A TEXT-BOOK

ON

ROOFS AND BRIDGES

PART II

GRAPHIC STATICS

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BY

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PREFACE TO THE FIFTH EDITION

THE course of instruction in ROOFS AND BRIDGES presented in this text-book consists of four parts. Part I deals with the computation of stresses in roof trusses and in all the common styles of simple bridge trusses. Part II treats of the determination of stresses by graphic methods. Part III presents the methods for the design of steel bridges, including proportioning of details and preparation of general drawings. Part IV discusses continuous, cantilever, movable, suspension, and arched bridges.

In the following pages the second part of this course is presented. The authors regard it as essential that students should completely work out a few typical cases like those here given in the four full-page plates; they also consider it as important that students should solve many practical problems like those given at the ends of most of the articles. In this volume, as in Part I, the minimum as well as the maximum stresses are determined for each case, and all varieties of loading are treated, thus training students to use all kinds of specifications.

For this edition the entire book has been rewritten. Obsolete examples and illustrations have been replaced by others in accordance with present-day practice. Influence lines have been introduced earlier. An article on the transverse bent and another on the viaduct tower have been added. Highway trusses have been treated separately and before railway trusses because of their more simple uniform loading.

Chapter VIII, upon influence lines for stresses in simple bridge trusses, employs a method which is perfectly general and may be applied as readily to a truss which is irregular in form or proportion as to any other. The method is graphic throughout, and hence it is not necessary to employ the equations of influence lines as in other methods. In modern practice the use of influence lines is restricted chiefly to trusses with inclined chords and with subdivided panels, and hence only four types of trusses are used in the illustrative examples. This method of treatment is especially adapted to finding the loading and stresses in trusses of a new, or unfamiliar, type like the K truss which has not yet come into common use in this country, but the use of which is increasing. One article is devoted especially to that purpose.

It is believed that Chapter IX, on deflection influence lines, contains valuable methods of determining the deflection of beams. The graphic methods are completely illustrated by examples. Especial attention has been paid to the units of measure employed in making the diagrams so as to be in full accord with the fundamental principles of constructing equilibrium polygons.

Grateful acknowledgments for photographs and other material are due to Joseph J. Yates, Clarence W. Hudson, Wm. K. Greene, O. F. Dalstrom, and the Iowa Highway Commission.

January, 1932

CONTENTS

Chapter I

PRINCIPLES AND METHODS

Chapter II

ROOF TRUSSES

Chapter III

HIGHWAY BRIDGE TRUSSES

Chapter IV

RAILWAY GIRDERS AND TRUSSES

CHAPTER V

RAILWAY TRUSSES WITH BROKEN CHORDS

CHAPTER VI

MISCELLANEOUS STRUCTURES

CHAPTER VII

ELASTIC DEFORMATION OF TRUSSES

Chapter VIII

INFLUENCE LINES FOR STRESSES

Chapter IX

DEFLECTION INFLUENCE LINES

Part II

GRAPHIC STATICS

CHAPTER I

PRINCIPLES AND METHODS

Art. 1. Introduction

Statics is the science which deals with forces in equilibrium. Graphic Statics is the solution of statical problems by means of geometric constructions. Numerous problems arise in the design of roofs and bridges which can be more conveniently solved by graphic constructions than by algebraic analysis. Other problems lend themselves to algebraic solutions and this method should be used. The main advantage of graphical solutions is that they can usually be made with a considerable saving of time. Often they also have the added advantage of being more clearly and more easily understood.

A 'force' is called by such various names as load, weight, pressure, or reaction, but can most easily be visualized as a push or a pull acting upon a body. A force is determined when its magnitude, direction, and line of action are known. These are represented graphically by the length, direction, and position of a straight line. Forces are usually given in pounds or kips (a kip equals 1000 lb.), whereas the lengths of lines are measured in inches. For convenience, an engineer's scale having the inch divided into 10, 20, 30, 40, 50 or 60 parts is used for measuring lengths of lines. For example, using a scale of 3000 lb. to 1 in., a force of 5460 lb. would be represented by a line 1.82 in. long. Using the scale having 30 divisions to the inch, this can be read directly as 54.6 divisions.

The direction of a force is usually determined by the angle it makes with the horizontal or vertical. This angle may be measured with a protractor but more accuracy can be obtained by using the tangent method. For example, a force makes an angle of 35° 20′ with the horizontal. The natural tangent of this angle is 0.709. Lay off 1000 units horizontally to the largest convenient scale, say 200 units to 1 in. From the ends of this line lay off 709 units vertically. The accuracy of graphic solutions depends upon the accuracy with which angles and lines are measured, so that any simple aids such as this are well worth while.

ART. 2. THE FORCE TRIANGLE

The resultant of two or more forces is a single force which produces the same effect as the forces themselves, and may therefore replace them. Let two forces P_1 and P_2 (Fig. 2*a*) which act in the same plane through the point *m* be represented in magnitude and position by the lines *mn* and *mp* and in direction by the arrows. Let the parallelogram be completed by drawing a line through *n* parallel to P_2, and a line through *p* parallel to P_1 and then let *m*

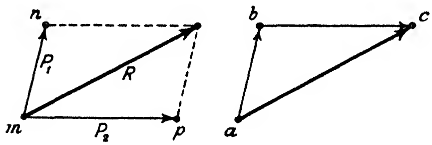


FIG. 2*a*.

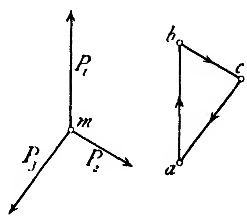


FIG. 2*b*.

be joined with their point of intersection. This line, designated by *R*, represents the resultant of the two given forces.

It will readily be seen that it is not necessary to construct the entire parallelogram, since the triangles on the opposite sides of the diagonal are equal. The triangle above the diagonal can be constructed by drawing a line through *n* parallel to P_2, laying off upon it the value of P_2 and then joining its end to *m*; similarly, the lower triangle can be independently drawn. Either of these triangles is called the 'force triangle.'

If the lines of action of the given forces form a part of a diagram upon which it is not desirable to construct the force triangle, any suitable point *a* may be selected and *ab* drawn parallel and equal to P_1; then *bc* is drawn through *b* parallel and equal to P_2. The line joining *a* and *c* represents the magnitude of the resultant *R* and is measured by the same scale as that used in laying off *ab* and *bc*. The direction in which the resultant acts is indicated by the arrow upon *ac*, and this is seen to be opposed to the directions of those upon *ab* and *bc* in following around the triangle. Finally, the line of action of the resultant R must pass through *m*, the point of intersection of the given forces P_1 and P_2. Hence the resultant R is found in magnitude, direction, and line of action by drawing a line through *m* equal and parallel to *ac*.

The combining of two or more forces into a single force is known as the 'composition of forces.' Conversely, the resolving of a single force into two or more forces (components) is known as the 'resolution of forces.' This may also be effected by the force triangle. For instance, let R in Fig. 2*a* be the given force, and let it be required to find its components in the directions of *mn* and *mp*. Let *ac* be drawn equal and parallel to R, and through its extremities let *ab* and *cb* be drawn parallel to the given directions; these lines intersect at *b*, and when they are measured by the scale the magnitude of the components P_1 and P_2 will be known. Lastly, through *m*, the point of application of R, let P_1 and P_2 be laid off in the given directions, equal to *ab* and *bc*, and the lines of action of the components are determined.

Several forces are said to be in equilibrium when no tendency to motion is produced in the body upon which they act. In Fig. 2*a* suppose a force P_3, equal and opposite to R, to be applied at *m*; then this force together with P_1 and P_2 will be in equilibrium, for the last two may be replaced by their resultant R, which by the conditions specified holds P_3 in equilibrium. The corresponding force triangle will be *abc* with the direction of *ac* reversed, so that all forces around the triangle have the same direction; hence, when three forces are in equilibrium, they form a closed force triangle.

When three forces whose lines of action lie in a plane and intersect at one point are in equilibrium, any one may be determined when two are given. In Fig. 2*b* let P_1 and P_2 be given, to find P_3. Let *ab* be laid off equal and parallel to P_1, and from *b* let *bc* be drawn equal and parallel to P_2; then *ca*, the closing side of the triangle, represents P_3 in magnitude and direction. As its line of action must also pass through *m*, the force P_3 is drawn equal and parallel to *ca*, and in the same direction, thus completing the solution.

Should only one force be given, together with the lines of action of the other two, their magnitudes and directions may be found. In Fig. 2*b*, let P_1 and the lines of action of P_2 and P_3 be given. Draw *ab* equal and parallel to P_1 and through its extremities draw lines parallel to P_2 and P_3; these lines intersect at *c*, and the length of *bc* gives the magnitude of P_2, its direction being from *b* to *c*, in the same direction around the force triangle as *ab*. P_3 is represented by *ca* in the same manner.

The force triangle is the foundation of the science of graphic statics. By it all problems relating to the composition and resolution of forces can be solved, when the forces are but three in number and act in the same plane upon a common point.

PROBLEM 2*a*.—Two forces of 25 lb. and 40 lb. make an angle of 68° with each other. Find the magnitude of their resultant and the angle it makes with each force.

PROBLEM 2*b*.—A vertical force of 2600 lb. acts upon a roof having a slope of 25°. Find the components of the force parallel and normal to the roof.

ART. 3. THE FORCE POLYGON

When it is required to find the resultant of a number of forces acting in the same plane and having a common point of application, the resultant of two of the forces may be found by the use of the force triangle as in Art. 2, a third force may be combined with this resultant to obtain a second resultant, and this operation continued until all the forces are combined. In Fig. 3*a*, the forces P_1, P_2, P_3, and P_4 act through the common point *m*. The force triangle *abc* is laid out combining P_1 and P_2 into the resultant R_1; the force triangle *acd* is next laid out combining R_1 and P_3 into the resultant R_2; finally, the force triangle *ade* is laid out combining R_2 and P_4 into the resultant R. Since all the forces have been combined, R is the resultant of the forces P_1, P_2, P_3, and P_4; its magnitude and direction are given by *ae* and its line of action is through *m*.

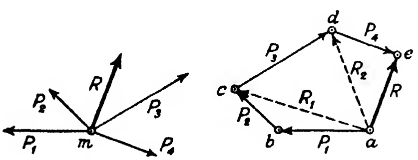


FIG. 3*a*.

It will readily be seen that it is not necessary to construct the resultants R_1 and R_2 in order to obtain the final resultant R, and they are generally omitted. The polygon *abcde* is known as the 'force polygon'; the resultant *R* forms its closing side and each of the other sides represents one of the given forces. The direction of the resultant is opposed to the direction of all the given forces in following around the sides of the polygon; thus the arrow on *ae* has the reverse direction of the other arrows.

The force polygon may therefore be constructed as follows:

> Draw in succession lines parallel and equal to the given forces, each line beginning where the preceding one ends, and extending in the same direction as the force it represents. The line joining the initial to the final point represents the resultant in direction and magnitude.

To produce equilibrium in Fig. 3*a*, suppose a force P_5, equal and opposite to *R*, to be applied at *m*. This added force in the force polygon is equal to *ea* with its former direction reversed, so

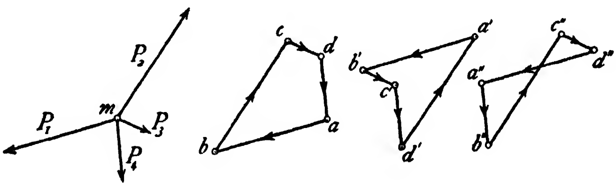


FIG. 3*b*.

the distance from the initial to the final point in the construction of the polygon becomes zero. Hence, if a number of forces lying in the same plane and having a common point of application are in equilibrium, they will form a closed force polygon, and in passing around it all the forces will have the same direction.

It makes no difference in what order the forces in the force polygon are arranged. In Fig. 3*a*, the sides of the force polygon were drawn in the order P_1, P_2, P_3, P_4, R; but the same value of *R*, both in intensity and direction, will be obtained if they are drawn in any other order, as for example, P_3, P_1, P_4, P_2, R. In Fig. 3*b*, the four forces meeting at *m* are in equilibrium. Taking them in the order P_1, P_2, P_3, P_4 the force polygon *abcd* is drawn; in the order P_1, P_3, P_4, P_2 the force polygon $a'b'c'd'$ results; and in the order P_4, P_2, P_3, P_1 the force polygon $a''b''c''d''$ is found, each of which graphically represents the given forces. In the last case it is seen that two of the lines cross each other; this is a frequent occurrence in practical problems.

The force triangle (Art. 2) is but a particular case of the force polygon, namely, when the forces are but three in number. Hence, the word polygon is often used in a general sense as including that of the triangle.

PROBLEM 3*a*.—Draw a force polygon for five forces in equilibrium, and prove that any diagonal of the polygon is the resultant of the forces on one side and holds in equilibrium those on the other side.

PROBLEM 3*b*.—Three forces of 120 lb., 175 lb., and 50 lb. make angles of 35°, 87°, and 238° with each other. Draw three different force polygons and determine from each the value of the resultant and the angle it makes with the 120-lb. force.

ART. 4. THE RESULTANT OF NONCONCURRENT FORCES

Article 3 deals entirely with forces that all intersect at a common point, or 'concurrent forces.' In Fig. 4*a*, a body is acted

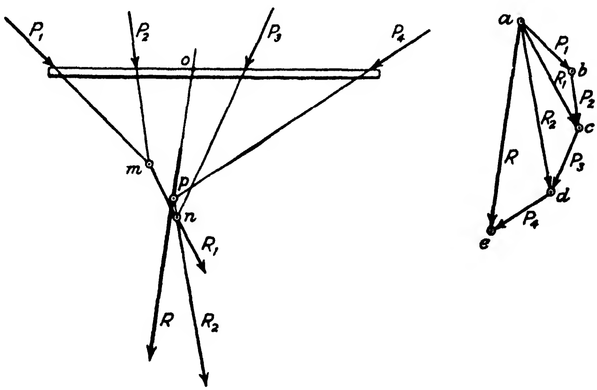


FIG. 4*a*.

upon by four forces P_1, P_2, P_3, and P_4 which do not intersect at a common point, or 'nonconcurrent forces.' It is desired to find the magnitude, direction, and line of action of the resultant. P_1 and P_2 may be combined into the resultant R_1 by drawing the force triangle *abc*, and its line of action must be through *m*, the intersection of P_1 and P_2; next R_1 and P_3 may be combined into the resultant R_2, by drawing the force triangle *acd*, and its line of action must be through *n*; finally, R_2 and P_4 may be combined into the final resultant R, by drawing the force triangle *ade*, and its line of action must be through p. By extending the line of action of R, the point of application on the body is found to be at o. The magnitude and direction of the resultant were obtained from the force polygon as in Art. 3, but to find its line of action it was necessary to construct the intermediate resultants R_1 and R_2.

When the forces acting upon a body are parallel or so nearly parallel that they do not intersect within the limits of the drawing, the line of action of the resultant cannot be determined as described above. In Fig. 4*b*, a body is acted upon by four nearly parallel forces. The system of marking the spaces between the forces instead of marking the forces themselves is used in Fig. 4*b*

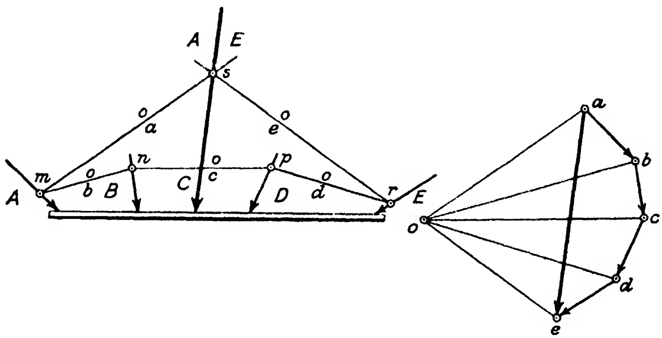


FIG. 4*b*.

and will be used in all subsequent figures except the very simplest. A force is designated in the space diagram by the upper-case letters on both sides of it, as *AB*, *BC*, etc., and in the force polygon by the lower-case letters at both ends, as *ab*, *bc*, etc. The magnitude and direction of the resultant may be determined from the force polygon as before and is equal to *ae*. Resolve the force *AB* into two components (Art. 2) by taking any point o and drawing the force triangle *oab*. *AB* may be replaced by its components *oa* and *ob* drawn from any point m on its line of action. The forces *ob* and *BC* may be combined into the resultant *oc*, acting through n, their point of intersection, by drawing the force triangle *obc*; next *oc* and *CD* may be combined into the resultant *od*, acting through p, by drawing the force triangle *ocd*; finally, *od* and *DE* are combined into the resultant *oe*, acting through r, by constructing the force triangle *ode*. All four forces have been combined into the two forces *oa* and *oe*. Their resultant, and hence the resultant of all the forces, must act through their intersection at *s*.

If in Fig. 4*b* there be applied through the point *s* a force equal and parallel to the resultant *AE*, but opposite in direction, the forces *AB*, *BC*, *CD*, *DE*, and *EA* are in equilibrium and the force polygon closes. The polygonal frame *mnprs* thus holds the given forces in equilibrium by the stresses of tension or compression acting in its members, in this case tension in *ms* and *rs* and compression in *mn*, *np*, and *pr*. The lines of this frame are hence called an 'equilibrium polygon.' If the equilibrium polygon be regarded as a structural frame supporting the external forces, the stresses in the members cut by any section hold in equilibrium the external forces on either side of the section.

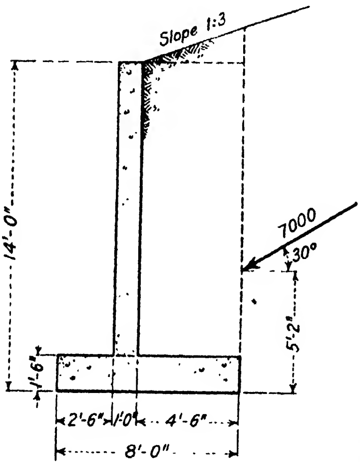


FIG. 4*c*.

The point *o* in the force polygon is called the 'pole,' and the lines *oa*, *ob*, etc., are called 'rays.' Since the position of the pole may be selected at pleasure it follows that for any given system of forces an indefinite number of equilibrium polygons can be constructed. It should be noted that in the equilibrium polygon the *oa* ray is always drawn across the *A* space; the *ob* ray across the *B* space; etc.

PROBLEM 4*a*.—Two parallel forces of 650 lb. and 370 lb. are 10 ft. apart and act in opposite directions. Find, by the force and equilibrium polygons, the magnitude and line of action of their resultant.

PROBLEM 4*b*.—Figure 4*c* shows the cross-section of a concrete cantilever retaining wall. The concrete weighs 150 lb. per cu. ft. and the earth fill 100 lb. per cu. ft. Consider the concrete and fill divided as shown by the broken lines and find the weight of each section for a wall 1 ft. long. The 7000-lb. force shown is the total pressure of the earth fill on a section of wall 1 ft. long. Find the resultant of all the forces acting on the wall and the point at which it passes through the base: (1) By using only force triangles; (2) by using only the force and equilibrium polygons; (3) by finding the resultant of all vertical forces and combining it with the inclined force.

ART. 5. CONDITIONS OF EQUILIBRIUM

When several forces lie in the same plane the necessary and sufficient conditions of static equilibrium are that there shall be no tendency to motion, either of translation or rotation. Analytically this is expressed by saying that the algebraic sum of the components, both horizontal and vertical, of the forces must be zero, and that the algebraic sum of the moments of the forces must also be zero; or as commonly expressed, $\Sigma H = 0$, $\Sigma V = 0$, and $\Sigma M = 0$.

When the given forces have a common point of application (concurrent forces), the graphic condition for equilibrium is that the force polygon must close, for, if it does not close, the line joining the initial with the final point represents the resultant of the given forces (Art. 3), and this resultant will cause motion; and if it does close there exists no resultant. Therefore, if the given forces which meet at a common point are in equilibrium, the force polygon must close; and conversely, if the force polygon closes, the given forces must be in equilibrium.

When several forces lying in the same plane have different points of application, so that their lines of action do not intersect in the same point (nonconcurrent forces), and are in equilibrium, the force polygon must close, since no resultant can exist. However, if the force polygon does close, it is not necessarily true that the forces are in equilibrium. For example, let a body be acted upon by three equal forces as shown in Fig. 5*a*, *AB*, and *BC* making an angle of 30° with the horizontal and *CA* being vertical. It is plain that equilibrium is here impossible, and yet the force polygon *abc* closes. The resultant of *AB* and *BC* would be a vertical force equal to *CA* acting downward midway between them. This resultant and *CA* form a couple so that equilibrium can be maintained only by another couple acting in the opposite direction. It is because the resultant of the forces of a couple is zero that the force polygon closes in this case; and it will be found that, in all cases of non-equilibrium where the force polygon closes, a couple is necessary to maintain equilibrium.

The three forces in Fig. 5*b* are equal in magnitude and make an angle of 120° with each other. The force polygon *abc* closes, but the forces are not in equilibrium because their lines of action do not intersect in the same point. Select a pole at any point *o* and draw the rays *oa*, *ob*, and *oc*. Now, from any point on the line of action of *AB*, draw the rays *oa* and *ob* parallel to their direction in the force diagram; from the point where *oa* intersects the line of action of *CA*, draw the ray *oc* parallel to its direction in the force diagram. The rays *ob* and *oc* intersect at *r*, which is not on the line of action of *BC*, hence the three given forces cannot be held in equilibrium by the equilibrium polygon. In this case it is

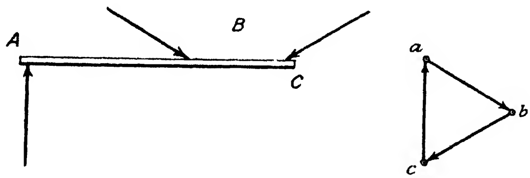


FIG. 5*a*.

said that the equilibrium polygon does not close. If, however, the force *BC* be moved parallel to itself until its line of action passes through *r*, the equilibrium polygon closes and the forces will be in equilibrium. Therefore, if the given forces which do not meet at a common point are in equilibrium, both the force polygon and the equilibrium polygon must close.

From the foregoing examples it can be seen that the closing

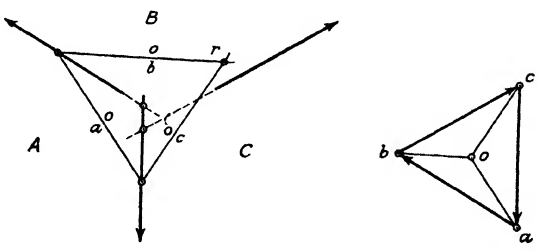


FIG. 5*b*.

of the force polygon is the graphic equivalent of $\Sigma H = 0$ and $\Sigma V = 0$, and that the closing of the equilibrium polygon is the graphic equivalent of $\Sigma M = 0$. In general, three unknowns may therefore be determined from the two polygons; two from the force polygon and one from the equilibrium polygon. A single force may have three unknowns (Art. 1), magnitude, direction, and line of action, as for example, the resultant of the given forces in Figs. 4*a* and 4*b*. In Figs. 2*a* and 3*a* the line of action of the resultant was known so that there were only two unknowns, both of which could be determined from the force polygon alone. The three unknowns may not all belong to the same force. For example, the line of action of one force and the direction and line of action of a second may be known; from the force and equilibrium polygons the magnitude and direction of the first and the magnitude of the second can be determined.

PROBLEM 5*a*.—A beam 10 ft. long carries a vertical load of 600 lb. 4 ft. from the left end and one of 400 lb. 2 ft. from the right end. Is the beam in equilibrium if vertical forces of 400 lb. at the left end and 600 lb. at the right end are acting upward?

ART. 6. REACTIONS OF BEAMS

The beam in Fig. 6*a* is acted upon by the three vertical forces *AB*, *BC*, and *CD*, and it is desired to determine the forces, or

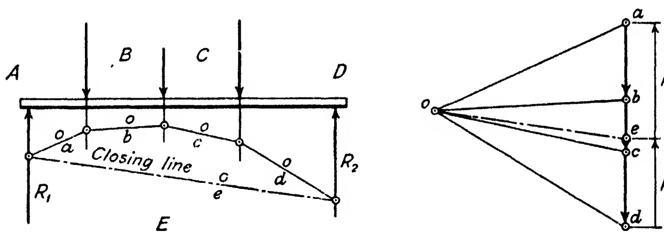


FIG. 6*a*.

reactions, *DE* and *EA* applied at the ends of the beam to hold it in equilibrium. Since the applied loads are vertical, both reactions will be vertical. Therefore the direction and line of action of each reaction is known and there will be only two unknowns, the magnitude of each. Lay off the given forces in the force polygon *abcd*. These forces represent the loads on the beam, so that their force polygon is called a 'load line.' Since the force polygon must close and the reactions are vertical, the line *da* represents the sum of the reactions, and the point *e* must lie at some place on that line. Select any pole *o* and draw the rays *oa*, *ob*, *oc*, and *od*. Starting with any point on the line of action of *AB*, construct the sides *oa*, *ob*, *oc*, and *od* of the equilibrium polygon. Since the equilibrium polygon must close, the side *oe*, or 'closing line,' can be drawn from the point where *oa* intersects *EA* to the point where *od* intersects *DE*. The point *e* can now be located in the force polygon by drawing a ray from *o* parallel to the closing line. The magnitudes of the reactions *DE* and *EA* are represented by *de* and *ea*, respectively.

In Fig. 6*b* an overhanging beam is acted upon by two vertical forces *AB* and *BC* and an inclined force *CD*; it is desired to determine the reactions *DE* and *EA* acting through the points *m* and *n*. The reaction at *m* is on rollers so that its direction must be vertical. The unknowns are three in number; the magnitude and direction of the reaction at *n* and the magnitude of the reaction at *m*. Lay off the load line *abcd* and draw the rays *oa*, *ob*, *oc*, and *od* to any

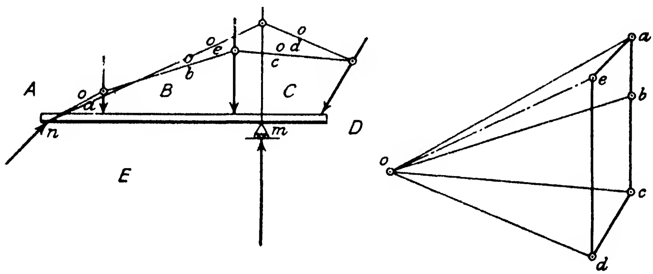


FIG. 6*b*.

pole *o*. The equilibrium polygon can now be constructed. The ray *oa* must be drawn from *AB* to *EA*. Since the direction of *EA* is not known, *n* is the only known point on its line of action, and the equilibrium polygon must be started there. The equilibrium polygon can be completed as before. The point *e* in the force polygon can be located by drawing the ray *oe* parallel to the closing line until it intersects a vertical line through *d*. The magnitude of *DE* is represented by *de*, and the magnitude and direction of *EA* by *ea*.

If the reaction *DE* in Fig. 6*b* had not been on rollers its direction would also have been unknown, making a total of four unknowns. This is one more than can be determined, so that in order to solve the problem it is necessary to make some assumption as to the distribution of the horizontal components between the reactions. An assumption frequently made is that the reactions are parallel; *da* (Fig. 6*c*) then represents the sum and direction of the two reactions. The point *e* must lie at some point on the line *da* and also on the ray through *o* parallel to the closing line. The magnitude and direction of the reactions *DE* and *EA* are represented by *de* and *ea*, respectively.

It will readily be seen by taking moments about either reaction that the vertical component of the other reaction is independent of the magnitude of the horizontal component. Therefore, in Fig. 6*c*, the point dividing the reactions must fall on a horizontal line through *e* regardless of the division of the horizontal components between the reactions. Assuming the right reaction to be vertical, as in Fig. 6*b*, the point falls at e'. Another assumption sometimes made is that the horizontal components of the reactions are equal, in which case the point falls at e'', and the reactions *DE* and *EA* are represented in magnitude and direction

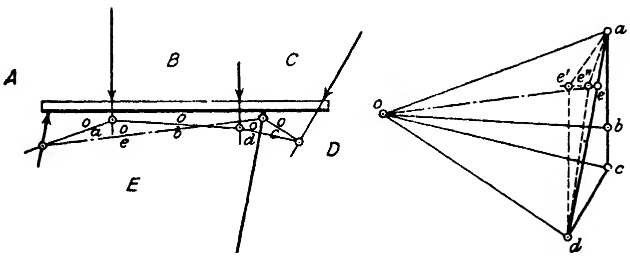


Fig. 6*c*.

by de'' and $e''a$, respectively. A solution for any other distribution of the horizontal components can be made in a similar manner.

A direct solution for the reactions for any given distribution of the horizontal components can be made in two ways. The first method is to pass the equilibrium polygon through the point of application of one reaction and resolve the other reaction into its vertical component and assumed horizontal component. A second method is to choose the pole in such a position that the equilibrium polygon will pass through the points of application of both reactions. The method for passing an equilibrium polygon through two given points will be explained in Art. 8.

Problem 6*a*.—An overhanging beam 12 ft. long carries a vertical load of 4000 lb. 4 ft. from the left end and one of 3000 lb. at the right end. One reaction is at the left end and the other 2 ft. from the right end: (1) Determine the reactions for the beam carrying each load separately and add them together. (2) Determine the reactions for the beam carrying both loads at once. (3) Determine the resultant of the two loads and find the reactions for it.

ART. 7. GRAPHICAL DETERMINATION OF MOMENTS

The beam in Fig. 7*a* is in equilibrium under the five forces shown. The bending moment is desired at the point *m*. Lay out the force polygon *abcde* and with any pole *o* draw the rays *oa*, *ob*, etc. The equilibrium polygon can now be drawn. The two forces *EA* and *AB* are to the left of the point *m*, and from the force polygon their resultant is *eb*, or *R*. This resultant will act through the intersection of the *ob* and *oe* rays of the equilibrium polygon at *s*. The resultant of the forces to the right of *m* would also act through *s* and be equal to *R* but opposite in direction.

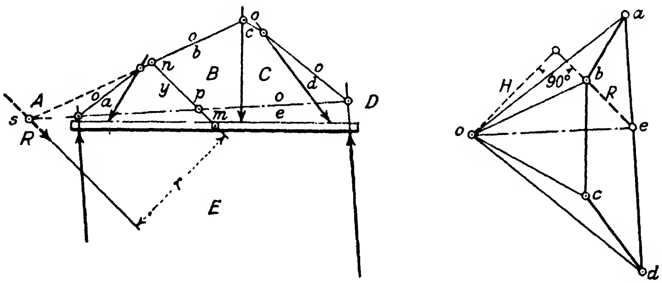


FIG. 7*a*.

The bending moment at *m* is evidently equal to the resultant *R* multiplied by *r*, its normal distance from *m*, or

$$M = Rr$$

Draw a line through *m* parallel to *R* and let the ordinate *np* intercepted by the equilibrium polygon be called *y*. Also erect a perpendicular to *R* in the force polygon and let it be called *H*. The triangle *snp* is similar to the triangle *obe* since their sides are parallel and

$$r : y :: H : R, \quad \text{or} \quad Rr = Hy$$

Therefore the bending moment of the external forces on the left of *m* (and also of those on the right) is

$$M = Hy$$

The force *H* is the component of the stresses *ob* and *oe* perpendicular to their resultant and should be measured by the same scale as the other forces in the force polygon. The following theorem can hence be stated:

> The bending moment of a system of forces in equilibrium, about any point, is equal to the intercept y in the equilibrium polygon on a line through the point parallel to the resultant of the forces on either side of the point, multiplied by the perpendicular distance H from the resultant to the pole in the force polygon.

When all the forces are parallel y becomes the ordinate of the equilibrium through the point parallel to the forces, and H becomes the perpendicular distance of the pole from the load line and is the same for all points.

PROBLEM 7a.—The beam in Fig. 6a is 16 ft. long. The loads are AB = 1200 lb., BC = 800 lb., and CD = 1200 lb. applied at 3, 7, and 11 ft., respectively, from the left end. Determine the bending moment at each load. Choose a different pole and again determine the bending moments.

ART. 8. EQUILIBRIUM POLYGON THROUGH TWO OR THREE POINTS

In Art. 6 one case was noted where it was desirable to draw an equilibrium polygon through two given points. Numerous other

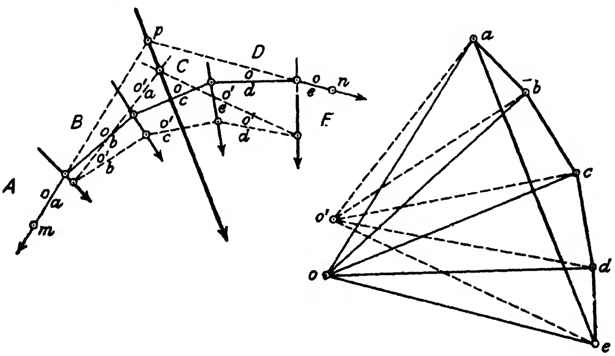


FIG. 8a.

cases arise so that two solutions of this problem will now be described.

It is desired to construct an equilibrium polygon passing through the points m and n in Fig. 8a. Lay out the load line $abcde$ and with any pole o' construct an equilibrium polygon locating the resultant of the given forces. The resultant will have the same location regardless of where the pole is chosen or where the equilibrium polygon is drawn. The outside rays must intersect on the resultant. Therefore, if any point *p* on the line of action of the resultant is chosen and the lines *mp* and *np* drawn, these lines may be taken as the sides *oa* and *oe* of an equilibrium polygon that will pass through *m* and *n*. The location of the pole *o* will be at the intersection of the rays *oa* and *oe* in the force diagram drawn parallel to their position in the equilibrium polygon. The other sides of the equilibrium polygon can now be completed. Since *p* was taken at any point on the line of action of the resultant, an indefinite number of equilibrium polygons can be constructed

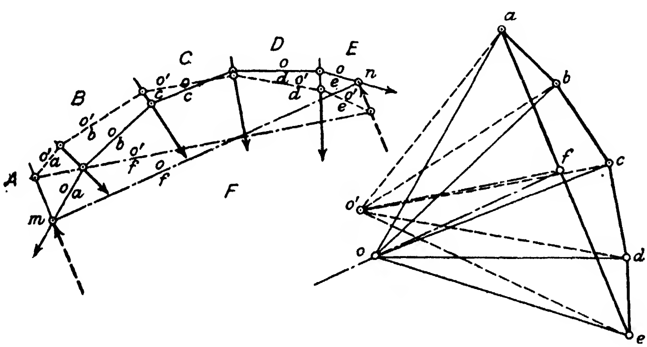


FIG. 8*b*.

that will pass through the two given points. In case it is desired to have the equilibrium polygon pass through a point between some of the given forces, only the forces between the points should be considered in locating a pole.

Another method of passing an equilibrium polygon through two points is shown in Fig. 8*b*. The given forces are assumed to be acting on a beam supported at the given points *m* and *n* by reactions parallel to the resultant. These reactions are determined as described in Art. 6 by constructing the force polygon and rays from any pole *o'*, then constructing the equilibrium polygon and locating point *f* by drawing a ray from *o'* parallel to the closing line. Obviously the value of the reactions and the location of *f* are not dependent upon the pole location so that any closing line must strike the point *f* in the force polygon. For an equilibrium polygon to pass through m and n the closing line must be the line mn. Therefore, if a pole is chosen at any point o, on a line through f parallel to mn, the equilibrium polygon will pass through m and n.

Passing an equilibrium polygon through three given points is just an extension of the two-point problem. In Fig. 8c the three given points are m, n, and p. Consider first only the points m and n and the forces between them. If a pole is chosen at any place on the line through j parallel to mn the equilibrium polygon will pass through m and n. Similarly, considering only the points n and p and the forces between them, if a pole is chosen at any place on the line through h parallel to np the equilibrium polygon

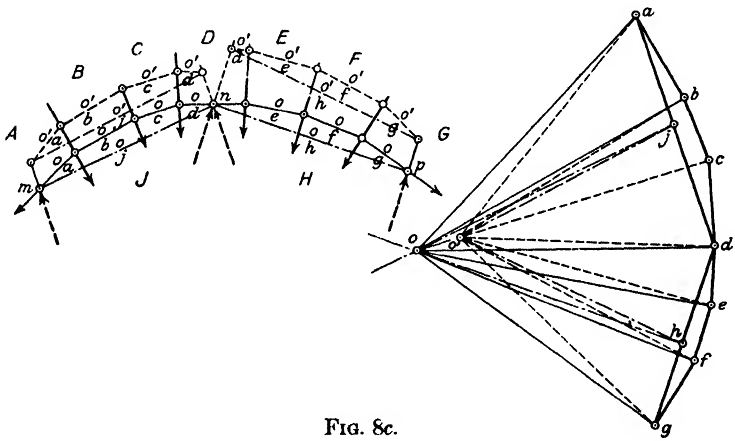


FIG. 8c.

will pass through n and p. Therefore, for the equilibrium polygon to pass through all three points, the pole must be at o, the intersection of the two lines. In this case there is only one possible location of the pole.

PROBLEM 8a.—A footbridge is carried by suspension cables. The distance between tower supports is 266 ft., and the sag of the cable is 30 ft. The walkway is level and supported from the suspension cables by 18 hangers spaced 14 ft. apart. The load on each hanger is 12 kips, and the center hangers are 4 ft. long. Determine the shape of the cables, the length of the hangers, and the maximum tension in the cables. (Since the bridge is symmetrical, the cables will be horizontal at the center and only half of the bridge need be considered.)

PROBLEM 8b.—Determine the shape of the cables for the bridge in Problem 8a if one end of the bridge is 12 ft. higher than the other end.

ART. 9. SIMPLE BEAMS UNDER CONCENTRATED LOADS

By applying the principles of the preceding articles the vertical shears and the bending moments may be found for all sections of a beam having only two supports and subject to any number of concentrated loads. For example, consider a simple beam 20 ft. long, carrying five loads whose positions and weights in pounds are shown in Fig. 9*a*. The reactions of the supports are found by laying off the load line *af*, taking the loads in order from left to right, *AB*, *BC*, *CD*, etc.; selecting a pole *o* and drawing the rays

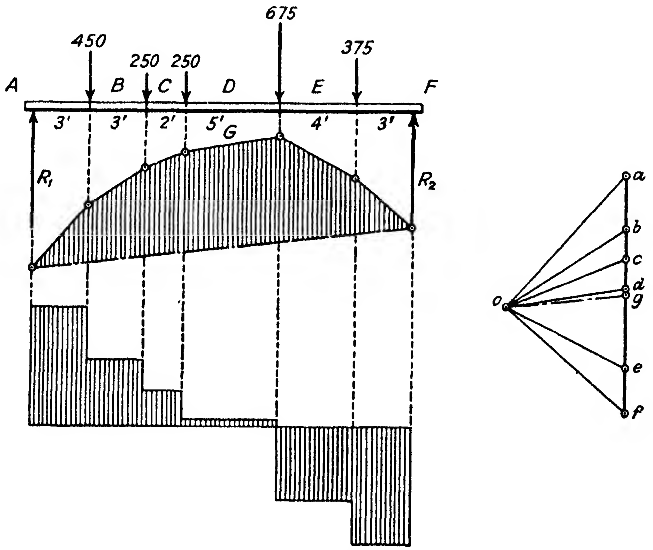


FIG. 9*a*.

from *o*; and constructing the equilibrium polygon, starting from any point since the directions of both reactions are known to be vertical. The point *g* is located on the load line by drawing a ray from *o* parallel to the closing line of the equilibrium polygon. The right reaction is then represented by *fg* and the left reaction by *ga*. Using the same scale as for the loads, each reaction is found to be 1000 lb.

Between the left support and the first load the vertical shear equals the reaction *GA*; between the first and second loads the shear is reduced by the amount of the load *AB* and therefore equals bg on the load line; between the second and third loads it is $GA-AB-AC = cg$; and so on. At the fourth load the shear changes from positive to negative and at the right support the value is the reaction FG. The diagram in Fig. 9a gives the shear for every point on the beam and is known as the 'shear diagram.' Its construction is apparent, each step being one of the loads laid off in the direction of its action. The shear at any point is considered positive if the resultant of the forces to the left of the point has an upward direction. Positive shears are plotted above the base line and negative shears below.

The bending moment at any point in the beam equals the vertical ordinate of the equilibrium polygon multiplied by the pole distance. Since the pole distance is constant for parallel loads, the equilibrium polygon shows the bending moment at any point in the beam if the proper scale is used and can therefore be used as the 'moment diagram.' The maximum moment occurs under the load DE, which is also the point where the shear passes through zero. Therefore the important relation is obtained that the maximum bending moment occurs at the section where the vertical shear passes through zero. A positive bending moment is considered as one which produces tension in the bottom fibers of the beam and is plotted above the base line. If the pole is chosen to the left of the load line, the positive moments will fall above the closing line and negative moments below.

In making the actual construction of Fig. 9a the linear scale used in laying off the beam and the positions of the loads was 5 ft. to an inch, and the force scale used in the force polygon was 800 lb. to an inch. The pole distance was taken as 1000 lb., hence, the moment scale was 5000 ft.-lb. to an inch. Any ordinate in the shear diagram, measured by the force scale, gives the vertical shear in pounds; thus, between the second and third loads the shear is +300 lb. and between the fourth and fifth loads it is −625 lb. Any ordinate in the moment diagram, measured by the moment scale, gives the bending moment in foot-pounds; thus, the maximum bending moment is +5500 ft.-lb. Figure 9a, however, as here printed, has been considerably reduced from the original construction.

PROBLEM 9a.—Construct the shear and moment diagrams for the beam in Problem 7a.

ART. 10. SIMPLE BEAMS UNDER UNIFORM LOADS

The correct reactions for a beam carrying a uniform load may be obtained by considering the entire load as concentrated at its center. This will not give the correct values for the bending moment, as will be shown later, but an approximate solution may be obtained by dividing the uniform load into a number of concentrated loads. This usually requires more work than an exact algebraic solution and for that reason uniform loads are generally treated analytically.

Let the simple beam in Fig. 10*a* whose span is l be uniformly loaded with the weight w per linear unit; then each reaction is equal to half of the total load or $\frac{1}{2}wl$. The load may be represented graphically by the shaded rectangle on the beam whose base is l and altitude w.

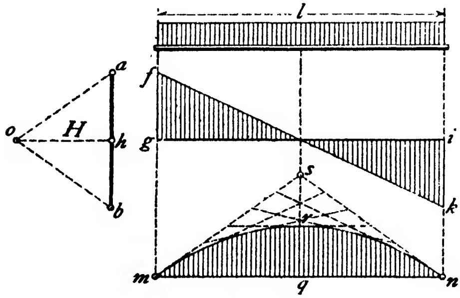


FIG. 10*a*.

For any section at a distance x from the left support the vertical shear is $V = \frac{1}{2}wl - wx = w(\frac{1}{2}l - x)$; if V be an ordinate corresponding to an abscissa x this is the equation of a straight line. Thus when $x = 0$, $V = \frac{1}{2}wl$; when $x = \frac{1}{2}l$, $V = 0$; and when $x = l$, $V = -\frac{1}{2}wl$. The shear diagram is constructed by laying off gi equal to the span, making gf and ik equal to $\frac{1}{2}wl$ and joining f and k.

The bending moment in a section distant x from the left support is $M = \frac{1}{2}wlx - \frac{1}{2}wx^2 = \frac{1}{2}w(lx - x^2)$. This is the equation of a parabola; for $x = 0$ and $x = l$ the value of M is 0; for $x = \frac{1}{2}l$, M reaches its maximum value of $\frac{1}{8}wl^2$. The moment diagram may be constructed by laying off mn equal to the span, drawing qr at the middle equal to the maximum moment, and then constructing the parabola mrn. To do this the lines ms and ns are drawn, rs being made equal to qr; these are divided into the same number of equal parts and the points of division joined as shown, thus determining tangents to the parabola.

If the entire load on the beam were concentrated at the middle, *ab* would be the load line, and *bh* and *ha* the two reactions. Now let *o* be a pole having the pole distance H, and let the equilibrium polygon *msn* be constructed. Then from the similar triangles *oah* and *msq*,

$$H : \tfrac{1}{2}wl :: \tfrac{1}{2}l : qs$$

Hence if H be equal to unity on the scale of force, the ordinate *qs* has the value of $\frac{1}{4}wl^2$, and since *qr* is $\frac{1}{8}wl^2$ the maximum moment for a single concentrated load at the middle is twice as great as that due to the same load when uniformly distributed.

ART. 11. OVERHANGING BEAMS

Let a beam be taken with one overhanging end and carrying a number of concentrated loads as shown in Fig. 11*a*. The loads

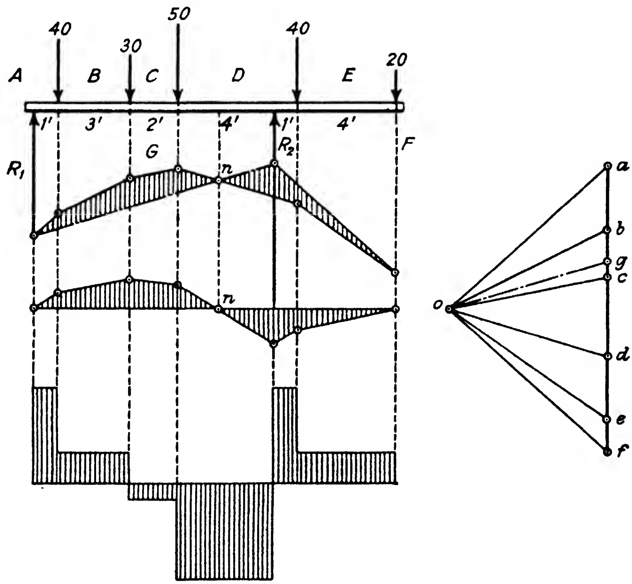


FIG. 11*a*.

are given in pounds and the distances in feet. If on the load line the loads be laid off successively in the order in which they are on the beam, and the equilibrium polygon be constructed, the ray *og* drawn parallel to the closing line will determine the reactions *fg* and *ga*. The sides of the equilibrium polygon are found to cross each other at n, and the ordinates to the right of this point lie on the opposite side of the closing line from those on the left. The ordinates on the left being regarded as positive, those on the right are negative, and they give the bending moments for all sections in the beam. The point where the bending moment is zero is called the 'inflection point'; on the left of this point the lower fibers are in tension whereas on the right the upper fibers are in tension.

In order to construct a moment diagram having a horizontal base line, the vertical ordinates of the equilibrium polygon may be measured and laid off from the base line of the new moment diagram. The method of constructing the shear diagram will be understood without further explanation than that given in Art. 9.

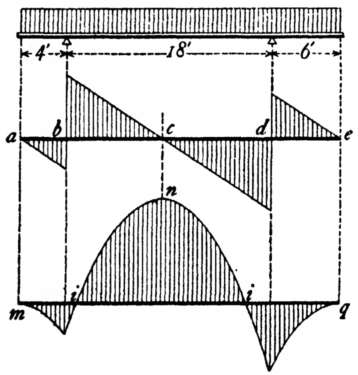


FIG. 11*b*.

It is seen that the shear passes through zero at two points, one where the maximum positive moment occurs, and the other at the right support where the negative moment is a maximum.

The linear scale used in the actual construction of Fig. 11*a* was 4 ft. to an inch, and the force scale was 60 lb. to an inch; the pole distance being 100 lb., the moment scale was 400 ft.-lb. to an inch. In the figure as printed the scales have been considerably reduced. By measurement it is found that the maximum shear is 60 lb., the maximum positive moment 120 ft.-lb., and the maximum negative moment 140 ft.-lb.

For the case of a uniform load a shear diagram and moment diagram may be constructed by computing the maximum ordinates and then drawing the straight lines for the shear diagram and the parabolas for the moment diagram. Thus, let Fig. 11*b* represent a beam 28 ft. long with overhanging ends of 4 ft. and 6 ft., and loaded with a uniform load of 40 lb. per linear ft. The left reaction is found by computation to be 497.8 lb. and the right reaction to be 622.2 lb.; these might also be obtained graphically by the equilibrium polygon, regarding the load on the entire beam as concentrated at its center. The shear diagram can next be drawn by starting at either end. The shear is found to pass through zero at each support and at a point 8.45 ft. from the left support. The moments for these points of zero shear are found by computation to be −320 ft.-lb. at the left support; −720 ft.-lb. at the right support; and +1106 ft.-lb. near the center of the beam. These maximum moments being laid off by scale, the curves can be constructed by the method given in Art. 10, it being known that the end parabolas have their vertices at the ends of the beam, and that the middle parabola has its vertex at the point of zero shear. The inflection points are equally distant from the point of maximum positive moment, this distance being 7.45 ft. in Fig. 11*b*. The diagrams thus furnish full information regarding the distribution of the shears and moments in the beam.

PROBLEM 11*a*.—An overhanging beam 18 ft. long has its left support 3 ft. from the left end and its right support 4 ft. from the right end. It carries concentrated loads of 600 lb. at each end and a uniform load of 200 lb. per linear foot between supports. Construct the shear and moment diagrams.

ART. 12. CENTER OF GRAVITY OF CROSS-SECTIONS

In problems relating to the strength of beams it is necessary to find the position of the neutral axis. The neutral axis passes

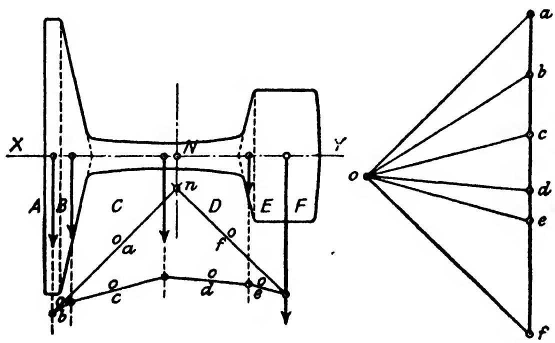


FIG. 12*a*.

through the center of gravity of the cross-section, and in finding the position of the center of gravity the equilibrium polygon may be employed, the forces being laid out to represent areas. For example, let the cross-section of the rail shown in Fig. 12*a* be taken. As this cross-section has an axis of symmetry *XY* the center of gravity must lie at some place on this axis. The area is divided into simple geometrical figures or narrow strips by lines perpendicular to the axis, and at the centers of gravity of these parts forces proportional to their areas are applied. The force and equilibrium polygons are constructed in the usual manner. The extreme sides of the equilibrium polygon parallel to *oa* and *of* are produced until they intersect at *n*, giving the position of the resultant of the forces. The center of gravity must lie on the resultant and also on the axis *XY*, therefore it must lie at *N*, their intersection. The best intersection of the closing rays is obtained by taking the pole *o* near the center of the load line and at such a distance out that the outside rays are nearly at right angles to each other. If the surface is very irregular in outline it should be divided into strips so narrow that the area of each one is equal to the product of its mean length by its width without appreciable error. If there is no axis of symmetry, the process described above must be repeated for another direction, preferably at right angles to the first, and the center of gravity will lie at the intersection of the two resultants found.

PROBLEM 12*a*.—Determine the center of gravity of a 12-in. channel weighing 20.7 lb. per lin. ft. Get dimensions from a manufacturer's handbook.

ART. 13. MOMENT OF INERTIA OF CROSS-SECTIONS

For beams under flexure the bending moment *M* for any section equals the resisting moment $\frac{SI}{c}$ with reference to the neutral axis in that section in which *S* is the unit stress in the most remote fiber distant *c* from the axis and *I* is the moment of inertia of the cross-section with reference to the same axis.

In Art. 12 the method is given for finding the center of gravity through which the neutral axis passes. A method will now be derived for determining the moment of inertia *I* from the same construction.

Let it be required to find the *I* of the T-shaped section shown in Fig. 13*a*. The cross-section *A* is 4.50 sq. in., and by the force and equilibrium polygons the neutral axis through *N* is found to be 1.52 in. from the base of the T. The pole distance *oh* was taken equal to $\frac{1}{2}A$. The side of the equilibrium polygon *of* was produced until it intersected the neutral axis at *t*. The triangles

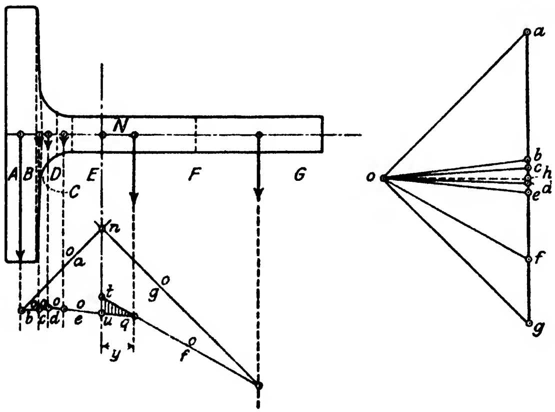


FIG. 13a.

qtu and ofe are similar, as their sides are mutually parallel. Let y be the distance from q to the neutral axis; then

$$tu : y :: ef : oh$$

But ef equals the area EF laid off to scale, and the pole distance oh was made equal to $\frac{1}{2}A$; hence

$$tu \cdot \tfrac{1}{2}A = EF \cdot y$$

Multiplying this equation by y, and remembering that $\frac{1}{2}tu \cdot y$ is the area of the triangle qtu, gives,

$$\text{area } qtu = \frac{EF \cdot y^2}{A}$$

The other triangles composing the area of the equilibrium polygon may be expressed in a similar manner. If each division of the cross-section were of differential width dy and area dA, the area of its triangle in the equilibrium polygon would be $\dfrac{dA \cdot y^2}{A}$. But $dA \cdot y^2$ is also the moment of inertia of the differential area dA about the parallel axis distant y, hence the area A' of the entire equilibrium polygon is

$$A' = \frac{\Sigma(dA \cdot y^2)}{A} = \frac{I}{A} \quad \text{or} \quad I = AA'$$

in which case the broken line *ob, oc, . . . , of* is a curve which is tangent to the sides *oa* and *og* at the extreme limits of the given cross-section. This curve may be drawn and the area A' determined either by dividing it into strips or by a planimeter.

By perfonning the above operations on a three-quarter size drawing of the T-section shown in Fig. 13*a*, the area A' was found by dividing it into strips to be 2.42 sq. in., hence the moment of inertia is

$$I = 4.50 \times 2.42 = 10.9 \text{ in.}^4$$

This agrees very well with the value of 10.8 in.^{4} given in the manufacturer's handbook.

PROBLEM 13*a*.—Determine the moment of inertia of the Am. Soc. C. E. rail section, 90 lb. per linear yd., about the axis through the center of gravity and normal to the web. The area and dimensions of this section may be found in a manufacturer's handbook.

CHAPTER II

ROOF TRUSSES

Art. 14. Definitions and Principles

A roof truss is a structure composed of separate members generally so arranged that they are subject only to tensile or compressive stresses. It lies in a vertical plane and is usually supported at its ends by columns or by the side walls of the building. For stability the elementary figures composing a truss must be triangles, since a triangle is the only polygon which cannot change its shape without altering the length of its sides.

The points where the center of gravity lines of adjacent members meet are called 'joints' or 'panel points.' All joints of the truss are assumed to be perfectly flexible, and the external forces, consisting of loads and reactions to be applied only at the joints.

The 'span' of a truss is the distance between end joints or the centers of the supports, and the 'rise' is the distance from the highest joint, or peak, to the line on which the span is measured. The 'pitch' is the ratio of the rise to the span. The 'upper chord,' or 'top chord,' consists of the upper line of members extending from one end of the truss to the other. The lower line of members is known as the 'lower chord,' or 'bottom chord.' The interior members connecting the joints of the upper chord with those of the lower chord are known as 'web members,' and may be either vertical, diagonal, or radial. Any member which takes compression is called a 'strut,' and one that takes tension is called a 'tie.' The upper chord and some of the web members are subject to compression while the lower chord and the rest of the web members are in tension.

The notation employed in this chapter for designating loads is the same as given in Art. 4, that is, placing a letter in each space instead of on each load. In addition, each space in the truss is given a letter so that each member is also designated by the letters between which it is situated. Thus, in Fig. 14*a*, *AC* and *BC* are the upper chord members and *CD* is the lower chord, while *AB* designates the load at the peak and *BD* and *DA* are the reactions.

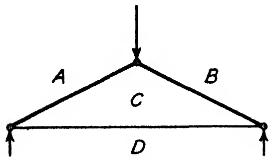


FIG. 14*a*.

The fundamental principles of Graphic Statics as given in Chapter I apply to the determination of stresses in trusses under any given condition of loading. The reactions for a truss are found in the same manner as for a beam. The stress in each member at any joint is a force whose line of action is known to be through the joint with a direction parallel to the member, hence the only unknown is its magnitude. Since two unknowns can be determined from the force triangle, or force polygon, the stresses in all members at any joint having only two unknown stresses can be determined. For example, the stresses in *AC* and *CD* can be found from a force triangle after the value of the reaction *DA* is known.

PROBLEM 14*a*.—The span of the simple triangular roof truss of Fig. 14*a* is 24 ft. and the pitch one-third. Find the stress in each member due to a load at the peak of 3000 lb.

ART. 15. DEAD AND SNOW LOADS

Four kinds of loads and various possible combinations thereof are to be considered in determining the stresses in a truss: the weight of the truss itself, the weight of the roof covering, the snow, and the wind. The first two of these make up the 'dead load' of the roof.

