REFRIGERATION AND AIR CONDITIONING ENGINEERING

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To

NORMAN DOUGLAS

PREFACE

This book is intended to serve as a reference for practicing engineers and as a text for engineering students. The treatment is rigorous and is restricted to fundamental principles and rational procedures; refrigeration and air conditioning are treated as sciences rather than as arts. Such treatment seems desirable since the scientific background of both these fields of engineering is relatively unchanging whereas the arts of refrigeration and air conditioning are subject to rapid development.

The theoretical treatment has the additional advantage of embracing not merely the equipments and methods which are in use today, but those also which are of the future. The potentialities and limitations of such relatively new applications as the heat pump, radiant heating, and porous-coil humidifying are implicit in existing theory; thus the engineer with a sound psychrometric and thermodynamic background can use his knowledge to foresee some of the patterns of the future.

For the above reasons this book contains neither descriptive material nor performance data on actual equipments; nor does it reproduce any appreciable amount of the empirical data which are available in such standard handbooks as the American Society of Heating and Ventilating Engineers *Guide* and the American Society of Refrigerating Engineers *Data Book*. Proficiency in the *art* of refrigeration and air conditioning must be gained largely through experience rather than study and the authors therefore visualize that in a college course the text will be supplemented with laboratory work, inspection trips, and catalogue studies.

Such claim to originality as may be advanced for the material presented here must be with respect to the emphasis placed on graphical procedures. Wherever possible, graphical solutions have been devised for the more important rational equations. Such solutions are given for polytropic processes, dual-effect compression, ventilation volume requirements, the equivalent radiation coefficient, duct friction losses, and duct-fitting head losses. Similarly, charts are developed for use in rapidly selecting the most efficient fan for any given service. The entire subject of refrigeration is treated in terms of pictorial representation of cycles, actual as well as ideal, on the pressure-enthalpy chart and a new method of graphically solving cycle problems, using a superimposed trace sheet, is developed; *ph* charts and trace sheets, in pairs, are provided for eight of the most widely used refrigerants.

The influence of transient and periodic heat transfer on load determination is given extended treatment with emphasis on methods for obtaining approximate graphical solutions. Detailed illustrative examples are given to show how the approximate methods can be applied to practical problems without recourse to advanced mathematics.

All air-vapor processes and cycles are treated in terms of the psychrometric chart. New material in this section includes the treatment of critical loading conditions (Chapter X) and the visualization of health as well as comfort zones on the psychrometric chart. The latter item (Figure 13·2) is one of increasing importance to designers who are concerned with the many industrial problems requiring careful differentiation between conditions which are definitely unhealthy and those which are uncomfortable but not hazardous to health.

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Berkeley, California
January, 1945

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CHAPTER I

THERMODYNAMIC PRINCIPLES

Three major divisions of subject matter in refrigeration and air conditioning — psychrometrics, heat transfer, and reversed cycle analysis — are special applications of thermodynamics. A fourth important division, fluid·flow, requires an understanding of hydrodynamics which, in turn, makes use of the same basic concepts. Before proceeding with the specialized study of these four branches of engineering the fundamentals of thermodynamics must be thoroughly understood. The present chapter is intended as a brief review with particular emphasis on those definitions, concepts, and basic thermodynamic equations which find specific application in the fields of refrigeration and air conditioning.

ENERGY

Almost every process performed in refrigeration, ventilation, or air conditioning has as its purpose the supply, removal, transformation, or transfer of energy. Within the limited scope of these specialized fields of engineering, energy can be defined as the ability to do work, or, more generally, the ability to produce an effect. Before attempting to design or analyze a refrigeration or ventilation system one must be able to supply satisfactory answers to four fundamental questions concerning energy:

1. In what forms can energy be transferred and how can the transfer rate be expressed?

2. In what forms can energy be stored and how can the quantity in storage be evaluated?

3. What are the quantitative relationships among the various forms of energy?

4. Under what conditions and to what extent can energy be transformed from one form to another?

Of the above questions, the first three are discussed in this section whereas the fourth is taken up in a later section on *Energy Equations.*

1·1. Energy in Transition. Two forms in which energy can be transferred are as heat, Q, and as work, W. Heat refers to any energy transfer occasioned by a temperature difference between the source and receiver, and work refers to energy transfers occurring for reasons other than a temperature difference. In refrigeration and air conditioning, work always appears as energy transfer through a shaft or as " flow " of energy through a steady and continuous column of fluid either in a pipe or in a cylinder.

(*a*) Work. Whatever its method of appearance, work is measured as the action of a force through a distance. Since the engineering units of force and distance are the pound and foot, one unit of work (foot-pound) can be defined as the quantity of energy transferred during the action of a 1-lb force over a distance of 1 ft. The rate of work, power, is expressed in units of foot-pounds per second, or horsepower (1 hp = 550 ft-lb per sec).

Shaft work is a measure of the flow of energy through a shaft and is expressed in terms of a circumferential force times the distance through which that force would travel if it remained fixed at one point on the circumference and acted through the distance represented by the travel of that point. Thus a force of 10 lb acting on the circumference of a shaft 2 ft in diameter would do $10(\pi \times 2)$ ft-lb of work per revolution. If the shaft were making 200 revolutions per minute the total work for one minute would be

$$10 \times \pi \times 2 \times 200 = 12{,}600 \text{ ft-lb or } 12{,}600/(550 \times 60) = 0.38 \text{ hp}$$

The concept of energy " flowing " through a shaft arises from the fact that there is no external evidence of the passage of energy and the condition of the shaft is not affected by the quantity of energy transferred; the energy is not stored and in no way affects the properties of the shaft during passage through it. Distortion of a shaft under load is caused not by work, but by turning moment. A large turning moment frequently occurs when the work rate increases, but this moment is not necessarily a measure of the rate of energy transfer. Thus of two shafts through which energy is flowing at the same rate, one could be turning at high speed with small moment and the other at low speed with large moment; the deflection in these cases would vary with moment rather than with the rate of energy transfer. The most important fact to visualize concerning the flow of energy as work is that it in no way affects the material, whether solid or fluid, through which it is passing.

In refrigeration and air conditioning the only appearance of a term for energy transfer as shaft work is in such equipments as the fan, pump, and compressor. In each of these cases work is done on the fluid in order to raise its pressure.

Flow work is comparable to shaft work in every respect except that the transfer of energy occurs by steady and continuous action of a fluid rather than by rotation of a solid shaft. In Fig. 1·1, consider a piston located at *d* and acted on by a working fluid, at a pressure of P pounds per square foot absolute, or p pounds per square inch absolute. The fluid pressure is resisted by a force acting through the piston rod. Taking the area of the piston as A square feet, the force acting on each side of the piston is $F = PA = 144pA$. If the piston is moved to the left a distance L, causing discharge at constant pressure of fluid from the cylinder, the work input to the shaft would have to be equal to FL foot-pounds. If now (with constant force F, applied to the right side of the piston) additional fluid were pumped into the cylinder and the piston returned to its original position, the work performed by the entering fluid on the piston would likewise be equal to FL. But $FL = PAL$, and AL is the volume of fluid, V', entering the cylinder so the work performed by the entering fluid (and transmitted through the shaft) is PV'. Assuming that w pounds of fluid enter and that the volume of each pound is v, the total work done by the fluid on the piston is wPv, and the work done per pound of entering fluid is Pv; this is the quantity of energy transferred through the fluid as flow work for each pound of fluid passing through the system.

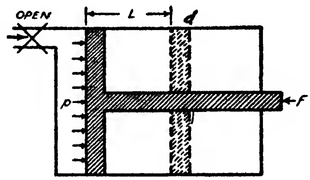

FIG. 1·1. Flow Work during Admission.

In the general case of any fluid moving in steady continuous motion past any cross section of a pipe, a quantity of flow work Pv is associated with each pound of fluid; this energy quantity is independent of the velocity, temperature, or viscosity of the fluid. Flow work is present in constant quantity whenever steady continuous fluid flow exists at constant pressure and specific volume; when flow ceases, flow work no longer exists. The requirement of flow as an accompaniment to flow work sometimes leads to confusion in that the product Pv can have the same numerical value independent of the fact that motion of the pound of fluid may or may not exist. The numerical value of the Pv product is always in foot-pound units, but the term has significance as a measure of energy in transition *only* when steady continuous flow exists.

Flow work is energy in transition and not energy stored in the fluid. Therefore it is not possible to multiply the total weight of fluid in a pipe by the flow work associated with one pound to obtain the total energy in transition. Considering friction losses and heat transfer as negligible, the quantity of energy present as flow work (for an incompressible fluid) is the same at all cross sections of a pipe; the same not merely in numerical value, but literally the identical energy. The flow work at any given cross section of such a pipe is the total energy in transition for the entire column of fluid. Flow work can be considered as the energy input at the pipe entrance needed to inject the fluid, or an energy dissipation at the pipe outlet representing the work done in discharging the fluid against the discharge pressure.

Work during expansion or compression of a given weight of fluid in a cylinder is of basically different origin from work realized from flow ener transformed during admission or discharge. In Fig. 1·2a consider eight, w, of fluid in the cylinder. Assume that energy is added to t fl d, causing an expansion under constant pressure with consequent d placement of the piston to a new position. During the expansion fl can be considered as occurring through the cylinder cross section at whi

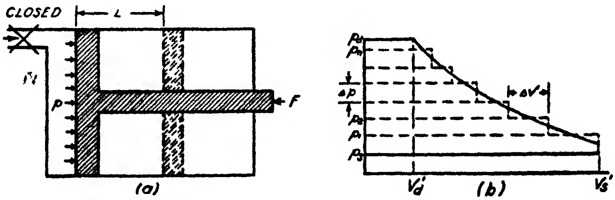

FIG. 1·2. Work during Expansion or Compression.

the piston was first located; the flow work dissipated in moving the pisto would be equal to $144pV'$ where V' is the volume of fluid passing the cro section. But V' is obviously equal to the difference between the final an original volumes of the fluid and the external work performed by the flui during constant pressure expansion is therefore equal to $144p(v_2 - v_1$ foot-pounds per pound of fluid.

When the pressure varies, as during the compression process in mechan cal refrigeration, the work can be determined by visualizing the com pression path as consisting of many short constant-pressure processes con nected by constant-volume processes (Fig. 1–2b). The work for such path is

$$W_{in} = 144[p_1(\Delta V') + p_2(\Delta V') + p_3(\Delta V') + \cdots + p_n(\Delta V')]$$

$$= 144 \sum_{p_1}^{p_n} p(\Delta V')$$

$$= -144 \int_{V_s'}^{V_d'} p dV' \qquad (1·1$$

where V_s' and V_d' are the volumes at beginning and end of compression anc the minus sign precedes the integral because the volume decreases during compression. Equation 1·1 gives the work in foot-pounds required during the actual process of compression (exclusive of admission or exhaust) anc indicates that this work can be numerically evaluated when a known func tional relationship exists between the pressure and volume.

Work equivalent of external latent heat is a special form of work during expansion. Whenever a fluid expands against constant pressure, as dur ing the evaporation of water to steam, external work is done and the energy equivalent of this work must either be extracted from storage in the fluid or supplied from an outside source. The specific volumes of water and steam at 212°F and atmospheric pressure (14.696 psia) are 0.01672 and 26.78 cu ft/lb respectively (from Keenan-Keyes *Steam Tables*), so the energy which must be supplied to 1 lb of such water to [illegible] the external work of expansion is 14.696 × 144 × (26.78 − 0.01672) = 56,700 ft-lb or 73 Btu. This energy is called the external latent heat of vaporization. It is not a quantity of energy which goes into storage in the fluid during evaporation but is energy passing through the fluid during the evaporative process; it is supplied to the fluid from an outside source (usually as heat) and dissipated by the fluid to an outside recei er (usually as work).

(*b*) HEAT. Energy transfer as heat occurs by conduction when flow is through a series of contiguous molecules and by radiation when the transfer is without reference to a tangible medium. A third mechanism of transfer, convection, need not be as heat since it may occur as a result of a purely mechanical mixing process rather than due to the temperature difference which defines an energy transfer as heat. Usually, however, all forms of energy exchange which occur when molecules at one temperature are physically transported into a region at different temperature are grouped under the term convective heat transfer. Equations defining the conditions under which a flow of energy may occur as heat are established in Chapter VI.

Unit of heat is the British thermal unit, Btu, defined as 1/180 of the quantity of energy transferred to 1 lb of water in raising its temperature (at standard atmospheric pressure) from the freezing to the boiling point; 1 Btu is approximately equal to the energy required to raise the temperature of 1 lb of water 1°F. Since heat and work are but two forms of energy, the arbitrarily chosen units must be related in a fixed way. The relationship, expressed as the mechanical equivalent of heat, is

$$1 \text{ Btu} = 778 \text{ ft-lb}$$

Thus,

$$778Q = W \quad \text{or} \quad JQ = W$$

where Q is a heat quantity expressed in Btu, W is the equivalent work quantity expressed in foot-pounds, and $J = 778$.

The rate of heat transfer, expressed in Btu per hour, is equivalent to the rate of work and is therefore an expression for power. Though the Btu per hour is the customary unit for power transfer as heat, equally correct expressions for rate of heat transfer in terms of horsepower or of watts may be used; thus,

$$1 \text{ hp} = 550 \text{ ft-lb/sec} = \tfrac{550}{778} \text{ Btu/sec}$$

Although the above equation expresses a quantitative relationship between units of energy transferred in different forms, it does not give any information concerning the possibility of achieving a transformation from one form to another.

1·2. Energy in Storage. Of the factors determining energy in storage, those which are of greatest importance include: velocity, gravity, internal molecular energy, chemical energy, capillarity, electricity, and magnetism. For problems in refrigeration and air conditioning, velocity, gravity, and molecular activity are the only ones that need be considered. A tangible body can store energy either in mechanical form, with respect to other tangible bodies, or in internal form by virtue of molecular activity within the body.

Mechanical energy, ME, consists of the sum of energies possessed by a tangible body owing to its position and velocity with respect to an arbitrary reference. For most problems in refrigeration and air conditioning, changes in the mechanical energy of the working substance are so small with respect to changes in internal energy that the former can usually be neglected. In the few cases where changes in either elevation or velocity are sufficient to suggest a need for consideration of mechanical energy the change (expressed as a reduction) of this energy quantity, expressed in foot-pounds per pound of working fluid, is given by

$$\Delta ME = \Delta Z + \frac{V_a^2 - V_b^2}{64.4} \text{ ft-lb/lb} \tag{1·2}$$

where ΔZ is the decrease in elevation in feet of the fluid between the two points a and b for which energies are being compared, and V is the velocity in feet per second measured at the same points.

Specific internal energy, u, like mechanical energy, consists of a sum of potential and kinetic energies. In this case, however, the potential energy term is with respect to the system of molecules within the tangible body; likewise the kinetic energy term refers to energy contained within the molecular system. For these reasons internal energy is actually stored *within* the single tangible body while mechanical energy is stored within a *system* consisting of the tangible body and some reference body, usually the earth.

The kinetic fraction of internal energy manifests itself as temperature. Any increase in the quantity of kinetic internal energy within a body results in a rise of temperature whereas a decrease is accompanied by a corresponding drop in temperature. The concept of an absolute temperature scale is based on the proportionality existing between change in the kinetic internal energy of a body and its degree of "hotness." The expression "sensible heat" refers to internal kinetic energy; it is so called because the temperature associated with storage of such energy permits recognition of its presence by the senses. Use of the word heat in connection with any form of internal energy is common but unfortunate since heat in the thermodynamic sense refers only to a particular form of energy in transit. Thus, strictly speaking, a hot body possesses not "heat," but kinetic internal energy; the energy which flows from such a hot body by virtue of temperature difference is, however, heat.

The potential fraction of internal energy is the energy storage required to maintain the molecules in their positions with respect to one another against the forces of mutual attraction. Because the forces responsible for potential internal energy are not subject to direct or rational evaluation in terms of the measurable properties of the tangible body, this fraction of internal stored energy is frequently called the internal latent heat.

The total internal energy of 1 lb of fluid, u, is the sum of the kinetic and potential fractions and is expressed in Btu per pound.

(c) Combination Energy Terms. The basic classifiable energy terms are those for energy in transition or storage. For convenience of calculation, additional relationships are established which represent combinations of one or more of the basic terms. The combined relationships permit necessary and desirable simplifications, but their use requires greater care because the physical significance of the energy quantities is not always self-evident.

Latent heat of vaporization, h_{fg}, is the total energy which must be supplied to accomplish evaporation of 1 lb of liquid at constant pressure and temperature. During the process of evaporation the quantity of energy stored in internal potential form increases and, simultaneously, a further quantity of energy must be temporarily supplied to the fluid in order to accomplish the external work of expansion. The latent heat of vaporization is equal to the sum of the energies which flow *into* the fluid (to storage) and *through* the fluid (to dissipation as external work). The term h_{fg} therefore represents energy in transition *during* the process of evaporation, but does not represent the gain in storage experienced by the fluid. The statement that steam at 212°F and 14.696 psia possesses a latent heat of 970.3 Btu/lb means that this quantity of energy had to be supplied to achieve evaporation, but it does not mean that the 970.3 Btu are present in 1 lb of steam; 73.0 Btu passed through the fluid during evaporation and were dissipated as work of expansion (see page 5), the remaining 897.3 Btu remain in the fluid as stored internal potential energy. Thus the latent heat is, strictly speaking, not a true energy term since it refers to real energy quantities *only* during the actual process of evaporation (or condensation); at all other times the latent heat does not refer to a particular energy quantity, but expresses a condition — addition or subtraction of energy — which would be realized if the fluid were undergoing either evaporation or condensation. The latent heat of vaporization is numerically equal to the increase in internal energy plus the work of expansion and can therefore be written,

$$h_{fg} = u_2 - u_1 + \frac{P}{778}(v_2 - v_1)$$

where the subscripts 1 and 2 refer to conditions of the fluid before and after evaporation. By regrouping terms,

$$h_{fg} = \left(u_2 + \frac{Pv_2}{778}\right) - \left(u_1 + \frac{Pv_1}{778}\right) = h_2 - h_1 = \Delta h \qquad (1{\cdot}3)$$

where h is the specific enthalpy, as defined below.

Specific enthalpy, h, is a combined energy term defined by the equation

$$h = u + \frac{Pv}{J}\ \text{Btu/lb} \qquad (1{\cdot}4)$$

The sum of internal energy and the Pv work term appear so frequently in thermodynamic calculations that selection of a single letter to represent it is a definite convenience; since enthalpy is defined in terms of properties of the fluid it must also be a property.

Enthalpy appears as an energy relationship in either of two basically different ways. First, if the fluid is not undergoing steady and continuous flow the enthalpy remains a property, but does not represent energy. Referring to equation 1·4, the first term, u, is stored internal energy, but the second term, $Pv/778$, is merely a product of properties of the fluid and happens to have the dimensions of Btu; it is neither energy in storage nor in transition. For the case of non-steady, non-continuous flow, $Pv/778$ is not an energy term and therefore enthalpy cannot be an energy term. However, the difference in enthalpies, Δh, between two conditions of the fluid for which the pressure is the same is seen from equation 1·3 to correspond to the latent heat of vaporization and therefore to have the same energy significance as does the term h_{fg}. In a later section it will be shown that the energy significance of Δh exists even when pressure is not constant provided there are no thermal or mechanical losses.

The second use of enthalpy occurs where the fluid is flowing steadily and continuously. In such cases the second term of equation 1·4, $Pv/778$, is equal to specific flow work; enthalpy is then a true energy term, but not a homogeneous one since it is made up of the sum of two terms one of which is for energy in storage and the other for energy in transition. Almost without exception, use of enthalpy in refrigeration problems is with respect to its significance as a combination energy term for steady and continuous flow conditions.

Specific heat is defined as the number of Btu which must be added to 1 lb of fluid under specifically defined conditions in order to bring about a temperature rise of 1°F. Innumerable different conditions may obtain at the time of heat addition so the specific heat can therefore have many different values.

When heat addition takes place without change in volume of the fluid no external work is done and all energy added as heat remains stored in the fluid as internal kinetic energy. The temperature increase of any material is proportional to the increase in internal kinetic energy so it follows that, per unit of heat added, maximum temperature rise will occur when the process of heat addition takes place at constant fluid volume. But specific heat is the inverse of temperature rise per unit of heat added, so the minimum specific heat will therefore be that which is realized during a constant-volume process of heat addition. This particular specific heat, c_v, is a true energy term since it relates the increase of stored energy to the increase of temperature. All other values of the specific heat, for different heat addition processes, take the form

$$c = c_v + c_w \tag{1·5}$$

where c is the specific heat for any given process, c_v is the specific heat of the same fluid for a constant-volume process, and c_w is the number of Btu expended as external work (or as change of potential internal energy) for each degree temperature rise of the fluid when heated by the process for which the specific heat is c.

The term c_w in equation 1·5 varies widely with the process which the fluid undergoes. Maximum external work is done when expansion occurs at constant pressure; for this condition, c_w is a maximum and the specific heat at constant pressure c_p therefore represents the largest obtainable value of c. Between c_p and c_v, there are an infinite number of possible values of c for a given fluid, but in the fields of refrigeration and air conditioning c_p and c_v alone constitute the values of specific heat which find application. Aside from c_v, specific heats are not true energy terms since they do not represent energy either stored in or passing through the fluid, but serve merely to indicate the energy addition which must have occurred at the time of the temperature rise; part of the energy then added, c_v, remains as kinetic internal energy stored within the fluid, but the balance, the c_w fraction, has usually passed from the fluid and been dissipated as external work. In many respects the specific heat resembles the latent heat in that it refers to an energy quantity which was of significance at the time of the process occurrence, but which is without meaning when applied to the fluid in its new condition.

FLUIDS, PROPERTIES, AND PROCESSES

1·3. Working Substances. Previous sections have investigated the storage, transfer, and transformation of energy. With the exception of energy transfer by radiation, all these occurrences take place in, or with respect to, some tangible medium which can be caused to pass through the series of processes constituting a thermodynamic cycle. The fluid in which, and on which, energy changes occur is the working substance. In the fields of refrigeration and air conditioning three basic types of working substances find use: gases, vapors, and gas-vapor mixtures.

In air conditioning, selection of the fluid is not left to the discretion of the engineer; the gas-vapor mixture of air with associated water vapor is the working substance which must be used. In refrigeration a choice is possible, but the thermodynamic and practical advantages of vapors so greatly outweigh those of the other two classes of materials that vapors are used almost exclusively in such systems. Paradoxically, the absence of a rational method of expressing vapor properties serves to simplify the calculations involved in analysis of such systems; because mathematical expression of the behavior of vapors is not accurately possible, tables and charts of properties have been prepared from which the engineer can directly obtain information which, for a perfect gas, he would have to calculate. Because of the almost universal acceptance of liquefiable vapors as refrigerating working substances the treatment throughout this section will be devoted exclusively to fluids of this type.

(*a*) Selection and Comparison of Working Substances. The selection of a particular liquefiable vapor for use as the working fluid in a given refrigerating system is based upon many considerations. Although not having a place in a textbook on engineering principles, such practical considerations as toxicity, stability, non-corrosiveness, low viscosity, fire and explosion hazard, high thermal conductivity, interaction effects with lubricants are all factors of great importance which may more than offset the thermodynamic advantages or disadvantages of a particular refrigerant.

Thermodynamically, the characteristics most desired in a refrigerant are low saturation pressure corresponding to the condenser temperature, high (preferably above atmospheric) saturation pressure corresponding to the desired evaporator pressure, high and low specific heats of the vapor and liquid respectively, large latent heat, a saturated vapor line which approaches as closely as possible an isentropic and a high coefficient of performance. In addition, the refrigerant should have a specific volume-specific enthalpy relationship such that the required piston displacement corresponding to a given load will be small.

Figure 1·3 shows saturation lines, drawn to uniform scales, for many of the common refrigerants. Examination of this figure brings out the wide range of characteristics and indicates that selection of a refrigerant on the basis of one particular characteristic may require acceptance of unsatisfactory conditions for some of the others. Ammonia, for example, has a large latent heat, but a saturated vapor line which departs widely from the isentropic (see *ph* chart for ammonia). Dichlorodifluoromethane and methyl chloride have more satisfactory saturated vapor lines but cannot compare with ammonia in terms of latent heat. Similar comparisons of other refrigerants and other thermodynamic characteristics will provide some of the data needed in making a selection. Complete data on the properties and essential characteristics of refrigerants are available in the

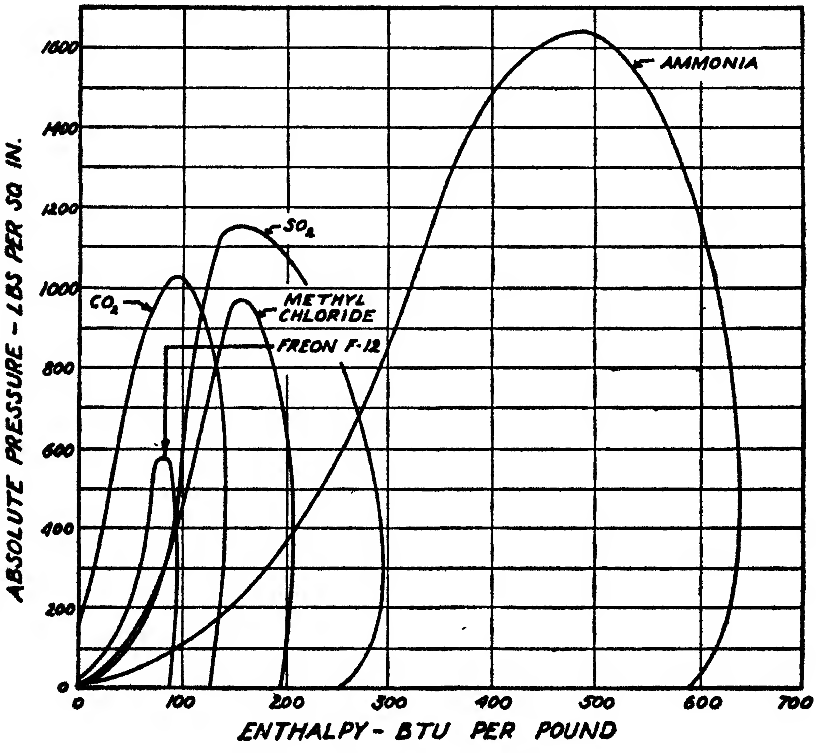

FIG. 1·3. Comparison of Refrigerants.

literature and should be studied carefully before a selection is attempted.

Based on a temperature range from 5°F to 86°F the coefficient of performance of the ideal reversible system (Carnot cycle efficiency) is 5.74. The coefficients of performance of some typical refrigerants, when used in a cycle without either superheating at compressor suction or subcooling at condenser discharge are given in Table 1·1 together with the percentage of Carnot performance realized (in the stated temperature range) by each refrigerant:

TABLE 1·1*

	COP	% CARNOT
Dichloroethylene	5.14	89.5
Dichloromethane	4.9	85.4
Propane	4.88	85.0
Methyl chloride	4.84	84.3
Monofluorotrichloromethane	4.82	84.0
Ammonia	4.76	82.9
Sulfur dioxide	4.73	82.4
Butane	4.63	80.6
Dichlorodifluoromethane	4.61	80.3
Water	4.10	71.4
Carbon dioxide	2.56	44.6

* For evaporation at 5°F; condensation at 86°F; saturation cycle.

The relative order of refrigerants with respect to coefficient of performance must not be overemphasized; many refrigerants having a low coefficient possess other qualities which may far outweigh this disadvantage, whereas refrigerants possessing a high coefficient may, for other reasons, be unsatisfactory.

1·4. Properties. The fundamental or measurable properties of a working substance are pressure, specific volume, and temperature. Composite or derived properties are:

a. Internal Energy. Since this term represents energy stored within the fluid when at a given state, it must be a property and is therefore independent of the process followed in reaching that state. For the working substances customarily used in compression refrigeration systems (liquefiable vapors) the functional relationship between internal energy and the measurable characteristics is complex; no mathematical statement of this relationship is needed, however, as tables and charts are available for all the common refrigerants and from such sources the numerical value of the internal energy can be read or indirectly calculated for any known condition of the fluid.

b. Enthalpy. As previously discussed, enthalpy is an energy term only during steady and continuous flow, but it is always a property of the fluid defined in terms of pressure, specific volume, and internal energy.

c. Entropy. This is a useful composite property the physical significance of which is unimportant in refrigeration problems. Likewise, the absolute value of entropy is not important, interest being centered on the entropy change which occurs in passing from one state to another. The change of specific entropy is defined by the equation,

$$ds = \left(\frac{dQ}{T}\right)\frac{1}{w} \quad \text{or} \quad \Delta s = \frac{1}{w}\int\frac{dQ}{T} \tag{1·6}$$

where w is the weight of fluid in pounds.

Equation 1·6 applies only when the state change occurs without friction and with uniformity of temperature (not necessarily constancy of temperature) throughout the fluid. The change of entropy in passing from one state to another is the same regardless of what reversible path or series of paths is followed; entropy is therefore fixed when the state of the working substance is fixed and it is a property of the fluid.

Work and *heat* represent transient forms of energy, are determined by conditions exterior to the fluid, and are not expressible in terms of the measurable properties of the fluid; work and heat are therefore not properties and consequently cannot be evaluated in terms of the initial and final states alone.

(*a*) Representation of Properties. Since each of the six properties of a working substance has a unique value at any given state, a knowledge of any two properties will suffice to establish the state and hence the numerical value of all six properties. The only exception to this statement occurs when the working substance is a non-homogeneous mixture of liquid and vapor; in this case charts and tables customarily show equivalent properties of the mixture which do not represent true properties at a real state, but fictitious values based on the ratio of weights of material present at each of the true states corresponding to a saturated liquid and a saturated vapor.

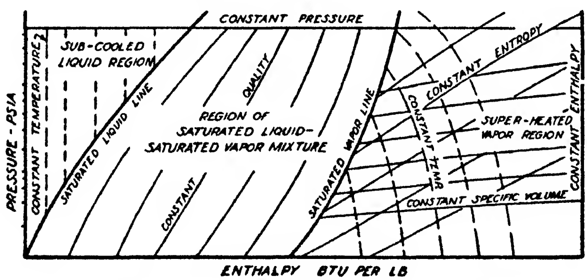

Fig. 1·4. Typical Pressure-Enthalpy Diagram for a Refrigerant.

Figure 1·4 shows a typical pressure-enthalpy (ph) chart of the type which will be used throughout this book. The saturated liquid line is the locus of states at which the fluid is in liquid form, but saturated with heat; flow of heat, while pressure is constant, to a fluid at a state on this line must result in evaporation. Similarly, the saturated vapor line is the locus of states for which the vapor is saturated with heat; heat flow, while pressure is constant, to such a vapor must result in superheating. In the regions to the left of the saturated liquid line and to the right of the saturated vapor line the fluid is present as a subcooled liquid or a superheated vapor, respectively.

The region between the two saturation lines includes the equivalent states which can be realized as a result of mechanically mixing saturated liquid with saturated vapor; in this region the pressure and temperature are not independent variables. With the above exception, a knowledge of any two properties will suffice to determine the state of the fluid at any point on the chart and hence permit evaluation — from the chart — of all other properties. Pressure, specific volume, temperature, enthalpy, and entropy are given on the chart; internal energy is usually not plotted but can be readily calculated once the pressure, specific volume, and enthalpy are known.

Other types of charts such as the *pv, Ts,* and *hs* are sometimes used, but for refrigeration problems the *ph* plotting has definite advantages.

1·5. Processes. A process is defined as an occurrence during which a working substance undergoes a change of state. When it takes place with little control, then fluid friction, turbulence, and heat transfer within the body of the fluid cause a variation of the properties throughout the mass and make it impossible to define the state at successive time intervals during the process. In such cases the fluid is not homogeneous, the change of state does not follow a continuous path, and the process is not subject to mathematical description or analysis; processes of this type are said to be irreversible.

When the state change occurs without fluid friction and with uniformity of all properties throughout the mass of the working substance, it becomes possible to express the state of the fluid at any point along the path; such a process is subject to complete definition, occurs with complete control, and is said to be reversible. Reversible processes are subject to mathematical description and analysis.

All real processes evidence some degree of non-homogeneity caused by mechanical or fluid friction. Real processes are therefore partially irreversible and are not subject to exact mathematical analysis. Fortunately, however, the degree of irreversibility is very small for most heat engine processes, and the analysis can be carried out as for a reversible process. The compression process in mechanical refrigeration usually approaches so closely to reversibility that it can be regarded as a reversible process and analyzed accordingly; the error resulting from this assumption is rarely great enough to be of any practical importance. Throughout this book all compression processes will be analyzed on the assumption that they are reversible.

(*a*) Polytropic Compression. Experience has shown that the compression process in mechanical refrigeration systems usually follows a path defined by the empirical relationship,

$$pv^n = \text{Constant} \tag{1·7}$$

where the exponent n depends upon the heat transfer from the cylinder during compression. Reversible processes which follow a path defined by equation 1·7 are said to be polytropic.

The work required during the process of polytropic compression can be evaluated by substituting the pressure-volume relationship of equation 1·7 in the basic work equation,

$$W_{\text{in}} = -144 \int_{v_s}^{v_d} p\,dv = -144pv^n \int_{v_s}^{v_d} \frac{dv}{v^n} = -\left.\frac{144pv}{1-n}\right|_{v_s}^{v_d}$$

$$= 144 \left(\frac{p_d v_d - p_s v_s}{n-1}\right) \text{ ft-lb/lb} \tag{1·8}$$

The total work required by a reciprocating compressor is equal to the work necessary for the process of compression plus that required to impart flow work to the leaving fluid less the flow work given up by the entering fluid. The entering and leaving flow work quantities are $144p_s v_s$ and $144p_d v_d$ respectively so the total work required by the compressor is

$$W_{\text{in}} = 144 \left[\frac{p_d v_d - p_s v_s}{n-1} + p_d v_d - p_s v_s\right] \text{ ft-lb/lb}$$

$$= \frac{144n}{n-1} (p_d v_d - p_s v_s) \text{ ft-lb/lb} \tag{1·9}$$

Equation 1·8 gives the work required, in foot-pounds, for the non-flow process of compression within a cylinder whereas equation 1·9 gives the total work required for the cycle of operations involving flow from line to cylinder, compression, and flow from cylinder into the discharge line. Solution of either equation requires a knowledge of the state at the start of compression, the discharge pressure, and either the specific volume at discharge or the polytropic exponent, the last two terms being functionally related by the defining equation of a polytropic process.

The problems usually encountered in compressor analysis require use of the polytropic equation for either of two determinations:

a. When the exponent n and the state of the refrigerant at the start of compression are known, analysis is necessary to determine the state of vapor at the compressor discharge. Applying equation 1·7,

$$p_s v_s^n = p_d v_d^n$$

so

$$v_d = v_s \left(\frac{p_s}{p_d}\right)^{1/n} \tag{1·10}$$

Use of equation 1·10 is facilitated by the graphical solution given in Fig. 1·5. If the pressure ratio p_d/p_s, the polytropic exponent n, and the specific volume v_s at the suction state are known, the graph gives directly the

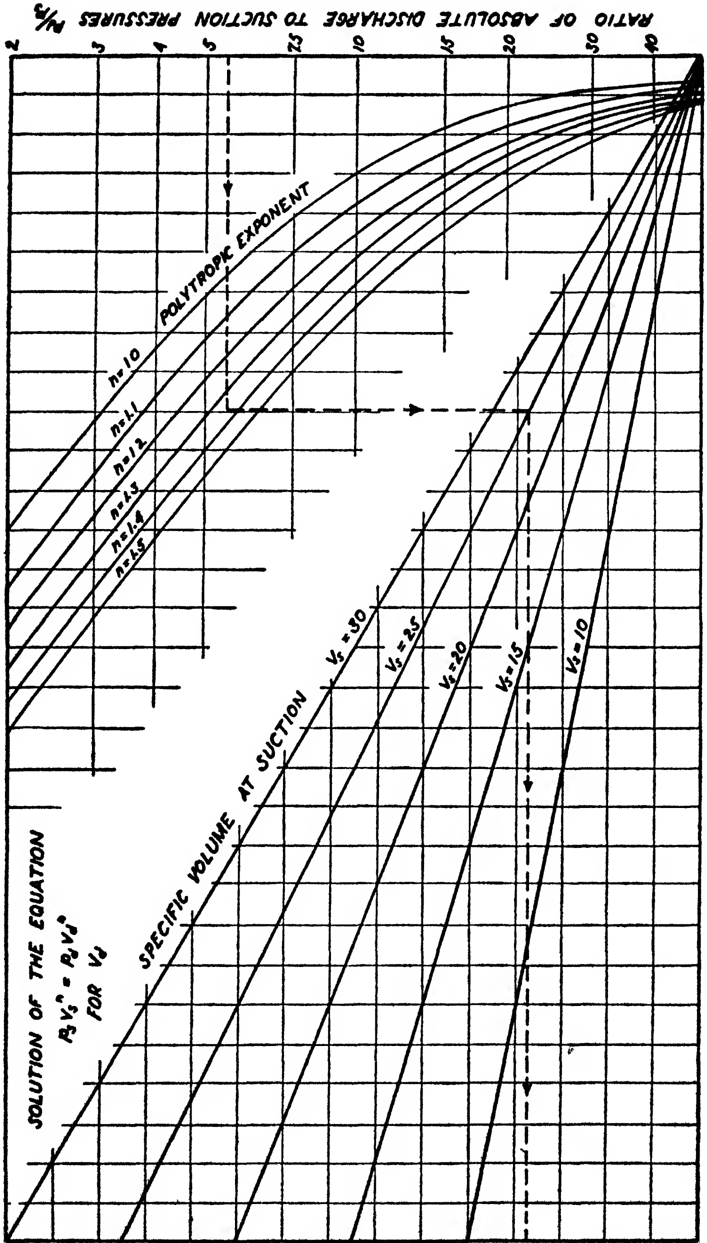

FIG. 1·5. Graphical Determination of State Point after Polytropic Compression.

specific volume at discharge from the compressor, v_d; p_d and v_d together define the discharge state.

b. In many problems the polytropic exponent is unknown, but the states at suction and discharge are determinable from experimental data. In such cases it is necessary to calculate the exponent in order to proceed with the analysis. Rewriting equation 1·7,

$$\left(\frac{v_d}{v_s}\right)^n = \frac{p_s}{p_d}$$

or

$$n = \frac{\log_{10}(p_d/p_s)}{\log_{10}(v_s/v_d)} \tag{1·11}$$

A graphical solution of equation 1·11 is given in Fig. 1·6. If the pressure ratio and the ratio of specific volumes before and after compression are known, the numerical value of the exponent for a polytropic path between the end points of the process is obtained directly from the graph.

ENERGY EQUATIONS

When the various forms in which energy can manifest itself have been established, it becomes necessary to fix relationships among those forms such that the engineer can determine the rate of transfer, storage, or transformation of energy in the course of a given process or operation. The equations relating energy quantities are of three kinds: (1) for a non-flow process, (2) for a process in which the fluid undergoes steady and continuous flow, and (3) for a process in which the fluid passes from a condition of steady continuous flow to one of non-flow, or vice versa. In the field of refrigeration the second, or flow equation, is the one that finds greatest use and on which the thermodynamic analysis of cycles is based; occasional use is also made of the non-flow equation for investigating and expressing conditions within the compressor during the compression stroke and of the flow to non-flow equation for establishing cylinder conditions at the start of a dual-effect compression.

1·6. The Simple (Non-flow) Energy Equation. Consider a given weight of fluid within, and just filling, a fixed horizontal cylinder fitted with a piston. Energy can be added to or removed from the fluid as heat or as shaft work and can enter or leave storage as a change of internal energy, but supply, removal, or change of storage of energy in this system cannot occur in any form other than heat, work, or internal energy. The *First Law of Thermodynamics* states that energy can neither be created nor destroyed, therefore an exact balance can be established between the energy transferred to or from the fluid and the increase or decrease in

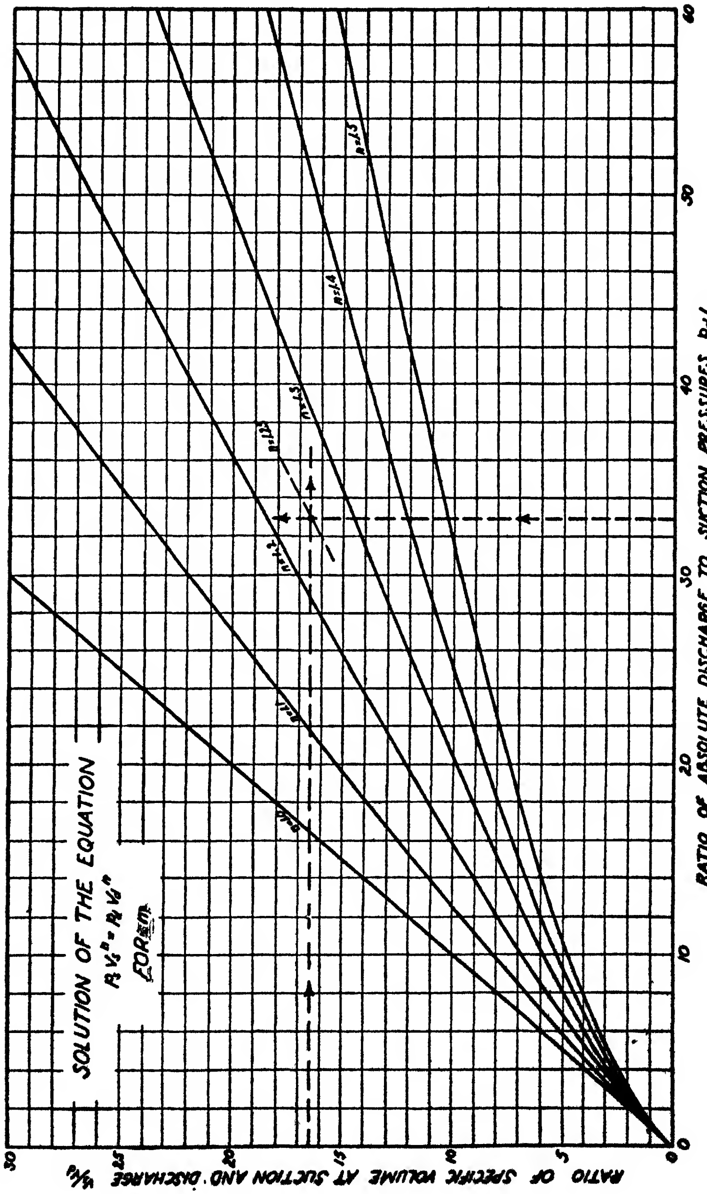

Fig. 1-6. Graphical Determination of Polytropic Exponent.

internal energy stored within the fluid. Thus,

$$Q_{in} + \frac{W_{in}}{J} = Q_{out} + \frac{W_{out}}{J} + w\,\Delta u \text{ Btu} \qquad (1\cdot12)$$

where Q is the heat transfer in Btu, W is the work transfer in foot-pounds, w is the weight of fluid (pounds) within the cylinder, and Δu is the increase in internal energy of the fluid in Btu per pound.

If equation 1·12 is applied to the process represented by compression, there must be a transfer of energy to the fluid as shaft work and, depending on the initial temperature, there will probably be a net transfer of energy, as heat, from the fluid. Then,

$$\frac{W_{in}}{J} = Q_{out} + w\,\Delta u \text{ Btu} \qquad (1\cdot12a)$$

Equation 1·12*a* is the usual form of the *Simple Energy Equation* for non-flow processes; it is a mathematical statement of the first law of thermodynamics, stating explicitly that when work is done on a fluid an equal quantity of energy must appear either as heat or as an increase of internal energy.

The principal application of the simple energy equation in refrigeration problems is in evaluation of the work required for compression. Unfortunately, this equation merely expresses a condition which must be met whenever an energy process occurs; it does not provide any information concerning the likelihood of a process taking place or the relationship between heat, work, and internal energy during such a process. These reasons make desirable the establishment of some functional relationship between the internal energy gain and the transfer of heat energy during the compression process. But heat, like work, depends on the process followed. Thus evaluation of heat and work both require knowledge of the compression process.

Three types of compression processes are customarily used in the analysis of mechanical refrigeration problems. They are:

1. When the compression is known to occur along a polytropic path for which the exponent is available, work of the process can be determined from equation 1·8 and the energy transfer as heat from the refrigerant during compression is then

$$Q_{out} = \left[\frac{144(p_d v_d - p_s v_s)}{J(n-1)} - (u_d - u_s)\right] w \text{ Btu} \qquad (1\cdot13)$$

where all terms on the right side of the equation are known or calculable.

2. When the initial and final states are known, but it is not known that compression follows a polytropic path:

a. The assumption may be made that compression is polytropic and the exponent then determined from Fig. 1·6; knowing *n*, work and heat are calculated as in the first case.

b. A more direct solution is obtained if it is assumed that compression follows a path along which the variation of temperature with entropy is a straight line; for most practical cases this assumption gives a result which very closely approximates that obtained by considering the compression process to be polytropic. When temperature varies as a linear function of entropy, the arithmetical average temperature, $(t_d + t_s)/2$, is equal to the mean temperature, t_m, acting during compression. But from equation 1·6,

$$Q_{\text{out}} = -w \int_{s_s}^{s_d} T ds \text{ Btu}$$

so by substitution of the mean temperature,

$$Q_{\text{out}} = -T_m w \int_{s_s}^{s_d} ds = (t_m + 460)(s_s - s_d)w \text{ Btu} \tag{1·14}$$

Since all terms in equation 1·14 are directly determinable from properties at the known suction and discharge states, solution of this equation can be carried out without determination of the polytropic exponent. Once Q_{out} is known, the work can be calculated from the simple energy equation,

$$W_{\text{in}} = w[(t_m + 460)(s_s - s_d) + (u_d - u_s)]J \text{ ft-lb} \tag{1·15}$$

The difference in accuracy of the approximations involved in methods *a* and *b* is so small that it can be considered insignificant for most practical problems; the selection of method is determined purely on a basis of convenience or facility in calculation. The second method has the advantage of greater simplicity and permits immediate determination of all necessary data from the properties at the two known end points. However, with the graphical solution of Fig. 1·6 available, the amount of calculation necessary to use the first method is not great. If a solution were to be obtained without help of the graphs, the second method would, in general, be found most rapid and direct.

3. The third type of process important in refrigeration analysis is that in which the compression occurs so rapidly that no significant transfer of energy as heat takes place. For a reversible process without heat transfer there is, from equation 1·6, no change in entropy; such a process is called an *isentropic*. For this case the simple energy equation reduces to

$$W_{\text{in}} = w(u_d - u_s)J \tag{1·16}$$

so the work required can be directly calculated from known values of the refrigerant properties at the end points of the process.

1·7. The General (Steady Flow) Energy Equation. When there is a steady flow of energy to or from a system, the first law of thermodynamics permits establishing an energy balance. Systems of the type used in refrigeration usually do not undergo a change in the quantity of stored energy once equilibrium has been established, and for such cases the energy balance can be obtained by equating the sum of all incoming energy quantities to the sum of all outgoing energy quantities. For systems in which there is no flow of working fluid the energy balance becomes

$$Q_{\text{in}} + \frac{W_{\text{in}}}{J} = Q_{\text{out}} + \frac{W_{\text{out}}}{J} \tag{1·17}$$

Equation 1·17 is applicable to the typical mechanical refrigeration system when considered as a whole. Energy enters as heat flowing to the evaporator and as work delivered to the compressor. Neglecting losses and assuming isentropic compression, the only energy leaving the system is as heat from the condenser. The balance therefore becomes

$$Q_{\text{in}} + \frac{W_{\text{in}}}{J} = Q_{\text{out}} \tag{1·17a}$$

which shows that the condenser in a refrigeration system must be capable of transferring a larger quantity of heat than the evaporator, since the condenser must dispose not only of the heat picked up by the evaporator, but also the heat equivalent of the work of compression.

When there is a flow of one or more working substances through the system for which an energy balance is to be established the equation must include terms to represent the energy stored in, or associated with, the fluids. For steady and continuous flow of one working fluid through a system at a weight rate of w pounds per minute, the general equation becomes

$$\frac{w}{J}\left(Z_1 + \frac{V_1^2}{2g} + P_1v_1 + Ju_1\right) + Q_1 + \frac{W_1}{J} =$$
$$\frac{w}{J}\left(Z_2 + \frac{V_2^2}{2g} + P_2v_2 + Ju_2\right) + Q_2 + \frac{W_2}{J} \tag{1·18}$$

or

$$\frac{w}{J}\left[(Z_2 - Z_1) + \frac{V_2^2 - V_1^2}{2g} + J(h_2 - h_1)\right] = \Delta Q_{\text{in}} + \frac{\Delta W_{\text{in}}}{J} \tag{1·18a}$$

where ΔQ_{in} and $\Delta W_{\text{in}}/J$ represent the net input of heat and shaft work each expressed in Btu per minute.

In most refrigeration problems the change in mechanical energy of the working fluid is small in comparison with other energy changes. For such cases equation 1·18a becomes

$$w(h_2 - h_1) = \Delta Q_{in} + \frac{\Delta W_{in}}{J} \tag{1·19}$$

or

$$wh_1 + Q_{in} + \frac{W_{in}}{J} = wh_2 + Q_{out} + \frac{W_{out}}{J} \tag{1·19a}$$

and it is in one or the other of these simplified forms that the *General Energy Equation* finds greatest use in the analysis of mechanical refrigeration processes and cycles.

Solution of equation 1·19 requires knowledge of a functional relationship between shaft work and either the heat transfer or the change of enthalpy. An expression for total work associated with a polytropic compression has been obtained in terms of the properties of the fluid at states 1 and 2 (equation 1·9); substituting this expression in equation 1·19 gives

$$w(h_2 - h_1) + \Delta Q_{out} = \left(\frac{144n}{n-1}\right)\frac{w}{J}(p_2v_2 - p_1v_1) \tag{1·20}$$

from which the quantity of heat flowing from the compressor cylinder during a reversible polytropic compression with exponent n can be determined as soon as the initial and final states are known. This equation is established for conditions of steady and continuous flow and must therefore be applied using reference points 1 and 2 at cross sections in the suction and discharge piping rather than in the compressor cylinder. An alternative solution of equation 1·19 is obtainable by using the approximate expression of equation 1·14 to evaluate the heat leaving the compressor. Application of the general energy equation to reciprocating machines is permissible only when the speed is sufficient to maintain a reasonable approach to steady and continuous flow conditions; this requirement is closely realized in compressors of the type and speed used in refrigeration.

For the case of isentropic compression Q_{out} becomes zero, and the steady flow energy equation simplifies to

$$\frac{W_{in}}{J} = w(h_2 - h_1)\ \text{Btu} \tag{1·21}$$

1·8. Flow to Non-flow Energy Equation. When flow occurs to or from a receiver, an energy balance can be established between the total energy possessed by 1 lb of fluid in the line under steady flow conditions and the same pound after it has come to rest in the receiver. By assuming that flow occurs without heat transfer,

$$Z_l + \frac{V_l^2}{2g} + Ju_l + P_lv_l = Z_r + Ju_r \tag{1·22}$$

where the subscripts l and r refer to line and receiver, respectively. In many cases the difference in elevation between line and receiver is negligible, and the velocity in the line is small in comparison with the flow work; when these conditions apply, equation 1·22 reduces to

$$Ju_l + P_l v_l = Jh_l = Ju_r \tag{1·22a}$$

which shows that flow from a line to a receiver is accompanied by an increase in the specific internal energy of the working fluid equal to the flow work which it possesses when in the line

$$\Delta u = \frac{P_l v_l}{J} = \frac{144 p_l v_l}{J} \tag{1·22b}$$

The increase is seen to be completely independent of the final receiver pressure, and the internal energy of the fluid in the receiver therefore has a fixed value for any receiver pressure up to that of the line. This fact can be most readily visualized by considering that the pressure drop occurs while the fluid is still in motion and that it takes place, in effect, through an adiabatic expansion valve. Under such conditions the enthalpy of fluid in the line would be constant and independent of the line pressure; the transformation of flow work from the moving fluid to internal energy in the stationary fluid would then occur at constant pressure.

The flow to non-flow equation does not apply to entrance of refrigerant into the cylinder of the usual type of compressor because in that case the flow work possessed by the fluid in the line is dissipated as shaft work performed by the entering fluid as it forces the piston through the cylinder. With multi-effect compression, however, the flow of high-pressure suction vapor into the cylinder usually occurs after the piston has almost reached dead center. For this vapor, no appreciable amount of energy leaves the cylinder as shaft work, and the flow work of the entering fluid therefore reappears as an increase of internal energy. This case will be investigated in detail in a later section on dual effect compression.

1·9. Energy Equation Examples.

Example 1·1. Saturated ammonia vapor enters a compressor at 25 psia and is discharged at 165 psia and 200°F. Assume polytropic compression and calculate: (*a*) work input during the compression *process;* (*b*) work input during the compression *cycle;* (*c*) rate of heat loss from the compressor. (All calculations to be in terms of a 1 lb/min refrigerant circulating rate.)

Solution. From *ph* chart for NH_3 (in envelope on rear cover) at $p_1 = 25$ and saturation, $v_1 = 11$ cu ft/lb; $t_1 = -8$°F, $h_1 = 609$, $s_1 = 1.35$. From chart at $p_2 = 165$ and $t_2 = 200$°F: $v_2 = 2.4$, $h_2 = 708$, $s_2 = 1.32$.

a. Pressure ratio = 165/25 = 6.60

Volume ratio = 11/2.4 = 4.58

From Fig. 1·6 read $n = 1.24$

By equation 1·8,

$$W_{\text{process}} = \left(\frac{144}{778}\right)\left[\frac{(165 \times 2.4) - (25 \times 11)}{(1.24 - 1)}\right] = 93.3 \text{ Btu/min}$$

b. By equations 1·8 and 1·9,

$$W_{\text{cycle}} = 1.24 \times 93.3 = 115.7 \text{ Btu/min}$$

The difference between the results of parts *a* and *b* is the net work done by the piston during constant-pressure admission and discharge.

Check. Net constant-pressure work $= \dfrac{p_2v_2 - p_1v_1}{778}$

$$= [(165 \times 2.4) - (25 \times 11)]\frac{144}{778}$$

$$= 22.4 \text{ Btu/min}$$

For this case the compression *cycle* requires $(22.4 \times 100)/93.3 = 24.0$ per cent more work than the compression *process*.

c. By equation 1·19,

$$Q_{\text{out}} = 115.7 - (708 - 609) = 16.7 \text{ Btu/min}$$

Example 1·2. Solve Example 1·1 on the assumption that the compression process occurs along a path for which the variation of temperature with entropy is linear.

Solution.

a. By equation 1·14,

$$Q_{\text{out}} = \left(\frac{200 - 8}{2} + 460\right)(1.35 - 1.32) = 16.7 \text{ Btu/min}$$

b. By equation 1·19,

$$W_{\text{cycle}} = (708 - 609) + 16.7 = 115.7 \text{ Btu/min}$$

c. $W_{\text{process}} = 115.7 - (165 \times 2.4 - 25 \times 11)\dfrac{144}{778}$

$$= 93.3 \text{ Btu/min}$$

The agreement between the answers of Examples 1·1 and 1·2 indicates that either assumption concerning the path of the compression process is acceptable.

Example 1·3. Saturated ammonia vapor (1 lb/min) is isentropically compressed to 165 psia from 25 psia and saturation. Calculate: (*a*) the cycle work, (*b*) the process work.

Solution.

a. From the *ph* chart at $p_2 = 165$ and $s_2 = s_1 = 1.35$, read $h_2 = 729$ and $v_2 = 2.55$. By equation 1·21,

$$W_{\text{cycle}} = 729 - 609 = 120 \text{ Btu/min}$$

b. $W_{\text{process}} = 120 - [(165 \times 2.55) - (25 \times 11)]\dfrac{144}{778}$

$$= 120 - 27 = 93 \text{ Btu/min}$$

Example 1·4. Saturated ammonia vapor is compressed from 25 psia to 165 psia at 275°F. Assume linear temperature-entropy relationship during compression and calculate: (*a*) work of the cycle, (*b*) work of the process, (*c*) compare results with those from Examples 1·2 and 1·3.

Solution. From the *ph* chart at $p_2 = 165$ and $t_2 = 275°F$ read $s_2 = 1.385$, $h_2 = 752$, $v_2 = 2.7$.

a. By equation 1·14,

$$Q_{\text{out}} = \left(\frac{275 - 8}{2} + 460\right)(1.350 - 1.385)$$

$$= -20.80 \text{ Btu/min}$$

By equation 1·19,

$$W_{\text{cycle}} = (752 - 609) - 20.20$$

$$= 122.2 \text{ Btu/min}$$

b. $$W_{\text{process}} = 122.2 - [(165 \times 2.7) - (25 \times 11)]\frac{144}{778}$$

$$= 122.2 - 31.5 = 90.7 \text{ Btu/min}$$

c. Comparison of results from Examples 1·2, 1·3, and 1·4 is shown in Table 1·2.

TABLE 1·2

	Example 1·2 (Cylinder Cooling)	Example 1·3 (Isentropic)	Example 1·4 (Cylinder Heating)
W_{process}	93.3 Btu/min	93.0 Btu/min	90.7 Btu/min
$W_{\text{adm. and discharge}}$	22.4 "	27.0 "	31.5 "
W_{cycle}	115.7 "	120.0 "	122.2 "
Q_{in}	−16.7 "	0.0 "	+20.8 "
Δh	99.0 "	120.0 "	143.0 "
Energy in	115.1 "	120.0 "	143.0 "

PROBLEMS

1. A liquid is passing through a pipe under conditions of steady and continuous flow. The pressure is 10 psia, but the specific volume of the fluid is unknown. Calculate the flow work in foot-pounds associated with each cubic foot of fluid.

2. Between cross sections *a* and *b* of a frictionless pipe line 1 lb of a refrigerant loses 2 Btu of internal energy but gains 1500 ft-lb of kinetic energy and 16 ft-lb of flow work. If there are no gains or losses of heat between cross sections, determine the increase or decrease in elevation of the pipe line.

3. A certain gas has specific heats at constant volume and at constant pressure of 0.2 and 0.3 Btu/(°F) (lb) respectively. If 40 Btu are added to 2 lb of gas during a constant-pressure process (155.6 psia), calculate the accompanying change in specific volume.

4. Work is done on the gas within a cylinder at the rate of 0.118 hp for each pound per minute of gas passing through the compressor. Heat loss from the cylinder amounts to 360 Btu/hr, and the specific enthalpy of the gas increases by 2 Btu. Calculate the pounds per minute of gas handled.

5. A gas at 25 psia has a specific enthalpy of 609 Btu/lb and a specific volume of 11 cu ft/lb. If this gas flows from a line to a receiver which is 77.8 ft lower than the line, calculate its specific internal energy on entrance to the receiver. (Neglect the line velocity.)

6. A gas having specific volume of 20 cu ft/lb is compressed polytropically ($n = 1.25$) from 16 psia to 32 psia. Determine the specific volume at discharge.

7. During compression a refrigerant is reduced in volume to one-fifteenth of its original value while its pressure is increased twenty-six times. Determine the polytropic exponent.

8. Add an additional row to Table 1·2 giving the polytropic exponent for each of the three examples.

9. Saturated ammonia vapor is compresesd polytropically ($n = 1.20$) from 25 psia to 165 psia. Calculate the work of the cycle and the heat flow to or from the cylinder. Compare results with the examples given in Table 1·2.

10. Solve the *b* part of illustrative Example 1·4 by use of equation 1·13 and compare result.

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CHAPTER II

FUNDAMENTAL CYCLES OF VAPOR REFRIGERATION SYSTEMS

2·1. The Basic Cycle. All vapor refrigeration systems, whether actuated by a reciprocating compressor, a centrifugal compressor, a steam jet, a secondary absorbing fluid, or an adsorbing material, operate because of a difference in pressure which permits the collection of heat by the refrigerant at a low saturation temperature and discard at a higher saturation temperature. The suction vapor pressure for any such system must have a value sufficiently low so that the corresponding saturation temperature will be below the temperature of the space to be cooled by a margin sufficient to establish adequate heat flow from the cooled space to the refrigerant. Similarly, the discharge pressure must be sufficiently high so that the corresponding saturation temperature will exceed the temperature of the available cooling medium by a margin ample to insure the necessary rate of heat transfer from the refrigerant to the cooling fluid. The requirement of all refrigeration systems is to provide means of establishing the necessary difference between the suction and discharge pressures p_s and p_d.

Regardless of what type of device is used to accomplish the pressure increase, three basic equipments are always found in refrigeration systems. These are the condenser, expander, and evaporator. Analysis of the thermodynamic processes occurring in each of these equipments can be accomplished by applying the general energy equation in the form established in equation 1·19:

a. For the condenser (with no work in or out and heat out only),

$$Q_{\text{out}} = w(h_{\text{in}} - h_{\text{out}}) \tag{2·1}$$

b. For the expander (with no heat in or out, no work in, and reversible expansion),

$$\frac{W_{\text{out}}}{J} = w(h_{\text{in}} - h_{\text{out}}) \tag{2·2}$$

Equation 2·2 indicates that a reversible adiabatic expansion of the vapor yields work which can be returned to the compressor, thereby reducing the overall energy requirements of the system. With the exception of the rarely used dense-air refrigerating system, reversible expansion is not used (because of the complexity of mechanical equipment required); instead, the refrigerant passes through an expansion valve irreversibly. The work term in equation 2·2 then becomes zero and the equation for such an irreversible expansion is

$$h_{\text{in}} = h_{\text{out}} \tag{2·2a}$$

c. For the evaporator (with no heat out and no work in or out),

$$Q_{\text{in}} = w(h_{\text{out}} - h_{\text{in}}) \tag{2·3}$$

d. When compression occurs isentropically, the energy equation for a reciprocating or centrifugal compressor is (equation 1·21),

$$\frac{W_{\text{in}}}{J} = w(h_{\text{out}} - h_{\text{in}}) \tag{2·4}$$

Examination of the above equations shows that complete analysis of the system is possible in terms of the enthalpies at entrance and exit from each equipment. Further, the ideal system is considered to have no piping losses, either of pressure or of heat; thus the enthalpy at exit from any piece of equipment is the same as at entrance to the following piece. Three values of enthalpy therefore provide all necessary data for analysis of the entire ideal cycle; these are:

h_s = enthalpy at compressor suction.
h_d = enthalpy at compressor discharge.
h_e = enthalpy leaving condenser, entering or leaving expansion valve and entering evaporator.

Since there are no pressure losses in the ideal system, the discharge pressure p_d must apply at all points from the compressor to the expansion valve and the suction pressure p_s at all other points in the system; this division of the refrigeration system into a " high side " and a " low side " is shown in Fig. 2·1*a*.

The starting point in analysis of a refrigeration system is a knowledge of the required suction and discharge pressures and the load which is to be handled. Load is commonly expressed in tons, T, where 1 ton is equal to a cooling rate sufficient to freeze 2000 lb per day of 32°F water to 32°F ice. Since the heat of fusion of water is 144 Btu/lb, a 1-ton load corresponds to a heat extraction rate of 144 × 2000 = 288,000 Btu/day = 12,000 Btu/hr = 200 Btu/min. If T is the load in tons carried by the evaporator, the weight rate of refrigerant passing through the system is given by

$$w = \frac{200T}{(h_s - h_e)} \text{ lb/min.} \tag{2·5}$$

and the work of compression (noting that 42.42 Btu/min = 1 hp) is

$$\text{hp} = \frac{w(h_d - h_s)}{42.42} = \frac{200T(h_d - h_s)}{42.42(h_s - h_e)} \tag{2·6}$$

The coefficient of performance of a refrigerating system is defined as the ratio of desired effect to the energy required to produce that effect,

$$\text{cop} = \frac{\text{Heat absorbed in evaporator}}{\text{Energy supplied to compressor}} = \frac{h_s - h_e}{h_d - h_s} \tag{2·7}$$

After a particular type of refrigeration system has been selected, many equipment arrangements are possible, giving different thermodynamic cycles of operation. The selection of a particular cycle depends on many factors such as availability of equipment, first cost, and complexity of control, and one object of system design is to determine that cycle which will give the highest coefficient of performance with reasonable first cost and practicable operating conditions. In the sections which follow, a series of possible thermodynamic cycles will be investigated, and the method of analysis for each cycle established. In all cases the discussion will be based on an ideal system for which:

1. There is no pressure change in condenser, evaporator, connecting piping, or through compressor valves.
2. There is no transfer of heat to or from the system at any point except in the condenser and evaporator.
3. Compression follows an isentropic path.

Once the method of cycle analysis has been established for ideal cycles, extension or correction to include actual cycles can be readily accomplished; such corrections will be discussed in a later section.

2·2. Simple Saturation Cycle. Referring to Fig. 2·1*a*, consider a system for which the suction and discharge pressures are known, vapor leaves the evaporator in a saturated state, and liquid leaves the condenser saturated. Once the principal state points have been established the complete cycle can be readily shown on the pressure-enthalpy diagram for the particular refrigerant and the analysis completed by use of data available from the *ph* chart. The cycle is plotted as follows.

(*a*) Compressor. The refrigerant at entrance to the compressor is known to be a saturated vapor at pressure p_s so its state on the *ph* chart (Fig. 2·1*b*) is on the constant-pressure line p_s and also on the saturated vapor line; the only point on the chart meeting these two conditions is the point of intersection *s*, which must therefore represent the state of the refrigerant at compressor entrance. Compression occurs isentropically so the path of the compression process can be drawn on the *ph* chart as a line of constant entropy starting at *s* and ending at intersection with the constant-pressure discharge line p_d. The intersection *d* establishes the state of the superheated vapor leaving the compressor. The enthalpies h_s and h_d are then read from the chart by traveling vertically downward (or upward) from the states s and d to the enthalpy scale.

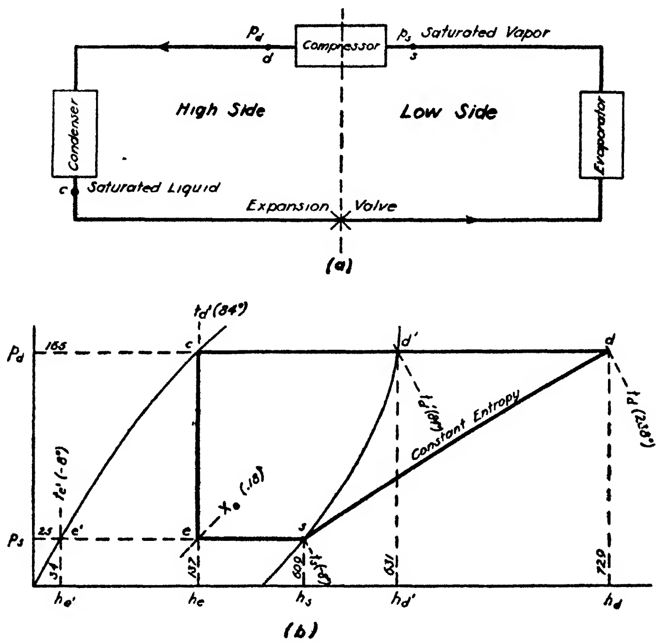

FIG. 2·1. Simple Saturation Cycle.

(*b*) CONDENSER. The vapor leaving the compressor at state d passes through the condenser at constant pressure. The vapor at d is superheated so the first process taking place in the condenser must be desuperheating to the saturated vapor state d' where the temperature has its saturation value $t_{d'}$, corresponding to p_d. From d' to c condensation occurs at constant pressure and constant temperature. Note that of the total heat extracted by the condenser, a substantial fraction,

$$\frac{h_d - h_{d'}}{h_d - h_e}$$

represents desuperheating rather than condensation.

(*c*) EXPANSION VALVE. By equation 2·2*a*, the expansion valve process occurs at constant enthalpy and is therefore shown on the *ph* diagram as a vertical line from state c to intersection with the suction pressure p_s. The intersection establishes the state e at entrance to the evaporator. The temperature at point e has the saturation value t_e, corresponding to

p_s, and can be read from the chart either at point s on the saturated vapor line or at point e' on the saturated liquid line.

At discharge from the expansion valve the refrigerant is partially vaporized, its quality (per cent present as saturated vapor) being given by the constant quality lines x which appear on some forms of ph charts (as on Fig. 1·4). Strictly speaking, points, as e, located between the saturated liquid and vapor lines, are not true states since the refrigerant is partly liquid and partly vapor and therefore is not homogeneous and does not possess uniform properties. However, equivalent properties as given by the ph chart for points in this region are advantageous in that they can be used in the solution of problems where knowledge of the true properties is unnecessary. At state e, for example, the chart gives an equivalent enthalpy of h_e. Actually, there is present at e, x_e pounds of saturated vapor at state s and $(1 - x_c)$ pounds of saturated liquid at state e'. The specific enthalpy of the mixture is

$$h_e = x_e h_s + (1 - x_e) h_{e'} \tag{2·8}$$

which can be solved for x_e, thereby permitting evaluation of the quality in terms of the known enthalpies at e', e, and s,

$$x_e = \frac{h_e - h_{e'}}{h_s - h_{e'}} \tag{2·9}$$

The "flash" or partial evaporation which occurs during passage through the expansion valve provides the cooling effect needed to reduce the temperature of the refrigerant from its value leaving the condenser, $t_{d'}$ to the evaporator temperature t_s. One pound of liquid refrigerant at state c possesses h_e Btu per pound, whereas liquid at state e contains only $h_{e'}$ Btu per pound; thus the required rate of heat removal to reduce 1 lb of saturated liquid from $t_{d'}$ to t_s is $(h_e - h_{e'})$ Btu. But the expansion takes place adiabatically so the enthalpies before and after expansion are the same; an amount of energy equal to that leaving the liquid must therefore appear as latent heat of vaporization in the vapor formed during expansion. The latent heat of vaporization of saturated vapor at p_s is $(h_s - h_{e'})$ Btu. Since there must be an energy balance between the loss of energy of the liquid and the gain of energy by the vapor,

$$x_e(h_s - h_e) = (1 - x_e)(h_e - h_{e'})$$

or

$$x_e = \frac{h_e - h_{e'}}{h_s - h_{e'}} \tag{2·9}$$

(*d*) **Evaporator.** The evaporator process takes place at constant pressure from state e to state s with an enthalpy gain of $(h_s - h_e)$ Btu per pound of refrigerant passing through the system. Heat gain in the evaporator is from the refrigerated space and the enthalpy difference $(h_s - h_e)$ is therefore referred to as the refrigerating effect. At state e a fraction of the refrigerant is already saturated vapor and therefore is incapable of providing any refrigerating effect. This fraction can be by-passed around the evaporator, as shown in Fig. 2·2, thereby reducing the weight of refrigerant passing through the evaporator to $(1 - x_e)$ pounds per pound circulating, but leaving the refrigerating effect unchanged at

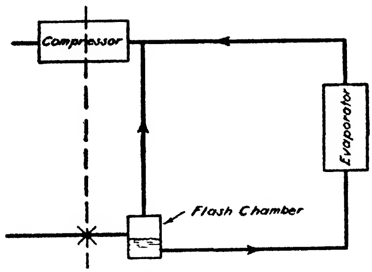

FIG. 2·2. Evaporator with Flash Chamber.

$$(1 - x_e)(h_s - h_{e'}) = (h_s - h_e) \text{ Btu/lb circulating} \qquad (2\cdot10)$$

The alternative arrangements of Figs. 2·1*a* and 2·2 have no effect on the thermodynamics of the cycle, but practical operating considerations frequently dictate selection of one method in preference to the other.

Example 2·1. A simple saturation cycle using ammonia carries a load of 100 ton. The suction and discharge pressures are 25 psia and 165 psia. Calculate: (*a*) the weight rate of circulating refrigerant, (*b*) the isentropic horsepower (*c*) the coefficient of performance, (*d*) heat transferred by the condenser, Btu/hr, and (*e*) quality of refrigerant leaving the expansion valve.

Solution.

a. By equation 2·5 (data from Fig. 2·1*b*),

$$w = \frac{200 \times 100}{(609 - 137)} = 42.4 \text{ lb/min}$$

b. By equation 2·6,

$$\text{hp} = \frac{42.4(729 - 609)}{42.42} = 120$$

c. By equation 2·7,

$$\text{cop} = \frac{609 - 137}{729 - 609} = 3.93$$

d. By equation 2·1,

$$Q_{\text{condenser}} = 42.4 \times 60(729 - 137) = 1{,}506{,}000 \text{ Btu/hr} \quad \textit{Ans.}$$

e. By equation 2·9,

$$x = \frac{137 - 34}{609 - 34} = 0.18$$

(*e*) SUCTION PRESSURE. The importance of maintaining the highest suction pressure consistent with carrying the required load cannot be overemphasized. Refer to Fig. 2·3 and consider a simple saturation system operating around the cycle *sdce* with cop $= (h_s - h_e)/(h_d - h_s)$. Suppose that the suction pressure is reduced to $p_{s'}$ with the new cycle becoming $s'd'ce'$. The coefficient of performance changes to

$$\text{cop} = \frac{h_{s'} - h_e}{h_{d'} - h_{s'}} = \frac{(h_s - h_e) - (h_s - h_{s'})}{(h_d - h_s) + (h_s - h_{s'}) + (h_{d'} - h_d)} \tag{2·11}$$

which shows that a reduction in suction pressure results both in a reduction in refrigerating effect and an increase in work of compression with consequent drastic reduction in the coefficient of performance.

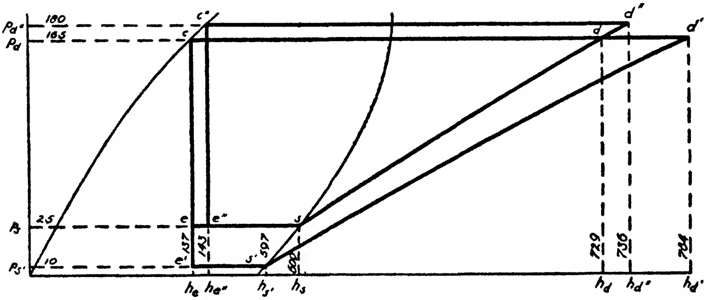

FIG. 2·3. Effect of Suction and Discharge Pressures on Performance.

Example 2·2. If the suction pressure in Example 2·1 were reduced from 25 psia to 10 psia, calculate the per cent change in the coefficient of performance.

Solution. By equation 2·11 (data from Fig. 2·3),

$$\text{cop} = \frac{597 - 137}{784 - 597} = 2.46$$

$$\text{Reduction} = \frac{3.93 - 2.46}{3.93} \times 100 = 37.4\%$$

(*f*) DISCHARGE PRESSURE. The discharge pressure is determined by the temperature of the cooling medium available for receiving heat discarded in the condenser. In Fig. 2·3, an increase in the discharge pressure from p_d to $p_{d''}$ results in changing the cycle from *sdce* to $sd''c''e''$ with a consequent reduction in refrigerating effect of $(h_{e''} - h_e)$ and an increase in the work of compression amounting to $(h_{d''} - h_d)$. The coefficient of performance for the new cycle becomes

$$\text{cop} = \frac{h_s - h_{e''}}{h_{d''} - h_s} = \frac{(h_s - h_e) - (h_{e''} - h_e)}{(h_d - h_s) + (h_{d''} - h_d)} \tag{2·12}$$

Comparing equations 2·11 and 2·12 the effect of decreasing the suction pressure is evidently more severe than that of increasing the discharge pressure since it increases the work of compression at both the suction and discharge ends of the compression path.

Example 2·3. If the discharge pressure in Example 2·1 is raised from 165 psia to 180 psia, calculate the per cent change in the coefficient of performance.

Solution. By equation 2·12 (data from Fig. 2·3),

$$\text{cop} = \frac{609 - 143}{736 - 609} = 3.67$$

$$\text{Reduction} = \frac{3.93 - 3.67}{3.93} = 6.6\%$$

A comparison of the results of Examples 2·2 and 2·3 shows that the effect of a $15/(165 - 25) = 10.7$ per cent change in the pressure range is more than five times as serious when the suction pressure is reduced as when the discharge pressure is raised.

2·3. Simple Subcooling Cycle. By installing a subcooler in the connecting piping between condenser and expansion valve (Fig. 2·4*a*) it is

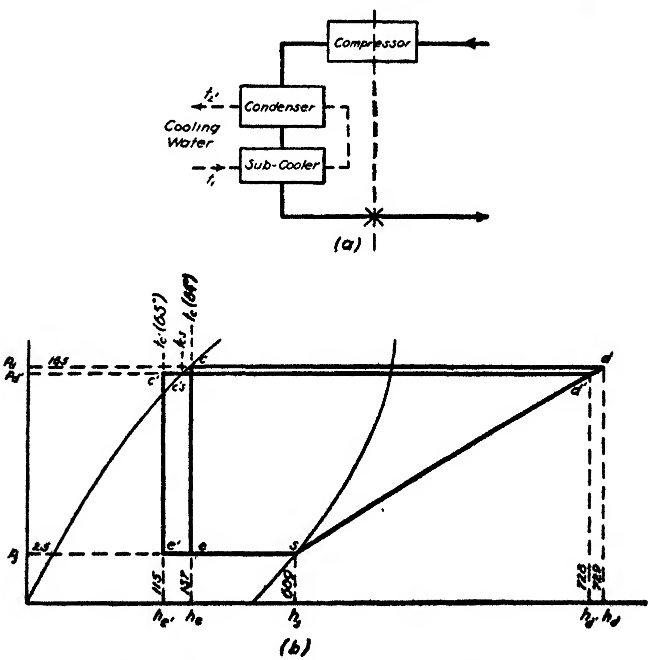

FIG. 2·4. Saturation versus Subcooling Cycle.

possible to reduce the temperature of the liquid refrigerant to within a few degrees of the temperature of the entering cooling water. This reduces the percentage of refrigerant lost due to flash in passing through the expansion valve and accordingly increases the refrigerating effect. The question arises, however, as to whether the extra cooling water circulated through the subcooler could not be used to better advantage by passing it directly through the condenser. To evaluate this possibility refer to Fig. 2·4*b* and consider the saturation cycle *sdce* and the subcooling cycle *sd'c'e'*, each of which is realized with the same total weight rate of cooling water; the water passes directly through the condenser for the saturated cycle, but through the subcooler (countercurrent) and the condenser in series for the subcooled cycle.

Consider that cooling water is available at a temperature t_1 and that the subcooler reduces the refrigerant temperature to within Δt degrees of t_1 so that

$$t_1 = t_{c'} - \Delta t$$

Similarly, the condensate in the saturation cycle will be considered as leaving at a temperature Δt greater than that of the exit cooling water (t_2); the exit temperature of cooling water in the subcooling cycle, $t_{2'}$, will be taken as Δt less than the condenser temperature. Then,

$$t_2 = t_c - \Delta t \quad \text{and} \quad t_{2'} = t_{c's} - \Delta t$$

The temperature rise of the cooling water in the two cycles is then,

For the saturating cycle

$$t_2 - t_1 = t_c - t_{c'} \qquad (2\cdot13a)$$

For the subcooling cycle

$$t_{2'} - t_1 = t_{c's} - t_{c'} \qquad (2\cdot13b)$$

Consider that the subcooling cycle is known and that it is desired to determine the coefficient of performance of the corresponding unknown saturation cycle which uses the same rate of cooling water. The cooling water rate for the subcooling cycle is

$$w = \frac{h_{d'} - h_{e'}}{t_{c's} - t_{c'}} \text{ lb/lb}$$

where $h_{d'}$, $h_{e'}$, $t_{c's}$ and $t_{c'}$ are all directly determinable from the *ph* chart (Fig. 2·4*b*).

