TELEGRAPHY AND TELEPHONY

TELEGRAPHY AND TELEPHONY

INCLUDING WIRELESS

An Introductory Textbook to the Science and Art of the Electrical Communication of Intelligence

◆

BY

E. MALLETT

D.Sc. (Eng.), London

REPRINTED

LONDON

CHAPMAN & HALL LTD.

11 HENRIETTA STREET, W.C.2

1943

First published 1929
Reprinted - 1941
Reprinted - 1943

THE PAPER AND BINDING
OF THIS BOOK CONFORM
TO THE AUTHORIZED
ECONOMY STANDARD

Printed in Great Britain by Lowe & Brydone Printers Ltd., London, N.W.10
Bound by G. & J. Kitcat Ltd., London; Flexiback Binding Patent No. 441294

PREFACE

THE communication of intelligence electrically, either by code or by the spoken word, and either by connecting wires between the signalling points or without them, is an extensive subject of great importance, and one which occupies the attention of an increasingly large number of highly-trained engineers. It is natural, therefore, that there are many textbooks dealing with the subject. On the one hand, there are those written for the telegraphist or the telephone lineman, containing an elementary treatment of the principles of electricity and magnetism as well as their practical application to telegraphy or to telephony. These books are mainly descriptive of practical apparatus used in the Post Office. On the other hand, there are many advanced treatises on various branches, such as Telephone Line Transmission, Common Battery Exchanges, Automatic Exchanges, Submarine Cables, Wireless Telegraphy and Telephony, etc., which are hardly suitable for a first approach to the subject. There appears to be no single book containing an outline exposition of the application of scientific principles to the whole art of Electro-communications, which serves as an introduction to the specialised books, and does not deal largely with the teaching of electricity and magnetism, which is better done elsewhere. It is this apparently well-defined gap in the literature of the subject that the author has attempted to fill.

The book is designed, therefore, to meet the needs of a student of a University or Technical College who has studied the principles of electricity and magnetism up to a second year standard, and of those junior engineers who find themselves engaged in "Light" Electrical Engineering without having had any instruction in the subject at college. As far as examinations are concerned, it may prove to be a suitable textbook for the telegraphy and telephony papers in the final examinations for the B.Sc. (Eng.) of the

University of London, in the higher grade technological examinations of the City and Guilds Institute, in the Institution of Electrical Engineers' Entrance Examination, and in the Civil Service Commission's Examinations.

The actual arrangement of the book has one unusual feature; the unit of subdivision is the section, and the sections are numbered consecutively throughout. The section number appears at the top of each page, the page number at the foot, and the section number is used in all references to mathematical equations. Thus (39.06) and (39.32) refer to the sixth and thirty-second equations respectively in section 39, but where the reference is to an equation number in the same section the section number is omitted. Thus in section 39, the above equations would be referred to as (6) and (32) respectively.

No attempt has been made to give detailed references or to make detailed acknowledgments. These will be found in the textbooks and papers recommended for further reading given at the end of each chapter. A general acknowledgment is made here for information derived from those works. The drawings are the work of Mr. D. W. Hopkin, and the author's thanks are due to him, not only for the excellence of the resulting illustrations, but also on account of his skill in reading the author's generally very rough sketches.

E. M.

City and Guilds (Eng.) College,
London, S.W. 7.

CONTENTS

PART I.—LINE TELEGRAPHY

PART II.—LINE TELEPHONY

Chapter VI.—Alternating Quantities

Chapter VII.—Speech Apparatus

Chapter VIII.—Line Transmission

Chapter IX.—Valves in Telephony

Chapter X.—Signalling and Switching

PART III.—WIRELESS TELEGRAPHY AND TELEPHONY

Chapter XI.—Electromagnetic Waves

TELEGRAPHY AND TELEPHONY

ERRATA (REPRINT) 1943

May we suggest, for the convenience of the reader, that these corrections be made in the text.

P. 6. Equations should include S as follows:—

$$\frac{2}{8\pi} \times \frac{16\pi^2}{100} \times \frac{N^2I^2}{4l^2} \times S = \frac{\pi}{100}.\frac{N^2}{l^2}.S\ .I^2 \text{ dynes}$$

$$I_1^2 = T_o \times \frac{100l^2}{\pi N^2 S}$$

$$I_2^2 = (T_o + Sx).\frac{100}{\pi N^2 S}(l - x)^2$$

P. 17. Line 5. Change *less* to *greater*.

P. 37. Line 6. Change *(22) and (23)* to (20) and (21).
Line 9. Change *(20) and (21)* to (22) and (23).

P. 42. Second expression should be:

$$= I_r \frac{R_o \cosh \alpha l / \sinh \alpha l + R_r}{R_o / \sinh \alpha l}$$

P. 47. Equation between (5) and (6) should be:

$$(I_{n-1} - I_n) - (I_n - I_{n+1}) = G\left\{(V_n - V_{n+1}) - \frac{R}{2}(I_{n+1} - I_n)\right.$$

P. 58. Equation (14). Change cosh *pt* to cosh *βt*.

P. 60. Fig. 44. The discharge curve for q is inverted. It should be the same shape as the charge curve for i.

P. 61. Equation (3) should be $Q = EC\epsilon^{-\frac{1}{RC}t}$.

P. 64. Fig. 46. Change C = 875 μF to 2C = 875 μF.

P. 89. Middle equation should be $-\frac{\delta i}{\delta x} = C\frac{\delta v}{\delta t}$.

P. 115. Equation (4) should be $\frac{\delta^2 x}{\delta t^2} = \frac{e}{\rho_o}\frac{\delta^2 x}{\delta y^2}$.

P. 123. Equation (2) third term should be $Y_2 \sin(2\omega t + \phi_2)$.

P. 145. Equation (4) first term should be $\frac{1}{2}I_o\left(\frac{r_o k}{R + r_o}\right)^2$

P. 149. Line 7. Insert *magnet* after *permanent*.

P. 151. Line 7. For *total* write *alternating*.

P. 155. Equation between (18) and (19). Last term on left-hand side should be $(s - pa_1)x$.
Equation (22). Second equation should be $S'' = S - S'$.

P. 158. Line (8) should be $S = S'' + S' = \ldots$

P. 164. Equation (8). Delete E from denominator.

P. 176. Equation before (1) should be $i - \left(i + \frac{\delta i}{\delta x}\delta x\right) = $ etc.

P. 177. Equation before (7). Insert V in second term of left-hand side.

P. 181. Equation (17). Write $\alpha^2 + \beta^2 = \sqrt{(\alpha^2 - \beta^2)^2 + 4\alpha^2\beta^2} =$ etc.

P. 182. After equation (25) read . . . infinitely long *distortionless* line . . .

P. 183. Line before equation (26). For 8·10 write 8·11.

P. 203. Four lines after equation (1). The root should also cover the bracket.

P. 211. Line 15. For *traction electric* write *electric traction.*

P. 217. Equations (3), (4) and (5). Write x_{dB} for xdB.

P. 227. Expression after equation (4) should be $\frac{\mu^2}{4R} Vg^2$.

P. 230. Equation (16). Write $((\delta V_g)^2$ for $(\delta V_a)^2$.

P. 231. Treatment of push pull amplifier is approximate, not exact. In the expression for I_a' and I_a'' I_a is taken to be the same, but actually it is different in the two cases. The result is "even" harmonics are eliminated but "odd" harmonics remain.

P. 240. In (7) read $1{\cdot}77/\sqrt{RC}.(\sqrt{f_1} - \sqrt{f_2})$.

P. 242. Equation (8) should be $w = \sqrt{\frac{1}{LC}\left(1 + \frac{R}{R_a}\right)}$.

P. 249. Line 6. Replace 45·16 by 46·16.

P. 286. Line 6. Non-directional aerials. Line 12, delete 4π *times*.

P. 294. Austin-Cohen should read $E = \frac{377Ih}{\lambda r}{}^{\cdot 0.0015r/\sqrt{\lambda}}$

P. 302. Replace table by the following :—

Frequency.	R^1/R. Dia. of wire 2·0 mm.	Dia. of wire 0·2 mm.
10^3	1·000	1·000
10^4	1·144	1·000
10^5	2·626	1·000
10^6	7·70	1·095
10^7	23·8	2·626

P. 310. Equation between (4) and (5) should be $I_1 = \frac{Z_2}{jwM} I_2$.

P. 315. Equation (24) should be $w' = \frac{1}{\sqrt{1 + r}} w_0$

$$w'' = \frac{1}{\sqrt{1 - r}} w_0.$$

P. 331. Line 13. For *study* write *steady*.

P. 342. Equation (24). For α write r. Line 11, after current insert *density*.

P. 359. Line 7. Instead of 6 (1·33) write 61·33.

P. 364. Equation (3). Last term should be $\frac{v^3}{3!} \cdot \frac{d^3}{dV_0^3} f(V_0)$.

P. 396. Line 13 *et seq.* should be represented by OP′ and OQ′ respectively drawn equal to and in the opposite direction to OP and OQ.

PART I

LINE TELEGRAPHY

CHAPTER I

SIMPLE APPARATUS AND SYSTEMS

(1) The Morse Code

The letters of the alphabet, figures and punctuation signs are represented in a code by various combinations of two distinct signals, arranged on the principle that the most commonly used letters are formed by the shortest groups of signals. This is the Morse code, which is invariably used for hand-operated as distinct from machine telegraphy in all countries using the Roman alphabet,

FIG. 1.—" Dots and Dashes " in the Morse Code.

although some of the letters are slightly different in the code as used in America.

The difference between the two signals is usually one of duration; one signal, the dash, lasts three times as long as the other, the dot. The interval between two signals has the same duration as the dot, that between letters lasts as long as three dots, and that between words as long as five dots. The actual dots and dashes are termed marks and the intervals spaces. Whatever the rate at which messages are sent, the relative lengths of dots, dashes and spaces must be kept accurate for good reception. Fig. 1 shows how " dots and dashes " would be sent in the Morse code.

The unit of time in the Morse code is thus the dot. A good key speed is thirty words a minute. Reckoning the average word as containing five letters, and the average letter as containing 10 units, this speed represents 1,500 units a minute, or 25 units a second, or the duration of the unit is 0·04 second.

Dots and dashes may be represented by short and long marks on a slip of paper (in a recording instrument) exactly as shown in Fig. 1, or by short and long intervals of time between two sounds (in a sounder), or by short and long periods of noise (buzzer telegraphy and spark wireless). The Morse code may also be represented by deflections to the right or left of the needle of a galvanometer, or by different sounds, one for the dot and the other for the dash.

(2) The Simplest Telegraph Circuit

The telegraph key, or Morse key, or single-current key, shown in Fig. 2, is a very simple instrument, but it is important for rapid and correct sending that it should be well balanced and work easily, and that the fingers should not be cramped in holding the knob. The movement of a brass lever A is limited by two platinum-tipped contact stops C and D, known as the front and back stops respectively, one on each side of the pivot B, known as the bridge. An adjustable spring E holds the lever normally against the back stop, which is adjustable in order that the play of the lever may be regulated. Terminal screws are mounted on the brass plates carrying the front stop, the bridge and the back stop.

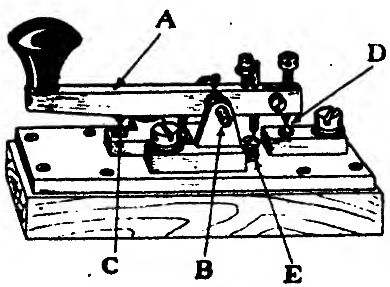


FIG. 2.—Morse Key.

Fig. 3 shows the construction of the ordinary type of Post Office sounder, or "pony sounder." A soft iron armature A is carried on a bell crank lever B pivoted at C. The armature is opposite the ends of two soft iron cores D, the other ends of which are joined by a soft iron yoke E. The cores carry coils which are joined in series and connected to the two terminal screws. The lever is normally held against the stop F by the spring H, the tension in which can be adjusted by the milled-headed screw J. When current flows through the coils, magnetic flux passes right round the electro-magnet across the air gaps and through the armature, as shown by the dotted line, creating a pull on the armature, and if the current is strong enough the armature will move towards the cores against the action of the spring. The armature is prevented from touching the cores and sticking by the stop G striking the elbow K. The sound made by G striking K when current flows indicates the commencement of the Morse signal, and the sound made by the lever striking F on its restoration by the spring on the cessation of the current indicates the end.

The shorter the air gap the more sensitive will the sounder be, but, on the other hand, the sound emitted will be greater, the greater the play of the lever between the stops F and G, and a larger play necessitates a longer air gap.

The spring tension should be adjusted so that the spring exerts the same force in restoring the armature that the current exerts in attracting it, so that the time of transit and the sound emitted is the

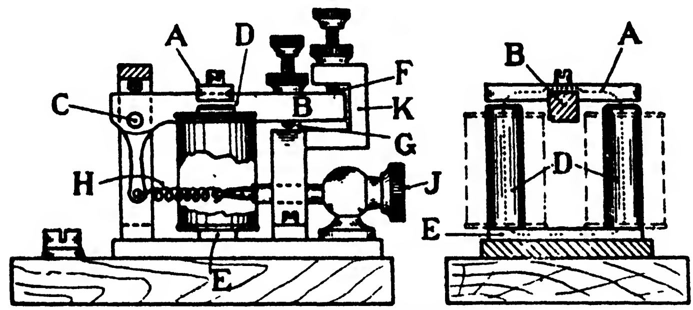


FIG. 3.—Telegraph Sounder.

same in each case. The adjustment will, therefore, be different for each adjustment of the stops F and G, and for each value of the current received, and is best done when signals are being received.

The resistance of the two coils is 21 ohms, and there is a shunting resistance of 420 ohms, so that the combined resistance is 20 ohms. The instrument will work with 55 milliamperes, but with 100 milliamperes good, strong signals are ensured.

Another pattern has a winding of 1,000 ohms shunted by a non-inductive resistance of 10,000 ohms, and works off a 24-volt battery.

A simple theory of the sounder may easily be written down. Let the total number of turns on the two coils be N, the length of the air gap when the armature is in the up position be l cms., the movement to the down position x cms., the cross-sectional area of each gap S sq. cms., the initial upward pull on the armature due to coil bobbins AA, Fig. 6, fixed side by side about ⅛ in. apart. On a horizontal axle between them swings a soft iron needle B of U shape, pivoted at the bottom. This needle is magnetised by a pair of permanent magnets, M, placed below it, and swings with its free ends uppermost, being kept in this position partly by the repulsion of the permanent magnets, and partly by the weight of the lower end of the pointer, P, attached to it and swinging in front of a dial (not shown). The peculiar shape of the needle ensures that the torque shall not greatly decrease as the deflection increases.

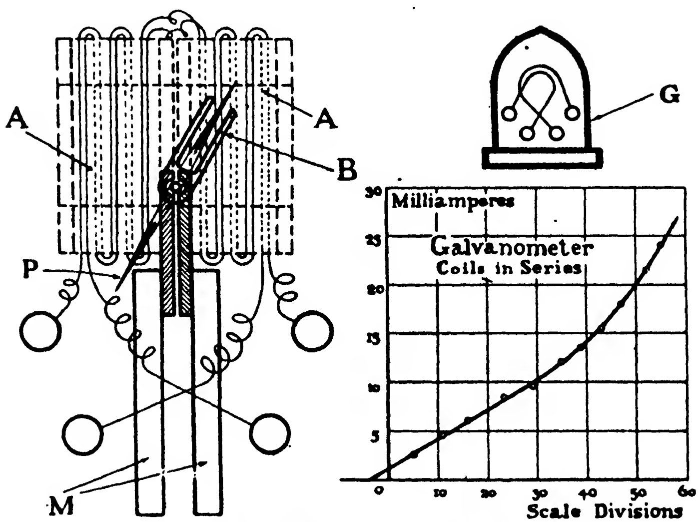


FIG. 6.—Telegraph Galvanometer and Calibration Curve.

The coil bobbins are usually wound with two coils, half of each coil on each bobbin, which are brought out to separate terminals. The magnetic action of the coils is well balanced so that the instrument can be used differentially; that is, if equal currents flow in opposite directions through the two coils, no deflection of the needle is produced. The coils can also be connected either in series or in parallel; the magnetic effect of a given current passing through the instrument being twice as great in the former case.

The scale of the instrument is marked in degrees. In Fig. 6 is given a calibration of a particular instrument. It is seen that for deflections up to 30 degrees the deflection is proportional to the current, and that there is a zero error of about 2 mA. The calibration for current in the reverse direction is almost symmetrical about the actual zero. The coils have each a resistance of 50 ohms; and each is shunted by a non-inductive resistance of 300 ohms in order to reduce the inductance of the instrument to make it more suitable for high-speed working. Current passing through the instrument from left to right deflects the needle from left to right. Instruments with a single-coil winding of 30 ohms are used on simplex circuits.

Actually the simple telegraph circuit, in addition to the provision of galvanometers, differs a little from the one shown in Fig. 5. It is arranged so that whatever station is sending, the current shall always flow in the same direction in the line, and therefore through the coils of the sounders or other receiving instruments. In the present case the sounder will work if the current is reversed, but in other systems signals depend upon the current flowing in the right direction, and so the rule is made to apply to all.

Stations are called up or down (or intermediate) on the same principle as that adopted on the railways, the station nearer to London being the "up" station, and the one further away the "down," and the convention adopted is that a marking current always flows in the line from the up to the down station.

This arrangement is carried out in the simple diagram of Fig. 5 by reversing the line and earth connections at the down station. Fig. 7a shows the diagram of connections for the modified circuit. It is known as a "single current circuit, direct working," and is the simplest system in practical use. Intermediate stations can readily be inserted as shown in Fig. 7b. Each station works all the others, each with its own battery.

(3) **Relays**

The direct working system of Fig. 7 can only be used over short lines with a battery of reasonable voltage. The resistance of each galvanometer is 30 ohms, and that of the sounder is 20 ohms, making 80 ohms for the instrument resistances. Ten miles of 150 lb. copper wire have a resistance of 119 ohms, so that for 100 milliamperes to flow, the battery voltage must be $0{\cdot}1\,(80 + 119) =$ 20 volts. There is, moreover, leakage of current along the open lines in general use for telegraphy, which becomes more and more important as the line is increased in length.

On longer lines, therefore, the sounder is replaced by a relay, which works with much less current. It does not itself give readable signals, but closes and opens a "local circuit" containing the sounder and a battery. The relay has a further advantage in that it will work efficiently with a wide range of current strength, and the sounder signals will, of course, be all of equal strength.

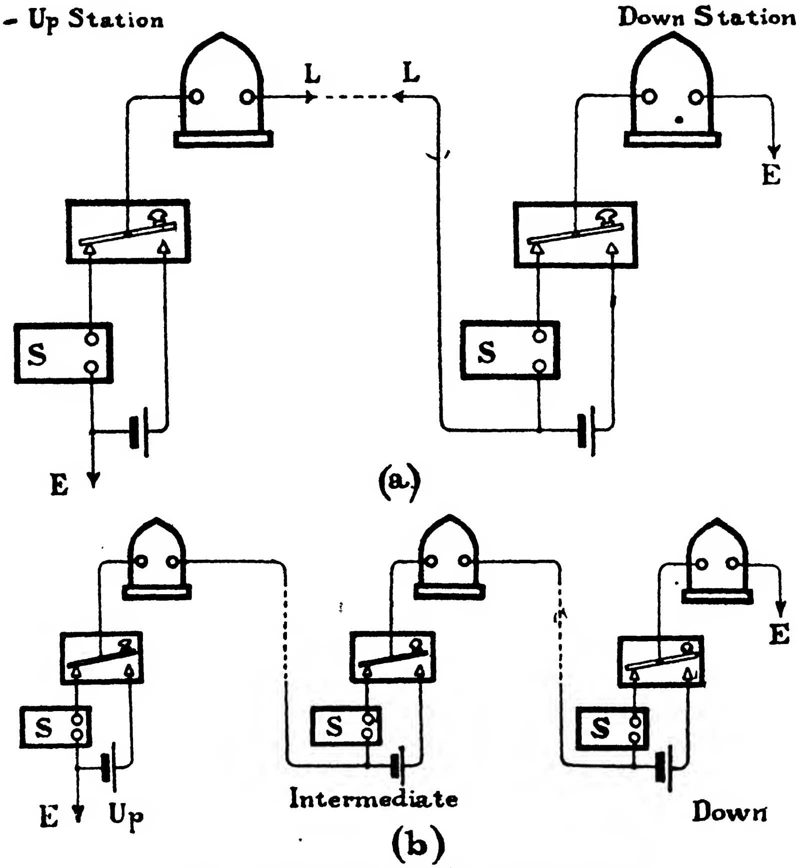


FIG. 7.—Single Current Simplex, direct working.

Telegraph relays may be divided into two classes, non-polarised and polarised. Each is differentially wound with two coils, half of each on the two soft iron cores of an electro-magnet. Fig. 8 shows the essential details of the construction of a non-polarised relay. The two soft iron armatures A are mounted on a pivoted brass spindle B. The armatures are split by a brass strip to prevent the possibility of a closed magnetic circuit causing excessive residual magnetism. The spindle at its upper end carries a light German silver tongue T, which plays between two adjustable platinum-tipped contact stops M (marking), and S (spacing). The tongue is normally held over to the spacing stop by the spring C, the tension of which can be adjusted by the screw D. The tension of the spring is determined by the same considerations as in the case of the sounder.

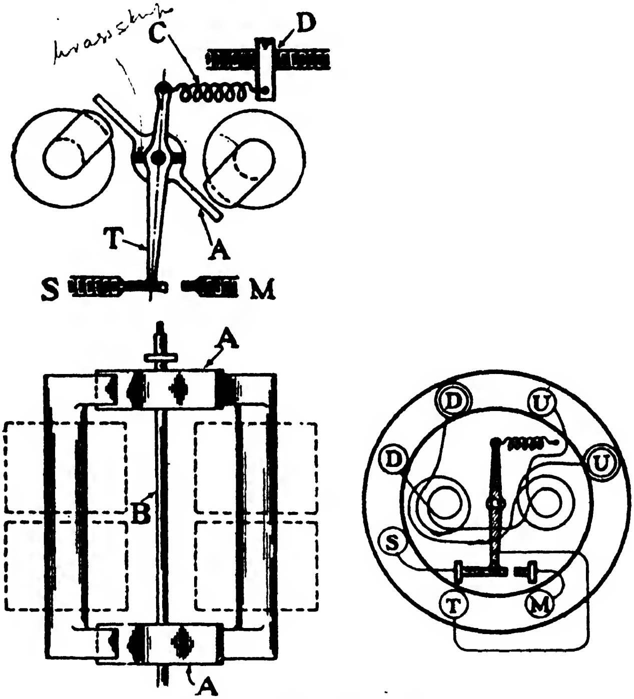


Fig. 8.—Non-polarised Relay.

A cylindrical case contains the whole relay. The terminal screws of the coils are marked as shown, D U for one coil, and D U for the other.

Two patterns of the relay are in use ; relay " B " will work with a current of 6 milliamperes, but the spring may be set so that it will not work with 15 mA, or as it is usually put, it may be biased against 15 mA, while relay " C " will work with $4\frac{1}{2}$ mA, and may be biased against 10 mA.

The magnetic circuit is round the iron cores and across the four air gaps in series, and the theory is the same as that of the sounder. The pull on the armatures is proportional to the square of the current, and the sensitivity inversely proportional to the air gap length. In adjustment, therefore, the stops S and M should be as close together as is consistent with definitely opening and breaking contact between T and M, and the armatures should be made to approach the cores as closely as possible without touching.

The polarised relay is of similar general construction, but armatures and cores are polarised by a strong permanent horse-shoe

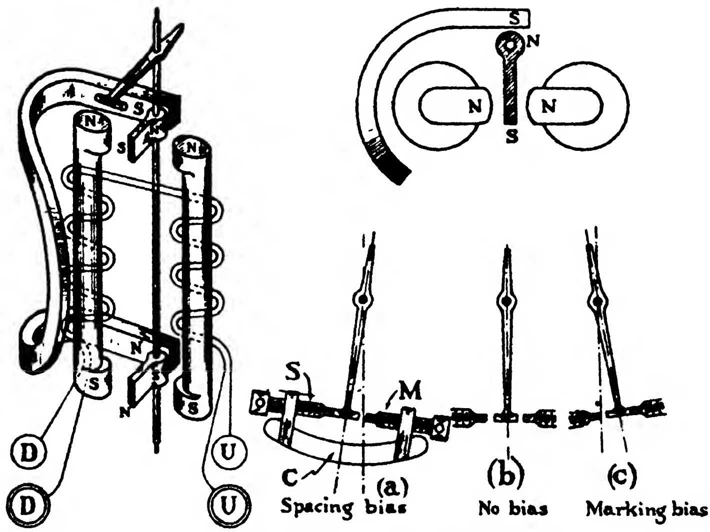


FIG. 9.—Polarised Relay.

magnet which is doubled round a bobbin in order that it may go into the cylindrical case. Fig. 9 indicates the general arrangement. Flux from the N-pole of the horse-shoe magnet passes through the lower armature, divides to pass up through the two cores, and reunites to pass through the upper armature to the S-pole, inducing poles as shown. If the armatures were exactly central, the pulls to right and left would be equal and no movement would result, but if the armatures are ever so little to one side, the flux through that side is increased and the armatures move right over. The winding of the coils is such that a current through them causes flux to pass up one core and down the other and across the gaps, thus weakening the flux on one side and strengthening it on the other and causing the armatures to be pulled to one side or the other, according to the direction of the current. The winding is such that a current entering from the left-hand terminals pulls the armatures to the left.

The movement of the tongue, and therefore of the armatures, is limited by the stops as in the non-polarised relay, but the stops themselves are mounted on a carriage, C (Fig. 9a), which can be rotated through a small angle about the spindle by a milled-headed screw, so that while the play of the tongue remains the same, the position of the tongue (and armatures), with respect to the cores, can be altered at will. If the carriage is central, the tongue will stay on either stop (Fig. 9b), but if it is displaced to one side (Fig. 9c), so that when the armatures are deflected by the current, the movement is limited so that the central line is not crossed, the armatures will return to their original position when the current ceases. This movement of the carriage is known as giving "bias" —spacing bias when moved in the direction shown in Fig. 9a, marking bias when moved to the opposite side, as shown in Fig. 9c.

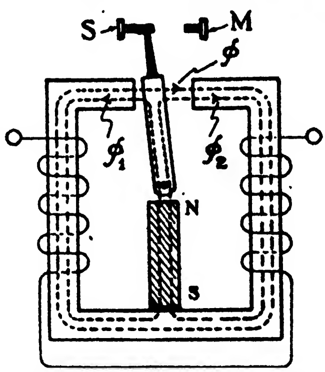


FIG. 10.—Theory of Polarised Relay.

The simplified device of Fig. 10 will be recognised to be the same theoretically as the polarised relay, with the exception that there is only one moving armature instead of two. In the diagram Φ_1 and Φ_2 are the fluxes in the left and right-hand gaps respectively due to the permanent magnet NS; Φ is the flux due to the current. The pull on the armature to the right is proportional to $(\Phi_2 + \Phi)^2$ and that to the left to $(\Phi_1 - \Phi)^2$. The resultant pull to the right is thus proportional to

$$\begin{aligned}&(\Phi_2 + \Phi)^2 - (\Phi_1 - \Phi)^2\\ &= \Phi_2^2 - \Phi_1^2 + 2\Phi(\Phi_2 + \Phi_1)\\ &= (\Phi_2 + \Phi_1)\{\Phi_2 - \Phi_1 + 2\Phi\}\end{aligned}$$

proportional to the permanent flux and to the flux Φ produced by the current, and therefore nearly proportional to the current itself, instead of to the square of the current as in the non-polarised relay. This is indicated in Fig. 11, where the curve n shows the pull plotted against the current for a non-polarised relay, and p is the corresponding curve for a polarised relay. The latter is far more sensitive for small currents. There are two instruments in general use, the "A" which has two coils of 200 ohms each, and the "B" in which the coils are each 100 ohms. Each instrument will work with 0·5 mA with the coils in series, and 1·0 mA with the coils in parallel.

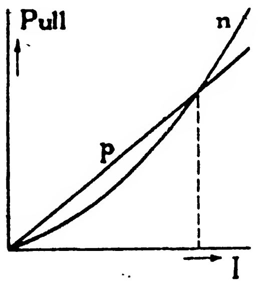


Fig. 11.—Variation of pull in Armature with current.

The curves of Fig. 12 show the results of some tests made on a polarised relay. The currents required to actuate the relay are plotted against the position of the carriage as measured by the number of turns of the biasing screw. The neutral setting is that which makes the marking current equal to the spacing current. As the carriage is moved in the marking direction (to the left on the diagram) the current required to mark is reduced and that required to space is increased, until at A, half a screw turn from the neutral position, no current is required to mark. Beyond A the tongue will return to the marking stop when the spacing current is reduced below the value indicated on the marking current curve. For instance, when the carriage is at the setting 2, that is, one turn from the neutral in the marking direction, a current of about 12½ milliamperes is required to move the tongue to the spacing stop, but it will stay there until the current is reduced to about 4 milliamperes, when it will return to the marking stop. Similarly with spacing bias. The critical position is at B. If the carriage is between A and B, the tongue will remain on whichever stop it is placed. To the right of B it will always return to the spacing stop.

former, spacing bias must be given by means of the carriage so that when the current ceases to flow, the tongue returns to the spacing stop.

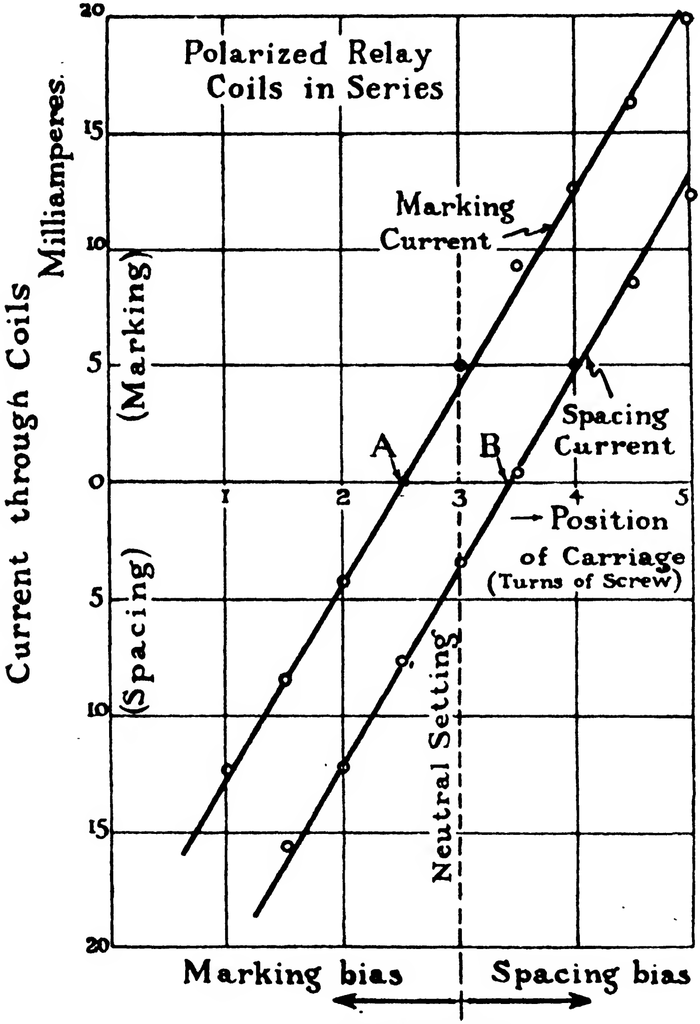


FIG. 12.—Test on Polarised Relay.

The reason for the 420 ohm shunt across the sounder coils is seen from the use of the sounder in the local circuit. With the sudden cessation of the current when the local circuit is broken by the tongue, considerable sparking at the contacts would take place owing to the large inductance of the sounder, and the large voltage produced in consequence at break. Instead of this, however, the induced electro-motive-force sends a current through the non-

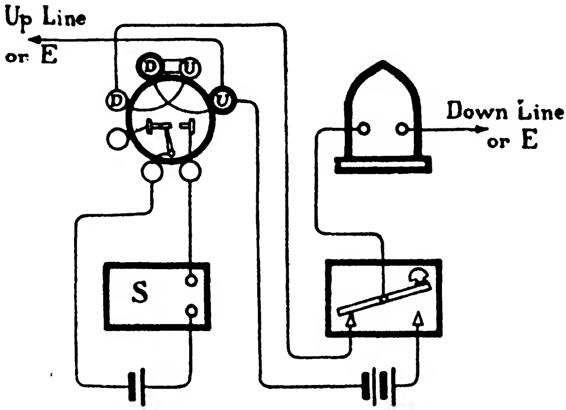


FIG. 13.—Single Current Simplex, relay working.

inductive resistance. The voltage rise cannot therefore exceed $0{\cdot}1 \times 420 = 42$ volts with a current of 100 milliamperes.

The coils of the relay can be joined either in series or in parallel. To find which is the better arrangement, let V = voltage of battery, R = resistance of each coil and N the number of turns, R_o = resistance of the rest of the circuit, I_s the current when the coils are in series, and I_p when in parallel.

Then $I_s = V/(R_o + 2R)$ and $I_p = V/\left(R_o + \frac{R}{2}\right)$.

The ampere turns in the two cases are $2NI_s$ and $2 \times \frac{I_p}{2} \times N$ respectively.

Thus

$$\frac{\text{magnetic effect in series}}{\text{magnetic effect in parallel}} = \frac{2NI_s}{NI_p}$$

$$= \frac{2\left(R_o + \frac{R}{2}\right)}{R_o + 2R} = \frac{2 + \frac{R}{R_o}}{1 + \frac{2R}{R_o}}$$

If $R = R_0$ equal results are obtained, if $R > R_0$ the parallel arrangement is better; if $R < R_0$ the series arrangement is better.

Extended to circuits where there are a number of relays and galvanometers, a rule may be stated: "use parallel connection if half the resistance of all the relays and galvanometers is ~~less~~ than the resistance of the rest of the circuit; otherwise use series connection."

It may, however, for high speed working, be better to use the parallel arrangement, as in this case the inductance of the relay is approximately only one-quarter of its value with the coils in series.

The differential winding of the coils of the relay is utilised in duplex working, by which messages in each direction along a line

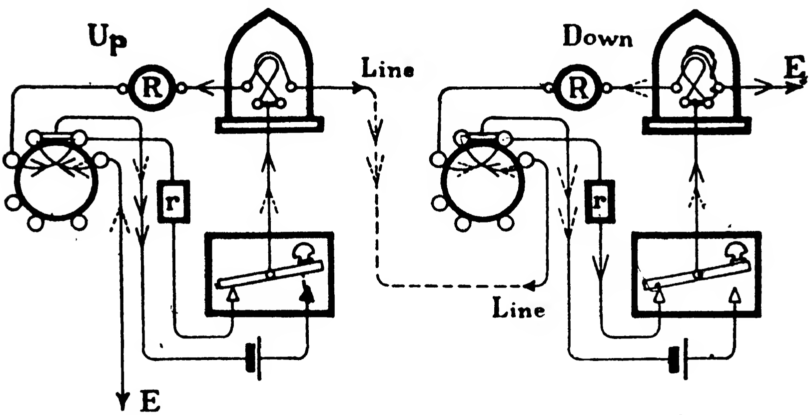


Fig. 14.—Single Current Duplex.

can be sent simultaneously. The manner in which this is achieved will be clear from Fig. 14. The local circuits are omitted for simplification. The resistances R are of such a value that when either key is depressed (say the up key) the current which flows through one coil of the galvo to line, and then through one coil of the down station relay, the small resistance r (equal to the internal resistance of the battery), the back stop of the down key, one coil of the down galvo, to earth and back to the battery through one coil of the up relay, is equal to the current which flows through the other coil of the up galvo, the resistance R, and the other coil of the up relay. Equal currents are then flowing in opposite directions through the two coils of the up station relay, and there is no magnetic effect, *but the current through one coil of the down station relay causes it to be operated in the marking direction. These currents are indicated by firm arrows.* Corresponding currents (indicated by dotted arrows) flow from the down battery when the down key is depressed, with the result that when both keys are depressed, both relays are operated. When the down key is half-way down, so that it makes no contact, current from the up battery flows through the two coils of the down relay in series, through the down balancing resistance R, through the two coils of the galvanometer and so to earth and back to the up station. The current will have half its normal value, but since it is passing through the two coils of the down relay, it will produce the normal magnetic effect, and the signal sent from the up to the down station will not be disturbed.

The galvanometers are used in adjusting R.

This circuit is known as single current differential duplex, combination method. In another arrangement, the opposition method, the battery voltages oppose each other when both keys are depressed, and no current flows in the line, although both relays are operated by the home batteries sending currents through the resistances R.

The circuits through the resistances R are known as the compensating circuits, and the resistance R as the duplex or line balance. Actually, as will appear later, on all but short lines the balance cannot be effected by a single resistance, and it is necessary to replace R by a network of condensers and resistances.

(5) Double Current systems

In double current working the restoration of the receiver is effected by a current in the reverse direction. The key used must on depression send a current to line in one direction to "mark," and on release a current in the reverse direction to "space." Further, the receiving apparatus must be polarised, so that the movement depends upon the current direction.

The double current key is shown in Fig. 15. Actually it is a key and switch combined, the switch having "send" and "receive" positions. There are five terminals, the connections among which for different positions of the key and the switch are shown in the figure. Fig. (*a*) gives a general diagram of connections, showing the send and receive switch and the double bridge piece A—B, in which the parts A and B are insulated from each other and play between two pairs of contacts CD and EF. With the switch in the send position the connections are seen to be as in Fig. (*b*) with the key down, and as in Fig. (*c*) with the key up. With the switch in the receive position the connections are as shown in Fig. (*d*). Details

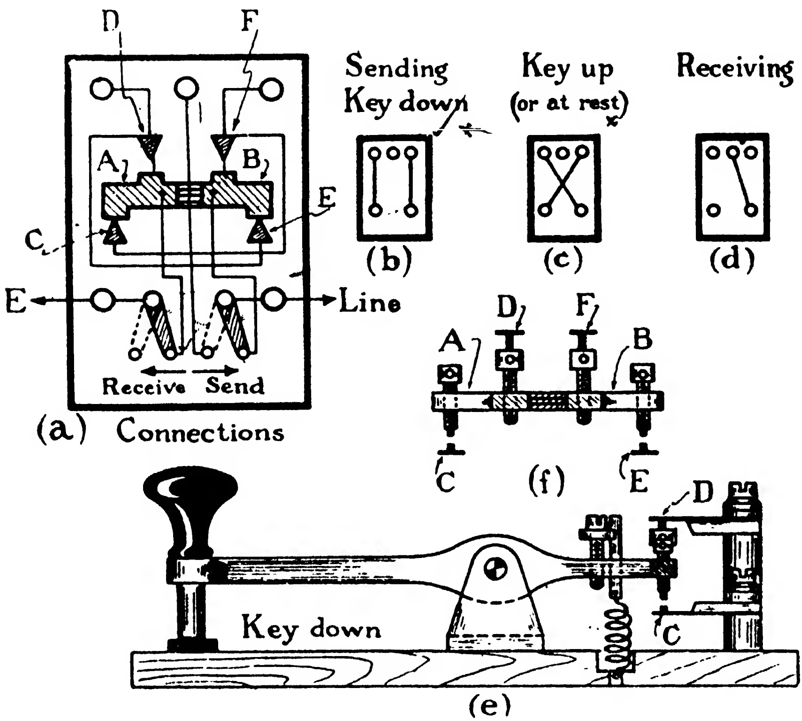


Fig. 15.—Double Current Key.

of the actual key are shown in Figs. (*e*) and (*f*), in which the lettering corresponds to that in Fig. (*a*).

The polarised relay with neutral adjustment is the usual receiver. The connections of a double current simplex circuit are shown in Fig. 16. With the keys in the receive position no current flows to line. When either operator wishes to send (say, the up station) he throws his switch to the send position. With his key at rest, current is sent to earth, through the galvanometer and relay coils in series at the down station, through the line and galvanometer at the up station back to the battery. The current has passed through the down station relay in the spacing direction and the tongue is held or pulled over to the spacing stop. When the up key is depressed the current is reversed and the down station relay marks.

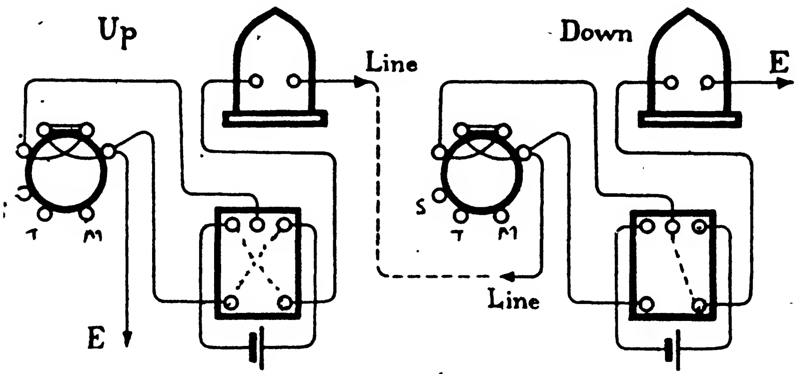


Fig. 16.—Double Current Simplex Working.

Before the up station operator can receive a reply he must throw his key switch to the receive position.

The double current system has several advantages over the single current system. In the first place, the neutral adjustment of the polarised relay renders it more sensitive than when set with magnetic bias. Another advantage is that the hour-to-hour relay adjustment occasioned in single current working, as the line current varies with changing line insulation, is not necessary in double current working, as, of course, the marking and spacing currents are equally affected, and equal forces in the spacing and marking directions are ensured. A third is that the reversal of the current through the coils of the receiving apparatus very considerably reduces the possibility of trouble from residual magnetism. A fourth is concerned with the effects of line capacity, and is dealt with in Chapter III.

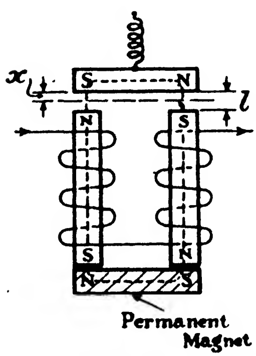


Fig. 17.—Magnetic Circuit of Polarised Sounder.

An economy may be effected on short lines by using a polarised sounder direct in the line instead of the polarised relay with a sounder in the local circuit. The polarised sounder is of quite similar construction to the pony sounder of Fig. 3, but the magnetic circuit is polarised by a powerful horse-shoe magnet as is indicated in Fig. 17. The tension of the spring is adjusted so that the armature will stay either in the up or in the down position. This is

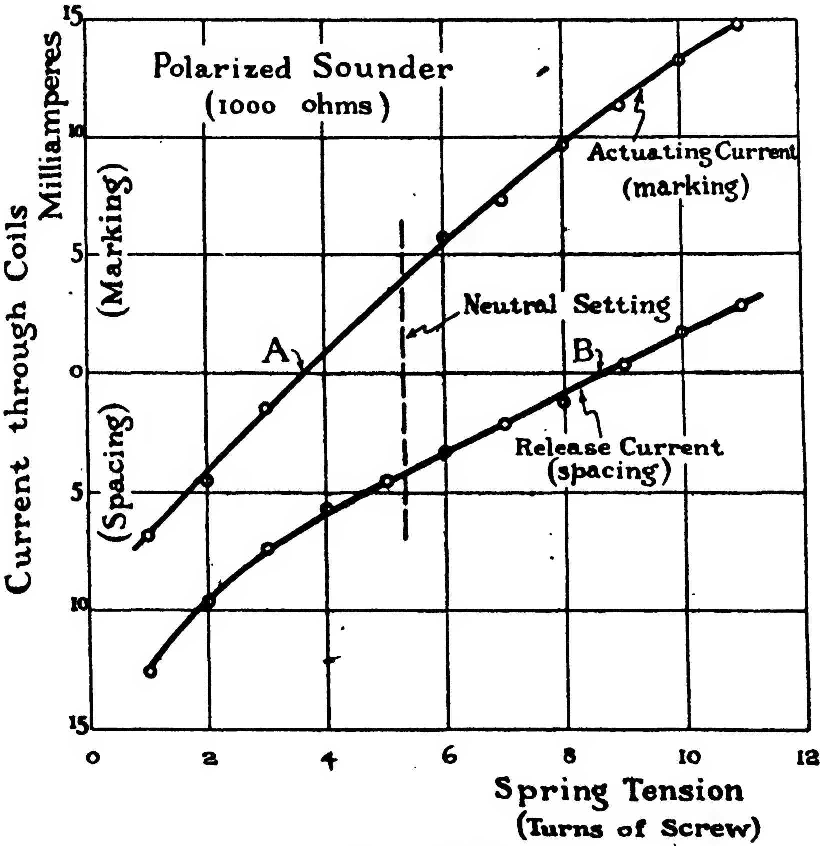


FIG. 18.—Test of Polarised Sounder.

possible since the flux varies roughly inversely as the air-gap length and the pull varies as the square of the flux, whereas the increase in the spring tension depends directly upon the armature movement. When current flows in the marking direction the flux round the magnetic circuit is increased, and the increased pull resulting overcomes the spring tension and pulls down the armature.

When the current is reversed the flux is weakened and the spring pulls the armature back against the weakened magnetic pull.

Fig. 18 shows some curves obtained from a 1,000 ohm polarised sounder, by finding the currents required to mark and to space, that

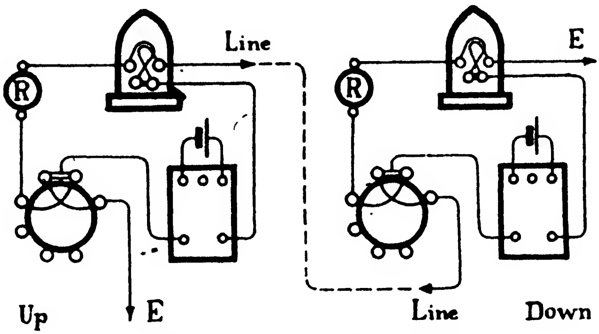


Fig. 19.—Double Current Duplex.

is to move the armature down and up, with different tensions of the spring. The general similarity with the polarised relay curves will be noted. The correct spring tension with the particular adjustment of the stops used is that which makes the marking current equal to the spacing current, and is shown by the vertical dotted

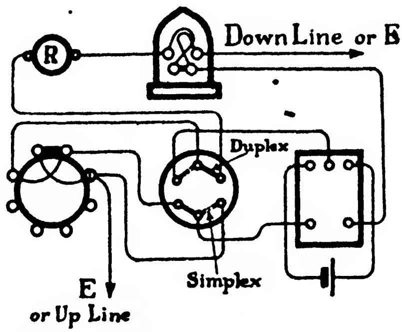


Fig. 20.—Double Current, Simplex or Duplex.

line. To the left of A the armature will always be down, and to the right of B it will always be up, when no current is flowing.

Double current duplex is carried out as shown in Fig. 19. The receive terminal of the double current key is not used, and current is always flowing whether messages are being sent or not. Accordingly a switch having six terminals and two positions is always inserted in the circuit so that the connections can be altered to simplex, key switches in receive position, when there is no traffic. This modification is shown in Fig. 20. The action of this double current duplex is exactly the same as the single current duplex described in section 4, and the tracing of the currents flowing with different key positions can safely be left to the reader. Single current duplex has practically dropped out of use in favour of double current duplex.

Still further use is made of a single line in quadruplex working, in which two messages are sent in each direction simultaneously, while intermediately there is a system known as diplex, in which two messages are sent in one direction simultaneously. The results achieved by the different systems are summarised in Fig. 21. In simplex working *either* A sends to B *or* B sends to A. In duplex, A sends to B and D sends to C. In diplex, A sends to B and C sends to D. In quadruplex A sends to B, D to C, E to F and H to G.

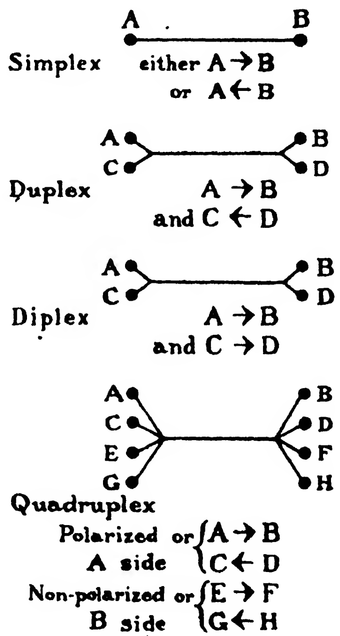


FIG. 21.—Simplex, Duplex, Diplex and Quadruplex.

Diplex is effected by a combination of polarised and non-polarised relays at the receiving station. The first works when the current direction is changed, the second being unaffected, while the second works when the current strength is increased, being biased against the current strength normally used for the first, the increased current not affecting the operation of the first. At the sending station there are two keys, one to reverse the current and one to alter its strength. Quadruplex is diplex duplexed.

Fig. 22 is in illustration of the principle of quadruplex working. The part of the circuit working with current reversals is known as the A side, and the part with current increments as the B side. The A key is shown as an ordinary double current key, the B key is shown as a single current key. The B battery has twice the voltage of the A battery.

(i.) When all the keys are at rest the conditions are very similar to those of a double current duplex; the batteries at either end combine to produce in the line a current (say 20 mA) twice as great as that in either of the compensating circuits (10 mA), the out of balance current being in the spacing direction in the polarised

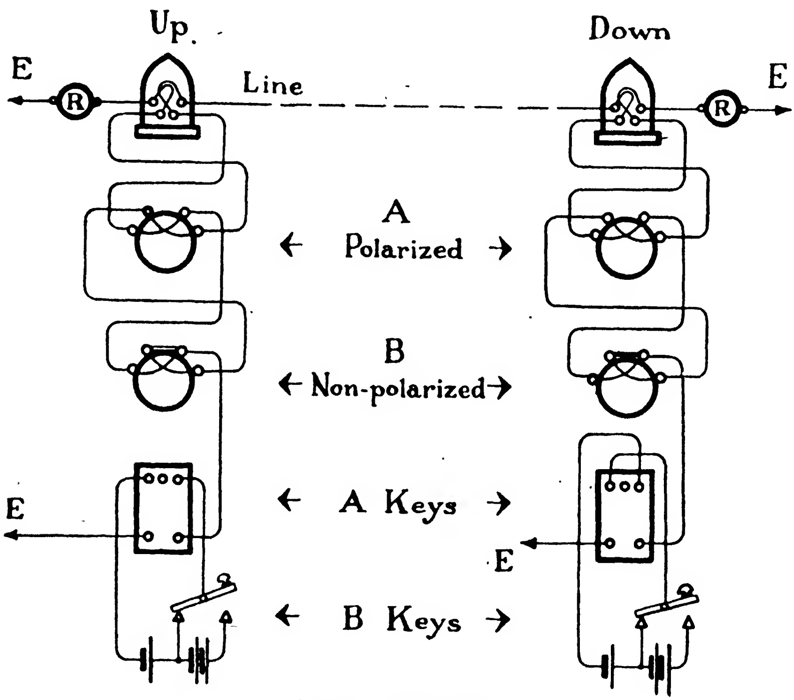


FIG. 22.—Quadruplex.

relays. The non-polarised relays are not operated, as the springs are set against the 10 mA available. (ii.) With both A keys depressed the currents are everywhere reversed, and the A relays mark. (iii.) With one A key depressed there is no line current, and the A relays are marking or spacing by the compensation circuit currents. (iv.) With both B keys depressed line current is 60 mA, and the compensating currents 30 mA, so that the B relays mark as the out of balance currents, 30 mA, are sufficient to move the armatures against the springs. (v.) It is seen now that signalling can continue on the A side with both B keys depressed without affecting the B side, provided that the current reversals are instantaneous, so that the actuating current in the B relays is at any instant 30 mA. (vi.) If only one B key is depressed, say at the up station, the line current is 40 mA. At the up station the compensating current is 30 mA and the up B relay does not operate, but the up A relay spaces. At the down station the compensating current is 10 mA, and the down B relay operates with an out of balance of 30 mA. The A relay spaces. (vii.) If now, with the up B key down, the up A key is depressed, a current of 20 mA flows to line in the marking direction. The up compensating current is 30 mA in the spacing direction, so that the up A relay spaces by 10 mA, but the B relay is not moved. At the down

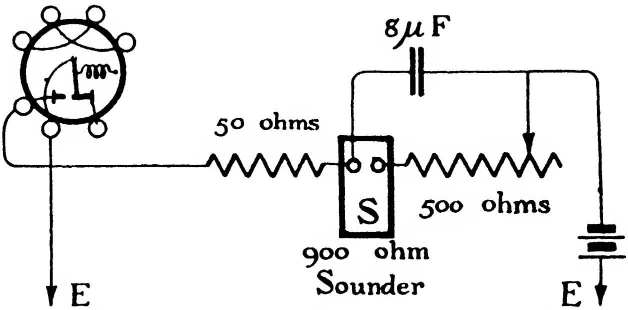


FIG. 23.—Condenser discharge to overcome B-kick.

station the marking current is 20 mA, and the spacing current (through compensating circuit) is 10 mA and the down A relay marks. There is 20 mA through the line coil of the B relay and 10 mA through the compensating circuit coil in the same direction, and the down B relay operates.

Two difficulties may be noted. The battery is entirely disconnected from the circuit while the B key is passing from the back stop to the front stop. This is overcome by using a special "increment key," with contacts similar to those of the double current key, but adjusted so that the back contact is not broken until the front contact is made. Similarly with the A key. A special key is used with the same adjustment.

The second difficulty is known as the "B-kick," and occurs when a B relay is marking and the current is reversed by an A key. Since this reversal cannot be instantaneous there is an interval of time during which sufficient current to mark is not available. The spring pulls the tongue off the marking stop for an instant and the signal is split. This difficulty is overcome by providing a condenser discharge current to hold over the armature of the sounder in the local circuit during the short time required for the current to be reversed, by the arrangement shown in Fig. 23.

(6) Universal Battery

The circuit arrangements described up to now have all been for use with a separate battery, usually of primary cells, for each

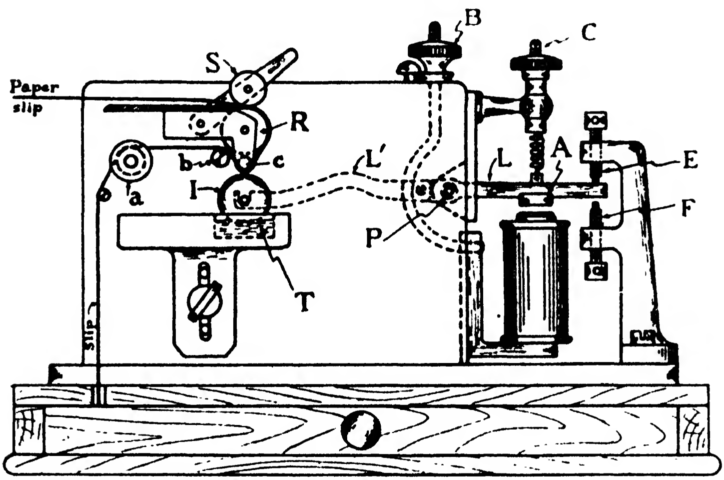


FIG. 24.—Morse Inker.

circuit. In large offices, however, it is usual to use one battery of large secondary cells to supply all the line circuits in the office, while a second battery supplies all the local circuits. The use of such a "universal battery" involves some modification in the diagrams for individual battery working to avoid short-circuiting the battery. It is evident, for instance, that an up and a down single current circuit cannot be worked off one battery without shorting it, for on depressing the two keys, in the one case the negative terminal of the battery is joined to earth, and in the other case the positive terminal.

The circuit modifications necessary are easily understood, involve no change of principle and are given in detail in the Telegraph Circuit Diagram Note-book published by the Stationery Office.

(7) Other Apparatus and Systems

When it is required to have a record of a message a Morse recorder or inker may be used direct in a single current circuit instead of the sounder, or in the local circuit of a relay. Electrically this instrument is the same as the pony sounder, but the lever which is moved by the armature carries on an extension an inking wheel I, which dips normally into an ink trough T, and is raised against a slip of paper as the armature is pulled down. The slip of paper is moved forward continuously by clockwork, and so dots and dashes are marked as such on the paper. The instrument is shown diagrammatically in Fig. 24. The armature A is fixed to a lever L pivoted at P. The bent arm L′ which carries the inking wheel is rigidly connected to L. The wheels R and I are driven by clockwork, speed-controlled by an expanding fan, while the roller S is pressed on to R by a spring. The paper slip is carried on a revolving drum in a drawer in the base of the instrument, is brought out through a slit, passed over the roller *a*, over the rod *b*, under the rod *c*, and then passed between R and S. In this way while the clockwork is working the paper is fed forward by the driving wheel R, but the inking wheel I does not touch it until the armature is attracted to the electromagnet. The screws B and C are for adjusting the position of the core with regard to the armature and the tension on the spring respectively. The lever plays between the adjustable stops E and F. The resistance of the coils is 300 ohms, and the current required is from 15 to 20 mA. There is also a polarised pattern of the instrument working on the same principle as the polarised sounder.

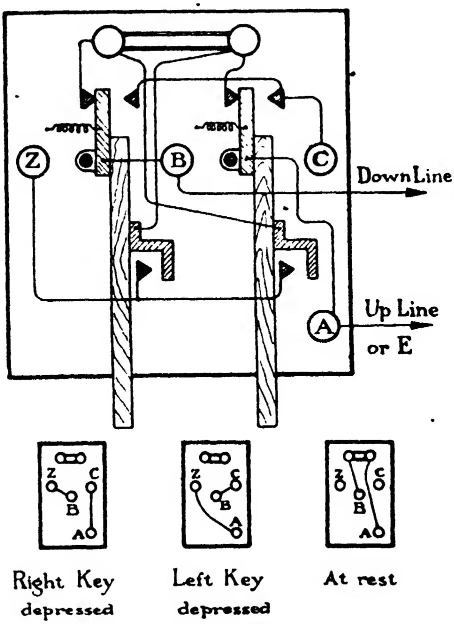


FIG. 25.—Commutator.

In the single needle system the two distinctive signals of the Morse code, instead of being the dot and the dash, are deflections of a galvanometer needle to the left or to the right. The movement of the needle is limited by ivory stops on the dial of the instrument. The signalling device, known as a commutator, has two keys, one of which is pressed to send current in one direction, and the other to send current in the reverse direction. The connections are shown in Fig. 25.

This system is very little used now except on the railways, as it is far more tiring to read visual signals than aural ones. In fact, it became the custom of the operator to put a tin sheath on one of the

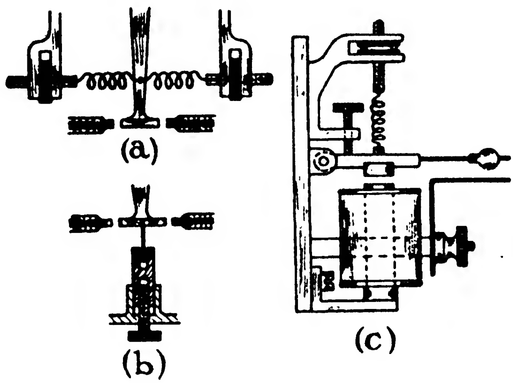


FIG. 26.—(a) and (b) Neutral Tongue Relays. (c) Sounder.

ivory stops and to read the message by the different sounds made by the needle in striking the tin and the ivory. This led to the introduction of the double sounder system, in which two sounders with different notes are operated, one for the "dot" and the other for the "dash." The sounders are placed in the local circuits of a neutral tongue polarised relay, *i.e.*, one in which the tongue is held neutrally in the rest position by means of springs in either of the manners (*a*) or (*b*), Fig. 26 ; (*c*) shows the type of sounder used, and the full connections are given in Fig. 27.

In all these systems using the Morse code skilled operators are required, but there are many circuits where the traffic is so light as to render the cost of the employment of skilled operators prohibitive. In many of these cases the telephone is replacing the telegraph apparatus, messages being spelt out over the telephone circuit. Before the advent of the telephone, however, Wheatstone had invented a system known as the ABC, which involved little or no skill on the part of the operator, and this, with a modified form of receiver, is still in use.

The transmitter, or communicator as it is called, has thirty keys arranged in a circle, each corresponding to a particular letter or number. A magneto generator worked by a handle has a pointer pivoted at the centre of the circle geared to its spindle, so that as the handle is turned and an alternating electromotive force generated, the pointer moves round and points to each key in turn. A word is always started from the zero key. When any key is pressed and the handle turned, the pointer rotates and an alternating current is sent to line until the pressed key is reached, when the pointer is stopped and the generator disconnected from the line. Thus the number of half-waves sent to line is determined by the key pressed. The receiver or indicator works on the same principle as the polarised relay, but the vibrations of the armature are caused through an escapement wheel arrangement to rotate a pointer over a dial with lettering identical with that of the keys of the communicator. The indicator is initially set at zero, and each half-wave received moves the pointer one letter forward. All the receiving operator has to do, therefore, is to read off the letters at which the pointer stops.

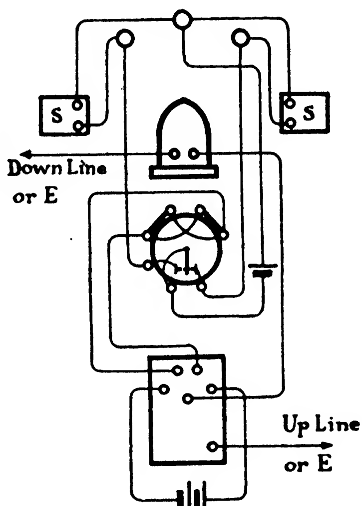


Fig. 27.—Double Sounder System.

The Stelges recorder, which has replaced the indicator, prints typed characters on a slip of paper. In addition to a polarised electromagnet which rotates a type wheel until the signalled letter is opposite the paper slip, there is a non-polarised electromagnet whose armature is attracted while the signal impulses are being received, but released when they cease, and the release of the armature causes the type wheel to strike the paper. The restoration of the type wheel which automatically follows carries the paper forward one letter space.

A further development is the Rabesi typewriting telegraph, which is also the invention of W. S. Stelges. A standard typewriter keyboard replaces the circular keyboard of the communicator, batteries are used instead of the magneto, and the receiver prints either on tape or page as required. A typist at the transmitting end produces typed sheet at the receiving end.

REFERENCES FOR FURTHER READING

"Connections of Telegraphic Apparatus and Circuits." Stationery Office.

"Instruction in Army Telegraphy and Telephony," Vol. I. "Instruments." Stationery Office.

A. E. Stone.—"A Text-book of Telegraphy."

T. E. Herbert.—"Telegraphy."

CHAPTER II

SHORT LINES

(8) Leaky Line Theory

Overhead telegraph lines are more or less leaky, that is current escapes along the line from line to earth to return to the sending

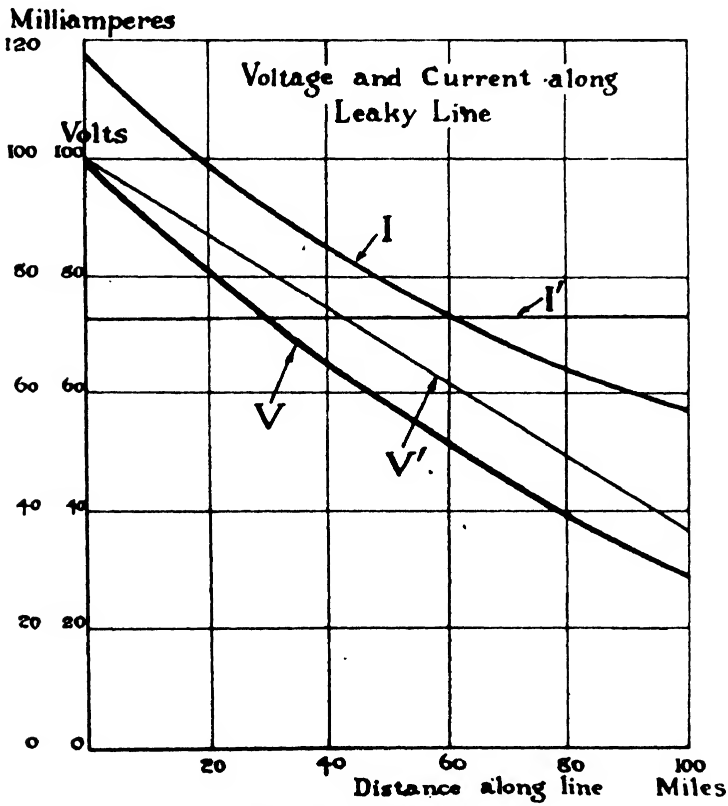


FIG. 28.—Voltage and Current along leaky line.

end battery, instead of passing through the apparatus at the receiving end. The amount of this leakage current depends upon the insulation resistance of the line. If the insulation resistance of one mile of the line is Ω ohms, then the reciprocal $1/\Omega$ is called the leakance of the line, and is denoted by G. The resistance of the line per mile is R ohms. In Fig. 28 the straight lines V′ and I′ show how the potential to earth and the line current vary along a perfectly insulated line with apparatus at the receiving end, while V and I show the variations with a leaky line. It is required to find how the curves V and I depend upon R and G, and particularly what value of current will be sent through the receiving end apparatus by a given voltage applied at the sending end.

Let AB (Fig. 29) represent a short length, δx miles, of the line distance x miles from the sending end. The resistance of the length δx is $R\delta x$ ohms and the conductance from it to earth is $G\delta x$ mhos. If the voltage at A is V, that at B is $V + \frac{dV}{dx} \cdot \delta x$, and if the current at A is I, that at B is $I + \frac{dI}{dx} \cdot \delta x$.

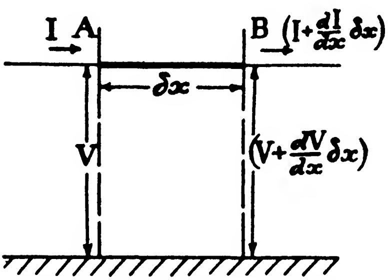


Fig. 29.—Fall of voltage and current along short length of line.

If the length δx is very short, the current I through it may be taken as constant in calculating the voltage difference between A and B, which is accordingly given by

$$IR\delta x = V - \left(V + \frac{dV}{dx}\delta x\right).$$

Similarly the voltage may be taken as constant in calculating the difference of the currents at A and B, giving

$$VG\delta x = I - \left(I + \frac{dI}{dx}\delta x\right).$$

These equations reduce to

$$\left.\begin{aligned} \frac{dV}{dx} &= -RI \quad \ldots\ldots\ldots (1) \\ \text{and} \qquad \frac{dI}{dx} &= -GV \quad \ldots\ldots\ldots (2) \end{aligned}\right\}$$

Differentiation of the first with regard to x and substitution from the second, and differentiation of the second and substitution from the first yield the following:—

$$\left.\begin{aligned}\frac{d^2V}{dx^2} &= RGV\\ \frac{d^2I}{dx^2} &= RGI\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (3)$$

As may be verified by substitution, the solutions of these equations may be written

$$V = A \cosh \sqrt{RG}\, x + B \sinh \sqrt{RG}\, x \quad . \quad . \quad . \quad (4)$$

$$I = C \cosh \sqrt{RG}\, x + D \sinh \sqrt{RG}\, x \quad . \quad . \quad . \quad (5)$$

where A B C and D are constants. C and D are not, however, independent of A and B.

For differentiating (5)

$$\frac{dI}{dx} = C\sqrt{RG} \sinh \sqrt{RG}\, x + D\sqrt{RG} \cosh \sqrt{RG}\, x$$

$$= -GA \cosh \sqrt{RG}\, x - GB \sinh \sqrt{RG}\, x$$

by (2) and (4). Since this must be true for all values of x, the coefficients of the sinh terms must be equal,

giving $$C\sqrt{RG} = -GB$$

or $$C = -B\sqrt{\frac{G}{R}},$$

and the coefficients of the cosh terms must be equal,

giving $$D\sqrt{RG} = -GA$$

or $$D = -A\sqrt{\frac{G}{R}}.$$

Writing $\sqrt{RG} = a$ and $\sqrt{\frac{R}{G}} = R_o$, ... (6)

equations (4) and (5) become

$$V = A \cosh ax + B \sinh ax \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$I = -\frac{B}{R_o} \cosh ax - \frac{A}{R_o} \sinh ax \quad . \quad . \quad . \quad . \quad (8)$$

These equations give the voltage to earth V and the line current I at a point distant x miles from the sending end. The values obtained depend on two constants, A and B, which are determined by the conditions at the ends of the line.

(i.) When the line is infinitely long, both $V = 0$ and $I = 0$ when $x = \infty$. Since for large values of ax, $\sinh ax$ and $\cosh ax$ become nearly equal, this is only possible by making $A = -B$. At the sending end, where $x = 0$, $V = V_s$, the sending end voltage, and $V_s = A$ from (7) and $I_s = V_s/R_0$ from (8).

Thus

$$\left.\begin{aligned} V &= V_s \cosh ax - V_s \sinh ax = V_s \epsilon^{-ax} \\ I &= \frac{V_s}{R_0} \cosh ax - \frac{V_s}{R_0} \sinh ax = \frac{V_s}{R_0} \epsilon^{-ax} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (9)$$

The fall of potential and current along the infinite line follow a logarithmic law, and the extent of the decay is determined by a; a is called the *attenuation constant* of the line. The ratio of the potential to the current at any point along the infinite line is equal to R_0: R_0 is called the *characteristic resistance* of the line.

Since the resistance at any point looking towards the infinitely distant end is R_0, it is clear that cutting the line at $x = l$ and earthing it through a resistance R_0 will make no alteration to the potential and current on the first l miles of the line. In other words, the potential and current along a line earthed at the far end through its characteristic resistance are the same as though the line were infinitely long.

(ii.) When the far end of a line length l miles is insulated, $A = V_s$ and $B = -I_sR_0$ from (7) and (8) when $x = 0$, and from (8) with $x = l$,

$$0 = -\frac{B}{R_0} \cosh al - \frac{V_s}{R_0} \sinh al,$$

giving

$$B = -V_s \tanh al$$

and

$$I_s = \frac{V_s}{R_0} \tanh al \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The apparent resistance of the line at the sending end is

$$R_f = \frac{V_s}{I_s} = R_0 \coth al \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The voltage at any point along the line is found by substituting the values of A and B in (7), giving

$$\begin{aligned} V &= V_s \cosh ax - V_s \tanh al \sinh ax \\ &= V_s \frac{\cosh a(l - x)}{\cosh al} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12) \end{aligned}$$

and the current by substitution in (8), giving

$$I = \frac{V_s}{R_o} \tanh al \cosh ax - \frac{V_s}{R_o} \sinh ax$$

$$= \frac{V_s}{R_o} \frac{\sinh a(l-x)}{\cosh al} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

(iii.) When the far end is earthed,

$A = V_s$ and $B = -I_sR_o$ as before, but when

$x = l$, $V = 0$, and from (7)

$0 = V_s \cosh al + B \sinh al$.

$\therefore B = -V_s \coth al$,

and $$I_s = \frac{V_s}{R_o} \coth al \quad . \quad . \quad . \quad . \quad . \quad (14)$$

The apparent resistance of the line from the sending end is

$$R_g = \frac{V_s}{I_s} = R_o \tanh al \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Substitution in (7) and (8) for the voltage and current along the line gives

$$V = V_s \frac{\sinh a(l-x)}{\sinh al} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

$$I = \frac{V_s}{R_o} \cdot \frac{\cosh a(l-x)}{\sinh al} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

Equations (11) and (15) can be used to obtain the resistance and leakance of a line by resistance measurements with a Wheatstone bridge at the sending end, with the far end first insulated and then earthed.