The weight of a truss depends upon its span and depth, the distance between adjacent trusses, the load to be carried, the material of which the truss is composed, the allowable unit stresses used in design, and various other elements of design. The actual weight cannot be ascertained until after a design has been completed. Since the truss must be designed to carry its own weight it is necessary to assume a weight to use in calculating dead-load stresses. The best method of assuming a weight is by comparison with other trusses designed to fit similar conditions. Where other trusses are not available for comparison an approximation can be made from the following formula for steel trusses:

$$w = \frac{P}{100}\left(1 + \frac{L}{10}\right)$$

in which w is the weight of the truss in pounds per square foot of horizontal area covered, P is the capacity of the truss in pounds per square foot of horizontal projection, and L is the span of the truss in feet. The weight of wooden trusses having steel tension web members, in accordance with the usual practice, will be about three-fourths of that given by the above formula. For ordinary conditions the weight of the truss is seldom more than 15 per cent of the total load, so that a relatively large error in the assumed weight would cause a small error in the final stresses. For large spans or unusual conditions it may be necessary to revise the dead-load stresses after the actual weight of the truss has been found.

The roof covering usually consists of a surface of tin, slate, tiles, corrugated steel, shingles, tar and gravel, or prepared roofing resting upon a 'deck' of wood, steel, concrete, or gypsum. The deck in turn is supported on 'purlins' or beams running longitudinally between the trusses and attached to them at the upper joints. In large roofs the deck is supported on 'rafters' running parallel to the upper chord, the rafters resting on the purlins. The actual weight of any roof covering can be determined only by computing the weight of the several elements of which it is composed. The roof covering is designed before the trusses are designed so that its actual weight is available for determining the dead load on the trusses.

The approximate weight of a number of roofing materials is given below. All weights are in pounds per square foot of roof area.

Coverings:

Tin	1 to $1\frac{1}{2}$
Corrugated steel	1 to 2
Wooden shingles	2 to 3
Slate shingles	5 to 10
Tiles	8 to 25
Tar and gravel	5 to 10
Prepared roofing	1 to 2

Decks:

1-in. wooden sheathing	3 to 4
2-in. concrete	25
Gypsum	12 to 25
Steel with insulation	3 to 5

Purlins:

Wood	1 to 3
Structural steel	2 to 4
Steel joists	1 to 2

The snow load varies with the latitude, humidity of the climate, and pitch of the roof, being about 30 lb. per horizontal sq. ft. in Northern New England and Canada, about 20 lb. in the latitude of New York City and Chicago, about 10 lb. in the latitude of Baltimore and Cincinnati, and rapidly diminishing southward. On roofs having an inclination to the horizontal of 60° or more this load may be neglected, as it might be expected that the snow would slide off.

For the purpose of securing uniformity in the solution of examples and problems in this book, the following average values will be used, unless otherwise specified:

For truss weight—compute from the above formula.
For the roof covering—12 lb. per sq. ft. of roof surface.
For the snow load—15 lb. per sq. ft. of horizontal area.

The span of the wooden roof truss shown in Fig. 15*a* is 48 ft., the rise 10 ft., and the distance between trusses 12 ft. The length of each half of the top chord is $\sqrt{24^2 + 10^2} = 26.00$ ft. and is divided into three equal parts called 'panels.' The weight of the roof covering, and of the snow which may be upon it, is brought by the purlins to the panel points of the upper chord. The weight of the truss itself is also generally regarded as concentrated at the panel points. At each joint of the upper chord there is hence a load called a 'panel load,' and it may be a 'dead panel load' or a 'snow panel load.' From the formula the truss weight is found to be 750 lb. The weight of the roof covering on the top chord is $2 \times 26 \times 12 \times 12 = 7488$ lb. The total dead load is $750 + 7488 = 8238$ lb. and the dead panel loads BC, CD, DD', $D'C'$, and $C'B'$ are each one-sixth of the total load or 1373 lb., while the end loads AB and $B'A'$ are each one-half as much, or 686 lb. The weight of the snow supported by each truss is $48 \times 12 \times 15 = 8640$ lb. The intermediate snow panel loads are 1440 lb., and the end loads 720 lb.

If the panels are of unequal lengths, the load at any panel point is found by considering that the weights brought to it by the purlins are those upon the rectangular area extending in each direction half-way to the adjacent panel points.

When a truss and the loads it carries are symmetrical the reactions of the supports are equal, each being one-half of the total load. For unsymmetrical conditions, the reactions are found in the same way as for a beam with concentrated loads. In the

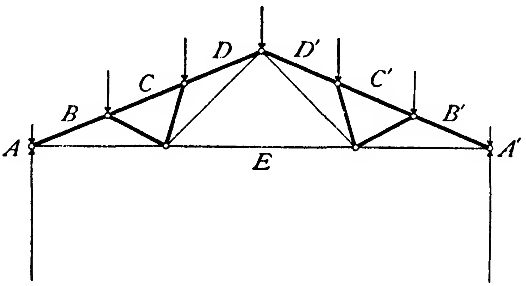


FIG. 15a.

above example each dead-load reaction is 4119 lb. and each snow-load reaction is 4320 lb. The half-panel load acting at each support is carried directly into the reaction, causing no stresses in the truss, and therefore may be omitted entirely from consideration in determining stresses. The net effective reaction is then one-half of the full panel loads, or for the snow load in the above problem is $\frac{1}{2}(5 \times 1440) = 3600$ lb.

PROBLEM 15a.—A steel roof truss like Fig. 15a has a span of 60 ft., a rise of 15 ft., and distance between trusses of 18 ft. Find the panel loads and reactions due to the dead and snow loads.

ART. 16. STRESSES DUE TO DEAD AND SNOW LOADS

Figure 16a shows the truss of Art. 15 with the dead loads and reactions determined there. The truss diagram composed of the center of gravity lines of the members, or 'working lines,' is carefully drawn to as large a scale as convenient, each joint being marked by a fine needle point and surrounded by a small circle to limit the lines drawn toward the point.

The forces acting upon the truss are in equilibrium and hence form a closed force polygon (Art. 5). Since these forces are parallel the resulting polygon, or 'load line,' is a straight line. The load line is constructed by taking the panel loads in regular order from left to right, or in a clockwise direction about the truss, and laying them off in succession on the vertical line aa', thus: ab is laid off equal to the load AB or 686 lb., bc equal to the load BC or 1373 lb., and so on for the other loads and reactions. Greater accuracy can be obtained by laying off each point on the load line as the accumulated distance from a, as for example, ac equals $AB + BC =$ 2059 lb., ad equals $ac + CD = 3432$ lb., and so on.

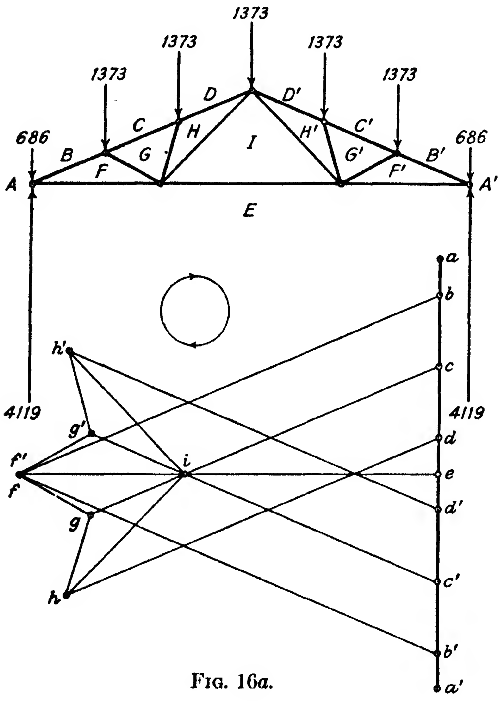


FIG. 16a.

Each joint of the truss is in equilibrium under the action of the external forces applied at the joint and the stresses in the truss members. Therefore all the forces acting on each joint must form a closed force polygon. Beginning with the joint at the left reaction, the known external forces are the reaction EA and the half-panel load AB. These are held in equilibrium by the unknown stresses in the members BF and FE. These stresses must act parallel to the members and through the joint so that the only unknown of each is its magnitude. The total unknowns being only two in number they can be determined from the force polygon (Art. 5). The polygon for the joint is constructed by first laying off the known forces *ea* and *ab* equal to *EA* and *AB*, respectively, and then through *b* drawing a line parallel to *BF* and through *e* drawing a line parallel to *EF*. The intersection of these lines locates the point *f*, and the lengths *bf* and *ef*, measured by the same scale used on the load line, give the stresses in the members. The character of the stresses is found by following around the polygon starting with the known forces, that is, from *e* to *a*, *a* to *b*, *b* to *f*, and *f* to *e*. Transferring these to the joint considered, the stress in *BF* acts toward the joint and is therefore compression, whereas that in *FE* acts away from the joint and is tension.

The first joint on the top chord is considered next as it has only two unknown stresses, *CG* and *GF*. The force polygon *fbcg* is constructed and the magnitude and character of the unknown stresses determined as before. The force polygons for the remaining joints are constructed in the same manner, proceeding each time to a joint having only two unknown stresses. All these force polygons taken together form the 'stress diagram.' The closing of the force polygon for the last joint gives a check on the force polygons for all other joints, and for that reason the entire stress diagram should always be drawn, though only half of it is necessary for a symmetrical truss. An additional check should be made by computing analytically the stress in one member conveniently located, as for example, *IE*. This not only checks the accuracy of the graphical work but also guards against measuring the stresses with a scale different from that used in laying off the load line.

The method given above for finding the character of the stresses requires passing around the perimeter of every force polygon contained in the stress diagram, unless the truss and its loadings are symmetrical, in which case only half of the polygons are so used. The line representing the stress in each member must in all cases be traced twice, as it forms a side of two polygons. If no external forces act at a joint the direction of passing around the corresponding force polygon must be obtained from the character of the stress in one of its connected members already found. Thus, for the left joint on the bottom chord the stress in *EF* is known to be tension and therefore acts away from the joint, that is, toward the left. The force polygon must therefore be followed around in the direction *efghie*.

In finding the character of the stresses above it will be noticed that a clockwise direction, as indicated by the circular arrow, was followed around all polygons. This will always be the case if the load line is laid off by taking all external forces in a clockwise order around the truss, and it makes it possible to determine the character of the stress in any member without reference to that in any other member. For example, consider the member *BF*. Passing clockwise around the joint at the left reaction the member is read *BF* and the corresponding stress in the stress diagram is read from *b* to *f*, which, transferred to the joint, acts toward it and is therefore compression. Passing clockwise around the adjacent top chord joint this same member is read *FB* and the stress *f* to *b* again acts toward the joint considered so that it makes no difference which joint is considered.

It is evident that any truss, whatever the arrangement of web members may be, supported as in Fig. 16*a* will have its top chord in compression and its bottom chord in tension so that it usually remains to find only the character of the stresses in the web members. In this case they may all be found in one-half of the truss by considering only one of the lower chord joints. Since the truss is symmetrical the stress in the corresponding members in the other half will be the same kind.

As soon as a stress is measured on the stress diagram it should be recorded on the corresponding member in the truss diagram together with its character. For convenience a tension stress is denoted by a plus sign and a compression stress by a minus sign. These signs can be fixed in mind by remembering that a plus stress (tension) tends to increase the length of a member whereas a minus stress (compression) tends to decrease its length.

All lines of the stress diagram should be drawn with a hard, well-sharpened pencil pressed lightly on the paper so as to produce a fine, distinct line. As soon as an intersection is obtained it should be marked with a needle point, enclosed with a small circle, and designated by the proper letter; other lines drawn to or from that point should not pass within the circumference of this circle. The triangle and straight-edge used in drawing parallel lines should be so arranged as to require the triangle to be moved the shortest possible distance. A drafting machine or T-square should not be used for graphic constructions, as they cannot be depended upon to move absolutely parallel at all times. Special care should be taken to hold the pencil at the same inclination from the beginning to the end of each line, or the line will not be strictly parallel to the edge of the triangle.

The snow-load stress diagram is constructed in exactly the same way as that for the dead load. Since the distribution of the snow load is the same as for the dead load, the two stress diagrams will be similar. The snow-load stresses may therefore be obtained by multiplying the dead-load stresses by the ratio of their panel loads; in this case $\frac{1440}{1373} = 1.048$.

The following table gives the dead- and snow-load stresses in all members of the truss as scaled from the stress diagrams:

Member	Dead Load	Snow Load
$BF = B'F'$	−8950	−9390
$CG = C'G'$	−7400	−7750
$DH = D'H'$	−7920	−8290
$EF = E'F'$	+8250	+8650
EI	+4960	+5200
$FG = F'G'$	−1620	−1700
$GH = G'H'$	−1620	−1700
$HI = H'I$	+3300	+3460

When but a slight difference is found between the lengths of any pair of symmetrical lines in the stress diagram the average of the two should be used; if the discrepancy is large the entire stress diagram should be redrawn.

PROBLEM 16a.—The steel truss shown in Fig. 16*b* has a span of 80 ft., a pitch of one-fifth, and a distance between trusses of 19 ft. Find the dead-

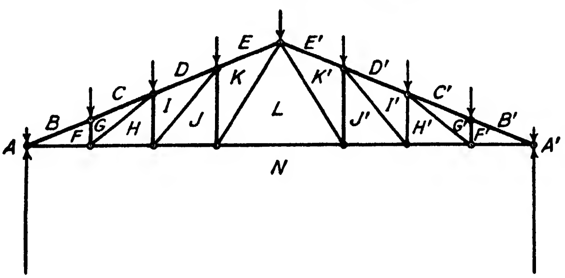


FIG. 16*b*.

and snow-load stresses. Prove that the stresses in the three members cut by a vertical plane through *DI* hold the external forces on either side of the plane in equilibrium.

ART. 17. WIND LOADS

The wind usually acts horizontally or nearly so. Experiments have shown that the pressure produced by the wind varies approximately as the square of its velocity. A hurricane at 100 miles per hour exerts a pressure of probably 40 lb. per sq. ft. of surface normal to its direction; a pressure of 30 lb. per sq. ft. corresponds to a velocity of about 85 miles per hour; 20 lb. per sq. ft. to about 70 miles per hour; and 10 lb. per sq. ft. to about 50 miles per hour. Most building codes in the United States specify a wind pressure of 20 to 30 lb. per sq. ft. horizontally. Unless otherwise noted, a pressure of 30 lb. per sq. ft. will be used in problems in this book.

The pressure produced by the wind on a roof depends on the direction and velocity of the wind, the inclination and shape of the roof, and the height of the building. Only the component of the pressure normal to the roof produces stresses in the truss. Several empirical formulas have been derived from experiments for determining this component. The formula most generally used was developed by DUCHEMIN. It is

$$P_n = P\frac{2 \sin \alpha}{1 + \sin^2 \alpha}$$

where P is the pressure in pounds per square foot on a surface normal to the direction of the wind, P_n is the component of the pressure normal to the roof, and α is the angle that the roof surface makes with the horizontal. The table below shows the normal pressure as given by Duchemin's formula for a horizontal wind pressure of 30 lb. per sq. ft.:

Inclin.	Nor. Press.	Inclin.	Nor. Press.	Inclin.	Nor. Press.
5°	5.2	25°	21.5	45°	28.3
10°	10.1	30°	24.0	50°	29.0
15°	14.6	35°	25.9	55°	29.4
20°	18.4	40°	27.3	60°	29.7

For inclinations exceeding 60° the normal pressure is equal to the horizontal pressure. For horizontal pressures other than 30 lb. per sq. ft. the normal pressure will change in the same ratio.

The wind panel loads will be determined for the truss shown in Fig. 17*a*. The inclination of AB is found to be 50° 40′, and that of BC, 16° 40′. From the above table the normal wind pressures are respectively 29.0 and 15.9 lb. per sq. ft. For the trusses 12 ft. apart the total normal wind pressure on AB is $\sqrt{9.6^2 + 11.7^2} \times 12 \times 29.0 = 5266$ lb., one-half of which is applied at A and one-half at B, as shown. In the same way the wind upon BC produces normal panel loads of 1433 lb. at B and C. The two-

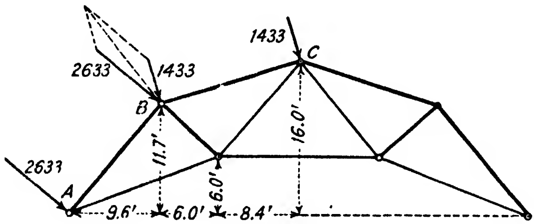


FIG. 17*a*.

panel loads at B are combined by a force triangle, the resultant being 3910 lb.

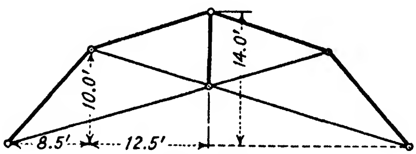


FIG. 17*b*.

PROBLEM 17*a*.—The trusses shown in Fig. 17*b* are 20 ft. apart. Determine the wind panel loads for the wind from the right.

ART. 18. A TRUSS WITH FIXED ENDS

Roof trusses of short span, especially wooden trusses, generally have both ends firmly 'fixed' to the supporting walls. Both reactions caused by the wind pressure are therefore inclined, and their horizontal components tend to overturn the walls of the building. Since the magnitude and direction of each reaction is unknown there is a total of four unknowns. As explained in Art. 6, this is one more than can be determined, so that it is necessary to make some assumption as to the distribution of the horizontal component between the reactions. The most common assumption is that the reactions are parallel. Another assumption sometimes made is that the horizontal components of the two reactions are equal.

With either assumption the reactions can be determined as explained for beams.

The truss in Fig. 18*a* has both ends fixed, the span is 40 ft., the rise of the top chord 10 ft., the rise of the bottom chord 2 ft., the distance center to center of trusses 12 ft., and the reactions are assumed parallel. Each half of the top chord is found to be 22.36 ft. long and its inclination is 26° 34′. The normal wind pressure is therefore 22.4 lb. per sq. ft. of roof surface. The total wind load on each truss is 22.36 × 12 × 22.4 = 6010 lb. = 6.00 kips. The

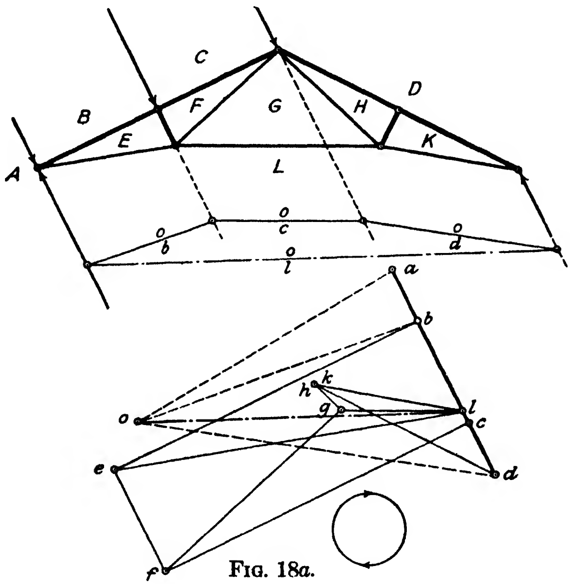


FIG. 18*a*.

panel load *BC* is 3.00 kips and the panel loads *AB* and *CD* are 1.50 kips each.

The reactions are found by means of the equilibrium polygon. It will be noticed that for wind loads the load line is inclined. Since the reactions are assumed parallel, the point *l* must fall on the resultant or in this case on the load line itself. The results are 4.12 kips for the left reaction and 1.88 kips for the right reaction. These will now be checked analytically. The perpendicular distance from the right support to the left reaction measures 35.8 ft. Taking moments about the right support,

$$LA \times 35.8 - 1.50 \times 35.8 - 3.00 \times 24.6 - 1.50 \times 13.4 = 0,$$

whence $LA = 4.12$ kips. By taking moments about the left reaction in a similar manner, the reaction DL is found to be 1.88 kips. Another check on the reactions is obtained by the closing of the stress diagram. The same result is obtained, both graphically and analytically, by replacing the wind panel loads by their resultant, 6.00 kips, applied at the middle of the left half of the top chord.

The stress diagram is drawn on the same load line used in finding the reactions. The procedure is the same as for vertical loads. Since the points h and k coincide, the line hk is of zero

Member	Stresses in Kips	
	Wind from Left	Wind from Right
BE	−8.79	−5.23
CF	−8.79	−5.23
DH	−5.23	−8.79
DK	−5.23	−8.79
EL	+9.15	+3.90
GL	+3.15	+3.15
KL	+3.90	+9.15
EF	−3.00	0
FG	+6.21	+0.98
GH	+0.98	+6.21
HK	0	−3.00

length, and the stress in HK is zero when the wind blows from the left. With scales of 3 ft. to an inch and 1 kip to an inch the results given in the table were obtained. The stress diagram for the wind loads applied on the right side of the truss is the same as that in Fig. 18*a* revolved about a vertical axis, therefore only one stress diagram is required. Accordingly the stress in any member in the truss for the wind from the right is the same as the stress in the corresponding member in the other half of the truss for the wind from the left.

Problem 18*a*.—Find the reactions and stresses for the truss in Fig. 18*a* assuming the horizontal components of the reactions equal.

Art. 19. A Truss with One End Free

Temperature changes cause expansion and contraction of roof trusses which, if both ends are fixed, give rise to certain stresses in the trusses and supporting walls. The coefficient of expansion of steel is about twice that of wood, so that for long-span steel trusses it is necessary to relieve these temperature stresses. This is usually accomplished by fastening only one end of the truss to the wall, the other being merely supported, or 'free,' so that it may move horizontally in the direction of the length of the truss. The free end may rest on a smooth steel plate upon which it slides but this is not a very satisfactory arrangement, as the friction between the truss and the plate must be overcome and this may be large for heavy trusses, especially if the plate becomes rusty. A better method is to support the free end on a rocker or on rollers so that friction is reduced to a minimum.

If no friction exists at the free end, the reaction there is vertical for the wind from either side. Therefore, in determining stresses due to wind it is necessary to construct two stress diagrams, one for the wind blowing on the fixed side and the other for the wind load on the free side.

The truss in Fig. 19*a* has a span of 76 ft. a rise of top chord of 19 ft., a rise of bottom chord of 4 ft., and a distance between trusses of 15 ft. The web members consist of verticals and diagonals as shown, the top chord being divided into eight equal parts. The inclination of the top chord is the same as that in the last article, hence the normal wind pressure is 22.4 lb. per sq. ft. of roof surface. The wind panel loads *BC*, *CD*, and *DE* are each $\frac{1}{4} \times 42.48 \times 15 \times 22.4 = 3568$ lb. $= 3.60$ kips. The panel loads *AB* and *EF* are each one-half as much, or 1.80 kips.

The reactions are found graphically by means of the equilibrium polygon to be $FU = 4.02$ kips and $UA = 10.95$ kips. It was necessary to start the equilibrium polyon at the left support as this is the only known point on the line of action of the left reaction. Since the right reaction is vertical it can easily be checked analytically. Taking moments about the left support and considering the resultant of the wind loads, $14.40 \times 21.24 - FU \times 76 = 0$, whence $FU = 4.02$ kips. A graphical check is also easily obtained by extending the two reactions and the resultant of the wind loads to see that they intersect at the common point, y.

The stress diagram is constructed in the usual manner, starting at the left support and using the same load line laid off for finding the reactions. It will be noticed that the points n, p, q, r, s, and t

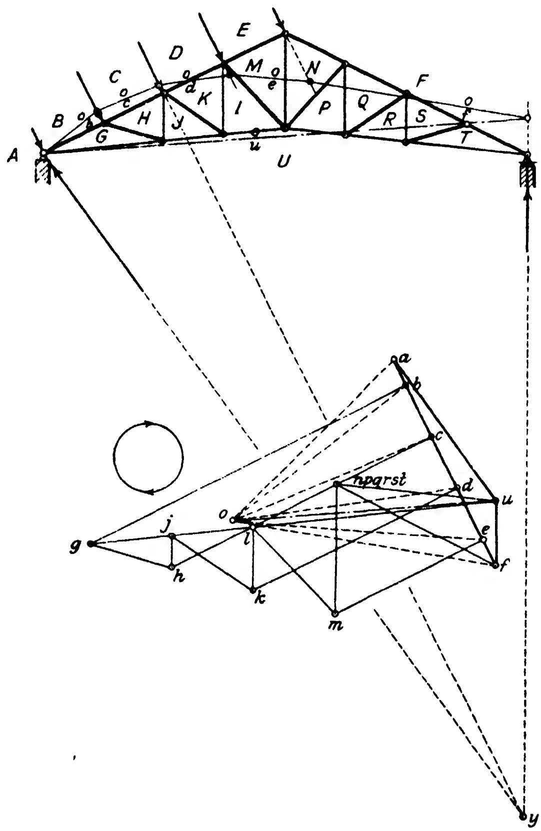


FIG. 19a.

all coincide so that the stress in all web members to the right of the center is zero. This may also be shown by beginning the stress diagram at the right support where the reaction FU is held in equilibrium by the stresses in UT and TF. The force triangle fut represents this relation, ut being the stress in UT and tf the stress in TF. Passing to the next joint on the upper chord it is required to draw a force triangle having two sides parallel to the straight upper chord and the third parallel to *TS*. This causes the two sides to coincide and the third side to disappear, hence the stress in *SF* equals that in *FT* and the stress in *TS* is zero. The same condition occurs at each joint on the upper and lower chords to the right of the center of the truss.

If the stress diagram is accurately drawn, the point *l* marks

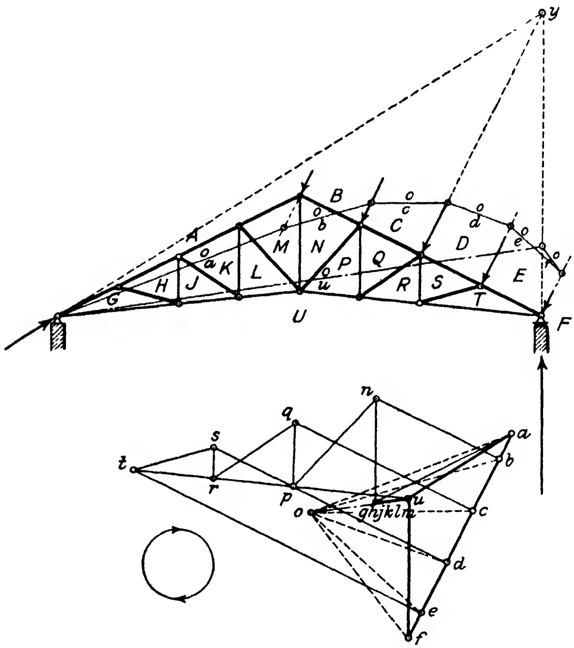


FIG. 19*b*.

the intersection of *ch*, *ug*, *kl*, and *lm*. Also the points *g*, *h*, *k*, and *m* will lie on a straight line and be equidistant.

When the wind blows upon the free side of the roof, as in Fig. 19*b*, the panel loads are the same as before. The reactions are determined graphically by means of the equilibrium polygon to be *FU* = 8.85 kips and *UA* = 7.60 kips. The right reaction was checked analytically and found to be 8.86 kips. A graphical check of the reactions is also obtained by extending the two reactions and the resultant of the wind loads to their common intersection at *y*. The stress diagram was drawn as usual. It will be noted again that all the web members to the left of the center of the truss carry no stress. The stress in the top chord of the entire left half of the truss is represented by *am* and that in the bottom chord of the left half of the truss by *um*.

The actual construction of the diagrams for this example was made to scales of 6 ft. to an inch and 4 kips to an inch. The stresses are shown on the skeleton truss diagrams in Fig. 19*c* for

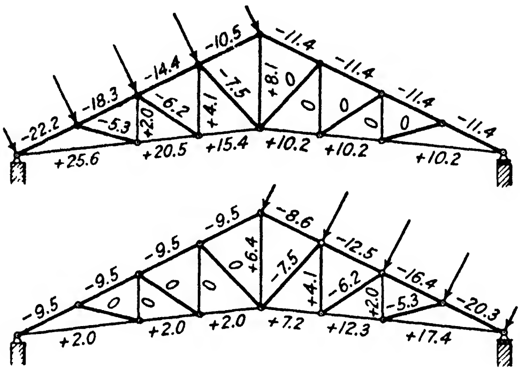


FIG. 19*c*.

convenient comparison. The greatest stresses are produced in the chords and in the center vertical when the wind blows on the fixed side. The stresses in the web on the windward side, except the center vertical, are the same for the wind blowing from either side, while the stresses in the web members on the leeward side are all zero. An analytical check gives the stress in *FN*, for the wind blowing from the left, of 11.40 kips, and the stress in *AM*, for the wind blowing from the right, of 9.48 kips.

PROBLEM 19*a*.—Determine the wind stresses for the truss in this article assuming both ends of the truss fixed and each reaction taking half of the horizontal component of the wind. Compare these stresses with the stresses shown in Fig. 19*c*.

ART. 20. ABBREVIATED METHODS FOR WIND STRESSES

It was shown in Art. 19 that the wind stresses in the web members of the leeward half of a triangular truss are always zero. The upper truss in Fig. 20*a* represents the same truss as Fig. 19*a* with the leeward web members omitted. The lower truss in

Fig. 20*a* differs from the upper one merely in having the rollers transferred to the left support. It corresponds to the truss in Fig. 19*b* turned end for end. The bottom diagram in Fig. 20*a* is a combination of the stress diagrams of Figs. 19*a* and 19*b*, the full lines being for the free end on the right and the broken lines for the free end on the left.

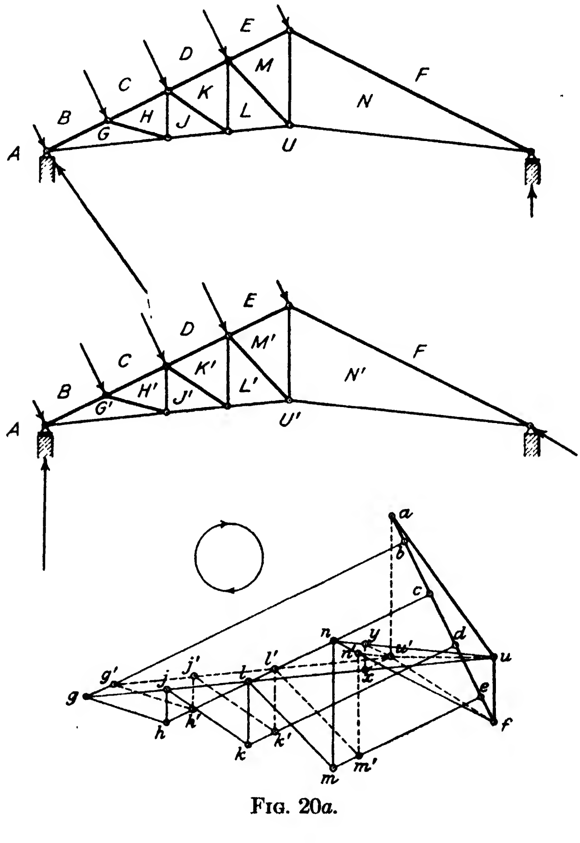


FIG. 20*a*.

Since the magnitude of the vertical components of the reactions is independent of the horizontal components (Art. 6), the line joining *u* and *u'* is horizontal. Draw *u'x* parallel to *bg* meeting *ug* at *x*, and *u'y* parallel to *fn* meeting *un* at *y*. Since *ug* and *un* make equal angles with the horizontal line *uu'*, *u'y* is equal to *u'x* and *xy* is vertical. The stresses in the top chord are changed when the rollers are transferred from the right support to the left support by the amount $gg' = hh' = kk' = mm' = nn' = u'x = u'y$, those in the bottom chord are changed by $ux = uy$ and that in MN by xy, while the remaining stresses are unaltered.

Applying the scale of forces, $u'x$ and $u'y$ are each found to be 1.9 kips; ux and uy, 8.2 kips; and xy, 1.7 kips. In the following table the first line contains the stresses as obtained from Fig. 19*a*, and after subtracting the changes of stress found above the same results are obtained as from Fig. 19*b*:

STRESSES FOR WIND ON THE LEFT

Truss Members.	BG	CH	DK	EM	FN	UG	UJ	UL	UN	MN
Rollers on right.	22 2	18.3	14.4	10.5	11.4	25.6	20.5	15.4	10.2	8.1
Change in stress.	1.9	1.9	1.9	1.9	1.9	8.2	8.2	8.2	8.2	1.7
Rollers on left. .	20.3	16.4	12.5	8.6	9.5	17.4	12.3	7.2	2.0	6.4
Truss Members.	BG'	CH'	DK'	EM'	FN'	UG'	UJ'	UL'	UN'	MN'

When the lower chord is horizontal, $u'x$, $u'y$, and xy each equal zero, so that the stresses in the upper chord and in all the web members are the same for the wind blowing on either the fixed side or the free side of the truss. The lower chord change, $ux = uy$, becomes equal to uu', the horizontal component of the wind loads.

For any other type of truss the changes of stress are easily obtained in a similar manner. When the middle panel of the bottom chord is horizontal, as in the example given on Plate I, the form of the auxiliary polygon $u'xzy$ is somewhat different from the preceding one. The change of stress in the horizontal part of the bottom chord is measured by kz. The following values were obtained from the original diagram which was made to a scale of 4 kips to an inch: $u'x = u'y = 1.7$ kips, $kx = ky = 8.1$ kips, $kz = 7.9$ kips, and $xz = yz = 0.7$ kip.

PROBLEM 20*a*.—Prepare a table of wind stresses similar to the above for the example given in Plate I.

PROBLEM 20*b*.—Find the wind stresses for a steel roof truss like that in Fig. 19*a*, except that the bottom chord is horizontal, having a span of 65 ft., a rise of 14 ft., and trusses spaced 18 ft. apart.

ART. 21. AMBIGUOUS CASES

When, in the determination of stresses in the Fink truss shown in Fig. 21*a*, the joint at the middle of the left half of the top chord is reached, the load *BC* and the stresses in *BG* and *GH* are known, leaving three unknowns, namely, the stresses in *CL*, *LK*, and *KH*. Since the resultant of the known forces cannot be resolved into

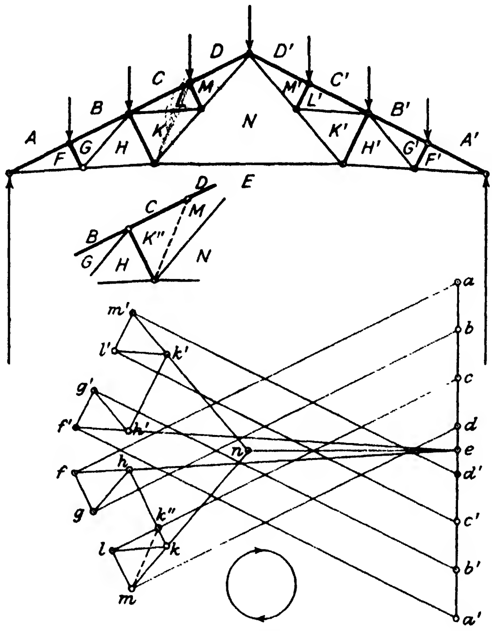


FIG. 21*a*.

more than two given directions some other procedure becomes necessary.

If the loads *AB* and *CD* are equal, as in this example, the symmetrical relation of *GH* and *LK* causes them to have equal stresses, and therefore *fg* and *lm* are equal and lie in the same straight line. The polygon *hgbclkh* is then readily completed, and the stresses in the remainder of the truss found without difficulty. If the loads *AB* and *CD* are unequal, the panels on the upper chord remaining equal, the polygon *hgbclkh* may be drawn by noting that the point *k* must lie midway between the parallel lines *cl* and *dm*. This follows from the fact that *LM* is normal to the upper chord and that KL and MN are of equal length and make equal angles with LM as well as with the upper chord. The triangle lkm is hence isosceles.

If CL and DM are of unequal lengths then both of the above methods fail. A general solution of this problem can be made by temporarily changing the webbing of the truss. The stress in NE would be determined analytically by taking moments about the peak, and it will readily be seen that its value is independent of the arrangement of the web members. If the web members KL and LM are removed and the diagonal $K''M$ substituted, as shown in Fig. 21*a*, it becomes possible to proceed with the stress diagram. First the polygon $hgbck''h$ can be completed, then the polygon $k''cdmk''$, and finally the polygon $ehk''mne$ giving the correct value of ne, as explained above. The original webbing may now be restored and the polygon $nehkn$ completed

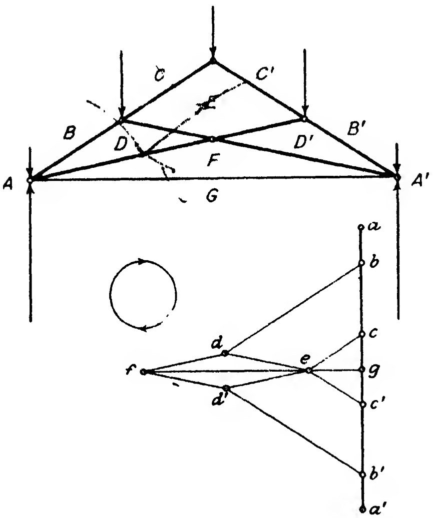


FIG. 21*b*.

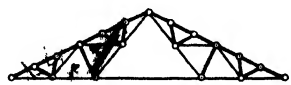


FIG. 21*c*.

since only the two stresses, HK and KN, remain unknown. The remainder of the stress diagram can be constructed without difficulty.

Other trusses of this general type, as for example the Fan truss, Fig. 21*c*, or the Compound Fink truss, may require two or more temporary members, but the procedure will be the same.

In the truss whose outline is given in Fig. 21*b* it is not possible to start the stress diagram in the usual manner, by considering the forces acting at the left support, since the known reaction and half-panel load are held in equilibrium by three unknown stresses. The load CC' at the peak is supported by only two members so that the stress diagram may be started there by constructing the force triangle *ecc'e*. Next the joints on each side of the peak may be considered and the polygons *bcedb* and *ec'b'd'e* constructed. Passing to the joint below the peak the polygon *ded'fd* is drawn. At the left support the stress in *FG* is the only remaining unknown. As equilibrium exists at this joint the polygon must be closed by the line *fg*, which is also parallel to *FG*. This completes the stress diagram. By following around the polygons all the stresses are found to be compression except *fg*, which is tension.

PROBLEM 21*a*.—The steel fan truss shown in Fig. 21*c* has a span of 60 ft., a rise of 14 ft., and a distance between trusses of 19 ft. Determine the maximum and minimum stresses.

ART. 22. COMPLETE STRESSES FOR A TRIANGULAR TRUSS

On Plate I are given the dimensions of a steel roof truss together with the specified loads. The web struts are normal to the top chord as shown on the skeleton outline of the truss. All the stress diagrams required to determine the stresses due to dead, snow, and wind loads are shown and are constructed in the manner explained in the preceding articles of this chapter. The stresses as measured by scale (4 kips to an inch on the original) are arranged in tabular form.

The preliminary computations give the following results:

Length of half top chord	41.15 ft.
Weight of truss	3.60 kips
Weight of roof covering	20.40 kips
Total dead load	24.00 kips
Dead panel load	3.00 kips
Dead-load reaction	12.00 kips
Snow panel load	2.28 kips
Ratio of snow load to dead load	0.76
Inclination of roof surface	25° 57′
Normal wind pressure per square foot of roof	22.0 lb.
Total wind load	14.96 kips
Wind panel load	3.74 kips

The reactions for the wind load are obtained graphically as explained in Art. 6. Assuming both ends of the truss to be fixed and the reactions to be parallel, the reactions due to wind from

Art. 22 Plate I

TRIANGULAR ROOF TRUSS

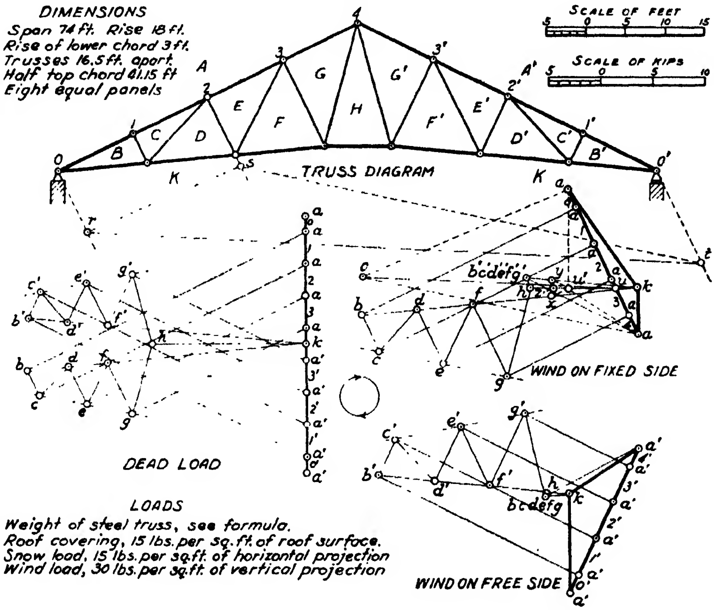


TABLE OF STRESSES

TRUSS MEMBERS	AB	AC	AE	AG	KB	KD	KF	KH	BC	DE	FG	CD	EF	GH
Dead load	-29.5	-28.2	-23.2	-18.4	+26.6	+22.8	+19.0	+14.7*	-2.7	-4.1	-5.4	+3.9	+4.5	+6.8
Snow load	-22.4	-21.4	-17.7	-14.0	+20.2	+17.3	+14.4	+11.2	-2.1	-3.1	-4.1	+2.9	+3.4	+5.2
Wind on fixed side	-22.8	-22.8	-17.9	-13.0	+26.4	+21.1	+15.8	+10.3*	-3.7	-5.6	-7.4	+5.3	+6.2	+8.4
Wind on free side	-10.0	-10.0	-10.0	-10.0	+2.5	+2.5	+2.5	+2.4*	0	0	0	0	0	+1.0
Maximum without wind	-51.9	-49.6	-40.9	-32.4	+46.8	+40.1	+33.4	+25.9	-4.8	-7.2	-9.5	+6.8	+7.9	+12.0
Maximum with wind	-74.7	-72.4	-58.8	-45.4	+73.2	+61.2	+49.2	+36.2	-8.5	-12.8	-16.9	+12.1	+14.1	+20.4
Minimum	-29.5	-28.2	-23.2	-18.4	+26.6	+22.8	+19.0	+14.7	-3.7	-4.1	-5.4	+3.9	+4.5	+6.8

TRUSS MEMBERS	A'B'	A'C'	A'E'	A'G'	KB'	KD'	KF'	KH	B'C'	D'E'	F'G'	C'D'	E'F'	G'H
Dead load	-29.5	-28.2	-23.2	-18.4	+26.6	+22.8	+19.0	+14.7	-2.7	-4.1	-5.4	+3.9	+4.5	+6.8
Snow load	-22.4	-21.4	-17.7	-14.0	+20.2	+17.3	+14.4	+11.2	-2.1	-3.1	-4.1	+2.9	+3.4	+5.2
Wind on fixed side	-11.7	-11.7	-11.7	-11.7	+10.5	+10.5	+10.5	+10.3	0	0	0	0	0	+0.3
Wind on free side	-21.2	-21.2	-16.2	-11.3	+18.3	+13.0	+7.7	+2.4	-3.7	-5.6	-7.4	+5.3	+6.2	+7.7
Maximum without wind	-51.9	-49.6	-40.9	-32.4	+46.8	+40.1	+33.4	+25.9	-4.8	-7.2	-9.5	+6.8	+7.9	+12.0
Maximum with wind	-73.1	-70.8	-57.1	-44.1	+65.1	+53.1	+43.9	+36.2	-8.5	-12.8	-16.9	+12.1	+14.1	+19.7
Minimum	-29.5	-28.2	-23.2	-18.4	+26.6	+23.8	+19.0	+14.7	-2.7	-4.1	-5.4	+3.9	+4.5	+6.8

+ = Tension - = Compression * Computed Values { +14.80, +10.24, +2.38 } the left are *au* and *ua* in the stress diagram marked 'wind on fixed side.' The pole is at *o* and the equilibrium polygon is *rst*, the wind panel loads being considered concentrated at the center of the left half of the top chord. But since the right end of the truss rests on rollers the reaction *AK* of the right support must be vertical and is represented by *ak*, or the vertical component of *au*. The closing side *ka* of the force polygon represents the reaction of the left support. Applying the scale the value of *ak* is found to be 4.15 kips, which is also the value of the vertical component of the left reaction when the wind blows on the right side. The value of *ka* is found to be 11.35 kips.

Most specifications allow a higher unit stress in the design of truss members when the wind-load stresses are combined with the dead- and snow-load stresses than for the dead- and snow-load stresses without the wind stresses. For this reason it is necessary to compute two maximum stresses for each member, one considering the wind load and one without. As the wind can blow on only one side of the roof at a time, only one of the wind stresses is combined with the vertical loads. The snow load always produces stresses of the same character as the dead load and hence is used only in obtaining the maximums. In the example of Plate I the wind load also produces stresses of the same character as the dead load. This is not always the case, and when the stresses are of different character the minimum stress is obtained by subtracting one from the other.

The stresses in the table on Plate I are given in kips. It will be noticed that the maximum chord stresses are greater on the fixed side than on the free side, whereas the maximum stresses in the web members are the same on both sides of the truss.

PROBLEM 22*a*.—A steel truss of the type shown in Fig. 21*a* has a span of 60 ft., rise of peak 16 ft., and rise of bottom chord 2 ft. The trusses are 20 ft. apart and their right ends rest on rollers. Find the maximum and minimum stresses in all members.

ART. 23. UNSYMMETRICAL LOADS AND TRUSSES

Figure 23*a* shows the same truss as Fig. 18*a* with a load applied on the bottom chord. This load might be caused by machinery attached to the truss or by a balcony being suspended from it. Such unsymmetrical loads are frequently encountered, but they present no unusual difficulties. The reactions, of course, are unequal, but they may readily be determined either graphically or analytically. Care should be exercised in laying out the load line so that the clockwise order of the forces is maintained. In Fig. 23*a* the load *AB* is laid off first. The next force encountered in passing clockwise around the truss is the left reaction *BC*, and finally the right reaction *CA*. The stress diagram is constructed

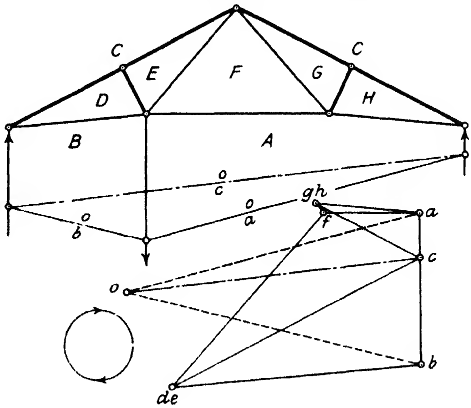


FIG. 23*a*.

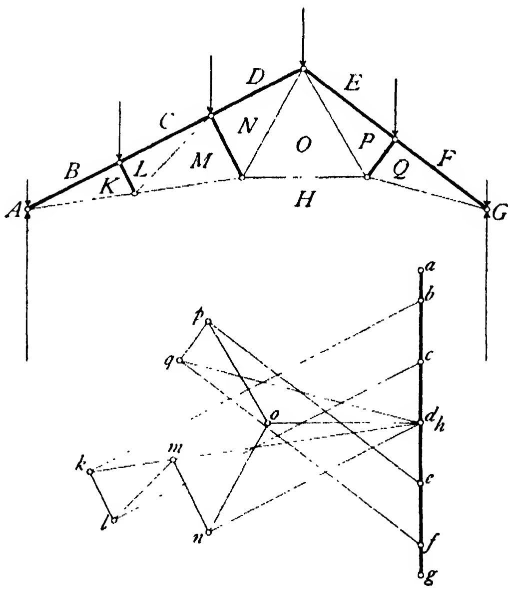


FIG. 23*b*.

in the usual manner. The load *AB* produces no stress in *DE* and *GH*, while it causes stresses in the other members of the same character as those due to the dead, snow, and wind loads, and hence their maximum stresses are increased. In certain cases such suspended loads may change some of the maximum stresses due to other loads from compression to tension or from tension to compression.

When a ceiling is attached to the lower chord of a truss it becomes a part of the dead load and needs no separate stress diagram. In constructing the load line the top chord loads are first laid off downward in a clockwise order, or from left to right; next the right reaction is laid off upward; then the lower chord loads are laid off downward, from right to left; and finally the left reaction is laid upward, closing the polygon. It will be noticed that portions of the load line overlap.

Another unsymmetrical condition frequently encountered is presented in Fig. 23*b*. Here an unsymmetrical truss is shown under the action of its dead load.

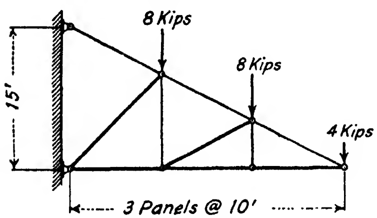


FIG. 23c.

The stress diagram for an unsymmetrical truss or loading is necessarily unsymmetrical and hence has fewer checks upon its construction. The main check lies in the closing of the stress diagram after working from each reaction toward the peak. An analytical check should be made of the stress in at least one member. The labor required to determine the stresses analytically in a truss under unsymmetrical conditions is materially greater than for symmetrical conditions, whereas with the graphic method there is little or no difference.

PROBLEM 23*a*.—Let the load *AB* in Fig. 23*a* be 5 kips, and the dimensions of the truss the same as given in Art. 18. Find the stresses in all members.

PROBLEM 23*b*.—Find the reactions and stresses for the cantilever truss shown in Fig. 23c.

ART. 24. THE TRANSVERSE BENT

Roof trusses are frequently supported on columns. A truss and the columns supporting it form a frame, called a 'transverse bent,' which must act as a unit in resisting lateral loads. Where the truss is of triangular form it is necessary to add members, known as 'kneebraces,' between the truss and columns (Fig. 24*a*).

The transverse bent is an indeterminate structure as it is really a two-hinged arch if the columns are free to rotate at their bases, or a fixed arch if the column bases are fixed. However, approximate solutions are generally used to determine the stresses due to both vertical and lateral loads and have been found satisfactory. For vertical loads the columns and kneebraces are disregarded and the truss treated as though it rested on solid supports.

For lateral loads it is necessary to assume a point of contra-flexure (point of zero moment) in the columns and a distribution of the horizontal reactions between them. The location of the point of contra-flexure depends upon the connections of the column to the truss and kneebrace and to the footing. If both ends of the column were rigidly fixed it would lie about mid-way between the base and the kneebrace connection. The lower the point of contra-flexure is taken, the greater are the stresses in both truss and columns. A conservative practice is to assume the point of contra-flexure to lie above the base one-third of the distance to the kneebrace connection. The proportion of the total shear, or the horizontal component of the reaction taken by each column, depends upon its relative rigidity. Where both columns are the same size and length it is reasonable to assume that they will carry equal shears.

The transverse bent in Fig. 24*a* consists of a triangular roof truss of 36-ft. span and 9-ft. rise supported on columns 20 ft. high. The kneebrace connections are 5 ft. from the top of the columns, and the points of contra-flexure are assumed to be 5 ft. above the base. The bents are 20 ft. apart. The wind pressure is assumed as 30 lb. per sq. ft. horizontally, and the component normal to the roof is found to be 22.4 lb. per sq. ft. by the formula of Art. 17. The wind panel loads on the truss are $6.7 \times 20 \times 22.4 = 3.0$ kips. The horizontal wind load at the top of the column is $\frac{1}{2} \times 5 \times 20 \times 30 = 1.5$ kips; that at the kneebrace is $\frac{1}{2} \times 20 \times 20 \times 30 = 6.0$ kips; and that at the base of the column is $\frac{1}{2} \times 15 \times 20 \times 30 = 4.5$ kips. The last load causes no stresses in the bent and is therefore not shown in Fig. 24*a*.

The reactions are most readily found algebraically but may also be found graphically as follows: Since the horizontal components of the two reactions are assumed to be equal the point *h* must lie on a vertical line through h'', which is at the mid-point of the resultant ag. The vertical components are the same as though both reactions were parallel to the resultant (Art. 6). These reactions, ah' and $h'g$, are easily found by means of the small equilibrium polygon shown in Fig. 24a, oh' being drawn parallel to the closing line oh. The point h must also lie on a horizontal line through h' and hence is at the intersection of this line with the

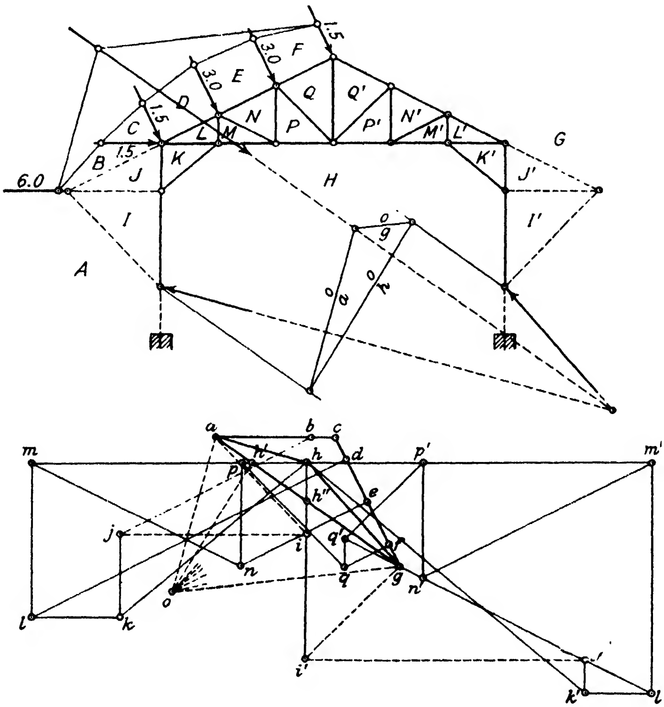


FIG. 24a.

vertical through h''. A check on the reactions is for them to intersect on the line of action of the resultant.

Before the stress diagram can be constructed it is necessary either to resolve the horizontal component of each reaction into two horizontal components, one applied at the bottom of the knee-brace and the other at the top of the column, or to add some auxiliary members to complete a truss system. The latter method is easier and, as it does not change the stresses in the members of the truss, will be used. The auxiliary members are shown in Fig. 24*a* in broken lines. The stresses are given in the table. The

Member	Wind from Left	Wind from Right
DL	−21.6	+17.7
EN	− 8.8	+ 1.6
FQ	− 3.0	− 3.7
LK	+ 5.6	− 4.4
MH	+16.9	−21.6
PH	+ 4.1	− 7.3
LM	+ 9.5	−14.4
MN	−14.3	+16.1
NP	+ 6.4	− 7.2
PQ	− 9.2	+ 6.8
QQ'	+ 1.7	+ 1.7
KH	+14.8	−22.5

stresses for the wind from the right are also scaled from Fig. 24*a* since they are the same as the stresses in the right half of the truss with the wind from the left. It will be observed that the character of the stresses in several members is opposite to that for the vertical loads.

The maximum moment in each column occurs at the kneebrace connection and is equal to the horizontal component of the reaction times the distance to the point of contra-flexure = 5.8 × 10 = 58.0 ft.-kips.

PROBLEM 24*a*.—Determine the stresses in the transverse bent of this article assuming the columns hinged at their bases.

CHAPTER III

HIGHWAY BRIDGE TRUSSES

Art. 25. Introduction

Bridge trusses differ from roof trusses only in general form and type of loads carried. Practically all simple bridges may be classified into three types, namely: deck, through, and pony truss bridges. A 'deck' bridge carries the roadway above the trusses and hence can be used only where there is ample clearance below the roadway. This type has the advantage of being economical, and it produces a rigid construction. A bridge that carries the roadway between the trusses and near the bottom chords is known as a 'through' truss bridge. Through bridges which are not deep enough to allow bracing over the roadway are called 'pony' truss bridges. Usually spans of less than 100 ft. are of the pony type whereas spans of 100 ft. or more are of the through type.

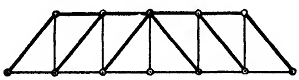


Fig. 25a.

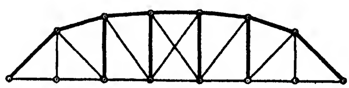


Fig. 25b.

The roadway of a modern highway bridge is usually made of reinforced concrete. The roadway is supported on 'stringers' running lengthwise of the bridge. The stringers are supported on 'floor-beams' running crosswise of the bridge and the floor-beams in turn are supported by the trusses. The stringers and floor-beams are usually of I-beam section and together constitute the 'floor system.' The bracing in the plane of the bottom chords of the trusses is known as the 'bottom lateral system'; that in the plane of the top chords is known as the 'top lateral system.' Bracing in a vertical plane between trusses is called 'cross bracing.'

Numerous types of trusses are used for bridges but only a few of the most common will be mentioned here. Other more complex types are shown in Chapter VI. The Howe truss, a type much used for wooden bridges, is shown in Fig. 25*a*. The Pratt truss is formed by reversing the inclination of the interior diagonals of the Howe truss. A Pratt truss with a curved upper chord, Fig. 25*b*, is known as a Parker truss. The Pratt and Parker trusses are popular for through trusses with spans up to about 200 ft. The Warren truss illustrated in Fig. 25*c* is much used for pony trusses and other relatively short spans. A type

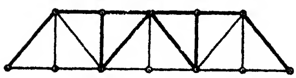


FIG. 25*c*.

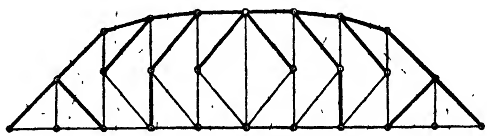


FIG. 25*d*.

having increasing use for longer spans is the K-truss, Fig. 25*d*. By breaking the length of the diagonals their inclinations are kept at a more economic angle for deep trusses.

ART. 26. DEAD LOADS

The dead load of a bridge consists of the weight of the trusses, bracing, floor system, floor, wearing surface, and any other permanent load that may be placed upon it. This weight depends upon the span, width, type of bridge and trusses, depth of trusses, type of floor and wearing surface, live load, unit stresses used in design, and various other details of design. The most variable factor of the dead load is the weight of the floor. A concrete floor with a wearing surface may weigh as much as 125 lb. per sq. ft. and account for 75 per cent or more of the total dead load whereas a timber floor may weigh as little as 15 lb. per sq. ft.