For the unknown saturation cycle the refrigerating effect is reduced by $(h_e - h_{e'})$ and the rate of refrigerant circulation is then $(h_s - h_{e'})/(h_s - h_e)$ pounds for each pound used in the subcooling cycle. Since the total cooling water rate is the same in both cycles the rate per pound of refrigerant passing through the saturation cycle becomes $w(h_s - h_e)/(h_s - h_{e'})$ where all terms are known except h_e. But the cooling water required in the saturation cycle is also given by the equation $(h_d - h_e)/$

$(t_c - t_{c'})$ where h_d, h_e, and t_c are unknowns. Equating these two expressions for cooling water rate per pound of refrigerant circulating through the saturation cycle,

$$\frac{w(h_s - h_e)}{(h_s - h_{e'})} = \frac{(h_d - h_e)}{(t_c - t_{c'})} \tag{2·14}$$

Since all three unknowns of equation 2·14, h_e, h_d, and t_c are functionally related, selecting a value of the saturation temperature t_c permits direct reading of the corresponding values of h_d and h_e from the *ph* chart (see Fig. 2·4*b*). Equation 2·14 can therefore be solved by trial and error, assuming values of t_c and testing for equality.

When the saturation cycle is known and it is desired to determine the corresponding subcooling cycle, an equation similar to 2·14 can be derived,

$$\frac{w'(h_s - h_{e'})}{(h_s - h_e)} = \frac{(h_{d'} - h_{e'})}{(t_{c's} - t_{c'})} \tag{2·15}$$

where h_s, h_e, are known and w' can be calculated from the equation

$$w' = \frac{h_d - h_e}{t_c - t_{c'}} \tag{2·16}$$

(where $t_{c'}$ is equal to the entering water temperature plus Δt). The value of $h_{e'}$ in equation 2·15 is also known (since $t_{c'}$ is known) so that $h_{d'}$ and $t_{c's}$ are the only unknowns. However, these two terms are dependent so by assuming values of $t_{c's}$ and reading the corresponding values of $h_{d'}$ from the *ph* chart, the solution of equation 2·15 can be obtained by trial and error.

When the state points for each of the two cycles have been determined, the respective coefficients of performance can be calculated in the usual way. Though the relatively complex empirical relationship between vapor properties prevents obtaining a simple mathematical equation to show the effect of subcooling on performance, calculation of a few numerical problems will show that the subcooling cycle has a definite advantage and in every case results in a higher coefficient of performance. When the absolute cooling water rate is held constant, subcooling will decrease the condenser pressure, increase refrigerating effect and increase the coefficient of performance.

In actual systems the possibility of reheating the subcooled liquid refrigerant in the piping between subcooler and expansion valve makes it desirable to locate the subcooler just ahead of the expansion valve rather than next to the condenser. Where this is done, it may not be feasible to have the water leaving the subcooler pass through the condenser; for parallel operation of these two equipments the conditions assumed in the derivation of equations 2·14 and 2·15 should be re-examined.

Example 2·4. A saturation cycle operating between 25 psia and 165 psia uses cooling water at an entering temperature of 60°F. The water leaves the condenser at a temperature 5°F below that of the saturated liquid. If a subcooler were placed in series with the condenser and, by counterflow, the refrigerant were cooled to within 5°F of the entering water temperature, determine: (*a*) the condenser pressure for the subcooled cycle (water leaves the condenser 5°F below the unknown saturated liquid temperature), (*b*) the coefficient of performance for the subcooled cycle, and (*c*) the per cent change in the coefficient of performance resulting from use of subcooler.

Solution.

a. (Refer to Fig. 2·4*b* for data):

$$t_{c'} = 60 + 5 = 65°\text{F}$$

By Eq. 2·16,

$$w' = \frac{729 - 137}{84 - 65} = 31.15 \text{ lb water per lb } NH_3$$

By Eq. 2·15,

$$\frac{31.15(609 - 115)}{609 - 137} = \frac{h_{d'} - 115}{t_{c's} - 65} = 32.60$$

The above equation must be solved by trial and error.

If $t_{c's} = 86$, then $h_{d'} = 731$ and $\dfrac{731 - 115}{86 - 65} = 29.3.$

If $t_{c's} = 84$, then $h_{d'} = 729$ and $\dfrac{729 - 115}{84 - 65} = 32.3$

If $t_{c's} = 82$, then $h_{d'} = 726$ and $\dfrac{726 - 115}{82 - 65} = 35.95.$

The correct value of $t_{c's}$ must therefore be between 82° and 84° and can be determined by interpolation,

$$t_{c's} = 84 - \left(\frac{32.6 - 32.3}{35.95 - 32.3}\right)(84 - \overset{82}{80}) = 84 - 0.3 = 83.7°\text{F}$$

b. cop = (609 − 115)/(728 − 609) = 4.15.

c. $\text{Increase} = \dfrac{4.15 - 3.93}{3.93} = 5.6\%.$

2·4. Simple Superheating Cycle. Figure 2·5 shows the *ph* diagram *s'd'ce* for a cycle with superheated vapor entering the compressor, as compared with the simple saturation cycle *sdce*. Superheating can occur as a result of any one of three conditions:

1. Automatic control of the expansion valve so that the refrigerant will leave the evaporator at state *s'*. For this case the refrigerating effect is increased by $(h_{s'} - h_s)$ Btu per pound, and the required work of compression is increased appreciably since *s'd'* is less steep than *sd*. The co-

efficient of performance, as compared with a saturation cycle at the same suction pressure, can be greater, less, or unchanged, depending on the difference between p_s and p_d.

2. Pick-up of superheat by the vapor after leaving the evaporator, but through piping located within the cooled space. In this case the superheat also represents a gain in refrigerating effect, and the cycle is thermodynamically identical with that for superheat gain within the evaporator.

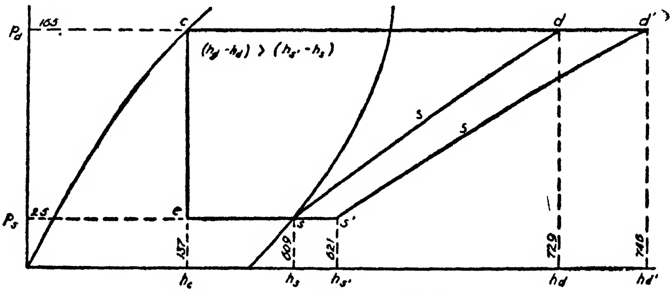

FIG. 2·5. Simple Superheating Cycle.

3. Pick-up of superheat in connecting piping outside of the cooled space. In th's case there is no increase in refrigerating effect, but an increase in work of compression and a consequent reduction of the coefficient of performance. Heat gain of this kind is also undesirable in that it increases the specific volume of the vapor at the compressor suction thereby increasing the required displacement; further, all heat entering the system in the low-pressure side must be discarded by the condenser so the added heat throws additional load on the condenser and increases the required supply rate of cooling water. If additional water is not available, or if the condenser does not have excess heat transfer surface, the effect of the added load may be to increase the discharge pressure with consequent increase in the work of compression and decrease in refrigerating effect.

When superheat occurs either in the evaporator or in piping within the cooled space, the suction pressure is somewhat lower than the value corresponding to a saturation temperature approaching that of the cooled space. Immediately the question arises as to whether or not better economy could be realized by increasing the suction pressure until the saturation temperature corresponds to the temperature of vapor leaving the evaporator in the superheated cycle. The extent to which superheat can be abandoned in favor of a higher suction pressure depends on many practical operating considerations, but best economy is usually realized when a saturation cycle is used to replace a cycle operating with superheat at the evaporator exit.

Example 2·5. A system operates between 25 and 165 psia with saturated liquid from the condenser and vapor leaving the evaporator with 20°F of superheat. Compare th^ coefficient of performance with that of the simple saturation cycle.

Solution. By equation 2·7, with data from Fig. 2·5,

$$\text{cop} = \frac{621 - 137}{746 - 621} = 3.87$$

$$\text{Reduction} = \frac{3.93 - 3.87}{3.93} = 1.52\%$$

2·5. Subcooling with Vapor. Previous sections have shown the advantage of subcooling and the disadvantage of superheating. In this section attention will be directed to the possibility of achieving subcooling of the

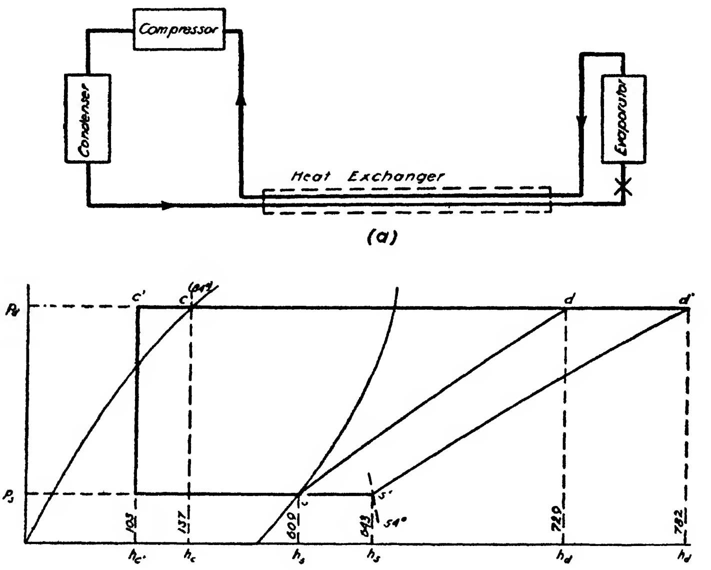

FIG. 2·6. Subcooling with Vapor.

liquid leaving the condenser by passing it through a heat exchanger parallel to the vapor leaving the evaporator. Figure 2·6*a* shows a similar equipment arrangement, but for counterflow, and Fig. 2·6*b* gives the *ph* chart for the parallel flow cycle. The refrigerant leaves the condenser at state *c*, temperature t_c and passes through a heat exchanger supplied with saturated vapor from the evaporator at temperature t_s. In the ideal case, both fluids leave the exchanger at a common temperature t_m. Since the specific heat of the vapor is less than that of the liquid, the temperature rise of the vapor is greater than the temperature reduction of the liquid. The weights of liquid and vapor are the same, so by an energy balance,

$$h_{s'} - h_s = h_c - h_{c'} \tag{2·17}$$

but

$$t_{s'} = t_{c'} \tag{2·17a}$$

The above equations can be solved by trial and error thereby establishing states c' and s'. The power required for this cycle is

$$\text{hp} = \left(\frac{200T}{h_s - h_{c'}}\right)\left(\frac{h_{d'} - h_{s'}}{42.42}\right) \tag{2·18}$$

and the loss over the corresponding simple saturation cycle is

$$\text{hp}_{\text{loss}} = \frac{200T}{42.42}\left(\frac{h_{d'} - h_{s'}}{h_s - h_{c'}} - \frac{h_d - h_s}{h_s - h_c}\right) \tag{2·19}$$

Even with a theoretical loss resulting from vapor precooling, there are many actual installations in which the use of precooling is economically justifiable because of superheating which would otherwise occur in the piping between the evaporator and compressor; if such superheating cannot be avoided, use of a precooler will increase the refrigerating effect while reducing both the compressor and condenser loads.

Example 2·6. A simple saturation system operating between 25 and 165 psia uses vapor for subcooling. Calculate the effect of the subcooling on the coefficient of performance.

Solution (Data from Fig. 2·6*b*). Assume $t_{c'} = 54$; then $h_{c'} = 103$, $t_{s'} = 54$, $h_{s'} = 643$, $h_{d'} = 782$. By equation 2·17,

$$643 - 609 = 137 - 103$$

or

$$34 = 34$$

so the assumed $t_{c'}$ is correct.

$$\text{cop} = \frac{609 - 103}{782 - 643} = 3.64$$

$$\%\ \text{change} = \frac{3.93 - 3.64}{3.93} = 7.38\%\ \text{reduction}$$

2·6. Subcooling with Liquid Refrigerant. An apparent method of achieving maximum subcooling is represented by the equipment arrangement of Fig. 2·7*a* in which liquid leaving the condenser is cooled to the evaporator temperature by refrigerant diverted from the downstream side of the expansion valve; the *ph* chart (Fig. 2·7*b*) shows the thermodynamic cycle. The weight of refrigerant required for precooling is $w' = (w + w')(h_c - h_{c'})/(h_s - h_{c'})$ which reduces to $w' = w(h_c - h_{c'})/(h_s - h_c)$, where

w is the weight of refrigerant passing through the evaporator. This equation shows that the weight of refrigerant needed in the precooler is exactly equal to the weight of flash that would occur in a simple saturation cycle operating between the same pressure limits. Obviously, therefore, no advantage accrues from use of such an arrangement; it is thermodynamically identical with the simple saturation cycle. Note however that the appearance of this cycle on the *ph* chart (Fig. 2·7*b*) is deceptive.

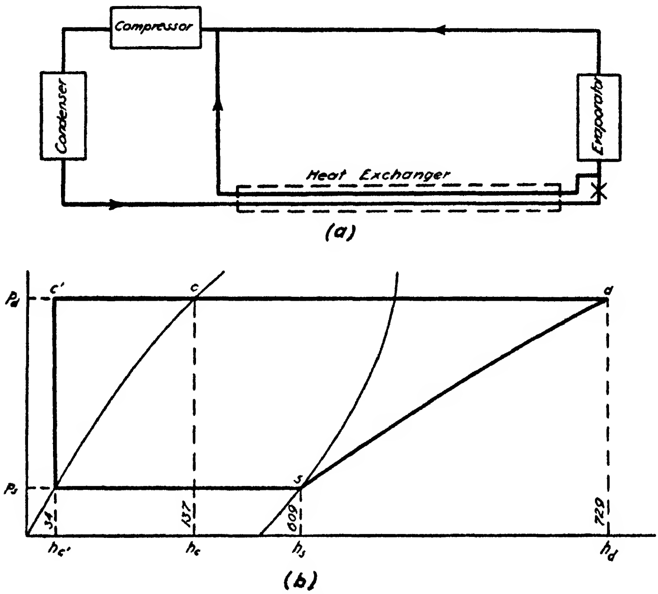

FIG. 2·7. Subcooling with Liquid Refrigerant.

2·7. Clearance and Conventional Volumetric Efficiency. Selection of a compressor to carry the load of a given system requires knowledge of the total volume of vapor to be handled at suction conditions and determination of the volume of suction vapor that will enter the cylinder during the admission period of each stroke. When the weight rate of refrigerant passing through the system has been determined, the specific volume at the suction state can be read from the *ph* chart, and the volume of vapor to be handled per unit time can be calculated immediately. If the volume of vapor entering the cylinder per stroke were known, the required operating speed could then be directly determined.

Refer to Fig. 2·8 and consider a compressor constructed so that there is no appreciable clearance between the cylinder end and the piston when the latter is at the end of its stroke. As soon as the piston starts moving, vapor will be drawn in from the suction line, and this admission process will continue until the piston reaches the other end of its stroke. The volume admitted will be equal to the piston displacement and can be represented by the line *es* of Fig. 2·8*a*. Compression will then occur from *s* to *d* and discharge of the high-pressure vapor from *d* to *d'*. Since no vapor remains in the cylinder at the end of the discharge stroke, the pressure ahead of the piston will instantaneously drop to p_s when the return stroke starts. The line *d'e* has no thermodynamic significance.

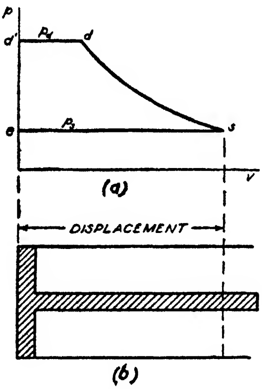

FIG. 2·8. Zero Clearance.

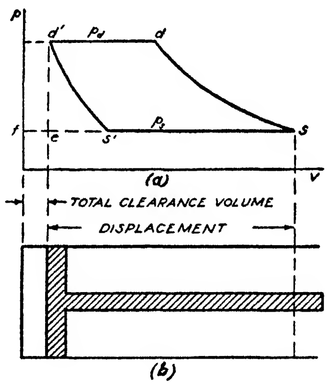

FIG. 2·9. Clearance and Re-expansion.

All compressors, to a greater or less degree, must provide some clearance between the dead center piston position and the end of the cylinder. In addition, there is certain to be some space in ports, valve pockets, and clearance chambers which will remain filled with high-pressure vapor after the discharge stroke has been completed. For convenience of representation Fig. 2·9*b* shows a cylinder with clearance and with an additional equivalent clearance volume to represent the effect of ports, valve pockets, and clearance chambers. The total clearance volume is shown as *ef* in Fig. 2·9*a* while the actual piston displacement is represented by a volume corresponding to line *es* in the same figure. For this operating cycle, compression would occur from *s* to *d*, discharge from *d* to *d'*, re-expansion of the vapor trapped in the clearance chamber from *d'* to *s'*, and admission of vapor from the suction line from *s'* to *s*. Thus the volume of fresh vapor handled during each stroke would be only a fraction, *s's*/*es* of the compressor displacement.

Clearance, C, of a refrigerant compressor is defined as the percentage of piston displacement which is represented by the total clearance volume; the clearance in Fig. 2·9 is

$$C = \frac{100(ef)}{es} \tag{2·20}$$

Conventional volumetric efficiency, CVE, is defined as the percentage of piston displacement which is represented by the volume of fresh vapor drawn into the compressor at each stroke; in Fig. 2·9 the conventional volumetric efficiency is

$$\text{CVE} = \frac{100(s's)}{es} = \frac{100(s's)}{D} \tag{2·21}$$

where D is the displacement in cubic feet per stroke and is known for a given compressor.

Evaluation of the conventional volumetric efficiency requires determination of the volume $s's$ as a function of the displacement and clearance, both of which are known characteristics of a given compressor. But $s's$ is the volume of fresh vapor admitted and must be equal to the total equivalent cylinder volume less the volume of re-expanded clearance vapor,

$$s's = fs - fs' = D\left(1 + \frac{C}{100}\right) - fs'$$

Since the weight of vapor in the cylinder does not change between states d' and s' an equality can be written between the quotient of total volume at each of these states and the respective specific volumes

$$\frac{fe}{v_{d'}} = \frac{fs'}{v_{s'}}$$

The state d' is identical with d (since $d'd$ represents a discharge of fluid from the cylinder rather than a change of state of a constant weight of fluid) so $v_{d'} = v_d$; similarly, $v_{s'} = v_s$. Then by substituting for $v_{d'}$ and $v_{s'}$ and solving for fs',

$$fs' = fe\left(\frac{v_s}{v_d}\right) = C\left(\frac{v_s}{v_d}\right)\left(\frac{D}{100}\right)$$

and

$$ss' = \frac{D\left[(100 + C) - C\left(\frac{v_s}{v_d}\right)\right]}{100}$$

so

$$\text{CVE} = 100 + C\left(1 - \frac{v_s}{v_d}\right) = 100 - C\left(\frac{v_s}{v_d} - 1\right) \tag{2·21a}$$

Equation 2·21*a* gives the conventional volumetric efficiency in terms of the known clearance and of the specific volumes at entrance and discharge from the compressor; from the suction and discharge states on the *ph* chart the specific volumes can be read directly. Then dividing the total volume of vapor to be handled per unit time by the conventional volumetric efficiency will give the total swept volume per unit time; if displacement is known, the speed can then be calculated; if speed is selected, the required displacement can be calculated.

Clearance, in the ideal cycle, has no effect on the power required to carry a given load. Re-expansion of the clearance vapor is considered as taking place isentropically with a return to the piston of an amount of work, per pound of clearance vapor, equal to that required for its compression. The principal disadvantage of clearance is that it reduces the capacity corresponding to a fixed displacement. For a given clearance the capacity decreases progressively as the pressure range becomes greater until a point is reached at which the compressor will not discharge, but will continuously re-expand and recompress a fixed quantity of vapor (with theoretically zero power).

In actual systems clearance is likely to have some effect on the required power because the re-expansion curve may not have the same polytropic exponent as the compression curve. When re-expansion occurs with a net heat gain by the vapor from the cylinder walls, the re-expanded clearance volume is greater than that indicated by the assumption of isentropic re-expansion; the conventional volumetric efficiency for this system is therefore less than for the ideal system. Thus the capacity of the plant is reduced, but the net power required is also less since the work returned during re-expansion is greater, on a per pound basis, than that required during the isentropic compression. The work delivered during a re-expansion with net heat flow to the vapor exceeds work delivery during isentropic re-expansion by the shaded area of Fig. 2·10. Conversely, if there were a net heat flow from the vapor to the cylinder the work delivered during re-expansion would be less than if the process occurred isentropically.

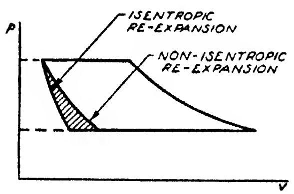

FIG. 2·10. Polytropic Re-expansion.

PROBLEMS

1. Twenty thousand pounds of milk are to be cooled to 40°F from an initial temperature of 85°F. If the cooling must be acomplished in a 1½-hr period, how many tons of refrigeration are required? (Specific heat of milk = .90.)

2. If the load of problem 1 occurs at 12-hr intervals, but if refrigeration is accomplished by using brine which is cooled continuously, calculate the tons of refrigeration required.

3. A plant was designed for an evaporator temperature of 5°F and a temperature of liquid ammonia to the expansion valve of 80°F. Changing conditions necessitate a liquid ammonia temperature of 60°F. (*a*) Calculate the percentage gain in refrigerating effect. (*b*) Determine the number of degrees of liquid cooling required to bring about a 1 per cent increase in refrigerating effect.

4. A plant designed to manufacture 100 tons of ice in 24 hr operates with one double-acting, single-cylinder, single-effect compressor the stroke of which is equal to 1.65 times the piston diameter. The compressor speed is 60 rpm, its volumetric efficiency is 75 per cent, and the mechanical efficiency of compressor and motor (as one unit) is 85 per cent. The cost of power is 0.9¢ per kwh. Losses of refrigerating effect as a result of heat infiltration, etc., are such that the plant requires 1.7 tons of refrigeration to make one ton of ice. Cooling water gains 10°F in passing through the condenser. The compressor discharges ammonia vapor at a gage pressure of 185.3 lb/sq in. Between compressor and condenser the vapor is cooled 20°F as a result of heat losses from the piping. The absolute suction pressure is 36 lb/sq in., and the ammonia vapor is superheated 25°F between the evaporating coils and the compressor. Liquid ammonia leaves the condenser at 80°F. Assume isentropic compression.

a. Determine the temperature of the ammonia vapor discharged from the compressor and the amount of heat removed: (1) to extract heat of superheat, (2) to liquefy the ammonia vapor, and (3) to cool the liquid ammonia.

b. Determine the useful refrigeration per pound of ammonia and the loss due to superheating in the suction line.

c. Calculate: (1) pounds of ammonia circulated per minute per ton of refrigeration, (2) pounds of ammonia circulated per minute for each ton of ice per day, (3) volume of ammonia vapor passing through compressor suction per minute for each ton of ice per day.

d. Determine the number of gallons per minute of cooling water required for the plant. If the heat losses between compressor and condenser were increased by 50 per cent, what consequent per cent reduction would occur in cooling water requirements?

e. Determine diameter and stroke of compressor. Determine power requirements in horsepower per ton of refrigeration and the size of motor (hp) required to drive the compressor.

f. Calculate the compressor clearance.

g. Neglecting maintenance and repair and assuming a 20 per cent annual return on investment, determine the permissible cost of improvements to the plant (insulation, etc.) which would reduce the refrigeration requirements from 1.7 tons/ton to 1.4 tons/ton.

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CHAPTER III

COMPLEX COMPRESSION CYCLES FOR SINGLE-LOAD SYSTEM

The use of a complex cycle in a system carrying a single evaporator load can rarely be justified (economically), except where compound compression with intercooling is used. In practice, however, problems frequently arise in which combinations of loads can be carried in such a way that the higher evaporator pressure becomes the intermediate pressure of the low-pressure cycle. Thus, although the multi-compression cycles analyzed in this chapter might well be impractical in a single evaporator system, they could be used to advantage if two or more evaporator temperatures were needed. The equations developed for a single evaporator system are equally applicable to combined systems and have the advantage over specific combined equations that they remain valid for all possible combinations. Typical combined systems are described and analyzed in section 3·7.

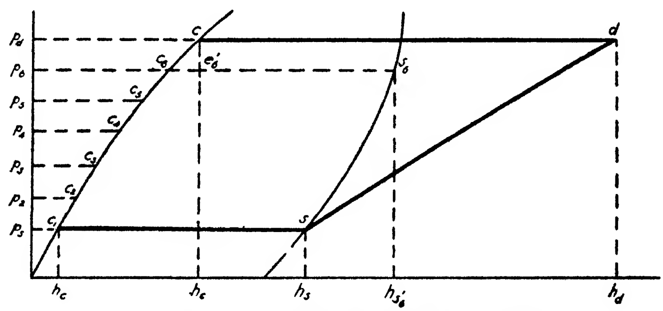

FIG. 3·1. Multiple Expansion Valves.

3·1. Sectionalized Compression with Multiple Expansion Valves. For the simple saturation cycle the work of compression increases as the difference between suction and discharge pressures. Although both pressures are fixed by operating conditions there remains a possibility of reducing the pressure range of that fraction of the refrigerant which flashes during passage through the expansion valve. In the *ph* diagram of Fig. 3·1, as soon as refrigerant starts through the expansion valve, vaporization begins. When the pressure has dropped the finite increment from p_d to p_6, a small fraction of the refrigerant is present as saturated vapor (at s_6'), and it is no longer capable of providing a refrigerating effect. If it were possible to extract this vapor and recompress it, the required work of compression would obviously be much less than is the case when this material is first expanded to p_s and then recompressed through the entire range.

If the saturated vapor were extracted at s_6', the refrigerant remaining would be a saturated liquid at state c_6. Allowing this material to expand through the finite pressure difference from p_6 to p_5 and again extracting the saturated vapor would leave saturated liquid at state c_5. Continuing the process of pressure reduction by finite intervals would lead to subsequent states of the remaining refrigerant at c_4, c_3, c_2, etc. If the pressure

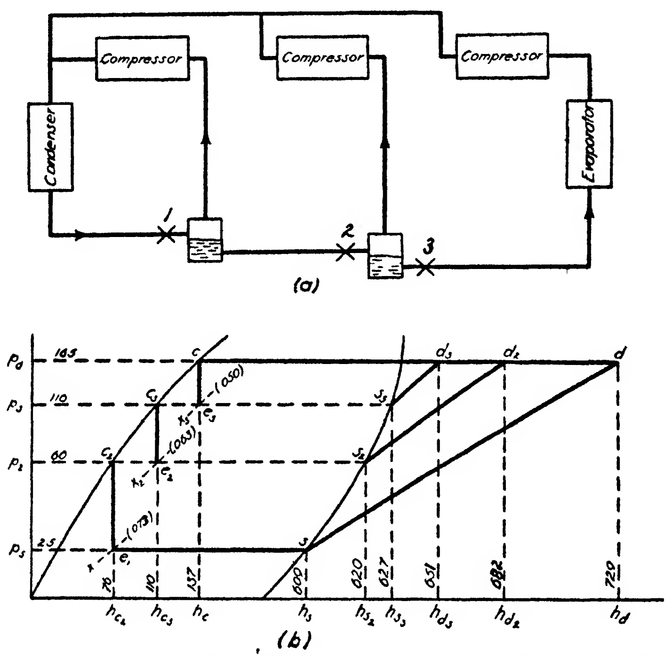

FIG. 3·2. Sectionalized Compression with Multiple Expansion Valves.

stages became infinite in number, saturated vapor would be continuously extracted during expansion, and the remaining liquid would then follow a path represented by the saturated liquid line. For sectionalized compression with an infinite number of stages the path followed by that fraction of the circulating refrigerant which passes through the evaporator would be $sdcc_1s$. The thermodynamic cycle for sectionalized compression with infinite stages resembles in many respects the cycle for regenerative feed water heating (in a steam power plant) with an infinite number of heaters.

In practice, continuous extraction of saturated vapor is obviously impractical, and it is rare that the first cost of more than two or three stages of compression can be justified. Figure 3·2*a* shows an equipment arrangement for a three-unit compression system where the only effect of the sectionalizing is to take advantage of the extraction of saturated vapor during expansion. Each pound of saturated liquid leaves the condenser at state c and passes through expansion valve 1 where its pressure is reduced from p_d to p_3; leaving the expansion valve at state e_3 (Fig. 3·2*b*) the material has a quality x_3. The saturated vapor is extracted at s_3, and the remaining saturated liquid, $(1 - x_3)$ pounds at state c_3, then passes through expansion valve 2 where its pressure is reduced to p_2. Leaving this valve at point e_2, with quality x_2, the saturated vapor is extracted at point s_2, and the remaining weight of saturated liquid, $(1 - x_2)(1 - x_3)$ pounds, at state c_2, then passes through the last expansion valve where its pressure is dropped to p_s and its new state, at entrance to the evaporator is e_1, with quality x_1. This material then passes through the evaporator providing a refrigerating effect of $(h_s - h_{e_1})$ Btu per pound, which would mean $(1 - x_2)(1 - x_3)(h_s - h_{e_1})$ Btu per pound of liquid leaving the condenser.

The weight of circulating refrigerant required for this system is (by analogy with equation 2·5)

$$w = \frac{200T}{(1 - x_2)(1 - x_3)(h_s - h_{e_1})} \tag{3·1}$$

The total power required (by analogy with equation 2·6) is

$$\text{hp} = \frac{w}{42.42}[x_3(h_{d_3} - h_{s_3}) + x_2(1 - x_3)(h_{d_2} - h_{s_2}) + (1 - x_2)(1 - x_3)(h_d - h_s)] \tag{3·2}$$

If the same load were carried with a simple saturation cycle the weight of circulating refrigerant would be, as in equation 2·5

$$w' = \frac{200T}{h_s - h_c}$$

and the power required to compress all the refrigerant in one compressor would be

$$\text{hp} = \frac{w'}{42.42}(h_d - h_s) \tag{3·3}$$

By analogy, the comparable equations for sectionalizing with n stages can be written,

$$w = \frac{200T}{(1 - x_2)(1 - x_3)(1 - x_4)\cdots(1 - x_n)(h_s - h_{e_1})} \tag{3·1a}$$

$$\text{hp} = \frac{w}{42.42}[x_n(h_{d_n} - h_{s_n}) + x_{n-1}(1 - x_n)(h_{d_{n-1}} - h_{s_{n-1}}) + \cdots$$
$$+ x_2(1 - x_3)\cdots(1 - x_n)(h_{d_2} - h_{s_2})$$
$$+ (1 - x_2)(1 - x_3)\cdots(1 - x_n)(h_d - h_s)] \qquad (3{\cdot}2a)$$

Example 3·1. An NH_3 system operating between 25 and 165 psia has multiple expansion valves with vapor extraction at 60 psia and 110 psia. The load is 100 tons. Calculate the total power required and compare the coefficient of performance with that of a simple saturation cycle operating between the same pressure limits.

Solution (Data from Fig. 3·2*b*). By equation 3·1,

$$w = \frac{200 \times 100}{(1 - 0.063)(1 - 0.05)(609 - 76)} = 42.15 \text{ lb/min}$$

By equation 3·2,

$$\text{hp} = \frac{42.15}{42.42}[0.05(651 - 627) + 0.063(1 - 0.05)(682 - 620)$$
$$+ (1 - 0.063)(1 - 0.05)(729 - 609)]$$
$$= \frac{42.15}{42.42}(1.20 + 3.68 + 106.82) = 111.7$$

Power required for simple saturation cycle = 120 hp (Example 2·1). Then per cent increase in the coefficient of performance is

$$\frac{120 - 111.7}{111.7} \times 100 = 7.4\%$$

3·2. Compound Compression with Water Intercooler and Single Expansion Valve. The slope of constant entropy lines on the *ph* chart decreases as the degree of initial superheat increases. By reducing superheat and operating on the steep isentropics near the saturated vapor line the increase in enthalpy corresponding to unit pressure rise is held to a minimum. Compound or stage compression is one effective method of operation on isentropics close to the saturation line.

Refer to Fig. 3·3*ab*; consider that compression occurs in stages through three compressors operated in series. If no further change were made in the system, the thermodynamic cycle would be unchanged from the simple system since discharge from the first stage would occur at state s', but admission to the second stage would occur at the same state; similarly, s'' would represent both discharge from the second stage and admission to the third. Consider, however, that a cooling fluid is available which can be used in intercoolers located between the compressors to reduce the temperature of the vapor to its saturation value at each discharge pressure.

Then the operating cycle becomes $ss's_2s_2's_3d'ce$, and the work of compression is

$$\text{hp} = \frac{200T[(h_{d'} - h_{s_3}) + (h_{s_2'} - h_{s_2}) + (h_{s'} - h_s)]}{42.42(h_s - h_e)} \tag{3·4}$$

Cooling water often is not available at a temperature sufficiently low to desuperheat completely the vapor leaving the first, and sometimes the

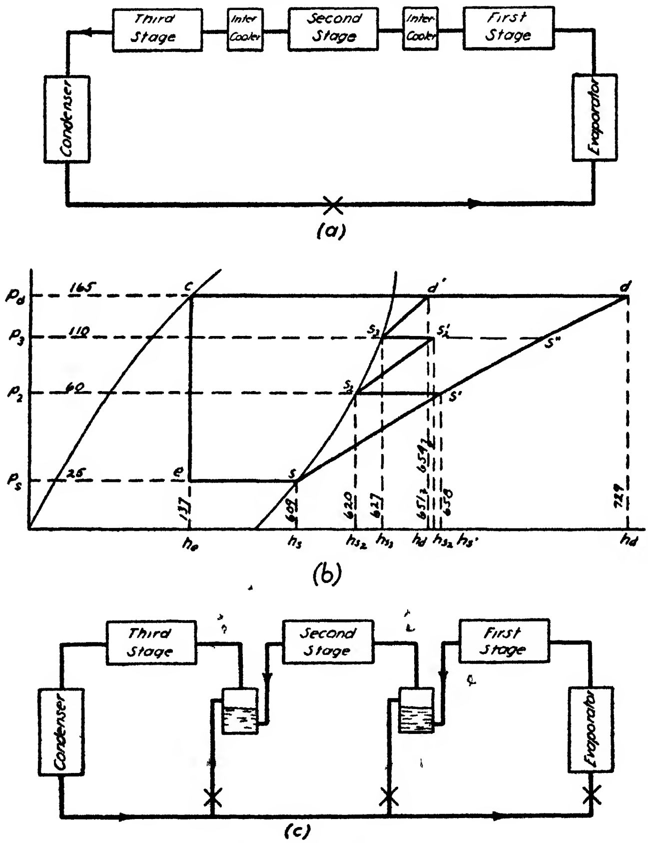

FIG. 3·3. Compound Compression Cycle.

second, stage. In such cases the superheated vapor will be cooled to within a few degrees of the water temperature and will enter the next stage of compression in a superheated rather than saturated state. The equation for work determination for such a system remains equation 3·4 but with states s_2 and s_3 taken at the compressor suction rather than on the saturation line.

The saving in power through use of water intercooling is given by the equation,

$$\text{Reduction in hp} = \Delta hp = \frac{200T[(h_d - h_{s'}) - (h_{d'} - h_{s_3}) - (h_{s_2'} - h_{s_2})]}{42.42(h_s - h_e)} \tag{3·5}$$

Example 3·2. A 100-ton system operates between 25 and 165 psia with three-stage compression and intermediate pressures of 60 psia and 110 psia. Intercooling is by a fluid supplied independently of the refrigeration cycle; vapor leaves both intercoolers in a saturated state. Calculate the horsepower and compare the coefficient of performance with that for a simple saturation cycle.

Solution (Data from Fig. 3·3*b*). By equation 3·4,

$$\text{hp} = \frac{200 \times 100\,[(651 - 627) + (654 - 620) + (658 - 609)]}{42.42(609 - 137)} = 106.9$$

$$\text{Increase in cop} = \frac{120 - 106.9}{106.9} = 12.3\%$$

3·3. Compound Compression with Flash Intercooling and Single Expansion Valve. The entire intercooling process could be carried out by expanding liquid refrigerant to the pressure of the vapor entering each of the later stages of compression and allowing the superheated vapor from the preceding stage to bubble through the liquid refrigerant and thereby become desuperheated (see Fig. 3·3*c*). The thermodynamic diagram for this cycle is the same as for the case of saturation water-intercooling (Fig. 3·3*b*) except that the process lines no longer indicate the path of a constant weight of circulating refrigerant. Since liquid used in the intercoolers represents an added load on the system, it is necessary to determine whether or not this added load is justified by a saving in power.

Let w_e represent the weight of refrigerant passing through the evaporator

$$w_e = \frac{200T}{h_s - h_e}$$

The weight of refrigerant which is used in the intercooler after the first stage of compression is

$$w_1 = \frac{w_e(h_{s'} - h_{s_3})}{h_{s_3} - h_e}$$

The refrigerant needed in the intercooler after the second stage is

$$w_2 = \frac{(w_e + w_1)(h_{s_2'} - h_{s_2})}{h_{s_2} - h_e}$$

Then the power required for stage compression with vapor intercooling is

$$\text{hp} = \frac{(w_e + w_1 + w_2)(h_{d'} - h_{s_3}) + (w_e + w_1)(h_{s_2'} - h_{s_2}) + w_e(h_{s'} - h_s)}{42.42} \tag{3.6}$$

The disadvantage of using flash intercooling in place of water intercooling (if water were available at a temperature low enough to permit its use) is represented by the added power required to recompress the vapor evaporated in the intercoolers; this amounts to

$$\Delta hp = \frac{(w_1 + w_2)(h_{d'} - h_{s_3}) + w_1(h_{s_2'} - h_{s_2})}{42.42} \tag{3·7}$$

Frequently it is possible partially to desuperheat the between-stage vapor with cooling water and then complete the desuperheating by means of flash intercoolers; whenever any part of the desuperheating load can be switched from refrigerant to cooling water, there will be an obvious improvement in the coefficient of performance.

Example 3·3. If the system of Example 3·2 were used with flash intercooling, compare the horsepower and coefficient of performance with values for water intercooling (Example 3·2).

Solution (Data from Fig. 3·3*b*). Refrigerant needed in intercooler after first stage of compression is

$$w_1 = \left(\frac{200 \times 100}{609 - 137}\right)\left(\frac{658 - 620}{620 - 137}\right) = 42.40 \times 0.0787 = 3.33$$

and

$$w_2 = (42.40 + 3.33)\left(\frac{654 - 627}{627 - 137}\right) = 2.52$$

By equation 3·7,

$$\Delta hp = \frac{(3.33 + 2.52)(651 - 627) + 3.33(654 - 620)}{42.42} = 5.98$$

So the total horsepower with flash intercooling is $106.9 + 5.98 = 112.9$. The coefficient of performance will be $100\left(\frac{120 - 112.9}{112.9}\right)$ or 6.3 per cent greater than for a simple saturation system, but $100\left(\frac{123 - 112.9}{123}\right)$ is 8.2 per cent less than with water intercooling.

3·4. Compound Compression with Multiple Expansion Valves, but No Intercooling. Instead of directly recompressing flash from multiple expansion valves in separate compressors, as was done in Fig. 3·2*a*, the two additional compressors can be placed in series with the machine evacuating the evaporator. The three units then form a three-stage compound system (Fig. 3·4*a*), and the flash from the multiple expansion valves provides a slight additional advantage by partially desuperheating the high-temperature vapor leaving the first- and second-stage compressors. The *ph* diagram for this cycle is shown in Fig. 3·4*b*.

The weight of refrigerant passing through the condenser (by equation 3·1) is

$$w = \frac{200T}{(1 - x_3)(1 - x_2)(h_s - h_{e_1})} \tag{3·8}$$

and the weights of vapor leaving at points s, s_2, and s_3 are, respectively, $(1 - x_3)(1 - x_2)w$, $x_2(1 - x_3)w$, and $x_3 w$.

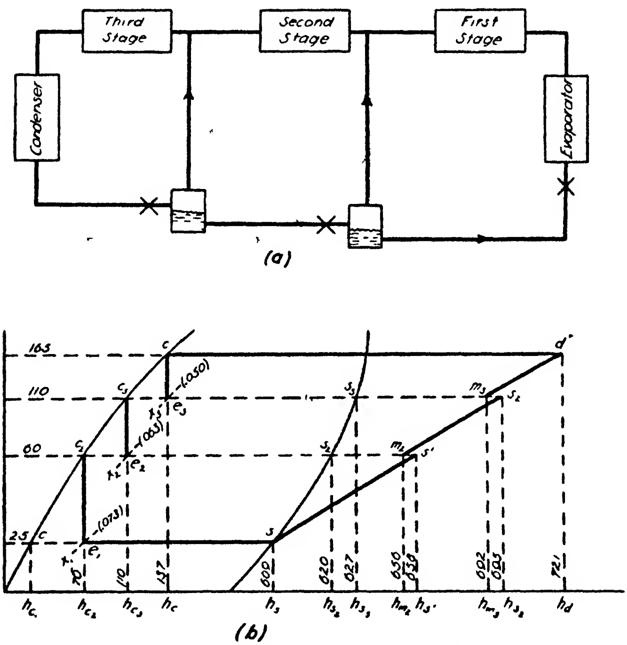

Fig. 3·4. Compound Cycle, Multiple Expansion Valves, No Intercooling.

The state resulting from mixture of superheated vapor at s' with saturated vapor at s_2 can be determined by establishing an energy balance and solving for the unknown enthalpy of the mixture, h_{m_2}, as it enters the second stage of compression,

$$h_{m_2} = \frac{(1 - x_2)(1 - x_3)h_{s'} + x_2(1 - x_3)h_{s_2}}{(1 - x_2)(1 - x_3) + x_2(1 - x_3)} \tag{3·9}$$

Knowing h_{m_2} determines the state at entrance to the second stage and permits fixing the state at discharge from the second stage. The enthalpy of the mixture entering the third stage is calculated from a similar energy balance,

$$h_{m_3} = \frac{[(1 - x_2)(1 - x_3) + x_2(1 - x_3)]h_{s_2'} + x_3h_{s_3}}{(1 - x_2)(1 - x_3) + x_2(1 - x_3) + x_3} \tag{3·10}$$

The work of compression is given by the equation,

$$\begin{aligned}\text{hp} = \frac{w}{42.42}\{(1 - x_2)(1 - x_3)(h_{s'} - h_s) + [(1 - x_2)(1 - x_3) \\ + x_2(1 - x_3)](h_{s_2'} - h_{m_2}) + [(1 - x_2)(1 - x_3) \\ + x_2(1 - x_3) + x_3](h_{d'} - h_{m_3})\}\end{aligned} \tag{3·11}$$

Equations 3·9, 3·10, and 3·11 are given in full in order to establish the procedure for applying this method of analysis to cycles in which there are more than three stages. For a system with only three stages these equations can be simplified by noting that

$$(1 - x_2)(1 - x_3) + x_2(1 - x_3) = 1 - x_3$$

and

$$(1 - x_2)(1 - x_3) + x_2(1 - x_3) + x_3 = 1$$

Then,

$$h_{m_2} = \frac{(1 - x_2)(1 - x_3)h_{s'} + x_2(1 - x_3)h_{s_2}}{1 - x_3} \tag{3·9a}$$

$$h_{m_3} = (1 - x_3)h_{s_2'} + x_3h_{s_3} \tag{3·10a}$$

$$\begin{aligned}\text{hp} = \frac{w}{42.42}[(1 - x_2)(1 - x_3)(h_{s'} - h_s) + (1 - x_3)(h_{s_2'} - h_{m_2}) \\ + (h_{d'} - h_{m_3})]\end{aligned} \tag{3·11a}$$

Example 3·4. A system similar to that of Example 3·2 operates without intercooling other than that from mixing of vapor leaving multiple expansion valves. Calculate the horsepower and compare the coefficient of performance with that of a simple saturation cycle.

Solution (Data from Fig. 3·4*b*). By equation 3·8,

$$w = \frac{200 \times 100}{(1 - 0.05)(1 - 0.063)(609 - 76)} = 42.15$$

By equation 3·9*a*,

$$h_{m_2} = \frac{(1 - 0.063)(1 - 0.05)658 + 0.063(1 - 0.05)620}{(1 - 0.05)} = 656$$

By equation 3·10*a*,

$$h_{m_3} = (1 - 0.05)695 + (0.05 \times 627) = 691.4$$

By equation 3·11*a*,

$$\text{hp} = \frac{42.15}{42.42}\{(1 - 0.063)(1 - 0.05)(658 - 609) + (1 - 0.05)(695 - 656) + (721 - 692)\}$$

$$= \frac{42.15}{42.42}[43.6 + 33.3 + 29] = 105.2$$

$$\text{Increase in cop over simple saturation cycle} = \frac{120 - 105.2}{105.2} = 14.1\%$$

3·5. Compound Compression with Multiple Expansion Valves and Flash Intercooling. Previous sections have shown the advantage of multiple expansion valves and the like advantage of intercooling with

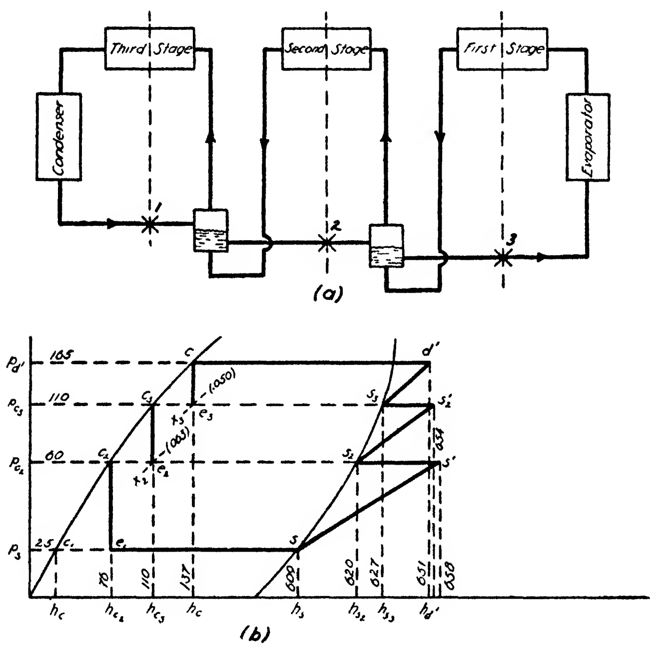

FIG. 3·5. Compound Compression with Flash Intercooling.

stage compression. Since the equipment arrangements for operation with multiple expansion valves or with compound intercooled compression are strikingly similar, there is reason for considering a system which combines them. The arrangement for such a system is shown in Fig. 3·5*a* and the corresponding thermodynamic cycle in Fig. 3·5*b*.

The refrigerant passing through expansion valve 1 is reduced in pressure from p_d to p_{c_3}, the flash passing directly to the suction of the third stage of compression and the saturated liquid proceeding through expansion valve 2. The receiver at the discharge side of valve 1 acts as a bubbling-type flash intercooler. Vapor from the second stage of compression is brought into the receiver and allowed to bubble up through the liquid refrigerant where in the process of desuperheating it evaporates some of the liquid; the evaporated refrigerant plus the vapor from the second stage passes to the third stage. The same process is repeated at the discharge side of expansion valve 2.

The weight of the refrigerant passing through the evaporator is

$$w_e = \frac{200T}{(h_s - h_{e_1})} \tag{3·12}$$

The weight of the vapor passing through the second stage of compression is the sum of that from the first stage plus flash resulting from expansion through valve 2 plus the evaporation occurring in the intercooler located between the first and second stages,

$$w_2 = w_e\left[1 + \frac{x_2}{1 - x_2} + \frac{h_{s'} - h_{s_2}}{h_{s_2} - h_{e_2}}\right] \tag{3·13}$$

The weight of the vapor through the third stage is that through the second plus flash from expansion valve 1 plus refrigerant evaporated in the intercooler between second and third stages,

$$w_3 = w_2\left[1 + \frac{h_{s_2'} - h_{s_3}}{h_{s_3} - h_{e_3}}\right] + \frac{w_e x_3}{(1 - x_2)(1 - x_3)} \tag{3·14}$$

The power required for this system is

$$\text{hp} = \frac{w_e(h_{s'} - h_s) + w_2(h_{s_2'} - h_{s_2}) + w_3(h_{d'} - h_{s_3})}{42.42} \tag{3·15}$$

where w_e, w_2, and w_3 are calculated from equations 3·12, 3·13, and 3·14, and all other terms are obtained directly from the ph diagram. The economy of the cycle would obviously be improved if cooling water were used for partially desuperheating and the flash intercooling were limited to that part of the total process which occurs below the cooling water temperature. The equations established in this section can be used directly in analysis of the combined water and vapor intercooler system by changing $h_{s'}$ in equation 3·13 and $h_{s'}$ in equation 3·14 to $h_{s_w'}$ and $h_{s_{w_2}'}$ where the latter terms represent the respective enthalpies of the partially desuperheated vapor leaving the water intercoolers after the first and second stages of compression. The terms $h_{s'}$ and $h_{s_2'}$ in equation 3·15 remain unchanged.

Example 3·5. Consider the system of Example 3·2 operating with multiple expansion valves and with flash intercooling. Compare the horsepower and coefficient of performance with the simple saturation cycle.

Solution (Data from Fig. 3·5*b*). By equation 3·12,

$$w_e = \frac{200 \times 100}{609 - 76} = 37.5$$

By equation 3·13,

$$w_2 = 37.5\left[1 + \frac{0.063}{1 - 0.063} + \left(\frac{658 - 620}{620 - 110}\right)\right]$$

$$= 37.5(1 + 0.067 + 0.075) = 42.8$$

By equation 3·14,

$$w_3 = 42.8\left(1 + \frac{654 - 627}{627 - 137}\right) + 37.5\left[\frac{0.05}{(1 - 0.063)(1 - 0.05)}\right] = 47.2$$

By equation 3·15,

$$\text{hp} = \frac{[37.5(658 - 609) + 42.8(654 - 620) + 47.2(651 - 627)]}{42.42} = 104.3$$

$$\text{Increase in cop over simple saturation cycle} = \frac{120 - 104.3}{104.3} = 15.0\%$$

3·6. Dual- or Multi-effect Compression. The advantage of multiple expansion valves can sometimes be realized without an extra compressor. By selecting a unit equipped with two sets of suction valves, Fig. 3·6*a*, expansion of the refrigerant can be accomplished in two steps with the intermediate-pressure vapor being compressed in the same cylinder as the low-pressure vapor. As the piston moves toward the right (Fig. 3·6*b*), vapor enters the cylinder at the evaporator pressure p_s. As this vapor enters it does work on the piston, the amount being equal to the flow work possessed by the refrigerant in the suction line. Writing an energy balance on the vapor before and after passing through the suction valve,

$$h_{\text{line}} = h_s = u_s + \frac{P_s v_s}{J} = u_{\text{cylinder}} + \frac{W_{\text{out}}}{J} \tag{3·16}$$

where subscript s refers to the state leaving the evaporator (see Fig. 3·6*e*) and W_{out}/J is the shaft work done by the entering vapor on the piston. But $W_{\text{out}} = P_s v_s$, so $u_s = u_{\text{cylinder}}$, and since the pressure has not changed, the state in the cylinder must be the same as that in the line; h_{cylinder} is therefore numerically equal to h_s even though the property h_s of the vapor in the cylinder no longer represents a quantity of energy.

When the piston approaches the right end of the cylinder (Fig. 3·6*c*) the first set of suction valves closes, trapping in the cylinder a given weight of low-pressure vapor at state s. The high-pressure suction valves then open (Fig. 3·6*d*), allowing vapor to flow irreversibly into the cylinder. The entering vapor does no work on the piston and the flow work associated with the higher-pressure suction vapor in the line must therefore be dissipated as internal energy in the receiver. Thus for the high-pressure vapor, $h_{s_2} = u_{\text{cylinder}}$, and the resultant internal energy of the mixture in

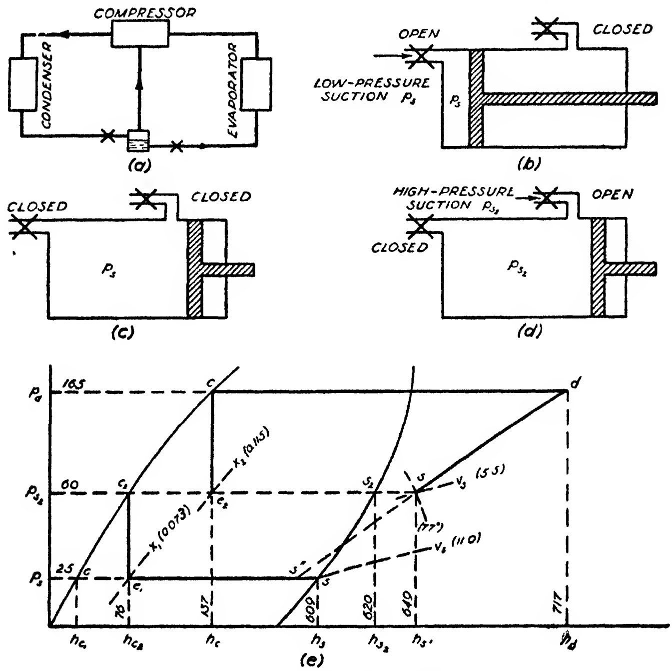

FIG. 3·6. Dual-effect Compression.

the cylinder is the sum of the internal energy of the low-pressure vapor plus the enthalpy of the entering high-pressure vapor,

$$u_{s'}(1 + R) = u_s + Rh_{s_2}$$

where R is the ratio of weight of high-pressure vapor admitted to low-pressure vapor in the cylinder when high-pressure admission begins. Then,

$$h_{s'} = \frac{\left(h_s - \dfrac{P_s v_s}{J}\right) + Rh_{s_2} + (1 + R)\dfrac{P_{s'} v_{s'}}{J}}{1 + R} \tag{3·17}$$

where $h_{s'}$ and $v_{s'}$ (properties of the mixture at start of compression) are unknown. Equation 3·17 is valid for a compressor with or without clearance provided that, if clearance exists, clearance vapor when re-expanded to the lower suction pressure is at the same state as the low-pressure vapor in the line. This condition is approached in practice.

The volume of low-pressure vapor in the cylinder before high-pressure admission begins is equal to the volume of mixture after admission,

$$v_s = (1 + R)v_{s'}$$

or (3·18)

$$R = \frac{v_s}{v_{s'}} - 1$$

By substitution in equation 3·17,

$$h_{s'} = \frac{v_{s'}}{v_s}\left[h_s - \frac{P_s v_s}{J} + \frac{P_{s'} v_{s'}}{J} + \left(\frac{v_s}{v_{s'}} - 1\right)\left(h_{s_2} + \frac{P_{s'} v_{s'}}{J}\right)\right]$$

$$= h_{s_2} + (h_s - h_{s_2})\frac{v_{s'}}{v_s} + \frac{v_{s'}(p_{s'} - p_s)144}{J} \quad (3·19)$$

Inspection of equation 3·19 shows that it contains, as unknowns, the two properties $h_{s'}$ and $v_{s'}$ each of which has a fixed value for any given value of the other. A solution of the equation can therefore be accomplished by trial and error; selecting an assumed state of the mixture at start of compression and reading $h_{s'}$ and $v_{s'}$ from the *ph* chart for that state, substitute these values in equation 3·19 to test the equality and repeat the process until a solution is obtained.

A more direct solution of equation 3·19 is possible if the empirical relationship between $h_{s'}$ and $v_{s'}$ is obtained from the *ph* chart for the particular refrigerant and graphs established. Such a procedure is not justified if interest is limited to a single problem, but it may be very helpful in cases where frequent problems based on use of the same refrigerant occur. Figures 3·7 and 3·8 present such graphical solutions for the dual-effect compression of ammonia. Figure 3·7 is for the simple cycle with both suction vapors at saturation states whereas Fig. 3·8 is applicable to the general problem in which one or both of the suction vapors are superheated and throttling occurs through the suction valves. Both graphs give the solution by fixing the temperature of the mixture at start of compression and therefore (since $p_{s'}$ is known) establishing the state.

When the state at start of compression has been determined, the discharge state *d* is readily fixed by following an isentropic to intersection with the discharge pressure. The work of compression is obtained by subtracting the total enthalpy of the entering vapor from the enthalpy of the vapor leaving,

$$\text{hp} = \frac{(w + w')h_d - wh_s - w'h_{s_2}}{42.42} \quad (3·20)$$

The work of compression for this system cannot be determined by subtracting the enthalpy at start of compression from that at discharge because irreversibility during admission of the high-pressure suction vapor is responsible for the energy loss already noted.

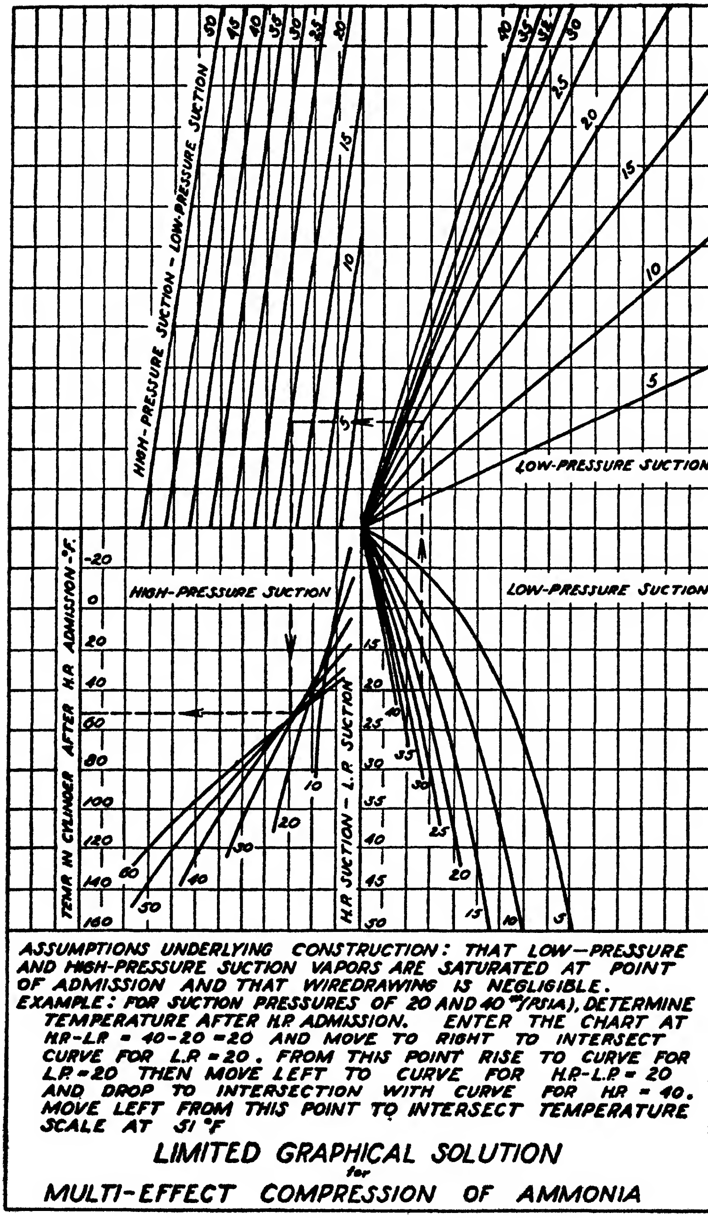

FIG. 3·7. (Redrawn from "A Graphical Solution for Multi-Effect Compression Problems," by F. W. Hutchinson in *Refrigerating Engineering*, July, 1941, by permission of ASRE.)

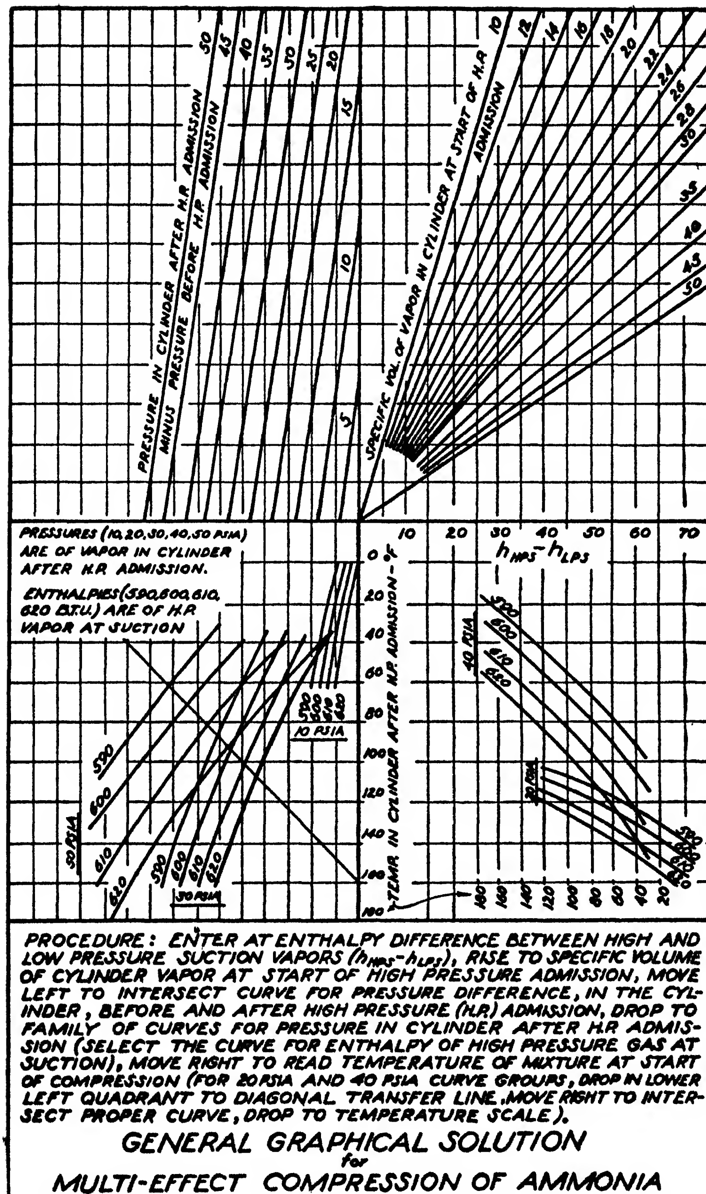

FIG. 3·8. (Redrawn from "A Graphical Solution for Multi-Effect Compression Problems," by F. W. Hutchinson in *Refrigerating Engineering*, July, 1941, by permission of ASRE.)

The terms w and w' (equation 3·20) are the pounds per minute of vapor passing through the low-pressure and high-pressure suction respectively; w is determined from the load on the system,

$$w = \frac{200T}{(h_s - h_{e_1})} \quad \text{(Fig. 3·6}e\text{)} \tag{3·21}$$

and w' is fixed as a function of w by the equation,

$$w' = \frac{wx_2}{1 - x_2} \quad \text{(Fig. 3·6}e\text{)} \tag{3·21a}$$

But the volumetric relationship of equation 3·18 must also be satisfied,

$$v_s = (1 + R)v_{s'} = \left(1 + \frac{w'}{w_t}\right)v_{s'} = \left(1 + \frac{w'}{w_c + w}\right)v_{s'} \tag{3·22}$$

where w_t is the total weight per minute of vapor in the cylinder at the start of high-pressure admission and w_c is the weight of clearance vapor. The solution for a dual-effect cycle is therefore based on the condition that

$$\frac{w'}{w + w_c} = \frac{v_s - v_{s'}}{v_{s'}} = R \tag{3·22a}$$

where the right-hand side can be evaluated as soon as the state at start of compression has been determined.

For a dual compressor with no clearance w_c is zero, and the unit can operate only if $w' = Rw$; this required ratio frequently does not exist, and dual compression (with zero clearance) cannot then be used. When predetermined refrigerant rates w and w' must be carried, a dual compressor capable of handling these quantities can be selected by solving equation 3·22a for w_c and multiplying this rate by the specific volume of vapor at the discharge state, thereby obtaining the clearance volume w_cv_d in cubic feet per minute, which a satisfactory compressor must possess. The necessary clearance (expressed as a per cent of the displacement) is

$$\text{Clearance} = \left(\frac{w_cv_d}{(w_c + w)v_s}\right)100$$

Multiple-effect compressors are usually equipped with some means of varying the clearance — as by clearance pockets — in order to permit changing the weight ratio of high- to low-pressure vapor; note, however, that the state of the mixture at start of compression depends only on the suction pressures and is independent of the weight ratio.

For a single load at suction pressure p_s, to be carried by a dual-effect compressor with fixed clearance, there is only one value of the high suction pressure at which the system can operate. In practice, this intermediate pressure is automatically attained when the system reaches equilibrium, but for purposes of analysis it must be determined by a trial-and-error solution of equation 3·22*a*. Assume a value of p_{s_2} fixing w' and thereby determining the numerical value of the left side of equation 3·22*a*; for the assumed value of p_{s_2} solve equation 3·19 for the state at start of compression, read $v_{s'}$ from the *ph* chart, and substitute in the right side of equation 3·22*a* to test the equality; repeat until a solution is realized.

The assumption that the re-expanded clearance vapor returns to the state of the low-pressure suction vapor is worthy of examination. From Fig. 3·6*e*, if re-expansion occurs isentropically from point *d*, the final state is at s'' in the mixture region rather than at the assumed state *s*. However, the cylinder walls are hot from contact with the high-temperature discharged vapor, and heat will flow from the cylinder to the clearance vapor during the re-expansion process. The effect of such a heat transfer is to cause point s'' to move toward the right, approaching *s* as a limit; because of this heating effect and lack of more exact knowledge of the true re-expansion process, the assumption that re-expansion occurs to a final state *s* is reasonable as well as convenient.

Example 3·6. A 100-ton NH_3 system operates on a simple saturation cycle between 25 and 165 psia with a dual-effect compressor. The intermediate pressure is 60 psia. Calculate: (*a*) the work, (*b*) the coefficient of performance, (*c*) the compressor clearance.

Solution (Data from Fig. 3·6*e*).

a. From Fig. 3·7 determine the temperature of vapor in the cylinder at start of compression as 77°F and read (from *ph* chart): $h_{s'} = 649$ and $h_d = 717$.

$$w = \frac{200 \times 100}{609 - 76} = 37.5 \text{ lb/min}$$

By equation 3·21*a*,

$$w' = \frac{0.115 \times 37.5}{1 - 0.115} = 4.87 \text{ lb/min}$$

By equation 3·20,

$$\text{hp} = \frac{(37.5 + 4.87)717 - (37.5 \times 609) - (4.87 \times 620)}{42.42} = 106.6$$

b. $\text{cop} = \dfrac{200 \times 100}{4540} = 4.40.$

c. By equation 3·22*a*,

$$w_c = \left(\frac{w'v_{s'}}{v_s - v_{s'}}\right) - w = \frac{4.87 \times 5.5}{11 - 5.5} - 37.5 = -32.63$$

The negative clearance vapor shows that operation under the conditions specified would not be possible. The actual intermediate pressure in this system would automatically fall to the lower value at which a balance would be realized. To investigate the possibility of operating the above system with an intermediate pressure of 30 psia: From Fig. 3·7, $t_{s'} = 12°F$. Then, $h_{s'} = 619.5$ and $h_d = 728$; $v_{s'} = 9.5$; $h_{e'} = 42$; $x_2 = 0.17$.

$$w = \frac{200 \times 100}{609 - 42} = 35.3$$

$$w' = \frac{0.17 \times 35.3}{0.83} = 7.23$$

$$w_c = \frac{7.23 \times 9.5}{11.5 - 9.5} - 35.3 = 34.4 - 35.3 = -0.9$$

From this result it is evident that a compressor with zero clearance would operate with an equilibrium intermediate pressure slightly less than 30 psia. This problem emphasizes that dual compression decreases in effectiveness as the ratio R decreases.

3·7. Combined Systems. All the equations developed in the foregoing sections have been for refrigeration systems in which the entire load is carried at one evaporator pressure. In many plants the load is divided

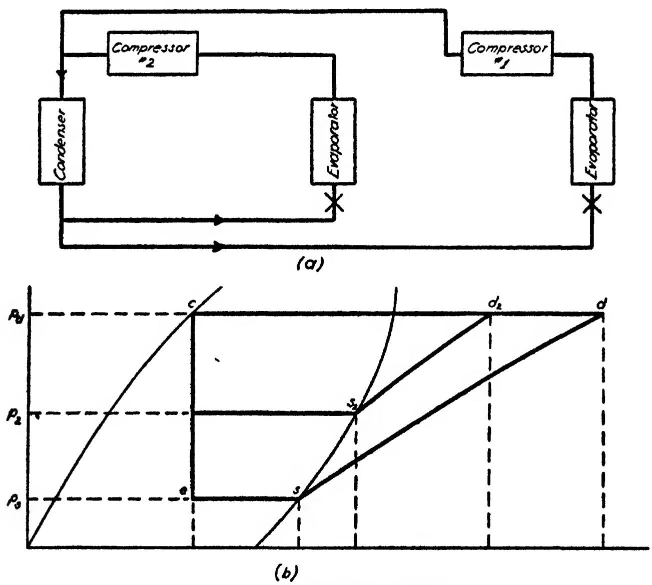

FIG. 3·9. Parallel Operation.

into parts each of which must be carried at a different temperature. By properly proportioning the area of the evaporators the entire system can operate at a suction pressure corresponding to the lowest temperature load. Operation under these conditions means, however, compression of vapor from the other evaporators through a pressure range greater than necessary and hence at the cost of additional power. When equipment is available, each evaporator should be operated at the highest possible pressure and some form of multi-compression used to raise the different suction pressure vapors to the common discharge pressure.

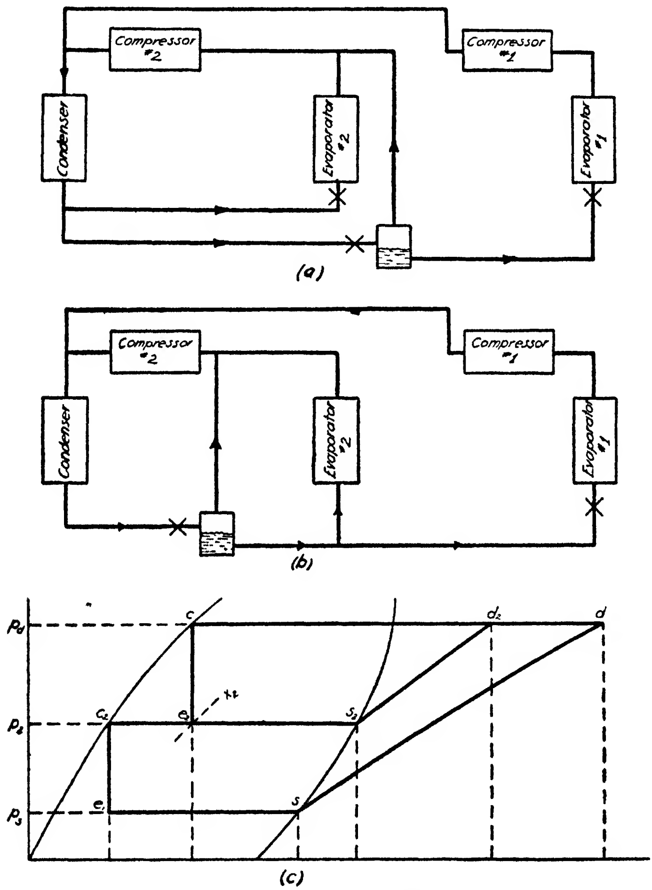

FIG. 3·10. Sectionalizing.

(*a*) PARALLEL OPERATION. The simplest and usual method of combining loads is by placing two or more simple systems in parallel, each system having its own expansion valve, evaporator, and compressor, but utilizing a common condenser. The equipment arrangement and cycle diagram are shown in Fig. 3·9*ab*. Analysis is carried out for each cycle as though it were completely separate; thermodynamically a combined system of this type is identical with two separate simple systems.

(*b*) SECTIONALIZING. The availability of a suction header (in Fig. 3·9*a*) at a pressure between the discharge and suction pressures of the low-pressure cycle suggests the possibility of operating the low-pressure evaporator on a multiple expansion cycle with intermediate pressure equal to that of the high-pressure evaporator. Figure 3·10*ab* shows two equipment arrangements which give the desired compound system. The only change between the parallel operation of Fig. 3·9*a* and the arrangement of Fig. 3·10*a* is the insertion of an additional expansion valve ahead of the low-pressure evaporator and of a flash chamber between expansion valves, with piping to discharge the flash vapor into the suction line from the high-pressure evaporator. Figure 3·10*b* simplifies the system of 3·10*a* by eliminating one expansion valve, but thermodynamically the arrangements of these two figures are identical. The *ph* diagram corresponding to Fig. 3·10*ab* is given by Fig. 3·10*c*. Thermodynamically, this combined system can be considered equivalent to two separate cycles, one at low pressure with multiple expansion valves and the other a higher-pressure simple saturation cycle. Use of equations developed in earlier sections permits determination of the weight of circulating refrigerant and horsepower requirements for each cycle, the total for the system being obtained by addition. Note, however, that this method does not give the individual horsepower requirements of the separate compressors; if these are required, the weight of refrigerant passing through each compressor must be determined and the horsepower calculated by the usual expression,

$$\text{hp} = \frac{w(h_{\text{discharge}} - h_{\text{suction}})}{42.42}$$

(*c*) COMPOUNDING. The two evaporator loads of Fig. 3·9 could be carried in a combined system with the lower evaporator operated on a compound compression cycle. To analyze the most involved arrangement, consider that the lower evaporator is to operate with compound compression, flash intercooling, and multiple expansion valves; the high-pressure evaporator will then remain on the equivalent of a simple saturation cycle. Figure 3·11*abc* shows two possible equipment arrangements and the *ph* diagram for the system. The total power required for the system is calculated by determining the power to carry the low-pressure load on a compound, intercooled, multiple expansion cycle and adding to it the power required for the high-pressure load operating on a simple compression cycle. (If the power requirement of the second stage of compression is needed, it can be most simply obtained by calculating the first stage power and subtracting it from the total system power requirement.)

(*d*) DUAL-EFFECT COMBINATIONS. Analysis of combined dual-effect systems requires a different treatment. The artifice of considering a combined system as equivalent to two parallel high- and low-pressure cycles cannot be used in this case because the vapor from the high-pressure evaporator enters the dual-effect compressor irreversibly and consequently suffers a loss of available energy. The preferable method of analysis is to treat the combined system as equivalent to a single dual-

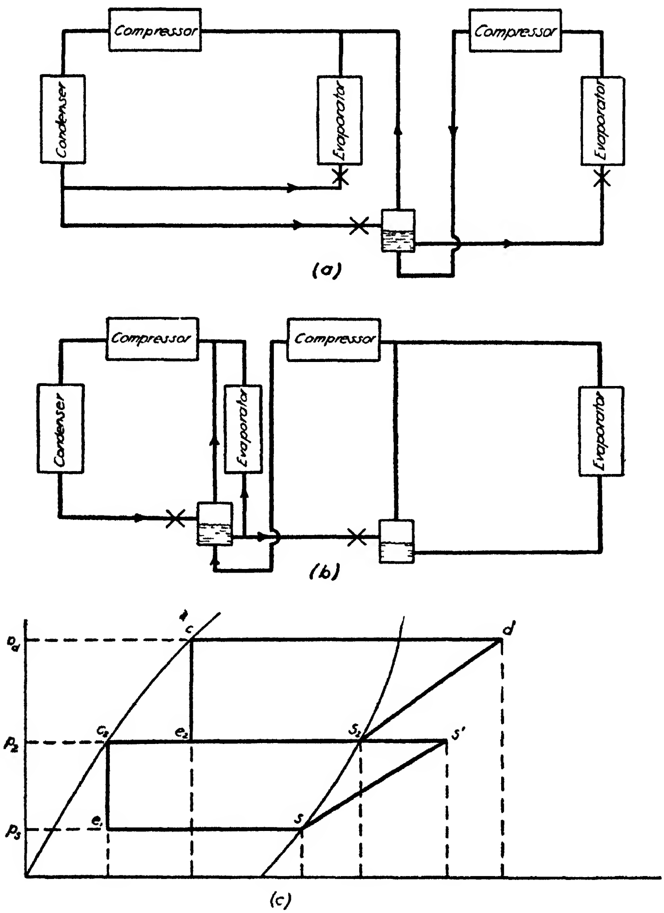

FIG. 3·11. Compounding.

effect cycle with a correction applied to the rate of flow of high suction pressure vapor to account for refrigerant required in the high-pressure evaporator.

Figure 3·12*ab* shows the equipment arrangement and the *ph* diagram for a dual system using two evaporators, each with single expansion valve. The state of the mixture in the cylinder at start of compression is calculated from equation 3·19 (note that this state point is independent of the weight rates through the evaporators), and the power required is calculated from equation 3·20 with w and w' taken as equal to the weight rates of refrigerant passing through the low- and high-pressure evaporators respectively.

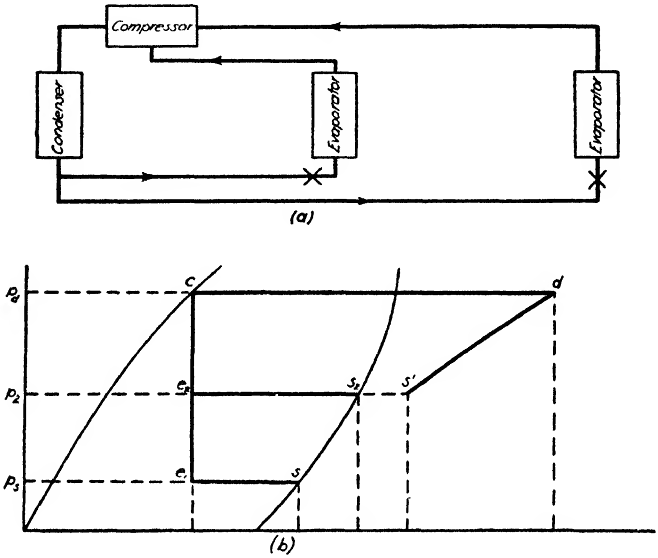

Fig. 3·12. Sectionalizing with Multi-effect Compression.

A combined dual-effect system using multiple expansion to the low-pressure evaporator is shown in Fig. 3·13*a* with 3·13*b* and 3·13*c* as alternative equipment arrangements, all of which are thermodynamically identical; the *ph* cycle for this system is the same as in Fig. 3·6*e*. The state at start of compression is determined from equation 3·19 and the power required calculated from equation 3·20 except that in the latter equation w' is taken as the sum of refrigerant through the high-pressure evaporator and the weight of vapor extracted at the intermediate pressure of the low-pressure system.

(*e*) Multiple Combinations. Obviously the evaporators in a multi-load plant can be combined in a wide variety of ways. Regardless of what type of combination is used, it is always possible to break the system down into equivalent separate thermodynamic cycles, analyze each cycle by the methods already established, and add the separate power requirements. As a typical example of a three-unit complex combination consider the equipment arrangement of Fig. 3·14*a* with corresponding *ph* diagram in Fig. 3·14*b*.

