Elementary Theory and Design

of

FLEXURAL MEMBERS

By Thomas C. Shedd and Jamison Vawter

THEORY OF SIMPLE STRUCTURES

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PREFACE

It is essential that the student beginning the study of structural design obtain a thorough grounding in theory before he becomes involved in the intricacies of the details he will encounter in putting the pieces of members and the parts of structures together, just as a thorough understanding of statics is a necessary foundation for the study of structures in all the various related fields. If, in the early stages of the study of design, too much attention is given to the details of the structure, the underlying principles of design are likely to become obscured in the student's mind.

This is a textbook on the basic theory of flexure as applied to the design of members in bending. It is necessary to give some attention to details, as they are an important part of design, but the purpose has been to keep such attention down to a minimum. If at any point the details outweigh the basic design concepts, the authors apologize.

The book covers a beginning course in design as taught by the authors and their co-workers during the past several years. The course is a result of many years of constant development of the curriculum and was established in order to utilize the students' time more effectively by covering the common materials. The first five chapters are covered in the course. The first five articles of Chapter 6 are also included so that the student, in beginning his study of design, will have brought to his attention the fact that the common application of the flexural formula is true only for symmetrical members, for members restrained in their movements, or in bending about certain axes. The authors have also endeavored, in their presentation, to avoid unnecessary use of formulas since they believe that the student will obtain a clearer picture of the behavior of the member if he avoids excessive formularization.

Although it is necessary, in following specifications, to have separate chapters for discussing the use of different materials, this textbook endeavors to show that the same physical laws govern any of the common materials, and that there are no fundamental differences in the analysis if the material is properly applicable. However, certain peculiarities of the material must be taken into account. For example, in Chapter 5 mention is made of the effect of time yield in the design of reinforced concrete, in order to agree with present specifications, but in the course as taught there is no deviation from the method of transformed sections.

This is believed to be the best method for the beginner. The other concepts should be reserved for the advanced undergraduate courses in structural design. In all such advanced courses in building and bridge design taught by the authors and their co-workers, steel, timber, and reinforced concrete are covered.

The book is therefore appropriate for an elementary design course that comes between strength of materials and stresses on the one hand, and courses in building and bridge design on the other, or it can be used to precede separate advanced courses in steel and reinforced concrete. In curricula in which structures are not emphasized, the book will fill the need for a first course in design, to be followed by a single course in bridge and building design. It is also suitable for a single course for non-civil engineers.

The authors wish to acknowledge their indebtedness to Professor Thomas C. Shedd who directed the establishment of the course for which the book was written and who has assisted in its development. They also wish to acknowledge their indebtedness to their co-workers who have made use of the rough class notes that formed a basis for this book, and who have made many valuable suggestions for its improvement.

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CONTENTS

INTRODUCTION

1. Design of Structures. In the design of any structure, whether it be a simple beam or a member (or members) of a more complicated character, it is first necessary to take account of the function of the structure and the loads that must be provided for. The characteristics of the material used, the stresses to be permitted, and the manner in which the various parts are connected to each other are all part of the problem of design.

This is an elementary textbook in the theory and design of flexural members, but the student in elementary design should have a background in mechanics of materials and should have had a course in structural theory (or stresses). A thorough background in these two subjects is essential for anyone attempting to go very far in designing structures.

2. Dead Load. The two general classes of loads that a structure has to support are the dead load and the live load (including the effect of any impact that may exist).

The dead loads are the loads that are always acting on the structure and that remain constant in magnitude. The weight of the material making up the structure and the weight of any permanent equipment that is built into the structure constitute the dead loads. Dead loads are the result of the action of gravity.

It is necessary to make an estimate of the dead loads before designing a structure. A beginning designer may not be able to make a very close estimate and may have to correct his design for these changed loads. The experienced designer will be able to make a very close estimate and will rarely have to make corrections. He is, of course, fortified by data accumulated during his years of experience.

3. Live Loads. Live loads are not of constant magnitude, and they are not always acting. They can be movable, such as goods on a warehouse floor, or similar loads; or moving, such as trains, highway traffic, cranes, etc. Other live loads, both movable or moving, are: building occupants, furniture, snow, ice, wind, any stored goods, machinery, etc. Moving live loads often produce an effect greater than they would produce in a fixed position, and this effect is called impact.

A more detailed discussion of various types of loads and weights of stored goods, together with building code recommendations for loads, can be found in Chapter I of *Theory of Simple Structures* by Shedd and Vawter.

4. Impact. The vibration effects caused by a load (or loads) moving along a structure, due to imperfect balance, irregularities in the contact surfaces, or blows, result in stresses greater than those that would be produced if the load were in a static position. These additional stresses are called impact stresses and are usually expressed in terms of a percentage of the live load stresses. A great deal of experimentation and study has been made regarding impact, but the values used are largely empirical and vary greatly with different authorities. More knowledge of the subject is desirable.

The amounts of impact for different structures and parts of structures are given in the various technical specifications for design.

5. Other Forces. There are other forces, in addition to the loads previously mentioned, that will be encountered in the design of structures. Forces due to traction and stresses due to temperature changes are common examples. Their existence, or importance, will depend on such factors as the type of loading, the size of the structure, and the likelihood of extreme temperature changes. The consideration of temperature stresses often necessitates a knowledge of statically indeterminate structures. The designer must be able to recognize when they are important and make proper provision for their effect.

Forces due to earthquake must also be included in localities where earthquakes are encountered.

6. Handbooks and Specifications. It is essential that anyone designing a structure of steel, wholly or in part, have ready access to a steel handbook. The manual of the American Institute of Steel Construction, *Steel Construction*, is the one in most general use. The 1949 printing has been utilized in the general discussion in this volume. Therefore the student using this textbook in designing steel flexural members should have that, or a similar, handbook at his disposal.

The handbook contains the "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings," as revised in 1949 and adopted by the American Institute of Steel Construction (A.I.S.C.). The authors make general use of these specifications in their discussion of steel design, and the illustrative examples are based on them, with certain minor modifications.

In this discussion the authors refer to the "Standard Specifications for Highway Bridges" (1949) of the American Association of State Highway Officials, and also to the "Specifications for Steel Railway

Bridges" (1947) of the American Railway Engineering Association. These are the two leading specifications in their fields.

The student should also have at his disposal the "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete," as submitted in 1940 by the Joint Committee on Standard Specifications for Concrete and Reinforced Concrete, or the "Building Regulations for Reinforced Concrete," as adopted by the American Concrete Institute in 1946; preferably both. The authors have made more reference to the Joint Committee report in their discussion as it is the basis of practically all reinforced concrete specifications in current use.

Handbooks with sets of elaborate tables and diagrams are not recommended for the beginning student. They are confusing and largely useless, and they detract the student's mind from the simple principles involved in reinforced concrete design. Dependence on such material will destroy initiative and independent thought on the part of the student. Experienced designers of reinforced concrete structures naturally develop and acquire tables and charts that are valuable and time saving. The "Proposed Manual of Standard Practice for Detailing Reinforced Concrete Structures" published by the American Concrete Institute, and mentioned in Chapter 5, has important information for detailing.

Although, as stated in the Preface, timber is a third important structural material that is widely used, it is not treated separately in this text. All the information necessary for the design of timber beams, or beams composed of timber and some other material, can be found in Chapters 1 and 2. The student will need information on timber details and connections. *Modern Timber Engineering* by Scofield and O'Brien, published by the Southern Pine Association (third edition, 1949), is recommended for this purpose. Timber design is illustrated in the Appendix.

The authors have made frequent reference to *Theory of Simple Structures* by Shedd and Vawter, published by John Wiley and Sons (second edition, 1941), and *Structural Design in Steel* by Shedd, also published by John Wiley and Sons (1934). Other references can just as well be used by the student, depending on his preference. Appropriate references are given at the end of Chapters 3 and 5, and in the Appendix, as well as in the body of the text.

CHAPTER 1

PURE BENDING IN HOMOGENEOUS, SYMMETRICAL SECTIONS

7. Types of Members. There are three types of members encountered in structural design:

(*a*) Beams or members resisting transverse loads, often called flexural members.

(*b*) Tension members, i.e., members resisting a pulling force.

(*c*) Compression members, generally called columns, which are designed to resist compressive stresses.

Since this book will be devoted primarily to a discussion of flexural members, there will be no further discussion of beams at this time.

Tension members are so proportioned as to have sufficient area to resist the applied load without exceeding the proper working stress for the material. Where it is necessary to reduce the area of the member because of details (rivets, threads, etc.), the net section of the member must be considered. In considering the tension side of beams it is also necessary to take account of the net area.

Unless the compression members are short, some type of column reduction formula is required in computing the allowable stress for their proportioning. Since compression members are very commonly subjected to flexure also, the elementary principles in their design will be considered later.

Although there are only three separate types of members, as listed above, many members are subjected simultaneously to transverse and axial loading, and some members are subjected, under varying conditions of loading, to either tension or compression, or both. Classification of a member under any one type depends primarily on the kind of load that produces the critical stresses in the design of the member.

8. Bending in Symmetrical Beams. The principles discussed now and later, except for Chapter 6, are applicable only to beams that have at least one axis of symmetry. Bending in unsymmetrical sections is discussed in Chapter 6.

The beam AB, shown in Fig. 1, is subjected to a load P which causes the beam to bend. In bending, the beam takes a shape somewhat as shown in Fig. 1*d*. In the discussion of members in flexure, it is assumed that the stresses are within the elastic limit of the material, that there is a linear distribution of stress, that stress is proportional to strain, and that plane sections that are normal to the axis before bending remain planes after bending.

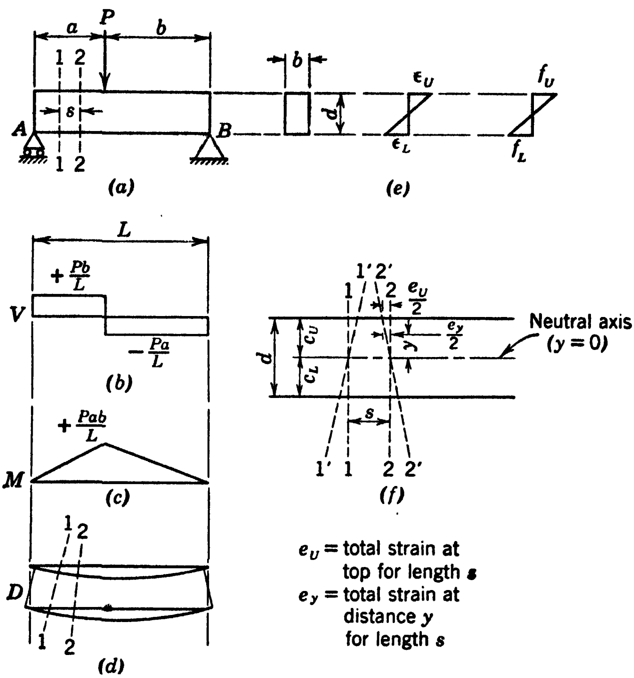


FIG. 1

If two parallel planes are taken, such as 1–1 and 2–2, a distance s apart, as shown in Fig. 1*a*, and the beam is considered as being bent as in Fig. 1*d*, then the planes take a position such as shown in Fig. 1*f*. The fibers in the top of the beam have been shortened, and those in the bottom of the beam lengthened. Those in the top of the beam are therefore in compression, and those in the bottom of the beam in tension. Somewhere there is a fiber that has not changed in length. This is called the neutral axis or the axis of zero stress. Since the plane sections remain planes, the change in length of the fibers varies as the distance from the neutral axis, and therefore the unit stress varies as the same distance.

9. Location of Neutral Axis. In Fig. 2 is shown the cross section of a beam symmetrical about the vertical axis.

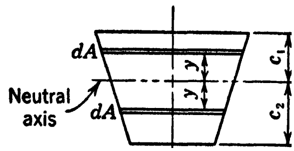


FIG. 2

Let dA = a small element of area either above or below the neutral axis.
y = the distance from the neutral axis to dA.
f = the unit fiber stress on this element of area.
c_1 = the distance from the neutral axis to the top of the beam.
c_2 = the distance from the neutral axis to the bottom of the beam.
C = the total compression at the section.
T = the total tension at the section.

Then

$$C = \int_0^{c_1} f\,dA \quad \text{and} \quad T = \int_0^{c_2} f\,dA$$

If f_1 = the fiber stress at the top of the beam and f_2 = the fiber stress at the bottom of the beam, then

$$f = \frac{f_1}{c_1} \cdot y \quad \text{or} \quad f = \frac{f_2}{c_2} \cdot y$$

Therefore $f_1/c_1 = f_2/c_2$.

$$C = \frac{f_1}{c_1} \int_0^{c_1} y\,dA \quad \text{and} \quad T = \frac{f_2}{c_2} \int_0^{c_2} y\,dA$$

From statics (when there is pure bending), $C = T$; therefore,

$$\frac{f_1}{c_1} \int_0^{c_1} y\,dA = \frac{f_2}{c_2} \int_0^{c_2} y\,dA$$

and, since $f_1/c_1 = f_2/c_2$, then

$$\int_0^{c_1} y\,dA = \int_0^{c_2} y\,dA$$

or the neutral axis is at the gravity axis of the section when there are bending stresses only on the section.

10. Bending Stresses in Rectangular Beams. The cross section of the rectangular beam shown in Fig. 1*e* is shown again in Fig. 3*a*, the section being taken at section 1–1.

The stresses in the beam at this section can be evaluated from the moment of the resisting couple from $\int f \times b\,dy \times y$, but in the simpler shapes it is usually easier to calculate the stresses directly by means of the relations previously obtained.

If M equals the bending moment at the section, then the resisting moment is equal to $C \times d_e$ or $T \times d_e$, in which C and T are the result-

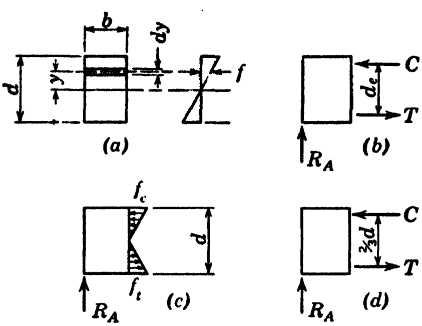


Fig. 3

ants of the compressive and tensile stresses, respectively, and d_e is the effective depth, or the distance between these resultants.

Therefore

$$M = C \times d_e \quad \text{or} \quad T \times d_e$$

Since we have a linear distribution of stress, the unit stress varies as shown in Fig. 3*c*. If the unit compressive stress in the top fiber of the beam is f_c, then the average unit stress is $f_c/2$, the total compression is

$$C = \frac{f_c}{2} \times b \times \frac{d}{2} = \frac{f_c \times bd}{4}$$

and the resultant of these compressive stresses is (where there is a constant width) at the centroid of the triangle between the neutral axis and the top.

Likewise, if f_t represents the unit tensile stress in the bottom fiber of the beam,

$$T = \frac{f_t}{2} \times b \times \frac{d}{2} = \frac{f_t}{4} \times bd$$

and the resultant is at the centroid of the lower triangle.

Therefore $d_e = \frac{2}{3}d$, and we can evaluate f_c and f_t, the two being equal in this case.

These are not formulas to be remembered, but in each instance the stresses should be evaluated from the given data, as shown in the following illustrative examples.

11. Illustrative Example. For a rectangular beam:

At section x–x: Area in tension = Area in compression = $20 \times 16 = 320$ sq in.

Total tension $= T = \dfrac{320 f_t}{2}$ = Total compression $= C = \dfrac{320 f_c}{2} = 160 f_c$ (lb).

Distance between T and $C = 40 \times \frac{2}{3} = 26.7$ in.

Resisting moment $= 160 f_c(26.7) = 4500$ in.-kips.

$$f_c = 1054 \text{ lb per sq in.} = f_t$$

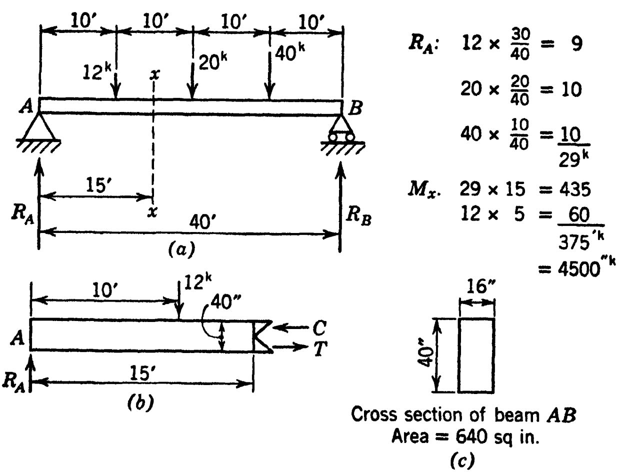


FIG. 4

12. Beams Symmetrical about One Axis Only. The method for finding stresses (or resisting moments) in rectangular beams, as shown above, is usually the most direct approach in similar calculations involving beams having only one axis of symmetry, particularly when they have rather regular cross sections. The examples given in the illustrative examples in the next article are intended to show these applications. Necessary explanations are given with the examples.

13. Illustrative Examples. For nonrectangular beams and beams with holes:

A. Use the same span and loads as shown in Fig. 4, but replace the cross section of beam in Fig. 4 with the one shown in Fig. 5, which has the same depth and area.

Neutral axis, or centroid:

$$\int y\, dA = 0$$

Take reference axis at top:
Moment of area around reference axis:

$$10 \times 32 \times 5 = 1600$$
$$40 \times 8 \times 20 = 6400$$
$$8000 \div 640 = 12.5 \text{ in.}$$

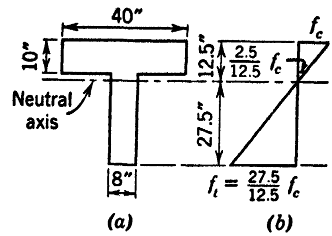


FIG. 5

Area in compression = $10 \times 40 + 2.5 \times 8 = 40 \times 12.5 - 32 \times 2.5 = 420$ sq in.

Area		Average Stress	Force	Arm about T	Moment
$40 \times 12.5 =$	500	$\dfrac{f_c}{2}$	$250f_c$	$\frac{2}{3} \times 40$ *	$6667f_c$
$-32 \times 2.5 =$	-80	$\dfrac{2.5}{25}f_c$	$-8f_c$	$\frac{2}{3} \times 30$	$-160f_c$
	420		$242f_c = C$		$6507f_c$
$8 \times 27.5 =$	220	$\dfrac{27.5}{25}f_c$	$242f_c = T$		

$$6507f_c = 4500$$

$$f_c = 0.691 \text{ kip per sq in.} = 691 \text{ lb per sq in.}$$

$$f_t = 2.2 \times 0.691 = 1.52 \text{ kips per sq in.}$$

* $\frac{2}{3} \times 12.5 + \frac{2}{3} \times 27.5 = \frac{2}{3} \times 40.$

B. Use the same span and loads as shown in Fig. 4, but replace the cross section of beam in Fig. 4 with the one shown in Fig. 6, which has the same depth and area.

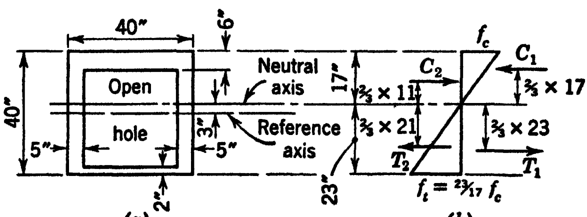


FIG. 6

Neutral axis: Take reference axis at mid-depth:
Moment of hole around reference axis:

Moment of hole $= -30 \times 32 \times -2 = +1920 \quad \div 640 = +3$ in.

Area		Average Stress	Force	Arm	Moment	
$40 \times 17 =$	680	$\frac{f_c}{2}$	$340.0f_c$	$\frac{2}{3} \times 17$	$3850f_c$	
$30 \times 11 =$	-330	$\frac{11}{34}f_c$	$-106.8f_c$	$\frac{2}{3} \times 11$	$-784f_c$	
	350		$233.2f_c = C$			$3066f_c$
$40 \times 23 =$	920	$\frac{23}{34}f_c$	$622f_c$	$\frac{2}{3} \times 23$	$9540f_c$	
$30 \times 21 =$	-630	$\frac{21}{34}f_c$	$-389f_c$	$\frac{2}{3} \times 21$	$-5450f_c$	
	290		$233f_c = T$			$4090f_c$
						$7156f_c$

$$7156f_c = 4500$$

$$f_c = 0.629 \text{ kip per sq in.} = 629 \text{ lb per sq in.}$$

$$f_t = \tfrac{23}{17} \times 0.629 = 850 \text{ lb per sq in.}$$

14. Flexural Formula. Consider the cross section of the beam shown in Fig. 2, and assume a bending moment M at the given section.

Since the bending moment is equal to the moment of the resisting couple, we can write:

$$M = \int_0^{c_1} f\,dA \cdot y + \int_0^{c_2} f\,dA \cdot y$$

or

$$M = \frac{f_1}{c_1}\int_0^{c_1} y^2\,dA + \frac{f_2}{c_2}\int_0^{c_2} y^2\,dA$$

using the same nomenclature as before.

If I = moment of inertia of the area about its gravity axis,

$$I = \int_0^{c_1} y^2\,dA + \int_0^{c_2} y^2\,dA$$

and, since $f_1/c_1 = f_2/c_2$, we can write

$$M = \frac{f_1}{c_1} \times I \quad \text{or} \quad f_1 = \frac{Mc_1}{I}$$

Likewise

$$f_2 = \frac{Mc_2}{I}$$

We also have $f = My/I$, by substituting in the expressions for f in Art. 9.

This is the flexural formula, widely used in beam design; it is very convenient in the design of sections where the moment of inertia is readily computed, and especially so in the design of steel beams where the values of I are furnished in the steel handbooks. Its use is not always advantageous, and the authors believe that sections such as those shown in Art. 13 can best be solved by the method shown in that article.

15. Shear in Beams. Since the student is presumed to have a working knowledge of strength of materials, he knows that the intensity

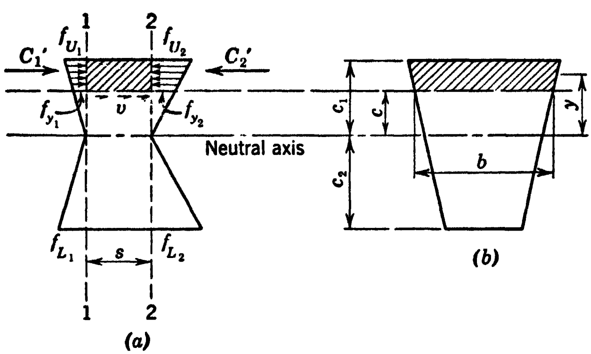


FIG. 7

of vertical shear is equal to the intensity of horizontal shear and that, if the unit horizontal shear can be evaluated, the unit vertical shear will be known. This relation will not be further discussed in detail, therefore, and the same symbol (v) will designate either kind of unit shear.

In Fig. 7*a* is shown a longitudinal view of a length of beam s between sections 1–1 and 2–2, and in Fig. 7*b* is shown a cross section of the same beam. The shaded area shown in Fig. 7*b* is acted upon by the compressive force C_1' at section 1–1 and C_2' at section 2–2.

Let M_1 = the bending moment at section 1–1.
M_2 = the bending moment at section 2–2 = $M_1 + \Delta M$.

If f = the fiber stress of any fiber,

$$C_1' = \int_c^{c_1} f\,dA = \frac{M_1}{I}\int_c^{c_1} y\,dA \quad \text{since} \quad f = \frac{My}{I} \tag{1}$$

$$C_2' = \int_c^{c_1} f\,dA = \frac{M_2}{I}\int_c^{c_1} y\,dA \tag{2}$$

Substituting $M_2 = M_1 + \Delta M$ in equation 2 and subtracting equation 1 from equation 2, we have

$$C_2' - C_1' = \frac{\Delta M}{I} \int_c^{c_1} y\, dA = \frac{\Delta M}{I} \times Q$$

Q being the static moment about the neutral axis of the shaded area above "c" shown in Fig. 7b. This is the difference in compressive stresses acting on the shaded areas at the two sections and is resisted by the horizontal shearing stress on the horizontal area $s \times b$, where b is the width of the beam at the horizontal section where the shear is being computed.

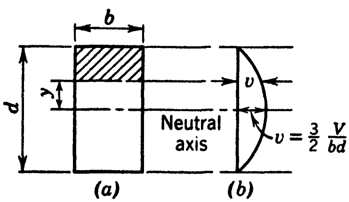


FIG. 8

The average unit shearing stress will then be $\frac{\Delta M \times Q}{I \times s \times b}$.

If s is made very small, then $\Delta M/s$ represents the rate of change of moment, and from statics this is known as V, or the total vertical shear at the section. Making this substitution, $v = VQ/Ib$, which is the general expression for unit horizontal shear at any certain distance above or below the neutral axis. The same expression can be derived from the tensile side of the beam as well as from the compressive. The unit vertical shear will be the same as the unit horizontal shear at the same place.

As can be seen from the expression, at the top or bottom fiber Q is equal to zero, and consequently the unit shearing stress at that point is zero.

The average unit horizontal shearing stress between two vertical sections a small distance apart may be obtained at any elevation by computing the change in total stress between those vertical sections for the area either above or below the given elevation and dividing by the area on which the shearing stress acts.

If the unit horizontal shear is calculated for a rectangular beam, the section being shown in Fig. 8a, the value of Q is $\frac{b}{2}\left(\frac{d^2}{4} - y^2\right)$. This is the equation of a parabola, and the value of unit shear in a beam of this cross section varies as the ordinate to a parabola. At the neutral axis, $y = 0$, and $v = \frac{3}{2} \times \frac{V}{bd}$.

16. Illustrative Examples. Shear for regular or irregular beams: The cross section shown in Fig. 9 is the same as that in Fig. 4. With data from Fig. 4:

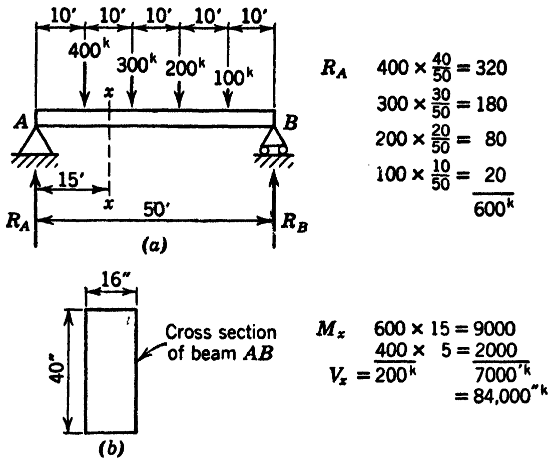


FIG. 9

At section x–x:

$$160 \times f_c \times 26.7 = 84{,}000 \text{ in.-kips}$$

$$f_c = 19.7 \text{ kips per sq in.}$$

or, from $f = Mc/I = 6M/bd^2$ for a rectangle,

$$f_c = \frac{6 \times 84{,}000}{16 \times 40 \times 40} = 19.7 \text{ kips per sq in.}$$

Shear:

$$v = \frac{VQ}{Ib} = \frac{200 \times 20 \times 16 \times 10}{\frac{1}{12} \times 16 \times 40^3 \times 16} = 0.469 \text{ kip per sq in. at the neutral axis}$$

For a rectangle

$$v = \frac{3}{2} \times \frac{V}{A} = \frac{3}{2} \times \frac{200}{640} = 0.469 \text{ kip per sq in.}$$

The ΔM between section x–x and a section 15 ft 1 in. from A is 200 in.-kips.

$$C_x = \frac{84{,}000}{26.7}$$

$$C_{x+1} = \frac{84{,}000 + 200}{26.7}$$

$$v = \frac{C_{x+1} - C_x}{b \times s} = \frac{200/26.7}{16 \times 1} = 0.469 \text{ kip per sq in.}$$

The cross section shown in Fig. 10 is the same as that in Fig. 5*a* but is used with the span and load of Fig. 9*a*. The computations for unit shear follow.

The calculations shown below Fig. 5 indicate the total resisting moment of this cross section to be $6507f_c$. Also, the total compressive force is shown to be $242f_c$, which, of course, is equal to the total tensile force.

At section x–x in Fig. 9*a*, the bending moment is 84,000 in.-kips; therefore $6507f_c = 84{,}000$, and $f_c = 12.9$ kips per sq in. The maximum tensile stress is $(27.5 \div 12.5) \times 12.9$, or 28.4 kips per sq in. For com-

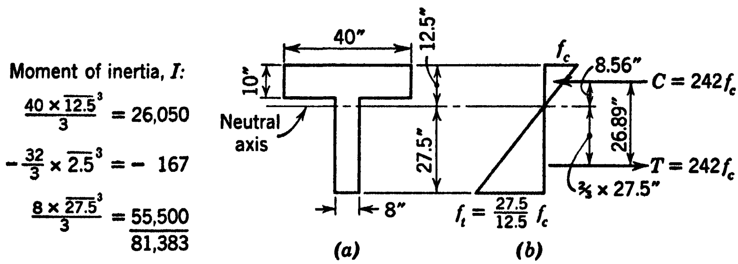


FIG. 10

parative computations, using $f_c = Mc/I$, $f_c = (84{,}000 \times 12.5) \div 81{,}383 = 12.9$ kips per sq in.

If the compressive force $= 242f_c$ and the resisting moment $= 6507f_c$, then the vertical distance from the center of compression to the center of tension (distance between C and T) is $6507f_c/242f_c = 26.89$ in. The center of compression, then, is $26.89 - \frac{2}{3} \times 27.5 = 8.56$ in. above the neutral axis.

The intensity of shearing stress at the neutral axis for this cross section at section x–x can be obtained from the expression $v = VQ/Ib$.

$$v = \frac{200 \times 27.5 \times 8 \times 27.5/2}{81{,}383 \times 8} = 0.929 \text{ kip per sq in.}$$

The same value can be obtained more easily if the change in the total compressive force is divided by the area of the shearing surface. The change in the total compressive force is equal to the change in moment divided by the distance between the centroid of tension and centroid of compression. The change in moment per unit of length is the shear; therefore,

$$\Delta C = \Delta T = \frac{\Delta M}{26.89} = \frac{200}{26.89} = 7.44 \text{ kips}$$

and

$$v = \frac{7.44}{8 \times 1} = 0.929 \text{ kip per sq in.}$$

If the cross section of Fig. 6 and the calculations for it are used in a similar manner with the span shown in Fig. 9*a*, the values of f_c and v can be obtained as follows:

$$7156f_c = 84{,}000$$

$$f_c = 11.74 \text{ kips per sq in.}$$

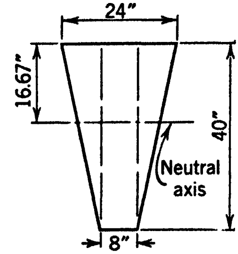


FIG. 11

The distance between the center of tension and the center of compression is $7156f_c/233f_c = 30.7$ in., and the change in the total compressive force is $200 \div 30.7 = 6.52$ kips. At the neutral axis, $v = 6.52 \div (10 \times 1) = 0.652$ kip per sq in. Again the beam span and loading from Fig. 9*a* will be used, but the section will be as shown in Fig. 11, which has the same area and depth as the previous cross section.

Dividing the area into two triangles and a rectangle and taking the references axis at the top, the following computations result.

Area = A	Arm to Reference Axis = y_{ra}	Ay_{ra}
$8 \times 40 = 320$	20	6,400
$2 \times \frac{8}{2} \times 40 = 320$	13.33	4,267
640		10,667
		$\div\ 640 = 16.67 = \bar{y}$

or, by using a common formula,

$$\bar{y} = \frac{d}{3} \times \frac{2b + b_1}{b + b_1} = \frac{40}{3} \times \frac{8 \times 2 + 24}{8 + 24} = \bar{y} = 16.67 \text{ in.}$$

About the reference axis,

$$I_{ra} = \frac{8 \times \overline{40}^3}{3} + \frac{16 \times \overline{40}^3}{12} = 256{,}000 \text{ in.}^4$$

About the neutral axis,

$$I_{na} = 256{,}000 - 640 \times \overline{16.67}^2 = 78{,}200 \text{ in.}^4$$

$$f_c = \frac{84{,}000 \times 16.67}{78{,}200} = 17.9 \text{ kips per sq in.}$$

$$f_t = \frac{84{,}000 \times 23.33}{78{,}200} = 25.0 \text{ kips per sq in.}$$

$$v = \frac{VQ}{Ib}$$

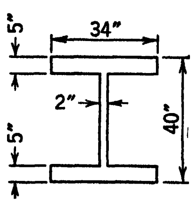
$$b \text{ at the neutral axis} = 8 + \frac{23.33}{40} \times 16 = 17.33 \text{ in.}$$

$$Q = \frac{8 + 17.33}{2} \times 23.33 \times \left(\frac{23.33}{3} \times \frac{2 \times 8 + 17.33}{8 + 17.33}\right) = 3020 \text{ in.}^3$$

$$v = \frac{200 \times 3020}{78{,}200 \times 17.33} = 0.445 \text{ kip per sq in. at the neutral axis.}$$

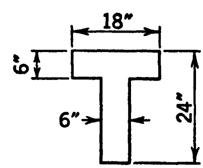
PROBLEMS

1. In the figure is shown the cross section of a beam. (*a*) For a bending moment of 8000 ft-kips, what is the maximum intensity of flexural stress? (*b*) For an external shear of 750 kips, what is the maximum intensity of shearing stress?



Prob. 1

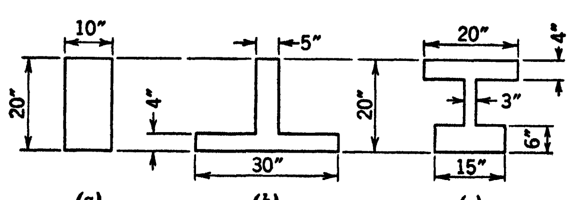
2. (*a*) In Problem 1, if the shear of 750 kips is assumed to be resisted by the web only and the shearing stress is assumed to be constant throughout the depth of the web, what is the shearing stress in the web? (*b*) What is the percentage of error in the answer for Problem 2*a* as compared with Problem 1*b*, if the depth of web is taken as the distance between the centroids of the flanges?



Prob. 3

3. In the figure is shown the cross section of a beam. (*a*) For a bending moment of 1200 ft-kips, what is the maximum compressive stress? the maximum tension stress? (*b*) For an external shear of 600 kips, what is the maximum shearing stress? How far from the top does it occur?

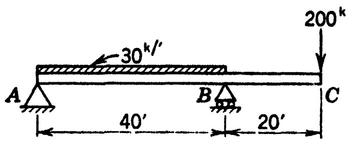
4. In the figure, *a*, *b*, and *c* show the cross sections of three beams. Each cross section is 20 in. deep and has an area of 200 sq in. (*a*) If the allowable $f_c = f_t =$

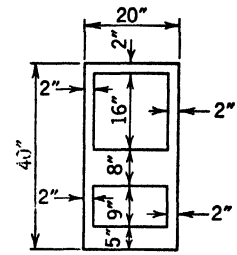


Prob. 4

1600 lb per sq in., what is the maximum resisting moment for each of the beams shown above? (*b*) If the allowable shearing stress = 500 lb per sq in., what is the maximum total shear that each of the beams can resist?

5. (*a*) Draw the shear and moment diagrams for the structure shown. (*b*) The cross section of beam *ABC* is as shown to the right with two rectangular holes. What is the maximum flexural stress in the beam *ABC*? Where is this stress with respect to the span? At this section, what is the stress in the bottom fibers?



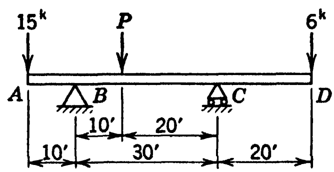


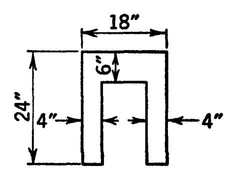
Cross section of *ABC*

Prob. 5

6. (*a*) With the data of Problem 5, what is the maximum shearing stress in beam *ABC*? Where is the maximum stress with respect to the cross section? with respect to the span? (*b*) What is the maximum shearing stress at the neutral axis? Where is this stress with respect to the span?

7. The figure shows the loading and cross section of a beam *ABCD*. If the allowable flexural stress is 2000 lb per sq in. and the allowable shearing stress is 300 lb





Cross section of *ABCD*

Prob. 7

per sq in., what is the maximum value that the load *P* can safely have (neglecting the weight of the structure)?

CHAPTER 2

BENDING IN NONHOMOGENEOUS, SYMMETRICAL BEAMS

17. Nonhomogeneous Beams. The beams considered previously have all been homogeneous beams, i.e., beams composed of one material throughout the entire section which has, for the common materials employed in structures, the same elastic properties in both tension and compression. Composite, or nonhomogeneous beams, can be handled in a similar manner if cognizance is taken of the relation of the elastic properties of the different materials that go to make up the beam. The most common combination of materials is that of concrete and steel, although timber and steel are often used together, and brick and steel have been combined. It is apparent that many combinations of two or more materials can be made that are useful or economical.

The different materials in a nonhomogeneous beam can be located in horizontal layers; or they can be side by side as when timber members are bolted together with steel plates on the outside, or between, or both. These latter combinations are often called flitched beams.

18. Relation between Modulus of Elasticity and Stress. The student is familiar with the following expression from his physics and mechanics, $E = f/\epsilon$, in which f = unit stress, ϵ = unit strain, and E = modulus of elasticity and is expressed in the same units as f.

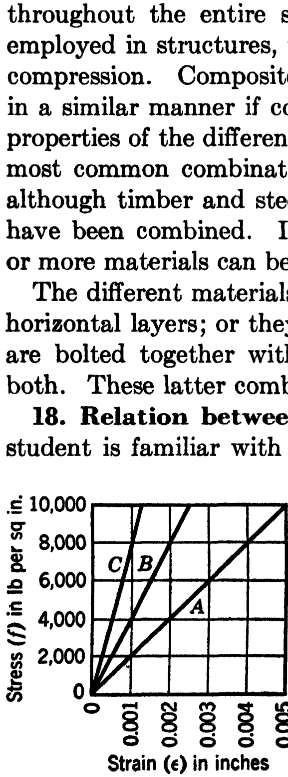


FIG. 12

For an example, take three materials whose moduli are as follows:

$$E_A = 2{,}000{,}000 \text{ lb per sq in.}$$

$$E_B = 4{,}000{,}000 \text{ lb per sq in.}$$

$$E_C = 8{,}000{,}000 \text{ lb per sq in.}$$

These values represent the stress, relative to strain, in the materials A, B, and C, respectively. If the stress-strain relationships are plotted, we have the curves shown in Fig. 12, these curves being straight lines below the proportional limit of the materials.

For a composite beam composed of these materials, the strain varies linearly from top to bottom, being zero at the neutral axis; but the stress varies linearly, only within the confines of each given material. From the figure, or from the relationship $f = E\epsilon$, it is seen that for a given strain the unit stress in material A is one-fourth the unit stress in material C, or in accordance with the ratio E_A/E_C. It follows then that for two beams of identical dimensions, one composed of material A and one of material C, the beam composed of material C is capable of sustaining a load four times as great as the beam composed of material A with the same amount of strain deflection, or the beam composed of material C is equivalent to one of material A with a width of four times that of either original beam.

19. Transformed Sections. The above relationship can be indicated better by means of a study of the cross section of an assumed beam composed of materials A, B, and C. In Fig. 13*a* is shown the cross section of a beam composed of the same three materials discussed above. The force on any differential area, dA $(= b \times dy)$, is equal to the stress (f) times the area; $dF = f \times dA$. Since strain is a linear variation, the variations for ϵ are as shown in Fig. 13*b*, and, since $f = E\epsilon$, Fig. 13*c* shows the variation in f.

If the moment of the resisting stresses about the neutral axis is equated to the bending moment, we have $M = \int f\,dA \cdot y$ or $M = \int fb\ dy \cdot y$. From Fig. 13*c* it can be seen that, at any given distance from the neutral

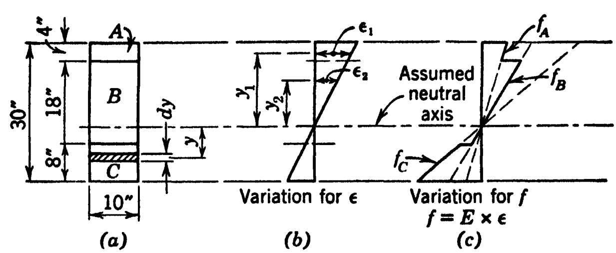


FIG. 13

axis, $f_C = (E_C/E_A)f_A$, or $f_B = (E_B/E_A)f_A$. The stress on an area dA in material C is then $f_C\,dA$ or $f_Cb \cdot dy$. This can be written (E_C/E_A) $b \cdot f_A\,dy$, and therefore, if we wish to assume a linear distribution of stress (such as f_A) over the entire section, it will be necessary to increase the width by the multiple E_C/E_A or E_B/E_A within the confines of materials C and B, respectively, if we wish to maintain the same total stress in the area dA as in the original material. If we wish to maintain the same resisting moment, there must be no change in the dimensions perpendicular to the neutral axis of the section; only the dimensions parallel to this axis are changed.

This procedure of transforming one material in terms of another, in the ratio of their moduli, is usually more convenient than employing a variation of unit stress that is other than a continuous straight line. This is usually referred to as the "method of transformed sections." The transforming can be done in terms of either material, as is illustrated in Fig. 14. In Fig. 14*b* the section has been transformed in terms

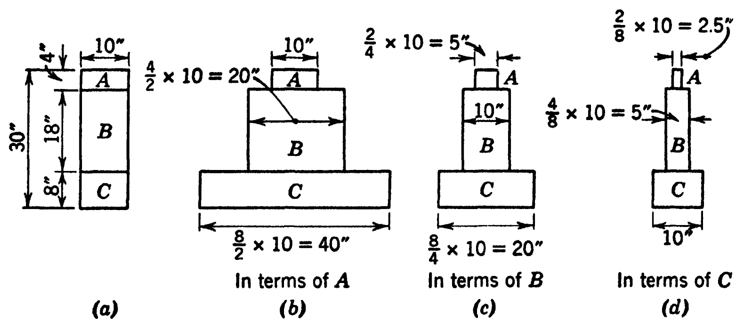


FIG. 14

of material A, in terms of material B in Fig. 14*c*, and in terms of material C in Fig. 14*d*. If unit stresses are computed for the section shown in Fig. 14*b*, the computed stresses are the correct ones for material A, and one-half and one-fourth the correct stresses for materials B and C, respectively. Similar relations hold for the other sections.

20. Illustrative Example. In Fig. 15 is shown a beam of span M–N having a cross section the same as in Fig. 14*a* and the same values

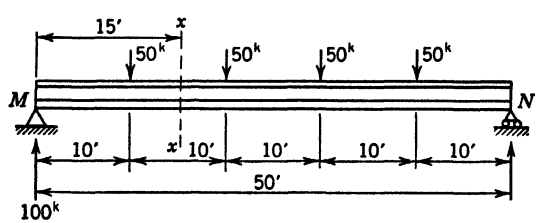


FIG. 15

of E_A, E_B, and E_C. The bending moment at section x–x is 1250 ft-kips or 15,000 in.-kips. The shear is 50 kips.

In Fig. 16*a* the transformed section of the above beam is shown in terms of material B, and below are the computations for the location of the neutral axis. The variation in stress is shown in Fig. 16*b*. The variation in strain is linear and continuous, whatever the shape and whatever the materials, according to the assumptions made. The

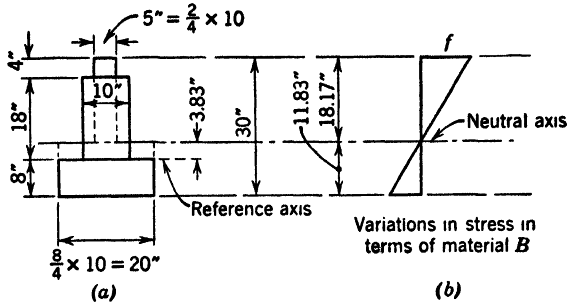


FIG. 16

variation of stress is linear, whatever the shape and whatever the materials; however, this variation is discontinuous at sections where materials are discontinuous.

$5 \times 4 =$	20	$\times\ 20 =$	$+\ 400$	
$10 \times 18 =$	180	$\times\ 9 =$	$+1620$	
$20 \times 8 =$	160	$\times -4 =$	$-\ 640$	
	360		$+1380$	$\div\ 360 = +3.83$ in.

An inspection of the cross sections shown in Figs. 14*a*, *b*, and *c* shows that the neutral axis is in the same location, irrespective of which section is used in the computations.

The computations for determining the unit stress at various points in the different materials are given below. They should be self-explanatory.

	Area	Average Stress	Force	Arm about Neutral Axis	Moment about Neutral Axis	
$5 \times 18.17 =$	90.85	$\frac{f}{2}$	$45.42f$	$\frac{2}{3} \times 18.17$	$550f$	
$5 \times 14.17 =$	70.85	$\frac{7.08}{18.17}f$	$27.60f$	$\frac{2}{3} \times 14.17$	$261f$	
	161.70		$73.02f$			$811f$
$20 \times 11.83 =$	236.6	$\frac{5.92}{18.17}f$	$77.00f$	$\frac{2}{3} \times 11.83$	$606f$	
$-10 \times 3.83 =$	$-\ 38.3$	$\frac{1.92}{18.17}f$	$-\ 4.05f$	$\frac{2}{3} \times 3.83$	$-\ 10f$	
	197.3		$72.95f$			$596f$
						$1407f$

$$1407f = \text{Bending moment} = 15{,}000 \text{ in.-kips}$$

$$f = 10.67 \text{ kips per sq in. at top}$$

(For material A, this would be $10.67 \times \frac{2}{4} = 5.33$ kips per sq in.)

$$\frac{11.83}{18.17} \times 10.67 = 6.95 \text{ kips per sq in. at bottom}$$

(For material C, this would be $6.95 \times \frac{8}{4} = 13.90$ kips per sq in.)

$$\frac{14.17}{18.17} \times 10.67 = 8.31 \text{ kips per sq in. at 4 in. below top in material } B$$

21. Shear in Nonhomogeneous Beams. The student will recall from the discussion on shear in Chapter 1 that the total horizontal shear on an area $s \times b$ (for length s of a beam) is equal to the change in stress on the area of the beam cross section above (or below) the horizontal plane, the unit shearing stress being equal to the total horizontal stress divided by $(s \times b)$. In calculating the change in stress it is usually more convenient to calculate the change per unit length (or rate of change), which, divided by the width of beam, gives the unit shearing stress. Since the stress in the beam is a function of the bending moment, it follows that the rate of change of stress is a function of the rate of change of moment, or a function of the shear.

Since the unit stress is computed by means of the transformed section, and since the computed unit stress times the beam width is the same at any definite point, irrespective of whether we are transforming in terms of material A, B, or C, as shown in Fig. 14, it follows that we can compute the rate of change of total stress in terms of either material, and this divided by the actual width of beam gives the unit shearing stress. The unit shearing stress 4 in. below the top of the beam shown in Fig. 13 is therefore the same in either material A or material B.

Shearing stress in nonhomogeneous beams can also be computed by means of the expression $v = VQ/Ib$, although there is usually no advantage in doing so in this type of beam. The student should recognize that in this expression VQ/I represents the change in total stress per unit length on the area above the horizontal plane where shearing stress is being calculated, Q being the static moment of this area about the neutral axis.

If

I_a = the moment of inertia in terms of material A,
I_b = the moment of inertia in terms of material B, and
I_c = the moment of inertia in terms of material C,

and if

Q_a = the static moment of the area in terms of material A,
Q_b = the static moment of the area in terms of material B, and
Q_c = the static moment of the area in terms of material C,

it follows, since the widths are changed in proportion to the modulus ratio in computing both Q and I, that

$$\frac{Q_a}{I_a} = \frac{Q_b}{I_b} = \frac{Q_c}{I_c}$$

This is merely a restatement of the one made previously that it makes no difference which transformed section is utilized in the computations as long as the change in stress is divided by the actual width.

Computations for shear are given in the following illustrative example, using the same beam as in Art. 20.

22. Illustrative Example. In Fig. 17*a* is shown the cross section of the beam in Fig. 15, in Fig. 17*b* the transformed section in terms of material B, and in Fig. 17*c* the variation in unit shearing stress. The intensity of shear varies as a parabola, although the equation of the

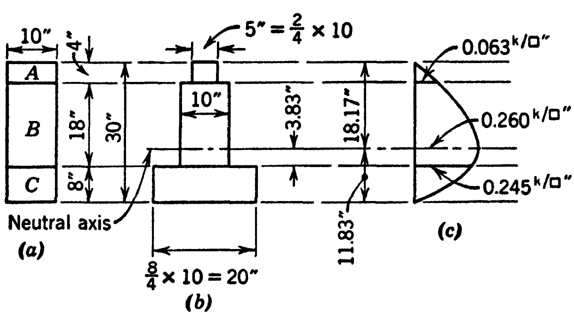


FIG. 17

parabola changes at points 4 in. below the top and 8 in. above the bottom of the beam. The moduli have the same values as before.

The shear at section x–x in the beam in Fig. 15 is 50 kips, and that value will be used here. If δM = the rate of change of moment = V, and δf = the rate of change of unit stress in the top fiber, then $V = 1407\,\delta f$, since, from the example in Art. 20, $M = 1407f$.

$1407\,\delta f = 50.$
$\delta f = \frac{50}{1407} = 0.0356$ kip per sq in. $\times \frac{2}{4} = 0.0178$ kip per sq in. in material A.
Average rate $= \frac{16.17}{18.17} \times 0.0178 = 0.0158$ kip per sq in.
Change in stress in material $A = 0.0158 \times 10 \times 4 = 0.632$ kip.

Shear in material A 4 in. below top $= v_a = \dfrac{0.632}{10 \times 1} = 0.0632$ kip per sq in.

Change in stress above (or below) neutral axis $= 73.02 \times 0.0356 = 2.60$ kips. (See calculations for total stress in Art. 20.)

Shear in material B at neutral axis $= v_b = \dfrac{2.60}{10 \times 1} = 0.260$ kip per sq in.

Average rate in material $C = 0.0356 \times \dfrac{7.83}{18.17} \times \dfrac{8}{4} = 0.0306$ kip per sq in.

Change in stress in material $C = 0.0306 \times 10 \times 8 = 2.448$ kips.

Shear in material C 8 in. above bottom $= v_c = \dfrac{2.448}{10 \times 1} = 0.2448$ kip per sq in.

These same shearing stresses will now be calculated from the expression $v = VQ/Ib$.

$$\begin{aligned}
\tfrac{1}{3} \times 5 \times 18.17^3 &= 9{,}990 \\
\tfrac{1}{3} \times 5 \times 14.17^3 &= 4{,}730 \\
\tfrac{1}{3} \times 20 \times 11.83^3 &= 11{,}050 \\
\tfrac{1}{3} \times 10 \times 3.83^3 &= -\ \ 190 \\
\hline
&\ 25{,}580 \text{ in.}^4 = I_b
\end{aligned}$$

$$I_a = 25{,}580 \times \tfrac{4}{2} = 51{,}160 \text{ in.}^4$$

$$I_c = 25{,}580 \times \tfrac{4}{8} = 12{,}790 \text{ in.}^4$$

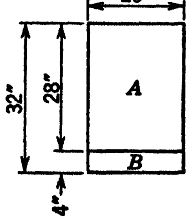
$$v_a = \frac{50 \times 5 \times 4 \times 16.17}{25{,}580 \times 10} = 0.063 \text{ kip per sq in.}$$

$$v_b = \frac{50\left(20 \times 16.17 + 10 \times 14.17 \times \dfrac{14.17}{2}\right)}{25{,}580 \times 10} = 0.260 \text{ kip per sq in.}$$

$$v_c = \frac{50 \times 20 \times 8 \times 7.83}{25{,}580 \times 10} = 0.245 \text{ kip per sq in.}$$

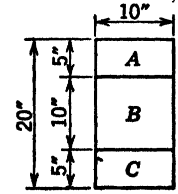
PROBLEMS

1. In the accompanying figure, the material B has a modulus of elasticity which is eight times as much as that for material A, $\left(\dfrac{E_B}{E_A} = \dfrac{8}{1}\right)$. (*a*) For a bending moment of 1000 ft-kips, what is the maximum flexural stress in material A? in material B? (*b*) If the external shear is 200 kips, what is the maximum shearing stress? In which material is this stress?



PROB. 1

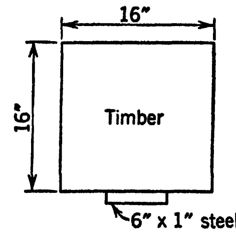
2. The figure shows the cross section of a beam composed of three materials. The moduli of elasticity are 12,000,000 lb per sq in. for A, 3,000,000 lb per sq in.



PROB. 2

for B, and 6,000,000 lb per sq in. for C. For a bending moment of 500 ft-kips, compute the maximum flexural stress in each material.

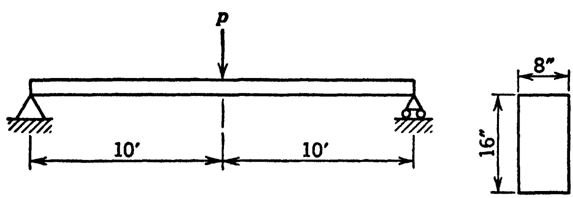
3. A 6 by 1 in. steel plate is attached to the bottom of a 16 by 16 in. timber as shown. It is assumed that $E_s/E_t = 20$ and that the allowable flexural stresses are 18,000 lb per sq in. for steel and 1600 lb per sq in. for timber. What is the maximum bending moment that this composite beam can safely resist?



PROB. 3

4. (*a*) Using the data from Problem 3, what is the maximum shear the composite beam can resist for a shearing stress of 150 lb per sq in. at the neutral axis? (*b*) Is the unit shearing stress greater at the neutral axis or at the attachment between the timber and steel? (*c*) What is the maximum shearing stress in the steel for the shear found in part *a*?

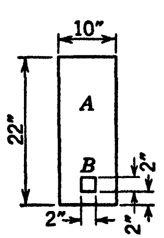
5. A load P is applied to an 8 by 16 in. beam of southern pine (No. 1 structural longleaf) at the center of a 20-ft span. The allowable flexural stress = 1600 lb per sq in., the allowable horizontal shearing stress = 120 lb per sq in., and the modulus of elasticity = 1,600,000 lb per sq in. If the weight of the beam is neglected, what is the maximum safe value for the load P?



PROB. 5

6. In Problem 5, if a steel plate 8 by $\frac{1}{4}$ in. were attached to the bottom of the 8 by 16 in. beam, what would be the maximum safe value for the load P? For the steel plate, assume that the allowable flexural stress is 20,000 lb per sq in. and E = 29,000,000 lb per sq in. For this value of P, what is the shearing stress between the steel plate and the wood?

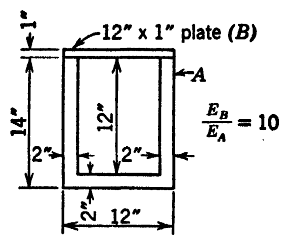
7. In Problem 5, if a steel plate 8 by $\frac{1}{4}$ in. were attached to the bottom of the 8 by 16 in. beam and another 8 by $\frac{1}{4}$ in. steel plate were attached to the top, what would be the maximum safe value for the load P? What is the shearing stress in the wood for this value of P?