In modern highway bridges up to 200 ft. in span with reinforced-concrete floors, the trusses, floor system, and bracing make up only 15 to 30 per cent of the dead load. The balance of the dead load consists mainly of the floor and wearing surface, both of which are completely designed before the dead-load stresses in the trusses are determined. It is therefore necessary to assume a weight for only the trusses, floor system, and bracing. The best method of assuming this weight is by comparison with other bridges designed for similar conditions. Where a comparison of this kind is not feasible an approximate value can be obtained from the following formulas:

For pony trusses, $w = 140 + 12b + 0.2bL - 0.4L,$

For through trusses, $w = 200 + 12b + 0.2bL - 0.4L,$

in which w is the weight of two trusses, floor system, and bracing in pounds per linear foot, b is the width of the roadway in feet (including sidwalks, if any), and L is the span in feet. The values given by the formulas should be sufficiently accurate for design purposes since a relatively large error would cause only a small error in the total dead load. However, the actual weight should be checked against the estimated weight after the design of the bridge has been completed and the dead-load stresses revised if necessary.

ART. 27. DEAD-LOAD STRESSES

Figure 27*a* represents a 200-ft. span bridge with a 20-ft. roadway as designed by the Iowa Highway Commission. A photograph of two of these spans is shown in Fig. III*a*. The floor of the bridge consists of an 8-in. reinforced-concrete slab, 1 in. of which was considered as wearing surface. In addition 38 lb. per sq. ft. was allowed for future wearing surface.

The floor system as designed was found to average 480 lb. per linear ft. of bridge. By comparison with similar designs, the trusses and bracing were estimated to weigh 880 lb. per linear ft. of bridge. The total dead load was:

Wearing surface	760
Floor and curbs	2280
Floor system	480
Trusses and bracing	880
Total per foot of bridge	4400 lb.

The dead panel loads for each truss were $\frac{1}{2} \times 4400 \times 25 = 55\,000$ lb. = 55 kips. The dead load computed from the completed design was 4360 lb. per ft. so that no revision of the dead-load stresses was necessary.

The dead-load stress diagram was drawn as explained in Chapter II, passing around the truss in a clockwise direction. The original stress diagram was drawn to a scale of 30 kips to an inch, and the stresses, in kips, as scaled from it are shown on the right half of the truss in Fig. 27*a*. A check on the web dead-load stresses will be obtained when the live-load stresses are determined in Art. 30.

In the above example the entire dead load was considered concentrated on the lower chord. An analysis of the dead load

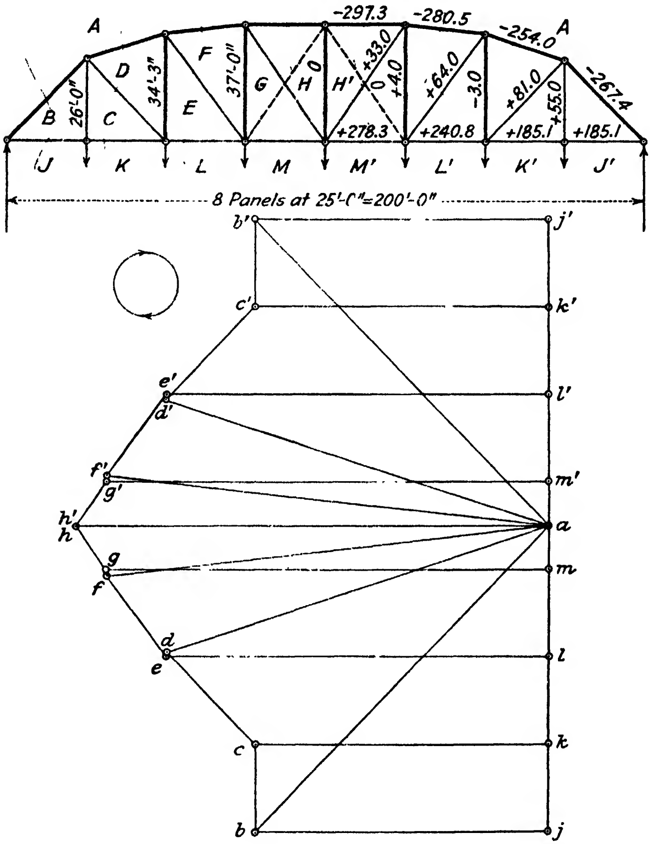


FIG. 27*a*.

shows that the trusses and bracing account for 20 per cent of the total. If half of this amount, or 10 per cent, amounting to 5.5 kips in this case, were applied at the top chord a more accurate determination of stresses would be obtained. It will readily be seen that the only stresses affected would be those in the verticals, each being changed by −5.5 kips.

As a further illustration let a through Pratt truss bridge be taken having seven panels of 18 ft. each, a span of 126 ft., and a height of 20 ft. (Fig. 27*b*). The roadway is 20 ft. wide and consists of a 7-in. concrete slab with an 8-in. curb on each side extending 8 in. above the floor slab. Allowance is also made for a wearing

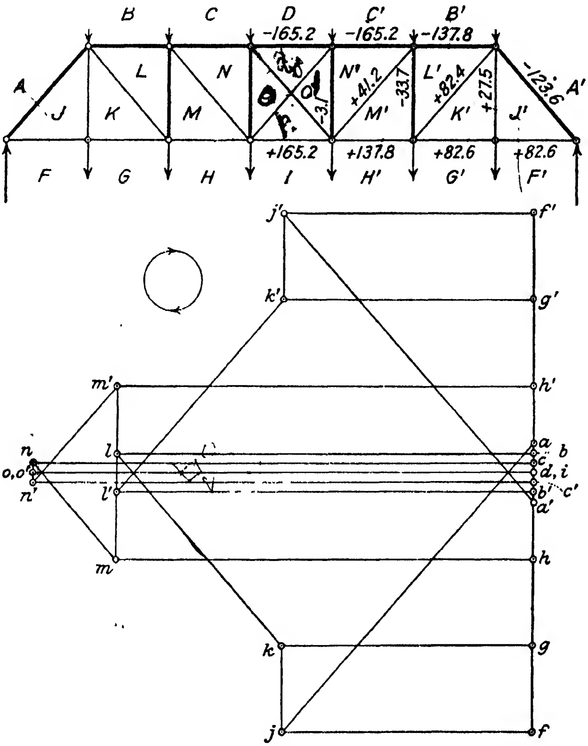


FIG. 27*b*.

surface weighing 25 lb. per sq. ft. By the formula in Art. 26, the weight of the steel is found to be 894 lb. per ft. of bridge. The dead load is therefore:

Wearing surface 25 × 20	= 500
Floor 0.583 × 20 × 150	= 1750
Curbs 2 × 0.667 × 1.250 × 150	= 250
Trusses, floor system, and bracing	= 894
Total per foot of bridge	= 3394 lb.

The total dead panel loads are $\frac{1}{2} \times 3400 \times 18 = 30\,600$ lb. $= 30.6$ kips. If about 10 per cent of the total dead panel loads are applied at the upper panel points, the upper panel loads become 3.1 kips each, and the lower panel loads 27.5 kips each.

The load line is constructed by taking the loads and reactions in a regular order clockwise around the truss, starting with the load AB. The force polygon is therefore $abcdc'b'a'f'g'h'ihgfa$. The stresses are shown on the right half of the truss.

An examination of Fig. 27*b* shows that the vertical component of aj is af, the reaction; the vertical component of $kl = af - ab - fg$; $lm = af - ab - fg - gh$; and so on. Therefore the following principle is established:

> For trusses with horizontal chords the vertical component of the stress in any web member equals the reaction minus all loads on the left; that is, equals the vertical shear for that member.

The only exception to this is the stress in the end vertical JK, which is equal to the lower panel load FG. . The above principle may also be derived from the relation existing between the stresses in any section of a truss and the external forces on either side of that section.

The diagram also shows that the difference between the magnitudes of the stresses in any two chord members equals the sum of the horizontal components of the stresses in the web members situated between them. For instance, the difference between oi and hm is the horizontal component of mn, which also equals the difference between cn and hm or between oi and bl. The horizontal component of any diagonal is called a 'chord increment' and forms the base of a right triangle whose height is the vertical shear in that diagonal.

Figure 27*c* shows one of the trusses of the Hill-to-Hill highway bridge at Bethlehem, Pa. (see Fig. III*b*) and its dead-load stress diagram. It has a span of 171 ft., and its depth at the various panel points is shown on the diagram. The upper dead panel loads are 25 kips each and the lower ones 214 kips each. The bridge is wide and has a concrete floor with granite block paving which accounts for the large dead panel loads. In constructing the stress diagram it is found that there are four members at both the upper and lower panel points at the right end of the first panel. This can be overcome by calculating the stress in the lower chord at the center, which also equals that in every panel

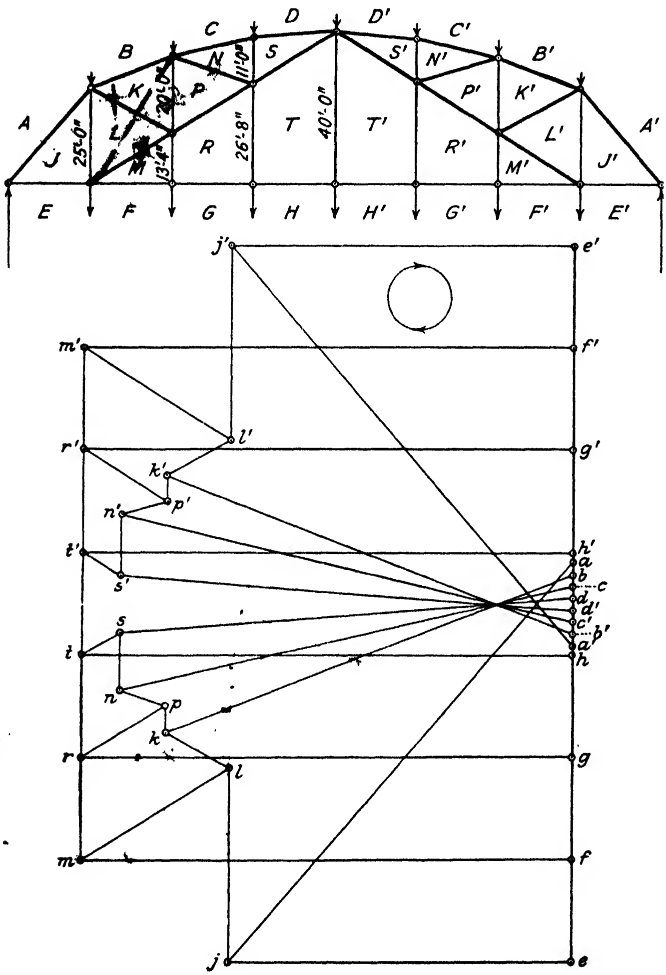


FIG. 27c.

except the end ones, and laying it off on the stress diagram. The remainder of the diagram is simple. This case is similar to that of the Fink truss described and illustrated in Art. 21 and may be solved in a similar manner by temporarily changing the left half of the truss into a Parker truss or some other simple web system.

PROBLEM 27*a*.—Determine the dead-load stresses in the Hill-to-Hill bridge, shown in Fig. 27*c*, by the entirely graphical method suggested in the last paragraph.

ART. 28. LIVE LOADS

The live load on a highway bridge consists of automobiles, motor trucks, wagons, and pedestrians. The density and character of the traffic depend upon the location of the bridge and the width of the roadway. The motor trucks produce the maximum stresses, especially for short spans. For longer spans there is less probability of the entire roadway being loaded at one time so that a uniform load is usually specified, diminishing with an increase in the length of roadway loaded. Highway-bridge live loads have become fairly well standardized, and those specified by the Iowa Highway Commission are given below.

Bridges are classified on the basis of location and traffic as follows:

Class A—bridges on primary and important county roads.
Class B—permanent bridges on unimportant county and township roads.
Class C—temporary bridges on township roads.
Class D—bridges carrying a traffic of exceptionally heavy load units.

All bridges, except class C, are designed to carry concrete floors. The live load for each class is specified as a uniform load as given by Fig. 28*a* or the following typical trucks (Fig. 28*b*), whichever gives the greater stress: class A, two 15-ton trucks; classes B and C, one 15-ton truck; and class D, two 20-ton trucks. Where two trucks are used they are to be placed side by side and headed in the same direction.

The stress produced by a moving load is greater than that produced by the same load when stationary. This increase in stress is called 'impact' and is due to the roughness of the floor and to the rapid application of the moving load. No rational formula for impact exists, but most empirical formulas take into consideration the loaded length since a short length can be loaded more rapidly than a long length and also because the impact effect is greater close to the load. The Iowa Highway Commission specifies an impact stress equal to $33\frac{1}{3}$ per cent of the live-load stress for I-beam spans and the floor systems of trusses and girders;

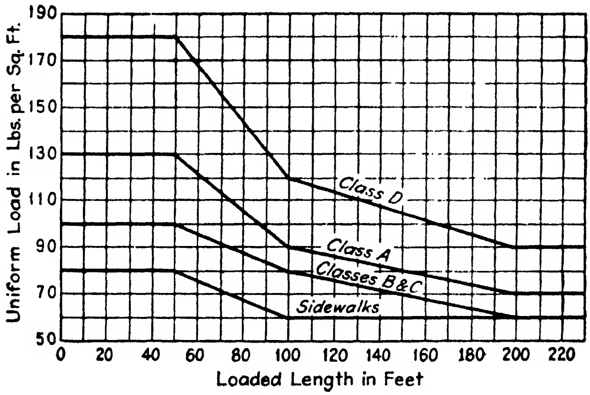


FIG. 28*a*.

and $66\frac{2}{3}$ per cent for floor-beam hangers. For main members of trusses and through girders the impact stress is given by the formula

$$I = \frac{75}{L + 200}S$$

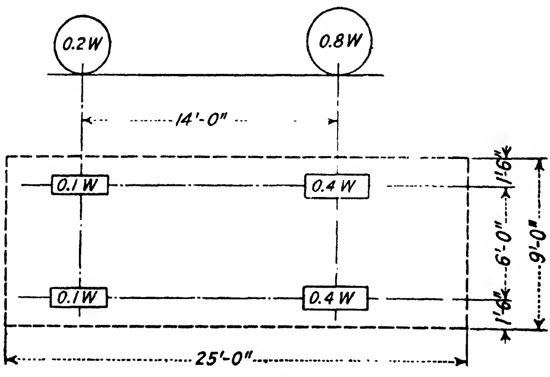


FIG. 28*b*.

in which I is the impact stress, L is the loaded length in feet producing the maximum live-load stress in the member, and S is the computed maximum live-load stress.

The above specifications, except those for impact, agree very closely with those adopted by the American Society of Civil Engineers and the American Association of State Highway Officials.

The snow load on a highway bridge may amount to as much as 20 lb. per sq. ft. but is usually neglected as it is not probable that a full live load would come on the bridge while a heavy fall of snow rests upon it. If the snow-load stresses are required a separate stress diagram is not necessary since the snow load is uniform and the stresses caused by it are proportional to those caused by the dead load, if the latter be taken only on the chord supporting the floor, or to those caused by a full live load.

PROBLEM 28*a*.—Construct a graph showing the relation of the percentage of impact to the loaded length as given by the impact formula. Use values of L up to 300 ft.

ART. 29. LIVE-LOAD STRESSES IN A WARREN TRUSS

As every load placed upon a bridge truss produces compression in the upper chord and tension in the lower chord, the greatest chord stresses produced by a live load occur when every panel point of the chord supporting the floor-beams is loaded. The chord stresses due to a uniform live load are hence obtained from

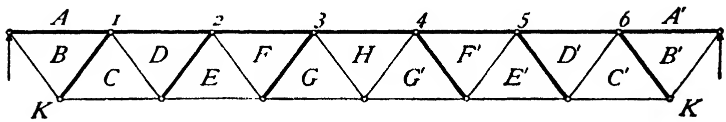


FIG. 29*a*.

a diagram exactly similar to that for a dead load applied only upon one chord. Hence the stress in any chord member, due to a uniform live load, bears the same ratio to the dead-load stress as that of the corresponding panel loads, and accordingly either stress may be derived from the other by using this constant ratio.

In order to investigate the effect of live load on the web members, let a deck Warren truss of seven panels be taken, the panel lengths being 18 ft., the span 126 ft., the depth 12 ft., and the roadway 20 ft. wide. From Fig. 28*a*, the uniform live load for a class A bridge of 126-ft. span is 85 lb. per sq. ft. The live panel load per truss is then $85 \times 10 \times 18 = 15.3$ kips. As stated in the specifications of Art. 28, the uniform live load depends upon the loaded length and not upon the span of the truss. A method of varying the panel load with the loaded length will be given in Art. 34, but in this article the panel loads will be kept constant. Constant panel loads have frequently been used in the past but their use is gradually being discontinued.

Placing a panel load at panel point 1 in Fig. 29*a*, the stresses due to this single load are obtained by drawing the stress diagram, Fig. 29*b*, in the usual manner. The reaction *a'k* is one-seventh of the panel load. The stresses in the web members are found to be alternately compression and tension each way from the load, and on either side the stresses are the same in magnitude from the load to the support, their vertical components being equal to the reaction on that side.

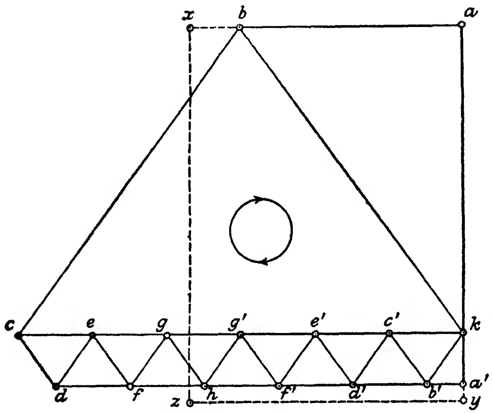


FIG. 29*b*.

For a panel load at panel point 2 the reaction of the right support will be twice as great as for the load at 1, and hence the stress in all web members on the right of panel point 2 will be twice as large; for a load at panel point 3 the stresses on its right will be three times as great as for the load at 1, and so on. Again, a load at panel point 6 will produce the same stresses on its left as the load at 1 caused on its right, and a load at 3 will produce stresses in the web members on its left equal to four times those due to the load at 6. The stress in each web member due to a single live panel load at any panel point may therefore be obtained by taking a simple multiple of the stress for that member as given by Fig. 29*b*.

In the following table the first and sixth lines are thus filled out directly with the results scaled from the stress diagram (which was originally drawn to a scale of 5 kips to an inch), and the other lines by taking multiples of these as indicated above. The stresses in each column are then combined so as to give the maximum and minimum stresses and those due to a live panel load at all panel points.

Web Members	*KB*	*BC*	*CD*	*DE*	*EF*	*FG*	*GH*
Live panel load at:							
1	+16.38	−16.38	− 2.73	+ 2.73	− 2.73	+ 2.73	− 2.73
2	+13.65	−13.65	+13.65	−13.65	− 5.46	+ 5.46	− 5.46
3	+10.92	−10.92	+10.92	−10.92	+10.92	−10.92	− 8.19
4	+ 8.19	− 8.19	+ 8.19	− 8.19	+ 8.19	− 8.19	+ 8.19
5	+ 5.46	− 5.46	+ 5.46	− 5.46	+ 5.46	− 5.46	+ 5.46
6	+ 2.73	− 2.73	+ 2.73	− 2.73	+ 2.73	− 2.73	+ 2.73
Max. live-load stresses	+57.33	−57.33	+40.95	−40.95	+27.30	−27.30	+16.35
Min. live-load stresses	0	0	− 2.73	+ 2.73	− 8.19	+ 8.19	−16.35
Total live-load stresses	+57.33	−57.33	+38.22	−38.22	+19.11	−19.11	0

It is found that for any given diagonal all the loads on one side of it cause one kind of stress, whereas those on the other side cause the opposite stress. The maximum stress is hence produced in a web member when the live load covers the larger segment of the span, and the minimum stress when the smaller segment is loaded.

In the construction of stress diagrams for a truss with horizontal chords and equal panels it is not necessary to draw the skeleton outline of the truss to a large scale. If in this example *ax* be laid off by the linear scale equal to some convenient multiple of the half panel length and *ay* equal to the same multiple of the depth of the truss, *xy* will give the direction of half the web members, and in transferring this direction the triangle will require very little shifting along a straight-edge, thus promoting accuracy. The line *ay* should be longer than *ak*. Completing the rectangle *xayz*, the direction of the remaining web members will be given by *az*.

The results in the line 'total live-load stresses' in the table should be the same as those derived from a stress diagram made for a live panel load at every panel point, and may thus be checked. As such a diagram is required for the chord stresses it will also be used for this purpose.

Problem 29*a*.—Determine the maximum and minimum live-load chord stresses for the above example by the same method used for the web members. Check all total live-load stresses by constructing a stress diagram for a panel load at every panel point.

ART. 30. LIVE-LOAD STRESSES IN A PARKER TRUSS

The 200-ft. Parker truss of Art. 27 will be used to illustrate the application to broken-chord trusses of the method given in Art. 29 for determining live-load web stresses. The chord stresses are of the same character for every load placed upon the bridge

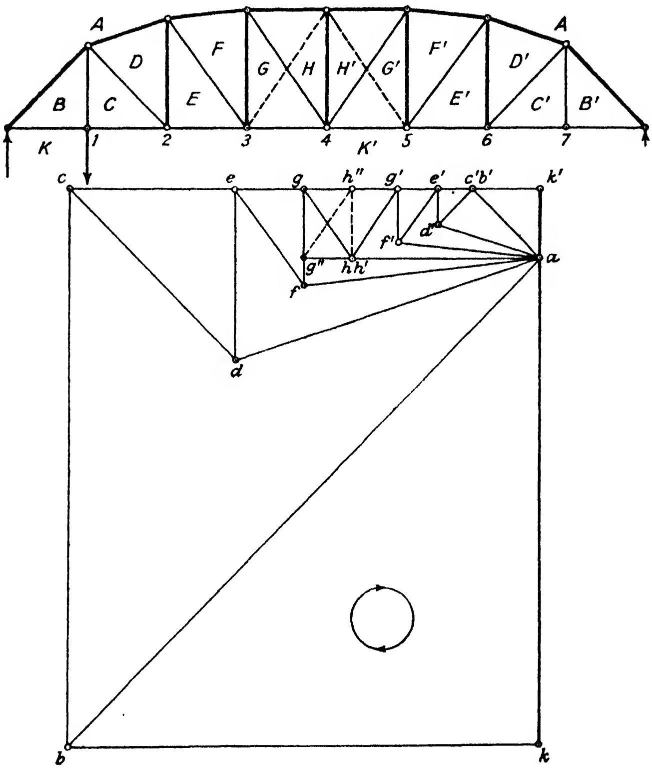


FIG. 30a.

so that the greatest live-load stresses occur with a panel load at every lower panel point. The stress diagram is therefore exactly similar to the dead-load stress diagram of Art. 27, and the stresses are proportional to their corresponding panel loads.

From Fig. 28a the uniform live load for a class A bridge of 200-ft. span is 70 lb. per sq. ft. The live panel load is 70 × 10 ×25 = 17.5 kips per truss. In Fig. 30a, a panel load is placed at panel point 1 and the stress diagram for this single load is drawn. The diagonals of this truss are too long to be designed economically to resist compression, therefore, an additional diagonal, sloping in the opposite direction and known as a 'counter,' is placed in each of the two center panels. The counters are represented in Fig. 30*a* by the broken lines. Only the left counter is stressed for the position of the load shown. The change in the stress diagram to accommodate the diagonal *GH* being replaced by its counter is shown by the broken lines. The other diagonals are also subject to compression from the live load, but this is more than offset by the tension caused by the dead load. It is found that the stresses in the adjacent verticals are changed when the counters are acting. The chord stresses in the same panel as the counter are also changed but need not be considered since the maximum chord stresses occur only under full live load.

The right reaction *a'k* is one-eighth of the panel load (Fig. 30*a*). For a load at panel point 2 the right reaction will be two-eighths, hence the stresses in all web members to the right of panel point 2 will be twice as large; similarly, for a load at panel point 3, the stresses on its right will be three times as great as for the load at 1, and so on.

The following table is filled out as explained in Art. 29, the first and seventh lines being scaled directly from the original drawing of Fig. 30*a* and the other lines taken as multiples of these. A column headed *G''H''* is included for the counter.

Web Members	*BC*	*CD*	*DE*	*EF*	*FG*	*GH*	*G''H''*	*HH'*
Live panel load at:								
1	+17.50	− 7.50	+ 5.35	− 3.72	(+ 3.04) + 0.81	(− 2.64) 0	(0) + 2.64	(0) − 2.19
2	0	+ 9.48	+10.70	− 7.44	(+ 6.08) + 1.62	(− 5.28) 0	(0) + 5.28	(0) − 4.38
3	0	+ 7.90	− 5.65	+10.55	(+ 9.12) + 2.43	(− 7.92) 0	(0) + 7.92	(0) − 6.57
4	0	+ 6.32	− 4.52	+ 8.44	− 6.80	+10.56	0	0
5	0	+ 4.74	− 3.39	+ 6.33	− 5.10	+ 7.92	0	0
6	0	+ 3.16	− 2.26	+ 4.22	− 3.40	+ 5.28	0	0
7	0	+ 1.58	− 1.13	+ 2.11	− 1.70	+ 2.64	0	0
Max. live-load stresses	+17.50	+33.18	−16.95	+31.65	−17.00	+26.40	(0) +15.84	(0) −13.14
Min. live-load stresses	0	− 7.50	+16.05	−11.16	(+18.24) + 4.86	(−15.84) 0	0	0
Total live-load stresses	+17.50	+25.68	− 0.90	+20.49	+ 1.24	+10.56	0	0

The stresses in parenthesis are those that would be produced if the counter $G''H''$ were not acting and must be used in finding the total live-load stresses.

It is found that the only panel load causing stress in the end vertical BC is the load at 1. It is also found that the center vertical HH' is stressed only when one of the counters is stressed. The rest of the web members follow the same general laws determined for parallel chord trusses, except that stresses caused by a single load are not equal in magnitude owing to the inclination of the top chord members and the difference in inclination of the web members. A check on the total live-load stresses can be made by multiplying the corresponding stresses of Fig. 27*a* by $\frac{17.5}{55}$, the ratio of their panel loads.

PROBLEM 30*a*.—Check the total live-load stresses in the above table from the dead-load stresses by the method explained.

ART. 31. STRESSES IN A BOWSTRING TRUSS

Bowstring trusses of the forms shown in Figs. 31*a*, *b*, *c*, and *d* have frequently been used for highway bridges in the past but their use has been practically discontinued. However, there are still many of these old structures in use which must be investigated from time to time in order to determine their ability to carry new types of floors or modern live loads. Also, the bowstring type is now much used for roof trusses.

The panel points of the broken chord lie either upon an arc of a circle or upon a parabola. When the web members are arranged as in Figs. 31*a* and *d*, the diagonals take only tension while the verticals take either tension or compression. In the truss in Fig. 31*c*, all the web members are designed to sustain either kind of stress. The same is true of the truss given in Fig. 31*b* with the exception of the middle and end verticals, which are subject to tension only. The form of truss shown in Fig. 31*d* is known as the 'lenticular' truss. The broken lines show the roadway and its connection to the trusses, the vertical end pieces being heavy posts and the others tension members.

As an example, let a truss like Fig. 31*a* be taken whose upper panel points lie in the arc of a circle. The truss has eight panels of 14 ft. each on the bottom chord, with a depth at the center of 16 ft. The depths of the truss at the first, second, and third panel points are 7.32, 12.23, and 15.07 ft., respectively. The bridge has a concrete roadway 22 ft. wide and two sidewalks each 5 ft. wide.

The weight of steel in the bridge is found by the formula of Art. 26 to be 1200 lb. per linear ft. to which must be added 2500 lb. per linear ft. for the weight of the floor and sidewalks. The dead panel loads are therefore, $\frac{1}{2} \times 14 \times 3700 = 25\ 900$ lb., or 26 kips, all of which are applied on the lower chord. From Fig.

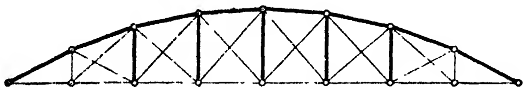


FIG. 31*a*.

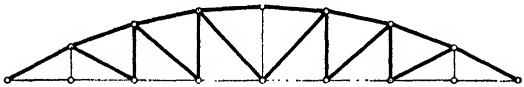


FIG. 31*b*.

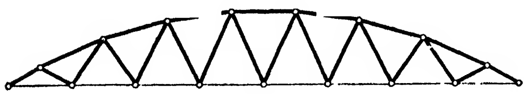


FIG. 31*c*.

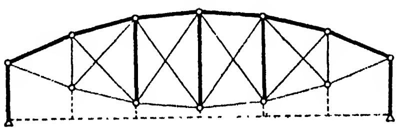


FIG. 31*d*.

28*a*, the class A uniform live load for the roadway is 88 lb. per sq. ft. and that for the sidewalks is 60 lb. per sq. ft. The live panel loads are $\frac{1}{2}(88 \times 22 + 60 \times 10)14 = 17\ 750$ lb., or 18 kips. The impact stresses are found from the formula of Art. 28 and amount to 24.0 per cent of the live-load stresses when the full length of the bridge is loaded.

The truss diagram shown in Fig. 31*e* contains only the main diagonals in the left half and only the counters in the right. The stress diagram obtained for the dead load is shown in the lower part of Fig. 31*e*, that for a live panel load at panel point 1 is shown in Fig. 31*f*, and that for a live panel load at panel point 7 in Fig. 31*g*. As a check on the construction of the dead-load diagram it is observed that *bc* and *b'c'* are in the same vertical line. The same is true of *de* and *d'e'* and of *fg* and *f'g'*. In general, the lines representing stresses in verticals equally distant from the center of the truss lie in the same vertical line. In Fig. 31*f* the line

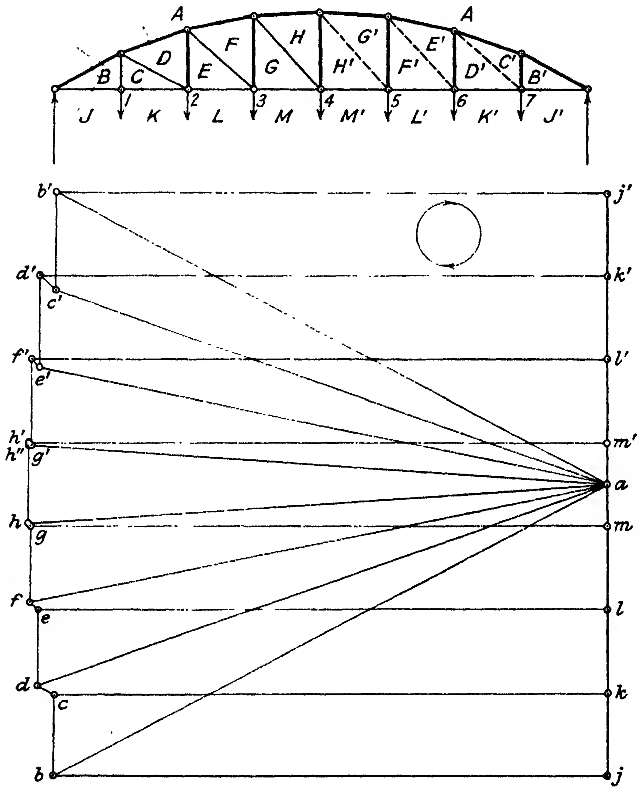


FIG. 31*e*.

de is the same distance from the load line as *d'e'* in Fig. 31*g*, also *d'e'* in Fig. 31*f* and *de* in Fig. 31*g* are similarly situated. The same relation exists between the lines representing the stress in any two verticals occupying symmetrical positions in the truss. Again, if in Fig. 31*g*, *f'g'* be produced to meet *ae'* and the intersection be called *g''*, then *f'g''* will be equal to *fg* in Fig. 31*f*, and *d'g''* will be equal to *ef* in Fig. 31*f*.

In Fig. 31*e*, *hh''* represents the stress in the vertical *HH'* when the main diagonal is inserted on its right side instead of the counter shown, the point *h''* being the intersection of the lines *ag'*

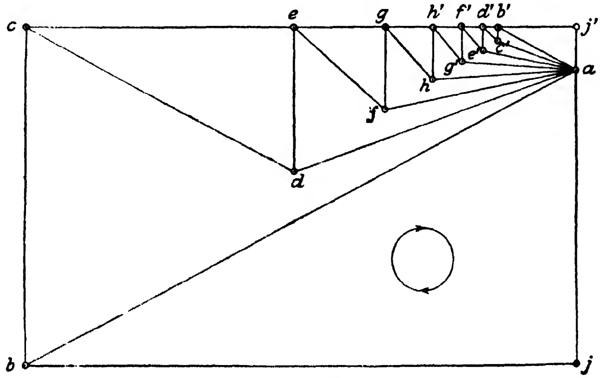


FIG. 31*f*.

and *hh'*. Since the maximum chord stresses occur under full live load, only the stresses in the web members are scaled from Figs. 31*f* and *g*. The live-load stresses in the chord members are found by proportion from the dead-load stresses, the ratio being 18 to

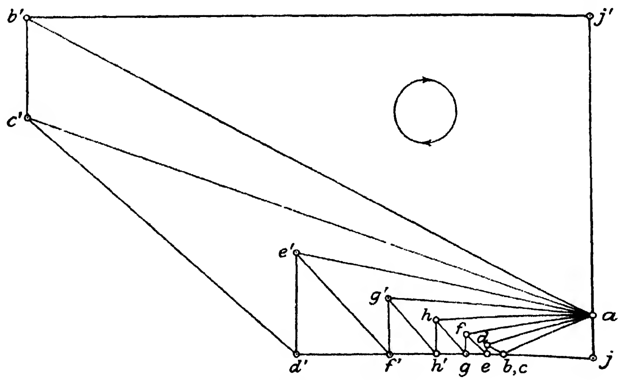


FIG. 31*g*.

26. The full live-load stresses for the web members may also be checked by proportion from the dead-load stresses.

The results expressed in kips are given in the following tables, and the maximum and minimum stresses, including impact, are obtained. The effect of lateral loads has been omitted.

	Upper Chord				Lower Chord		
	AB	*AD*	*AF*	*AH*	*BJ* = *CK*	*EL*	*GM*
Dead load....	−196.0	−189.1	−185.0	−182.3	+174.0	+178.8	+181.2
Live load......	−135.8	−131.0	−128.0	−126.2	+120.5	+123.8	+125.7
Impact........	− 32.6	− 31.4	− 30.7	− 30.3	+ 28.9	+ 29.7	+ 30.2
Maximum.....	−364.4	−351.5	−343.7	−338.8	+323.4	+332.3	+337.1
Minimum.....	−196.0	−189.1	−185.0	−182.3	+174.0	+178.8	+181.2

	Main Diagonals			Counters		
	CD	*EF*	*GH*	*G′H′*	*E′F′*	*C′D′*
Live panel load at:						
1...............	−16.6	− 6.6	− 3.8	− 2.5	− 1.7	− 1.1
2...............	+ 5.8	−13.3	− 7.6	− 4.9	− 3.4	− 2.2
3...............	+ 4.8	+ 7.5	−11.4	− 7.4	− 5.1	− 3.3
4...............	+ 3.9	+ 6.0	+ 9.4	− 9.8	− 6.8	− 4.4
5...............	+ 2.9	+ 4.5	+ 7.1	+11.9	− 8.5	− 5.5
6...............	+ 2.0	+ 3.0	+ 4.7	+ 7.9	+14.8	− 6.6
7...............	+ 1.0	+ 1.5	+ 2.4	+ 3.9	+ 7.4	+19.4
+Total........	+20.4	+22.5	+23.6	+23.7	+22.2	+19.4
−Total........	−16.6	−19.9	−22.8	−24.6	−25.5	−23.1
Uniform live load....	+ 3.8	+ 2.6	+ 0.8	− 0.9	− 3.3	− 3.7
Impact load........	+ 5.3	+ 6.1	+ 6.7	+ 7.1	+ 7.1	+ 6.6
Dead load..........	+ 6.0	+ 3.4	+ 1.0	− 1.0	− 3.7	− 6.7
Maximum..........	+31.7	+32.0	+31.3	+29.8	+25.6	+19.3
Minimum..........	0	0	0	0	0	0

The impact stresses in web members were calculated for a loaded length extending one-half panel past the last panel load. For example, the maximum tension occurs in the diagonal *GH* with panel points 4 to 7 inclusive loaded. The load length is therefore four and one-half panels, or 63 ft. No difficulty is encountered in finding the maximum and minimum stresses in the chords, diagonals, and counters as they carry only one kind of stress. The verticals carry both tension and compression stresses and care must be exercised in combining the live- and dead-load stresses to see that they can occur simultaneously.

	Verticals						
	BC	*DE*	*FG*	*HH'*	*F'G'*	*D'E'*	*B'C'*
Live panel load at:							
1	+18.0	+ 7.7	+ 4.4	+ 2.8	+ 1.8	+ 1.2	+ 0.7
2	0	+15.4	+ 8.7	+ 5.6	+ 3.6	+ 2.4	+ 1.5
3	0	− 2.1	+13.0	+ 8.4	+ 5.5	+ 3.6	+ 2.2
4	0	− 1.7	− 4.0	+11.2	+ 7.3	+ 4.8	+ 2.9
5	0	− 1.3	− 3.0	− 5.3	+ 9.2	+ 6.0	+ 3.6
6	0	− 0.8	− 2.0	− 3.5	− 6.0	+ 7.2	+ 4.4
7	0	− 0.4	− 1.0	− 1.8	− 3.0	− 5.4	+ 5.1
+Total	+18.0	+23.1	+26.1	+28.0	+27.4	+25.2	+20.4
−Total	0	− 6.3	−10.0	−10.6	− 9.0	− 5.4	0
Uniform live load	+18.0	+16.8	+16.1	(+16.8) +17.4	+18.4	+19.8	+20.4
+Impact load	+ 5.1	+ 4.9	+ 3.9	(+ 4.0) + 8.0	+ 2.1	+ 2.4	+ 3.9
−Impact load	0	− 1.7	− 2.9	− 3.2	− 2.9	− 1.8	0
Dead load	+26.0	+23.6	+23.7	(+24.3) +25.2	+26.9	+28.8	+30.8
Maximum	+49.1	+46.5	+43.7	+45.1	+36.5	+39.0	+47.8
Minimum	+26.0	+15.6	+10.8	+11.4	+15.0	+21.6	+30.8

The table shows a maximum live-load stress in the vertical *BC* of +18.0 kips with a load at panel point 1. Actually this stress cannot occur under this loading since the adjacent diagonal *CD* would then be subject to compression, which it cannot carry, and the counter would be brought into action. However, under a uniform live load the diagonal *CD* would be in tension and the stress could occur. It can be seen from the table that tension would still exist in *CD* if the live loads were removed from panel points 5, 6, and 7. The loaded length for calculating the impact stress is then four and one-half panels.

The maximum live-load stress of 23.1 kips tension for the vertical *DE* in the line marked '+total' when combined with that due to the impact and dead loads cannot actually occur because the adjacent diagonals, *CD* and *EF*, are not brought into action simultaneously. The greatest tension in *DE* will therefore occur under full live load, unless one or more of the live panel loads on the right may be removed without causing either *CD* or EF to cease acting. From the table it is seen that the live load may be removed from panel points 6 and 7, and the corresponding live-load stress will be $+16.8 - (-0.4 - 0.8) = +18.0$ kips. The loaded length is five and one-half panels, and the impact stress 27.1 per cent of $18.0 = +4.9$ kips. The maximum stress is therefore $+23.6 + 18.0 + 4.9 = +46.5$ kips.

The greatest live-load compression in the vertical DE is shown by the table to be due to the panel loads 3 to 7 inclusive. This is a real stress because the adjacent diagonals CD and EF are then acting, both of them receiving almost their maximum stress. The loaded length for calculating the impact stress is again five and one-half panels. The minimum combined stress is then $+23.6 -6.3 - 1.7 = +15.6$ kips.

In a similar manner the values of $+39.0$ and $+21.6$ kips are obtained for $D'E'$, and these are its maximum and minimum stresses under the condition that the adjacent counters $C'D'$ and $E'F'$ are both acting. On account of the symmetry of the truss the maximum and minimum stresses in DE have the same values provided the counters are acting in each adjacent panel.

Two more conditions for DE require attention. The first is that when the main diagonal acts on its right and the counter on its left. The table indicates that this condition cannot exist under any combination of the given loads.

The second condition occurs when the main diagonal acts on the left of DE and the counter on its right. This one is possible and requires an additional tabulation. The live load at panel point 1 produces a tension in DE of 2.28 kips, and that at panel point 7 of 0.77 kip. The former value is obtained from Fig. 31f by measuring the distance from d to the point where the vertical de meets af produced; the latter is obtained from Fig. 31g by measuring the distance from d to the intersection of af with the vertical de produced. In the same way, Fig. 31e gives the corresponding dead-load stress of $+26.2$ kips. From the stresses due to the live panel loads at 1 and 7 those produced by the loads at panel points 2 to 6 inclusive are found by the method used in the above tabulation to be $+4.6$, $+3.8$, $+3.1$, $+2.3$, and $+1.5$, when expressed to the nearest tenth of a kip. Since all these stresses are tension, it is clear that the maximum will be caused by the dead load combined with as many of the live panel loads as possible without bringing the main diagonal on the right of DE into action. The table for the diagonals indicates that this occurs when the live panel loads are placed at panel points 1 to 4 inclusive, and the resulting live-load, dead-load, and impact stress is +2.3 + 4.6 + 3.8 + 3.1 + 26.2 + 3.9 = + 43.9 kips. If a similar tabulation were made for $D'E'$ when the counter acts on its left and the main diagonal on its right, the same result of +43.9 kips would be obtained.

On comparing the maximum stresses in DE under the three conditions above described and investigated, the first value obtained is seen to be the greatest in magnitude, and hence the true maximum to be used for both DE and the corresponding vertical $D'E'$ in the other half of the truss. As the range of stress from this maximum of +46.5 to +15.6, the minimum in DE, is greater than to the minimum of +21.6 in $D'E'$, the true minimum stress to be used for both DE and $D'E'$ is +15.6 kips.

The stresses in FG when the main diagonal acts on its left and the counter on its right are found to be +1.4, +2.8, +4.2, +3.3, +2.5, +1.7, and +0.8 for the live panel loads 1 to 7 inclusive and +24.4 for the dead load. The maximum stress occurs when the live load is at panel points 1 to 6 inclusive, combined with the dead-load and impact stresses, and is +44.4 kips. On comparing this stress with the values given in the table for FG and $F'G'$, the true maximum for both these verticals is seen to be +44.4 kips and the true minimum +10.8 kips.

The maximum stress in the vertical HH' when the main diagonal acts on its left and counter on its right occurs with live load at panel points 2 to 7 inclusive and is +43.6 kips, including dead-load and impact stresses. The minimum stress occurs with live panel loads at points 5, 6, and 7 and is +11.4 kips. The real maximum for the vertical HH' occurs under full live load with both adjacent main diagonals acting. The value of the dead-load stress shown in parenthesis in the table is represented in Fig. 31*e* by hh'', as previously explained. The uniform live-load stress, also shown in parenthesis, is obtained by proportion from the dead-load stress. The combined dead-load, live-load, and impact stresses amount to +45.1 kips.

The true maximum and minimum stresses for the vertical $B'C'$ are equal to those for the vertical BC.

PROBLEM 31*a*.—A deck bowstring truss has six panels each 15 ft. long, the depth at the first and fifth panel points being 7.5 ft., at the second and fourth panel points 11.7 ft., and at the center 13 ft. The dead panel load is 5.0 kips and the live panel load 12.5 kips. Find the maximum and minimum stresses due to these loads only.

ART. 32. THE PARABOLIC BOWSTRING TRUSS

When the panel points of the broken chord of a bowstring truss lie upon a parabola whose vertex is midway between the supports, the stress diagrams become simpler. Let a parabolic bowstring truss be taken with the same general dimensions and loads as given in the preceding article. In the diagram like Fig. 31*e* the broken lines *bcd* . . . *d'c'b'* become a straight line, and the points *c* and *d*, *e* and *f*, . . . , *c'* and *b'*, coincide. This shows that under a uniform load the stress in the horizontal chord is the same throughout, the diagonals are not stressed at all, and each vertical carries only the panel load applied at its lower panel point. In the diagrams similar to 31*f* and 31*g*, the points *b*, *d*, *f*, *h*, *g'*, *e'*, and *c'* lie upon a straight line which intersects the load line produced at a distance from *j* or *j'* equal to the smaller reaction, thus checking the construction.

In the tabulated live-load stresses for the web members the sum of the '+total' and the '−total' will give zero for the diagonals and +18.0 for the verticals, provided the work be done with the utmost precision. With diagrams like Figs. 31*f* and 31*g*, made to a scale of 5 kips to an inch, the stresses obtained by tabulation for uniform live load averaged 0.02 kip in magnitude for the diagonals, two being tension and three compression, and those in the verticals varied on an average 0.04 kip from the true result, some being too large and others too small.

The final results in kips are given in the following table:

Chords	Maximum Stresses	Minimum Stresses	Diagonals	Maximum Stresses	Verticals	Minimum Stresses
AB	−378.2	−203.5	*CD*	+22.3	*BC*	+26.0
AD	−359.0	−193.0	*EF*	+26.6	*DE*	+18.9
AF	−346.1	−186.1	*GH*	+29.9	*FG*	+14.4
AH	−339.2	−182.5	*G'H'*	+31.2	*HH'*	+12.7
BJ, *CK*, *EL*, *GM*	+338.3	+182.0	*E'F'*	+30.7	*F'G'*	+14.1
			C'D'	+28.0	*D'E'*	+18.5
					B'C'	+26.0

The minimum stresses in all diagonals are zero, and the maximum stress in each vertical equals $+26.0 + 18.0 + 4.3 = 48.3$ kips, or the sum of the dead, live, and impact panel loads.

The properties of the parabola are such as to provide a very simple and abridged construction for obtaining directly the maximum and minimum stresses due to dead, live, and impact loads. The stress in the horizontal chord due to the dead load is equal to,

$$\frac{91 \times 56 - 78 \times 28}{16} = 182.0 \text{ kips}$$

Similarly that due to the live load is 126.0 kips and that due to impact is 30.3 kips.

Now in Fig. 32*a* on the horizontal line *ad*, let *ab* be laid off to

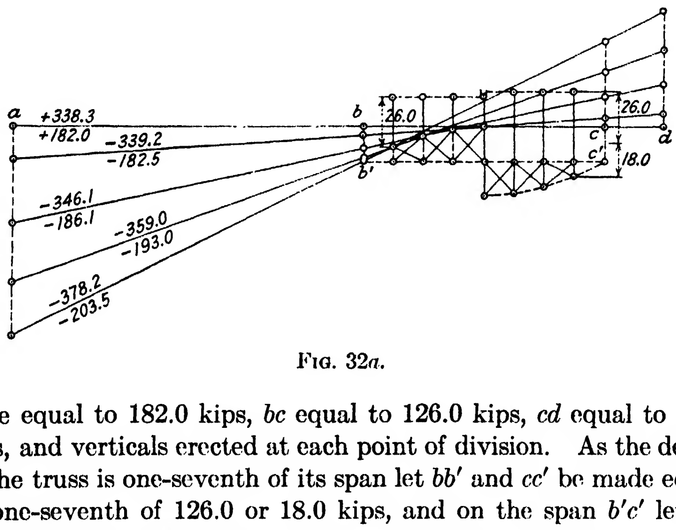


FIG. 32*a*.

scale equal to 182.0 kips, *bc* equal to 126.0 kips, *cd* equal to 30.3 kips, and verticals erected at each point of division. As the depth of the truss is one-seventh of its span let *bb′* and *cc′* be made equal to one-seventh of 126.0 or 18.0 kips, and on the span *b′c′* let an outline diagram be drawn similar to the truss diagram. In the figure one half is drawn as a through truss and the other half as a deck truss. The maximum and minimum stresses in the chord members are shown in Fig. 32*a* and those in the web members in the table on page 80.

By measuring the diagonals with the scale of force their maximum live-load stresses are obtained, to which must be added the proper percentage for impact stresses, depending on the loaded length. The dead-load stresses and the minimum live-load stresses in all diagonals are zero. The maximum and mini-

Diagonals	D.-L. + L.-L. Stresses	Impact Stresses	Maximum Stresses	Through Truss Verticals	D.-L. + L.-L. Stresses	Impact Stresses	Minimum Stresses	Deck Truss Verticals	D.-L. + L.-L. Stresses	Impact Stresses	Maximum Stresses
CD	+17.7	+4.6	+22.3	*BC*	+26.0	0	+26.0	*BC*	−44.0	−6.1	−50.1
EF	+20.9	+5.7	+26.6	*DE*	+20.4	−1.5	+18.9	*DE*	−49.6	−7.5	−57.1
GH	+23.3	+6.6	+29.9	*FG*	+17.0	−2.6	+14.4	*FG*	−53.0	−8.1	−61.1
G'H'	+24.0	+7.2	+31.2	*HH'*	+15.8	−3.1	+12.7	*HH'*	−54.2	−7.8	−62.0
E'F'	+23.3	+7.4	+30.7	*F'G'*	+17.0	−2.9	+14.1	*F'G'*	−53.0	−7.3	−60.3
C'D'	+20.9	+7.1	+28.0	*D'E'*	+20.4	−1.9	+18.5	*D'E'*	−49.6	−6.1	−55.7
......	......	......	......	*B'C'*	+26.0	0	+26.0	*B'C"*	−44.0	−4.4	−48.4

mum stresses in all verticals and diagonals occur under the same positions of the live panel loads as in Art. 31, so that the loaded length corresponding to each stress can be obtained from the tables of that article.

On the through truss draw the horizontal base line 26.0 kips (the value of the dead panel load) above the first panel point on the upper chord. By measuring the verticals extending from this base line to each panel point on the upper chord, upward being compression and downward tension, the minimum stresses due to dead and live loads in the verticals are found. The difference between the minimum stress in any vertical and the dead panel load is the minimum live-load stress on which the impact stress is based. The maximum stresses in all verticals occur under full live load and are equal to the sum of the dead, live, and impact panel loads.

On the deck truss let a similar base line be drawn 44.0 kips (the sum of the dead and live panel loads) above the first panel point from the support on the lower chord, and the verticals measured from the panel points to the base line; thus are found the maximum stresses in the verticals due to the dead and live loads, all of them being compression. The impact stresses are based on the live-load stresses which are again the difference between the maximum dead- and live-load stresses and the dead panel load. The minimum stresses in all verticals are −26.0 kips, the dead panel load. It will be observed that the differences between the minimum dead- and live-load stresses in the verticals of the through truss and the maximum dead- and live-load stresses in the same members of the deck truss equal twice the dead panel load plus the live panel load in all cases.

Let the chord members be prolonged until they meet the verticals through *a* and *d*. Each of these lines is divided into three parts by the four verticals, these parts giving the stresses due to dead, live, and impact loads, respectively. For example, the dead-load stress in the horizontal chord is represented by *ab*, the live-load stress by *bc*, and the impact stress by *cd*. The maximum stress in the same member is hence *ad*, or 338.3 kips, and the minimum stress is *ab*, or 182.0 kips. The maximum and minimum chord stresses shown in Fig. 32*a* are for the through truss. Those for the deck truss are equal in magnitude but opposite in character.

PROBLEM 32*a*.—A deck parabolic bowstring truss of 10 panels has a span of 120 ft. and a depth of 15 ft. at the center. Find the maximum and minimum stresses, including impact, for a dead panel load of 4.0 kips and a live panel load of 9.0 kips.

ART. 33. INFLUENCE LINES

An influence line is a line which shows the variation at a given point in a structure of a reaction, vertical shear, bending moment, stress, deflection, or any other function, when a single concentrated load moves across the span of the structure. The variation of the reaction of one of the supports of a beam or bridge, as a single load P crosses from one end to the other, may be exhibited by a line called 'a reaction influence line.' To draw it, the values of the reaction for several positions of the load P are laid off as ordinates at these positions, and the line joining their tops is the desired influence line. For example, let it be required to draw the influence line for the right reaction of the simple beam shown in Fig. 33*a*. If z is the distance of P from the left support and l is the span, the right reaction $R_2 = Pz/l$, and the value of this is P when the load is at the right support, $\frac{1}{2}P$ when the load is at the middle of the span, and zero when the load is at the left support. The line joining the tops of the ordinates is a straight line as shown in the figure, each ordinate giving the value of the reaction R_2 for a load P directly above it.

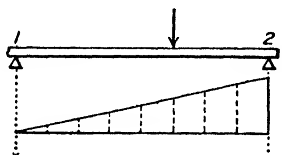


FIG. 33*a*.

An influence diagram is usually constructed for a unit load of 1 lb. or 1 kip. For a concentrated load the value of the function under consideration is obtained by multiplying the load by the value of the ordinate. If the structure is loaded with a uniform load the value of the function is obtained by multiplying the load per foot by the area of the influence diagram for the loaded portion. If Fig. 33*a* is constructed for a unit load, the ordinate at the right support is unity. The area of the influence diagram is then $\frac{1}{2}l$ and the right reaction for a load of w per foot over the entire length is $w \times \frac{1}{2}l = \frac{1}{2}wl$.

The variation of the vertical shear at any given section of a beam or bridge, as a single load P crosses from one end to the other, may be exhibited by a line called 'the shear influence line.' To draw it the values of the vertical shear for several positions of the load are laid off as ordinates at those positions. For

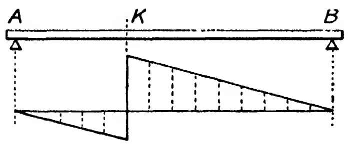


FIG. 33*b*.

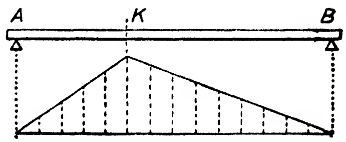


FIG. 33*c*.

example, the vertical shear at the section K of the simple beam in Fig. 33*b* due to a load P at the distance kl from the left support is $+P(1 - k)$ when the load is on KB, and $-Pk$ when the load is on AK. Hence, when P is at A or B the ordinate is zero, when P is passing K the ordinates are $+P(1 - k_1)$ and $-Pk_1$, the distance AK being k_1l. These lines clearly show how loads should be placed in order to give respectively the greatest positive and negative shears at the given section K.

The variation of the bending moment at a given section may also be represented by a line called 'the moment influence line.' Thus, for the section K at the distance k_1l from A, in the simple beam of Fig. 33*c*, the moment due to a load P at a distance kl from A is $+Pk_1l(1 - k)$ for a load on the right of K, and $+Pkl(1 - k_1)$ for a load on the left of K. Since the bending moment at the given section varies as the first power of kl in both cases, the influence diagram consists of two straight lines, and these are readily drawn after erecting an ordinate at the given section to represent the moment $Pk_1l(1 - k_1)$ due to a load at that point. This diagram shows that the maximum bending moment at any section of a simple beam occurs when the span is fully loaded, and when the heavy loads are near the section.

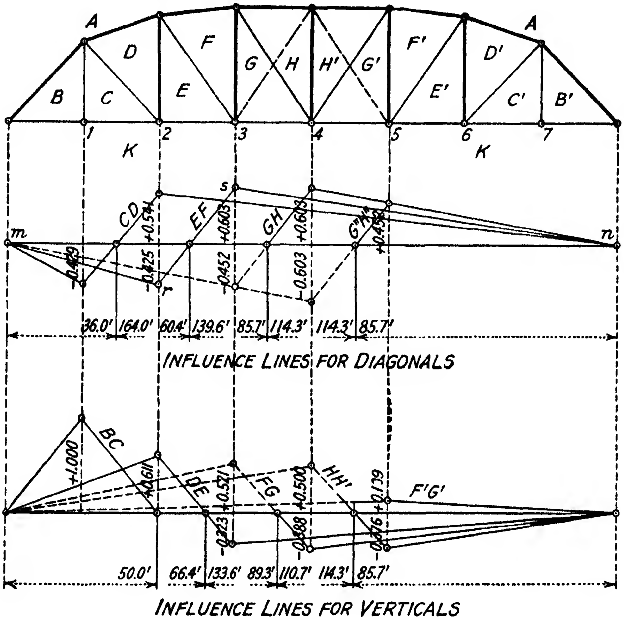
It is very important to distinguish clearly the difference between a 'bending moment diagram' and a 'moment influence diagram.' In the former, the load or loads are fixed in position and each ordinate represents the bending moment in a section of the beam having the same location as the ordinate. In the latter, the given section has a fixed location and each ordinate represents the bending moment in that section of the beam when a single concentrated load occupies the same position as the ordinate. Stated in another way, in the former, the diagram gives the bending moments in the different sections of the beam, for loading in one position; in the latter, the diagram gives the bending moment in only one section of the beam for the different positions of a single moving load.

PROBLEM 33*a*.—An overhanging beam 24 ft. long has one support at the left end and the other 6 ft. from the right end. Construct the influence lines for both reactions and for the shear and moment at a point 8 ft. from the left support.

