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ELECTRICAL TRANSMISSION AND DISTRIBUTION

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BY

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PREFACE

THIS volume is based on the notes compiled by the author in lecturing on the subject to Technical College students and is intended to serve as an introduction to the specialized treatises on this branch of Electrical Engineering. It covers the more theoretical parts of the professional examination syllabuses in Transmission and Distribution.

Grateful acknowledgement is made to the many firms who have kindly lent material for the illustrations, and to the Institution of Electrical Engineers, the University of London, and the City and Guilds of London Institute for permission to reproduce examination questions. All are duly acknowledged in the text.

The author's best thanks are due to Mr. C. W. Shaw for valuable assistance in preparing most of the drawings and in checking many of the examples, and to another student, Dr. A. N. Mosses, for reading the proofs. Acknowledgement is also made to Mr. C. N. Frank for his very helpful literary criticism of the manuscript.

E. T. A. R.

April, 1933.

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NOTE

Vector quantities and vector operators are printed throughout in Clarendon type, thus: **V, I, Z.**

Naperian logarithms are denoted by the symbol: logh.

The letters 'ab' prefixed to the name of a practical unit indicate the corresponding absolute unit.

CHAPTER I

INTRODUCTION

THE electrical system of Great Britain consists of a number of large efficient generating stations operating at pressures ranging from 6·6 to 33 kV., interconnected through a 132 kV. network, called the 'Grid'. At any point near a load centre a supply may be taken from this network and transformed down to any standard pressure from 6·6 to 66 kV., at which voltages it may be transmitted along secondary lines to

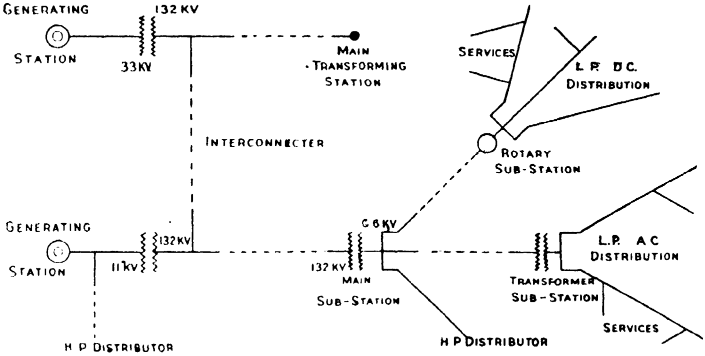

FIG. 1. Typical transmission and distribution system.

(1) high-pressure distribution systems for supplying large power consumers directly;
(2) transformer stations, at which it is transformed down to 400/230 volt A.C. for supplying small consumers;
(3) rotary or rectifier sub-stations, at which it is converted to D.C. for supplying a 460/230 volt three-wire system.

Supplies may also be obtained directly from a main generator for distribution over short distances depending upon the generator voltage.

A typical arrangement is indicated in Fig. 1.

Advantages of High Voltage Transmission.

Consider a line of length l, cross-sectional area a, transmitting power P at a voltage V, the power-factor, if the system is A.C., being $\cos\phi$.

Then since $P = VI\cos\phi$, $I = \dfrac{P}{V\cos\phi}$, and the power loss in the line is

$$p = I^2R = \frac{P^2R}{V^2\cos^2\phi}, \qquad (1)$$

where R is the total line resistance.

The percentage regulation* $= \dfrac{100I}{V}(R\cos\phi + X\sin\phi)$

$$= \frac{100P}{V^2}(R + X\tan\phi), \qquad (2)$$

where X is the total line reactance.

Again, since $R = \dfrac{\rho l}{a}$, $a = \dfrac{\rho l}{R}$. The volume of copper in the line is

$$al = \frac{\rho l^2}{R} = \frac{\rho l^2 P^2}{p}\,\frac{1}{V^2\cos^2\phi}, \qquad (3)$$

since $R = \dfrac{pV^2\cos^2\phi}{P^2}$ from (1).

Hence in the transmission of power P it is seen that

(1) the power loss on a given line is inversely proportional to the square of the transmitting voltage, and, if the supply is alternating, is also inversely proportional to the square of the power factor;

(2) the regulation on a given line is inversely proportional to the square of the voltage;

(3) the volume and therefore the cost of the conductor material in a line of given length working at a given efficiency, i.e. at a given power loss, is inversely proportional to the square of the voltage and to the square of the power factor.

This indicates the necessity of transmitting power at high voltages, and, if the system is A.C., at high power factors.

Comparison of Conductor Costs in D.C. and A.C. Transmissions.

We will consider here only the two-wire D.C. and the three-wire three-phase A.C. systems.

In order to compare the two systems, assume the same power transmitted over the same distance with equal power losses in

---

* See Chapter III.

the conductors, and the same insulation. The latter implies the same maximum voltage to earth in an overhead system, and the same maximum voltage between conductors in an underground system employing multicore cables.

Let V be the voltage to earth of the D.C. system, which is earthed at the mid-point so that the transmission voltage is $2V$, and let V_a be the voltage to earth of the three-phase alternating current system (Fig. 2).

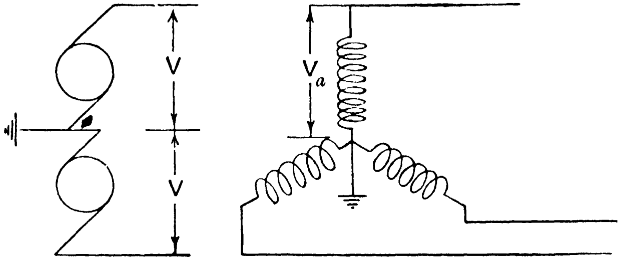

FIG. 2.

Then if I and I_a are the direct and alternating currents respectively, for the same power we have

$$2VI = 3V_a I_a \cos\phi$$

i.e.

$$\frac{I}{I_a} = \frac{3V_a \cos\phi}{2V}.$$

For the same power loss, $2I^2R = 3I_a^2 R_a$, where R and R_a represent the resistance per line of the respective systems.

$$\therefore \quad \frac{R_a}{R} = \frac{2I^2}{3I_a^2} = \frac{2}{3}\cdot\frac{9V_a^2\cos^2\phi}{4V^2}$$

$$= \frac{3\cos^2\phi}{2}\left(\frac{V_a}{V}\right)^2. \tag{4}$$

Now the cost of a conductor is proportional to its area and, neglecting the skin effect, is therefore inversely proportional to its resistance.

Hence the ratio of the total conductor costs in the two systems is

$$\frac{C}{C_a} = \frac{2R_a}{3R} = \left(\frac{V_a}{V}\right)^2 \cos^2\phi. \tag{5}$$

(i) *Overhead System.*

Assuming a sinusoidal alternating current, then for the same insulation we have $V = V_a\sqrt{2}$, so that

$$\frac{V_a}{V} = \frac{1}{\sqrt{2}} \quad \text{and} \quad \frac{C}{C_a} = \frac{\cos^2\phi}{2}. \tag{6}$$

(ii) *Underground System.*

We have $$2V = V_a\sqrt{2}\sqrt{3}.$$

$$\therefore \quad \frac{V_a}{V} = \frac{2}{\sqrt{2}\sqrt{3}} = \frac{\sqrt{2}}{\sqrt{3}}.$$

Hence $$\frac{C}{C_a} = \tfrac{2}{3}\cos^2\phi. \tag{7}$$

Since $\cos\phi$ can never be greater than unity, it will be observed that, on the basis of the assumptions made, the D.C. system is more economical in conductor material than the A.C.

ALTERNATING CURRENT *v.* DIRECT CURRENT, FOR TRANSMISSION AND DISTRIBUTION

This question can be discussed best by setting forth the advantages and disadvantages of the alternating current system as compared with the direct current system.

Advantages of Alternating Current.

(1) Alternating voltages can be easily and efficiently transformed up to a high transmission voltage, which is limited only by considerations of insulation, and can be transformed down at the receiving end to the working voltage with equal facility.

(2) Voltages up to 36,000 volts can be generated compared with a direct voltage per commutator of only 10,000 volts for large units or 20,000 volts for units up to 20,000 kW.

(3) Smaller and cheaper sub-stations and equipment are necessary, and this reduces capital outlay and rate valuation.

(4) Transformer sub-stations are more efficient and require less maintenance than rotary or rectifier sub-stations.

Disadvantages of Alternating Current.

(1) Conductor costs on a given line operating at a given efficiency are greater than those with direct current.

(2) The effective resistance of the line is increased, due to skin effect.

(3) Losses occur in the cable sheaths.

(4) Inductance and capacitance of the line increase the regulation.

(5) The corona limit is lower than with D.C.

(6) The line construction is not so simple as for D.C.

(7) Turbines and generators must run at a speed which is not necessarily the most economical.

(8) It is not possible to use a battery in parallel with the low voltage distributors, nor is it possible to use an A.C. supply for electrolytic work.

(9) Supplies operating in parallel have to be synchronized.

On the whole, it will be seen that A.C. is advantageous for generation and distribution, but that D.C. has the advantage for transmission.

It is possible that in the future all energy will be generated and distributed as A.C., but transmitted as D.C. by the use of mercury arc rectifiers at the transmitting ends and thyratons at the receiving ends. A thyraton is an inverted mercury arc rectifier having an auxiliary electrode called the grid and is capable of converting from D.C. to A.C.

EXAMPLES ON CHAPTER I

Q. 1. Compare the relative weights of copper required for a distribution network on the D.C. three-wire and the three-phase four-wire system. Assume in both cases the same voltage at consumers' terminals, the same copper losses, that the loads are balanced, and unity power factor in the three-phase case. Neglect the losses in the neutrals. . C.G.

Q. 2. Compare the copper used on a three-phase four-wire system with that used on a two-wire direct-current system. Assume the same voltage and losses and that the load is balanced. Take the cross-section of the neutral wire to be half that of the outers.

CHAPTER II

DISTRIBUTION

DISTRIBUTION refers to the conveyance of electrical energy from sub-stations to the various consumers' premises. This distribution may be effected at low pressure to small consumers or at high pressure to large power consumers. The classification of pressures adopted by the Electricity Commissioners is as follows:

(*a*) low pressure—up to 250 volts;
(*b*) medium pressure—between 250 and 650 volts;
(*c*) high pressure—between 650 and 3,000 volts;
(*d*) extra high pressure—above 3,000 volts.

By a regulation of the Electricity Commissioners the pressure of a supply delivered to any consumer is not allowed to exceed the limit of low pressure, i.e. 250 volts, except for special purposes.

Three systems of distribution are in use:

(1) the direct-current two-wire system;
(2) the direct-current three-wire system;
(3) the alternating current three-phase system with four wires (three 'lines' and the neutral).

Each consumer is connected to the 'service' mains which are supplied from the 'distributors': these are supplied by 'feeders', the function of which is to maintain the feeding-points at definite voltages. A variation of not more than ±4 per cent. of the declared voltage is permitted by the Board of Trade in the supply to the consumer.

(1) DIRECT-CURRENT TWO-WIRE SYSTEM

The size of the feeders may be calculated by allowing a maximum volt-drop of 10 per cent. at full load; all feeders should have approximately the same drop whatever their lengths, so that the feeding-points are maintained at approximately the same potential. It is advisable to keep the feeding-point voltage at 4 per cent. above the declared consumer's pressure, this being indicated by pilot wires run back from the feeding-points to the station. A maximum drop of 7·5 per cent. may then be allowed in the distributors when working at full load, and no consumer's pressure would vary more than 4 per cent. above or below the declared value. In practice the actual volt-drop is kept well within this limit.

It will be seen from Fig. 3 that a distributor may be fed from both ends, which may or may not be at the same potential, or at one point only, as in the case of the branch distributor.

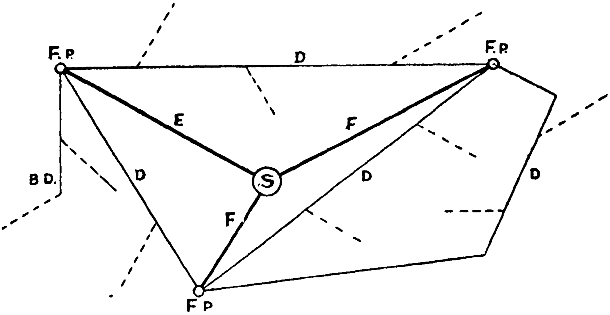

FIG. 3. Typical distribution system. BD = Branch distributor. D = Distributor. FP = Feeding points. F = Feeder. S = Sub-station. Services shown dotted.

(i) *Distributor fed at One Point.*

Consider the circuit shown in Fig. 4, where I_1, I_2, I_3, I_4, and I_5 represent the loads in amperes tapped off at B, C, D, E, and F, distant d_1, d_2, d_3, d_4, and d_5 miles respectively from the end A. This may be simplified as shown in Fig. 5, where only one side of the system is considered. The resistances of the sections between the loads are regarded as the sum of the resistances of the outgoing and return conductors of those sections.

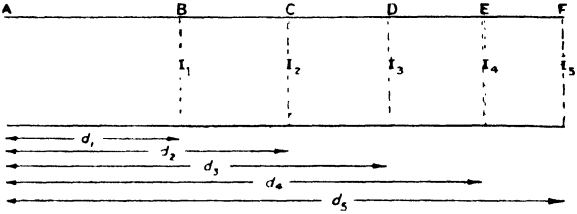

FIG. 4.

(*a*) Assume the distributor fed at A. Then F is the point of lowest potential, and the volt-drop from A to F is given by

$$v = r(I_1 d_1 + I_2 d_2 + I_3 d_3 + I_4 d_4 + I_5 d_5), \tag{1}$$

where r is the resistance per loop mile of the distributor, which is assumed to be uniform. Hence, as the permissible volt-drop is known, the value of r, and hence the size of the cable, may be determined. If this cable will carry the maximum current safely, then it is the size adopted: if not, the cable having the appropriate current-carrying capacity is chosen.

The current loading and volt-drop diagrams are given in Fig. 5*a*.

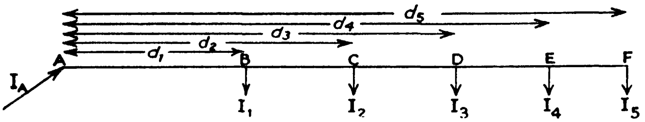

FIG. 5. Distributor fed at one point.

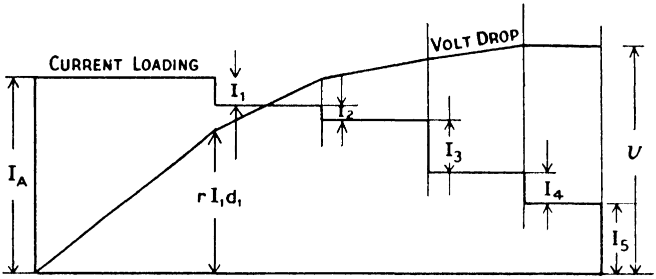

FIG. 5*a*.

(*b*) Assume the distributor fed at D only.

Points B and F are now points of lowest potential and the volt-drop from D to B is $v_1 = r\{I_2(d_3-d_2)+I_1(d_3-d_1)\}$. The volt-drop from D to F is $v_2 = r\{I_4(d_4-d_3)+I_5(d_5-d_3)\}$.

It will be clear that in this instance the value of r to confine the greater of these two values within the specified limit will be greater than in the preceding case, so that a smaller cable is permissible.

In general it is advantageous to feed at or near the point of densest load.

(ii) *Distributor fed from Both Ends at the Same Potential.*

Consider a distributor AB (Fig. 6) loaded at C, D, E, and F, and fed at the ends A and B which are at the same potential.

Each of the loads produces a volt-drop in respect to A, while the feed current I_B produces a rise in potential in respect to A. Since, however, the potentials of A and B are equal, the sum of the volt-drops due to the loads must be equal to the potential increase due to I_B, and we have

$$rI_B l = r\{I_1 a_1 + I_2 a_2 + I_3 a_3 + I_4 a_4\}.$$

$$\therefore \quad I_B = \frac{I_1 a_1 + I_2 a_2 + I_3 a_3 + I_4 a_4}{l} = \frac{\sum Ia}{l} \tag{2}$$

and

$$I_A = I_1 + I_2 + I_3 + I_4 - I_B.$$

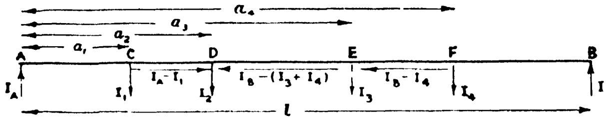

FIG. 6. Distributor fed from both ends.

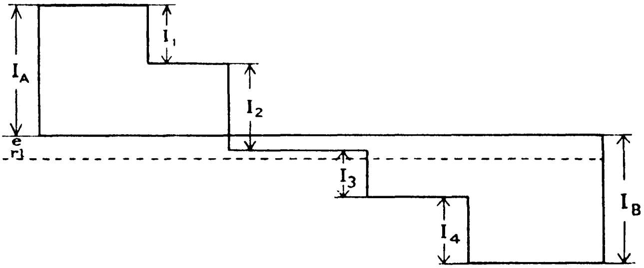

FIG. 7. Loading diagram.

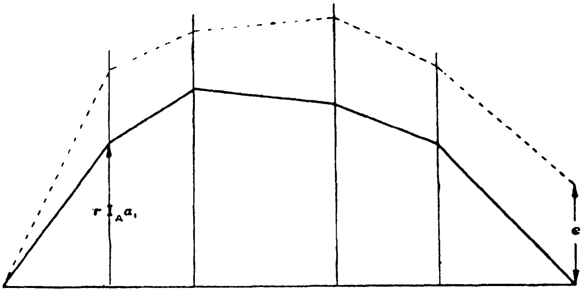

FIG. 7*a*. Diagram of volt-drop in respect to *A*.

The current distribution along the line can now be found: the current in *BF* is I_B; in *FE* it is $I_B - I_4$; in *ED* it is $I_B - (I_3 + I_4)$, and so on. That load point at which two currents meet is the point of lowest potential, and the maximum volt-drop can be found by adding the volt-drops in each section up to that point.

The current loading diagram is shown in Fig. 7 and the volt-drop diagram in Fig. 7*a*, the maximum volt-drop occurring at that point of the distributor where the current changes direction, i.e. at D in Fig. 6. The volt-drop is represented by the area of this diagram either above or below the base line, if the loads are drawn to a current scale and the distances between them are drawn to a resistance scale.

(iii) *Distributor fed at Both Ends not at the Same Potential.*

Suppose now that the potential of A is raised above that of B by e. Then a current of $\frac{e}{rl}$ will flow from A to B and will be superimposed upon the current distribution found when the potentials of A and B were equal. In the loading diagram this is equivalent to lowering the base line by an amount $\frac{e}{rl}$, and it will be seen that in this case the point of lowest potential has been moved from D to E.

An example will make this clear.

A two-wire distributor is fed at both ends with a 480-volt direct-current supply. The resistances and loads are as follows:

0·004 0·0085 0·005 0·005 0·0025 ohms
A 40 20 30 50 amperes B
0·004 0·0085 0·005 0·005 0·0025 ohms.

Determine the position and value of the maximum drop in volts and the magnitudes and the directions of the currents in the various parts of the distributor. If the voltage at the feeding-point A happens to be 480·5 volts, while that at B is held constant at 480 volts, find the redistribution of currents in the distributor on the assumption that the load currents remain unaltered. (Lond. Univ. 1923, Trans.)

Referring to Fig. 8 we have

$$I_B \times 0{\cdot}05 = (40\times 0{\cdot}008)+(20\times 0{\cdot}025)+(30\times 0{\cdot}035)+(50\times 0{\cdot}045)$$
$$= 0{\cdot}32+0{\cdot}5+1{\cdot}05+2{\cdot}25 = 4{\cdot}12.$$

$$\therefore\quad I_B = \frac{4{\cdot}12}{0{\cdot}05} = 82{\cdot}4 \text{ amp.}$$

$$\therefore\quad I_A = 40+20+30+50-82{\cdot}4 = 57{\cdot}6 \text{ amp.}$$

The current distribution is as shown in Fig. 8, and the point of minimum volts is at *D*: the loading diagram is shown in Fig. 9, and the volt-drop diagram is given in Fig. 9*a*.

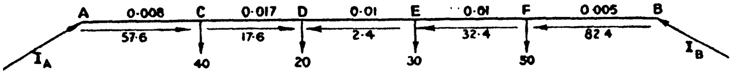

FIG. 8.

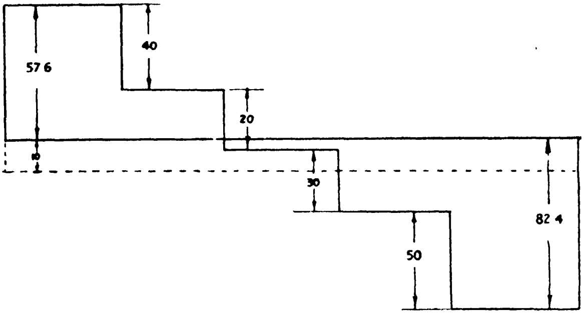

FIG. 9. Current loading diagram.

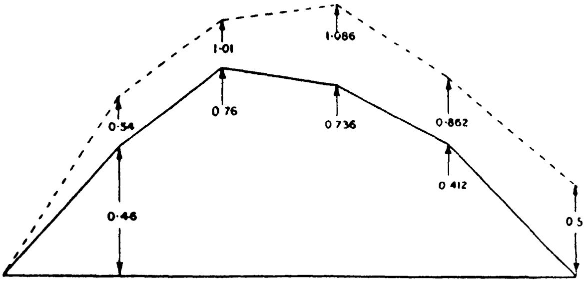

FIG. 9*a*. Volt-drop in respect to *A*.

The maximum volt-drop (at *D*) is

$$40\times0{\cdot}008+17{\cdot}6\times0{\cdot}025 = 0{\cdot}76 \text{ volt.}$$

Raising the potential of *A* by 0·5 volt causes an additional current of $\dfrac{0{\cdot}5}{0{\cdot}05}=10$ amp. to flow from *A* to *B*, and to be superimposed on the existing current distribution.

The new distribution becomes as indicated in Fig. 10, 67·6

FIG. 10.

amp. being fed from A, and 72·4 amp. from B; the point of lowest potential is now at E. The loading diagram is modified in this case by moving the base line 10 amp. downwards as shown in Fig. 9. The new volt-drop line is shown dotted in Fig. 9*a*.

The voltage at E below that at B is

$$(50\times 0{\cdot}005)+(22{\cdot}4\times 0{\cdot}015) = 0{\cdot}586 \text{ volt.}$$

The voltage at E below that at A is then

$$0{\cdot}586+0{\cdot}5 = 1{\cdot}086 \text{ volt.}$$

(iv) *Ring Distributor.*

The problem of a ring distributor (Fig. 11) can be dealt with in the same way as a distributor fed from both ends at the same potential. This will be clear if the ring is imagined to be broken at the feeding-point and opened out so that the points a and b form the left and right extremities of the distributor respectively.

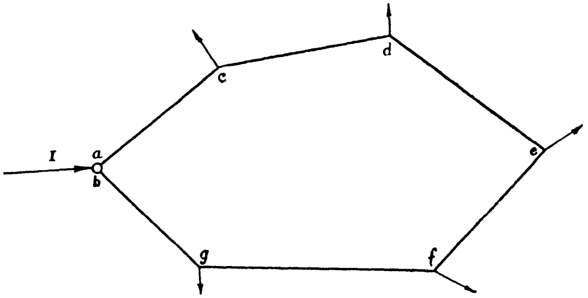

FIG. 11. Ring distributor.

(v) *Double Ring Distributor or Ring Distributor with Interconnector.*

Consider now the case of a ring distributor with an interconnector ae (Fig. 12).

Let I be the total feed current at a, and let x and y be the currents along ac and ag respectively. The current distribution can be obtained by Kirchhoff's first law, which states that at any point $\sum I = 0$. Applying now to any two meshes the second law, which states that in any mesh $\sum IR = \sum E$, we can obtain two simultaneous equations in x and y, from which these two quantities can be obtained. The current distribution and the point of minimum voltage can then be determined.

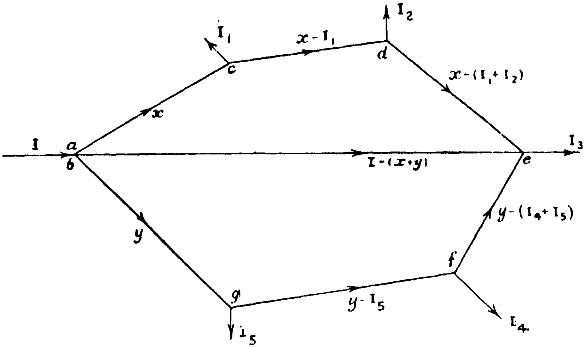

FIG. 12. Double-ring distributor.

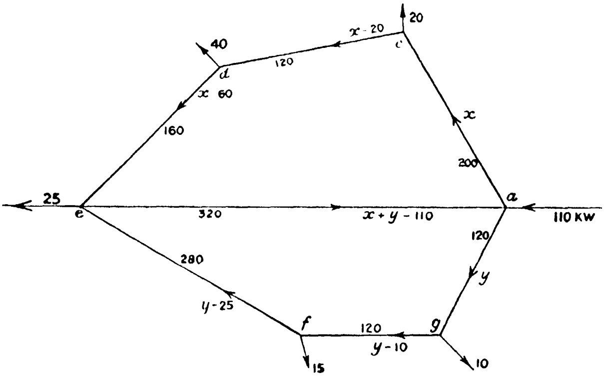

FIG. 13.

EXAMPLE. *Consider the double ring distributor shown in Fig.* 13, *the loads being in kW. and the distances in yards. The total load is* 110 *kW. which is fed in at a.*

If the loads on *ac* and *ag* are x and y respectively, then the load on the interconnector, assumed to flow from *e* to *a*, is $x+y-110$.

Applying Kirchhoff's first law to each of the load points, the load distribution is found in terms of x and y. Applying the second law to the upper and lower meshes respectively, and assuming that the resistances of the cables are proportional to their lengths, we have

$$200x+120(x-20)+160(x-60)+320(x+y-110) = 0.$$

$$\therefore\quad 5x+2y = 295. \tag{3}$$

$$120y+120(y-10)+280(y-25)+320(x+y-110) = 0.$$

$$\therefore\quad 21y+8x = 1085. \tag{4}$$

Solving equations (3) and (4) we have

$$x = 45{\cdot}2 \text{ kW.};\qquad y = 34{\cdot}4 \text{ kW.}$$

The interconnector load is $x+y-110 = -30{\cdot}4$ kW., the negative sign indicating that it flows in the reverse direction to that assumed.

The load distribution is therefore as shown in Fig. 14, the point of lowest voltage being at *d*. Assuming the supply voltage to be 230, then, if *r* is the loop resistance per 1,000 yards, the volt-drop is

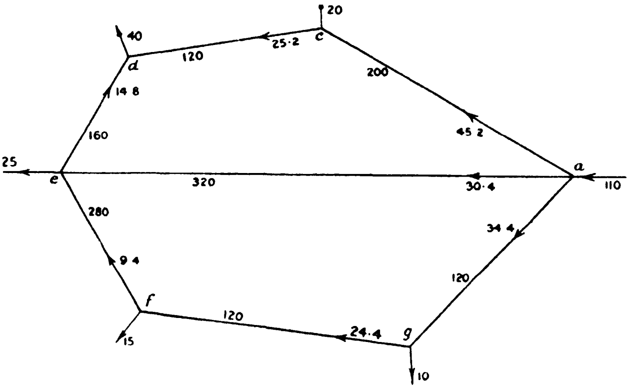

FIG. 14.

$$v = \frac{1000r}{230}\{45{\cdot}2\times 0{\cdot}2+25{\cdot}2\times 0{\cdot}12\}$$

$$= 52{\cdot}5r.$$

Allowing a maximum drop of, say, 15 volts,

$$r = \frac{15}{52{\cdot}5} = 0{\cdot}286 \text{ ohm.}$$

The permissible cable resistance is therefore 0·143 ohm per 1,000 yards.

A 0·2 sq. in. paper-insulated cable has a resistance of 0·125 ohm per 1,000 yards and a carrying capacity of 390 amp., i.e. 90 kW.: this cable is therefore suitable for each section.

(vi) *Uniformly Loaded Distributor.*

The case of a distribution cable laid along a long street with consumers at regular short intervals approximates closely to that of a uniformly loaded distributor, which may be treated as follows.

Let AB (Fig. 15) be a distributor fed at both ends at the same potential, and loaded with a current i per unit run, and let the length be l. Then the feed current at each end is $\frac{1}{2}il$ and the load on the distributor falls linearly from this value to zero at the middle as indicated in the loading diagram, Fig. 16.

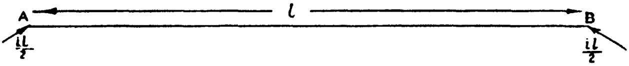

Fig. 15. Uniformly loaded distributor.

Fig. 16. Loading diagram.

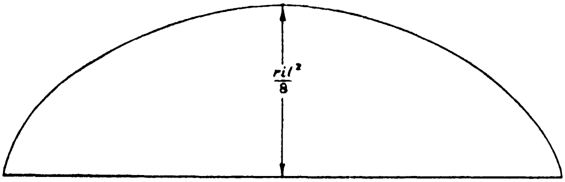

Fig. 16*a*. Volt-drop diagram.

Consider a length δx distant x from the end A. The load current along this element is $i\delta x$ and the volt-drop produced by the load current flowing through length x is $\delta v = rix\,\delta x$, so that the total volt-drop at any point x ($< \frac{1}{2}l$) from A is $\int rix\,dx = \frac{1}{2}rix^2$.

Current flowing from B produces a similar fall of voltage, so that the curve of volt-drop is a parabola as indicated in Fig. 16*a*. The maximum volt-drop occurs at the mid-point and is

$$\frac{ri}{2}\left(\frac{l}{2}\right)^2 = \frac{ril^2}{8}, \qquad (5)$$

where r is the resistance per unit loop.

The solution for a distributor fed from one end only is given by considering one-half of the above distributor, its length being then $\frac{1}{2}l$.

(2) DIRECT-CURRENT THREE-WIRE SYSTEM

Three wires are employed in this system, viz. one 'neutral' which is earthed at the generating or sub-station, and two 'outers' which are maintained at the distribution pressure above and below the neutral respectively. Each side may be loaded independently, the neutral wire carrying the out-of-balance current.

A 10 per cent. drop may be allowed in the feeders as in the case of the two-wire system. It is assumed that the station balancer deals with the out-of-balance current in the feeder. The middle wire need only be about one-third the section of each outer.

Comparing the three-wire distribution with the two-wire system on the basis of the same total power distributed, we find that, since the voltage is doubled, the current is halved. The permissible volt-drop on each side of the system is the same as that for a two-wire system, and assuming a balanced load, this drop takes place in one wire only, i.e. in the outer, since there is no current in the neutral. Hence the permissible resistance of each outer may be four times that of each of the two-wire conductors and the section need only be one-quarter.

The neutral wire of the distributor has usually half the cross-section of each outer, so that we have

$$\frac{\text{copper in the three-wire system}}{\text{copper in the two-wire system}} = \frac{2\times 0{\cdot}25+0{\cdot}125}{2\times 1} = \frac{0{\cdot}625}{2}$$

$$= 0{\cdot}3125.$$

In actual practice, however, any out-of-balance current will increase the current on the heavily loaded side and also cause a current and therefore a volt-drop in the neutral. To keep the volt-drop on the heavily loaded side within the specified limit, slightly larger conductors than this would be required, the above ratio being increased to about 0·4. This still represents an appreciable economy in copper.

As an example, compare the copper required in the distribution of 88 kW. a distance of 1,000 yards, at a consumer's pressure of 220 volts (*a*) on a two-wire system, (*b*) on a three-wire system.

$$(a)\ \text{Maximum current} = \frac{88000}{220}$$

$$= 400 \text{ amp.}$$

Since the consumers are usually distributed fairly evenly along the whole length of the distributor, the average current may be assumed to be 200 amp.

$$\text{Permissible volt-drop} = 7{\cdot}5 \text{ per cent. of } 220 \text{ volts}$$

$$= 16{\cdot}5 \text{ volts.}$$

$$\text{Resistance of each conductor per } 1{,}000 \text{ yards} = \frac{16{\cdot}5}{2\times 200}$$

$$= 0{\cdot}04125 \text{ ohm.}$$

A 0·6 cable (i.e. one having a cross-sectional area of 0·6 sq. in.) has a resistance of 0·0404 ohm per 1,000 yards and a carrying capacity of 760 amp. and is therefore the one chosen.

$$\text{Hence total cross-sectional area} = 2\times 0{\cdot}6$$

$$= 1{\cdot}2 \text{ sq. in.}$$

(*b*) Assuming no out-of-balance current, maximum current

$$= \frac{88000}{440}$$

$$= 200 \text{ amp.}$$

$$\text{Average current} = 100 \text{ amp.}$$

$$\text{Resistance of each outer} = \frac{16{\cdot}5}{100}$$

$$= 0{\cdot}165 \text{ ohm per } 1{,}000 \text{ yards.}$$

A 0·15 cable has a resistance of 0·165 ohm per 1,000 yards: the middle wire would then be a 0·075 cable.

$$\text{Hence total copper section} = 2\times 0{\cdot}15 + 0{\cdot}075$$

$$= 0{\cdot}375 \text{ sq. in.}$$

Assuming now a 15 per cent. out-of-balance, the average current in the heavily loaded outer will be increased to 107·5 amp. (Fig. 17) and the volt-drop will be increased to

$$1{\cdot}075\times 16{\cdot}5 = 17{\cdot}75 \text{ volts.}$$

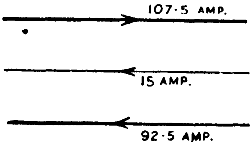

FIG. 17.

The current in the neutral will be 15 amp., and the volt-drop in it becomes

$$16{\cdot}5\times2\times\frac{15}{100}=4{\cdot}95 \text{ volts.}$$

$\therefore$ Total volt-drop $= 22{\cdot}7$ volts, which represents an increase of about 38 per cent, and necessitates a corresponding increase in the conductor section.

The actual main used in this case would therefore be $0{\cdot}2\times0{\cdot}2\times0{\cdot}1$, representing a total copper section of 0·5 sq. in.

(3) ALTERNATING-CURRENT DISTRIBUTION

In this case the problem is complicated by the presence of reactance as well as resistance in the line and by the fact that the load currents may not be in phase with the voltages.

(i) *Single-phase Two-wire Distributor.*

Consider a distributor fed from one end A and loaded at B, C, and D as shown (Fig. 18) with currents of I_1 lagging ϕ_1, I_2 lagging ϕ_2 and I_3 lagging ϕ_3 respectively. The current I_1 may be represented by $\mathbf{I}_1 = I_1\cos\phi_1 - jI_1\sin\phi_1$, where $I_1\cos\phi_1$ represents the active component and $I_1\sin\phi_1$ the reactive component. The magnitude of the current is given by

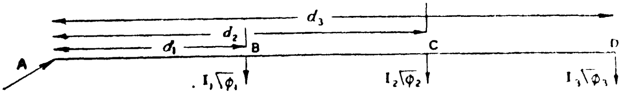

FIG. 18.

$$I_1 = \sqrt{(I_1\cos\phi_1)^2+(I_1\sin\phi_1)^2}.$$

Similarly, $\mathbf{I}_2 = I_2\cos\phi_2 - jI_2\sin\phi_2$

and $\mathbf{I}_3 = I_3\cos\phi_3 - jI_3\sin\phi_3.$

The current in CD is $\mathbf{I}_3$ and that in BC is the vector sum of $\mathbf{I}_2$ and $\mathbf{I}_3$ which is given by

$$\mathbf{I}_2+\mathbf{I}_3 = (I_2\cos\phi_2+I_3\cos\phi_3) - j(I_2\sin\phi_2+I_3\sin\phi_3).$$

Similarly, the current in AB is given by

$$\mathbf{I}_3+\mathbf{I}_2+\mathbf{I}_1 = (I_3\cos\phi_3+I_2\cos\phi_2+I_1\cos\phi_1) - \\ -j(I_3\sin\phi_3+I_2\sin\phi_2+I_1\sin\phi_1).$$

If, now, the resistance and reactance per loop mile of conductor are r and x respectively, the volt-drop is obtained from the approximate expression*

$$Ir\cos\phi \pm Ix\sin\phi,$$

where $I\cos\phi$ represents the active current component and $I\sin\phi$ represents the reactive component. The positive sign is used in the case of lagging currents and the negative for leading currents.

Then the volt-drop from A to D is approximately

$$v = r(I_1 d_1 \cos\phi_1 + I_2 d_2 \cos\phi_2 + I_3 d_3 \cos\phi_3) + \\ + x(I_1 d_1 \sin\phi_1 + I_2 d_2 \sin\phi_2 + I_3 d_3 \sin\phi_3). \quad (6)$$

EXAMPLE. *A* 230-*volt distributor* 300 *yards long is loaded with currents of* 100 *amp. at power factors of* 0·707 (*lagging*), *unity, and* 0·8 (*lagging*) *respectively at equal intervals from the feeding-point end. Find the cable size allowing a drop of* $7\frac{1}{2}$ *per cent. at full load.*

The loading is indicated in Fig. 19. If r and x are the resistance and reactance respectively per 1,000 yards loop, the total volt-drop is from (13)

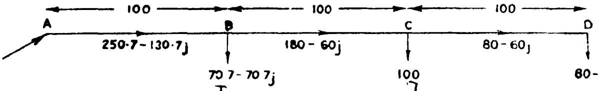

FIG. 19.

$$v = r(0{\cdot}1\times70{\cdot}7+0{\cdot}2\times100+0{\cdot}3\times80)+x(0{\cdot}1\times70{\cdot}7+0{\cdot}3\times60)$$
$$= 51{\cdot}07r+25{\cdot}07x.$$

Since there is no simple relationship between r and x, the correct cable size must be found by trial and error. Taking a 0·2 paper-insulated cable, the resistance and reactance per 1,000 yards loop are 0·25 ohm and 0·125 ohm respectively.

Then $\quad v = 51{\cdot}07\times0{\cdot}25+25{\cdot}07\times0{\cdot}125$

$$= 15{\cdot}9 \text{ volts},$$

which corresponds to about 7 per cent.

Hence the 0·2 cable would be chosen, its carrying capacity being 296 amp.

The current in $CD = \mathbf{I}_D = 80-60j$

„ „ „ $BC = \mathbf{I}_C+\mathbf{I}_D = 180-60j$

„ „ „ $AB = \mathbf{I}_B+\mathbf{I}_C+\mathbf{I}_D = 250{\cdot}7-130{\cdot}7j.$

---

* See Chapter III.

The magnitudes of these currents are therefore

$$\text{in } CD = \sqrt{80^2+60^2} = 100 \text{ amp.}$$

$$\text{in } BC = \sqrt{180^2+60^2} = 190 \text{ amp.}$$

$$\text{in } AB = \sqrt{250{\cdot}7^2+130{\cdot}7^2} = 282 \text{ amp.}$$

(ii) *Three-phase Four-wire Distributor.*

In this case each phase is considered separately, and the volt-drop computed in regard to the neutral.

For example, consider a three-phase four-wire 400/230 volt main loaded as follows: 200 amp. at 0·9 P.F. off the red phase at A, 100 yards from the feeding-point; 150 amp. at 0·8 P.F. off the white at B, 200 yards; and 250 amp. at 0·7 P.F. off the blue at C, 300 yards from the feeding-point. The resistance per 100 yards of single conductor is 0·01 ohm and per 100 yards of the neutral is 0·016 ohm: the reactance per 100 yards of line and neutral is 0·0127 ohm. Find the volt-drop in each phase.

Red Phase.

Since $\cos\phi = 0{\cdot}9, \qquad \sin\phi = 0{\cdot}436,$

$$\therefore \quad \mathbf{I}_A = 180-87{\cdot}2j.$$

Resistance per 100 yards of line and neutral $= 0{\cdot}01+0{\cdot}016$

$$= 0{\cdot}026 \text{ ohm.}$$

$$\therefore \quad \text{Pressure drop } v_A = 180\times0{\cdot}026+87{\cdot}2\times0{\cdot}0127$$

$$= 4{\cdot}68+1{\cdot}108$$

$$= 5{\cdot}788 \text{ volts.}$$

White Phase.

Since $\cos\phi = 0{\cdot}8, \qquad \sin\phi = 0{\cdot}6,$

$$\therefore \quad \mathbf{I}_B = 120-90j.$$

$$\therefore \quad \text{Pressure drop } v_B = 2(120\times0{\cdot}026+90\times0{\cdot}0127)$$

$$= 2(3{\cdot}12+1{\cdot}143)$$

$$= 8{\cdot}53 \text{ volts.}$$

Blue Phase.