PROB. 8

8. As shown in the figure, a beam 10 in. wide and 22 in. deep is composed of material A with a hole 2 in. by 2 in. filled with material B. Material B is attached to material A and has a modulus of elasticity which is twelve times that of A. For a bending moment of 115 ft-kips, what is the maximum flexural stress in material A? in material B?

9. In the figure shown is the cross section of a beam which is composed of a U-shaped member of material A with a plate of material B attached to the top of the legs. The inside of the U (8 by 12 in.) is open. (*a*) For a bending moment of 250 ft-kips, what is the maximum flexural stress in *A*? in *B*? (*b*) For an external



PROB. 9

shear of 90 kips, what is the maximum shearing stress in *A*? What is the shearing stress at the attachment of *B* to *A*?

CHAPTER 3

ELEMENTARY DESIGN OF STEEL BEAMS AND GIRDERS

23. The most common specifications for steel structures are those of the American Institute of Steel Construction (A.I.S.C.), the American Association of State Highway Officials (A.A.S.H.O.), and the American Railway Engineering Association (A.R.E.A.). These were referred to in the Introduction. Each of these specifications has been revised from time to time, and further revisions of the current ones are being made constantly. Since it is desirable that designers be able to apply the structural principles involved in a design to any set of specifications and since, where specifications are followed, it is further desirable that the most recent ones be used, no formal set of specifications is incorporated in this text. The three specifications above, or others, are available, depending on the purpose for which the members are intended.

Many different shapes of steel sections are manufactured (rolled) by various companies. Each shape is rolled in different sizes and is identified by linear dimensions (such as depth, width, or both) and also by weight. The designating dimensions are nominal dimensions only. The actual dimensions, allowing for permissible tolerances, are given completely in a handbook or catalog published by the company producing the shape. These dimensions include thicknesses, tangent lengths, and rivet gages, as well as widths and depths in both fractional and decimal values. Weights, areas, radii of gyration, section moduli, and moments of inertia are also tabulated. Some of the shapes rolled include plates, angles, zees, tees, H sections, channels, and I beams of two types, American Standard and wide flange. Information concerning the available lengths of some of these shapes is also available in handbooks.

In the design of rolled sections to be used as beams, or sections composed of fabricating two or more of these shapes together, as shown in Fig. 18, the student will find some type of handbook indispensable. As mentioned in the Introduction, the one most generally available is the handbook *Steel Construction*, published as a manual of the American Institute of Steel Construction. This handbook contains data on the shapes rolled by practically all the steel companies. The latest edition

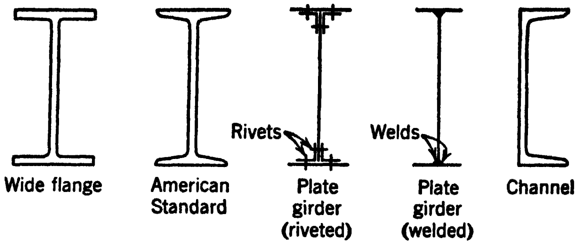


FIG. 18

at this writing is the fifth edition (ninth printing, 1949), containing the 1949 A.I.S.C. design specifications used largely in this text, with some modifications.

24. Rolled Beams. Rolled steel beams, usually referred to as I beams from their shape, play an important role in structural design. The two general types ordinarily met with in design are the American Standard beams, rolled by practically all rolling mills, and the wide-flange (WF) beams, any of which can be obtained promptly from the Carnegie-Illinois Steel Company and the Bethlehem Steel Company. The wide-flange beams have wider and heavier flanges and are rolled with greater depths, and are thus capable of resisting greater bending moments. American Standard beams are rolled in depths from 3 in. to 24 in., and wide-flange beams from 8 in. to 36 in. The wide-flange beams have, with some exceptions listed in the handbooks, flanges of uniform thickness, whereas the American Standard beams have tapered flanges. For wide-flange beams of a given nominal size such as $30 \times 10\frac{1}{2}$, as the weights change from 108 lb per ft to 116 to 124 to 132 lb per ft, the heavier beams are obtained by increasing the distance between the rolls laterally to make the web thicker and the flange wider and increasing the distance between the rolls vertically to make the flanges thicker. For American Standard beams of a given nominal size such as 24×7, the heavier beams are obtained by spreading the rolls only in a lateral direction, thus keeping the depth a constant (24 in.). In both the wide-flange beams and the American Standard beams the increase in web thickness is always equal to the increase in flange width as weights are increased within any nominal size group.

When beams deeper than the above are required, they are obtained by building up an I-shaped section by means of plates and angles. This type of section is called a plate girder; the design principles involved will be discussed later in this chapter.

Another type of rolled section often found advantageous when a light beam is desired is the channel. In the channel section the flange is on one side of the web only. They are often found more economical in the design of purlins for carrying the load to roof trusses.

25. Design of Rolled Beams by Section Modulus. In the design of rolled steel beams, best use can be made of the previously derived expression, $f = Mc/I$. This can be expressed as $M/f = I/c$, in which $I/c =$ the section modulus (called S in the handbooks). It is a geometric property of the cross-sectional area, independent of span or loading. Thus, if the maximum bending moment on any span is divided by the allowable unit stress, the resulting value is the required section modulus of the beam section. Any beam having a section modulus not less than the required value will have a unit stress not greater than the allowable. It will be seen that the depth does not enter directly into the requirements of a suitable beam (within the limits of the available section moduli), but it is true that deeper beams will have lighter weights and smaller deflections than more shallow beams.

Taking as an example a simple beam span of 20 ft with a uniformly distributed load of 6 kips per ft of span, the maximum bending moment is $(6 \times \overline{20}^2) \div 8 = 300$ ft-kips $= 3600$ in.-kips. If the allowable unit stress is 20,000 lb per sq in. in either tension or compression, then 3600 in.-kips divided by 20 kips per sq in. $= 180$ in.^{3} $=$ the required section modulus. A study of the handbook will show that the lightest weight beams that will give this section modulus will be a 24-in. I at 90 lb or a 24-in. wide-flange at 84 lb. Since the wide-flange beams are priced at a higher cost per pound, it is quite probable in this case that the 90-lb I beam will be the cheaper. Also, deeper beams are usually priced at a higher cost per pound than more shallow ones; therefore, at the point of price change it is possible that a slightly shallower beam will be the cheaper.

Because of limitations in headroom, or other factors, it sometimes is necessary to employ a shallower beam than the least weight section. Reference to the handbook will show that the least depth that can be used in the above example will be a 12-in. wide-flange at 133 lb. In general, the least depth beam is found to be uneconomical. The most economical beam, considering all factors, may be one with a weight intermediate between that of least weight and that of least depth.

A shallower beam can also be used, or a given beam can be made to carry more load, by riveting or welding a cover plate on both flanges of the beam. This increases the moment of inertia, and consequently the section modulus, with very little increase in the depth of the beam.

The addition of cover plates to the flanges of rolled beams adds little or nothing to the shearing strength of the beam. However, except for heavy loadings and short spans, shear is seldom critical for rolled beams;

but, when cover plates are necessary to increase the flexural strength of a rolled beam, the likelihood of critical shear is increased.

26. Effect of Lateral Support. In the beams considered in the previous article it was assumed that the allowable unit stress was the same in both the tension and compression flange, namely, 20 kips per sq in. Where the compression flange is supported against lateral movement, this is permitted by the various specifications and is entirely logical. If the beam flange is not restrained against lateral movement, there may be additional stresses produced in the flange, as it acts essentially as a column.* Any imperfection in the make-up of the flange, any eccentricity in the application of load, or several other possible causes will tend to accentuate this lateral deflection.

To take account of this additional stress due to lack of lateral support, various formulas are available. They are derived from the basic expression

$$f = f_1 + f_2 \tag{1}$$

in which f = the maximum unit compressive stress.

f_1 = the intensity of stress due to bending caused by applied loads.

f_2 = the intensity of stress due to lateral deflection or buckling.

The above expression may be written

$$f_1 = f - f_2 \tag{2}$$

and, since

$$f_1 = \frac{Mc}{I}$$

the stress f can be kept within any stated value, such as a maximum allowable unit stress, provided a satisfactory expression for f_2 can be found.

All expressions for f_2 used heretofore contain some form of the value L/b, in which L = the distance between points of lateral support, and b = the width of the compression flange, both in inches or in the same unit of measure. Where L/b is used, the formula takes the form

$$f_1 = f - k\frac{L}{b} \tag{3}$$

which is a straight-line reduction formula. The most widely adopted formula of this type for many years was

* Two major differences between the compression flange of a beam and a column are (1) the compression flange of a beam has a varying axial load and (2) it is attached to the web of the beam.

$$f_1 = 16{,}000 - 150\frac{L}{b} \tag{4}$$

This formula appeared in many specifications.

If L^2/b^2 is used, a curve is produced that more nearly follows a curve representing column action than is obtained from the straight-line formula. Expressing f_2 in terms of L^2/b^2, we have

$$f_1 = f - k_1\frac{L^2}{b^2} \tag{5}$$

which is the formula for a parabola

A reduction formula of this type,

$$f_1 = 18{,}000 - 5\frac{L^2}{b^2} \tag{6}$$

is found in the current American Railway Engineering Association's "Specifications for Steel Railway Bridges," and also in "Standard Specifications for Highway Bridges" of the American Association of State Highway Officials. The base design stress in both specifications is 18,000 lb per sq in.

Since f_1 is a function of the total stress on the compression flange, many designers feel that f_2 is therefore a function of f_1 and, accepting that relation, we can rewrite formula 5 as,

$$f_1 = f - k_2 f_1\frac{L^2}{b^2} \tag{7}$$

or

$$f = f_1 + k_2 f_1\frac{L^2}{b^2}$$

from which we obtain

$$f_1 = \frac{f}{1 + k_2(L^2/b^2)} \tag{8}$$

which is the general form of the Rankine-Gordon formula. In the A.I.S.C. specifications printed in the 1941 edition of *Steel Construction*, this appears as

$$\frac{22{,}500}{1 + (L^2/1800b^2)} \tag{9}$$

The value of f in the formula is often taken at a value that is higher than the maximum allowable unit stress, and this is compensated for in the value of k. In formula 9 the maximum allowable unit stress is 20,000 lb per sq in., which agrees with an L/b of 15. Beyond 15 the formula is used with a maximum allowable unsupported length of 40 times the flange width.

A formula of a different type is given in the 1949 A.I.S.C. specifications. This is

$$f_1 = \frac{12{,}000{,}000}{\dfrac{L \times d}{b \times t}} \tag{10}$$

in which d = depth of the beam, and t = thickness of the compression flange, L and b having the same meaning as before. This formula was derived from studies made on a series of rolled beams. It is given in the specifications for girders as well, but the authors believe that there is some doubt regarding the general application of this formula. This will be discussed further under girders.

The use of this formula gives some uneconomical weights in certain cases (at least compared with older satisfactory methods of design). Specifications should be consistent within themselves, but it will be found that formula 10, for some beams, gives a less allowable unit stress than is obtained by applying the column reduction formula in the same specifications to the top flange acting alone as a column. Column reduction formulas are discussed in the next chapter.

Referring to the beam of 20-ft span in the previous article (M = 3600 in.-kips), and assuming no lateral support for the top flange for the entire 20 ft, it will be necessary to use a larger beam. The allowable unit stress in the top flange will control the design.

A 24-in. wide-flange at 94 lb has a flange width of 9.061 in., giving an L/b of 240 ÷ 9.061 = 26.5, and applying formula 9, an allowable unit stress of 16.19 kips per sq in. 3600 ÷ 16.19 = 222. Since this beam has a section modulus of 220.9, it is satisfactory.

With an American Standard beam, a heavier section is required. A 24-in. I beam at 120 lb has a flange width of 8.048 in. (240 ÷ 8.048 = 29.8) and an allowable unit stress of 15.07. 3600 ÷ 15.07 = 239. This is satisfactory (S = 250.9).

Formula 10 applied to the 24-in. wide-flange at 94 lb gives an allowable unit stress of

$$\frac{12{,}000{,}000}{\dfrac{240 \times 24.29}{9.061 \times 0.872}} = 16.28 \text{ kips per sq in.}$$

For the 24-in. I at 120 lb, the formula gives

$$\frac{12{,}000{,}000}{\dfrac{240 \times 24}{8.048 \times 0.798}} = 13.38 \text{ kips per sq in.}$$

The two formulas agree very well in one case but not in the other.

The student should understand that the value of f_1, as obtained from these formulas, is the average unit stress in the top fiber over the width of the top flange. This stress plus the possible maximum unit stress due to lateral bending or eccentricity of the total flange stress is presumed to be equal to the maximum allowable unit stress, which is represented by f in the basic formula.

27. Shear in Rolled Beams. In Chapter 1 the value of the unit shearing stress in beams was expressed in the form $v = VQ/Ib$. In rolled beams this is usually expressed as $v = VQ/It$, in which t = the thickness of the web. It can be seen from this expression that Q (the static moment of the area above the point at which unit shear is being computed, about the neutral axis) varies within rather narrow limits since the large concentration of area is in the flange. It is therefore assumed in steel beam design that the total shear divided by the web

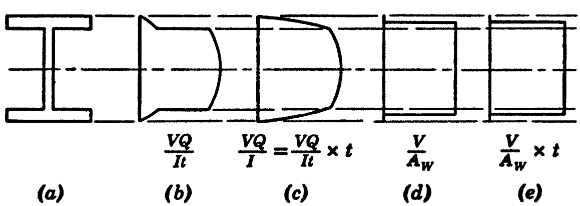


Fig. 19

area gives a value of unit shearing stress close enough to the maximum value so that the error involved is small enough to be neglected as far as design purposes are concerned. The student should compute the more exact values to satisfy himself that the error is small.

For rolled beams the difference between the value obtained by dividing the external shear by the area of the web and the value obtained from the expression VQ/It is even smaller if the area of the web is taken as the thickness times the depth between flange centroids. Figure 19*c* shows the shear resisted per inch of depth, which is the intensity of shearing stress times the thickness. The summation of the shear per inch of depth for the depth of the beam must be equal to the external shear, or area of Fig. 19*c* = area of Fig. 19*e* = external shear. Considering these two figures, the two areas are more nearly equal if the depth of web is taken as the distance between the centroids of the flanges. However, specifications invariably give an allowable intensity of shearing stress which is based on the gross area of the web and the assumption that the unit shearing stress is uniform. This allowable stress is less than would be permitted for a computed maximum unit shearing stress; and consequently, in the design and analysis of I-shaped members resisting shearing forces, the overall depth of the web is used.

Shear normally is not a factor in the design of an I beam, but it is in plate girder design, as will be seen later. In short-span beams carrying a very heavy load, shear might control the size of the section. In a beam of 20-ft span carrying a load of 6 kips per ft, the maximum end shear is 60 kips. The A.I.S.C. specifications allow a unit shearing stress of 13,000 lb per sq in. If 60 is divided by 13, a required web area of 4.62 sq in. is obtained, which is considerably less than the web area available in all the beams previously considered.

28. Net Section. Whenever it is necessary to have holes in the flanges of I beams, for bolts or any other such purpose, the beam must be designed for the moment of inertia of the net area. This can be easily obtained by multiplying the area of each hole by the square of its distance to the neutral axis and subtracting the values from the moment of inertia of the entire section. The neutral axis is at the gravity axis of the gross area since the holes occur at regular or infrequent intervals and the neutral axis cannot move up and down like the corrugations in a washboard. It should be recalled that the neutral axis is the axis of zero stress, with all that that implies. From this a little study will give the following rules for computing net sections of beams when the holes are not symmetrical in both flanges. If unfilled holes are in the compression flange only, holes should be deducted from both flanges in obtaining the net moment of inertia for computing the stress in the compression flange. If the holes are in the tension flange only, the holes should be deducted from both flanges in computing the tensile stress, but the gross area of the beam should be used to compute the compressive stress.

Holes are generally punched $\frac{1}{16}$ in. larger than the diameter of the rivet or bolt, but, on account of damage to adjacent metal in punching, the diameter of the hole deducted is $\frac{1}{8}$ in. larger than the diameter of the rivet or bolt. Driven rivets or turned driven bolts are presumed to fill the holes, and no deduction is made for net area where compressive stress is being computed. Full deduction should be made in computing tensile stress according to most specifications and in the opinion of the authors; however, the 1949 A.I.S.C. specifications do not require any deduction for rivet holes in beams or girders if the reduction in the area of the flange for these rivet holes does not exceed 15 per cent of the gross flange area. Full deduction is made for computing compressive stress where ordinary bolts are used.

In Art. 39, the calculation of net area (or net width) for members having two or more lines of rivets is discussed and computations for a typical member are shown.

29. Lateral Forces. Beams are sometimes subjected to lateral forces as well as to vertical loads. These can be due to inclined loads on a vertical beam, to vertical loads on an inclined beam (such as purlins on a pitched roof), or to a combination of vertical loads and lateral thrust. From data on strength of materials, or as shown in Chapter 6, the general expression for bending in two directions for symmetrical sections is given as

$$f = \frac{M_1 c_1}{I_1} \pm \frac{M_2 c_2}{I_2} \tag{11}$$

the values of M_1, I_1, and c_1 being about the horizontal axis 1–1, and the values of M_2, I_2, and c_2 being about the vertical axis 2–2.

The above expression for symmetrical sections applies only if the resultant of the applied load acts through the center of gravity of the section; that is, there is no axial load or torsion on the section. In beams the lateral force, or resultant, is usually applied to one flange only. A precise analysis of this case would be extremely complicated, but sufficiently accurate results can be obtained by considering that the lateral forces are resisted by the one flange only and the vertical forces are resisted by the entire section. In using the above formula on this assumption for rolled beams, the section modulus of the flange in question, about the axis 2–2, is conveniently taken as one-half of I_2/c_2 for the entire beam. The accuracy of the assumption that the lateral force is entirely resisted by the flange to which it is applied is obviously dependent on the dimensions of the beam. For deep beams with thin webs it would be essentially correct, but for shallow heavy beams some error would be involved.

In applying formula 11 it is customary to assume a beam that appears to be of proper size and by means of the formula to calculate the maximum stress in the flange. If this does not exceed, and is sufficiently close to, the allowable stress, the beam is satisfactory. When the lateral force is appreciable it is generally necessary to choose a wide-flange beam.*

For economy in weight a section such as that shown in Fig. 20 is often used. This consists of an American Standard I beam combined with a channel. The channel and top flange of the I beam is assumed to resist the lateral load. The entire composite section resists the vertical load. This type of section is often employed for light crane girders.

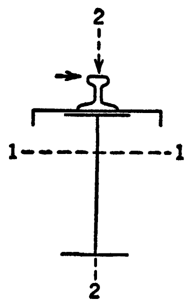


FIG. 20

Other types of sections for resisting forces in two directions will be considered in Chapter 6 on unsymmetrical sections.

30. Illustrative Example. Alternative designs for a crane beam (or crane girder) as a rolled wide-flange section and a smaller rolled beam with a channel added to the top flange are given below. The assumptions made in the computations for selecting the sections to support a 15-ton Whiting crane are as follows. The width of the building (or more accurately the span for the traveling crane) is 60 ft. The span for the beam supporting the crane is 30 ft. This crane has a truck at each end (each side of the building), and each truck is supported by two wheels whose loads (not including impact) and spacing are shown in Fig. 21*a*. The rails on which the wheels run are shown in Fig. 21*b* or *c*, and the weight of the

* On page 110 of *Structural Design in Steel*, Professor Shedd derives the following formula for determining the size of a rolled section to resist bending in two directions.

$$S_1 = \frac{\left(M_1 + M_2 \frac{S_1}{S_2}\right)}{s}$$

where M_1 = bending moment about the major axis 1–1.
M_2 = bending moment about axis 2–2.
S_1 = section modulus about axis 1–1.
S_2 = section modulus about axis 2–2.
s = maximum allowable unit fiber stress.

Where the lateral force is resisted by one flange only, the value of S_2 should be taken as one-half the section modulus of the entire beam about axis 2–2. This formula is of value in selecting the section that will resist bending in two directions. It is necessary to know the ratio S_1/S_2. After a ratio is assumed and the approximate size of beam is obtained, a close value of the ratio can be obtained from the values of the two section moduli in the handbook. The ratio is, of course, doubled when one flange resists the entire lateral force. The student should try proportioning a beam by means of this expression.

rail is 60 lb per yd. The live load is the load on the crane wheels,* which includes the crane weight (approximately 55 kips for the entire crane) and the lifted load of 15 tons. The critical position for the wheels on

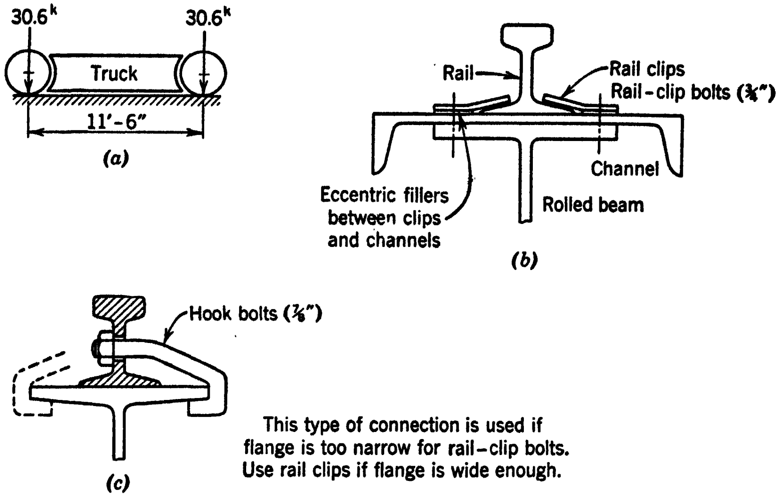


FIG. 21

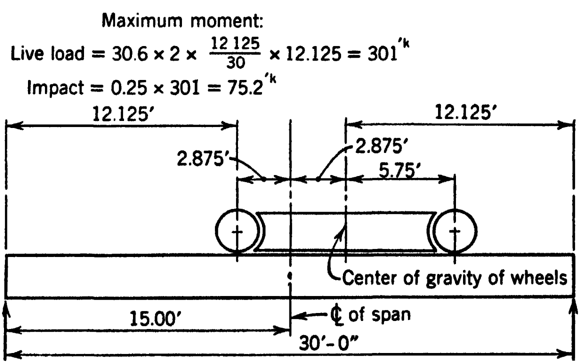


FIG. 22

the beam is shown in Fig. 22. The maximum live load moment occurs under the left wheel a distance 2.875 ft to the left of the centerline of the span.

* For data concerning crane sizes, capacities, weights, and dimensions, a designer generally refers directly to the company producing the crane to be used. The Whiting Corporation published a book entitled *Crane Engineering,* from which the data used here were taken. This book is now out of print.

According to A.I.S.C. specifications, the effect of impact is 25 per cent of the live load, and the bending moment for impact therefore is 0.25 of that for live load. The lateral load is taken as 20 per cent * of the sum of the lifted load and weight of the crane trolley (30 + 11 = 41 kips) and is equally distributed between the four wheels. The lateral load per wheel is therefore $(0.2 \times 41) \div 4 = 2.05$ kips. The critical position for the 30-ft span is the same as shown in Fig. 22 for live load, and the maximum lateral moment can be obtained by taking the ratio of the lateral to the vertical wheel loads times the maximum live load moment, or $(2.05 \div 30.6) \times 301 = 20.2$ ft-kips.

If it is assumed that the beam and rail attachments weigh 160 lb per ft, the total dead load for beam, attachment, and rail then is $160 + (60 \div 3) = 180$ lb per ft, and the dead load bending moment at the center of the span is $(0.180 \div 8)(30)^2 = 20.3$ ft-kips. The dead load moment has been computed at the center of the span, which is the general practice of designers as a matter of convenience. If the dead load moment were computed at the point of maximum live load moment, the value would be 19.5 ft-kips instead of 20.3 ft-kips. This difference is too small to be concerned with, and for longer spans and heavier cranes the percentage difference would be even less.

In the selection of a section to support the crane girder, first a wide-flange shape will be considered, and then a rolled beam with a channel attached to the top flange. For both it is assumed that the top flange alone resists the lateral bending moment. The rail is attached by bolts that do not have a tight fit, and, therefore, the rail is assumed to transmit the vertical and lateral loads to the beam but not to participate in resisting either the vertical or lateral bending moment.

Vertical Bending Moment		Lateral Bending Moment
L.L. =	301 ft-kips	20.2 ft-kips
Impact =	75.2	
D.L. =	20.3	
	396.5 ft-kips	

The ability of a designer to choose the proper section to check increases with his design experience. The beginner probably will have to check a few sections before he can determine the most economical one. The

* The A.I.S.C. specifications provide that the above lateral force shall be 20 per cent of the sum of the lifted load and the weight of the crane trolley. This crane trolley has a weight of approximately 11 kips.

method mentioned in the footnote on page 36 probably will decrease the number of trials.

Checking a 30 × 15 WF that weighs 172 lb per ft:

$$f = \frac{396.5 \times 12}{528.2} + \frac{20.2 \times 12}{73.4 \times \frac{1}{2}} = 9.01 + 6.60 = 15.61 \text{ kips per sq in.}$$

The flange is wide enough for rail clips; consequently 2—$\frac{7}{8}$-in. holes should be deducted for the 2—$\frac{3}{4}$-in. bolts in the top flange.

$$\text{Gross area of flange} = 1.065 \times 14.985 = 15.96 \text{ sq in.}$$

$$\text{Net area of flange} = 1.065(14.985 - 2 \times \tfrac{7}{8}) = 14.10 \text{ sq in.}$$

Correcting for holes:

$$f = 9.01\left(\frac{15.96}{14.10}\right) + 6.60 = 10.20 + 6.60 = 16.80 \text{ kips per sq in.}$$

$$\text{Allowable } f = \frac{12{,}000{,}000}{30 \times 12 \times 1.87} = 17.83 \text{ kips per sq in.}$$

The section is satisfactory, since 16.80 is less than 17.83.*

To check a composite section composed of a rolled beam with a channel attached to the top flange:

Required section modulus for vertical bending moment only equals 396.5 $\times \frac{12}{20} = 238$.

* Some designers would check flanges subject to both vertical and lateral bending by the stress ratio method. Taking the position that the top flange has an average stress of 10.20 kips per sq in. due to vertical bending and an allowable stress of 17.83 kips per sq in., the ratio of its useful capacity that is utilized is 10.20 ÷ 17.83 = 0.572. However, for the lateral bending moment only half the top flange is in compression while the other half is in tension; also, only the extreme fibers have a stress of 6.60 kips per sq in. For these reasons, the allowable stress for lateral bending should be 20 kips per sq in., and the ratio of capacity which is utilized is 6.60 ÷ 20 = 0.330. The sum of these ratios is 0.572 + 0.330 = 0.902. Since this sum is less than unity, the section is satisfactory.

A 27 WF 177 is also satisfactory if a shallower beam is desirable. The computed stresses, including the correction for holes, are 11.04 and 6.58, giving a total of 17.62 kips per sq in. The allowable stress is the maximum of 20 kips per sq in. (the formula gives a value of 20.45).

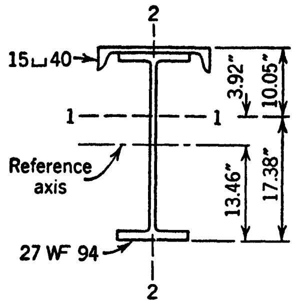


FIG. 23

A 27 WF 94 has a section modulus of 242.8, so the first trial will be this beam with a 15-in. channel at 40 lb, as shown in Fig. 23.

		Area	y	Ay	$A\bar{y}^2$	I_g	I_{2-2}
27-in. WF	94 lb	27.65	0	0	0	3266.7	$57.5 = \frac{115.1}{2}$
15-in. ⊔	40 lb	11.70	13.20	154.4	2038	9.3	346.3
	134 lb	39.35		154.4	2038	3276.0	403.8 in.^{4}

$154.4 \div 39.35 = 3.92 \text{ in.} = \bar{y}$

$$2038 + 3276 = 5314$$

$$39.35 \times \overline{3.92}^2 = -605$$

$$I_{1-1} = 4709 \text{ in.}^4$$

$$f = \frac{396.5 \times 12}{4709} \times 17.38 = 17.56 \text{ kips per sq in.} \quad \text{(tension at bottom)}$$

$$f = \frac{396.5 \times 12}{4709} \times 10.05 + \frac{20.2 \times 12 \times 7.5}{403.8} = 10.15 + 4.50$$
$$= 14.65 \text{ kips per sq in.}$$

This composite beam will also have bolt holes for the attachment of the rail clips, and the correction is as follows:

$$\text{Gross area} = (9.99 \times 0.747) + 11.70 = 19.16 \text{ sq in.}$$

$$\text{Net area} = 19.16 - 2(0.747 + 0.520) \times \tfrac{7}{8} = 19.16 - 2.22 = 16.94 \text{ sq in.}$$

$$f = 10.15\left(\frac{19.16}{16.94}\right) + 4.50 = 11.48 + 4.50 = 15.98 \text{ kips per sq in.}$$

The rivet holes in the top flange are filled with rivets to attach the channel and the beam flange together in order to make the two shapes act as a composite member. Since these holes are filled, no correction is necessary for compressive stress. The spacing of such rivets will be taken up later in the discussion of rivet pitch.

$$\text{Allowable } f = \frac{22{,}500}{1 + \frac{1}{1800}\left(\frac{360}{15}\right)^2} = 17.05 \text{ kips per sq in.}$$

This formula for allowable stress in a flange without lateral support will be used for all sections except rolled beams. Although the compressive stress is less than the allowable, a smaller beam would give a tensile stress greater than the allowable, and with a 33.9-lb channel the computed compressive stress, corrected for holes, is 17.32 kips per sq in.

Although the expression $12{,}000{,}000 \div (ld/bt)$ was not developed for built-up flanges, some designers have suggested a modification of this expression in order that it can be used for composite flanges. The authors believe that the value, $12{,}000{,}000 \div (ld/bt)$, should not be applied to composite flanges (including plate girder flanges), but, for the sake of comparison with the allowable stress above, this modification can be made by substituting for the quantity $b \times t$ the quantity $12I_y/b^2$ where I_y is the moment of inertia of the top flange about a vertical axis and b is the width of the top flange. For a rectangular shape,

$$\frac{12I_y}{b^2} = \frac{12(b^3t/12)}{b^2} = bt$$

and for the composite flange of this example

$$\frac{12I_y}{b^2} = \frac{12 \times 403.8}{(15)^2} = 21.54$$

$$\text{Allowable } f = \frac{12{,}000{,}000}{\frac{360 \times 27.43}{21.54}} = \frac{12{,}000{,}000}{458} \text{ lb per sq in.}$$

but, since ld/bt is less than 600, the allowable stress to be used is the base stress, 20,000 lb per sq in.

The dead load assumed for the beam and rail attachments was 160 lb per ft, but no correction was made for the 172-lb wide-flange beam or for the composite beam which weighed 134(= 94 + 40) lb per ft. Such corrections would be extremely small and would not affect the design; however, most design offices desire that the calculations filed with all designs be made with the correct dead loads including the weight of detail material (attachments, etc.).

Although the composite section would save 38 lb per ft, it is composed of two members riveted (or welded) together, and the additional fabrication costs would offset to some extent the saving in weight. Where the percentage of saving in weight is much less, the added fabrication costs could make the composite section the more expensive one.

31. Plate Girders. When a member is required that is larger than is available in rolled beams, it is necessary to build up a section which, for the sake of economy, has a general shape of an I beam. This built-up I beam is called a plate girder. Three common types of plate girders are shown in Fig. 24. In Fig. 24*a* the I beam is formed from a plate, called the web plate, and four angles which compose the flanges. Fig. 24*b* is similar to Fig. 24*a*, the main exception being that additional area has been obtained in the flanges by adding cover plates at the top and bottom. In Fig. 24*c* the girder is composed of plates only and is a form of plate girder widely used in welded construction. The flange plates are welded to the top and bottom of the web plate.

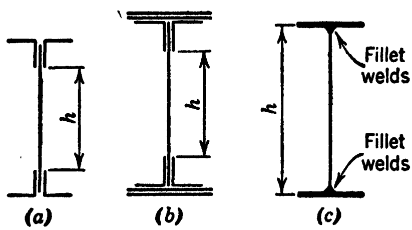


FIG. 24

In welded plate girders, each flange should consist in general of a single plate rather than two or more plates superimposed. A series of shorter plates may be laid end to end and butt-welded at their junctions to form the single plates. However, more than one flange plate can be utilized in a welded girder, and when this is done the outer plates should be made wider, or narrower, than the inside plates in order to facilitate the welding.

When the web depth is large or the web thickness is small, stiffener angles are necessary for the girders shown in Fig. 24*a* and *b*, and a stiffener plate would have to be welded to the web in Fig. 24*c*.

The girders shown in Fig. 24 are types that can be employed in ordinary construction where extremely large bending moments are not encountered. Where heavy girders are required the flanges will be more complicated, as far as shape is concerned, by side plates; by four angles and cover plates; by built-up channel sections; or by other variations. Details of the heavier types are discussed more properly in textbooks on steel design. The principles involved are no different from those discussed here in connection with the design of the more common types.

In proportioning plate girders it is necessary to make them of sufficient size to properly resist the shearing and bending stresses.

In resisting shearing stresses it is assumed, as it was for I beams, that the shear is resisted entirely by the web and that the allowable unit stress is the average shearing stress in the web.

For many years, in resisting bending, plate girders were proportioned on the assumption that the tension and compression in the flanges were concentrated at their respective centers of gravity, that the average stress in the flange was the allowable unit stress on the extreme fiber (this being equivalent to assuming that the unit stress was uniform over the flange), and that the web resisted its portion of the bending as a rectangular beam with its extreme fiber stress being equal to the flange unit stress. This method of proportioning, called the flange area method of design, gave fairly satisfactory results for a number of years, although the stress in the extreme fiber would be higher than the allowable unit stress. For deep girders this error was small, but for shallow girders carrying heavy loads the error could be appreciable.

Present specifications require that plate girders be proportioned by the net moment of inertia of the section or the gross moment of inertia as in the A.I.S.C. specifications. The flange area method will not meet these specifications, but the student will find that proportioning and designing a girder by the moment of inertia method is a rather cumbersome procedure. Article 33 presents a method, called the modified flange area method of design, that has all the advantages of the flange area method and gives results that are closer to those obtained with the moment of inertia method, and slightly conservative. It is a satisfactory substitute, conforming to all specifications calling for the moment of inertia method. This modified method was published first by Professor Thomas C. Shedd in his *Structural Design in Steel*, and it has been used by the authors ever since its introduction. They have found it satisfactory, and it is given here as the preferred method.

32. Economical Depth. The least weight depth of plate girders generally met with in practice varies from about $\frac{1}{14}$ to $\frac{1}{8}$ of the length of the span. For building girders an average value is about $\frac{1}{12}$ of the span length, and about $\frac{1}{10}$ for bridge girders. For heavy loads the economical depth is increased, and for long spans this ratio is decreased. An experienced designer is able to estimate the required depth for economy within very close limits, taking into consideration the various factors that enter into the proper choice.

In Art. 84 of *Structural Design in Steel*, Professor Shedd has derived several formulas for economical depth of plate girders under various conditions and has drawn several diagrams for the value of his factor k.

He suggests two approximate formulas as:

$$d = 5.5 \sqrt[3]{\frac{M}{f}}$$

for girders with intermediate stiffeners, and

$$d = 4.6 \sqrt[3]{\frac{M}{f}}$$

for girders without intermediate stiffeners. In these formulas M = the maximum bending moment in inch-kips, and f = the allowable intensity of bending stress in kips per square inch.

These two formulas prove very satisfactory for obtaining an estimate of girder depth. For a given loading and span, if a curve were plotted with depth as the abscissa and cost (or weight) as the ordinate, on the basis of comparative designs, it would be found to be rather flat over a fair range in depth at the low point of the curve. There would therefore be considerable latitude in choosing the depth, but the expressions above will prove to be a useful guide. The web slenderness ratio and other factors will have some bearing on the depth chosen.

33. Modified Flange Area Method of Design.* After determining the approximate depth of a plate girder, the design is carried forward in the following order:

(*a*) Proportion the web.
(*b*) Proportion the flange.
(*c*) Check the depth and modify it if necessary.

The web must have sufficient area to resist the maximum shear, and it must also have a great enough thickness so that the requirements for minimum thickness of material, or the proper slenderness ratio (h/t), shall be met. This latter requirement is more likely to determine the size of the web plate used rather than the area required to resist shear. It is a usual requirement in specifications that the thickness of plate girder webs (t) be not less than $\frac{1}{170}$ of the unsupported distance between flanges (h in Fig. 24). It is common practice to place the backs of the flange angles $\frac{1}{4}$ in. beyond the edge of the web plate, and, if a preliminary estimate can be made of the size of the required angles, the unsupported distance will be known for the assumed depth. If, in assuming a depth

* As stated in Art. 31, this method was first introduced by Shedd in his *Structural Design in Steel.*

of web, it is found that the unsupported depth is slightly more than that permitted for a given thickness to meet the $\frac{1}{170}$ ratio, it will be more economical to reduce the depth the small amount necessary to use the given web than to use a web $\frac{1}{16}$ in. thicker, provided the web area requirements are satisfied. As an example, suppose the expression for least weight depth gave a depth of web of 88 in. and 6-in. flange angles were to be used, then h, the clear depth between flanges, would be $88 + 2 \times \frac{1}{4} - 2 \times 6 = 76\frac{1}{2}$ in. The minimum thickness of web is therefore $76\frac{1}{2} \div 170 = 0.45$ in., or $\frac{1}{2}$ in. If the depth of web were de-

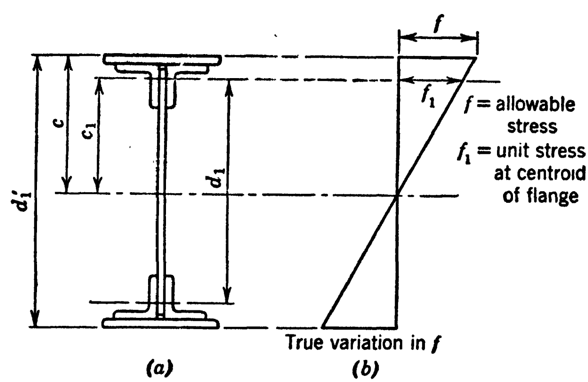


Fig. 25

creased to 86 in., the web could have a thickness of $\frac{7}{16}$ in. The depths and thicknesses of webs, as well as other parts, should be consistent with readily available sizes. The A.I.S.C. specifications permit a minimum thickness of $\frac{1}{4}$ in. on interior construction. The authors are not inclined to favor a web this thin except in light girders where the clear depth is considerably less than $170 \times \frac{1}{4}$ in.

The flange is proportioned so as to be able to resist the bending moment with the assistance the web provides as a rectangular beam. In proportioning the flange it is assumed that the average unit stress in the flange is that at the center of gravity of the flange, and this value must be such that the unit stress at the extreme fiber of the girder is not greater than the allowable value given in the specifications. It is assumed that the unit stress varies as its distance from the neutral axis, as shown in Fig. 25. In order to assume correctly the average unit stress, it is necessary to know the ratio between the distance between centers of gravity of the flanges (the effective depth) and the overall depth of the girder. The correctness of the design depends on the correctness with which the assumed effective depth is chosen. The procedure can be as follows:

Let d_1' = the assumed overall depth.
d_1 = the assumed effective depth.

Then $f_1 = f(d_1/d_1')$ = the estimated average unit stress.

If A_1 = the estimated flange area, including the portion of the web effective as flange area, then

$$A_1 = \frac{M}{d_1 \times f_1}$$

The dimensions for the assumed girder are shown in Fig. 25.

The flange can be proportioned on the basis of A_1, and the design can then be checked by calculating the location of the center of gravity, the overall depth, and the new average stress $f_2 = f(d_2/d_2')$, in which d_2' and d_2 are the overall and effective depths, respectively, for the girder as proportioned. The final required flange area A_F would then be

$$A_F = A_1 \times \frac{d_1}{d_2} \times \frac{f_1}{f_2}$$

If there were very little difference, it would not be necessary to recalculate the location of the center of gravity. Where cover plates are not required, it probably will be possible to locate the center of gravity and overall depth on the first calculation accurate enough for final design. An experienced designer is quite likely to estimate his values closely enough so that a recalculation of area is not necessary, but the student probably will find it necessary to go through one or more trials in the process of his design.

Practically all specifications require that the tension flange be proportioned for the net area, i.e., that rivet holes be deducted, and they also provide that the compression flange cannot be smaller than the tension flange. This means that the design of the tension flange controls the design of the girder if the compression flange has sufficient lateral support. The above discussion applies with equal force, however, to the design of either flange. The computed center of gravity is the center of gravity of the gross area, no matter which flange is being considered.

Where the compression flange does not have lateral support, it is possible that the allowable unit stress is reduced to a low enough value so that the required compression flange is larger than the tension flange, even though it is figured on the basis of gross area. In this case the center of gravity of the girder is above the center of the web, and the portion of the web effective as compression flange area is $\frac{1}{3}$ of the area above the center of gravity rather than $\frac{1}{6}$ of the total web area. The student should be able to show why this is so after he has studied the development presented in the next paragraph. The formulas for the reduction in allowable stress, where there is lack of lateral support, are usually the same for girders as for beams, except that for girders the authors feel that more consistent results can be obtained with the older formulas than with the 12,000,000 ÷ (ld/bt) formula. This formula was derived from a study of rolled beams, and in welded or riveted girders, especially where the webs are deep, the results are not con-

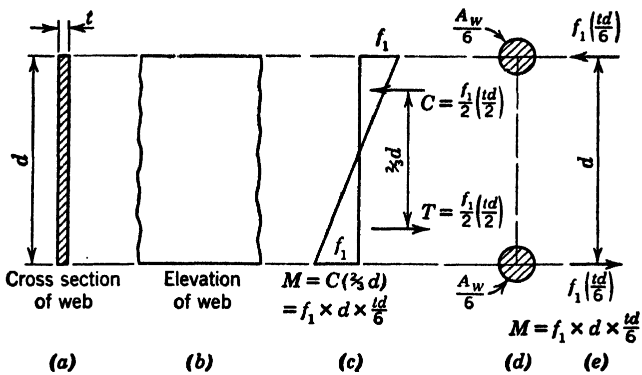


FIG. 26

sistent with the generally used column formulas. Since the base stress used in this chapter is 20,000 lb per sq in., the authors believe that the expression

$$f = \frac{22{,}500}{1 + \dfrac{L^2}{1800b^2}}$$

should be used to compute the allowable stress for compression flanges without continuous lateral support. The maximum stress permitted is 20,000 lb per sq in., which corresponds to an L/b ratio of 15.

The web is a rectangular beam with a section modulus of $d^2t/6$ and therefore can resist a moment equal to $f_1(d^2t/6)$, where d is the depth of web, t is the thickness, and f_1 is the stress at the extreme fiber of the web. This can be written $f_1(dt/6) \times d$. If f_1 is assumed to be the same as f_2, and therefore d = the effective depth, it will be found that the web is as effective in resisting moment as if $\frac{1}{6}$ of its area were concentrated at the center of gravity of the flange, and consequently it can be assumed that $\frac{1}{6}$ of the area of the web can be subtracted from the required flange area in proportioning the flange. This is illustrated in Fig. 26.

This value of $\frac{1}{6}$ is based on the gross area of the web. Where the flange is being proportioned for net area, as is common for the tension flange, a suitable deduction should be made from the area of the web. The greatest concentration of rivet holes in the web occurs at web splices or stiffener angles. The maximum deduction normally encountered in design is about $\frac{1}{4}$ the web area. It is customary to assume that the net area of the web is $\frac{3}{4}$ of the gross area, and therefore $\frac{3}{4} \times \frac{1}{6} = \frac{1}{8}$; and in proportioning the tension flange $\frac{1}{8}$ of the web area is considered effective. Where the flange is proportioned for gross area, $\frac{1}{6}$ the area of the web is effective.

This method differs from the flange area method outlined in Art. 31 only in that the unit stress used in proportioning the flange is that occurring at the center of gravity of the flange with the extreme fiber stressed at the allowable working stress. In the flange area method it is assumed that the working stress is distributed uniformly over the entire flange.

The method of proportioning a girder, as outlined above, can best be exemplified by means of an illustrative example. Article 34 gives a discussion of the steps taken, as well as the design calculations.

34. Illustrative Example. Assume a girder span of 48 ft carrying a uniform load of 8 kips per ft of span, including the weight of the girder itself.

$$M = \frac{8 \times \overline{48}^2}{8} = 2{,}304 \text{ ft-kips} \qquad V = 8 \times 24 = 192 \text{ kips}$$
$$= 27{,}650 \text{ in.-kips}$$

$\frac{7}{8}$-in. rivets will be used, giving a computed hole diameter of 1 in. The following allowable stresses will be used:

Tension on extreme fiber = 20,000 lb per sq in. on net area.
Compression on extreme fiber = 20,000 lb per sq in., with lateral support.
Shear on webs = 13,000 lb per sq in. on gross area.

The compression flange is assumed to have lateral support.

$$5.5\sqrt[3]{\frac{27{,}650}{20}} = 61 \text{ in.} \qquad \frac{192}{13} = 14.77 \text{ sq in. required web area}$$

With a 60-in. web and 6 by 6 in. flange angles,

$$\frac{14.77}{60} = 0.246 \text{ in.} \qquad \frac{60.5 - 12}{170} = 0.286 \text{ in.}$$

A thickness of not less than $\frac{5}{16}$ in. will be required.

$$1 \text{ web } 60 \times \tfrac{5}{16} = 18.75 \text{ sq in.}$$
$$\times \tfrac{1}{8} = 2.34 \text{ sq in.}$$

The overall depth will be assumed to be 62 in., and the effective depth 59.5 in.

$$20 \times \frac{59.5}{62} = 19.19 \text{ kips per sq in.}$$

$$\frac{27{,}650}{59.5 \times 19.19} = 24.22 \text{ sq in.}$$

Trial section:

$$\begin{array}{llll}
1 \text{ web } 60 \times \frac{5}{16} & = 18.75 \text{ sq in. gross} \times \frac{1}{8} & & = 2.34 \\
2 \text{ bottom } \angle\text{s } 6 \times 6 \times \frac{5}{8} & = 14.22 \text{ sq in.} & - 2.5 & = 11.72 \\
1 \text{ bottom cover } 14 \times \frac{1}{2} & = 7.00 & - 1.0 & = 6.00 \\
1 \text{ bottom cover } 14 \times \frac{3}{8} & = 5.25 & - 0.75 & = 4.50 \\
\hline
 & & & 24.56 \text{ sq in. net}
\end{array}$$

$$\begin{array}{lr}
14.22 \times 1.73 = & 24.60 \\
12.25 \times 0.438 = & 5.36 \\
\hline
26.47 &)19.24 \\
\hline
 & 0.73 \text{ in.}
\end{array}$$

$$\begin{array}{r}
60.5 \\
1.0 \\
0.75 \\
\hline
\end{array}$$

Overall depth = 62.25 in.

Effective depth = 60.5 − 1.46 = 59.04 in.

$$20 \times \frac{59.04}{62.25} = 18.97 \text{ kips per sq in.} \qquad \frac{27{,}650}{59.04 \times 18.97} = 24.69 \text{ sq in. net}$$

The section is satisfactory and is shown in Fig. 27. If the outside plates were made 14 by $\frac{7}{16}$ the net area would be 25.31 sq in.

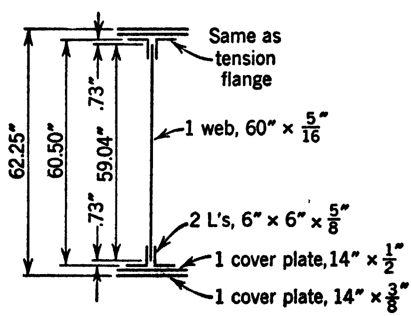


Fig. 27

The specifications require that the center of gravity of the flange lie within the flange angles. This can usually be met, in flanges of the type in this example, if the area of the angles is from 33 to 40 per cent of the required flange area, or if the cover plates do not exceed 60 to 70 per cent. Thinner angles and thicker plates could be used and still meet this requirement. Some specifications limit the cover plate area, in moderate-sized girders of this type, to 50 per cent of the required flange area. This flange meets this requirement.

The student should design this girder with a shallower web and check on the total required weight as compared with the weight of the above girder. Using a different size of rivet would also be good practice. The approximate weight of this girder can be computed as follows:

Maximum top flange net area	= 22.22
Maximum bottom flange net area	= 22.22
Web gross area	= 18.75
Details, 60 per cent of web (approximate)	= 11.25
	74.44 sq in. at 3.4 lb per ft = 253 lb per ft *

Some designers prefer to use 6 by 4 in. flange angles rather than 6 by 6 in. angles, although there are certain advantages in the latter as far as shop work is concerned. For 6 by 4 in. angles (the long legs out), the effective depth of the girder is increased a small amount, thus resulting in a slightly less required net area. For this particular girder, however, the extra thickness of material required results in a slightly greater gross area.

35. Design by Moment of Inertia. Although most specifications require that plate girders be proportioned by the moment of inertia of the net section, and the A.I.S.C. specifications allow the gross section, there is no direct method of approach in proportioning a girder to meet some predetermined moment of inertia or section modulus. Any direct attack of the problem would entail a design by some method comparable to the one above and then computation of the moment of inertia for a check on the specification requirement. The above method should prove satisfactory without the moment of inertia check.

The computations for checking the girder in Art. 34 by the net moment of inertia are as follows:

* In these calculations the common assumption is made that where cover plates are cut off the average gross area of a flange is approximately equal to the net area at point of maximum moment. One square inch of steel weighs 3.4 lb per ft of length.

Top flange	2×24.2	$= 48.4$	
	$14.22 \times \overline{28.52}^2$	$= 11,566$	
	$12.25 \times \dfrac{\overline{0.875}^2}{12}$	$= 0.8$	
	$12.25 \times \overline{30.69}^2$	$= 11,538$	
Bottom flange	(same)	$= 23,153$	
Web		$= 5,625.0$	
			51,931
Web holes *	$\frac{1}{7} \times 2968$	$= 424$	
2 holes	$1 \times 1\frac{9}{16} \times \overline{28.00}^2$	$= 2,450$	
4 holes	$1 \times 1\frac{1}{2} \times \overline{30.38}^2$	$= 5,537$	
			8,411
		$I =$	43,520

$$\text{Section modulus} = \frac{43,520}{31.13} = 1398.$$

$$\text{Required section modulus} = \frac{27,650}{20} = 1383.$$

36. Cover Plate Length. When cover plates are used in a plate girder flange, their area is proportioned so that in combination with the flange angles and web they can resist the maximum bending moment on the span. At other points the maximum flange area is not required, and, as the bending moment decreases toward the end of a simple span, one or all the cover plates can be omitted. On outside construction, specifications require that at least one cover plate on the top flange should extend the full length of the span, and some specifications require that one on the bottom flange should extend the full length also. On inside construction, it is permissible for all cover plates to be cut off when they are no longer needed to resist stress.

If we assume that the effective depth of the girder is a constant, which is essentially true, and if we assume that the allowable stress at the center of gravity of the flange is a constant, then the required area of the flange will be directly proportional to the bending moment in the girder. This provides a very simple way in which to determine the points at which the cover plates are no longer needed to resist stress. These points are called the theoretical points of cover plate cutoff.

* These are the holes for the stiffener rivets that go through the web only. If the maximum spacing of 8 diameters is used, it will be 7 in. For a 1-in. hole every 7 in., the deduction will be approximately $\frac{1}{7}$ of the moment of inertia of a plate, $48\frac{1}{2} \times \frac{5}{16}$.

If the moment curve for a girder is drawn as in Fig. 28, where the moment curve for the girder previously designed is shown, and the maximum ordinate (in this case the middle ordinate) is divided in proportion to the various parts that make up the area of the flange, the theoretical points of cutoff of the cover plates can be obtained by drawing horizontal lines through the ordinates corresponding to the relative area of the respective cover plates. From the points where these horizontal lines intersect the moment curve, the theoretical length of cover plates L_1 and L_2, can be obtained. If one prefers, he can choose a scale

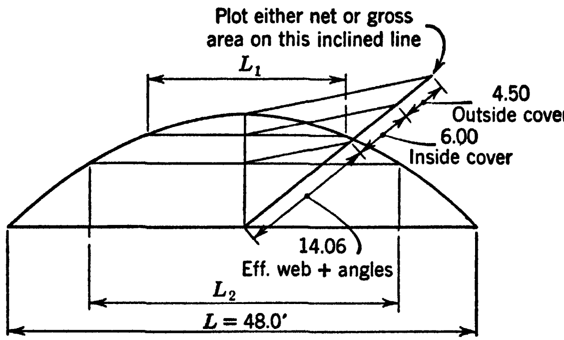


FIG. 28

for the flange area so that the ordinate for the total flange area is equal to the ordinate for the maximum moment, thus eliminating the inclined line in Fig. 28.

Although the moment curve shown in Fig. 28 is a regular symmetrical curve, the method of obtaining cover plate length shown there can also be applied where the moment curve is unsymmetrical. As a matter of fact, the method in Fig. 28 has its greatest measure of applicability when the moment curve is unsymmetrical or consists of a small number of straight lines.

Where the moment curve is a parabola, as in Fig. 28, it is usually more satisfactory to employ a simple expression, that utilizes the properties of the parabola, in finding the theoretical cover plate length.

If L = span length,
L_c = theoretical length of the cover plate in question,
a = area of the cover plate in question plus area of all cover plates outside it,
A_F = total flange area including the effective web area ($\frac{1}{8}$ of the web area for net areas, $\frac{1}{6}$ for gross areas).

Since the offsets to a parabola, from a tangent, vary as the squares of the distances along the tangent, the following relation holds:

$$\frac{(L_c/2)^2}{(L/2)^2} = \frac{a}{A_F}$$

Therefore

$$L_c = L\sqrt{\frac{a}{A_F}} \tag{12}$$

A cover plate length obtained by either of the two methods above is the theoretical length, i.e., the point where cover plate area no longer is required. Since there is stress in the cover plate up to this point, it is necessary to extend the cover plate some additional length in order that the rivets through the cover plate shall not be greatly overstressed. Specifications vary as to the requirements for this additional length. Some of the variations are that it must be at least 1 ft beyond the theoretical point of cutoff, and that it is sufficient to allow two rows of rivets at the regular pitch beyond the theoretical point; and many girders have been designed with the cover plates having an additional length beyond the theoretical point that permits enough rivets to develop in the cover plates the average flange stress at this theoretical point. No matter how far the cover plate extends, there will be some rivets overstressed unless the plate extends to a point where the flange stress is very low. The authors believe that an extension of some convenient amount, approximately 1 ft at each end, makes good practice, considering all points concerned in the problem.

If expression 12 is applied to the girder designed in Art. 34, the following values would be obtained.

$$L_1 = 48\sqrt{\frac{4.50}{24.56}} = 20.54 \text{ ft}$$

$$L_2 = 48\sqrt{\frac{10.50}{24.56}} = 31.38 \text{ ft}$$

Lengths of 23 ft and 33 ft are satisfactory.