ART. 34. LIVE-LOAD STRESSES BY METHOD OF INFLUENCE LINES

From Art. 30 it was found that a load at panel point 2 causes compression in the diagonal *EF* whereas a load at 3 causes tension. Also, a load at panel point 1 causes half as much compression as one at 2 and likewise the tension caused by loads to the right of panel point 3 decreases with a straight line variation to zero at the support. The influence diagram (Fig. 34*a*) for the stress in *EF* then consists of the straight line *mr* to the left of 2 and the straight line *sn* to the right of 3. A load applied between panel points is carried to the adjacent panel points by the floor system. Since the stress in *EF* changes from compression for a load at panel point 2 to tension for a load at 3, there must be some point between 2 and 3 at which a load can be applied that will produce no stress in *EF*. This point will evidently be located so that the reaction of the floor system at 3 times the ordinate to the influence line at that point is numerically equal to the reaction at 2 times the ordinate at that point. This location will be where the line connecting *r* and *s* intersects the axis *mn*. The complete influence diagram is therefore *mrsn*.

The influence diagrams for the other diagonals are drawn in the same manner as explained for EF. The diagram for the diagonal GH is drawn by ignoring the counter in that panel, but as the diagonal cannot resist compression the part of the diagram below



MEMBER	MAXIMUM STRESS	MINIMUM STRESS
CD	$+\frac{1}{2}\times 164.0\times 0.541\times 770=+34.2$ Kips	$-\frac{1}{2}\times 36.0\times 0.429\times 1300=-10.0$ Kips
EF	$+\frac{1}{2}\times 139.6\times 0.603\times 820=+34.5$	$-\frac{1}{2}\times 60.4\times 0.425\times 1210=-15.5$
GH	$+\frac{1}{2}\times 114.3\times 0.603\times 870=+30.0$	
G″H″	$+\frac{1}{2}\times 85.7\times 0.452\times 1020=+19.8$	
BC	$+\frac{1}{2}\times 50.0\times 1.000\times 1300=+32.5$	
DE	$+\frac{1}{2}\times 66.4\times 0.611\times 1170=+23.8$	$-\frac{1}{2}\times 133.6\times 0.323\times 830=-17.9$
FG	$-\frac{1}{2}\times 110.7\times 0.388\times 880=-18.9$	$+\frac{1}{2}(200\times 0.139-114.3\times 0.127)1020=+6.8$
HH′	$-\frac{1}{2}\times 85.7\times 0.376\times 1020=-16.5$	

FIG. 34a.

the axis mn is shown with broken lines. Likewise, the influence diagram for the counter $G''H''$ (broken line in truss diagram) is drawn ignoring the diagonal $G'H'$. The ordinates to the influence diagrams were obtained by proportion from the table of Art. 30.

The construction of the influence diagrams for the verticals is similar to that for the diagonals. It is found that only the load applied at panel point 1 causes stresses in the vertical *BC*.

The maximum and minimum stresses in the web members, given in the table of Fig. 34*a*, are found by multiplying the corresponding areas of the influence diagrams by the uniform load per foot as given by Fig. 28*a* for class A loading. Each truss carries 10 ft. of roadway. The effect of the two 15-ton trucks on the stress in the vertical *BC* must also be investigated. In this case it is found to be 31.0 kips, which is slightly less than that caused by the uniform load.

The combined dead-load, live-load, and impact stresses are given in the following table. The maximum live-load stresses in the chords occur under full live load and are found by proportion from the dead-load stresses. The impact stresses are found by the formula of Art. 28 for all members except *BC*, which is classed as a floor-beam hanger and must therefore be given an impact stress of $66\frac{2}{3}$ per cent of the live-load stress. Where the live-load stress in a member is of different character from the dead-load stress only two-thirds of the latter is combined with the live-load and impact stresses.

Member	D.-L. Stress	L.-L. Stress		Impact Stress		Combined Stress	
		Max.	Min.	Max.	Min.	Max.	Min.
AB	−267.4	−85.0	0	−15.9	0	−368.3	−267.4
AD	−254.0	−80.7	0	−15.1	0	−349.8	−254.0
AF	−280.5	−89.2	0	−16.7	0	−386.4	−280.5
AH	−297.3	−94.5	0	−17.7	0	−409.5	−297.3
BK	+185.1	+58.9	0	+11.0	0	+255.0	+185.1
CK	+185.1	+58.9	0	+11.0	0	+255.0	+185.1
EK	+240.8	+76.6	0	+14.4	0	+331.8	+240.8
GK	+278.3	+88.5	0	+16.6	0	+383.4	+278.3
BC	+ 55.0	+32.5	0	+21.7	0	+109.2	+ 55.0
CD	+ 81.0	+34.2	−10.0	+ 7.1	−3.2	+122.3	+ 40.8
DE	− 3.0	+23.8	−17.9	+ 6.7	−4.0	+ 28.5	− 24.9
EF	+ 64.0	+34.5	−15.5	+ 7.6	−4.5	+106.1	+ 22.7
FG	+ 4.0	−18.9	+ 6.8	− 4.6	+1.8	− 20.8	+ 12.6
GH	(0) + 33.0 (− 33.0)	+30.0	0	+ 7.2	0	+ 70.2	0
G″H″	0	+19.8	0	+ 5.2	0	+ 3.0	0
HH′	0	−16.5	0	− 4.3	0	− 20.8	0

As a means of comparing the results obtained by the method of a single panel load at each panel point, used in Art. 30, with those obtained by influence lines, the following table of live-load stresses is prepared, using a uniform load of 70 lb. per sq. ft. for all loaded lengths.

Web Members	*BC*	*CD*	*DE*	*EF*	*FG*	*GH*	*G″H″*	*HH′*
Influence lines, maximum.....	+17.5	+31.0	−15.1	+29.4	−15.0	+24.2	+13.6	−11.3
Influence lines, minimum.....	0	− 5.4	+14.2	− 9.0	+ 4.7	0	0	0
Panel loads, maximum......	+17.5	+33.2	−17.0	+31.7	−17.0	+26.4	+15.8	−13.1
Panel loads, minimum.......	0	− 7.5	+16.1	−11.2	+ 4.9	0	0	0

It is found that in all cases, except for the vertical *BC*, the method of influence lines gives smaller stresses than the method of panel loads. This is due to the fact that it is not possible to have a full panel load at one panel point without having some load at the adjacent panel point. For example, consider the diagonal *EF* in Fig. 34*a*. The influence diagram shows that the maximum compression occurs when the live load covers 60.4 ft. at the left end of the bridge. This load corresponds to a full panel load at 1 but only about eight-tenths of a panel load at 2. In addition there is about one-tenth of a panel load at 3 causing tension, which further decreases the compression in *EF* below what would be obtained for full panel loads at panel points 1 and 2 only. The errors of the panel-load method are on the side of safety, but the method is not readily adaptable to varying the live load with the loaded length.

PROBLEM 34*a*.—Determine the live-load web stresses for the truss of Fig. 29*a* by means of influence lines and compare them with those obtained by the panel-load method.

ART. 35. LATERAL LOADS AND STRESSES

The lateral forces to be considered in the design of a highway bridge are wind and lateral vibrations. Centrifugal forces must also be considered on rare occasions. The specification of the Iowa Highway Commission is representative of present-day practice. It provides that, "The force due to wind and lateral vibrations shall consist of a horizontal moving load equal to 30 lb. per sq. ft. on the side area of any exposed floor construction, the side area of all railings, and 1½ times the side area of one truss, girder, or arch. In addition to the foregoing, a moving load of 150 lb. per linear ft. shall be considered as acting in the plane of the loaded chord on highway bridges and 300 lb. per linear ft. upon bridges for combined highway and electric railway service. However, in the case of structures having a reinforced-concrete floor slab effectively anchored to the supporting structure, this additional chord load need not be considered."

The lateral systems are generally of the Pratt type, the chords of the trusses acting as chords of the lateral system. In the loaded chord the floor-beams act as posts. The diagonals are usually too long to be designed economically to resist compression so that all panels have counter diagonals. Since the wind may blow from either side of the bridge, the two diagonals in each panel are designed for the same maximum stress. The top chord loads are carried to the end posts by the top lateral system whence they are carried to the bearings by the portal bracing in the plane of the end posts.

The allowable unit stresses are usually increased 25 per cent when lateral stresses are combined with dead-load, live-load, and impact stresses. The lateral stresses in chords and floor-beams never equal 25 per cent of the stresses due to vertical loads so that the only stresses required in the lateral systems are those in the diagonals and in the struts of the unloaded chord.

As an example, let the 200-ft. Parker truss of Art. 27 be taken. The side area of the top chord and half of the web is found to average 2.9 sq. ft. per linear ft. of bridge. The top chord lateral load is therefore $2.9 \times 1\frac{1}{2} \times 30 = 130$ lb. per ft. and the panel loads 3.3 kips. The average side area of the bottom chord and half of the web members is 2.6 sq. ft. and that of the floor construction and railing 3.3 sq. ft. per ft. of bridge. The bottom chord lateral load is therefore $(2.6 \times 1\frac{1}{2} + 3.3) \times 30 = 215$ lb. per ft. and the panel loads 5.4 kips. If the floor were not of reinforced concrete, an additional lateral force of 150 lb. per ft. would be applied at the lower chord.

Figure 35*a* represents the top lateral system with a panel load at panel point 6. The load is considered as moving from right to left. For the conditions as shown, the diagonals in the left half of the truss are main members and those in the right half are counters. With the wind on the other side of the bridge and moving from left to right these conditions are reversed, the main diagonals being on the right. For other conditions of loading the other set of diagonals in either half of the truss may become the main diagonals. Therefore, the only stresses required in the lateral system are the maximum tension stresses in the main diagonals and the maximum compression stresses in the struts.

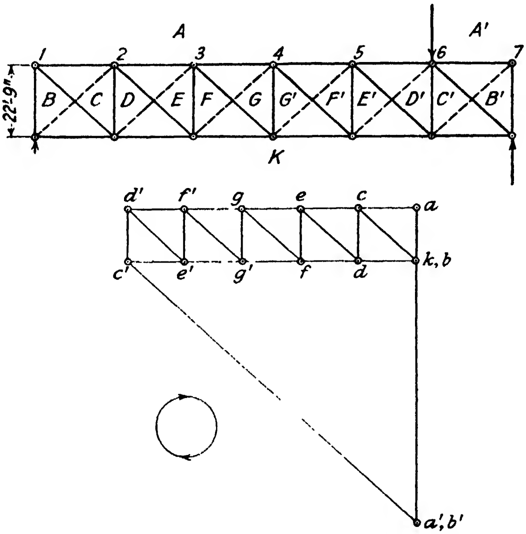


FIG. 35a.

The following table is compiled as in Art. 29, the stresses for the panel load at 6 being scaled directly from Fig. 35a:

Member	*AB*	*BC*	*CD*	*DE*	*EF*	*FG*	*GG'*
Panel load at:							
2..................	−2.75	+ 4.10	−2.75				
3..................	−2.20	+ 3.28	−2.20	+3.28	−2.20		
4..................	−1.65	+ 2.46	−1.65	+2.46	−1.65	+2.46	−1.65
5..................	−1.10	+ 1.64	−1.10	+1.64	−1.10	+1.64	−1.10
6..................	−0.55	+ 0.82	−0.55	+0.82	−0.55	+0.82	−0.55
Maximum stress.......	−8.25	+12.30	−8.25	+8.20	−5.50	+4.92	−3.30

Owing to the slope of some of the top chord members, the stresses in the corresponding top lateral diagonals must be corrected by dividing the tabulated values by the cosine of their angle of slope.

The bottom chord lateral system is shown in Fig. 35b together

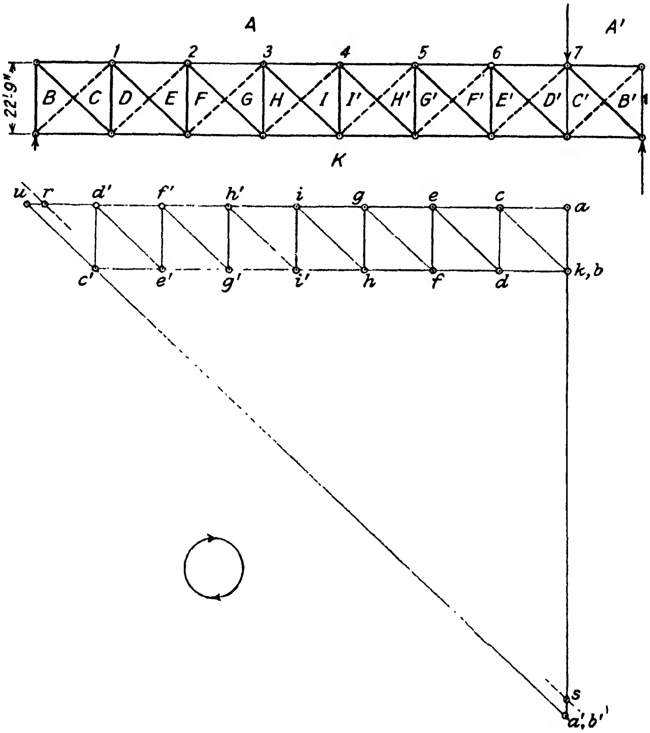


FIG. 35*b*.

with the stress diagram for a panel load applied at panel point 7. Only the stresses in the diagonals are tabulated since the floor-beams act as struts and their stresses are not required.

Member	*BC*	*DE*	*FG*	*HI*
Panel load at:				
1............	+ 7.00			
2............	+ 6.00	+ 6.00		
3............	+ 5.00	+ 5.00	+ 5.00	
4............	+ 4.00	+ 4.00	+ 4.00	+ 4.00
5............	+ 3.00	+ 3.00	+ 3.00	+ 3.00
6............	+ 2.00	+ 2.00	+ 2.00	+ 2.00
7............	+ 1.00	+ 1.00	+ 1.00	+ 1.00
Maximum stress..	+28.00	+21.00	+15.00	+10.00

In Fig. 35*b*, *ar* is laid off equal to a panel length and *as* equal to the distance center to center of trusses with as large a scale as convenient. The line *rs* therefore gives the slope of the diagonals much more accurately than the small diagram of the lateral system. Since *ak* is equal to one-eighth of the panel load *aa'*, the short diagonals of the stress diagrams are each equal to one-eighth of the diagonal *ua'* and the verticals are each equal to *ak*. All the stresses required may therefore be found by laying out the triangles *ars* and *aua'*.

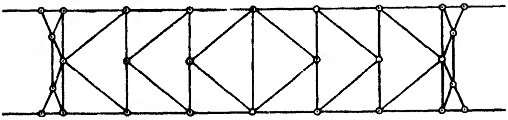


FIG. 35*c*.

PROBLEM 35*a*.—Figure 35*c* represents a *K*-system of top lateral bracing. The trusses have eight panels of 20 ft. each and are 32 ft. apart. Determine the maximum and minimum stresses due to a lateral load of 125 lb. per linear ft. of bridge.

ART. 36. APPLICATION OF THE EQUILIBRIUM POLYGON

In the preceding articles of this chapter the method of the force polygon has been employed exclusively. To illustrate the application of the equilibrium polygon in the determination of stresses let a through Pratt truss be taken having eight panels, a span of 176 ft., a depth of 26 ft., and a 20-ft. roadway. It is required first to find the chord stresses and then the web stresses due to the dead load.

From the formula of Art. 26 the weight of the steel in the bridge is 1075 lb. per ft. The weight of the concrete roadway and curbs is 2000 lb. per ft., making a total dead load of 3075 lb. per ft. of bridge. The dead panel loads are $\frac{1}{2} \times 3075 \times 22 = 33.8$ kips. Let the truss diagram be drawn to a scale of 30 ft. to an inch, and the panel loads laid off successively on the load line *j'j* in Fig. 36*a* to a scale of 100 kips to an inch (considerably reduced as here printed). The effective reactions are *ja* and *aj'*, the load line being bisected at *a*. Let the pole *o* be taken on a horizontal line through *a*, the pole distance *H* being made equal to 104 kips, and the equilibrium polygon constructed (Art. 4). The ordinates at the vertices of this polygon when measured by the linear scale and multiplied by *H* give the bending moments in foot-kips at the corresponding sections of the truss (Art. 7).

The chord stresses are obtained by dividing these moments by 26 ft., the depth of the truss. For instance, the ordinate nn' measures 42.9 ft., whence the stress in AD and EL is

$$\frac{42.9 \times 104}{26} = 171.6 \text{ kips}$$

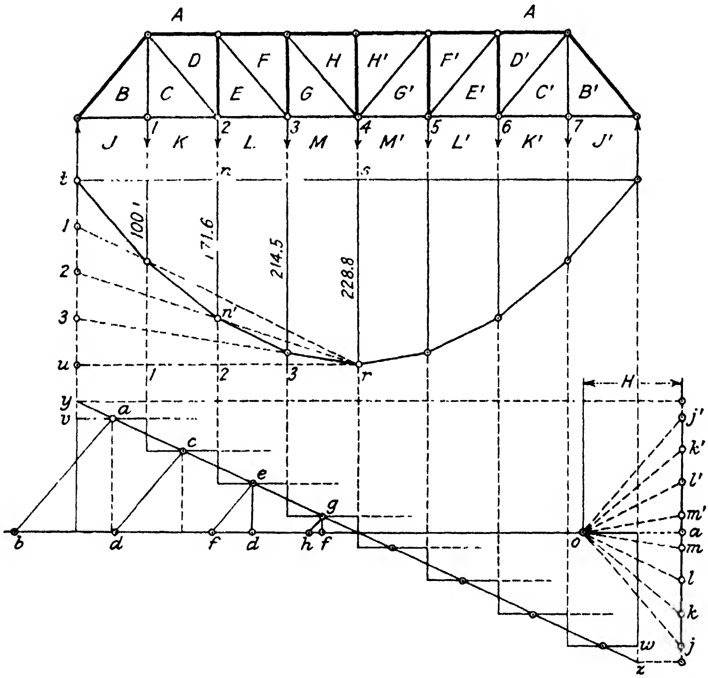


FIG. 36a.

The chord stresses may therefore be directly obtained by measuring the ordinates with a scale of $30 \times \frac{104}{26} = 120$ kips to an inch, the results being marked on the diagram.

If the pole distance could have been laid off equal to 26 kips, the ordinates would have been measured by a force scale equal to the linear scale, or 30 kips to an inch, to obtain the chord stresses. Again, if H be laid off by a linear scale equal to double the depth of the truss, then the ordinates are to be measured by double the scale of force, or 200 kips to an inch.

As the vertices of the equilibrium polygon lie upon a parabola whose vertex is at r, the ordinates may be obtained without drawing the equilibrium polygon. The chord stress at the center of a uniformly loaded truss (having an even number of panels) is

$$S = \frac{Wl}{8d}$$

in which W includes the half-panel load at each support. Let the middle ordinate rs be made equal to

$$S = \frac{8 \times 33.8 \times 176}{8 \times 26} = 228.8 \text{ kips}$$

and let tu be made equal to rs and divided into the same number of parts as ur, in this case four. Drawing radial lines from r to these points of division their intersections with the corresponding verticals give the required points.

For a truss with an odd number of panels tu should be divided into as many parts as there are panels in the entire truss, only the alternate points of division from t to u being used. The ordinate at the vertex of the parabola is given by the above formula, but since the vertex lies at the center of a panel the ordinate is larger than the chord stress in that panel. For a uniform live load the same method may be employed as that here given, or the dead and live loads may be combined in one diagram.

The shear diagram for dead load is shown below the moment diagram in Fig. 36*a*, the ordinates representing the vertical shear being limited by the line forming a series of steps from v to w. If the load were not concentrated at the panel points, but uniform throughout, the ordinates for shear would be measured to the straight line yz, which intersects the former at the center of each panel. The straight inclined line is the more convenient to use. The lines ab, cd, ef, and gh are the stresses in the diagonals, and de and fg in two of the verticals. To avoid confusing the diagram, cd, ef, and gh are turned toward the left, but have the same inclination as the diagonals of the truss. The stress in BC is one panel load and that in HH' is zero. If part of the dead panel load be taken on the upper chord, a compression of that amount is to be added to each of the above stresses in the verticals.

In order to determine the maximum live-load shear in any panel another shear diagram is necessary. On the horizontal axis AG in Fig. 36*b* let the position of the panel loads be marked, and their magnitude laid off on the load line *ag*. Let the pole *o* be placed in a horizontal line through the beginning of the load line, the pole distance *H* being made equal to the span of the truss by the linear scale, and the equilibrium polygon $A'B'C' \ldots H'$ constructed. Now let the span be so placed that its right support comes at *F*, then every panel point from 3 to 7 inclusive is loaded. This position of the load gives the maximum live-load shear in *EF* and *DE* of Fig. 36*a*. The ordinate $F'F''$ is equal to the reaction of the left support and hence equals the vertical shear in the members named, for the ordinate being contained between the sixth side of the equilibrium polygon and the first side pro-

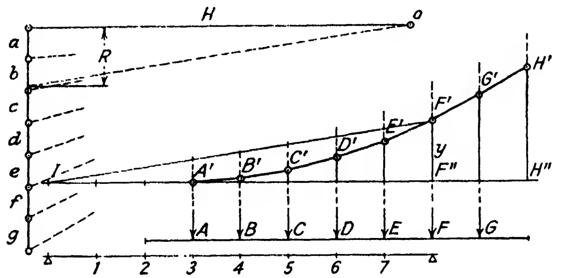


FIG. 36*b*.

duced measures the sum of the moments of all the loads between them with reference to the section through F' (Art. 7). Calling the value of the ordinate *y*, the sum of these moments equals $y \times H$. But the section at F' is at the right support of the truss, and hence the sum of the moments also equals the moment of the left reaction with reference to this support. Therefore

$$y \times H = R \times l$$

and since *H* was made equal to *l*,

$$y = R$$

This may also be proved by drawing through the pole a parallel to the closing line $F'I$ of the equilibrium polygon forming a triangle which is equal to the triangle $IF'F''$ since one side *H* is equal to its parallel IF'' and all the sides of both triangles are mutually parallel. $F'F''$ is hence equal to its parallel *R*.

The ordinates taken in succession from H' to A' measured by the scale of force give the maximum live-load shears in each panel of the truss from left to right. The stresses in the diagonals from these shears are obtained in the manner indicated in Fig. 36*a*. The results from this method are found to be the same as those given by the method of panel loads in Art. 29.

For trusses with inclined chords the moment diagram gives only the horizontal component of any chord stress, the ordinate being measured in a section passing through the center of moments of the chord member. The shear diagram is not applicable in such cases except for the purpose of finding the reaction from which the stress in the required web member may be found by the method of the force polygon.

It will be observed by the student that the method of the equilibrium polygon does not indicate the character of the stresses as in the method of the force polygon. Whether a member is in tension or compression has to be determined by cutting it by a plane, noting its direction and the sign of the shear, as was done in the analytic method in Part I.

PROBLEM 36*a*.—Find the maximum and minimum stresses in the truss of Art. 29 due to the dead and live loads. Assume the weight of the roadway to be 1800 lb. per ft. of bridge and the weight of the trusses from the formula of Art. 26.

CHAPTER IV

RAILWAY GIRDERS AND TRUSSES

Art. 37. Dead Loads

Railway bridges are of the same general types as the highway bridges described in Art. 25 with the exception that pony trusses are used very seldom. Bridges of 100-ft. span, or less, are practically always of the plate girder type, and numerous plate girder bridges have been constructed with spans longer than 100 ft. (see Art. 41 and Fig. IV*b*). A few long-span bridges of the Warren type with broken upper chords and subverticals at each panel point have been built recently (see Fig. VI*a*).

Plate girder bridges of the deck type have the ties supported directly on the girders so that no floor system is required. Consequently, they are more economical than the through plate girder and are used wherever clearances permit.

A railway bridge is composed of the floor system, lateral bracing, and trusses or girders together with the pieces that unite and stiffen them. The weight of steel in a bridge depends upon the span, number of tracks, type of bridge, type of floor, live load, and unit stresses adopted in design, and varies considerably with individual cases. The best method of estimating the weight of steel in a bridge is by comparison with similar designs, the actual weights of which are known. Where such designs are not available, an approximate value for single-track bridges, designed for Cooper's E60 loading (Art. 38), with an ordinary wooden tie deck laid directly on the floor system can be obtained from the following formulas:

$$w = 14L + 150, \text{ for deck plate girders,}$$

$$w = 12L + 1000, \text{ for through plate girders,}$$

$$w = 12L + 500, \text{ for trusses,}$$

in which w is the total weight of steel in pounds per linear foot of bridge and L is the span of the bridge in feet.

The above formulas give only the weight of steel in the bridge. To this must be added the weight of the floor to obtain the entire dead load for which the bridge must be designed. The approximate weight of a wooden tie floor, including rails and guard rails, is 400 to 450 lb. per linear ft. of bridge. The weight of a concrete floor or ballasted floor varies considerably but will be several times as heavy as a wooden tie floor. In any event, the floor is designed first so that its weight can be computed accurately and used in determining the dead load on the bridge.

An E50 loading reduces the weight of steel about 10 per cent from that given by the formulas. The weight of a double-track bridge depends upon the construction but is increased about 80 per cent where two or three girders or trusses are used.

ART. 38. LIVE LOADS

In the last chapter it was found that the maximum stresses in the members of a highway truss were usually produced by the specified uniform load rather than by the concentrated loads. Owing to the number and magnitude of the concentrations of railroad locomotives, it is more difficult to treat them as a uniform load although it is sometimes done. The live load for railroad bridges is usually specified as one or two typical locomotives followed by a uniform train load of a given weight per linear foot of track. The locomotive axle loads constitute a system of concentrated loads whose relation to each other and to the uniform load following them remain unchanged while passing over the bridge.

A few railroads design their bridges to carry actual locomotives in use on their roads. Locomotives used by the different railroads vary considerably and are also subject to change from time to time. In order to have a standard of comparison, typical loadings have been devised and adopted by the American Railway Engineering Association and most of the railroads. Figure 38*a* represents two typical locomotives followed by a uniform train load according to Cooper's standard loading, class E60. The numbers above the wheels show the axle loadings in kips and the numbers between them show their distances apart in feet. Other classes of Cooper's series have the same spacing of axles, while the loads on the corresponding axles and the uniform load are directly proportional to their class numbers. Alternate loads on two axles, 7 ft. apart, are also specified for each class, the load on each axle being 75 kips for class E60 loading. At the time Cooper's standard loading was presented (1894), the heaviest locomotives in use were closely represented by class E40. At the present time the American Railway Engineering Association specifies class E60 for main lines and class E50 for branch lines.

Types of locomotives now in use vary somewhat in wheel groupings and weight distribution from the consolidation locomotives in Cooper's standard. Moreover, the uniform train load has not increased as fast as locomotive loadings. To care for these discrepancies bridge specifications frequently vary the loading classification with the span, for example, specifying E70 or E75 for short spans and E60 for long spans. Another method sometimes employed is to give the locomotives a different classi-

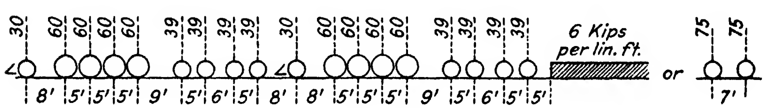


FIG. 38a.

fication from that of the train load. For example, a class E70 locomotive might be followed by a class E60 train load. The Suisun Bay Bridge (see Fig. VI*a*) recently built by the Southern Pacific Railroad was designed for two locomotives equivalent to Cooper's class E90, followed, preceded, or both, by a uniform train load of 7500 lb. per linear ft.

The variations of the Cooper loadings outlined above give satisfactory results but are nevertheless makeshift. For that reason attempts have been made to establish new typical loadings that would more nearly conform to actual conditions, but they have not been generally adopted, owing mainly to the wide use of the Cooper standard and to the inconvenience of changing to a new standard. Not only each bridge capacity is evaluated in terms of this series, but also each locomotive in use on the railroad. Also, loads are changing so fast that in a short time any new loading might be nearly as approximate as the Cooper loadings. Locomotive loadings have increased faster than train loadings. Tender-axle loadings have increased faster than driver-axle loadings until at the present time locomotives are being built with about as much weight on each tender axle as on each driver axle (see Fig. IV*a*). Cooper's loadings may be replaced for design purposes, but they will remain the standard of comparison for some time.

An 'equivalent uniform load' is sometimes used in place of the standard locomotives and train loads described above. An equivalent uniform load is one that will produce the same stress at the section considered as the given concentrated loads. This load necessarily varies with the span and for different sections of the same span and is not the same for moment as for shear. Therefore, for this method to be of value, it is necessary to have tables or diagrams giving the equivalent uniform loads for various conditions. Where this method is used, the stresses in a truss may be determined as in Chapter III.

Still another method of calculating stresses due to locomotive loadings is to determine first the stresses produced by the uniform train load and then superimpose an additional or 'excess load' consisting of a uniform load, or one or more concentrated loads. The excess load represents the difference between the locomotive load and the uniform train load, and the stresses due to it are added to those produced by the uniform load.

Numerous formulas have been devised for determining the stresses due to impact, but only two will be considered here. The first one is included in the 1920 specifications of the American Railway Engineering Association. It is

$$I = S\frac{300}{300 + \frac{L^2}{100}}$$

where I is the impact stress; S is the live-load stress and L is the loaded length in feet which produces the maximum live-load stress. The second formula is given in the 1923 specifications of the American Society of Civil Engineers and is

$$I = S\frac{2000 - L}{1600 + 10L}$$

Both formulas give results practically equal for spans of 50 ft. to 100 ft., but the latter gives somewhat larger values for longer or shorter spans.

ART. 39. MOMENTS DUE TO WHEEL LOADS

The live load on deck girder bridges and on through girder bridges with solid floors may be applied at any point along their length whereas that on girders and trusses with floor-beams can be applied only at the floor-beams. Girders of the former type will be considered first.

Figure 39*a* is the moment influence diagram for any point C on the girder AB. This diagram is easily constructed by making the ordinates a' and b' equal numerically to the segments a and b, respectively, but not necessarily to the same scale. The ordinates to the influence diagram must be measured with the same scale

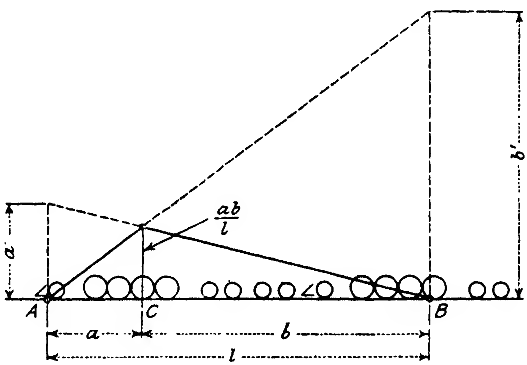


FIG. 39*a*.

used in laying out a' and b', and will be in foot-pounds if a and b are in feet and the unit load is 1 lb., and in foot-kips if the unit load is 1 kip.

The moment produced at C by a concentrated load at any point on the girder is equal to the product of the load and the ordinate to the influence line at that point (Art. 33). The moment produced by a train loading is equal to the numerical sum (since all ordinates have the same sign) of the products of the individual concentrated loads and their ordinates, plus the product of the area of that portion of the influence diagram covered by the uniform load and its weight per linear foot. The influence diagrams should be drawn to a large scale so that the ordinates can be scaled with precision. Some arithmetical labor can be saved by adding together all the ordinates for loads of the same magnitude before multiplying by the loads. This addition may be performed graphically. Cooper's loading has wheel loads of only three different magnitudes so that not more than three multiplications are necessary to determine the moment (or shear) at any section due to these loads. Where the uniform load is also on the structure another multiplication will be required.

The maximum moment at section C will occur when the train load is in such a position that the sum of the products of the individual loads and their ordinates is a maximum. First consider wheel 1 at the section. As the loads are moved to the left the ordinate for wheel 1 decreases while the ordinates for all other loads on the girder are increasing and the total moment at the section is probably increasing. As soon as wheel 2 passes the section its ordinate also decreases, and the difference between the rate of increase in moment due to the loads on the right of the section and the rate of decrease due to the loads on the left of the section becomes smaller. Each load that passes the section increases the rate of decrease, and it soon becomes larger than the rate of increase. The maximum moment at the section will therefore occur with that wheel at the section which causes the rate of decrease to become larger than the rate of increase. This wheel is generally one of the heaviest wheels and may be on either locomotive. The position giving the maximum moment usually occurs with nearly the entire length of the girder loaded and with a little experience can readily be located by trial.

The criterion for the position of the wheel loads which produces the maximum moment at any section of a girder supporting the load directly is

$$P' = \frac{l'}{l}W$$

in which W is the whole load on the girder, P' is the part of the load to the left of the section, l' the distance from the section to the left support, and l the span. This is the same formula deduced in Part I which applies to vertical sections through the panel points of trusses with vertical posts, and to the panel points of the loaded chord of those with inclined web members. In plate girders with floor systems the criterion applies only to the sections at the floor-beams.

To apply this criterion graphically, construct an equilibrium polygon $ABC'D'$ and a load line $ABCD$, composed of a series of steps, as shown in Fig. 39*b*. The rise of any step in the load line indicates the magnitude and position of the corresponding wheel load. Any ordinate to the load line gives the sum of all the loads on its left, and its value can be scaled off directly. The first side AB of the equilibrium polygon is horizontal and coincides with the axis AX. The total ordinate at any point represents the sum of the moments of all the wheels on the left of the point with

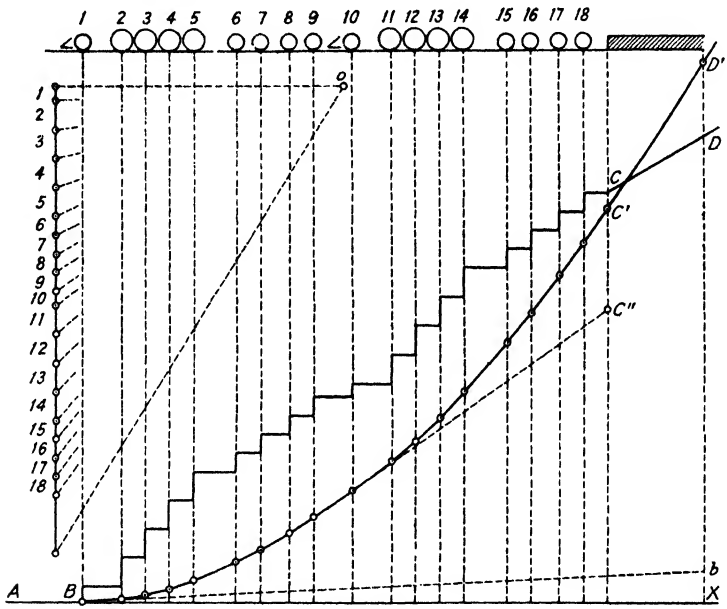


Fig. 39*b*.

reference to the point as a center. If the side of the polygon directly in front of the second locomotive be produced to C'', the portion of the ordinate above this point represents the moment of the entire second locomotive about the head of the train; the portion below represents the moment of the first locomotive about the same point, the values of these moments being read directly from the diagram (Art. 7).

Since the diagram may be used for a number of problems it should be constructed with ink on heavy paper. The scales adopted should be as large as possible without making the drawing too unwieldy. The diagram is usually constructed for one rail only so that the wheel loads are each half of the axle loads shown in Fig. 38*a*. A convenient-sized drawing for office use is obtained by constructing an E60 diagram with a linear scale of 10 ft. to an inch, a force scale of 50 kips to an inch, and a pole distance of 300 kips. The scale of moments is therefore $10 \times 300 = 3000$ ft.-kips to an inch.

On a sheet of tracing paper let the span *ab* of the girder be laid out to exactly the same linear scale as the above diagram, and let the half span be divided into the required number of equal parts and indefinite ordinates erected at these points. Figure 39*c* shows the completed diagram for a span of 80 ft. placed in position on the load line. It is very important that, when the base lines of the two diagrams coincide, their ordinates shall be truly parallel.

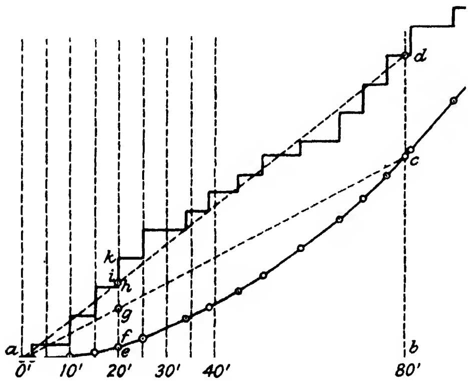


FIG. 39*c*.

The live load used in this example is Cooper's E60. The sections are 5 ft. apart, and the 20-ft. section is made to coincide with wheel 4. If this be the proper position for the maximum moment in the 20-ft. section, the equation $P' = \frac{l'}{l}W$ must be satisfied. Remembering that the horizontal axis on the left of *a* is to be considered as a part of the load line, connect by a straight line the points *a* and *d* where the ordinates at the supports intersect the load line. Now the ordinate *bd* equals *W*, *ab* equals *l*, *ae* equals *l'*, and hence $\frac{l'}{l}W$ is represented by the ordinate *ei*. If wheel 4 is just on the right of the section the ordinate *eh* represents *P'*; if just on the left the ordinate *ek* is the load *P'*; and when it is at the section the load *P'* may be regarded as having any value between these limits. The condition is therefore satisfied and this position will probably give the maximum moment at the 20-ft. section. Another wheel (or wheels) may also satisfy the criterion, and if so the maximum moment must be found by trial. All the possible positions for all the sections can be determined in a few minutes by using a fine silk thread, shifting the tracing paper in each case so as to bring a wheel over the section, stretching the thread as indicated above and noting whether it intersects that wheel on the load line. The graphical method of determining position by means of a load line was first published by WARD BALDWIN in *Engineering News*, Vol. XXII, pp. 295 and 345. See also letter of Dec. 11, 1889, on p. 615.

The value of the maximum bending moment at the 20-ft. section may now be obtained by drawing the closing line *ac* (Fig. 39*c*) and measuring the ordinate *fg*. Where the left end *a* of the closing line is on the axis *ab*, greater precision can be obtained by reading the ordinate *bc*, multiplying it by the ratio of l' to l (in this case one-fourth), and subtracting the ordinate *ef*, which is known and usually recorded on the diagram. In plate girders, however, *a* is frequently not on the axis. It is therefore better to make the scale large enough to insure the requisite precision when the ordinates are read off directly.

For the above girder the probable positions for maximum moment at the various sections are shown in the following table. Other positions satisfy the criterion for several of the sections but can be eliminated by inspection.

Section	Wheel at Section	Section	Wheel at Section
5′	2–3	25′	4–12–13
10′	2–3–4	30′	12–13
15′	3–4	35′	12–13
20′	4–12	40′	5–6–12–13

Where more than one position satisfies the criterion, the use of the dividers will show which produces the largest moment, and its value alone need be carefully read and recorded.

In order to enable a larger vertical scale to be used, the equilibrium polygon may be constructed as shown in Fig. 39*d*, with the side of the equilibrium polygon directly in front of the pilot wheel of the second locomotive horizontal. If the linear and force scales are taken the same as recommended for Fig. 39*b* and the pole distance one-half as great, or 150 kips, the moment scale will be twice as large, or $10 \times 150 = 1500$ ft.-kips to an inch. By reference to the table of positions on p. 103 it is seen that most of

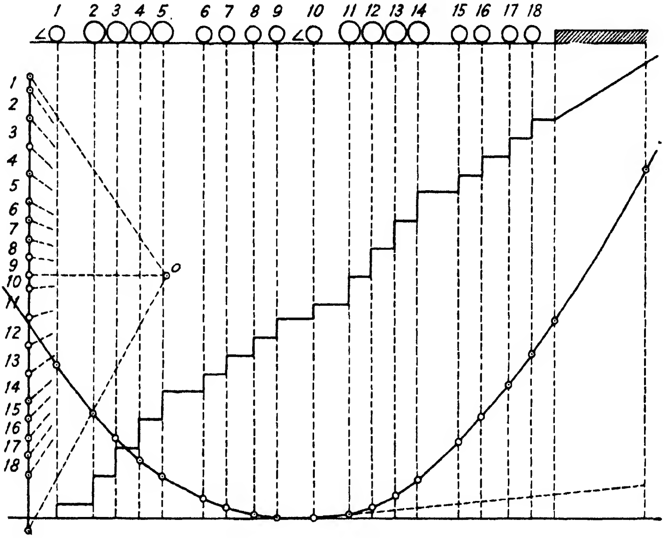


FIG. 39*d*.

the ordinates to be measured lie in the left half of the diagram so that acute intersections of the ordinates with the polygon are avoided. If this diagram were not also to be used for obtaining shears, a further improvement could be made by inclining the axis downwards toward the right, thus bringing the closing lines still more nearly horizontal.

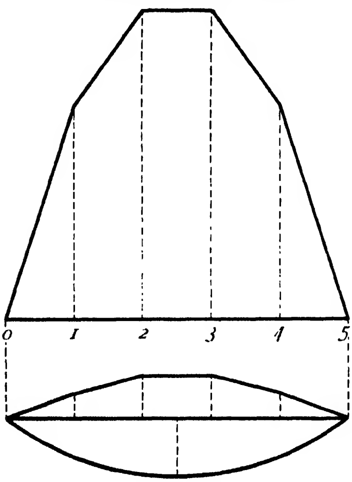


FIG. 39*e*.

Referring again to Fig. 39*c* in order to call attention to the relation between the analytic and graphic methods, it should be observed that, if no load is off the girder at the left, the ordinate *eg* is the moment of the left reaction and *ef* is the moment of the loads on the left of the section, the center of moments being at some point in the section. The difference between these moments, or fg, is the moment at the section. If, however, some loads have passed beyond the left support, then the corresponding point e will lie on that side of the equilibrium polygon produced which is intersected by the vertical at the left support.

If a plate girder has floor-beams and stringers, its bending moment diagram for the live load will be bounded by irregular curves between the moment ordinates at the floor-beams, or panel points. These curves may be either concave or convex but for all practical purposes may be replaced by straight lines, as in the upper diagram of Fig. 39e. An accurate representation of the dead-load moment diagram consists of two parts, one due to the weight of the floor system, which is concentrated at the panel points and which is plotted above the axis in the lower diagram of Fig. 39e; the other due to the uniformly distributed weight of the girder itself, and is plotted below the axis.

PROBLEM 39a.—Construct an equilibrium polygon and load line, like Fig. 39b, for Cooper's E60 loading and check the positions given in the table. Find the maximum moment at each section.

ART. 40. SHEARS DUE TO WHEEL LOADS

Figure 40a represents the influence diagram for the left reaction, and also the shear at the left end, of the girder AB. The ordinate for each load on the girder increases as the loads are

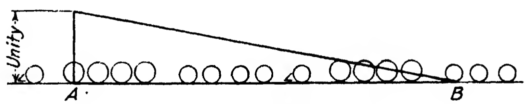


FIG. 40a.

moved to the left, therefore, the reaction and shear also increase until one load is moved off the girder. The shear then drops suddenly, but increases again as the remaining loads, and any additional loads brought onto the right end of the girder, are moved to the left. With practice, the position giving the maximum value can readily be found by trial.

The shear influence diagram for an intermediate point C on the girder AB is shown in Fig. 40b. The ordinates to the left of the point are negative and those to the right are positive. The shear due to any group of loads is the algebraic summation of the products of the individual loads and their ordinates. The individual ordinates and the total shear at the section increase as the loads are brought onto the girder from the right and moved to the left, until the first wheel reaches the section. As the first wheel is moved across the section its ordinate suddenly changes from positive to negative, the algebraic value of the change being unity. The shear at the section is therefore decreased by the magnitude of the first wheel load. As the loads are again moved to the left, the positive ordinates increase and the negative ordinates decrease, both of which increase the total shear at the section. The maximum shear at any section will therefore occur with a wheel at the section.

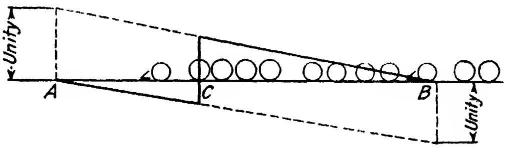


FIG. 40b.

The position of the wheel loads that produces the maximum shear at any section will usually be with the heaviest loads just to one side of the section. For most sections the maximum shear due to Cooper's loading (Fig. 38a) will occur with the second wheel at the section, but for sections near the right end the maximum shear may occur with the first wheel at the section. For short spans the maximum shear is caused by the special loading shown at the right in Fig. 38a.

The position of the loads which produces the maximum shear at any section may be determined graphically by means of the equilibrium polygon. It was shown above that the maximum shear occurs when one of the wheels near the head of the locomotive is at the section. Let M_1 be the moment of all the loads on the girder about the right support when wheel 1 is at the section and M_2 when wheel 2 is at the section. The corresponding values of the vertical shear are

$$V_1 = \frac{M_1}{l}, \quad \text{and} \quad V_2 = \frac{M_2}{l} - P_1$$

P_1 being the load on wheel 1. As the load is directly supported by the girders there will be some section where the shear due to both positions is equal. Equating these values and transposing,

$$M_2 - M_1 = P_1 l$$

In Fig. 40c let cd be the moment M_2 and ab the moment M_1. The distance ac, between these moment ordinates is equal to the distance between wheels 1 and 2. If $V_1 = V_2$, $de = cd - ab = M_2 - M_1 = P_1 l$. The position of the section where $V_1 = V_2$ may then be found as follows: Place the girder diagram (constructed on the tracing paper as described in Art. 39) on the locomotive diagram (Fig. 39b) with its left support at wheel 1, and mark on its section or ordinate at the right support the distance de to the line Bb, which is the side of the equilibrium polygon on

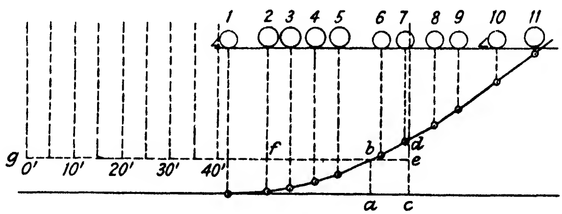


Fig. 40c.

the right of wheel 1 produced. This ordinate is $P_1 l$ (Art. 7). Move the girder diagram to the left until the section at the right support is at wheel 2, and mark the position of wheel 1. The two points marked are shown in Fig. 40c at d and b, respectively. Now move the tracing with the point d remaining on the equilibrium polygon and with its axis horizontal until a position is found where b is also on the polygon. Mark the position of wheel 2 at f. It is easily remembered what wheel is to be marked by noticing that the right-hand ordinate cd is M_2, which by the notation is the moment at the support when wheel 2 is at the section. At every point, therefore, between f and the right support the greatest shear will be produced when wheel 1 is at the section, and for all sections between f and the left support when wheel 2 is at the section.

In the girder under consideration the shears under the first two wheels are equal at a section about 50 ft. from the left support.

Plate girders are not used for spans at which wheel 3 will cause a greater shear than wheel 2.

The load is now placed in position for the different sections successively, and the corresponding moments at the right support read off. When these are divided by the span the left reactions are obtained, and from the reactions the shears are found by subtracting P_1 in the case of those sections for whose position P_1 lies between the section and the left support. For the section at 15 ft. the moment at the right support is 11 200 ft.-kips. This gives a reaction of 11 200 ÷ 80 = 140.0 kips, and a shear of 140.0 −15.0 = 125.0 kips. For the section 0 wheel 1 is off the girder,

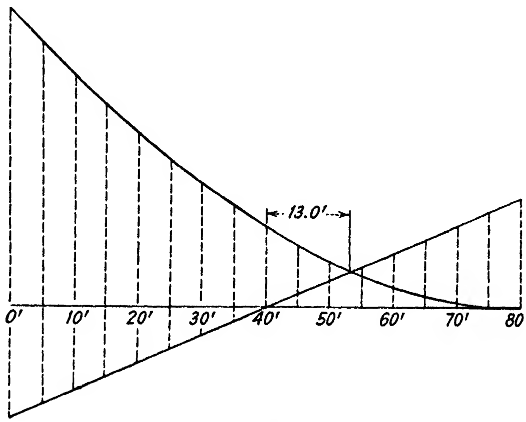


FIG. 40*d*.

and hence the ordinate below the line *Bb* in Fig. 39*b* must be deducted from the moment at the right support. The shear at 0 is found to be (16 200 − 1300) ÷ 80 = 186.3 kips. The values of the shears from sections 0 to 40 ft. inclusive are 186.3, 165.5, 144.7, 125.0, 107.3, 91.8, 77.6, 63.8, and 50.5, all expressed in kips. For the 20-, 25-, and 30-ft. sections, slightly greater shears are produced by a single locomotive followed by a train load than by the load with two locomotives, their values being 108.0, 92.4, and 77.7 kips, respectively.

Usually only the shears in one-half of the girder are required, but in order to show the variation of shear across the entire girder as well as the minimum values, the shear diagram due to dead and live load is shown in Fig. 40*d*. The greatest live-load shears (all positive) are laid off above the axis and the dead-load positive shears below the axis, so that the diagram combines their values. The point of zero shear is at 53.0 ft. or at 13 ft. from the middle of the girder. The shear may therefore change sign at all points within this distance on each side of the middle. If the live load were uniformly distributed, the shear curve would be a parabola, with its vertex at the right end of the girder.

Where the locomotive loading has two pairs of pilot wheels it is possible that for some spans the section where $V_2 = V_3$ may be on the right of that where $V_1 = V_2$, in which event it is necessary to find where $V_1 = V_3$. No position for maximum shear then requires wheel 2 to be at any section.

If the girder is divided into panels by floor-beams the criterion for position for maximum shear is the same as for trusses with parallel chords. The necessary formula was deduced in Part I, but its graphical application will be deferred to Art. 43. The live-load shear diagram would be transformed into a series of steps like that in Fig. 36*a*, and the dead-load shear diagram would consist of two parts, one for the floor system which is concentrated at the panel points, and the other for the weight of the girder itself, which is uniformly distributed.

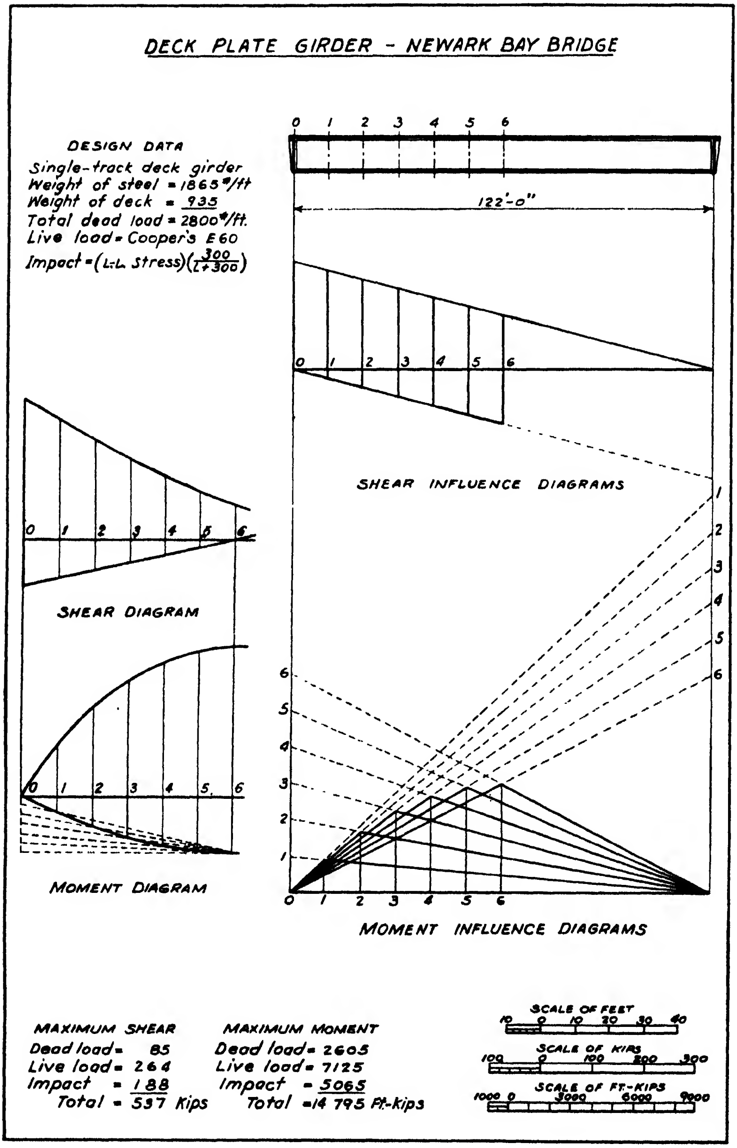
PROBLEM 40*a*.—Find where $V_1 = V_2$ for a plate girder of 100-ft. span, using the loading shown in Fig. 38*a*. Divide the girder into 10-ft. sections and find the maximum shear at each section.

ART. 41. ANALYSIS OF A PLATE GIRDER

Plate II shows the analysis of a plate girder used by the Central Railroad Company of New Jersey in the approach spans of a bridge across Newark Bay. The structure was completed in 1926 and carries four tracks, each supported by two girders. There are 36 spans requiring a total of 288 girders, all exactly alike. The girders have an over all length of 124 ft. 6 in. and a span of 122 ft. 0 in. center to center of bearings. The distance center to center of piers is 125 ft. 0 in. The weight of steel in a complete span for one track is 232 025 lb., or 1865 lb. per linear ft.

In the example of Arts. 39 and 40 the shears and moments were found by means of the equilibrium polygon, but in this article influence diagrams will be used. The construction of the influence diagrams and the methods of finding the positions for maximum

Art. 41 Plate II



shear and moment were explained in the last two articles. The influence diagrams should be made to a large scale on tracing paper so that they can be placed in the desired positions over the locomotive loading and the ordinates read off directly.

As an example of the calculations involved the maximum end shear is computed below. Wheel 2 is at the left end of the girder which places 21 ft. of uniform load on the right end of the girder.

		Kips
Pilot,	15.0(0.606)	= 9.1
Drivers,	30.0(1.000 + 0.959 + 0.918 + 0.877 + 0.541 + 0.500 + 0.459 + 0.418)	= 170.2
Tender,	19.5(0.803 + 0.762 + 0.713 + 0.672 + 0.344 + 0.303 + 0.254 + 0.213)	= 79.3
Uniform load,	$3.0 \times \frac{1}{2} \times 21 \times 0.172$	= 5.4
	Total end shear	= 264.0

Where all wheels in each group of equal loads are on the same segment of the influence diagram some labor can be saved by using the average ordinate for each group and multiplying it by the load on the entire group. If this method had been used in the above example, only two ordinates would have been required for the drive wheels and two for the tender wheels.

The absolute maximum moment does not occur at the center of the span but for relatively long spans the moment at the center may be used. In this example the difference amounts to only 19 ft.-kips, or 0.3 per cent. For short spans the center of gravity of the loads producing the maximum moment at the center should be found and the loads moved so that the center of the span is midway between the center of gravity and the load previously at the center of the span.

PROBLEM 41*a*.—Draw the shear and moment influence diagrams for the 80-ft. girder of Arts. 39 and 40.

ART. 42. SIMULTANEOUS MOMENTS

In designing the riveting of the flanges to the web of a plate girder it is necessary to have the horizontal shear between them or the increments of flange stress between the sections for which the bending moments and vertical shears were found. If the sections are a distance dx apart, the difference of bending moments

dM is a maximum when the load is so placed as to cause the vertical shear to be a maximum, since from mechanics $dM = Vdx$. When the distance between the sections is greater than dx the difference of bending moments is a maximum when the loads are so placed that the vertical shear is a maximum in the section nearer the middle of the span, and this holds true for uniform loads until the sections are separated a distance somewhat less than half the span. As the sections are not taken farther apart than the depth of the girder, which even in short spans is generally less than one-eighth of the span, the exact value of the limiting distance referred to need not be determined.

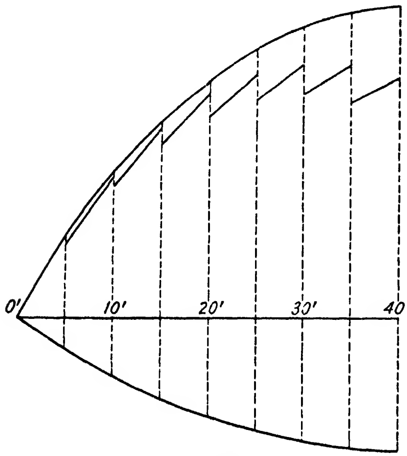


FIG. 42a.

The following table contains the simultaneous bending moments in each pair of adjacent sections of the girder used in Arts. 39 and 40 when the live load is so placed as to produce the maximum shear in the section nearer the middle of the girder. The moments are expressed in foot-kips. The differences in this table are to be

Distance of Section from Support	Load in Position for Maximum Shear at Section								Distance of Section from Support
	5′	10′	15′	20′	25′	30′	35′	40′	
0′	0	.....	.....	.....	2090	.....	.....	.....	20′
5′	830	755	.....	.....	2550	2270	.....	.....	25′
10′	.....	1480	1355	.....	.....	2655	2320	.....	30′
15′	.....	.....	1980	1795	.....	.....	2640	2250	35′
20′	.....	.....	.....	2330	.....	.....	.....	2500	40′
Difference	830	725	625	535	460	385	320	250	Difference

increased for impact and added to the corresponding differences in the dead-load bending moments. The moments themselves are laid off as ordinates in Fig. 42a, those of each pair being joined by a right line. The upper curve gives the maximum live-load bending moments. Those due to dead load are laid off below the axis.

PROBLEM 42*a*.—Find the simultaneous moments in the sections of the plate girder used in Art. 41 (see Plate II).