The low- and intermediate-pressure evaporators can be treated as a dual-effect cycle with multiple expansion. The state at start of the dual compression is given by equation 3·19, and the horsepower required by the dual-effect compressor is obtained from equation 3·20, using w' as the

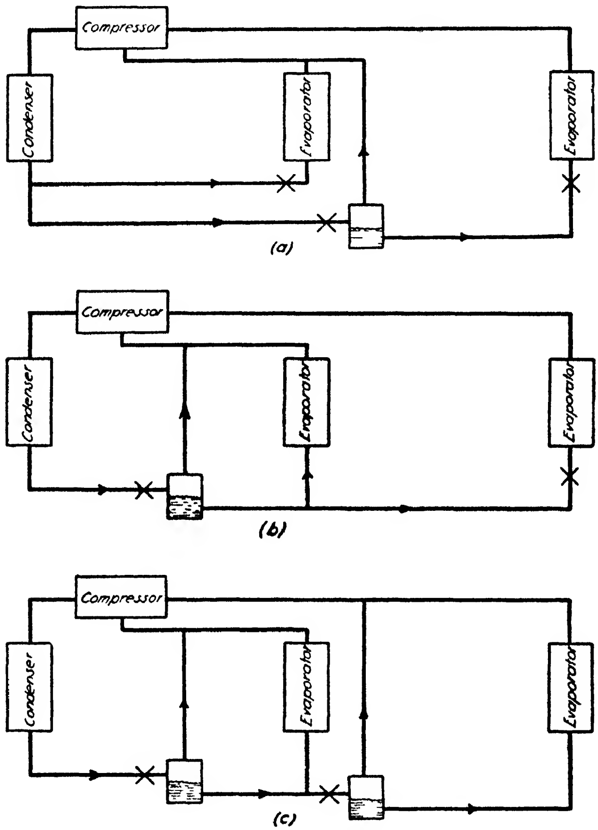

FIG. 3·13. Alternate Multi-effect Arrangement (Thermodynamically Idealized).

sum of refrigerant passing through the intermediate-pressure evaporator and the weight of vapor extracted at the intermediate pressure of the low-pressure cycle.

The power required for the second stage is determined from the equation,

$$\text{hp} = \frac{w_3(h_{d'} - h_{s_3})}{42.42}$$

where w_3 is the total weight rate of vapor passing through this compressor.

The simplest and most direct method of solving problems involving multi-load complex systems is, in every case, to reduce the complex system to separate equivalent single load cycles and solve each of these components by the methods which have already been outlined. Superimposing the equivalent cycles and adding the separate power requirements will then give the power for the actual combined system.

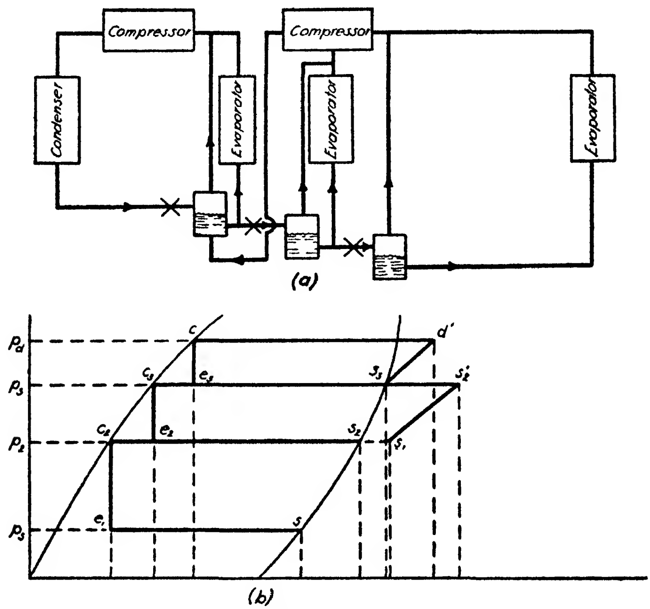

FIG. 3·14. Typical Multiple Load Cycle.

3·8. Graphical Analysis. The first step in the solution of any refrigeration-cycle problem is to establish the ideal cycle on the *ph* chart for the proper refrigerant. Once this has been done the specific enthalpies at all states can be read from the chart and the analysis completed by substitution of these values into the proper equations. For all simple cycles, and for many of the complex ones, the first of the above steps will suffice to give a complete analysis, as will be shown, without recourse to *any* computations.

Since the *ph* chart is constructed with a linear enthalpy scale and since both refrigerating effect and horsepower per pound circulating (for assumed isentropic compression process) are directly proportional to enthalpy differences, it follows that refrigerant rate and power requirements must bear some lineal relationship to distances measured horizontally on the *ph* chart. This relationship has been used in constructing a series of graphical solutions for use with *ph* charts of the corresponding refrigerant to permit direct determination of the refrigerant rate and power requirements. Large *ph* charts and accompanying graphical solutions (trace sheets) will be found in pairs in the envelope inside the back cover of this book. Sets are included for each of the following refrigerants: Freons* 11, 12, 21, and 113, sulfur dioxide, carbon dioxide, ammonia, and methyl chloride.

Use of the trace sheets is illustrated by the following typical cases.

1. Figure 3·15*a* shows a skeleton trace sheet for ammonia. Its use in solving a simple saturation problem (Fig. 3·15*b*) is illustrated in Fig.

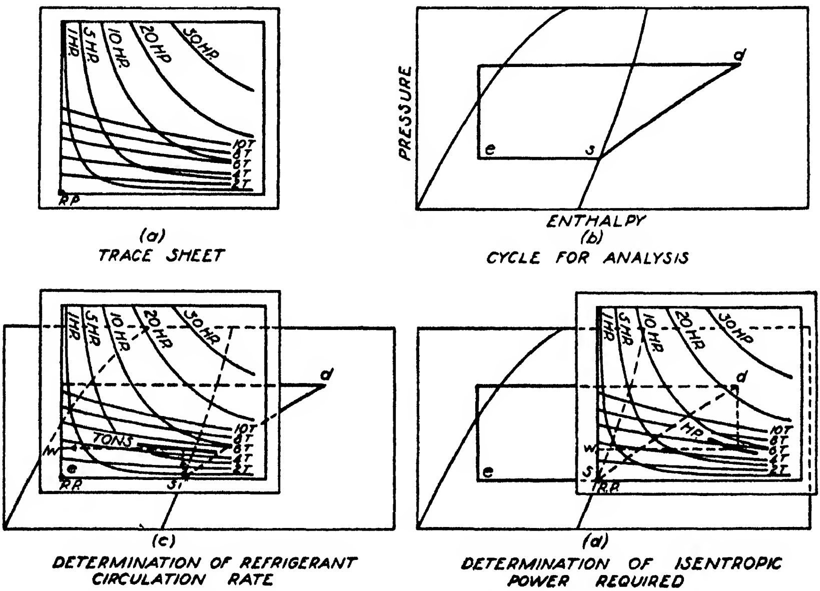

FIG. 3·15. Graphical Solution of Cycle by Use of *ph* Chart and Trace Sheet.

3·15*cd*. Place the trace sheet over and parallel with the *ph* chart in such a way that the state *e* of refrigerant entering the evaporator is directly under the reference point *RP* on the trace sheet (Fig. 3·15*c*). Move right from *RP* until directly over state *s* of the refrigerant leaving the evaporator; from this point follow a vertical line to intersection with the curve corresponding to the load carried by the evaporator and from this intersection move horizontally left to read the rate of refrigerant circulation from the vertical scale at left side of the trace sheet.

Now move the trace sheet to the position shown in Fig. 3·15*d* where the reference point *RP* is directly over state *s*. From state *d* move downward to intersection with the previously determined horizontal line representing

* Trade name.

refrigerant rate. The power required by the system is given by the horsepower curve passing through this intersection.

In a similar manner the refrigerant rate and power required for any simple system can be determined from the trace sheet for any particular refrigerant as soon as the cycle has been plotted on the *ph* chart.

Systems involving multiple expansion valves or flash intercooling are not amenable to simple graphical solution. Dual-effect compression systems cannot be analyzed by the trace sheet method because the enthalpy difference during compression is not, for such systems, directly proportional to the power required. Any other type of ideal combined system can, however, be analyzed graphically by breaking it down to simple components.

2. Consider a two-stage compound single-load system with partial desuperheating between stages through use of cooling water. The horse-

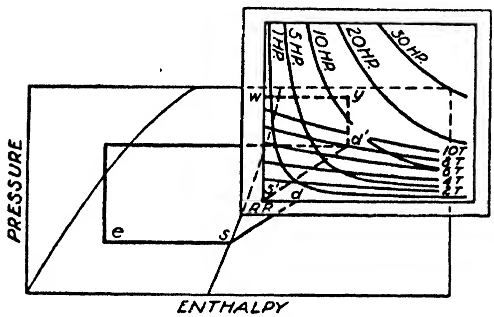

FIG. 3·16. Graphical Analysis of Compound Compression Cycle.

power needed at the first stage is obtained from the trace sheet by the same method outlined for the problem of Fig. 3·15 while the horsepower of the second stage is given at point y of the trace sheet after moving it to the position shown in Fig. 3·16.

3. The two-load system shown in Fig. 3·17*a* consists of a low-pressure evaporator, vapor from which is compressed to the intermediate pressure, water intercooled to state point s' and then mixed with saturated vapor from the high-pressure evaporator before entering the second stage of compression at the unknown state point s_m. The graphical analysis of this system is carried out by analyzing the first stage of compression in the manner shown in Fig. 3·15*cd*; the weight of refrigerant through the high-pressure evaporator is then determined as shown in Fig. 3·17*b*. Knowing the weight through each of the evaporators the state of the mixture at intermediate pressure is fixed by the mixture enthalpy, which is

$$h_m = \frac{wh_{s'} + w'h_{s_2}}{w + w'}$$

The horsepower required by the second stage is obtained by placing the trace sheet with *RP* over s_m (Fig. 3·17*c*) and rising vertically from state point d' to intersection with the horizontal line corresponding to the total refrigerant rate; the power is given by the horsepower curve passing through this intersection.

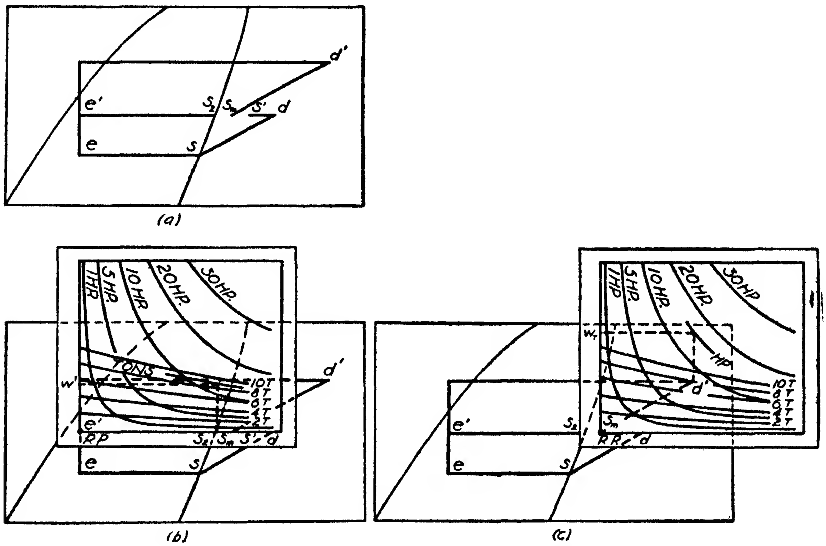

FIG. 3·17. Twin-load Cycle Analysis.

Once facility has been established in the use of trace sheets, many possibilities for time-saving combinations of graphical and analytical methods will become evident. Even with the most complicated systems involving multiple expansion, flash intercooling, and vapor precooling, use of the graphical method makes directly possible the determination of evaporator refrigerant rates, and power requirements; once total weight rates have been established, the need for calculations is reduced to determination of weight rates through multiple valves, intercoolers, or precoolers, and the determination of the states resulting from vapor mixing.

PROBLEMS

1. In a certain cold storage plant four ammonia suction mains are connected to three separate compressor cylinders. The mains are:

(*a*)	Ice floor	25 psia
(*b*)	No. 1 brine cooler	35 psia
(*c*)	No. 2 brine cooler	16 psia
(*d*)	Water forecooler	35 psia

All compressor cylinders discharge into a common condenser main in which pressure is maintained at 165 psia on a day when loads are as follows:

(*a*) 105 Tons of refrigeration.
(*b*) 185 " " "
(*c*) 85 " " "
(*d*) 25 " " "

Assuming compressions to be from saturation and that subcooling (leaving the condenser) is to 60°, find the per cent saving in isentropic compressor power when running as a sectionalized plant, compared with isentropic compressor power when running as a non-sectionalized plant on a single suction main at 16 psia.

2. For the load conditions of problem 1 assume all three compressors placed in series, the superheated vapor from each stage then being discharged into the suction line (where it mixes with saturated vapor) of the next stage. Calculate the required total power.

3. Assume that the compressors of problem 1 are placed in series with desuperheaters (using liquid refrigerant) located between stages. Calculate the total power.

4. Determine the power for the load conditions of problem 1 if only two compressors are used and the system is arranged as in Fig. 3·14*a*.

5. Solve illustrative Example 3·2 by the graphical method.

6. Work illustrative Example 3·6 using 28 psia as the intermediate pressure. Would such a system be economically practical? Calculate the conventional volumetric efficiency of the compressor.

7. A simple saturation Freon-12 cycle operates between 20°F and 90°F. For a load of 7.5 tons, determine graphically the refrigerant circulating rate and the power required.

8. For a compressor having zero clearance and operating under the conditions given in the example shown on Fig. 3·7, determine the relative weight rates of low- and high-pressure suction vapors which pass through the machine.

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CHAPTER IV

ANALYSIS OF ACTUAL CYCLES

The methods of analysis developed in preceding sections have been established for ideal systems in which there are no heat or pressure losses and for which the compression process is isentropic. In actual refrigeration systems losses of both heat and pressure do occur and the process of compression may follow neither an isentropic nor a polytropic path. If sufficient data were available on the state of the refrigerant at intervals throughout the cycle, analysis of actual systems could be carried out by rational means. Usually, however, such experimental data are not available and cannot be easily obtained, so an approximation to the real cycle must be established either through use of empirical methods based on experience with systems similar to the one under consideration, or by using the rational equations for an ideal cycle with corrections and adjustments to account for the more important and recognizable deviations.

The most important deviations of the actual cycle result from periodic heat transfer between the vapor and cylinder wall, departure of the compression process from isentropic due to heat loss to the surrounding air or cooling water, and throttling or wiredrawing during passage through suction and discharge valves. The last-named cause is often responsible for wide departure of the actual from the theoretical power requirements and a correction for this factor should always be made before starting the cycle analysis. The other effects are somewhat less important and, fortunately, tend to offset one another so that the net departure of the actual from the ideal power requirement is frequently so small as to be of little practical significance.

As a general summary of the procedure recommended in adapting the actual cycle to analysis by the methods developed for the ideal cycle, the following suggestions, more fully discussed and explained in subsequent sections, are listed.

1. Establish on the *ph* chart the states *c* and *s* of vapor entering the expansion valve and leaving the evaporator in the real cycle (Fig. 4·1).

2. Through the above points draw the ideal process lines for evaporator and for expansion valve.

3. From point *s* follow a constant enthalpy line through the pressure range $p_s - p_{s'} = \Delta p_{w_s}$, where Δp_{w_s} is the pressure loss due to wiredrawing through the suction valves, and is either known or estimated. This determines the actual state, s', at start of compression (neglecting superheating

of the vapor during admission or assuming that it will be compensated for during discharge).

4. If the temperature at discharge is known, fix the actual discharge state d on the ph chart. From d rise through the pressure range, $p_{d'} - p_d = \Delta p_{w_d}$, where Δp_{w_d} is the wiredrawing through discharge valves, thereby establishing the state d' of vapor in the cylinder at end of compression. Solve for the polytropic path through points s' and d' and calculate the corresponding work.

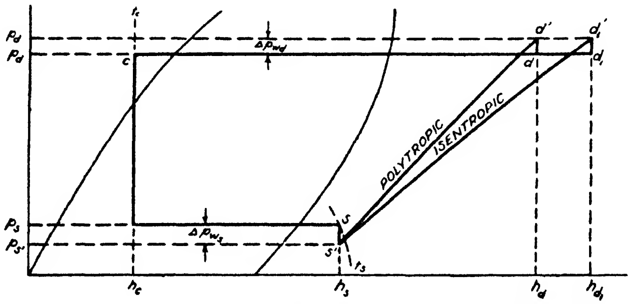

Fig. 4·1. Throttling Losses at Suction Discharge Valves.

5. If the polytropic exponent is known, but other data are not available, calculate the specific volume at the unknown state d', thereby fixing d' and permitting determination of d.

6. If no information is available concerning either the compression process or the discharge state, assume isentropic compression from s', thereby fixing $d_{t'}$ and d_t. In many problems the assumption of isentropic compression can be justified even though data on the actual compression process are available; the error is not likely to be great unless a large quantity of heat leaves the compressor in the jacket cooling water.

7. When isentropic compression is assumed the actual cycle can be analyzed by applying the theoretical equations and methods to an equivalent ideal cycle passing through states s' and $d_{t'}$.

Following sections discuss in more detail the effects on the ideal cycle of the deviations already listed. An understanding of the way in which these departures from ideal conditions affect the analysis will assist in understanding the above recommendations and in determining the probable degree of inaccuracy likely to result from ignoring any of these factors in a given problem.

4·1. Effects of Periodic Cylinder Wall Temperature Variation. The assumption of isentropic compression requires that no transfer of heat occur between the vapor passing through the compressor and the cylinder walls. Realization of this condition would require that the cylinder walls and all surfaces making up the clearance volume follow a temperature-time cycle exactly the same as that followed by the vapor during compression and re-expansion. In actual compressors the cylinder wall temperature follows a pattern similar, in general contour, to the temperature-time curve for the vapor, but with a very much smaller amplitude of temperature variation and with the maxima and minima of the wall curve lagging the maxima and minima of the vapor curve. The wall temperature varies periodically around a mean value which is between the suction and discharge temperatures of the vapor. The cylinder receives heat from the vapor during part of each cycle and returns heat during a later period. If the cylinder is insulated or if the mean temperature of the inside surface is equal to that of the compressor room, there will be no appreciable net transfer of heat to or from the vapor in the cylinder. Under these circumstances the quantity of heat transferred from cylinder walls to vapor must equal the quantity returned by the vapor at a later period.

Flow of heat from cylinder walls to vapor may occur during re-expansion, or admission, and during the first part of the compression process. Heat gain during re-expansion can be neglected as it is partially offset by a heat loss at the start of the re-expansion process. Further, gain of heat during re-expansion increases the work returned to the shaft by the clearance vapor; the effect of neglecting this transfer is therefore conservative in that it indicates a greater total work of compression than is actually needed. The principal importance of heat gain by the clearance vapor during re-expansion results from the increase in expanded volume and consequent reduction in the weight of vapor handled by the compressor during each stroke.

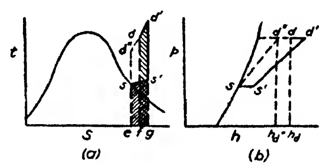

FIG. 4·2. Heat Transfer during Admission and Discharge.

As a first approximation in determining the effect on power requirements of cylinder-vapor heating, consider that the entire flow of heat to the vapor occurs during the constant-pressure admission process, and the return to the cylinder of an equal heat quantity occurs during the process of constant-pressure discharge; the actual process of compression will be assumed isentropic. Figure 4·2*ab* shows this cycle on the *Ts* and *ph* charts respectively. On the *Ts* chart, vapor enters the compressor at state *s* and is superheated at constant pressure along *ss′* due to heat received from the cylinder walls. By the equation defining entropy, unit area on the *Ts* chart represents the heat transferred so the total heat received by the vapor during admission is represented by the area *ss′ge*. Isentropic compression occurs from *s′* to *d′* and is followed by the constant-pressure process *d′d* during which heat is returned from the vapor to the cylinder. But the heat returned must equal that received so areas *d'dfg* and *ss'ge* must be equal. Since heat return occurs at a higher temperature level, the entropy change during return must be less than that during heat addition. The overall result of the heat transfer between vapor and cylinder is therefore an entropy increase represented by $s_d - s_s$. The same cycle is also shown on the *ph* chart where it will be noted that an increase, $h_d - h_{d''}$, in the required work of compression has resulted from the periodic heat flow. Since there is no net heat transfer during this compression cycle, the work is given by the difference in enthalpy between final and initial states and is equal to $h_d - h_s$.

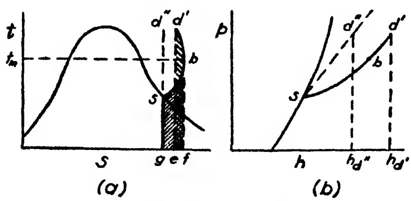

FIG. 4·3. Heat Transfer during Compression Process.

Refer to Fig. 4·3*ab* and consider that admission and discharge occur adiabatically, but heat flows to the vapor during the first part of the compression process and to the cylinder walls during the latter part. On the *Ts* chart, vapor is considered as entering the cylinder at state *s* and receiving heat, with consequent increase in entropy, during that part of the compression process in which the vapor temperature is less than the mean cylinder temperature t_m. As compression proceeds, the path increases in slope until, when the vapor temperature is equal to t_m, heat transfer ceases and the compression process is, at that instant, isentropic; this point is shown at *b* in Fig. 4·3 and the total heat received from the cylinder is then represented by the area *sbfg*. At pressures exceeding that corresponding to a vapor temperature of t_m, heat is returned to the cylinder at an ever-increasing rate (as shown by the rate of change of slope between *b* and *d'*) until, when discharge occurs at *d'*, the quantity of heat which has been returned, represented by area *bd'ef*, is equal to the heat quantity shown by *sbfg*. Since heat transfer from the vapor occurs at a higher average temperature than its reception by the vapor, the change of entropy from *b* to *d'* must be less than from *s* to *b*, so the compression must therefore occur with an entropy increase, $s_{d'} - s_s$. The equivalent compression process is shown on the *ph* chart (Fig. 4·3*b*) with an evident increase in the work of compression, over isentropic, of $h_{d'} - h_{d''}$.

From the above cases it is evident that heat interchange between vapor and cylinder, when the net heat transfer is zero, results in an increase in the work of compression regardless of whether the exchange takes place during the actual compression process or during the processes of admission and discharge. Further, since the increased work is done on the refrigerant during a process in which no net heat is lost from the cylinder, the additional energy must also appear as an added load on the condenser.

The combined effects of periodic heat flow during all steps in the passage of vapor through a compressor are shown in the *Ts* and *ph* charts of Fig. 4·4*ab*. The work of compression is the enthalpy difference between suction and discharge states (since $Q_{out} = 0$) and is seen to be considerably greater than for an isentropic compression. Fortunately, however, other departures of the actual from the ideal cycle serve to compensate for the apparent work increase. The discussion of preceding paragraphs was based on the assumption that the mean cylinder wall temperature is equal to the temperature of the surroundings in the compressor room. For most pressure ranges the mean temperature is considerably greater than this so an appreciable net heat loss occurs from the compressor. Referring back to Fig. 4·3*a*, a compressor heat loss requires that area *bd'ef* exceed *sbfg*, and that point *d'* therefore move to the left giving either a smaller entropy increase or even a decrease for the actual compression process. In recalling, however, that the work of compression during a non-adiabatic process exceeds the enthalpy gain, it is evident that if sufficient heat were removed to displace point *d'* to the left of the theoretical discharge state *d''*, there would then be a good chance that the actual work of non-adiabatic compression might equal or very closely approach the ideal isentropic work.

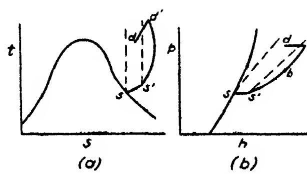

FIG. 4·4. Heat Transfer throughout Compression.

(*a*) JACKETING. When water jacketing is used, the heat dissipation from the compressor is likely to be sufficiently great to reduce the power requirements somewhat below those for isentropic compression; in such cases, lacking experimental data on the actual suction and discharge states, the calculation of work based on assumed isentropic compression will usually be sufficiently accurate for practical purposes and will always give an error on the conservative side. If necessary, a closer approximation to the true work can be obtained by assuming that the path of compression is polytropic. Solution on this basis requires a knowledge of the polytropic exponent (assumed from experience or estimated), or the state of the vapor leaving the compressor (from experimental data), or a knowledge of the rate of heat dissipation from the compressor. If the latter term is found by test it must be remembered that the total heat loss from the compressor includes energy derived from the friction horsepower; this fraction of the heat loss is not part of the Q_{out} term as used in the energy equation.

Jacketing is beneficial from the standpoint of power consumption, but even when this saving is not great, jacketing is necessary to protect the valves and the lubricant from the high superheat temperatures which might otherwise exist in the cylinder. At the other extreme, if jacketing is used with low-temperature cooling water there is a possibility of reducing the mean cylinder wall temperature below the saturation temperature corresponding to the discharge pressure; then condensation would occur on the surface of the clearance space with consequent large increase in weight of re-expanded clearance vapor and reduction in the capacity of the compressor.

4·2. Effects of Valve Throttling or Wiredrawing. The ideal cycle does not make allowance for pressure changes other than during compression or expansion. Actual systems involve pressure losses in the condenser, evaporator, and piping, due to fluid and skin friction as well as losses in the compressor valves from the same cause and from turbulence due to sudden expansion or abrupt change of direction. For all practical purposes the entire pressure loss from the discharge side of the expansion valve to the start of the compression stroke can usually be considered as occurring during passage of the vapor through the suction valves of the compressor. Similarly, all losses from the end of the compression process to entrance into the expansion valve can be grouped as an equivalent pressure drop through the discharge valves.

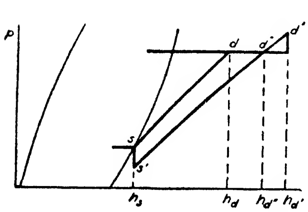

FIG. 4·5. Power Losses from Wire-drawing.

Noting that the suction and discharge valves precede and follow, respectively, the constant-pressure processes during which vapor is admitted and discharged from the compressor, it is evident that the values of the suction and discharge pressure for use in the work equations should be taken on the compressor side of each set of valves. Figure 4·5 shows the effect of throttling. Vapor leaving the evaporator at state s would, without throttling, be isentropically compressed along the line sd with a work requirement of $h_d - h_s$. Due to throttling through the suction valve the state at start of compression becomes s', thereby displacing the discharge state point to d'' with an increase in the required work of $h_{d''} - h_d$. Similarly, throttling through the discharge valve is responsible for a further work increase of $h_{d'} - h_{d''}$. The work required for a system in which there is throttling is exactly the same as work for a similar non-throttling system operating over the pressure range $p_{s'}$ to $p_{d'}$. In addition to increasing the power requirements, suction valve losses cause an increase in specific volume with a consequent reduction in the capacity of the compressor. A similar effect results from leakage past the valves and the piston, but this is relatively small for well-designed compressors.

SPECIAL COMPRESSION SYSTEMS

4·3. Centrifugal and Rotary Compression. The preceding discussions have been for systems in which the compression process is accomplished in a compressor of the reciprocating type. When rotary or centrifugal compressors are used the analysis remains the same except in so far as data may not be available on the actual process followed, and the assumption of polytropic compression may not be justified. The efficiency of centrifugal compressors is usually expressed as that fraction of the actual enthalpy gain during compression which is equal to the energy input that would have been required had the compression process been isentropic. When the efficiency is known, determination of the state at compressor discharge, the work of compression, and the coefficient of performance can be readily calculated by the methods already established.

4·4. Steam Jet System. Compression by means of a steam jet ejector is limited to systems in which water is used as the refrigerant. The principal advantages of the steam jet are that it uses energy having a lower degree of availability than that required for mechanical compression and that there are no moving parts. The jet equipment is so highly specialized that system analysis can most satisfactorily be accomplished by use of manufacturer's data rather than by establishing a rational thermodynamic analysis of an ideal system.

(*a*) Direct Water Cooling by Steam Jet System. Where the object of the refrigeration system is to provide cold water, the steam jet has been used both as an indirect system with the water to be cooled passing through heat transfer surfaces located in the body of the evaporator and as a direct system with water flash-cooled as it enters the evaporator. In the latter case the circulating water pump must be able to take suction at the low absolute pressure corresponding to the saturation temperature of the evaporator. The recooling of a circulating stream of water by direct flash can lead to an unsatisfactory condition of the water owing to an increase in the concentration of such impurities as are present in the make-up; for this reason, direct flash cooling of circulating drinking water is likely to be undesirable. This condition can be investigated as follows. Referring to the system shown in Fig. 4·6, let:

C = concentration of non-volatile impurity (as common salt) in the water leaving the flash chamber.

Z = concentration of non-volatile impurity in the make-up water.

M = the quantity of make-up water added in pounds per minute.

R = the quantity of recirculated water entering the flash chamber in pounds per minute.

$E = M + R$, the total weight of water passing through the flash chamber, pounds per minute.

L = fraction of E not vaporized in the flash chamber (then LE is the weight rate of cold water leaving the cooler).

n = the number of passes through the flash chamber.

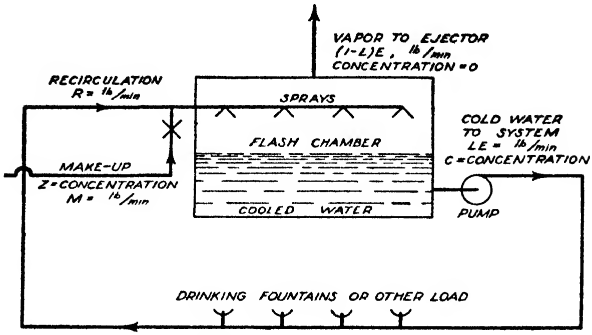

FIG. 4·6. Steam Jet Direct Water Cooling.

For the first pass through the flash chamber all the entering water is make-up, so

$$C = \frac{EZ}{LE} = \frac{Z}{L} \tag{4·1}$$

For $n = 2$

$$C = \frac{\dfrac{ZR}{L} + MZ}{LE} \tag{4·2}$$

For $n = 4$

$$C = \frac{\dfrac{\left[\left(\dfrac{\frac{ZR}{L} + MZ}{LE}\right)R + MZ\right]R}{LE} + MZ}{LE} = \frac{ZR^3}{L^4E^3} + \frac{MZR^2}{L^3E^3} + \frac{MZR}{L^2E^2} + \frac{MZ}{LE}. \tag{4·3}$$

For $n = n$

$$C = \frac{ZR^{n-1}}{L^nE^{n-1}} + \frac{MZR^{n-2}}{L^{n-1}E^{n-1}} + \frac{MZR^{n-3}}{L^{n-2}E^{n-2}} + \cdots + \frac{MZR^{n-n}}{L^{n-(n-1)}E^{n-(n-1)}}$$

$$= \frac{ZE}{R}\left(\frac{R}{LE}\right)^n + \frac{MZ}{R}\left[\left(\frac{R}{LE}\right)^{n-1} + \left(\frac{R}{LE}\right)^{n-2} + \cdots + \left(\frac{R}{LE}\right)^{n-(n-1)}\right] \tag{4·4}$$

Equation 4·4 gives the concentration as a function of the number of passes through the flash chamber and can be used to establish the concentration versus time curve for a given system during the transient interval before the equilibrium concentration is reached. For most practical problems, however, interest centers on the value of C which will result after equilibrium is established. This concentration can be directly obtained by setting up a mass balance on the system or from equation 4·4 by letting the number of passes become very large. As n approaches infinity, $(R/LE)^n$ approaches zero so the first term of equation 4·4 must also approach zero. But the remaining series can be expressed as

$$\left(\frac{R}{LE}\right)^{n-1} + \left(\frac{R}{LE}\right)^{n-2} + \cdots + \left(\frac{R}{LE}\right)^{n-(n-1)} = \frac{\left(\frac{R}{LE}\right)\left[\left(\frac{R}{LE}\right)^n - 1\right]}{\frac{R}{LE} - 1} - \left(\frac{R}{LE}\right)^n \qquad (4\cdot5)$$

where the right side of the equation approaches $R/(LE - R)$ as n approaches infinity. Then substituting in equation 4·4 for the limit $n = \infty$ gives

$$C = \frac{MZ}{R}\left(\frac{R}{LE - R}\right) = \frac{MZ}{LE - R} \qquad (4\cdot6)$$

The concentration in the system at equilibrium is therefore equal to the concentration in make-up multiplied by the ratio of weight rate of make-up to weight rate of consumption.

4·5. Heat Pump. An ideal heat engine having for its objective the transformation of heat energy into mechanical energy would operate with a maximum efficiency equal to that realized by a machine having complete reversibility. The efficiency of all reversible machines is the same, and an equation for this efficiency, derived from the Carnot cycle, can be shown to be

$$\text{Efficiency} = \frac{T_s - T_r}{T_s} \qquad (4\cdot7)$$

where T_s and T_r are the temperatures, degrees Fahrenheit absolute, at which the engine obtains heat from a source and rejects heat to a receiver. The efficiency of such a heat engine must always be less than 100 per cent.

Applying the same criterion to effectiveness of a heat machine having for its objective the *removal of heat,* the coefficient of performance is found to be:

$$\text{cop} = \frac{T_{\text{evaporator}}}{T_{\text{cond.}} - T_{\text{evap.}}} \qquad (4\cdot8)$$

Experience shows that coefficients of performance of actual machines are obtainable in excess of 4.00 (see Table 1·1, Chapter I). That is, each unit of energy supplied to the system as work is capable of causing the extraction of four or more units of energy from the low-temperature source.

Consider a heat engine having for its objective the *supply of heat* to a high-temperature receiver. The desired result would in this case be equal to the total heat rejected by the machine which is the sum of heat

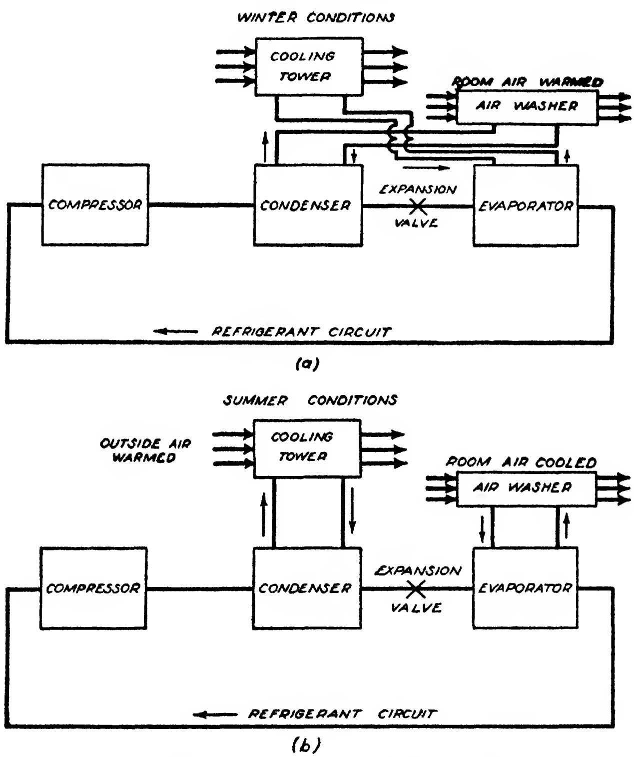

FIG. 4·7. Heat Pump Using Atmospheric Air as Source (Winter) or as Receiver (Summer).

extracted from the source and that supplied to the system as mechanical work. The coefficient of performance would be greater for a device of this type than for a refrigerating machine. The expression is

$$\text{cop} = \frac{T_{\text{cond.}}}{T_{\text{cond.}} - T_{\text{evap.}}} \tag{4·9}$$

All refrigerating systems are heat pumps in the sense that they serve to raise heat from a low to a high energy level. When the quantity of energy lifted by the pump is of paramount importance, the machine is said to be operating as a refrigerating unit; when the quantity of energy discharged at the higher temperature level is of greater importance, the machine is said to be acting as a warming engine or heat pump.

Any refrigerating machine arranged so that its evaporator is extracting heat from an available low-temperature source and its condenser dis-

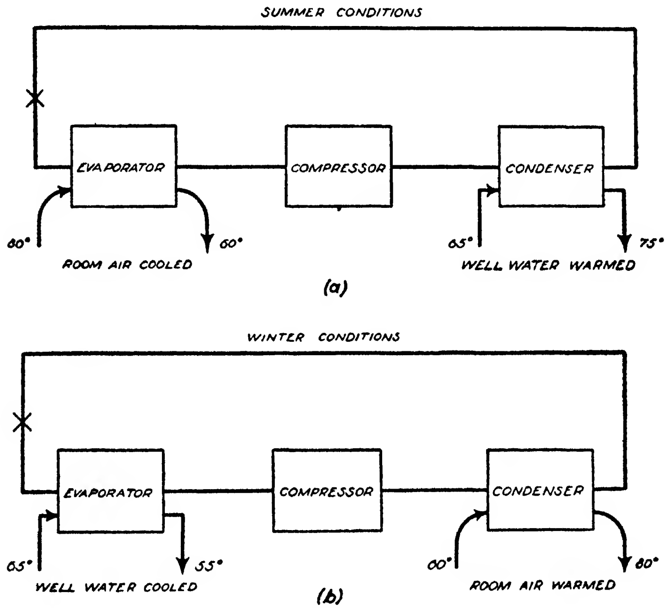

FIG. 4·8. Heat Pump Using Well Water.

charging this heat to a space in which a quantity of high-temperature energy is needed is likely to be delivering five or more units of energy for each unit entering as shaft work. One unit of electrical energy consumed for direct space heating provides less than one-fifth as much heat as the same unit when used to operate a heat pump. At first glance it might therefore appear that a heat pump is an energy-creating mechanism, but this is obviously not true; like the pump in a hydraulic system, a heat pump acts merely to change the level (with respect to temperature) of the energy passing through it. In spite of the high coefficient of performance, the use of heat pumps is somewhat restricted because of the higher first cost in comparison with other types of heating equipment and because, though little energy is required, that which is needed must be supplied with a high degree of availability and is therefore many times more expensive than the low-grade energy used in direct heating. The equation for coefficient of performance (4·9) shows that heat pumps, like refrigerating systems, have better performance characteristics when operated between a low condenser temperature and a high evaporator temperature. For this reason the most effective use is in localities where the winter outside temperature (or the temperature of whatever source is used as a heat supply for the evaporator) is high. In this respect California affords a climate in which high coefficients can be expected, and there are now many heat pump installations in that state. From the standpoint of first cost, reverse cycle systems cannot compete with direct heating installations, but when year-round air conditioning is to be installed, the added cost of converting the refrigeration system for use as a heat pump is likely to be no more than the cost of a separate heating plant. Figures 4·7 and 4·8 show, diagrammatically, the flow circuit for two year-round heat pump installations.

The customary heat source in heat pump installations is the outside air. In localities where the air temperature is very low, other less extreme heat sources, such as well water, must be used. The extraction of heat from the earth at a level for which extremes of weather would have little influence is one possibility, and extraction of the latent heat of fusion of ice (32°F) is another.

PROBLEMS

1. Saturated ammonia vapor at 30 psia undergoes a 4-psia pressure drop while passing through the suction valves of a compressor. The pressure in the discharge line is 170 psia, and the wiredrawing loss in the discharge valves is 8 psia. Consider the compression process to be isentropic and determine the percentage error which would be present in calculated power if both valve losses were neglected and the compression process assumed isentropic from suction line state to discharge line pressure.

2. Saturated NH_3 vapor at 30 psia enters a compressor without valve loss, but receives 10 Btu/lb heat transfer from the cylinder walls during the constant-pressure admission process. A like amount of heat is returned to the cylinder walls during the constant-pressure discharge. Determine the exponent of the polytropic compression process and the rate of net heat flow from vapor to cylinder *during compression* if the entropy of the vapor is the same in suction and discharge lines.

3. Make-up water at 60°F is supplied to a steam jet unit which is carrying an exterior load of 5 tons with an evaporator temperature of 35°F. Calculate the weight rate of make-up and the volume of vapor in cubic feet per minute which must be extracted. What is the total refrigerating capacity, in tons, of the unit?

4. A direct steam jet drinking water unit sends out water at 38°F and receives returns at 45°F. Consumption is 10 gal/hr and is replaced with 15 gal/hr of 60°F make-up. Calculate the weight rate of flow from the unit. If the NaCl concentration in the make-up water had the same value as in the tap water for your locality, calculate the equilibrium NaCl concentration in the circulated water. Would this be objectionable?

5. A 10-ton Freon-12 system operates on a simple saturation cycle with evaporator and condenser temperatures of 15°F and 85°F. If the system were used as a heat

pump, what would be its rated capacity in Btu per hour? If reversed and used as a heat engine, what would be its rating in horsepower? Determine the actual coefficient of performance for each of these three uses and compare with the corresponding coefficients of performance for a Carnot engine.

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CHAPTER V

THE ABSORPTION CYCLE

The cost of energy having the degree of availability necessary to operate a compressor of the usual type is so great that many efforts have been made to find a method of compression using heat in place of shaft work. Examining the problem from the thermodynamic standpoint, the greater part of the energy required for compression is necessary because the material is compressed as a vapor and therefore undergoes a very great change in volume during the process. If means were available for raising the pressure of the refrigerant without appreciably altering its volume, the work requirement would be enormously reduced. The magnitude of the possible reduction is shown by the following example.

A refrigeration system using ammonia operates with saturated vapor leaving the evaporator at 0°F and saturated liquid leaving the condenser at 85°F. The work of isentropic compression is the difference of enthalpy between suction and discharge which, from the *ph* chart, is 720 − 612 = 108 Btu/lb of refrigerant. If the saturated vapor leaving the evaporator were absorbed by cold water at evaporator pressure (30 psia) until the weight concentration of ammonia in the water reached 40 per cent, and if this mixture (weighing 2½ lb/lb of ammonia) were raised to the pressure of the condenser (165 psia), the total work of compression (neglecting the slight change in specific volume from the initial value of 0.0184 cu ft/lb), would be given by

$$\text{Heat equivalent of work of compression} = \frac{(P_d - P_s)v_s w}{778}$$

$$= (165 - 30) \times 144 \times 2\tfrac{1}{2} \times \frac{0.0184}{778}$$

$$= 1.15 \text{ Btu/lb of ammonia}$$

The slight error in the calculated work which results from assuming that the water-ammonia solution does not undergo a change in specific volume is on the conservative side since the actual volume would slightly decrease and the work therefore would be slightly less than calculated.

The above example shows that the required work could be reduced to 1 per cent or less of the work of vapor compression provided means were available to place the refrigerant in solution before compression and to remove it from solution following compression. The absorption cycle achieves this objective but does so at the expense of a much greater total energy requirement, though in the relatively low-grade form of heat rather than shaft work.

5·1. Equipment Arrangement. In simplest form, an ideal absorption system is identical with a mechanical compression refrigeration system, except that the compressor is replaced by a group of four elements in which the absorbent fluid completes its cycle independently of all other parts of the refrigeration system. An arrangement of this kind is shown in Fig. 5·1. Vapor from the evaporator enters the absorber where it is brought into contact with absorbent fluid of low concentration which is held at a low temperature by cooling water coils. The refrigerant goes into solution in the absorbent, raising its concentration, and the resultant strong liquor is pumped through a heat exchanger into the generator. Heat is supplied to the generator, releasing the refrigerant as a high-

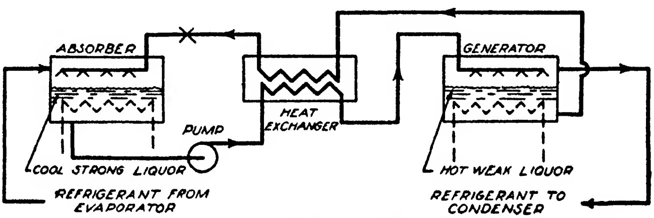


FIG. 5·1. Elements of Ideal Absorption System.

pressure vapor which then passes to the condenser; the hot, weak absorbent flows back through the heat exchanger and through a pressure-reducing valve to the absorber where the cycle is again started. The heat required at the generator is approximately equal to the heat extracted in the absorber; this, in turn, is the quantity of heat needed initially to cool the absorbent plus the heat liberated in the unit during the process of absorption minus the cooling effect of the entering refrigerant vapor.

Actual systems differ from the ideal in two important respects: The heat exchanger is not completely effective and, in consequence, a greater amount of energy must be removed in the absorber and supplied in the generator. Secondly, the absorbent partially vaporizes in the generator and some of it therefore flows to the condenser with the refrigerant; special equipments must be installed to remove, in so far as practicable, the absorbent from the vapor mixture leaving the generator.

Figure 5·2 shows diagrammatically the flow circuits for a typical absorption system. Liquid refrigerant (with a small amount of entrained absorbent) leaves the condenser at point *b*, passes through a precooler where it gives up heat to the vapor from the evaporator, and then enters the evaporator. Leaving the evaporator, the refrigerant vapor and such entrained absorbent as it may contain, passes through the exchanger and into the absorber where contact is established with the low-temperature weak absorbent. The absorbent increases in concentration and leaves the absorber at point *f*, passing through the pump, receiving heat in the exchanger (from the absorbent liquor leaving the generator) and then passing through the analyzer into the generator. Heat supplied in the generator raises the temperature of the concentrated solution, thereby driving off vapor having a high concentration of refrigerant. The hot, weak absorbent then leaves the generator and passes through the heat exchanger where it is cooled by the cool strong absorbent going to the generator. The weak liquor flows from the exchanger to the absorber where the cycle is again started.

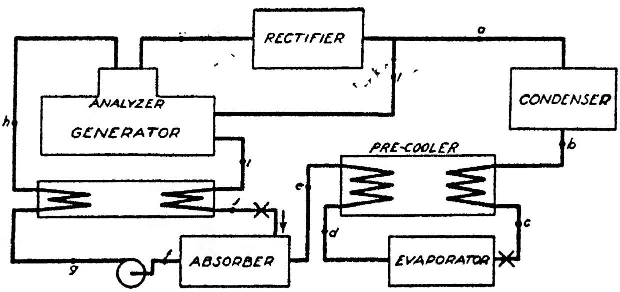


Fig. 5·2. Elements of Actual Absorption System.

The strong hot vapor driven off in the generator passes upward through the analyzer where it flows countercurrent to the rain of cool strong liquor coming in from the pump. Cooling of the vapor occurs with consequent partial condensation, the condensate having a higher concentration of absorbent (refer to Fig. 5·3) and the cool vapor leaving the analyzer with a higher concentration of refrigerant. From the analyzer the vapor passes to the rectifier where it is further cooled by passage over water coils. Additional condensation occurs, and the concentration of refrigerant in the vapor is further raised. However, each small weight of absorbent extracted from the vapor as it passes through the rectifier carries away with it some of the refrigerant, and it is therefore imperative that rectification not be carried too far. Highly concentrated refrigerant vapor enters the condenser while condensate from the rectifier flows back to the generator and repeats its cycle.

From the above discussion it follows that there are three separate flow circuits followed by refrigerant or by absorbent during the actual cycle of an absorbent refrigeration system. These are: (1) the main cycle of the refrigerant (with a small weight of associated absorbent) as it through the condenser, precooler, evaporator, exchanger, absorber, pump, exchanger, analyzer, generator, analyzer, and rectifier; (2) the main cycle of the absorbent (with a small weight of refrigerant) as it passes through absorber, pump, exchanger, analyzer, generator, and exchanger; (3) the cycle followed by the small weight of absorbent and associated refrigerant passing from the generator through analyzer and rectifier back to the generator. These three cycles are not closed, so fluids can pass from one to another, but for a system in steady state operation the weight rate of material through each of the cycles must remain constant. This fact is of great help in analyzing the system.

5·2. Thermodynamic Relationships. Investigation of the effectiveness of various equipment arrangements and analysis of the processes actually occurring in real systems requires an understanding of the thermodynamic relationships which exist between the absorbent and the refrigerant when these materials are in solution, or are present as vapor over a solution and in equilibrium with it.

(*a*) Boiling Point Diagram. A typical boiling point diagram of a refrigerant-absorbent mixture is shown in Fig. 5·3; the abscissa represents the weight fraction of refrigerant present in the mixture while the ordinate shows a temperature scale. Points *A* and *B* give the saturation temperatures (at the particular total pressure for which the diagram is constructed) of the pure absorbent and pure refrigerant respectively. The lower curve connecting *A* and *B* is a locus of equilibrium temperatures corresponding to weight concentrations between 0 and 100 per cent of the refrigerant in the liquid. At any point, as x, on the lower curve, a solution containing x per cent of refrigerant would reach equilibrium with the vapor above it at a temperature of t_x. Any solution containing x per cent of refrigerant would be a subcooled liquid at temperatures below t_x. The entire area below the lower curve therefore represents states of subcooled solutions.

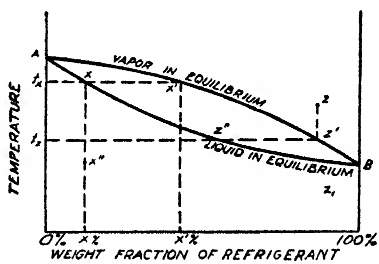


Fig. 5·3. Boiling Point Diagram.

The upper curve is a locus of states for vapor over a solution and in equilibrium with it. At point x, for example, a solution containing x per cent of refrigerant will be in equilibrium with vapor in which the refrigerant concentration is x′ per cent. For the ideal system of Fig. 5·1 the absorbent would never vaporize but would simply undergo a change of temperature while the refrigerant passed into or out of solution. Figure 5·3 shows, however, that when vapor is formed over a solution it must contain some absorbent as well as refrigerant, the respective concentrations being determined from the boiling point diagram by entering at the concentration of refrigerant in the solution, rising to intersection with the curve for liquid in equilibrium, moving right horizontally to intersection with the curve for vapor in equilibrium and then dropping vertically to read the concentration of refrigerant in the vapor from the scale along the abscissa.

The area between the liquid and vapor equilibrium curves represents a region of liquid-vapor mixtures whereas the area above the vapor curve includes the states of superheated vapor mixtures. At states on the vapor equilibrium curve the refrigerant and absorbent vapors are not saturated but superheated because the vapor pressure over a solution is somewhat depressed. The superheat of the absorbent is less than that of the refrigerant so when superheated mixtures are cooled, condensation of the absorbent occurs more rapidly; for this reason some of the absorbent carried out of the generator in vapor form can be extracted differentially from the mixture in the analyzer and in the rectifier.

Consider highly superheated vapor at point z of Fig. 5·3: If heat were removed the state would move down along the vertical line zz' until the equilibrium vapor state at z' would be reached. Additional heat removal would cause condensation of both vapors in proportions such that the condensate would have the state point z'' with a higher concentration of absorbent than was originally present in the vapor. During condensation the state of the vapor would move to the right from z' along the equilibrium vapor line, but with an accompanying increased concentration of refrigerant in the condensate until, if condensation were carried to completion, the final condensate would have the same concentration as the original superheated vapor. Thus complete removal of absorbent from vapor passing to the condenser of an actual absorption system cannot be achieved. This is unfortunate because the presence of even a small amount of absorbent raises the boiling point of the mixture and therefore necessitates a lower evaporator pressure in order to maintain the required evaporator temperature.

(*b*) Mixture Effects. For all practical purposes the assumption can be made that no interaction of vapors takes place when superheated ammonia and water vapor are mixed. The volume of the mixture is calculated by assuming that Dalton's law holds, and the enthalpy of the mixture can be taken as the sum of the enthalpies of the separate vapors, the heat of solution being considered zero.

Special mixture effects must be considered, however, when ammonia goes into solution in water. The volume of an aqua-ammonia solution is somewhat less than the sum of the volumes of its constituents and can be considered as approximately equal to the volume of the water which it contains plus 85 per cent of the contained liquid ammonia.

When ammonia is absorbed by an aqua solution in which the weight concentration is less than 59.4 per cent there is an accompanying liberation of the heat of solution in amount as given by the empirical equation proposed by Mollier,

$$Q = 345\left(1 - \frac{x_f}{2}\right) - 133.3x_f^2 \tag{5·1}$$

where Q is the heat liberated per pound of ammonia absorbed as the solution concentration is changed from an initial value of $x = 0$ to a final value of $x = x_f$. An equal quantity of energy is absorbed when ammonia leaves solution. Absorption into a solution in which the concentration is in excess of 59.4 per cent takes place without heat liberation.

The enthalpy of an aqua-ammonia solution is equal to the sum of the enthalpies of the anhydrous ammonia and the water less the heat of absorption. Thus, at any state z_1 where the concentration of the solution is x_{z_1}, the solution specific enthalpy is given by the equation

$$h_{z_1} = h''_{z_1} + h'_{z_1} - x_{z_1}Q_{z_1} \tag{5·2}$$

where the h'_{z_1} and h''_{z_1} are the respective specific enthalpies of the anhydrous ammonia and water, and the term Q_{z_1} is the average heat of absorption over the concentration range from $x = 0$ to $x = x_{z_1}$ and is given by equation 5·1.

The most complex enthalpy determination needed for the standard system is that of the equilibrium vapor-liquid mixture (sometimes a superheated vapor mixture) entering the absorber (point e in Fig. 5·2). In order to permit operation with a reasonably high evaporator pressure, it is necessary to purge the evaporator by discharging entrained liquid in the leaving vapor. The purging rate depends on the concentration of solution entering the evaporator and on the operating temperature. Since the concentration of refrigerant in the entrained liquid is greatly in excess of 59.4 per cent, enthalpy determination necessitates separating the stream of purge material into two fictitious parts: one containing the ammonia for which there is a heat of liberation and the remainder that for which no such heat exists. The accompanying flow diagram shows such a breakdown. The weight of equilibrium purge solution is w_{se} pounds and is associated with w_e total pounds of mixture at point e. The concentrations of ammonia in the saturated vapor and saturated liquid are taken as x_{ve} and x_{se} respectively.

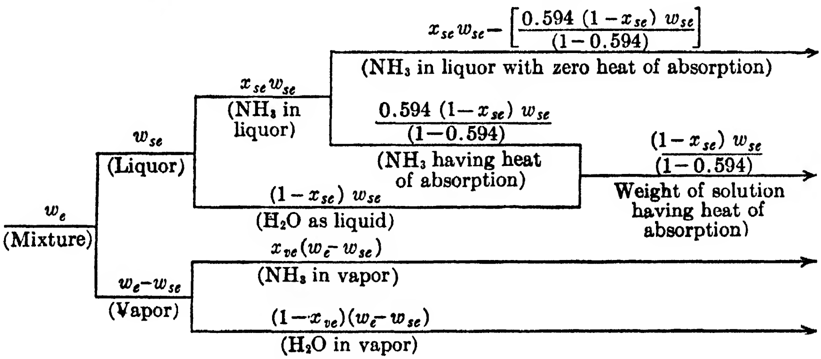
From the diagram on page 94,

$$h_e = (1 - x_{ve})(w_e - w_{se})h''_{ve} + (1 - x_{se})w_{se}h''_{se} + x_{ve}(w_e - w_{se})h'_{ve} + x_{se}w_{se}h'_{se} - \frac{0.594(1 - x_{se})w_{se}Q_m}{(1 - 0.594)} \tag{5·3}$$

where Q_m is the maximum average heat of absorption obtained from equation 5·1 with $x_f = 0.594$.

The terms h'_{se} and h''_{se} are the specific enthalpies of saturated anhydrous liquid ammonia and water at the temperature existing at point e. (Values from steam and anhydrous ammonia tables or charts.)

The terms h'_{ve} and h''_{ve} for NH_3 and H_2O, respectively, are the specific enthalpies of superheated vapor at t_e and at the partial pressures of the mixture at e (values of enthalpy from steam and anhydrous ammonia tables after obtaining partial pressure of water vapor in mixture).



The above analytical method of enthalpy determination is indirect because it requires previous knowledge of the equilibrium concentration of ammonia in both the liquid and vapor; likewise experimental data are needed on the partial pressure of water vapor over an aqua-ammonia solution. Additional disadvantages arise due to deviations from Dalton's law. The solution by this method is therefore only approximate. A less involved and more accurate method of solving aqua-ammonia energy problems is by direct reference to tables or charts giving the properties of equilibrium solutions.

(c) Aqua-ammonia Chart. The chart shown in Fig. 5·4 permits direct reading of the more important properties of aqua-ammonia equilibrium solutions and the vapors in contact with them. For an equilibrium vapor-solution mixture at known total pressure p and temperature t, the chart gives:

x_v = weight concentration of ammonia in the vapor, pound per pound
x_s = weight concentration of ammonia in the solution, pound per pound
h_v = specific enthalpy of the vapor, Btu per pound
h_s = specific enthalpy of the solution, Btu per pound

Data, as plotted, are for equilibrium mixtures only, but the enthalpy of a subcooled solution can be read from the chart, with sufficient accuracy for engineering calculations, as that of an equilibrium solution at the same

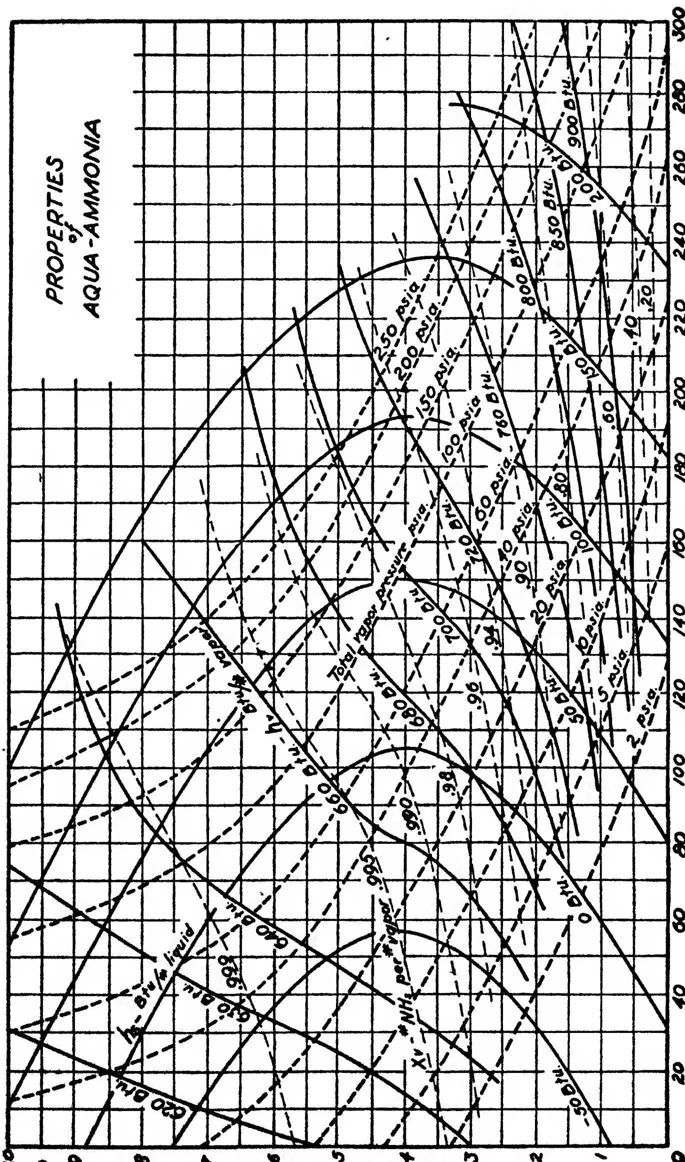


FIG. 5-4. Equilibrium Aqua-ammonia Properties. (Plotted from data published by ASRE.)

temperature and weight concentration. The enthalpy of a highly superheated vapor mixture cannot be determined from Fig. 5·4; for that case calculation from steam and anhydrous ammonia superheat tables is necessary.

Reference to Fig. 5·4 will show the need for purging some liquid with the vapor leaving the evaporator. Consider a system operating with an evaporator temperature of 30°F. If anhydrous ammonia were supplied to the evaporator the pressure would be (Fig. 5·4) 60 psia. Suppose now that the concentration of solution entering the evaporator is reduced to 99.9 per cent; the new evaporator pressure for 30°F and with no purging would be 35 psia, and the equilibrium concentration of solution in the evaporator would drop to 62 per cent. By continuous purging of liquid from the evaporator the pressure (at constant temperature) could be raised. In most practical systems, particularly where the evaporator temperature must be low, purging is essential.

5·3. Complete Absorption Cycle Analysis. The complete analytical procedure for establishing a heat balance and investigating performance characteristics of a typical absorption cycle will be illustrated by making a step-by-step analysis of the system shown in Fig. 5·2. Each of the twelve different states assumed by the working substances at various points in the cycle will be identified by letters as shown on this figure. Analysis will be carried out with the help of ammonia tables, steam tables, and the data of Fig. 5·4. To assist in making cross references each step of the analysis will be assigned a number.

TABLE 5·1

Point	Pressure	Temperature	Weight	Concentration	Specific Enthalpy
a	Given	Given	12	14	14
b	Given	16	Given	14	16
c	Given		12	14	25
d	Given	Given	12	14	23
e	Given	16	12	14	22
f	Given	Given	19	13	17
g	Given		19	13	21
h	Given	6	19	13	13
i	Given	Given	19	15	15
j	Given		19	15	24
k	Given	8	20	18	18
l	Given	t_a	20	14	14

The known data required for establishing a heat balance are listed in Table 5·1. The letters appearing in the first vertical row refer to corresponding points on Fig. 5·2, whereas the numbers in the table refer to the step in the analysis where the property in question is evaluated.

The procedure is as follows.

Known or Assumed

1. The pressure at each point in the system. For the ideal cycle the only pressure changes in the system occur in the pump and the expansion valve. Thus $p_a = p_b = p_c = p_g = p_h = p_i = p_j = p_k = p_l =$ the condenser pressure, and $p_d = p_e = p_f =$ the evaporator pressure.
2. The temperature of the vapor (with entrained liquor) leaving the evaporator, t_d.
3. The temperature of the strong liquor (not saturated with NH_3) leaving the absorber, t_f.
4. The temperature of the equilibrium vapor and the drips leaving the rectifier, $t_a = t_l$.
5. The temperature of the equilibrium liquid leaving the generator, t_i.
6. That the strong liquor in passing through the exchanger is heated to within Δt_1 of the weak liquor entering temperature. Then, $t_h = t_i - \Delta t_1$.
7. That the strong liquor leaves the exchanger saturated at equilibrium concentration.
8. That the equilibrium vapor leaving the analyzer is Δt_2 degrees hotter than the entering strong liquor, $t_k = t_h + \Delta t_2$.
9. That the mixture of vapor and entrained liquor passing through the precooler is heated to within Δt_3 degrees of the liquor entering from the condenser, $t_b = t_e + \Delta t_3$.
10. That the liquor leaving the condenser is at equilibrium concentration. (If this condition is not met the validity of the analysis is unaffected provided the condenser temperature is known.)
11. Base calculations on a flow through condenser of 1 lb/min.
12. By observation $w_a = w_b = w_c = w_d = w_e$.

From Fig. 5·4

13. For equilibrium liquid at p_h, t_h, read x_h, h_h. Note $x_h = x_f = x_g$.
14. For equilibrium liquid and vapor leaving rectifier at p_a, t_a, read x_a, x_l, h_a, h_l. Note $x_a = x_b = x_c = x_d = x_e$.
15. For equilibrium liquid at p_i, t_i, read x_i, h_i. Note $x_i = x_j$.
16. For equilibrium liquid (10) at p_b, x_b, read t_b, h_b. Then $t_e = t_b - \Delta t_3$ (9).
17. For subcooled liquid at state f, read h_f as for a saturated liquid at x_f, t_f.
18. For equilibrium vapor at p_k, t_k, read x_k, h_k.
19. Establishing a mass balance on the absorber,

$$w_f = w_j + w_e$$

and

$$x_f w_f = x_j w_j + x_e w_e$$

Substituting for w_f from the first equation and solving for w_j,

$$w_j = \frac{w_e(x_f - x_e)}{x_j - x_f}$$

and by observation,

$$w_i = w_j$$

Knowing w_j and w_e, it is now possible to solve the first equation above for w_f and, by observation, $w_f = w_g = w_h$.

20. By mass balance on the rectifier,

$$w_k = w_a + w_l$$

and

$$x_k w_k = x_a w_a + x_l w_l$$

Substituting for w_k and solving,

$$w_l = \frac{(x_a - x_k)w_a}{x_k - x_l}$$

Then, knowing w_l and w_a, solve the first equation above for w_k.

21. Calculation of work input to the circulating pump.

$$W = \frac{144(p_g - p_f)w_f}{778\rho e}$$

where ρ is the density of the aqua-ammonia passing through the pump, and e is the pump efficiency. The density, ρ, is the reciprocal of the specific volume v_f, which is equal to the volume of water in 1 lb of solution plus 85 per cent of the volume of liquid ammonia in the solution or

$$v_f = (1 - x_f)v_f'' + 0.85x_f v_f'$$

where v_f' and v_f'' are obtained from the ammonia and steam tables, respectively for saturated liquid at temperature t_f.

$$h_g = h_f + W$$

22. To determine the weight of entrained liquor, w_{se} in w_e:

a. The total weight of ammonia at state point e is the sum of weights of ammonia in the liquor and in the vapor or,

$$x_e w_e = x_{se} w_{se} + (w_e - w_{se})x_{ve}$$

With p_e and t_e known and the mixture in equilibrium, x_{se}, x_{ve}, h_{se}, h_{ve} are read from Fig. 5·4.

b. Solving for w_{se} in terms of known quantities,

$$w_{se} = \left(\frac{x_{ve} - x_e}{x_{ve} - x_{se}}\right) w_e$$

c. Then,

$$h_e = w_{se} h_{se} + (w_e - w_{se})h_{ve}$$

d. In some cases the value of w_{se} calculated from the above equation will be negative. Such a result shows that the mixture passing through the heat exchanger completely vaporizes and leaves superheated. Since Fig. 5·4 is applicable only to *equilibrium* mixtures, the enthalpy of superheated material at e must be determined otherwise. The procedure is as follows:

(1) From an empirical equation proposed by Wilson,* the following equation is derived,

$$p_e'' = \frac{17p(17 - 0.8x_s)(1 - x_s)}{(17 + x_s)^2}$$

where p_e'' is the partial pressure of the water vapor over an aqua-ammonia solution which is in equilibrium with vapor at x_e, p_e; p_e'' is therefore approximately equal to the partial pressure of the water in the superheated vapor-mixture at state e, p is the saturation pressure of pure water at the temperature of equilibrium vapor at x_e, p_e; x_s is the ammonia concentration in a solution in equilibrium with vapor at x_e, p_e.

(2) From the steam tables read the enthalpy of superheated steam, h_e'', at p_e'' and t_e.

(3) The partial pressure of ammonia in the vapor is $p_e' = p_e - p_e''$. Then from the ammonia *ph*-chart read the enthalpy of superheated ammonia vapor, h_e', at p_e', t_e.

(4) The enthalpy of the mixture at e is then calculated by the equation, $h_e = x_e h_e' + (1 - x_e)h_e''$.

23. Read x_{vd}, x_{sd}, h_{vd}, h_{sd} at p_d, t_d. Then,

$$w_{sd} = \left(\frac{x_{vd} - x_d}{x_{vd} - x_{sd}}\right) w_d$$

and

$$h_d = w_{sd}h_{sd} + (w_d - w_{sd})h_{vd}$$

24. $w_i(h_i - h_j) = w_g(h_h - h_g)$

or

$$h_j = h_i - \left(\frac{w_g}{w_i}\right)(h_h - h_g)$$

25. $h_c = h_b - \Delta h_{\text{precooler}} = h_b + h_d - h_e.$

The twenty-five steps outlined above provide all data necessary for filling in Table 5·1. With these data a heat balance can be established and the energy requirements investigated for the various pieces of equipment which make up the system. The heat balance on page 100 gives all energy quantities in Btu per pound of material passing through the condenser. Since some entrained water is carried over into the condenser, it should be noted that the energy is therefore not expressed in terms of one pound of refrigerant. The total enthalpy, H, is the product of specific enthalpy, h, and weight of fluid at that state.

If the coefficient of performance of an absorption system is calculated by the method used for the compression cycle, that is, taking the inverse ratio of energy supplied to the system (except in evaporator) to the heat

* T. A. Wilson, *Bulletin 146*, Engineering Experiment Station, University of Illinois.

Refrigerating effect	$= H_d - H_c$
Heat added in the generator	$= H_k + H_i - H_l - H_h$
Energy added at the pump	$= W$
Total energy added during the cycle	$= H_d + H_i + H_k + W - H_c - H_h - H_l$
Heat rejected in the absorber	$= H_e + H_j - H_f$
Heat rejected in the condenser	$= H_a - H_b$
Heat rejected in the rectifier	$= H_k - H_a - H_l$
Total heat rejected	$= H_e + H_j + H_k - H_b - H_f - H_l$

picked up in the evaporator, the result will show that such a system requires approximately eight times as much energy as does a compression system. The coefficient for any absorption system can be obtained from the equation

$$\text{cop} = \frac{H_d - H_c}{H_k + H_i + W - H_l - H_h} \tag{5·4}$$

However, a direct comparison of coefficients of performance of absorption and compression systems is not likely to be of value because the difference in availability of energy required for the two systems is great; the ratio of total energy requirement is therefore not a good indication of the probable ratio of operating costs.

Example 5·1. An ammonia absorption system operates under conditions such that all pressures are known (see values in Table 5·2) and the following temperatures are known: t_a, t_d, t_f, t_i (Table 5·2). Also:

$$\Delta t_i = 20°\text{F (6)}; \quad \Delta t_2 = 10°\text{F (8)}; \quad \Delta t_3 = 10°\text{F (9)}; \quad e = 70\%$$

TABLE 5·2

Point	Pressure	Temperature	Weight	Concentration	Specific Enthalpy	Enthalpy
a	158	125	1	0.995	664	664
b	157	84	1	0.995	135	135
c	155		1	0.995	92.4	92.4
d	22	15	1	0.995	615.6	615.6
e	20	74	1	0.995	657.2	657.2
f	19	85	11.25	0.38	−20	224
g	165		11.25	0.38	−11.9	
h	162	195	11.25	0.38	102	1148
i	160	215	10.25	0.32	130	1331
j	157		10.25	0.32	4	41
k	160	205	1.16	0.94	740	858
l	158	125	.16	0.60	48	7.7

(*a*) Determine the concentration, specific enthalpy, and total enthalpy at each point in the system. (*b*) Establish a heat balance for the cycle. (*c*) Calculate the coefficient of performance.

Solution.

a. Follow the step-by-step procedure using Fig. 5·4 and recording data in Table 5·2 as obtained.

19. $w_f = \dfrac{1(0.38 - 0.995)}{0.32 - 0.38} = 10.25.$

20. $w_l = \dfrac{1(0.9945 - 0.940)}{0.940 - 0.60} = 0.16.$

21. From steam tables and ammonia tables, v_f'' and v_f' are, respectively, 0.01609 and 0.02687. Then,

$$v_f = (1 - 0.38)0.01609 + 0.85 \times 0.38 \times 0.02687 = 0.01866$$

and

$$\rho = \frac{1}{0.01866} = 53.6$$

so

$$W = \frac{144(165 - 19)11.25}{778 \times 53.6 \times 0.70} = 8.1$$

and

$$h_g = (-20) + 8.1 = -11.9$$

22. $w_{se} = (0.99 - 0.995)/(0.99 - 0.37)$, and since the result is negative the mixture at e is superheated and its enthalpy must therefore be determined by the method outlined in item 22*d*. Thus:

(1) From Fig. 5·4 determine the temperature of equilibrium vapor at x_e, p_e as 60°F; then from steam tables the pressure of saturated vapor at 60° is $p = 0.2563$.

From Fig. 5·4 at $x_v = 0.995$ and $p = 20$, read $x_s = 0.42$.

Then,

$$p_e'' = \frac{(17)(0.2563)(17 - 0.8 \times 0.42)(1 - 0.42)}{(17 + 0.42)^2} = 0.141$$

(2) From tables for superheated steam at p_e'' and t_e, read $h_e'' = 1081.1$. (This value can otherwise be read as for saturated steam at p_e''.)

(3) The partial pressure of ammonia vapor in the superheated vapor mixture is

$$p_e' = 20 - 0.141 = 19.86$$

From *ph*-chart for ammonia at $p_e' = 19.86$ and $t_e = 74°$, read $h_e' = 655$.

(4) Then the enthalpy of the mixture at e is

$$h_e = (0.995)(655) + (0.005)(1081.1) = 657.2$$

23. $w_{ed} = \left(\dfrac{0.9995 - 0.995}{0.9995 - 0.625}\right) = 0.012$ and $h_d = 0.012(-75) + (1 - 0.012)624 = 615.6.$

24. $h_j = 130 - (11.25/10.25)(102 + 11.9) = 4.$
25. $h_c = 134 + 615.6 - 657.2 = 92.4.$

b. From data in Table 5·2 the heat balance for the system is:

Refrigerating effect	$= 615.6 - 92.4$	$=$ 523.2
Head added in generator	$= 858 + 1331 - 7.7 - 1148$	$=$ 1033.3
Energy added at pump	$= 8.1$	$=$ 8.1
Total energy added	$=$	1564.6
Heat rejected in the absorber	$= 657.2 + 41 + 224$	$=$ 922.2
Heat rejected in the condenser	$= 664 - 135$	$=$ 539
Heat rejected in the rectifier	$= 858 - 664 - 7.7$	$=$ 186.3
Total heat rejected	$=$	1647.5

c. $\text{cop} = \left(\dfrac{523.2}{1041.4}\right) = 0.50$ Non-balance $= 5\%$

5·4. Special Systems. The principles which have been explained in connection with the conventional aqua-ammonia absorption system can be applied in a multitude of ways leading to cycles which differ to a greater or less degree from the cycle which has been described. In general, the purpose of such changes is to permit use of a more satisfactory refrigerant, or absorbent, or to re-arrange the cycle in such a way that the coefficient of performance will be improved.

Descriptions are available in the literature of systems using dichloromonofluoromethane and lithium chloride as working fluids, one advantage over aqua-ammonia systems being the reduced toxicity. Other systems propose the use of absorbent materials having thermodynamic characteristics such that the concentration of absorbent in the vapor leaving the generator is very much less than when water is used.

The use of an inert third fluid serves in one type of system to eliminate need for a pump by maintaining a constant total pressure (except for friction losses) throughout the cycle. In this system the partial pressure of the refrigerant varies, as in the conventional system, between the evaporator minimum and the generator maximum, but a compensating variation in partial pressure of the third fluid permits maintenance of the same total pressure in both equipments.

A resorption system, proposed by Maiuri, makes use of a second circuit of absorbent which reabsorbs the high-pressure vapor leaving the generator and carries it directly to the evaporator. Multi-stage absorption systems have also been developed, as have combined systems using conventional absorption with a preliminary stage of mechanical compression.

Similar in principle to the absorption cycle is a system in which vapor leaving the evaporator is condensed on the surface of a solid adsorbent as a result either of capillarity or of surface attraction. In this system a pump is not used; the adsorbent is alternately cooled and heated, thereby establishing a sequence of operations during which the refrigerant flows from the evaporator and is adsorbed, then discharged to the condenser by application of heat to the "saturated" adsorbent. The obvious disadvantage of the system is that operations are not continuous. This objection can be partially overcome by using two chambers containing adsorbent, one of which is receiving refrigerant from the evaporator while the other is discharging vapor to the condenser. Silica gel, activated charcoal, and lithium chloride are the most common adsorbents.

PROBLEMS

1. Calculate the heat liberated when 2 lb of ammonia vapor are absorbed by 10 lb of water. Repeat the calculation for absorption of 2 lb by 3 lb of water.

2. Liquid aqua-ammonia at 100°F and 40 psia is in equilibrium with its vapor. From Fig. 5·4 determine: the concentrations of ammonia in the liquid and vapor; the specific enthalpy of the vapor. Calculate the specific enthalpy of the vapor-liquid mixture provided 20 per cent by weight is in vapor form.

3. A vapor mixture having a concentration of 90 per cent is in equilibrium with its liquid in a container which is in boiling water. Determine (from Fig. 5·4) the pressure within the container.

4. For the conditions of problem 3 calculate the partial pressure of the water vapor.

5. A solution of aqua-ammonia at 100°F is under a pressure of 100 psia. The concentration is 30 per cent. Determine the specific enthalpy.

6. Aqua-ammonia vapor having a concentration of 98 per cent is subject to a temperature of 150°F when the pressure is 40 psia. Determine the specific enthalpy.

7. Using the given data of illustrative Example 5·1 except that t_a is to be taken as 165°F: (*a*) Determine the concentration, specific enthalpy, and total enthalpy at each point in the system. (*b*) Establish a heat balance for the cycle. (*c*) Calculate the coefficient of performance. (*d*) Compare results with those of the illustrative example and critically discuss the effect of absorbent carry-over on the performance of the system.

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CHAPTER VI

STEADY STATE HEAT TRANSFER

The transfer of energy from a hot body to a cold body can occur by the mechanisms of conduction, convection, and radiation. Even though the three phenomena are strikingly different, facility in computation is best realized by expressing the transfer coefficients for radiation and convection in a form suitable for insertion in the fundamental conduction equation. There is need of some such unified procedure since most practical problems involve transfer by at least two mechanisms.

A general classification of heat transfer problems can be established with respect to time as a variable. The first group is that in which conditions do not change with time; problems of this type occur in systems which are in equilibrium and are generally referred to as *steady state heat transfer systems.* A second group is that in which the heat transfer rate varies with time from an original boundary value toward some new boundary value. Problems of this type frequently occur in both air conditioning and refrigeration; typical examples are an insulated refrigerant pipe during the interval between starting the system and reaching equilibrium, or the problem of establishing heating and cooling curves for an intermittently heated structure. Such problems come under the heading of *transient heat flow* (Chapter VIII) and require a more complex treatment than that needed for steady state problems. The third group, *periodic heat transfer* (Chapter VII), covers the type of problem resulting from the load variation due to diurnal changes in the outside temperature, wind velocity, or humidity. A satisfactory method of analysis for such cases is needed to determine the maximum load and the time at which it will occur.

This chapter takes up the problems which come under the first group in the above classification. Steady state equations are presented and methods developed for expressing all three mechanisms of heat transfer in terms of the conduction equation.

6·1. Conduction. The basic equation for the instantaneous rate of heat transfer by conduction is given by Fourier's law,

$$\frac{dQ}{d\theta} = q = -kA\,\frac{dt}{dx} \tag{6·1}$$

where dQ = heat transfer, Btu, during the time interval $d\theta$.
q = instantaneous rate of heat transfer, Btu/hr.

k = thermal conductivity, Btu/(hr) (sq ft) (°F/ft), a function of the temperature. Values of k for materials used in air conditioning and refrigeration are available in standard textbooks and handbooks. A few representative values are given in Table 6·1.

dt = change in temperature along the path length dx.

A = area, square feet, normal to the path. The area may be a function of path length.

The minus sign on the right-hand side of equation 6·1 indicates that the temperature is decreasing in the direction in which heat is flowing.