For $R_fR_g = R_o^2$ and $\frac{R_g}{R_f} = \tanh^2 al$, two equations from which a and R_o are found, and hence R and G from

$$R = aR_o. \qquad G = \frac{a}{R_o} \text{ from (6).}$$

As an example, the measurements on an 800-mile line were 5912 and 4434 ohms respectively.

$$R_o = \sqrt{5912 \times 4434} = 5120 \text{ ohms.}$$

$$al = \tanh^{-1}\sqrt{\frac{4434}{5912}} = \tanh^{-1} 0{\cdot}866 = 1{\cdot}317.$$

$$\therefore a = 1{\cdot}317/800 = {\cdot}001646,$$

and $R = 5120 \times \cdot 001646 = 8{\cdot}428$ ohms per mile,

$$G = \frac{\cdot 001646}{5120} = 0{\cdot}3215 \times 10^{-6} \text{ mho per mile,}$$

or the insulation resistance was 3·11 megohms per mile.

(iv.) When the far end is earthed through a resistance R_r,

$$A = V_s \text{ as before, and } V_r = I_r R_r$$
$$= V_s \cosh al + B \sinh al$$

from (7), while from (8)

$$I_r = -\frac{B}{R_o}\cosh al - \frac{V_s}{R_o}\sinh al.$$

Hence $B\left(\sinh al + \frac{R_r}{R_o}\cosh al\right) = -V_s\left(\cosh al + \frac{R_r}{R_o}\sinh al\right)$

Substitution in (7) gives

$$V = V_s\left\{\cosh ax - \frac{\cosh al + \frac{R_r}{R_o}\sinh al}{\sinh al + \frac{R_r}{R_o}\cosh al}\sinh ax\right\}$$

$$= V_s \frac{\sinh a(l-x) + \frac{R_r}{R_o}\cosh a(l-x)}{\sinh al + \frac{R_r}{R_o}\cosh al} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and in (8) similarly

$$I = \frac{V_s}{R_o} \cdot \frac{\cosh a(l-x) + \frac{R_r}{R_o}\sinh a(l-x)}{\sinh al + \frac{R_r}{R_o}\cosh al} \quad . \quad . \quad . \quad . \quad . \quad (19)$$

It should be noted that when $R_r = R_o$, these equations reduce to $V = V_s\epsilon^{-ax}$ and $I = (V_s/R_o)\epsilon^{-ax}$ as in the infinitely long line.

The equations may be put into a simpler form. If $R_r < R_o$, by writing

$$\tanh\theta = R_r/R_o,$$

$$V = V_s\frac{\sinh\{a(l-x)+\theta\}}{\sinh(al+\theta)} \quad . \quad . \quad . \quad . \quad . \quad (20)$$

and
$$I = \frac{V_s}{R_o}\frac{\cosh\{a(l-x)+\theta\}}{\sinh(al+\theta)} \quad . \quad . \quad . \quad . \quad . \quad (21)$$

while if $R_r > R_o$, by writing

$$\tanh \theta = R_o/R_r,$$

the equations become

$$V = V_s \frac{\cosh\{a(l-x)+\theta\}}{\cosh(al+\theta)} \quad . \quad . \quad . \quad . \quad (22)$$

$$I = \frac{V_s}{R_o} \frac{\sinh\{a(l-x)+\theta\}}{\cosh(al+\theta)} \quad . \quad . \quad . \quad . \quad (23)$$

Comparing (22) and (23) with (16) and (17) for the earthed line, it is seen that the effect of the resistance is equivalent to an increase of the length of the line by θ/a, while a similar statement holds with regard to (20) and (21) and (12) and (13) for the insulated line.

The curves of Fig. 28 are plotted from (20) and (21) for a line 100 miles long having a resistance of 8·8 ohms per mile and a leakance of 10 micro-mhos per mile when the voltage applied to the line at the sending end is 100 volts and the resistance of the receiving apparatus is 500 ohms. $R_o = 940$ ohms, $a = \cdot 0094$ per mile, $\tanh \theta = 500/940 = 0{\cdot}532$, giving $\theta = 0{\cdot}593$ and the equivalent increase of length of line $= 0{\cdot}593/\cdot 0094 = 63$ miles.

(9) Equivalent Networks

All leaky line problems can be solved by the use of equations 8·07 and 8·08, but when interest lies only in the voltages and currents at the ends, a simplification can be effected by replacing the line by an equivalent network. The two simplest networks are the T and the Π.

(i.) Equivalent T.

First find (from 8·07 and 8·08) the voltage V_r and the current I_r at the receiving end of the line, in terms of V_s and I_s.

When $\qquad x = 0,\ A = V_s$ and $-\dfrac{B}{R_o} = I_s,$

and when $\qquad x = l,\ V = V_r$ and $I = I_r$.

Hence

$$V_r = V_s \cosh al - R_o I_s \sinh al \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$I_r = I_s \cosh al - \frac{V_s}{R_o} \sinh al \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Referring to Fig. 30, it is required to find the values of R_1 and R_2, so that equations (1) and (2) shall apply. From Fig. 30,

$$V_s = I_s(R_1 + R_2) - I_r R_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$0 = I_r(R_1 + R_2) - I_s R_2 + V_r \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

(3) rearranged is

$$I_r = I_s \frac{R_1 + R_2}{R_2} - \frac{V_s}{R_2},$$

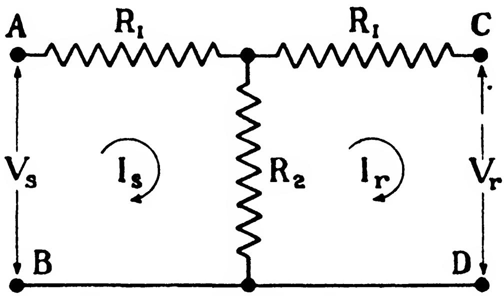


FIG. 30.—Equivalent T of leaky line.

and this is identical with (2) if

$$\frac{R_1 + R_2}{R_2} = \cosh al \text{ and } \frac{1}{R_2} = \frac{\sinh al}{R_o},$$

i.e.,

$$\left.\begin{aligned} R_2 &= \frac{R_o}{\sinh al} \\ R_1 &= \frac{R_o}{\sinh al}(\cosh al - 1) \quad . \quad . \\ &= R_o \tanh \frac{al}{2}. \end{aligned}\right\} \quad . \quad . \quad . \quad (5)$$

Substituting (5) in (4) gives

$$V_r = I_s \frac{R_o}{\sinh al} - I_r \,.\, R_o \frac{\cosh al}{\sinh al},$$

and using (2) to eliminate I_r, this becomes

$$V_r = I_s \frac{R_o}{\sinh al} - \frac{R_o \cosh al}{\sinh al}\left\{I_s \cosh al - \frac{V_s}{R_o} \sinh al\right\}$$

$$= V_s \cosh al - I_s R_o \sinh al,$$

i.e., equation (1) is obtained.

Evidently, therefore, equations (5) for the values of the resistances of the T circuit of Fig. 30 do lead to results identical to those obtained for the smooth line as far as the sending and receiving end voltages and currents are concerned.

If the line is very short, so that al is very small, $\sinh al = \tanh al = al$ very nearly,

and $$R_1 = R_o\frac{al}{2} = \frac{1}{2}\cdot\sqrt{\frac{R}{G}}\sqrt{RG}\,.\,l = \frac{1}{2}Rl,$$

and $$R_2 = \frac{R_o}{al} = \sqrt{\frac{R}{G}}\cdot\frac{1}{\sqrt{RG}\,l} = \frac{1}{Gl}.$$

The series resistance of the T from A to C is equal to the total line resistance, while the shunting conductivity Gl is equal to the total leakance of the line. Such a T is known as a "nominal" T; it is only useful in the case of short lines.

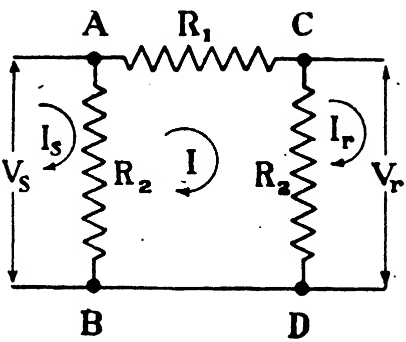


FIG. 31.—Equivalent Π of leaky line.

As a very simple example of the use of the equivalent T, if a voltage V_s is established across AB, and CD is insulated, the current flowing is $V_s/(R_1 + R_2)$ and the voltage across CD = voltage across $R_2 = V_sR_2/(R_1 + R_2)$

$$= \frac{V_s\,.\,R_o}{\sinh al}\cdot\frac{\sinh al}{R_o\cosh al} = \frac{V_s}{\cosh al},$$

in agreement with the result obtained from equation (8·12) by putting $x = l$.

(ii.) Equivalent Π.

Fig. 31 shows the equivalent Π circuit, and the following equations may be written down by Kirchhoff's Laws :—

$$\left.\begin{aligned} V_s &= (I_s - I)\,R_2 \\ 0 &= I\,(R_1 + 2R_2) - I_sR_2 - I_rR_2 \\ 0 &= I_rR_2 - IR_2 + V_r \end{aligned}\right\}$$

Elimination of I from the second and third by means of the first yields

$$\left.\begin{aligned} I_r &= I_s\frac{R_1+R_2}{R_2} - \frac{V_s}{R_2^2}(R_1+2R_2) \\ \text{and} \qquad V_r &= -V_s + I_sR_2 - I_rR_2 \end{aligned}\right\} \quad \begin{aligned} &\ldots\ldots (6) \\ &\ldots\ldots (7) \end{aligned}$$

To make (6) identical with (2), write $\dfrac{R_1+R_2}{R_2} = \cosh al$ and $\dfrac{R_1+2R_2}{R_2^2} = \dfrac{\sinh al}{R_o}$, from which

$$\left.\begin{aligned} R_1 &= R_o \sinh al \\ \text{and} \qquad R_2 &= \frac{R_o}{\sinh al}(\cosh al + 1) \\ &= R_o \coth\frac{al}{2} \end{aligned}\right\} \quad \ldots\ldots (8)$$

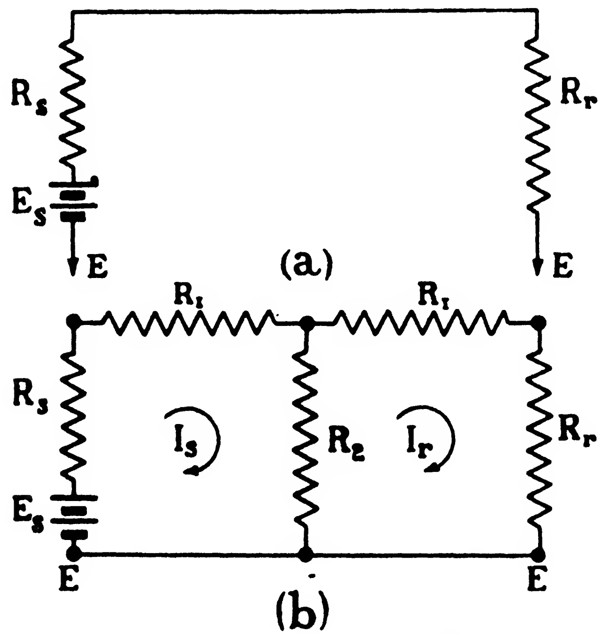


FIG. 32.—General case of leaky line.

are obtained as the values of the resistances in the equivalent Π. It may be verified that substitution of (8) in (7) and elimination of I_r by (2) gives (1).

When the line is very short,

$$R_1 = R_o\, al = \sqrt{\frac{R}{G}}\,\sqrt{RG}\,.\,l = Rl$$

$$R_2 = R_o\,.\,\frac{2}{al} = \sqrt{\frac{R}{G}}\,.\,\frac{2}{\sqrt{RG}l} = \frac{2}{Gl}$$

The whole of the resistance of the line is between A and C, while half the leakance is between A and B and half between C and D. This is the "nominal" Π.

The equivalent T network can now be used to find the sending end and receiving end currents in the most general case, that is, when there is apparatus of resistance R_s at the sending end and R_r at the receiving end, and a battery of electromotive force E_s at the sending end. The line shown in Fig. 32 (a) can be replaced by the network of Fig. 32 (b), where R_1 and R_2 have the values given by equations (5).

The circuital equations are :—

$$E_s = I_s\,(R_s + R_1 + R_2) - I_rR_2$$
$$0 = I_r\,(R_1 + R_2 + R_r) - I_sR_2$$

Putting the value of I_s from the second into the first gives

$$E_s = I_r\left\{\frac{(R_1 + R_2 + R_r)\,(R_1 + R_2 + R_s)}{R_2} - R_2\right\}$$

$$= \frac{I_r}{R_2}\left\{(R_1 + R_2)^2 + (R_r + R_s)\,(R_1 + R_2) + R_rR_s - R_2^2\right\}$$

$$= I_r\frac{\sinh\,al}{R_o}\left\{R_o^2\frac{\cosh^2 al}{\sinh^2 al} + (R_r + R_s)\,R_o\frac{\cosh al}{\sinh al} + R_rR_s - \frac{R_o^2}{\sinh^2 al}\right\}$$

$$= I_r\frac{\sinh\,al}{R_o}\left\{R_o^2 + R_rR_s + (R_r + R_s)\,R_o\frac{\cosh\,al}{\sinh\,al}\right\}$$

$$= I_r\left\{\left(R_o + \frac{R_rR_s}{R_o}\right)\sinh al + (R_r + R_s)\cosh al\right\}$$

$$\therefore I_r = \frac{E_s}{(R_s + R_r)\cosh al + \left(R_o + \dfrac{R_sR_r}{R_o}\right)\sinh al} \quad . \quad . \quad . \quad . \quad (9)$$

Also, since
$$I_s = I_r \frac{R_1 + R_2 + R_r}{R_2}$$
$$= I_r \frac{R_0 \cosh al + R_r}{R_2/\sinh al}$$
$$= I_r \frac{R_0 \cosh al + R_r \sinh al}{R_0},$$

the sending end current is given by
$$I_s = E_s \frac{R_0 \cosh al + R_r \sinh al}{R_0(R_s + R_r) \cosh al + (R_0^2 + R_s R_r) \sinh al} \quad . (10)$$

It is interesting to note that when the line is very long, so that $\cosh al = \sinh al = \epsilon^{al}$ very nearly, $I_s = \dfrac{E_s}{R_s + R_0}$, a result to be expected since an infinitely long line offers a resistance R_0 at the sending end.

Also when $R_r = R_0$ with any length of line, the expression for the sending end current reduces to $I_s = \dfrac{E_s}{R_s + R_0}$, showing that a line closed through its characteristic resistance behaves as an infinitely long line, a fact already noted on pages 34 and 36.

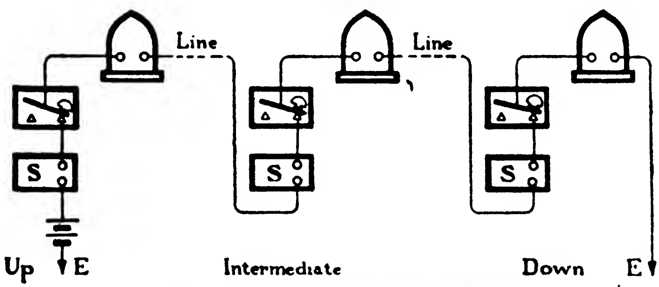


FIG. 33.—Continuous Working Systems.

(10) Central Battery Telegraphy

In central battery working one battery only for all the stations on a line is required. The battery is located at the head office, and a very considerable saving in battery maintenance is effected. The simplest system is the closed-circuit or continuous working system shown in Fig. 33, which is used to some extent in some other countries and in the Army. Its advantage, apart from the centralisation of the battery, is that since current is always flowing a fault is immediately indicated by the deflections of the galvanometers. Its disadvantage is the waste of battery power. The keys at each station are normally held down, so that current flows and pulls down the armatures of all the sounders. Raising the key at any station breaks the current, and signalling proceeds as usual.

A much more satisfactory arrangement is shown in Fig. 34. The apparatus at each out station consists simply of a key, a polarised sounder and a condenser (4 microfarad). At the head office there is in addition a battery of voltage E_s, a series resistance R_s, and a galvanometer. Depressing any key places an earth on the line and causes all the condensers to discharge through the polarised sounders; this discharge current is arranged to be in the

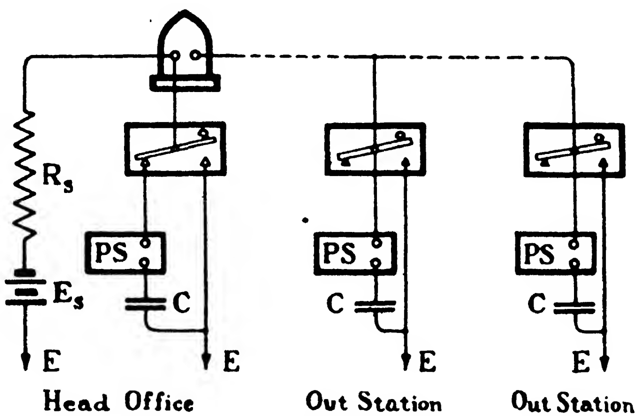


Fig. 34.—Central Battery Simplex.

marking direction and the sounder armatures are attracted to the cores. On releasing the key the condensers charge up again from the H.O. battery, the charging current passing through the sounders in the spacing direction and restoring the armatures. At each out station the home sounder works as well as the others, so that no galvanometer is needed. At the H.O. the sounder is connected to the back stop of the key, and so does not work with the home key. This is done to avoid unnecessary noise, and a galvanometer is provided to indicate that current is being sent to line.

The actuating currents depend upon the differences of potential across the condensers when a key is at rest and depressed, and these differences are smaller the lower the insulation resistance of the line. Conditions are most unfavourable at the H.O. Let the last station

(B) on the line be l miles from the H.O. It is required to find the difference of the potential difference across the condenser at the H.O. when B's key is at rest and when it is depressed. The current I_s' flowing to line when B's key is at rest is found by putting $R_r = \infty$ in (9·10), giving

$$I'_s = E_s \frac{\sinh al}{R_o \cosh al + R_s \sinh al} \quad . \quad . \quad . \quad . \quad (1)$$

Similarly the current I_s'' flowing when B's key is depressed is found by putting $R_r = 0$, giving

$$I_s'' = E_s \frac{\cosh al}{R_s \cosh al + R_o \sinh al} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The required voltage difference δV is thus

$$\delta V = R_s (I_s'' - I_s')$$

$$= E_s \left\{ \frac{\sinh al}{R_o \cosh al + R_s \sinh al} - \frac{\cosh al}{R_s \cosh al + R_o \sinh al} \right\}$$

$$= \frac{E_s}{\cosh 2\,al + \dfrac{R_s^2 + R_o^2}{2R_sR_o} \sinh 2\,al} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This is a maximum when $(R_s^2 + R_o^2)/2R_sR_o$ is a minimum, *i.e.*, when

$$\frac{d}{dR_s}\left(\frac{R_s}{2R_o} + \frac{R_o}{2R_s}\right) = \frac{1}{2R_o} - \frac{R_o}{2R_s^2} = 0,$$

or when $R_o = R_s$.

The best value of the series resistance at the H.O. is thus the characteristic resistance of the line, and when this resistance is used the available voltage is from (3)

$$\delta V = \frac{E_s}{\cosh 2\,al + \sinh 2\,al} = E_s \epsilon^{-2al} \quad . \quad . \quad . \quad (4)$$

The available voltage at the out station can be calculated in a similar manner. When the H.O. key is depressed the voltage is zero. When it is open the voltage is by (8·12) (putting $x = l$) $V_s/\cosh al$.

But (8·10) $$I_s = \frac{V_s}{R_o} \tanh al \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and
$$V_s = E_s - I_sR_s$$
$$= E_s - \frac{V_sR_s}{R_o}\tanh al,$$
$$\therefore \quad V_s = \frac{E_s}{1 + \frac{R_s}{R_o}\tanh al},$$

and
$$\delta V = \frac{V_s}{\cosh al} = \frac{E_s}{\cosh al + \frac{R_s}{R_o}\sinh al} \quad . \quad . \quad . \quad (6)$$

This is evidently greater than the voltage available (equation 3) at the H.O. When $R_s = R_o$ it becomes
$$\delta V = E_s\epsilon^{-al} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Actually, with the apparatus used, δV in practice must be at least 11 volts, and with the open lines in general use for such circuits, and using a battery of 80 volts, the limiting length is about sixty miles. R_s is usually made 1,000 ohms.

(11) **Artificial Leaky Line**

The characteristics of a leaky line may be closely simulated by an artificial line or chain built up of T sections or links as in Fig. 35,

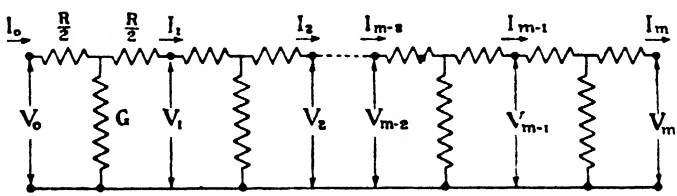


FIG. 35.—Artificial line of T links.

or of π sections as in Fig. 36. In Fig. 35 there are m sections; the resistance of the horizontal part of the T is R ohms, and that of the vertical part 1/G ohms. A voltage V_o is established at the beginning of the line. If the voltage and current at the end are V_m and I_m respectively, the voltage across the last leak is $V_m + \frac{R}{2}I_m$ and the current flowing to earth through the leak is $\left(V_m + \frac{R}{2}I_m\right)G$.

The current I_{m-1} entering the link is therefore $I_m + \left(V_m + \frac{R}{2} I_m\right) G$, and the voltage V_{m-1} at the beginning of the link is $V_m + \frac{R}{2}(I_m + I_{m-1})$. In this way the following equations may be written down :—

$$\begin{aligned}
V_{m-1} - V_m &= \frac{R}{2}(I_m + I_{m-1}) \\
V_{m-2} - V_{m-1} &= \frac{R}{2}(I_{m-1} + I_{m-2}), \\
V_{m-3} - V_{m-2} &= \frac{R}{2}(I_{m-2} + I_{m-3}) \\
&\vdots \\
V_o - V_1 &= \frac{R}{2}(I_1 + I_o) \quad . \quad . \quad . \quad . \quad . \quad (1)
\end{aligned}$$

$$\begin{aligned}
I_{m-1} - I_m &= \left(V_m + \frac{R}{2} I_m\right) G \\
I_{m-2} - I_{m-1} &= \left(V_{m-1} + \frac{R}{2} I_{m-1}\right) G \\
I_{m-3} - I_{m-2} &= \left(V_{m-2} + \frac{R}{2} I_{m-2}\right) G \\
&\vdots \\
I_o - I_1 &= \left(V_1 + \frac{R}{2} I_1\right) G \quad . \quad . \quad . \quad (2)
\end{aligned}$$

Write as a tentative solution

$$\left.\begin{aligned} V_n &= A \cosh n\gamma + B \sinh n\gamma \\ I_n &= C \cosh n\gamma + D \sinh n\gamma \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Inserted in (1) this gives

$$A \{\cosh (n-1)\gamma - \cosh n\gamma\} + B \{\sinh (n-1)\gamma - \sinh n\gamma\}$$
$$= \frac{R}{2}[C \{\cosh (n-1)\gamma + \cosh n\gamma\} + D \{\sinh (n-1)\gamma + \sinh n\gamma\}],$$

or $$A \sinh \frac{1}{2}(2n-1)\gamma \sinh\left(-\frac{\gamma}{2}\right) + B \cosh \frac{1}{2}(2n-1)\gamma \sinh\left(-\frac{1}{2}\gamma\right)$$
$$= \frac{1}{2} R \left\{ C \cosh \frac{1}{2}(2n-1)\gamma \cosh\left(-\frac{1}{2}\gamma\right) + D \sinh \frac{1}{2}(2n-1)\gamma \cosh\left(-\frac{1}{2}\gamma\right) \right\} \quad . \quad . \quad (4)$$

Since this must be true for all values of n, the coefficients of the variable cosh and sinh terms on each side may be evaluated separately, giving

$$A \sinh\left(-\frac{1}{2}\gamma\right) = \frac{1}{2} RD \cosh\left(-\frac{1}{2}\gamma\right)$$

and $$B \sinh\left(-\frac{1}{2}\gamma\right) = \frac{1}{2} RC \cosh\left(-\frac{1}{2}\gamma\right)$$

or $$\left.\begin{aligned} D &= -A\frac{2\tanh\frac{1}{2}(\gamma)}{R} \\ C &= -B\frac{2\tanh\frac{1}{2}(\gamma)}{R} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

There are only two arbitrary constants.

Considering two successive equations in the series (2),

$$(I_{n-1} - I_n) - (I_n - I_{n+1}) = G\left\{(V_{n-1} - V_n) - \frac{1}{2} R (I_{n-1} - I_n)\right\}$$

and using the corresponding equation from (1),

$$I_{n-1} + I_{n+1} = (2 + RG) I_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Substitution from (3) gives

$$2C \cosh n\gamma \cosh(-\gamma) + 2D \sinh n\gamma \cosh(-\gamma)$$
$$= (2 + RG)(C \cosh n\gamma + D \sinh n\gamma)$$

whence $$\cosh \gamma = 1 + \frac{1}{2} RG \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Write also $$R_0 = \frac{R}{2 \tanh \frac{1}{2}(\gamma)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and the solutions (3) are

$$V_n = A \cosh n\gamma + B \sinh n\gamma \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$I_n = -\frac{B}{R_o} \cosh n\gamma - \frac{A}{R_o} \sinh n\gamma \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Comparison with the solutions (equations 8·07 and 8·08) in the case of the real line shows the form to be identical. R_o is evidently the characteristic resistance of the chain and γ its attenuation constant per link.

If the number of links to represent a given line is made very large so that γ becomes very small, (7) may be written $1 + \frac{1}{2}\gamma^2 = 1 + \frac{1}{2}RG$, or $\gamma = \sqrt{RG}$, and (8) $R_o = \frac{R}{2 \times \frac{1}{2}\sqrt{RG}} = \sqrt{\frac{R}{G}}$; the chain of links merges into the actual line (see 8·06).

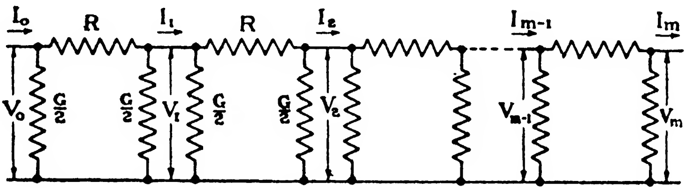


FIG. 36.—Artificial line of п links.

Since the forms of the general solutions are identical, all the particular results obtained for the real line can be applied direct to the chain line, using R_o as defined by (8), n instead of x, m instead of l, and γ as defined by (7) instead of a.

Consideration of the chain of п links (Fig. 36) follows on similar lines. The series resistance of each link is R ohms, and the conductance of each of the leaks of the п is G/2 mhos. The currents and voltages in the links are related by the following equations :—

$$I_{m-1} - I_m = \frac{1}{2} G (V_m + V_{m-1})$$

$$I_{m-2} - I_{m-1} = \frac{1}{2} G (V_{m-1} + V_{m-2})$$

$$\vdots$$

$$I_o - I_1 = \frac{1}{2} G (V_1 + V_o) \quad . \quad . \quad . \quad (11)$$

$$V_{m-1} - V_m = R\left(I_{m-1} - \frac{1}{2} GV_{m-1}\right)$$
$$V_{m-2} - V_{m-1} = R\left(I_{m-2} - \frac{1}{2} GV_{m-2}\right)$$
$$\vdots$$
$$V_o - V_1 = R\left(I_o - \frac{1}{2} GV_o\right) \quad . \quad . \quad . \quad . \quad (12)$$

The solution (3) substituted in (11) gives

$$C\{\cosh(n-1)\gamma - \cosh n\gamma\} + D\{\sinh(n-1)\gamma - \sinh n\gamma\}$$
$$= \frac{1}{2} G\Big[A\{\cosh n\gamma + \cosh (n-1)\gamma\}$$
$$+ B\{\sinh n\gamma + \sinh(n-1)\gamma\}\Big]$$

or $C \sinh \frac{1}{2}(2n-1)\gamma \sinh\left(-\frac{\gamma}{2}\right) + D \cosh \frac{1}{2}(2n-1)\gamma \sinh\left(-\frac{1}{2}\gamma\right)$

$$= \frac{G}{2}\Big\{A \cosh \frac{1}{2}(2n-1)\gamma \cosh\left(-\frac{\gamma}{2}\right)$$
$$+ B \sinh \frac{1}{2}(2n-1)\gamma \cosh\left(\frac{\gamma}{2}\right)\Big\}$$

whence $\qquad C \sinh\left(-\frac{\gamma}{2}\right) = \frac{G}{2} . B \cosh\left(-\frac{\gamma}{2}\right)$

and $\qquad D \sinh\left(-\frac{\gamma}{2}\right) = \frac{G}{2} . A \cosh\left(-\frac{\gamma}{2}\right)$

or
$$\left.\begin{aligned} D &= -A \frac{G}{2 \tanh \frac{\gamma}{2}} \\ C &= -B . \frac{G}{2 \tanh \frac{\gamma}{2}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

Two successive equations in (12) give

$$(V_{n-1} - V_n) - (V_n - V_{n+1}) = R\left\{I_{n-1} - I_n - \frac{1}{2} G(V_{n-1} - V_n)\right\}$$

and on substitution from (11)

$$V_{n-1} + V_{n+1} = (RG + 2)V_n \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

which gives on substitution from (3)

$$\cosh \gamma = 1 + \frac{1}{2} \mathrm{RG} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

while from (3) and (13)

$$\mathrm{R}_o = \frac{2 \tanh \frac{\gamma}{2}}{\mathrm{G}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The propagation constant of the chain of Π links is the same as the chain of T links, but the characteristic resistance is different. R_o, however, when the number of links is very large, so that γ is small, becomes $\sqrt{\mathrm{R/G}}$ as before.

It is a simple matter to construct an artificial leaky line, and useful laboratory experiments may be made with one. A direct current voltage is applied at one end, and the voltages along the line, that is at the end of each section, measured with an electrostatic voltmeter. In this way equation (8·18), and (8·20) or (8·22) may be verified with different terminal resistances, and the constants R and G may be calculated from resistance measurements with the far end open and closed.

In one particular T line there are five sections with $\mathrm{R} = 700$ ohms and $\mathrm{G} = 1/3500$ mhos. $\mathrm{Cosh}\,\gamma = 1 + \frac{1}{2}\mathrm{RG} = 1{\cdot}10$ and $\gamma = 0{\cdot}444$. $\sqrt{\mathrm{RG}} = 0{\cdot}446$. $\mathrm{R}_o = \mathrm{R}/\left(2\tanh\frac{1}{2}(\gamma)\right) = 700/\left(2\tanh 0{\cdot}222\right) = 1600$. $\sqrt{\frac{\mathrm{R}}{\mathrm{G}}} = \sqrt{700 \times 3500} = 1560$. So that the error involved in looking upon the line as smooth is at the most about 3 per cent.

REFERENCES FOR FURTHER READING

A. L. KENNELLY.—"Hyperbolic Functions Applied to Electrical Engineering."

A. L. KENNELLY.—"Artificial Electric Lines."

CHAPTER III

TRANSIENTS

(12) **Transients in Telegraphy**

So far little or no attention has been paid to the time that elapses between pressing the key at the sending end of a line and the striking of the sounder lever against the lower stop at the receiving end, and the time that elapses between releasing the key and the striking of the sounder lever against the upper stop. It has been tacitly assumed that these intervals of time are quite negligible. Even if they are not negligible but are equal, the dot and dash lengths will remain unaltered, and no distortion of the signal will ensue.

Three main causes contribute to the total time interval: (i.) the current will not, owing to the capacity and resistance of the line, immediately rise to its final value at the receiving end; (ii.) the current will not, owing to the inductance of the receiving instruments, immediately rise to its final value in the instruments themselves; and (iii.), owing to the inertia of the moving parts of the instruments, time will elapse between the rise of the current and the completion of the movement.

The capacity of the line causes other transient effects. On pressing the key the initial current flowing may be considerably greater than the final steady value. This affects the balance in the duplex systems, while the time taken to reverse the current at the distant end, when an A key is pressed, leads to the "B" kick experienced in quadruplex.

(13) **Instrument Inductance**

The inductance of telegraph receiving instruments is quite large. While depending to some extent upon the current flowing, average figures for a few instruments are :—

Sounder, 20 ohms . . .	Armature up,	0·18 henry.
	Armature down,	0·24 ,,
Sounder, 900 ohms . . .	Armature up,	18 henries.
	Armature down,	22 ,,

Polarised relay, 200 + 200 ohms or 100 + 100 ohms.	Coils in parallel, 0·85 henries. Coils in series, 3·4 ,,
Non-polarised relay, 200 + 200 ohms.	Coils in parallel, 0·8 ,, Coils in series, 3·2 ,,

In Fig. 37 let L be the inductance of the instrument and R the resistance of the instrument plus any other resistance in series. If the circuit is closed by the key at time $t = 0$, and if i is the instantaneous current flowing at time t, while $I = E/R$ is the final current flowing, then at any instant the voltage across the inductance plus that across the resistance must equal the battery e.m.f., or

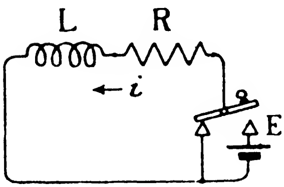


FIG. 37.—Rise of current in Inductive Circuit.

$$L\frac{di}{dt} + Ri = E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

One solution is evidently $i = E/R$; another is to be found by writing

$$L\frac{di}{dt} + Ri = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

If in this $i = A\epsilon^{at}$, $LAa\epsilon^{at} + AR\epsilon^{at} = 0$, or $a = -\frac{R}{L}$.

Thus the complete solution of (1) is

$$i = \frac{E}{R} + A\epsilon^{-\frac{R}{L}t}$$

and since when $t = 0$, $i = 0$, $A = -E/R$, and finally

$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

If the key is released at any instant $t = 0$ after the full current I is flowing, and assuming that the back contact is made before the front contact is broken, the differential equation is (2) with solution $i = A\epsilon^{-\frac{R}{L}t}$, and since in this case when $t = 0$, $i = \frac{E}{R}$, the final solution is

$$i = \frac{E}{R}\epsilon^{-\frac{R}{L}t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This last equation can be arrived at in a slightly different manner. If, instead of releasing the key to the backstop, an additional battery of e.m.f. $-$ E is inserted in the circuit at time $t = 0$, the electrical conditions will evidently be identical.

The current flowing at time $t = 0$ is $i_1 = \frac{E}{R}$, and after $t = 0$ the additional current due to the e.m.f. $-$ E is by (3)

$$i_2 = -\frac{E}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right).$$

The total current at any instant is the sum of i_1 and i_2, which is the same as (4).

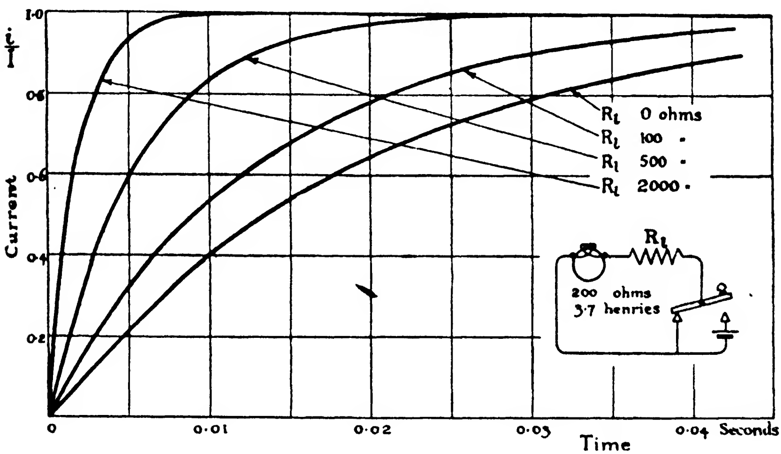


Fig. 38.—Effect of Instrument Inductance on Rise of Current.

This procedure can be carried out at any time, not necessarily after the current has reached its final value, and will be found of value in building up the form of the current curve from Morse signals.

Fig. 38 gives some curves (calculated from formula (3)), showing the rise of current in a Standard B relay with the coils in series. The resistance of the relay is 200 ohms and its inductance 3·7 henries, and the curves are drawn with different values of the line resistance, up to 2,000 ohms. It is seen how much steeper the curves are with the greater line resistance, and it is clear that, provided that sufficient resistance can be inserted in the line, trouble from instrument inductance can be minimised to any desired extent. But the insertion of resistance means the increase of sending end voltage in order to obtain sufficient working current, and the practical limit to increase of voltage is soon reached.

With the 500-ohm line at key speed signalling the current will have reached its final value before the end of a dot signal, which can be taken to last 0·04 second at thirty words a minute. On releasing the key to the backstop, assuming that the circuit is not broken, the current will fall in the same way as it rises, and the shape of the dot signal, as received, will be as shown in Fig. 39. A study of this figure shows how distortion of the signal may be produced by the inductance. For instance, if the bias is set so that the relay marks when the current strength is represented by a_1, and spaces when the current falls to b_1, the relay will be operated

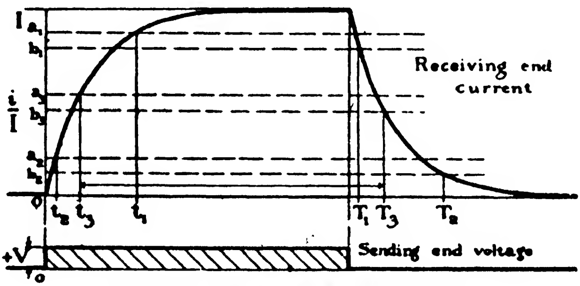


FIG. 39.—Dot signal, single current.

from t_1 to T_1, a shorter time than the time of the dot as sent. If the bias is such that the relay marks at a_2 and spaces at b_2, the time of operation t_2T_2 will be longer than the dot. It is only when a_3 and b_3 represent the currents required for marking and spacing that the time t_3T_3 is equal to the dot time, and this occurs when the mean of oa_3 and ob_3 is about equal to one-half I, the final current. Any alteration of received current will alter the correct setting of the relay.

This only applies to single current working. In double current working the received dot signal will be as shown in Fig. 40. With neutral adjustment, if the relay marks when the current strength is oa, it will space when the current strength is $-oa$, and it is clear from the symmetry of the curves that the time of operation t T is exactly the dot length whatever the actual value of oa, that is, whatever the received current. The relay is self-adjusting.

When the time of the signal is so short (or the value of R/L so small) that the current does not sensibly reach its final value before the signal is closed, the condition of affairs is different, and no setting of the relay can avoid distortion in either single or double current working. Suppose, for example, that in the case of the 500-ohm line the speed of signalling is increased (by machine transmission) to 240 words a minute. The time of a dot is now 0·005 second, and when the key is released at the end of a dot, the current has only reached 0·6 of its final value. The current begins to die away, but before it is zero the next signal starts, and so on. The form of the received current from three dots (letter "s") is found in Fig. 41, and is shown by the firm lines. It is derived graphically from the rise of current curve in the manner indicated above.

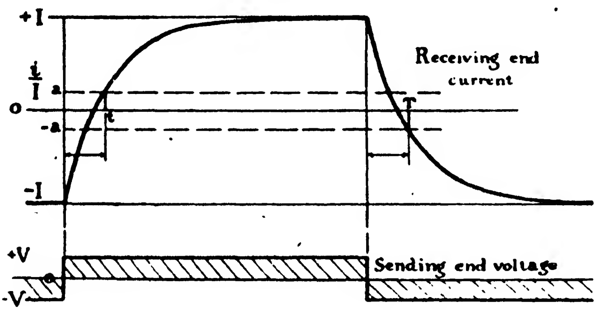


Fig. 40.—Dot signal, double current.

At time $t = 0$ the key is closed, and an e.m.f. E_1 is inserted in the line. The current rises along the curve ABc_1. At $t = T =$ 0·005 second (the dot time), an e.m.f. $E_2 = -E_1$ is inserted in the line. The current curve due to this is $a_2\,b_2\,c_2$. The first dot has now been sent, and the current curve, obtained by adding at every instant the ordinates of the two curves $A\ B\ c_1$ and $a_2\ b_2\ c_2$, is A B C D′ E′.

After a further interval T an e.m.f. $E_3 = E_1$ is inserted, leading to the curve $a_3\,b_3\,c_3$, and so on, $E_4 = -E_1$, $E_5 = E_1$ and $E_6 = -E_1$ leading to curves $a_4\ b_4\ c_4$, $a_5\ b_5\ c_5$ and $a_6\ b_6\ c_6$, all at intervals of T seconds, and the letter "s" is completed. The received current found by adding all the six curves together is A B C D E F G. It will be observed how very different in shape it is from the curve of the voltage applied to the line. Two extreme cases may be noted. If the relay operates when $i/I = 0{\cdot}2$, the three dots will be received as a single very long dash, while if a value of 0·6 is necessary to operate the relay the first dot will be missed altogether. The best setting would be roughly for the relay to mark at $i/\mathrm{I} = 0·5$, and to space at $i/\mathrm{I} = 0·4$. The dots will then occupy times t_1t_2, t_3t_4, and

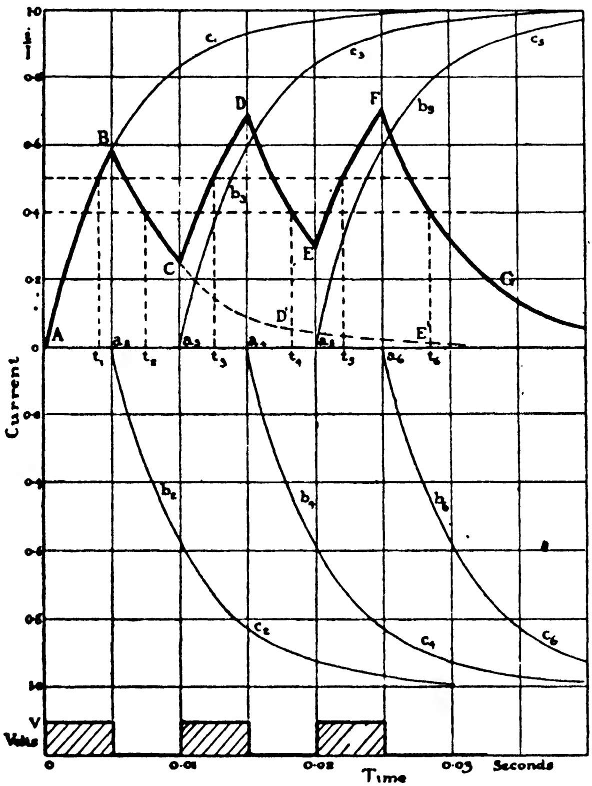


Fig. 41.—Letter S at high speed.

t_5t_6, equal to 0·0035, 0·00575 and 0·0065 second in order, and the spaces are correspondingly distorted.

(14) The Reading Condenser

The deleterious effects of instrument inductance can be reduced to some extent by the use of a "reading" or "shunted condenser," which, as shown in Fig. 42, is a condenser of capacity C shunted by a resistance R put in series with the instrument of inductance L, and total resistance (including line) R_1. It can be seen that, generally speaking, on closing the key the charging of the condenser through the inductance will help to build up the current quickly, while on opening the key the condenser discharge will take place partly through the inductance, and as the discharge current is in the reverse direction, the decay of the instrument current will be hastened.

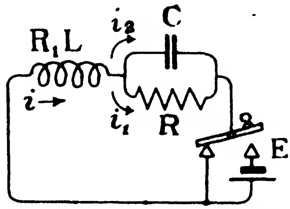


FIG. 42.—Shunted Condenser Circuit.

The effect naturally depends upon the values given to C and R, and it is usually taken that the best values are those which ensure that on raising the key the total quantity of electricity flowing through the inductance is *nil*. If at any time t after the removal of the battery the current in the inductance is i, that in the resistance R i_1 and that in the condenser i_2, then equating the total voltage round the circuit to zero gives

$$R_1 i + L\frac{di}{dt} + R i_1 = 0, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

but

$$i_1 + i_2 = i$$

hence

$$R_1 i + L\frac{di}{dt} + R(i - i_2) = 0 \quad . \quad . \quad . \quad . \quad . \quad (2)$$

or

$$(R_1 + R)\, i\, dt + L\, di - Ri_2\, dt = 0. \quad . \quad . \quad . \quad . \quad (3)$$

Integrating between $t = 0$ and $t = \infty$, which is the same as between $i = I$ and $i = 0$, gives

$$(R_1 + R)\int_0^\infty i\, dt + L\int_I^0 di - R\int_0^\infty i_2\, dt = 0. \quad . \quad . \quad . \quad (4)$$

But $\int_0 i\, dt$ is the total quantity which flows through the instrument after the battery is removed, and in order that this may be zero

$$L\int_I^0 di = R\int_0^\infty i_2\, dt = -RQ, \quad . \quad . \quad . \quad . \quad (5)$$

where Q is the initial charge on the condenser = RIC, where I is the initial steady current (= E/(R + R_1)

Also $L\int_I^0 di = -LI$.

Hence

$$-LI = -R^2CI,$$

or

$$L = R^2C \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

is the desired relationship.

The actual delineation of the transient current i flowing in the circuit of Fig. 42 is a considerably more difficult matter than in the case of Fig. 37. The following equations may be written down for the various currents at time t after closing the key :—

$$i = i_1 + i_2 \qquad (7)$$

$$R_1 i + L\frac{di}{dt} + Ri_1 = E \qquad (8)$$

$$\frac{1}{C}\int i_2\,dt = Ri_1 \qquad (9)$$

From (9)

$$\frac{1}{C}i_2 = R\frac{di_1}{dt} \qquad (10)$$

and using (7)

$$i - i_1 = RC\frac{di_1}{dt} \qquad (11)$$

(8) gives

$$Ri_1 = E - L\frac{di}{dt} - R_1 i \qquad (12)$$

and substitution of (12) in (11) and rearrangement yields

$$RLC\frac{d^2i}{dt^2} + (R_1RC + L)\frac{di}{dt} + (R_1 + R)\,i = E. \qquad (13)$$

The solution of this equation (see Appendix 2) is

$$i = \frac{E}{R + R_1}\left[1 - \epsilon^{-at}\left\{\cosh \beta t - \frac{R_1 + R - aL}{\beta L}\sinh \beta t\right\}\right] \qquad (14)$$

where
$$a = \left(\frac{R_1}{2L} + \frac{1}{2RC}\right)$$

and
$$\beta = \sqrt{\left(\frac{R_1}{2L} - \frac{1}{2RC}\right)^2 - \frac{1}{LC}}$$

If β is imaginary, so that $\omega = j\beta = \sqrt{\frac{1}{LC} - \left(\frac{R_1}{2L} - \frac{1}{2RC}\right)^2}$

the solution is more conveniently written

$$i = \frac{E}{R + R_1}\left[1 - \epsilon^{-at}\left\{\cos \omega t - \frac{R_1 + R - aL}{\omega L}\sin \omega t\right\}\right] \qquad (15)$$

The result of some calculations from this formula are given in Fig. 43.*

* A. B. Morice, P.O.E.E.J., vol. xviii., pp. 1 and 304.

Curve O A B C shows the rise of current in the inductance with $R_1 = 300$ ohms, $R = 700$ ohms, $L = 3{\cdot}5$ henries, and $C = L/R^2 = 7{\cdot}14\mu F$, while the curve C D E F shows the fall when the key is released after 0·04 second. The dotted curve O *a* *b* shows for comparison the rise of current (calculated from formula (13·03)), in the circuit when there is no condenser. The shapes of the dot signals in the two cases have also been drawn with a dot duration of

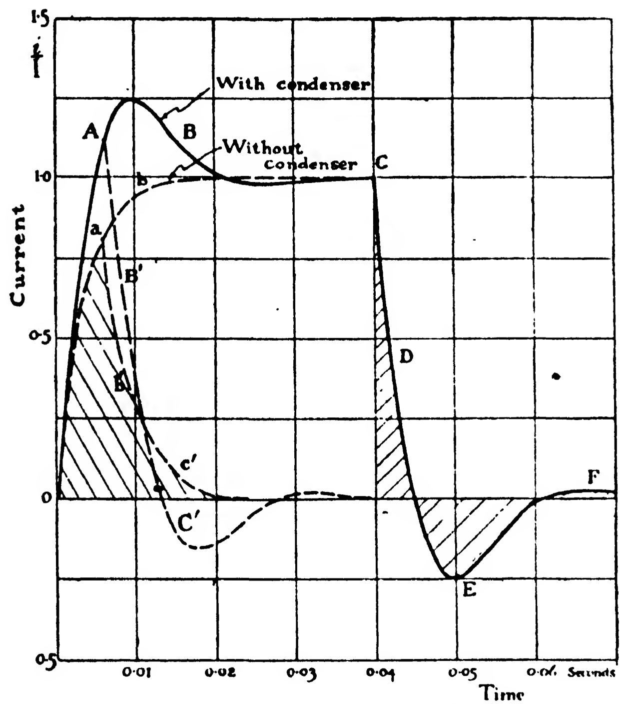


FIG. 43.—Effect of Shunted Condenser on rise and fall of Current.

0·006 second (200 words a minute); the curves are O A B′ C′ with the condenser and O *a* *b′* *c′* without it. It will be observed how the condenser has increased the steepness of the rise of current and increased the height as well as improved the shape of the dot.

In the discharge curve the shaded area is zero. This is the condition noted above that the total quantity flowing through the inductance should be zero.

All the above calculations and curves of the effects of instrument inductance in delaying the rise of current must be looked upon as illustrative rather than as exact. The inductance of any instrument containing iron in its magnetic circuit is not a constant quantity, but owing to the non-linear form of the magnetisation curve, it varies somewhat with the current flowing. There will, moreover, be effects due to eddy currents and hysteresis in the iron delaying the penetration of the flux to the centre of the iron cores.

(15) Capacity and Resistance

The capacity to earth of a telegraph line is of great importance in its transient effects. The subject may be approached by a considera-

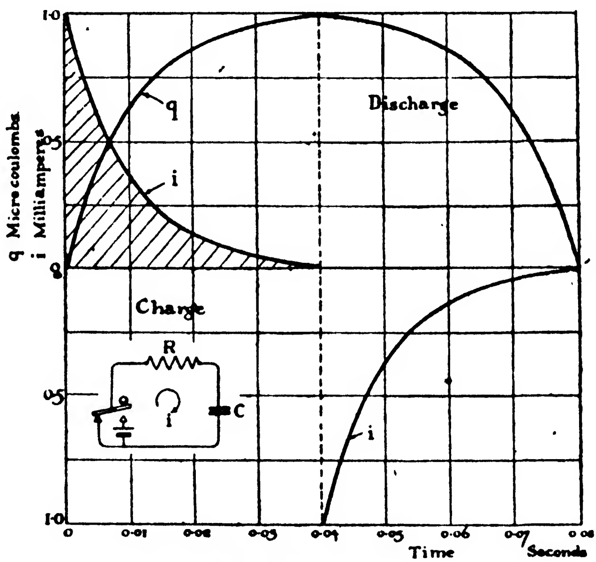


FIG. 44.—Charge and Discharge of Condenser.

tion of the simple circuit of Fig. 44, in which a condenser of capacity C farads is charged and discharged through a resistance of R ohms. If the battery e.m.f. is E and the current at any time t after pressing the key is i, and the quantity of electricity on the condenser plates at time t is q, then $q = \int_0^t i dt$, and the voltage across the condenser is q/C, so that at any instant

$$E = Ri + \frac{1}{C}\int i dt,$$

or

$$R\frac{dq}{dt} + \frac{q}{C} = E \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This equation is of exactly the same form as (13·01) and its solution can be written down by analogy with (13·03) as

$$q = EC\left(1 - \epsilon^{-\frac{1}{RC}t}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Following the analogy to (13·04), it is clear also that on discharge

$$q = EC\,\epsilon^{-\frac{1}{RC}t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

The curves of charge are exactly similar to those of the rise of current in an inductive circuit drawn in Fig. 38 ; the greater the value of the product CR the longer will the condenser take to charge.

The current flowing during charge and discharge is obtained by differentiating (2) and (3).

On charge

$$i = \frac{E}{R}\epsilon^{-\frac{1}{RC}t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

On discharge

$$i = -\frac{E}{R}\epsilon^{-\frac{1}{RC}t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The curves are logarithmic, the initial current being E/R. Fig. 44 shows the curves of q and i for charge and discharge (after 0·04 second) of a 10 μF condenser through a 1,000-ohms resistance with E = 1 volt.

Consider now the slightly more complicated circuit of Fig. 45. If the circuital currents are i_1 and i_2 as shown, then on closing the key

$$\left.\begin{aligned} E &= Ri_1 + \frac{1}{C}\int i_1\,dt - \frac{1}{C}\int i_2\,dt \\ 0 &= Ri_2 + \frac{1}{C}\int i_2\,dt - \frac{1}{C}\int i_1\,dt \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

which on differentiating become

$$0 = RC\frac{di_1}{dt} + i_1 - i_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and

$$0 = \mathrm{RC}\frac{di_2}{dt} + i_2 - i_1 \quad . \quad . \quad . \quad (8)$$

Substituting i_1 from (8) in (7) and integrating, gives

$$\mathrm{RC}\frac{di_2}{dt} + 2i_2 = 0.$$

One solution for i_2 must be E/2R (the final value of the current). Another is $\mathrm{A}\epsilon^{-\frac{2}{\mathrm{RC}}t}$, as may be confirmed by substitution. The whole solution is, therefore,

$$i_2 = \frac{\mathrm{E}}{2\mathrm{R}} + \mathrm{A}\epsilon^{-\frac{2}{\mathrm{RC}}t},$$

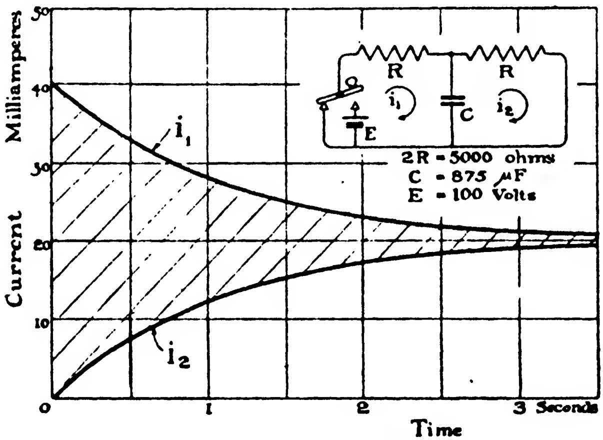


FIG. 45.—Currents in nominal T of line.

and since when $t = 0$, $i_2 = 0$ (the second resistance is shorted by the initial infinite conductance of the condenser), $\mathrm{A} = -\mathrm{E}/2\mathrm{R}$ and

$$i_2 = \frac{\mathrm{E}}{2\mathrm{R}}\left\{1 - \epsilon^{-\frac{2}{\mathrm{RC}}t}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

which on substitution in (8) leads at once to

$$i_1 = \frac{\mathrm{E}}{2\mathrm{R}}\left\{1 + \epsilon^{-\frac{2}{\mathrm{RC}}t}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Currents calculated from (9) and (10) putting 2R = 5,000 ohms and C = 875 μF are drawn in Fig. 45.

The difference of the ordinates of i_1 and i_2 is, at any instant, the current flowing into the condenser, and this integrated up to the instant is the charge in the condenser, and on the diagram is the shaded area to the left of the ordinate. The whole shaded area is the final charge on the condenser.

This from (9) and (10) is

$$\int_o^\infty (i_1 - i_2)\, dt = \frac{E}{2R}\int_o^\infty 2\epsilon^{-\frac{2}{RC}t}\, dt,$$
$$= \frac{EC}{2},$$

as would be expected, as the final voltage across the condenser is E/2.

In Fig. 46 another link is added and the circuital equations are

$$\left.\begin{aligned} E &= Ri_1 + \frac{1}{C}\int i_1\, dt - \frac{1}{C}\int i_2\, dt \\ 0 &= 2Ri_2 + \frac{2}{C}\int i_2\, dt - \frac{1}{C}\int i_1\, dt - \frac{1}{C}\int i_3\, dt \\ 0 &= Ri_3 + \frac{1}{C}\int i_3\, dt - \frac{1}{C}\int i_2\, dt \end{aligned}\right\} \quad . \quad . \quad . \; (11)$$

These are solved in Appendix 3 with the following results :—

$$\left.\begin{aligned} i_1 &= \frac{E}{4R}\left\{2\epsilon^{-\frac{1}{RC}t} + \epsilon^{-\frac{2}{RC}t} + 1\right\} \\ i_2 &= \frac{E}{4R}\left\{-\epsilon^{-\frac{2}{RC}t} + 1\right\} \\ i_3 &= \frac{E}{4R}\left\{-2\epsilon^{-\frac{1}{RC}t} + \epsilon^{-\frac{2}{RC}t} + 1\right\} \end{aligned}\right\} \quad . \quad . \; (12)$$

The currents i_1 i_2 and i_3 have been calculated from these equations with $4R = 5{,}000$ ohms and $2C = 875\ \mu F$, that is, with the same total resistance and capacity as before, and are plotted in Fig. 46. The shaded area between i_1 and i_2 is the charge on the first condenser, and that between i_2 and i_3 the charge on the second condenser.

The circuit of Fig. 45 may be looked upon as the nominal T of a line having only resistance and capacity. In Fig. 46 the representation of the line is taken a step further by dividing the line into two halves, and representing each by its nominal T. A closer representation would be by three T's, and so on. The actual line is an

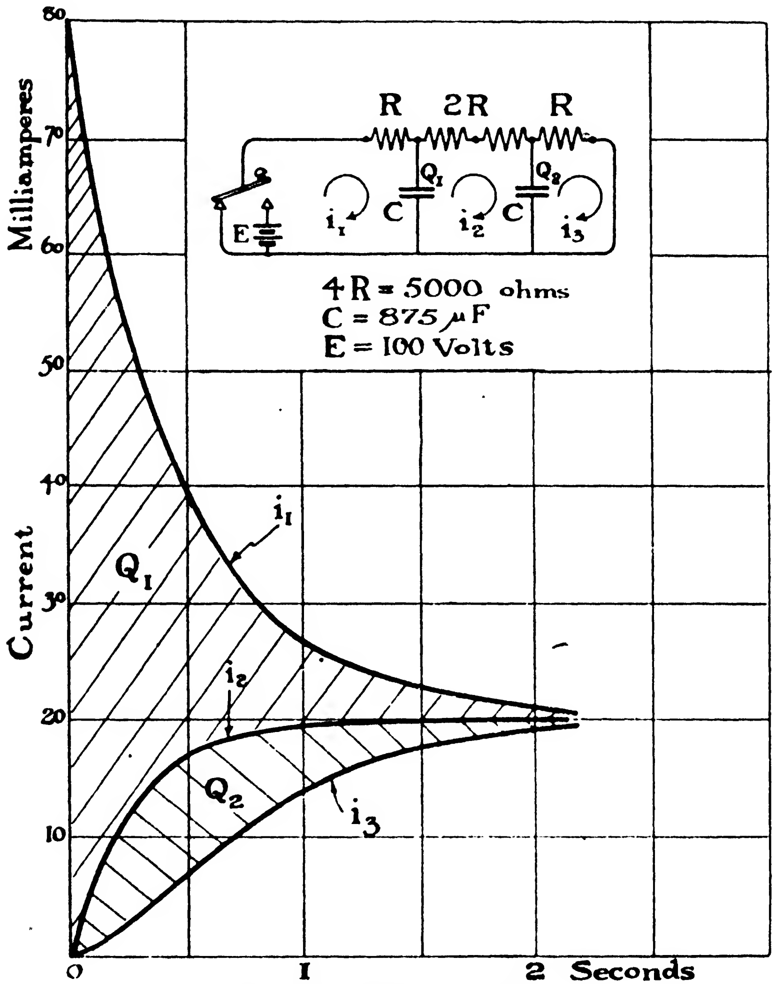


FIG. 46.—Artificial line of 2 T's.

infinitely large number of T's, with the total resistance equal to the total line resistance and the total capacity equal to the total capacity of the line to earth.

The initial rush of current at the sending end, far in excess of the final steady current, and the slow rise to the final value at the receiving end, are to be noted. In shorting the receiving end the resistance of the apparatus has been neglected.

(16) Line Capacity

The effects of the resistance of the line and its capacity to earth are of the same nature as those considered in Figs. 45 and 46, but the whole phenomenon is more complicated because the resistance and the capacity are uniformly distributed along the line. It is clear that the rise of current through the terminal apparatus is likely to be considerably delayed. On the other hand, there will be a rush of current into the line on pressing the key, and the initial current will be considerably greater than the final current. The greater the resistances and the greater the capacities, the greater these transient effects will be.

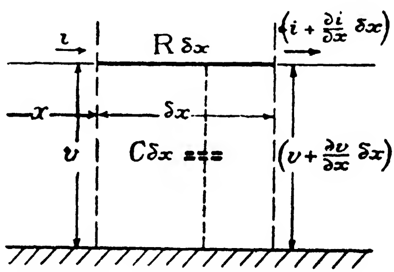


FIG 47.—Line with resistance and capacity.

An equation will now be obtained for the actual case of the line. Inductance and leakance of the line are neglected. This involves no serious error in the cases of underground cables and submarine cables, in which capacity effects are most pronounced. Let C be the capacity to earth of the cable in farads per mile, and R its resistance in ohms per mile, and consider (Fig. 47) the current and voltage changes over a length of the cable δx miles of a current being sent from left to right. The resistance of the length δx is $R\delta x$ and its capacity to earth is $C\delta x$. At the beginning of the length the current and potential are i and v, at the end at the same instant they are $i + \frac{\partial i}{\partial x}\delta x$ and $v + \frac{\partial v}{\partial x}\delta x$.

When δx is very small the current may be taken as sensibly constant in calculating the voltage drop along the length, which is accordingly given by

$$iR\delta x = v - \left(v + \frac{\partial v}{\partial x}\delta x\right) = -\frac{\partial v}{\partial x}\delta x$$

or
$$\frac{\partial v}{\partial x} = -\mathrm{R}i \quad \ldots \ldots \ldots (1)$$

The current flowing to earth through the capacity of the length is $i = \partial q/\partial t = \partial \mathrm{C}\delta x v/\partial t = \mathrm{C}\delta x(\partial v/\partial t)$, since the voltage may be taken as constant in estimating the loss of current. Accordingly

$$\mathrm{C}\delta x \frac{\partial v}{\partial t} = i - \left(i + \frac{\partial i}{\partial x}\delta x\right)$$

or
$$\frac{\partial i}{\partial x} = -\mathrm{C}\frac{\partial v}{\partial t} \quad \ldots \ldots \ldots (2)$$

Differentiating (1) with regard to x and substituting from (2) gives

$$\frac{\partial^2 v}{\partial x^2} = \mathrm{CR}\frac{\partial v}{\partial t} \quad \ldots \ldots \ldots (3)$$

Differentiation of (1) with regard to t and (2) with regard to x and elimination of $\frac{\partial^2 v}{\partial x \partial t}$ gives

$$\frac{\partial^2 i}{\partial x^2} = \mathrm{CR}\frac{\partial i}{\partial t} \quad \ldots \ldots \ldots (4)$$

(3) and (4) are the telegraph equations in their simplest form as first given by Lord Kelvin in 1855. The solution of these equations is given in Appendices (4) and (5). Interest in telegraphy lies chiefly in the rise of current at the receiving end of a cable of length l miles when a voltage E is applied at time $t = 0$ at the sending end. The resistance of the apparatus is neglected, and the required solution is

$$i_r = \frac{\mathrm{E}}{\mathrm{R}l}\left[1 + 2\sum_{m=1}^{m=\infty} \epsilon^{-\frac{m^2\pi^2 t}{\mathrm{CR}l^2}} . \cos m\pi\right] \quad \ldots (5)$$

where m is any integer.

The current at the sending end is also of interest in connection with the duplex balance. Again neglecting apparatus resistance, it is given by

$$i_s = \frac{\mathrm{E}}{\mathrm{R}l}\left[1 + 2\sum_{m=1}^{m=\infty} \epsilon^{-\frac{m^2\pi^2 t}{\mathrm{CR}l^2}}\right] \quad \ldots (6)$$

The form that these currents take is shown in Fig. 48, which is drawn for a long submarine cable having a total resistance (Rl) of 5,000 ohms and a total capacity (Cl) of 875 μF. The difference of the ordinates of the current flowing into the line and current flowing out of the line represents the rate of accumulation of charge on the line. The actual charge on the line is the shaded area multiplied by the final current I, since i/I is plotted against t in each case.

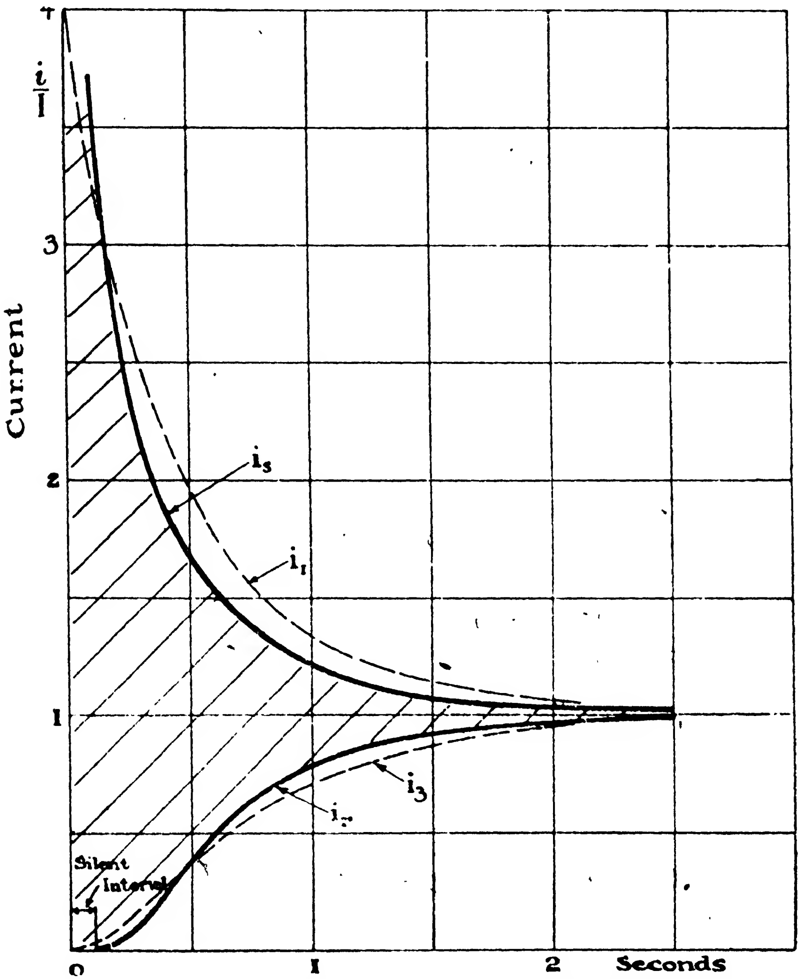


FIG. 48.—Sending End and Receiving End Currents in Submarine Cable.

The curves for i_1 and i_3 (from Fig. 46) are also plotted (dotted) on Fig. 48 for comparison. They were drawn with the same total resistance and capacity as the cable, and show that an artificial line of even two T sections only gives a rough approximation to the actual cable as far as the sending end and receiving end transient currents are concerned.

The curve of the current at the receiving end plotted against the time is known as the arrival curve. Its equation (5) can be put into a somewhat different form by writing

$$u = \frac{\pi^2}{CRl^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Equation (5) then becomes

$$i_r = \frac{E}{Rl}\left[1 + 2\sum_{m=1}^{m=\infty} \epsilon^{-m^2ut} \cos m\pi\right]$$

$$= \frac{2E}{Rl}\left[\frac{1}{2} - \epsilon^{-ut} + \epsilon^{-4ut} - \epsilon^{-9ut} + . \quad . \quad . \quad .\right]$$

or writing
$$f(ut) = \frac{1}{2} - \epsilon^{-ut} + \epsilon^{-4ut} - \epsilon^{-9ut} + . \quad . \quad (8)$$

$$i_r = \frac{2E}{Rl} f(ut) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The value of $f(ut)$ varies from zero when ut is zero to 1/2 when ut is infinity. For values of ut up to 0·23, $f(ut)$ is very nearly zero; no measureable current is received. The time corresponding to this value is known as the silent interval. In Fig. 48, $u = \pi^2/5000 \times 875 \times 10^{-6} = 2{\cdot}26$, and the silent interval is 0·23/2·26 = 0·102 second. A table of the values of $f(ut)$ is given in Appendix (5), and from this the arrival curve for any cable in which the inductance and leakance may be neglected can be drawn.

The current has reached within 1½ per cent. of its maximum value when ut = 5·0. If it is desired that the current should reach this value within the dot, that is, at key speeds when t = 0·04 second, the smallest permissible value of u is 5·0/·04 = 125. From this may be found the longest length of line. With 150 lb. copper aerial wire, for which R = 5·86 ohms and C = 0·0147 μF per mile, $CRl^2 = 0{\cdot}0147 \times 10^{-6} \times 5{\cdot}86\, l^2 = \pi^2/125$, and l = 960 miles.

With 40 lb. underground screened paper core cable, R = 21·956 ohms and $C = 0{\cdot}125 \times 10^{-6}$ μF per mile, $CRl^2 = 0{\cdot}125 \times 10^{-6} \times 21{\cdot}956\, l^2 = \pi^2/125$, and l = 170 miles.

For four times this speed of signalling the lengths must be halved, and so on. The speed of signalling is inversely proportional to CRl^2, or to the product of the total capacity and the total resistance of the line. This was known as the "KR" law—K being the symbol for capacity in the early days.

The effect of line capacity on the distortion of signals is quite similar to that of instrument inductance. The arrival curve is somewhat similar in shape to the inductive rise curves of Fig. 38, at any rate so far as its effect on the relative merits of single current and double current working from the point of view of distortion of signal is concerned. Curves similar to those of Figs. 39 and 40, but drawn from the values of $f(ut)$ given in Appendix (5), bring this out quite clearly.

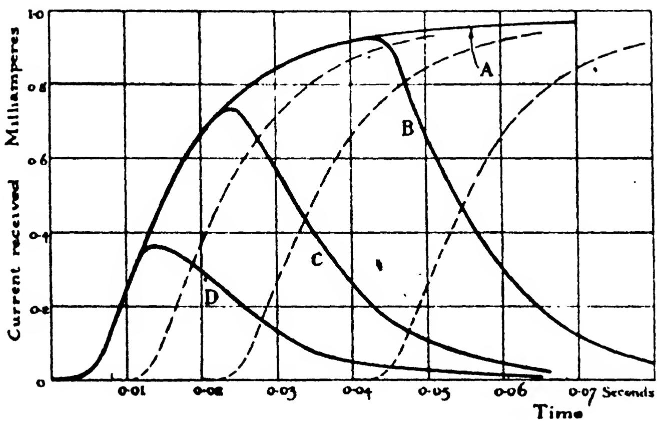


FIG. 49.—Dots of various lengths.