ART. 43. SHEARS IN TRUSSES

In Part I the criterion for the position of the live load producing the greatest vertical shear in any section of a truss was found to be $P'' = \frac{1}{m}W$, in which W is the whole load on the truss, P'' the wheel loads (one or more) on the panel cut by the section, and m the number of panels in the truss. This equation may be transformed into $W = mP''$. When the truss diagram is placed on the load line (Fig. 39*b*) the value of W can be read off at once on the ordinate at the right support for any given position of the truss with respect to the loads. If wheel 2 be placed just on the right of a panel point W must be equal to mP_1, and if just to the left of the same panel point then W must be $m(P_1 + P_2)$. The condition will therefore be satisfied if wheel 2 is at the panel point, and the value of W is found to lie between the values of mP_1 and $m(P_1 + P_2)$, and similarly for any other wheel.

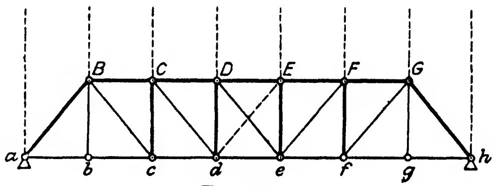


FIG. 43*a*.

Figure 43*a* shows a single track Pratt truss having seven panels of 20 ft. each and a depth of 24 ft. The live load is Cooper's E60. In this example, $m = 7$, $P_1 = 15$ kips, $P_1 + P_2 = 45$ kips, and $P_1 + P_2 + P_3 = 75$ kips. The following table is then arranged like that shown in Part I for the corresponding analytic method:

No. of Wheel at Right End of Panel	Values of P'', Kips	Corresponding Values of mP'', Kips
1	0–15	0–105
2	15–45	105–315
3	45–75	315–525

A truss diagram is constructed on tracing paper to the same linear scale as the moment diagram and load line, but with the verticals extended as high as the load line. The values of mP'' from the table above are marked off on the vertical through the right end of the truss. To find the position for maximum shear in *Cc* and *Cd* due to this load, shift the truss diagram on that of the load line until wheel 3 is at *d*. If this is the correct position the load on the truss must be between 315 and 525 kips and the load line must cross the vertical through the right end of the truss between $m(P_1 + P_2)$ and $m(P_1 + P_2 + P_3)$. This is found to be the case, its value being 367.5 kips. For the greatest positive shear in *Fg* (which equals the greatest negative shear in *Bc*) wheel 2 is placed at *g*. The load W is then 135 kips, which meets the condition. Where the value of W falls at or near one of the limits mP'' the adjacent wheel should also be tried at the section. The positions are all recorded in the following table:

Panel	Wheel at Right End of Panel	Moment about Right Support	Moment about Right End of Panel	Shear	Stress	Web Member
ab	3	35 630	345	237.3	−309.0	*aB*
bc	3	26 280	345	170.5	+222.0	*Bc*
cd	3	18 060	345	111.8	−111.8	*Cc*
......	......	....	....	....	+145.6	*Cd*
de	2	9 800	120	64.0	− 64.0	*Dd*
......	......	....	....	....	+ 83.4	*De*
ef	2	5 240	120	31.4	− 40.9	*eF*
fg	2	1 920	120	7.7	− 10.0	*fG*

In practice it is preferable to obtain all the positions before reading off any moments from the equilibrium polygon, that is, to deal with only the load line at first and with the equilibrium polygon afterwards.

The moments in the third column of the table may now be read directly from the diagram. Those in the next column were computed and marked on the diagram when it was constructed. They are all expressed in foot-kips. Since the span is 140 ft. and the panel length is 20 ft., the shear in *Cc* and *Cd* is $(18\,060 \div 140) - (345 \div 20) = 111.8$ kips. As it is not necessary to know the reaction separately, and as frequently the panel length is not such a simple number as the above, it is better to multiply the moments in the fourth column by the number of panels in the truss, subtract the products from the moments in the third column, and divide the remainder by the span. Thus the shear in *Cc* and *Cd* is $(18\,060 - 7 \times 345) \div 140 = 111.8$ kips. The stresses are obtained from the shears either graphically or analytically, as may be more expeditious.

The method described above is the exact graphic equivalent of the analytic method given in Part I, and the student should make a careful comparison between them. The criterion for position used in this article applies to all trusses with horizontal chords and single systems of webbing, like the Howe and the Warren as well as the Pratt truss. When one or both of the chords are inclined, the maximum shear does not give the maximum web stresses, since the chord takes some of the shear. The method of finding the position of the live load for trusses with inclined chord members will be given in Chapter V.

PROBLEM 43*a*.—The Pratt trusses of a single-track through railroad bridge have nine panels of 21 ft. 4 in. each and a depth of 28 ft. 0 in. Find the shears and stresses in all web members except the hangers due to Cooper's E60 loading.

ART. 44. FLOOR-BEAM REACTIONS

In order to find the maximum stress in the hanger *Bb* (Fig. 43*a*), which is equal to the floor-beam reaction at *b*, it is necessary to deduce additional formulas. Let R_a be the stringer reaction at *a*, and R_b the sum of adjacent stringer reactions, or the floor-beam reaction, at *b*. Let *P* be the whole load on the two stringers of equal spans *ab* and *bc* of length *p*, and *g* the distance of the center of gravity from *c*; let P' be the load on *ab*, and g' the distance of its center of gravity from *b*. Since the sum of the moments of loads and reactions about *b* is zero,

$$R_a p - P'g' = 0, \quad \text{or} \quad R_a p = P'g'$$

Taking moments about *c*,

$$R_a 2p + R_b p - Pg = 0$$

Substituting and reducing,

$$R_b = \frac{Pg - 2P'g'}{p}$$

If the loads be moved a distance dx to the left both g and g' will receive an increment dx, and R_b an increment

$$dR_b = \frac{Pdx - 2P'dx}{p}$$

Placing the derivative equal to zero gives

$$P = 2P'$$

That is, when the live load in both panels is double that in the panel ab the resulting value of R_b is a maximum. This is the same condition as for finding the maximum bending moment in the middle of a girder whose span is ac.

The use of the load line gives the position very quickly, and the moments Pg and $P'g'$ can be read off directly on the moment diagram. If M_c be the moment ordinate at c, and M_b that at b, the value of R_b may be more conveniently expressed and remembered as

$$R_b = \frac{M_c - 2M_b}{p}$$

In the case of bridges where the two panels at the end, p_1 and p_2, are not equal, the value of R_b deduced in a similar manner is

$$R_b = \frac{p_1 M_c - (p_1 + p_2)M_b}{p_1 p_2}$$

the criterion for loading being that which produces the maximum moment at b in a girder whose span is ac, p_1 being the span of stringer ab, and p_2 the length bc.

In applying this condition for loading, one of the heaviest loads should be placed at b and as large a load brought on the two panels from a to c as possible. When wheel 4, in the example used in the previous article, is placed at b and the thread is stretched to unite the intersections of the ordinates at a and c with the load line, it crosses the step representing wheel 4 and hence satisfies the condition. The moment at c is 3405, and that at b is 720 ft.-kips, whence $Bb = (3405 - 2 \times 720) \div 20 = 98.2$ kips. When wheel 13 is at b the condition is also satisfied, and the stress is found to be the same as for the other position since they are corresponding positions on the two locomotives.

PROBLEM 44*a*.—Find the stress in the hangers in Problem 43*a*, using the method of this article. Also find the stress by use of an influence diagram.

ART. 45. MOMENTS IN TRUSSES

The condition of loading for maximum moment in a truss is expressed by a formula deduced in Part I, which, in slightly modified form, was given in Art. 39 and its application to a girder fully explained. As there stated, it applies only to the bending moments in vertical sections through the panel points of the loaded chords of trusses. The positions are recorded in the following table for the required sections of the left half of the truss used in Art. 43:

Section	Wheel at Section	Moment at Right Support	Moment at Section	Bending Moment	Stress	Chord Members
Bb	3	35 640	350	4740	197.5	*ab* = *bc*
Cc	5 6	30 800 35 140	1 250 2 460	7550 7580	315.8	*BC* = *cd*
Dd	9 10 11	33 190 37 150 39 130	5 240 6 950 7 810	8980 8970 8960	374.2	*CD* = *de*
Ee	12	33 670	10 060	9180	382.5	*DE* = *EF*

For a truss having panels of equal length, as in this example, time may be saved as well as increased precision secured by not drawing the closing lines and reading the bending moments directly, but by reading the moments at the right support and at the given section. The latter, being at a wheel, has its value marked on the diagram, and is hence quickly obtained. The moment of any wheels off the left end of the span must be subtracted from both of the above moments.

The computation for the bending moment is very simple for trusses with equal panels, as it is not necessary to obtain the value of the reaction separately. For wheel 6 at c the bending moment in the section Cc is

$$\tfrac{2}{7}(35\,140) - 2460 = 7580 \text{ ft.-kips}$$

and when divided by the depth of the truss the chord stress is $7580 \div 24 = 315.8$ kips.

Since there is no dead-load shear in the middle panel, it is necessary to find which one of the diagonals is acting for each of the positions for sections *Dd* and *Ee*. The shear equals the left reaction of the truss minus the loads from *a* to *d*, minus the reaction at *d* of the stringer *de*. By producing the side of the equilibrium polygon immediately on the left of *d*, when wheel 9 is at *d*, and reading the moment intercepted above this line at *e*, the moment of the stringer reaction at *d* about *e* as a center is obtained. Its value is found to be 690 ft.-kips. The shear is therefore

$$\frac{33\,190 - 7 \times 690}{140} - 193.5 = +\,9.1 \text{ kips}$$

As the diagonals can take only tension, this shear calls *De* into action, and hence the bending moment for the section *Dd* gives the chord stress in *CD* and *de*.

Similarly, for wheel 12 at *e* the shear in the middle panel is

$$\frac{33\,670 - 7 \times 345}{140} - 213 = +\,10.3 \text{ kips}$$

Since this also stresses the diagonal *De*, the moment for the section *Ee* gives the chord stress in *DE* and *EF*.

The sign of the shear without its magnitude may be more quickly determined for each of these positions by drawing the closing line of the equilibrium polygon and with the aid of the dividers finding whether the bending moment at *d* is greater or less than the simultaneous moment at *e*. In the former case the shear is negative and in the later positive (see Art. 9).

If the live loading for the greatest moments at the sections *Dd* and *Ee*, respectively, had caused shears of unlike signs in the middle panel, the required stress in one of the chords of that panel would not have been given by either loading. In such a case it becomes necessary to shift the load to some intermediate position for which the bending moment ordinates at both ends of the middle panel are equal, and the shear is therefore zero. The required stress may then be obtained from the moment at either section. However, the stresses in both chords of the middle panel are nearly equal and in practice are generally assumed to be equal.

PROBLEM 45*a*.—Find the greatest live-load stresses in the chords of Problem 43*a*.

ART. 46. ANALYSIS OF A PARALLEL-CHORD TRUSS

Plate III shows the complete analysis of a subdivided Warren truss bridge under vertical loads. The dead load was calculated from the formula of Art. 37 and the stress diagram constructed for the entire dead panel loads applied on the lower chord. In the table of stresses one-third of the dead load was considered at the upper panel points, thus changing the stress in each vertical by −8.0 kips.

The live-load stresses were obtained by the methods of Arts. 43, 44, and 45, although influence lines could have been used with equal facility. The length used in the impact formula should be the actual length loaded to produce the maximum live-load stress. For example, the maximum live-load stress in the diagonal *EF* occurs with wheel 3 at panel point 3, and the loaded length is therefore 103 ft. The maximum stresses in the end post *AB* and in all the chord members occurs with the live load covering nearly all the bridge so that in practice it is customary to consider the entire span loaded when figuring impact. It should be noted that for the floor-beam hangers, *BC* and *FG*, only the loads on the panels adjacent to the hangers produce stresses in them. The length to be used in the impact formula is therefore two panels, or in this case 36 ft.

PROBLEM 46*a*.—Determine the live-load stresses in the trusses of Plate III by the use of influence lines.

ART. 47. LATERAL LOADS AND STRESSES

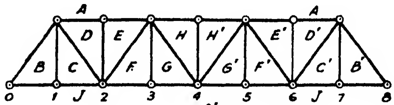
The lateral forces acting on a railway bridge are wind on the structure itself, wind on the train, side sway of the locomotive and train, and lateral vibration of the bridge. Centrifugal force must also be considered if the bridge is on a curve.

None of the lateral forces can be accurately determined, consequently a rather wide variation in specified loads has resulted. The 1920 specification of the American Railway Engineering Association is most used at the present time. It provides that,

"The wind force on the structure shall be a moving load of 30 lb. per sq. ft. on $1\frac{1}{2}$ times its vertical projection on a plane parallel with its axis, but not less than 200 lb. per linear ft. at the loaded chord or flange, and 150 lb. per linear ft. at the unloaded chord or flange.

Art. 46 Plate III

SUBDIVIDED WARREN TRUSS UNDER LOCOMOTIVE LOADING



DESIGN DATA

Single-track through bridge.
Span 144 ft. Depth 24 ft. 8 panels.
Weight of steel = 2225 #/ft.
Weight of deck = 450
Total dead load = 2675 #/ft.
Dead panel loads = 24 kips.
Live load - Cooper's E 60

$$\text{Impact} = (\text{L-L. Stress})\left(\frac{300}{300+\frac{L^2}{100}}\right)$$

FLOOR-BEAM REACTION

Wheel 3 at floor-beam

M_c = 2325 ft.-kips

M_b = 345 ft.-kips

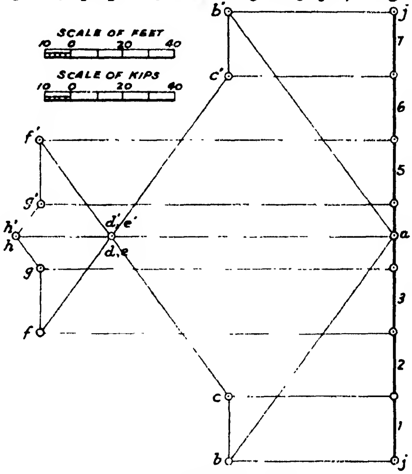
$R_b = \frac{2325-690}{18} = 90.8$ kips

Wheel 4 at floor-beam

M_c = 3080 ft.-kips.

M_b = 720 ft.-kips.

$R_b = \frac{3080-1440}{18} = 91.0$ kips.



D-L STRESS DIAGRAM

STRESSES IN WEB MEMBERS

PANEL	WHEEL AT RIGHT END OF PANEL	MOMENT AT RIGHT SUPPORT	MOMENT RIGHT END OF PANEL	SHEAR	STRESS	WEB MEMBER
0-1	3	38680	345	249.4	311.8	AB
1-2	3	29880	345	188.3	235.4	CD
2-3	3	22010	345	133.7	167.1	EF
3-4	2	13440	120	86.6	108.3	GH
4-5	2	8320	120	51.1	63.9	G'H'
5-6	2	4470	120	24.4	30.5	E'F'
6-7	2	1650	120	4.8		
	1	720	0	5.0	6.3	C'D'

STRESSES IN CHORDS

SECTION	WHEEL AT SECTION	MOMENT AT RIGHT SUPPORT	MOMENT AT SECTION	BENDING MOMENT	STRESS	CHORD MEMBER
1	3	38680	345	4490	187.1	BJ=CJ
2	5	34650	1245	7418	309.1	AD=AE
3	8	35640	4280	9085	378.5	FJ=GJ
4	11	37150	8770	9805	408.5	AH=AH'

TABLE OF COMBINED STRESSES

MEMBER	AD=AE	AH	BJ=CJ	FJ=GJ	AB	CD	EF	GH	BC=FG	DE=HH'
Dead load	-108.0	-144.0	+ 63.0	+135.0	-105.0	+ 75.0	- 45.0	+ 15.0	+ 16.0	- 8.0
Live load - max.	-309.0	-408.5	+187.1	+378.5	-311.8	+235.4	-167.1	+108.3	+ 91.0	0
Live load - min.	0	0	0	0	0	- 6.3	+ 30.5	- 63.9	0	0
Impact - max.	-182.7	-241.6	+110.6	+223.8	-184.4	+158.4	-123.4	+ 89.3	+ 87.2	0
Impact - min.	0	0	0	0	0	- 6.1	+ 28.6	- 56.6	0	0
Maximum	-599.7	-794.1	+360.7	+737.3	-601.2	+468.8	-335.5	+212.6	+194.2	- 8.0
Minimum	-108.0	-144.0	+ 63.0	+135.0	-105.0	+ 62.6	+ 14.1	-105.5	+ 16.0	- 8.0

"The wind force on the train shall be a moving load of 300 lb. per linear ft. on one track, applied 8 ft. above the base of rail.

"The lateral force to provide for the effect of the sway of the engines and train in addition to the wind loads specified above, shall be a moving load equal to 5 per cent of the specified live load on one track, but not more than 400 lb. per linear ft. applied at the base of rail."

The wind load on the train being applied above the rail does not affect the stresses in the laterals otherwise than as though it were applied at the rail, but it does produce vertical reactions at the ends of the floor-beams which must be carried by the trusses. For E60 locomotives 5 per cent of the average load is slightly greater than the maximum specified load so that the force to provide for the effect of sway may be taken as a uniform moving load of 400 lb. per linear ft. All the above loads are uniform moving loads which may be combined and the stresses in laterals found as explained for highway bridges in Art. 35.

The stresses in trusses, the floor, and lateral systems of a bridge due to the curvature of the track are treated in a paper by WARD BALDWIN in *Transactions* of the American Society of Civil Engineers, Vol. XXV, p. 459, Nov., 1891, entitled "Stresses in Railway Bridges on Curves." The paper contains a practical example in which the stresses are computed.

PROBLEM 47*a*.—The trusses of Plate III are 17 ft. apart, and the bottom chord lateral bracing is of the type shown in Fig. 35*b*. Find the stresses in the bottom chord laterals due to the lateral loads specified in this article.

CHAPTER V

RAILWAY TRUSSES WITH BROKEN CHORDS

Art. 48. Points of Division in Panels

For the truss in Fig. 48*a* the position of locomotive wheel loads, or any other class of live load, which produces the greatest stress in any chord member is found by the same criterion as if the chords were both horizontal. The same statement would be true if both chords were broken or curved. On the other hand,

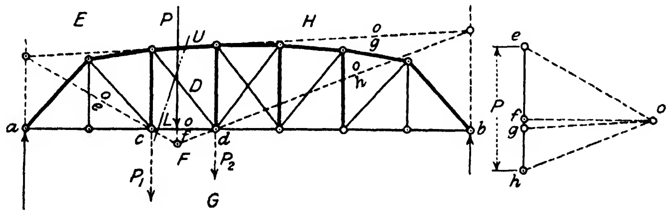


Fig. 48*a*.

the stress in any web member for the position of the live load which causes the maximum shear in the section is not the greatest in this case, because the inclination of any chord member cut by the section causes it to take some of the shear. It will be necessary, therefore, to find the condition of loading which is required by the web members of trusses with inclined chords.

The section shown in Fig. 48*a* cuts the upper chord member U, the diagonal D, and the lower chord member L. By the method of moments the center of moments for D is at the intersection of U and L, some distance beyond the figure on the left. If a single concentrated load P_1 be placed at c or at any point on the left of c, it will cause compression in the diagonal D. This is readily seen to be the case, since the stress in D (which is directed away from the section, and hence downward) holds in equilibrium the forces on the left of the section, and therefore their resultant. The resultant of P_1 and the reaction at a is a downward force at the right support

b, and hence its moment is positive. If a single load P_2 be placed at d or at any point on the right of d, it will then cause tension in the diagonal, for the stress in D holds in equilibrium only the upward reaction at a whose moment is negative.

When a load P is placed between c and d, the floor system of the bridge transfers a portion P_1 to the panel point c of the truss and the remaining portion P_2 to the panel point d, that is, the load P is replaced on the truss by its components P_1 and P_2. As one of these causes compression in D and the other tension, there must be some position of the load P for which the resulting stress in D is zero.

Let U be produced to meet the verticals through a and b, let these points of intersection be joined with c and d, and the lines produced until they meet. Let P be placed directly over this latter intersection. On the right of the figure is shown a force polygon, in which eh is laid off by scale equal to P, and the rays drawn parallel to the corresponding strings on the space diagram. The strings oe, oh, and og form an equilibrium polygon, and hence eg equals the reaction at a and gh the reaction at b. Portions of the sides oe and oh and the line cd, or of, form another equilibrium polygon, and therefore ef represents P_1 and fh represents P_2. Consequently oe, of, oh, and og form an equilibrium polygon for the loads P_1, P_2, and the reactions at a and b, respectively. The line of action of the resultant of P_1 and the reaction at a must pass through the intersection of of and og (Art. 7), which by construction coincides with the center of moments. The moment of the resultant is therefore zero, whence it follows that the stress in D is also zero.

The position of a concentrated load P causing no stress in any web member can therefore be found by the following rule:

> Pass a section cutting the web member and a member of each of the chords. Produce the unloaded chord member to an intersection with the verticals through the supports. Join these points with the panel points at the end of the loaded chord member. The intersection of these lines gives the required position.

From the manner in which the above investigation was made it is clear that this rule applies to a truss in which both chords are curved, and for webbing whose posts are not vertical. The rule is therefore stated in its general form and applies to deck as well as to through bridges.

The manner in which the section must be cut to obtain the stresses in the posts depends upon which diagonals are acting in the adjacent counter-braced panels (see Art. 31, and also Part I). The position of P which produces no stress in the vertical on the left of D in Fig. 48a when both the adjacent main diagonals are acting is somewhat nearer to panel point d.

The results of this investigation also show that if the live load consists of panel loads, all the panel points on the right of P are to be loaded for the greatest tension in D, as was found in Art. 29. If one excess load is employed it must be placed at d. For the greatest compression in D the load is similarly placed at c, and the panel points on the left. This loading, it will be observed, does not differ from the corresponding one for horizontal chords.

If the live load is uniformly distributed it must extend from the right support to the position of P for the greatest tension in D, and from the left support to the position of P for the greatest compression. When the construction shown in Fig. 48a is applied to trusses with horizontal chords it gives the position for true maximum live-load shear which was determined by the analytic method in Part I. The position of locomotive wheel loads which produces the greatest stress in D will be found in the next article.

Problem 48a.—Find the position of P for all the diagonals and the second, third, and fourth verticals of the bowstring truss in Fig. 31e.

Art. 49. Position of Wheel Loads

In Fig. 49a the position of P (in Fig. 48a) which causes a stress of zero in the diagonal is indicated by the vertical line marked z. Let the stress in the diagonal, lower chord, and upper chord, cut by a section through the panel cd be denoted by S, S_1, and S_2, respectively, and the depths of the truss at c and d by h_1 and h_2. Let the total weight of the wheels (one or more) on the panel cd be P''', and the distance of its center of gravity from d be g''', W being the weight of the entire live load on the truss, and g the distance of its center of gravity from the right support. Let the bending moments at the upper and lower extremities of D be M_1 and M_2, respectively. The remaining terms employed in the discussion are shown in the figure.

Let the segment of the truss on the left of the section cutting S_2, S, and S_1, be considered. Resolving horizontally,

$$S_2 \cos \alpha + S \sin \theta + S_1 = 0$$

The lever arm of S_2 makes the same angle α with the vertical h_2 as S_2 makes with the horizontal. Taking moments about d,

$$M_2 + S_2 h_2 \cos \alpha = 0, \text{ whence } S_2 \cos \alpha = - M_2 \div h_2$$

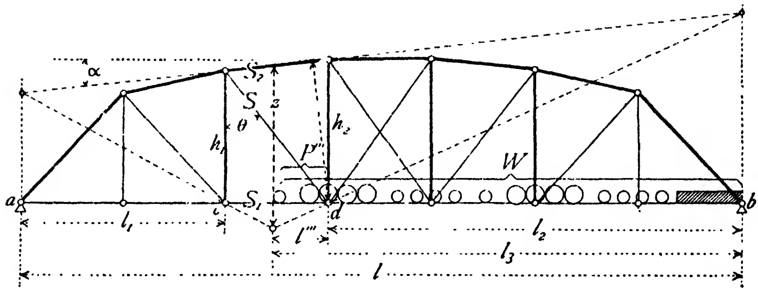


FIG. 49a.

and taking moments about the panel point at the upper end of D,

$$M_1 - S_1 h_1 = 0, \text{ whence } S_1 = M_1 \div h_1$$

After substituting these values above, there follows,

$$-\frac{M_2}{h_2} + \frac{M_1}{h_1} + S \sin \theta = 0, \quad \text{or} \quad S \sin \theta = \frac{M_2}{h_2} - \frac{M_1}{h_1}$$

The last equation is an important one, and indicates that the horizontal component of the stress in any web member equals the difference of the quotients obtained by dividing the simultaneous bending moments at the extremities of the member by the corresponding depths of the truss at those points.

The values of the bending moments are,

$$M_1 = \frac{Wgl_1}{l}, \quad \text{and} \quad M_2 = \frac{Wg}{l}(l - l_2) - P''' g'''$$

From similar triangles,

$$h_1 : z = l_1 : l - l_3, \text{ and } h_2 : z = l_2 : l_3$$

whence

$$h_1 = \frac{l_1 z}{l - l_3}, \qquad \text{and} \qquad h_2 = \frac{l_2 z}{l_3}$$

Substituting these values of M_1, M_2, h_1, and h_2, and reducing, and finally replacing $(l_3 - l_2)$ by l''',

$$S = \frac{Wgl''' - P'''g'''l_3}{l_2 z \sin \theta}$$

If the load advance a distance dx, both g and g''' receive an increment of dx, and the stress S receives an increment of

$$dS = \frac{(Wl''' - P'''l_3)dx}{l_2 z \sin \theta}$$

Placing the derivative equal to zero gives the condition which makes S a maximum, which is $Wl''' - P'''l_3 = 0$, or, when put into more convenient form for use,

$$P''' = \frac{Wl'''}{l_3}$$

This formula is very convenient to use graphically, and as it is similar in form to that for maximum moment (Art. 39) it is to be treated in like manner. Referring to Fig. 49*b*, which illustrates the truss diagram (drawn on tracing paper) placed in position on the live-load moment diagram, *bg* represents the total load W, *ob* the distance l_3, and *od* the distance l'''. The ordinate *di* is therefore equal to $Wl''' \div l_3$, and this must equal

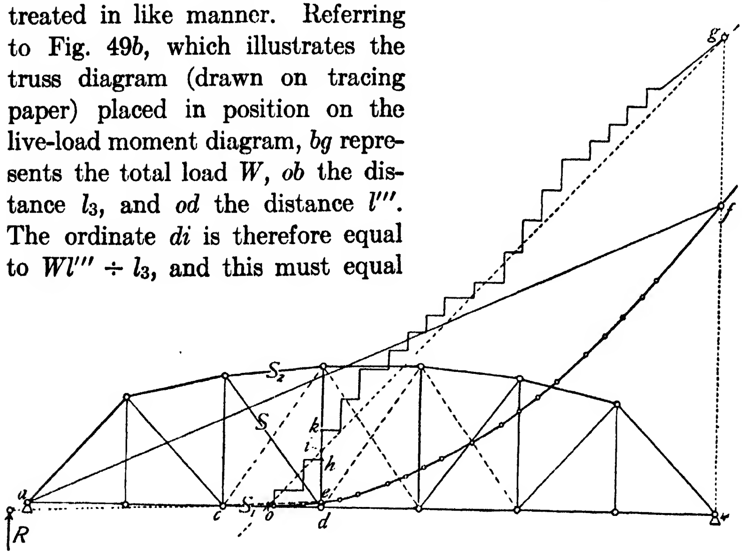


FIG. 49*b*.

P''' if the position is correct. When the load is so placed that a wheel is just on the right of the panel point d, the load P''' is represented by *dh*, and if just to the left of it by *dk*; hence if *i* lies between *h* and *k*, or, in other words, if a thread stretched from *o* to *g* cuts the load placed at the panel point, the criterion for position is satisfied. A wheel must always be placed at the panel point, and although usually the first wheel is at the right of *o*, it may sometimes happen that the condition is met when the first wheel is a little to the left of *o*. After the right position is found the moment ordinates *bf* and *de* are read off as usual.

PROBLEM 49*a*.—A double-track through railroad bridge has trusses of the type illustrated in Fig. 49*b*. There are twelve panels each 30 ft. long. The depths at panel points 1 to 6 inclusive are 29 ft. 0 in., 41 ft. 0 in., 49 ft. 5 in., 53 ft. 4 in., 58 ft. 10 in., and 60 ft. 0 in., respectively. The four center panels have counter diagonals. Find the position of Cooper's E60 loading which shall produce the greatest stresses in the main and counter diagonals. Also compare these positions with what they would be if the truss had parallel chords.

ART. 50. RESOLUTION OF THE SHEAR

In Fig. 49*b* in the preceding article the stresses S_2, S, and S_1 hold in equilibrium the external forces on the left of the section

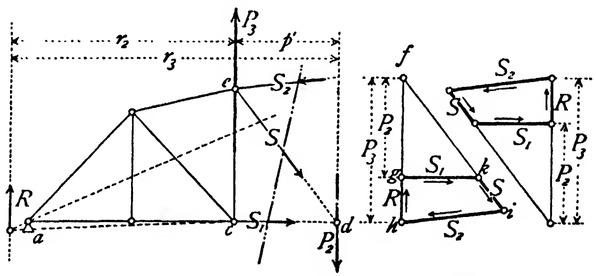


FIG. 50*a*. FIG. 50*b*. FIG. 50*c*.

cutting these members. These external forces consist of an upward reaction at *a* and a downward force at *c* equal to the left reaction of the stringer in the panel *cd*. The resultant *R* of these two forces is an upward force whose line of action is a little to the left of the support *a*. Its position may be readily determined, if desired, by drawing the closing line *af* and the chord *ce* of the equilibrium polygon and producing them to their intersection.

Referring now to Fig. 50*a*, let the resultant *R* be replaced by two forces P_2 and P_3, the former acting downward at panel point d and the latter acting upward at e. The points d and e are at the extremities of the diagonal cut by the section. Taking moments about e, and remembering that the bending moment at e is M_2,

$$Rr_2 = P_2 p', \quad \text{and} \quad P_2 = \frac{Rr_2}{p'} = \frac{M_2}{p'}$$

Similarly taking moments about d,

$$Rr_3 = P_3 p', \quad \text{and} \quad P_3 = \frac{Rr_3}{p'} = \frac{M_3}{p'}$$

Taking vertical components,

$$R = P_3 - P_2 = \frac{M_3}{p'} - \frac{M_2}{p'}$$

Since R is equal to the vertical shear in the section, the last member of the preceding equation affords a useful method of obtaining the vertical shear when the simultaneous moments are known.

In Fig. 50*b* the force triangle *hfi* gives the magnitude and direction of a force acting in the diagonal which is in equilibrium with P_3 and S_2, while the superimposed force triangle *fgk* gives the magnitude and direction of a force acting in the same diagonal which is in equilibrium with P_2 and S_1. The polygon *hfgkih* must therefore express the relation of equilibrium between P_3, P_2, S_1, S, and S_2, or the polygon *hgkih* that between R, S_1, S, and S_2. Following around the triangle in the direction of the known force R as indicated by the arrows, and transferring these directions to the truss diagram in Fig. 50*a*, S_1 and S are found to be tension and S_2 compression. It will be observed that the forces in the polygon *hgki* follow each other in the same order as they are found when passing counterclockwise around the segment of the truss. Figure 50*c* shows the same construction when the forces are laid off in the reverse order. As will be illustrated later by an example, it is sometimes preferable to use the one and sometimes the other order.

It is evident on inspection that the most convenient and economical construction of the force polygon in Fig. 50*b* (or in Fig. 50*c*) would be to draw it directly on a large-scale truss diagram. In Fig. 50*d* one such force polygon is drawn in the third panel. The notation shown is well adapted to promote rapid construction and freedom from confusing the stresses. The panels are numbered from left to right, and the corresponding numbers are placed at the panel points on their right. The upper chord members are denoted by U, the diagonals by D, and the lower chord members by L, the subscripts being those of the panels containing them. The verticals V necessarily have the subscripts of the panel points. The forces $M_2 \div p$ and $M_3 \div p$, equal respectively to the forces P_2 and P_3 in Fig. 50*a*, are laid off as indicated, and the sides of the force polygon drawn parallel respectively to the truss members whose names are placed by their sides. The panel length p in this case is equal to the horizontal projection p' of the diagonal.

In order to obtain the stress in the vertical V_2, for example, the values of $M_2 \div p$ and $M_3 \div p$ are found for the proper posi-

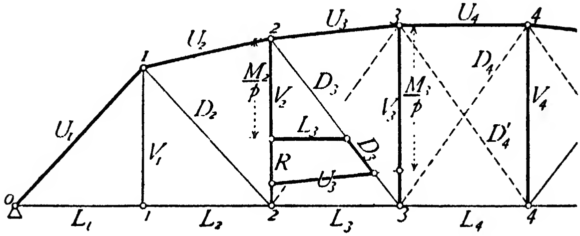


FIG. 50*d*.

tion of the live load (which may not be the same as for D_3) and a force polygon drawn as in Fig. 50*d*. This gives U_3 and D_3 for that position of the load, and on constructing the force polygon for the upper panel point 2 the stress in V_2 is determined. The latter polygon may be drawn directly on the former so as to avoid redrawing the sides U_3 and D_3. To avoid confusion it is omitted in the figure.

This method may also be applied with advantage to determine the stresses in the chords when the moments have been found after placing the live load in its proper position. In this case it will be desirable to place the force polygon on the other side of the diagonal, the values of $M \div p$ being laid off upward from the lower panel points. This will give a polygon like that in Fig. 50*c*.

PROBLEM 50*a*.—Find the maximum live-load stress in the counter tie of the fourth panel, and the minimum live-load stress in the second vertical, of the truss in Problem 49*a*.

ART. 51. EXAMPLE—MAXIMUM CHORD STRESSES

Let the truss in Fig. 50*d* and on Plate IV be that of a single-track through railroad bridge, having seven panels each 27 ft. long, and depths at the panel points 1, 2, and 3, of 29, 35, and 38 ft., respectively. Let the live load consist of Cooper's class E60 (Art. 38). The dead load is estimated from the formula of Art. 37 to be 1600 lb. per linear ft. per truss, of which 400 lb. is assumed to be applied on the upper chord. This makes the panel loads on the lower chord 32.4 kips and on the upper chord 10.8 kips. Since the dead-load stresses in all the members of the truss except the verticals are the same whether the dead load is all applied on the lower chord or divided between the chords, and the stresses in the verticals differ by the amount of the upper panel loads (Art. 27), the stresses will be obtained for only lower panel loads of 43.2 kips, and the maximum and minimum stresses in the verticals afterward corrected by adding to each a compression of 10.8 kips.

In constructing the load line and equilibrium polygon for the live load, it was found convenient to use the weights on one rail only, as in Art. 39. Scales were also adopted as specified for Fig. 39*b*. This diagram may be used for a double-track bridge by expressing the stresses obtained in tons instead of in kips.

The dead-load stress diagram for panel loads of 43.2 kips is shown in Plate IV, and the stresses are marked on the diagram. The character of each web stress is also indicated as referred to the small truss diagram. The computation of the bending moments at the panel points may be arranged as follows, when the panels are all equal: The product of the panel load and the panel length is $43.2 \times 27 = 1166.4$. The half products of the number of panels in each segment into which the panel points respectively divide the truss are $\frac{1}{2}(1 \times 6) = 3$, $\frac{1}{2}(2 \times 5) = 5$, $\frac{1}{2}(3 \times 4) = 6$; and the bending moments are

$$M_1 = 3 \times 1166.4 = 3499, \quad M_2 = 5 \times 1166.4 = 5832,$$

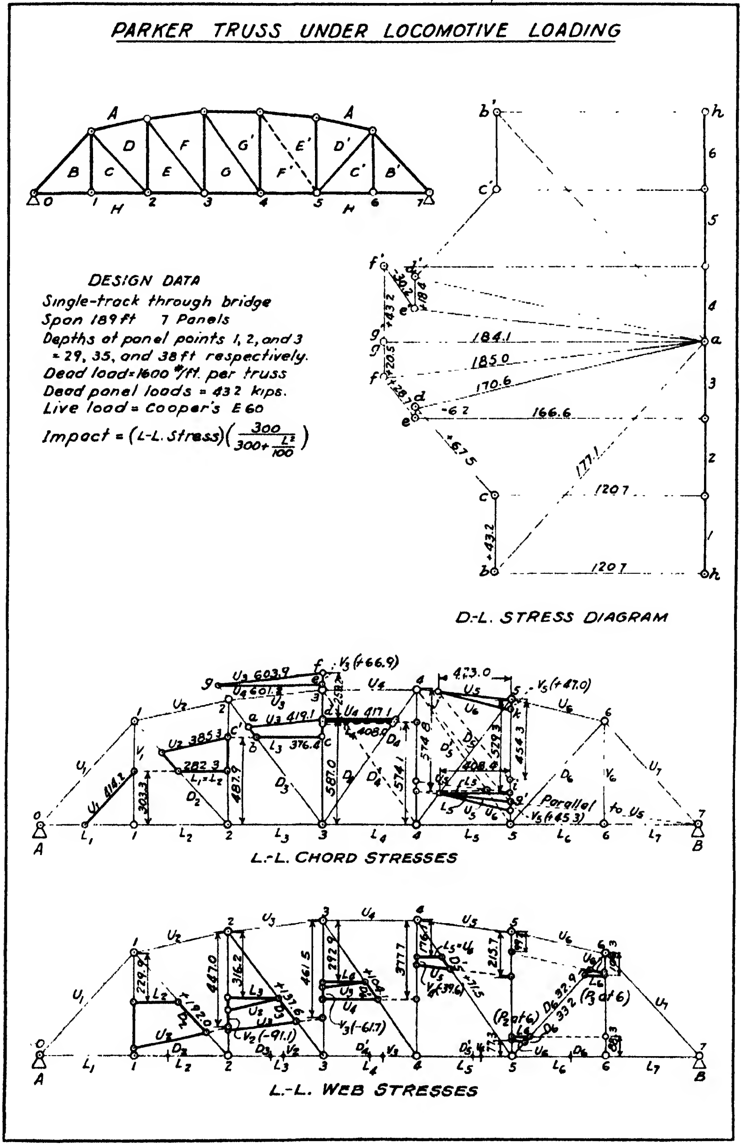
and

$$M_3 = 6 \times 1166.4 = 6998 \text{ ft.-kips.}$$

Since $p = 27$ ft. the corresponding values of $M \div p$ are:

$$\frac{M_1}{p} = 129.6, \quad \frac{M_2}{p} = 216.0, \quad \text{and} \quad \frac{M_3}{p} = 259.2 \text{ kips.}$$

Chap. V Plate IV



These values may be used on a large truss diagram like those in Plate IV for checking the dead-load stresses, or the dead-load stress diagram may be omitted entirely. The following table shows the position of the live load obtained by means of the load line. The moments in the third column were read from the moment diagram; those in the fourth column were copied from the same diagram; and the quantities in the fifth and sixth columns were computed from those in the preceding columns. The moments are expressed in foot-kips.

Center of Moments	Wheel at Section	Moment at Right Support	Moment at Section	Bending Moment (M)	$\frac{M}{p}$	Remarks
1	4	62 360	720	8 188	303.3	
2	8	61 090	4 280	13 173	487.9	
3	12	60 450	10 060	15 849	587.0	D_4 acts
4	14	50 180	13 090	15 583	577.1	D'_4 acts
	15	55 510	16 220	15 500	574.1	D_4 acts

It is necessary to know which diagonal acts in the middle panel for the last three positions in order to determine the center of moments for the chords of that panel. As explained in Art. 45, the vertical shears are found as follows:

$$(60\,450 - 7 \times 2190) \div 189 - 258 = -\ 19.3 \text{ kips};$$
$$(50\,180 - 7 \times 1240) \div 189 - 213 = +\ 6.6 \text{ kips};$$
$$(55\,510 - 7 \times 1980) \div 189 - 228 = -\ 7.6 \text{ kips}.$$

These results enable the remarks to be inserted in the last column of the table.

The values of $M \div p$ are now laid off on the truss diagram in Plate IV, as there indicated, and the force polygons completed as explained in the preceding article. The scales of the original drawing were 6 ft. and 120 kips per in., respectively. The values of the stresses are marked on the polygons. The special attention of the student is called to the fact that since U_3 has its center of moments at the lower panel point 3, the side of the polygon *ad* parallel to U_3 must be drawn through *d*, the extremity of $M_3 \div p$ laid off on the vertical ordinate passing through the center of moments. Similarly, as L_3 has its center of moments at the upper panel point 2, the side *bc* must be drawn parallel to L_2 through *c'*, which lies on the vertical through its center of moments. Strict attention to this statement is especially required when the upper panel points are not directly above the lower ones, in which case the panel points should be numbered in regular order from left to right, no matter on which chord they lie. Each chord should then have the same subscript as its center of moments.

The side *ab* of the polygon *abcd* is not the stress in the diagonal D_3, because the moments used at 2 and 3 in its construction are not simultaneous.

If it be desired to find by this method whether D_4 or D'_4 acts when the moment is a maximum at panel point 3, it can be done by finding the simultaneous value of $M_4 \div p$. It is found to be 567.6 kips. If D_4 be assumed to act, the side L_4 will lie below U_4. It is shown as a broken line. By referring again to Fig. 50*a*, the direction around the polygon is toward the right of L_4, upward on D_4, and toward the left on U_4. Upward on D_4 means also toward the right, or away from the section, and therefore tension, which proves the assumption to be correct. Again since $(M_3 \div p) - (M_2 \div p) = R$, which equals the vertical shear in the section indicated in Fig. 50*a*, the vertical shear in the middle panel is $567.6 - 587.0 = -19.4$, the difference of 0.1 from the value given above being due to the neglect of decimals. Usually the value of the vertical shear is not desired, but simply its sign, in which case it may be known as soon as it is seen whether M_4 is greater or less than the simultaneous value of M_3 (see also Art. 9).

As the end post receives its maximum stress under the same position of the live load as L_1, its stress may be found in connection with the chords. Indeed, it may be regarded as an upper chord member, the polygon of forces becoming a straight line, as shown on the drawing (Plate IV).

The following table gives the maximum stresses in the end post and the chords due to dead load, live load, and impact, expressed in kips. All impact percentages are calculated for the entire span loaded. The minimum stresses in all cases equal the dead-load stresses.

Chord Members	U_1	U_2	U_3	U_4	$L_1 = L_2$	L_3	L_4
Dead load......	−177.1	−170.6	−185.0	−184.1	+120.7	+166.6	+184.1
Live load.......	−414.2	−385.3	−419.1	−417.1	+282.3	+376.4	+408.0
Impact.........	−189.1	−175.9	−191.3	−190.4	+128.9	+171.8	+186.3
Maximum......	−780.4	−731.8	−795.4	−791.6	+531.9	+714.8	+778.4

PROBLEM 51*a*.—A truss of the same type as that in the example given in this article has nine panels, each 24 ft. 9 in. long. The depths at its panel points 1, 2, 3, and 4 are 27, 36, 41, and 43 ft., respectively. Assume a dead load from the formula of Art. 37. The live load and impact are as specified on Plate IV. Find the maximum and minimum stresses in the chords and end post due to these loads.

ART. 52. EXAMPLE—MAXIMUM STRESSES IN DIAGONALS

The first step in finding the live-load web stresses is to find the point of division in each panel where a concentrated load will produce no stress in the diagonal (see Art. 48). In practice only that portion of each triangle lying below *cd* in Fig. 48*a* need be drawn after the vertices on the ordinates at *a* and *b* are marked off. These points are shown in Plate IV. In case more than one point is shown in a panel the left-hand one belongs to the diagonal. The data in the following table are obtained in the same manner as for the chords after the positions are determined. That relating to the first panel is inserted here, although it was also included in the table in the preceding article, the end post being treated as a chord member. The moments are expressed in foot-kips. The panels are indicated by the panel points in this table as a guide to the subscripts which properly belong to the corresponding values of M.

Panel	Wheel at Right End of Panel	Moment at Right Support	Moment at Right End of Panel	Bending Moment (M) at Left End of Panel	Bending Moment (M) at Right End of Panel	Left $\frac{M}{p}$	Right $\frac{M}{p}$
0–1	4	62 360	720	0	8 188	0	303.3
1–2	3	43 450	345	6207	12 069	229.9	447.0
2–3	3	29 880	345	8537	12 461	316.2	461.5
3–4	3	18 450	345	7907	10 198	292.9	377.7
4–5	2	8 320	120	4754	5 823	176.1	215.7
5–6	3	3 750	345	2679	2 869	99.2	106.3
	2	2 920	120	2086	2 383	77.3	88.3

Attention is again called to the fact that the vertical shear in any panel may be found by taking the difference between the corresponding quantities in the last two columns.

In testing for position in panel 5–6 it was noticed that the thread just touched the edge of the step when wheel 3 was placed at panel point 6. This position places wheel 1 a little to the left of the point of division, but the condition of loading is satisfied. If the chords were parallel the positions would be 4, 3, 3, 3, 2, and 2 in the successive panels, no panel having two positions of the live load.

The values of $M \div p$ are next laid off on the verticals through the panel points in Plate IV. Both values belonging to each panel are marked inside of the panel to guard against confusion. This danger is not great, however, as it will be noticed that at each vertical the ordinate referring to the panel on the right is considerably less than that for the panel on the left. After completing the force polygons the stresses in the diagonals are scaled off and marked on the diagram. As the portion of the truss on the left of the section through any diagonal is considered, and the lower chord is always in tension, the direction of passing around the polygon is toward the right on L and toward the left on U, and therefore if the direction along D is toward the right it indicates tension. This is seen to be the case for all the polygons on the plate, except those in panel 5–6.

In the sixth panel two polygons are drawn, the right one for wheel 3 at panel point 6, and the left one for wheel 2 at 6. The latter is placed at the bottom of the truss to avoid interference. This position is not convenient for diagonals, as will be shown in the next article.

The maximum and minimum stresses, expressed in kips, are given in the following table, the end post being omitted as its stresses were given in the preceding article:

Diagonals	D_2	D_3	$D_4(=D'_4)$	$D'_5(=D'_3)$
Dead load	+ 67.5	+ 28.7	0	− 30.2
Live load—maximum	+192.0	+137.6	+104.1	+ 71.5
Live load—minimum	− 33.2	0	0	0
Impact—maximum	+111.0	+ 92.4	+ 80.5	+ 62.1
Impact—minimum	− 31.9	0	0	0
Maximum	+370.5	+258.7	+184.6	+103.4
Minimum	+ 2.4	0	0	0

PROBLEM 52*a*.—Find the maximum and minimum stresses in the diagonals in Problem 51*a*.

ART. 53. EXAMPLE—MAXIMUM STRESSES IN VERTICALS

The position for the maximum stress in V_1 is either wheel 4, wheel 12, or wheel 13 at panel point 1 (Art. 44). When wheel 13 is at 1, the same wheels of the second locomotive are placed on the first two panels and in the same position as those of the first locomotive when wheel 4 is at panel point 1, together with one additional wheel; hence it is not necessary to find the stress due to the latter position. The greatest stress occurs when wheel 13 is at 1, and equals

$$[5700 - (2 \times 1227)] \div 27 = 120.2 \text{ kips tension.}$$

If it is desired to employ a similar method for V_1 as that prescribed in the preceding article, let the first two panels be regarded as a separate truss, and the load placed in proper position for maximum moment at panel point 1. The bending moments are then $M_0 = 0$, $M_1 = \frac{1}{2}(5700) - 1227 = 1623$, and $M_2 = 0$, $M_1 \div p = 1623 \div 27 = 60.1$. The vertical shear in each panel (disregarding signs) is therefore 60.1, and the floor-beam reaction, or the stress in V_1, equals their sum, or $60.1 + 60.1 = 120.2$ kips.

The respective points of division in the third, fourth, and fifth panels, where a concentrated load produces no stress in the vertical at the left of the panel, are the right-hand ones shown in Plate IV. The position and other necessary data given in the following table are found in exactly the same way as for the diagonals:

Panel	Wheel at Right End of Panel	Moment at Right Support	Moment at Right End of Panel	Bending Moment (M) at Left End of Panel	Bending Moment (M) at Right End of Panel	Left $\frac{M}{p}$	Right $\frac{M}{p}$
2–3	2	27 610	120	7889	11 713	292.2	433.8
3–4	2	16 590	120	7110	9 360	263.3	346.7
4–5	2	8 320	120	4754	5 823	176.1	215.7

If the greatest live-load stress in V_2 were due to the same position of the load as for D_3, it would only remain to draw (on the diagram in the third panel of the truss in Plate IV) the line marked U_2 parallel to that member of the truss in order to complete the force polygon for the upper panel point 2. The magnitude and character of the simultaneous stress in V_2 is marked on the diagram. If a force polygon like that one be drawn for the values of $M \div p$ in the first line of the above table, the stress in V_2 is found to be -91.5 kips. The construction is omitted on the plate to avoid confusion, as it would partly cover the diagram already drawn. In the same way the greatest live-load stress in V_3 is found to be -62.6 kips. As the stress in V_4 (-39.6) is less than that in V_3, and since V_3 and V_4 are symmetrically located in the truss, the compression to be used for V_4 is the same as for V_3 and will occur when the live load comes on the bridge from the left. The compression in the verticals is usually not required on the right of the middle of the truss.

It will be noticed that all values of $M \div p$ are laid off downward, except for the one polygon in the sixth panel. The reason that this is desirable is that the side of the polygon giving the stress in any vertical lies on that vertical. For example, if the values of $M \div p$ were laid off upward in the third panel, the side of the polygon giving the stress in V_2 would lie on the vertical V_3 instead of on the vertical V_2 as now drawn.

The maximum stresses in kips are given in the following table, the correction being applied on account of having taken the dead panel loads as explained in Art. 51. Attention is again called to the fact that the impact stress in V_1 is calculated as though it were a member of a truss only two panels in length.

Verticals	V_1	V_2	V_3
Dead load	+ 43.2	− 6.2	+ 20.5
Correction for division of dead panel loads	− 10.8	− 10.8	− 10.8
Live load	+120.2	− 91.5	− 62.6
Impact	+109.9	− 63.1	− 49.6
Maximum	+262.5	−171.6	−102.5

PROBLEM 53a.—Find the maximum stresses in the verticals in Problem 51a.

ART. 54. EXAMPLE—MINIMUM STRESSES IN VERTICALS

In Art. 31, as well as in other places, attention has been called to the fact that the stresses in the verticals of a truss with counter-braced panels depends upon the diagonals which are acting simultaneously in the adjacent panels. The influence of the diagonals affects not only the magnitude and character of the stress for any given position of the live load, but also the rate of change in the stress as the live load passes across the bridge. In order to determine what position of the live load will produce the minimum stresses in the verticals of the truss employed in the three preceding articles, let the complete cycle of changes in the stress in V_5 (Fig. 54a) be traced as the locomotives and train pass across the bridge from right to left.

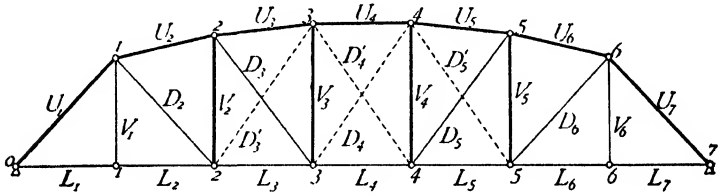


FIG. 54a.

When wheel 1 is at the right support, the stress in V_5 is simply that due to the dead load. As the live load advances, the combined dead- and live-load stress in V_5 gradually diminishes at an increasing rate, until a position is reached when the stresses in both diagonals D_5 and D'_5 are zero. Meanwhile the stress in the vertical V_5 has passed through zero from compression into tension. The tension increases at a reduced uniform rate until the stresses in D_6 and D'_6 are both zero. If this is not possible, then the tension increases until D_6 becomes a minimum. As the load advances, the tension in V_5 at first diminishes and afterwards increases until the stresses in D_6 and D'_6 again become zero (if possible). During this interval the stress in V_5 has passed through zero twice, so that it is again tension, but larger in magnitude than before. The rates of change at the beginning and end of the period are also more rapid than in either the preceding or the succeeding one. The tension now increases at a reduced uniform rate until the stresses in both D_5 and D'_5 are again zero. As the load advances until it covers the entire bridge, the stress in V_5 diminishes and passes through zero the fourth time and into compression. The rate of decrease was itself a decreasing one, being greater at the beginning of this period and nearly if not quite zero at the end. It will now be very slightly reduced until the head of the train arrives at the left support, when it will remain constant until the rear of the train begins to pass over the bridge. As the train continues to pass off the bridge the compression in V_5 increases at a variable rate until it reaches the maximum value, and then gradually diminishes again to the value of the dead-load stress. The absolute maximum compression in V_5 was not reached in this passage of the live load, but will occur when it crosses the bridge from left to right.

During this cycle there were several periods during which the diagonals whose upper extremities are at panel point 5 were not acting, and when the stress in V_5 was therefore to be obtained by drawing the force polygon for that upper panel point. It is evident then that the tension in V_5 is the greatest when the compression in U_5 and U_6 is the largest possible without calling D_5 into action. As the maximum stresses in U_5 and U_6 occur when the entire bridge is covered with the live load, the required position may be obtained from this one by moving the train backwards until the main diagonal in the panel which is adjacent to the vertical, and on the side toward the middle of the bridge, shall just cease to act. For deck bridges this statement would, of course, need modification. In the present example, D'_6 does not act under any position of the live load and is therefore omitted, but the statement in the preceding paragraph was so framed as to apply also to V_4 by making the corresponding changes in the subscripts of D and D'.

The required position for the greatest tension in V_5, or its minimum stress, was found by trial to be that when wheel 1 is $3\frac{1}{2}$ ft. to the left of panel point 4. The moment at the right support is 15 000, and those at panel points 4 and 5 are 52 and 2257 ft.-kips, respectively. The live-load bending moments at these points are therefore 8522 and 8460. Adding those due to dead load (Art. 51), $M_4 = 15\,520$, and $M_5 = 14\,292$. When divided by the panel length of 27 ft., the quotients are 574.8 and 529.3 kips. When these are laid off on Plate IV the resulting force polygon is reduced to two straight lines, indicating that there is no stress in D_5. The corresponding polygon for D'_5 is drawn in broken lines. On drawing a parallel to U_6, as shown, and thus completing the force polygon for the upper panel point 5, the stress in V_5 may be measured by scale. The direction of passing around the polygon is evident, since U_5 and U_6 are both known to be in compression. The combined stress in V_5 is +45.3 kips. If M_4 is divided by 38 ft. and M_5 by 35 ft., the quotients are both 408.4 kips, which being the horizontal components of U_5 and U_6 shows also that there is no stress in the diagonals and checks the graphic construction.

After some experience this position can be found with but few trials, and will not require much time if all the operations are performed by graphics. In Fig. 54*b* let the depths of the truss at panel points 4 and 5 (38 and 35 ft., respectively) be laid off on one side of an angle, and some convenient number, as 50, on the other side to the same scale. Join *a* and *b* with *f*. With the same scale which was used in drawing the equilibrium polygon for the wheel loads, lay off the bending moments M_4 and M_5 due to the dead load. Now assume a position of the live load, draw the closing line of the equilibrium polygon, and with the dividers transfer the bending moments due to the live load and lay them off above the others. If the position is correct, the lines *ce* and *de*, parallel respectively to *af* and *bf*, will intersect each other on the line *of*. If the head of the locomotive is too far to the left, they will intersect below the line. If *oe* be measured by the scale of moments and divided by 50 ft. (assumed as above), the quotient will be the horizontal component of the stresses in U_5 and U_6.

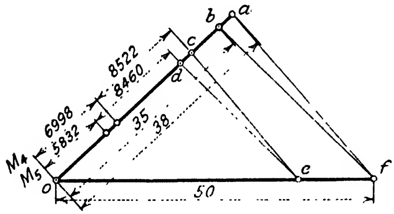


FIG. 54*b*.

If portions of two trains cover certain panels at each end of the bridge, a stress will be caused in V_5 which is a little larger than the value given above, and which can be found as follows: Let the required positions of two trains approaching each other be that illustrated in Fig. 54*c*. The diagonals in the fifth panel are omitted, since there must be no stress in the diagonals of that panel for a maximum tension in V_5, as proved in the preceding portion of this article. Let P_0 be the resultant of the loads transferred to the truss at panel points 5 and 6 by the floor system, together with the dead panel loads at these points, and g its distance from the right support. Let c be the intersection of the chord members U_5 and L_5, and d the intersection of U_6 and L_5. If a section be passed through U_5 and L_5, the stresses in those members hold in equilibrium the forces P_0 and the reaction B, and therefore their resultant. The resultant of the stresses in U_5 and L_5 passes through c; and therefore the resultant R of P_0 and B must be equal and opposite to it and applied at the same point. The value of R is readily found by taking moments about 7, whence $R = P_0 g \div r$.

If a section now be passed cutting U_6, V_5, and L_5, the stresses in these members hold in equilibrium the same forces P_0 and B as the stresses in U_5 and L_5, since the dead load at the upper

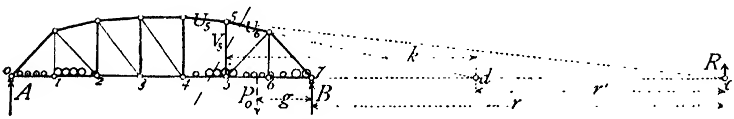


FIG. 54c.

panel 5 is zero in this case. Substituting R for P_0 and B, denoting the stress in V_5 by S, and taking moments about d, there results $-Rr' + Sk = 0$, whence $S = Rr' \div k = P_0 gr' \div kr$. But k, r, and r' are constant; therefore the position of the live load to give the maximum tension in V_5 must be such as to render $P_0 g$ a maximum. This shows that the stress in V_5 is independent of the distribution of the train load on the left, and it may therefore consist of the rear portion of a preceding train.