It will be observed that in obtaining these lengths the areas, a and A_F, were net areas. Either the net areas or the gross areas could be used, or each could be used in separate solutions to obtain two values. This is also true for a graphical solution, such as Fig. 28. Since the net area of the tension flange usually determines the size of both flanges, it is generally customary to calculate the cover plate lengths for the tension flange and make the lengths for the compression flange the same. If the gross areas had been taken, the following results would have been obtained.

$$L_1 = 48\sqrt{\frac{5.25}{29.60}} = 20.22 \text{ ft}$$

$$L_2 = 48\sqrt{\frac{12.25}{29.60}} = 30.88 \text{ ft}$$

This has no material effect on the lengths used; 22 ft and 33 ft are satisfactory.

Flange plate lengths for welded girders can be obtained by either the graphical method or the arithmetical method described here. Because current specifications permit the same intensity of stress for butt welds in tension and in compression, the tension flange plates and compression flange plates will be the same in length if the compression flange has sufficient lateral support.

It is possible to write an equation (or series of equations) for an unsymmetrical moment diagram and solve for the points of theoretical end of cover plates, but the work involved is such that the graphical method shown previously is more satisfactory. Where the moment diagram is a parabola (or where it is the result of a large number of equal and equally spaced concentrated loads so that it is essentially a parabola), the expression in formula 12 is more advantageous.

37. Stiffener Spacing. Shear was not discussed to any great extent under rolled beams since it is not important in their design, except in rare cases. Diagonal compression was not mentioned. Both shear and diagonal compression are of importance in the design of plate girders because of the deep slender web plates encountered. Where shear occurs alone, as at the neutral axis of a flexural member, the vertical and horizontal shear can be combined into tensile or compressive principal stresses making angles of ± 45 deg with the neutral axis. These values will be discussed in more detail in Chapter 5, where the value of diagonal tension is important in the design of reinforced concrete members. The diagonal compression is important in steel girders as it acts as a compressive force on a thin diagonal column, and, if not properly reinforced where necessary, the girder can fail by diagonal buckling in the web.*

At the neutral axis the compressive principal stresses are equal to the unit shearing stresses. On the compressive side of the neutral axis they

* A method is presented in Chapter 7 whereby a girder can be designed safely even though the web may buckle.

are greater than the shearing stresses and act at an angle with the neutral axis of less than 45 deg, and on the tensile side they are less than the shearing stress and act at an angle greater than 45 deg. The actual value is of no importance since methods of spacing stiffener angles to stiffen the web against buckling are based on formulas that include the value of the shear and, although semirational in derivation, are largely empirical in fact. It is somewhat beyond the scope of this book to discuss the detailed background of the various formulas. It is sufficient to say that they have been based on the idea of column action in diagonal strips of the web, but attention here will be given only to formulas in current specifications.

Where carbon steel is used, specifications generally state that intermediate web stiffeners shall be provided where the clear depth of the web between flanges is greater than 60 times the web thickness (the A.I.S.C. specifications use 70). Since this ratio in rolled beams is less than 60, it is seen that buckling is not a problem in normal beam design.

The A.I.S.C. specifications provide that, if h/t is equal to or greater than 70, intermediate stiffeners shall be required at all points where v exceeds

$$\frac{64{,}000{,}000}{(h/t)^2}$$

in which h = clear depth between the flanges in inches.
t = thickness of the web in inches.
v = unit shear in pounds per square inch.

It further provides that where intermediate stiffeners are required the clear distance between them, d, shall not exceed 84 in. or the value given by the formula

$$d = \frac{11{,}000t}{\sqrt{v}}$$

The A.A.S.H.O. (American Association of State Highway Officials) specifications are somewhat more conservative. They require stiffeners, for girders of carbon steel, if h/t is equal to or greater than 60, and they give for spacing a maximum of 6 ft, or the clear unsupported depth of the web, or

$$d = \frac{9000t}{\sqrt{v}}$$

The A.R.E.A. (American Railway Engineering Association) specifications give

$$d = \frac{10{,}500t}{\sqrt{v}}$$

with a maximum of 6 ft.

Taking the girder designed in Art. 34 and assuming the dead load at 300 lb per ft and the live load as a moving uniform load of 7700 lb per ft, the following values of maximum shear * and unit shearing stress are obtained:

At the end $V = 192$ kips $v = \dfrac{192{,}000}{18.75} = 10{,}240$ lb per sq in.

At the quarter point $V = 107.6$ kips $v = \dfrac{107{,}600}{18.75} = 5{,}740$ lb per sq in.

At the center $V = 46.2$ kips $v = \dfrac{46{,}200}{18.75} = 2{,}460$ lb per sq in.

$$\frac{64{,}000{,}000}{(48.5 \div \frac{5}{16})^2} = 2660 \text{ lb per sq in.}$$

Since this is so near the maximum shear at the center, the authors are inclined to have stiffeners there also.

The following spacing is obtained.

At the end $\dfrac{11{,}000 \times \frac{5}{16}}{\sqrt{10{,}240}} = 34.0$ in.

At the quarter point $\dfrac{11{,}000 \times \frac{5}{16}}{\sqrt{5740}} = 45.4$ in.

At the center $\dfrac{11{,}000 \times \frac{5}{16}}{\sqrt{2460}} = 69.3$ in.

The easiest way to take advantage of these values would be to plot a curve of stiffener spacing as shown in Fig. 29 and choose stiffener spacings that conform to the required value and also make the total of the

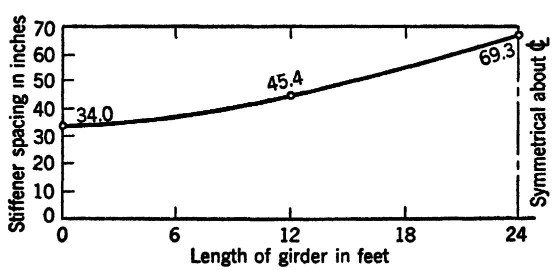


FIG. 29

* The maximum shear at any given point, for the live load, is obtained by placing the live load on that part of the span between the given point and the far reaction, for a girder span without floorbeams. Dead load covers the entire span. A curve of maximum shears is constructed for the girder span with floorbeams in the illustrative design at the end of this chapter.

spaces equal the span of the girder. It will probably be found easier to draw a smooth curve if the stiffener spacing is computed for at least four points rather than three as shown in the figure. The following stiffener spacing, as shown for the left half of the span, is satisfactory: 3 at 36 in., 2 at 42 in., and 2 at 48 in. = 24 ft 0 in. It will be observed that an end spacing of 36 in. is indicated although the curve shows 34 in.

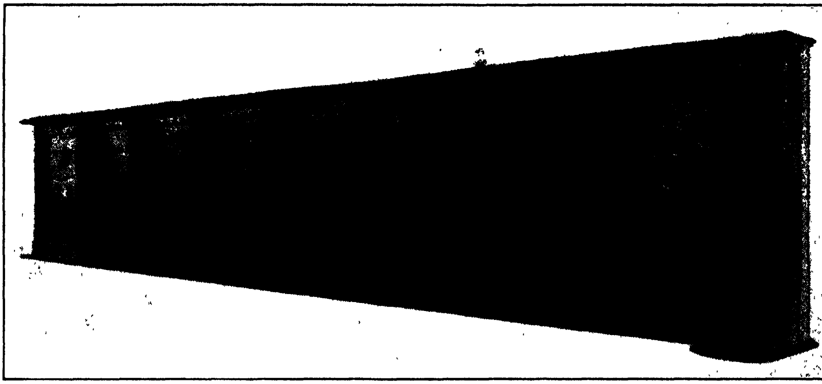


FIG. 30. Plate girder for Chinese National Railways, showing rivet spacing, crimped stiffeners, and bearing stiffeners. (Courtesy of American Bridge Company.)

The formula is for clear spacing whereas the spacing above is center-to-center spacing measured from the end of the span.

If this girder is for a highway bridge and $d = (9000 \times t)/\sqrt{v}$ is used, the following values are obtained.

At the end	27.8 in.
At the quarter point	37.1 in.
At the center	56.7 in., with a maximum of 48.5 in.

For proportioning the stiffener angles the A.I.S.C. specifications provide that the moment of inertia of intermediate stiffeners I_s about the centerline of the web be equal to $0.00000016H^4$, where H is the total depth of web. This expression permits the use of absurdly small angles for moderate-size and small girders. A rule for proportioning intermediate stiffeners given in some specifications and followed by many designers is that the outstanding leg of the stiffener angle be not less than 2 in. plus $\frac{1}{30}$ the depth of the girder, and its thickness be not less than $\frac{1}{16}$ the width of the outstanding leg. For the girder just discussed this is $2 + (62.5 \div 30) = 4.08$ in. Strictly speaking, this should call for 5 by 3 by $\frac{5}{16}$ in. stiffener angles. It is satisfactory to use 4 by 3 by $\frac{1}{4}$ in. angles.

Intermediate stiffener angles may be crimped over the flange angles and fit directly against the web. Where they function also as connection angles, they should not be crimped but should have filler plates between them and the web. The thickness of the filler plates should be equal to the thickness of the flange angles. Most specifications require that stiffener angles be attached to the web in pairs with one angle on each side, but the A.I.S.C. specifications allow them to be staggered. The authors prefer intermediate stiffeners to be placed in pairs; however, if they are staggered, each angle should be larger than when they are placed in pairs.

Where concentrated loads are applied to the flange of the girder, the intermediate stiffeners at or near that point should be replaced by a pair of bearing stiffeners under the load. Bearing stiffeners will be discussed in Art. 40.

38. Rivet Pitch. Where there is a change of total stress in the flange of a beam or girder, there must be some means available to hold this flange to the web; otherwise the unbalanced force will have a tendency to cause the flange to slide with respect to the web. In beams this is

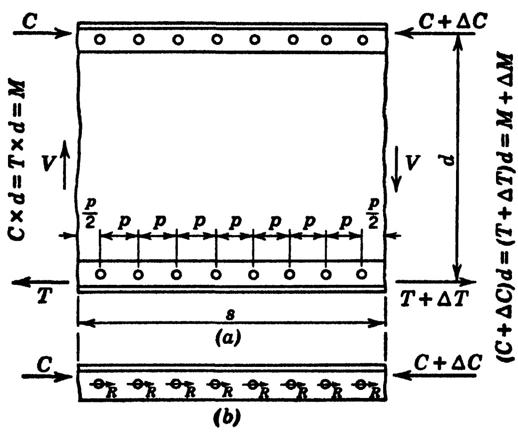


FIG. 31

taken care of by the horizontal shearing stress, as discussed previously; in a riveted plate girder the flange rivets perform this function; and in a welded girder the welds, intermittent or continuous, perform this same function.

Figure 31 shows a short length s of a plate girder having a moment M at the left section and a moment $M + \Delta M$ at the right section. If the length s is very small, or if there are no vertical loads applied between the sections, the shears at the two sections may be taken as equal, as shown on the figure. If one assumes all the flexural stresses to be concentrated in the flanges, it follows that

$$C = T = \frac{M}{d}$$

$$C + \Delta C = T + \Delta T = \frac{M + \Delta M}{d}$$

$$\Delta C = \frac{\Delta M}{d}$$

ΔC being the change in total flange stress in the distance s. Therefore, $\Delta C/s$ = the rate of change of total stress in the flange = $\Delta M/(s \times d)$.

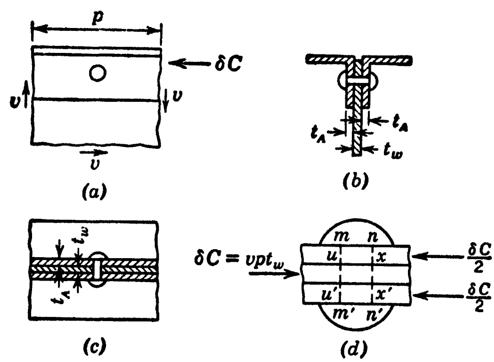


Fig. 32

Since $\Delta M/s$ = the rate of change of moment = V, the relation, V/d = the rate at which the total flange stress is changing per unit length, is obtained. This also represents the tendency of the flange to slide along the web and is equal to the horizontal shearing force, at the flange, per unit length in the web. It also represents the load on whatever medium attaches the flange to the web; in this case, rivets.

If R represents the allowable load on one rivet and p represents the required spacing of the rivets in order that the allowable load be maintained, then this relationship can be written,

$$R = \frac{V}{d}p \quad \text{or} \quad p = \frac{Rd}{V} \tag{13}$$

The value of R is the allowable load on the rivet in single shear, double shear, or bearing, whichever is the smaller value. These values are given in *Steel Construction* and in other handbooks.

In Fig. 32*a* is shown in elevation a short length p of a compression flange including the top of the web. Figure 32*b* shows a cross section of the flange, and Fig. 32*c* shows a bottom view. Figure 32*d* is an enlarged view at the rivet, showing the flange angle thickness as mu, the web thickness as uu', and the rivet diameter as mn. If the change in total flange stress for a length p is taken as δC, $\delta C/2$ is the change in total stress in each flange angle and also the bearing load on one angle or the shear load on one shear plane (ux or $u'x'$). The total load δC bears on the web. The bearing area for each flange angle is the thickness of the angle times the diameter of the rivet. This is single shear bearing (bearing area with a plane of shear on only one side), and according to the 1949 A.I.S.C. specifications the allowable stress is 32,000 lb per sq in. The shearing area for each plane of shear is the cross-sectional area of the rivet. The bearing area for the web is the thickness of the web times the rivet diameter. This is double shear bearing (bearing area with a plane of shear on both sides), and the specifications above permit 40,000 lb per sq in.

In the above discussion it was assumed that all the flexural stress is in the flange. Since the web resists some of the bending, an area equivalent to $A_w/6$ can be assumed, concentrated at the compression flange and acting as a part of the compression flange. This is a proportional part of the change in total flange stress that is already in the web and does not need to be transmitted to it through rivets. The rate of change to be transmitted is $\left(\frac{V}{d} \cdot \frac{A}{A + \frac{A_w}{6}}\right)$, where A is the gross area of the flange exclusive of $A_w/6$.

Therefore, the relation for p is as follows:

$$R = p\left(\frac{V}{d} \cdot \frac{A}{A + \frac{A_w}{6}}\right)$$

$$p = \frac{Rd}{V}\left(\frac{A + \frac{A_w}{6}}{A}\right) \quad \text{or} \quad p = \frac{Rd}{V}\left(1 + \frac{A_w}{6A}\right) \tag{14}$$

For the tension flange with the net area $= A_n$, this is

$$p = \frac{Rd}{V}\left(\frac{A_n + \frac{A_w}{8}}{A_n}\right) \quad \text{or} \quad p = \frac{Rd}{V}\left(1 + \frac{A_w}{8A_n}\right) \tag{15}$$

The values obtained from formulas 14 and 15 differ very little in value but are somewhat larger than the values from formula 13. It is usual to calculate the pitch for one flange only, generally the tension flange, and to have the same pitch in both.

The values of V, d, A, and A_n should be those at the section where p is being computed. No serious error results, however, if the values of d, A, and A_n at the point of maximum moment are used, which is common practice with some designers.

The rivet pitch will be computed for the girder previously designed, first by the simple relation shown in formula 13. The computations will be made for the pitch at the end, the quarter point, and the center, using the shears computed for these points in Art. 37, 192 kips, 107.6 kips, and 46.2 kips, respectively. The following are the allowable loads on a $\frac{7}{8}$-in. rivet.

$$40{,}000 \times \tfrac{5}{16} \times \tfrac{7}{8} = 10.94 \text{ kips (bearing on web)}$$

$$32{,}000 \times \tfrac{5}{8} \times 2 \times \tfrac{7}{8} = 35.0 \text{ kips (bearing on two angles)}$$

$$15{,}000 \times \frac{2\pi(\frac{7}{8})^2}{4} = 18.04 \text{ kips (double shear)}$$

The bearing on the web will govern. These values can be obtained directly from *Steel Construction*. If account is taken of the cover plate lengths as obtained in Art. 36, there will be no cover plate at the end where the effective depth of the girder will be 57.04 in., there will be one cover plate at the quarter point where the effective depth will be 58.34 in., and there will be two cover plates at the center where the effective depth will be 59.04 in.

$$\text{At the end} \qquad p = \frac{10.94 \times 57.04}{192} = 3.25 \text{ in.} \quad (\text{use } 3\tfrac{1}{4} \text{ in.})$$

$$\text{At the quarter point} \qquad p = \frac{10.94 \times 58.34}{107.6} = 5.92 \text{ in.} \quad (\text{use } 5\tfrac{3}{4} \text{ in.})$$

$$\text{At the center} \qquad p = \frac{10.94 \times 59.04}{46.2} = 14.0 \text{ in.} \quad (\text{use } 6\tfrac{1}{4} \text{ in.})$$

$$16 \times \tfrac{5}{8} = 10 \text{ in.} \qquad\qquad 20 \times \tfrac{5}{16} = 6.25 \text{ in.}$$

The A.I.S.C. specifications provide that the maximum pitch be limited to 16 times the thickness of the thinnest outside plate or 20 times the thickness of the thinnest inside plate (including the web) with a maximum of 12 in.

If formula 15, derived for the tension flange, is used:

At the end $\quad p = 3.25\left(1 + \frac{18.75}{8 \times 11.72}\right) = 3.90 \quad$ (use $3\frac{3}{4}$ in.)

At the quarter point $\quad p = 5.92\left(1 + \frac{18.75}{8 \times 17.72}\right) = 6.70 \quad$ (use $6\frac{1}{4}$ in.)

At the center $\quad p = 14.0\left(1 + \frac{18.75}{8 \times 22\ 22}\right) = 15.5 \quad$ (use $6\frac{1}{4}$ in.)

The above computations are based on the areas and effective depth that are obtained where the cover plates have been cut off. If the area

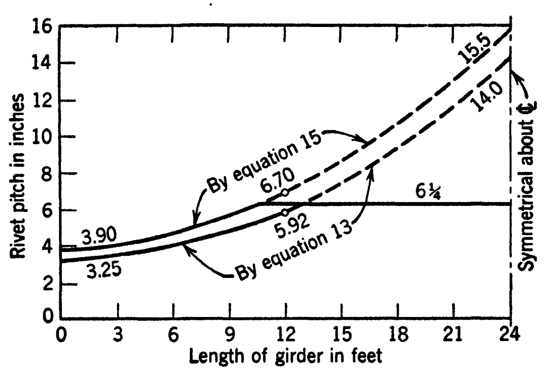


FIG. 33

and effective depth at the point of maximum moment are used, the computations are as follows:

At the end $\quad p = \frac{10.94 \times 59.04}{192}\left(1 + \frac{18.75}{8 \times 22.22}\right) = 3.73 \quad$ (use $3\frac{3}{4}$ in.)

At the quarter point $\quad p = \frac{10.94 \times 59.04}{107.6}\left(1 + \frac{18.75}{8 \times 22.22}\right) = 6.65 \quad$ (use $6\frac{1}{4}$ in.)

At the center $\quad p = \frac{10.94 \times 59.04}{46.2}\left(1 + \frac{18.75}{8 \times 22.22}\right) = 15.5 \quad$ (use $6\frac{1}{4}$ in.)

If a flexural member does not have panels (or in the rare case where the variation of shear within a panel is considerable), the value of pitch should be computed at a sufficient number of points to draw a diagram of maximum rivet pitch such as the diagram shown in Fig. 33 for this girder.

Cover plates are attached to the flange angles by means of rivets. If it is assumed the stress is uniform over the flange, the required pitch will be that necessary to transfer the change in total stress in the cover plates to the angles, or

$$p = \frac{Rd}{V}\left(\frac{A_F}{A_c}\right)$$

where A_c is the area of the cover plates in one flange at the section, and A_F is the total area of that flange at that section. This relation holds true for cover plates that extend the entire length of the girder and also for cover plates shorter than the girder, provided that the plates extend beyond the point of theoretical cutoff and have sufficient rivets at the end to properly develop the stress in the plate. After the stress in the plate is developed, the above relation holds.

As stated in Art. 36, there is some disagreement as to how far a cover plate should extend beyond the point of theoretical cutoff. If the common specification requirement of two rows of rivets at normal pitch is used, there is no doubt but what the end rivets will be greatly overstressed. If the cover plate is extended a sufficient amount to develop in it the average flange stress with rivets at the normal pitch, it will usually require a rather long extension. It can be safely said that in any case where the cover plates are cut off, even if they are extended, some of the end rivets will be overstressed, unless the point of cutoff occurs where the average flange stress is very low. Frequently, cover plates are extended 1 ft or so beyond the theoretical point of cutoff, and the rivets in the cover plates are given the same pitch as those in the vertical legs of the flange angles.

For a discussion of rivet pitch covering these points, the student is referred to *Structural Design in Steel* by Shedd. Particular attention should be given to the footnotes on pages 124, 126, 139, and 141.

The above discussion covers the requirement that sufficient rivets are needed to transmit the change in stress from the flange to the web (or provide for horizontal shear), but the specifications also require that provisions must be made for any vertical load applied directly to the flange. This latter is taken care of by obtaining an expression for p that includes the resultant of the horizontal load on the rivet together with the vertical load. The occasion for these combined loads is rare and will not be further discussed here. A full treatment can be found on page 140 of Shedd's *Structural Design in Steel*, or in other advanced textbooks on steel design.

Cover plate rivets are generally placed symmetrically with respect to the web. For each rivet on the near side there is one on the far side, and the pitch is determined by taking a value of R equal to the capacity of both rivets. Since each rivet is in single shear, the two rivets provide two areas the same as a single rivet in double shear. For thin cover plates the maximum pitch may be limited by the value equal to 16 times the thickness of the thinnest outside plate.

Rivet pitch is measured in common fractions of an inch, usually to the nearest $\frac{1}{4}$ in., although some shops work to the nearest $\frac{1}{8}$ in. In actual fabrication the spacing between rivets is not changed for every $\frac{1}{4}$-in. change in computed rivet pitch but rather at more convenient increments so as to have a constant pitch over a greater length for economy in shop work. Where stiffeners are used, the change in pitch usually occurs at the stiffeners. In order to make the sum of the rivet spaces equal the distance between stiffeners, it is sometimes necessary to have an odd pitch immediately adjacent to the stiffener.

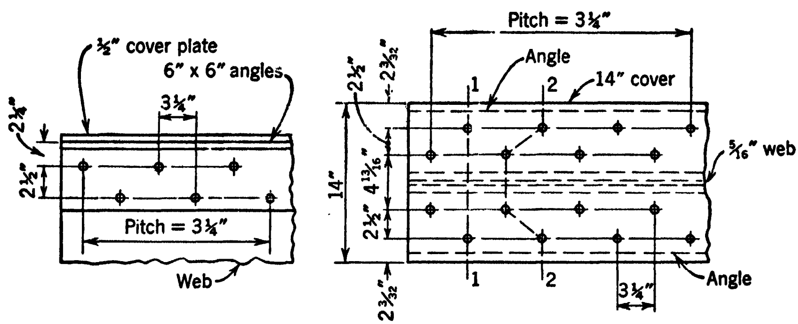
The methods discussed relative to spacing of rivets apply equally to the design of welds in a welded girder. If they are intermittent welds of a given size and length, the treatment is practically the same as the foregoing. A similar treatment can be used for a continuous weld of varying size. Welds will be further discussed in Art. 43 in which a welded girder is designed.

39. Net Width. When the legs of angles are 5 in. or more in width, they may have two lines of rivets (two gage lines); in fact, the A.A.S.H.O. specifications require two lines of rivets in legs of flange angles which have widths of 5 inches or greater. When there are two lines, the rivets are normally staggered. Standard dimensions for the location of these gage lines, or the lines along which the rivets are placed, are given for the various sizes of angles in many handbooks, including *Steel Construction*. These gages are the ones usually used, but any convenient dimension that conforms to the spacing, edge distance, and driving standards is acceptable.

In Fig. 34*a* is shown the elevation of a girder flange, and in Fig. 34*b* is shown the top view. The dimensions shown in the figure are for the girder that has been discussed in the previous articles, where calculations indicate that the rivet pitch near the end is $3\frac{1}{4}$ in. and also that the inside cover plate length can be 33 ft. For exterior construction the first cover plate should extend to the ends of the girder. In the following discussion it is assumed that this plate does have the full length of 48 ft, also that the pitch of the rivets connecting it to the flange angles is the same as that for angle to web ($3\frac{1}{4}$ in.). This figure is for illustrative purposes only. Net width is seldom computed for a compression flange or near the end of a tension flange in a simple beam.

The matter of net section was discussed in Art. 28, but it is now desirable to discuss net area, or net width, when the rivets are staggered, and where, if they are closely spaced, a tensile failure could occur along a zigzag rather than a straight line across the section.

In Fig. 34*b* a section 1–1 straight across the cover plate is shown. The net area of the cover plate, along this section, is the net width of the plate times the thickness; the net width is (for $\frac{7}{8}$-in. rivets or 1-in. holes) 14 in. − 2 in. = 12 in. The specifications provide: "In the case of a chain of holes extending across a part of any diagonal or zigzag line, the net width of the part shall be obtained by deducting from the gross



Elevation of flange
(*a*)

Top view of flange
(*b*)

Fig. 34

width the sum of the diameters of all the holes in the chain, and adding, for each gage space in the chain, the quantity

$$\frac{s^2}{4g}$$

where s = longitudinal spacing (pitch) in inches of any two successive holes.

g = transverse spacing (gage) in inches of the same two holes.

The critical net section of the part is obtained from that chain which gives the least net width."

The above quotation is taken from the A.I.S.C. specifications, but it represents present practice in practically all specifications.

In applying this to section 1–1 in Fig. 34*b*, there are no diagonal or zigzag lines, and therefore only the two holes are deducted, giving a net width of 12 in. as calculated above. If section 2–2 is taken there are four holes and two zigzag lines. The net width is computed as follows:

$$14 - 4 + \frac{(3\frac{1}{4})^2}{4 \times 2\frac{1}{2}} + \frac{(3\frac{1}{4})^2}{4 \times 2\frac{1}{2}} = 12.11 \text{ in.}$$

The critical net width of the cover plate is therefore 12 in. If the pitch had been 3 in. the critical net width would have been

$$14 - 4 + \frac{3^2}{4 \times 2\frac{1}{2}} + \frac{3^2}{4 \times 2\frac{1}{2}} = 11.8 \text{ in.}$$

For angles the width is taken as the sum of the two legs less the thickness of the angle. In the handbook *Steel Construction* is a chart, Net Section of Riveted Tension Members, giving the solution of the expression $s^2/4g$. This expression is also used in calculating the net width of shapes making up a tension member as found in a truss or frame.

It is often desirable to obtain the pitch that can be used and still maintain a net width equal to the width minus the number of rivet holes on a straight section. This can be easily done by means of the chart in the handbook. In the above example it will be found to be between 3 in. and $3\frac{1}{4}$ in.

In designing the girder in Art. 34, the number of holes deducted was that on a straight section through the angles and cover plates. This was done because, at the point of maximum moment, the rivet pitch was the maximum and the zigzag net width would not govern.

40. Bearing Stiffeners. As stated at the end of Art. 37, bearing stiffeners are attached to webs at points where concentrated loads are applied in order to strengthen the webs against buckling or crippling. In Fig. 35*a* the concentrated load is an end reaction. As shown, four end bearing stiffeners distribute the end reaction to the web and prevent excessive crippling stresses in the web. Stiffeners occur in pairs; here, two pairs (total of four angles) are used, but in other cases it can be one to four pairs. The end reaction is a concentrated load and is applied to the girder through a bearing plate. The bearing stiffeners must have sufficient contact area with the outstanding legs of the bottom flange angles to resist the end reaction load without exceeding the allowable bearing stress. The contact area between the milled or fitted end of a stiffener and the horizontal leg of the flange angle is limited to the thickness of the stiffener leg times the length that the stiffener leg extends beyond the fillet of the flange angle, as shown as *e* in Fig. 35*b*. The amount of the stiffener leg that is deducted on account of the fillet in the flange angles is usually $\frac{5}{8}$ in. for 8-in. flange angles, $\frac{1}{2}$ in. for 6-in. or 5-in. angles, and $\frac{3}{8}$ in. for 4-in. or smaller angles. This reaction load which is applied to the outstanding legs of the stiffener angles is distributed to the web by means of the rivets connecting the web and the stiffeners.

The A.I.S.C. specifications require that the bearing stiffeners be investigated as a column having a length equal to $\frac{3}{4}$ of the depth of the girder although many designers use $\frac{1}{2}$ of the girder depth as the length. It is extremely rare that the action as a column will control the design of a bearing stiffener.

There must be sufficient rivets through the bearing stiffeners to transmit the entire load to the web. Where tight fillers are employed (a tight filler being one extending beyond the stiffener angles and having an independent row of outside rivets), there must be enough rivets through

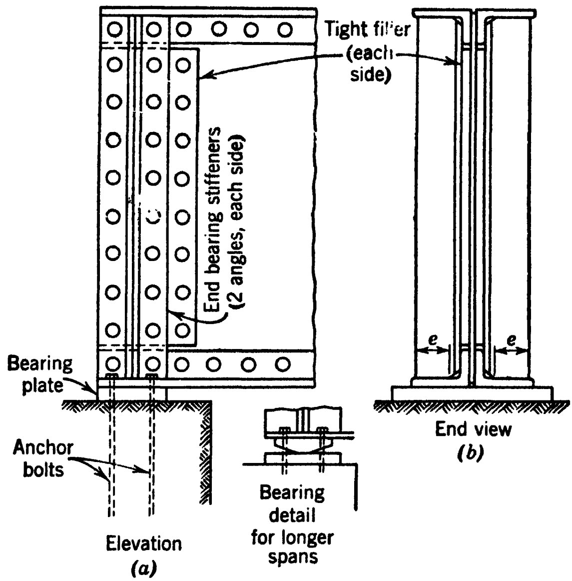


FIG. 35

the stiffener angles to transmit the load by means of double shear, and there must be sufficient rivets through the filler plates (including those through the angles) to transmit the load through bearing on the web. Where the filler is loose, the required number of rivets is increased because of bending in the rivets, and, in general, the number is increased from $\frac{1}{3}$ to $\frac{1}{2}$.

It is assumed that the girder designed in Art. 34 rests on a bearing plate at one end and that the other end frames into a carrying girder by means of connection angles. The bearing stiffeners will be designed to transmit the end reaction of 192 kips into the web. Tight fillers will be used.

It is assumed that the stiffeners will be fitted rather than milled, and the allowable bearing value is therefore 27 kips per sq in. The required contact area is then $192 \div 27 = 7.11$ sq in. If 5-in. bearing angles are used, allowing $\frac{1}{2}$ in. for the fillet of the flange angle, 7.11 ÷ 4.5 = 1.58 in., the total thickness required in the bearing angles. If four ∠s 5 by 3 by $\frac{7}{16}$ in. are used, the area of contact will be $4 \times 4\frac{1}{2} \times \frac{7}{16} = 7.88$ sq in. If $\frac{3}{8}$-in. angles are used, the area will be 6.75 sq in. With $\frac{7}{8}$-in. rivets having a value of 10.94 kips for bearing on a $\frac{5}{16}$-in. web and 18.04 kips in double shear, the total number of rivets required through the filler plate will be 192 ÷ 10.94 = 17.6, or eighteen rivets, and the number required through the angles only will be 192 ÷ 18.04 = 10.6, or eleven rivets.

One arrangement is to have three rows of six rivets each, making eighteen through the filler plate and twelve through the angles. This does not take account of the four rivets at the top and bottom of the stiffener angles that are also in the flange angles. Although these rivets transmit some load from the bearing stiffeners into the web, they are usually not included in the calculations. If account were taken of these four rivets, it would probably be found satisfactory to have three rows of five rivets each, making a total of fifteen rivets between the flange angles.

The specifications provide that the maximum pitch in stiffeners shall be 8 times the diameter of the rivets used. Since the rivets are $\frac{7}{8}$ in. in diameter, this permits a maximum pitch of 7 in. For that reason the bearing stiffeners designed above would require two rows of seven rivets each in the four angles, and for simplicity in shop work the outside row of rivets through the filler plate should also have seven.

Although intermediate stiffeners can be crimped when they are not used as connection angles, bearing stiffeners are never crimped.

If checked in accordance with the A.I.S.C. specifications, the capacity of these bearing stiffeners acting as a column will be 286 kips.

The design of a bearing stiffener occurring under a concentrated load applied to the flange at a point other than an end reaction follows the same procedure. Such stiffeners are designed to have sufficient contact area with the loaded flange, rivets to the web, and column strength to resist the concentrated loads.

41. Beam and Girder Connections. In building construction, the most common method of supporting beams at their ends is by means of connection angles fastening them to columns, other beams, or girders. When beams are supported by girders, advantage is often taken of the fact that the connection angles can serve also as stiffener angles and thus replace a pair of stiffener angles.

In Fig. 36*a* is shown the end view of a beam with the outstanding legs of its connection angles, in Fig. 36*b* an elevation with a cross section of the beam to which it is attached, and in Fig. 36*c* two beams of different size framing into a larger beam. Beams also frame into the flange or webs of columns as shown in Fig. 36*d* and *e*, respectively.

The number of rivets required in the outstanding legs of connection angles is usually determined by the allowable stress in single shear, especially when a beam from only one side frames into another beam or girder, although bearing may govern where the beam frames into a thin web. Where two beams frame into a web at the same place but

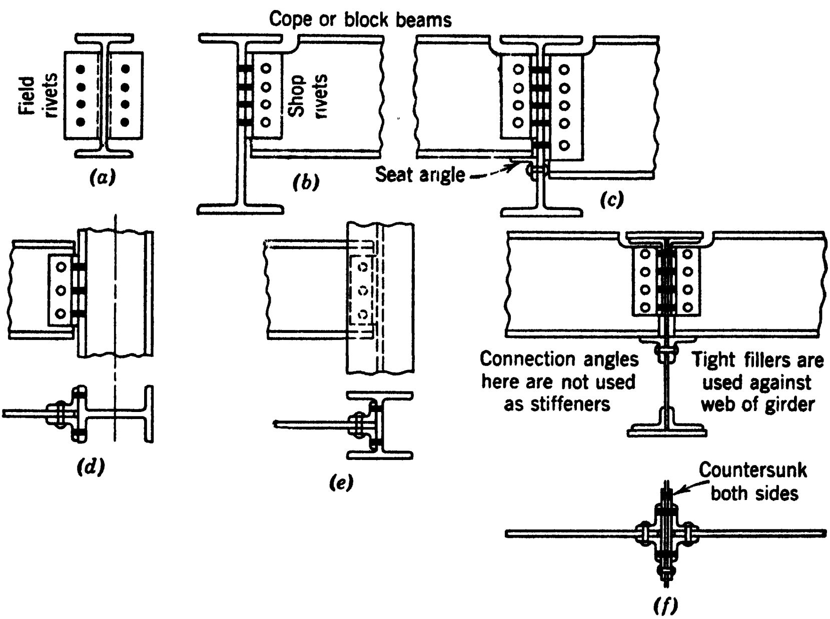


FIG. 36

from opposite sides, it is usually found that bearing on the web governs. The value of one rivet, in single shear or bearing, divided into the value of the end reaction, gives the total number of rivets required in the two outstanding legs. Where beams are framed opposite, as in Fig. 36*c* or *f*, the bearing value is taken for one-half the web into which they frame.

The rivets through the angle legs along the web of the beam, as shown in Fig. 36*b*, are in double shear, and this value, or bearing on the enclosed web, governs.

In connecting rolled beams it is customary to use standard connections in order to eliminate a multiplicity of design and shop work. All fabricating shops keep a stock of standard connections on hand, making it cheaper and easier to use them. Standard connections for the various size beams are shown in *Steel Construction.* They are shown in six series, A, H, HH, B, K, and KK. Series A, H, and HH are for $\frac{7}{8}$-in. rivets, series A for ordinary connections, and series H or HH where connections of greater capacity are desired. Series B, K, and KK are the corresponding series, respectively, for $\frac{3}{4}$-in. rivets. The tables accompanying the standard connections give their series number and size for the various size beams and also give the capacity of the connection in shear or bearing on various thicknesses of web. These standard connections have a uniform pitch and gage for the rivets and therefore can be used when beams of different depth frame opposite, as in Fig. 36*c*.

If $\frac{3}{4}$-in. rivets are used in the connection angles for the beam designed in Art. 25, using the 24-in. I at 90 lb, the web has a thickness of 0.624 in. so that the double shear value of 13.25 kips governs. The end reaction is 60 kips; and $60 \div 13.25 = 4.53$, or five rivets. In the 24 WF 84 the web is thinner (0.470 in.), but double shear still governs, making the computation the same. If it is assumed that the outstanding legs of the connection angles frame into a web (or plate) so thick that single shear governs, then ten rivets will be required in the two outstanding legs.

Turning to the B series of connections in *Steel Construction* it will be found that the B–6 connection for 24-in. I or wide-flange beams has six rivets in the leg of the angles along the web and also in each outstanding leg. The table gives the capacity of this connection in shear as 79.5 kips. If two of these 24-in. beams frame from opposite sides into a $\frac{5}{16}$-in. web, the two connections would be limited to 112.5 kips total, or 56.25 kips per connection, which is less than the 60 kips required. The calculation for twelve rivets in double shear bearing is:

$$40 \times \tfrac{5}{16} \times \tfrac{3}{4} \times 12 = 112.5 \text{ kips}$$

In Fig. 37*a* and *b* is shown how this connection would be made in order to act also as an intermediate stiffener of a girder; in Fig. 37*c* the intermediate stiffeners have been shortened, and a seat angle is attached for ease in erection. Regular standard connection angles are used, and a stiffener angle has been placed below the seat angle on each side of the girder web. The loose fillers are shop-riveted to the girder in Fig. 37*a*, and enough rivets are in the connection angle, which acts also as a stiffener, so that bending in the rivets because of the loose filler is not critical. It is doubtful that a loose filler under the connection angle in Fig. 37*c* is always economical since it is difficult to hold in place in field erection and since additional rivets are sometimes needed because of the bending in the rivet.

Where connection angles are like those above on either beams or girders, they are considered as simple supports as they are unable to resist any appreciable amount of bending moment. For that reason they are not made thick enough to make them unduly stiff. They should be thick enough so that shear will govern, instead of bearing on the angle itself.

The design of connection angles for girders follows the procedure outlined above for beams. For the girder designed in Art. 34, using $\frac{7}{8}$-in.

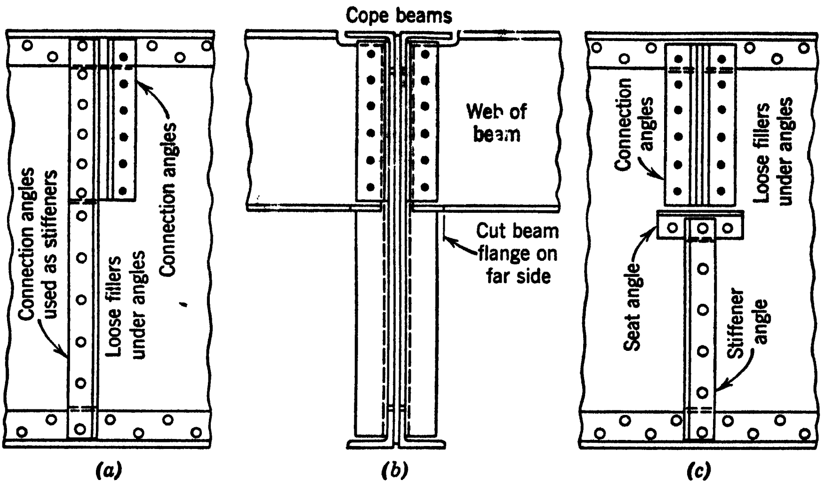


FIG. 37

rivets, the number of rivets required in double shear would be 192 ÷ 18.04 = 10.6, or eleven rivets. Since the web is $\frac{5}{16}$ in. thick it will be necessary to have a tight filler or a larger number of rivets in the connection angles. The number of rivets required in both the angles and filler will be 192 ÷ 10.94 = 17.6, or eighteen rivets.

The rivet lines in the web should be kept to a minimum and be such that all rivet holes can be punched during a single pass through a multiple punch machine. To accomplish these conditions, the gage lines for end connections, stiffener angles, intermediate connections, and web splices should be as similar as possible. If the maximum spacing of 7 in. ($8 \times \frac{7}{8}$) is used for intermediate stiffeners, then a spacing of $3\frac{1}{2}$ in. ($\frac{1}{2} \times 7$) would probably be most economical for this end connection. The connection is shown in Fig. 38. If, in accordance with a rather common practice, the two rivets that also go through the flange angles are not considered, the connection angles have thirteen rivets, or an excess of two more than required. The total for bearing on the web is twenty rivets, and bending in the rivets would not be critical.

If this girder frames into a carrying girder, it will probably be necessary to have a tight filler on the carrying girder in order that the 4-in. leg can be used on the outstanding legs of the connection angles. Then, it is desirable to have one connection angle shop-riveted to the carrying girder, the other being loose, since neither angle can be shop-riveted to this girder. In this case, the end row of rivets would be field rivets.

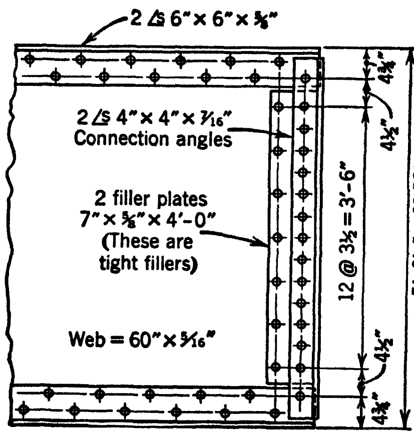


FIG. 38

Seat connections serve sometimes for supporting beams carrying small loads. However, since the more complex connections, such as stiffened seat connections, moment connections, etc., are beyond the scope of this book, the student who is interested in such connections should refer to *Structural Design in Steel* by Shedd or to similar steel design textbooks.

When rolled beams are supported by masonry or reinforced concrete walls as shown in Fig. 39, the length of bearing, a, should be enough to prevent the web from crippling. In *Steel Construction* the following procedure is recommended for computing the bearing capacity of an unstiffened web:

$$\text{Capacity} = 24{,}000t(a + k)$$

where allowable stress = 24,000 lb per sq in. on effective area.
t = web thickness.
k = distance from outside of flange to toe of fillet.
a = length of bearing (bearing plate).

Where a concentrated load is applied at an intermediate point, the bearing capacity of the web is $24{,}000t(a_1 + 2k)$, as shown on the right in Fig. 39. If stiffener angles are used on the web of the rolled beam, the procedure is the same as discussed in Art. 40.

The above method is only approximate and obviously cannot be considered exact. It gives fairly satisfactory results. In *Structural Design in Steel*, Shedd presents a method of computing web crippling which is a little advanced for an elementary textbook such as this but which is preferred by the authors. The interested student is referred to it.

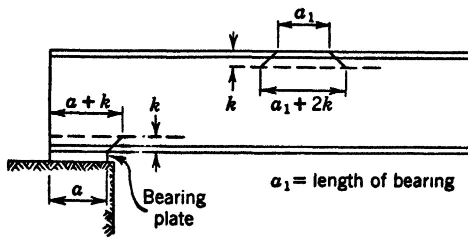


FIG. 39

42. Flange and Web Splices. Normally it is unnecessary to splice the flanges or the web of a plate girder, and this should never be done unless circumstances demand it. Circumstances of erection procedures, the means of shipment, or the available lengths of plates and shapes would be most likely to require splices.

Since angles and cover plates can be obtained in adequate lengths for all but the largest girders, flange splices are seldom encountered. They are necessary in long girders that are continuous over the supports. The specifications usually provide that, where practicable, flanges shall not be spliced at points of greatest moment, and also that, in general, not more than one part shall be spliced at the same cross section.

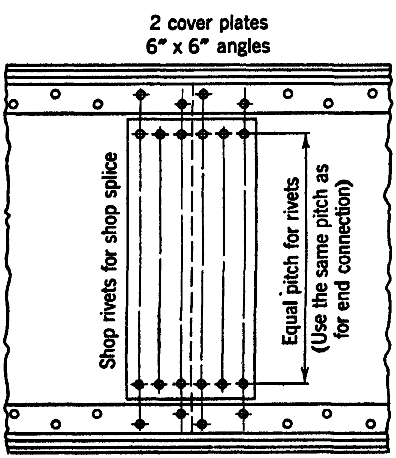


FIG. 40

Web splices are more common than flange splices since, where a deep web is encountered, the plate may not be available in the length needed for the girder. Some mills could not furnish the plate shown in the web of the girder designed in Art. 34. When webs are spliced, the splice should be designed to transmit all the shear and moment that the web is calculated to resist. In other words, the splice should be no weaker than the required web; however, the authors believe that the splice should be equal in strength to the web that is used. The general appearance of a web splice is shown in Fig. 40.

There are two splice plates, one on each side of the web. Some designers proportion these splice plates so that their combined moment of inertia equals that of the web and they can resist the same bending as the web with the same bending stress as in the adjacent portions of the web. This provides more than sufficient area for shear. The rivets have to be sufficient in number to resist both the shearing and bending stresses.

A better way to proportion this splice, preferred by some designers, is to make the combined splice plates equal to or greater than the web and to consider that it replaces that part of the web under the splice plates. The rest of the shear and moment in the web under the flange angles is resisted by an additional plate on the flange or, at points other than maximum moment, by additional area in the flange.

There are other types of web splices, but the second type mentioned above is generally the most satisfactory.

The student is referred to more advanced texts in steel design for a complete treatment of the various types of flange and web splices.

43. Welded Girders. The advantages and disadvantages of riveted fabrication as compared with welded fabrication will not be discussed in detail here. It is sufficient to say that in many cases there is no marked advantage in either riveting or welding, whereas in other cases there is a distinct advantage of one method over the other, depending on many various factors. The design procedure for a welded girder follows the same order as that outlined in previous articles, taking proper cognizance of the different means employed to tie the members together.

The welding symbols shown on drawings for welded structures were developed by the American Welding Society. They are now standardized and are shown in Fig. 41 with the Society's permission. These symbols are used in the figures accompanying this article.

The specifications of the A.I.S.C. (1949) apply to both welded and riveted building structures, and the specifications on welding are in agreement with those of the American Welding Society for similar structures.

The first step in proportioning the web is to make some determination of the depth. The authors believe that, for the average web, a good approximation of the least weight depth can be obtained from the relation

$$d = 5\sqrt[3]{\frac{M}{f}}$$

where there are stiffener plates and a variable thickness of the flange plates. In this expression,

d = the depth of the girder in inches.
M = the maximum bending moment in inch-kips.
f = the allowable intensity of bending stress in the critical flange in kips per square inch.

The value, 5, in the above expression was obtained by following a procedure similar to that followed by Shedd in obtaining the expressions given in Art. 32.

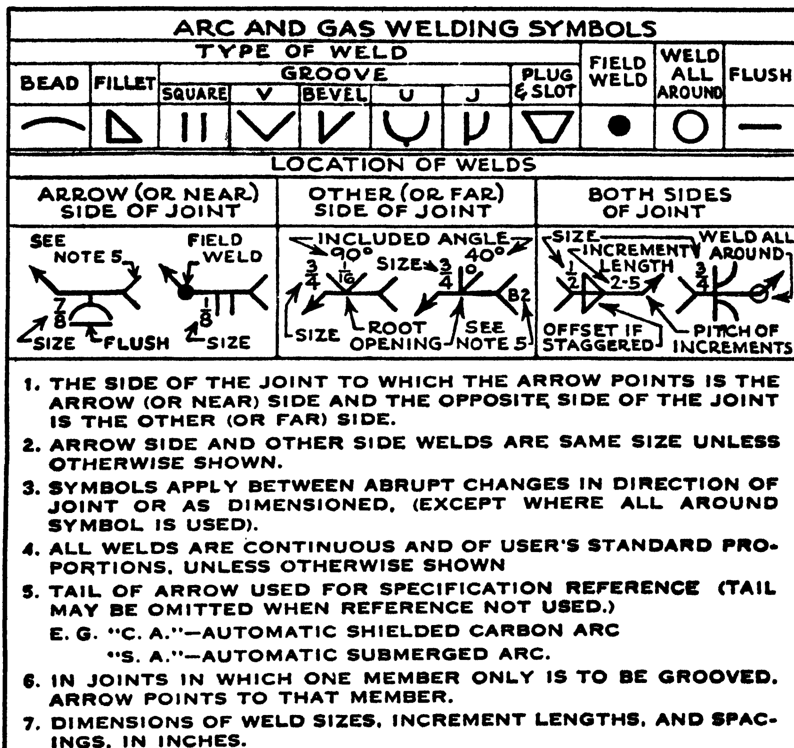


FIG. 41. Taken from "Standard Specifications for Welded Highway and Railway Bridges," American Welding Society, 1947. (Courtesy of American Welding Society.)

The web must have a sufficient area so that the allowable shearing stress is not exceeded, but the thickness is usually determined by the slenderness ratio (h/t). The unsupported distance h between the flanges of a welded girder is the distance between the flange plates and can be taken as the depth of the web, as shown in Fig. 42*a*. Since there are no rivet holes, $\frac{1}{6}$ the area of the web and the gross area of the flange plates are effective as flange area in either the tension or compression flange.

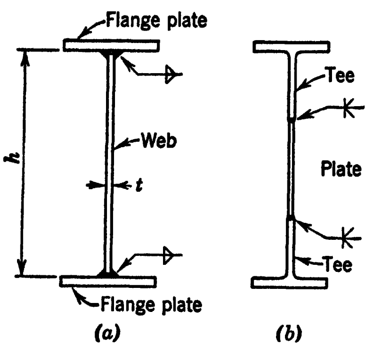


Fig. 42

Welded girders sometimes are composed of two tees butt-welded to the ends of a plate, as shown in Fig. 42*b*. Structural tees are obtained by splitting the webs of wide-flange beams. The strength of this type of girder can be increased by adding one or more cover plates to each flange by means of fillet or plug welds.

The girder for which design computations were made in Art. 34 will be redesigned here as a welded girder.

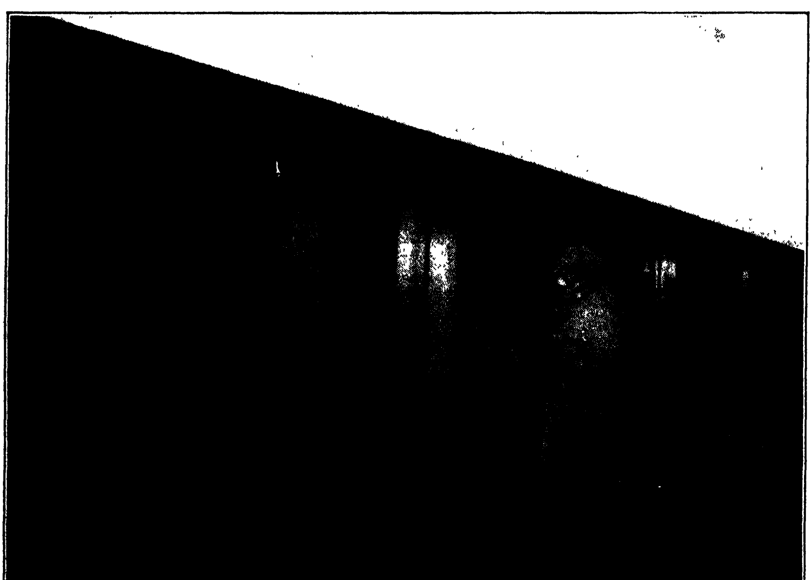


Fig. 43. Welded girder for Housatonic River Bridge, Shelton-Derby, Conn., showing single-flange plates and plate stiffeners. (Courtesy of American Bridge Company.)

With the same span and loads, $M = 2304$ ft-kips, and the maximum end shear = 192 kips.

$$\text{Approximate value of } d = 5\sqrt[3]{\frac{2304 \times 12}{20}} = 55.7 \text{ in.}$$

assuming the compression flange to have lateral support. A depth a few inches greater or less than this value might give a girder with a lesser total weight, but the difference would be small and a complete design of all parts would be necessary for final determination of this point. For these computations a depth of web of 56 in. is assumed.

$$\text{Web area required} = \tfrac{192}{13} = 14.77 \text{ sq in.}$$

$$\text{Minimum thickness based on shear} = \frac{14.77}{56} = 0.264 \text{ in.}$$

$$\text{Minimum thickness based on slenderness ratio} = \tfrac{56}{170} = 0.330 \text{ in.}$$

$$\text{Use a web } 56 \times \tfrac{3}{8} \text{ in.} = 21.00 \text{ sq in.}$$

The overall depth is assumed to be 60 in., and the effective depth 58 in.

$$20 \times \tfrac{58}{60} = 19.33 \text{ kips per sq in.}$$

$$\frac{2304 \times 12}{58 \times 19.33} = 24.6 \text{ sq in. required}$$

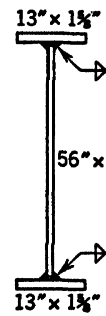


Fig. 44

Trial section (see Fig. 44):

1 web $56 \times \frac{3}{8}$	= 21.00 sq in. gross $\times \frac{1}{6}$ =	3.50 sq in.
1 top plate $13 \times 1\frac{5}{8}$	= 21.13	
1 bottom plate $13 \times 1\frac{5}{8}$	= 21.13	21.13
		24.63 sq in. gross

$$\frac{57.62}{59.25} \times 20 = 19.45 \text{ kips per sq in.} \qquad \frac{2304 \times 12}{57.62 \times 19.45} = 24.67 \text{ sq in.}$$

Although the area supplied is slightly less than that required, the difference is well within allowable limits in design. The student should be cautioned in this connection that the modified flange area method as applied to welded girders is slightly on the unsafe side, since $\frac{1}{6}$ the area of the web is assumed to be acting at the center of the flange, which is always outside the edge of the web. In this instance the area involved is $3.50 \div 24.63 = 0.1421$, or 14.21 per cent, and the difference in moment arm is $(57.62 - 56) \div 56 = 0.029$ or 2.9 per cent. The error involved is $0.1421 \times 0.029 = 0.00412$, or about 0.4 per cent. This percentage is slightly reduced by the fact that the center of stress in the flanges is outside their center of gravity.

Although the moment of inertia of a welded girder is not difficult to compute at various sections, the above method is simple and direct and sufficiently accurate. This method is also convenient for determining the location of butt welds where the flange plate is reduced in thickness.

Whether the saving obtained by reducing the thickness of the flange offsets the cost of the butt welds is an economic problem involving labor costs and other factors. Three flange segments per flange would probably be justified; five might not be. However, for illustrative purposes

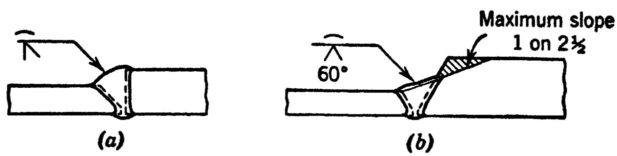


FIG. 45

it will be assumed that there are five flange segments 13 in. wide, two being $\frac{5}{8}$ in. thick, two being $1\frac{1}{8}$ in. thick, and the middle one being $1\frac{5}{8}$ in. thick, as obtained in the above design.

The procedure is the same as for determining the length of cover plates. For unsymmetrical or concentrated loads, the determination can be made graphically. For the uniform load, equation 12 of Art. 36 is more convenient. With this equation, the following values are obtained:

$$L_1 = 48\sqrt{\frac{6.50}{24.63}} = 24.7 \text{ ft} \quad \text{(use 26 ft)}$$

$$L_2 = 48\sqrt{\frac{13.00}{24.63}} = 34.9 \text{ ft} \quad \text{(use 36 ft)}$$

The $\frac{5}{8}$-in. plates are each $\frac{48 - 36}{2} = 6$ ft long

The $1\frac{1}{8}$-in. plates are each $\frac{48 - 12 - 26}{2} = 5$ ft long.