The arrival curve can be used to build up the form of the received current for any transmitted signal by the same method as that employed in Fig. 41, that is, by imagining that the cessation of a mark is brought about by the insertion of a counter e.m.f. In Fig. 49, A is the arrival curve for a cable 346 miles long, having a capacity of 0·3 μF per mile and a resistance of 3 ohms per mile. Thus $\mu = 92$ and the silent interval is 0·0025 second. In 0·06 of a second the current has practically reached its final value. The curves B, C and D show the dot signal as it arrives: B for a dot length of 0·04 second, C 0·02 second, and D 0·008 second; curves drawn by adding an arrival curve from a negative voltage (shown dotted in each case, but above the axis to save space) starting at the end of the dot period. It will be observed how the amplitude of the signal is reduced as its duration is reduced, and how it departs more and more in shape from the square-topped wave of the sending end volts. In Fig. 50 the single dot of 0·008 seconds is used to build up the shape of the received signal for the letter S three dots. The individual dots are shown by dotted curves, and the firm line curve is the sum of these and is the actual current received. Reception of the signal by the methods described above is impossible with this form of signal. If the first dot is received at all, the remaining two dots run into each other and form a long dash. If the second two dots are to be separated, the first one must be missed altogether. The speed of signalling in this example is

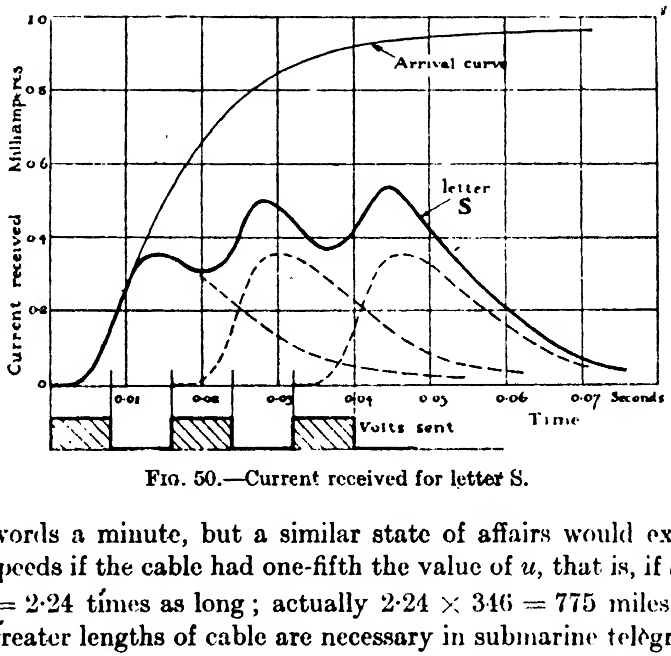


FIG. 50.—Current received for letter S.

150 words a minute, but a similar state of affairs would exist at key speeds if the cable had one-fifth the value of u, that is, if l were $\sqrt{5} = 2{\cdot}24$ times as long; actually $2{\cdot}24 \times 346 = 775$ miles long. Far greater lengths of cable are necessary in submarine telegraphy, and methods different from those hitherto described have to be adopted. These are described in the next chapter.

(17) The Duplex Balance

The wave form of the current (i_s) flowing into the line after the key is depressed is shown in Fig. 48. Consider this curve in conjunction with the duplex diagram of Figs. 14 or 19. While the i_s curve of Fig. 48 has been obtained on the assumption of no terminal apparatus, the influence of the latter will not alter the general shape. Evidently then, immediately after depressing the key, the current flowing into a long line will be greatly in excess of the current flowing in the compensating circuit through the resistance R, and it will continue to be in a gradually lessening excess for an appreciable time. During this time the home station relay may be operated by the out-of-balance current through its two coils, and the duplex arrangement fails. It is necessary to replace R by some circuit or network having as nearly as possible the same characteristic as the line with regard to the transient current flowing into it, so that the current balance in the two coils of the relay is preserved at every instant. Two such networks have been examined in Figs. 45 and 46. The greater the transient effect, that is the longer the line, the more the network must be subdivided into meshes in order sufficiently to simulate the line. The mesh of Fig. 45 is sufficient for long aerial lines and shorter underground lines, while that of Fig. 46 is sufficient for the longer underground lines. For very long underground lines and short submarine lines the network is divided into four meshes.

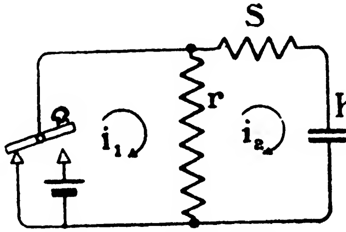


FIG. 51.—Duplex Balance.

These particular networks are not, however, used in practice. Instead, those shown in Figs. 51 and 52 are invariably used instead of those of Figs. 45 and 46 respectively, and they involve identical transient currents from the battery provided the correct values of the resistances and condensers are chosen.

Comparing Figs. 45 and 51, for instance, it is clear that in order that the final currents may be identical, $r = 2\mathrm{R} =$ total line resistance. Also from inspection and equation (15·04) the current i_1 in Fig. 51 must be given by

$$i_1 = \frac{\mathrm{E}}{r} + \frac{\mathrm{E}}{s}\epsilon^{-\frac{1}{s\mathrm{K}}t}$$

For this equation and equation (15·10) to be identical it is necessary that

$$r = s = 2\mathrm{R}$$

and $$\frac{\mathrm{RC}}{2} = s\mathrm{K} = 2\mathrm{RK}, \text{ or } \mathrm{K} = \frac{\mathrm{C}}{4}$$

The networks of Figs. 46 and 52 will have identical i_1/t curves if the following relations hold :—

$$r = 4\,R$$

$$s_1 = \frac{4}{3}R \qquad K_1 = \frac{9}{16}C$$

$$s_2 = \frac{32}{3}R \qquad K_2 = \frac{1}{16}C$$

as is proved in Appendix (6).

It may further be shown that for every additional condenser in the T line an additional mesh must be added to the network of Fig. 52 for equivalence. There is no apparent reason why the balance should not take the form of the artificial line, but the standard method is as given in Figs. 51 and 52. The resistance r is known as the rheostat, S_1 as the retardation coil, and S_2 as the condenser coil. The resistances and condensers are adjustable, as the balance on long aerial lines varies from day to day owing to changing insulation resistance. The differential galvanometer in the duplex circuit is used in adjusting the balance.

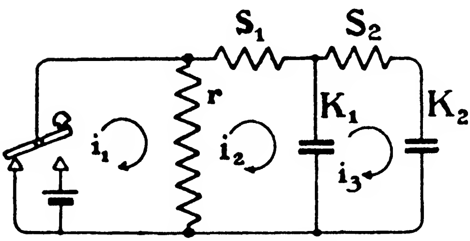


FIG. 52.—Duplex Balance.

For use on very long lines, such as submarine cables, where a very high degree of balance is required, another form of compensation circuit known as the grid has been devised. Each unit of this is made up (see Fig. 53) of successive layers of a sheet of paraffined paper, a sheet of tin-foil, a sheet of paraffined paper, a grid of tin-foil, paraffined paper, tin-foil, and so on. All the grids are joined in series and all the sheets of tin-foil are connected to earth. In this way a perfect representation of a cable is made as far as resistance and capacity are concerned.

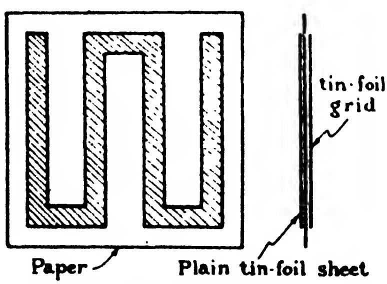


FIG. 53.—Grid type Duplex Balance.

(18) **The Quadruplex B-kick**

The B-kick noticed on page 25 in describing quadruplex working can now be explained more fully. When an A side key is depressed, the battery applied to the line at that end is reversed. At the other end the reversal of the current takes place in accordance with the

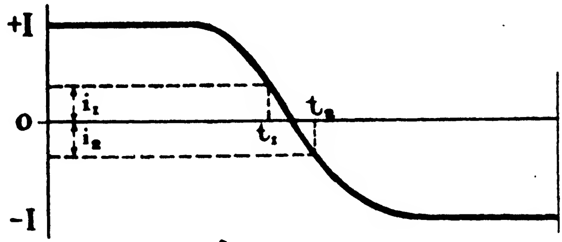


FIG. 54.—Quadruplex B-kick.

theory given above along an arrival curve as shown in Fig. 54. When at time t_1 the current has fallen to the value i_1, against which the non-polarised relay is biased, the spring pulls the tongue off the stop, and it is not restored until time t_2, when the current reaches the value i_2 (a little greater than $-i_1$) in the opposite direction. During the time from t_1 to t_2, therefore, the B-side relay is not marking.

REFERENCES FOR FURTHER READING

F. BREISIG.—"Theoretische Telegraphie."

A. B. MORICE.—"The Shunted Condenser." *Post Office Electrical Engineers Journal*, Vol. XVIII., pp. 1 and 304. Also Correspondence by J. G. Hill, pp. 177 and 387.

H. W. MALCOLM.—"The Theory of the Submarine Telegraph and Telephone Cable."

A. B. MORICE.—"The Duplex Balance." *Post Office Electrical Engineers Journal*, Vol. IX., p. 253.

CHAPTER IV

LONG LINES

(19) Repeaters

In the last chapter it was shown how very seriously the capacity effects on long lines limited the speed of signalling, owing to the distortion of the signal. The speed was shown to be inversely proportional to the square of the length of a cable of given constants. If therefore any line is divided into two, and at the end of the first section the distorted signal is received and re-transmitted

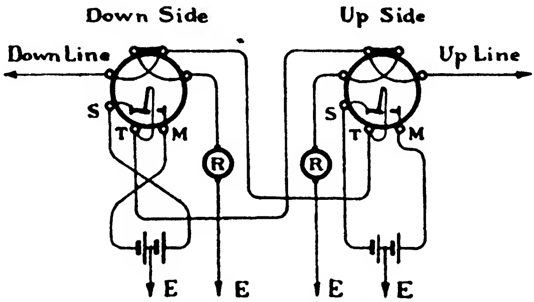


Fig. 55.—Double Current Duplex Repeater.

along the second section of the line in its original form, the possible speed of the line will have been increased four times, provided, of course, that the limit of instrument speed has not been reached. The assembly of apparatus necessary for this purpose is known as a repeater. Naturally there may be several repeaters in a line, and the speed of the whole line will be the speed of the longest section.

Fig. 55 shows the principle of a double-current duplex repeater. The incoming signals are received on polarised relays, and sent out from batteries connected to the "local" contacts. The outgoing currents divide through the relays, the circuits R being compensating circuits as in ordinary duplex. The up side relay acts as the key working to the down line (and *vice versâ*), and since the outgoing currents through the down side relay are equal and in opposite directions through the two coils, the relay is not actuated by them.

The actual circuit is more complicated than that shown, owing to the necessity of being able to call the operator at the repeater station from either end, and of the operator being able to " listen in " to make any necessary adjustments. This involves incidental apparatus, such as automatic switches, and sounder silencers to reduce the noise.

(20) **The Gulstad Relay**

It has been noted in Fig. 50 that when the speed of signalling is sufficiently high, or the cable sufficiently long, the dots as received

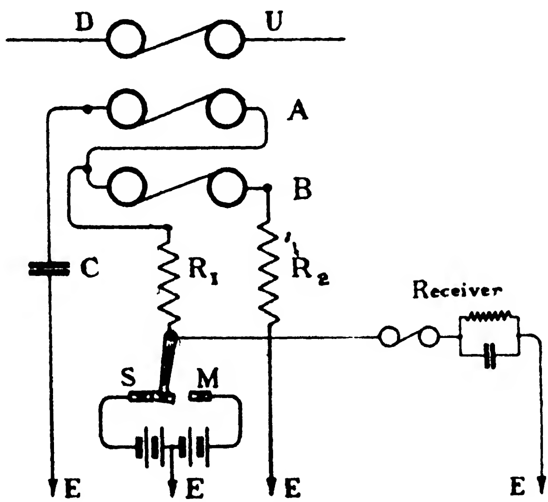


FIG. 56.—Gulstal Relay.

by an ordinary relay will necessarily run into each other. The dashes, however, being three times as long as the dots, will not be affected in this way until the speed is increased to a much higher value. The same is true of the longer spaces. The vibrating relay invented by Gulstad takes advantage of this by recording dots all the time unless a dash or long space is received ; the line provides the dashes, the relay the dots.

In general construction the vibrating relay is similar to a polarised relay, but there are two additional windings, A and B, on the cores as shown diagrammatically in Fig. 56. A battery earthed in the middle is connected to the marking and spacing stops, while resistances R_1 and R_2 and a condenser C are connected as shown. With these connections and with suitable values of the resistances, the condenser and the battery voltage, the tongue will vibrate continuously between the marking and the spacing stops in the absence of any current through the line coils D U. For suppose the tongue is on the spacing stop when the battery is switched on. The condenser charging current flows through the resistance R_1 and the A coils in the spacing direction, holding the tongue firmly on the stop. As this current dies away, a second current through R_2 the coils B and R_1 is rising in value. This second current is in the marking direction through the coils B, and eventually the tongue is pulled off the spacing stop. Directly contact is broken, the condenser commences to discharge through R_2 and the coils B and A, and the direction being marking through each, the movement of the tongue is further accelerated. On reaching the marking stop the condenser charges from the other half of the battery in the reverse direction, *i.e.*, marking through A, and thus holds the tongue firmly on the stop and prevents chattering. But an opposing current through the coil B in the spacing direction eventually preponderates, and pulls the tongue off the stop, and so on; the tongue vibrates continuously. The coils A are known as the accelerating coils and the coils B as the opposition coils. The rate of vibration depends chiefly on the values of C, R_1 and E. It is increased by reducing C and R_1 or by raising the voltage, and is adjusted to synchronise with the speed of transmission of the dot.

The resistance R_2 determines the current through the opposition coils, and is adjusted so that the line current takes control when dashes and long spaces are being received. The actual reception is on any suitable form of recording instrument connected, as shown. through the usual shunted condenser.

(21) **Submarine Cable**

In submarine telegraphy the lengths between repeater stations are usually much greater than in land lines, so that the total resistance and total capacity are much greater, and the sending end voltage is usually limited to 50 volts in order to safeguard fully against any danger of damaging the insulation of the cable. Thus the currents received are small and the distortion is great, and on each account the ordinary telegraph relays are quite unsuitable.

Signals are received by actually tracing the received current-time curve on a slip of paper by means of an instrument known as the siphon recorder. The dots and dashes are distinguished by oppositely directed currents as in single-needle working. This results in faster sending and in better defined signals, and is known as cable code. If the dot signal lasts 0·04 second, the rate of signalling is about forty words a minute. The hand signalling key is similar to the commutator of Fig. 25.

The siphon recorder is essentially a moving-coil galvanometer of the d'Arsonval type. The coil C, Fig. 57, is suspended with its vertical sides in a strong magnetic field maintained by a powerful permanent magnet, the reluctance of the flux path being reduced

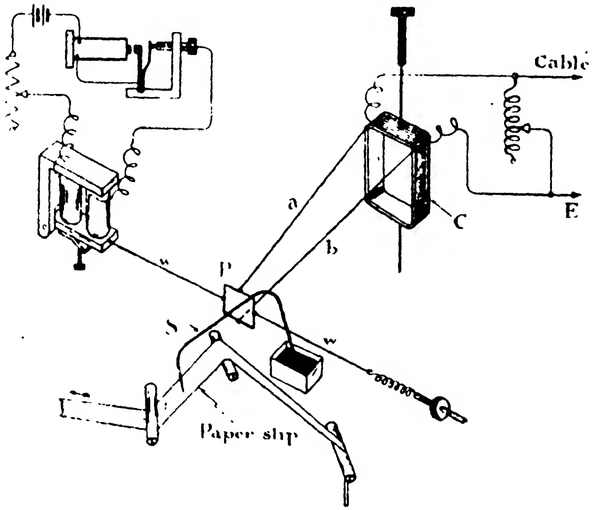


FIG. 57. Siphon Recorder.

by a soft iron block within the coil. The fine glass tube siphon S is fixed by wax to a light aluminium plate, P, carried on a stretched wire. *w*. The top and bottom of the plate are joined by silk fibres, *a* and *b*, to the left- and right-hand corners of the coil. One end of the siphon dips into a small inkwell. the other hardly touches a paper slip which is drawn forward by a small electric motor. When the coil is deflected by the incoming current. the plate P is tilted, and the marking end of the siphon is carried across the paper. To reduce friction the wire *w* is vibrated electrically at one end, so that the siphon touches the paper only intermittently, and the current curve is traced out by a series of small dots as in the figure. The telegraphist deciphers the message from the current curve received.

In the case of Fig. 50 this would be a simple matter. But this was for a short cable of only 346 miles, with a value of $u = 92$, although with a dot signal of 0·008 second corresponding in cable code to a speed of $40 \times 0{\cdot}04/0{\cdot}008 = 200$ words a minute. To obtain the same shape of signal with the cable for which the arrival curve of Fig. 48 was drawn would necessitate a far lower speed of signalling. For this cable $u = 2{\cdot}26$, and for the same shape the

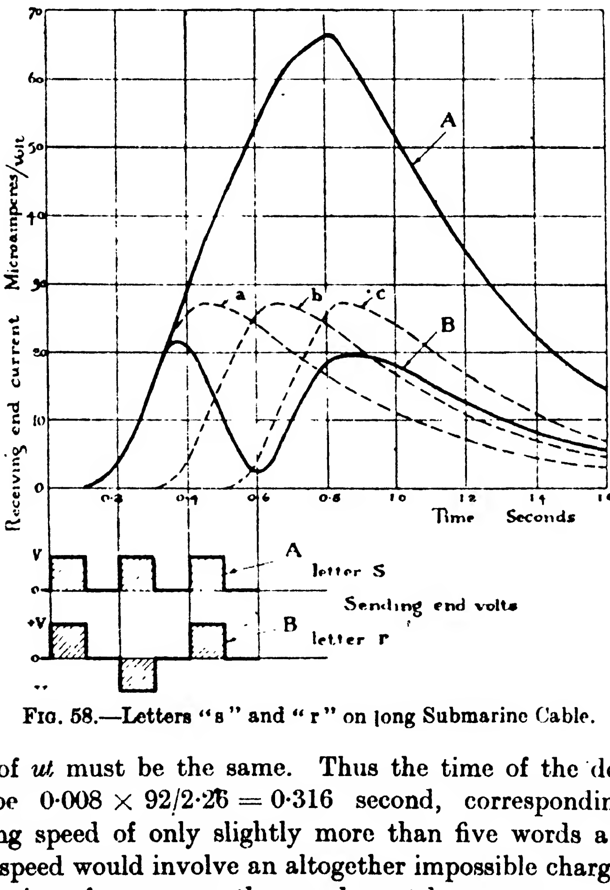


FIG. 58.—Letters "s" and "r" on long Submarine Cable.

values of ut must be the same. Thus the time of the dot signal must be $0{\cdot}008 \times 92/2{\cdot}26 = 0{\cdot}316$ second, corresponding to a signalling speed of only slightly more than five words a minute. Such a speed would involve an altogether impossible charge for the transmission of messages: the speed must be greater.

In Fig. 58 the speed for this cable has been increased to sixteen words per minute by making the dot length 0·1 second. The single-dot signal a is drawn as before from the arrival curve of Fig. 48. Although the final value of the received current would be 200 microamperes per volt at the sending end, the current only rises to a value of 27 microamperes per volt. Two other dots, *b* and *c*, are then added at intervals of 0·2 second to give the curve A for the received current, or siphon recorder record, for the letter "s." The shape now is very nearly, but not quite, hopelessly distorted. A skilled telegraphist would read the signal without difficulty, but the speed

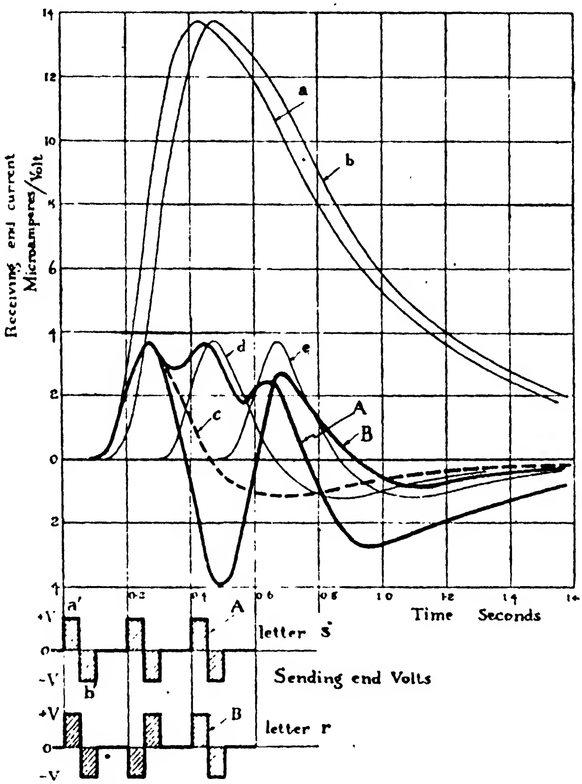


FIG. 59.—Improvement of Shape by Curbing.

of signalling is somewhat near the limit for intelligibility. On the other hand, the letter "r"—dot, dash, dot—is still quite clearly defined. It is drawn in the figure by giving the curve *b* a negative sign and adding as before. Curve B results. Letters in which dots and dashes alternate are known as cross-letters, and are far more easily read than the non-cross ones. The application of the negative voltage quickens the discharge of the line, or diminishes in amplitude the long tail of the elementary dot signal, thus making the line electrically more neutral and ready for the next signal.

Recognition of this difference led to the introduction of "curbed" signalling: each signal is followed immediately by a reversal of voltage for a longer or shorter time, the reversed voltage acting as a "curb" to help to clear the line of the charge occasioned by the signal. The effect of a 50 per cent. curb on the signals of Fig. 58 is worked out in Fig. 59. Curve a is a dot of 0·05 second duration obtained from the arrival curve of Fig. 48 in the usual way. A similar curve b drawn after an interval of 0·05 second, and reckoned negative, is the current curve of the curbing signal b'. Adding a and b algebraically (actually subtracting the ordinates of b, as drawn, from those of a) gives the curve c, which is the elementary dot signal as received from the transmitted curbed dot, a' b'. Two other curves, d and e, the same as c, are drawn at 0·2 second intervals. The letter "s" signal, curve A, is obtained by adding the curves c, d and e, and the letter "r" signal, curve B, by adding c and e and subtracting d.

Comparison with the corresponding curves of Fig. 58 shows how very considerably curbing has improved the shape of the signals; "s" is now perfectly intelligible, while "r" is distributed about the zero line in much the same manner as a dot, dash. dot should be. But amplitude has been lost. The current scale in Fig. 59 is five times that of Fig. 58. The uncurbed letter "s" rises to an amplitude of 66μA per volt, the curbed letter "s" to an amplitude of only 3·6μA per volt.

A condenser was first placed in series with the line at the receiving end of the cable to prevent parasitic earth currents flowing through the receiving apparatus, but it was soon discovered that the presence of the condenser led to an increase in the possible working speed of the cable, and that the insertion of another series condenser at the sending end led to a still further increase. The use of a receiving condenser is known as single-block working, and of a condenser at each end as double block. The double-block is always used.

The arrival curves with single- and double-block on the cable considered previously are drawn in Fig. 60 as A and B respectively. The calculation of these curves is due to Malcolm, is not easy, and a consideration of the method (see Appendix 5) should be deferred until a study is made (in Part II.) of alternating currents in cables.

With single-block the current maximum is only 30·5μA per volt, instead of 200μA per volt without the condenser, and a long tail to the curve remains. With double block the current maximum has fallen to 7μA, but the tail has to a large extent disappeared. These

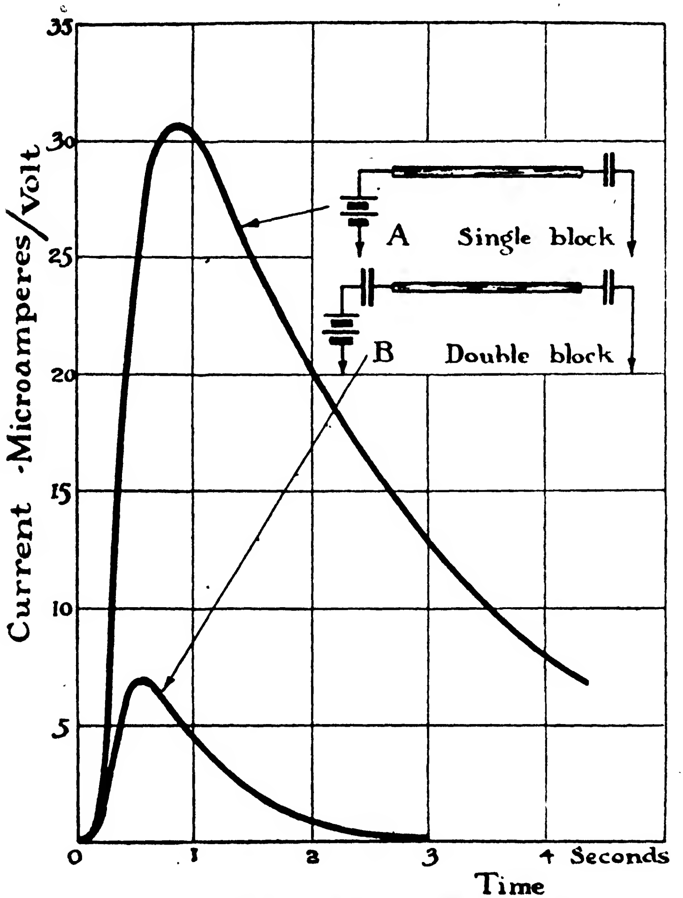


FIG. 60.—Arrival Curves with Series Condensers.

curves are drawn for condenser values of one-tenth of the total capacity of the line, which is about the usual working value. They may be looked upon as a combination of the usual arrival curve of Fig. 48 and the condenser charging curve of Fig. 44.

The action of the condensers in increasing the working speed is quite similar to that of curbing. The similarity of shape between the double-block arrival curve B, Fig. 60, and the 0·05 elementary dot curve (*a*), Fig. 59, will be noticed. The double-block curve is for a single sustained depression of the key. Releasing the key after, say, 0·1 second produces a similar curve, but drawn negatively and spaced 0·1 second to the right. The dot signal is the sum of these two, and will evidently be of similar shape to the elementary

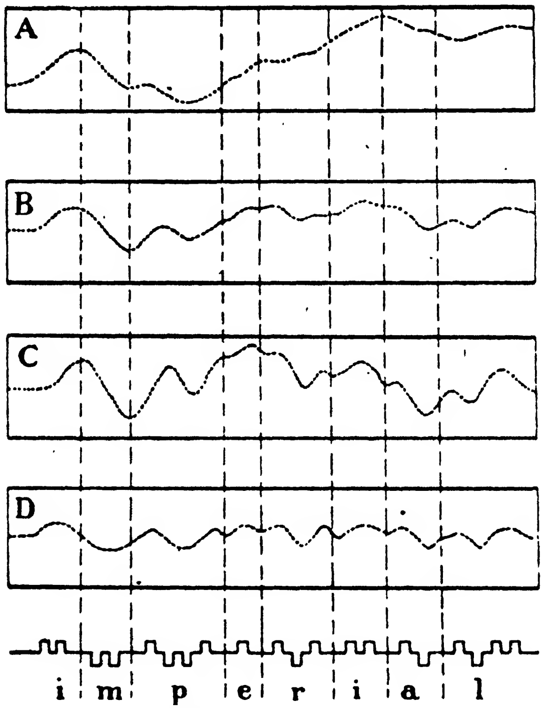


FIG. 61.—A—No Condensers, no Curb. B—Curb, no Condensers. C—Double Block, no Curb. D—Double Block and Curb.

curbed dot signal shown as curve *c*, Fig. 59. Hence also the complete signals will be similar in shape to the signals curbed by battery.

These two methods of reducing distortion are illustrated in Fig. 61, in which are shown reproductions of recorder slip of the word "Imperial" taken on an artificial line representing a Pacific cable. The sending end voltages were adjusted so as to make the amplitude fit the slip. Slip A was taken with direct sending without either condensers or curb, slip B with curb but no condensers, slip C with double block but no curb, and slip D with double block and curb. In the last curve the adjustment of the curb is approximately correct—the signal lies more or less uniformly about the zero line.

No account has been taken in the above curves of the receiving instrument resistance. Its general effect is to flatten the curves somewhat, but not to alter the general shape. A further improvement in the definition of the signals is effected by shunting the receiving apparatus by a large inductance, which causes the arrival voltage to rise far more steeply than would otherwise be the case. Malcolm shows that the best value of the inductance of the shunt is CRl^2R_r/π^2, where R_r is the resistance of the receiver. In the case considered, with a siphon recorder resistance of 500 ohms, this would be an inductance of 220 henries.

(22) **Apparatus**

The adoption of curbed signalling obviously necessitates a special transmitting device. Each dot consists of the application to the cable of first a battery in one direction and then a battery in the reverse direction, and the relative duration of the two applications must be adjustable while keeping the total time of the dot the same. Similarly with the dash, signalled by reversing each battery.

This is effected by an automatic clockwork or electric motor-driven machine known as the curb transmitter. The message is punched out on a slip of strong paper by means of a perforator. The perforator has three keys; the middle (space) key punches a small centre hole, the left-hand (dot) key punches a centre hole and a larger one above it, while the right-hand (dash) key punches a centre hole and one below it. After pressing any key the slip is moved forward automatically. The appearance of the slip punched for the letters "s" and "r" is shown in Fig. 62 (*a*).

The centre holes are used to feed the slip through the transmitter, the other holes allow upward movement to the rods R and R′ which are pivoted at the ends of levers L_1 and L_1', the other ends of which are bent over to touch levers L_2 and L_2'. Only one set, R L_1 and L_2, of rods and levers is shown in Fig. 62 (*b*); the other set is immediately behind. Pins P_1 and P_2 working on the levers oscillate vertically in slots; while P_2 is moving up, P_1 moves down. A spring S constrains the movement of L_1, while a jockey wheel J on a spring renders impossible any intermediate rest position of L_2; the contact is either on x or y. There is a third contact lever L_3

working between stops K_1 and K_2 to give the curb. The pins P_1 and P_2 and the lever L_3 are driven from cams on the main shaft. The cam for L_3 is long and varied in cross section, so that the time of the movement of L_3 can be altered by moving a carriage holding L_3 and its stops along the cam by means of a milled-headed screw.

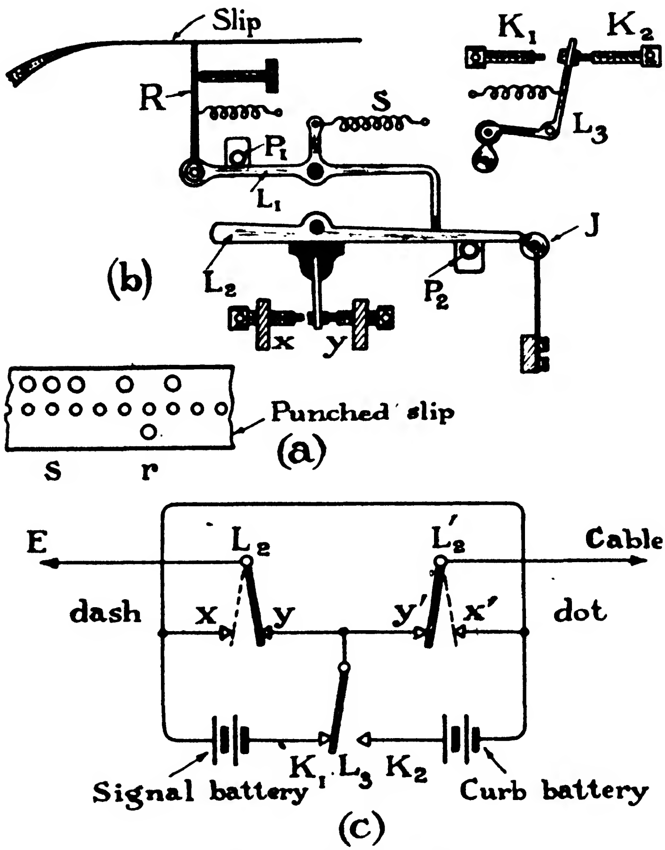


Fig. 62.—Curb Transmitter.

The connections are as shown in Fig. 62 (c). With no slip running through the transmitter, the levers L_1 L_1' and L_2 L_2' follow the movements of the pins P_1 and P_2, contact is made alternately on x and x', and on y and y', and in each case the cable is connected to earth. When slip is passed through, in the absence of any dot or dash perforation the slip prevents the upward movement of the rods R and R′ and the levers L_2, L_2' are not pushed down; contact remains on y and y' and the cable is earthed. When a dot perforation appears opposite R′ this rod is allowed to move upward; L_1' follows by the action of the spring S′ and L_2' is pushed down, changing the contact from y' to x'. The positive terminal of the signalling battery is now connected to cable and the negative terminal to earth, until the oscillation of the pins restores the contact from x' to y' and so again earths the cable. A dot has been sent. At some time, determined by the position of the carriage along the cam, during the time the dot is being sent, the lever L_3 is thrown over from contact K_1 to K_2. This brings the curb battery with reversed

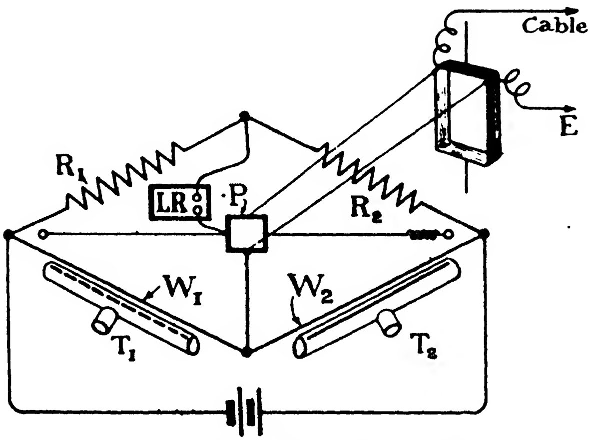


FIG. 63.—Heurtley Hot Wire Amplifier.

polarity on to the line instead of the signalling battery. Similarly with a dash perforation R moves up followed by L_1, and L_2 is moved to change the contact from y to x, L_2' remaining on y', since rods R and R′ cannot both move up together when slip is running through. The negative terminal of the signal battery is connected to cable and the positive terminal to earth, until the lever L_3 substitutes the curb battery with reversed polarity.

The means for obtaining greater definition of signals have all resulted in signals of considerably reduced amplitude. The siphon recorder needs at least 50 microamperes for its operation, and the sending end voltage is limited to 50 volts by considerations of safety. Some amplifying device becomes, therefore, a necessity.

Many have been proposed and used to some extent, but the Heurtley hot-wire amplifier and the Orling jet relay have had more success than most.

The Heurtley arrangement is illustrated in Fig. 63. The small aluminium plate P which is rocked by the moving coil of the receiving galvanometer carries two fine platinum wires W_1 and W_2, which with resistances R_1 and R_2 form the four arms of a Wheatstone bridge. The local recorder LR takes the place of the galvanometer of the bridge, while the battery heats the fine wires. Air blown into the tubes T_1 and T_2 emerges from long narrow slits and just misses the two hot wires. But a clockwise movement of

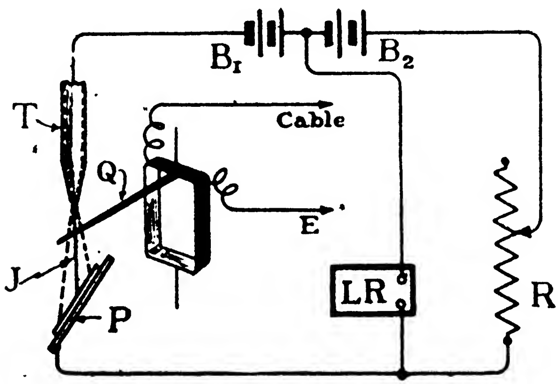


FIG. 64.—Orling Jet Relay.

the coil and consequent upward movement of the wires brings W_2 opposite the slit in T_2 and hence into the cool air stream, while a counter-clockwise movement brings W_1 into a cool air stream. When either wire is cooled its resistance is reduced and the balance of the bridge disturbed. Current flows through the local recorder in a direction determined by which of the two wires is cooled. This amplifier is capable of receiving at seventy-five words a minute with a current of 0·7 microampere.

In the Orling jet relay (Fig. 64) the moving coil carries a fine quartz rod Q, which moves in a jet of salt water J, which falls from a glass tube T drawn out to a nozzle and fed from a small tank. The jet finally falls on to an inclined plane P carrying a number of glass siphon recorder tubes. The battery B_1 sends current through the jet and a local recorder LR; the battery B_2 sends current through the adjustable resistance R in the opposite direction through the local recorder, so that when the quartz is not deflected no current flows through LR. The deflection of the quartz, however, deflects the jet so that it strikes the inclined plane at a higher or a lower point according to the direction of the deflection, and so reduces or increases the jet resistance. This causes a comparatively large current to flow in one direction or the other through LR. It is stated that good signals can be obtained with a current of 0·01 microampere.

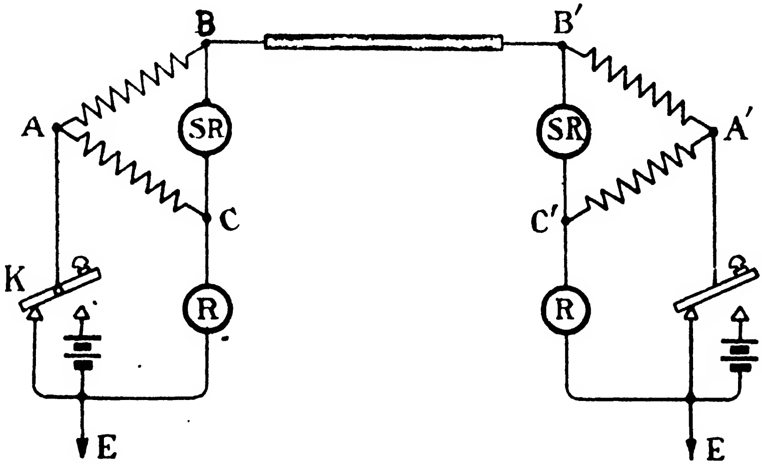


Fig. 65.—Principle of Bridge Duplex.

Duplex working on long submarine cables is carried out by means of the bridge duplex system, so called because it is practically a double Wheatstone bridge. This will be seen from Fig. 65, in which the principle is illustrated. On pressing the key K current flows to line through the resistance AB and to earth through the resistances AC and R. The balancing resistance R is so adjusted that the currents to line and to earth are equal, and then with equal resistances in AB and AC there is no potential difference between B and C and no current through the siphon recorder SR at the transmitting end. At the receiving end only part of the received current passes through the siphon recorder and the resistance C′A′; more than half is shunted by the resistance B′A′.

In practice with long submarine cables signalling condensers replace the resistance arms of the bridge, but a small resistance is left at the apex to assist in reducing and balancing the large initial current surge. Fig. 66 is a complete diagram of one end. The balancing resistance is replaced by a duplex balance which may be of the grid type (Fig. 53); the usual shunted condenser is shown in series with the siphon recorder, while a large inductance shunts recorder and shunted condenser.

Out of balance of the duplex results in currents through the sensitive siphon recorder or magnifier which may be very serious. The greater the sensitivity of the receiving instrument the greater the necessity for an exact representation of the line in the duplex balance. Distortion of signal is hardly the speed limiting factor with

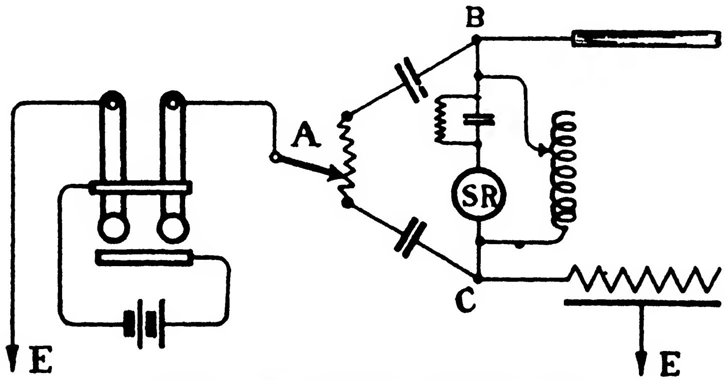


FIG. 66.—Bridge Duplex on Submarine Cable.

present-day methods of correction and amplification; its place has been taken by the limit to the accuracy of the duplex balance. On account of this difficulty, the duplex speed of handling messages is considerably less than twice the simplex speed.

(23) **Loading**

It has already been noted that the transient effects due to inductance may be reduced by introducing capacity into the circuit, and conversely that the transient effects due to cable capacity may be reduced by the introduction of terminal inductance. But the introduction of inductance into the cable itself offers a means of improvement of signal speed, by improving definition without such a great loss in amplitude, so valuable as far to outweigh any other means, and to render possible the use of relays similar in principle to the Gulstad on even the longest submarine cables, provided that sufficient inductance can be added.

The inductance of any wire can be increased by covering it with a close helix of iron tape or thin wire, a process known as continuous loading. Or the inductance of a line may be increased by the insertion of inductive coils at intervals along the line. This is known as coil loading. The practical difficulties in the way of coil loading a submarine cable are extensive, and with ordinary iron sufficient inductance to obtain any marked benefit cannot be added by continuous loading. So that although the great advantages in submarine telegraphy to be obtained by sufficient loading were fully understood and explained by Malcolm, it was not until the invention and manufacture on a commercial scale of an alloy of nickel and iron (in the proportion of about 8 to 2) having remarkably large permeability at low flux densities that any practical steps were taken. But now long cables loaded with this alloy are working simplex at five times the speed of similar unloaded cables working duplex, and using instruments as robust as those in use on land lines. There are two such alloys in actual use, known by the trade names of permalloy and mumetal respectively.

An outline explanation of these results may be attempted. Taking into account an inductance of L henries per mile, equations (16·01) and (16·02) become

$$\left.\begin{aligned} -\frac{\partial v}{\partial x} &= \mathrm{L}\frac{\partial i}{\partial t} + \mathrm{R}i \\ \text{and}\qquad -\frac{\partial i}{\partial t} &= \mathrm{C}\frac{\partial v}{\partial t} \end{aligned}\right\}$$

leading to

$$\frac{\partial^2 i}{\partial x^2} = \mathrm{LC}\frac{\partial^2 i}{\partial t^2} + \mathrm{RC}\frac{\partial i}{\partial t},$$

and a similar equation in v.

These equations are those of a wave motion, or propagation of an electrical disturbance with finite velocity, instead of the diffusion of electricity in the electrostatic propagation of the Kelvin arrival curve, which involves an infinitely fast arrival of an infinitely small current which steadily increases. The exact solution of the equation is a matter of considerable difficulty, and the following approximate ideas must suffice.

The velocity of propagation is $1/\sqrt{LC}$ miles per second, and the height of the current wave front is $E\sqrt{\frac{C}{L}}$ at the sending end. The current wave decreases logarithmically along the line according to the equation $I = E\sqrt{\frac{C}{L}}\,\epsilon^{-\frac{Rx}{2}\sqrt{\frac{C}{L}}}$, and at the receiving end (earthed) is doubled by the reflection. The initial received current is therefore $i_0 = 2E\sqrt{\frac{C}{L}}\,\epsilon^{-\frac{Rl}{2}\sqrt{\frac{C}{L}}}$, the decay of the wave front being so pro-

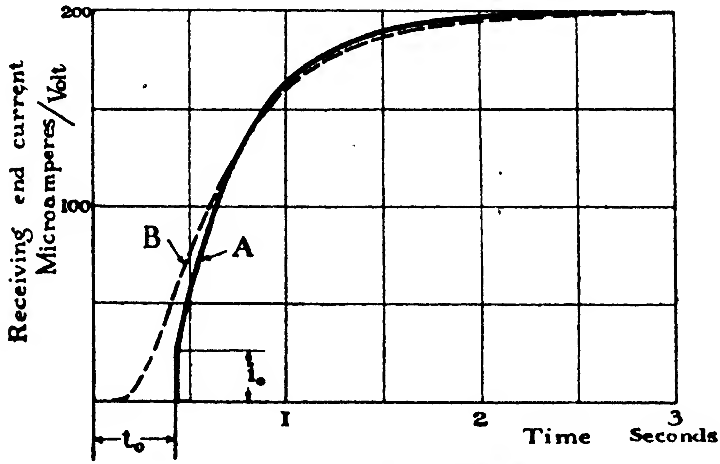


FIG. 67.—Arrival Curve of Loaded Cable.

nounced that no further reflections need be considered. This current i_0 is received at time $t = l\sqrt{LC}$ seconds after the sending of the impulse. After the initial wave front has been received, the current rises gradually to its final value in much the same way as in the unloaded cable.

Actually the wrapping of alloy round the copper conductor increases the effective resistance of the latter to transient currents, and the inductance of the wire depends upon the current strength, so that the whole phenomenon is very complicated. There is, moreover, an " absorption " of electricity phenomenon in the dielectric. But making the simplifying assumptions of unaltered resistance, constant inductance and no leakance, Malcolm has calculated the arrival curve for the cable of Fig. 48 when the inductance added is 0·095 henries per nautical mile, or a total of 216 henries. This is shown in Fig. 67, as curve A, while the arrival curve of the unloaded cable is drawn for comparison as the dotted curve B. If the added inductance is much smaller, or the cable length much greater, the initial steep rise of current becomes very small, and the two curves very nearly coincide. This explains the lack of success of loading

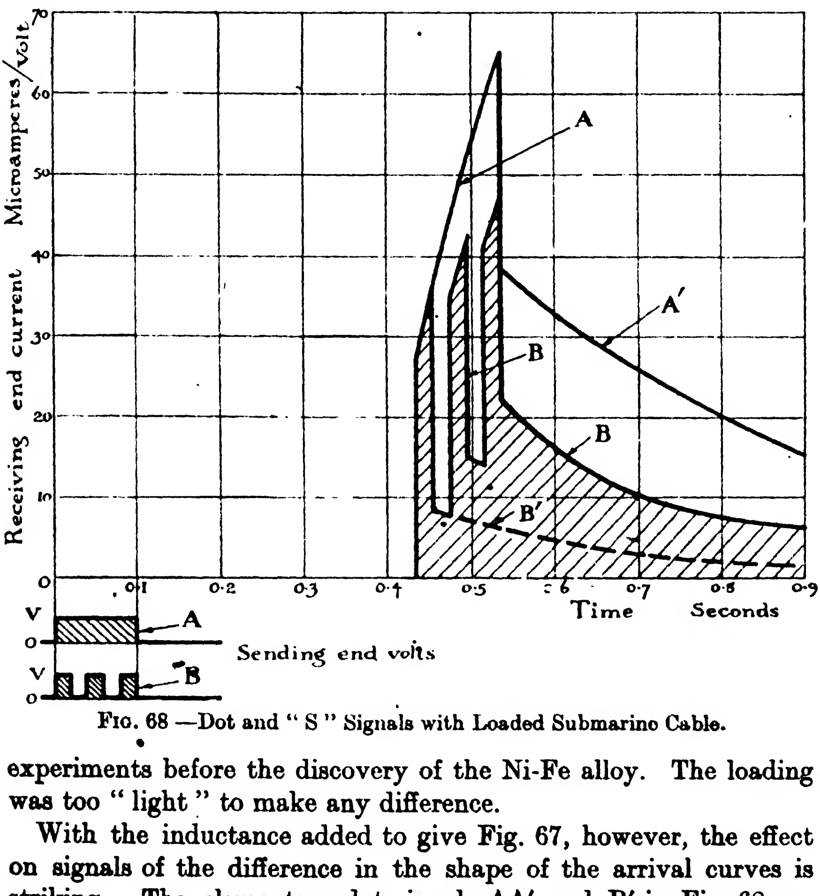


FIG. 68 —Dot and " S " Signals with Loaded Submarine Cable.

experiments before the discovery of the Ni-Fe alloy. The loading was too " light " to make any difference.

With the inductance added to give Fig. 67, however, the effect on signals of the difference in the shape of the arrival curves is striking. The elementary dot signals AA′ and B′ in Fig. 68 are drawn in the usual way from the arrival curve A, Fig. 67, for dots of 0·10 and 0·02 second duration respectively. There is a sharp, very well defined peak in each case, but the tail with the shorter dot is much smaller than the tail with the longer. Since it is the "tail" that leads to distortion, it is seen that in this case the greater the speed of signalling the less is the distortion. The curve BB is the letter "s" with dot units of 0·02 second. It would be quite practicable to receive this after amplification on an ordinary telegraph relay.

Not only is the dot far better defined, but it has a far greater amplitude than in the case of the curbed dot or dot with blocking condensers on the unloaded cable: 30μA in the loaded cable compared with 4μA in the unloaded cable. The old "KR" law ceases to apply at all. The speed of signalling is limited not by distortion, but by amplitude. This is chiefly determined by the exponential ϵ^{-al} in the expression for the initial current, where $a = \frac{R}{2}\sqrt{\frac{C}{L}}$. a is known as the attenuation constant, and it will be seen later that this a governing the attenuation of the wave front of the current transient is the same with the same assumptions as the attenuation constant of the loaded telephone cable with periodic currents. In this way the problems of the loaded telegraph and the loaded telephone cable merge into one.

REFERENCES FOR FURTHER READING

F. Breisig.—"Theoretische Telegraphie."

H. W. Malcolm.—"The Theory of the Submarine Telegraph and Telephone Cable."

R. L. Hughes.—"Recent Developments in Submarine Cable Design." *J.I.E.E.*, Vol. LXVI., p. 140.

W. T. Palmer and H. J. Josephs.—"Arrival Curves and Theoretical Telegraph Speeds." *Post Office Electrical Engineers' Journal*, Vol. XXI., p. 199.

O. E. Buckley.—"High-speed Ocean Cable Telegraphy." *Bell System T. J.*, Vol. VII., p. 225, or Bell Telephone Laboratories, Reprint B. 310.

A. E. Foster, P. G. Ledger and A. Rosen. "The Continuously Loaded Submarine Telegraph Cable," *J.I.E.E.*, 1928.

CHAPTER V

HIGH-SPEED APPARATUS AND SYSTEMS

(24) Introduction

ECONOMIC considerations demand that expensive long lines shall carry as much traffic as possible, and many inventions have been directed towards increasing the message-carrying capacity of a line. These schemes may be divided into two main classes, those in which a single channel is worked at high speed and those in which multiple channels are used, each working at moderate or even key speeds. Either scheme may be duplexed, and duplexing is hardly counted as a multiple scheme, as it is so much the usual practice on any line. But quadruplex is a multiple system and has already been described. The high-speed system in considerable use in this country is the Wheatstone Automatic, while the Baudot is typical of the multiple systems. The Wheatstone is in general use for newspaper traffic, but is not so suitable for ordinary messages, owing largely to the difficulty and delay arising in answering queries. With a multiple system queries can be dealt with at once.

(25) Wheatstone Automatic

The Wheatstone Automatic is essentially a high-speed double-current system working on the Morse code. The apparatus is capable of working up to a speed of 400 words a minute. The *transmitter* is driven by gravity through a falling weight or by an electric motor. The speed is controlled by an expanding fan device, and the signals sent out are determined by holes punched in a slip of strong paper by a *perforator*. The *receiver* is a combination of a polarised relay with an inking device of type similar to that illustrated in the local inker of Fig. 24. The message is received therefore as dots and dashes on a slip of paper, and this has to be transcribed by an operator and written out or typed.

The transmitter works on a similar principle to the curb transmitter in submarine work, but the arrangement is necessarily different in detail, as the former has to send dots and dashes of long and short duration instead of the oppositely directed currents of the latter, and the curb is not required.

The slip as punched is as shown in Fig. 69 at (*a*); the space key punches a centre hole only, the dot key punches a centre hole and two larger holes, one above and one below the centre, and the dash key punches two centre holes and two larger holes, one above the left-hand centre hole and one below the right-hand centre hole.

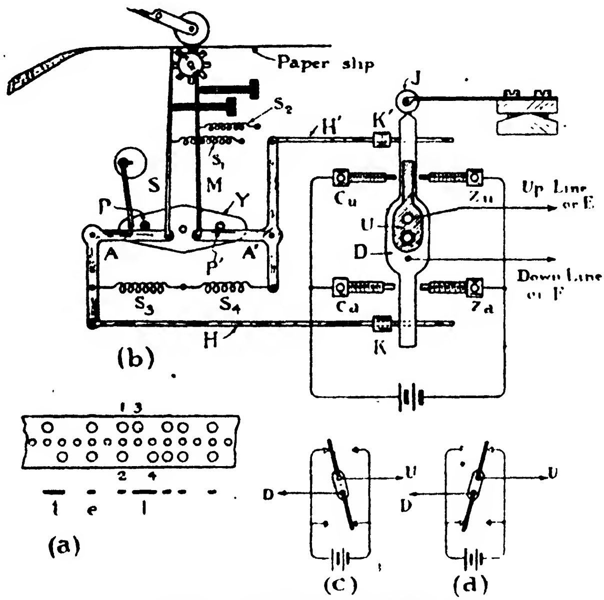


Fig. 69.—Wheatstone Transmitter.

The slip is moved forward the correct distance automatically after each perforation has been made.

The essential part of the transmitter mechanism is shown at (*b*) Fig. 69. There are two levers, A and A′, pivoted as shown, which in the absence of slip are held by the springs S_3 and S_4 against the pins P and P′. These pins are carried on a rocker Y, which is vibrated when the transmitter is run, thus moving the pins up and down alternately. When P rises and P′ falls, the rod H carried by A is moved to the right, and the rod H′ carried by A′ moves to the left. The collet K on H pushes the divided lever DU over to make contact on Zd and Cu; the collet K′ on H′ has moved to the left to allow the lever DU to move. The electrical connections are now as shown at (c); the positive terminal of the battery is connected to U (up line or earth) and the negative terminal to D (earth or down line); spacing current is being sent. When P falls and P′ rises, the collet K′ pushes the lever DU over on to the contacts Zu and Cd, the connections are as shown at (d), and marking current is being sent. The jockey wheel J on a spring ensures that the movement of the lever DU is smart and positive; contact is made definitely on one side or the other.

In the absence of slip, therefore, a train of dots, or "reversals," is sent to line. When the slip is fed forward by the star wheel the rods S attached to A and M attached to A′ come into play to

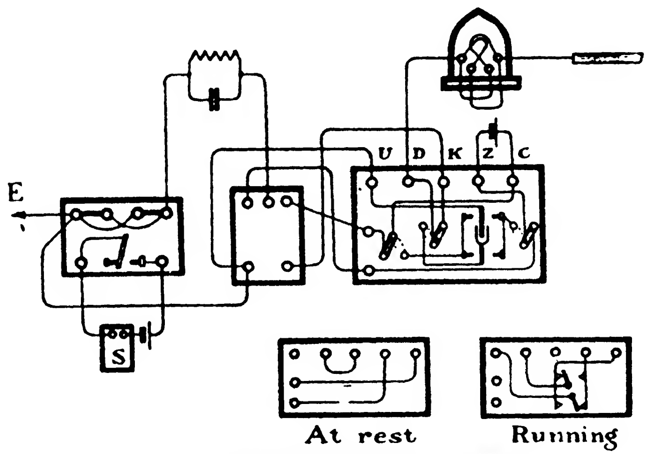


Fig. 70.—Wheatstone Simplex Up Station.

prevent the levers A and A′ following the pins P and P′, unless a perforation in the slip allows them to pass through. When there is no perforation, therefore, the lever DU is not moved. Now suppose a dot is passing over the star wheel, the first dot, say, of the *l* at (a). When M rises it is free to pass through the hole 1, and K′ pushes the lever DU over to the marking side. When in its turn P rises, S is free to pass through the hole 2, and K pushes the lever back to the spacing side. A dot has been sent. When P′ rises again, M passes through the hole 3, and the lever is again pushed over to marking. When P rises, there is no hole for S to enter, and the lever stays at marking until P rises the second time and finds the hole 4, when the lever is restored to spacing. A dash has been sent of duration three times the dot. The rods S and M travel forward a little with the slip when they enter a perforation; the springs S_1 and S_2 restore them to their normal position.

Fig. 70 is a diagram of connections at an up station arranged for simplex working of Wheatstone Automatic in both directions. The Wheatstone receiver has a sounder and battery connected to the local circuit of its polarised relay and a double-current key is provided so that the operators at the two ends may "speak" to each other by hand signalling. The start-stop switch on the transmitter works the three switches shown. In the running position the key and receiver are cut out. In the rest position the key and receiver are brought in, and to receive the key switch must be at the receive position. The shunted condenser acts as described in Section 14

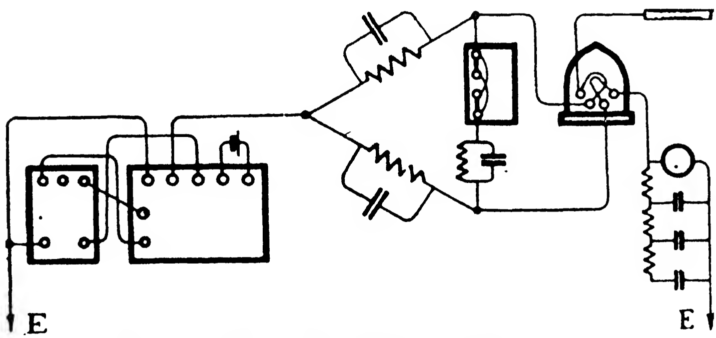


FIG. 71.—Wheatstone Automatic Bridge Duplex.

to minimise the effects of the receiver inductance. The down station is similar, line and earth connections being reversed. Intermediate stations are often inserted in a line with automatic receivers only, but with keys for speaking purposes.

The duplex working of a Wheatstone Automatic circuit can be carried out by the usual double current duplex arrangement; the receiver is differentially wound for this purpose. Special care must be given to the duplex balance owing to the high speed of working. On difficult lines it is found, however, that the bridge method gives higher speeds than the differential. Fig. 71 shows the connections at one end of a bridge duplex arranged for automatic working in each direction. Signalling condensers shunted by large resistances are used in the arms of the bridge, thus combining the advantages of the steep rise of current through the condensers with the large final value through the resistances. The galvanometer is inserted to aid in obtaining the balance.

When the line is sufficiently long or the speed sufficiently high, a Gulstad relay may be inserted in place of the Wheatstone receiver, and the receiver run off its local contacts. Or an undulator, a very robust form of siphon recorder with a neutral tongue polarised relay for the movement, and a silver instead of glass tube for the siphon, may be used instead of the receiver.

Developments of the Wheatstone Automatic system have been towards simplified and less skilled operating on the perforator and receiver; the transmitter remains unaltered. Thus for newspaper messages to be sent to several stations, a perforator using compressed air is in use for punching up to eight slips simultaneously. There are perforators (the Gell and the Kleinschmidt) for punching Morse slip from a standard typewriter keyboard. There are receiving mechanisms (the Creed and the Bille), which produce an exact replica at the receiving end of the perforated slip at the sending end, at speeds up to 200 words a minute. These slips are fed through a "telegraph printer" (Creed), which produces a typed message on paper slip which is cut up and pasted on to telegraph forms. There is also another machine (Creed) known as the translator, of value at the junction of land lines and submarine cables, which produces perforated cable code slip from Wheatstone Morse slip.

(26) Multiple Systems

There are various systems of multiple telegraphy which may be classified as follows:—

(i.) Systems in which the signalling currents vary in strength to actuate one relay and in direction to actuate another. The quadruplex, as described in Chapter I., is the simplest of these, but the quadruplex may also be arranged with two channels terminating at one station and two at another, with four channels to one station and two channels extended as one channel to a number of stations (omnibus circuit), with two channels extended to two others of a second quadruplex, with two channels extended to each of two different stations and with two channels extended to each of two other stations on a wire, by which they are working quadruplex. These circuits can be worked Wheatstone on the polarised side up to 150 words a minute, but on the non-polarised side a speed of twenty-five words a minute is seldom exceeded owing to the B-kick.

(ii.) Systems in which alternating signalling currents of different frequencies actuate special relays each sensitive only to one particular frequency. Direct current signalling provides an additional channel. In the Mercadier the frequencies range from 480 to 900 cycles per second, and good results are obtained. The only objection is the interference to telephone circuits, and on this account it is not used in this country. Another example on a limited scale is the phonopore, in which one extra channel is provided by buzzer signals, and which is used to some extent on the railways.

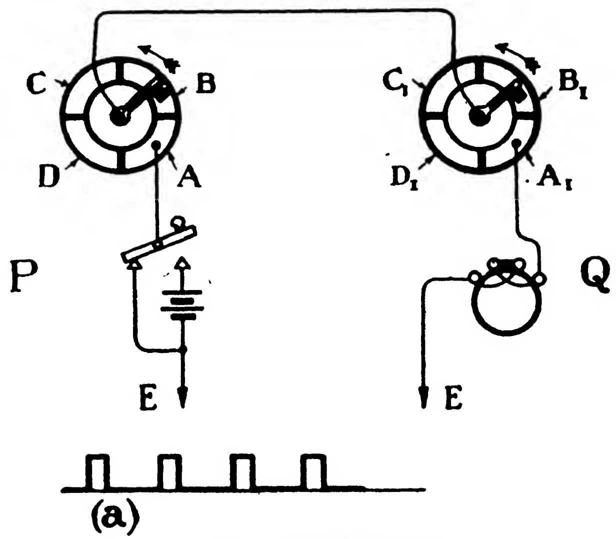


FIG. 72.—Delaney Multiplex.

In this class also fall the systems of multiplex telegraphy by voice frequency and by high-frequency currents, which are outlined in Section 49.

(iii.) Systems in which the use of the line is allotted to each of the channels in turn by means of synchronously running machines at the sending and receiving stations. The scheme is illustrated in Fig. 72 for four channels. The line is connected at each of the two stations P and Q to a rotating arm carrying at its extremity a gauze brush which sweeps over a flat brass ring divided into four insulated sections A, B, C and D. Means are provided to ensure that the arms rotate in synchronism, so that sections AA_1, BB_1, CC_1 and DD_1 are connected in turn, and never AB_1, BC_1, etc. Each of the sections A B C D is connected to a key and battery, and each of A_1 B_1 C_1 D_1

to a relay. When the A key is depressed the voltage applied to the line will be as shown at (*a*), and in the absence of inductance and capacity the current through the relay at A_1 will be of similar form. But by making the inductance of the relay high and shunting it by a condenser in a manner similar to the arrangement of Fig. 23 for avoiding the B-kick in quadruplex, the relay tongue may be kept continuously on the marking stop in spite of the interrupted voltage. It is only necessary that there should be sufficient contacts within the dot time to ensure its transmission. This necessitates a very high speed of rotation of the arms, a difficulty which is overcome by dividing the rings into a far larger number of sections, and connecting each fourth section together. Thus if there were eighty sections, numbers 1—5—9—13 . . ., 2—6—10—14 . . ., 3—7—11—15 . . ., and 4—8—12—16 . . ., would be connected, and each group at one end would be connected to a key and each group at the other end to a relay.

This is the principle of the Delaney system, at one time considerably used, especially on the shorter lines. Four channels have been taken in illustration, but any number up to six has been found practicable. Some difficulties may be noted. The current rise is necessarily delayed by the line, and it is necessary to move the whole ring at Q counter-clockwise (or the relative brush position clockwise) in order that the transmitted currents may arrive on the correct sections. On long lines this renders two-way working difficult if not impossible on the same distributors, as the devices with the rotating brushes are called, but the difficulty can be overcome by having separate sections for working in the reverse direction, mounted, say, behind the first and swept by a second brush rotated from the same axle. The back sections at P would be turned through a small angle to receive from Q. Duplex working is possible on the bridge principle, the transmitting plates replacing the keys and the receiving plates the receivers in Fig. 65 or Fig. 71. The most serious difficulty, however, is in maintaining the synchronous running of the arms at the two stations. The method used is to run the brush arms at each station by phonic motors driven from tuning fork vibrators, and to pass "correcting" currents from special sections of the distributor plate from one station to accelerate or retard the phonic motor at the other station according to whether the correcting current arrives early or late with respect to the brush arm position.

The Delaney has now, however, completely gone out of use, but its action has served as a useful introduction to the Baudot, which acts on the same principle of giving the line for part of the time to each channel, and which has many advantages, one of which is the printing of Roman characters on a slip of paper.

(27) **Baudot**

Baudot recognised the fact that the Morse code, with its letters of very unequal time duration, was unsuitable for systems of

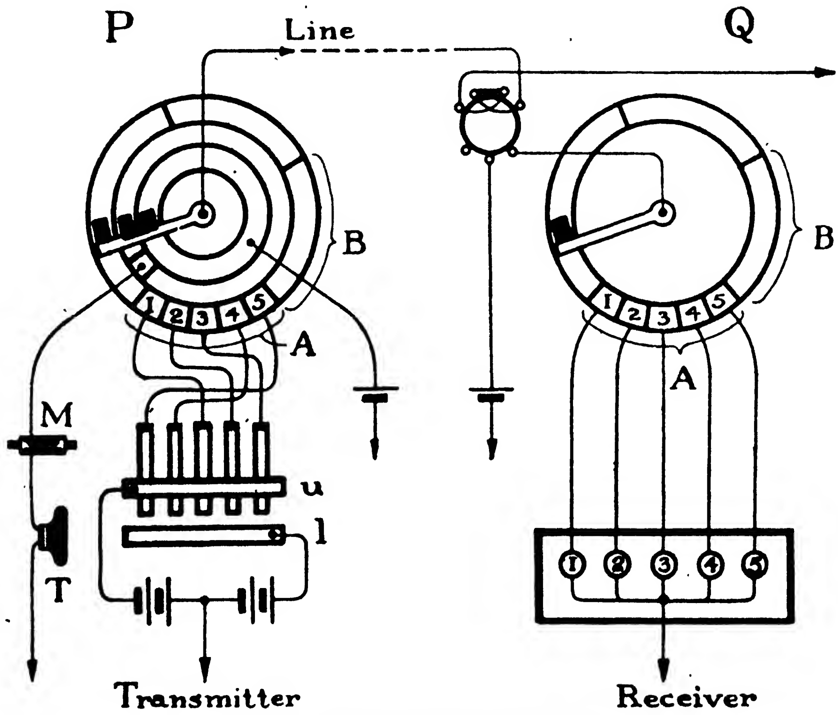


Fig. 73.—Principle of Baudot Multiplex.

printing telegraphy and multiple telegraphy, and adopted a five-unit code, in which every letter takes the same time and is signalled by permutations of five marks or spaces within this time. Thus " a " is signalled mark space space space space, " t " by mark space mark space mark, and so on. Since all letters occupy the same time, there is no need for a space between letters. The line speed of the five-unit code is considerably faster than Morse ; the ratio is about 8 to 5, so that a line having a working speed of 250 words a minute Morse could have a speed of 400 words a minute by the five-unit code.

Fig. 73, the general similarity of which to Fig. 72 is to be noted, illustrates the principle of operation for a four-channel system. Each of the four channel sections A B C D of the distributors is subdivided into five equal sections 1, 2, 3, 4, 5, and in addition to the twenty sections thus formed there are four left for speed correction and line retardation purposes. A transmitter is shown at P and a receiver at Q connected to the channel A; there are similar transmitters and receivers connected to each of the other channels B, C and D. Transmission is by double current. When the five keys of the transmitter are at rest they make contact through the upper bar *u* to the negative terminal of the battery, and negative current flows to line when the brush passes over the section to which the key is connected. When a key is depressed it is connected through the lower bar *l* to the positive terminal of the battery, and positive current flows to line when the brush passes over the section to which this key is connected. Key 1 will be depressed for the letter " a," and keys 1, 3 and 5 for the letter " t." Fig. 74 shows the voltage applied to line when " a " is signalled on channels A and C, and " t " on channels B and D. The currents are received on the polarised relay at Q, and current in the local circuit flows through the electromagnets 1, 2, 3, 4, 5 of the receiver as the brush at Q passes over the corresponding sections if a mark has been sent. Otherwise no current flows. The receiver magnets are not polarised; the relay is therefore necessary to convert from double current working on the line to single current working through the receiver. In the example taken, in the case of channel A only electromagnet 1 will be operated, in channel B electromagnets 1, 3 and 5, and so on. According to which electromagnets have been operated in the

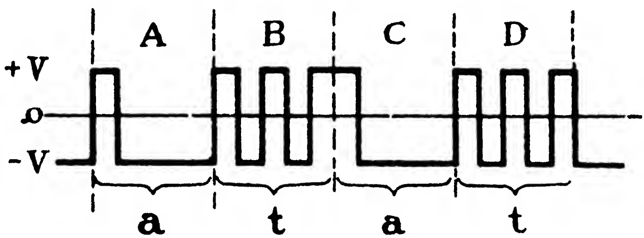


FIG. 74.—Line Current in 5-unit Code.

passage of the brush over the channel sections, a letter is printed on a paper slip by a most ingenious mechanical arrangement, in conjunction with a rotating type wheel, and the electromagnets are reset in readiness to receive another letter on the next passage of the brush. The mechanical operation of printing a letter in any channel thus takes place during the time that the brush is passing over the other channels. At the transmitting end the keys once depressed are held down until the passage of a " cadence " current through the electromagnet M releases them, at the same time giving a click in the telephone receiver T as a signal to the operator that he must press his keys to transmit the next letter. In this way signalling proceeds rhythmically at a letter speed predetermined by the revolutions per minute of the brush arm.

In the actual Baudot apparatus there are four rings on the front plate lettered F_1, F_2, F_4 and F_5 in order towards the centre, and six rings on the back plate lettered R_1, R_2, R_3, R_4, R_5 and R_6. These rings are shown developed in Figs. 75, 76 and 77. The front plate is movable as a whole with respect to the back plate. Two pairs of brushes sweep the front plate and three the back plate, connecting rings F_1 and F_4, F_2 and F_5, R_1 and R_4, R_2 and R_5, and R_3 and R_6. All these brush pairs are driven off the same shaft and are shown in line in the diagrams, but actually in order to obtain mechanical balance in rotation they are spaced 180 degrees apart on the front plate and 120 degrees apart on the back plate, and the section connections of the various rings are correspondingly shifted in phase.

The front plate is used for receiving and the back plate for sending and for synchronism adjustments. Fig. 75 shows the general layout of the connections at one station for a four-channel or quadruple set, but only one channel is drawn. With the switch at send currents determined by the position of the keys of the sender flow to line through R_2 and R_5. It is desirable, however, that there should be a record of the message sent. This is achieved through the leak relay connected, in series with a high resistance, between line (ring R_5) and earth. A duplicate of the line currents, therefore, flows through the leak relay coils, and when the current is in the marking direction, current in the local circuit flows *viâ* rings R_4 and R_1 through the corresponding electromagnets of the receiver.

When the switch is thrown to receive, the main battery is taken off the upper bar of the sender and the line relay connected instead. Since the keys are now all at rest, incoming line currents will flow *viâ* R_5 R_2 and the sender through the coils of the line relay to earth. The local circuit of the line relay sends double current *viâ* rings F_1 and F_4 to a third relay known as the combiner relay. The sections on F_1 are shortened to reduce the time of contact of the brushes, and the combiner relay has its coils shunted by a low resistance to make its action somewhat sluggish ; this combination counteracts the effect of line distortion and ensures signals of the proper length flowing in the local circuit of the combiner relay

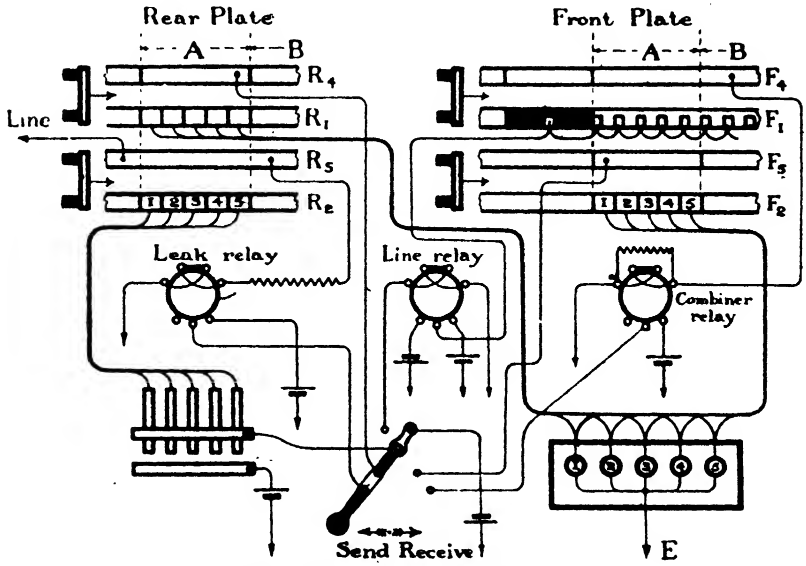


FIG. 75.—Baudot Quadruple Connections.

through the receiving electromagnets *viâ* the switch and rings F_2 and F_5.