Since $\cos\phi = 0{\cdot}7, \qquad \sin\phi = 0{\cdot}714,$

$$\therefore \quad \mathbf{I}_C = 175-178{\cdot}5j.$$

$$\therefore \quad \text{Pressure drop } v_C = 3(175\times0{\cdot}026+178{\cdot}5\times0{\cdot}0127)$$

$$= 3(4{\cdot}55+2{\cdot}265)$$

$$= 20{\cdot}45 \text{ volts.}$$

The vector diagram is shown in Fig. 20, where V_R, V_W, and V_B represent the potentials at the load points on the red, white

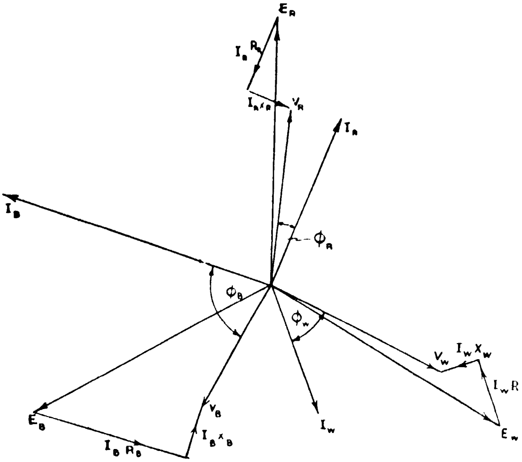

FIG. 20. Vector diagram of three-phase distributor.

and blue respectively, while E_R, E_W, and E_B represent the voltages across these phases at the feeding-points.

'Tapered' Lines or Cables.

It is sometimes advisable to 'taper' a long cable supplying loads at considerable distances apart instead of using a uniform cable. In this way the volume of copper in the line may be minimized for a given permissible volt-drop.

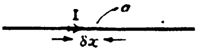

FIG. 21.

Consider an elemental length δx of the cable (Fig. 21). Let this element have a section a and carry a current I.

The volt-drop in the element $= I\rho\dfrac{dx}{a}$.

The total volt-drop in the cable $= \rho\displaystyle\int\frac{I}{a}\,dx = v$.

Since the volt-drop is fixed, it is independent of the variation of a, so that

$$\frac{dv}{da} = \rho\frac{I}{a}\frac{dx}{da} = 0. \tag{7}$$

Now the total volume of copper in the cable $= \int a\,dx$. For this to be a minimum, its differential coefficient with respect to

a must be zero, and we have

$$a\frac{dx}{da} = 0. \tag{8}$$

Hence from (7) and (8)

$$a\frac{dx}{da} \propto \rho\frac{I}{a}\frac{dx}{da} \quad \text{or} \quad a^2 \propto I,$$

i.e.

$$a \propto \sqrt{I}.$$

Hence the total weight of the copper is a minimum when the cable areas between consecutive load points are in the same proportion as the square roots of the currents in those sections.

Consider for example a distributor AD, fed at A and loaded at B, C, and D, as shown in Fig. 22, the permissible volt-drop up to D being 50 volts.

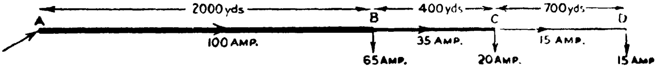

FIG. 22.

Now if a is the area of a section of the cable, length l and carrying a current I, $a \propto \sqrt{I}$, and the volt-drop

$$v \propto I\rho\frac{l}{a} \propto I\rho\frac{l}{\sqrt{I}} \propto \sqrt{I}.l.$$

Hence the volt-drop from A to B is given by

$$v_{AB} = \frac{2000\sqrt{100}}{2000\sqrt{100}+400\sqrt{35}+700\sqrt{15}} \times 50.$$

$$= \frac{20000}{25080}.50 = 39{\cdot}9 \text{ volts},$$

$$v_{BC} = \frac{400\sqrt{35}}{25080}.50 = 4{\cdot}7 \text{ volts},$$

$$v_{CD} = 50-(39{\cdot}9+4{\cdot}7) = 5{\cdot}4 \text{ volts}.$$

Then, remembering that the volt-drop takes place in both the feed and return, we have

$$R_{AB} \text{ per 1,000 yards} = \frac{39{\cdot}9}{2\times 2\times 100} = 0{\cdot}1 \text{ ohm},$$

$$R_{BC} \text{ per 1,000 yards} = \frac{4{\cdot}7}{2\times 0{\cdot}4\times 35} = 0{\cdot}168 \text{ ohm},$$

$$R_{CD} \text{ per 1,000 yards} = \frac{5{\cdot}4}{2\times 0{\cdot}7\times 15} = 0{\cdot}257 \text{ ohm}.$$

These resistances correspond to cable sections of 0·25, 0·15, and 0·1 sq. in. for AB, BC, and CD, respectively.

If a uniform cable had been used, of which r is the resistance per 1,000 yards of single conductor, then

$$2r(2\times 100+0{\cdot}4\times 35+0{\cdot}7\times 15) = 50$$
$$449r = 50$$
$$r = 0{\cdot}111 \text{ ohm.}$$

This necessitates a cable of 0·25 sq. in. cross-section throughout.

Comparing, then, the copper in the tapered cable with that in the uniform cable we have

$$\frac{C_t}{C_u} = \frac{2\times 0{\cdot}25+0{\cdot}4\times 0{\cdot}15+0{\cdot}7\times 0{\cdot}1}{3{\cdot}1\times 0{\cdot}25}$$
$$= \frac{0{\cdot}63}{0{\cdot}775} = 0{\cdot}81.$$

Tapering the cable represents in this case, therefore, an economy of nearly 20 per cent. in the cost of the copper in the conductor, although the actual cable cost will not be reduced to the same extent.

It will be observed that, since $a \propto \sqrt{I}$,

$$\frac{I}{a} \propto \frac{a^2}{a} \propto a;$$

i.e. the current density in any section is proportional to the cross-section of the conductor in that section.

It can also be shown that for a given I^2R loss, the weight of copper is a minimum when the current density is constant throughout.

Thus the I^2R loss in the element $= I^2\rho\dfrac{dx}{a}$.

The total I^2R loss in the cable $= \rho\displaystyle\int I^2\frac{dx}{a}$.

Since this is fixed we may write

$$\rho\frac{I^2}{a}\frac{dx}{da} = 0. \tag{9}$$

As before, for a minimum weight of copper we have

$$a\frac{dx}{da} = 0. \tag{10}$$

Hence from (9) and (10), $\rho\frac{I^2}{a} \propto a$, i.e.

$$\frac{I^2}{a^2} \text{ and therefore } \frac{I}{a} \text{ is constant.}$$

Hence in this case the weight of copper is a minimum when the current density is uniform.

Best Position for Supply Point.

Theoretically the best position for a given generating station, sub-station, or feeding-point is at the centre of gravity of the load, though, usually, practical considerations determine the actual position.

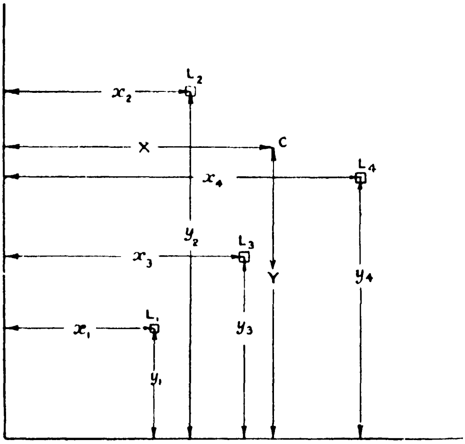

FIG. 23.

The procedure for determining the centre of gravity is similar to that used in mechanics. Thus let L_1, L_2, L_3, L_4 be the loads, whose coordinates referred to any two straight lines at right angles are x_1, x_2, x_3, x_4 and y_1, y_2, y_3, y_4 respectively (Fig. 23). Then the coordinates of the centre of gravity C are given by

$$X = \frac{x_1 L_1 + x_2 L_2 + x_3 L_3 + x_4 L_4}{L_1 + L_2 + L_3 + L_4}, \tag{11}$$

$$Y = \frac{y_1 L_1 + y_2 L_2 + y_3 L_3 + y_4 L_4}{L_1 + L_2 + L_3 + L_4}. \tag{12}$$

For example, find the most economical centre of distribution in the case of three sub-stations, *A*, *B*, and *C*, loaded with 1,000, 1,500 and 900 kW. respectively, and situated as indicated in Fig. 24.

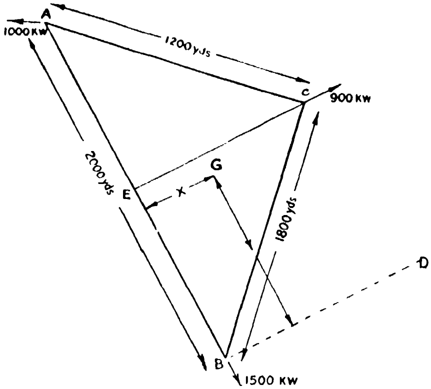

FIG. 24.

Let the reference lines be *AB* and *BD* at right angles to it. Then working in thousands of yards we have

$$\cos B = \frac{4+3{\cdot}24-1{\cdot}44}{2\times2\times1{\cdot}8} = 0{\cdot}8055$$

$$\sin B = 0{\cdot}5925$$

$$BE = 1800\times0{\cdot}8055 = 1450$$

$$CE = 1800\times0{\cdot}5925 = 1066$$

$$\therefore \quad X = \frac{900\times1066}{3400} = 282 \text{ yards.}$$

$$Y = \frac{1000\times2000+900\times1450}{3400}$$

$$= 973 \text{ yards.}$$

It should be understood that the distances between supply points are measured in cable distances and not, of course, as the crow flies.

Division of Load between Lines or Cables in Parallel.

Consider two parallel connected feeders of impedances $\mathbf{Z}_1$ and $\mathbf{Z}_2$, where $\mathbf{Z}_1 = R_1+jX_1$ and $\mathbf{Z}_2 = R_2+jX_2$: R_1 and R_2 are the resistances of the respective feeders and X_1 and X_2 are their reactances.

Then if $\mathbf{I}_1$ and $\mathbf{I}_2$ are the currents in the respective feeders and $\mathbf{I}$ is the total load current, it follows that

$$\mathbf{I}_1\mathbf{Z}_1 = \mathbf{I}_2\mathbf{Z}_2 = \mathbf{I}\mathbf{Z},$$

where $\mathbf{Z}$ is the combined impedance of the two feeders in parallel and is given by

$$\mathbf{Z} = \frac{1}{\frac{1}{\mathbf{Z}_1}+\frac{1}{\mathbf{Z}_2}} = \frac{\mathbf{Z}_1\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}.$$

$$\therefore \quad \mathbf{I}_1 = \mathbf{I}\frac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}$$

and

$$\mathbf{I}_2 = \mathbf{I}\frac{\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2}.$$

From these equations the current in each feeder may be found in magnitude and direction.

The magnitudes of the currents are given by

$$I_1 = I\sqrt{\frac{R_2^2+X_2^2}{(R_1+R_2)^2+(X_1+X_2)^2}} \tag{13}$$

$$I_2 = I\sqrt{\frac{R_1^2+X_1^2}{(R_1+R_2)^2+(X_1+X_2)^2}}. \tag{14}$$

The same method of solution may be adopted for two transformers in parallel.

If the feeders carry direct current, the impedance is simply the resistance, and we have

$$I_1 = \frac{IR_2}{R_1+R_2}$$

$$I_2 = \frac{IR_1}{R_1+R_2}.$$

EXAMPLE. *The full output* (5,000 *kW. at power factor* 0·8 *lagging*) *of a hydro-electric station is transmitted to a sub-station by two routes, the lines being connected in parallel. The respective resistances are* 1·5 *ohms and* 1·0 *ohm and the corresponding reactances are* 1·25 *ohms and* 1·2 *ohms. Determine the power transmitted by each route.* C. G.

Solution. Let the transmission pressure be V kV. and let the total current be

$$\mathbf{I} = I\lvert\overline{\cos^{-1}0\cdot8} = I\lvert\overline{36^\circ\ 52'}.$$

Then $\mathbf{Z}_1 = 1\cdot5+1\cdot25j = \sqrt{1\cdot5^2+1\cdot25^2}\lvert\underline{\tan^{-1}\frac{1\cdot25}{1\cdot5}} = 1\cdot953\lvert\underline{39^\circ\ 48'}$

$$\mathbf{Z}_2 = 1+1\cdot2j = 1\cdot562\lvert\underline{50^\circ\ 12'}$$

$$\mathbf{Z}_1+\mathbf{Z}_2 = 2\cdot5+2\cdot45j = 3\cdot5\lvert\underline{44^\circ\ 25'}.$$

Then $$\mathbf{I}_1 = \frac{I\lvert\overline{36^\circ\ 52'}\,.\,1\cdot562\lvert\underline{50^\circ\ 12'}}{3\cdot5\lvert\underline{44^\circ\ 25'}}$$

$$= 0\cdot4462\,I\lvert\overline{31^\circ\ 5'}.$$

$$\therefore\quad V\mathbf{I}_1 = 0\cdot4462\,VI\lvert\overline{31^\circ\ 5'}.$$

But $$I = \frac{5000}{0\cdot8V}.$$

$$\therefore\quad V\mathbf{I}_1 = \frac{5000}{0\cdot8}\times0\cdot4462\lvert\overline{31^\circ\ 5'} = 2789\lvert\overline{31^\circ\ 5'}.$$

Hence kVA. in first feeder = 2,789 kVA.

Power in first feeder = 2789 cos 31° 5′ = 2,389 kW.

Similarly, $$\mathbf{I}_2 = \frac{I\lvert\overline{36^\circ\ 52'}\,.\,1\cdot953\lvert\underline{39^\circ\ 48'}}{3\cdot5\lvert\underline{44^\circ\ 25'}}$$

$$= 0\cdot558\,I\lvert\overline{41^\circ\ 29'}.$$

$$\therefore\quad V\mathbf{I}_2 = \frac{5000}{\cdot8}\times0\cdot558\lvert\overline{41^\circ\ 29'}$$

$$= 3487\lvert\overline{41^\circ\ 29'}.$$

Hence kVA. in second feeder = 3,487 kVA.

Power in second feeder = 3487 cos 41° 29′ = 2,613 kW.

EXAMPLES ON CHAPTER II

Q. 1. A direct-current main 1,000 yards long is fed at each end at 440 volts. Loads are taken off as follows, the distances being measured from one end:

At 100 yards, 200 amp.
,, 300 yards, 400 amp.
,, 600 yards, 100 amp.
,, 800 yards, 500 amp.

The resistance of each conductor of the main is 15×10^{-6} ohm per yard.

Find the current in each section of the main and the voltage at each load. L.U. 1931.

Q. 2. A section of a two-wire distributor network is fed from both ends, 400 yards apart, and supplies the following loads at distances measured from one end:

Current:	30	10	25	15	20	10	80	50	30	10
Yards:	5	20	35	65	100	150	200	300	340	380

A constant P.D. of 220 volts is maintained at the two feeding-points. If the maximum voltage-drop in the section is not to exceed 3 per cent., find the cross-sectional area of each conductor. Resistance of copper wire 1 metre long and 1 sq. mm. cross-section is 1/58 ohm at 20° C. L.U. 1930.

Q. 3. A 460-volt two-wire distributor *AB*, 500 yards long, is fed at both ends at the same potential: the loads in kW. are 10, 15, 25, 40, and 20 at distances of 60, 120, 260, 340, and 400 yards from *A*. Find the position of the maximum volt-drop and the permissible cable resistance per 100 yards if this drop is not to exceed 30 volts.

Q. 4. A sub-station is fed from a power-house by two routes of equal length; on one route the reactance per mile is 50 per cent. greater than on the other route, a conductor of the same size being used. Show, graphically or analytically, how the load divides between the two routes. L.U.

Q. 5. A three-phase four-wire system with 440 volts between the phase conductors has a motor load of 500 kW. at power factor 0·8. The lamp loads connected between the several lines and neutral are 100, 120, and 250 kW. Find (*a*) the current in each phase conductor, (*b*) the current in the neutral, and (*c*) the power factor of the system. L.U.

Q. 6. A street main is 650 yards long from *A* to *B*, the power being supplied at *A*. At a point *C*, 250 yards from *A*, there is a branch main *CD*, 200 yards long. The equivalent loads which give the same fall of voltage over the sections concerned are

On the section *AC*, 40 amp. at 100 yards from *A*
„ „ *CB*, 15 „ 300 „ „ *C*
„ „ *CD*, 20 „ 80 „ „ *C*.

Determine the sections of the mains *AC*, *CD*, and *CB* for minimum total copper volume if the fall of voltage is to be 4 volts between both *A* and *B*, and *A* and *D*. L.U.

Q. 7. A ring main consisting of a single-circuit overhead line is fed from a power-station *P*, and has four sub-stations *A*, *B*, *C*, and *D* connected to it. The following simultaneous loads are taken by these sub-stations: *A*, 1,500; *B*, 2,500; *C*, 1,000; *D*, 3,000 kW. The distances between stations are as follows: *PA*, 5; *AB*, 10; *BC*, 8; *CD*, 15; *DP*, 6 miles. Find the power in each section of the line. I.E.E.

Q. 8. A sub-station is supplied by two feeders in parallel: the respective feeder resistances are 3 ohms and 2 ohms and the corresponding reactances are 2·5 ohms and 2·4 ohms. Find the power and the kVA. transmitted by each feeder when the station output is 10,000 kW. at 0·8 power factor lagging.

CHAPTER III

SHORT TRANSMISSION LINES

EVERY transmission line, consisting as it does of two or more long parallel conductors separated by a dielectric, possesses inductance and capacitance as well as resistance. Since the inductive reactance is in series with the resistance, its effects may be readily calculated; the capacitative reactance is, however, in parallel with the line and is, moreover, uniformly distributed along its length, and exact calculation of its effects involve complex mathematics.

Now the capacitance current or charging current $I = V\omega C$, where C is the line capacitance, varying directly as the length: the voltage used in practice also varies, to a first approximation, as the length, so that the charging current is approximately proportional to the square of the length. Hence, although the effects of capacitance are important on long lines, they may be neglected on overhead lines up to about 30 miles in length. The effects of capacitance in underground cables can rarely be neglected even on the shortest lines. We will proceed therefore to consider first the problem of short aerial lines, neglecting capacitance.

SHORT LINES: CAPACITANCE NEGLECTED

Single-phase Line.

The circuit diagram is indicated in Fig. 25.

Let I = line current,

V = voltage at the load,

ϕ = phase angle of the load,

R = resistance of one line,

X = reactance of one line,

E = voltage at the generator end.

In the vector diagram (Fig. 26), OI represents the line current and OV the load voltage. The resistance drop $VA = 2IR$ is drawn parallel to OI and the reactance drop $AE = 2IX$ is drawn 90° in advance of OI. Then OE gives the voltage E at the generator end.

Now
$$OB = V\cos\phi + 2IR$$
$$BE = V\sin\phi + 2IX.$$

Hence
$$E = \sqrt{(V\cos\phi+2IR)^2+(V\sin\phi\pm 2IX)^2},$$

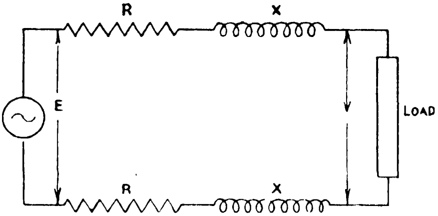

FIG. 25. Single-phase line.

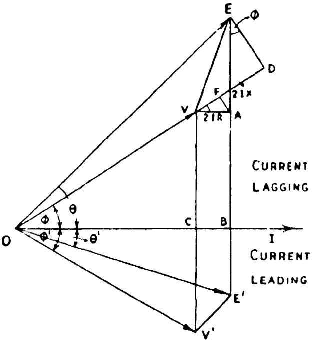

FIG. 26.

the positive sign being used when the current is lagging, and the negative sign when it is leading.

$$\therefore\quad E = V\left\{\left(\cos\phi+\frac{2IR}{V}\right)^2+\left(\sin\phi\pm\frac{2IX}{V}\right)^2\right\}^{\frac{1}{2}}$$
$$= V\left\{1+\frac{4I}{V}(R\cos\phi\pm X\sin\phi)+\frac{4I^2}{V^2}(R^2+X^2)\right\}^{\frac{1}{2}}$$
$$\doteqdot V\left\{1+\frac{2I}{V}(R\cos\phi\pm X\sin\phi)\right\},$$

neglecting the third term when I^2R^2 and I^2X^2 are negligible compared with V^2.

Hence the volt-drop

$$v = E-V \doteqdot 2I(R\cos\phi\pm X\sin\phi). \qquad (1)$$

This can be obtained directly from the geometry of Fig. 26 thus:

Produce OV and draw ED and AF perpendicular to it. Then since angle EOD is small, $OE \doteqdot OD$ and the volt-drop is

$$\begin{aligned} v &\doteqdot VD = VF+FD \\ &\doteqdot I(2R\cos\phi\pm 2X\sin\phi) \\ &\doteqdot 2I(R\cos\phi\pm X\sin\phi). \end{aligned}$$

The regulation $= \dfrac{100v}{V}$ per cent.

If θ is the angle of lag at the generator end, we have

$$\tan\theta = \frac{V\sin\phi\pm 2IX}{V\cos\phi+2IR}. \qquad (2)$$

The power loss in the line is $2I^2R$.

When the regulation or generator voltage is required for any given load current at various power factors, use is sometimes made of graphical constructions, e.g. the Kapp regulation diagram or the Mershon chart.

Graphical Construction.

Let OI be the datum line (Fig. 27) representing the phase of the current. OA represents the ohmic volt-drop $2IR$, and AB the reactive volt-drop $2IX$: then OB is the impedance drop which is constant in magnitude and phase for a given current. With O as centre and radius V, the load-voltage (taken as 30,000 volts in this Fig.), the circle VDG is drawn: with B as centre, and the same radius, the circle EFG is described. OD is divided into tenths to represent power factor, and also into voltage graduations through which concentric circles having O as centre are described.

To find the sending-end voltage required for any load power factor, a vertical CV is erected at the corresponding power factor (0·7 lagging in the figure). From V, VE is drawn parallel to OB to cut the eccentric circle in E. Then OE is the sending-end voltage and HE is the regulation which is usually expressed as a percentage of OV.

For leading power factors the vertical CV' is drawn downwards; the sending-end voltage OE' in this case is less than the received voltage by $E'H'$; i.e. the regulation is negative.

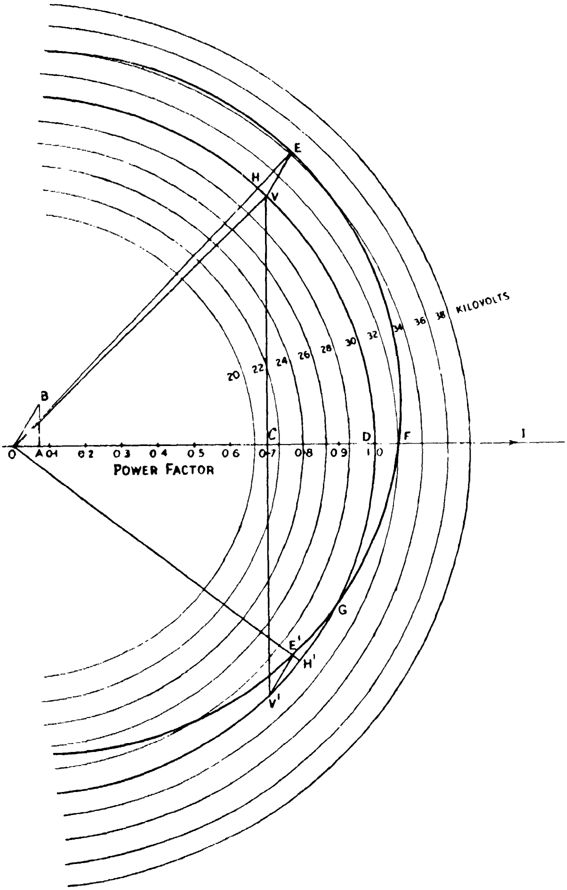

FIG. 27. Modified Mershon chart.

Three-phase Line.

In a three-phase line (Fig. 28) each phase is considered separately. With a balanced system there is no neutral current and therefore no drop in the neutral. Hence the circuit diagram of one phase is as indicated in Fig. 29.

Let I = line current,

V = receiving-end phase voltage,

$V_L = V\sqrt{3}$ = receiving-end line voltage,

R = resistance of one line,

X = reactance of one line,

E = sending-end phase voltage,

$E_L = E\sqrt{3}$ = sending-end line voltage.

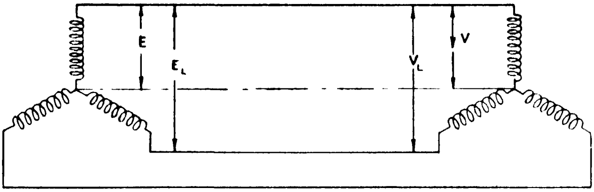

FIG. 28. Three-phase line.

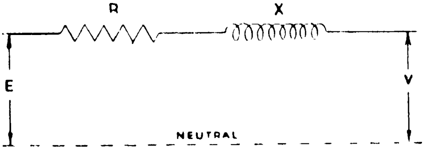

FIG. 29.

Then from similarity with the single-phase case, and noting that the drop takes place in only one line we have

$$E = \sqrt{(V\cos\phi + IR)^2 + (V\sin\phi \pm IX)^2}.$$

$$\therefore \quad E_L = E\sqrt{3} = \sqrt{3(V\cos\phi + IR)^2 + 3(V\sin\phi \pm IX)^2}$$

$$= \sqrt{(V\sqrt{3}\cos\phi + \sqrt{3}IR)^2 + (V\sqrt{3}\sin\phi \pm \sqrt{3}IX)^2}$$

$$= \sqrt{(V_L\cos\phi + \sqrt{3}IR)^2 + (V_L\sin\phi \pm \sqrt{3}IX)^2}.$$

The approximate volt-drop per phase is

$$I(R\cos\phi \pm X\sin\phi). \tag{3}$$

The approximate drop between phases is

$$\sqrt{3}I(R\cos\phi \pm X\sin\phi). \tag{4}$$

Also
$$\tan\theta = \frac{V_L\sin\phi \pm \sqrt{3}IX}{V_L\cos\phi + \sqrt{3}IR}. \tag{5}$$

The total power loss in the line is $3I^2R$.

EXAMPLES ON CHAPTER III

Q. 1. Find the fall in voltage along a 30-mile three-phase transmission line delivering 5,000 kVA. at a line voltage of 30,000 and a lagging power factor of 0·8. The resistance and reactance of each phase are 0·717 ohm and 0·6 ohm per mile respectively. L.U. 1925.

Q. 2. What distance can 300 kW. be transmitted at 0·8 power factor lagging on a three-phase system with a power loss in the lines of 30 kW. if the receiving pressure is 6,600 volts and the resistance per mile of single conductor is 1·3 ohms? Find also the approximate generated voltage necessary to maintain this voltage at the receiving end on full load if the reactance per mile of single conductor is 0·6 ohm.

Q. 3. 500 kW. at 11,000 volts are received from a three-phase transmission line, each wire of which has a resistance of 1·2 ohms and a reactance of 1 ohm. Calculate the supply pressure when the power factor is (*a*) unity, (*b*) 0·5 leading. C.G.

CHAPTER IV

LONG TRANSMISSION LINES. APPROXIMATE SOLUTION

LOCALIZED CAPACITANCE METHODS

In the preceding chapters the line capacitance has been ignored. We will now consider the effects of line capacitance by assuming that it is concentrated at one or two points in the line, although actually it is uniformly distributed along its length. This leads to a somewhat simplified problem, the solution to which may be obtained graphically or analytically and yields results which are sufficiently accurate for overhead lines of lengths up to 100 miles or so.

Nominal Π *Method.*

In this method half the line capacitance is assumed to be concentrated at each end of the line, as indicated in Fig. 30, which may represent a single-phase line with the total resistance and inductive reactance assumed to lie in one line, or one phase of a polyphase line. In the latter case the dotted line then represents the neutral.

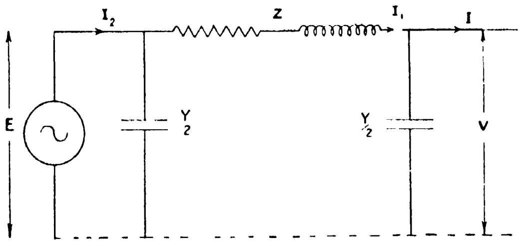

Fig. 30. Nominal Π.

Let I, I_1, and I_2 represent the currents in the load, the line, and the generator respectively. Let $\mathbf{Z} = R+jX$ be the series impedance of the line, and let $\mathbf{Y} = j\omega C$ be the shunt admittance, where C represents either the capacitance between lines of a single-phase system or the capacitance from one line to earth of a polyphase system.

The vector diagram is shown in Fig. 31, where I represents the load current lagging ϕ behind the load voltage V; $IA = \frac{V\omega C}{2}$ drawn at right angles to OV represents the current in the condenser at the load end. Then $OA = I_1$ gives the line current. $VB = I_1R$ drawn parallel to OA gives the ohmic drop in the line, while $BE = I_1X$ drawn at right angles to OA gives the inductive drop. Then $OE = E$ represents the generator voltage. Finally, $AD = \frac{E\omega C}{2}$ drawn at right angles to OE represents the sending-end condenser current, so that OD represents the sending-end current. The phase angle at the sending end is angle EOD.

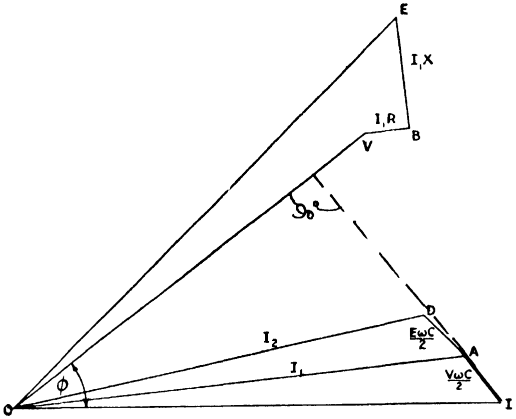

FIG. 31. Vector diagram for nominal Π.

The equations for an analytical solution are obtained as follows:

Now $$\mathbf{I}_1 = \mathbf{I}+\mathbf{V}\frac{\mathbf{Y}}{2}.$$

Impedance drop in the line $$= \mathbf{Z}\mathbf{I}_1 = \mathbf{Z}\left(\mathbf{I}+\frac{\mathbf{VY}}{2}\right).$$

Sending-end voltage $$\mathbf{E} = \mathbf{V}+\mathbf{Z}\left(\mathbf{I}+\frac{\mathbf{VY}}{2}\right)$$

$$= \mathbf{V}\left(1+\frac{\mathbf{YZ}}{2}\right)+\mathbf{ZI}. \qquad (1)$$

Again, sending-end current $\mathbf{I}_2 = \mathbf{I}_1+\mathbf{E}\frac{\mathbf{Y}}{2}$

$$= \mathbf{I}+\mathbf{V}\frac{\mathbf{Y}}{2}+\frac{\mathbf{Y}}{2}\left\{\mathbf{V}\left(1+\frac{\mathbf{YZ}}{2}\right)+\mathbf{ZI}\right\}$$

$$= \mathbf{VY}\left(1+\frac{\mathbf{YZ}}{4}\right)+\mathbf{I}\left(1+\frac{\mathbf{YZ}}{2}\right). \qquad (2)$$

This method of solution gives sending-end voltages which are too high.

Nominal T Method.

In this method the total line capacitance is assumed to be concentrated at the mid-point of the line, as indicated in Fig. 32.

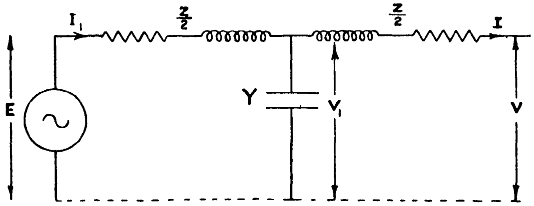

FIG. 32. Nominal *T* network.

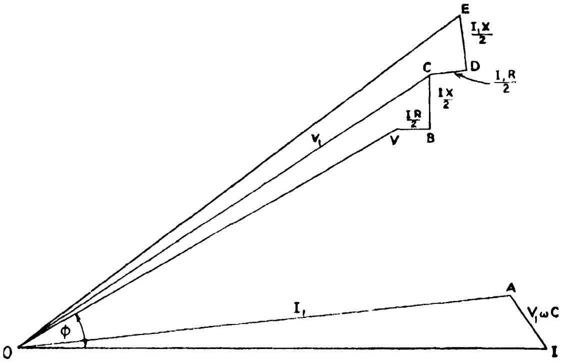

FIG. 33. Vector diagram for nominal *T*.

Let *OI* represent the load current *I* (Fig. 33) lagging ϕ behind the load voltage *OV*. The load current flows through the right-hand half of the series line impedance, in which the ohmic volt-drop is $VB = \frac{IR}{2}$ while the inductive volt-drop is $BC = \frac{IX}{2}$, drawn respectively parallel and at right angles to OI. The voltage at the mid-point of the line is therefore represented by $OC = V_1$. The condenser current leads this voltage by 90° and is represented by $IA = V_1\omega C$. OA therefore represents the sending-end current. This current flows through the remaining half of the series line impedance in which the ohmic drop is represented by $CD = \frac{I_1 R}{2}$ parallel to OA, and the inductive drop by $DE = \frac{I_1 X}{2}$ at right angles to OA. Then OE represents the sending-end voltage E, and angle EOA is the phase angle at the sending end.

The analytical solution is as follows:

Now $$\mathbf{V}_1 = \mathbf{V} + \mathbf{I}\frac{\mathbf{Z}}{2}.$$

The condenser current $$\mathbf{I}_C = \mathbf{V}_1\mathbf{Y} = \mathbf{Y}\left(\mathbf{V} + \frac{\mathbf{IZ}}{2}\right).$$

The sending-end current $$\mathbf{I}_1 = \mathbf{I} + \mathbf{I}_C = \mathbf{I} + \mathbf{Y}\left(\mathbf{V} + \frac{\mathbf{IZ}}{2}\right)$$

$$= \mathbf{I}\left(1 + \frac{\mathbf{YZ}}{2}\right) + \mathbf{VY}. \quad (3)$$

The sending-end voltage $$\mathbf{E} = \mathbf{V}_1 + \mathbf{I}_1\frac{\mathbf{Z}}{2}$$

$$= \mathbf{V} + \frac{\mathbf{IZ}}{2} + \frac{\mathbf{IZ}}{2}\left(1 + \frac{\mathbf{YZ}}{2}\right) + \frac{\mathbf{VYZ}}{2},$$

i.e. $$\mathbf{E} = \mathbf{V}\left(1 + \frac{\mathbf{YZ}}{2}\right) + \mathbf{IZ}\left(1 + \frac{\mathbf{YZ}}{4}\right). \quad (4)$$

This method of solution gives sending-end voltages which are too low.

EXAMPLE. *Find the sending-end characteristics of a* 110 *kV. three-phase overhead line which delivers* 8,400 *kW. at* 0·8 *power factor lagging, over a distance of* 100 *miles. The resistance, inductive reactance, and capacitative reactance per mile of single conductor are* 0·736, 1·41, *and* $1{\cdot}2 \times 10^5$ *ohms respectively.*

Now $R = 73{\cdot}6 \qquad X = 141.$

$$\therefore\quad \mathbf{Z} = 73{\cdot}6+141j = 159{\cdot}1\underline{|62^\circ\,26'}$$

$$Y = 100\frac{1}{1{\cdot}2\times 10^5} = 8{\cdot}333\times 10^{-4}.$$

$$\therefore\quad \mathbf{Y} = 8{\cdot}333\times 10^{-4}\underline{|90^\circ}.$$

Phase voltage at load end $V = \dfrac{110000}{\sqrt{3}} = 63520.$

Taking this voltage as the datum vector, $\mathbf{V} = 63520\underline{|0^\circ}.$

Line current in the load $I = \dfrac{8400}{\sqrt{3}\times 110\times 0{\cdot}8} = 55{\cdot}12$ amp.

$$\therefore\quad \mathbf{I} = 55{\cdot}12|\cos^{-1} 0{\cdot}8 = 55{\cdot}12|36^\circ\,52'.$$

Nominal Π *Method.*

$$1+\frac{\mathbf{YZ}}{2} = 1+\frac{0{\cdot}1326\underline{|152^\circ\,26'}}{2} = 0{\cdot}9412+0{\cdot}03068j$$

$$= 0{\cdot}9417\underline{|1^\circ\,52'}.$$

$$1+\frac{\mathbf{YZ}}{4} = 0{\cdot}9706+0{\cdot}01534j = 0{\cdot}9707\underline{|0^\circ\,54'}.$$

$$\therefore\quad \mathbf{E} = 63520(0{\cdot}9412+0{\cdot}03068j)+$$

$$+(159{\cdot}1\underline{|62^\circ\,26'})(55{\cdot}12\overline{|36^\circ\,52'})$$

$$= 59780+1948j+8770\underline{|25^\circ\,34'}$$

$$= 59780+1948j+7911+3785j$$

$$= 67691+5733j = 67940\underline{|4^\circ\,50'},$$

i.e. phase voltage at sending end $= 67{\cdot}94$ kV.

∴ Pressure drop to neutral $= 67{\cdot}94-63{\cdot}52 = 4{\cdot}42$ kV.

∴ Line voltage at sending end $= 67{\cdot}94\sqrt{3} = 117{\cdot}7$ kV.

$$\mathbf{I}_2 = 63520(8{\cdot}333\times 10^{-4}\underline{|90^\circ})(0{\cdot}9707\underline{|0^\circ\,54'})+$$

$$+(55{\cdot}12\overline{|36^\circ\,52'})(0{\cdot}9417\underline{|1^\circ\,52'})$$

$$= 51{\cdot}39\underline{|90^\circ\,54'}+51{\cdot}92\overline{|35^\circ}$$

$$= 41{\cdot}72+21{\cdot}59j = 46{\cdot}97\underline{|27^\circ\,22'}.$$

Note that the large capacity of the line causes the current to lead at the sending end.

$$\text{Power factor at sending end} = \cos|\underline{27^\circ\,22' - 4^\circ\,50'}$$
$$= 0{\cdot}9237 \text{ leading.}$$

$$\text{Sending-end power} = \sqrt{3}\times 117{\cdot}7\times 46{\cdot}97\times 0{\cdot}9237 = 8{,}845 \text{ kW.}$$

$$\therefore\ \text{Line efficiency} = \frac{8400}{8845} = 94{\cdot}97 \text{ per cent.}$$

Nominal T Method.

$$\mathbf{E} = 59780 + 1948j + (8770|\underline{25^\circ\,34'})(0{\cdot}9707|\underline{0^\circ\,54'})$$
$$= 59780 + 1948j + 8513|\underline{26^\circ\,28'}$$
$$= 67401 + 5742j = 67640|\underline{4^\circ\,52'}.$$

$$\text{Pressure drop per phase} = 67{\cdot}64 - 63{\cdot}52 = 4{\cdot}12 \text{ kV.}$$

$$\therefore\ \text{Line voltage at the sending end} = E\sqrt{3} = 117{\cdot}1 \text{ kV.}$$

$$\mathbf{I}_1 = 51{\cdot}92\,\overline{|35^\circ} + 52{\cdot}93|\underline{90^\circ}$$
$$= 42{\cdot}53 - 29{\cdot}78j + 52{\cdot}93j = 42{\cdot}53 + 23{\cdot}15j$$
$$= 48{\cdot}42|\underline{28^\circ\,34'}.$$

$$\text{Power factor} = \cos 23^\circ\,42' = 0{\cdot}9157 \text{ leading.}$$
$$\text{Power input} = \sqrt{3}\times 117{\cdot}1\times 48{\cdot}42\times 0{\cdot}9157 = 8991 \text{ kW.}$$
$$\text{Line efficiency} = \frac{8400}{8991} = 93{\cdot}44 \text{ per cent.}$$

REGULATION OF LONG LINES

The regulation of a line is defined as the percentage rise in pressure at the receiving end when the load is thrown off, the sending-end voltage remaining constant. For short lines this is the percentage volt-drop in the line for that load, since on throwing off the load the receiving-end voltage rises to the voltage at the generating end. On long lines, however, this is not so, since on no load there still flows the charging current of the line, which, flowing through the line reactance, causes the voltage at the receiving end to exceed that at the generator end. Thus, consider the vector diagram Fig. 34. OV represents the phase voltage at the receiving end when delivering a load current OI which is assumed to lag by an angle ϕ. Using the nominal Π conception, let OI_C represent the current through the load-end condenser, leading OV by 90°.

The vector sum of OI_C and OI gives the line current OA. The generator voltage OE may then be found as in Fig. 31, VE being the impedance voltage-drop. On throwing off the load, the capacitance current still flows, although slightly altered in magnitude and direction since the receiving-end voltage has changed. This capacitance current causes an ohmic volt-drop $DV_0 = I_C R$ and a reactance voltage $ED = I_C X$ respectively parallel and at right angles to I_C. The triangle EDV_0 is thus similar to triangle EBV, but is rotated anti-clockwise relatively to it through angle AOI_C. The induced voltage ED is thus practically in phase with OE, so that the no-load receiving-end voltage OV_0 is greater than the generator voltage OE.

