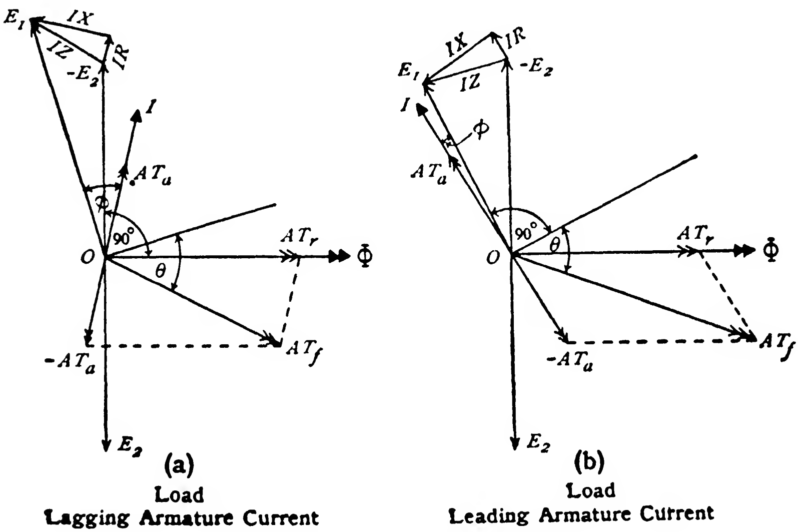


FIG. 300.—Complete Vector Diagram.

and induced back E.M.F. The armature current lags behind this impedance voltage by an angle α where $\tan \alpha = \frac{X}{R}$. From the ampere-turn diagram, it is seen that the flux, Φ, lags behind the neutralizing component of the applied E.M.F., $-E_2$, by 90°, and this flux is set up by the resultant ampere-turns, AT_r. The ampere-turns provided by the D.C. exciting current must therefore consist of two components (*a*) to neutralize the ampere-turns of the armature, and (*b*) to provide the flux, Φ. The field ampere-turns are represented by AT_f. Another way of viewing the problem is to regard the resultant ampere-turns producing the flux, Φ, as being the vector sum of (*a*) the ampere-turns due to the field current and (*b*) the ampere-turns due to the armature.

The power factor of the motor is given by $\cos \phi$, where ϕ is the angle of lag of the current, I, behind the applied voltage, E_1. The input to the armature is $E_1 I \cos \phi$, and the output, neglecting iron and friction losses, is $E_2 I \cos \psi$, where ψ is the angle between $-E_2$ and I.

Effect of Excitation.—The value of the excitation affects the power factor of the motor to an enormous extent. In a D.C. shunt motor a variation in the exciting current produces a variation in the speed, but in the synchronous motor this is not possible, and so a variation in the current is produced instead. But as the power supplied to the motor depends upon the load and not upon the exciting current (except in so far as the losses are varied), the variation in the current must be accompanied by a variation in the power factor. The effect can best be studied by a reference to the vector diagram in Fig. 300 (*a*) and (*b*). At (*a*) the motor is operating with a lagging current, I, the power output being proportional to the component of I in phase with $-E_2$. An increase in the exciting current brings about an increase in the field ampere-turns AT_f, and since, by supposition, $-E_2$ is kept constant, E_2 and also Φ remain constant. It follows, therefore, that the resultant ampere-turns, AT_r, remain constant. The ampere-turns of the armature are proportional to the armature current, and the only way in which AT_f can increase in magnitude without disturbing AT_r is for the phase of AT_a to advance, thus causing the armature current, I, to advance in phase. The locus of the vector $-AT_a$ is a straight line at right angles to E_2, since the active component of I is assumed to remain constant. (Owing to the change in armature copper loss with armature current, this locus is slightly concave with reference to the origin, since the active component of the armature current is a minimum at unity active factor.) If the field current is raised sufficiently, the armature current is made to lead, and a leading power factor is the result as shown in Fig. 300 (*b*).

It will be observed that, for the same value of Φ and, therefore, of $-E_2$, the magnitude of E_1 is less in Fig. 300 (*b*) than in Fig. 300 (*a*), owing to the shape of the diagram. Actual operating conditions, however, demand that E_1 shall be kept constant, and not $-E_2$, in which case $-E_2$ is found to increase slightly as the armature current is advanced in phase. As a result of this, the flux, Φ, increases slightly, thus demanding a slight increase in AT_r. In turn, this demands a slight increase in the exciting current to bring it about, but the effect is small compared with the effect due to the change in phase of AT_a. With synchronous motors possessing a large reactance, however, the point is by no means negligible.

It is usually sufficiently accurate to draw the diagram with $-E_2$ of constant length, the voltage scale being altered slightly for different conditions.

It thus appears that the phase of the current with respect to the applied voltage may be adjusted at pleasure by the simple expedient of varying the excitation, and the motor may be made to operate on any power factor with either a leading or a lagging current. There is, of course, a limit to this reduction of the power factor, due to the dangerous heating which is set up on account of the excessive current, or the motor may drop out of step. Synchronous motors have, however, been known to remain running even though the excitation has been wholly interrupted, the machines running on their residual magnetism.

Load Angle.—When load is applied, the rotor actually drops back in space phase, *i.e.* a point on the surface of the rotor does not reach a particular point on the surface of the stator until a little later in time. This is the exact opposite to the effect in an alternator, where the rotor must be advanced in space phase in order to enable it to take up load, when running in parallel with other alternators. As the load on the motor is increased, the magnitude of this load angle, as it is called, is increased. It is represented in the vector diagram in Fig. 300 by the angle θ. If the armature current be negligibly small, then AT_f becomes equal to AT_r both in magnitude and phase. Similarly, the vectors, E_1 and $-E_2$ now coincide, so that AT_f and E_1 are represented as being at right angles to each other. On the application of load, $-E_2$ falls behind E_1 in phase, thus retarding the flux, Φ, in phase by an equal angle. In addition, AT_f now falls behind AT_r, with the result that the angle θ takes on a substantial value. Its value depends very largely on the load, since this determines the magnitude of AT_a. Changes in power factor also affect the value of the load angle.

Direct measurements of the load angle can be made by means of an insulated disc attached to the end of the shaft. Two narrow contacts are inserted in the rim of the disc, on which a brush presses. The device is operated in conjunction with a Joubert contact (see page 107) which is adjusted so that circuit is made on the two discs simultaneously. The load angle can then be read off the Joubert contact directly.

V-curves.—If the excitation is varied, a number of armature currents can be obtained all relating to the same power output. Each current corresponds to a particular back E.M.F., which in turn corresponds to a particular excitation. The latter can be determined from the magnetization curve of the machine. By varying the excitation, a number of corresponding pairs of armature current and exciting current can be obtained. It is found that on plotting these figures the resulting curve takes the form of a V, as shown in Fig. 301 (*a*), and is known as a V-curve. When the excitation is small, the back E.M.F. is low, giving rise to a resultant voltage leading the applied voltage by a relatively small angle. This causes the current to lag behind the applied voltage by a considerable angle, and since the power factor is small the current is relatively large. As the excitation is increased, the back voltage is also increased, thus swinging the resultant voltage vector round and advancing it in phase. The current is also advanced in phase, its magnitude decreasing since the power factor is increased. When the current becomes in phase with the applied voltage it reaches a minimum value, after which it commences to increase again. The excitation corresponding to this minimum current is called the normal exciting current for that particular load. The effect of over-exciting a synchronous motor is to cause the current to lead the applied voltage due to the lengthening of the back E.M.F. vector. As the excitation is still further increased the armature current also increases and the power factor falls. On no-load, the point on the V-curve is sharply accentuated, but if the machine is loaded the tendency is to round off the point, this effect being more marked at the higher loads.

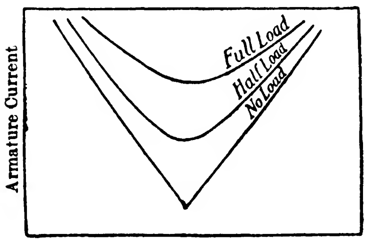


FIG. 301 (*a*).—V-curves.

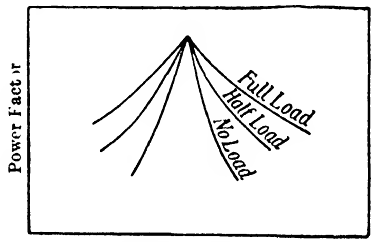


FIG. 301 (*b*).—Power Factor Curves.

Power Factor.—From the preceding paragraph it is seen that the armature current varies between wide limits for the same power output, and this causes the power factor to vary widely in accordance with it. The curves of power factor corresponding to the armature currents represented in Fig. 301 (*a*) are shown in Fig. 301 (*b*), where it is seen that they look like inverted current curves. Again, the no-load power factor curve is fairly sharp at the apex, whilst the others are less sharp, an increase in load tending to flatten out the curve. An examination of these power factor curves shows that particularly on no-load the armature current and the power factor are very susceptible to changes of excitation. The warming up of the field coils is quite sufficient to cause a material change in the armature current.

Example of V-curve.—Instead of assuming the angle θ to be constant for a given load, the power input may be assumed constant instead. In this case, since the applied voltage is fixed, the power component of the current remains unaltered, even although the excitation varies. In Fig. 302, where OE_1 represents the applied voltage and OI the armature current, a line drawn through I at right angles to OE_1 is the locus of the current vector. A line drawn from O to any point in this line represents a current the active component of which is OP. Taking any random value of OI, the resultant voltage producing it can be obtained by multiplying by the synchronous impedance. This voltage leads the current by an angle, α, equal to $\tan^{-1}\frac{X_s}{R}$. Then, knowing the value of this resultant voltage and the applied voltage, the back E.M.F. generated is obtained by subtraction. The excitation corresponding to this back E.M.F. is then read off the magnetization curve. Working backwards in this way, a series of vector diagrams can be constructed without making any assumption as to the magnitude of the angle θ. If a series of values of I are taken in this way, a series of values of E_r and E_2 can be obtained and the locus drawn in for each, this being done in Fig. 302. The whole diagram can then be repeated, if desired, for another load. Strictly speaking, the locus of the current for a given output is not a straight line, since the losses are increased when the current goes up, so that the input is not quite constant. This can be allowed for by making the current locus bend upwards a little on each side.

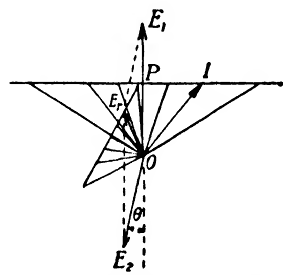


FIG. 302.—Locus Vector Diagram of Synchronous Motor.

Experimental Determination of V-curves.—The actual experimental determination of one of these curves is carried out by maintaining a constant load throughout a single series of observations, and by varying the excitation through as wide limits as the machine will permit without overheating or falling out of step. The applied voltage should be maintained constant throughout, and this will often be a source of trouble, because, if the driving alternator is comparable in output with the motor, the large current at the very low power factor obtained will necessitate a certain amount of field regulation in order to maintain the voltage constant.

A series of such curves can be obtained from the one motor, each one corresponding to a particular load.

Armature Reaction.—The armature reaction of a synchronous motor can be investigated in the same way as is adopted in the case of an alternator. Reference should be made to Figs. 232 and 233, remembering that in the synchronous motor the current is in phase with the applied voltage if the power factor be unity, and consequently leads the generated back E.M.F. by an angle $180° + \theta$. As a first approximation, therefore, if the currents in Figs. 232 and 233 be reversed, the diagrams will represent the action in the synchronous motor, and proceeding in this manner it is seen that when the motor is operating with a lagging current the magnetic flux is increased in magnitude, and when a leading current is taken the magnetic flux is decreased, this being the reverse of what happens in an alternator. In addition, the flux is, of course, distorted in both cases. A lagging current thus tends to increase the back E.M.F., whilst a leading current tends to decrease it.

Another way of looking at the problem is to consider a lagging current as a magnetizing one and a leading current as the opposite, viz. a demagnetizing one. A lagging current, therefore, provides a portion of the magnetizing ampere-turns, so that less are required from the D.C. field system, the reverse being the case with leading currents.

Overload Capacity.—If the applied voltage and the exciting current of a synchronous motor be kept constant, the effect of an increase of load is to retard the back E.M.F., thus causing an increased resultant voltage and an increased current. The power input, however, does not go on increasing indefinitely, since the gradual retardation of the back E.M.F. vector causes the current to lag more and more behind the voltage, and there comes a time when the power factor decreases at a greater rate than the current increases. A further increase of load then causes a further retardation of the back E.M.F., resulting in a decrease in the power absorbed and in the driving torque. The reduced driving torque retards the motor again, which reduces the power input, and so this action goes on until the motor falls out of step and comes to rest. The difference between the maximum load capable of being overcome and the normal full load of the motor, expressed as a percentage of the full load, is called the *overload capacity* of the motor.

An increase in excitation results in a roughly proportional increase in the overload capacity until magnetic saturation occurs.

The presence of reactance in the motor armature is also bad, and any reduction in this direction is accompanied by an increase in the overload capacity. Thus whilst reactance is beneficial from the synchronizing point of view, it is objectionable if the motor is called upon to withstand sudden heavy overloads.

Hunting.—Sudden changes of load on synchronous motors sometimes set up oscillations which are superimposed upon the normal rotation, giving rise to periodic variations in speed of a very low frequency. This effect is known as *hunting* or *phase-swinging*, and can be detected by ear on account of the different notes which are set up by the fluctuating speed of the motor. Occasionally the trouble is aggravated by the motor having a natural period of oscillation approximating to the hunting period, when it is possible for the motor to be phase-swung into the unstable region, thus causing it to fall out of step.

The first effect of a sudden increase of load is a retardation of the armature causing the vector E_2 [see Fig. 298 (*a*)] to take up some such position as is shown in Fig. 298 (*b*). But during this period of retardation the armature is running at a speed slightly less than that of synchronism, and when E_2 has reached a position such that the power input is exactly that required to overcome the load, it is still running at this slightly reduced speed. Owing to the inertia of the armature, however, this reduced speed is maintained for a very short interval longer, during which the angle θ continues to increase. The power input is now greater than is required for the load on the motor, and the armature ceases to be retarded and commences to accelerate. This causes the vector E_2 to gain on E_1 and the angle θ commences to decrease. When the stable position is reached the armature is running a trifle faster than the absolutely correct synchronous speed, and, due to its inertia again, it continues to run a little fast after this point has been reached. The angle θ has now become too small, resulting in a decreased power input which is no longer capable of maintaining the rotation against the resisting torque of the load. The armature is thus again subjected to a retardation and the whole cycle of events is repeated. The net result is that the motor perpetually increases and decreases in speed, the power input periodically varying in unison. The frequency of these changes is usually quite capable of being detected by ear and also by observation of the wattmeter pointer, which oscillates about a certain mean position on the scale.

In order to prevent hunting, modern synchronous motors are fitted with damping grids or amortisseurs (see page 273). Whenever any motion takes place, other than the absolutely synchronous rotation, the flux in the poles is distorted, and the movement of this flux across the bars of the damping grids sets up eddy currents which tend to damp out the superimposed oscillatory movement. If the rotation is absolutely uniform, there is no relative movement of the flux and the damping grid, and so no eddy currents are set up and no losses occur.

Another method of reducing hunting is to work with relatively strong fields, in which case a given change in the angle θ results in a greater change in the torque produced. This is illustrated in Fig. 303, where (*a*) represents the condition with a strong field as indicated by the value of AT_f, and where (*b*) represents the condition with a weak field. In each case the angle θ is increased by 10° to θ', the field current remaining unchanged. The armature current changes from I to I' and the power input from P to P' in each case. The percentage increase of this power input is obviously greater at the higher power factor. The percentage increase of P is still further magnified when the field current (proportional to AT_f) is raised so as to bring about a leading armature current. The force tending to pull the motor into step is thus greater when strong fields are employed than when the motor is worked with a weak excitation.

Starting.—As the ordinary synchronous motor is not self-starting, it is necessary to make arrangements for starting it by some auxiliary means.

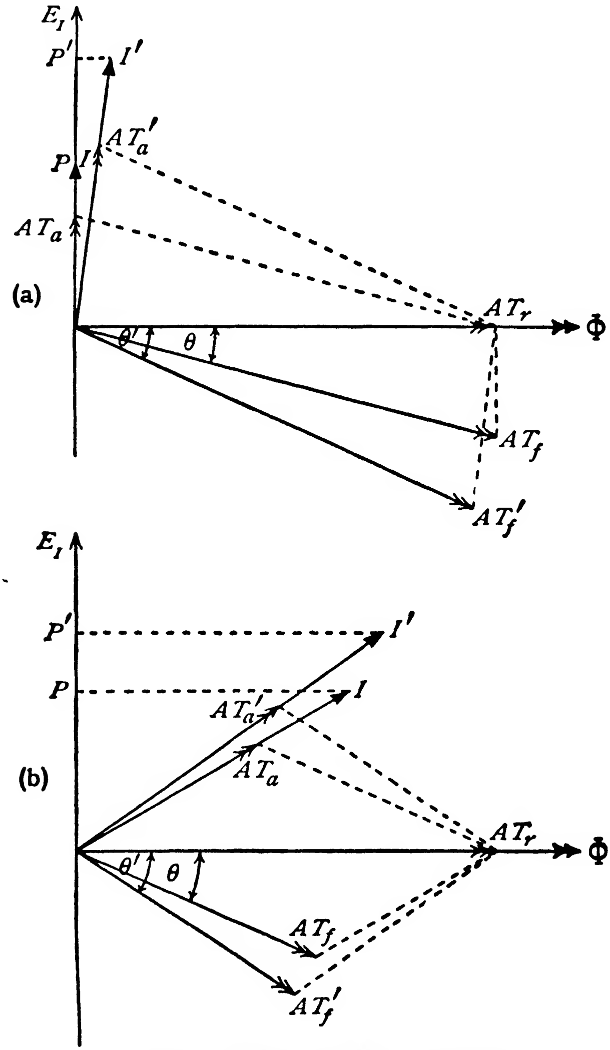


FIG. 303.—Effect of Excitation upon Stability.

One method of starting up is to employ a small auxiliary induction motor. As these motors always run at a speed slightly less than that of synchronism, taking into consideration the number of poles, it is seen that if the auxiliary motor has the same number of poles as the main synchronous motor the set can never be run up to the correct speed for synchronizing. In order to get over this difficulty, the auxiliary induction motor is often made with fewer poles than the main motor, so that the set can be synchronized as it runs through the correct speed.

Synchronous motors direct coupled to D.C. generators are sometimes used for transforming from H.T., A.C. to L.T., D.C. The D.C. bus bars are never dead except when a complete shut down occurs, and the motor-generator set can be run up from the D.C. side, using the generator as a motor. The correct speed for synchronizing is obtained by shunt regulation, and after the A.C. motor has been thrown on to the bus bars the field of the D.C. machine is strengthened, which tends to lower the speed. But since the A.C. machine must run at synchronous speed, it follows that the back E.M.F. of the D.C. machine when running as a motor becomes greater than the bus bar voltage, and so it commences to generate. In the event of a complete shut down, these sets would be unable to commence running again, and so one set at least must be provided with independent means for starting up.

Polyphase synchronous motors can also be started up by the addition of a special squirrel cage winding on the field system. This consists of a number of bars let into slots or holes in the pole shoes after the manner of damping grids, all the bars being joined at each end by a stout copper ring going right round the field system. The currents in the armature set up a rotating magnetic flux which cuts the squirrel cage winding on the field system and induces currents in it. A torque is developed and the motor runs up to a speed a *little less* than that of synchronism, as an induction motor. The D.C. exciting current is now switched on, and sets up definite poles on the rotor (the field system), these poles slowly slipping past the poles due to the rotating field set up by the stator (armature) currents. The relative speed of the two sets of poles is that of slip and decreases as the slip gets less. As the D.C. field strength is gradually increased, the two sets of poles suddenly lock with one another, the set thus pulling into synchronism automatically. This method of starting does away with the necessity for synchronizing gear, and in addition the squirrel cage winding acts like a damping winding when running and serves to prevent hunting. On account of the large starting current required by this method, it is usual to start up the motors on a reduced voltage through auto-transformers. The D.C. field circuit should also be broken in several places during the earlier part of the starting period, in view of the high voltages induced in it due to transformer action when the speed is low.

As is explained on page 332, the torque developed by a squirrel cage winding at a particular speed depends upon the value of the rotor resistance, so that if a self-starting synchronous motor is required to develop a large starting torque, it must be provided with a squirrel cage winding of relatively high resistance. In this event the normal speed to which the squirrel cage winding will run up the motor is rather less than would be the case if the resistance of the auxiliary winding were less. This means that the torque pulling the machine into synchronism is less with a high resistance squirrel cage winding than with a low resistance one, so that high starting torque involves a low " pull-in " torque and *vice versa.*

Self-synchronizing Motors.—An alternative method of connecting the starting motor is to place it in series with the main synchronous motor. In a three-phase case, the starting induction

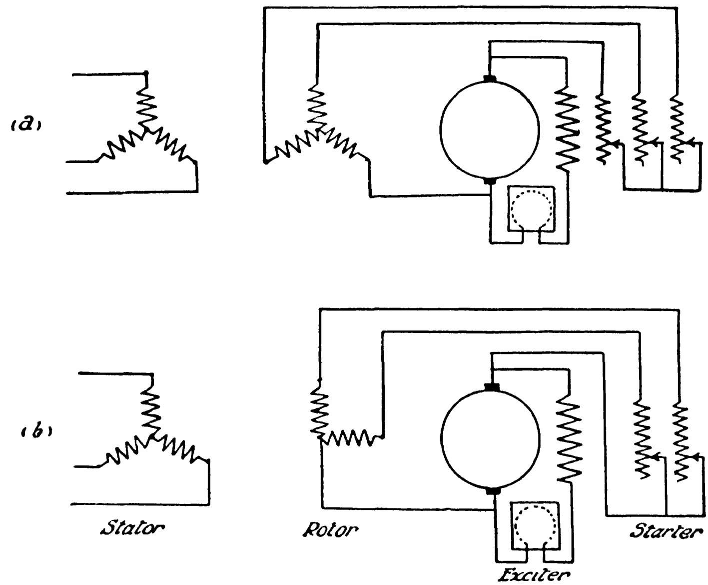


FIG. 304.—Connections of Synchronous Induction Motor.

motor has all six ends of its stator winding brought out, the three front ends being connected to the supply and the three rear ends to the main motor armature terminals. The rotor of the starting motor is of the short-circuited type and is frequently made of a solid steel cylinder without any slots or winding at all. The eddy currents induced in this rotor set up sufficient torque to enable the motor to start up.

When first switched in, the greater part of the voltage is thrown across the starting motor on account of the lower impedance of the main motor armature. As the set speeds up, however, these conditions are changed gradually and automatically, until when near synchronism nearly all the volts are across the main motor. The auxiliary starting motor now has its stator windings short-circuited, thus cutting it out and throwing the whole of the supply voltage on to the main motor which now pulls itself into synchronism.

Synchronous Induction Motor.—Synchronous motors are usually of the salient pole type, but this construction is not the only one. Alternators are built with a cylindrical type rotor, the field winding being placed in slots. Synchronous motors are also built in the same way. The machine now looks like an induction motor inasmuch as both stator and rotor are provided with slots. A further step in the development consists in using an induction motor construction entirely, the rotor being provided with a polyphase winding which is connected to the usual starting resistances during the starting period, these being finally short-circuited and the D.C. exciter substituted in their place. The connections are shown in Fig. 304 (*a*), where it is seen that the D.C. exciting current flows unequally through the three rotor phases, one receiving double the current of the other two. To obviate this, a two-phase three-wire rotor may be used as shown in Fig. 304 (*b*), the D.C. exciter feeding into the common point. The rotor currents are now equal.

A modification of this two-phase system has the exciter connected in series with one of the phases only, instead of feeding into the common point. The exciter armature shown in Fig. 304 (*b*) is now removed and inserted in the upper connection joining the rotor winding to the starter. When operating as a synchronous motor the rotor field is provided by one phase winding only, the other phase winding being short-circuited and acting as a damping winding. When starting as an induction motor the exciter armature carries A.C. as before.

Such a motor can be designed to pull into synchronism against full load torque, and by suitably regulating the D.C. field current can be made to operate if necessary on a leading power factor. If overloaded to the limit of stability as a synchronous motor, it falls out of step but continues to run as an induction motor with a slip of a few per cent., until finally the breakdown load is reached.

When a synchronous induction motor is liable to high momentary overloads, it is desirable to increase the D.C. exciting current to enable the motor to deal with this load. This can be accomplished automatically by providing the exciter with an additional field winding of the series type. This is supplied from a double wave metal rectifier which is fed from the secondary of a series transformer, the primary of which is connected in series with the stator winding.

The radial air-gap length of such motors is usually two or three times that of a pure induction motor of similar size, since the machine operates normally as a synchronous and not as an induction motor.

Examples

(1) A single-phase synchronous motor operates on 500 volts. Its synchronous reactance is 0·6 ohm and its resistance is 0·2 ohm. What is the back E.M.F. of the motor when operating on 0·8 power factor lagging, the armature current being 80 amperes?

(2) What is meant by "hunting" in a synchronous motor? Describe the principal methods adopted to overcome it.

(3) Describe in detail the operations for starting a large 16-pole, 50-cycle, three-phase synchronous motor, using an auxiliary starting motor.

(4) A single-phase synchronous motor generates a back E.M.F. of 250 volts, lagging by 210° behind the applied E.M.F. of 200 volts. The armature synchronous reactance is 2·5 times its resistance. What power factor is the motor operating on and does the current lag or lead?

(5) A single-phase synchronous motor has an armature resistance of 0·04 ohm and a synchronous reactance of 0·20 ohm. The applied and back E.M.F.'s are 100 and 80 volts respectively. Draw curves of current and power factor against power input and determine the current and power input for maximum power factor.

(6) Describe the principle of operation of the synchronous induction motor. Explain in detail what occurs when the motor is started up and pulls into synchronism.

CHAPTER XXIV

PARALLEL OPERATION OF ALTERNATORS

Synchronizing Current.—If two alternators generating exactly the same E.M.F. are perfectly synchronized, there is no resultant E.M.F. acting on the local circuit consisting of their two armatures. No current circulates between the two and no power is transferred from one to the other. There is, apparently, no force tending to keep them in synchronism, but as soon as the conditions are disturbed a synchronizing force is called into play, tending to keep the whole system stable. Suppose one generator falls behind a little in phase. The two alternator E.M.F.'s now produce a resultant voltage (see the vector diagram for a synchronous motor in Fig. 298), and this acts on the local circuit consisting of the two armature windings and the joining connections. In modern alternators the synchronous reactance is large compared with the resistance, so that the resultant circulating current is very nearly in quadrature with the resultant E.M.F. acting on the circuit. Fig. 305 represents a single-phase case, where E_1 and E_2 represent the two induced E.M.F.'s, the latter having fallen back slightly in phase. The

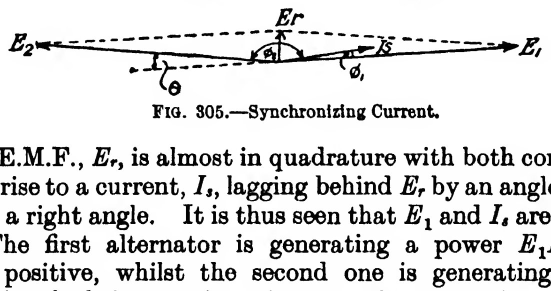


FIG. 305.—Synchronizing Current.

resultant E.M.F., E_r, is almost in quadrature with both components, and gives rise to a current, I_s, lagging behind E_r by an angle approximating to a right angle. It is thus seen that E_1 and I_s are almost in phase. The first alternator is generating a power $E_1I_s \cos \phi_1$, which is positive, whilst the second one is generating a power $E_2I_s \cos \phi_2$ which is negative, since $\cos \phi_2$ is negative. In other words, the first alternator is supplying the second with power, the difference between the two amounts of power representing the copper losses occasioned by the current I_s flowing through the circuit which possesses resistance. This power output of the first generator tends to retard it, whilst the power input to the second one tends to accelerate it, so that the action helps to keep both machines in stable synchronism. The current, I_s, is called the *synchronizing current.*

Synchronizing Power.—Suppose that one alternator has fallen behind its ideal position by an electrical angle θ, measured in radians. This corresponds to an actual geometrical angle of $\frac{2\theta}{p} = \psi$, where p is the number of poles. Since E_1 and E_2 are assumed equal, E_r is very nearly equal to θE_1. Moreover, since E_r is practically in quadrature with E_1, and I_s is practically in quadrature with E_r, E_1 and I_s may be assumed to be in phase as a first approximation. The synchronizing power may, therefore, be taken as $E_1 I_s = \frac{\theta E_1^2}{X}$ since $I_s = \frac{E_r}{X} = \frac{\theta E_1}{X}$, where X is the synchronous reactance of both armatures, the resistance being neglected. When one alternator is considered as running on a set of bus bars the power capacity of which is very large compared with its own, the combined reactance of the other sets connected to the bus bars is negligible, so that in this case X is the synchronous reactance of the one alternator under consideration. If $I_x = \frac{E}{X}$ is the steady short-circuit current of this alternator, then the synchronizing power may be written $\frac{\theta E^2}{X} = EI_x\theta$, although the current I_x does not actually flow.

In an m-phase case, the synchronizing power becomes $P_s = mEI_x\theta$ watts, E and I_x now being the phase values.

If the alternator drops behind its ideal position, it receives this power which tends to advance it in phase, whilst if it becomes ahead in phase it gives out this power which tends to make it fall back, until stable conditions are set up.

If the resistance of the armature be taken into account, then the synchronizing power is not quite the same when the machine is ahead in phase as when it is behind. When behind in phase it receives a power $-E_2 I_s \cos \phi_2$ (see Fig. 305), and when ahead it delivers to the bus bars a power $E_1 I_s \cos \phi_1$. Although E_1 and E_2 are assumed to be equal, yet on account of the armature resistance which causes I_s not to be in exact quadrature with E_r, $\cos \phi_1$ is greater than $-\cos \phi_2$, so that the synchronizing power is now greater. The difference is due to the copper losses in the circuit, which in the latter case are supplied by the alternator in question, whilst in the former they are supplied from the bus bars.

It should be borne in mind that the above is only an approximate representation of what happens, since the effects of armature reaction have been included with those of true leakage reactance to make up the synchronous reactance. This method of calculation usually gives too low a value for the synchronizing power, and the discrepancy may be considerable with salient pole machines. It may be allowed for by multiplying the synchronous reactance by a constant determined more or less empirically.

It will also be noticed that alternators with a large ratio of reactance to resistance are superior from a synchronizing point of view to those which have a smaller ratio, as then the synchronizing current cannot be considered as being in phase. Thus whilst reactance is bad from a regulation point of view it is good for synchronizing purposes.

Synchronizing Torque.—The accelerating or decelerating torque on the shaft produced by the synchronizing power depends upon the speed. Let T_s be this synchronizing torque in lb.-ft. and n = r.p.m. Then

$$\frac{2\pi n T_s}{33{,}000} = \frac{mEI_x\theta}{746}$$

and

$$T_s = \frac{33{,}000}{2\pi \times 746} \times \frac{mEI_x\theta}{n} = \frac{7{\cdot}04 mEI_x\theta}{n} \text{ lb.-ft.}$$

$$= \frac{0{\cdot}0587 mEI_x\theta p}{f} \text{ lb.-ft.}$$

Since all these quantities except θ are constants for a particular alternator operating at a given voltage, it follows that the synchronizing torque can be expressed as $\frac{0{\cdot}0587 mEI_x p}{f}$ lb.-ft. per electrical radian displacement, or $\frac{0{\cdot}0293 mEI_x p^2}{f}$ lb.-ft. per geometrical radian displacement, and this is a constant for a given machine. (It should be noted that a displacement of one radian is usually impossible.)

Inequality of Voltage.—Suppose two alternators to be running exactly in phase, but that their induced E.M.F.'s are not quite equal. Considering the local circuit, their E.M.F.'s are now in exact phase opposition, as shown in Fig. 306 (*a*), but they set up a resultant voltage E_r, now in phase with E_1, assumed to be the greater of the two. The synchronizing current, I_s, now lags by almost 90° behind E_1, so that the synchronizing power, $E_1 I_s \cos \phi_1$, is relatively small, whilst the synchronizing torque per ampere is very small. This lagging current, however, exerts a demagnetizing effect upon the alternator generating E_1, so that the effect is to reduce its induced E.M.F. Again, the other machine is, so far as this action is concerned, operating as a synchronous motor, taking a current leading by approximately 90°. The effect of this is to strengthen its field and so raise its voltage. The two effects combine to lessen the inequality in the two voltages and thus tend towards stability.

The power delivered by the first alternator now tends to retard it in phase, whilst the other machine is accelerated, and the new conditions are represented in Fig. 306 (*b*). A very small relative phase displacement of E_1 and E_2 is sufficient to cause a very considerable alteration in phase to E_r, so that I_s, lagging behind E_r by the same angle as before, is very quickly moved into the adjacent quadrant, when the direction of flow of synchronizing power is reversed. The tendency is now to bring the two machines into phase again, so that stable conditions are once more set up.

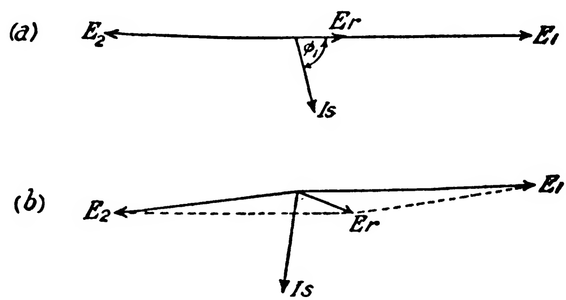


FIG. 306.—Effect of Unequal Voltages.

Inequality of voltage is, however, objectionable, since it gives rise to synchronizing currents which have a very large reactive component.

Load Sharing.—The amount of load taken up by an alternator running in parallel with others is governed by the power input to its prime mover. Alteration of the excitation produces a change in the kVA output but does not affect the kW output. It merely causes a change in the power factor by altering the reactive component.

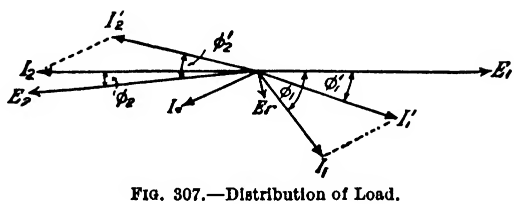


FIG. 307.—Distribution of Load.

Consider two single-phase alternators running in parallel, generating E_1 and E_2 volts respectively. If they are exactly in phase, the E.M.F.'s will appear in direct phase opposition when considered with respect to the local circuit. Now suppose their excitations and the powers developed by their prime movers to be so adjusted as to make them deliver I_1' and I_2' amperes at power factors of cos ϕ_1' and cos ϕ_2' respectively (see Fig. 307). The total load current is the vector sum of I_1' and I_2' (shown as the difference in the vector diagram). The power input to the second alternator is now increased, so that this set attempts to accelerate. As soon as it has got ahead in phase by a small angle as shown in Fig. 307, new conditions of load distribution are set up. The E.M.F.'s E_1 and E_2 now produce a resultant voltage E_r acting around the local circuit. This in its turn sets up a synchronizing current I_s lagging behind E_r by an angle which is not far short of 90°. This current I_s must now be added vectorially to the currents originally carried by the two armatures. It is generated by the second one and must therefore be added to I_2' to obtain the new armature current I_2. The first alternator receives this synchronizing current, which tends to lessen its current output, on account of its phase angle. Its resultant current is now I_1, being the vector sum of I_1' and I_s.

The effect of increasing the input to one prime mover is thus seen to make its alternator take an increased share of the load, the other one being relieved to a corresponding extent. The final power factors, cos ϕ_1 and cos ϕ_2, are also altered since the ratio of the reactive components of the load has been changed as well.

Effect of Change of Excitation.—Suppose two paralleled alternators of the same rating have the same input. Their kW outputs are therefore equal. The reduction of the field current of one of them causes its induced E.M.F. to be lowered. The resultant voltage around the local circuit now has the same phase as the E.M.F. of the more strongly excited machine, the synchronizing current lagging behind this by 90°. This gives rise to a demagnetizing armature reaction and so lowers the induced E.M.F. On the other hand, the synchronizing current leads by 90° the induced E.M.F. of the weaker machine, and so produces a magnetizing armature reaction. It therefore strengthens the E.M.F. of this machine. The result is a tendency to equalize the E.M.F.'s. Also the distribution of the reactive component of the total load has altered, the more strongly excited machine now taking a larger proportion. In fact, it is possible to make one alternator operate on a leading power factor by sufficiently disturbing the two excitations.

Effect of Difference in Wave Form.—If the open-circuit E.M.F. wave forms of two paralleled alternators be different, then a circulating current between the two will be set up, irrespective of any phase advance of one or other of the prime movers. Assuming the fundamental components of the E.M.F.'s to be equal and in exact phase opposition, they will neutralize each other around the local circuit, but this will not apply to the harmonics, which are, by assumption, different. The wave form of the circulating current will be very irregular in shape and will contain no fundamental. Such conditions are to be avoided. In other words, only alternators with the same wave form should be run in parallel, thus giving an additional reason for the choice of the sine wave as a standard.

There is usually a small change in the wave form as the load comes on, so that the circulating current will be different for different loads, and the division of the total load may be affected thereby. Fortunately, however, in modern machines, this change of wave form with change of load is not usually of any serious moment.

Free Oscillations.—The actual rotation of the rotor may be resolved into a uniform rotation together with a superimposed to-and-fro oscillation or phase-swinging which may be set up by faulty synchronizing, by change of load or by various other causes. A sudden increase in load causes the rotor to lag momentarily behind its synchronous position. The load is then reduced and a synchronizing torque is set up proportional to the retardation. When the retardation angle reaches a certain value, this torque becomes available for accelerating the rotor. After a further time it gets ahead of its synchronous position and acquires too great a proportion of the total load upon which it is subjected to a decelerating force. As the synchronizing torque is proportional to the phase displacement it can be shown that these oscillations obey the law of harmonic motion. If the oscillations are damped, they gradually die out and are not of great importance, but if, as on occasion happens, they gradually increase in amplitude, the machine ultimately is swung into an unstable position and falls out of step. For steady running the frequency of oscillation should be low.

Forced Oscillations.—Due to irregularities in the torque set up by the prime mover throughout its cycle, forced oscillations are impressed upon the rotor of the alternator.

With a single-cylinder (double-acting) reciprocating steam engine, there are two accelerations per revolution. The periodic time is thus $\frac{60}{2n} = \frac{30}{n}$ seconds, where n = r.p.m. For a two-crank engine with cranks at 90° and for three-crank engines with cranks at 120°, the periodic times are $\frac{15}{n}$ and $\frac{10}{n}$ seconds respectively. If this periodic time approximates to that of the free oscillations, then mechanical resonance is set up. The phase-swinging gradually increases in amplitude until the alternator falls out of step.

Practice has shown that the maximum displacement should not exceed 2·5° (electrical) either way. The corresponding mechanical angle is obtained by dividing by the number of pairs of poles. A low supply frequency is an advantage in this respect, as this reduces the number of poles for the same speed and allows a larger mechanical displacement, so that a smaller flywheel effect may be employed.

Speed Regulation.—There is a slight drop of a few per cent. in speed on the part of the prime mover, and the magnitude of this drop affects the distribution of load between the several paralleled alternators. Suppose that two machines are sharing the load and that the first suffers a drop of 2 per cent. and the second one of 4 per cent. from no-load to full load. Let them be arranged to share the total load equally at one particular value. An increase in total load now takes place. The drop in speed of both sets must be the same, for their frequencies must remain exactly the same. The drop in speed, however, corresponds to an increase in load of the 2 per cent. machine which is double that of the other, assuming straight line speed characteristics. The machine with the flatter speed curve will thus acquire two-thirds of the increase in load. The load will only be shared equally at one point. For lower loads, the first alternator will take less than its share, and for higher loads it will take more. It is therefore very desirable that alternators which are to run in parallel should have similar speed characteristics on the part of their prime movers.

Control of Bus Bar Voltage and Frequency.—If the bus bar voltage is low, the field currents of all the alternators must be increased. Raising the excitation of one only merely causes it to acquire reactive load.

If the bus bar frequency is low, the steam admission to all the alternators must be increased. Adjustment in the case of one only would merely cause it to acquire load at the expense of its fellows. Ultimately this would cause a rise in frequency, but the machine would probably be burnt out before any change in frequency was noticeable.

Disconnection from Bus Bars.—When it is desired to remove an alternator from the bus bars, the kW load should first of all be gradually removed by throttling the steam supply, after which the reactive load can be removed by reducing the excitation until the alternator is just floating on the bus bars. The main switch may now be opened with impunity, after which the prime mover may be brought to rest.

Current Limiting Reactors.—In modern large generating and transmission schemes the forces called into play when a breakdown occurs are very considerable, and for the purpose of protecting the plant, current limiting reactors are now largely employed. These serve to limit the current to more or less safe values by virtue of their reactance without wasting any appreciable amount of energy, since their resistance is kept small. They also tend to localize the effects of a fault by limiting the amount of power which can flow into it from other parts of the system, and thus they tend to protect the plant from a complete shut-down. Reactors may be introduced into the system at various points. They may be placed directly in series with individual generators, or they may be placed in the feeder circuits. They may also be inserted between different bus bar sections or in trunk mains acting as interconnectors between two generating stations.

Current limiting reactors are employed in practically all new large systems, but in small systems they tend to interfere to a certain extent with the regulation.

Rating of Reactors.—Reactors bring about a voltage drop, and this (when the rated current is flowing through the reactor at normal frequency) is usually expressed as a percentage of the voltage between lines on single-phase circuits, or as the voltage between line and neutral on three-phase circuits. For example, a reactor on a single-phase 2000 volt circuit, dropping 120 volts on full load, would be called a $\frac{120}{2000} \times 100 = 6$ per cent. reactor. To obtain the same percentage reactance on a three-phase circuit with a line pressure of 2000 volts, the voltage drop would have to be $\frac{2000}{\sqrt{3}} \times \frac{6}{100} = 69{\cdot}3$ volts.

The kVA rating of a reactor is obtained by multiplying the rated current and the pressure drop across it, the resulting product being divided by 1000.

Situation in System.—Reactors placed in series with the generators limit the current in the armature windings and protect the generators. They also protect the end turns (see page 194) from the extra electrical stresses to which they are subjected. In modern alternators the usual practice is to incorporate a portion of the necessary reactance in the design of the machine itself, and to provide the remainder by means of an external reactor.

Reactors are occasionally installed on the L.T. side of transformers feeding modern high-voltage transmission systems, but the transformers are usually designed with a comparatively high internal reactance so that they can safely withstand short-circuits without extra help.

Long feeders frequently have sufficient inherent reactance without the addition of artificial reactance, but in short feeders the latter is very common. Short-circuits are more common on feeders than on any part of the system, and the reactors tend to limit the area of disturbance.

Bus bar reactors may be connected in two ways, known as the *ring* and *star* methods. In the former, bus bar sections and reactors are connected alternately in series to form a complete ring, whilst in the latter the bus bar sections are connected through reactors to a common star point.

When reactors are inserted in trunk interconnector mains between two stations they may be treated very much in the same way as if they were connecting different bus bar sections.

Reactors in Bus Bars.—Reactors placed between sections of the bus bars tend to confine the trouble due to a fault on the mains to the particular section in which the fault occurs. This results in the various generators being operated with a certain angular displacement in their voltages. For example, consider the two alternators, A and B, in Fig. 308, connected to two feeder circuits, C and D, the bus bars being divided into two sections connected through the reactors XX. Assuming the power factor to be unity, the load on C greater than that on D, and the total load shared equally between the two generators, it follows that a certain amount of current must flow through the two reactors. Neglecting the losses in these, this current is in quadrature with the voltage across the terminals of the reactors. The vector diagram is shown in Fig. 309 (*a*), which represents the conditions for unity power factor. V_A and V_B represent the voltages of the two alternators and V_AV_B the voltage across the reactor. I_C and I_D represent the currents in the two feeder circuits. The current flowing through the reactor is determined by the magnitude of V_AV_B, and hence the phase displacement of the two alternators depends upon the difference in the loads on the two feeders. By subtracting I_AI_C (at right angles to V_AV_B) in the one case and adding I_BI_D in the other, these two currents being the same, the currents delivered by the two alternators, I_A and I_B, are obtained. When the current is a lagging one, the conditions are represented in Fig. 309 (*b*), the current $I_AI_C = I_BI_D$ flowing through the reactor being added to one alternator current and subtracted from the other to obtain the two feeder currents.

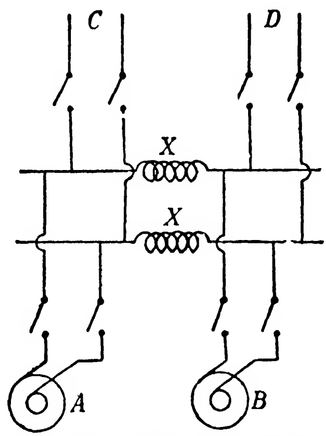


FIG. 308.—Reactors in Bus Bars.

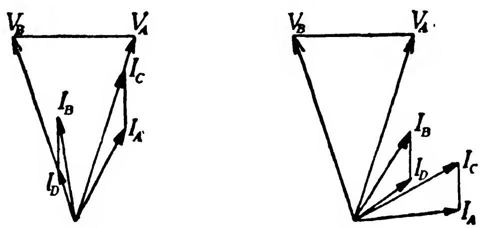
Effect on Power Factor and Regulation.—The insertion of additional reactance in the system tends to lower the general power factor, but not to a very great extent, particularly if the original power factor is near unity. The regulation is also affected in an adverse manner, but this is almost negligible when the power factor is high, although it may become serious if the power factor of the load is low.

Construction of Reactors.—When a short-circuit occurs on a current limiting reactor, the current flowing through its winding becomes very large. Take the case, for example, of a 5 per cent. reactor, which will allow 20 times the normal current to flow in the case of a dead short-circuit. Its reactance must not fall, which means that the iron (if it has an iron core) must not become saturated with 20 times normal current flowing through it. This is really understating the case, for abnormal current rushes are liable to take place (see page 581) if a short circuit occurs at or near to the zero point of the voltage wave. Normally, therefore, it must work at an extremely low flux density, which makes the use of iron uneconomical; in fact, the iron magnetic circuit would be larger than an air circuit. These reactors are, therefore, built with non-magnetic cores, or, alternatively, with an iron core but with an air-gap included in the magnetic circuit.

In one type a number of bare conductors are wound in conical coils which are alternately upright and inverted. These coils are embedded in concrete supports which are cast around the turns so that the whole construction is very rigid. This is necessary on account of the large forces which are suddenly called into play when the first rush of current occurs.

Losses in Reactors.—The losses in reactors are due to the I^2R and eddy current losses in the conductors, and to reduce the latter the conductors are stranded. The total loss is usually about 5 per cent. of the kVA rating of the reactor.

Interconnected Power Systems.—The practice of intercon-



(a) Unity Power Factor.

(b) Power Factor Less than Unity. Current Lagging.

FIG. 309.—Effect of Reactor in Bus Bars.

necting power systems is becoming more and more common, and is an extension of the idea of the parallel running of generators. It enables each station to supply the other with power in time of need. It is not necessary that each station should operate at the same voltage, as the stations may be linked through transformers. In any case the interconnector is usually worked at a very high voltage, and transformers are employed at each end to step up the generator pressure to that of the interconnector. A difference in the number of phases is also no bar to this interlinking as phase-changing groups of transformers may be used, as, for example, Scott-connected groups in changing from two- to three-phase or *vice versa*. Frequency changers must be installed when there is a difference of frequency in the two supplies.

If the two stations are in phase and the bus bar voltages are exactly the same, no current will flow in the interconnector. This corresponds to the case of two alternators in parallel exactly balanced. There is no synchronizing torque. But directly one station gets ahead of the other in phase a synchronizing current flows in the interconnector, and this keeps the two stations in step.

In order to cause power to be transmitted from one station to the other, the two station bus bar voltages must be displaced in phase, and usually one is required to be boosted so as to overcome the impedance drop in the interconnector.

Induction Regulator.—The induction regulator is a piece of apparatus which enables a continuously variable boost to be applied to a circuit. It consists of a stator and rotor wound as an induction motor, the primary being connected across the supply and the secondary in series with it. In order to avoid the use of six slip rings in a three-phase case, the secondary is wound on the stator. In some cases the use of slip rings is avoided altogether by making connection to the rotor by means of flexible connections, for the rotor does not rotate in the ordinary way. The rotor is geared to a handwheel through a rack and pinion, so that its position relative to the stator can be varied at will. The magnitude of the voltage induced in the secondary is constant provided the primary voltage

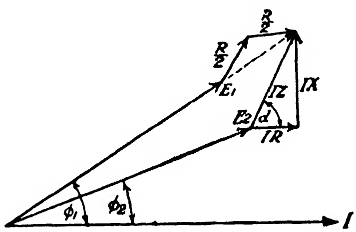


FIG. 310.—Induction Regulator in Interconnector.

is constant, but its phase depends upon the position of the rotor. This phase can be varied at will throughout the full 360°, so that a continuously variable boost is obtained. The maximum and minimum voltages obtainable are the arithmetical sum and difference of the supply and boost voltages respectively. The phase of this resultant voltage varies somewhat from maximum to minimum, and to avoid this a double regulator is sometimes employed. The second one displaces the phase of the resultant voltage in the opposite direction to the first, so that the regulated and non-regulated voltages have always the same phase. Instead of two regulators, one only may be employed, if its series winding be divided into two halves, these being connected in reverse series.

Induction Regulator in Interconnector.—As an example to show how an induction regulator operates in an interconnector, consider the vector diagram in Fig. 310, where E_1 and E_2 are the voltages at the sending and receiving ends respectively. Each half of the double induction regulator boosts up the sending voltage by an amount $\frac{R}{2}$. The vector difference between this boosted voltage and that at the receiving end of the line is equal to the impedance drop in the interconnector and the connected apparatus between the two stations. The impedance drop is resolved into its two components IR and IX, the former being in phase with the current I flowing through the interconnector. The power factors at the sending and receiving ends are cos ϕ_1 and cos ϕ_2 respectively, whilst the power factor of the interconnector itself is given by cos α.

On altering the phase of $\frac{R}{2}$ by adjustment of the rotor of the induction regulator (the two vectors $\frac{R}{2}$ move in opposite directions) the magnitude of IZ can be controlled, thus controlling the amount of power transmitted from one station to the other. But as α depends upon the resistance and reactance of the interconnector it remains constant, so that both ϕ_1 and ϕ_2 are altered, the power factor of the transferred load varying accordingly.

Examples

(1) Two three-phase star-connected 50-cycle alternators of equal capacity are synchronized, the terminal voltage being 6500 volts and the external load zero. The resistance and synchronous reactance of each generator is 1·4 ohms and 7·0 ohms per phase respectively. Determine the synchronizing current for a phase displacement of 2·5° (electrical), the excitations being the same.

(2) Assuming that each of the above machines runs at 1500 r.p.m., determine the synchronizing torque per geometrical radian displacement.

(3) Explain the factors which affect the sharing of load between two alternators running in parallel.

(4) Two single-phase alternators each have a terminal pressure of 2000 volts and are linked through an 8 ohm reactor the resistance of which is negligible. A load of 500 kW at unity power factor is taken off the bus bars at one particular point. What are the kVA loads on the two alternators assuming equal inputs?

(5) Two three-phase stations each have a bus bar voltage of 6600 volts and are linked by an interconnector which has a resistance and reactance of 2·0 and 5·0 ohms per line respectively. A double induction regulator gives a boost of 400 volts per phase to one station voltage without altering its phase, whereupon the current per interconnector line is found to be 150 amperes. What is the phase displacement between the two station bus bar voltages?

CHAPTER XXV

ROTARY CONVERTERS

Methods of Transformation.—A number of pieces of apparatus are at present on the market for converting alternating into direct current. The main application of such machinery is in the equipment of a sub-station where A.C. is received from the E.H.T. mains and an L.T., D.C. is delivered to the consumers. There are four chief types of converting plant, viz. motor generators, rotary converters, motor converters and mercury arc rectifiers. Then there are the electrolytic rectifier or electric valve, the thermionic valve, metal rectifiers, and lastly mechanical rectifiers.

Principle of the Rotary Converter.—The ordinary D.C. generator really generates an alternating E.M.F. which is made direct by the action of the commutator. If a pair of slip rings be mounted on the armature of such a generator at the non-commutator end, these slip rings being connected to conductors situated diametrically opposite each other in a bipolar case, the machine will also act as an A.C. generator. Moreover, it is possible to make it generate both D.C. and A.C. at the same time by mechanically driving it. An A.C. generator is, however, capable of acting as a synchronous motor, so that the rotation may be produced by supplying the A.C. end with power in an electrical form instead of driving the machine mechanically. Driven in this way, the machine is known as a *rotary converter*. The rotation of the armature induces an alternating back E.M.F. in it, and this is converted into a direct E.M.F. by the action of the commutator. If no current is delivered by the D.C. end, the motoring current taken by the A.C. end is only that required to overcome the losses of the machine and maintain its rotation. When the D.C. end is connected to some form of load resistance, a drag on the armature conductors is produced and is the equivalent of putting a load on the motoring A.C. end, which consequently takes a larger current. The power input to the rotary converter is obviously equal to the power output together with the power wasted in its losses.

This machine must not be confused with a motor-generator employing a single armature core and field system. The latter machine has a double armature winding and the conductors carrying the motoring current are quite distinct electrically from those carrying the generated current. In a rotary converter the same conductors carry both currents superimposed on each other, and since, generally speaking, the motoring current is flowing in the opposite direction to the generated current, the resultant current at any instant is the difference of the two.

Inverted Rotary Converter.—Instead of supplying the A.C. end with a motoring current and generating a direct E.M.F. at the D.C. end, the reverse may be done. The commutator is then supplied with D.C., in which case the machine runs as a D.C. motor and acts like an A.C. generator. When run in this reverse manner, the machine is known as an *inverted rotary converter.*

Polyphase Rotary Converter.—If an inverted rotary converter be supplied with three slip rings connected to conductors situated one-third of a cycle apart it will generate a three-phase supply. Conversely, when run in the ordinary way it may be supplied with a polyphase current if suitable slip ring connections are made. The number of slip rings necessary for the various polyphase supplies are the same as in the case of a rotating armature alternator or synchronous motor.

Ratio of Transformation.—The brushes on the commutator are placed so as to obtain the maximum voltage generated, and, in a single-phase bipolar machine, once during each half-revolution or half-cycle the conductors connected to the slip rings come under the brushes as shown diagrammatically in Fig. 311. In every other position the voltage across the slip rings is less than it is at this instant. In the position shown, therefore, the voltage across the slip rings is also a maximum, and, neglecting the losses in the machine, this is equal to the commutator voltage. The maximum value of the A.C. voltage is thus equal to the D.C. voltage in a single-phase rotary converter, and, assuming a sinusoidal wave form, the R.M.S. value of the A.C. voltage is equal to $\frac{1}{\sqrt{2}}$ times the D.C. voltage.

Still neglecting the losses, the watts at the two ends of the machine are equal, or

$$E_D I_D = E_A I_A \cos \phi,$$

and

$$I_A = I_D \times \frac{E_D}{E_A}\frac{1}{\cos \phi}$$

$$= I_D \times \frac{\sqrt{2}}{1} \times \frac{1}{\cos \phi}.$$

If the machine is operating on unity power factor the alternating current is equal to $\sqrt{2}$ or 1·414 times the direct current.

In the case of a two-phase rotary converter (bipolar machine) there are four slip rings connected to conductors situated 90° apart (see Fig. 312). Since the armature winding is necessarily of the distributed type as used in D.C. machines, the two phases are linked together in the armature of the rotary converter itself, and consequently the phases of the supply must be linked at the centre and not at one end. The three-wire system of transmission is therefore inadmissible. The two slip rings belonging to each phase being connected to conductors situated diametrically opposite, the voltage per phase is the same as in the single phase case, viz. $\frac{1}{\sqrt{2}}$ or 0·707 times the D.C. voltage.