It will be realised that the synchronising arrangements are of vital importance. The distributors at the end of the line and the receivers connected to each distributor must all run at identically the same speed and with the correct phase. The distributors are driven by phonic motors off vibrating reeds (or by falling weight), and at one station (the "correcting") the speed is considered standard and the other station (the "corrected") and all the receivers are made to conform. The distributor phonic motor at the corrected station is set to run a little fast, and its speed is reduced by correcting currents received every revolution from the correcting station. The electrical arrangements are outlined in Fig. 76, where P is the correcting and Q the corrected station. The correcting currents are sent from sections 23 and 24 of R_2 at P; 21 and 22 are connected to the line relay for another reason, to be explained shortly. At Q these currents are received on sections 21,

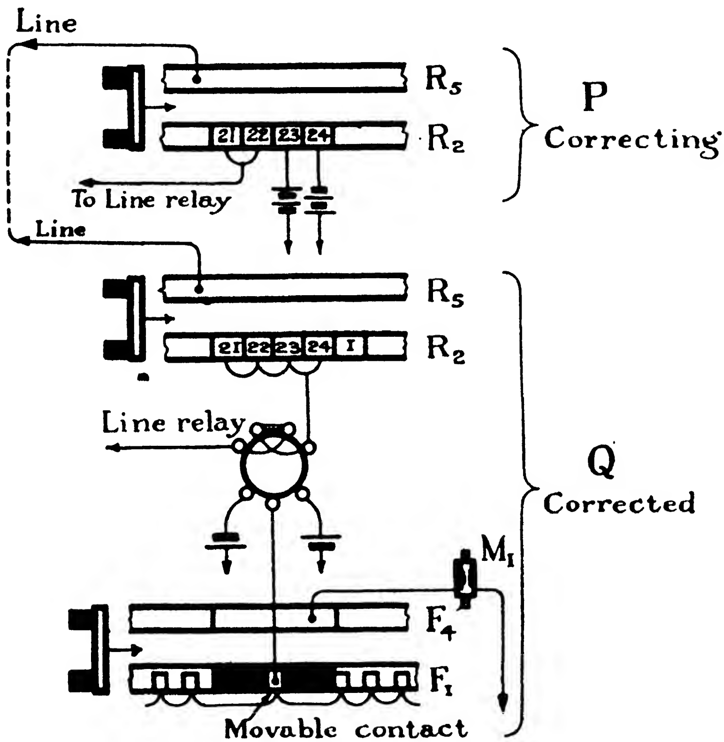


FIG. 76.—Main Synchronising Arrangement.

22, 23, and 24, which are all connected to the line relay. The local circuit is completed through F_4, a movable contact in the break in F_1 between sections 20 and 1, and an electromagnet M_1. If the distributor at Q is in synchronism with that at P, the brush reaches the movable contact just after the correcting signal has passed and M_1 is not operated. But if the brush at Q is ahead of its proper position the correcting signal is received and M_1 is operated for a time depending upon how far ahead the brush is. While M_1 is operated, the axle carrying the brushes is caused to slip on the shaft of the phonic motor, and the required retardation in phase takes place.

Each distributor sends local synchronising currents through its own receivers. The receivers are run a little too fast by electric motors, and the synchronising current actuates an electromagnet M_2 (Fig. 77) and applies a brake to the type-wheel shaft. This local synchronisation is carried out from rings R6 and R3, which also provide for the key release of the senders and the cadence signals. The latter is done from sections 22 and 23, 3 and 4, etc.,

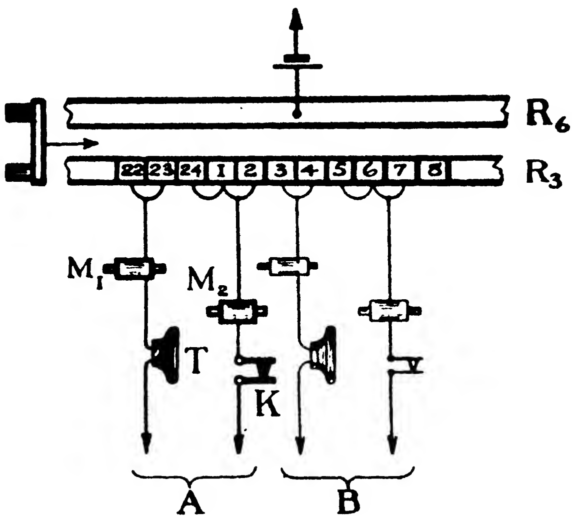


Fig. 77.—Local Synchronising Arrangement.

the former from sections 24—1—2, 5—6—7, etc. The battery connected to R_6 sends a current through M_2 only if the contact K is closed. K is controlled by a cam on the type-wheel shaft, so arranged that if the rotation is of the correct phase it is open while the brush passes over sections 24, 1 and 2 (for the A channel), but if the type wheel is a little ahead, the contact has not been opened when the brush passes, and current flows through M_2 and the brake is applied, for a time depending upon how far ahead the type-wheel is.

No further explanation is needed if all four channels are being worked in the same direction from P to Q. The main correcting currents will ensure that the distributor at Q is lagging behind that at P by an amount corresponding to the line retardation. But suppose channels A and B are being used in the direction P to Q, and C and D in the direction Q to P, and suppose, for example, that the line retardation is equivalent to one section, so that the distributor at Q is running one section behind that at P. Reception of A and B will be quite straightforward, but signals from C and D at Q will arrive at P on R_2 two sections behind, actually on sections 13 to 22 instead of on sections 11 to 20. It is for this reason that sections 21 and 22 (R_2, Fig. 76) are connected to the line relay at the correcting station P ; the currents arriving on 21 and 22 go straight to the line relay instead of through the upper bar of the channel D key. But, following the received currents through on Fig. 75, there will be the further effect that they would normally be received on F_2 two sections late throughout. This is avoided by rotating the whole of the front plate forward (*i.e.*, in the direction of motion of the brush arm) two sections.

Baudot has been duplexed successfully by using differentially wound line relays and the usual double current differential duplex circuit arrangement.

The speed of working each channel has been increased by automatic transmission somewhat in the same way as the Wheatstone transmission. Holes are perforated across the slip by a keyboard perforator and the slip is passed through a transmitter whose speed is controlled by the cadence signal of the distributor. A further development is page printing by the receiver instead of printing on slip as in the original Baudot. This avoids the cutting up and gumming of the slip on telegraph forms.

REFERENCES FOR FURTHER READING

W. A. J. O'Meara.—" The Various Systems of Multiple Telegraph." Inst. Post Office E.E., Professional Papers, No. 41.

A. C. Booth.—" The Baudot Telegraph System." Inst. Post Office E.E., Professional Papers.

A. C. Booth.—" Baudot System Duplexed." *Post Office Electrical Engineers' Journal*, Vol. III., p. 336.

A. C. Booth.—" Machine Telegraphy." Inst. Post Office E.E., Professional Papers, No. 63.

A. Crotch.—" The Hughes and Baudot Telegraphs."

H. H. Harrison.—" Printing Telegraph Systems and Mechanisms."

PART II

LINE TELEPHONY

CHAPTER VI

ALTERNATING QUANTITIES

(28) Sound

In telegraphy communication of intelligence is by means of the written word signalled by a code employing generally direct currents of electricity; in telephony the actual sound of the word spoken at one station is reproduced by electrical means at the distant station.

The word "sound" is used in three quite distinct ways. It means generally a sensation produced in the brain, but is used by the physicist in connection with the air wave or air vibration that causes the movement of the eardrum that gives rise to the sensation, and also in connection with the vibrating body that sets up the air vibration. The air vibration is called a sound wave, and the vibrating body a sounding body or source of sound.

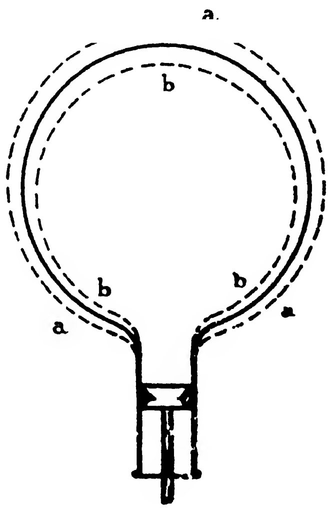


Fig. 78.—Toy Balloon illustrating production of sound waves.

Imagine that a small toy balloon has been partially inflated and fitted with a tube in which a piston can be moved up and down as indicated in Fig. 78. As the piston is moved upwards very slowly the balloon expands gradually to the dotted line *a a a* and the air outside will be pushed away; as the piston is moved very slowly downwards the balloon contracts to the line *b b b* and the air outside moves inwards. During this very slow movement of the piston the air in the neighbourhood of the balloon is accordingly moving outwards and inwards; each air particle moves backwards and forwards along a line drawn from the particle to the centre of the balloon. If the movement were infinitely slow and the balloon in an infinitely large air-filled space this is all that would happen; the actual air movement would decrease as the square of the distance from the centre of the balloon, since the total movement of air across any concentric sphere would be the same, and the surface of a sphere is proportional to the square of its radius. But air has mass, which gives it inertia, and it has elasticity. When the upward motion of the piston is not infinitely slow, the inertia of the air a little away from the balloon delays its movement; and the air close to the balloon is accordingly compressed between the walls of the balloon and the stationary air. Similarly when the piston moves down the air close to the balloon is rarefied, as the air further away does not immediately move in to fill the space created. This compressed or rarefied air then acts on the air outside it in the same way as the walls of the balloon acted on it, and so on. A wave of compressed air travels outwards from the balloon for each upward movement of the piston, and one of rarefied air for each downward movement. The movement of the air particles for each piston stroke is now confined between two concentric spheres, the distance between the spheres remaining the same as they enlarge. The movement accordingly varies inversely as the distance from the centre of the balloon instead of as the square of the distance, as in the case of the infinitely slow motion; it is because of this that wave motions can be appreciated at such great distances.

The actual mechanism of the motion is made clearer by a model such as is illustrated in Fig. 79. A number of heavy balls are suspended by long light threads an equal distance apart, and between each is fixed a spiral spring. If ball 1 is quickly pushed a little to the right, ball 2 will not at once move owing to its inertia, but the spring 1—2 will be compressed. Ball 2 will now start to move under the action of the compressed spring, and in moving will compress the spring 2—3, and so on. The movements of the balls take place one after another; the disturbance travels along the line of balls from left to right. Similar action takes place when ball 1 is moved to the left, but this time the springs are extended. This model may be taken to represent the motion of the air along any line drawn outwards from the centre of the balloon. The mass of the balls corresponds to the mass of the air particles, a compression of the springs to a compression of the air or a crowding of the air particles together, and an extension of the springs to a rarefaction of the air or a separation of the air particles apart.

If the last spring of the model is fixed to a rigid body, a pressure or pull will be exerted on the body when the wave reaches this spring, and the wave will be reflected and travel backwards from right to left. If, for instance, the last spring has been compressed, it will a little later push the last ball back, resulting in a compression and subsequent movement of the last ball but one, and so on. Similarly when the air wave from the balloon reaches a rigid body it causes a push or pull on the body in being reflected. If the air wave strikes the ear this push or pull produces a movement of the eardrum which causes a sensation of sound in the brain.

If the piston of the balloon is moved continuously instead of just once, it is evident that the air movements along the radial lines will follow the initial movement, but at a progressively later time and reduced amplitude as the distance from the balloon is increased. If the piston movement is quite irregular the corresponding sound sensation will be of noise; but if the movement repeats itself time

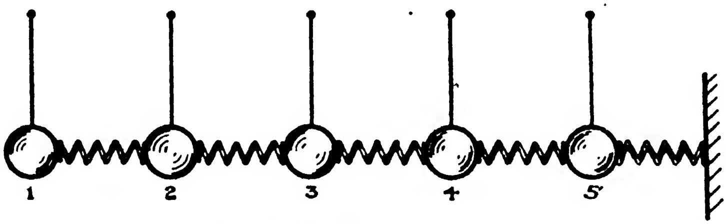


FIG. 79.—A Mechanical Model illustrating wave motion.

after time regularly, the sensation may be of music or speech, depending only upon the nature of the movements, that is, upon the curve of the displacement plotted against time. This curve is called the "wave form." The simplest possible continuous movement in sound is that which has a sinusoidal wave form, and this gives rise to a sensation known as a simple or pure tone. Such a sound is given by a tuning-fork or by a lightly blown organ pipe. The time T in seconds that is taken for the movement of an air particle from its extreme position in one direction to its extreme position in the opposite direction and back again is called the period, the whole of the movement taking place during the period is called the vibration or alternation, and the number of vibrations in a second is called the frequency f. Evidently $f = 1/T$. Half the distance between extreme positions is called the amplitude a. The amplitude determines the loudness of a tone, the frequency its pitch. Sound sensations are only produced by vibrations of frequency between 24 and 16,000 cycles per second, though these limits vary greatly in individuals.

The sound wave emitted by a sounding body similar to the very small toy balloon is a spherical wave, but at considerable distances away, over a limited area of the sphere and for limited distances the wave approximates to a plane wave, in which the air particle movements are in parallel lines and there is no reduction in amplitude with distance. The movement of a body sounding a pure tone can be represented by the curve drawn in Fig. 80, in which the displacement is plotted against the time according to the equation $x = a \sin(2\pi ft + \phi)$. ϕ is the "phase angle," which determines the instant from which the time is reckoned. This curve can be drawn from a table of sines. It can also be drawn in

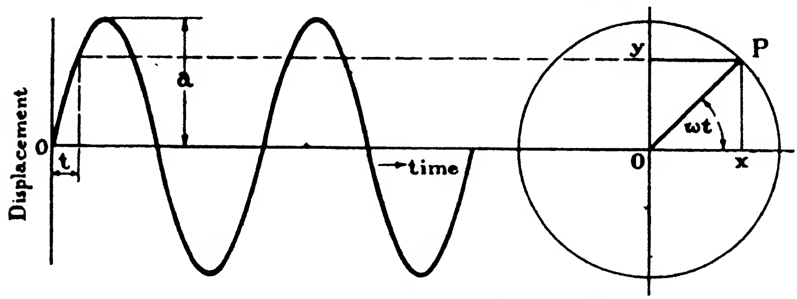


FIG. 80.—Displacement of a sounding body producing a pure tone.

the manner indicated in the figure by projecting from a line OP which is rotated in a counter-clockwise direction about O at an angular velocity ω such that $\omega = 2\pi f$. The projection of OP on the vertical axis $= Oy = \text{OP} \sin \omega t = a \sin 2\pi ft$ if OP is made equal to the amplitude a. In this diagram ϕ has been taken to be zero. The same diagram represents also the movement of any particle of air influenced by the disturbance, provided that the displacement scale is suitably modified to take account of the reduction in amplitude (*i.e.*, a suitably altered) and provided also that the time taken for the wave to reach the particle is taken account of in placing actual times on the time base (*i.e.*, ϕ suitably altered). If now instead of considering what happens to one particle as time goes on, a bird's-eye view is taken of all the particles in a straight line drawn outwards from the source, the view being taken at one instant of time only, the displacement plotted against the distance from the source will give the curve of Fig. 81. Each wave is repeated again and again, but with decreasing amplitude. The distance between similar points in the curve, say from a to a', is known as the wavelength λ. At a little later time than the instant chosen, the positions of the particles will be represented by the dotted curve. The disturbance has moved to the right. After a time equal to the period the dotted line will coincide again with the firm line. The disturbance has moved a distance λ in time T. Its velocity is therefore $c = \frac{\lambda}{T} = \lambda f$. The velocity of sound is 331 metres per second at 0° C.

The reduction of amplitude of displacement with distance is more

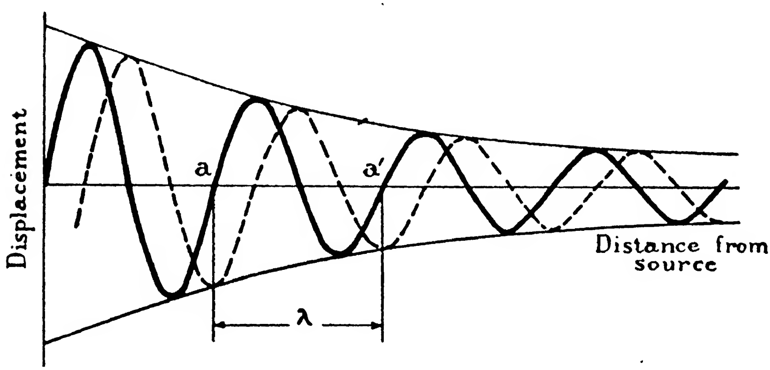


FIG. 81.—Showing relationship at a particular time between displacement and distance from source.

rapid near the source than further away. If the source is a very small one, the " point source " of mathematical physics, then close up to the source the amplitude varies inversely as the square of the distance, but when the distance is great the amplitude varies inversely as the distance. The amplitude of the pressures produced in the sound wave always varies inversely as the distance (see p. 119).

(29) Sound Waves

The ideas of the previous section may be made more definite by putting them into mathematical form. As the simplest case consider an imaginary tube of unit cross-section lying in the direction of propagation in a plane sound wave. Let (Fig. 82) the section AB of the tube distant y from some fixed reference point move to A′B′ distant x from AB in time t as the wave passes. In the same time section CD will move to C′D′, and if the thickness of the slab AC is δy, the distance CD moves is $x + \frac{\partial x}{\partial y}\delta y$, and the increase of thickness of the slab is

$$\left(x + \frac{\partial x}{\partial y}\delta y\right) - x = \frac{\partial x}{\partial y}\delta y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and this in a tube of unit cross-section is the increase in volume, which gives rise to a reduction in pressure.

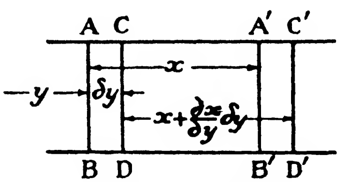


FIG. 82.—Element of imaginary tube of unit cross-section in plane sound wave.

The changes of volume in a sound wave are so rapid that no transfer of heat takes place, and the adiabatic law of expansion holds, *i.e.*, $p_0 v_0^{\gamma_0}$ = constant, where p_0 is the pressure, v_0 is the volume and γ_0 the ratio of the specific heat at constant pressure to the specific heat at constant volume. Differentiation gives

$$\frac{dp_0}{dv_0}v_0^{\gamma_0} + p_0\gamma_0 v_0^{\gamma_0 - 1} = 0,$$

or

$$-\frac{v_0 dp_0}{dv_0} = \gamma_0 p_0 = e.$$

where e is the elasticity of the gas (= stress/unital strain = $dp_0/(dv_0/v_0)$. The negative sign appears because an increase of pressure causes a decrease of volume). This is more conveniently written

$$-\frac{v_0 p}{v} = \gamma_0 p_0 = e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where v_0 and p_0 are the steady volume and pressure, and v and p are the alterations of volume and pressure.

The movement of the slab AC is caused by the difference of pressure on the faces AB, CD, and this is given by the expression

$$-\frac{\partial p}{\partial y}\delta y = -\frac{\partial}{\partial y}\left(-\frac{ev}{v_0}\right)\delta y \quad . \quad . \quad . \quad . \quad \text{by (2)}$$

$$= e\frac{\partial}{\partial y}\left(\frac{\partial x}{\partial y}\right)\delta y,$$

since v from (1) $= \frac{\partial x}{\partial y} \delta y$, and v_0 is δy.

Hence $$-\frac{\partial p}{\partial y} \delta y = e \frac{\partial^2 x}{\partial y^2} \delta y. \qquad (3)$$

This is the force (in a tube of unit cross-section) producing the motion of the slab AC whose mass is $\rho_0 \delta y$, where ρ_0 is the density of the air, and whose acceleration is $\frac{\partial^2 x}{\partial t^2}$. Hence the equation of motion is

$$e \frac{\partial^2 x}{\partial y^2} \delta y = \rho_0 \frac{\partial^2 x}{\partial t^2} \delta y$$

or $$\frac{\partial^2 x}{\partial t^2} = \frac{e}{\rho_0} \frac{\partial^2 y}{\partial y^2} \qquad (4)$$

If the sound wave is that of a simple tone the variations will be sinusoidal both in space and time, and a solution of the form

$$x = \text{A} \cos (\omega t + my) \qquad (5)$$

is suggested, with $\omega = 2\pi f$.

Differentiating (5) twice with regard to t gives

$$\frac{\partial^2 x}{\partial t^2} = -\omega^2 x,$$

and with regard to y gives

$$\frac{\partial^2 x}{\partial y^2} = -m^2 x,$$

and substitution in (5) gives

$$-\omega^2 = -\frac{e}{\rho_0} m^2$$

or $$m = \pm \omega \sqrt{\frac{\rho_0}{e}}.$$

The solution of (4) may accordingly be written

$$x = \text{A}_1 \cos \omega \left(t - \sqrt{\frac{\rho_0}{e}}\, y\right) + \text{A}_2 \cos \omega \left(t + \sqrt{\frac{\rho_0}{e}}\, y\right) \qquad (6)$$

This represents the sum of two waves, the first travelling from left to right and the second from right to left. For consider the first term. An increase in t and an increase in y leave $\left(t - \sqrt{\frac{p_o}{e}}\, y\right)$ unaltered if $y/t = \sqrt{\frac{e}{\rho_o}}$. Therefore $\sqrt{\frac{e}{\rho_o}}$ is the velocity of a wave propagated from left to right. In the second term, on the other hand, in order to obtain the same value of x, y must be decreased as t is increased. The wave is propagated from right to left.

The source of sound being situated to the left, the second term represents a reflected wave. If there is no reflecting surface $A_2 = 0$ and this term disappears.

The velocity of the wave is from (2)

$$c = \sqrt{\frac{e}{\rho_o}} = \sqrt{\frac{\gamma_o p_o}{\rho_o}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$\gamma_o = 1{\cdot}40$
$p_o = 1{\cdot}015 \times 10^6$ dynes/cm.^{2} } For air at 0° C. and 760 mm.
and $\rho_o = 0{\cdot}00129$ gm./cm.^{3} } pressure.

Hence $c \doteq 33{,}100$ cm. per second.

From (2) and (1) the alternating pressure of the air particles is

$$p = -e\frac{v}{v_o} = -e\frac{\partial x}{\partial y}.$$

whence from (6), with no reflected wave,

$$p = -e\omega\sqrt{\frac{\rho_o}{e}}\,A_1 \sin\omega\left(t - \sqrt{\frac{\rho_o}{e}}\,y\right)$$

$$= -\omega\sqrt{\gamma_o p_o \rho_o}\,A_1 \sin\omega\left(t - \frac{y}{c}\right) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The particle velocity u in the sound wave is evidently $\partial x/\partial t$, and (6) gives

$$u = -\omega A_1 \sin\omega\left(t - \frac{y}{c}\right) \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The particle velocity and the alternating pressure in a plane wave are therefore in phase, and the following relation exists between them :—

$$\frac{p}{u} = \sqrt{\gamma_o p_o \rho_o} = 42{\cdot}8, \quad . \quad . \quad . \quad . \quad . \quad (10)$$

when the alternating pressure is measured in dynes per square centimetre and the particle velocity in centimetres per second.

The flow of energy per second or flux of energy through the tube is given by the product of the alternating pressure and the particle velocity, *i.e.* by

$$\omega^2 A_1^2 \sqrt{\gamma_o p_o \rho_o} \sin^2 \omega\left(t - \frac{y}{c}\right),$$

and the average value by

$$\begin{aligned} W &= \frac{1}{2}\omega^2 A_1^2 \sqrt{\gamma_o p_o \rho_o} \\ &= \frac{1}{2}\omega^2 A_1^2 \rho_o c \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11) \end{aligned}$$

This is the power in ergs per second that must be exerted by the source in an unlimited medium for each square centimetre of the surface over which the sound wave is maintained.

Calculations in acoustics are frequently facilitated by the use of a mathematical expression called the "velocity potential," which is such that if the velocity potential at any point in a wave is Φ, then the particle velocity at that point in any direction r is

$$u = -\frac{\partial \Phi}{\partial r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and the alternating pressure at the point is

$$p = \rho_o \frac{\partial \Phi}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

In the plane wave considered above the velocity potential at any point y is given by

$$\Phi = cA_1 \cos \omega \left(t - \frac{y}{c}\right) \quad . \quad . \quad . \quad . \quad . \quad (14)$$

For
$$-\frac{\partial \Phi}{\partial y} = -\omega A_1 \sin \omega \left(t - \frac{y}{c}\right) = u$$

in agreement with (9), and

$$\begin{aligned} \rho_o \frac{\partial \Phi}{\partial t} &= -c\rho_o A_1 \omega \sin \omega \left(t - \frac{y}{c}\right) \\ &= -\omega \sqrt{\gamma_o p_o \rho_o} A_1 \sin \omega \left(t - \frac{y}{c}\right) = p \end{aligned}$$

in agreement with (8).

Consider now the spherical wave produced by the expansions and contractions of the toy balloon of Fig. 78, imagined to be very small. The wave will be symmetrical about the centre of the balloon and the velocity potential Φ will depend only on the distance r from the centre. The direction of the particle velocity at any point will by symmetry be along the radius at that point and is given from (12) by

$$u = -\frac{\partial \Phi}{\partial r}.$$

The flux, *i.e.* the rate of flow of air, outwards across the sphere of radius r is consequently

$$-\frac{\partial \Phi}{\partial r} . 4\pi r^2,$$

and the difference of flux across a spherical shell of thickness δr is

$$-4\pi \frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right) \delta r \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The difference of the flux across the shell is the rate of accumulation of air in the shell, that is

$$\frac{\partial}{\partial t}\left(\frac{\rho_1 - \rho_o}{\rho_o}\right) \times 4\pi r^2 \delta r,$$

and $(\rho_1 - \rho_o)/\rho_o = \rho_1/\rho_o - 1 = v_o/v_1 - 1 = (v_o - v_1)/v_o = \delta v/v_o = -p/\gamma_o p_o$ by (2). (The subscript $_1$ indicates the *altered* values of the density and volume.)

The rate of accumulation of air is consequently

$$-\frac{\partial}{\partial t}\left(\frac{p}{\gamma_o p_o}\right) . 4\pi r^2 \delta r = -4\pi r^2 \delta r . \frac{1}{\gamma_o p} . \frac{\partial}{\partial t}\left(\rho \frac{\partial \Phi}{\partial t}\right)$$

$$= -\frac{4\pi r^2 \delta r \rho}{\gamma_o p_o} . \frac{\partial^2 \Phi}{\partial t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

which equated to (15) gives

$$\frac{r^2 \rho}{\gamma_o p_o} . \frac{\partial^2 \Phi}{\partial t^2} = \frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right)$$

or

$$\frac{\partial^2 \Phi}{\partial t^2} = \frac{c^2}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{\partial \Phi}{\partial r}\right), \quad \text{using (7)},$$

and this may be written

$$\frac{\partial^2 (r\Phi)}{\partial t^2} = c^2 \frac{\partial^2 (r\Phi)}{\partial r^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

This equation is of the same form as (4) and indicates that $r\Phi$ is propagated unchanged, or that Φ varies inversely as r. The solution for sinusoidal vibrations will be (by analogy with (6))

$$r\Phi = B_1 \cos \omega\left(t - \frac{r}{c}\right) + B_2 \cos \omega\left(t + \frac{r}{c}\right) \quad . \quad . \quad . \quad (18)$$

the first term representing a diverging wave and the second a converging wave. In the absence of reflection, B_2 is zero and

$$\Phi = \frac{B_1}{r} \cos \omega\left(t - \frac{r}{c}\right) \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The particle velocity is from (12)

$$u = -\frac{\partial \Phi}{\partial r} = \frac{B_1}{r^2} \cos \omega\left(t - \frac{r}{c}\right) - \frac{B\omega}{rc} \sin \omega\left(t - \frac{r}{c}\right) \quad . \quad . \quad (20)$$

When r is very small the first term is far greater than the second and u varies inversely as the square of the distance from the source. When r is large the second term only matters, and u varies inversely as the distance.

As r approaches zero (20) gives

$$4\pi u r^2 = 4\pi B_1 \cos \omega t = A \cos \omega t,$$

where A is written for $4\pi B_1$.

$4\pi r^2 u$ is the rate of introduction of air at the source (from the piston in the toy balloon model); hence A is the amplitude of the rate of introduction of air, and the source is defined from (19) as

$$r\Phi = \frac{A}{4\pi} \cos \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

From such a source the particle velocity and alternating pressure in a diverging spherical wave are given from (12) and (13) by

$$u = \frac{A}{4\pi r^2} \cos \omega\left(t - \frac{r}{c}\right) - \frac{A}{4\pi r} \cdot \frac{\omega}{c} \sin \omega\left(t - \frac{r}{c}\right) \quad . \quad . \quad . \quad (22)$$

and $$p = \rho \frac{\partial \Phi}{\partial t} = -\frac{A\rho\omega}{4\pi r} \cdot \sin \omega\left(t - \frac{r}{c}\right) \quad . \quad . \quad . \quad . \quad (23)$$

and the vector diagram representing them is as shown in Fig. 83.

The velocity component which varies as $1/r^2$ is 90 degrees out of phase with the component varying as $1/r$, and the latter is in phase with the pressure.

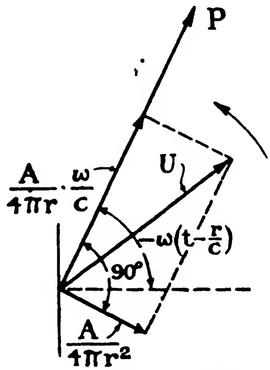


FIG. 83.—Vector diagram representing particle velocity and alternating pressure in a diverging spherical wave.

The root mean square values of the particle velocity and alternating pressure are given by

$$U = \frac{A}{4\pi\sqrt{2}}\sqrt{\frac{1}{r^4}+\frac{1}{r^2}\cdot\frac{\omega^2}{c^2}}. \quad . \ (24)$$

$$\text{and } P = \frac{A\rho\omega}{4\pi r\sqrt{2}} \quad . \ . \ . \ . \ . \ . \ (25)$$

The average flux of energy across each square centimetre of the sphere of radius r is given by the mean of the product of u and p (equations 22 and 23), that is, by

$$\frac{1}{2}\cdot\frac{A^2}{16\pi^2}\cdot\frac{1^1}{r^2}\cdot\frac{\omega^2}{c},$$

and the total flux of energy over the sphere is obtained by multiplying by the area of the sphere $4\pi r^2$, giving

$$W = \frac{A^2}{8\pi}\cdot\frac{\rho\omega^2}{c} \quad . \ . \ . \ . \ . \ . \ . \ (26)$$

This is the output power of the source in ergs per second if c.g.s. units are used throughout.

Measurements in connection with sound waves are difficult. In general three quantities are involved in a determination of the flux of energy across any area; the particle velocity, the alternating pressure and the phase angle between these two. The particle velocity may be measured by a Rayleigh disc, and the alternating pressure by a suitable microphone, but no ready means is available for measuring the phase angle.

The measurement of particle velocity depends upon the fact that a flat obstacle in a stream tends to turn its flat side towards the stream, as may be gathered from Fig. 84, in which a general idea is given of the stream lines round a disc suspended at the extremity of a diameter. In order to pass the disc the stream must divide, and it is evident that the larger part will go round the side B and the smaller part round the side A. Thus the centre of pressure on the disc will not be at the centre C, but at a point P somewhere as shown between C and A. Similarly the point of reunion of the stream lines on the down-stream side will be at Q, between C and B, and the centre of the forces of reaction will be at Q. Thus the effect of the stream on the disc will be to produce a torque tending to make the

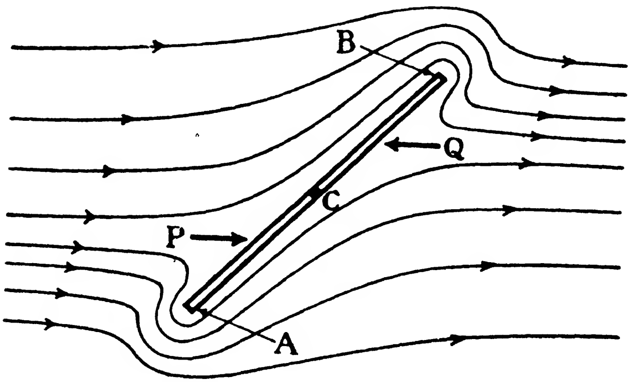


FIG. 84.—Action of Rayleigh Disc.

disc face the stream, and this torque will be unchanged if the direction of the flow is reversed.

The torque has been proved by Koenig to be given by

$$T = \frac{4}{3} U^2 a^3 \rho_o \sin 2\theta,$$

where a is the radius of the disc and θ is the angle its normal makes with the direction of the undisturbed stream. If the disc is suspended by an elastic thread such as a glass or quartz fibre of stiffness τ the angle δ it turns through will be such as to make

$$\delta\tau = \frac{4}{3} \rho_o a^3 U^2 \sin 2\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

and the disc may be calibrated to read particle velocities. To obtain the greatest deflections θ is made 45 degrees, and the angle is measured by the deflections of a beam of light reflected from a small mirror mounted on the disc.

The most suitable telephone to use for pressure measurements is the condenser transmitter (section 37), but the ordinary telephone receiver may be used successfully for many purposes. The alternating voltage produced by the sound wave acting on the transmitter is amplified by means of valves, and the resulting voltage is a measure of the alternating pressure with a suitable calibration.

Fig. 85 (*a*) shows the results of some measurements made in front of a resonator consisting of a tube, 5 cm. bore, inside which was mounted a small telephone receiver. Curve U shows the particle velocity as measured by a Rayleigh disc at various distances from the mouth of the tube along its axis, while curve E shows the

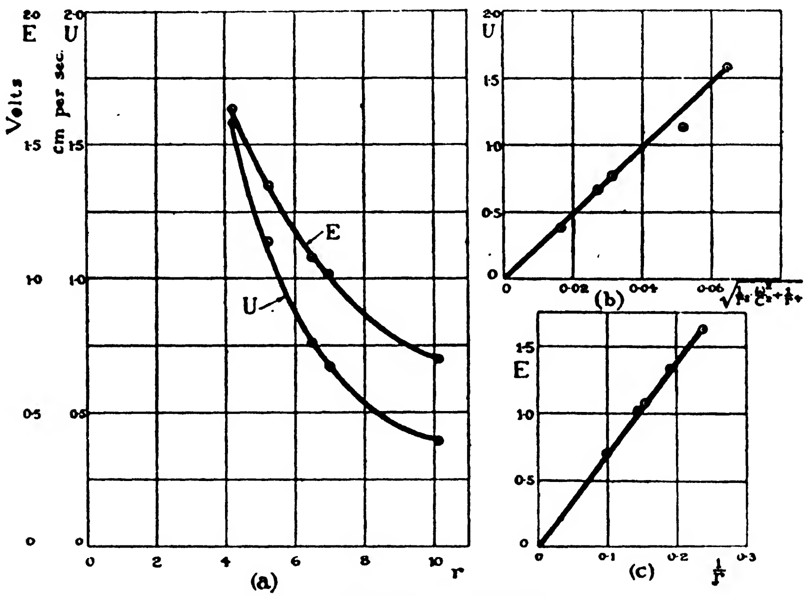


FIG. 85.—Sound Measurements.

amplified voltages obtained from a telephone receiver with its earcap fitted with a small-bore tube projecting into the sound field.

In Fig. 85 (*b*) U is plotted against $\sqrt{\frac{1}{r^2}\frac{\omega^2}{c^2}+\frac{1}{r^4}}$. The straight line obtained indicates that the wave is spherical (equation 24), and this is confirmed by Fig. 85 (*c*) in which E is plotted against $1/r$ (equation 25). The measured voltages E are proportional to the alternating pressures in the sound wave, and the curves may be used to calibrate the pressure measuring device against the Rayleigh disc. The calibration of the latter may be carried out in a steady air stream, or by calculation from (27), τ for the suspension being estimated from the period of vibration when the disc is replaced by a small rod of known moment of inertia.

(30) **Wave Forms of Speech and Music**

The wave form of most musical sounds and of speech is very different from the simple sine wave of the tuning fork or lightly blown organ pipe. The wave forms of notes of a violin and of an oboe taken by Professor D. C. Miller are shown in Fig. 86 as A and B respectively. In each case there is a wave form which is very complicated compared with the sine wave of Fig. 80, but which shows a repetition time after time of a certain cycle of displacements. The time of this cycle determines the pitch of the note; the actual wave form of the cycle determines the quality or timbre.

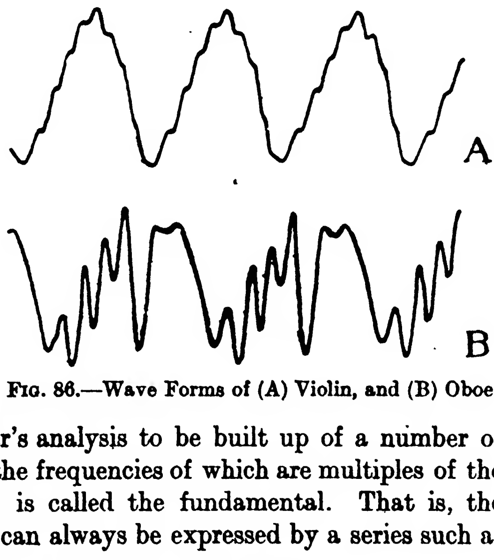


Fig. 86.—Wave Forms of (A) Violin, and (B) Oboe.

All sustained vibrations of this nature can be shown by Fourier's analysis to be built up of a number of sine and cosine waves, the frequencies of which are multiples of the pitch frequency, which is called the fundamental. That is, the wave form of the cycle can always be expressed by a series such as

$$y = A_0 + A_1 \sin \omega t + A_2 \sin 2\omega t + A_3 \sin 3\omega t + + B_1 \cos \omega t + B_2 \cos 2\omega t + B_3 \cos 3\omega t + \ldots \quad (1)$$

or

$$y = Y_0 + Y_1 \sin(\omega t + \phi_1) + Y_2 \sin(\omega t + \phi_2) + Y_3 \sin(3\omega t + \phi_3) + \ldots \quad (2)$$

where $\omega = 2\pi f = 2\pi/T$, T being the periodic time of the cycle, and $A_0\ A_1\ A_2 \ldots B_1\ B_2\ B_3 \ldots Y_0\ Y_1\ Y_2 \ldots$ are the amplitudes of the steady term (generally absent), the fundamental and the various harmonics, as the remaining terms are called. Thus the term in $2\omega t$ is the second harmonic, that in $3\omega t$ is the third harmonic, and so on. $\phi_1\ \phi_2 \ldots$ are phase angles determining the relative positions on the time base of the various harmonics. A knowledge of the amplitudes $Y_1\ Y_2\ Y_3$ . . . determines the character of the sound; it is generally agreed that the phase angles $\phi_1\ \phi_2$ . . . have very little, if any, influence.

The Fourier expression has a real physical significance. If the wave form, for instance, of the oboe is put into the form (2), and a number of tuning forks of angular frequencies ω, 2ω, 3ω . . . are sounded together with amplitudes $Y_1\ Y_2\ Y_3$ . . ., the sound sensation produced will be precisely that of the oboe.

Analysis of wave forms such as those of Fig. 84 into a series such as (1) is carried out in virtue of the following relationships :—

$$\left.\begin{aligned}
&\int_o^T \sin n\omega t\, dt = 0\\
&\int_o^T \sin n\omega t \sin m\omega t\, dt = 0\\
&\int_o^T \sin n\omega t \cos m\omega t\, dt = 0\\
&\int_o^T \sin n\omega t \cos n\omega t\, dt = 0\\
&\int_o^T \sin^2 n\omega t\, dt = \frac{1}{2}T
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and similar expressions for the cosines. In words, the average value of a sine or cosine wave, or the product of two sine or cosine waves of different frequencies, or the product of any sine and cosine wave is zero if taken over a complete period, but the average value of the square of a sine or cosine wave is one-half.

The average value of the ordinates of the wave form will in consequence at once give A_o in equation 1. The average value of $y \sin \omega t$ is equal to $A_1/2$, the average value of $y \cos \omega t$ is equal to $B_1/2$, the average value of $y \sin 2\omega t$ is equal to $A_2/2$, and so on.

Generally then

$$\left.\begin{aligned}
A_n &= \frac{2}{T}\int_o^T y \sin n\omega t\, dt\\
B_n &= \frac{2}{T}\int_o^T y \cos n\omega t\, dt
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and the computation is best done graphically in the majority of the cases met with in acoustics.

Although speech sounds are far less sustained than musical sounds, yet it appears that the vowel sounds and even some of the consonant sounds are periodic in the same way as musical sounds, and may be analysed in the same manner and even synthetised by tuning forks. Professor Miller has examined a large number of records of the wave form of vowels taken with various persons speaking or singing, and has found that whatever the note on which the vowel sound is sung or spoken, and whoever is the singer or speaker, a large part of the total energy of the sound is round about one particular frequency or two particular frequencies, that

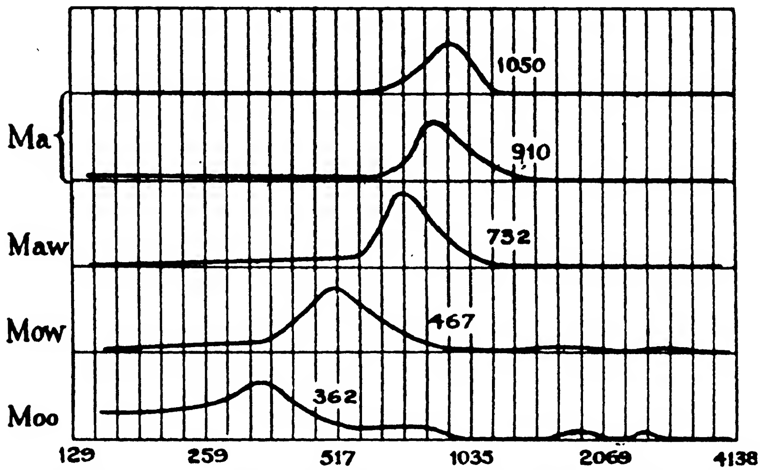


FIG. 87 (a).—Relative energy distribution of vowel sounds at different frequencies.

these frequencies are different with different vowel sounds, and that the vowel sound is definitely characterised or defined by these frequencies. The average of a large number of analyses showing the relative distribution of energy over the frequency range are set out for various vowel sounds in Fig. 87a and b, while Fig. 87c shows the characteristic frequencies of the vowels in ordinary musical notation.

It will be seen that the vowels may be divided into two classes according to whether they are characterised by one frequency or two. It will also be seen from the diagrams that the lower frequencies of the vowels of the second class are very nearly the same as the frequencies of the vowels of the first class. For instance, the "ee" sound in "three" (Fig. 87b) has frequencies of 308 and 3,100, while the "oo" sound in "two" (Fig. 87a) has a frequency of 326. If therefore the frequency of 3,100 were cut-off in some

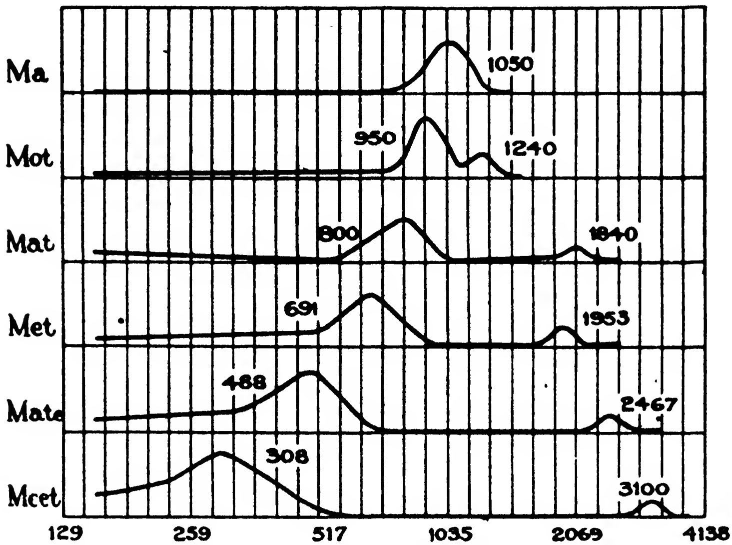


FIG. 87 (b).

way from the vowel sound "ee," the vowel would degenerate to "oo," and "three" would very possibly be heard as "two." This is clearly the reason for the rolling of the "r" in passing telephone

FIG. 87 (c).

calls, as all the apparatus in use in telephony has a tendency to cut off the higher frequencies.

Sir Richard Paget,* working synthetically from the shape of the cavities of the mouth and throat, and the manner in which they are

* *Phys. Soc.*, Vol. 36, p. 45, 1923.

modified by movements of the tongue, arrived at similar conclusions to Miller's, although he ascribes two frequencies to each vowel sound. This comparatively recent work would appear to settle the old controversy as to the nature of vowel sounds in favour of Helmholtz, who first stated the fixed pitch theory and produced vowel sounds mechanically by means of tuning forks in front of resonators.

Even the consonant sounds, although they can hardly be sustained, but are merely different ways of commencing and ending vowel sounds, are yet sufficiently periodic to be analysed by Fourier's analysis. C. Stumpf * finds that the frequency range of the consonants is very great, up to 9,300 cycles per second and down to 130.

Two conclusions may be drawn as the result of the foregoing. In the first place, in virtue of Fourier's analysis, in considering the properties of telephone apparatus and lines, it will be sufficient to make all calculations on the assumption of sinusoidal wave forms; each harmonic of the analysis may be considered separately. In the second place, the frequency range that has to be considered is very large for perfect reproduction of speech and music; up to 10,000 cycles per second probably, and down to, say, 16 for the organ pipe of lowest pitch.

In ordinary commercial telephony, however, the frequency range aimed at is very much more restricted. A range from about 200 to 2,100 cycles per second will give quite intelligible speech, because uncertainties are largely eliminated by the intelligence of the listener, though some of the naturalness of the speaker's voice will be lost. In broadcasting the standard aimed at is much higher.

(31) Vector Algebra and Simple Resonance Curves

It has been seen in Fig. 80 how the sine wave representing the displacement in a pure tone can be drawn from a line OP rotating with angular velocity ω. Once the length of the line is known and its angle at the instant from which time is reckoned, there is no need to draw the sine curve at all; the line OP gives all the information required. Such a line is known as a rotating vector, and is of the greatest value in simplifying calculations in alternating quantities. Rotating vectors are added graphically in the same way as forces in

* See K. W. Wagner E.T.Z., May 8th, 1924.

space are added; by a parallelogram construction. If there are two sources of sound producing displacements determined by the vectors OP_1 and OP_2, Fig. 88, differing in phase by an angle ϕ_1, then the resulting displacement will be determined by the rotating vector OR, found by drawing P_1R parallel to OP_2 and P_2R parallel to OP_1. That this is so can be verified by drawing out the sine waves corresponding to OP_1, OP_2 and OR and adding the ordinates of the first two at every point, when the curve obtained will be found to coincide with the third.

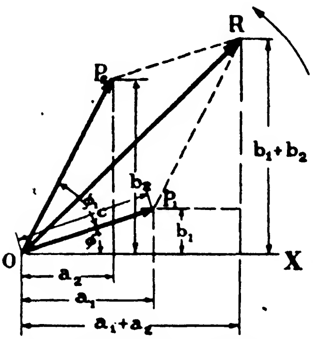


FIG. 88.—Vector Addition.

The vector OP, Fig. 89, may be drawn in two ways. In the first a length $Ox = a$ is marked off from O along the axis of X and a vertical $xP = b$ is set up at x. If by j is meant that the quantity which it multiplies is turned counter-clockwise through 90 degrees, the construction may be written

$$OP = a + jb \quad . \quad . \quad . \quad . \quad (1)$$

In the second way a line is drawn at an angle ϕ to the axis OX, and a length c is marked along it. Using the above notation this may be written

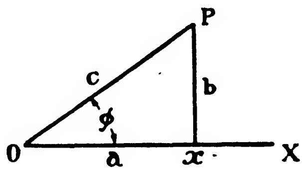


FIG. 89

$$OP = c\cos\phi + jc\sin\phi.$$

But $$\cos\phi = \frac{\epsilon^{j\phi} + \epsilon^{-j\phi}}{2}$$

and $$\sin\phi = \frac{\epsilon^{j\phi} - \epsilon^{-j\phi}}{2j}$$

Hence $$OP = c\epsilon^{j\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

This is very often abbreviated to

$$OP = c\,\underline{/\phi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

or $c\,\overline{\setminus\;\phi}$ if ϕ is a negative angle.

The two forms (1) and (2) are obviously related by the expressions

$$\left.\begin{aligned} c &= \sqrt{a^2 + b^2} \\ \text{and} \qquad \tan\phi &= \frac{b}{a} \end{aligned}\right\} \quad \ldots \ldots \quad (3)$$

In obtaining (2) j has been identified with $\sqrt{-1}$, and this identity leads to quite consistent results. Multiplying b by j twice, for instance, turns it counter-clockwise through two right angles; or b is now drawn in the negative direction along the axis. So that $j\{j(b)\} = -b$, from the geometrical consideration. Also $jjb = j^2b = -b$ from the algebraic point of view.

The addition of the two vectors OP_1 and OP_2 can be carried out algebraically as follows:—

$$\begin{aligned} OP_1 &= a_1 + jb_1 \quad OP_2 = a_2 + jb_2, \\ OP_1 + OP_2 &= a_1 + jb_1 + a_2 + jb_2 \\ &= a_1 + a_2 + j(b_1 + b_2) \\ &= OR \end{aligned}$$

as is clear on reference to Fig. 88. The rule for addition is clearly "add the real terms and the imaginary terms separately," the terms not multiplied by j being called the real terms and those multiplied by j the imaginary terms. The same rule applies to subtraction, due regard being paid to signs.

The product or ratio of two vectors is often required, and is best obtained by writing the vectors in the form (2).

$$\begin{aligned} \text{Thus} \qquad OP_1 &= c_1\epsilon^{j\phi_1} \quad OP_2 = c_2\epsilon^{j\phi_2} \\ OP_1 \times OP_2 &= c_1\epsilon^{j\phi_1} \,.\, c_2\epsilon^{j\phi_2} \\ &= c_1c_2\epsilon^{j(\phi_1+\phi_2)} = c_1c_2 \underline{/\phi_1 + \phi_2}. \end{aligned}$$

c is called the modulus or size of the vector, ϕ the argument or angle. Hence the rule "to multiply vectors, multiply the moduli and add the angles."

$$\frac{OP_1}{OP_2} = \frac{c_1\epsilon^{j\phi_1}}{c_2\epsilon^{j\phi_2}} = \frac{c_1}{c_2}\epsilon^{j(\phi_1-\phi_2)} = \frac{c_1}{c_2}\underline{/\phi_1 - \phi_2}.$$

The ratio of two vectors is obtained by dividing the moduli and subtracting the angles.

The products and ratios of rotating vectors are known as complex quantities.

As an example of the use of these rules, consider the motion of a mass m suspended by a long thread and attached by a spring s to a rigid support as shown in Fig. 90. Unital extension of the spring brings into play a restoring force s, and the thread is long enough and the stiffness great enough to make the effects of gravity negligible. If a force f acts on the mass which in consequence moves a distance x, the spring will exert an opposing force sx, there will be an opposing force equal to the mass multiplied by its acceleration, *i.e.* $= m\dfrac{d^2x}{dt^2}$, and there will be friction and losses in moving the air which are proportional to the velocity and constitute a third opposing force which may be written $r\dfrac{dx}{dt}$, where r is a coefficient of resistance. Thus the equation of motion may be written

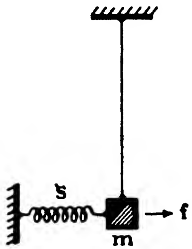


FIG. 90.—Mechanical Example.

$$m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + sx = f \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

It is, however, more often desired to know the velocity of the mass instead of its displacement. Writing $u = \dfrac{dx}{dt}$ for the velocity, equation 4 becomes

$$m\frac{du}{dt} + ru + s\int udt = f. \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Suppose now that the force f is a sinusoidally alternating one of angular frequency ω and amplitude $|F|$, and in consequence can be determined from a rotating vector expressible algebraically by $F\epsilon^{j\omega t}$. (F is written for the rotating vector and $|F|$ for its amplitude or modulus or size. But the vertical lines are often omitted when the context makes it clear that the size only is intended.) u also will vary sinusoidally, and may be expressed algebraically as $U\epsilon^{j\omega t}$, where U is to be found by substitution in (5). For the substitution

$$f = F\epsilon^{j\omega t}$$
$$u = U\epsilon^{j\omega t}$$
$$\frac{du}{dt} = j\omega U\epsilon^{j\omega t}$$

$$\int u\,dt = \frac{U}{j\omega}\,\epsilon^{j\omega t}$$

giving

$$j\omega m U\epsilon^{j\omega t} + rU\epsilon^{j\omega t} + \frac{sU}{j\omega}\,\epsilon^{j\omega t} = F\epsilon^{j\omega t}$$

or

$$U\left(j\omega m + r + \frac{s}{j\omega}\right) = F$$

or

$$U = \frac{F}{r + j\left(\omega m - \frac{s}{\omega}\right)} = \frac{F}{z} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where z is written for the complex quantity $r + j\left(\omega m - \frac{s}{\omega}\right)$. z can also be written $|z|\,\epsilon^{j\phi}$.

$$\tan\phi = \frac{\omega m - \frac{s}{\omega}}{r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The rotating vector representing the velocity of the mass under the impressed sinusoidal force can now be written, taking F as of phase angle zero; *i.e.*, drawing the rotating vector diagram at the instant when OF is horizontal, as shown in Fig. 91.

$$\left.\begin{aligned} U &= \frac{F\epsilon^{j\omega t}}{z} = \frac{|F|\,\epsilon^{j\omega t}}{|z|\,\epsilon^{j\phi}} \\ &= \frac{|F|}{|z|}\,\epsilon^{j(\omega t - \phi)} = |U|\,\epsilon^{j(\omega t - \phi)} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (8)$$

In drawing a rotating vector diagram (Fig. 91), therefore, to represent the vélocity u as well as the impressed force f, the vector U must lag behind the vector F by an angle ϕ determined by (7), and must be equal in length to

$$\frac{|F|}{|z|} = \frac{|F|}{\sqrt{r^2 + \left(\omega m - \frac{s}{\omega}\right)^2}} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

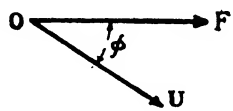


Fig. 91.—Vector diagram of vibromotive force and velocity.

f is known as the vibromotive force and z as the mechanical impedance of the system.

These terms are suggested by the analogous electrical problem of the circuit of Fig. 92. For the current i flowing through the inductance L resistance R and capacity C under an impressed electromotive force e the equation is

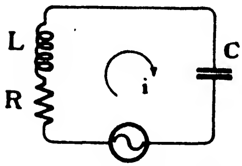


FIG. 92.—Analogous Electrical Circuit.

$$L\frac{di}{dt} + Ri + \frac{1}{C}\int idt = e \quad . \quad . \quad . \quad (10)$$

and this is exactly similar to (5), with L instead of m, R instead of r, $\frac{1}{C}$ instead of s, i instead of u and e instead of f. If then the electromotive force is alternating with sine wave form, so that e can be represented by $E\epsilon^{j\omega t}$,

$$\left.\begin{aligned} i &= \frac{E\epsilon^{j\omega t}}{Z} \\ \text{or } I &= \frac{E}{Z} \\ \text{where } Z &= R + j\left(\omega L - \frac{1}{\omega C}\right) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

or
$$\left.\begin{aligned} I &= |I|\epsilon^{j(\omega t - \phi)} \\ \text{where} \quad |I| &= \frac{|E|}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \\ \text{and} \quad \tan\phi &= \frac{\omega L - \frac{1}{\omega C}}{R} \end{aligned}\right\} \quad . \quad (12)$$

The complex quantity Z is called the electrical impedance, or more generally simply the impedance of the circuit, and the vector diagram will be similar to Fig. 91, with E instead of F and I instead of U.

The ratio of the rotating vector F (or E) to the rotating vector U (or I) is the complex impedance z (or Z), and it is clear that z alters both in size and angle as the frequency alters. If z is drawn as OP, Fig. 93, with different values of ω, all the points P lie in a vertical straight line drawn through R distance r from O. With ω very small, P is far below the axis, with ω very large P is far above the axis. As ω is gradually increased P travels up the line as indicated by the arrows. When

$$\omega m - \frac{s}{w} = 0 \quad \text{or} \ \omega = \sqrt{\frac{s}{m}} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

P coincides with R, the impedance is a minimum in size and has zero angle. This is known as the resonant condition, and

$$f_o = \frac{\omega_o}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{s}{m}}$$

as the resonant frequency. The velocity U is a maximum $\left(= \frac{F}{r}\right)$ and is in phase with the vibromotive force F.

The straight line is the locus of the impedance z with varying frequency. Let the vibromotive force be of standard phase, that is, of zero angle at the time considered, and hence represented by OF drawn along the axis, and let it be of constant amplitude though of varying frequency. The velocity for any frequency will then be obtained by dividing the vector OF by the corresponding complex quantity OP.

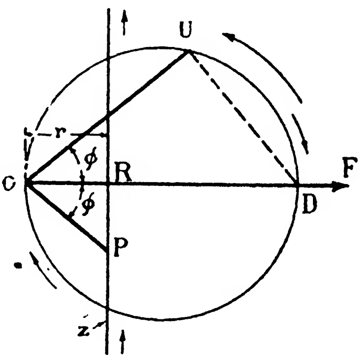


FIG. 93.—Circle Diagram of Particle Velocity.

$$U = \frac{F}{z} = \frac{F}{|z|\,\epsilon^{j\phi}} = \frac{F}{|z|}\,\epsilon^{-j\phi}.$$

The size of F is divided by the size of z, and a negative sign is applied to the angle. Thus OU, drawn above the axis with an angle UOF = FOP = ϕ, and of length = OF/OP, is the vector giving U for the frequency considered. If this construction is carried out for a number of points P lying on the straight line, the points U obtained will all be found to lie on a circle. This circle then is the locus of the velocity vector as ω is varied, and is described in the clockwise direction with ω increasing.

The proof that the locus is a circle is simple. Let OD be the diameter. Join DU. By construction $\tan\phi = \dfrac{\omega m - \frac{s}{\omega}}{r}$

$$\therefore \qquad \cos\phi = \frac{r}{\sqrt{r^2 + \left(\omega m - \frac{s}{\omega}\right)^2}}.$$

But also by construction

$$OU = \frac{F}{\sqrt{r^2 + \left(\omega m - \frac{s}{\omega}\right)^2}}$$

and $$OD = \frac{F}{r},$$

$$\therefore \qquad \frac{OU}{OD} = \frac{r}{\sqrt{r^2 + \left(\omega m - \frac{s}{\omega}\right)^2}} = \cos \phi.$$

Hence the angle OUD is a right angle and the locus is a circle.

The lengths of the vectors OU and the phase angles ϕ are plotted against ω in Fig. 94. The curve U shows the maximum velocity

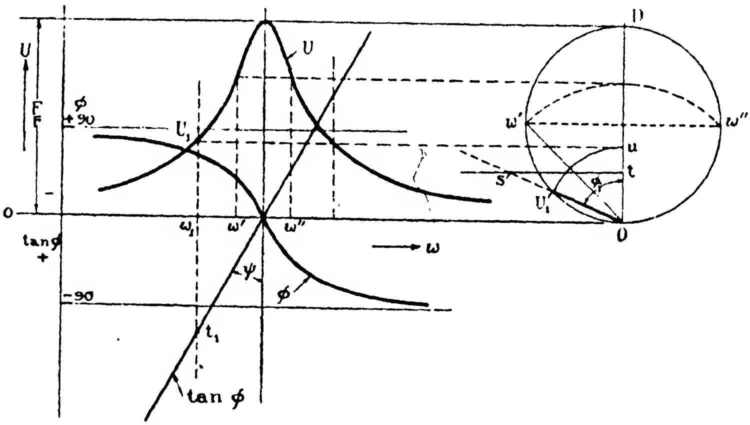


FIG. 94.—Graphical construction for determination of decay factor from resonance curve.

for each frequency, and is known as a resonance curve. The phase angle ϕ varies from $+ 90°$ when the frequency is very low through zero at resonance to $- 90°$ when the frequency is very high. It often happens that the maximum velocities can be measured experimentally, that is to say, the resonance curve can be obtained, but not the phase angles. In these cases the phase angles can be deduced from a knowledge of the circular locus of the velocity vector. For suppose the resonance curve determined experimentally is U as drawn in Fig. 94. A circle is drawn as shown with diameter OD equal to the height of the resonance curve. At any frequency ω_1, a vertical is drawn to meet the resonance curve in U_1, a horizontal is drawn through U_1 to meet the diameter OD in u, and with centre O and radius Ou an arc is drawn to meet the circle in U'_1. Then OU'_1 is evidently the velocity vector at the frequency $\omega_1/2\pi$, and the angle DOU'_1 is the required phase angle ϕ_1.

In the majority of resonances of interest the whole curve is described with only a small percentage variation in ω, and in these cases a very simple relation exists between ω and ϕ. In the case of the circle

$$\tan\phi = -\frac{\omega m - \dfrac{s}{\omega}}{r}.$$

Differentiating with respect to ω

$$\frac{d\tan\phi}{d\omega} = -\frac{m}{r} - \frac{s}{r\omega^2}$$
$$= -\frac{m}{r}\left(1 + \frac{s}{m}\cdot\frac{1}{\omega^2}\right)$$
$$= -\frac{m}{r}\left(1 + \frac{{\omega_0}^2}{\omega^2}\right).$$

by (13), where ω_0 is the resonant value.

When $\omega = \omega_0$, $\dfrac{d\tan\phi}{d\omega} \doteq -\dfrac{2m}{r}$. Also when the resonance is sharp ${\omega_0}^2/\omega^2$ is very nearly 1 over the whole range, and

$$\frac{d\tan\phi}{d\omega} = -\frac{2m}{r},$$

or
$$\frac{d\omega}{d\tan\phi} = -\frac{r}{2m} = -a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

a is called the decay factor of the system and is a quantity of considerable importance as determining the sharpness of the resonance.

Equation 14 gives a ready means of determining the decay factor from an experimentally determined resonance curve, by plotting the values of ω against the corresponding values of $\tan\phi$ and measuring the slope of the curve at resonance. This is most conveniently done as indicated in Fig. 94. The circle is drawn and a length Ot measured along the diameter OD equal to unity to any convenient scale. Then if OU'_1 or OU'_1 produced meets the horizontal through t in s', $s't$ is the tangent of the angle $U'_1OD = \tan\phi_1$, and this is measured off vertically from ω_1 to give a point t_1 on the required curve, which is practically a straight line in most cases. a is given as minus the slope of this curve at resonance, that is, by $-\tan\psi$.

This circle and straight line construction gives a useful check on the accuracy of the experimental results. When it is quite clear that the resonance curve is of the simple form described, the decay factor may be found more quickly. For when $\phi = \pm 45°$,

$$\tan\phi = \pm 1,\ \omega m - \frac{s}{\omega} = \pm 1 \text{ and } U = U_0/\sqrt{2} \text{ by (8) and (9),}$$

where U_0 is the resonance value of U. Draw a horizontal through the resonance curve at a height $= U_0/\sqrt{2}$, and drop verticals through the points of intersection to meet the ω axis in points ω' and ω''. Then from (14)

$$a = \frac{\omega'' - \omega'}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The corresponding ω points on the circle lie at the extremities of the diameter at right angles to OD, and are known as the quadrantal points.

Exactly similar considerations and constructions apply, of course, to the corresponding electrical case of Fig. 92. In all the above work steady state values only are intended. The transients taking place on the initial application of the vibromotive force or electromotive force are considered to have died out.

(32) Ball and Spring Model

An algebraic examination of the ball and spring model used to illustrate the propagation of sound waves in air is of interest in leading up to the similar electrical problem of the propagation of currents in networks and lines, as well as in providing an alternative idea of the propagation of sound waves through air.

Let, in Fig. 95, the terminal masses be $\frac{m}{2}$ and each of the remaining masses m, the stiffness of each spring be s and its mass negligible, let the movement of each mass be opposed by a mechanical resistance r ($r/2$ in the case of the terminal masses), and let the movements of the masses be x_0 x_1 x_2 . . . to the right as indicated under an impressed force f on the terminal mass. The effect of gravity is neglected.

The compression of the first spring is $x_o - x_1$ and the force brought into play $(x_o - x_1)\,s$. Hence for the motion of the terminal mass

$$f = \frac{m}{2}\frac{d^2x_o}{dt^2} + \frac{r}{2}\frac{dx_o}{dt} + (x_o - x_1)\,s.$$

The compression of the second spring is $x_1 - x_2$, producing a force $(x_1 - x_2)s$ to the left on mass 1. The compression of the first

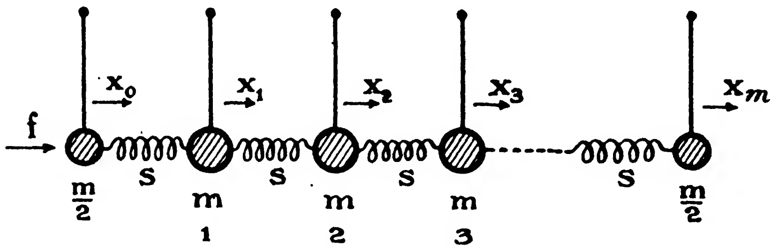


FIG. 95.—Ball and Spring Model.

spring produces a force to the right of $(x_0 - x_1)s$. The net force to the left is therefore $(x_1 - x_2)s - (x_o - x_1)s$, or $(2x_1 - x_o - x_2)s$.

The equation of motion of mass 1 is accordingly

$$0 = m\frac{d^2x_1}{dt^2} + r\frac{dx_1}{dt} + (2x_1 - x_0 - x_2)s$$

and so on.

For the nth mass

$$0 = m\frac{d^2x_n}{dt^2} + r\frac{dx_n}{dt} + (2x_n - x_{n-1} - x_{n+1})s,$$

and for the last

$$0 = \frac{m}{2}\frac{d^2x_m}{dt^2} + \frac{r}{2}\frac{dx_m}{dt} + (x_m - x_1)s$$

where there are m springs.

If the force f is sinusoidal and is determined by the rotating vector $F\epsilon^{j\omega t}$, the motions of all the masses will be sinusoidal and determined by vectors $X_o\epsilon^{j\omega t}$, $X_1\epsilon^{j\omega t}$ . . . $X_n\epsilon^{j\omega t}$.

And since $\frac{dX_n}{dt} = j\omega X_n \epsilon^{j\omega t}$ and $\frac{d^2X_n}{dt^2} = -\omega^2 X_2 \epsilon^{j\omega t}$ the equations may be written

$$\left.\begin{aligned}
F &= -\frac{\omega^2 m}{2} X_o + j\frac{\omega r}{2} X_o + (X_o - X_1)s \\
0 &= -\omega^2 m X_1 + j\omega r X_1 + (2X_1 - X_o - X_2)s \\
&\vdots \\
0 &= -\omega^2 m X_n + j\omega r X_n + (2X_n - X_{n-1} - X_{n+1})s \\
&\vdots \\
0 &= -\frac{\omega^2 m}{2} X_m + j\omega\frac{r}{2} X_m + (X_m - X_{m-1})s
\end{aligned}\right\} \quad (1)$$

Try $\quad X_n = A \cosh n\gamma + B \sinh n\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$
as a general solution for these equations.

That is, it is assumed that the steady state of the motion has been reached; the transients occurring when the vibromotive force is first applied have died away.

Substitution in the general equation gives

$$\begin{aligned}
0 &= (A \cosh n\gamma + B \sinh n\gamma)(-\omega^2 m + j\omega r + 2s) \\
&- sA(\cosh \overline{n-1}\gamma + \cosh \overline{n+1}\gamma) - sB(\sinh \overline{n-1}\gamma + \sinh \overline{n+1}\gamma \\
&= (A \cosh n\gamma + B \sinh n\gamma)(-\omega^2 m + j\omega r + 2s) \\
&- 2sA \cosh n\gamma \cosh \gamma - 2sB \sinh n\gamma \cosh \gamma
\end{aligned}$$

This must be true for all values of n, and hence the sinh and cosh coefficients may be equated to zero independently, yielding

$$-\omega^2 m + j\omega r + 2s = -2s \cosh \gamma$$

or
$$\cosh \gamma = 1 + \frac{-\omega^2 m + j\omega r}{2s} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

to determine γ.

The constants A and B depend upon the terminal conditions. If instead of consisting of m springs the system is infinitely long, when $n = 0$ in (2) $X_o = A$, and when $n = \infty$ in order that $X_n = 0$, $A = -B$, since as $n\gamma$ increases indefinitely sinh $n\gamma$ and cosh $n\gamma$ become more and more nearly equal.

For the infinitely long system therefore

$$\begin{aligned} X_n &= X_0 (\cosh n\gamma - \sinh n\gamma) \\ &= X_0 \epsilon^{-n\gamma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4) \end{aligned}$$

γ is thus shown to be a propagation constant of the wave, as it determines the displacement rotating vector at any point along the system in terms of that at the end at which the disturbance is initiated.

It is clear from (3) that in general γ will be complex, and may be written

$$\gamma = a + j\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Using this in (4)

$$\begin{aligned} X_n &= X_0 \epsilon^{-(a+j\beta)n} \\ &= X_0 \epsilon^{-an} . \epsilon^{-j\beta n}. \end{aligned}$$

a therefore determines the decrease of the size of the displacement vector along the system, and β determines its phase angle in relation to that of the sending end displacement vector.

In a wavelength the phase angle changes by 2π, and the number of sections n_λ to a wave length is therefore

$$n_\lambda = \frac{2\pi}{\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The velocity of the wave is its wavelength multiplied by its frequency, or

$$c = n_\lambda \times f = \frac{2\pi f}{\beta} = \frac{\omega}{\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If instantaneous values are required, the time element is introduced again by multiplying by $\epsilon^{j\omega t}$ to give

$$\begin{aligned} X_n \epsilon^{j\omega t} &= X_0 \epsilon^{-an} . \epsilon^{-j\beta n} . \epsilon^{j\omega t} \\ &= X_0 \epsilon^{-an} . \epsilon^{j(\omega t - \beta n)} \end{aligned}$$

If the displacement at the sending end is $x_0 = X_0 \cos \omega t$, *i.e.*, if the rotating vector is projected on to the horizontal axis, that at the nth mass is

$$x_n = X_0 \epsilon^{-an} . \cos (\omega t - \beta n)$$

Exactly similar equations hold if the mass velocities are considered instead of the mass displacements. For

$$u_n = \frac{dx_n}{dt} = \frac{d}{dt} X_n \epsilon^{j\omega t} = j\omega X_n \epsilon^{j\omega t}.$$

$\therefore \quad U_n = j\omega X_n$. The velocity vectors lead the displacement vectors by 90° and are ω times their size.