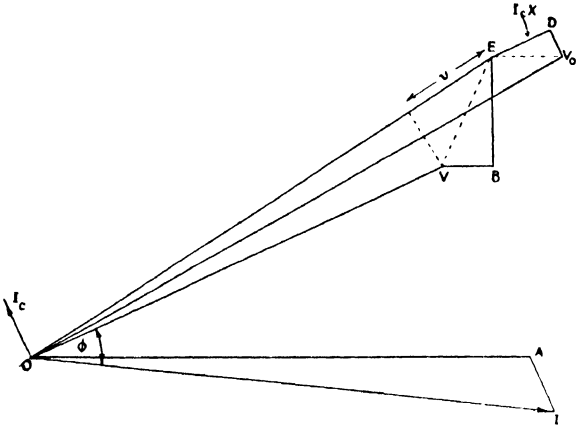

FIG. 34.

Ignoring the resistance drop DV_0,

$$V_0 - E \doteqdot ED = I_C X.$$

The regulation $= V_0 - V = V_0 - E + E - V$
$= I_C X + v$,

where v represents the pressure drop on load.

Now the induced voltage $I_C X = \frac{V_0 \omega C}{2} . \omega L$, where C and L represent the capacitance and inductance of the line respectively.

But for overhead lines,

$$C = \frac{0\cdot0388l}{d} \times 10^{-6} \text{ farads},$$

and $$L = \left(0{\cdot}08+0{\cdot}741\log\frac{d}{r}\right)\times 10^{-3}l \text{ henrys}$$

(see Chap. XI), where l is the length of the line in miles. It will be seen that the product LC is practically constant for all lines, and from tables it is found to be approximately $3{\cdot}1\times 10^{-11}l^2$.

$$\begin{aligned}\text{Hence induced voltage } &= \frac{V_0}{2}\omega^2\times 3{\cdot}1\times 10^{-11}l^2\\ &= 1{\cdot}53V_0\times 10^{-6}l^2\\ &\doteqdot 1{\cdot}5V_0\left(\frac{l}{1000}\right)^2.\end{aligned}$$

The regulation of the line at any load is therefore

$$v+1{\cdot}5V_0\left(\frac{l}{1000}\right)^2,$$

where v is the volt-drop for that load. This may be further simplified to approximately

$$v+1{\cdot}5E\left(\frac{l}{1000}\right)^2. \tag{5}$$

In the preceding example this becomes

$$(117{\cdot}7-110)+1{\cdot}5\times 117{\cdot}7\left(\frac{100}{1000}\right)^2 \text{ kV.},$$

using the figures for the nominal Π.

$$\begin{aligned}\text{i.e. regulation } &= 7{\cdot}7+1{\cdot}765 = 9{\cdot}465 \text{ kV.}\\ &= \frac{9{\cdot}465}{110}\times 100 \text{ per cent.}\\ &= 8{\cdot}6 \text{ per cent.}\end{aligned}$$

Capacity Effects on Open-circuited Lines.

It will be gathered from the foregoing that in a long open-circuited line if V_0 is the no-load receiving-end voltage, the load-end condenser current will be $I_C = \frac{V_0\omega C}{2}$ leading OV_0 by 90° (Fig. 34). The sending-end voltage will be OE, which is less than OV_0 by an amount represented approximately by

$$\begin{aligned}ED &= I_C X\\ &= 1{\cdot}5V_0\left(\frac{l}{1000}\right)^2.\end{aligned} \tag{6}$$

This phenomenon of voltage rise at the receiving end of an open-circuited or lightly loaded line is sometimes called the Ferranti effect.

The capacitance current flowing through the line resistance will also give rise to open-circuit copper losses. Thus if C is the total capacitance of the line, the total charging current is $I_C = V\omega C$, where V is the line voltage, assumed constant along its length for the present calculation. The current will increase uniformly from zero at the receiving end, and at any distance x from that end (Fig. 35) will be given by $I_C\frac{x}{l}$. If this current flows through an elemental length δx of the line, which is assumed to have a resistance r per unit length, the copper losses in that element will be

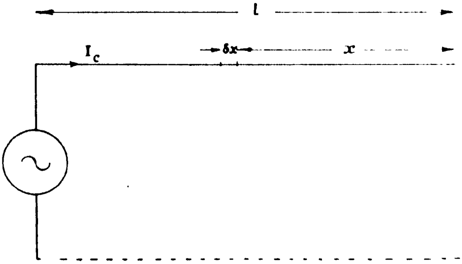

FIG. 35.

$$\left(I_C\frac{x}{l}\right)^2 r\,\delta x.$$

The total copper loss in the whole line will then be

$$\int_0^l \left(I_C\frac{x}{l}\right)^2 r\,dx = I_C^2\frac{r}{l^2}\int_0^l x^2\,dx$$

$$= \frac{I_C^2 rl}{3} = \frac{I_C^2 R}{3}, \qquad (7)$$

where R is the total line resistance.

EXAMPLES ON CHAPTER IV

Q. 1. Explain the phenomenon of voltage rise at the receiving end of lightly loaded transmission lines.

Find the approximate voltage on open circuit at the receiving end of a 50-frequency overhead transmission line 100 miles long, if the line voltage at the sending end be maintained constant at 66 kV. Assume one-half of the total capacitance of the line to be concentrated at each end. Neglect line leakage and resistance. L.U.

Q. 2. Find the full-load voltage at the generator end of a three-phase transmission line which delivers 6,000 kVA. at 0·8 power factor lagging to a balanced load at 33,000 volts. The resistance, inductive reactance, and capacitative reactance of the line are 15·21, 59·2, and 2,130 ohms respectively. Use the nominal Π method.

Q. 3. A three-phase transmission line delivers 20,000 kVA., power factor 0·8 lagging at 66 kV. at the receiving end. Each conductor has a resistance of 10 ohms, an inductive reactance (line to neutral) of 30 ohms, and an admittance due to capacitance between conductors (line to neutral) of 4×10^{-4} mhos. Calculate the voltage, current, and power factor at the transmitting end, and the efficiency of transmission, neglecting line leakance. L.U.

CHAPTER V

LONG TRANSMISSION LINES. EXACT SOLUTION

CONSIDER a long transmission line of length l miles, and let r and x be the resistance and inductive reactance respectively per mile of line. Let g be the leakage conductance per mile due to the line insulation and let b be the capacitative susceptance per mile of the line, i.e. $b = \omega c$, where c is the line capacitance per mile.

An elemental length ds of this line may then be represented as in Fig. 36 and may be further simplified as shown in Fig. 37,

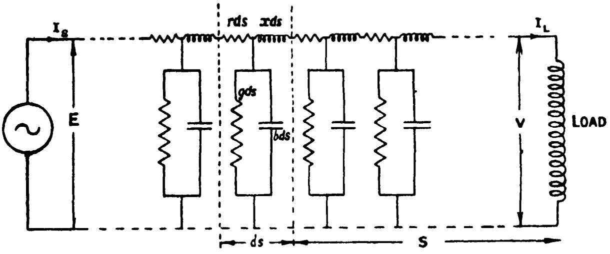

FIG. 36.

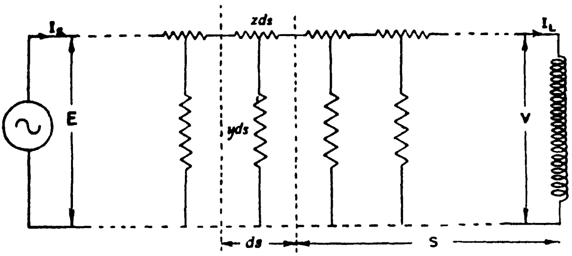

FIG. 37.

where the series impedance $\mathbf{z} = r+jx$ and the shunt admittance $\mathbf{y} = g+jb$. If $\mathbf{i}$ is the current in this section of the line and $\mathbf{e}$ is its potential, the voltage increase when proceeding from the load end is

$$d\mathbf{e} = \mathbf{i}\mathbf{z}\,ds.$$

The increase in current due to leakage and capacitance effects is

$$d\mathbf{i} = \mathbf{e}\;ds.$$

Hence $$\frac{d\mathbf{e}}{ds} = \mathbf{iz}, \tag{1}$$

$$\frac{d\mathbf{i}}{ds} = \mathbf{ey}. \tag{2}$$

Differentiating equation (1) with respect to s we have

$$\frac{d^2\mathbf{e}}{ds^2} = \mathbf{z}\frac{d\mathbf{i}}{ds}$$

$$= \mathbf{zye}. \tag{3}$$

The solution to the differential equation (3) is

$$\mathbf{e} = A\cosh\sqrt{\mathbf{yz}}\,s + B\sinh\sqrt{\mathbf{yz}}\,s$$

$$= A\cosh\mathbf{p}s + B\sinh\mathbf{p}s, \tag{4}$$

where A and B are constants; $\mathbf{p} = \sqrt{\mathbf{yz}}$ and is called the propagation constant of the line.

$$\cosh\mathbf{p}s = \frac{e^{\mathbf{p}s}+e^{-\mathbf{p}s}}{2},$$

$$\sinh\mathbf{p}s = \frac{e^{\mathbf{p}s}-e^{-\mathbf{p}s}}{2}.$$

Again, since from (1) $\mathbf{i} = \frac{1}{\mathbf{z}}\frac{d\mathbf{e}}{ds}$, then by differentiating (4) we have

$$\mathbf{i} = \frac{\sqrt{\mathbf{yz}}}{\mathbf{z}}\{A\sinh\sqrt{\mathbf{yz}}\,s + B\cosh\sqrt{\mathbf{yz}}\,s\}$$

$$= \sqrt{\frac{\mathbf{y}}{\mathbf{z}}}\{A\sinh\mathbf{p}s + B\cosh\mathbf{p}s\}. \tag{5}$$

Now when s, measured from the load end, is zero, $\mathbf{e} = \mathbf{V}$, the load voltage, and $\mathbf{i} = \mathbf{I}_L$, the load current: also $\cosh 0 = 1$ and $\sinh 0 = 0$.

Substituting these values in (4) and (5) we have

$$\mathbf{V} = A$$

$$\mathbf{I}_L = B\sqrt{\frac{\mathbf{y}}{\mathbf{z}}}, \quad \text{or} \quad B = \sqrt{\frac{\mathbf{z}}{\mathbf{y}}}\mathbf{I}_L.$$

Inserting these values in (4) and (5), and putting $s = l$ to give the sending-end values, we have:

$$\text{Sending-end voltage } \mathbf{E} = \mathbf{V}\cosh \mathbf{p}l + \sqrt{\frac{\mathbf{z}}{\mathbf{y}}}\,\mathbf{I}_L \sinh \mathbf{p}l. \tag{6}$$

$$\text{Sending-end current } \mathbf{I}_S = \mathbf{I}_L\cosh \mathbf{p}l + \sqrt{\frac{\mathbf{y}}{\mathbf{z}}}\,\mathbf{V} \sinh \mathbf{p}l. \tag{7}$$

If we replace the line constants per mile by the corresponding constants for the whole line, which are represented by capital letters, and note that the total series impedance of the line

$$\mathbf{Z} = \mathbf{z}l;$$

and the total shunt admittance of the line

$$\mathbf{Y} = \mathbf{y}l;$$

then

$$\mathbf{E} = \mathbf{V}\cosh \mathbf{P} + \mathbf{Z}_0\mathbf{I}_L\sinh \mathbf{P}, \tag{8}$$

$$\mathbf{I}_S = \mathbf{I}_L\cosh \mathbf{P} + \frac{\mathbf{V}}{\mathbf{Z}_0}\sinh \mathbf{P}, \tag{9}$$

where

$$\mathbf{P} = \mathbf{p}l = \sqrt{\mathbf{YZ}},$$

$$\mathbf{Z}_0 = \sqrt{\frac{\mathbf{z}}{\mathbf{y}}} = \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}},$$

and is called the characteristic impedance of the line.

If equations (8) and (9) are solved as simultaneous equations in $\mathbf{V}$ and $\mathbf{I}_L$, or if $-s$ is written for s in equations (4) and (5) to obtain expressions in terms of generating-end values we may write

$$\mathbf{V} = \mathbf{E}\cosh \mathbf{P} - \mathbf{Z}_0\mathbf{I}_S\sinh \mathbf{P}, \tag{10}$$

$$\mathbf{I}_L = \mathbf{I}_S\cosh \mathbf{P} - \frac{\mathbf{E}}{\mathbf{Z}_0}\sinh \mathbf{P}. \tag{11}$$

These equations are valid for a single-phase or a three-phase line if in the former case the constants r, g, x, and b are those for a loop mile, while in the case of the three-phase line they refer to the constants per mile of conductor. The voltages in the single-phase case are the line pressures, while in the three-phase case they refer to the phase volts, i.e. to neutral.

The most direct method of solving these equations involves the use of tables of complex hyperbolic functions. If these are not available the complex functions may be obtained either by expansion or by series.

The expressions necessary for these evaluations are

$$\begin{aligned}\cosh\mathbf{P} &= \cosh(\alpha+j\beta)\\ &= \cosh\alpha\cos\beta+j\sinh\alpha\sin\beta\\ &= 1+\frac{\mathbf{P}^2}{|\underline{2}}+\frac{\mathbf{P}^4}{|\underline{4}}+\frac{\mathbf{P}^6}{|\underline{6}}+\dots\\ &= 1+\frac{\mathbf{YZ}}{|\underline{2}}+\frac{(\mathbf{YZ})^2}{|\underline{4}}+\frac{(\mathbf{YZ})^3}{|\underline{6}}+\dots. \qquad (12)\end{aligned}$$

$$\begin{aligned}\sinh\mathbf{P} &= \sinh(\alpha+j\beta)\\ &= \sinh\alpha\cos\beta+j\cosh\alpha\sin\beta\\ &= \mathbf{P}+\frac{\mathbf{P}^3}{|\underline{3}}+\frac{\mathbf{P}^5}{|\underline{5}}+\frac{\mathbf{P}^7}{|\underline{7}}+\dots\\ &= \sqrt{\mathbf{YZ}}\left(1+\frac{\mathbf{YZ}}{|\underline{3}}+\frac{(\mathbf{YZ})^2}{|\underline{5}}+\frac{(\mathbf{YZ})^3}{|\underline{7}}+\dots\right). \qquad (13)\end{aligned}$$

$$\therefore\quad \mathbf{Z}_0\sinh\mathbf{P} = \mathbf{Z}\left\{1+\frac{\mathbf{YZ}}{|\underline{3}}+\frac{(\mathbf{YZ})^2}{|\underline{5}}+\frac{(\mathbf{YZ})^3}{|\underline{7}}+\dots\right\} \qquad (14)$$

$$\frac{1}{\mathbf{Z}_0}\sinh\mathbf{P} = \mathbf{Y}\left\{1+\frac{\mathbf{YZ}}{|\underline{3}}+\frac{(\mathbf{YZ})^2}{|\underline{5}}+\frac{(\mathbf{YZ})^3}{|\underline{7}}+\dots\right\}. \qquad (15)$$

Line on Open Circuit.

With the line on open circuit, we have, putting $\mathbf{I}_L = 0$ in (8),

$$\mathbf{E} = \mathbf{V}\cosh\mathbf{P},$$

or

$$\mathbf{V} = \frac{\mathbf{E}}{\cosh\mathbf{P}}. \qquad (16)$$

Putting $\mathbf{I}_L = 0$ in (9),

$$\mathbf{I}_S = \frac{\mathbf{V}}{\mathbf{Z}_0}\sinh\mathbf{P} = \frac{\mathbf{E}}{\mathbf{Z}_0}\tanh\mathbf{P}. \qquad (17)$$

This is the charging current of the line.

The impedance viewed from the sending end would be

$$\begin{aligned}\mathbf{Z}_S = \frac{\mathbf{E}}{\mathbf{I}_S} &= \frac{\mathbf{EZ}_0}{\mathbf{E}\tanh\mathbf{P}}\\ &= \mathbf{Z}_0\coth\mathbf{P}. \qquad (18)\end{aligned}$$

Line Earthed or Short-circuited.

If a single-phase line is short-circuited, or a three-phase line is earthed at the load, $\mathbf{V} = 0$.

Making this substitution in (8),

$$\mathbf{E} = \mathbf{Z}_0\,\mathbf{I}_L \sinh \mathbf{P}$$

or

$$\mathbf{I}_L = \frac{\mathbf{E}}{\mathbf{Z}_0 \sinh \mathbf{P}}. \tag{19}$$

Putting $\mathbf{V} = 0$ in (9),

$$\begin{aligned} \mathbf{I}_S &= \mathbf{I}_L \cosh \mathbf{P} \\ &= \frac{\mathbf{E}}{\mathbf{Z}_0} \coth \mathbf{P}. \end{aligned} \tag{20}$$

The impedance viewed from the sending end would be

$$\mathbf{Z}_S = \frac{\mathbf{E}}{\mathbf{I}_S} = \mathbf{Z}_0 \tanh \mathbf{P}. \tag{21}$$

If the line is earthed or short-circuited at some point distant x from the sending end, then

$$\mathbf{I}_x = \frac{\mathbf{E}}{\mathbf{Z}_0 \sinh \mathbf{p}x},$$

$$\mathbf{I}_S = \frac{\mathbf{E}}{\mathbf{Z}_0} \coth \mathbf{p}x,$$

$$\mathbf{Z}_S = \mathbf{Z}_0 \tanh \mathbf{p}x.$$

EXAMPLE. *Find the sending-end current and voltage in the example of Chapter IV.*

Now $\mathbf{E} = \mathbf{V} \cosh \mathbf{P} + \mathbf{Z}_0\,\mathbf{I}_L \sinh \mathbf{P}$.

$$\begin{aligned} \cosh \mathbf{P} &= 1 + \frac{\mathbf{YZ}}{\underline{|2}} + \frac{(\mathbf{YZ})^2}{\underline{|4}} \\ &= 1 + \frac{0{\cdot}1326\,\underline{|152^\circ\ 26'}}{2} + \frac{0{\cdot}0176\,\overline{|55^\circ\ 8'}}{24} \\ &= 0{\cdot}9417 + 0{\cdot}03j, \end{aligned}$$

$$\mathbf{V} \cosh \mathbf{P} = 63520(0{\cdot}9417 + 0{\cdot}03j) = 59810 + 1906j.$$

$$\begin{aligned} \mathbf{Z}_0\,I_L \sinh \mathbf{P} &= \mathbf{ZI}_L\left(1 + \frac{\mathbf{YZ}}{\underline{|3}} + \frac{(\mathbf{YZ})^2}{\underline{|5}}\right) \\ &= 159{\cdot}1\,\underline{|62^\circ\ 26'} \times 55{\cdot}12\,\overline{|36^\circ\ 52'}\,(1 - 0{\cdot}0196 + \\ &\qquad + 0{\cdot}0102j + 0{\cdot}0001 - 0{\cdot}0001j) \\ &= 8772\,\underline{|25^\circ\ 34'}\,(0{\cdot}9805 + 0{\cdot}0101j), \end{aligned}$$

$$= 7719+3792j.$$

$$\therefore\quad \mathbf{E} = 59810+1906j+7719+3792j,$$

$$= 67760\underline{|4^\circ\ 50'}.$$

$$\mathbf{I}_S = \mathbf{I}_L\cosh\mathbf{P}+\frac{\mathbf{V}}{\mathbf{Z}_0}\sinh\mathbf{P}$$

$$= 55{\cdot}12\overline{|36^\circ\ 52'}(0{\cdot}9417+0{\cdot}03j)+$$

$$+63520\mathbf{Y}\left(1+\frac{\mathbf{YZ}}{\underline{|3}}+\frac{(\mathbf{YZ})^2}{\underline{|5}}\right)$$

$$= 51{\cdot}94\overline{|35^\circ\ 2'}+51{\cdot}89j-0{\cdot}5346$$

$$= 47{\cdot}5|28^\circ.$$

EXAMPLES ON CHAPTER V

Q. 1. A three-phase transmission line delivers 20,000 kVA., power factor 0·8 lagging at 66kV. at the receiving end. Each conductor has a resistance of 10 ohms, an inductive reactance (line to neutral) of 30 ohms, and an admittance due to capacitance between conductors (line to neutral) of 4×10^{-4} mhos. Calculate the voltage, current, and power factor at the transmitting end and the efficiency of transmission, neglecting line leakance. L.U.

Q. 2. Find the full-load voltage at the generator end of a three-phase transmission line which delivers 6,000 kVA. at 0·8 power factor lagging to a balanced load at 33,000 volts. The resistance, inductive reactance, and capacitative reactance of the line are 15·21, 59·2, and 2,130 ohms respectively.

CHAPTER VI

ECONOMIC PRINCIPLES OF TRANSMISSION

Economics of Transmission.

IN a distribution network the size of conductor is determined by the permissible volt-drop, but in a transmission system the conductor size for a given current is governed by economic principles. The best section is that for which the annual cost is a minimum. The larger the section, the greater the initial cost of the copper and the greater therefore is the annual cost of interest and depreciation on the line. On the other hand, the larger the conductor section the smaller the resistance and the I^2R losses, and the smaller therefore is the annual cost of energy wasted in the line. It will be clear therefore that there will be some section for which the total annual cost is a minimum.

The total initial expenditure on the complete line installation is made up of the cost of the copper, which is directly proportional to the area, and the costs of towers, insulation, and labour of installation, which tend to increase with the conductor size. At a given rate of interest and depreciation the annual charges on this initial outlay may be represented therefore by

$$C_1 = PA+Q,$$

where A is the conductor area and P and Q are constants. The annual cost of the ohmic energy wasted is proportional to the resistance of the line and is therefore inversely proportional to its cross-sectional area, and hence may be represented by

$$C_2 = \frac{K}{A},$$

where K is a constant.

The total cost is therefore

$$C = PA+Q+\frac{K}{A},$$

which is a minimum when

$$\frac{dC}{dA} = 0,$$

i.e. when $$P-\frac{K}{A^2} = 0,$$

i.e. $$P = \frac{K}{A^2}$$

or $$PA = \frac{K}{A}.$$

Hence the most economical conductor area is that for which the annual cost of energy wasted in the line is equal to the annual cost of interest and depreciation on that part of the initial outlay which may be considered proportional to the conductor area. This is Kelvin's law as modified by Kapp. In practice, that part of the initial outlay proportional to the conductor area is usually considered as the cost of the line copper itself. The law is illustrated graphically in Fig. 38.

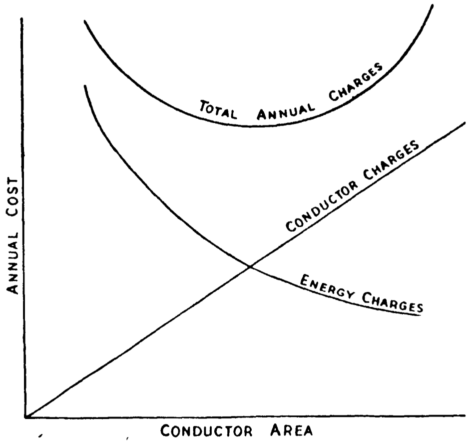

Fig. 38. Graphical representation of Kelvin's law.

Economic Area of Conductor.

Let C_c = cost in pence per mile of conductor of 1 sq. in. section,

C_e = cost of energy losses in pence per Kelvin,

r = rate of interest and depreciation per cent. per annum,

R = resistance of 1 mile of conductor 1·0 sq. in. cross-section, in ohms,

I = R.M.S. current in the conductor during the year,

A = conductor area in sq. in.

Then,

Annual charge for interest and depreciation per mile of conductor $= \dfrac{rAC_c}{100}$ pence.

Power lost in the line $= \dfrac{I^2R}{A}$ watts.

Energy wasted per annum in the line $= \dfrac{I^2R}{A} \cdot \dfrac{8760}{1000}$ kWh.,

where 8760 represents the number of hours in a year.

Annual cost of energy wasted in the line $= \dfrac{8{\cdot}76 I^2 R C_e}{A}$ pence.

Equating the annual charges, we have

$$\frac{8{\cdot}76 I^2 R C_e}{A} = \frac{rAC_c}{100},$$

or

$$A = I\sqrt{\frac{876 R C_e}{rC_c}}.$$

Taking $R = 0{\cdot}0445$ for copper,

$$A = 6{\cdot}25 I\sqrt{\frac{C_e}{rC_c}}. \qquad (1)$$

Taking $R = 0{\cdot}073$ for aluminium,

$$A = 8I\sqrt{\frac{C_e}{rC_c}}. \qquad (2)$$

The value of I may be obtained by dividing the maximum load current by the crest factor of the probable load curve for the year. If this curve is not available, I may be taken as $k_f k_l I_m$, where I_m is the maximum current in the line, k_l is the load factor, i.e. the ratio of the average to the peak load on the line, and k_f is the form factor, i.e. the ratio of the R.M.S. to the average current in the line. This factor may be obtained from the curve (Fig. 39) giving the average form factor values of typical systems for various load factors.

The formula is only approximately true since no account has been taken of those portions of the capital cost, other than the cost of the cable, which vary with the size of the conductor. Further, the value of C_e cannot be accurately assessed, since it is greater than the generated cost on account of the annual charges on the increase in the size of the cable and station plant necessary to transmit the losses, the load factor of which is less than that of the load. These considerations may raise the value of C_e to 1·3 times the cost of generation.

For extra high voltage cables (above 20 kV.) the formula cannot be used, since the dielectric hysteresis losses may be commensurate with the ohmic losses and must be taken into account. In this case the total annual cost of the I^2R losses, dielectric losses, and interest and depreciation on the installed cable must be worked out for a number of conductor areas, the one chosen being that for which the total cost is a minimum.

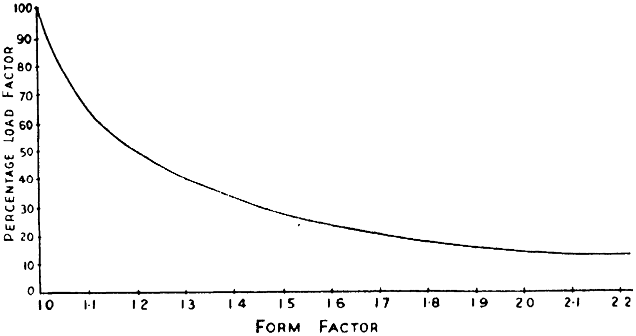

FIG. 39.

It must be remembered also that the economic section obtained from the preceding considerations is subject to the limitations of safe current density, permissible volt-drop, mechanical strength, and corona loss.

EXAMPLE. *Find the economic size of copper conductor necessary to transmit a maximum loading of* 5,000 *kVA. over a three-phase overhead line operated at* 33 *kV., the annual load factor being* 50 *per cent. Allow* 10 *per cent. for interest and depreciation.*

Now $\sqrt{3}VI_m = 5{,}000$ kVA.

Hence $I_m = \dfrac{5000}{33\sqrt{3}} = 88$ amp.

For a 50 per cent. load factor, $k_f = 1{\cdot}2$.

$$\therefore \quad I = 88 \times 0{\cdot}5 \times 1{\cdot}2 = 53 \text{ amp.}$$

Taking the cost per Kelvin of the energy losses as $\frac{1}{2}d.$, and assuming the cost per mile of 1 sq. in. conductor to be about £1,600, we have

$$A = 6{\cdot}25\times 53\sqrt{\frac{1}{2\times 10\times 1600\times 240}}$$

$$= \frac{6{\cdot}25\times 53}{1600\sqrt{3}},$$

i.e. economic conductor area $= 0{\cdot}12$ sq. in.

Economic Voltage.

The best voltage to be employed on any transmission system is again governed by considerations of economics. We have seen in Chapter I that the higher the voltage the smaller are the transmission losses and the conductor copper required. The cost of the terminal apparatus and insulation, however, increases rapidly at the higher voltages and imposes the limit to the pressure which may be adopted.

No definite rule can be given for the best voltage to employ, which can be arrived at only by estimating the total annual cost of the transmission losses and the interest and depreciation on the conductors, transformers, switch-gear, and insulation for various voltages. The economic voltage is that for which this total cost is a minimum.

As a guide for preliminary estimates the line pressures for cables and for overhead lines up to 20 miles in length may be taken as 1,000 volts per mile of transmission. For longer distances on overhead lines an empirical value for the approximate economical line pressure in kilovolts is

$$5{\cdot}5\sqrt{L+\frac{\text{kVA.}}{150}}, \qquad (3)$$

where L is the distance of transmission in miles.

The standard transmission voltages in this country are:

3,300 volts
6,600 „
11,000 „
33,000 volts
66,000 „
132,000 „

EXAMPLES ON CHAPTER VI

Q. 1. Find the most economical resistance per mile of copper conductor for the transmission of 8,000 kW. at 132 kV., three-phase, power factor 0·8.

Use the following data, and prove any formula used:

Cost of copper = 8*d.* per pound.
Density of copper = 8·9 gm. per cu. cm.
Specific resistance of copper = $1{\cdot}78 \times 10^{-6}$ ohm per cm. cube.
Annual charges for interest and depreciation = 10 per cent.
Cost of energy loss = £4 per kW.-year. L.U.

Q. 2. On a 250-volt D.C. system a conductor is to carry 60 kW. The average losses in the conductor are 18 per cent. of their maximum value. Assume that copper costs £83 per ton, energy costs 0·2*d.* per kWh., and capital charges are at the rate of 10 per cent. The resistance of a conductor having 1 sq. in. cross-section is 0·023 ohm per 1,000 yards. A cubic foot of copper weighs 560 lb. Find the economical area. A.M.I.E.E.

Q. 3. A 500-volt two-core feeder cable 2 miles long is required to supply a maximum current of 200 amp. The annual load factor is 50 per cent. The resistance of 1 mile of copper conductor 1 sq. in. in cross-sectional area is 0·046 ohm and the cost of cable including cost of installation is £(6A+1·2) per yard of cable, where A is the cross-sectional area in square inches. Interest on capital expenditure and depreciation charge total 10 per cent. of this cost, and the cost of energy wasted is 0·5*d.* per unit. Find the most economical size of cable, and discuss the result from the point of view of cable heating and voltage-drop. L.U.

Q. 4. Determine the resistance per mile of the most economical copper conductor to transmit 10,000 kW. a distance of 125 miles on a three-phase line at 0·8 power factor. Cost of copper per lb. 8*d.* Annual interest and depreciation charges on conductor 12·5 per cent. Transmission voltage 132,000. Estimated cost of energy losses per kW.-year £5.

Q. 5. Find the most economical size of copper conductor required to transmit a maximum current of 150 amp. at 40 per cent. load factor, allowing 10 per cent. interest and depreciation on capital expenditure. Take the cost of energy to be 0·5*d.* per Kelvin, the cost of 1 mile of 1·0 sq. in. conductor to be £1,650, and the form factor corresponding to a load factor of 40 per cent. to be 1·3.

CHAPTER VII

OVERHEAD LINES. MECHANICAL PRINCIPLES

In the design of an overhead transmission line care must be taken to ensure that the line is so tensioned that at no subsequent time will the safe stress in the conductor be exceeded. The maximum stress will occur at the lowest temperature, when the line has contracted and is also possibly covered with ice or sleet. Allowance must also be made for possible wind pressures on the line. These considerations involve a knowledge of the sag of the conductor, which also governs the height and strength of the supporting towers, and the length of the span.

Statics of the Line.

Consider a line AOB of length L feet, suspended freely from two supports at the same level spaced l feet apart (Fig. 40). Let w be the weight of the conductor in lb. per foot run, T (lb.) the tension of the line at any point P, and T_0 the tension at the lowest point O, which is taken as the origin.

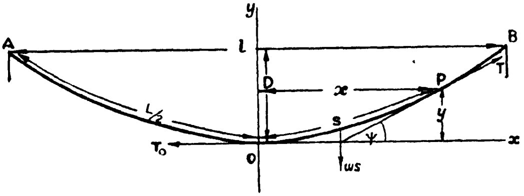

FIG. 40.

The portion OP of the conductor is in equilibrium under the tension T at P, the weight ws acting vertically downwards, and the horizontal tension T_0 (Fig. 41).

Now $\tan\psi = \dfrac{ws}{T_0} = \dfrac{dy}{dx}$, where x and y are the coordinates of the point P.

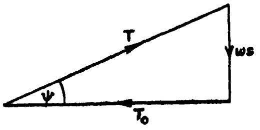

FIG. 41.

But $$\left(\frac{ds}{dx}\right)^2 = 1+\left(\frac{dy}{dx}\right)^2 = 1+\left(\frac{ws}{T_0}\right)^2.$$

$$\therefore \quad dx = \frac{ds}{\sqrt{1+\left(\dfrac{ws}{T_0}\right)^2}}.$$

$$\therefore \quad x = \int \frac{ds}{\sqrt{1+\left(\frac{ws}{T_0}\right)^2}} = \frac{T_0}{w}\sinh^{-1}\frac{ws}{T_0} + A.$$

When $x = 0$, $\quad s = 0. \quad \therefore \quad A = 0.$

$$\therefore \quad s = \frac{T_0}{w}\sinh\frac{wx}{T_0}.$$

When $x = \frac{l}{2}$, $\quad s = \frac{L}{2} = \frac{T_0}{w}\sinh\frac{wl}{2T_0}.$

$$\therefore \quad L = \frac{2T_0}{w}\sinh\frac{wl}{2T_0} \tag{1}$$

$$= \frac{2T_0}{w}\left\{\frac{wl}{2T_0} + \left(\frac{wl}{2T_0}\right)^3\frac{1}{\underline{|3}} + \ldots\right\}$$

$$\doteqdot l\left(1 + \frac{w^2l^2}{24T_0^2}\right). \tag{2}$$

Now $\quad dy = \frac{ws}{T_0}\,dx = \sinh\frac{wx}{T_0}\,dx.$

$$\therefore \quad y = \int \sinh\frac{wx}{T_0}\,dx$$

$$= \frac{T_0}{w}\cosh\frac{wx}{T_0} + B.$$

If $x = 0$, $\quad y = 0. \quad \therefore \quad B = -\frac{T_0}{w}.$

$$\therefore \quad y = \frac{T_0}{w}\left(\cosh\frac{wx}{T_0} - 1\right).$$

This is the equation of a catenary.

$$\therefore \quad y = \frac{T_0}{w}\left\{1 + \left(\frac{wx}{T_0}\right)^2\frac{1}{\underline{|2}} + \ldots - 1\right\}$$

$$\doteqdot \frac{wx^2}{2T_0}. \tag{3}$$

Putting $x = \frac{1}{2}l$, we have maximum sag or dip

$$D = \frac{wl^2}{8T_0}. \tag{4}$$

The dip at any point P distant x from O is

$$D-y = \frac{w}{8T_0}(l^2-4x^2)$$
$$= D\left(1-\frac{4x^2}{l^2}\right).$$

Combining equations (2) and (4), we may write

$$L = l+\frac{8D^2}{3l}. \qquad (5)$$

Again $$T = \sqrt{T_0^2+(ws)^2} = \sqrt{T_0^2\left(1+\sinh^2\frac{wx}{T_0}\right)}$$
$$= T_0\cosh\frac{wx}{T_0}. \qquad (6)$$

When $x = \frac{1}{2}l$, the tension at B is

$$T_B = T_0\cosh\frac{wl}{2T_0},$$

which from (3) gives

$$T_B = T_0+wD. \qquad (7)$$

Line covered with Ice and subjected to Wind.

The line must be designed to withstand the stress which may occur when it is covered with ice, and subjected to wind, representing the most severe condition.

The ice covering has the effect of increasing the dead weight per foot of the line; the weight of ice is 57 lb. per cu. ft.

Let d be the conductor diameter and i the radial thickness of the ice, both in inches (Fig. 42).

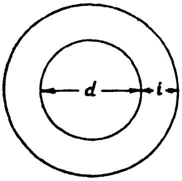

FIG. 42.

Area of ice section $= \frac{1}{4}\pi\{(d+2i)^2-d^2\}$
$= \pi i(d+i)$ sq. in.

Hence weight of ice per foot run

$$w_i = \frac{\pi i(d+i)}{144}\times 57 \text{ lb.}$$
$$= 1{\cdot}244i(d+i) \text{ lb.} \qquad (8)$$

The effect of wind is usually allowed for by assuming the wind to blow horizontally across the line and to exert a pressure of 8 lb. per sq. ft. on the projected surface of the ice-covered line. This corresponds to a wind velocity of 50 miles per hour.

Hence the pressure exerted per foot of line is given by

$$P = \tfrac{8}{12}(d+2i) = \tfrac{2}{3}(d+2i) \text{ lb.}$$

The resultant force W on the line is the vector sum of $(w+w_i)$ acting vertically downwards and P acting horizontally (Fig. 43).

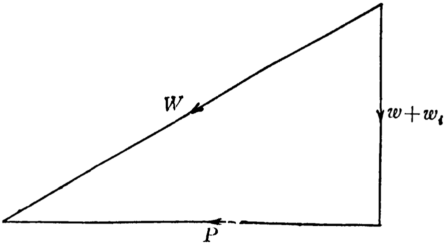

FIG. 43.

Then the total pressure per foot of the line is given by

$$W = \sqrt{P^2+(w+w_i)^2}. \tag{9}$$

The vertical component of the sag due to this load is $\frac{w+w_i}{W}D$, where D is the sag obtained from eqn. (4) if w is replaced by W.

Calculation of Erection Sag.

According to the regulations of the Electricity Commissioners, overhead lines must be designed so that at a temperature of 22° F. with the line covered with ice to a radial thickness of $\frac{3}{8}$ in. and subjected to a wind of 50 miles per hour at right angles to the line, the factor of safety is 2: i.e. under these conditions the line tension is half the breaking load.

The line must be erected so that at the conditions then prevailing, usually a higher temperature and no ice, the above regulation is complied with.

Let w = the weight in lb. per foot of conductor,

W = the resultant pressure in lb. per foot of line when covered with ice and subjected to wind,

L_c = the line length in feet under these conditions,

T_c = the tension of the line when cold in lb., being half the breaking load,

L = the line length in feet during erection, assuming no ice and still air,

T = the tension at erection,

A = the cross-sectional area of the line in sq. in.,

E = the modulus of elasticity in lb./in.^{2},

α = the expansion coefficient per degree F.,

t = the erection temperature in excess of 22° F.

Then $L_c = l\left(1+\frac{W^2l^2}{24T_c^2}\right)$ from (2), assuming the tension to be constant throughout the line.

The line will expand when the temperature rises t° F. by an amount $L_c\,\alpha t$.

The line will also contract elastically when the ice and wind loads are removed by an amount

$$\frac{L_c}{AE}(T_c-T).$$

Hence the line length at erection is given by

$$L = L_c\left(1+\alpha t-\frac{T_c-T}{AE}\right)$$

$$= l\left(1+\frac{W^2l^2}{24T_c^2}\right)\left(1+\alpha t-\frac{T_c-T}{AE}\right) = l\left(1+\frac{w^2l^2}{24T^2}\right).$$

If we ignore terms involving the product of small quantities, this may be reduced to the form

$$T^3+T^2\left\{AE\left(\alpha t+\frac{W^2l^2}{24T_c^2}\right)-T_c\right\}-\frac{w^2l^2AE}{24} = 0. \qquad (10)$$

This cubic equation in T may be solved by a graph or by Newton's approximation, when the erection dip $D = \frac{wl^2}{8T}$ can be obtained.

EXAMPLE. *Find the sag in still air at* 122° *F. for a steel core aluminium conductor* 0·77 *in. diameter on a* 900*-foot span. The factor of safety is to be* 2 *when the conductor is loaded with ice to a radial thickness of* $\frac{3}{8}$ *in., and subjected to a simultaneous wind pressure of* 8 *lb. per sq. ft. of projected area at* 22° *F. What would be the corresponding sag on an equivalent copper conductor?*

(*a*) *Steel Core Aluminium.*

Assume the stranding is 30/0·110 aluminium and 7/0·110 steel.

Take $E = 13{\cdot}31 \times 10^6$

$\alpha = 10{\cdot}24 \times 10^{-6}$.

Breaking load = 17,720 lb.

$\therefore\ T_c = 8{,}860$ lb.

$w = 0{\cdot}571$

$A = 0{\cdot}3515$.

Temperature rise $t = 100°$ F.

Diameter with ice loading $= 0{\cdot}77 + 0{\cdot}75 = 1{\cdot}52$.

Wind load per foot run $P = \frac{2}{3}(1{\cdot}52) = 1{\cdot}01$.

Weight of ice per foot $w_i = 1{\cdot}244 \times 0{\cdot}375(1{\cdot}145)$

$= 0{\cdot}535$.

Hence $W = \sqrt{(1{\cdot}01)^2 + (1{\cdot}106)^2} = 1{\cdot}491$.

Substituting in equation (10) this becomes

$$T^3 + 457T^2 - 514{\cdot}7 \times 10^8 = 0.$$

An approximate solution to this is 3600. By Newton's approximation, a more exact solution is obtained by subtracting

$$\frac{T^3 + 457T^2 - 514{\cdot}7 \times 10^8}{3T^2 + 914T}$$

when T is 3600, the denominator of this fraction being the differential coefficient of the numerator.

This gives $T = 3600 - 26 = 3574$ lb.

Hence the erection dip $= \dfrac{0{\cdot}571 \times 81 \times 10^4}{8 \times 3574} = 16{\cdot}17$ ft.

(*b*) *Copper.*

Assume the stranding is 7/0·180.

Take $E = 18 \times 10^6$ lb./in.^{2}

$\alpha = 9{\cdot}22 \times 10^{-6}$

Diameter = 0·540.

Breaking load = 9,920 lb.

$\therefore\ T_c = 4{,}960$ lb.

$w = 0{\cdot}698$ lb.