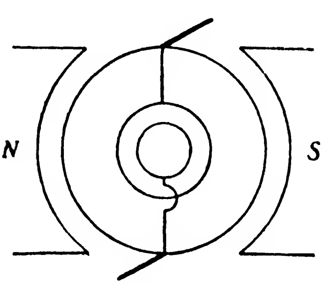


FIG. 311.—Armature Position for Maximum Voltage. Single-Phase.

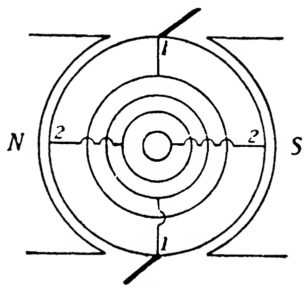


FIG. 312.—Two-Phase Rotary Converter.

Equating the D.C. and A.C. power, we get

$$E_D I_D = 2E_A I_A \cos\phi$$

and

$$I_A = I_D \times \frac{E_D}{E_A} \times \frac{1}{2\cos\phi}$$

$$= I_D \times \frac{1}{\sqrt{2}\cos\phi}.$$

On unity power factor, the A.C. line current is therefore 0·707 times the direct current.

In a three-phase rotary converter, the conductors connected to any two slip rings cannot be connected to the brushes at the same time, since the slip ring connections are spaced 120° apart instead of 180°, as in the single- and two-phase machines. The position of the slip ring connections for maximum voltage must therefore be determined. In the single-phase case it was seen that this position was the one where the slip ring conductors enclosed the maximum number of lines of force. In other words, the vertical distance between a and b in Fig. 313 must be a maximum. Calling the radius of the circle unity, this distance is $1 + \sin 30° = 1·5$ in Fig. 313 (a) and $2 \sin 60° = 1·732$ in Fig. 313 (b). The latter position will be found to give the maximum value for all the various positions, the length of the vertical line being $\frac{\sqrt{3}}{2}$ times the full diameter of the circle.[1]

The maximum A.C. line voltage is therefore $\frac{\sqrt{3}}{2}$ times the D.C. voltage, and the R.M.S. value of the A.C. voltage is $\frac{\sqrt{3}}{2\sqrt{2}} = 0{\cdot}612$ times the D.C. voltage.

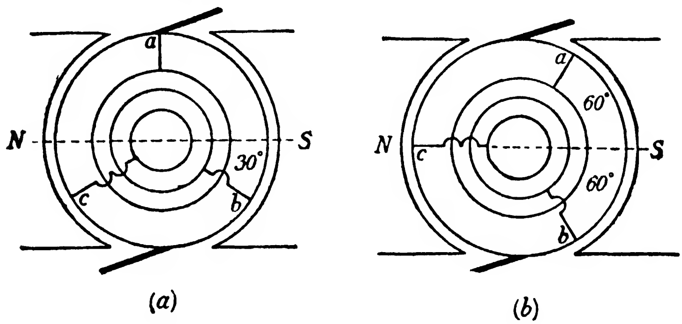


FIG. 313.—Three-Phase Rotary Converter.

Equating the D.C. and A.C. power again, we get

$$E_D I_D = \sqrt{3} E_A I_A \cos\phi$$

and

$$I_A = I_D \times \frac{E_D}{E_A} \times \frac{1}{\sqrt{3}\cos\phi}$$

$$= I_D \times \frac{2\sqrt{2}}{\sqrt{3}} \times \frac{1}{\sqrt{3}\cos\phi}$$

$$= I_D \times \frac{0{\cdot}943}{\cos\phi}.$$

Since the winding is distributed in many slots per pole, the vector diagram of the voltages per conductor takes the form of a semicircle

[1] This can be shown by the calculus as follows :—

Vertical distance $= \sin\theta + \sin(120^\circ - \theta)$

$= \sin\theta + \sin 120^\circ \cos\theta - \cos 120^\circ \sin\theta$

$= \frac{3}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta = y.$

For $y = \max.,\ \frac{dy}{d\theta} = 0,$

$$\frac{dy}{d\theta} = \frac{3}{2}\cos\theta - \frac{\sqrt{3}}{2}\sin\theta = 0.$$

$$\frac{3}{2}\cos\theta = \frac{\sqrt{3}}{2}\sin\theta,$$

$$\sqrt{3}\cos\theta = \sin\theta,$$

$$\tan\theta = \sqrt{3} \text{ and } \theta = 60^\circ.$$

(see Fig. 211). The resultant voltage when acting as a single-phase rotary converter is given by the diameter of the circle, the breadth factor being $\frac{2}{\pi}$ for this case. When acting as a three-phase machine, the slip ring tappings are situated two-thirds of a pole pitch, or 120°, apart, and the ratio of the three-phase to the single-phase voltage is given by $\frac{AC}{AB}$ in Fig. 314. From the geometry of the diagram, $\frac{AC}{AB} = \frac{\sqrt{3}}{2}$, and the three-phase voltage is equal to $\frac{\sqrt{3}}{2}$ times the single-phase voltage, or $\frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$ $= 0{\cdot}612$ times the D.C. voltage, assuming no losses, the same as before.

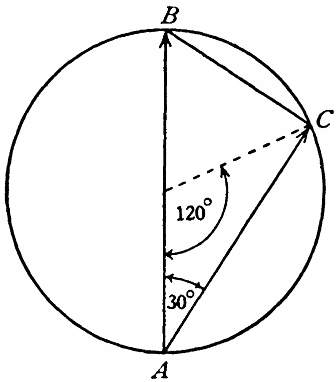


FIG. 314.—Construction for Voltage Ratio of Three-Phase Rotary Converter.

Similar calculations can be worked out for six- and twelve-phase rotary converters.

The various ratios for different numbers of phases, assuming 100 volts and 100 amperes at the D.C. end, are tabulated as follows :—

	D.C.	Single-Phase.	Two-Phase.	Three-Phase.	Six-Phase.	Twelve-Phase.
Volts between Slip Rings	100	70·7	70·7	61·2	35·4	18·3
Current per Slip Ring at Unity Power Factor ...	100	141·4	70·7	94·3	47·2	23·6

These ratios of transformation are somewhat affected by the ratio of pole arc to pole pitch, just as this ratio affects the E.M.F. generated in an alternator (see page 268). As a general rule, the A.C. voltage is raised by reducing the ratio of pole arc to pole pitch and *vice versa*, this being done at the expense of the wave form. Displacing the brushes from the no-load neutral position on the commutator alters the distribution of the flux on load and also affects the voltage ratio. The ratios calculated above have been worked out on the assumption of a sinusoidal wave form which involves a ratio of pole arc to pole pitch of approximately 0·7.

Ratio of Transformation.—General Case.—Consider the general case of a rotary converter with *m* slip rings. The number of phases is also equal to *m*, except in the single- and two-phase cases, where *m* is 2 and 4 respectively. The slip ring tappings are now situated

$\alpha = \frac{2\pi}{m}$ radians apart. (This is an electrical angle; to obtain the geometrical angle, $\frac{2\pi}{m}$ must be divided by the number of pairs of poles.)

The voltage between slip rings is given by AC in Fig. 315, AB representing the single-phase voltage, or $\frac{E_D}{\sqrt{2}}$. If a line be drawn from D to meet AC at right angles, it is seen that $\frac{AC}{2} \div \frac{AB}{2} = \sin\frac{\alpha}{2}$, so that $\frac{AC}{AB} = \sin\frac{\pi}{m}$. Let E_{SR} represent the R.M.S. voltage between slip rings, and E_n the voltage between slip ring and neutral. Then

$$E_{SR} = \frac{E_D}{\sqrt{2}}\sin\frac{\pi}{m},$$

and

$$E_n = \frac{E_D}{2\sqrt{2}}.$$

Let I_A represent the A.C. component of the current per conductor, I_{SR} the current per slip ring, and I_D the D.C. line current. Then, assuming no losses and unity power factor,

$$mE_nI_{SR} = E_DI_D$$

d

$$I_{SR} = \frac{E_DI_D}{mE_n} = \frac{2\sqrt{2}I_D}{m}.$$

Also

$$mE_{SR}I_A = E_DI_D,$$

$$m\frac{E_D}{\sqrt{2}}\sin\frac{\pi}{m}.I_A = E_DI_D,$$

$$I_A = \frac{\sqrt{2}\,I_D}{m\sin\frac{\pi}{m}}.$$

But

$$I_D = \frac{m}{2\sqrt{2}}I_{SR}$$

so that

$$I_A = \frac{\sqrt{2}}{m\sin\frac{\pi}{m}} \times \frac{m}{2\sqrt{2}}I_{SR}$$

$$= \frac{I_{SR}}{2\sin\frac{\pi}{m}}.$$

The above relations must be modified somewhat in practice on account of (1) efficiency, (2) power factor, and (3) the ratio of pole arc to pole pitch.

Let hI_{SR} be the active component of the slip ring current, allowing for iron loss, friction, excitation, etc. With 96 per cent. efficiency, for example, $h = \frac{1}{0 \cdot 96} = 1 \cdot 04$. Also let kI_{SR} be the reactive component of the slip ring current. Then the total slip ring current is

$$I_{AC} = I_{SR} \times \sqrt{h^2 + k^2}.$$

The ideal current must be multiplied by

$$H = \sqrt{h^2 + k^2}$$

where h is slightly greater than unity, and $k = h \tan \phi$.

Effect of Output on Ratio of Transformation.—Owing to the presence of resistance and reactance in the armature, the ratio of transformation does not remain constant for all values of the load. A certain amount of voltage is lost in this way, the magnitude depending upon the value of the load. If the A.C. supply voltage be maintained constant, the D.C. voltage gradually falls as the load comes on, after the manner of a shunt generator, and the relation between the D.C. load current and D.C. terminal voltage is called the characteristic of the rotary converter.

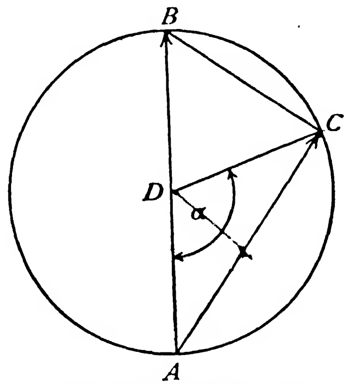


FIG. 315.—Ratio of Transformation.—General Case.

Construction.—The construction of a simple rotary converter differs very little from that of a D.C. generator, with the exception that slip rings are mounted at the non-commutator end of the armature. They are, however, sometimes complicated by the addition of boosters for the purpose of improving the regulation, the two armatures being carried on the same shaft. Interpoles are also always employed on modern machines for the purpose of improving the operation. Owing to the fact that the alternating and direct currents tend to neutralize each other in the armature, the heating is considerably reduced, which means that a larger output can be obtained from a particular armature with a given temperature rise than would be the case if the currents flowed in separate conductors instead of being superimposed on one another in the same bar. The size of a rotary converter for a given output and speed is thus less than that of the corresponding D.C. generator. Another advantage lies in the fact that a single magnetic field system serves the machine, whilst two field systems are required in the case of a motor-generator set.

Number of Poles.—The relation between the number of poles and the frequency and speed is the same as in the case of a synchronous motor or an alternator. For a given frequency the speed depends to a large extent upon the armature diameter, and, as this goes up with the output, the speed must come down with a corresponding increase in the number of poles. The following table shows the number of poles which may be expected for various outputs :—

Output in kW.	25 Cycles.	50 Cycles.
250	4 Poles	4 Poles
500	6 ,,	6 ,,
1000	6 ,,	8 ,,
1500	8 ,,	12 ,,
3000	12 ,,	16 ,,

Armature Windings.—Drum wound armatures are employed with windings exactly like those in D.C. generators. Either two-circuit

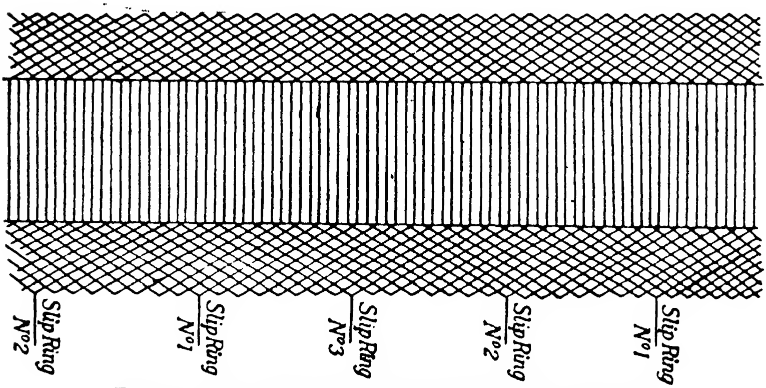


FIG. 316.—Three-Phase Armature Winding for Rotary Converter.

(wave) or multiple circuit (lap) windings may be used, but it is the usual practice to adopt the two-circuit windings for outputs up to about 100 kW, whilst multiple-circuit windings are used for the larger outputs.

When two-circuit windings are adopted there is only one connection required per slip ring, corresponding to the single brush spindle in the D.C. generator. When multiple-circuit windings are adopted there must be as many connections to each slip ring as there are pairs of poles, corresponding to the number of brush spindles in parallel in the D.C. generator. The total number of tappings required is therefore equal to the number of slip rings in a two-circuit winding and the number of slip rings multiplied by the pairs of poles in a multiple-circuit winding.

Consider as an example the case of a 6-pole multiple-circuit armature having 144 conductors. Used on single-phase, there would be 6 tappings spaced 24 conductors apart; as a two-phase machine there would be 12 tappings spaced 12 conductors apart; as a three-phase machine there would be 9 tappings spaced 16 conductors apart; and as a six-phase machine there would be 18 tappings spaced 8 conductors apart.

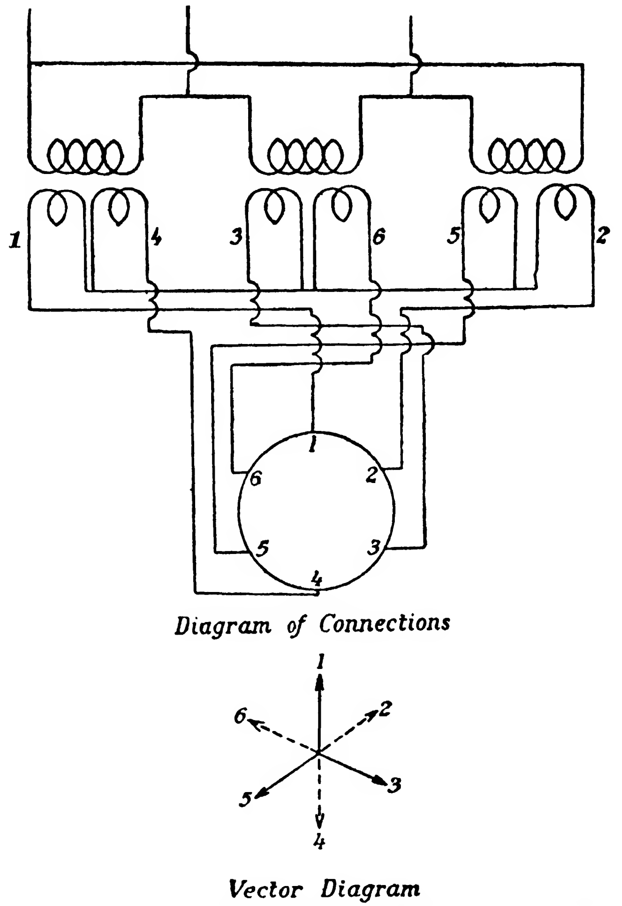


FIG. 317.—Double Star Connections for Six-Phase Rotary Converter.

A development of a portion of this winding when used for three phases is shown in Fig. 316.

Transformer Connections for Six-Phase Rotary Converter.—Owing to their improved performance, six-phase rotary converters are largely adopted on three-phase systems, since the six phases can be produced merely by connecting up the single three-phase or the three single-phase transformers in certain ways.

The first method is called the double star method of connection and is occasionally employed. Double secondary windings are required, each set being connected in simple star. The two star points are joined together and the six free ends connected as shown in Fig. 317, which also shows the vector diagram. The transformer secondary voltage per phase is now $\frac{0{\cdot}612}{\sqrt{3}}E_D = 0{\cdot}354E_D$ and if the

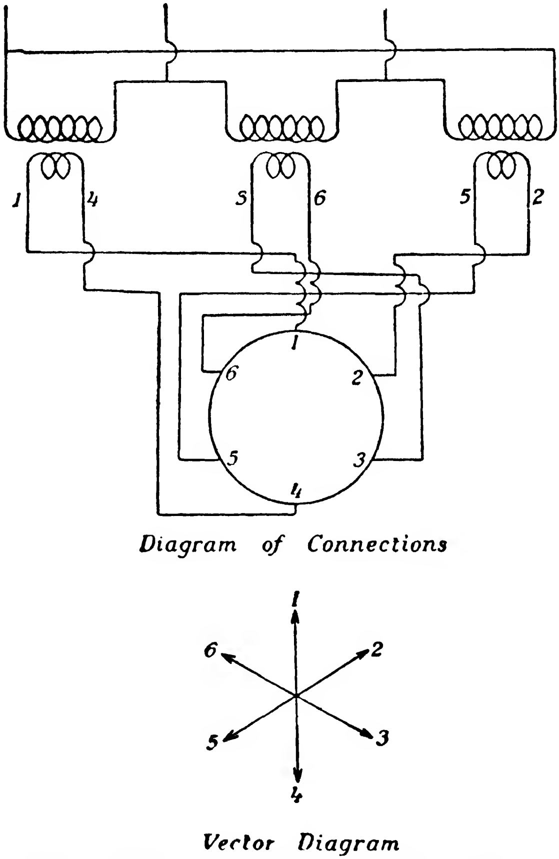


FIG. 318.—Diametric Connections for Six-Phase Rotary Converter.

primaries are connected in delta, the transformer ratio must be made $\frac{R}{0{\cdot}354} = 2{\cdot}83R$.

The next method, called the *diametric method*, is really a development of the previous one and is now standard practice. Referring to Fig. 317, it is seen that each pair of secondaries can be replaced by a single winding, the middle points being taken to a common star point. But this latter connection is unnecessary, as in a three-phase transmission scheme the armature itself fixes the neutral point by its various connections. This method, therefore, only requires three single secondaries instead of six, these being connected as indicated in Fig. 318, which shows the vector diagram in addition. The latter also shows that the middle points of the three secondaries are all at the same potential. Since the single secondary takes the place of two in the second method, the trans-

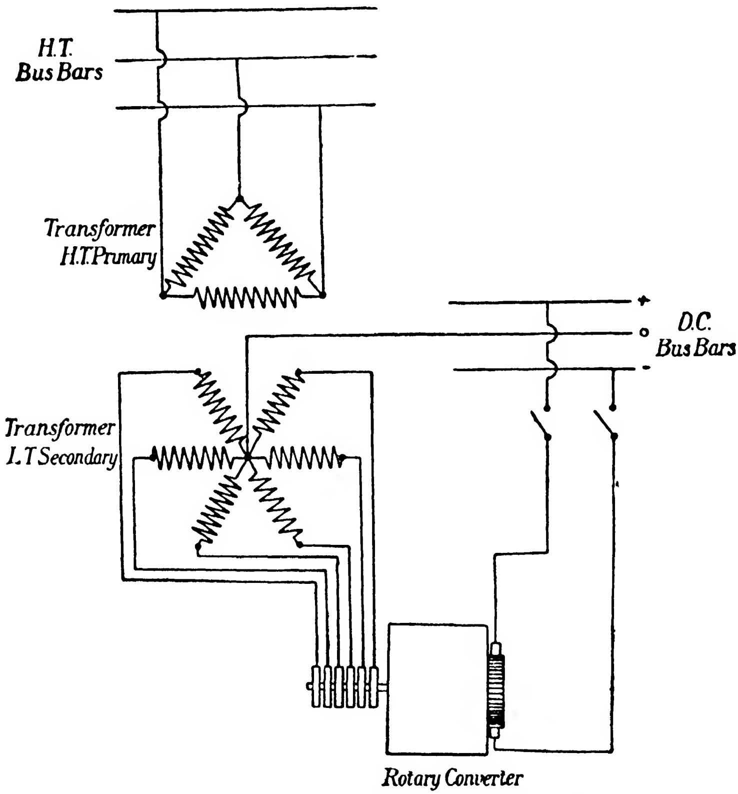


FIG. 319.—Connections for Three-Wire Rotary Converter.

former secondary voltage must be doubled, thus halving the transformer ratio, which now becomes $1{\cdot}41R$.

Three-Wire Rotary Converters.—Rotary converters can be used to supply a three-wire D.C. system without employing separate balancers, provided that the neutral point of the transformer low tension windings is available. With a balanced load, this neutral point maintains a potential midway between the potentials of any two slip rings connected to opposite points on the armature. The potential of this neutral point is thus constant, and is also midway between the potentials of the two sets of brushes on the commutator. The middle wire of the D.C. system is therefore connected to the neutral of the A.C. side as shown in Fig. 319, which represents a six-phase three-wire rotary converter. The out-of-balance current of the three-wire system is thus led back to the star point of the transformer secondaries, where it distributes itself amongst the various phases, producing an unbalanced effect similar to an ordinary unbalanced load. This disturbs the regulation to a certain extent, but out-of-balance currents up to 25 per cent. of the full load current can be dealt with in this manner without detriment. If close regulation is desired, the series field windings and the interpole windings can be split into two halves, one being connected on the positive side of the machine and the other on the negative.

In practice, where the diametric connections are used for six-phase rotary converters (see Fig. 318), it is found to be quite sufficient if the middle wire of the D.C. side is connected to the centre of one phase only of the transformer.

Static Balancers.—If the neutral point of the transformer secondaries is not available, an artificial neutral point can be made by a system of choking coils connected across the slip rings. Such an arrangement is called a *static balancer*. In the case of a single-phase machine a single choking coil is all that is necessary, the middle point being connected to the middle wire [see Fig. 320 (*a*)]. In a two-phase machine two choking coils are necessary, their middle points being joined together to form the neutral point [see Fig. 320 (*b*)]. Three- and six-phase rotary converters require three choking coils [see Fig. 320 (*c*), representing a three-phase machine]. Strictly speaking, a six-phase machine requires six choking coils, but three are sufficient to provide the neutral point, the other three being neglected as far as the static balancer is concerned. The neutral point of the static balancer takes up a constant potential midway between those of the brushes on the commutator, exactly as in the three-wire rotary converter, where the neutral point of the transformer secondaries is available. Any out-of-balance current is dealt with in the same way as described in the previous paragraph.

Three-phase static balancers are wound with interconnected or zigzag windings (see page 199) as this results in better balancing. Any out-of-balance current now causes E.M.F.'s to be induced in the corresponding winding in the other phase and this tends to preserve the voltage balance of the system.

Balancing at the neutral of the transformer is, however, to be preferred in the majority of cases, on account of the lower resistance of the windings.

Effect of Excitation.—When run from the D.C. side a rotary converter acts as a D.C. shunt motor, the speed varying with the excitation if the A.C. side is on open-circuit. When driven from the A.C. side, it runs as a synchronous motor without any speed variation. When connected to the supply on both sides, either may act as a motor driving the other part as a generator, depending upon the voltages and the excitation. If running with a weak field, the D.C. side may try to run faster than the speed of synchronism, the result being to make the A.C. side act as a generator. If over-excited, the D.C. side tries to run below the speed of synchronism, and is consequently driven by the A.C. side, which is now motoring whilst the D.C. side is generating. At one excitation, therefore, the D.C. current will die down to zero and then reverse in direction as the excitation is still further increased, whilst just before this point is reached it will be found that both sides are motoring, neither being strong enough to make the other side generate. Raising the D.C. line voltage is the equivalent of weakening the field and has the same effect. The variations in the main currents for different excitations are shown in Fig. 321.

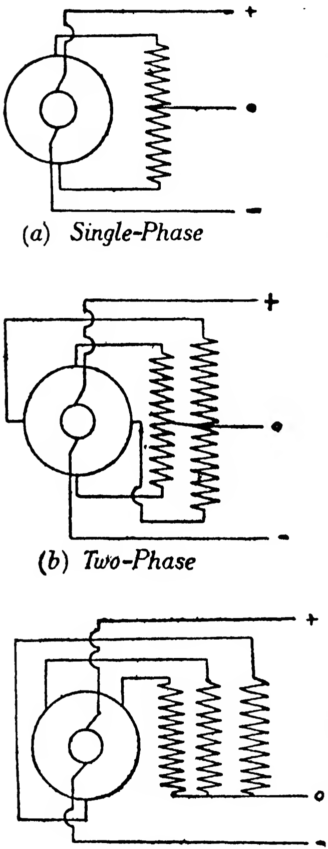


FIG. 320.—Connections of Static Balancers.

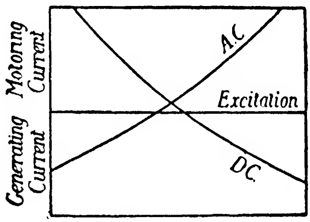


FIG. 321.—Effect of Excitation.

Armature Current and Heating.—The resultant current in the armature of a rotary converter is the difference between the direct and the alternating currents flowing at the instant under consideration. This current will vary from instant to instant and also will be different in different conductors, depending upon their proximity to the slip ring tappings.

Consider a single-phase bipolar case. The A.C. is a maximum when the conductors connected to the slip rings come under the commutator brushes, assuming this current to be in phase with the

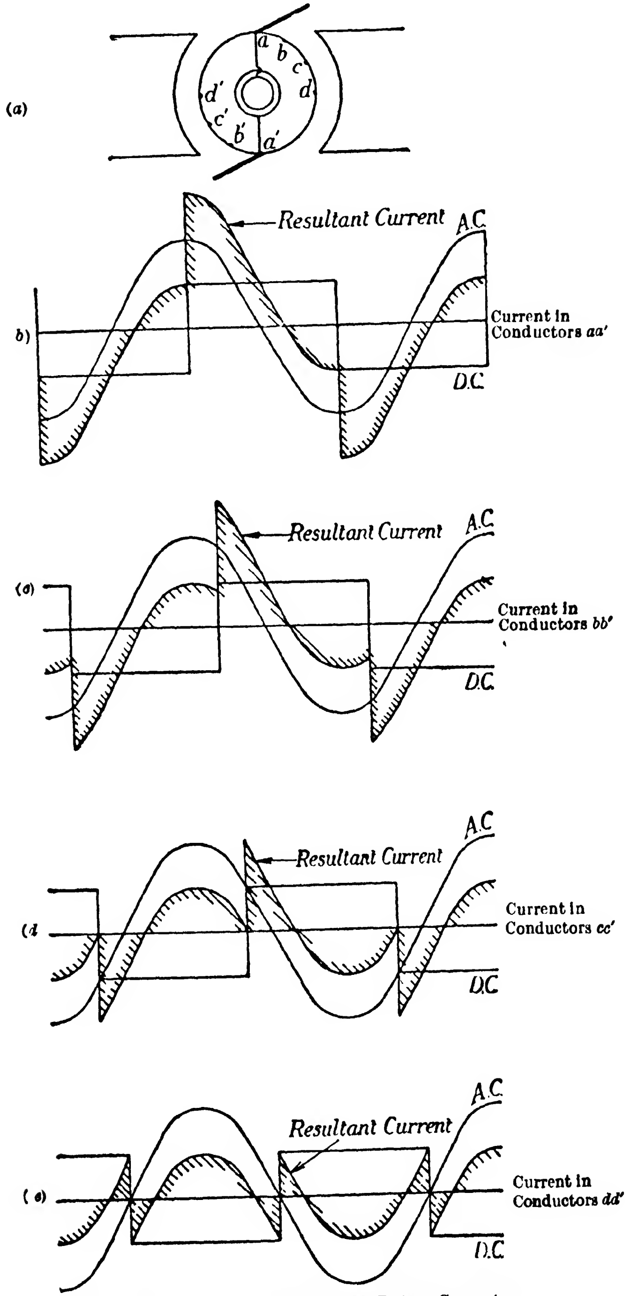


FIG. 322.—Armature Currents in Single-Phase Rotary Converter.

E.M.F. In the position shown in Fig. 322 (*a*), therefore, the conductors *aa'* are just carrying their maximum A.C., but at this instant these conductors pass under the brushes and the D.C. flowing through them reverses. The A.C. and D.C. components of the current in the conductors *aa'*, together with their resultant, are shown in Fig. 322 (*b*). For an efficiency of 100 per cent., the R.M.S. value of the A.C. is $\sqrt{2}$ times and the maximum value double that of the D.C. In the case of the conductors *aa'*, the D.C. reversal occurs at the peak of the A.C. wave, and consequently the resultant current takes on a momentary value far in excess of the normal D.C., but this state of affairs does not apply to all the other conductors. For example, the conductors *dd'* also have the maximum A.C. flowing in them in the position shown in Fig. 322 (*a*), but the D.C. reversal does not take place until a quarter of a period later. The resultant current in these conductors is shown in Fig. 322 (*e*), whilst the currents in intermediate conductors such as *bb'* and *cc'* are shown in Fig. 322 (*c*) and (*d*). The maximum height of the A.C. wave remains the same throughout, but the different positions at the time of the D.C. reversal cause vast changes in the resultant armature current. The most complete neutralization occurs at those conductors situated midway between the slip ring tappings, and these conductors carry the minimum resultant current. As the slip ring tappings are approached, the R.M.S. value of the resultant current increases until it attains a maximum at the conductors nearest to the tappings themselves.

Next consider a three-phase bipolar case. In Fig. 323 (*a*) maximum voltage exists between the slip rings (1) and (2) in the position shown. Again assuming unity power factor, the current in this section of the delta-connected armature will also be a maximum in the same position. (The fact that the line current is 30° out of phase with the voltage across slip rings does not matter. It is the armature current that is desired.) The conductor *a*, immediately to the right of slip ring No. 1, will therefore come under the brushes 30° after the point of maximum A.C. The D.C. and A.C. components of the current in this section of the armature, together with their resultant, are shown in Fig. 323 (*b*). Considering conductor *b* situated 30° behind *a*, the maximum A.C. occurs at the same instant as in *a*, but the D.C. reversal in this case occurs 60° after the instant of maximum A.C. The currents are shown in Fig. 323 (*c*). Conductor *c* is situated 60° behind *a* and is midway between the two slip ring tappings. The component currents and their resultant in this case are shown in Fig. 323 (*d*). Here the most complete neutralization is obtained, and, in consequence, this conductor carries the minimum current. In fact, whatever the number of phases, the conductor situated midway between two slip ring tappings will carry the minimum current.

The maximum height of the A.C. wave compared with the height of the D.C. curve is calculated from the voltage ratio. The R.M.S. line voltage is $\frac{\sqrt{3}}{2\sqrt{2}}$ times the D.C. voltage, and the R.M.S. line current is, therefore, $\frac{2\sqrt{2}}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{2\sqrt{2}}{3}$ times the D.C. The maximum line current is $\frac{2\sqrt{2}}{3} \times \sqrt{2} = \frac{4}{3}$ times the D.C., and the

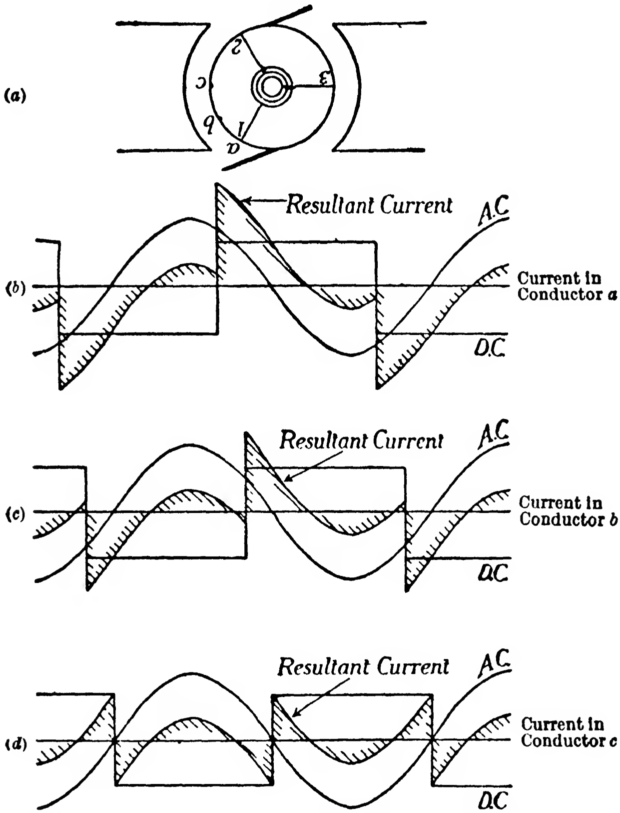


FIG. 323.—Armature Currents in Three-Phase Rotary Converter.

maximum armature A.C. is $\frac{4}{3\sqrt{3}} = 0{\cdot}77$ times the line value of the D.C. and 1·54 times the armature value.

On studying Figs. 322 and 323 it is seen that the further the conductor is situated from the midway position the larger is the current which it has to carry. In this respect the three-phaser has an obvious advantage over the single-phase machine, particularly as the heating effect is proportional to the square of the current.

The armature currents for various positions relative to the poles can be studied further by referring to Fig. 324, which represents the armature of a bipolar rotary converter in various positions, the poles being supposed to lie along a horizontal axis. Fig. 324 (*a*) represents the A.C. only in a single-phase winding. In Fig. 324 (*b*) the D.C. is superimposed, the resultant current being shown.

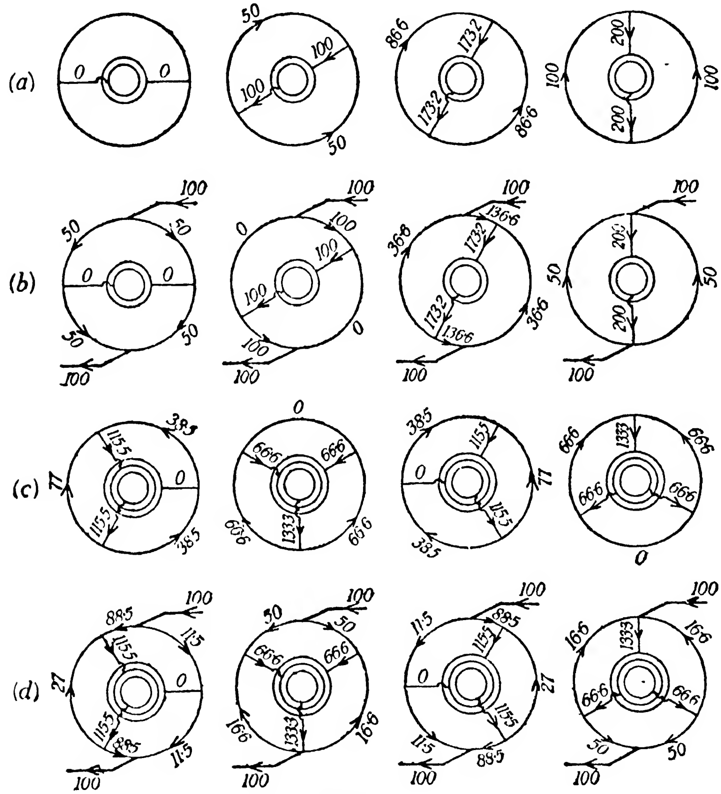


Fig. 324.—Armature Currents in Rotary Converter.

(*a*) Single-Phase. A.C. only. (*b*) Single-Phase. D.C. on A.C.
(*c*) Three-Phase. A.C. only. (*d*) Three-Phase. D.C. on A.C.

Fig. 324 (*c*) represents the A.C. only in a three-phase winding, whilst Fig. 324 (*d*) shows the resultant effect of superimposing the D.C. on the A.C.

One conclusion to be drawn is that the closer the slip ring tappings are made the less is the armature heating.

Effect of Number of Slip Rings on Output.—It was shown above that the greater the number of slip rings the less was the amount of heating produced in the armature conductors. This is equivalent to saying that for the same temperature rise of the armature the output is greater the greater the number of slip rings, or, conversely, for a given output and temperature rise, a six-phase machine is smaller than one with fewer phases. This is the reason why six-phase rotary converters are now standard practice. Twelve-phase machines have been used, but the extra benefits do not compensate for the additional complications involved.

Starting.—There are three methods of starting up a rotary converter, viz. :—

(1) By means of an auxiliary starting motor.
(2) From the D.C. side.
(3) From the A.C. side.

The first method is to employ a small auxiliary induction motor, exactly as in the case of a synchronous motor. After being synchronized, the D.C. side is paralleled on to the bus bars.

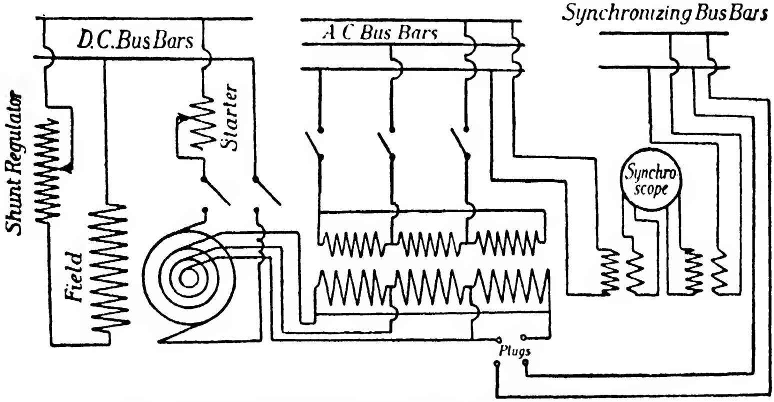


FIG. 325.—Three-Phase Rotary Converter started from D.C. Side.

The second method is to run the set up from the D.C. side as an ordinary shunt motor, synchronizing when the correct speed is attained. When this method is employed in a sub-station a complete shut-down renders it impossible to start up again, since the D.C. bus bars would be dead. To obviate this danger, it is the usual practice to have at least one of the rotary converters started up by means of an auxiliary motor. In addition, the sub-station is usually linked up to others from which it is possible to obtain the necessary D.C. supply.

In order to avoid the large currents in the L.T. circuit it is usual to synchronize on the H.T. side of the transformers. Fig. 325 shows the chief connections for starting up a rotary converter in this manner. When the load is subjected to heavy fluctuations a somewhat different procedure is commonly adopted. The rotary converter is run up to a speed slightly above that of synchronism. The D.C. side is then opened and the A.C. side immediately closed.

The machine then pulls itself into step, when the D.C. switches are again closed.

The third method is applicable to polyphase rotary converters, the procedure being the same as in the case of self-starting synchronous motors (see page 411). A squirrel cage winding is provided, this time on the stationary field system. Unfortunately, however, the polarity of the D.C. side is not definite, this depending upon the position of the armature at the instant of switching in. To see the reason for this, consider a machine started up and ready to be synchronized. If the armature be now suddenly retarded so that it drops back through a pole pitch, the current being reversed at the same instant, the conditions for synchronizing are still maintained, but the polarity on the commutator is also reversed. When the armature is started up by means of a squirrel cage winding, either of these two conditions may be set up, depending upon the moment at which synchronism is reached. To determine the polarity on the commutator, a moving coil voltmeter is employed, the pointer of which will indicate a voltage of a very low frequency (the difference between that of the supply and that of the machine) when near synchronism. The field switch is then closed when the voltmeter indicates that the polarity is correct and the voltage is a maximum. Another method of correcting the polarity is to provide the field with a double pole change-over switch. If the D.C. voltage is found to be reversed, this switch is thrown over and then back again. Since the rotation is maintained in the same direction, the first reversal causes the exciting current to demagnetize the field, whilst the second change-over causes it to be built up in the other direction, the armature having been retarded half a period in the meantime.

In order to avoid the heavy currents on starting in this method, it is usual to start from tappings on the secondary side of the transformer. The starting current is of the order of 100 per cent. full load current when one-half tappings are used, and of the order of 50 per cent. full load current when one-third tappings are employed.

A combination of the first and third methods is now commonly adopted for large rotary converters. An auxiliary polyphase induction motor is used for starting purposes, this being connected in *series* with the rotary converter. This induction motor frequently has a solid rotor with absolutely no winding on it at all, and it operates in the same manner as a squirrel cage motor. On being switched in the auxiliary motor takes the bulk of the voltage on account of its higher impedance, but as the set speeds up, the back E.M.F. generated by the rotary converter causes the voltage to be gradually transferred to the latter machine. Finally the auxiliary starting motor is short-circuited and the set is paralleled as before. The starting current with this method is of the order of 30 to 40 per cent. full load current.

Inverted Running.—When converting D.C. to A.C. the machine is called an *inverted* rotary converter, in which case its speed depends upon its field strength if running alone. If it is running in parallel with synchronous alternators of considerably greater capacity it must, however, run at the same frequency. When running alone a weakened field causes a rise in speed, and if the load be inductive, the lagging current will weaken the field and the speed may rise to a dangerous degree. To overcome this difficulty the excitation is obtained, not directly from the commutator, but from a separate exciter coupled directly to the main armature. An increase in the speed now brings about a greater increase in the exciter voltage, thus strengthening the field and checking the speed variation.

Inverted rotary converters are usually compound wound as well, so that any increase in reactive current will bring about a counteracting increase in the field strength.

Speed Limiting Device.—All large rotary converters are now fitted with a speed limiting device. This causes the machine to be thrown out of circuit should the speed exceed a predetermined figure. The device is operated by centrifugal force and consists of an arm, mounted on the shaft and revolving with it, which tends to fly outwards against the resistance of a spring. At the critical speed this arm trips a trigger which closes the tripping circuit of the D.C. circuit breaker.

Parallel Running.—Rotary converters driven from the same A.C. system are often required to run in parallel on the D.C. side. If they are directly paralleled at the commutator and slip rings, however, a complete local circuit is formed by two converters by way of the A.C. and D.C. bus bars, and large cross currents are liable to be set up in this local circuit, due to slight differences in the operation of the two machines. To avoid this, the rotary converters are usually operated from separate transformers. When the same bank of transformers is used, the rotary converters should be operated from independent secondaries.

For good parallel running it is desirable that there should be a comparatively large voltage drop from no-load to full load, as in the case of alternators. When run in conjunction with a battery, it is usual to include a reverse current cut-out on the D.C. side to prevent the rotary converter running back should a large drop in voltage occur on the A.C. side.

Hunting.—Rotary converters, like synchronous motors, are subject to hunting troubles, and the way in which hunting is set up has already been described (see page 408). The armature currents of a polyphase converter set up a field which, under perfect conditions, is stationary in space. When hunting occurs, however, this field oscillates to and fro and sets up an E.M.F. in the coils undergoing short-circuit by the brushes on the commutator, and thus is the cause of violent sparking. The squirrel cage starting winding now operates as a series of damping grids (see page 273) and any oscillation of the flux across the pole face, produced by irregularities in the speed, generates an E.M.F. in these damping grids, setting up eddy currents which tend to damp out the oscillations which produce them.

In addition to adopting the above expedients, the wave form employed should be as close an approximation to a sine wave as possible.

Overload Capacity.—Well-designed rotary converters have a high overload capacity, since the heating is less than in a corresponding D.C. generator. Also the armature reaction due to the motoring and generating action is a differential effect, and as interpoles are used the sparking is not so acute as it otherwise might be. A good machine will have an overload capacity of 100 per cent. for short periods, this being an important fact when considering the spares required in a sub-station.

Power Factor.—The power factor of a rotary converter depends upon its excitation, just as n the case of a synchronous motor, and, as a general rule, it is advisable to operate it at as near unity power factor as possible. If there is other apparatus connected to the line, taking a lagging current, the resultant power factor of the system can be improved by over-exciting the rotary converter so as to make it take a leading current at a power factor of, say, 0·9. Unfortunately, however, this affects the armature heating to a serious extent, particularly in the neighbourhood of the slip ring tappings. In fact, in a six-phase converter, the heating in such a coil is increased by about 80 per cent. if the power factor is changed from unity to 0·9 leading. As this is the equivalent of a very considerable overload, it is very necessary to operate the machine at as near unity power factor as possible when on or about full load.

It is thus seen that from a practical standpoint the rotary converter is not nearly so good as a synchronous motor-generator set for the purpose of improving the power factor.

Compounding.—Rotary converters may be compounded in just the same way as D.C. shunt generators, this winding being in addition to that of the interpoles. In such a case, the D.C. bus bars should include an equalizing bar for the purpose of paralleling the various series coils of the different converters running in parallel. When operating a D.C. three-wire system, half the compound winding is connected on one side and half on the other.

Armature Reaction.—In a D.C. generator the armature reaction acts in such a direction as to require a forward lead of the brushes, whilst in a motor the armature reaction is in the opposite direction. In the rotary converter the D.C. and A.C. currents may be considered as being superimposed on one another in the armature so that there are two armature reactions tending to neutralize one another. In a three-phase machine the A.C. and D.C. currents are roughly of the same magnitude, so that the two reactions are more or less equal. Successful commutation may therefore be obtained for all loads up to full load, and even for overloads, by fixing the brushes in the no-load neutral position. This neutralization of the armature reactions has the effect of considerably raising the limiting load from the sparking point of view, this being much higher than would be the case if the machine were used as a D.C. generator.

The armature reaction, measured in ampere-turns, may be regarded as being composed of three components, viz. (1) the D.C. armature reaction, (2) that due to the active component of the A.C., and (3) that due to the reactive component of the A.C.

Consider the case of a bipolar six-phase rotary converter. The total armature ampere-conductors due to the D.C. are $\frac{I_D}{2}N$, where N is the total number of armature conductors. The total ampere-turns per pole, due to the D.C., are, therefore, $\frac{I_D N}{8} = 0{\cdot}125\, I_D N$, or, say, $0{\cdot}122\, I_D N$ allowing for the coils undergoing short-circuit during commutation. Now considering the A.C. active component, it was shown on page 432 that $I_A = \frac{\sqrt{2} I_D}{m \sin \frac{\pi}{m}}$, and since $m = 6$, this becomes $\frac{\sqrt{2} I_D}{6 \sin 30^\circ} = \frac{2\sqrt{2} I_D}{6} = 0{\cdot}47\, I_D$. The mean value of the ampere-turns oscillates between $0{\cdot}407\, I_A N$ and $0{\cdot}47\, I_A N$, having an average value of $0{\cdot}44\, I_A N$ or $0{\cdot}22\, I_A N$ ampere-turns per pole. But $I_A = 0{\cdot}47 I_D$, so that the armature reaction due to the active component of the A.C. becomes $0{\cdot}22\, I_A N = 0{\cdot}22 \times 0{\cdot}47\, I_D N = 0{\cdot}103\, I_D N$ ampere-turns per pole. Allowing 4 per cent. for losses ($h = 1{\cdot}04$), this now becomes $0{\cdot}107\, I_D N$ ampere-turns per pole. If the rotary converter is working at unity power factor, the resultant armature reaction is, therefore, $0{\cdot}122\, I_D N - 0{\cdot}107\, I_D N = 0{\cdot}015\, I_D N$ ampere-turns per pole. The D.C. reaction is approximately 15 per cent. greater than the A.C. reaction. This difference is too small to cause appreciable field distortion, and permits heavy overloads to be taken without serious commutation difficulties. The interpoles are therefore much smaller than on a D.C. generator.

If the rotary converter is not operating at unity power factor, there is an additional armature reaction due to the reactive component of the A.C. These ampere-turns are in quadrature with those discussed above, and since the reactive component of the armature current is kI_A, where $k = h \tan \phi$, the armature reaction due to this component becomes

$$\frac{k}{h} \times 0{\cdot}107\, I_D N = 0{\cdot}107\, I_D N \times \tan \phi.$$

Voltage Regulation.—The D.C. voltage obtained from a given rotary converter depends upon the impressed A.C. voltage and drops slightly as the load comes on. In an ordinary generator this can be corrected by adjusting the field strength, but in the present instance this is of no avail, since it merely changes the power factor on the A.C. side. In order to obtain a certain amount of voltage regulation on the D.C. side, various methods have been devised of which the following are the chief :—

(1) Reactance regulation.
(2) Booster regulation.
(3) Induction regulator control.

Reactance Regulation.—A change in the excitation of a rotary converter does not affect its D.C. voltage, but it alters the power factor and the armature current. If a choking coil be placed in series with the rotary converter it absorbs a variable voltage depending upon the value of the current. The phase of this voltage

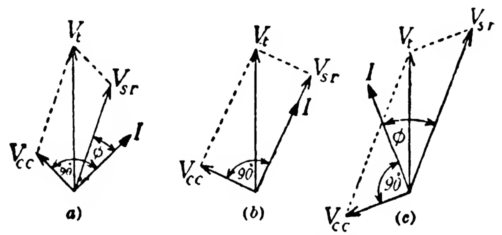


FIG. 326.—Reactance Regulation.

also depends upon the current in the converter armature, which in turn depends upon the excitation. The terminal voltage of the converter is the vector difference of the supply voltage and that absorbed by the choking coil, and by adjusting the excitation the phase and magnitude of the voltage absorbed by the choking coil can be regulated so that a practically constant voltage is obtained at the D.C. end for all loads.

In order to maintain a constant D.C. voltage, the voltage across the converter slip rings must rise slightly as the load comes on, and the way in which this is brought about is shown in Fig. 326, which represents the vector diagram of the quantities concerned. Fig. 326 (*a*) is drawn for a weak excitation, the armature current, I, lagging behind the slip ring voltage, $V_{s.r.}$. The voltage drop across the choking coil is represented by $V_{c.c.}$ and leads the current by 90°, whilst the voltage supplied by the transformer secondary is represented by V_t and is assumed to be constant throughout. V_t is the vector sum of $V_{s.r.}$ and $V_{c.c.}$. The weak excitation and lagging armature current correspond to the conditions existing on light loads. Fig. 326 (*b*) represents the conditions on a larger load.

The excitation of the rotary converter has been increased so as to bring the power factor up to unity. $V_{c.c.}$ still leads the current by 90°, whilst its magnitude is reduced on account of the improved power factor, but is increased on account of the increased load. It is the displacement in phase, however, which is the chief cause of the increased slip ring volts $V_{s.r.}$, as is seen from the diagram. Fig. 326 (*c*) represents the conditions on overload. The rotary converter is now over-excited, so that its armature current leads the slip ring voltage. The voltage absorbed by the choking coil is now increased, both on account of the increased load and the decreased power factor. But, again, the chief feature of the diagram is the advance in phase of this voltage which still leads the current by 90°. The result is clearly shown to be a further increase in the voltage supplied to the slip rings, and this increase can, by a suitable design, be made to counteract the increased drop in voltage due to the load coming on. It is also possible to obtain a rising D.C. voltage, as in the case of an over-compounded D.C. generator, by increasing the size of the reactance. By making the rotary converter compound wound, the increase in field strength is obtained automatically.

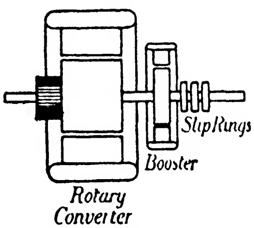


FIG. 372.—Rotary Converter with Booster.

The desired reactance may be obtained in two ways. A special reactance coil may be inserted in each phase between the transformer and the slip rings, or, alternatively, the reactance may be supplied by the transformer itself. For this purpose special packets of stampings are placed at intervals in the path of the secondary leakage flux, thus reducing the reluctance of these paths.

The disadvantage of this method of control is that the rotary converter cannot always be worked on the best power factor, since the regulation depends to a large extent upon the angle of phase difference between the slip ring voltage and the armature current. This method of voltage control is suitable for voltage variations up to about 15 per cent.

Booster Regulation.—The additional voltage required at the slip rings of the rotary converter is here obtained by means of a booster, which is carried on the main shaft and is situated between the slip rings and the armature of the rotary converter as shown in Fig. 327. The booster consists of a rotating armature A.C. generator having the same number of phases and poles as the converter itself. Each phase of the booster armature is connected in series with one of the slip ring leads, the far end being taken to one of the tappings on the converter armature. The function of the booster is to generate a small voltage proportional to the D.C. load, this small voltage being added to that at the slip rings in order to counteract the natural drop as the load comes on. To effect this the booster receives its excitation from the D.C. end of the converter, its fields being connected in series with the load, like the compounding coils on the converter itself. (The booster is usually compound wound in practice.) As the load increases the field current increases and the generated E.M.F. boosts up the supply so as to maintain a practically constant voltage at the commutator for all loads. If desired, a larger booster voltage may be induced, so that the D.C. terminal voltage rises with the load, as in the case of an over-compounded generator.

With booster control, the power factor is independent of the load, so that unity power factor can be obtained, or, if desired, a leading current can be drawn from the mains. This method of voltage control is suitable for voltage variations up to about 25 per cent.

Induction Regulator Control.—The simplest method of hand regulation is to employ a small boosting transformer, as shown in

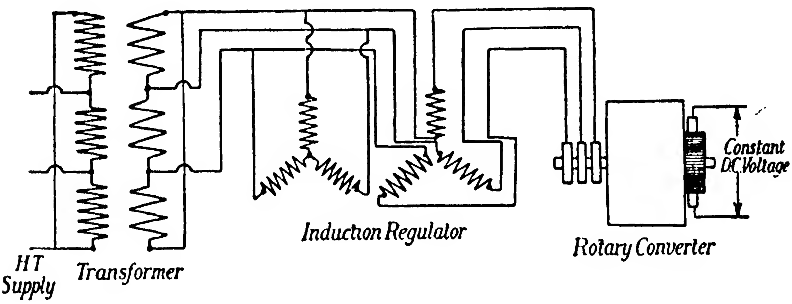


FIG. 328.—Induction Regulator Control.

Fig. 179, in addition to the main transformers, so that boost can be applied gradually as the load comes on. Unfortunately, either very heavy currents or very high voltages have to be dealt with, according as the boost is obtained on the H.T. or L.T. side of the main transformers. The method is therefore comparatively expensive, but close regulation can be obtained by it, such as is desired if the rotary converter is operating on a lighting load.

Instead of using a boosting transformer of the type shown in Fig. 179, an *induction regulator* may be employed (see page 425).

Connections for a three-phase rotary converter are made as shown in Fig. 328. The phase of the voltage generated in the induction regulator depends upon the position of the rotor, so that by slowly rotating it by hand the main transformer voltage can be boosted up or down at will. The two extreme positions of the induction regulator are those in which the boost voltage is directly added and subtracted, intermediate voltages being obtained in between. In order to prevent any alteration in the power factor, a variation in the boost voltage must be accompanied by a hand adjustment of the field strength.

Examples

(1) A six-phase rotary converter is required to give 480 volts at the commutator. The supply is 6600 volts 3 phase. The primaries are connected in delta, and the secondaries diametrically. Determine the ratio of transformation of the transformers, neglecting drops.

(2) Show how a six-phase rotary converter can be operated from three-phase mains, illustrating by means of a skeleton diagram of connections. If the D.C. side supplies a three-wire system, show how the transformers may be made to deal with the out-of-balance current.

(3) Describe two methods of obtaining automatic voltage regulation with rotary converters.

(4) A three-phase rotary converter with reactance control operates at 0·85 power factor (lagging) when the slip ring current is 200 amperes per ring, and the slip ring voltage is 550 volts. The choking coils have a reactance of 0·25 ohm each and a negligible resistance. What power factor must the set work at with 400 amperes per ring, in order to obtain a slip ring voltage of 570 volts? Assume a constant voltage of supply.

(5) Draw a diagram of connections for a rotary converter, showing starting arrangements with a pony motor in series.

(6) Describe how a rotary converter can be made to give a constant D.C. voltage for all loads, by means of reactance control.

CHAPTER XXVI

MOTOR CONVERTERS

General Arrangement.—The motor converter consists of an ordinary induction motor with a wound rotor and a D.C. generator, rigidly coupled together. In addition to the mechanical coupling, the two rotating elements are also connected together electrically, a hollow shaft being employed for the purpose of carrying the connecting leads.