TABLE 6·1

SELECTED VALUES* OF k IN BTU/(HR) (SQ FT) $\left(\frac{F°}{FT}\right)$

Air, dry, at 1 atm, at 64°, approx	0.0147
Hydrogen, dry at 1 atm, at 64°, approx	0.1050
Water, as steam at 212°, approx	0 0126
Cork, granulated, grains $\frac{1}{32}$ in. to $\frac{1}{8}$ in., density 10.05, dry, at 32°	0.0208
Glass wool, fibers 0.0003 to 0.0006 in. in diam., density 1.5	0.0225
Cork, ground and made into board with asphalt binder, at 32°	0.0292
Magnesia, 85%, and asbestos, density 13.5, 50° to 1100°, average	0.0475
Wood, yellow pine, 16% moisture, across grain	0.0831
Plaster, gypsum	0.275
Water, at 64°, liquid	0.336
Brick, common	0.416
Glass, ordinary window, soda base	0.433
Concrete, solid, with ordinary stone aggregate, average	1.00
Granite, average	1.17
Aluminum, at 64°	83.0
Copper, at 64°	225.
Silver, at 64°	243.

* Various sources.

For steady state conditions q is a constant and, providing variation of thermal conductivity with temperature is negligible, equation 6·1 takes the form,

$$q = \frac{kA_m}{L}(t_2 - t_1) = C'(t_2 - t_1) = \frac{1}{R'}(t_2 - t_1) \tag{6·2}$$

where A_m = the mean area in the path of heat flow. For systems in which the cross section varies directly with the length of path A_m is the logarithmic mean area, given by the expression

$$A_m = \frac{A_2 - A_1}{\log_e\left(\frac{A_2}{A_1}\right)} \tag{6·3}$$

where A_2 and A_1 are the larger and smaller areas respectively. For systems in which the area varies as the square of the path length A_m is equal to the geometric mean area = $(A_1A_2)^{1/2}$.

t_2 = the temperature of the hot surface, °F.
t_1 = the temperature of the cold surface, °F.
L = the length of path along which heat is flowing, feet.
C' = the conductance of the wall = kA_m/L.
R' = the resistance to heat flow for the system = $L/kA_m = 1/C'$.

Equation 6·2 is seldom directly applicable to problems in air conditioning and refrigeration since such problems usually involve transfer across fluid films on one or both sides of the material through which heat is flowing. In such cases the surface temperatures t_1 and t_2 differ from the fluid temperatures and cannot be readily determined. An equation must therefore be established for transfer across the fluid boundary; this is a problem in convection.

6·2. Convection. Rational evaluation of convective heat transfer is complicated by the fact that two, and sometimes three, separate and distinct regions exist between the surface to or from which heat is flowing and the main body of the passing fluid stream (Fig. 6·1). Immediately next to the surface is a sublayer in which there is laminar or streamline flow; in this region heat transfer is entirely by the mechanism of conduction and could be evaluated by equation 6·2, if data were available on the thickness of the layer and the temperature difference across it. Such data are not directly available and cannot be easily obtained.

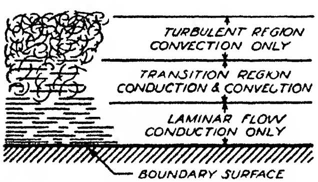

FIG. 6·1. Heat Transfer from a Fluid to a Solid.

Beyond the region of laminar flow is a transition layer of variable thickness across which there is a temperature drop and through which heat is transferred partly by conduction and partly by the convection resulting from eddy currents. In this region the relative importance of the factors influencing heat transfer becomes increasingly difficult to evaluate. The transition layer is of greatest importance when flow is with a high degree of turbulence; as the main body of the fluid approaches streamline flow the transition region tends to disappear.

The third heat transfer mechanism is represented by conditions in the core of a turbulent stream. Heat entering or leaving the core must result in an increase or a drop in its temperature, but eddy current activity is so great in this region that no appreciable temperature gradient exists, and heat transfer occurs almost entirely as a result of mechanical mixing. Core conditions can be roughly visualized by considering that in a heated room the temperature in a horizontal plane does not appreciably vary until the walls are closely approached; the entire temperature drop from room air temperature to inside surface temperature occurs within the thin laminar and transition layers adjacent to the wall surface.

Heat transfer by convection is greatly influenced by those factors which make for greater or less turbulence within the fluid. An equation giving convective transfer rates would have to include terms involving all those properties of the fluid which affect its flow characteristics. Such an equation would be very complex and would have to include empirical relationships which might be expected to vary with different fluids and with different types of fluid flow. To reduce this equation to a more convenient form, all relationships involving the thermal or flow characteristics of the system are grouped in one empirical term, the film coefficient of heat transfer, h, Btu/(hr) (sq ft) (°F). The convective heat transfer rate is then given by Newton's law of cooling, which possesses the additional advantage of resembling the basic conduction equation

$$q = hA(t_f - t_s) = C'_f(t_f - t_s) = \frac{t_f - t_s}{R'_f} \tag{6·4}$$

where A = the area of the boundary surface across which the heat transfer rate is being established.

t_f = the temperature of the core or main stream of passing fluid.

t_s = the temperature of the surface to which heat is flowing.

C'_f and R'_f = respectively, the conductance and thermal resistance of the film through which heat is flowing.

The accurate evaluation of h is difficult. No general solution is available and the existing empirical and semi-rational equations are multitudinous and beyond the scope of this book. Whenever the importance of the problem warrants it, the heat transfer literature should be consulted for the particular equation most directly applicable to the conditions of the individual problem. Lacking such an equation, or other more particularized data, an approximate evaluation of the film coefficient for a subcooled liquid, a superheated vapor, or a gas flowing turbulently within a pipe can be obtained from Nusselt's equation,

$$h = 0.023\frac{k}{D}\left(\frac{DV\rho}{\mu}\right)^{0.8}\left(\frac{c_p\mu}{k}\right)^{0.4} \tag{6·5}$$

where k, μ, ρ, and c_p are the thermal conductivity, viscosity, density, and specific heat respectively of the fluid at its mean temperature; D is the pipe diameter; V is the fluid velocity. Any set of consistent units can be used. Equation 6·5 is not applicable to condensing vapor or evaporating liquid.

The fluids of greatest interest in air conditioning and refrigeration are, of course, air, water, and refrigerants. Simplified expressions for determining h for some of these fluids under restricted conditions are available in standard engineering handbooks. (See last reference in bibliography.)

6·3. Conduction and Convection. The steady state problem of fundamental importance in air conditioning is that of evaluating the rate of heat transfer, from air to air, across a wall. Consider the applicability of the equations already obtained to calculation of the transfer rate through the single-layer, homogeneous wall of Fig. 6·2. Equation 6·2 can be written for the rate q_w of heat transfer between wall surfaces, whereas equation 6·4 can be written twice, once each for the transfer rates q_i and q_o across inside and outside air films. But the three values of q obtained from these equations must be the same, since otherwise heat would be entering or leaving storage in the wall, and the assumed condition of steady state could not exist.

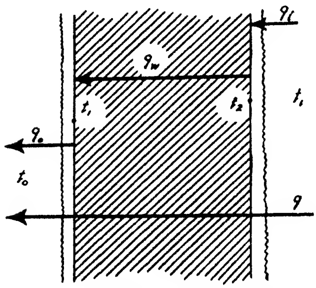

FIG. 6·2. Air-to-air Heat Transfer.

However, no one of the three equations can be solved because each includes at least one unknown wall surface temperature. What is needed is a single equation giving the rate of heat transfer in terms of the air to air temperature difference, $t_i - t_o$, and an "overall" coefficient of heat transfer, U'. Assuming the existence of such an equation, it is then possible to write

$$q = U'(t_i - t_o) \tag{6·6}$$

$$U'(t_i - t_o) = C_i'(t_i - t_2) \tag{6·6a}$$

$$U'(t_i - t_o) = C_w'(t_2 - t_1) \tag{6·6b}$$

$$U'(t_i - t_o) = C_o'(t_1 - t_o) \tag{6·6c}$$

Solving each of these three equations for the unknown temperature difference in the right-hand side and adding the resultant set of three equations gives

$$U'(t_i - t_o)\left(\frac{1}{C_i'} + \frac{1}{C_w'} + \frac{1}{C_o'}\right) = (t_i - t_2) + (t_2 - t_1) + (t_1 - t_o) = t_i - t_o$$

or,

$$U' = \frac{1}{\dfrac{1}{C_i'} + \dfrac{1}{C_w'} + \dfrac{1}{C_o'}} = \frac{1}{R_i' + R_w' + R_o'} = \frac{1}{R_t'} \tag{6·7}$$

where R_t' is the total resistance of the system $= 1/U'$.

By analogy with equation 6·7, the value of U' for a plane wall made up of any number of sections can be written directly, one resistance term being included for each homogeneous wall section and one for each film coefficient. Equations 6·6 and 6·7 therefore establish a general solution of any steady state heat transfer problem in which the heat flow is by combined conduction and convection. The equations are valid for any shape of wall and require only a knowledge of the film coefficients, the thermal conductivities, length of path through each solid section, mean area of each wall section, and area of each film. The most common type of problem encountered in the fields of refrigeration and air conditioning is that of unidirectional flow through a plane wall; in this case the mean wall area is the same as the area of each film and the equations can be simplified by factoring out the area and using unprimed values of U, R, and C to indicate the overall-coefficient, resistance, and conductance, respectively, based on unit area. Thus,

$$q = UA(t_i - t_o) \tag{6·8a}$$

where

$$U = \frac{1}{\dfrac{1}{h_i} + \dfrac{L}{k} + \dfrac{1}{h_o}} = \frac{1}{\dfrac{1}{C_i} + \dfrac{1}{C_w} + \dfrac{1}{C_o}}$$

$$= \frac{1}{R_i + R_w + R_o} = \frac{1}{R_t} \tag{6·8b}$$

where R_t is the total resistance, per unit area, of the system $= 1/U$. There are two types of wall construction for which equation 6·8*b* does not seem directly applicable. The first is that represented by non-homogeneous constructions such as hollow tile or mixed rock. The actual heat flow through sections of this type is not unidirectional and cannot be rationally expressed except in certain special cases and then at the cost of a very complex equation. For practical purposes, such walls can be treated as equivalent to homogeneous sections for which the conductance has been determined; reference to standard textbooks or handbooks will give values of the equivalent C for most of the types of non-homogeneous construction which are used in practice.

The second type of special construction is that represented by air spaces located between homogeneous wall sections. If the space is of sufficient width to insure the presence of a turbulent core, the heat transfer across it will be the same as that across two air films in series and the conductance of the space, C_s, will be equal to $\dfrac{1}{(1/C_i + 1/C_o)}$ where C_i and C_o are the film coefficients on either side of the space and may, or may not, be equal; more simply, $R_s = R_i + R_o$. Usually, space width is not great enough to insure a turbulent core, and conditions in the space are such that the total resistance is less than the sum of the two film resistances. The conductance of such spaces cannot be calculated but must be taken from available data determined experimentally. As the space becomes narrower, the interaction effects become greater and the conductance increases. Further, as the temperature difference across the space increases, circulation is set up, which again increases the conductance. The variation of C_s with respect to width and temperature is shown graphically in Fig. 6·3.

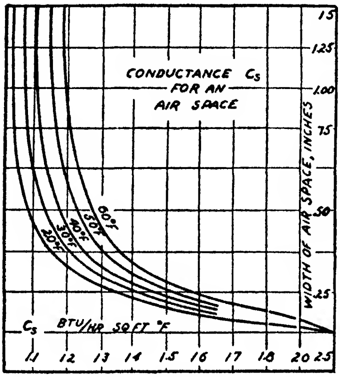

FIG. 6·3. (From tabular data in *ASHVE Guide*, p. 92, 1944.)

6·4. Radiation. Evaluation of the heat flow to or from a surface depends not only on the transfer by conduction and convection through the air film, but also on the energy interchange by radiation between the wall surface and its surroundings. The fundamental equation for radiant exchange between surfaces separated by a non-absorbing medium is the Stefan-Boltzmann law,

$$q = 0.173AF_{ae}\left[\left(\frac{T_h}{100}\right)^4 - \left(\frac{T_c}{100}\right)^4\right] \tag{6·9}$$

where A = the area of one of the bodies between which radiant energy is flowing.

F_{ae} = a term dependent on the relative geometrical position of one body with respect to the other and the emissivity characteristics of the two surfaces.

T_h and T_c = the temperatures in °F absolute of the hot and cold surfaces.

q = the net rate of heat transfer in Btu per hour.

To assist in computations, the term F_{ae} is replaced by the product $F_a \times F_e$ where F_a is expressed in terms of the geometry of the system and F_e in terms of the surface emissivities only; these two new terms are mutually dependent and each has numerical values ranging from 0 to 1.0. Detailed tabulations of F_a and F_e are available in texts on heat transmission; typical values of emissivity, important in air conditioning and refrigeration, are listed in Table 6·2. For a small body enclosed by a large body, the most important single case, $F_a = 1$.

Equation 6·9 is not in a form readily adapted for use with the combined conduction-convection equation. To permit solution of the entire wall transfer problem by means of one equation it is desirable to establish a radiant transfer coefficient which can be used in combination with the film coefficient for convection. Such a coefficient could be added to h for

TABLE 6·2

EMISSIVITIES OF TYPICAL MATERIALS*

Highly polished copper plate	0.02–0.03
Highly polished iron plate	0.05–0.06
Aluminum surfaced roofing	0.2
Aluminum paint	0.3–0.5
Clean smooth sheet iron	0.55–0.65
Sheet steel with oxide layer	0.80
White enamel fused on iron	0.90
Red brick, rough	0.93
Smooth glass	0.94
Oil paints, all colors	0.92–0.96
Water	0.95–0.96
Lampblack	0.96–0.98

* In temperature range from 30° to 110°F; various sources.

the air film to give a single combined surface transfer coefficient h' such that

$$q = h'A(t_i - t_2) = (h_f + h_r)A(t_i - t_2) \tag{6·10}$$

where h_f and h_r are the film coefficient by convection and the equivalent surface coefficient for radiant exchange with the surroundings. From equations 6·9 and 6·10, h_r can be evaluated in terms of t_i, t_2 and the uniform temperatures t_{s_1}, t_{s_2}, t_{s_3}, etc., of different sections of the surroundings by the equation,

$$h_r = 0.173A_2\left\{F_{a_1}F_{e_1}\left[\left(\frac{T_{s_1}}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] + F_{a_2}F_{e_2}\left[\left(\frac{T_{s_2}}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right] + \cdots + F_{a_n}F_{e_n}\left[\left(\frac{T_{s_n}}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]\right\} \div A_2(t_i - t_2)$$

$$= \sum_{n=1}^{n=n}\left\{\frac{0.173F_{a_n}F_{e_n}\left[\left(\frac{T_{s_n}}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{t_{s_n} - t_2}\right\}\left(\frac{t_{s_n} - t_2}{t_i - t_2}\right)$$

where F_{a_n} and F_{e_n} are evaluated with respect to A_2. Then,

$$h_r = \sum_{n=1}^{n=n}\left[h'_{r_n}\left(\frac{t_{s_n} - t_2}{t_i - t_2}\right)\right] \tag{6·11}$$

where

$$h'_{r_n} = \frac{0.173F_{a_n}F_{e_n}\left[\left(\frac{T_{s_n}}{100}\right)^4 - \left(\frac{T_2}{100}\right)^4\right]}{t_{s_n} - t_2} \tag{6·12}$$

The coefficient h'_r is based on the surface to surface temperature difference and satisfies the equation,

$$q_r = h'_rA_2(t_s - t_2) \tag{6·13}$$

Figure 6·4 gives a graphical solution of equation 6·12. Enter the graph at the hot surface temperature t_s, rise to intersection with the cold surface temperature t_2, move horizontally to intersection with the curve corresponding to the shape factor F_a, of the hot surface with respect to the cold (based on the cold surface area), then move vertically to intersection with the proper F_e curve, then move horizontally to the left to read the numerical value of h'_r. Similarly, determine a value of h'_r for each uniform temperature section of the enclosure.

A serious difficulty arises, however, in calculating h_r from equation 6·11. The values of h'_r, determined from the graph, are based on known or assumed temperatures of the wall surfaces, and since these temperatures usually are not known until a heat balance has been established the assumptions may be in considerable error. Fortunately, small inaccuracy in t_s does not seriously affect h'_r so the values determined from Fig. 6·4 can be used with accuracy well within the limit of other heat transfer data even though h'_r is obtained on the basis of an assumed t_s which may be in error by a few degrees. In calculating h_r, however, the correction applied to h'_r is a ratio of assumed temperature differences and has a direct and important effect on the value of h_r; in equation 6·11 the fractional error in h_r will be directly proportional to the error in this ratio. Since the temperature differences are likely to be very small — a matter of but a few degrees — it is obvious that even a small error in the assumed value of t_s or t_2 may seriously affect the accuracy of the term h_r.

The only satisfactory way to obtain an accurate heat balance by the above method is to assume values of t_2, t_{s_1}, t_{s_2}, etc., determine the corresponding h_r, then use this term in the heat balance equations to investigate the accuracy of the assumptions; if the results do not agree, revise assumptions and repeat the process until agreement is reached.

Many problems lend themselves to a simplification which eliminates the cut-and-try process outlined above and thereby reduces the amount of work necessary to obtain a solution. In the average room, with exposure on only one or two sides, most of the surfaces surrounding and exchanging radiation with the exterior wall are inside partitions or floor or ceiling surfaces which are at an equilibrium temperature very close to that of the air in the room. If it can be considered that all such surfaces are at a uniform temperature equal to that of the room air, the series of equation 6·11 reduces to a single term in which F_a is equal to unity (provided A_2 lies in one plane), the ratio of temperature differences is also unity, and the value of h'_r determined from Fig. 6·4 is therefore equal to the desired coefficient h_r. Except in special problems, such as those which occur in panel heating or in convection heated rooms where there is an unusually large area of glass, the assumption that $t_i = t_{s_n}$ and hence that $h_r = h'_r$ is usually accurate within the limits required for engineering calculations.

Based on the above, or other similar conditions, empirical relationships

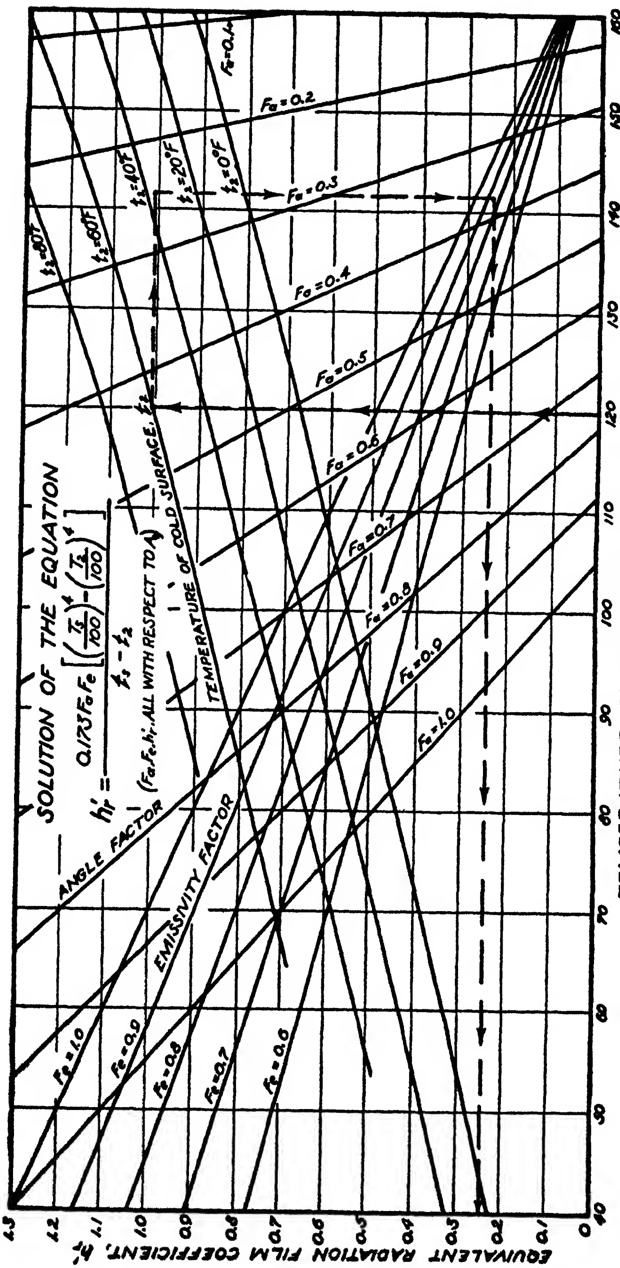

FIG. 6·4. Graphical Determination of Equivalent Radiation Coefficient.

have been developed for directly calculating the combined transfer coefficient h'. For air conditioning calculations the most widely used empirical relationships of this type are those obtained by Houghten and McDermott and published by the ASHVE:

$h' = 1.4 + 0.28V'$ for very smooth surfaces such as glass. (6·14*a*)

$h' = 1.6 + 0.3V'$ for smooth surfaces such as planed wood and plaster. (6·14*b*)

$h' = 2.0 + 0.4V'$ for moderately rough surfaces such as finished concrete and smooth brick. (6·14*c*)

$h' = 2.1 + 0.5V'$ for rough surfaces such as stucco and rough brick. (6·14*d*)

where V' = the wind velocity over the surface, expressed in miles per hour.

6·5. Surface and Interface Temperatures. The need for an overall coefficient of heat transfer, U', arose from lack of data or adequate practical means of determining the value of wall surface temperatures. Knowing U', however, it is now possible to return to equation 6·6*a*, rewrite it in terms of resistances and solve for the unknown inside wall surface temperature t_2,

$$\frac{t_i - t_o}{R_t'} = \frac{t_i - t_2}{R_i'} \tag{6·15}$$

or

$$\frac{t_i - t_2}{t_i - t_o} = \frac{R_i'}{R_t'}$$

which states that the ratio of temperature drop across the inside film is to the total air-to-air temperature difference as the resistance of the inside film is to the total air-to-air resistance. Solving for t_2,

$$t_2 = t_i - \frac{R_i'}{R_t'}(t_i - t_o)$$

and, by analogy, the equation for the temperature at any point x in the wall is

$$t_x = t_i - \frac{R_x'}{R_t'}(t_i - t_o) = t_i - R_x'U'(t_i - t_o) \tag{6·16}$$

where R_x' is the resistance from the inside air to the point in question. Equation 6·16 is applicable to a wall or body of any shape or curvature.

A graphical method of determining surface or interface temperatures is shown in Fig. 6·5. An insulated pipe is subject to a flow of heat from the surrounding atmosphere through an outside air film, through the insulation (wall section *a*), the pipe wall (section *b*), and the inside fluid film to the main body of fluid passing through the pipe at temperature t_i. A longitudinal cross section of the pipe and insulation is drawn to scale (left side of Fig. 6·5), then an arbitrary scale of temperature is set up on a vertical line, SS, to the right of this cross section. Moving to the right from SS a distance $1/h_oA_o$ gives the equivalent thickness in feet of an air film of unit conductance and establishes line 2–2, which represents the outside surface of the insulation. Continuing to the right a distance $L_a/k_aA_{m_a}$ sets up a section equivalent to the actual insulation, but of unit conductance, and serves also to establish the location of the interface, line 3–3, between the insulation and the pipe wall. In the same way the width of an equivalent pipe wall of unit conductance, $L_b/k_bA_{m_b}$, is laid off, establishing line 4–4; to the right of this is set off the equivalent inside film of width $1/h_iA_i$.

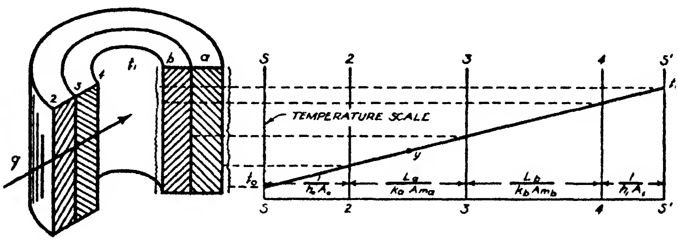

FIG. 6·5. Graphical Temperature Determination.

The resultant equivalent wall of width SS' (including both surface films) is of unit resistance throughout and must therefore have a straight and continuous temperature gradient. From the temperature scale establish the point t_o on line SS and t_i on $S'S'$, where t_o and t_i are the fluid temperatures outside and inside, respectively, of the pipe. A straight line connecting these two points is the temperature gradient through the equivalent wall. From the intersection of this line with surfaces 2–2 and 4–4 and with the interface 3–3, the temperatures on these lines can be readily determined by direct reading from the temperature scale. The points can then be transposed back to the cross section to permit interpretation of the steepness of the temperature gradient through the different wall sections. However, the true gradient through the scale drawing of the pipe cannot yet be drawn since the area is variable and this gradient, between surfaces or interfaces, is therefore curved; thus the point y located midway between lines 2–2 and 3–3 of the equivalent wall does not correspond to a point midway through the insulation, but rather to that point in the insulation at which the resistance is one-half of the total value for the insulating layer. By breaking down the insulation into sections each having a separate value of A_m, as many points on the temperature gradient can be found as may be desired; usually, however, a determination of surface and interface temperatures will alone provide all information needed in the solution of practical problems.

For plane walls the difficulty of variable resistance does not occur (except when the thermal conductivity varies appreciably with temperature), and immediate determination of the temperature at any point in the wall is therefore directly possible by the graphical method. Surface and interface temperatures are fixed as on the scale drawing in Fig. 6·5, but it is now possible to connect such points by means of straight lines, thereby fixing the true gradient and permitting the determination of any desired point temperature by direct transposition of the gradient intersection to the temperature scale. If the temperature at but one point in a wall is required, calculation by equation 6·16 usually gives the most rapid and direct solution, but if the complete temperature gradient across a wall consisting of more than one homogeneous section is needed, the graphical method is more direct and less time consuming.

PROBLEMS

1. A wall having a surface of 450 sq ft transmits 11,000 Btu/hr by conduction. The inside surface is smooth and the outside very rough. Wind velocity is 20 mph with still air on the inside. The conductivity of the homogeneous material of which the wall is made is 7 Btu/(hr) (sq ft) (°F/in.). What is the wall thickness in feet if inside and outside air temperatures are 73° and 32°F?

2. A wall of construction identical with the above passes 27,000 Btu/hr and is ¼ in. thick. What is the approximate wind velocity? (Temperatures as in problem 1.)

3. The air velocity on one side of a $\frac{1}{16}$-in. smooth copper sheet is equivalent to a 15-mph wind while still air conditions exist on the other side. The conductivity of copper is 222 Btu/hr (sq ft) (°F/ft). If the copper is replaced with a sheet of metal for which the conductivity is only 10 per cent that of copper, what per cent reduction in heat transfer will occur?

4. A cold storage room measures 10 ft by 8 ft by 7 ft high. It is surrounded on three sides and on top and bottom by enclosures in which the temperature is 85°F. The temperature in a quick-freezing room adjoining one 10-ft side of the storage room is −10°F. If the resistance through the freezer room wall is one-fifth of that through other walls, floors, and ceilings, what will be the temperature in the cold storage chamber?

5. A fluid is passing through a pipe having an inside diameter of 2 in. The velocity is 4.5 fps, the density, specific heat, viscosity, and thermal conductivity of the fluid (at main body temperature) are, respectively, 60 lb/cu ft, 0.65 Btu/(lb) (°F), 2.0 centipoises [viscosity in centipoises × 0.000672 = viscosity in lb/(sec) (ft)], and 0.08 Btu/(hr) (sq ft) (°F/ft). If heat is being added to the fluid, through the pipe wall, calculate the thermal resistance of the inside fluid film.

6. A refrigerant line having an inside diameter of 0.5 in. is insulated with 85 per cent magnesia (density 13.5 lb/cu ft) to an outside diameter of 3 in. At a particular cross section the refrigerant is at −20°F and room air is at 70°F. Taking the inside and outside film coefficients as 20 and 2 Btu/(hr) (sq ft) (°F) and neglecting the thermal resistance of the thin walled tubing: (*a*) Calculate the rate of heat gain (at that cross section) in Btu/hr. (*b*) Determine the temperature at the mid-point and on the outside surface of the insulation.

7. Solve (*b*) of problem 6 by the graphical method.

8. A small blackened electric heating plate at 150°F is losing energy by radiation in a room where all interior surfaces are at a uniform temperature of 40°F. Determine the equivalent radiation film coefficient and calculate the rate of net radiant energy loss.

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CHAPTER VII

PERIODIC HEAT TRANSFER

7·1. Introduction. The general differential equation expressing the rate of change of temperature with respect to time at any point in any body is

$$\frac{\partial t}{\partial \theta} = \frac{\partial}{\partial x}\left(\frac{k_x}{\rho_x c_x}\frac{\partial t}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{k_y}{\rho_y c_y}\frac{\partial t}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{k_z}{\rho_z c_z}\frac{\partial t}{\partial z}\right) \tag{7·1}$$

where t is the temperature, θ the time, x, y, and z distances along axes centered at the point in question, k_x, k_y, and k_z the thermal conductivities expressed as functions of position and temperature, ρ_x, ρ_y, and ρ_z the densities expressed as functions of position, c_x, c_y, and c_z the specific heats at constant pressure expressed as functions of both position and temperature.

If the body is of uniform material with properties which are the same in all directions and if variations with temperature of thermal conductivity and specific heat are negligible, equation 7·1 reduces to the form

$$\frac{\partial t}{\partial \theta} = \alpha\left[\frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 z}{\partial z^2}\right]. \tag{7·2}$$

where α, the thermal diffusivity, is equal to $k/\rho c$. For unidirectional heat flow,

$$\frac{\partial t}{\partial \theta} = \alpha \frac{\partial^2 t}{\partial x^2} \tag{7·3}$$

which is the form of the unsteady state heat transfer equation finding greatest application in refrigeration and air conditioning. Analytical solutions of equation 7·3 are available in the literature for many common boundary conditions, but application of this equation to cases for which a type solution is not available frequently becomes complex. For the particular type of transient and periodic problem of greatest interest in air conditioning, a simpler and more satisfactory solution is frequently obtained by resorting either to approximate methods based on the assumption that instantaneous departures from equilibrium conditions are not great, or to the more exact graphical method of finite differences.

7·2. Method of Finite Differences. The method presented in this section is based on an equation developed by E. Schmidt* and extended by Nessi and Nisolle.† Consider a finite section of homogeneous wall, Δx of

* E. Schmidt, *Foppls Festschrift*, p. 179, Springer, Berlin, 1924.

† A. Nessi and L. Nisolle, *Méthodes graphiques pour l'étude des installations de chauffage et de refrigeration en régime discontinu*, Dunod, Paris, 1929.

Fig. 7·1*a*, across which the temperature gradient at a particular time is given by the dashed line. At that time, the gradient at the left boundary is tangent to the dashed line and the gradient at the right boundary is likewise tangent to the true section temperature gradient. Draw these tangent lines, establish a pair of construction lines (Fig. 7·1*b*) at distance $p = K\Delta x$ on either side of the mid-plane of the Δx section, and label the intersections *a*, *b*, and *c* as shown in the figure. Then extend *ab* to inter-

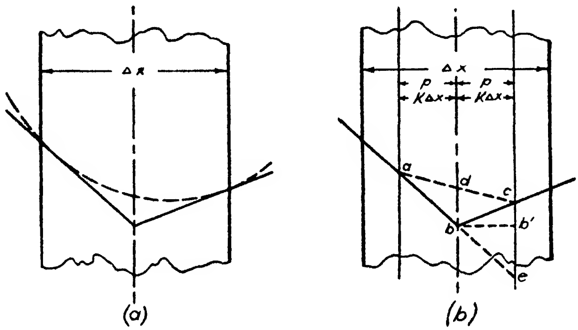

FIG. 7·1. Method of Finite Differences

sect the right construction line at point *e* and connect *a* and *c* with a straight line which intersects the mid-plane of the finite area at point *d*.

The instantaneous rate of heat flow into the finite element from the left (considering it to be of unit depth) is

$$q = \frac{\Delta Q_l}{\Delta\theta} = -kA\,\frac{\Delta t}{\Delta x} = -kA\ (\text{slope } ab) = +kA\,\frac{b'e}{p} \tag{7·4}$$

and the corresponding rate of heat flow in from the right is

$$q = \frac{\Delta Q_r}{\Delta\theta} = -kA\,\frac{\Delta t}{\Delta x} = +kA\ (\text{slope } bc) = +kA\,\frac{b'c}{p} \tag{7·4a}$$

The net heat gain by the element during the finite time interval $\Delta\theta$ is equal to the sum of the heat entering from left and right, but can also be expressed in terms of the change in mean temperature, Δt, which must occur during the same time interval,

$$\Delta Q_l + \Delta Q_r = kA(\Delta\theta)\left(\frac{b'e}{p} + \frac{b'c}{p}\right) = \rho c(\Delta t)(\Delta x)A \tag{7·5}$$

or

$$\frac{b'e}{p} + \frac{b'c}{p} = \frac{\rho c(\Delta t)(\Delta x)}{k(\Delta\theta)} = \frac{b'e + b'c}{p} = \frac{ce}{p} \tag{7·5a}$$

By similar triangles, $bd = \frac{1}{2}ce$, giving $ce/p = 2(bd)/p$, which on substitution into equation 7·5a [noting that $p = K(\Delta x)$] gives

$$\frac{\rho c(\Delta t)(\Delta x)}{k(\Delta\theta)} = \frac{2(bd)}{K(\Delta x)}$$

or

$$bd = \frac{1}{2}\frac{\rho c}{k}(\Delta t)\left(\frac{\Delta x^2}{\Delta\theta}\right)K = \Delta t\left[\frac{1}{2a}\frac{K(\Delta x)^2}{(\Delta\theta)}\right]$$

Now if a relationship is established among Δx, $\Delta\theta$, and K such that

$$\frac{1}{2a}\cdot\frac{K(\Delta x)^2}{(\Delta\theta)} = 1 \tag{7·6}$$

then the mean temperature change of the element during the time interval $\Delta\theta$ would be represented by the distance bd. Knowing the temperature gradient abc at a particular time, the temperature at the mid-plane of the element $\Delta\theta$ hours later would be directly determined by drawing line ac and noting the intersection at point d on the mid-plane; d would give the mid-plane temperature at the end of the $\Delta\theta$ interval.

In the relationship of equation 7·6, the element of wall width, Δx, *must* have a numerical value such that it will be equally divisible into the actual width of wall. The time interval, $\Delta\theta$, should have a numerical value that will be equally divisible into the period for which the analysis is to be conducted; this latter condition is essential when the problem is one involving periodic variation. When both the width and time intervals are arbitrarily selected, the necessary value of K must be determined by solution of equation 7·6. Experience shows, however, that accurate results cannot be expected if K is very much greater than unity since this requires extrapolation of the temperature gradient beyond the boundaries of the finite secton. For any particular problem there is an optimum value of K, usually between ¼ and 1, for which the greatest accuracy will be realized, but this value cannot be determined except by trial and error, and the process is usually too time consuming to be justified. The error involved in using a value of K other than the optimum (providing it is within the range $\frac{1}{4} < K < 1$) is usually negligible.

An alternative method of realizing any required degree of accuracy and, at the same time simplifying the graphical work, is arbitrarily to fix $K = 1$, select a value of Δx sufficiently small so that change in rate of change of temperature gradient across the section will not be great, and calculate the required value of $\Delta\theta$ needed to satisfy equation 7·6. This has the enormous advantage of eliminating all the construction lines on either side of the finite section center lines, since p is now equal to Δx (see Fig. 7·2). The temperature at the mid-plane of a finite section, $\Delta\theta$ hours after a known temperature gradient abc, is fixed by drawing a straight line connecting the intersections of the original temperature gradient at the mid-planes of sections to the left and right of the one for which the temperature is to be determined; the desired temperature is given at the intersection of this straight line with the dashed mid-plane line of the section being investigated. Thus a solution is obtained in terms only of the finite section center lines and the temperature gradient at the start of the time interval $\Delta\theta$.

Before setting up a procedure for applying this method to actual problems, it is necessary to provide some means of treating the surface films so that there will not be a discontinuity in the graphical construction at the wall boundaries. If the film transferred heat entirely by conduction and had a thermal conductivity equal to that of the adjacent wall there would be no discontinuity in the temperature gradient at the wall surface. Such a condition can be readily obtained by establishing an imaginary film having the same conductance as the actual film, but having a thermal conductivity equal to that of the wall; the thickness of the equivalent film is then $L = k/h$. By establishing a film of this type on each side of the wall, the entire system can be set up as an equivalent homogeneous wall. The equivalent film thickness is a function of the actual film coefficient, which may vary with respect to time; in that event, the equivalent thickness would likewise vary, but the continuity of gradient at the wall surface would still be realized at all times.

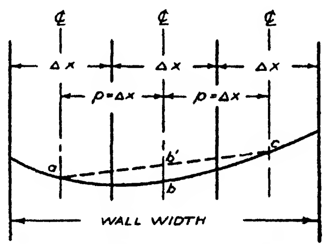

FIG. 7·2. Construction for $K = 1$.

7·3. Periodic Variation. The method of carrying out a graphical analysis of the temperature gradient for a structure subject to periodic daily variation of inside and outside air temperatures will be established for the general case. The data from such a solution can be used to calculate the heating or cooling load in the conditioned space at any hour of the day and to determine the time lag between the incidence of extreme conditions exterior to the structure and the subsequent appearance, within the structure, of the actual load. In many problems involving variable inside temperature (as intermittent heating of buildings) the time-temperature curve may not be available, but later sections will develop methods whereby this curve can be approximated and the accuracy of the approximation checked. The graphical method will be outlined for two cases: first, that of a single-section, homogeneous wall with K taken as unity; second, a composite wall made up of a number of homogeneous sections for each of which K will have a different value.

CASE I. *Periodic Flow through a Homogeneous Wall.*

Consider a homogeneous wall with inside air at fixed temperature t_i, and outside air varying periodically over a known diurnal time-temperature curve. The inside film coefficient h_i will be considered constant, while the outside coefficient h_o varies as a function of periodic diurnal changes in the wind velocity. The graphical construction leading to a complete solution for the temperature-time relationship at all points in this wall is given in Fig. 7·3*abcde*, each part of which will be explained.

Figure 7·3*a*. Draw a vertical line S_oS_o'' and on it, starting at some point, as S_o'', establish a 24-hr linear scale of time, $S_o''S_o'$, beginning at the hour for which maximum outside temperature occurs (the solution does not require that the scale begin and end at the time of maximum temperature, but this is a convenient starting point and assists in estimating the first approximation to the temperature gradient). Normal to $S_o'S_o''$ and through S_o' draw a line BS_o'; starting with zero at S_o' lay off, to the left along this line, a linear scale covering the range of values of k_w/h_o which exist throughout the 24-hr period. At any point such as B far to the left on BS_o', draw a vertical BB', and with point B as any convenient value less than the minimum outside air temperature, establish a linear temperature scale along this line. From B, along BS_o' establish a linear time scale similar to $S_o''S_o'$.

From weather bureau records, direct observation, or other source, obtain data on the outside air temperature as a function of time and plot the time-temperature curve as shown in the figure. A vertical line from any point on the horizontal time scale to intersection with the curve, then transferred horizontally to the left, fixes the outside air temperature for that particular time. Since the time-temperature curve is periodic, each temperature between the maximum and minimum must be realized at least twice during the 24 hours so any horizontal line gives the temperature at two particular times during the day.

From the weather bureau, or other sources, data is obtained on the variation of wind velocity with time. The value of the film coefficient at a given time can then be calculated, and the curve plotted for k_w/h_o as a function of time. Any line from the vertical time scale to intersection with this curve, then transferred vertically upward, represents the value of k_w/h_o for that particular hour of the day. For a given time, the intersection of the proper temperature and k_w/h_o lines (as x and y) establishes the position of a reference point (p). Reference points are established at intervals of $\Delta\theta$ hours throughout the day; each such point fixes the outside air temperature and the equivalent outside film thickness at a particular time.

Figure 7·3*b*. Allow the line S_oS_o' to represent the outside surface of the wall and establish the requisite reference points with respect to S_oS_o' by the method of Fig. 7·3*a*. Then move to the right from S_oS_o' a distance equal to the wall width in feet and measured to the same scale as that used for k_w/h_o (the units of which are also feet). This establishes the position of the inside surface of the wall S_iS_i'. For the example illustrated neither t_i nor h_i vary with time so a single inside reference point RP_i will suffice; this can be fixed by moving to the right from the inside wall surface a dis-

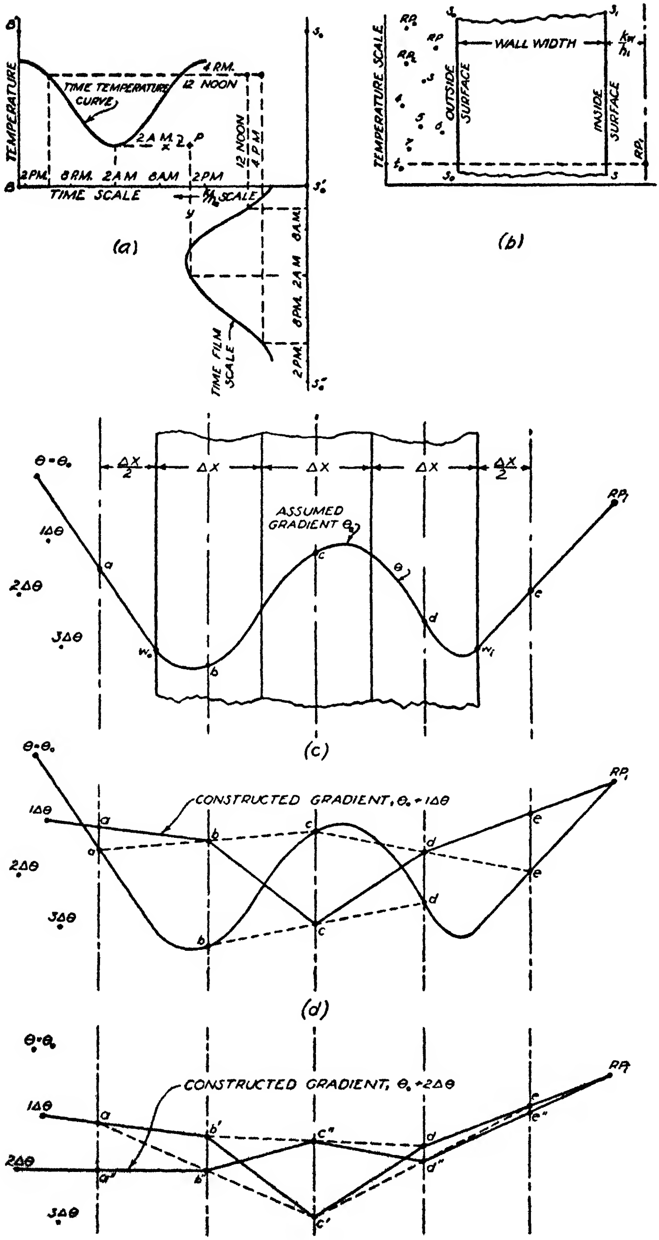

Fig. 7·3. Graphical Construction for Homogeneous Wall.

tance k_w/h_i and locating on this vertical the point RP_i, corresponding to the inside air temperature t_i. If either t_i or h_i varied with time additional inside reference points would be established by the method of Fig. 7·3*a*.

Figure 7·3*c*. The sections Δx, of width satisfying equation 7·6 for the assumed condition of $K = 1$ and for $\Delta\theta$ having the value selected in establishing the reference points, are now marked off on the wall. Note that care must be exercised to assure Δx having a value that is equally divisible into the actual wall width; before fixing reference points, the investigation of this condition should be made to verify the possibility of using the desired value of $\Delta\theta$.

In many cases the Δx corresponding to a desired $\Delta\theta$ is too large (in comparison with the total wall width) to permit an accurate solution whereas in other cases (notably diurnal periodicity) selection of a small Δx may give a $\Delta\theta$ so small that an impractical number of steps would have to be followed in completing the 24-hr cycle. This basic inflexibility of the $\Delta x, \Delta\theta$ relationship is the most serious fault of the graphical method. Whenever accuracy requires a Δx less than that corresponding to a time interval of ½ hr, the graphical method will require so many steps that it will be cumbersome in construction. Fortunately, this difficulty does not often exist in structures having a thermal capacity great enough to require analysis of diurnal periodic variation.

Dashed center lines are now drawn through each Δx section and similar lines are established a distance $\Delta x/2$ out from each wall surface. If the temperature gradient through the wall at any particular time is known, this gradient is established on Fig. 7·3*c*; if no such gradient is known, one is estimated or assumed for an arbitrarily selected starting time. The need for an assumed initial gradient does not introduce any error into the final result, for regardless of how poor the estimate may be, a correct result, accurate to within the limits of the chosen Δx, is unavoidable provided that directions for the graphical construction are carefully followed. Even if the estimator were so lacking in judgment as to assume that heat flow occurs from low to high temperature the construction would automatically lead to a correct solution; skill in estimating the initial gradient is repaid, however, with a more rapid solution.

Having fixed an initial temperature gradient, its intersections with center lines of the Δx sections are marked (*b, c, d* of Fig. 7·3*c*) and straight lines are drawn from w_o to the outside reference point for the time of the initial gradient and from w_i to the inside reference point; these lines intersect the film construction lines at points *a* and *e*, giving as the initial gradient through the system of wall and two equivalent films the line *abcde*. If *abcde* were the true gradient at the initial time, the first step in the construction would give the correct gradient $\Delta\theta$ hours later, and subsequent steps would give the correct gradients at the initial time plus multiples of

$\Delta\theta$; continuing in this way the constructed temperature gradient after 24 hr would superimpose on the known gradient with which the construction had started.

Since the initial gradient is usually assumed and therefore not correct, subsequent gradients determined in the process of analysis will likewise be in error and at the end of the 24-hr period the indicated gradient will differ from the one which was originally assumed. It can be shown that the new gradient is a better approximation to the true value than was the assumed one. The procedure is then to repeat the periodic analysis, using as an initial gradient the result obtained from the first construction. The result for the second 24-hr analysis will be a third approximation, and this can be used as the starting point for a third repetition of the construction. In this way the analysis proceeds until close superposition of the initial and final gradients indicates that a solution has been realized. In most practical problems, a reasonably good estimate of the general location and curvature of the initial temperature gradient will permit realizing an accurate solution in not more than three cycles and frequently in two. The process of correcting the initial assumption can be accelerated somewhat by carefully scrutinizing the difference in shape and position of the assumed gradient and the first constructed approximation; from the direction and magnitude of the changes, it is often possible to anticipate the need for further correction and to estimate such corrections when drawing the initial gradient at the start of the second cycle.

Figure 7·3*d*. The only parts of Fig. 7·3*c* which are used in the actual construction are the section center lines, the two dashed lines located beyond the wall surfaces, the inside and outside reference points, and the assumed initial temperature gradient. This material is shown in Fig. 7·3*d* with the accompanying construction lines for determination of the gradient at time interval $\Delta\theta$ after the start. To determine the new temperature gradient, connect points a and c to establish b', connect b and d to establish c', c and e for d'. Then connect b' with the outside reference point corresponding to a time $1\Delta\theta$ after that at the start, thereby fixing a'; similarly connect d' with the inside reference point to fix e'. The line $a'b'c'd'e'$ then represents the temperature gradient $1\Delta\theta$ hours after the time for which *abcde* was drawn.

Figure 7·3*e*. The gradient $a'b'c'd'e'$ is used for establishing the gradient $2\Delta\theta$ hours after the starting time. Connecting alternately primed points gives the new double primed points $b''c''d''$. A line connecting b'' with the outside reference point for $2\Delta\theta$ plus the initial starting time fixes a'' and a similar construction determines e''. The new gradient is then $a''b''c''d''e''$. The same construction is repeated for subsequent intervals until at the end of 24 hours a gradient is available for comparison with the one originally assumed; if agreement is close, a solution has been obtained; if not, the cycle is repeated starting with the gradient determined from the first cycle.

Figure 7·4 gives the results of a graphical analysis for a typical problem. Construction lines used in passing from one gradient to another are not shown in this figure, on which appear only the 49 temperature gradients (since $\Delta\theta$ was taken as ½ hr), resulting from the last cycle of the analysis. From such a solution it is immediately possible to read the temperature at any point in the wall for any time.

Since the graphical analysis is based on relative rather than absolute values, one solution is valid for an infinite number of cases. The particular problem of Fig. 7·4, for example, was for a wall subjected to sinusoidal outside temperature variation between 30°F and 70°F when the inside air temperature remained fixed at 70°F and the maximum outside temperature occurred at 2 P.M. Consider the following extensions of this one solution:

1. If the time of the maximum temperature were to change, other conditions remaining the same, the solution would remain valid and the only needed change in the figure would be to displace the time scale and note that the numbers on the temperature gradients are for the number of $\Delta\theta$ time intervals which have passed since the time of the maximum outside temperature.

2. If other conditions remain the same, but the range of outside temperature increases or decreases (with fixed maximum of 70°F), the solution is valid subject only to numerical adjustment of the temperature scale. If any change occurs in the maximum temperature the solution still holds provided the inside temperature remains constant at a value equal to the maximum outside temperature.

3. For summer conditions the same solution applies, subject to the single limitation that the inside air temperature must be constant at a value equal to the minimum outside temperature and the outside temperature variation must follow the same *shape* temperature-time curve as that which applied for winter.

4. The solution is likewise valid for any wall of the same width, whatever its material or physical properties, provided its thermal diffusivity $(k/\rho c)$ has the same value as for the wall of Fig. 7·4 and the equivalent film widths, k/h_i and k/h_o, are the same.

Figure 7·5 groups the temperature-time curves of the outside air, the inside air, and the inside wall surface; the latter curve is plotted from data read directly from Fig. 7·4. Similar curves could readily be established for the temperature-time variation at any selected planes in the wall. From Fig. 7·5 it is evident that the periodic temperature variation becomes less, and the time at which the maximum temperature occurs comes later, as the inside surface is approached. This time lag between

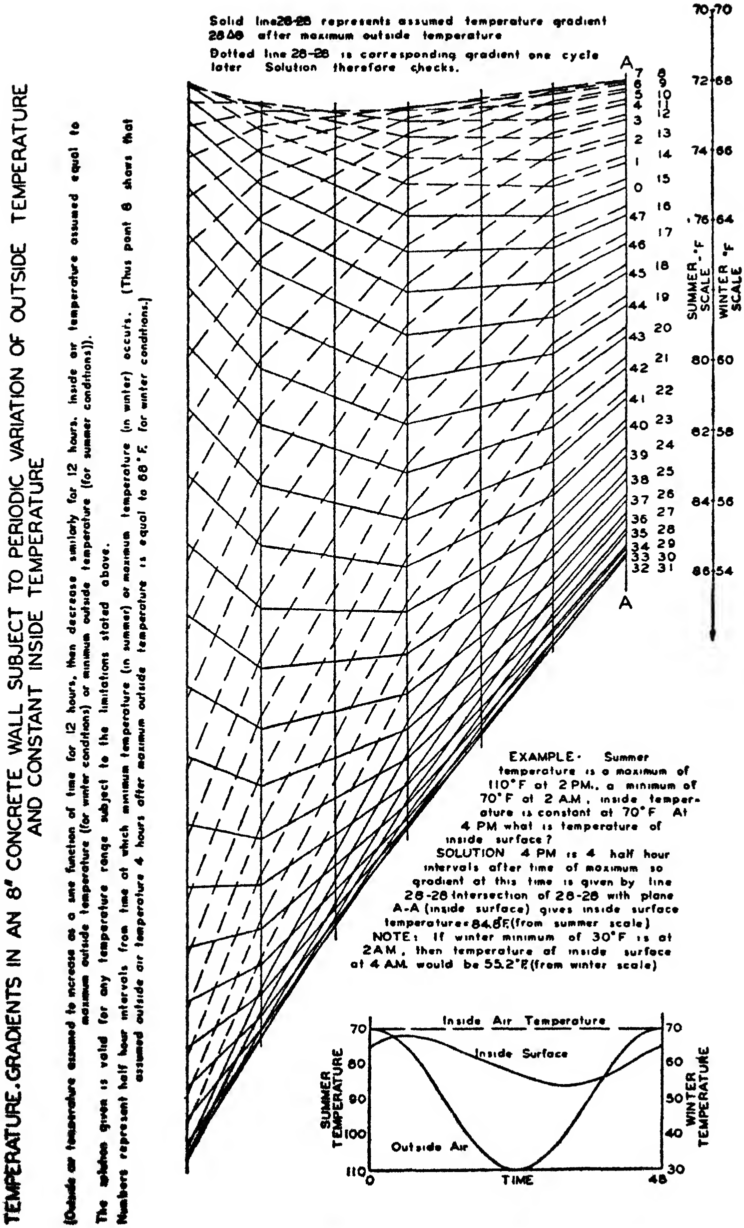

Fig. 7·4. (*Upper left*). Graphical Solution of Homogeneous Wall Problem.
Fig. 7·5. (*Lower right*). Time-temperature Curve.

(Figures 7·4 and 7·5 are from "Graphical Analysis of Periodic Loads," by Raber and Hutchinson, in *Heating, Piping, and Air Conditioning*, April, 1942.)

incidence of extreme conditions outside and appearance of the maximum inside surface temperature is of great practical importance because it permits determination of the load-time curve for the structure. Assuming constancy of the inside film coefficient, the rate of heat transfer from room air to the wall is directly proportional to the air-to-wall surface temperature difference; thus the vertical distance between the curves for inside air and inside wall surface temperature (Fig. 7·5) is directly proportional to load. Since the inside air temperature does not vary, a load scale can be established beside the temperature scale; zero load occurs at 70°F on the winter temperature scale and maximum load at the point on the scale corresponding to the minimum inside wall surface temperature.

The graphical analysis, as carried out above, will include the effect of solar radiation, provided the film coefficient used is h', the combined coefficient based on $(t_i - t_2)$. From handbook data on the amount of solar radiation absorbed by the surface in question, h_r and hence h' can be determined as a function of time. The method loses its validity, however, when the solar absorption reaches a value such that the wall surface temperature exceeds that of the outside air; in such cases it is simplest to resort to analytical or empirical methods of solution.

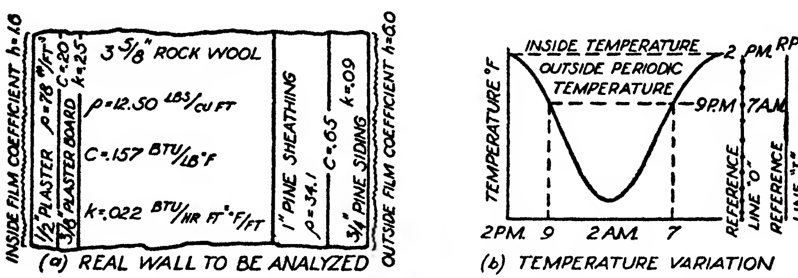

FIG. 7·6ab. Graphical Construction for Composite Wall.

Figure 7·6 is from "Analysis of Periodic Loads in Composite Walls," by Raber and Hutchinson, *Heating, Piping, and Air Conditioning*, June, 1942.

CASE II. *Periodic Flow through a Composite Wall.*

Application of the graphical analysis to a non-homogeneous wall requires slight modification of the fundamental equation and some increase in the work of construction. To avoid abrupt changes in the slope of the temperature gradient at interfaces between sections of different material the artifice is adopted of setting up an equivalent wall having uniform conductivity throughout. This necessitates modification of the equation 7·6 relating Δx and $\Delta\theta$,

$$\Delta\theta = \frac{K(\Delta x)^2}{2a}$$

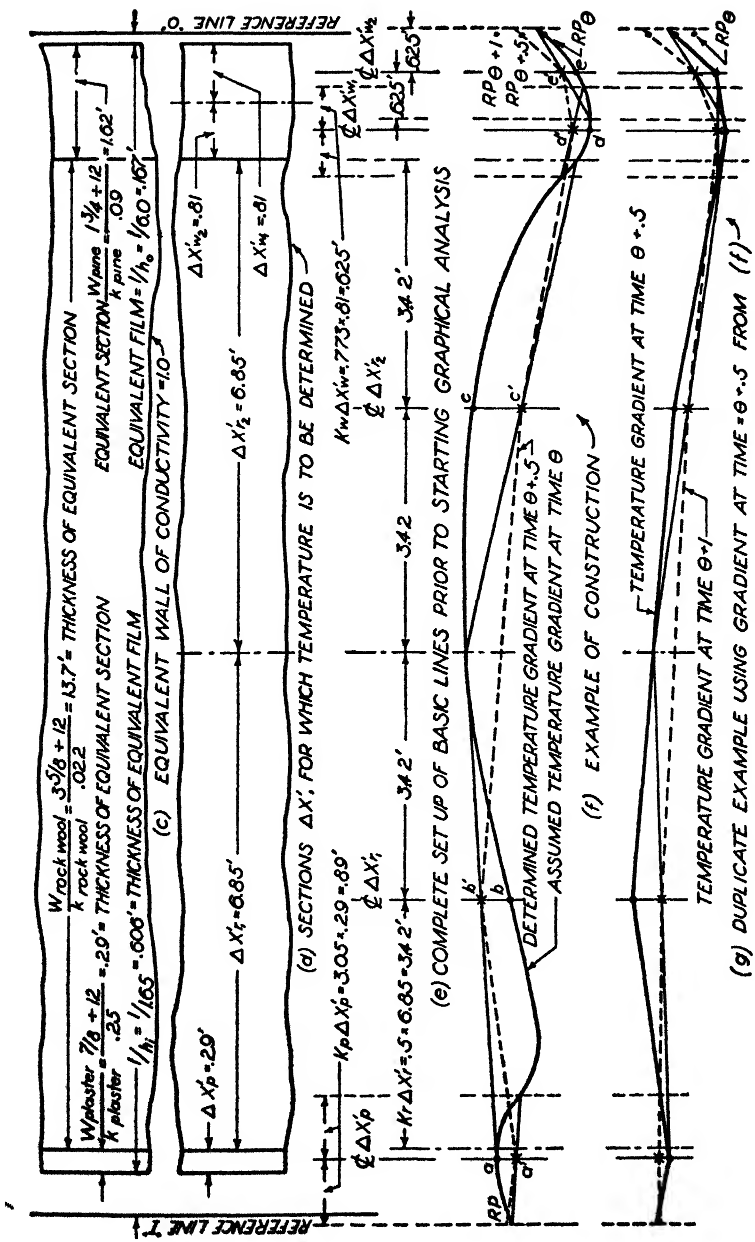

FIG. 7·6cdefg. Graphical Construction for Composite Wall.

The total equivalent width of each homogeneous section of the composite wall will be $k_e w_a / k_a$ where k_e is the uniform conductivity of the fictitious wall, k_a the true conductivity of section a which has width w_a. The term k_e can be selected arbitrarily and may therefore be taken as unity. Then the equivalent total width of section a becomes w_a / k_a, and the equivalent width $\Delta x'$ of the finite element Δx of section a is

$$\Delta x' = \frac{\Delta x}{k_a} \quad \text{or} \quad \Delta x = k_a \Delta x' \tag{7·7}$$

Substituting in equation 7·6 for k and for Δx,

$$\Delta\theta = \frac{K(k_a \Delta x')^2}{\dfrac{2k_a}{\rho c}} = \tfrac{1}{2}\rho c k_a K (\Delta x')^2 \tag{7·8}$$

Equation 7·8 must be satisfied for graphical analysis of composite walls.

The procedure for solution of a composite wall problem will be illustrated by setting up the construction for a typical case. Consider the wall of Fig. 7·6*a* made up of an inside layer of ½-in. plaster on ⅜-in. plaster board, followed by 3⅝ in. of rock wool and 1¾-in. pine.

For this problem both the inside and outside film coefficients are taken as constant with respect to time, and the inside temperature is also considered to hold a fixed value. The outside temperature varies as shown in Fig. 7·6*b*; by the usual method, inside and outside reference points are established with respect to the two wall surfaces. Figure 7·6*c* shows the set-up for the equivalent wall of unit conductivity.

The next step in the procedure is to establish $\Delta x'$ for each of the three materials and set up the corresponding construction lines. Starting with the plaster, select a value of $\Delta x'_p$ which is equally divisible into the equivalent width of the plaster wall. Since this width is not great $\Delta x'_p$ was taken, in the example, as equal to the section width, that is, only one $\Delta x'$ was used for the plaster. The value of $\Delta x'_p$ is therefore 0.29 ft (Fig. 7·6*d*). A time interval of ½ hr was arbitrarily selected so the value of the constant K_p for the plaster is calculated from equation 7·8 as, $K_p = (2 \times 0.5)/(78 \times 0.20 \times 0.25 \times 0.29^2) = 3.05$. As already discussed, values of K greater than unity are to be avoided as they require extrapolation of the temperature gradient to such an extent that substantial loss of accuracy may result. In the present case, however, the difficulties attendant on a reduction in K are so great and the plaster such a small part of the entire wall structure that a change does not seem justified. Figure 7·6*e* shows the center line for the plaster section with construction lines established a distance $K_p \Delta x'_p$ out on either side.

Similarly a value of $\Delta x'_r$ for the rock wool is chosen (in this case $\Delta x'_r$ was taken as one-half the equivalent section width) and K_r is calculated in the usual way,

$$K_r = \frac{2 \times 0.5}{12.5 \times 0.157 \times 0.022 \times 6.85^2} = 0.52 = 0.5 \text{ (approx)}$$

The center lines of each of the $\Delta x_r'$ sections are shown in Fig. 7·6*e* with construction lines located a distance $K_r \Delta x_r' = 0.5 \times 6.85 = 3.42$ on either side. The wood is likewise split into two $\Delta x_w'$ sections of width 0.81 for which the calculated value of K_w is $(2 \times 0.5)/(34.1 \times 0.65 \times 0.09 \times 0.81^2) = 0.773$. Both center lines and all four construction lines for the wood are shown in Fig. 7·6*e*.

The next step is to record the known or assumed temperature gradient at the time for which the construction is to be started. This is done in Fig. 7·6*f* which also shows the construction leading to the temperature gradient $\Delta\theta$ hours after the start of the analysis. The assumed gradient intersects the center lines at points *a*,*b*,*c*,*d*,*e* which give the mean temperature of each $\Delta x'$ section at the starting time. The same gradient also intersects the construction lines located to the right and left of each center line. A straight line connecting the latter intersections crosses the corresponding center line at a point which represents the temperature of this $\Delta x'$ section one $\Delta\theta$ time interval after the time of the original gradient.

In the same way temperatures at the center of all $\Delta x'$ sections are established. Connecting the resultant series of points $a'b'c'd'e'$ and joining a' and e' with the respective reference points for time $\Delta\theta$ gives the temperature gradient $\Delta\theta$ hours after the start, provided the original gradient was correct. If, as is practically certain, the assumed initial gradient is in error, the constructed gradient is also in error, and the entire cyclic construction must be repeated a sufficient number of times — as explained in Case I — to obtain the correct initial gradient. Figure 7·6*g* shows the graphical procedure for fixing the gradient $2\Delta\theta$ hours after the start; the construction duplicates that described for Fig. 7·6*f* except that the starting line is the constructed gradient for $\Delta\theta$ hours rather than the assumed initial gradient. In a similar manner the procedure is repeated as many times as may be required.

For the particular problem of Fig. 7·6 the construction lines at the extreme right and left are located outside of the reference lines. This condition frequently occurs and calls merely for extrapolation of the gradients from points *a* and *e* through the proper reference points to intersection with the construction lines.

The complete solution of the problem of Fig. 7·6 is presented in Fig. 7·7*a* while Fig. 7·7*b* gives a comparison of temperature-time curves for the outside and inside air and wall surface temperatures.

7·4. Graphical Treatment of Air Spaces. The presence of an air space between sections of a homogeneous wall does not require that the wall be treated as composite; if air spaces occur in a composite wall, the additional complication is less than that represented by a section of solid material. The method of graphical solution for air spaces will be illustrated for a particular case, but the procedure is the same for all types of walls.

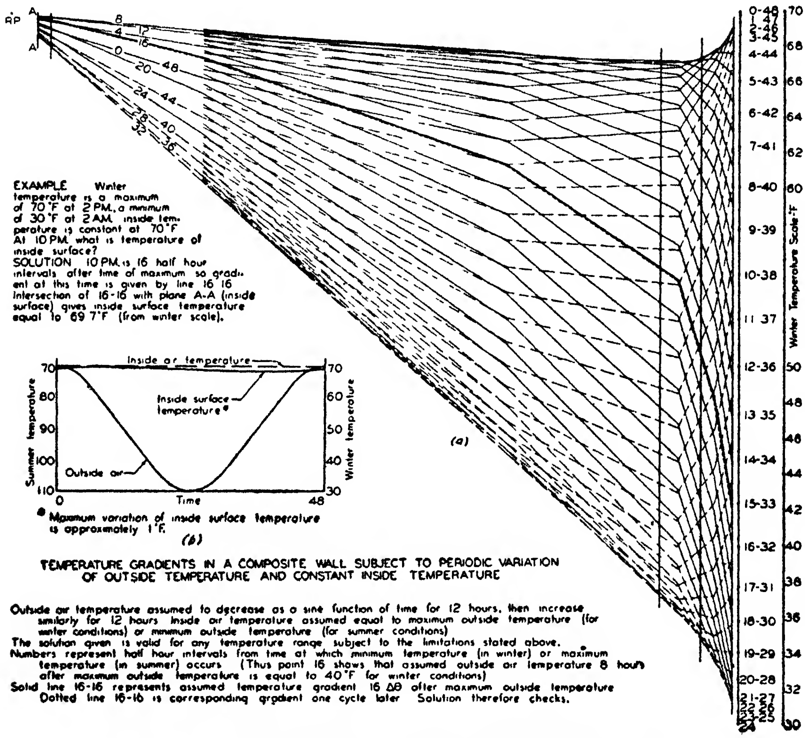

FIG. 7.7. Graphical Solution of Composite Wall Problem.

From "Analysis of Periodic Loads in Composite Walls," by Raber and Hutchinson, *Heating, Piping, and Air Conditioning*, June, 1942.

Figure 7·8*a* is a scale drawing of two sections of homogeneous wall with included air space. The usual construction is used in setting up equivalent films on inside and outside surfaces (Fig. 7·8*b*) and an equivalent air space, of conductivity equal to that of the wall, is established between wall sections. The assumed temperature gradient is then drawn in (Fig. 7·8*c*), noting, however, that the air space has a thermal capacity so low that the gradient through it can be taken as a straight line. By the usual method of construction the temperature at the mid-point of each section of the wall is established (giving points b', c', d' and g', h', i'). Since the gradient through the equivalent air space is a straight line and there is no change in gradient at the wall surface on either side, it follows that a straight line joining d' and g' will represent the temperature gradient between these points and will determine points e' and f' (Fig. 7·8d). Repetition of the same procedure determines gradients at subsequent times.

7·5. Graphical Solution for Non-plane Walls. The possibility of curvature being great enough and walls thick enough to necessitate a correction for non-plane surfaces is so remote that it need not be considered in connection with air conditioning problems. In refrigeration, however, this problem may be of importance in determination of the load represented

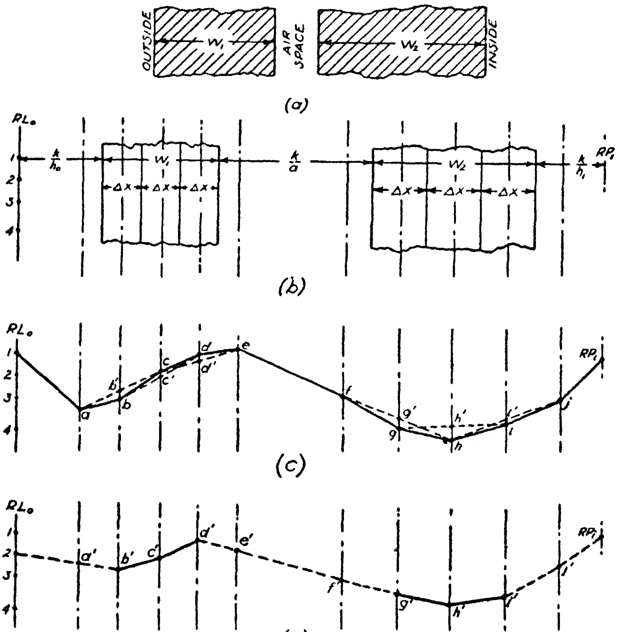

FIG. 7·8. Graphical Construction for Wall with Included Air Space.

by the cooling of insulation around refrigerant piping during the interval following start-up. A graphical solution for this case is available in the literature.*

PROBLEMS

1. The results of applying the graphical method to the solution of a typical periodic heat flow problem are shown in Fig. 7·4. With data as given in the statement of the problem and with an assumed linear temperature difference from 70°F inside air to 30°F outside air (at 2 A.M. in winter) go through a complete graphical analysis and compare results with Fig. 7·4.

* See reference 2 on page 134.

2. From the result of problem 1 (or by scaling from Fig. 7·4) plot the temperature-time curves for each of the five Δx mid-planes. Draw a line connecting the minimum point of each of the five curves and discuss its shape.

3. A structure having the same construction and film coefficients as the one of Fig. 7·4 is maintained at an inside air temperature of 80°F during a cooling season in which the outside air temperature varies from 120°F at 4 P.M. to 80°F at 4 A.M.; the outside temperature-time curve has the same shape as that of Fig. 7·4. From the solution as given, determine the temperature at mid-point of this wall at 6 P.M. and at 10 A.M.

4. Go through the detailed construction leading to the graphical solution of the heterogeneous periodic transfer problem as shown in Fig. 7·6.

5. From Fig. 7·6 discuss the relative importance, as determining factors in the solution, of each of the homogeneous wall sections.

6. Obtain a graphical solution for the wall (and the temperature conditions) of Fig. 7·4, but with the assumption that the wall is divided into two sections of equal thickness by an air space having a conductance of 2 Btu/ (hr) (sq ft) (°F).

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CHAPTER VIII

TRANSIENT HEAT FLOW

For application to air conditioning problems the periodic and transient heat flow analyses are used for opposite purposes. The periodic analysis leads to a determination of the actual load as a function of time; frequently it makes possible a *reduction* in the size of the equipment over that which would be required if maximum load were assumed to have steady state value and to occur at the time of most extreme outside conditions. The transient analysis, on the contrary, is used to determine the necessary *increase* in capacity of the heating plant to bring the structure to the desired inside temperature in a reasonable heating-up time. The transient analysis also assists in relating the operating saving due to intermittent heating with the added fixed charges resulting from the larger heating plant.

When the heating system in a particular structure is shut off, transmission and ventilation losses lead to a reduction in inside air temperature at a rate dependent on the thermal capacity and the heat transfer characteristics of the structure and its contents. The cooling curve for the structure depends on wall temperature gradients, but the gradients are in turn dependent on the shape of the cooling curve. Thus the problem of intermittent heating is one in which neither the cooling curve nor the wall gradients are at first known.

A simple, but time-consuming method of trial-and-error solution is to assume a cooling curve and apply the graphical method (Chapter VII) to a determination of the temperature gradients corresponding to it. From the gradients so established, the load-time curve can be drawn and from this curve, together with knowledge of the heat stored in the furnishings and air within the room, a new cooling curve can be calculated. If the new curve differs from the one assumed, it can then be used as a second approximation, and the process can be continued until agreement is reached between the curve used as a basis for the gradient determination and the curve calculated from the gradients so determined.

An approximate method, proposed by Holme,* is much more rapid than the construction described above. Even in cases where exact results are essential, it can be advantageously used to determine an approximate cooling curve which may then be taken as a starting assumption in the trial-and-error solution. The Holme method neglects departures of the

* H. Holme, "Wärme," October 17, 1931.

wall temperature gradient from a straight line and assumes that, at any time, conditions across the wall correspond to those of equilibrium. This assumption would be closely justified in a structure for which the thermal capacity of the furnishings was large in comparison with the capacity of the wall, but it would become increasingly inaccurate as the thermal capacity of the wall increased. Derivations of the heating and cooling curves, based on the straight line gradient assumption, are given in the following sections.

8·1. Cooling Equation. Arbitrarily selecting 0°F as the temperature from which stored heat (actually *internal energy*) will be measured, the total heat stored within the furnishings, air, and inside partitions (assumed at inside air temperature) is

$$Q_t = (V_f\rho_f c_f + V_a\rho_a c_a + V_i\rho_i c_i)t_i \tag{8·1}$$

where V is the volume, ρ the density, c the specific heat, and the subscripts f, a, and i refer to furnishings, air, and inside partitions respectively. Since Q_t is a linear function of t_i, the heat entering or leaving storage per °F change in t_i is independent of the absolute value of t_i, so we can write

$$Q_t' = V_f\rho_f c_f + V_a\rho_a c_a + V_i\rho_i c_i \tag{8·1a}$$

where Q_t' is the total heat flowing to or from the inside wall surface for each 1°F change in inside air temperature.

As the inside air temperature changes, the gradient through the wall must become either steeper or flatter (based on straight line variation and continuous equilibrium) so the mean temperature of the wall and hence the heat stored within the wall must vary as a function of t_i. The temperature at the mid-point of the wall is (equation 6·16)

$$t_m = t_i - R_m'U'(t_i - t_o)$$

which, for a plane wall, becomes

$$t_m = t_i - R_mU(t_i - t_o) = t_o + R_m''U(t_i - t_o) \tag{8·2}$$

where R_m'' is the resistance from mid-point to outside air.

For each 1°F change in t_i, the mean wall temperature changes by $R_m''U$, and the heat which enters or leaves storage in the wall per 1°F change in t_i is then,

$$Q_w' = V_w\rho_w c_w R_m U = \rho_w c_w A_w \left(\frac{L_w}{12}\right) R_m U \tag{8·3}$$

where L is the wall thickness in inches.

The total flow of heat to or from the outside surface of the wall Q' is equal to the sum of the quantities entering or leaving storage within the structure and its contents,

$$Q' = Q_t' + Q_w' \tag{8·4}$$

where Q_t' and Q_w' are from equations 8·1a and 8·3.

The rate of heat transfer through the wall will vary with the air-to-air temperature difference. During the interval in which the inside air temperature drops from t_{i_1} to t_{i_2}, the log mean air-to-air temperature difference is

$$\Delta t_m = \frac{(t_{i_1} - t_o) - (t_{i_2} - t_o)}{\log_e \dfrac{t_{i_1} - t_o}{t_{i_2} - t_o}} \tag{8·5}$$

As the inside temperature changes from t_{i_1} to t_{i_2}, the total quantity of heat which must leave storage is $Q'(t_{i_1} - t_{i_2})$. The average rate of heat loss during this interval is $q = q'\Delta t_m$ where q' is the unit rate of loss, Btu/(hr)(°F). The number of hours required for cooling to t_{i_2} is given by

$$\Delta\theta_c = \frac{Q'(t_{i_1} - t_{i_2})}{q'\Delta t_m}$$

$$\Delta\theta_c = \frac{Q'}{q'} \log_e \frac{t_{i_1} - t_o}{t_{i_2} - t_o} \tag{8·6}$$

Equation 8·6 is the cooling equation and gives the number of hours, $\Delta\theta_c$, required for the inside temperature to drop from t_{i_1} to t_{i_2} when the outside temperature is constant at t_o and the heat transfer and heat storage characteristics of the structure are as given by terms q' and Q'.

8·2. Heating Equation. The rapidity with which a structure can be heated depends on the rate of energy input over that required to supply instantaneous equilibrium heat losses. Considering that the heating plant operates at full capacity throughout the heat-up period and letting R represent this capacity in Btu per hour, the energy available for raising the temperature level is at any time equal to $(R - q)$ where q is the rate of heat loss from the structure for equilibrium conditions at that time. When heating starts from an inside temperature of t_o, the initial transmission rate is zero and the entire output of the plant R goes to storage. At a later time when t_i has risen to t_{i_2}, the transmission loss is $q'(t_{i_2} - t_o)$ and the rate of heat flow to storage has been reduced to $R - q'(t_{i_2} - t_o)$. The logarithmic mean rate of heat flow to storage for the time interval during which the inside temperature increases from t_o to t_{i_2} is

$$q_{\text{average to storage}} = \frac{(R) - [R - q'(t_{i_2} - t_o)]}{\log_e \left[\dfrac{R}{R - q'(t_{i_2} - t_o)}\right]} \tag{8·7}$$

and the number of hours needed to raise the inside temperature from t_o to t_{i_2} is

$$\Delta\theta_h = \frac{Q'}{q'} \log_e \left[\frac{R}{R - q'(t_{i_2} - t_o)}\right] \tag{8·8}$$

Equation 8·8 is the heating equation (for t_o constant) and permits direct calculation of the number of hours required to raise the inside temperature from an original value of t_o. If heating does not start from $t_i = t_o$, equation 8·8 cannot be used directly, but it can be used to determine the time of heating to the original t_{i_1} from t_o and to the final t_{i_2} from t_o; the difference between these two times is then the heating period required to go from t_{i_1} to t_{i_2}.

8·3. Analysis of Intermittent Heating.* The principal air conditioning application of the transient heat flow equations is in the design and investigation of intermittently operated heating and cooling systems. In

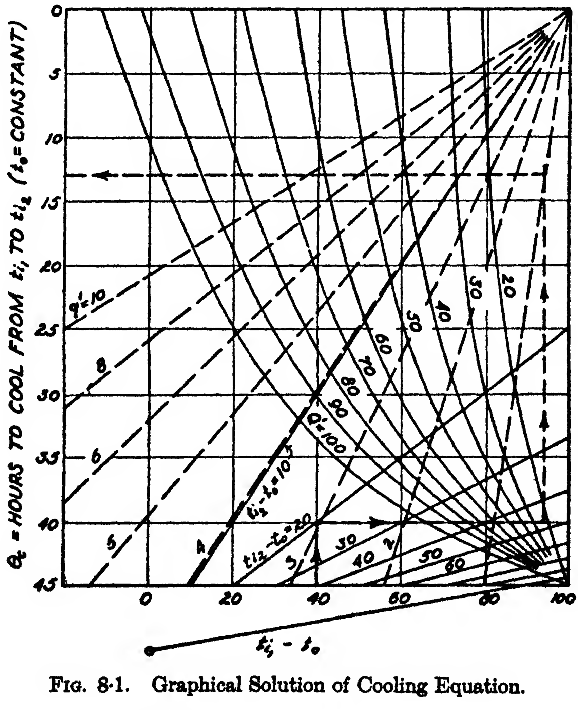


FIG. 8·1. Graphical Solution of Cooling Equation.

the order of their practical importance, as well as simplicity of application, the basic equations (8·6, 8·8) will be considered as starting points for graphical methods of analyzing intermittently operated systems.

(*a*) GRAPHICAL SOLUTION USING HEATING AND COOLING EQUATIONS. Both of these equations (8·6, 8·8) are subject to direct graphical solution. Figure 8·1 presents such a solution for the cooling equation whereas Fig.

* The illustrative material in this section is based on, and the numerical example in section 8·4 is taken from, the following reference: Hutchinson, *Proceedings of the National District Heating Association*, Vol. XXXII, pp. 102–115, 1941.

8·2 does the same for the heating equation. Use of either graph requires knowledge of the heat transfer and heat storage constants q' and Q' of the particular structure. These can be determined by calculation or by graphical solution. Figure 8·3 illustrates a graphical method of determining both q' and Q' for walls finished inside and out in any one of five specific ways; similar curves could be readily established for any other type of wall construction. The method of using Figs. 8·1, 8·2, and 8·3 is shown by the dashed path drawn on each figure.