$A = 0{\cdot}178$ sq. in.

Diameter with ice loading = 1·29 in.

$$P = \tfrac{2}{3}\times 1{\cdot}29 = 0{\cdot}86$$

$$w_i = 0{\cdot}426$$

$$W = \sqrt{0{\cdot}86^2+1{\cdot}125^2} = 1{\cdot}416.$$

Substituting in equation (10) we have

$$T^3+6810T^2-52{\cdot}7\times 10^9 = 0.$$

An approximate solution is 2400.

The correction is $-\dfrac{3{\cdot}44\times 10^8}{0{\cdot}5\times 10^8} = -6{\cdot}8.$

Hence $T = 2393$ lb.

$$\therefore\quad D = \frac{0{\cdot}698\times 81\times 10^4}{19140} = 29{\cdot}54 \text{ ft.}$$

These calculations show the advantage of steel cored aluminium over copper from the point of view of sag reduction.

Line supported at Different Levels.

Consider a line *AOP* (Fig. 44) freely supported at two points *A* and *P*, differing in level by *h* ft, and spaced *l* ft. apart. Let

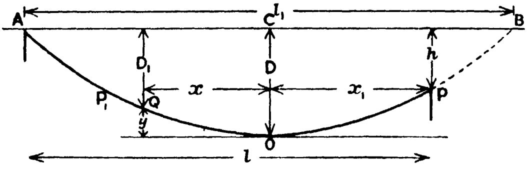

FIG. 44.

AOB be the complete catenary of which *AOP* forms a part and let $AB = l_1$.

Now *D* is the sag of *AOB* and $D-h$ is the sag of *P'OP*. Also $x_1 = l-\frac{1}{2}l_1$ so that $P'P = 2l-l_1$.

$$\therefore\quad \frac{wl_1^2}{8T}-h = \frac{w(2l-l_1)^2}{8T},$$

from which

$$l_1 = l+\frac{2Th}{wl}. \tag{11}$$

Then the lowest point sags below the higher support by

$$D = \frac{wl_1^2}{8T}. \qquad (12)$$

At any point Q distant x from O, the dip D_1 below the higher support is given by

$$D_1 = D - y, \qquad \text{where } y = \frac{wx^2}{2T},$$

$$= D\left(1 - \frac{4x^2}{l_1^2}\right). \qquad (13)$$

Now $\quad AC = l_1/2.$

Hence $\quad x_1 = l - \frac{l_1}{2} = \frac{l}{2} - \frac{Th}{wl}. \qquad (14)$

If this is negative, then the lowest point of the imaginary catenary lies outside the actual span (Fig. 45).

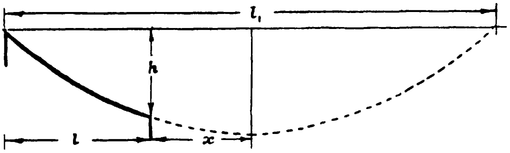

FIG. 45.

Spans of Unequal Length.

When a line is constructed of spans of unequal length, each span should theoretically be tensioned to suit its own length: with suspension insulators this is not possible, since the insulator strings would swing so as to equalize the tension in each span. In practice, therefore, the calculations are based on the length of a hypothetical equivalent span, and the tension so obtained is that adopted for each span of the section.

Let l_e be the length of the equivalent span,
n the number of spans,
l_1, l_2, l_3, etc., the lengths of the actual spans.

Then since the length of the actual line must be the same as that of the equivalent line we have

$$n\left(l_e + \frac{w^2 l_e^3}{24T_0^2}\right) = \sum l + \frac{w^2 \sum l^3}{24T_0^2}.$$

But $nl_e = \sum l$,

$$\therefore \quad nl_e^3 = \sum l^3,$$

$$l_e^2 = \frac{\sum l^3}{nl_e} = \frac{\sum l^3}{\sum l},$$

i.e.

$$l_e = \sqrt{\frac{l_1^3+l_2^3+l_3^3+\dots}{l_1+l_2+l_3+\dots}}.$$

The line tension is estimated using this equivalent span length, and the sag for each actual span is then calculated from

$$D = \frac{wl^2}{8T}.$$

Line Erection.

In practical erection the line is first paid out from drums by a steel running line attached to the end of the conductor. The hauling off is effected in steps, being stopped as the conductor end reaches each tower, when the conductor itself is raised and laid over snatch-blocks hung from the cross-arm on a level with the ends of the insulator strings. At the point selected for pulling up, the conductor is carried down over the last snatch-block to blocks and tackle and a dynamometer. By means of man-power or mechanical power, according to the size of the line, the latter is pulled up to a 'killing tension' somewhat in excess of the correct tension: this is held for about five minutes, when the conductor is slackened off until the tension reaches the correct value for the temperature prevailing at the time. The sag is checked either by fixing a telescopic level to the tower at the correct height, when the lowest point of the conductor should be sighted through it, or by fixing a horizontal batten to each of two adjacent towers so that the line of sight joining them is tangential to the lowest point of the conductor. The line is then permanently anchored off and transferred from the snatch-blocks to the insulator clamps.

The height from the ground of any conductor at any part of the span at 122° F. must not be less than the following:

20 ft. for voltages up to 66,000
21 ft. ,, ,, ,, 110,000
22 ft. ,, ,, ,, 165,000.

This rule governs the height of the towers employed.

EXAMPLES ON CHAPTER VII

Q. 1. Calculate the sag in an 80-yard span of a 0·025 sq. in. overhead line when covered with ice to a radial thickness of $\frac{3}{8}$ in. in a wind which exerts a pressure of 8 lb. per sq. ft. Take the breaking load as 1,520 lb. and weight of conductor per foot 0·1 lb. Allow a safety factor of 2.

Q. 2. Find the sag in still air at 122° F. for a steel core aluminium conductor 0·77 in. diameter, and also for the equivalent copper conductor, on a 900-ft. span. Allow a safety factor of 2 when the conductor is loaded with ice to a thickness of 0·5 in. and subjected to a wind pressure of 8 lb. per sq. ft. at 22° F.

Q. 3. Find the sag in a 100-yard span of a 0·075 sq. in. overhead line when covered with $\frac{3}{8}$ in. of ice in a wind pressure of 8 lb./ft.^{2} Take the breaking load as 4,250 lb., weight of conductor per foot 0·3 lb., and safety factor 2. Find also the sags and tensions in still air with no ice for temperatures of 22° F. and 122° F.

Q. 4. Estimate the sag of a 19/14 copper conductor having a diameter of 0·4 in., an area of 0·0976 sq. in., and weighing 0·384 lb. per foot in a 500-ft. span:

(*a*) at datum temperature with an ice covering of 0·5 in. and wind pressure of 8 lb. per sq. ft.
(*b*) in still air at datum temperature with ice loading.
(*c*) in still air at datum without ice.
(*d*) in still air at 100° F. above datum.

The conductor stress is not to exceed 25,000 lb./in.^{2} Modulus of Elasticity 18×10^6 lb./in.^{2}

CHAPTER VIII

OVERHEAD LINE INSULATORS

Materials.

MODERN overhead line insulators are made principally of porcelain or steatite. The basis of porcelain is china clay, a naturally occurring aluminium silicate, which is mixed by firing at a high temperature with exact proportions of plastic kaolin, quartz, and felspar. The most important supplies of these materials are found in Cornwall and Dorset. The insulators are carefully finished with a hard, smooth glaze; this allows the surface to be washed clean by rain. The tensile strength of porcelain is about 2·5 to 3·5 tons per sq. in., and the compressive strength from 15 to 18 tons per sq. in.

Steatite is a magnesium silicate which is found native in many parts of the world. The relative proportions of magnesium oxide and silica vary considerably. The only combination suitable for electrical insulators is that obtained from the mines in Bavaria. Steatite has very much higher tensile and bending strengths than porcelain, and is therefore suitable for cases where the ceramic part is subjected to high tensile or bending stresses.

Pin-Type Insulators.

In this type the insulator is screwed on and firmly attached by hemp or other means to a steel bolt or pin secured to the cross-arm of the supporting tower. The conductor is supported in a groove at the top or side of the insulator to which it is bound by annealed wire of the same material as the conductor.

An insulator may fail electrically by a puncture through the material or by a flash-over across its surface. The former is prevented by providing adequate thickness of porcelain between the end of the pin and the conductor. Since a failure is therefore invariably due to a flash-over across the surface, the insulator must be shaped to secure a uniform potential gradient along this surface. This is achieved by making the body of the insulator conform to the lines of the electrostatic field.

The flash-over voltage of an insulator when wet is always lower than when it is dry, since a wet surface is practically conducting. Insulators are therefore provided with one or more rain sheds, so that when the outer surfaces are wet with rain the inner surfaces remain sensibly dry. In order that these rain sheds shall not alter the voltage distribution, they are shaped as far as possible perpendicular to the lines of electrostatic force, so that they lie in planes of equal potential (Fig. 46). A single-part triple-shed pin-type insulator, hemp-fixed to its pin, is shown in Fig. 47. The hemp fixing is effected by winding

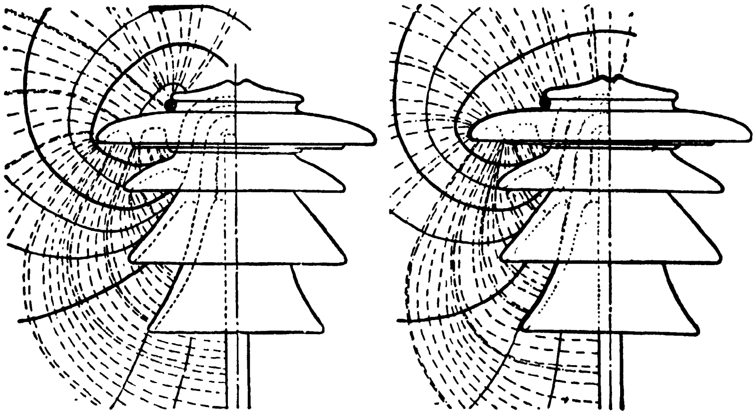

FIG. 46. Diagrams of the lines of electrostatic force surrounding a pin-type insulator, ascertained by the straw fibre method.

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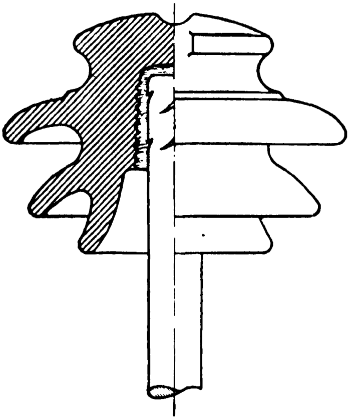

FIG. 47. Fixing the insulator to the pin by means of hemp.

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around the end of the pin a long skein of hemp, making a packing which is smoothed over and saturated with oil or red lead: after an elastic washer of felt or asbestos has been placed in the pin-hole, the insulator is screwed on to the pin. The fixing may also be effected by a lead head cast on the end of the pin, or by a sheet-steel or zinc thimble fitted into the pin-hole of the insulator.

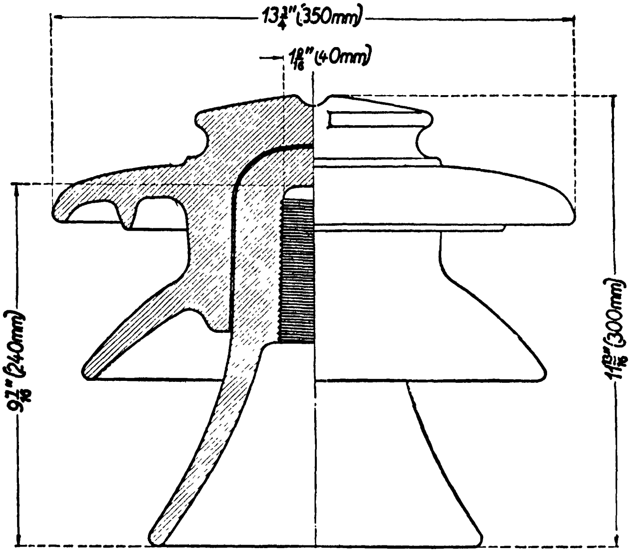

Fig. 48. 55 kV. Multi-part pin insulator. Dry flash-over, 158 kV. Wet flash-over, 140 kV.

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For operating voltages greater than 33 kV., the thickness of a single-part insulator becomes excessive and leads to faulty manufacture. Hence a multipart construction is adopted in which the insulator is made of two or more parts which are fired individually and joined together by cement or preferably by an elastic layer of hemp. Fig. 48 shows a two-part insulator for use on a line voltage of 55 kV. The dry flash-over voltage is 158,000 and the wet flash-over 140,000.

The mechanical characteristics of modern pin-type insulators are such that the limit of mechanical stress which may be placed on the insulator is determined only by the bending strength of the pin.

Since the dimensions and weights of high-tension pin insulators increase fairly rapidly with the working voltage, they are not recommended for pressures above 55 kV. Above this value suspension insulators are used and are frequently adopted for pressures as low as 33 kV.

Suspension Insulators.

In this type of insulator the conductor is held in a clamp which is suspended from the cross-arm of the tower by a string of insulator units connected to each other by metal links or pins. The advantages of this arrangement are:

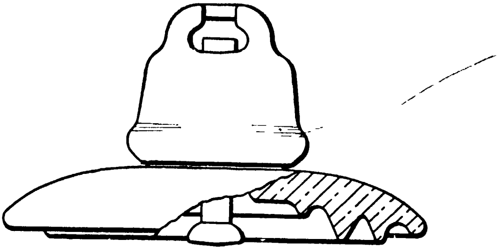

Fig. 49. Cap-and-pin disk insulator.

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(1) Since each unit can be designed for about 11 kV., the line can be insulated for any operating voltage by providing a sufficient number of units in the string, instead of increasing the size of the unit as with the pin-type insulator. This results in greater economy and reliability in manufacture.

(2) Increased flexibility is given to the line, the insulator string being free to swing in any direction: the tensions in successive spans are thus equalized.

The conductor spacing must be slightly increased with the suspension-type insulator to allow for the possible swinging of the line towards the support.

In one form of cap-and-pin type suspension insulator unit, Fig. 49, the head of the porcelain disk is surmounted by a galvanized cast-iron cap to which it is cemented. A galvanized forged-steel pin is cemented in a hollow cylindrical cavity in the insulator head: the enlarged end of this pin fits into a socket in the cap of the next lower insulator forming a ball-and-socket connexion.

In the spring-ring insulator, Fig. 50, a spiral spring ring of steel wire of high tensile strength carried on the stem of the pin is forced through the neck of the hollow cylindrical space in the head of the insulator and allowed to expand into the wider cavity, the shape of which is specially designed to conform in curvature with that of the spring itself. The spring is retained in position by means of a second spring and clip. In order to protect it from the weather, to prevent any possibility of relative movement of the components, and to secure a better distribution of the mechanical and electrical stresses over the insulator, the interspace is filled with a lead alloy. The substitution of this alloy for cement avoids the cracking of the insulator which frequently occurs owing to the expansion of the cement. It is impossible without actually breaking away the whole of the porcelain for the pin to come out of the insulator and allow the conductor to fall. The ball-and-socket joint is used to connect the respective units.

FIG. 50. Spring-ring insulator. Dry flash-over, 90 kV.
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In the Hewlett suspension insulator no cement is employed in the insulator assembly, each unit consisting of a single piece of porcelain: two lead-covered steel U-links are threaded through the insulator, each lying in one of two curved channels at right angles to each other. In this way the stress in the porcelain is uniformly distributed and is compressive; if the porcelain breaks, the line is still supported by the steel links.

Voltage Distribution over a String of Suspension Insulators.

A suspension insulator consists of a string of alternate insulator units and metal fittings by which the units are connected. This constitutes, therefore, a number of condensers in series, the capacitance of each condenser being that between successive metal connectors with the porcelain as dielectric. There are, in addition, air capacitances from each connector to the supporting cross-arm or tower, i.e. to earth, and also to the line. The latter are usually negligible, but the former are not, and cause a non-uniform distribution of potential over the string.

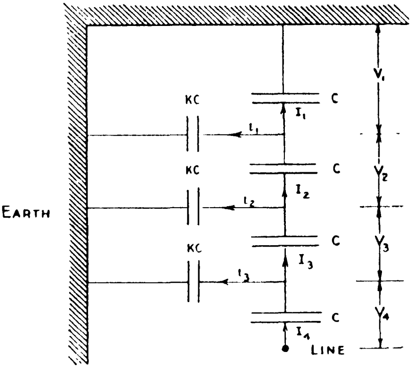

FIG. 51.

Thus, consider a four-unit suspension insulator, Fig. 51, and let C represent the self-capacitance of each unit and kC the capacitance of each metal connector to earth. Then, if V_1 is the voltage across the top unit, we have

$$I_1 = V_1\omega C,$$

$$i_1 = kV_1\omega C,$$

$$I_2 = I_1+i_1 = (1+k)V_1\omega C,$$

$$V_2 = I_2/\omega C = (1+k)V_1,$$

$$i_2 = k\omega C(V_1+V_2) = k(2+k)V_1\omega C,$$

$$I_3 = (1+3k+k^2)V_1\omega C,$$

$$V_3 = (1+3k+k^2)V_1,$$

$$i_3 = (V_1+V_2+V_3)k\omega C = k(3+4k+k^2)V_1\omega C,$$

$$I_4 = (1+6k+5k^2+k^3)V_1\omega C,$$

$$V_4 = (1+6k+5k^2+k^3)V_1.$$

Hence, if $k = 0{\cdot}1$, we have

$$\begin{aligned}
\text{Voltage across top unit} &= V_1 &&= 0{\cdot}198E,\\
\text{,, ,, second ,,} &= 1{\cdot}1V_1 &&= 0{\cdot}217E,\\
\text{,, ,, third ,,} &= 1{\cdot}31V_1 &&= 0{\cdot}259E,\\
\text{,, ,, bottom ,,} &= 1{\cdot}651V_1 &&= 0{\cdot}326E.\\
\text{Total voltage}\quad E &= 5{\cdot}061V_1.
\end{aligned}$$

It will be seen, therefore, that in this case the earth-end insulator takes only one-fifth of the total voltage, while the line-end insulator takes nearly one-third.

The insulator string will therefore flash over when the voltage across the line-end insulator reaches its flash-over voltage, i.e. $0{\cdot}326\,E$ in the case of a four-unit string. The line voltage will then be E and not $4\times 0{\cdot}326\,E$ as would have been expected if the potential had been uniformly distributed across the string. The string efficiency of the insulator has been defined as

$$\frac{\text{flash-over voltage of } n \text{ units}}{n\times\text{flash-over voltage of one unit}}. \qquad (1)$$

In the above example the string efficiency is

$$\frac{E}{4\times 0{\cdot}326E} = 0{\cdot}767 = 76{\cdot}7 \text{ per cent.}$$

This efficiency decreases as k increases and as the number of units in the string increases.

Grading Rings.

The potential distribution along the string can be improved by the use of a grading or guard ring, which consists of a large metal ring surrounding the line insulator and connected electrically to the line. This introduces capacitances between the metal connectors and the line, which can be made to balance the earth capacitances.

Thus, consider a suspension insulator of n units, and let c represent the capacitances of the links to earth, assumed to be all equal (Fig. 52). Let C_1, C_2,..., C_{n-1} be the capacitances of the links to the guard ring which is at line-potential nV. If each unit has to take the same voltage V, the charging current I through each must be the same.

Hence
$$i_1 = I_1$$
$$i_2 = I_2$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot$$
$$i_{n-1} = I_{n-1}.$$

If $i_1 = I_1$, $\quad Vωc = (n-1)VωC_1. \quad \therefore \quad C_1 = \dfrac{c}{n-1}.$

If $i_2 = I_2$, $\quad 2Vωc = (n-2)VωC_2. \quad \therefore \quad C_2 = \dfrac{2c}{n-2}.$

If $i_{n-1} = I_{n-1}$, $(n-1)Vωc = VωC_{n-1}. \quad \therefore \quad C_{n-1} = (n-1)c.$

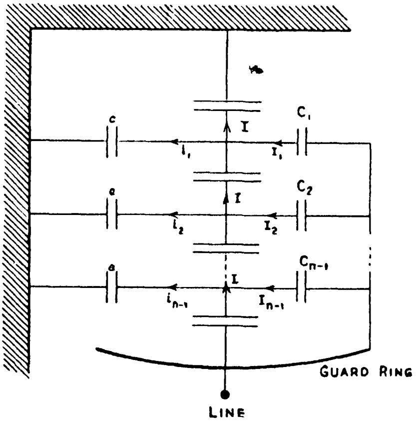

FIG. 52.

In general, the capacitance from the ring to the mth link from the top is given by

$$C_m = \frac{mc}{n-m}. \tag{2}$$

If these ideal values can be obtained, the potential distribution will be uniform and the string efficiency will be unity.

The original purpose of guard rings in equalizing the voltage distribution is now secondary to the main function of protecting the conductor and insulators from damage by power arcs. At the top or cross-arm end of the string is fitted an arcing horn. The distance from the horn to the guard ring forms the shortest flash-over distance. A power arc due to a sudden over-voltage will therefore strike from horn to ring and will be kept away from the insulators.

A transient over-voltage or surge, due to a switching or other disturbance within the system, or to lightning, may cause a flash-over: the danger to the insulation lies not so much in this transient voltage, which is usually of high frequency, as in the power arc which follows the short-circuit path so formed. An insulator may withstand a transient flash-over, but cannot withstand the intense heat of a prolonged power arc over its surface.

Typical suspension sets are shown in Figs. 53 and 54. In the latter, which employs double W-horns, the arc forms on the lowest point of a horn where there is a large radius and is carried to the top by convection currents of hot ionized air, and is automatically broken almost instantaneously by bowing out, without the assistance of wind. The provision of four points to the horn ensures that the wind, whatever its direction, will not blow the arc on to the insulators.

Testing of Insulators.

The following basic tests are usually carried out:

(1) *Dry spark-over voltage test.* The dry and clean insulator is mounted in a manner approximating as closely as possible to practical conditions. A power-frequency voltage is applied and gradually increased until flash-over occurs.

(2) *Wet spark-over voltage test.* This is usually carried out with a spray of water of resistance 20,000 ohms per cm. cube at 15° C., falling at an angle of 45° with the vertical at the rate of 1 in. in five minutes.

(3) *Puncture test.* The insulator is immersed in oil to eliminate surface spark-over, and the voltage is applied gradually until puncture occurs.

(4) *Impulse and/or high-frequency tests.* The most important source of over-voltage is lightning, the voltages induced being of an impulsive nature, although by reflection from terminal apparatus oscillatory voltages may also be built up. An impulse is characterized by its very rapid increase of voltage to the maximum value, followed by a less rapid decrease to zero. The voltage may rise from zero to maximum in one-millionth of a second, and is said to have a steep wave front. Higher spark-over and puncture figures are usually obtained with impulsive voltages than with those of power frequency, due to the

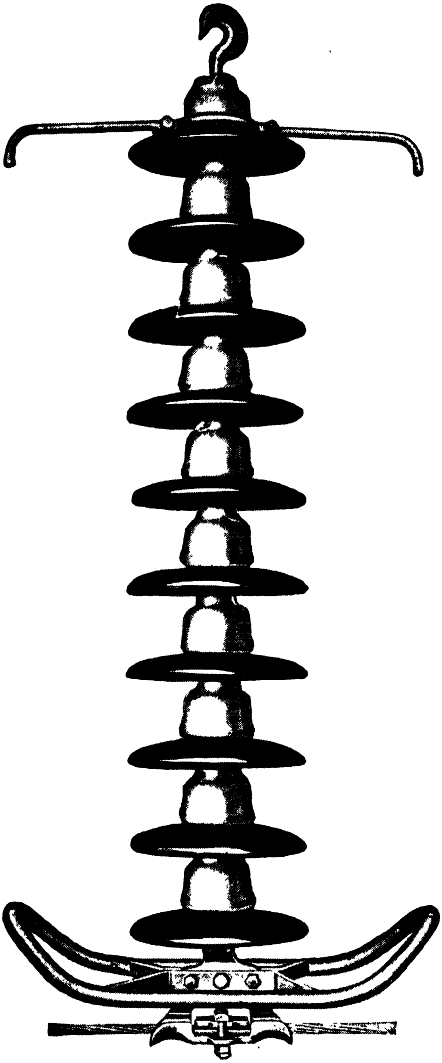

FIG. 53. Typical suspension set.

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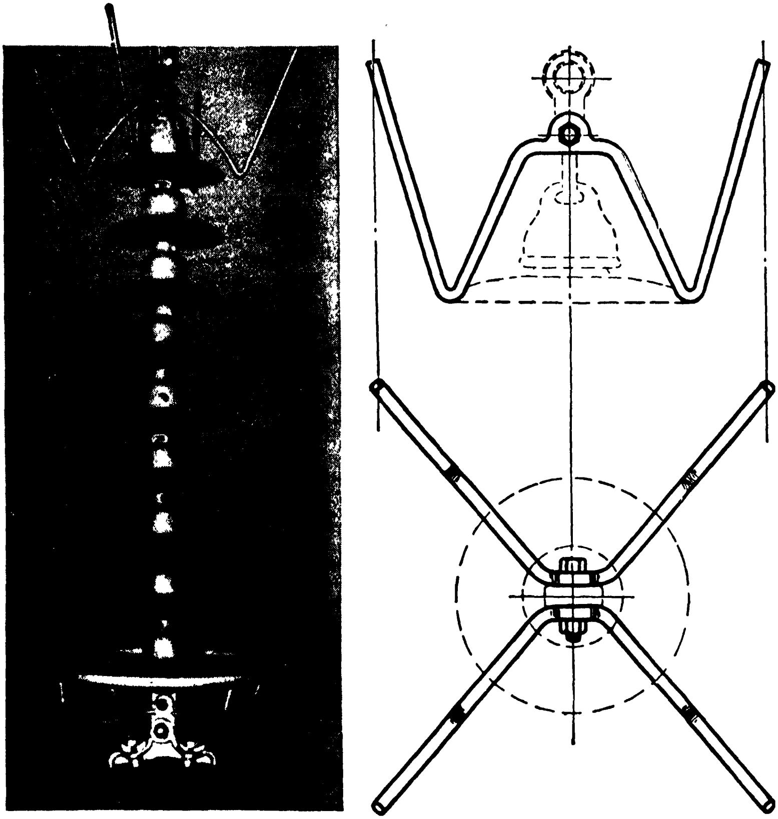

FIG. 54.

(*a*)
A complete suspension string as used on the 132 kV. Grid lines.

(*b*)
W-type arcing horns.

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relatively short time of application of the former. The impulse ratio is the ratio

$$\frac{\text{impulse spark-over voltage}}{\text{power-frequency spark-over voltage}}$$

and its order of magnitude is about 1·3 to 1·5 for pin-type insulators and 1·2 to 1·4 for suspension insulators. Special equipment for the generation of impulsive and high-frequency voltages is used for this test.

(5) *Fog or mist test.* Insulators in special air-tight rooms are

tested for flash-over under conditions of heavy fog, mist, salt spray, artificial fouling by smoke, chemicals, cement dust, coal dust, etc. Special designs of insulators, provided with very long creepage surfaces, have been designed for operations in positions exposed to such fouling.

(6) *Mechanical strength test.* This consists of a tensile test on suspension and straining insulators, and a bending or cantilever test on pin-type insulators.

(7) *Temperature cycle test.* The insulator is subjected to sudden changes of temperature by transferring it from a hot- to a cold-water bath and vice versa. The time of immersion should be sufficient to ensure that the insulator attains a sensibly uniform temperature throughout its mass.

(8) *Porosity test.* The insulator is immersed in some liquid for a given period, after which the increase in weight is measured, or the insulator is immersed in a solution of fuchsin or eosin for a given period, and then fractured to observe the depth to which the dye has penetrated the material.

Other tests which may be applied are electro-mechanical tests involving simultaneous electrical and mechanical stresses, vibration and fatigue tests, shock tests, corona tests, and voltage-distribution tests.

EXAMPLES ON CHAPTER VIII

Q. 1. Explain what is meant by the string efficiency of a suspension insulator consisting of a number of units. Describe one method of adjusting the voltage distribution over such a string. Find the voltage distribution and the string efficiency of a three-unit suspension insulator if the capacitance of the link pins to earth and to the line are respectively 20 per cent. and 10 per cent. of the self-capacitance of each unit. L.U.

Q. 2. A 30 kV. overhead line is suspended by means of three-unit suspension type insulators. The capacitance of each link pin to earth is $0{\cdot}2C$, where C is the self-capacitance of each unit.

(*a*) Determine the potential distribution over the string.

(*b*) If a guard ring introduces capacitances from the line of $0{\cdot}4C$ and $0{\cdot}1C$ to the bottom and second link pin respectively, find the new distribution.

CHAPTER IX

UNDERGROUND CABLES

Current-carrying Capacity.

It has been seen that although the size of a cable is determined by the permissible volt-drop on a low-pressure system and by economic considerations on a high-pressure system, in each case the cross-section so obtained is subject to the safe current-carrying capacity: the latter forms the sole criterion for extra high voltage cables, since on account of their high cost they must be loaded up to the safe heating limit.

The safe current-carrying capacity is determined by the maximum permissible temperature rise, and this in turn is dependent on the losses in the cable, which manifest themselves as heat. The losses taking place are:

(1) copper losses in the conductors,
(2) hysteresis losses in the dielectric, and
(3) eddy current losses in the sheath.

The safe working conductor temperature is 65° C. for armoured cables and 50° C. for lead-sheathed cables in ducts, these values permitting conductor rises of 50° C. and 35° C. respectively above earth temperature which is assumed to be 15° C. The maximum steady temperature will be obtained when the heat generated in the cable is equal to the heat dissipated by it. The heat generated in the conductor is dissipated by conduction through the insulation to the sheath, from which the total losses, including dielectric and sheath losses, are conducted to the earth. The rate of dissipation therefore entails a knowledge of the thermal conductivity of the insulation and of the soil. The thermal conductivity of a body is defined as the quantity of heat which passes per second between the opposite faces of a centimetre cube of the material, when these faces are maintained at a temperature difference of 1° C. The reciprocal of this is the thermal resistivity. In electrical units the quantity of heat per second is measured in watts, so that the conductance is measured in watts/°C. per cm., and the resistance in °C./watt per cm.

THERMAL RESISTANCE OF CABLE DIELECTRIC

Single-core Cable.

Let r be the radius of the core in cm.,
r_1 the inside radius of the sheath,
k the thermal resistivity of the insulation.

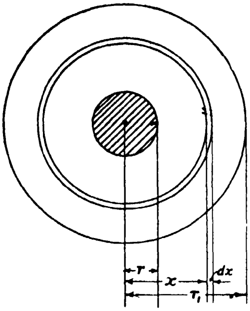

FIG. 55. Single-core cable.

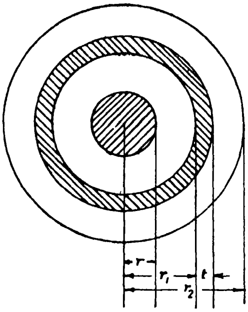

FIG. 56. Concentric cable.

Consider 1 cm. length of cable. Then the thermal resistance of the element radius x, thickness dx (Fig. 55) is

$$dS_1 = \frac{k\,dx}{2\pi x}. \tag{1}$$

The thermal resistance of the cable dielectric is then

$$S_1 = \int_r^{r_1} \frac{k\,dx}{2\pi x}$$

$$= \frac{k}{2\pi} \operatorname{logh} \frac{r_1}{r} \text{ °C./watt per cm.} \tag{2}$$

Concentric Cable.

Let r_1 be the inner radius of the outer conductor (Fig. 56),
t the strand diameter or thickness of the outer conductor,
r_2 the inner radius of the sheath.

Assuming that both conductors have the same heat loss, half the heat due to conductor loss flows through the inner layer of insulation, while the total heat flows across the outer layer.

Hence the thermal resistance is

$$S_1 = \frac{k}{2\pi}\left[\tfrac{1}{2} \operatorname{logh} \frac{r_1}{r} + \operatorname{logh} \frac{r_2}{r_1 + t}\right].$$

It t is negligible compared with r_1, then

$$S_1 = \frac{k}{2\pi}\left[\operatorname{logh}\sqrt{\frac{r_1}{r}}.\frac{r_2}{r_1}\right],$$

$$= \frac{k}{2\pi}\operatorname{logh}\frac{r_2}{\sqrt{rr_1}}. \qquad (3)$$

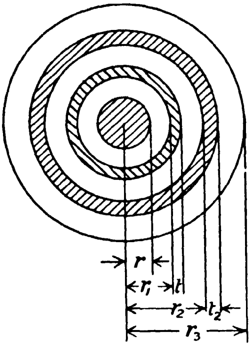

FIG. 57. Triple concentric cable.

For a triple concentric cable, Fig. 57, let

r_2 be the inner radius of the outside conductor,
t_2 be the strand diameter or thickness of the outside conductor,
r_3 be the inner radius of the sheath.

Then $$S_1 = \frac{k}{2\pi}\left[\tfrac{1}{3}\operatorname{logh}\frac{r_1}{r} + \tfrac{2}{3}\operatorname{logh}\frac{r_2}{r_1+t} + \operatorname{logh}\frac{r_3}{r_2+t_2}\right].$$

If the thicknesses are negligible we have

$$S_1 = \frac{k}{2\pi}\operatorname{logh}\frac{r_3}{\sqrt[3]{rr_1r_2}}. \qquad (4)$$

Three-core Cable.

For the belted type of three-core cable the following formula due to Russell gives results which are correct to within approximately 10 per cent.

$$S_1 = \frac{k}{6\pi}\operatorname{logh}\frac{R^6-a^6}{3R^3a^2r}\ \text{°C./watt per cm.,} \qquad (5)$$

where R is the sheath radius,
r is the core radius,
a is the radius of the pitch axis through the conductor centres.

No simple formula has been developed for the 'H' and 'SL' types of cable (see p. 95), since the problem is further complicated by the fact that the heat can travel along the metal layer as an additional outlet. An approximate solution has been given by Beavis.*

The value of k is usually taken as 750 for cables up to and including 2,200 volts, and 550 for cables above 2,200 volts.

Thermal Resistance of Protective Covering.

In armoured cables the lead is protected from the armour by a bedding which may consist of hessian tape or jute strands. If S_2 is the thermal resistance of this protective covering, then

$$S_2 = \frac{k_1}{2\pi} \operatorname{logh} \frac{R_2}{R_1}, \tag{6}$$

where R_1 = external radius of lead sheath,
R_2 = external radius of outer covering of the cable.

The value for k_1 may be taken as 300.

Thermal Resistance of the Soil.

The thermal resistance of the soil cannot be calculated accurately, since the thermal resistivity depends upon the type of soil and the amount of moisture present. Moreover, the nature of the heat flow through the soil is somewhat obscure. Kennelly assumes that the surface of the ground above the cable is a plane isothermal, and thus calculates the thermal resistance of the soil to be

$$G = \frac{g}{2\pi} \operatorname{logh} \frac{2h}{R_2},$$

where g = thermal resistivity of the soil,
h = depth of the cable axis below ground level,
R_2 = overall radius of the cable.

An empirical correction factor of 2/3 is usually introduced to

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* 'Thermal resistance of E.H.T. cables,' *Electrical Times*, 6 Aug. 1931, p. 201.

make the expression correspond more nearly to practical conditions of buried cables.

Hence $$G = \frac{g}{3\pi} \operatorname{logh} \frac{2h}{R_2}. \tag{7}$$

The values of g for various soils are given in the following table, the figures being due to Melsom and Fawcett.

Thermal Resistivity.	*Percentage Moisture Content.*		
g	*Sandy Loam.*	*Crushed Chalk.*	*Heavy Clay.*
100	13	18	..
150	7	13	..
200	4	9	14
250	2·3	7·2	8·5
300	1	3·5	4
350	..	1·6	1

The resistances of the thermal circuit add up like electrical resistances in series, so that the total thermal resistance $S = S_1 + S_2 + G$. The thermal and electrical circuits are in fact analogous, the P.D. corresponding to the temperature difference and the current corresponding to the thermal flux in watts. It will be noted that the thermal resistance of the lead sheath has been ignored, since this is so small as to be negligible.

Permissible Current Loading.

Writing down the thermal law corresponding to Ohm's law for the electrical circuit we have

$$\text{watts dissipated} = \frac{\text{temperature rise}}{\text{thermal resistance}}.$$

Neglecting the dielectric and sheath losses we have

$$nI^2R = \frac{T}{S}$$

or $$I = \sqrt{\frac{T}{nRS}}, \tag{8}$$

where I = permissible current per conductor,

n = number of conductors,

R = effective resistance per cm. of conductor at the working temperature.

Where sheath losses are important, the formula is modified as follows:

Let W be the sheath losses, the heat corresponding to which has to flow through the covering and the soil.

Then
$$(nI^2R)S+W(S_2+G) = T,$$

or
$$I = \sqrt{\frac{T-W(S_2+G)}{nRS}}. \qquad (9)$$

Similarly, if the dielectric losses w are not negligible we have

$$I = \sqrt{\frac{T-w\left(\frac{S_1}{2}+S_2+G\right)}{nRS}} \qquad (10)$$

If both losses are important, then

$$I = \sqrt{\frac{T-w\frac{S_1}{2}-(W+w)(S_2+G)}{nRS}}. \qquad (11)$$

It should be noted that when cables are laid in proximity to each other, the permissible current is reduced further on account of mutual heating.

Example. *Find the permissible current loading of a* 0·5 *sq. in. paper-insulated concentric cable, single wire armoured and served, and laid direct to a depth of* 18 *in. Take the thermal resistivity of the soil as* 120 *and allow a temperature rise of* 50° *C.*

The dimensions of this cable are as follows, which represent diameters over

core	0·93 in.	$= 2r$
first dielectric	1·13 ,,	$= 2r_1$
outer conductor	1·45 ,,	$= 2(r_1+t)$
second dielectric	1·65 ,,	$= 2r_2$
lead sheath	1·89 ,,	$= 2R_1$
finished cable	2·73 ,,	$= 2R_2$.

Then
$$S_1 = \frac{750}{2\pi}\left[\tfrac{1}{2}\,\text{logh}\,\frac{1\cdot 13}{0\cdot 93}+\text{logh}\,\frac{1\cdot 65}{1\cdot 45}\right]$$
$$= 27\cdot 05.$$

$$S_2 = \frac{300}{2\pi}\operatorname{logh}\frac{2{\cdot}73}{1{\cdot}89}$$
$$= 17{\cdot}56.$$
$$G = \frac{120}{3\pi}\operatorname{logh}\frac{36}{1{\cdot}36}$$
$$= 41{\cdot}71.$$
$$\therefore \quad S = S_1+S_2+G = 86{\cdot}32.$$

Now resistance per 1,000 yards at 60° F. (15·6° C.) = 0·04193 ohm.

Resistance per cm. at 60° F. (15·6° C.) $= 0{\cdot}5374\times 10^{-6}$.

Resistance per cm. at 65° C. $= 0{\cdot}5374\times 10^{-6}(1+0{\cdot}004\times 49{\cdot}4)$

$$= 0{\cdot}6439\times 10^{-6}.$$

Hence permissible current loading

$$I = \sqrt{\frac{50\times 10^6}{2\times 0{\cdot}6439\times 86{\cdot}32}}$$
$$= 671 \text{ amp.}$$

Dielectric Loss.

When an alternating voltage V is applied to a cable, a displacement or charging current I flows, leading the voltage by

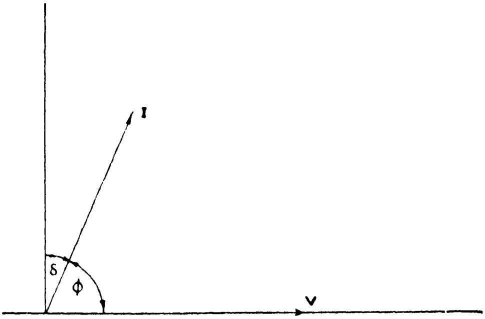

FIG. 58. Charging current of a cable.

nearly 90°. The actual angle of lead ϕ is $(\frac{1}{2}\pi-\delta)$, where δ is called the dielectric loss angle (Fig. 58). There is thus a power loss in the dielectric

$$W = VI\cos\phi = VI\sin\delta$$
$$\doteqdot VI\delta \doteqdot VI\tan\delta$$

when δ is small.

This loss is due to dielectric hysteresis and appears to represent the energy consumed in reversing the stresses in the dielectric.

Now $$I = V\omega C,$$

and hence $$W = V^2\omega C \cos\phi. \tag{12}$$

The dielectric losses are therefore proportional to the square of the applied voltage and become important for operating pressures of over 22 kV.

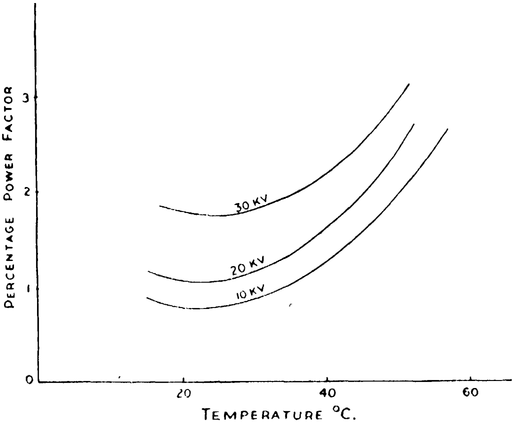

FIG. 59. Power factor curves for a paper cable.