The butt welds between the $\frac{5}{8}$-in. plate and the $1\frac{1}{8}$-in. plate and between the $1\frac{1}{8}$-in. plate and the $1\frac{5}{8}$-in. plate can be single-bevel butt welds, as shown in Fig. 45*a*. The joint is to be welded from both sides, but there is a single bevel on the thinner plate. For a structure designed for repeated loads, a single-V butt joint, welded both sides, should be used with a maximum slope of 1 on $2\frac{1}{2}$, meaning that the thicker plate will probably have to be tapered.

In order to compute the stiffener spacing at any given point in the span, it is necessary to know the maximum shear that can exist at that point. Taking the values in Art. 37 for maximum shear, the unit shearing stresses are as follows:

At the end $\quad V = 192 \text{ kips} \quad v = \dfrac{192{,}000}{21} = 9150 \text{ lb per sq in.}$

At the quarter point $\quad V = 107.6 \text{ kips} \quad v = \dfrac{107{,}600}{21} = 5120 \text{ lb per sq in.}$

At the center $\quad V = 46.2 \text{ kips} \quad v = \dfrac{46{,}200}{21} = 2200 \text{ lb per sq in.}$

$$\frac{64{,}000{,}000}{(56/\frac{3}{8})^2} = 2860 \text{ lb per sq in.}$$

This indicates that stiffeners are not needed at the center of the span; however, the maximum stiffener spacing is computed below for all three points.

At the end $\quad \dfrac{11{,}000 \times \frac{3}{8}}{\sqrt{9150}} = 43.1 \text{ in.}$

At the quarter point $\quad \dfrac{11{,}000 \times \frac{3}{8}}{\sqrt{5120}} = 57.5 \text{ in.}$

At the center $\quad \dfrac{11{,}000 \times \frac{3}{8}}{\sqrt{2200}} = 88.0 \text{ in.}$ (maximum spacing where required = 84 in.)

Although stiffeners are not required at the center of the span, if they are used there the spacing should not exceed the maximum spacing permitted, which is 84 in.

Using a stiffener plate width of 2 in. plus $\frac{1}{30}$ the depth of the girder $= 2 + (59.25 \div 30) = 3.98$ in. (or 4 in.), and a thickness $\frac{1}{16}$ of the width $= \frac{4}{16} = \frac{1}{4}$ in., the stiffener plates will be 4 by $\frac{1}{4}$ in., attached in pairs, one on each side of the web. According to the A.I.S.C. requirement for stiffeners, namely, $I_s = 0.00000016H^4$, a pair of stiffener plates, each $1\frac{7}{8}$ by $\frac{1}{4}$ in., or a single plate, $2\frac{5}{8}$ by $\frac{1}{4}$ in., can be used. The authors are inclined to prefer the 4 by $\frac{1}{4}$ in. plates in pairs as representing better design.

With intermittent fillet welds * of $\frac{1}{4}$-in. size to attach the stiffener plates to the girder web and with the length of each weld as $1\frac{1}{2}$ in., the

* Intermittent fillet welds can be replaced by a smaller continuous fillet weld of equal strength which can be made by an automatic process at a saving in cost; however, when sequence welding is required to prevent shrinkage stresses or distortion, the use of intermittent welds may result in an overall economy.

clear spacing between these welds is limited by 12 in., or 16 times the thickness of the thinner plate joined. In this case, $\frac{1}{4}$-in. intermittent fillet welds, each $1\frac{1}{2}$ in. long, would be spaced at $5\frac{1}{2}$-in. centers, as shown in Fig. 46, giving a clear distance between welds of 4 in. ($16 \times \frac{1}{4}$); however, some specifications permit the welds to be staggered on the two sides of each stiffener plate so that the distance between welds on each side would be 11 in.

The same maximum shears govern in calculating the attachment of the flange plate to the $\frac{3}{8}$-in. web as were used for spacing the stiffeners.

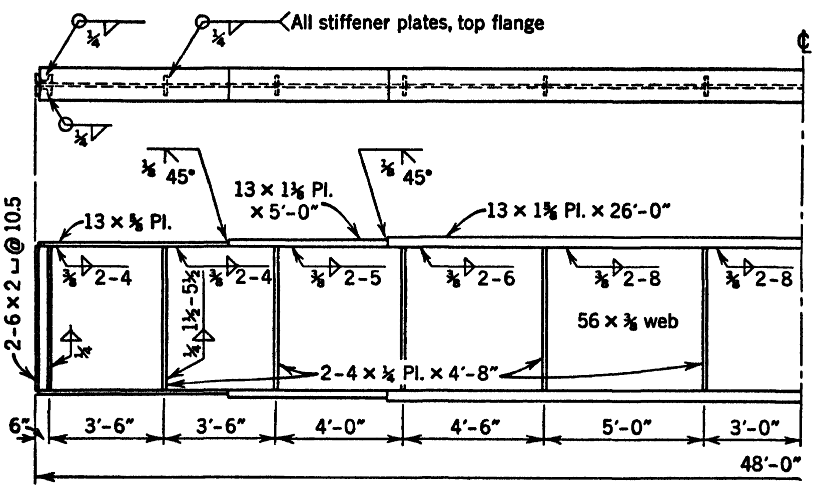


FIG. 46

The allowable stress for shear on a section through the throat of a fillet weld is 13,600 lb per sq in. For a 45-deg fillet weld the throat dimension is 0.707 times the fillet size; therefore the capacity of a $\frac{1}{8}$-in. fillet weld is $13{,}600 \times 0.707 \times \frac{1}{8} = 1200$ lb, and for any fillet weld it is 1200 lb per $\frac{1}{8}$ in. of size.

Specifications indicate the minimum size of fillet weld to be used on a given thickness of material. For a $1\frac{5}{8}$-in. plate, the minimum size for fillet welds is $\frac{3}{8}$ in. The minimum length of fillet welds is $1\frac{1}{2}$ in., or 4 times the nominal size. The capacity of a $\frac{3}{8}$-in. fillet weld is $3 \times 1200 = 3600$ lb per in. of length, and if $\frac{3}{8}$-in. fillet welds are placed on each side of the web the capacity is 7200 lb per in. of length. The capacity of the $\frac{3}{8}$-in. web in shear, however, is only $\frac{3}{8} \times 13{,}000 = 4880$ lb per in. and would control. If the fillet welds are intermittent and staggered, the 3600 lb per in. controls.

Using $\frac{3}{8}$-in. intermittent fillet welds on each side of the $\frac{3}{8}$-in. girder web and opposite, the capacity of each 2-in. segment is 4880 × 2 = 9760 lb. If equation 14 of Art. 38 is used (R in the equation being 9760 lb) the following spacing between the centers of these 2 in. segments is obtained:

At the end $$p = \frac{9760 \times 56.62}{192{,}000}\left(1 + \frac{3.50}{8.13}\right) = 4.12 \text{ in.} \quad \text{(use 4 in.)}$$

At the quarter point $$p = \frac{9760 \times 57.62}{107{,}600}\left(1 + \frac{3.50}{21.13}\right) = 6.10 \text{ in.} \quad \text{(use 6 in.)}$$

At the center $$p = \frac{9760 \times 57.62}{46{,}200}\left(1 + \frac{3.50}{21.13}\right) = 14.2 \text{ in.} \quad \text{(use 8 in.)}$$

At the center the maximum clear distance between segments is limited to 6 in. (16 × $\frac{3}{8}$), and therefore the center-to-center distance cannot be over 8 in.

FIG. 47. Erection view of ramp-type garage, Cincinnati, Ohio, showing beam and girder construction. (Courtesy of American Bridge Company.)

44. Illustrative Design of a Typical Floor Panel. Figure 48 is a line diagram of the plan of the second-floor framing for a warehouse building. The structural floor consists of a reinforced concrete slab supported on rolled steel beams which frame into the webs of steel plate girders. The interior columns are spaced longitudinally at 54-ft intervals which is therefore the span of the carrying girders. Although the spacing of interior columns at 27-ft intervals would result in a saving in total steel weight, the authors have assumed that the purpose of the building makes a 54-ft spacing desirable along this line of columns; they also desire to bring into this illustrative design a larger girder to proportion

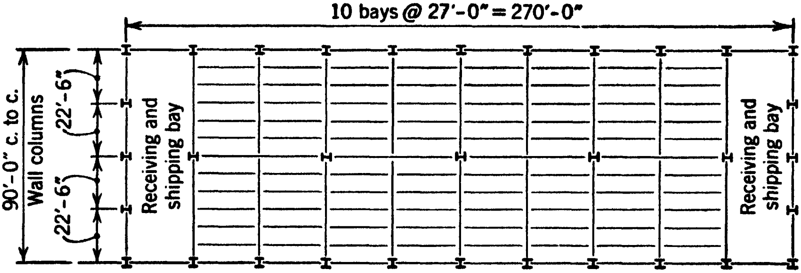


FIG. 48

and one with a different loading condition. The reinforced concrete slab has a wearing surface consisting of wood blocks, 3 in. thick, resting on a sand cushion $\frac{1}{2}$ in. thick.

The design will be based on the 1949 A.I.S.C. specifications (with modifications *) and a live load of 360 lb per sq ft. It is assumed that the nature of the stored material and the manner in which it is handled is such that the effect of impact can be neglected.

Slab. In order to proceed with the design of the steel beams and girders it is necessary to know the weight of the slab. The design of reinforced concrete members will not be discussed until Chapter 5. The method given here of estimating the weight of the slab is based on a relationship involving a factor, R, that will not be developed until that chapter.

The distance between the beams is 7 ft 6 in., and the span of the slab will therefore be taken as that amount. The slab is continuous over the beam supports, and for an interior panel the maximum moment at the support due to live load will be assumed equal to $\frac{1}{8}\, w_L L^2$, and that due to dead load as $\frac{1}{12}\, w_D L^2$.

* The modifications are those applying to net section and compression flanges without continuous lateral support. These points were discussed previously in this chapter.

If the slab is assumed to be 5 in. thick and the wood block and sand cushion to weigh 20 lb per sq ft of floor area, the dead load is:

$$\begin{aligned} \tfrac{5}{12} \times 150 &= 63 \\ &\ 20 \\ &\ \overline{83 \text{ lb per sq ft}} \end{aligned}$$

Therefore:

$$\begin{aligned} \tfrac{1}{12} \times 83 \times \overline{7.5}^2 &= 388 \\ \tfrac{1}{8} \times 360 \times \overline{7.5}^2 &= 2250 \\ \text{Moment at support} &= 2638 \text{ ft-lb} \end{aligned}$$

This is the bending moment on a strip of slab 12 in. or 1 ft in width. The slab is proportioned on the basis of the moment on a typical strip.

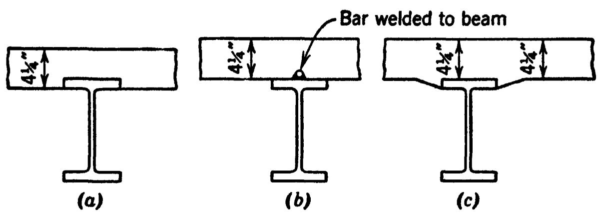


Fig. 49

The relation mentioned above is $M = Rbd^2$, in which R is a constant for given permissible unit stresses, and b and d are the width and depth of the beam, respectively, d being the distance from the compressive face to the center of the tension steel. For concrete having an ultimate strength of 3000 lb per sq in. at 28 days and an allowable unit stress of 1350 lb per sq in., and, allowing a unit stress of 18,000 lb per sq in. in the steel, the value of R is 248 lb per sq in. Then

$$d = \sqrt{\frac{M}{Rb}}$$

and, using inch units,

$$d = \sqrt{\frac{2638 \times 12}{248 \times 12}} = 3.26 \text{ in.} \quad (\text{use } 3\tfrac{1}{4} \text{ in.})$$

If one additional inch of concrete is used to provide protective covering for the steel, the overall depth of the slab is $3\frac{1}{4} + 1 = 4\frac{1}{4}$ in.

Beams. The continuous slab is constructed in such a manner that it gives lateral support to the compression flanges of the interior beams as shown in Fig. 49*a*, *b*, or *c*. The wall beams in the receiving and shipping bays, however, have no lateral support between the end connections.

Floor cover =	20 lb per sq ft
$4\frac{1}{4}$-in. slab =	53
Live load =	360
	433 lb per sq ft
$\times$ 7.5 =	3250 lb per ft of beam
Assumed weight of beam =	100
	3350 lb per ft of beam

$$\tfrac{1}{8} \times 3.35 \times 27^2 = 305 \text{ ft-kips} = 3660 \text{ in.-kips}$$

$$\frac{M}{f} = \frac{3660}{20} = 183 \text{ in.}^3 = S$$

Any of the following are satisfactory for interior floorbeams:

24 WF 84	$S = 196.3$ in.^{3}
24 I 90	$S = 185.8$ in.^{3}
18 WF 96	$S = 184.4$ in.^{3}

If headroom on the first floor is controlled by the girder depth, the lightest weight beam (24 WF 84) will probably be preferred, although the price per pound might give the 90-lb standard I a slight cost advantage. The 84-lb beam will be chosen here.

The load on the wall beam in the receiving and shipping bay will be its own weight and the weight of the wall. The total weight is assumed to be 500 lb per ft of beam.

$$M = \tfrac{1}{8} \times 0.500 \times \overline{27}^2 = 45.6 \text{ ft-kips for the 27-ft span}$$

The allowable compression stress is:

$$f = \frac{12{,}000{,}000}{\dfrac{L \times d}{b \times t}}$$

Trying a 10 WF 39,

$$\frac{d}{b \times t} = 2.36 \qquad \frac{Ld}{bt} = 27 \times 12 \times 2.36 = 764$$

$$f = \frac{12{,}000{,}000}{764} = 15{,}720 \text{ lb per sq in.} = 15.72 \text{ kips per sq in.}$$

$$\frac{45.6 \times 12}{15.72} = 34.8 \text{ in.}^3 \qquad S \text{ for beam} = 42.2 \text{ in.}^3$$

$$\Delta \text{ (at center)} = \frac{5}{384}\frac{WL^4}{EI} = \frac{5}{48}\frac{ML^2}{EI} = \frac{5}{48}\frac{45.6 \times 12 \times 27^2 \times 144}{29{,}000 \times 209.7} = 0.98 \text{ in.}$$

(The A.I.S.C. manual, *Steel Construction*, recommends $E = 29{,}000{,}000$ lb per sq in.)

$$\frac{\Delta}{L} = \frac{0.98}{27 \times 12} = 0.00303 = \frac{1}{330}$$

If it is desired to limit this ratio to 1 in 360 (depending on the likelihood of cracking in the wall), a slightly stiffer beam should be selected.

Girder. The beams frame into the interior girders from both sides. The girder loads, the shear and moment diagrams, and the necessary calculations for a typical interior girder are shown in Fig. 50. The

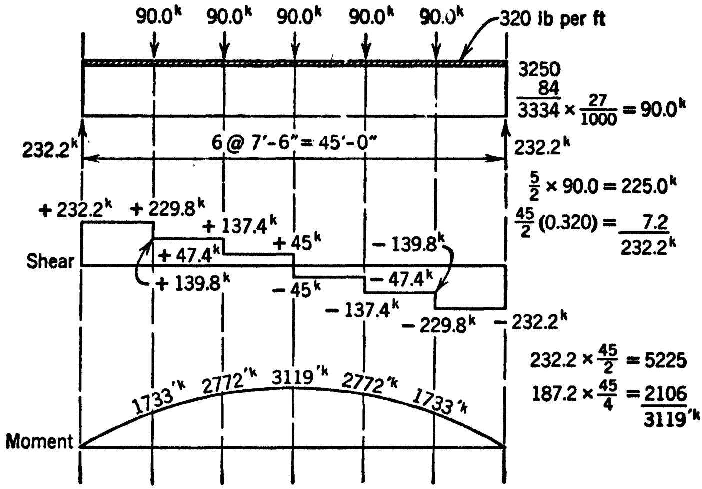


FIG. 50

girder is assumed to weigh 320 lb per ft. The top flange has lateral support provided by the beams.

$$\text{Least weight depth} = 5.5\sqrt[3]{\frac{3119 \times 12}{20}} = 67.8 \text{ in.}$$

$$\text{Required area of web} = \frac{232.2}{13} = 17.86 \text{ sq in.}$$

Using 6 by 6 in. angles and a 68-in. web:

$$\frac{17.86}{68.0} = 0.263 \text{ in.} \qquad \frac{68.5 - 2 \times 6}{170} = 0.333 \text{ in.}$$

A $\frac{3}{8}$-in. web is required.

With a 64-in. web, $\frac{64.5 - 2 \times 6}{170} = 0.309$ in.

or a $\frac{5}{16}$-in. web would be adequate.

$$64 \times \tfrac{5}{16} = 20.0 \text{ sq in.}$$

A larger number of rivets is required with the thinner web, but the 64-in. web is the more economical. Assume the effective depth to be 64 in.

Assume $(d_1/d_1') \times 20 = 18.5$ kips per sq in.

$$\frac{3119 \times 12}{64} = 585 \text{ kips} \qquad \frac{585}{18.5} = 31.62 \text{ sq in. net}$$

Using $\frac{7}{8}$-in. rivets and deducting 1-in. holes for net section, the following section will be obtained:

1 web 64 × $\frac{5}{16}$ = 20.00 sq in. × $\frac{1}{6}$	= 3.33 sq in. gross × $\frac{1}{8}$		= 2.50 sq in. net
2 bottom ∠s 6 × 6 × $\frac{3}{4}$	= 16.88	− 3.00	= 13.88
1 bottom plate 16 × $\frac{9}{16}$	= 9.00	− 1.13	= 7.87
1 bottom plate 16 × $\frac{1}{2}$	= 8.00	− 1.00	= 7.00
	37.21 sq in. gross		31.25 sq in. net

The girder section is shown in Fig. 51, and since the compression flange cannot be smaller than the tension flange the sections will be

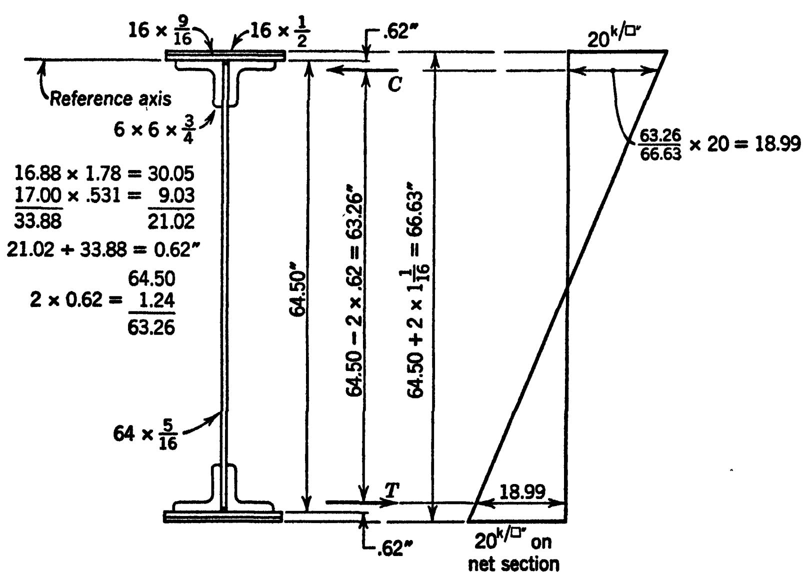


FIG. 51

the same. Figure 51 also shows the calculations for the location of the center of gravity of the flanges, the effective depth, and the allowable stress at the center of gravity of the tension flange.

Checking the weight of the section:

Top flange gross area (average)	= 28.75 sq in.	(net area at maximum section)
Bottom flange gross area	= 28.75	
Web gross area	= 20.00	
Details—60% of web (approximate)	= 12.00	
	89.50 sq in.	

at 3.4 lb per ft = 305 lb per ft

This is less than the 320 lb per ft assumed, but correcting for the smaller weight will have only a negligible effect on the design. From the previously computed moment:

$$\frac{3119 \times 12}{63.26} = 592 \quad \div 18.99 = 31.17 \text{ sq in. net, required area}$$

The net area in the above section is 31.25 sq in. This is so close to the required area that no check * is necessary to determine whether a $\frac{1}{16}$-in. thinner cover plate would be satisfactory even if the smaller dead load weight were used.

In Fig. 52 are shown sketches of the top flange at the points where there are no cover plates, where there is one cover plate, and where

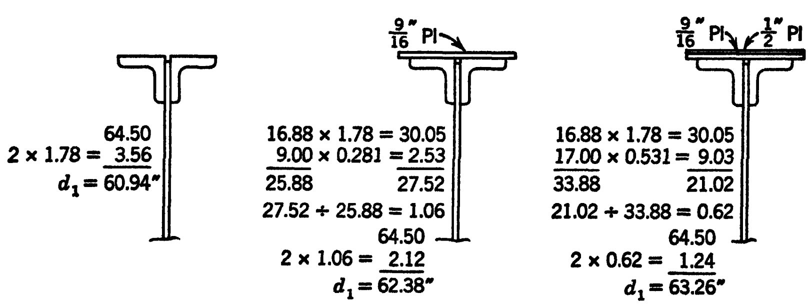


Fig. 52

there are two. Also are shown the calculations for the location of the centers of gravity for these three conditions and the corresponding effective depths.

Cover Plate Lengths. When a uniform load is applied to several panel points, the resulting moment curve is almost identical with a parabola,

* The student should make this calculation in order to satisfy himself that for the same moment a change in cover plate thickness of $\frac{1}{16}$ in. has practically no effect on the allowable unit stress and therefore will not modify the required area; also that the change in dead load makes no significant change in the design moment.

since the value of the moments at the panel points all lie in the arc of a parabola and are the same as the corresponding values for a span without floorbeams. Therefore formula 12 in Art. 36 is the most convenient and direct way to determine the theoretical lengths of cover plates.

For the outside plate,

$$L_1 = 45\sqrt{\frac{7.00}{31.25}} = 21.3 \text{ ft} \quad (\text{use } 24 \text{ ft})$$

For the inside plate,

$$L_2 = 45\sqrt{\frac{14.87}{31.25}} = 31.0 \text{ ft} \quad (\text{use } 33 \text{ ft})$$

Curve of Maximum Shears. In Fig. 53 are shown the influence lines for shear for the panels of the girder, the dead loads, the shear diagram for dead loads, and the curve (or diagram) of maximum shears. The weight of the girder is taken as 305 lb per ft rather than the 320 lb per ft as originally assumed. The live load = 360 × 27 = 9.72 kips per ft of girder. The student will recall from his previous work with influence lines that for maximum live load shear in panel 0–1 the entire span is loaded, for the maximum in panel 1–2 the load extends from C to B, and for the maximum in 2–3 it extends from D to B. The live load shears are:

Panel 0–1 $\frac{1}{2} \times \frac{5}{6} \times 45 \times 9.72 = 182.3$ kips

Panel 1–2 $\frac{1}{2} \times \frac{2}{3} \times 36 \times 9.72 = 116.6$ kips

Panel 2–3 $\frac{1}{2} \times \frac{1}{2} \times 27 \times 9.72 = 65.6$ kips

Rivet Pitch. For $\frac{7}{8}$-in. rivets, the following values are obtained from the table of rivet values in the handbook:

Bearing on $\frac{5}{16}$-in. web at 40,000 lb per sq in. = 10.9 kips.
Bearing on angles at 32,000 lb per sq in. = 42 kips.
Double shear = 18.04 kips.
R = 10.9 kips.

Panels 0–1 and 5–6:

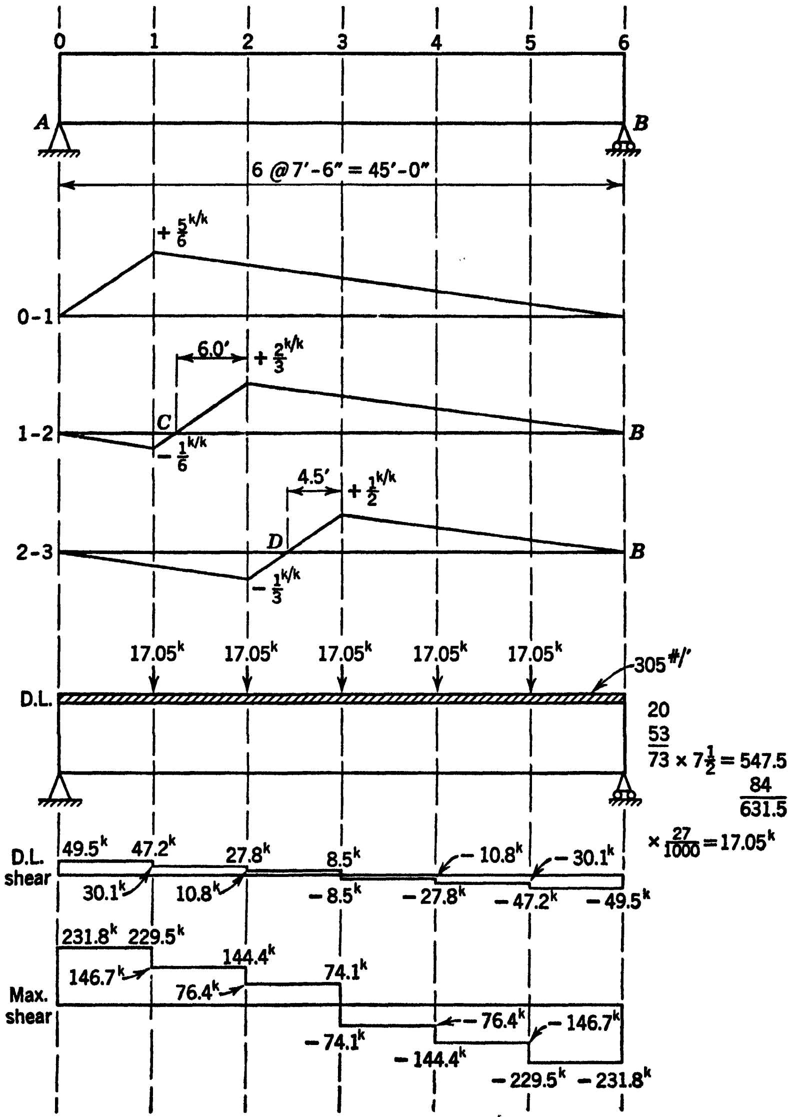
d = 60.94 in. with no cover plate

d = 62.38 in. with 16 × $\frac{9}{16}$-in. plate

$$p = \frac{10.9 \times 60.94}{231.8}\left(1 + \frac{2.50}{13.88}\right) = 2.86 \times 1.18 = 3.37 \text{ in.}$$

$$p = \frac{10.9 \times 62.38}{231.8}\left(1 + \frac{2.50}{21.75}\right) = 2.93 \times 1.115 = 3.27 \text{ in.}$$

Use $3\frac{1}{4}$-in. pitch in panel.



0 1 2 3 4 5 6
A
B
6 @7'-6" = 45'-0"
+ 5/6 k/k
0-1
6.0'
+ 2/3 k/k
C
1-2
− 1/6 k/k
4.5'
+ 1/2 k/k
D
2-3
− 1/3 k/k
17.05k 17.05k 17.05k 17.05k 17.05k
305#/'
D.L.
20
53
73 × 7½ = 547.5
84
631.5
× 27/1000 = 17.05k
D.L. shear
49.5k 47.2k 27.8k 8.5k −10.8k −30.1k
30.1k 10.8k −8.5k −27.8k −47.2k −49.5k
Max. shear
231.8k 229.5k 144.4k 74.1k
146.7k 76.4k −76.4k −146.7k
−74.1k −144.4k −229.5k −231.8k

Fig. 53

Panels 1–2 and 4–5:

$$p = \frac{10.9 \times 62.38}{146.7}\left(1 + \frac{2.50}{21.75}\right) = 4.63 \times 1.115 = 5.16 \text{ in.}$$

$$p = \frac{10.9 \times 63.26}{146.7}\left(1 + \frac{2.50}{28.75}\right) = 4.70 \times 1.087 = 5.11 \text{ in.}$$

Use 5-in. pitch in panel.

Panels 2–3 and 3–4:

$$p = \frac{10.9 \times 63.26}{76.4}\left(1 + \frac{2.50}{28.75}\right) = 9.02 \times 1.087 = 9.81 \text{ in.}$$

$$20 \times \tfrac{5}{16} = 6.25 \text{ in.}$$

$$16 \times \tfrac{9}{16} = 9 \text{ in.}$$

Use $6\frac{1}{4}$-in. pitch in panel.

The same pitch will be used in the cover plates as in the vertical legs of the flange angles. Cover plate thickness will not control, since $16 \times \frac{1}{2} = 8$ in.

Stiffeners. Following the recommended practice given in Art. 37, intermediate stiffeners will consist of 5 by $3\frac{1}{2}$ by $\frac{5}{16}$ in. angles,* placed in pairs on opposite sides of the web.

$$\frac{64{,}000{,}000}{\left(\frac{52.5}{0.3125}\right)^2} = 2270 \qquad \frac{76.4}{20} = 3820 \text{ lb per sq in.}$$

It is seen that intermediate stiffeners are required in all panels.

Panel 0–1:

$$\frac{231.8}{20} = 11{,}590 \text{ lb per sq in.}$$

$$\frac{11{,}000 \times \frac{5}{16}}{\sqrt{11{,}590}} = 31.8 \text{ in.}$$

Panel 1–2:

$$\frac{146.7}{20} = 7335 \text{ lb per sq in.}$$

$$\frac{11{,}000 \times \frac{5}{16}}{\sqrt{7335}} = 40.1 \text{ in.}$$

Panel 2–3:

$$\frac{76.4}{20} = 3820 \text{ lb per sq in.}$$

$$\frac{11{,}000 \times \frac{5}{16}}{\sqrt{3820}} = 55.5 \text{ in.}$$

* This size for stiffener angles is more than adequate to meet the A.I.S.C. specifications that $I_s = 0.00000016H^4$.

The center-to-center spacing of beams framing into the girder is 7 ft 6 in., or 90 in. If it is desired to have stiffeners at all connections, the spacing is as follows: 6 at 30 in., 4 at 45 in., and 6 at 30 in., a total of 15 intermediate stiffeners. On the basis of the computations above, the following could be used: 3 at 30 in., 2 at 40 in., 4 at 50 in., 2 at 40 in., and 3 at 30 in., a total of 13 intermediate stiffeners. In the latter spacing, two of the beams are connected apart from the stiffeners, necessitating separate connection angles. These two connections would be fairly close to stiffener angles and might give difficulty in erection, and the authors prefer the first spacing given. There should be little difference in the cost of the two arrangements.

Connections. Figure 37 illustrates two methods by which the beams can be connected to the girder. For the full-length stiffener angle shown in Fig. 37*a* and *b*, a special connection angle with a 5-in. outstanding leg is used. This connection needs to resist the total load of 90 kips, or 45 kips on each side of the girder. If the connection is as shown in Fig. 37*c*, standard connection angles can be placed above the seat angle, and the erection will be somewhat easier. The authors normally favor this latter connection.

The girder connection, either to the column or the carrying girder, should be designed for an end reaction of 231.8 kips, following the procedure given in Art. 41.

Carrying Girder. Only sufficient computations will be given as are necessary to determine the section.

Uniform load on girder:

$$\begin{array}{r}
360 \\
20 \\
\underline{53} \\
433 \times 7.5 = 3250
\end{array}$$

$$\begin{array}{rrlll}
 & & & V & M \\
\text{Estimated weight} = & \underline{590} & & & \\
 & 3840.0 & \times \frac{54}{2} & = 103.7 \text{ kips} & \\
 & & \times \frac{54^2}{8} & & = 1400 \text{ ft-kips} \\
231.8 \times 2 = & 463.6 & \times \frac{1}{2} & = 231.8 & \\
 & & \times \frac{54}{4} & & = \underline{6260} \\
 & & & 335.5 \text{ kips} & 7660 \text{ ft-kips}
\end{array}$$

$$335.5 \div 13 = 25.81 \text{ sq in.} \quad \text{(minimum web area)}$$

$$5.5 \sqrt[3]{\frac{7660 \times 12}{20}} = 91.4 \text{ in.} = \text{least weight depth}$$

A 90-in. web is a convenient value, but it requires a $\frac{7}{16}$-in. thickness, even if the flange angles have 8-in. legs. As a matter of fact, a $\frac{7}{16}$-in. web barely meets the $\frac{1}{170}$ requirement. It is generally found that, where a thinner web can be obtained by a few inches further decrease in depth, the girder should be no heavier and possibly lighter. In this case a shallower girder will give more headroom. With 8 by 8 in. angles, a 79-in. or 78-in. web with a thickness of $\frac{3}{8}$ in. is satisfactory, although this girder may be slightly heavier.

For a depth of web = 78 in.:

$$\begin{array}{l} 78.5 \\ 16 \\ \hline 62.5 \div 170 = 0.368 \text{ in. (use } \tfrac{3}{8} \text{ in.)} \end{array}$$

$$\frac{78.4}{83.0} \times 20 = 18.89 \text{ kips per sq in.} \qquad \frac{7660 \times 12}{78.4 \times 18.89} = 62.07 \text{ sq in. net}$$

Using $\frac{7}{8}$-in. rivets:

1 web 78 × $\frac{3}{8}$ = 29.25 sq in. × $\frac{1}{6}$	= 4.88 sq in. gross × $\frac{3}{4}$	=	3.66 sq in. net
2 ∠ 8 × 8 × $\frac{3}{4}$	= 22.88	− 3.00 =	19.88
2 plates 20 × $\frac{3}{4}$	= 30.00	− 3.00 =	27.00
1 plate 20 × $\frac{11}{16}$	= 13.75	− 1.38 =	12.37
	71.51 sq in. gross		62.91 sq in. net

$$\begin{array}{lr} 22.88 \times 2.28 = & 52.17 \\ 43.75 \times 1.094 = & 47.86 \\ \hline 66.63 &)\ 4.31 \\ & \overline{0.065 \text{ in.}} \end{array} \qquad \begin{array}{rr} & 78.5 \\ 2 \times 0.065 = & 0.13 \\ \hline & 78.37 \text{ in.} \end{array} \qquad \begin{array}{r} 78.5 \\ 4.375 \\ \hline 82.875 \text{ in.} \end{array}$$

$$\frac{78.37}{82.88} \times 20 = 18.91 \text{ kips per sq in.} \qquad \frac{7660 \times 12}{78.37 \times 18.91} = 62.02 \text{ sq in. net}$$

This is satisfactory. Decreasing the outside cover plate from 20 by $\frac{11}{16}$ to 20 by $\frac{5}{8}$ provides a total net area of 61.79 sq in., and the required area again is 62.07 sq in. net. This section is also satisfactory.

The carrying girder was designed, as to section, so as to illustrate the proportioning of a heavier girder flange. The calculations for cover plate cutoff, rivet pitch, and stiffener spacing have not been made, as they would illustrate no new principles. It should be noted that, since the moment diagram consists of two parabolic arcs meeting at the center of the span with a large angle between their slopes, the graphic method illustrated in Fig. 28 should be used in determining the lengths of cover plates. Also, since the change in shear on the half span is less than

$\frac{1}{3}$ of the end shear, there will not be a great difference in the rivet pitch or stiffener spacing at the end and center, nor many changes in their values. The student should complete the design of all parts of this girder.

PROBLEMS

In the following problems, unless otherwise specified, the allowable stresses for steel members are those of the latest "Specification for the Design, Fabrication and Erection of Structural Steel for Buildings" of the American Institute of Steel Construction.

1. For a simple span of 30 ft, a rolled steel beam is to be used. The live load is 4 kips per ft of beam. There is no impact, and the dead load is 500 lb per ft of beam, plus the beam weight. The compression flange has continuous lateral support. Select a rolled beam and indicate by calculations that it is adequate.

2. A short heavily loaded beam has a span of 6 ft and a total load (including its own weight) of 30 kips per ft. This is a simple span, and the compression flange has lateral support. Select a wide-flange beam that is adequate; select also an American Standard beam. Show calculations.

3. From the data of Problem 1, except that the compression flange has lateral support at the ends only, select a rolled beam that satisfies the requirements, using as an allowable stress the following expressions:

(*a*) $$f = 18{,}000 - 5\left(\frac{L^2}{b^2}\right)$$ (A.R.E.A., 1947; A.A.S.H.O., 1949)

(*b*) $$f = \frac{22{,}500}{1 + \dfrac{L^2}{1800b^2}}$$ (A.I.S.C., fourth edition)

(*c*) $$f = \frac{12{,}000{,}000}{(Ld/bt)}$$ (A.I.S.C., fifth edition)

4. In Art. 30 alternate designs for a crane beam are presented as an illustrative example. Using the data from this example, design a crane beam for a span of 40 ft (*a*) as a rolled wide-flange section, and (*b*) as a rolled beam with a channel added to the top flange.

5. A girder has a span of 42 ft and is loaded with 7.5 kips per ft (not including the weight of the girder). (*a*) Assuming that the compression flange has lateral support, what is the rolled beam of least weight that would be satisfactory? (*b*) Assuming that the compression flange has lateral support, determine the section required for a plate girder with a web 54 in. deep and $\frac{7}{8}$-in. rivets. (*c*) Assuming that the compression flange has lateral support at the ends only, determine the section required for a plate girder with a web 54 in. deep and $\frac{7}{8}$-in. rivets.

6. If, in Problem 5*b*, 6 by 6 by $\frac{5}{8}$ in. flange angles are used, a cover plate is required. What is the length required for this cover plate?

7. An illustrative design of a typical floor panel is shown in Art. 44. From the data given, what is the rivet pitch at the end of the carrying girder?

8. Redesign the girder in Art. 44 as a welded plate girder.

REFERENCES

T. C. Shedd, *Structural Design in Steel*, John Wiley and Sons, 1934.

H. Sutherland and H. L. Bowman, *Structural Design*, John Wiley and Sons, 1938.

C. D. Williams and E. C. Harris, *Structural Design in Metals*, The Ronald Press Company, 1949.

L. E. Grinter, *Design of Modern Steel Structures*, The Macmillan Company, 1941.

G. A. Hool and W. S. Kinne, *Steel and Timber Structures*, McGraw-Hill Book Company, Second Edition. Revised by the late R. R. Zipprodt and D. M. Griffith, 1942.

C. T. Bishop, *Structural Drafting*, John Wiley and Sons, 1941.

CHAPTER 4

ELEMENTARY DESIGN OF STEEL COLUMNS WITH BENDING

45. Design of Compression Members. A detailed discussion of the design of steel compression members would be foreign to the general subject matter of this book. For such a discussion the student is referred to the many excellent textbooks on steel design, one of them being *Structural Design in Steel* by Shedd.

In the design of members subjected to bending it is often found that such members are subjected to axial loads also. Where such axial loads are compressive, it becomes necessary to take cognizance of the principles of design of compression members as well as those of flexural members.

In the design of a short compression member there is no great difficulty involved. It is necessary only to divide the total stress (or load) by the allowable unit stress in compression and thus determine the required area. It is assumed that the member will be proportioned in such a manner that the thicknesses of all parts (webs and flanges) meet the requirements of the specifications and that the slenderness ratio of the entire section is less than that for which the specifications require a column reduction formula. In the design of longer compression members, column action is involved, and it therefore becomes necessary to take account of the properties and general shape of the section. Some assumptions as to these values must be made as the first step in design.

46. Column Reduction Formulas. Only a very brief treatment of column reduction formulas will be attempted in this article—just enough to give the ones most commonly used, both now and in former years, and a little of their background. The discussion will closely parallel that in Art. 26 of Chapter 3 regarding the design of compression flanges that do not have full lateral support.

The same basic expressions

$$f = f_1 + f_2 \quad \text{or} \quad f_1 = f - f_2$$

are equally applicable in the study of columns as in that of compression flanges. In the present instance,

f = the maximum unit compressive stress, as before.
f_1 = the average intensity of stress on the column = P/A.
f_2 = the intensity of stress due to eccentricities in load application, whether caused by imperfections in manufacture or column action, including lateral displacements and end rotations.

In the above, P is the axial load, and A is the cross-sectional area of the member.

All common expressions for f_2 contain some form of the ratio L/r, in which

L = the length of the column in inches.
r = critical radius of gyration of the column section in inches.

The value of r is about that axis which gives the maximum value of L/r.

Where L/r is used, the expression takes the form $f_1 = f - k(L/r)$, which is the formula for a straight line. The most widely adopted straight-line formula for many years was

$$f_1 = 16{,}000 - 70\frac{L}{r} \tag{1}$$

with a maximum of 14,000 lb per sq in. allowed. This appeared in specifications written for a base stress of 16,000 lb per sq in. There were a number of others utilized to a somewhat limited extent.

Where L^2/r^2 is used, the basic expression takes the form

$$f_1 = f - k_1\frac{L^2}{r^2}$$

which gives a parabolic formula or a Rankine-Gordon type of formula, depending on whether or not k_1 is made a function of f_1.

The 1949 A.I.S.C. specifications use a parabolic formula for values of L/r less than 120, namely,

$$f_1 = 17{,}000 - 0.485\frac{L^2}{r^2} \tag{2}$$

The base stress in these specifications is 20,000 lb per sq in., but the above formula limits the stress in a column to some value less than 17,000 lb per sq in.

The current A.R.E.A. and A.A.S.H.O. bridge specifications give

$$15{,}000 - \frac{1}{4}\frac{L^2}{r^2} \quad \text{for columns with riveted ends} \tag{3}$$

and

$$15{,}000 - \frac{1}{3}\frac{L^2}{r^2} \quad \text{for columns with pin ends} \tag{4}$$

Both specifications have a base stress of 18,000 lb per sq in.

If k_1 is replaced by $k_2 f_1$, the Rankine-Gordon formula is obtained.

$$f_1 = \frac{f}{1 + k_2 \dfrac{L^2}{r^2}}$$

There have been many forms of this formula in recent years, but the current specifications usually require a parabolic formula.

For bracing and secondary members having a value of L/r greater than 120, the A.I.S.C. specifications give

$$f_1 = \frac{18{,}000}{1 + \dfrac{L^2}{18{,}000r^2}} \tag{5}$$

This was also the formula for all values of L/r from their earliest specifications in 1923 until 1936, during which time the base stress was 18,000 lb per sq in. Formula 5 has given very good satisfaction in previous years.

In the design of a compression member it is necessary to make some estimate of the probable value of r and obtain the resulting allowable stress. Working from that starting point the column is proportioned.

47. Bending in Columns. Bending in columns * may be due to wind and other lateral loads, to the fact that beams are framed continuous with the column, or to an eccentricity of the applied axial load. The latter is often due to the fact that the axes of building truss members meet in the connection angles at the ends of the truss, thus causing an eccentricity of the reaction between the truss and the column. A common type of lateral force, other than wind, is the horizontal force from crane girders.

The computation of bending moments in columns, due to lateral forces, is a problem in statically indeterminate structures. They are often computed by means of simplifying assumptions. Where the

* In reinforced concrete columns especially, but also for steel columns in some instances, large changes in temperature may be the cause of critical bending moments. Advanced textbooks treat this subject as well as the bending moments caused by shrinkage in reinforced concrete. In earthquake areas, the bending moments in columns caused by earthquake forces should be considered.

columns are of equal size and height, a common assumption for the distribution of stress due to wind loads is that the plane of contraflexure occurs at the mid-height of the column, and that the shear for the forces above the plane of contraflexure is equally distributed between the columns. The foregoing is based on the assumption that the column is not free to rotate at the bottom of the truss or at the ground, and it applies to normal building bents. Proper account must be taken of columns that are hinged at the base or have different sections or lengths. For a discussion of approximate methods of computing stress and moments in columns, due to lateral loads, the student is referred to Chapter IX of *Theory of Simple Structures* by Shedd and Vawter. A more exact treatment can be obtained by applying the methods discussed in Chapter X of the same book.

Where a single lateral force is applied at some mid-point of a column, as a side thrust from a crane girder, it is customary to treat the column as a fixed end beam with a single concentrated load. The end moments for this case can be obtained from the relation,

$$M_A = \frac{Pab}{L}\frac{b}{L} \quad \text{and} \quad M_B = \frac{Pab}{L}\frac{a}{L}$$

where a and b are the two segments into which the length L is divided by the load, a being the segment on the A end, and b the segment on the B end.

Where a column supports a beam framed integrally with the column, it is necessary to use one of the methods of solving statically indeterminate structures to obtain a proper solution.

48. Design of Columns Subjected to Bending. If it is assumed that a column has at least one axis of symmetry and that bending is about the axis of symmetry or the axis perpendicular to it, the following expression for stress is obtained:

$$f = \frac{P}{A} + \frac{Mc}{I}$$

where P = the total axial load.
A = the cross-sectioned area of the column.
M = the bending moment.
c = the distance from the gravity axis to the extreme fiber on the compression side, due to bending.
I = the moment of inertia about the axis perpendicular to the plane of bending.

The above expression gives the computed stress in the member if the additional stress due to column action or due to lack of lateral support in the compression flange is neglected. Allowance is made for these additional stresses by the value of the allowable computed stress, when a compression member or a beam is being designed, and can similarly be made for the combined action.

Since $I = Ar^2$, the above expression can be written

$$A = \frac{P}{f} + \frac{Mc}{fr^2} \tag{6}$$

In this equation f is the allowable unit stress, whether due to axial load or bending. It is more satisfactory from a rational standpoint to account separately for the allowable stress due to column action and for the stress due to bending; equation 6 can then be written

$$A = \frac{P}{f_1} + \frac{Mc}{f_2 r^2} \tag{7}$$

where f_1 = the allowable stress from the column reduction formula, using the least radius of gyration (or that for maximum L/r), and

f_2 = the allowable stress in the compression flange of a flexural member, taking account of any lack of lateral support.

In the application of equation 7, it is necessary to make some assumption regarding the physical dimensions of the cross section of the member in order to obtain approximate values of f_1 and f_2 and also to estimate r. The design is entirely by trial, and the number of trials is determined by the accuracy of the estimates. Experience is the best teacher in making these assumptions, and the experienced designer can make a fairly accurate estimate on his first trial. A table of approximate radii of gyration is of aid in estimating these properties. A useful table of this character may be found on page 421 of *Structural Design in Steel* by Shedd. This table covers most column sections.

The 1949 A.I.S.C. specifications provide that members subjected to both bending and axial stresses shall be proportioned so that

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} \text{ shall not exceed unity}$$

where F_a = axial unit stress permitted if axial stress only existed.

F_b = bending unit stress permitted if bending only existed.

f_a = computed axial unit stress = P/A.

f_b = computed bending unit stress = Mc/I.

The above relationship is merely a restatement of equation 7. The authors believe that equation 7 provides as direct a method of proportioning as any other and recommend its use.

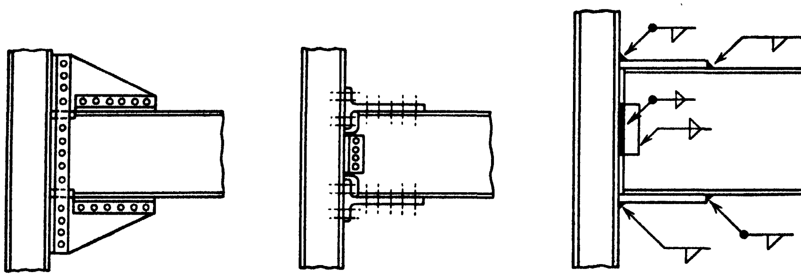


FIG. 54

FIG. 55. Erection view of United Nations Secretarial Building, New York, showing girder to column connections, including connections to resist moment. (Courtesy of American Bridge Company.)

If there is bending about both rectangular axes, a third quantity, similar in form to the second quantity in equation 7, can be added to include the effect of the bending moment about the other axis.

Although the maximum axial load that a column must resist is an important design condition, it is also true that, if a column connects to other structural members by means of connections capable of resisting bending moments, a lesser axial load with an accompanying bending moment may be a more critical design condition. Connections sometimes are designed to resist bending moments caused by unsymmetrical vertical loads, or lateral forces, or both. Three of the many possible types of moment connections are shown in Fig. 54.

Where the members are attached by welding, the connections may be designed to resist bending moments even though the effect of lateral forces is unimportant. Although it is not necessary that welded connections be designed to resist bending moments, sometimes it is economical to do so.

Even the simplest riveted connection designed to resist shearing force will resist a small bending moment. Generally, however, the amount of bending that exists at a riveted shear connection is unimportant in the design of either member being connected. One exception would be a deep girder framed into a short column by means of long connection angles attached to the flange of the column. Here the bending moment in the connection could be of sufficient magnitude to materially influence the design of the column. Figure 56 illustrates this.

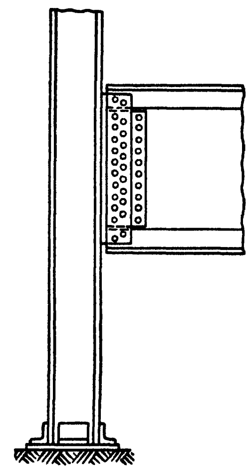


FIG. 56

49. Illustrative Example. In Fig. 57*a* is shown a column supporting a girder on each side and two beams, which frame into the web of the column. Figure 57*b* shows the condition of maximum vertical load with no bending, and Fig. 57*c* shows the condition of maximum bending with the accompanying vertical load. The bending moment is the result of a partial floor loading. Lateral forces such as the wind loads would also cause bending in the column; however, when wind loads are included along with the live load, impact, and dead load, the unit stresses may be increased by $33\frac{1}{3}$ per cent, according to the A.I.S.C. specifications. For this example it is assumed that wind loads are not critical.

The design conditions to be considered, as shown in Fig. 57, are an axial load of 500 kips or an axial load of 350 kips, plus a bending moment of 90 ft-kips. The length L used in computing L/r is taken as the distance from the column base to the mid-depth of the beam. $L = 19.50$ ft. For the axial load of 500 kips, assuming an allowable stress of about 15 kips per sq in.,

$$A = \frac{500}{15} = 33.33 \text{ sq in.}$$

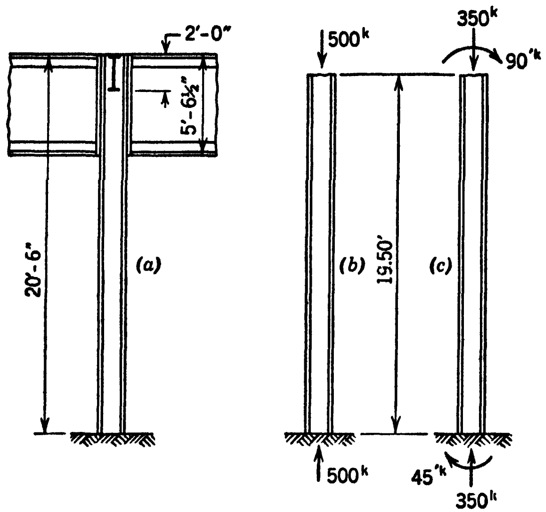


FIG. 57

Using a 14 × 14½ WF 119, A = 34.99 sq in., r_{x-x} = 6.26 in., and r_{y-y} = 3.75 in.

$$\frac{L}{r} = \frac{19.50 \times 12}{3.75} = 62.4$$

$$f_1 = 17{,}000 - 0.485(62.4)^2 = 15{,}110 \text{ lb per sq in}$$

$$A = \frac{500}{15.11} = 33.1 \text{ sq in.}$$

For the axial load of 350 kips and a bending moment of 90 ft-kips, using equation 7 and f_1 = 17,000 lb per sq in. and f_2 = 20,000 lb per sq in., since at the point where this bending moment occurs the column is supported in both directions,

$$A = \frac{350}{17} + \frac{90 \times 12 \times 7.25}{20 \times (6.26)^2} = 20.6 + 10.0 = 30.6 \text{ sq in.}$$

Use a 14 × 14½ WF 119.

A section taken a sufficient distance below the beams so that reduced allowable stresses should be used would have a bending moment so much less than the maximum that this section would not be critical. The 14 × 14½ WF 119 is the least weight section, since the next smaller section has an area of only 32.65 sq in.

CHAPTER 5

ELEMENTARY DESIGN OF REINFORCED CONCRETE BEAMS AND COLUMNS

50. Concrete. Concrete is a structural material, as are steel, aluminum, timber, brick, stone, etc. It must conform to certain definite strength specifications. It differs from other materials in that it is manufactured on the job, or hauled from a central mixing plant in the vicinity, and placed immediately after the manufacture, or mixing, while still in the semifluid state.

The strength of concrete is dependent on the size, the gradation, and the type of aggregate; the proportioning of the aggregates; the amount and quality of cement; the ratio of the water content to the cement; and the time and type of curing and mixing. Methods of depositing also influence the strength. The strength of concrete to be used in any structure is normally given in the specifications for that structure. General specifications, notably those of the American Society for Testing Materials, tell how concrete of a given specified strength can be obtained. Also, test cylinders are made on the job, and these cylinders are tested after a given number of days in order to determine whether concrete of the desired strength is being obtained. The time interval between pouring and making the cylinder tests is usually seven days for one group of cylinders and twenty-eight days for the remaining cylinders.

The details of selecting and grading the aggregate, and of proportioning, mixing, and depositing the concrete are somewhat beyond the scope of this chapter. These details can be found in the specifications of the various societies and in textbooks on plain concrete. The material will be treated herein as if it is a finished product, of specified strength, and ready to be used with proper reinforcing material.

In Chapter 1, Art. 8, it was assumed that within the elastic limit of the material there would be a linear distribution of stress in a member in flexure, or that stress is proportional to strain. In concrete, if the stress-strain curve were plotted it would be a curve somewhat like a parabola in shape. Within the limits of the allowable design stress this curve will deviate slightly from the straight line, and, after repeated loads, the stress-strain curve will be essentially straight. At this stage the concrete will not entirely recover its initial length upon removal of the load, and it first appears that the secant line would prove to be the correct stress-strain relation for some given stress. The phenomenon of time yield, which will be discussed later, also enters the problem; therefore there is no justification in considering the stress-strain relation as other than a straight line, on the basis of values that have proved satisfactory by years of application and by experiment. The building code of the American Concrete Institute recommends for the modulus of elasticity of concrete a value of 1000 f_c', in which f_c' = the ultimate strength of concrete after twenty-eight days.

51. The Use of Concrete with Steel. Although concrete is one of the more important materials used in various types of structures, it has the disadvantage of being extremely weak in tension although relatively strong in resisting compression. This strength deficiency in tension can be overcome by the addition of some other material that is capable of resisting the tensile forces; the most common material is steel. These two in proper combination make reinforced concrete, the bond between the concrete and the steel causing the two materials to act together. The steel resists the tensile stresses in general but frequently assists the concrete in resisting the compressive stresses.

The combination of steel and concrete is not confined to the tensile side of flexural members or to tension members; as will be seen later, it has an important application in compression. The combination has one great advantage in that the two materials have about the same coefficient of thermal expansion, and, consequently, the stresses due to temperature changes are small. Concrete does shrink in setting, causing some initial stresses, but these normally can be ignored in reinforced concrete as they are immaterial compared to the normal working stresses. Sound concrete also protects the steel against corrosion and to some extent against fire hazards.