To find the displacements and velocities in terms of the impressed force, substitute from (4) in the first equation of (1) to obtain

$$F_o = -\omega^2\frac{m}{2}X_o + j\omega\frac{r}{2}X_o + (X_o - X_o\epsilon^{-\gamma})s$$

Using (3) this becomes

$$\begin{aligned} F_o &= X_o s(\cosh\gamma - \epsilon^{-\gamma}) \\ &= X_o s \sinh\gamma \end{aligned}$$

Writing $U_o = j\omega X_o$

$$U_o = \frac{F_o}{\dfrac{s\sinh\gamma}{j\omega}} = \frac{F_o}{z_o} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where
$$z_o = \frac{s\sinh\gamma}{j\omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

is a mechanical impedance determining the sending end mass velocity produced by a given vibromotive force.

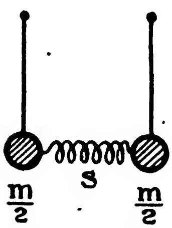


FIG. 96.—Link of Chain.

This mechanical system may be looked upon as a chain, each link of which comprises two balls, each of mass $\frac{m}{2}$, connected by a spring of stiffness s as shown in Fig. 96. In forming the chain of Fig. 95 from the links of Fig. 96, the balls are rigidly connected to form the masses m. The force F_n acting to the right on the nth link will then be given by

$$\begin{aligned} F_n &= (X_n - X_{n-1})s - \frac{1}{2}\omega^2 m X_n + j\omega\frac{r}{2}X_n \\ &= \left\{(1 - \epsilon^{-\gamma})s - \frac{1}{2}\omega^2 m + j\omega\frac{r}{2}\right\}X_n \\ &= (\cosh\gamma - \epsilon^{-\gamma})sX_n = s\sinh\gamma\, X_n \end{aligned}$$

by (3).

Hence
$$\frac{F_n}{U_n} = \frac{F_n}{j\omega X_n} = \frac{s\sinh\gamma}{j\omega} = z_o \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and
$$F_n = z_o U_n = z_o U_o \epsilon^{-n\gamma} = F_o \epsilon^{-n\gamma} \quad . \quad . \quad . \quad (11)$$

where F_o is written for the impressed vibromotive force. It is seen that the vibromotive force and the ball velocity vary along the infinitely long chain according to the same law, and that the ratio of the two at any link is the complex mechanical impedance z_0.

The analogous electrical circuit is drawn in Fig. 97 and will be considered more fully later in connection with filters. The similarity of the equations above with those developed in section 11 for the artificial leaky line is to be noted. The treatment of the

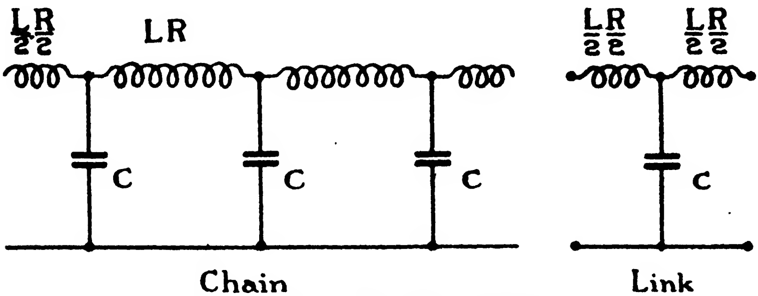


FIG. 97.—Analogous Electrical Circuit.

electrical case is extended to the case of a finite number of links in section 40, and enough has been written to enable the results obtained there to be applied direct to the corresponding mechanical case.

If, while keeping the total mass and spring in the system the same, the balls are made smaller and smaller and correspondingly more numerous, γ will be a small quantity. Equation (3) may be written

$$1 + \frac{\gamma^2}{2} = 1 + \frac{-\omega^2 m + j\omega r}{2s},$$

whence

$$\gamma = \sqrt{\frac{-\omega^2 m + j\omega r}{s}}.$$

If m, r and s now refer to unit length of the system, $n\gamma$ must be replaced by Px in the above equations, where x is the length of the system from the sending end and P is the propagation constant per unit length.

If, further, an ideal system is considered with no dissipative forces, so that $r = 0$,

$$\mathrm{P} = \alpha + j\beta = \sqrt{-\frac{\omega^2 m}{s}} = j\omega\sqrt{\frac{m}{s}}.$$

Hence $\qquad \alpha = 0$

and $\qquad \beta = \omega\sqrt{\dfrac{m}{s}} \qquad \ldots \ldots \ldots (12)$

The velocity of propagation becomes from (7)

$$c = \frac{\omega}{\beta} = \sqrt{\frac{s}{m}} \qquad \ldots \ldots \ldots (13)$$

and the characteristic impedance from (9)

$$z_0 = \frac{s \sinh \gamma}{j\omega} = \frac{sj\beta}{j\omega} = \sqrt{ms} \qquad \ldots \ldots \ldots (14)$$

Such a system is found in the imaginary tube in a plane sound wave. If the cross section of the tube is S, the mass per unit length is ρ_0S, and the stiffness is $Se = S\gamma_0 p_0$ by equation 29·02.

Hence by substitution in (13)

$$c = \sqrt{\frac{S\gamma_0 p_0}{S\rho_0}} = \sqrt{\frac{\gamma_0 p_0}{\rho_0}}$$

in agreement with (29·07)

and $\qquad z_0 = S\sqrt{\rho_0\gamma_0 p_0}$

where z_0 can be looked upon as the *acoustical impedance* in a plane sound wave.

The use of this acoustical impedance to determine the particle velocity u in terms of the impressed force $f = p$S, gives

$$u = \frac{f}{z_0} = \frac{pS}{S\sqrt{\rho_0\gamma_0 p_0}} = \frac{p}{\sqrt{\rho_0\gamma_0 p_0}}$$

in agreement with (29·10).

REFERENCES FOR FURTHER READING

Sir J. A. Fleming.—" Problems in Telephony, Solved and Unsolved." *J.I.E.E.*, Vol. LXI., p. 613.

Sir Richard Paget, Bart.—" The Nature and Reproduction of Speech Sounds " (Vowels). *J.I.E.E.*, Vol. LXII., p. 963.

Sir Oliver Lodge.—" The History and Development of the Telephone." *J.I.E.E.*, Vol. LXIV., p. 1098.

D. C. Miller.—" Science of Musical Sounds."

H. Lamb.—" The Dynamical Theory of Sound."

E. Mallett.—" The Determination of Resonant Frequencies and Decay Factors." *J.I.E.E.*, Vol. LXII., p. 517.

CHAPTER VII

SPEECH APPARATUS

(33) The Microphone

THREE main steps may be distinguished in the reproduction of sound at the distant end of a telephone line. They are :—

(1) The production at the sending end of electrical voltages of wave form as nearly as possible the same as the wave form of the sound to be reproduced, or rather having the same constituent frequencies with as nearly as possible the same relative amplitudes (relative phases do not matter).

(2) The propagation of currents due to these voltages along the telephone line preserving as nearly as possible the relative amplitudes of the constituent frequencies.

(3) The production of sound waves by causing the currents arriving at the receiving end to flow through suitable apparatus.

The apparatus used in the production of the electrical voltages from sound waves is known as a microphone or transmitter, and that for the production of sound waves from electrical currents as a receiver or loud speaker. The one corresponds to the key in telegraphy and the other to the sounder.

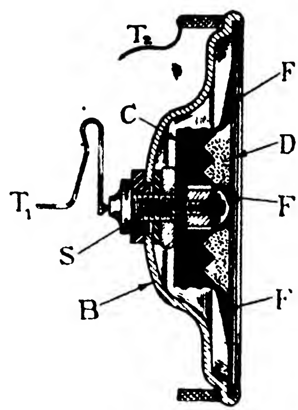


FIG. 98.—Inset Microphone.

Various different principles of microphone action have been tried from time to time, but the one in universal use for commercial telephony depends upon the change of resistance of loose granules of carbon when they are vibrated. There are two forms extensively used, the " inset " and the " solid back," shown in section in Figs. 98 and 99 respectively.

The inset microphone (Fig. 98) consists of a case B with an insulated screw S carrying a carbon electrode C, the front face of which is cut to form pyramid-shaped projections. The carbon diaphragm D is held round its circumference in the case B, and the active granules are loosely held between the diaphragm and the electrode by the pieces of flannel F. Current entering at the terminal T_2 flows through the diaphragm D, through the granules and the electrode C to the terminal T_1. The microphone is " inset " in a robust iron case with mouthpiece mounted on a pedestal, or in the case of a hand micro-telephone. It is used in local battery circuits (see Chapter X.).

The " solid back " is a far more robustly made instrument and is used in common battery circuits. The carbon granules are contained in a small brass box A, the front covering of which is a flexible mica disc H carrying a polished carbon electrode C_2, through which the current enters the granules from the terminal T_1. The current leaves by a second carbon electrode C_1 at the back of the box and the terminal T_2. The box is very firmly held in the heavy framework B, which also carries the face F with the mouthpiece M. The aluminium diaphragm D is mounted in a rubber ring and held firmly against the back of the face F by rubber protected springs S. The front electrode C_2 is rigidly clamped to the diaphragm by the screw and nut G. The instrument is mounted on a pedestal which also carries the receiver on a switch hook.

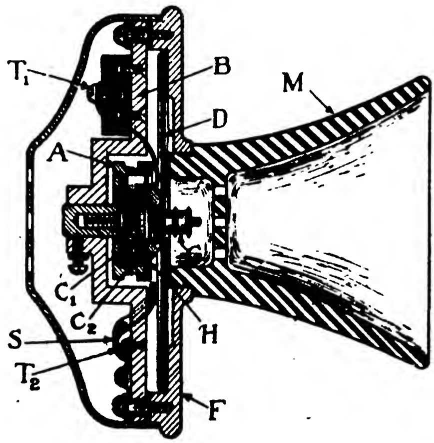


FIG. 99.—Solid Back Microphone.

Various theories have been advanced for the microphone action, such as the existence of minute arcs between the carbon surfaces, a silent discharge through an air film at the surfaces, a variation of the internal resistance of the carbon, and a variation of the areas of contact with pressure. In any case, what happens is that in some way oı other the resistance of the microphone changes as the sound waves fall upon the diaphragm.

In Fig. 100 let r be the resistance at any instant of the microphone M, R the remaining resistance in the circuit and E the electromotive force of the battery. Suppose the resistance r varies sinusoidally with an impressed sinusoidal pressure on the diaphragm, and that the variation of resistance can be expressed by the equation

$$r = r_0(1 + k \sin \omega t) \quad . \quad . \quad . \quad . \quad (1)$$

k is a coefficient (which must be less than unity), depending upon the alternating air pressure and upon the granules, and determining the maximum alteration of resistance. It may be termed a modulation coefficient. r_0 is the mean resistance of the microphone.

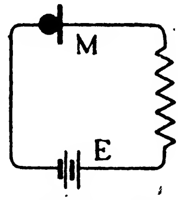


FIG. 100.—Microphone Theory.

The current flowing is evidently

$$I = \frac{E}{R + r_0(1 + k \sin \omega t)}$$

$$= \frac{E}{(R + r_0)\left\{1 + \frac{r_0 k \sin \omega t}{R + r_0}\right\}}$$

which expanded by the binomial theorem gives

$$I = \frac{E}{R + r_0}\left\{1 - \frac{r_0}{R + r_0} k \sin \omega t + \left(\frac{r_0}{R + r_0}\right)^2 k^2 \sin^2 \omega t - \right\} \quad . \quad (2)$$

The mean current flowing is

$$I_0 = \frac{E}{R + r_0},$$

and the alternating ripple superposed on this is $I - I_0$, *i.e.*

$$i = -\frac{I_0 r_0}{R + r_0} k \sin \omega t + I_0 \left(\frac{r_0}{R + r_0}\right)^2 k^2 \sin^2 \omega t - \quad . \quad (3)$$

The first term is the alternating current required, the second and remaining terms involve distortion by the introduction of currents of higher frequencies than that of the sound wave. For $\sin^2 \omega t = \frac{1}{2} - \frac{1}{2} \cos 2\omega t$, and the second term is

$$\frac{1}{2} I_0 \left(\frac{r_0}{R + r_0}\right)^2 - \frac{I_0}{2}\left(\frac{r_0 k}{R + r_0}\right)^2 . \cos 2\omega t \quad . \quad . \quad . \quad (4)$$

The first term of this expression gives an increase of the steady current flowing which does not matter, but the second term gives a current of double frequency. Similarly the terms of higher powers of sin ωt in (3) give currents of still higher frequencies.

In order to make these distorting currents small it is necessary to make $r_0 k$ small in comparison with $R + r_0$, a condition which also makes the amplitude of the useful current small. Design is therefore a compromise between sensitivity and distortion.

The distortion noticed with a single tone is harmonic, and would not produce unpleasant effects. But suppose two tones are being impressed on the diaphragm so that $k \sin \omega t$ is replaced by $(k_1 \sin \omega_1 t + k_2 \sin \omega_2 t)$, and $k^2 \sin^2 \omega t$ in the second term of (3) becomes

$$(k_1 \sin \omega_1 t + k_2 \sin \omega_2 t)^2 = k_1^2 \sin^2 \omega_1 t + k_2^2 \sin_2 \omega_2 t + 2k_1 k_2 \sin \omega_1 t \sin \omega_2 t \quad . \quad . \quad (5)$$

The first two terms on the right produce harmonic distortion as in (4); the third may be written

$$k_1 k_2 \left\{ \cos \overline{\omega_1 - \omega_2}\, t - \cos \overline{\omega_1 + \omega_2}\, t \right\} \quad . \quad . \quad . \quad (6)$$

Currents leading to difference and summation tones of frequencies $(\omega_1 - \omega_2)/2\pi$ and $(\omega_1 + \omega_2)/2\pi$ are introduced, and the effects may be very unpleasant indeed.

Assuming, then, that by suitable arrangements the distortion is kept low, the first term of (3) is the resulting alternating current, and it may be written (omitting the $-^{ve}$ sign as determining phase only)

$$i = \frac{I_0 r_0 k \sin \omega t}{R + r_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$$= \frac{\text{alternating potential difference at microphone terminals}}{\text{total resistance of microphone circuit}} \quad . \quad (8)$$

The microphone, when supplied by a suitable direct current and when a sound wave of given frequency falls on its diaphragm, may thus be looked upon as a source of alternating electromotive force e, the source having an internal resistance equal to the average resistance r_0 of the microphone.

The alternating power absorbed in the resistance R is

$$W = i^2 R = \frac{e^2 R}{(R + r_0)^2}.$$

This is a maximum when $\frac{dW}{dR} = 0$, or when

$$\frac{1}{(R + r_o)^2} - \frac{2R}{(R + r_o)^3} = 0$$

i.e., when $$R = r_o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The primary of a telephone transformer or induction coil is actually connected in the circuit instead of the resistance R, and the secondary of the induction coil is connected to line (Fig. 101). If the effective resistance to alternating currents of the primary is R and its inductance is L, and its direct current resistance is R_o, then

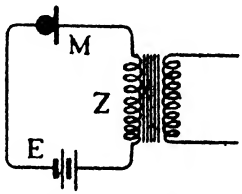


FIG. 101.—Simple Microphone Circuit.

$$I_o = \frac{E}{R_o + r_o},$$

but it may be inferred from (8) that

$$i = \frac{e}{r_o + R + j\omega L} = \frac{e}{r_o + Z} \quad . \quad . \quad . \quad . \quad (10)$$

and the distortional higher harmonics are more effectively suppressed than with a pure resistance load, as the impedance of the microphone circuit will be increased with the frequency.

The amplitude of the microphone e.m.f. would appear from (7) to increase directly with the steady current I_o, and experiment shows this to be true to some extent, as indicated by the results given in Fig. 102, in which the alternating current voltage on the secondary of the induction coil is plotted against the direct current through a solid back microphone when excited by a constant tone from a telephone receiver. But a limit to the permissible increase in I_o is soon reached by the heating of the granules and the formation of small arcs between them, resulting in the production of noises in the telephone receiver known as "frying." The microphone is a somewhat unstable device, and it is seldom that the curve of Fig. 102 is repeated exactly on a second attempt.

Various tests have been made on the carbon microphone, but the complete theory has yet to be written. The physical action of the granules is by no means clear, and the microphone diaphragm of the solid back, with its rubber cushioning and damping springs, is a very complicated mechanical device, whose response to tones of different frequencies is very different, and the problem is still further complicated by the mouthpiece. The extent to which the microphone voltage varies with frequency is well shown in Fig. 103,* in which the relative volts output for constant acoustical pressure

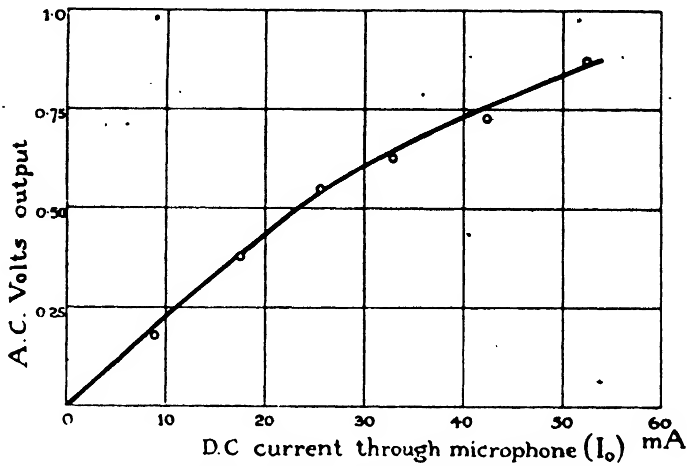


FIG. 102.—Effect of steady current through microphone.

on the diaphragm is plotted against the frequency. It might be thought that such a response curve would hardly give com-

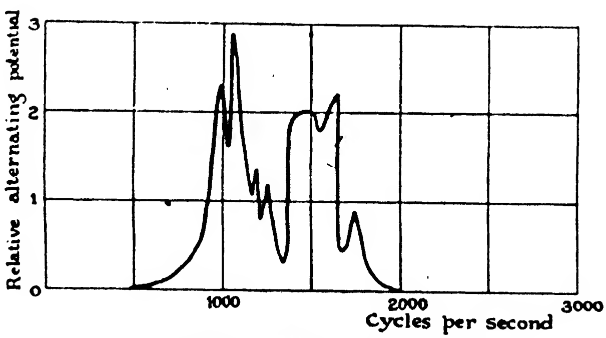


FIG. 103.—Variation of sensitivity with frequency.

mercial speech; it certainly will not give sufficiently faithful reproduction for use in broadcasting music, and for this purpose other means are adopted.

* B. S. Cohen, *J.Inst.E.E.*, Vol. 66, p. 184.

(34) The Telephone Receiver

The usual telephone receiver or Bell receiver is shown in section in Fig. 104. It consists essentially of a powerful elongated horse-

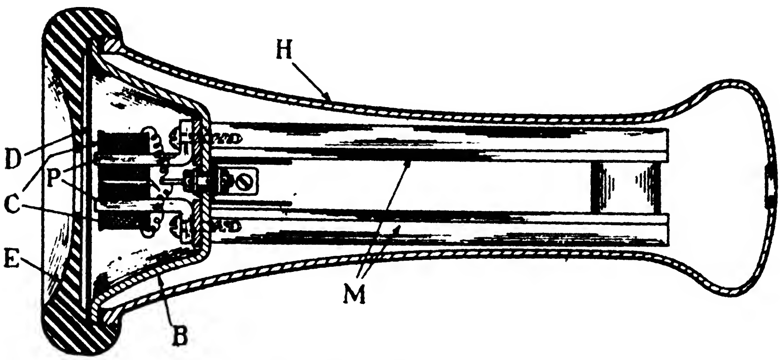


Fig. 104.—Bell Receiver.

shoe magnet M with two pole pieces P carrying coils C, and a diaphragm D rigidly clamped to the cup B by the ebonite earpiece E. Two small airgaps are thus formed between the diaphragm and the pole shoes, and the flux path of the flux due to the permanent is across the two gaps in series. The coils are so wound that the flux due to a current through them also passes across the two gaps in series, thus either helping or opposing the permanent flux in each gap according to the direction of the current. The whole receiver is mounted in a holding case H which is ebonite covered and serves as a handle by which the earpiece is held closely against the ear.

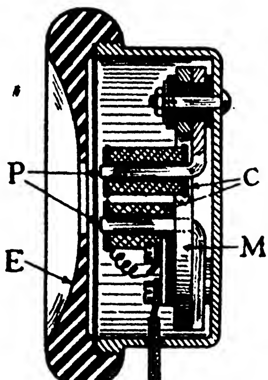


Fig. 105.—Watch Case Receiver.

Another pattern, used very largely in wireless telephony and telegraphy, is the watch receiver shown in Fig. 105, in which the permanent magnet is of ring shape magnetised across a diameter. Otherwise the construction is similar to that of Fig. 104, and the lettering corresponds. Two of these receivers are generally mounted on a steel band which fits on the head and holds one receiver against each ear.

The theory of the telephone receiver is concerned mainly with the amplitude of the vibrations produced in the diaphragm by a given alternating current through the coils. As in use the receiver is held closely against the ear, no travelling sound wave is normally produced, but the alterations of air pressure in the cavity between the diaphragm and the eardrum affect the latter directly.

Since the equations developed must necessarily involve both electrical and mechanical quantities, confusion of units is best avoided by measuring the former in absolute electromagnetic units in the C.G.S. system, and the latter in the C.G.S. system. Thus I is the current rotating vector in absolute units, called abamperes by Professor A. E. Kennelly—to whom the theory of the telephone receiver is mainly due—and is taken as of standard phase. The resistance of the coils R is similarly measured in abohms and the voltage across them V in abvolts.

If B_o is the flux density in the air gaps due to the permanent magnet and b is that due to the current, then the total pull on the diaphragm is given by

$$\frac{2S}{8\pi}(B_o + b)^2,$$

where S is the cross-sectional area of one pole, and the 2 allows for the two poles. If $b = B \sin \omega t$ this becomes

$$\text{Pull} = \frac{2S}{8\pi}\left\{B_o^2 + 2B_oB \sin \omega t + B^2 \sin^2 \omega t\right\} \quad . \quad . \quad . \quad (1)$$

The first term is the normal steady pull, the second the alternating pull or vibromotive force of the frequency that is required, and the third introduces harmonic distortion in the same way as was noted in section 33 in the microphone theory. If, moreover, two or more alternating fluxes of different frequencies are acting together, sum and difference tones will be produced according to the third term as in the microphone case. It is necessary, therefore, to keep B^2 small in comparison with B_oB—overloading the receiver must produce distortion. The larger the value of B_o, that is, the more powerful the permanent magnet, the greater is the sensitivity of the receiver, as is seen from the second term; a limit is, however, reached when the steady pull of the first term causes the diaphragm to buckle on to the poles. The magnetic system of the receiver is very similar to that of the polarised relay or sounder described in sections 3 and 5.

Assuming, then, that the distortion term can be neglected, the vibromotive force is given from (1) by

$$F = \frac{1}{2}\frac{S}{\pi} B_o B \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where F and B are rotating vectors.

If R is the reluctance of the magnetic circuit to alternating fluxes, and N the total number of turns on the two coils, then the magnetomotive force is $4\pi NI$ and the total flux in the air gaps is

$$BS = \frac{4\pi NI}{R}$$

and

$$F = \frac{2B_o N}{R} I = AI \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where

$$A = \frac{2B_o N}{R} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and is a " force factor " determining the vibromotive force F dynes acting on the diaphragm produced by a current I abamps.

The diaphragm is a somewhat complicated mechanical system with distributed mass and stiffness, and is capable of resonant vibrations with an indefinitely large number of different frequencies of the applied force. At the lowest resonant frequency each point of the diaphragm is moving in the same direction at the same time. This is called the fundamental mode of vibration. But at the higher resonant frequencies at the instant that some points of the diaphragm move in one direction others move in the reverse direction, and nodes are formed where there is no movement. The nodes take the form of lines and circles and may be delineated by sand pictures.*

For frequencies round about the resonant frequency of each mode the diaphragm may, with regard to its movement at the centre where the vibromotive force is applied, be considered as a simple mechanical system as in Fig. 90, with an equivalent mass m grams, an equivalent stiffness s dynes per centimetre, and an equivalent mechanical resistance r dynes per centimetre per second, which will include the resistance due to the useful air movements produced. These considerations lead to equations

$$U = \frac{F}{r + j\left(\omega m - \frac{s}{\omega}\right)} = \frac{A}{z} I \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

* J. T. MacGregor-Morris and E. Mallett, *J.I.E.E.*, Vol. 61, p. 1134.

for the determination of the velocity of the diaphragm, exactly as in the case of the simple system in section 31. There will be a resonance curve of the velocity derived from a circle for each mode of vibration, and the response of the receiver will vary largely with different frequencies.

The diaphragm vibrating in front of the magnet produces an alteration of the flux due to the magnet through the coils, and a consequent back electromotive force which can be looked upon as modifying the impedance of the coils. R being the reluctance of the path of the varying flux through the coils, a movement x of the diaphragm towards the poles will reduce the reluctance by x/S in each gap, the reluctance will accordingly be $R - 2x/\mathrm{S}$, and the total flux Φ will be given by

$$\Phi = \mathrm{B_oS}\left(\frac{R}{R - \frac{2x}{\mathrm{S}}}\right)$$

$$= \mathrm{B_oS}\left(1 + \frac{2x}{R\mathrm{S}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

(by the binomial theorem) for small movements. The back e.m.f. E is accordingly

$$\mathrm{E} = \mathrm{N}\frac{d\Phi}{dt} = \frac{2\mathrm{B_oN}}{R} \cdot \frac{dx}{dt}$$

$$= \mathrm{AU} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

If V is the potential across the receiver coils and Z_d is the impedance of the coils with the diaphragm quiescent or "damped," then

$$\mathrm{V} = \mathrm{Z}_d\mathrm{I} + \mathrm{E}$$

$$= \mathrm{Z}_d\mathrm{I} + \mathrm{AU} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$= \mathrm{Z}_d\mathrm{I} + \frac{\mathrm{A}^2}{z}\mathrm{I} \qquad \text{by (5)}$$

$$= \left(\mathrm{Z}_d + \frac{\mathrm{A}^2}{z}\right)\mathrm{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

The effective impedance of the receiver is accordingly increased by an amount $\frac{\mathrm{A}^2}{z}$ due to the motion of the diaphragm, and this has been called the "motional impedance" Z_m by Kennelly. The total impedance Z of the receiver is accordingly

$$\mathrm{Z} = \mathrm{Z}_d + \mathrm{Z}_m \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Z_m can readily be determined experimentally at any frequency by making bridge measurements of the complex impedance of the receiver with the diaphragm free (Z) and with the diaphragm prevented from vibrating (Z_d) and subtracting the two. The result plotted vectorially should give a circle (see section 31), with the diameter Od through the origin horizontal as shown dotted in Fig. 106. The actual result is very approximately a circle, but with the diameter OD depressed below the horizontal by an angle XOD. The diameter of the circle is $\frac{A^2}{r}$, and it would thus appear that A must be a complex quantity with an angle θ determined by $2\theta = X\hat{O}D$. Referring to equation (3), this means that the vibromotive force lags behind the current by an angle θ, because (equation (4)) the reluctance must be looked upon as complex with a positive angle θ, and the current produced flux lags behind the current by an angle θ.

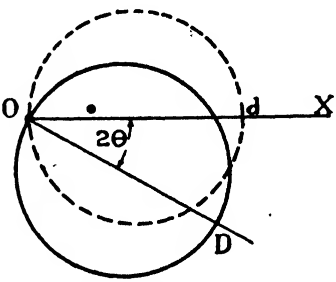


Fig. 106.—Motional Impedance Circle.

This may be explained in part by hysteresis and in part by eddy currents in the pole shoes and metal on the face of the formers on which the coils are wound. Equations 8 and 5 are

$$\left.\begin{aligned} V &= Z_d I + AU \\ 0 &= -zU + AI \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and these may usefully be compared with the equations

$$\left.\begin{aligned} V &= Z_1 I_1 + j\omega M I_2 \\ 0 &= Z_2 I_2 + j\omega M I_1 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

for the electrical circuit of Fig. 107.

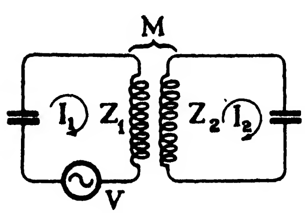


Fig. 107.—Equivalent Electrical Circuit.

The eddy currents and hysteresis losses may be looked upon as occurring in a closed circuit of impedance Z_2 coupled with the receiver coils by mutual inductance M and with the diaphragm by a real force factor b, the force factor of the receiver coils being a real quantity a. If the current in the coil is I_1 and that in the eddy current circuit is I_2, and Z_1 is written instead of Z_d for the impedance of the coils, the equations are

$$\left.\begin{aligned} V &= Z_1 I_1 + j\omega M I_2 + aU \\ 0 &= Z_2 I_2 + j\omega M I_1 + bU \\ 0 &= -zU + aI_1 + bI_2 \end{aligned}\right\} \quad \ldots\ldots (13)$$

Using the second to eliminate I_2 from the first and third yields

$$\left.\begin{aligned} V &= \left(Z_1 + \frac{\omega^2 M^2}{Z_2}\right) I_1 + \left(a - \frac{j\omega M}{Z_2} b\right) U \\ 0 &= -\left(z + \frac{b^2}{Z_2}\right) U + \left(a - \frac{j\omega M}{Z_2} b\right) I_1 \end{aligned}\right\} \quad \ldots (14)$$

Comparing (14) with (11) it is seen that the impedance of the coils, the mechanical impedance of the diaphragm and the force factor are all modified by the presence of the eddy current circuit. In all the above equations up to (11), therefore,

$$Z_d = Z_1 + \frac{\omega^2 M^2}{Z_2}$$

$$z \text{ should be } z + \frac{b^2}{Z_2}$$

and

$$A = a - \frac{j\omega M b}{Z_2} = |A| \overline{\diagdown \theta}$$

Another modification remains to be noticed. It has been seen how the alterations of flux brought about by the vibration of the diaphragm bring into play a back e.m.f., but these alterations also cause an additional vibromotive force F_x to act on the diaphragm. The actual alteration of flux density is from (6)

$$\frac{2B_o x}{RS}$$

and this inserted instead of B in equation (2) gives

$$F_x = \frac{1}{2\pi} \frac{B_o^2}{R} . 2X \quad \ldots\ldots (15)$$

which can be put into the form

$$F_x = pAX \quad \ldots\ldots (16)$$

where $A = \frac{2B_o N}{R}$ as before,

and $$p = \frac{A}{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

where L is the inductance (complex) of the coils given by

$$L = \frac{4\pi N^2}{R}$$

The equation of motion of the equivalent simple system must now be written

$$-\omega^2 mX + j\omega rX + sX = AI + pAX \quad . \quad . \quad . \quad (18)$$

instead of equation (4) of section 31, and writing $A = a_1 - ja_2$, this becomes

$$-\omega^2 mX + j\omega\left(r + \frac{pa_2}{\omega}\right)X + (s - pa_2)X = AI$$

whence

$$U = \frac{A}{r + r' + j\left(\omega m - \frac{s - s'}{\omega}\right)} I \quad . \quad . \quad . \quad . \quad (19)$$

with $$r' = \frac{pa_1}{\omega} = \frac{p|A|}{\omega}\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

$$s' = pa_2 = p|A|\cos\theta \quad . \quad . \quad . \quad . \quad . \quad (21)$$

The "r" and "s" of the circle will accordingly be altered to

$$\left.\begin{aligned} r'' &= r + r' \\ \text{and} \quad s'' &= s + s' \text{ respectively} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (22)$$

The fact that r'' varies with the frequency will distort the circle somewhat, but the distortion will not be serious as the major part of the circle is usually described with only a small percentage change of frequency.

The total mechanical power put into the diaphragm at resonance is $U_0^2 r''$, where U_0 is the root mean square value of the diaphragm velocity at resonance. Of this $U_0^2 r'$ is used up in electric and magnetic losses due to hysteresis and eddy currents, and $U_0^2 r$ is used in vibrating the diaphragm.

The electrical input power at resonance is $I^2 R_0$, where R_0 is the resistance of the receiver windings at resonance with the diaphragm free, and the gross mechanical efficiency is

$$\eta_1 = \frac{\text{total mechanical power}}{\text{electrical input power}}$$

$$= \frac{U_o^2 r''}{I^2 R_o} = \frac{(|A|^2/r'')I^2}{I^2 R_o} \quad \text{by (5)}$$

$$= \frac{\overline{OD}}{R_o} \quad \text{by (9)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

where $\overline{OD}$ is the diameter of the motional impedance circle.

The mechanical efficiency of the diaphragm is

$$\eta_2 = \frac{\text{power used in vibrating diaphragm}}{\text{total power input to diaphragm}}$$

and this at resonance is

$$\eta_2 = \frac{U_o^2 r}{U_o^2 r''} = \frac{r}{r + r'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The net mechanical efficiency is

$$\eta'_1 = \eta_1 \eta_2 = \frac{\overline{OD}}{R_o} \cdot \frac{r}{r + r'} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

but this is not the overall electrical acoustical efficiency, as most of the net power used in vibrating the diaphragm is used in internal mechanical friction.

For the determination of the four receiver constants A, r'', m and s'' three relations can be obtained from the experimentally drawn motional impedance circle.

The angle of A is half the angle of XOD.

$$OD = \frac{|A|^2}{r''}$$

$$\omega_o = \sqrt{\frac{s''}{m}}$$

and ω_o is determined by the value of ω at D.

$$a = \frac{r''}{2m}$$

and a is determined from the quadrantal frequencies of the circle or by the slope of the tan a/ω curve at resonance (see section 31).

Kennelly obtained a necessary fourth relation by measuring, by means of a tiny mirror at the centre of the diaphragm, the amplitude $|X_o|$ at resonance with a current of amplitude I., then from (4)

$$U = \omega X_o = \frac{|A|}{r} I$$

Finally, r'' and s'' are split up by using the inductance L and the force factor A to find p from (17), and finding r' and s' from (20) and (21).

As an example, the following figures from one of Kennelly's experiments on a standard 60-ohm receiver are used to calculate the various constants :—

Decay factor = 149.
Resonant frequency = 1015.
Motional impedance at resonance = 140 ohms.
Angle of depression of motional impedance circle = 50·6°.
Maximum velocity of diaphragm at resonance with current of 2·04 milliamperes (r.m.s.) through receiver coils = 6·6 cm./sec.
Inductance at resonance = 36·5 millihenries.
Resistance at resonance = 257·3 ohms.

From these results the following identities may be written down :—

$$a = \frac{r''}{2m} = 149 \quad \text{. . . . .} \quad (a)$$

$$\omega_o = \sqrt{\frac{s''}{m}} = 2\pi \times 1015 \quad \text{. . . . .} \quad (b)$$

$$\overline{OD} = \frac{|A|^2}{r''} = 140 \times 10^9 \text{ abohms} \quad \text{. . . . . .} \quad (c)$$

$$U_o = \frac{AI}{r''}$$

$$\therefore \frac{A}{r''} = \frac{U_o}{I} = \frac{6{\cdot}6}{2{\cdot}04 \times \sqrt{2} \times 10^{-4}} \left(\frac{\text{max. cm./sec.}}{\text{max. abamps}}\right) \quad . \quad (d)$$

$(c)/(d)$ gives $|A| = 6{\cdot}12 \times 10^6$ dynes/abamp. . . . . (e)

angle of $A = \frac{1}{2}$ angle of depression of circle $= -25{\cdot}3°$.

(e) in (c) gives $r'' = 268$ dynes/cm./sec. . . . . . . (f)

(f) in (a) gives $m = 0{\cdot}902$ gm. . . . . (g)

and (g) in (b) gives $s'' = 36{\cdot}7 \times 10^6$ dynes/cm. . . . . . (h)

From (17), $$p = \frac{A}{L} = \frac{6{\cdot}12 \times 10^6}{36{\cdot}5 \times 10^6} = 0{\cdot}168$$

and from (20)

$$r' = \frac{0{\cdot}168 \times 6{\cdot}12 \times 10^6}{2\pi \times 1015} \times \sin 25{\cdot}3^\circ$$
$$= 68{\cdot}8 \text{ dynes/cm./sec.}$$

and from (21)

$$s' = 0{\cdot}168 \times 6{\cdot}12 \times 10^6 \times \cos 25{\cdot}3^\circ$$
$$= 0{\cdot}928 \times 10^6 \text{ dynes/cm.}$$

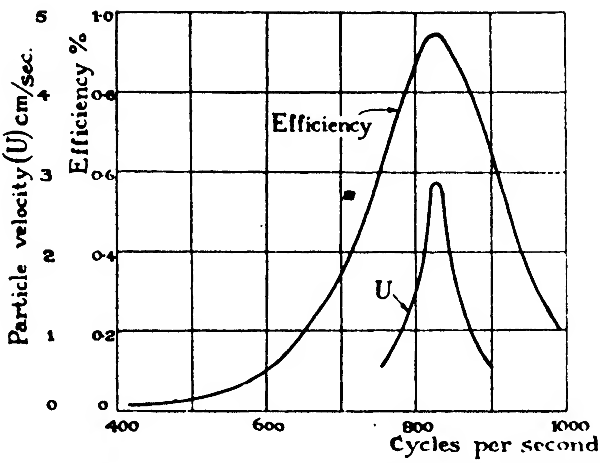


FIG. 108.—Efficiency and Diaphragm Resonance Curve of Receiver.

Hence from (22)

$$r = r'' - r' = 199 \text{ dynes/cm./sec.}$$
$$s = s'' - s' = 37{\cdot}63 \times 10^6 \text{ dynes/cm.}$$

The efficiencies are from (23), (24) and (25)

$$\eta_1 = \frac{140}{257{\cdot}3} = 0{\cdot}544$$

$$\eta_2 = \frac{199}{268} = 0{\cdot}74$$

$$\eta'_1 = \eta_1\eta_2 = 0{\cdot}40$$

The overall acoustical electrical efficiency of the receiver when working into the open air has been measured * directly. Fitted

* E. Mallett and G. F. Dutton, *J.I.E.E.*, Vol. 63, p. 515.

with a large flange the sound wave emitted was found in the manner described in section 29 to be hemispherical, and the acoustical power accordingly calculable (from 29·26) from Rayleigh disc or pressure measurements. The electrical power input was measured by the three-voltmeter method, using valve voltmeters. The result for a 60-ohm receiver of which the earcap had been cut away to expose the diaphragm is given in Fig. 108, which also shows the diaphragm resonance curve U as measured by the Rayleigh disc. The decay factor given by the latter curve agreed well with that obtained from impedance measurements.

The overall efficiency at resonance is seen to be only 0·01 ; calling

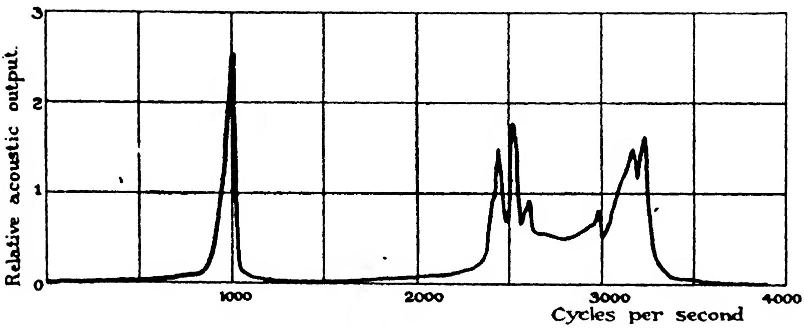


Fig. 109.—Variation of acoustic output with frequency.

this η and the acoustical-vibrational efficiency of the diaphragm η_3,

$$\eta = \eta_1 \eta_2 \eta_3$$

and using Kennelly's figures

$$\eta_3 = \frac{0{\cdot}01}{0{\cdot}544 \times 0{\cdot}74} = 0{\cdot}025.$$

Only one-fortieth of the power actually used in vibrating the diaphragm is converted into sound waves ; the remainder is used in internal friction.

The manner in which the acoustic output from the receiver varies with the frequency is shown in Fig. 109.* The effect of the fundamental resonance at 1,000 cycles is clearly seen, but the higher diaphragm resonances are confused by acoustic resonances between the diaphragm and the earcap. When the receiver is held against

* B. S. Cohen, A. J. Aldridge and W. West, *J.I.E.E.*, Vol. 64, p. 1023.

the ear the damping is increased and the various resonance curves are flatter and spread over a wider frequency range. The lower characteristic frequencies of the vowels fall within the fundamental resonance, the upper frequencies within the range covered by the two diameter and one circle modes. The receiver as designed by trial and error methods extending over a long time thus accentuates the vowel sounds.

(35) Impedances and Transformers

A problem that frequently arises in telephone circuits is the suitable design of apparatus with regard to its impedance. A case has already been noticed in the induction coil of the microphone circuit; others are the design of the receiver to work at the end of a line and of transformers to work in valve circuits. In all cases impedance matching is necessary in order that the maximum available power may be absorbed where it is required.

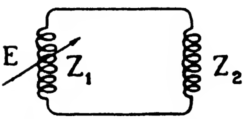


FIG. 110.—Simple Electrical Circuit.

An example will probably make the principle clear. Suppose (Fig. 110) that an alternating electromotive force represented by a rotating vector E is acting in an impedance Z_1 and that it is required to obtain power in some apparatus of impedance Z_2. The vector diagram is drawn in Fig. 111. OA is the complex impedance Z_1 with an angle ϕ_1, AB is Z_2 with an angle ϕ_2 and OB is the sum of these and is the total impedance of the circuit. Thus if OE lying on OB is the electromotive force, vector OI lying along the axis is the current vector, of length given by

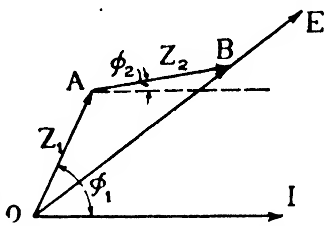


FIG. 111.—Vector Diagram for simple electrical circuit.

$$|I| = \frac{|E|}{|Z_1 + Z_2|} = \frac{|E|}{OB}$$

The power in the impedance Z_2 is

$$\frac{1}{2}|I||V| \cos \phi_2$$

where $|V|$ is the voltage across it.

$$|V| = |I| \times |Z_2|$$

Thus the required power is

$$W = \frac{1}{2}|I|^2|Z_2|\cos\phi_2$$

$$= \frac{1}{2}|E|^2\frac{|Z_2|}{|Z_1 + Z_2|^2}\cos\phi_2$$

Take the value of the electromotive force as 1 volt root mean square. Then $|E| = \sqrt{2}$ volts and

$$W = \frac{|Z_2|}{|Z_1 + Z_2|^2}\cos\phi_2 \text{ watts} \quad . \quad . \quad . \quad (1)$$

In the example from which the curves of Fig. 112 are drawn Z_1 is taken as 1,000 ohms with an angle of $-45°$, and Z_2 is taken first

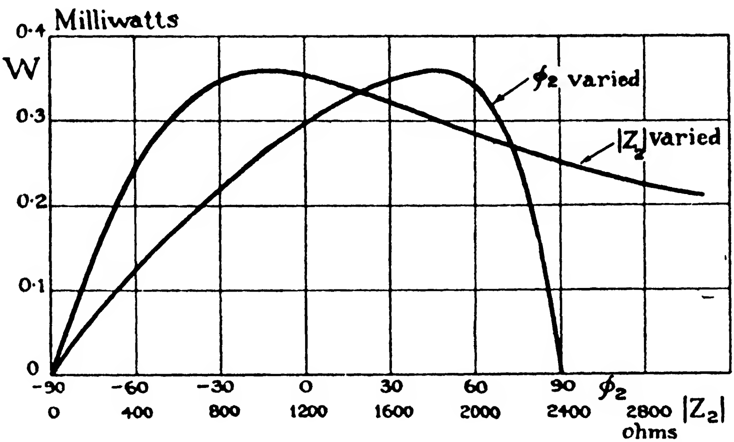


FIG. 112.—Effect of departure from matched impedance conditions.

with a size of 1,000 ohms and variable angle. $|Z_1 + Z_2|$ is found graphically in the same way as in Fig. 111, and the power is calculated from equation (1) and plotted against the phase angle ϕ_2. It is seen that the power in Z_2 is a maximum when ϕ_2 is 45°, that is, when $\phi_2 = -\phi_1$, but that the peak is by no means sharp, and increasing the angle by 25° and decreasing it by 45° only decreases the power by about 16 per cent. If the angles cannot be matched it is better to have the load angle smaller rather than larger than the optimum.

In a similar manner the second curve is drawn with varying values of $|Z_2|$, keeping the angle at the optimum value of 45°. It is seen that maximum power is obtained when $|Z_2| = 1{,}000$ ohms $= |Z_1|$, but that again the peak is quite blunt. A decrease of 50 per cent in the optimum value of $|Z_2|$ or an increase of 80 per cent only involve a decrease in the power of 16 per cent. It is clearly better to err on the side of making $|Z_2|$ too large rather than too small.

This result may be arrived at alternatively as follows. The power W in Z_2 is given by

$$W = \frac{1}{2}|E|^2 \frac{R_2}{|R_1 + jX_1 + R_2 + jX_2|^2}$$

where R_1R_2 and X_1X_2 are the resistances and reactances of Z_1 and Z_2.

Clearly W is a maximum when $X_1 = -X_2$, and is then

$$W = \frac{1}{2}|E|^2 \frac{R_2}{|R_1 + R_2|^2}$$

which is a maximum when $R_1 = R_2$, as in (33.09).

Thus maximum power is obtained when $R_1 = R_2$ and $X_1 = -X_2$, which is the same as saying when $|Z_1| = |Z_2|$ and $\phi_1 = -\phi_2$.

Where the impedances cannot be matched transformers may be introduced to give the same effect. The circuit is now as indicated in Fig. 113, where Z_pZ_s are the impedances of the primary and secondary windings of the iron-cored transformer respectively, and M is the mutual inductance between them.

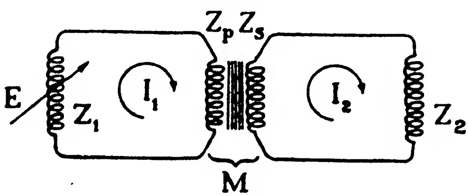


FIG. 113.—Impedance Matching with Transformers.

The equations for drawing the vector diagram may be written

$$\left.\begin{aligned} E &= (Z_1 + Z_p)I_1 - j\omega MI_2 \\ 0 &= (Z_2 + Z_s)I_2 - j\omega MI_1 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Whence
$$E = \left\{Z_1 + Z_p + \frac{\omega^2M^2}{Z_2 + Z_s}\right\} I_1 \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Now let the transformer be ideal, that is, without losses, expressed by writing $Z_p = j\omega L_p$ and $Z_s = j\omega L_s$ (L_p and L_s being the inductances of the primary and secondary windings respectively), without magnetic leakage, expressed by writing $M = \sqrt{L_p L_s}$, and with no magnetising current, expressed by taking the impedances $j\omega L_p$ and $j\omega L_s$ as infinitely large.

Equation (3) now becomes

$$E = \left\{ Z_1 + \frac{(Z_2 + j\omega L_s) j\omega L_p + \omega^2 M^2}{Z_2 + j\omega L_s} \right\} I_1$$

$$= \left\{ Z_1 + \frac{j\omega L_p Z_2 - \omega^2 L_p L_s + \omega^2 M^2}{Z_2 + j\omega L_s} \right\} I_1$$

$$= \left\{ Z_1 + \frac{L_p}{L_s} Z_2 \right\} I_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

on equating $L_p L_s$ to M^2 and neglecting Z_2 in the denominator in comparison with $j\omega L_s$.

The inductances of the windings are proportional to the squares of the number of turns $N_p N_s$ of the primary and secondary windings. Hence (4) becomes

$$E = \left\{ Z_1 + \left(\frac{N_p}{N_s}\right)^2 Z_2 \right\} I_1 . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The effective impedance of Z_2 with regard to the primary circuit has been increased (or reduced) by multiplication by $\left(\frac{N_p}{N_s}\right)^2$, and if the windings of the transformer are so designed that $\left(\frac{N_p}{N_s}\right)^2 |Z_2| = |Z_1|$ the condition for obtaining maximum power into the transformer and hence into the impedance Z_2 will have been fulfilled as far as the size of the impedance is concerned, and this is usually the more important consideration.

The same result is obtained by referring the primary circuit impedance and voltage to the secondary circuit. Elimination of I_1 from equations (2) gives

$$E = \left\{ \frac{(Z_1 + Z_p)(Z_2 + Z_s)}{j\omega M} - j\omega M \right\} I_2 \quad . \quad . \quad . \quad (6)$$

$$= \frac{Z_1(Z_2 + Z_s) + Z_2 Z_p}{j\omega M} I_2$$

$$= \left\{ Z_1 \sqrt{\frac{L_s}{L_p}} + Z_2 \sqrt{\frac{L_p}{L_s}} \right\} I_2$$

on the assumption as before of an ideal transformer. Thus

$$\sqrt{\frac{L_s}{L_p}}\,E = \left\{ Z_1 \frac{L_s}{L_p} + Z_2 \right\} I_2$$

or

$$\frac{N_s}{N_p} E = \left\{ \left(\frac{N_s}{N_p}\right)^2 Z_1 + Z_2 \right\} I_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

N_s/N_p times the voltage acts in a circuit of impedance $\left(\frac{N_s}{N_p}\right)^2$ times the internal impedance of the generator, plus the load impedance, and the maximum power is obtained in the latter when

$$\left(\frac{N_s}{N_p}\right)^2 |Z_1| = |Z_2| \text{ as before.}$$

The current ratio I_1/I_2 can be obtained from (5) and (7) as

$$\frac{I_1}{I_2} = \frac{\left(\frac{N_s}{N_p}\right)^2 Z_1 + Z_2}{\frac{N_s}{N_p} E \left\{ Z_1 + \left(\frac{N_p}{N_s}\right)^2 Z_2 \right\}} = \frac{N_s}{N_p} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

The ratio of the voltages V_p/V_s across the transformer terminals is given by

$$\frac{V_p}{V_s} = \frac{I_1 Z_p - j\omega M I_2}{-I_2 Z_s + j\omega M I_1} = \frac{I_1 L_p - M I_2}{-I_2 L_s + M I_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Putting $V_p/V_s = a$ and $I_2/I_1 = b$, equation (9) becomes

$$a = \frac{\frac{L_p}{M} - b}{-b\frac{L_s}{M} + 1} = \frac{\frac{N_p}{N_s} - b}{-b\frac{N_s}{N_p} + 1}$$

$$\therefore \qquad -ab\frac{N_s}{N_p} + a = \frac{N_p}{N_s} - b$$

$$b\left(1 - a\frac{N_s}{N_p}\right) = \left(\frac{N_p}{N_s} - a\right) = \frac{N_p}{N_s}\left(1 - a\frac{N_s}{N_p}\right)$$

$$\therefore \qquad 1 - a\frac{N_s}{N_p} = 0 \qquad \text{or } a = \frac{N_p}{N_s} = \frac{V_p}{V_s} \quad . \quad . \quad . \quad (10)$$

and

$$b = \frac{N_p}{N_s} = \frac{I_2}{I_1}$$

When the resistances of the windings and magnetic leakage are to be taken into account the complete expressions (3) and (6) must be used for the currents in the primary and secondary circuits respectively. In many cases a simplification of the calculations

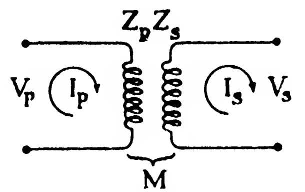


FIG. 114.—Transformer Circuit.

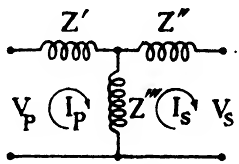


FIG. 115.—Equivalent T Network of Transformer.

may be effected by replacing the transformer by an equivalent network. The equations for the transformer circuit of Fig. 114 are

$$\left.\begin{aligned} V_p &= Z_p I_p - j\omega M I_s \\ 0 &= Z_s I_s - j\omega M I_p + V_s \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and those for the asymmetrical T network of Fig. 115 are

$$\left.\begin{aligned} V_p &= (Z' + Z''')I_p - Z'''I_s \\ 0 &= (Z'' + Z''')I_s - Z'''I_p + V_s \end{aligned}\right\} \quad . \quad . \quad . \quad (12)$$

Equations (11) and (12) are identical if

$$Z_p = Z' + Z''',\ Z_s = Z'' + Z''',\ \text{and}\ Z''' = j\omega M$$

that is, if the impedances of the T are

$$\left.\begin{aligned} Z' &= Z_p - j\omega M \\ Z'' &= Z_s - j\omega M \\ \text{and} \quad Z''' &= j\omega M \end{aligned}\right| \quad . \quad . \quad . \quad (13)$$

Actually there are additional losses due to hysteresis and eddy currents in the iron core. These losses may, as in the case of the telephone receiver, be considered to occur in a third closed circuit of impedance Z_t, coupled with the primary winding by an inductance M_p and with the secondary by an inductance M_s as indicated in Fig. 116. The circuital equations are now

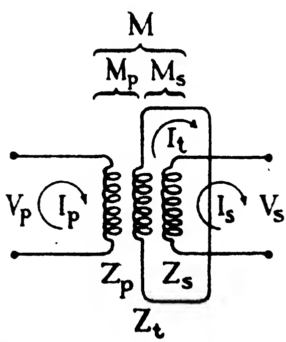


FIG. 116.—Addition of tertiary winding to account for hysteresis and eddy current losses.

$$\left.\begin{aligned} V_p &= Z_p I_p - j\omega M I_s - j\omega M_p I_t \\ 0 &= Z_t I_t - j\omega M_p I_p + j\omega M_s I_s \\ 0 &= Z_s I_s - j\omega M I_p + j\omega M_s I_t + V_s \end{aligned}\right\} \quad . \quad . \quad . \quad (14)$$

and on using the second to eliminate I_t from the first and third

$$\left.\begin{aligned} V_p &= \left(Z_p + \frac{\omega^2 M_p{}^2}{Z_t}\right) I_p - j\omega\left(M - \frac{j\omega M_p M_s}{Z_t}\right) I_s \\ 0 &= \left(Z_s + \frac{\omega^2 M_s{}^2}{Z_t}\right) I_s - j\omega\left(M - \frac{j\omega M_p M_s}{Z_t}\right) I_p + V_s \end{aligned}\right\} \quad . \quad (15)$$

Comparing these with equation (11), the impedances Z_p and Z_s have been altered to $Z_p + \omega^2 M_p{}^2/Z_t$ and $Z_s + \omega^2 M_s{}^2/Z_t$ respectively, and instead of a pure mutual inductance M there appears a complex quantity $M - j\omega M_p M_s/Z_t$. In the equivalent network of Fig. 115 therefore

$$\left.\begin{aligned} Z' &= Z_p + \frac{\omega^2 M_p{}^2}{Z_t} - j\omega\left(M - j\omega\frac{M_p M_s}{Z_t}\right) \\ Z'' &= Z_s + \frac{\omega^2 M_s{}^2}{Z_t} - j\omega\left(M - j\omega\frac{M_p M_s}{Z_t}\right) \\ Z''' &= j\omega\left(M - \frac{j\omega M_p M_s}{Z_t}\right) \end{aligned}\right\} \quad . \quad . \quad . \quad (16)$$

In each case the effect of the eddy currents is to increase the resistance and to reduce the inductance. Fig. 117 shows the effect on the primary impedance. $Oa = Z_t$ and $Ob = \omega^2 M_p{}^2$. Oc, drawn with $cOb = aOb$ and length $Oc = Oc/Oa$ is $\omega^2 M_p{}^2/Z_t$. $O'O = Z_p$ and $O'c$ the modified primary impedance $(Z_p + \omega^2 M_p{}^2/Z_t)$. Dropping perpendiculars from O and c on to the axis through O′ at X and X′, it is clear that the inductance has been reduced from OX/ω to cX'/ω, and that the resistance has been increased from O′X to O′X′.

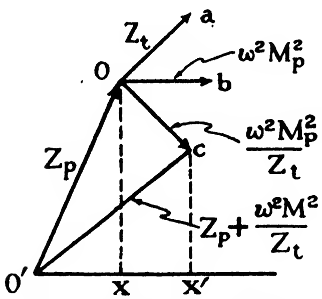


FIG. 117.—Effect on Primary Impedance.

Fig. 118 shows the effect on the mutual inductance. $Oa = M$, $Ob = 1/Z_t$, $Oc = -j\omega M_p M_s$ and $Od = -j\omega M_p M_s/Z_t$. af is drawn equal and parallel with Od, so that $Of = M - j\omega M_p M_s/Z_t$. Drawing fa' perpendicular to the axis, it is seen that the effective mutual inductance has been reduced from Oa to Oa'. Z''' is obtained by multiplying Of by $j\omega$ to give Og. Drawing gg' perpendicular to the axis, it is seen that Z''' now contains a resistance term Og', whereas without the eddy current circuit it was a pure reactance.

In some transformers, such as those used in valve circuits, where the number of turns is very large, an additional complication arises owing to the turn to turn capacity of the winding. This may be taken account of to a first approximation by supposing that there is a condenser shunting the terminals of the winding. The effects are most felt at the higher frequencies, and the possibility arises of resonance within the frequency band used in telephony.

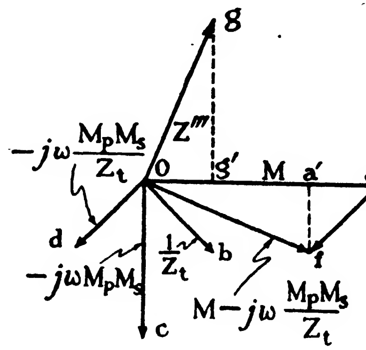


FIG. 118.—Effect on Mutual Inductance.

Generally speaking the alternating currents and fluxes in telephone impedances and transformers are very small, the hysteresis losses are very small and the permeability is small. The iron is worked on the initial straight part of the magnetisation curve and the permeability and hence the inductances are practically constant with different current strengths. This is true even when the alternating currents and fluxes are superposed on a direct current and steady flux as in the case of the inductance coils used with microphones, provided the direct current does not produce saturation. The flux alternations in this case do not follow the main magnetisation curve, but take place along a line more or less parallel to the initial line of the curve. The inductance of a coil is practically constant over the frequency range used, although the effective resistance increases with frequency. When capacity effects are felt, however, the effective inductance is reduced as the frequency is raised; in fact, the whole resonance phenomenon described in section (31) and Part III. will be experienced.

(36) Simple Circuit

The simplest telephone circuit in which speech is possible in each direction without switching can now be drawn as in Fig. 119 (a). At each end the battery B, usually two leclanché cells, is connected in series with the primary winding of the induction coil T and the microphone M of the local battery type. The telephone receiver R is connected in series with the secondary of the induction coil and the line. The latter almost invariably consists of two wires, "go" and "return," instead of the earth "return" usual in telegraphy. This is necessary to avoid interference from one circuit to another when as is usual more than one line is run along the same route. The speaker evidently hears his own speech in his own receiver; as "side tone." Side tone affords a ready means of testing that all is well with the circuit; if the speaker blows quietly into his microphone he should hear a rustling sound in his receiver.

The induction coil is of the open iron circuit type to avoid saturation of the iron by the direct current flowing. Two designs are in common use; in one the primary resistance (D.C.) is 1 ohm

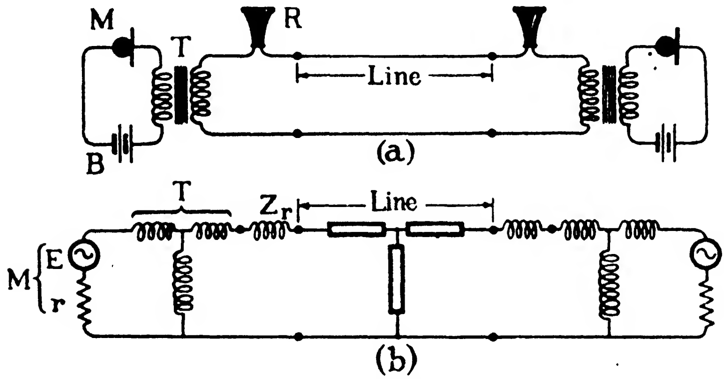


FIG. 119.—Simple Telephone Circuit and its Equivalent Network.

and the secondary 25 ohms, and in the other the resistances are 1 ohm and 150 ohms respectively.

The simple circuit may be replaced by the equivalent circuit of Fig. 119 (b). The microphones M become speech current alternators of electromotive force E and internal resistance r_0, and the induction coils are replaced by their equivalent T circuits. The line is also shown by an equivalent T; the calculation of this is described in the next chapter.

(37) Other Transmitters

Various other devices for producing electric currents from sound waves have been used and still are used for special purposes. The telephone receiver can be used as a transmitter, and, in fact, when the telephone was first invented by Graham Bell identical instruments very similar to the receiver in use to-day were used at each end of the line, one as the transmitter and the other as the receiver.

When used as a transmitter the alternating pressure of the sound wave causes vibrations of the diaphragm in front of the permanent magnet, the reluctance of the magnetic circuit and the flux through the coils are accordingly varied and electromotive forces are set up in the coils. The energy of the sound wave is accordingly converted into electrical energy, and the response is feeble. In this the receiver differs strikingly from the carbon microphone, in which the energy of the sound wave is used only to vibrate the diaphragm, all the electrical energy coming from the battery. A modification of the telephone receiver is used as a transmitter in some broadcasting stations.

A wire heated by an electric current will change its resistance when in a sound wave, as the particles of air in their movements to and fro will cool the wire. A device consisting of a grid of very fine platinum wire in the neck of a Helmholtz resonator, in which the particle velocity of sound of a particular frequency is very much increased, is used for sound ranging purposes.* It is hardly suitable for telephony, however, as the response falls off considerably with increasing frequency owing to thermal lag, and when used with a resonator it is essentially a single frequency device.

A modification (the Marconi-Reisz) of the solid back microphone in which the carbon granules are held in a heavy marble container and the diaphragm is a stretched silk membrane or a thin mica plate is used in some broadcasting stations.

Another instrument much used for broadcasting is the magnetophone (Marconi-Sykes), in which an electromotive force is induced in a winding mounted on a diaphragm which is caused to vibrate by the sound wave in a powerful magnetic field. The diaphragm is annular in shape and very light, the winding is of aluminium and the annular field is produced by a powerful electro-magnet. The whole device is held in a rubber cradle to avoid extraneous vibrations, and the movements of the diaphragm are damped by cotton wool or by immersion in oil.

In the condenser microphone † the vibrating diaphragm is one plate of a condenser and the other plate is massive and very close to the vibrating plate. The capacity of the condenser varies as the moving plate vibrates. the charge due to a battery of somewhat

* W. S. Tucker and E. T. Paris, *Trans. Royal Soc.* 221A, pp. 389-430, March, 1921.

† E. C. Wente, *Phys. Review*, 1917, Vol. 10, p. 39.

high potential varies accordingly, and the resulting charging and discharging currents have much the same wave form as the sound wave producing the vibrations. By placing the two plates very close together the air damping is considerable and a particularly level response-frequency curve is obtained. This instrument suitably calibrated is used in acoustic measurements as well as in some broadcasting stations.

The eddy current transmitter (C. W. Hewlett) consists of a tightly stretched aluminium foil diaphragm between two concentrically wound flat coils of wire wound with annular air spaces for the passage of sound waves. The current through the coils produces a radial field between them. The movement of the foil in this field sets up eddy currents which in turn produce a variation of voltage in the coils. The back of the instrument is enclosed by a box loosely filled with cotton waste to prevent reflection of the sound waves.*

In all of these transmitters the voltage obtained from a given sound wave is very much less than that from the solid back microphone, and none of them is suitable for use in ordinary commercial telephony. But for special purposes, such as in broadcasting stations, where valve amplifiers (to be described later) can be used to any desired extent, this feebleness of response is not a serious drawback at all, and is far more than offset by the more uniform response obtained over a large frequency range.

(38) **Loud Speakers**

The considerations underlying the design of loud speakers to produce sound waves audible throughout a room or larger space are very different from those concerned with the telephone receiver, which has only to produce pressure variations in a confined space between the diaphragm and the drum of the ear. The loud speaker has to produce far more acoustical energy, it must be as free from resonances as possible, and the output for a given input must be as constant as possible over a wide range of frequencies in order that music may be faithfully reproduced. Further, the amplitude of vibration must have a linear relationship with the exciting current, otherwise distortion by the introduction of unwanted frequencies will result, as seen in the cases of the microphone and telephone receiver. Since for equal acoustical energies the

* B. S. Cohen, *P.O.I.E.E.J.*, Vol. 19, p. 237.

amplitude required varies inversely as the frequency (equation 29.26), at low frequencies the amplitude must be large and the linear relationship is difficult to achieve.

The loud speaker can conveniently be divided into two parts, (1) the movement or motor in which mechanical vibrations are produced from the electrical currents and (2) the acoustical converter, in which the mechanical vibrations are converted into sound waves.

The telephone receiver movement made on a larger scale and with a larger diaphragm has been extensively used for loud speakers

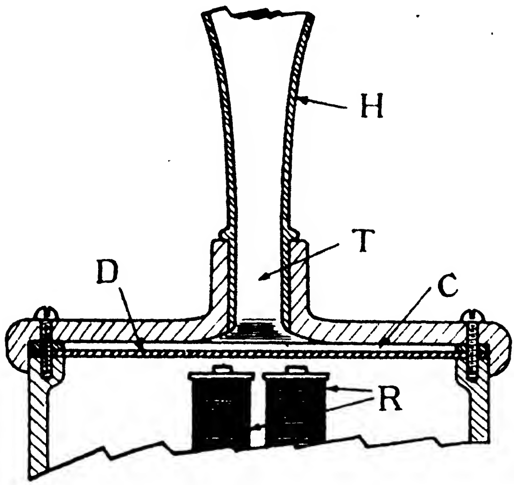


FIG. 120.—Receiver Movement.

(Fig. 120). Three others are illustrated in Figs. 121, 122 and 123. In Fig. 121 a powerful permanent magnet M establishes flux through pole pieces P, and across two air gaps to a reed R rigidly clamped at A to a massive ring. The speech current passes through coils C on the pole pieces and the reed is accordingly set into vibration, the theory being exactly the same as that of the ordinary receiver. In each of the movements of Figs. 120 and 121 the position of the permanent magnet system with regard to the reed or diaphragm is adjusted in order that the air gap length may be varied according to the amplitude of vibration required.

In Fig. 122 the vibrating member is an armature A held by a spring S_1 in a central position in the gaps between two pairs of pole

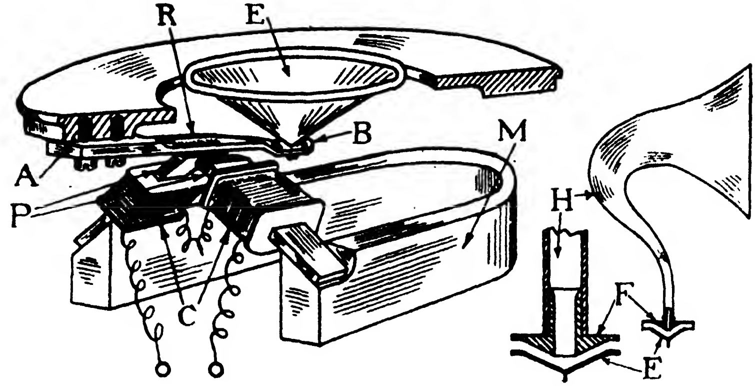


FIG. 121.—Reed Movement.

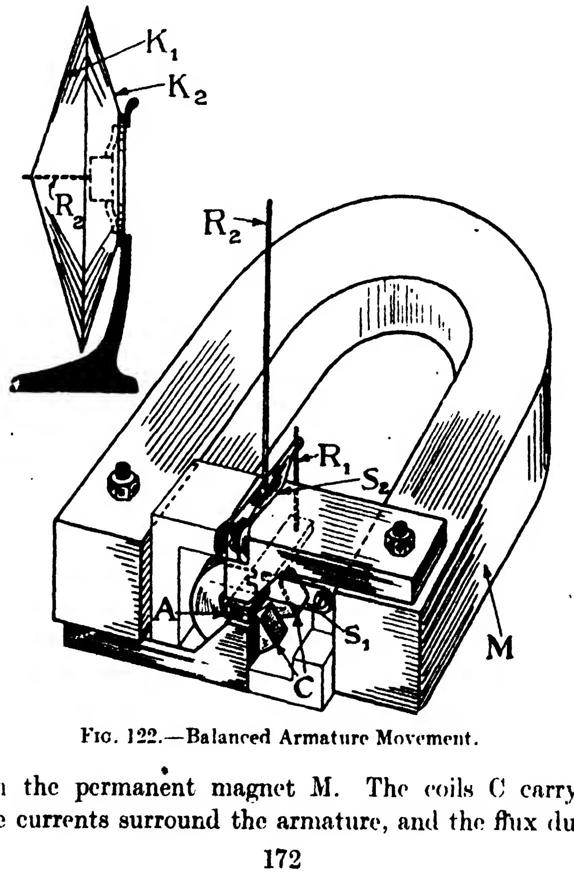


FIG. 122.—Balanced Armature Movement.

pieces on the permanent magnet M. The coils C carrying the telephone currents surround the armature, and the flux due to the currents divides at the armature ends, strengthening and weakening the permanent flux in opposite gaps and so producing a torque which vibrates the armature about its centre, and hence vibrates the rod R_1 and through the spring S_2 the rod R_2. There are four air gaps all helping to produce torque, and the device is very similar in principle to the neutral tongue relay. and the slight necessary modification of the telephone receiver theory to meet this case is obvious. The movement is known as the "balanced armature." The further the armature moves from the neutral position the greater the force due to the permanent flux tending to increase the movement still more, but this is balanced by the restoring force exerted by the spring.

The movement of Fig. 123 is quite different. Here the coil carrying the telephone current vibrates, and the movement is known as the "moving coil." The thin cylindrical coil C is situated in the annular air gap of a powerful electromagnet M excited by a winding W. The movement is shown in section, and the flux paths by the dotted lines. The current through the turns of the coil is everywhere at right angles to the flux, and the force produced is at right angles to both flux and current, that is, parallel to the core of the electro-magnet. The coil thus vibrates along the air gap instead of across it as in the other models, and clearly greater movement is possible without loss of linearity.

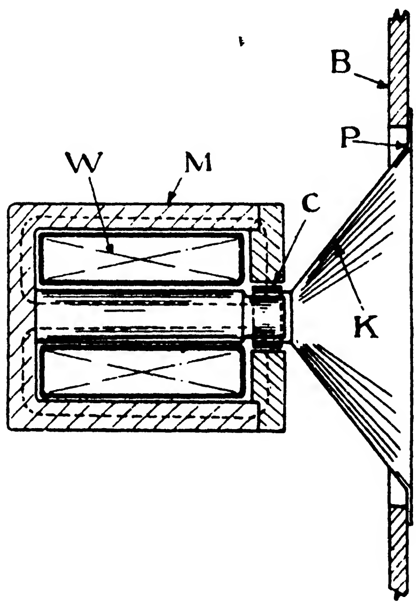


FIG. 123.—Moving Coil Movement.

The great difficulty in mechanical-acoustical convertion of power is the provision of sufficient acoustical impedance. For efficient transfer of power the considerations obtaining are similar to those in the electrical case—a certain amount of impedance matching is necessary as between the mechanical impedance of the moving part of the " motor " and the acoustical impedance of the air it " drives." Methods for approximating to this fall into two types, the horn and the large diaphragm.