The power factor of a cable dielectric varies with temperature, decreasing up to about 40° C., after which it increases again. Typical curves for an impregnated paper cable are shown in Fig. 59, the curves being generally known as dielectric 'V' curves.

The power factor of modern high-voltage cables is quite small, varying from a minimum of about 0·3 per cent. at 40° C. to 2 or 3 per cent. at 60° C.

Dielectric Strength.

The dielectric strength of an insulating layer is its resistance to puncture or disruptive breakdown, and is usually expressed in kV. per cm. thickness. It is a function of the duration of the stress, the voltage V to produce breakdown varying with the time of application T approximately according to the law

$$V = \frac{a}{\sqrt[n]{T}},$$

where a and n are constants for a given dielectric. For a 33 kV. cable, a and n are approximately 240 and 10 respectively. A typical time-voltage curve is shown in Fig. 60.

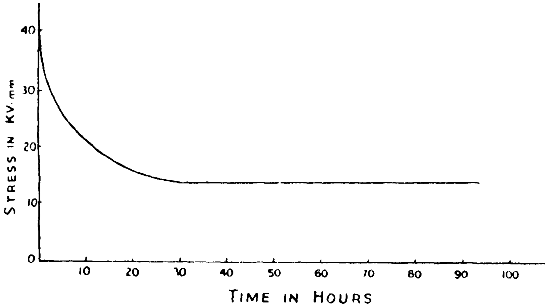

FIG. 60. Typical time-voltage curve for a 50 kV. single-core cable.

The dielectric strength of impregnated paper in a direction parallel to the surface is only about 10 per cent. of that in a normal direction.

Dielectric Stress in a Single-core Cable.

Consider 1 cm. length of cable (Fig. 61) and let the conductor be given a charge of q electrostatic units.

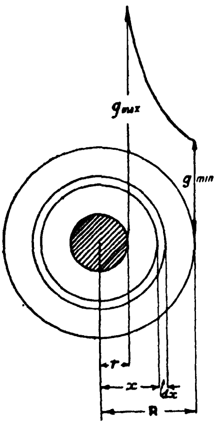

FIG. 61.

The total electrostatic flux emanating radially from this charge is $4\pi q$.

The flux density over an element of radius x is

$$\frac{4\pi q}{2\pi x} = \frac{2q}{x}.$$

If the permittivity of the dielectric is ϵ, the electrostatic stress or field intensity is

$$\frac{2q}{\epsilon x}.$$

This is also the potential gradient g at this radius, so that

$$g = \frac{2q}{\epsilon x}.$$

Now the voltage across the annulus of thickness dx is $g\,dx$, and hence the total voltage between core and sheath is

$$E = \int_r^R g\,dx = \int_r^R \frac{2q}{\epsilon x}\,dx$$

$$= \frac{2q}{\epsilon}\Big[\operatorname{logh} x\Big]_r^R = \frac{2q}{\epsilon}\operatorname{logh}\frac{R}{r}$$

$$= gx \operatorname{logh}\frac{R}{r},$$

or

$$g = \frac{E}{x \operatorname{logh}\dfrac{R}{r}} \text{ volts/cm.} \tag{13}$$

This is clearly a maximum when $x = r$, and hence

$$g_{\text{max}} = \frac{E}{r \operatorname{logh}\dfrac{R}{r}}. \tag{14}$$

Similarly,

$$g_{\text{min}} = \frac{E}{R \operatorname{logh}\dfrac{R}{r}}.$$

It will be seen that the maximum potential gradient occurs at the conductor surface. Normal working values for g_{max} range from 40 to 50 kV. per cm. for paper-insulated cables.

Economical Core Diameter.

For a given voltage E and a given overall diameter $2R$ of cable the maximum potential gradient will be a minimum when $r \operatorname{logh}\frac{R}{r}$ is a maximum, r being regarded as the variable.

This may be shown to occur when

$$\operatorname{logh}\frac{R}{r} = 1$$

or

$$r = \frac{R}{2{\cdot}718},$$

when the conductor radius

$$r = \frac{E}{g_{\text{max}}}. \tag{15}$$

In high-voltage cables this radius of copper conductor will in general involve a cross-sectional area too great for the required current-carrying capacity. Hence, in practice, an aluminium conductor might be used, or the copper may be stranded in tubular form around a 'dummy' of jute or hemp.

The preceding theory has been worked out on the assumption that the conductor surface is smooth. The comparatively small radius of the individual wires will give rise to a concentration of stress greater than that indicated by the above formula. Hence in practical high-voltage cables the core is frequently enclosed by a thin lead, or other metal, sheath to provide a smooth cylindrical surface.

GRADING OF CABLES

(i) *Capacity Grading.*

In a cable with a homogeneous dielectric the permissible voltage between cable and sheath is governed by the maximum potential gradient at the conductor surface, the remainder of the insulation being considerably understressed. A more economical conductor would result if the potential gradient could be maintained constant throughout the insulation. This may be achieved by grading the insulation so that the permittivity of any layer is inversely proportional to its radius, i.e. $\epsilon \propto 1/x$, when ϵx and $2q/\epsilon x$ would be constant.

Consider the above economical cable having a core radius r, sheath radius $R = er$, and applied voltage E. The maximum potential gradient $= E/r$.

Assume the insulation to be replaced by a graded dielectric in which the potential gradient is maintained at this maximum value.

Then permissible voltage between cable and sheath

$$= \frac{E}{r}(R-r)$$

$$= E\left(\frac{R}{r}-1\right) = E(e-1)$$

$$= 1{\cdot}718\,E.$$

Hence with ideal grading the voltage can be increased 72 per cent.: this assumes that each layer has the same dielectric strength. In practice the manufacturing costs limit the number of layers to two or three, but even so the heterogeneous nature of the dielectric is not satisfactory.

(ii) *Grading by Intersheaths.*

In this method the potential gradient is controlled by providing the cable with one or more metallic intersheaths, the potentials of which are 'anchored' by connecting these sheaths to tappings on the secondary of the power transformer.

Consider a cable designed for a voltage E, and let it be provided with one intersheath of radius r_1 having potentials of E_1 and E_2 to the conductor and outer sheath (earth) respectively (Fig. 62).

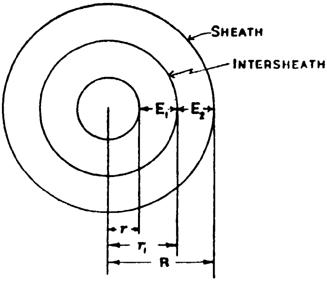

FIG. 62.

Then $$E = E_1+E_2.$$

We have from (15), $$r = \frac{E_1}{g}$$

and $$r_1 = re = \frac{E_1 e}{g},$$

where g is the permissible potential gradient.

Also $$g = \frac{E_2}{r_1 \,\text{logh}\,\dfrac{R}{r_1}} \text{ from (13).}$$

$$\therefore\quad \text{logh}\,\frac{R}{r_1} = \frac{E_2}{gr_1} = \frac{E_2}{E_1 e},$$

i.e. $$R = r_1 e^{\frac{E_2}{E_1 e}} = \frac{E_1 e}{g} e^{\frac{1}{e}\left(\frac{E}{E_1}-1\right)}.$$

For a minimum overall diameter, $\dfrac{dR}{dE_1} = 0$,

i.e. $$\frac{d}{dE_1}\left(E_1 e^{\frac{E}{eE_1}}\right) = 0 \text{ since } e \text{ and } g \text{ are constants.}$$

This gives
$$E_1 = \frac{E}{e}, \tag{16}$$
$$r = \frac{E}{eg}, \tag{17}$$
$$r_1 = \frac{E}{g}, \tag{18}$$
and
$$R = r_1 e^{\frac{E_2}{E}}. \tag{19}$$

EXAMPLE. *Estimate the minimum overall diameter of a single-core metal-sheathed* 66 *kV. cable provided with one metallic intersheath: the permissible potential gradient of the dielectric is* 40 *kV. per cm. Indicate also the voltage at which the intersheath must be maintained.*

Now conductor radius $r = \frac{66}{40 \times 2{\cdot}718} = 0{\cdot}607$ cm.

Intersheath radius $r_1 = \frac{66}{40} = 1{\cdot}65$ cm.

$$E_1 = \frac{66}{2{\cdot}718} = 24{\cdot}3 \text{ kV.}$$

Hence voltage at which intersheath must be maintained is
$$E_2 = 66 - 24{\cdot}3 = 41{\cdot}7 \text{ kV.}$$

Overall radius $R = 1{\cdot}65\, e^{\frac{41\cdot7}{66}} = 3{\cdot}105$ cm.

$\therefore$ overall diameter = 6·21 cm.

Comparing this with the economical conductor without an intersheath, we have

conductor radius $r = \frac{66}{40} = 1{\cdot}65$ cm.

$$R = 2{\cdot}718 \times 1{\cdot}65$$
$$= 4{\cdot}485.$$

$\therefore$ overall diameter $= 9$ cm.,

which is 44 per cent. greater than that for the intersheath cable.

Ionization.

The limitations in regard to maximum stress and operating temperature of cables with normal impregnation are due to their imperfect homogeneity and particularly to the enclosure of gas-filled and vacuous spaces in the insulation. These spaces may occur between the wrappings of the paper or may be formed during the process of manufacture by loosely-wrapped paper or improper filling with the impregnating compound. They may also be produced by the expansion of the impregnating material due to heat during its operation, causing vacuous spaces to be formed in the neighbourhood of the conductor. Such a gas or vacuous space is liable to ionization, partly because, owing to its low permittivity, it tends to take a relatively high difference of potential, and partly because its dielectric strength is only about 10 per cent. of that of the solid insulation. If the ionization is strong enough, chemical effects follow, which may lead to the gradual destruction of the dielectric at that point.

The design of modern extra high tension cables is largely influenced by the necessity of minimizing or preventing such ionization effects. The 'H' type cable, the oil-filled cable, and the pressure cable are examples of practical attempts to solve this problem.

TYPES OF CABLE

A twin cable contains two cores not arranged concentrically. A three-core cable contains three cores not arranged concentrically. A concentric cable has two separate conductors arranged concentrically and insulated from each other. A triple concentric cable has three concentric conductors.

A twin concentric cable has three separate conductors, two of which are arranged as in a twin cable, enclosed by the third conductor which is of annular form. A lead-covered cable is provided with a sheath of lead to exclude moisture. An armoured cable is provided with one or two layers of steel wires or tape over the lead for mechanical protection. A bedding of hessian tape or jute strands protects the sheath from the armour, while the same materials are used as an overall serving.

(i) *Belted Cable.*

In the 'belted' type of three-core cable, each conductor is insulated with a number of oil-impregnated paper wrappings: an outer layer of paper, called the belt, surrounds the three insulated cores, and the interstices between them are filled with a fibrous insulating material. Fig. 63 shows such a cable, each conductor consisting in this case of a number of wires of different diameters selected to give a sector formation to avoid undue waste space in the cable. Fig. 64 shows diagrammatically the lines of electrostatic force in such a cable (with circular cores) at the instant when the voltage of conductor *A* is zero. It will

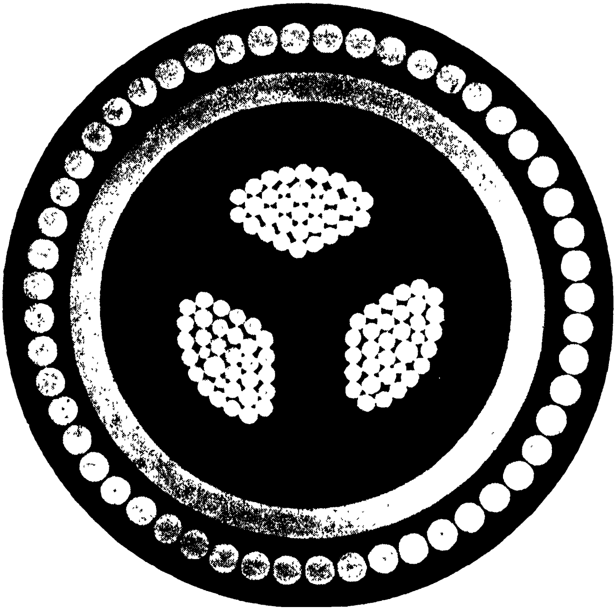

FIG. 63. Belted type cable.
Messrs. Siemens Bros. & Co., Ltd.

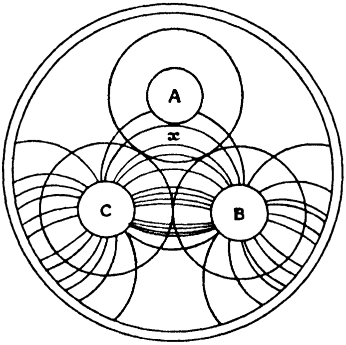

FIG. 64. Electrostatic field in a belted cable.

be noticed that the paper insulation at x is subjected to a pure tangential stress, which may cause breakdown at high voltages, since the dielectric strength of paper to tangential stresses is only about 10 per cent. of that to radial stresses. Further, it will be noticed that considerable electrical stresses exist in the fillers: the latter, however, may contain air pockets or voids formed during manufacture or after being heated up on load. This is due to the fact that the relatively non-elastic lead sheath remains distended after cooling. In the case of cables working at pressures of 22 kV. and over, this may lead to ionization and gradual breakdown of the insulation, particularly during prolonged off-load periods when the contraction of the oil and the cores has left vacuous spaces in the fillers.

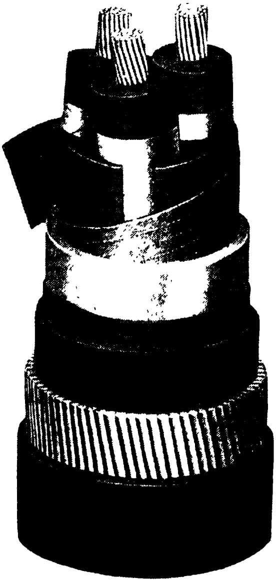

FIG. 65. 'H' type 33,000 volt three-core paper-insulated cable: lead covered, served, single wire armoured and served and compounded overall.

Messrs. W. T. Henley's Telegraph Works Co., Ltd.

(ii) *'H' type Cable.*

The 'H' type cable (Fig. 65), due to M. Hochstadter, has no belt insulation. Each conductor is insulated to the desired thickness in the ordinary way, and over this insulation is wound a layer of perforated metallized paper having the same coefficient of expansion and contraction as the dielectric. All these sheaths are in contact with each other and also with an outer binder of cotton interwoven with fine copper wires. The cable is impregnated (this process is facilitated by the perforations in the metal foil), and finally it is lead-sheathed. The lead sheath, the binder, and the metallized foils are all at earth potential, so that the electrical stresses are confined to the insulation between the conductors and the foils, the fillers being outside the electric field: moreover, these stresses are purely radial (Fig. 66). The insulation can be wound on the conductor so that air pockets are less likely to occur either during manufacture or when on load than is the case with the belted cable.

Additional advantages are that the metallized foil increases the rate of heat dissipation, and that there are no sheath losses.

Such cables may be employed for pressures up to 66 kV. and are capable of carrying 14 per cent. more current than the equivalent belted cable.

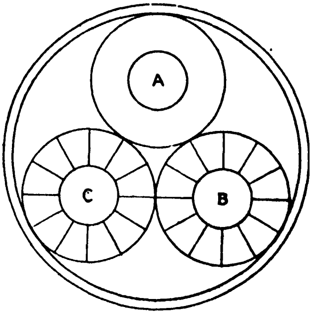

FIG. 66. Electrostatic field in an 'H' type cable.

(iii) *'SL' Cable.*

In the 'SL' (separate lead) type (Fig. 67), the individual cores are first impregnated and lead covered, after which they are laid up and armoured, there being no lead sheath over the assembled cores. This type of cable gives similar results to those of the 'H' type.

(iv) *'HSL' Cable.*

The 'HSL' type of cable (Fig. 68) is a combination of the 'H' and 'SL' types: each conductor is insulated, sheathed with metallized paper tape, and then lead-sheathed. The three cores are then laid up with fillers, served and armoured overall. This construction results in a simpler impregnation process than with the 'H' type, and the three small lead sheaths are more satisfactory mechanically than one large sheath.

(v) *The Oil-filled Cable.*

In the Pirelli oil-filled cable (Fig. 69) the conductors are stranded around a hollow cylindrical spiral of plain, narrow metal strip: this strip is made of copper in the case of the smaller cables and of steel in the larger cables to give greater mechanical strength. The core is paper-insulated, lead-sheathed

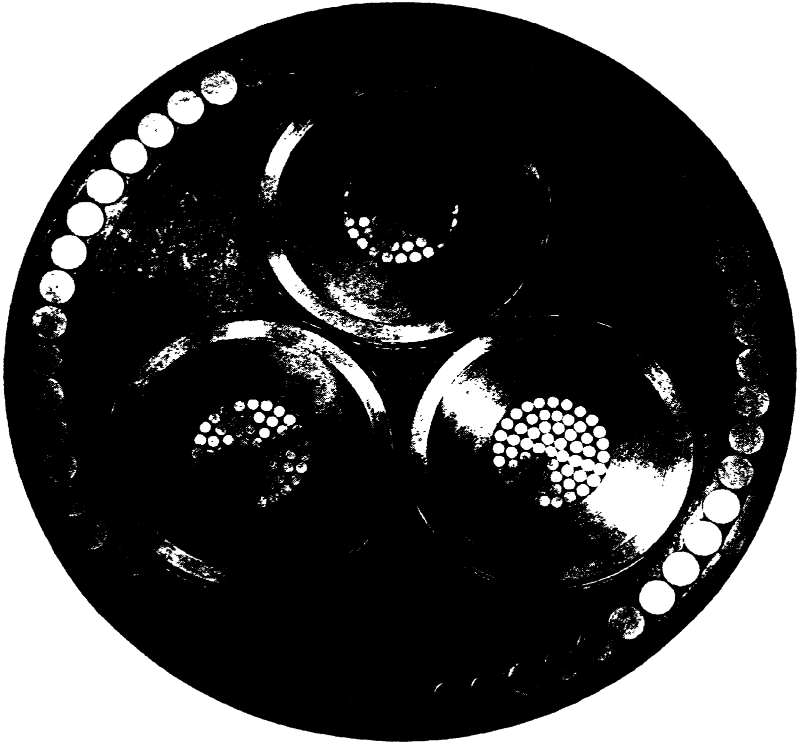

FIG. 67. 'SL' type cable (33 kV. three-core).
Messrs. Siemens Bros. & Co., Ltd.

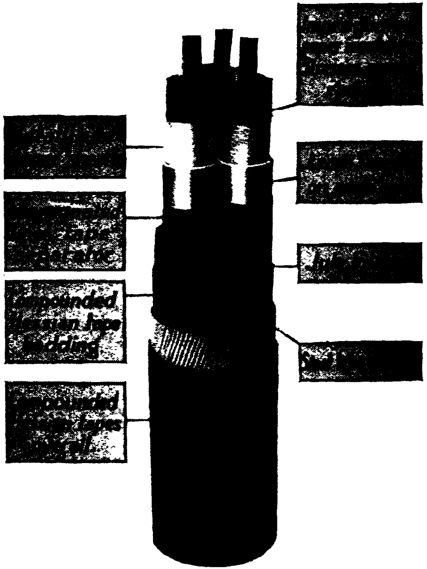

FIG. 68. 'HSL' type cable for super-tension power transmission.
Messrs. Johnson and Phillips, Ltd.

and impregnated, after which the cable is lapped around with impregnated cloth tape and is then brass-tape armoured to prevent distension of the lead. This is insulated with paper and cloth tape, after which a second lead sheath is applied, the cable being served overall with two impregnated papers and two compounded hessian tapes. The central cavity is filled with thin oil at the factory, and the cable is dispatched in lengths wound on drums which are provided with tanks filled with oil under pressure; this ensures the maintenance of good impregnation during transit or storage.

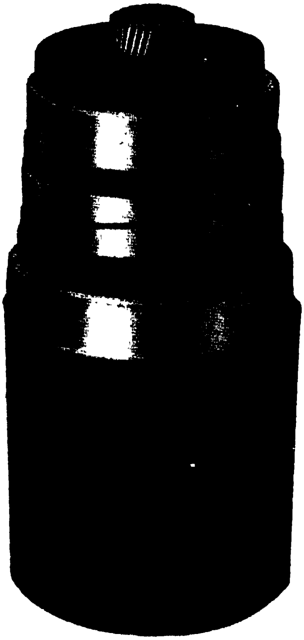

FIG. 69*a*. Section of single-core oil-filled cable.
General Electric Co., Ltd.

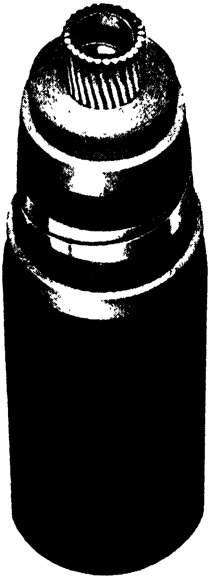

FIG. 69*b*. 66 kV. single-core oil-filled cable.
Messrs. W. T. Henley's Telegraph Works Co., Ltd.

FIG. 69*c*. Henley 33,000 volt three-core oil-filled cable: Pirelli system.
W. T. Henley's Telegraph Works Co., Ltd.

The cable installation is divided hydrostatically into a number of sections by installing at each end of a cable section a stop joint, which cuts off oil communication in the central cavity from that in adjacent sections. Each section is connected to an oil reservoir, which feeds oil to the cable during cooling periods and accommodates the expanded oil when the cable heats up on load. The oil in the tank is at atmospheric pressure and care is taken to exclude external air. To maintain the necessary oil pressure each reservoir or feeding tank is situated at a height above ground depending upon the length of the working section: the latter is increased by installing at some distance from the reservoir a pressure tank containing oil under gas pressure. The pressure variations that occur within the cable are thus reduced. A typical lay-out is indicated in Fig. 70. An average length of cable section of the order of 2,000 to 2,500 ft. is the most practical and economical. Alternative constructions of the cable are to use a triple conductor with oil channels in the filler spaces, or a single conductor with oil channels directly under the lead sheath. The permissible voltage gradient is 80 kV. per cm., and the dielectric power factor is about 0·5 per cent. at 15° C.

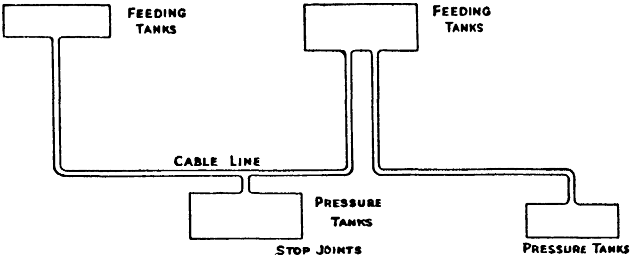

FIG. 70. Typical lay-out of oil-filled cable.

Oil-filled cables are in use in this country on pressures of 132 kV., and a 0·15 sq. in. cable is capable of carrying 300 amp. at this voltage.

*The Pressure Cable.**

This cable, due to M. Hochstadter, W. Vogel, and E. Bowden, is very similar to a normal three-core cable except that the lead sheath has only about 75 per cent. of the usual thickness. It is protected by a thin metal tape to prevent the formation of local excrescences. The cable is installed in a steel pipe about 10 to 20 mm. larger in diameter than itself, and may be drawn in without difficulty in lengths up to 550 yards. The pipe is filled with nitrogen at a pressure of 12 to 15 atmospheres. The cable

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* *Roy. Soc. Arts J.*, 80, pp. 84–125; 11 Dec. 1931.

is thus compressed radially in such a way that radial 'breathing' occurs, which is reversible at all temperatures. Such vacuous spaces as tend to form are closed by the compression, while the pressure in the gas-filled spaces is raised to such an extent that no ionization takes place. The steel pipes are coated with a special paint and protected against chemical action by a covering of impregnated wool felt. Fig. 71 shows an example of a 'triangular' cable installed as a pressure cable. The absence of the bedding and serving causes a considerable reduction of the heat resistance and helps to increase the carrying capacity of the cable. It is claimed that the current loading may be 1·4 to 1·6 times, the voltage twice, and the power transmitted 2·4 to 3·2 times that for a normal cable: it is capable of the same performance as the oil-filled cable without departing from the usual methods of manufacture and impregnation. The dielectric power factor is about 0·6 per cent. at 15° C. at a potential gradient of 100 kV. per cm.

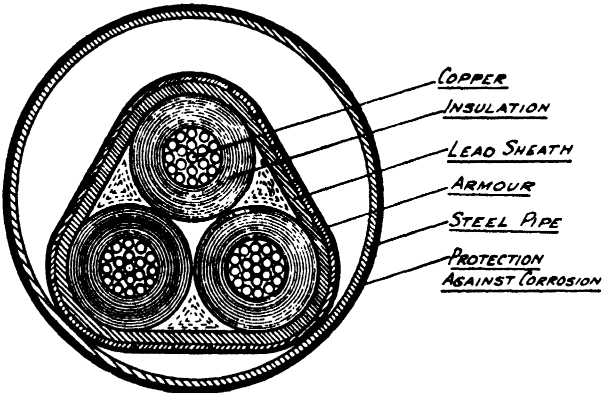

FIG. 71. 'HSO.' cable as pressure cable in pipe line.
By courtesy of the Royal Society of Arts.

While the cost of the steel pressure tube increases the capital expenditure, this is more than compensated for by the higher permissible operating currents and pressures above 30 kV. The additional advantages offered by this cable are that the pressure tube presents an ideal method of laying the cable, while the nitrogen contained therein forms a quenching medium in the case of flames caused by faults.

EXAMPLES ON CHAPTER IX

Q. 1. A concentric cable has a conductor diameter of 1 cm. and an insulation thickness of 1·5 cm. Find the maximum field strength when the cable is subjected to a test pressure of 33 kV. L.U.

Q. 2. The diameter over the insulation of a single-core cable for an 80,000 volt three-phase system is 7·4 cm. Find the diameter over the inner conductor to give a minimum dielectric stress, and find the value of this stress.

Q. 3. Calculate the permissible current loading of a 660 volt 0·1 sq. in. single-core paper-insulated armoured cable buried at a depth of 18 in. Diameter of conductor 0·415 in.: thickness of dielectric 0·08, of sheathing 0·07 and of armouring and serving 0·3 in. Take the thermal resistivities of the dielectric, covering, and soil to be 750, 300, and 120 respectively, and allow a temperature rise of 50° C.

Q. 4. The equivalent star capacitance of a 66 kV. three-core cable is 0·534 microfarad per mile. The dielectric loss at a frequency of 50 is 0·045 watts per cm. Find the power factor of the cable.

Q. 5. A single-core metal-sheathed 66 kV. cable is to be designed for minimum overall diameter by the use of one metallic intersheath. Find the voltage at which the intersheath must be maintained, and the diameters of the conductor, intersheath, and outer sheath if the maximum permissible potential gradient of the dielectric is 60 kV. per cm.

CHAPTER X

FEEDER PROTECTION

Introduction.

If a fault occurs on any section of a distribution or transmission system it is essential that the faulty section should be rapidly isolated automatically from the remainder of the network, in order to limit the damage at the point of fault, and to localize the area of disturbance. This branch of the subject is called protection, and the ideal characteristics of protective gear are that:

(1) It must be sufficiently sensitive to detect the presence of a fault.

(2) It must be absolutely reliable in operation, to which end the design should aim at simplicity and robustness.

(3) It must discriminate with absolute certainty between currents fed to a fault in its section and currents fed through it to a fault in some other section, in order to prevent the isolation of healthy feeders.

(4) It must operate in the shortest possible time.

Protective systems may be divided into two broad classes, viz. that which employs auxiliary or pilot wires and that which does not.

In general, pilot systems are more simple and reliable than pilotless systems, but the cost of the pilots is prohibitive on long-distance transmission.

PILOT SYSTEMS

(1) *Voltage Balance Protection.*

(i) *Mertz-Price System.* This method is based on the fact that the current entering one end of a healthy feeder is equal to that leaving the other end. If a fault occurs on the feeder, this equality is not maintained, and the difference between the two currents is arranged to operate relays and so to isolate the faulty section. The application to a single feeder is shown in Fig. 72. C, C are two identical current transformers whose primaries are connected in series with the line L at its extremities. The secondaries are connected together through relays R by means of the pilot wires P, P, so that under healthy conditions the secondary voltages balance and no current flows through the relays. If a fault occurs on the line, the currents at its extremities are no longer equal and the secondary voltages are unbalanced, causing a current to flow through both relays. Each relay closes its local circuit, energizing the trip-coil *T*, which opens the circuit breakers *B*. In order that the transformers shall balance as regards both voltage and phase angle for all primary currents up to, say, 10,000 amp., it is necessary to prevent saturation of the iron core by providing a number of distributed air-gaps in the iron circuit. This enables a straight line voltage-current characteristic to be obtained.

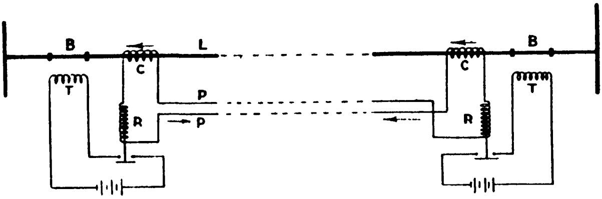

Fig. 72. Mertz-Price system.

In a three-phase line each conductor has a pair of current transformers and relays which are connected by a three-wire pilot, the other ends of the transformers being connected in star.

The pilot circuit usually consists of a three-core 7/0·029 cable generally laid at the same time as the main cable it protects.

The relay employed is usually of the Fawssett-Parry type: this is similar in principle to a moving-iron ammeter. It has rectangular stampings with a rectangular space cut away from the centre, this space being bridged near the centre with fixed and moving irons in such a way as to leave air-gaps; these irons in the centre are surrounded by three concentric coils, one carrying the fault current, and the other two the tripping current for tripping the switch. The moving iron is attached to a pivoted spindle carrying a switch arm, the whole being controlled by a spiral spring. When a fault occurs, the out-of-balance current causes the moving member to be repelled from the fixed iron and attracted into a slightly tapering gap until the switch arm closes the tripping circuit through the outer two coils in series. This puts extra torque on the movement to make a good contact on the switch arm, and when this contact is closed the outer coil is cut out, leaving in just enough turns to keep the tripping circuit closed. By this means a maximum number of switches can be tripped simultaneously by a given battery. The tripping of the main switch breaks the trip circuit which resets the relay.

The advantages of this system are that:

(*a*) No potential transformers are required.

(*b*) The discrimination is ideal.

(*c*) The operation is practically instantaneous.

(*d*) The method is applicable to all kinds of feeders, both overhead and underground.

(*e*) It operates for all types of fault, whether to earth or between phases.

(*f*) The operation is reliable.

The disadvantages are that:

(*a*) Principally, in common with all pilot schemes, the cost of the pilots is heavy.

(*b*) Heavy through-currents may induce in the transformer secondaries pressures of several hundred volts, causing currents to flow through the relays via the capacitances which shunt each pair of pilot wires. This may cause operation of the relays, unless the latter are given a high minimum fault setting, of the order of 100 per cent. full load.

(*c*) Frequent testing of the pilot circuit is necessary, since no warning would be given of a break in the pilots as these normally carry no current.

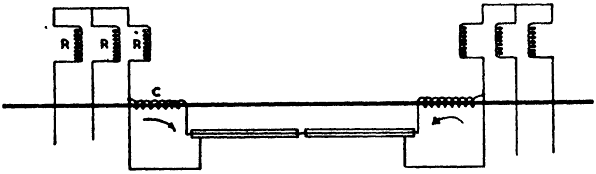

FIG. 73. Beard-Hunter system.

(ii) *Beard-Hunter System.* This employs a special pilot cable, in which each conductor is surrounded by a metallic screen or sheath, which is divided at its mid-point to form two conductors of equal length (Fig. 73).

The pilot capacitance currents now flow in the local circuit formed by the current transformer, the pilot, and its capacitance to the sheath as indicated by the arrow, and do not traverse the relay windings. Hence lower fault settings and a more sensitive relay can be used than in the Mertz-Price system. Its disadvantage is the higher cost of the sheathed pilot, although this is offset by the possibility of embodying telephone wires which would be screened by the sheath.

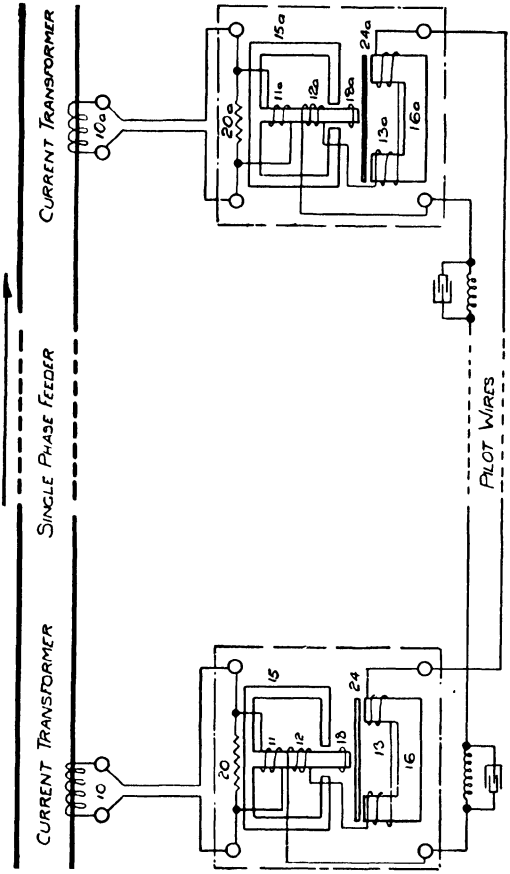

FIG. 74. Simplified scheme of connexions for protection of single-phase feeder by the translay system.

Messrs. Metropolitan Vickers Electrical Co., Ltd.

(iii) *The Translay System.* This method, due to Metropolitan Vickers, takes its name from the use of a relay incorporating a transformer feature, and is indicated in Fig. 74, the tripping circuits being omitted for clearness.

The line current transformer 10 energizes the primary winding 11 of the transformer element 15; this sets up a flux, part of which follows leakage paths through the non-magnetic metal disk 24. In addition a voltage is induced in the secondary winding 12.

As long as the feeder is healthy, the transformers 10 and 10*a* at opposite ends of the feeder carry equal currents, and the coils 11 induce equal e.m.f.'s in the windings 12 and 12*a*. The latter are connected in opposition by means of the pilot wires in series with the operating windings 13 and 13*a*, so that no current flows in this circuit under healthy conditions.

In the event of a fault on the protected feeder, the current transformer at one end of the feeder carries a greater current than that at the other end, the voltages of the windings 12 and 12*a* become unbalanced, and a small current flows round the pilots and the operating windings 13. Each of the latter generates in its electromagnet 16 a flux which is out of phase with that in 15. These two fluxes combine to produce a rotating field, which induces eddy currents in the disk: the latter thus rotates according to Lenz's law in the direction of the rotating field against the braking effect of a permanent magnet. A contact-making device, mounted on the disk spindle and normally held in an inoperative position by a spring, rotates through a small arc and closes the trip circuit of the circuit breaker. The relay automatically resets when the fault is removed.

The pilot capacitance currents normally generate a flux in the magnet 16 approximately in phase with the leakage flux from 15 and hence cause no torque on the disk. To adjust the leakage flux and bring it exactly in phase with the flux from 16, closed copper loops 18 are provided. In this way the relays are prevented from operating with pilot capacitance current. The loops 18 also provide a means of counteracting lack of balance between the line current transformers.

The advantages of this system are that:

(*a*) Ordinary current transformers without air-gaps are used.

(*b*) Pilot capacitance currents are compensated.

(*c*) Robust relays are used as the power available, for their operation is not limited by the pilot current which is only one of two currents causing operation.

(*d*) Inexpensive pilot cables without sheaths are used and the pilot voltage is limited by saturation to 130 volts.

(2) *Circulating Current Protection.*

(i) *Split Pilot System.* The application of this principle to a single-phase system is indicated in Fig. 75. The secondaries of the two distributed-air-gap current transformers C_1, C_2 are connected together through the pilot wires so that their voltages assist instead of oppose as in the voltage-balance systems. X is

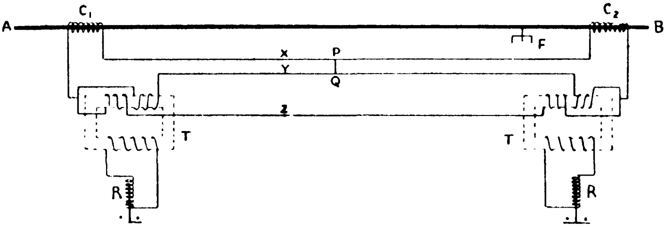

Fig. 75. Single-phase split pilot system.

the common pilot, while Y and Z form the split pilot and are wound differentially on the split pilot transformers T, the secondary windings of which are connected to the relays R. Under healthy conditions equal voltages are induced in the secondaries of transformers C_1, C_2, and the points P and Q are at equal potentials, so that no current flows through the tripping connexion PQ. The current which, however, circulates round the pilots divides equally between the splits Y and Z and has no effect on the transformer T.

In the event of a fault at F fed from A, the induced voltage in C_1 exceeds that in C_2, producing a P.D. between P and Q and causing a current to flow through the tripping connexion. This current disturbs the balance in the split pilots, producing tripping currents in the secondaries of transformers T which operate the relays; these may be of the Fawssett-Parry type. The tripping effects in each relay are always equal.

For three-phase operation the three current transformers at each end of the line are connected in delta, each transformer having a different turns ratio, in such proportions as 4, 5, and 6 (Fig. 76). This enables discrimination for earth and phase faults to be obtained, and subjects the pilot to a minimum resultant voltage for a given straight-through current. Tuning condensers connected between the two split pilots at each end make the transformers insensitive to currents of higher frequency than the third harmonic: the relays are electrically tuned to be responsive only to normal-frequency fault currents. The principal advantage of the system is that capacitance and other harmful currents are rendered self-balancing by the use of the split conductor principle.

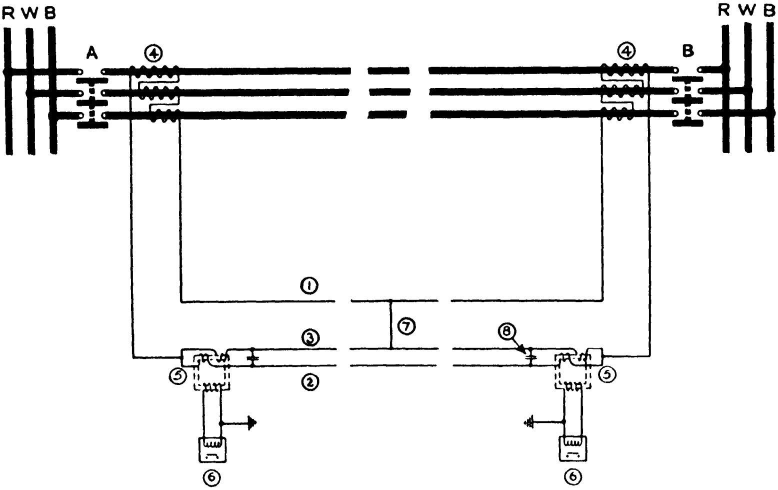

FIG. 76. 'Split pilot' protective system for voltages up to 44,000 volts.

(1) Common pilot no. 1.
(2) Split pilot no. 2.
(3) Split pilot no. 3.
(4) Air-gap transformers connected in discriminating delta.
(5) Split pilot transformer.
(6) Relay.
(7) Mid-point tripping connexion.
(8) Small tuning condenser.

Messrs. A. Reyrolle & Co., Ltd.

(ii) *McColl Protection.* This method is indicated in Fig. 77. The secondaries of the current transformers at each end of the line are connected, through the two pilot wires, in series with each other, and with the restraining coils of two beam relays. The operating coil of each relay is fed from the transformer secondary through a duplicate circuit having the same resistance as one of the pilot wires. The relay is mechanically biased in favour of the restraining coil by giving the latter a greater leverage.

With a healthy feeder each transformer secondary has the same induced voltage and, on account of the symmetrical arrangement of the resistances, equal currents circulate through the duplicate and pilot-wire circuits. The operating and restraining coils therefore balance and the relays remain stable.

In the event of a fault fed from *A*, the transformer secondary voltage at this end will exceed that of the transformer at *B*.

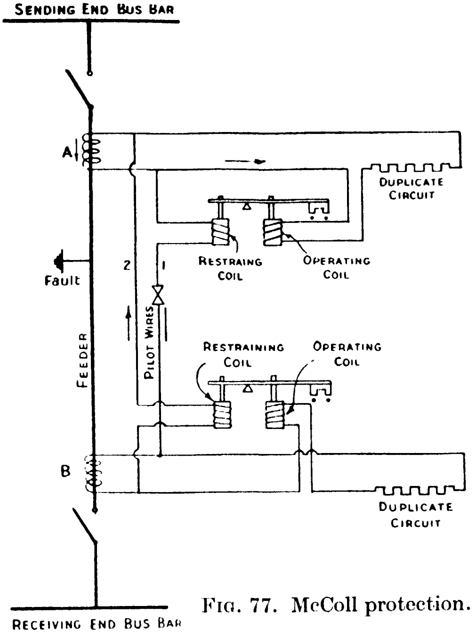

FIG. 77. McColl protection.