Connections.—The diagram of connections for a three-phase motor converter is shown in Fig. 329, which represents a set having two poles on both A.C. and D.C. sides. The stator of the induction motor is wound for the same number of phases as the supply, but the rotor is usually provided with a twelve-phase winding. When running normally, these twelve phases are connected in star, the outer ends being connected to twelve equidistant points on the D.C. armature, from the commutator of which the main direct current is collected. This end of the set usually is provided with interpoles, and may be either shunt or compound wound. When starting up, only three, or in the largest sizes six, phases of the rotor of the induction motor are used, these being connected to slip rings and thence to a starting resistance in the usual manner with induction motors. In addition, a short-circuiting ring is mounted at the end of the slip rings for the purpose of short-circuiting the whole of the inner ends of the twelve phases when the normal running conditions have been attained. A synchronizing voltmeter, which is used during starting up, is also connected across two of the slip rings, as shown in the diagram.

Principle of Action.—Assuming that the combined set is running at some definite speed, the rotor of the induction motor has E.M.F.'s induced in it corresponding in phase and magnitude to its slip. These E.M.F.'s produce currents in the armature of the D.C. generator, which, in addition, has another series of E.M.F.'s induced in it on account of its rotation in its own magnetic field. The frequency of the E.M.F.'s supplied from the induction motor is that of slip, whilst the frequency of the E.M.F.'s due to pure generator action is determined by the speed. For these two frequencies to be the same, the induction motor must run with a slip of 50 per cent., or, in other words, the set must run at half synchronous speed.

If the speed differs from this value by some small amount, one of these frequencies goes up and the other down, so that circulating currents will flow round the two rotating elements, just as in the case

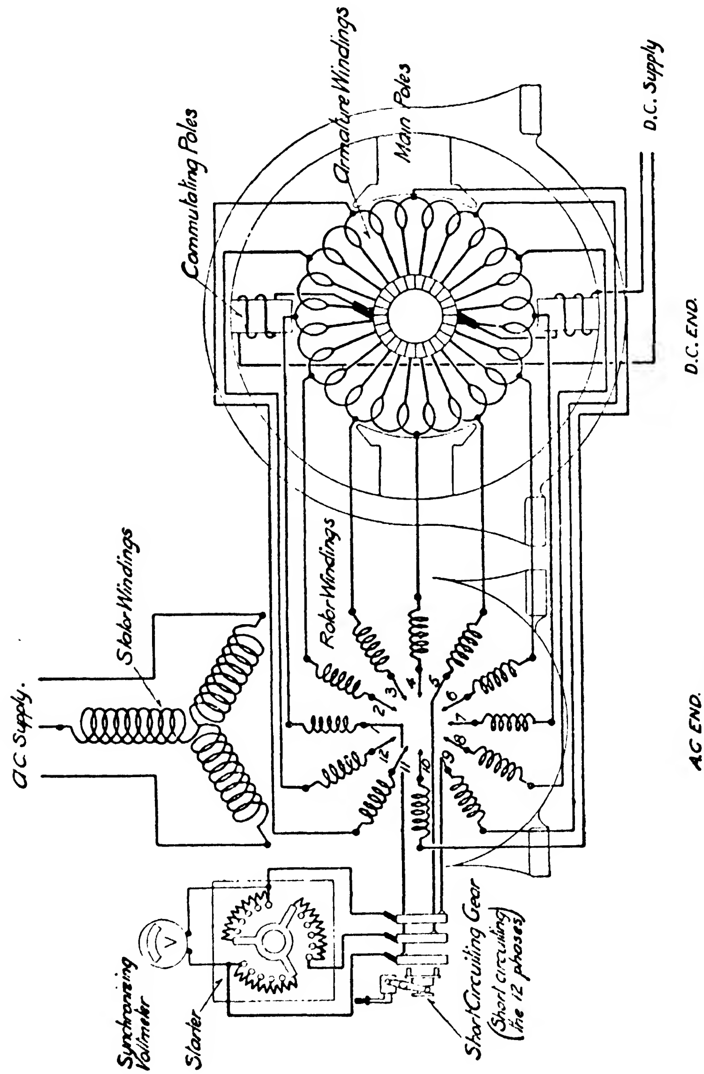


FIG. 329.—Connections of Motor Converter.

of two alternators, and a large synchronizing force is produced tending to make the two frequencies equal. The normal running speed of the combination is thus half the speed of synchronism, and it behaves as a single synchronous machine.

As the rotor of the induction motor is rotating with a slip of 50 per cent., it follows that it is running with an efficiency of 50 per cent. (neglecting the stator losses). Half the power input is transformed into mechanical form by the ordinary motor action, and the remaining half is transmitted to the rotor by transformer action. The mechanical power developed goes towards driving the D.C. generator through the rigid coupling, whilst the electrical power developed in the rotor is supplied to the armature of the D.C. generator and helps to drive it by rotary converter action. The power given out at the commutator in D.C. form is due to the sum of these two effects, one-half of it having been converted by motor-generator action, and the other half by transformer and rotary converter action. Thus the A.C. end operates half as an induction motor and half as a transformer, whilst the D.C. end operates half as a generator and half as a rotary converter.

Construction.—The construction of the two machines forming a complete unit differs very little from the standard practice with induction motors and D.C. generators. In the larger sizes three bearings are employed, whilst for the smaller sets only two are used. The induction motor is provided with three or six slip rings, for the purpose of starting, and a short-circuiting ring to short-circuit the twelve phases when the speed of synchronism is attained. The air-gap is made much larger than is customary with induction motors of similar size. The object of employing the small air-gaps usually adopted is to keep the magnetizing current down as much as possible, thereby raising the power factor, but this does not apply in the case of a motor converter, since the magnetizing current is drawn from the armature of the D.C. end. Larger clearances can therefore be used without introducing any harmful effects.

On the D.C. end the currents are fed into the rotating element through the hollow shaft direct from the rotor of the induction motor.

Speed, Frequency and Number of Poles.—It has been shown that the speed of a motor converter is half the synchronous speed of the A.C. end when both machines have only two poles each. This is also true for multipolar sets when both machines have the same number of poles. The general formula connecting the speed and the numbers of poles is obtained by considering the frequency of slip of the induction motor and the frequency of generation of the D.C. generator and equating them. If n be the speed in r.p.m., f and sf the frequency of supply and slip respectively, and p_a and p_d the number of poles on the A.C. and D.C. ends respectively, the speed of the induction motor is equal to

$$\frac{f - sf}{p_a} \times 120$$

and $$sf = f - \frac{np_a}{120}.$$

The frequency of the D.C. generator is equal to

$$sf = \frac{np_d}{120}.$$

Therefore $$sf = f - \frac{np_a}{120} = \frac{np_d}{120},$$

$$np_a + np_d = 120f$$

and $$n = \frac{120f}{p_a + p_d}.$$

The set, therefore, runs at a speed equal to the synchronous speed of an induction motor having as many poles as both A.C. and D.C. ends together. An increase in the number of poles at either end causes a reduction in speed.

The frequency of the currents in the D.C. generator is

$$sf = \frac{120f}{p_a + p_d} \times \frac{p_d}{120}$$

$$= f \times \frac{p_d}{p_a + p_d}.$$

The D.C. end is thus always operating on a frequency considerably lower than that of the supply, which is advantageous, since the performance of rotary converters is always better on low frequencies.

In an ordinary induction motor, the number of rotor turns is not rigidly fixed, but in the present case the D.C. generator must be supplied with a definite voltage, depending upon the value of the D.C. line voltage required and the number of phases. The rotor at the A.C. end must therefore have the correct number of turns per phase to generate this voltage when running with its normal slip.

Power converted Mechanically and Electrically.—Considering the A.C. end, the output of the induction motor is given by

$$\frac{\text{Mechanical rotor output}}{\text{Electrical rotor input}} = \frac{\text{Actual speed}}{\text{Synchronous speed}},$$

whence

$$\text{Mechanical rotor output} = \text{Rotor input} \times \frac{\text{Actual speed}}{\text{Synchronous speed}}$$

$$= \text{Total input} \times \frac{\text{Actual speed}}{\text{Synchronous speed}},$$

neglecting the stator losses.

But the actual speed is $n = \dfrac{120f}{p_a + p_d}$, and the synchronous speed of the induction motor by itself is $\dfrac{120f}{p_a}$. Therefore the mechanical output of the rotor is

$$\text{Total input} \times \frac{120f}{p_a + p_d} \times \frac{p_a}{120f}$$

$$= \text{Total input} \times \frac{p_a}{p_a + p_d}.$$

The power received by the D.C. end in electrical form is equal to the power developed electrically in the rotor of the induction motor, and this is obtained from the equation

$$\frac{\text{Electrical power developed in rotor}^{1}}{\text{Rotor input}}$$

$$= \frac{\text{Speed of slip}}{\text{Synchronous speed}} = \frac{sf}{f}.$$

But $$sf = f \times \frac{p_d}{p_a + p_d} \text{ (see above).}$$

Therefore the electrical power developed in the rotor

$$= \text{Rotor input} \times \frac{f \times \dfrac{p_d}{p_a + p_d}}{f}$$

$$= \text{Total input} \times \frac{p_d}{p_a + p_d}.$$

Thus $\dfrac{p_a}{p_a + p_d}$ is the fraction of the total power converted by motor-generator action, and $\dfrac{p_d}{p_a + p_d}$ is the fraction of the total power converted by transformer and rotary converter action.

Starting.—Motor converters can be started up from either the D.C. or the A.C. end.

In starting up from the D.C. end, an ordinary motor starter is employed, the A.C. end being synchronized like an alternator when the correct speed is attained.

Starting up from the A.C. end is very simple in the actual operations gone through. Three, or, in the largest sizes, six, of the twelve rotor phases are brought out to slip rings, the other ends of these phases being permanently connected to the D.C. armature. The

[1] In an ordinary induction motor this is the rotor loss.

slip rings are connected to a rotor starter in the usual manner, and a synchronizing voltmeter, *V* (see Fig. 329), is connected across two of the rotor slip rings, this comprising the whole of the synchronizing gear. The D.C. end being unexcited, the rotor starting switch is closed, and the set commences to run up to speed as an induction motor. Since the synchronous speed of the motor converter is only half the synchronous speed of the induction motor acting by itself (assuming the same number of poles at both ends), the rotor starter allows the set to run up to a higher speed than that of synchronism. When a speed of about 10 per cent. above the normal running speed is obtained, the shunt regulator at the D.C. end is adjusted until the rotary converter begins to excite as a D.C. dynamo. When this occurs, the speed commences to fall and approaches that of synchronism. The voltmeter, *V*, now begins to be affected by two different voltages of slightly different frequency, viz. the voltage induced in the induction motor rotor having a frequency equal to that of slip, and the voltage induced in the rotary converter armature which is connected to the voltmeter through the induction motor rotor, the frequency of this voltage being determined by the speed. The voltmeter pointer now commences to pulsate after the manner of the synchronizing lamps in the case of alternators, the pulsations gradually becoming slower as the synchronous speed is approached. The correct speed of synchronism is reached when the pulsations cease and the voltmeter is at zero. The starter is now short-circuited and the remaining rotor phases are all connected together by the short-circuiting ring mounted near the slip rings. The set is now ready for running on load.

The starting current varies from one-quarter to one-third of the normal full load current, and depends upon the magnetizing current of the A.C. end.

Three-Wire Motor Converters.—When motor converters are used to supply a three-wire system the D.C. outers are connected to the brushes on the commutator of the rotary converter, whilst the middle wire is connected to the neutral point of the induction motor rotor. The potential of this point is midway between the potentials of the outside ends of any two diametrically opposite phases, and consequently is midway between the potentials at opposite points on the D.C. armature. The potential of the middle wire is therefore midway between the potentials of the brushes on the commutator, and any out-of-balance current finds its way back to the armature of the rotary converter by way of the neutral of the induction motor rotor. The actual connection is made on the A.C. starter, as shown in Fig. 330, where three leads are brought out to the starter through a triple pole change-over switch. When starting up, the middle wire of the three-wire system is open-circuited, and is only connected to the motor converter when the starting switch is thrown over to the running position after the set has been brought into synchronism.

The out-of-balance current is dealt with in the same way as in the case of three-wire rotary converters, where the middle wire is connected to the star point of the transformer secondary.

Motor Converter with Synchronous Booster.—Motor converters are sometimes provided with a direct-coupled synchronous booster for the purpose of regulation, just as in the case of a rotary converter. This booster is practically an alternator with a rotating armature, and it must be wound for as many phases as the rotor of the A.C. end. The various windings are connected directly in series between the rotor at the A.C. end and the tappings on the D.C. armature.

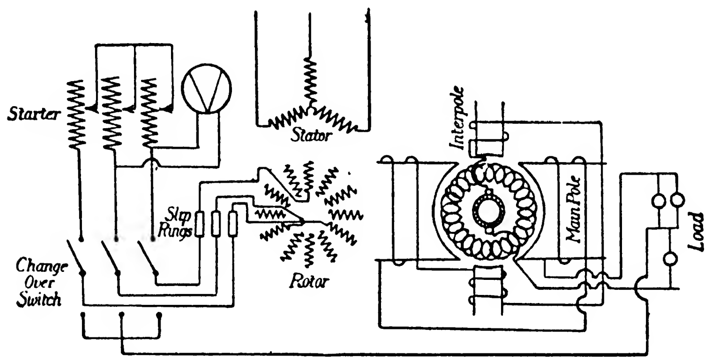


Fig. 330.—Motor Converter for Three-Wire System.

Examples

(1) Draw a skeleton diagram of connections of a motor converter operating off three-phase mains. The induction motor rotor is to be wound with 12 phases.

In a particular motor converter, the induction motor has 4 poles and the rotary converter 6 poles. The frequency of supply is 50 cycles per second. Calculate the synchronous speed of the set, together with the rotor frequency. Prove any formulæ used.

(2) Compare the motor converter with the rotary converter together with its transformers, and also with the motor generator.

(3) Derive an expression to show what proportion of the power in a motor converter is converted by motor generator action and what proportion by transformer and rotary converter action.

The induction motor end of a 700 kW motor converter has six poles and the rotary converter end eight poles. The frequency of supply is 50 cycles per second. Determine the speed of the set, the frequency of the rotary converter, and the amount of power transmitted by the middle portion of the shaft.

CHAPTER XXVII

RECTIFIERS

Mercury Arc Rectifier.—The essential part of a mercury arc rectifier consists of an arc which is struck between a graphite or an iron anode (+) and a mercury cathode (−). Within a certain range of pressure, in practice from about 0·0001 to about 0·03 mm. of mercury, it is found that a current will only flow one way and that when the mercury is positive no appreciable current flows at all, the current being due to electrons emitted from the mercury cathode. This is on account of the speed of the negative ions (electrons) which is much greater than that of the heavier positive ions (protons). The disparity in the relative masses is such that,

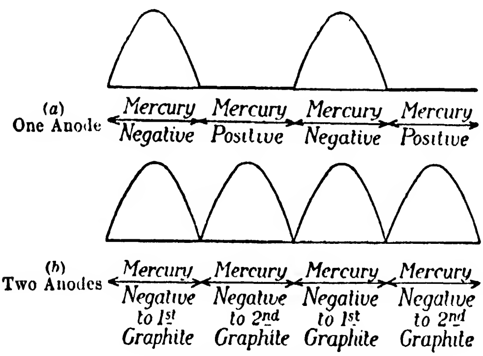


FIG. 331.—Current Wave Form.

whereas the electrons can move very quickly from the cathode to a positively charged anode, the acceleration of protons to a negatively charged anode is relatively so slow as to result in a current of only a few micro-amperes. Since the electrons consist of negative charges, they constitute a current in the opposite direction when moving from cathode to anode.

If such an arc is set up by an alternating E.M.F., one-half of the current wave is cut off, leaving a unidirectional current having a wave form similar to that shown in Fig. 331 (*a*). In order to make use of both half-waves, a second anode is provided, this being connected to the other A.C. line, a common cathode being employed. With this arrangement the theoretical wave form obtained is that shown in Fig. 331 (*b*).

Description of Apparatus.—The arc is formed in an exhausted glass vessel, T, called the rectifier tube (see Fig. 332). This contains two graphite anodes, AA, each connected with the exterior by means of a platinum wire fused through the glass. At the bottom is a pool of mercury, C, forming the cathode, also connected with the outside by a platinum wire fused through the glass. Unfortunately, however, the arrangement is not self-starting, so an auxiliary electrode, S, is used as a temporary anode for starting the arc. This consists of another little pool of mercury lying close to the main pool as shown, and is connected to one of the main anodes through a resistance, R. In order to start the arc and bring about the necessary emission of electrons from the cathode, the

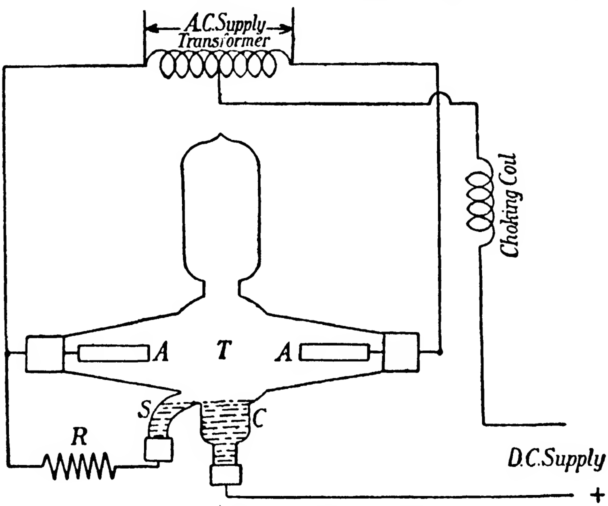


FIG. 332.—Mercury Arc Rectifier.

whole glass bulb is tilted to one side, thus causing momentary contact between the mercury in the two pools. An arc is thus struck producing sufficient mercury vapour to start the main arcs from the graphite anodes. The resistance, R, is included to limit the violence of the starting arc.

The A.C. supply is brought to the two ends of an auto-transformer, each end being further connected to one of the anodes. One of the D.C. supply leads is connected to the mercury cathode, whilst the other is connected through a choking coil to the middle point of the transformer. The choking coil is for the purpose of smoothing out the irregularities of the current wave form, and does not cause any loss, since it is connected on the D.C. side. The D.C. supply is thus obtained from the left- and the right-hand side of the transformer alternately.

Three- and Six-Phase Rectifiers.—In the case of a three-phase rectifier, the bulb is provided with three anodes, the common cathode forming the positive of the D.C. supply, and the neutral point of the star-connected transformer secondaries forming the D.C. negative as shown in Fig. 333.

An increase in the number of phases results in the rectified currents being more uniform in character, and six-phase rectifiers are most usual, although for outputs up to about 50 kW three anodes may be employed.

The D.C. voltage obtained from a mercury arc rectifier is not steady like that obtained from a battery of accumulators, but contains a certain amount of *ripple*, *i.e.* rapid fluctuation of relatively small amplitude. The use of a greater number of phases results in a reduction of this ripple, but this advantage is offset by

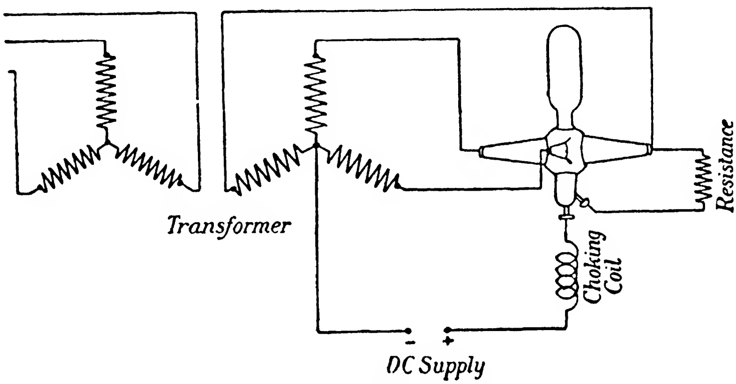


FIG. 333.—Three-Phase Mercury Arc Rectifier.

the shorter operating time of each anode per cycle, and by complications of construction. For a bulb of a given size there is an increased risk of a short-circuit between adjacent anodes on account of their relative nearness. In the largest sizes (up to 10,000 amperes), 18 anodes are sometimes employed, three in parallel per phase.

The effect on the supply of a larger number of phases can be obtained by connecting rectifiers in parallel in different ways, *e.g.* a 30° phase shift can be obtained by the use of delta-connected primary windings for one rectifier and star-connected primary windings for the next. This gives the effect on the aggregate A.C. and D.C. supplies of twelve-phase connections when two six-phase rectifiers are used.

All-metal Mercury Arc Rectifier.—In the larger sizes the glass bulb is replaced by an air-tight steel container. In the former case the various connecting wires can be sealed directly into the glass, but when a metal container is used, insulators must be employed to carry the leads into the interior of the rectifier, and difficulty is experienced in making these parts air-tight. A high degree of vacuum is desirable for efficient operation, the normal working range being from 0·01 to 0·001 mm. of mercury when the arc is not in action. Higher pressures obtain, of course, when the arc is in operation. A further factor adding to the difficulty of the vacuum problem is the desirability of being able to open the container for the purpose of inspection and cleaning.

Mercury Seal.—In order to carry a conductor through the steel wall, a brass bolt is passed through a porcelain insulator bedded down on to the steel by means of a vulcanized india-rubber or asbestos washer. The space between the bolt and the bushing portion of the insulator, and between the latter and the steel wall, is filled with mercury. The vacuum on the inside of the chamber tends to keep this mercury in position. The device has the additional advantage that if any of the mercury is drawn into the interior, it mixes with the other mercury there without doing any damage.

Construction of All-metal Rectifier.—The arc operates in a large welded steel cylinder above which is a narrow condensing cylinder. These are joined together by a plate which carries the anodes in a ring, these pointing downwards into the main cylinder. At the bottom of the main arc chamber the mercury cathode is situated, and the whole rectifier is mounted on insulators.

The six main anodes are mounted on insulators, attached to which are six tubular arc guides open at their lower ends. These guide the arcs towards the cathode, and prevent flashing-over between adjacent anodes. In a number of cases the anodes are provided with external radiator coolers.

At the top of the condensing cylinder there is a solenoid called the ignition coil. When the rectifier is switched into circuit this becomes energized and gives a downward movement to a central rod, this being opposed by a helical spring. At the lower end of this rod an auxiliary *ignition anode* is situated. When the central rod is depressed sufficiently it makes contact with the mercury cathode. The solenoid is now cut out of circuit automatically, upon which the spring draws the rod upwards. This causes an arc to be struck from the surface of the mercury, creating sufficient vapour to enable the main anodes to come into action.

There are two further auxiliary anodes called *excitation anodes*. Their function is to maintain the arc should the load fall below a certain value, for which purpose they are connected to a small artificial load.

In order to limit the temperature rise, water cooling is adopted. This water is passed through a pipe round the base of the cathode and also through a water jacket round the main cylinder. Cooling water also circulates around the anode bases. The condensing cylinder is provided with another water jacket. The mercury vapour is here condensed, the mercury trickling back into the cathode pool in the form of drops.

Vacuum Pump.—An all-metal rectifier does not keep its vacuum in the same way that a glass one does, so that it is necessary to instal a vacuum pump. It is not necessary to have this pump working continuously, an occasional half-hour being all that is necessary.

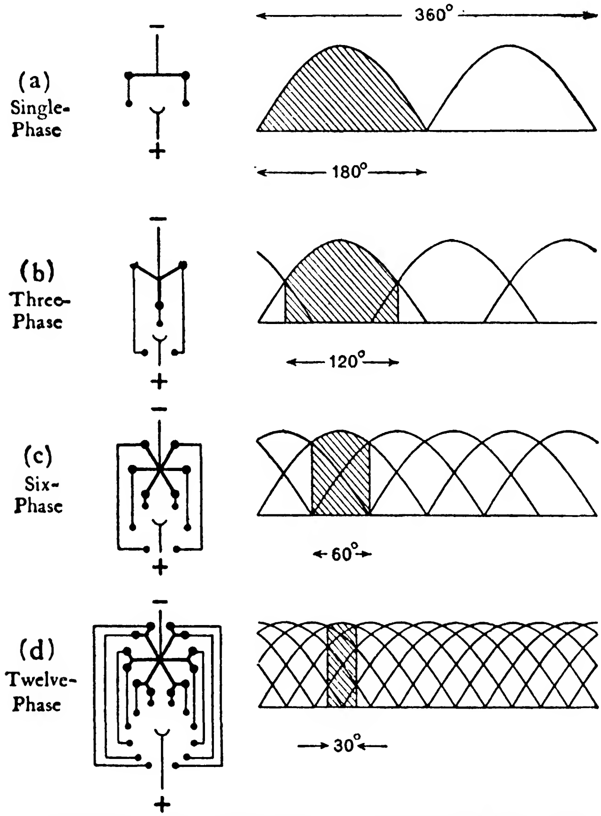


FIG. 334.—Output Voltage Waves for Different Numbers of Phases.

Output Voltage Waves.—In the case of a single-phase rectifier with two anodes, each anode works theoretically for half of each cycle and, as a first approximation, the D.C. output voltage may be assumed to take the form shown in Fig. 334. The working time for one anode is shown shaded. The output voltage varies between a maximum and zero, and the preponderant ripple has a frequency of 100 cycles per second on a 50-cycles supply.

In a plain three-phase rectifier, each anode fires theoretically for one-third of the total time. During the interval, the potential of the firing anode is higher than that of either of the other two, with respect to the cathode, and the electrons emitted from the cathode are therefore directed to this particular anode. The working time for one anode is now represented by 120°, and is again shown shaded in the diagram. The preponderant ripple now has a frequency of 150 cycles per second on a 50-cycles supply.

In a six-phase rectifier, each anode fires for a time equal to 60° (again shown shaded), and the preponderant ripple has a frequency of 300 cycles per second.

In a twelve-phase rectifier, the firing time is reduced to 30°, whilst the frequency of the preponderant ripple has risen to 600 cycles per second. The method of connecting the transformer secondary windings should be noted.

It is apparent that, as the number of phases is increased, the amplitude of the ripple is decreased but that its frequency is progressively increased.

Voltage and Current Relations in Six-Phase Rectifier.—The case of a six-phase rectifier will be studied, the transformer primaries being connected in delta and the transformer secondaries in six-phase star with the neutral ends solidly connected together. (This is also called the *diametric* connection.) These connections are shown in Fig. 335.

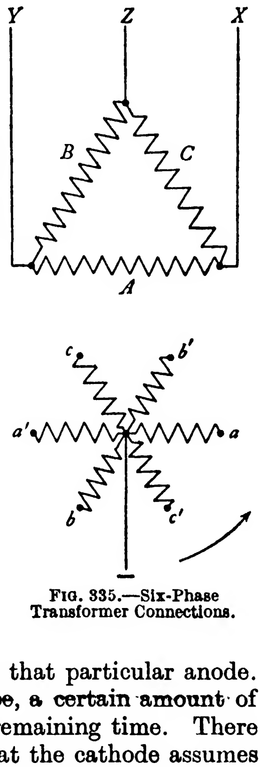


FIG. 335.—Six-Phase Transformer Connections.

Each of the six anodes in turn acquires a higher positive potential than the others for one-sixth of a cycle, and when this takes place the arc is assumed to be transferred to that particular anode. (In practice, owing to the effects of reactance, a certain amount of overlap occurs.) It is inactive during the remaining time. There is a drop of about 15 volts in the arc, so that the cathode assumes a potential about 15 volts below that of the working anode, this becoming the D.C. positive terminal. The transformer secondary neutral point forms the D.C. negative terminal.

The voltages between the various anodes and the neutral point are shown at (*a*) in Fig. 336, which also shows the output voltage.

The maximum D.C. voltage is therefore $\sqrt{2}$ times the R.M.S. secondary phase voltage minus the arc drop, whilst the minimum D.C. voltage is $\frac{\sqrt{3}}{2}$ times the maximum value, again minus the arc drop, since the point of intersection of two anode voltage waves occurs 30° away from the maximum value, and $\cos 30° = \frac{\sqrt{3}}{2}$. The mean value between these two extremes is 1·35 times the R.M.S. phase voltage, neglecting the voltage drop in the arc.[1]

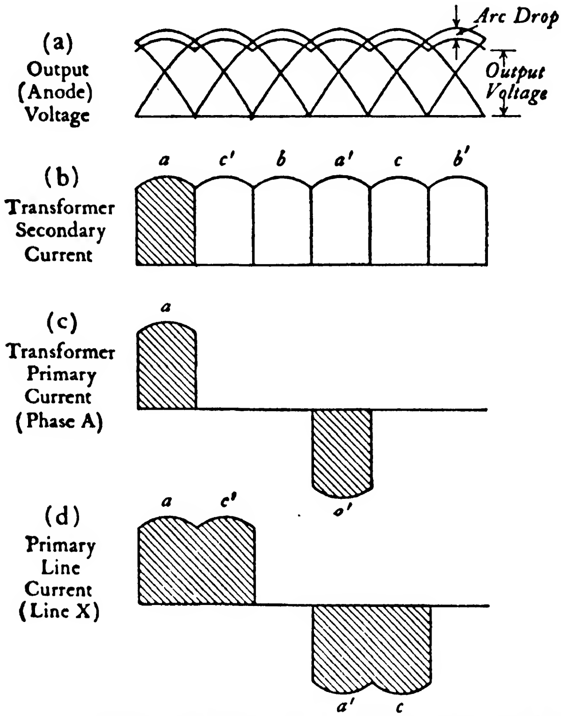


FIG. 336.—Six-Phase Rectifier Wave Forms without Interphase Transformer.

Neglecting voltage drops, the current wave form for each anode consists of the central 60° portion of a sine wave, rising from zero

[1] This can be proved as follows :—

Mean voltage = Area of voltage wave during firing time, divided by firing time.

$$= \frac{1}{\frac{2\pi}{6}} \int_{-\frac{\pi}{6}}^{+\frac{\pi}{6}} \sqrt{2}E \cos\theta \, d\theta$$

$$= \frac{6}{2\pi}\left[\sqrt{2}E \sin\theta\right]_{-\frac{\pi}{6}}^{+\frac{\pi}{6}}$$

$$= \frac{6}{2\pi}\left[\sqrt{2}E\{\tfrac{1}{2} - (-\tfrac{1}{2})\}\right]$$

$$= \frac{3\sqrt{2}}{\pi}E = 1{\cdot}35E.$$

30° before the maximum value is reached, and falling again to zero 30° after the maximum value has been passed, as shown at (*b*) in Fig. 336. In practice, reactance prevents the current in any anode from rising so rapidly, and also causes it to die away more slowly. There is thus a gradual transfer of current from anode to anode, or overlap, and this modifies the anode current wave form considerably.

The current in the primary *phase* windings of the transformer has the same wave form as that on the secondary side, except that each primary feeds two secondaries exactly opposite in phase. Thus, in Fig. 335, the primary winding A feeds the two secondary windings, a and a'. The primary phase current of the transformer is shown at (*c*) in Fig. 336.

The primary line current is obtained by subtracting two phase currents. Thus the current in line X is equal to (phase current A) $-$ (phase current C). In Fig. 336 (*c*) the current for the secondary a was shown above the zero line, and that for a' below the zero line. Since the phase current C has to be subtracted, the current for the secondary c is drawn *below* the zero line, whilst that for c' is shown *above* the zero line. The resultant primary line current is now represented at (*d*) in Fig. 336, and this is seen to be very far from sinusoidal.

The 300-cycle ripple is apparent in the D.C. output voltage, and the A.C. line current is now seen to contain various harmonics of varying magnitude. The D.C. ripple and the A.C. harmonics can be choked down by the introduction of reactance, and this is always done in practice. Since there is no energy storage in the arc, any modification of the D.C. output current must produce precisely the same effect on the A.C. input.

Interphase Transformer.—A disadvantage of the plain six-phase rectifier discussed above is that the utilization of the active material is not efficient, since each anode carries the full current for only one-sixth of the total time. The six-phase rectifier will now be compared with two three-phase rectifiers, each carrying half the total current and operating 180° out of phase with each other.

Let R be the resistance of each secondary winding in the six-phase case, and let the output be I amperes. The heat involved in a time t is now

$$6I^2R \times \frac{t}{6} = I^2Rt.$$

If the same transformer be utilized to feed two three-phase rectifiers in parallel (from alternate secondary phases), the load on each rectifier being $\frac{I}{2}$ amperes, each anode would fire for one-third of a cycle instead of one-sixth. The heat involved is now

$$2 \times 3\left(\frac{I}{2}\right)^2R \times \frac{t}{3} = \tfrac{1}{2}I^2Rt.$$

Hence, for the same copper loss the resistance could be doubled and the volume of copper halved. In addition to reducing the cost of the transformer, this arrangement also results in an improved voltage regulation.

Instead of having two separate three-phase rectifiers, the same effect can be obtained in one chamber by separating the two three-phase neutral points, and connecting them to the ends of a mid-point auto-transformer as shown in Fig. 337. The mid-point forms the D.C. negative terminal. The auto-transformer is called

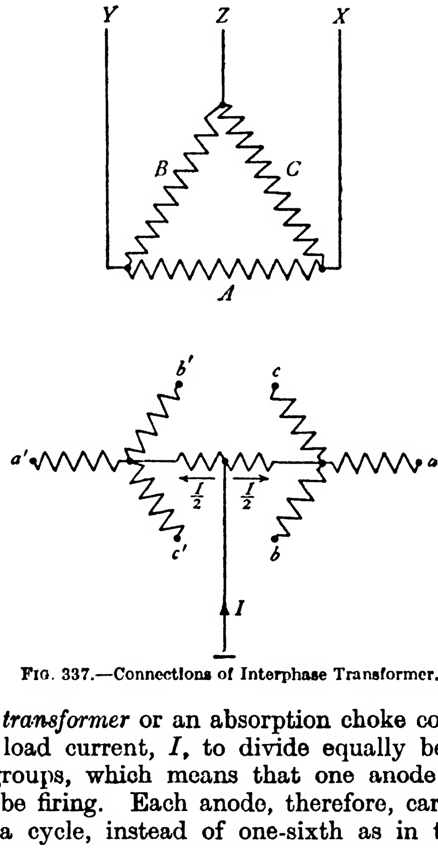


FIG. 337.—Connections of Interphase Transformer.

an *interphase transformer* or an absorption choke coil. Its effect is to cause the load current, I, to divide equally between the two three-phase groups, which means that one anode of each group must always be firing. Each anode, therefore, carries current for one-third of a cycle, instead of one-sixth as in the case of the six-phase rectifier without an interphase transformer. On the other hand it only carries half the total load current.

When two anodes are firing simultaneously, one in each group, their voltages to the cathode must be the same. The difference in the voltages across the corresponding transformer phase windings appears across the interphase transformer, the mid-point of which assumes a value half-way between the two.

The graphs of the different anode voltages are shown at (*a*) in Fig. 338, together with the D.C. output voltage which is the average of the two voltages of those anodes which are firing simultaneously. This mean value can be obtained in the following manner :—

Let the voltage of the anode *a* be represented by $\sqrt{2}E \sin \omega t$, and that of the anode *b'* by $\sqrt{2}E \sin (\omega t + 60°)$. The average of these two values is

$$\frac{\sqrt{2}E}{2}[\sin \omega t + \sin (\omega t + 60°)]$$

$$= \frac{\sqrt{2}E}{2} \times 2 \sin (\omega t + 30°) \cos 30°$$

$$= \frac{\sqrt{3}}{2} \times \sqrt{2}E \sin (\omega t + 30°).$$

The D.C. output voltage thus consists of the crests of sine waves having a maximum height of $\frac{\sqrt{3}}{2}$ times the maximum A.C. phase voltage, or $\frac{\sqrt{3}}{2} \times \sqrt{2}$ times the R.M.S. value. The mean value of the D.C. output voltage is, as before, $\frac{3}{\pi}$ times the maximum value, or $\frac{3}{\pi} \times \frac{\sqrt{3}}{2} \times \sqrt{2} = 1{\cdot}17$ times the R.M.S. phase voltage. Without the interphase transformer this value was (see p. 466) 1·35 times the R.M.S. phase voltage. (The voltage drop in the arc is neglected in each case.)

The voltage across the terminals of the interphase transformer is represented in Fig. 338 (*a*) by the vertically shaded portions between two anode voltage waves, and this can be shown to be the same as the horizontally shaded portions on the same diagram. This is practically a wave of triangular shape and of triple frequency, and the voltage between one terminal and the mid-point of the interphase transformer is of the same wave form but of half the amplitude.

The anode current, and therefore the transformer secondary current, has the same characteristic form as the D.C. output voltage and is represented at (*b*) in Fig. 338. Each primary winding has to serve two secondary windings, so that the current in the primary phase *A* has a positive portion on account of the current in the secondary phase *a*, and a negative portion on account of the secondary phase *a'* which operates half a cycle later. The primary phase current for phase *A* is shown at (*c*) in Fig. 338.

The current in line *X* (see Fig. 337) consists of the difference of the currents in phase *A* and phase *C*. In obtaining the primary line current, therefore, the primary current due to the secondary c is drawn *below* the zero line, and that due to the secondary c' is drawn *above* the zero line. This primary line current, for line X,

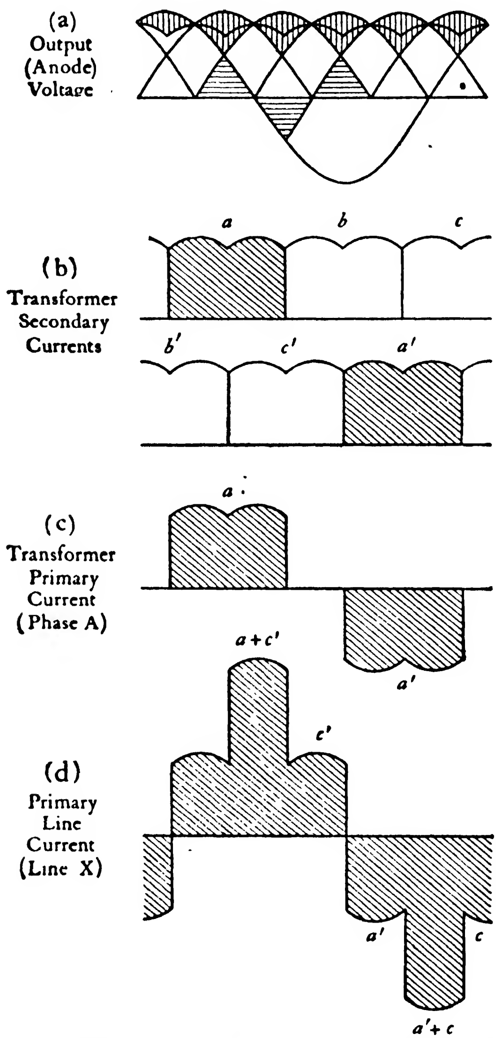


Fig. 338.—Six-Phase Rectifier Wave Forms with Interphase Transformer.

is shown at (*d*). Incidentally, this is a better wave form than that shown in Fig. 336 (*d*).

Magnetizing Current of Interphase Transformer.—If the currents in the two halves of the interphase transformer are exactly equal, the resultant ampere-turns are zero, but in practice a certain amount of magnetizing current must flow in order to set up the flux. This flux chokes down the current in the more heavily loaded half of the winding, and increases that in the more lightly loaded section, thus tending to maintain equality of the currents in the two halves. The magnetizing current may be regarded as flowing straight through the winding, the circuit being completed by those main transformer secondary windings which are in operation and the corresponding arcs in the rectifier itself. The voltage producing this magnetizing current is the voltage existing between the two ends of the interphase transformer windings.

The magnetizing current aids the main load current in one half of this winding and opposes it in the other. If the main load current falls below a certain value, therefore, the magnetizing current cannot flow, since this would involve reversing the current in one of the main arcs, and this cannot take place on account of the valve action. Below this load, called the *transition load*, the interphase transformer no longer functions, and the rectifier reverts to simple six-phase working, showing a rise in D.C. voltage of about 15 per cent. at the point of change-over.

Again, on heavy overloads, the anodes begin to work over the full 180°, on account of overlap, and when this occurs, the arc is not transferred from anode to anode, but from one anode to that diametrically opposite. Here, again, the interphase transformer becomes inoperative. From the transition load up to full load there is a further small drop in voltage of 2½—5 per cent. depending upon the transformer reactance. Compensation for this can be made by arranging the transformer with on-load tap-changing gear, or by providing the rectifier with grid control.

Performance.—The rectified voltage falls gradually as the load comes on, due to the reactance of the transformer, etc., but this drop does not usually exceed 5 per cent. In addition, there is the voltage drop in the arc which is of the order of 15—20 volts. This voltage drop is independent of the D.C. output voltage, so that higher efficiencies are obtained at the higher voltages, and overall efficiencies of 95 per cent. are obtainable. The power factor of the transformer primary varies from about 0·90 up to 0·95 at full load.

A mercury arc rectifier can stand overloads up to 100 per cent. for a few minutes, whilst the permissible momentary overload is very high. This makes them very suitable for traction supply, where the ripple is not of great consequence.

When two or more rectifiers are, for reasons of economy, fed from the same transformer, a paralleling reactance coil is connected in series with each anode circuit to ensure the proper division of the load.

Grid Control.—The current in a mercury arc rectifier can be varied by means of grids situated in the discharge paths between the anodes and the cathode, just as in the case of a three-electrode valve. The instant at which the arc discharge takes place is controlled by applying potential impulses to these grids, and this can be conveniently brought about by employing an alternating grid potential, of the same frequency as the main anode voltage, but of adjustable phase. As an alternative to varying the phase, a steady D.C. potential may be superimposed on the A.C. grid potential, the control being obtained by varying the magnitude of this D.C. bias potential.

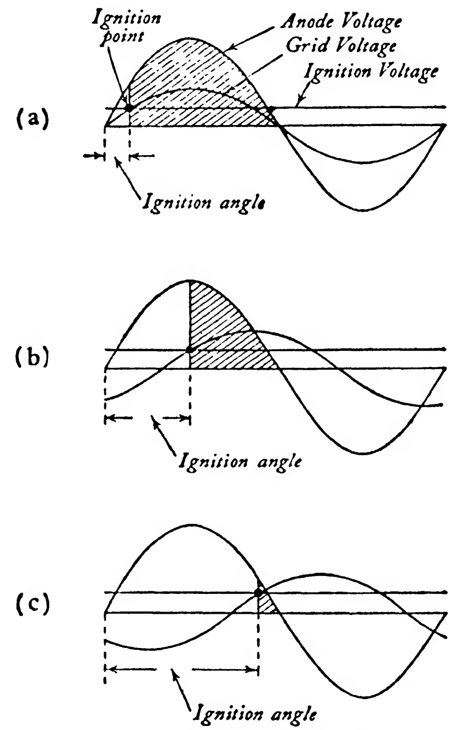


FIG. 339.—Grid Phase Control of Mercury Arc Rectifier.

Phase Control.—The grid has to be raised to a certain minimum potential before an arc can be established between anode and cathode, this being known as the *ignition voltage*. If an alternating voltage be applied to the grid, therefore, the main arc is struck at the instant that this grid voltage reaches the value of the ignition voltage. This is illustrated in Fig. 339. At (*a*) the grid voltage is in phase with the anode voltage, and the arc is struck very early. The *ignition angle* is small, this being the angle between the instant when the anode voltage is zero and the instant when the arc is struck. This arc is maintained until the voltage of the main anode falls below the ignition value. The shaded portion of the curve represents the time during which the arc is established, and the D.C. output voltage depends upon the average value of this useful part of the anode voltage wave.

At (*b*) the grid voltage has been retarded in phase, so that the duration of the arc is only about half what it was at (*a*). The

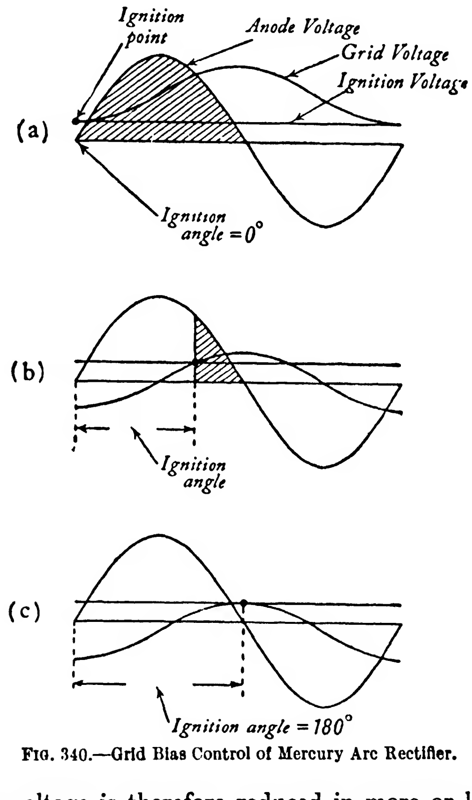


FIG. 340.—Grid Bias Control of Mercury Arc Rectifier.

D.C. output voltage is therefore reduced in more or less the same proportion.

At (*c*) the grid voltage is almost in phase opposition, and the ignition angle is almost 180°. The shaded portion of the anode curve is now extremely small, and the D.C. output voltage has been reduced almost to zero.

The phase control of the three grid voltages (in a three-phase rectifier) is effected by supplying them through an induction regulator (see p. 425). The primaries of this induction regulator are connected to the A.C. supply, and the secondaries, which are star-connected, are taken through current-limiting resistances to the grids. The neutral point of this star is connected to the cathode, so as to fix the potentials of the grids with respect to the cathode.

Bias Control.—The control of the ignition point is here effected by applying an adjustable D.C. bias potential to each grid, in addition to the A.C. potential discussed above. This D.C. grid bias is obtained by means of a regulating potentiometer energized by a number of cells, as in the case of the grid bias used in connection with thermionic valves. The mid-point of the cells is connected to the cathode and the sliding connection on the potentiometer is taken to the neutral point of the star-connected induction regulator secondaries.

The method is illustrated in Fig. 340. At (*a*) the maximum positive grid bias is shown, the ignition point occurring with zero anode potential. The arc is now established over the full 180°, and the D.C. output voltage is a maximum.

At (*b*) the grid bias is reduced to zero, and the ignition point is retarded by more than 90°. The main arc is now only established for less than half the time shown at (*a*) and the D.C. output voltage is accordingly reduced.

At (*c*) a negative grid bias is applied, with the result that the ignition angle is increased to 180°. The ignition is now retarded to its maximum, and the D.C. output voltage is reduced to zero.

Other methods of grid control are employed, but the two types outlined above are sufficient to illustrate the principle.

Wave Form of Grid Voltage.—In order to control the exact time of the ignition point more exactly, the wave form of the voltage applied to the grid is made very sharply peaked. This is achieved by interposing a highly saturated transformer in series with a resistance in the grid circuit, the grid itself being connected to the secondary. A very flat-topped flux wave is obtained in the transformer, this inducing a secondary voltage which is in the nature of a voltage impulse, as shown in Fig. 341 (*a*). The sinusoidal anode voltage is shown in Fig. 341 (*b*), together with the peaked grid voltage, where it is seen that the ignition point is much more sharply defined than it is with a sine wave of grid voltage. The addition of a negative D.C. bias to the grid results in the grid voltage curve being lowered throughout by a constant amount as shown in Fig. 341 (*c*). The desired ignition voltage is now only just exceeded for a very short time thus giving a very sensitive control of the D.C. output voltage by phase control of the grid voltage.

Hot Cathode Rectifier.—A two-electrode discharge tube containing gas or vapour, and operated with a hot cathode acts as a rectifier. The anode and cathode are sealed into a partially exhausted vessel, the atmosphere consisting of mercury vapour or an inert gas such as argon, neon or helium. These rectifiers differ from ordinary thermionic valves in this respect, since the latter are highly evacuated. The introduction of vapour or gas modifies the characteristics.

In the *thyratron* the current flowing between the anode and cathode is controlled by a third electrode or grid. In the case of the thermionic valve used for radio work, the grid potential controls the amount of current passing between anode and cathode, but in the *thyratron*, the only function of the grid is to prevent or control the starting of the arc. After this takes place, the grid has

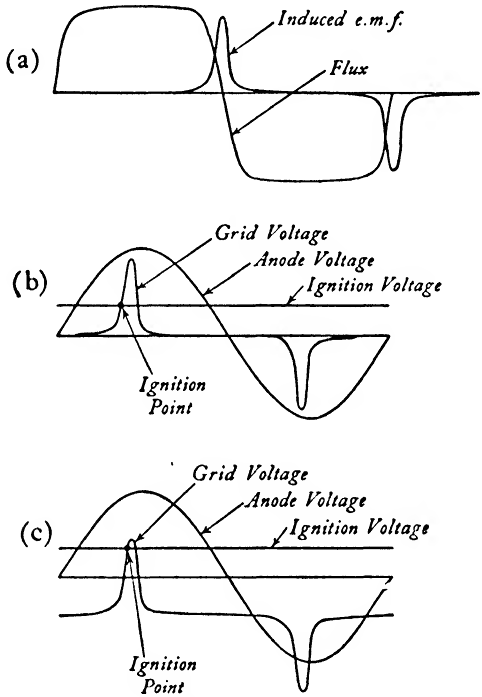


FIG. 341.—Peaked Wave Form of Grid Voltage.

no further effect. The arc ceases when the anode potential is reduced to zero or given a negative value.

The cathode may be directly heated, as in the filament type, or it may be indirectly heated by means of a separate tungsten heater energized from a separate transformer winding.

The anode current is controlled by adjustment of the phase of the grid voltage, or by applying a steady bias voltage, as in the case of the mercury arc rectifier.

Copper Oxide Rectifier.—This rectifier consists of a disc of copper upon which has been formed, by heat treatment, a layer of copper oxide. Contact is made with the outer surface of the oxide by pressing a metal disc against it, or by means of a metal sprayed on to the oxide surface. A number of these rectifier elements are formed in series, the number depending upon the voltage applied. Cooling fins are fitted between the elements to aid in the dissipation of the heat produced by the internal resistance drop.

The resistance of such a rectifier is low measured in the direction

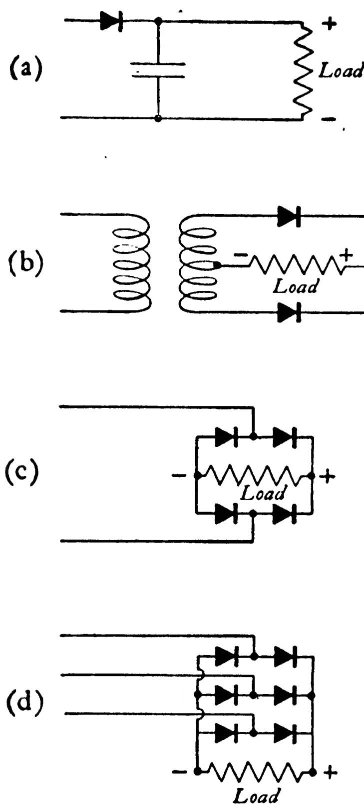


FIG. 342.—Connection of Copper Oxide Rectifiers.

of oxide to copper, whilst in the opposite direction it is very high. The current in one direction is approximately proportional to the applied voltage, whilst in the other direction it is practically zero. The rectifier action thus permits current to flow one way only.

For high voltages a number of elements are connected in series, whilst for heavy currents, they are connected in parallel.

The simplest method of connection, called half-wave rectification, is illustrated in Fig. 342 (*a*). Rectified current flows through the load circuit during one-half of each cycle only, but a condenser is frequently added as shown in the diagram, in order to maintain the output during the idle half-cycles.

Full-wave rectification is obtained by the arrangement shown in Fig. 342 (*b*). The secondary winding of the transformer has a centre tapping, so that the upper and lower halves of this winding carry load current during alternate half-cycles. The current through the load is unidirectional, but consists of a succession of half-waves, always in the same direction and without any intervals.

An alternative method of obtaining full-wave rectification is shown in Fig. 342 (*c*). This is known as the bridge connection, from its similarity to the connections of a Wheatstone bridge. When the upper supply lead is positive the top right-hand and the bottom left-hand rectifiers carry current, the other two being idle, this being reversed during the half-cycle that the bottom supply lead is positive. No transformer is required with this arrangement, unless the supply voltage is too high. The theoretical wave form of the rectified load current is the same as for Fig. 342 (*b*).

Fig. 342 (*d*) shows six rectifiers connected for rectification of a three-phase supply. When any particular supply lead is positive, current flows through the appropriate right-hand rectifier, whilst when a supply lead is negative, current returns to the supply by way of the appropriate left-hand rectifier. The resultant rectified output current fluctuates much less in this case than in any of the previous cases mentioned.

Examples

(1) Explain how a uni-directional current can be obtained from an A.C. supply by the aid of a mercury arc.

(2) Draw a diagram of connections for two three-phase mercury arc rectifiers arranged for six-phase working.

(3) Describe the construction of an all-metal mercury arc rectifier.

(4) Explain the action of an interphase transformer as used with mercury arc rectifiers.

(5) Describe two methods of grid control of mercury arc rectifiers.

CHAPTER XXVIII

FREQUENCY CHANGERS

Interlinking of Power Systems.—The parallel running of power systems is now resorted to, so that energy may be transferred from one to the other, either in normal or abnormal conditions such as take place when a fault occurs (see page 424). When the frequency of the two stations differs, the paralleling must be performed through the medium of a frequency changer. The voltage or phase adjustment can still be carried out by the aid of transformers, induction regulators and the like.

Synchronous Frequency Changers.—When the ratio of the two frequencies must be kept rigidly constant, the most obvious method of changing the frequency is by means of a synchronous motor-alternator set. When run on constant frequency, the synchronous motor is a constant speed machine and thus enables the alternator to generate at a constant frequency. It will be observed that no deviation in speed is permissible. (Of course hunting troubles may still be encountered.) From this it follows that any small change in frequency of the one system must be reflected by an exactly proportional change in the frequency of the other system, as otherwise the conditions are not fulfilled. If a small rise occurs, then the whole of the power necessary to accelerate all the synchronized machines in the other station must be transmitted by the frequency changer. Corresponding conditions obtain when one station suffers a small drop in frequency. This results in there being a lower limit to the kVA capacity of the frequency changer linking the two stations, since if this were too small, it would run the risk of being burnt out by the heavy load which it would be called upon to carry, even although this is only of short duration.

Possible Speeds.—The frequency of a synchronous machine running at a constant speed is proportional to the number of poles, so that the ratio of the number of poles on the component elements of the frequency changer must be the same as the ratio of the two frequencies. This considerably limits the choice of speed in certain cases. No disadvantage accrues on this score in a 25—50 cycle conversion, for a bipolar 25-cycle machine running at 1500 r.p.m. will operate with a 4-pole 50-cycle machine. The conditions are rather more awkward, however, with a 25—60 cycle conversion.

The pole ratio is now 5 : 12 and the minimum numbers of poles are 10 and 24 respectively, the corresponding speed being 300 r.p.m. For a given output such a set would be relatively expensive, as compared with the 25—50 cycle set running at 1500 r.p.m., so that it is seen that some frequency changes are more expensive to effect than others, the powers being the same in each case.

The speeds quoted above are the maximum ones; lower speeds can also be obtained, these being found by dividing the maximum speeds by any whole number.

The following table shows the maximum speeds and the corresponding numbers of poles for several of the more common frequency changes :—

Frequency Change.	Maximum Speed in r.p.m.	Numbers of Poles.
25 to 40	300	10 and 16
25 „ 50	1500	2 „ 4
25 „ 60	300	10 „ 24
40 „ 50	600	8 „ 10
40 „ 60	1200	4 „ 6
50 „ 60	600	10 „ 12

Angular Displacement.—When an alternator, which is running in parallel with others, is loaded, the phase of its induced E.M.F. must be advanced on account of its synchronous reactance drop (see page 289). To effect this the rotating element must be advanced in position at a given instant of time. On the other hand, in a synchronous motor the rotor drops behind as the load comes on (see Fig. 298).

When a synchronous motor drives an alternator, these two displacements are in the same direction. It follows, therefore, that the correct position of the rotor of the alternator is different when on load from what it is on no-load. On this account the stator of one machine is mounted on a cradle, so that it can be rotated through a small angle, either by hand or by means of a small motor. It does not matter which stator is moved, since a forward displacement of the one is equivalent to a backward displacement of the other.