Having established the heating and cooling curves for a structure, their use in determining the optimum start-up time for intermittent operation

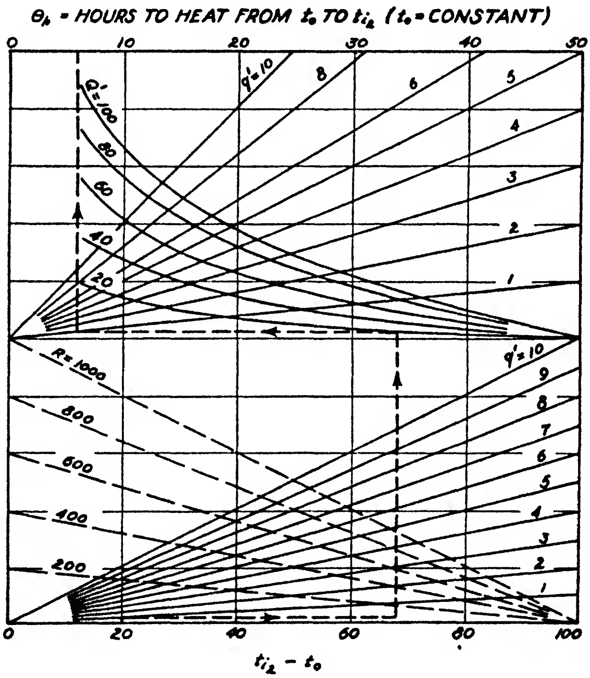
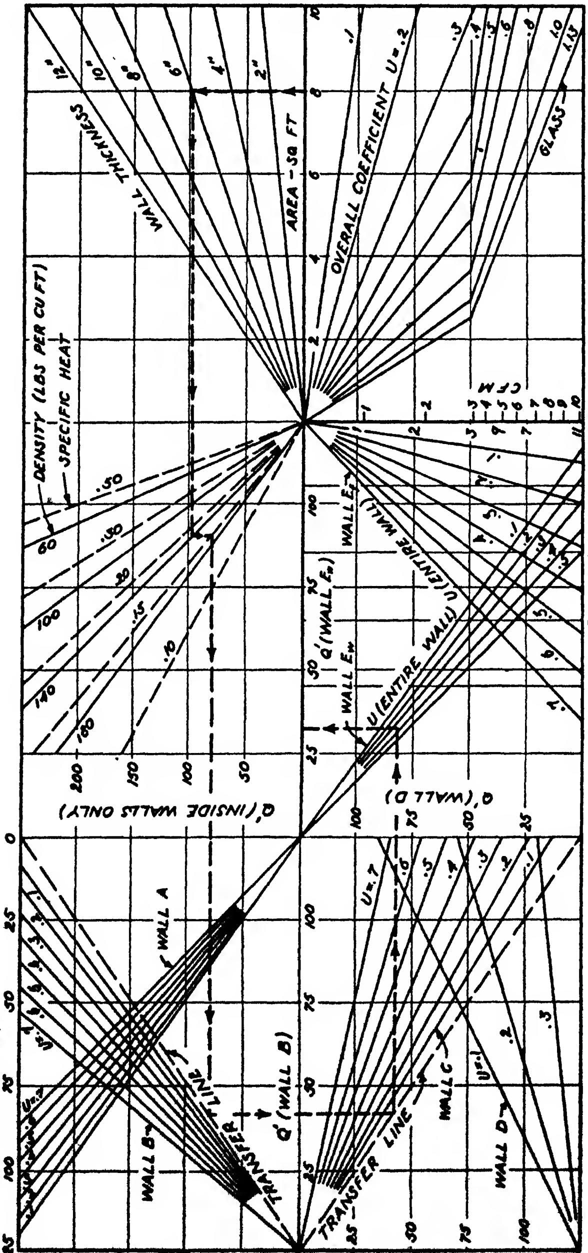


FIG. 8·2. Graphical Solution of Heating Equation.

and the amount of the saving to be realized by intermittent operation is illustrated in Fig. 8·4. For the case shown, the heating plant is shut down at 4:30 P.M., and cooling proceeds along the cooling curve as indicated. The structure is required to be up to temperature by 9:00 A.M. the following morning so heating must be started at such time as will assure the desired inside temperature at 9:00 A.M., but preferably no earlier than this. If the heating curve is now placed on the graph in such manner that the 70°F point corresponds to 9:00 A.M., the curve will show



Wall A Plain; unfinished surface inside and outside.
Wall B Plain outside; ½-in. plaster on inside.
Wall C Plain outside; ¾-in. plaster on metal lath (furred) inside.
Wall D Plain outside; ¾-in. plaster (furred) inside, with ½-in. insulation in furred space.
Wall E_w Same wall as C but with 4-in. cut-stone veneer. Q' is for wall exclusive of veneer.
Wall E_f Same as E_w, but Q' is for veneer only.

Fig. 8·3. Graphical Evaluation of Heat Storage and Transfer Characteristics, Q' and q'.

the temperature at all previous times during the heating-up period. The intersection of the heating and cooling curves gives the optimum time for starting the heating plant. If started earlier, the inside temperature would reach occupancy level before the required time with consequent increased loss of heat, but if started later, the desired temperature would not be attained until after 9:00 A.M.

Coordinates of the heating and cooling curves are degrees and hours, so unit area on either of these curves is degree-hours and can readily be shown to be proportional to the heating load. The area *abc* of Fig. 8·4 is then proportional to the saving represented by intermittent rather than continuous heating. Anything which increases *abc* increases the saving and vice versa. Note that selection of a larger heating plant makes the heating curve steeper and thereby effects a greater operating saving due to intermittent heating. Whether or not the saving is sufficient to justify the added first cost is a problem in economics; obviously, however, there

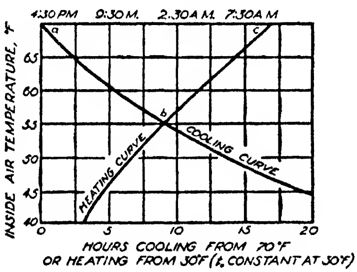


FIG. 8·4. Typical Heating and Cooling Curves.

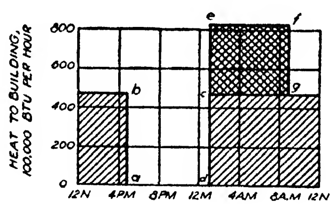


FIG. 8·5. Typical Time-Load Relationship.

will be an optimum size of heating plant for any particular system. A poorly insulated structure would cool more rapidly, thereby increasing the area above curve *ab*, but the heating-up rate would be less rapid (for the same size heating plant) so the area above curve *bc* would be reduced. The net effect of heat transmission losses on the economy of intermittent heating would therefore have to be investigated for each case.

Figure 8·5 shows the curve of heat input plotted against time. During the hours when equilibrium conditions exist the output of the heating plant remains constant at the value needed to supply energy losses. During cooling there is, of course, no energy input to the structure, while during the heating-up period the plant operates at constant maximum capacity. The saving due to intermittent heating can be determined from this graph (since unit area is equal to Btu) by subtracting the area *cefg* from area *abcd*.

(*b*) GRAPHICAL SOLUTION FOR COMPLEX HEATING AND COOLING CONDI-

TIONS. In many problems the variation of outside temperature with time necessitates determination of the heating and cooling curves from equations other than 8·6 and 8·8. Once these curves have been determined, however, the method of analysis for intermittent operation of the heating plant is exactly as outlined in the preceding section.

A more serious difficulty arises in those problems for which the thermal capacity of the outside walls is great in comparison with that of the contents of the structure. Fortunately, such problems are unusual, but when they do occur a special method of solution is needed as the assumption of instantaneous equilibrium is no longer tenable. For special cases of this kind, the heating and cooling equations can only be used as a means of obtaining a first approximation to the respective curves.

These approximate heating and cooling curves are then used in establishing the inside reference points for the periodic analysis (Chapter VII). Result of such an analysis is the complete set of temperature gradient curves which would have to exist in the wall if the approximate heating and cooling curves were correct. From the gradients, the inside wall surface temperature can be determined as a function of time. The mean wall temperature during any time interval can then be estimated with considerable accuracy, and a cooling curve consistent with the temperature gradients can then be determined from the equation

$$\Delta\theta_c = \frac{(V_f\rho_f c_f + V_a\rho_a c_a + V_i\rho_i c_i)(t_{i_1} - t_{i_2}) + \rho_w c_w A_w (L_w/12)\Delta t_m}{\dfrac{h_i A_w[(t_{i_1} - t_{s_1}) - (t_{i_2} - t_{s_2})]}{\log_e\left(\dfrac{t_{i_1} - t_{s_1}}{t_{i_2} - t_{s_2}}\right)}} \tag{8·9}$$

where t_s is the inside surface temperature of the wall through which heat is flowing, h_i is the inside film coefficient, and the subscripts 1 and 2 correspond to the beginning and end of the cooling time interval $\Delta\theta$. A similar heating equation can be readily set up.

In solving equation 8·9, values of t_{i_1}, t_{s_1}, t_{s_2}, and t_m are taken from the periodic solution for a selected time interval $\Delta\theta$; the only unknown is then t_{i_2} which can be determined by trial and error. In this way a number of values of t_{i_2}, corresponding to various values of $\Delta\theta$, are determined and a cooling curve constructed; if it agrees with the curve obtained from equation 8·6 a solution has been realized, but if not, the constructed curve is used as a second approximation and the periodic analysis repeated.

In problems where the outside temperature does not appreciably change during that part of the day for which the inside air temperature is maintained at a fixed value, the wall is very likely to reach equilibrium before the cooling period begins; for such cases the analysis of the cooling and heating sections of the 24-hr curve can be carried out as a problem in transient rather than periodic heat flow. In this case the gradient at the start of cooling is known to be a straight line and the graphical method does not involve an initial assumption. One construction gives the solution.

The establishment of heating and cooling curves for a given structure is helpful not merely in determining the most economical maximum capacity of the heating plant, but also in setting up an operating schedule to permit realizing the greatest possible saving as a result of intermittent heating. Noting that the operating saving varies directly with the over-capacity of the heating plant, it is evident that continuous heating is required if the plant selected has no capacity in excess of that needed to carry maximum equilibrium load.

Example 8·1.* The intermittent heating analysis (assuming instantaneous equilibrium) is to be applied to a large industrial building constructed as follows:

1. Exterior surfaces
   a. 8000 sq ft of exterior wall (exclusive of glass). Wall is of 6-in. concrete with 4-in. cut stone facing outside and is finished inside with ¾-in. plaster on metal lath, furred.
   Concrete: Density = 140 lb/cu ft
   Specific heat = 0.15 Btu/lb/°F
   Facing: Density = 160 lb/cu ft
   Specific heat = 0.20 Btu/(lb)(°F)
   Overall coefficient of heat transfer = 0.36
   b. 2500 sq ft of roof consisting of 4-in. concrete slab unfinished on both sides ($U = 0.9$). Storage and transmission characteristics of basement neglected.
   c. 2000 sq ft of window area ($U = 1.13$).
2. Interior surfaces
   a. 10,000 sq ft of interior (non-transmitting surface) floors consisting of 4-in. concrete slab.
   b. 12,500 sq ft of interior partitions consisting of 4-in. clay tile plastered (½ in.) on both sides.
   Density: 130 lb/cu ft; specific heat = 0.22.
   Note: Storage characteristics of furniture are not taken into account in this example, but the methods of including a term for this effect should already be clear.
3. Ventilation requirement of 4000 cfm of outside air.
4. The heating system has a maximum output of 820,000 Btu/hr.

To assist in establishing an operating procedure for the heating system, the following information is required:

1. The thermal storage constant, Q', of the structure.
2. The heat transfer constant, q', of the structure.
3. Time required to cool from 70° to 50° inside air temperature when outside temperature remains constant at 30°.

* Example from Hutchinson, *Proceedings of the National District Heating Association*, Vol. XXXII, pp. 102–115, 1941.

4. Time required to raise inside air temperature from 50° to 70° when outside temperature is constant at 30°.

5. The cooling curve.

6. The optimum time for starting to heat if occupancy is such that temperature must be held at 70° from 9:00 A.M. to 4:30 P.M., but can vary in any way during the night hours. (Based on $t_o = 30°$).

7. The heat required when operating as in 6 above, compared with requirements for continuous heating.

Solution. (In this solution the methods of using the graphs are explained, but the numerical results are actually obtained from the equations. The graphs, as drawn, are for illustrative purposes only, but a similar set sufficiently accurate for engineering computations can be constructed readily.)

1. Determination of thermal storage coefficient, Q'.
   a. Q'_w for exterior wall consists of two parts.
      Q'_c for the concrete and plaster.
      Q'_f for the stone facing.
      (1) To obtain Q'_c enter Fig. 8·3 at 8 sq ft, rise to 6-in. wall thickness, move horizontally left to intersect density line for 140 lb/cu ft, drop to intersect specific heat = 0.15, then move to left (see dotted line as example) to transfer line, drop to transfer line in lower left quadrant, move right to intersection with $U = 0.36$ for wall E_w and then rise to read $Q'_c = 32.5$. Since the actual area is 8000 sq ft rather than 8 sq ft, it follows that $Q'_c = 32.5 \times 10^3$.
      (2) Similarly for Q'_f enter at 8, rise to 4 in., cross to 160, rise to 0.2, cross to transfer line, drop to transfer line, cross to $U = 0.36$ for E_f, and drop to read $Q'_f = 8.13$ or (corrected for 8000 sq ft), $Q'_f = 8.13 \times 10^3$.
      (3) Then $Q'_w = Q'_c + Q'_f = (32.5 + 8.13)10^3 = 40.6 \times 10^3$.
   b. Q'_r for roof is obtained by entering at 2.5 sq ft, rising to 4 in., crossing to 140, dropping to 0.15, crossing to $U = 0.9$ for wall A, then rising to read $Q'_r = 5$ or (corrected for 2500 sq ft) $Q'_f = 5 \times 10^3$.
   c. Q'_g for window area: negligible.
   d. Q'_{if} for interior, floors: Enter at 10, rise to 4 in., cross to 160, drop to 0.15, cross to scale for inside walls where read $Q'_{if} = 70$ or (corrected for 10,000 sq ft) $Q'_{if} = 70 \times 10^3$.
   e. Q'_{ip} for interior partitions: Enter at 1.25, rise to 4 in., cross to 130, rise to .22, cross to scale for inside walls where read $Q'_{ip} = 11$ or (corrected for 12,500 sq ft), $Q'_{ip} = 110 \times 10^3$.
   f. Total thermal storage coefficient for the structure,

$$
\begin{aligned}
Q' &= Q'_w + Q'_r + Q'_{if} + Q'_{ip} \\
&= (40.6 + 5.0 + 70.0 + 110.0)10^3 \\
&= 226 \times 10^3.
\end{aligned}
$$

2. Determination of heat transfer coefficient (q'),
   a. To obtain q'_w for exterior wall enter at 8, drop to $U = 0.36$, move left to read $q' = 2.9$ or (for 8000 sq ft) $q'_w = 2.9 \times 10^3$.

b. For q'_g of glass enter at 2, drop to 1.13, cross to read $q' = 2.3$ or $q'_g = 2.3 \times 10^3$.

c. For q'_r of roof enter at 2.5, drop to 0.9, cross to read $q' = 2.3$ or $q'_r = 2.3 \times 10^3$.

d. For q'_{air}, opposite 4 on cfm scale (Fig. 8·3) read $q' = 4.2$ or (corrected for 4000 cfm) $q'_{air} = 4.2 \times 10^3$.

e. Total transmission coefficients

$$q' = q'_w + q'_g + q'_r + q'_{air}$$
$$= (2.9 + 2.3 + 2.3 + 4.2)10^3 = 11.7 \times 10^3$$

3. Determination of time required to cool from 70° to 50°F when outside temperature is constant at 30°F. Enter Fig. 8·1 at $(t_{i_1} - t_o) = 70 - 30 = 40$°F, rise to $(t_{i_2} - t_o) = 50 - 30 = 20$°F, cross to right to intersect $Q' = 22.5$, rise to $q' = 1.17$, cross to left and read cooling time as 13.3 hours.
4. Determination of time required to heat from 50° to 70°F with outside temperature constant at 30°F.
   (*a*) The time to heat from 30° to 50°
      (1) Note that the constants q', Q', R, having actual values of 11.7×10^3, 226×10^3 and 820×10^3, can be conveniently rearranged to the relative values 1, 19.3 and 70.
      (2) Enter Fig. 8.2 at $(t_{i_2} - t_o) = 50 - 30 = 20$, rise to $q' = 1$, cross right to $R = 70$, rise to $Q' = 19.3$, cross left to $q' = 1$, rise to read time as 6.5 hr.
   (*b*) Similarly the time to heat from 30° to 70° is found to be 16.5 hr.
   (*c*) Thus required heating time from 50° to 70° is $16.5 - 6.5 = 10$ hr.
5. Cooling curve for the structure. By the method of part 3 above, points on the cooling curve can be determined and the curve drawn. This is done in Fig. 8·4.
6. Heating curve for the structure. Using the method of part 4 above, any required number of points on the heating curve can be determined (for constant t_o and given R) and the curve plotted. This, also, is done in Fig. 8·4.
7. Optimum time to start heating. The inside temperature need not be maintained after 4:30 P.M. so at that time heat will be turned off and the structure allowed to cool; thus the time zero (Fig. 8·4) will now be called 4:30 P.M. At 9 A.M. — 16½ hr after the heat is turned off — the temperature must again be 70°; thus the heating curve must be so placed that it crosses the 70° line 16½ hr. after cooling begins. By pure coincidence the position of the heating curve in Fig. 8·4 already meets this condition and is therefore ready for use. A time scale is now placed across the top of the figure (8·4) and from this the temperature at any time during the night hours can be determined; examination of the figure reveals that the optimum time for starting the heating system is 1:30 A.M. If heating were started later than 1:30 A.M., the building would not be at 70° by 9 A.M., while if heating were started before 1:30 A.M. the temperature would reach 70° before 9 A.M., and consequently unnecessary energy would be dissipated.
8. Energy requirements for intermittent and continuous heating. Fig. 8·5 shows the rate of heat supply to the structure as a function of time for an

operating schedule in accordance with Fig. 8·4. From 9 A.M. to 4:30 P.M. the structure is assumed at steady state and heat is required at a rate of $(70 - 30)11.7 \times 10^3 = 468{,}000$ Btu/hr. At 4:30 P.M. heating ceases and no further energy is supplied until 1:30 A.M. at which time heating comes on at maximum capacity ($R = 820{,}000$ Btu/hr) and continues at this rate until the air temperature reaches 70° at 9 A.M. at which time the thermostat acts to reduce the supply of heat. All the energy which would have been supplied between 4:30 and 1:30 A.M. if operation were continuous ($468{,}000 \times 9 = 4{,}212{,}000$ Btu) is not saved since the output from 1:30 A.M. to 9 A.M. exceeds the value for continuous heating by

$$(820{,}000 - 468{,}000)7\tfrac{1}{2} = 2{,}490{,}000$$

The net reduction in daily heat requirement as a result of intermittent heating is, therefore,

$$4{,}212{,}000 - 2{,}490{,}000 = 1{,}722{,}000 \text{ Btu}$$

which is

$$\frac{1{,}722{,}000}{468{,}000 \times 24} = 15.3\%$$

The saving can be determined from Fig. 8·5 as area *abcd* minus area *cefg*.

PROBLEMS

A storage room 10 ft by 10 ft by 10 ft (inside) is made with uniform walls, floor, and ceiling and with the same exposure on all of these surfaces. The construction is 8 in. thick, density 100 lb/cu ft, specific heat 0.18, and overall coefficient of heat transfer is 0.7 Btu/(hr) (sq ft) (°F). The inside and outside surfaces are unfinished.

1. The above room, empty, is at an initial inside temperature of 70°F when the outside temperature is 20°F and the heating system is turned off. If the outside air temperature does not change: (*a*) Use Figs. 8·1 and 8·3 to determine the number of hours before inside temperature will drop to 40°F. (*b*) Determine the cooling time (to 40°F) by use of the analytical expressions.

2. If the above storage room were filled with 600 cu ft of a material having very high thermal conductivity, density of 140 lb/cu ft and specific heat of 0.22 Btu/(lb) (°F) determine: (*a*) the time to cool from 70°F to 40°F (with outside air constant at 20°F); (*b*) the cooling curve from an initial inside temperature of 70°F to a final temperature of 30°F.

3. Calculate the size of heating plant needed if the inside temperature is to be raised from 40° to 70°F in 3½ hr: (*a*) for the empty room of problem 1, (*b*) for the filled room of problem 2.

4. Plot the heating curve for the conditions of the above problem and by examination of heating and cooling curves determine the required start-up time if heat is turned off at 5 P.M. and if the room is to be at 70°F by 8 A.M.: (*a*) for empty room, (*b*) for full room.

5. Compare intermittent and continuous heating costs (operating only) for the conditions of problem 4, *a* and *b*.

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CHAPTER IX

PSYCHROMETRIC PRINCIPLES

Air and water vapor, the normal atmosphere, is used as a thermodynamic working substance in all heating, ventilating, and air conditioning operations. Whether the designer is concerned with the exhaust system from a foundry, the comfort air conditioning system for a residence, the condensation problem in a cold storage room, or humidity control in a textile mill, the basic working substance which he must use to carry heat, moisture, oxygen, or odor into or out of the conditioned space is in every case atmospheric air plus its attendant load of water vapor. Whether the equipment to be selected is a fan, a grille, a duct, an extended surface coil, a radiator, or an air washer, the designer must first know the physical properties of the gas-vapor mixture as it enters, passes through, and leaves that equipment. Obviously, therefore, a thorough and sound understanding of the physical and thermal properties of the ambient atmosphere is a fundamental requirement for any engineer working in the field of heating, ventilating, and air conditioning.

The following treatment of psychrometrics starts with a review of basic principles, from which the construction of a typical psychrometric chart is developed, and in subsequent sections the assumptions and approximations involved in the use of such a chart are critically examined.

9·1. Basic Principles. The ambient atmosphere, constituting the working substance used by the ventilating engineer, consists of a mechanical mixture of gases plus a variable quantity of water vapor. Taken together, the mixture of gases acts as, and has specific properties equivalent to, a single homogeneous gas. Essentially, therefore, the atmosphere can be treated thermodynamically as a gas-vapor mixture. This requires some knowledge of the thermodynamic properties of a vapor, of a gas, and of a mixture of the two.

For all psychrometric purposes dry air can be considered a perfect gas. Within the temperature range used in heating and cooling, either for comfort or industrial purposes, the specific heat of dry air, for constant pressure, is substantially constant ($c_p = 0.2411$ Btu/lb/°F). All other data needed by the ventilating engineer for psychrometric calculations involving dry air are obtainable from the perfect gas law,

$$Pv = RT \tag{9·1}$$

where P = pressure in pounds per square foot.
v = specific volume, cubic feet per pound.
T = temperature in degrees Fahrenheit absolute = °F + 460.
R = gas constant (= 53.3 for dry air).

If the atmosphere with which the engineer worked were free of water vapor, there would be little or no need for tables or charts of air properties since the relationship of pressure, specific volume, and temperature is readily obtainable from equation 9·1, and the specific enthalpy (base 0°F) would in every case be equal to 0.2411 multiplied by the air temperature in degrees Fahrenheit. Since water vapor is present, a less direct method of property evaluation is necessary.

The properties of water vapor cannot be expressed so simply as those of dry air. Though empirical equations are available for expressing the variation of vapor properties over limited ranges, greater facility and accuracy in calculations are possible through use of vapor tables or graphical representations. A complete pressure-enthalpy chart of the properties of water vapor is given in Fig. 9·1; from this figure it is evident that the isothermals for low-pressure vapor (less than 1 psia) in the superheat region are substantially parallel to the lines of constant enthalpy. This fact is of enormous value to the ventilating engineer because it enables him to determine the enthalpy of superheated water vapor by reading directly from Fig. 9·1 the corresponding enthalpy of saturated vapor at the same temperature. It also shows that low-pressure superheated vapor closely obeys the perfect gas law. For psychrometric purposes, therefore, a table or chart of saturated steam properties provides all the data needed for a thermodynamic evaluation of the properties of such vapor as is present in the atmosphere.

Before evaluating the properties of the mixture, one additional thermodynamic point must be considered: the effect, if any, of the presence of a gas on the properties of a vapor. Fortunately, this influence is so small that for psychrometric purposes it can be considered negligible, and evaluation of properties can be accurately carried out on the assumption that the gas and vapor jointly and independently occupy the same volume. The only additional equation needed in dealing with such mixtures is Dalton's law of partial pressures, which states that the total pressure of the mixture is equal to the sum of the partial pressures of the gas and vapor present,

$$p_t = p_g + p_v \tag{9·2}$$

where the subscripts refer to total, gas, and vapor respectively.

Summarizing, all data essential to a complete thermodynamic and psychrometric analysis of an air-with-water atmosphere are available from two equations, a table of saturated vapor properties, and two physical constants (R and c_p). A recognition of the simplicity of these basic data is essential to the engineer, since otherwise he may fail to comprehend that the sole justification for a psychrometric chart is that it can, if properly constructed, reduce to a minimum the effort needed to calculate atmospheric properties.

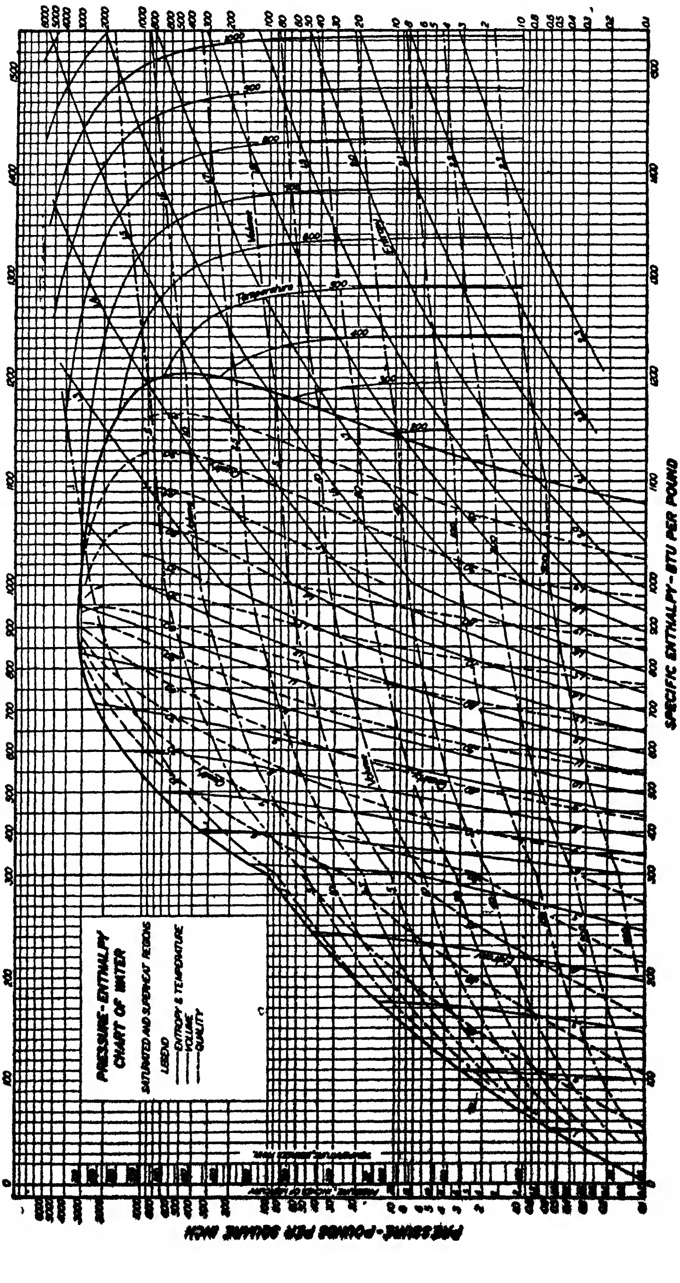
PRESSURE-ENTHALPY CHART OF WATER
SATURATED AND SUPERHEAT REGIONS
LEGEND
ENTROPY & TEMPERATURE
VOLUME
QUALITY
SPECIFIC ENTHALPY-BTU PER POUND
PRESSURE-POUNDS PER SQUARE INCH

FIG. 9·1.

CONSTRUCTION OF THE PSYCHROMETRIC CHART

9·2. Thermodynamic Properties. Many forms of psychrometric chart are in use at the present time. Differences in construction occur as a result of varying choice in the selection of basic variables, or because consideration has been given to the dynamic characteristics of processes involving the humidification or dehumidification of a given atmosphere. The typical chart presented in Fig. 9·5 (page 161) is constructed on the basis of the simplest and most fundamental thermodynamic considerations.

The chart is constructed for standard atmospheric pressure (14.696 psia) and properties are expressed in terms of a basic arbitrarily defined atmospheric unit of one pound of dry air plus the weight of water vapor associated with it. For convenience, the x axis is selected as the *scale of temperature* and the y axis as the *scale of moisture content*, or *specific humidity* W, expressed in grains per pound of dry air (7000 grains = 1 lb).

(*a*) Saturation Line. As a first state to be fixed on the chart, consider an atmosphere at 70°F and saturation. The vapor pressure of saturated water vapor at 70° is 0.3628 psia (from a standard steam table). Then, by equation 9·2, the pressure of the dry air in this atmosphere is 14.696 − 0.3628 = 14.3332, and the volume of 1 lb of such dry air is (by equation 9·1)

$$v = \frac{53.3(70 + 460)}{14.3332 \times 144} = 13.68 \text{ cu ft}$$

The weight of saturated vapor associated with a pound of dry air (reading the specific volume at 70° as 869 cu ft/lb from steam table) is therefore,

$$W' = \frac{13.68}{869} = 0.01574 \text{ lb vapor/lb dry air}$$

or

$$W = 0.01574 \times 7000 = 110.2 \text{ grains vapor/lb dry air}$$

Accordingly, the state of 70° saturated air is given at the intersection (a' on Fig. 9·2) of a horizontal line through 110.2 on the specific humidity scale and a vertical line through 70° on the temperature scale. In exactly the same way enough additional points are determined to permit fixing the saturation line.

(*b*) Constant Specific Volume Lines. The second step is the construction of lines representing the loci of states for which the specific volume of the atmosphere is constant. The method can be demonstrated by determining that state of an 80°F atmosphere for which the specific volume is the same as that of the 70° saturated atmosphere (v = 13.68). The necessary dry air pressure, from equation 9·1, is

$$p_a = \frac{53.3(80 + 460)}{13.68 \times 144} = 14.60 \text{ psia}$$

and the vapor pressure must be 14.696 − 14.60 = 0.096 psia. The desired state is therefore on the 80°F vertical at intersection with a constant-pressure line for water vapor at 0.096 psia. To fix this intersection (b on Fig. 9·2) requires information as to the location, on the chart, of constant vapor pressure lines. Noting that along any horizontal line the total pressure, weight of the mixture, weight of vapor, and weight of dry air

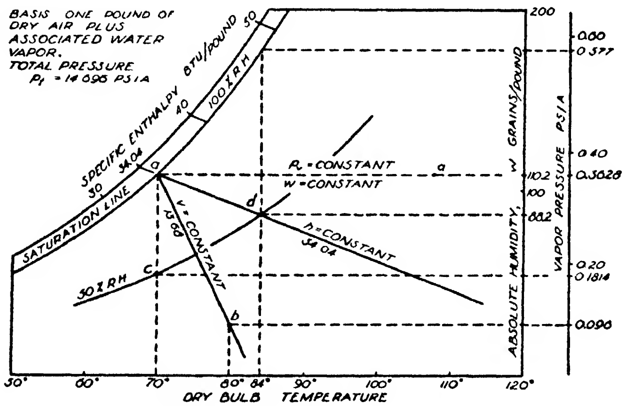

Fig. 9·2. Thermodynamic Properties on Psychrometric Chart.

are all constant, application of the perfect gas law (involving the assumption that the vapor present obeys this law) leads immediately to the conclusion that the vapor pressure, p_v, must likewise be constant along any horizontal line. But values of the vapor pressure are known for points on the saturation line and can therefore be recorded on the vertical scale. Thus, any horizontal line on the chart now represents a particular moisture content (exact) per pound of dry air and also a fixed and related vapor pressure (approximate) corresponding to the saturation temperature for that moisture content.

One difference between the grains per pound and pressure scales is significant. The former, being arbitrarily selected as one of the two basic coordinates, was laid off to a linear scale; the latter is not a constructed scale, but a transposition of data from points on the saturation line to the vertical axis. The pressure scale is not linear (compare scales in Fig. 9·5) but varies according to an empirical equation determined by steam table relationships.

By the method already described sufficient points are established having the same specific volume to permit drawing a curve through them ($a'b$ on Fig. 9·2). Additional lines are established to provide adequate coverage of the entire chart area and permit ready interpolation of the specific volume of points not falling directly on a plotted line.

(*c*) Relative Humidity Lines. Relative humidity (RH) is defined as the ratio of the actual vapor pressure in an air-vapor mixture to the vapor pressure which would exist in a saturated mixture at the same temperature. The saturation line on the chart is therefore the line of 100 per cent RH. Lines on the chart for any desired value of the relative humidity can be constructed as follows: Taking 50 per cent RH as an example, the point on the 70°F line corresponding to this relative humidity must be at the intersection (*c* on Fig. 9·2) with the line representing a vapor pressure of $0.5 \times 0.3628 = 0.1814$ psia. In a similar manner states on the 50 per cent line at other temperatures are determined, and the line is then drawn through these points (Fig. 9·2). By proceeding in this way as many lines of constant relative humidity can be established as may be needed.

(*d*) Lines of Constant Enthalpy. The specific enthalpy of an air-vapor mixture, based on one pound of dry air plus associated water vapor, is commonly taken as including the enthalpy of the air above 0°F, plus the enthalpy of the vapor above 32°F. The energy in the water vapor includes that necessary to raise the liquid from 32°F to the temperature corresponding to the saturation pressure, plus the latent heat of vaporization at saturation pressure, plus energy required to superheat the vapor from the saturation temperature to the actual temperature of the mixture. As already pointed out, the fortunate circumstance that constant-enthalpy lines in the low-pressure region of the pressure-enthalpy chart are approximately parallel to constant temperature lines, greatly simplifies the psychrometric construction since it permits fixing the specific enthalpy of vapor in a mixture by determining the specific enthalpy of saturated vapor at the temperature of the mixture; thus, on any vertical line on the psychrometric chart the specific enthalpy of the associated water vapor is constant and is equal to the specific enthalpy of the saturated vapor which would exist at the intersection of that vertical with the saturation line. This relationship (exact only for a perfect gas) permits determination of lines of constant mixture enthalpy on the chart without need of taking into account the variation of latent heat of vaporization with pressure.

As an example, consider the state at 70°F and saturation which was previously investigated. The enthalpy of the dry air at 70°F is $0.2411 \times 70 = 16.88$ Btu/lb. The weight of vapor present was calculated as 0.01574 whereas the enthalpy of saturated vapor at 70°F (steam tables) is 1090.8 Btu/lb. The enthalpy of water vapor associated with one pound of dry air is then $0.01574 \times 1090.8 = 17.16$ and the total enthalpy of the air-vapor mixture is $16.88 + 17.16 = 34.04$ Btu/lb of dry air present.

To fix the state for some temperature other than 70°F, at which the enthalpy of the mixture is also 34.04, note that the dry air enthalpy at the new temperature (taking 84°F as an example) is $0.2411 \times 84 = 20.24$, and the enthalpy of associated vapor at the unknown state point must then be $34.04 - 20.24 = 13.80$. But the specific enthalpy of vapor at any state on the 84°F line is the same as that of saturated vapor at 84°, having a pressure of 0.577 psia (from psychrometric chart). The specific enthalpy of vapor at the unknown state must therefore (from steam tables at 0.577 psia) be 1097.1 Btu/lb. The weight of vapor present is then $(13.80/1097.1) \times 7000 = 88.2$ grains/lb, and the state is fixed at the intersection (*d* on Fig. 9·2) of the 88.2 grains/lb horizontal and the 84°F vertical. In the same way other points having a mixture enthalpy of 34.04 Btu/lb can be determined and the constant enthalpy line thereby located on the chart. Other lines of constant enthalpy can be similarly located.

For convenience in psychrometric measurement the constant-enthalpy lines as plotted on the chart are not for even values of the enthalpy but are for arbitrarily selected values corresponding to even increments in the temperature of the saturated mixture. Thus a constant-enthalpy line is shown passing through 70°F saturated (Fig. 9·5) where the enthalpy is 34.04 rather than through the lower fractional saturated temperature for which the enthalpy would be an even 34.00; the reason for such an apparently irrational procedure will be discussed in detail in the later section on methods of psychrometric measurement. Because of the above condition, the numerical values of enthalpy are not marked on the chart lines but can be interpolated from the enthalpy scale which appears just above the saturation line (Fig. 9·5).

(*e*) Summary. All needed thermodynamic properties of an air-vapor mixture have now been appropriately indicated on the chart. A knowledge of any two of the following properties will permit fixing the state and consequent determination of all other properties: temperature, grains per pound of dry air or vapor pressure, specific volume, relative humidity, specific enthalpy.

9·3. Psychrometric Properties. In order for the chart of thermodynamic properties to become a tool useful to the ventilating engineer in investigating proposed processes, there is obvious need for some method of experimentally determining the numerical value of at least two of the above mentioned properties; it was with respect to this adaptation of the thermodynamic relationships to interpretation from psychrometric measurements that Carrier* made his important contribution to the advance of air conditioning with a paper presented to the American Society of Mechanical Engineers in 1911. In his paper the concepts of *wet bulb*

* W. H. Carrier, "Rational Psychrometric Formulae," *Transactions ASME*, 1911.

temperature and *dew point temperature* were developed and their relationship to the basic chart properties indicated.

(*a*) Dry Bulb Temperature. Of the five properties recorded on the chart the only one amenable to direct measurement is temperature. For many practical purposes the temperature can be measured with sufficient accuracy by inserting a thermometer in the air-vapor mixture. To distinguish this reading from others to be discussed, it is referred to as the *dry bulb temperature* (dbt).

Where the temperature of walls enclosing the mixture differs from that of the mixture, the unshielded thermometer gives an erroneous reading due to radiant transfer between the bulb and the surrounding surfaces. In precise work the radiation error may be too large to neglect, and it then becomes necessary to use either a specially shielded thermometer or to apply a correction to the reading of the unshielded instrument. Based on recent experimental work, Dropkin* recommends the following equation for correcting unshielded dry bulb temperature readings,

$$t_a = t - \left[\left(\frac{T_w}{100}\right)^4 - \left(\frac{T}{100}\right)^4\right]\left(\frac{d}{0.235}\right)^{0.44}\left(\frac{0.3105}{V_m^{0.56}}\right) \tag{9·3}$$

where t_a = true temperature of the air, °F.
t = temperature of the thermometer bulb exposed to radiation, °F.
T_w = temperature of the wall of the enclosure, °F absolute.
T = temperature of the thermometer bulb exposed to radiation, °F absolute.
d = diameter of the thermometer bulb, inches.
V_m = air velocity, feet per minute at a barometric pressure of 29.92 in. Hg and temperature of 70°F.

(*b*) Dew Point Temperature. To fix the state of an air-vapor mixture on the psychrometric chart at least one property in addition to the dry bulb temperature must be known. A second such measurable characteristic is the *dew point temperature* (dpt). If the pressure of a superheated vapor remains constant and heat is removed, the temperature decreases until desuperheating is complete; further heat extraction causes condensation. Determination of the temperature at which condensation starts therefore fixes the vapor pressure of the unsaturated mixture. Instruments for measuring dew point temperature are commercially available.

With the dry bulb and dew point temperatures known (t_a and $t_{a'}$, respec-

* David Dropkin, "Effect of Radiation on Psychrometric Readings," *Bulletin 26*, Cornell, 1939.

tively, in Fig. 9·3) the state of the original mixture is fixed at the intersection x of the dry bulb vertical with the line of constant vapor pressure which originates at the dew point temperature on the saturation line.

(*c*) Temperature of Adiabatic Saturation. An air-vapor mixture at a state, as z (Fig. 9·3), when allowed to remain in contact with a body of water (at temperature in excess of the dew point of the mixture) will undergo an increase in humidity until eventually it reaches a saturated state. When the water is supplied at a temperature equal to the final

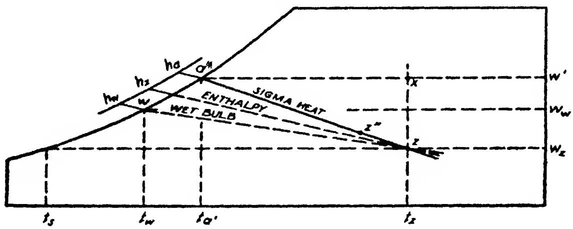

Fig. 9·3. Psychrometric Properties.

temperature of the saturated mixture, provided no energy enters or leaves the system during the humidifying process, the final mixture temperature is defined as the *temperature of adiabatic saturation*. Refer to Fig. 9·3 and let a' represent the final state of a mixture originally at some unknown state such as z, from which it was adiabatically humidified to a' as a result of contact with water supplied at temperature $t_{a'}$. For an energy balance on the system the enthalpy of the mixture at z plus heat of the liquid evaporated must equal the enthalpy at a'.

$$h_z + (W'_{a'} - W'_z)(t_{a'} - 32) = h_{a'} \tag{9·4}$$

From this equation the state at z, corresponding to any particular initial value of the specific humidity, can be calculated. As an example, consider an air-vapor mixture at unknown state z which contains 50 grains/lb and after adiabatic saturation reaches state a' at 72°F. The enthalpy and humidity at a' are (from the chart) 35.9 Btu/lb and 118 grains/lb. Then, from equation 9·4,

$$\begin{aligned} h_z &= 35.9 - \frac{(118 - 50)(72 - 32)}{7000} \\ &= 35.9 - 0.389 \\ &= 35.5 \text{ Btu/lb} \end{aligned}$$

and the per cent change in enthalpy during the process of adiabatic saturation is. for this example, 0.389/35.5 = 1 1 per cent For most practical problems 1 per cent accuracy is well within the limit of experimental error, and in such cases it is permissible to regard the constant enthalpy lines as equivalent to lines of constant temperature of adiabatic saturation.

If greater accuracy were required it would be readily possible to plot true lines of adiabatic saturation (from equation 9·4 by the same method as in the above example). Psychrometric charts have been published which show adiabatic saturation in place of constant enthalpy lines.

Regrouping the terms of equation 9·4,

$$h_z - W_z'(t_{a'} - 32) = h_{a'} - W_{a'}'(t_{a'} - 32) \tag{9·5}$$

Referring again to Fig. 9·3, suppose that the mixture at z is adiabatically humidified to any non-saturated state z'' as a result of contact with water at the temperature of adiabatic saturation, $t_{a'}$. By a heat balance on the system,

$$h_z + (W_{z''}' - W_z')(t_{a'} - 32) = h_{z''} \tag{9·6}$$

and by rearranging terms,

$$h_z - W_z'(t_{a'} - 32) = h_{z''} - W_{z''}'(t_{a'} - 32) \tag{9·7}$$

Comparing equations 9·5 and 9·7 and noting that the state z'' was taken as *any* point on the line of adiabatic saturation, it is evident that this line is characterized by constancy of the term,

$$h - W'(t' - 32) = \Sigma \tag{9·8}$$

where h is the enthalpy of any mixture having a specific humidity, W, and a temperature of adiabatic saturation, t'. The above equation states that the enthalpy of an air-vapor mixture less the heat of its associated liquid at the temperature of adiabatic saturation is constant along any path of adiabatic humidification. This term appears so frequently in air conditioning calculations that it has been given a name, the *sigma heat content,* Σ.

Equation 9·8 is usually written in different form. The enthalpy of the mixture is equal to the enthalpy of the dry air ($c_{pa}t$) plus that of associated water vapor ($W'h_v$). But the specific enthalpy of the superheated vapor at z is approximately the same as at x (Fig. 9·3), or,

$$h_{v_z} = h_{v_x} = (t_{a'} - 32) + h_{f'g} + c_{ps}(t_z - t_{a'}) \tag{9·9}$$

where $h_{f'g}$ is the latent heat of vaporization at the pressure corresponding to a saturation temperature of $t_{a'}$, and c_{ps} is the specific heat at constant pressure of superheated steam. Then substituting in equation 9·8,

$$\begin{aligned}\Sigma &= c_{pa}t_z + W_z'[(t_{a'} - 32) + h_{f'g} + c_{ps}(t_z - t_{a'})] - W_z'(t_{a'} - 32)\\ &= c_{pa}t_z + W_z'[h_{f'g} + c_{ps}(t_z - t_{a'})] \end{aligned} \tag{9·10}$$

Equating the sigma heat contents for points z and a' and rearranging the terms, an equation is realized similar to that originally established by Carrier,

$$h_{f'g}(W'_{a'} - W'_z) = c_{pa}(t_z - t_{a'}) + W'_z c_{ps}(t_z - t_{a'})$$

$$h_{f'g}(W'_{a'} - W'_z) = (c_{pa} + W'_z c_{ps})(t_z - t_{a'}) \tag{9·11}$$

Equation 9·11 is applicable only when the final state is on the saturation line.

The left side of the above equation is a quantitative statement of the amount of energy that must be obtained during humidification from the sensible heat present in the mixture at z (Fig. 9·3), but it is not equal to the difference in energy stored as latent heat between state points a' and z. The true latent heat difference is

$$h_{f'g}W'_{a'} - h_{fg}W'_z$$

but this quantity is less than the left side of equation 9·11 by an amount equal to W'_z times the difference between the increased heat of the liquid (from z to a') in Btu per pound and the decreased superheat in Btu per pound; thus,

$$h_{f'g}(W'_{a'} - W'_z) = (h_{f'g}W'_{a'} - h_{fg}W'_z) + W'_z[(t_{a'} - t_s) - c_{ps}(t_{a'} - t_s)] \tag{9·12}$$

where t_s is the dew point temperature of air at state z.

Summarizing, lines of constant enthalpy (as on the chart of Fig. 9·5) are not identical with lines representing constancy of the temperature of adiabatic saturation, but the deviation is of the order of 1 per cent and for most practical problems can be safely neglected. The sigma heat content Σ is constant along an adiabatic path (provided water is supplied at the temperature of adiabatic saturation).

The temperature of adiabatic saturation is the third measurable characteristic from which a thermodynamic property can be established. Refer to Fig. 9·3 and assume that an air-vapor mixture has known dry bulb temperature t_z, but its state point z is not known. By adiabatically saturating this atmosphere it would come to a temperature which, as already shown, would very closely approach the dry bulb temperature of a saturated atmosphere of equal enthalpy. Thus the state of any atmosphere can be closely approximated by measuring its dry bulb temperature, then adiabatically humidifying a sample of the atmosphere and measuring its new dry bulb temperature; entering the chart at the temperature of adiabatic saturation (on dry bulb temperature scale), rising to saturation line, and then moving down a constant-enthalpy line to intersection with the original dry bulb temperature fixes the state.

(*d*) Wet Bulb Temperature. The principal disadvantage of using the temperature of adiabatic saturation as a psychrometric measurement is the obvious difficulty of isolating and adiabatically humidifying a sample of the air-vapor mixture. To solve this problem, the concept of *wet bulb temperature* (wbt) has been established. If an unsaturated mixture (z on Fig. 9·3) is passed over a wetted surface, as the wick-covered bulb of a thermometer, a condition of dynamic equilibrium must be reached such that the sensible heat flowing from the passing air-vapor stream will be exactly equal to the latent heat carried from the surface in the diffusing vapor. If M' (lb/hr) is the rate at which diffusion occurs, the conditions necessary for a dynamic balance will be

$$h_{fg_w}M' = hA(t_z - t_w) \tag{9·13}$$

where h_{fg_w} = latent heat of vaporization, Btu per pound at the saturation pressure corresponding to t_w.
t_z = dry bulb temperature of passing air stream.
t_w = temperature of the wetted surface when dynamic equilibrium has been reached; t_w is, by definition, the *wet bulb temperature* of the passing stream.
h = film coefficient of heat transfer by convection, corrected for radiation between the wetted surface and its surroundings, Btu/(sq ft)(hr)(°F).

But in order to satisfy the diffusion equation,

$$M' = KA(p_w - p_z) \tag{9·14}$$

where K = diffusion coefficient through the gas film on the wetted surface in pounds per square foot of surface, hour, unit difference in vapor pressure across the film.
p_w = pressure (psia) of saturated vapor at t_w.
p_z = pressure of vapor in the passing stream.

Substituting from 9·14 into equation 9·13,

$$h_{fg_w}K(p_w - p_z) = h(t_z - t_w) \tag{9·15}$$

or by re-expressing the diffusion coefficient in terms of specific humidity differences,

$$h_{fg_w}K'(W'_w - W'_z) = h(t_z - t_w) \tag{9·15a}$$

Comparing equation 9·11 (for the temperature of adiabatic saturation, $t_{a'}$) and equation 9·15*a* (for the wet bulb temperature, t_w) it is evident that if h/K' were equal to the humid specific heat ($c_{pa} + W'c_{ps}$), the wet bulb temperature would always be exactly equal to the temperature of adiabatic saturation. There is no theoretical reason for expecting the above equality to hold, but experiment has shown that for water vapor in air it does very closely apply. Fortuitously, therefore, the wet bulb temperature of an air-vapor mixture is a close approximation to the true temperature of adiabatic saturation and can be used as such in psychrometric measurements. Thus the dynamic equilibrium temperature reached by a wet bulb thermometer over which an air-with-vapor stream is passing at moderate velocity can be used as a third psychrometric measurement from which to determine the thermodynamic properties requisite to fixing the state of such a mixture.

For some special types of problems arising in industrial air conditioning the difference between wet bulb and adiabatic saturation temperatures may be of great importance. Consider, for example, the air-vapor mixture in an oil storage tank; since the vapor present is not water the fortuitous h/K' relationship no longer holds, and the humid specific heat may not even closely approach equality with h/K'. For atmospheres of this kind, a special psychrometric chart must be constructed and consideration given to the deviation of wet bulb from adiabatic saturation temperatures.

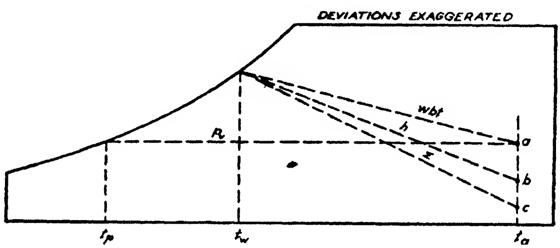

FIG. 9·4. State-point Determination.

9·4. Summary. The psychrometric chart (Fig. 9·5) is a plotting of the basic thermodynamic properties of air water-vapor mixtures at fixed total pressure. A necessary preliminary to use of the chart is determination of the state of the atmosphere in question; this requires a knowledge of at least two independent properties.

The only thermodynamic property subject to direct and simple measurement is temperature. This is usually determined with an ordinary thermometer and, when necessary, the experimental reading is corrected for radiation; the corrected reading is called the dry bulb temperature and appears as the x coordinate of the chart. Referring to Fig. 9·4, knowledge of the dry bulb temperature t_a fixes the state of an unknown atmosphere as somewhere along the vertical through t_a.

The dew point temperature (t_p in Fig. 9·4) fixes the specific humidity and locates the state of the mixture at point a.

The enthalpy is not subject to direct measurement but has been shown to be approximately constant along a line representing a fixed value of the temperature of adiabatic saturation; the latter temperature has also been shown to remain approximately constant along a line of fixed wet bulb temperature. Thus measuring the wet bulb temperature fixes the approximate enthalpy of the mixture. By referring to Fig. 9·4, the known wet bulb temperature t_w fixes the mixture state at a if lines of constant

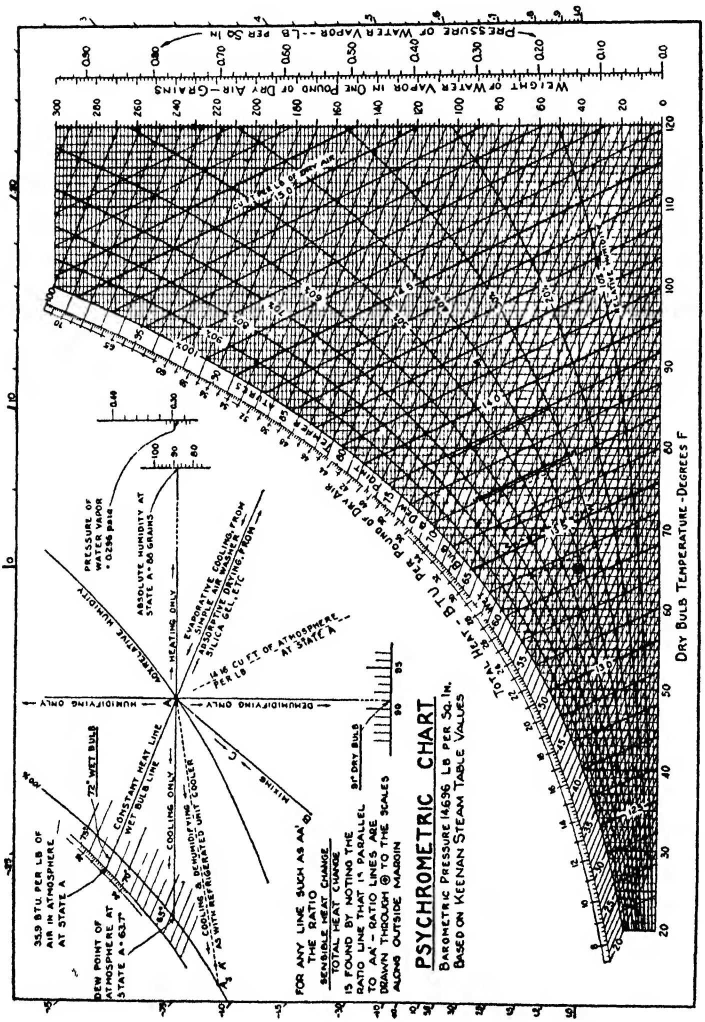

FIG. 9·5.
(See larger chart in pocket in rear cover of book.)

wet bulb temperatures are available on the chart. If lines of constant sigma heat content are used, the state fixed from the wet bulb temperature will be at c, and the error ac will be due to the deviation of the true temperature of adiabatic saturation from the wet bulb temperature. If lines of constant enthalpy are used, the state will be fixed at *b* with a consequent error of *ab*. As already discussed, the error *ab* is usually negligible for mixtures of water vapor in air, but may be very serious for mixtures of other vapors and gases.

Summarizing, the state of any air-water vapor mixture can be fixed on the chart with accuracy sufficient for most air conditioning problems by means of any two of the three following measurable psychrometric characteristics: dry bulb temperature, wet bulb temperature, dew point temperature. For most practical purposes the first two characteristics are used because they are simplest to determine experimentally.

PROBLEMS

1. A ¼-in. thermometer bulb reads 62°F when in an enclosure having a uniform inside surface temperature of 90°F. The air velocity past the bulb is 85 ft/min. Calculate the temperature of the air in the enclosure.

2. An air-vapor mixture containing 100 grains/lb is adiabatically saturated (using water at 80°F) to a temperature of 80°F (155 grains/lb). (*a*) Calculate the enthalpy of the mixture before and after saturation and determine the percentage change. (*b*) Calculate the initial temperature of the mixture.

Figure 9·5 should be used in solution of the following problems.

3. Calculate the sigma heat content of an atmosphere at 95° dbt with relative humidity of 40 per cent.

4. Air at 35° dbt and 100 per cent RH has its state changed to 85°F and 50 per cent. Is moisture added or removed? How much?

5. Air is at 60° dpt and contains 30 Btu/lb. What will be its wbt if the dpt is decreased to 40°F without change in enthalpy? Its dbt?

6. The inside temperature in a living room is maintained constant at 72°F when the outside temperature is 40°F and wind is at some steady, but unknown velocity. Condensation starts to form on the windows when inside RH reaches 40 per cent. Calculate the rate of heat loss through the windows per square foot of surface.

7. Assume inside and outside temperatures as in problem 6 but take the overall coefficient for the windows as 1.13 Btu/(hr) (sq ft) (°F). At what RH will condensation commence?

8. In problem 7, what inside velocity would be required to make possible an increase of 16 per cent in the RH at which condensation begins? Would such a velocity be practical from an electric fan of the usual type?

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CHAPTER X

SUPPLY STATE OF CONDITIONED AIR

The preceding chapter presented the theoretical background of the psychrometric chart and critically examined the methods which are used for establishing the thermodynamic state from psychrometric measurements. Having fixed the state on the chart, the problem becomes one of establishing the paths of such processes as may be needed to " condition " the working substance prior to supplying it to the air-conditioned space.

In this chapter consideration will first be given to methods of determining the necessary supply states for the five critical loading conditions which must be considered in the design of a complete air conditioning system. From critical loads, the discussion will proceed to an evaluation of supply state requirements for all loading conditions.

10·1. Sensible Load-Total Load, Ratio. Second in importance to knowledge of the total load acting on a given system is information as to the fraction of that load which is due to the moisture loss or gain (latent fraction) and the part due to the direct transfer of sensible heat. Sensible heat change varies directly as the length of horizontal path on the psychrometric chart whereas latent load is proportional to vertical change of state. For any given set of loading conditions there is a ratio of sensible heat to total heat which determines the relative position of the initial and final states of air passing from the conditioning equipment to the conditioned space.

A ratio scale has been added to the psychrometric chart (Fig. 9·5) to facilitate investigation of load ratios. As a first step in the solution of a process problem the sensible and total loads are determined and the ratio of sensible to total load calculated. The marginal scale, at the value of this ratio, is entered (Fig. 10·1), and a line is drawn to the reference point printed on the chart at 65°F and 40 grains/lb. A second line is then drawn, parallel to the first and passing through the state r, for air in the conditioned space; this line is the locus of possible supply states of air going to the conditioned space. From any state on this line, the ratio of sensible to total heat gain or loss, as the air approaches r, is the same; conversely, there is no state, not on this line, from which air could be supplied to the conditioned space to meet both sensible and latent heat requirements. If air were supplied from some state to the left of the line (as s') at a weight rate sufficient to remove the requisite sensible heat, the corresponding latent heat removal would be insufficient, and the actual room state would necessarily move up to a value of higher humidity. If supplied from a point to right of the line (as s''), an air volume sufficient to meet sensible heat requirement would extract too great a quantity of latent heat and point r would therefore move down to a state of lower specific humidity.

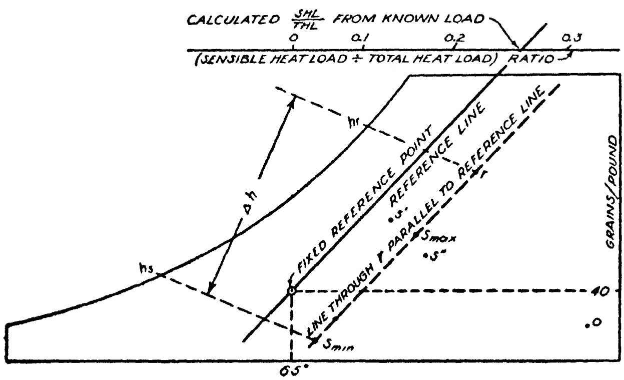

FIG. 10·1. Ratio of Sensible Heat Load to Total Heat Load.

10·2. Rate of Supply. Prerequisites to use of the psychrometric chart in solving a particular problem is knowledge of the state r to be maintained in the conditioned space, the sensible and latent heat loads, and the state o of outside air which is to be supplied to the space. The two states, r and o, are fixed on the chart, and the ratio of sensible to total heat load is then used to fix the direction of the dashed line through the room state (Fig. 10·1). This locus is completely independent of the outside state, except in so far as outside conditions have entered into the load determination. Thus in an inside room where the sensible and latent loads are due entirely to equipment and occupants, the supply state is at all times independent of changes in outside temperature or humidity.

For any given state on the direction line, there is only one volume of supply air for which load conditions can be met. Either the state or the volume, but not both, can be arbitrarily selected. The selection of a particular volume, or of an allowable range of volumes, is usually governed by general considerations concerning the effective distribution of the supply air into the conditioned space. In many problems, particularly in comfort systems, avoidance of drafts and of undesirable temperature differences in the space requires that the difference in temperature between supply and room air not exceed a fixed value. For such cases, the minimum permissible volume of supply air can be calculated from the equation

$$\text{cfm} = \frac{SHL \times v_s}{60 \times 0.24(t_s - t_r)} \tag{10·1}$$

where SHL = the sensible heat load in the room in Btu per hour.
t_s = supply air dry bulb temperature.
v_s = supply air specific volume.
t_r = room air dry bulb temperature.

Since v_s varies with the state of the supply air, equation 10·1 can more conveniently be written in terms of the weight rate of air supply M_s in pounds per minute,

$$M_s = \frac{SHL}{0.24(t_s - t_r)60} \tag{10·2}$$

The supply state is now determinable since the *total* heat load divided by M_s gives the enthalpy gain Δh experienced by one pound of air in passing through the room. From point r follow a constant-enthalpy line to the saturation curve (Fig. 10·1), move down to $h_s = h_r - \Delta h$, then proceed down the constant-enthalpy line h_s to intersection with the direction line, the intersection giving the required supply air state point, $s_{\min}$.

The maximum weight rate of air supply is limited either by room distribution problems, as production of drafts, or by the economic limitation on the volume of air which can be handled through the equipment. As the volume is increased either the fixed charges (larger equipment) or the operating cost (greater resistance losses) must increase. Once the maximum weight rate is established the corresponding supply state can be fixed ($s_{\max}$, Fig. 10·1) just as in the previous case. If the equipment for a particular air conditioning system is capable of modulating the delivered air volume, any supply state on the direction line between $s_{\max}$ and $s_{\min}$ can be used in meeting the imposed load.

10·3. Critical Loading Conditions. The five critical loading conditions are determined by the maximum values of the three heating loads (sensible, latent, and total) and by the maximum and minimum values of the sensible to total heat ratio (that is, by the extreme slopes of the direction line). In Fig. 10·2, let the direction line marked $SH_{\max}$ correspond to the sensible heat-total heat ratio when sensible heat load is a maximum. The maximum change in sensible heat per pound of supply air that can occur under any load conditions is $\Delta SH_{\max}$, which determines the lowest dry bulb temperature, t_d, that will ever be required at the supply state when operating with fixed weight rate of supply.

Similarly, the direction line marked $TH_{\max}$ has a slope determined by the ratio when total heat is a maximum. Taking the weight rate at the value previously determined for maximum sensible heat load gives a supply state for which the total enthalpy gain, ΔTH_{max}, has its greatest value; the corresponding sensible heat gain is less than in the previous case. The third critical load occurs at the condition for which latent heat gain is a maximum. For this case, the direction line is marked LH_{max}, and the supply state (for same weight rate as before) is such that the latent heat gain ΔLH_{max} is greater than for either of the preceding cases, though neither sensible nor total loads are as great. Again in Fig. 10·2,

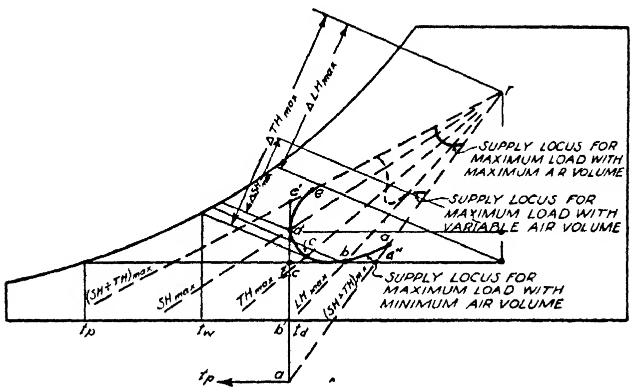

FIG. 10 2. Five Critical Loading Conditions.

when the weight rate of air for the first three critical loading conditions is taken as the system minimum, the lowest supply *dry bulb temperature, t_d,* will occur at maximum *sensible heat load,* the lowest *dew point temperature, t_p,* at maximum *latent heat load,* and the lowest *wet bulb temperature, t_w,* at maximum *total heat load.* A curve through these three maximum load supply states represents a locus of states which must be produced by the system if it is to maintain the room at state *r* for any combination of maximum loads occurring at the assigned constant minimum weight rate. Minimum weight rate will, of course, be fixed in terms of maximum sensible heat load. A similar locus can be readily plotted for constant maximum weight rate.

The two remaining loading conditions frequently occur at partial load. By referring to Fig. 10·2 and drawing the direction line for the load at which maximum ratio of sensible heat-total heat occurs, the supply state is determined in the usual way. Similarly, the direction line for minimum ratio and the corresponding state of supply are established. Continuing the locus line already obtained to include the extreme ratio states gives the solid curve of Fig. 10·2 as the complete locus of all critical loading conditions which occur with constant minimum weight rate Similarly the locus based on constant maximum weight rate can be constructed. An infinite number of loci, one for each possible weight rate, fall between those for maximum and minimum. If the conditioning equipment is capable of handling air at variable rates between minimum and maximum, complete satisfaction of all critical loading conditions can be realized by supplying air along any single, continuous line of states terminating, at either end, on one of the extreme direction lines; the dashed line in Fig. 10·2 represents such a path.

The minimum weight rate, as defined previously, has been determined for conditions of maximum sensible heat load. Since the criterion used in selection of this minimum was limitation of temperature differential between supply and room air, it follows that the permissible minimum decreases as the sensible heat load decreases. Thus in Fig. 10·2 the curved locus, *abcde,* for constant weight rate equal to the minimum at maximum sensible load, would become a vertical line, *a′b′c′de′*, if the absolute minimum (variable for each critical loading condition) could be used; actually, the section *a′b′* of this locus represents imaginary supply states. The obvious objection to selecting the absolute minimum weight rate at each critical condition (in cases where such a selection is possible) is that the required minimum supply dew point decreases from t_p to $t_{p'}$ with a consequent increase in the size and operating cost of necessary equipment. An alternative arrangement is to use the absolute minimum down to a dew point corresponding to that for maximum latent heat load, then increase the weight rate to hold this dew point constant for remaining critical loading conditions; the locus for this case is line *a″c′de′* in the figure.

Separate investigation of the five critical loading conditions is the most complex case which can face the designer. In most conditioning problems one or more of the critical conditions will automatically drop out, with a resultant simplification in the method of analysis. Thus when design conditions are based on a day when both dry bulb and wet bulb temperatures are high, the three maximum heat loads will probably occur together; that is, the equipment would be supplying air at minimum wet bulb, dry bulb, and dew point temperatures for a single supply state. However, when the sensible and latent loads in the structure are due to equipments within the conditioned space, the relationship between a single design condition and critical loading conditions may be slight or even non-existent. The greatest care should in every case be exercised to assure adequate capacity of equipment to meet all five critical loads rather than limiting the design to an analysis of the condition of maximum heat gain (or loss) alone.

Summarizing, the psychrometric analysis which must precede an investigation of possible processes to be followed and equipments to be used in solving a particular air conditioning problem has as its objective the fixing of the required supply state or the locus of states which will have to be met during critical loading. The supply state is determined as a function of room state, the sensible heat ratio, and the volume of air to be handled.

10·4. Ventilation Load. The maximum loads which have been discussed above are sensible and latent heat gains or losses *within* the room and are not in any direct way indicative of the total loads which may have to be carried by the air conditioning equipment. As an example consider an inside room which is to be maintained at 70°F dry bulb and 50 per cent RH when the outside state is 100°F and 70 per cent RH. If there is no transmission load and no source of moisture or heat within the room, the state of supply air would correspond to room conditions, and

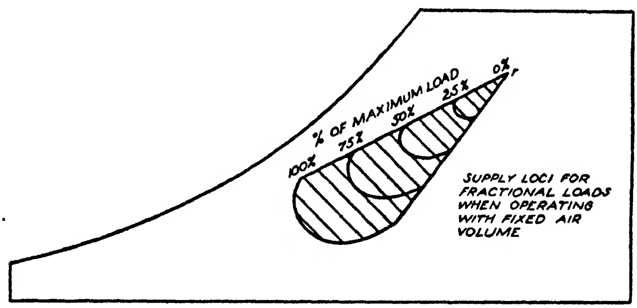

FIG. 10·3. Supply Loci for Fractional Loads.

the room heat load would be zero. However, if ventilation requirements were such that a large volume of fresh air had to be supplied, then the load carried by the equipment, in reducing outside air to the room state, would be very large. In short, the load carried by the air conditioning equipment in heating or cooling outside air needed for ventilation requirements is not part of the room heat or moisture load and does not in any way affect the critical loading conditions which have been discussed in this section. Ventilation air and its attendant heating or cooling load must be considered in the investigation of conditioning processes, but it does not enter into the analysis leading to a determination of the supply state.

10·5. Supply State at Partial Load. Consider the effect on the supply state of operation under reduced load. If in Fig. 10·3 it is assumed that the supply volume is held constant, the locus of supply states must move toward the room state as the load becomes smaller; for zero load the supply state, regardless of volume, would be at room conditions. Thus the *supply state, for any loading condition, or load, must be located in the area* swept by the maximum load locus as it passes through the segment bounded by the minimum and maximum ratio lines and approaches the apex r. Air conditioning equipment to meet all loading conditions when operating with constant weight rate equal to the minimum must therefore be capable of supplying this air at *any* state point within the shaded area. When the load is determined by outside conditions — the usual case — the problem is somewhat simplified since there is then one outside state for each particular state within the supply area and the problem becomes one of selecting equipment which can condition air along specific paths.

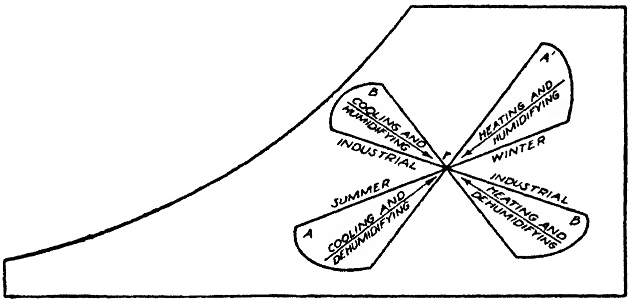

FIG. 10·4. Four Types of Supply Problems.

Where the room load is fixed by other than outside conditions, the problem remains general, and it is then necessary to have equipment capable of meeting any point in the area from any of the possible outside states.

Figure 10·4 shows the four basic types of air conditioning problems; lobes A and A' correspond, respectively, to the two most common comfort air conditioning problems, summer cooling with accompanying dehumidification and winter heating with accompanying humidification. Lobes B and B' are, respectively, for heating with dehumidification and cooling with humidification, problems that do not commonly occur in conditioning for comfort, but may be of great importance in industrial systems. The shape and position of the lobes depend, of course, on conditions for the particular problem.

10·6. Mixing. For systems in which the total volume and the per cent recirculation are fixed, the entire supply area on the chart must be considered in selecting equipments. Where either of these volumes is subject to change, the engineer can reduce the equipment supply area to a single line through use of mixing processes. Mixing is a mechanical rather than a thermodynamic process, and since it can be usefully employed with all types of air conditioning systems, the analysis will be taken up in this chapter rather than in the following one on equipments.

As an example, consider Fig. 10·5 where the shading represents the required supply area; conditioning equipment must be capable of providing a fixed weight rate of air, M, at any point in this region. Let point o represent outside conditions, but assume that the available equipment is only capable of providing M at the single state point e. Since e is outside the supply area, air leaving the equipment cannot be directly admitted to the room. Assume further, that the equipment is of the type for which the state of leaving air varies along the straight line ee' as the rate is altered; thus if the weight rate through the equipment were reduced from M, the resultant exit state would be to the left of e on the line ee'.

Consider the case in which all of M passes through the equipment; the state leaving is then e. If, however, no air is passed through, but all is by-passed around the equipment, the supply state is o. Obviously, as an

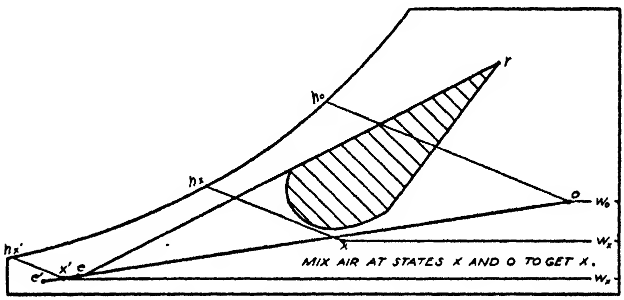

FIG. 10·5. Straight Line (Approximate) Mixing.

increasing percentage of M is allowed to pass through the conditioning equipment, the state at discharge from the unit will move left from e toward e', and the supply state, after mixing with by-passed air, will travel from o toward e. There must, therefore, be a curve connecting o and e which is the locus of mixture states. It is necessary to determine the location of this locus and investigate the magnitude of its deviation from the straight line oe.

Let M_{bp} represent the weight of air by-passed. Then $(M - M_{bp})$ is the weight of air passing through the equipment. By the law of conservation of mass,

$$\frac{M_{bp}W_o + (M - M_{bp})W_{x'}}{M} = W_x \tag{10·3}$$

The second property of the mixture which can be used to fix its state is the enthalpy. By the law of conservation of energy,

$$\frac{M_{bp}h_o + (M - M_{bp})h_{x'}}{M} = h_x \tag{10·4}$$

In Fig. 10·5, the mixture state is located at intersection of W_x and h_x lines and is found to be slightly below the straight line connecting e and o. The divergence can be readily explained by noting that the humid specific heat at x', $(c_{pa} + W_{x'}c_{ps})$, is less than the humid specific heat at o, $(c_{pa} + W_o c_{ps})$, since $W_{x'}$ is less than W_o; in consequence the temperature drop per pound of dry air at o, per Btu reduction in sensible heat is less than the gain in temperature per Btu increase experienced by each pound of air at x'. The mixture temperature is therefore greater than would be the case if mixing occurred along the straight line. Maximum deviation occurs when the states of the mixed materials are widely separated and mixing proportions approach 1 to 1.

Example. Mixing equal weights of dry air at states of 40°, zero humidity and 120°, 269 grains per pound, gives a true mixture temperature of 81° rather than the 80° which would be found if mixing were assumed to follow the straight connecting line. The deviation in this case is much larger than would be found in practical problems; except where great precision is needed the assumption that the state of the mixture falls on the line connecting the states of the materials mixed is sufficiently accurate, and most psychrometric investigations make use of this approximation. In all discussions which follow, the mixture will be considered as falling on such a line.

Returning to Fig. 10·5, it is now evident that equipment capable of supplying air at state e can also, when equipped with a by-pass,* supply the same, or any lesser air volume at any state between e and o. In Fig. 10·6, note that the weight available at point x can be varied at will from M to zero. By installing a second mixing damper to permit blending the air at point x with return air at state r, the mixing analysis again applies, and it is now possible to obtain the required weight M at any state as y, on the straight line connecting x and r. Since x is free to move along eo and y free to move along xr, the combination of two mixing dampers has enabled the equipment to supply air at the required rate, M pounds per minute, at any state in the shaded area.

The minimum conditions necessary to permit satisfaction of all possible supply needs are that the line eo be tangent to the supply area and the line er be a continuation of the ratio line established for maximum sensible heat-total heat ratio. When these two conditions are satisfied, the point e'' is established as the minimum single state of exit air from which all supply states can be met without aid of additional equipment other than two mixing dampers. The double shaded area (Fig. 10·6), below and to the left of e'', is the section of the chart from which air at any single state can be successfully mixed with varying quantities of outside and room air to give the required supply volume at any state in the re-

* Refer to Auditorium Conditioning Corporation patents.

quired supply lobe. Conversely, there is no single state outside of the double shaded area from which all supply conditions can be met. From the design standpoint, the simplest way to solve the supply problem is to locate point e'' on the chart and then provide whatever equipment is needed to deliver air at e'' for any weight rate up to a maximum equal to the fixed weight rate of supply to the room.

The obvious objection to reducing the supply area to point e'' is the requirement, evident from the figure, that this state must have lower dry bulb, wet bulb, and dew point temperatures than any point actually required for supply conditions. Lowered temperatures mean increased first costs and operating costs and should therefore be avoided whenever possible. Since the locus of critical-loading conditions includes the most ex-

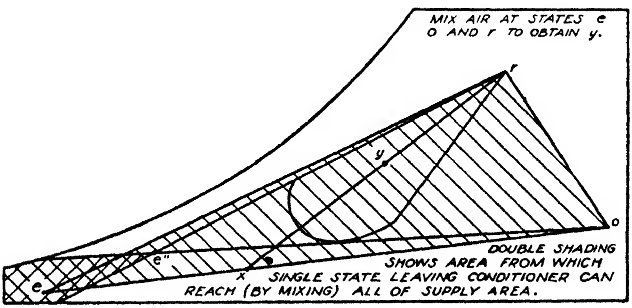

FIG. 10·6. Permissible Area for Single-point Supply.

treme temperatures ever required at the supply state, this line is also a locus of the absolute minimum of state requirements which must be met by the equipment. Use of this locus as a criterion of basic design is justified by the fact that any state in the supply area can be realized from the locus by mixing with room air.

There is no first cost penalty in working from the maximum loading locus since it represents a minimum of permissible performance for the equipment. The possibility remains, however, that additional facilities permitting air to exit from the equipment at higher temperatures under reduced load would result in an operating economy. Usually the latter variation can be achieved by adjustment at the equipment itself, though in some cases there may be justification for carrying out a special analysis of processes most economically fitted to partial load operation.

10·7. Summary. The first step in the psychrometric analysis leading to a selection of equipment for an air conditioning installation is to plot on the chart the five critical loading conditions defining minimum values of the dry bulb, wet bulb, and dew point temperatures for supply air and the minimum and maximum ratios of sensible heat load to total heat load.

The extreme supply air states, defined by the five critical loading conditions, appear as a family of curves the position of each depending on the constant supply air rate. If supply air rate were maintained constant for partial as well as for extreme loads the envelope of all required supply states would be fixed by the two extreme ratio lines through the room air state point together with the locus of maximum loading conditions at the fixed supply rate.

In its most general form the designer's problem is to select equipment which will condition the fixed volume of air to any state within the supply area. Actually, however, the fundamental problem of meeting extreme loading conditions is in itself one solution of the entire problem because mechanical mixing of air from the critical locus with room or outside air will, in every case, assure ability to realize any state within the supply area.

The first criterion of design is to choose thermodynamic processes which can be followed by the equipment in bringing air from the intake condition to any state on the critical locus. Secondary design criteria may then be investigated in an effort to reduce the partial-load operating cost, though this extra care in design cannot reduce the first cost as determined by the first criterion (since critical loads must still be met) and may substantially increase first cost through need for greater complexity of the equipments and controls.

Summarizing, the data required on the chart before the process investigation and subsequent equipment selection can be started are: (1) the room state, fixed by comfort or storage conditions beyond control of the designer; (2) the outside state, or range of states, fixed by climate and weather; (3) the locus of critical loading conditions at a selected weight rate of air supply to the conditioned space.