Referring now to Fig. 54d, which shows the truss diagram in position on the load line and moment diagram, the ordinate bf is the moment at the right support of the truss due to the locomotives of the right-hand train. If the chord $4i$ be produced to e, be will represent the moment of the live panel load at 4 about b as a center. The ordinate ef will therefore represent $P_0 g$, less the moment due to the dead panel loads at 5 and 6. As this last moment is constant, ef must be made a maximum. It is also evident that heavy loads should be placed at 5, and usually the head of the locomotive will not pass beyond the panel. The possible positions are therefore quite limited, and on applying the test it is found that when wheel 3 is at panel point 5, $ef = 9544 - (3 \times 345) = 8509$. In this equation 9544 equals bf as read from the diagram, and 345 equals the ordinate $5i$. Similarly, for wheel 4 at 5, $ef = 10\,924 - (3 \times 720) = 8764$, and for wheel 5 at 5, $ef = 12\,454 - (3 \times 1245) = 8719$. In the required position, therefore, wheel 4 must be placed at panel point 5.

Assuming that the train on the left is also in its right position, the closing line of the equilibrium polygon is hf; if the train is

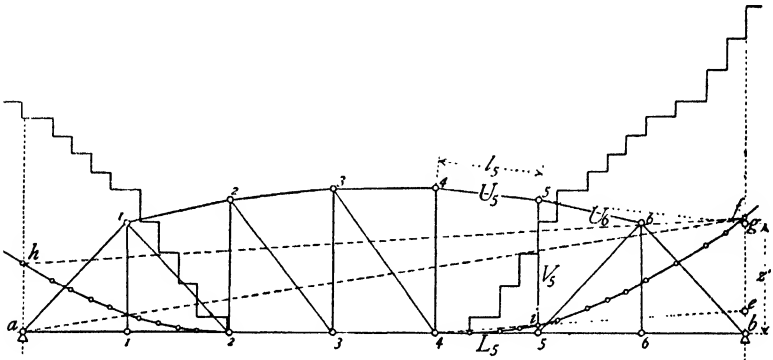


FIG. 54d.

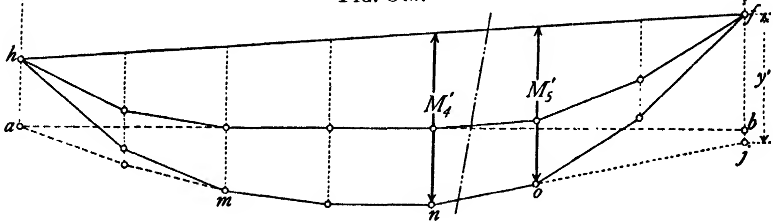


FIG. 54e.

off the bridge the closing line is af. The equilibrium polygon for the truss is shown in Fig. 54e, the ordinates at the panel points being the same as in Fig. 54d, and all the sides straight lines. By adding the moments due to dead load below those due to live load the polygon becomes $hmnofh$. By Art. 7 the intersection of the sides no and hf produced (Fig. 54e) is on the line of action of the resultant R of the forces on the right of the section, and therefore this intersection lies in the same vertical as the intersection of U_5 and L_5. The position of R was shown in Fig. 54c. Let the intercept bg at the right support, between chords U_5 and L_5 produced

(Fig. 54*d*), be denoted by z', and that in the same vertical between *hf* and *no* produced (Fig. 54*e*) by y', and the depths of the truss at 4 and 5 by h_4 and h_5. There follows, $M'_4 : h_4 = M'_5 : h_5 = y' : z'$, whence

$$\frac{M'_5}{h_5} = \frac{y'}{z'}$$

If the stress in U_5 be denoted S', and the angle between U_5 and a horizontal by α, and the length of U_5 by l_5, p being the panel length,

$$S' = \frac{M'_5}{h_5 \cos \alpha}, \quad \text{and} \quad \frac{p}{\cos \alpha} = l_5$$

Substituting,

$$S' = \frac{y'}{z' \cos \alpha} = \frac{py'}{pz' \cos \alpha} = \frac{l_5}{z'} \cdot \frac{y'}{p}$$

An inspection of Fig. 54*e* shows that in order to determine y' it is not necessary to know M'_4 and M'_5, and therefore not necessary to consider either the weight or the position of the train on the left. When the train is not on the bridge the closing line is *af*, and therefore the corresponding bending moments M_4 and M_5 will also determine y'. Remembering that the moment *bf* (Fig. 54*d*) for wheel 4 at panel point 5 was found to be 10 924, that the moment 5*i* is 720, and that the bending moments at 4 and 5 for dead load are 6998 and 5832 ft.-kips. respectively,

$$M_4 = \tfrac{4}{7} \times 10\,924 + 6998 = 13\,240 \text{ ft.-kips}$$

and

$$M_5 = \tfrac{5}{7} \times 10\,924 - 720 + 5832 = 12\,915 \text{ ft.-kips}$$

As M_4 and M_5 are respectively 3 and 2 panel lengths from the right support,

$$\frac{y'}{p} = \frac{(3 \times 12\,915) - (2 \times 13\,240)}{27} = 454.3 \text{ kips}$$

The stress in U_5 can now be found by graphics in the following manner: On Plate IV draw the line $7g'$ parallel to U_5 intersecting the vertical V_5 at g'; join g' with upper panel point 4; lay off $y' \div p = 454.3$ on V_5 as indicated, and draw *ij* parallel to $g'4$.

The line $j5$ represents the stress in U_5. The force polygon for the upper panel point 5 can next be obtained by drawing jk parallel to U_6. On measuring $5k$ by scale the stress in V_5 is found to be $+47.0$ kips.

This result may be checked as follows: Let a stress diagram be drawn giving the stress in D'_5 (Fig. 54a) when the reaction at the right support is 1.0. It is found to be 1.017. By the method described in the preceding article, let the stress in D'_5 be found for the above values of M_4 and M_5. Its value is $+ 35.6$ kips. To reduce this stress to zero the reaction at b (Fig. 54d) must be increased by $35.6 \div 1.017 = 35.0$ kips. This requires a moment at the left support a of $35.0 \times 189 = 6615$ ft.-kips. If this is to be produced by a train approaching from the left, its wheel 1 must be about half a foot on the right of panel point 2, as shown in Fig. 54d. If, however, it be produced by the rear end of a preceding train, the train must cover a distance of $66\frac{1}{2}$ ft. from the left support. The bending moment $M'_4 = 13\,240 + \frac{3}{7} \times 6615 = 16\,075$ ft.-kips, and $M'_5 = 12\,915 + \frac{2}{7} \times 6615 = 14\,805$ ft.-kips. If these values are respectively divided by the depths $h_4 = 38$ and $h_5 = 35$ ft., each quotient gives the same horizontal component of 423.0 kips for U_5 and U_6 (see Plate IV).

The stresses in the diagonals in the center panel become zero the second time when the live load covers the entire bridge, and therefore the greatest tension in V_4 or in V_3 occurs when U_4 has its maximum stress. By laying off $M_3 \div p = 259.2$, which is due to the dead load, above d, and constructing the triangle fge, the stress in V_3 is found to be $+66.9$ kips. If it were attempted to apply the method outlined above, $y' \div p$ would be 907.4, which would give a stress in U_4 greater than that under full load, which is not possible; and if the position of the train approaching from the left, which reduces the stress in D_4 and D'_4 to zero, were determined, it would be found to conflict with that of the other train. The maximum tension occurs under full load only for the vertical adjacent to a center panel, or for the middle vertical of a truss with an even number of panels.

The minimum stress in V_1 occurs under dead load only. The accompanying table gives the final minimum stresses after applying the correction on account of dividing the dead panel loads. The stresses due to dead and live loads have been separated so that the impact stress could be computed.

Verticals	$V_1 = V_6$	$V_2 = V_5$	$V_3 = V_4$
Dead load	+43.2	+18.4	+20.5
Correction for division of dead panel loads	−10.8	−10.8	−10.8
Live load	0	+28.6	+46.4
Impact	0	+24.4	+25.0
Minimum stress	+32.4	+60.6	+81.1

If the number of panels in the truss were nine or more, the second vertical from the right support would be adjacent to two panels requiring no counter bracing. In such cases the minimum stress in the vertical is obtained in exactly the same way as the maximum, except that the load covers only the smaller segment of the span.

It is apparent that to secure precise results the methods outlined in the example of the four preceding articles, and illustrated in Plate IV, must be drawn to a large scale. Results which shall answer all the purposes of design, however, may be readily secured with reasonable care on drawings which are not unwieldy in size.

PROBLEM 54*a*.—Find the minimum stresses in the verticals in Problem 51*a*.

CHAPTER VI

MISCELLANEOUS STRUCTURES

Art. 55. Trusses without Verticals

When the center of moments for any chord member of a truss is not in the same vertical as a floor-beam, the method of determining the position of the locomotive wheel loads and the corresponding maximum moment described in Art. 45 does not apply. In Part I the required criterion for position,

$$P' + \frac{q}{p}Q = \frac{l'}{l}W$$

was deduced, in which Q is the load in the panel cut by the vertical through the center of moments, P' the load on the left of this panel, W the whole load on the truss, q the horizontal distance from the center of moments to the left end of the panel containing Q, p the panel length, l' the distance of the center of moments from the left support, and l the span of the truss.

In order to satisfy this criterion a wheel has in most cases to be placed at the left end of the panel containing the aggregate load Q, although it will often be satisfied when a wheel is placed at the right end of this panel.

A truss of this type, now encountered only in the investigation of existing structures, is the Pegram truss. It has a broken upper chord, and posts whose angles with the vertical increase from the middle of the truss to its ends. The horizontal projection of the upper chord is about one and one-half panel lengths shorter than the lower chord, but both chords are divided into the same number of panels. The panel points of the upper chord lie upon the arc of a circle. The form, proportion, and relative economy of this type of truss are discussed by the inventor in *Engineering News*, Vol. XVIII, pp. 414 and 432, Dec. 10 and 17, 1887.

Figure 55*a* shows the left end of a Pegram truss in position on the load line and moment diagram of the wheel loads for the maximum moment at the panel point 3. The line *au* produced passes through the point where the vertical at the right support cuts the load line. Wheel 4 is at the floor-beam at panel point 2. The ordinate $4r$ represents the load $P' + Q$. If wheel 4 be just to the right of the floor-beam the ordinate $2i$ or the equal ordinate mj represents P', and md represents $P' + \frac{q}{p}Q$. If, however, wheel 4 be just on the left of the floor-beam, the ordinate $2h$ equals P' and mf equals $P' + \frac{q}{p}Q$. The ordinate mc equals $\frac{l'}{l}W$, and the

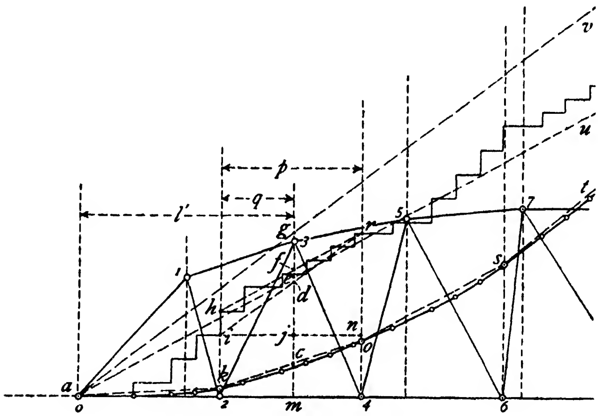


FIG. 55*a*.

position is therefore correct when wheel 4 is at the floor-beam and the point *e* falls between the points *d* and *f* where the lines *ri* and *rh*, respectively, cut the vertical through the center of moments 3. The point *r* is at the intersection of the load line with the vertical through panel point 4.

With the load in this position the equilibrium polygon for the truss is composed of the straight sides *ak*, *ko*, *os*, *st*, etc., and the closing line *av*. The bending moment is therefore given by the ordinate *cg*. In a similar manner the positions of the live load and the corresponding moments are obtained for all the panel points of the upper chord of the given truss.

The positions of the live load and the bending moments for the sections through the panel points of the lower chord are determined in the manner described in Art. 45. For those points the second term of the left-hand side of the above criterion for position becomes zero.

ART. 56. THE PENNSYLVANIA TRUSS

This type of truss is illustrated in the skeleton diagram of Fig. 56*a*. It is derived from the Pratt truss with a curved upper chord by subdividing its panels by means of subverticals and short diagonals. The vertical broken lines indicate struts which support the upper chord members at their middle points, and the corresponding horizontal lines serve to give lateral support in the plane of the truss to the long vertical posts. These are not real

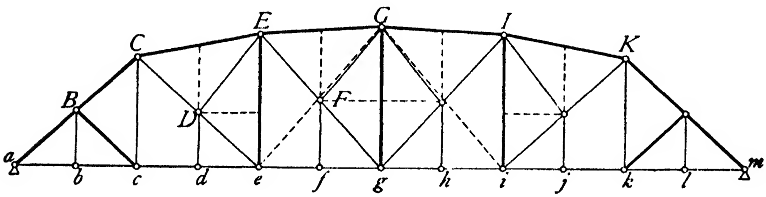


FIG. 56*a*.

truss members, and are omitted in the diagrams employed in finding the stresses in the truss. In this case the counter diagonal *eG* does not coincide with the short diagonal *FG*, although in a number of trusses which have been erected the panel *eEGg* is counter-braced by connecting the points *e* and *F* with a tie.

The stress in *ef* due to locomotive wheel loads is found in the same manner as for the Pratt truss, the center of moments being at *E*. The stress in *fg* equals that in *ef*, as may be seen from the force polygon for the panel point *f*.

By the method employed in Part I, a criterion for the position of the live load may be obtained which will give the stress in *EG*, and which indicates that the wheel loads from *a* to *e*, plus twice those from *e* to *f*, shall equal $Wl' \div l$, W being the whole load on the truss, l the span, and l' the distance from the left support to the center of moments g. To satisfy this criterion a wheel load must usually be placed at f, although sometimes it may be satisfied when a wheel is at e. In view of the examples given in Chapter V and in the preceding article, the student should have no difficulty in making the graphic construction required by this criterion.

As the section cutting *EG* and only two other members must pass on the left of *f*, the bending moment for *EG* must equal the moment of the left reaction of the truss, minus the moments of the loads transmitted by the floor system to the truss at the panel points *a* to *e* inclusive; or, in other words, the required bending moment exceeds that at the vertical section through *g* by the moment of the panel load at *f*. In the graphic determination, if the line joining the points where the verticals through *e* and *f* meet the moment curve of the live-load diagram, be produced to the vertical at *g*, and the ordinate from this point of intersection to the closing line be read off, the required moment will be obtained.

For the main diagonal *EF*, the position is found by the method given in Arts. 48 and 49, auxiliary lines *oe* and *oh* of Fig. 48*a* being drawn in this case through the points *e* and *f* in Fig. 56*a*. The force polygon is then constructed as in Art. 50 by using the moments at *E* and *g*, the points where the diagonal *EF* meets the upper and lower chords, respectively.

The maximum stress in *Ee* occurs also when the head of the locomotive is in the panel *ef*, and hence there will be no simultaneous live-load stress in *DE*. The section through *Ee* must therefore cut *CE* and *ef*, and in finding the point of division in *ef* the chord member *CE* must be produced as in Fig. 48*a*. If in any case the position for *Ee* should be the same as for *EF*, the stress in the former may be obtained from the force polygon already constructed for the latter by completing the polygon for the stresses meeting at the panel point *E*. If not, then a new force polygon for the simultaneous stress in *EF* must be drawn.

The panel load at *f* is suspended from *F* by the subvertical *Ff*, while the members *EG*, *EF*, and *FG* form a secondary truss which serves to transfer this panel load to the panel points *E* and *G*. As the panels are equal, one-half of the load at *f* is transferred to *E* and *G*, respectively. Since the posts *Ee* and *Gg* are both vertical, the stress in *Fg* is exactly the same as if the wheel loads in the panels *ef* and *fg* were transferred to the panel points *e* and *g* by a stringer whose span is *eg*. The stress in *Fg* is therefore found by the method given in Chapter V, after considering the members *Ff* and *FG* removed. The greatest stress in the counter *eG* (or *Gi*) is obtained in a similar manner to that in *Fg*.

The preceding statement also shows that the maximum stresses in *Ff* and *FG* occur when the floor-beam reaction is a maximum. This usually requires the second or third locomotive driver to be placed at the floor-beam. If the panels are long, the reaction will be greater under the corresponding position of the second locomotive, for one or two of the tender wheels of the first locomotive may then be brought on the panel at the left. The tension in *FG* equals one-half of the floor-beam reaction multiplied by the secant of the angle which *FG* makes with the vertical.

The stress in the vertical *Cc* depends not only upon the floor-beam reaction at *c*, but also upon that at *b*, one-half of the latter being transferred to *c* by the secondary truss *abcB*. By employing the same method as in Art. 44, the following formula may be deduced for the stress in *Cc* due to the locomotive loads:

$$S = \frac{M_d - \frac{3}{2}M_c}{p},$$

in which M_d is the moment of all the loads on the first three panels about *d* as a center; M_c the moment of the loads on the first two panels about *c* as a center; and *p* the panel length of the three equal panels *ab*, *bc*, and *cd*. The values of M_c and M_d can be read off directly from the live-load moment diagram. The corresponding position of the load requires the wheel loads in the first two panels to be equal to two-thirds of the load on the three panels. It will be observed that this is the same position as that for the maximum moment at *c* of a beam or truss whose span is *ad*. As the live-load diagrams are always constructed with the head of the locomotives toward the left, the maximum stress should also be found in *Kk* and compared with that in *Cc*, the larger value being used for both members. The tension in *Bb* equals the floor-beam reaction at *b*.

If instead of the short tie *FG* a short strut *eF* be inserted, the auxiliary truss will then be *efgF*, which will transfer the panel load at *f* to the points *e* and *g*. The moment of the stress in *fg* about the center *E* will then be the bending moment in the vertical section through *E* plus the moment of the panel load at *f*. The corresponding criterion for position will require the wheel loads from *a* to *f* minus the wheels from *f* to *g* to equal $Wl' \div l$, in which W, l, and l' have the same significance as before, whence l' equals the horizontal distance from the left support to *E*, which is the center of moments. The stress in *EG* is the same as if the secondary truss were omitted, and the stringer extended from *e* to *g*. The methods described above for finding the stresses in *EF* and *Fg* will now have to be exchanged.

A combined railway and highway bridge at Vicksburg, Miss., in which three spans of this type are used, is shown in skeleton diagram in *Engineering News-Record*, Vol. 105, p. 182, July 31, 1930, together with three highway bridges employing spans of the same type.

The construction of the stress diagram for the dead load offers no difficulty in either case, and will therefore not be illustrated.

The form of this truss as shown in Fig. 56*a* is sometimes modified by reducing the two panels at each end to one, thereby omitting the subverticals at *b* and *l*. Another modification was adopted in a railway bridge across the Ohio River at Louisville, Ky., whereby the panel point *B* was raised so as to bring it into the curve of the upper chord. See *Engineering News-Record*, Vol. 103, pp. 358–362, Sept. 5, 1929. This bridge is also of interest because of its unusual bottom lateral bracing.

Problem 56*a*.—The truss in Fig. 56*a* has a span of 283 ft., the depths at *C*, *E*, and *G* being 42, 47, and $48\frac{3}{8}$ ft., respectively. Find the stresses in all the members of the truss due to Cooper's E60 loading, the bridge having a single track.

Art. 57. The Baltimore Truss

The Baltimore truss is a special case of the Pennsylvania truss when the upper chord is horizontal. Such a truss with a web system similar to that shown in Fig. 56*a* is illustrated in the *Engineering News-Record*, Vol. 101, pp. 583–586, Oct. 18, 1928. Another Baltimore truss in which short strut diagonals are used in place of the short ties is shown in *Engineering News-Record*, Vol. 101, p. 310, Aug. 30, 1928.

The chord stresses for both the Baltimore and Pennsylvania trusses are found in exactly the same manner. The method used for the web stresses of the Pennsylvania truss also applies to the Baltimore truss, but for most of the web members it is preferable to make the comparison with the methods employed for the Pratt truss.

If the upper chord in Fig. 56*a* were horizontal the stresses in *EF* and in *Ee* would be the same as if the truss were of the Pratt type with fourteen panels, the load on the truss being fourteen times the load in the panel *cf*. For the stress in *Fg* the load on the truss must equal seven times the load in the panel, and the stress would be the same as if the truss were of the Pratt type with only seven panels. The stresses in *Ff* and *Bb* are equal to those in the hangers of a Pratt truss having the same panel lengths; those in *Cc* and *FG* are the same as for the Pennsylvania truss.

PROBLEM 57*a*.—A single-track Baltimore truss through bridge has the same span, number of panels, and live load as the truss in Problem 56*a*, its depth being 47 ft. 2 in., the counters *Gc* and *Gi* are omitted, however, and the diagonals *Eg* and *Ig* are designed to carry compression as well as tension. Find the live-load stresses in all the members.

ART. 58. UNSYMMETRICAL TRUSSES

Figure 58*a* represents the side elevation of the two unsymmetrical Pratt trusses of a through railroad bridge, together with

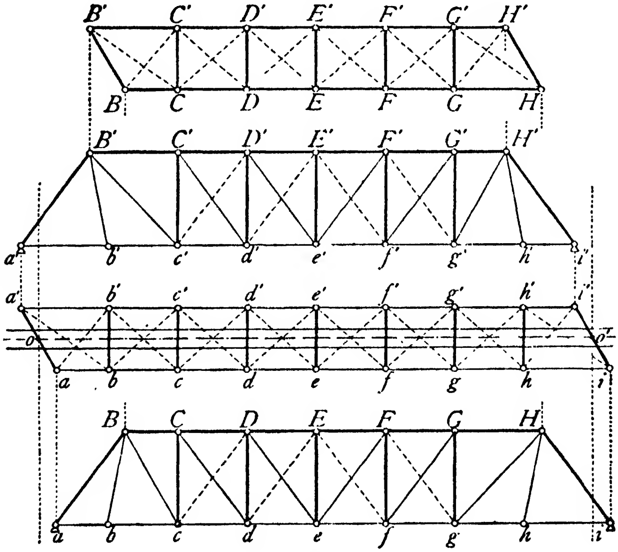


FIG. 58*a*.

the plans of the upper and lower lateral systems. The floor-beams are perpendicular to the center line of the bridge, and they are placed at equal distances apart from each other, and from the points mid-way between the bearings of the end stringers. The end posts are inclined so that their horizontal projections are equal to the distance between the floor-beams, and this necessitates shortening the end panel of the upper chord at one end of the truss, and lengthening that at the other end by an amount equal to one-half the longitudinal component of the skew. The end panels of both chords are therefore equal at the same end of each truss, and the hangers are inclined. Trusses of skew spans have been built whose end posts are not equally inclined, but this complicates the portal bracing and is very unusual.

As the two trusses are equal, but have their ends reversed, it is necessary to find the stresses in all the members of one truss for all the loads. Since the load line and equilibrium polygon for the locomotive wheel loads are usually arranged for the load advancing from the right, the live-load stresses are found in all the members of the left half of each truss, except the counters whose stresses are found in the right half.

The dead-load stresses in the trusses are as readily determined by the graphic method as for symmetrical trusses, whereas in the analytic method the numerical work of computation is materially increased.

The stresses in the lateral systems due to the lateral loads on the trusses are found in a manner similar to that used in Art. 35, but extended to all the members of each system. The relation between the values in the different lines of the table are, however, not as simple as for trusses whose panels are all equal. For instance, if the nearer truss *ai* is on the windward side, the diagonals of the lower system brought into action are *ab'bc'cd'de'e* . . . *hh'i*, and a lateral panel load at *g* will cause stresses in members on its left equal to those due to a panel load at *h* multiplied by the ratio of *gi* ÷ *hi*.

Another method, which is preferable in most cases, is to draw a diagram giving the stresses in all members for a reaction of 1.0 at the left support *a*, and then to multiply the stress in each diagonal by the corresponding reaction produced by the panel loads which make its stress a maximum. The reactions are most quickly determined by means of an equilibrium polygon whose closing line will shift as one panel load after another is taken away from the full load for which it is at first constructed.

The position of the wheel loads is obtained in the same way as for symmetrical trusses, it being assumed, however, that the loads are distributed along the center line oo' of the track as shown in Fig. 58*a*, and that the trusses have their supports at o and o'. Although this method is approximate, it generally gives the correct position. In case a very slight shifting of the loads would dissatisfy the criterion, it may be well also to find the stress when the next wheel is placed at the corresponding panel point and compare the results.

In the bridge represented in Fig. 58*a* let the span be 146 ft., $bc = cd = de = ef = fg = gh = 18$ ft. 3 in., $ab = BC = 13$ ft. $7\frac{1}{2}$ in., $hi = GH = 22$ ft. $10\frac{1}{2}$ in., the width 16 ft. between centers of trusses, and the depth 24 ft. For the greatest stress in the

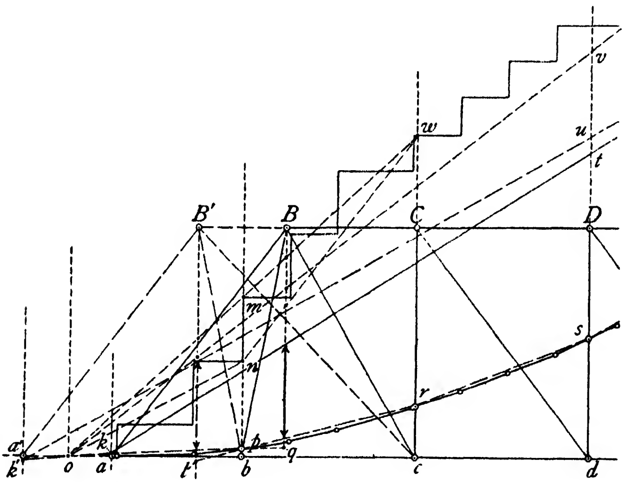


FIG. 58*b*.

chord members *ab* and *bc* due to Cooper's E60 loading (Art. 38) the required position is that with wheel 3 at panel point *b*. Fig. 58*b* shows the left portion of the truss diagram placed in this position on the load line and moment diagram of the live load. The intersection of the line *ov* with the vertical through the center of moments *B* is seen to lie between the points where *mw* and *nw* cross the same vertical. As the floor system distributes the load to the panel points of the truss the lower sides of the equilibrium polygon for the truss under this position of the load are the right lines, or chords, *op*, *pr*, *rs*, etc., the points *o*, *p*, *r*, and *s* being on the live-load polygon. The point *o* is the regular panel length of 18 ft. 3 in. on the left of *b*. The closing line of the polygon must end on a vertical through the left support of the truss at *a*, and therefore at the intersection *k* of the vertical *ka* with the line *op*. In a similar manner the right end of the closing line (not shown in Fig. 58*b*) is located at the intersection of the chord or side of the equilibrium polygon whose extremities lie on the verticals through *h* and *o'* (see Fig. 58*a*) with the vertical through *i*. The bending moment in the section through *B* is measured by the full line ordinate with arrows at its extremities (Fig. 58*b*). This moment divided by the depth of the truss gives the stress in *bc*. As the stresses in *ab* and *aB*, however, hold in equilibrium only the reaction at *a*, the moment of the stress in *ab* about *B* is equal to the moment of the reaction, and hence is measured by the ordinate when produced to *q*, its intersection with the side *op* (or *kp*) produced.

The stress in the end post *aB* is preferably obtained by dividing the stress in *ab* by the sine of the angle which *aB* makes with the vertical. For the remaining web members, except the hangers, the only modification required of the method described in Art. 43 for trusses with equal panels is that the moment at the right support must be read to the point corresponding to that described in the preceding paragraph as the right end of the closing line of the equilibrium polygon for the truss.

The floor-beam reaction is the same as if the panel *ab* of the truss were equal to *bc*, for the deduction of the formula in Art. 44 indicates that the panel lengths introduced are really the spans of the corresponding stringers, and although usually they are equal to the panel lengths of the truss, this is not always the case. The tension in *Bb* equals the product of the floor-beam reaction by the secant of the angle which *Bb* makes with the vertical.

Figure 58*b* also shows the left-hand portion of the diagram of the truss *a'B'H'i'* superimposed upon the other. The closing line is *k'u*, *k'* being at the intersection of *po* produced with the vertical *a'k'*. The moment of the chord stress in *a'b'* (*b'* coincides with *b*) is measured by the ordinate below *B'* and is indicated by arrows at its extremities. By producing this ordinate to *l*, on the chord *rp* produced, it gives the moment of the stress in *b'c'*.

It will be observed that the same position of the live load was used for the end chord members of both trusses. If the point *o* had been moved to *a'* the criterion would not have been satisfied by placing wheel 3 at b, but only by putting wheel 4 at b. The moment for the latter position, however, is less than that for the former.

PROBLEM 58*a*.—Find the maximum and minimum stresses in the above example due to the given live load and a dead load of 1350 lb. per linear ft. per truss, one-fourth to be taken on the upper chord.

ART. 59. MULTIPLE WEB SYSTEMS

For trusses having more than one system of webbing an approximate stress analysis is made by assuming that each system is affected only by the loads it carries.

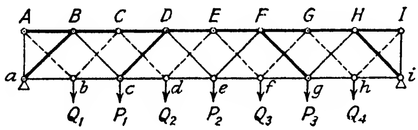


FIG. 59*a*.

In the double intersection Warren truss in Fig. 59*a*, the loads P_1, P_2, and P_3 are carried by the full line diagonals, and the loads Q_1, Q_2, Q_3, and Q_4 by the diagonals drawn in broken lines. The truss is therefore regarded as composed of two separate trusses having common chords. The stresses in each system may then be determined and the results combined. In this case, however, either the dead-load stresses in all the members or the live-load stresses in the chords may be found by means of one diagram.

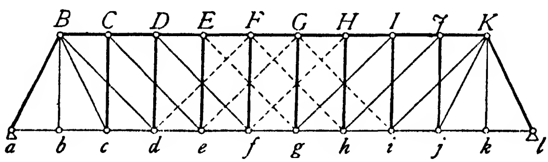


FIG. 59*b*.

The reaction at the left support is in equilibrium with the stresses in Aa, Ba, and ab, but the compression in Aa is known, since it equals the reaction due only to the loads Q, thus leaving but two unknown stresses. The stress diagram may therefore be readily constructed. The maximum live-load stresses in the diagonals are obtained by considering each system separately.

For the Whipple truss in Fig. 59*b* (which is a double intersection truss of the Pratt type) both dead- and live-load stresses must be found for each system, the division into systems, however, being somewhat different for the required stresses in the chords and web members. For the chord stresses the division may be made into the two symmetrical systems shown in Fig. 59*c*, provided the live load is uniform throughout. If the live load is not uniform, or if it consists of excess loads combined with a uniform train load, the division may be made similar to that shown in Fig. 59*d*, care being taken to insert those diagonals near the middle which are in tension under combined dead and live loads. If the stresses are obtained only for the left half of the truss, the excess loads may require an additional diagonal to slope downward toward the left in the lower diagram of Fig. 59*d*. It is clear that only those dead- and live-load stresses in any chord member may be added together, which were obtained under the same conditions;

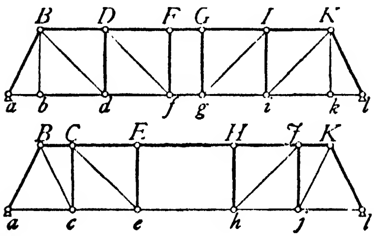


FIG. 59*c*.

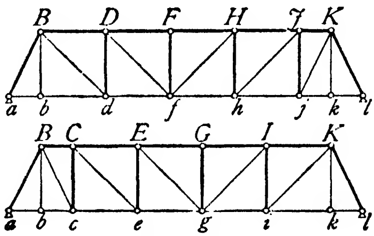


FIG. 59*d*.

that is, with the same diagonal of the given panel acting in each case.

When the division is made into systems, or component trusses, which are unsymmetrical, there is some ambiguity in the stresses owing to the fact that the hangers are attached to the panel points *B* and *K* which are common to both systems. As reasonable an assumption as any is to regard the panel loads at *b* and *k* as equally divided between the two systems. The same stresses will, however, be obtained if both hangers be considered as a part of only one and the same system, and this arrangement is also more convenient in finding the stresses.

If these two methods of division be compared for a uniform load throughout, the greatest difference in chord stresses is found to be not quite 4 per cent, most of them being much less. The difference may be reduced one-half by considering the hanger *Bb* as belonging only to that component truss which contains the adjacent diagonal *Bc*, and the hanger *Kk* as being a part of the system containing the diagonal Kj. This arrangement reduces the shear at the middle to the minimum value possible in each case, and requires the construction of only one stress diagram for the chord members, since one system equals the other with its ends reversed.

For the stresses in the web members the systems are divided as in Fig. 59*e*, all the diagonals sloping one way except those near the right end, where it is certain that no counters are needed. Only the loads supported by one of the systems are considered in finding the stresses due to both dead and live loads in any web member of that system. The method employed is that of Art. 29, the labor being materially lessened by noticing the general statements made in that article in regard to the maximum and minimum stresses in web members. On account of the ambiguity in stresses the panel loads at b and k may be placed on either system, so as to produce the maximum and minimum stresses in any given web member.

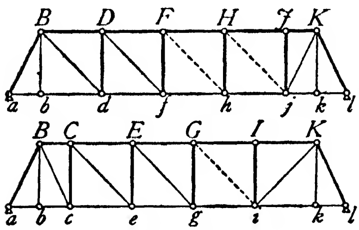


FIG. 59*e*.

When one excess load is used in connection with uniform panel live loads, it must be placed on that system which gives the greatest chord stresses; and for two excess loads the first may be on one system and the second on the other, depending upon their distance apart and the panel length. For maximum stresses in the web members the excess loads are always placed at the head of the train.

If concentrated wheel loads are to be employed it will be best always to place the first driver at the panel point. Each system is regarded as acting independently, and as being strained only by the loads transferred to it by the stringers and floor-beams. As part of the weight of the pilot wheel is carried by the stringers to the other system, that part is disregarded in obtaining the stresses in the web members. For the chord stresses the locomotives are so placed as to produce the greatest moment at the middle of the truss, and the loads transferred to each system are used only in determining the chord stresses for that system. For all the above types of loading, influence lines may be used to good advantage but they are especially valuable in obtaining an exact solution for excess loads and locomotive loads.

Problem 59*a*.—A double system deck Warren truss of 120-ft. span has ten panels and is 12 ft. deep. The dead load per linear foot per truss is 1200 lb. Find the maximum and minimum stresses in all members due to a uniform live load of 3000 lb. per linear ft per truss.

Art. 60. Wind Stresses in a Viaduct Tower

Figure 60*a* represents one bent of a viaduct tower. Each tower is composed of two such bents spaced 30 ft. apart and braced

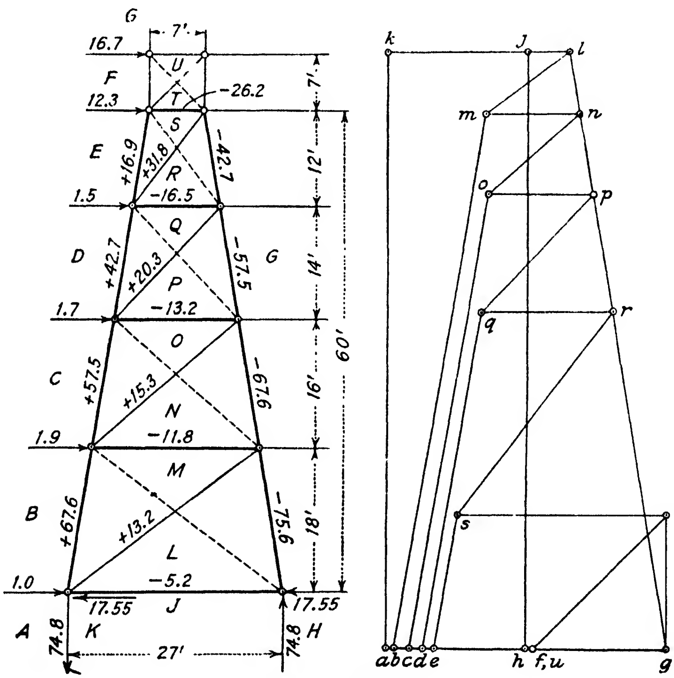


Fig. 60*a*.

together longitudinally. The distance between towers is 60 ft. so that each bent carries $\frac{1}{2}(30 + 60) = 45$ ft. The tower is 60 ft. high and supports deck plate girder spans 7 ft. deep.

A common specification requires viaducts to be designed for a wind load of 50 lb. per sq. ft. on one and one-half times the vertical projection of the structure unloaded, or 30 lb. per sq. ft. on the same surface plus 400 lb. per linear ft. of structure applied 7 ft. above the rail for assumed wind force on train when the structure is loaded with empty cars (on the leeward track, if double track) assumed to weigh 1200 lb. per linear ft., whichever gives the larger stress.

Figure 60*a* gives the loads and stresses for the first condition of loading. Assuming the depth of ties and guard rail to be $1\frac{1}{2}$ ft. and the width of columns to be $1\frac{1}{2}$ ft., the loads are:

$$
\begin{aligned}
AB &= 1.5 \times 50 \times 1.5 \times 9 &&= 1.0 \text{ kip}\\
BC &= 1.5 \times 50 \times 1.5 \times 17 &&= 1.9 \text{ kips}\\
CD &= 1.5 \times 50 \times 1.5 \times 15 &&= 1.7 \text{ kips}\\
DE &= 1.5 \times 50 \times 1.5 \times 13 &&= 1.5 \text{ kips}\\
EF &= 1.5 \times 50(3.5 \times 45 + 1.5 \times 6) &&= 12.3 \text{ kips}\\
FG &= 1.5 \times 50 \times 5 \times 45 &&= 16.7 \text{ kips.}
\end{aligned}
$$

The cross-bracing of the deck girder span is considered as part of the bent in order to transfer the load *FG* to the bent. For the other condition of loading two auxiliary members are necessary to transfer the wind load on the train to the bent.

It is necessary to make some assumption as to the division of the horizontal loads between the two reactions. A reasonable assumption and the one used in Fig. 60*a* is that the horizontal components of the two reactions are equal. It will be noticed that the stress diagram must be drawn by starting at the top of the bent and it therefore is not necessary to determine the reactions first. In fact, the reactions can be determined from the stress diagram.

PROBLEM 60*a*.—Determine the stresses in the bent of Fig. 60*a* for the second condition of loading given above.

PROBLEM 60*b*.—Determine the stresses in the bent of Fig. 60*a* for the loads shown, assuming the left reaction to be vertical. Compare these stresses with those shown in the figure.

ART. 61. A FERRIS WHEEL WITH TENSILE SPOKES

The skeleton diagram of a small Ferris wheel with eight apexes and supported at the hub is given in Fig. 61*a*. The broken circular line indicates the rack where the power is applied to rotate the wheel. The spokes are designed to take only tension, and equal loads are applied at all the apexes. The crown of the wheel tends to sag under the influence of the load at apex 1; but as this would produce compression in the spoke *IB* it will not act, and therefore the load is supported by the segments *AI* and *AB* of the rim. These stresses are obtained by constructing the force polygon for apex 1 as shown in the stress diagram at the right. The force polygons for the remaining apexes may now be

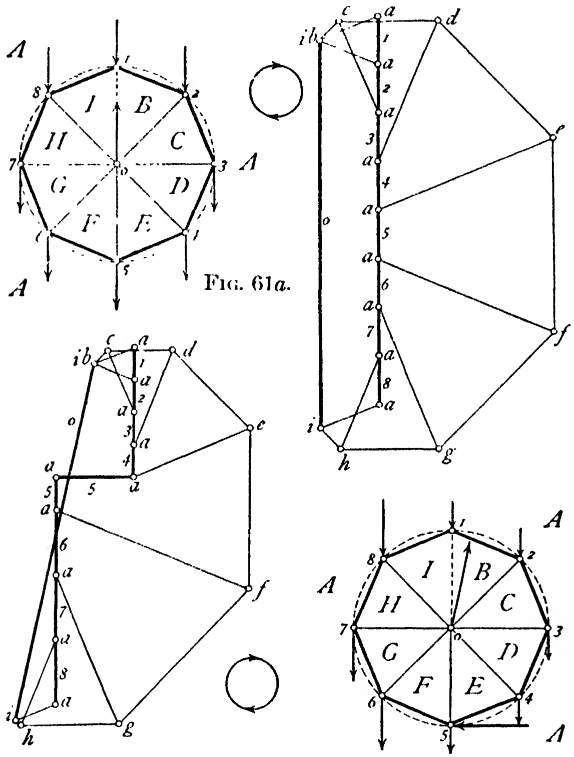


Fig. 61*a*.

Fig. 61*b*.

constructed in regular order. The long vertical *ib* is the reaction of the support, and the diagram shows that it is in equilibrium with the stresses in all the spokes, which truly expresses the relation of the forces at the hub. In this article the resistance due to friction in the bearings is not taken into account.

An examination of this diagram shows that under uniform load the stresses in the spokes and in the segments of the rim gradually increase from the top to the bottom of the wheel, those in the rim being compression throughout. When a segment of the rim is in a horizontal position at the top of the wheel its stress is $-1.0P$, P being the apex load. The stress diagram for this position is not given. As it revolves its stress gradually increases to the maximum value of $-3.92P$ when the segment reaches the position of AE; the stress now remains unchanged until the position of AF is reached, and then gradually diminishes until the segment is again horizontal at the top. If the wheel had sixteen spokes the compression in any segment of the rim would vary between the limits $2.414P$ and $7.689P$.

The stress in any spoke remains zero during the interval between its two upper positions which make an angle with the vertical equal to one-half the angle between the spokes, and then gradually increases until it reaches its maximum value of $+4P$, when the spoke is below the hub in a vertical position. It is interesting to observe that the maximum stress in any spoke is independent of the number of spokes when that number is not less than four. The construction of the diagram also indicates that the stresses in both rim and spokes are independent of the size of the wheel, except so far as the apex loads may depend upon it.

Figure 61*b* gives the wheel and stress diagrams for the case when only three of the observation cars are occupied. The horizontal reaction applied on the circular rack which is used to rotate the wheel is found by equating to zero the sum of the moments of all the external forces with reference to the center of rotation of the wheel. In the stress diagram the points i and b which coincide indicate no stress in the vertical spoke IB, while the inclined line ib is the reaction IB of the supports, for, since the spoke has no stress, the letter I really applies to the entire space on the left of the inclined arrow designating the reaction. It will be noticed that most of the stresses on the loaded side are considerably greater than those on the other side.

The Ferris wheel at the World's Columbian Exposition in 1893 was 250 ft. in diameter and had thirty-six spokes. An article on this subject applying the graphic method to water wheels of a given type as well as to other structures, and including a description of the main features of a wheel of the same magnitude as that at the Exposition, but designed so that its stresses should be statically determinate, may be found in *Zeitschrift für Bauwesen*, Vol. XLIV, p. 586 (1894).

Bicycle wheels are usually constructed with spokes of very light steel rods and stiff rims of metal or wood. The number of spokes is generally thirty-two. The load on a bicycle wheel is applied at the hub, and there is an equal reaction at the point where the wheel rests upon the ground. The stresses in the rim and spokes may be found in a manner similar to that used for the Ferris wheel since the reaction tends to produce compression in the spoke attached to the rim at that point, but as it can take only tension it will not act, and hence may be considered as removed for the time being. The stress diagram is then readily constructed.

When the wheel rests on the ground at a point between the spokes the rim acts as a beam to transfer the reaction to the adjacent apexes or panel points of the wheel truss. The direct stresses in the wheel members may then be found by replacing the reaction by its two components, and considering the spokes at those panel points temporarily removed.

PROBLEM 61*a*.—Determine the stresses in all members of a Ferris wheel with eight tensile spokes for a load of P at each apex, when one of the spokes is vertical, and also when one of the rim segments is horizontal.

PROBLEM 61*b*.—Find the stresses in a bicycle wheel having thirty-two spokes under a load of 150 lb. when the reaction is applied at a spoke, and also when it is applied mid-way between spokes.

ART. 62. HORIZONTAL SHEAR IN A BEAM

Let it be required to construct a diagram showing the distribution of horizontal shear in a beam. This may be conveniently illustrated by an example such as occurs in the design of a deepened beam. A deepened beam consists of two timbers of rectangular cross-section placed one above the other and united by keys or brace blocks so as to make the timbers act as a single stick. By this means the combined strength of the timbers is double that secured when they act separately.

Let the loads to be supported, exclusive of the weight of the beam, consist of three concentrated loads of 4000, 8000, and 6000 lb., respectively, and a uniformly distributed load of 2000 lb. per linear ft., extending over a portion of the span as indicated in Fig. 62*a*. A single bending moment diagram for both concentrated and uniform loads may be constructed by treating separately the portions of the uniform load which lie between the

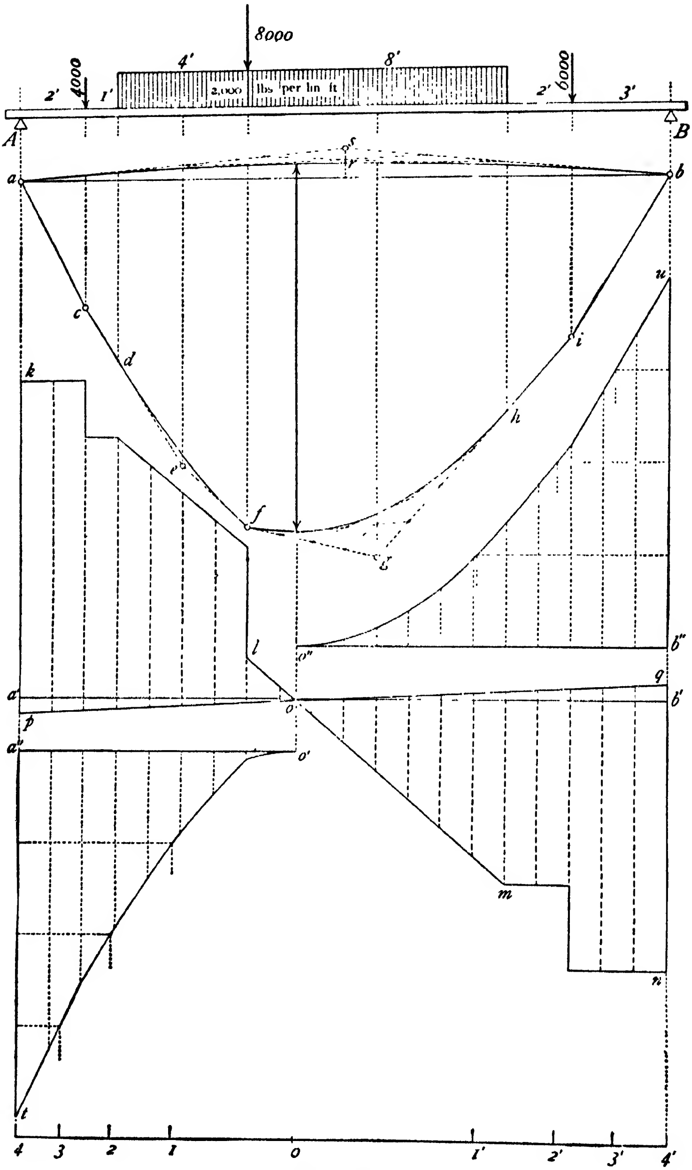

8000
4000
6000
4'
8'
2'
1'
2'
3'
2,000 lbs per lin ft
A
B
a
b
c
d
e
f
g
h
i
k
l
m
n
o
o'
o"
p
q
r
s
t
u
a'
b'
a"
b"
4
3
2
1
0
1'
2'
3'
4'

FIG. 62a.

concentrated loads. The loads are taken in succession from left to right and laid off on the load line (not shown). The left reaction A is found by computation to be 22 900 lb., and by laying this off on the load line the closing line of the equilibrium polygon may be made horizontal, if so desired, by taking the pole directly opposite the point of division of the reactions (see Arts. 9 and 11). The equilibrium polygon *acefgiba* is first drawn by regarding the portions of the uniform load as concentrated at their centers of gravity. The final form is then obtained from this by constructing a parabola (Art. 10), tangent to the right lines *ce* and *ef* at the points d and f, respectively, these points being directly below the extremities of the 4-ft. portion, and a second parabola tangent to *fg* and *gi* at the points f and h.

The vertical shear diagram $a'k \ldots lm \ldots nb'$ is drawn next. The shear passes through zero where *lm* crosses $a'b'$, and on measuring the moment ordinate directly above this it is found to be 126 400 ft.-lb. If b be the breadth and d the depth of the rectangular section of the beam, and 875 lb. per sq. in. the working unit stress in the outer fibers,

$$bd^2 = (126\,400 \times 12 \times 6) \div 875 = 10\,400 \text{ in.}^3$$

If b be assumed as about $\frac{1}{2}d$, the beam will require two timbers 14 by 14 in. in section.

The weight of these timbers for a length equal to the span, at 3 lb. per ft. board measure, equals 1960, or say 2000 lb. The bending moment at the middle of the span due to this weight is 5000 ft.-lb. The corresponding moment diagram is drawn as explained in Art. 10 by making the parabola *arb* tangent to *as* and *sb*, the ordinate at s being $2 \times 5000 = 10\,000$ ft.-lb. The shear diagram $a'pqb'$ for the weight of the beam is added to the other by laying off the reactions in the opposite direction from the axis $a'b'$. The shear due to all the loads passes through zero at o where *lm* crosses *pq*, which is a little to the right of the point found before, and is 8.52 ft. from the left support. The maximum moment directly above this point is now measured, and its value of 131 300 ft.-lb. obtained. As the resisting moment of the beam slightly exceeds this amount no correction is necessary.

If S_h is the unit horizontal shear, and V the total vertical shear in any section of the beam, b and d the breadth and depth of the rectangular cross-section, the following relation is given by mechanics (Mechanics of Materials).

$$S_h = \frac{3}{2}\cdot\frac{V}{bd}$$

In order to obtain the horizontal shear for a distance dx along the beam, S_h must be multiplied by bdx, giving

$$S_h bdx = \frac{3}{2d}\cdot Vdx$$

If the total horizontal shear be required between any two sections of the beam, it is necessary to integrate this expression between the given limits of x. V is a function of x, and the integral of Vdx is the area of the vertical shear diagram between the given sections.

The total horizontal shear between a' (the left support) and o is $\frac{3}{2d}\cdot M_{max}$, since $\int Vdx = \int dM = M$. Substituting the values found above, the total horizontal shear for either the portion $a'o$ or ob' equals (3 × 131 300 × 12) ÷ (2 × 28) = 84 420 lb. If four keys or brace blocks of equal strength are to be employed to resist this shear, each block must be designed to take a pressure of 21 105 lb. In order to determine their location, it is necessary to divide the vertical shear diagram on each side of the zero shear into four equal parts. This is most readily done by dividing it into narrow strips, say a foot wide by scale, finding the area of each one, and beginning at the point o, adding each area to the sum of the preceding ones. The total area should equal the maximum moment. These areas are laid off as ordinates on the axis $a''o'$ in such a way that the length of the ordinate at any section of the beam represents the area of the vertical shear diagram from that section to the point of zero shear. By dividing the last ordinate $a''t$ into four equal parts and drawing parallels to $a''o'$ through these points as indicated by broken lines, their intersection with the curve passing through the extremities of the ordinates gives the positions required.

The corresponding diagram for the right-hand portion is drawn above the axis $o''b''$. All the positions of the brace blocks are marked on the bottom line of Fig. 62*a*. Numbers 1, 2, and 3 are respectively 4 ft. 9 in., 2 ft. 11 in., and 1 ft. $4\frac{1}{2}$ in. from 4, which is at the left support; while numbers 1′, 2′, and 3′ are 5 ft. $10\frac{1}{2}$ in., 3 ft. 5 in., and 1 ft. $7\frac{1}{2}$ in. from 4′ at the right support. It will be observed that in the middle portion of the beam no brace blocks are required in this case for almost one-half of the span.

When moving loads are substituted for stationary loads the length of the middle space is materially reduced, for in that case the maximum horizontal shear in any section does not occur when the load covers the entire beam except for the sections at the supports.

If the cross-section of the beam is not uniform, it is necessary to construct the diagram so that the ordinates shall represent the corresponding sums of the horizontal shears directly. By using the general form of the equation the distribution of the horizontal shear in a beam of any cross-section may be similarly shown by a diagram.

Problem 62*a*.—Two deepened beams having an effective span of 22 ft. carry a single-track railway across a culvert. The weight of the track is to be assumed at 450 lb. per linear ft., and the live load at 5000 lb. per linear ft. The beams are to be of timber weighing 45 lb. per cu. ft., and with an allowable unit stress in the outer fibers of 1500 lb. per sq. in. Find the positions of the brace blocks or keys, provided eight are used in each beam.

CHAPTER VII

ELASTIC DEFORMATION OF TRUSSES

Art. 63. The Displacement Diagram

The change in length λ of any member of a truss which is subject to given loads and reactions may be computed by the well-known formula (Mechanics of Materials)

$$\lambda = \frac{Pl}{AE}$$

in which l is the length of the given member, A the area of its cross-section, E the modulus of elasticity of the material of which it is composed, and P the total stress in the member. In case stresses due to temperature are to be taken into account the above value of λ must be combined with the quantity ϵtl, in which ϵ is the coefficient of linear expansion for a change of one degree, and t the rise or fall of temperature expressed in degrees.

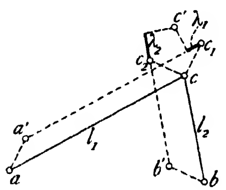


Fig. 63a.

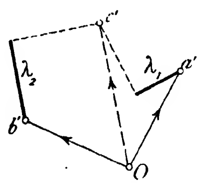


Fig. 63b.

As a truss is composed of triangles, the method of finding the displacement of its panel points due to any given loads may be illustrated by showing how to determine the displacement of one panel point when two others with which it is connected by truss members are known. Let the panel point c in Fig. 63a be connected with a and b by members whose lengths are l_1 and l_2, respectively. Let the stress in ac be a compression which produces a shortening of λ_1 in its length, while the stress in bc is tension and λ_2 is the corresponding elongation. The known magnitudes and directions of the displacements of a and b are represented by the lines aa' and bb'.

Let $a'c_1$ be drawn parallel and equal to ac. Since the stress in ac is compression, λ_1 must be laid off from c_1 towards the point a', which for the moment is regarded as fixed. This shortening, indicated by a heavy full line, is very much exaggerated in the figure, for if it were laid off to the same scale as $a'c_1$ it would not be visible. With a' as a center and the reduced length $l_1 - \lambda_1$ as a radius let an arc be described. The panel point c must lie somewhere on this arc. Because the elastic deformations of the truss members are, however, very small, the tangent to the arc may be substituted for the arc itself. A perpendicular to $a'c_1$ is therefore drawn at the end of the line marked λ_1. Similarly, $b'c_2$ is drawn parallel to bc, and its length increased by its elongation λ_2 and a perpendicular erected at its extremity. The point c' is therefore at the intersection of these two perpendiculars, and the line cc' (not drawn) represents the displacement of c in magnitude and direction.

In view of the exceedingly small values of λ as compared with l it is desirable to exclude from the diagram that portion which contains the lines representing the lengths of the members themselves. This can readily be done, as it is seen that the lines cc_1 and cc_2 representing the displacement of a and b, the lines λ_1 and λ_2, and the perpendiculars at the extremities of λ_1 and λ_2 form a closed polygon. In Fig. 63*b* it is drawn separately to three times the scale of that in Fig. 63*a*, the pole O in the former replacing the point c in the latter. The displacement of the panel points a, b, and c all radiate from the pole O.

Such a diagram is called a displacement diagram. In its construction especial care must be exercised in observing the directions in which the values of λ are to be laid off, constantly referring to the panel points of the truss diagram, which for the time being are considered as fixed. The lengths of the perpendiculars whose intersections locate the successive panel points need not be measured.

PROBLEM 63*a*.—A weight of 15 kips is suspended from a ceiling at points 10 ft. apart by means of two steel bars, one being 1 in. square and 9 ft. long, and the other $\frac{3}{4}$ in. square and 10 ft. long. Find the displacement of the point where the weight is attached to the bars.

ART. 64. DEFORMATION OF A TRUSS

It is required to find the displacements of the panel points of a wooden king-post truss whose span is 16 ft. and depth 8 ft., which carries a load of 12 000 lb. at panel point b (Fig. 64*a*). The data required for the construction of the displacement diagram are given in the following table. In computing λ the value of E was assumed as 1 500 000 lb. per sq. in. For the sake of illustration let the

Member	Stress, Lb.	Length, In.	Area, Sq. In.	λ, In.	Member, No.
$ab = bc$	+ 6 000	96.0	36	+0.0107	1 and 5
$aB = Bc$	− 8 490	135.7	64	−0.0120	2 and 4
Bb	+12 000	96.0	36	+0.0213	3

point *a* be fixed, and the point *c* be regarded as perfectly free to move horizontally, although in practice such a short span is fixed at both supports, since the horizontal movement of *c* due to the loads is too small to require a movable support.

The displacement diagram may be constructed by beginning at any panel point and regarding as fixed its own position as well as the direction of one member attached to it. Let the point *a* (which is actually fixed) and the direction of *ab* be so regarded. In Fig. 64*c* the point *a′* will therefore coincide with the pole *O*, and λ_1 will be laid off toward the right, that is, in the direction of *a* toward *b* on the truss diagram. For convenience the values of λ are marked on the truss diagram, and when they are laid off in Fig. 64*c* they are marked by the same numbers as the corresponding members in Fig. 64*a*. After *b′* is thus determined the displacement of *B* is next found by regarding *a* and *b* in the triangle *aBb* as fixed. The elongation λ_3 is laid off upward from *b′*, and the shortening λ_2 downward from *a′*, the intersection of the perpendiculars giving *B′*. In the same way *c′* is located. The lines *OB′* and *Oc′* are the displacements of *B* and *c*.