Consider the case of a 25—50 cycle set, motoring on the lower frequency and running at 1500 r.p.m. For unity power factor, let the phase of the induced E.M.F. of the alternator on full load be 20° ahead of its no-load position, and let the motor fall back by 25° behind its synchronous position at the same time, the angles being measured in electrical degrees. When running light the load on the motor is assumed to be 10 per cent. of its full load value, and assuming the motor displacement angle to be proportional to the load, this is now 2·5°. On the application of full load, therefore, the motor falls back by 22·5°, and to neutralize this, the pole ratio being two to one, the alternator must be advanced by $22{\cdot}5 \times 2$

= 45°. In addition, the alternator must be advanced by 20° in order to enable it to take up the load, so that the total electrical angle of advance is 65°. This being measured on the 4-pole 50-cycle machine, it corresponds to an actual stator displacement of 32·5°.

If the set be motoring from the 50-cycle side, the required stator displacement is different. The motor phase angle is now 20° on load, the alternator angle being 25°. When running light the motor angle is 10 per cent. of 20° or 2°, so that the additional displacement is 18°. To neutralize this the 25-cycle alternator must be advanced by 9°, to which must be added its own 25°, making a total of 34°. These are electrical degrees, but being measured on a bipolar machine the angle is the same as the required actual stator displacement.

Regulation of Load by Stator Setting.—Since the displacement angles of both motor and generator are affected by the load, it follows that a change of load can be effected by adjustment of the stator setting. Whilst this is the only adjustment necessary when several frequency changers are running in parallel, it must be accompanied by adjustment of the prime mover outputs in the two stations concerned when the frequency changer is the only connecting link. In order to make one particular machine act as the generator, its stator must be racked round against the rotation, as this makes its armature conductors come under the influence of the flux a little earlier. Conversely, racking the stator in the direction of rotation makes the machine into a motor.

Alteration of the excitation of either motor or generator merely affects the power factor of the machine concerned and does not, to any appreciable extent, alter the power load.

Starting of Synchronous Frequency Changers.—An auxiliary induction motor may be used for starting, usually built with two less poles than the synchronous motor so as to enable it to reach synchronous speed. Self-starting or self-synchronizing synchronous motors are, however, commonly used (see page 412). The synchronous induction motor may also be employed.

Synchronizing.—The fact that the motors of two identical synchronous frequency changers are synchronized does not necessarily imply that the generators are also in synchronism, even if both sets are running light.

Considering a 25—50 cycle set with two and four poles respectively, if the 25-cycle machine be synchronized first, the other machine must be in its proper phase, assuming a correct stator setting, for there is only one rotor position in the whole revolution for synchronizing a bipolar machine. If, however, the 50-cycle machine be synchronized first, the 25-cycle machine may or may not be in its correct position, for either of the two north poles of the former set may be opposite to a given point on the stator at a given instant of time. One of these positions permits of synchronizing, but the other corresponds to a phase displacement of 180°, since a shift of a double pole pitch on the 50-cycle side is equivalent to a shift of only a single pole pitch on the 25-cycle side. There is, therefore, an even chance of the generator coming into the correct position for synchronizing.

With other frequency ratios this chance may be considerably reduced. The conversion of 25 to 60 cycles is a case in point. There are now 10 and 24 poles respectively. The synchronous motor may be synchronized in any one of five positions, and the total number of electrical degrees in one revolution on the 24-pole side is $12 \times 360 = 4320°$. The five possible positions thus differ in phase from each other by $\frac{4320}{5} = 864°$, or 144°. Assuming the first position to be the correct one, the succeeding positions correspond to phase angles of 144°, 288°, 72°, 216°, 0°, etc. There is thus one chance in five that the generator is in the correct phase. If the 60-cycle machine be paralleled first, the chance of obtaining correct synchronism is one in twelve. The correct phase angle is actually obtained by pole-slipping, as explained with rotary converters (see page 445). It is, however, an advantage to synchronize the high frequency machine first, as then a given error in stator position corresponds to a smaller electrical angle.

Induction Frequency Changers.—An induction motor, preferably of the slip ring type, is now made to drive an alternator of the synchronous type. The motor works with a certain slip, and this affects the frequency ratio. Consider again the case of 25—60 cycles. With the synchronous type the maximum speed is 300 r.p.m., but in the present case a speed of 720 r.p.m. may be adopted, employing 4 and 10 poles respectively. The speed of a 10-pole 60-cycle synchronous machine is $\frac{120 \times 60}{10} = 720$ r.p.m. The synchronous speed of a 4-pole 25-cycle induction motor is $\frac{120 \times 25}{4} = 750$ r.p.m., but with a slip of 4 per cent. this is also reduced to 720 r.p.m. If such a set be run in parallel with one of the synchronous type, the frequency ratio is definitely fixed, and therefore the slip of 4 per cent. must be maintained, thus determining the load carried by it.

The induction frequency changer can be made to transmit power in the reverse direction, the induction machine running as the generator, but in this case there must be a synchronous machine running in parallel with it, so as to supply the magnetizing current to its stator. The induction machine now runs above synchronous speed, the negative slip varying with the load.

If an induction machine be driven at a speed considerably higher than that of synchronism, the frequency of the rotor currents becomes appreciable. A frequency changer may consist, therefore, of a driving motor coupled to an induction machine, the stator of which is supplied from the motor side. The generator rotor may be driven either in the same direction as that of the rotating field or against it. In the former case the rotor frequency is equal to

$$\text{Stator frequency} \times \frac{(\text{Synchronous speed} - \text{Operating speed})}{\text{Synchronous speed}}.$$

If the rotor is driven against the field, the frequency of its current becomes

$$\text{Stator frequency} \times \frac{(\text{Synchronous speed} + \text{Operating speed})}{\text{Synchronous speed}}.$$

As an example, suppose the supply frequency to be 25 cycles, and the rotor to be driven at 140 per cent. of synchronous speed in the opposite direction to that in which it tends to rotate as a motor. The rotor frequency is now $25 \times (1 + 1{\cdot}4) = 60$ cycles. Part of the energy is transferred by transformer action (25/60th of the total) and part by motor-generator action (35/60th of the total), somewhat after the manner of the motor converter. In this way a variable frequency can be obtained by varying the speed of the driving motor.

Comparison of Synchronous and Induction Frequency Changers.—If the frequency ratio must be kept constant under all conditions, or if reversible operation is desired without the aid of paralleled synchronous machines, it is necessary to adopt synchronous frequency changers. On the other hand, from the point of view of operation, the paralleling and the suitable load distribution is simpler in the case of the induction type. The power factor of the synchronous type, however, is under control, whereas this is not so in the other case, unless a phase advancer be also installed (see page 533).

Frequency Changers for Operating High-Speed Motors.—Certain industrial operations demand speeds in excess of 3000 r.p.m., the highest that can be obtained from an ordinary synchronous or induction motor working on 50 cycles per second. Frequency changers of the induction type can be designed to give output frequencies ranging from 75 to 450 cycles per second, and giving motor speeds from 4500 to 27,000 r.p.m.

A frequency changer of this type consists of an A.C. generator, or alternator, driven by an induction motor. The generator is also constructed in the manner of an induction motor, having stator and rotor windings. The stator winding is fed from the source of supply, and sets up a rotating field in the air-gap of the generator. The speed of this rotating flux is that of synchronism, corresponding to the frequency of supply and the number of poles on the generator. The rotor is driven by a squirrel cage induction motor in the *opposite* direction to that of the rotating field. At zero speed, the rotor windings have E.M.F.'s induced in them at a frequency equal to that of supply. As the speed of the rotor is increased, the frequency of the generated E.M.F.'s is also increased, the output frequency being equal to the sum of the frequency of supply and that of speed.

In general, the high frequency is given by

$$\text{Output frequency} = \text{Excitation frequency} + \frac{pn}{120},$$

where p is the number of poles on the generator and n is the speed, *i.e.* that of the driving squirrel cage induction motor.

For example, if a bipolar induction motor runs at 2900 r.p.m. on a 50-cycle supply, and the generator has 6 poles, the output frequency is

$$50 + \frac{6 \times 2900}{120} = 195 \text{ cycles per second.}$$

Various combinations and the resulting speeds are given in the following table, a supply frequency of 50 cycles per second being assumed.

Driving Motor.	poles	8	4	4	2	2	2	2
	r.p.m.	750	1,500	1,500	3,000	3,000	3,000	3,000
Frequency Changer.	poles	4	4	6	4	6	10	14
	cycles per second	75	100	125	150	200	300	400
High Frequency Motor.	r.p.m.	4,500	6,000	7,500	9,000	12,000	18,000	24,000

Frequency changers of the above type have a voltage regulation of about 10—15 per cent. at a power factor of 0·8.

Triple-Frequency Welding.—A specially connected transformer is employed to give a single-phase output from a three-phase supply, this output being at three times the supply frequency (see Fig. 343). A three-phase transformer has its primaries connected in star, the secondaries forming an open delta which is only closed through the arc welding circuit. A high flux density is employed in the transformer core, which necessitates a third harmonic in the magnetizing current for sinusoidal magnetization. Since no third harmonic magnetizing currents can flow in the primary star, the induced flux is not sinusoidal. Consequently, third harmonic E.M.F.'s are induced in the secondary windings in addition to the normal E.M.F.'s of fundamental frequency (see p. 200). The E.M.F.'s of fundamental frequency cancel out round the secondary delta, but the third harmonic E.M.F.'s are in phase. They thus produce a resultant which supplies current at triple frequency to the welder. This current can be regulated by means of the tapped inductance as shown. The power factor is rather low, but this can be improved by incorporating condensers on the primary side, the load of which is balanced.

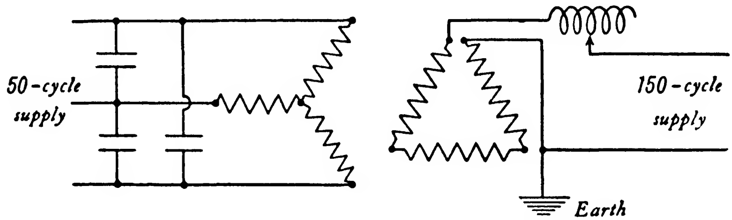


FIG. 343.—Triple-Frequency Welding Circuit.

EXAMPLES

(1) Determine the minimum numbers of poles and the maximum speed of a synchronous frequency changer to operate on 30—85 cycles.

(2) The full load displacement angles from the neutral position of the component machines in a 25—60 cycle synchronous frequency changer are 15° and 24° respectively, the stator of the latter machine being adjustable. The speed is 300 r.p.m. and the power required when running light is 16 per cent. of full load. Determine the angle from the neutral position through which the adjustable stator must be racked in order to obtain full load (*a*) when this machine is generating, and (*b*) when it is motoring.

(3) An induction frequency changer is to convert from 40 to 50 cycles and the slip is to be less than 10 per cent. What is the maximum speed possible and the corresponding numbers of poles?

(4) An induction frequency changer set for operating high speed motors is designed to give an output frequency of 350 cycles per second on no-load. The supply frequency is 50 cycles per second and the driving motor has 2 poles. How many poles must the frequency changer have? (Neglect slip.)

CHAPTER XXIX

SINGLE-PHASE SERIES MOTORS

Simple Series Motor.—The simple single-phase series motor is very similar to the D.C. series motor in its general arrangement, the armature A and the field winding F being connected in series as shown in Fig. 344. The brushes on the commutator make connection with conductors in the neutral zone, and since these motors are largely used for traction work where reversible rotation is required, the brushes are kept in the no-load neutral position for all loads. The field system is laminated throughout, as otherwise there would be prohibitive iron losses. The salient pole construction is now abandoned in favour of a stator like that of an induction motor, the field winding being placed in slots as in the rotor of a turbo-alternator, a uniform air-gap being the result.

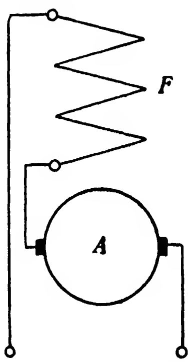


FIG. 344.—Connections of Simple Series Motor.

E.M.F.'s induced in the Armature.—In addition to the E.M.F.'s induced in the armature due to rotation in the magnetic field, there are also E.M.F.'s induced by transformer action, the field winding acting as the primary and the armature winding as the secondary. These E.M.F.'s will now be considered in detail.

The resultant magnetic field is due to the combination of the fluxes set up by the main field and the armature winding respectively. The M.M.F.'s due to the field and armature windings act on different axes in space, and it is convenient to consider the resulting fluxes as though each had a separate existence. That due to the main field winding (acting vertically in Fig. 344) is called the *main flux*, and that due to the armature winding (acting horizontally in Fig. 344) is called the *cross flux*.

First, consider the effect of rotation in the main field. All the conductors from A to A' down the left-hand side of the armature (Fig. 345) will have E.M.F.'s induced in the same direction, and all the conductors from A to A' down the right-hand side of the armature will have E.M.F.'s induced in the opposite direction. The E.M.F. per conductor will be a maximum at B and B' and zero at A and A', but it is across these latter two points that the maximum p.d. is set up in the armature due to this cause, and it is at these two points that the brushes are situated. When the current reverses the magnetic field reverses, and thus the back E.M.F. between the brushes is reversed, since the rotation is unchanged. This E.M.F. is therefore permanently opposed to the applied E.M.F., no matter what value the speed may have. The frequency of this back E.M.F. is equal to that of the field and therefore to that of the supply, and is independent of the speed of rotation. Its magnitude, however, depends upon the strength of the field and the rate of cutting lines of force. The magnitude of this E.M.F. is given by the same formula as used for D.C. machines, viz.

$$E_{av} = \frac{n\Phi_{av}N}{60 \times 10^8} \times \frac{p}{q},$$

where E_{av} is the average value of the E.M.F., n is the speed in r.p.m., N is the number of conductors, Φ_{av} is the average value of the flux per pole, p is the number of poles, and q is the number of parallel circuits on the armature. With sinusoidal distribution the average value of the flux per pole, Φ_{av}, is equal to $\frac{2}{\pi}$ times the maximum value of the flux per pole, Φ, and if T be the number of turns in series, then

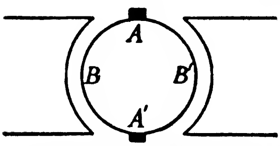


FIG. 345.—To illustrate E.M.F.'s induced in Armature.

$$T = \frac{N}{2q}$$

and

$$E_{av} = \frac{n \times \frac{2}{\pi}\Phi \times 2qT \times p}{60 \times 10^8 \times q}$$

$$= \frac{\frac{4}{\pi} \times n\Phi Tp}{60 \times 10^8}.$$

The R.M.S. value of the voltage is given by

$$E = \frac{\pi}{2\sqrt{2}}E_{av} \text{ (see page 13)},$$

and hence

$$E = \frac{\pi}{2\sqrt{2}} \times \frac{4}{\pi} \times \frac{n\Phi Tp}{60 \times 10^8}$$

$$= \frac{\sqrt{2}n\Phi Tp}{60 \times 10^8} \text{ volts.}$$

This E.M.F. is in phase with the flux which produces it. As will be shown later, the power factor of the motor is less than unity, and consequently the back E.M.F., which is in phase with the flux, and, approximately, the current, is not in phase with the applied terminal voltage.

The second E.M.F. induced in the armature is due to rotation in the armature reaction field (the cross flux), and the axis of this flux is a line joining the brushes. This flux is geometrically at right angles to the main field in a bipolar motor, and consequently the maximum p.d. occurs between the points B and B'. As far as this E.M.F. is concerned, the points A and A' are equipotential, and therefore this E.M.F. has no effect upon the total back E.M.F. induced. Unfortunately, however, the conductors at A and A' are situated so as to have the maximum voltage induced in them, and since the turns in this region are short-circuited by the action of the brushes on the commutator, this results in a tendency to spark.

There are next the E.M.F.'s set up by transformer action. Considering the armature as stationary for the time being, the maximum E.M.F. is induced in those turns which embrace the greatest area in a plane at right angles to the flux. These turns are situated at AA', and consequently this effect will also tend to produce sparking at the brushes. The conductors between which the maximum p.d. exists are situated at BB' (at right angles to AA'), and consequently the cumulative effect of the whole winding is neutralized at the brushes, since the E.M.F. induced in the conductors between B and A is balanced by the E.M.F. induced in the conductors between B and A'; similarly on the other side of the armature. The phase of this induced voltage is 90° behind the flux, since it is proportional to minus the rate of change of flux, and its frequency is the same as that of the flux, which is the frequency of supply. Since the magnitude of this voltage is only dependent upon the rate of change of the flux and the number of turns, it follows that it is independent of the speed of rotation, and consequently the same E.M.F. is induced due to this cause, whether the armature is rotating or not. The ordinary formula for the induced E.M.F. in a transformer is

$$E = \sqrt{2}\pi f \Phi T \times 10^{-8} \text{ volts (see page 177),}$$

but in this case a breadth factor $\frac{2}{\pi}$ must be introduced to account for the fact that all the turns do not link with all the flux. Therefore

$$E = \frac{2}{\pi} \times \sqrt{2}\pi f \Phi T \times 10^{-8}$$

$$= 2\sqrt{2} f \Phi T \times 10^{-8} \text{ volts.}$$

If the field is not uniform, or if the armature winding is differently arranged, the value of the constant $\frac{2}{\pi}$ is affected somewhat. In the case of a multipolar wave wound armature, T is the number of turns in series between the brushes.

The ratio of this transformer E.M.F. to the rotation E.M.F. is

$$\frac{\text{Transformer E.M.F.}}{\text{Rotation E.M.F.}} = \frac{2\sqrt{2}f\Phi T \times 10^{-8}}{\frac{\sqrt{2}n\Phi Tp}{60 \times 10^{8}}} = \frac{120f}{np}.$$

But the synchronous speed is

$$n_s = \frac{120f}{p}.$$

Therefore

$$\frac{\text{Transformer E.M.F.}}{\text{Rotation E.M.F.}} = \frac{n_s}{n} = \frac{\text{Synchronous speed}}{\text{Actual speed}}.$$

The fourth E.M.F. induced in the armature is one due to transformer action set up by the armature reaction cross flux. This is produced in the same way as the transformer E.M.F. due to the main field, except that it acts along an axis AA' instead of BB', since the armature reaction cross flux is at right angles to the main field. The phase of this induced E.M.F. is again 90° behind the flux, and requires the application of an E.M.F. at the brushes 90° ahead of the flux to overcome it. This is only another way of saying that the armature possesses reactance, since the E.M.F. induced is an E.M.F. of self-induction, and the voltage required to overcome it, together with the E.M.F. required to overcome the resistance of the armature and brushes, forms the impedance voltage of the armature.

Torque.—The torque developed depends upon the instantaneous product of the armature current and the field. The field is proportional to the current, neglecting the effect of change of permeability, so that the torque is nearly proportional to the square of the current. When the current reverses in direction the field does so at the same instant, since they are practically in phase with each other, and so the torque is always developed in the same direction. Owing to the variation in the strength of the current, the value of the torque is constantly varying between zero and a maximum value at a frequency equal to double that of the current, so that although the torque is uni-directional in character it is pulsating in magnitude.

Speed.—Assuming for the moment that the power factor and efficiency remain constant for all loads, then the output is proportional to the current for a constant applied voltage. But since the torque is proportional to the square of the current, the speed, which is proportional to $\frac{\text{output}}{\text{torque}}$, is inversely proportional to the current and to the output. Actually this proportionality does not hold good exactly, but it is sufficient to show that the speed decreases considerably with increase of load and that the speed-load characteristic is of the same general shape as in the case of the D.C. series motor. In general it is called a series characteristic, to distinguish it from that of the D.C. shunt motor, known as a shunt characteristic.

Vector Diagram.—In drawing the vector diagram of the series motor, it is convenient to start with the flux vector. In Fig. 346, Φ represents the main magnetic flux entering the armature and

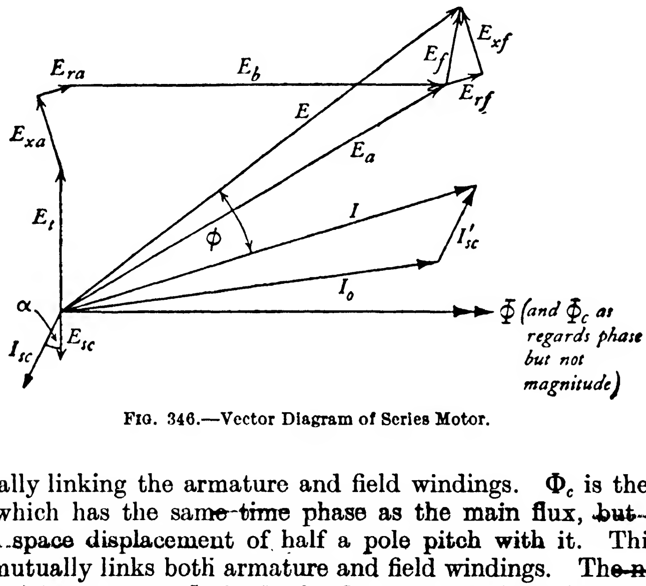


FIG. 346.—Vector Diagram of Series Motor.

mutually linking the armature and field windings. Φ_c is the cross flux which has the same time phase as the main flux, but which has a space displacement of half a pole pitch with it. This flux also mutually links both armature and field windings. The no-load magnetizing current, I_0, leads the flux by a small angle dependent upon the hysteresis loss and the eddy current loss. This is relatively much larger than in the case of a transformer since there is a relatively small number of turns in the exciting circuit, and on account of the two air-gaps in the magnetic circuit. Another component of the current is introduced on account of the current flowing through the coils short-circuited by the brushes on the commutator. These coils have E.M.F.'s induced in them by the transformer action of the main flux. The resultant of these E.M.F.'s is represented by E_{sc}, lagging behind the flux by 90°. Since these coils possess both resistance and reactance, the short-circuit current, I_{sc}, flowing through them, lags behind this E.M.F.

by some angle α. To balance the ampere-turns set up in this way, an equivalent number must be supplied by the main armature current, exactly opposite in phase. The neutralizing component which effects this is I'_{sc}. On adding this to I_o, vectorially, the total armature current, I_r is obtained.

The total voltage across the armature is the resultant of several components. There is an E.M.F. induced between the brushes, due to the transformer effect in the mutual cross flux, Φ_c. This E.M.F. demands the application of an equal and opposite E.M.F., E_t, leading the cross flux, Φ_c, by 90°. Another voltage, E_{xa}, must also be applied to neutralize the E.M.F. induced in the armature due to self-inductance associated with leakage flux, *i.e.* flux linking with the armature but not with the field winding. This E.M.F. is in quadrature with the current, I. There is a small IR drop, E_{ra}, on account of the resistance of the armature winding and brushes in phase with the current, I; and finally there is the E.M.F., E_b, applied to the armature winding to neutralize the back E.M.F. induced by rotation in the main flux, Φ. This is in phase with Φ, and is relatively large compared with the other components. The resultant of these four components gives E_a, which is the total voltage applied to the armature.

The field winding has a relatively small IR drop, E_{rf}, in phase with the main current, and a larger reactive drop, E_{xf}, due to the reactance of the field winding, this component leading the current by 90°. The resultant of these two components gives E_f, the total voltage applied to the field winding.

The total voltage applied to the motor terminals, E, is the vector sum of E_a and E_f. The angle of lag of the total current behind the applied voltage is given by ϕ, and cos ϕ is the power factor of the motor.

It is thus seen that for a high power factor, the reactances should be small and the rotation E.M.F. large. The latter is obtained by running the motor at a high speed, above that of synchronism. The motors previously dealt with have not been able to run at hypersynchronous speeds, but there is no fundamental reason why this should be impossible in the present case.

At the moment of starting the rotation E.M.F. is zero, and this advances the phase of the terminal volts with respect to the current, thus making the power factor worse. The power factor gradually improves as the speed increases. For heavy loads, the power factor comes down again, since the speed falls as the load goes up.

Circle Diagram.—Considering the motor to have a constant voltage applied to its terminals, this voltage can be split up approximately into two main components 90° out of phase with each other. The component in phase with the current consists of the resistance drops in the armature and field windings and the rotation voltage. The latter is not strictly in phase with the current, as shown in the vector diagram, but is assumed so for the purposes of the simple circle diagram. The voltage in quadrature with the current consists of the reactance drops in the armature and field windings. This again is only an approximation, since the vector diagram showed the voltage overcoming the transformer effect of the armature 90° out of phase with the flux instead of the current. These assumptions are made, however, in order to split up the constant applied voltage into two components at right angles

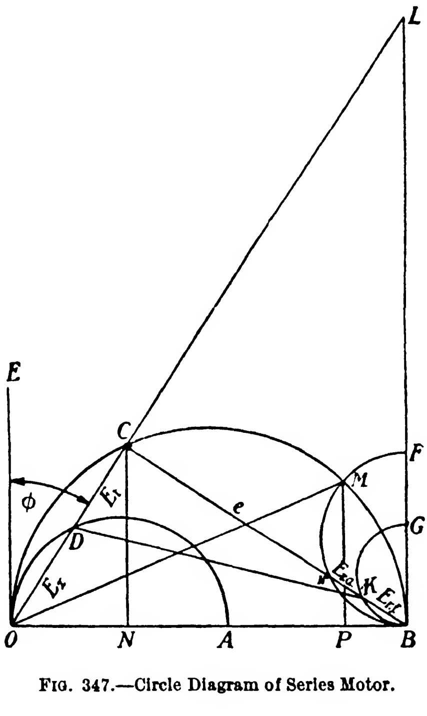


FIG. 347.—Circle Diagram of Series Motor.

to each other. In Fig. 347, if *OB* represents the constant applied voltage, *OC* represents the voltage overcoming the total reactance of the motor and *CB* represents the total voltage in phase with the current, and since *OB* is constant and the angle *OCB* is a right angle, it follows that the point *C* moves in a semicircle erected on *OB* as a diameter (see Fig. 44). Since the reactance of the motor may be assumed constant, the current is proportional to the voltage across the reactance of the motor, and is represented to scale by *OC*. The phase of the current, however, is represented by *CB* and the angle of lag is given by the angle *CBO*, which is equal to the angle EOC. The power factor, therefore, is given by $\frac{CB}{OB}$ or cos EOC. OB is now divided at A, so that

$$\frac{OA}{AB} = \frac{\text{reactance of field}}{\text{reactance of armature}} = \frac{E_x}{E_l} \text{ (Fig. 346)},$$

and another semicircle is erected on OA as a diameter. Now wherever the point C may be, OC is divided by this second semicircle at D in the same ratio as OB is divided at A, since the triangles OBC and OAD are similar. (The line AD is omitted in the diagram for the sake of clearness.) OD therefore represents the voltage drop in the reactance of the field and DC the voltage drop in the reactance of the armature. As in Fig. 49 (which see), a vertical line is erected at B and a length BF is measured off along it such that

$$\frac{BF}{OB} = \frac{\text{Total resistance}}{\text{Total reactance}} = \frac{E_{rf} + E_{ra}}{E_x + E_l}.$$

Another semicircle is now erected on BF as a diameter and this is cut by the line CB at H. Now the angle HBF equals the angle COB, and therefore

$$\frac{BH}{BF} = \frac{OC}{OB}$$

and $$BH = OC \times \frac{BF}{OB}$$

$$= \text{Total reactance voltage drop} \times \frac{\text{Total resistance}}{\text{Total reactance}}$$

$$= \text{Total reactance} \times \text{Current} \times \frac{\text{Total resistance}}{\text{Total reactance}}$$

$$= \text{Current} \times \text{Total resistance}$$

$$= \text{Total resistance voltage drop.}$$

The line BF is now divided at G, and another semicircle erected on BG as a diameter, in the same way that OB was divided at A. In the present instance $\frac{BG}{GF}$ is made equal to $\frac{\text{Resistance of field}}{\text{Resistance of armature}}$, and the line CB is now split up at H and K, so that BK represents the voltage drop due to the resistance of the field, E_{rf}, and KH represents the voltage drop due to the resistance of the armature, E_{ra}. The remaining part of the voltage in phase with the current, HC, must therefore represent the voltage, e, induced in the armature due to its rotation in the magnetic field. The voltage drop across the brushes on the armature is given by the vector sum of e, E_{ra}, and E_t, and is represented by DK in the circle diagram. Since e is proportional to the flux × speed, the speed is proportional to $\frac{e}{\text{flux}}$ or $\frac{e}{\text{current}}$, assuming the flux to be proportional to the current. In the circle diagram, therefore, the speed is represented by $\frac{CH}{OC}$. Let OC and BF be produced until they meet at L. Now $OC =$

$$OB \cos COB = OB \cos CBL$$
$$= OB \times \frac{CB}{BL} = OB \times \frac{CH}{FL}.$$

Therefore the speed is proportional to

$$\frac{CH}{OC} = \frac{CH}{OB \times \frac{CH}{FL}} = \frac{FL}{OB},$$

and since OB is constant, the speed is represented to scale by FL.

As in the case of the induction motor, the input is measured by CN to a scale of watts. The copper losses can be obtained by drawing vertical lines from the points H and K, the vertical height of H representing the total copper loss and the vertical height of K the copper loss in the field only. The output (neglecting iron and friction loss) is thus obtained by subtracting the vertical height of H above the base line from the input CN, from which the electrical efficiency can be obtained. The torque is proportional to the instantaneous product of current and flux, and, neglecting the phase difference between these two quantities and also the effect of magnetic saturation, the torque may be taken as being approximately proportional to the (current)^2. Now in the circle diagram

$$\cos COB = \frac{OC}{OB} = \frac{ON}{OC}$$

and therefore

$$OC^2 = ON \times OB.$$

The torque being proportional to OC^2 and OB being constant, it follows that the torque is represented to scale by ON.

At the moment of starting e is zero, and if the full voltage be applied the starting current will be represented by OM and the starting torque by OP.

Power Factor.—From the circle diagram in Fig. 347 it is seen that the power factor is better on light loads than on heavy loads, since the vector OC is swung round to the right as the current increases, thus increasing the angle of lag. When the point C reaches the top of the semicircle, the power input has reached a maximum and the power factor has dropped to $\frac{1}{\sqrt{2}} = 0{\cdot}707$. Further increase of load reduces the power factor still more, so that when M is reached the power factor is at its worst. This, however, is the condition at starting when the back induced voltage, e, is zero, so that when the motor is first switched into circuit the power factor is very poor, but it gradually improves as the motor gains speed. Also, the higher the speed the greater does e become, thus improving the power factor. If reference be made to the vector diagram in Fig. 346, which is rather more accurate than the circle diagram, it will be seen that if e be made sufficiently large it is possible for the motor to draw a leading current from the supply. This is due to the action of the short-circuited coils under the brushes, which set up a number of ampere-turns and cause a phase displacement between the flux and the main armature current. It is not desirable to magnify this effect deliberately, however, although it

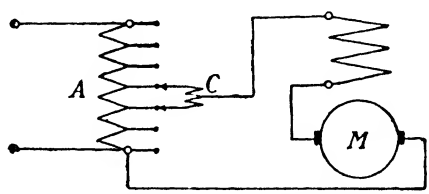


FIG. 348.—Auto-transformer for starting Series Motor.

would improve the power factor, since it would do it at the expense of the commutation and the efficiency.

The best way of obtaining a high power factor is to neutralize the armature reactance (which will be dealt with in the case of the compensated series motor) or to reduce the working field strength. This reduces the reactance of the field winding and brings about the high speeds which are desirable.

It is also seen that a low frequency is advantageous, since this results in relatively low reactances and high power factors.

Full load power factors of 0·8 to 0·95 are met with in practice.

Starting.—The usual starting device for a single-phase series motor consists of an auto-transformer with a number of tappings on the winding, so as to enable the voltage across the motor terminals to be gradually raised. This arrangement allows a reduced voltage to be applied to the motor and also relieves the line to a large extent of the excessive current which the motor is allowed to take at the moment of switching on. The connections are shown in Fig. 348, where M represents the motor, A the auto-transformer, and C a choking coil known as a *preventive coil* or *mid-point reactor*. One terminal of the motor is connected to one end of the auto-transformer and the other to the middle point of the choking coil. The two ends of the choking coil are connected to two moving fingers which in some positions lie on the same contact and in others bridge over two contacts. When both fingers are on the same contact, the currents flow in opposite directions in the two halves of the choking coil, thus rendering it practically non-inductive, so that the drop in volts is inappreciable. When bridging two contacts, the impedance of the choking coil is sufficient to prevent the short-circuiting of the particular section of the auto-transformer concerned. The choking coil thus appears non-inductive to the motor circuit and yet acts like a highly inductive resistance to the local circuit across the contacts.

Compensated Series Motor.—One of the methods of improving the power factor of the series motor consists in counteracting the leakage reactance of the armature. The latter tends to set up a distorting field, the axis of which is a line drawn through the brushes in a

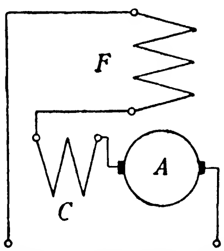


FIG. 349.—Compensated Series Motor.

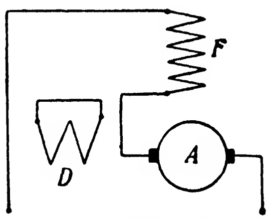


FIG. 350.—Compensated Series Motor with Damping Coil.

bipolar case. These cross ampere-turns can be neutralized by means of an auxiliary field winding placed midway between the main field windings in much the same way that interpoles neutralize the cross ampere-turns of the armature in a D.C. motor. The general arrangement is shown in Fig. 349, where *A* represents the armature, *F* the main field winding, and *C* the compensating winding. This compensating winding is arranged to set up as many ampere-turns as the armature, and since it acts in direct opposition it practically eliminates the leakage cross flux and renders the armature non-inductive. The addition of the compensating winding also suppresses to a large extent the rotation E.M.F. which is induced by the armature cross flux in the short-circuited coils under the brushes. The compensating winding, therefore, has a beneficial effect on the commutation as well as effecting an improvement in the power factor.

An alternative arrangement to that shown in Fig. 349 consists in short-circuiting the compensating winding on itself and disconnecting it from the main circuit altogether. The auxiliary winding is now often called a *damping coil*, and the connections of this type of motor are shown in Fig. 350, where D represents the damping coil and A and F have the same significance as before. This produces the same result as the former method of connection, although the action is somewhat different. The damping coil acts like the short-circuited secondary of a transformer the primary of which is formed by the armature winding, whilst the flux linking the two consists of the armature cross leakage flux. The E.M.F. induced in the short-circuited damping coil causes a current to flow which sets up ampere-turns sufficient to neutralize the ampere-turns of the armature, thus suppressing the cross flux and rendering the armature tolerably non-inductive.

The difference between the two methods of compensation lies in the fact that in one case the neutralizing ampere-turns are obtained directly from the main circuit (forced), whilst in the other they are obtained by transformer action from the armature (induced).

Series Motor with Transformer.—As series motors are usually limited by their design to line pressures of about 300 volts, it is necessary to supply a transformer when higher voltages are dealt with. There is, however, no objection to the field winding being connected to the H.T. supply, and so only the armature is supplied from the L.T. side of the transformer, which is connected in series with the field winding as shown in Fig. 351, in which T represents the transformer. This transformer can also be used for starting purposes and for variable speed work by providing the secondary with a number of tappings, so that the increase in the apparatus is not as much as would appear at first sight.

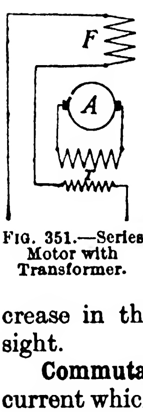


FIG. 351.—Series Motor with Transformer.

Commutation.—In addition to the usual process of reversing the current which takes place in D.C. motors, there are the extra E.M.F.'s induced in the short-circuited coils to be dealt with. This adds to the difficulty of commutation and leads to special features in the design of the commutator. This is made of larger diameter than in the corresponding D.C. motor, so that it more nearly approaches the diameter of the armature. A large number of commutator bars is also essential, the maximum number possible for the particular armature winding being put in. Narrow brushes are also employed, so as to avoid short-circuiting more turns than absolutely necessary at a given instant. Since the current is large at starting, commutation is more difficult at this time and sparking may be anticipated, but as the motor gains speed the current decreases and the commutation improves. As tolerably good commutation can be obtained over a wide range of speeds, it follows that the series motor is suitable for variable speed work. The addition of interpoles also exerts a very great effect on the commutation as in D.C. motors, and they are now always employed.

Interpoles.—In D.C. motors interpoles fulfil a double purpose. They act as a compensating winding for the purpose of neutralizing the armature cross flux, and they also aid commutation by introducing a reverse flux into the armature so as to cut the conductors coming under the brushes. In A.C. motors these two functions are separated. The action of the compensating winding has already been discussed, but it must be remembered that this winding must embrace a complete pole pitch, since the M.M.F. set up by the armature acts over the whole of this region. The commutating winding, on the other hand, is only required to produce a local effect on those conductors undergoing commutation. For this reason, the interpole (or commutating pole) winding is concentrated over a narrow area, and one large tooth is provided on the stator core to receive this winding. The main field winding and the compensating winding are wound in the same sized slots, displaced half a pole pitch with respect to each other, whilst the interpole winding is situated on the centre line of the compensating winding and is wound on a single tooth of larger dimensions than its fellows.

Sphere of Application.—Owing to its variable speed properties, the series motor, in common with the other types of single-phase commutator motors, is particularly adapted to traction work, and for this purpose has been built in sizes up to 3000 B.H.P. For this class of work a single-phase system has obvious advantages over a polyphase system in the transmission and collection of the current, since an earthed return can be adopted and only one live wire need be used. The various types of single-phase commutator motors have to compete, therefore, with the single-phase induction motor and have now practically displaced the latter from the market in this class of work.

Examples

(1) Explain the origin of the various E.M.F.'s acting in the armature of an A.C. series motor.

(2) A single-phase series motor takes a current of 100 amperes at 0·85 power factor. The flux lags behind the applied voltage by 40°. The resistances of the armature and field winding are 0·3 and 0·35 ohm respectively. The corresponding reactances are 0·40 and 0·60 ohm respectively. What are the values of the applied E.M.F. and the induced back E.M.F.?

(3) What is meant by forced and induced neutralization of the armature reaction in an A.C. series motor?

(4) Describe the vector diagram for a single-phase series motor, and show how the neutralizing of the armature reaction flux results in an improvement of the power factor. How is the diagram modified at the moment of starting?

(5) Draw a vector diagram for a single-phase series motor, and explain its construction. Explain the effect of the neutralizing winding on the proportions of this diagram.

Describe two methods of connecting the neutralizing winding.

(6) What are the functions of the various windings on the stator of an A.C. series motor and how are they wound?

CHAPTER XXX

REPULSION MOTORS

Simple Repulsion Motor.—The simple repulsion motor consists of a field system and an armature with a commutator just like a series motor. The difference between the two motors lies in their connections and action. The field system or stator does not possess salient poles, but is built up of toothed laminations as in the series motor, thus producing a uniform air-gap. The field winding is wound in the stator slots, the position of the poles being determined by the winding. The armature possesses an ordinary distributed winding connected to the commutator in the usual way, but the brushes are set at an angle of about 70° from the position of maximum p.d. on the armature. These brushes are short-circuited and are not connected electrically to the main circuit at all. The stator winding producing the main field is connected across the supply. The connections of a simple repulsion motor are represented diagrammatically in Fig. 352.

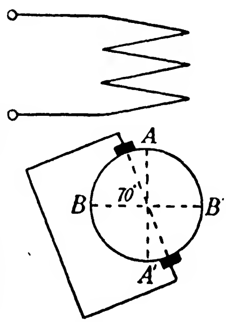


FIG. 352.—Connections of Simple Repulsion Motor.

Theory of Operation.—Considering the motor as a transformer in which the stator winding forms the primary and the rotor winding the secondary, the maximum p.d. is set up between the points AA'. If the brushes be placed on this line the maximum current will flow in the armature, but no torque will be produced, since the axis of the main field coincides with the axis of the flux set up by the armature. To obtain the maximum torque these two fluxes should be at right angles to each other in a bipolar case. The position corresponding to this is along the line BB', but if the brushes be placed in this position they short-circuit two equipotential points on the armature, and thus no armature current is produced. As it is essential that both an armature current should flow and a phase displacement should exist between the axes of the two fluxes, an intermediate position between AA' and BB' is chosen for the brushes. In these circumstances, neither the armature short-circuit current nor the phase displacement is a maximum, but their instantaneous product has a definite value, whilst in the two extreme positions it is zero.

In the series motor, the rotation is set up by the armature current producing poles along the axis of the brushes, these poles being attracted to those of the main field of opposite sign. In the present instance, however, the armature tends to set itself in such a position that it expends the minimum amount of electrical energy, just as a sheet of copper suspended in the field of an alternating electro-magnet endeavours to set itself so as to produce the minimum eddy currents. This means that the armature shown in Fig. 352 tends to rotate in a counter-clockwise direction. Owing to the apparent repelling action instead of the usual attracting action, the motor is called a *repulsion motor*. This action does not take place in the series motor, although the transformer E.M.F. is present, since the brushes are set in the neutral position.

If the brushes had happened to lie on the other side of the vertical line AA', the induced rotation would have been in the opposite direction, so that in order to reverse the direction of rotation, all that is necessary is to move the brush rocker backwards through 40° against the original direction of rotation.

Owing to the fact that the armature is not connected to the main circuit, it is possible to design these motors to work directly on H.T. systems without the help of a transformer, since only the stationary stator winding is connected to the line. In this respect the simple repulsion motor has the advantage over the series motor, but this advantage is lost when the repulsion motor becomes compensated, as will be seen when the connections are studied (see Fig. 357).

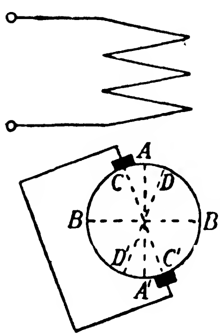


FIG. 353.—To illustrate E.M.F.'s induced in Armature.

E.M.F.'s induced in the Armature.—The flux set up by the stator winding cuts the armature in a vertical direction in Fig. 353. The resulting short-circuit current of the armature sets up a field of its own, the axis of which is the axis of the brushes. This latter field can be resolved into two components at right angles to each other. One of these components may be considered as acting in a vertical direction along the line AA' so as to oppose the main stator flux, whilst the other component produces a horizontal flux along the line BB'. This horizontal component of the armature flux can be regarded, therefore, as a cross flux in quadrature with the now reduced main flux. The armature ampere-turns can also be split up into the two components which produce the back and the cross flux. The ampere-turns producing the back flux are obtained from the conductors lying between C and D' and between D and C', whilst the ampere-turns producing the cross flux are obtained from the conductors lying between C and D and between D' and C'. Imagining the armature winding to be divided in this manner, each portion has two voltages induced in it. The back turns CD' and DC' have one E.M.F. induced in them by the transformer action of the main field and another E.M.F. due to rotation in the cross field. The transformer action of the cross field and the rotation in the main field produce no resultant E.M.F. in these turns. The cross turns CD and $D'C'$ have a transformer E.M.F. induced in them by the cross field and a rotation E.M.F. due to the main field. In these turns the transformer E.M.F. of the main field and the rotation E.M.F. due to the cross field have no resultant value. The transformer E.M.F.'s lag behind their respective fluxes by 90° whilst the rotation E.M.F.'s are in phase with them.

In addition to these E.M.F.'s in the armature, there is also the back E.M.F. induced in the stator winding, due to the transformer effect of the original stator flux.

Torque.—The torque developed by the armature conductors is due to the fact that the armature tends to set itself in such a position that the currents flowing through it are reduced to a minimum, since in this way it reduces its losses to a minimum. In the motor shown in Fig. 352 this torque is acting in a counter-clockwise direction. But the brushes on the commutator short-circuit a number of turns independently of the main short-circuiting lead, and these turns tend to set up a flux of their own at right angles to the plane of their coils, *i.e.* at right angles to the brush axis. This flux interacts with the main stator flux and tends to produce rotation in a *clockwise* direction. This torque therefore opposes the main driving torque, which is weakened by this differential effect.

Atkinson Repulsion Motor.—The stator field can be imagined to be split up into two components at right angles to each other, one of these acting along the axis of the brushes and the other in quadrature with it. The former component is the one producing the armature current by transformer effect, whilst the latter produces that component of the field which may be said to develop the torque. The actual stator winding can thus be replaced by two windings at right angles to each other and connected in series, as shown in Fig. 354, F being called the field winding and T the transformer winding. This modification of the simple repulsion motor is known as the Atkinson repulsion motor. In reality, of course, the two stator windings do not produce separate fluxes, but only one resultant, so that there is no theoretical difference between the simple repulsion motor and the Atkinson motor. One

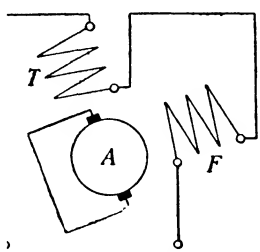


FIG. 354.—Atkinson Repulsion Motor.

advantage possessed by the latter, however, lies in the fact that the direction of rotation can be reversed by reversing one of the stator windings without touching the brush rocker.

The connection between the repulsion and the series motor can also be explained by a consideration of Fig. 354. The power which ultimately appears as useful output is transferred to the rotor from the stator winding, T. If this power is supplied directly from the stator, by conducting the current from the winding F to the armature brushes, and thence to the machine terminal, the short-circuit on the armature being removed at the same time, the winding T is eliminated, and the motor becomes a series motor having exactly the same connections as shown in Fig. 344.

Vector Diagram.—The approximate vector diagram can be most easily studied by referring to the Atkinson modification on account of the splitting up of the stator flux, but it also applies to the simple repulsion motor, since there is no theoretical difference between

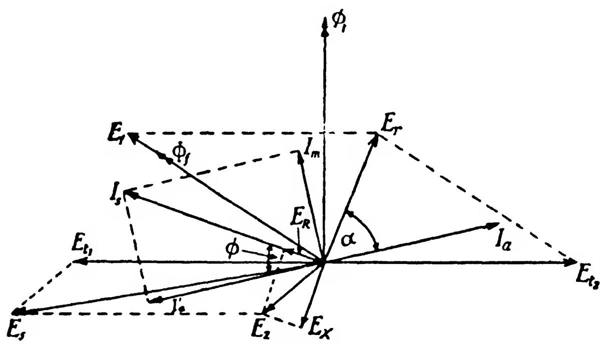


FIG. 355.—Vector Diagram of Repulsion Motor.

the two. The transformer winding, T, in Fig. 354 produces a flux parallel to the brush axis, which is represented by Φ_t in the vector diagram in Fig. 355. This induces a voltage, E_{t_2}, in the armature by transformer action, this voltage lagging behind Φ_t by 90°. The stator winding T, acting as the primary of the transformer, has a corresponding voltage induced in it, this being counterbalanced by the application of an equal and opposite voltage, E_1, this voltage leading the flux by 90°. Now the stator current lags behind this E.M.F. by a small angle, and this stator current has to produce the flux Φ_f when flowing through the field winding, F. Owing to the hysteresis and eddy current loss, however, the stator current leads this flux by a further small angle, since it includes a small active component, with the result that the flux Φ_f is not in quadrature with the flux Φ_t, but leads it by rather less than 90°. The armature now has a second E.M.F. induced in it, due to its rotation in the flux Φ_f. This E.M.F. is in phase with the flux Φ_f, and is represented by E_f. The resultant armature E.M.F. acting along the line of the brushes is the vector sum of E_{t_2} and E_f, and is represented by E_r. The secondary or armature current, I_a, lags behind this resultant voltage, E_r, by a definite angle, α, which depends upon the relative magnitudes of the armature resistance and leakage reactance. The primary or stator current, I_s, consists of two components. The first is the magnetizing current necessary to produce the flux Φ_t. In an ideal case this lags behind the voltage, E_{t_1}, by 90° and is in phase with Φ_t, but owing to the presence of hysteresis and eddy current losses this current leads the flux by a small angle and is represented by I_m. The second component of the stator current is that required to balance the ampere-turns of the secondary or armature circuit and is exactly opposite in phase to I_a. This component is represented by I'_a. The resultant stator current, I_s, is obtained by adding vectorially the two components I_m and I'_a. At first sight it may appear as if another magnetizing component of the stator current is required for the field winding, F, but this is not so, since the phase of the current is settled by the above two components and the field winding, F, has to take whatever current the transformer winding permits. It is seen from the vector diagram that the flux Φ_f lags behind the current, I_s, by a small angle, this being due to the iron losses caused by this flux. Although the field winding, F, does not introduce any additional magnetizing current, yet it brings about a certain voltage drop in the stator, and this will be dealt with next. The resistance of the complete stator circuit brings about a voltage drop, E_R, in phase with the current, I_s, and the reactance of the field winding, F, brings about another voltage drop, E_X, leading the stator current, I_s, by 90°. The voltage associated with the flux, Φ_t, in the transformer winding, T, has already been dealt with and is represented by E_{t_1}. Combining E_R and E_X, the impedance voltage, E_Z, is obtained, and combining this with E_{t_1} the total applied stator voltage, E_s, is obtained. The power factor of the motor is given by cos ϕ, where ϕ is the angle of phase difference between the applied stator voltage, E_s, and the stator current, I_s.

Supply of Energy to Rotor.—The energy which appears in the rotor is transmitted to it by transformer action across the air-gap. This comes from the transformer winding T and not from F. The latter acts in much the same manner as the field winding of a D.C. motor, the only power associated with this winding being that necessary to supply its own iron losses.

Starting.—Repulsion motors can be started up by means of a starter like that used for a D.C. series motor, or they may be started up by means of an auto-transformer. The latter method is superior, inasmuch as it relieves the line to a certain extent of the excess current which flows during the starting period.

When the motor is first switched into circuit the rotation E.M.F.,

E_f (Fig. 355), is zero. The resultant E.M.F., E_r, acting in the rotor circuit is now coincident with E_{t_2}. This has the effect of retarding the rotor current in phase considerably, which in its turn causes the stator current to lag behind the stator voltage by a larger angle, thus reducing the power factor. As the rotor gains speed the rotation E.M.F. is gradually increased and the rotor current is gradually advanced in phase. This reacts on the stator circuit so that the power factor improves as the speed increases.

Owing to the very large current taken at starting, the starting torque is high, notwithstanding the poor power factor. This can be seen from Fig. 355, for the torque is proportional to the instantaneous product of the rotor current and the resultant flux. The phase of the resultant flux lies in between that of Φ_t and Φ_f, and at the moment of starting the rotor current is very nearly in phase opposition. As the rotor current is also large at starting, the resulting torque is very great. As the motor gains speed the rotor current swings round in phase to the position shown in the vector diagram, and this produces a drop in the torque, apart from the reduction in the actual magnitude of the current.

Commutation.—The commutator of a repulsion motor is characterized by its relatively large diameter and high number of bars, just as is the series motor. The brushes also are made very narrow. In the short-circuited coils there are two distinct E.M.F.'s induced in addition to the usual reactance voltage due to the reversal of the current. The two E.M.F.'s are due to the transformer action of the field winding, F (Fig. 354), and the generator action of the transformer winding, T. The transformer E.M.F. lags behind the flux due to F by 90°, whilst the rotation voltage is 180° out of phase with the flux due to T. Since the two fluxes are very nearly in quadrature, it follows that the two E.M.F.'s are very nearly in phase opposition and tend to neutralize each other. This neutralization is most complete at the synchronous speed, so that the commutation is best at speeds in this neighbourhood.

Déri Motor.—The torque developed is influenced by the position of the brushes, so that speed regulation can be obtained by moving the brushes. With the ordinary repulsion motor, however, a small movement of the brushes produces a large variation in the torque. In addition to this, the armature coils are subjected to an excessive short-circuit current when the brushes are in the extreme position

In the Déri type of repulsion motor these objections are overcome by employing a double set of brushes as shown in Fig. 356. One set is fixed permanently in position, whilst the other set is capable of movement round the commutator. The number of brush spindles is now equal to twice the number of poles. The fixed brush spindles are set along the axis of the single stator winding, and each of these is connected to one of the movable brush spindles. The resultant axis of the armature M.M.F.'s is now that indicated by the dotted line, and a given movement of the brushes results in a much smaller displacement of this resultant axis than when single brush spindles are employed. In this way a much finer speed control is obtained.

Compensated Repulsion Motor.—In the Atkinson repulsion motor, the stator winding was split up into two components, one acting along the line of the brushes and the other at right angles to it. The flux due to the latter component can be produced directly from the armature, by transformer action, without the aid of this second winding, which can be done away with altogether. The flux due to the winding, *F*, in Fig. 354 is proportional to the stator current. If this stator current were passed through the armature by means of two additional sets of brushes on the commutator arranged along the axis of the winding *F*, the resulting armature ampere-turns set up along this axis would act in the same direction as the ampere-turns of the stator winding *F*. The diagram of connections of this new variation, which is called the *com-*

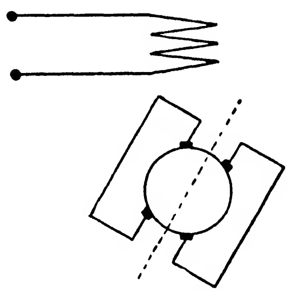


FIG. 356.—Déri Motor.

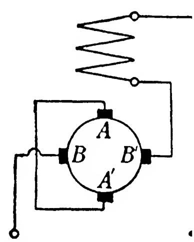


FIG. 357.—Compensated Repulsion Motor.

pensated repulsion motor, is shown in Fig. 357. An important point to notice is that there are four brush spindles, although it is only a two-pole machine. One pair of brushes, *AA′*, is set along the axis of the single stator winding. These brushes are short-circuited and carry what is called the *armature* or the *short-circuit current*. The other pair of brushes, *BB′*, is set along a line at right angles to the first pair and is connected in series with the stator winding. The current passed through these brushes is the stator, or *exciting current*, as it is termed. This current, by passing through the armature, sets up a number of ampere-turns which produce a flux in the direction *BB′*. This flux combines with the flux set up by the stator winding directly, producing a resultant flux which acts along an axis in between *AA′* and *BB′*. If the motor be regarded in this manner, it is seen to be the equivalent in principle of the simple repulsion motor.

One result of this modification is that the second stator winding of the Atkinson repulsion motor is done away with, and as this winding produced the main field flux, its removal leads to the elimination of the reactance associated with it. In the simple repulsion motor, the one stator winding fulfilled the same functions as both the stator windings of the Atkinson repulsion motor, so that the stator reactance of this motor is reduced as well. The effect on the behaviour of the motor can be studied by means of the vector diagram in Fig. 355. The voltage drop due to the reactance of the field winding, F, is there represented by E_X, and if this component be eliminated the stator voltage, E_s, is brought more nearly into phase with the stator current, I_s. It is thus seen that the compensation of the motor leads to an improvement in the power factor which is maintained at a high value over a wide range of hypersynchronous speeds.

If the brushes AA' be removed, it is seen that the motor changes into a simple series motor, but this alteration causes a profound change in its working, since the two motors are quite different in their action.

Winter-Eichberg Motor.[1]—One of the advantages possessed by the simple repulsion motor over the series motor lay in the fact

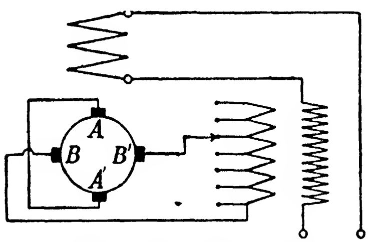


FIG. 358.—Winter-Eichberg Motor.

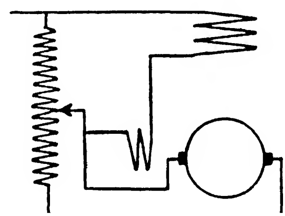


FIG. 359.—Series-Repulsion Motor.

that the armature was not connected to the stator winding, which could therefore be wound for high voltages. This advantage is lost in the compensated repulsion motor, since the armature is connected in series with the stator across the brushes BB'. The improvement in the power factor is thus obtained at the expense of limiting the voltage on which the motor may be used. If the armature brushes, BB', are fed from the secondary of a transformer, however, this advantage may be regained, and this is the arrangement adopted in the Winter-Eichberg motor which is illustrated diagrammatically in Fig. 358. By providing the transformer with a number of tappings it may be used for starting the motor in addition to its normal use, the stator being switched straight on to the supply.