PROBLEMS

1. A structure to be air conditioned has a sensible heat load of 20,000 Btu/hr at a time when the total load is 100,000 Btu/hr. If the inside state is to be at 80°F, 50 per cent RH, is it possible to meet the load conditions by supplying air to the room at 100°F and 60 per cent RH? If not, discuss the direction in which the inside state would be expected to move if such air were supplied.

2. The summer load on a structure is 50,000 Btu/hr sensible heat when the total load is 60,000 Btu/hr; 30,000 Btu/hr latent heat when the total load is 50,000 Btu/hr; 40,000 Btu/hr sensible load when the total load is 65,000 Btu/hr. If the maximum permissible temperature difference between supply air and room air is 20°F and if the room is to be held at 80° dbt and 50 per cent RH, calculate: (*a*) the minimum permissible year-round weight rate of air supply, (*b*) the minimum dbt of supply air when system operates with weight rate as calculated in (*a*), (*c*) the minimum wbt for operation at minimum weight rate, and (*d*) the minimum dpt for operation at minimum weight rate.

3. Supply air at 110° dbt and 84 wbt is supplied to an enclosure at a constant rate

sufficient to maintain the inside state at a fixed value of 70° dbt and 65° wbt. If the sensible load is 56,000 Btu/hr, determine the latent and total loads.

4. The most economical *single* supply state from which a system can meet *any* state in a given supply area is 64° dbt and 35° dpt. Room state is 90° wbt and 210 grains/lb. Determine the maximum *SHL/THL* ratio for the system.

5. Mixing occurs between 600 cfm of air at 70° dpt and 14.5 cu ft/lb specific volume and 400 cfm of air at 45° wbt and vapor pressure of 0.10 psia. Determine the dbt and wbt of the resultant mixture: (*a*) by the precise analytical method, (*b*) on the assumption that the state of the mixture is on a line connecting the states of the two air volumes.

6. For the conditions of problem 5 determine the volume of the mixed air and compare with the sum of the volumes of air going into the mixture.

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CHAPTER XI

PSYCHROMETRIC PROCESSES

The intent of this chapter is to investigate the thermodynamic process lines followed by air in passing through each of the different types of equipment used in air conditioning. Such equipment will be analyzed with referenee to its limitations in reaching all parts of any required supply area. The latter part of the chapter takes up combinations of equipments which may permit achieving supply conditions otherwise unobtainable, or may make possible more economical operation at supply points otherwise costly to attain.

11·1. Equivalent By-pass Concept. Most equipments used in air conditioning, as heating coils, wet or dry cooling coils, or adiabatic humidifiers; have one characteristic in common: the air to be conditioned is brought into contact with a surface (liquid or solid) which is maintained at a substantially uniform temperature. Heat or vapor transfer to or from the air then occurs across this relatively uniform surface. The rate at which heat and vapor are transferred depends on the nature and extent of the surface, its position with respect to the air stream, and the relative velocity and degree of turbulence. All these factors are fixed, directly or indirectly, by the design of the equipment; it therefore follows that effectiveness is largely determined by the original design and is beyond control by the application engineer.

Transfer effectiveness is largely governed by the degree of contact. That part of the air stream which makes and maintains contact is brought to surface conditions while the fraction which does not may undergo no change; the resultant state of the air mass leaving such an equipment is derived from the mixing of fractional quantities of air from all states between inlet and surface conditions. Although there would be great difficulty in estimating the distribution of fractional states, the overall effect of such a mixing process can be readily described in terms of two equivalent masses of air, one at inlet conditions and the other at surface conditions. Applying the mixture analysis of the last chapter (section 10·6), the state of material leaving the conditioning equipment can be expressed in terms of these two masses. The actual equipment is thus considered equivalent to a smaller conditioner (100 per cent effective) working in conjunction with a by-pass. Effectiveness can then be expressed in terms of an equivalent by-pass factor.

The by-pass concept is helpful not only in permitting use of a simple factor to express effectiveness, but also because it reduces the problem of analyzing processes of actual equipments to the simpler one of analyzing only ideal (100 per cent effective) equipments and expressing the actual performance in terms of the ideal. This method will be used in the sections which follow.

11·2. Simple Heating or Cooling. Referring to Fig. 11·1*a*, if air at state *o* is passed over any dry surface at a temperature other than t_o, but greater than the dew point temperature t_d, the dry bulb temperature of the air stream would approach that of the surface while the specific

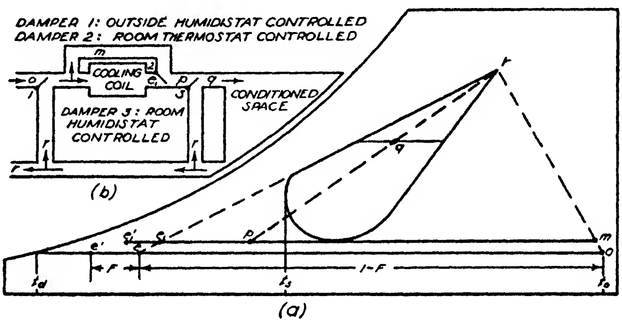

FIG. 11·1. Fixed Surface Temperature.

humidity would remain unchanged; thus a horizontal line through point *o* is the locus of all possible states resulting from a simple heating or cooling process. Using the supply area of the figure as an example, three methods of operation are available to provide air at any required supply state. The simplest, but most costly, is to fix the surface temperature at a value sufficiently low so that all supply conditions can be met without need for adjustment. A second and more economical method is to control the humidity of supply air by mixing ahead of the conditioner and control exit air temperature by adjustment of surface temperature in accordance with a fixed relationship between this temperature and the specific humidity. The third method is to control the humidity, as in case two, but vary the surface temperature as a function of the required dry bulb temperature of the supply state; this last is the most economical method of applying simple transfer surfaces to the solution of a complex supply requirement. The three methods will be discussed in detail in following paragraphs.

CASE I. *Fixed Surface Temperature.* If all outside air were used, the maximum leaving dry bulb temperature from which air could be supplied to any point in the lobe would be at the intersection, point *e*, of a horizontal through *o* with the extension of the maximum sensible heat-total heat ratio line. For a surface having a by-pass factor of F, the surface temperature needed to provide an exit air temperature of t_e would be (refer to Fig. 11·1 for nomenclature),

$$t_{e'} = \frac{t_e - Ft_o}{1 - F} \tag{11·1}$$

Thus any surface capable of delivering the required weight rate of air with by-pass factor F could provide supply air at any required supply state if the surface temperature were maintained at $t_{e'}$.

Low surface temperature requires low refrigerant temperature. As discussed in Chapter III, best economy is realized when heat extraction occurs at the highest permissible refrigerant temperature. For this reason greater economy of operation would be realized for the conditions of this problem if a sufficient fraction of room air were used in the mixture going to the cooling coil so that the cooling process line could be raised to a condition of tangency, me_1', with the supply area boundary. Then a higher surface temperature, $t_{e_1'}$, could be maintained, and exit air at e_1 would represent the most economical single state of air from the cooling surface for which all supply conditions could be satisfied with constant surface temperature. Strictly speaking, the terminal state of exit air t_{e_1} would move to the left for rates of air flow across the surface below the design value (because of lesser F due to greater time for contact), but in all such cases terminal air would be mixed with sufficient by-passed air at state m to provide the design supply quantity, and the mixture state would fall between e_1 and m, so the effect of change in terminal conditions would not enter into the problem.

A system capable of operating under the conditions of Case I is shown in Fig. 11·1b. Mixing damper (1) is set in a fixed position (for a given outside state) such that the state of the mixture, m, is always the same. Mixing damper (2) takes a position determined by a thermostat in the conditioned space; as the sensible heat load increases the thermostat requires a larger proportion of air at state e_1 and a correspondingly smaller part at state m, giving a lower dry bulb temperature at p with a resultant increase in the capacity of supply air, at constant specific humidity, to extract sensible heat from the conditioned space. Damper (3) is controlled by a humidistat in the conditioned space. When the relative humidity tends to depart from the desired value the humidistat acts to re-position this damper and thereby vary the proportion of high humidity recirculated air. Other operating arrangements satisfying Case I are also possible.

Case II. *Variable Surface Temperature as Function of Latent Load.* The fixed surface temperature of Case I had to be low enough to provide air at a dry bulb temperature substantially less than the minimum t_e required for supply conditions. The loss in refrigerating efficiency attending this unnecessary reduction in temperature can be overcome by designing the system for operation with a variable-temperature working fluid so that the surface temperature will never be less than the minimum value needed to meet the most extreme sensible heat load which can obtain at a particular value of the latent heat load.

In Fig. 11·2*a*, let W_x be the required specific humidity for a particular load condition. Corresponding to this humidity there is a particular value of the minimum dry bulb supply temperature, t_e. The by-pass factor, F, mixture temperature, t_m, and minimum exit temperature, t_e, together fix the unique value of the required surface temperature, $t_{e'}$, corresponding to the particular specific humidity W_x.

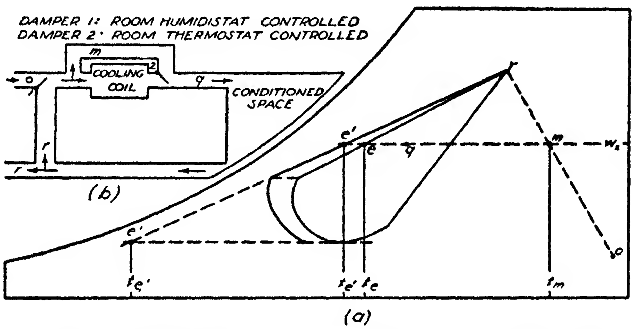

FIG. 11·2. Variable Surface Temperature (Humidity Controlled).

Figure 11·2*b* shows an equipment arrangement for carrying out Case II. Mixing damper (1) takes a variable position in response to the room humidistat, thereby causing point m to move along the straight line ro to fix the specific humidity of supply air. Mixing damper (2) is actuated by a room thermostat which causes point q to move along the line em. The surface temperature $t_{e'}$ is controlled to that value corresponding to the particular setting of mixing damper (1).

For Case II the locus of surface temperatures is a path which varies with specific humidity along the heavy line of Fig. 11·2*a*. The upper section of this locus constitutes one leg of an angle with apex at point r and base along the maximum ratio line for sensible to total heat. To compare Cases I and II, the fixed surface temperature of Case I is indicated at e_1'. The economic advantage of operation as in Case II cannot be estimated from the figure, since it is largely dependent on the hours per season of operation with supply air at various states within the lobe; obviously, if 90 per cent of the operating time were at 10 per cent partial load, the advantage of Case II would be greater than if 90 per cent of operating hours were at full load.

CASE III. *Variable Surface Temperature as a Function of Sensible Load.* Though Case II avoids some unnecessarily low dry bulb exit temperature, it still requires, for a given humidity, continuous operation of the cooling equipment at the minimum surface temperature ever required for a supply state of that humidity. Further improvement would therefore be expected if the surface temperature were varied as a function of the required dry bulb supply temperature. If Fig. 11·3*a* is referred to and if any possible supply point *q* is considered, the requisite surface temperature at this state has some such value as *e*′. At first glance one might suppose that the surface temperature would remain constant so long as the dry bulb temperature of the supply air did not change, but examination of the figure shows that the required surface temperature is

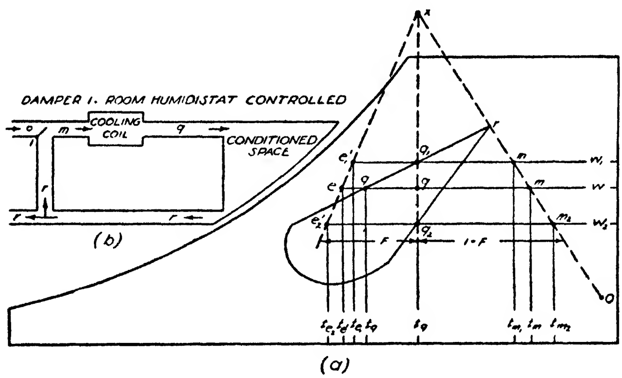

FIG. 11·3. Variable Surface Temperature (Temperature Controlled).

influenced by the dry bulb temperature of the mixture entering the cooling equipment and is therefore indirectly dependent on the humidity at the supply state as well as on the temperature.

Considering the three supply states, q_1, q, and q_2, the humidity of the mixture entering the cooler decreases from W_1 to W_2 with a consequent increase in the entering dry bulb temperature from t_{m_1} to t_{m_2} and a resultant decrease in the required surface temperature from $t_{e_1'}$ to $t_{e_2'}$. If the by-pass factor were constant, the locus of surface temperatures corresponding to a fixed value of the supply air dry bulb temperature could be graphically determined by noting that it must be a straight line passing through the intersection, *x*, of the extended dry bulb supply line and the extended line connecting *r* to *o*. Thus, knowing the required surface temperature at *e*′, the locus of surface temperature is a straight line connecting *e*′ with *x*.

Case III differs from Case II only in that the position of *q* on the line *qm* is fixed by varying the surface temperature rather than by mixing air at states q' and m. Since mixing is always a thermally irreversible process it is evident that Case III possesses an inherent thermodynamic advantage. An equipment arrangement suitable for use with Case III is shown in Fig. 11·3*b*. Mixing damper (1) is actuated by the room humidistat to give an entering mixture at state m having the necessary specific humidity. Neither by-pass around the cooling unit nor mixing with return air after the cooling unit is needed. The surface temperature is controlled from a room thermostat.

Case III is thermodynamically the most effective of the systems by means of which a simple cooling process can be made to provide air at any necessary state in the supply area; it also represents the simplest possible arrangement of control equipment.

Comparison of Cases I, II, and III. Regardless of what kind of system is designed around a simple (dry) cooling surface, two predominant facts must always be recognized: (1) The moisture load of the conditioned space is met by blending air at states r and o, regardless of whether the mixing occurs through damper (1) or damper (3) and regardless of which of the three cases is employed, there is one and only one proportion of recirculated to outside air which will meet the requirements of the humidity load at a given supply state. (2) When the weight rate of air supply to the conditioned space is held constant at all loads, the cooling load for outside air is a function of the specific enthalpy at the supply state and is the same for all three cases.

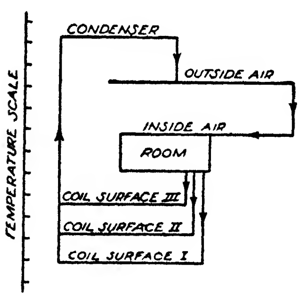

FIG. 11·4. Energy Lift Diagram.

Setting up a heat balance on the conditioned space shows that the total load carried by the cooling equipment is fixed by the supply state and must be the same for all arrangements of equipment. However, the average temperature of the working fluid which carries heat from the cooling surface is not the same, and the work required to carry the load is greatest for Case I and least for Case III. This can be visualized by referring to Fig. 11·4 and noting that the quantity of heat flowing through the system is unchanging (for a given supply state), but the temperature level of the receiver into which this heat flows is greatest for Case III. Since energy collected in the working fluid must be returned to the atmosphere (or to some receiver — as condenser cooling water — at a substantially constant temperature) the work required to dispose of this heat will vary directly as the difference between receiver and outside temperature and will therefore be least for the system arranged to operate as in Case III.

Aside from variations in thermodynamic effectiveness, the three methods of using simple cooling coils differ in other important respects. Case I is unique in that the cooling surface operates independently of the distribution equipment; its surface temperature is held constant so there is no direct connection between the refrigerating unit and the conditioned space. Case I is also interesting in that, though the least effective of the three, it uses the greatest number (three) of mixing dampers. Case II, with two dampers, is more effective, while Case III, with but one such damper, has the greatest effectiveness. If it is recalled that mechanical mixing is a thermally irreversible process, the above result can be generalized to an approximate rule to the effect that if two systems serve the same purpose their relative effectiveness will vary inversely with the number of mechanical mixings that occur.

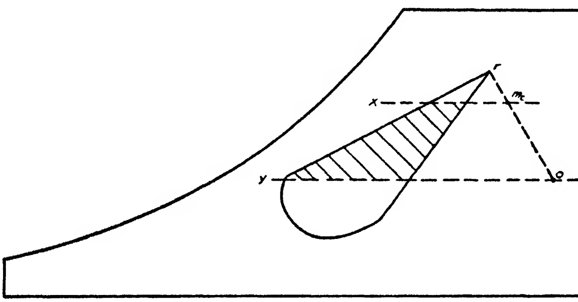

FIG. 11·5. Humidity Limitation with Simple Cooling.

11·3. Limitations of Simple Heating or Cooling. An obvious limitation of the simple heating and cooling process is that it does not change the specific humidity of the conditioned air and can therefore be effectively used only when some other method of humidity control is available. For the examples already considered, conditions have been such that mixing could provide complete humidity control, and the combination of mixing dampers with the cooling equipment then provided adequate coverage of the entire supply area. For many problems such a combination will not suffice.

Refer to Fig. 11·5 and consider the situation in the event that the specific humidity of the outside air exceeds that for some of the supply points. By equipments arranged as in Cases I, II, or III all supply points in the area above the horizontal line *yo* could be attained, but none of these systems could provide air at states below this line.

A second and similar limitation sometimes arises. When the total rate of air supply to the system is low and the occupancy high, ventilation requirements may necessitate a minimum volume of outside air which is a substantial fraction of the total supply volume. Under such conditions a critical mixture point, m_c, is reached. This state represents the maximum allowable fraction of recirculated air and therefore fixes the maximum specific humidity available at any attainable supply air state. Under these conditions, no system using only simple cooling equipments combined with mixing could realize any state in that part of the supply area which lies above the horizontal line xm_c; the limited area which could successfully be covered by simple cooling and mixing is shaded in the figure.

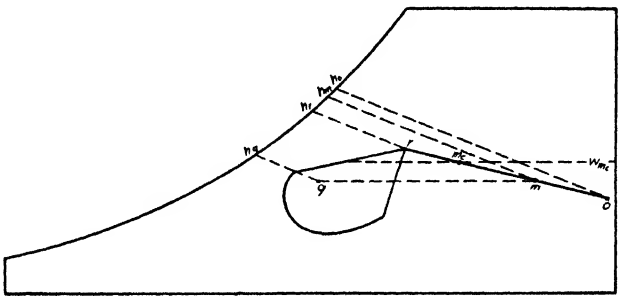

FIG. 11·6. Economic Limitation of Simple Cooling.

Aside from inability to meet all supply requirements, simple cooling is sometimes possible, but uneconomical. In Fig. 11·6, when the outside state is at o, the minimum heat extraction per pound of air supplied at point q is $h_m - h_q$. Since the enthalpy of outside air is in this case greater than that of room air, the enthalpy of any mixture along line ro must also exceed that at r and the excess energy requirement of such a system is $(h_m - h_r)$ Btu per pound over that needed if equipment were available to recondition 100 per cent room air. If ventilation requirements establish a critical mixture state, m_c, then simple cooling would be possible for all specific humidities below W_{m_c}, but would be more costly than direct reconditioning; a similar analysis can be readily developed for the heating season.

The reverse of the above situation exists when the enthalpy of outside air, regardless of its dry bulb temperature, is less (for cooling season) than the enthalpy of the air in the conditioned space, or more (for heating season) than the room enthalpy. For such cases, simple heating or cooling requires a lesser change in enthalpy than does direct reconditioning of air from the room.

Summarizing, three arrangements are possible for combining simple heating and cooling equipment with mixing dampers to provide supply air of controlled temperature and humidity. Such systems are of varying degrees of effectiveness, but greatest thermodynamic advantage is always realized by the arrangement which permits operation with the maximum surface temperature consistent with the required dry bulb temperature of the supply air. Regardless of the arrangement selected, inherent limitations arising from inability of the conditioning equipment to add or extract moisture from air passing through it make any system of this type inoperative when outside air is not available at an absolute humidity below the minimum needed at any supply state, or when ventilation requirements reduce the per cent allowable recirculation to a point such that the maximum specific humidity of such a mixture is less than the maximum humidity required at any supply state.

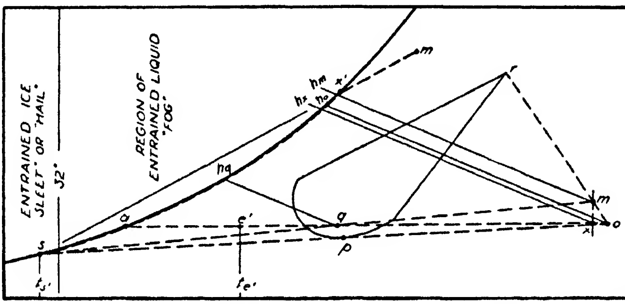

FIG. 11·7. Dehumidifying Coil Performance.

11·4. Surface Cooling with Dehumidification. The preceding section has emphasized the thermodynamic advantage to be gained by keeping the surface temperature of a cooling coil as high as practicable. In many problems other factors more than offset the thermodynamic consideration and low surface temperatures may be used to advantage. Where lower temperatures permit use of smaller surfaces with higher by-pass factors, the saving in first cost may justify reduced thermal effectiveness. Of even greater significance is the possibility of securing an actual reduction in the total heat load on the system through use of psychrometric processes made possible by lower surface temperatures. Where this can be done, the expense of removing fewer Btu at greater unit cost will often be less than that of removing more heat at lower unit cost.

(*a*) THEORY OF SURFACE DEHUMIDIFICATION. Consider the supply area of Fig. 11·7 with room and outside states as shown at *r* and *o*. The absolute humidity of outside air exceeds that of many required supply points; so, as already discussed, a simple cooling system would be unable to provide supply air for all loading conditions. Suppose now that the coil surface temperature were reduced below the dew point of the outside air, to some such value as $t_{s'}$. The temperature of the coil surface air film would then approach $t_{s'}$ and as the vapor pressure in the film could not exceed the saturation pressure corresponding to the film temperature, condensation would have to take place on the coil. The passing air stream would then be contacting a liquid surface having both a lower temperature and a lower vapor pressure than the main body of air. Each air mass contacting the surface would lose both heat and vapor, and air would leave the conditioner with a lower specific humidity than on entrance *even though the period of contact might be insufficient to reduce the dry bulb temperature of the entire volume to a value as low as its original dew point temperature.*

The heat transfer and vapor transfer processes are completely independent of one another since only sensible heat is transferred through the air film; there is a mass transfer of vapor toward the surface, but the latent heat is extracted *at* the surface and is not transferred across the air film. The heat transfer from the surface to the refrigerant is equal to the total heat extraction from the air stream, but the transfer across the exterior vapor film is only the sensible fraction of the total.

The concept of simultaneous heat and vapor transfer is vitally important to an understanding of the process followed by air passing over a low-temperature coil since from it one can visualize that condensation can, and does, occur from an *unsaturated* air stream passing over a wet surface which is at a temperature below the dew point of the air. If simultaneous transfer did not occur, the process line on the psychrometric chart would be a combination of simple cooling followed by further cooling with accompanying dehumidification along the saturation line; for the example of Fig. 11·7 the processes would then be indicated by the path *oas'*. Actually, however, no part of the process follows a simple cooling path, for as soon as sensible heat transfer starts there is a simultaneous vapor transfer with consequent dehumidification. The true process is therefore representable by some continuous path connecting *o* with *s'* and falling, at every point, below the line *oas'*. The position of the actual curve is determined by the relative rates of heat and vapor transfer.

Fortuitously, the resistance of the air film is approximately the same for the passage of vapor as for the transfer of heat so the state of the leaving mixture is very nearly on the straight line connecting the entering state and the state of saturated air at the surface temperature. Actually, the resistance to mass transfer is usually slightly less than that for heat transfer so the final state is somewhat below the line *os'* for a dehumidification process and somewhat above it for humidification (as by means of an air washer). However, experience has shown that the final state so closely approaches this line that for most commercial surfaces the deviation can be safely neglected. With the exception noted in the following paragraph, the leaving state of an air stream will be taken as on a straight line connecting the entering air state and the state for saturated air at surface temperature; the exact position of the leaving state can be readily determined when the by-pass factor of the surface is known.

(*b*) Fog and Ice Processes. A special case and exception to the above analysis occurs when the process line crosses the saturation curve on the psychrometric chart. In Fig. 11·7 consider an entering state at m' and a coil at temperature $t_{s'}$. The initial process followed by air passing over the surface would be along $m's'$ to the intersection x' with the saturation line. From this point on, however, the straight line process would not apply. At any point between m' and x' the passing air is unsaturated and can therefore lose sensible heat to the coil without causing condensation in the air stream; such condensation as occurs during this part of the process is localized at the coil surface. After reaching x', however, the air is in a saturated condition and hence cannot lose further sensible heat without simultaneous condensation in the air stream. Cooling beyond x' therefore occurs with formation of a fog. From this point on the process becomes more complex because in addition to the sensible heat and mass transfer through the surface air film, it is also necessary that the latent heat of that part of the vapor which condenses in the main body of air be transferred across this film. The process from x' to s' is then along the saturation line (assuming removal of the liquid present in the main air stream) and the complete process is shown by the broken line $m'x's'$. Since the process is not a straight line, the equivalent by-pass method does not apply, and the state of the leaving air is not on the process line $m'x'$, but below it in the area bounded by curve $m'x's'$, a tangent from m' to the saturation line, and that part of the saturation line which falls between the tangent's intersection and point s'.

A further complication occurs if $m's'$ crosses the saturation line when $t_{s'}$ is below 32°F. In this event the process from m' to the point on the saturation curve corresponding to 32°F is as before, except that the latent heat transfer through the surface includes the heat of fusion as well as the heat of vaporization. As the air stream reaches temperatures below 32°F, the fog freezes and hail or snow is precipitated; in this region the heat transfer through the air film includes sensible heat plus a fraction of the heat of fusion and that part of the total heat of vaporization representing vapor precipitated directly in the main body of air. Operation of a practical system under such conditions is extremely unlikely, but the case is presented in order to complete the concept of psychrometric processes which can be followed by an air stream as it passes over a surface.

(*c*) Limitations of Analysis. Throughout the above analysis the assumption has been made that the surface is at a uniform and constant temperature. With commercial surfaces uniformity is frequently not realized, but if constancy exists, the analysis can be applied on the basis of an "equivalent" uniform surface temperature. In cases where the surface temperature is not constant, or where it is non-uniform along the path of air flow, the analysis is applicable at a given place and time, but integration must be used to determine the final state. Cases of this kind will be given detailed consideration in a later section.

11·5. Application of Dehumidifying Surface to Conditioning Problems. Returning to the problem considered in connection with simple cooling, investigate a dehumidifying surface as a possible solution. Connecting the outside air state with the required supply locus by the tangent line *op* (Fig. 11·7) it is evident that state *p* could be realized by means of a cooling surface operating at temperature $t_{s'}$. The entire supply area could then be covered by a system utilizing simple cooling for supply states above *oa* and dehumidifying cooling by means of a wetted surface for states below.

But consider a required supply state such as *q*, and compare the effectiveness of simple and dehumidifying conditioning to this point. With simple cooling all outside air would have to be used, and the required heat removal per pound would be $h_o - h_q$; the surface temperature would be (based on by-pass factor of F and operation as for Case III) $t_{e'}$. If the same state were to be reached with a dehumidifying coil operating at its minimum surface temperature ($t_{s'}$) the state of the mixture going to the coil would be at intersection *m* of an extended straight line through *s'* and *q* with the line *ro*. The heat removed per pound of air would then be $h_m - h_q$ and the loss over simple cooling would be $(h_m - h_o)$ Btu per pound. Since the weight rate of supply air is the same in both cases, the per cent loss for the system would be the same as for each pound.

Changing the state of the entering air from *o* to *m* *increases* latent heat load by $(h_m - h_x)$ while simultaneously *decreasing* the sensible heat load by the lesser amount $(h_o - h_x)$. The overall effect of using a dehumidifying coil to reach *q* is to increase the total load handled by the conditioning equipment. When the outside state is so located that h_o exceeds h_r, then $(h_o - h_m)$ Btu per pound would be a saving realized by use of a dehumidifying coil in place of a simple cooling coil. Even in such a case, however, the advantage of dehumidification would be subject to question since the reduction in total heat load would be at the expense of a greater temperature differential and consequently a greater work requirement per Btu handled at the cooling surface. The choice between high- or low-temperature operation must be made on the basis of work requirements and a decision cannot be reached from heat load considerations alone. In many cases the most economical operating procedure results when the surface temperature is at some value between the minimum and maximum values; for such cases the arrangement of the system represents a compromise between simple and dehumidifying coil procedure.

Although the state of air leaving the dehumidifying coil is on the line $s'm$, its exact position is determined by the by-pass factor of the coil and is not subject to variation. In the example of Fig. 11·7, the by-pass factor of the surface might be so great that the leaving state would be between points q and m; in that event no supply state could be realized which had a lower dry bulb temperature than the air leaving the coil. On the other hand, a low by-pass factor would place the exit air dry bulb temperature between s' and q; in this event a mixing damper could blend leaving air with air at state m such that state q could be reached. The effect of such a mixing damper is to increase the equivalent by-pass factor of the cooling surface.

11·6. Cooling Surfaces in Series. Three possibilities exist for the processes followed by air when passing through a series of cooling surfaces. Each of the cases will be treated separately with a brief discussion of the practical conditions which might lead to its adoption and a study of the path followed during passage through the conditioner.

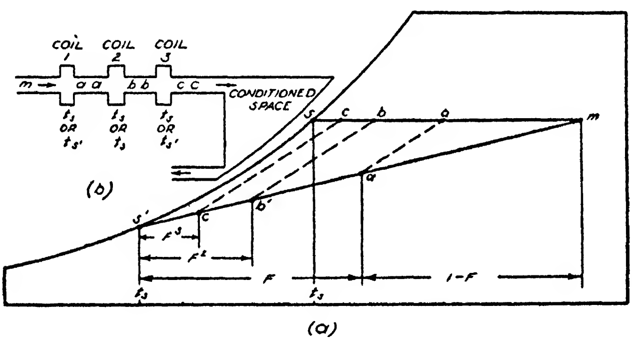

Fig. 11·8. Constant-temperature Coils in Series.

Case I. *A Series of Surfaces at Constant and Uniform Temperature.* The analysis is the same whether the surfaces are wet or dry. If the by-pass factor is the same for each section the states leaving will be as shown in Fig. 11·8*ab*: Approaching coil 1 is a mixture at m which leaves at state a for the dry coil or state a' for the wet coil. (In the example illustrated, the by-pass factor for each section is taken as 50 per cent so states a and a' are at dry bulb temperatures midway between the surface temperature and the incoming air temperature.) Similarly, the states b and b' leaving the second section are at dry bulbs between the surface and incoming air temperatures. The equivalent by-pass factor for the first two sections is then $F \times F = F^2$. In the same way the condition leaving the third section is equivalent to that of air which had passed through a single coil having a by-pass factor of F^3. By analogy, if there were n such coils in series the state leaving the last coil would be fixed by an equivalent by-pass factor for the system of F^n.

Case I is common in practice and occurs when the cross section of the conditioning unit is fixed by velocity or other requirements, and the only available method of realizing the required cooling capacity is by placing the requisite number of standard surfaces in series. Exact uniformity of surface temperature is not realized in such cases because the progressively decreasing air temperature causes a corresponding reduction in coil surface temperature. Usually, however, this non-uniformity is very small.

Case II. *A Precooling Coil Operating as a Simple Cooling Unit and followed by a Dehumidifying Coil.* This condition arises when some working fluid (as sea water) is available at a temperature below the dry bulb of the entering air, but not sufficiently low to care for the dehumidification requirements. In large installations where a sectionalized refrigeration plant is installed there is also an economic advantage in splitting the total cooling load into two parts, one of which can be handled by refrigerant at higher temperature.

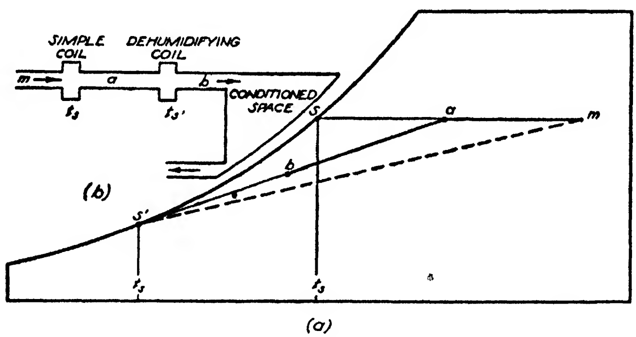

Fig. 11·9. Precooling and Dehumidifying Coils in Series.

The process for a series installation of this kind is shown in Fig. 11·9. Air enters the simple cooling unit at state m and leaves without humidity change at some state as a, depending on the by-pass factor. Entering the dehumidifying coil at a, the air is conditioned along the straight line as' to some leaving state as b. The final state can be readily determined by applying the mixing analogy successively to the first and second sections of the surface (when by-pass factors are known), but it obviously does not lie on the line ms', connecting initial state and second-surface saturated state; this case is not subject to solution in terms of an equivalent by-pass factor for the system as a whole.

Case III. *Dehumidifying Coils in Series, but Operating at Different Surface Temperatures.* As shown in Fig. 11·10 this case is analyzed in exactly the same manner as Case II; the final state does not fall on the line ms''' and cannot be determined from an equivalent system by-pass factor. Possible reasons for using two such coils in series are the same as for Case II.

The special importance of Case III arises from the fact that it suggests a method of analyzing the performance of single cooling units in which the surface temperature progressively and continuously changes as the air stream passes along the surface. Suppose, for example, that cold water is the working substance and the weight rate of water flowing through the coil is so low that a significant temperature rise occurs. Assuming counterflow, the average cross-sectional surface temperature is greatest where the entering air stream first makes contact and progressively decreases as the air travels through the unit. At a given cross section the average temperature is assumed constant with respect to time.

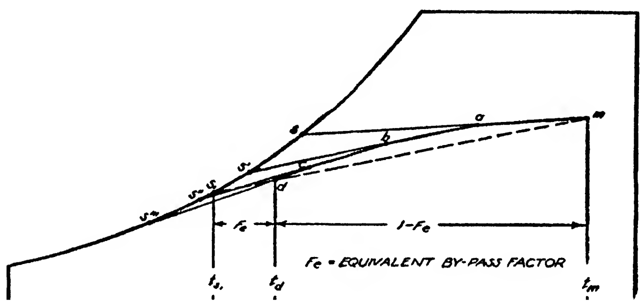

Fig. 11·10. Equivalent By-pass Factor.

The path of the process at any cross section can be determined from the air and surface temperatures at that cross section, but since contact occurs only for an infinitesimal time, the change of state for that section would also be infinitesimal. Refer to Fig. 11·10 and let m be the state of entering air and s the surface temperature at the cross section where the first contact occurs; the process would then be along the line ms. If now, we consider a very short distance along the path of air flow and neglect the small drop of surface temperature which would occur through this length, we have a finite period of contact and therefore a determinable, though large, by-pass factor for this short section of the unit. Knowing the by-pass factor we can fix the state of air leaving this section, as at point a in the figure.

Proceeding in the same way, the process line through the next small length of section for which average surface temperature is s', is along the line as' and the state of air leaving this section is at some such point as b. Similarly, the entire length of the cooling surface can be divided into sections of small but finite length and the end point for each such section determined. The end point of air from the last section is obviously the state leaving the unit; the process for the unit as a whole is then represented by the curve drawn through the end points for each section. In Fig. 11·10 this process is described by the curve *mabcd*; the leaving state is d and a straight line from m through d, extended to intersection with the saturation curve, gives the equivalent uniform surface temperature t_{s_1}; an equivalent overall by-pass factor, $F_e = (t_d - t_{s_1})/(t_m - t_{s_1})$, can then be calculated.

The above analysis can also be conducted without knowledge of the sectional by-pass factor since each end point is calculable from the law of conservation of energy provided only that the entering and leaving temperatures of the cooling fluid and the weight rates of the cooling fluid and the air stream are known. From these data one can calculate the heat absorbed by the cooling fluid and hence the loss of energy, in Btu per pound experienced by the air passing through the unit,

$$\text{Enthalpy change of air stream, Btu/lb} = \frac{M_f c_f (t_{f_l} - t_{f_e})}{M_a} \qquad (11\cdot2)$$

where M_f = weight rate of flow, cooling fluid, lb/min.
M_a = weight rate of flow, air stream, lb/min.
c_f = specific heat of cooling fluid, Btu/(lb)(°F).
t_{f_e} = entering temperature of cooling fluid.
t_{f_l} = leaving temperature of cooling fluid.

The entering state of the air and process line for the finite section are known, so the end point is fixed at intersection of the calculated enthalpy with the process line.

The surface temperature of all cooling units is affected to a greater or less degree by the rate of heat transfer as well as by the temperature of the cooling fluid. Since heat transfer is a function of temperature difference, the surface temperature varies with variation in the temperature of the air stream even though the cooling fluid temperature remains constant (as for an evaporating refrigerant). Consequently, the true process path of air passing through almost any type of cooling coil is a curve of the type shown in Fig. 11·10. Fortunately, the variation in surface temperature is so small for direct expansion coils or for coils using a moderate weight rate of cold water that it can safely be neglected; the analysis for such cases can be carried out in terms of the simpler straight line equivalent by-pass concept.

11·7. Surface Heating with Humidification. From the thermodynamic standpoint, the process lines investigated for cooling and dehumidifying surfaces should be equally applicable for heating and humidifying provided a supply of water were available to keep the heating surface in a wetted condition. In Fig. 11·11, if air at state o were passed over a dry surface at temperature t_s, the process would be simple heating, and the final state would be somewhere on the line os. If this hot surface were wetted, the water film would be at surface temperature and the vapor in the air film would have a saturation pressure corresponding to temperature t_s. There would now be a vapor pressure gradient as well as a temperature gradient toward the passing air so heating and humidification would occur simultaneously. Then, for the same reasons advanced in dis-

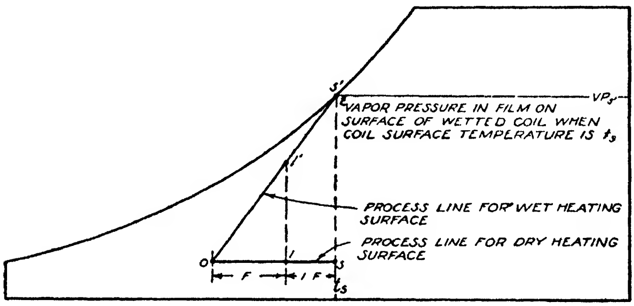

FIG. 11·11 Wet and Dry Heating Surface Processes.

cussing dehumidification, the state of leaving air would be on the straight line os'. Simultaneous heating and humidification by means of wetted surfaces has not been used commercially, but the principle is applicable and may, in the future, be developed; the greatest difficulty is in the design of a surface to which water can be supplied without previous passage through the air stream.

11·8. Direct Humidification. Many types of equipment are available for humidifying an air stream. Essentially, however, the thermodynamic and psychrometric processes followed by such equipments are based upon one of three fundamental paths.

CASE I. When an air stream is brought into contact with a body of water maintained at the entering dry bulb temperature of the air, humidification takes place without change in the dry bulb temperature. Under these conditions humidification occurs because of the mass transfer of vapor from the saturated air film into the passing air stream. The film is saturated at the temperature of the air stream so its vapor pressure exceeds that in the passing unsaturated air. Since there is no temperature difference, heat is not transferred, and all energy needed to vaporize the water used in the humidifying process must be supplied from the main body of water and must therefore be added to this water from an outside source. The psychrometric path followed during a process of this kind would be simple humidification without heating, and would appear as a vertical line on the chart.

A variation of Case I is the ordinary humidifier consisting of a body of water in which a heating coil is used to raise the temperature and supply the latent heat of vaporization. The principal reason for the raised temperature is to increase the vapor pressure gradient and thereby accelerate the mass transfer of vapor to the air stream. Accompanying the raised temperature, however, is a transfer of heat to the air stream with a resultant temperature increase. The process for such a humidifier is exactly the same as that already taken up for humidification with surface heating (line *os′* of Fig. 11·11).

Case II. In many cases humidification is realized by the direct injection of water into the conditioned space. Usually this is accomplished by passing the water through a nozzle or discharging a small, high-velocity stream against a hard surface to break it up into a fine spray or mist. With injection systems the need is for complete and rapid evaporation of all liquid entering the room and since evaporation occurs from the liquid surface, the process is hastened if the transfer area is increased. The purpose of creating a spray is to increase the surface-volume ratio. Though the effectiveness of a given humidifier in achieving complete evaporation of the injected water is largely determined by the surface-volume ratio, the final state (for complete evaporation) is always the same and is independent of the effectiveness of the atomizing equipment. The principal factor influencing the final state of the conditioned air is the temperature of the injected water, but, whatever this temperature may be, the method of analysis is always the same.

Consider an atmosphere at outside state *o*, the specific humidity of which is to be increased, by direct water injection, from W_o to W_m. Per pound of dry air in the conditioned space it is necessary to add $(W_m - W_o)/7000$ lb of water. The state of the air after humidification can be determined by application of the laws of conservation of mass and of energy. This state must be on the horizontal line through W_m and must also be on the line for which the enthalpy is h_o plus $[(W_m - W_o)(t_w - 32)]/7000$ where t_w is the temperature of the injected water. Noting that an increase in absolute humidity of 70 grains/lb represents an unusually high value and a supply water temperature of 100°F is conservatively high, a calculation from the above equation shows that the enthalpy increase likely to occur during humidification by injection will be less than 1 Btu/lb for any injection water temperature up to 100°F. Thus for all such equipments the final state of the conditioned air can be considered (approximately) as falling on a constant enthalpy line through the initial state; if greater accuracy is needed the final enthalpy can be determined by an exact analysis as discussed in the following paragraphs.

From the thermodynamic standpoint the paths followed by humidification processes are largely dependent, in the early stages, on the initial water temperature, but the final section of the process is usually one of adiabatic saturation. Refer to Fig. 11·12 and consider, as a first case, that injection water is supplied at the wet bulb temperature of the entering air, t_s. Each entering water drop is surrounded by a film of vapor at pressure VP_s; a simultaneous transfer occurs of vapor from the drop to the air stream and of sensible heat from the air stream to the drop. The drop remains at the equilibrium wet bulb temperature, and the process of humidification proceeds along a constant sigma line, or, approximately, a constant enthalpy line.

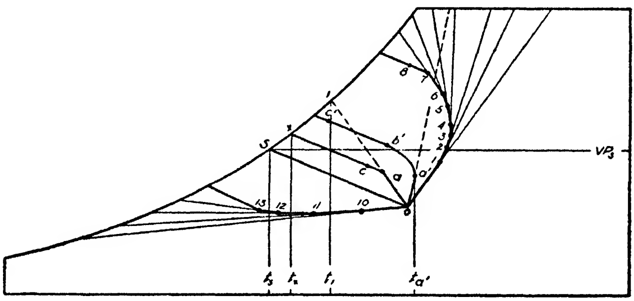

FIG. 11·12. Spray Dehumidification Processes.

As a second case, let injection water be supplied at t_1, any temperature greater than the wet bulb temperature of the air, but less than its dry bulb. When the water first enters the air stream the rate of heat transfer to the drop is less than for equilibrium while the rate of mass transfer from drop to air stream is greater. Then the latent heat needed to sustain evaporation must come in part from the sensible heat of the liquid, and the liquid temperature will drop until it reaches the equilibrium value corresponding to the final wet bulb temperature t_x of the air stream. During this transient period the air stream will be gaining latent heat in excess of its loss of sensible heat so its enthalpy will increase. The complete psychrometric path consists of a brief transient process, *oa*, followed by the equilibrium process of adiabatic partial saturation *ac*.

The extreme example of the above case occurs when water is injected at a temperature greater than the dry bulb of entering air. In that event, the transient process is itself divided into two parts: Initially, heat as well as vapor flows from the droplet to the air (line *oa′*, Fig. 11·12); the cooling rate is very great so the droplet and air are soon reduced to a common temperature, *ta′*. At this point there is no heat transfer either to or from the drop, but vapor is still being transferred to the passing air and its latent heat must be supplied by sensible cooling of the unevaporated water; the drop temperature falls below the dry bulb temperature of the air, and the remaining part of the transient period $a'b'$ is the same as in the previous case. By assuming uniformity of all droplets, the true process curve $oa'b'c'$ for humidification under such conditions can be determined graphically by using the equivalent mixture concept and determining instantaneous states by the method previously established for dehumidifying coils in series. Curve o,1,2,3,4,5,6,7,8 shows a typical graphical solution.

The third case occurs when injection water enters at a temperature lower than the wet bulb in the conditioned space. The analysis is essentially the same as for the preceding case, but with preheating of the droplets during the transient period. If the initial water temperature is above the dew point, heat flows to the droplet at a rate greater than that corresponding to equilibrium while vapor enters the air stream, but at a lower rate than for equilibrium. There is thus a gain in total heat at the droplet (since sensible heat gain exceeds latent heat loss) and a resultant temperature rise until equilibrium conditions have been established. The extreme example of this case occurs when injection water is supplied below the dew point of the room air. Under such conditions there is an initial transfer of both heat and vapor *to* the drop so that the initial period of the process is actually one of dehumidification rather than humidification. During this period the drop temperature rapidly rises as it is gaining both sensible and latent heat. When the dew point temperature is reached, the flow of vapor ceases, but the drop continues to gain sensible heat, its temperature rises, and the vapor flow now starts in the opposite direction; Fig. 11·12, Curve 10,11,12,13, shows a graphical solution of this extreme case.

Case III. The obvious disadvantage of water injection as a method of humidification is that, as has been shown, the process inevitably becomes one approaching adiabatic humidification with a consequent temperature reduction of the conditioned air. This disadvantage can be overcome by injecting steam rather than water into the conditioned space. For many industrial humidification problems steam injection is the simplest and the most economical method (both in first and in operating cost) of raising humidity. For comfort installations, however, direct injection of steam is less desirable because of the possibility of introducing objectionable odors into the conditioned space.

The thermodynamic processes followed by injection steam are shown on the temperature entropy diagram of Fig. 11·13a. Moist steam at state l passes through a nozzle, undergoing a constant-enthalpy pressure reduction to b, where it is saturated, and beyond to a final pressure at c corresponding to the desired vapor pressure in the conditioned space. From state c the highly superheated vapor follows a constant-pressure line, while losing heat to the air, and ends at d, corresponding to the state of vapor in the humidified space. The difference in enthalpy between states c and d is the energy, in Btu per pound of steam added, which appears as a sensible heat gain in the conditioned space. On the psychrometric chart of Fig. 11·13b, point d corresponds to the same point on the temperature

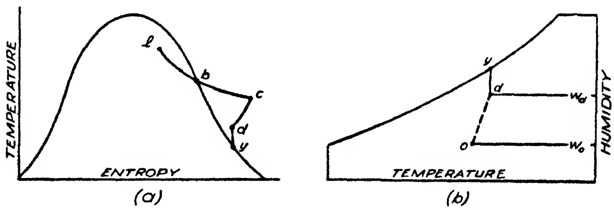

FIG. 11·13. Steam Jet Humidification.

entropy diagram. Thus the energy available for sensible heating is the difference in enthalpy between line steam and saturated steam at state y. The specific humid heat of the atmosphere at o is $c_{pa} + W_o c_{ps} = s$, so the temperature gain due to humidification is

$$(t_d - t_o) = \frac{[(h_l' - h_y')(W_d - W_o)]/7000}{s} \tag{11·3}$$

There are two independent unknowns in this equation (t_d and h_y') so it must be solved by trial and error. The enthalpies h_l' and h_y' are from the steam table, *not* the psychrometric chart. The sensible heating effect resulting from injection of steam is usually not negligible and should, in every case, be investigated by equation 11·3. Humidification by steam injection cannot be shown as a process line on the psychrometric chart because the humidification occurs instantaneously and not along a path; the initial and final states can, however, be shown and the dotted line connecting them considered as an equivalent process.

11·9. Air "Washing" Processes. Many of the psychrometric processes already described, as well as others, can be carried out directly from a controlled liquid transfer surface. The term "air washer," as used here, refers to any equipment which produces a large liquid transfer surface by bringing drops of water into contact with an air stream in the conditioning equipment. Commonly this is done by spraying water into the air stream and at a later cross section installing baffles or eliminators to prevent entrained droplets from being carried into the conditioned space.

For purposes of process analysis, air washers can be classified as simple or complex. Simple washers are non-energy-transferring equipments in which the spray water is recirculated without treatment; complex washers operate in conjunction with heat transfer apparatus designed to add or remove energy from the recirculated spray water. Complex units can be further classified according to the ratio of weight rate of spray water to weight rate of air.

(*a*) Simple Washing Process. The only energy entering the air stream as a result of passage through a simple air washer is the heat of the liquid of that water which is evaporated; the process in such an equipment closely approaches that of adiabatic saturation. The simple washing process is an effective method of humidification, provided the associated dry bulb temperature reduction is not objectionable or can be restored by passing the cooled air stream over a reheating surface. As a cooling process the adiabatic saturation method has the great advantage that it does not require a low-temperature heat receiver (as a refrigerant); simple air washing, where it can be used, is probably the most direct and least costly method of achieving a dry bulb temperature reduction.

The use of adiabatic humidification as a means of sensible cooling is, however, subject to two severely restrictive limitations. Since the maximum possible dry bulb reduction is equal to the depression of wet bulb below dry bulb temperature of the incoming air, it follows that a substantial cooling effect can be realized only when relatively dry air is available at the washer entrance. For problems where the sensible cooling load has to be carried under conditions of high outside humidity, adiabatic humidification therefore cannot be used. A second objection is that the sensible cooling may be attained at the cost of room relative humidities so high as to constitute a possible health hazard.

(*b*) Complex Washing Processes: Water Temperature Substantially Constant. When the weight rate of water supplied to the sprays is very great with respect to the weight rate of air flow, the change in temperature of the water during passage through the conditioner is negligible. For such cases, the function of the washer is identical with that of a wetted solid surface maintained at constant surface temperature, and the analysis is therefore the same as that which has already been presented (section 11·7) for surface heating or cooling accompanied by humidification or dehumidification. The concept of such air washing processes as special cases of the general surface transfer process should assist materially in simplifying the visualization of paths followed and of the limitations inherent in washing equipment.

Figure 11·14 shows the various paths which can be followed by air in passing through a complex washer. The determining factor responsible for a particular path is the energy exchange process which occurs during recirculation of the spray water. Thermodynamically, washing processes differ from solid surface processes only in that the energy transfer occurs continuously over the solid surface, but intermittently in the washer; heat flows to or from storage in the wash water and is later (during conditioning of the recirculated water) discharged or regained in an energy exchange outside of the washer. Since all energy passing to or from the air stream must be temporarily stored in the wash water, a high rate of water flow is necessary if the water temperature is to remain substantially constant throughout the cycle. Note, however, that the requirement of a high water rate is dictated solely by the constant surface temperature consideration; equipment can, and does, operate effectively with low water rates — the process lines for such cases will be considered in the next section.

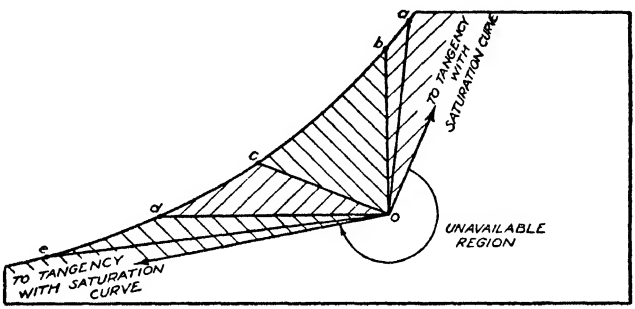

FIG. 11·14. Washer Processes: High Water-air Ratio.

If Fig. 11·14 is referred to, a complex washer of the type under consideration is seen to be capable of accomplishing any of the following processes:

1. Heating with humidification (*oa*). By supplying spray water in adequate quantity at the desired leaving air temperature the process will be a straight line and the end point of the leaving air will be somewhere on this line, the exact position being determined by the equivalent by-pass factor of the washer. The limit of leaving state which can be attained by this method is fixed by the maximum available water temperature. Since the saturation curve rises rapidly, the effectiveness (in increasing attainable supply states) of a given water temperature rise becomes less as the temperature increases. When the initial relative humidity of the air is high, the area in which supply states can be attained becomes very small.

2. Humidification without change in dry bulb temperature (*ob*).

3. Cooling with humidification (*oc*). The region in which this is possible must be divided into two parts. Any state in the area where the final enthalpy exceeds that of the initial air (shaded area *bcob*) requires that energy be *supplied* to the spray water before recirculation, whereas

states in the area of reduced enthalpy (area *cdoc*) require the removal of energy prior to recirculation.

4. Cooling without change in humidity (*od*).

5. Cooling with dehumidification (*oe*). Exactly as with surface dehumidification, this process requires that the working fluid be maintained at a temperature below the dew point of the entering air. The area (*deod*) in which supply states can be attained is limited by the temperature of the wash water and decreases rapidly as the initial state of the air approaches the saturation line.

Summarizing, complex washing processes with high water supply rates can be used to reach *any* point in the " cooling and humidifying " quadrant; some points in the " cooling and dehumidifying " and " heating and humidifying " quadrants; no points in the " heating and dehumidifying " quadrants. Thus the complex air washer duplicates the function of either a humidifier or a simple cooling surface and overlaps the range of the wetted surface for heating and cooling. In Fig. 11·14 the unshaded area on the chart includes all states which cannot be attained by a complex air washer.

(*c*) Complex Washing Processes: Water Temperature Varying. Assuming that the variation in water temperature occurs as a function of longitudinal position in the washer, the process followed by the air in passing through the unit can be determined by the method previously developed for coils in series. The analysis is identical for the two cases, but the conditions are usually strikingly different in that parallel flow *must* occur in the washer while counterflow is *possible* and is commonly used in the coil series. The effect of this difference is to change the curvature of the path from convex, for the coils, to concave for the washer. Figure 11·15 shows the graphical determination of path for a cooling and dehumidifying washer and compares it with the path for a series of coils operating between the same end points. This difference in path means little if the equipment, in either case, is capable of reducing the passing air to saturation at surface temperature, but it does alter the location of intermediate states for systems operating with equivalent by-pass factors.

Figure 11·15 also shows the supply area attainable with complex washers of this type. The straight dashed lines indicate the maximum and minimum water temperatures, t_w and $t_{w'}$, needed to realize the area limits shown by the solid lines; states *a, b, c,* and *d* correspond to the same points in Fig. 11·14. Note that the adiabatic saturation line is the limit approached from either direction; this is the one process line for a complex washer that is independent of the weight rate of water flow; the only effect of water rate on the adiabatic process is to alter the equivalent by-pass factor and thus the end point of the air. All other lines increase in curvature as the weight rate decreases and approach the straight condition as the rate increases.

The graphical determination of a path is time consuming and subject to many inaccuracies resulting from the required assumptions that temperature is constant with respect to time and is uniform over a given cross section. For these and other reasons, the graphical analysis can serve only to illustrate the type of path which might be expected from equip-

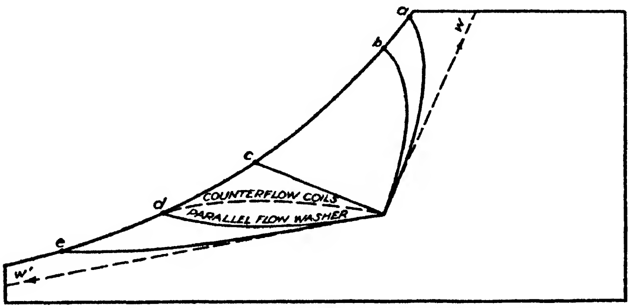

FIG 11·15: Washer Processes: Low Water-air Ratio.

ment of this kind. Fortunately, most commercial equipments of the complex washer type bring the air stream very close to a saturated state at the leaving temperature of the spray water; in consequence, the exact path is not of practical importance as the final state of air, or the temperature drop of the water, can be determined from a simple energy balance on the washer.

11·10. Dehumidification. The processes so far discussed have not included any means of dehumidifying in conjunction with heating. Of the two principal methods in commercial use, one, adsorption, is based on the mechanics of capillary attraction and the other, absorption, on the chemistry of solutions; in each case the function of the dehydrating device is to reduce the vapor pressure at the dehumidifying surface and thereby establish a pressure gradient from the main body of the air stream to the surface at which condensation is to occur. The two methods differ principally with respect to the nature of the forces responsible for bringing about vapor removal at the active surface.

(*a*) ADSORPTION. Capillary attraction is responsible for drawing vapor condensed at the surface of the adsorbent into capillary openings, thereby reducing the vapor pressure at the surface, causing a pressure gradient and hence a mass transfer from the passing air stream to the working surface. As the capillaries fill with water, the attraction decreases and the rate of dehumidification falls off. For this reason the problem of maintaining a fixed state of air leaving an adsorber becomes difficult and by-pass control, to assure a fixed condition irrespective of the capillary condition (in the working range), may be necessary.

Thermodynamically, an adsorption process resembles reverse-acting adiabatic humidification. As air passes over the transfer surface, vapor flows through the air film to the surface, condenses and releases its latent heat of vaporization which raises the adsorbent temperatures above the dry bulb of the air stream and thereby induces a flow of sensible heat back to the passing air stream. When equilibrium is reached the sensible heat given up to the air per unit time must equal the energy released at the adsorbent surface per unit time. If the only energy release were that due to condensation, the sensible and latent heat transfers in opposite directions would be equal, and the process line would be an adiabatic. Actually, however, an exact balance between sensible heat and latent heat is not realized because of an accompanying "heat of adsorption." With the adsorbents used commercially, silica gel and activated alumina for example, the heat evolved during adsorption may be significantly large. The exact value varies with the condition of the adsorbent, but for silica gel is of the order of magnitude of 200 Btu/lb of vapor adsorbed. At equilibrium, therefore, the sensible heat gain of the air exceeds the loss of latent heat and the temperature of the air at exit from the adsorber is greater than it would be if adiabatically dehumidified.

The adsorption process, like that for the simple washer, reaches a condition of heat transfer equilibrium; unlike the washer process, the vapor transfer in adsorption does not reach equilibrium conditions as there is no way of continuously extracting from the system the moisture which collects in the adsorbent. In consequence, some method is needed to extract, at intervals, the moisture from the adsorbent and hence regenerate it. Certain commercial installations do this by using two dehumidifiers, one of which operates while the other, by application of heat, is being reconditioned or "dehumidified"; other systems use one dehumidifier arranged to rotate slowly so that part of the adsorbent is subject to regeneration at the same time that the remainder is in active service.

(*b*) Absorption. Some salts, when dissolved in water, have the capacity of reducing the vapor pressure over the resultant solution below the saturation pressure of pure water for the same temperature. The usefulness of this property arises from the fact that such solutions can be used as dehumidifying sprays in air washers without first reducing the spray temperature below the dew point of the passing air. The critical temperature for such a spray, that is, the temperature at which the vapor pressure in the air film around the drop is equal to the vapor pressure in the main body of air, depends on the spray temperature, the concentration of the absorbent and the dew point temperature of the air. If a highly concentrated absorbent spray were used at a high weight rate and at the critical temperature, with incoming air of high relative humidity, it would conceivably be possible to heat the air without change in humidity; this is the only possible means of attaining simple heating from an air washer. Similarly, highly concentrated absorbent spray flowing at high weight rate through air of high relative humidity and with air dry bulb equal to the spray temperature could conceivably accomplish either simple humidification or simple dehumidification.

As dehumidification occurs, the concentration of the absorbent decreases with a constant rise in vapor pressure and decrease in effectiveness. Some method is needed for maintaining the concentration. This is sometimes done by regenerating the fluid by means of evaporation, or, less economically, by overflowing solution at the same rate at which water is extracted from the air and then restoring the concentration of remaining fluid by addition of the necessary quantity of salt. Salts commonly used in absorbent solutions are lithium chloride and calcium chloride.

The psychrometric process for an absorption system is similar to that for adsorption, the principal difference being that the extra increase in dry bulb temperature of the air is due in one case to the heat of wetting and in the other to the heat of solution.

PROBLEMS

1. Air at 120° dbt and 60 grains/lb enters a simple cooling coil having a surface temperature of 75°F; the leaving air is at 100° dbt. Assuming that the by-pass factor of the coil does not change, determine the coil surface temperature needed to cool air from 110° dbt and 135 grains/lb to a leaving temperature of 100°F.

2. A system operates as in Fig. 11·2 with room state at 70° dbt, 50 per cent RH, outside state at 90° dbt, 55° wbt and supply state at 60° dbt, 35° dpt. What percentage of recirculated air is used in the supply stream?

3. A dehumidifying coil with a by-pass factor of 10 per cent receives outside air at 100° dbt, 40 per cent RH, and operates with a surface temperature of 40°F. If 500 lb/min of supply air are required with a dbt of 70°F, calculate the weight of outside air which must be by-passed around the cooling unit.

4. Six cooling coils operate in series; all coils have the same constant and uniform surface temperature (40°F), and the same equivalent by-pass factor (15 per cent). If air enters the first coil at 100° dbt, 40 per cent RH, determine the state leaving: (*a*) the second coil, (*b*) the fifth, (*c*) the sixth.

5. In problem 4 consider that 30 per cent by weight of the air from the second coil is by-passed around the third, and an additional 20 per cent by weight of air from second coil is by-passed around both the third and fourth coils. Determine the state leaving the sixth coil and calculate the equivalent by-pass factor for this system and compare it with the equivalent by-pass factor for the system of problem 4.

6. Three coils, each having a 12 per cent by-pass factor, are in series and receive outside air, counterflow, at 100° dbt, 40 per cent RH. The first coil operates with a surface temperature of 70°, the second with 60°, the third with 50°F. Calculate the state of air leaving the third coil.

7. In problem 6 if the last coil operated with a 20°F surface temperature estimate

the approximate state of the leaving air-vapor mixture and discuss, qualitatively, the kind or kinds of entrained material.

8. Air at 100° dbt, 40 per cent RH passes, in parallel flow, over a series of five water-cooled plates. Water enters the first plate at 50°F and leaves at 54°F; enters the second at 54°F and leaves at 57°F; enters the third at 57°, leaves at 59°; enters the fourth at 59°, leaves at 60°; enters the fifth at 60°, leaves at 60.5°F. If the flow rate is 1.5 lb air/lb water and if each plate is assumed to operate at a uniform surface temperature equal to the average temperature of the water passing through it, calculate the state of air leaving each plate and plot the approximate cooling process on the psychrometric chart.

9. Water at 90°F is injected into an air stream (at 80° dbt, 55° wbt) in an amount just sufficient to raise the absolute humidity by 20 grains/lb. (*a*) Calculate the exact state of the humidified air and compare with the approximate state determined from the psychrometric chart for the assumption of a constant-enthalpy humidification process. (*b*) From the exact state calculated in (*a*), sketch a curve on the chart showing, qualitatively, the humidification process.

10. Saturated steam at 10 psi is injected into a passing air stream in amount sufficient to raise the absolute humidity from 40 to 100 grains/lb. If the air enters the humidifier at 70 dbt, determine its leaving state.

11. Repeat problem 6 for parallel flow.

12. Repeat problem 8 for counterflow.

13. An air washer (assume parallel flow) receives air at 90° dbt, 65° wbt, and water at 95°F. Assume zero by-pass factor and determine the ratio of water to air flow rates necessary to condition the air to a saturated state at 75°F.

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CHAPTER XII

PROCESS COMBINATIONS

Most equipments used singly in air conditioning systems are capable of supplying air at a given state, or, in a few cases, along a particular process line, but not at *any* state in an arbitrarily assigned area. Yet the latter condition is the one which must be met if the air conditioning system is to function satisfactorily for all load conditions. In this chapter consideration will be given to some of the various equipment combinations which permit realization of the flexibility requisite to meeting all required states.

12·1. Heating and Humidifying Processes. As an example of the method of combining equipments to obtain a greater range of possible supply states, consider that outside air is at state *o* and is to be conditioned by some system made up of heating coils, air washers, and damper-equipped by-passes; the arrangement of equipment is to be left to the designer's judgment. The following systems illustrate arrangements of increasing effectiveness (weight rate of supply assumed constant in all cases).

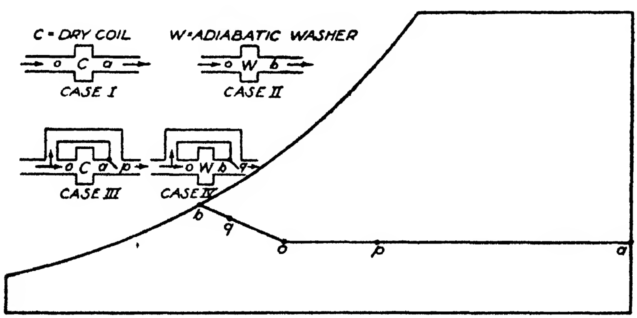

Fig. 12·1. Tempering Coil and Adiabatic Washer.

Case I. *Equipment:* Dry heating coil operating at a fixed temperature.

Supply: At state *a* (Fig. 12·1).

Comment: This arrangement represents a minimum of flexibility. There is no control over the humidity, and the leaving air temperature would obviously vary with changing outside conditions.

CASE II. *Equipment:* Adiabatic saturating air washer.

Supply: At state *b*.

Comment: Like Case I, this arrangement is inflexible since both supply temperature *and humidity* are dependent on the outside conditions.

CASE III. *Equipment:* Dry heating coil operated at variable temperature or fixed temperature coil equipped with by-pass. (Henceforth such units to be called *controlled coils.*)

Supply: Any state along line *oa*.

Comment: The controlled coil permits realizing any supply temperature between t_a and the outside dry bulb temperature, but does not provide humidity control.

CASE IV. *Equipment:* Adiabatic saturating air washer with by-pass, or washer with variable by-pass factor (*controlled air washer*).

Supply: Any state along line *ob*.

Comment: This system permits realizing any absolute humidity between the values corresponding to initial dew point and wet bulb temperatures respectively, but does not provide control of supply temperature.

CASE V. *Equipment:* Controlled adiabatic air washer followed by controlled reheating coil.

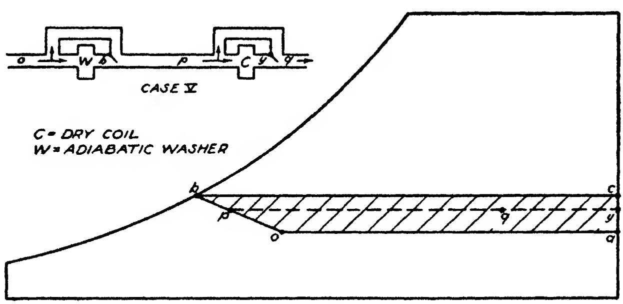

FIG. 12·2. Adiabatic Washer and Reheat Coil.

Supply: Any state in the area *obcao* (Fig. 12·2).

Comment: A damper in the by-pass around the washer would be controlled to give the desired absolute humidity; the reheating coil controlled to give desired supply temperature. This system provides much better flexibility than any so far considered, but is severely limited as to range of obtainable humidities.

CASE VI. *Equipment:* Same as in Case V, but with the coil preceding (preheating or " tempering " coil) the washer.

Supply: Any state in area *obdao* (Fig. 12·3).

Comment: Control would be exactly as for Case V, the only difference between these systems being that use of the heating surface ahead of the washer permits variation of the wet bulb temperature of air entering the washer and hence greatly extends the range of humidity variation.

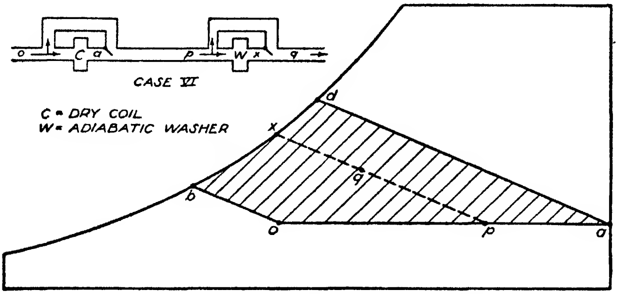

FIG. 12·3. Dry Coil Preceding Adiabatic Washer.

Aside from the greater supply area which can be reached by installing the heating coil ahead of the washer, this arrangement has the additional advantage — important in regions where winters are severe — of providing tempering of the outside air before admission to the washer. Such preheating is necessary when outside air temperatures are substantially below freezing as otherwise there would be, under conditions of low humidifying load, a hazard of freezing in the washer piping and sprays.

CASE VII. *Equipment:* Controlled preheating coil followed by controlled adiabatic air washer followed by controlled reheating coil.

Supply: Any state in area *obdeao* (Fig. 12·4).

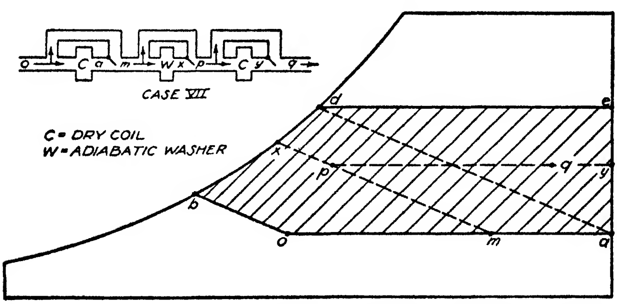

FIG. 12·4. Tempering Coil, Adiabatic Washer, and Reheat Coil.

Comment: Use of the reheating coil increases the area in which supply states can be reached by adding area *adea* to that obtainable from preheat coil and washer alone. The use of a reheating coil for this purpose requires provision of extra surface rather than a mere division of the surface previously required for the single preheating coil.

This case represents the arrangement having greatest possible flexibility of supply when a simple air washer is the only equipment available for changing humidity. Series operation of a second air washer following the reheat coil would permit attainment of state points above the line *de*, but the first cost and the complexity of such an arrangement are usually prohibitive. Where supply humidities greater than that corresponding to the maximum wet bulb temperature of air leaving the preheating coils are required, it is necessary to use either direct humidifiers or complex air washers with heated spray water.

CASE VIII. *Equipment:* Controlled preheating coil followed by air washer with heated spray water, followed by controlled reheating coil. The washer is supplied with spray water at temperature $t_{f'}$ and saturates the air at the leaving water temperature t_f.

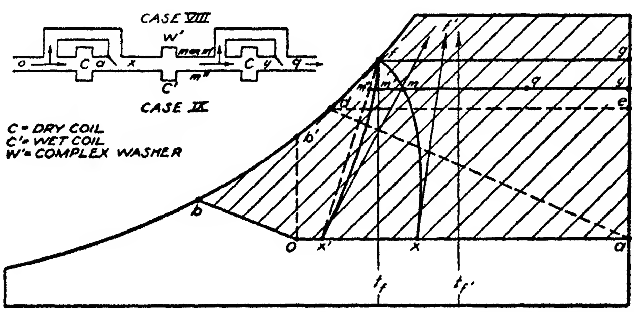

FIG. 12·5. Tempering Coil, Complex Washer (or Wet Surface), and Reheat Coil.

Supply: Any point in the area *obfgao* (Fig. 12·5).

Comment: The increased area of possible supply is *dfge*. Unlike the preceding cases, there is not a definite and fixed value of the state of air leaving the preheater and corresponding to a given supply state; the state *q* could be reached by preheating to *x* and following path *xf* through the washer, or, equally well, by preheating to *x'* and following path *x'f* through the washer (the spray water rate being the same for the two cases). The selection of point *x* is determined purely on the basis of load distribution and bears no functional relationship with the supply state. If it were desired to use a saturating complex washer with a bypass, the resultant state of the mixture would be on the straight line *x'm''f* rather than on the curved washing path *x'm'f*.

CASE IX. *Equipment:* Controlled heating coil followed by surface humidifier supplied with water at dry bulb temperature of supply state.

Supply: Any point in area *ob'fgao* (Fig. 12·5).

Comment: Theoretically, this arrangement can duplicate any state possible with Case VIII except those which fall in area *obb'o*, but actually there is little possibility that a practical system could realize states in the area above line *de* without requiring a prohibitively large area of heated water surface. The only feasible method of achieving adequate evaporating surface is by using a fine spray which, of course, means a return to use of a complex air washer.

CASE X. *Equipment:* Controlled heating coil followed by direct steam injection humidifier.

Supply: Any state having a dry bulb temperature and humidity greater than that of the air at point *o* (Fig. 12·6).

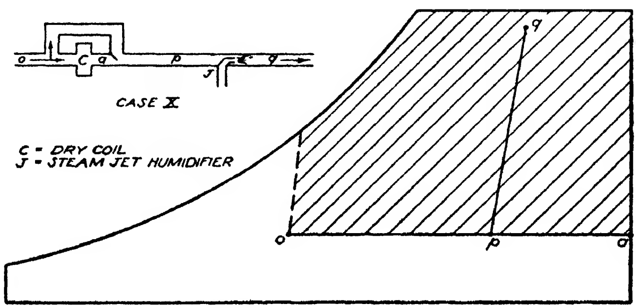

FIG. 12·6. Tempering Coil and Steam Jet Humidifier.

Comment: When the steam supplied to the humidifying nozzle is saturated at pressures less than 5 lb gage, the rise in dry bulb temperature of the air during process of humidification is rarely more than 10°F. The exact value of the temperature rise depends on state of the steam and on the humidity increase which is desired; it therefore varies with change in outside conditions. The most simple control arrangement for a system of this kind is to have the heating coil operated from a room thermostat and the steam nozzle from a room humidistat.

12·2. Cooling and Dehumidifying Processes. In most climates a heating load is inevitably accompanied by a humidifying load. In the cooling season no comparable rule is applicable; sometimes cooling and simultaneous humidification may be needed though in many installations cooling with dehumidification is required. In all cases the object of the designer is to realize the desired condition with the least expenditure for energy and for equipment.

CASE XI. *Equipment:* Controlled adiabatic air washer in series with by-passed cooling coil.

Supply: At any state in the shaded area of Fig. 12·7.

Comment: For cooling service the supply area realized from series operation of these two pieces of equipment is the same regardless of which equipment comes first. If the system is to be used for year-round operation, however, the coil should precede the washer since, as was shown in Figs. 12·2 and 12·3, this greatly increases the area of possible supply states for winter operation. Aside from the supply area problem, operating effectiveness dictates location of the coil ahead of the washer for summer operations because this permits extracting heat from the air at a higher temperature. By referring to Fig. 12·7 and noting that the two possible paths to be followed in going from o to s are $oa's$ or $ob's$, it is evident that

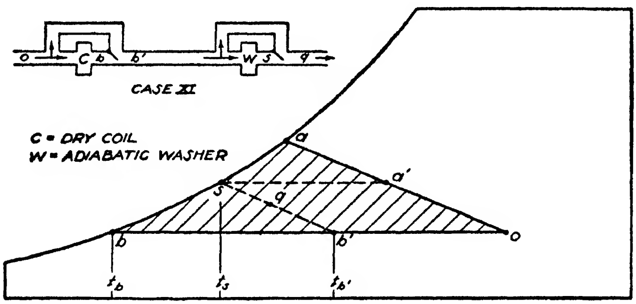

Fig. 12·7. Dry Cooling Coil and Adiabatic Washer.

the refrigerant in the cooling coils must be introduced at a temperature below t_s if cooling occurs from a' to s, while the minimum refrigerant temperature can be somewhat higher ($t_{b'}$ if zero by-pass factor) when path $ob's$ is followed. The difference between these two temperatures may be great enough, in some instances, to permit use of available cooling water without refrigeration in going from o to b' while necessitating mechanical or other artificial refrigeration to provide a cooling fluid at sufficiently low temperature to extract heat along the line $a's$. Conversely, for a by-passed coil discharging saturated air the refrigerant temperature would have to be lower for path $ob's$ than for $oa's$.

Case XII. *Equipment:* Controlled complex saturating air washer.

Supply: Any state in area oab (Fig. 12·8).

Comment: The limiting minimum temperature, t_b, is determined both by the capacity of the spray cooling equipment, and by the position of point o with respect to the saturation line. The lowest permissible value is fixed by a straight line through o at its point of tangency with the saturation line; this condition is shown in Fig. 12·8. The upper temperature limit a is at the wet bulb temperature of the entering air; the use of heated spray water would permit raising state a, but for most cooling problems the complex washer operating with either cooled or unheated recirculating water will suffice and heated spray water is unnecessary.

CASE XIII. *Equipment:* Controlled dehumidifier in series with controlled simple cooling coil.

Supply: *A*. When cooling coil precedes dehumidifier: Supply at any state in area *oabco* (Fig. 12·9). As drawn on this figure the lower limit

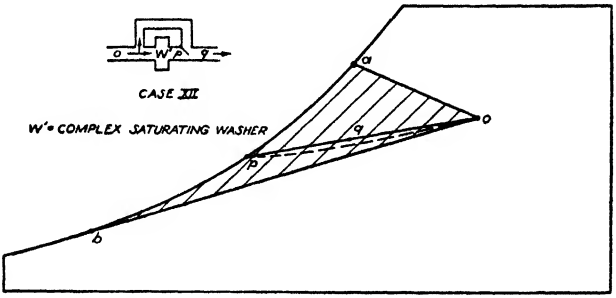

FIG. 12·8. Cooling by Complex Saturating Air Washer.

bc is horizontal, thus representing a fixed minimum value of the absolute humidity that could be realized at exit from the dehumidifier. Actually the minimum humidity might vary with the dry bulb temperature (and the relative humidity) of the entering air and therefore be greater or less at point *c* than at point *b*; this possibility is indicated by the alternative areas (*oab′co* and *oab″co*). Further, the exact slope of the dehumidifying

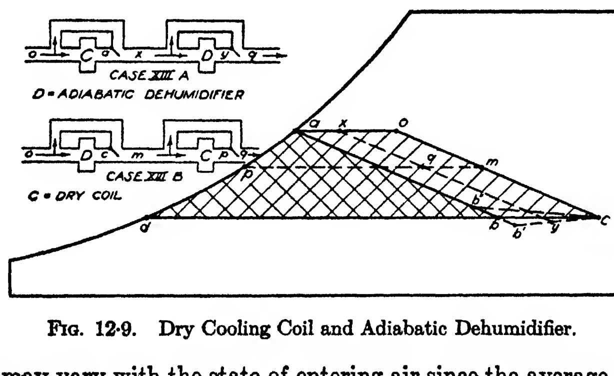

FIG. 12·9. Dry Cooling Coil and Adiabatic Dehumidifier.

process may vary with the state of entering air since the average dry bulb temperature in the dehumidifier will affect the rate of heat loss and therefore influence the departure of the process from a true adiabatic; thus the slope of lines *oc*, *xy*, and *ab* may not be the same.

B. When cooling coil follows the dehumidifier: Supply at any state in the area *oadco*. Note that the effect of changing the location of the coil has been to increase the possible supply area by the amount *abd*.

Though the possible supply area has been extended by this change, it should be clear that the extension could not be realized in practice unless the coil had more capacity than the one used in the previous arrangement; the enlarged supply area is a *possible* extension, but its realization depends on adequacy of transfer surface.

Comment: The above conditions emphasize the difference in arrangement sequence for adiabatic washers and dehumidifiers: For operation of a coil and air washer, the coil should always precede the washer; for operation of a coil and dehumidifier, the coil usually should be installed last. Another important difference is that there is no psychrometric advantage to be attained for the dehumidifier series from splitting the coil surface into precooling and recooling sections; the entire possible area is realized with all coil surface after the dehumidifier.

Case XIV. *Equipment:* Controlled dehumidifier followed by controlled dehumidifying recooling coil (Fig. 12·10).