Since the concrete is weak in tension, it cracks long before the steel is stressed to its normal working stress in tension. For this reason the steel has to take all the tension in a tension member of concrete and steel. In a flexural member the steel is assumed to take all the tension as the small amount of uncracked concrete on the tensile side adjacent to the neutral axis does not resist enough bending moment to justify its inclusion in the computations.

52. Placement of Steel. Since the stresses in concrete and steel are calculated on the assumption that the steel is to occupy a certain space, it is important that extreme care be exercised in placing the steel so that these stresses are at least approximately realized. The placing of the reinforcement in reinforced concrete structures should be done by experienced workmen.

A number of various types of bar supports and spacers are becoming increasingly more common compared to small blocks of concrete and similar means of support. Welding of the reinforcement before it is

FIG. 58a. View of reinforcing steel in place. Note W stirrups in foreground, slab steel in background. (Courtesy of Portland Cement Association.)

placed helps to keep it in proper location. Beam and column steel is often wired or welded into units before being placed in the forms.

Where beams frame into columns, it is often necessary to spread the steel in the top of the beam in order to avoid interference with the column steel, possibly placing some in the slab beyond the side of the beam. Bottom steel may be slightly raised in order to avoid steel in the bottom of the beam on the other side of the column. Such details of steel placement have been purposely omitted. The student is referred to more advanced textbooks for the complete details. The *Proposed Manual of Standard Practice for Detailing Reinforced Concrete Structures,*

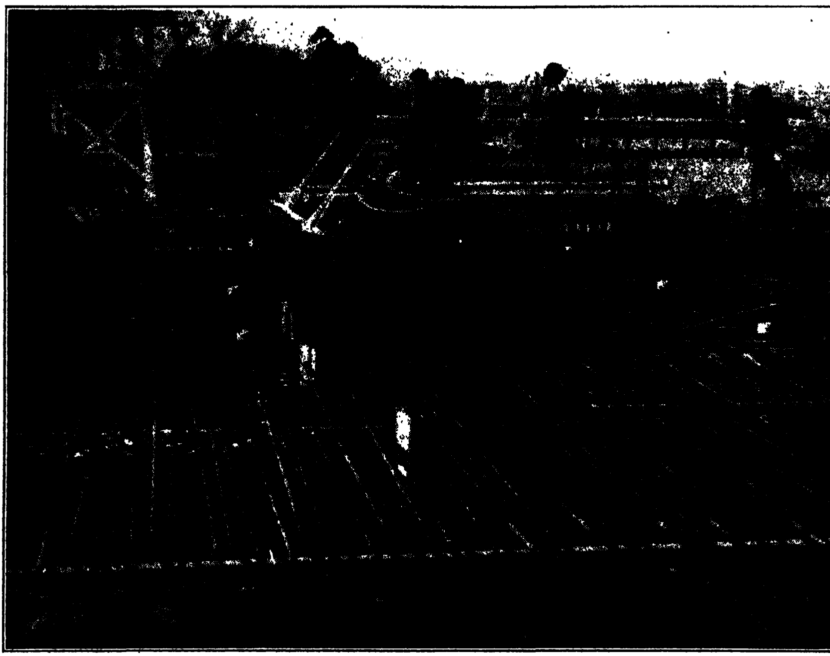


FIG. 58*b*. Close-up view of slab reinforcing steel in place. (Courtesy of Portland Cement Association.)

published by the American Concrete Institute, has valuable information on the placement of steel.

53. Transformed Sections. The combination of concrete and steel in a beam forms a nonhomogeneous beam, and the necessary computations can best be made by means of the method of transformed sections similar to the method followed in Chapter 2. Many handbooks contain complicated formulas and extensive and elaborate tables and diagrams for the design of reinforced concrete. It is believed that the beginning student should ignore these and confine his work to the application of the method of transformed sections to the use of steel with concrete. For transformed sections there are certain common expressions that will be obtained and that can be easily remembered if frequently used, but the student should not try to remember them, since anyone familiar with the fundamental relations in reinforced concrete design should be able to obtain all the necessary expressions in five to ten minutes. After the student has become thoroughly familiar with the fundamental procedures in the design of reinforced concrete, he is in a position to choose which tables or diagrams might be valuable to him.

The following nomenclature is common in the literature on reinforced concrete; more will be added as needed.

E_c = modulus of elasticity of concrete in compression.
E_s = modulus of elasticity of steel.
f_c = unit stress in extreme fiber of concrete.
f_c' = ultimate strength of concrete at the end of twenty-eight days.
f_s = unit tensile stress in steel reinforcement.
f_s' = unit compressive stress in steel reinforcement.
A_s = area of tensile steel.
A_s' = area of compressive steel.
b = width of beam.
d = depth of beam to center of steel reinforcement.
kd = distance from compression face of beam to neutral axis.
jd = moment arm of resisting couple, or distance between resultant compressive and tensile forces.
$n = E_s/E_c$.*
$p = A_s/bd$, the steel ratio (sometimes expressed as percentage of steel).

Figure 59 presents a reinforced concrete beam with dimensions as shown. In Fig. 59*b*, the transformed section has been drawn, following the same principles as in Chapter 2. It will be noted that, since the

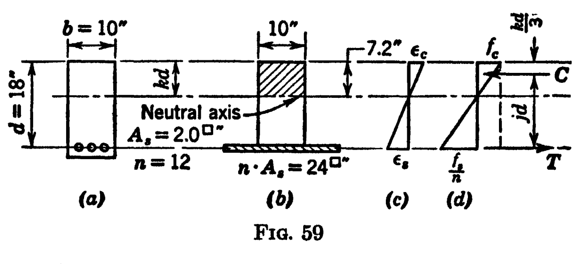


FIG. 59

* According to the 1940 report of the Joint Committee, if

f_c' = 2000–2400, n = 15.
= 2500–2900, n = 12.
= 3000–3900, n = 10.
= 4000–4900, n = 8.
= above 5000, n = 6.

concrete is considered incapable of taking tension, the only material considered on the tensile side of the beam is the transformed steel area. Since the amount of concrete considered in the transformed section is dependent on the location of the neutral axis, a quadratic equation is involved in finding this value. The calculations for locating the neutral axis of the beam in Fig. 59 are as follows:

Taking moments about the neutral axis ($kd = x$) and equating the first moment of the compressive area to the first moment of the tensile area:

$$10 \times x \times \frac{x}{2} = 12 \times 2(18 - x)$$

$$5x^2 = 432 - 24x$$

$$x^2 + 4.8x = 86.4$$

By completing the square,

$$x^2 + 4.8x + 5.76 = 92.16$$

$$x + 2.4 = \pm 9.6$$

$$x = 7.2 \quad \text{or} \quad -12.0$$

therefore,

$$kd = 7.2 \text{ in.}$$

The deformations for the fibers in the beam shown in Fig. 59*c* are based on the assumption that plane sections remain planes after bending; the stress distribution shown in Fig. 59*d* is based on the assumption that we have a linear distribution of stress. It should be noted that the only stress below the neutral axis is that in the steel, the unit value being f_s in the steel or f_s/n in the transformed area. The assumption that we have a linear distribution of stress is not entirely correct as far as the concrete is concerned, but within the values of the working stresses normally permitted this assumption is exact enough for design purposes.

A study of Fig. 59*d* also shows a definite relation between kd, f_c, f_s, and n. From the geometry of the figure, we can write

$$\frac{kd}{f_c} = \frac{d}{f_c + (f_s/n)} \quad \text{or} \quad k = \frac{f_c}{f_c + (f_s/n)}$$

also

$$jd = d - \frac{kd}{3} \quad \text{or} \quad j = 1 - \frac{k}{3}$$

This value of k is a relation that will always hold true in any reinforced concrete member subjected to bending and having both tensile and compressive stresses. It is not an alternate method for locating the neutral axis to be used where the physical dimensions of the beam are given, as in Fig. 59*a*. Where the complete physical dimensions of the beam are given, including the area of the steel, these locate the neutral as was done with the quadratic equation. However, once the location of the neutral axis is determined, there is also determined a definite ratio between the stresses f_c and f_s. Also, where a beam is being proportioned for certain working unit stresses to be completely realized, the neutral axis is thereby determined according to the relations above. When a beam is proportioned for certain definite values of the stresses f_c and f_s, it is called a "balanced design."

54. Design of Rectangular Beams and Slabs with Tension Steel. Figure 60 shows a beam similar to the one shown in Fig. 59, except that it has not been assigned finite dimensions. If it is desired to proportion this beam to resist a certain bending moment M, there are certain working stresses that must not be exceeded. The ideal condition would be to realize these stresses f_c and f_s simultaneously, and where this is done a balanced design is achieved, as discussed in the previous article. This is not hard to attain where it is not necessary to consider the effect of shear or where the beam is not limited in its dimensions. If these

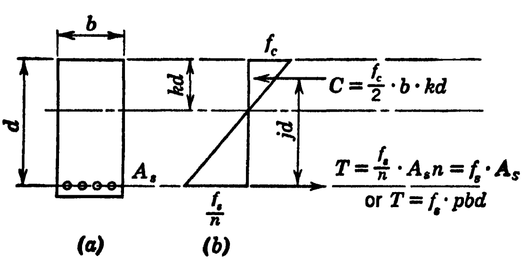


FIG. 60

stresses are realized simultaneously, there are certain relations that must be maintained, one of which is the location of the neutral axis, or determination of k, as discussed in the previous article.

Since the total compressive force C must equal the total tensile force T for $\Sigma H = 0$,

$$C = \frac{f_c}{2} \times b \times kd = T = f_s \times pbd$$

or

$$p = \frac{f_c \times k}{2f_s}$$

which gives a definite relation between the area of steel and the area of concrete. Also,

$$M = C \times jd = \frac{f_c}{2} \times b \times kd \times jd = \left(\frac{f_c}{2} \times kj\right) bd^2$$

and

$$M = T \times jd = f_s \times pbd \times jd = (f_s pj)bd^2$$

These two expressions $[(f_c/2) \times kj]$ and $(f_s pj)$ are equal and are commonly called R in the concrete literature. Definite values of R are obtained for any combination of f_c, f_s, and n, and, where the values of b and d satisfy the expression $M = Rbd^2$, a balanced design is obtained.

These are not formulas to be remembered but are merely relations that can be easily obtained and that must be observed if it is desired to secure a balanced design. A balanced design is not always obtained in beams, but it is usually fairly well observed in the design of slabs.

Another common problem is to check an existing beam as to its capacity or as to the stresses produced by given loads. Although this does not involve any new principles beyond those discussed under flexure and nonhomogeneous beams, the critical stresses in both concrete and steel will be computed for the beam shown in Fig. 59*a* for a bending moment of 50 ft-kips.

$$d = 18 \text{ in.}, \; b = 10 \text{ in.}, \; kd = 7.2 \text{ in.}, \; A_s = 2.0 \text{ sq in.}$$

$$jd = 18 - \frac{7.2}{3} = 15.6 \text{ in.}$$

$$C = \frac{M}{jd} = \frac{50{,}000 \times 12}{15.6} = 38{,}500 \text{ lb}$$

$$\frac{f_c}{2} \times 10 \times 7.2 = 38{,}500 \text{ lb} \qquad f_c = 1070 \text{ lb per sq in.}$$

$$T = 38{,}500 \text{ lb} = 2.0 f_s \qquad f_s = 19{,}250 \text{ lb per sq in.}$$

55. Illustrative Example. Figure 61*a* shows a reinforced concrete beam in elevation. The span is 20 ft, and the live load is 1500 lb per ft of length. Taking $f_c' = 2500$ lb per sq in., $n = 12$, and $f_s = 20{,}000$ lb per sq in., the beam can be designed on the basis of the discussion in the preceding articles. The maximum bending moment for this simple span will be at the center, and, if we assume the beam to weigh 300 lb per ft of length, the center bending moment is $\frac{1}{8} \times 1800 \times 20^2 = 90{,}000$ ft-lb = 1,080,000 in.-lb. The allowable compressive stress * f_c for

* Allowable stresses, protective covering, bar spacing, etc., for this illustrative example and for the remainder of this chapter follow closely the "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete" submitted in 1940 as a report of the Joint Committee on Standard Specification for Concrete and Reinforced Concrete, and also follow closely "Building Regulations for Reinforced Concrete," published by the American Concrete Institute in 1946.

concrete in flexure is 45 per cent of f_c', and here $f_c = 0.45 \times 2500 = 1125$ lb per sq in. For balanced design, the calculations can be made as follows:

$$k = \frac{f_c}{f_c + (f_s/n)} = \frac{1125}{1125 + (20{,}000 \div 12)} = 0.403$$

$$j = 1 - \frac{k}{3} = 0.866$$

$$M = Cjd = \frac{f_c}{2} kjbd^2 = Rbd^2 \quad \text{for } R = \frac{f_c}{2} kj$$

$$R = \frac{1125}{2} \times 0.403 \times 0.866 = 196 \text{ lb per sq in.}$$

$$M = Rbd^2 \quad \text{or} \quad d = \sqrt{\frac{M}{Rb}}$$

For a given b, d is definite; therefore b can be assumed, and, for each

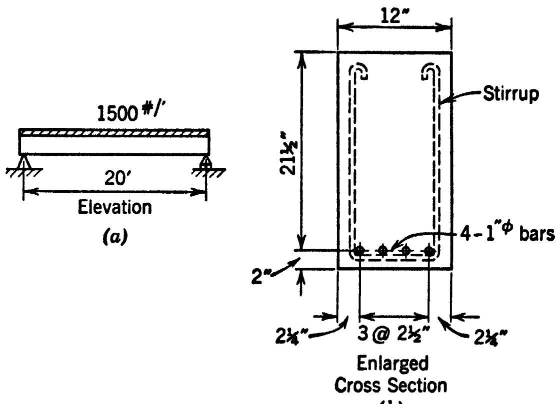


FIG. 61

value, the corresponding value of d can be determined. From the equation $d = \sqrt{\dfrac{1{,}080{,}000}{196b}}$, Table 1 can be obtained. It is not recom-

TABLE 1

b	d
8	26.2
10	23.5
12	21.4
13	20.6
14	19.8
16	18.6

mended that such a table be made. It is shown here merely to indicate that a designer does have a choice which should be governed by other factors than just those in the equation. For an economical design, the ratio b/d should be about $\frac{1}{2}$ to $\frac{3}{4}$ for small beams and $\frac{1}{4}$ to $\frac{1}{3}$ for large beams.

The weight and cost of the beam depend on the overall depth of the beam which is the distance d plus the thickness of the protective covering. The protective covering insures adequate bond between steel and concrete, helps prevent corrosion of the steel, and mainly protects the steel from damage by fire. The amount of protective covering is dependent on the type of structure and the estimated duration of fire. Average values would be $\frac{3}{4}$ in. to 1 in. for slabs and 1 in. to 2 in. for beams and girders.

If $b = 12$ in., $d = 21\frac{1}{2}$ in., and the protective cover is 2 in., the weight of the beam would be

$$12 \times 23.5 \times \tfrac{150}{144} = 294 \text{ lb per ft of length}$$

if the usual assumption is made that reinforced concrete weighs 150 lb per cu ft. This checks closely enough with the assumed 300 lb per ft.

The required steel area A_s is obtained as follows:

$$A_s = \frac{M}{f_sjd} = \frac{1{,}080{,}000}{20{,}000 \times 0.866 \times 21.5} = 2.90 \text{ sq in.}$$

Reinforcing steel bars are readily available with diameters from $\frac{1}{4}$ in. to 1 in., in increments of $\frac{1}{8}$ in. for round bars, and with the side dimensions of 1 in., $1\frac{1}{8}$ in., and $1\frac{1}{4}$ in. for square bars. For this problem any of the following would provide sufficient steel area:

$$4\text{—1-in. round} = 4 \times 0.785 = 3.14 \text{ sq in.}$$
$$3\text{—1-in. square} = 3 \times 1.00 = 3.00 \text{ sq in.}$$
$$5\text{—}\tfrac{7}{8}\text{-in. round} = 5 \times 0.601 = 3.01 \text{ sq in.}$$
$$7\text{—}\tfrac{3}{4}\text{-in. round} = 7 \times 0.442 = 3.09 \text{ sq in.}$$

Table 2 lists the areas and perimeters of bars.

Reinforcing steel must be placed, or spaced, in such a manner that the concrete can surround it so that sufficient bond can be developed between the two materials to permit them to act together. The clear distance between bars should not be less than $1\frac{1}{2}$ times the maximum size of coarse aggregate, with a minimum of 1 in. for beams, and the center-to-center distance for bars should not be less than $2\frac{1}{2}$ diameters for round bars or 3 times the side distance for square bars.

TABLE 2

Cross-Sectional Area of Bars *

Size in Inches	Number of Bars									
	1	2	3	4	5	6	7	8	9	10
Round bars										
1/4	0.05	0.10	0.15	0.20	0.25	0.29	0.34	0.39	0.44	0.49
3/8	0.11	0.22	0.33	0.44	0.55	0.66	0.77	0.88	0.99	1.10
1/2	0.20	0.39	0.59	0.79	0.98	1.18	1.37	1.57	1.77	1.96
5/8	0.31	0.61	0.92	1.23	1.53	1.84	2.15	2.45	2.76	3.07
3/4	0.44	0.88	1.33	1.77	2.21	2.65	3.09	3.53	3.98	4.42
7/8	0.60	1.20	1.80	2.41	3.01	3.61	4.21	4.81	5.41	6.01
1	0.79	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85
Square bars										
1	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00
1 1/8	1.27	2.53	3.80	5.06	6.33	7.59	8.86	10.13	11.39	12.66
1 1/4	1.56	3.13	4.69	6.25	7.81	9.38	10.94	12.50	14.06	15.63

Perimeter of Bars *

Size in Inches	Number of Bars									
	1	2	3	4	5	6	7	8	9	10
Round bars										
1/4	0.79	1.57	2.36	3.14	3.93	4.71	5.50	6.28	7.07	7.85
3/8	1.18	2.36	3.53	4.71	5.89	7.07	8.25	9.42	10.60	11.78
1/2	1.57	3.14	4.71	6.28	7.85	9.42	11.00	12.57	14.14	15.71
5/8	1.96	3.93	5.89	7.85	9.82	11.78	13.74	15.71	17.67	19.63
3/4	2.36	4.71	7.07	9.42	11.78	14.14	16.49	18.85	21.21	23.56
7/8	2.75	5.50	8.25	11.00	13.74	16.49	19.24	21.99	24.74	27.49
1	3.14	6.28	9.42	12.57	15.71	18.85	21.99	25.13	28.27	31.42
Square bars										
1	4	8	12	16	20	24	28	32	36	40
1 1/8	4.5	9	13.5	18	22.5	27	31.5	36	40.5	45
1 1/4	5	10	15	20	25	30	35	40	45	50

* See note on page 166.

The number of bars that can be placed in a single horizontal row depends on the spacing of the bars, the width of the beam, and the amount of protective covering at each side. Shearing stresses in concrete cause diagonal tension stresses that will be discussed later. Vertical steel stirrups, shown as dotted lines in Fig. 61*b*, are employed to resist these diagonal tension stresses, and protective covering is required for these stirrups which lie outside the longitudinal reinforcing steel bars. If 4—1-in. round bars are spaced $2\frac{1}{2}$ in. from center to center and the vertical stirrups are $\frac{3}{8}$-in. round bars, the minimum width of beam would be as follows:

$$b(\text{minimum}) = (1 + \tfrac{3}{8} + \tfrac{1}{2})2 + (3 \times 2\tfrac{1}{2}) = 11\tfrac{1}{4} \text{ in.}$$

However, a width of 12 in. was used. For 5— $\frac{7}{8}$-in. round bars,

$$b(\text{minimum}) = (1\tfrac{3}{8} + \tfrac{7}{16})2 + (4 \times 2\tfrac{1}{4}) = 12\tfrac{5}{8} \text{ in.}$$

or 13 in. could be used. If $b = 13$, then d of 20.6 in. is the minimum allowed.

Taking $d = 20.75$ in.,

$$A_s = \frac{1{,}080{,}000}{20{,}000 \times 0.866 \times 20.75} = 3.01 \text{ sq in.}$$

Although this equals the area of 5— $\frac{7}{8}$-in. round bars and would be satisfactory, the section will be as shown in Fig. 61*b*; $b = 12$ in., $d = 21\frac{1}{2}$ in. (overall depth $= 23\frac{1}{2}$ in.), and 4—1-in. round bars.

56. Moment Coefficients in Continuous Beams. In the illustrative design of the beam in Art. 55 it was assumed that the beam was simply supported at the ends, and consequently the maximum moment was at the center and equal to $wL^2/8$. In steel construction most beams are simply supported; in reinforced concrete construction the reverse is true. Simply supported reinforced concrete beams and slabs are commonly encountered in the construction of short-span bridges, but in building construction the beams and slabs are normally poured continuous with each other. They therefore have to be treated as continuous beams in their design, and the maximum bending moments occur at the supports and are negative in sign.

The codes and specifications provide that in this type of construction the members shall be designed to resist the moments and shears as computed from the principle of continuity. They also permit certain arbitrary moment coefficients when loads are uniformly distributed and the spans are equal in length, or nearly so. In general, the principles of continuity should be followed in computing the moments and shears in continuous structures although the arbitrary coefficients give very good results within the limitations named. Since the beginner in beam design will have had little, if any, training in the theory of indeterminate structures, the authors will make use of arbitrary moment coefficients in the illustrative examples of continuous beams. Most designers employ arbitrary moment coefficients in their preliminary designs, and many final designs have been based on them. These coefficients are based on the supposition that the beams or girders are generally much stiffer than the members that support them and can thus be considered as being essentially continuous over knife-edge supports.

The specifications of the 1940 Joint Committee give separate coefficients for dead load and live load, which is entirely logical since the maximum moments in each case are produced by different loadings. They are given in decimal form but would average about as shown in Table 3 for beams of four or more spans. The coefficient shown is to be

TABLE 3

MOMENT COEFFICIENTS

	End	Center of First Span	First Interior Support	Interior Span Center	Interior Span Support
D.L.	$-\frac{1}{25}$	$\frac{1}{12}$	$-\frac{1}{9}$	$\frac{1}{24}$	$-\frac{1}{12}$
L.L.	$-\frac{1}{25}$	$\frac{1}{10}$	$-\frac{1}{8}$	$\frac{1}{12}$	$-\frac{1}{9}$

multiplied by wL^2, where w is the uniformly distributed load. The coefficients do not apply to concentrated loadings unless the concentrations are sufficient in number and equally spaced so as to be essentially a distributed loading. The span length L is the distance center to center of supports unless the supports restrain the rotation of the ends. It is stated that these coefficients are for continuous slabs, but the authors see no reason why they cannot be used for beams as long as the beams conform to the requirements of span length and uniformity of load. At supports, the moments are obtained for the centerline of the support.

The "A.C.I. Building Code" permits the use of the coefficients shown in Table 4 where one span does not exceed an adjacent span by more than 20 per cent and where the unit live load does not exceed three times

TABLE 4

A.C.I. MOMENT COEFFICIENTS

Center of First Span	First Interior Support	Interior Span Center	Interior Span Support
$\frac{1}{14}$	$-\frac{1}{10}$	$\frac{1}{16}$	$-\frac{1}{11}$

the unit dead load. These values are for more than two spans. The value of shear in the end span at the first interior support is to be increased 15 per cent over the value for a simple beam. The span length L' is to be the clear span. The coefficients are multiplied by $w(L')^2$, and at the supports they represent the moment at the face of the support. The dead load and live load together equal w.

The authors recognize that the recommended values of the Joint Committee represent more conservative practice and will use similar values when computing the moments separately due to dead load and live load. They agree with the increase of 15 per cent for the total shear in the end span, but they also believe that the live load shear in the interior spans should be increased by at least 10 per cent over that of a simple beam. Shear should be computed on the clear span.

Where the center to center of supports is taken as the span length in computing the negative moment at the end of the beam, the designer is justified in computing the moment at the face of the support, allowing for a normal moment diagram, in proportioning the steel at the support.

57. Illustrative Example—Slab Design. In Fig. 62*a* is shown a reinforced concrete slab that is continuous over several supports. The elevation view in cross section shows three span lengths of the slab. The spans are equal, and the center-to-center distance between the supporting beams is 8 ft. The live load for which the slab will be designed is 300 lb per sq ft, and the weight of the floor covering (wood blocks on a sand cushion) is 20 lb per sq ft, giving a total of 320 lb per sq ft exclusive of the weight of the slab itself.

The width of the slab normal to the slab span, i.e., the span of the supporting beams, is not shown. If this width is large relative to the slab span, the entire load can be assumed to be resisted by the strength of the slab in the direction of the span only. Each foot of width of the slab should have the same strength as any other foot of width of slab for a uniform load. If the uniform load = w pounds per square foot, and for purposes of design we take a 12-in. or 1-ft width of slab, then w is the load per foot of slab span in pounds.

The span length for computing bending moment will be 8 ft, or the distance, center to center, of the beams. The negative moment at the face of the support will be assumed $\left(-\frac{w_L}{9} - \frac{w_D}{12}\right)L^2$, which actually is for the centerline of the support, and the positive moment at the center of the span is $\left(+\frac{w_L}{12} + \frac{w_D}{24}\right)L^2$, where dead load = w_D and live load = w_L. It is understood that these moments occur with different groups of spans loaded. The span length would depend on the torsional rigidity of the supporting beams and for a completely fixed support would be the clear span, but the span above agrees with general practice for beams of normal size.

The bending moment at the support, being the greater, controls the depth of the slab. The tension steel is at the top of the slab over the support. If the

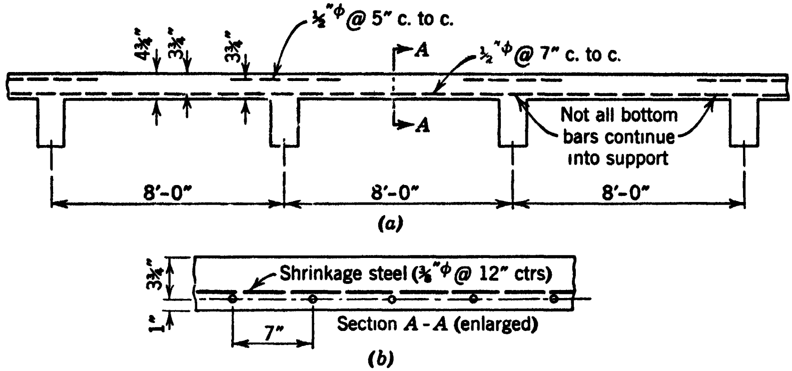


FIG. 62

overall depth of the slab is assumed to be 5 in., the weight per foot for a strip 1 ft wide is $\frac{5}{12} \times 150 = 63$ lb. The total $w = 320 + 63 = 383$ lb per ft.

$$M = (-\tfrac{300}{9} - \tfrac{83}{12})\overline{8}^2 = -2580 \text{ ft-lb} = -30{,}960 \text{ in.-lb}$$

Using the same stresses and value of R as in Art. 55,

$$d = \sqrt{\frac{2580}{196 \times 1}} = 3.62 \text{ in.} \quad (\text{use } 3\tfrac{3}{4} \text{ in.})$$

If the minimum cover of $\frac{3}{4}$ in. were used outside the steel, the overall depth would be $4\frac{3}{4}$ in., with $\frac{1}{2}$-in. bars. The required area of steel A_s per foot width of slab would then be $M/f_s jd$. For rectangular beams and for slabs, it is quite common to assume $\frac{7}{8}$ as the value of j, as this represents a fair average value. Then

$$A_s = \frac{30{,}960}{20{,}000 \times \frac{7}{8} \times 3.75} = 0.472 \text{ sq in. per ft of width}$$

Bars $\frac{1}{2}$ in. round require $0.472 \div 0.20 = 2.36$ bars per ft of width, or one bar every $12 \div 2.36 = 5.09$ in. Spacing over support = $\frac{1}{2}$-in. round bars spaced every 5 in.

At the center of the span $M = (\frac{300}{12} + \frac{83}{24})8^2 = 1820$ ft-lb.

$$\text{At the center } A_s = \frac{21{,}840}{20{,}000 \times \frac{7}{8} \times 3.75} = 0.334 \text{ sq in. per ft.}$$

$$\frac{0.334}{0.20} = 1.67 \text{ bars per ft of width}$$

$$\frac{12}{1.67} = 7.18 \text{ in.}$$

Spacing of bottom tension steel = $\frac{1}{2}$-in. round bars spaced every 7 in. The authors do not recommend bent-up bars in any slabs less than 5 in. in depth, especially if the bars are $\frac{1}{2}$ in. or more in diameter.

The cross section of the slab is shown in Fig. 62*b*. The overall depth of slab is $\frac{1}{4}$ in. less than the assumed depth. The change in moment is negligible. With $\frac{1}{2}$-in. round bars, the protective covering is $4\frac{3}{4} - 3\frac{3}{4} - \frac{1}{2} \times \frac{1}{2} = \frac{3}{4}$ in., which is sufficient for slabs designed for short periods of resistance to fires. The steel bars shown normal to the main reinforcing steel are known as shrinkage (or temperature) steel; they prevent the opening up of cracks that are parallel to the longitudinal steel and are caused by shrinkage or temperature changes. The quantity of shrinkage steel is generally determined by using an area of steel between 0.2 per cent and 0.3 per cent of the slab area. The minimum overall depth of slab should not be less than 4 in., the minimum clearance between bars is the same for slabs as for beams, and the maximum spacing between main longitudinal steel bars should preferably not exceed about $1\frac{1}{2}$ to 2 times the slab depth and in no case be more than 3 times the slab depth.

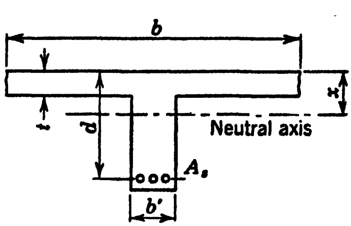


FIG. 63

Bond stresses, shearing stresses, and length and embedment of reinforcement in beams will be discussed later in this chapter. Shear is, in general, not of great importance in slabs.

58. Design of T Beams. The term T beam is applied to beams that are shaped such as the beam shown in Fig. 63. They most commonly occur as a part of a floor system, the slab not only acting in the function discussed in Art. 57 but also composing the flange of the T beam. A T beam can occur as a separate member, but that is rare. In either event the flange provides additional area of compressive concrete, and, if there is sufficient steel to resist the tension, a greater moment can be resisted than could be done with only a rectangular beam whose width is just that of the stem, b'.

The location of the neutral axis in a T beam is rather indefinite and does not have a great deal of meaning. Although a value for x (or kd) can be obtained for the beam in Fig. 63 by equating the first moment of the area above the axis to that of effective area below as

$$b \times t\left(x - \frac{t}{2}\right) + b'(x - t)^2 \times \frac{1}{2} = n \times A_s(d - x)$$

it has little meaning except for an isolated beam. Where a T beam occurs as a part of a floor system, the flange width b is rather a nebulous quantity.

Specifications require that b be limited to the smallest of the following values: $\frac{1}{4}$ the span length of the beam, 16 times the thickness of the slab plus the thickness of the web (where there is slab on both sides of the beam), or the distance from center to center of the beams. The first two are empirical requirements, but the last one is entirely rational as it permits a length of the slab (for each side of the flange) equal to $\frac{1}{2}$ the distance to the next beam to be considered as flange area.

Balanced design does not commonly occur in the design of T beams since the flange area usually supplies more compressive area than is needed. The slab is designed to perform its primary function where its compressive stress is parallel to the span of the slab and its thickness is determined on that basis. The action of the slab as a flange of a T beam, with the compressive stress perpendicular to the span of the slab, is entirely incidental and has no bearing on the selection of the slab thickness. If a balanced design did occur in a T beam, the location of the neutral axis would still be the same function of the allowable stresses as previously developed:

$$k = \frac{f_c}{f_c + \dfrac{f_s}{n}}$$

It is usually quite convenient to treat a T beam as if it were a balanced design when a check is made of the compressive stresses. The steel will be stressed to its allowable value. If it is assumed that the concrete at the top is also stressed at its allowable value, it is very easy to compute the corresponding location of the neutral axis and then compute the amount of flange necessary to resist the compression, using the allowable stress. It will usually be found that the required area is much less than the flange area available.

In proportioning the tension steel the total tension and total compression form a couple $M = Tjd = Cjd$, the same as in any other beam Since $T = A_s f_s$, $A_s = M/f_s jd$. The exact value of j is a rather uncertain quantity, but within the range of stresses that occur j varies within rather narrow limits. An approximate value is all that is justified, and a value of 0.9 is commonly taken. Experience has shown that this value gives satisfactory results, and computations show that the value can hardly be less than 0.9. In some unusual cases the computed value of j may be as large as 0.95.

Where there is negative moment at the support of a T beam, the compressive stresses are in the bottom of the beam and the flange is in tension. The beam is now a rectangular beam whose width is that of the stem (or web) of the T beam. Since a certain amount of the bottom steel is required, by specifications, to extend into the support, this is also a rectangular beam with compressive reinforcement (see Art. 60).

59. Illustrative Example. In the T beam shown in Fig. 64 it is assumed that $b = 60$ in. and $t = 4$ in., and the maximum bending moment is 500 ft-kips or 6000 in.-kips.

The allowable stresses are as follows:

$f_c = 0.45f_c' = 0.45 \times 2800 = 1260$ lb per sq in.
$f_s = 18{,}000$ lb per sq in.
$n = 12$, according to the specifications (1940 Joint Committee) for an f_c' value of 2800

The area of a beam or the beam stem ($b \times d$ for a rectangular beam, and $b' \times d$ for a T beam) is usually determined by the maximum vertical shear. The effect of shearing stresses will be considered in Art. 64, so at this point it will be assumed that the maximum shear value is such that an area of 620 sq in. (value of $b'd$) will be required. This will be satisfied if $b' \times d = 17$ in. $\times$ 37 in. $= 629$ sq in.

Since the value of j for T beams is usually 0.90 or more, a value of 0.90 will be assumed.

$$T = \frac{M}{jd} = \frac{6000}{0.90 \times 37} = 180.4 \text{ kips}$$

$$\frac{180.4}{18} = 10.02 \text{ sq in.} \quad \text{(use 10—1-in. square bars)}$$

Ten bars would require two layers of steel; the cross section of the T beam and the arrangement of the steel are shown in Fig. 64. The width of 17 in. was chosen as it is the minimum width that will permit adequate cover and bar spacing. Similar considerations require an overall depth of 41 in. if the vertical center-to-center distance of the rows is 3 in.

Some designers would separate these rows by a 1-in. bar, making the center-to-center distance 2 in. and the overall depth 40.5 in.

The computations for the location of the theoretical neutral axis and the value of j are given below. If this were an isolated T beam of the dimensions given in Fig. 64, it could be assumed that the compressive stress across the flange on fibers a given distance from the neutral axis is fairly constant. If the flange shown is a portion of a continuous slab whose span is normal to the beam span, this stress varies. For such a

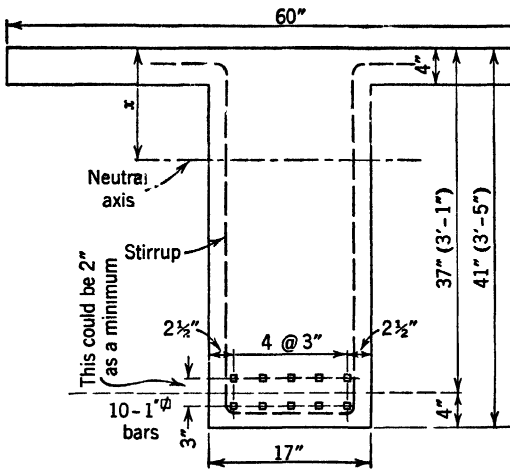


Fig. 64

flange the computations below are indicative, but obviously they cannot be correct unless the assumed effective flange width is correct.

To locate the neutral axis:

$$\frac{60x^2}{2} - \frac{43(x - 4)^2}{2} = 10 \times 12(37 - x)$$

$$30x^2 - 21.5x^2 + 172x - 344 = 4440 - 120x$$

$$8.5x^2 + 292x = 4784$$

$$x^2 + 34.4x = 563$$

$$(x + 17.2)^2 = 859$$

$$x = kd = 29.3 - 17.2 = 12.1 \text{ in.}$$

To determine the value of j:

$$60 \times 12.1 = 726 \times \frac{f_c}{2} = 363f_c \times \tfrac{2}{3} \times 12.1 = 2925f_c$$

$$-43 \times 8.1 = -349 \times \frac{4.05}{12.1} f_c = -117f_c \times \tfrac{2}{3} \times 8.1 = -630f_c$$

$$246f_c \qquad 2295f_c$$

$$37 - 12.1 = 24.9$$

$$\frac{2295f_c}{246f_c} = 9.3$$

$$34.2 \div 37 = 0.924 = j$$

To determine the value of f_s and f_c:

$$f_s = \frac{6000}{0.924 \times 37 \times 10} = 17.6 \text{ kips per sq in.}$$

$$T = A_s f_s = 10 \times 17.6 = 176 \text{ kips} = C$$

$$f_c = \frac{176{,}000}{246} = 715 \text{ lb per sq in.}$$

60. Design of Beams with Compression Reinforcement. The combination of steel and concrete to resist tension has been discussed in Art. 54. Steel is also very effective when combined with concrete to resist compressive stresses. When a compressive load is applied to a reinforced concrete member, the two materials will have equal strains since there is bond between them, and therefore the ratio of the unit stress in the steel to that in the concrete will be the same as the ratio of their moduli of elasticity. If a constant value is assumed for E_c, the modulus of elasticity of concrete, for stresses below the allowable working stress, then the ratio of these moduli, $E_s/E_c = n$, is considered a constant.

It would not be economical to substitute compression steel for concrete merely because the steel can resist more stress. The cost of the steel would be more than that of the concrete replaced. If the beam dimensions are limited by some requirement such as headroom or architectural features, compression steel may be necessary to resist the bending moment. In continuous beams the tensile reinforcement is at the top of the beam over the support, but reinforced concrete specifications require that a certain amount of the bottom tensile steel, which resists the positive bending moment at the center of the span, extend into the support at the bottom of the beam. If this bottom reinforcement is properly anchored or bonded, it acts as compression steel and therefore is available.

These represent two examples where compression steel is justified or economical. In floor construction where the beam and slab constitute a T beam, the moment at the support is negative, and compression is therefore in the bottom of the beam. The flange is on the tension side of the beam, and, since the concrete does not resist tension, the beam at this location for structural considerations is a rectangular beam with compression reinforcement. In computing the transformed section of this beam, the transformed area of the compression steel will extend sideways and have somewhat the same effect as a T beam, since this transformed area constitutes added compression area.

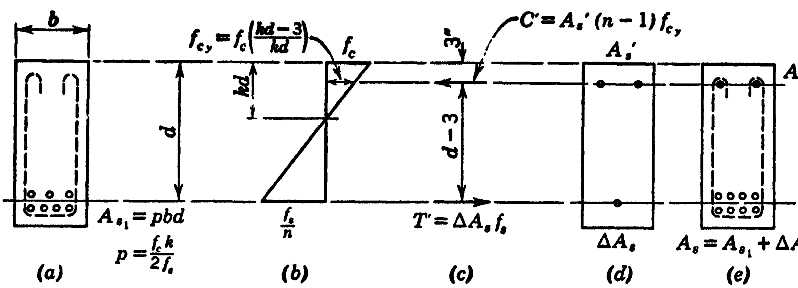


FIG. 65

In Fig. 65*a* is shown a rectangular beam, the area of which ($b \times d$) has been determined by the amount of shear to be resisted, or other requirements. This beam is to resist a bending moment equal to M, which is a greater moment than M_1, which is the bending moment the beam could resist if balanced design existed, that is, if the allowable stresses f_c and f_s were realized simultaneously, as shown in Fig. 65*b*. For balanced design, the tensile steel required to resist the bending moment M_1 is designated as A_{s_1} and is equal to $p \times b \times d$, or $A_{s_1} = M_1/f_s jd$. The derivation of p (the steel ratio) for balanced design was presented in Art. 54, and from it was obtained the expression

$$p = \frac{f_c k}{2f_s}$$

If $\Delta M = M - M_1$, some provision must be made to take care of this increment of moment. This can be done if the compression and tensile areas are increased, which can be accomplished by putting compression steel A_s' in the top of the beam and by increasing the area of the tensile steel by ΔA_s. If the compression steel is placed 3 in. below the top of the beam, and if C' = the added compression due to the compression steel and T' = the added tension in the tensile steel, then,

$$\frac{\Delta M}{d - 3} = C' = T'$$

The stress in the concrete adjacent to the compression steel is $\frac{kd - 3}{kd} \times f_c (= f_{c_y})$, and if C' is divided by this value the additional transformed area required to resist ΔM will be obtained. The transformed area of the compression steel is nA_s', but since the steel replaces an equal area of effective concrete the additional effective transformed area in compression is $(n - 1)A_s'$. Therefore

$$A_s' = \frac{C'}{f_{c_y}(n - 1)}$$

and

$$\Delta A_s = \frac{T'}{f_s} \quad \text{or} \quad A_s = A_{s_1} + \Delta A_s$$

The added steel is indicated in Fig. 65*d*. The complete beam is shown in Fig. 65*e*.

It should be obvious that the addition of this steel, both A_s' and ΔA_s, makes no change in the location of the neutral axis. In obtaining the required areas of the added steel, it was assumed that f_c and f_s were unchanged, and, since $k = \frac{f_c}{f_c + (f_s/n)}$, there is no appreciable change in this value as long as the actual added amount of steel is close to the required amount.

This method of analysis for beams with compression reinforcement is simple, direct, and equally effective whether the beam is being designed or being checked for sufficiency of steel. The method is not approximate as it is where some assumed value of j is used (generally 0.9 for beams of this type, although that would probably be large for shallow beams). In checking the sufficiency of the compression steel, as at a support, it is necessary only to determine the amount required to resist the bending moment. If more is needed, add it; if there is an excess, the beam is satisfactory.

61. Illustrative Problem. If we assume a continuous beam with a negative bending moment at the support of 350 ft-kips = 4200 in.-kips and an end shear requiring an area of 464 sq in., the computations for its proportioning would be as below. The area of 464 sq in. required for shear was determined in accordance with the principle presented in Art. 64. The positive moment at the center is assumed to be less than the end moment.

The allowable stresses are as follows:

$$f_c' = 3000 \text{ lb per sq in.}$$

$$f_s = 20{,}000 \text{ lb per sq in.}$$

$$f_c = 1350 \text{ lb per sq in.}$$

$$n = 10$$

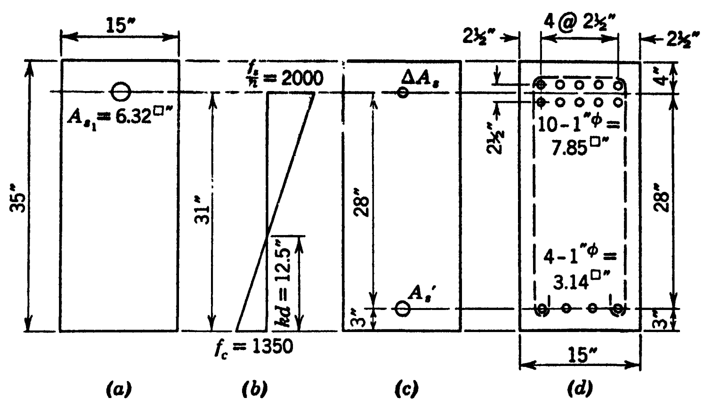


FIG. 66

Take $b = 15$ in., and $d = 31$ in. $b \times d = 465$ sq in. A beam of these dimensions is shown in Fig. 66*a*, and the stress variation is shown in Fig. 66*b*. The proportioning for a balanced design with no compression steel would be:

$$k = \frac{1350}{1350 + 2000} = 0.403 \qquad kd = 0.403 \times 31 = 12.5 \text{ in.}$$

$$j = 1 - \frac{0.403}{3} = 0.866 \qquad jd = 26.8 \text{ in.}$$

$$C = \frac{1350}{2} \times 12.5 \times 15 = 126{,}500 \text{ lb} = T$$

$$A_{s_1} = \frac{126{,}500}{20{,}000} = 6.32 \text{ sq in.}$$

The bending moment this beam can resist is

$126.5 \times 26.8 = 3390$ in.-kips $= Rbd^2$, or moment for balanced design

$4200 - 3390 = 810$ in.-kips

If the compression steel is 3 in. above the bottom,

$$\Delta A_s = \frac{810}{28 \times 20} = 1.45 \text{ sq in.}$$

$$A_s = 6.32 + 1.45 = 7.77 \quad (10\text{—1-in. round} = 7.85 \text{ sq in.})$$

$$f_{c_y} = \frac{9.5}{12.5} \times 1350 = 1026 \text{ lb per sq in.}$$

$$A_s' = \frac{810}{28(10 - 1)1.026} = 3.13 \quad (4\text{—1-in. round} = 3.14 \text{ sq in.})$$

The indicated additional steel and moment arm are shown in Fig. 66*c*, and the total steel areas are shown in Fig. 66*d*. Because the actual areas exceed the required areas, the theoretical location of the neutral axis for the areas and dimensions shown is computed below, although this calculation is unnecessary.

$$\frac{15x^2}{2} + 9(3.14)(x - 3) = 78.5(31 - x)$$

$$x = 12.54 \text{ in.}$$

As expected, the neutral axis is, for all practical purposes, in the original location dictated by balanced design. It should be pointed out here that the compression steel is not stressed to its capacity. Its stress is determined by the strain in the concrete to which it is bonded.

62. Effect of Time Yield. When a load is maintained on concrete over a period of time, there is a slow gradual increase in the deformation. This increase in deformation, although small, may continue over a period of several years. This is known as time yield or plastic flow. It should be clear that the effect of this progressive yield causes the steel to take an increasing amount of the load or stress to be resisted.

The effects of shrinkage and volume changes have bearing on the stresses. However, a detailed discussion of this phenomenon is beyond the scope of this volume; therefore, application of the more or less empirical methods recommended in the specifications is all that will be attempted.

To take care of this increased stress in the steel, the Joint Committee recommends that in a beam or girder having compression reinforcement the effectiveness of such reinforcement in resisting bending may be taken at twice the value indicated from the calculations; i.e., twice n times the stress in the adjacent concrete, as discussed in Art. 60. "In no case should a stress in compression reinforcement greater than 16,000 * lb per sq in. be allowed."

* The "A.C.I. Building Code" (1946) permits this value to be the same as in tension.

If this recommendation is followed in the design of the beam for which computations were made in Art. 61, the following results are obtained.

With the same data and Fig. 66, the section shown in Fig. 66*a* can resist a bending moment of 3390 in.-kips as calculated in the previous article, leaving a moment of 810 in.-kips to be resisted by the compressive and added tensile steel. From this set of computations the stress in the compression steel 3 in. from the bottom is 10,260 lb per sq in.; 10,260 × 2 = 20,520 lb per sq in. Since this is greater than 16,000 lb per sq in., the latter is the allowable unit stress in the steel. Let n' = the ratio between the allowable steel stress and the stress in the adjacent concrete. Then

$$n' = \frac{16{,}000}{1026} = 15.59$$

$$A_s' = \frac{810}{28 \times 14.59 \times 1.026} = 1.93 \text{ sq in.} \quad (3\text{—}1\text{-in. round} = 2.36 \text{ sq in.})$$

A similar line of reasoning is that each square inch of steel resists 16,000 − 1026 = 14,974 lb more than 1 sq in. of concrete.

$$A_s' = \frac{810}{28 \times 14.974} = 1.93 \text{ sq in.}$$

The 3—1-in. round bars give more than the required area but are apparently the best combination.

$$\Delta A_s = 1.45 \text{ sq in.} \quad \text{as previously calculated.}$$

The beam will be the same as that shown in Fig. 66*d* except that 3—1-in. round bars will be used instead of the 4—1-in. round bars as shown in the figure.

63. Limited Depth Beams with Tension Steel. Where a beam is limited in depth and width but must resist a larger bending moment than can be resisted by this section as a balanced design, a greater resistance to bending can be accomplished by increasing the area of the tensile steel. This increase in steel area changes the location of the neutral axis and thus increases the area of the compressed concrete. This also tends to decrease the value of jd, but the increased compression area offsets this to the extent of increasing the carrying capacity of the section.

The authors do not recommend this procedure as being at all economical in the normal design problems. In such instances it will be found that, if some of the additional steel is added to the compression side of the beam, a smaller total amount is required to resist the additional moment than if the total addition is made to the tensile steel. For very shallow beams or slabs, however, it may be found economical to add tensile steel only. This is because any compression steel, with proper embedment, will be so close to the neutral axis that it will not be sufficiently stressed to have much effect in resisting the moment.

Where a slab is proportioned for proper depth in an interior span, it is often desirable to maintain the same depth in the end span. Since this will not be as deep as that needed for a balanced design, on account of greater moments in the end span, the additional moments can be resisted with compression steel or by adding only to the tensile steel.

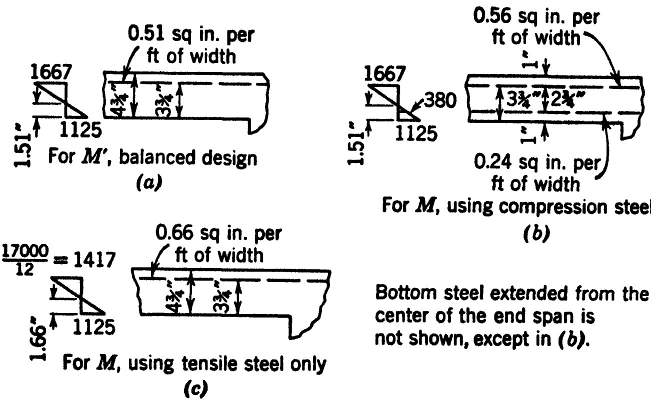


FIG. 67

Figure 67 shows the depth and location of the steel for the slab designed in Art. 57. At the first interior support the moments are (using the same data as in Art. 57)

$$M = \left(-\frac{300}{8} - \frac{83}{9}\right)8^2 = 2990 \text{ ft-lb} = 35{,}880 \text{ in.-lb}$$

This slab has a depth of 3.75 in., but the depth required for a balanced design for the above moment is

$$d = \sqrt{\frac{2990}{196 \times 1}} = 3.91 \text{ in.}$$

For the slab shown in Fig. 67 with the steel located as shown, the calculations for compression steel required are as follows:

$$kd = 0.403 \times 3.75 = 1.51 \text{ in.} \qquad jd = 3.25 \text{ in.}$$

$$\frac{1125}{1.51} \times 0.51 = 380 \text{ lb per sq in.} = \text{stress in concrete adjacent to compression steel}$$

$$C' = \tfrac{1125}{2} \times 1.51 \times 12 = 10{,}200 \text{ lb} = T'$$

$$M_1 = 10{,}200 \times 3.25 = 33{,}150 \text{ in.-lb}$$
$$M = 35{,}880 \text{ in.-lb}$$

$$\text{Moment to be provided for} = 2{,}730 \text{ in.-lb}$$

$$\frac{10{,}200}{20{,}000} = 0.51 \text{ sq in.} = A_{s_1}$$

$$\frac{2730}{2.75 \times 20{,}000} = 0.05 \text{ sq in.} = \Delta A_s$$

$$0.56 \text{ sq in.} = \text{required } A_s$$

$$\frac{2730}{380 \times 11 \times 2.75} = 0.24 \text{ sq in.} = A_s'$$

$$0.80 \text{ sq in.} = \text{total steel required}$$

To increase the compression area by means of added tensile steel it first is necessary to determine the compression area required. Let $x = kd$.

$$C = \tfrac{1125}{2} \times 12x = 6750x$$

$$6750x\left(3.75 - \frac{x}{3}\right) = 35{,}880$$

Solving this quadratic equation, $x = kd = 1.66$ in.

$$C = \tfrac{1125}{2} \times 12 \times 1.66 = 11{,}200 \text{ lb} = T$$

$$f_s = \frac{1125}{1.66} \times 2.09 \times 12 = 17{,}000 \text{ lb per sq in.}$$

$$\frac{11{,}200}{17{,}000} = 0.66 \text{ sq in.} = A_s$$

The total steel required in this case is 0.66 sq in. per ft of width of slab rather than the total of 0.80 sq in. required when compression steel was used. This is an example of economy in increasing the area of the tensile steel. The fact that the tensile steel would be carried through to the support and thus give all the compression steel needed detracts somewhat from the illustration, but in some instances there would be a possible economy in strengthening a beam or slab in this manner.

64. Shear and Diagonal Tension. Shear was discussed in Art. 15 of Chapter 1 and in Art. 21 of Chapter 2; therefore here it will be discussed only with regard to its distribution in a beam of reinforced concrete.

The beam shown in Fig. 68*a* is subjected to a positive bending moment with the tension on the bottom and with no compression steel. Since V is equal to the rate of change of moment, it also represents the increment of change in moment in a unit length of beam; consequently the value of ΔT in Fig. 68*b* is equal to V/jd. Since it is assumed that there is no tension in the concrete below the neutral axis, ΔT is also equal to the

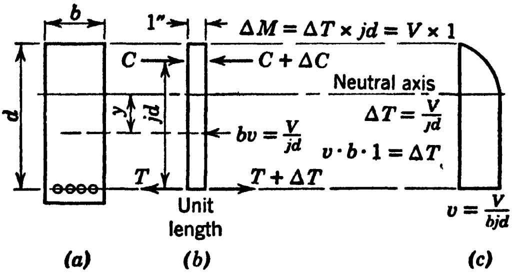


FIG. 68

total horizontal shear on a horizontal area any distance y below the neutral axis of this unit length of beam. Therefore it can be stated that the value of the unit horizontal shear on the tension side of the neutral axis of a reinforced concrete beam can be expressed by $v = V/bjd$. As stated in Chapter 1, this is also the value of the unit vertical shear.

Since ΔT represents the total change in tension on the tensile side of the neutral axis, it follows that the unit shear will be a constant on that

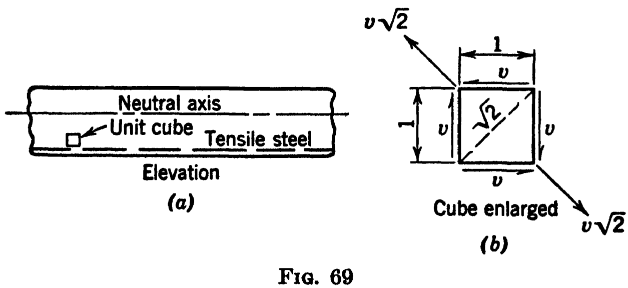


FIG. 69

side and will be as represented in the lower part of Fig. 68*c*. This is also true for T beams and beams with compression steel. Where there is no compression steel, as in Fig. 68, the action above the neutral axis is no different from the action on the compression side of any other rectangular beam; therefore Fig. 68*c*, with the parabolic distribution shown on top, gives the correct unit shear distribution for the beam shown.

In Fig. 69*a* is shown a portion of a beam in elevation, and in Fig. 69*b* a small unit cube taken from the tensile side of the beam with the shearing stresses shown thereon. Shear itself is not a matter of concern in concrete, but the tensile principal stresses are, since the concrete is weak in tension, and these tensile principal stresses will be a maximum where the shear is a maximum and where there is no compression. Consequently, the computations have been made at the location shown. Since the concrete is considered as taking no direct tension, this location has the added advantage of simplifying the work. It is believed that the relations shown in Fig. 69*b* are clear. The shearing forces can be combined into resultant tensile forces, each equal to $v\sqrt{2}$, and, since this force acts on an area equal to $\sqrt{2}$, the unit diagonal tension is v, which is the same value as the unit shearing stress.