The excess voltage will cause superimposed currents to flow in the pilot-wire circuit and the duplicate circuit at *A* in the inverse ratio of their resistances, i.e. in the ratio of 1 to 3. If the fault current exceeds a certain value, the operating coil will overcome the restraining coil and the bias, actuating the relay and tripping the circuit breaker at the end *A*.

The advantages of this system are that:

(*a*) Since the relay operates on percentages, a fault setting down to 5 or 10 per cent. of normal full-load current can be employed without the risk of faulty tripping due to heavy through-currents.

(*b*) Ordinary current transformers can be used.

(*c*) The pilot capacitance currents flow through the restraining coils and actually produce a stabilizing effect.

(*d*) Frequent testing of the pilot circuit is unnecessary, since a break in it would interrupt the normal circulating current and cause operation of the relays.

(3) *D.C. Pilot Protection.*

(i) *Interlock System.* This method is illustrated in its application to a three-phase system in Fig. 78 and makes temporary use of an existing telephone pair as pilots.

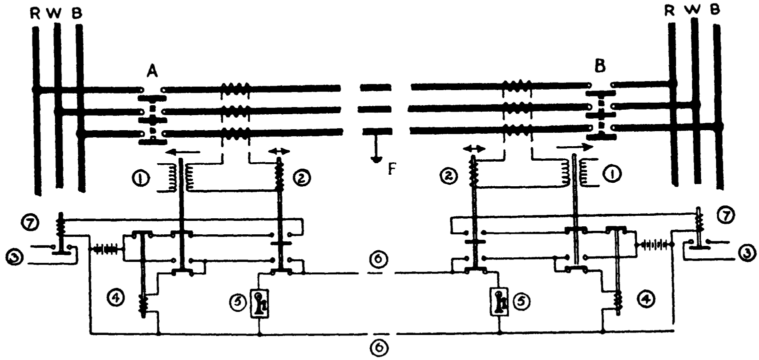

Fig. 78. Diagram illustrating 'interlock' protective system, utilizing an existing pilot circuit.

(1) Three-phase directional stabilizing element.
(2) Three-phase over-current element.
(3) Trip circuit.
(4) Interlock relay.
(5) Telephone.
(6) Pilot.
(7) Tripping element with time lag.

Messrs. A. Reyrolle & Co., Ltd.

On the occurrence of a fault at *F*, assumed to be fed from both ends of the feeder, the over-current elements 2 operate to isolate momentarily the telephone circuit 6, and energize instantaneously the tripping elements 7; these close the circuit-breaker trip circuits 3 after a time lag of 0·3 second, thus isolating the faulty feeder.

On the occurrence of a fault external to the feeder at *B*, the over-current elements instantaneously borrow the telephone circuit as before: the straight-through fault current at the end *B* of the feeder nearer the fault is now in such a direction as to operate the directional stabilizing element 1 at that end. The latter breaks the tripping-element circuit and connects its battery to the pilots at *B*; this causes a D.C. signal to be transmitted over the pilot circuit to operate the interlock relay 4 at *A*, thus interrupting the tripping-element circuit at that station. Thus the trip element at each end is de-energized before it has time to operate, so preventing the isolation of the healthy feeder.

When no pilot wires are available, the signal for stabilizing the interlock system is transmitted by carrier currents superimposed upon the main conductors.

The system is universally applicable and has the following advantages:

(*a*) Its small time hesitation is uniform on all feeders.

(*b*) Its momentary use of pilots either eliminates their cost or reduces it to a mere fraction by reason of the other services for which they may be normally used.

(*c*) Severance of the pilot in the faulty section cannot prevent proper operation of the gear.

(ii) *Duplex Protection.** This system is illustrated in Fig. 79. On the occurrence of a fault fed through station A, the over-current relay X on the faulty phase opens and, if the fault current is fed in the requisite direction, the directional relay Y closes. The over-current relay Z on the faulty phase then closes, energizing the sending relay. The contacts of this relay are brought to the upper position, and the bottom half of the differential relay is energized from the local battery; an alternative path for the current from this local battery is provided through the upper half of the differential relay, in the opposite direction through the pilot wire, and through the upper winding of the differential relay at the remote station. The compensator resistance is adjusted so that the current divides equally in opposite directions through the local differential relay, which is therefore not energized. The current flows in one direction only through the remote differential relay, energizing this and so completing the trip circuit at this station.

If the fault is also fed through station B, a similar sequence of operations occurs at that station; both sending relays being operated, the battery voltages oppose each other in the circuit completed through the upper portions of the differential relays, whilst at each station a local circuit is completed through the lower portion of the differential relay. Each differential relay closes its contacts, both trip circuits are completed simultaneously, and the faulty section is isolated.

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* See 'Faults, and their clearance on large networks'. R. O. Kapp and C. G. Carrothers. *I.E.E.J.* 71, p. 696, November 1932.

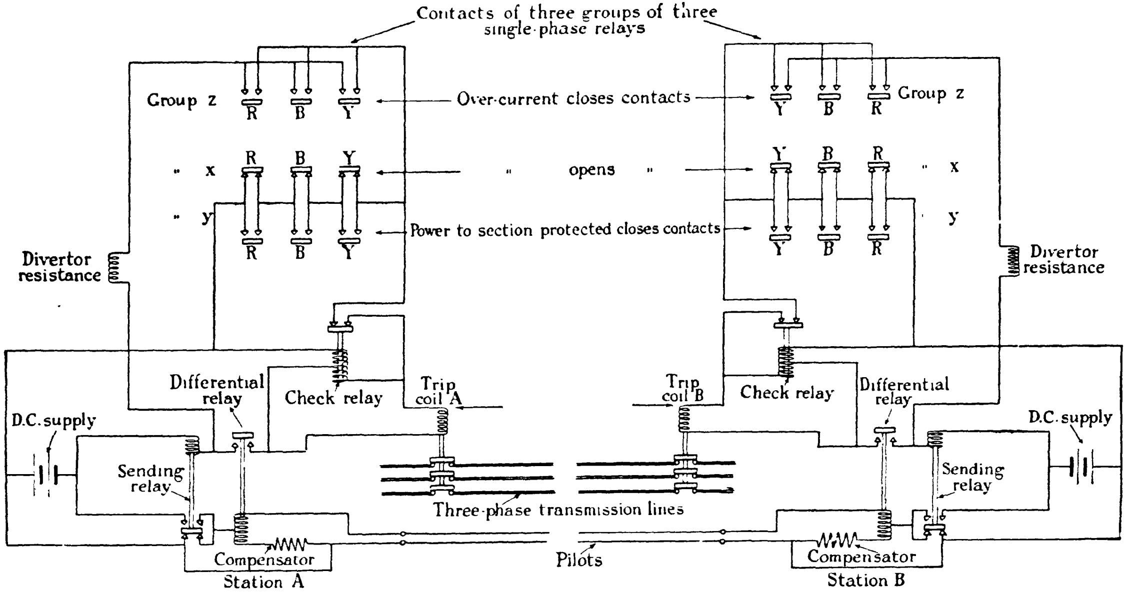

FIG. 79. Duplex protective apparatus for three-phase transmission line.

By courtesy of the Institution of Electrical Engineers.

If one station, say B, is dead ended, i.e. there is no generating plant to feed the fault at this end, the relay contacts X remain closed, but Y and Z remain open at this station. The closing of B's differential relay contacts by the tripping impulse from the remote station energizes its trip coil and sending relay. The former clears station B immediately, while the latter sends a tripping impulse to station A in quick succession.

The function of the check relay is to prevent isolation of a sound section in a ring main.

The system derives its name from the fact that signals may be sent simultaneously in opposite directions over the pilot circuit as in duplex telegraphy.

PILOTLESS SYSTEMS

(1) *Simple Over-current Protection.*

The application of this method to a single three-phase feeder is indicated in Fig. 80. The three current transformer secondaries are connected to three over-current relays R which are joined in star. On the occurrence of an excess current in any conductor, the corresponding relay operates to close the circuit of the trip coil TC which opens the breaker. The system will respond to faults between phases or from phase to earth.

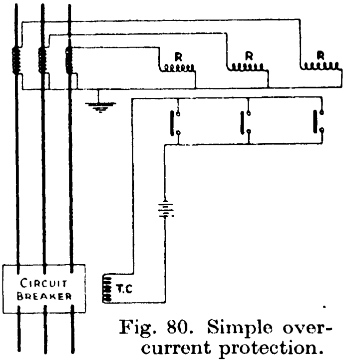

Fig. 80. Simple over-current protection.

The relay employed is of the induction type and has an inverse time characteristic, i.e. its operating time is inversely proportional to the current, so that the greater the overload the quicker the feeder is isolated.

On an unearthed system only two current transformers and two relays are necessary, since at least two phases must be earthed to produce an overload, in which case at least one relay will function when the predetermined magnitude of current is exceeded.

The advantages of this system are that:

(*a*) In common with all pilotless systems it eliminates the cost of the pilot cables and is therefore not expensive.

(*b*) Accurately balanced transformers are not necessary.

The disadvantages are that:

(*a*) The system is not sensitive, since the relays must have an initial setting which is far in excess of normal load current; otherwise they may operate on momentary harmless overloads. The setting is usually about 150 to 200 per cent. of full-load current.

(*b*) It is applicable only to a single radial feeder.

(2) *Graded Time Over-current Protection.*

(i) *Radial Feeder System.* In the protection of radial feeders in series some modification of the preceding method must be used in order to prevent the simultaneous operation of all the relays through which the fault current passes. This consists in the use of discriminating relays which isolate only those sections of the feeder beyond the fault. Such a relay has an inverse time characteristic, but has a definite minimum time of operation which can be adjusted from instantaneous up to 100 cycles (2 seconds). A typical curve is shown in Fig. 81.

The application of the method is shown in Fig. 82, in which the generating station *GS* feeds four sub-stations in series: the sub-stations are protected by relays, the time-settings of which decrease with increasing distance from the main station. Thus a fault on the *SS*4 outgoing feeder would operate relay *e* instantaneously; a fault on the *SS*3-*SS*4 feeder would operate relay *d* in a minimum time of 0·4 second, and so on. The time to clear a fault is the sum of the times occupied in operating the relay, energizing the trip coil, moving the circuit-breaker parts and extinguishing the arc. It is possible to clear a fault in 8 cycles with instantaneous relays. It is to be noted that the operating times indicated in Fig. 82 apply to short-circuit conditions; less severe faults will operate the relays in longer times, which still retain the same approximate relative proportions.

An over-current relay with inverse and definite minimum time characteristics is indicated in Fig. 83. The primary of the upper electromagnet is connected to a current transformer in the line to be protected, and is tapped at intervals so that the setting can be varied; the secondary is connected to the winding of the lower electromagnet. The rotating field produced by the leakage flux from the upper electromagnet and the flux from the lower magnet causes the metallic disk to rotate. The disk spindle carries a moving contact which bridges the trip terminals when the disk is rotated through an angle which is adjustable between 0° and approximately 360°.

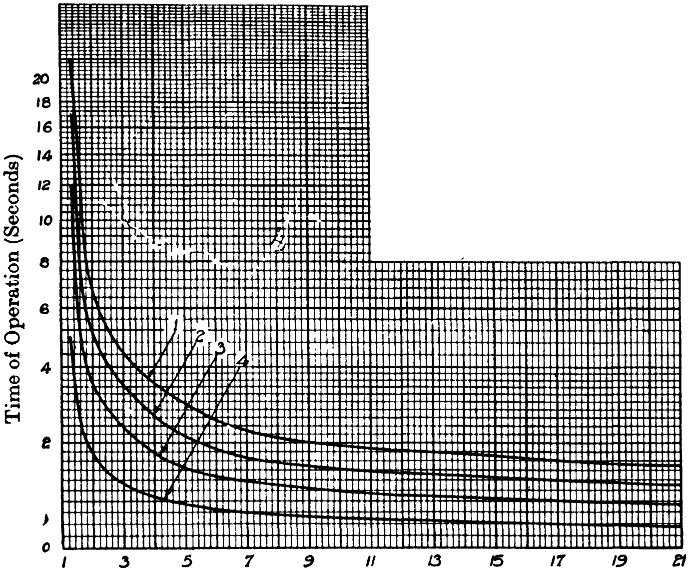

Fig. 81. Characteristic time-current curves for an over-current relay. Definite minimum—50 periods.
Curve 1 Relay set at 1·0.
Curve 2 Relay set at 0·75.
Curve 3 Relay set at 0·50.
Curve 4 Relay set at 0·25.

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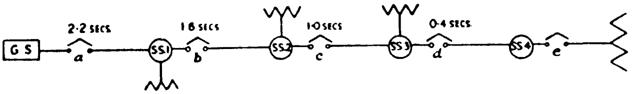

Fig. 82. Protection of radial feeders in series.

The iron circuit of the upper electromagnet is designed to be saturated beyond a certain value of primary current, so that the relay has a definite minimum time of operation.

The disadvantages of this system are that:

(*a*) It is not very sensitive.

(*b*) To obtain proper discrimination the minimum operating relay times should differ by about 0·5 second. This limits the number of relays in series to five, since a short circuit should not be sustained on the generator for longer than about 2 seconds.

(*c*) The maximum fault currents generally occur at the generating end of the feeder where the need for high speed is the greatest, but where actually the time delay is the greatest.

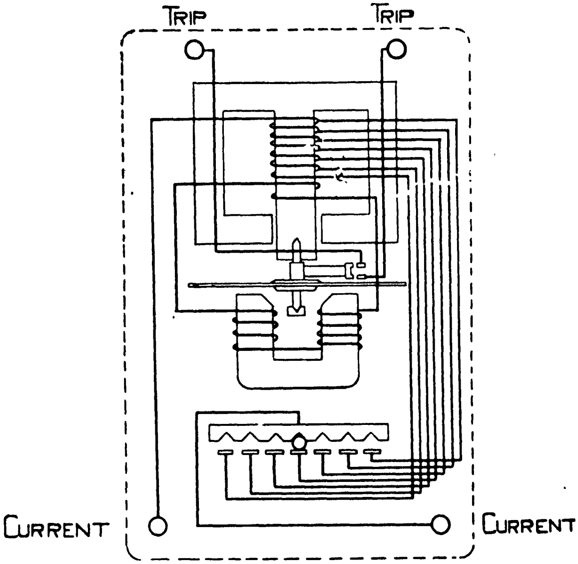

FIG. 83. Diagram of internal connexions of an over-current relay.
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(ii) *Ring Main System.* Fig. 84 shows the application of discriminative time protection to a ring main. Since a fault at any point can be fed in two directions, over-current relays which incorporate directional features must be used. Each sub-station is protected by two relays, the one with the lower time setting being directional, and operating only for fault currents in the direction of the arrow.·

With a fault at *P*, relay *g* operates first, since all other relays with equal or shorter time settings are sensitive only to currents in the opposite direction. This leaves the fault fed through *SS*4, and after an additional one second, relay *h* operates, isolating the faulty section completely.

With a fault at *Q*, relays *e* and *f* operate simultaneously.

A directional over-current relay is shown in Fig. 85. The

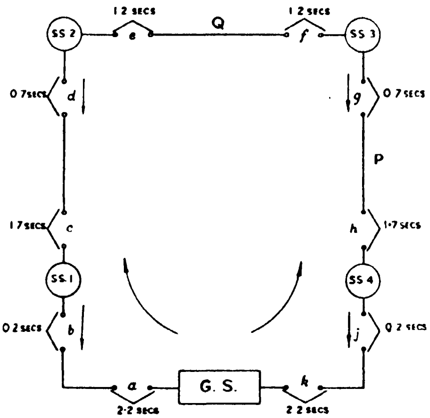

Fig. 84. Graded time protection for a ring main.

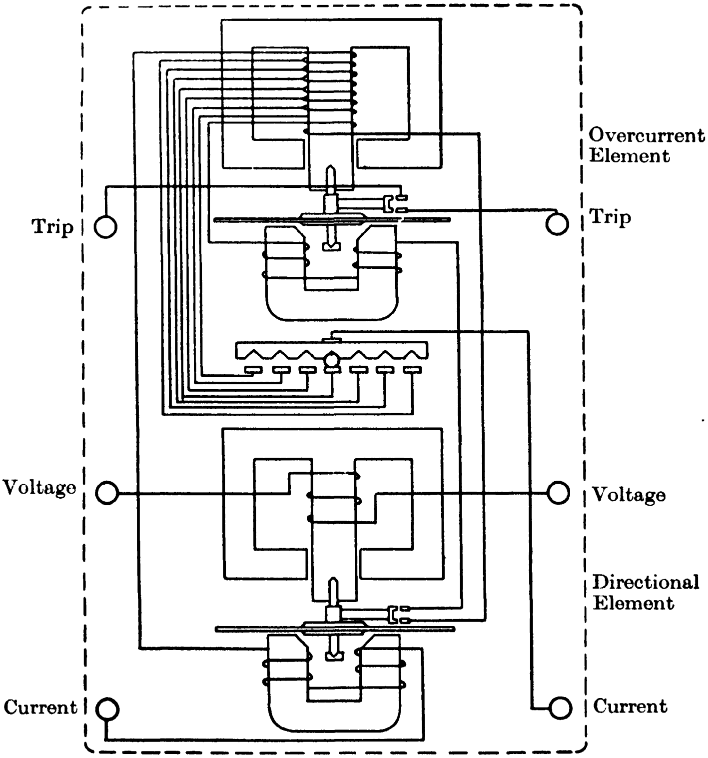

Fig. 85. Diagram of internal connexions of directional over-current relay.

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over-current element is the same as the non-directional relay (Fig. 83) except that the secondary winding of the upper magnet is connected to the lower magnet winding in series with the contacts of the directional element. The latter is a sensitive wattmeter element, consisting of voltage and current coils and a metallic disk; the disk can rotate only in a clockwise direction and when the fault current is in the desired direction. A small lever attached to the disk spindle then instantaneously closes the contacts of the directional element and completes the circuit of the over-current element which thus commences to operate.

This method has all the disadvantages of the preceding system, and suffers from the additional disadvantage that potential transformers are necessary.

(3) *Leakage Protection.*

The core-balance method of leakage protection is shown in Fig. 86 *a*. The three current transformers *CT* are joined in star

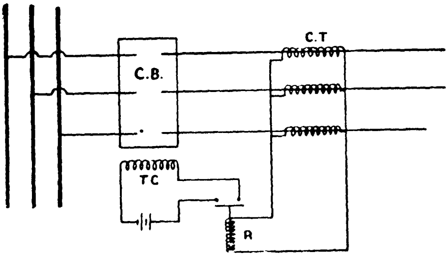

Fig. 86 *a*. Leakage protection.

at both ends, the two star points being connected through the leakage relay *R*. If there is no earth fault on the line, the instantaneous sum of the three currents is zero, and no current passes through the relay. If, however, an earth fault occurs on the system this no longer holds, and a current passes through the relay to close the trip circuit.

The method may be applied to a three-core cable, Fig. 86 *b* by the use of a Ferranti-Field current transformer; the core of this transformer is in two halves which are clamped over the outside of the cable from which the armouring is first removed. A single secondary is connected to the relay *R*. With a sound feeder there is no flux in the transformer core and no secondary current; an earth fault gives rise to a resultant flux and an induced secondary voltage which causes relay operation.

The advantages of this system are that:

(*a*) It is simple and inexpensive.

(*b*) It is sensitive, disconnexion being ensured with a leakage current of only 5 per cent. of full-load current.

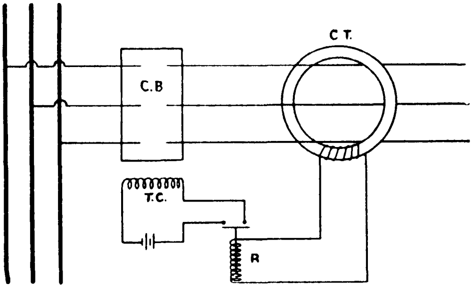

FIG. 86 *b*. Leakage protection.

The disadvantages are that:

(*a*) It does not protect against phase faults, but only against faults to earth.

(*b*) On all except large systems the neutral point must be earthed to provide a return path for the fault currents: on large systems the feeder capacitance may provide a path of sufficiently low impedance.

(*c*) It is not suitable for use with parallel feeders.

(4) *Combined Leakage and Overload Protection.*

Protection can be obtained against faults to earth or between phases as shown in Fig. 87, in which two overload relays and one leakage relay are used. The two overload relays are sufficient to protect all three phases, since a phase fault must affect at least two lines: the leakage relay protects against earth faults, since it receives the resultant current of the three phases, which is zero except when there is a fault to earth.

(5) *Parallel Feeder Protection.*

This system depends upon the principle that two or more identical feeders connected in parallel will carry equal currents which may be balanced against each other. To ensure stability on heavy overloads, biased overload relays are used as shown in Fig. 88.

The current transformers *CT* are connected up for circulating

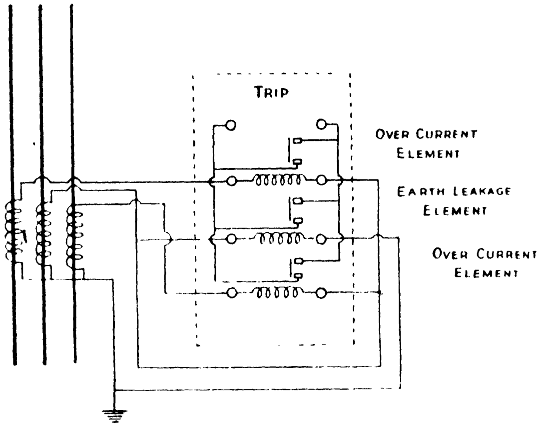

FIG. 87. Combined leakage and overload protection.

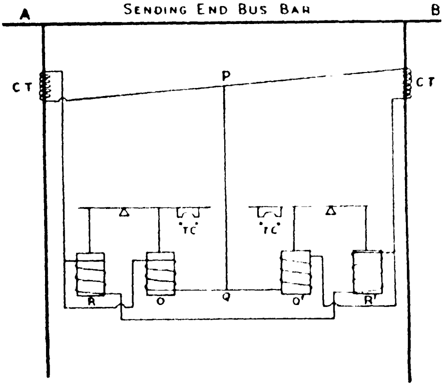

FIG. 88. Parallel feeder protection.

current which divides into the two parallel paths formed by the two operating coils and the two restraining coils respectively of the relays. With equal loads on feeders *A* and *B*, the points *P* and *Q* are at equal potentials and no current flows through the tripping connexion *PQ*. If feeder *B* carries a heavier load than that in *A*, the unbalanced voltage of its current transformer causes an additional current to circulate through the local circuit formed by the operating coil *O′* and the tripping connexion *PQ*. The pull of *O′* overcomes that of the restraining coil *R′*, causing the arm of the relay to close immediately the tripping contacts *TC*, thus opening the circuit breaker of feeder *B*. The relays are biased in favour of the restraining coils by about 10 per cent. to prevent false operation of the relays due to differences in the current transformer characteristics or in the feeder impedances.

At the receiving end of the feeders, interlocked directional relays are used for protection against reversal of power in the lines: the interlocking of the relays prevents false operation due to momentary power surges fed back from synchronous machinery on the system.

(6) *Distance Protection.*

(i) *Impedance-Time System.* This system of protection depends upon the time graded principle, the time delay being proportional to the impedance between the relay and the fault, so that the more distant the relay the slower will be its operation. This is achieved by making the time lag directly proportional to the voltage at the fault, and inversely proportional to the fault current passing.

Then the time delay $t \propto E/I \propto Z \propto d$, where d is the distance of the fault from the relay.

The impedance-time relay (Fig. 89) consists of a current-driven induction element and a voltage restraint element. The induction element comprises a non-magnetic disk 9 capable of rotating between two electromagnets 3 and 12. The primary of the upper electromagnet is connected to the secondary of a current transformer in the line to be protected, and is provided with tappings so that the current setting may be varied on a plug bridge 14. The secondary winding energizes the coil on electromagnet 12. The rotating field produced by the two electromagnets reacts with the eddy currents induced in the non-magnetic disk, and produces a torque tending to rotate the latter against the braking effect of a permanent magnet: the disk speed is directly proportional to the driving torque.

The spindle of the disk is geared to a countershaft to which is attached the inner end of a spiral spring. The outer end of this spring is attached to a bent lever, the movement of which controls the position of an insulated arm mounted on a separate shaft which is coaxial with the countershaft. This second shaft also carries a flat soft iron armature which is normally held against the face of the restraining magnet energized by the voltage coil.

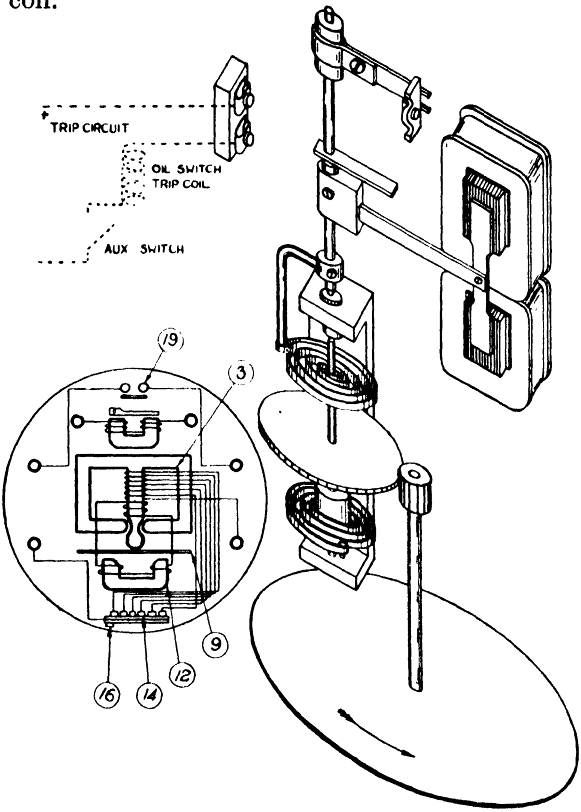

FIG. 89. Diagram to illustrate action of the impedance element of impedance-time relays.
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On the occurrence of a fault on the line, the disk starts to rotate at a speed dependent on the value of the fault current and winds up the spiral spring against the pull of the restraining element on the armature. Movement of the armature cannot take place until the torque due to the spring tension exceeds that due to the pull of the restraining magnet, when the fixed contacts 19 are closed instantly by the insulated arm: this closes the trip circuit. The contacts are thus closed in a time which varies inversely as the fault current and directly as the voltage across the line at the relay. The relays nearest the fault will operate first, thus isolating that section of the line on which the fault occurs.

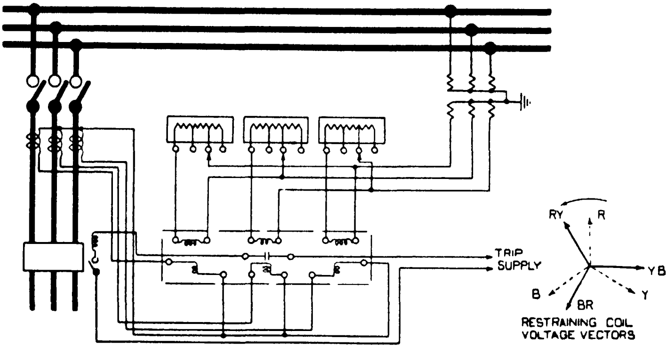

FIG. 90 Connexions for phase fault protection.
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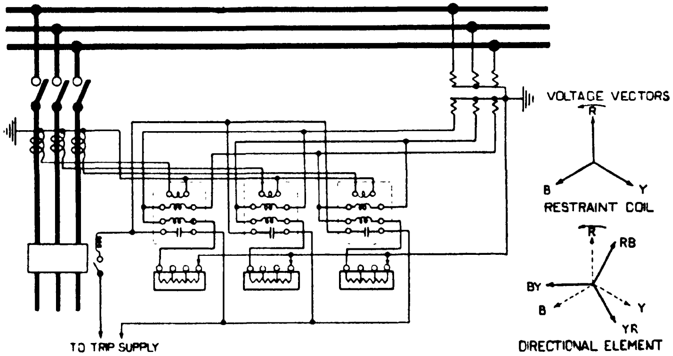

FIG. 91. Connexions for earth fault protection.
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When the network is connected to two or more power sources, directional impedance-time relays are used. Such a relay comprises a sensitive wattmeter directional element, the contacts of which are connected in series with the lower electromagnet of the impedance element, so that the latter remains inoperative until the contacts are closed.

Fig. 90 shows non-directional relays connected for phase fault protection on a three-phase system, and Fig. 91 shows directional relays connected for earth fault protection. The advantages of this system are that:

(*a*) Relays adjust their time automatically according to their distance from the fault.

(*b*) Accurate discrimination is obtainable even when the line impedance between switching points is only 5 per cent.

(*c*) No balancing of transformers is necessary.

(*d*) Networks may be extended without revising the relay settings.

The disadvantages are that:

(*a*) In common with other methods depending on the distance principle, this system cannot be applied safely to lines less than 10 miles in length.

(*b*) Owing to variation in the resistance of an arcing fault, impedance measurements may lead to incorrect location of the fault with consequent isolation of the wrong section.

(*c*) Time delays, which may reach two seconds near the feeder ends, occur.

(*d*) Voltage elements are necessary involving either costly potential couplers or compensating circuits permitting the potential to be derived from the lower-voltage side of the power transformers.

(ii) *Ratio-Balance System.* An impedance relay may discriminate wrongly, since the impedance which it measures may represent not the real geographical distance of a fault from it but the increased distance of what may be called an apparent fault due to the resistance of the earth connexions.

In the ratio-balance system relay operation is governed by the reactance of the line. The elements comprising the system divide into two groups, one dealing with phase faults and the other with earth faults. On the occurrence of an earth fault, for example, the directional starting element (Fig. 92) and the phase selector on the faulty phase operate simultaneously. These energize the voltage-coil and current-coil circuits respectively (Fig. 93) by closing contact (*a*) and opening contact (*b*). A flux is produced in the magnet system B of the distance-measuring element which is proportional to the current I. The flux produced in the magnet system A is proportional to the vector sum of two fluxes, one of which is proportional to and in phase with the current I, the other being proportional to and

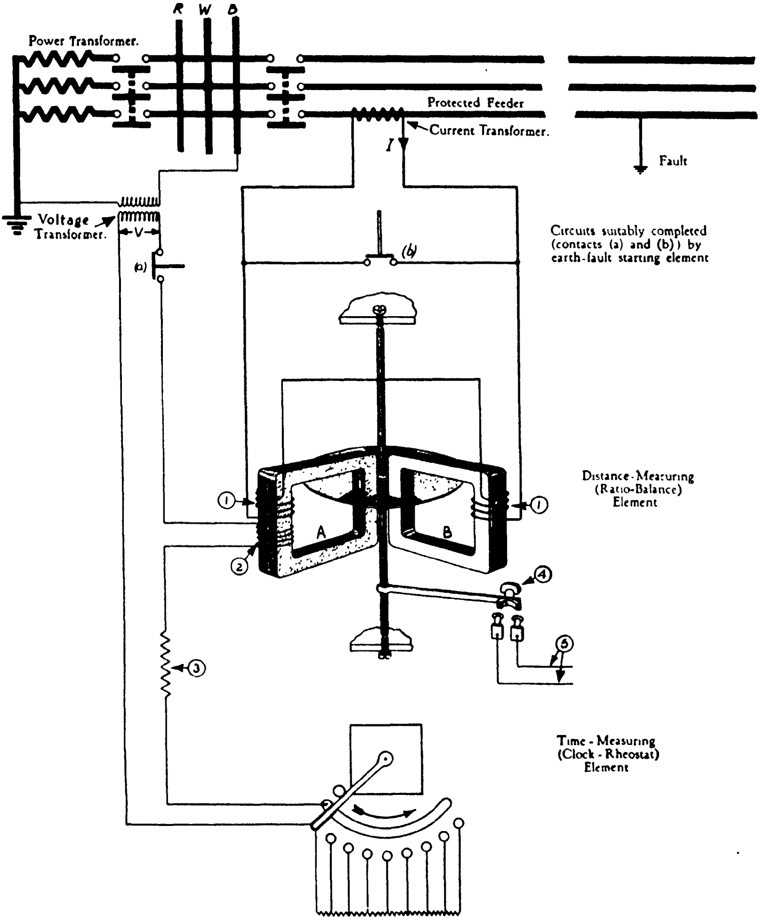

FIG. 92. Diagram illustrating the principle of the time-distance discriminating unit of the reactance type 'ratio-balance' protective systems.

(1) Current coil.
(2) Voltage coil.
(3) Resistance.
(4) Stop.
(5) Trip circuit.

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lagging approximately 90° behind the voltage V: the flux in A may thus be made proportional to $I - \frac{V}{k} \sin \phi$, where k is a constant and ϕ is the phase angle between V and I. The torque on the movable induction member—shown as a disk, but consisting in practice of a double aluminium rotor—is proportional to the product of these fluxes, i.e. to $I^2 - \frac{VI}{k}\sin\phi$.

If the fault is in the instantaneous zone, the relay instantaneously operates and closes the tripping contacts when the first term exceeds the second, i.e. when $I^2 > \frac{VI}{k}\sin\phi$ or $\frac{V}{I}\sin\phi < k$. The relay therefore operates at or below a constant value of $\frac{V}{I}\sin\phi$, i.e. a constant reactance. If the fault is outside

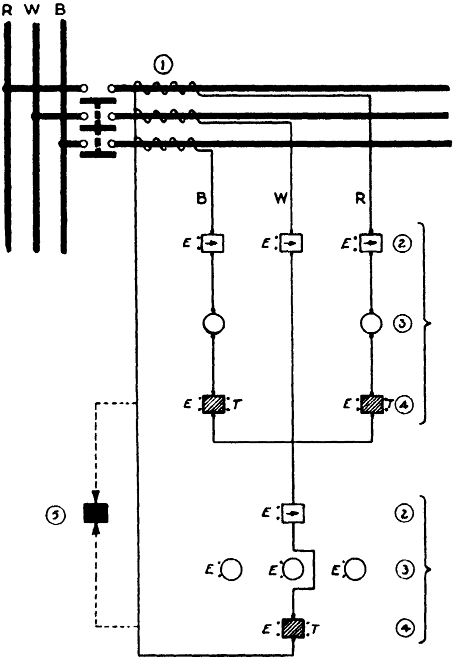

FIG. 93. Key diagram of elements forming 'ratio-balance' protective system.

(E) Indicates a voltage connexion.
→ Indicates a directional feature.
(T) Indicates a tripping-circuit connexion.
(1) Current-transformers.
(2) Starting-elements.
(3) Phase-selectors.
(4) Distance-measuring (ratio-balance) elements.
(5) Time-measuring (clock-rheostat) element.

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the instantaneous zone, the time-measuring element, which is also started up on the occurrence of the fault, gradually reduces the flux produced by V by inserting in the voltage-coil circuit a resistance whose value is proportional to the time t. The flux produced by the voltage coil is therefore inversely proportional to t, and the relay operates when $\frac{V \sin\phi}{tI} < k$ or $t > \frac{1}{k} \cdot \frac{V}{I} \sin\phi$. Thus the tripping time is proportional to the reactance.

The principal application of the ratio-balance system is to protect long overhead transmission lines. Its chief advantage is that relay operation is independent of the fault arc resistance. It suffers from the disadvantages that the apparatus is delicate and complicated, and that errors may be introduced by the appliances to separate out the reactive component of the fault current and by the phase angle of the instrument transformers.

(7) *Carrier Current Protection.**

This method is illustrated in Fig. 94 and employs superimposed high-frequency current over the power line as the pilot for an interlock system.

In the event of a fault on the interconnector as shown, the heavy current flowing in at each end towards the fault will operate the over-current and earth-leakage relays and isolate the section. The stabilizing relays remain inoperative owing to their directional characteristic. Thus the system may be regarded as simple overload protection for internal faults.

When a fault occurs outside the section, a straight-through fault current operates the over-current and earth-leakage relays at each end. In addition, however, the directional stabilizing relay at the end nearer the fault where the current is flowing outwards is also energized. The operation of this relay breaks a contact in the circuit-breaker tripping connexion, thereby preventing the switch opening. The same relay action changes over the high-frequency line connexion from the receiver to the transmitter. This causes a high-frequency impulse to be sent to the receiver at the other end, which closes the circuit of a lock-out relay and prevents the tripping of the circuit breaker at that station. By duplication of the equipment at either end it is

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* H. W. Clothier, 'Metal-clad switchgear, etc.', *I.E.E.J.* 71, p. 312, August 1932.

assured that a straight-through fault current in either direction will leave the switches of the healthy section unaffected.

The high-frequency transmitter consists of a four-valve oscillator-amplifier tuned to about 130 kc. per second. The receiver, which is normally connected to the line, except when the stabilizing relay operates, is a simple one-valve unit with a relay connected in the anode circuit. In superimposing the high-frequency current on to extra high tension lines a coupling condenser is necessary to prevent direct connexion of the high voltage on to the protective gear. At each end of a section a resonant circuit tuned to 130 kc. is connected to restrain the high-frequency currents to within the section under control.

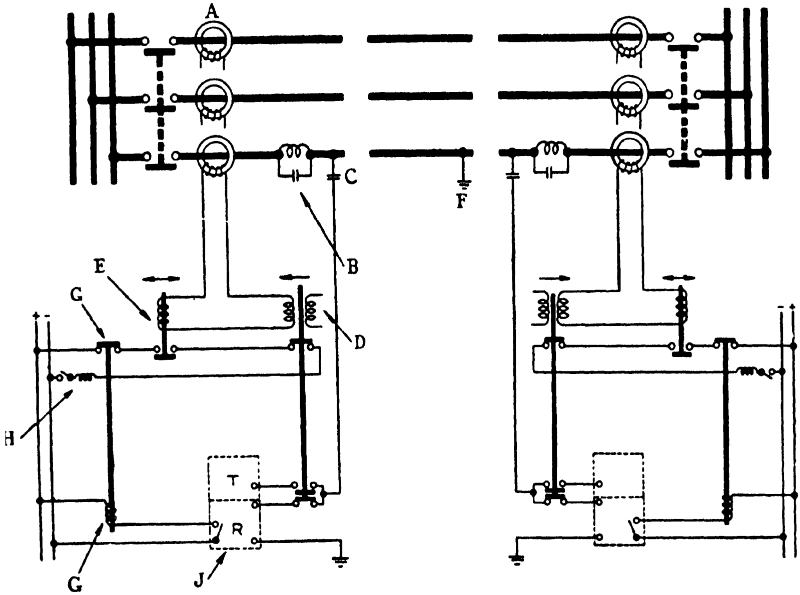

Fig. 94. Carrier current protection.

A.—Current transformers.
B.—High-frequency choke.
C.—Condenser coupler.
D.—Directional stabilizing elements.
E.—Over-current tripping-element with time-margin.
F.—Fault.
G.—Lock-out relay.
H.—Trip coil.
J.—High-frequency transmitter and receiver.
R.—High-frequency receiving apparatus.
T.—High-frequency transmitting apparatus.

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CHAPTER XI

PRIMARY CONSTANTS OF TRANSMISSION LINES

RESISTANCE

THE direct-current resistance of a conductor is determined by its resistivity, which for soft annealed copper is 1·694 microhms per cm. cube and for hard-drawn copper is 1·73 microhms per cm. cube at 60° F. The resistivity of hard-drawn aluminium is 2·793 microhms per cm. cube at 60° F.

Skin Effect.

The resistance of a conductor to an alternating current is slightly greater than that to direct current for the following reason. A solid conductor may be imagined to be composed of a large number of conducting filaments parallel to its axis. Each filament will carry a share of the current, which, if alternating, will produce an alternating flux. Now a filament at the centre of the core will be linked with all the flux, while a filament at the periphery will link only with the external flux. Thus a central filament will have a higher inductance and therefore a higher reactance than a surface filament, with the result that the current is not uniformly distributed over the cross-section but has a greater density at the surface than at the centre. The effective resistance of the conductor to alternating current is therefore greater than that to direct current, since the current tends to confine itself to the outside or the skin of the conductor. This is called the skin effect, which increases with the area of the conductor, the conductivity and the permeability of the material, and the frequency of the current.

The same effect is present in stranded conductors.

The inner conductor of a large concentric cable is usually built up of segmental copper strips around a hemp core; this reduces the skin effect.

In an aluminium conductor the skin effect is the same as that in a copper conductor of equal conductivity.

Fig. 95 shows the percentage increase in resistance in round stranded copper conductors at 50 cycles. It can be seen that the effect may be ignored for conductors of diameters less than 0·75 in.

In steel-cored conductors the effect is also small since the core carries no appreciable current.