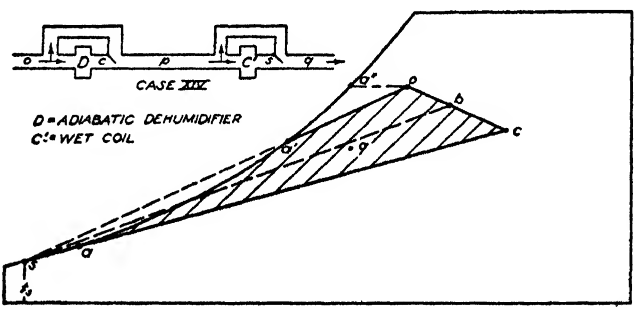

Fig. 12·10. Adiabatic Dehumidifier and Wet Cooling Coil.

Supply: For a cooling coil operating at fixed temperature t_s, supply can be from any state in the area *oa′aco,* as well as on the line *as.*

Comment: Since coil surface temperature t_s is less than dew point of entering air, states in the region *oa″a′o* cannot be attained.

12·3. Special Cycles.

Case XV. *The Over-dehydration Cycle.*

Equipment: Dehumidifier, followed by simple controlled cooling coil followed by simple, controlled saturating air washer.

Supply: Any state in area *oabcdo* (Fig. 12·11).

Comment: Psychrometrically this cycle has no evident advantages over those which have already been discussed, but it is of particular interest from the standpoint of the thermodynamics of energy transfer from a working substance. Consider outside air at state *o* to be conditioned to a supply state point *s.* The most direct method of conditioning would be to pass the air over a simple cooling coil reducing its temperature at constant absolute humidity. However, to do this there must be a cooling fluid available at a temperature less than t_s, hence well below the outside temperature. In many cases no natural supply of low-temperature working substance (as well water) is available and the simple cooling process *os* would therefore require some kind of refrigerating equipment to permit collection of heat at the low temperature and disposal of it at some temperature greater than t_o.

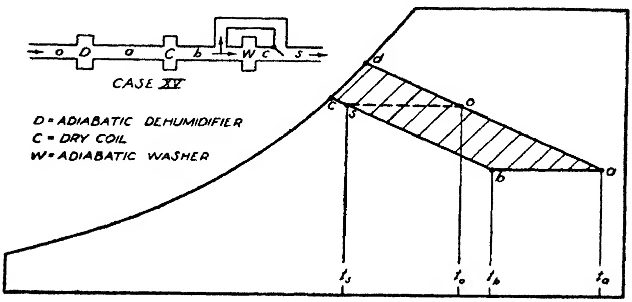

FIG. 12·11 The Over-dehydration Cycle.

In the cycle of Fig. 12·11, the dehumidification process *oa* is not strictly adiabatic, but can be so considered. The washer process *bs* is adiabatic. The only process in the cycle in which there is an appreciable energy transfer is therefore the cooling line *ab* and the heat which is to be extracted along *ab* is effectively equal to that which would have been removed along the simple cooling line *os*. The effect of the dehumidifier and washer is then merely to displace the heat removal process into a temperature range in which cooling can be more economically carried out. Temperatures t_a and t_b are both sufficiently far above the outside dry bulb so that cooling can be accomplished by means of outside air.

This cycle is thermodynamically similar to a heat pump since heat is picked up in the room (as in an evaporator) along the line *so*, raised in temperature level (as in a compressor) along the line *oa*, discarded (as from a condenser) along *ab*, and the temperature of the working substance reduced by a constant-enthalpy process (as in an expansion valve) along the line *bs*. The total load is practically unchanged, but the need for refrigeration has been obviated.

A practical disadvantage of the over-dehydration cycle is the need for supplying a relatively large amount of heat to the dehumidifier during the regenerative process. Except in cases where heat is available from exhaust steam or some other low-cost source, the cost of supplying regenerative energy to the dehumidifier is likely to be prohibitive.

CASE XVI. *The Run-around Cycle.*

Equipment: Reheating coil in series with and following a precooling coil with a simple saturating air washer located between them.

Supply: Any state in the area $ob'a's$ (Fig. 12·12).

Comment: When a saturating air washer is used without a by-pass, the only available method of controlling the final absolute humidity is through control of the wet bulb temperature of air entering the washer. Considering that outside air at state o is to be conditioned to a supply state point s, which lies on the constant-enthalpy line through o, the simple air washer could not alone provide the desired state since there would be no way of controlling the humidity at discharge. If a by-pass were available it would, of course, be possible to blend air from the washer with a by-passed fraction to get any desired state point on the line oa. Without a by-pass some other method of controlling the humidity at discharge is essential; the method most commonly used is to precool the air

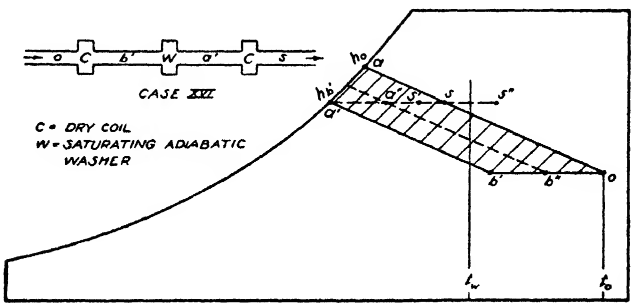

FIG. 12·12. The Run-around Cycle.

going to the washer until its wet bulb temperature is numerically equal to the dew point temperature desired at the supply state. The air is then saturated in the simple washer and reheated along line $a's$ to the required dry bulb temperature. In the event that the air washer had an equivalent by-pass factor, precooling would be stopped at a point as b'' and reheating carried out along $a''s$.

The precooling and reheating processes are obviously responsible for poor economy of operation since there is no theoretical need for any energy transfer into or out of the system, yet a quantity of heat equal to $(h_o - h_{b'})$ is extracted from the system at the expense of cooling water or refrigerant whereas an equal heat quantity is later returned to the system from steam or other heated working substance.

The run-around cycle is a mechanical arrangement for using the same working fluid alternately to cool and heat the air stream without addition or removal of energy from the system. Consider that a relatively large supply of water is available at temperature t_w and that the temperature change of this water due to reception or rejection of energy from the air stream is negligible. Then the water could first be circulated through the precooling coil reducing the air temperature from t_o to $t_{b'}$ while the water underwent a very small temperature rise; it could then be passed through the reheating coil returning to the air stream the quantity of heat extracted in the precooler and simultaneously cooling the water to its original temperature t_w.

For the case illustrated the energy requirements for precooling and reheating happened to be the same. Usually, however, the supply state is either to the left, s', or to the right, s'', of the adiabatic line. In either case the coil loads are not equal, and it then becomes necessary to provide an

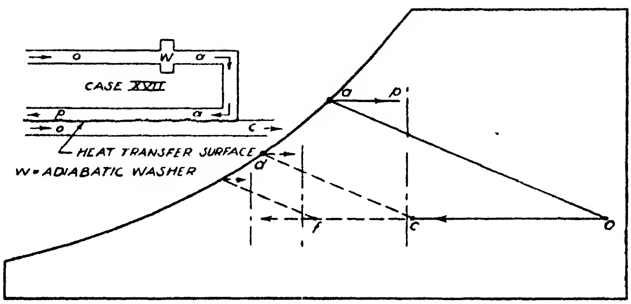

FIG. 12·13. Indirect Evaporative Cooling.

extra heating or cooling coil through which energy is added to or carried from the system. The complexity of the run-around system frequently increases the first cost to a point such that the operating saving does not justify the investment.

CASE XVII. *Indirect Evaporative Cooling.*

Equipment: Simple air washer and plate type air cooler.

Supply: Any state along line oc (Fig. 12·13).

Comment: In many installations the supply dry bulb temperature is greater than the outside wet bulb temperature, but direct evaporative cooling cannot be used because of the attendant rise in absolute humidity. When this is the case cooling can sometimes be effected by an indirect evaporative system. Outside air enters the washer at state o and leaves saturated at temperature t_a. This air is then carried through a plate type heat exchanger where it receives heat from a counterflow of outside air which is sensibly cooled along the line oc; air for supply to the room, at c, is then available at reduced dry bulb temperature, but without having undergone the absolute humidity increase that is associated with passage through an air washer. Examination of Fig. 12·13 shows that a second cycle could be carried out in series with the first: air entering the second washer at state c, leaving at d, passing through an exchanger which cools a second fraction of outside air from t_o to t_f. Similarly, other cycles could be added indefinitely; the limiting state of the conditioned supply air is at a dry bulb temperature corresponding to the initial dew point of the outside air. Thus, indirectly, evaporative cooling can be used to reduce air to its dew point rather than its wet bulb temperature. In practice, the cost of equipment to reduce the dry bulb below the initial wet bulb is almost always prohibitive, but an understanding of the method does assist in visualizing the operating limits of evaporative systems.

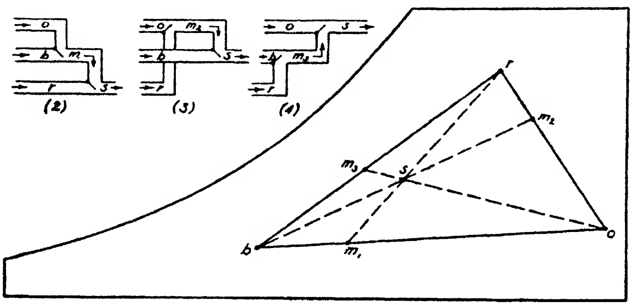

FIG. 12·14. Three-way Mixing.

12·4. Three-way Mixing. Many combinations of equipment are possible with which control can most advantageously be attained through mixing of conditioned air, recirculated air, and by-passed air. Psychrometrically the state resulting from three-way mixing can be determined by any of four methods (Fig. 12·14):

1. By calculation. If the weight rates of air at states o, b, and r are M_o, M_b, and M_r, then the specific humidity at the supply state, s, must be equal to $(M_oW_o + M_bW_b + M_rW_r)/(M_o + M_b + M_r)$. Similarly, the enthalpy at s can be calculated and, knowing two properties, the state fixed.

2. From the chart by visualizing that mixing occurs first along ob to fix point m_1, then along m_1r to fix point s.

3. From the chart by visualizing that mixing first occurs along or to fix m_2, then along m_2b to fix point s.

4. From the chart by visualizing that mixing first occurs along br to fix m_3, then along m_3o to fix point s.

12·5. Effect of Recirculation: Winter. In all the arrangements which have been considered, the state of air going to the conditioner has been taken at point o. In every case it has been considered that each equipment in the system would handle the same weight rate of air (that is, the sum of air weights passing through an equipment and its by-pass would be a constant of the system). Where recirculation is permitted this assumption need not be true. Refer to Fig. 12·15*a* and consider that the minimum outside air requirement is such that if direct mixing of

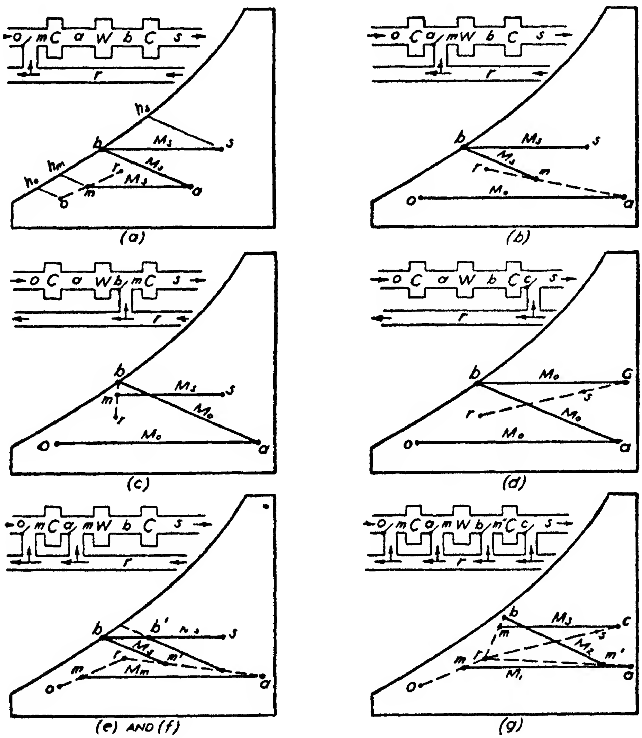

Fig. 12·15. Recirculation for Heating Season.

outside and recirculated air occurred, the state of the mixture would be at point *m*. Then for any arrangement of equipment, and regardless of the place or places in the cycle where the actual mixing of recirculated air occurred, the total heat required to be added by the system is equal to $(h_s - h_m)$ Btu per pound of supply air; the method of bringing recirculated air back into the supply system does not in any way affect the total load.

Although immaterial as to load, the method of introducing recirculated air may have a very important effect on the first cost of the system, its flexibility, and ease of control. The designer must therefore consider the various possibilities in terms of load distribution between individual equipments and general operating effectiveness. Figure 12·15 illustrates seven possible methods of introducing room air into the supply duct of a heating and ventilating system made up of a preheating coil, simple saturating air washer, and a reheating coil in series. The states of outside, supply, and room air are the same in all figures and are designated as *o, s,* and *r* respectively.

a. Referring to Fig. 12·15*a,* the introduction of recirculated air ahead of the tempering coil reduces the total heat requirement from that for all outside air by $(h_m - h_o)/(h_s - h_o)$. In addition to the operating saving there is also a saving in the first cost of needed equipment since the load on the preheat coil has been substantially reduced, and the mass transfer requirement at the air washer is likewise modified. For operation under the arrangement of this figure the load and operating conditions at the reheating coil are exactly the same as would have been true if all outside air were used. Though the preheating coil can be smaller for this type of recirculating operation, the size does not decrease as rapidly as the load since the average air temperature is greater and the mean temperature difference between heating fluid and air is therefore less; with lower temperature difference, a greater area of coil surface is needed for each Btu per hour transferred.

b. When the recirculated air is introduced after the preheating coil (Fig. 12·15*b*) the system is split into two separate parts and care must be exercised when taking data from the psychrometric chart to be certain that all data are for a section of the system in which the weight is constant. Thus, the weight of air handled by the preheating coil is equal to the total fresh air supply M_o, and the heating load on this coil is therefore $M_o(h_a - h_o)$. The weight of air passing through the air washer and the reheating coil is constant and equal to the total supply quantity M_s, so the heat load on the reheating coil is $M_s(h_s - h_b)$. Note, in particular, that the difference in enthalpy between the outside state and the supply state, $h_s - h_o$, has no significance with respect to the total heating load of the system; this enthalpy difference does represent the heat added to each pound of *outside air,* but it is not indicative of total load. The size of the reheating coil and the load which it carries are the same as if all outside air had been used.

c. Another method of introducing the recirculated air is shown in Fig. 12·15*c.* In this case outside air is taken through the preheating coil and the air washer before mixing with return air. The mixture then passes through the reheating coil and is admitted to the conditioned space. The effect of this arrangement is to transfer some of the load from the reheating coil to the other equipments, thus permitting use of a smaller coil; this is particularly advantageous in installations where zoning is used and individual reheat or "booster" coils are located at each zone entrance.

The size of preheating coil will increase more rapidly than the load because of the greater average air temperature and consequent reduction in the mean temperature difference across the coil surface. In this case the weight M_o is constant through preheating coil and washer, but increases to M_s at entrance to the reheating coil.

d. When the introduction of return air is delayed until after the reheating coil, Fig. 12·15*d*, system performance is again based on constant weight rate of air flow and analysis is therefore the same as for a nonrecirculating system. This arrangement is sometimes used in order to allow the conditioning equipment to operate with a minimum amount of air; the recirculated quantity is then added to reduce the temperature difference between supply and room air. The state of air leaving the reheating coil must fall on a straight line through points r and s. Note that the heat added per pound of conditioned air $(h_c - h_o)$ is greater than would be needed if all outside air were used, but the total load on the conditioning equipment, $M_o(h_c - h_o)$, is less than the total load for a nonrecirculating system.

e. Return air is sometimes introduced in more than one place. In Fig. 12·15*e*, some recirculated air mixes with the outside air prior to going over the preheating coil, and the remaining room air is returned just ahead of the air washer. The psychrometric analysis of this system would necessitate considering process ma, with weight M_m, separately from processes $m'b$ and bs, for each of which the weight is M_s.

f. When the air washer is not 100 per cent effective the wet bulb temperature of air to washer must be increased so that the humidity of unsaturated air from the washer will have the desired value. Figure 12·15*f* is for a system similar to that of Fig. 12·15*e*, but with an equivalent washer by-pass such that the air leaves at b' and is reheated along the shorter path $b's$. The effect of the non-saturating washer is to shift part of the heating load from the reheating to the preheating coils. This fact is of paramount importance since it shows clearly that if both coils were accurately and closely sized to operate with a saturating washer and if the actual washer were then found to be non-saturating, the system would be incapable of carrying the load even though the combined rating of the two coils remained adequate. In this case the excess capacity then available at the reheat coil would in no way assist.

The above difficulty brings out the fact that split heating surface has two entirely independent functions: (1) To provide for the sensible heat load: this requirement will always be met if the total surface is of adequate size; it is in no way affected by the way in which the split occurs or by inadequacy or overcapacity of intervening adiabatic equipment. (2) To provide an entering state at the washer such that the requisite humidity increase will be realized at the discharge side and to condition the leaving air by reheating to the desired final dry bulb temperature. This requirement necessitates a correct ratio between the preheat and reheat sections of surface; if the preheat is inadequate the design supply humidity cannot be reached; if the reheat is inadequate the design dry bulb temperature cannot be obtained.

g. The most complex arrangement is that in which fractions of the return air are introduced ahead of each of the three equipments and after the reheating coil. This case is shown in Fig. 12·15*g*.

12·6. Effect of Recirculation: Summer. The previous discussion has not taken into account the possibility of using return air to modify the load distribution among cooling and dehumidifying equipments. As was

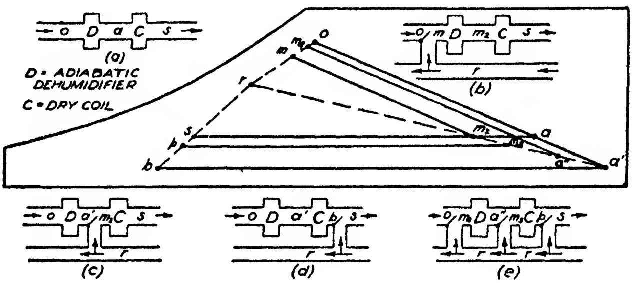

FIG. 12·16. Recirculation for Cooling Season.

true for the various heating and humidifying cycles, so too for summer conditions return air can be introduced in various places and in various proportions to achieve the desired load distribution. Figure 12·16 shows five methods of blending return air in a system consisting of a dehumidifier followed in series by a simple cooling coil.

a. No recirculation. Load equals weight rate of conditioned air times enthalpy difference between state points *o* and *s*.

b. Return air introduced before outside air enters dehumidifier. Load equals weight rate of conditioned air times enthalpy difference between state points m_1 and *s* (considering adiabatic dehumidification).

c. Return air introduced between dehumidifier and cooling coil. Load equals weight rate of conditioned air times enthalpy difference between state points m_2 and *s*. Note that (when dehumidification is adiabatic) the enthalpy must be the same at points m_1 and m_2 since for a given weight rate of supply air and a given fraction of return air the load cannot change.

d. Return air introduced after the cooling coil. Load equals weight rate of *outside* air multiplied by the enthalpy difference between state points *o* and *b*.

e. Return air introduced in three fractions: ahead of dehumidifier, ahead of and after the cooling coil. If dehumidifier is considered adiabatic, the load is equal to weight rate of air entering the cooling coil times enthalpy difference between states m_3 and p.

12·7. Zoning. A central air conditioning plant, either summer or winter, is frequently called upon to supply air to sections of the structure in which the loads may vary independently. Satisfactory operation of such systems requires control of the state or the volume of air to each individual section. Arrangements to provide simultaneous supply at more than one state (or at variable volume) are known as zoning.

The present discussion is limited to an investigation of the psychrometric processes represented by the different types of zoned systems. Since the principles are the same whether the system is for winter or for summer operation, the examples given will be based on winter operation only; extension to summer conditions is readily possible. The most direct method of zoning is through use of independent parallel-operated conditioning systems; such systems involve no new psychrometric processes or combinations and will therefore not be considered. Other methods are:

a. Zoning by volume control at constant supply state. Since air is supplied at fixed state, there is no psychrometric evidence of the zoning process. The specific enthalpy of air entering the conditioned space does not change with heating load, but the volume does, so adjustment of incoming heat to meet energy requirements is purely by variation in the quantity of air entering the enclosure. (In this connection it is well to note that the psychrometric chart shows changes in the quality only; quantity variations are never evident from processes or cycles as drawn on the chart.)

The objection to this method is that variation in volume of air admitted to the conditioned space may result in serious disturbance to the planned distribution since the " throw " of the air stream from the grilles will vary with velocity and hence with volume. When volume control is used for zoning, some arrangement such as a damper stop must be provided to insure that the volume will never be reduced below that critical value which represents the ventilation requirement. A zoning system psychrometrically similar to volume control, but without the disadvantage of variable air flow at the grilles, is obtained in some installations by introducing sufficient return air into the supply duct (ahead of the grille) to maintain constant discharge volume.

b. Reheating coils are widely used for zoning. Air leaving the central conditioner passes over a zone reheat coil where its temperature is raised, at constant humidity, to the desired value. If separate zone fans are used, recirculated air can be introduced before passing over the heating coil thereby providing means for controlling, within limits, the humidity as well as the temperature of the supply air.

c. A third commonly used, but more expensive (first cost) method of zoning is to run two ducts, one for heated and one for unheated or tempered air, from the central conditioner to the entrance of each zone. The two air streams are mixed in accordance with the demand, and a constant volume of air is admitted to the conditioned space. The advantage of this dual-duct system is that each room becomes, in effect, a separate zone, and conditions in it can be controlled to the individual preference of the occupants.

d. Occasionally, industrial conditioning problems require control of both temperature and humidity by zones. In such cases complete control can be realized by combining the dual duct and reheat systems. The two ducts now carry air streams of different absolute humidity; mixing of the streams at each zone gives the desired supply humidity. Reheating of the mixture then raises the dry bulb temperature to the requisite value.

12·8. Selection of Equipment. Throughout the preceding discussion no reference has been made to the practical problem of selecting a particular piece of air conditioning equipment and investigating its capacity, rating, and ability to function throughout the necessary range of operating conditions. With air washers, for example, the engineer must investigate the equivalent by-pass factor as a function of load, entering air state, and velocity through the washer. Similarly, in selecting wet surface coils the engineer is faced with the task of determining the equivalent by-pass factor, and capacity, as a function of all the variables which are important to the system.

Two essentially different methods are in common use for the selection of equipment. The first is to investigate in terms of the rational heat transfer and mass transfer equations the expected performance of an unknown equipment and to compare it with known equipments which have also been theoretically investigated. This method leads to a sound rational evaluation of each equipment and is of great value to the engineer whose task it is to design and construct units for carrying out the psychrometric processes.

The second method of selection is based on use of empirical data available in the manufacturer's rating tables and data sheets. From such data it is frequently very difficult to evaluate the fundamental effectiveness or soundness of basic design. However, the air conditioning engineer is primarily interested in the ability of a particular equipment to meet the specific conditions of the problem with which he is concerned, and if this can be done at reasonable cost, theoretical shortcomings or limitations of the particular equipment which he has selected are unimportant. For this reason the second method of selection is the one usually adopted.

The entire problem of equipment selection is outside the province of a book on principles. Once the psychrometric cycle has been fixed and the arrangement and relative capacity of the individual equipments determined, the engineer is in a position to go to manufacturers' catalogues, or consult sales representatives, for details as to the relative abilities of different commercial apparatus to meet the requirements which he has specified.

12·9. Summary. In this chapter the psychrometric processes of Chapter XI have been combined to give many possible cycles representative of the arrangements which can be used in air conditioning systems. The two basic psychrometric problems facing the air conditioning engineer have now been covered: (1) to locate on the chart the point, or line, or area, for which the equipment used for air conditioning must be able to supply air; (2) to arrange a series of psychrometric processes which can be followed by commercial equipments, giving an operating cycle that will make it possible for the system, as installed, to supply air at any of the required states.

A third problem, outside the scope of this book, is to select particular equipments to follow satisfactorily the processes making up the cycle. This latter problem is essentially one of economics based on variations in first cost, probable maintenance, obsolescence, and repair charges. A satisfactory solution of this problem can most effectively be realized from mechanical and structural examination of the competitive units rather than from an attempt at rational interpretation of their thermodynamic or psychrometric merits.

PROBLEMS

1. Outside air at 85° dbt, 65° wbt is to be conditioned to 90° dbt, 80 grains/lb by passage through a by-passed simple saturating air washer in series with and followed by a reheating coil. What percentage of air must be by-passed around the air washer?

2. Air at 53.5° dbt, 45° wbt is conditioned to 101.5°F, 20 per cent RH as a result of passage through a tempering coil, a controlled adiabatic air washer, and a reheating coil. Determine: (*a*) the state leaving the tempering coil, (*b*) the total latent heat addition and sensible heat addition for the system, (*c*) the fraction of load carried by each of the coils.

3. Sketch an equipment arrangement which would permit attaining *any* required supply state from *any* possible outside state.

4. Ten thousand cfm of air at 115° dbt, 70° wbt are to be cooled at constant absolute humidity to a dbt of 61°. A two-stage indirect evaporative cooler is used in which each air washer saturates the stream passing through it and each plate type heat exchanger reduces the dbt of the outside air to within 5°F of the temperature of air entering from the corresponding washer. Calculate the weight of air, pounds per minute, passing through each washer (assuming that washer air gains 5°F as it passes through the plate type exchanger).

5. Three air streams at 110° dbt (10 per cent RH), 90° dbt (50 per cent RH), and

50° (10 per cent RH) are mixed to give a resultant state of 95° dbt, 68° wbt. Calculate the percentage of mixture air (by weight) from each of the three streams.

6. Outside air at 60° dbt, 45° wbt is blended with recirculated room air at 70° dbt, 50 per cent RH and the mixture then passed through a tempering coil, adiabatic saturating washer, and a reheating coil. Air from the reheating coil is mixed with an equal weight of recirculation, and the resultant mixture is admitted to the room at a supply state of 88° dbt, 70° wbt. Calculate the percentage of supply air which represents recirculated air added ahead of the tempering coil. Sketch the processes of this system on a psychrometric chart. Calculate the heat added in tempering and reheat coils and check by an overall computation based on the enthalpy change of the outside air. Take dbt of air to washer as 115°F.

7. Air at 96° dbt, 60° wbt is to be heated to 110° dbt and the same absolute humidity by means of an unlimited supply of water which is available at 80°F. Devise an equipment arrangement by means of which this can be accomplished.

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CHAPTER XIII

DESIGN STATE AND DESIGN VOLUME

Many ventilating systems are designed to circulate unconditioned outside air. A common and important example of this type is an occupied space in which the entire ventilation need can be attributed exclusively to human occupancy. The first step in the analysis of any such system is to investigate the air volume needed to accomplish each of the five basic ventilation objectives. Four of these are:

1. Supply oxygen and remove carbon dioxide.
2. Reduce odor concentration.
3. Remove body heat dissipated by the occupants.
4. Remove body moisture dissipated by the occupants.

The fifth purpose of ventilation is to provide sufficient air movement in the occupied space to increase the rate of heat loss from the exposed parts of the body and to relieve the monotony which would exist in still air; this objective, however, can be realized equally well by recirculating room air.

Examination of the above list immediately discloses that ventilation is essentially a negative process since four of its objectives are concerned with removal of undesirable characteristics from the inside air rather than the actual need for fresh outside air. The estimation of volume requirements is based on investigation of each of the basic objectives; the one requiring the greatest air quantity will obviously determine the supply rate for the system. In the following sections each of the criteria will be discussed and data presented to permit direct determination of volume requirements for a given system.

13·1. Oxygen Supply. Like all other machines which convert chemical energy into mechanical work, the human body requires an adequate supply of oxygen to sustain the process of combustion, and it dissipates, as "exhaust gas," carbon dioxide. The first requirement of ventilation is to supply the requisite oxygen and dispose of the exhaled carbon dioxide. Fortunately, this requirement is very simply met so the influence of oxygen supply as a criterion of ventilation design is extremely slight. Any ventilating system which effectively accomplishes odor, temperature, or moisture control will automatically supply more than sufficient air to meet all oxygen requirements.

Under normal comfort conditions the average individual consumes approximately 2 cu ft of oxygen per hour. In the same interval he produces approximately 0.6 cu ft of carbon dioxide. Thus a definite relationship exists between the concentrations of oxygen and carbon dioxide in the air of an occupied enclosure; the rise in concentration of CO_2 is therefore an index of oxygen consumption. Because of this relationship, it was thought for many years that carbon dioxide was harmful and should never be allowed to reach a concentration in room air greater than 2 per cent. Actually, the CO_2 concentration in the *true* atmosphere in which man lives, that in the lungs, is close to 6 per cent and must be maintained near this concentration for physiological reasons. Dill* has pointed out that one of the most undesirable attributes of intemperate deep breathing exercises is that the carbon dioxide concentration in the lungs can be reduced so far below normal that dizziness and other severe physiological consequences may result. In any concentration likely to occur in an occupied enclosure, carbon dioxide is, in itself, physiologically unimportant; the only deleterious effect of a high concentration is that it serves to indicate an oxygen shortage.

Experiment has shown that when the CO_2 concentration in an occupied space exceeds 2 per cent, the partial pressure of oxygen will have been reduced to a value such that breathing will be more difficult; the respiratory rate will then increase. When the concentration reaches 6 per cent extreme discomfort is likely, but not until 10 per cent will loss of consciousness occur. The relative insignificance of oxygen needs can be demonstrated from a consideration of occupancy conditions in a space which represents the most extreme case likely to occur in practice.

Consider that the density of occupancy in a ventilated space is such that there is but 180 cu ft of air per person. If the carbon dioxide concentration in outside air is approximately 3 parts in 10,000, the time during which all occupants could remain in such an enclosure with *no* fresh air whatsoever can be readily calculated as follows:

$$\text{Per cent concentration of } CO_2 \text{ after } X \text{ hours} = 0.03 + \frac{0.6 \times 100}{180} X$$

$$= 0.03 + 0.333X$$

Rapid breathing would first be noticed (2 per cent CO_2) after 6 hr; marked discomfort would not occur until 18 hr; loss of consciousness would not occur until more than a full day (30 hr). The danger of a health hazard existing in any ventilated room due to inadequate oxygen supply is seen, then, to be extremely remote. This fact is even more clearly recognized when one visualizes that a single occupant could be sealed in an airtight box 10 ft by 10 ft by 10 ft for a week before loss of

* David Dill, *Life, Heat, and Altitude*, Harvard University Press, 1938.

consciousness would be expected from oxygen deficiency. Note that the above examples are based on considerations of oxygen alone; other factors might lead to serious physiological disorders, even death, in a much shorter time than the examples would seem to indicate.

Approaching this same problem from a different point of view, assume that it is desired to supply only sufficient outside air to the room considered above (180 cu ft air space per occupant) to prevent the carbon dioxide concentration from exceeding 2 per cent. Establishing a volume balance between the CO_2 entering and leaving the enclosure,

$$0.0003 \times \text{cfm} \times 60 + 0.60 = 0.02 \times \text{cfm} \times 60$$

$$\text{cfm/occupant} = 0.507$$

In an ordinary enclosure supplied with the absolute minimum required outside air to prevent objectionable odor, the CO_2 concentration would be less than 0.1 per cent. Thus any ventilation system capable of meeting the second requirement, odor control, will automatically supply more than an ample quantity of oxygen.

13·2. Odor Removal. Odor control is essentially a problem of diluting the odor-producing agents released by occupants of an enclosed space. Since odor is a sensation resulting from exposure of the olfactory bulb to contaminated air, the degree of dilution required is dependent on the concentration corresponding to the threshold at which the sensation of odor first appears. The threshold concentration varies somewhat with different individuals, but average values have been well established.

The air quantities needed for dilution obviously depend on the rate at which odoriferous material is added to the room air. For the usual type of ventilation problem the principal sources of air contamination are body odors resulting from breathing, insensible perspiration, and the products of organic decomposition. Contamination of the air occurs either directly from the bodies of the occupants or from their clothes. The seriousness of the odor problem varies with the personal cleanliness, occupation, and degree of exertion of the occupants, but the basic requirement of ventilation air for odor dilution exists, and is of real significance, even though the occupants of the space under consideration are sedentary and given to the practice of a high degree of personal cleanliness.

The effective rate of air contamination is now known to be a function of the air space available per occupant. Many of the products causing odor are self-oxidizing and disappear spontaneously when the time interval and the mixing volume are sufficient; thus body odor disappears from a closed room within a short time after the occupants leave. Because of self-oxidation the required ventilation rate decreases as the air space per occupant is increased; conversely, the ventilation requirement for odor control becomes particularly severe as the density of occupancy increases.

Figure 13·1 gives outside air supply rates based on recommendations of the American Society of Heating and Ventilating Engineers for sedentary adults of average socio-economic status. The same society gives 4 cfm/occupant as the requirement for summer operation when using a spray type dehumidifying air conditioner operated with sufficient recirculated air to provide a total ventilation rate of 30 cfm/occupant.

Control of odor in occupied spaces is a health as well as a comfort necessity. Even in rooms where the occupants are unaware of odor, the psychological and physiological effects persist and later evidence themselves in loss of appetite and other signs of decreased health or working effectiveness. The sensation of odor is controlled by the response of the olfactory bulb; as time of exposure increases, olfactory fatigue occurs, and the threshold of odor detection is raised to a higher concentration. For this reason occupants of a poorly ventilated space can be unaware of an extremely high odor concentration which would be very disagreeable to someone entering from the outside. But as already mentioned, subconscious effects on appetite and general physiological condition make contaminated air undesirable even if the occupants do not find it unpleasant.

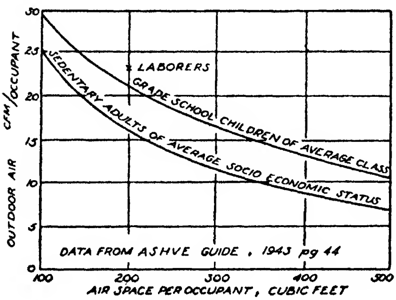

FIG. 13·1. Outside Air Ventilation Requirement for Odor Control.

The ventilation rates given in Fig. 13·1 are frequently, and erroneously, used as indices of the general and all-inclusive effectiveness of a ventilation system. These rates are significant only with respect to one objective, odor control; systems designed on this basis might well be inadequate when judged in terms of their ability to meet the other criteria of effective ventilation. Note, however, that *any* ventilating system capable of controlling odor will supply a more than adequate quantity of oxygen.

13·3. Heat Removal. The analogy between the human body and a machine for converting thermal energy into mechanical work can be extended to account for the normal heat loss (approximately 400 Btu/hr) from a person at rest. The thermal effectiveness of the human mechanism is approximately 20 per cent; thus for each unit of mechanical work realized five units of thermal energy must be expended and of this amount four units must then be dissipated or rejected from the body as waste heat. When doing no external work man must still do sufficient internal work, as pumping of blood through the arterial system and muscular work needed for respiration, to sustain life. Since all internal work must be degraded to heat, the *total* energy input for the sedentary individual must be dissipated as waste heat.

The significance of the heat quantities which must be disposed of can be visualized in terms of the room (already considered in terms of oxygen requirements) in which there is 180 cu ft of space per occupant, no ventilation and no heat transfer to or from an outside source. This room contains 13.4 lb of *standard* air (68° dbt and 50 per cent RH) per occupant and since the specific heat of air is 0.24, the room temperature will rise approximately 0.31°F for each Btu released. Assuming that the increased room temperature did not affect the rate of heat production of the occupants, the temperature rise in such an enclosure would amount to 0.31×300 Btu/hr (sensible body heat loss) = 93°F per hour. Though the numerical answer of this example is not of significance (because physiological changes would occur as soon as the air temperature started to rise) it nevertheless serves to demonstrate that, in a thermally insulated, sealed room, the need for outside ventilation air would occur in a matter of minutes, in contrast to the much longer time before an oxygen shortage would be experienced.

In practice, no actual room is likely even to approach the assumed condition of thermal insulation, but not infrequently the heat loss through the walls of rooms will be too small to assist appreciably in dissipating body heat. In such cases the supply of ventilation air must be great enough to carry the load without allowing a temperature rise sufficient to make conditions in the room unhealthy or extremely uncomfortable. Obviously, however, the limiting room conditions for any simple ventilation system are the same as existing outside conditions.

Equilibrium conditions in a ventilated, occupied, thermally insulated room are independent of room size and of the air space available per occupant; they depend only on the ventilation rate. For a ventilation rate adequate to care for both oxygen and odor requirements (approximately 25 cfm/occupant) the temperature rise is more than 11°F. The adequacy of such ventilation obviously depends on the weather; when the outside temperature is less than 59° the room temperature would be too low for comfort though at all outside temperatures above 59°, room conditions would be above 70°F.

The need for a small temperature difference between inside and outside remains relatively unimportant so long as the inside temperature is in the seventies. As this temperature enters the eighties physiological and psychological effects become significant and at 85°F a critical point is reached. When the temperature reaches 90°F, with a relative humidity of 70 per cent, physiological changes start to occur; the pulse increases about one beat a minute per hour and body temperature increases approximately 0.1°F per hour. Above this state (either in temperature or humidity) the body is incapable of adapting itself without severe discomfort and possible injury to health.

The extent of the hazard represented by excessive temperature depends on the period of exposure. Conditions very much worse than the critical exist in many industrial and marine installations and have no immediate deleterious effect on the occupants. The objective of ventilation design is, however, to establish — where it is economically or physically possible to do so — an atmosphere in which the occupants can live or work without danger of developing chronic ailments, or of suffering loss of efficiency due to difficulty in making physiological adjustments to compensate for abnormal environmental conditions.

13·4. Moisture Removal. The moisture loss from men comfortably at rest in air at 70°F is approximately 0.1 lb/hr or 700 grains/hr. The increase in specific humidity of air passing through a ventilated occupied space at the rate recommended for control of odor is approximately 6.2 grains. If the room air temperature were 70°, this moisture gain would correspond to an increase in relative humidity within the room of from 5 to 6 per cent. As the humidity increases the body's ability to dispose of heat by evaporation decreases so an increase in humidity has the same qualitative effect as an increase in air temperature. In addition to the objection of greater difficulty in disposing of body heat, high humidity reduces the apparent " freshness " of air in an enclosed space and seemingly accentuates odor. In general, adequate ventilation should, where possible, supply an air volume sufficient to maintain the relative humidity in the enclosure at a value less than 70 per cent. This figure, unlike the critical value set for temperature, is based almost entirely on considerations of comfort rather than health. In this connection it is especially important to note that the response of individuals within the enclosure is not proportional to the absolute need for improvement in conditions. Thus in a room where the air temperature and relative humidity are 85°F and 100 per cent respectively, the feeling of discomfort, aside from the sensation of warmth, would be decidedly more pronounced than in an enclosure with air at 100°F and relative humidity of 33 per cent; yet the degree of physiological adjustment needed to maintain a normal balance between rate of heat production and loss would be nearly the same for the two cases.

13·5. Air Motion. The influence of air motion on comfort is relatively complex, but for design purposes two particular effects stand out:

1. Increased air velocity decreases the resistance to heat transfer by reducing the thickness of the air film adjacent to the body or its clothing. When the ambient air is at a temperature lower than that of the body surface (or clothing surface) the effect of increased velocity is to promote body heat loss and thereby alleviate the discomfort due to temperature

or humidity. If, however, the air temperature exceeds that of the body or clothing surface, the sensible heat transfer is in the opposite direction and tends to accentuate the already uncomfortably warm conditions. At the same time a second factor, frequently the controlling one, must be considered: increased air velocity reduces the thickness of the layer of saturated vapor near the body surface and hence promotes evaporation; when the dew point is below 85° the cooling effect due to the increased latent heat loss by evaporation is usually greater than the heating effect from convection so that increased air velocity is advantageous. For air with high temperature and excessive dew point, both the sensible and latent heat transfer are to the body, with the inevitable consequence that (barring exposure to cold walls) body temperature must rise with resultant serious effect upon occupants. With high air temperatures, the effectiveness of air movement as a means of achieving cooling varies inversely with absolute humidity. From negligible cooling effect at 85° and saturation, an air movement of 100 fpm is equivalent to a temperature reduction in excess of 20°F in dry air at 120°F.

Control of air velocity is usually not a satisfactory method of establishing cooling. Wherever possible, however, the air movement in an uncomfortably warm working space should be kept above 25 fpm; if possible a directional flow of 50 to 80 fpm is to be preferred.

2. The second important effect of air movement is to establish sufficient turbulence within the enclosure to mix thoroughly the room air and thereby prevent undue stratification or by-passing of the incoming ventilation air.

Air movement without good distribution is responsible for the local cooling sensations known as drafts. Even when the temperature of room air and the moving air stream are the same, a feeling of local cooling will be experienced because of the greater rate of heat loss accompanying the higher velocity. When the incoming air stream differs in temperature as well as velocity the draft will be especially severe. Available data now seem to indicate that differential air velocities of less than 25 fpm associated with differential temperatures less than 1½°F do not result in noticeable drafts, but velocities greater than 40 fpm with temperature differences of 2°F represent uncomfortably drafty conditions. Physiologically the importance of drafts seems to be very much greater when the local condition which they represent is one of undue cooling rather than when the draft effect is to reduce locally the discomfort of a hot environment. At present there do not seem to be data adequate enough to permit stating that summer drafts are definitely harmful, but from the standpoint of good engineering, the unbalanced distribution conditions which are indicated when drafts are present should be avoided.

13·6. Effective Temperature. From the discussion in the above sections it is evident that the sensation of warmth experienced by an individual when subjected to a given ambient condition depends on the combined effect of air temperature, humidity, and motion. The average air velocity in an occupied space usually is of the order of magnitude of 15 to 20 fpm. As a matter of convenience this value is arbitrarily taken as a defining condition in expressing the effective temperature of a given ambient condition. Arbitrarily, the effective temperature is defined as the temperature of saturated air traveling at 20 fpm for which the sensation of warmth would be the same as that experienced in the ambient condition which is to be compared.

On the basis of extensive experimental work the American Society of Heating and Ventilating Engineers has established lines on the psychrometric chart along which the feeling of warmth does not vary; such lines represent conditions for which the effective temperature is constant. Within the range of environmental conditions for which comfort is attained, effective temperature lines very nearly coincide with lines of

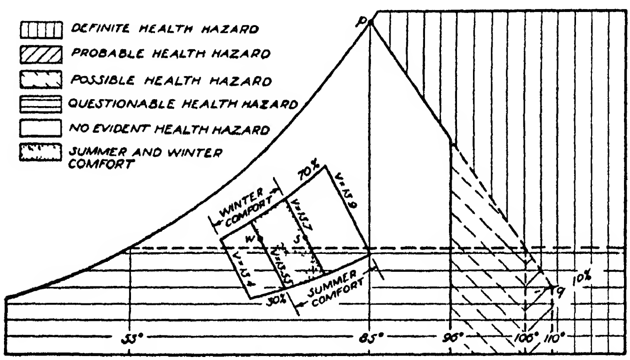

FIG. 13·2. Comfort and Health Zones on Psychrometric Chart. (Comfort zone adapted from ASHVE data)

constant specific volume. It is therefore possible to say, with reasonable accuracy, that any two atmospheres for which the specific volume is the same will have equal cooling powers provided the subject is capable of establishing the requisite physiological balance between convective, radiant, and evaporative heat loss.

13·7. Design Conditions — Comfort. Figure 13·2 summarizes the comfort and health data needed for purposes of design. The comfort zones for summer and winter are based on ASHVE findings, slightly modified to permit greater ease in locating them on the standard chart. The comfort zones for both summer and winter are seen to be bounded by the 30 and 70 per cent relative humidity lines; at relative humidities outside these limits the occupant is likely to find the environment unpleasant even though its cooling power is adequate.

The winter comfort zone includes all states at which a majority of normal people would experience reasonable comfort. It is defined at the lower limit by the constant specific volume line for an atmosphere occupying 13.4 cu ft/lb and at the upper limit by one having a specific volume of 13.7 cu ft/lb. At an intermediate condition of 13.55 cu ft/lb is located the cooling rate for which the greatest number of people (97 per cent) can be satisfied, whereas on either side of this line the percentage of satisfied occupants decreases at almost a linear rate. The winter zone includes acclimatization effects for occupants adapted to outside conditions similar to those found in the eastern part of the United States; where the climate is greatly different from this, some adjustment should be made to find the effective temperature of maximum comfort. Departures from the chart due to acclimatization are not, however, of very great design importance because their effect is usually to reduce the inside temperature in winter and raise it in summer.

The summer zone is limited on the lower side by a specific volume of 13.55 and on the upper side by the specific volume line for 13.9 cu ft/lb; the greatest percentage of subjects are comfortable in summer when the specific volume is 13.7 (corresponding to the upper limit for winter comfort of the majority of occupants).

Summarizing, the comfort zones marked on Fig. 13·2 give recommended design inside conditions. As an aid to memory and as a statement of optimum design conditions, it is suggested that 70°F, 60 per cent relative humidity be taken as a basic reference for winter design and 76°, 50 per cent as a similar reference state for summer inside design conditions. These points are shown in Fig. 13·2 as w and s respectively. The exact boundaries (including slight deviation from constant specific volume lines) of the summer and winter comfort zones are shown on the psychrometric chart of Fig. 9·5.

13·8. Design Conditions — Health. The comfort zones provide a simple and convenient method of determining the inside design range within which the designer must operate when his object is to establish conditions of optimum comfort. Though comfort is the criterion of design in the greatest number of installations, there are many, particularly industrial applications, for which the designer's problem is quite different. Here he must estimate the extent to which extreme environmental conditions constitute a health hazard and from this deduce the minimum air conditioning or ventilation requirements to alleviate the hazard. In such cases comfort is not the design criterion since it is frequently impractical or economically prohibitive to provide a system of adequate capacity to realize comfort. Design conditions should be selected which, though still uncomfortable, will not constitute a danger to the health and effectiveness of the occupants.

The determination of limiting conditions beyond which there is a health hazard is obviously a problem for the medical rather than the engineering profession. Owing to the great differences that exist in the physiological capacities of different persons to meet external conditions, no exact statement can be made as to a particular demarcation between conditions which are definitely hazardous and those which are not; to account for personal differences there must, inevitably, be a rather indeterminate region in which some people will be capable of establishing effective regulation while others will not. The engineer should, therefore, use the greatest caution in generalizing as to the relative health hazard associated with a given set of environmental conditions.

Figure 13·2, in addition to showing the comfort zones, indicates the areas on the psychrometric chart in which a definite health hazard is known to exist and others in which the probability or possibility of such a hazard is unusually great. The sections of the psychrometric chart which include neither comfort zones nor zones of health hazard are either areas in which it is possible to avoid danger to health by taking simple precautions (as the area in the low-temperature region where extra clothing is a simple means of avoiding the danger of excessive body heat loss) or areas in which there may be positive discomfort, but without an established hazard to health. The latter condition occurs in the region where the body is capable of maintaining a very close balance between heat production and heat loss by means of evaporative regulation; in this region discomfort is experienced and sweating may be profuse, but there is no residue of waste heat going to storage in the body tissues and hence no rise in body temperature. The limit of the zone of evaporative regulation is determined either through the limiting sweat secretion of the individual, or the falling off of this rate if the exposure is prolonged.

The basis of a health hazard is inability of the body to dispose of waste heat. Obviously, therefore, the danger represented by a given environment will vary directly with the degree of exertion of the occupants. Contrary to popular opinion, high temperatures are less hazardous in sleeping quarters than in working spaces. Since the effectiveness of a given environment in maintaining thermal equilibrium in the body is a function of the rate at which the occupant is doing work, the area or zone of hazard should properly be varied as a function of exertion. To avoid undue complexity and yet to provide reasonable protection for normal occupations, the area of health hazard in Fig. 13·2 is defined in terms of moderate work by an occupant who is lightly clothed and in an enclosure where the air movement is slight (15 to 20 fpm) with average wall surfaces at approximately air temperature. In the event that the latter condition is not realized an approximate correction can be obtained by considering that the "equivalent" air temperature in a room has a value halfway between the actual air temperature and the average temperature of the surfaces surrounding the occupant.

In certain sections of the psychrometric chart, conditions exist which are healthful but uncomfortable. Conversely, there is one region within the comfort zone which may be unhealthful. Sufficient data have not yet been collected to permit a definite assertion as to the health significance, but it is now known that definite drying of the membranes in the nose and throat occurs whenever the dew point temperature is less than 53°F. The area within the comfort zones in which such drying occurs is represented in Fig. 13·2 by horizontal shading. Even though definite proof of a health hazard does not exist for states in this area it is recommended that, whenever a choice is practical or economically permissible, the engineer select an operating state above the dashed horizontal line.

The region on the chart in which there is an unquestionable hazard is shown with vertical shading. It starts at 85°F saturated and is bounded by a straight line through this point to 110°F, 10 per cent, then dropping vertically to dry air at the same temperature 110°. The discontinuity in the boundary of this region is due to the change from heat transfer to physiology as the controlling factor at 110°, 10 per cent. The line pq is a locus of states for which the cooling power of the atmosphere is constant, and the slope of this line is determined solely from heat transfer considerations. At p the evaporative heat loss from the body is less than at any other point along pq; as the state moves toward q the total rate of heat dissipation remains constant, but the convective fraction decreases (becoming zero when body surface and air temperatures have the same value, and then increasing negatively), and the evaporative fraction becomes greater. But to attain a balance at states approaching q, the body must supply an ever-increasing quantity of liquid to be evaporated. In order to establish a balance at state q, the supply of liquid must be at a rate approximating one quart per hour. This rate represents the maximum output of the average person when, by acclimatization, he has developed his ability to secrete sweat to the utmost. In exceptional cases rates in excess of this have been attained, but they are not typical and cannot be expected. For all practical purposes 110°F is the maximum temperature to which an acclimated individual can be continuously exposed without damage to health.

Persons who are not acclimated are incapable of secreting sweat at a rate sufficient to permit a heat balance at 110°F. Further, many persons are physiologically incapable of secreting at adequate rate even when they do reach their maximum degree of acclimatization. For these reasons there is a secondary zone on the chart (shown with diagonal shading in the figure) in which a definite hazard exists for the unacclimated person and a *possible* hazard for some acclimated individuals. A third zone, representing the area in which unacclimated persons *may* be endangered, is shown on the chart with dashed shading between the 96°F and the 106°F temperature lines. At 96°F, with light work, there is danger to anyone who is incapable of secreting sweat at a rate greater than 1 pint per hour while at 106°F the hazard exists for those whose secretion rate is limited to 1½ pints per hour. The latter value is the one representative of the average person's ability and should therefore be used as a design criterion. Sweat is a chemically complex fluid having a salt concentration which varies among different people and, for the same person, with degree of acclimatization. Even when the rate of secretion is sufficient to permit a body heat balance, there may be danger of physiological damage due to the depletion of salt in the body; for this reason salt capsules should be available at drinking fountains located in working spaces where conditions are such that the occupants maintain a body heat balance by means of evaporative regulation.

From Fig. 13·2 it is evident that there is a large area above the comfort zone, but below the region that is unhealthful. The object of many industrial installations is to bring the enclosure state down from the unhealthful region into this area of safe, but uncomfortable conditions. No numerical scale is available for evaluating the degree of discomfort experienced in atmospheres outside of the comfort zone, but comparisons of such atmospheres can be roughly determined by considering that the cooling power varies inversely as the specific volume. This approximate relationship is particularly valuable in that it permits realizing a minimum of discomfort by operating the available conditioning equipment to provide the best environment compatible with its heat-removing capacity.

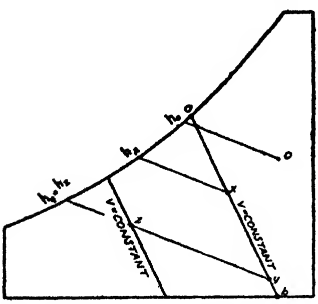

FIG. 13·3. Economics of Comfort Cooling.

Referring to Fig. 13·3, recall that the sensation of warmth does not greatly change along a line of constant specific volume and note that the enthalpy at states along such a line progressively decreases as the air becomes less humid. Thus the cost, in Btu, of conditioning an uncomfortably warm atmosphere decreases as the absolute humidity of the conditioned atmosphere increases.

As an example (Fig. 13·3) consider an atmosphere at state o which is to be reduced to a condition not worse than that existing along the line ab. If state x is selected as the inside design condition, the required cooling load will be $(h_o - h_x)$, but if state y is selected the cooling load will be greater by $(h_x - h_y)$ Btu per pound. In this case, if the equipment were capable of carrying the load $(h_o - h_y)$, and if operation at such load could be justified economically, there would be an obvious advantage in operating at a new state z for which the load remains $(h_o - h_y)$, but the cooling power of the atmosphere is increased to a value closer to the comfort zone. In many cases operation at higher rather than lower humidities will actually permit bringing the conditioned atmosphere inside the comfort zone without altering the total load on the conditioning equipment. It must be remarked, however, that the ability of a system to extract a given number of Btu total load does not necessarily mean that the equipment is capable of extracting the same quantity of heat if the ratio of sensible to total heat load is altered.

The above discussion has been in terms of summer operation. Conversely, for winter operation, the minimum heating load corresponding to a given atmospheric cooling power is realized at low humidities. As already pointed out, the recommended minimum humidity is that corresponding to a dew point temperature in the conditioned space of 53°F. Insufficient data are available to permit evaluation of the health effects of relative humidities greater than 70 per cent, but some of the existing data indicate the strong possibility that there may be a health hazard in this region. Wherever possible relative humidities below 70 per cent are therefore recommended.

13·9. Operation Conditions — Acclimatization. The discussion so far has been limited to determination of the inside atmospheric state for use in system design. A second use of the comfort zone is to determine the adjustment which should be made to the inside state as the outside temperature varies, or as the degree of acclimatization of the occupants varies. Referring to Fig. 9·5, there is a 5°F effective temperature difference between the most favorable summer and winter conditions. This arises from the fact that nature has given man the ability to readjust his thermal control mechanism if exposure to changed conditions is prolonged.

As a result of this acclimatization the physiological strain resulting from prolonged exposure is reduced and, at the same time, the condition at which comfort would be realized is likewise altered to a value closer to that representing normal outside conditions. Acclimatization can be taken into account by adjusting the inside temperature as a function of the outside. Based on practice in the United States, it now seems probable that the inside air temperature, for summer, should be kept at 70°F plus one-third to one-half the temperature difference between outside conditions and 70°.

Similar in effect to acclimatization, but of different origin, is the condition of shock which occurs as a person passes from one environment to another. Obvious, for example, is the shock effect which would occur if, in winter, one were to leave a heated space and go outside without first adding extra clothing. In summer the occasion for shock, a temperature difference between inside and outside conditions, still exists, yet people commonly pass from a cool air-conditioned atmosphere to the hot outdoors without any change in clothing or other attempt to assist the body in making the readjustment which is necessary to meet the new conditions.

Shock and acclimatization always operate to make a change of conditions seem more severe; because of them, cold outside air seems colder or warm outside air warmer. In this respect it is somewhat paradoxical that the one defense mechanism which nature provides to man as partial protection against extremes of climate is a pronounced obstacle in the path of the air conditioning engineer whose object is also to provide protection from the same extremes. If it were not for the protection afforded by acclimatization, the discomfort caused by summer heat would be greater, but the shock attendant upon the use of summer air conditioning would not be so great. The same can be said of extreme winter conditions and the heating systems which alleviate them, except that, for winter, the occupants do somewhat reduce the severity of shock effects by use of protective outdoor clothing.

PROBLEMS

1. Five persons occupy a 3000-cu ft unventilated room. Determine the CO_2 concentration in the room 6 hr after occupancy starts.

2. What ventilation rate, cfm, is needed in the room of problem 1 to give an equilibrium CO_2 concentration of 0.07 per cent? If the room volume were reduced to 100 cu ft would the necessary equilibrium ventilation rate, in cfm/occupant, be altered?

3. Thirty grade school children occupy a 12,000-cu ft classroom. If sixty children were to occupy the same room, what increase, in cfm, of required ventilation would be necessary?

4. A cabin in the crew's quarters of a naval vessel contains 150 cu ft of air space per ocupant. If it is located in the interior of the ship, no transmission heat losses being possible, calculate the temperature rise which would occur within 10 min if the ventilation system failed.

5. For the case of problem 4 what ventilation rate, cfm per occupant, is necessary if the cabin temperature is to be 5°F greater than the outside air temperature? What would then be the difference in absolute humidity between inside and outside air?

6. The atmosphere in a foundry is at 110°F, 5 per cent RH. No mechanical refrigerating equipment is available, but there is an ample supply of water at 110°F. What equipment purchase would you recommend and on what grounds would you justify it?

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CHAPTER XIV

VENTILATION SYSTEMS

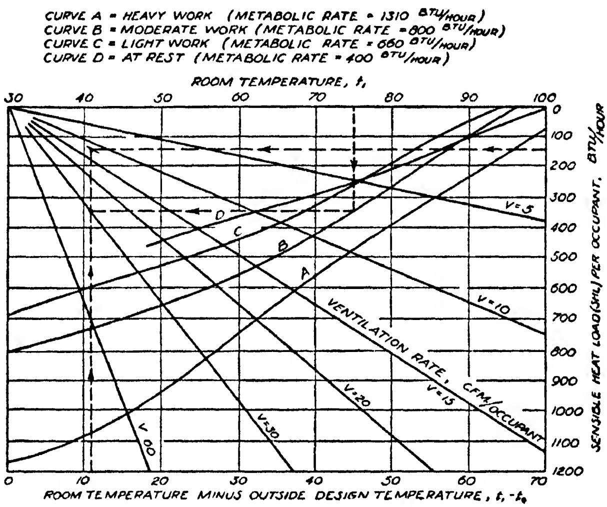
Ventilation is the process of supplying outside air to an enclosed space. The most widely used type of ventilating system is that in which untreated (except possibly filtered) outside air is caused to pass through an enclosure. Systems of this kind, involving neither intentional change in temperature nor humidity, will henceforth be referred to as "simple" ventilating systems in contrast with the complex systems which provide conditioning of the air before its admission.

Mechanical ventilating systems usually are classified as of the pressure type, in which fresh outside air is forced into the enclosure and room air, in equal volume, is forced out through cracks or special openings, or of the exhaust type, in which the room air is extracted and fresh air then allowed to enter through cracks or special openings. Pressure systems are commonly used in all installations except those, as kitchens, washrooms, and ventilated closets, where it is necessary to have a positive flow of air from the ventilated space to avoid possible leakage of contaminated air out into the surrounding enclosures. Where large-volume enclosures are to be exhausted, resort is frequently made to combined pressure-exhaust systems. By carefully balancing such systems a negative pressure can be established in the enclosure low enough to prevent exfiltration, yet sufficiently close to atmospheric to avoid undue pressure differentials at the entrances and to permit drawing the greater part of the ventilation air in from the supply duct rather than by infiltration from surrounding enclosures.

Since simple ventilation consists in supplying untreated outside air, the best conditions that can possibly be realized in the enclosed space (summer operation) are those existing in the outside atmosphere. In practice no ventilating system can maintain inside conditions identical with those outside. Consequently, if the outside atmosphere is uncomfortable, conditions in the ventilated space will be somewhat more uncomfortable — if outside conditions are satisfactory, a well-designed ventilating system should be able to establish a comparable inside condition, provided the space does not have excessive heat or humidity gain.

14·1. Volume of Air Required: Simple System. The volume of air required for ventilation is a function of the design outside conditions, the allowable difference between inside and outside conditions, and the heat and humidity loads which occur in the ventilated space. Heat and humidity loads are determined by the density of occupancy, by other sources, as electric lights and fans within the enclosure, and by losses or gains from transmission of heat to or from adjoining spaces.

Not infrequently the fraction of load in a simple ventilation system due to the occupants is so great that other sources or losses of heat and humidity can be neglected. After having investigated a few typical rooms in any structure the designer will usually be able to estimate the relative error likely to result from use of occupancy load alone; in any event, the neglect of transmission losses in simple ventilating systems usually leads to a conservative result.



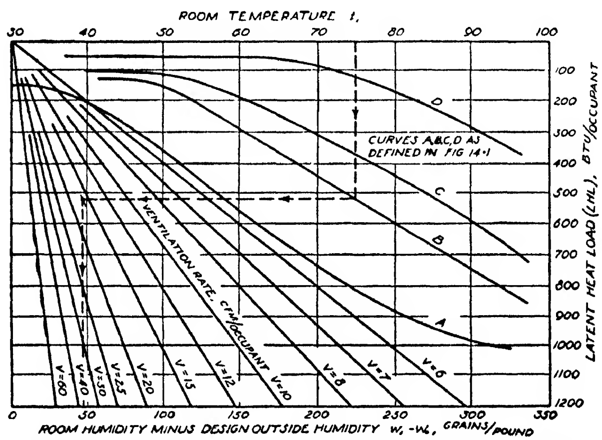
Example 1. To maintain a room at 75°F when outside air is at 64°F and entire heat load is from occupants working at a moderate rate. Enter at $t_i = 75$°F. Drop to curve *B*. Move left to intersection with vertical from $t_i - t_o = 75 - 64 = 11°$ where read ventilation requirement as 30 cfm/occupant.

Example 2. Heat sources in room produce 1850 Btu/hour, and if transmission losses are 4000 Btu/hour, determine the net required ventilation for rooms of Example 1 when there are fifteen occupants. Net heat loss from source and transmission is $(4000 - 1850)/15 = 143$ Btu/hour. Enter at SHL = 143, move left to intersect vertical from $t_i - t_o = 11$°F where read $v = 15$ cfm. The net required ventilation is $30 - 13 = 17$ cfm/occupant $= 15 \times 17 = 255$ cfm.

FIG. 14·1. Ventilation Requirements for Temperature Control. (Data for curves *A*, *B*, *C*, *D* from *ASHVE Guide*, p. 55, 1944.)

Figure 14·1 is a selection chart for determining the volume requirements in terms either of the occupancy alone, or, if data are available, the Btu load in the space to be ventilated. If the design outside temperature is known, the chart gives directly the volume of outside air per occupant which is needed to maintain the inside temperature at any specified value. When the load due to sources other than the occupants is known, it can be divided by the number of occupants to obtain additional load per occupant for which, from the chart, one can read the additional required volume of outside air in cubic feet per minute per occupant. A correction for external transmission losses is obtained by dividing the total rate of transmission loss by the number of occupants, determining from the chart the outside air volume, per occupant, which this load represents and *subtracting* this volume from the sum of occupancy and source volumes to get the actual rate at which outside air must be supplied.

A second and related criterion of effectiveness in a simple ventilating system is the resultant inside relative humidity. The determination of this property does not lend itself to direct graphical solution since it de-



Example. **If humidity of outside air is 38 grains/lb for Example 2 of Fig. 14·1, determine the humidity in the room. Enter at 75°F, drop to curve *B*, move left to intersection with $v = 17$ (from previous example), drop to read humidity gain as 45 grains/lb, then room absolute humidity = 38 + 45 = 83 grains/lb and relative humidity (from psychrometric chart at 75°F and 83 grains) is 64.**

FIG. 14·2. Ventilation Requirements for Odor Control. (Data for curves *A*, *B*, *C*, *D* from *ASHVE Guide*, p. 55, 1944.)

pends not merely on the temperature of the outside air, but on its specific humidity as well. Figure 14·2 gives the increase in specific humidity of the inside air over the outside as a function of the ventilation rate in cubic feet per minute per occupant. Knowing the outside state and the ventilation rate, the inside temperature is obtained from Fig. 14·1 and the increase in absolute humidity from Fig. 14·2. It is then a simple matter to fix the inside state and determine the attendant inside relative humidity.

14·2. Volume of Air Required: Supply Air Cooled, but Humidity Unchanged. When the supply air is cooled prior to admittance to the enclosure, Fig. 14·1 can still be used to obtain the required volume in terms of the permissible temperature difference between supply and room states. For systems of this kind the outside state does not enter into the calculations except in so far as it determines the transmission gain or loss and the load to be carried by the conditioning equipment. To use Fig. 14·1 in solving this case, enter from the vertical scale at the known load, Btu/(hr) (occupant), and the horizontal scale at the desired temperature difference; the intersection of these two lines then fixes the volume. Similarly, if the available volume were initially known, the requisite temperature difference tc carry the load would be determined by entering at the load scale, passing over to intersection with the curve for assigned volume and dropping to read the temperature difference from the scale at lower edge of the figure.

14·3. Summer-Winter Ventilation. Since the selection of a design air volume can be based on a number of possible criteria, there is frequently a need for adjusting this volume as the governing criterion changes. Thus a system providing a volume adequate to establish a reasonable degree of summer comfort would probably be supplying much more air than would be needed during the winter months when odor control is the principal reason for ventilation. In such cases it is particularly helpful if the designer has provided some means, as a variable-speed fan, for reducing the ventilation volume in accordance with changing outside conditions. Usually it is impractical always to operate with the absolute minimum volume. The most commonly used method of control is that of reducing air volume at the operating condition for which heating becomes necessary. Thus a given system would supply a large volume at all times when artificial heating is not used, but would drop to a lesser volume as soon as the heating system goes into operation. The resultant saving would be twofold since power at the fan is a function of volume, and heating requirements obviously are reduced as the quantity of ventilation air decreases. The latter reduction would be realized whether heating was accomplished by directly warming the incoming air or by indirect air heating from radiators or convectors located within the room.

14·4. Refrigeration vs. Simple Ventilation. In the many problems where some relief would be afforded by ventilation, but conditioning for comfort is not justified, an investigation should be conducted to determine whether or not the same relief is available at less cost (or better conditions at equal cost) from air cooling rather than simple ventilation. The difference in operating costs of these two methods is based on the reduction in energy requirements resulting from circulation of a smaller air volume through the ducting as against the increased energy requirements needed to provide the necessary refrigeration.

Aside from the matter of operating costs, thought should be given to the choice between simple ventilation and air cooling because of a number of intangible factors. Among these are:

1. The simple ventilating system has the advantage of simplicity. It contains no parts which are not also required when cooling is used, yet does not have refrigerating equipment, cooling coils, etc. Maintenance and repair would therefore be less.

2. Relative first cost depends on the length of ducting in the system, the size, arrangement, and location of refrigeration equipment. Use of cooling obviously reduces the size of fan and ducting, a factor which may be quite large in an installation where the ducting is tortuous or passes through spaces in which the installation problem is severe.

3. Determination of yearly charges may allow credit for the saving in space occasioned by the smaller ducts which are permissible with cooled air.

4. Other factors for consideration include a probable operating saving with cooling resulting from the fact that the simple ventilation system usually is designed for operation with constant air volume independent of outside conditions, whereas with the cooling system refrigeration can be readily controlled to shut off at any desired inside temperature and, from then on, the only operating cost is for power to force the smaller air volume through the system. On the other hand, the available extra capacity of the cooling system at times when outside conditions are not extreme can be taken advantage of in providing inside conditions closer to those representing optimum comfort.

The decision between refrigeration or simple ventilation rests largely on the relative costs of cooling one cubic foot of air or of circulating a cubic foot through a duct and distribution system; these costs vary with each system and the comparison is possible only after preliminary designs have been completed for the particular system in question.

14·5. Miscellaneous Load Considerations. When, as should always be the case, the cooling load is calculated as part of the original design problem, the minor load items discussed in this section will appear as part of the original calculations and need not again be considered. Frequently, however, ventilation is provided with the single objective of preventing disagreeable odors. In such cases no load analysis is made and no consideration given, in the design, to methods of attaining maximum cooling effect from the design air volume. When this is true the operation of the system, for cooling purposes, can often be improved by studying the minor load factors and, wherever possible, either reducing or eliminating the extra heat sources. Some of the more common items of miscellaneous load are considered in the following sections.

(*a*) Duct Pick-up. For maximum effectiveness the supply air in a simple ventilation system should enter the room at the same state as that outside. Ducts connecting the outside air intake with the room grille frequently pass through hot regions with a consequent gain in temperature of the air prior to arrival at the point of supply. In such cases, the cooling effect of the ventilation air is obviously reduced. This pick-up of heat can be avoided most effectively by a careful selection of the path to be followed by the ducts and by use of adequate duct insulation on those sections which pass through regions where the ambient temperature is high. Selection of insulation and calculations leading to a determination of its effectiveness are conducted by applying the steady state heat transfer equations (Chapter VI).

(*b*) EXCESS FRICTION. Visualizing that duct friction, either mechanical or fluid, is responsible for a conversion of kinetic energy into internal energy (with resulting temperature rise), it is evident that the cooling effect is decreased when friction losses increase. The general subject of duct losses is considered technically in Chapter XVI, but for present purposes it is sufficient to caution that every effort should be made to permit realizing the required volume of ventilation air with a minimum pressure rise across the supply fan. This factor is of particular importance in systems for which an over-conservative selection of the duct friction losses has resulted in the need of dampering as a means of reducing the air volume to the design value. When this is the case, best cooling is realized either by opening the dampers and allowing increased air flow (when feasible) or by opening the dampers and reducing fan speed to bring the volume down to the design value. The effectiveness of pressure reduction as a means of increasing cooling effect can be roughly estimated by taking a 1°F air temperature rise as corresponding to each 1 in. of water pressure differential across the fan.

(*c*) ELECTRIC FANS. One of the most common sources of added heat load occurs because of the indiscriminate use of desk type electric fans. Whenever a fan is in operation, the energy input is almost entirely dissipated within the room, and, although fan operation must result in a slight *increase* in inside air temperature, the greater air velocity does increase the cooling effect and will therefore assist in making the atmosphere more comfortable. However, since cooling effect is based on a heat balance at the surface of the occupant, it should be clear that there is no advantage in operating such a fan in an unoccupied room, but only the positive disadvantage of increased heat load with resultant greater air temperature.

(*d*) ILLUMINATION. As with portable fans, electric lights dissipate as heat practically all the energy supplied to them. Every effort should therefore be made to realize a maximum of lighting effectiveness and to keep lights turned off during periods of non-occupancy.

14·6. Ventilation of Hot Working Spaces. The problem of establishing a satisfactory working atmosphere in industrial enclosures where the heat load is very great is often extremely difficult. The density of occupancy in such spaces is usually low, and the cost of providing adequate cooling for the entire space is likely, therefore, to be prohibitive. The basis of design for all such high heat load occupied spaces is to provide working conditions which, though uncomfortable, are not sufficiently extreme to constitute a hazard to health or an impediment to reasonable working efficiency.

Two basically different methods of improving conditions find application. One is to provide sufficient general ventilation, or cooling, to reduce the entire atmosphere of the working space to a condition which is satisfactory, and the other method is to provide some form of local ventilation intended to make conditions in the immediate vicinity of the worker more comfortable though not attempting to alter the average condition in the enclosure. The first method is obviously preferable whenever conditions are such that it can be employed to a reasonable degree of effectivness and at moderate cost; the design procedure for this method is the same as for a simple ventilation system. The second method is usually limited to working spaces where conditions are extremely severe or where the density of occupancy is so low that even moderate improvement of the general working space cannot be justified.

Of the many possible methods of realizing local cooling all have in common the objective of isolating the worker from the surrounding high-temperature atmosphere and providing only sufficient cooling effect to neutralize such heat as may reach him through whatever type of barrier is used. Three methods have been used successfully from this purpose: solid barriers, fluid barriers, and ventilated clothing. These will be discussed in order.

(*a*) Solid Barriers. Where all the functions of the worker are carried out in a restricted section of the working space, or where he is but infrequently called upon to leave such a restricted area, it may be possible to partition off the section in which he works and provide general ventilation and cooling within this area. This can be done in cases where the actual work operations are performed in one place or where the workman's principal task is to observe conditions throughout the entire work space from a single point of vantage. Control board operators can frequently be protected by providing glassed-in observation chambers within which they can work and yet have good visibility over the entire working space. Aside from the advantage in ventilation, closed compartments of this kind permit reduction of the working noise level to a very much lower value; the improved working effectiveness from this cause may alone justify the use of such an enclosure.

(*b*) Fluid Barriers. The effect of a hot atmosphere on an occupant is largely due to convective transfer from air to clothing or exposed body surface. If it were possible to prevent the hot air from establishing contact, the heating effect would be avoided. One way of doing this is to interpose a cooled atmosphere between the worker and his surroundings such that air flows outward from him in all directions and thereby prevents the hot air from establishing contact. This method requires that a source of air be provided near the worker and that the volume be sufficient to cause an outward velocity great enough to offset circulation by convection within the enclosure and transfer of heat by conduction and convection through the cooled layer of air.

Effective local ventilation has long been used in industry in the form of directed streams of ventilating air so located that they flow over the position most frequently occupied by the worker. Recently basic experimental work has been done to determine the effectiveness of such air streams as a function of the velocity, volume, and state of the entering air. The data which follow (both for fluid barriers and for ventilated clothing) are taken from a report* of research conducted at the ASHVE Research Laboratory

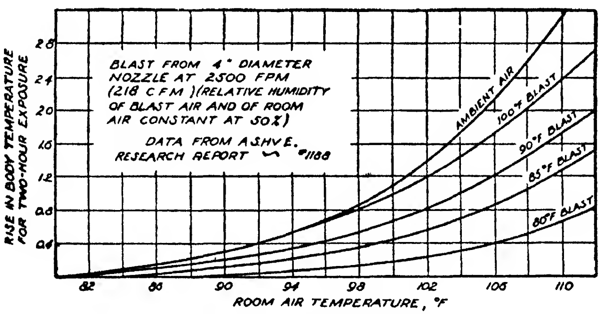


FIG. 14·3. Local Cooling by Air Blast.

The data were obtained for subjects who spent 80 per cent of the time standing, or seated at rest in the cool air stream, and the remaining 20 per cent doing moderate work in the ambient atmosphere; work was carried on for approximately one minute out of each five. When at rest the subjects were allowed to choose preferred positions with respect to a 4-in. nozzle located close to them and discharging at a velocity of 2500 fpm. The investigators found that the subjects preferred to have the nozzle so located that the air stream did not pass over the head or face; the position suggested as being most practical was at shoulder level with the air stream flowing downward so as to cover as large a part of the body surface as possible.

Figure 14·3 gives data permitting evaluation of the equivalent atmosphere corresponding to a given ambient temperature and a given tempera-

* Houghten, Ferderber, and Gutberlet, "Local Cooling of Workers in Hot Industry," *ASHVE Transactions*, Vol. 47 (1941).

ture of air blast. Increasing the volume of air in the blast, improving the contact between blast air and the subject, reducing the ratio of ambient air entrained by the cooled air stream are all ways of increasing the effectiveness of the local cooling effect and thereby mitigating the effect of the hot atmosphere.

(c) Ventilated Clothing. An unusual and highly effective method of local cooling is realized by supplying a small volume of air (20 cfm) directly into a loose-fitting airtight garment, as a pair of coveralls, worn by the subject. The air flows in from a small connection flexibly attached at the back, near the waist, and passes through the garment, escaping around the neck, ankles, and wrists. Figure 14·4 gives the experimental

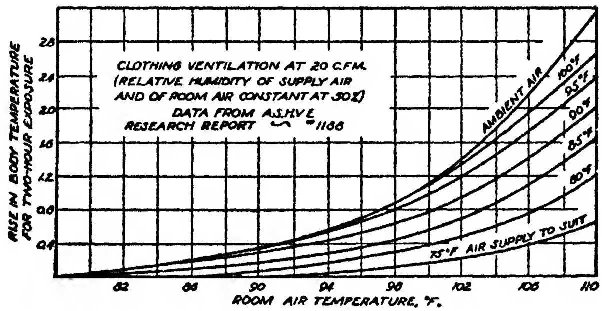


Fig. 14·4. Local Cooling by Ventilation of Clothing.

results from tests with local cooling of this kind. Note that the air volume for the ventilated clothing method is much less than that required to attain equivalent effectiveness with an air blast; it is therefore evident that the ventilated suit is an economical as well as an effective means of local cooling.

Because the ventilation volume is so small, the cooling effect with clothing ventilation is realized with less general cooling of the surroundings than results with other systems. This characteristic is advantageous in some industrial enclosures where it is desired to maintain a high general temperature level. An additional advantage of this method over local cooling by air blast is that protection is provided against heat gain by radiation from the enclosure surfaces.

When radiation is a large factor the clothing should have some insulating value in order to retard the transfer of heat absorbed at the outside surface through to the body. The outside surface may then rise above the ambient air, and a heat balance may be established at this surface such that a large part of the energy received by radiation is given up by convection to the ambient air while the smaller remaining portion which does get through is then picked up by the ventilation air passing through the clothing.

An important consideration in the industrial use of ventilated clothing is provision for immediate and rapid detachment of the supply air inlet in the event of emergency. This can be readily achieved by designing the hose connection so that detachment will be automatic in the event of extreme movement by the worker.

(*d*) SUMMARY. All methods of local ventilation have as their objective the realization of working conditions within the high-temperature space that are more acceptable than those which could be obtained with the same air volume or the same cooling capacity if it were directed toward general ventilation rather than local cooling. In some forms, the idea of local cooling for human comfort is so new as to seem impracticable, but before reaching such a conclusion one should give consideration to the fact that similar cooling systems for particular pieces of mechanical or electrical equipment have long been commonly used. The idea of bringing a supply of ventilation air to a motor or generator for the purpose of local cooling is not sufficiently new to warrant objection; the idea of bringing a similar special supply of ventilation air to a workman is based upon the same principle of maximum economy and, in many instances, is equally as practical. In particular, the method of ventilated clothing affords surprising possibilities for both economy and effectiveness in overcoming the hazards of extremely hot working spaces which are not amenable to practical or economical treatment by conventional methods.

PROBLEMS

1. The sensible heat *gain* in a working space is 600 Btu/occupant; latent heat *loss* is 400 Btu/occupant. The inside state is at 70° dbt with 90 grains/lb, and the occupants are doing heavy work. Conditioned air is supplied at 50° dbt. (*a*) Determine the requisite ventilation rate expressed in cfm/occupant. (*b*) Determine the required absolute humidity of supply air.

2. An enclosure in which there is a substantial heat source must be maintained at a temperature 10°F greater than that of the outside air. Calculate the cubic feet per minute of unconditioned ventilation air which correspond to a cooling effect of one ton of refrigeration.

3. Based on a 10°F inside-to-outside temperature difference, determine the cfm of ventilation air required to offset the heating effect of a 100-watt electric light.

4. The occupant of an unventilated enclosure experiences a rise in body temperature of 1.2°F after 2 hr of exposure. If a blast type local cooling unit were installed what would have to be the supply temperature in order to reduce his 2-hr body temperature rise to 0.5°F?

5. For the conditions of problem 4, if ventilated clothing were used, what would be the necessary air supply temperature?

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CHAPTER XV

FAN PERFORMANCE AND SELECTION

The purpose of a fan in an air conditioning system is to provide the motive power whereby the working substance will receive the energy needed to establish flow. In all systems, real or ideal, the working substance is transmitted through ducts or conduits the cross sections of which determine the velocity for a given weight or volume of the fluid. The primary purpose of a fan in an ideal system is to impart to the air the kinetic energy needed to establish the required duct velocity; the measure of kinetic energy is velocity pressure. This energy quantity represents work done on the air by the fan in raising its velocity from zero to the duct value. In the ideal, frictionless system, this alone would constitute the total work of the fan.