In order to show the deformed truss under the conditions assumed (that *a* and the direction of *ab* are fixed), the displacements are laid off on Fig. 64*a* to one-tenth of the scale employed in Fig. 64*c*, and the corresponding points joined by broken lines. The student will observe that the deformation shown is greatly exaggerated, and hence the members seem to have unduly altered their lengths.

The primary conditions of the problem, however, require that *c* shall move only in a horizontal line, and therefore the entire truss must be revolved about *a* as a center until c_1 falls into the horizontal through c. As the arc thus described is very small compared with the radius ac_1 which in turn differs but very little from ac, a perpendicular to ac from c_1 may be substituted for the arc without appreciable error.

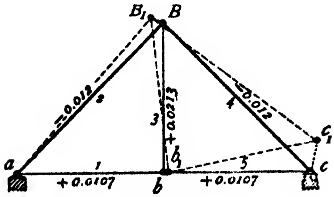


FIG. 64a.

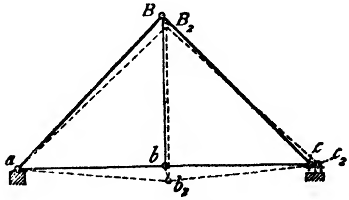


FIG. 64b.

In Fig. 64d, to which were transferred the displacements Ob', OB', and Oc' without the construction lines, the corresponding path of rotation of c is represented by $c'c'''$, which is drawn perpendicular to ac in Fig. 64a, and continued until it meets the line

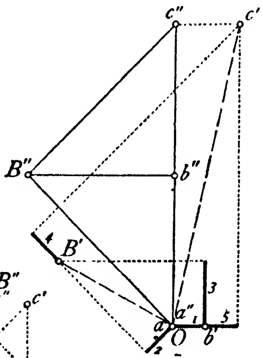


FIG. 64c.

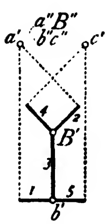


FIG. 64e.

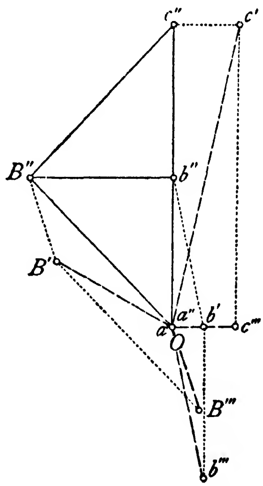


FIG. 64d.

Oc''', which is drawn parallel to the direction in which the panel point c is free to move. In this example that direction is horizontal, and happens to coincide with the line ac, but the statement here given is so framed as to apply equally to inclined lines of motion of panel points supported by expansion rollers or rockers.

When the successive displacements Oc' and $c'c'''$ are combined, the resultant displacement is Oc'''. The displacement of B due to the rotation of the truss is $B'B'''$, which is perpendicular to aB (in Fig. 64a), and whose length is proportional to its distance from the center a. That is, $B'B''' : c'c''' = aB : ac$, from which the length of $B'B'''$ may be conveniently found by similar triangles. Similarly, as ab equals one-half of ac, $b'b'''$ equals one-half of $c'c'''$. The resultant displacements are then represented in direction and magnitude by OB''', Ob''', and Oc''', and as a is the center of rotation, a''' coincides with a', and Oa''' is zero.

In Fig. 64b the final position of the deformed truss is shown in broken lines, the resultant displacements BB_2, bb_2, and cc_2 being laid off parallel to and equal to one-tenth of the lengths of OB''', Ob''', and Oc''' in Fig. 64d. If the deformation were not exaggerated the truss diagram $aB_2c_2b_2$ in Fig. 64b would be equal to $aB_1c_1b_1$ in Fig. 64a in both form and dimensions.

For the purpose of simplifying the construction let the three parallelograms in Fig. 64d be completed, and the points a'', B'', c'', and b'' joined by lines as indicated. The lines $B''a''$, $b''a''$, and $c''a''$ (parallel and equal to $B'B'''$, $b'b'''$, and $c'c'''$) represent the displacement of panel points B, b, and c due to rotation about a, and are respectively perpendicular and proportional to aB, ab, and ac of the truss diagram, and therefore it follows that the diagram $a''B''c''b''$ is similar to $aBcb$, and all their lines are mutually perpendicular. This important fact furnishes a means of determining the final displacements on Fig. 64c in a very simple manner as follows: Through c' draw $c'c''$ parallel to the constrained line of motion of the panel point c, and draw $a''c''$ perpendicular to ac in Fig. 64a, and intersecting $c'c''$ at c''. The line $c''a''$ represents the arc of rotation of c_1. On $a''c''$ draw a diagram similar to the truss diagram. The required displacements are then given by the directions and distances of the points B', b', and c' from B'', b'', and c'', respectively. It is thus seen that the points B'', b'', and c'' in Fig. 64c correspond to what may be regarded as successive positions of the shifted pole O in Fig. 64d, which conception aids the memory in reading the directions correctly.

It is very desirable in practice to reduce the displacement diagrams to their most compact form. It will both diminish the errors due to slight inaccuracies in the directions of the intersecting perpendiculars as well as allow increased precision by the use of a larger scale. This result may be secured by beginning the construction with the line which suffers the minimum change in direction under the influence of the given loads. In simple trusses some line may be found at the middle which does not change its direction at all, provided the truss and the loading are both symmetrical with reference to a vertical section at the middle, or which changes but little in unsymmetrical trusses. For a bridge truss with an even number of panels, the middle vertical is such a member, or the chords of the middle panel when the number of panels is odd. As an illustration of the effect thus produced, the displacement diagram for the above truss when the direction of the middle vertical and the position of either of its extremities is assumed to be fixed, is given in Fig. 64*e* to the same scale as that in Fig. 64*c*. Since a perpendicular to *ac* through *a′* meets a horizontal through *c′* at *a′*, the diagram *a″B″c″b″* is thereby reduced to zero. If the scale were doubled, the vertical dimension of this diagram would not be quite equal to that of the one previously drawn. With a larger number of panels the difference is still greater. In this case the diagram makes a direct comparison between the displacements of all the panel points.

On applying the scale and protractor to the original drawing the displacements of *B* and *b* were found to be 0.0296 and 0.0501 in., their directions being inclined $21\frac{1}{4}°$ and $12\frac{1}{4}°$ to the vertical. The angles were read only to the nearest quarter degree. As the lower chord is horizontal, the displacement of *c* is the sum of the elongations λ_1 and λ_5, which equals 0.0214 in.

In computing the change in length of a tension member it is customary to use the gross cross-sectional area, although the area is reduced for part of the length by rivet holes in riveted shapes or by mortices or other cuts in timber. Gusset plates and other connecting details increase the areas of the members where they occur and tend to compensate for the use of the gross sections of tension members.

PROBLEM 64*a*.—The members of the highway truss shown in Fig. 27*a* have the following cross-sectional areas in square inches: $AB = 45.90$, $AD = 37.30$, $AF = 40.90$, $AH = 43.40$, $BJ = CK = 17.58$, $EL = 23.27$, $GM = 27.64$, $BC = 11.72$, $DE = FG = HH' = 8.94$, $CD = 18.00$, $EF = 15.88$, and $GH = 11.44$. Use a value of $E = 29\,000\,000$ lb. per sq. in. Find the displacement of all the panel points of the truss due to the dead load: (1) by starting at the center of the truss, and (2) by starting at one end.

ART. 65. DEFLECTION AND CAMBER OF A TRUSS

While the displacement diagram gives the actual displacement of the panel points in the plane of the truss, their vertical components only are generally required. When bridge trusses are erected they are 'cambered' so that under their maximum load none of the panel points of the loaded chord will fall below a horizontal line joining the panel points at the supports. For highway trusses the camber is frequently increased so that under dead load the roadway will lie on a vertical curve which is tangent to the roadways of the approaches.

For long trusses this camber is secured by shortening the tension members by an amount equal to (or proportional to) the elastic elongation due to the sum of the live- and dead-load stresses, when the live load is so placed as to produce the maximum moment at the middle of the truss, plus an allowance for clearance in the case of pin-connected joints. The compression members are lengthened in a similar way. The maximum live-load stresses must not be employed throughout because they are not simultaneous stresses. For shorter spans the camber is usually obtained by increasing the length of the top chord members. This requires a corresponding increase in the length of the diagonals, but the verticals and bottom chord members are not changed. A common figure used for railway trusses is to increase the top chord $\frac{1}{8}$ in. for each 10 ft. of length. For highway trusses where a permanent camber is desired the elongation will have to be greater. The elongation may be computed from the following approximate formula, the panel points of each chord being assumed to lie on arcs of concentric circles,

$$E' = \frac{8CD}{SN}$$

in which E' is the panel increase in inches; C is the desired camber in inches; D is the depth of the truss in feet; S is the span of the truss in feet; and N is the number of panels.

A displacement diagram may be drawn for these changes of length of the members to determine the elevation of each panel point, or 'blocking diagram,' when the truss is erected on falsework. If to these changes of length there be added the values of λ caused by the dead load only, due regard being paid to their respective signs, and the corresponding displacement diagram drawn, the vertical components of the displacements will give the elevation of the several panel points when the bridge supports only the dead load, and their values may be used for comparison with the observed elevations after the false-work has been removed. Roof trusses supporting horizontal ceilings are cambered in a similar manner.

In the example used in the preceding article, the deflection of b is found to be 0.0489 and of B is 0.0276 in. The diagram shows that when two panel points are directly above each other their deflections should differ by the change in length of the member connecting them. This may serve as a useful test of the accuracy of the drawing.

PROBLEM 65*a*.—The truss in Fig. 27*a* was cambered by increasing the length of each top chord member $\frac{3}{4}$ in., and the length of the end post and diagonals $\frac{3}{16}$ in. each. Construct a displacement diagram and determine the blocking diagram for erecting the truss on false-work.

ART. 66. TRUSS DEFLECTION UNDER LOCOMOTIVE WHEEL LOADS

The most convenient way to find the stresses in the truss members whether the chords are both horizontal, or either one or both of them broken, is the following: Let the position of the live load be found which produces the maximum moment at the panel point at, or nearest to, the middle of the truss (Art. 45 or 51), and with the truss diagram in this position on the equilibrium polygon let the closing line be drawn as well as the chords of the polygon whose horizontal projections equal the successive panel lengths in magnitude and position. The extremities of these chords connect the points two and two where the verticals of the truss diagram intersect the equilibrium polygon. By drawing rays parallel to these chords through the pole they will cut off on the vertical load line the panel loads for this position of the wheel loads, and a ray parallel to the closing line will divide the reactions. The stress diagram, which is similar to that for dead load, may now be completed in the usual manner.

The following table gives the required data for determining the deflection of the double-track through bridge No. 77 of the second division of the Baltimore and Ohio Railroad, due to the specified live load of two B. & O. typical consolidation locomotives and train.

Member	Stress Kips	Length In.	Area Sq. In.	λ In.	Member No.
ab = *bc*	+287.0	321.0	36.00	+0.0882	11, 8
cd	+360.0	321.0	46.00	+0.0866	4
de	+356.0	321.0	46.00	+0.0856	14
ef = *fg*	+266.5	321.0	36.00	+0.0819	18, 21
BC	−369.0	329.0	65.12	−0.0717	6
CD = *DE*	−414.0	321.0	71.52	−0.0715	2, 12
EF	−365.0	329.0	65.12	−0.0709	16
aB	−413.0	464.7	76.44	−0.0966	10
Bb	+140.0	336.0	17.52	+0.1033	9
Bc	+106.0	464.7	23.56	+0.0804	7
Cc	+ 12.5	408.0	30.60	+0.0064	5
Cd	+ 87.5	519.1	34.00	+0.0514	3
Dd	0	408.0	20.60	0	1
dE	+ 94.5	519.1	34.00	+0.0555	13
Ee	+ 5.0	408.0	30.60	+0.0024	15
eF	+130.0	464.7	23.56	+0.0986	17
Ff	+106.0	336.0	17.52	+0.0782	19
Fg	−386.5	464.7	76.44	−0.0903	20

The form of truss is shown in Fig. 66*a*. The lower chord consists of eye-bars of medium open-hearth steel, whereas the upper chord and web members are built up of shapes of soft steel. There are no counters. The stresses were found in the manner described above, the stress diagram being drawn to a scale of 50 kips to the inch. The value of the modulus of elasticity was assumed as 26 000 000 for soft and 29 000 000 lb. per sq. in. for the medium steel.

The displacement diagram, shown in reduced size in Fig. 66*b* was constructed by assuming the position of *d* and the direction of *dD* as fixed. It is the best one that can be drawn since *dD* changes its direction actually less than any other member, and requires very little rotation of the truss. The truss members are numbered in the order in which their values of λ were used in constructing the diagram. As the left end of the truss is fixed and the right end rests on expansion rollers, a'' coincides with a', and g'' lies in a horizontal through g' directly above a'. The distance $a''g''$ is too small a span to permit a diagram similar to the truss diagram to be drawn without confusion of points. As, however, only the deflections are desired, the necessity for this diagram may be obviated by the following construction of a deflection polygon.

Let a_1 be obtained by projecting a' across on the vertical through a, and similarly for g_1. The intersection of the line joining a_1 and g_1 with any vertical, as for example that through c, gives a point c_2 whose height is the same as c'' would have been if the diagram $a'' \ldots g''$ had been drawn. By projecting $b'c'd'e'$

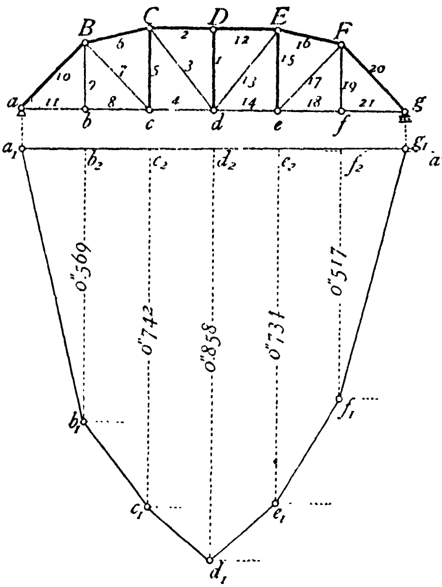


FIG. 66a.

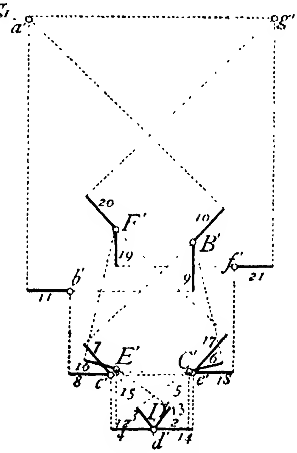


FIG. 66b.

and f' on the corresponding verticals and joining them as indicated, a polygon will be obtained whose ordinates at the panel points represent the corresponding deflections of the panel points of the lower chord. The values of the deflections in inches are marked on the diagram. The scale of the original displacement diagram was 0.060 in. to an inch. A deflection polygon for the panel points of the upper chord might also be drawn, if desired; but as these points are united to the lower chord by verticals, their deflections may be obtained by subtracting the elongations of the verticals from the corresponding deflections marked on the diagram.

PROBLEM 66a.—What change in the deflection of the truss used in this article would result by constructing all members of structural grade steel having a modulus of elasticity of 29 000 000 lb. per sq. in. and substituting built-up shapes for the eye-bar members of the lower chord having areas of 39.28 sq. in. for *ab*, *bc*, *ef*, and *fg*, and 49.86 sq. in. for *cd* and *de*?

ART. 67. SPECIAL CONSTRUCTION FOR CENTER PANEL

When a simple truss has a center panel and the loading is symmetrical both of the diagonals in that panel have no stress and hence no linear deformation. If either one of the diagonals is omitted the resulting deflection polygon is not quite symmetrical. If the other diagonal is omitted the deflections of corresponding panel points are interchanged and hence the true deflection of each panel point is the mean of these two values. The necessity of constructing a displacement diagram for more than half of the

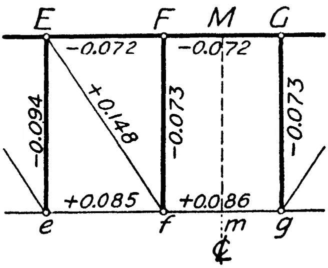


FIG. 67*a*.

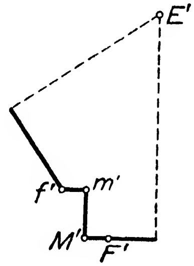


FIG. 67*b*.

truss may be avoided by means of the following expedient: Let an imaginary vertical be inserted in the middle of the center panel, with a shortening equal to that of the vertical on each side of the panel. The vertical will not change its direction when the truss deflects under symmetrical loading. Hence the displacement diagram is drawn by taking the direction of this vertical as fixed. If the diagram were drawn for the entire truss it would be symmetrical and the perpendicular truss diagram (which represents the rotation of the given truss, if required) would be reduced to a point as in Fig. 64*e*. Hence the displacement diagram for only one-half of the truss is required.

In Fig. 67*a* the imaginary vertical *Mm* joins the middle points of the upper and lower chord members of the center panel. Taking *m* and the direction of *Mm* as fixed, the point *m'* in Fig. 67*b* is located in any convenient position. From *m'* the deformation of *Mm* (equal to that of *Ff*) is laid off parallel to *Mm*. It is laid off downward since *M* moves downward toward the fixed end *m*, when the member is shortened.

Moreover, since *FG* does not change its direction when the truss deflects, the point F' may be located directly by laying off toward the right from *M* and parallel to *FM* (or *FG*) one-half of the deformation of the chord *FG*. Since M' was located before F', *M* is regarded now as fixed; as the member is shortened, *F* moves toward the right, and hence the change of length is laid off toward the right from M'. For the same reason $m'f'$ is drawn parallel to *fm* (or *fg*) and equal to one-half of the elongation of *fg*. It is laid off toward the left as *f* moves toward the left from the fixed end *m* when the chord elongates. The rest of the construction is the same as was fully explained in Art. 64, only a small portion of which is shown in Fig. 67*b*.

PROBLEM 67*a*.—Referring to Fig. 22*b*, which member must be taken as fixed in direction in order to secure a symmetrical displacement diagram? Must the entire diagram be constructed?

CHAPTER VIII

INFLUENCE LINES FOR STRESSES

ART. 68. INFLUENCE LINES FROM A DEFLECTION DIAGRAM

IN Art. 33 influence lines were constructed for various functions by finding the value of the function at one or more positions of a load P, laying them off as ordinates, and joining their tops. Another method of constructing a stress influence line is by means of a deflection diagram for the truss due to a unit change in length of the given member. Referring to the truss in Fig. 68a let a load P be applied at panel point e, and let Δ be the deflection of

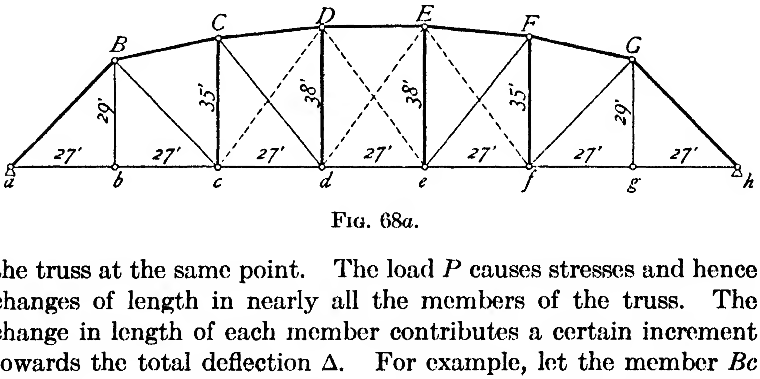


FIG. 68a.

the truss at the same point. The load P causes stresses and hence changes of length in nearly all the members of the truss. The change in length of each member contributes a certain increment towards the total deflection Δ. For example, let the member Bc be taken. Let S be its stress due to the load P, let λ be its corresponding change in length or its linear deformation, and let δ be the increment of deflection at e due to the change in length of Bc only. By equating the external and internal work (see Part I) there is obtained the equation

$$\tfrac{1}{2}P\delta = \tfrac{1}{2}S\lambda, \quad \text{or} \quad P\delta = S\lambda$$

Now if P be made equal to a unit load, and λ be made equal to a linear unit, the equation becomes

$$1 \cdot \delta = 1 \cdot S$$

which shows that the numerical magnitude of the deflection equals that of the stress S, provided each is expressed in the proper units. The same relation exists for any position of the load. It follows, therefore, that the deflection diagram of the truss due to a unit change in length of the given member is identical with the influence diagram for the stress in the member.

To construct the deflection diagram which is to be used as a stress influence diagram the given truss member is assumed to have a unit linear contraction, while all the other members remain unchanged in length. The member is assumed to be shortened instead of lengthened in order that positive ordinates, that is, those which are measured upward from the closing line or axis, may denote tension, while negative ordinates, or those measured downward from the closing line, may denote compression.

The deflections of the panel points of the truss are found by means of a displacement diagram as explained in Arts. 63, 64, and 65. The displacement diagram may be constructed by considering both the position of any joint of the truss and the direction of any member attached to it as temporarily fixed. In order to make the diagram as simple as possible for members in the left half of the truss, it is best to commence at a joint at the right end of the panel in which the member is situated, and to regard the position of this joint and the direction of a vertical member attached to it as fixed. For members in the right half of the truss it is best to begin at a joint at the left end of the panel containing the member.

As explained in Art. 64, the final displacement of each joint of a truss is the resultant of two component displacements. The first of these is due to changes in the lengths of the truss members under an assumed condition that the position of a certain joint and the direction of a member attached to it are both fixed. Since this assumed condition often does not agree with the actual limitations of the movements of certain truss joints it becomes necessary to rotate the truss about a certain joint to make it conform to these limitations. The second component displacement of each joint is derived from this rotation of the truss. The graphic methods for determining these two sets of displacements were introduced, respectively, by WILLIOT and MOHR, and hence the combined displacement diagram is often called a WILLIOT-MOHR diagram.

The method of constructing stress influence lines by means of displacement diagrams as described in this article is absolutely general, applying to any bridge truss with constant or variable depth, and with any kind of web system. By its use the necessity of remembering many different rules for constructing influence lines for different chord or web members is, therefore, avoided. Stress influence lines may accordingly be drawn without previously requiring an analysis of stress relations in various members of a truss, thus making this method especially useful in determining the variations of stresses in the members of new or uncommon types of trusses.

The general method of developing the subject of influence lines from the viewpoint of their identity with deflection diagrams, and some applications of this principle were first published in an article, "Influence Lines as Deflection Diagrams," by D. B. STEINMAN, in *Engineering Record*, Vol. 74, p. 648, Nov. 25, 1916.

PROBLEM 68*a*.—Construct the influence diagram for the chord member *ab* in Fig. 64*b*.

ART. 69. INFLUENCE LINES FOR A PARKER TRUSS

In Fig. 69*a* are shown influence lines for the stresses in an upper chord member, lower chord member, main diagonal, counter diagonal, and a vertical in a Parker truss, as well as for a vertical under a special condition imposed upon an adjacent diagonal. The Parker truss is the same one used as an example in Chapter V, its diagonals taking only tension. Its dimensions and loading are given in Art. 51.

The displacement diagram shown at the left of the stress influence line for *cd* is constructed by the method given in Art. 64 as follows: The member *cd* is assumed to be shortened a linear unit or 1 in., while the lengths of all the other members remain unchanged. The position of the panel point *d* and the direction of the vertical *dD* are assumed to be fixed. Since there is no deformation in *dD* the point *D'* on the displacement diagram will coincide with the point *d'*, which was first located at any convenient position on the drawing paper. From *D'* the deformation in the member *DC* is laid off equal to zero and a line drawn at its extremity (also *D'*) in a direction perpendicular to the member *DC*; while from *d'* the deformation of the member *dC* is laid off equal to zero and a line drawn at its extremity (also *d'*) in a direction perpen-

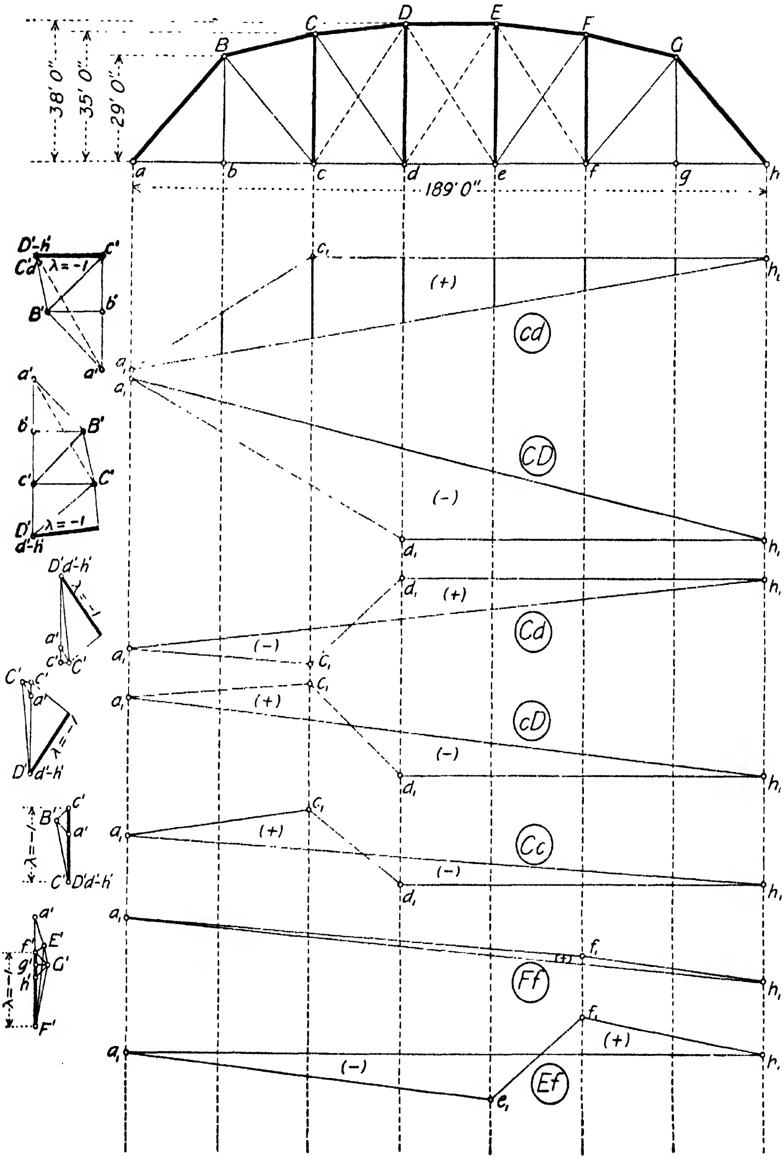


FIG. 69a.

dicular to dC; the intersection of these two perpendiculars gives the location of the point C'. In this particular case, since D' coincides with d', the point C' also coincides with both. For the same reason E', e', F', f', G', g', and h' coincide with d'.

To locate c', the deformation of Cc and of cd must be laid off in the proper directions, perpendiculars erected at their extremities and their intersections found. The zero deformation of Cc is laid off from C', and at its extremities (also C') a line is drawn perpendicular to the member Cc. Since the member cd is shortened, its left end c will move toward the right when its right end d is held in position, hence the deformation $\lambda = 1$ in. is laid off by scale to the right from d' in the displacement diagram and parallel to the truss member cd, and at its extremity a line is drawn perpendicular to it. The point c' is located at the intersection of both perpendiculars. At C' the zero deformation of BC is laid off and at its extremity a perpendicular to BC is drawn; from c' the zero deformation of Bc is laid off and a perpendicular to Bc is drawn; their intersection gives the point B'. In a similar manner the points b' and a' are located. It must be remembered that each perpendicular referred to above represents an arc of rotation for one end of a truss member about the other end whose corresponding point in the displacement diagram has been previously located (see Art. 63).

The deflection diagram may be drawn next by projecting the points a', b', c', . . . h' horizontally across to the corresponding verticals drawn through the panel points a, b, c, . . . h, of the truss, and joining these points as shown. The line a_1h_1 forms the closing line or axis of the diagram. This closing line takes the place of the auxiliary truss diagram whose members are respectively perpendicular to those of the given truss, when only vertical deflections are required. If the value of any deflection is desired the ordinate must be measured by the same scale as that used in laying off the shortening of the member cd. The ordinate below c measures 1.102 in. According to the relation developed in the preceding article the deflection diagram is also the influence diagram for the stress in cd. If a load of 1 kip is applied at c the stress in cd is therefore 1.102 kips.

If there are several panels to the left of the one containing the member for which an influence line is to be constructed, a short cut may be employed to avoid locating all the corresponding points on the displacement diagram. For example, in the displacement diagram for the shortening of *cd*, after the points C' and c' are located, the point a' may be located next by considering the panel point *a* to be connected directly to *C* and *c* by two members *aC* and *ac*. These members have zero deformations since the members replaced by them had no changes in length. Hence $C'a'$ is drawn perpendicular to *Ca* and $c'a'$ perpendicular to *ca*, thus locating a' by their intersection. If preferred, this method may be used to check the location of a' when determined by the method previously described.

In a similar manner, the influence lines for the members *CD*, *Cd*, *cD*, and *Cc* are constructed. In the diagram for *Cd* the position of the point of division *o* is located where the line c_1d_1 crosses the closing line a_1h_1. The diagram also shows that any load on the left of the point of division causes compression provided there is no counter in the panel and *Cd* is designed to take both compression and tension. Similarly the diagram for *Cc* shows that any load on the left of its point of division causes tension in the member provided the main diagonal *Cd* is acting, since the displacement diagram was constructed for this condition.

The greatest stress in *Cc* occurs when the live load covers the portion of the span on the right of the point of division. For this loading the diagonal *Cd* is acting. All points of division should be checked by the method given in Art. 48.

The sixth influence diagram in Fig. 69*a* was drawn for the corresponding vertical *Ff* in the right half of the truss when the counter diagonal *Ef* is assumed to act. The displacement diagram was drawn by assuming the position of *f* and the direction of *Ff* to be fixed. It would have been a little simpler in form if *e* and the direction of *eE* had been assumed as fixed. The line a_1f_1 would then be horizontal. The influence diagram shows that any load on the span causes tension in *Ff* under the condition assumed for its construction; that is, provided the diagonal *Ef* is acting, or when *Fe* is not acting. The influence line for *Ef* is placed directly below that for *Ff*.

If in any given case it is desired to secure a higher degree of precision than can be obtained from the measured ordinates of an influence line constructed by means of a displacement diagram, the stresses represented by the maximum positive and negative ordinates may be computed by the analytic method. For example, let it be required to find the value of the ordinate at f_1 for the vertical Ff. Referring to the dead-load stress diagram on Plate IV, as well as to the live-load stress triangle at the upper chord U_5, it will be seen that the most convenient method of computing the stress in the vertical when the counter acts on its left and the main diagonal on its right, is to find the ratio of its stress to the horizontal components of the stresses in the adjacent upper chords. This ratio is one-ninth, as may be seen by referring to Art. 51, in which the lengths of the lower chords and verticals are given. If a line be drawn through E parallel to FG it will intersect Ff at a distance of 3 ft. below F, which is one-ninth of the panel length of 27 ft. For a load of 1 kip at f the left reaction is $\frac{2}{7}$ kip. Taking the moment of the reaction about f and dividing by the length of Ff the horizontal component of the stress in EF or FG is found to be $\frac{270}{245}$ kips. The stress in Ff is therefore one-ninth of this or $\frac{30}{245} = 0.1225$ kip.

PROBLEM 69a.—Construct the influence diagram for the hanger Bb in Fig. 69a.

ART. 70. STRESSES IN A PARKER TRUSS

The value of any ordinate of a stress influence diagram for a given truss member when measured by the proper scale of loading gives the magnitude of the stress when a load of 1 kip or of 1 lb. occupies the corresponding position. If the specified live load on the truss consists of equal panel loads, it is only necessary to measure all the ordinates below the loaded panel points, and to multiply their sum by the value of a panel load.

For example, let the dead-load stress in cd be found from the influence line in Fig. 69a. The ordinate at c_1 measures 1.102 kips. The sum of all the ordinates below the panel points is $3.5 \times 1.102 = 3.857$ kips, and since the dead panel load is 43.2 kips the stress equals $43.2 \times 3.857 = + 166.6$ kips. This agrees with the value given on Plate IV.

However, if the panel loads are unequal then the magnitude of each ordinate must be multiplied by the corresponding panel load, and the sum of these products obtained. Due regard must be paid to whether the ordinates are positive or negative.

When locomotive axle loads are specified it is not necessary to determine the panel loads for any given position of the live load, since the same result will be obtained by measuring the ordinate at the position of each axle load, multiplying its magnitude by the corresponding axle load and adding the products. Since locomotive axle loads are usually divided into several groups of equal loads, the number of products may be reduced by adding the values of the ordinates for each group of loads and multiplying the sum by the value of one of the axle loads in that group (see Arts. 33 and 41).

If a uniform train load covers a part of the span the corresponding stress is found by computing the partial area of the influence diagram for the given member, and multiplying this by the load per linear foot, as illustrated in Art. 41.

The form of the influence diagram shows how to determine the position of axle loads graphically by stretching a thread when a tracing of the truss diagram is shifted over a stepped load line as illustrated in Fig. 49*b* for the diagonal *Cd*. The position of the thread is shown by the broken line *og*, and it cuts the stepped load line at the point *i*. Referring now to the influence line in Fig. 69*a*, the points *o*, *i*, and *g* in Fig. 49*b* correspond in position to the points *o*, d_1, and h_1. The influence line for *cd* shows that the load must cover the entire span, an axle load must be above the panel point *c*, and the thread which joins the two points of the stepped load line which are on the verticals through the end supports must cut the step above *c*, as illustrated in Fig. 39*c*. The points *a*, *i*, and *d* in that figure correspond to a_1, c_1, and h_1 on the influence line. Whenever the influence line on one side of the closing line or axis forms a triangle the position of the axle loads may be similarly determined by stretching a thread.

Let it be required to find the largest tension in *Ff* which will be the minimum stress in *Cc* as well as in that of *Ff*. For a load of 1 kip at *f* the tension in *Ff* is 0.1225 kip (see Art. 69), which equals the measured value of the ordinate at f_1. The sum of all the ordinates under the panel points is 3.5 times the ordinate at f_1, and hence the dead-load stress is $3.5 \times 0.1225 \times 43.2 = +18.5$ kips, the panel loads being 43.2 kips. On Plate IV the value of this stress is given as +18.4 kips. Since 10.8 kips of the total dead panel load is applied at the upper panel point, this stress must be corrected, making it $+18.5 - 10.8 = +7.7$ kips.

The influence line for *Ff* shows that its greatest tension would occur when the live load covers the entire span, provided the counter *Ef* were acting, since this condition was assumed in constructing its influence line. As *Ef*, however, cannot take compression, the greatest tension in *Ff* will occur when the live load comes on from the right and extends just far enough to reduce the total stress in *Ef* (and also *eF*) to zero. By the method employed in Chapter V the required position was found to be that which placed axle 1 at a distance of $3\frac{1}{2}$ ft. to the left of panel point *e*, as stated in the fourth paragraph of Art. 54. For this position the stress in *Ef* is found as follows, by means of its influence line in Fig. 69*a*. The ordinates at e_1 and f_1 are 0.5808 and 0.5007 kip, respectively. The sums of the positive ordinates under the pilot, driver, and tender axles, respectively, are 0.264, 0.682, and 1.650 kips, and the stress due to these loads is $15 \times 0.264 + 30 \times 0.682 + 19.5 \times 1.650 = +56.6$ kips. The negative ordinates under the pilot and driver axles are 0.562 and 0.601 kip, and the stress is $15 \times 0.562 + 30 \times 0.601 = -26.5$ kips. The total live-load stress is $+56.6 - 26.5 = +30.1$ kips. The dead-load stress obtained by the influence line is $(-1.452 + 0.751)\,43.2 = -30.3$ kips. The combined live- and dead-load stress is therefore $+30.1 - 30.3 = -0.2$ kip, showing that the stress in the counter is reduced practically to zero. This position of the loads makes the stress in the vertical *Ff* equal to $15 \times 0.160 + 30 \times 0.554 + 19.5 \times 0.404 = +26.9$ kips. Adding the dead-load stress the total stress is $+26.9 + 7.7 = +34.6$ kips. This checks within 0.1 kip the combined stress of +45.3 kips, given in the fourth paragraph of Art. 54, before the correction of −10.8 kips was applied on account of the division of the dead panel load between the upper and lower panel points. A value of +36.2 kips was obtained in the latter part of Art. 54 for a different loading.

If the assumed position of the live load does not reduce the stress in *Ef* to zero, the correct position may be found by trial. For example, let the axle loads be moved $\frac{1}{2}$ ft. to the right. The live-load stress in *Ef* becomes $+56.8 - 25.3 = +31.5$ kips and the sum of the live- and dead-load stresses is $+31.5 - 30.3 = +1.2$ kips. Therefore, the loads should have been moved only about 0.1 ft.

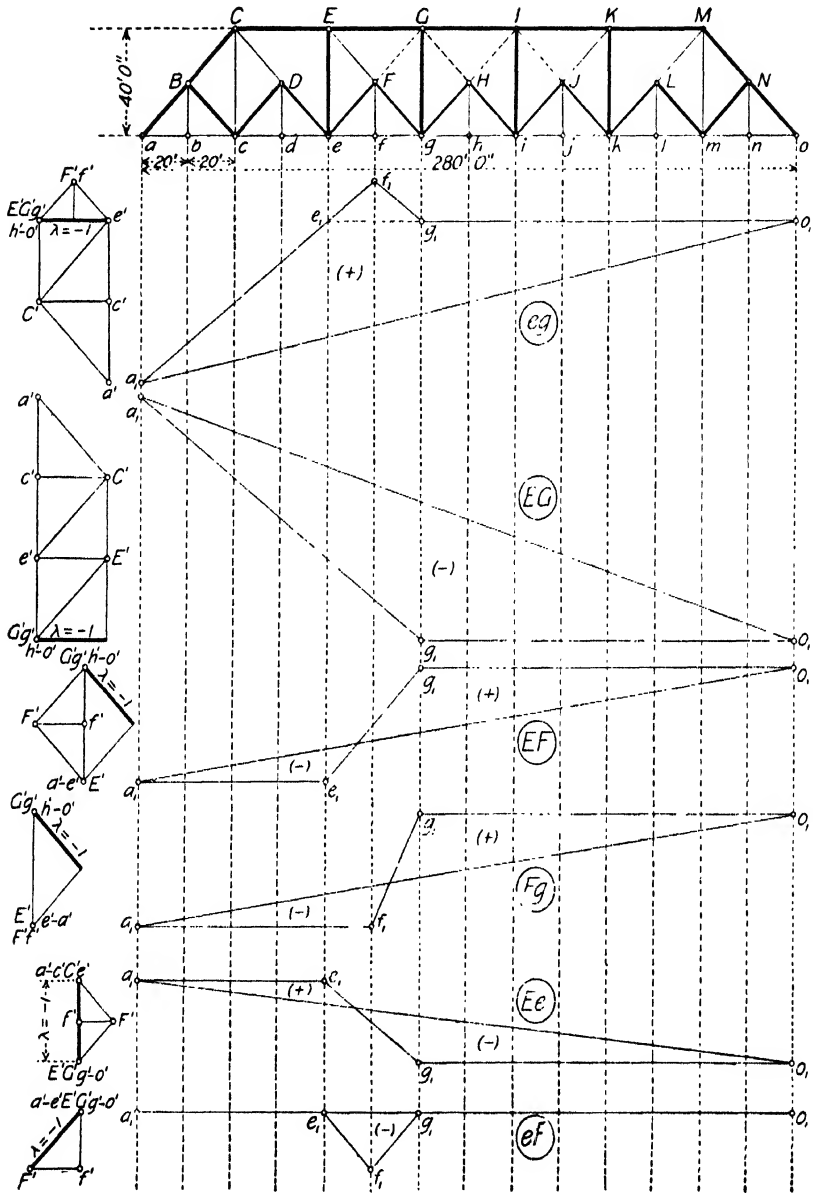
An approximate value of the minimum stress in *Ff* may be obtained as follows: Let a stress polygon be constructed for the member meeting at the joint *F* for an assumed live-load compression in eF equal in magnitude to its dead-load stress of +28.7 kips. This polygon gives a stress Ff of +49.0 kips. The dead-load stress in Ff when eF acts and the proper correction is made for a part of the panel load being applied at the upper panel point is −6.2 − 10.8 = − 17.0 kips. The combined stress is +49.0 − 17.0 = + 32.0 kips, which is 4.2 kips less than the correct minimum, without impact, given in the table in Art. 54.

PROBLEM 70*a*.—By means of its influence line check the greatest tension in Ff for the position of the live load illustrated in Fig. 54*d*, as well as the corresponding minimum stress.

ART. 71. INFLUENCE LINES FOR BALTIMORE TRUSS

Figure 71*a* shows the stress influence lines for most of the members in the subdivided panel from e to g, and the displacement diagrams by means of which they are constructed. It will be observed that the influence diagrams for EG, EF, and Ee are exactly the same as if the panels were not subdivided, or like those of a Pratt truss with seven panels. That for Fg has the same form as if the truss were a Pratt truss with fourteen panels. The influence line for eF has the same form as if the secondary truss eFg were acting independently. The influence line for the short hanger Ff has the same form as that for eF except that the vertical ordinate at f_1 measures 1 kip by the scale.

In constructing the displacement diagrams, the point g and the direction of Gg are assumed to be fixed as recommended in Art. 68. Since their general construction was explained in Art. 69, only those features are given special attention in this article which depend upon the effect of subdivided panels. After locating the points g' and G', the points E' and e', representing panel points of a large or primary panel of the truss, are to be located before F' and f' which represent panel points of the secondary truss. In the displacement diagram for EF it will be noted that, since there is no deformation in Fg, the deformation in Eg is also $\lambda = -1$ in., and hence E' is located as readily by laying off deformations from g' and G' and drawing perpendiculars at their extremities as though the intermediate panel point F did not exist. As indicated in Fig. 71*a* it is not necessary to locate the points b' and d' for the secondary trusses on the left of the panel containing the member whose linear deformation of 1 in. is laid off.


C
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(+)
eg
EG
(-)
(+)
EF
(-)
(+)
Fg
(-)
Ee
(+)
(-)
eF
(-)
λ = -1

Fig. 71a.

The influence diagram for *eg* shows the effect of subdividing the panel *eg*. If the line o_1g_1 is produced toward the left it intersects the line a_1f_1 at e_1, directly below the panel point *e*. Accordingly, the diagram may be regarded as a combination of the two triangles $a_1e_1o_1$ and $e_1f_1g_1$. The former is the influence line for the chord member *eg* when the panel *eg* is not subdivided, while the latter is the influence line for *eg* as a member of the secondary truss *eFg*, thus indicating that the stresses for these two conditions may be obtained separately and added together, if desired. When locomotive axle loads are employed the same position of the loads must be used in both causes. A criterion for the correct position of the loads may be found by deducing a formula in a similar manner to that employed in Part I. In this case it is desirable to use separate resultants for the loads from *a* to *e*, *e* to *f*, and *f* to *g*. The criterion is as follows:

The load from *a* to *f* minus the load from *f* to *g* must equal $Wl' \div l$, in which *W* is the entire load on the truss, l' the distance from the left reaction to the center of moments, and *l* the span of the truss. For the analytic computations of the stress, the section would be passed vertically between *f* and *g*, hence the left-hand member of this equation may be expressed in more general terms as the load in the panels on the left of the section minus the load in the panel cut by the section.

To satisfy this criterion the load must practically cover the entire span, an axle load being placed at *f*. It is also desirable to bring the heavier loads as near to *f* as possible while the preceding condition remains fulfilled. The graphic method of applying this criterion is as follows: Place the tracing containing the truss diagram over the sheet containing the stepped load line. Let a_2 be the intersection of the vertical at the left support *a* with the load line, and o_2 the corresponding point over the right support *o*. Let g_2 be the intersection of the vertical at *g* with the load line, and e_3 the point where the vertical at *e* intersects a thread stretched from a_2 to o_2. Next stretch the thread from g_2 through the top of the step in the load line above *f* and mark the point where it crosses the vertical at *e*; mark a second point after passing the thread similarly through the bottom of the step. If e_3 lies between the last two points the criterion is satisfied.

To construct the influence diagram for a counter diagonal *FG*; or for *eF* when the counter *FG* is acting, and the main diagonal *EF* is not acting; or for *Gg* when either *FG* or *GH*, or both, are acting; it is desirable to re-draw a portion of the truss diagram in accordance with the proper conditions, and then construct the displacement diagram to correspond with it. To attempt to do so without re-drawing the truss diagram involves too large a risk of errors in construction.

If the Baltimore truss in Fig. 71*a* had sub-diagonal ties instead of struts it would change the form of several influence diagrams. The effect of this change is illustrated in the next article, which gives the stress influence diagrams for a Pennsylvania truss.

PROBLEM 71*a*.—Draw the displacement diagram and influence line for the stress in the hanger *Cc*.

ART. 72. INFLUENCE LINES FOR PENNSYLVANIA TRUSS

The influence diagrams shown in Fig. 72*a* are constructed for most of the members in one panel of a Pennsylvania truss corresponding to those of a Baltimore truss as illustrated in the preceding article. In this case, however, the sub-diagonals act as ties. The differences in the forms of the displacement diagrams are due both to the curved upper chord, and the kind of stress for which the sub-diagonals are designed. The influence line for the lower chord *eg* has the simple triangular form which is the same as if the truss panels were not subdivided. That for the upper chord member *EG* forms a combination of the two triangles $a_1g_1p_1$ and $e_1f_1g_1$, since the secondary truss carries the panel load at *f* to the panel points *E* and *G*.

The criterion for loading for the greatest live-load stress in *EG* is indicated by the following formula:

The load from *a* to *e* plus twice the load from *e* to *f* equals $Wl' \div l$. The left-hand member of the equation may be expressed in more general terms as the load in the panels on the left of the section plus twice the load in the panel cut by the section. In this case the vertical section is passed between *e* and *f* and the center of moments is at *g*. To satisfy the criterion an axle load must be placed at *f*. Using a notation similar to that employed in the preceding article the graphic method of applying this criterion is as follows: Let a_2, e_2, and p_2 be the respective points of intersection with the stepped load line of the verticals from *a*, *e*, and *p*. Let g_3 be the point where the vertical at *g* intersects a thread stretched from a_2 to p_2. Next stretch the thread from e_2 through the top of the load line step above f and mark its intersection with the vertical at g; mark a second point after changing

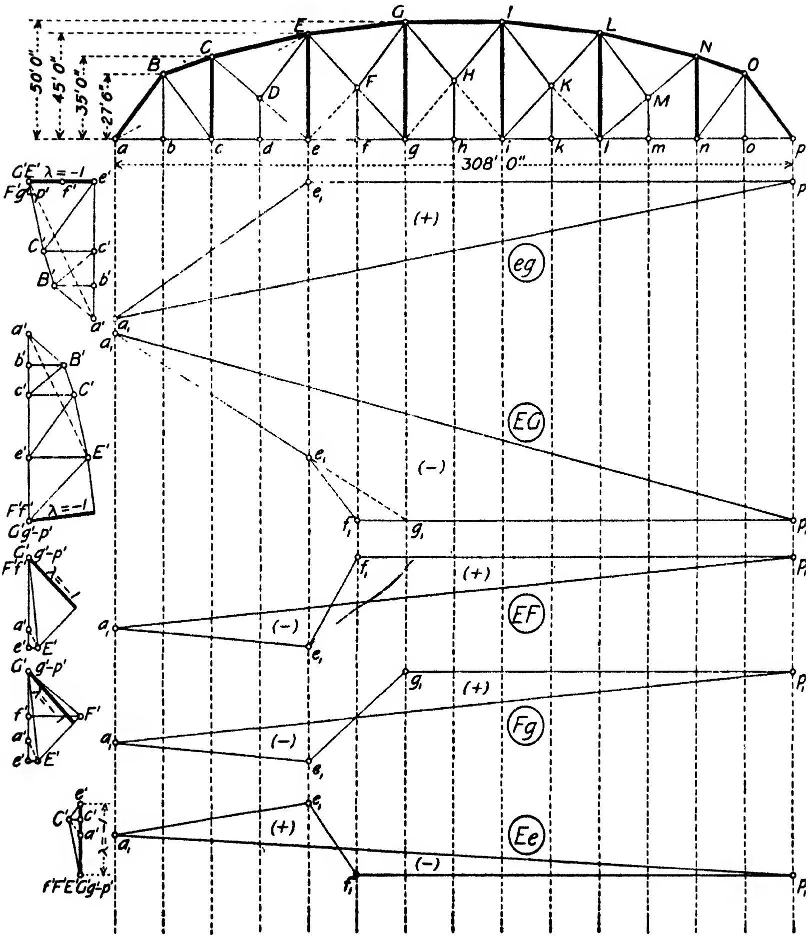


FIG. 72*a*.

the thread from e_2 through the bottom of the step. If g_3 falls between the last two points the criterion is satisfied.

In finding the stress in *GI* located in the middle panel of the truss it is impossible to tell in advance whether the counter diaggonal *Hg* or *Hi* is acting. Assuming *Hi* to act the load is placed in position to satisfy the criterion given in the preceding paragraph. If it then be found that *Hg* acts for this position of the loads, the loading must be tested by the criterion given in the preceding article (for *eg* of the Baltimore truss in which the section is also passed through the panel *fg*), and another determination made as to whether *Hg* still acts or not.

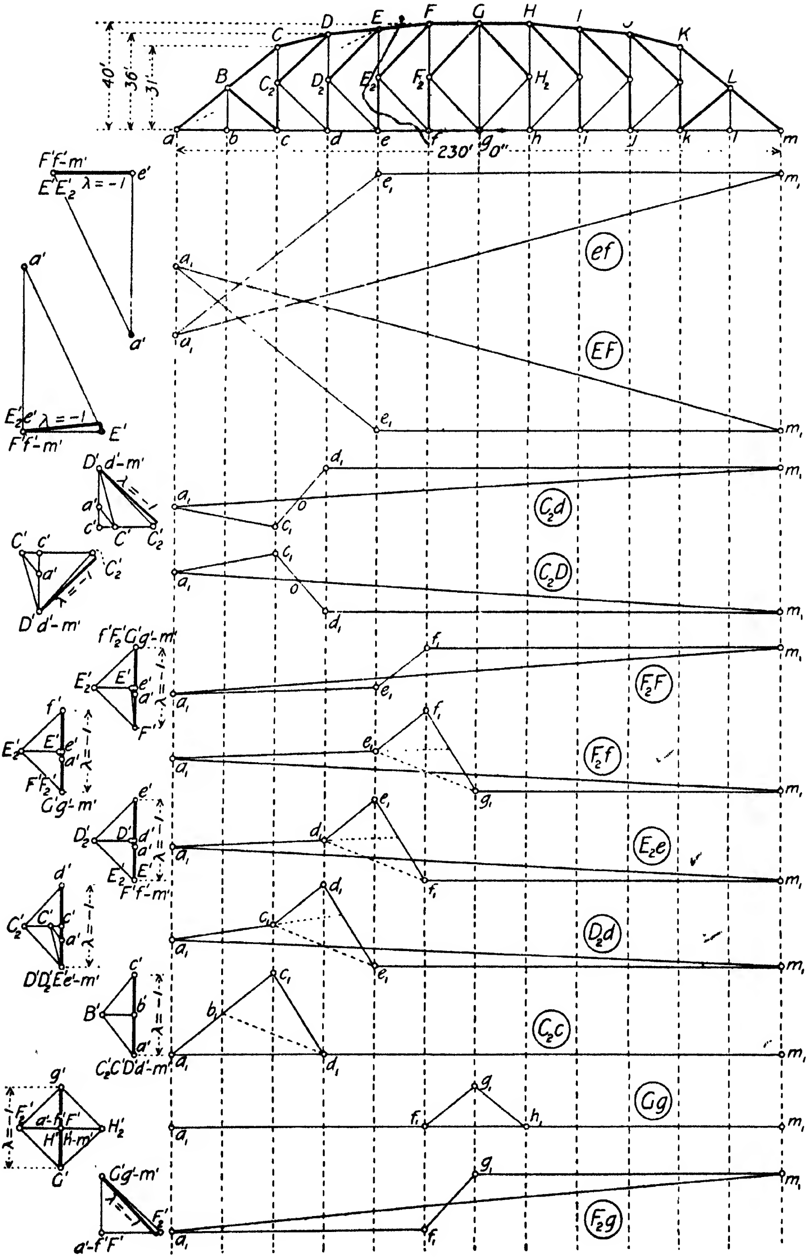
Referring to the influence lines for the diagonals *EF*, *Fg*, and the vertical *Ee*, it will be observed that the horizontal projection of the middle influence line occupies one, two, and one panels, respectively, whereas for the Baltimore truss in which sub-struts were used, it occupies two, one, and two panels, respectively, or just the reverse arrangement.

PROBLEM 72*a*.—Construct the displacement diagram and influence line for *Ee* when the counter *Fe* on its right is acting, and check the three larger ordinates by computation.

ART. 73. INFLUENCE LINES FOR THE K TRUSS

The skeleton diagram of the K truss in Fig. 73*a* represents the trusses in the bridge of the Atchison, Topeka, and Santa Fe Railway over the Arkansas River at Pueblo, Colo. It is the first simple truss span constructed in this country having the K form of web system, and was erected in 1915. A brief description of it was published in *Engineering News*, Vol. 76, p. 104, July 20, 1916. A half-tone illustration is given in Fig. VIII*a*. K trusses were used in two bridges by the Pittsburgh and West Virginia Railway, for which a considerable saving in cost was obtained over what would have been required for Warren or Baltimore type trusses. Illustrations and a brief description were published in *Engineering News-Record*, Vol. 106, p. 553, April 2, 1931.

The displacement diagrams are constructed in the same manner as those in the preceding articles of this chapter. The direction of the vertical at the right end of the panel which contains the member and one of the extremities of that vertical are assumed to be fixed. For any of the vertical members it is necessary to extend the usual construction from panel point to panel point for a full panel to the left of the member before the end panel point can be located directly by means of an imaginary diagonal. For example, in the displacement diagram for F_2f, the position of



A B C D E F G H I J K L
C2 D2 E2 F2 H2
a b c d e f g h i j k l m
40'
36'
31'
230' 0"
ef
EF
C2d
C2D
F2F
F2f
E2e
D2d
C2c
Gg
F2g
λ = -1

FIG. 73a.

g and the direction of gG are assumed as fixed; then the points g', G', F'_2, F', f', E'_2, e', and E' are located successively; and finally a' is located by inserting an imaginary diagonal Ea, $E'a'$ being perpendicular to Ea, and $e'a'$ perpendicular to ea.

The influence lines for ef and EF show the characteristic triangular form for all the chord members, and in this respect are like those of the Parker truss in Fig. 69*a*. The diagrams show that the centers of moments for both chords in the same panel lie in the same vertical. Since the upper chord member FG is horizontal, the magnitude of its stress for any loading is the same as that of the lower chord member fg in the same panel.

The influence lines for C_2d and C_2D give the typical form for all the diagonals except those in the first two panels, which have influence lines like the corresponding diagonals in the Baltimore truss. The influence diagrams for C_2d and C_2D have exactly the same ordinates, since the inclinations of these members are equal. Their stresses, however, are opposite in character. Their points of division o may be checked in the same manner as those of the diagonals in the Parker truss. The truss was designed so that the two diagonals in each panel have the same slope. Since the horizontal components of the diagonals in each panel must be equal, it is preferable to incline them equally and thus make the magnitudes of their stresses equal for any kind of loading.

The influence diagrams for D_2D and E_2E are similar in form to that shown for F_2F, the upper part of a vertical. It will be observed that its point of division is in the panel on its left instead of on the right as in the Parker truss. This is due to the fact that both the diagonals adjacent to F_2F slope downward to the left.

The most interesting influence diagrams for this truss are those for the lower parts of the verticals, and hence all of them are given in Fig. 73*a*, viz.: C_2c, D_2d, E_2e, and F_2f. In all these except C_2c, which is directly below the hip C of the truss, there is a break in the influence line on the left of the corresponding member similar to one of the lines for eg in Fig. 71*a*, and for EG in Fig. 72*a*. The dotted lines sloping downward to the right are drawn to show an interesting fact that the middle ordinate in the triangles $b_1c_1d_1$, $c_1d_1e_1$, $d_1e_1f_1$, and $e_1f_1g_1$ are all equal and measure 0.75 kip. This indicates that the lower verticals are influenced by a subsidiary truss action, like the chords in Figs. 71*a* and 72*a* referred to above.

An examination of the displacement diagrams shows that the elevation of the left vertex of each triangle is mid-way between those of the other two vertices. This is due to the fact that both diagonals in the same panel have the same slope. The points of division for D_2d, E_2e, and F_2f are located about the same distance from one end in the corresponding panels. It will be noticed that these points of division occur in the panel on the right of the respective members, since the adjacent diagonals slope downward to the right. A criterion for loading for the greatest tension in these members could be readily deduced, but since the maximum ordinate is so close to the point of division it will be unnecessary. As this distance is about 14.4 ft., axle 3 of the Cooper loading should be placed at the panel point indicated by the maximum ordinate, the live load coming on from the left, thus bringing the pilot axle about 1.4 ft. from the point of division. It will also be noticed that the hanger C_2c receives no stress from any load transmitted to the truss at all panel points on the right of d.