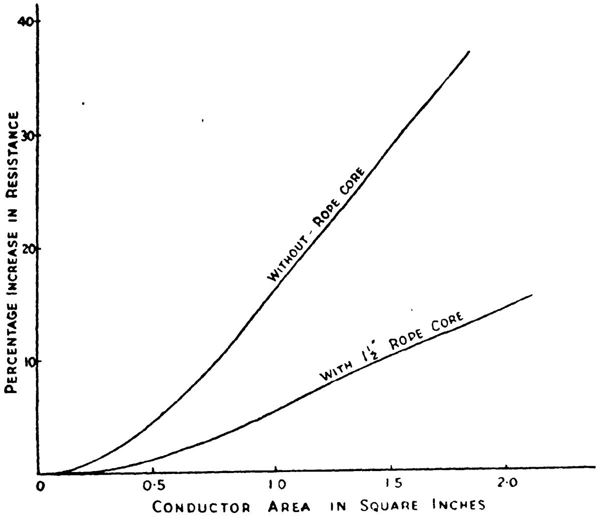

FIG. 95. Increased resistance due to skin effect.

Insulation Resistance.

The insulation resistance of a single-core or concentric cable may be calculated as follows.

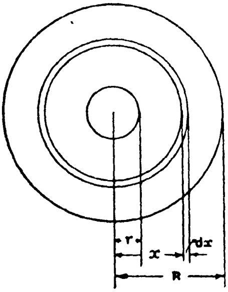

FIG. 96.

Let r be the radius of the core (Fig. 96),

R the inside radius of the sheath or of the outer conductor,

l the length of the cable in cm.,

ρ the resistivity of the dielectric in ohms per cm. cube.

If the core is assumed to be positive, then the leakage current will flow radially outwards from it.

The resistance of the element whose annular length is dx and whose area is $2\pi xl$ is

$$\frac{\rho\, dx}{2\pi xl}.$$

Hence the resistance of the cable is

$$\int_r^R \frac{\rho\, dx}{2\pi x l} = \frac{\rho}{2\pi l} \operatorname{logh} \frac{R}{r} \text{ ohms.}$$

If l is expressed in miles, this becomes

$$\text{resistance} = \frac{\rho}{2\pi \times 30{\cdot}48 \times 5280 l} \operatorname{logh} \frac{R}{r}$$

$$\doteqdot \frac{\rho \times 10^{-6}}{l} \operatorname{logh} \frac{R}{r} \text{ ohms.} \qquad (1)$$

If the resistivity is expressed as P megohms per cm. cube, this reduces to

$$\text{insulation resistance} = \frac{P}{l} \operatorname{logh} \frac{R}{r} \text{ ohms.} \qquad (2)$$

P varies for impregnated paper insulation from about 5 to about 8 megohms per cm. cube.

INDUCTANCE

(i) *Single-phase Circuit.*

Consider a pair of parallel round solid conductors A, B of overall radius r cm., spaced a distance d cm. apart in air (Fig. 97). Each conductor will, when carrying a current, produce lines of force which take the form of concentric circles inside and outside the conductor.

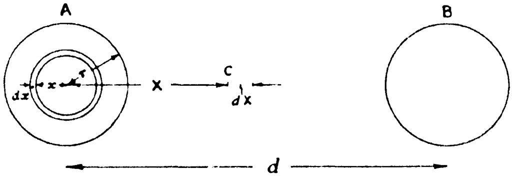

FIG. 97.

Consider the flux set up by 1 cm. length of conductor A when traversed by unit current of 1 abamp. Dealing first with the outside of the conductor, the field at C due to 1 abamp in A is by Laplace's law

$$H = \frac{2}{X}.$$

The flux threading an elemental width dX at this point is therefore

$$\frac{2}{X}\,dX.$$

The total flux between A and $B = \int_{r}^{d-r} \frac{2}{X}\,dX$

$$= 2\,\mathrm{logh}\frac{d-r}{r}.$$

Since these lines of force link with a single-turn circuit, this expression represents the linkages per unit current and therefore the inductance in abhenries due to the external flux.

Turning now to the flux inside the conductor, the field strength at radius x is $h = \frac{2I}{x}$, where I represents the current flowing in the section of conductor of radius x.

Then since the total current is unity, and assuming it to be distributed uniformly over the section we have

$$I = \frac{x^2}{r^2}.$$

$$\therefore \quad h = \frac{2x^2}{xr^2} = \frac{2x}{r^2}.$$

If the permeability of the conductor is μ, the flux density is $B = \frac{2\mu x}{r^2}$, and the flux in an elemental width δx is $\frac{2\mu x}{r^2}\,\delta x$.

This flux does not link with the whole of the conductor but with only a fraction $\frac{x^2}{r^2}$ of its cross-section.

Hence the linkages are $\frac{2\mu x}{r^2}\frac{x^2}{r^2}\,\delta x = \frac{2\mu x^3}{r^4}\,\delta x$.

The total linkages and therefore the inductance due to the internal flux is

$$\int_{0}^{r} \frac{2\mu x^3}{r^4}\,dx = \frac{\mu}{2}.$$

$\therefore$ Total inductance per cm. of single line

$$= 2\,\mathrm{logh}\frac{d-r}{r} + \frac{\mu}{2} \text{ abhenrys.}$$

Inductance per cm. of loop line

$$= 4\,\mathrm{logh}\frac{d-r}{r}+\mu \text{ abhenrys.}$$

If the conductor is non-magnetic, $\mu = 1$, and we have

$$L = \left(4\,\mathrm{logh}\frac{d-r}{r}+1\right)\times 10^{-9} \text{ henrys.}$$

If the spacing is large compared with the conductor radius, as it usually is for overhead lines,

$$L = \left(4\,\mathrm{logh}\frac{d}{r}+1\right)\times 10^{-9} \text{ henrys per loop cm.} \tag{3}$$

$$= 0{\cdot}161\left(4\,\mathrm{logh}\frac{d}{r}+1\right) \text{ millihenrys per loop mile.} \tag{4}$$

This is worked out on the assumption of a solid conductor, but stranding affects the constant term, which instead of unity becomes

1·03 for a 61-strand conductor.
1·11 „ 19 „ „
1·28 „ 7 „ „
1·55 „ 3 „ „

(ii) *Three-phase Circuit.*

In a three-phase circuit each conductor is linked not only with its own flux, but with part of the flux due to each of the other two conductors. The algebraic sum of the currents in the conductors at any instant must be equal to zero and any one conductor can therefore be regarded as carrying the outgoing current, while the remaining conductors together carry the return current.

It can be shown that the inductance of each wire is the same as that obtained per conductor in the single-phase case, i.e.

$$L = 0{\cdot}0805\left(4\,\mathrm{logh}\frac{d}{r}+1\right) \text{ millihenrys per mile.} \tag{5}$$

In the case of symmetrical spacing, i.e. where the three conductors are situated at the corners of an equilateral triangle, *d* is the distance between any two wires.

When the wires are spaced irregularly as in Fig. 98 but are transposed, i.e. are given one complete rotation at intervals, so that each conductor occupies each position for one-third of the total line length, then $d = \sqrt[3]{abc}$. (6)

In the case of regular flat spacing as in Fig. 99,

$$d = \sqrt[3]{2a^3} = 1{\cdot}26a. \tag{7}$$

The presence of the steel core in a steel-cored aluminium cable affects the inductance, particularly when there is only one layer of aluminium over the steel core, in which case the inductance is also a function of the current density. With two or more aluminium layers the effect is not so marked and is independent of the current density. The inductance values are best obtained from the manufacturer's tables.

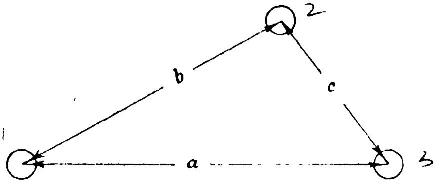

FIG. 98. Irregular spacing of conductors.

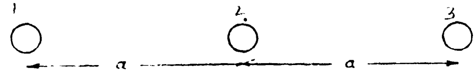

FIG. 99. Regular flat spacing.

CAPACITANCE

Overhead Lines.

(i) *Single-phase Circuit.* Let A and B (Fig. 100) represent two round parallel conductors in section, separated in air by a distance d cm., and maintained at a potential difference V.

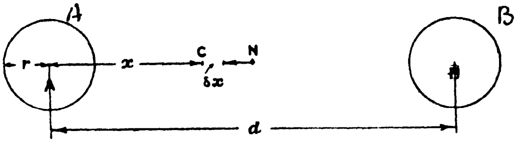

FIG. 100.

Let the two conductors carry charges of $+q$ and $-q$ electrostatic units respectively per cm. length, and assume that these charges are distributed uniformly over the surfaces of the conductors.

Then since $4\pi q$ lines of dielectric flux emanate radially from a charge q, the dielectric flux density at C due to A is

$$\frac{4\pi q}{2\pi x}=\frac{2q}{x}.$$

This is also the electric intensity or potential gradient due to A at C.

Similarly the flux density and therefore the potential gradient at C due to B is

$$\frac{2q}{d-x}.$$

Now the potential of the point C is the sum of the potentials of this point due to each charge.

Thus $$V=\int_{r}^{d-r}\left(\frac{2q}{x}+\frac{2q}{d-x}\right)dx$$

$$=2q[\operatorname{logh} x-\operatorname{logh}(d-x)]_{r}^{d-r}$$

$$=2q\left[\operatorname{logh}\frac{x}{d-x}\right]_{r}^{d-r}=4q\operatorname{logh}\frac{d-r}{r}.$$

Hence the capacitance per cm. of the two conductors is given by

$$\frac{q}{V}=\frac{q}{4q\operatorname{logh}\frac{d-r}{r}}=\frac{1}{4\operatorname{logh}\frac{d-r}{r}}\text{ e.s.u.}$$

The capacitance of each wire to the mid-point N which will be at zero potential and may therefore be regarded as the neutral is, then,

$$C=\frac{1}{2\operatorname{logh}\frac{d-r}{r}}\text{ e.s.u. per cm.}$$

If r is small compared with d, this becomes approximately

$$C=\frac{1}{2\operatorname{logh}\frac{d}{r}}\text{ e.s.u. per cm.}\qquad(8)$$

$$=\frac{30{\cdot}48\times5280}{2\times9\times10^{5}\operatorname{logh}\frac{d}{r}}\text{ microfarads per mile}$$

$$=\frac{0{\cdot}0894}{\operatorname{logh}\frac{d}{r}}=\frac{0{\cdot}0388}{\log_{10}\frac{d}{r}}\text{ microfarads per mile.}\qquad(9)$$

This ignores the shunting capacitance of the conductor to the earth's surface; this, however, is usually so small as to be negligible.

The charging current of a line of capacitance C, supplied with a voltage V at a pulsatance ω, is given by

$$I = V\omega C. \tag{10}$$

(ii) *Three-phase Circuit.* The capacitance to neutral of a symmetrical balanced three-phase line, i.e. one in which the wires are situated at the corners of an equilateral triangle, is given by the preceding formula, i.e.

$$C = \frac{0{\cdot}0388}{\log_{10}\frac{d}{r}} \text{ microfarads per mile,}$$

where d represents the side of the triangle.

When unsymmetrical spacing is used, but the conductors are transposed, the average capacitance to neutral per mile of line is the same for each conductor and it may be shown that, as in the case of the inductance, $d = \sqrt[3]{abc}$ and

$$C = \frac{0{\cdot}0388}{\log_{10}\frac{\sqrt[3]{abc}}{r}} \text{ microfarads per mile,}$$

where a, b, and c are the conductor spacings.

Underground Cables.

(i) *Single-core or Concentric Cables.*

Let r be the radius of the core (Fig. 101),
R the radius over the insulation,
ϵ the permittivity of the insulation.

Then if there are charges of $+q$ and $-q$ on the core and sheath respectively, the dielectric flux density at radius x is $\frac{2q}{\epsilon x}$.

The voltage is therefore $V = \int_r^R \frac{2q}{\epsilon x}\, dx$

$$= \frac{2q}{\epsilon} \operatorname{logh} \frac{R}{r}.$$

$$\therefore \quad C = \frac{\epsilon}{2\,\text{logh}\dfrac{R}{r}} \text{ e.s.u. per cm.}$$

$$= \frac{0{\cdot}0388\epsilon}{\log_{10}\dfrac{R}{r}} \text{ microfarads per mile.} \qquad (11)$$

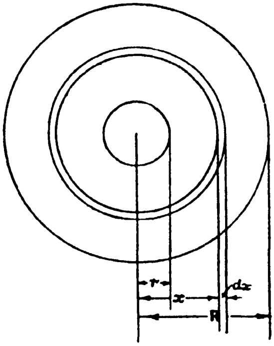

FIG. 101.

Taking the permittivity of impregnated paper insulation at 60° F. as 3·6, this gives

$$C = \frac{0{\cdot}14}{\log_{10}\dfrac{R}{r}} \text{ microfarads per mile.}$$

This formula is applicable to single core and concentric paper-insulated cables.

(ii) *Three-core Cables.* The preceding formula is valid for three-phase cables in which each core has a separate earthed sheath.

For three-phase three-core paper-insulated cables with round conductors when the individual cores are not separately sheathed, the capacitance to neutral of each conductor is given approximately by the empirical formula

$$C = \frac{0{\cdot}16}{\log_{10}\left(1+\dfrac{d_1+d_2}{d}\right)} \text{ microfarads per mile,} \qquad (12)$$

where d = diameter of conductor,

d_1 = thickness of dielectric between conductors,

d_2 = thickness of dielectric between conductor and sheath.

For shaped conductors this is increased by about 8 per cent.

EXAMPLES ON CHAPTER XI

Q. 1. Find the insulation resistance per mile of a concentric cable whose inner conductor has a diameter of 0·504 in., the dielectric thickness being 0·08 in. Take $P = 5 \times 10^8$ megohms per cm. cube.

Q. 2. Calculate the inductance per mile of a single 7/0·193 copper conductor whose diameter is 0·579 in. when spaced (*a*) 12 in., (*b*) 60 in.

Q. 3. Calculate the inductance per mile per conductor of a 7/0·064 three-core cable if the diameter of each core is 0·192 in. and the thickness of the dielectric between cores is 0·3 in.

Q. 4. Calculate the inductance, capacitance, and charging current per conductor of a 132 kV. three-phase overhead line 20 miles long. The overall conductor diameter is 0·77 in. and three conductors are arranged with a regular flat spacing of 12 ft.

Q. 5. Calculate the capacitance per conductor of a 0·0225 three-core three-phase 660-volt paper-insulated cable. Its conductor diameter is 0·192 in., and the dielectric thickness between conductors and between conductor and sheath is 0·08 in.

Q. 6. Find the capacitance per mile of a 0·2 three-core three-phase 3,300-volt paper-insulated cable. Its conductor diameter is 0·581 in., and the dielectric thickness between conductors is 0·14 in. and between conductor and sheath 0·11 in.

Q. 7. The insulation resistance of a mile of cable having a conductor diameter of 1·5 cm. and an insulation thickness of 1·5 cm. is 500 megohms. What would be the insulation resistance if the thickness of the insulation were increased to 2·5 cm.? Prove any formula used. L.U.

Q. 8. A single-core high-voltage cable has a conductor diameter of 0·15 in., the diameter of the cable over the insulation being 1 in. Calculate the insulation resistance per mile of cable, given that the insulating material has a specific resistance of 5×10^{10} ohms per cm. cube. C.G.

CHAPTER XII

TRAVELLING WAVES

When any sudden change is made in a transmission line circuit due to switching operations in the system or to breakdown of the line or of the terminal apparatus through faulty insulation, the current and the voltage cannot take instantly their permanent values as determined by the new conditions on account of the inductance and capacitance of the line. During the time required for conditions to become steady, there will therefore exist transient currents and voltages in addition to the permanent currents and voltages. A transient quantity is one whose amplitude decreases sensibly to zero in a short time. Such a transient initiates waves of current and voltage which travel along the line from the point of disturbance with a high velocity depending upon the line constants. Atmospheric disturbances which occur during thunderstorms may also produce such travelling waves.

Natural Velocity of Propagation.

Let the wave advance from the point of origin with a velocity v cm. per second.

Then in a time dt sec. it will travel a distance $v\,dt$ cm. along the line.

If L is the line inductance per cm. length, the inductance of this distance is $Lv\,dt$ and the flux associated with it will be $LIv\,dt$, where I is the line current.

The e.m.f. induced in the line due to the establishment of this flux in time dt is

$$E = \frac{LIv\,dt}{dt} = LIv.$$

$$\therefore \quad \frac{E}{I} = Lv. \qquad (1)$$

Again, if C is the line capacitance per unit length, the capacitance of the section under consideration is $Cv\,dt$ and the charge on it due to a voltage E is $Q = ECv\,dt$.

But since current is the rate of change of quantity we have

$$I = \frac{ECv\,dt}{dt} = ECv.$$

$$\therefore \quad \frac{E}{I} = \frac{1}{Cv}. \tag{2}$$

Hence from (1) and (2)

$$Lv = \frac{1}{Cv},$$

and

$$v = \frac{1}{\sqrt{LC}}. \tag{3}$$

Now for air lines, the inductance per loop cm., neglecting the internal inductance, is $L = 4 \log_h \frac{d}{r} . 10^{-9}$ henrys, and the capacitance per loop cm. is

$$C = \frac{1}{9 \times 10^{11}} \frac{1}{4 \log_h \frac{d}{r}} \text{ farads.}$$

Hence

$$v = \frac{1}{\sqrt{\frac{10^{-9}}{9 \times 10^{11}}}}$$

$$= 3 \times 10^{10} \text{ cm./sec.},$$

which is the velocity of light.

The actual velocity of propagation in overhead lines is slightly less than this on account of the internal inductance of the conductor. The higher permittivity of the insulation of cable lines reduces the velocity to about half that for air lines.

It will be seen that the velocity of propagation of travelling waves is very great, and since also the duration of a transient is very short it may be assumed that the power frequency voltage and current are sensibly constant during its duration.

Surge Impedance.

Combining equations (1) and (3) we have

$$\frac{E}{I} = Z = \sqrt{\frac{L}{C}}. \tag{4}$$

This is called the natural impedance or surge impedance of the line.

This relationship may also be derived from energy considerations. Thus the energy of a wave is partly electromagnetic and partly electrostatic. When the current is zero, the energy is wholly electrostatic, and when the voltage is zero, the energy is wholly electromagnetic.

Then if E and I are the maximum values of the voltage and current respectively, the electrostatic energy per unit length of line will be $\frac{1}{2}CE^2$ and the electromagnetic energy per unit length is $\frac{1}{2}LI^2$. Neglecting the power losses in the circuit, these two expressions must be equal, and we have

$$\tfrac{1}{2}CE^2 = \tfrac{1}{2}LI^2$$

i.e.
$$\frac{E}{I} = \sqrt{\frac{L}{C}} = \sqrt{\frac{\left(4\,\mathrm{logh}\,\frac{d}{r}\right)^2 \times 9 \times 10^{11}}{10^9}}$$

$$= 120\,\mathrm{logh}\,\frac{d}{r}. \qquad (5)$$

For overhead lines the surge impedance lies between 400 and 600 ohms, while for cable lines where the inductance is smaller and the capacitance larger, the surge impedance is approximately one-tenth of that for overhead lines.

Transmission and Reflection of Travelling Waves.

When a travelling wave arrives at a point of discontinuity in the line, i.e. at a point where the impedance suddenly changes, the wave is partly transmitted in the forward direction and partly reflected backwards. Such a discontinuity would occur at the junction of an overhead line with a cable or at a load point. In a forward wave the current and voltage are taken to be of the same sign, so that $I = \frac{E}{Z}$, while in a reflected wave the current and voltage are of opposite sign, so that $I = -\frac{E}{Z}$. When two waves travelling in opposite directions along a transmission line meet, the actual current and voltage at the meeting-point are the algebraic sums of the currents and voltages respectively of the separate waves.

Consider now a line of surge impedance Z_1 connected at its far end to a load impedance Z_2. Imagine the line to be suddenly switched on to the supply voltage, causing a pressure wave to travel towards the load end. Let E and I represent the voltage and current values of the incident wave, E_T and I_T the corresponding values of the wave transmitted to and absorbed by the load, and E_R and I_R the corresponding values for the reflected wave.

Then we have

$$E = IZ_1, \quad E_T = I_T Z_2, \quad E_R = -I_R Z_1. \tag{6}$$

Now the voltage transmitted to the load must be the same as that in the line at the junction, which is the sum of the incident and reflected waves, i.e.

$$E_T = E+E_R. \tag{7}$$

Similarly

$$I_T = I+I_R. \tag{8}$$

Substituting (6) in (7), $I_T Z_2 = IZ_1 - I_R Z_1$.

Multiplying (8) by Z_1, $I_T Z_1 = IZ_1 + I_R Z_1$.

Adding,

$$I_T = \frac{2Z_1}{Z_1+Z_2} I. \tag{9}$$

$$\therefore \quad I_R = I_T - I = \frac{Z_1-Z_2}{Z_1+Z_2} I. \tag{10}$$

Again,

$$E_T = I_T Z_2 = \frac{2Z_1 Z_2}{Z_1+Z_2} \cdot \frac{E}{Z_1}$$

$$= \frac{2Z_2}{Z_1+Z_2} E. \tag{11}$$

$$\therefore \quad E_R = E_T - E = \frac{Z_2-Z_1}{Z_1+Z_2} E. \tag{12}$$

Thus it follows that the transmitted waves are always positive. The reflected current is positive if $Z_1 > Z_2$, in which case the voltage is negative, and the reflected current is negative if $Z_2 > Z_1$, when the voltage is positive.

Now if $Z_2 = 0$, i.e. the line is short-circuited at the receiving end, we have from equations (9) to (12)

$$I_T = 2I, \quad I_R = I, \quad E_T = 0, \quad E_R = -E.$$

These relationships may be verified by the following reasoning: since the receiving end is short-circuited, the voltage at that point must be zero, i.e. $E_T = 0$; this necessitates a reflected voltage equal and opposite to the incident voltage, i.e. $E_R = -E$. But this gives rise to a reflected current

$$I_R = -\frac{E_R}{Z_1} = \frac{E}{Z_1} = I,$$

so that the transmitted current, being the sum of the incident and reflected waves, must be $2I$.

If $Z_2 = \infty$, i.e. the line is open circuited at its far end, we have

$$I_T = 0, \qquad I_R = -I, \qquad E_T = 2E, \qquad E_R = E.$$

The transmitted current is now zero, since the line is open at the far end, and the current is wholly reflected. Since

$$E_R = -I_R Z_1 = IZ_1 = E,$$

the reflected voltage is equal to and in the same direction as the incident voltage, giving a transmitted voltage $E_T = 2E$. This shows that the voltage at the open-circuited end of a line may rise to twice the maximum value of the supply voltage at the moment of switching in.

If $Z_2 = Z_1$, i.e. the line is closed through an impedance equal to its surge impedance,

$$I_T = I, \qquad I_R = 0, \qquad E_T = E, \qquad E_R = 0.$$

Hence in this case there is no reflection.

Junction of Overhead and Cable Lines.

On account of the abrupt change in surge impedance at the junction of an overhead with an underground line, reflection will occur; the formulae (9) to (12) are applicable to this case, Z_1 being the surge impedance of the line in which the pressure wave originates, and Z_2 being that of the line to which the wave is transmitted.

If the surge originates in an overhead line, $Z_1 > Z_2$, and there will be a reduction in the transmitted pressure, but a rise in the transmitted current wave.

If, on the other hand, the surge originates in the cable, $Z_1 < Z_2$, and the transmitted current wave decreases, while the pressure wave transmitted to the overhead line increases; this pressure surge may again be doubled if the remote end is open circuited.

The preceding analysis may be extended to the case where three lines meet at a point, such as at a T-junction or at a point where one line forks.

Transients due to Sudden Interruption of Load.

Hitherto we have considered the effects of a sudden application of voltage to a line. Important effects may also be produced in switching out a load. If a current I in a line is suddenly interrupted by switching off the load, a transient pressure rise is produced of $E = IZ$, where Z is the surge impedance of the line. This pressure rise is dependent on the value of the current at the instant of breaking the circuit, which, however, is usually zero in a modern oil circuit breaker. Trouble may, however, be experienced due to arcing at the switch contacts which will initiate a pressure wave equal in magnitude to the voltage across the switch at the moment of arcing.

Breakdown of Line Insulation.

If the insulation of a line breaks down either to earth or between phases the voltage at the fault immediately falls to zero. If the instantaneous voltage of the line at the moment of fault is E, then a negative pressure wave of magnitude E moves out in both directions from the fault, giving rise to current waves of magnitude E/Z. These waves obey the laws of reflection previously investigated and are dangerous, not so much because of their magnitudes, as on account of their steep wave fronts. The steepness of a wave front is determined by the time taken by the surge to reach its maximum value. Fig. 102 shows a typical steep fronted wave.

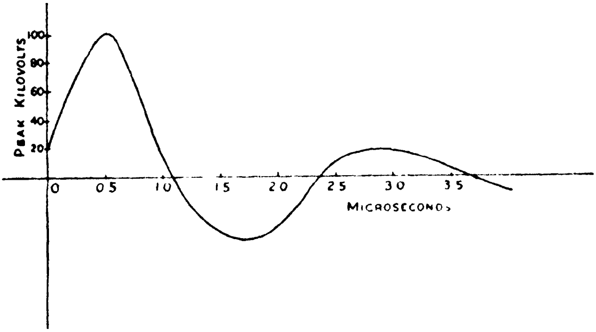

Fig. 102. Oscillatory transient with steep wave front.

Arcing Grounds.

On an unearthed system a line may break down to earth either by the failure of an insulator or of a length of cable. The fault arc reduces the potential of the faulty line to earth causing the voltage of the other lines suddenly to rise. At the same time the arc is extinguished, when the potential of the faulty line rises again until spark-over occurs once more. This is repeated, causing trains of voltage oscillations to be impressed on the system and resulting in a cumulative building up of voltage on the sound lines. These may also break down at one or more places, resulting in a short circuit on the system with possible damage to conductors. Such intermittent flashings to earth are called arcing grounds.

Lightning.

According to the theory of G. C. Simpson* water cannot fall relatively to the air faster than about 8 metres per second, and therefore no rain can fall through air which is ascending with a vertical velocity component greater than 8 metres per second. Violent thunderstorms are characterized by such currents, and in any region *B* (Fig. 103) of a cloud where they occur the raindrops are broken up. Every time a drop breaks the water of which the drop is composed receives a positive charge, the corresponding negative charge being given to the air, and is immediately absorbed by the cloud particles.

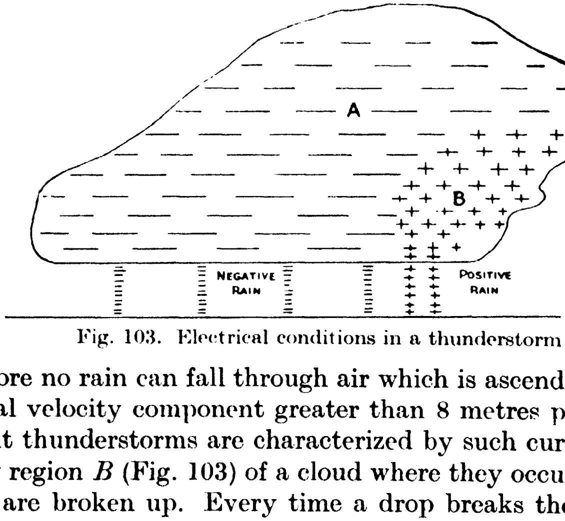

Fig. 103. Electrical conditions in a thunderstorm

The accumulated water in *B* thus becomes highly charged with positive electricity, the negatively charged air passing out into the main cloud *A*. These two charges reside on a non-conducting cloud floating within a conducting atmosphere. With increasing accumulation of charge the electric field-strength increases, until ultimately the air breaks down: a discharge then takes place within the cloud, or from a positive region of a cloud

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* G. C. Simpson, 'Lightning', *I.E.E.J.*, vol. 67, p. 1269, Nov. 1929.

to an induced negative charge on the ground, or from a point on the ground to a negatively charged region of a cloud. Although discharges from positive sections of cloud are the more frequent, the greater damage is done on the earth by the discharges to negative sections of the cloud. The quantity of electricity discharged in an average lightning-flash varies between 10 and 50 coulombs; the potential reached in a thunder-cloud before a discharge of about 20 coulombs is of the order of 10^9 volts. The resistance of the air-channel limits the potential at the earth's surface, but peak values of $2\frac{1}{2}$ million volts have actually been measured in a direct stroke to earth. Now the average duration of a lightning discharge is greater than 0·001 second: taking this minimum figure, a discharge of 20 coulombs gives a mean current of 20,000 amp., although instantaneous values of the order of 100,000 amp. may be reached. The danger, however, does not lie in the current alone, but in the voltage and in the steepness of its wave front, which causes the voltage to pile up across the end sections of terminal apparatus such as transformers.

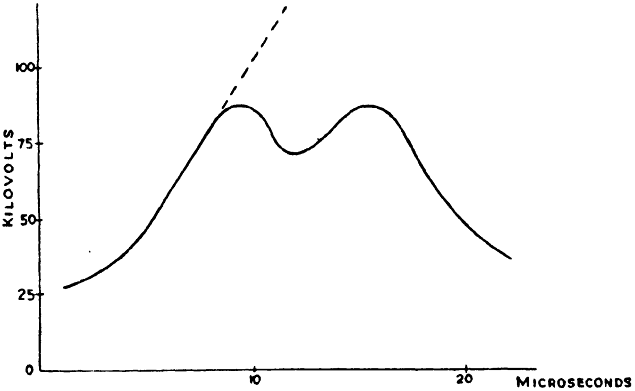

FIG. 104. Example of a lightning discharge.

The main discharge in a lightning-flash consists of a unidirectional current which starts from zero, rises to a maximum and then decreases more or less rapidly to zero again. Fig. 104 shows the beginning of a typical lightning surge with a wave front of about 20 kV. per microsecond.

Lightning surges may also be produced by induction. Thus a charged cloud induces an opposite charge on a neighbouring transmission line: as soon as the cloud discharges to another cloud-section or to ground, the bound charge on the line is released and immediately travels in both directions along the line with the speed of light, The energy of such a surge is, however, relatively small and the pressure wave has a sloping front, so that such surges are rarely, if ever, dangerous.

Protection against Abnormal Increases in Pressure.

(1) *Earth Wires.* It is now common practice to use high conductivity earth wires carried on the main towers above the conductors. Such an earth wire serves the double purpose of acting as a definite return for fault currents and as an electromagnetic screen against lightning discharges.

(2) *Surge Absorbers.* A Ferranti surge absorber consists essentially of an air-core inductance connected in series with the transmission line or cable close to the apparatus to be protected. Adjacent to the conductors of the coil but insulated from them is a high-resistance metallic sheet called the dissipator, which is connected to earth. The arrangement is illustrated diagrammatically in Fig. 105.

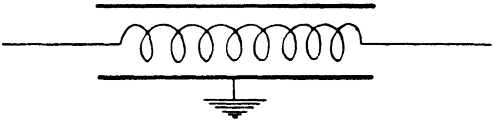

FIG. 105. Illustrating the Ferranti surge absorber.

For voltages above 66 kV., a horizontal coil is mounted in an oil tank, the dissipator consisting of a metal cylinder inside the coil. For lower voltages (33 kV.) the coil consists of two or more disk sections contained between two flat metal sheets forming the dissipator. For still lower voltages down to 400 a cylindrical coil is contained within a sheet-iron case.

The inductance coil of the absorber acts as the primary winding of a transformer, and the energy dissipator forms a secondary winding of one short-circuited turn. The energy of an incoming surge is absorbed partly by the losses due to the secondary current induced in the dissipator, and partly by the eddy currents in the latter. The effect of such a device is to reduce the steepness of a wave by abstracting its energy and by flattening its wave front.

(3) *Petersen Coil.* This is used on the Continent to suppress the arc in arcing grounds on unearthed systems. It consists of a reactance connected between the neutral point of the system and earth. When a fault occurs on one phase of a three-phase system, the potentials of the other two phases rise and a charging current flows, which in the faulty phase leads the phase voltage by 90°. The current through the Petersen coil lags almost 90° behind its voltage, which is the voltage of the faulty phase. With a suitable value of coil reactance the resultant fault current may be limited to a small watt component, and the arc is immediately suppressed.

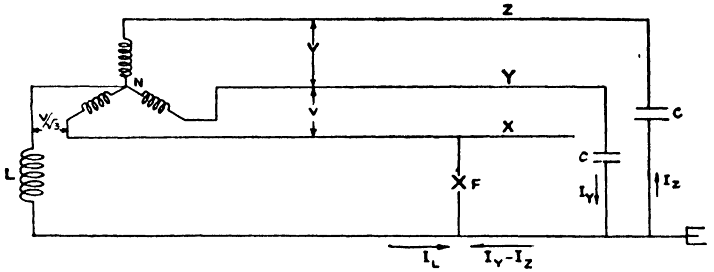

FIG. 106. Connexions for a Petersen coil.

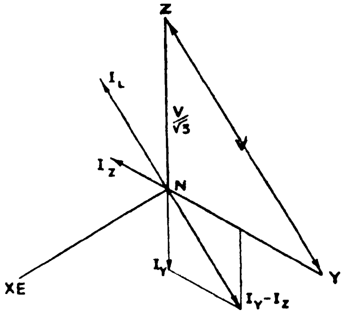

Fig. 107. Vector diagram for Petersen coil.

In Fig. 106, X, Y, and Z represent three air lines, of which X is assumed to have developed a fault to earth at F: the Petersen coil is represented by L. Let C be the capacitance of each of the conductors Y and Z to earth when X is earthed. This will not be the capacitance of each conductor to earth under normal conditions, but will be a function of the height of the conductor. Then referring to the vector diagram, Fig. 107, the total capacitance current flowing to earth is $I_Y - I_Z = \sqrt{3} I_Y = \sqrt{3} V \omega C$. The current I_L through the Petersen coil is $\frac{V}{\sqrt{3}} \cdot \frac{1}{\omega L}$.

Hence, in order that the current through the fault is a minimum,

$$\sqrt{3} V \omega C = \frac{V}{\sqrt{3} \omega L}.$$

$$\therefore \quad 3\omega^2 LC = 1.$$

$$\therefore \quad \text{Inductance of coil is } L = \frac{1}{3\omega^2 C},$$

or the reactance of the Petersen coil is

$$X = \omega L = \frac{1}{3\omega C}. \tag{13}$$

(4) *Other Methods.* Lightning arresters have been used in the past but are not being adopted in new systems in this country. In general, such a device consists of a high resistance having in series one or more air-gaps and is connected between the line and earth. A lightning surge of sufficient amplitude jumps the gap and dissipates its energy in the resistance. Travelling waves may, however, be set up at the moment of flash-over so that the line may not actually be relieved of over-potential. Further, it is difficult to prevent the flow of power frequency current when once the earth connexion is established.

Arcing horns and rings are used on the insulators to provide a definite path for transient discharges and to protect the insulators and conductors from the effects of power arcs.

The termination of an overhead line in a length of cable affords protection for terminal apparatus, since an incoming surge voltage is reduced, as we have seen, and, further, the attenuation of surges in cables is high on account of the high dielectric losses in cables at high frequencies.

By working a line near the critical corona voltage a discharge path is provided to high voltage surges. Sometimes the main line is terminated in a short length of line, the critical voltage of which is near the operating voltage, so that any surges are attenuated before they reach the terminal apparatus. The attenuation due to corona is fairly rapid since the power loss is proportional to the square of the excess of the surge voltage above the disruptive critical voltage.

EXAMPLES ON CHAPTER XII

Q. 1. An underground cable, having a self-inductance of 1 millihenry per mile and a capacitance of 0·4 microfarad per mile, terminates in an air line having a self-inductance of 2·4 millihenrys per mile and a capacitance of 0·015 microfarads per mile. Compute the instantaneous voltage and current transmitted to the line due to a transient pressure wave of 18 kV. originating in the cable.

Q. 2. An overhead transmission line has a capacitance of 0·0125 microfarad per mile and an inductance of 1·5 millihenrys per mile. It terminates in an underground cable having a capacitance of 0·3 microfarad per mile and an inductance of 0·25 millihenry per mile. Calculate the rise in voltage produced at the junction of the line and cable by a wave of crest value 50 kV. originating in the cable.

Q. 3. An overhead line of impedance 350 ohms terminates in a transformer of impedance 3,000 ohms. Find the amplitudes of the current and pressure waves transmitted to the transformer due to an incident pressure wave of 20 kV.

Q. 4. Determine the reactance at 50 cycles of a Petersen coil installed on a three-phase transmission line consisting of 120 miles of 33 kV. cable having a capacitance of 0·3 microfarad per mile per core to neutral.

A.M.I.E.E.

Q. 5. Find the surge impedance of the 'grid' lines, of which the inductance between phases is 0·0042 henry per mile and the electrostatic capacity is 0·007 microfarad per mile.

Q. 6. A cable line of surge impedance 50 ohms links up with an air line of surge impedance 400 ohms. Calculate the value of the transient voltage produced at the open end of the air line due to a surge of 20 kV. originating in the cable.

CHAPTER XIII

CORONA

Formation of Corona.

If a gradually increasing voltage is applied to a pair of parallel conductors, the spacing of which is not large compared with their diameters, the potential gradient will also increase, and will be a maximum at any time at the conductor surface. When the potential gradient exceeds the breakdown strength of air, the latter becomes conducting and the lines spark across.

If, however, the spacing is greater than about fifteen times the conductor diameter, as is usual on overhead lines, and if a wattmeter is connected in the circuit, a power reading will be observed when the breakdown gradient is reached at the conductor surface, but spark-over does not occur; the voltage at which this takes place is called the disruptive critical voltage. At a higher potential difference each conductor becomes surrounded by a faint violet halo-like glow, which is called corona: the voltage at which this occurs is called the visual critical voltage. The appearance of corona is accompanied by a hissing sound, an odour of ozone, and vibration of the conductors.

This luminous envelope is brightest at those parts where the conductor surface is rough or dirty. As the voltage increases the luminous regions increase in diameter and brightness, becoming more definite and extending the whole length of the conductor, until finally at a still higher voltage the conductors spark across.

If a direct voltage is used, the positive conductor has a more uniform glow than the negative.

This phenomenon becomes of practical importance on lines operating above 66 kV.

A physical explanation of the effect is as follows: the gaseous molecules of which the air is composed may be uncharged, or may have positive charges due to the loss of one or more electrons. In an electric field free electrons and charged ions are subjected to forces causing motion: the velocity of these particles is dependent upon the potential gradient, which, if sufficiently high, will result in the displacement, by collision, of electrons from other uncharged particles, which then also acquire a velocity. At high voltages the process is cumulative and the air around the conductor becomes conducting, increasing the effective diameter of the conductor and reducing the field intensity beyond the conducting layer. A corona envelope is thus formed beyond which the air retains its insulating properties.

The critical voltages and the power loss accompanying corona may be calculated from the following formulae which are based on the experiments of F. W. Peek and J. B. Whitehead.

Disruptive Critical Voltage. The potential gradient G at the surface of each of two parallel conductors, radius r, spacing d, assuming that the charge is uniformly distributed over the surface, has been shown to be

$$G = \frac{E}{r \operatorname{logh} \frac{d}{r}},$$

where E is the voltage to neutral.

$$\therefore \quad E = Gr \operatorname{logh} \frac{d}{r}. \tag{1}$$

Now the breakdown strength G_0 of air at 76 cm. of barometric pressure and 25° C. temperature is approximately 30 kV. per cm.

Thus the critical voltage at these conditions for clean, smooth wires is

$$E_{0\,\max} = 30r \operatorname{logh} \frac{d}{r} \text{ kV.},$$

where r is expressed in cm.

Assuming a sinusoidal voltage, the R.M.S. value is therefore given by

$$E_0 = 21{\cdot}1 r \operatorname{logh} \frac{d}{r}.$$

Now the disruptive gradient is proportional, over a considerable range, to the density of the air. Thus, if δ is the ratio of the density of the air at a temperature θ and barometric pressure b cm. to that at normal temperature and pressure, then

$$E_0 = 21{\cdot}1\, r\delta \operatorname{logh} \frac{d}{r},$$

where $$\delta = \frac{b}{76} \cdot \frac{273+25}{273+t} = \frac{3{\cdot}92b}{273+t}.$$

With stranded and dirty wires, breakdown occurs at a lower voltage than this, and we have finally for the disruptive critical voltage

$$E_0 = 21{\cdot}1\, m_0 r\delta \operatorname{logh}\frac{d}{r} \text{ kV. to neutral,} \tag{2}$$

r being expressed in cm., where the irregularity factor

$$m_0 = 0{\cdot}93\text{–}0{\cdot}98 \text{ for roughened weathered wires,}$$
$$= 0{\cdot}8\text{–}0{\cdot}87 \text{ for stranded cables.}$$

In terms of common logarithms, and with r expressed in inches, this becomes

$$E_0 = 123\, m_0 r\delta \log\frac{d}{r} \text{ kV. to neutral.} \tag{3}$$

This value applies to fair-weather conditions only: for storm conditions the value of E_0 is usually taken as 0·8 of that for fair-weather conditions.

Visual Critical Voltage. Visual corona is produced not when the breakdown gradient occurs at the conductor surface but when it occurs at $0{\cdot}3\sqrt{r\delta}$ cm. radially beyond the surface of the conductor. Consequently the visual voltage E_v is greater than the disruptive voltage E_0, and we have

$$E_v = 21{\cdot}1\, m_v r\delta\left(1+\frac{0{\cdot}3}{\sqrt{r\delta}}\right)\operatorname{logh}\frac{d}{r} \text{ kV. to neutral,} \tag{4}$$

where r is expressed in cm., and m_v is an empirical surface factor.