Both the stator winding and the transformer supplying the armature have good power factors, since they act like loaded transformers, so that the resulting power factor of the motor is very good.

[1] This motor is sometimes termed the Latour-Winter-Eichberg motor, since it was invented by Latour, working independently of Winter and Eichberg, about the same time.

In common with the simple repulsion motor, its power factor is poor at the moment of starting, but this improves rapidly as the motor gains speed.

Series-Repulsion (or Doubly Fed) Motor.—In the series motor, power is fed directly into the rotating element, whilst in the repulsion motor the power is supplied to the stator and is transferred to the rotor by inductive action. In the series-repulsion motor useful power is fed into both stator and rotor. The connections are shown in Fig. 359. As the rotor is in series with the upper stator winding, there is a series motor action, but the rotor may also be considered as operating on a closed circuit of its own, and the combination of the two stator windings enables it to operate as a repulsion motor.

Repulsion-Start Single-Phase Induction Motor.—This motor is constructed like a repulsion motor, having an armature, commutator, and brushes which are short-circuited in the normal way. During the starting period the brushes are in contact with the commutator and the motor develops a large starting torque as a repulsion motor. When a certain speed is attained a centrifugal governor operates a short-circuiting device which short-circuits the whole of the commutator, at the same time removing the brushes from the commutator, thus eliminating a large amount of wear. This converts the motor into a single-phase induction motor, and as such it continues to run.

Examples

(1) How does change of brush position affect the performance of a repulsion motor?

(2) What are the advantages gained by employing four sets of brushes on the armature of a bipolar machine, as in the case of the Déri motor?

(3) Explain the principle of action of a compensated repulsion motor suitable for use on a high voltage.

(4) Draw a diagram of connections for a compensated repulsion motor with a variable ratio auto-transformer for starting purposes. Explain the action of the motor.

Show that such a motor can be used directly on a high-voltage supply.

(5) Draw the vector diagram for a single-phase repulsion motor. Explain how this diagram is derived, and show, by its aid, why the motor develops a large starting torque.

CHAPTER XXXI

SINGLE-PHASE SHUNT MOTORS

Simple Shunt Motor.—Both the series and the repulsion motor have series characteristics, and for certain operations a shunt characteristic is desirable. A simple shunt motor, with a laminated frame, however, has no commercial value for several reasons. Owing to the many turns on the field winding, this circuit is very inductive, giving rise to an extremely low power factor. The large phase angle between the armature and field currents results in a greatly reduced torque and a low weight efficiency. Sparking at the commutator is also troublesome. The operation is greatly improved if a two-phase supply is available, the field being supplied from the leading phase. but in that event a two-phase induction motor is preferable.

Shunt-induction Motor.—The connections of this motor are represented in Fig. 360, the phase angle trouble now being overcome. The stator winding sets up a flux lagging by 90° behind the stator E.M.F. and, assuming rotation, there is an E.M.F. induced in the rotor across the commutator brushes, XX, in phase with the flux. These brushes being short-circuited, the resulting current, called the exciting current, lags by an angle approximating to 90°, so that the rotor flux due to this is almost in phase opposition with the supply voltage. (In the simple shunt motor the flux lags behind the supply voltage by a very large angle.) The working armature current is induced by transformer action and flows between the second pair of short-circuited brushes, YY, lying along the axis of the stator flux. This current is approximately in phase opposition with the stator applied E.M.F., and therefore in phase with the rotor flux due to the exciting current along the axis, XX, and the torque is due to the interaction of these two. As the exciting current is proportional to the speed, the torque is zero at standstill, so that the motor is not self-starting. It must, therefore, be started either as a series or as a repulsion

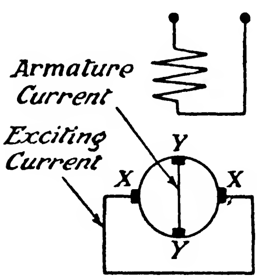


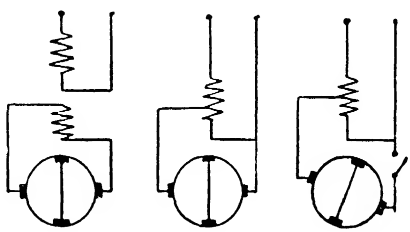
FIG. 360.—Shunt induction Motor

motor. The magnitude of the exciting flux is, however, independent of the armature current and the load, so that the motor has a shunt characteristic.

With a lap winding there are two sets of brushes per pole, but with a wave winding there need not ever be more than four sets.

This motor has the same characteristics as the single-phase induction motor, viz. a relatively constant speed, low power factor and no starting torque. The efficiency is less since there are additional brush and commutator losses.

Compensated Shunt-induction Motor.—In order to improve the power factor, the phase of the exciting current must be advanced, or rather it must be prevented from falling behind. The stator E.M.F. leads the E.M.F. induced between XX, so that phase compensation can be provided by injecting into the rotor circuit an E.M.F. having the phase of the stator E.M.F. This is done in Fig. 361 (a), which shows a secondary winding on the stator connected to the exciting brushes, the main stator winding acting as



(a) (b) (c)

FIG. 361.—Compensated Shunt-induction Motor.

the primary of a transformer in order to provide the necessary E.M.F. In Fig. 361 (b) the exciting brushes are connected directly across a section of the stator winding, which now acts as an autotransformer.

In order to make the motor self-starting, the brushes are tilted by 15° and an automatic switch is placed in the rotor exciting circuit as shown in Fig. 361 (c). With this switch open the motor starts as a repulsion motor. When a certain speed is attained the switch can be closed either by hand or automatically by means of a centrifugal device.

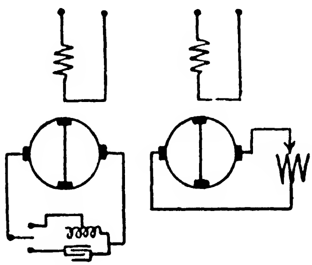
If this switch is omitted, the flux producing acceleration is materially reduced and the starting torque is less. The motor is now known as a repulsion-induction motor.

In one design the commutator is completely short-circuited by a centrifugal device when running, the motor then operating as a single-phase induction motor (see page 507).

Instead of tilting the brushes the same result can be attained by providing an additional winding on the stator in space quadrature with the main winding as in the Atkinson repulsion motor (see Fig. 354). Starting can now be effected by controlling the magnitude of the main armature current.

Speed Regulation by Field Control.—The working field is controlled by the strength of the exciting current. By adding inductance in this circuit, the exciting current is reduced, thus raising the speed, and by adding capacitance the exciting current is strengthened, thus lowering the speed. The connections are shown in Fig. 362 (*a*).

As an alternative the field control can be obtained by the aid of an auxiliary winding on the stator energized from the armature as shown in Fig. 362 (*b*). The armature acts as the primary of a transformer and the auxiliary winding as the secondary, and the E.M.F. of the latter modifies the value of the exciting current. This



(*a*) (*b*)

FIG. 362.—Speed Regulation by Field Control.

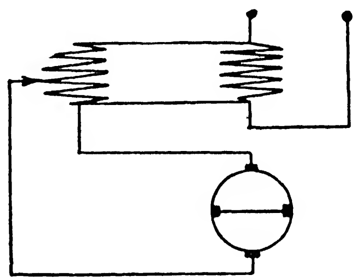


FIG. 363.—Speed Regulation by Voltage Control.

auxiliary winding may also be used for starting purposes by disconnecting it and re-connecting it in the main stator circuit. The exciting brushes now being open-circuited, the motor starts as a repulsion motor.

Speed Regulation by Voltage Control.—Instead of varying the field strength, the desired speed regulation may be obtained by controlling the main armature current. To do this, the voltage across the main brushes is regulated by the introduction of an additional E.M.F. as shown in Fig. 363. The motor is now a type of doubly-fed motor, energy being fed into the rotor both directly and inductively. This particular variation is known as the series-repulsion shunt motor.

EXAMPLES

(1) Tabulate, by diagrammatic sketches, the chief types of single-phase commutator motors, with brief notes relating to each.

(2) What are the means adopted for starting shunt-induction motors, and how is speed regulation effected?

CHAPTER XXXII

THREE-PHASE COMMUTATOR MOTORS

Rotating Field in Commutator Motor.—The polyphase winding on the stator sets up a synchronous rotating field, and this is acted upon by another rotating field due to the rotor. The frequency of the E.M.F.'s induced in the rotor is proportional to the rate at which the rotor conductors cut the flux, *i.e.* the difference between the synchronous speed of the rotating field and the speed of the rotor, or the speed of slip. In general this slip may be either positive or negative, so that the speed is not confined to sub-synchronous values. The action of the commutator adds to this frequency a further number of cycles per second which is proportional to the speed. The resultant frequency at the brushes is equal to the sum of these two, or,

Stator frequency = Frequency of slip + Frequency of speed,

just as in the case of the induction motor. The rotating field due to the rotor rotates at slip frequency with respect to the rotor and, considering the actual speed of the latter, at synchronous speed with respect to the stator. The M.M.F.'s due to the stator and rotor rotating with the same speed and having a space displacement with respect to each other, there is a torque set up the magnitude of which depends upon the ampere-turns in the two windings and upon the space angle referred to above. The direction of this torque is independent of the direction of rotation of the rotating field; it only depends upon whether the rotor pole is to the right or to the left of the stator pole. It is, however, usual for the motor to rotate in the same direction as the rotating field, as better operating characteristics are then obtained.

Three-phase commutator motors can be designed with either series or shunt characteristics, and variable speeds can be obtained without great loss of efficiency.

Three-Phase Series Motor.—Each one of the three stator phase windings is connected in series between the commutator brushes and the supply, as shown in Fig. 364. In order to enable the motor to be used on high voltages a transformer is interposed between the supply and the stator [see Fig. 364 (*a*)]. This transformer must be rated for the full power of the motor. An alternative method of connection is to place the transformer between the stator and rotor winding [see Fig. 364 (*b*)]. This is permissible since the stator may be wound for high voltages, whereas the rotor may not. The transformer is now much reduced in size, since it only deals with the portion of the power which is fed directly into the rotor. A large portion of the power is fed into the rotor inductively across the air-gap, and this power does not pass through the transformer. On account of commutating conditions, the transformer is necessary for all voltages higher than about 200 volts.

The space displacement angle between the stator and rotor M.M.F.'s can be varied throughout 180° by altering the position of the brushes, and as it is the resultant of these M.M.F.'s that produces the flux, this will therefore vary with the brush position. The flux also depends upon the current flowing through the windings.

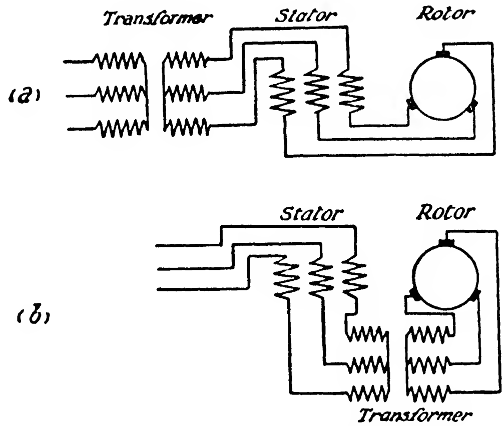


FIG. 364.—Three-Phase Series Motor.

The rotating field induces a back E.M.F. in the stator winding proportional to the stator frequency and the flux. There is also a back E.M.F. induced in the rotor winding proportional to the frequency of slip and the flux. This rotor back E.M.F. is zero at synchronous speed and a maximum at standstill. In addition to these E.M.F.'s there is also an impedance voltage which is approximately proportional to the current. The flux being determined by the brush position and the current, the back E.M.F. is determined by the same factors, so that given values of these correspond to one definite constant speed. With a given current the speed can be altered by change of brush position.

Light Load Conditions.—On low loads the current and the flux are weak, so that the speed rises to a high value in order to generate the necessary back E.M.F. There is now a relatively small voltage drop across the stator and a correspondingly large one across the rotor, and in order to prevent racing on light loads the transformer supplying the rotor is designed so that it becomes saturated when it supplies this large voltage. In consequence the magnetizing current of this transformer is greatly increased, and as this current circulates through the stator windings, it increases the flux and thus prevents the speed from attaining a dangerously high value. The no-load speed is usually designed to be about 50 per cent. above synchronism, and as the motor becomes somewhat unstable in its operation below a speed of 50 per cent. of synchronous speed, a speed range of 3 : 1 is obtained. The speed-torque curve is like that of an induction motor with added rotor resistance, except that hypersynchronous speeds are reached on the low loads.

The stability of this motor can be improved by connecting it in delta for the high speeds and in star for the lower ones.

Series Motor with Fixed and Movable Brushes.—Two sets of brushes are now employed, one fixed and one movable as shown in Fig. 365. Each movable brush is connected to the other end of the transformer winding to which the corresponding fixed brush is connected. If the two sets of brushes touch the same commutator bars, the transformer secondaries are short-circuited, and no rotation ensues. If the movable brushes are rotated through one-third of a pole pitch, the transformer secondaries are connected in simple delta. Speed regulation is obtained by brush movement.

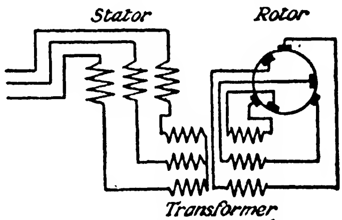


FIG. 365.—Series Motor with Fixed and Movable Brushes.

Three-Phase Shunt Motor.—The stator is made like that of an induction motor and is connected to the supply, whilst the rotor is made like a D.C. armature and is connected through a transformer, also to the supply [see Fig. 366 (*a*)]. Tappings on this transformer enable the armature voltage to be varied at will. Alternatively, an induction regulator may be used instead of the transformer, voltage variation being obtained by movement of the rotor. The stator winding itself may be used as the transformer, the armature being supplied from tappings on this, as shown in Fig. 366 (*b*).

The phase of the currents supplied to the rotor can be altered by using an interconnected winding on the transformer (see Fig. 171). Each phase winding contains a coil in series with it from another phase. In this way a great improvement in power factor is obtained. When the stator winding itself is used as the transformer the connections shown in Fig. 366 (*c*) may be used. There is now a small delta in the middle of the voltage star and this alters slightly the phase of the voltage between line and the true star point.

The stator winding sets up a rotating field, and at standstill this flux cuts the rotor conductors at synchronous speed, inducing in them a voltage proportional to the supply frequency, the magnitude of the flux and the number of rotor turns. If the rotor be allowed to rotate in the same direction as the flux, the rate of cutting lines will be reduced and the rotor induced E.M.F. will fall. The frequency of this E.M.F. in the winding itself will also be lower, but as the brushes are stationary, the frequency external to the brushes will still be that of supply. The magnitude of the voltage across the brushes is a maximum at standstill, zero at synchronous speed, and of opposite sign for speeds higher than that of synchronism. With reverse rotation the voltage is increased from standstill.

The current flowing in the armature is due to the resultant voltage acting. This has three components, viz. the voltage impressed on the armature by the transformer, that induced in the winding due to its rotation in the flux, and the impedance drop. On light load the current is small, so that the induced voltage is practically equal to the impressed voltage. To effect this the motor must run at a certain speed. As the load increases the impedance drop increases, so that a vector difference appears between the values of the impressed and the induced E.M.F.'s. It follows, therefore, that the motor has a shunt characteristic, since the impedance of the armature circuit is low. Speed variation can be obtained by altering the transformer tappings, the number of speeds being equal to the number of tapping points.

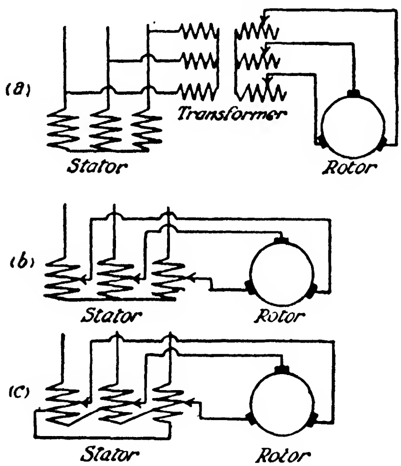


FIG. 366.—Three-Phase Shunt Motor.

Neglecting the relatively small armature impedance, the impressed voltage is zero at synchronous speed. If the tapping points on the transformer are continued beyond the star point, reverse voltages are impressed on the armature and the direction of slip is reversed. In this way hypersynchronous speeds are obtained. This can also be applied to the tappings on the stator winding, when a separate rotor transformer is not used.

The power required to run the motor at synchronous speed is transmitted to the rotor inductively from the stator across the air-gap. The transformer only deals with the slip power, so that it is relatively small. This slip power is reversible. Above synchronous speed, power is supplied through the transformer to the rotor, but below synchronous speed, power is returned to the mains from the rotor. The efficiency is thus maintained at a relatively high figure.

Schrage Motor.—This motor, which is a variable speed one with shunt characteristics, has two sets of brushes on its commutator. These are moved in opposite directions, the rockers being geared together for this purpose. The front end of each stator phase winding is connected to a brush in one set, whilst the rear end of the same phase winding is connected to the corresponding brush in the other set. The main primary winding is placed on the rotor in this motor and is fed through slip rings. In addition there is a special regulating winding on the rotor, this being connected to the various commutator bars. This winding is wound in the same slots as the main primary winding. Speed regulation is obtained by moving the brushes, each position of which corresponds to a definite speed and a definite commutator voltage (see Fig. 367).

As with the three-phase shunt motor and the induction motor with a phase advancer, sub-synchronous speeds are obtained by impressing on the rotor an E.M.F. which has the same frequency but an opposite sense to the induced E.M.F. In order to obtain hypersynchronous speeds, the impressed E.M.F. is reversed. This necessitates a reversal of the induced E.M.F., which is brought about by making the conductors cut the flux in the opposite direction. In other words, the slip becomes negative.

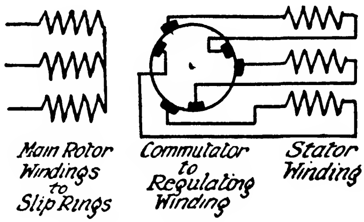


FIG. 367.—Schrage Motor.

Theory of Operation.—In order to study the operation of this motor, assume the primary winding to be stationary, together with the commutator, the brushes rotating together with the secondary winding, at the speed of the motor. The regulating winding is dispensed with for the moment, the commutator being connected to the main primary winding. The connections now appear as shown in Fig. 368. The voltage between any two brushes such as a_1 and b_1, due to the primary winding, is proportional to the angular displacement between them. If these brushes are made to rotate around the stationary commutator, the magnitude of the voltage will not be altered, but the frequency will vary with the speed. With the brushes stationary, the frequency of the voltage between each pair is the full supply frequency, but this falls to zero when synchronous speed of the brushes is attained. For still higher speeds the frequency commences to rise again, being always the frequency of slip. The induced E.M.F. in the secondary winding also has a frequency equal to that of slip.

Summarizing the above, the magnitude of the voltage between the pairs of brushes is proportional to the angular displacement, the frequency to the speed of slip, whilst the actual phase depends upon the position of the mean brush axis.

If a_1 and b_1 touch the same bars, then the motor is equivalent to an induction motor with a short-circuited secondary. Separating the brushes causes an E.M.F. to be injected into the rotor circuit, so that the motor is now equivalent to an induction motor with a phase advancer. This produces a drop (or alternatively a rise) in speed of such a magnitude that the injected E.M.F., the induced E.M.F. and the impedance drop add up vectorially to zero. A different speed is thus obtained for each angle of brush separation. By interchanging the positions of a_1 and b_1 hypersynchronous speeds are obtained, through the reversal of the injected E.M.F. and the consequent reversal of the induced E.M.F.

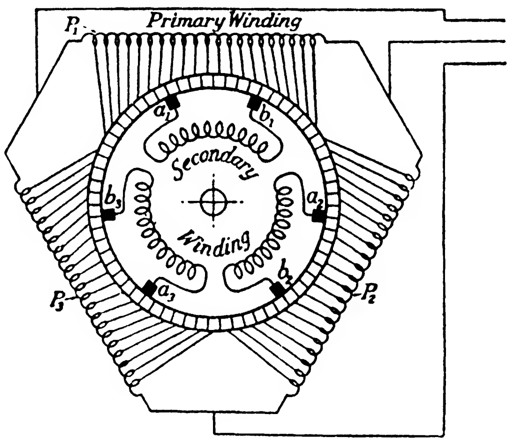


FIG. 368.—Windings in Simple Schrage Motor.
(By courtesy of the British Thomson-Houston Co., Ltd.)

The practical speed range is from about 50 per cent. above to about 50 per cent. below synchronism, thus giving a 3 : 1 ratio.

When load is applied, the rotor current rises and the impedance voltage drop is increased, so that the motor has a shunt characteristic.

Actual Arrangement of Windings.—There is a practical objection to having rotating brushes, inasmuch as the control of the displacement is difficult. The primary is therefore wound on the rotor, being fed through slip rings. There is now a rotating commutator and stationary brushes. Another practical objection is that for the ordinary voltages in use, the volts between adjacent commutator bars are too high with a single primary rotor winding. In order to overcome this, a second rotor winding is used, wound in the same slots as the main primary winding, but with less turns, this second, or regulating winding, as it is termed, being energized by transformer action from the main winding. The commutator and brush gear are thus operated at a lower voltage. The main primary winding is placed at the bottom of the slots, the regulating winding being placed at the top of the same slots. The brush gear is mounted on two yokes which are moved in opposite directions by gearing. The arrangement of the windings is shown in Fig. 369.

Action of Regulating Winding.—The primary current sets up a rotating field with respect to the rotor, which rotates in the opposite direction like an inverted induction motor. E.M.F.'s are induced in the secondary winding on the stator at slip frequency, and in the regulating winding on the rotor at supply frequency. The latter are changed at the commutator also to slip frequency.

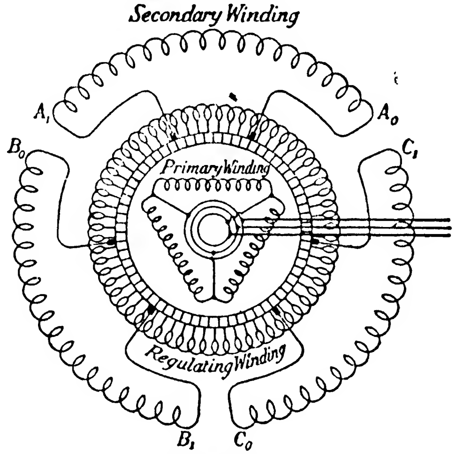


FIG. 369.—Arrangement of Windings in Schrage Motor.
(By courtesy of the British Thomson-Houston Co., Ltd.)

The voltage between commutator segments is constant for all speeds, and so the motor is superior in this respect to the plain three-phase shunt motor where the voltage between segments is a maximum at standstill and zero at synchronism. The commutation of the Schrage motor is therefore better at starting, and it can be designed with a higher starting torque. A further advantage possessed by the Schrage motor is that a gradual speed change is obtained by change of brush position, instead of a change in definite steps by alteration of transformer tappings.

Vector Diagram.—Assume the motor to be running at sub-synchronous speed with the brushes equally displaced on either side of the neutral axis, so that the phase of the regulating volts is exactly the same as that of the back induced E.M.F. in the appropriate primary winding. In Fig. 370, let Φ represent the flux set up by the magnetizing current, I_m, flowing in the primary winding. The secondary induced E.M.F., E_2, lags behind this flux by 90°.

The frequency of this voltage is really that of slip, but taking the actual rotation into account as well, it has the same effect as the supply frequency when considered with respect to a fixed point on the rotor, which carries the primary winding. (The same conditions arise in the vector diagram of the induction motor.) The E.M.F. induced in the regulating winding has the same phase as E_2 and is represented by E_R. The vector difference between E_2 and E_R is the resultant voltage producing current in the secondary circuit, and as this possesses reactance as well as resistance, the secondary current, I_2, lags as shown in the vector diagram. The voltage difference between E_2 and E_R is shown split up into its two components, I_2R_2 and I_2X_2, in phase and in quadrature with the current respectively. In order to balance the ampere-turns of the secondary current, an equivalent current, I'_1, is taken from the supply by the primary winding, and this, when combined with the magnetizing current, I_m, forms the total primary current, I_1. The flux

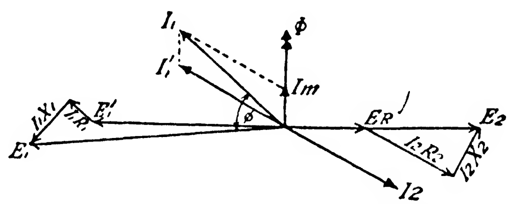


FIG. 370.—Vector Diagram of Schrage Motor with Neutral Brush Axis.

Φ also causes a back E.M.F. to be induced in the primary winding, and this necessitates the application of a voltage, E'_1, in order to neutralize it. To this voltage must be added the resistance and reactance drops, I_1R_1 and I_1X_1 respectively, in order to obtain the resultant applied slip ring voltage, E_1. The power factor is given by cos ϕ.

When the speed is below that of synchronism, the current in the regulating winding is flowing in opposition to the induced voltage in it, so that it is being supplied with power from the secondary winding. This power is transferred back to the main primary winding by transformer action with the regulating winding, so that a reduction in speed is accompanied by a corresponding drop in power taken from the mains. This is not the case with the plain induction motor.

At synchronous speed the regulating winding is cut out, since the several pairs of brushes touch the same commutator bars. E_R now disappears in the vector diagram and, neglecting losses, no power is transferred either way by the reg ating winding.

For hypersynchronous speeds the b s are displaced in the reverse direction, thus reversing the se of the E.M.F. injected into the secondary winding. The resultant secondary E.M.F. is now increased by the action of the regulating winding, and as a consequence more current flows. This current now has the same sense as the E.M.F. in the regulating winding, so that electrical power is being given out by it, this power being obtained by transformer action from the main primary winding. In other words, extra power is drawn from the supply on account of the increased output at the higher speed.

The regulating winding and the commutator only deal with the slip power, or the difference between the powers corresponding to synchronism and the actual speeds, and consequently are of relatively small magnitude.

Power Factor Adjustment.—If all the brushes be moved bodily round the commutator, the phase of the regulating voltage is altered, and by this means it is possible to improve the power factor of the motor. If this be done the vector diagram takes the form shown in Fig. 371, where E_R is shown lagging behind E_2. The secondary

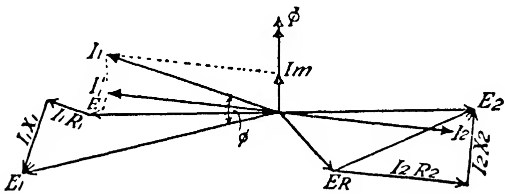


FIG. 371.—Effect of Alteration of Brush Axis.

impedance triangle is tilted round and the current, I_2, is brought more nearly into phase with E_2. This improvement is reflected in the primary circuit with a consequent rise in power factor. Whilst this is true for subsynchronous speeds for which the vector diagram in Fig. 371 is drawn, the reverse is the case for hypersynchronous speeds. This can be seen by reversing E_R, when the vector completing the voltage triangle is found to lag behind E_2, thus causing the current to fall behind in phase instead of being advanced. Thus whilst the power factor is raised for subsynchronous speeds by this means, it is made worse at hypersynchronous speeds. In the former case the power factor is inherently rather low, whilst in the latter case it approaches unity, so that on the whole a certain amount of permanent brush displacement is desirable, particularly as this also improves the starting torque. By operating the two sets of brushes through a differential gear, a gradual change of brush axis is obtained, so as to counteract the adverse effect at the higher speeds.

Compensated Induction Motor.—If the brushes in a Schrage motor be advanced through 90°, a leading E.M.F. is impressed on the secondary stator windings from the regulating winding. This causes the phase of the secondary currents to be advanced, and this again is reflected by a corresponding advance in phase on the part of the primary currents in the main rotor windings which are fed from the supply by way of the slip rings. A rise in power factor is thus brought about. The motor is now worked with a fixed brush position, starting being effected by means of three variable starting resistances each connected between a secondary (stator) winding and the corresponding brush on the commutator.

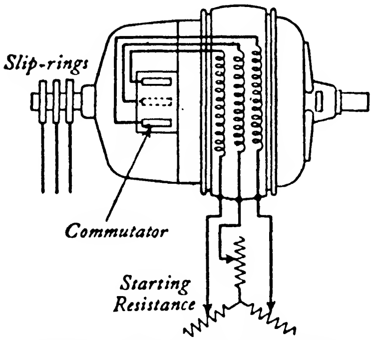


FIG. 372.—Three-Phase No-Lag Motor.

As an alternative, the stator windings may be connected in star, the three ends being taken to three sets of fixed brushes, the other three sets of brushes being now omitted. This motor, known as the No-Lag motor, is started by opening the star point of the stator windings and inserting an ordinary three-phase induction motor resistance starter, which is gradually cut out as the motor gains speed. A diagram of connections of this motor is shown in Fig. 372.

EXAMPLES

(1) What steps are taken to prevent a three-phase series motor from racing on light load?

(2) Draw a diagram of connections to show how speed variation can be obtained in a three-phase shunt motor.

(3) Compare the Schrage motor with (*a*) a three-phase induction motor, with and without a phase advancer, and (*b*) a three-phase shunt motor.

(4) Draw a diagram of connections of a three-phase Schrage motor, and show why an alteration of the distance between brush spindles produces a change in speed.

(5) Draw a vector diagram of a Schrage motor with the phase of the regulating voltage displaced by 30° from the neutral position (*a*) for subsynchronous and (*b*) for hypersynchronous speed.

(6) A plain three-phase series motor is to have a speed regulation of 3 : 1, this being obtained by means of an auto-transformer. Show that if the latter is connected between stator and rotor windings, it need only have one-third of the kVA rating which would be necessary if it were connected between the stator windings and the supply.

(7) A three-phase commutator motor has its stator windings connected in star across the bus-bars. The three sets of brushes on the commutator are connected to tappings on the secondary windings of a three-phase transformer, the primary windings of which are also connected to the bus-bars. Show that such a motor has a "shunt" characteristic. How is speed regulation obtained, and why?

CHAPTER XXXIII

POWER FACTOR CONTROL

Effect of Low Power Factor.—In view of the fact that the great majority of loads contain more inductance than capacitance, the current in most transmission lines lags behind the voltage, giving rise to power factors of less than unity. This means that the line current for a given power transmitted is greater than it need be and causes two results. First, the losses in transmission are increased, or, conversely, the size of wire is increased, leading to the employment of larger quantities of copper, and, second, the voltage regulation is made poorer, due to the increased voltage drop in the line with the larger currents. To illustrate this fact, the case will be considered where a given amount of power is transmitted at a power factor of 0·7, and comparison will be made with what would be the case if the power factor were unity. The current is inversely proportional to the power factor and is $\frac{1}{0{\cdot}7}$ of its minimum value, or 43 per cent. greater than it need be. If the size of wire be kept constant throughout, the losses, being proportional to the square of the current, are $\left(\frac{1}{0{\cdot}7}\right)^2$, or approximately double their value in the ideal case. If the losses are kept the same, approximately double the amount of copper must be employed. The latter arrangement leaves the regulation unaltered, but the former results in an increased voltage drop. These figures, which are quite practical ones, will suffice to show the magnitude of the effect and the desirability for improving the power factor wherever possible.

General Method of Improvement.—The power factor of a system can be improved in two main ways. The first is to use only that apparatus which works at approximately unity power factor, and some supply authorities make it a rule not to allow motors of more than a given output and working below a certain power factor to be connected to their mains. In general, however, this method is impracticable. The second method is to add to the existing load, apparatus which will take a leading current of such a magnitude as to neutralize the lagging current brought about by the general character of the load. This method is now widely adopted.

Magnitude of the Required Reactive Current.—For the purposes of investigation the load current, which is imagined to lag behind the line voltage, can be split up into an active component and a reactive component, the latter lagging by 90° behind the line voltage. In Fig. 373, I represents the load current lagging behind the line voltage E by an angle ϕ such that $\cos\phi$ is equal to the power factor. The active component of the current is equal to $I_P = I\cos\phi$ and the reactive component is equal to $I_L = I\sin\phi$. In order to neutralize this reactive current, it is necessary to add a leading current, I_C, of the same magnitude. The vector sum of I and I_C is then given by I_P, which is less than I. The current is thus brought into phase with the voltage, raising the power factor to unity, and its magnitude is decreased at the same time without any reduction in the amount of power transmitted. The magnitude of the required leading reactive current is, therefore,

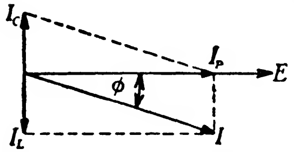


FIG. 373.—Showing Effect of Added Leading Current on Power Factor.

$$\begin{aligned} I_C &= I\sin\phi \\ &= I \times \sqrt{1-\cos^2\phi} \\ &= I \times \sqrt{1-(\text{power factor})^2}. \end{aligned}$$

The relation between the power factor and the value of I_C expressed as a percentage of the line current I is represented in

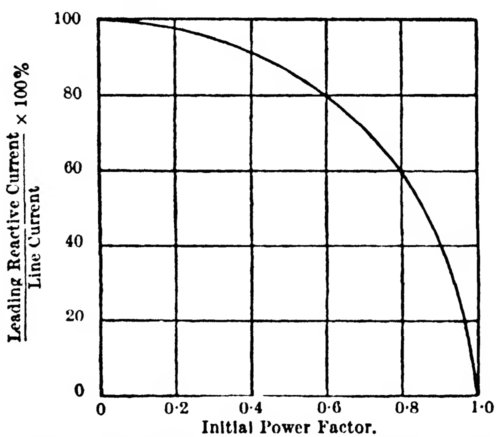


FIG. 374.—Relation between Added Leading Current and Initial Power Factor.

Fig. 374. The varying steepness of this curve shows that different amounts of leading current are required at different power factors in order to effect the same improvement. When in the neighbourhood of unity power factor a greater amount of additional leading

current is required for a given improvement than is the case at low power factors. This really means that it is easier to raise the power factor when it is very low than when it is high.

The above reasoning applies equally well to a polyphase system, provided that in Fig. 374 the added leading reactive current and the line current refer to the same circuit.

kVA Diagram.—In a simple series circuit the voltage E may be split up into an active component E_P and a reactive component E_L (or E_C). On multiplying each side of this voltage triangle by the current I and dividing throughout by 1000, a kVA diagram is obtained as shown in Fig. 375. Vertical lines drawn above and below the horizontal represent lagging and leading reactive kVA respectively, the one neutralizing the other. This diagram may represent the total effect of all phases in a polyphase case, but a balanced circuit must then be assumed.

Improvement by Condensers.—The first obvious way of providing a leading current in addition to the existing load current is to connect a number of condensers in parallel across the mains at the

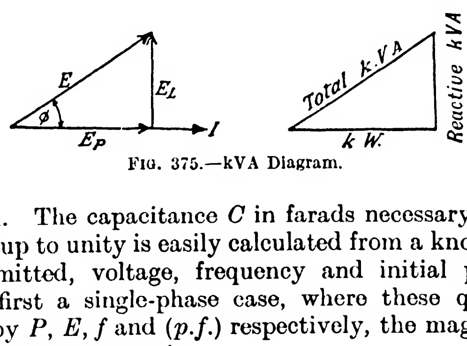


FIG. 375.—kVA Diagram.

receiving end. The capacitance C in farads necessary to bring the power factor up to unity is easily calculated from a knowledge of the power transmitted, voltage, frequency and initial power factor. Considering first a single-phase case, where these quantities are represented by P, E, f and $(p.f.)$ respectively, the magnitude of the required condenser current is

$$I_C = I \times \sqrt{1 - (p.f.)^2}.$$

Then

$$2\pi f C E = I \times \sqrt{1 - (p.f.)^2}$$

$$= \frac{P}{E\,(p.f.)} \times \sqrt{1 - (p.f.)^2}$$

$$= \frac{P}{E} \times \sqrt{\frac{1}{(p.f.)^2} - 1}$$

and

$$C = \frac{P}{2\pi f E^2} \times \sqrt{\frac{1}{(p.f.)^2} - 1}.$$

It is thus seen that a high frequency is desirable when condensers are used for improving the power factor, together with as high a voltage as the condensers will stand. For very high voltages, the actual value of the voltage does not much matter, for the cost per microfarad is approximately proportional to the (voltage)^2. This is also true when L.T. condensers are arranged in series, for the resultant capacitance of a group is again inversely proportional to the square of the number connected in series.

In a three-phase transmission, the power is given by

$$P = \sqrt{3}EI \times (p.f.),$$

where E and I are the line voltage and line current respectively. The reactive component of the line current is equal to

$$\frac{P}{\sqrt{3}E \times (p.f.)} \times \sqrt{1-(p.f.)^2}$$

$$= \frac{P}{\sqrt{3}E} \times \sqrt{\frac{1}{(p.f.)^2} - 1}.$$

Assuming the condensers to be connected in delta across the lines, the current per condenser circuit is $2\pi fCE$. The condenser current per line is the vector sum of the currents of two condenser circuits, and is, therefore, $\sqrt{3} \times 2\pi fCE$. If the final power factor is to be unity, this must be equal to $\frac{P}{\sqrt{3}E} \times \sqrt{\frac{1}{(p.f.)^2} - 1}$, and, therefore,

$$\sqrt{3} \times 2\pi fCE = \frac{P}{\sqrt{3}E} \times \sqrt{\frac{1}{(p.f.)^2} - 1}$$

and

$$C = \frac{P}{6\pi fE^2} \times \sqrt{\frac{1}{(p.f.)^2} - 1}.$$

Since there are three condenser circuits, the total capacitance required is

$$\frac{P}{2\pi fE^2} \times \sqrt{\frac{1}{(p.f.)^2} - 1},$$

which is the same as in the single-phase case for the same line voltage.

If the condensers are connected in star instead of delta, the voltage across each condenser is reduced to $\frac{1}{\sqrt{3}} = 0{\cdot}577$ times its former value. But the current per condenser circuit is now equal to $2\pi fC\frac{E}{\sqrt{3}}$, and this must neutralize a lagging reactive current of

$$\frac{P}{\sqrt{3}E} \times \sqrt{\frac{1}{(p.f.)^2} - 1}.$$

The capacitance per condenser circuit is, therefore,

$$C = \frac{P}{2\pi fE^2} \times \sqrt{\frac{1}{(p.f.)^2} - 1},$$

and the total capacitance required is

$$\frac{3P}{2\pi f E^2} \times \sqrt{\frac{1}{(p.f.)^2} - 1},$$

or three times as much as when the condensers were connected in delta. Provided the condensers will stand the voltage, therefore, the delta connection is the preferable of the two.

Condenser Installations.—Where the voltage is too low to be economical, the condensers are fed from the secondary side of a step-up auto-transformer. This increases the current per microfarad, this current being further magnified on the supply side of the auto-transformer. Another method is to resonate the supply voltage up to the required value by means of series inductances.

The condensers are usually oil-immersed in the larger sizes and enclosed in tanks like transformers. The switches controlling them are fitted with discharge resistances like field-break switches on account of the sparking which would otherwise ensue on opening the circuit.

Over-Voltages Caused by Condensers.—It was shown on p. 274 that an alternator becomes self-exciting, even if the D.C. field circuit be interrupted, provided a sufficiently large capacitance be connected to its terminals. It may, in fact, build up its voltage to a value exceeding the normal line voltage. In the same way an induction motor may be made self-exciting by the help of condensers connected across its terminals. The leading current taken by the condensers may be regarded as a lagging (magnetizing) current delivered by the condensers to the induction motor. The magnitude of this condenser current depends upon the capacitance of the condensers and, in fact, may be given any value. On the other hand, an increase in the magnetizing current taken by the induction motor, at a fixed frequency, corresponds to an increase in terminal voltage. So long as the motor is connected to the line, no variation in its voltage is permitted, the line voltage being kept constant. If, however, the motor and its condenser installation be disconnected from the line, an appreciable rise in voltage may occur, if the capacitance is large enough. The relatively large condenser current supplies the induction motor stator winding with magnetizing current sufficient to cause the machine momentarily to excite, and set up a voltage across its terminals. This rise in voltage may, in extreme cases, be as much as 30 per cent.

The duration of the over-voltage is very short, since the machine speed falls away rapidly, and whilst it may be of no consequence so far as the insulation is concerned, it may wear out any lamps still connected to the motor in a very short time indeed.

It is thus seen that over-compensation of particular motors with the intention of improving the power factor of neighbouring machines is undesirable. Each motor should, preferably, be provided with its own individual equipment.

Synchronous Motor as Synchronous Condenser.—Since a synchronous motor can be made to take a leading current by over-exciting it, such a motor might be employed to produce the required leading current for the purpose of improving the power factor. The magnitude of the current taken by a synchronous motor depends to an enormous extent upon the excitation, and full load current can easily be attained even when the motor is running light. The function of a synchronous motor used in this manner is to act as a variable rotary or synchronous condenser. The motor is not for the purpose of overcoming any load, although in practical cases it may be utilized for this purpose as well, but only in a subsidiary way. The primary object is to produce a leading current to neutralize the existing lag.

Considering first an ideal motor with no losses, the current taken leads the voltage by 90° if it is over-excited, and the magnitude of this current depends upon the value of the excitation. The current can thus be adjusted to suit the load and the power factor by varying the excitation of the machine, this being much more convenient than varying the number of condensers, which would be necessary if they were employed.

If the losses of the synchronous condenser are taken into consideration, it means that a certain active component must be added, vectorially, to the leading current already mentioned, the result being that the motor current leads the voltage by an angle which is less than 90°. The extra power taken from the mains is, of course, wasted.

This method is equally applicable to the case of polyphase systems, being, in fact, cheaper in first cost, since polyphase synchronous motors cost less per kVA than single-phase ones. Since the construction of these motors is the same as for alternators, it follows that they can be designed for any line voltages which are obtained directly from alternators.

One of the disadvantages of this method of power factor control lies in the starting of the motor. Unless a self-synchronizing motor be adopted, some auxiliary means of running up the motor must be provided.

Situation of Synchronous Condenser.—The correct situation for the synchronous condenser is at the receiving end of the line as close to the load as possible. If placed at the generator end of the line, it would improve the power factor of the generators and reduce their current, but it would not relieve the line at all, nor would it affect the voltage regulation of the line. When placed at the receiving end of the line, however, it not only improves the power factor of the generators and reduces their armature currents, but it reduces the main line current and improves the regulation. The ideal spot for the condenser is alongside the actual load taking the lagging current, but this is impracticable in the great majority of cases. In the case of high voltage transmissions where the voltage is stepped down and distributed from a number of sub-stations, the synchronous condenser is placed in the sub-station. Its effects are thus felt throughout the whole of the high voltage system, but the low voltage network is unaffected.

Reactive Current Generator.—When a synchronous motor is employed as a condenser, it may be regarded as a reactive current generator. Considered as a motor, it receives a small in-phase current which supplies the losses of the machine together with a reactive current leading the voltage by 90°. This leading motoring current may be regarded as a generator current by reversing its direction, so that the synchronous motor may be said to receive an in-phase motoring current to supply its losses and to generate a reactive current lagging by 90° behind the voltage. There is, of course, no power associated with this reactive current which supplies the

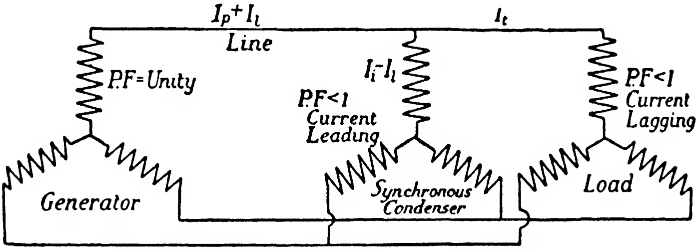


FIG. 376.—Synchronous Condenser in Transmission Line.

purely inductive portion of the load. The main generators supplying the power may thus be regarded as working on a non-inductive load. There is a local circuit formed by the inductive portion of the load and the synchronous motor in which resonance is set up, this resonating current being superimposed on to the main in-phase current flowing between the generators and the load. The general idea of the system applied to a three-phase case is shown in Fig. 376, where the generator is working at a power factor of unity and is delivering a current $(I_p + I_l)$, the second term representing the active current supplied to the synchronous motor to overcome its losses. The latter is working at a very low power factor and receiving an active current I_l, whilst generating a purely lagging current I_i. The load, the power factor of which is less than unity, is receiving a current I_t which is the vector sum of I_p and I_i, the angle of lag being determined by the power factor.

kVA Capacity of Synchronous Condenser.—Since a synchronous condenser is only required for the purpose of producing a purely reactive current, the only power which it receives is that necessary to overcome its losses. The machine also works on a very low power factor, so that it is obviously unfair to rate it on a kilowatt basis. The proper unit for specifying the capacity [1] of the machine is the kilovolt-ampere. In a single-phase case, neglecting the losses, the reactive current is given by $I \times \sqrt{1 - (p.f.)^2}$ and the total kVA capacity of the synchronous condenser is

$$EI \times \sqrt{1 - (p.f.)^2} \times 10^{-3}.$$

Expressed as a percentage of the kVA capacity of the alternators, this is

$$\sqrt{1 - (p.f.)^2} \times 100,$$

and this expression applies equally to the case of a polyphase system.

The size of the synchronous condenser is thus seen to be unaffected by the value of the current or the voltage, provided the power transmitted and the power factor are constant. A high voltage is associated with a low current and *vice versa*, but it is their product which determines the size of the machine, just as in the case of alternators.

Effect of Motor Losses.—If the synchronous motor is running unloaded the only power which it takes is that required to overcome its losses. The angle of lead of the current is thus not quite 90°, but rather less. The heating of the machine is dependent upon the losses, and since it is to be heavily over-excited, special care must be taken in the design of the field system to prevent over-heating. The fact that the power factor is greater than zero affects the kVA capacity to a slight extent and also modifies the vector diagram, the general form of which is shown in Fig. 377, which is drawn for a single-phase case.

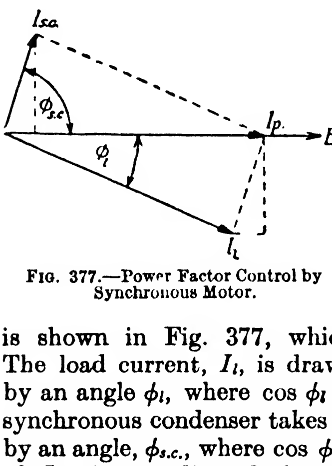


FIG. 377.—Power Factor Control by Synchronous Motor.

The load current, I_l, is drawn lagging behind the line voltage, E, by an angle ϕ_l, where $\cos \phi_l$ is the power factor of the load. The synchronous condenser takes a current, $I_{s.c.}$, leading the line voltage by an angle, $\phi_{s.c.}$, where $\cos \phi_{s.c.}$ is its power factor. The magnitude of $I_{s.c.}$ is so adjusted that the vector sum of I_l and $I_{s.c.}$ gives a current in the mains, I_p, in phase with the voltage. In order to bring this about, the leading reactive current of the synchronous condenser must be equal to the lagging reactive current of the load, so that

$$I_{s.c.} \sin \phi_{s.c.} = I_l \sin \phi_l$$

and

$$I_{s.c.} = I_l \frac{\sin \phi_l}{\sin \phi_{s.c.}}$$

[1] The term "capacity" here has no reference whatever to the condenser effect, the unit of which is the farad.

The kVA capacity of the synchronous motor is thus

$$EI_{s.c.} \times 10^{-3}$$
$$= EI_l \frac{\sin \phi_l}{\sin \phi_{s.c.}} \times 10^{-3}$$
$$= EI_l \sqrt{\frac{1 - (p.f._l)^2}{1 - (p.f._{s.c.})^2}} \times 10^{-3}.$$

The power factor of the synchronous motor is usually somewhere about 0·1, so that the expression $\sqrt{1 - (p.f._{s.c.})^2}$ becomes equal to $\sqrt{1 - 0{\cdot}1^2} = 0{\cdot}995$. The effect of taking the losses into account is thus seen to increase the kVA capacity by only 0·5 per cent., and it can, therefore, be neglected in the majority of cases.

As an actual example, the case will be taken where a load of 1000 kW is transmitted at a power factor of 0·7, the synchronous condenser running at a leading power factor of 0·05 so as to bring the resultant power factor up to unity. The total kVA, excluding the condenser, is $\frac{1000}{0{\cdot}7} = 1430$, and the reactive kVA is $\sqrt{1430^2 - 1000^2} = 1022$. If the power factor is 0·05, then $\sin \phi_{s.c.} = \sqrt{1 - 0{\cdot}05^2} = 0{\cdot}999$, and the capacity of the synchronous condenser is $\frac{1022}{0{\cdot}999} = 1023$ kVA, the power taken by it being $1023 \times 0{\cdot}05 = 51$ kW. The total power transmitted is now 1051 kW instead of 1000 kW, an increase of 5·1 per cent., but the total current in the mains is reduced in the ratio of 1430 to 1051, and as the copper losses are proportional to the square of the current they are reduced to $\left(\frac{1051}{1430}\right)^2$, or 0·54 of their initial value. There is, therefore, a saving of 46 per cent. of the copper losses in the mains against an additional loss of 5·1 per cent. of the total useful load. The question as to whether there is a resultant saving or not depends upon the resistance of the mains and must be settled for each individual case.

Partial Improvement of Power Factor.—Instead of raising the power factor from its initial value to unity, it might be raised to some such value as, say, 0·9 by the employment of a smaller synchronous condenser. A considerable improvement is still effected if this be done, and it is found to be much more expensive to raise the power factor by a given amount when it is near unity than when it has a much lower value. It is thus possible to instal a synchronous condenser having a kVA capacity insufficient to raise the power factor to unity, but sufficient to raise it to some predetermined lower value. This machine will be considerably smaller than one designed to raise the power factor to unity and may be more economical to instal on account of the reduced capital outlay required.

The kVA capacity of the synchronous condenser for a partial improvement can be calculated by determining the reactive component of the load before and after the synchronous condenser has been installed, remembering that in the latter case the kW losses of the synchronous condenser have to be added to the kW of the load to obtain the total kW on the system.

The kVA of the synchronous condenser to raise the power factor of a system from 0·7 to different final power factors is given in the following table, the power factor of the synchronous condenser itself being taken as 0·1 and the total load as 100 kW.

Final power factor	0·75	0·80	0·85	0·90	0·95	1·00
kVA of synchronous condenser	13·4	26·1	39·2	52·5	68·2	102·2
Additional kVA	—	12·7	13·1	13·3	15·7	34·0
Reduction of line copper losses, per cent.	13	23	32	40	46	51

The above table also shows the reduction in the line copper losses in the various cases.

The example on page 529 will now be further considered, and the kVA capacity of a synchronous condenser to bring the resultant power factor up to 0·9 instead of unity will be calculated. The total kVA, excluding the condenser, is 1430, and the reactive kVA is 1022 as before. The kVA of the load after the synchronous condenser has been installed is $\frac{1000}{0{\cdot}9} = 1111$, and the reactive kVA is $\sqrt{1111^2 - 1000^2} = 484$. The difference between the reactive kVA of the load before and after the synchronous condenser has been installed gives the required reactive kVA of the synchronous condenser, and this is $1022 - 484 = 538$. As before, $\sin \phi_{s.c.} = 0{\cdot}999$, so that the total kVA capacity of the synchronous condenser is $\frac{538}{0{\cdot}999} = 538$ kVA. The power taken by it is $538 \times 0{\cdot}05 = 26{\cdot}9$ kW. It will be noticed that this machine is only approximately half the size of the one necessary to raise the power factor to unity. The vector diagram is similar to that shown in Fig. 377, except that I_p now lags behind E by an angle of 26°, the cosine of which is 0·9.

Most Economical Final Power Factor.—The question as to what is the most economical final power factor is a very important and practical one and is ultimately decided by considerations of cost. When the power factor is raised it involves an extra expenditure on account of the synchronous condenser, but there is a reduction in the cost of the mains, or, alternatively, they are capable of transmitting a greater load. If the load is expanding it may be more economical to instal a synchronous condenser in preference to laying down additional mains.

Starting from the low power factor due to the load and gradually raising it, the saving in the mains at first far outweighs the extra cost of the synchronous condenser in the majority of cases, but as the power factor is raised still further its cost begins to approximate to the saving in the mains, and finally any additional saving in the mains is only obtained by a greater expenditure in increasing the size of the synchronous condenser. There is a point, therefore, beyond which it is not economical still further to improve the power factor, and this usually occurs at a power factor somewhere about 0·95.

Synchronous Condenser with Mechanical Load.—A synchronous motor used in the above way can be utilized to develop mechanical power in the same way as an ordinary synchronous motor. All that is necessary is that the motor should be over-excited to a certain extent. The reactive component of the motor current tends to eliminate the lagging reactive current of the remainder of the load, whilst the active component of the motor current serves to develop the torque required for the load. Whilst the motor adds to the useful power load on the supply, it materially increases the total power factor.

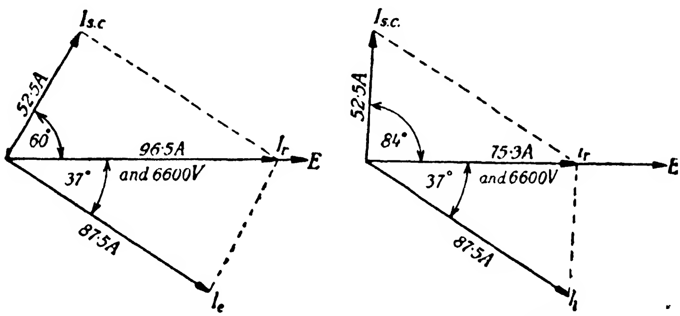
An example will be worked out wherein a 600 kVA synchronous motor developing 360 B.H.P. is installed to improve the power factor of a three-phase transmission line having a load of 1000 kVA at a power factor of 0·8, the line voltage being 6600.

The line current is, excluding the synchronous motor,

$$\frac{1000 \times 10^3}{\sqrt{3} \times 6600} = 87{\cdot}5 \text{ amperes,}$$

and lags behind the phase volts by an angle of 37°, since $\cos 37° = 0{\cdot}8$ [see Fig. 378 (*a*)]. Assuming an efficiency of 0·9 for the synchronous motor, its input is $\frac{360 \times 0{\cdot}746}{0{\cdot}9} = 298{\cdot}4$ kW, and its power factor is $\frac{298{\cdot}4}{600} = 0{\cdot}50$. The motor current is $\frac{600 \times 10^3}{\sqrt{3} \times 6600} = 52{\cdot}5$ amperes, and leads the phase volts by an angle of 60°, since $\cos 60° = 0{\cdot}50$. The lagging reactive current of the load is $87{\cdot}5 \times \sin 37° = 52{\cdot}5$ amperes, and the leading reactive current of the synchronous motor is $52{\cdot}5 \times \sin 60° = 45{\cdot}5$ amperes. The resultant reactive current (lagging) is $52{\cdot}5 - 45{\cdot}5 = 7{\cdot}0$ amperes. The resultant active component of the current is $87{\cdot}5 \times 0{\cdot}8 + 52{\cdot}5 \times 0{\cdot}5 = 96{\cdot}25$ amperes. The resultant line current is $\sqrt{96{\cdot}25^2 + 7^2} = 96{\cdot}5$ amperes, and the final power factor is $\frac{96{\cdot}2}{96{\cdot}5} = 0{\cdot}997$, say, unity. It is thus seen that for an increase of $\frac{96{\cdot}5 - 87{\cdot}5}{87{\cdot}5} \times 100 = 10{\cdot}3$ per cent. in the current the power transmitted has been increased by $\frac{800 + 298\cdot4}{800} \times 100 = 37\cdot3$ per cent., the efficiency of transmission being considerably improved.

The case will now be considered where the synchronous motor is run unloaded, assuming it to run at a power factor of 0·1, thus taking 60 kW to drive it light. The motor current is $\frac{600 \times 10^3}{\sqrt{3} \times 6600} = 52\cdot5$ amperes, the same as before, but the reactive component is now $52\cdot5 \times \sqrt{1 - 0\cdot1^2} = 52\cdot2$ amperes, and the active component $52\cdot5 \times 0\cdot1 = 5\cdot25$ amperes. The resultant



(a) Loaded. (b) Unloaded.