This diagonal tension is a tensile stress that must be resisted by the concrete, or by steel, and, since the concrete is weak in tension, this weakness will act as a limit on the allowable shear. The specifications generally assume that the concrete can safely resist a unit tensile stress of $0.02f_c'$, and any diagonal tension beyond that value must be resisted by web reinforcement. Where the reinforcing steel has ordinary anchorage the maximum allowable unit diagonal tension, or shear, is usually given as $0.06f_c'$. This will be discussed more fully when web reinforcement is considered.

65. Bond and Anchorage. When there is a change in stress in the reinforcing steel in a reinforced concrete beam, this change is transferred from the steel to the concrete by means of the bond between the two materials, and this bond is analogous to the rivets or welding in the flanges of a plate girder. These bond stresses are resisted by the horizontal shear in the concrete.

Figure 68 shows that the value of ΔT for a unit length of beam is V/jd. If this value of ΔT is divided by the area of contact, which is the sum of the perimeters of the bars times the length in question, the expression $u = V/\Sigma ojd$ is obtained, in which u is the unit bond stress and Σo is the sum of the perimeters of the tensile steel at the section where the value of the total shear is V. In most specifications the allowable value of u is given as $0.05f_c'$ for deformed bars.

In the end anchorage of steel in flexural members or in members in direct stress, the bond stress is assumed to be uniformly distributed over the embedded length. For bars with bent or hooked ends the length of bend or of hook is considered as a part of the embedded length. The dimensions for hooks are given in the various specifications and in the *Proposed Manual of Standard Practice for Detailing Reinforced Concrete Structures*, published by A.C.I.

The bond stress in the tensile steel of the beam shown in Fig. 66 will be computed, assuming the total shear at the face of the support as

75 kips. The value of j will be assumed as $\frac{7}{8}$. For 10—1-in. round bars, $\Sigma o = 31.42$.

$$u = \frac{75{,}000}{31.42 \times \frac{7}{8} \times 31} = 88 \text{ lb per sq in.}$$

The allowable value is $0.05 \times 3000 = 150$ lb per sq in.

Since this steel is at a point of maximum moment, it is also necessary to check for the length of embedment required in order to secure proper anchorage.

$$\frac{20{,}000 \times 7.85}{31.42 \times 150} = 33.3 \text{ in. embedment}$$

The extension of the compression steel should be checked for proper embedment as well as the tensile steel.

$$\frac{10{,}260 \times 3.14}{12.57 \times 150} = 17.1 \text{ in.}$$

66. Stirrup Spacing. Where the unit diagonal tension exceeds the value of $0.02f_c'$, the specifications require that web reinforcement be provided to resist all diagonal tension above this value, the concrete resisting all diagonal tension up to $0.02f_c'$. As stated in Art. 64, the specifications limit the maximum unit shear (and diagonal tension) to $0.06f_c'$ for ordinary anchorage but permit higher values where there is special anchorage. However, the 1940 Joint Committee specifications provide that, where the values are greater than $0.06f_c'$ under conditions of special anchorage, web reinforcement must be provided to resist all the diagonal tension, the concrete not being considered as acting. Special anchorage (called end anchorage by the 1940 Joint Committee) is defined in the specifications but will not be further discussed here; in all following discussions it will be assumed that ordinary anchorage is provided.

The most common type of stirrups are vertical U stirrups as they are easier to place and the labor costs are consequently lower. Stirrups of this type are shown in Figs. 64, 65, and 66. The hooks may be turned in or out, and the angle of bend may be 90 deg or 135 deg or any value between 90 deg and 180 deg. The principles involved in computing the proper spacing for vertical stirrups are extremely simple and will therefore be discussed first. Discussion of the proper spacing for inclined stirrups will follow.

In calculating the proper spacing for vertical stirrups it is assumed that the vertical component of diagonal tension, above that resisted by the concrete, is resisted by the stirrups and that the horizontal component of diagonal tension is resisted by the longitudinal steel. Since the unit horizontal component is equal to the vertical, they are both equal to v, and therefore the total horizontal component for a unit length and width b is equal to V/jd, or the change in stress in the steel. There is, therefore, always a sufficient area of longitudinal steel to resist this horizontal component of diagonal tension.

In Fig. 70 is shown a short longitudinal elevation of a beam having vertical stirrups with a spacing s. Let

V' = the total vertical shear to be resisted by the web reinforcement ($V-V'$ being that resisted by the concrete).

s = the stirrup spacing corresponding to the value of V'.

A_v = the area of the stirrup (area of 2 bars for a U stirrup).

$$v' = \frac{V'}{bjd}.$$

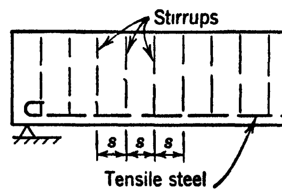


FIG. 70

Therefore

$$v'bs = A_v f_s$$

or

$$s = \frac{A_v f_s jd}{V'}$$

As stated previously in regard to other examples, this is not a formula to be remembered but is merely a statement of relationship in finding the proper spacing for vertical stirrups. The principles and relationships for vertical stirrups are so simple that it hardly seems necessary to show them in this manner. The basic principle is simply that the vertical

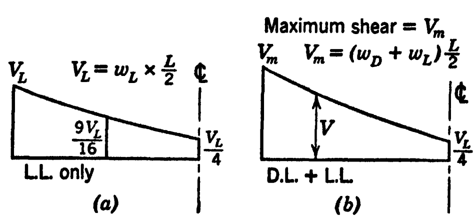


FIG. 71

stirrups will resist that part of the vertical component of diagonal tension that is in excess of what can be safely resisted by the concrete, and that the stirrups must be so spaced that the steel will not be overstressed. The student should not assume that this is an exact procedure. It is purely approximate, but it gives results that have proved satisfactory in the past and represent standard practice in design.

The value of shear to be used in computing stirrup spacing is the maximum shear at the section considered. In Fig. 71 are shown curves of maximum shears for the left half of a simple beam. As seen from the figure, the maximum shear is represented by a curve that is concave upward. In spacing stirrups, however, the common practice is to consider the maximum shear curve to be a straight line between the end and the center, and the curve of maximum shears shown in Fig. 71*b* then appears as the one shown in Fig. 72*a*. In Fig. 72*b* the shear is

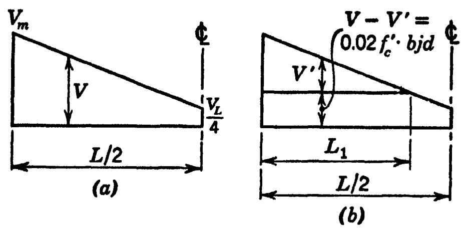


FIG. 72

shown separately as that representing the diagonal tension to be resisted by the stirrups (V'), and that to be resisted by the concrete. L_1 represents the length, on each end of the beam, where stirrups are required.

Figure 73*a* is similar in shape to Fig. 72*b*, but the values shown represent the total vertical component of diagonal tension across a width of beam b, per unit length. In Fig. 73*b* the values are given for the beam

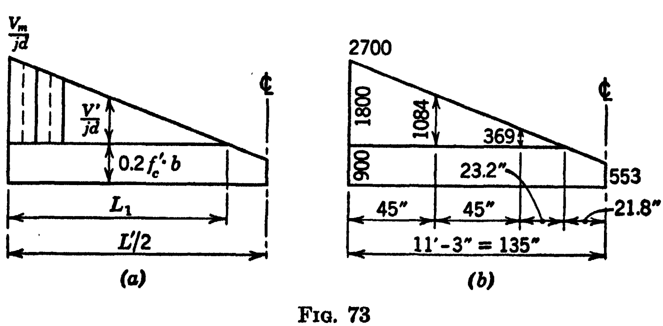


FIG. 73

shown in Fig. 66, assuming a dead load of 1 kip per ft, a live load of 5 kips per ft, and a span of 24 ft between centerlines of supports or 22.5 ft between faces of supports. The dead load has been assumed much larger than the weight of the beam to allow for weights of other parts of the structure.

Some texts show methods of computing stirrup spacing by dividing the upper triangle in Fig. 73*a* into a series of equal-area trapezoids (two of which are shown in the figure), each area being equal to the value of one stirrup, the stirrup being considered as being in the center of the trapezoid. Since stirrups are not thus placed, it is not considered that this method has any advantage over the simple method illustrated below.

For the beam used in Fig. 73*b* the end shear is $(1.10w_L + w_D)\frac{L'}{2} = (1.10 \times 5 + 1)\frac{22.5}{2} = 73.13$ kips (11.25 kips D.L. and 61.88 kips L.L.), and the shear at the center would be 15 kips $= (5 \times \frac{24}{2} \times \frac{1}{4})$. The values shown in the figure are obtained as follows:

$$\frac{73{,}130}{\frac{7}{8} \times 31} = 2700 \text{ lb per in.}$$

$$\frac{15{,}000}{\frac{7}{8} \times 31} = 553 \text{ lb per in.}$$

The approximate value of $\frac{7}{8}$ was assumed for the value of j as it represents conservative practice. This is the usual value for rectangular beams, and 0.9 is a common value for T beams. Rectangular beams with compression reinforcement, such as this, are frequently computed for $\frac{7}{8}$, although 0.9 or the computed value is preferred by some designers.

The vertical component of diagonal tension resisted by the concrete is $0.02f_c'$, or $0.02 \times 3000 = 60$ lb per sq in. ($60 \times 15 = 900$ lb per in.). The diagonal tension to be resisted by the stirrups per inch length of beam is $2700 - 900 = 1800$ lb per in. at the end of the beam. Assuming a straight-line distribution of shear and computing the ordinates at the sixth and third points of the beam span, 1084 and 369 are obtained for the values to be resisted by the stirrups at these points, respectively. Assuming $\frac{5}{8}$-in. round U stirrups and a stress of 16,000 lb per sq in. (the Joint Committee specifies 16,000 lb per sq in., regardless of the grade of steel; however, A.C.I. permits the same for web reinforcement as is used in main tensile steel), the capacity of one U stirrup is $0.61 \times 16{,}000 = 9760$ lb.

$$9760 \div 1800 = 5.42 \text{ in.}$$

$$9760 \div 1084 = 9.00 \text{ in.}$$

$$9760 \div 369 = 26.45 \text{ in.}$$

These represent the required spacings at the respective points. From these values the curve shown in Fig. 74 is obtained. The curve has been drawn through the three computed points. The horizontal line at the ordinate of $15\frac{1}{2}$ represents the maximum allowable spacing which is usually limited by the specifications to one-half the depth of the beam. The distance from the center where stirrups are no longer needed can be computed thus:

2700	900
553	553
2147	347

$$\frac{347}{2147} \times 135 = 21.8 \text{ in.}$$

For this beam, the end spacing would be assumed to be $5\frac{1}{4}$ in., and the first stirrup placed $2\frac{1}{2}$ in. from the left end. There would be three or four stirrups put in at this spacing, then a second group at the spacing permitted where this

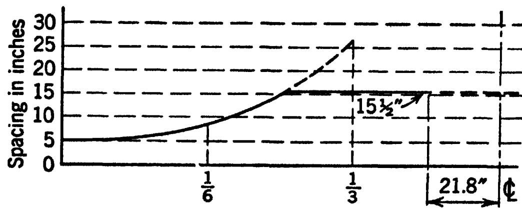


FIG. 74

new group starts, and so on to the point of maximum permissible spacing of $15\frac{1}{2}$ in. On account of the labor costs involved in the bending and the greater ease of placing concrete where the spacing is larger, it is desirable not to have stirrups too small in diameter, but they must be consistent with proper stress in bond. If the diameter of the stirrup is no larger than $\frac{1}{50}$ of the depth d, this latter requirement can usually be met if hooked ends of normal size are used and if f_c' is 2500 lb per sq in. or more. On account of the bending involved it is desirable that about $\frac{3}{4}$ in. be the limit of diameter of the stirrups. If this gives too small a spacing in large beams, W stirrups can be substituted.

67. Inclined Stirrups. It is believed by some designers that inclined stirrups are more efficient than vertical stirrups, especially those at an angle of 45 deg; this idea is based on the fact that the maximum diagonal tension acts at an angle of 45 deg. Since the vertical stirrups together with the horizontal steel are capable of resisting both components of the diagonal tension, there is no reason why they should not be entirely satisfactory. It is true that for higher shear stresses in cases of special anchorage inclined stirrups better meet the requirements of the specifications for the number of stirrups to be intersected by any 45-deg line extending downward and toward the nearest support from the mid-depth of the beam.

The 1940 Joint Committee specifications state that inclined stirrups are assumed to contribute to diagonal tension resistance only the component of the total stirrup strength that lies in a direction of 45 degrees with the axis of the beam. On this assumption the following formula is obtained:

$$s = \frac{A_v f_s jd(\cos X + \sin X)}{V'}$$

in which the nomenclature is the same as that previously given, except that X = the angle the stirrups make with the axis of the beam, s still being the longitudinal spacing.

The above formula is a general expression for stirrup spacing if one should be desired. If

$$X = 90 \text{ deg} \qquad s = \frac{A_v f_s jd}{V'}$$

and if

$$X = 45 \text{ deg} \qquad s = \frac{A_v f_s jd \times 1.414}{V'}$$

The last value shows that for stirrups inclined at 45 deg the horizontal spacing is 1.414 times that of vertical stirrups of the same size, but, since the length of the inclined stirrup is 1.414 times that of the vertical stirrup, the same volume of steel is required. Inclined stirrups are more expensive to place than vertical stirrups.

68. Bending Tensile Reinforcement. For any beam of constant depth the value of jd is essentially a constant throughout the entire length of span, and therefore the area of tensile steel required is a direct function of the bending moment. For a simple beam the points of theoretical cutoff of the tensile steel bars can be obtained from the moment diagram, the same as was done for cover plates in a plate girder. The problem is analogous. Each bar must extend beyond its theoretical point a sufficient distance to develop the stress in the bar in bond.

Since most concrete beams are continuous or have restrained ends, it is desirable to have the tension steel continuous throughout the beam, and the usual problem is to find the points where the bars can be bent. In Fig. 75 are shown the curves of maximum positive and negative moment for a span of a continuous beam. If at the major positive and negative ordinates of these moments there are laid off, to some scale, the required areas of tensile reinforcement (or the number of bars where all bars are equal in size), the point where any bar or pair of bars can be bent up or down can be found by drawing horizontal lines through the proper points. The bars bent up should match some corresponding bars to be bent down. Bars are usually bent in pairs, as it is desirable to keep the steel symmetrical about the vertical longitudinal axis. The points of bend are usually adjusted so that the angle of bend is the same throughout the beam. The possibility of negative moment at the center of a span should be investigated.

Where bars are bent, as in Fig. 75, the inclined rods can be considered as inclined stirrups, and vertical stirrups can be omitted for the length of

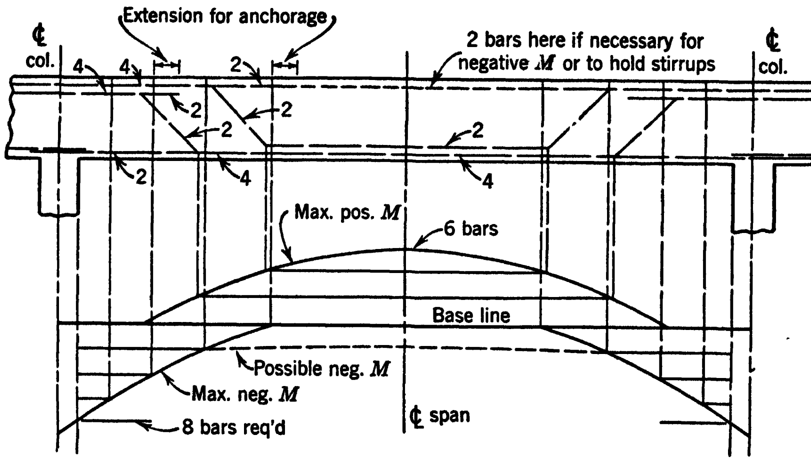


FIG. 75

beam where the inclined stirrups are effective. Where this length is small, there is no real economy in the omission of the one or two vertical stirrups.

69. Design of Reinforced Concrete Columns for Axial Loads. The design of members subjected to compression only does not properly belong in a text on flexure. This was discussed in Chapter 4 in connection with steel columns. Like steel columns, reinforced concrete columns are often subjected to flexure, and it is necessary to study the effect of compressive loads alone before combining these stresses with the stress due to bending. In fact, it is very rare that flexural stresses can be avoided in reinforced concrete columns.

Reinforced concrete columns may have a variety of shapes in cross section, but they are usually either square, rectangular, or round. The reinforcement consists of longitudinal bars spaced around the perimeter of the column and inside the surface a distance of 2 in. to 3 in. under normal conditions. These longitudinal bars are held in position by lateral ties or a helix (commonly called spiral reinforcement) and are tied to them by means of wiring, welding, or fasteners. The spiral is used for a circular arrangement of the longitudinal reinforcement, and lateral ties for rectangular arrangements, as shown in Fig. 76.

The lateral ties are encircling bars ($\frac{1}{4}$ in. to $\frac{3}{8}$ in. in diameter), but they are not continuous like the spiral (helix), and they are spaced at intervals that prevent buckling of the longitudinal bars. The spirals have a closer spacing than the ties, and spiral columns are designed to resist larger loads than tied columns of the same areas.

If, as before, the concrete is assumed to have a constant modulus of elasticity E_c for stresses within the working range, a reinforced concrete column can be designed on the basis of transformed sections. Some specifications still provide for this method of design, although those of the

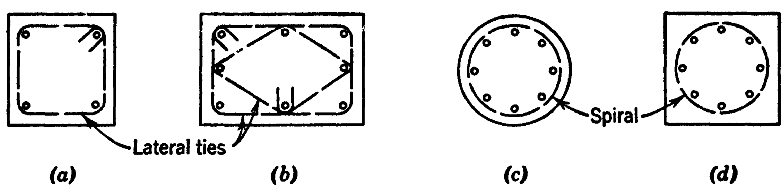


FIG. 76

Joint Committee and the American Concrete Institute require the steel to resist a larger portion of the stress. The first part of the following discussion will be based on transformed sections as that is the foundation for the entire theory of reinforced concrete design. Afterward the recent modifications to take account of time yield will be discussed and illustrated.

The list of definitions of symbols is given below.

A_g = overall or gross area of the column.
f_a = permissible compressive stress in concrete (usually about one-half of allowable stress in flexure).
f_s = working stress in longitudinal steel.
A_s = area of longitudinal steel.
p_g = ratio of longitudinal steel to gross area of column = A_s/A_g.
n = E_s/E_c.
P = axial load on the column.

The transformed area of the column is the area of the concrete $(A_g - A_s)$ plus the transformed area of the steel (nA_s), or $A_g + (n - 1)A_s$.

The load a column can resist is then

$$P = f_a[A_g + (n - 1)A_s]$$

Since

$$A_s = p_g \times A_g$$

$$P = f_a[A_g + (n - 1)p_g \times A_g] = f_a A_g[1 + p_g(n - 1)]$$

or

$$A_g = \frac{P}{f_a[1 + p_g(n - 1)]} \tag{1}$$

This expression is valuable in determining the required gross area of a column for any desired value of p_g within its allowable range, or it can be applied in solving for f_a for a given column with a given load under the assumption, $f_s = nf_a$. It is applicable for either a tied column or a spiral column, with the proper value of f_a in each case, although in some of the older specifications the area of the core would be substituted for A_g for spiral columns. The area of the core is defined by the Joint Committee as the area within the out-to-out dimensions of the spirals. A column with lateral ties could be either square or rectangular, using the proper dimensions to obtain the value of A_g.

The following relations and requirements are illustrative of those in recent specifications. The ratio of steel area to gross area p_g should lie between 0.01 and 0.04 for lateral ties, and between 0.01 and 0.08 for spirals. The minimum-size bar for longitudinal reinforcement should be $\frac{5}{8}$ in. round, and at least 4 are needed with ties and 6 with spirals. When more than 4 bars are used with lateral ties, additional ties should be employed to hold the additional bars properly in position, as in Fig. 76*b*. Lateral ties should be at least $\frac{1}{4}$ in. in diameter and spaced apart not over 16 bar diameters, 48 tie diameters, or the least dimension of the column. The longitudinal reinforcement should be placed with a clear distance from the column surface of at least $1\frac{1}{2}$ in. plus the diameter of the tie. For spirals the core diameter is the distance out to out of spirals. Spiral bars may be $\frac{1}{4}$ in. in diameter for core diameters up to 18 in., and they should be $\frac{3}{8}$ in. round for core diameters above 18 in. The center-to-center spacing of the spirals should not exceed one-sixth of the core diameter, with a clear spacing not over 3 in. nor less than $1\frac{3}{8}$ in. or $1\frac{1}{2}$ times the maximum size of coarse aggregate.

This discussion of the transformed section method should be clear in its application and need no further illustration. The more recent requirements in column design, as given in the Joint Committee specifications, will be discussed and illustrated below. For both requirements it is assumed that the column is in the short column range and will not require a reduction in allowable stress. For short columns the length should not exceed 10 times the least lateral dimension.

The Joint Committee requirements for column design have been modified in their 1940 specifications as follows for spirally reinforced columns.

$$P = 0.225 f_c' A_g + A_s f_s \qquad (2)$$

f_s = 16,000 lb per sq in. for intermediate grade steel, and 20,000 lb per sq in. for hard grade or rail steel.

A_g and A_s are as defined above, and f_c' is the ultimate strength at twenty-eight days, as previously defined.

For tied columns the values are to be 80 per cent of those given in formula 2 above.

It will be observed that the area of the steel has not been subtracted from the gross area in formula 2. The Joint Committee states that the effect of this small area was taken into account in arriving at the coefficient 0.225.

70. Illustrative Example. An interior column supporting a floor structure has a total axial load of 540 kips with no bending when all adjacent panels are loaded. A tied column will be used.

The capacity of the tied column is given by

$$P = 0.180 f_c' A_g + 0.8 A_s f_s$$

or

$$P = 0.180 f_c' A_g + 12{,}800 p_g A_g$$

if intermediate grade steel is used. Assuming f_c' = 3000 lb per sq in., and estimating $p_g = 0.03$,

$$A_g = \frac{540{,}000}{0.18 \times 3000 + 12{,}800 \times 0.03} = \frac{540{,}000}{540 + 384} = 584 \text{ sq in.}$$

$$A_s = 0.03 \times 584 = 17.52 \text{ sq in.}$$

	540,000 lb
12—1¼-in. square bars = 18.75 sq in. × 12,800 =	240,000
	300,000 lb

$$\frac{300{,}000}{540} = 556 \text{ sq in.} \qquad \sqrt{556} = 23.6 \text{ in.}$$

A square column 23¾ by 23¾ in. is satisfactory, or a rectangular column (20 × 28 = 560) if it best fits the dimensions of the beams and girders framing into the column. Obviously, a column 24 by 24 in. can be used, and some designers would; however, it requires more concrete and does not agree with the nominal size of form lumber any better than a column 23¾ by 23¾ in.

The cross section of the column is shown in Fig. 77. If it is desirable to have more steel, the size of the column should be reduced; and if it is desired to use

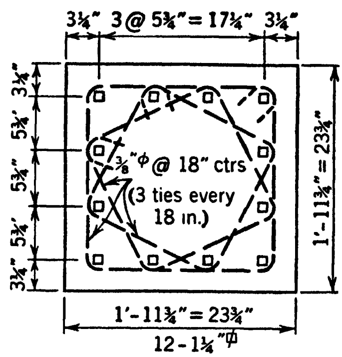


FIG. 77

only 8 bars, a larger column is required. This decision can be made after the first trial computation is made for A_g.

With $\frac{1}{4}$-in. round ties:

$48 \times \frac{1}{4} = 12$ in. (use 12 in. center to center)

$16 \times 1\frac{1}{4} = 20$ in. Least dimension of column $= 23\frac{3}{4}$ in.

With $\frac{3}{8}$-in. round ties:

$48 \times \frac{3}{8} = 18$ in. (use 18 in. center to center)

$16 \times 1\frac{1}{4} = 20$ in. Least dimension of column $= 23\frac{3}{4}$ in.

An alternate manner for tying 12 bars can be found on p. 45 of *Proposed Manual of Standard Practice for Detailing Reinforced Concrete Structures,* published by A.C.I.

71. Design of Reinforced Concrete Columns with Bending. Bending in columns can be caused by lateral forces, by the column acting as a part of a continuous framework, or by an actual eccentricity of the axial load. Temperature changes in structures exposed to great changes in temperature may cause appreciable bending moments in columns, but the distribution of shears and moments is a problem in statically indeterminate structures and is not a problem of elementary theory and design. Whatever the cause of the bending moment in the column, it can be expressed, if desired or convenient, as an axial load times some eccentricity. The effect in the design of the column will be the same.

From the general expression for combined axial load and bending about one axis:

$$f = \frac{P}{A} + \frac{Mc}{I}$$

the following can be obtained:

$$f = \frac{P}{A_t} + \frac{Mc}{A_t r^2} \tag{3}$$

or

$$A_t = \frac{P}{f_a} + \frac{Pec}{f_c r^2} \tag{4}$$

where A_t = transformed area = $A_g[1 + (n - 1)p_g]$.
P = axial load.
e = eccentricity of the resultant load, measured from the gravity axis ($M = Pe$).
r = radius of gyration of the transformed area.
c = distance from the gravity axis to the extreme fiber in compression.
f_a = average allowable stress on an axially loaded column.
f_c = allowable stress in bending (usually $0.45f_c'$).

Formula 4 can be used with any values of f_a and f_c the designer chooses. The Joint Committee recommends

$$f_a = \frac{0.225f_c' + f_s p_g}{1 + (n - 1)p_g}$$

for spiral columns, obtained by dividing the carrying capacity of a column ($P = 0.225f_c'A_g + A_s f_s$), as given in Art. 70, by the transformed area. For tied columns, 0.8 of the above is specified.

If P is divided by the value of A_t as given in formula 4, the average stress on the transformed section is obtained for the column having the combined axial load and bending. If this average stress is multiplied by $[1 + (ec/r^2)]$, the following is obtained:

$$f_m = f_a \frac{1 + \dfrac{ec}{r^2}}{1 + \dfrac{f_a}{f_c}\dfrac{ec}{r^2}} \tag{5}$$

in which f_m = the maximum allowable compressive fiber stress.

A convenient expression can be obtained for the extreme fiber stress in the concrete from formula 3 by replacing M with Pe and A_t with $A_g[1 + (n - 1)p_g]$, obtaining

$$f = \frac{P}{A_g} \frac{1 + \frac{ec}{r^2}}{1 + (n - 1)p_g} \tag{6}$$

Expressions 5 and 6 are in the Joint Committee specifications. The authors have made a slight modification in symbols. These expressions, together with formula 4, can be applied to the design of reinforced concrete columns with bending with the values of allowable unit stress as recommended by the Joint Committee, or with any other specified values. Formula 6 gives the computed stress, which should not exceed the allowable stress as expressed by formula 5.

In preliminary design, for medium-size columns, it will be satisfactory to replace ec/r^2 by $6e/d$ for rectangular columns and $8e/d$ for round columns, with a fair degree of accuracy (d = overall depth of section).

The previous discussion in this article is based on an uncracked section where there is compression over the entire area or where the tension in the concrete is not greatly in excess of $0.02f_c'$. It gives very satisfactory results as long as the resultant load lies within the column, although some cracked concrete can be expected when $e = d/2$. Where larger eccentricities are encountered it will probably be necessary to ignore the cracked concrete. The determination of the neutral axis, in this situation, involves the solution of a cubic equation. A thorough discussion of the design of columns with appreciable tension in the steel is beyond the scope of this book, and the student is referred to one of the complete textbooks on reinforced concrete design for such a treatment.

Bending about two axes will not be discussed here as it is also beyond the scope of an elementary discussion of reinforced concrete members.

72. Illustrative Example. The example in Art. 70 was assumed to be an interior column with all four panels loaded. If only two adjacent panels on the same side of the column are loaded, the axial load will be reduced but there will be bending in the column. If only one panel is loaded, the load will be further reduced and there will be bending about two axes. This latter does not normally control in the design of the column. However, the reduced load with bending about one axis often does control in the design.

The column in Art. 70 is assumed, in the following calculation, to have an axial load of 380 kips and a bending moment of 1200 in.-kips.

$$e = \tfrac{1200}{380} = 3.16 \text{ in.}$$

Using the column designed in Art. 70 to check these conditions:

$$p_g = \frac{18.75}{23.75 \times 23.75} = \frac{18.75}{564} = 0.0332$$

$$f_a = \frac{0.18 \times 3000 + 12{,}800 \times 0.0332}{1 + 9 \times 0.0332} = \frac{965}{1.30} = 742 \text{ lb per sq in.}$$

If the approximate form of formula 4 is used,

$$A_t = \frac{380}{0.742} + \frac{1200 \times 6}{1.350 \times 23.75} = 737 \text{ sq in.}$$

$$A_g = \frac{737}{1.30} = 567 \text{ sq in.}$$

This is almost exactly the area used. A more precise check would give:

$$\begin{aligned} 564 \quad &\times \frac{23.75^2}{12} = 26{,}500 \\ 12.50 \times 9 = 113 \quad &\times 8.62^2 = 8{,}400 \\ 6.25 \times 9 = 56 \quad &\times 2.88^2 = 460 \end{aligned}$$

$$A_t = 733 \text{ sq in.} \qquad I_t = 35{,}360 \text{ in.}^4 \div 733 = 48.2 = r^2$$

Required

$$A_t = \frac{380}{0.742} + \frac{1200 \times 11.88}{1.350 \times 48.2} = 731 \text{ sq in.}$$

$$\frac{731}{1.30} = 562 \text{ sq in.} = \text{required } A_g$$

The column is satisfactory.

Instead of proportioning for area, some designers prefer to check a column of prescribed dimensions for the fiber stress. From formula 5 the maximum allowable stress would be

$$f_m = 742 \frac{1 + \dfrac{3.16 \times 11.88}{48.2}}{1 + \dfrac{742}{1350}\,\dfrac{3.16 \times 11.88}{48.2}} = 742 \frac{1.779}{1.428} = 924 \text{ lb per sq in.}$$

The computed stress on the extreme fiber from formula 6 is

$$f = \frac{380}{564} \frac{1 + \dfrac{3.16 \times 11.88}{48.2}}{1 + 9 \times 0.0332} = \frac{380 \times 1.779}{564 \times 1.30} = 0.922 = 922 \text{ lb per sq in.}$$

These computations show that the design chosen for the axial load is almost exactly that necessary for the reduced load combined with bending of 1200 in.-kips. The same design is satisfactory for an intermediate condition of bending moment and axial load. If a bending moment larger than 1200 in.-kips had occurred with the axial load of 380 kips, it would have been necessary to increase the concrete or steel area, or both.

Sometimes it is necessary to design a reinforced concrete column for an axial load combined with bending where an axial load alone does not enter into the design. An approximate area can be obtained from the simplified form of formula 4.

From the same data, $P = 380$ kips and $M = 1200$ in.-kips, a spirally reinforced column will be designed. It should be realized that the amount of bending moment that a column must resist increases as the stiffness of the column increases, and a decrease in column stiffness results likewise in a decrease in bending moment; however, the same load and moment are being assumed in the following calculations.

Assuming a value of p_g of about 0.04,

$$f_a = \frac{0.225 \times 3000 + 16{,}000 \times 0.04}{1 + 9 \times 0.04} = \frac{1315}{1.36} = 967 \text{ lb per sq in.}$$

It is necessary to make some estimate of the diameter of the column. Assuming $d = 25$ in.,

$$A_t = \frac{380}{0.967} + \frac{1200 \times 8}{1.350 \times 25} = 677 \text{ sq in.}$$

$$A_g = \frac{677}{1.36} = 498 \text{ sq in.}$$

This requires a diameter of $25\frac{1}{4}$ in. and a steel area of $0.04 \times 498 = 19.92$ sq in.

13—$1\frac{1}{4}$-in. square bars = 20.32 sq in.

16—$1\frac{1}{8}$ in. square bars = 20.25 sq in.

20—1-in. square bars = 20.00 sq in.

A column 25 in. in diameter with 20—1-in. square bars as shown in Fig. 78 will be checked. This column has a transformed area of 670.9 sq in. In Fig. 78 the column is shown as a square, 25 by 25 in. A spirally reinforced concrete column may be built as a square, octagonal, or other shaped section of the same least lateral dimension; however, in such cases, the allowable load, the gross area considered, and the required percentages of reinforcement should be taken as those of the circular column.

Computing the radius of gyration:

$$\frac{\pi}{64}(25)^4 = 490.9 \times \frac{625}{16} = 19{,}180$$

$$9 \times 20 \times \frac{\text{radius}^2}{2} = 180 \times \frac{\overline{10.06}^2}{2} = 9{,}110$$

$$670.9 \qquad 28{,}290 \div 670.9 = 42.2 = r^2$$

Checking stress:

$$p_g = \frac{20}{490.9} = 0.0407$$

$$f_a = \frac{0.225 \times 3000 + 16{,}000 \times 0.0407}{1 + 9 \times 0.0407} = \frac{1326}{1.366} = \text{ 971 lb per sq in.}$$

$$f = \frac{380}{490.9}\,\frac{1 + \dfrac{3.16 \times 12.5}{42.2}}{1.366} = \frac{380}{490.9} \times \frac{1.936}{1.366} = \text{ 1097 lb per sq in.}$$

$$f_m = 971\,\frac{1.936}{1 + \frac{971}{1350}\,(0.936)} = 971\,\frac{1.936}{1.673} = \text{ 1124 lb per sq in.}$$

The allowable stress (1124) is slightly greater than the computed (1097); therefore the column shown in Fig. 78 is satisfactory.

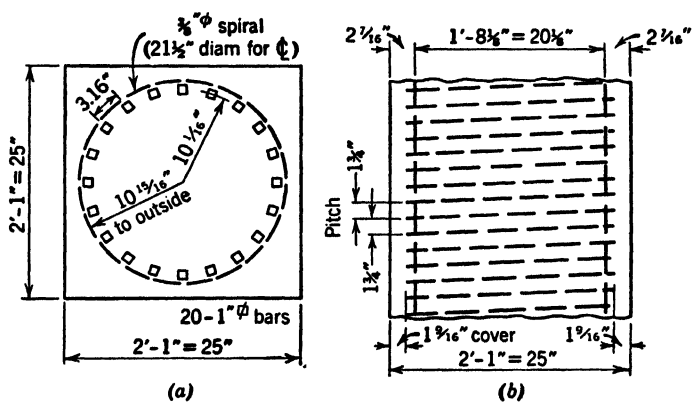


FIG. 78

Since the column has a core diameter ($21\frac{7}{8}$ in. out to out of spiral) greater than 18 in., a $\frac{3}{8}$-in. round spiral will be used. The ratio of spiral reinforcement p' should not be less than that shown in Formula 7.

$$p' = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f_c'}{f_s'} \tag{7}$$

where p' = ratio of volume of spiral reinforcement to volume of concrete core (out to out of spirals).

$\frac{A_g}{A_c}$ = ratio of gross area to core area of column.

f_s' = useful limit of stress of spiral reinforcement to be taken as 40,000 lb per sq in. for hot rolled rod of intermediate grade, 50,000 lb per sq in. for hard grade, and 60,000 lb per sq in. for cold drawn wire.

$$\text{Core area} = \frac{\pi \overline{21.88}^2}{4} = 376 \text{ sq in.}$$

$$p' = 0.45\left(\frac{490.9}{376} - 1\right)\frac{3000}{40{,}000} = 0.01031 \quad \text{(intermediate grade spiral)}$$

Volume of spiral per inch of column height = 0.01031 × 376 = 3.88 cu in.
Volume of spiral for one complete turn = $0.11 \times 21\frac{1}{2}\pi = 7.43$ cu in.
Maximum pitch = 7.43 ÷ 3.88 = 1.915 in., or $21\frac{7}{8} \div 6 = 3.65$ in., or 3 in. clear. The 1.915 in. controls.
Minimum clear distant between spirals = $1\frac{3}{8}$ in. Use pitch of $1\frac{3}{4}$ in., as shown in Fig. 78*b*.
The spacing of the 1-in. square bars around the spiral = $(20\frac{1}{8} \times 3.14) \div 20$ = 3.16 in., which is greater than the minimum requirement of 3 in.

FIG. 79. Hecht Company Garage, Washington, D. C., showing slab, beam, and girder construction in reinforced concrete. (Courtesy of Portland Cement Association.)

73. Illustrative Design of a Typical Floor Panel. The floor designed in steel in Art. 44 of Chapter 3 will be redesigned here as a reinforced concrete structure. The first three bays are shown in Fig. 80. It will be noted that additional columns are desired, making the interior panels 22 ft 6 in. by 27 ft 0 in. The floorbeams have been turned the other way (perpendicular to the length of the building) as it is generally preferable not to have the span of the floorbeams greater than that of the girders. The floorbeams are spaced 6 ft 9 in. This spacing should be slightly more economical than a spacing of 9 ft 0 in., although comparative designs would be necessary to be absolutely certain.

The recommendations for moment coefficients and shear given in Art. 56 of this chapter will be followed, as will other recommendations

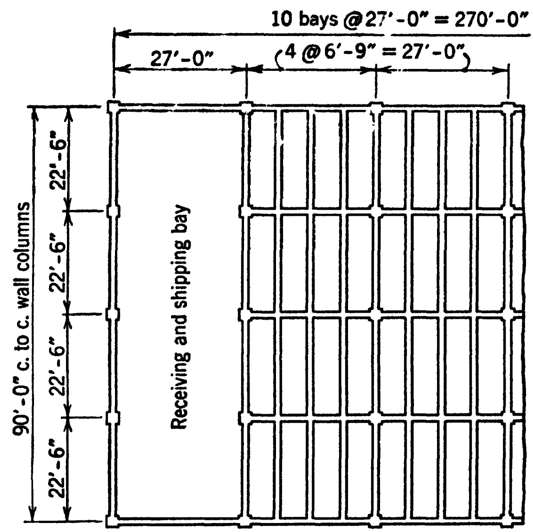


Fig. 80

given in articles following Art. 56. Aside from such deviations preferred by the authors, the recommendations of the 1940 Joint Committee will be followed. One deviation is the use of the transformed section, neglecting the effect of time yield. The effect of time yield will be briefly discussed at the end of this design, but the authors do not wish to detract from the application of the basic principles in the illustrative design.

The following values will be used:

$f_c' = 3000$ lb per sq in.
$n = 10.$
$f_c = 1350$ lb per sq in.
$f_s = 20{,}000$ lb per sq in. (intermediate grade steel)

Live load = 360 lb per sq ft.
Impact is negligible.
Floor cover = 20 lb per sq ft.
Allowable tension in web reinforcement = 16,000 lb per sq in.

From these:

$$k = \frac{1350}{1350 + 2000} = 0.403$$

$$j = 1 - \frac{0.403}{3} = 0.866$$

$$R = \tfrac{1350}{2} \times 0.403 \times 0.866 = 236$$

$$p = \frac{1350 \times 0.403}{2 \times 20{,}000} = 0.0136$$

Slab: assuming a 4-in. slab:

$$\begin{aligned} \tfrac{4}{12} \times 150 &= 50 \\ \text{Floor cover} &= 20 \\ &\overline{} \\ &70 \text{ lb per sq ft} \end{aligned}$$

		Moment center of support		Moment center of span
D.L. moment	$= \dfrac{70 \times \overline{6.75}^2}{12}$	= 266 ft-lb	$\times \dfrac{12}{24}$ =	133 ft-lb
L.L. moment	$= \dfrac{360 \times \overline{6.75}^2}{9}$	= 1823	$\times \dfrac{9}{12}$ =	1367
		2089 ft-lb		1500 ft-lb

$$d = \sqrt{\tfrac{2089}{236}} = 2.98 \quad \text{use 3 in.}$$

Cover 1 in.

Depth of slab = 4 in.

$$jd = 3.00 \times 0.866 = 2.60 \text{ in.}$$

Tensile steel at support:

$$\frac{2089 \times 12}{2.60 \times 20{,}000} = 0.482 \text{ sq in. per ft}$$

$\tfrac{1}{2}$-in. round at $4\tfrac{3}{4}$ in. = 0.495 sq in. per ft

Tensile steel in center of span:

$$\frac{1500 \times 12}{2.60 \times 20{,}000} = 0.346 \text{ sq in. per ft}$$

$\tfrac{1}{2}$-in. round at $6\tfrac{3}{4}$ in. = 0.348 sq in. per ft

The value of 2.60 in. for jd is conservative with the smaller moment.

Shrinkage steel: $\frac{3}{8}$-in. round at 12 in. = 0.11 ÷ (4 × 12) = 0.23 per cent of slab area.

Beam: assume 18-in. width for girder.

Beam span = 22 ft 6 in. center to center = 21 ft 0 in. clear span.

Computing shear at the first interior column:

$$70 \times 6.75 = 473 \text{ lb per ft}$$
$$360 \times 6.75 = 2430$$
$$2903 \text{ lb per ft} \times \frac{21.0}{2} \times 1.15 = 35{,}000 \text{ lb}$$
$$\text{Estimated stem weight} = 240 \times \frac{21.0}{2} \times 1.15 = 2{,}900 \text{ lb}$$
$$V = 37{,}900 \div (0.875 \times 180)$$
$$= 241 \text{ sq in.}$$

$$10 \times 25 = 250 \text{ sq in.}$$
$$\text{Cover} = 3\tfrac{1}{2} \text{ in. for two rows of bars}$$
$$28\tfrac{1}{2} \text{ in. overall depth}$$

The 10-in. width will not be sufficient to take 4 bars in one row, which will probably be required, and, since it is not desirable to decrease the depth, a width of $11\frac{1}{2}$ in. will be taken, making the beam $11\frac{1}{2} \times 25 = 287$ sq in.

$$\text{Weight of stem} = \frac{11.5 \times 24\frac{1}{2}}{144} \times 150 = 294 \text{ lb per ft}$$

Recalculating the shear,

$$35{,}000 \text{ lb}$$
$$294 \times \frac{21.0}{2} \times 1.15 = 3{,}600$$
$$38{,}600 \text{ lb}$$

The shear in an interior panel will be:

$$473$$
$$294$$
$$767 \times \frac{21.0}{2} = 8{,}100 \text{ lb}$$
$$2430 \times \frac{21.0}{2} \times 1.10 = 28{,}100$$
$$36{,}200 \text{ lb}$$

The same-size beam will be used for interior spans as for end spans, however.

The computations for bending moments can be made very easily as follows:

$$\begin{array}{l} 473 \\ 294 \\ \hline 767 \times \overline{22.5}^2 = 388{,}000 \text{ ft-lb} \qquad 2430 \times \overline{22.5}^2 = 1{,}230{,}000 \text{ ft-lb} \end{array}$$

TABLE OF BENDING MOMENTS

(Values in Foot-Pounds)

wL^2	Location				
	Center of Interior Support	Center of First Interior Support	Center of Interior Span	Center of First Span	End of End Span
Dead load 388,000	$\times \frac{1}{12} =$ 32,400	$\times \frac{1}{9} =$ 43,100	$\times \frac{1}{24} =$ 16,200	$\times \frac{1}{12} =$ 32,400	$\times \frac{1}{25} =$ 15,500
Live load 1,230,000	$\times \frac{1}{9} =$ 136,700	$\times \frac{1}{8} =$ 153,800	$\times \frac{1}{12} =$ 102,500	$\times \frac{1}{10} =$ 123,000	$\times \frac{1}{25} =$ 49,200
Total	169,100	196,900	118,700	155,400	64,700

As stated in Art. 56, the negative moment at the face of the support may be used in proportioning the steel at a support. This is also permitted in Section 808(*d*) of the 1940 Joint Committee specifications, where a recommendation for obtaining the value of this moment is given. In this floor the greater number of the beams are supported by the girders, and, in order to obtain the moments at the face of a girder, it is necessary to know the width of the girder. At this point, preliminary calculations for the girder indicate that a size of 18 by 46 in. is satisfactory.* This agrees with the value of 18 in. previously assumed.

The specifications of the 1940 Joint Committee, Section 808(*d*), state that the moment at the face of a support may be obtained approximately from the moment at the centerline by subtracting a quantity $Va/3$, where V is the shear at the face of the support, and a is the width of the support. The authors believe that this approximation is both conservative and simple to apply. It is used as follows:

* Approximate computations indicate a shear of about 130 kips, requiring an area of girder of 825 sq in., and a negative moment of about 800 ft-kips. $18 \times 46 = 828$ sq in. The steel required is about 11.8 sq in. Three rows of 5 bars each = 15 1-in. round = 11.78 sq in. The 18-in. width is more than sufficient for this number of bars.

$$\text{Moment at an interior support} = 169{,}100 - \frac{36{,}200 \times 1.5}{3} = 151{,}000 \text{ ft-lb}$$

$$\text{Moment at first interior support} = 196{,}900 - \frac{38{,}600 \times 1.5}{3} = 177{,}600 \text{ ft-lb}$$

The steel required at an interior support could be calculated as follows:

$$\begin{aligned} 25 \times 0.403 &= 10.08 \text{ in.} \\ 25 \times 0.866 &= 21.65 \text{ in.} \\ 25 \times 0.9 &= 22.5 \text{ in.} \\ 0.0136 \times 287 &= 3.90 \text{ sq in.} \end{aligned}$$

$$\begin{aligned} 151{,}000 \times 12 &= 1{,}812{,}000 \text{ in.-lb} \\ 3.90 \times 20{,}000 \times 21.65 &= 1{,}689{,}000 \\ \hline & \quad 123{,}000 \text{ in.-lb} \end{aligned}$$

$$\div (22\tfrac{1}{2} \times 20{,}000) = 0.27$$
$$3.90$$

$$A_s = 4.17 \text{ sq in. at face of support}$$

$$\frac{1350}{10.08} \times 7.58 = 1015 \text{ lb per sq in.}$$

$$\frac{123{,}000}{1015(10 - 1)22\frac{1}{2}} = 0.60 \text{ sq in.} = A_s'$$

At center of interior span:

$$\frac{118{,}700 \times 12}{22.5 \times 20{,}000} = 3.16 \text{ sq in.} = A_s \text{ at center of span}$$

The concrete is not overstressed in compression.

Figure 81 shows a sectional view at the face of the support and at the center of the span.

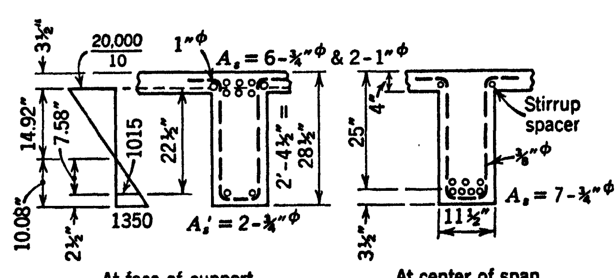


Fig. 81

For the end span:

$$
\begin{array}{l}
177{,}600 \times 12 = 2{,}131{,}000 \text{ in.-lb} \\
\qquad\qquad\qquad\ \underline{1{,}689{,}000} \\
\qquad\qquad\qquad\quad 442{,}000 \text{ in.-lb} \quad (2 \text{ rows comp. steel, arm} = 21\tfrac{1}{2} \text{ in.}) \\
\qquad\qquad\qquad\quad \div (21\tfrac{1}{2} \times 20{,}000) = 1.03 \\
\qquad\qquad\qquad\qquad\qquad\qquad\qquad\ \underline{3.90} \\
\qquad\qquad\qquad\qquad\qquad\quad A_s = 4.93 \text{ sq in. at face of first interior support}
\end{array}
$$

$$\frac{1350}{10.08} \times 6.58 = 881 \text{ lb per sq in.}$$

$$\frac{442{,}000}{881(10 - 1)21\frac{1}{2}} = 2.59 \text{ sq in.} = A_s'$$

$$\frac{155{,}400 \times 12}{22.50 \times 20{,}000} = 4.15 \text{ sq in.} = A_s \text{ at center of span}$$

The concrete is not overstressed, since the flange of the T beam is available.

$$\frac{64{,}700 \times 12}{21.65 \times 20{,}000} = 1.79 \text{ sq in.} = A_s \text{ at end}$$

The following steel arrangements will be used:

End span:

7—$\frac{7}{8}$-in. round = 4.21 sq in. at center. Bend 3 up.
4—$\frac{7}{8}$-in. round = 2.41 sq in. in bottom, and 3—$\frac{7}{8}$-in. round = 1.80 sq in. in top at end.
3—$\frac{7}{8}$-in. round, 3—$\frac{3}{4}$-in. round, and 2—1-in. square (added straight bars) = 5.13 sq in. in top at first interior support.
4—$\frac{7}{8}$-in. round and 2—$\frac{3}{4}$-in. round = 3.29 sq in. in bottom at first interior support.

Interior span:

7—$\frac{3}{4}$-in. round = 3.09 sq in. at center. Bend 3 up, cut 2.
6—$\frac{3}{4}$-in. round and 2—1-in. round (added straight bars) = 4.22 sq in. in top at interior support.
2—$\frac{3}{4}$-in. round (extended from center of span) = 0.88 sq in. in bottom at support.

The perimeters of bars are given in Table 2. Checking the bond stress for an interior beam at the face of the support, the average stress is equal to $\frac{36{,}200}{(14.14 + 6.28)\frac{7}{8}(25)} = 81$ lb per sq in. The larger the bar, the greater the stress in bond; therefore the 1-in. round bars have a bond stress greater than 81 lb per sq in. (but less than the allowable of 150 lb per sq in.), and the $\frac{3}{4}$-in. round bars have a bond stress less than 81 lb per sq in. The intensity of bond stress should be checked at sections near a support where bars are bent or cut off, especially if the bond stress at the support is close to the allowable.

Stirrups: $\frac{3}{8}$-in. round U stirrups will be used.

$$A_s = 0.22 \text{ sq in.} \qquad 0.22 \times 16{,}000 = 3520 \text{ lb}$$

Shear at center:

$$\frac{2430 \times 21.0}{2} \times \frac{1}{4} = 6380 \quad \div \left(\tfrac{7}{8} \times 25\right) = 292 \text{ lb per in.}$$

$$\frac{36{,}200}{\frac{7}{8} \times 25} = 1655 \text{ lb per in.}$$

$$\frac{-292}{1363 \times \tfrac{1}{3} = 454.3 \text{ lb per in.}}$$

Shear resisted by concrete = $60 \times 11.5 = 690$ lb per in.

$$\frac{-292}{398 \text{ lb per in.}}$$

At end, $1655 - 690 = 965$ lb per in.
At $\frac{1}{6}$ point, $1201 - 690 = 511$ lb per in.
At $\frac{1}{3}$ point, $746 - 690 = 56$ lb per in.

$3520 \div 965 = 3.65$ in. (use $3\frac{1}{2}$-in. spacing at end)
$\div 511 = 6.89$ in. (use $6\frac{3}{4}$-in. spacing at $\frac{1}{6}$ point)
$\div 56 = 62.8$ in. $\left(\text{maximum allowable} = \frac{d}{2} = 12\tfrac{1}{2} \text{ in. at } \tfrac{1}{3} \text{ point}\right)$

Within $\frac{(10.5 \times 12)398}{1363} = 36.8$ in. from center, stirrups are not required.

A curve of required spacing should be drawn, as shown in Fig. 82, as an aid in spacing the groups of stirrups. The stirrups can be placed in groups of the same spacing, using Fig. 82 for the allowable spacing and placing the first stirrup not more than $1\frac{3}{4}$ in. from the face of the support. If this first stirrup were $1\frac{1}{2}$ in. from the girder, the stirrup spacing to the beam centerline could be 7 at $3\frac{1}{2}$ in., 3 at $4\frac{1}{2}$ in., 4 at 6 in., and 5 at $12\frac{1}{2}$ in.

With $\frac{1}{2}$-in. round stirrups, the calculated spacing would be 6.47 in. at the end and 12.21 in. at the $\frac{1}{6}$ point. Although more stirrups are needed with the $\frac{3}{8}$-in. stirrups, there will be a saving in steel since the maximum spacing is reached at a point much nearer the end if the $\frac{1}{2}$-in. round are

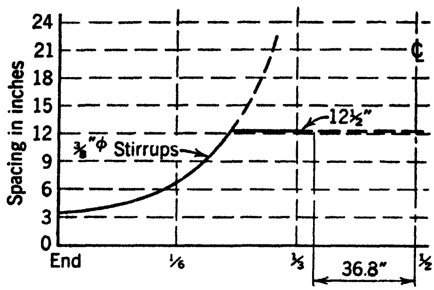


FIG. 82

used. Most designers would carry the stirrups through the center of the beam at the $12\frac{1}{2}$ in. spacing rather than discontinue them 36.8 in. from the center.

Girder calculations:

Girder span = 27 ft 0 in. center to center of columns. Columns assumed to be 18 by 18 in.

Shear:

	Interior Support		First Interior Support
$8100 \times 2 = 16{,}200$			
$\times \frac{3}{2}$	$= 24{,}300$	$\times 1.15 =$	$27{,}900$
$2430 \times \frac{21.0}{2} \times 2 = 51{,}000$			
$\times \frac{3}{2} \times 1.10 =$	$84{,}100$		
$\times \frac{3}{2} \times 1.15 =$			$88{,}000$

For 18-in. width of load over girder:

	Interior Support		First Interior Support
$360 \times 1.5 \times \frac{25.5}{2} = 6900$			
$\times 1.10$	$= 7{,}600$		
$\times 1.15$	$=$		$7{,}900$

Weight of girder and floor surface:

	Interior Support		First Interior Support
$1000 \times \frac{25.5}{2}$	$= 12{,}800$	$\times 1.15 =$	$14{,}700$
	128,800 lb		138,500 lb

$\div (\frac{7}{8} \times 180) = 818$ sq in.	$\div (\frac{7}{8} \times 180) = 879$ sq in.
$18 \times 46 = 828$ sq in.	$18 \times 49 = 882$ sq in.
$\times 0.0136 = 11.26$ sq in.	$\times 0.0136 = 12.00$ sq in.

Corresponding moments:

			Interior Support, ft-lb	First Interior Support, ft-lb
$16{,}200 \times \frac{3}{2} \times 13.5 =$	328,000			
$16{,}200 \times 6.75 =$	109,300			
	218,700			
$1{,}000 \times \frac{27^2}{8} =$	91,100	$309{,}800 \times \frac{8}{12} =$	206,500	
		$\times \frac{8}{9} =$		275,400
$51{,}000 \times \frac{3}{2} \times 13.5 =$	1,032,700			
$51{,}000 \times 6.75 =$	344,200			
	688,500			
$360 \times 1.5 \times \frac{27^2}{8} =$	49,200	$737{,}700 \times \frac{8}{9} =$	655,700	737,700
		1,047,500 ft-lb	862,200	1,013,100

Approximate computations indicate that the column will be a little larger than the 18 by 18 in. estimated, but this value will be used in obtaining the moment at the face of the supports.