The horn method is illustrated in Figs. 120 and 121. In Fig. 120 there is a cavity C to produce sufficient loading on the diaphragm connecting with the throat T of the horn H. The horn acts somewhat as an acoustical transformer connecting the regions of high pressures in the throat and low pressures at the mouth. In Fig. 121 the reed at the free end B is rigidly connected to the apex of a very light cone of aluminium E, flexibly connected at the rim to the face A. Above the cone is a piece F of similar shape acoustically

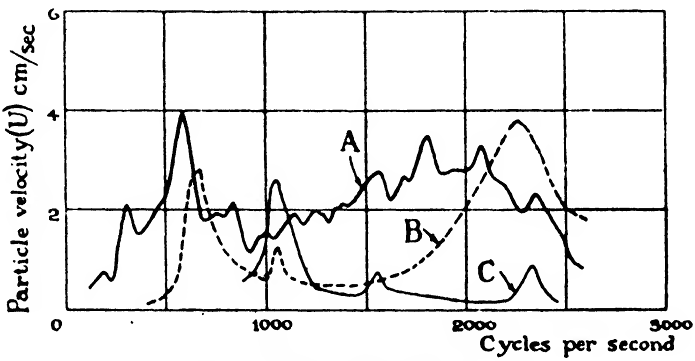


FIG. 124.—Effect of Acoustical Loading.

connected to the horn H by a hole through the centre. The air cavity between E and F is the region of high acoustical pressure.

The effect of this transformer system in providing acoustical loading is well shown by the relative particle velocity measurements made with a Rayleigh disc at various frequencies (Fig. 124). Curve A was taken with the system complete, curve B after the removal of the horn, and curve C after the removal of the member F. It will be noted how the amplitude of response is levelled up and extended to lower frequencies by the complete system.

The horn has resonances of its own and a lower cut-off frequency (*i.e.*, it has a high pass filter characteristic) both according to its length. Long horns are essential if the lower tones are to be preserved, and the best results are obtained if the cross-sectional area of the horn increases logarithmically along its length.*

* O. R. Hanna and J. Slepian, *J.A.I.E.E.*, March, 1924.

The large diaphragm method operates by moving against a large area of air, but the difficulty is to obtain a diaphragm sufficiently light and at the same time sufficiently stiff. For this reason the diaphragm is usually in the form of a cone, as is illustrated in Figs. 122 and 123, made of stiff paper treated to render it non-hygroscopic. In Fig. 122 a large double cone is employed, the two cones being K_1 and K_2, and the movement is mounted in between. The front cone K_1 is vibrated by the rod R_2. In Fig. 123 the cone K is attached at its apex direct to the moving coil, and at its rim by a flexible connection P to a large baffle board B. The baffle prevents the flow of air round the rim instead of forward to form a sound wave, and is essential for the proper reproduction of low frequencies.

REFERENCES

A. E. KENNELLY.—" Electrical Vibration Instruments."

B. S. COHEN, A. J. ALDRIDGE and W. WEST.—" The Frequency Characteristics of Telephone Systems and Audio Frequency Apparatus and their Measurement." *J.I.E.E.*, Vol. LXIV., p. 1023.

D. W. DYE.—" The Performance and Properties of Telephonic Frequency Inter-valve Transformers." *Expl. Wireless*, September, October, November, 1924.

H. J. GREGORY.—" Equivalent T's for Telephone Transformers." *P.O.E.E.J.*, Vol. XXI., p. 127.

O. R. HANNA and J. SLEPIAN.—" The Function and Design of Horns." *Journal of the A.I.E.E.*, March, 1924.

CHAPTER VIII

LINE TRANSMISSION

(39) Line Equations

Any line has four primary constants, resistance, inductance, leakance and capacity, which determine the propagation of electromagnetic waves along it. Considering a line consisting of go and return wires, let these constants per mile of loop be R ohms, L henries, G mhos and C farads in order, and let AB, Fig. 125, represent a short length, δx miles, of the line distant x miles from the sending end. The resistance of the loop AB is $R\delta x$, its inductance is $L\delta x$, the leakance from wire to wire is $G\delta x$, and the capacity from wire to wire is $C\delta x$. If the voltage across the loop at A is v, that at B is $\left(v + \frac{\partial v}{\partial x}\delta x\right)$; and if the current in the loop at A is i, that at B is $\left(i + \frac{\partial i}{\partial x}\delta x\right)$. Making the length δx very short, the current in it is sensibly constant as far as voltage calculations are concerned, and the voltage across the wires is sensibly constant as far as current calculations are concerned.

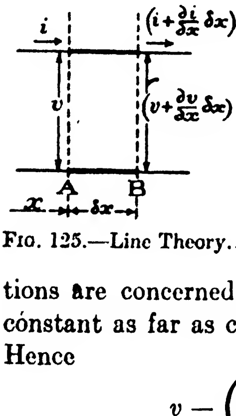


Fig. 125.—Line Theory.

Hence

$$v - \left(v + \frac{\partial v}{\partial x}\delta x\right) = R\delta x i + L\delta x\frac{\partial i}{\partial t}$$

and

$$i - \left(i - \frac{\partial i}{\partial x}\delta x\right) = G\delta x v + C\delta x\frac{\partial v}{\partial t}$$

Whence

$$-\frac{\partial v}{\partial x} = Ri + L\frac{\partial i}{\partial t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$-\frac{\partial i}{\partial x} = Gv + C\frac{\partial v}{\partial t}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Differentiating (1) with regard to x gives

$$-\frac{\partial^2 v}{\partial x^2} = R\frac{\partial i}{\partial x} + L\frac{\partial^2 i}{\partial t\partial x} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and differentiating (2) with regard to t gives

$$-\frac{\partial^2 i}{\partial x \partial t} = G\frac{\partial v}{\partial t} + C\frac{\partial^2 v}{\partial t^2} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Substitution of (2) and (4) in (3) gives

$$\frac{\partial^2 v}{\partial x^2} = CL\frac{\partial^2 v}{\partial t^2} + \left(CR + GL\right)\frac{\partial v}{\partial t} + RGv \quad . \quad . \quad . \quad (5)$$

Similarly

$$\frac{\partial^2 i}{\partial x^2} = CL\frac{\partial^2 i}{\partial t^2} + \left(CR + GL\right)\frac{\partial i}{\partial t} + RGi \quad . \quad . \quad . \quad (6)$$

Equations (5) and (6) are the telegraph equations in their most general form, on the assumption of primary constants which do not vary with frequency. Two particular cases have already been considered : (1) the short leaky telegraph line, in which L = O and C = O, that is, consideration of the steady state for direct currents (in section 8), and (2) Kelvin's treatment of the submarine cable taking L = O and G = O, that is, the consideration of the direct current transients with these assumptions (in section 16).

In studying the propagation of telephone currents, it has already been noted that in virtue of Fourier's analysis it is necessary only to consider sinusoidal variations, but it is further assumed that the steady state condition is very rapidly reached and is of the greatest practical interest, what happens during the transient period in which the final steady state is built up being of only minor if of any importance. Calculations made on this assumption appear to be justified by actual experimental results.

Equations (5) and (6) can then be transformed into rotating vector equations by writing $v = V\epsilon^{j\omega t}$, $\frac{\partial v}{\partial t} = j\omega V\epsilon^{j\omega t}$, and $\frac{\partial^2 v}{\partial t^2} = -\omega^2 V\epsilon^{j\omega t}$. when equation (5) becomes

$$\frac{d^2V}{dx^2}\epsilon^{j\omega t} = -\omega^2 CLV\epsilon^{j\omega t} + j\omega(CR + GL)\epsilon^{j\omega t} + RGV\epsilon^{j\omega t}$$

or

$$\frac{d^2V}{dx^2} = (R + j\omega L)(G + j\omega C)V \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and similarly (6) becomes

$$\frac{d^2I}{dx^2} = (R + j\omega L)(G + j\omega C)I \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Time has disappeared from the equations, but can be re-introduced at any stage by multiplication throughout by $\epsilon^{j\omega t}$. Equations (7) and (8) enable the vector diagram to be drawn; multiplication by $\epsilon^{j\omega t}$ rotates the diagram for the purpose of ascertaining instantaneous values.

Comparison of equations (7) and (8) with equation (3) of section 8 shows them to be of the same form exactly, with $(R + j\omega L)$ instead of R, and $(G + j\omega C)$ instead of G, and all the results obtained for the leaky line case can be applied at once to the telephone line case by making these alterations.

Instead of an attenuation constant

$$a = \sqrt{RG}, \quad . \quad . \quad . \quad . \quad (8{\cdot}06)$$

there will be a complex *propagation constant*

$$P = \sqrt{(R + j\omega L)(G + j\omega C)} \quad . \quad . \quad . \quad . \quad (9)$$

and instead of a characteristic resistance

$$R_o = \sqrt{\frac{R}{G}} \quad . \quad . \quad . \quad . \quad (8{\cdot}06)$$

there will be a complex *characteristic impedance*

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

and the solutions of (7) and (8) can be written down from 8·07 and 8·08 as

$$V = A \cosh Px + B \sinh Px \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and
$$I = -\frac{B}{Z_o} \cosh Px - \frac{A}{Z_o} \sinh Px \quad . \quad . \quad . \quad . \quad . \quad (12)$$

where A and B are voltage vectors determined by conditions at the ends of the line.

When the line is infinitely long, from 8·09

$$\left.\begin{aligned} V &= V_s \epsilon^{-Px} \\ I &= \frac{V_s}{Z_o} \epsilon^{-Px} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

This will enable a physical interpretation to be given to the complex propagation constant P. Writing

$$P = a + j\beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

(13) gives

$$V = V_s\, \epsilon^{-ax} . \epsilon^{-j\beta x} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13a)$$

a determines the size of the vector $V_s\epsilon^{-ax}$ which represents the voltage V at the point x, while β determines the phase angle $-\beta x$ through which it is rotated from the vector representing V_s. The distance $x = \lambda$ which makes the angle 2π is the wavelength of the propagation. Hence

$$\lambda = \frac{2\pi}{\beta} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

α is called the *attenuation constant* and β the *wavelength constant.* These constants, together with the propagation constant P and the

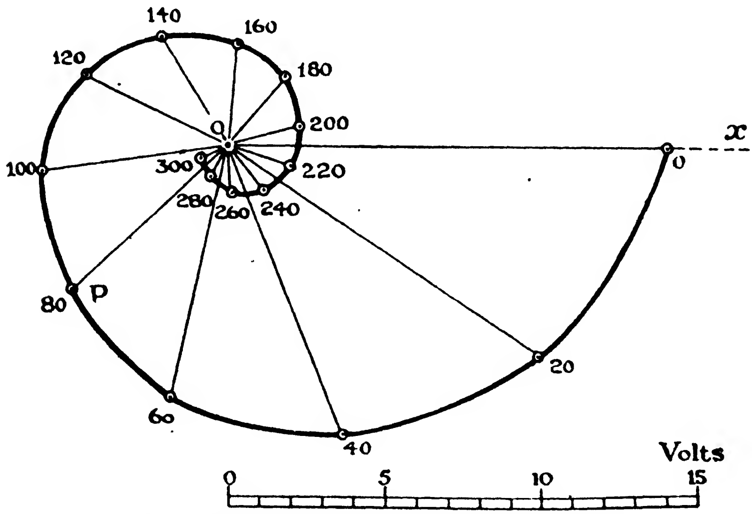


FIG. 126.—Vector Voltage along infinitely long line.

characteristic impedance Z_0, are known as the *secondary constants* of the line. R, L, G and C are the *primary constants.*

As an example of the use of these constants, consider the case of a 150-lb. aerial loop, for which $P = 0{\cdot}009 + j0{\cdot}03$ and $Z_0 = 725 \overline{\backslash 15{\cdot}5^\circ}$ approximately at 800 cycles per second. Let a potential difference of 10 volts (r.m.s.) be maintained at one end of a line which may be considered infinitely long. Equation (13a) gives for the vector diagram of the fall of voltage along the line

$$V = 10\sqrt{2}\,\epsilon^{-0{\cdot}009x} \,.\, \epsilon^{-j0{\cdot}03x},$$

i.e.,
$$V = 14{\cdot}14\,\epsilon^{-0{\cdot}009x} \overline{\Big\backslash 0{\cdot}03 \times \frac{180^\circ}{\pi} \times x}$$

Fig. 126 is constructed from this equation. The ends of the vectors representing the voltage at various points along the line (the figures are miles from the sending end) lie on a logarithmic spiral. As an example of the calculation of the curve, take $x \doteq 80$ miles. $\epsilon^{-0{\cdot}009x} = \epsilon^{-0{\cdot}72} = 0{\cdot}487$. $14{\cdot}14 \times \epsilon^{-0{\cdot}009x} = 6{\cdot}89$ volts = the length of the vector OP. $\diagdown\overline{0{\cdot}03 \times \frac{180}{\pi} \times x} = \diagdown\overline{137^\circ}$. The angle xOP is accordingly made $- 137^\circ$.

The instantaneous values of the voltage are to be found by

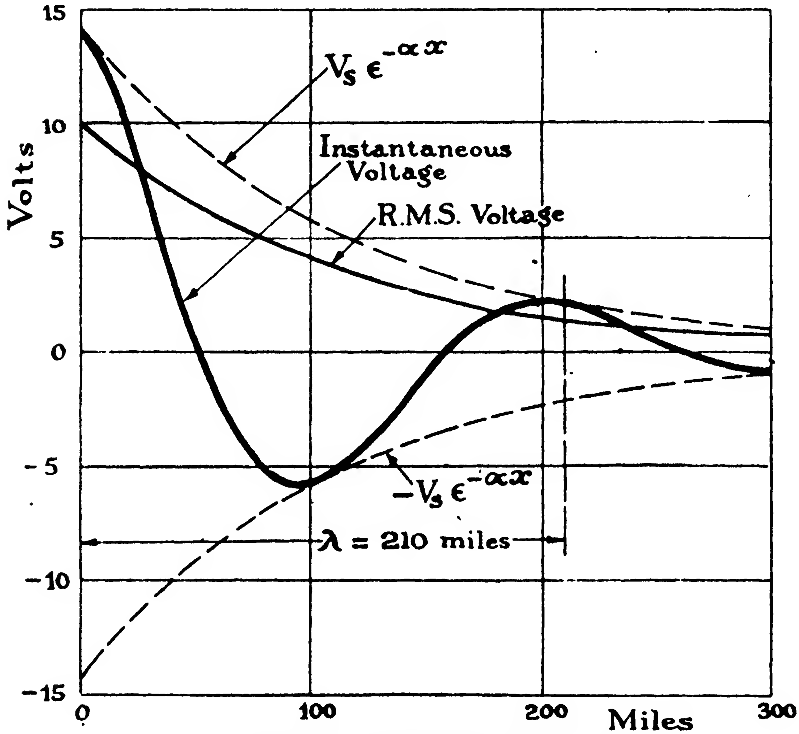


FIG. 127.—Instantaneous Voltage along infinitely long line.

projection on to any line through O, the position of the reference line being determined by the instant chosen. If this is the instant at which the voltage at the sending end is a maximum, the vectors are projected into the axis Ox; the distances from O to the foot of the projections then give the instantaneous values, and these are plotted in Fig. 127. These values are evidently $V_s\epsilon^{-\alpha x}\cos\beta x$. At 80 miles this gives $6{\cdot}89 \times \cos 137 \doteq -5$ volts.

The curve of instantaneous values lies between the curves $y = V_s\epsilon^{-\alpha x}$ and $y = -V_s\epsilon^{-\alpha x}$, and shows the wave shape. The wavelength $= 2\pi/\beta = 2\pi/0{\cdot}03 = 210$ miles. The r.m.s. voltage is drawn as $1/\sqrt{2}$ times the maximum at each point.

The vector current locus is also a logarithmic spiral (from equation (13)), leading the voltage spiral by 15·5°, and having vector length determined at each distance by V/725 amps.

The secondary constants can be calculated by complex algebra from the primary constants by equations (9), (10) and (14). α and β can, however, be obtained directly, though generally with more arithmetic, as follows. P^2 from (9) and from (14) gives the identity.

$$\alpha^2 + 2j\alpha\beta - \beta^2 = RG + j\omega(LG + RC) - \omega^2LC$$

and equating real and imaginary terms,

$$\alpha^2 - \beta^2 = RG - \omega^2LC \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$
$$2\alpha\beta = \omega(LG + RC)$$

But
$$\alpha^2 + \beta^2 = \sqrt{(\alpha^2 - \beta^2)^2 - 4\alpha^2\beta^2}$$
$$= \sqrt{(RG - \omega^2LC)^2 + \omega^2(LG + RC)^2}$$
$$= \sqrt{(R^2 + \omega^2L^2)(G^2 + \omega^2C^2)} \quad . \quad . \quad . \quad (17)$$

Hence, by addition and subtraction of (16) and (17),

$$\alpha = \sqrt{\frac{1}{2}\left\{\sqrt{(R^2 + \omega^2L^2)(G^2 + \omega^2C^2)} + (RG - \omega^2LC)\right\}} \quad . \quad (18)$$

$$\beta = \sqrt{\frac{1}{2}\left\{\sqrt{(R^2 + \omega^2L^2)(G^2 + \omega^2C^2)} - (RG - \omega^2LC)\right\}} \quad ..(19)$$

Equation (18) may be used to find the condition of minimum attenuation. From (18)

$$2\alpha^2 = \sqrt{(R^2 + \omega^2L^2)(G^2 + \omega^2C^2)} + (RG - \omega^2LC).$$

Differentiating with regard to L and equating to zero gives

$$\frac{\omega^2L(G^2 + \omega^2C^2)}{\sqrt{(R^2 + \omega^2L^2)(G^2 + \omega^2C^2)}} = \omega^2C,$$

i.e., $\omega^4L^2(G^2 + \omega^2C^2)^2 = \omega^4C^2(R^2 + \omega^2L^2)(G^2 + \omega^2C^2)$

whence $$LG = RC \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

This relationship actually never holds, as the product RC is always much greater than the product LG. If it did hold, however, substitution in (18) and (19) would give

$$\alpha = \sqrt{RG} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

and

$$\beta = \omega\sqrt{LC} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The attenuation, which generally is greater the greater the frequency, leading to line distortion, is in this case independent of the frequency. The condition (20) which leads to minimum attenuation also leads, therefore, to distortionless transmission as far as the amplitudes of the various frequencies involved are concerned.

From (22) and (15) the wavelength in this special case is

$$\lambda = \frac{2\pi}{\omega\sqrt{LC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

and the velocity of transmission c is

$$c = \lambda f = \frac{1}{\sqrt{LC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

The line is distortionless in this respect also : that all the waves of various frequencies arrive at the receiving end at the same time.

The characteristic impedance is also independent of frequency. From (10)

$$Z_o = \sqrt{\frac{(R + j\omega L)(G - j\omega C)}{G^2 + \omega^2C^2}}$$

$$= \sqrt{\frac{RG + \omega^2LC}{G^2 + \omega^2C^2}} = \sqrt{\frac{R\left(G + \omega^2\dfrac{LC}{G}\right)}{G\left(G + \omega^2\dfrac{C^2}{G}\right)}}$$

$$= \sqrt{\frac{R}{G}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Z_o is a pure resistance ; the voltage and current in an infinitely long line are in phase.

It is of interest to note that the formulæ derived give for an ideal open line a velocity of propagation equal to the velocity of light. With $R = 0$ and $G = 0$ (18) and (19) become $\alpha = 0$ and $\beta = \omega\sqrt{LC}$, and

$$c = \frac{1}{\sqrt{LC}},$$

as in the distortionless case.

If the wires of the loop are of radius r cm., and the distance apart of the wires (large compared with r) is d cm., the wire to wire capacity per centimetre is given by

$$C = \frac{\kappa}{4 \log_e \frac{d}{r}} \quad \text{electrostatic units}$$

$$= \frac{\kappa}{4 \log_e \frac{d}{r}} \times \frac{1}{9 \times 10^{11}} \text{ . farads,}$$

and the inductance of the loop per centimetre by

$$L = 4\mu \log_e \frac{d}{r} \quad \text{electromagnetic units}$$

$$= 4\mu \log_e \frac{d}{r} \times 10^{-9} \quad \text{henries.}$$

Hence $$c = \frac{1}{\sqrt{\kappa\mu}} \, . \, 3 \times 10^{10} \quad \text{cms./second,}$$

and with $\kappa = 1$ and $\mu = 1$ as in air, this is the velocity of light.

To measure the primary constants of a telephone line, complex impedance measurements may be made at one end of a length l with the far end insulated (Z_f) and with the far end shorted (Z_g). By 8·10 and 8·15

$$\left.\begin{aligned} Z_f &= Z_0 \coth Pl \\ Z_g &= Z_0 \tanh Pl \end{aligned}\right\} \quad \ldots \ldots \quad (26)$$

Hence $$Z_0 = \sqrt{Z_f Z_g}$$

and $$\left.\tanh Pl = \sqrt{\frac{Z_g}{Z_f}}\right\} \quad \ldots \ldots \quad (27)$$

determine Z_0 and P from the measurements.

Finally, $$\left.\begin{aligned} Z_0 P &= R + j\omega L \\ \frac{P}{Z_0} &= G + j\omega C \end{aligned}\right\} \quad \ldots \ldots \quad (28)$$

(from (9) and (10)) are used to determine R L G and C by equating real and imaginary terms.

Equations for the fall of potential and current along a line under different end conditions can be written down from the corresponding equation in section 8 as follows :—

(i.) Line insulated at end (8·12 and 8·13).

$$V = V_s \frac{\cosh P(l-x)}{\cosh Pl} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

$$I = \frac{V_s}{Z_0} \frac{\sinh P(l-x)}{\cosh Pl} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

(ii.) Line shorted at end (8·16 and 8·17)

$$V = V_s \frac{\sinh P(l-x)}{\sinh Pl} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

$$I = \frac{V_s}{Z_0} \frac{\cosh P(l-x)}{\sinh Pl} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

(iii.) Line closed at end through impedance Z_r (8·18, 8·19, 8·20 and 8·21).

$$V = V_s \cdot \frac{\sinh P(l-x) + \dfrac{Z_r}{Z_0} \cosh P(l-x)}{\sinh Pl + \dfrac{Z_r}{Z_0} \cosh Pl} \quad . \quad . \quad . \quad (33)$$

$$I = \frac{V_s}{Z_0} \cdot \frac{\cosh P(l-x) + \dfrac{Z_r}{Z_0} \sinh P(l-x)}{\sinh Pl + \dfrac{Z_r}{Z_0} \cosh Pl} \quad . \quad . \quad . \quad (34)$$

or

$$V = V_s \frac{\sinh \{P(l-x) + \theta\}}{\sinh (Pl + \theta)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35)$$

$$I = \frac{V_s}{Z_0} \frac{\cosh \{P(l-x) + \theta\}}{\sinh (Pl + \theta)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

where

$$\tanh \theta = \frac{Z_r}{Z_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

If $Z_r = Z_0$ the variations of potential and current are the same as in the infinitely long line.

Equations 8·22 and 8·23 are not required, as a value for θ can always be found from (37).

Equivalent T and Π networks of the telephone line consisting of impedances determined by the line constants can be found by the methods of section 9. The equivalent T has (Fig. 128)

$$\left.\begin{aligned} Z_2 &= \frac{Z_0}{\sinh Pl} \\ Z_1 &= \frac{Z_0}{\sinh Pl}(\cosh Pl - 1) = Z_0 \tanh \frac{Pl}{2} \end{aligned}\right\} \quad . \quad . \quad . \quad (38)$$

from 9·05, while the equivalent Π (Fig. 129) has (from 9·08)

$$\left.\begin{aligned} Z_1 &= Z_0 \sinh Pl \\ Z_2 &= \frac{Z_0}{\sinh Pl}(\cosh Pl + 1) = Z_0 \coth \frac{Pl}{2} \end{aligned}\right\} \quad . \quad . \quad . \quad (39)$$

These equivalent circuits are of general value when only the voltages and currents at the ends of a line are of interest.

If an alternating e.m.f. E_s acts through an impedance Z_s at the

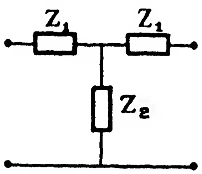


FIG. 128.—Equivalent T.

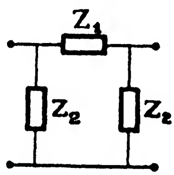


FIG. 129.—Equivalent Π.

sending end, the currents at the receiving and sending ends are given by (9·09 and 9·10)

$$I_r = \frac{E_s}{(Z_s + Z_r)\cosh Pl + \left(Z_0 + \frac{Z_s Z_r}{Z_0}\right)\sinh Pl} \quad . \quad . \quad (40)$$

$$I_s = E_s \frac{Z_0 \cosh Pl + Z_r \sinh Pl}{Z_0(Z_s + Z_r)\cosh Pl + (Z_0^2 + Z_s Z_r)\sinh Pl} \quad . \quad (41)$$

Numerical values from these equations involving hyperbolic functions of complex quantities can be obtained by expanding the function. For instance, equation (29) can be written

$$V = V_s \frac{\cosh \alpha(l - x)\cos\beta(l - x) + j\sinh\alpha(l - x)\sin\beta(l - x)}{\cosh\alpha l\cos\beta l + j\sinh\alpha l\sin\beta}$$

involving hyperbolic functions and circular functions of real quantities only, and readily evaluated from ordinary mathematical tables.

But the work is enormously simplified by the use of special tables and charts that have been prepared by Professor A. E. Kennelly, the use of which is explained in Appendix 7.

Using these tables and equation (29), Figs. 130 and 131 have been drawn for the aerial line of the previous example (Figs. 126 and 127) with an impressed voltage of 10 volts maximum, open circuited at 180 miles. The curve of Fig 130 is the locus of the potential rotating vector V as the distance along the cable is varied ; 0—0, 0—20, 0—40, etc., being the vectors at distances 0, 20, 40, etc.,

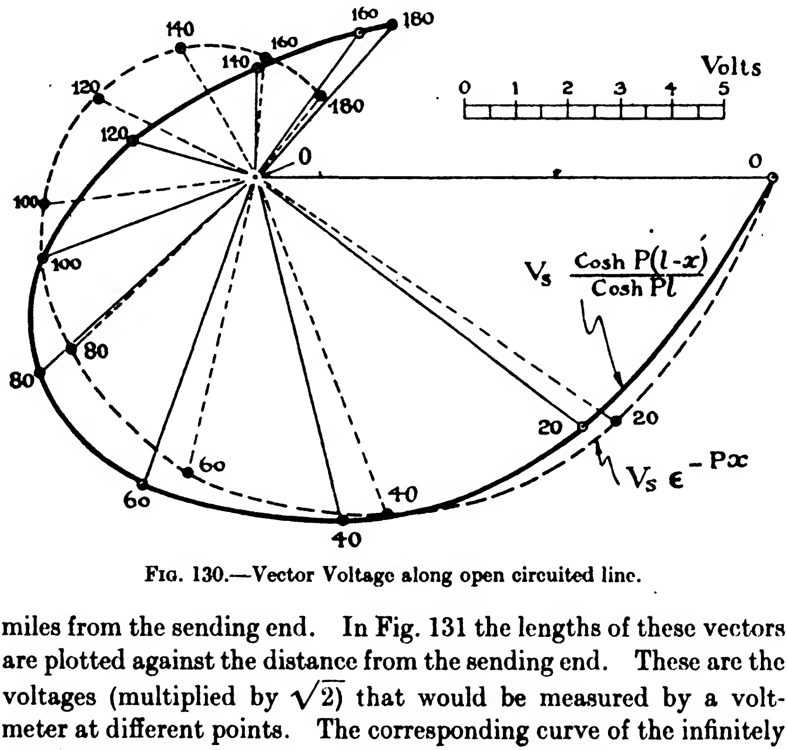


FIG. 130.—Vector Voltage along open circuited line.

miles from the sending end. In Fig. 131 the lengths of these vectors are plotted against the distance from the sending end. These are the voltages (multiplied by $\sqrt{2}$) that would be measured by a voltmeter at different points. The corresponding curve of the infinitely long line is drawn dotted for comparison in each figure.

The distortion of the logarithmic spiral and of the logarithmic curve is due to reflections from the ends. On arrival at the end of the line the voltage has become $V_s\epsilon^{-Pl}$ by equation (13). At the open end it is reflected without change of sign and travels back again towards the sending end, its propagation being determined by $V_s\epsilon^{-P(2l-x)}$. Thus at any point x the incident wave voltage is

$V_s\epsilon^{-Px}$ and the reflected wave voltage is $V_s\epsilon^{-P(2l-x)}$, and the actual resultant is the sum of the two. This is brought out in Fig. 132. 100′—120′—140′—160′—180′ is the logarithmic spiral of the incident wave, and 180″—160″—140″—120″—100″ is the logarithmic spiral of the reflected wave. The resultant vector locus is found in the usual way by adding by the parallelogram construction the two vectors at each point. Thus at 180 miles (the end of the line) the addition is a straight line one, and the voltage is

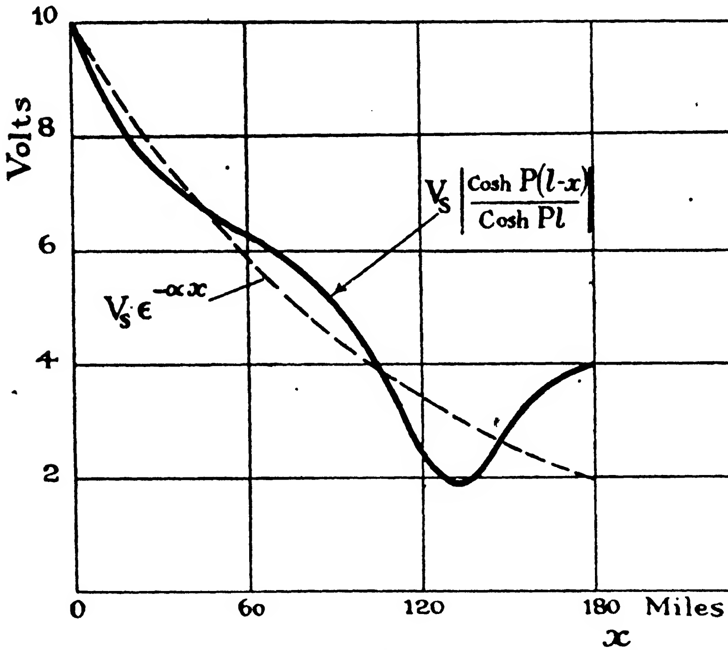


FIG. 131.—Maximum Voltage along open circuited line.

double that of the infinite line and in phase with it, at 160 miles 0—160′ added vectorially to 0—160″ gives 0—160, and so on. The resulting locus 100—120—140—160—180 is seen to be of nearly the same shape as the locus of Fig. 130 as determined from formula (29).

What difference there is is due to further reflections. On arrival at the sending end the reflected wave is again reflected, this time with change of sign, and arrives at the point x as $-V_s\epsilon^{-P(2l+x)}$. At the open end it is again reflected without change of sign, and arrives at the point x as $-V_s\epsilon^{-P(4l-x)}$. Thus an infinite number of vectors have to be added to find the resultant vector at x, the series being

$$V = V_s\epsilon^{-Px} + V_s\epsilon^{-P(2l-x)} - V_s\epsilon^{-P(2l+x)} - V_s\epsilon^{-P(4l-x)}$$
$$+ V_s\epsilon^{-P(4l+x)} + V_s\epsilon^{-P(6l-x)} \quad . \quad . \quad . \quad .$$

This can be re-written

$$V = V_s\epsilon^{-Px}\{1 - \epsilon^{-2Pl} + \epsilon^{-4Pl} \quad . \quad . \quad . \quad . \quad . \quad . \}$$
$$+ V_s\epsilon^{+Px}\{\epsilon^{-2Pl} - \epsilon^{-4Pl} + \epsilon^{-6Pl} \quad . \quad . \quad . \quad . \quad . \}$$

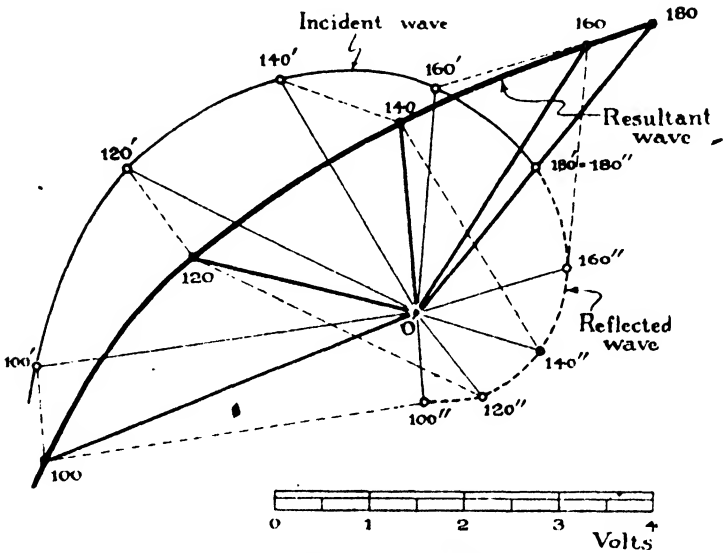


FIG. 132.—Reflection at open end.

which, on summing the geometric series within the brackets, becomes

$$V = V_s\epsilon^{-Px} \cdot \frac{1}{1+\epsilon^{-2Pl}} + V_s\epsilon^{Px}\,\frac{\epsilon^{-2Pl}}{1+\epsilon^{-2Pl}}$$

$$= V_s\epsilon^{-Px} \cdot \frac{\epsilon^{Pl}}{\epsilon^{Pl}+\epsilon^{-Pl}} + V_s\epsilon^{Px} \cdot \frac{\epsilon^{-Pl}}{\epsilon^{Pl}+\epsilon^{-Pl}}$$

$$= V_s\,\frac{\epsilon^{P(l-x)}+\epsilon^{-P(l-x)}}{\epsilon^{Pl}+\epsilon^{-Pl}}$$

$$= V_s\,\frac{\cosh P(l-x)}{\cosh Pl}$$

a result identical with equation (29).

In the same way it could be shown that all the equations involving the hyperbolic functions of complex quantities take account of the multiple reflections at the ends of the line.

When the line is so long electrically that only the first reflection is of significance the equations simplify. For instance, equation (29) with $x = l$, that is, for the voltage at the receiving end of the line, is

$$V = V_s \cdot \frac{1}{\cosh Pl}$$

$$= V_s \cdot \frac{2}{\epsilon^{Pl} + \epsilon^{-Pl}}.$$

If Pl is great, ϵ^{-Pl} is negligible in comparison with ϵ^{Pl}, and

$$V = 2V_s\epsilon^{-Pl};$$

the voltage has twice the value it would have at the same point in the infinitely long line. In other words, the voltage has been doubled by reflection. In a similar manner, if the line is short-circuited at the end, equation (32) shows that the current is doubled by reflection. If on the other hand the line is closed through its characteristic impedance there is no reflection, and the current has the value it would have in the infinitely long line. It often happens in practice that the line is long enough and the closing impedance is sufficiently near the characteristic impedance for calculations made on the assumption of no reflection to give a sufficiently close approximation. In this way the total attenuation of a number of lines joined in series is generally given with sufficient accuracy by the sum $\Sigma(al)$ of the attenuation of the various sections.

(40) Artificial Lines and Filters

Artificial lines simulating telephone lines may be built up in the manner described in section 11 for the leaky telegraph line, and in Figs. 35 and 36. R is replaced by $(R + j\omega L)$ and G by $(G + j\omega C)$, and the whole of the equations in the section apply with these alterations, involving also a complex propagation constant γ per link or section, and a complex characteristic impedance Z_0 instead of R_0.

Such networks have an interest quite apart from their use as artificial telephone lines, as when suitably designed they may be used as frequency filters.

(i.) *Low Pass Filter.*

Consider, for instance, the network of Fig. 133, built up of T links with series inductance L and resistance R, and shunting capacity C. For substitution in the equations of section 11, $R + j\omega L$ must be written for R, and $j\omega C$ for G. The propagation constant per link γ is therefore determined (equations 11·07) by

$$\cosh \gamma = 1 + \frac{1}{2}(R + j\omega L)(j\omega C)$$

$$= 1 - \frac{1}{2}\omega^2 LC + \frac{1}{2}j\omega CR \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Writing $\gamma = \alpha + j\beta$ and expanding $\cosh \gamma$ gives

$$\cosh \alpha \cos \beta + j \sinh \alpha \sin \beta = 1 - \frac{1}{2}\omega^2 LC + \frac{1}{2}j\omega CR \quad . \quad . \quad (2)$$

In order to consider the effect of the frequency on the attenuation the resistance R may be made negligibly small, and equating real and imaginary terms,

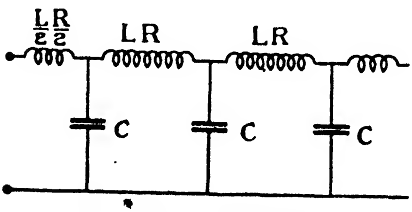


FIG. 133.—Low Pass Filter.

$$\cosh \alpha \cos \beta = 1 - \frac{1}{2}\omega^2 LC \quad (3)$$

$$\sinh \alpha \sin \beta = 0 \quad . \quad . \quad . \quad (4)$$

The second of these may be solved by either $\sinh \alpha = 0$ or $\sin \beta = 0$. If $\sinh \alpha = 0$, $\alpha = 0$ and $\cosh \alpha = 1$, and the first becomes

$$\cos \beta = 1 - \frac{1}{2}\omega^2 LC \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This is possible for low frequencies, but impossible when ω becomes great enough to make $\cos \beta$ more negative than -1. The limiting value ω_1 of ω is accordingly given by

$$1 - \frac{1}{2}\omega_1^2 LC = -1$$

or

$$\omega_1 = \frac{2}{\sqrt{LC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Below this frequency the attenuation is zero, and β is increasing with frequency from 0 to π. Above, β remains at π, so that $\sin\beta = 0$ is the solution of (4); and from (3)

$$-\cosh a = 1 - \frac{1}{2}\omega^2 LC$$

a accordingly increases rapidly with further increase of ω.

The curve of a plotted against ω is as shown by a, Fig. 134. The effect of including the resistance R is to round off the sharp corner to give the curve b. The greater the resistance the less marked is the change of attenuation at the frequency ω_1.

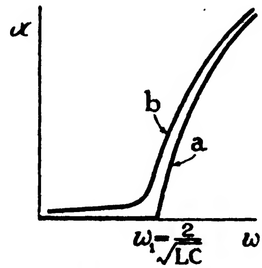


FIG. 134.—Attenuation of Low Pass Filter.

The general effect is to allow all frequencies below ω_1 to pass readily through the network, but greatly to attenuate all frequencies above ω_1. The network is accordingly called a *low pass filter*, and the frequency ω_1 is called the *cut-off frequency*.

(ii.) *High Pass Filter.*

The network of Fig. 135, on the other hand, allows high frequencies to pass readily, but offers a high attenuation to low frequencies. In this case the series impedance is $-j/\omega C$ and the shunting admittance is $1/(R + j\omega L)$ and γ is determined by

$$\cosh\gamma = 1 + \frac{1}{2}\left(-\frac{j}{\omega C}\right)\left(\frac{1}{R + j\omega L}\right) \quad . \quad . \quad . \quad (7)$$

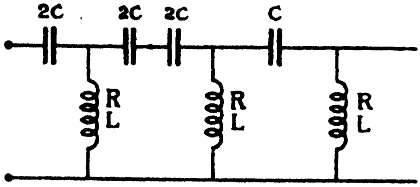


FIG. 135.—High Pass Filter.

Proceeding as before by expanding $\cosh\gamma$, putting $R = 0$, and equating reals and imaginaries,

$$\cosh a \cos\beta = 1 - \frac{1}{2\omega^2 LC} \quad (8)$$

$$\sinh a \sin\beta = 0 \quad . \quad . \quad . \quad (9)$$

(9) can be solved by $\sinh a = 0$, which makes $a = 0$, $\cosh a = 1$ and (8) becomes

$$\cos\beta = 1 - \frac{1}{2\omega^2 LC} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

provided that ω is large enough to make (10) possible. The limiting value is given by

$$1 - \frac{1}{2\omega_1^2 LC} = -1$$

or

$$\omega_1 = \frac{1}{2\sqrt{LC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

As ω decreases from a very large value to the cut-off frequency ω_1, $\cos\beta$ changes from 1 to -1, and β from 0 to π. Below the cut-off, β remains π, and a increases as shown at a in Fig. 136. The effect of resistance is as before to round off the sharp corner as at b.

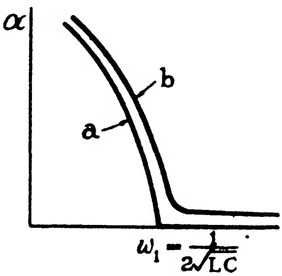


FIG. 136.—Attenuation of High Pass Filter.

(iii.) *Band Pass Filter.*

It is clear that if two networks such as 133 and 135 are joined in series, the cut-off frequencies can be so arranged as to pass a band of frequencies of any desired width, and cut off all frequencies both above and below the band. The same result may be achieved more simply by the network of Fig. 137. In this case equation 11·07 becomes

$$\cosh\gamma = 1 + \frac{1}{2}\left(R + j\omega L - \frac{j}{\omega K}\right)(j\omega C)$$

$$= 1 - \frac{1}{2}\omega^2 LC + \frac{1}{2}\frac{C}{K} + \frac{1}{2}j\omega CR \quad . \quad . \quad . \quad . \quad (12)$$

Proceeding as before, cut-off is determined by

$$\cosh a \cos\beta = 1 - \frac{1}{2}\omega^2 LC + \frac{1}{2}\frac{C}{K} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$\sinh a \sin\beta = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

With $a = 0$,

$$\cos\beta = 1 - \frac{1}{2}\omega^2 LC + \frac{1}{2}\frac{C}{K} \quad . \quad . \quad . \quad . \quad (15)$$

which is evidently impossible with very high and very low values of ω. There are two cut-off frequencies, which are determined from

$$\cos\beta = 1 = 1 - \frac{1}{2}\omega_1^2 LC + \frac{1}{2}\frac{C}{K}$$

whence

$$\omega_1 = \frac{1}{\sqrt{LK}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and
$$\cos\beta = -1 = 1 - \frac{1}{2}\,\omega_2{}^2LC + \frac{1}{2}\frac{C}{K}$$

whence

$$\omega_2 = \sqrt{\frac{4}{LC} + \frac{1}{LK}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The attenuation curve is as shown in Fig. 138 ; a is the curve with $R = 0$, and b shows how resistance rounds off the sharp corners.

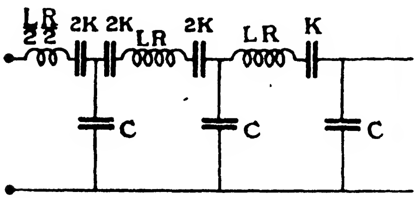


FIG. 137.—Band Pass Filter.

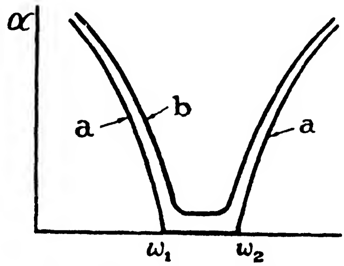


FIG. 138.—Attenuation of Band Pass Filter.

In the ideal case of $R = 0$, as ω increases from 0 to ω_1, $\sin\beta = 0$ and $\beta = 0$. From ω_1 to ω_2, β changes from 0 to π, and from ω_2 upwards β remains π.

(iv.) *Chain of Resonant Circuits.*

A chain of resonant circuits (Fig. 139) also has band pass characteristics. As shown in section 35, the coupled coils can be replaced by equivalent T's, and the condensers C may be replaced by two series condensers 2C, and the T network of Fig. 140 results, with series impedance

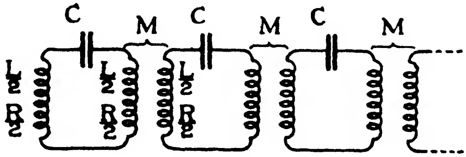


FIG. 139.—Chain of Circuits.

$$R + j\omega\,(L - 2M) - j/\omega C$$

and shunting admittance $1/j\omega M$. Equation 11·07 gives therefore

$$\cosh \gamma = 1 + \frac{1}{2}\left\{ R + j\omega(L - 2M) - \frac{j}{\omega C} \right\} \left(\frac{1}{j\omega M}\right)$$

$$= 1 + \frac{L - 2M}{2M} - \frac{1}{2\omega^2 CM} + \frac{1}{2}\frac{R}{j\omega M}$$

$$= \frac{L}{2M} - \frac{1}{2\omega^2 CM} + \frac{1}{2}\frac{R}{j\omega M} \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

With $R = 0$,

$$\left.\begin{aligned} \cosh \alpha \cos \beta &= \frac{L}{2M} - \frac{1}{2\omega^2 CM} \\ \sinh \alpha \sin \beta &= 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

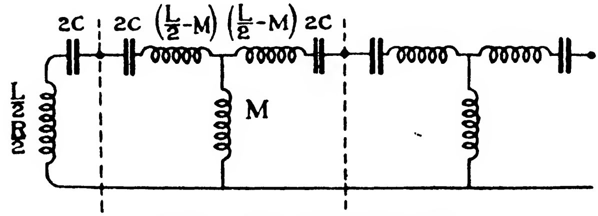


Fig. 140.—Equivalent to Fig. 139.

and the cut-off frequencies are obtained from

$$\frac{L}{2M} - \frac{1}{2\omega^2 CM} = \pm 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

or

$$\left.\begin{aligned} \omega &= \frac{1}{\sqrt{C(L \pm 2M)}} \\ \textit{i.e.,} \qquad \omega_1 &= 1/\sqrt{C(L + 2M)} \\ \text{and} \qquad \omega_2 &= 1/\sqrt{C(L - 2M)} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

and the curve obtained is of the same shape as that of Fig. 138.

In all these four cases, having determined γ for any frequency, Z_0 is found from 11·08, and with terminal impedances Z_s and Z_r, and with a chain of m links, the current I_r in the impedance Z_r due to an electromotive force E_s acting in Z_s is given (equation 9·09) by

$$I = \frac{E_s}{(Z_s + Z_r)\cosh m\gamma + \left(Z_0 + \frac{Z_s Z_r}{Z_0}\right)\sinh m\gamma} \quad . \quad . \quad . \quad (22)$$

Writing for a moment R′ for the series impedance of the T link, and G′ for its shunting admittance, from 11·08

$$\left.\begin{aligned}
Z_o &= \frac{R'}{2\tanh\frac{1}{2}\gamma} = \frac{R'}{2}\,\frac{\sqrt{\cosh\gamma+1}}{\sqrt{\cosh\gamma-1}} \\
&= \frac{R'}{2}\,\frac{\sqrt{2+\frac{1}{2}R'G'}}{\sqrt{\frac{1}{2}R'G'}} = \frac{1}{G'}\sqrt{R'G' + \frac{1}{4}R'^2G'^2} \\
&= \frac{1}{G'}\sqrt{\left(1+\frac{1}{2}R'G'\right)^2 - 1} \\
&= \frac{1}{G'}\sqrt{\cosh^2\gamma - 1} = \frac{\sinh\gamma}{G'}
\end{aligned}\right\} \quad (23)$$

Examination of the termination of the chain of Fig. 140 shows that

$$Z_s = Z_r = \frac{R'}{2} + \frac{1}{G'} = \frac{R'G'+2}{2G'} = \frac{1}{G'}\cosh\gamma. \quad . \quad . \quad . \quad (24)$$

Using (23)

$$Z_o + \frac{Z_sZ_r}{Z_o} = \frac{1}{G'}\sinh\gamma + \frac{1}{G'}\frac{\cosh^2\gamma}{\sinh\gamma} = \frac{1}{G'}\cdot\frac{\cosh 2\gamma}{\sinh\gamma} \quad . \quad . \quad . \quad . \quad . \quad (25)$$

and substitution of (23), (24) and (25) in (22) gives

$$\begin{aligned}
I_r &= \frac{E_s}{\frac{2}{G'}\cosh\gamma\cosh m\gamma + \frac{1}{G'}\,\frac{\cosh 2\gamma}{\sinh\gamma}\sinh m\gamma} \\
&= E_s\,.\,\frac{G'\sinh\gamma}{\sinh(m+2)\gamma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)
\end{aligned}$$

In the case of the chain of Fig. 140, $G' = 1/j\omega M$, and

$$I_r = \frac{E_s\sinh\gamma}{j\omega M\sinh(m+2)\gamma} \quad . \quad . \quad . \quad . \quad . \quad (27)$$

In applying this to the chain of resonant circuits of Fig. 139 it must be noticed that m is one less than the number of resonant links.

The form of solution (26) for the received current will hold in the other cases also provided (24) is satisfied. This involves in Fig. 133 a termination of L/2, R/2 and C in series, in Fig. 135 a termination of 2C, L and R in series, and in Fig. 137 a termination of R/2, L/2, 2K and C in series.

(41) Loading

Oliver Heaviside first pointed out (in 1887) that an increase in the inductance of a telephone line would result in a decrease of its attenuation constant and a decrease in the frequency distortion, as noted in section 39, but no practical use of his mathematical investigations was made until M. I. Pupin, in 1899 and 1900, showed that beneficial results could be obtained by the insertion of inductive coils at intervals along the line, provided that the distance apart of the coils was not too great. To-day long underground telephone conductors are almost invariably "loaded" by the insertion of iron-cored "loading coils."

The effects of the insertion of loading coils of impedance Z_l ohms

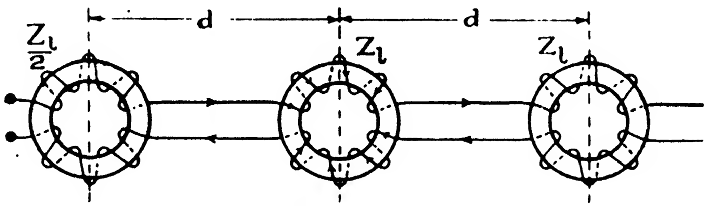


FIG. 141.—Loaded line with half-coil termination.

(resistance R_l ohms and inductance L henries) at distances d miles apart in a telephone line of propagation constant P per mile and characteristic impedance Z_0 can be examined by the results of the last section. Two cases arise, the first, Fig. 141, in which the termination is by a half-loading coil, and the second, Fig. 143, in which the termination is by a half-loading section length of cable.

The loading coils, as indicated in Fig. 141, are wound on an iron core of toroidal shape, and the coils are evenly distributed in the two lines of the loop, the windings being such that the magnetic effects of the "go" and "return" currents are additive. Z_l is the total impedance of the two windings.

(i.) *Half-coil Termination.*

The network of Fig. 142 results on replacing the lengths of cable d by their equivalent T's, where the impedances Z_1 and Z_2 are obtained from 39·38 as

$$Z_1 = \frac{Z_0}{\sinh \mathrm{P}d}(\cosh \mathrm{P}d - 1) \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$Z_2 = \frac{Z_0}{\sinh \mathrm{P}d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Fig. 142 is thus a chain conductor with a T link (shown between the dotted lines) having

$$\mathrm{R}' = \frac{2Z_0}{\sinh \mathrm{P}d}(\cosh \mathrm{P}d - 1) + Z_l \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and
$$\mathrm{G}' = \frac{\sinh \mathrm{P}d}{Z_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

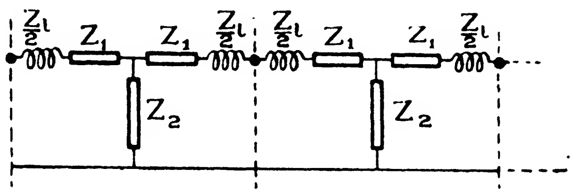


FIG. 142.—Equivalent network of Fig. 141.

Calling γ the propagation constant of the chain per link (*i.e.*, of the loaded line per d miles), γ is given from 11·07 and (3) and (4) by

$$\cosh \gamma = 1 + \frac{1}{2}\mathrm{R}'\mathrm{G}'$$

$$= 1 + \frac{1}{2}\left\{\frac{2Z_0}{\sinh \mathrm{P}d}(\cosh \mathrm{P}d - 1) + Z_l\right\}\left\{\frac{\sinh \mathrm{P}d}{Z_0}\right\}$$

$$i.e., \cosh \gamma = \cosh \mathrm{P}d + \frac{Z_l}{2Z_0}\sinh \mathrm{P}d \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

This formula was first given by G. A. Campbell,* and is frequently referred to as Campbell's formula. Upon examination it shows that the loaded line has a low pass filter characteristic. For writing $\gamma = \alpha_1 + j\beta_1$, and substituting for P and Z_0 their values in terms of the primary line constants, 39·09 and 39·10, (5) becomes

$$\cosh \alpha_1 \cos \beta_1 + j \sinh \alpha_1 \sin \beta_1 = \cosh d\sqrt{(\mathrm{R} + j\omega \mathrm{L})(\mathrm{G} + j\omega \mathrm{C})}$$

$$+ \frac{Z_l}{2}\sqrt{\frac{\mathrm{G} + j\omega \mathrm{C}}{\mathrm{R} + j\omega \mathrm{L}}}\sinh d\sqrt{(\mathrm{R} + j\omega \mathrm{L})(\mathrm{G} + j\omega \mathrm{C})} \quad . \quad (6)$$

* *Phil. Mag.*, Vol. V., p. 319 (1903).

Taking a no-loss line as before in order to determine the cut-off frequency, *i.e.*, putting $\mathrm{R} = 0$, $\mathrm{G} = 0$ and $\mathrm{Z}_l = j\omega\mathrm{L}_l$, (6) reduces to

$$\cosh a_1 \cos \beta_1 + j \sinh a_1 \sin \beta_1 = \cos \omega\sqrt{\mathrm{LC}}\,d - \frac{\omega \mathrm{L}_l}{2}\sqrt{\frac{\mathrm{C}}{\mathrm{L}}}\sin \omega\sqrt{\mathrm{LC}}\,d \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and equating reals and imaginaries,

$$\cosh a_1 \cos \beta_1 = \cos \omega\sqrt{\mathrm{LC}}\,d - \frac{\omega \mathrm{L}_l}{2}\sqrt{\frac{\mathrm{C}}{\mathrm{L}}}\sin \omega\sqrt{\mathrm{LC}}\,d \quad . \quad (8)$$

$$\sinh a_1 \sin \beta_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

whence it is clear that a_1 can be zero if

$$\cos \beta_1 = \cos \omega\sqrt{\mathrm{LC}}\,d - \frac{\omega \mathrm{L}_l}{2}\sqrt{\frac{\mathrm{C}}{\mathrm{L}}}\sin \omega\sqrt{\mathrm{LC}}\,d \quad . \quad . \quad . \quad (10)$$

Now $\omega\sqrt{\mathrm{LC}}\,d$ is in a practical case a small angle, and $\cos \omega\sqrt{\mathrm{LC}}\,d$ may be replaced by unity and $\sin \omega\sqrt{\mathrm{LC}}\,d$ by $\omega\sqrt{\mathrm{LC}}\,d$, leading to

$$\cos \beta_1 = 1 - \frac{\omega^2 \mathrm{L}_l \mathrm{C}\,d}{2}$$

This is possible for small values of ω, and the limiting value ω_1 is given when $\cos \beta_1 = -1$, *i.e.*,

$$-1 = 1 - \frac{\omega_1{}^2 \mathrm{L}_l \mathrm{C} d}{2}$$

whence

$$\omega_1 = \frac{2}{\sqrt{\mathrm{L}_l \mathrm{C}\,d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

Above this frequency the attenuation increases rapidly in a similar manner to the attenuation in a low-pass filter (Fig. 134), and it is necessary to design the loading so that $\omega_1/2\pi$ is not less than the highest frequency essential to commercial speech, which is usually taken to be 2,000 cycles per second.

It is of interest to notice that theoretically equation (10) will give rise to an infinitely large number of critical frequencies, and the line will transmit an infinitely large number of bands of frequencies. But in practice these bands are all too high to be of any importance.

Provided that ω is well below the cut-off frequency, γ as well as $\mathrm{P}d$ are small quantities, and (5) may be written

$$1 + \frac{\gamma^2}{2} = 1 + \frac{\mathrm{P}^2 d^2}{2} + \frac{\mathrm{Z}_l}{2\mathrm{Z}}\,\mathrm{P}d$$

whence
$$\left(\frac{\gamma}{d}\right)^2 = \mathrm{P}^2 + \frac{\mathrm{Z}_l}{\mathrm{Z}_o d}\mathrm{P}$$
$$= (\mathrm{R} + j\omega\mathrm{L})(\mathrm{G} + j\omega\mathrm{C}) + \frac{\mathrm{Z}_l}{d}(\mathrm{G} + j\omega\mathrm{C})$$

Or, writing P′ for the propagation constant per mile of the loaded line,

$$\mathrm{P}' = \frac{\gamma}{d} = \sqrt{\left(\mathrm{R} + j\omega\mathrm{L} + \frac{\mathrm{Z}_l}{d}\right)(\mathrm{G} + j\omega\mathrm{C})}$$
$$= \sqrt{\left\{\mathrm{R} + \frac{\mathrm{R}_l}{d} + j\omega\left(\mathrm{L} + \frac{\mathrm{L}_l}{d}\right)\right\}\{\mathrm{G} + j\omega\mathrm{C}\}} \quad . \ (12)$$

showing that the calculation of P′ may be made by the ordinary line formula 39·09, with the assumption that the loading coil resistance and inductance are spread out uniformly along the line.

In order to calculate the characteristic impedance Z_o' of the loaded line, the most convenient formula (from 40·23) is

$$\mathrm{Z}_o' = \sqrt{\mathrm{R}'\left(\frac{1}{\mathrm{G}'} + \frac{\mathrm{R}'}{4}\right)} \quad . \ . \ . \ . \ . \ . \ (13)$$

which gives on substituting from (3) and (4)

$$\mathrm{Z}_o' = \sqrt{\left\{\frac{2\mathrm{Z}_o}{\sinh \mathrm{P}d}(\cosh \mathrm{P}d - 1) + \mathrm{Z}_l\right\}}$$
$$\times \sqrt{\left\{\frac{\mathrm{Z}_o}{\sinh \mathrm{P}d} + \frac{\mathrm{Z}_o}{2\sinh \mathrm{P}d}(\cosh \mathrm{P}d - 1) + \frac{\mathrm{Z}_l}{4}\right\}}$$

reducing to

$$\mathrm{Z}_o' = \mathrm{Z}_o\sqrt{1 + \frac{\mathrm{Z}_l^2}{4\mathrm{Z}_o^2} + \frac{\mathrm{Z}_l}{\mathrm{Z}_o}\coth \mathrm{P}d} \quad . \ . \ . \ . \ (14)$$

This allows an accurate calculation. Replacing coth Pd by 1/Pd, as Pd is small, (14) gives

$$\mathrm{Z}_o' = \sqrt{\mathrm{Z}_o^2 + \mathrm{Z}_l\frac{\mathrm{Z}_o}{\mathrm{P}d} + \mathrm{Z}_l^2}.$$

and neglecting Z_l^2 as small compared with the other terms,

$$\mathrm{Z}_o' = \sqrt{\frac{\mathrm{R} + j\omega\mathrm{L}}{\mathrm{G} + j\omega\mathrm{C}} + \frac{\mathrm{Z}_l}{d}\cdot\frac{1}{\mathrm{G} + j\omega\mathrm{C}}},$$

i.e.,
$$\mathrm{Z}_o' = \sqrt{\frac{\left(\mathrm{R} + \frac{\mathrm{R}_l}{d}\right) + j\omega\left(\mathrm{L} + \frac{\mathrm{L}_l}{d}\right)}{\mathrm{G} + j\omega\mathrm{C}}} \quad . \ . \ . \ . \ (15)$$

or the characteristic impedance, as well as the propagation constant, of the loaded line can be calculated approximately from the usual line formula with the assumption that the loading coil resistance and inductance are spread out uniformly along the line.

(ii.) *Termination by Half loading-coil Section*

This case is shown in Fig. 143, and the simplest equivalent circuit leading to symmetrical links is drawn in Fig. 144. Each

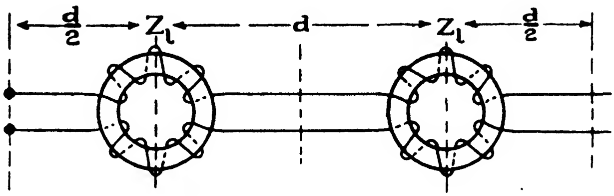


FIG. 143.—Loaded line with half loading-coil section termination.

link consists of the equivalent T of a length $d/2$ miles of the cable, the loading coil impedance Z_l, and another T representing a length $d/2$ of the cable, all in series. The values of Z_1 and Z_2 are given from 39·38 by

$$\left.\begin{aligned} Z_1 &= \frac{Z_0}{\sinh\frac{Pd}{2}}\left(\cosh\frac{Pd}{2} - 1\right) \\ Z_2 &= \frac{Z_0}{\sinh\frac{Pd}{2}} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (16)$$

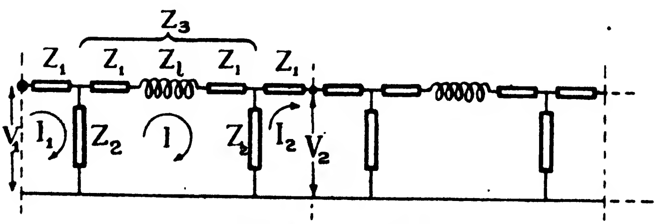


FIG. 144.—Equivalent network of Fig. 143.

The chain of Fig. 144 can be replaced by a chain of T links as in Fig. 145. To find the values of Z' and Z'', the equations connecting the voltages and currents at the beginning and end of a link of

Fig. 144 must be identical with the corresponding equations for the T-link of Fig. 145.

In the first case

$$\left.\begin{aligned} V_1 &= (Z_1 + Z_2)I_1 - Z_2 I \\ 0 &= (2Z_2 + Z_3)I - Z_2(I_1 + I_2) \\ 0 &= (Z_2 + Z_1)I_2 - Z_2 I + V_2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (17)$$

(where Z_3 is written for $2Z_1 + Z_l$) and eliminating I.

$$\left.\begin{aligned} V_1 &= \left(Z_1 + Z_2 - \frac{Z_2^2}{2Z_2 + Z_3}\right) I_1 - \frac{Z_2^2}{2Z_2 + Z_3} I_2 \\ 0 &= \left(Z_1 + Z_2 - \frac{Z_2^2}{2Z_2 + Z_3}\right) I_2 - \frac{Z_2^2}{2Z_2 + Z_3} I_1 + V_2 \end{aligned}\right\} \quad . \quad (18)$$

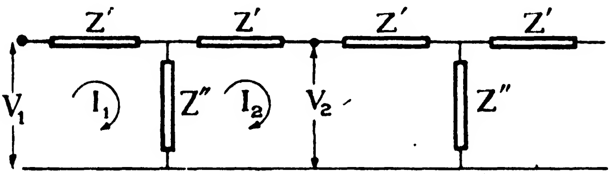


FIG. 145.—Equivalent network of Fig. 144.

In the second case

$$\left.\begin{aligned} V_1 &= (Z' + Z'')\, I_1 - Z'' I_2 \\ 0 &= (Z' + Z'')I_2 - Z'' I_1 + V_2 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (19)$$

In order that (18) and (19) may be identical

$$\left.\begin{aligned} Z'' &= \frac{Z_2^2}{2\,(Z_1 + Z_2) + Z_l} \\ Z' &= Z_1 + Z_2 - \frac{2Z_2^2}{2\,(Z_1 + Z_2) + Z_l} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad (20)$$

The propagation constant of the chain is found as before from

$$\cosh \gamma = 1 + \frac{1}{2} R'G' = 1 + \frac{1}{2} \frac{(2Z')}{Z''},$$

which, on substitution from (20), gives

$$\cosh \gamma = \frac{(Z_1 + Z_2)\, \{2(Z_1 + Z_2) + Z_l\}}{Z_2^2} - 1$$

and on substitution from (16)

$$\cosh\gamma = \frac{Z_o \coth\frac{Pd}{2}\left(2Z_o \coth\frac{Pd}{2} + Z_l\right)}{Z_o^2/\sinh^2\frac{Pd}{2}} - 1$$

$$= \cosh\frac{Pd}{2}\left(2\cosh\frac{Pd}{2} + \frac{Z_l}{Z_o}\sinh\frac{Pd}{2}\right) - 1$$

i.e.,
$$\cosh\gamma = \cosh Pd + \frac{Z_l}{2Z_o}\sinh Pd \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

which is Campbell's formula as obtained before.

For the characteristic impedance Z_o' of the loaded line (13) can be used again.

$$Z_o' = \sqrt{R'\left(\frac{1}{G'} + \frac{R'}{4}\right)}$$

$$= \sqrt{2Z'\left(Z'' + \frac{Z'}{2}\right)}$$

which on substitution from (20) gives

$$Z_o' = \sqrt{2\left\{Z_1 + Z_2 - \frac{2Z_2^2}{2(Z_1 + Z_2) + Z_l}\right\}\left\{\frac{Z_1 + Z_2}{2}\right\}}$$

$$= \sqrt{(Z_1 + Z_2)^2 - \frac{2(Z_1 + Z_2)(Z_2^2)}{2(Z_1 + Z_2) + Z_l}}$$

and this on substitution from (16) gives

$$Z_o' = \sqrt{Z_o^2\coth^2\frac{Pd}{2} - \frac{2Z_o^3\coth\frac{Pd}{2}/\sinh^2\frac{Pd}{2}}{2Z_o\coth\frac{Pd}{2} + Z_l}}$$

$$= Z_o\sqrt{\frac{2Z_o\cosh^2\frac{Pd}{2}\coth\frac{Pd}{2} + Z_l\cosh^2\frac{Pd}{2} - 2Z_o\coth\frac{Pd}{2}}{2Z_o\cosh\frac{Pd}{2}\sinh\frac{Pd}{2} + Z_l\sinh^2\frac{Pd}{2}}}$$

$$= \sqrt{\frac{2Z_o\sinh\frac{Pd}{2}\cosh\frac{Pd}{2} + Z_l\cosh^2\frac{Pd}{2}}{2Z_o\sinh\frac{Pd}{2}\cosh\frac{Pd}{2} + Z_l\sinh^2\frac{Pd}{2}}}$$

or
$$Z_o' = Z_o \sqrt{\frac{2Z_o \sinh Pd + Z_l(\cosh Pd + 1)}{2Z_o \sinh Pd + Z_l(\cosh Pd - 1)}} \quad . \quad . \quad . \quad (22)$$

When Pd is small, $\cosh Pd = 1$ and $\sinh Pd = Pd$, and

$$Z_o' = Z_o \sqrt{\frac{2Z_oPd + 2Z_l}{2Z_oPd}} = Z_o\sqrt{1 + \frac{Z_l}{Z_oPd}}$$
$$= \sqrt{\frac{R + j\omega L}{G + j\omega C} + \frac{Z_l/d}{G + j\omega C}}$$
$$= \sqrt{\frac{R + j\omega L + \frac{Z_l}{d}}{G + j\omega C}}$$

and the loading coil impedance can be looked upon as smoothed out uniformly along the line.

(42) **Loading** (*continued*).

Calculation of the propagation constant of loaded cables can be considerably simplified by making use of the "smoothing" approximation of the last section, and of the fact that in a loaded cable ωL is much greater than R, and in all practical cases ωC is much greater than G. The symbols now refer to the "smoothed" constants of the loaded cable and

$$P = \sqrt{(R + j\omega L)(G + j\omega C)} \quad . \quad . \quad . \quad . \quad (1)$$

with the new meaning of R and L is identical with 41·12.

Equation (1) may be written

$$P = \sqrt{\left(1 + \frac{R}{j\omega L}\right)\left(1 + \frac{G}{j\omega C}\right) j\omega L \,.\, j\omega C}$$
$$= j\omega\sqrt{LC}\left\{1 + \frac{R}{j\omega L} + \frac{G}{j\omega C}\right\}$$

on neglecting the product of the two small quantities $R/j\omega L$ and $G/j\omega C$,

$$= j\omega\sqrt{LC}\left\{1 + \frac{R}{2j\omega L} + \frac{G}{2j\omega C}\right\}$$

on expanding by the binomial theorem.

Thus

$$P = j\omega\sqrt{LC} + \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}}$$

and for the loaded line

$$\alpha = \frac{R}{2}\sqrt{\frac{C}{L}} + \frac{G}{2}\sqrt{\frac{L}{C}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\beta = \omega\sqrt{LC} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

With a very well insulated cable, in which $G = 0$, $\alpha = \frac{R}{2}\sqrt{\frac{C}{L}}$, a result which has already been used in section 23 in connection with the continuously loaded submarine telegraph cable. When G is not zero, an increase in L, while reducing the first term of equation (2), will increase the second term. and with a large leakance or poor insulation resistance the effect of loading may actually be to increase the attenuation constant. It is for this reason that the loading of aerial lines has not been very successful in the varying climatic conditions in this country, and only underground cables are loaded.

In underground cables the natural inductance is very small, and no serious error is involved in taking the L of equations (2) and (3) to be that contributed by the loading coils. Equation 41·11 for the cut-off frequency is

$$\omega_1 = \frac{2}{\sqrt{L_l C d}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and replacing L_l by Ld gives

$$\omega_1 = \frac{2}{d\sqrt{LC}}, \text{ or } d = \frac{2}{\omega_1\sqrt{LC}} \quad . \quad . \quad . \quad . \quad (5)$$

Since (equation (3)) $\beta = \omega\sqrt{LC}$ and (equation 39·15) the wave-length $\lambda = \frac{2\pi}{\beta}$, at the cut-off frequency

$$\lambda = \frac{2\pi}{\omega_1\sqrt{LC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

From (5) and (6)

$$d = \frac{\lambda}{\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

or the limit of efficient transmission in a coil loaded cable is reached when there are π coils per wavelength. This is clearly brought out in the following table, showing approximately the percentage by which the true attenuation calculated from formula 41·05 exceeds the attenuation as calculated from formula (2), the wavelength being calculated from formula (3).

Number of coils per wavelength	9	8	7	6	5	4	3
Percentage difference of attenuation constant . . .	0	1	2	3	7	16	200

If 2,000 cycles per second is taken as the maximum frequency that is necessary for commercial telephony, and L_l' is the inductance in *millihenries* of each coil, and C' is the capacity in *microfarads* of a mile of loop, (4) gives

$$2\pi \times 2000 = \frac{2}{\sqrt{L_l' \times 10^{-3} \times C' \times 10^{-6} \times d}}$$

or

$$C'L_l'd = 25{\cdot}4$$

The usual practical formula for the design of loaded cables is

$$C'L_l'd = 25 \qquad \qquad (8)$$

which gives a cut-off frequency of just above 2,000 cycles per second.

Another practical formula concerns the attenuation. The ratio R_l/L_l of the resistance of the inductance coil to its inductance depends upon the design of the coil and can, of course, vary widely in value. But it is found that for a suitable size of coil the ratio is about 50, while the ratio of G/C with air-core paper insulated underground cables is about 20, the capacity C being about 0·065 microfarads per mile.