The influence diagram for Gg shows that its stress is due only to loads in the adjacent panel on each side of it. The maximum ordinate measures 0.5 kip. This indicates that the panel load at g is equally divided among the four diagonals in the two adjacent panels. The ordinate at g_1 in the influence line for F_2g measures a value equal to 0.25 kip multiplied by the secant of the angle which F_2g makes with the vertical. This angle is 45°.

It may be added that the verticals in a K truss, though having to resist some compression, are primarily tension members. The upper diagonal on the left of a vertical corresponds to the compression diagonal of a Howe truss, whereas the lower diagonal corresponds to the tension diagonal of a Pratt truss; hence the vertical combines the functions of verticals in the web systems of both Howe and Pratt trusses.

PROBLEM 73a.—Construct displacement diagrams and influence lines for the stresses in C_2C and BC of the truss in Fig. 73a.

CHAPTER IX

DEFLECTION INFLUENCE LINES

Art. 74. Deflections of Beams

The methods of graphic statics are well adapted to obtain the deflection of any beam or girder, with cross-sections having either constant or variable moments of inertia, and with any kind of loading, whether concentrated, distributed, or both combined.

The fundamental equation in mechanics for the vertical deflection of a horizontal beam at any given point (see Mechanics of Materials) is

$$f = \int MM'\delta x \div EI \tag{1}$$

in which M is the bending moment in any section of the beam due to the given loads, M' the bending moment due to an assumed load unity placed at the given point, δx a differential horizontal distance, E the modulus of elasticity, and I the moment of inertia of any cross-section.

Sometimes it is required to find the deflection of a beam at different points due to any given loading in a fixed position. In this case the bending moment diagram is first drawn and by treating it in turn as a loading diagram and drawing another equilibrium polygon to conform to certain conditions, this becomes a deflection diagram. The methods for doing this will be described and illustrated in subsequent articles.

At other times it is required to find the deflections at one point of a beam under different sets of loads, or under a set of moving loads. In this case it is necessary to construct a deflection influence line for the given point. The theoretical relations upon which the construction of deflection influence lines depend will now be presented.

Figure 74a shows a cantilever beam with a load P at b and an assumed load of unity at a. The deflection of the beam at a, due to the load P only, can be found by means of equation (1) of this article. For the sake of distinction let subscripts be added to M and M' caused by P and unity, respectively, thus: M_1 and M'_1. Now let the loads P and unity exchange positions as in Fig. 74b, the corresponding bending moments being designated M_2 and M'_2. Since the bending moment in any section due to a load in either position is proportional to the magnitude of the load, $M_2 = PM'_1$, and $M'_2 = M_1 \div P$; hence also $M_2M'_2 = M_1M'_1$. For the same beam, equation (1) shows that the only variable is the quantity $\int MM'$. Therefore the deflection at a due to P located at b is exactly the same as the deflection at b due to P located at a. The same relation holds wherever b is located on the span.

Furthermore, if a graphic method is adopted the deflections of all points in the span are obtained by the same diagram whose construction is required to find that of any one point only. If the load at the end is made unity as in Fig. 74c, and the deflection diagram constructed by one of the methods given in the following articles, the diagram becomes a deflection influence line for a point at the end of the beam. To obtain the deflection at that point due to a concentrated load in any position, it is only necessary to measure the deflection ordinate at that position and to multiply its value by the magnitude of the given load.

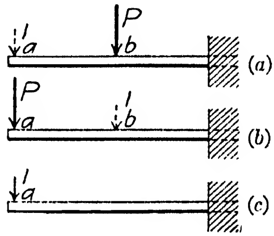


FIG. 74.

The methods referred to require special care with respect to the units in which different terms are to be expressed. The unit in which the deflection f is given by equation (1) when the units for all other terms are known may be found conveniently by cancellation. Let M be expressed in inch-pounds, M' in inches since the assumed load is an abstract unity, E in pounds per square inch or pounds divided by square inch, and I in inches^4. The following equation may then be written:

$$\frac{\text{in.-lb.} \times \text{in.} \times \text{in.}}{(\text{lb.} \div \text{in.}^2)\ \text{in.}^4} = \frac{\text{in.-lb.} \times \text{in.} \times \text{in.} \times \text{in.}^2}{\text{lb.} \times \text{in.}^4} = \text{in.}$$

If one of the distances involved were expressed in feet it would change the result to feet. The same method may be employed in other combinations of terms to see whether all the terms have been expressed in the proper units to give the result required.

It is to be remembered that equation (1) in this article gives the deflection due only to the bending moments in a beam and hence is in some degree approximate, although usually it fully satisfies the requirements of practice. If very precise values of deflections are required in any case, it is necessary to determine also the deflection due to shear (see Mechanics of Materials).

ART. 75. DEFLECTION OF A CANTILEVER GIRDER

For example, let it be required to find the deflection at the end of a railroad turntable due to Cooper's E50 loading (see Art. 38). When a locomotive is balanced on the turntable preparatory to turning it, as illustrated in Fig. IV*a*, each one of the pair of plate girders becomes a double cantilever, and each half of a girder is a cantilever with its neutral axis fixed in a horizontal direction over the middle support. Figure 75*a* gives an elevation of one of these cantilevers on which the positions of the tender axle loads are indicated. The stiffeners of the plate girder are omitted on this diagrammatic representation. Figure 75*b* is a diagram in which the ordinates represent the corresponding values of I, the moment of inertia of the cross-sections of the girder. The values of I are computed from the cross-section areas, and it is assumed that I does not change abruptly at the end of a cover plate but increases gradually to its full value in a distance of about 2 ft.

In a cantilever in which the horizontal distances x are measured from the free end $M' = -x$ in equation (1) of the preceding article, and if the span of the beam be divided into lengths Δx so as to apply graphic methods, the deflection may be determined with sufficient precision by equation (1) after making these substitutions, giving $f = \Sigma Mx \cdot \Delta x \div EI$. This may be changed into the form

$$f = \frac{\Delta x}{E} \sum \frac{M}{I} \cdot x \qquad (2)$$

This summation can be most conveniently made by constructing an equilibrium polygon or bending moment diagram by treating the values of $M \div I$ as loads. In order to draw the deflection influence line for this cantilever girder it is necessary to place a load of 1 kip at its free end, since it is desired to find the deflection

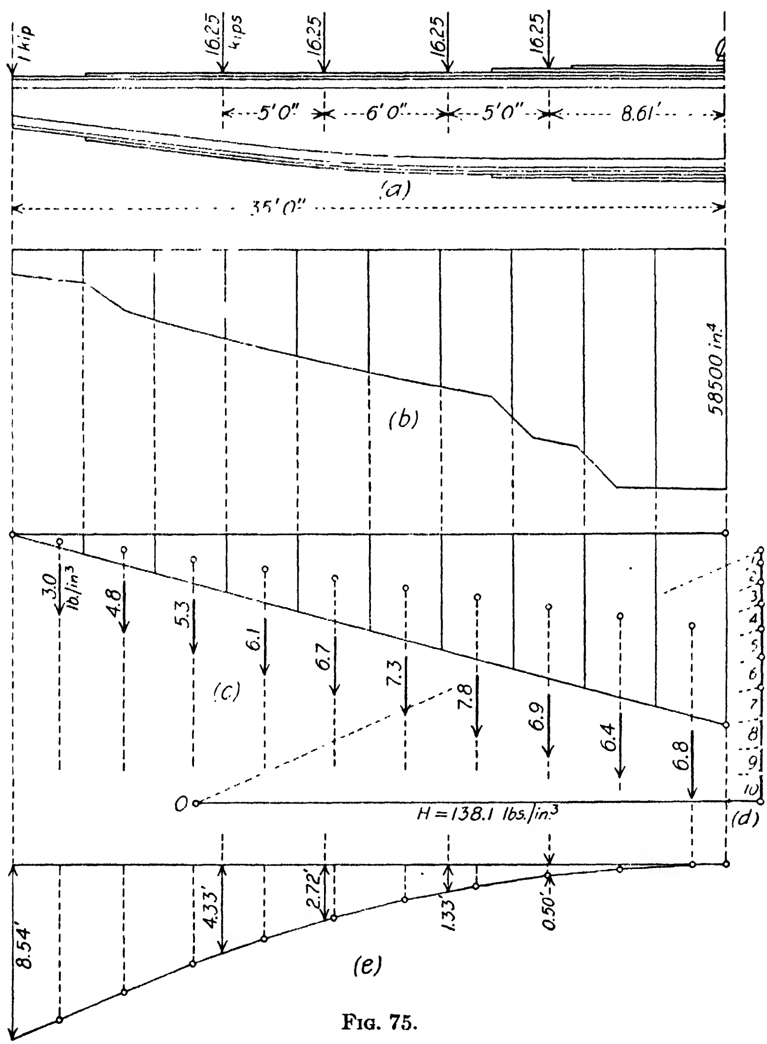


FIG. 75.

at that point, and then to construct the deflection polygon for that load, the girder being assumed as fixed at the support.

Figure 75c shows the bending moment diagram for this load of 1 kip at the end of the girder. The span of the girder is divided into ten equal divisions of 3.5 ft. = 42 in. each. The diagrams for I and M are both divided by vertical lines spaced the same distance apart. The average values of M, I, and $M \div I$ for each division are given in the following table:

Division, No.	M, In.-lb.	I, In.^{4}	$M \div I$, Lb. ÷ in.^{3}
1	21 000	7 000	3.0
2	63 000	13 200	4.8
3	105 000	19 800	5.3
4	147 000	24 200	6.1
5	189 000	28 000	6.7
6	231 000	31 600	7.3
7	273 000	34 800	7.8
8	315 000	45 600	6.9
9	357 000	56 000	6.4
10	398 000	58 500	6.8

Since $M \cdot \Delta x$ represents the area of one division of the moment diagram, the loads $M \div I$ are to be applied at the centers of gravity of the division areas. If desired, the loads may be taken as $M \cdot \Delta x \div I$ instead of $M \div I$, but this involves ten times as many multiplications by Δx.

The next step is to find the proper value of the pole distance H, remembering that the fundamental theory of the equilibrium polygon requires H to be laid off with the same scale as the loads, and hence H must be expressed in the same units. The same theory also gives $\Sigma(Mx \div I) = Hz$, in which z is the ordinate of the equilibrium polygon (which in this case becomes a deflection polygon) at the limit to which the summation is made. Since the value of any deflection f is very small it is desirable that the ordinates z shall give the deflection as magnified n times, so as to be measured with greater precision; that is, $z = nf$. Making these substitutions in equation (2) there is obtained $f = (\Delta x \div E)Hnf$ whence

$$H = E \div (\Delta x \cdot n)$$

The value of E is 29 000 000 lb. per sq. in., and $\Delta x = 42$ in.; hence the numerical value of the pole distance is

$$H = 29\,000\,000 \div (42n) = 690\,500 \div n$$

expressed in pounds divided by inches[3], which agrees with the units in which the values of $M \div I$ are expressed in the table. As H has to be laid off with the same scale as the load line which has a total value of 61.1 lb. per in.[3] it is found that a convenient value for n is 5000, thus making $H = 138.1$ lb. per in.[3] In Fig. 75*d* the loads are laid off in regular order and the pole is taken opposite the extremity of load 10 in order that the axis of the deflection polygon may be horizontal. The equilibrium polygon Fig. 75*e* is drawn in the usual manner, remembering that any side which lies between the lines of action of two forces must be parallel to the ray whose extremity lies between the same two forces in the force polygon.

Since the ordinate of an equilibrium polygon must be measured by the same scale of distances which is used to locate the positions of the forces on the beam, the deflection ordinate z at the left end is found to be 8.54 ft. = 102.5 in., and hence the deflection $f = 102.5 \div 5000 = 0.0205$ in., for a load of 1 kip applied at the end. As the deflection diagram is also a deflection influence line, the ordinates under each axle load are measured by the linear scale and found to be 4.33, 2.72, 1.33, and 0.50 ft., respectively, their sum being 8.88 ft. The end deflection therefore for these four axle loads of 16.25 kips each is $8.88 \times 12 \times 16.25 \div 5000 = 0.3463$ in. This value is practically the same as 0.3459 in., which was obtained by analytic computation, the lengths of division being 2 ft. except at each end of the span where they were a little longer.

If it be desired to make the divisions of the span unequal in order to conform somewhat to the variations of the moment of inertia, then the areas of the bending moment divisions must be obtained and the loads to be used for the deflection diagram are $M \cdot \Delta x \div I$. The deflection influence line is really a curve which is tangent to the deflection polygon at the points of division. However, where the divisions are as short as in this example, it is not necessary to draw the curve, since it coincides so closely with the polygon.

PROBLEM 75*a*.—Determine the deflection of the extremity of the other half of the turntable girder referred to in this article, due to the axle loads which it supports.

ART. 76. ALTERNATIVE METHOD

In Fig. 76*b* the diagram showing the variation of the moment of inertia is reproduced from Fig. 75*b* and a series of similar isosceles triangles is drawn so as to divide the span of the cantilever girder into lengths Δx which have a constant ratio to their respective average values of I. The relation follows from the constant ratio of the base to the altitude of each triangle. Hence the formula in the preceding article may be changed to

$$f = (\Delta x \div EI)\Sigma Mx$$

The summation of Mx is again made most conveniently by constructing an equilibrium polygon or moment diagram in which the bending moment M is treated as a load.

The bending moment diagram due to the assumed load of 1 kip at the end is shown in Fig. 76*c*. Each value of M is the average value for its division and when treated as a load for the construction of another equilibrium polygon is applied at the center of gravity of its corresponding area. These middle ordinates may be transferred with the dividers to the load line without measuring their values by scale.

Before constructing this equilibrium polygon and its tangent curve which represents the deflection curve of the beam, it is necessary to determine the proper value of the pole distance H in Fig. 76*d*. Since $\Sigma Mx = Hz$ in which z is any ordinate at the limit for which the summation is made, and $z = nf$, as in the preceding article, there is obtained by substitution in the preceding equation $f = (\Delta x \div EI)Hz = (\Delta x \div EI)Hnf$, and hence

$$H = EI \div (n \cdot \Delta x)$$

Since $E = 29\,000\,000$ lb. per sq. in., I in the largest division is $53\,000$ in.^{4}, and the length of that division is $\Delta x = 11.8$ ft. $= 141.6$ in.,

$$H = \frac{29\,000\,000 \times 53\,000}{n \cdot 141.6} = \frac{10\,855\,000\,000}{n}\text{ in.-lb.} = \frac{904\,600}{n}\text{ ft.-kips}$$

As H has to be laid off to the same scale as the load line, it is found that a convenient value for n is 5000, making $H = 180.9$ ft.-kips. With this value of H, the force polygon Fig. 76*d* is drawn, and the equilibrium polygon Fig. 76*e* constructed. The end ordinate measured by the linear scale used in laying off the length of the beam is 8.46 ft. = 101.5 in. The deflection is therefore 101.5 ÷5000 = 0.0203 in. The ordinates below the axle loads measure 4.30, 2.74, 1.34, and 0.52 ft., or a sum of 8.90 ft. = 106.8 in.;

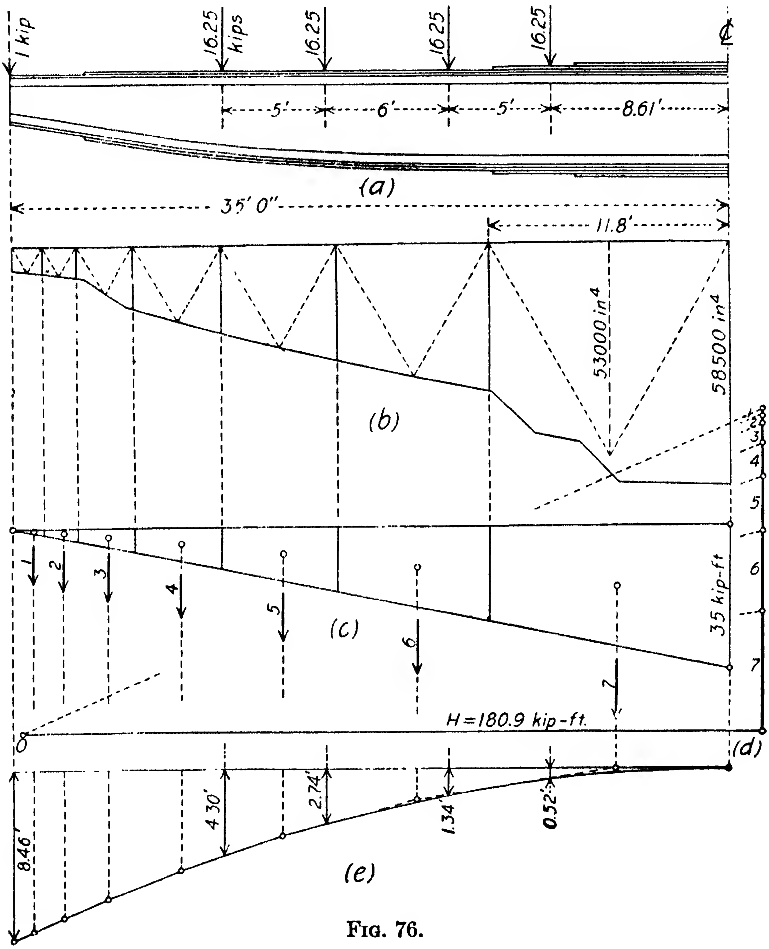


Fig. 76.

hence the deflection at the end due to these loads is 106.8 × 16.25 ÷5000 = 0.3471 in.

As the deflection polygon in this case has such long sides it is necessary to draw a smooth curve tangent to the sides respectively at the points of division, before the ordinates under the axle loads are measured. Only a part of this curve is shown in Fig. 76*e* on account of the reduced size.

In Art. 75 the bending moments M were divided by the respective moments of inertia I, before being laid off on the load line. In this article the span is divided into unequal divisions Δx so as to make the ratio of $\Delta x \div I$ constant. There remains a third method of procedure in which the pole distance H is changed in direct proportion to the average value of I for each division.

In case it is desired to find the deflections for one or more loads, which are fixed in position at a number of different points on the span, then the simplest procedure is to draw first the bending moment diagram for the given loading, and afterward to construct the deflection diagram by either of the three methods referred to in the preceding paragraph. The magnitude of any ordinate will give the total deflection at the location of that ordinate. By using the bending moment diagram for the four axle loads in the example considered in this article, and adopting the third method of drawing the deflection polygon, the deflection at the end of the girder was found to be 0.343 in., and under the axle loads 0.210, 0.146, 0.078, and 0.032 in., respectively. In this case the moment diagram was divided into only five divisions of unequal length.

Usually the method described and illustrated in Art. 75 is the most advantageous, since the average values of I are more readily obtained than in the other methods. Where the values of I change more or less abruptly it requires a number of trials to divide the span so as to make $\Delta x \div I$ constant, and to make I really the average value for its division. Under some conditions, however, this method may be preferable.

When the moment of inertia is constant the construction is simplified still further, since I is eliminated from the summation. For some cases of special loading the analytic method may then be more advantageous than the graphic method.

PROBLEM 76*a*.—Check the value of the deflection obtained in Problem 75*a* by the method explained in this article.

ART. 77. DEFLECTION OF A SIMPLE BEAM

The same reciprocal relation which was shown in Art. 74 to exist between the deflections and loads at any two points on a cantilever beam apply likewise to any other type of beam. Hence the simplest graphic determination of the deflection of a simple beam at any given point under any loading consists in the construction of a deflection polygon for a unit load placed at the given point. This polygon is also the deflection influence line by means of which the deflection may be readily obtained for any other loading.

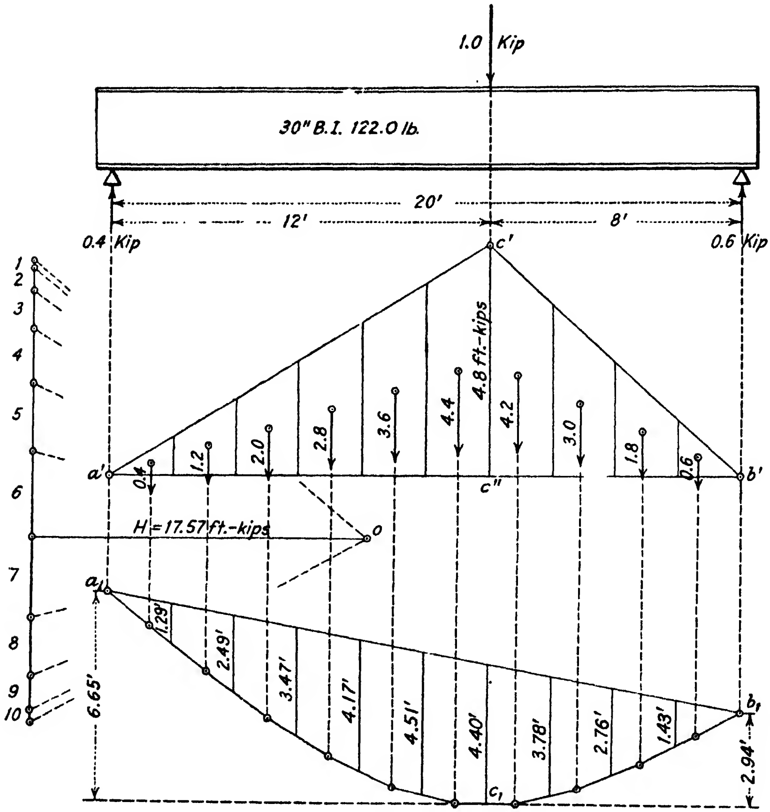


FIG. 77a.

In Fig. 77a a simple beam is shown with a load of 1 kip applied at six-tenths of its span from the left end. The reactions are therefore 0.4 and 0.6 kip, respectively. Let the tangent of the neutral axis of the beam at a point under the load be assumed as fixed in a horizontal direction. The two parts of the beam may then be regarded as cantilever beams with upward loads of 0.4

and 0.6 kip applied at their respective ends. The deflection polygons for both cantilevers are drawn according to the method given in Art. 75. Since the bending moments are positive the deflections of the ends are upward from the fixed tangent or axis, as indicated in the lower diagram of Fig. 77*a*. But according to the actual condition of the beam the supports are fixed in elevation, making the deflection of the beam at those points equal to zero; hence the required vertical deflection at any point must be measured from the closing line a_1b_1 as an axis.

The entire deflection polygon with its closing line is found to be identical with an equilibrium polygon constructed by treating the entire bending moment diagram for the simple beam as a load diagram. It was assumed that the tangent to the neutral axis of the beam under the load was fixed in a horizontal direction. The assumed direction is really immaterial, since the magnitudes of the vertical ordinates in an equilibrium polygon are independent of the position of the pole in the force polygon, provided the pole distance H remains the same (see Art. 7). If the axis a_1b_1 be made horizontal and the same end ordinates laid off as before the true inclination of the tangent at c_1 will be obtained.

Let the beam represented in Fig. 77*a* be a 30-in. Bethlehem I-beam weighing 122.0 lb. per linear ft., and having a moment inertia of cross-section of 5235.7 in.^{4} It is required to find its deflection for a span of 20 ft., under a uniform load of 9.3 kips per linear ft., at a section 12 ft. distant from the left support. The load includes the weight of the beam itself.

The span is divided into ten equal parts, making each division 2 ft. long. A load of 1 kip is placed at the point whose deflection is to be determined and the bending moment diagram constructed. The maximum ordinate $c'c''$ equals $1 \times 12 \times 8 \div 20 = 4.8$ ft.-kips. The value of the middle ordinate of each division of the bending moment diagram is given on the drawing expressed in foot-kips. The average or middle ordinate in each division is to be treated as a load applied at the center of gravity of the division in constructing the deflection polygon. The sum of those on the left of c'' is 14.4 ft.-kips, and of those on the right 9.6 ft.-kips.

In order to make the direction of the deflection polygon horizontal at c_1 or under the load of 1 kip, the pole is located in a horizontal ray through the extremity of load 6 on the load line.

Since $H = EI \div (n \cdot \Delta x)$, its numerical magnitude is

$$H = \frac{29\,000\,000 \times 5235.7}{24n} = \frac{6\,326\,500\,000}{n} \text{ in.-lb.} = \frac{527\,200}{n} \text{ ft.-kips}$$

In order to secure a deflection polygon of good proportions it is found convenient to take $n = 30\,000$, making $H = 17.57$ ft.-kips. Upon constructing the deflection polygon and measuring the ordinates at a_1 and b_1 by the linear scale it is found that the left end of the beam deflects upward $6.65 \div n$ ft. and the right end $2.94 \div n$ ft. from the tangent to the neutral axis at c. Upon drawing the closing line a_1b_1 and measuring the ordinates at the ends of the divisions, their values are found to be 0, 1.29, 2.49, 3.47, 4.17, 4.51, 4.40, 3.78, 2.76, 1.43, and 0 ft., respectively.

Since the load is uniformly distributed it is necessary to find the area of the deflection influence diagram or the sum of the average ordinates in divisions of 1 ft. each. By means of Simpson's rule the area is found to be 57.04 sq. ft., or the sum of the ordinates is 57.04 ft. for the 1-ft. divisions. The deflection for a load of 9.3 kips per linear ft. is therefore

$$f = 57.04 \times 9.3 \div 30\,000 = 0.01768 \text{ ft.} = 0.2122 \text{ in.}$$

By means of the equation of the elastic line for this case in which $x = 0.6l$ (see Mechanics of Materials), the deflection is computed to be 0.2100 in., which differs from the value obtained graphically by a little over 1 per cent. This difference could have been reduced by using larger scales. The scales employed on the original diagram for Fig. 77a are 3 ft. to 1 in. and 5 kips to 1 in. The maximum deflection of this beam at the center of the span computed by means of the ordinary formula is 0.220 in.

PROBLEM 77a.—A simple beam having a span of 20 ft. consists of a 20-in. I-beam weighing 65.4 lb. per ft., and having a moment of inertia of 1169.5 in.^4 The load varies uniformly from zero at one end of the span to 11 kips per ft. at the other end. Find the deflection at intervals of 2 ft. throughout the span by means of a deflection polygon.

ART. 78. DEFLECTION OF A CRANE GIRDER

The simple beam in Fig. 78a represents one of a pair of box girders in a traveling crane similar to that illustrated in Fig. IXa. It has two web plates $\frac{1}{4}$ in. thick. The upper cover plate is 20 by $\frac{1}{2}$ in., and the lower one 20 by $\frac{3}{8}$ in. The two upper flange angles are 5 by 3 by ½ in., and the lower ones 5 by 3 by ⅜ in., their longer legs being vertical. The short flange angles at each end are 3½ by 2½ by ½ in., and 48¾ in. long. Some of the dimensions of the girder are given on the diagram. Stiffeners and

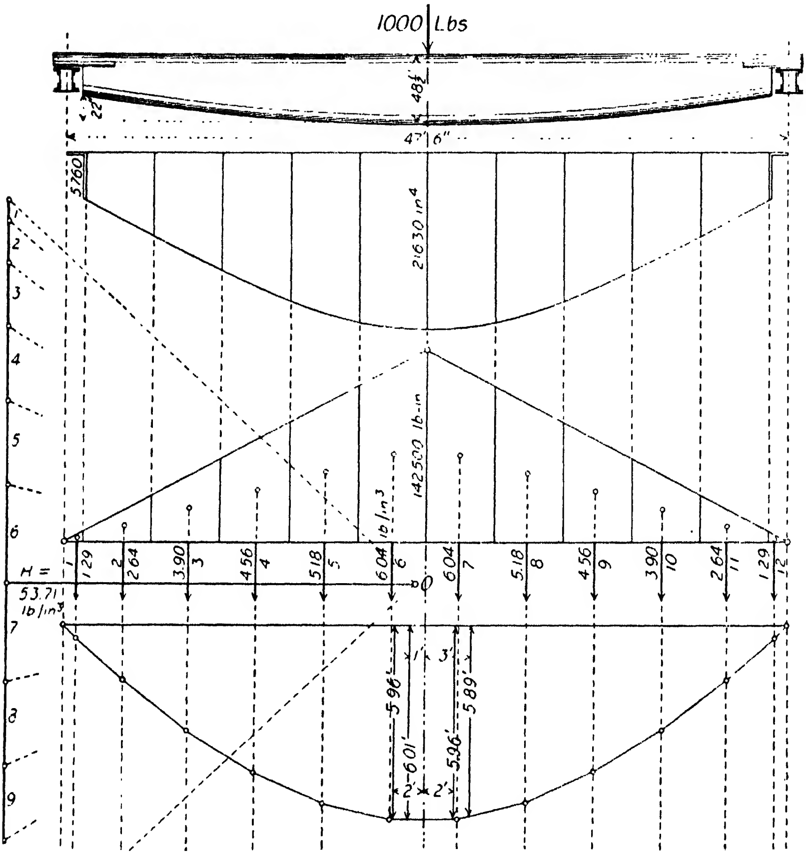


FIG. 78a.

minor details are omitted. The lower flange has a parabolic curve. The depth back to back of flange angles is 7⅞ in. above the supporting girders, 26½ in. at the end of the curved bottom flange, and 48½ in. at the center of the span. The moments of inertia at these sections are 250, 5760, and 21 630 in.[4] The distance between the second and third of these sections was divided into five equal parts, and the moments of inertia computed at the ends of divisions, giving the values 9970, 14 460, 18 340, and 20 760 in.^{4} The deflection of the girder at the center of the span is to be found due to a load of 30 kips on each girder.

On account of the abrupt change of section next to the supporting girders the span is not divided into equal parts throughout. All divisions except one at each end are 4.5 ft. long, the end divisions being 1.25 ft. in length. The values of the middle ordinates in the divisions of the bending moment diagram due to a load of 1 kip at the center of the span are given in the following table. The table also contains the average values of the moments of inertia for the respective divisions and the corresponding values of $M \div I$.

M, In.-lb.	I, In.^{4}	$M \div I$, Lb. $\div$ in.^{3}	Loads, No.
3 750	810	4.63	1, 12
21 000	7 960	2.64	2, 11
48 000	12 320	3.90	3, 10
75 000	16 460	4.56	4, 9
102 000	19 700	5.18	5, 8
129 000	21 360	6.04	6, 7

Since the end divisions are only 15 in. long instead of 54 in., the first $M \div I = 4.63$ lb. $\div$ in.^{3} must be multiplied by $15 \div 54$, giving 1.29 lb. $\div$ in.^{3}, before it is laid off on the load line in constructing the deflection influence line. The magnitude of the pole distance is

$$H = \frac{29\,000\,000}{54n} = \frac{537\,060}{n} \frac{\text{lb.}}{\text{in.}^3}$$

Since the total load line measures 47.22 units, a convenient value to assume for n is 10 000, making $H = 53.71$ units. The deflection polygon is constructed in the same manner as described previously. The electric hoist and its live load of 60 kips are carried by a four-wheeled trolley which is supported by the rails on top of the pair of crane girders. The two axles of the trolley are spaced 4 ft. apart, hence it is necessary to measure two ordinates with this spacing on the deflection influence line. If both ordinates are

2 ft. from the center of the span they measure 5.96 ft., whereas if one is 1 ft. from the center and the other 3 ft. on the other side of the center they measure 6.01 and 5.89 ft. The larger sum is 11.92 ft., and therefore the deflection at the center due to a load of 15 kips on each trolley wheel is

$$f = 11.92 \times 12 \times 15 \div 10\,000 = 0.215 \text{ in.}$$

In this formula the load is introduced as 15, since 15 kips is 15 times as large as the load of 1 kip which was placed at the point whose deflection was to be found.

To find the total deflection of the crane girder it is necessary to consider the weight of the trolley and the electric hoist as well as the distributed weight of the girder itself, the squaring shaft, the bridge walk and brackets which carry it.

PROBLEM 78*a*.—Find the greatest deflection of the above girder at the quarter point of the span due to the same load.

ART. 79. DEFLECTION INFLUENCE LINES FOR TRUSSES

In Chapter VII a method was given for finding the deflection of the different panel points of a truss under a load which is fixed in position. As stated there, this method is used in determining the elevation of the blocking required for the erection of bridge trusses, in which case the change of length for each truss member is computed for the sum of its stresses due to both dead load and full live load. The position of the live load is that which causes the greatest bending moment at the center of the span. It is also used in finding the deflection of different panel points due to a given live load when a truss bridge is being tested by observing the actual deflections under that live load in a specified position, in order to compare the results with the theoretic values previously found.

If, on the other hand, it is desired to find the deflection of any given panel point under different loads, or under a moving live load, it is necessary to construct a deflection polygon for a load of 1 kip placed at that point. This polygon is also a deflection influence line for the given panel point, on account of the reciprocal relation between loads and deflections, as shown for beams in Art. 74. The same relation between the bending moments M and M' indicated in that article exists likewise between the stresses

S and T, since the formula for the deflection of a truss involves the product ST (see Roofs and Bridges, Part I) just as the formula for the deflection of a beam includes the product MM'.

The most convenient method for finding the stresses in the truss members due to a load of 1 kip at the given panel point is to construct an ordinary stress diagram, unless the chords are both horizontal, in which case the analytic method furnishes the quickest solution. After the values of the changes in length λ due to these stresses are known, the displacement diagram is constructed and afterwards the deflection diagram, or deflection influence line, as described and illustrated in Arts. 66 and 69.

PROBLEM 79*a*.—Construct the deflection influence line for panel point c of the truss in Fig. 66*a*.

FIG. IIa.—Interior View of a Freight Shed Showing Steel Roof Trusses with Wide Spacing and Riveted Truss Purlins.

Fig. II*b*.—Interior View of Power House of the Niagara Falls Power Co., Showing Fink Roof Trusses Supporting Purlins of King-post Trussed Beams. The roof sheathing is supported directly by the purlins.

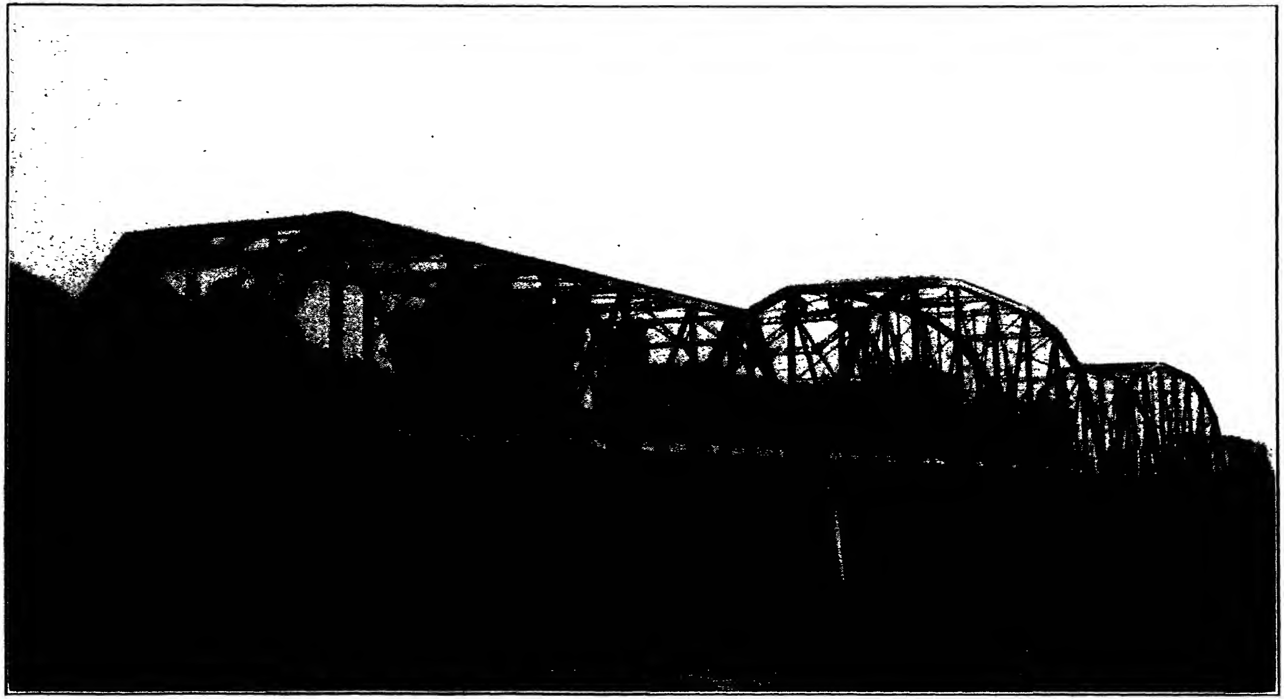


FIG. III*a*.—Two 120-ft. Pratt trusses and two 200-ft. Parker trusses on Lincoln Highway over the Des Moines River near Boone, Iowa. Completed in 1928.

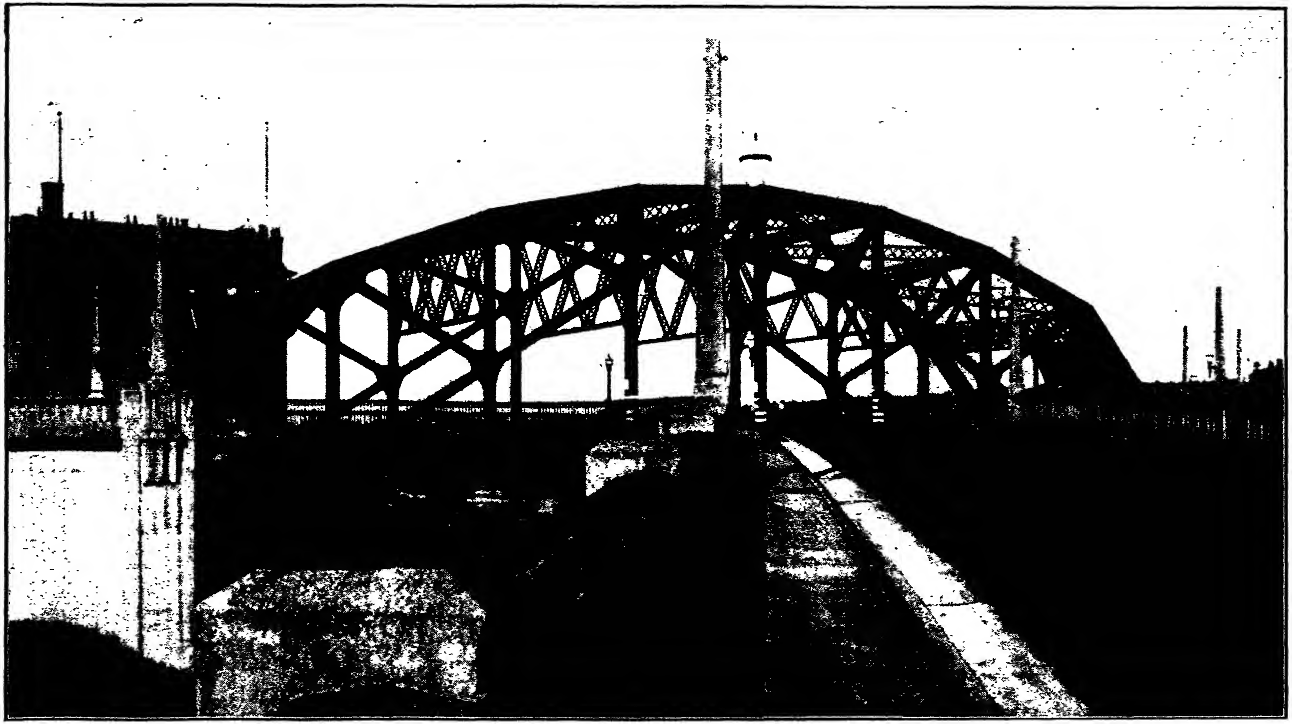


Fig. IIIb.—Through-truss span over the Philadelphia & Reading and Lehigh Valley Railroads, where the Second-Street ramp enters the Hill-to-Hill Bridge, Bethlehem, Pa. Completed in 1924.

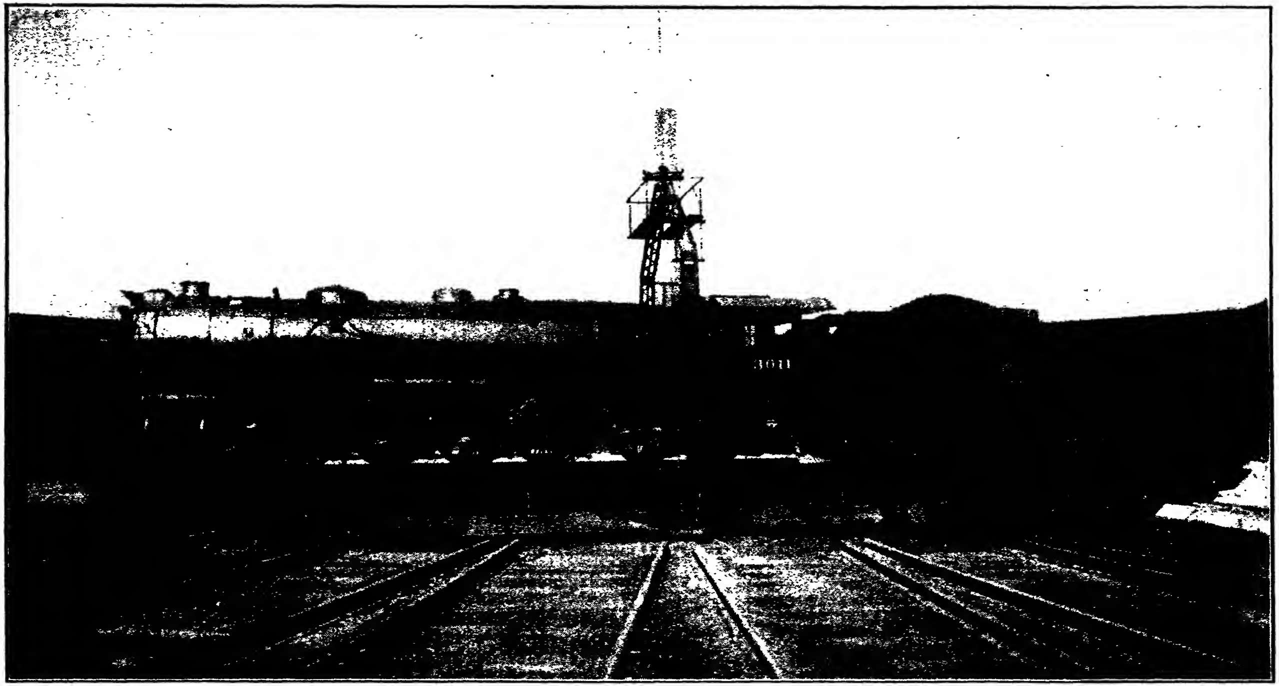


Fig. IVa.—Class H Locomotive of the Chicago & Northwestern Railway in position on a 110-ft. turntable. The total weight of the engine and tender is 818 kips with an average weight on each driving axle of 72 kips. It has a length over couplers of 103 ft. The locomotive is used for passenger and fast freight service.

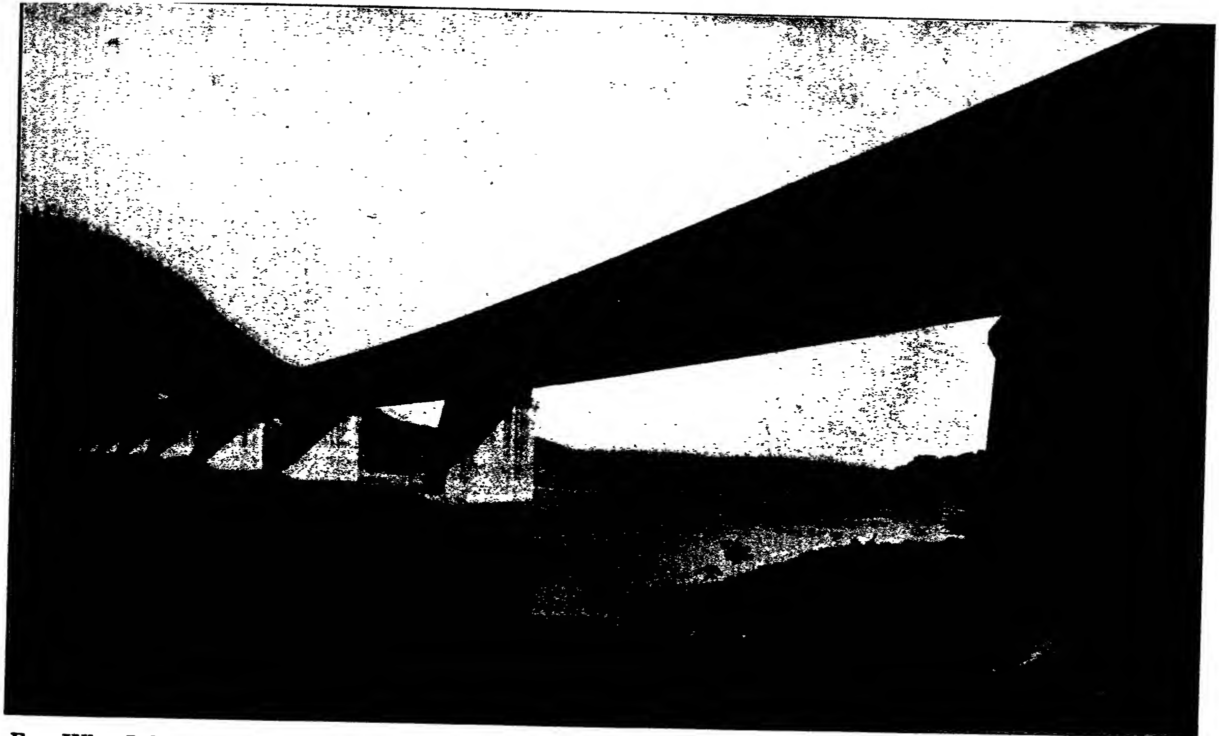


Fig. IVb.—Lehigh Valley Railroad Bridge over the Susquehanna River at Towanda, Pa. It contains thirteen double-track deck plate-girder spans 129.5 feet long, and one span 120 feet long. Completed in 1907.

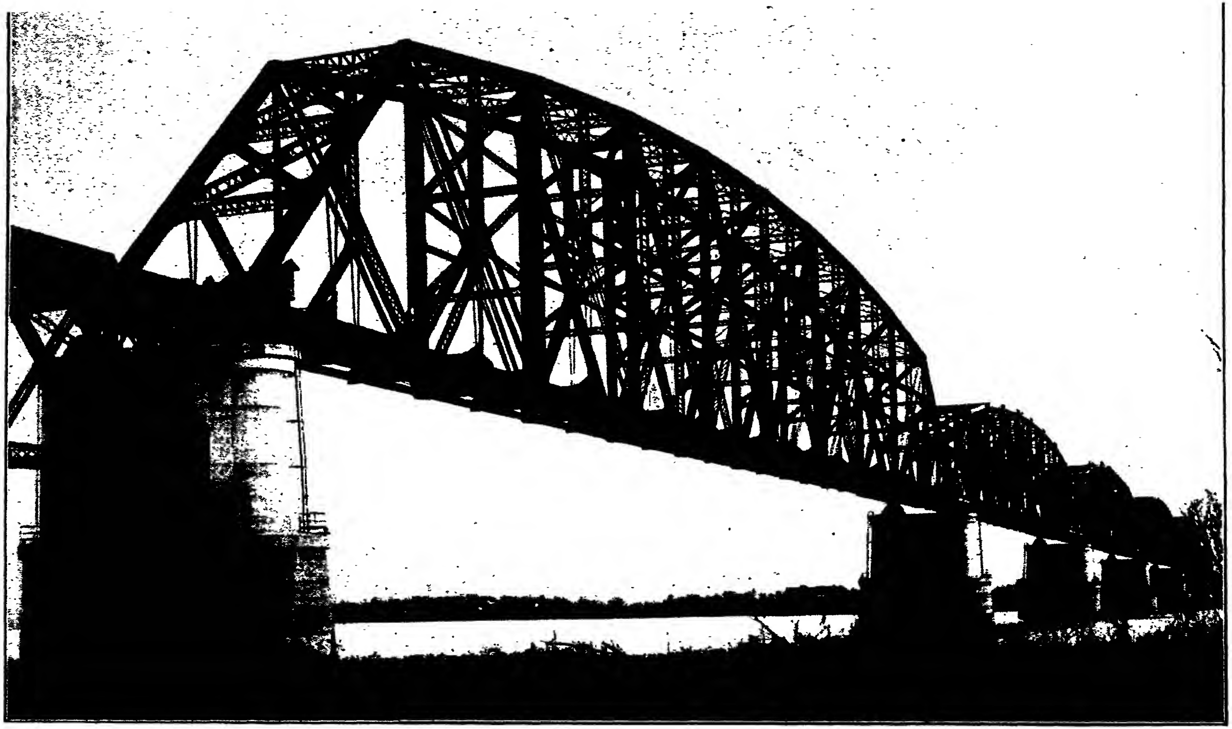


FIG. Va.—C., B. & Q. R. R. and N. C. & St. L. Ry. Bridge over the Ohio River at Metropolis, Ill. Completed in 1917. The span in the foreground is the longest simple truss span in the world, its length being 720 feet.

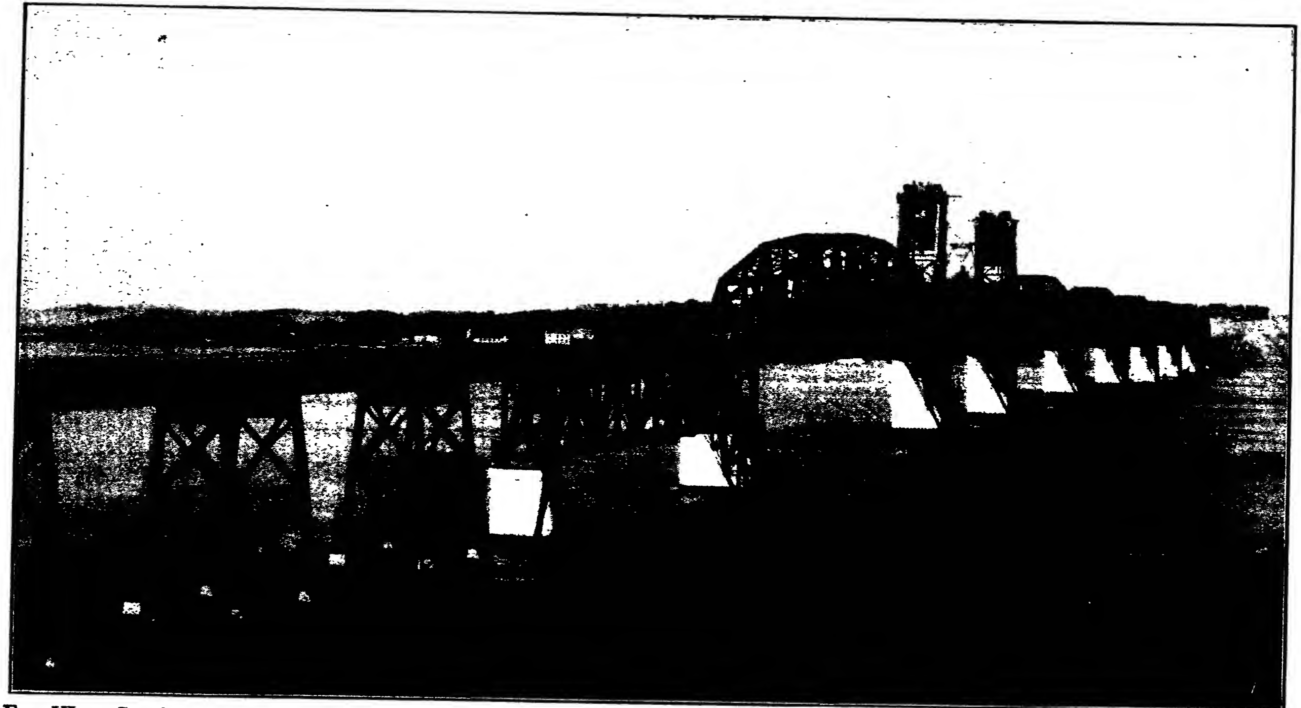


FIG. VIa.—Southern Pacific Railway Bridge over Suisun Bay between Martinez and Benicia, Calif. It is a double-track structure over a mile in length, the main spans being simple trusses of the Warren type bearing on piers 531 ft. apart and with viaduct construction and deck trusses used in the approaches. Completed in 1930 at a cost of about $12,000,000.

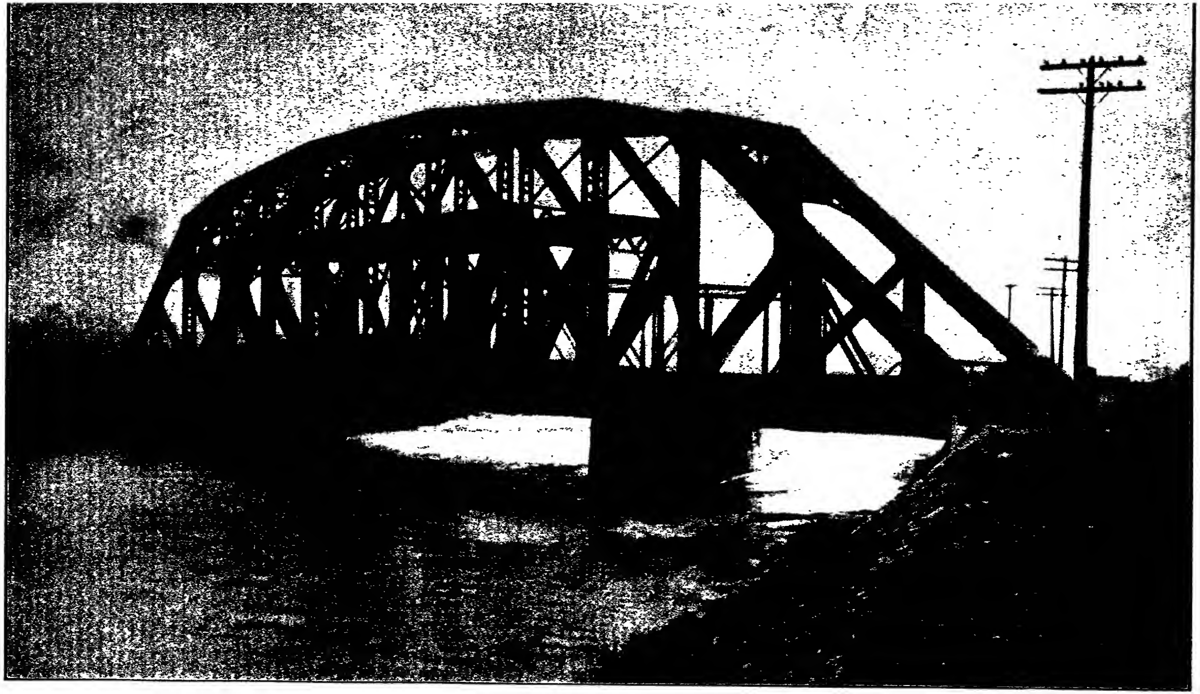


FIG. VIII*a*.—First Simple Truss Bridge with K System of Web Members Built in America. Atchison, Topeka and Santa Fe Railway Bridge over Arkansas River at Pueblo, Colo. Erected in 1915. Span 230 feet. The pier in the river supports the Denver & Rio Grande Railroad bridge shown directly behind the K-truss bridge. A city highway bridge is also shown in the background.

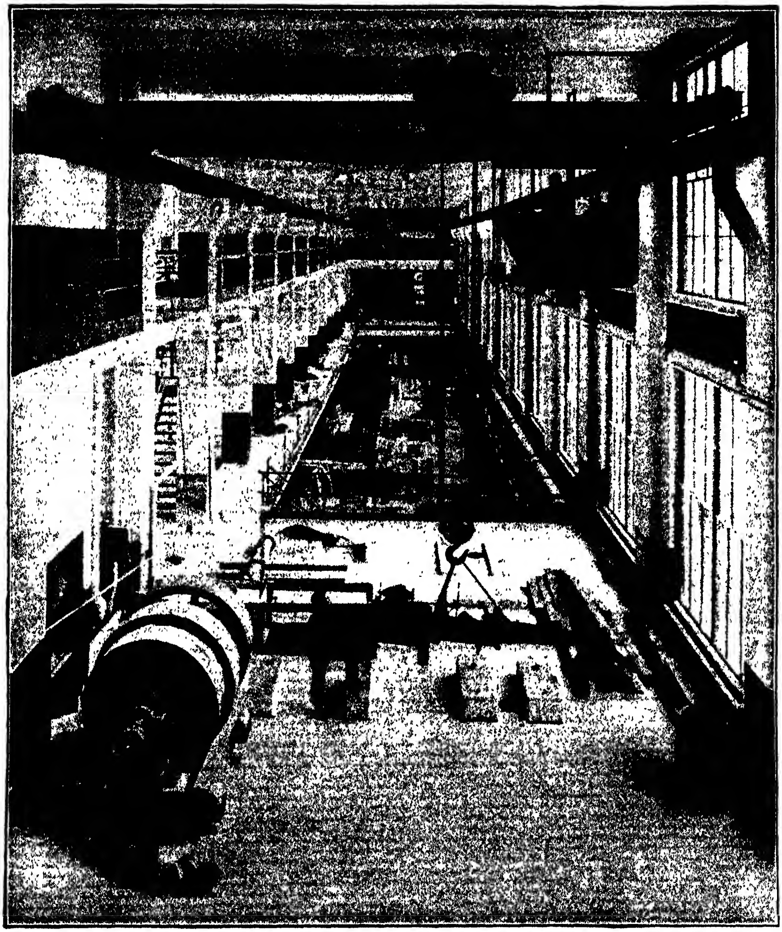


Fig. IXa.—Electric Traveling Crane in the Hydraulic and Steam Laboratory of the Massachusetts Institute of Technology, 1916.

INDEX