For stranded cables m_v is about 0·70 to 0·75 for local corona when the effect is first visible, and 0·80 to 0·85 for decided corona when the effect is definite along the whole conductor length.

In terms of common logarithms and with r expressed in inches, we have

$$E_v = 123\, m_v r\delta\left(1+\frac{0{\cdot}189}{\sqrt{r\delta}}\right)\log\frac{d}{r} \text{ kV. to neutral.} \tag{5}$$

Power Loss due to Corona. The power loss due to corona is given by the formula

$$P = \frac{390.10^{-5}}{\delta}(f+25)(E-E_0)^2\sqrt{\frac{r}{d}} \text{ kW.} \tag{6}$$

per mile of conductor.

This is applicable to single-phase and symmetrically spaced three-phase lines, where

f = frequency,
E = voltage to neutral,
E_0 = disruptive critical voltage to neutral.

This formula is valid only after the decided corona point has been reached.

Practical Importance of Corona.

It is not advisable in practice to operate lines with decided corona present on account of the losses to which this would give rise and also on account of the triple frequency harmonics which are set up and which may cause interference with communication circuits. This may, for a given spacing, limit the working-line voltage, but the latter can be increased by employing steel-cored aluminium conductors in preference to copper conductors on account of their greater diameter.

It may, on the other hand, be an advantage to work near the corona point, since then any excess pressures due to lightning or other transients may be relieved by their discharge through corona, which thus acts as a safety valve.

In general, it is advisable that the disruptive critical voltage should exceed the actual voltage by 20 per cent. to allow for fog and rain.

EXAMPLES ON CHAPTER XIII

Q. 1. Calculate the disruptive critical voltage at normal temperature and pressure on a three-phase line, the conductors having a diameter of 0·5 in. and a spacing of 10 ft. Take $m_0 = 0{\cdot}83$.

Q. 2. Estimate the disruptive critical voltage on a three-phase line, the conductors having a diameter of 0·6 in. and spacing of 12 ft.,

(*a*) at sea-level and normal temperature;
(*b*) at 5,000 ft. above sea-level.

Assume that the barometer falls 2·8 cm. per 1,000 ft. rise and that the temperature falls 1° F. per 300 ft. rise.

Q. 3. Find the disruptive critical voltage on the grid system under normal atmospheric conditions: take the diameter of the conductor as 0·77 in., and the spacing 12 ft.

Q. 4. An overhead transmission line operates at 220 kV. between phases at 50 cycles. The conductors are arranged in a 12 ft. delta. What is the minimum diameter of conductor that can be used for no corona loss under fair-weather conditions ? Assume an air density factor of 0·95, an irregularity factor of 0·85, and take the critical voltage to be 230 kV. Find also the power loss under storm conditions.

Q. 5. What is the minimum safe spacing between solid rod conductors 0·4 in. diameter in an outdoor sub-station operating at 110 kV., three-phase, 50 cycles, at standard temperature and pressure, if the conductors are arranged in a symmetrical delta ?

CHAPTER XIV

THE BRITISH 'GRID' SYSTEM

Introduction.

THE British 'Grid' consists of a system of main transmission lines, constituting a great bus bar, which interconnects the principal generating stations throughout the country. Of the 474 generating stations existing at the end of 1931, 119 have been selected as generating centres and 16 new stations are being erected. The remaining stations will ultimately cease to generate and will then take a supply in bulk from the Grid.

The main factors in determining the selected stations are:

(1) The cost of coal delivered to the station.

(2) The abundance of water for condensing purposes.

(3) Technical characteristics of the station, such as type and size of the plant units, steam pressure, etc.

(4) Proximity to the load.

(5) The possibilities of the site for further expansion.

The whole of Great Britain is divided into ten areas, each of which is administered by a local staff. Members of local staffs, together with the London head office, form the Central Electricity Board.

All the power produced by the selected stations is purchased by the Board which then sells it to the various distributing authorities. A distributing authority owning a selected generating station re-purchases from the Board the electricity required for its own undertaking. The purchase price from the Board in such a case is either the National Grid price, or the actual cost of generation at the station, whichever is the lower. The advantages gained by such interconnexion are that:

(1) Large generating plants are cheaper per kilowatt installed than small plants.

(2) Smaller percentage reserve plant is required, since the stations are reserves to each other.

(3) The overall load factor is improved because the maximum demands in different districts occur at different times.

(4) The larger stations are more efficient.

(5) The light load of an interconnected network can be sup-

plied from a few large generating stations operating at high efficiency on full load, and the smaller stations are closed down.

(6) The site of new stations can be chosen to obtain plentiful water and cheap coal.

(7) The additional rural load picked by the interconnectors augments the load and improves the load factor.

The map (Fig. 108) shows the Grid so far as projected up to January 1932. It consists of some 3,000 miles of three-phase overhead lines operating at a frequency of 50 cycles per second and a pressure of 132,000 volts between phases, and 240 miles of cable lines operating at 66,000 and 33,000 volts.

Every authorized undertaking is supplied in duplicate, so that either connexion can provide the whole of the authority's requirements, the second connexion giving the 100 per cent. stand-by. To obtain greatest security single-circuit ring mains are used wherever possible in preference to double circuit lines.

Conductors.

The difficulty of obtaining way-leaves and tower sites necessitates long spans, calling for conductors of great strength. Steel-cored aluminium conductors are therefore used consisting of a central core of seven strands of galvanized-steel wire each of 0·11-in. diameter, surrounded by thirty strands of aluminium wire of the same diameter. The overall diameter is 0·77 in., which relatively large value sets the critical corona voltage at 184,000 volts, and ensures that corona losses are negligible. To provide an electrostatic screen and a low resistance path for returning fault currents, a high conductivity earth wire consisting of seven steel and twelve aluminium strands of 0·1-in. diameter is run above the conductors.

The capacity of a single-circuit line is 50,000 kVA., and the current corresponding to this at 132 kV. is 219 amp.

Towers.

The towers employed are constructed of galvanized steel and are of the wide base straight-line type: the average height is 70 ft. for single-circuit units and 80 ft. for double-circuit towers. Figs. 109 and 110 show a standard single-circuit and a standard double-circuit suspension tower respectively. The widths at the base are 15 ft. and 18 ft. 6 in. respectively, permitting a 2° deviation in the line. Wider bases up to 20 ft. and 22 ft. are

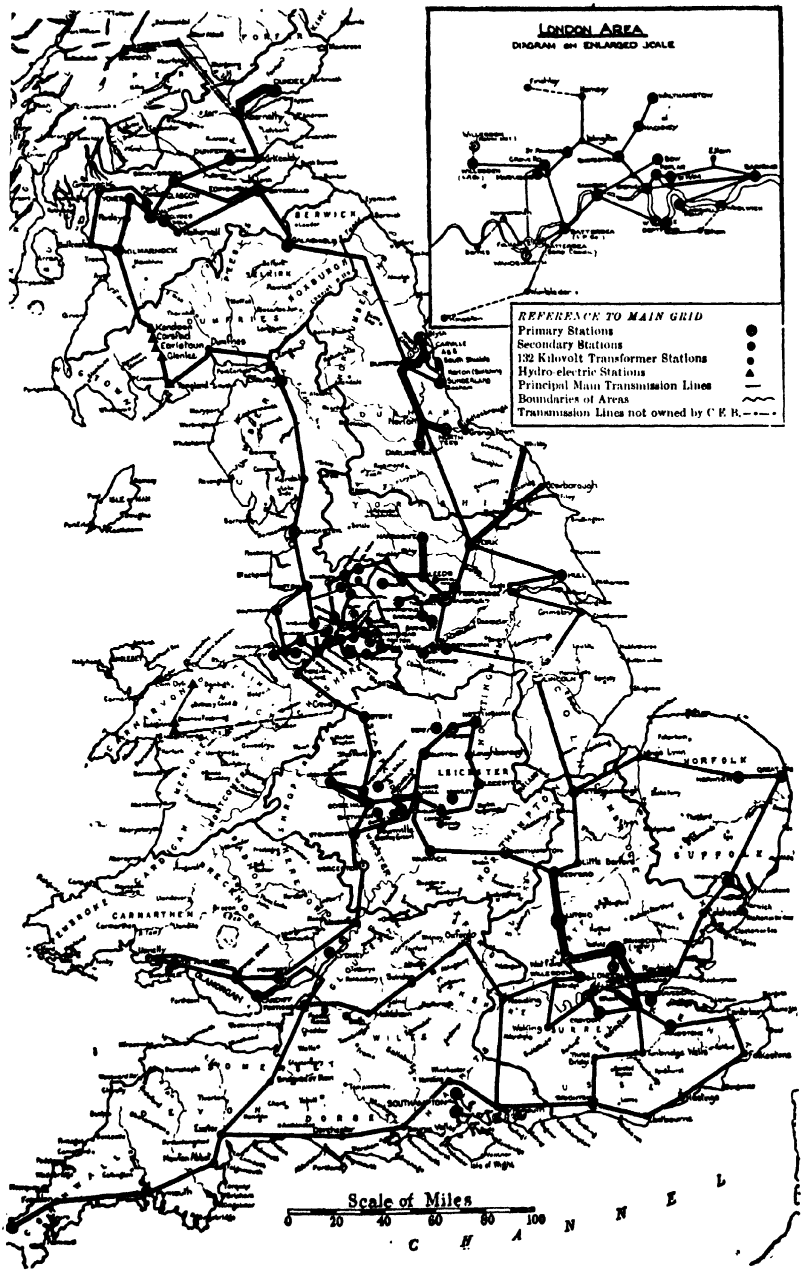

FIG. 108. The Main Grid. *By courtesy of the Central Electricity Board.*

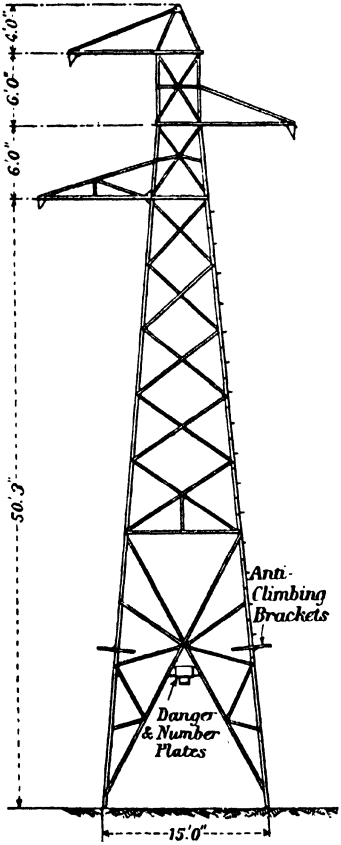

FIG. 109. Single-circuit tower.
By courtesy of the Central Electricity Board.

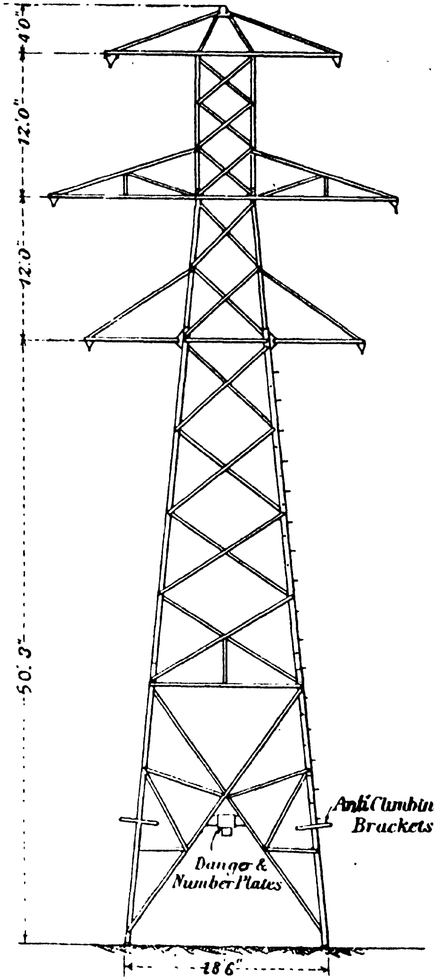

FIG. 110. Double-circuit tower.
By courtesy of the Central Electricity Board.

standardized for angles of 60°. The clearance of the lowest line conductor to earth at a temperature of 50° C. is 22 ft., while over buildings the minimum clearance is 12 ft. The vertical distance between conductors is 12 ft. and the projected horizontal distance is 3 ft. There is one complete rotation of the phases in each line section to minimize interference with communication lines, and special transposition towers are provided for this purpose. The normal span between the towers is 900 ft., so that there are roughly six towers to the mile.

The type of foundation used for the towers depends upon the nature of the subsoil: they may be of excavated earth, concrete-ball, or concrete-block construction. The last two require independent earthing by means of galvanized iron pipes. The concrete-ball foundation is made by drilling a cylindrical hole for each tower-leg and expanding the lower end of the hole to a spherical form by exploding in it a charge of dynamite. The tower-leg stubs are fitted into position by means of a template, and the cavities are filled in with concrete. Sample towers are tested to destruction to establish definitely the factors of safety obtained.

Insulators.

The suspension insulators used are of the cap-and-pin type with ball and socket fittings. Each insulator unit is 10 in. in diameter, and there are 9 units to the string in open country remote from industrial districts. Where there is considerable pollution, suspension chains of 10 or 11 units are used. At tension-points two or even three strings may be used in parallel with 10 to 12 units per string. The minimum wet flash-over voltage of a 9-unit string is 385 kV. Robust arcing rings and horns are fitted capable of withstanding the effect of power arcs for an adequate time.

Protection.

On short lines up to about 10 miles in length current-balance protection employing pilot cables is used as well as carrier-current interlock protection. On longer lines reliance is placed on impedance-time relays and reactance relays. Lightning arresters are not used on the main Grid. For protection against abnormal increases of potential due to lightning or to abrupt changes of load, reliance is placed on the provision of the high conductivity earth wire, on arcing rings and horns, on the reinforcement of transformer end-turn insulation, and on the earthing of the neutral point at every sub-station.

Sub-stations.

These are of the outdoor type. In cases where there is plenty of ground room, the 'low' or single-deck type is used, while the 'high' or double-deck type is used where space is limited. Three-circuit-breaker and double bus-bar patterns are designed of each type: the former is capable of reconstruction without structural alteration to form the double bus-bar arrangement and also to be extended if desired to include more circuits by the addition of further bays. For supplies to comparatively small undertakings a so-called one-circuit-breaker station may be used.

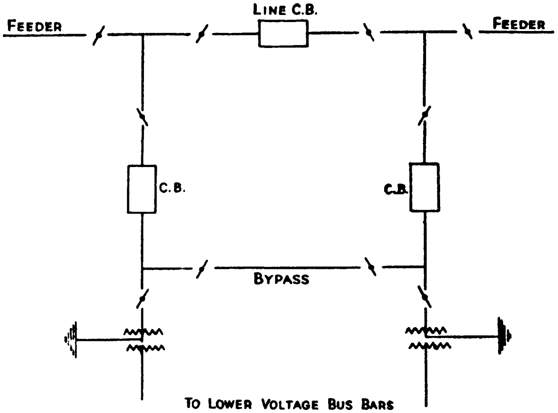

FIG. 111. Connexions of a three-circuit-breaker sub-station.

Fig. 111 shows one phase of a typical three-circuit-breaker sub-station, the e.h.t. winding of each transformer being star-connected and the neutral point earthed. It will be observed that there are three circuit-breakers. The remaining switches are isolating switches, which are interlocked with the breakers so that they cannot make or break current but can only isolate sections of the circuit which are carrying no current. The by-pass connexion is normally kept open. In the event of a fault on a line the transformer circuit-breaker on that side and the line circuit-breaker are opened by the protective gear. One transformer is thus temporarily disconnected, but can be reconnected after opening the line isolator. A faulty transformer is cut out of circuit by the operation of the high-tension circuit-breaker and also by the opening of a circuit-breaker on the low-tension side.

In emergencies a circuit-breaker can be taken out of service for overhaul, and a supply taken through both transformers with the by-pass connexion closed. When the station is to be used for working with one transformer only, the three circuit-breakers are kept in use and the by-pass connexion is closed. The transformer which is out of use is disconnected by means of its isolator switch and low-tension circuit-breaker.

In the case of line failures, the line circuit-breaker and the transformer circuit-breaker adjacent to it operate, leaving the transformer connected to the sound line through the remaining circuit-breaker.

Switch Gear and Transformers.

The main oil circuit-breakers are built up of single-phase units and their rated rupturing capacity is 1½ million kVA. The three cylindrical tank units for the three phases are operated by one electromagnet excited from a 110-volt storage battery.

Isolating switches are of the horizontal rotating centre-pillar type, the three phases being operated by the one handle.

The transformers are of the outdoor type and range in size from 7,500 to 75,000 kVA.: those under 30,000 kVA. are three-phase, while larger transformers consist of single-phase units. On-load tap changing gear is fitted for voltage control, giving a ratio variation of ±10 per cent. in at least 14 steps. The lower voltage windings are delta-connected, and the 132 kV. windings are star-connected, the neutral point being connected permanently and directly to earth.

Conservator tanks fitted with calcium-chloride breathers are provided to ensure that there shall be no danger of moisture getting to the oil. Provision is made for fitting Buchholz relays in the connector between the conservator and the main tanks.

Frequency Standardization.

This involves doubling the frequency in the Birmingham, Glasgow, and Gorseinon areas and raising the frequency in the north-east coast and north-western districts from 40 to 50 cycles per second. The former is carried out by rewinding or replacing all turbo-alternators, rotary converters, and motors, while the latter involves a change of speed and requires new plant. The procedure is to arrange for a nucleus supply at the standard frequency by rewinding one or more existing alternators or installing new ones: this supply is reinforced by advance sections of the Grid, after which the change-over of motors on consumers' premises is begun.

Metering Arrangements.

Accurate metering of the power input to and output from the Grid is essential. Stations connected to the Grid may be divided into two main types according to whether they have generating plant in operation or not. Fig. 112 shows the metering scheme for stations without generating plant. Each incoming circuit from the Grid system is equipped with a kWh. meter and a reactive kVAh. meter, the purpose of the latter type being to determine, in conjunction with the energy meters, the power factor. In addition there are summator meters for determining the total energy input and the maximum power demand and the corresponding reactive quantities.

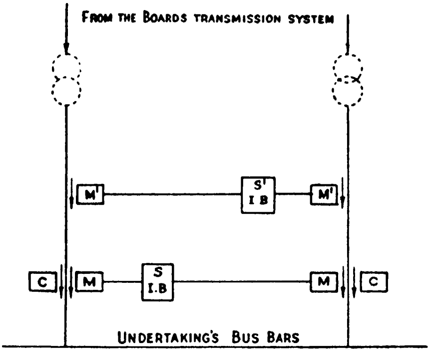

FIG. 112. Metering scheme for stations without generating plant.

$\frac{S}{I.B.}$ Summator import from the Board (kW.).
$\frac{S'}{I.B.}$ Summator import from the Board (Reactive kVA.).
M. Main integrating meter with summating attachment (kWh.).
M′. Main integrating meter with summating attachment (Reactive kVAh.).
C. Check integrating meter (kWh.).

For a station having its own generating plant, the metering arrangements are such as to give the reactive kVA. as well as the active kVA., both for the quantity generated and the quantity imported from or exported to the Grid. In addition, the maximum demands both of the wattless and the power kVA.s are registered. The Grid price thus takes into account not only the maximum demands but also the power factor, so that supply authorities are compelled to establish their own charges to private consumers on a similar basis.

Operation of the Grid.

There is established in each of the ten districts a control room in charge of the district engineer. Each district control engineer has telephonic communication with all power and transforming stations in his area, and all operations are carried out by the local engineers in accordance with the decision of the district engineer. The control room is provided with remote indication instruments which show:

(1) The operating positions (whether open or closed) of all circuit-breakers, isolating switches, and earthing switches.

(2) The transformation ratio in use on each transformer bank.

(3) The load which is being imported by or exported from each station, together with the current and power factor.

(4) The current, voltage, and power factor in each feeder circuit.

(5) The plant in operation and that available for operation.

These indications are recorded on special manual and automatic diagrams mounted on the indicating boards. The control engineer is thus enabled to operate the system to the best advantage. The local operating engineers are responsible for the direct and detailed control of the plant.

SUMMARY OF FORMULAE

CONDUCTOR COSTS ON D.C. AND ON A.C.

PAGE

$\frac{C}{C_a} = \left(\frac{V_a}{V}\right)^2 \cos^2\phi$ 3

$= \frac{\cos^2\phi}{2}$ for overhead systems 4

$= \frac{2}{3}\cos^2\phi$ for underground systems . . . 4

where C = cost of copper on D.C. system (two-wire)

C_a = cost of copper on A.C. system (three-phase, three-wire).

DISTRIBUTORS

Direct Current.

Fed at One Point.

Volt-drop $= \sum rId$ 7

where I = load currents

d = distances of load points from the feed points (miles)

r = resistance per loop mile of distributor.

Fed from Both Ends A and B.

Feed current $I_B = \frac{\sum Ia}{l}$ 9

where a = distance of load points from end A

l = total length of distributor.

Uniformly loaded, fed at Both Ends.

Maximum volt-drop $= \frac{ril^2}{8}$ 15

Tapered Cables.

For a given volt-drop, minimum copper is obtained when $a \propto \sqrt{I}$. 22

For a given I^2R loss, minimum copper is obtained when $a \propto I$. 24

where a = cross-sectional area.

Alternating Current.

Volt-drop $= \sum rId\cos\phi + \sum xId\sin\phi$ 19

where x = reactance per loop mile of conductor.

BEST POSITION FOR SUPPLY POINT

$X = \frac{\sum xL}{\sum L}$ 24

$Y = \frac{\sum yL}{\sum L}$ 24

where L = loads

x, y = coordinates referred to any two lines at right angles.

DIVISION OF LOAD

$\mathbf{I}_1 = \mathbf{I}\dfrac{\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2}$ 26

$\mathbf{I}_2 = \mathbf{I}\dfrac{\mathbf{Z}_1}{\mathbf{Z}_1+\mathbf{Z}_2}$ 26

where $\mathbf{I}_1, \mathbf{I}_2$ = the respective vector currents
$\mathbf{I}$ = the total vector current
$\mathbf{Z}_1, \mathbf{Z}_2$ = the respective vector impedance operators.

SHORT TRANSMISSION LINES

Single-phase.

Volt-drop $\doteqdot 2I(R\cos\phi \pm X\sin\phi)$ 32

where R = resistance per line
X = reactance per line
I = line current
V = load-end voltage between lines.

The positive sign is used for lagging currents and the negative sign for leading currents.

Phase angle at generator end $\theta = \tan^{-1}\left(\dfrac{V\sin\phi \pm 2IX}{V\cos\phi + 2IR}\right)$ . . . 32

Power loss $= 2I^2R$.

Three-phase.

Volt-drop between phases $v = \sqrt{3}I(R\cos\phi \pm X\sin\phi)$ . . . 34

$\theta = \tan^{-1}\left(\dfrac{V_L\sin\phi \pm \sqrt{3}IX}{V_L\cos\phi + \sqrt{3}IR}\right)$ 34

where V_L = receiving-end line voltage.
Power loss $= 3I^2R$.

LONG TRANSMISSION LINES

Approximate Solution.

Nominal Π *method.*

$\mathbf{E} = \mathbf{V}\left(1+\dfrac{\mathbf{YZ}}{2}\right)+\mathbf{ZI}$ 37

$\mathbf{I}_2 = \mathbf{VY}\left(1+\dfrac{\mathbf{YZ}}{4}\right)+\mathbf{I}\left(1+\dfrac{\mathbf{YZ}}{2}\right)$ 38

where $\mathbf{V}$ = load-end phase voltage
$\mathbf{E}$ = sending-end phase voltage
$\mathbf{I}$ = load current
$\mathbf{I}_2$ = sending-end current
$\mathbf{Z}$ = series impedance of the line
$\mathbf{Y}$ = shunt admittance of the line.

Nominal T method.

$\mathbf{I}_1 = \mathbf{I}\left(1+\dfrac{\mathbf{YZ}}{2}\right)+\mathbf{VY}$ 39

$\mathbf{E} = \mathbf{V}\left(1+\dfrac{\mathbf{YZ}}{2}\right)+\mathbf{IZ}\left(1+\dfrac{\mathbf{YZ}}{4}\right)$ 39

Regulation.

Regulation per phase $= v + 1{\cdot}5E\left(\frac{l}{1000}\right)^2$ 43

where l = length of line in miles
v = volt-drop per phase.

Regulation of open circuited line $= 1{\cdot}5V_0\left(\frac{l}{1000}\right)^2$ . . 43

where V_0 = no-load receiving-end phase voltage.

Copper loss of open-circuited line $= \frac{1}{3}I_C^2R$ 44

where $I_C = \frac{V_0\omega C}{2}$

and C = total capacitance of one line to earth.

Exact Solution.

$\mathbf{E} = \mathbf{V}\cosh\mathbf{P} + \mathbf{Z}_0\mathbf{I}\,\sinh\mathbf{P}$ 48

$\mathbf{I}_S = \mathbf{I}_L\cosh\mathbf{P} + \frac{\mathbf{V}}{\mathbf{Z}_0}\sinh\mathbf{P}$ 48

$\mathbf{V} = \mathbf{E}\cosh\mathbf{P} - \mathbf{Z}_0\mathbf{I}_S\sinh\mathbf{P}$ 48

$\mathbf{I}_L = \mathbf{I}_S\cosh\mathbf{P} - \frac{\mathbf{E}}{\mathbf{Z}_0}\sinh\mathbf{P}$ 48

Propagation constant $\mathbf{p} = \frac{\mathbf{P}}{l} = \frac{\sqrt{\mathbf{YZ}}}{l} = \sqrt{\mathbf{yz}}$ 48

Characteristic impedance $\mathbf{Z}_0 = \sqrt{\frac{\mathbf{z}}{\mathbf{y}}} = \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}}$ 48

where $\mathbf{E}$ = sending-end voltage
$\mathbf{V}$ = load-end voltage
$\mathbf{I}_L$ = load current
$\mathbf{I}_S$ = sending-end current
$\mathbf{Z}$ = $R+jX$ the total series impedance of the line
$\mathbf{z}$ = the series impedance per mile
$\mathbf{Y}$ = $G+jB$ the total shunt admittance of the line
$\mathbf{y}$ = the shunt admittance per mile.

These equations hold for single-phase or three-phase lines if the constants refer to both lines and the voltages are line values in the former case, and the constants refer to one line and the voltages are phase values in the three-phase case.

$\cosh\mathbf{P} = 1 + \frac{\mathbf{YZ}}{\underline{|2}} + \frac{(\mathbf{YZ})^2}{\underline{|4}} + \ldots$ 49

$\mathbf{Z}_0\sinh\mathbf{P} = \mathbf{Z}\left\{1 + \frac{\mathbf{YZ}}{\underline{|3}} + \frac{(\mathbf{YZ})^2}{\underline{|5}} + \ldots\right\}$ 49

$\frac{1}{\mathbf{Z}_0}\sinh\mathbf{P} = \mathbf{Y}\left\{1 + \frac{\mathbf{YZ}}{\underline{|3}} + \frac{(\mathbf{YZ})^2}{\underline{|5}} + \ldots\right\}$ 49

Open-circuited line.

$\mathbf{E} = \mathbf{V}\cosh\mathbf{P}$ 49

$= \mathbf{Z}_0\mathbf{I}_S\coth\mathbf{P}$ 49

Sending-end impedance $\mathbf{Z}_S = \mathbf{Z}_0\coth\mathbf{P}$ 49

Line earthed or short-circuited.

$\mathbf{I}_x = \dfrac{\mathbf{E}}{\mathbf{Z}_0\sinh\mathbf{p}x}$ 50

$\mathbf{I}_S = \dfrac{\mathbf{E}}{\mathbf{Z}_0}\coth\mathbf{p}x$ 50

$\mathbf{Z}_S = \mathbf{Z}_0\tanh\mathbf{p}x$ 50

where x = distance of earth or short circuit from sending end.

ECONOMICS OF LINE

Economic Conductor Area.

$A = 6{\cdot}25I\sqrt{\dfrac{C_e}{rC_c}}$ for copper 54

$= 8I\sqrt{\dfrac{C_e}{rC_c}}$ for aluminium 54

where I = R.M.S. value of load current during year
C_c = cost in pence per mile of conductor of one square inch section.
C_e = cost of energy losses in pence per Kelvin
r = annual percentage interest and depreciation.

Economic Voltage.

$V = 5{\cdot}5\sqrt{L+\dfrac{\text{kVA.}}{150}}$ 56

where L = transmission distance in miles.

SAGS AND TENSIONS

Unloaded Line.

$D = \dfrac{wl^2}{8T_0}$ 59

$L = l+\dfrac{8D^2}{3l}$ 60

$T_B = T_0+wD$ 60

where L = length of line in feet
l = distance between supports in feet
D = maximum sag or dip in feet
w = weight of conductor in lb. per foot
T_0 = tension at lowest point
T_B = tension at support.

Line loaded with Ice and Wind.

Weight of ice per foot $w_i = 1{\cdot}244i(d+i)$ lb. 60

Wind pressure per foot $P = \frac{2}{3}(d+2i)$ lb. 61

Total pressure per foot $W = \sqrt{P^2+(w+w_i)^2}$ lb. 61

where i = radial thickness of ice in inches

d = conductor diameter in inches.

Erection Conditions.

Erection tension T may be calculated from

$$T^3+T^2\left\{AE\left(\alpha t+\frac{W^2l^2}{24T_c^2}\right)-T_c\right\}-\frac{w^2l^2AE}{24}=0 \quad . \quad . \quad . \quad . \quad 62$$

where T_c = maximum tension in line when cold

t = temperature during erection in °F.

α = expansion coefficient per degree F.

A = cross-sectional area of line in sq. inches

E = modulus of elasticity in lb. per sq. inch.

Erection dip $D = \dfrac{wl^2}{8T}$ 62

Line supported at Different Levels.

Sag below the higher support $D = \dfrac{wl_1^2}{8T}$ 65

where $l_1 = l+\dfrac{2Th}{wl}$

h = difference in levels of supports.

Spans of Unequal Length.

Equivalent span $l_e = \sqrt{\dfrac{\sum l^3}{\sum l}}$ 66

INSULATORS

String efficiency $= \dfrac{\text{flash-over voltage of } n \text{ units}}{n(\text{flash-over voltage of one unit})}$ . . . 74

For uniform potential distribution $C_m = \dfrac{mc}{n-m}$ 75

C_m = capacitance of mth link from the top to guard-ring

c = capacitance of each link to earth

n = number of units in the string.

CABLES

Thermal Resistance.

Single-core.

$S_1 = \dfrac{k}{2\pi}\operatorname{logh}\dfrac{r_1}{r}$ °C./watt per cm. 81

where r = radius of core

k = thermal resistivity of the insulation

r_1 = inside radius of sheath.

Concentric.

$$S_1 = \frac{k}{2\pi}\left[\tfrac{1}{2}\operatorname{logh}\frac{r_1}{r} + \operatorname{logh}\frac{r_2}{r_1+t}\right] \qquad \ldots\ldots \qquad 81$$

$$\doteqdot \frac{k}{2\pi}\operatorname{logh}\frac{r_2}{\sqrt{rr_1}} \qquad \ldots\ldots \qquad 82$$

where r_1 = inner radius of outer conductor
r_2 = inner radius of sheath
t = strand diameter or thickness of outer conductor.

Triple Concentric.

$$S_1 = \frac{k}{2\pi}\left[\tfrac{1}{3}\operatorname{logh}\frac{r_1}{r} + \tfrac{2}{3}\operatorname{logh}\frac{r_2}{r_1+t} + \operatorname{logh}\frac{r_3}{r_2+t_2}\right] \qquad \ldots \qquad 82$$

$$\doteqdot \frac{k}{2\pi}\operatorname{logh}\frac{r_3}{\sqrt[3]{rr_1r_2}} \qquad \ldots\ldots \qquad 82$$

where r_2 = inner radius of outer conductor
r = inner radius of sheath
t_2 = strand diameter or thickness of outside conductor.

Protective Covering.

$$S_2 = \frac{k_1}{2\pi}\operatorname{logh}\frac{R_2}{R_1} \qquad \ldots\ldots \qquad 83$$

where R_1 = external radius of lead sheath
R_2 = external radius of outer covering of the cable
k_1 = thermal resistivity of the covering.

Soil.

$$G = \frac{g}{3\pi}\operatorname{logh}\frac{2h}{R_2} \qquad \ldots\ldots \qquad 84$$

where g = thermal resistivity of the soil
h = depth of cable axis below ground level
R_2 = overall cable radius.

Current Loading.

Permissible current per conductor $I = \sqrt{\dfrac{T}{nRS}}$ $\qquad \ldots \qquad$ 84

where n = number of conductors
R = effective resistance per cm. of conductor
T = permissible temperature rise in °C.
$S = S_1+S_2+G$.

Dielectric Loss.

$$W = VI\sin\delta \doteqdot VI\delta \qquad \ldots\ldots \qquad 86$$

where I = charging current
δ = dielectric loss angle.

Potential Gradient.

Maximum gradient $g_{max} = \dfrac{E}{r \operatorname{logh} \dfrac{R}{r}}$ 89

where E = voltage between core and sheath
R = sheath radius.

Economical conductor radius $r = \dfrac{E}{g_{max}}$ 89

For minimum overall diameter with one intersheath:

Potential of intersheath to conductor $E_1 = \dfrac{E}{e}$ . . . 92

Radius of core $r = \dfrac{E}{eg}$ 92

Radius of intersheath $r_1 = \dfrac{E}{g}$ 92

Radius of sheath $R = r_1 e^{E_2/E}$ 92
where E_2 = potential of intersheath to outer sheath (earth)
$= E - E_1$.

LINE CONSTANTS

Insulation Resistance.

Resistance for single-core or concentric cable $= \dfrac{P}{l} \operatorname{logh} \dfrac{R}{r}$ ohms. 131

where P = dielectric resistivity in megohms per cm. cube
R = inner radius of sheath or outer conductor
l = length of cable in miles.
r = radius of core.

Inductance.

Single-phase.

$L = \left(4 \operatorname{logh} \dfrac{d-r}{r} + \mu\right) \times 10^{-9}$ henrys per loop cm. . . 133

$\doteqdot 0{\cdot}161 \left(4 \operatorname{logh} \dfrac{d}{r} + 1\right)$ millihenrys per loop mile . . 133

where μ = permeability of conductor
d = conductor spacing.

Three-phase.

Inductance per phase $L = 0{\cdot}0805 \left(4 \operatorname{logh} \dfrac{d}{r} + 1\right)$ millihenrys per mile 133

For delta spacing d = distance between wires.
For irregular spacing $d = \sqrt[3]{abc}$.
For regular flat spacing $d = 1{\cdot}26a$ 134

Capacitance.

Overhead Lines.

Single-phase.

Capacitance of each wire to mid-point or neutral is

$$C = \frac{1}{2\,\text{logh}\,\frac{d-r}{r}} \text{ e.s.u. per cm.} \quad . \quad . \quad . \quad . \quad 135$$

$$\doteqdot \frac{0{\cdot}0388}{\log_{10}\frac{d}{r}} \text{ microfarads per mile} \quad . \quad . \quad . \quad 135$$

where d = conductor spacing.

Three-phase.

$$\text{Capacitance per phase} = \frac{0{\cdot}0388}{\log_{10}\frac{d}{r}} \text{ microfarads per mile} \quad . \quad 136$$

For delta spacing d = distance between wires.
For irregular spacing $d = \sqrt[3]{abc}$.
For regular flat spacing $d = 1{\cdot}26a$.

Underground Cables.

Capacitance of single-core or concentric cables

$$C = \frac{0{\cdot}0388\epsilon}{\log_{10}\frac{R}{r}} \text{ microfarads per mile} \quad . \quad . \quad 137$$

$$\doteqdot \frac{0{\cdot}14}{\log_{10}\frac{R}{r}} \text{ microfarads per mile} \quad . \quad . \quad 137$$

for paper when $\epsilon = 3{\cdot}6$.

TRAVELLING WAVES

$$\text{Surge impedance } Z = \sqrt{\frac{L}{C}} \quad . \quad . \quad . \quad . \quad . \quad . \quad 140$$

$$= 120\,\text{logh}\,\frac{d}{r} \quad , \quad . \quad . \quad . \quad 141$$

Reflection.

$$\text{Reflected current wave } I_R = \frac{Z_1-Z_2}{Z_1+Z_2}I \quad . \quad . \quad . \quad . \quad 142$$

$$\text{Transmitted current wave } I_T = \frac{2Z_1}{Z_1+Z_2}I \quad . \quad . \quad . \quad . \quad 142$$

$$\text{Transmitted voltage wave } E_T = \frac{2Z_2}{Z_1+Z_2}E \quad . \quad . \quad . \quad . \quad 142$$

$$\text{Reflected voltage wave } E_R = \frac{Z_2-Z_1}{Z_1+Z_2}E \quad . \quad . \quad . \quad . \quad 142$$

where E, I are the voltage and current values of the wave incident in a line of surge impedance Z_1.

PETERSEN COIL

Reactance of coil $X = \dfrac{1}{3\omega C}$ 149

where C is the capacitance of each healthy conductor to earth when the third is earthed.

CORONA

R.M.S. disruptive critical voltage

$$E_0 = 123 m_0 r\delta \log\frac{d}{r} \text{ kV. to neutral} \quad . \quad . \quad 153$$

R.M.S. visual critical voltage

$$E_v = 123 m_v r\delta\left(1+\frac{0{\cdot}189}{\sqrt{r\delta}}\right)\log\frac{d}{r} \text{ kV. to neutral} \quad 153$$

$$\text{where } \delta = \frac{3{\cdot}92(\text{barometric height in cm.})}{273+\text{temperature °C}}$$

m_0 = irregularity factor

m_v = surface factor.

(r is expressed in in.)

$$\text{Corona power loss } P = \frac{390\times 10^5}{\delta}(f+25)(E-E_0)^2\sqrt{\frac{r}{d}} \text{ kW. per mile} \quad 153$$

where f = frequency

E = voltage to neutral

E_0 = disruptive critical voltage to neutral.

ANSWERS

CHAPTER I

1. D.C. Copper = 1·07 A.C. Copper.
2. D.C. Copper = 3·43 A.C. Copper.

CHAPTER II

1. 600; 400; 0; 100; 600 amp.; 438·2; 435·8; 435·8; 436·4 volts.
2. 0·106 sq. in.
3. At the 25 kW. load; 0·08 ohm.
5. (*a*) 1,160; 1,232; 1,713 amp. (*b*) 556 amp. (*c*) 0·886.
6. 0·29; 0·045; 0·126 sq. in. respectively.
7. 4,136; 2,636; 136; 864; 3,864 kW. respectively.
8. First feeder, 4,780 kW.; 5,580 kVA.
Second feeder, 5,220 kW.; 6,975 kVA.

CHAPTER III

1. 4,670 volts.
2. 7 miles; 7,160 volts.
3. (*a*) 11,055 volts, (*b*) 10,988 volts.

CHAPTER IV

1. 67 kV.
2. $41800\ \underline{|9^\circ\,44'}$.
3. 73·65 kV; 165·3 amp; 0·8; 94·84 per cent.

CHAPTER V

1. 73·66 kV; 165·3 amp.; 0·8; 94·82 per cent.
2. $41750\ \underline{|9^\circ\,43'}$.

CHAPTER VI

1. 0·627 ohm per mile.
2. 0·2 sq. in.
3. 0·061 sq. in.
4. 0·5 ohm per mile.
5. 0·173 sq. in.

CHAPTER VII

1. 6·77 feet.
2. 18·85 feet for steel-cored aluminium; 32·75 feet for copper.
3. 5·31; 2·99; 4·86 feet.; 1131; 695 lb.
4. (*a*) 17·08; (*b*) 16·1; (*c*) 14·4; (*d*) 17·0 feet.

CHAPTER VIII

1. 0·31 V.; 0·303 V.; 0·387 V. across the line end insulator; 86·13 per cent.
2. (*a*) 7,800; 9,400; 12,800 volts. (*b*) 10,000 volts each.

CHAPTER IX

1. 47·61 kV. per cm.
2. 2·72 cm.; 34 kV. per cm.
3. 370 amp.
4. 0·014.
5. 41·7 kV.; 0·81; 2·2; 4·14 cm.

CHAPTER XI

1. 138 megohms.
2. (*a*) 1·3 millihenrys; (*b*) 1·82 millihenrys.
3. 0·64 millihenry.
4. 0·0412 henry; 0·29 microfarad; 6·95 amp.
5. 0·607 microfarad per mile.
6. 1·03 microfarad.
7. 667 megohms.
8. 0·094 megohms.

CHAPTER XII

1. 32 kV.; 80 amp.
2. 42·4 kV.
3. 11·9 amp.; 35·8 kV.
4. 29·5 ohms.
5. 770 ohms.
6. 71·1 kV.

CHAPTER XIII

1. 118 kV.
2. (*a*) 145 kV.; (*b*) 124 kV.
3. 180 kV.
4. 1·12 in.; 25·8 kW. per mile.
5. 10 feet.

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