Equation (2) may be written

$$a = \frac{R + \frac{G}{C}L}{2}\sqrt{\frac{C}{L}} \qquad \qquad (9)$$

If now R_1 is the resistance of the unloaded line per mile, the contribution of the coils to the resistance per mile is $\frac{R_l}{L_l}.L$,

$R = R_1 + \frac{R_l}{L_l}L$, and (9) becomes

$$a = \frac{R_1 + \left(\frac{R_l}{L_l} + \frac{G}{C}\right)L}{2}\sqrt{\frac{C}{L}}$$

and putting in the above-mentioned ratios for R_l/L_l and G/C,

$$a = \frac{R_1 + 70L}{2}\sqrt{\frac{C}{L}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Some usual loading figures may be given to show the effect on the attenuation of various cables of various degrees of loading. The three main types of loading are known as light, medium and heavy. In light loading the coils have each 135 mH, and are spaced at 2·6 mile intervals, in medium loading the inductance is 175 mH and the spacing 1·6 miles, and in heavy loading the inductance is 250 mH and the spacing 1·125 miles. The following table gives the attenuation constants per mile of underground air-core cables with these different loadings :—

Conductor lbs./mile.	Unloaded.	Light.	Medium.	Heavy.	Continuous.
10	0·175	0·095	0·070	0·052	—
20	0·118	0·050	0·037	0·028	—
40	0·080	0·027	0·020	0·017	0·034
70	0·059	0·016	0·013	0·012	0·023
100	0·047	0·012	0·010	—	0·018
150	0·037	0·009	0·008	—	0·013
200	0·029	0·007	—	—	—

It is seen that the lighter the conductor the heavier the loading that can be employed with advantage. On the heavier conductors only a relatively small advantage is gained by using the heavier types of loading.

The last column shows the results that can be obtained with a continuous loading of ordinary soft iron. These figures are very greatly reduced by the use of the special iron-nickel alloys referred to in section 23, but these alloys have not up to the present been used for telephone cables.

(43) **Superposing and Interference.**

There is a system of telegraphy (used in the Army) depending upon alternating currents of telephone frequency, produced by a buzzer or vibrator working on the same principle as the house trembler bell, but at a much higher frequency. The starting and stopping of the direct current through the vibrator produces an alternating electromotive force on a secondary coil wound over the main coils of the vibrator. The secondary coil is connected to line, the Morse code is sent by a key in the usual way, and received by long or short "buzzes" in a telephone receiver.

Both this system and the normal direct current telegraph system can be used simultaneously on the same line by the arrangement of Fig. 146. The choke L and condenser C_1 ensures the rounding off of the sharp corners of the direct current dots and dashes, so that loud clicks are not received in the telephone receiver of the vibrator set, while the condenser C_2 prevents the direct currents of the

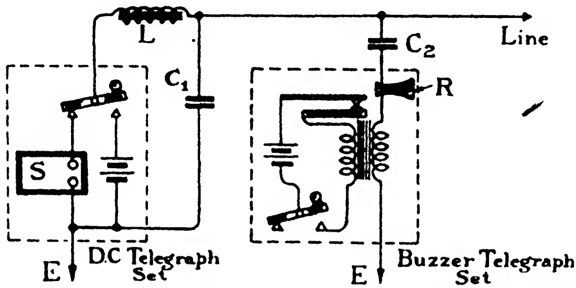


FIG. 146.—Buzzer and direct current telegraph sets superposed.

sounder set flowing to earth through the buzzer set. Intermediate vibrator stations may be inserted.

If the vibrator is replaced by a microphone circuit simultaneous speech and Morse signalling is obtained. The system is then known as the "phonopore," and is in considerable use on the railways.

Working on a similar principle is Van Rysselberghe's system of superposing a telegraph channel on each of the two wires of a telephone loop. Only one telegraph set is shown at A in Fig. 147; there are identical sets at B, C and D. The condensers C_1 and the chokes L_1 and L_2 smooth the telegraph signals, while the condensers C_2 prevent direct currents flowing through the telephone sets.

In Fig. 148 a telegraph circuit is shown superposed on a telephone loop by means of differentially wound telephone transformers. The telegraph sets are connected to the "peaks" of the secondaries of the transformers and equal direct currents flow through the two wires of the loop in parallel. The magnetomotive forces produced

P

in the two halves of the secondary coils are equal and opposite, and no currents are induced in the telephone circuit. This is only true if the impedances of the two wires are identical. The alternating currents of the telephone sets T, on the other hand, produce an

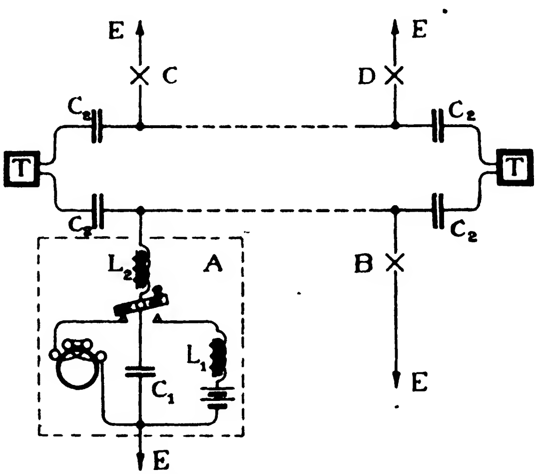


FIG. 147.—Two telegraph circuits superposed on a telephone loop.

electromotive force across the secondaries, and telephone currents flow round the loop and are duly received.

In the same manner a third telephone loop can be obtained by superposition on two other telephone loops, as is shown in Fig. 149.

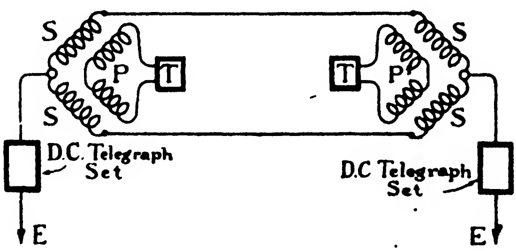


FIG. 148.—Superposed telegraph and telephone circuits.

T_1 and T_2 speak round the loop AB, T_3 and T_4 round the loop CD, and T_5 and T_6 round a loop composed of the wires A and B in parallel and the wires C and D in parallel: the first two circuits are known as the side circuits, the third as the superposed or phantom or plus (+) circuit.

The phantom circuit can be loaded by double wound coils, as shown in Fig. 150, where S_1 and S_2 are the side circuit coils and P is the phantom coil. The side circuit currents flow differentially through the phantom coil and *vice versâ,* so the phantom coil adds no inductance to the side circuits, and the side circuit coils add no

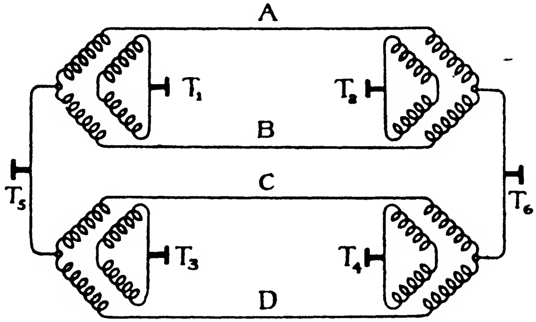


FIG. 149.—Superposed Telephone Circuit.

inductance to the phantom circuits, but each adds resistance to the other and reduces the efficiency on that account. This is, however, on long lines far more than offset by the saving effected in obtaining three circuits on four wires. The resistance of the phantom circuit

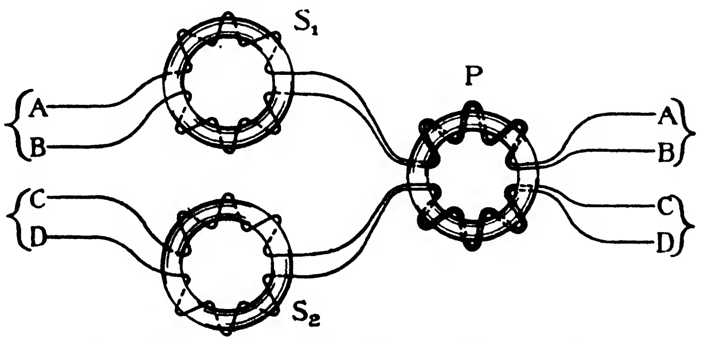
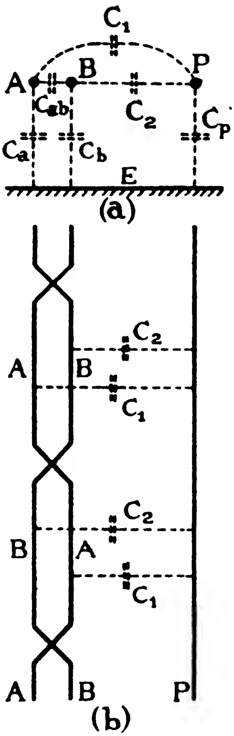


FIG. 150.—Loading coils for side and phantom circuits.

is only half that of the side circuits, and although its capacity is greater, a smaller inductance is required to load the phantom than the side circuits, in order that both may have the same attenuation constant.

Aerial telegraph and telephone circuits are run parallel with each other on the same route for many miles, and unless precautions were taken to prevent it, interference among the circuits would render speech impossible. The same effect is produced by the harmonics of overhead power lines running parallel with the telephone line. Underground telephone cable circuits are only subject to interference from circuits in the same cable.

Interference can be either magnetic or electric. Let P, Fig. 151 (*a*) be a single-phase power line with earth return, and A and B the wires of a telephone loop. There will be mutual inductance M henries per mile between P and the loop AB, and a magnetically induced e.m.f. $j\omega MI$ in the loop to send current through the apparatus at the ends, where ω is the frequency and I the amplitude of the harmonic of the power current. If, however, the wires of the telephone loop are crossed over or "transposed" at regular intervals as shown at (*b*) the induced e.m.f. will have a negative value in alternate sections, and no current will be produced in the loop. The same result may, of course, be attained by twisting the wires continuously as indicated in Fig. 152.



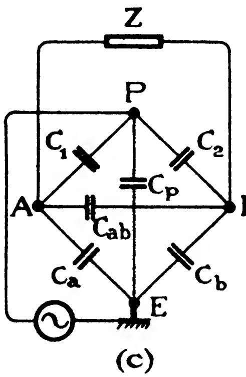


FIG. 151.—Interference from power line.

Electric interference arises from the various wire to wire and wire to earth capacities. Let these be C_1C_2 between P and A and B respectively, C_{ab} between A and B, and C_a C_b C_p between A B and P to earth. The currents flowing on any short length of the line may now be obtained from the equivalent diagram of Fig. 151 (c). If C_a and C_b are equal and the power line is sufficiently far away to make C_1 and C_2 practically equal, there will be no potential difference between A and B and no current will flow round the loop. If C_a and C_b are equal but C_1 and C_2 are unequal, and the telephone line is transposed, the magnitudes of C_1 and C_2 will change over in alternate sections and the resultant potential difference driving current round the loop can be made very small. But the voltage at any point on the telephone line can remain quite large, being equal to C_1/C_a and C_2/C_b times the power line voltage on the A and B wires respectively.

Exactly the same considerations apply in the case of a telegraph wire run on the saddle of the pole route. The currents and voltages in the telegraph line are much smaller than in the power line, but the wire is much closer to the telephone loop. The wires of the

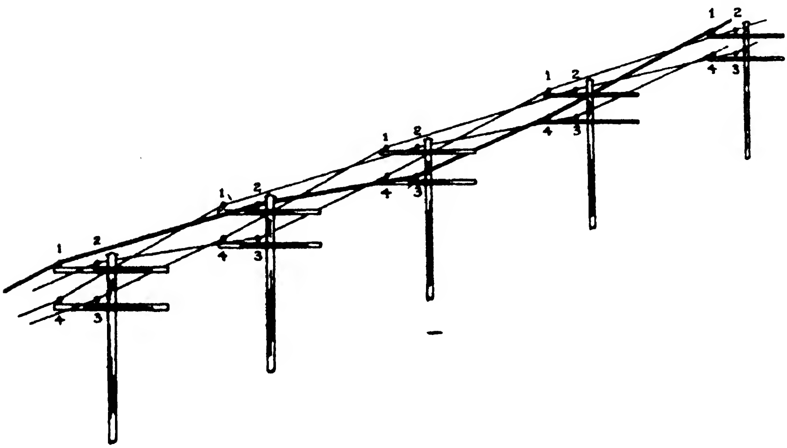


FIG. 152.—Twisting to avoid interference.

telephone loop must be balanced not only with regard to capacity (and leakance) to earth, but also with regard to impedance in order that there shall be minimum interference.

The single-phase power line with earthed return is found in traction electric, and direct current traction offers the same problem owing to the pronounced ripple usually present. Power transmission is usually by three-phase lines, however, and if the three wires have the same capacities to earth, and are close enough together to be considered as one in comparison with the distance to the telephone loop, there will be no resultant voltage to cause electric interference. Similarly if there is no earth on any power line there will be no resultant current to cause magnetic interference. The vector diagram of the voltages is in this case as shown by the firm lines meeting in O in Fig. 153. But if the capacities to earth were unequal the voltages to earth might be represented by the dotted lines meeting in O_1, and there would, considering the three wires lumped together, be a residual voltage equal to three times the length OO_1 to cause interference in the same way as in the single-phase earthed return line. Similarly magnetic interference results from out-of-balance currents due to any earth on the power system. Generally speaking, the wires of a power transmission line have unequal capacities to earth, and they may then be transposed to equalise them.

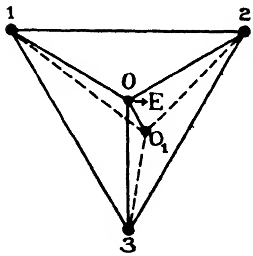


FIG. 153.—Voltages in Three-phase Transmission Line.

To minimise interference among the various telephone circuits along the same aerial route they are twisted and transposed or simply transposed, the lengths between transpositions being different with different circuits to obtain the best balance possible. In cables the wires are twisted in pairs and pairs twisted together to form quads, and the quads are laid up to form the cable. The pitch of the twist is varied among the various quads. These are multiple twin cables, and are generally used for long-distance circuits. Another method is to twist four wires together to form a quad, the cable so formed being a multiple quad.

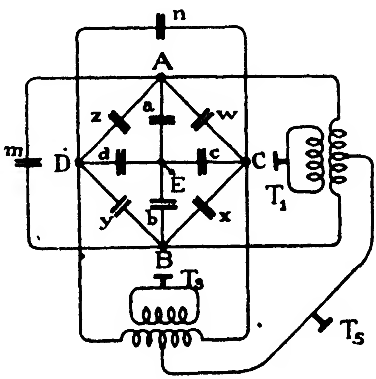


FIG. 154.—Capacities among the four wires of a quad and earth.

Superposing is carried out on the two twins of a quad. Here there is an additional interference problem, for there may be (i.) interference between the side circuits themselves, or (ii.) between either side circuit and the phantom, as well as (iii.) between either circuit and any other in the cable. Interferences (i.) and (iii.) are called cross-talk and (ii.) overhearing. The various wire to wire and wire to earth capacities in a quad ABCD may be represented as in Fig. 154, where *abcd* are the capacities of the four wires to earth, $w\,x\,y\,z\,m\,n$ are wire to wire capacities as shown, and T_1T_3 are the side circuit telephones, and T_5 the phantom circuit telephone at one end. When T_3 is in use a potential difference is established across C and D; in order that there may be no cross-talk it is necessary that there shall be no potential difference between A and B, and in order that there may be no overhearing the mean potential between C and D must be the same as the mean potential between A and B. Similar statements hold when the other telephones are in use. The following relations among the capacities must exist to fulfil these conditions :—

$$w - x + \frac{c(a-b)}{\Delta} = 0$$

$$z - y + \frac{d(a-b)}{\Delta} = 0$$

$$w - z + \frac{a(c-d)}{\Delta} = 0$$

$$x - y + \frac{b(c-d)}{\Delta} = 0$$

where $\Delta = a + b + c + d$, and these hold if $w - x = z - y = w - z = x - y = a - b = c - d = 0$. (See Appendix 8.)

After sections of a cable have been laid these capacity differences are measured by a special bridge, and the quads of the sections joined by selection so as to make the residual out-of-balance capacity as small as possible. It is found that if the cable is balanced in this way for capacity, out-of-balances of resistance, inductance, mutual inductance and leakance are at the same time allowed for. There remains the possibility of out-of-balance in the loading coils and terminal apparatus. The windings of loading coils are always in four sections, and opposite sections are connected together in series in each line, and terminal apparatus is always as far as possible arranged to be balanced in each line. Filters, for instance, where they are used in actual lines, are made of ⌶ instead of T links, and of ☐ instead of Π links.

(44) Transmission Measurements and Units.

The merit of a telephone circuit depends on the volume of sound received, the distortion that the wave has undergone, the amount of interference from power lines and telegraph or other telephone circuits, as well as upon extraneous noise at the transmitting and receiving ends and upon any noise created in the circuit itself, as by a " frying " microphone or a loose contact.

For the purposes of volume measurements the circuit is divided into terminal apparatus and line, and transmission over the line is compared with transmission over a " standard " line. The line chosen as " standard " is an air-core cable of conductor weighing 20 lb. per mile, with $R = 88$ ohms, $C = 0{\cdot}054\mu F$, $L = 1$ mH and $G = 1$ μM per loop mile. If speech over any line is found to be

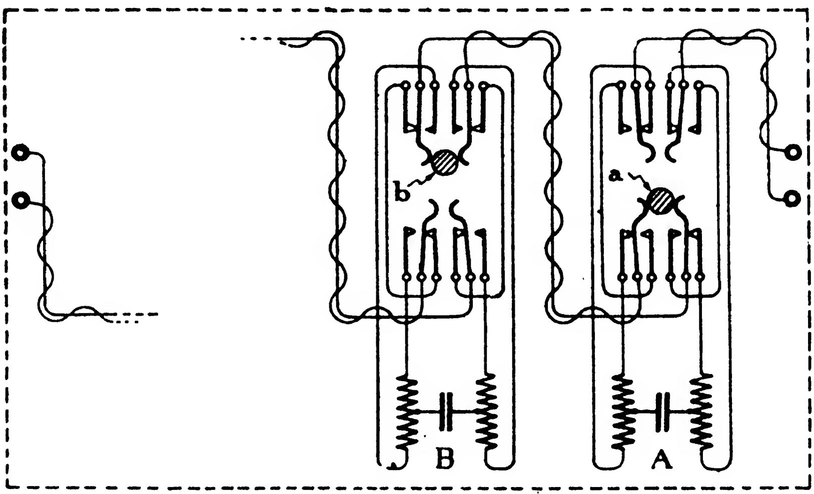


FIG. 155.—Standard Cable Box.

equal in volume to that over x miles of the standard cable, the transmission equivalent of the line is x miles of standard cable (m.s.c.).

For the actual measurement an artificial standard cable box (Fig. 155) is used, in which various networks all of the same impedance and having attenuation constants corresponding, for instance, to 10, 10, 10, 5, 2, 2, 1 miles can be put in series by means of telephone switch keys. In the figure the network A is in series with the corresponding key in the position shown at a, but the network L is cut out with the key at b. The circuit for speech testing is arranged as in Fig. 156. By means of the switches speech between A and B takes place alternately over the line under test and through the standard cable box, and the keys of the latter are adjusted by A until a volume balance is obtained. With a line of approximately the same impedance as the standard cable, if l_1 miles of the line balance l_o miles of the standard cable, it is possible to write

$$a_1 l_1 = a_o l_o,$$

where a_1 a_o are the attenuation constants of the line under test and of the standard cable respectively, at a frequency which may be looked upon as a *mean telephone frequency*. The results of many speech tests show this to be about 800 cycles second (an ω value of approximately 5,000), for which a_o is 0·107. The standard cable

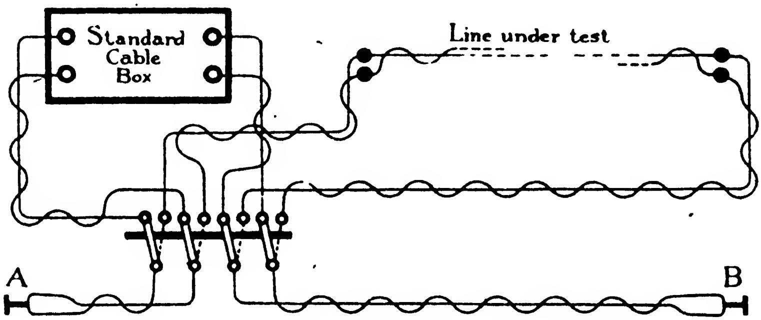


FIG. 156.—Speech test for standard cable equivalent.

equivalent of l_1 miles of any line having at $\omega = 5{,}000$ an attenuation constant of a_1 is therefore $l_1 a_1/0{\cdot}107$ m.s.c.

The "standard" terminal apparatus used at A and B in such speech tests is as shown in Fig. 157. The impedance of this apparatus matches approximately the impedance of unloaded underground cables, but is much less than that of loaded underground cables. Consequently the results of speech tests on various lengths of the same cable will, in the case of an unloaded cable, give a straight line A (Fig. 158) passing through the origin, but in the case of the loaded cable the straight line B does not pass through the origin, but cuts the s.c.e. axis at a point R. OR is known as the reflection or terminal loss. In Fig. 158 it is about 4 m.s.c. It can, however, always be reduced by the use of terminal transformers to match impedances (section 35).

Instruments and other telephone circuits are tested by reference to the standard circuit with "standard" instruments in a similar manner. Speech takes place over thirty miles of standard cable in each case, and sufficient cable is added either to the standard side

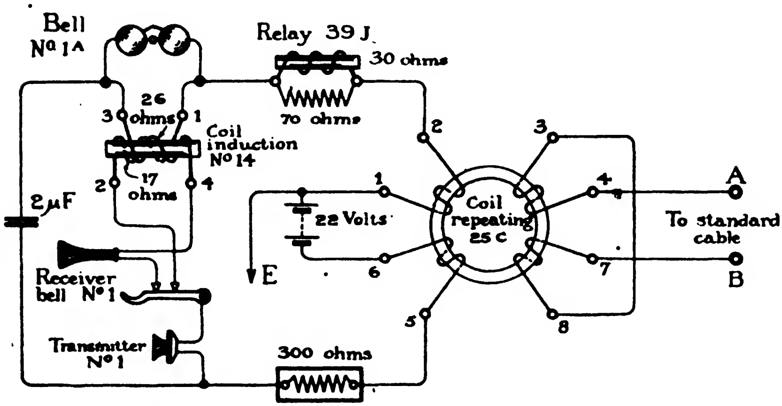


Fig. 157.—Standard Terminal Apparatus.

or the test side as may be required to secure a balance. In this way the transmission merit of the microphone or receiver or circuit under test is measured as so many m.s.c. loss or gain.

Since attenuation along a telephone line proceeds according to a

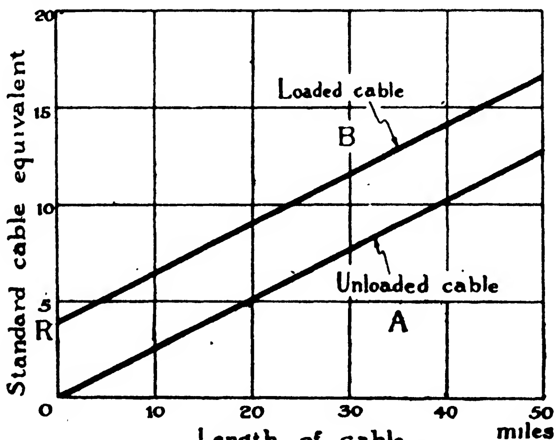


Fig. 158.—Showing reflection loss.

logarithmic law, the mile of standard cable is a logarithmic unit. There is an advantage in this, as it is generally agreed that sound sensations also follow a logarithmic law with regard to the air pressures producing them. But there are disadvantages attached to the m.s.c. as a unit, chief among which appears to be the fact that it varies with frequency.

If one circuit has an equivalent of x m.s.c. loss compared with another, the ratio of the currents received in the two cases if there is no reflection loss is

$$\frac{I_1}{I_2} = \epsilon^{-\alpha x} \quad \text{or} \ldots \left|\frac{I_2}{I_1}\right| = \epsilon^{\alpha x}.$$

Since $\alpha = 0{\cdot}107$ this gives

$$x_{\text{m.s.c.}} = \frac{1}{0{\cdot}107} \log_\epsilon \left|\frac{I_2}{I_1}\right|$$

$$= 21{\cdot}5 \log_{10} \left|\frac{I_2}{I_1}\right| \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The ratio of the powers in the two cases is similarly

$$\left|\frac{W_2}{W_1}\right| = \epsilon^{2\alpha x},$$

from which

$$x_{\text{m.s.c.}} = 10{\cdot}75 \log_{10} \left|\frac{W_2}{W_1}\right| \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Either of the expressions (1) or (2) could have been used to define the mile of standard cable.

A "transmission unit" (written TU) is being widely used in replacement of the m.s.c. It is defined by an expression of the same form as (2), actually

$$x dB \doteq 10 \log_{10} \left|\frac{W_2}{W_1}\right| \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

from which follows the expression corresponding to (1).

$$x dB = 20 \log_{10} \left|\frac{I_2}{I_1}\right| \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

This unit is of nearly the same size as the m.s.c., and is by its manner of definition independent of frequency. It has been decided to call it a decibel, the bel being a unit ten times as large, defined by

$$\left.\begin{aligned} xB &= \log_{10} \frac{W_2}{W_1} \\ &= 2 \log_{10} \frac{I_2}{I_1} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Yet another unit is in use, the total attenuation al. This is called the néper, and can be defined by

$$\left.\begin{aligned} x_{\text{népers}} &= \log_e \frac{I_2}{I_1} \\ &= \frac{1}{2}\log_e \frac{W_2}{W_1} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (6)$$

To make measurements in dB's direct, it is only necessary to replace the standard cable networks in the box of Fig. 155 by H networks of non-inductive resistances, designed in accordance with the formula of section 40 to have a suitable characteristic impedance for general purposes (say, 600 ohms) and attenuations of 1·15, 1·15, 1·15, 0·58, 0·23, 0·23 and 0·11, to give dB's of 10, 10, 10, 5, 2, 2, 1 in order. These figures are arrived at as follows. For 10 dB's equation (4) gives

$$\log_{10}\left|\frac{I_2}{I_1}\right| = 0{\cdot}5$$

If a is the attenuation required

$$\left|\frac{I_2}{I_1}\right| = \epsilon^a$$

Whence $$\log_{10}\frac{I_2}{I_1} = \frac{a}{2{\cdot}30} = 0{\cdot}5$$

and $$a = 1{\cdot}15$$

and so on.

With such a box measurements can be made at any single frequency from a suitable sound source for overall acoustic-electric-acoustic measurements, or a suitable alternating current source for purely electrical measurements, and it would be possible by a series of such measurements over the telephone frequency range to find the distortion of the circuit. But it would at present be difficult to interpret the results of such tests in terms of (i.) the *intelligibility*, or (ii.) the *articulation* of the circuit, two terms generally taken to mean (i.) the percentage of total ideas received from connected speech, and (ii.) the percentage of detached meaningless monosyllables received. The latter quantity is readily measured by speech tests; the former is inferred from the results. If, for instance, the articulation is 50 per cent., the intelligibility is estimated to be 90 per cent. Fig. 159 * gives a relation between the two quantities.

Cross-talk and overhearing can be measured in m.s.c. or dB's by

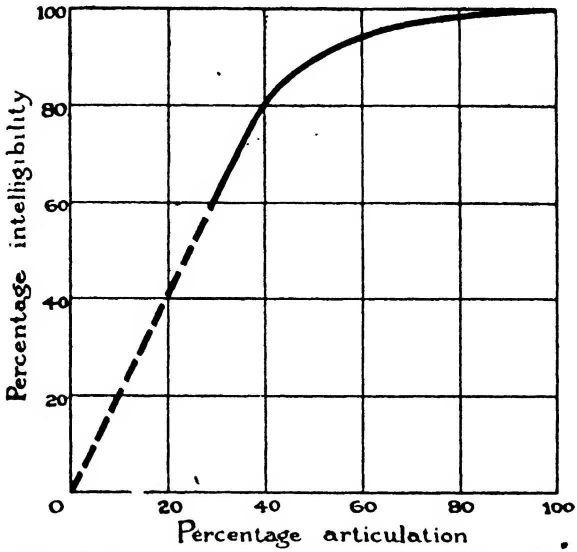


FIG. 159.—Relation between intelligibility and articulation.

means of a suitable network box. Speech takes place (Fig. 160) over the interfering circuit A from the telephone T, and listening is

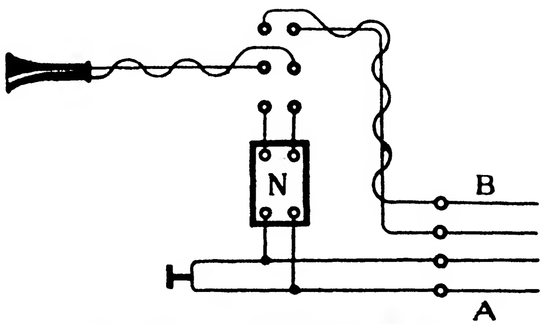


FIG. 160.—Measurement of Cross Talk.

either on the interfered-with circuit B or directly from T through the network box N. N is adjusted until a balance is obtained, and

* B. S. Cohen, J.I.E.E., Vol. 66, p. 169.

the m.s.c. or dB's of the disturbance read direct. Measurements may also be made by a cross-talk meter, a universal shunt arrangement in which by a calibrated rotating arm a known fraction of the current sent to line is passed through the telephone receiver.

REFERENCES

O. Heaviside.—"Electromagnetic Theory."

Sir J. A. Fleming.—"The Propagation of Telephone and Telegraph Currents along Electric Lines and Cables."

A. E. Kennelly.—"Application of Complex Hyperbolic Functions to Electrical Engineering."

A. E. Kennelly.—"Artificial Electric Lines."

P.O. Engineering Department.—Technical Instruction XIX.

E. Mallett.—"Chains of Resonant Circuits." *J.I.E.E.*, Vol. LXVI., p. 968.

S. C. Bartholomew.—"Power Circuit Interference with Telegraphs and Telephones." *J.I.E.E.*, Vol. LXII., p. 817.

A. Rosen.—"Interference between Circuits in Continuously Loaded Telephone Cables." *J.I.E.E.*, Vol. LXIV., p. 849.

B. S. Cohen.—"Apparatus Standards of Telephone Transmissions, etc." *J.I.E.E.*, Vol. LXVI., p. 165.

CHAPTER IX

VALVES IN TELEPHONY

(45) The Thermionic Valve

WHEN a wire is heated electrons ("atoms" of negative electricity) are emitted or boiled off according to the law

$$N = A\sqrt{\theta}\,\epsilon^{-\frac{b}{\theta}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Where N = the number of electrons per second per square centimetre of heated surface,

θ = the absolute temperature of the surface. and A and b are constants depending, especially in the case of A, upon the surface material.

Directly an electron breaks away from the surface a positive charge is induced on the surface, and there is an electric field tending to cause it to return. If the wire is in a vacuum there will be a cloud of electrons round the wire constituting a "space charge," and a stable state is reached in which as many electrons return to the wire per second as leave it. If now the wire F, Fig. 161, is surrounded by a cylindrical anode A. and an electric field is maintained by a battery V_a in opposition to the field of the space charge some of the electrons will be drawn right across from the filament to the anode and the galvanometer will indicate that a current I_a is flowing. The current will increase with the voltage V_a, for a particular value of the filament temperature, until the space charge field is completely neutralised and all the electrons that are emitted are drawn across. The current now flowing is known as the saturation current. For tungsten $A = 1{\cdot}6 \times 10^{26}$ and $b = 5{\cdot}3 \times 10^{4}$ (in 1),

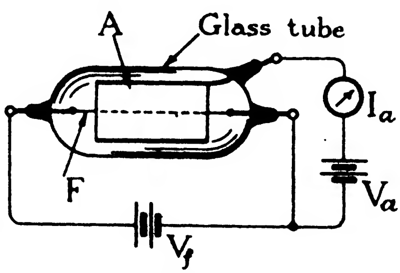


FIG. 161.—Thermionic Valve.

and since an ampere is a stream of $6{\cdot}3 \times 10^{18}$ electrons per second, the saturation current $= 2{\cdot}6 \times 10^7 \sqrt{\theta}\, \epsilon^{-\frac{5{\cdot}3 \times 10^4}{\theta}}$ amperes per square centimetre of the heated wire.

The variation of this anode current with anode battery voltage is shown in Fig. 162. I_a' is the saturation current and V_a' the least voltage necessary to produce it. The law of the initial part of the curve is theoretically $I_a = kV_a^{\frac{3}{2}}$, but in actual valves the index is nearer 2 than 3/2. The direction of the current, according to the usual convention, is round the external anode circuit from the filament to the anode, and internally from the anode to the filament. If the direction of the anode battery is reversed the space charge field is assisted instead of being opposed, and no current flows. Hence the valve action.

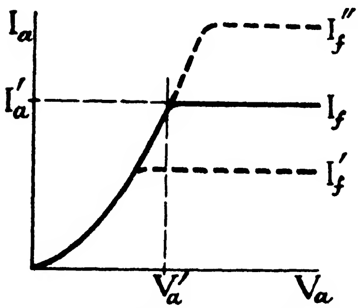


FIG. 162.—Characteristic Curves of Two-electrode Valve.

If the temperature of the wire or filament F is increased by

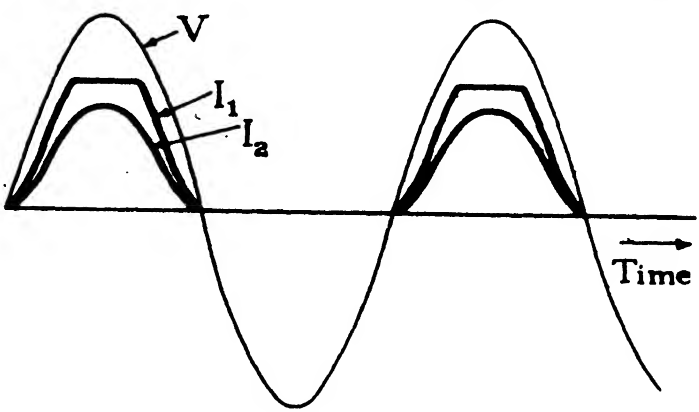


FIG. 163.—Rectification by Two-electrode Valve.

increasing the voltage of the battery V_f and hence the filament current I_f the saturation current is also increased and the curve of Fig. 163 is extended as shown by the upper dotted line. For each value of the filament current I_f (determining θ) there is a corresponding saturation current I_a' and minimum voltage to give saturation V_a'.

If the battery V_a is replaced by an alternator current will flow during the half-cycle that makes the anode positive, but not during the half-cycle that makes the anode negative. This is indicated in Fig. 163, where V is the sinusoidal voltage of the alternator and I_1 and I_2 are the curves of anode current. In I_1 the voltage rises to a value sufficient to produce the saturation current of the valve. The curve I_2 is produced by a smaller voltage; the saturation current is not reached. In each case the ammeter will indicate the mean anode current flowing. The alternating current has been rectified, and the two electrode valve used for this purpose is called a diode.

Suppose now a third electrode in the form of a cylindrical wire grid is introduced between the filament and the anode and maintained at a potential V_g. The space charge is acted on by two fields, that due to V_g acting direct, and that due to V_a acting through the grid. The effect of V_a is thus lessened to V_a/μ, say, where μ is greater than unity, and the abscissæ of Fig. 162 are the values of $(V_g + V_a/\mu)$ instead of V_a, and the initial part of the curve is $I_a = k\,(V_g + V_a/\mu)^{\frac{3}{2}}$, or, by a change of constant,

$$I_a = k_1\,(V_a + \mu V_g)^{\frac{3}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The curves of I_a plotted against V_g for various values $V_a'V_a''$ . . . of the anode voltage (Figs. 164 and 165) are known as the characteristic curves of the valve. While the grid has a negative potential there is no current in the grid circuit, but when the grid is positive electrons are attracted to it and there is grid current I_g. The circuits of the triode, as the three-electrode valve is often called, are generally arranged so that the grid is always negative and the complication of grid currents is avoided. Fig. 166 shows diagrammatically the circuit arrangement for drawing the characteristic curves. A is the anode of the valve (frequently called the plate), G the grid, and F the filament. The negative terminal of the anode battery is connected to the negative terminal of the filament battery, and one terminal (usually the positive) of the grid battery is connected to the same point. The curves of Fig. 164 are for a small triode with a tungsten filament, or a "bright emitter." It is found, however, that rare earths such as barium or strontium

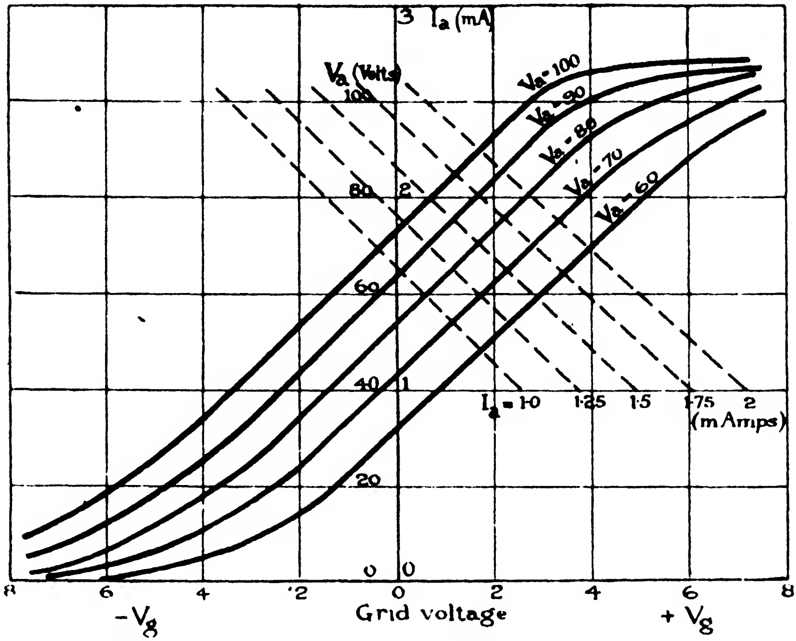


FIG. 164.—Characteristic Curves of Bright Emitter Triode.

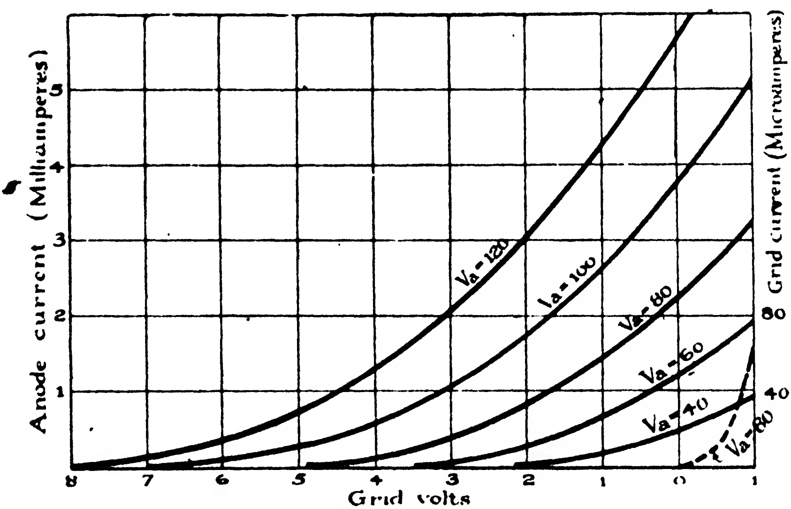


FIG. 165.—Characteristic Curves of Dull Emitter Triode.

oxide have a very large value of A in equation (1), and filaments coated with these substances will give the emission necessary in small valves, although run at comparatively low temperatures. These valves are called "dull emitters," and their characteristic curves (Fig. 165) do not show the saturation noticed in Fig. 165. It will be seen also that the characteristics are curved over a wider range of grid potential than those of the bright emitter; they follow approximately a square law.

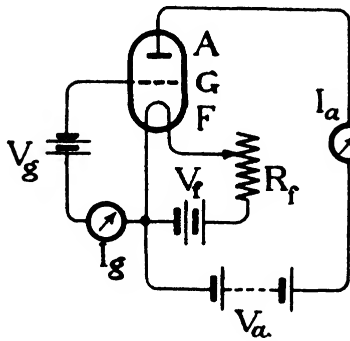


Fig. 166.—Circuit for Taking Valve Characteristics.

Reference to Fig. 164 shows that a considerable portion of the characteristic curves approximate to straight lines. Where this is so, the equation to the curves can be written (compare equation (2)) as

$$I_a = a\,(V_a + \mu V_g + c) \qquad (3)$$

where a and c are constants.

If in (3) V_a is altered by δV_a and V_g by δV_g, the corresponding alteration δI_a is found from

$$I_a + \delta I_a = a\{V_a + \delta V_a + \mu(V_g + \delta V_g) + c\} \qquad (4)$$

and subtracting (3) from (4)

$$\delta I_a = a\delta V_a + \mu a\delta V_g \qquad (5)$$

Differentiating (3) with regard to V_a and V_g gives

$$\frac{\partial I_a}{\partial V_a} = a, \text{ whence } \frac{\partial V_a}{\partial I_a} = \frac{1}{a} = R_a, \text{ say} \qquad (6)$$

and

$$\frac{\partial I_a}{\partial V_g} = a\mu = g = \frac{\mu}{R_a} \qquad (7)$$

R_a as defined by (6) is clearly the *internal resistance* between anode and filament of the valve to changes of anode current, while a is the *internal conductance*, and g as defined by (7) is the slope of the characteristic curves of Fig. 164, and is called the *mutual conductance*. It is clear from (3) and (5) that the effect on the anode current of a change of V_g in the grid voltage is the same as a change of μV_g in the anode voltage. μ is accordingly called the *amplification factor* of the valve.

Using these definitions, (5) may be written

$$\delta I_a = \frac{1}{R_a}\left\{\delta V_a + \mu\delta V_g\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

or

$$\delta I_a = a\delta V_a + g\delta V_g \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In Fig. 164 the dotted lines are the curves connecting V_a and V_g with various constant anode currents. If in (3) both V_a and V_g are regarded as variables, differentiation with regard to V_g gives

$$\frac{\partial I_a}{\partial V_g} = a\left(\frac{\partial V_a}{\partial V_g} + \mu\right)$$

which in the case of the dotted curves, where I_a is constant, and $\frac{\partial I_a}{\partial V_g} = 0$, leads to

$$\frac{\partial V_a}{\partial V_g} = -\mu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

The slope of the dotted lines accordingly gives μ, the slope of the firm lines is g by (7), $a = g/\mu$ from (7), and $R_a = 1/a$ from (6); all the constants of the valve are accordingly found.

The constants may also be found experimentally, without drawing out the whole of the characteristic curves, by bridge methods.*

(46) **The Valve Amplifier**

The circuit of Fig. 167 can be used to amplify voltage. The voltage to be amplified is applied as δV_g between grid and filament, and the amplified voltage is found across the load resistance R. Suppose δV_g is varying sinusoidally and can accordingly be written $V_g \epsilon^{j\omega t}$, δI_a and δV_a will also vary sinusoidally and may be written $I_a \epsilon^{j\omega t}$ and $V_g \epsilon^{j\omega t}$ respectively. Equation (45·08) becomes, therefore (on dividing throughout by $\epsilon^{j\omega t}$), the vector equation

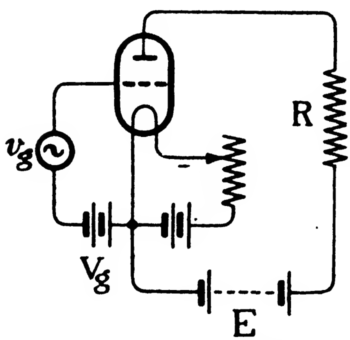


FIG. 167.—Valve Amplifier.

$$I_a = \frac{1}{R_a}(V_a + \mu V_g) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now a rise of anode current involves a fall of anode potential

* L. Hartshorn, *Phys. Soc. Proc.*, 41, p. 113.

owing to the increase of potential drop in the resistance R. Hence

$$V_a = -RI_a$$

and (1) becomes

$$I_a = \frac{1}{R_a}(-RI_a + \mu V_g)$$

$$I_a\left(1 + \frac{R}{R_a}\right) = \frac{\mu V_g}{R_a}$$

or

$$I_a = \frac{\mu V_g}{R_a + R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The alternating component of the anode current is thus the current that would be obtained in a simple circuit from an alternator of voltage μV_g and internal resistance R_a connected to a load of resistance R.

The amplified voltage available is

$$I_a R = \frac{\mu R}{R_a + R} V_g, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and is greater the greater the value of R. The amplification approaches μ as R becomes indefinitely great.

The alternating power put into the load is

$$I_a^2 R = \frac{\mu^2 R}{(R_a + R)^2} V_g^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

is a maximum when $R_a = R$ and is then

$$\frac{\mu^2}{4R^2} V_g^2$$

To obtain the alternating voltage given by (3) the D.C. voltages of the circuit must be properly adjusted so that the valve is working on the straight part of its characteristics, and the input voltage V_g must be limited in amount so that the grid swing neither makes the grid positive nor makes it so negative that the curved part of the characteristic is reached.

From the curves of Fig. 164, $g = 250$ μM, $\mu = 10$, and $R_a = \mu/g = 40{,}000$ ohms. If the load resistance is 40,000 ohms and it is desired to have an anode voltage of 100 volts, with -2 volts on the grid, the anode current will be about 1·3 *m*A, and the drop in the resistance R will be $1{\cdot}3 \times 10^{-3} \times 40{,}000 = 52$ volts. The voltage of the anode battery must accordingly be $100 + 52 = 152$ volts. The grid swing may have an amplitude of 2 volts, or an r.m.s. value of 1·4 volts before grid current flows. The valve could, however, be used with greater grid swings by making the anode voltage greater than has been done in drawing the curves, with a consequent shifting of the curves bodily to the left.

The voltage amplification obtained is given by (13) as

$$\frac{10 \times 40{,}000}{40{,}000 + 40{,}000} = 5.$$

Greater amplifications involve larger load resistances R, and, in consequence of the ohmic drop in the resistance, larger anode battery voltages.

For amplifying telephone currents the valve amplifier circuit is usually arranged as in Fig. 168, with an input transformer T_1 to step up the voltage available to as high a value as possible (the ratio is only limited by the self capacity of the windings of the secondary of T_1), and an output transformer T_2 matching the valve impedance on the primary side and the load impedance on the secondary side. If Z_a is the effective impedance of the primary of T_2, it may be inferred from (2) that

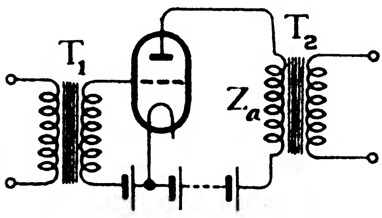


FIG. 168.—Amplifier for Telephone Currents.

$$I_a = \frac{\mu V_g}{R_a + Z_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The d.c. resistance only of the impedance Z_a causes loss of anode potential; while the impedance at the frequency involved determines the amplification obtained.

There is a possible difficulty however. The small capacity between the anode and the grid results in some part of the alternating anode voltage appearing at the grid and causing further anode current changes. The effect may be cumulative and the whole circuit may be unstable and burst into sustained oscillations. This condition is investigated in Appendix 9.

Non-linearity of the characteristic curves of a valve leads to the introduction of harmonic distortion, a result of non-linearity seen previously also to hold in the cases of the microphone and telephone receiver.

Many valves (especially dull emitters, see Fig. 165) have characteristics which follow quite closely a square law, so that the direct current characteristics may be written

$$I_a = a(V_a + \mu V_g + c)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

giving

$$\frac{\partial I_a}{\partial V_g} = 2a\mu\,(V_a + \mu V_g + c)$$

$$\frac{\partial I_a}{\partial V_a} = 2a\,(V_a + \mu V_g + c)$$

Whence

$$\frac{\partial I_a}{\partial V_g} \Big/ \frac{\partial I_a}{\partial V_a} = \mu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and

$$\frac{\partial V_a}{\partial I_a} = R_a = \frac{1}{2a\,(V_a + \mu V_g + c)} \quad . \quad . \quad . \quad (8)$$

Comparing these with 45.06 and 45.07 it is seen that the amplification factor is still a constant, but that the internal resistance varies as the anode current varies.

If I_a, V_a and V_g are all varied (6) gives

$$I_a + \delta I_a = a(V_a + \delta V_a + \mu V_g + \mu\delta V_g + c)^2 \quad . \quad . \quad (9)$$

and subtracting (6) from (9) (remembering that the difference of the squares of two quantities is the product of the sum and the difference of the quantities) gives

$$\delta I_a = a(\delta V_a + \mu\delta V_a)\,(2V_a + 2\mu V_g + 2c + \delta V_a + \mu\delta V_g) \quad (10)$$

If now the anode load is a resistance R as in Fig. 167, so that $\delta V_a = -R\delta I_a$, and using (8), (10) becomes

$$\delta I_a = (-R\delta I_a + \mu\delta V_g)\left(\frac{1}{R_a} - aR\delta I_a + a\mu\delta V_a\right)$$

$$= -\frac{R}{R_a}\,\delta I_a + \frac{\mu}{R_a}\,\delta V_g + aR^2\,\delta I_a{}^2 + a\mu^2\delta V_g{}^2 - 2a\mu R\delta I_a\delta V_g \quad (11)$$

Writing

$$\delta I_a = a_1\delta V_g + a_2\delta V_g{}^2 + \quad . \quad . \quad . \quad . \quad (12)$$

and substituting gives

$$\begin{aligned} a_1\delta V_g + a_2\delta V_g{}^2 + \ldots = & -\frac{R}{R_a}\,a_1\delta V_g - \frac{R}{R_a}\,a_2\delta V_g{}^2 - \ldots \\ & + \frac{\mu}{R_a}\,\delta V_g \\ & + aR^2a_1{}^2\delta V_g{}^2 + 2aR^2a_1a_2\delta V_g{}^3 + \ldots \\ & + a\mu^2\delta V_g{}^2 \\ & - 2a\mu R a_1\delta V_g{}^2 - \quad . \quad . \quad . \quad . \quad (13) \end{aligned}$$

Equating the coefficients of δV_g, δV_g^2, etc., on each side of (13) gives

$$a_1 = -\frac{R}{R_a}a_1 + \frac{\mu}{R_a}$$

whence
$$a_1 = \frac{\mu}{R_a + R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$a_2 = -\frac{R}{R_a}a_2 + aR^2a_1^2 + a\mu^2 - 2a\mu Ra_1$$

$$a_2\left(1 + \frac{R}{R_a}\right) = aR^2\frac{\mu^2}{(R_a + R)^2} + a\mu^2 - 2a\mu^2\frac{2R}{R_a + R}$$

$$= \frac{a\mu^2R_a^2}{(R_a + R)^2}$$

whence
$$a_2 = \frac{a\mu^2R_a^3}{(R_a + R)^3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and so on.

Hence from (12), (14) and (15),

$$\delta I_a = \frac{\mu}{R_a + R}\delta V_g + \frac{a\mu^2R_a^3}{(R_a + R)^3}(\delta V_a)^2 + \quad . \quad . \quad . \quad (16)$$

Suppose now that the fluctuations of the grid potential are sinusoidal. Writing $\delta V_g = V_g \cos \omega t$, (16) gives

$$\delta I_a = \frac{\mu}{R_a + R}V_g \cos \omega t + \frac{a\mu^2R_a^3}{(R_a + R)^3}V_g^2 \cos^2 \omega t + \quad . \quad . \quad (17)$$

The first term gives the amplified anode current as in (2), the second and subsequent terms involve harmonic distortion, and sum and difference terms when potentials of more than one frequency act together on the grid.

The even terms also introduce a rectifying action, in that they involve an alteration of the mean direct current through the valve. Considering the second term, for instance, since

$$V_g^2 \cos^2 \omega t = V_g^2\left(\frac{1}{2} + \frac{1}{2}\cos 2\,\omega t\right)$$

the mean direct current through the valve is increased by

$$\frac{a\mu^2R_a^3}{2(R_a + R)^3}V_g^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

which is, therefore, the rectified current; the valve may be used in this way to produce a change of direct current for an impressed alternating electromotive force.

Two valves of identical square law characteristics may be used together in the "push-pull" arrangement (Fig. 169) to amplify without distortion. If $\delta V_g'$ is the grid voltage change on one valve and $\delta V_g''$ that on the other, then the arrangement of the input transformer ensures that $\delta V_g' = -\delta V_g''$, while the secondary of the output transformer has a voltage which is proportional to $I_a' - I_a'' = 2\delta I_a$, say. If R is the impedance (considered non-inductive for simplicity) of each half of the primary of the output transformer, equation (6) gives for the two valves

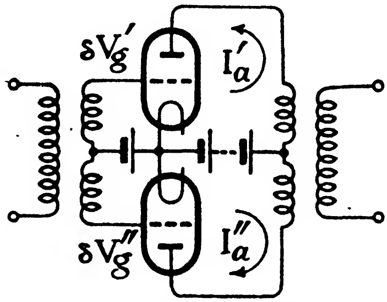


FIG. 169.—Push-pull Amplifier.

$$I_a' = a(V_a - R\delta I_a + \mu V_g + \mu\delta V_g + c)^2$$

$$I_a'' = a(V_a + R\delta I_a + \mu V_g - \mu\delta V_g + c)^2$$

and $2\delta I_a = I_a' - I_a'' = 4a(V_a + \mu V_g + c)(-R\delta I_a + \mu\delta V_g)$
and using (8)

$$2\delta I_a = \frac{2}{R_a}(-R\delta I_a + \mu\delta V_g)$$

whence
$$\delta I_a = \frac{\mu\delta V_g}{R_a + R}$$

or, with δV_g a sinusoidally varying quantity, the vector equation is

$$I_a = \frac{\mu V_g}{R_a + Z}$$

for the contribution of each valve, as with linear characteristics (equation 5), where Z is written instead of R for the effective impedance of each half of the primary of the input transformer. There is no introduction of currents of unwanted frequencies.

Larger amplification can be obtained by the use of two or more valves in cascade. One practical arrangement is shown in Fig. 170.

Common filament and anode batteries are used for the two valves V_1 and V_2. The amplified voltage from V_1 across R_1 is put across the grid and filament of the valve V_2, but a series condenser C is inserted to prevent the anode battery voltage E appearing at the

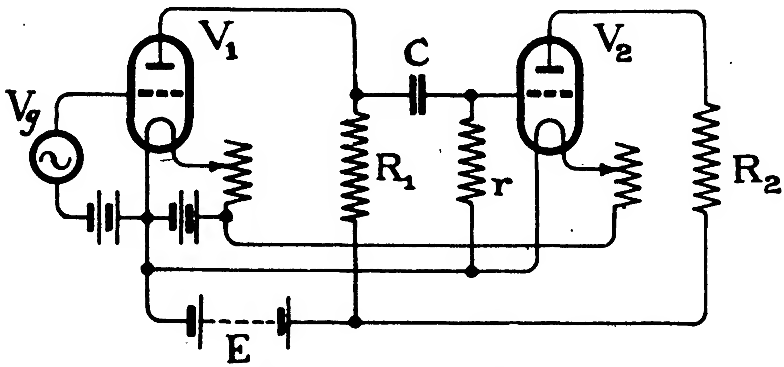


FIG. 170.—Resistance Coupled Two-stage Amplifier.

grid of V_2. This necessitates the use of a high resistance grid leak *r* (one megohm or more) to ensure that the grid does not accumulate a large negative charge through electrons striking it.

To reduce the voltage of the anode battery the resistances R_1 and R_2 may be replaced by iron-cored chokes, or transformers may be

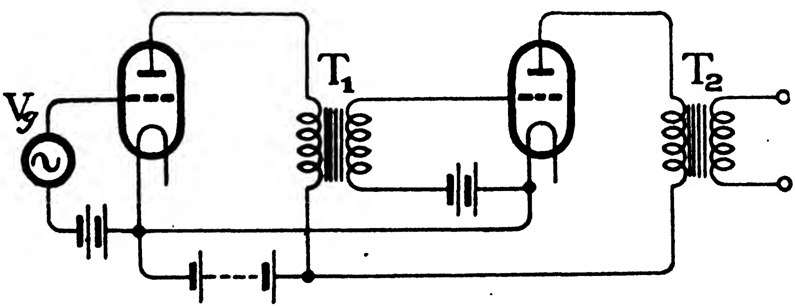


FIG. 171.—Transformer Coupled Two-stage Amplifier.

used as in Fig. 171, with the result that the condenser and grid leak are rendered unnecessary. Chokes and transformers, however, have frequency characteristics; the impedance and, therefore, the amplification at low frequencies is less than that at higher frequencies, while at still higher frequencies the capacity of the windings may again cause a falling off of the amplification. Where distortionless amplification is of primary importance, therefore, the resistance capacity coupling arrangement of Fig. 170 is to be preferred.

The amplification obtained with such two-stage amplifiers is the square of that obtained with a single stage. With three stages the amplification is cubed, and so on. But every stage increases the danger of feed back to the grid of the first valve and the chances of instability.

(47) Telephone Repeaters

Any of the valve amplifier arrangements of the last section may be used to amplify speech currents and so function as a telephone "repeater." But since they essentially work in one direction only, some modification is necessary in order to obtain the both-way working necessary in telephony.

Fig. 172 shows the circuit of a single telephone repeater at the centre of a uniform line. The input transformer T_1 is differentially wound on the primary or line side; the secondary is connected, with a suitable grid biasing battery, across the grid and filament of the valve.

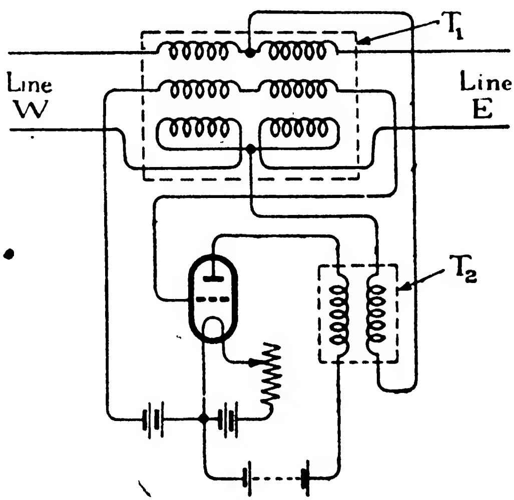


Fig. 172.—Telephone Repeater.

The secondary of the output transformer T_2 is connected across the "peaks" of the transformer T_1, so that equal currents flow east and west and no voltage is induced in the secondary of T_1. With current, however, coming from either east or west a voltage is induced in the secondary of T_1, and amplified currents from the valve are sent to line. For success it is essential that the impedance of the line to the east must be the same as the impedance of the line to the west at all frequencies; for otherwise there will be out of balance currents in T_1 and an e.m.f. induced on the grid. In other words, the output of the valve will feed back to the input, and spontaneous oscillations may occur leading to a "howl" on the circuit.

An estimate may be made of the improvement in transmission to be effected by such a repeater. The power given by the valve to the output transformer is

$$W_o = I_a^2|Z| = \frac{\mu^2 V_g^2 |Z|}{(R_a + |Z|)^2}$$

where $|Z|$ is the size of the impedance (the angle is assumed to be zero in this estimate) of the primary winding of the output transformer. This is designed so that $R_a = |Z|$, and then

$$W_o = \frac{\mu^2 V_g^2}{4R_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The secondary winding is designed to make its impedance $|Z_2|$ match that offered at the peaks of T_1 by making the transformation ratio $K_2 = \sqrt{|Z_2|/R_a}$.

If the input transformer ratio is K_1, the voltage across the incoming line is V_g/K_1 and the incoming power is

$$W_1 = \frac{V_g^2}{K_1^2\,|Z_1|} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where Z_1 is the primary impedance.

Only half W_o goes in the direction required, so that the power magnification M is given from (1) and (2) as

$$M = \frac{\mu^2 K_1^2\,|Z_1|}{8R_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the gain in transmission units is from 44·03

$$\text{Gain} = 10 \log_{10} M\ dB \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For instance, with $R_a = 26{,}000$ ohms, $|Z_1| = 800$ ohms, $K_1 = 40$, and $\mu = 8{\cdot}7$,

$$M = \frac{8{\cdot}7^2 \times 40^2 \times 800}{8 \times 26{,}000} = 460$$

representing a gain in T.U's. of $10 \log_{10} 460 = 27dB$

Two such repeaters are not used in tandem along a line. For since half the repeater power output goes in each direction, if there were another repeater further along the line, the two would "talk" to each other and howling might again result. To prevent this an artificial line balance may be constructed for each side of the line, and a separate valve provided for working in each direction. That half of the energy which does not go in the desired direction is absorbed in the artificial line LB, and no reflection of energy between repeaters is possible with proper balances. The arrangement is shown in Fig. 173, in which the circuit differs from that of Fig. 172 in another respect; the output transformer is differentially wound or balanced instead of the input. This involves no change in principle, but in practice the balanced output transformer is easier to construct than the balanced input with its huge number of secondary turns. Another modification is the introduction of low-pass filters F. The line balance is made sufficiently good up to, say, 2,000 cycles per second, and frequencies above this are cut out by

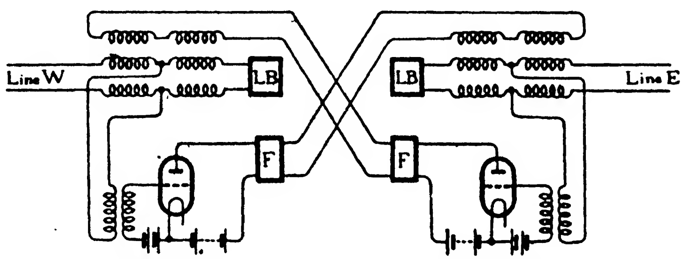


FIG. 173.—Two-valve Telephone Repeater.

the filters. Hence even though howling might take place owing to out of balance above 2,000 cycles, the filters prevent it.

Any number of such two-valve repeaters can be placed in tandem along a long line, but each involves two line balances and filter circuits, and where several repeaters are necessary the "four-wire system" may be adopted, in which (Fig. 174) speech in one direction takes place over one pair of wires and speech in the other direction takes place over a second pair. Line balances are only required at the ends of the circuit, and the "repeaters" are simple one-way amplifiers (either single stage, Fig. 168, or two-stage, Fig. 171) working in the directions shown by the arrows. Thus in the figure speech from west to east takes place over the lower loop, through the amplifiers $A_1A_2 \ldots A_n$, and speech from east to west over the upper loop through the amplifiers $B_n \ldots B_2B_1$. A_1B_1 are located at one repeater station, A_2B_2 at the next, and so on.

Three difficulties become prominent on very long telephone lines:

attenuation distortion, phase angle distortion and echo effects. The first is due to the unequal attenuation by the line of the components of the speech wave. The higher the frequency of the component the greater the attenuation. This distortion is corrected by inserting at the repeater stations artificial lines or networks having series capacities and shunting inductances, which attenuate the lower frequencies more than the higher.

Phase distortion (due to the variation of β from the distortionless relation $\beta = \omega\sqrt{LC}$) results in the velocity of propagation ($c = \omega/\beta$) varying with frequency, with the result that on very long lines the different frequency components of speech arrive at the receiving end at times sufficiently different to cause appreciable distortion.

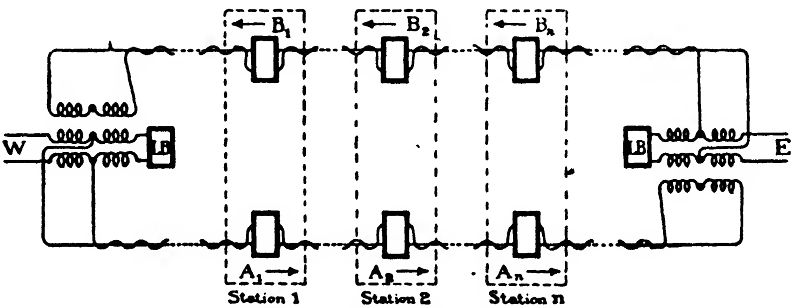


FIG. 174.—Four-wire System of Telephone Repeaters.

The correction of this distortion is more difficult, but special networks have been used with some success.

Echo effects are due to imperfections of the line balances. For instance, speech from the W (Fig. 174) arrives at E along the lower loop, but owing to imperfection in the line balance at E travels back along the upper loop through the B amplifiers, and is heard at W as an echo of the speaker's speech. Further, owing to the imperfections of the line balance at W, the speech wave travels back again along the lower loop and is heard as an echo by the listener at E. On a particular loaded line 1,000 km. long the time of propagation is 85 milliseconds, and the first listener's echo is heard after 170 milliseconds, about the time of an echo in a room 30 metres long. Echoes are eliminated by throwing the return loop out of action while speech is taking place, either by making the grid of one of the amplifiers in the return loop so negative that no current flows through the valve, or by causing relays to short-circuit the line.

The first method is illustrated by Fig. 175, showing an echo suppressor inserted at station 1 (Fig. 174) to put the amplifier B_1 out of action when speech is passing from W to E along the lower loop through A_1. The amplified currents from A_1 are further amplified by V_1. V_2 acts as a rectifier or diode, the grid and anode being joined together. The network N serves to smooth out the current pulses and provide a necessary delay before the resulting

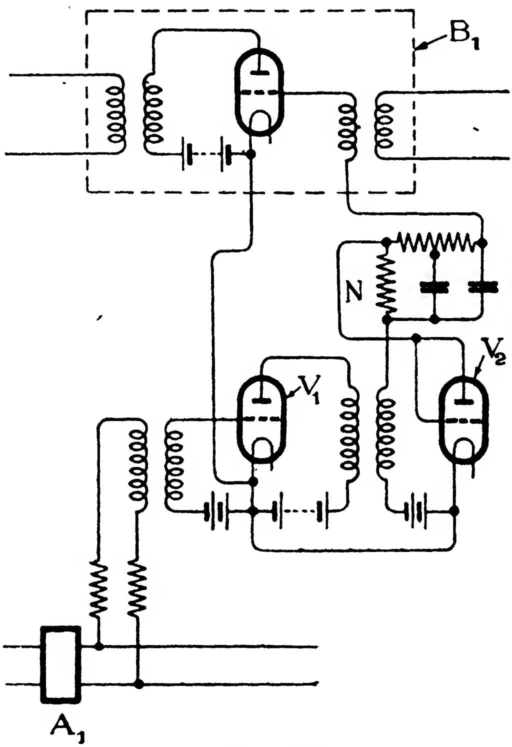


FIG. 175.—Echo Suppressor.

negative potential is put on to the grid of the valve of B_1. A similar suppressor, but acting in the opposite direction, is placed at another station. The relay suppressor is speech controlled in a similar manner, but the valve V_2 closes a relay which shorts the upper loop in front of B_1. The delay is obtained from the relay system.

The successful operation of the repeaters of Figs. 173 and 174 depends very largely upon the success with which an artificial balancing network can be constructed. The requirement is that at all frequencies below the cut-off frequency of the filters, the impedance of the network shall be the same, both in magnitude and angle, as the impedance of the line, both being measured from the line transformer of the repeater.

The impedance of a perfectly uniform unloaded or continuously loaded line, either infinitely long or closed through its characteristic impedance, is given by

$$Z_o = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

and both the magnitude and the angle, or the resistance and react-

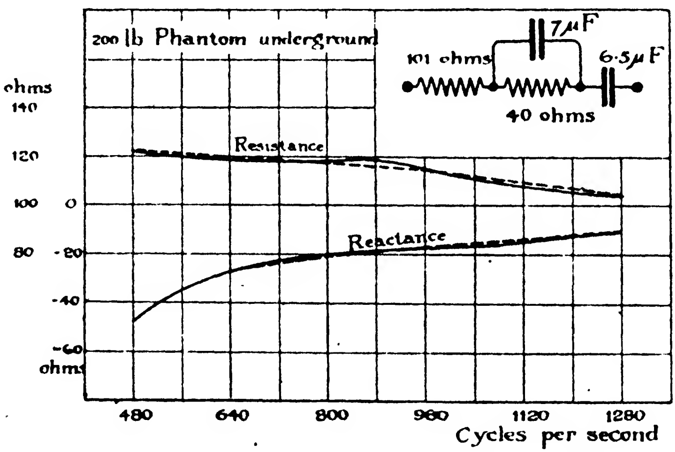


FIG. 176.—Impedance/frequency Curve of Unloaded Line.

ance terms, when plotted against the frequency give smooth curves, and the construction of an artificial network of resistances and condensers to give similar curves is not a difficult matter.

The firm lines in Fig. 176* show the resistance and reactance components of the impedance of a 200-lb. underground unloaded phantom circuit; the dotted lines show the impedance of the balancing network drawn in the inset.

When, however, the line is not perfectly uniform, or is not effectively infinitely long or closed through its characteristic impedance, the impedance-frequency curves will present a wavy

* C. Robinson and R. M. Chamney, P.O. Professional Papers. No. 76.

appearance due to partial reflections, as shown in Fig. 177.* If on arrival at the repeater station the voltage of the reflected wave is in opposition to that of the transmitted wave the effective impedance will be a maximum ; if the reflected voltage is in phase the impedance will be a minimum, and for intermediate phase angles the impedance will have intermediate values. If l is the distance from the repeater station to the point of discontinuity, and the change of phase angle on reflection is ϕ, the phase of the reflected wave will have changed by $2\beta l + \phi$ on reaching the station, and maximum impedance will occur when this is equal to $(2n + 1)\pi$, when n is any integer. Suc-

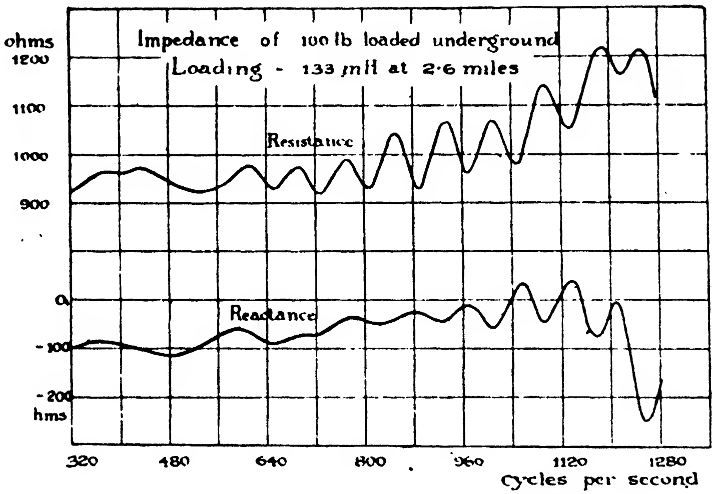


FIG. 177.—Impedance, frequency Curve of Loaded Line.

cessive points of maximum impedance on the impedance/frequency curve occur when the values of $2\beta l + \phi$ differ by 2π.

If the impedance is a maximum at successive frequencies f_1 and f_2, and if β_1 β_2 are the corresponding wave-length constants, and ϕ_1 ϕ_2 the phase-angle changes at reflection, then

$$(2\beta_2 l + \phi_2) - (2\beta_1 l + \phi_1) = 2\pi \quad . \quad . \quad . \quad (5)$$

It appears from practical measurements that ϕ_1 and ϕ_2 can be assumed to be equal, when (5) gives

$$l = \frac{\pi}{\beta_2 - \beta_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

* C. Robinson and R. M. Chamney, P.O. Professional Papers, No. 76.

For an unloaded line (from equation (39·19) putting L = O and G = O)

$$\beta \doteq \sqrt{\pi RCf}$$

and
$$l = \frac{1 \cdot 77}{\sqrt{RC(f_2 - f_1)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For a loaded line (equation (42·03))

$$\beta = 2\pi f \sqrt{LC}$$

and
$$l = \frac{1}{2(f_2 - f_1)\sqrt{LC}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Equations (7) and (8) can be used to localise a fault from impedance measurements.