Moment at face of an interior support:

$$862{,}200 - \frac{128{,}800 \times 1.5}{3} = 797{,}800 \text{ ft-lb}$$

Positive moment in interior span:

$$309{,}800 \times \tfrac{8}{24} = 103{,}300$$
$$737{,}700 \times \tfrac{8}{12} = 491{,}800$$
$$595{,}100 \text{ ft-lb}$$

Moment at face of first interior support:

$$1{,}013{,}100 - \frac{138{,}500 \times 1.5}{3} = 943{,}900 \text{ ft-lb}$$

Positive moment in end span:

$$309{,}800 \times \tfrac{8}{12} = 206{,}500$$
$$737{,}700 \times \tfrac{8}{10} = 590{,}200$$
$$796{,}700 \text{ ft-lb}$$

With the floorbeams as shown in Fig. 80, the moment curve for the girder for a simple span is so close to a parabola that the moment coefficients above give results accurate enough for design purposes. Since there are six interior spans, the girder will not have a constant depth, as the beam did, but will be as follows.

Interior span:

$$d = 46 \text{ in.} \qquad \frac{18 \times 51}{144} \times 150 = 956 \text{ lb per ft}$$

$$\text{Cover} = 5 \qquad 980 \text{ assumed}$$

$$51 \text{ in.}$$

End span:

$$d = 49 \text{ in.} \qquad \frac{18 \times 54}{144} \times 150 = 1012 \text{ lb per ft}$$

$$\text{Cover} = 5 \qquad 980 \text{ assumed}$$

$$54 \text{ in.}$$

For an interior span:

$$0.403 \times 46 = 18.54 \text{ in.}$$
$$\times 49 \qquad = 19.75 \text{ in.}$$

$$0.866 \times 46 = 39.84 \text{ in.}$$
$$\times 49 \qquad = 42.43 \text{ in.}$$

$$0.9 \times 46 = 41.40 \text{ in.}$$
$$\times 49 \qquad = 44.10 \text{ in.}$$

$$0.0136 \times 828 = 11.26 \text{ sq in.}$$
$$\times 882 \qquad = 12.00 \text{ sq in.}$$

$$797{,}800 \times 12 = 9{,}574{,}000 \text{ in.-lb}$$
$$11.26 \times 20{,}000 \times 39.84 = 8{,}972{,}000$$
$$602{,}000$$

$$\div (43 \times 20{,}000) = 00.70$$
$$11.26$$

$$\text{At face of support, } A_s = 11.96 \text{ sq in.}$$

$$\frac{1350}{18.54} \times 15.54 = 1132 \text{ lb per sq in.}$$

$$\frac{602{,}000}{1132(10 - 1)43} = 1.38 \text{ sq in.} = A_s'$$

$$\frac{595{,}100 \times 12}{41.40 \times 20{,}000} = 8.62 \text{ sq in.} = A_s \text{ at center of span}$$

Figure 83 shows a sectional view at the face of the support for an interior span.

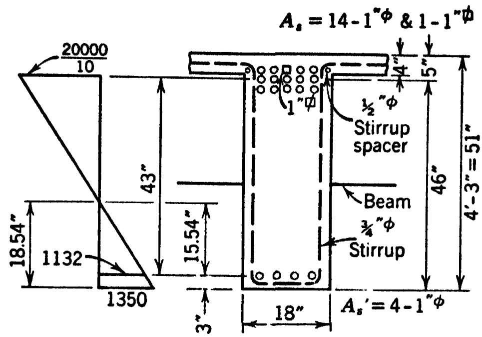


FIG. 83

For the end span:

$$\frac{796{,}700 \times 12}{44.10 \times 20{,}000} = 10.84 \text{ sq in.} = A_s \text{ at center}$$

$$943{,}900 \times 12 = 11{,}330{,}000 \text{ in.-kips}$$

$$12.00 \times 20{,}000 \times 42.43 = 10{,}183{,}000$$

$$\overline{1{,}147{,}000} \text{ in.-kips}$$

$$\div (46 \times 20{,}000) = 1.25$$

$$12.00$$

$$\overline{13.25} \text{ sq in.} = A_s \text{ at first interior support}$$

$$\frac{1350}{19.75} \times 16.75 = 1145 \text{ lb per sq in.}$$

$$\frac{1{,}147{,}000}{1145(10 - 1)46} = 2.42 \text{ sq in.} = A_s'$$

$$\frac{1{,}047{,}500 \times \frac{8}{25} \times 12}{44.10 \times 20{,}000} = 4.56 \text{ sq in.} = A_s \text{ at end}$$

The steel arrangements will be as follows:

End span: 11—1-in. square bars = 11.00 sq in. Bend 7 up.

At end: 7—1-in. square = 7.00 sq in.

First interior support:

7—1-in. square	=	7.00 sq in.
7—1-in. round	=	5.50
1—1-in. square (straight)	=	1.00
		13.50 sq in.

A_s' = 4.00 sq in. in bottom at support

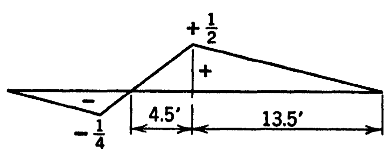
Interior span: 11— 1-in. round bars = 8.64 sq in. Bend 7 up.
Interior support:

14— 1-in. round	= 11.00
1— 1-in. square straight	= 1.00
	12.00 sq in.

$A_s' = 3.14$ sq in. in bottom at support

Since the steel required at the centers of the spans is less than the critical moments for balanced design (11.26 and 12.00 sq in.), there is no compression steel required in the top of the beam, nor is any of the slab required as flange area.

The depth of the girder would be the same on both sides of the first interior column. For that reason the bottom of the girder in the first



$$\tfrac{1}{2} \times \tfrac{1}{2} \times 18 = +4.5$$
$$\tfrac{1}{2} \times \tfrac{1}{4} \times 9 = -1.12$$
$$3.38$$

FIG. 84

interior span should be given a flat taper so as to increase the overall depth from 51 in. to 54 in. at the face of the column.

Girder stirrups for an interior span:
Shear in end panel = 128,800 lb maximum.

This is reduced 1540 × 5.48 = 8400 lb at the first floorbeam, but the difference is so small that the stirrups have a constant spacing between floorbeams.

For the second panel the influence line is as shown in Fig. 84, using centers of supports for convenience.

D.L. shear:	8100	
1000 × 3.38 =	3380	
		11,480
L.L. shear:		
360 × 22.5 × 4.5 =		36,450
		47,930 lb

With ¾-in. round U stirrups: *

* At the ends of beams or girders, if sufficient length for bond is provided, the stirrups may be turned up (as they would be in the center of the span) for convenience of placing the longitudinal bars.

$A_s = 0.88$ sq in. $\qquad 0.88 \times 16{,}000 = 14{,}080$ lb

$\dfrac{128{,}800}{\frac{7}{8} \times 46} = 3200$ lb per in. $\qquad \dfrac{47{,}930}{\frac{7}{8} \times 46} = 1192$ lb per in.

$18 \times 60 = 1080 \qquad\qquad 1080$

2120 lb per in. $\qquad\qquad$ 112 lb per in.

$14{,}080 \div 2120 = 6.64$ in. (use $6\frac{1}{2}$-in. spacing)

In the second panel $\frac{1}{2}$-in. round stirrups spaced at the maximum spacing of 23 in. is satisfactory.

For dimensioning the bars and other details, the student is referred to more advanced texts devoted to reinforced concrete design. The points where bars may be bent up or down can be obtained by the method outlined in Art. 68.

Effect of Time Yield. The above design has been made on the basis of transformed sections. As was explained in Art. 62, current specifications for reinforced concrete take account of the effect of time yield by permitting an increase in the compressive stress in the steel. If this were done in the present instance, the design of the compressive steel at the supports would be modified as follows:

At an interior support of the beam:

$$n' = \frac{16{,}000}{1015} = 15.76$$

$$A_s' = \frac{123{,}000}{1015 \times 14.76 \times 22.5} = 0.36 \text{ sq in.}$$

This is less than the 0.60 sq in. obtained in the previous calculations, but 2—$\frac{3}{4}$-in. round bars would still be used, since the specifications require that one-quarter of the center steel extend into the support.

At the first interior support:

$$n' = \frac{16{,}000}{881} = 18.16$$

$$A_s' = \frac{442{,}000}{881 \times 17.16 \times 21.5} = 1.36 \text{ sq in.}$$

as compared with 2.59 sq in. obtained previously.

At an interior support of the girder:

$$n' = \frac{16{,}000}{1132} = 14.13$$

$$A_s' = \frac{602{,}000}{1132 \times 13.13 \times 43} = 0.94 \text{ sq in.}$$

as compared with 1.38 sq in. obtained previously.

At the first interior support:

$$n' = \frac{16{,}000}{1145} = 13.97$$

$$A_s' = \frac{1{,}147{,}000}{1145 \times 12.97 \times 46} = 1.68 \text{ sq in.}$$

as compared with 2.42 sq in. obtained previously.

There will be no change in the steel arrangement as given before because of the requirement that not less than 25 per cent of the steel at the center should be carried to the support.

Interior column:

Girder stem: 881 × 27.0 =	23,800	
4 × 294 × 21 =	24,700	
70 × 22.5 × 27.0 =	42,500	91,000 D.L.
360 × 22.5 × 27.0 =		218,700 L.L.
		309,700
		5,300 est. weight of column
		315,000 lb = total load

On basis of transformed sections, assuming $p = 0.03$,

$$f_c = 675 \text{ lb per sq in.}$$

$$\frac{315{,}000}{675(1 + 0.03 \times 9)} = 367 \text{ sq in.}$$

$$\times\, 0.03 = 11.01 \text{ sq in.}$$

8—1⅛-in. square = 10.13 sq in.		315,000 lb
	× 9 × 675 =	61,500
		253,500 lb

$$\frac{253{,}500}{675} = 376 \text{ sq in.} \qquad \sqrt{376} = 19.39 \text{ in.}$$

19½ by 19½ in. is satisfactory.

Using the Joint Committee recommendation as illustrated in Art. 70 for a tied column, assuming $p_g = 0.03$:

$$\frac{315{,}000}{540 + 384} = 341 \text{ sq in.}$$

$$\times\ 0.03 = 10.23 \text{ sq in.}$$

$$8\text{—}1\tfrac{1}{8}\text{-in. square} = 10.13 \text{ sq in.} \qquad 315{,}000 \text{ lb}$$

$$\times\ 12{,}800 = \qquad 129{,}700$$

$$185{,}300 \text{ lb}$$

$$\frac{185{,}300}{540} = 343 \text{ sq in.} \qquad \sqrt{343} = 18.52 \text{ in.}$$

$18\tfrac{3}{4} \times 18\tfrac{3}{4} = 352$ sq in. is satisfactory.

Approximate computations indicate that when the live load is on one side only the moment at the top of the column should not exceed 50 ft-kips = 600 in.-kips. This is based on a clear length of column of 10 ft or more to the bottom of the girder.

The total load at the top of the column is,

D.L.	91,000 lb
L.L.	109,350
	200,350 lb (use 201 kips)

According to the method illustrated in Art. 72:

$$e = \tfrac{600}{201} = 2.985 \text{ in.}$$

$$p_g = \frac{10.13}{352} = 0.0288$$

$$f_a = \frac{0.18 \times 3000 + 12{,}800 \times 0.0288}{1 + 9 \times 0.0288} = \frac{909}{1.259} = 722 \text{ lb per sq in.}$$

$$A_t = \frac{201}{0.722} + \frac{600 \times 6}{1.350 \times 18.75} = 278 + 142 = 420 \text{ sq in.}$$

$$\frac{420}{1.259} = 334 \text{ sq in.} \qquad \sqrt{334} = 18.27 \text{ in.}$$

$$18\tfrac{1}{4} \times 18\tfrac{1}{4} = 333 \text{ sq in.}$$

The column $18\tfrac{3}{4}$ by $18\tfrac{3}{4}$ in. is satisfactory.
If $\tfrac{1}{4}$-in. round ties are used:

$$48 \times \tfrac{1}{4} = 12 \text{ in.}$$

$$16 \times 1\tfrac{1}{8} = 18 \text{ in.}$$

$$\text{Column width} = 18\tfrac{3}{4} \text{ in.}$$

The spacing for the $\frac{1}{4}$-in. round ties is 12 in. Since 8—$1\frac{1}{8}$-in. square bars are used for vertical steel, two ties are required in each 12 in. of column height. A sectional view of the column is shown in Fig. 85.

The student will have observed that various values of jd were used in the design of the beam and girder. The reasons for these various values have been mentioned at appropriate points in this chapter but will be summarized here.

In proportioning the steel at the support where there was compressive steel, the value of j was that for a balanced design. The moment arm

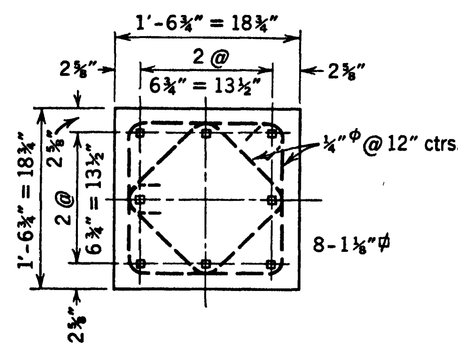


FIG. 85

for obtaining the additional tensile steel and the required compressive steel was the distance between these two and, with stresses consistent with the stress diagram, this gives the correct value of the area of steel required.

For the T beam and the girder at the center of the span, a value of 0.9 was assumed for j, representing conservative practice in normal design.

A value of $\frac{7}{8}$ was used in determining the size of the beam, the bond stress, and the stirrup spacing. This is common practice and is conservative. A value of 0.9 is permissible where an actual T beam occurs at the point of maximum shear.

The bond stresses for the girders were not checked, as it is obvious by inspection that the perimeter of the bars is much greater than $\frac{6}{5}$ the width of the girder. The student should check the bond stress for his own satisfaction.

A typical interior column was designed on the basis of an axial load, when all panels were loaded, and then checked for moment, where loaded for maximum eccentricity about only one axis. This illustrates all essential points in elementary column proportioning. If all maximum loading conditions are considered, it is possible to obtain a larger axial load, also bending about two axes, bending in end columns, and bending in corner columns, which involve the most indeterminant types of unsymmetrical loading conditions. The student will encounter these in his advanced design work, and for proper solution of these problems he will need to be fortified with a thorough course in statically indeterminate structures.

Specifications do not require a designer to investigate all the loading conditions implied above. Some of these conditions are so unlikely that increased allowable stresses would be justified.

In making the detail drawings for this design, it will be necessary to consider the items mentioned in Art 52 concerning the placement of steel. The arrangement of steel as shown in the various cross sections in this article may require modification to facilitate the placement of steel for a given construction plan.

An alternate design in timber, with a reduced live load and closer column spacing, is shown in the Appendix. These calculations are in the form of design plates.

PROBLEMS

1. A simple beam of reinforced concrete has a width of 12 in. and an overall depth of 24 in. If the center of the steel is $2\frac{1}{2}$ in. from the bottom face, $A_s = 2.41$ sq in., and $n = 10$, what is the value of kd?

2. If $f_c' = 3200$ lb per sq in. and $f_s = 20{,}000$ lb per sq in. (*a*) What is the value of n, according to the 1940 Joint Committee? According to A.C.I.? (*b*) Taking the value of n according to the 1940 Joint Committee and assuming balanced design, what is the value of k? (*c*) If in (*b*) there is no compression steel or flange, what is the value of j? of p?

3. Taking the data of Problem 1, with $f_c' = 3000$ lb per sq in., and $f_s = 20{,}000$ lb per sq in., what is the maximum bending moment permissible for the simple beam?

4. If the depth from the compression face to the center of tension steel is limited to $23\frac{1}{2}$ in. for a beam, how wide must the beam be to resist a shearing force of 48,000 lb, if $f_c' = 2800$ lb per sq in. and stirrups are to be used?

5. In Problem 4, what would be the maximum spacing for $\frac{3}{8}$-in. stirrups? Would $\frac{1}{2}$-in. stirrups permit a more practical spacing?

6. A reinforced concrete T beam has an effective flange width of 55 in., a flange thickness of 5 in., and the depth to the center of the tension steel is 24 in. If the stem is 12 in. wide, $A_s = 6.00$ sq in., and $n = 12$, what are the stresses f_c and f_s for a bending moment of 200 ft-kips?

7. A reinforced concrete beam is 20 in. wide and 40 in. deep to the tension steel which has an area of 16.00 sq in. For $f_c' = 3200$ lb per sq in. and $f_s = 18{,}000$ lb per sq in., what area of compression steel is needed 3 in. from the compression face of the beam in order to develop the allowable stresses in the concrete and tension steel? What moment would this reinforced beam be capable of resisting?

8. If no compression steel were added in Problem 7, what would be the permissible bending moment?

9. A reinforced concrete girder has a bending moment of 2400 ft-kips and a shear of 280 kips. For $f_c' = 3000$ lb per sq in. and $f_s = 18{,}000$ lb per sq in., determine the width and overall depth for the girder and the size and number of reinforcing bars.

10. In Problem 9, what would be the spacing for $\frac{3}{4}$-in. (W) stirrups?

REFERENCES

H. Sutherland and R. C. Reese, *Introduction to Reinforced Concrete Design*, John Wiley and Sons, 1943.

C. W. Dunham, *The Theory and Practice of Reinforced Concrete*, McGraw-Hill Book Company, Second Edition, 1944.

F. W. Taylor, S. E. Thompson, and E. Smulski, *Concrete, Plain and Reinforced*, Vol. 1, *Theory and Design of Concrete and Reinforced Structures*, John Wiley and Sons, 1925.

F. E. Turneaure and E. R. Maurer, *Principles of Reinforced Concrete Construction*, John Wiley and Sons, 1935.

Dean Peabody, Jr., *The Design of Reinforced Concrete Structures*, John Wiley and Sons, 1946.

G. A. Hool and H. E. Pulver, *Reinforced Concrete Construction*, Vol. 1, *Fundamental Principles*, McGraw-Hill Book Company, Fourth Edition, 1937.

Note: After this book had gone to press it was announced by the steel industry that the manner of designating steel reinforcing bars is being changed from the use of diameters to numbers. Also, the square bars are being replaced by equivalent round bars. The new standard designations are numbered from 3 to 11, inclusive, in which numbers 3 to 8 are identical in area and perimeter to the $\frac{3}{8}$-in. round to 1-in. round as shown in Table 2 on page 113. Numbers 9, 10, and 11 replace the 1-in., $1\frac{1}{8}$-in., and $1\frac{1}{4}$-in. square bars with round, having the same areas as the square bars and having perimeters of 3.544, 3.990, and 4.430 in., respectively. This change was made at the same time the deformations were being changed to conform with the A.S.T.M. Standard A305-49 which would permit higher bond stresses. Some mills are still rolling square bars, and there have been no changes at this time in the design specifications for reinforced concrete referred to in this chapter.

CHAPTER 6

BENDING IN UNSYMMETRICAL SECTIONS

74. In Chapter 1 it was stated that the principles discussed there, and in later chapters, were applicable only to beams with at least one axis of symmetry and that bending in unsymmetrical sections would be discussed in Chapter 6. It will be seen later in this chapter that the foregoing statement could have been amplified to state that the principles discussed in Chapter 1 were applicable only where bending was about the principal axis, but that would have presupposed a knowledge of matters yet to be discussed. Since the sections previously discussed and designed were for types of members that normally have at least one axis of symmetry, the statement was sufficient for that place.

There are many cases where the functional shape of the member is such that a section can have no axis of symmetry, and where the member is not restricted in the direction in which it deflects. Some unsymmetrical crane girders, some eave struts, and most lintels would be examples where there was little or no restraint against deflection in any direction. Where deflection is restrained to one direction only, then bending would be about a neutral axis perpendicular to the movement of the member, and the principles developed in Chapter 1 would apply for that axis. The subject of bending in unsymmetrical members whose deflections are not restrained is important since, where the principles of unsymmetrical bending apply, serious errors can result if these principles are ignored. The engineer should have a concept of a general expression for stress; it is also an advantage in certain methods of analysis in advanced structural theory.

The matter of torsion and twisting, a discussion of shear center, and such other subjects that belong in an advanced textbook on strength of materials will not be covered in this chapter.

75. Properties of Plane Areas. As a preliminary to a general discussion of bending in unsymmetrical sections, it is desirable to review briefly the properties of plane areas, giving particular attention to their second moments, commonly called moments of inertia.

In Fig. 86 is shown an irregular area with axes X and Y which are perpendicular to each other and both of which pass through the centroid of the area at 0. If the summations of the second moments of a differential area dA are made, the moments of inertia of the area about the X axis I_x and that about the Y axis I_y can be expressed as follows:

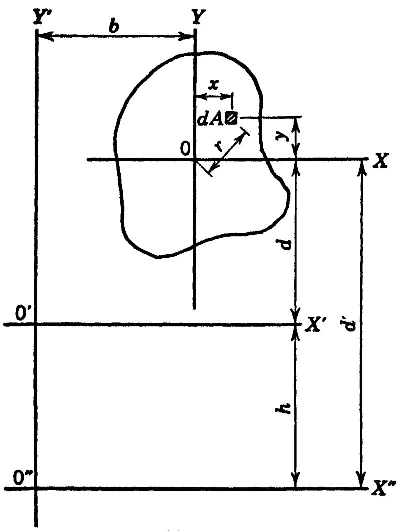


FIG. 86

$$I_x = \int y^2 \, dA$$

$$I_y = \int x^2 \, dA$$

If it is desired to obtain the moment of inertia about some axis X', which is a distance d from the X axis, it can be obtained by taking the second moment about the X' axis, or

$$I_{x'} = \int (y + d)^2 \, dA = \int (y^2 + 2yd + d^2) \, dA$$

$$= \int y^2 \, dA + 2d \int y \, dA + d^2 \int dA$$

Since X is a centroidal axis $\int y \, dA = 0$, therefore

$$I_{x'} = \int y^2 \, dA + d^2 \int dA = I_x + Ad^2$$

In like manner,

$$I_{y'} = I_y + Ab^2$$

If it is desired to transfer the moment of inertia from one noncentroidal axis to another noncentroidal axis, it cannot be done directly as above but must be transferred through the gravity axis. The moment of inertia about the axis X'', a distance d' from X and a distance h from X', would be

$$I_{x''} = I_x + A(d')^2 = I_{x'} - Ad^2 + A(d')^2 = I_{x'} + A[(d')^2 - d^2]$$

not $I_{x''} = I_{x'} + Ah^2$, which is incorrect. Since $h = d' - d$, h^2 cannot equal $(d')^2 - d^2$.

The polar moment of inertia of the area is

$$\int r^2\,dA = \int (y^2 + x^2)\,dA = \int y^2\,dA + \int x^2\,dA = I_x + I_y$$

Since I_x and I_y are the moments of inertia about any pair of rectangular centroidal axes, it follows that the sum of the moments of inertia will remain a constant if the axes remain rectangular. This is also true of noncentroidal axes, so the statement can be made: "The sum of the moments of inertia about any pair of rectangular axes through a given origin is equal to the polar moment of inertia referred to the same origin, and the sum is therefore a constant."

There is a quantity analogous to moment of inertia that is called product of inertia, obtained by multiplying each elementary area by the product of its coordinates. It is designated variously by I_{xy}, K_{xy}, P_{xy}, or J_{xy}. The authors will use I_{xy} to denote the product of inertia of an area.

$$I_{xy} = \int xy\,dA$$

Although the moment of inertia is always positive, the product of inertia can be either positive or negative, and care should be taken to determine the proper sign when computing the value. This expression is equal to zero for any area that has an axis of symmetry, since any elementary area on the positive side of the axis of symmetry has an equal elementary area on the negative side of the axis.

The product of inertia of an area is always referred to a pair of rectangular axes. They do not have to be centroidal axes. If the product of inertia about the centroidal axes is known and it is desired to obtain the product of inertia about parallel axes, as X' and Y' in Fig. 86,

$$I_{x'y'} = \int (x + b)(y + d)\,dA = I_{xy} + Abd$$

the values of $\int x\,dA$ and $\int y\,dA$ being zero.

76. Location of Principal Axes. Although principal axes, or axes of maximum and minimum moments of inertia, can be determined for any point of origin, in calculations for flexure the bending occurs about a centroidal axis, as developed in Chapter 1. Consequently, principal axes of the cross-sectional area of a member will be used in this discussion as the perpendicular axes through the center of gravity of the area about which the moments of inertia I_x and I_y are a maximum and a minimum. The absolute minimum moment of inertia for the area will be about one of the principal axes through the center of gravity.

In Fig. 87 the rectangular centroidal axes X and Y are rotated through some angle ϕ into the position indicated by the new axes X_1 and Y_1.

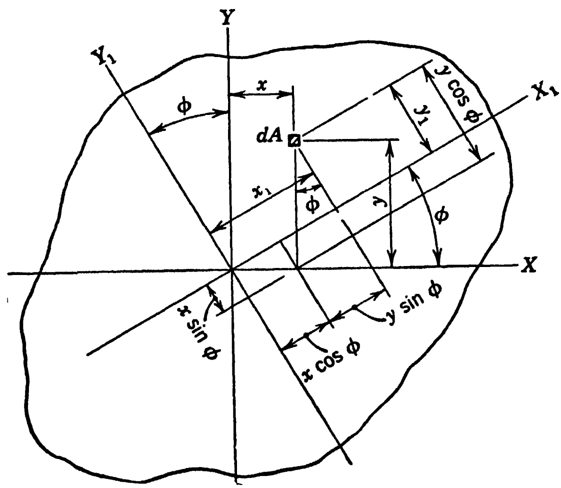


FIG. 87

The moments of inertia about the axes X_1 and Y_1 can be expressed as follows:

$$I_{x_1} = \int y_1^2 \, dA$$

$$I_{y_1} = \int x_1^2 \, dA$$

For an elementary area of dA the coordinates x_1 and y_1 can be written in terms of x and y and as functions of the angle ϕ, as shown in Fig. 87.

$$y_1 \doteq y \cos \phi - x \sin \phi$$

$$x_1 = y \sin \phi + x \cos \phi$$

These relations hold for an angle ϕ falling within any quadrant as long as proper care is shown in using the correct algebraic signs for x, y, and the sine and cosine of ϕ.

The values of I_x, I_y, and I_{xy} are known. Then

$$I_{x_1} = \int y_1^2\,dA = \int (y\cos\phi - x\sin\phi)^2\,dA$$

$$= \int y^2\cos^2\phi\,dA + \int x^2\sin^2\phi\,dA - \int 2xy\sin\phi\cos\phi\,dA$$

Since $2\sin\phi\cos\phi = \sin 2\phi$,

$$I_{x_1} = I_x\cos^2\phi + I_y\sin^2\phi - I_{xy}\sin 2\phi \tag{1}$$

$$I_{y_1} = \int x_1^2\,dA = \int (y\sin\phi + x\cos\phi)^2\,dA = \int y^2\sin^2\phi\,dA$$

$$+ \int x^2\cos^2\phi\,dA + \int 2xy\sin\phi\cos\phi\,dA$$

or

$$I_{y_1} = I_x\sin^2\phi + I_y\cos^2\phi + I_{xy}\sin 2\phi \tag{2}$$

If equations 1 and 2 are added,

$$I_{x_1} + I_{y_1} = I_x + I_y \tag{3}$$

The same relation was obtained by means of the polar moment of inertia in Art. 75. This relation must hold true, since the sum of the moments of inertia remain a constant for rectangular axes through the same origin.

Subtracting I_{y_1} from I_{x_1},

$$I_{x_1} - I_{y_1} = (I_x - I_y)(\cos^2\phi - \sin^2\phi) - 2I_{xy}\sin 2\phi$$

or, since $\cos^2\phi - \sin^2\phi = \cos 2\phi$,

$$I_{x_1} - I_{y_1} = (I_x - I_y)\cos 2\phi - 2I_{xy}\sin 2\phi \tag{4}$$

Formulas 3 and 4 are sometimes more convenient than formulas 1 and 2 for solving for I_{x_1} and I_{y_1} when the sections are regular.

To obtain a value for $I_{x_1y_1}$,

$$I_{x_1y_1} = \int x_1y_1\,dA = \int (y\sin\phi + x\cos\phi)(y\cos\phi - x\sin\phi)\,dA$$

$$= \int y^2\sin\phi\cos\phi\,dA - \int x^2\sin\phi\cos\phi\,dA$$

$$+ \int xy(\cos^2\phi - \sin^2\phi)\,dA$$

or

$$I_{x_1y_1} = (I_x - I_y)\frac{\sin 2\phi}{2} + I_{xy}\cos 2\phi \tag{5}$$

Since the principal axes are those about which the moments of inertia are a maximum and a minimum, the location of these axes can be obtained by considering the angle ϕ a variable, differentiating one of the values of I with respect to ϕ, and equating to zero. From formula 1,

$$\frac{dI_{x_1}}{d\phi} = -I_x 2 \sin \phi \cos \phi + I_y 2 \sin \phi \cos \phi - I_{xy} 2 \cos 2\phi = 0$$

$$(I_y - I_x) \sin 2\phi - 2I_{xy} \cos 2\phi = 0$$

and therefore

$$\tan 2\phi = \frac{2I_{xy}}{I_y - I_x} \tag{6}$$

The same value would be obtained by differentiating I_{y_1}, which is apparent since both sets of axes are rectangular and the Y axis must be rotated through the same angle as the X axis.

If either axis X or Y is an axis of symmetry, the value of I_{xy} is zero, and therefore the value of ϕ for obtaining maximum and minimum values of I, or principal axes, is zero. From this it follows that, if an area has an axis of symmetry, this axis and the centroidal axis perpendicular to it constitute principal axes.

A study of Fig. 87 should show that if the axes X and Y are rotated through an angle of 90 deg, the values of x and y for any area dA would be interchanged, and the algebraic sign of one would be changed. This will hold true for an area dA in any quadrant. Therefore, for a rotation of 90 deg, $I_{x_1y_1} = -I_{xy}$. From this it follows that somewhere during the rotation the value of the product of inertia must be zero.

If, in formula 5, $I_{x_1y_1}$ is equated to zero,

$$(I_x - I_y) \frac{\sin 2\phi}{2} + I_{xy} \cos 2\phi = 0$$

$$(I_x - I_y) \sin 2\phi = -2I_{xy} \cos 2\phi$$

and

$$\tan 2\phi = \frac{2I_{xy}}{I_y - I_x} \tag{6}$$

This is formula 6 as previously derived for principal axes, and it is therefore seen that the axes about which the product of inertia is zero are the same pair of axes about which the moments of inertia are a maximum and a minimum; or the product of inertia about principal axes is zero.

77. General Expression for Stress. The expressions for stress in members with one or two axes of symmetry have been derived and discussed in previous chapters, and it was stated in Chapter 1 that the principles discussed were applicable only if the members had one axis of symmetry. These same principles hold if the bending is about principal axes. Many members such as zees, unequal leg angles, and built-up shapes have no axis of symmetry, but it is more convenient

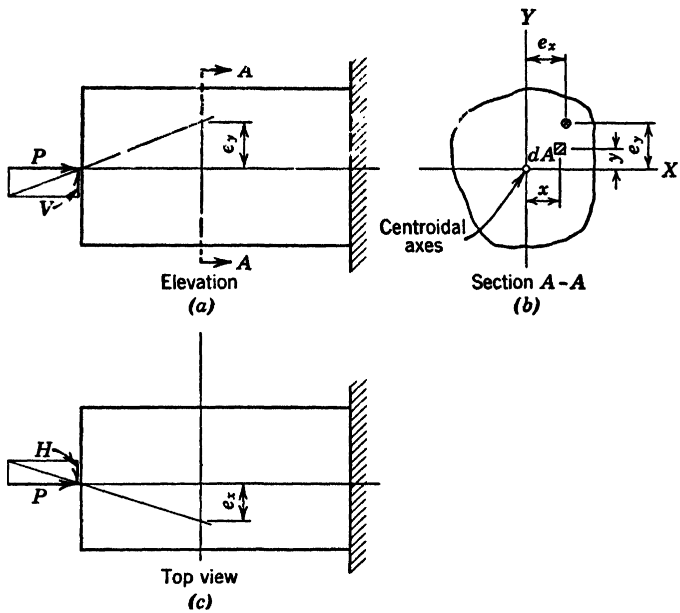


FIG. 88

to choose axes parallel to certain parts of the section rather than to rotate for principal axes. For such axes a general expression for stress can be derived.*

In Fig. 88 is shown an unsymmetrical section of a member having an axial load P and subjected to bending moments about the X and Y axes equal to M_x and M_y, respectively. These moments can be due to transverse loads or to eccentricities of the applied load (e_y and e_x), M_x being equal to Pe_y and M_y being equal to Pe_x. The effect of the moments would be the same for the computation of stresses, whatever the cause.

* This expression was first introduced by Swain in *Strength of Materials;* it is also given in substantially the same form as shown here in *The Column Analogy* by Hardy Cross, Bulletin 215 of the University of Illinois, Engineering Experiment Station.

If the unit stresses in the member are below the elastic limit of the material, then according to the assumptions in Art. 8 of Chapter 1 they will have a planar distribution, and the unit stress at any point, such as the elementary area dA, can be expressed by $f = a + bx + cy$, where f is the intensity of stress at any fiber and x and y are the distances to the fiber from the Y and X axes, respectively. $f\,dA$ is the total stress on the area dA.

Since P is the total load on the section,

$$P = \int f\,dA = a\int dA + b\int x\,dA + c\int y\,dA$$

Since the X and Y axes are centroidal axes,

$$\int x\,dA = 0 \qquad\qquad \int y\,dA = 0$$

and

$$P = a\int dA = aA$$

or

$$a = \frac{P}{A}$$

A being the area of the section. Also,

$$M_x = \int f\,dA\cdot y = a\int y\,dA + b\int xy\,dA + c\int y^2\,dA$$

and

$$M_y = \int f\,dA\cdot x = a\int x\,dA + b\int x^2\,dA + c\int xy\,dA$$

With

$$\int x\,dA \text{ and } \int y\,dA \text{ being zero,}$$

$$M_x = bI_{xy} + cI_x$$
$$M_y = bI_y + cI_{xy}$$

Solving these simultaneous equations to eliminate c,

$$M_xI_{xy} = bI_{xy}^2 + cI_xI_{xy}$$
$$M_yI_x = bI_xI_y + cI_xI_{xy}$$

Subtracting the first and dividing by $I_xI_y - I_{xy}^2$,

$$b = \frac{M_yI_x - M_xI_{xy}}{I_xI_y - I_{xy}^2}$$

In like manner,

$$c = \frac{M_x I_y - M_y I_{xy}}{I_x I_y - I_{xy}^2}$$

These values for a, b, and c give the following general expression for stress:

$$f = \frac{P}{A} + \frac{M_y I_x - M_x I_{xy}}{I_x I_y - I_{xy}^2} x + \frac{M_x I_y - M_y I_{xy}}{I_x I_y - I_{xy}^2} y \qquad (7)$$

If both the numerator and the denominator of the coefficient of x are divided by I_x and those of the y coefficient by I_y, the following form is obtained:

$$f = \frac{P}{A} + \frac{M_y - M_x \dfrac{I_{xy}}{I_x}}{I_y - \dfrac{I_{xy}}{I_x} I_{xy}} x + \frac{M_x - M_y \dfrac{I_{xy}}{I_y}}{I_x - \dfrac{I_{xy}}{I_y} I_{xy}} y \qquad (8)$$

Many designers prefer formula 8 as it does not involve the difference between large quantities; but either formula constitutes the general expression for stress.

It is convenient to follow an arbitrary convention of algebraic signs in applying the above general expression. The most convenient one is to assume a compression axial load as positive, and x as positive to the right and y as positive upward. M_x is positive when the eccentricity is above the origin, and M_y when it is to the right (or when compression is in the top fiber or the right fiber, due to bending). A positive value of f will be compression, and a negative value tension. The sign for M_x agrees with the convention for bending moment signs when bending is about a horizontal axis.

If the most convenient axes chosen are principal axes, then $I_{xy} = 0$, and formula 8 becomes

$$f = \frac{P}{A} + \frac{M_y}{I_y} x + \frac{M_x}{I_x} y$$

If the axes are principal axes and there is no bending about the Y axis,

$$f = \frac{P}{A} + \frac{M_x}{I_x} y$$

If there is no bending at all,

$$f = \frac{P}{A}$$

If the axes are principal axes and there is only bending about the X axis,

$$f = \frac{M_x}{I_x} y$$

which is the flexural formula previously derived.

78. Illustrative Example. An unequal leg angle, acting as a lintel, is shown in Fig. 89. It is assumed that the angle resists a bending moment of 350 in.-kips about the X axis. There is no bending moment about the Y axis, and no axial load. The initial calculations of the properties of this cross section are first taken relative to reference axes X' and Y', and later referred to centroidal axes X and Y. The angle will be divided into two parts as shown in the figure, V being 4 by 1 in. and U being 1 by 8 in.

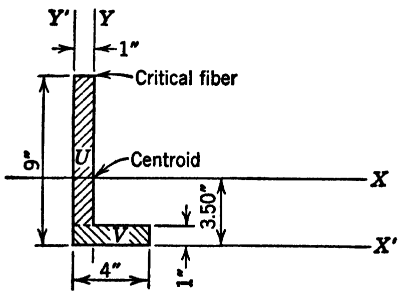


FIG. 89

The computations for the properties are tabulated below and should be self-explanatory except that I_{x_0} and I_{y_0} are the moments of inertia of each part about the centroidal axes of that part.

Part	Area (A)	y'	Ay'	x'	Ax'	$A(y')^2$	I_{x_0}	$A(x')^2$	I_{y_0}
V	$4 \times 1 = 4$	$+\frac{1}{2}$	$+2.00$	$+2$	$+8.00$	1.00	0.33	16.00	5.33
U	$1 \times 8 = 8$	$+5$	$+40.00$	$+\frac{1}{2}$	$+4.00$	200.00	42.67	2.00	0.67
	12		$+42.00$ $\div 12 =$ $+3.50$ in.		$+12.00$ $\div 12 =$ $+1.00$ in.	201.00	43.00	18.00	6.00

$$\begin{array}{r} 43.00 \\ 201.00 \\ \hline 244.00 \text{ in.}^4 = I_{x'} \end{array} \qquad \begin{array}{r} 6.00 \\ 18.00 \\ \hline 24.00 \text{ in.}^4 = I_{y'} \end{array}$$

The centroidal axes X and Y are 3.50 in. above and 1.00 in. to the right, respectively, of the reference axes, X' and Y'.

$$\begin{array}{rr} & 244.00 \\ 12 \times (3.50)^2 = & -147.00 \\ \hline I_x = & 97.00 \text{ in.}^4 \end{array} \qquad \begin{array}{rr} & 24.00 \\ 12 \times (1.00)^2 = & -12.00 \\ \hline I_y = & 12.00 \text{ in.}^4 \end{array}$$

The product of inertia of the angle about the centroidal axes can be determined by multiplying the areas of the parts V and U by the product of their coordinates. The product of inertia of each part about its own centroid is zero.

$$\begin{aligned} 4 \times (1.00)(-3.00) &= -12.00 \\ 8 \times (-0.50)(+1.50) &= -\ \ 6.00 \\ \hline I_{xy} &= -18.00 \text{ in.}^4 \end{aligned}$$

Substituting these values in formula 8, the general expression for stress is

$$f = 0 + \frac{0 - 350\left(\frac{-18}{97}\right)}{12 - \frac{18}{97}18} x + \frac{350 - 0\left(\frac{-18}{12}\right)}{97 - \frac{18}{12}18} y$$

$$f = \frac{64.9}{8.66} x + \frac{350}{70} y = 7.49x + 5.00y$$

It is apparent that the maximum intensity of stress occurs at one of the corners of the angle, and by inspection it will be seen that the maximum positive value of the above expression will be obtained at the upper right-hand corner of part U and that this point also has the absolute maximum value of stress.

The coordinates are zero and 5.50:

$$f = 5.00 \times 5.50 = 27.50 \text{ kips per sq in.} \quad \text{(compression)}$$

If this angle had been proportioned under the erroneous assumption that the expression $f = Mc/I$ could be applied for such unsymmetrical sections, the following stress would have been obtained in the same fiber:

$$f = \frac{350 \times 5.50}{97} = 19.85 \text{ kips per sq in.} \quad \text{(compression)}$$

The excess is

$$\begin{array}{r} 27.50 \\ -19.85 \\ \hline 7.65 \end{array} \div 19.85 = 0.385 \text{ or } 38.5 \text{ per cent on the unsafe side}$$

79. Unsymmetrical Beams with Lateral Loads. A composite crane beam consisting of a wide flange was designed in Art. 30 of Chapter 3 and is shown in Fig. 21 of that chapter. These composite beams are sometimes formed as shown in Fig. 90, with the channel riveted to the web of the beam. This arrangement forms an unsymmetrical beam.

In Fig. 90 is shown a 16-in. wide-flange beam at 50 lb combined with a 10-in. channel at 15.3 lb. Various combinations of wide-flange beams and channels are given in the A.I.S.C. handbook, *Steel Construction,* and the dimensions shown are largely taken from there, including the location of the center of gravity.

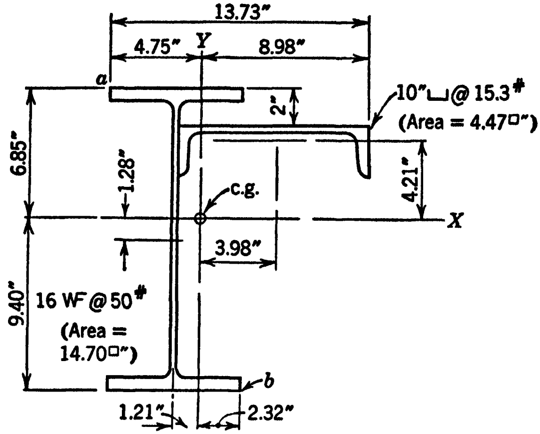


FIG. 90

The computations for the moments of inertia and product of inertia can be made from the values of section moduli given in the handbook, or they can be developed as follows:

	655.4		34.8		
$14.70 \times \overline{1.28}^2 =$	24.1	$14.70 \times \overline{1.21}^2 =$	21.5	$14.70 \times (-1.21)(-1.28) =$	22.8
	2.3		66.9		
$4.47 \times \overline{4.21}^2 =$	79.2	$4.47 \times \overline{3.98}^2 =$	70.8	$4.47 \times 3.98 \times 4.21 =$	75.0
$I_x =$	761.0	$I_y =$	194.0	$I_{xy} =$	97.8

It will be assumed that this beam is subjected to vertical loads, causing a bending moment about the X axis of $+1460$ in.-kips, and to a lateral load acting from the left, causing a bending moment about the Y axis of 75 in.-kips. It will be further assumed that the lateral force is so acting that the entire section will resist the load with no torsion. According to the convention of signs given in connection with the general expression for stress, they will be as follows:

$$M_x = +1460 \text{ in.-kips} \qquad M_y = -75 \text{ in.-kips}$$

$$\frac{I_{xy}}{I_x} = \frac{97.8}{761} = 0.129 \qquad \frac{I_{xy}}{I_y} = \frac{97.8}{194} = 0.504$$

Substituting in the general expression for stress and remembering $P = 0$,

$$f = \frac{-75 - 1460 \times 0.129}{194.0 - 0.129 \times 97.8}x + \frac{1460 + 75 \times 0.504}{761 - 0.504 \times 97.8}y$$

$$f = \frac{-263.3}{181.4}x + \frac{1497.8}{711.7}y = -1.451x + 2.105y$$

It can be seen by inspection that the above equation has its maximum positive value at the point marked a in Fig. 90 and its maximum negative value at the point marked b. Therefore maximum compression is at a, and maximum tension at b.

At a,

$$\begin{aligned} -1.451 \times (-4.75) &= 6.90 \\ 2.105 \times 6.85 &= 14.42 \\ \hline f &= 21.32 \text{ kips per sq in. (compression)} \end{aligned}$$

At b,

$$\begin{aligned} -1.451 \times 2.32 &= -\ 3.37 \\ 2.105 \times (-9.40) &= -19.79 \\ \hline f &= -23.16 \text{ kips per sq in. (tension)} \end{aligned}$$

These stresses are too high. If this beam had been proportioned by the erroneous assumption that bending was about the two axes perpendicular to the loads, X and Y, the computations would have been as follows:

$$f = \frac{M_x}{I_x} y + \frac{M_y}{I_y} x$$

or

$$\tfrac{1460}{761}\, 6.85 + \tfrac{75}{194}\, 4.75 = 13.14 + 1.84 = 14.98$$

$$f = 14.98 \text{ kips per sq in.} = \text{compression at } a$$

$$\tfrac{1460}{761}\, 9.40 + \tfrac{75}{194}\, 2.32 = 18.03 + 0.89 = 18.92$$

$$f = 18.92 \text{ kips per sq in.} = \text{tension at } b$$

The actual stresses are in excess of the above; the excess is:

$$\frac{6.34}{14.98} = 0.423, \text{ or } 42.3 \text{ per cent for compression}$$

$$\frac{4.24}{18.92} = 0.224, \text{ or } 22.4 \text{ per cent for tension}$$

80. Bending about Principal Axes. In Art. 77 it was shown that, when bending is about the principal axes of a section, the value of I_{xy} is zero, and the expression for stress becomes

$$f = \frac{P}{A} + \frac{M_y}{I_y} x + \frac{M_x}{I_x} y$$

For symmetrical sections the location of the principal axes is apparent; for unsymmetrical sections it is necessary to determine their location as was illustrated in Art. 76.

Some designers prefer to compute the stress in bending of unsymmetrical sections about the principal axes. The authors prefer the general expression for stress with convenient axes since the stress can usually be obtained more easily than with principal axes. Also, in most cases, the point at which maximum stress occurs is obvious by inspection of the equation derived from the general expression.

It may be more convenient to use the principal axes for these computations, if it is also desired to obtain the deflections of a member having an unsymmetrical section. However, these deflections can be obtained

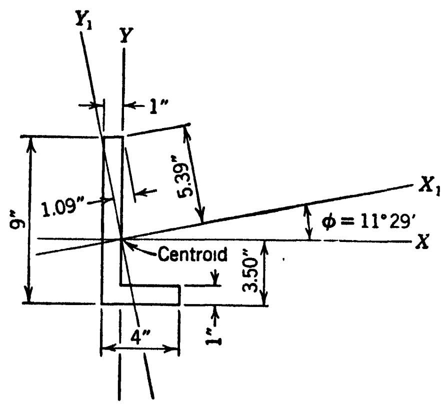


FIG. 91

by locating the neutral axis of the section in question, and this question of deflection will be discussed in Art. 83.

The determination of stress in the angle of Art. 78 and the composite beam of Art. 79 will be discussed below, using principal axes.

Unequal Leg Angle. The same moment of 350 in.-kips will be used.

$$I_x = 97 \text{ in.}^4 \qquad I_y = 12 \text{ in.}^4 \qquad I_{xy} = -18 \text{ in.}^4$$

From formula 6,

$$\tan 2\phi = \frac{2(-18)}{12 - 97} = \frac{-36}{-85} = 0.4235$$

$$2\phi = 22 \text{ deg } 57 \text{ min} \qquad \phi = 11 \text{ deg } 29 \text{ min}$$

$$\sin 2\phi = 0.3899 \qquad \sin \phi = 0.1991$$

$$\cos 2\phi = 0.9208 \qquad \cos \phi = 0.9800$$

The position of the principal axes X_1 and Y_1 are shown in Fig. 91. The values of I_{x_1} and I_{y_1} can be computed by means of formulas 3 and 4.

$$I_{x_1} + I_{y_1} = 97 + 12 = 109$$

$$I_{x_1} - I_{y_1} = 85 \times 0.9208 + 2 \times 18 \times 0.3899 = 92.31$$

$$2I_{x_1} = 201.31 \qquad 2I_{y_1} = 16.69$$

$$I_{x_1} = 100.65 \qquad I_{y_1} = 8.35$$

$$M_{x_1} = M_x \cos\phi = 350 \times 0.9800 = 343 \text{ in.-kips}$$

$$M_{y_1} = M_x \sin\phi = 350 \times 0.1991 = 69.7 \text{ in.-kips}$$

Both are positive, according to the sign convention given with the general expression for stress.

$$f = \frac{69.7}{8.35} x_1 + \frac{343}{100.65} y_1 = 8.35x_1 + 3.41y_1$$

The maximum intensity of stress occurs at the same point as was determined in Art. 78, although it is not always so easy to determine the proper point by inspection with the rotated axes. However, it is necessary to calculate the coordinates of the point with reference to the principal axes, X_1 and Y_1.

$$x_1 = y \sin\phi + x \cos\phi = 5.50 \times 0.1991 + 0 = +1.09$$

$$y_1 = y \cos\phi - x \sin\phi = 5.50 \times 0.9800 - 0 = +5.39$$

$$8.35 \times 1.09 = +\ 9.10$$
$$3.41 \times 5.39 = +18.38$$

$$f = +27.48 \text{ kips per sq in. (compression)}$$

Composite Beam. The moments are the same as in Art. 79.

$$M_x = 1460 \text{ in.-kips} \qquad M_y = -75 \text{ in.-kips}$$

$$I_x = 761 \text{ in.}^4 \qquad I_y = 194 \text{ in.}^4 \qquad I_{xy} = 97.8 \text{ in.}^4$$

From formula 6,

$$\tan 2\phi = \frac{2 \times 97.8}{194 - 761} = \frac{195.6}{-567} = -0.3450$$

$$2\phi = -19 \text{ deg } 2 \text{ min} \qquad \phi = -9 \text{ deg } 31 \text{ min}$$

$$\sin 2\phi = -0.3261 \qquad \sin\phi = -0.1653$$

$$\cos 2\phi = 0.9453 \qquad \cos\phi = 0.9862$$

The positions of the principal axes X_1 and Y_1 are shown in Fig. 92. The angle of 160 deg 58 min has a tangent of -0.3450, and the X axis could have been rotated through an angle of 80 deg 29 min, which would place it where the Y_1 axis is. The small rotation of X is more convenient.

From formulas 3 and 4:

$$I_{x_1} + I_{y_1} = 761 + 194 = 955$$

$$I_{x_1} - I_{y_1} = 567 \times 0.9453 - 195.6(-0.3261) = 600$$

$$2I_{x_1} = 1555 \qquad 2I_{y_1} = 355$$

$$I_{x_1} = 777.5 \qquad I_{y_1} = 177.5$$

The signs for the sine and cosine have no real significance in moment computations. Following the convention given with the general expres-

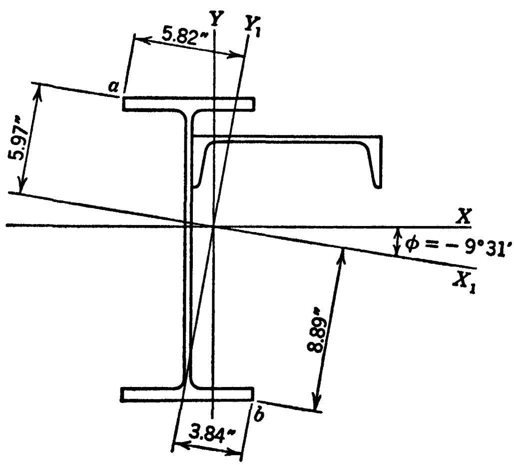


FIG. 92

sion for stress, compression on the top or right-hand fibers is positive moment; this convention should be followed in giving signs to M_{x_1} and M_{y_1}. If an axial load were present and M_x and M_y were expressed as Pe_y and Pe_x, an arbitrary convention of signs could be followed in the algebraic signs of the angle functions and e_{y_1} and e_{x_1}, but taking cognizance of the direction of bending is the easiest and safest method for determining the sign of a bending moment.

$$1460 \times 0.9862 = 1440 \qquad 1460 \times 0.1653 = -241$$

$$-75 \times 0.1653 = -\ 12 \qquad -75 \times 0.9862 = -\ 74$$

$$M_{x_1} = 1428 \text{ in.-kips} \qquad M_{y_1} = -315 \text{ in.-kips}$$

$$f = \frac{-315}{177.5}x_1 + \frac{1428}{777.5}y_1 = -1.775x_1 + 1.836y_1$$

At a:

$$x_1 = 6.85(-0.1653) - 4.75 \times 0.9862 = -1.13 - 4.69 = -5.82$$

$$y_1 = 6.85 \times 0.9862 + 4.75(-0.1653) = 6.76 - 0.79 = 5.97$$

$$\begin{aligned} -1.775 \times (-5.82) &= 10.33 \\ 1.836 \times 5.97 &= 10.96 \\ \hline f &= 21.29 \text{ kips per sq in. (compression)} \end{aligned}$$

This checks the computed value in Art. 79 with reasonable accuracy.

At b:

$$x_1 = -9.40(-0.1653) + 2.32 \times 0.9862 = 1.55 + 2.29 = 3.84$$

$$y_1 = -9.40 \times 0.9862 - 2.32(-0.1653) = -9.27 + 0.38 = -8.89$$

$$\begin{aligned} -1.775 \times 3.84 &= -6.82 \\ 1.836 \times (-8.89) &= -16.32 \\ \hline f &= -23.14 \text{ kips per sq in. (tension)} \end{aligned}$$

81. Location of Neutral Axis. The neutral axis was defined in Chapter 1 as the axis where there was no change in lengths of the fibers (or no strain) and consequently as the axis of zero stress. If there is no axial load ($P = 0$), one point of zero stress is the centroid of the cross section, and the angle the neutral axis of the cross section makes with a reference axis through the centroid can be determined from the general expression for stress. If the general expression is equated to zero (where there is no axial load) and the ratio y/x is obtained, this is the tangent of the angle ϕ_{na}, which the neutral axis X_{na} makes with the reference axis X.

Using the form in equation 7 with $P = 0$,

$$f = \frac{M_y I_x - M_x I_{xy}}{I_x I_y - I_{xy}^2} x + \frac{M_x I_y - M_y I_{xy}}{I_x I_y - I_{xy}^2} y = 0$$

$$(M_x I_y - M_y I_{xy}) y = -(M_y I_x - M_x I_{xy}) x$$

$$\frac{y}{x} = \tan \phi_{na} = -\frac{M_y I_x - M_x I_{xy}}{M_x I_y - M_y I_{xy}} \tag{9}$$

It was stated in Chapter 1 that one of the basic assumptions in flexure is that plane sections that are normal to the longitudinal axis of members of constant section remain planes after bending. If this is true, the bending in any unsymmetrical section would be about its neutral axis, and, since stress is proportional to strain, the stress would have a linear variation as shown in Fig. 93.

Since these relations are identical with those obtained when the flexural formula Mc/I was developed in Chapter 1, Art. 14, a similar relation must hold in this case.

Let M_{na} = the resultant bending moment about the neutral axis.
I_{na} = the moment of inertia of the section about the neutral axis.
y_{na} = a coordinate perpendicular to the neutral axis.

Then

$$f = \frac{M_{na}}{I_{na}} y_{na} \quad \text{(for axial load} = 0) \tag{10}$$

Although it is true that there is normally a moment about a centroidal axis perpendicular to the neutral axis and, in unsymmetrical sections, a

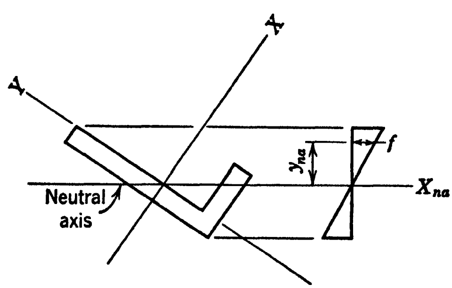


FIG. 93

product of inertia for the two axes, still the simple expression of formula 10 gives the same stress in a fiber that is obtained by applying the general expression for stress and using the neutral axis and its perpendicular axis.