FIG. 378.—Synchronous Motor as Synchronous Condenser.

reactive current (lagging) is $52\cdot5 - 52\cdot2 = 0\cdot3$ ampere, and the resultant active component is $87\cdot5 \times 0\cdot8 + 5\cdot25 = 75\cdot25$ amperes. The final line current is $\sqrt{75\cdot25^2 + 0\cdot3^2} = 75\cdot3$ amperes, the power factor being unity. The line current has thus been reduced by $\frac{87\cdot5 - 75\cdot3}{87\cdot5} \times 100 = 14$ per cent. at the expense of an additional power loss of $\frac{60}{800} \times 100 = 7\cdot5$ per cent. of the total power load.

The vector diagram for one phase, both loaded and unloaded, is shown in Fig. 378 (*a*) and (*b*).

Phase Adjuster.—The voltage at the far end of a transmission line can be maintained at a constant value by ensuring that the voltage drop in the line remains constant. This can be effected by controlling the phase of the current, which is done by varying the excitation of an unloaded over-excited synchronous motor, now called a phase adjuster. An increase in load causes an increased voltage drop in the line, and this is counterbalanced by an increase in the leading reactive kVA taken by the phase adjuster, obtained by increasing its excitation. The increase in the leading reactive kVA neutralizes a larger portion of the lagging reactive kVA in the line, and so reduces the line current by raising the power factor. This, in turn, reduces the voltage drop, counterbalancing its natural increase due to increase of load. By suitable adjustment of the reactive kVA load on the phase adjuster, obtained by field control, the voltage at the receiving end of the line can be maintained at a constant value for all loads.

Asynchronous Condenser.—The No-Lag machine (see page 520) can also be made to operate as an asynchronous condenser, at almost zero leading power factor. The leading kVA drawn from the line is adjusted by movement of the brushes.

Phase Advancers.—A further type of machine has been developed with the object of improving the power factor of individual motors, but so far the method has only been applied practically to induction motors. These phase advancers, as they are called, are really A.C. exciters and fulfil the same function with induction motors as do D.C. exciters with synchronous motors.

If a synchronous motor were run without its exciter it would take a very large lagging current from the A.C. supply. The magnetizing ampere-turns are now provided from an A.C. source instead of by means of D.C. Ordinarily an induction motor is magnetized from the A.C. supply, and this brings about a drop in its power factor. If the magnetizing ampere-turns can be provided from some other source, the stator winding will be relieved of the magnetizing current, and the power factor can be raised to unity. The A.C. exciter, or phase advancer, is connected in the rotor circuit of the induction motor and provides the magnetizing ampere-turns at slip frequency. It may even be arranged to provide more than are required, the result being that the induction motor operates on a leading power factor just as does an over-excited synchronous motor.

The provision of a given M.M.F. requires a fixed number of magnetizing ampere-turns, irrespective of the frequency, but the volt-amperes corresponding to this are directly proportional to the frequency, for the voltage required to overcome the inductive drop ($2\pi fLI$) is proportional to the frequency. This is even true in the case of the synchronous motor which is excited by means of D.C., for the magnetizing volt-amperes of the field current are here equal to zero, the whole of the power being wasted in I^2R loss in the field circuit.

In the case of the induction motor the rotor frequency is much less than that of the stator, so that it is obviously desirable to supply the magnetizing ampere-turns from the rotor at slip frequency, rather than from the stator at supply frequency. If this is done completely, then the stator will work on unity power factor. Over-exciting from the rotor produces a leading power factor in the stator of the induction motor.

Leblanc Exciters.—If an armature with a commutator be placed in a field system and excited with A.C., the frequency of the currents obtained at the brushes is the same as that of the exciting field current. The field system must be laminated throughout the same as the armature in order to reduce the iron losses, and actually the field (stator) winding is wound in slots, in the same way as is done in the case of the cylindrical type rotor for a turbo-alternator.

The phase of the E.M.F. induced in the armature is the same as that of the field current (neglecting losses), so that if an armature be excited from the current in a leading phase, and then connected in series with the main rotor winding of the induction motor, the latter will have a leading E.M.F. injected into it.

A number of exciters are coupled to the shaft of the main motor,

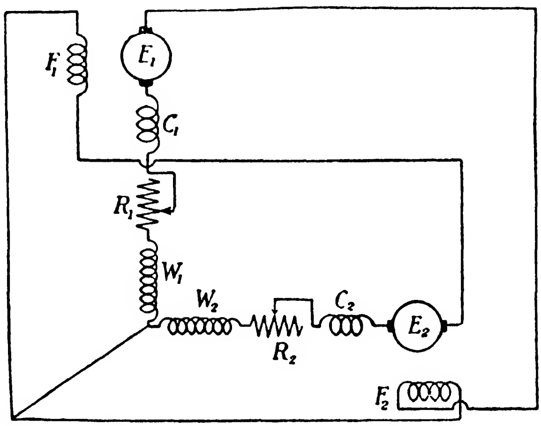


FIG. 379.—Leblanc Exciters for Two-Phase Rotor.

one for each phase. These exciters are made like single-phase series motors provided with compensating windings. The action can be most easily followed in the case of a two-phase rotor (see Fig. 379). W_1 and W_2 represent the two-phase windings of the rotor and E_1 and E_2 the armatures of the two exciters. The current from the phase W_1 passes from the slip ring through the compensating winding C_1, through the armature E_1, and then through the field F_2 of the exciter in the second phase, and thence to the neutral point. The current from the phase W_2 passes through the compensating winding C_2, through the armature E_2, and then through the field F_1 of the exciter in the first phase and to the neutral point. The resistances R_1 and R_2 are those of the starter for the main motor, the exciters being short-circuited during starting up. When the motor is running, the armature E_1 has generated in it an E.M.F. which is in phase with the current in the second phase. If the polarity of the poles is suitably arranged this E.M.F. will lead the current in the first phase by 90°. In a similar manner, the armature

E_2 generates an E.M.F. in the second phase, again leading the current by 90°. The introduction of this leading E.M.F. advances the phase of the whole rotor current, which in turn advances the stator current, so that, if desired, the induction motor can be made to take a leading current from the supply.

In the case of a three-phase rotor three exciters are required, and this constitutes the main drawback to the method. In this case each exciter receives its magnetizing current from the next phase, as shown in Fig. 380, which shows the connections of the main field and armature windings.

In its practical form, sometimes called the Scherbius phase advancer, the three exciters are combined into one machine which consists of an ordinary D.C. armature and commutator, with three

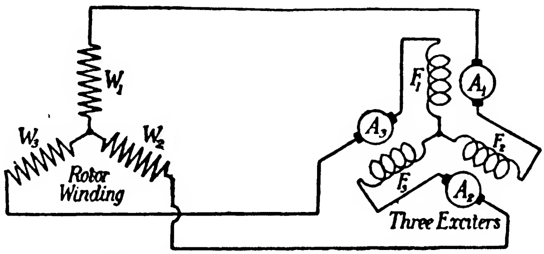


FIG. 380.—Leblanc Exciters for Three-Phase Rotor.

sets of brushes spaced 120° apart in a bipolar case, and with three sets of brushes per pair of poles in a multipolar case. These brushes are connected to the main induction motor slip rings, so that the three-phase rotor currents flow through the phase advancer armature and set up a rotating field which rotates *with respect to the phase advancer armature* at a speed corresponding to the slip frequency of the rotor currents. This rotating flux cuts the armature conductors and does away with the necessity for any stator windings on the phase advancer. The armature is rotated, either by a direct drive from the main induction motor or by means of a separate auxiliary motor, in the same direction as this rotating field. If driven at a slower speed than the rotating field, a back E.M.F. is induced in the phase advancer armature, lagging by 90° behind the current, but if the actual speed of rotation is faster than that of the rotating field, then the relative direction of cutting of flux by the armature conductors is reversed, and the phase of the induced E.M.F. is also reversed. The induced E.M.F. now leads the current by 90°, since the magnetic axis of the armature is at right angles to a section of the armature winding when the latter has zero current passing through it. The armature thus behaves as if it were a condenser, and its effect is to inject into the main rotor circuit an E.M.F. which leads the current by 90°.

The main rotor current is thus advanced in phase, and this is reflected by a corresponding advance in phase on the part of the induction motor stator current (see the vector diagram of the transformer), thus bringing about a rise in power factor.

The stator has no windings on it, and its function is merely to provide a low reluctance path for the magnetic flux. If a winding with closed slots is employed, the stator iron may be made to rotate with the rotor. In other words, the rotor slots become tunnels situated in a ring the diameter of which is midway between the inner and outer diameters of the stampings. The stator is now dispensed with entirely, the rotor being driven from a shaft extension of the main motor.

In certain cases the stator stampings are retained, and the phase advancer may be driven by means of a separate motor.

Walker Phase Advancer.—In the Leblanc phase advancer the voltage injected into the induction motor rotor leads the inducing current by 90°, and this is not always desirable, inasmuch as it may introduce a component which is in opposition to the slip E.M.F. of the induction motor. For this reason a smaller angle of lead of the injected E.M.F. is often required. This is brought about in the Walker phase advancer, which re-introduces the stator winding in order to effect this.

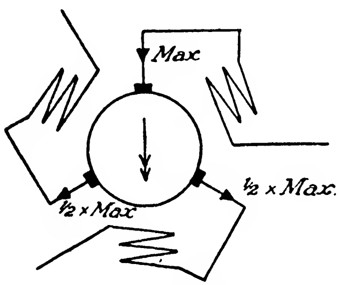


FIG. 381.—Walker Phase Advancer.

Consider the three-phase bipolar machine shown in Fig. 381. At the instant when the current entering by the top brush is a maximum, the current leaving each of the other two brushes is at half its maximum value. The current in the bottom section of the winding is zero and that in each of the other two sections is half its maximum value. The magnetic axis of the armature is vertical at this instant. These three currents are now passed through a three-phase stator winding, the magnetic axis of which may be anything depending upon the relative positions of the stator coils and the brushes on the commutator.

If the stator magnetic axis be vertical at the same instant as that of the rotor, then the conductors in the bottom section of the winding in Fig. 381 would be cutting the stator flux at the maximum rate and would have their maximum E.M.F. induced in them at this instant. But this is the instant when the current flowing through this section is zero, so that the induced voltage is in quadrature with the current. Whether the volts lead or lag depends upon the direction of rotation.

If now the resultant magnetic axis be displaced by 60° in the correct direction, the current in the bottom section will lead by only 30° instead of by the original 90°. The three free ends of the stator windings are now connected to the induction motor slip rings as before.

This phase advancer, which is series-connected, is connected as shown in Fig. 382. On light loads the main rotor current is low, so that the exciting ampere-turns are greatly reduced, with the result that there is very little flux and very little injected E.M.F. On light loads, therefore, the improvement in power factor is much less than in the neighbourhood of full load.

Scherbius Phase Advancer.—This phase advancer, which is provided with shunt excitation, does not suffer from the disadvantage of low injected E.M.F. at light loads, the phase advancing effect being independent of load. The connections are shown in Fig. 383.

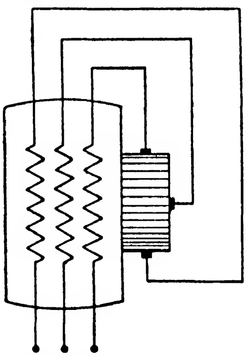


FIG. 382.—Series Phase Advancer (Walker).

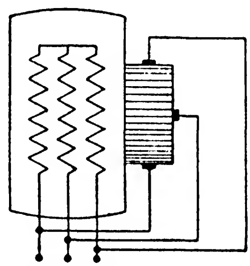


FIG. 383.—Shunt Phase Advancer (Scherbius).

The flux set up by the stator windings of the phase advancer is now proportional to the E.M.F. applied to the terminals, *i.e.* to the slip E.M.F. of the main induction motor, and inversely proportional to the frequency. As the slip increases, so does the slip E.M.F., but the slip frequency increases in the same proportion, so that the flux remains constant. The phase advancer E.M.F., which is injected into the rotor circuit, is therefore constant and independent of the slip, and so is independent of the load. By altering the position of the brushes, the phase of the injected E.M.F. can be controlled, so that this can be given any desired angle with respect to the current.

During the starting period, the phase advancer is cut out of circuit, and the induction motor slip rings are connected to an ordinary rotor starter by means of a three-pole change-over switch. A typical diagram of connections is illustrated in Fig. 384 which shows the phase advancer driven by an auxiliary induction motor.

When the motor reaches full speed, the change-over switch is thrown over into the other position. The field regulator in the stator circuit now enables the power factor to be adjusted as desired.

An alternative method of starting employs only a simple three-pole switch in the rotor circuit in addition to the rotor starter. The slip rings of the induction motor are now permanently connected to the starter terminals, and the motor is brought up to speed by gradually cutting out the starter resistance. The rotor switch is then closed, this connecting the phase advancer in parallel with the starter. The starter handle is then moved back again, thus gradually inserting the phase advancer in the rotor circuit.

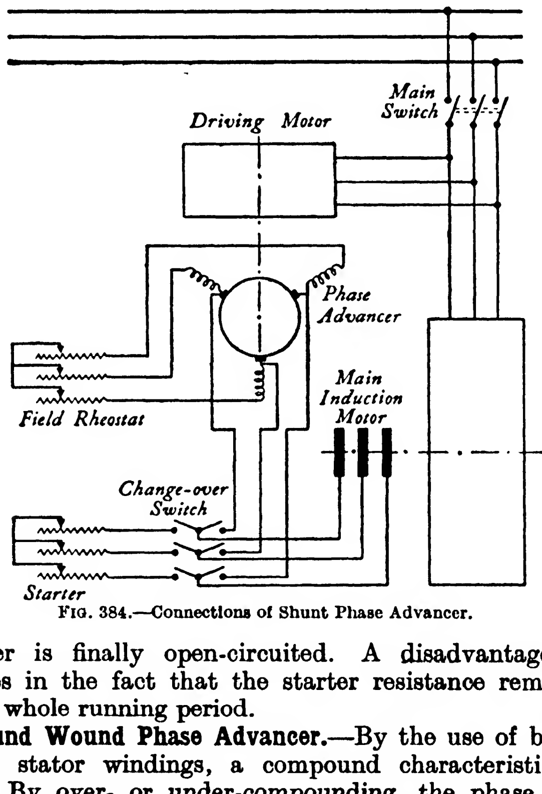


FIG. 384.—Connections of Shunt Phase Advancer.

The starter is finally open-circuited. A disadvantage of this method lies in the fact that the starter resistance remains alive during the whole running period.

Compound Wound Phase Advancer.—By the use of both series and shunt stator windings, a compound characteristic can be obtained. By over- or under-compounding, the phase advancer kVA can be made to increase or decrease with the load.

Frequency Converter as Phase Advancer.—If a phase advancer armature be provided with three slip rings (in addition to the commutator) instead of a stator winding, these slip rings being connected to the main supply, the slip ring currents set up a rotating field which rotates at (supply) synchronous speed *relative to the armature conductors.* This frequency is converted at the commutator into (stator frequency − frequency of speed) = slip frequency (see page 482). Whatever the speed of the main motor, this frequency is always exactly that of slip. The magnitude of the converted E.M.F. depends upon the slip ring voltage of the phase advancer, and since this is constant, the injected E.M.F. into the induction motor rotor windings is also constant in magnitude. The phase of the injected E.M.F. can be varied at will by altering the brush position. This frequency converter is therefore seen to have the same general characteristics as a shunt phase advancer of the Scherbius type.

Vector Diagram.—In Fig. 385 (*a*) let S_a represent the induced slip E.M.F. in the case where the E.M.F. of the phase advancer leads the current by 90°. This applies to the Leblanc type. The phase advancer E.M.F. is E_L, and the resultant E.M.F. acting on the rotor circuit after the phase advancer is installed is E_a. Neglecting all external resistances and reactances, the whole of this E.M.F. is available for producing the rotor current I_a leading the slip E.M.F. by an angle ψ. Without the phase advancer, and for the same torque developed by the induction motor, the rotor current is I_b, this being equal to the active component of I_a. The slip E.M.F. necessary to produce I_b is S_b, where $\frac{S_b}{E_a} = \frac{I_b}{I_a}$.

The rotor current has been advanced by an angle ψ and the lag of the stator current has been reduced by the same angle, which is not quite the same as raising the power factor by $\cos \psi$.

The case of the Walker phase advancer in which the E.M.F. leads the current by 30° is shown in Fig. 385 (*b*). All the other quantities in this vector diagram have the same significance as before, only E_w leads the current by 30°, whereas in the Leblanc case E_L led by 90°. It will be noticed that S_a is now less than S_b instead of greater. In order to make the two diagrams comparable the same currents have been chosen in both cases, necessitating the same slip E.M.F.'s with the phase advancer disconnected.

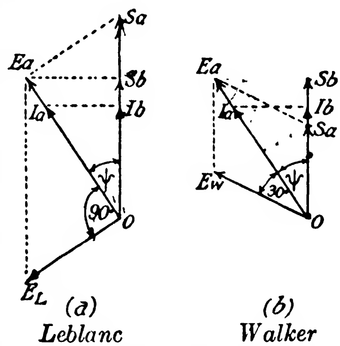


FIG. 385.—Vector Diagram of Phase Advancer.

Effect on Slip.—The slip of the main induction motor is affected by the introduction of the phase advancer. In the Leblanc case the slip E.M.F. has to be increased in the ratio $\frac{S_a}{S_b}$, and this brings about a similar increase in the slip. Therefore

$$\frac{\text{Slip after advancer is connected}}{\text{,, before ,, ,, ,,}} = \frac{S_a}{S_b} = \frac{1}{\cos^2 \psi}.$$

In the Walker case, where the angle of lead is 30°, the slip is reduced, and from the geometry of the figure it is found that now

$$\frac{S_a}{S_b} = \frac{1}{2\cos\psi\cos(60^\circ - \psi)}$$
$$= \frac{1}{\cos^2\psi + \frac{\sqrt{3}}{2}\sin 2\psi}.$$

The power necessary to increase the speed of the main motor comes from the phase advancer, the current of which now has an active component instead of being wholly reactive as in the Leblanc case. In fact the power factor of the phase advancer is cos 30° = 0·866.

The ideal power factor for the phase advancer to work on is $\cos(90^\circ - \psi)$, for this gives the minimum value for the injected E.M.F. and the smallest size of phase advancer.

kVA Capacity of Phase Advancer.—The output of the Leblanc phase advancer per phase is $E_L I_a$ volt-amperes, and

$$E_L = S_a \sin\psi$$
$$= S_b \frac{\sin\psi}{\cos^2\psi}.$$

Also $I_a = \frac{I_b}{\cos\psi}$.

Therefore $$\sqrt{3}E_L I_a = \sqrt{3}S_b I_b \times \frac{\sin\psi}{\cos^3\psi}.$$

But $\sqrt{3}S_b I_b$ is the copper loss in the main motor and is equal to motor input × fractional slip without phase advancer (see page 340). The kVA capacity of the phase advancer is therefore equal to

$$\text{kW input of main motor} \times \text{normal fractional slip} \times \frac{\sin\psi}{\cos^3\psi}.$$

The size of the phase advancer goes up rapidly as ψ is increased, as is shown in the following table, so that with this type it is inadvisable to attempt to set up too large an angle of lead.

ψ	0°	10°	20°	30°	40°	50°	60°
$\frac{\sin\psi}{\cos^3\psi}$ (Leblanc) ...	0	0·18	0·41	0·77	1·43	2·88	6·93
$\frac{\tan\psi}{\cos\psi\sin(30^\circ+\psi)}$ (Walker) ...	0	0·28	0·51	0·77	1·15	1·88	3·46

The output of the Walker phase advancer is $\sqrt{3}E_wI_a$ volt-amperes and, from Fig. 385 (*b*),

$$E_w = S_b \times \frac{\tan \psi}{\sin (30^\circ + \psi)}.$$

As before

$$I_a = \frac{I_b}{\cos \psi},$$

so that

$$\sqrt{3}E_wI_a = \sqrt{3}S_bI_b \times \frac{\tan \psi}{\cos \psi \sin (30^\circ + \psi)}.$$

The kVA capacity of the phase advancer is now equal to

$$\text{kW input of main motor} \times \text{normal fractional slip} \times \frac{\tan \psi}{\cos \psi \sin (30^\circ + \psi)}$$

The values of $\frac{\tan \psi}{\cos \psi \sin (30^\circ + \psi)}$ are also shown in the above table. For values of ψ less than 30° the Leblanc advancer is the smaller, but for the larger angles the advantage lies with the Walker or with a Scherbius (shunt) machine.

The main advantage possessed by the phase advancers described above over the over-excited synchronous motor is that they are machines of relatively small output. The reason for this is that the phase advancer stands in the same relation to an induction motor as an exciter does to a synchronous motor. An exciter of comparatively small capacity can over-excite a synchronous motor so as to make it supply a reactive load fifty times as great, measured in kVA, as the rating of the exciter. A phase advancer of only 30 kVA capacity is capable of effecting a total change of 1000 reactive kVA in the main circuit, since it only deals with the rotor circuit of the induction motor, and the kVA in the rotor circuit is small compared with that drawn by the stator circuit from the supply.

Examples

(1) A synchronous motor having an efficiency of 86 per cent. delivers a load of 120 B.H.P. Its power factor is 0·6 (leading), and it runs in parallel with a group of motors taking a total of 300 kW at 0·75 power factor (lagging). What is the resultant power factor of the whole system?

(2) A 10 B.H.P., 220 volt, three-phase, 50 cycle induction motor has a full load efficiency of 85 per cent. and power factor of 0·82. The power factor is to be raised to unity by static condensers connected in delta across the supply mains. Calculate the total capacitance required.

(3) A sub-station takes a load of 10,000 kVA at 0·75 power factor. It is desired to raise this to 0·9 by the aid of a synchronous condenser the losses of which are 7 per cent. of its kVA output. Determine the capacity of this set.

(4) A 500 B.H.P. induction motor runs normally with a 2 per cent. slip and an efficiency of 93 per cent., and is to be fitted with a phase advancer either of the Leblanc or of the Walker type. Determine the kVA capacity of this in order to make the rotor current lead by 40°. Also determine the new values of the slip.

(5) A three-phase induction motor takes a line current of 20 amperes at 415 volts, the power factor being 0·8. An over-excited synchronous motor is to be run light at 0·2 leading power factor, in parallel with the induction motor. What must be the capacity of the synchronous motor in order to bring the final total power factor up to 0·95 lagging?

(6) A 200 H.P., three-phase induction motor has a full load efficiency and power factor of 91 per cent. and 0·9 respectively. It is proposed to connect a synchronous motor to the same mains. This is to be over-excited so as to bring up the resultant power factor to unity. The synchronous motor is also to deliver a useful load, its power input to be 50 kW in these circumstances. The line voltage is 500 volts, and the resistance per phase and synchronous reactance per phase of the synchronous motor are 0·05 and 0·2 ohm respectively. What is its induced back E.M.F. per phase?

(7) What is the object in using an auto-transformer in conjunction with a bank of static condensers used for power factor improvement?

(8) Describe the construction of a phase advancer with no stator windings. Show that such a machine generates an E.M.F. in quadrature with the current, when driven above its own synchronous speed.

(9) Derive, from first principles, a formula to give the relative slips of an induction motor with and without a phase advancer of the type which generates an E.M.F. leading the current by 90°.

If the addition of the phase advancer makes the rotor current of the induction motor lead by 30°, calculate the slip, if the slip is 3 per cent. with the phase advancer removed from the circuit.

CHAPTER XXXIV

PROTECTION OF A.C. SYSTEMS

Classification of Protective Devices.—The importance of automatic protective devices has grown with the size of generating plants, and their systematic employment is now almost a necessity. Their functions are twofold, viz. (1) to protect the plant from damage in the event of a fault occurring, and (2) the localization of the fault and the preservation of the continuity of supply.

Protective devices may be classified under the following headings :—

(1) Devices which operate when the current or voltage exceeds or falls below certain predetermined limits, *e.g.* overload, minimum load, over-voltage and no-voltage relays.

(2) Devices which operate when the current or power flows in the reverse direction to that which is normal, *e.g.* reverse current and reverse power relays.

(3) Devices which operate when the current flows through abnormal paths, *e.g.* core balance and leakage relays.

(4) Devices which operate when the relative values of the current in different parts of the circuit become abnormal, *e.g.* the Merz-Price and Merz-Hunter split-conductor systems.

Oil Circuit Breakers.—High voltage A.C. circuits are broken by oil circuit breakers. They possess the great advantage of breaking the circuit near to the zero point on the current wave. When the switch is opening the arc goes out as the current falls to zero and the voltage has to rise to a certain value before the oil between the contacts is punctured.

In small and medium sizes all the poles are located in one oil tank, but in the larger sizes each pole is mounted in a separate tank. As the time of operating is usually not less than about one-tenth or one-twentieth second, it is seen that a number of complete cycles elapse before the circuit is broken, during which period the switch is subjected to large stresses, and it must be made strong enough mechanically to resist these.

Automatic breakers are opened by trip coils energized by means of relays. The tripping mechanism consists of a system of toggles and triggers so constructed that only a slight pressure is needed to open the breaker.

Relays.—Relays used for the purpose of operating circuit breakers have their connections divided into three circuits. The first is the primary circuit (usually E.H.T.) which is to be protected, connected in series with which is the primary of a current transformer. This primary winding very often consists of a single turn, or even of a main conductor merely linked through an iron core. The secondary circuit consists of the secondary winding on the current transformer and the relay operating winding. The third or tripping circuit may be either D.C. or A.C. (although it is usually D.C.), and consists of a source of supply, the trip coil on the circuit breaker and the relay contacts which are closed by a plunger operated by the secondary winding as explained above.

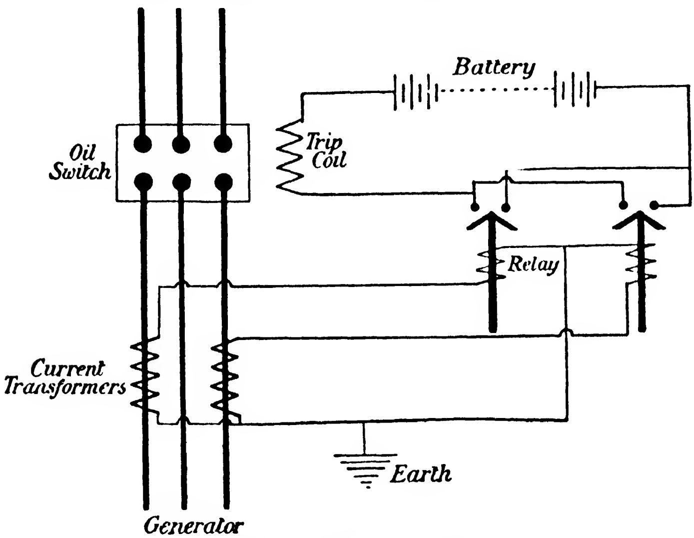


FIG. 386.—Relay Connections.

A typical set of connections for a three phase-circuit is shown in Fig. 386, which refers to a plain overload relay. It should be observed that only two of the three lines are directly protected, for if an excessive current flows in the other line it must return by one of the two protected lines, thus causing the relay to operate.

Time Setting.—Certain breakers are designed to operate practically instantaneously, but for a number of purposes this is undesirable. In the event of a momentary fault or a temporary overload the circuit should not be disturbed, so a time-lag is introduced into the relay. In these cases the time required to cause the relay to operate becomes shorter the greater the overload, and the time-lag varies almost inversely as the current. An increase in the current beyond a certain point, however, should not affect the time of operation.

A development consists in having a time-limit fuse shunted across the trip coil of the breaker, the trip coil being connected directly across the operating current transformer. Practically the whole of the current from the transformer normally passes through the fuse on account of the relatively high impedance of the trip coil. When the fuse blows, the current is diverted through the trip coil and immediately trips the switch. The time-lag is brought about by the fuse.

Overload Relays.—The usual type of overload relay operates on

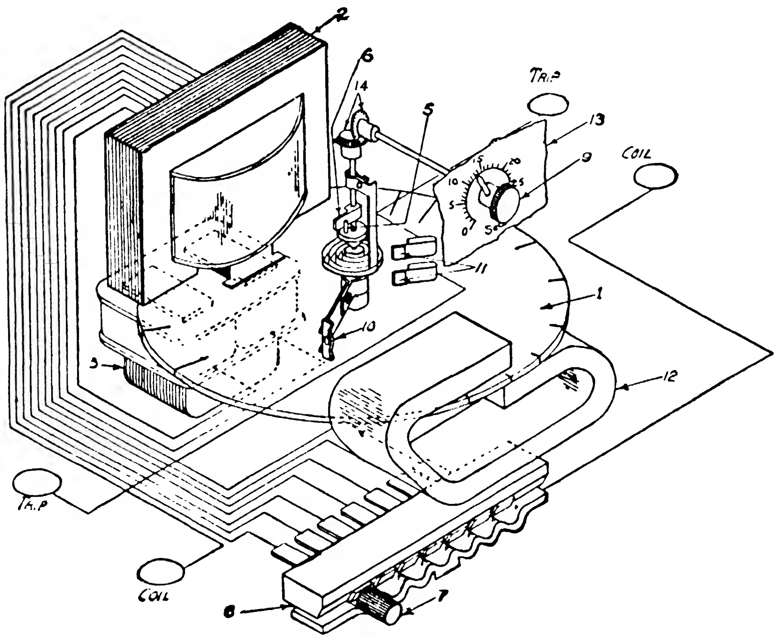


FIG. 387.—Induction Type Overload Relay. (B.T.H.)

the induction principle, a typical example being shown in Fig. 387. It consists essentially of a metallic disc (1) which is free to rotate between the poles of two electromagnets (2) and (3). The spindle of this disc carries a self-aligning moving contact (10) which bridges two fixed contacts (11) when the disc is rotated through an angle that is adjustable between 0° and 300°. The disc is normally held in the "off" position by a flat spiral spring, a pin (5) engaging with an adjustable stop (6). The time-setting of the relay is altered by means of the bevel gear (14), this being performed by turning the knob (9), a pointer indicating the position on the scale (13). An adjustable time-lag is thus obtained. A permanent magnet (12) provides the necessary damping.

The upper electromagnet (2) has a primary and secondary winding. The former is connected to the secondary of a current transformer in the line to be protected, this winding being tapped at intervals for adjustment of the current setting, by means of the pin (7) engaging in the plug bridge (8). The secondary winding is connected to the winding on the lower electromagnet, and the resultant flux produces the operating torque in conjunction with the eddy currents induced in the disc.

Reverse Power Relays.—The wattmeter principle is frequently adopted for reverse power relays, since the deflection of a wattmeter pointer reverses when the direction of transference of power is reversed. In the relay the pointer is replaced by a disc which rotates, thereby raising or lowering the trip coil switch contact. A diagram of connections of such a relay is given in Fig. 388. The

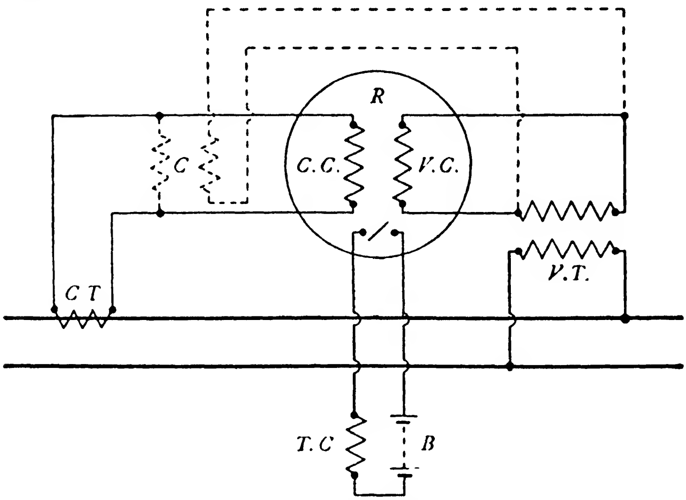


FIG. 388.—Connections of Compensated Reverse Current Relay.

relay, *R*, has a series and shunt coil fed from a current transformer, *C.T.*, and a voltage transformer, *V.T.*, respectively. When the relay contacts are closed the battery, *B*, sends a current through the trip coil, *T.C.*, so as to operate the circuit breaker and thus isolate the generator.

If a serious fall in line voltage should take place when a fault occurs, the current through the voltage coil, *V.C.*, will fall appreciably and the relay may fail to operate. Compensation for this is obtained in the following way. An auxiliary compensating transformer, *C*, has one winding connected across the secondary of the current transformer, *C.T.*, and its other winding across the secondary of the voltage transformer, *V.T.* (These connections are shown dotted in Fig. 388.) Normally the turns of these two windings are so proportioned that only a small current of predetermined value flows through the winding in the circuit of the current transformer, *C.T.* This current is diverted from the current coil, *C.C.*, the current through which is reduced accordingly. If, now, a fall in line voltage should occur, the voltage operating the voltage transformer, *V.T.*, is reduced, and the voltage across the second winding of the compensating transformer is also reduced. The current through the compensating transformer is therefore reduced, and since the total current delivered by the current transformer, *C.T.*, is unchanged, it follows that a larger current is actually passed through the current coil, *C.C.*, of the relay. The increase in the current through the current coil, *C.C.*, is designed to compensate for the reduction in current through the voltage coil, *V.C.* Such a compensated relay is now a reverse *current*, rather than a reverse *power*, relay.

Parallel Feeders and Ring Mains.—The feeders which serve to distribute energy to a number of points may be connected in several ways. The simplest way is to connect the point to the source of supply through a single independent feeder. (Really such a feeder would consist of a number of lines corresponding to the number of phases, but it is convenient for the present purposes to consider it as a single feeder.) The next way is to employ duplicate feeders in parallel, this permitting the employment of certain types of protection. When duplicate feeders are applied to overhead lines, the two conductors at the same potential may be carried on the same insulator, being only lightly insulated from each other, or the poles may be provided with six insulators mounted on three cross-arms. The upper and lower insulators on one side, together with the middle insulator on the other side, form a delta arrangement, this being duplicated by the other three lines.

The supply to two neighbouring points may be duplicated by the aid of an interconnector between them. The direction of energy-flow in this interconnector depends upon the distribution of the load. Trunk mains linking up two generating stations are really interconnectors.

The ring main is another method of connection, the various distribution points being connected to one another and to the source of supply at each end, so that each line forms a closed ring. This system is really a combination of feeders and interconnectors, the latter forming the connections between any two of the distribution points.

Core Balance Leakage Protection.—To protect a feeder or a generator against leakage to earth a core balance leakage relay is employed. A three-phase generator with an earthed neutral is connected to a feeder, the three cores of which are surrounded by the iron circuit of a current transformer. This has a single secondary winding which is connected to the relay (see Fig. 389). If there is no earth fault on the system, the sum of the three main currents is zero and there is no resultant flux in the core of the current transformer, no matter how much the load may be out of balance. The sum of the three main currents, however, is no longer zero if an earth fault appears on any phase, in which case a current is induced in the transformer secondary, causing the relay to operate. Sometimes core balancing is obtained by the use of three separate current transformers, the three secondaries being connected in parallel. This arrangement enables the same transformers to be employed as are used to operate the overload relays and the instruments.

If the earth lead from the generator is also passed through the core balancing transformer, then the relay is unaffected by a feeder fault, but is still affected by a fault on the generator or on the machine side of the transformer.

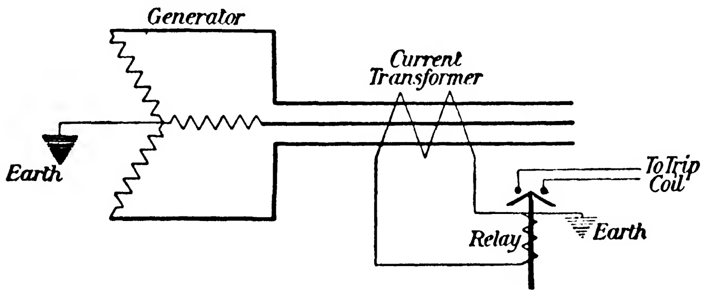


FIG. 389.—Core Balance Leakage Protection.

Instead of passing the earth lead through the current transformer, the six ends of the alternator armature winding may be brought out. The two ends of each phase are then passed through a current transformer, three of which are now required. A star point is formed from three ends, this being earthed, whilst the other three form the main leads. The three secondaries are connected in star and are taken to the relay, which also has three windings connected in star. This forms a very efficient protection for the generator.

An earthed neutral system can also be protected from leakage faults by means of a current transformer linking with the earth lead, the secondary of which is connected to the relay.

Combined Core Balance and Overload Protection.—The core balance protection outlined above suffers from the disadvantage that if the fault either starts as, or immediately develops into, a short-circuit between phases, there is no out-of-balance current and the relay will not operate. This disadvantage is overcome in the following method of combined core balance and overload protection. Two overload relays and one leakage relay are connected as shown in Fig. 390. These two overload relays are sufficient to protect all three phases (see page 544), whilst the leakage relay shown in the diagram receives the resultant current from all three transformers, this being zero unless there is a leakage fault on the system.

The two overload relays are each shunted with fuses. This imparts a certain amount of time-lag to the relay, but if the fuse

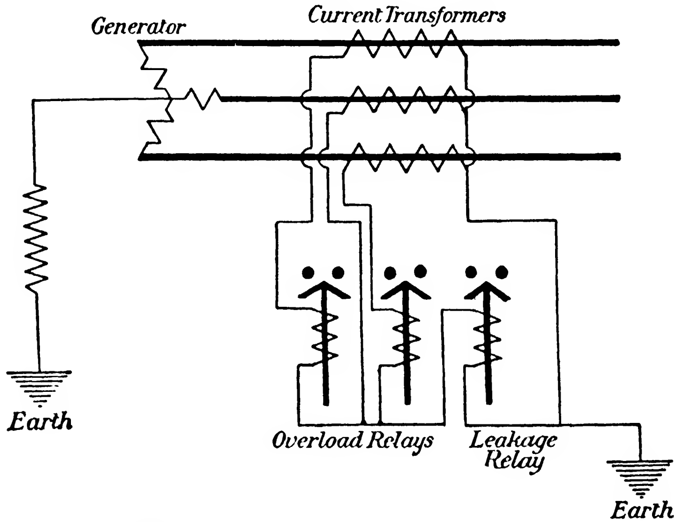


FIG. 390.—Combined Core Balance and Overload Protection.

blows, the full current is diverted through the relay which immediately operates.

Merz-Price Gear.—The Merz-Price protective gear has for its object the isolation of a faulty feeder when a breakdown occurs, the relays not operating in the case of a surge or a temporary overload. The feeder to be protected has a small current transformer

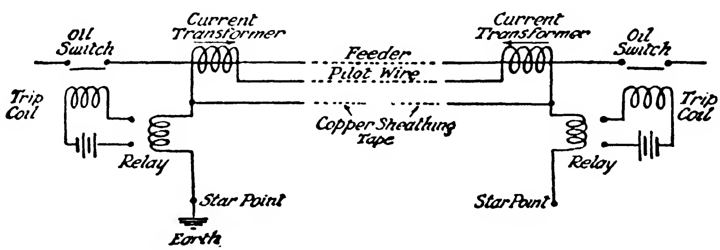


FIG. 391.—Arrangement of Merz-Price Gear. One phase only shown.

placed in series with it at each end, the two secondaries being joined in series with one another, but in opposition, and in series with two relays, as shown in Fig. 391. This diagram really refers to one phase of a three-phase system, but in its simplest form the two star points would be joined by another pilot wire. This necessitates a small auxiliary cable joining the two relays, which are situated at opposite ends of the feeder. Since the two current transformers are similar and in opposition, they balance one another for all loads, and the relays do not open the circuit breakers even on an overload. But if a fault develops on the feeder, the current flowing out at the far end is no longer equal to the current flowing in at the near end, and the two secondaries no longer balance each other. The resulting current in the relay circuit now operates the circuit breaker and isolates the faulty feeder. In the case of a feeder linking up two sub-stations, the current may flow either way in normal circumstances, and thus a reverse current relay would not be admissible, whilst this apparatus would protect the circuit.

In the case of a three-phase system, each feeder has its own pair of current transformers and relays, these being connected by a three-core pilot cable. The other ends of the current transformers are connected in star, the neutral point being earthed at one end only so as to prevent any current flowing between the star points, as this might operate the relays.

Beard-Hunter Shielded Pilot Cable.—The ordinary Merz-Price protective gear sometimes operates wrongly in the case of long feeders, as a heavy momentary overload might set up a capacitance current in the pilot wire of sufficient magnitude to operate the relays. This difficulty is overcome by surrounding each core of the pilot wire with a metal sheath in which is an open circuit (see Fig. 391). The current transformers are directly connected to the pilot wire on the one side and to the relays on the other, and the sheath is connected at each end to the relay side of the current transformer. Any induced capacitance current now flows from the core to the sheath and is entirely diverted from the relay. The open circuit in the sheath, however, does not prevent the operation of the gear in the event of a fault.

Merz-Price Generator Protection.—The Merz-Price voltage balance system is largely used for the protection of cables and can also be used for the protection of generators. For the latter purpose, however, the circulating current system is better suited. In this system the current transformers are connected in such a manner that their secondary currents are circulated in the pilot wires as shown in Fig. 392. The relays are connected across equipotential points in the pilot wires connecting each pair of current transformers. There are an infinite number of these, but unless the pilots are very long, their impedances are negligibly small compared with the internal impedances of the current transformers, and consequently the equipotential points need not be carefully chosen. The same current transformers can be used to actuate the working coils of instruments, the latter being connected in series with the pilot wires, but in this case it must be remembered that since an instrument so connected represents an additional impedance, this must be balanced by a corresponding impedance in the other phases.

As long as the generator winding is sound, each current transformer circulates the same secondary current and the equipotential points are maintained. Consequently no current flows in the relay coil. If an earth occurs in the generator winding, however, the secondary currents differ, the equipotential conditions are upset, and a current flows in the relay winding which actuates the oil switch mechanism. The generator can also be protected against

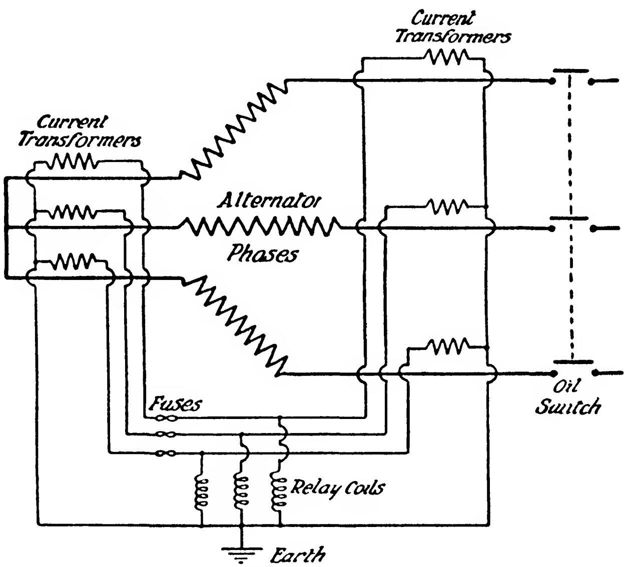


FIG. 392.—Merz-Price Circulating Current System.

overloads by the addition of fuses in the pilot wires. Since the latter carry currents proportional to those in the generator windings, the fuses melt when a predetermined current passes through the generator windings and after the lapse of a suitable time interval.

This system can also be applied to transformer protection, although complications arise due to the magnetizing currents and, where present, the phase difference on the two sides, as in delta-star transformers.

Merz-Hunter Split Conductor System.—This is another system for automatically protecting feeders from the effects of faults. Each phase of the feeder is divided into two equal conductors, known as splits, each of which is supposed to carry half the current. The connections are shown in Fig. 393, the three phases being split at the oil switch. The two split conductors forming one phase are then taken to a current transformer, *C.T.*, having two primaries linked with the core so as to be in opposition. As long as there are equal currents in the two conductors, the two primary windings balance each other and no current flows through the secondary winding of the transformer. The relay, *R*, therefore, is unaffected. Should any leakage occur on one of the splits, however, the currents are no longer equal and the secondary winding operates the relay. As the equipment shown in Fig. 393 is provided at each end of the feeder, the latter is isolated from the remainder of the system.

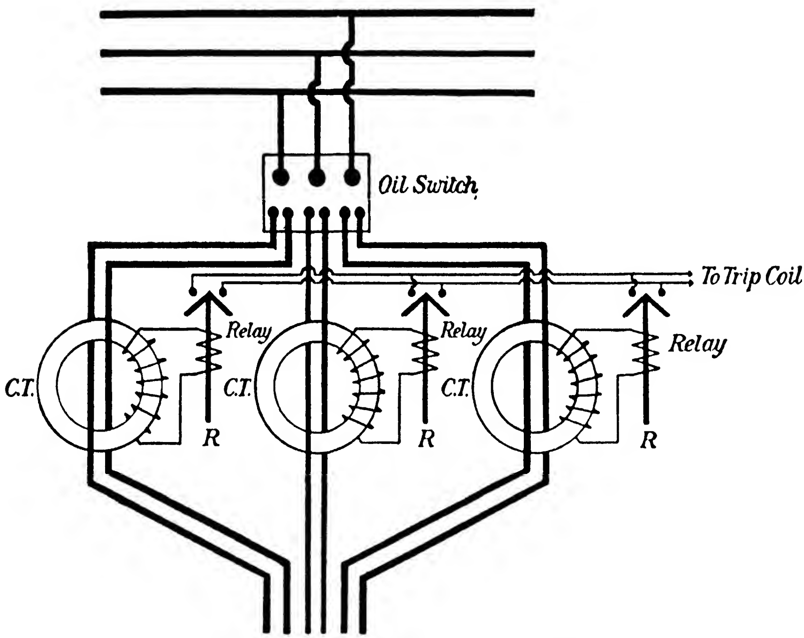


FIG. 393.—Merz-Hunter Split Conductor System.

The splits are carried into the oil switch contacts for the following reason. Suppose this were not done and that a fault develops very near to the far end of the cable. The impedance of the differential current transformer at this end may be insufficient to unbalance seriously the total current carried by each split conductor, but the oil switch at this end would operate because the current in the faulty split would reverse. The fault current would then be divided practically equally between the two splits, since they are assumed to be solidly connected at the oil switch. As a consequence the oil switch remote from the fault would not operate. This does not occur if the splits are disconnected in the oil switch itself, since the fault current is confined to the faulty split after the opening of the near oil switch. The opening of the remote oil switch then follows. In any case the split oil switches conduce to the more sensitive operation of the gear even if the impedance of the differential current transformers is sufficient to unbalance the currents in the two splits.

A three-phase feeder is often made with the splits arranged concentrically (they are really ovals), the three pairs then being placed as in a three-core cable. In another design the three-phase feeder is made with six equal cores and the two splits forming each phase are situated at opposite ends of a diameter. This avoids the possible danger that a fault on one split might spread to the other split of the same phase, and thus bring about failure owing to equal faults being developed on both splits. If the fault does spread it affects the other phases, thus bringing the other relays into action. A further advantage of the diametrical arrangement is that it tends to reduce any inequality in the currents in the two splits due to any difference in their impedance. Precise equality of impedance cannot altogether be secured, and if the difference is not kept small the relays may operate on a momentary overload, even on a healthy feeder.

This system possesses the advantage that cables of standard design can be used and that no pilot wires are required.

Impedance Protection.—The theory of the impedance type relay depends upon the fact that, since the impedance of a given feeder or section of line has a constant value, the relation between the fault current on a dead earth and the voltage existing between any two points on the faulty line or lines is known. The impedance between the relay and the fault depends upon the electrical distance between them, and is measured by the ratio of the voltage at the relay to the fault current flowing.

In its simplest form, the impedance relay has an operating force proportional to the fault current and a restraining force proportional to the line voltage at the relay. The relay operates whenever the ratio of this voltage to the fault current falls below a certain value, *i.e.* whenever a fault occurs within a given predetermined distance from the relay. Such a relay is, therefore, suitable for discriminative protection, since it operates on the occurrence of a fault within the section for which the relay is set, and is inoperative with faults occurring outside this section. It is, however, essential to have a controlled time-lag, so that the relay nearest to the fault operates first, thus localizing the disturbance. If this relay fails to clear the fault, the next nearest relay then operates, and so on. The time-lag is made proportional to the distance from the fault by designing the relay so that it has a time-lag characteristic that varies directly as the line voltage at the relay, and inversely as the fault current passing.

When a short-circuit occurs, there is a steady fall of voltage along the line between the source of supply and the fault. This voltage gradient can be utilized to grade the time-lags of the relays controlling a number of switches along the line. The relays automatically adjust their time of operation to their distance from the fault. The time taken to clear a fault is thus very low, and in this respect is superior to the system employing plain current overload relays. In the latter case, the time taken to clear a fault

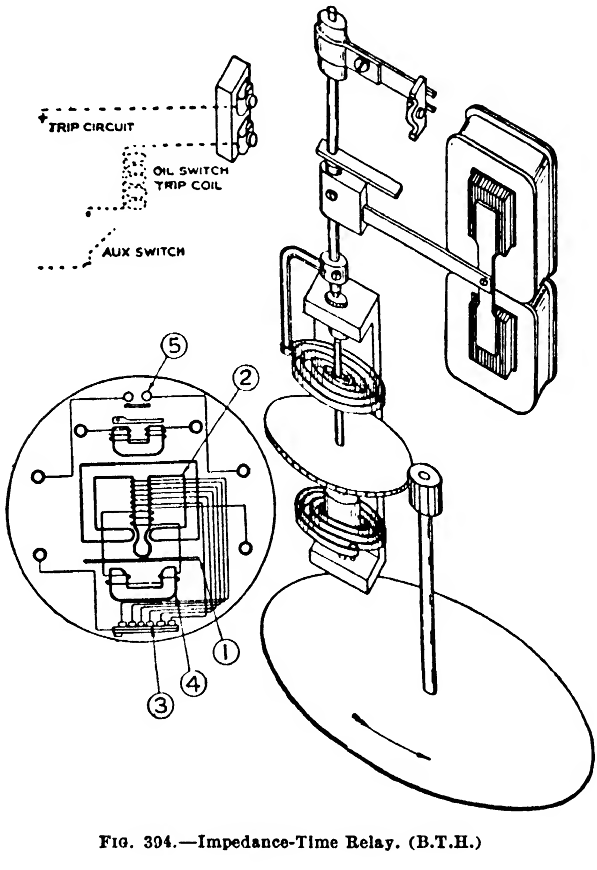


FIG. 394.—Impedance-Time Relay. (B.T.H.)

varies with its position, being a maximum at the power station. Again, if graded overload relays are used with a large number of switching points, the maximum time-lag tends to become larger than desirable.

Impedance-time Relay.—The impedance-time relay consists of two distinct parts, viz. a current driven induction type operating element, and a voltage-operated restraining element. The arrangement is shown in Fig. 394. The induction type element consists of a non-magnetic metallic disc (1), which can rotate between two electromagnets (shown in left-hand portion of diagram). The upper electromagnet (2) has two separate windings, the primary being connected to the secondary of a current transformer in the line to be protected. This primary winding is provided with a number of tappings, so that the current setting may be varied, these tappings being connected to a number of plugs (3). The secondary winding on the upper electromagnet energizes the coil on the lower electromagnet (4). The resultant magnetic flux due to these two electro-magnets reacts with the eddy currents induced in the disc and sets up a torque tending to rotate the disc against the braking effect of a permanent magnet. The speed of the disc is therefore proportional to the driving torque (see page 158).

The spindle of the disc is geared to a countershaft (see right-hand portion of diagram), and to this is attached the inner end of a spiral spring. The outer end of this spring is attached to a bent lever controlling the position of another separate but co-axial shaft. Carried by this second shaft is a flat armature of soft iron normally held against the face of the restraining magnet, the latter being shown at the top of both portions of the diagram. This magnet is energized by the voltage coil.

The function of the spring is to provide the time discrimination which is one of the most important features of the relay. The pull of the spring acts in opposition to the holding-on force provided by the voltage element, and the latter varies with the voltage gradient due to the fault current. The relay contacts (5) are shown at the top, and when these are closed, the trip coil of the oil switch is operated.

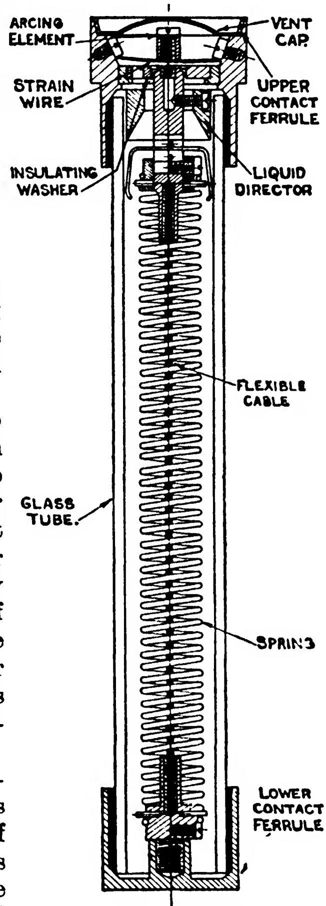


FIG. 395.—Carbon Tetrachloride Fuse.

E.H.T. Fuses.—The great difficulty with E.H.T. fuses lies in effectually extinguishing the arc which is formed when they operate. For this purpose very long fuses are employed, often encased in porcelain tubes which sometimes reach a length of several feet. These porcelain tubes are apt to become broken by the violence of the explosion on a sudden short-circuit, and to reduce the chance of this they are sometimes lined with plaster of Paris, which serves as a buffer.

The carbon tetrachloride fuse is very effective, the general construction being shown in Fig. 395. The fuse wire, under the constant tension of a spring, is mounted inside a sealed glass tube filled with carbon tetrachloride, the object of which is to extinguish the arc formed when the fuse blows.

The use of E.H.T. fuses is now practically confined to voltage circuits.

Protection of Current Transformers.—To protect current transformers against the effects of over-voltage due to a surge or other cause, they are sometimes shunted by a non-inductive resistance. Owing to the reactance of the transformer windings, a sudden rush of current passes very largely through the resistance and leaves the current transformer unharmed.

Instruments.—For voltmeters in use on E.H.T. circuits the usual practice is to operate them through voltage transformers [see Fig. 178 (*a*)], the scales of the instruments being marked so as to indicate the line voltage. Each voltmeter must be calibrated in conjunction with its own particular voltage transformer.

In a similar way, ammeters are run from the secondaries of current transformers [see Fig. 178 (*b*)], so that the instruments themselves are not in electrical contact with the E.H.T. system.

In the case of wattmeters and watt-hour meters, the voltage coils are treated like voltmeters, being run through voltage transformers, and the current coils are treated like ammeters, being run through current transformers.

Earthing.—The advisability of earthing A.C. systems has given rise to much discussion. In the case of single-phase railway systems an earthed return is used, the line wire being insulated. In polyphase transmission schemes it is the neutral point which is earthed except in those cases where the whole system is left insulated. With perfectly balanced loads the neutral point automatically assumes the earth potential, but these conditions are by no means always fulfilled. The potential of the neutral point then tends to assume a value other than that of the earth, and this is prevented by putting the two points in electrical contact. This tends to maintain equality in the capacitance currents to earth in the different phases and prevents the consequent unbalancing of the voltages. It also tends to maintain equality in the maximum stress in the insulation to earth, the magnitude of which would be increased if the potential of the neutral point were allowed to alter.

On the other hand, a single earth fault on the system is sufficient to cause an interruption of the supply, which is not the case if the whole system is insulated throughout in normal circumstances. In order to limit the short-circuit current resulting from such a fault, it is usual to insert a resistance or a choking coil in the earth connection from the neutral point.

The earthing resistance may be connected in several ways. The simplest way is to connect the neutral points of all the generators to a common bar which is connected to earth through a common resistance. The disadvantage of this method is that there is no check on the triple harmonic current which may flow between the various generators. This can be overcome by providing each generator with its own earthing resistance. The disadvantage of this method is that the value of the resistance to earth depends upon the number of generators which are running. Both the above disadvantages can be overcome to a large extent by a combination of the two methods. The neutral point of each generator is connected to a common bar through individual resistances, this bar being earthed through a common resistance. A further method is to use one earthing resistance only, this being connected to one of the running generators. This method involves the risk of the operator forgetting to transfer the resistance to another generator when one is shut down, thus leaving the system insulated.