In addition to supplying the ideal kinetic energy requirement, a fan operating as part of a real system must impart to the air stream an additional quantity of energy equal to the losses occasioned by mechanical friction and by turbulence in the duct, the fittings, and at the entrance and exit from the system. The measure of this additional energy quantity of a real system is the static pressure. The total energy supplied by the fan must in all cases be proportional to the sum of the velocity pressure and the static pressure, that is, to the total pressure. In any system the total pressure must be directly proportional to the energy requirements of an ideal system having a duct of the same size as the actual system plus the actual energy losses, fluid and mechanical, of the real system. The first problem in connection with fans is then to investigate the manner in which a fan establishes velocity pressure and static pressure.

BASIC EQUATIONS

15·1. Velocity Pressure. The necessary kinetic energy of the air at a cross section of area A in the fan discharge duct can be determined directly from the required velocity which, in turn, is known as a function of the necessary rate of flow, Q (cfm), and the duct area, by the equation,

$$Q = Av \quad \text{or} \quad v = \frac{Q}{A} \tag{15·1}$$

The kinetic energy which must be imparted to the fluid by the fan is given by

$$\text{KE} = \frac{V^2}{2g} = \frac{V^2}{64.4} = h_{va} \tag{15·2}$$

where h_{va} is the velocity pressure expressed in feet of head corresponding to one pound of air. As a convenience in measurement, fan pressures are usually expressed in inches of water equivalent to the actual pressure in feet of air; likewise, air velocity is customarily expressed in feet per minute, v, rather than in basic units of feet per second, V. These changes can be made in equation 15·2 by noting that

$$h_{va} = \left(\frac{VP}{12}\right)\left(\frac{\rho_w}{\rho_a}\right)$$

where VP is the velocity pressure expressed in inches of water, and ρ_w, ρ_a are the respective densities of the water in the manometer and the air in the duct. Then,

$$VP = \left(\frac{12}{64.4}\right)\left(\frac{\rho_a}{\rho_w}\right)\left(\frac{v}{60}\right)^2 = 0.0000518\left(\frac{\rho_a}{\rho_w}\right)v^2 \qquad (15\text{·}2a)$$

For water at 68°F and air at standard conditions (68°F at 50 per cent relative humidity) the above equation is approximately equal to

$$VP = 0.0000000623v^2 = \left(\frac{v}{4005}\right)^2 \qquad (15\text{·}3)$$

or

$$v = 4005(VP)^{1/2} \qquad (15\text{·}3a)$$

which shows that a velocity pressure of one inch of water corresponds to a velocity of 4005 fpm.

Total pressure is measured by inserting an open tube into the air stream with the tube mouth facing upstream. The fluid upstream from the mouth is decelerated as it approaches and undergoes a virtual isentropic compression so that the increase in pressure at the tube mouth (over that due to static pressure alone) is a definite function of the stream velocity and is numerically equal to the term VP in equations 15·3 and 15·3a.

Velocity is imparted to the air passing through a centrifugal fan as a result of centrifugal force acting on the air which enters axially and is then thrown out radially through the blades and also by the rotary movement of the blades which causes the air to leave the blade tip with a component of velocity tangent to the path and equal to the tip speed. The average area of the fan housing enclosing the rotor is considerably less than that of the discharge duct so the velocity of the air as it leaves the blade tip is greater than duct velocity; thus some of the energy present in kinetic form in the air leaving the blades must be converted to flow work by the time the air stream reaches the duct. The scroll type housing used with centrifugal fans serves as a means of reducing the velocity and converting some of the stored kinetic energy in the air leaving the blade tips to transitory flow work in the air passing through the duct.

15·2. Static Pressure. The pressure range over which an average centrifugal fan operates is relatively small when expressed in pounds per square inch or in inches of water, but it must be remembered that the head should be thought of in terms of the fluid handled by the fan. In such terms the actual head on a typical fan system is not at all inappreciable since it amounts to approximately 70′ of air for each 1 in. of water pressure. On the pressure-volume diagram, however, the pressure change appears very small and the corresponding change in volume (Fig. 15·1) is likewise small. The work done on the air during the actual process of compression is insignificant in comparison with the work quantities representing entrance and discharge from the fan.

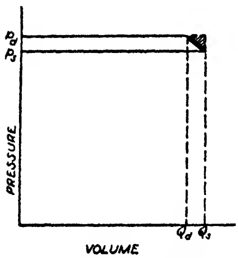

Fig. 15·1. Fan Work.

Neglecting the change in volume of the air during compression, the net work input to the fan, exclusive of that required to impart velocity to the air stream, is given by

$$W_{\text{compression only}} = 144(p_d - p_s)Q_s = 144\Delta pQ \qquad (15\cdot4)$$

In Fig. 15·1, the error resulting from use of equation 15·4 is indicated by the work quantity corresponding to the shaded area; if Q_d were used in place of Q_s, the work calculated by this equation would be slightly closer to the exact value, but the error would no longer be conservative since the calculated work would be less than the actual work.

Expressed in terms of the static pressure (duct " bursting " pressure) in inches of water, SP, equation 15·4 becomes

$$W = 5.19(\Delta SP)Q \qquad (15\cdot4a)$$

Neglecting changes in the volume and temperature of the air passing through the fan the static pressure increase is thus seen to be directly proportional to the flow work imparted to the air stream by the fan.

15·3. Velocity Pressure-Static Pressure Relationship. The static pressure produced by a fan occurs in part as a result of centrifugal compression of the air passing through the fan and in part as a result of transformation of kinetic energy to flow work in the fan casing. The principal difference between the pressure characteristics of different types of fan blading systems is due to variation in the fraction of duct static pressure derived from centrifugal compression. In general, losses are greater during the process of conversion than during compression so the efficiency of a fan usually can be expected to increase as the fraction of static pressure coming from compression becomes greater. Figures 15·2abc illustrate the three basic types of blade arrangement and show the resultant tip velocity of air as it leaves each type of blade.

Figure 15·2*b* shows a radial blade rotating clockwise about the fan axis. The air passing through the fan travels radially along the blade, leaving the tip with a velocity with respect to the blade of u_2. At the same time air at the tip has a velocity component, u_2', in the direction of blade travel. The absolute velocity of air leaving the radial blade is then equal to the resultant V_2, as shown in the figure. For the same numerical value of the u and u' velocity components, the resultant velocity of the air stream at the blade tip is greatest for a forward blade (Fig. 15·2*a*) and least for a backward blade fan (Fig. 15·2*c*). Considering that all three types of blades are radial at the heel and allowing for viscosity drag effects at entrance, conditions at the heel would be the same for all types; the entering component of velocity with reference to the wheel being u_1, the component in a direction tangential to the path of the heel being u_1' and the absolute velocity of the entering air being V_1, where all velocities are expressed in feet per second.

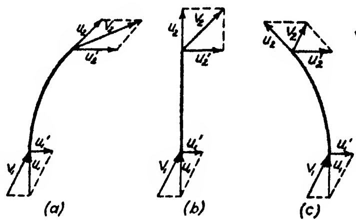

FIG. 15·2. Fan Blade Velocity Vectors.

The change in total pressure of the air passing through the fan blades is equal to the sum of the increases in static and velocity pressures across the blades and, for an ideal system, is given by

$$TP = SP_{\text{blades}} + VP_{\text{blades}}$$

$$= \frac{(u_2')^2 - (u_1')^2}{64.4} + \frac{u_1^2 - u_2^2}{64.4} + \frac{V_2^2 - V_1^2}{64.4} \qquad (15\text{·}5)$$

The first two terms represent the gain in static pressure across the blades, the first term being expressed in terms of the change in heel to tip circumferential velocity and representing a pressure gain from centrifugal force, and the second term as a static gain due to change in the relative velocity of the fluid with respect to the wheel. Note that, for all three blade types of Fig. 15·2 the static pressure gain is the same.

The last term of equation 15·5 gives the increase in kinetic energy, or the velocity pressure increase, occurring during passage of the air through the fan wheel. This term depends on the absolute leaving velocity and hence is governed by the type of blading. Though the static pressure increase across the blades of all three types shown in Fig. 15·2 would be the same, the velocity pressure and hence total pressure gains would be greatest for the forward blade fan and least for the backward blade type. Thus as the blade moves progressively from a forward angle to a backward angle, the ratio of static pressure gain to total pressure gain increases.

Some of the velocity pressure possessed by the air leaving the blade tips can be converted to static pressure in the casing, but as already mentioned this conversion is a less efficient method of realizing a static gain than the direct methods which occur during passage through the fan wheel. Referring again to the blading systems of Fig. 15·2, the total pressure of the backward blade can be raised to equal that of the forward blade unit by increasing the speed. But it is unnecessary for the speed increase to be great enough to raise the absolute leaving air velocity V_2 to the same value for backward as for forward because the static pressure gain across the higher speed backward blade fan would exceed that of the forward blade fan; thus a smaller fraction of the total pressure would have to come from kinetic energy of the air leaving the blades. Therefore, for equal air volumes supplied in the discharge duct with equal static and velocity pressures, a backward blade fan would have to operate at higher speed than a forward or radial blade fan of the same size, but the absolute velocity of air leaving the backward blade unit would be lower than for either of the other fans and the per cent of static pressure gain occurring across the blades (and therefore most efficiently) would be greatest for the backward blade fan.

Conversely, if forward and backward blade fans are operated at relative speeds such that the absolute air velocity leaving the blades is the same and if the fraction of velocity pressure converted to static pressure in the scroll is identical for the two fans, the backward blade will supply air at a greater ratio of static to total pressure. Since the static pressure is used in overcoming resistance, it follows that this higher ratio is advantageous in that it reflects the greater capacity of the air from a backward blade fan to overcome friction in the ducts and fittings.

15·4. Air Horsepower. From equation 15·4a (note that 33,000 foot-pounds per minute are one horsepower), the ideal horsepower which must be delivered to the air to produce the static pressure increase SP is

$$AHP_s = \frac{5.19(SP)Q}{33{,}000} = \frac{(SP)Q}{6350} \tag{15·6}$$

where AHP_s is the static air horsepower.

Similarly the total air horsepower is determined from the equation,

$$AHP_t = \frac{(TP)Q}{6350} \tag{15·7}$$

where TP is the total pressure gain of the air passing through the fan. A change in density does not alter the air horsepower per unit weight of air since the density corrections to pressure and volume are compensating. Conversely, as the density varies, the weight of air handled by a given fan and duct system will vary, but the volume handled will remain the same; thus the actual air horsepower required by a fixed system operating with air of variable density would vary directly with the density.

Fan efficiencies are usually expressed in terms of the static pressure and the total pressure, as follows:

$$\text{Static efficiency} \quad e_s = \left(\frac{AHP_s}{BHP}\right) 100 \tag{15·8a}$$

$$\text{Total efficiency} \quad e_t = \left(\frac{AHP_t}{BHP}\right) 100 \tag{15·8b}$$

The range of efficiency for centrifugal fans of the usual type is almost independent of size (for a given set of fans) and varies in the approximate range from 40 to 80 per cent.

Fan ratings are sometimes expressed in terms of capacity at point of maximum total efficiency and sometimes in terms of capacity at maximum static efficiency. Though neither method has gained universal acceptance, the latter is more likely to be useful as it defines a rating point closer to the actual point of operation of most commercial fan types; the rating based on total efficiency may occur outside the usual operating range and, in some cases, cannot be readily fixed.

FAN PERFORMANCE

15·5. Fan Types and Characteristics. The performance characteristics of forward blade, radial, and backward blade fans are shown in Figs. 15·3, 15·4, and 15·5. Forward blade fans are characterized by a point of inflection in the static pressure curve (Fig. 15·3) and by a steadily rising power curve. The inflection is not important when such a fan is operating alone on a given airway, but may lead to operating difficulties when two such fans are placed in parallel.

Of greater importance is the characteristic rising power curve of such a fan. Note that when operating under shut-off conditions the static pressure and power have fixed values; as the resistance of the system is decreased (as by opening dampers) the static pressure first falls, then rises, and finally falls again as the volume of air handled and the power requirements both steadily increase. Consideration of this relationship between static pressure and power leads to the conclusion that a forward blade fan must be equipped with a sufficiently oversize motor to handle the load corresponding to the minimum resistance against which the fan might discharge. Frequently the possibility of accidental operation against resistances very low in comparison with the rated resistance of the system makes necessary the selection of a much larger motor than the service would normally justify.

A further disadvantage of forward blading is that the static pressure curve is relatively flat and its maximum occurs in or near the usual operating range. This means that a large change in capacity may occur as a result of minor pressure adjustments or changes in the system. A slight error in estimating the resistance of a given airway may thus lead to selection of a fan whose output will depart considerably from the desired capacity.

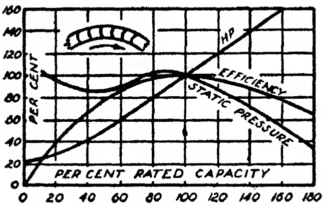

FIG. 15·3. Forward Blade Characteristics.

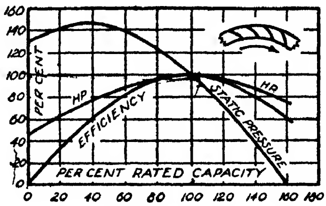

FIG. 15·4. Backward Blade Characteristics.

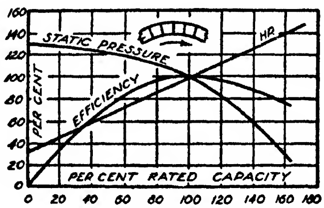

FIG. 15·5. Radial Blade Characteristics.

The advantages of the forward blade type are that it provides a maximum air volume for a given wheel size and speed and is therefore lighter, smaller, and responsible for less noise due to mechanical vibration than other types. Offsetting the effect of lower speed on noise reduction is the higher absolute velocity of air leaving the blade tips of a fan of this type; noise originating as a result of fluid motion and turbulence is greatest from a forward blade type fan, and since this source of noise is the controlling factor in most commercial fans it follows that the forward blade type is likely to be more noisy than either of the others. As already mentioned the greater conversion of velocity to static pressure which takes place in forward blade fan casings results in a loss of efficiency.

The backward blade type of fan (Fig. 15·4) does not have a point of inflection in its static pressure curve, and does have a power characteristic of the limit load type; maximum power is required at a point close to the rated capacity, so a motor capable of providing rated output will be more than capable of handling any other load which can occur at fixed speed. The steeper static pressure curve of the backward blade fan means that only minor changes in volume will accompany small changes in system resistance; failure to predict accurately the resistance of a new airway will therefore have less effect on the actual volume of air handled by the fan if it is of the backward type. For the same total pressure and air volume

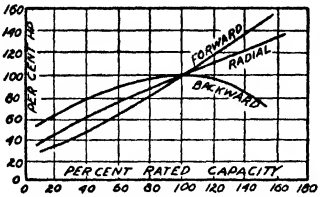

FIG. 15·6. Comparative Horsepower Curves.

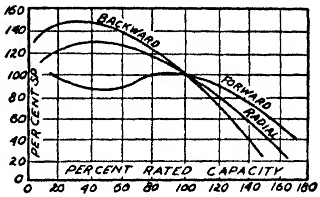

FIG. 15·7. Comparative Static Pressure Curves.

a backward blade fan must operate at approximately twice the speed of a forward blade fan of equal size.

The characteristics of fans having radial blades (Fig. 15·5) are between those of the forward and backward types. Figures 15·6 and 15·7 show the assembled horsepower and air static pressure curves, respectively, of the three types of fans. Note that the positions of the horsepower curves with respect to one another are based on the rated values for each fan; the absolute values of horsepower would not have the same value for all three fans operating at the same absolute capacity.

15·6. Series Operation. The combined characteristics for series operation of two identical low-pressure centrifugal fans are shown in Fig. 15·8. Neglecting the difference in density of air passing through the two fans, the velocity pressure for the combination, VP_c, must be the same as the velocity pressure of either fan, VP_1, when operating alone. The combined total pressure is twice the total pressure of a single fan, so the static pressure of the combination, h_c, must be given by

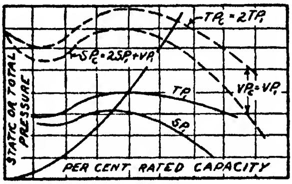

FIG. 15·8. Series Operation of Fans.

$$SP_c = TP_c - VP_c = 2TP_1 - VP_1 \tag{15·9}$$

and also by

$$SP_c = TP_1 + (TP_1 - VP_1) = TP_1 + SP_1 = 2SP_1 + VP_1 \tag{15.9a}$$

Thus, for the same capacity, two identical fans in series will develop more than twice the static pressure of a single fan; the total pressure developed by all but the first fan of a group in series appears entirely as a static pressure increase for the combination. The determination of the pressure characteristics of any group of fans in series can be investigated by the method already outlined even if the fans are not identical; it can also be extended to include high-pressure fans if correction is made for the density change of the air passing through the different fans of the series.

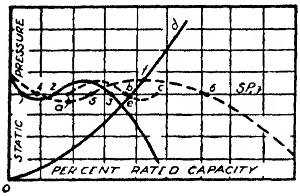

FIG. 15·9. Parallel Operation of Fans.

15·7. Parallel Operation. Figure 15·9 shows the single and combined characteristics of two identical low-pressure centrifugal fans operating in parallel. For a given static pressure the capacity of the combination must be twice that of a single fan. If both fans are operating at the same point on their characteristics their individual deliveries will necessarily be equal and the operating point of the combination will then be such that $Q_c = 2Q$; on this basis the equal load characteristic, 4,5,6 of Fig. 15·9, is established.

But a forward blade fan having a characteristic such as shown in Fig. 15·9 has three possible points of operation corresponding to each particular value of the static pressure over a limited range; at SP_1, for example, the single fan can deliver Q_1 or Q_2 or Q_3. This multiplicity of capacities at a fixed static pressure does not have any important effect on a fan operating alone, because the airway characteristic will automatically fix the one and only value of Q which can be delivered against a fixed static pressure; when operating in parallel, however, the stabilizing influence of the airway is only partially effective, as will be seen.

Consider two fans having the individual characteristic shown in Fig. 15·9, each operating against a static pressure of SP_1, but one fan delivering Q_1 and the other Q_2; the operating point for combined parallel operation would then be

$$Q_a = Q_1 + Q_2$$

which is not a point on the uniform load characteristic, 4,5,6. Similarly, one fan might be delivering Q_1 and the other delivering Q_3 to give an operating point

$$Q_b = Q_1 + Q_3$$

or one fan delivering Q_2, the other Q_3 to give

$$Q_c = Q_2 + Q_3$$

The points a, b, and c are possible points on the combined characteristic for parallel operation, but they do not represent equal load distribution and do not fall on the stable operating characteristic, 4,5,6. In a similar manner other non-equal load points can be found, thereby establishing the dotted line characteristic *a,b,c* of the figure.

Consider two fans operating in parallel on a system defined by the airway characteristic *od* of Fig. 15·9. Either of the two points *e* and *f* fall on both the airway and the combined fan characteristic and therefore represent possible operating points of the system. Since both fans are operating at the same constant speed there is no apparent difference between them and no way of determining whether they will, in combination, share the load equally or unequally. This condition therefore represents unstable operation, and it is impossible either to prophesy or assure operation of the system at either point *e* or *f*. Further, *e* and *f* are not functionally related, so a change in operation of the system from one point to the other would be abrupt rather than continuous.

Aside from discontinuity of the combined characteristic, parallel operation at non-equal loads is undesirable because the efficiency of both fans is reduced, and the power requirements are therefore greater, for the same capacity, than would be the case for equal load. In many cases the shift from stable to unstable operation is accompanied by a reduction in capacity, static pressure, and air horsepower, but an increase in brake horsepower. Because of the unstable characteristic accompanying non-equal load distribution, forward blade fans are not, in general, satisfactory for parallel service.

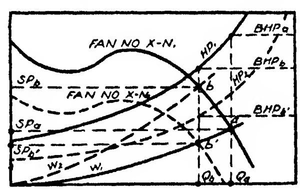

Fig. 15·10. Volume Control.

15·8. Adaptation of Fan to Airway. The resistance curve of a given airway consisting of ducts, fittings, filters, air conditioning equipment, etc., is, in general, parabolic in shape, the resistance increasing approximately as the square of the volume handled. For a particular airway there is one and only one resistance curve, and the fan delivering air through that airway must be operating at some point on the curve. Refer to Fig. 15·10 and consider a particular fan, identified as No. X, operating at speed N_1 and delivering air through an airway having the resistance curve indicated by W_1. The point of operation must fall on both the fan and airway characteristic curves and is therefore point a; the static pressure, capacity, and power requirements of this operating point can be read directly from the figure. Suppose that it is now desired to reduce the flow rate to a new value, Q_b; this can be accomplished by either of two methods or by any combination of them.

1. Increasing the resistance of the airway (as by partially closing a damper) establishes a new airway curve W_2 which intersects the fan characteristics at point b for which the capacity has the desired value; corre-

sponding static pressure and horsepower can be determined from the figure.

2. Reducing the fan speed to N_2 changes the fan characteristic and permits operation on the original airway with reduced volume. The operating point is then b' of Fig. 15·10 with corresponding values of the static pressure and horsepower. Note that the power required at point b' is much less than that at b, indicating that speed reduction is a more economical method of decreasing the volume than is alteration of the airway. For maximum operating effectiveness (as measured by power requirements) the airway resistance should at all times be kept at a minimum and volume adjustments made by varying the fan speed.

15·9. Fan Laws. Fan laws are statements concerning the functional relationships existing among capacity, pressure, power, speed, and size of homologous fans which are operating at the same point on their characteristic curves; that is, fans geometrically similar and operating at the same efficiency. When size is not a variable and the density of air handled does not change, three basic laws can be developed for operation of a given fan at variable speed:

1. Capacity is proportional to the product of area by velocity (Equation 15·1). Since all areas of a given size fan remain constant and velocities through such areas are proportional to the speed,

$$\frac{Q_1}{Q_2} = \frac{N_1}{N_2} \tag{15·10}$$

where N is the fan speed in revolutions per minute.

2. For operation at the same point on the characteristic curve all property ratios must remain the same. Thus the ratio of static pressures must be equal to the ratio of velocity pressures at any fixed point within the fan. The velocity pressure leaving the blades (equation 15·5) is proportional to the square of the tip speed and therefore to the square of the fan speed in revolutions per minute. Thus,

$$\frac{SP_1}{SP_2} = \left(\frac{N_1}{N_2}\right)^2 \tag{15·11}$$

3. The horsepower (either brake or air) is proportional to work, which is proportional to the product of capacity and total pressure; thus (from equations 15·10 and 15·11),

$$\frac{HP_1}{HP_2} = \left(\frac{N_1}{N_2}\right)^3 \tag{15·12}$$

The three above equations are established for the specific condition represented by operation at the same point on the fan characteristic; in their derivation nothing is said as to the possibility of applying them to actual conditions represented by a fan discharging through a particular airway. Fortunately, however, the same equations are applicable to the latter case. This can be shown as follows: For flow through a given airway the resistance varies approximately as the square of the volume. Thus the static pressure required must increase with capacity by the equation

$$\left(\frac{Q_1}{Q_2}\right)^2 = \left(\frac{SP_1}{SP_2}\right)_{\text{required}} \tag{15·13a}$$

But from equations 15·10 and 15·11, the available static pressure varies with capacity by

$$\left(\frac{Q_1}{Q_2}\right)^2 = \left(\frac{SP_1}{SP_2}\right)_{\text{available}} \tag{15·13b}$$

From equations 15·13*a* and *b* it is evident that, as the speed changes, the point of intersection of the fan characteristic and airway characteristic will always occur at the same per cent of rating; equations 15·10, 15·11, and 15·12 are therefore applicable to a variable speed fan operating on a fixed airway with air of constant density.

Variations in performance resulting from density changes can be readily determined. The capacity, Q, and the pressure measured in feet of air do not change with density whereas the horsepower, the head in inches of water, and the weight of air handled vary directly with density. These relationships can be combined with the fan laws to obtain the effect of a density change.

Three additional fan laws are frequently of use in comparing the constant-speed performance of fans differing only in size. These are:

4. $$\frac{Q_1}{Q_2} = \left(\frac{D_1}{D_2}\right)^3 \tag{15·14}$$

where D is the diameter of the fan wheel or any other dimension or number which is proportional to wheel diameter. This law follows from equation 15·1, noting that A is proportional to D^2 and v proportional to D.

5. $$\frac{SP_1}{SP_2} = \left(\frac{D_1}{D_2}\right)^2 \tag{15·15}$$

This equation is obtained by the same reasoning used to establish equation 15·11; velocity is proportional to diameter, and head is proportional to velocity squared, so head must be proportional to diameter squared.

6. $$\frac{HP_1}{HP_2} = \left(\frac{D_1}{D_2}\right)^5 \tag{15·16}$$

Equation 15·16 follows from 15·14 and 15·15, just as equation 15·12 followed from 15·10 and 15·11.

The possibility of applying equations 15·14, 15·15, and 15·16 to fans operating on the same airway can be investigated by combining equations 15·14 and 15·15 to determine the variation of capacity as a function of available static pressure, giving

$$\frac{Q_1}{Q_2} = \left(\frac{SP_1}{SP_2}\right)^{3/4}_{\text{available}} \tag{15·17}$$

But equation 15·13*a* shows that the capacity-pressure relationship for a fixed airway differs from that of equation 15·17, so equations 15·14, 15·15, and 15·16 cannot be used to determine performance of two fans operating on an airway of fixed characteristic. The difference in application between equations 15·10, 15·11, 15·12 and 15·14, 15·15, 15·16 is illustrated in Fig. 15·11. A fan D_1, operating at speed N_1, is connected to a given airway and delivers air at point *a*. If the fan speed were increased to N_2, the new operating point on the fan characteristic would also fall on the airway characteristic (by equations 15·13*a* and *b*) and the fan would therefore deliver air at point *b*. If, however, the original fan were replaced by one of larger size, D_2, but operating at the same speed, N_1, the same point on the characteristic curve of the larger fan, when operating against static pressure SP_2, would be a point *c* which does not lie on the original airway characteristic. Alteration of the airway characteristic is then necessary in order to permit operation of the larger fan at the same fan characteristic point; otherwise the larger fan will operate at a new point, *c′*, on its characteristic curve, and in this event equations 15·14, 15·15, and 15·16 are not applicable.

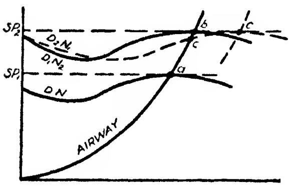

FIG. 15·11. Airway Adaptation.

15·10. Selection of Centrifugal Fans. The problem of selecting a particular type and size of centrifugal fan for use with a given airway system is complicated by the need for considering many economic and other factors which are not susceptible to simple quantitative evaluation. Weight and space limitations may dictate the type even when operating economy or performance characteristics might better be realized with some other selection. First cost may necessitate selection of a smaller fan to operate at a point beyond rated load and hence at reduced efficiency. The system characteristics may require a fan with a relatively flat static pressure-capacity curve even when such a selection means greater power requirement. For all these reasons and for many others, tangible and intangible, the use of selection charts or of other more or less mechanical methods of choosing the "best" fan for a particular service must be carried out with considerable care, and the final selection must, in any case, depend very largely on the judgment and experience of the engineer.

The particular hazard connected with charts or graphs for fan selection is that such devices, when used incautiously, overemphasize some factors at the expense of others and lead, therefore, to erroneous generalizations concerning the relative merits of the fans from which selection is to be made. A fan selection chart can easily be designed from which one can choose directly the size and type of fan which will operate on a known airway and deliver the required air volume with minimum power requirements; the hazard is in assuming that the fan requiring the least power is necessarily the "best" for the service in question. In many cases it will be the logical choice, but not infrequently factors other than operating cost will outweigh in importance the aerodynamic efficiency of the fan. Recognizing these limitations, some form of selection chart is nonetheless extremely helpful in evaluating the relative performance of fans and, particularly, in selecting the most efficient size and type of fan from a group which, in other respects, may be equally well suited for the service.

Basically, methods of fan selection are of two kinds: (1) those in which the selection system is intended to show the point on the rating curve at which different sizes of the same type of fan would operate; (2) those intended to permit determination of the correct size of any of a number of different types of fans each to operate at its point of maximum efficiency. Selection charts of the first kind are particularly useful in representing complete performance data for a series of homologous fans, and special forms of such charts have been developed and are used by certain of the fan manufacturers; all such charts are constructed on the basis of some form of generalized variable derived from the fan laws and used as a coordinate against a static efficiency curve. One such generalized variable is the so-called specific speed N_s, which is a constant for all sizes of a given type of fan when operating at the same rating point; from the fan laws the specific speed can be derived as

$$N_s = \frac{NQ^{1/2}}{(SP)^{3/4}} \tag{15·18}$$

Other generalized variables have been obtained and ingenious methods established for graphically evaluating them in terms of the basic characteristics of the fan and airway.

The second kind of selection chart is of greater value to the engineer who wishes to investigate the relative performance of fan types as well as sizes, but it is subject to the disadvantage, not present with the first type, that selection is limited to the point on the rating curve corresponding to maximum efficiency. Frequently, however, this limitation is not serious as the efficiency differences indicated at best rating point are likely to be maintained throughout the section of the characteristic performance curve within which the selection is to be made. Selection charts of the second kind can be constructed on the basis of the specific speed concept or directly from a consideration of the fundamental fan laws. The task of construction is largely a matter of plotting the fan law relationships for one type of homologous fan series and then using this basic background for comparing the performance characteristics of as many other types of fans as may be desired; the graphical construction for a single fan type will suffice for showing comparative performance of a limitless number of other types of fans. While the time needed to set up the original selection chart may be out of proportion to the value of data which it gives concerning the one fan type, the same chart can be used as a basis for comparison of as many other fan types as may be desired. In the paragraphs which follow, complete details are given for the construction and use of a fan selection chart based on the fan laws and intended to permit selection of the most efficient size and type of fan for any system for which the capacity and static pressure are known.

15·11. Maximum Rating Point Fan Selection Chart. Experimental data necessary to the construction of a fan selection chart of this type consist of the static pressure, capacity, and brake horsepower of any size fan of a given type when operating at any one speed, but at the rating point corresponding to maximum static efficiency. From this one set of performance data a selection chart can be constructed and used to determine the size and operating speed of a fan from the same homologous series which will carry any particular load at a rating point closer to maximum static efficiency than could any other size fan from the same homologous series. The required basic experimental data for any given type of fan can be obtained from the manufacturer's catalogues or rating tables by one of the following methods:

1. If a set of characteristic curves (similar to Figs. 15·3, 15·4, or 15·5, but with absolute rather than per cent ratings) is available for any one size and speed of fan the values of Q_m, SP_m, and BHP_m corresponding to the maximum static efficiency can be read directly from the curves.

2. Characteristic curves with absolute rating scales may not be available, but per cent rating curves similar to Figs. 15·3, 15·4, and 15·5 are often included in fan catalogues. From such a set of curves the values of Q_m, SP_m, and BHP_m can be calculated providing values of Q_1, SP_1, and BHP_1 are available for any operating point of any size fan of the series, at any speed. The latter set of data can be obtained from multi-rating tables, rated-capacity tables, or any other tabular or graphical source of specific operating information.

Knowing Q_1, SP_1, and BHP_1 for any rating point, the static air horsepower is calculated by equation 15·6 and the corresponding static efficiency then determined from equation 15·8*a*. Entering the ordinate of the characteristic curve at the calculated static efficiency, the per cent of rated capacity representing Q_1 is read on the abscissa, and the capacity at maximum static efficiency is then calculated by dividing Q_1 by its corresponding per cent of rating. In the same manner SP_m and BHP_m are calculated.

3. Frequently no form of characteristic curve is available and specific information is not given in the catalogue concerning the position on the rating curve of experimental points for which data are published. In such cases the available data can be used to establish a characteristic curve and the values of Q_m, SP_m, and BHP_m then determined as in the first method described above. The characteristic curve is established as follows:

a. If data are available for rating points at the same speed these can be plotted directly to give a rating curve.

b. If constant-speed data are not available, a convenient value of the speed can be arbitrarily selected, and data from such points as are available can be extrapolated by equations 15·10, 15·11, and 15·12 to the assigned speed; these points can then be plotted to give the desired characteristic curve, and the values of Q_m, SP_m, and BHP_m can be read therefrom. By a judicious selection of data from the multirating or other tables, the necessary correction for selected speed can be kept small.

4. Multirating tables sometimes indicate in bold-face type the operating points, at a given static pressure and outlet velocity, which most closely approach the rating point for maximum static efficiency. In such cases the actual points indicated will individually depart slightly from maximum rating, but they can be used to determine maximum efficiency by a simple graphical method based on the assumption that, on the average, departure of individual points from maximum occurs equally both above and below 100 per cent rating. This method will be described in a later section.

The use of bold-face rating points to determine the point of maximum efficiency is, of course, limited to those cases where the bold-face ratings are given as the most efficient operating points for the corresponding pressure and outlet velocity; not infrequently such ratings are given as the manufacturer's recommendation for selection of fans to operate at some point beyond 100 per cent rating; that is, to sacrifice operating effectiveness in order to reduce first cost by permitting selection of a fan somewhat smaller than would be needed if maximum efficiency were to be realized. Selection at points beyond 100 per cent rating is also desirable in order to take advantage of a static-pressure curve slope best adapted to the particular service; in all such cases the criterion of maximum static efficiency which forms the basis of this form of fan selection chart does not hold. The actual selection chart, once it has been constructed, can be used to select undersize fans, but the bold-face data from the multirating tables cannot be used to provide the needed rating data used in the chart construction.

Having determined Q_m, SP_m, and BHP_m by any one of the above methods the actual construction of the complete selection chart can now be carried out in five steps:

1. As an abscissa establish a linear capacity scale extending from zero to the maximum number of cubic feet per minute for which the selection chart is to be used. On the vertical coordinate establish a speed scale, expressed in revolutions per minute, from zero to the maximum revolu-

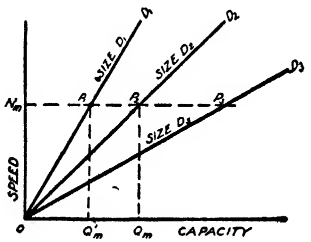

Fig. 15·12.

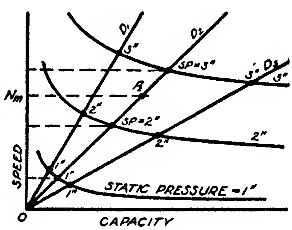

Fig. 15·13.

tions per minute at which fan operation is to be considered. Entering the capacity scale at Q_m (Fig. 15·12) rise to intersection with the speed N_m for which the rating data apply; this intersection, P_2, is one operating point for the size of fan for which the data were obtained. From a consideration of the first fan law, equation 15·10, the capacity is directly proportional to the speed, so a straight line, OD_2 of Fig. 15·12, from the origin through operating point P_2 is the locus of all maximum efficiency rating points of this particular size and type of fan.

2. The operating locus of other size fans from the same homologous series can now be readily determined by application of the fourth fan law, equation 15·14, which permits direct calculation of the optimum capacity Q'_m of a fan having wheel diameter D_1 when operating at the same speed N_m as the original unit of the diameter D_2. The intersection of Q'_m and N_m, P_1, gives an operating point for the second fan size and permits drawing the locus OD_1 for this unit. In the same manner loci can be constructed for as many sizes of fans of the same homologous series as may be desired. Thus optimum performance of all sizes of one type of fan can be shown by a series of radial lines passing through the origin.

3. The static pressure corresponding to point P_2 is known, but in all probability it does not happen to be an integer. As a first step in constructing constant static pressure lines on the chart it is necessary to determine the points on the line OD_2 where the static pressure has non-

fractional values, as 1 in., 2 in., 3 in. This is done by applying the fan law for static pressure change as a function of speed with fan size constant (equation 15·11); thus,

$$N = N_1 \left(\frac{SP}{SP_1}\right)^{1/2}$$

By calculating the speed corresponding to each desired value of the static pressure, points along the line OD_2 can now be readily located (Fig. 15·13).

Determination of static pressure points on lines for other size fans follows from a combination of equations 15·11 and 15·15, giving

$$N_1 = N_2 \frac{D_1}{D_2}$$

for constant static pressure. Having used this equation to determine points of constant static pressure for a number of different fan sizes (as D_1, D_2, and D_3 of Fig. 15·13), lines of constant static pressure can be drawn through these points thereby establishing a static pressure background on the fan selection chart.

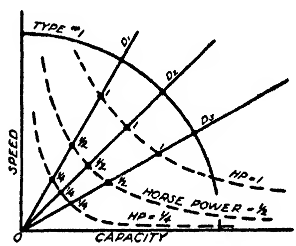

FIG. 15·14.

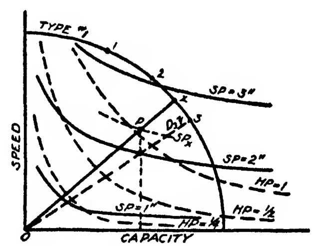

FIG. 15·15.

4. The brake horsepower corresponding to operation of size D_2 at point P_2 is known from the original data. Using the fan law (equation 15·12) for variation of HP with N when size is constant, points can be determined along the line OD_2 at which the HP has selected values, as ¼, ½, 1, or 2 (see Fig. 15·14). Location of corresponding horsepower points on operating lines for other size fans can be accomplished by combining equations 15·12 and 15·16 to obtain the relationship between size and speed when power is constant,

$$N_1 = N_2 \left(\frac{D_1}{D_2}\right)^{5/3}$$

From this equation points are determined at a sufficient number of sizes to permit drawing in the dashed lines of constant horsepower as shown in Fig. 15·14.

5. To complete the fan selection chart for the one type of fan the static pressure and horsepower curves of Figs. 15·13 and 15·14 should be superimposed to constitute a complete background for the different fan size locus lines. The resulting three sets of curves together with horizontal and vertical backgrounds for speed and capacity would be confusing; a great simplification can be realized by using points instead of lines to show the relative location on the chart of the different size fans. Refer to Fig. 15·14 and draw an arc of any convenient radius through the origin of coordinates and intersecting the locus lines for different size fans at points labeled D_1, D_2, D_3, etc. This arc, by definition, constitutes the locus of base points for all sizes of fan type 1 and the arc alone need be shown on the selection chart.

Figure 15·15 shows the complete selection chart for type 1. As an example of its use consider that it is desired to select a fan of this type to deliver Q_x cubic feet per minute against a static pressure of SP_x inches of water. Entering the chart at Q_x rise to intersection with curve for SP_x, thereby establishing the desired operating point P. From the origin draw a straight line through P and extend it to intersect the type 1 arc at point x. This intersection shows that fan size D_3 is too large and size D_2 too small; D_3, however, is the fan which will come closest to operating at maximum efficiency. Drawing in the fan locus line OD_3 shows that fan D_3 would deliver the required air volume Q_x' at maximum static efficiency provided the airway characteristic were altered so that the static pressure at required capacity would be 2 in. instead of SP_x. If the airway cannot be altered no fan of this type (unless specially constructed at a size corresponding to point x) is capable of meeting the required load while operating at its maximum rating point. The exact horsepower required by fan D_3 when operating on the unaltered airway cannot be determined from the selection chart as the chart data are applicable only for operating points corresponding to maximum rating. However, the slope of the efficiency curve in the close vicinity of optimum rating is usually quite flat, so reasonably accurate approximations to non-optimum power can be read from the chart.

Use of the basic selection chart for fan types other than the one for which it was constructed can be readily accomplished by addition of a new speed scale for each additional type and determination of a correction coefficient to be applied to the values of horsepower given for the basic type. As an example consider that data are available for one optimum operating point of some one fan of type 2; these data include Q', SP', N', BHP'. Enter the basic chart on the capacity scale at Q' (Fig. 15·16) and rise to intersect the static pressure curve corresponding to SP'; this establishes the operating point R, corresponding to the given data for the second type of fan, but the speed and BHP which appear on the chart for point R do not agree with the speed N' and the horsepower BHP' for the fan.

To construct a corrected speed scale, first move to the left from O along the capacity scale any convenient distance OO'. From point O' draw straight lines to points on the vertical speed scale; each such line is then a constant-speed line having the value indicated on the vertical scale (see Fig. 15·16). Now from point R extend the horizontal to the left to intersect the constant-speed line for N' and through this intersection draw a vertical line which will then be the correct speed scale for all fans of type 2.

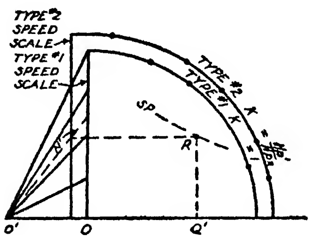

FIG. 15·16.

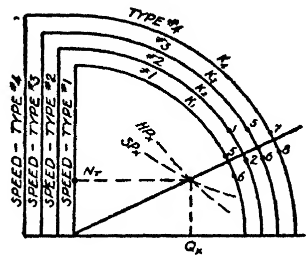

FIG. 15·17.

The correction coefficient for brake horsepower is obtained by dividing the known power requirement at point R, BHP', by the chart value; the quotient is a multiplying correction coefficient which can be used with chart values of the horsepower for all type 2 fans. This coefficient is conveniently marked on the arc representing the locus of base points for all fans of type 2, and the arc is extended to connect with the type 2 speed scale.

As many types of fans as may be desired can be similarly shown on the single fan selection chart. The background is the same for all fan types, each type added requiring only a speed scale, selection arc with fan sizes marked thereon and the correction coefficient for calculating the actual brake horsepower requirements from the values shown on the power lines of the background. The type of fan for which the power coefficient is smallest will obviously be the one having greatest efficiency. The value of the power coefficient is, of course, a constant for each fan type, and the variation of this coefficient between types is a good first indication of the relative desirability of the respective fans. As previously mentioned, operating efficiency is not alone a sufficient criterion for fan selection, but with differences of over 20 per cent sometimes appearing in the power coefficients of competitive centrifugal fans the efficiency is a factor that, in some cases, may easily be of major importance.

Figure 15·17 shows diagrammatically a selection chart including four types of fans. If the problem is to select a unit capable of delivering Q_x cubic feet per minute against a static pressure of SP_x, the required operating point is located on the chart and a line through that point and the origin then drawn and extended to intersect the four arcs. From the figure it is seen that type 1, size 5 is closest of its type, but slightly too small to permit operating at maximum efficiency on the design airway; type 2, size 2 is closest of its kind, but somewhat too large; type 3, size 6 permits maximum efficiency of operation (for its kind); type 4, size 7 operates at close to maximum efficiency, but is slightly too small. Assuming that the loss of efficiency of all types due to use of a slightly offsize fan is negligible, the horsepower of each of the sizes and types of fans considered is obtained by multiplying the chart horsepower, read at the operating point, by the respective correction coefficients, K_1, K_2, K_3, and K_4. Similarly, the approximate required operating speed of each of the fans can be obtained by moving horizontally left from the operating point to intersection with the speed scale for each of the respective fan types. If the airway characteristic were to be altered to permit each of the fans to operate at maximum efficiency the exact horsepowers and speeds would be found for each fan by drawing a line from the origin through the fan size point on the arc, noting its intersection with the required capacity line, reading actual static pressure and horsepower at the intersection, correcting the power by use of the power coefficient, and determining the speed corresponding to the actual operating point by going to the correct speed scale at the left side of the fan selection chart.

For most practical purposes the basic selection chart as constructed above will suffice for the investigation of relative efficiencies attendant upon choice of a fan for some particular service. Where efficiency comparisons of fans operating at points other than that of maximum static efficiency (as maximum total efficiency, or some rating point ahead of or beyond that corresponding to optimum static efficiency), a new chart can be constructed or the same basic chart used with a new set of speed scales, one scale for each desired rating point of a given type of fan. Thus if enough speed scales were added the one selection chart would permit choice of any type of fan to operate at any per cent of rated capacity.

Another item of operating information which can be readily obtained from the chart is tip speed. This is proportional to revolutions per minute times diameter and therefore remains constant (for operation at a fixed point on the fan characteristic curve) along any constant static pressure line. The *SP* lines which constitute part of the selection chart background are therefore constant tip speed lines and can be so labeled. The recorded values of tip speed will be for the basic fan type (for which the

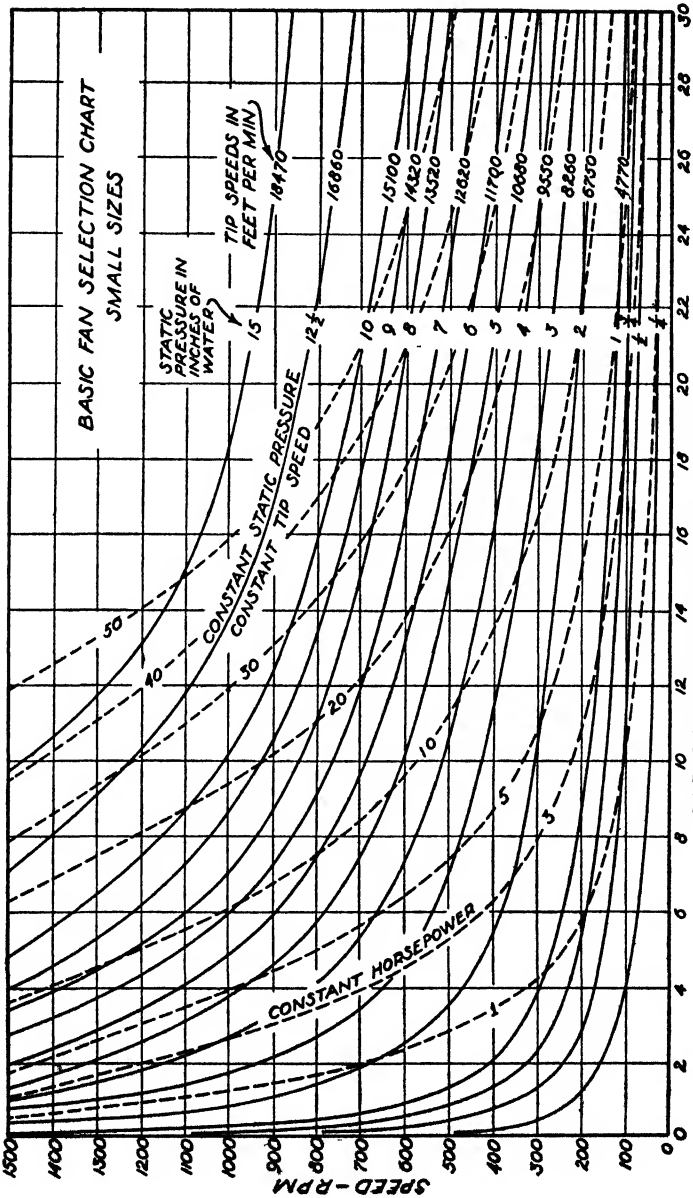
BASIC FAN SELECTION CHART
SMALL SIZES
TIP SPEEDS IN FEET PER MIN.
18470
16860
15100
14320
13520
12620
11700
10680
9550
8260
6750
4770
STATIC PRESSURE IN INCHES OF WATER
15
12½
10
9
8
7
6
5
4
3
2
1
CONSTANT STATIC PRESSURE
CONSTANT TIP SPEED
50
40
30
20
10
5
3
1
CONSTANT HORSEPOWER
CAPACITY - THOUSANDS OF CFM
0
2
4
6
8
10
12
14
16
18
20
22
24
26
28
30
SPEED - RPM
1500
1400
1300
1200
1100
1000
900
800
700
600
500
400
300
200
100
0

Fig. 15·18.

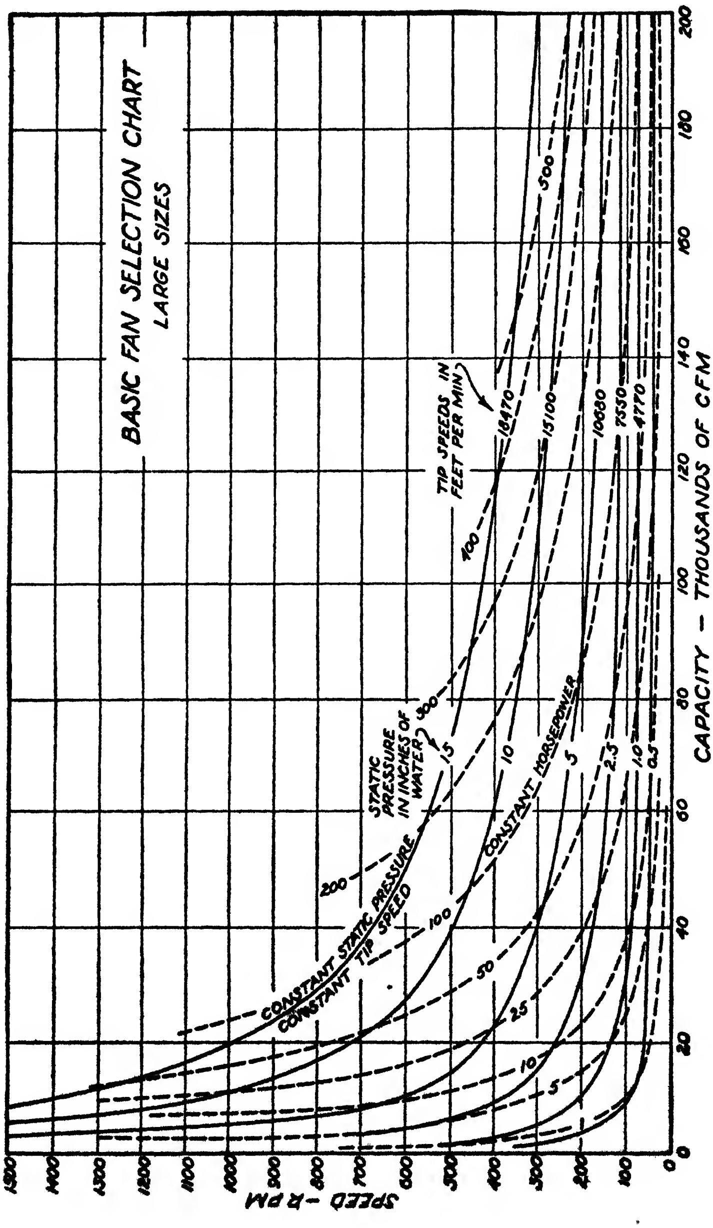
BASIC FAN SELECTION CHART
LARGE SIZES
TIP SPEEDS IN FEET PER MIN
18470
15100
10680
7550
4770
STATIC PRESSURE IN INCHES OF WATER
15
10
5
2.5
1.0
0.5
CONSTANT HORSEPOWER
500
300
200
100
50
25
10
5
CONSTANT STATIC PRESSURE
CONSTANT TIP SPEED
CAPACITY - THOUSANDS OF CFM
0
20
40
60
80
100
120
140
160
180
200
SPEED - R PM
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500

FIG. 15·19.

chart was originally constructed), and correction coefficients can then be determined (as for horsepower) for all other fan types which are included on the selection chart.

To assist in establishing a selection method for the fan types of greatest interest to a particular user, Figs. 15·18 and 15·19 are included. Each of the figures gives a working size fan selection background on which the reader can readily establish size reference points for the fan types of interest to him. Knowing operating conditions of a given type and size of fan when running at its optimum point on the characteristic curve, the intersection of Q and SP lines on the background fixes the rating point and permits establishing the speed scale, type arc, size points and correction coefficient for horsepower and for tip speed. Figure 15·18 is a standard selection background for small fans while Fig. 15·19 is for larger fans. In many cases it will be found advantageous to redraw the selection background with logarithmic coordinates, thereby giving straight lines for the constant static pressure and constant horsepower curve families. One additional advantage of log-log plotting is that it extends the capacity scale sufficiently to permit use of a single chart for fans of all sizes.

As the user gains familiarity with the chart, other possible extensions of its usefulness will become evident and he can add or remove items until the chart form is that best suited to the needs of his particular selection problems. In any event this type of fan selection chart, or any other graphical or mechanical device for evaluating relative performance of different types of fans, will be found to be a most effective tool for reducing the calculations and the time needed to determine the relative performance characteristics of a number of competitive fans when considered for a particular installation.

PROBLEMS

1. The velocity head of an air stream at 68°, 50 per cent RH, is 95 ft of air. Calculate the velocity in feet per minute.

2. A fan isentropically raises the static pressure of 6000 cfm of standard air from atmospheric to 3 in. of water. Calculate the work of the compression process (exclusive of work during admission and discharge) and express this as a percentage of the total work.

3. Calculate the exact static air horsepower for the conditions of problem 2 and compare with AHP_s calculated from equation 15·7.

4. A forward blade fan (similar to the one of Fig. 15·3) is selected for operation at 100 per cent of rated capacity. When installed and operated on the airway it is observed that the actual static pressure developed (at design speed) is 20 per cent less than the design value. What is the consequent percentage difference between actual and design air volumes?

5. If the fan of problem 4 were of the backward blade type, what then would be the per cent difference in actual and design air volumes (assuming that actual SP, at design speed, is 20 per cent below the design value)?

6. A fan is rated at 3 in. total pressure and 2 in. static pressure when delivering 4000 cfm. If four such units were placed in series what would be the velocity pressure and static pressure in the discharge line (for 4000 cfm delivery)?

7. For two parallel-operating identical forward blade fans (of type shown in Fig. 15·3) establish the stable and unstable pressure-capacity curves.

8. A fan delivers 10,000 cfm with a static pressure of 2½ in. What per cent change in speed would permit this fan to deliver 16,000 cfm through the same airway, and what would be the corresponding static pressure? What per cent change in static *AHP* would occur?

9. A size 10 fan operating on a fixed airway is found to be inadequate. If it is replaced with a size 20 (same type), can performance of the larger fan, operating on the same airway, be calculated by means of the fan laws? If not, explain.

10. A fan delivers 18,000 cfm against 1 in. static pressure when operating at 400 rpm and with a brake horsepower of 4.67. Calculate the static efficiency.

11. The speed of the fan in problem 10 is increased enough to permit delivery of 25,000 cfm through the same airway. For the changed operating condition calculate the speed, horsepower, and static pressure. Has the static efficiency changed?

12. For two or more fan types for which you have manufacturer's rating data, establish type locus lines, size points, and speed scales on Fig. 15·18 or 15·19. Calculate the power and tip speed correction coefficients for each type.

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CHAPTER XVI

AIRWAY DESIGN

16·1. Duct Losses. If an inviscid fluid were circulated through a closed frictionless conduit there would be no need to supply energy to the system so long as the velocity at any given cross section remained constant. The total pressure throughout the circuit would be constant, and the static and velocity pressures would therefore vary inversely. Static pressure would be a dependent variable subject to change only as a result of change in velocity pressure (such as would occur if the cross-sectional area of the conduit were to vary).

If air were an inviscid fluid and were circulated through a frictionless conduit open at both ends to the atmosphere, the energy requirements would be determined by the work necessary to impart velocity to the stationary fluid. The total pressure would have a value equal to the velocity pressure at the entrance and would, at any other cross section of the system, be equal to the sum of velocity and static pressures. Changes in static pressure would occur only as a result of area variation.

Since air does possess viscosity and is circulated through ducts having varying degrees of roughness, fluid frictional losses (dynamic) occur as a result of turbulence, and skin friction losses as a result of drag along the duct surface. These two types of losses, the one due to friction within the main body of the fluid, the other due to conditions near the boundary, do not have the same physical origin and cannot properly be evaluated by a single equation. Both losses, however, have the same overall effect on the flow and, for straight duct sections, it is customary to evaluate the total friction by means of the Fanning equation,

$$\Delta h = (f)(L)\left(\frac{P}{A}\right)(VP) \tag{16·1}$$

where Δh = change in total pressure due to dynamic and skin friction losses in straight duct, inches of water.

L = duct length, feet.

P = duct perimeter, feet.

A = duct cross-sectional area, square feet.

VP = velocity pressure, inches of water.

f = friction factor = ϕ (Reynolds' number, roughness).

The above equation is for isothermal, turbulent flow of an incompressible fluid, but it is applicable with good accuracy to air flowing through ducts under conditions such as are found in ordinary ventilating and air conditioning systems. Note that the friction loss, Δh, is expressed as a change in total pressure, even though the direct effect of both dynamic and skin friction losses is to tend to decelerate the fluid, transforming kinetic energy into internal energy. But the equation of continuity requires that the velocity of an incompressible fluid remain constant so long as the duct area does not change; thus deceleration is not possible, and the loss of kinetic energy must be equaled by a transfer of static pressure to velocity pressure at a rate such that the kinetic energy and velocity pressure at a given cross section will remain unchanged. All friction losses are therefore reflected in a decrease of both static and total pressure along all sections of constant area duct and a decrease of total pressure along *all* sections of duct.

Examination of equation 16·1 shows that friction increases directly with the ratio of contacted surface per unit stream area. It is evident, therefore, that, for the same velocity, a duct of circular cross section will have minimum loss whereas rectangular ducts of high aspect ratio will afford much larger losses. Since the first cost of the duct will likewise increase as the cross section goes from circular, to square, to rectangular, it is evident that every effort should be made to approach, as closely as conditions will permit, to a duct of minimum perimeter.

For a round duct $P/A = \pi D/(\pi D^2 \div 4) = 4/D$. Making this substitution in equation 16·1 and substituting for VP from equation 15·2a,

$$\Delta h = \frac{4fL}{D}\left[0.0000518\left(\frac{\rho_a}{\rho_w}\right)(v^2)\right] \qquad (16\cdot2)$$

where ρ_a and ρ_w are the weight densities, pounds per cubic feet, of air (in duct) and water (in manometer), respectively. For standard conditions, substitution for VP in equation 16·1 can be made directly from equation 15·3a, giving

$$\Delta h = 4f\frac{L}{D}\left(\frac{v}{4005}\right)^2 = \frac{1}{C}\frac{L}{D}\left(\frac{v}{4005}\right)^2 \qquad (16\cdot3)$$

where D is the duct diameter, feet, and $C\left(=\frac{1}{4f}\right)$ is the length of duct, expressed as a multiple of the diameter, in which the pressure loss would be equal to one velocity head. The term C varies with velocity, diameter, duct roughness, and with the physical properties of the fluid, but for approximate calculations it is frequently assigned a conservatively low value and considered a constant. In the range of velocities from 1000 fpm to 2500 fpm, for ducts varying from 12 in. to 48 in. in diameter, C has an average value of 67 when the fluid is air at usual temperatures. For ordinary straight sheet metal ducting with the usual number of joints, values of C ranging from 40 to 75 have been recommended by different writers. With such a wide variation (88 per cent) a safer procedure is to retain the friction factor in equation 16·2 as a variable and evaluate it in terms of the conditions for each particular problem.

The form of equation 16·2 would be more convenient if v were expressed in terms of the volume of air handled, Q cubic feet per minute, and the diameter d, in inches,

$$v = \frac{Q}{A} = \frac{Q}{(\pi D^2/4)} = \frac{4 \times 144Q}{\pi d^2} \tag{16·4}$$

Substituting for v^2 in equation 16·2, replacing D with d and evaluating friction loss for 100 ft of straight duct gives

$$\Delta h_{100'} = \frac{12 \times 4 \times 0.0000518 \times 100 \times (4 \times 144)^2}{\pi^2} \frac{fQ^2}{d^5} \frac{\rho_a}{\rho_w}$$

But for a range of water temperatures from 40° to 100°, ρ_w can be taken as approximately constant at 62.3 lb/cu ft (maximum error of 1 per cent) and ρ_a can be expressed as a function of temperature by the equation

$$\rho_a = 0.0001507(565.3 - t)$$

Then by substituting,

$$\Delta h_{100'} = \frac{0.0000518 \times 100 \times (4 \times 12)^3 \times 144 \times 0.0001507}{62.3\pi^2} \frac{fQ^2}{d^5} (565.3 - t)$$

$$= 0.0202(565.3 - t)\frac{fQ^2}{d^5} \tag{16·5}$$

Use of equation 16·5 to evaluate pressure loss first requires selection of, or determination of, the proper value of the friction coefficient. This term varies with the roughness of the duct and with a dimensionless term, Reynolds' number, which is expressed in terms of system characteristics and fluid properties by the equation

$$\text{Reynolds' number} = \frac{DV\rho}{\mu g} \tag{16·6a}$$

where D, V, ρ are as previously defined, $g = 32.2$ ft/sec², and μ is the absolute viscosity in slugs/cu ft. Substituting for $\mu g/\rho = y$, the kinematic viscosity (ft²/sec), and expressing V(fps) in terms of Q' (cu ft/sec), and area,

$$\text{Reynolds' number} = \frac{DQ'}{(\pi y D^2/4)} = \frac{4Q'}{\pi y D} \tag{16·6b}$$

Then rewriting in terms of Q and d, and substituting for y the empirical expression, $y = 0.0593 \times 10^{-5}\ (196 + t)$,

$$\text{Reynolds' number} = \frac{4 \times 12}{60\pi}\frac{Q}{d}\left(\frac{1}{196 + t}\right)\frac{10^5}{.0593}$$

$$= 429{,}400\frac{Q}{d(196 + t)} \qquad (16\text{·}6c)$$

16·2. Duct Friction Chart. Rather than attempt to establish an empirical equation giving the friction factor in terms of Reynolds' number and roughness, a more direct procedure will be followed. From equation 16·6*c*, Reynolds' number can be evaluated in terms of Q, d, and t. Then f can be read from an empirical curve of f vs. Reynolds' number at a given value of roughness. Knowing f, $\Delta h_{100'}$ can be determined from equation 16·5 as a function of f, Q, d, and t. Thus equations 16·5, 16·6, and experimental curves of f as a function of Reynolds' number and roughness permit direct evaluation of $\Delta h_{100'}$. To facilitate this solution a completely graphical method has been devised for evaluating $\Delta h_{100'}$ in terms only of Q, d, and t. The construction, developed by D. Corcoran, is as follows:

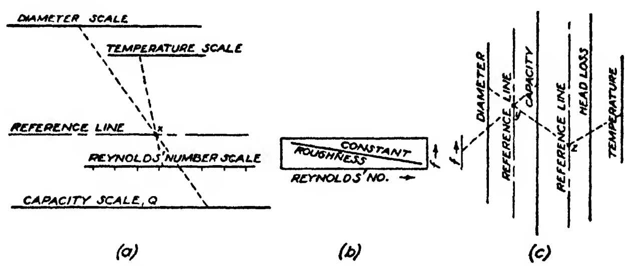

FIG. 16·1. Detail of Friction Chart Construction.

1. A nomogram is constructed (Fig. 16·1*a*), solving equation 16·6*c* for Reynolds' number in terms of known volume, duct diameter, and temperature of air passing through the duct. Connecting the known points on Q and d scales with a straight line gives intersection x with the reference scale. From x a straight line through the known point on temperature scale extrapolates to intersect the Reynolds number scale at the value for flow conditions of the particular problem.

2. Using the Reynolds number scale of the above nomogram as one coordinate and the friction factor as a second (Fig. 16·1*b*) establishes a set of empirical curves, one for each value of roughness. Roughness can be expressed as a ratio of depth of surface imperfections to diameter of pipe, but the imperfections are approximately the same for all sizes of one type of pipe so roughness can therefore be correlated as an inverse func-

tion of diameter. Curves can be established for pipe sizes from 3 in. to 60 in. Rising from the value of Reynolds' number established as in Fig. 16·1*a*, intersect the roughness curve for correct pipe diameter, then move horizontally right to intersection with the friction factor scale.

3. By starting with the f scale of Fig. 16·1*b* a nomogram is constructed solving equation 16·5. Connect known points on f and Q scales (Fig. 16·1*c*), intersecting reference line at point y. From y draw a straight line to known point on diameter scale and extrapolate to intersection with reference line at point z. Connect z with known point on temperature scale and read the answer at intersection of this line with the friction loss scale.

4. By placing in series the nomograms of Fig. 16·1*a* and 16·1*c* with the curves of Fig. 16·1*b*, a continuous, once through, solution is realized and the need for numerical evaluation of either Reynolds' number or f is obviated. A complete graphical solution of this kind is shown in Fig. 16·2. This figure is sufficiently large to permit its use in solving practical problems. As an additional help, a velocity scale has been added so $\Delta h_{100'}$ can be evaluated in terms of known Q, d, and t, or Q, v, t, or known v, d, t. A typical example is given on the figure and the solution shown by the dashed line.

Friction losses in rectangular ducts can be obtained indirectly from Fig. 16·2 by first calculating the equivalent diameter d of the rectangular duct having sides b and a, where b is the shorter. The exact relationship between b, a, and d leads to a complex equation, but a direct and reasonably accurate approximate solution is shown in the inset curve at top center of Fig. 16·2. To use this curve, enter at the ratio a/b, rise to intersection with curve, then move left to read corresponding a/d ratio; knowing equivalent diameter and actual volume, friction loss is then determined from Fig. 16·2.

16·3. Duct Design. Effective design of any airway system requires judgment and experience, in lieu of which no mathematical or graphical system of analysis can be substituted. The "best" design for a particular installation is determined from considerations of noise, strength, fire hazard, first cost, operating cost, and other factors, among which duct friction is significant, but by no means all important. Whenever possible, therefore, the sizing of ducts for an airway should be accomplished by assigning velocities in each main, branch, or riser on the basis of the designer's experience. Knowing the required volumes and having the assigned velocities, duct diameters can be readily calculated, and the friction loss in each straight section of the airway system can then be evaluated by means of Fig. 16·2. When this method is used the total calculated loss will probably not be the same in parallel sections of the system. Since the total pressure drop from fan to discharge must be the

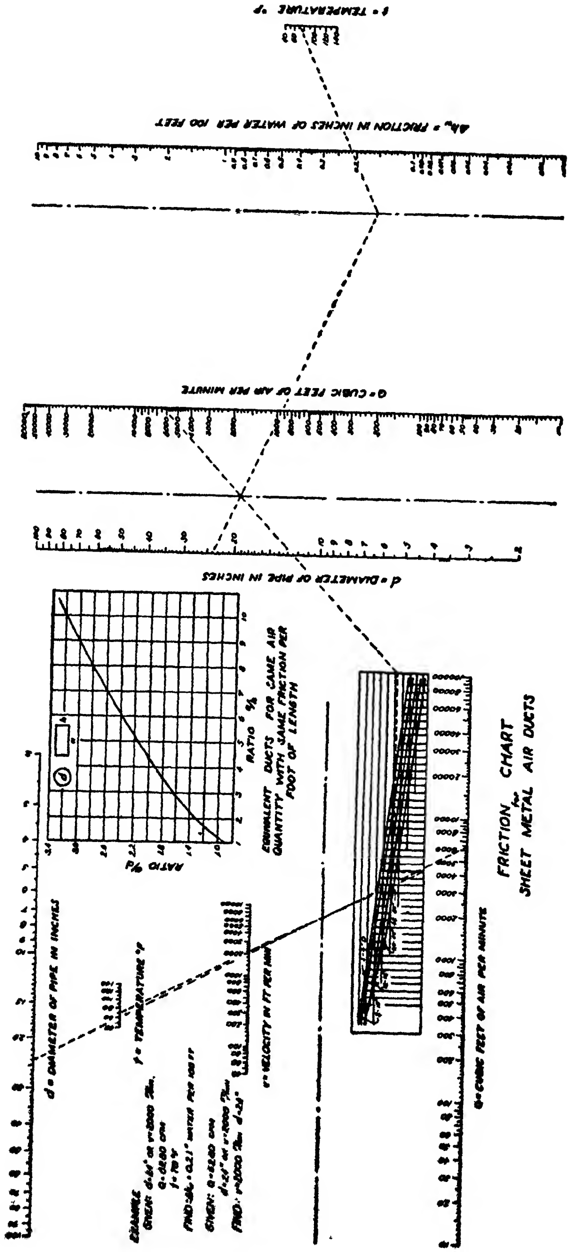

Fig. 16-2.
(See enlarged chart in pocket in rear cover of book.)

same for all branches, dampers would have to be used in all sections except that one having greatest loss.

Lacking the background of experience required in arbitrarily assigning duct velocities, the designer can refer to any standard handbook for tables of recommended air velocities for ducts of different type or service. The usual range is from 500 fpm for residential branch risers to 2000 fpm for main ducts in industrial installations, but velocities as high as 6000 fpm are frequently used in special industrial or marine installations.

Two arbitrary design procedures which are often used are the following:

1. Constant-pressure drop. Assigning the velocity in the main duct, the corresponding friction loss per 100 ft is obtained from Fig. 16·2. The same loss per unit length is then assumed as a basis for sizing all other ducts in the system. This method has the advantage of speed and simplicity, but the disadvantage of giving uneconomically low velocities in the smaller size branches.

2. Constant velocity. Assigning a fixed velocity throughout the system is a design method now justly obsolete. Such a procedure inevitably results either in undue noise from relatively high velocity in branch ducts, or increased first cost from relatively low velocity in main ducts.

A design procedure based essentially on establishing optimum grille or outlet performance is to size the airway so that the velocity pressure will decrease at a rate equal to the head loss. By this method the static pressure can be maintained constant throughout the system with the consequent advantage of a uniform pressure drop across all outlets. When designed in this way a duct of usual size operates with the characteristics ordinarily found only in large plenum chambers. Continuous and uniform reduction of cross section is usually not feasible, but a comparable design can be readily achieved by reducing the velocity pressure at the entrance to each branch of duct by an amount sufficient to compensate for the friction loss expected in that branch.

As a matter of interest and as a striking example of the basic principles, the plenum design described above could lead to an airway in which the air flow was in a direction of *increasing* static pressure. This would occur if the velocity reduction per unit length of duct were greater than that needed to provide static regain. Thus the usual conception of air flow from a region of high static pressure to low static pressure is in need of analysis when the design is of the static regain type.

All the above design methods lead to evaluation of the total friction loss and the consequent load for which a fan must be selected. In unusual cases a fan may already be available and a design required for which the airway resistance will not exceed the fan rating. The procedure is then to calculate an available $\Delta h_{100'}$ from the fan rating and the overall duct length; sizing of the duct is then carried out by means of the graphical solution.

Refinements of design such as sizing branches to give an equal rate of friction loss on either side of a connection to the main, or reducing velocity at a rate for which static regain will equal the friction loss, will commend themselves to the designer of a system requiring either precise performance or high engineering efficiency. Detailed discussion of such design is not required here as the methods do not differ in principle from those already covered, and the detailed procedure can only be established in terms of the particular design problem.

16·4. Fitting Losses. In addition to the friction losses in straight ducts, additional losses, very largely of dynamic origin, occur at entrance and exit and whenever the air stream undergoes a change of velocity or of direction. Many forms have been used for expressing such fitting losses: as equivalent length of straight duct, fraction of a velocity head, or loss in inches of water. To establish uniformity and provide the designer with a maximum of data from a single source, Fig. 16·3 has been prepared. On this figure friction losses for forty types of commonly used fittings are arranged on a log scale with a velocity background grid such that the actual head loss in inches of water can be obtained for any of the fittings at any velocity between 400 fpm and 6000 fpm.

To use the chart, move down the left-hand side to the horizontal line for the particular fitting of interest. Then move right to the point at which this line intersects the vertical corresponding to duct velocity; at the intersection read the head loss from the scale marked on the horizontal. As an example, consider a round elbow with ratio of radius of curvature to diameter equal to 1.0 and velocity of 800 fpm. This fitting is represented by the second horizontal from the top; at intersection with vertical for 800 fpm read friction loss as 0.01 in. Whenever a change of velocity occurs during passage through a fitting the chart values are based on the *velocity in the smaller pipe.* Brief discussion of the fitting groups follows:

a. The three lines for round elbows are based on 90° ells followed by a short section of straight pipe. For elbows of angle between 90° and 180° the head loss can be computed as proportional to the angle through which the stream is deflected.

b. Rectangular ells are grouped according to ratio of width to depth where the width, for a vertical ell, is considered as the horizontal dimension. For each W/D-ratio scales are provided for the same three R/D ratios which were used with round ells.

c. Gradual expansion losses are arranged in groups based on velocity ratio, with three scales in each group to cover the usual range of angles likely to be used in transition pieces. The transition angle is evaluated

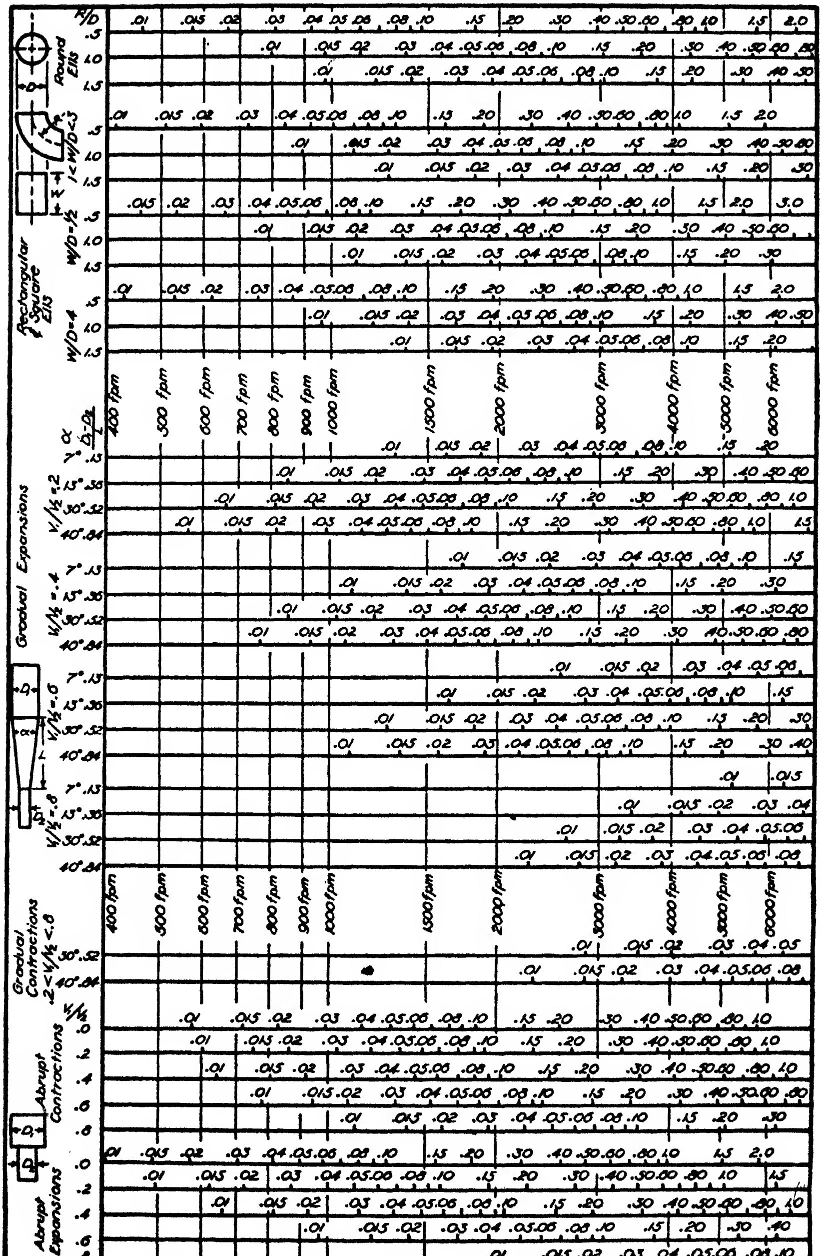

FIG. 16·3. Friction Loss in Duct Fittings.

in degrees and also by means of the term $(D_1 - D_2)/L$. Where detail drawings are available the angle can most readily be taken off with a protractor; lacking such drawings the $(D_1 - D_2)/L$ term will be of greater convenience.

d. Gradual contraction losses are negligible up to a transition angle of 30°. This loss is also practically independent of the velocity ratio in the range from $0.2 < V_1/V_2 < 0.8$.

e. Abrupt contraction head loss depends on velocity ratio and reaches a maximum for free entrance through a sharp-edged opening (scale for $V_1/V_2 = 0$). Free entrance through a bell mouth orifice can be taken as 10 per cent of the loss given by the chart scale.

f. Abrupt expansion results in exceptionally high head losses, the maximum occurring at free discharge (corresponding to $V_1/V_2 = 0$). For this case correction of exit conditions by use of a bell mouth or other orifice will have no effect on the loss as the entire kinetic energy of the leaving air stream must be dissipated through turbulence. The scale for free discharge gives a loss equal to one velocity head and this line can therefore be used as a convenient datum for evaluating the velocity pressure corresponding to any particular stream velocity.

g. Note that the chart gives the loss in total pressure which occurs in any fitting. This will be equal to the loss of static pressure *only* if the velocity through the fitting does not change. For the general case the loss of static pressure is given by the equation

$$\Delta SP = \Delta h_{100'} - (VP_1 - VP_2)$$

where $\Delta h_{100'}$ is the head loss taken from the chart for the fitting (based on higher of the two velocities), VP_1 is the velocity pressure at entrance (taken from chart for condition of free discharge), and VP_2 is the velocity pressure at fitting exit (also from chart for condition of free discharge). The $\Delta h_{100'}$ term is a loss and will always be positive. The term in parenthesis may be positive or negative depending on whether the velocity is decreasing or increasing. The ΔSP term is given as a loss and will usually be positive, but for a condition of great velocity decrease through the fitting it can be negative; negative ΔSP is equal to static regain.

The scale arrangement used in Fig. 16·3 is similar to that of a standard slide rule. The data could be readily transposed to a single scale and a slide rule constructed to consolidate all the information given on Fig. 16·3. Additional data for types of fittings not now included can be readily placed either on the slide rule or on added scales of Fig. 16·3.

16·5. Summary. In this chapter the basic data needed for the design of a duct system have been summarized in the two graphical solutions of Figs. 16·2 and 16·3. The first figure permits direct determination of friction loss in straight sheet metal duct from known values of capacity, temperature, and either velocity or diameter. The second figure gives the head loss due to passage of air at given velocity through any of forty types of fittings.

PROBLEMS

1. Ten thousand cfm of 70°F air at 2000 fpm pass through a 240-ft rectangular sheet metal duct which is 10 in. deep. Determine the duct width and the friction loss in inches of water.

2. Ten thousand cfm enter a 400-ft, 20-in. diameter duct. If 2000 cfm are extracted at intervals of 80 ft along the duct, determine the necessary diameter of each 80-ft section to permit realizing the same static pressure at each take off point.

3. Six thousand cfm of 70°F air are to be passed through 250 ft of round duct including 4 ells with $R/D = 1.5$ and an abrupt expansion (at discharge). If the total allowable pressure drop is 0.5 in. H_2O, calculate the required total pressure in the entering end of the duct.

4. Determine the size of each branch connection to the main duct of problem 2 if the linear rate of friction loss at the connection is to be the same in main and in branch. When branches are designed on this basis note the abrupt velocity change of air leaving the main and estimate the probable energy loss from this cause. Would the connections be more acceptable (from standpoint of performance and energy requirements) if they were designed for equal *velocity* in branch and in main?

5. Air at 5000 fpm passes through a rectangular elbow having $R/D = 1.5$ and $w/D = 0.5$. Determine the energy loss.

6. Determine the loss as 4000 cfm of 70°F air enter a 15-in. diameter exhaust grille located in the wall of a room.

7. Determine the loss due to gradual contraction [$(D_1 - D_2)/L = 0.52$ and $V_1/V_2 = 0.4$] of an air stream initially moving at 4000 fpm and compare with the loss for an equal abrupt contraction.

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