If there is an axial load P, then the unit stress at the centroid is P/A, or

$$\frac{P}{A} + \frac{M_yI_x - M_xI_{xy}}{I_xI_y - I_{xy}^2}x + \frac{M_xI_y - M_yI_{xy}}{I_xI_y - I_{xy}^2}y = \frac{P}{A}$$

Therefore the neutral axis has the same slope as given in formula 9, and its perpendicular distance from the centroid is

$$\frac{P/A}{M_{na}/I_{na}}$$

The determination of the neutral axis is very useful in computing deflections.

The neutral axis can be located with respect to the principal axis X_1 in a similar manner. The value of the tangent of the angle between the principal axis and neutral axis, θ, will be in a more simple form since the product of inertia is zero for principal axes.

$$f = \frac{M_{y_1}}{I_{y_1}} x_1 + \frac{M_{x_1}}{I_{x_1}} y_1 = 0 \quad \text{(for axial load = 0)}$$

$$\frac{M_{x_1}}{I_{x_1}} y_1 = - \frac{M_{y_1}}{I_{y_1}} x_1$$

$$\frac{y_1}{x_1} = \tan \theta = - \frac{M_{y_1} I_{x_1}}{M_{x_1} I_y} \tag{11}$$

The above expression assumes no axial load ($P = 0$). If there were an axial load, the effect would be similar to that discussed when locating the neutral axis with reference to the axis X.

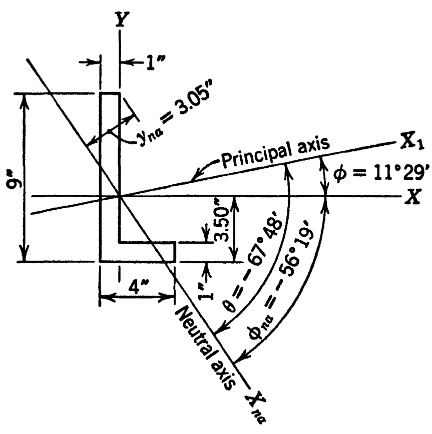


FIG. 94

82. Illustrative Example. The angle used in Art. 78 is shown in Fig. 94. With the same data as in Art. 78:

$$\tan \phi_{na} = - \frac{0 \times 97 - 350(-18)}{350 \times 12 - 0(-18)} = - \frac{6300}{4200} = -1.500$$

$$\phi_{na} = -56 \text{ deg } 19 \text{ min}$$

$$\sin \phi_{na} = -0.8321$$

$$\cos \phi_{na} = +0.5546$$

$$\sin 2\phi_{na} = -0.9230$$

$$M_{na} = M_x \cos \phi_{na} = 350 \times 0.5546 = +194.1 \text{ in.-kips}$$

$$I_{na} = \overline{0.5446}^2 \times 97 + \overline{0.8321}^2 \times 12 - (-0.9230)(-18)$$

$$= 29.83 + 8.31 - 16.61 = 21.53 \text{ in.}^4$$

$$y_{na} = 5.50 \times 0.5546 - 0(-0.8321) = +3.05$$

$$f = \frac{194.1}{21.53} \times 3.05 = 27.50 \text{ kips per sq in.}$$

If it is desired to locate the neutral axis with reference to the principal axis X_1, using the data for the unequal leg angle from Art. 80,

$$\tan \theta = -\frac{(+69.7)(+100.65)}{(+343)(+8.35)} = -2.4494$$

$$\theta = -67 \text{ deg } 48 \text{ min}$$

In Art. 80, the value of the angle between the reference axes and the principal axes ϕ was found to be +11 deg 29 min.

$$-67 \text{ deg } 48 \text{ min} + 11 \text{ deg } 29 \text{ min} = -56 \text{ deg } 19 \text{ min}$$

This checks with the value of ϕ_{na}.

The values of M_{na}, I_{na}, and y_{na} could be determined with respect to the principal axes using $\theta = -67$ deg 48 min, and the same values would be obtained.

83. Deflection of Unsymmetrical Members. The deflection of an unsymmetrical member can be obtained with the data for bending about the principal axes, or for bending about the neutral axis. The computations are generally more direct in the latter, but it must be remembered that the location of the neutral axis is a function of the moments (or loads) as well as the physical properties of the section, and it changes with a change of relative loading. The principal axes are determined by the physical properties of the section, and once computed can be used with any changes in both the vertical and lateral loads.

The deflection of the unequal leg angle will be computed: first, from the data of Art. 80; and second, from the data from Art. 82.

It will be assumed that the angle has a span of 12 ft, or 144 in., and a uniformly distributed load of 1.62 kips per ft. The bending moment is

$$M_x = \frac{1.62 \times 12^2}{8} \times 12 = 350 \text{ in.-kips}$$

which is the same moment previously used.

For a uniform load,

$$\delta = \frac{5}{384}\frac{wL^4}{EI} = \frac{wL^2}{8}\frac{5L^2}{48EI}$$

or

$$\delta = M\frac{5L^2}{48EI}$$

all units being in inches.

$$E = 29{,}000 \text{ kips per sq in.}$$

From the data for principal axes from Art. 80, the downward deflection perpendicular to the principal axis X_1 is

$$343 \times \frac{5 \times 144^2}{48 \times 29{,}000 \times 100.65} = 0.2538 \text{ in.}$$

and the deflection parallel to the principal axis X_1 and toward the left is

$$69.7 \times \frac{5 \times 144^2}{48 \times 29{,}000 \times 8.35} = 0.6217 \text{ in.}$$

The tangent is 0.6217 ÷ 0.2538 = 2.4496, or the line of deflection makes an angle of 67 deg 48 min with the axis Y_1. The resultant deflection is $(\overline{0.2538^2 + 0.6217^2})^{1/2} = 0.6715$ in., as shown in Fig. 95.

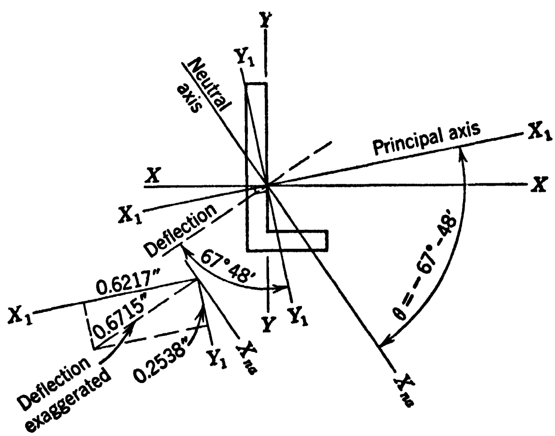


FIG. 95

From the data for neutral axis from Art. 82:

$$194.1 \times \frac{5 \times 144^2}{48 \times 29{,}000 \times 21.53} = 0.6715 \text{ in.}$$

downward to the left in a direction perpendicular to the neutral axis.

CHAPTER 7

BENDING IN SPECIAL BEAMS INCLUDING THOSE WITH THIN WEBS

84. Introduction. In aircraft structures, semi-tension-field beams * are quite common, and frequently the diagonal compression that can be resisted by the webs is so small that the beams are practically complete tension-field beams. In other structures, such as those previously discussed and designed, the webs of beams are completely shear resistant; however, stiffeners are often added, especially in plate girders, to prevent the web from buckling due to the diagonal compressive stresses. This book thus far has dealt only with beams having shear-resistant webs.

In tension-field beams the diagonal compressive stresses exceed the capacity of the web in buckling at smaller applied loads than those for which the beam is designed. In complete tension-field beams, any applied load, however small, produces diagonal compressive stresses beyond the buckling strength of the web. (Obviously a complete tension-field beam is never completely realized.) For a semi-tension-field beam, small applied loads do not cause web buckling, and the beam acts the same as one with a shear-resistant web, but increased applied loads cause web buckling.

It was found in Chapters 1 and 5 that the horizontal or vertical maximum unit shearing stress (f_s) † in a beam subjected to flexural stresses only occurs at the neutral axis and is equal to the unit diagonal tensile stress f_t and likewise to the unit diagonal compressive stress f_c, or $f_s = f_t = f_c$. This relationship is shown in Fig. 96, where an extremely

* The procedures presented in this chapter follow closely the theories and design procedures presented by Professor Herbert Wagner of Danzig, Germany (as translated in publications of the National Advisory Committee for Aeronautics), and Paul Kuhn in publications of the N.A.C.A.

† The symbols in this chapter differ from those in the previous chapters in order to agree better with the major part of the printed literature on this subject which follows "Strength of Metal Aircraft Elements" (ANC-5a), issued in May 1949, by the Munitions Board, Aircraft Committee. For example, f_s here is the same as v in the previous chapters. This should cause no confusion, however, since each new symbol is defined the first time it appears.

small unit cube is shown at the neutral axis. The shearing forces acting on it and the forces acting on the diagonal are shown in Fig. 96*b*.

In a beam having a very small web thickness t_w and a very large flange area A_f, the moment of inertia I of the beam can be taken as equal to $2\left(A_f \frac{h^2}{4}\right)$ without appreciable error. In the expression for unit shearing stress previously developed, $f_s = VQ/It_w$. In this expres-

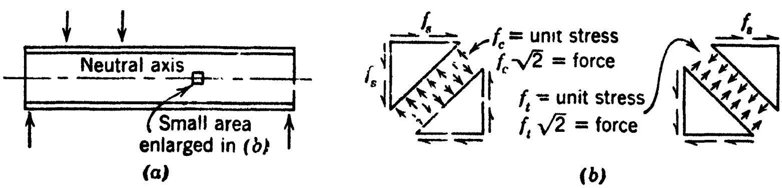


FIG. 96

sion, Q = the first moment of area about the neutral axis, and, since the web is so thin, the first moment of any web area about the neutral axis is negligible compared to that of the flange and can be ignored. This gives $Q = A_f(h/2)$ for any section between the neutral axis and the flange for which shear is to be computed. Using these values of I and Q in the computations shown in Fig. 97, and taking the web area A_w as

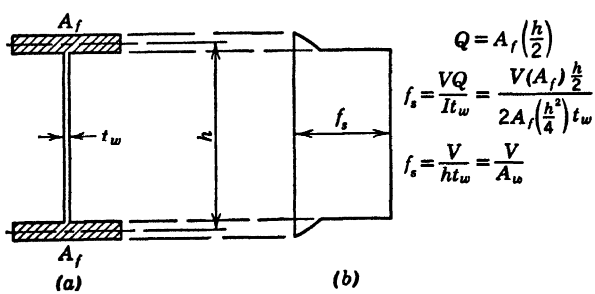


FIG. 97

$t_w \times h$, the unit shearing stress within the depth of the web is found to be a constant, or $f_s = V/A_w$.

If the small unit cube of Fig. 96*b* is taken as shown in Fig. 98*a* (the faces of the cube perpendicular to the web making an angle of 45 deg with the neutral axis), the same relationship between f_s, f_t, and f_c exists. This is shown in Fig. 98*b*. If the web can resist no compressive stresses (a complete tension-field beam), the relationship between f_s and f_t is as shown in Fig. 98*c*; that is, f_t is now twice as large. Also, the horizontal component of the diagonal tension is resisted by tensile stresses on the vertical face, the unit value of these tensile stresses being equal to f_s. These tensile stresses on the vertical face of the web are in addition to any other strrsses due to bending.

Another way of indicating how the diagonal tensile forces are doubled when the diagonal compressive forces are zero is shown in the single

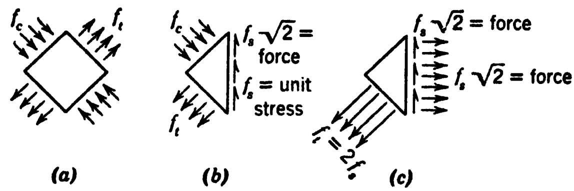


FIG. 98

panel truss in Fig. 99. For the panel in Fig. 99*a* with both tension and compression diagonals, the applied load P, which is the shear in the panel, will be resisted by the joint action of the two diagonals. If the members AB, BD, DC, and CA are infinite in area and stiffness as compared to the diagonals, then these members will not change in length under load. A view of the deflected structure (vastly exaggerated) is shown in Fig. 99*b*, with AB parallel to CD, and AC parallel to BD. For

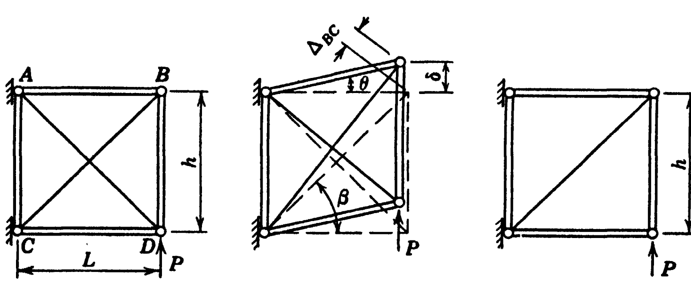


FIG. 99

small deflections as occur in structures, and consequently small angle changes, the increase in length of CB equals the decrease in length of AD. For diagonals of the same area and the same material, the tension in CB equals the compression in AD, or the vertical component of each is equal to $P/2$.

If, as shown in Fig. 99*c*, the compression member is removed, the vertical component in the tension member is equal to P. The compression in AB is $PL/2h$ in Fig. 99a but PL/h in Fig. 99c, and the tension in CD is $PL/2h$ in Fig. 99a, but zero in Fig. 99c. The change in stress in both members when the compression diagonal is removed is equivalent to adding a compression of $PL/2h$. Such compressive stresses exist in the flanges of tension-field beams and resist the horizontal tension shown on the vertical face in Fig. 98c.

85. Complete Tension-Field Beams. In Fig. 100a is shown a beam with a web that is assumed to be incapable of resisting compression.

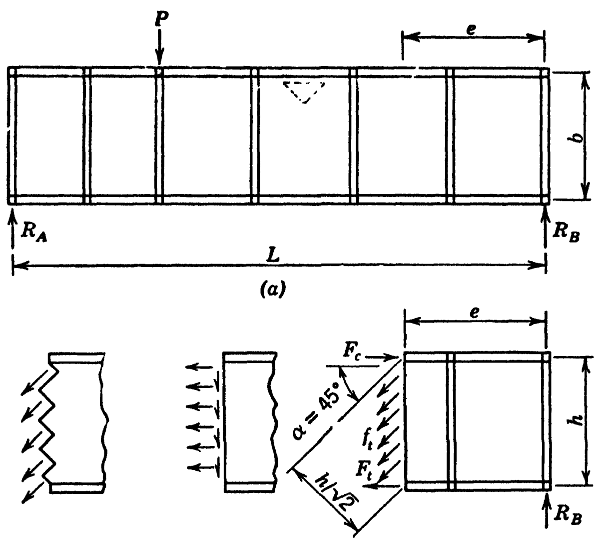


FIG. 100

A vertical section between stiffeners, at a distance e from the right support R_B, is shown in Fig. 100b. The forces on the left vertical face of the right segment of the beam can be considered as acting in either of the three manners shown.* If a saw-tooth vertical section is shown, only the faces slanting downward to the right are stressed, as the other faces would have compressive stresses and the web can resist no compression. If, as in the center section, the shearing stresses parallel to

* In this discussion, as well as in the previous article where these web stresses were developed, all tension (as well as compression) in the web due to flexural action of the beam has been neglected. In very thin webs this flexural tension would be negligible in resisting bending, as compared with the flange stresses, and would be small in its effect on the other stresses in the web. It is on the basis of this assumption that the resultants of the web stresses act at an angle of 45 deg at all points.

the vertical face are shown, there must also be the normal tension stress, as previously shown in Fig. 98*c*.

In the right diagram of Fig. 100*b* the resultant diagonal tensile stresses f_t are shown, acting downward and to the left. Where the tension due to bending is neglected, as where the web is extremely thin, this diagonal tensile stress acts at an angle of 45 deg with the horizontal and on a projected area (normal to the line of action of f_t) of $(h/\sqrt{2}) \times t_w$. The total diagonal tension is $f_t(h/\sqrt{2})t_w$, its vertical component is $0.707f_t(h/\sqrt{2})t_w = (f_t h t_w)/2 = V$ (in this instance $V = R_B$). From the previous article, $f_s = V/A_w = V/ht_w$. Therefore $f_t = 2f_s$, as previously shown in Fig. 98*c*.

86. Flange Compression and Tension. Since in Fig. 100*b* the diagonal tension f_t is acting at an angle of 45 deg and the vertical component of the total diagonal tension is equal to V, then the horizontal component (or the total of the normal web tension) is also equal to V. F_c and F_t, as shown in Fig. 100*b*, represent the total stresses in the compression flange and the tension flange, respectively. Taking moments about F_t, and remembering that V can represent the total web tension normal to the section,

$$F_c h - V\frac{h}{2} = M \quad \text{or} \quad F_c = \frac{M}{h} + \frac{V}{2}$$

If moments are taken about F_c,

$$F_t h + V\frac{h}{2} = M \quad \text{or} \quad F_t = \frac{M}{h} - \frac{V}{2}$$

This means that each total flange stress, as computed from the bending moment at a section as for ordinary girders, would be modified by a compressive force equal to one-half the external shear at the section. This, of course, increases the total compression and decreases the total tension by that amount.

87. Flange Bending. The small triangular element shown dotted and greatly enlarged in Fig. 100*a* is shown further enlarged in Fig. 101*a*. The segment is taken just below the top flange, with the sides making an angle of 45 deg with the horizontal. The small area of the horizontal face has a unit value. The area of each inclined face is $1/\sqrt{2}$. The stress values shown are unit stresses. Since the web is assumed to be incapable of resisting compression, the only forces acting on the inclined faces are those from diagonal tension. Along the horizontal face the shearing stresses f_s act to the right, and in order that the vertical forces be in equilibrium there must be a vertical tensile stress f_n acting on this horizontal face. Therefore, since the flange is pulling on the web, the web exerts a downward pull on the flange. From a similar segment at the bottom flange, it should be clear that the web exerts an upward pull. From the equilibrium of the vertical forces,

$$f_n = \frac{f_t}{\sqrt{2}} \times 0.707 = \frac{f_t}{2}$$

Therefore $f_n = f_s$. This is an average value for f_n, and is conservative. The flange does not have an infinite stiffness, and its deflection causes the value of f_n to increase near the stiffener and decrease midway between

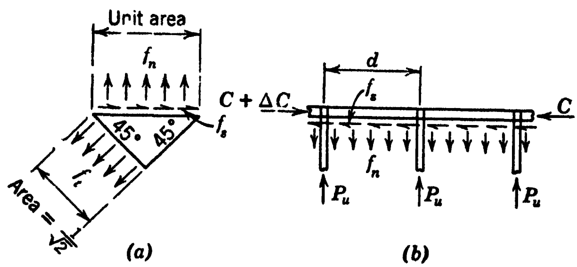


FIG. 101

stiffeners; however, near the stiffeners the web is shear resistant (or almost so), and for this reason the effect of f_n decreases near the stiffener.

In Fig. 101*b* is shown a portion of the top flange together with the forces that are acting on it. The horizontal web shearing stress f_s is acting to the left, and the vertical stress f_n is acting downward. If it is assumed that the stiffeners are uniformly spaced in this region of the beam, the spacing being the distance *d*, the flange would act as a flexural member continuous over supports (the stiffeners). For such a condition the maximum moment would be approximately equal to $\frac{1}{12}wd^2$, where w (the load per unit length) $= f_n t_w$. The maximum moment occurs at the interior stiffeners and is comparable to the negative moment at the supports of a continuous beam. Since the web can resist no compression and is itself pulling on the flange, the flexural stresses in the flange will be a factor of the moment of inertia of the flange I_F about its own gravity axis, and the distance c from the gravity axis to the appropriate extreme fiber. In the compression flange the inside fiber is the critical one, whereas the outside fiber is the one to consider in the the tension flange. This stress due to bending in the flange is $wd^2c/12I_F$ and is added to the unit stress in the flange due to the action of the beam.

88. Stiffener Loads. From Fig. 101*b* it is readily seen that the compressive load on an interior stiffener P_u is $f_n t_w d = Vd/h$, since $f_n = f_s$. With the web unable to resist compressive stresses, this load P_u is the required strength of the attachment of the stiffener and the flange and remains a constant load in the stiffener between flange attachments.

Where a load is applied to the flange at a stiffener, account must be taken of this load in the strength of the flange attachment, and the load in the stiffener is also modified on account of the change in shear V. This modification is also true when the load is applied to the beam through a connection to the stiffener.

End stiffeners in tension-field beams are subjected to bending due to the horizontal component of the diagonal tensile stresses. The attachment of the stiffener to the flange is such that the end stiffener for these flexural loads is considered as a simple span between flange attachments.

Loads applied to tension-field beams should be applied only at points where stiffeners are located unless the resulting bending in the flange is considered.

89. Rivet Spacing. In a beam with a shear-resistant web, the load on a rivet R is measured by the shearing stress f_s, as shown in Fig. 101, over the distance between rivets p. In the tension-field beam there is also the normal force f_n, as shown in Fig. 101, the intensity of which is equal to f_s. The total load on a rivet in a complete tension-field beam in a distance p is therefore the resultant of the two forces, $pf_s t_w$ and $pf_n t_w$, and therefore is equal to $pf_s t_w \sqrt{2}$.

If there is a shear-resistant web,

$$p = \frac{Rh}{V}$$

R being the capacity of the rivet.

In a complete tension-field beam,

$$p = \frac{Rh}{\sqrt{2}V}$$

Since the web is connected to the flange by means of rivets which require holes in the web, the capacity of the web is reduced to the capacity of the net section of the web, since tension is the controlling stress.

90. Semi-Tension-Field Beams. The design of semi-tension-field beams (sometimes called incomplete tension-field beams) requires, among other things, the knowledge of experimental data obtained from physical tests. An adequate treatment of the design procedure for this type of beam can be found in textbooks on the design of aircraft structures, but a lengthy treatment is inappropriate here. Some of the factors affecting such designs will be briefly mentioned.

For tension-field beams with webs that can take small compressive stresses, the beam acts as one with a shear-resistant web, where $f_t = f_s$, when the applied loads produce diagonal compressive stresses within the capacity of the web in compression. For applied loads producing diagonal compressive stresses greater than the web can resist it is assumed that the diagonal compressive stresses continue to exist at a value equal to the capacity of the web in compression. For the smaller first portion of such an applied load the web is shear resistant, but for the remainder it is tension field, and the total diagonal tensile stress f_t is greater than f_s but less than $2f_s$. Also, the angle that the diagonal tensile stresses make with the horizontal is other than 45 deg when the web can resist compression, whether it be the result of shear or of flexure.

The compression change in the total stress in the flange of a semi-tension-field beam is less than $V/2$ if the web resists some of the shear as a shear-resistant web.

The flange bending stresses are reduced in a semi-tension-field beam because the vertical component of the diagonal tension is less and also because some of the web is acting with the flange as a flexural member.

For similar reasons the stiffener loads are decreased and the required rivet spacings increased.

In all cases the web can take flexural stresses in tension. This fact is neglected in a complete tension-field beam because the effect on the design is extremely small. For a semi-tension-field beam the effect of the flexural stresses in both tension and compression should be included in the design.

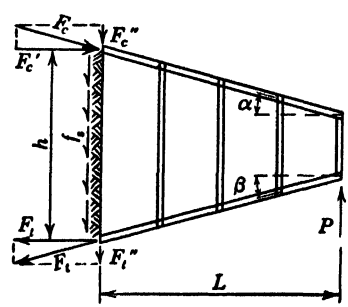


FIG. 102

91. Tapered Beams. Tapered beams (beams with nonparallel flanges) are found in bridges, buildings, and many other structures, although their use is not extensive. Because of the desirability of saving weight, they are more common in aircraft structures. A tapered beam can have one sloped flange, or both can be sloped as shown in Fig. 102. An extensive discussion of tapered beams will not be attempted in this article. The main interest here is in connection with lightweight beams, or beams with thin webs, and the effect of the tapered flange on the shear in the web which is shear resistant.

A short tapered cantilever beam is shown in Fig. 102, with an upward load P at the right end and an effective depth h at the left support. If a section is taken at the left support and the flexural stress in the web is neglected, the horizontal component of stress in the compression flange F_c' and the horizontal component of stress in the tension flange F_t' will each be approximately equal to PL/h. The neglect of the flexural stress in the web is conservative for any size of web, and the difference is negligible for very thin webs. The total stress in the compression flange due to flexure in the beam is $PL/h \sec \alpha$, and the total stress in the tension flange is $PL/h \sec \beta$. PL, of course, represents the bending moment at the section. These values are safe and rather accurate for values of α and β, and $(\alpha + \beta)$, that are not too large. For larger angles the error is disproportionately greater, but it is always on the conservative side. This method of analysis is simple and sufficiently accurate for ordinary design purposes. More accurate analyses can be found in advanced treatments on strength of materials and other technical publications. An excellent theoretical paper on this subject is "A Theory of Flexure for Beams with Nonparallel Extreme Fibers," by W. R. Osgood, in the *Journal of Applied Mechanics*, September 1939, Vol. 6, No. 3, page A-122.

These total flange stresses have vertical components (F_c'' and F_t'', as shown in Fig. 102). These vertical components act in the direction shown and reduce the shearing stresses in the web. The total shear in the web in this case is $P - F_c'' - F_t'' = P - \frac{PL}{h}(\tan \alpha + \tan \beta)$. The unit shearing stress in the web is, therefore,

$$\frac{P - \frac{PL}{h}(\tan \alpha + \tan \beta)}{ht_w} = \frac{P}{ht_w}\left[1 - \frac{L}{h}(\tan \alpha + \tan \beta)\right]$$

There is a small amount of shear in the flanges that is ignored in these expressions. This is the same as the shear in the flanges of beams and girders that is also ignored where the web is assumed to resist all the shear.

In calculating the unit compression in the flange, the student should realize that the horizontal component F_c' is acting on a vertical section through the flange, the vertical flange area being A_c'. The total stress F_c also acts on this same area, but to compute the unit stress F_c should be divided by the cross-sectional area of the flange which is $A_c \doteq A_c' \cos \alpha$. Therefore,

$$f_c = \frac{\dfrac{PL}{h} \sec \alpha}{A_c' \cos \alpha} = \frac{PL}{hA_c'} \sec^2 \alpha = \frac{PL}{hA_c} \sec \alpha$$

Similarly

$$f_t = \frac{PL}{hA_t'} \sec^2 \beta = \frac{PL}{hA_t} \sec \beta$$

in which f_c and f_t are the unit flexural stresses in the compression and tension flanges, respectively.

It will be seen from these relations that, if the total stress in the flange is divided by the cross-sectional area perpendicular to the slope of the flange, the result will be the unit stress. If the area of the vertical section is used, the value will have to be further multiplied by the secant of the angle of slope of the flange. Consequently, if the unit stress were computed by the flexural formula Mc/I (I being the moment of inertia of the vertical section), the correct unit stress would be very closely $(Mc/I) \sec^2 \alpha$ (or $\sec^2 \beta$ for the bottom flange in Fig. 102).

92. Illustrative Example. The specifications for the design of aircraft structures differ from those for the design of bridges and buildings. The stresses produced by design loads in aircraft structures are compared to the stresses that the structures can resist at ultimate strength; * however, the design loads are obtained by multiplying the applied loads by some factor greater than 1 (frequently 1.50). Also, the stresses resulting from applied loads are required to be less than the yield stress.

Because the examples of design given thus far have been based on specifications for the design of buildings, the following example conforms for the most part to specifications for buildings. Since tension-field beams are not allowed under current building specifications, it will be necessary to deviate slightly from these specifications, especially from a clause such as the one that states: "Plate girder webs shall have a thickness of not less than $\frac{1}{170}$ of the unsupported distance between flanges."

As an example problem, a steel plate girder with a span of 100 ft will be considered. The distance between the centroids of area of the flanges will be taken arbitrarily as about 10 ft. The total load to be supported is shown in Fig. 103. The choice of span, depth, and loads as well as the cross-sectional dimensions and arrangements of the members which will be suggested are not typical for buildings or aircraft structures but are

* For parts of some structures, the ultimate strength is equal to or less than the area times the yield stress, in which cases the stresses caused by design loads are thus limited to lower stresses.

given here merely as a simple means of explaining the application of the theory of tension-field beams.

The maximum total stresses in the flanges will be equal to the algebraic sum of the bending stresses and those due to diagonal tension in the web,

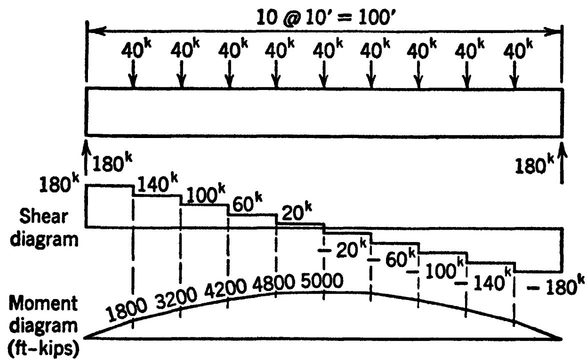


FIG. 103

as shown below. It is the algebraic sum of the moment divided by the effective depth (115.76 in., see Fig. 104*a*), and the shear divided by 2.

TABLE 5

Panel	Compression Flange			Tension Flange		
	Bending	Shear	Total	Bending	Shear	Total
0–1	−187	−90	−277	+187	−90	+ 97
1–2	−332	−70	−402	+332	−70	+262
2–3	−435	−50	−485	+435	−50	+385
3–4	−497	−30	−527	+497	−30	+467
4–5	−518	−10	−528	+518	−10	+508

The 40-kip loads are applied to the girder by means of connections to the stiffeners on the web; also, it is assumed that the girder has continuous lateral support for the upper flange, and the allowed stress in this flange is 20 kips per sq in. The allowable stress in tension for any member will be taken as 20 kips per sq in. on the net section.

If the web is in tension and has a net section of 75 per cent of the gross section, then the allowable tension stress on the gross section of the web is 75 per cent of 20 kips per sq in., or 15 kips per sq in. The maximum shear is 180 kips, the effective depth is assumed to be about 120 in., and the diagonal tensile stress f_t is twice the shearing stress f_s; therefore f_s is limited to 7.5 kips per sq in., and the web thickness t_w

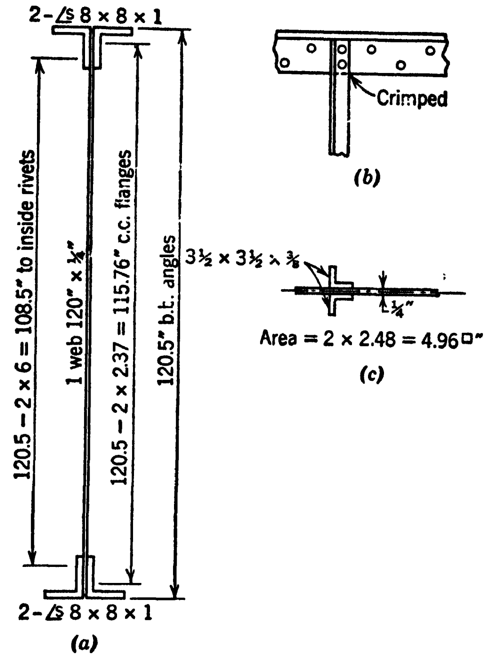


FIG. 104

must be at least $180 \div (120 \times 7.5) = 0.200$ in. The web thickness will be $\frac{1}{4}$ in.

The trial section shown in Fig. 104*a* will be checked, using 1-in. round rivets (in the flanges) which have a capacity in double shear of $0.785 \times 15 \times 2 = 23.56$ kips and 10 kips in bearing on a $\frac{1}{4}$-in. web.

Using both gage lines (see Fig. 104*b*), the attachment of the stiffener angles to the flange angles has a capacity of $2 \times 23.56 = 47.12$ kips ($\frac{3}{8} \times 32 \times 4 = 48$ kips) if the stiffener angles are $\frac{3}{8}$ in. or more in thickness. The load on a stiffener (P_u) is Vd/h, or $d = P_u h/V = (47.12 \times 115.76) \div 180 = 30.3$ in. in the end panel, and 39.0 in. in the second panel, 54.6 in. in the third panel, 91.0 in. in the fourth panel, and 273 in. in the center panel, if the spacing is determined by a P_u of 47.12 kips.

The stiffeners act much like columns hinged at the ends, with lengths each of which is equal to 108.5 in. Two angles $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{3}{8}$ in. separated by $\frac{1}{4}$ in., as shown in Fig. 104c, have a radius of gyration of 1.56 in., an L/r of 108.5 ÷ 1.56 = 69, an area of 2 × 2.48 = 4.96 sq in., and a capacity in compression of 14.69 × 4.96 = 73 kips.

The moment of inertia of the flange I_F composed of 2 angles 8 by 8 by 1 in. and the 8 in. of ¼-in. plate between the angles is 194 in.⁴ The flange bending stress for an interior panel is $wd^2c/12I_F$, but at the first interior stiffener it is about $wd^2c/10I_F$, where $w = f_s(t_w) = (V/A_w)(t_w) = V/h$. If the end panel of 10 ft has a pair of stiffeners every 2½ ft, then d = 30 in., w = 180 ÷ 115.76 = 1.56 kips per in., and the flange bending stress just to the left of the first 40-kip load is $1.56(30)^2 5.63 \div (12 \times 194) = 3.38$ kips per sq in. The axial load in the flange at this first load is $-(M/d) - (V/2) = -[(1800 \times 12) \div 115.76] - (180 \div 2) = -277$ kips (see Table 5), and the flange area is 2 × 15 + 2 = 32 sq in. The maximum unit stress is −(277 ÷ 32) − 3.38 = −12.04 kips per sq in.

In the first panel where the shear is 180 kips, the diagonal tension stress $f_t = (2 \times 180) \div (115.76 \times \frac{1}{4}) = 12.4$ kips per sq in., f_s = 6.2 kips per sq in., and the spacing of 1-in. round rivets with a 10-kip capacity for bearing on a ¼-in. web is $10 \div \sqrt{2}(1.56) = 4.54$ in. (say 4½ in.).

For the second panel, the following calculations are made:

$f_s(t_w) = V/h = 140 \div 115.76 = 1.21$ kips per in.

Rivet pitch = $10 \div \sqrt{2}(1.21) = 5.85$ in. (say 5¾ in., unless 20 × ¼ = 5 in. is assumed to be maximum).

Stiffener spacing = 40 in. (3 at 3 ft 4 in. = 10 ft). (39 in. has previously been computed, based on the capacity of attachment of flange angles to stiffener angles, but some small amount of load can be transmitted vertically from the flange angles to the web which lies between the two stiffener angles.)

Flange bending stress = $1.21(40)^2 5.63 \div (12 \times 194) = 4.68$ kips per sq in.

Total axial stress = −402 kips.

Maximum unit stress = (402 ÷ 32) + 4.68 = 17.23 kips per sq in. (compression).

For the third panel:

$V/h = 100 \div 115.76 = 0.865$ kips per in.

Rivet pitch = $10 \div \sqrt{2}(0.865) = 8.19$ in. (If 20 × ¼ is a limit, then smaller rivets should possibly be used.)

Stiffener spacing = 40 in. (3 at 3 ft 4 in. = 10 ft); 3½ by 3½ by $\frac{5}{16}$ in. angles may be used in this panel.

Flange bending stress = $0.865(40)^2 5.63 \div (12 \times 194) = 3.34$ kips per sq in.

Total axial stress = −485 kips.

Maximum unit stress = (485 ÷ 32) + 3.34 = 18.49 kips per sq in. (compression).

For the fourth panel:

$V/h = 60 \div 115.76 = 0.518$ kips per sq in.

Stiffener spacing = 40-in., and 3½ by 3½ by ¼ in. angles.

Flange bending stress = 2.01 kips per sq in.

Total axial stress = −527 kips.

Maximum unit stress = (527 ÷ 32) + 2.01 = 18.49 kips per sq in. (compression).

For the center panels:

V/h = 20 ÷ 115.76 = 0.173 kips per sq in.

Stiffener spacing = 60 in. (2 at 5 ft = 10 ft), and $3\frac{1}{2}$ by $3\frac{1}{2}$ by $\frac{1}{4}$ in. angles.

Flange bending stress = $0.173(60)^2 5.63$ ÷ (12 × 194) = 1.50 kips per sq in.

Total axial stress = (−5000 × 12) ÷ 115.76 − (20 ÷ 2) = −528 kips.

Maximum unit stress = (528 ÷ 32) + 1.50 = 18.00 kips per sq in. (compression).

Net area of tension flange = 26.94 sq in.

Maximum flange stress = +508 ÷ 26.94 = +18.87 kips per sq in.

To this should be added the flange bending stress in tension. At the stiffener it is 1.50 × (2.37 ÷ 5.63) = 0.63 kip per sq in. At the center of the stiffener spacing (using a $\frac{1}{24}$-moment coefficient) it is 0.75 kip per sq in. The maximum unit stress is under 20 kips per sq in.

Smaller angles with cover plates can be used with a saving in weight if the cover plates are cut off.

If the end reactions are vertical forces applied to the bottom flange, the end bearing stiffeners must be designed to resist not only this load but also the bending due to the horizontal components of the diagonal tensile stresses in the end panel. If the end of this member frames into a column by means of an attachment to the web, then the column plus any attachments to the other side of the column are available to assist in resisting the bending.

Actually the web is capable of resisting some compression, and the girder is not a complete tension-field beam. Where the stiffeners have a spacing of 30 in., the web will not buckle and thus it will be shear resistant for shearing stresses less than 10.5 kips per sq in.* This means that the web will be shear resistant. However, if the diagonal compression is combined vectorially with the compression due to bending, this web will probably buckle, and it has been considered as the web of a complete tension-field beam. The fact that it is not a complete tension-field beam indicates that the calculations are unduly conservative and that the stiffener spacings and rivet pitches can be greater and the flanges smaller.

* This value is conservative, according to data given in recent publications of the National Advisory Committee for Aeronautics. It is obtained from the procedure suggested in the "Strength of Aircraft Elements" (ANC-5), issued December 1942 by the Army-Navy Civil Committee on Aircraft Design Criteria; that is, the shearing stress at which the web would buckle = $K_s E(t_w/d)^2$, where K_s is a constant depending upon the panel dimensions and the restraint the flanges and stiffeners offer to web rotation, and E is the modulus of elasticity.

The deflection of this girder will be greater than for a similar one with a thicker web. Whenever a web buckles, the deflection is increased considerably.

The web splice for a tension-field beam contains more rivets than one for a beam with a shear-resistant web, just as the number of rivets between the web and the flange must be increased for a tension-field beam.

APPENDIX

The design of timber beams involves no principles beyond those discussed in Chapter 1, unless one attempts to introduce the discussion of timber connectors and other details. There are many excellent textbooks, handbooks, and other publications covering these details, and a partial list of references is given below. Timber makes an excellent structural material for many purposes, but cognizance must be taken of certain properties peculiar to the material. As steel I beams are weak in diagonal compression in the web and reinforced concrete beams are weak in diagonal tension, so timber beams are weak in horizontal shear. Also, timber is much stronger in compression parallel to the grain than in compression perpendicular to the grain.

The floor system that was previously designed in steel and reinforced concrete is redesigned here as a timber floor. The computations are shown on the following pages about as they would be made by the designer. The design is limited to the subfloor, a typical interior beam, girder, and column. The live load has been reduced to 200 lb per sq ft. Although in the previous designs it was assumed that the roof was carried by steel roof trusses, 90 ft in span, supported by steel columns, it is here assumed that the roof is supported by timber trusses, 45 ft in span, supported by a line of columns down the center of the upper story. The bays have been made 13 ft 6 in. in length, and the girder spans 15 ft in length.

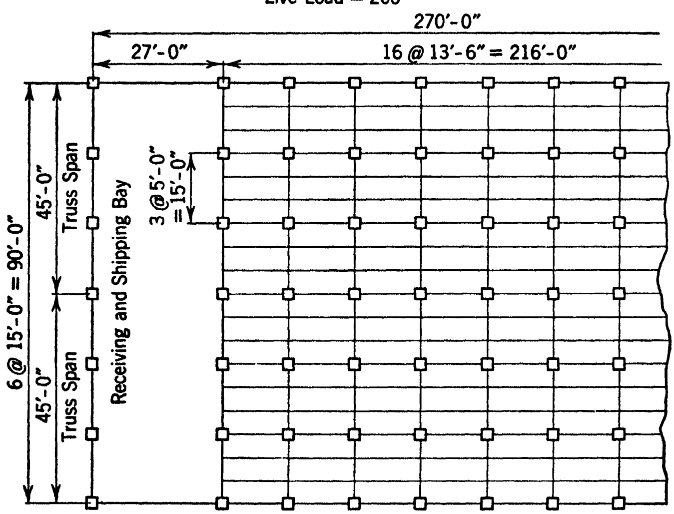
The working stresses are as shown in the calculations and are for good structural grade material. Since the tongue and groove flooring is continuous over the beams, the same moment coefficients are given for the floor as were previously recommended for a concrete slab. For simplicity, the beam and girder spans were taken as simple spans to the center of the supports instead of the centers of the bearing areas, although good practice permits the latter. The small difference makes no change in the design. The weight of the roof, purlins, and trusses, with snow load, was arbitrarily assumed as 50 lb per sq ft for the purpose of computing the load on the center line of columns. The roof columns are carried to the center columns by means of a cast iron pintle.

Timber Floor	
Interior Panel	
1950	J.V., J.G.C.
Sheet 1 of 3	

Flooring: 1″ T & G maple
Subfloor: T & G southern pine ($f = 1450$)
Beams and girders: Douglas fir ($f = 1700$)
or southern pine ($f = 1600$)

Species \ Allowable Stress	Flexure f	Horizontal Shear	Comp. ⊥ Grain	Comp. ∥ Grain	Modulus E
Southern pine	1450	125	390	1200	1,600,000
Southern pine	1600	120	455	1150	1,600,000
Douglas fir	1700	145	455	1325	1,600,000

Live Load = 200#/□′



For typical interior panel: Subfloor- Est. D.L. $8^{\#/\square'} \times \frac{5^2}{12} = 17'^{\#}$

$$\text{L.L. } 200^{\#/\square'} \times \frac{5^2}{9} = 556$$

Mom. per 12″ of width = $573'^{\#}$

$$\frac{573 \times 12}{1450} = 4.74 = \frac{I}{c} = \frac{12d^2}{6} \qquad d = \sqrt{2.37} = 1.54''$$

Use 2″ T & G subfloor (1⅝″ net) = $5.42^{\#/\square'}$
1″ T & G maple floor = 2.50
$7.92^{\#/\square'}$ Use $8^{\#/\square'}$

Timber Floor	
Interior Panel	
1950	J.V., J.G.C.
Sheet 2 of 3	

Beam

$$208 \times 5 = 1040^{\#/'}$$

$$\text{Est. wt. beam} = \underline{\quad 30}$$

$$1070^{\#/'} \times \frac{\overline{13.5}^2}{8} = 24{,}400'^{\#} = M$$

$$\times \frac{12.7}{2} = 6800^{\#} = V$$

$$\frac{24{,}400 \times 12}{1600} = 183 = \frac{I}{c} \qquad \text{Use } 8'' \times 14'',\ \frac{I}{c} = 228 \qquad \text{Wt.} = 28^{\#/'}$$

$$v = \frac{3}{2}\ \frac{6800}{7\frac{1}{2} \times 13\frac{1}{2}} = 101^{\#/\square''} \qquad \frac{1070 \times 13.5}{2 \times 455} = 16^{\square''} \text{ required for bearing}$$

Girder

$$1040$$

$$\underline{\quad 28}$$

$$1068 \times 13.5 = 14{,}400^{\#} \times 5 = 72{,}000'^{\#}$$

$$50 \times \frac{\overline{15}^2}{8} = \underline{\ 1{,}410}$$

$$M = 73{,}410'^{\#}$$

$$50 \times \frac{14.2}{2} = \begin{array}{r} 14{,}400 \\ \underline{355} \\ 14{,}755^{\#} \end{array}$$

$$\text{Use } V = 14{,}800^{\#}$$

$$\frac{73{,}410 \times 12}{1600} = 550 = \frac{I}{c}$$

$$\text{Use } 10'' \times 20''\ \frac{I}{c} = 602 \qquad \text{Wt.} = 51.4^{\#/'} \qquad v = \frac{3}{2}\ \frac{14{,}800}{9\frac{1}{2} \times 19\frac{1}{2}} = 120^{\#/\square''}$$

$$\frac{14{,}800}{455} = 33^{\square''} \text{ required for bearing}$$

Columns

Clearance to bottom of beams = 12′-0″

Length of column = 10′-4½″ = 10.37′

Minimum column size for girder width = 10″ × 10″

$$K = 0.702\sqrt{\frac{1{,}600{,}000}{1150}} = 26.1$$

$$1150\left[1 - \frac{1}{3}\left(\frac{10.37 \times 12}{26.1 \times 9.5}\right)^4\right] = 1150 \times 0.98 = 1127^{\#/\square''}$$

$$14{,}800 \times 2 = 29{,}600$$

$$14{,}400$$

$$\text{Est. col. wt.} \quad \underline{\quad 250}$$

$$44{,}250 \div 1127 = 39.3^{\square''} \qquad 9\frac{1}{2} \times 9\frac{1}{2} = 90.3^{\square''}$$

Checking this section for reduced load and bending

Reduced load = 44,250 − (200 × 13.5 × 5) = 30,750#

Required bearing length for girder = 33 ÷ 9.5 = 3.5″

Bending in column = (200 × 13.5 × 5)(4.75 − 1.75) = 40,500′#

$$\frac{\frac{P}{A}}{1127} + \frac{\frac{Mc}{I}}{1600} = \frac{30{,}750}{90.3(1127)} + \frac{40{,}500}{143(1600)} = 0.302 + 0.177 = 0.479 < 1.000$$

Checking center column with roof load

$$44{,}250$$

$$13.5 \times 45 \times 50 = 30{,}400$$

$$\text{Wt. roof col.} \quad = \underline{\quad 200}$$

$$74{,}850^{\#} \div 1127 = 66.4^{\square''} < 90.3 \quad 10'' \times 10'' \text{ satisfactory}$$

$$\frac{61{,}350}{90.3(1127)} + \frac{40{,}500}{143(1600)} = 0.603 + 0.177 = 0.780 < 1.000$$

The columns will have a length more than 11 times the thickness and the allowable unit stress is computed from the following:

$$c\left[1 - \frac{1}{3}\left(\frac{L}{Kd}\right)^4\right]$$

where $K = 0.702\sqrt{E/c}$.
c = the allowable stress in compression parallel to the grain, 1150 lb per sq in. in this case.
L = unsupported length in inches.
d = least dimension of the column in inches.
E = modulus of elasticity in pounds per square inch.

In computing the dead loads, the weight of all timber members has been assumed to be 40 lb per cu ft.

If large timber members, such as a 10 by 20 in. girder, are not available, two 10 by 10 in. timbers are satisfactory, with one on top of the other and with bolts and shear connectors provided to make the two members function as a structural unit.

On sheet 3 of the computations there is shown an alternate floor design in which the tongue and groove subflooring is replaced by a laminated floor (2 by 6 in. placed on edge and nailed together). The ends of these boards are staggered so that the subfloor will be continuous over the beams. The girders have been eliminated as the 15 ft. span for the floor is a little more economical than the 13 ft 6 in. span. The laminated floor will require more material than the beam and girder layout; however, it may not cost more. The deflection in the laminated floor will be greater than the suggested $\frac{1}{360}$th of the span where plastered ceilings are involved, but this should not be a material consideration in a building of this type.

Plates I and II are shown, following the computation sheets, to illustrate heavy timber framing. They are not illustrations of the floor systems designed here.

REFERENCES

W. F. Scofield and W. H. O'Brien, *Modern Timber Engineering*, Southern Pine Association, 1949.

H. J. Hansen, *Modern Timber Design*, John Wiley and Sons, 1948.

C. W. Dunham, *Planning Industrial Structures*, McGraw-Hill Book Company, 1948.

Typical Designs of Timber Structures, Timber Engineering Company, Washington, D.C., 1949.

Heavy Timber Construction Details, National Lumber Manufacturers Association, 1943.

Wood Handbook, prepared by The Forest Products Laboratory, United States Department of Agriculture, Washington, D.C., revised June 1940.

Timber Floor	
Interior Panel	
1950	J.V., J.G.C.
Sheet 3 of 3	

Alternate Design:

Use laminated subfloor to replace planks and girders.

Span of laminated floor $= 15'$

Maple floor $= 2.5^{\#/\square'}$

6″ subfloor $= \underline{18.3}$

$$20.8^{\#/\square'} \times \frac{\overline{15}^2}{12} = 390'^{\#}$$

$$200 \times \frac{\overline{15}^2}{9} = \underline{5000}$$

$$5390'^{\#} \times \frac{12}{1450} = 44.6$$

$$d = \sqrt{22.3} = 4.73''$$

Use 2″ × 6″ on edge

Beam:

Span $= 13'\text{-}6''$

$220.8 \times 15 = 3310^{\#/'}$

Est. wt. of beam $= \underline{\quad 50}$

$$3360^{\#/'} \times \frac{\overline{13.5}^2}{8} = 76{,}500'^{\#}$$

$$\times \frac{12}{1600} = 574 = \frac{I}{c}$$

Use $10'' \times 20''$; $\frac{I}{c} = 602$

Clear span $= 12.7'$

$$3360 \times \frac{12.7}{2} = 21{,}500^{\#}$$

$$\text{Design shear} = 21{,}500 \left(1 - \frac{2 \times 19.5}{12.7 \times 12}\right) = 16{,}020^{\#}$$

$$v = \frac{3}{2}\ \frac{16{,}020}{9.5 \times 19.5} = 130^{\#/\square''}$$

Use 1700f Douglas fir

allowable shear $= 145^{\#/\square''}$

Column:

Use $10'' \times 10''$

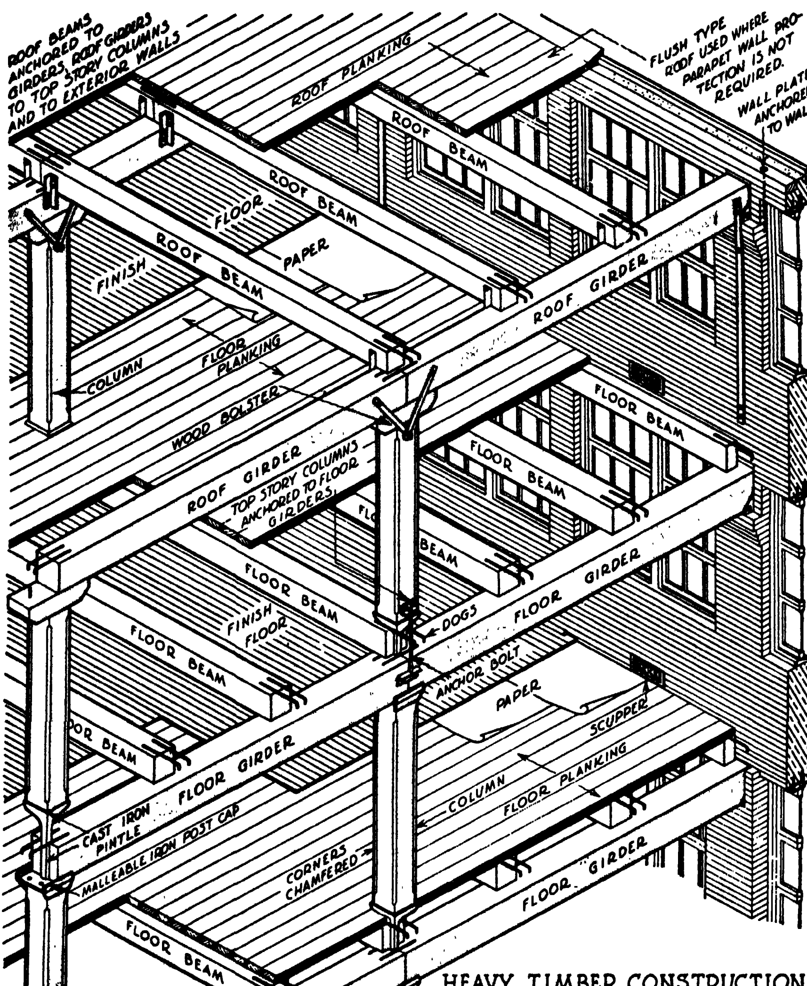
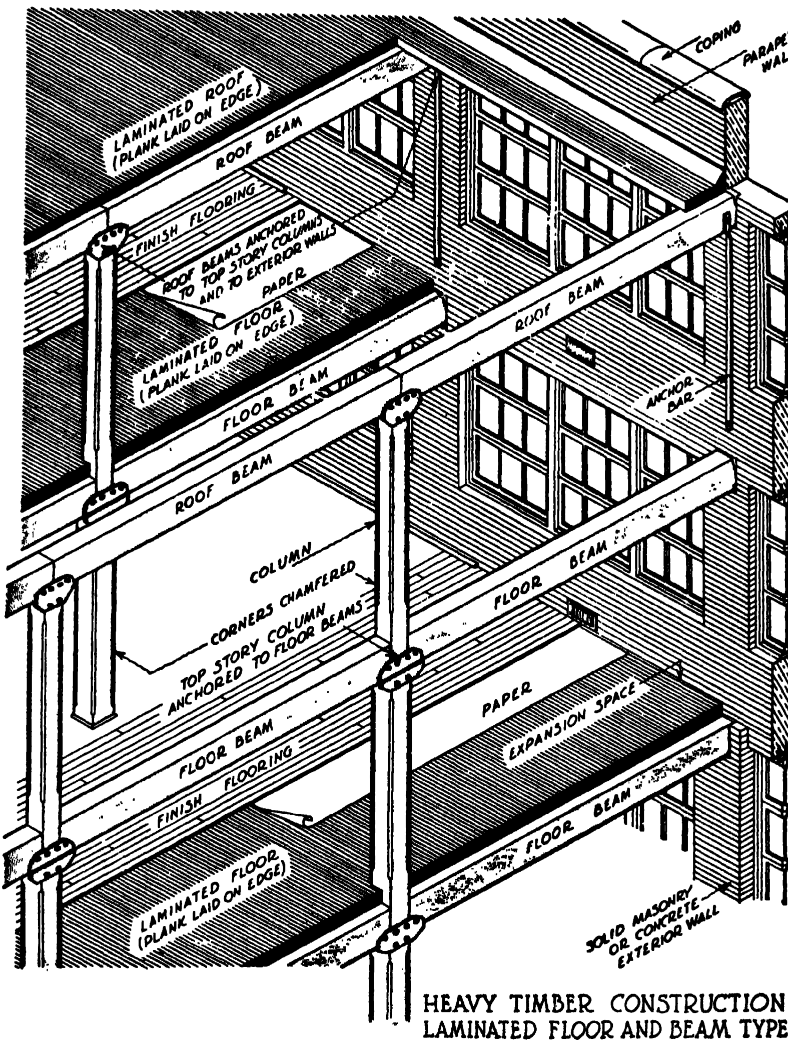


PLATE I

(Courtesy of National Lumber Manufacturers' Association.)



HEAVY TIMBER CONSTRUCTION
LAMINATED FLOOR AND BEAM TYPE

PLATE II

(Courtesy of National Lumber Manufacturers' Association.)

INDEX