Petersen Coil.—The Petersen coil, or arc suppression coil, consists of a specially designed reactor which is connected between the neutral point on the secondary side of a high voltage transformer feeding a transmission line, and earth, its object being to suppress any arc which may result from an earth fault developing on any one of the three line conductors. Each of these line conductors has a capacitance to earth distributed along its length, this being assumed to be the same for all three phases, and the normal conditions can be represented by Fig. 396 (*a*). The vector sum of the three capacitive currents, I_c, is equal to zero, and no current normally flows through the Petersen coil.

Now consider the effects of an earth fault at $A \cdots B$, as shown at Fig. 396 (*b*). If the Petersen coil were removed, leaving the star point isolated, one of the three hypothetical condensers is short-circuited, and the voltage across each of the other two has been increased $\sqrt{3}$ times, being now line voltage. Each of the corresponding capacitive currents is therefore increased $\sqrt{3}$ times, and since these are now 60° out of phase, their resultant is three times the original condenser current per line in healthy conditions. This current to earth, which leads the voltage in the faulty phase by 90°, returns to the system by way of the fault. If the Petersen coil be now added, it will take a current lagging by 90° on the voltage of the faulty phase. The value of the inductance of the Petersen coil is now adjusted so that the current flowing through it to earth, I_1, is equal to $3I_c$, and since it is in exact phase opposition, the resultant of I_1 and $3I_c$ is zero, so that no current now flows through the fault, and no arcing occurs.

The effect of the Petersen coil is to alter the *potentials* of the three lines with respect to earth, without altering the *potential differences* between them. The potential of the faulty line becomes that of earth, the potential of the star point acquiring a voltage to earth equal to the phase voltage. The potential to earth of each of the two healthy lines becomes $\sqrt{3}$ times the normal value.

Surges.—Transmission lines are sometimes subjected to transient rises of pressure called *surges.* These may be brought about

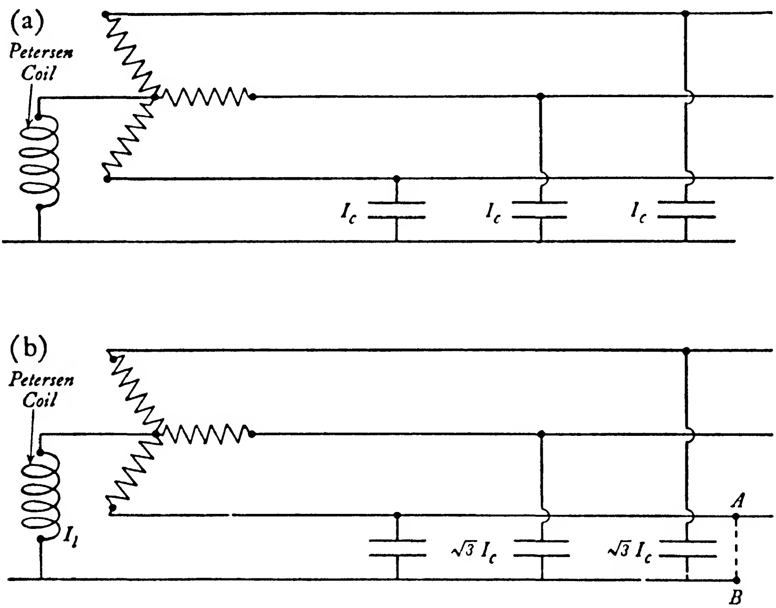


FIG. 396 (*a*) and (*b*).—Action of Petersen Coil.

by short-circuits, by sudden changes in the current or voltage on the line or by breaking the circuit, and are due to the sudden dissipation of the energy stored in the magnetic and electrostatic fields around the conductor. In the event of a short-circuit the line may be considered for the moment as consisting of a condenser in series with an inductance, the short-circuit being equivalent to closing the switch. Up to this point the condenser is charged, but no current flows. When the short-circuit occurs the condenser begins to discharge, causing a current to flow through the inductance, and this continues until the condenser is completely discharged. The energy stored up in the condenser, $\frac{1}{2}CE^2$ (see page 51), is now converted into an electromagnetic form and is equal to

$\frac{1}{2}LI^2$, but owing to the inductance of the line the current, I, cannot stop immediately, with the result that the condenser becomes charged up again, but this time in the reverse direction. The action is then repeated until all the energy concerned is frittered away in heating the circuit.

If, now, the short-circuit happens to occur at the moment when the current is at or near to its maximum value, the quantity, $\frac{1}{2}LI^2$, will be relatively large. This energy is transformed into an electrostatic form, $\frac{1}{2}CE^2$, and since C is fixed, the magnitude of the voltage to which the line is charged momentarily may be considerable. From this it follows that surges of a dangerous nature are more likely to occur on low voltage lines with heavy currents than on high voltage lines with relatively small currents.

The voltage and frequency set up by a surge depend upon the current flowing at the instant of interruption and upon the length of line connected.

Underground systems are not subjected to surges to the same extent as overhead lines on account of the fact that the capacitance of an underground line is usually greater than that of an overhead one, and hence E is less for a given value of $\frac{1}{2}CE^2$.

Line Disturbances.—In the case of overhead lines it is necessary to provide for a discharge to earth of potentials higher than that for which the line is intended, since the insulation of the various apparatus connected is insufficient. Disturbances may occur due to lightning, static charges or surges.

In the case of lightning the line may be either struck directly or it may be affected by a discharge near the line. A lightning discharge takes place extremely rapidly, and consists of a pulse, or pulses, with a very high rate of growth (steep wave front), and it may affect the line by inductive action. Owing to their frequency, induced E.M.F.'s due to this cause are very dangerous if allowed to act upon machinery, since ordinary inductances act practically as insulators, and the induced discharge frequently finds it easier to puncture the insulation than to overcome the impedance of the various windings.

Cases of a line being struck directly are fortunately very rare, since there is no adequate protection known. Cases of inductive effects due to discharges near the line are, however, frequent, but the ill effects of these can be warded off.

Static charges may accumulate on a line owing to the friction of snow or dust against the conductor and due to inductive effects from charged clouds. The same result can be set up by a sudden cooling of the atmosphere such as may occur at nightfall. Static charges usually accumulate slowly and are easier to deal with than lightning discharges on this account.

A third form of line disturbance is that brought about by surges (see page 558).

Lightning Arresters.—Protection against the above sources of trouble is afforded by the use of lightning arresters, the function of which is to provide an easy path to earth for high potentials and for high frequency currents without providing such a path for the ordinary working current.

Horn Arrester.—The simplest type of lightning arrester is the horn arrester, which consists of two metal rods bent into the form of horns and mounted in the same vertical plane on porcelain insulators so that the horns are close together at the base, the air-gap between them gradually increasing until the top is reached. One of these horns is connected to the line and the other one through a resistance to earth. The minimum air-gap is set so that a discharge does not take place at the ordinary working pressure, but at a predetermined excess voltage an arc is set up. This arc travels up the horns both on account of thermal and electromagnetic action, until finally it becomes so long that it breaks, thus automatically insulating the circuit again. Should the excess voltage on the line not be relieved, a second arc follows the first, and so on, until the line voltage becomes normal.

Lead Oxide Arrester.—In this arrester use is made of the change in resistivity which occurs in the case of certain inorganic salts on heating. The salt used is lead peroxide (PbO_2), which at ordinary temperatures has a resistivity of one ohm per inch cube. At 150° C. this peroxide is converted into red lead (Pb_3O_4), which has a resistivity of 24×10^6 ohms per inch cube. This property is made use of in the arrester, the practical form of which is shown in Fig. 397. Each cell consists of two iron electrodes of circular cross section, varnished on one side in such a manner that the varnish films break down when a voltage of approximately 400 volts is applied across the electrodes. The cells are stacked in columns, the number of cells depending upon the voltage setting required. When an over-voltage occurs on the line the varnish film breaks down at one or more points in each cell, and a discharge takes place through the lead peroxide, which, owing to the heating, is converted into red lead, after which the discharge ceases.

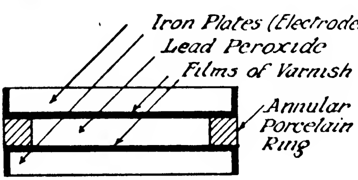


FIG. 397.—Lead Oxide Arrester.

In time the whole surface of the electrodes becomes covered with small plugs of red lead, which gradually increases the total resistance of the arrester, which has then to be re-conditioned. The dimensions are, however, so chosen that this takes some years with normal use.

In another type of construction a column of lead peroxide pellets is used, in series with a spark gap. When an arc occurs across this gap, due to an over-voltage, additional voltage is applied to the pellet column and a current flows to earth. After the surge has been dissipated, the voltage falls to normal. The resistance of the pellet column now increases until only a few milliamperes flow, this current being finally interrupted by the series spark gap.

Thyrite Arrester.—Thyrite consists of a particular kind of clay mixed with carborundum and fired over a definite temperature cycle. The variation in temperature throughout this cycle is extremely important, a very small deviation resulting in the loss of its special properties. This material possesses the remarkable property of being practically an insulator at one voltage and an excellent conductor at a higher voltage. Its electrical resistance appears to depend solely upon the voltage, the current increasing 12·6 times every time the voltage is doubled. Thyrite obeys this characteristic law apparently indefinitely and independently of the rate of application of the voltage, there being no time-lag.

The standard unit is rated for 11·5 kilovolts, and consists of eleven Thyrite discs in series, with a suitable arrangement of gaps, all assembled in a porcelain container. For higher voltages a number of such units are arranged in series.

Line Choking Coils.—Choking coils of a particular design and carrying the full line current are employed at the generator end of a transmission line for the purpose of protecting the apparatus behind it from the effects of a lightning discharge. A common design for such a choking coil consists of a simple helix of stout wire mounted in air upon two porcelain standards. At the normal frequencies of supply these are practically non-inductive and cause only a negligible voltage drop, but with the extremely high frequencies obtained in the case of a lightning discharge they produce a very strong choking effect, so that practically none of the induced current flows through them, but is dissipated by the lightning arresters instead.

Overhead Earthed Wire.—If one or more earthed conductors are run near to and parallel with the transmission line conductors, their effect is to reduce the line potential due to static charges and discharges. The capacitance between line and earth is increased by this expedient, whilst that between a charged cloud and the line remains unaltered. The static potential of the line is thus reduced, the overhead earthed wire acting as a valuable adjunct to the lightning arrester.

EXAMPLES

(1) What is meant by " reverse current " and " reverse power " ? Describe a relay which will operate under these conditions.

(2) Why is plain overload protection insufficient in the case of large generators ?

(3) Draw a skeleton diagram of connections showing how to protect a generator against overload, leakage and internal short-circuit.

(4) Describe two methods of feeder protection, one employing pilot wires and one with no pilots.

(5) Describe one form of lightning arrester, and explain the method of its operation.

CHAPTER XXXV

SYMBOLIC NOTATION

Symbolic Representation of Vectors.—Vector quantities can be expressed in algebraic form if the horizontal and vertical components of the vector are known. As the vector rotates these two alter in their relative value, according to the sine law, each becoming positive and negative alternately. In Fig. 398 let $OA = a$ represent to scale some quantity which, at the moment under consideration, has a positive value, since OA is drawn to the right from O. The same magnitude with a negative sign is represented by OA', or $OA' = -a$. To indicate that a is now negative, OA' (the length of which determines the arithmetic magnitude) is drawn to the left. By measuring off the same length in a vertical direction, other conditions may be implied. For example, in the ordinary vector diagram OB (which has the same length as OA) represents the same quantity, but advanced in phase by 90°. If OB is again advanced in phase by 90° it becomes equal to OA', and if yet another 90° be added to its phase, then OB' is obtained.

The j Operator.—To represent the fact that OA' is negative in direction, the minus sign is prefixed to the quantity, thus $OA' = -a$. In the same way let the symbol j be prefixed to the quantity to represent that it is drawn in the direction OB, or that it has been advanced in phase by 90°. Then $OB = ja$. Again advancing the phase by 90°, OA' is obtained, and this may be written $OA' = j \cdot OB = j(ja) = j^2a$. The symbol j is not a mere multiplier, but is called an *operator*, indicating that the quantity following must be treated in a certain way (*i.e.* rotated through 90° in an anti-clockwise direction). The symbol j^2 indicates that this must be done twice. This is the same as rotating the vector through 180°, or, in other words, reversing it. This again is equivalent to multiplying it by -1, so that $j^2 = -1$ and $j = \sqrt{-1}$. This is an imaginary quantity, since it cannot be evaluated algebraically, but nevertheless it is represented graphically in the diagram by $OB = ja$. In the same way OB', drawn vertically downwards, is equal to $-ja$, since it is obtained by adding 90° in phase to OA', or by multiplying $OA' = -a$ by j. Multiplying by $-j$ is equivalent to retarding the quantity in phase by 90°. Division by j gives the same result, since $\frac{1}{j} = \frac{j}{j^2} = \frac{j}{-1} = -j$.

All horizontal components are called real quantities and all vertical ones imaginaries. (Any line, however, may be considered as the horizontal base.) Generally a vector has both horizontal and vertical components, but it is a simple matter to resolve such a vector into its quadrature components as shown in Fig. 399, when OP may be written

$$OP = a + jb.$$

This indicates that the point P must be moved from O by a units in the horizontal direction to the right and by b units vertically upwards. The actual numerical value of OP is equal to $\sqrt{a^2 + b^2}$, and the inclination of the vector to the horizontal is given by $\tan\,\theta = \dfrac{b}{a}$. A vector in the second quadrant is illustrated by

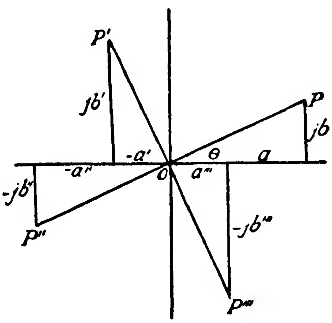


FIG. 398.—Real and Imaginary Quantities.

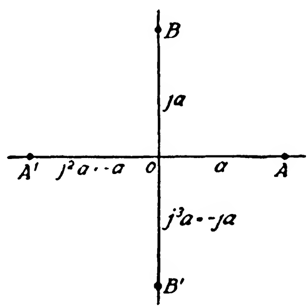


FIG. 399.—Real and Imaginary Components of Vectors.

$OP' = -a' + jb'$. Let OP' be obtained by rotating OP through 90° in an anti-clockwise direction. Then $OP' = j \cdot OP$ and

$$\begin{aligned} -a' + jb' &= j(a + jb) \\ &= ja + j^2 b \\ &= -b + ja. \end{aligned}$$

From this it is seen that $a' = b$ and $b' = a$. Similarly a vector in the third quadrant is expressed by $OP'' = -a'' - jb''$ (the real component being negative and the imaginary one also negative or drawn downwards). A vector in the fourth quadrant is expressed by $OP''' = a''' - jb'''$.

Again, let $r = \sqrt{a^2 + b^2}$, numerically.

Then $a = r\cos\theta$,

and $b = r\sin\theta$.

Therefore

$$\begin{aligned} OP &= a + jb \\ &= r(\cos\theta + j\sin\theta). \end{aligned}$$

Complex Quantities.—A quantity expressed in the $a + jb$ form is called a complex quantity, the a term being real and the jb term imaginary.

Addition of Complex Quantities.—This is performed by simply adding their real and imaginary parts respectively. Suppose $OA = a + jb$, and $OB = c + jd$ represent two vectors. Their sum is then given by

$$OA + OB = (a + c) + j(b + d).$$

The magnitude of the resultant is equal to

$$\sqrt{(a + c)^2 + (b + d)^2}$$

and the slope is given by

$$\tan\theta = \frac{b + d}{a + c}.$$

Subtraction of Complex Quantities.—The difference between two complex quantities is given by

$$OA - OB = (a - c) + j(b - d),$$

the magnitude of the resultant is

$$\sqrt{(a - c)^2 + (b - d)^2}$$

and the slope is given by

$$\tan\theta = \frac{b - d}{a - c}.$$

Multiplication by a Complex Quantity.—The multiplication of a vector by a complex quantity, $a + jb$, results in the original vector being lengthened in proportion to the factor $\sqrt{a^2 + b^2}$ (known as the *tensor*), and in being advanced in phase by an angle $\theta = \tan^{-1}\frac{b}{a}$ (this rotating factor being known as the *versor*).

Division by a Complex Quantity.—Dividing a vector by a complex quantity reduces its length in the ratio $\frac{1}{\sqrt{a^2 + b^2}}$, and retards it in phase by an angle $\theta = \tan^{-1}\frac{b}{a}$.

Polar Representation.—Instead of specifying a quantity in terms of rectangular co-ordinates, *i.e.* in terms of a and b, it is sometimes preferable to express it in terms of polar co-ordinates, *i.e.* in terms of r and θ, where $r = \sqrt{a^2 + b^2}$ and $\tan\theta = \frac{b}{a}$. It is then written in the form $r\underline{/\theta}$. For example, if $a = 1$ and $b = \sqrt{3}$, then $r = \sqrt{1^2 + (\sqrt{3})^2} = 2$, and $\theta = \tan^{-1}\frac{\sqrt{3}}{1} = 60°$, and it would be written $2\underline{/60°}$.

Symbolic Expression for Impedance.—If I be the R.M.S. value of the current, then the applied voltage (R.M.S. value) is given by $E = IZ$, and the voltage leads the current by an angle $\phi = \tan^{-1}\frac{X}{R}$. The voltage vector consists of two quadrature components IR and IX, so that the voltage may be expressed (see Fig. 400) by

$$\underset{\cdot}{E} = \underset{\cdot}{I}\underset{\cdot}{Z} = \underset{\cdot}{I}R + j\underset{\cdot}{I}X.$$

(The symbols with the dots underneath indicate that the quantities are expressed in symbolic form.) From this it follows that

$$\underset{\cdot}{Z} = R + jX = Z(\cos\phi + j\sin\phi).$$

The actual numerical value is $Z = \sqrt{R^2 + X^2}$. The latter value gives no information respecting the relative magnitudes of the two components, whereas the symbolic expression does give full information.

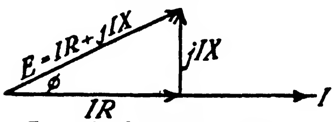


FIG. 400.—Impedance Diagram.

Impedances in series may not be added together numerically in order to obtain their resultant, if expressed in the ordinary way, but this is quite permissible when expressed in symbolic fashion. Thus

$$\underset{\cdot}{Z} = \underset{\cdot}{Z}_1 + \underset{\cdot}{Z}_2 + \ldots = (R_1 + R_2 + \ldots) + j(X_1 + X_2 + \ldots).$$

When capacitive reactance occurs, it must be regarded for the present purposes as negative in sign.

The phase of the current in Fig. 400 may have been such that the vector representing it must be drawn at some angle to the horizontal. In that case the current may be represented by

$$\underset{\cdot}{I} = I' + jI''.$$

The voltage is then represented by

$$\begin{aligned} \underset{\cdot}{E} = \underset{\cdot}{I}\underset{\cdot}{Z} &= (I' + jI'')(R + jX) \\ &= I'R + j^2I''X + jI'X + jI''R \\ &= I'R - I''X + j(I'X + I''R). \end{aligned}$$

Alternately, the current could be represented by

$$\underset{\cdot}{I} = I(\cos\theta + j\sin\theta).$$

The voltage is then given by

$$\begin{aligned} \underset{\cdot}{E} = \underset{\cdot}{I}\underset{\cdot}{Z} &= IZ(\cos\theta + j\sin\theta)(\cos\phi + j\sin\phi) \\ &= IZ[(\cos\theta\cos\phi - \sin\theta\sin\phi) + j(\cos\theta\sin\phi + \sin\theta\cos\phi)] \\ &= IZ[\cos(\theta + \phi) + j\{\sin(\theta + \phi)\}]. \end{aligned}$$

This shows clearly the lead of ϕ degrees resulting from the multiplication by Z.

Example of Series Circuits.—Assuming the data shown in Fig. 401, it is required to determine E_1 and E_2 in phase and magnitude.

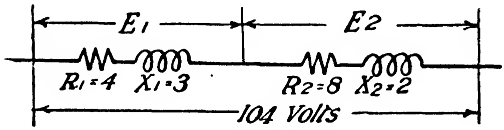


FIG. 401.—Example of Series Circuits.

$$Z = (4 + 8) + j(3 + 2) = 12 + j5.$$

$$I = \frac{104}{12 + j5}.$$

$$E_1 = IZ_1 = \frac{104}{12 + j5} \times (4 + j3)$$

$$= \frac{104(4 + j3)(12 - j5)}{(12 + j5)(12 - j5)}\text{[1]}$$

$$= \frac{104(48 + j36 - j20 + 15)}{144 + 25}$$

$$= 38{\cdot}7 + j9{\cdot}85$$

$$E_1 = \sqrt{38{\cdot}7^2 + 9{\cdot}85^2} = 40 \text{ volts.}$$

Phase angle $= \tan^{-1}\dfrac{9{\cdot}85}{38{\cdot}7} = 14{\cdot}25°.$

$$E_2 = IZ_2 = \frac{104}{12 + j5}(8 + j2)$$

$$= \frac{104(8 + j2)(12 - j5)}{169}$$

$$= \frac{8}{13}(96 + j24 - j40 + 10)$$

$$= \frac{8}{13}(106 - j16)$$

$$= 65{\cdot}2 - j9{\cdot}85$$

$$E_2 = \sqrt{65{\cdot}2^2 + 9{\cdot}85^2} = 66 \text{ volts.}$$

Phase angle $= \tan^{-1}\dfrac{-9{\cdot}85}{65{\cdot}2} = -8{\cdot}6°.$

These results should be compared with those obtained by ordinary vector diagram.

[1] When a complex number occurs in the denominator it is usually simpler to rationalize it.

Derivation of Admittance, Conductance and Susceptance.—In a series circuit

$$\dot{I} = \frac{\dot{E}}{\dot{Z}} = \frac{\dot{E}}{R + jX} = \dot{E} \times \frac{R - jX}{R^2 + X^2}$$
$$= \dot{E}\left[\frac{R}{R^2 + X^2} - j\frac{X}{R^2 + X^2}\right]$$
$$= \dot{E}[G - jB].$$

But $\dot{I} = \dot{E}\dot{Y}$,

so that $\dot{Y} = G - jB$.

$$G = \frac{R}{R^2 + X^2} = \text{conductance in mhos,}$$
$$B = \frac{X}{R^2 + X^2} = \text{susceptance ,, ,,}$$
$$Y = \sqrt{G^2 + B^2} = \text{admittance ,, ,,}$$

Parallel Circuits.—Let two parallel circuits possess admittances $\dot{Y}_1 = G_1 - jB_1$ and $\dot{Y}_2 = G_2 - jB_2$ respectively. Then $\dot{I}_1 = \dot{E}\dot{Y}_1$ and $\dot{I}_2 = \dot{E}\dot{Y}_2$. Also $\dot{I} = \dot{I}_1 + \dot{I}_2$.

$$= \dot{E}(\dot{Y}_1 + \dot{Y}_2)$$
$$= \dot{E}[(G_1 + G_2) - j(B_1 + B_2)]$$
$$= \dot{E}\left[\left(\frac{R_1}{R_1^2 + X_1^2} + \frac{R_2}{R_2^2 + X_2^2}\right) - j\left(\frac{X_1}{R_1^2 + X_1^2} + \frac{X_2}{R_2^2 + X_2^2}\right)\right]$$
$$I = E\sqrt{\left(\frac{R_1}{R_1^2 + X_1^2} + \frac{R_2}{R_2^2 + X_2^2}\right)^2 + \left(\frac{X_1}{R_1^2 + X_1^2} + \frac{X_2}{R_2^2 + X_2^2}\right)^2}.$$

The resultant current is thus obtained very simply. It should be noted that the minus sign now prefixes the imaginary part of the expression.

Example of Series-Parallel Circuit.—Consider the same circuit as formed the example on page 35 (see Fig. 26). The admittance of the two parallel branches is

$$\dot{Y}_1 = \frac{20}{20^2 + 30^2} - j\frac{30}{20^2 + 30^2} = 0{\cdot}0154 - j\,0{\cdot}0230$$

and
$$\dot{Y}_2 = \frac{5}{5^2 + 60^2} - j\frac{60}{5^2 + 60^2} = 0{\cdot}0014 - j\,0{\cdot}0166$$

respectively. The combined admittance of these two is

$$\dot{Y}_a = \dot{Y}_1 + \dot{Y}_2$$
$$= 0{\cdot}0154 + 0{\cdot}0014 - j\,(0{\cdot}0230 + 0{\cdot}0166)$$
$$= 0{\cdot}0168 - j\,0{\cdot}0396.$$

But since $G = \frac{R}{Z^2}$ and $B = \frac{X}{Z^2}$, $R = \frac{G}{Y^2}$ and $X = \frac{B}{Y^2}$.

Therefore $$R_a = \frac{0{\cdot}0168}{0{\cdot}0168^2 + 0{\cdot}0396^2} = 9{\cdot}1 \text{ ohms}$$

and $$X_a = \frac{0{\cdot}0396}{0{\cdot}0168^2 + 0{\cdot}0396^2} = 21{\cdot}4 \text{ ohms.}$$

The total impedance

$$\dot{Z} = 60 + 9{\cdot}1 + j(15 + 21{\cdot}4)$$
$$= 69{\cdot}1 + j\,36{\cdot}4$$

and $$Z = \sqrt{69{\cdot}1^2 + 36{\cdot}4^2} = 78{\cdot}1 \text{ ohms.}$$

The phase angle is obtained from $\tan^{-1}\frac{36{\cdot}4}{69{\cdot}1}$ as before.

Exponential Treatment.—It is shown in mathematical text-books that

$$\cos\theta = \tfrac{1}{2}(\varepsilon^{j\theta} + \varepsilon^{-j\theta})$$

and $$j\sin\theta = \tfrac{1}{2}(\varepsilon^{j\theta} - \varepsilon^{-j\theta}).$$

Therefore $$\cos\theta + j\sin\theta = \varepsilon^{j\theta}$$

and $$\cos\theta - j\sin\theta = \varepsilon^{-j\theta}.$$

Referring to Fig. 399, $OP = a + jb$

$$= r(\cos\theta + j\sin\theta)$$
$$= r\varepsilon^{j\theta}.$$

The addition of two vectors, $a + jb$, and $c + jd$, is now seen to be

$$r_1\varepsilon^{j\theta_1} + r_2\varepsilon^{j\theta}.$$

The product of two vectors is most conveniently expressed in this form, viz.

$$\text{Product} = r_1r_2\varepsilon^{j\theta_1}\varepsilon^{j\theta_2} = r_1r_2\varepsilon^{j(\theta_1+\theta_2)}.$$

Examples

(1) A current represented by $\dot{I} = 25 - j10$ flows through a circuit the resistance and reactance of which are 8 and 12 ohms respectively. Determine the components of the applied voltage.

(2) A circuit consisting of $R_1 = 4$ and $X_1 = 3$ is in parallel with another consisting of $R_2 = 8$ and $X_2 = 2$, the applied pressure being 100 volts. Determine, by symbolic notation, the current in each branch, the total current and the phase angles of the two branch currents with respect to the resultant current.

(3) A voltage represented by $\dot{E} = 50 + j\,75$ acts on a circuit consisting of 10 ohms resistance in series with 6 ohms capacitive reactance. Determine the current.

(4) A single-phase transmission line has an inductive reactance

of 7·5 ohms and a resistance of 25 ohms. It supplies current to a condenser the capacitive reactance of which is 100 ohms. The generator voltage is 20,000 volts. What is the terminal voltage at the condenser, and what is the phase difference between the voltage at the generator end and that across the condenser?

(5) Two parallel circuits have conductances of 0·1 and 0·08 mho respectively, and susceptances of 0·03 and 0·15 mho respectively. Determine, by symbolic notation, the total current when an E.M.F. of 100 volts is applied, the resultant power factor, and the phase angle between the currents in the two parallel circuits.

(6) Two impedances $4 + j5$ and $8 + j10$ are connected in parallel across 200-volt, 50-cycle mains. Find (*a*) the admittance, conductance and susceptance of each branch, and of the entire circuit; (*b*) the total current and its power factor.

What value of capacitance must be connected in parallel with this combination to raise the resultant power factor to unity?

CHAPTER XXXVI

TRANSIENTS

Occurrence of Transients.—Transients occur in all cases where a change in the quantity of stored energy takes place. This does not include the cases where energy is stored periodically in electromagnetic and electrostatic fields under the action of a permanent alternating current.

Transients are non-recurrent in their nature and represent the growth or the dying away of the quantity concerned, and in certain abnormal circumstances their destructive effects are of very serious moment. In the simplest cases they obey the exponential law, just as the permanent alternating currents obey the sine law.

Simplest Type of Transient.—This occurs when the effect is proportional to the cause. Assume that a current I in a circuit is allowed to die down to zero. The current-time curve obeys a particular law. After the lapse of a time t the current has died down to i. The initial rate of decay, or the initial slope of the curve, is $-\frac{dI}{dt_1}$, which is a constant. The slope of the curve at time t is $\frac{di}{dt}$, the magnitude of which depends upon the value chosen for t. If, now, the whole series of operations had commenced at the time t, the initial rate of decay would be $-\frac{di}{dt}$, the initial magnitude of the current now being i. Since proportionality is assumed between the cause and the effect, then

$$-\frac{dI}{dt_1} : -\frac{di}{dt} :: I : i.$$

Therefore $$-\frac{di}{dt} = -\frac{i}{I} \times \frac{dI}{dt_1} = \frac{i}{T}$$

where T is a constant.

Then $$\frac{di}{i} = -\frac{1}{T}dt.$$

Integrating, we get

$$\log_\epsilon i = -\frac{t}{T} + K$$

$$i = \epsilon^{-\frac{t}{T}+K} = A\epsilon^{-\frac{t}{T}}$$

where A is a constant. This is the natural law of decay. After the lapse of a time $t = T$, the value of i becomes $A\varepsilon^{-1} = A \times \frac{1}{\varepsilon}$, so that the constant T is equal to the time taken for the current to fall to $\frac{1}{\varepsilon} = 0{\cdot}368$ of its original value, since ε is approximately $2{\cdot}718$.

Short-Circuit of Resistance and Inductance carrying a Steady Current.—The current gradually dies down to zero, having the value i corresponding to a time t measured from the instant of short-circuit.

Then
$$Ri + L\frac{di}{dt} = 0$$
$$\frac{di}{i} = -\frac{R}{L}\,dt.$$
Integrating, we get
$$\log_\varepsilon i = -\frac{Rt}{L} + K$$
$$i = A\varepsilon^{-\frac{Rt}{L}}$$
$$= I_0\varepsilon^{-\frac{Rt}{L}},$$
since $A = I_0$, the initial value of the current, when $t = 0$. Comparing this with the general formula above, it is seen that $T = \frac{L}{R}$, T being measured in seconds and L and R in henries and ohms respectively.

Application of Steady Voltage to a Circuit.—Suppose that a steady direct voltage E is applied to a circuit consisting of a resistance of R ohms and an inductance of L henries in series. The current rises to a final value equal to $\frac{E}{R}$, but this does not occur instantly. On the contrary, it takes a quite definite time to accomplish. This time is usually a small fraction of a second.

The applied E.M.F. has to overcome the resistance and also the back E.M.F. set up due to the fact that the current is changing in an inductive circuit. The applied E.M.F. can be expressed as
$$E = Ri + L\frac{di}{dt},$$
where i is the instantaneous value of the current.

Therefore
$$Ldi = (E - Ri)dt$$
and
$$\frac{L}{E - Ri}\,di = dt.$$
Integrating both sides we get
$$-\frac{L}{R}\log_\varepsilon (E - Ri) = t + K$$
where K is the constant of integration.

When $t = 0$, $i = 0$, and $-\frac{L}{R}\log_\varepsilon E = K$.

Therefore

$$\frac{L}{R}\{\log_\varepsilon E - \log_\varepsilon(E - Ri)\} = t$$

$$\log_\varepsilon \frac{E}{E - Ri} = \frac{Rt}{L}$$

$$\frac{E}{E - Ri} = \varepsilon^{\frac{Rt}{L}}$$

$$E - Ri = E\varepsilon^{-\frac{Rt}{L}}$$

$$Ri = E(1 - \varepsilon^{-\frac{Rt}{L}})$$

$$i = \frac{E}{R}(1 - \varepsilon^{-\frac{Rt}{L}}).$$

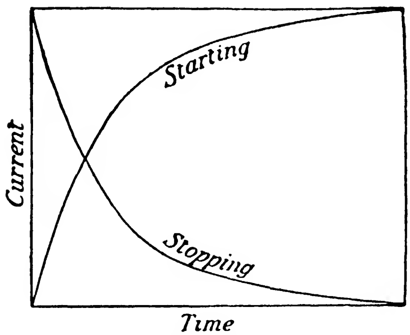


FIG. 402.—Current at Starting and Stopping.

The shape of the curves showing the relation between current and time when the current is being built up and when it is dying down is shown in Fig. 402. These two curves are similar if one of them is turned upside down.

As an example, consider a circuit having a resistance of 10 ohms and an inductance of 1 henry. If the applied voltage is 100, the expression for the current becomes

$$i = \frac{100}{10}(1 - \varepsilon^{-\frac{10t}{1}})$$

$$\doteq 10\ (1 - \varepsilon^{-10t}).$$

At the end of 0·1 second after the E.M.F. has been applied the value of the current becomes

$$i = 10\,(1 - 2{\cdot}718^{-1})$$

$$= 6{\cdot}32 \text{ amperes.}$$

At the end of 1 second the value of the current becomes

$$i = 10\,(1 - 2{\cdot}718^{-10})$$
$$= 9{\cdot}999 \text{ amperes.}$$

Time Constant.—The time constant, T, of a circuit is the time taken for the current to reach its final value, assuming the initial rate of increase to be maintained. This constant is equal to the ratio $\frac{L}{R}$, and the larger this ratio is, the greater is the time taken by the current in rising to its final value. Theoretically it takes an infinite time to rise to the value given by the ratio $\frac{E}{R}$, but as far as practical measurements are concerned the final value is attained very soon. For instance, in the example quoted above, the current has risen to within 0·01 per cent. of its final value at the end of 1 second.

The rate of increase of the current is

$$\frac{di}{dt} = \frac{E}{R} \cdot \frac{R}{L} \varepsilon^{-\frac{Rt}{L}}$$
$$= \frac{E}{L} \varepsilon^{-\frac{Rt}{L}}.$$

When $t = 0$, $\varepsilon^{-\frac{Rt}{L}} = 1$, and

$$\frac{di}{dt} = \frac{E}{L}.$$

If this rate of increase were maintained, the final current, $I = \frac{E}{R}$, would be reached in T seconds (by definition), so that

$$\frac{E}{L} \cdot T = I = \frac{E}{R}$$

and
$$T = \frac{L}{R}.$$

An alternative definition of time constant is the time taken for the current to rise to $\frac{1}{\varepsilon}$ of its full value.

Stopping a Direct Current.—The usual way of stopping a current is by the opening of a switch. What exactly goes on in the circuit during this operation is rather complicated, but looking at the problem from the simplest point of view, it may be considered that an enormous resistance is very rapidly introduced into the circuit, this resistance being the air gap between the fixed and the moving contacts of the opening switch. The result of this is to bring the current rapidly to zero, although the presence of inductance in the circuit retards the fall of the current to some extent. This effect is seen when the field winding of a generator or motor is suddenly open-circuited. The spark which ensues is much more vicious than would be the case if there were no inductance in the circuit. It must be remembered that inductance can be present in a direct current circuit, just as in an alternating one, since inductance is due to the linkage of magnetic lines of force with ampere-turns. The effects of inductance, however, are only noticeable when the current is varying. Another way of looking at this question of the broken field winding is to consider the large E.M.F. which is suddenly introduced into the circuit at the moment of opening the switch, this E.M.F. being due to the rapid change of linkages in the circuit. This large E.M.F. will cause a spark to persist across the retreating switch contacts for a longer time than would be the case if only the normal E.M.F. of the circuit were acting.

Starting and Stopping a Current—Graphical Construction.—The time constant, $T = \frac{L}{R}$, may be defined as the time taken for the current to reach its final value, provided the initial rate of increase were maintained. This initial rate of growth is obtained from the equation

$$E = Ri + L \times \text{Rate of change of current},$$

so that, since the initial value of i is zero,

$$\text{initial rate of change of current} = \frac{E}{L} = \frac{1}{T}\cdot\frac{E}{R} = \frac{I}{T}.$$

Shortly afterwards, when the current has reached a momentary value of i, the rate of change of current is given by

$$\frac{E - iR}{L} = \frac{I - i}{T}.$$

Finally, when i becomes equal to $I = \frac{E}{R}$, the rate of increase dies down to zero and steady conditions are set up.

In order to obtain the graph of the curve, a horizontal line is drawn through the point representing the final value of the current (see Fig. 403). A length is now marked off along this line equal to T seconds and the point obtained is joined to the origin. The slope of this line is $\frac{I}{T}$ amperes per second. When the current has reached a value of i amperes at point P, a horizontal line is drawn through P and a length PQ also equal to T marked off along it, a vertical being drawn through Q to cut the maximum line at R. The height QR is equal to $I - i$. The line joining P and R will now give a further portion of the curve, since its slope is $\frac{I - i}{T}$ amperes per second. Other triangles $P'Q'R'$, $P''Q''R''$, etc., can be constructed until the full curve is drawn in. In practice the points P, P', etc. should be taken fairly close together.

The graph representing the stopping of a current can be obtained in the same manner, the slopes, of course, being now negative.

Application of Steady Voltage to a Resistance and Condenser in Series.—At any instant the total applied voltage E is divided between the two, the voltage across the resistance being Ri and that

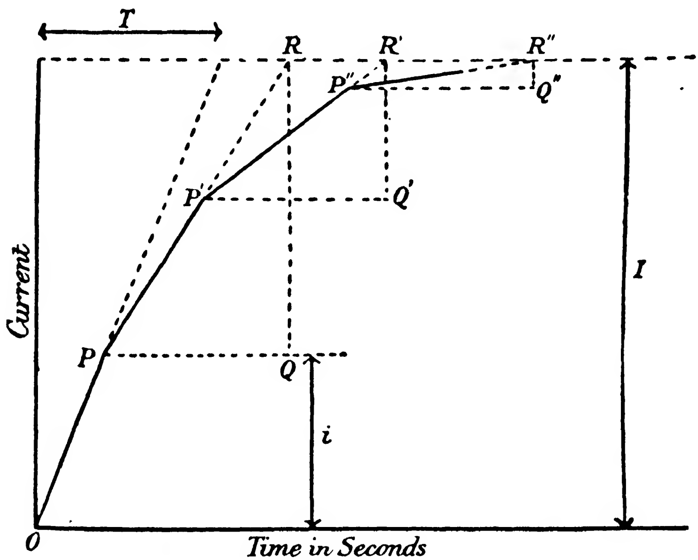


FIG. 403.—Graphical Construction for Current at Starting.

across the condenser being $e = E - Ri$. The current taken by the condenser is $C\frac{de}{dt}$, so that

$$i = C\frac{de}{dt} = \frac{E - e}{R}$$

$$\frac{de}{E - e} = \frac{dt}{RC}.$$

Integrating, we get

$$-\log_\epsilon (E - e) = \frac{t}{RC} + K.$$

Therefore $$E - e = \epsilon^{-\left(\frac{t}{RC} + K\right)} = A\epsilon^{-\frac{t}{RC}}$$

$$e = E - A\epsilon^{-\frac{t}{RC}}.$$

But $e = 0$ when $t = 0$; therefore $A = E$.

Finally, $$e = E(1 - \varepsilon^{-\frac{t}{RC}}).$$

The time constant is now equal to $T = RC$.

Short-Circuit of a Condenser through a Resistance.—The instantaneous value of the current is now given by

$$i = \frac{e}{R} = -C\frac{de}{dt}{}^{1}$$

$$\frac{de}{e} = -\frac{dt}{RC}.$$

Integrating, we get

$$\log_\varepsilon e = -\frac{t}{RC} + K$$

$$e = \varepsilon^{-\frac{t}{RC} + K} = A\varepsilon^{-\frac{t}{RC}}.$$

When $t = 0$, $e = E_0$; therefore $A = E_0$, so that

$$e = E_0\varepsilon^{-\frac{t}{RC}}.$$

Change of Current in a Circuit.—When the conditions of an electric circuit are changed so as to require a change in the stored

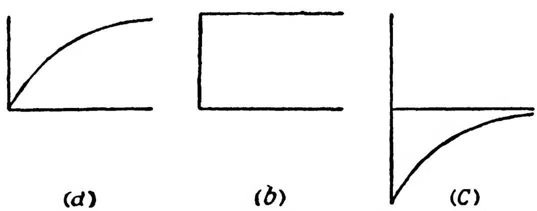


FIG. 404.—Resolution of Current into Permanent and Transient Components.

energy, a transient occurs. For example, in a theoretically non-inductive circuit, a change of resistance would result in the current attaining its final value instantaneously. When inductance is present, however, as it always must be to a greater or less extent, a transient occurs and the current takes an appreciable time to come to its ultimate value. This is due to the electromagnetic field associated with the inductance. During the transition period the current may be considered as consisting of two components, viz. a permanent and a transient current. The former may be either direct or alternating. For example, the current shown in Fig. 404 (*a*) may be considered as the resultant of the steady permanent current shown in Fig. 404 (*b*) and the transient current shown in Fig. 404 (*c*), which obeys a simple exponential law.

[1] The minus sign is here inserted since the condenser is discharging.

Change in Value of Alternating Current.—When an A.C. changes in value, its graph during the transition period can be obtained by drawing the final permanent current and, at the instant when the circuit conditions change, noting the difference between the instantaneous values of the original and the final permanent currents. The initial magnitude of the transient is equal to this difference (see Fig. 405), its rate of decay depending upon the time constant of the circuit. The resultant current is now obtained by adding together the permanent and transient components as shown by the dotted curve. The magnitude of the transient depends upon the point in the wave at which the change occurs. It is a maximum when the change occurs at the maximum point of the wave. When the change occurs at the zero point of the wave there is no transient at all and the change takes place smoothly.

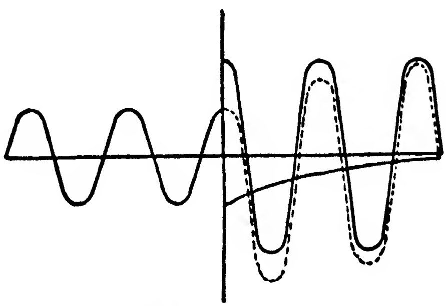


FIG. 405.—A.C. Transient.

Starting Transient in Balanced Three-Phase System.—When a three-phase current is switched on, the three currents must each start from zero, although it is impossible that the permanent components of these three currents can each be equal to zero in view of their phase difference. In general let the initial instantaneous values of the permanent (alternating) components of the three-phase currents be i_{p1}, i_{p2}, and i_{p3} respectively, these being all different. The corresponding transient currents are i_{t1}, i_{t2} and i_{t3} respectively, and since the sum of the permanent and transient components must be zero at this instant, it follows that $i_{p1} = -i_{t1}$, $i_{p2} = -i_{t2}$ and $i_{p3} = -i_{t3}$. The resultant currents are therefore all different, containing transients of different magnitudes but with the same time constant, since balance is assumed. Again, since $i_{p1} + i_{p2} + i_{p3} = 0$, therefore $i_{t1} + i_{t2} + i_{t3} = 0$ also, for the sum of the resultant currents must be zero at any instant.

If the circuit is switched in at an instant corresponding to the zero point on one current wave, then the transient in this phase is absent, those in the other two phases each being $\frac{\sqrt{3}}{2}$ times their maximum possible value. If, on the other hand, the current is switched on at the maximum point of the permanent current wave for one phase, the transient in this phase is a maximum, those in the other two phases each being half the maximum possible. Fig. 406 shows an example of the starting transients in a balanced three-

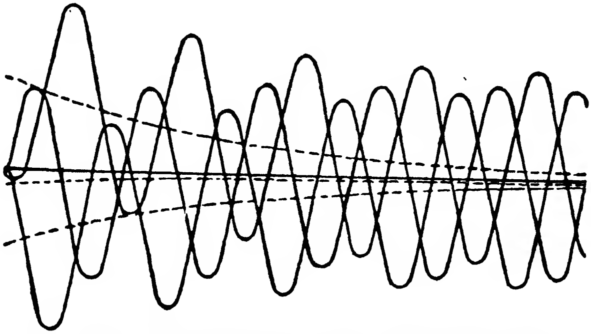


FIG. 406.—Starting Transients in Balanced Three-Phase Circuit.

phase case, the point of commencement corresponding to 10° after the zero point on the permanent component of one of the phase currents.

Starting Transient of Rotating Field.—Consider the currents in a three-phase case to be resolved into their permanent and transient components. The former give rise to a synchronously rotating

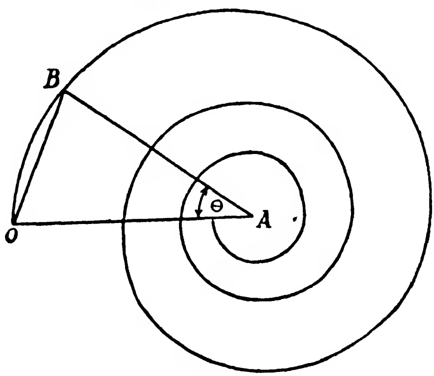


FIG. 407.—Rotating M.M.F. Locus.

M.M.F., but the latter being unidirectional must set up a M.M.F. along a fixed axis. At the commencement the resultant M.M.F. is zero, since the resultant currents must all start from zero. The resultant M.M.F. due to the transients must therefore be equal and opposite to the resultant M.M.F. due to the permanent components. As the flux rotates, the M.M.F. due to the permanent components remains constant in magnitude, but the M.M.F. due to the transients does not rotate, merely acting in a single direction along a fixed axis and gradually dying away. In order to determine the resultant M.M.F. curve, imagine the permanent M.M.F. to remain fixed in space, the transient M.M.F. rotating with respect to it at synchronous speed. The permanent M.M.F. can now be represented by a fixed line OA of constant length (see Fig. 407), whilst the transient M.M.F. is represented by a line such as AB rotating around A at synchronous speed, the length AB dying away according to an exponential law. At any instant of time corresponding to an advance of $\theta°$ from the commencement, the resultant M.M.F. is given by OB, the vector sum of OA and AB. The spiral curve represents the locus of the resultant M.M.F., one revolution corresponding to one cycle. It is seen that the resultant M.M.F. is alternately stronger and weaker than that due to the permanent com-

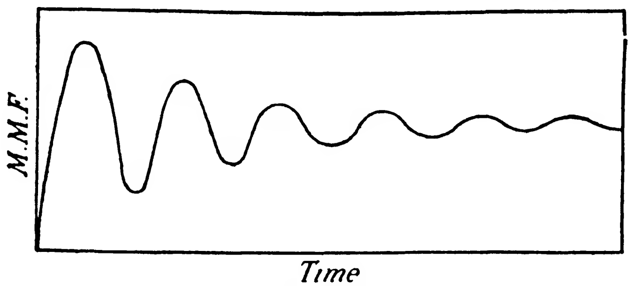


FIG. 408.—Rotating Field Transient.

ponents alone. When the length of the line OB is plotted against θ, the resultant M.M.F. curve takes on the shape shown in Fig. 408.

Starting Transient in Air-cored Inductance.—The current must start from zero, and, assuming no losses, this corresponds to the maximum point on the E.M.F. wave. If switched in at any other point than this there is a starting transient engendered. The worst conditions are obtained when the circuit is switched in at the zero point of the E.M.F. wave. Neglecting resistance, the applied E.M.F. must be equal to $L\frac{di}{dt}$, and if the voltage acts in the positive direction for the first half-cycle, then $\frac{di}{dt}$ must be positive for half a cycle, instead of for 90°, as it is from this point when permanent conditions are attained. The current, therefore, rises to double its normal value in the first half-wave, this being known as the *doubling effect*. Further, since

$$L\frac{di}{dt} = N \times 10^{-8}\frac{d\Phi}{dt},$$

where N is the number of turns, then the applied E.M.F.

$$e = N \times 10^{-8} \frac{d\Phi}{dt}$$

and
$$d\Phi = \frac{10^8}{N} edt = \frac{10^8}{N} E_m \sin \omega t \, dt.$$

Integrating both sides we get

$$\Phi_{\text{inst.}} = -\frac{10^8 E_m}{N\omega} \cos \omega t + K.$$

But $\Phi_{\text{inst.}} = 0$ when $t = 0$, so that $K = \frac{10^8 E_m}{N\omega}$

and
$$\Phi_{\text{inst.}} = \frac{10^8 E_m}{N\omega}(1 - \cos \omega t).$$

When $\omega t = 180°$, $\Phi_{\text{inst.}} = 2 \times \frac{10^8 E_m}{N\omega}$, or double its normal value.

The current is proportional to the flux, since there is no iron present, so that the current also rises to double its normal value and is

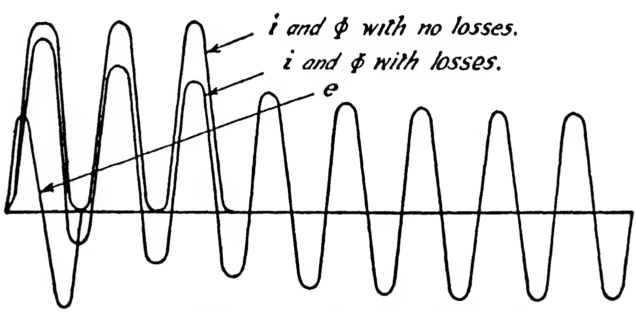


FIG. 409.—Starting Transients in Air-cored Inductance.

unidirectional in character. The sine wave representing the permanent current is displaced so as to be completely on one side of the zero line. If the circuit is completed at some intermediate point on the E.M.F. wave, then the current wave is displaced about its axis to a lesser degree.

Effect of Losses.—The general effect of the I^2R losses is to make the current wave gradually symmetrical. As the current rises to its maximum value, the IR drop increases and prevents the current from rising to as large a value as it would do otherwise (see Fig. 409). The second current wave, therefore, commences from a point slightly below the zero line, with the result that the second peak is again smaller. This action continues until the current wave is symmetrically placed about the zero line, the time taken depending upon the value of the time constant.

Effect of Iron Core.—When iron is present and saturation occurs, the distortion of the current wave at starting is much more marked. Theoretically the flux may rise to double its permanent maximum value, but as this often takes the iron beyond the knee of the magnetization curve, it follows that the magnetizing current is increased to a very much greater extent. The effect is again accentuated if the iron core happened to be magnetized initially in the reverse direction. In fact, the current rush at starting may reach, say, twenty or thirty times its final value. This current rush consists of very large positive half-waves, followed by very small negative half-waves, each succeeding wave approaching more nearly to the normal, until finally the two half-waves become equal (see Fig. 410).

Transformers are subject to these current rushes on switching in, the disturbance lasting on occasion for as long as two minutes, These current rushes are not accompanied by any voltage rises, but they may occasion large mechanical stresses between adjacent coils, which ought to be avoided.

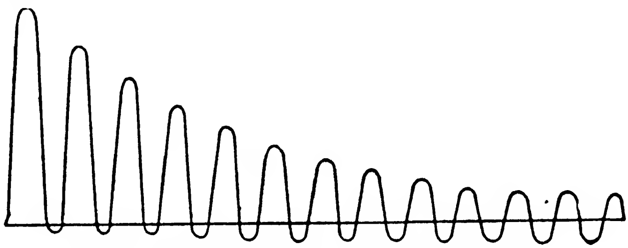


FIG. 410.—Starting Transient in Iron-cored Inductance.

Collapse of Magnetic Field.—Let a steady pressure, E_0, be applied to a coil possessing a resistance of R ohms and an inductance of L henries. A steady permanent current $I_0 = \frac{E_0}{R}$ is set up. This produces a flux $\Phi_0 = \frac{LI_0}{N} \times 10^8$. If now the coil be suddenly short-circuited, the field gradually collapses, the current dying down with it. Neither disappears instantaneously. As soon as the field commences to decrease, it induces an E.M.F. in the coil which, since it is short-circuited, sets up a current tending to prevent the collapse. These conditions are represented in Fig. 411. The induced E.M.F. is given by

$$e = N\frac{d\Phi}{dt} \times 10^{-8}.$$

Therefore

$$\int edt = N \times 10^{-8}\int d\Phi$$
$$= N \times 10^{-8} \times \Phi_0 = LI_0.$$

But $\int edt$ is the area of the E.M.F. curve from the instant of short-circuit. If the flux had continued to die away at its initial rate, the induced E.M.F. would have been constant and equal to E_0 for a time T equal to the time constant of the circuit. The area of this curve (shown dotted) is $E_0T = \frac{E_0L}{R} = LI_0$. In other words, the dotted rectangle has the same area as the exponential curve.

Strictly speaking the transient takes an infinite time to die down to zero, but the time of its duration is usually defined as T seconds, this being the time taken to dissipate the energy, assuming the initial rate of decrease to be maintained.

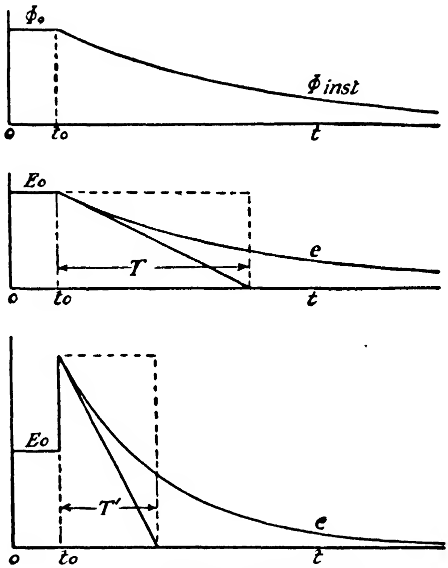


FIG. 411.—Collapse of Magnetic Field.

If the coil be short-circuited through a resistance, R', then the time constant is reduced to $T' = \frac{L}{R + R'}$. The same amount of energy ($\frac{1}{2}LI^2$) has now to be dissipated in a shorter time, so that the initial value of the induced E.M.F. must be increased as shown in Fig. 411. Before the current has had time to change, the induced voltage to produce this current must be increased in the ratio $\frac{R + R'}{R}$. The duration of the transient, however, is now only $\frac{R}{R + R'}$ times its former value.

If the added resistance be infinite, *i.e.* if the coil be open-circuited, then theoretically an infinite voltage lasts for an infinitesimal time, T being zero. Actually, however, the circuit cannot be broken instantaneously on account of the spark at the switch contacts. Generally, the more slowly the field is killed, the less is the strain on the insulation due to the induced E.M.F.

Sudden Short-Circuit on Alternator.—When an alternator is running under normal conditions the strength of the field is several times as great as when it is running under a steady short-circuit with the same excitation, this being due to the direct weakening action of the armature reaction. When a short-circuit takes place suddenly, therefore, the field has to decrease in magnitude to a very considerable extent. This weakening of the field does not occur instantaneously, but takes a certain time to accomplish, because every change in the flux causes a change in the number of lines of force linked with the field coils. As soon as the lines of force begin to die down, therefore, a voltage is induced in the field coils, giving rise to a current opposing the fall of the flux. Eddy currents are also induced in the iron itself, particularly in any solid portions, and this also helps to retard the fall of the flux, which thus takes a definite time to accomplish. On suddenly short-circuiting the armature, a large current flows before the armature reaction has had time to make its effects felt, and the first few current waves after the short-circuit indicate an abnormally high current. Whilst this is taking place, large mechanical forces are set up which may tear the end connections from their fastenings; hence the necessity for adequate clamping arrangements.

The current rush decreases fairly slowly and lasts for many cycles. A current rush also occurs in the field circuit. These transients can be predicted fairly accurately. Both armature reactance and reaction enter into the problem. Let the synchronous reactance, which is the combined effect of the two, be represented by X_s, and the true leakage reactance by X_l. The latter consumes a voltage X_lI which appears simultaneously with the current. On the other hand, the armature reaction takes an appreciable time in order to make its full effects operative, since it involves the killing of the field to a large extent.

Immediately after the short-circuit occurs, the armature current is equal to $I_l = \frac{E_0}{X_l}$, where E_0 is the induced E.M.F. before the short-circuit. Ultimately the current dies down to a steady value of $I_s = \frac{E_0}{X_s}$. Therefore $\frac{I_l}{I_s} = \frac{X_s}{X_l}$. The magnitude of the momentary short-circuit current is thus affected considerably by the ratio of reaction to reactance. A high reaction and low reactance means a large momentary short-circuit current, and *vice versa*. Let the original value of the flux be Φ_1, this dying down ultimately to Φ_0. Then $\Phi_1/\Phi_0 = \frac{E_0}{I_sX_l} = \frac{X_s}{X_l} = m$, and $\Phi_1 = m\Phi_0 = \Phi_0 + (m-1)\,\Phi_0$. The transient component of the flux, or the portion which has to die away, is $(m-1)\Phi_0$. This is usually a slow transient with a time constant of a minute or more, and gives rise to a current transient in the field circuit.

Examples

(1) A steady voltage of 100 volts is applied to a coil the resistance and inductance of which are 5 ohms and 2 henries respectively. Determine the magnitude of the current 0·4 second after the application of the voltage.

(2) By means of a graphical construction, draw the graph of the voltage across the condenser at starting of a 100 microfarad condenser in series with a 10,000 ohm resistance when connected to a 100 volt supply.

(3) The time constant of the stator windings of a three-phase 50 cycle induction motor is 0·1 second. Draw the starting transient of the rotating field in polar and rectangular form for the first five cycles.

(4) Prove that in an air-cored inductance supplied with A.C. the initial value of the current may rise to a value equal to double the permanent value. How does the presence of iron affect this?

(5) Write an account of what takes place in an alternator when it is suddenly short-circuited.

(6) Describe, with the aid of diagrams, what occurs when the supply is disconnected from an electromagnet. Assume a non-inductive resistance to be connected across the coil terminals, prior to the disconnection from the source of supply. Discuss the effect of the magnitude of this resistance upon the duration of the transient.

CHAPTER XXXVII

OSCILLATORY CIRCUIT

Oscillatory Circuits.—An oscillatory circuit is fundamentally the same as an alternating current circuit, but operates, of course, at a much higher frequency. The current may be undamped or damped. In the former case the amplitude remains constant, but in the latter case it gradually decreases until the current dies away. The current also may be modulated by the superimposition of another high frequency alternating current, so that the resultant amplitude rises and falls in various ways. A damped oscillating current follows the same general laws as a transient alternating current, discussed in Chap. XXXVI.

Discharge of Condenser through Inductance.—When a condenser is discharged into a circuit consisting of an inductance and a relatively high resistance, the current is unidirectional, gradually dying away to zero. If the resistance is relatively small, the current becomes oscillatory in character, flowing each way in turn, and gradually dying away. If there is no resistance at all in the circuit, no losses occurring, then the current would flow indefinitely without diminution in magnitude, and would be sinusoidal in wave form. The energy stored in the electrostatic field is transferred to the electromagnetic field due to the current, and then back and forth, the current maintaining an undiminished amplitude if there are no losses. The presence of losses causes this energy to be gradually frittered away as heat, and in consequence the amplitude of the current steadily decreases.

The above action is analogous to that which occurs when a weight is attached to a distended helical spring. The combination continues to oscillate until all the available energy has been expended as heat due to friction. The spring corresponds to the condenser, the weight to the inductance, and the friction to the resistance. If the friction is large, the energy is rapidly dissipated, and the combination quickly comes to rest. In such a case there may be no oscillation at all, but merely a unidirectional movement until equilibrium is again set up. If friction is entirely absent, the combination goes on oscillating indefinitely with undiminished amplitude.

Law of the Discharge.—When a condenser is discharged, free oscillations being set up, all the energy stored in the condenser,

$\frac{1}{2}CE^2$, is dissipated in the circuit during the oscillations. The condenser p.d. becomes less and less with each successive oscillation. If the only source of E.M.F. is that due to the charge in the condenser, then the algebraic sum of the various p.d.'s must be zero, or

$$L\frac{di}{dt} + Ri + \frac{q}{C} = 0,$$

where i is the instantaneous value of the current and q is the instantaneous value of the charge in the condenser.

Differentiating, and substituting i for $\frac{dq}{dt}$,

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0,$$

or

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0.$$

The same expression holds good for an ordinary A.C. circuit, except that in the latter case, the expression must be equated to E instead of to zero.

The solution of the above differential equation may take three different forms, depending upon whether $\frac{R^2}{4L^2}$ is greater than, equal to, or less than $\frac{1}{LC}$.

Case I. $\frac{R^2}{4L^2}$ is greater than $\frac{1}{LC}$.

$$i = -\frac{E}{\beta L}\varepsilon^{-\alpha t}\left(\frac{\varepsilon^{\beta t} - \varepsilon^{-\beta t}}{2}\right)$$

$$= -\frac{E}{\beta L}\varepsilon^{-\alpha t}\sinh \beta t,$$

where E is the initial voltage of the condenser, $\alpha = \frac{R}{2L}$, $\beta = \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$, and t is the time in seconds.

The discharge current rises to a maximum and falls to zero again, without any oscillatory discharge, as shown in Fig. 412 (*a*).

Case II. $\frac{R^2}{4L^2} = \frac{1}{LC}$.

$$i = -\frac{Et}{L}\varepsilon^{-\alpha t}.$$

The discharge current is now represented by the graph in

Fig. 412 (*b*). The current is critically damped so that an oscillatory discharge is just prevented.

Case III. $\frac{R^2}{4L^2}$ is less than $\frac{1}{LC}$.

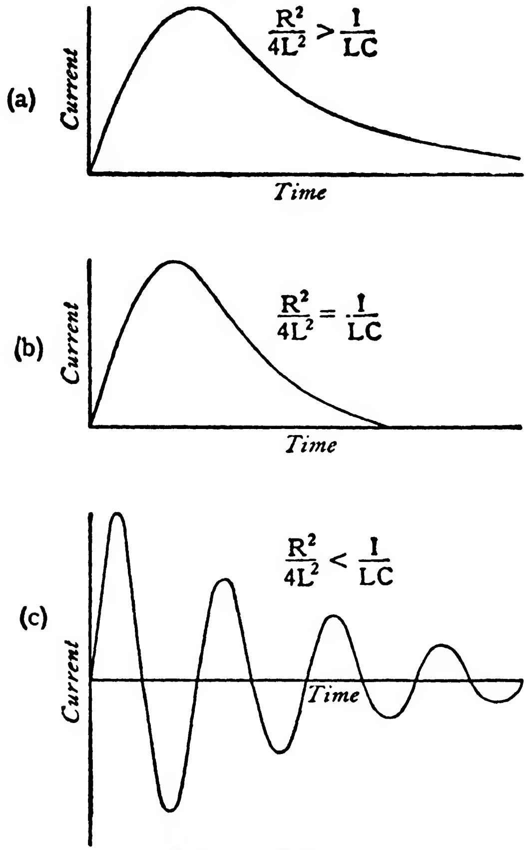


FIG. 412.—Discharge of Condenser through an Inductance.

In this case, $\beta = \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}}$ becomes the square root of a negative quantity, which is imaginary.

$$\beta = \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} = \sqrt{(-1)\left(\frac{1}{LC} - \frac{R^2}{4L^2}\right)}$$

$$= j\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} = j\omega,$$

where $\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$, and $j = \sqrt{-1}$.

Substituting in the general formula for the current,

$$i = -\frac{E}{j\omega L}\varepsilon^{-\alpha t}\left(\frac{\varepsilon^{j\omega t} - \varepsilon^{-j\omega t}}{2}\right)$$
$$= -\frac{E}{\omega L}\varepsilon^{-\alpha t}\left(\frac{\varepsilon^{j\omega t} - \varepsilon^{-j\omega t}}{2j}\right)$$
$$= -\frac{E}{\omega L}\varepsilon^{-\alpha t}\sin\omega t.$$

The current in this case is oscillatory, the theoretical maximum value being $-\frac{E}{\omega L}$, and the frequency being $f = \frac{\omega}{2\pi}$. The term $\varepsilon^{-\alpha t}$ shows the diminishing amplitude of the current, and $\alpha = \frac{R}{2L}$ is called the *damping factor* of the circuit. The discharge current now takes the form shown in Fig. 412 (*c*).

Frequency of Oscillation.—The frequency of the current in an oscillatory circuit is determined by the inductance and the capacitance, if the resistance is small. If resistance is entirely absent, the frequency of oscillation is the same as the resonating frequency with forced alternating currents. The damping, *i.e.* the decrease in amplitude, is determined by the relative values of resistance, inductance and capacitance.

From above,

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

and

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}.$$

In high frequency circuits, $\frac{R^2}{4L^2}$ is usually very small, so that in such cases,

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}},$$

which is the same as for a resonant circuit.

The frequency of the oscillatory discharge is called the *natural frequency* of the circuit, this being practically the same as the resonant frequency when the resistance is small.

If R is small, $\omega = 2\pi f = \sqrt{\frac{1}{LC}}$, and the maximum value of the current, one-quarter of a cycle after the commencement, is

$$I_m = -\frac{E}{\omega L}\varepsilon^{-\alpha t}\sin\omega t$$
$$= -\frac{E}{L} \times \sqrt{LC} \times \varepsilon^{-\alpha t}\sin\omega t$$
$$= -E\sqrt{\frac{C}{L}},$$

since $\varepsilon^{-\alpha t}$ and $\sin\omega t$ can both be taken as unity.

The same result can be obtained by equating $\frac{1}{2}CE^2$ and $\frac{1}{2}LI_m^2$, when, neglecting losses,

$$\tfrac{1}{2}CE^2 = \tfrac{1}{2}LI_m^2$$

and $$I_m = E\sqrt{\frac{C}{L}} \text{ as before.}$$

Damping and Decrement.—In the expression for a damped A.C., it is seen that the amplitude gradually dies away according to the factor $\varepsilon^{-\alpha t}$, where $\alpha = \frac{R}{2L}$. The natural or Napierian logarithm of the ratio of the maximum amplitude of an oscillation to the maximum amplitude in the same direction of the oscillation immediately succeeding it, is called the *logarithmic decrement*, sometimes contracted to *log. dec.* or *decrement* simply. It can be determined from a knowledge of successive amplitudes in the following manner. Let I_1, I_2 and I_3 represent the first three successive amplitudes in the same direction, and let I_0 be the constant amplitude assuming no damping at all. The maximum amplitude of the first cycle occurs one-quarter of a cycle from the commencement, when $t = \frac{T}{4}$, T being the periodic time and equal to $\frac{1}{f}$, so that

$$I_1 = I_0\varepsilon^{-\alpha t} = I_0\varepsilon^{-\frac{R}{2L}\cdot\frac{T}{4}}.$$

The maximum amplitude of the second cycle occurs one cycle later when

$$I_2 = I_0\varepsilon^{-\frac{R}{2L}\cdot\frac{5T}{4}}.$$

Similarly, $$I_3 = I_0\varepsilon^{-\frac{R}{2L}\cdot\frac{9T}{4}}.$$

Therefore, $$\frac{I_1}{I_2} = \frac{I_0\varepsilon^{-\frac{R}{2L}\cdot\frac{T}{4}}}{I_0\varepsilon^{-\frac{R}{2L}\cdot\frac{5T}{4}}} = \varepsilon^{\frac{RT}{2L}},$$

and also $$\frac{I_2}{I_3} = \frac{I_0\varepsilon^{-\frac{R}{2L}\cdot\frac{5T}{4}}}{I_0\varepsilon^{-\frac{R}{2L}\cdot\frac{9T}{4}}} = \varepsilon^{\frac{RT}{2L}}.$$

The actual decrement is equal to $\varepsilon^{\frac{RT}{2L}}$, and the Napierian logarithm of the ratio is $\frac{RT}{2L} = \frac{R}{2fL}$. The logarithmic decrement, δ, is therefore equal to $\frac{R}{2fL}$. In those cases where the natural frequency can be expressed by

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}},$$

the logarithmic decrement is given by

$$\delta = \frac{R}{2fL} = \frac{R}{L} \times \pi\sqrt{LC} = \pi R\sqrt{\frac{C}{L}}.$$

The general form of the curve of the discharge current is shown in Fig. 413, where $\frac{I_1}{I_2} = \frac{AB}{CD}, \frac{I_2}{I_3} = \frac{CD}{EF}$, etc. The log. dec. is now given by

$$\delta = \log_e \frac{AB}{CD} = \log_e \frac{CD}{EF}.$$

The *persistency* of the wave is the ratio of the amplitude of any half-cycle to that of its immediate predecessor in the same direction,

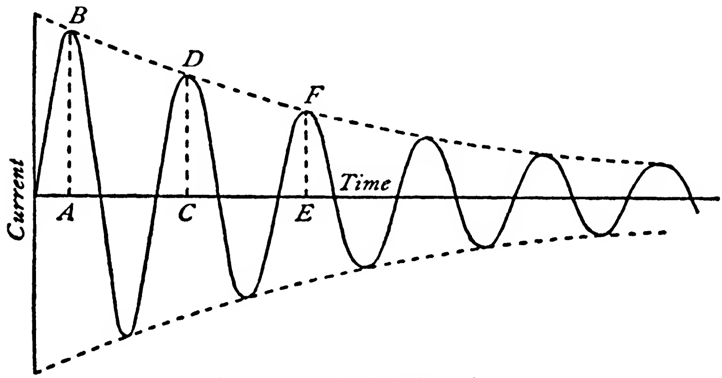


FIG. 413.—Damped Discharge Current.

and is expressed conveniently as a percentage. Defined in this way,

$$\text{Persistency} = \frac{CD}{AB} \times 100 \text{ per cent.}$$

In high frequency circuits the logarithmic decrement may vary from 0·02 to 0·2, the latter value being found in oscillatory circuits which include a spark gap. The logarithmic decrement is an important factor in determining the sharpness of the resonance curve, and hence the selectivity, of a receiving circuit.

Number of Oscillations in a Train.—A train of oscillations consists of a number of individual oscillations, commencing with a maximum amplitude and gradually dying away until they disappear. Such a train of oscillations can be obtained by discharging a condenser through a suitable circuit. The number of component oscillations is theoretically infinite, since $\varepsilon^{-\frac{Rt}{2L}}$ only becomes zero when t is infinite. In practice, however, it is usual to assume that a train of oscillations has ended when the amplitude has fallen to one per cent of the initial value. Defined in this way, the number of oscillations in a train can be calculated as follows.

Let I_1 be the initial amplitude and I_n the amplitude after an interval of time, t, when $\frac{I_1}{I_n} = 100$.

By definition,

$$\frac{I_1}{I_2} = \frac{I_2}{I_3}, \text{ etc.} = \varepsilon^{\delta}.$$

Therefore $$\frac{I_1}{I_n} = \left(\frac{I_1}{I_2}\right)^{n-1} = \varepsilon^{\delta(n-1)} = 100$$

so that $$\delta(n-1) = \log_{\varepsilon} 100 = 4{\cdot}6$$

and $$n = \frac{4{\cdot}6}{\delta} + 1$$
$$= \frac{4{\cdot}6}{\delta} \text{ approx.}$$

As an example, if the log. dec. $= 0{\cdot}046$, the number of oscillations in the train is $\frac{4{\cdot}6}{0{\cdot}046} = 100$.

Frequency and Wave Length.—When high frequency alternating currents are made to flow in an aerial circuit, a high frequency disturbance is produced in the ether, and electromagnetic waves are propagated. If the high frequency oscillations are damped, a train of damped waves is sent out. These waves all travel at the same rate, viz., 3×10^8 metres per second, and the wave length, λ, of a wave in any medium is equal to its speed, v, divided by its frequency, f, or

$$\lambda = v/f.$$

In the present case, the wave length in metres is $\frac{3 \times 10^8}{f}$, and if the resistance is small, the frequency is given by $\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$, so that

$$\lambda = \frac{v}{f} = 3 \times 10^8 \times 2\pi\sqrt{LC} \text{ metres}$$
$$= 1885 \times 10^6\sqrt{LC} \text{ metres,}$$

where L and C are in henries and farads respectively, or

$$\lambda = 1885\sqrt{LC} \text{ metres,}$$

where L and C are in microhenries and microfarads respectively.

The wave-length is proportional to $\sqrt{LC}$, and for a given wave length there is an infinite number of values that can be adopted for L and C; it is their product that is fixed in value. This product, LC, is known as the oscillation constant, or LC value, of the circuit.

Effective Value of Damped Current.—When a succession of trains of oscillations takes place, the energy associated with each train is practically all dissipated before the commencement of the next one. Each train has a very short duration, and the number of trains per second is comparatively large. The effective or R.M.S. value of the current is the value indicated by a hot wire or thermocouple ammeter, and it may be determined in the following way.

Let n be the number of trains of oscillations per second, I_0 the initial amplitude of the current at the commencement of a train, and W the energy dissipated per train. The latter is also the same as the energy associated with the magnetic field at the commencement of each train. Then, if I is the R.M.S. value of the current and R is the total resistance of the circuit,

$$I^2R = Wn$$

or $$W = \frac{I^2R}{n},$$

and also $$W = \tfrac{1}{2}LI_0^2.$$

Therefore, $$\frac{I^2R}{n} = \tfrac{1}{2}LI_0^2$$

$$I^2 = \frac{nLI_0^2}{2R}$$

and $$I = \sqrt{\frac{nLI_0^2}{2R}}.$$

Substituting $$I_0 = E\sqrt{\frac{C}{L}},$$

$$I = \sqrt{\frac{nL}{2R} \times \frac{E^2C}{L}} = E\sqrt{\frac{nC}{2R}}.$$

Alternatively, putting $\delta = \dfrac{R}{2fL}$ and $\dfrac{L}{R} = \dfrac{1}{2f\delta}$,

$$I = \sqrt{\frac{nI_0^2}{2} \times \frac{1}{2f\delta}} = I_0\sqrt{\frac{n}{4f\delta}}.$$

Coupled Circuits.—When two circuits are placed so that there is mutual inductance between them, they are said to be coupled. Two coils, each possessing a definite amount of self-inductance, may be more or less closely interlinked, the value of the mutual inductance, M, depending upon the closeness of this interlinking. They are said to be loosely or tightly coupled. Fig. 414 shows two circuits, loosely coupled at (*a*), the coupling gradually increasing until they are tightly coupled at (*d*).

The voltage induced in the second coil is $E_2 = -M \times$ rate of change of current in the first coil, and the voltage induced in the first coil is $E_1 = -M \times$ rate of change of current in the second coil, where M is the mutual inductance in henries.

Coupling Factor.—The self-inductance of a coil varies as the square of the number of its turns, and the mutual inductance of two coils varies as the product of the numbers of turns in the two coils, therefore

$$M^2 \propto (L_1 L_2)$$

and
$$M = k\sqrt{L_1 L_2},$$

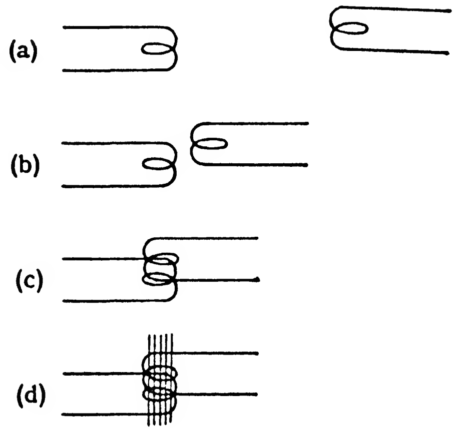


FIG. 414.—Loosely and Tightly Coupled Circuits.

where k is a constant called the *coupling factor*. If the coupling is infinitely tight, $k = 1$, and if it is infinitely loose, $k = 0$.

Double Frequency Effect.—When two coupled circuits are set in oscillation, and are allowed to react freely upon each other, oscillations of two frequencies are set up, the effect being more marked the tighter the coupling.

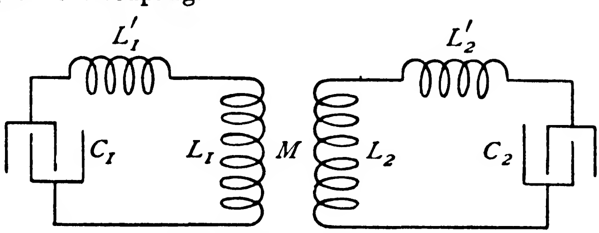


FIG. 415.—Two Coupled Circuits.

Suppose the two circuits shown in Fig. 415 are tuned to the same LC value. Let the primary circuit be set in oscillation by some means, the whole combination being allowed to oscillate freely. It might be imagined that each circuit will oscillate at a frequency given by

$$f = \frac{1}{2\pi\sqrt{L_1 C_1}} = \frac{1}{2\pi\sqrt{L_2 C_2}},$$

but this is not the case. Oscillations of two different frequencies are set up, one higher and the other lower than is given by the above formula. Let the LC value of either circuit when coupled to the other one be represented by $LC = \frac{1}{\omega^2}$, ω being equal to $2\pi f$, where f is the oscillating frequency. It will be shown that f has two distinct values.

The voltage induced in the secondary by the primary $= E_2 = \omega M I_1$.

The voltage induced in the primary by the secondary $= E_1 = \omega M I_2$.

These two voltages are equal (see page 24), or $E_1 = E_2$. Neglecting the resistances of both circuits,

$$I_1 = \frac{E_1}{\omega L_1 - \frac{1}{\omega C_1}},$$

and

$$I_2 = \frac{E_2}{\omega L_2 \quad \frac{1}{\omega C_2}}$$

$$= \frac{\omega M I_1}{\omega L_2 - \frac{1}{\omega C_2}}.$$

But

$$I_2 = \frac{E_1}{\omega M} = \frac{I_1\left(\omega L_1 - \frac{1}{\omega C_1}\right)}{\omega M}.$$

Therefore

$$\frac{\omega M I_1}{\omega L_2 - \frac{1}{\omega C_2}} = \frac{I_1\left(\omega L_1 - \frac{1}{\omega C_1}\right)}{\omega M}$$

and

$$(\omega M)^2 = \left(\omega L_1 - \frac{1}{\omega C_1}\right)\left(\omega L_2 - \frac{1}{\omega C_2}\right).$$

Dividing both sides by $\omega^2 L_1 L_2$, we get

$$\frac{M^2}{L_1 L_2} = \left(1 - \frac{1}{\omega^2 L_1 C_1}\right)\left(1 - \frac{1}{\omega^2 L_2 C_2}\right).$$

Substituting LC for $\frac{1}{\omega^2}$, and L_1C_1 for L_2C_2, we get

$$\frac{M^2}{L_1 L_2} = \left(1 - \frac{LC}{L_1 C_1}\right)\left(1 - \frac{LC}{L_2 C_2}\right)$$

$$= \left(1 - \frac{LC}{L_1 C_1}\right)^2$$

and $$\frac{M}{\sqrt{L_1 L_2}} = \pm\left(1 - \frac{LC}{L_1 C_1}\right)$$

or $$\frac{LC}{L_1 C_1} = 1 \pm \frac{M}{\sqrt{L_1 L_2}} = 1 \pm k,$$

where k is the coupling factor.

It is therefore evident that LC has two values, viz., $L_1C_1\ (1 + k)$ and $L_1C_1(1 - k)$, and the circuits oscillate at the two frequencies,

$$f' = \frac{1}{2\pi\sqrt{L_1C_1(1+k)}} = \frac{f}{\sqrt{1+k}}$$

and $$f'' = \frac{1}{2\pi\sqrt{L_1C_1(1-k)}} = \frac{f}{\sqrt{1-k}}.$$

When the coupling is infinitely loose, *i.e.* there is no mutual inductance at all, the two coils are independent, $k = 0$, and the two frequencies become equal. The tighter the coupling, the larger does k become, and the further apart are the two frequencies.

Resultant Current in Coupled Circuit.—Each of the coupled circuits discussed above has a current flowing in it which is really the sum of two alternating currents of slightly different frequency. Let these frequencies be represented by

$$f' = \frac{f}{\sqrt{1+k}} \text{ and } f'' = \frac{f}{\sqrt{1-k}},$$

and let $\omega' = 2\pi f'$ and $\omega'' = 2\pi f''$.

The first current can be represented by

$$i' = I' \sin \omega' t$$

and the second by

$$i'' = I'' \sin \omega'' t.$$

These two currents produce beats in the circuit, aiding one another at certain times, and neutralizing each other at other times when they are in opposition. It can therefore be assumed that $I' = I'' = I$, so that

$$\begin{aligned} i = i' + i'' &= I\,(\sin \omega' t + \sin \omega'' t) \\ &= I \times 2 \sin \frac{\omega' t + \omega'' t}{2} \cos \frac{\omega' t - \omega'' t}{2} \\ &= 2I \sin \alpha t \cos \beta t. \end{aligned}$$

The current can therefore be represented as a sine wave, the maximum amplitude of which gradually changes between positive and negative limits according to a cosine law. Such a curve is shown in Fig. 416, the envelope (shown dotted) appearing as a cosine curve. A similar curve to this is obtained when considering the synchronizing current of two alternators, the beats appearing in visual form in the synchronizing lamps.

Transfer of Energy in Coupled Circuits.—The energy, represented by $\frac{1}{2}LI^2$, first appears in the primary, and is transferred to the secondary. The secondary current grows and the primary current dies down. After a short interval of time, the whole of the energy

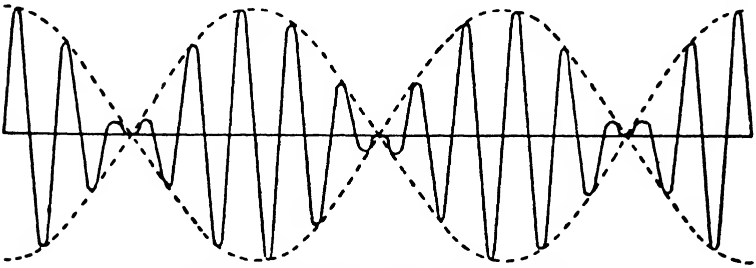


FIG. 416.—Resultant Current in Coupled Circuit.

appears in the secondary coil, neglecting losses, the primary current having died away. This energy is now re-transferred back again to the primary, the secondary current dying down and the primary current rising. This transfer of energy goes on, back and forth,

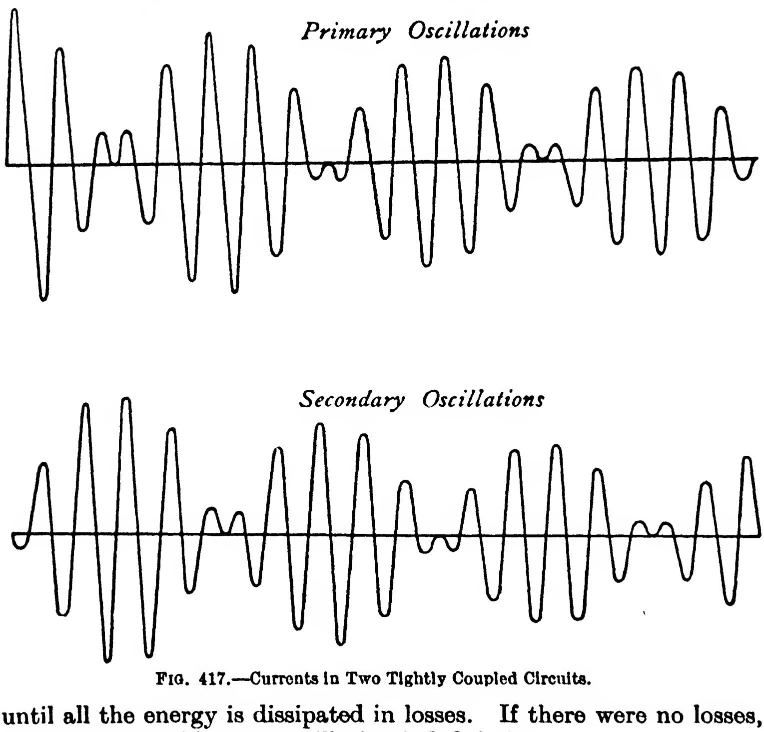


FIG. 417.—Currents in Two Tightly Coupled Circuits.

until all the energy is dissipated in losses. If there were no losses, the current would go on oscillating indefinitely.

The effect is shown in Fig. 417 which indicates the type of effect encountered with a relatively tight coupling. The beat frequency is fairly rapid, since there is an appreciable difference between the two frequencies f' and f''. The rate at which energy is transferred from one coil to the other depends upon the beat frequency, *i.e.* upon the cosine term in the above formula.

With a loose coupling the two frequencies are closer together, and the beat frequency (the difference between f' and f'') is lower. The maximum amplitude of the current is less, but the rate of transfer of energy from one coil to the other and back, is slower. Oscillations in two loosely coupled circuits are shown in Fig. 418 which should be compared with Fig. 417.

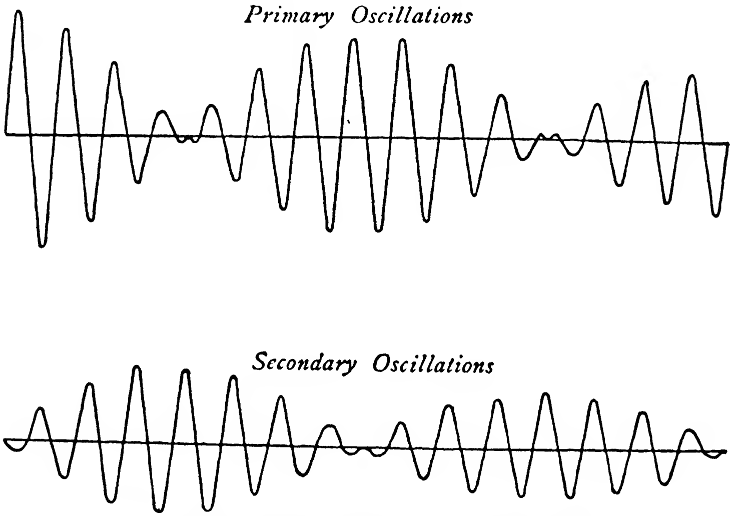


FIG. 418.—Currents in Two Loosely Coupled Circuits.

Acceptor and Rejector Circuits.—For the purpose of selecting a particular high frequency oscillation which it is desired to receive, others of different frequency being allowed to escape, the principle of resonance is used. There are two methods employed, corresponding to voltage resonance and to current resonance.

In the first method an inductance is connected in series with a variable condenser (see Fig. 419), the latter enabling the circuit to be tuned to the desired frequency. Such a circuit is called an *acceptor circuit* since the desired current is magnified, whilst the impedance of the circuit is high to currents of all other frequencies, which are consequently reduced to negligible proportions.

In the second method, a variable condenser is connected in parallel with an inductance (see Fig. 420), the circuit being tuned for current resonance. It now acts practically as an insulator for this desired frequency, and sets up a measurable p.d. across the combination which can be utilized for operating another circuit. Such a circuit is called a *rejector circuit.* All unwanted currents of other frequencies escape by way of the variable condenser.

A small amount of resistance must necessarily be present in both acceptor and rejector circuits, but the smaller this resistance the greater are the effects of resonance. The relations between current and frequency are shown in Fig. 421. As the resonant frequency is approached, the current becomes relatively very large

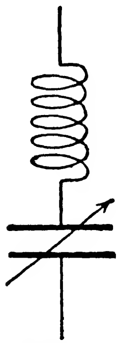


FIG. 419.—Acceptor Circuit.

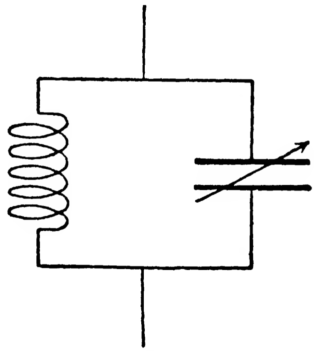


FIG. 420.—Rejector Circuit.

in an acceptor circuit and very small in a rejector circuit. In fact, if resistance and all other losses could be entirely eliminated, a rejector circuit would act as a perfect insulator. In an actual case, the lower the resistance, the sharper are the curves, and the circuits are said to be sharply tuned. They are very selective, *i.e.* frequencies differing slightly from the resonant frequency give rise to only very small currents in an acceptor circuit, whilst appreciable currents are permitted to flow in a rejector circuit. When the resistance is relatively low, the voltages across the component inductance and capacitance in the acceptor circuit are very much larger than the total voltage applied to the combination, whilst in the rejector circuit, the currents in each of the two parallel branches of the circuit are greatly in excess of the total current drawn from the supply.

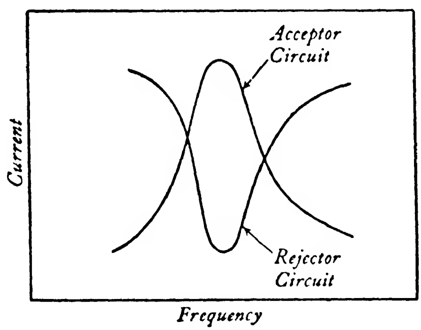


FIG. 421.—Effect of Frequency in Acceptor and Rejector Circuits.

High Frequency Resistance.—The increase in the effective resistance of a conductor due to skin effect has already been explained. The skin effect is enormous at very high frequencies, and this is of importance since it is desirable to keep the losses down to a minimum. In a cylindrical conductor, the current tends to flow only in the outer surface layers, whilst in a flat conductor the current is concentrated on the outer edges. In fact, surface becomes more important than cross sectional area. For this reason, hollow copper tubes are sometimes used as conductors in high frequency circuits.

Production of Damped Currents by Spark Discharge.—When a charged condenser is discharged through an inductance, a train of damped oscillations is produced, if the circuit conditions are suitable. In order to produce a succession of such trains, the condenser must be re-charged at the end of each train. Mechanical methods of switching are not satisfactory, but a spark gap can be arranged to act as an automatic switch. The circuit is shown in Fig. 422. A low tension, low frequency, alternating E.M.F. is applied to the primary of a transformer. (Alternatively, a D.C. source may be used with an interrupter and an induction coil.)

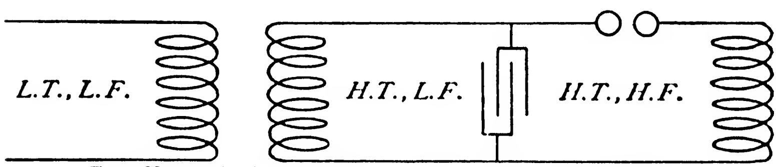


FIG. 422.—Production of High Frequency Currents by Spark Discharge.

The secondary of the transformer delivers a H.T. voltage, but still at low frequency, to the condenser, which becomes charged as the voltage rises. When the voltage reaches a sufficiently high value, the spark gap breaks down and the condenser gives rise to an oscillatory discharge through the spark gap and the inductance in series. It is enabled to do this because the air between the spark gap electrodes becomes conducting as soon as a spark occurs. When the condenser becomes discharged, the spark ceases and the air becomes an insulator once again. This oscillatory discharge is damped as already explained, a train of oscillations being set up for each spark.

There are really three circuits in Fig. 422, the source of supply and the primary of the transformer forming the first which is L.T., L.F. The second circuit consists of the transformer secondary and the condenser which is H.T., L.F. The third or oscillatory circuit consists of the condenser, the spark gap and the inductance and is H.T , H.F.

High Frequency Transformers.—When transformers are used in high frequency oscillatory circuits, they are made with an air core, since the use of iron is inadmissible on account of the prohibitive iron losses which would take place.

Quenched Spark Gap.—If the inductance in Fig. 422 be used as the primary of a further high frequency transformer, then the double frequency effect explained on page 594 is encountered. Energy is transferred back and forth from primary to secondary and back, but this is prevented if the primary circuit is broken after the first train of oscillations. This is accomplished by using what is called a *quenched spark gap*, which consists of a number of small gaps in series, large cooling fins being provided between each gap. The high frequency currents in primary and secondary now take the form shown in Fig. 423. During the time occupied by the train of oscillations in the primary, the secondary current is building up, but after the spark is extinguished and the primary circuit opened, the secondary current dies away according to the exponential law for a freely oscillating circuit.

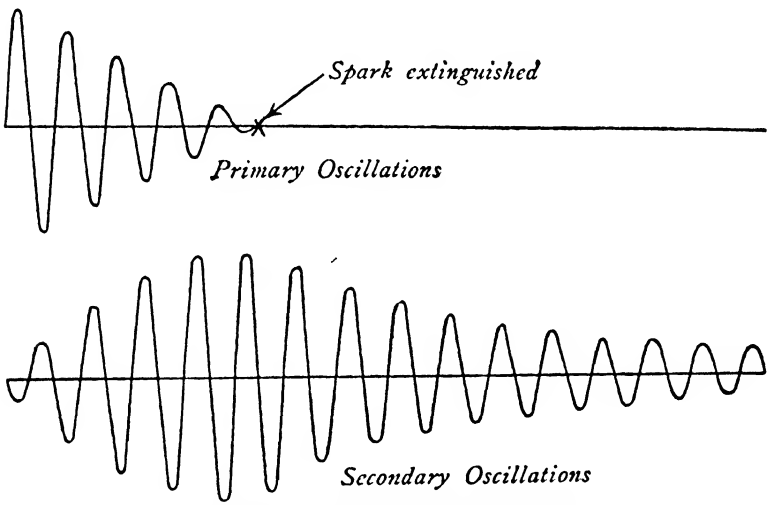


FIG. 423.—Currents with Quenched Spark Gap.

Production of Undamped High Frequency Currents.—There are three principal methods available for the production of undamped high frequency currents, viz., the high frequency alternator, the arc, and the three-electrode thermionic valve, but of these three the last-named is the one generally used.

Detection of High Frequency Currents.—These currents are always very small, and the only suitable instrument for their detection is the telephone. In view of the frequency, however, neither the telephone diaphragm nor the human ear will respond, and it becomes necessary to modulate the amplitude at audio-frequency and then to rectify the currents into unidirectional pulsating currents. These currents can now be made to vibrate the telephone diaphragm at the modulated frequency.

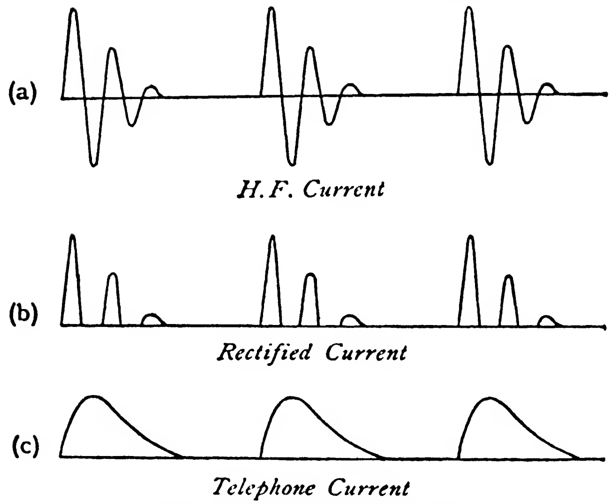


FIG. 424.—Detection of Damped Current.

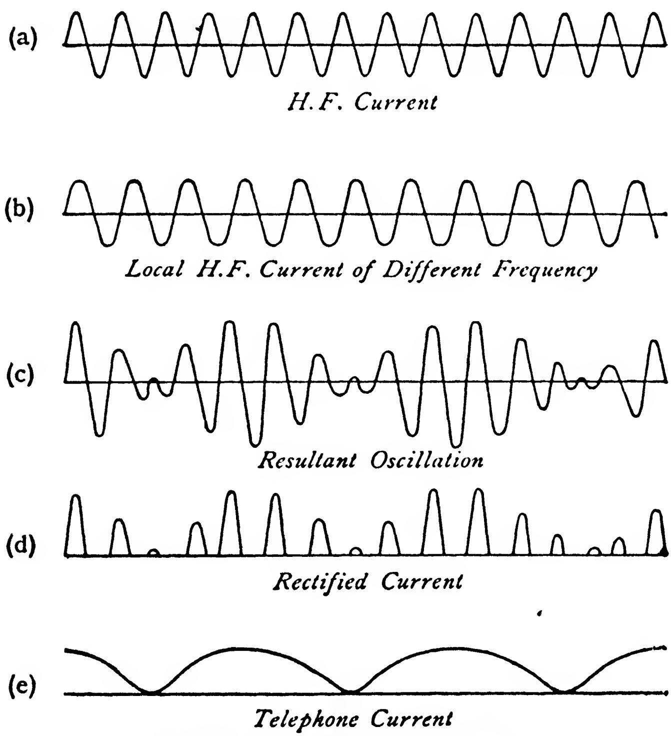


FIG. 425.—Detection of Undamped Current.

Rectification.—There are various ways of rectifying high frequency currents, but the method employing the three electrode thermionic valve is the one generally used in high frequency work. The theory and description of the thermionic valve, however, lie outside the scope of this work.

Detection of Damped Currents.—The general method adopted for the detection of damped high frequency currents is represented in Fig. 424. The trains of damped high frequency currents are shown at (*a*), the corresponding rectified currents being shown at (*b*). The telephone averages these rectified currents, the telephone current being shown at (*c*). The latter is a pulsating current of a much lower frequency, capable of being heard in the telephone.

Detection of Undamped Currents.—The manner of detecting undamped high frequency currents is represented in Fig. 425. The current to be detected is shown at (*a*), and superimposed on this is another current of slightly different frequency, usually obtained from a local source. The resultant oscillations are shown at (*c*), beats being obtained the frequency of which depends upon the *difference* of frequency of the two high frequency currents. After rectification, the current becomes of the type shown at (*d*), whilst the final telephone current is shown at (*e*).

EXAMPLES

(1) Assuming the equation for a damped alternating current, obtain an expression for the logarithmic decrement of an oscillatory circuit.

(2) A circuit tuned for a wave length of 300 metres has an inductance of 150 microhenries and an effective resistance of 10 ohms. Calculate the logarithmic decrement.

(3) Discuss the effect produced by a tight coupling with damped oscillations, and explain how a quenched spark gap minimizes this effect.

(4) What is (*a*) an acceptor and (*b*) a rejector circuit, and how do they operate?

(5) Explain how oscillations of two frequencies are set up in two tuned coupled circuits.

SOLUTIONS TO NUMERICAL EXAMPLES

I. (1) 34, 71, − 64 units.
(2) 1 volt.

II. (1) 81·6.
(2) 1·15.
(3) R.M.S. value = 72·7 amperes.
Average value = 59·2 amperes.

III. (1) 43·7 volts.
(2) $\frac{1}{300}$ second.
(3) (*a*) 27210 amperes per second.
(*b*) −31420 ,, ,, ,,
(*c*) 0.
(4) 51·2 volts.
(5) 57°, 38°.

IV. (1) 17·3 amperes.
(2) 175 ohms.
(3) $Y = 0{\cdot}128$, $G = 0{\cdot}124$, $B = 0{\cdot}032$.
(4) 4 ohms.
(5) $\frac{RX}{\sqrt{R_2 + X_2}}$.

V. (1) 20 watts, 0·031 p.f.
(2) 660 watts, 0·6 p.f.
(3) 14·9 amperes, 0·74 p.f.
(4) 0·104 joule.

VI. (1) 0·882 ampere, 93·6 volts, 7940 micro-coulombs.
(2) (*a*) 5·004 ohms, 19·99 amperes, 318 volts.
(*b*) 7·50 ohms, 13·33 amperes, 177 volts.
(3) 6·76μF.
(4) 36 joules.

VII. (1) 76·5 volts.
(2) 181 volts.
(3) 7·7 ohms.

VIII. (1) 244 watts.
(2) $0{\cdot}2 \times 10^6$ lines, 0·00947 henry.
(3) 1354 watts, 1440 watts.
(4) 205 watts.

IX. (1) 72·4 volts.
(2) $3{\cdot}19 \sin(\omega t - 90°) - 0{\cdot}266 \sin(3\,\omega t - 90°)$.
$3{\cdot}14 \sin(\omega t + 90°) - 2{\cdot}36 \sin(3\,\omega t + 90°)$.
(3) 31·7 ohms, 29·1 ohms.
(8) 268 ohms.
(9) 3·73 amperes.
(10) 10, 30 and 90 per cent.

X. (2) 48·2 amperes.
(3) 60 ohms, 8 amperes.
(4) 4400 watts. 0·44 power factor.
(5) 182, 342, 368 volts. 107 volts.

XI. (3) 1·88 times.

XII. (4) 0·76.

XIV. (1) 11550 volts, 11·45 amperes, 0·70 p.f., 86·5 per cent. efficiency, 15·5 per cent. regulation.
(2) $0{\cdot}2 \times 10^6$ lines.
(3) 1486 and 50 turns.
(6) 83 and 72 turns. 277 amperes.
(7) 30 and 20 amperes in 40 and 60 per cent. turns respectively.
(8) 22·8 : 1.

XV. (2) 2·1 per cent.
(3) 1·05 per cent., 2·62 per cent.
(4) 97·78 per cent. efficiency, 94·8 per cent. full load.
(5) 98·16 per cent. efficiency. Iron loss = 3·5 kW. Copper loss = 4 kW.
(6) 4·2 per cent.

XVI. (3) About 80 kVA.
(4) 88 and 78 cms.
(7) 158 and 32 turns; 0·26 sq. cm. and 1·25 sq. cms.
(8) Depth of stampings; 7·5 cms., 12·5 cms., 16·0 cms.
(9) 438 and 28 turns.
Width of yoke stampings, 30 and 18 cms.
(10) Width of core stampings, 18 and 11 cms.
" " yoke " 28 and 13·75 cms.

XVII. (3) 0·956. 5·8% less due to lower breadth factor.
(6) $15{\cdot}1 \times 10^6$ lines.
(7) $95{\cdot}6 \times 10^6$ lines.

XVIII. (3) 10·5% on unity power factor.
24·2% on 0·8 " "
(4) 22%.

XIX. (1) D = 200 cms., L = 30 cms.
(3) 72 slots, 5 conductors per slot.
(7) D = 175; L = 220; slots = 72 (or 75); conductors = 72.
(8) 144 slots and 16 conds. per slot, or
192 slots and 12 conds. per slot.

XX. (2) 71 lbs.-ft.
(3) 1·8 times full load current.
0·79 times full load torque.
(6) 1000, 600 and 375 r.p.m.
(7) 30 cycles per second. Speed is zero.
(8) 1500 r.p.m., 30 B.H.P.
1000 r.p.m., 20 B.H.P.
600 r.p.m., 20 B.H.P.
30 cycles.
(10) Transformer steps down in ratio $\sqrt{2}$: 1.

XXI. (1) 75 r.p.m.
(2) 725 r.p.m.
(3) 0·89 p.f., 0·88 p.f.
(5) 0·91 p.f., 106 B.H.P., 622 lbs.-ft.

XXII. (1) 27 B.H.P., 60 slots, 12 conductors per slot.
(3) 0·87 p.f.
(4) 0·84 p.f.

XXIII. (1) 460 volts.
(4) 0·88 p.f. leading.
(5) About 300 amperes, 27 kW.

XXIV. (1) 11·5 amperes.
(2) 27800 lbs.-feet.
(4) 258·8 kVA at 0·966 p.f. for each machine.
(5) 10°.

XXV. (1) 19·4 : 1.
(4) Unity power factor.

XXVI. (1) 600 r.p.m. 30 cycles per second.
(3) 428·6 r.p.m.; 28·6 cycles; 300 kW.

XXVIII. (1) 12 and 34 poles; 300 r.p.m.
(2) 4·5° and 4·7°.
(3) 750 r.p.m.; 6-pole motor and 8-pole generator.
(4) 12 poles.

XXIX. (2) 168 volts and 73 volts.

XXXIII. (1) 0·95 p.f.
(2) 405 μF.
(3) 2900 kVA.
(4) 11·5 kVA and 3·41 per cent. slip for the Leblanc type.
9·3 kVA and 1·39 per cent. slip for the Walker type.
(5) 4·7 kVA.
(6) 304 volts.
(9) 4 per cent.

XXXV. (1) $E = 320 + j\,220$.
(2) $I_1 = 16 - j\,12$, $I_2 = 11{\cdot}77 - j\,2{\cdot}94$, $I = 27{\cdot}77 - j\,14{\cdot}94$,
$\phi_1 = 8{\cdot}6°$, $\phi_2 = -14{\cdot}3°$.
(3) $I = 0{\cdot}37 + j\,7{\cdot}72$.
(4) 20900 volts, 15°.
(5) 25·5 amperes; 0·707 power factor; 45° 14′ phase angle.
(6) $Y_1 = 0{\cdot}1488$, $Y_2 = 0{\cdot}068$, $Y = 0{\cdot}234$.
$G_1 = 0{\cdot}0976$, $G_2 = 0{\cdot}0488$, $G = 0{\cdot}1464$.
$B_1 = 0{\cdot}122$, $B_2 = 0{\cdot}0610$, $B = 0{\cdot}183$.
I = 46·9 A; p.f. = 0·625 (lagging).
Capacitance = 582 μF.

XXXVI. (1) 12·6 amperes.

XXXVII. (2) 0·033.

APPENDIX *A*

LIST OF SYMBOLS

THE following list of rules and symbols is in accordance with the recommendations issued by the International Electrotechnical Commission, and is adhered to in the present book.

RULES FOR QUANTITIES

(*a*) Instantaneous values of electrical quantities which vary with time are represented by small letters.

(*b*) R.M.S. values of electrical quantities are represented by capital letters.

(*c*) Maximum values of periodic electrical and magnetic quantities are represented by capital letters followed by the subscript "*m*."

(*d*) Angles and ratios are represented by small Greek letters.

TABLE OF SYMBOLS

	Name of Quantity.	Symbol.
1.	Length	l
2.	Mass	m
3.	Time	t
4.	Angles	α, β, γ . . .
5.	Acceleration of Gravity	g
6.	Work	A
7.	Energy	W
8.	Power	P
9.	Efficiency	η
10.	Number of Turns in Unit of Time	n
11.	Temperature Centigrade	t
12.	Temperature Absolute	T
13.	Period	T
14.	$2\pi/T$	ω
15.	Frequency	f
16.	Phase Displacement	ϕ
17.	Electromotive Force	E
18.	Current	I
19.	Resistance	R
20.	Resistivity	ρ
21.	Conductance	G
22.	Quantity of Electricity	Q
23.	Flux Density, Electrostatic	D
24.	Capacitance	C

	Name of Quantity.	Symbol.
25.	Dielectric Constant	ϵ
26.	Self-Inductance	L
27.	Mutual Inductance	M
28.	Reactance	X
29.	Impedance	Z
30.	Reluctance	S
31.	Magnetic Flux	Φ
32.	Flux Density, Magnetic	B
33.	Magnetic Field	H
34.	Intensity of Magnetization	J
35.	Permeability	μ
36.	Susceptibility	κ
37.	Difference of Potential, Electric	V
38.	Magnetomotive Force	†

† Letter symbol to be proposed by the National Committees.

APPENDIX *B*

TRIGONOMETRICAL RATIOS

Angle.		Sine.	Cosine.	Tangent.	Angle.		Sine.	Cosine.	Tangent.
Deg.	Radians.				Deg.	Radians.			
0	0	0	1	0	46	0·8029	0·7193	0·6947	1·0355
1	0·0175	0·0175	0·9998	0·0175	47	0·8203	0·7314	0·6820	1·0724
2	0·0349	0·0349	0·9994	0·0349	48	0·8378	0·7431	0·6691	1·1106
3	0·0524	0·0523	0·9986	0·0524	49	0·8552	0·7547	0·6561	1·1504
4	0·0698	0·0698	0·9976	0·0699	50	0·8727	0·7660	0·6428	1·1918
5	0·0873	0·0872	0·9962	0·0875	51	0·8901	0·7771	0·6293	1·2349
6	0·1047	0·1045	0·9945	0·1051	52	0·9076	0·7880	0·6157	1·2799
7	0·1222	0·1219	0·9925	0·1228	53	0·9250	0·7986	0·6018	1·3270
8	0·1396	0·1392	0·9903	0·1405	54	0·9425	0·8090	0·5878	1·3764
9	0·1571	0·1564	0·9877	0·1584	55	0·9599	0·8192	0·5736	1·4281
10	0·1745	0·1736	0·9848	0·1763	56	0·9774	0·8290	0·5592	1·4826
11	0·1920	0·1908	0·9816	0·1944	57	0·9948	0·8387	0·5446	1·5399
12	0·2094	0·2079	0·9781	0·2126	58	1·0123	0·8480	0·5299	1·6003
13	0·2269	0·2250	0·9744	0·2309	59	1·0297	0·8572	0·5150	1·6643
14	0·2443	0·2419	0·9703	0·2493	60	1·0472	0·8660	0·5000	1·7321
15	0·2618	0·2588	0·9659	0·2679	61	1·0647	0·8746	0·4848	1·8040
16	0·2793	0·2756	0·9613	0·2867	62	1·0821	0·8829	0·4695	1·8807
17	0·2967	0·2924	0·9563	0·3057	63	1·0996	0·8910	0·4540	1·9626
18	0·3142	0·3090	0·9511	0·3249	64	1·1170	0·8988	0·4384	2·0503
19	0·3316	0·3256	0·9455	0·3443	65	1·1345	0·9063	0·4226	2·1445
20	0·3491	0·3420	0·9397	0·3640	66	1·1519	0·9135	0·4067	2·2460
21	0·3665	0·3584	0·9336	0·3839	67	1·1694	0·9205	0·3907	2·3559
22	0·3840	0·3746	0·9272	0·4040	68	1·1868	0·9272	0·3746	2·4751
23	0·4014	0·3907	0·9205	0·4245	69	1·2043	0·9336	0·3584	2·6051
24	0·4189	0·4067	0·9135	0·4452	70	1·2217	0·9397	0·3420	2·7475
25	0·4363	0·4226	0·9063	0·4663	71	1·2392	0·9455	0·3256	2·9042
26	0·4538	0·4384	0·8988	0·4877	72	1·2566	0·9511	0·3090	3·0777
27	0·4712	0·4540	0·8910	0·5095	73	1·2741	0·9563	0·2924	3·2709
28	0·4887	0·4695	0·8829	0·5317	74	1·2915	0·9613	0·2756	3·4874
29	0·5061	0·4848	0·8746	0·5543	75	1·3090	0·9659	0·2588	3·7321
30	0·5236	0·5000	0·8660	0·5774	76	1·3265	0·9703	0·2419	4·0108
31	0·5411	0·5150	0·8572	0·6009	77	1·3439	0·9744	0·2250	4·3315
32	0·5585	0·5299	0·8480	0·6249	78	1·3614	0·9781	0·2079	4·7046
33	0·5760	0·5446	0·8387	0·6494	79	1·3788	0·9816	0·1908	5·1446
34	0·5934	0·5592	0·8290	0·6745	80	1·3963	0·9848	0·1736	5·6713
35	0·6109	0·5736	0·8192	0·7002	81	1·4137	0·9877	0·1564	6·3138
36	0·6283	0·5878	0·8090	0·7265	82	1·4312	0·9903	0·1392	7·1154
37	0·6458	0·6018	0·7986	0·7536	83	1·4486	0·9925	0·1219	8·1443
38	0·6632	0·6157	0·7880	0·7813	84	1·4661	0·9945	0·1045	9·5144
39	0·6807	0·6293	0·7771	0·8098	85	1·4835	0·9962	0·0872	11·4301
40	0·6981	0·6428	0·7660	0·8391	86	1·5010	0·9976	0·0698	14·3007
41	0·7156	0·6561	0·7547	0·8693	87	1·5184	0·9986	0·0523	19·0811
42	0·7330	0·6691	0·7431	0·9004	88	1·5359	0·9994	0·0349	28·6363
43	0·7505	0·6820	0·7314	0·9325	89	1·5533	0·9998	0·0175	57·29
44	0·7679	0·6947	0·7193	0·9657	90	1·5708	1·0000	0	∝
45	0·7854	0·7071	0·7071	1·0000					

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