If the discontinuity is close to the repeater station the reflected energy is great and the waviness of the curves pronounced. If it is at the far end (due to terminal apparatus not being of characteristic impedance) it is still present, but not to the same extent, owing to attenuation of the reflected wave. Inequalities in the inductance of loading coils or in the distance they are spaced apart have the same effect.

The construction of a line balance is simplified by shunting the line with a small condenser, which results in smoothing out the peaks in the impedance curves and rendering their general shape suitable for simulation by a line-balancing network composed of resistances and condensers.

(48) The Valve Oscillator

It has already been mentioned that sustained oscillations may occur if the output energy from a valve "feeds back" in any way to cause a voltage to appear at the grid. Properly arranged valve circuits designed with the object of producing such oscillations are of the greatest value as a source of alternating current for telephone measurements, for use in voice frequency telegraphy, and in high frequency telegraphy and telephony or "wireless."

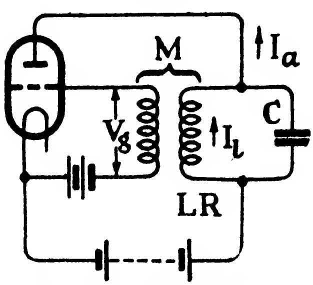


Fig. 178.—Valve Oscillator.

A very usual circuit arrangement is shown in Fig. 178. The anode load is an oscillatory circuit of inductance L, resistance R, and capacity C, and the feed back is by mutual inductance M to a grid coil. The frequency of the oscillations produced is very nearly the resonance frequency of the oscillatory circuit ($\omega = 1/\sqrt{LC}$) and is readily adjustable over a very wide range.

The load impedance is the "parallel impedance" Z_p of the oscillatory circuit which is given by

$$Z_p = \frac{(R + j\omega L)\left(-\frac{j}{\omega C}\right)}{R + j\omega L - \frac{j}{\omega C}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and if an alternating e.m.f. E angular frequency ω is introduced into the grid coil from an independent source, equation 46.05 gives for the alternating component of the anode current

$$I_a = \frac{\mu E}{R_a + \dfrac{(R + j\omega L)\left(-\frac{j}{\omega C}\right)}{R + j\omega L - \frac{j}{\omega C}}} \quad . \quad . \quad . \quad . \quad (2)$$

The current through the coil L is

$$I_l = \frac{-\dfrac{j}{\omega C}}{R + j\omega L - \dfrac{j}{\omega C}} \, . \, I_a \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the voltage induced on the grid by this current is $V_g = j\omega M I_l$. Hence E in (2) must be replaced by $E + V_g$, giving

$$I_a = \frac{\mu}{R_a + \dfrac{(R + j\omega L)\left(-\frac{j}{\omega C}\right)}{R + j\omega L - \frac{j}{\omega C}}} \left\{ E + j\omega M \frac{-\dfrac{j}{\omega C}}{R + j\omega L - \dfrac{j}{\omega C}} \, . \, I_a \right\}$$

Whence

$$I_a \left\{ R_a + \frac{(R + j\omega L)\left(-\frac{j}{\omega C}\right)}{R + j\omega L - \frac{j}{\omega C}} - \frac{\mu \dfrac{M}{C}}{R + j\omega L - \dfrac{j}{\omega C}} \right\} = \mu E$$

or
$$I_a \left\{ R_a \left(R + j\omega L - \frac{j}{\omega C} \right) - j\frac{R}{\omega C} + \frac{L}{C} - \frac{\mu M}{C} \right\}$$
$$= \mu E \left\{ R + j\omega L - \frac{j}{\omega C} \right\} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For oscillations to persist when the independent e.m.f. E is made equal to zero, the left-hand side of (4) must also be zero, and equating real and imaginary terms separately to zero gives

$$R_a R + \frac{L}{C} - \frac{\mu M}{C} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and
$$R_a \left(\omega L - \frac{1}{\omega C} \right) - \frac{R}{\omega C} = 0 \quad . \quad . \quad . \quad . \quad (6)$$

From (5) the least value of the mutual inductance necessary for oscillations to commence is given by

$$M = \frac{1}{\mu}(R_a RC + L) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and from (6) the frequency of the oscillations is given by

$$\omega = \frac{1}{\sqrt{LC\left(1 + \frac{R}{R_a}\right)}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Since R/R_a is usually a very small number, the frequency is very nearly that of the oscillatory circuit. Neglecting the small departure an approximate vector diagram can be drawn as in Fig. 179. The impressed e.m.f. E is drawn as OE of standard phase. With $\omega L = 1/\omega C$, (2) gives $I_a = \mu E / \left(R_a + \frac{L}{CR} \right)$ on neglecting R in comparison with ωL; OI_a represents the anode current in phase with E. I_l from (3) is $-jI_a/\omega CR$, and is drawn vertically downwards as OI_l, while $V_g = j\omega MI_l$ is OI_l rotated clockwise through 90°, bringing OV_g into phase with OI_a. Z_p from (1) is L/CR, neglecting R in comparison with ωL, and the anode voltage is $-Z_p I_a = -LI_a/CR$, and is drawn as OV_a, in phase opposition to OV_g. It is necessary

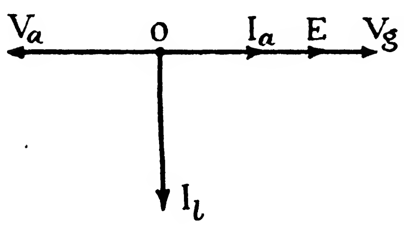


FIG. 179.—Vector Diagram of Valve Oscillator.

in order that oscillations may be maintained that there shall be components of the anode voltage and grid voltage in phase opposition. If, for instance, the connections to the grid coil are reversed, M is given a negative sign and the condition (5) cannot be fulfilled.

Oscillations can similarly be maintained by a valve in a mechanical oscillatory system, such as a tuning-fork or a quartz crystal. The circuit of a valve-maintained tuning-fork is outlined in Fig. 180. M_1 and M_2 are permanent magnets with coils (telephone receiver magnets for instance) connected respectively in the anode and grid circuits of the valve. Alternating current through M_1 will cause the fork to vibrate most vigorously if the frequency is that of the fork, and the vibrations will produce an e.m.f. in the coils of M_2. The theory of the forced vibrations of the fork is the same as that of the telephone receiver (section 34), the fork prong taking the place of the telephone diaphragm. Let A_1 and A_2 be the force factors of the magnets M_1 and M_2 respectively, Z_1 the damped impedance of the winding of M_1, and $z = r + j(\omega m - s/\omega)$ the mechanical impedance of the fork. Then for an external e.m.f. E induced in the grid

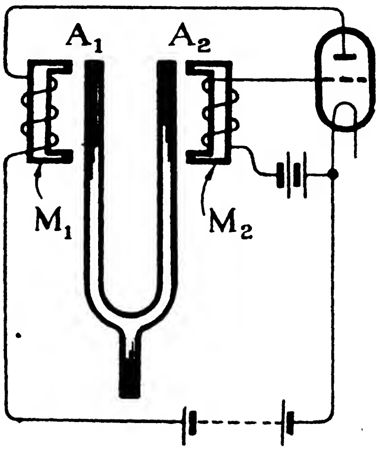


Fig. 180.—Valve-maintained Tuning Fork.

$$I_a = \frac{\mu E}{R_a + Z_1 + \dfrac{A_1^2}{z}} \quad \ldots \quad (9)$$

by equations 34·09 and 46.05.

The prong velocity U is given by

$$U = \frac{A_1 I_a}{z} \quad \ldots \quad (10)$$

and the induced voltage V_g on the grid is

$$= A_2 U = \frac{A_1 A_2}{z} I_a \quad \ldots \quad (11)$$

Hence as before

$$I_a = \frac{\mu}{R_a + Z_1 + \frac{A_1^2}{z}} \left\{ E + \frac{A_1 A_2}{z} I_a \right\}$$

or

$$I_a \left\{ R_a + Z_1 + \frac{A_1^2}{z} - \frac{\mu A_1 A_2}{z} \right\} = \mu E \quad . \quad . \quad . \quad (12)$$

For vibrations to persist when $E = 0$, the left-hand side must also be zero, and the exact frequency and condition for maintenance are thus determined. The frequency will differ very little from $\omega = \sqrt{s/m}$, in which case $z = r$, and if R_1 is the resistance of Z_1 the condition for maintenance is

$$R_a + R_1 + \frac{A_1^2}{r} - \frac{\mu A_1 A_2}{r} = 0 \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The anode voltage is $-\left(Z_1 + \frac{A^2}{r}\right) I_a$, and the vector diagram is as shown in Fig. 181. OA is Z_1, AB is A_1^2/r, and OB, therefore, the impedance of the coils of M_1, and OC drawn in the opposite direction is V_a. V_g is drawn as shown owing to the angles of the force factors A_1 and A_2. To improve the phase relationship between V_a and V_g condensers may be shunted across the coils, and to obtain better impedance matching, transformers may be inserted between M_1 and M_2 and the anode and grid circuits respectively.

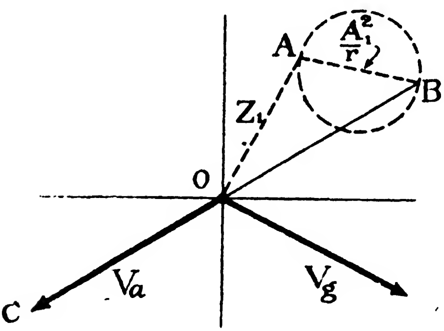


FIG. 181.—Vector Diagram of Valve-maintained Tuning Fork.

The amplitude of the oscillations depends upon the valve characteristics and upon the voltage of the anode battery. If the grid and anode potentials are adjusted so that in the absence of oscillations the steady anode current is half the saturation current I_s of the valve, the maximum alternating component of the anode current when the oscillations have reached their steady value must be $\frac{1}{2}I_s$. If the characteristics of the valve are straight lines terminating on horizontal straight lines ($I_a = 0$ and $I_a = I_s$) and the mutual inductance just satisfies (7), the variations of I_a will be sinusoidal and the current in the oscillatory circuit of Fig. 178 will be very nearly (from (3) and (8), but taking $\omega = 1/\sqrt{LC}$)

$$|I| = \frac{1}{2}\frac{I_s}{R}\sqrt{\frac{L}{C}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

On the other hand, the anode voltage may fall so that the saturation current may not be reached; the amplitude limit on this account is reached when the amplitude of the alternating component of the anode voltage is equal to the battery voltage.

The alternating anode voltage is

$$V_a = -Z_p I_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

From (1), assuming R small compared with ωL, and assuming $\omega = 1/\sqrt{LC}$,

$$Z_p = \frac{L}{CR} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and from (3),

$$I = -\frac{j}{\omega CR} I_a$$

or

$$|I_a| = \sqrt{\frac{C}{L}}\, RI \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

and using (16) and (17) in (15), and writing $V_a = V_o$, where V_o is the battery voltage, gives

$$|I| = V_o\sqrt{\frac{C}{L}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

The amplitude of the oscillations will be limited either by the saturation current of the valve (14) or by the anode voltage (18), whichever gives the smaller limit. Maximum amplitude will be reached when the two limits coincide, that is, when

$$\frac{1}{2}\frac{I_s}{R}\sqrt{\frac{L}{C}} = V_o\sqrt{\frac{C}{L}}$$

or

$$\frac{2V_o}{I_s} = \frac{L}{RC} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Equation (19) is simply the statement that the maximum oscillatory current is obtained when the internal resistance of the valve $\left(= \frac{V_o}{I_s/2}\right.$ with the ideal characteristics considered) is equal to the resistance of the load (see equation (16)). Condition (19) can be met by adjusting the ratio L/C, keeping the product LC constant as determining the frequency. An equivalent adjustment is by means of the anode tap (p. 333).

If (19) is satisfied, the oscillatory current is from (14) or (18)

$$|I| = \sqrt{\frac{V_o I_s}{2R}}$$

and the power expended in the oscillatory circuit is

$$W_1 = \frac{|I|^2 R}{2} = \frac{V_o I_s}{4} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The average power expended by the anode battery is

$$W_2 = \frac{V_o I_s}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

since the mean anode current is $I_s/2$ on the assumption made, whether the valve is oscillating or not. Hence the efficiency of the conversion from direct to alternating current is 50 per cent. When the valve is oscillating half the power supplied by the battery is converted into alternating current in the oscillatory circuit; the other half is dissipated in the internal resistance of the valve and appears as heat at the anode due to bombardment by the electrons. If the valve ceases to oscillate for any reason and the valve batteries remain connected, the whole of the power from the battery heats the anode, a condition which may result in the anode being destroyed.

(49) Multiple Systems

The army vibrator system of alternating current telegraphy has never been used generally on account of the interference it causes with telephone circuits. The same cause led to the disuse of the Mercadier system of multiple telegraphy, in which currents of different frequencies produced by a number of vibrating reeds selectively operate a number of receivers mechanically tuned to the reeds. But the advent of the valve as oscillator, amplifier and rectifier enables the line currents in a multiple system of voice frequency telegraphy to be kept of the same order of magnitude as telephone currents, and ordinary telegraph receiving apparatus to be used instead of telephone receivers, the selection or separation of the various channels being carried out by means of band pass filters.

The scheme is shown in outline in Fig. 182. At the transmitting end the telegraph sending apparatus controls the relays R_s, whose local circuits determine when voltage from the oscillators O is

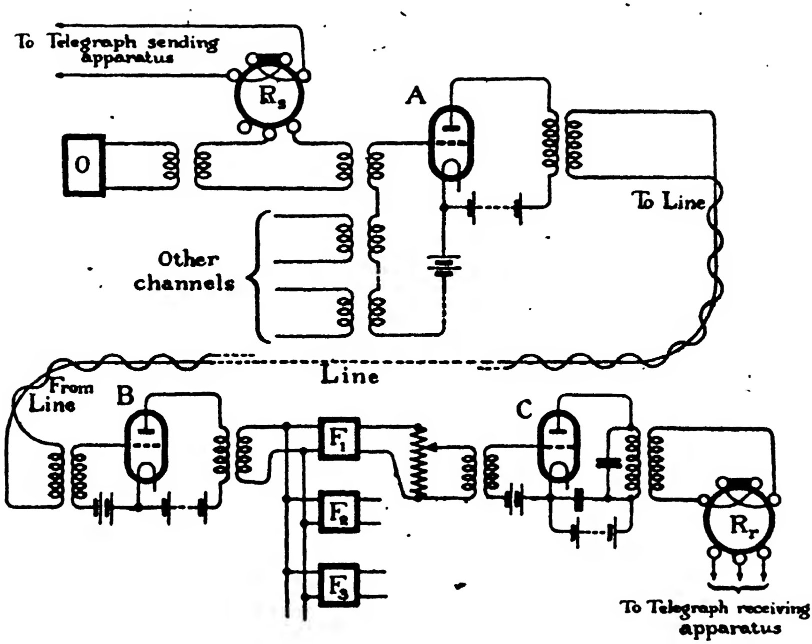


FIG. 182.—System of voice frequency multiple telegraphy.

impressed on the grid of the common amplifying valve A, the output side of which is connected to line. The oscillators O may be of the tuning-fork or oscillatory circuit type (Figs. 180 or 178). The line currents are received on another common amplifier B—the output side of which is connected through the various filters F to the rectifying valves C containing the receiving relays R_r in their output circuits. The increase of anode current through C on the receipt of a signal gives a marking impulse through R_r, and the reduction of anode current on the cessation of the signal gives a spacing impulse. The frequencies of the various channels are all below 2,000 cycles per second, so that the system can be worked on loaded lines. Different frequencies can be allocated for transmission in the two directions over a single loop, in which case the apparatus shown as sending and receiving in Fig. 182 would appear at each end of the line, the input to the valves A and B on the line side of the transformers being connected in series. Or a single frequency can be used for sending and receiving by duplexing with a differential line transformer similar to one side of the valve repeater of Fig. 173. Or separate lines may be used for sending and receiving as in the four-wire repeater of Fig. 174.

The design of the band-pass filters presents a mathematical problem of considerable complexity, involving as it does, when the speed of signalling is at all large, a consideration of the alternating current transients occurring in the building up and dying away of the signal. It has, however, been found quite possible to obtain on a moderate length of line (London-Manchester, with a repeater at Derby) the equivalent of a Wheatstone speed of 100 words a minute with six channels in operation. Also a single channel can be used for the operation of a multiple system of the Baudôt type, giving a multiple multiple system in which a very extensive use is made of the line.

There is nothing in principle to limit the frequency to 2,000 cycles per second except the cut off of the loaded line, and the excessive attenuation to higher frequencies of unloaded underground lines. But on aerial lines multiple systems of telegraphy, as well as of telephony, at much higher frequencies are in use. These are known as "wired wireless" or "carrier wave," and their operation will be clearer after a consideration of high frequency currents (in Part III.). But at present consider the circuit of Fig. 183, in which a voltage of comparatively high frequency $\omega_1/2\pi$ is impressed from the oscillator O through the transformer T_1 on the grid of the square-law valve (the grid bias is adjusted to bring the working point on to the

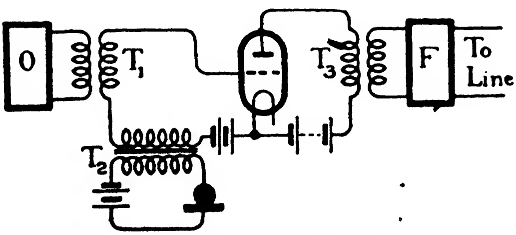


Fig. 183.—Production of voice modulated high frequency currents.

square law part of the curve), as well as a speech voltage from the microphone through the telephone transformer T_2. Considering one component of the speech voltage of frequency $\omega_2/2\pi$, the alternating voltage impressed on the grid is

$$V_g = V_1 \sin \omega_1 t + V_2 \sin \omega_2 t \quad . \quad . \quad . \quad . \quad (1)$$

and 45.16 gives for the alteration of anode current

$$\delta I_a = a_1(V_1 \sin \omega_1 t + V_2 \sin \omega_2 t) + a_2(V_1 \sin \omega_1 t + V_2 \sin \omega_2 t)^2 \quad . \quad . \quad . \quad . \quad (2)$$

where a_1 and a_2 are constants defined by 46.14 and 46.15. On expansion (2) may be written

$$\delta I_a = a_1 V_1 \sin \omega_1 t + a_1 V_2 \sin \omega_2 t + \frac{a_2}{2} V_1^2 - \frac{a_2}{2} V_1^2 \cos 2\omega_1 t + \frac{a_2}{2} V_2^2 - \frac{a_2}{2} V_2^2 \cos 2\omega_2 t + a_2 V_1 V_2 \cos (\omega_1 - \omega_2)t - a_2 V_1 V_2 \cos (\omega_1 + \omega_2)t \quad . \quad . \quad . \quad (3)$$

Besides the alteration of mean anode current, which produces only a transient effect on the line side of the output transformer T_3, there are six frequencies present in the anode current wave form, two low frequencies of angular values ω_2 and $2\omega_2$, and four high frequencies of angular values ω_1, $2\omega_1$, $(\omega_1 - \omega_2)$, and $(\omega_1 + \omega_2)$. If there were two low frequencies ω_2' and ω_2'' as well as the high frequency ω_1, there would be low frequency terms in $(\omega_2' - \omega_2'')$ and $(\omega_2' + \omega_2'')$ as well as the other terms. The air-core transformer T_3 more or less effectually cuts out all the low frequency terms and the band-pass filter F completes the process, as it is arranged to transmit to line only the band of high frequencies between ω_1 and $\omega_1 + \omega_2$, where ω_2 is, say, $2\pi \times 2{,}000$, corresponding to the highest frequency necessary for commercial speech. The frequency ω_1 is called the *carrier* frequency, and the bands of frequencies $\omega_1 - \omega_2$ and $\omega_1 + \omega_2$ the lower and upper *side bands* respectively; the wave consisting of the three together is known as the modulated wave.

At the receiving end of the line the wave of frequencies ω_1 and $\omega_1 + \omega_2$ is impressed on another valve with square-law characteristics, and the result, as before, is the production of currents of angular frequencies ω_1, $\omega_1 + \omega_2$, 2ω, $2(\omega_1 + \omega_2)$, $2\omega_1 + \omega_2$ and ω_2. All except the last are eliminated by a low-pass filter, and the result is a theoretically distortionless telephone reception, ω_2 comprising all the telephone frequencies.

Different channels are obtained by the use of different carrier frequencies, which are separated out by high frequency band-pass filters at the receiving end in much the same manner as in the voice frequency telegraph scheme of Fig. 182.

(50) **Picture Telegraphy**

The electrical transmission of pictures is successfully accomplished by means of carrier waves modulated by means of a photo-electric cell in accordance with the depth of tone of the picture. The photo-electric cell depends for its action on the emission of electrons under the influence of light from a cathode coated with potassium hydride, the number of electrons emitted being proportional to the illumination. An anode battery thus causes an anode current proportional to the illumination to flow through a suitable transformer, and the resulting voltage fluctuations can be amplified to any desired extent by valve amplifiers.

Light from a suitable electric lamp is focused to a very small dot on a cylinder containing the picture to be transmitted, and the cylinder is rotated and translated so that the spot of light traverses the whole surface of the picture in a close helix. The cylinder may be of glass and the picture a photographic negative, in which case the light passes through the negative to influence the photo cell. Or the picture may be positive and the cell influenced by reflected light. The light from the source is varied sinusoidally to provide the carrier wave by a rotating disc suitably notched, or the carrier wave may be provided by an independent oscillator, the modulations being carried out by causing the amplified output from the cell to act on the grid in the manner of Fig. 183.

At the receiving end a cylinder carrying a sensitised paper or film is caused to rotate and translate in exact isochronism with the cylinder at the sending end, and the incoming currents are caused to control the amount of light falling in a small dot on the film. This may be done by causing the rectified current to tilt an oscillograph mirror on which the beam of light falls, or by passing the rectified current through a strip conductor in a powerful magnetic field, the movements of the strip uncovering more or less of an aperture through which the beam of light falls on the cylinder. A third method depends upon the Kerr "light valve" cell. Light from the source is passed through a Nicol prism, the Kerr cell (consisting of electrodes of brass in a glass vessel containing nitro-benzol), and another Nicol prism in series before being focused on to the cylinder as a small spot. Normally the Nicol prisms are set so that they polarise the light in planes at right angles, and none is transmitted. But the rectified incoming current is caused to apply a potential across the Kerr cell with the result that the plane of polarisation is twisted, and light gets through to the cylinder in amount proportional to the voltage applied to the cell.

REFERENCES FOR FURTHER READING

H. J. Van der Bijl.—" The Thermionic Vacuum Tube."
S. Butterworth.—" The Maintenance of a Vibrating System by Means of a Triode Valve." *Phys. Soc. Proc.* 32, p. 345.
T. G. Hodgkinson.—" Valve Maintained Tuning Forks without Condensers." *Phys. Soc. Proc.* 38, p. 24.
H. Cruickshank.—" Voice Frequency Telegraphs." *J.I.E.E.*
E. S. Ritter.—" Picture Telegraphy." *J.P.O.I.E.E.*, Vol. XXI., p. 191.

CHAPTER X

SIGNALLING AND SWITCHING

(51) Apparatus

THE simplest circuit for speech in either direction over a line has been drawn in Fig. 119, and this is complete as far as speech is concerned. But other practical requirements arise. There must be a switch in the microphone circuit to avoid wasting the battery, and a means must be provided of calling the other station when speech is desired. Fig. 184 shows how these requirements may be met in a simple telephone circuit where the line is short. Calling is by an ordinary trembler or house bell C, which is rung by direct current from primary batteries. When the push button K is depressed the batteries B_1 and B_2 in series send a calling current to line. The receiver is normally carried on the hook H, and contact is thus made between the line terminals and the bell, and an incoming calling current flows through the bell and rings it. When the receiver is lifted from the hook to answer the call, the secondary S of the induction coil and the receiver R are connected across the line, and at the same time the microphone M, the primary P of the induction coil and the battery B_1 are connected in series. The connections are now identical with those of Fig. 119, and speech currents can be sent and received in either direction. When the receiver is replaced on the hook the microphone circuit is broken and the bell connected across the line ready to receive another call.

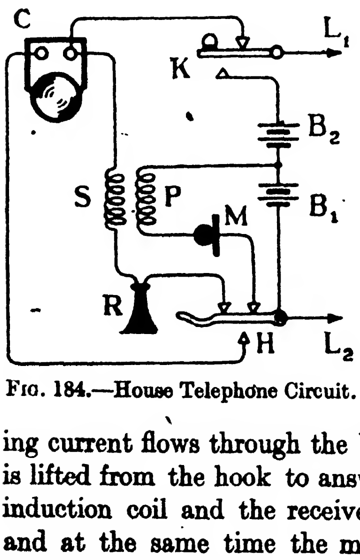


FIG. 184.—House Telephone Circuit.

When the line is anything but very short its resistance necessitates the use of large batteries B_2 of primary cells in order to obtain sufficient current to ring the bell, and "magneto ringing" is adopted, that is, a polarised bell is rung by low frequency alternating current generated by hand turning a small permanent magnet alternator. The bell is illustrated in Fig. 185. The two soft iron cores CC are connected at the bottom by a soft iron yoke B and an armature A at the top pivoted at its centre completes the magnetic circuit through two air gaps. The system is polarised by the permanent magnet NS, and the coils on the cores are connected so that current through them in series sends flux round the circuit which strengthens the permanent flux in one air gap and weakens it in the other, depending upon the direction of the current. With alternating current, therefore, the armature is rocked backwards and forwards, and the rod and striker which it carries rings the two bell domes DD. The theory of the action is practically identical with that of the telegraph polarised relay.

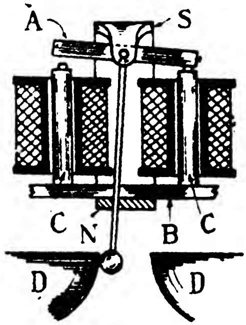


Fig. 185.—Polarised Bell.

The field of the magneto generator consists of three or more

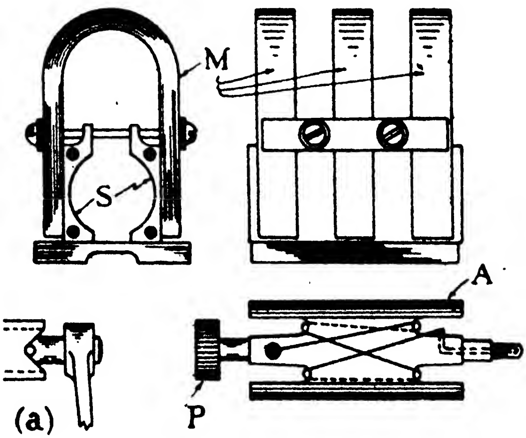


Fig. 186.—Magneto Generator.

permanent magnets M with two cast-iron pole pieces S (Fig. 186), and the H type armature A is hand rotated through a spur wheel and pinion P arrangement with a ratio of about 5 to 1. An automatic switching arrangement is devised by mounting the spur wheel loosely on the shaft carrying the turning handle, and causing it to be driven by a pin which rests against an inclined surface cut in the boss (shown at *a*, Fig. 186). When the armature is at rest a spring keeps the pin at the bottom of the recess, but when the shaft is turned the pin travels up the inclined surface and the shaft is moved outwards. This movement releases a contact spring as shown at C, Fig. 187, which gives one way in which a magneto ringing station may be arranged. A received calling current passes through the bell, armature and switch hook H, the last short circuiting the receiver R and secondary S of the induction coil. When the magneto handle is turned the contact C causes the bell to be short circuited and current is sent direct to line. When the receiver is lifted, the hook short circuits bell and armature, the microphone circuit is made and speech currents flow as in Figs. 184 and 119.

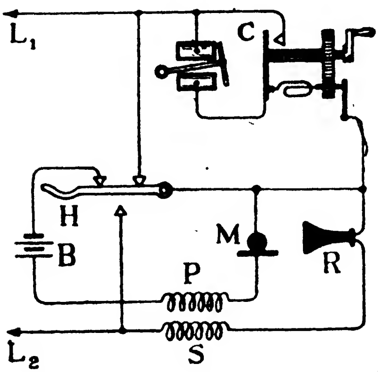


FIG. 187.—Magneto Telephone Circuit.

When the telephone line is not a direct one from one station to another, but is a line to an "exchange" at which connections are made as required among the various subscribers to the telephone system, some form of device to call the operator other than a bell is required, and in addition a means of signalling the completion of a call in order that the line may be cleared ready for another. The "indicator" is in common use in magneto ringing systems for these purposes. There are many patterns, but the general idea will be gathered from Fig. 188. The winding is on the soft iron core C, which is rigidly screwed in a tubular case B of soft iron. The disc armature A pivoted at P carries an extension F terminated by a small hook H, which normally prevents a shutter S from falling. When the armature is attracted, however, F is raised and H releases S which falls by gravity about its pivot J, and uncovers a label L on which is printed the subscriber's number. The fall of the shutter is stopped by its tail striking a bare wire W which runs along the whole row of indicators. All the shutters are connected electrically, so that a circuit between W and the shutters containing a battery and bell is closed when any shutter falls, and is used to give a ring when the operator is not in attendance.

The indicator of Fig. 188 has to be restored by hand. Other indicators are self-restoring, such as the eyeball indicator illustrated

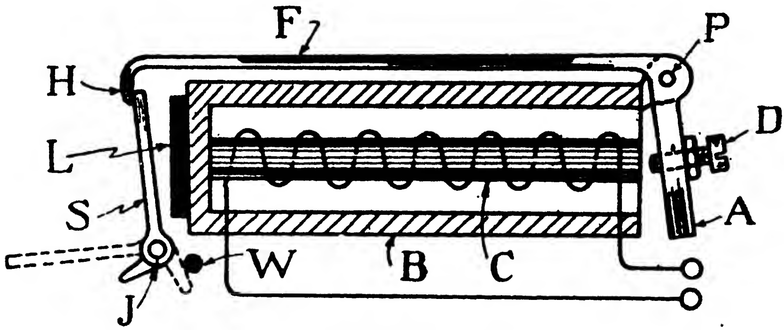


FIG. 188.—Drop Indicator.

in Fig. 189, shown at rest at (*a*) and operated at (*b*). The operation is similar to that of a moving iron ammeter, the increased cross-section of iron resulting from a movement of the armature A causing the eyeball B to rotate in a counter-clockwise direction about the spindle P to fill the space S when either alternating or

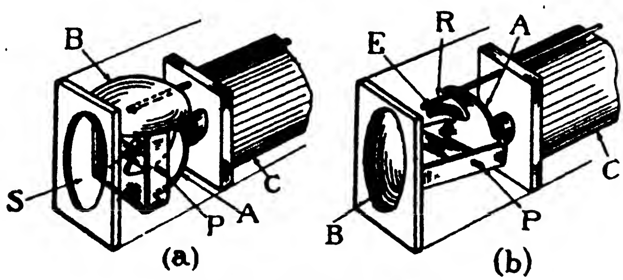


FIG. 189.—Eyeball Indicator.

direct current flows through the coil C. Contact for a local circuit is made between the rod R and an extension E on the armature. Restoration is by gravity.

Another calling arrangement in very extensive use is the lighting of a small electric lamp. The calling current actuates a relay and the lamp is in the local circuit of the relay. The relays used in telephone exchanges are of many different forms, are always non-polarised, and are designed for compactness, robustness, reliability and cheapness of manufacture rather than for extreme rapidity of action or sensitivity. Typical relays are drawn in Figs. 190 and 191. Fig. 190 shows a single contact relay. The armature A is

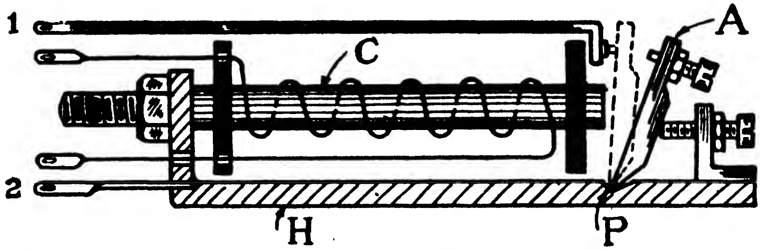


Fig. 190.—Single Contact Relay.

pivoted on a knife edge P and restoration is by gravity. When the armature is pulled up contact is made between terminals 1 and 2. The magnetic circuit is through the core C, the armature A and the heel H.

Fig. 191 shows a multiple contact relay. When the armature A is pulled up an extension A_1 pushes the spring S_1 up to make

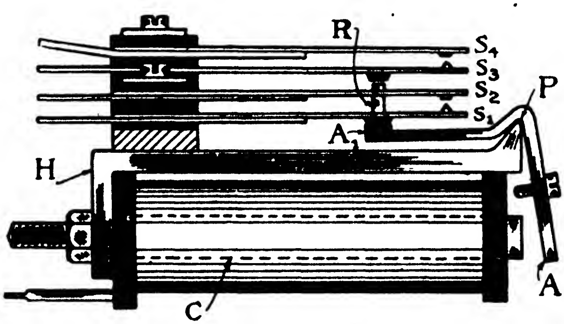


Fig. 191.—Multiple Contact Relay.

contact with S_2, S_1 and A_1 being insulated from each other. A_1 also carries a rod R which passes without contact through holes in S_1 and S_2 to force (through an insulated contact) the spring S_3 into contact with S_4. There may be three rods R and three sets of springs S_1, S_2, S_3, S_4 (behind each other in the section) so that the armature controls six contacts. This number may be increased up to eighteen by a different arrangement of the springs. Restoration is by the spring pressure.

The time of operation of these relays is about 5 milliseconds. For many purposes in automatic switching, relays are required which operate or release much more slowly. An additional winding closed through an external resistance will cause a delay in the operation and release of the relay. For by Lenz's law the current which flows in the closed coil will produce flux in the core in opposition to that produced by the operating coil, and the rate of rise of flux in the air gap will be reduced accordingly. The most efficient form of second winding from the point of view of delay action is a copper cylinder or sleeve fitting closely over the core between the core and the operating winding, and this will have a similar effect on both operation and release. In another form the winding is confined to one portion of the length of the core, and the remainder is filled with

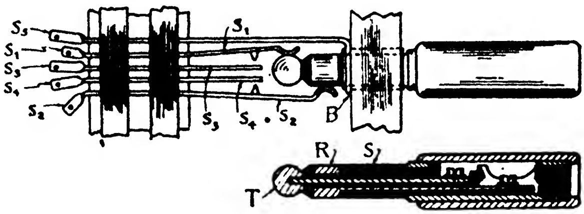


FIG. 192.—Jack and Plug.

a thick copper cylinder or "slug." If the slug is fitted at the armature end of the core, the presence of the opposing magneto-motive force due to the current in the slug will cause considerable leakage flux at the expense of the flux in the air gap; the relay will be slow to operate. The slug will also on breaking the operating current ensure a continuance of the flux in the air gap; the relay will also be slow to release. But if the slug is fitted at the heel end of the core, the relay will be quick to operate (leakage flux will now pass through the air gap), although, as before, slow to release. In this manner the time of operation may be increased from 5 to 300 milliseconds.

In all manually operated exchanges the incoming lines are terminated on jack switches, and connections are made by plugs connected to flexible cords. The construction of a simple jack and plug is shown in Fig. 192. The line is connected to the two outer springs S_1 S_2 of the jack, and the indicator to the two inner springs S_3 S_4. The bush B of the jack is connected to a terminal S_5. When the plug is inserted in the jack, the tip T makes contact with the shorter jack spring S_1, the ring R with longer jack spring S_2, and the sleeve S with the bush B. The cord has three wires connected to tip ring and sleeve, although in small exchanges only the tip and ring may be used. Not only is the line connected to the cord by the insertion of the jack, but the indicator is completely cut off from the circuit by the forcing apart of the two jack springs (see Fig. 195). The insertion of a similar plug at the other end of the cord into the jack of another subscriber connects the two lines together, but apparatus must be provided in the cord circuit to ring the second subscriber, to enable the operator to speak to either subscriber and to listen in, and to obtain a clearing signal.

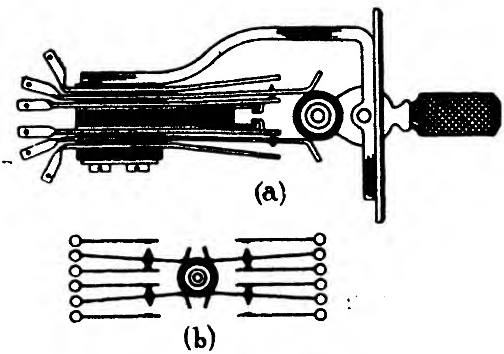


FIG. 193.—Key Switch.

Key switches are used in this connection. The construction of a typical key is shown at *a*, Fig. 193, and its diagrammatic representation at *b*. There are three positions of the key, and the connections corresponding to each are clear from *b*. There are many different spring contacting arrangements, and the springs may be so arranged that on one side the key may remain thrown, while on the other the key is restored to normal when released.

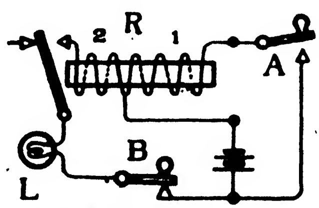


FIG. 194.—Holding-on Circuit.

A holding-on arrangement is frequently used in exchange circuits, in which a momentary signal is caused to give a permanent indication. In Fig. 194 closing the switch A causes current to flow through coil 1 of the relay R, the armature is pulled up and current flows through the coil 2 and the lamp. The armature is held up by the current, even though the switch A is now opened, and the lamp lights until switch B is opened.

(52) **Local Exchanges, Manual**

The arrangements for signalling and switching at a small exchange may now be explained. Fig. 195 gives an outline diagram of connections of a small magneto switchboard. All the lines terminate on jacks; and above each jack and connected to its inner springs is the corresponding indicator. The subscriber by turning the handle of his magneto sends an alternating current to line and drops the indicator shutter. The operator answers by inserting the answering plug of a cord in the jack. This disconnects the indicator from the line, and allows the operator's telephone set to be connected to the

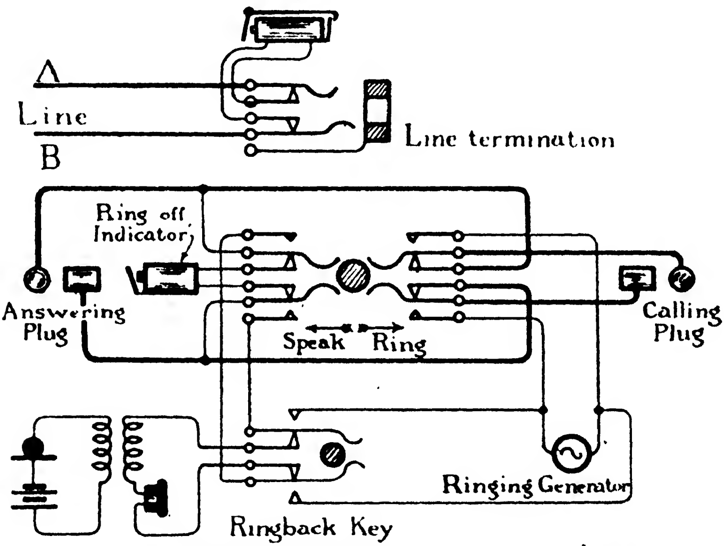


Fig. 195.—Magneto Exchange Line and Cord Circuits.

line when the speak and ring key is thrown to speak. Having ascertained the number of the subscriber required, the calling plug is inserted in the corresponding jack, and the key thrown to ring. This results in alternating current from the exchange generator flowing round the loop of the called subscriber and ringing his bell. The key is now thrown to speak again and the operator listens until the two subscribers are speaking to each other. If in the meantime the first subscriber has hung up his receiver, he is called by the depression of the ring-back key. When the connection is finally established the key is restored to the mid position, and a clearing ring from either subscriber is received on the ring-off indicator.

The plugs are now withdrawn, the shutters of the indicators are restored by hand (if self-restoring indicators are not fitted) and the cord circuit is available for another call.

Magneto generators are expensive instruments, and the provision and maintenance at the subscriber's premises of the large number of cells necessary for direct current signalling is also expensive. Both are avoided in the common battery signalling (C.B.S.) arrangement, shown in outline in Fig. 196, in which a 24-volt battery located at the exchange is used to supply the current required to signal the exchange, while the subscribers are rung by a magneto generator machine at the exchange. The connections of the sub-

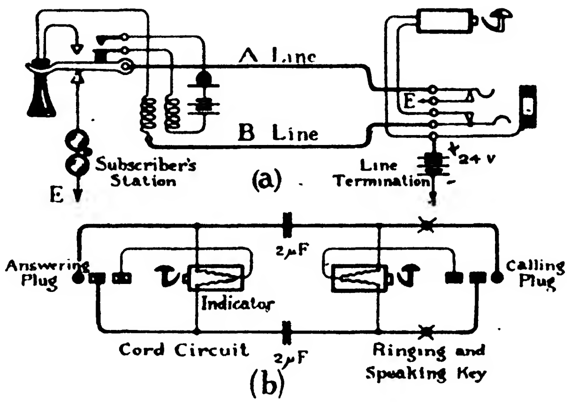


FIG. 196.—C.B.S. Exchange (*a*) Line Circuit, (*b*) Cord Circuit.

scriber's apparatus and the termination of his line at the exchange are shown at (*a*) and the cord circuit at (*b*). A three-wire cord is required, and the wires of the subscriber's loop are differentiated as A and B. When the subscriber lifts his telephone receiver off the hook, the earthed 24-volt battery at the exchange sends current through the eyeball indicator, jack springs, the B line, the secondary of the induction coil, the receiver, the A line and to earth through jack springs. The eyeball indicator is thus operated until the circuit is broken by the insertion of the answering plug into the jack. The cord circuit contains a speaking and ringing key exactly as in Fig. 195, and the required subscriber is rung over his A line to earth through his bell. The cord circuit is divided into two halves as far as direct currents are concerned by the 2μF condensers, which offer only a small impedance to speech currents, so that a clearing signal is obtained from each subscriber. The differentially wound eyeball clearing indicators which are bridged across the cord circuit offer a high impedance to speech currents. With the plug in a jack, the earthed 24-volt battery is connected to the mid point of the coils. When the subscriber replaces his receiver the battery sends current through one coil of the indicator to the A line and to earth through the subscriber's bell, and the indicator is actuated. When both indicators are actuated the operator knows that both subscribers have hung up their receivers, and the line can be cleared.

In large exchanges the use of a single battery of accumulators located at the exchange is extended to provide the direct current required for the subscriber's microphones as well as the necessary signalling currents; the exchanges are called common battery exchanges.

Two systems are in use, the Hayes and the Stone. The former is outlined in Fig. 197. Microphone current is supplied to the telephone set A through the windings 1 and 2 of a transformer, and to the telephone set B through windings 3 and 4. The windings have all the same number of turns and are wound on a common toroidal core; and the transformer is called a repeating coil. Looking upon the microphone at A as a speech current generator, the alternating currents flow from A round the windings 1 and 2 and through the battery, and the electromotive forces produced inductively in windings 3 and 4 cause speech currents to flow through the receiver at B. If the repeating coil were omitted, the battery would short circuit the alternating currents as far as the receiver at B is concerned. The battery is common to all the circuits, and must have very low internal resistance R, otherwise interference among the circuits would be caused by the alternating potential difference IR established across the battery, where I is the speech current.

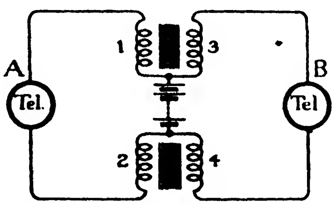


FIG. 197.—C.B. Telephony—Hayes' System.

The same result is achieved in the arrangement (Stone's) of Fig. 198 by means of impedances Z (instead of the repeating coil), the general effect of which is to cause the alternating currents to flow right round the loop instead of through the battery. Owing, however, to the direct current potential drop in the impedances, the microphone current received by any subscriber depends upon the resistance of the loop of the subscriber to whom he is connected.

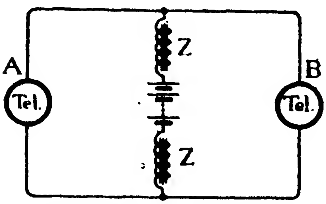


FIG. 198.—C.B. Telephony—Stone's System.

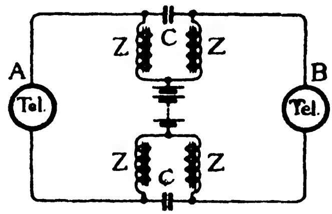


FIG. 199.—C.B. Telephony—Modified Stone System.

This difficulty is overcome as in Fig. 199 by having four impedances Z and providing a path of low impedance to the telephone current by the condensers C. The impedances Z may be the impedances of some of the signalling apparatus. The repeating coil of Fig. 197

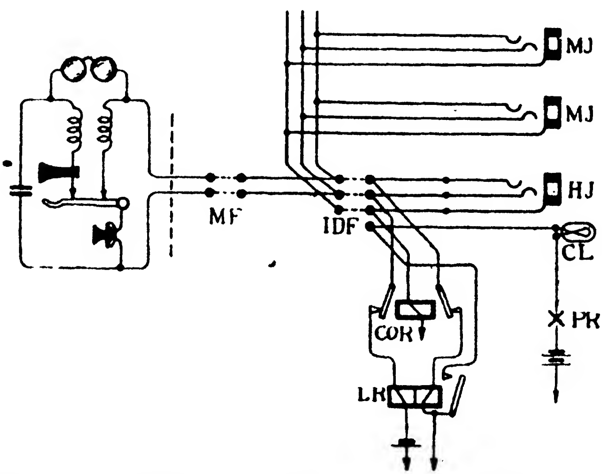


FIG. 200.—C.B. Exchange—Subscriber's Apparatus and Line Termination.

and the impedances and condensers of Fig. 199 are located in the cord circuit.

The same battery that is used to provide the microphone currents is used also for the signalling currents and the signalling is to a large extent automatic. Fig. 200 shows the apparatus and circuit that must be provided for each subscriber; to the left is the subscriber's telephone set and to the right the termination of his line at the exchange. At the main frame (MF) on which all the incoming lines terminate, the lines are cross connected ("jumpered") so that on the exchange side the lines are in numerical order, and protective devices such as fuses, lightning protectors and heat coils are introduced. At the intermediate distribution frame (IDF) the jumpering is used to give a uniform distribution of load among the operators.

The line relay LR is normally across the subscriber's loop in series with the 24-volt battery (although for convenience 24-volt batteries are shown in different parts of a circuit diagram, actually there is only one battery, always earthed at the positive terminal),

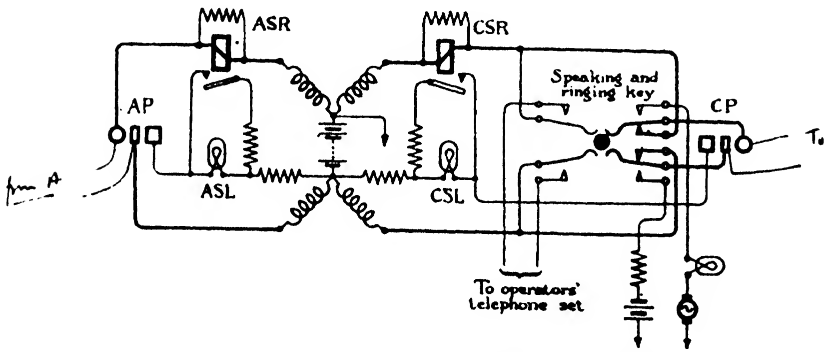


FIG. 201.—C.B. Exchange—Operator's Cord Circuit.

but the condenser of the subscriber's set prevents direct current flowing through the bell when the receiver is on the hook. When the receiver is lifted, however, current flows through the primary of the induction coil, the microphone and the two coils of LR which is operated. Current now flows through the home or calling lamp CL (and also through a pilot relay PR common to the operator's position which lights a larger lamp or rings a night alarm) which is placed just above the home jack HJ.

The operator replies by inserting the answering plug AP of a cord, the circuit of which is shown in Fig. 201. Current from the 24-volt battery connected to the sleeve now flows through the cut off relay COR and operates it, causing LR to be disconnected and CL extinguished. The line is now through to the cord circuit and all other apparatus is cut off, but the multiple jacks MJ remain. These are carried right round the switch room so that any operator can plug in to call any subscriber, but it is necessary first to ascertain whether the subscriber is already connected. The sleeves of all the multiple jacks (when a plug is in one or in the home jack) are above earth potential owing to the drop through the COR, and the operator hears a click in her receiver on touching the sleeve with the tip of the calling plug if the subscriber is "engaged." In the absence of the click the calling plug is inserted in the required MJ, the corresponding COR operates, and ringing takes place as described before, through the subscriber's bell and condenser in series.

The current which operates the COR's would also light the answering and calling supervisory lamps ASL and CSL, but these are short circuited by the operation of the corresponding supervisory relays ASR and CSR by the microphone currents sent to line. When the subscribers hang up their receivers the microphone circuits are broken, ASR and CSR restore and ASL and CSL light, and the circuit can be cleared.

A further requirement is the recording of the number of calls made by a subscriber. This is done on a "meter" similar to a bicycle cyclometer, which is operated by a ratchet and pawl arrangement driven electrically from the cord circuit when the operator presses a button. At the same time a "position meter" is operated which records all the calls dealt with by the operator. There is also a "non-effective meter"; so that the total load of the operator over any period can be ascertained. These metering arrangements are omitted from the circuit diagram for the sake of simplicity.

(53) Local Exchanges, Automatic

While in the common battery exchange signalling is very largely automatic, the operator has to ascertain the number required, inform the subscriber if the number is engaged, make the connection and ring the required subscriber, and clear the line on the completion of the call. In an "automatic" exchange all these operations are carried out by "machines," and the necessary intelligence must be provided by the calling subscriber.

While there are several successful machine switching systems in existence, in this country the Strowger or step-by-step scheme has been adopted as standard. The mechanical principle of the switch unit upon which the scheme is built is illustrated in Fig. 202. The end P of an arm W fixed to a shaft S can reach any point Q on a cylinder by two movements of the shaft S; the first a vertical translation to the required height or level, and the second a horizontal rotation through the required angle. If double contacts are mounted at P, and corresponding contacts are mounted on the cylinder, and if the contacts at P are connected to the incoming calling line and those on the cylinder to the line of the other subscribers on the exchange, the calling subscriber can in this way be connected to any other subscriber. This is the principle of the Strowger two-co-ordinate switch. There are ten vertical steps and ten rotatory steps, 100 contacts in all, and exchanges of up to 100 subscribers could be dealt with in this way. Each subscriber's line would terminate on the contacts P of a switch, and be multipled to the contacts Q on all the switches. The points P, therefore, correspond to the home jacks of the manual exchange, and the points Q to the multiple jacks, while the switch takes the place of the cord circuit. Another set of contacts reached by another arm W on the same shaft correspond to the third wires of the manual exchange.

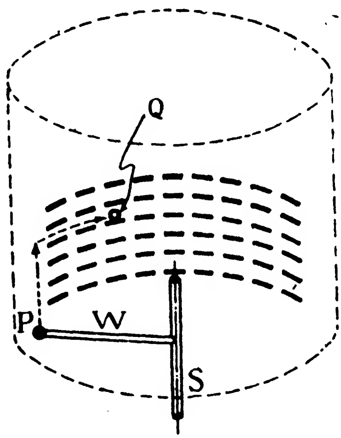


FIG. 202.—Step by Step Principle.

The actual operation of the switch is shown in more detail in Fig. 203. The vertical shaft S carries two racks VR and RR for the vertical and rotary motions respectively, engaged by two pawls VP and RP operated by the two electromagnets VM and RM. The vertical motion is against gravity and the rotary motion against the restoring spring RS. The shaft is held in the position attained by dogs D_1 and D_2 which are pressed against the racks by the springs S_1 and S_2. When, however, the release magnet Rel.M is operated the dogs D_1 and D_2 are withdrawn and the switch returns to its normal position in two motions, a rotation by the spring RS and a drop under gravity. At the first vertical movement the "off normal springs" ONS are operated.

The wiper W is shown as a double arm consisting of two springs P_1T_1 and P_2T_2 insulated from each other, and making contact on double projections insulated from each other numbered 1 to 0

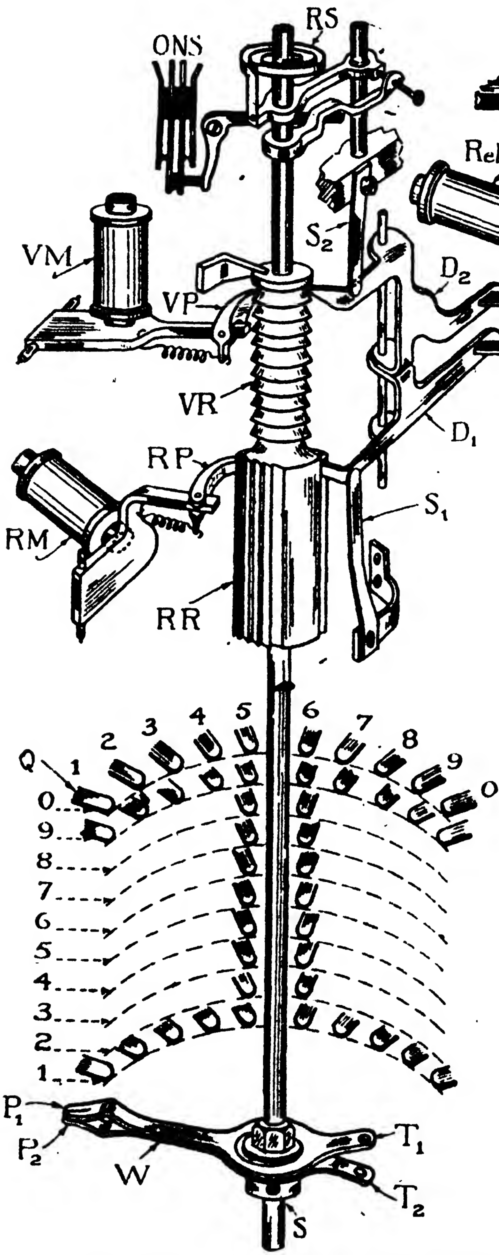


FIG. 203.—Strowger Two Co-ordinate Switch.

vertically and 1 to 0 horizontally. These contacts (corresponding to Q in Fig. 202) are known collectively as the "line bank."

In addition there is (not shown) another single contact wiper mounted on the same shaft S above the line wiper W, which makes contact on a second set of single contacts spaced in exactly the same way as the line bank and situated above it. These are known as the "private wiper" and "private bank" respectively.

Control of the switch magnets from the subscriber's telephone is effected by breaking the subscriber's loop circuit a number of times equal to the number of steps to be made by the switch, means being provided at the exchange for operating a stepping magnet once each time the loop is broken. Finally the loop circuit remains closed to provide the speech circuit.

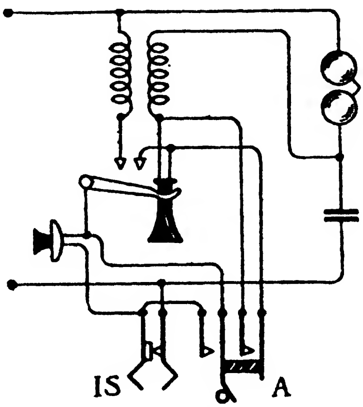


FIG. 204.—Subscriber's Telephone Circuit.

The necessary breaks are made by the subscriber by means of a "dial" mounted on his telephone. The electrical connections of the subscriber's telephone are shown in Fig. 204. The dial is rotated by hand through a number of steps corresponding to the number "dialled," and is spring returned at constant speed (controlled by a governor) to its original position. While the dial is in motion the springs A are operated to short circuit microphone and receiver, and while the dial is returning the springs IS are opened a number of times depending upon the number of steps originally dialled.

An outline of the circuit arrangements necessary for the operations of the two-co-ordinate or selector switch from the dialling impulses is given in Fig. 205. When the subscriber lifts his receiver current flows round the loop made and through the line relay A closing the contact *a* (the battery connections are shown as (+) and (—); the (+) is always earthed), and hence operating relay B (slow to release as indicated in the diagram by the cross hatching) and closing contact *b*. The subscriber now turns the dial, and on release the switch IS is opened, *a* releases and current is sent from +

through *b* springs 3 and 1 of ONS through relay C and the vertical magnet VM to —. Relay C operates closing contact *c* and VM is operated, raising the switch one level and operating the ONS to break contact between 1 and 3 and make contact between 2 and 3 and 4 and 5. Successive breaks at IS with consequent openings of the contact *a* result in current flowing from + *a* through *b* springs 3 and 2 of the ONS, through contact *c*, relay C and the VM. Relays B and C being slow to release hold up while the train of current interruptions passes, and the switch is raised to the required level. While the subscriber is setting the dial for the units figure relay C restores (its original circuit *viâ* springs 1 and 3 of ONS is now broken), so that the impulses provided by the return of the dial now pass through the rotating magnet RM, from + *a b* springs 3 and 2 of ONS, *c* and RM to —. In this way the switch is rotated through the correct number of steps. When the subscriber hangs

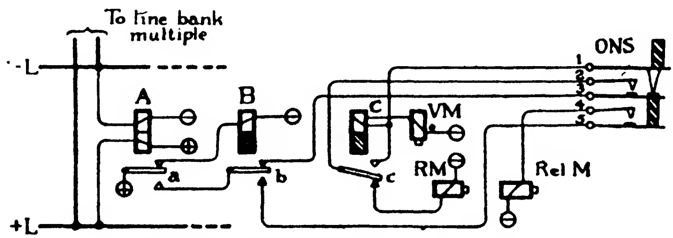


Fig. 205.—Operation of Strowger Switch.

up his receiver relays A and B release, and current flows from + *a b* springs 4 and 5 of the ONS through the release magnet Rel.M to —. The release magnet operates and the switch is restored to its normal position.

The two windings of the relay A act in the speech circuit as two of the necessary impedances Z of the modified Stone circuit of Fig. 199. The dotted lines in Fig. 205 carry the line through $2\mu F$ condensers (and other relay contacts required for engaged tests, ringing, ringing tone to calling subscriber and metering) to the line wipers of the selector. Between the wipers and the $2\mu F$ condensers another relay similar to A is bridged across the loop, and when the called subscriber lifts his receiver, microphone current is sent through the two windings of this relay, which act as the remaining two impedances Z required for the speech circuit.

The provision of a two-co-ordinate switch for every subscriber would entail an impossible expenditure, and it is no more necessary than the provision of a cord circuit for every subscriber in a manual exchange. Instead each subscriber is provided with a simple single co-ordinate switch known as a rotary line switch (or pre-selector) to the bank of which the two co-ordinate switches are connected, which automatically hunts for a free selector and connects the line thereto before dialling commences. The private line and bank corresponding to the bush and sleeve wire in the manual exchange multiple and cord circuit is used in this connection.

This hunting takes place quite automatically with no control by the subscriber, and is known as non-numerical selection. The stepping by dialling is known as numerical selection.

The line switch is described in Fig. 206. There are twenty-five sets of contacts over which the wipers W hunt. The wipers are stepped round by an electromagnet M with armature A acting through a pawl P and ratchet wheel R. Actually the movement does not take place on the pulling up of the armature, which moves the pawl forward and compresses the spring S. On release of the armature the spring S moves the wipers round. There are three wipers all mounted on the same shaft and working on the positive and negative line contacts and the private contacts. In addition there may be a "homing" arrangement in which a fourth wiper works on a continuous contacting arc except in the normal position. By means of this the line switch returns to normal by a completion of the rotation when the circuit is cleared. The wiper of the private contacts bridges the contacts while stepping from one to the other; the other wipers do not. In addition, there may be another wiper for use in connection with a meter for recording the number of calls made by a subscriber.

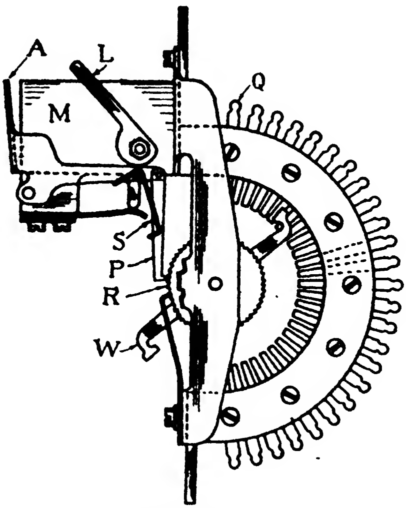


FIG. 206.—Line Switch.

The manner in which the line switch finds a free selector is illustrated in Fig. 207. When the subscriber's line is looped current flows from + through b_2 + L round the loop — L to b_1, through relay winding A to —, and A is operated, closing contacts a_1 and a_2. Two cases now arise, according to whether the outlet on which the wipers of the line switch are standing is "busy" or "free." If the former, the line relay C of the selector will have been operated to close the contact *c* and operate the relay D. + (earth) will have been placed on the private or test wire of the selected "trunk," through *d*. Current will, therefore, flow from + through *d*, test wire, private bank and wiper of line switch, b_3, a_1, interrupter spring and winding of SM to —. SM is the stepping magnet of the

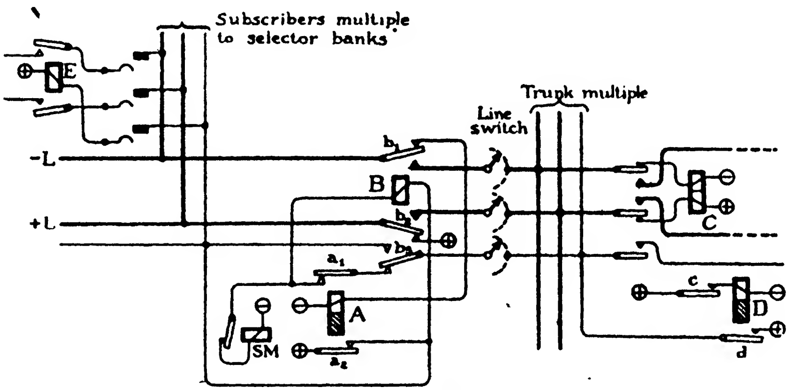


FIG. 207.—Hunting for Free Selector.

line switch, and the wipers are moved round one step before the interrupter springs of SM break the circuit. This action will be repeated until the wipers reach a free outlet, when current flows from + a_2 relay B SM to —, and the cut-off relay B operates to put the line through to the line relay C of the free switch, and dialling can proceed. The stepping magnet does not operate when in series with relay B.

The operation of C causes D to operate by closing *c*, and earth (+) is put on to the private wire of the trunk multiple to protect the switch from interference from other hunting line switches. The earth is also carried through the private wiper and b_3 to the private wire of the selector multiple by which protection is afforded in a similar manner against interference from any selector through the multiple. This protection was afforded during the line switch hunting by earth being put on to the private wire through a_2. The current flowing from + through d the test wiper, b_3, B and SM to − holds B operated, but is not sufficient to operate SM. A is slow to release so that B is held operated until this final holding current flows. It is necessary that a_1 should operate before a_2 otherwise B might operate prematurely over the circuit +, a_2, B, SM, −.

When the dialling is completed the subscriber is put through to the number he is calling and the latter is rung by the operation of other relays (not shown). The called subscriber on answering completes his loop, and it is clear that this would operate his line switch and so cause a selector to be seized quite unnecessarily if means were not taken to avoid it. By a mechanical interlocking of the armatures of relays A and B, B cannot be operated fully unless A is first operated; if A is not operated B can only be half operated, with the result that contacts b_1 and b_2 and b_3 are broken on each side and the line switch is completely disconnected. This happens when the switching relay E of the selector operates over the circuit +, E, test wiper, third wire of multiple, B, SM, to −.

The above very bare outline refers to a two-digit or 99-line exchange. If the exchange capacity is 999 lines, connections are made through three switches, the line switch, and two two-coordinate switches known as the first and final selectors respectively. The three digits are dialled in the order hundreds, tens, units. A group of first selectors is allotted to a group of subscribers, the numbers depending upon the traffic conditions at the exchange, and in the same way the first selectors have access to a group of final selectors.

When the subscriber's receiver is lifted the line switch hunts for a free first selector. When the hundreds digit is dialled the first selector steps vertically to the correct level, and then automatically hunts (by a similar circuit arrangement to that of the line switch) by rotation for a free final selector, to which the line is connected. When the tens and units digits are dialled the final selector connects to the required line, all the subscriber's lines in the particular hundred concerned being multiplied to the bank of the final selector.

Similarly with a four-digit exchange there are three two-coordinate switches, first selector, second selector and final selector, as well as the line switch. In the first and second selectors the vertical steps are numerical, controlled by the dialled thousands and hundreds digits respectively, but the rotary steps are non-numerical, being automatic hunting for free second and final selectors respectively. The subscriber's lines are multipled to the final selector, on which the vertical and rotary steps are both numerical, controlled by the tens and units digits respectively.

Microphone current for the calling subscriber is provided at the first selector (through relay C in Fig. 207) and for the called subscriber at the final selector. By this arrangement the subscriber always receives his microphone current from his own exchange, even if the call necessitates the use of a junction line.

(54) **Junctions and Trunks.**

The size of local exchanges is limited by two considerations; the increasing complexity of the apparatus for dealing with very large numbers of subscribers, and the increasing length, and therefore cost, of the average subscriber's line. In densely populated areas, therefore, such as the large cities, there may be several exchanges within a short distance of each other, or even two quite separate exchanges housed in the same building, and the wires which run from exchange to exchange to enable subscribers connected to one exchange to speak to subscribers connected to another are termed junctions. The same term applies to the lines between towns not very far apart, up to, say, fifty miles. But the long distance lines that run between large cities are termed trunks, and they terminate in special exchanges called trunk exchanges, to which access is obtained by other exchanges in the area by trunk junctions. Service over junction lines is given on a no-delay basis, but calls over trunks are ordered beforehand and completed in rotation. Intermediately between junctions and trunks is a "toll" service, also working on a no-delay basis, but through a special toll exchange.

For the purposes of telephone service the country is divided into a number of zones, the zone centres being London, Bristol, Cardiff, Birmingham, Cambridge, Manchester, Leeds, Glasgow and Dublin. The backbone trunk system runs between the various zone centres, and these trunks have a standard cable equivalent of from eight to ten miles. The general standard for long-distance transmission is thirty-five miles s.c., but over shorter distances better transmission is expected. Between stations in the same urban area the figure is twenty miles s.c., between stations in the same zone or in adjacent zones twenty-five miles s.c., but if the stations are more than 200 miles apart this is increased to thirty miles. To attain these figures the combination of circuits by which a subscriber reaches his zone centre for a trunk call must not be more than ten miles s.c.

These figures include the losses due to signalling apparatus at the exchanges through which the call is wanted, and these losses may be taken roughly as one mile s.c. per exchange.

The number of junction circuit diagrams is very large, as different signalling arrangements are necessitated between the different types of local exchange, magneto CBS and CB.

A very simple arrangement is shown in Fig. 208. The middle point of an impedance Z bridging the line is connected to an indicator I, which is normally earthed through additional jack springs. When a plug is inserted in the jack, battery is put on to the B line through the indicator and finds earth through the indicator at the far exchange. Both indicators are thus operated. When a plug is inserted in the jack at the far exchange an equal opposing battery causes both indicators to restore. When either plug is withdrawn the indicators are again actuated and a clearing signal given.

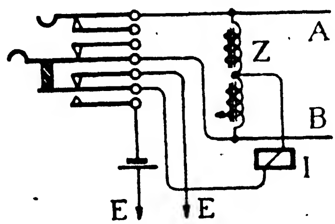


FIG. 208.—Simple Junction Circuit.

In large C.B. exchanges where there are a number of junctions to the same exchange, they are divided into incoming and outgoing junctions and worked by means of "order wires," separate operator's circuits connecting the two exchanges. The outgoing junctions are multipled to the operator's ("A") positions, and at the incoming end they terminate at a separate junction operator's ("B") position. At the outgoing end the "A" operator tells the "B" operator over the order wire the number required, and the "B" operator replies by giving the number of the junction to be used, and inserts the plug on which that junction is terminated into the multiple jack of the subscriber required. Ringing may then be automatic, and the subsequent control of the call remains with the "A" operator.

The circuit arrangement is shown in Fig. 209. The jack to the left of the dotted line is the junction multiple jack into which the "A" operator inserts the answering plug of her cord. This results in current flowing through the 12,000 ohm coil of the relay F and operating it, although the current is not sufficient to operate the CSR of the "A" cord (Fig. 201), and the CSL remains lit. While this is happening the "B" operator, having ascertained that the required subscriber is disengaged in the usual way (T is connected to a tertiary winding on the induction coil of the operator's telephone set), has inserted the plug P into the required subscriber's jack. Current then flows from the battery through the clearing lamp CL, relay A sleeve of plug and bush of jack to COR (Fig. 200) to earth. Relays A and the COR are operated. Current now flows from the battery through relay B, contacts c_1, d, a_2, f, relay A and the COR

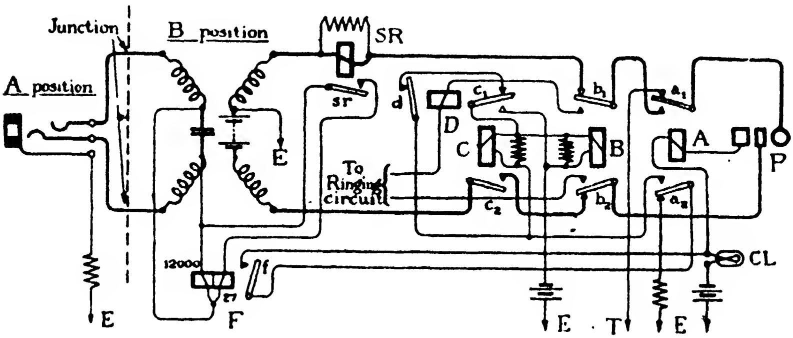


FIG. 200.—Ringing Junction.

to earth, and relay B is operated, and the lamp CL extinguished. Ringing current now flows to line through relay D, contact b_1 and a_1, tip of plug round the loop through the subscriber's bell and condenser back to the ring of the plug, and through contact b_2, but relay D does not operate. When, however, the subscriber lifts his receiver the ringing current is increased sufficiently to operate D, with the result that relay C, which previously was short circuited through contact d, now operates and short circuits relay B through c_1, so that B restores. The ringing circuit is now broken at b_1 and b_2, and the speaking circuit made through a_1, b_1, relay SR, c_2, and b_2. A path is thus provided through contact *sr* for current from the "A" exchange to flow through the 27 ohm winding of relay F, and the increased current operates the CSR and extinguishes the calling supervisory lamp CSL (Fig. 201). The call is now complete.

When the called subscriber hangs up his receiver, SR restores, the 27 ohm winding circuit of F is broken, and the CSL lights. The " A " operator removes the plugs with the result that F restores and the shunt on CL is removed. The CL accordingly lights and the " B " operator withdraws the plug.

If on test the required subscriber is found by the " B " operator to be engaged, the plug is inserted into a " busy back " jack, the springs of which are connected through an interrupter to an alternator giving the " busy " tone, which the calling subscriber hears through the operation of relays A and C as before.

In an area where there is a single automatic exchange and a large proportion of the calls are " local," the junction calls would be dealt with manually by operators, called by dialling a specially reserved number.

Where there are several automatic exchanges in an area, additional digits may be allocated to the subscriber's numbers according to the exchange they are connected to, the dialling of the first digit or two digits leading to the selection of a junction and the remainder of the dialling proceeding in the usual way. This method involves a fixed routing of the call from exchange to exchange, and junctions between every pair of exchanges. There are, moreover, difficulties during the transition period when the exchanges are being converted from manual to automatic, so that some are automatic and others still manual, and the directory has to be altered to show the additional digits.

All these difficulties are very much increased in a densely populated area like London, and for a long time it was not at all clear that the London exchanges could economically be converted to automatic working. A solution of the routing difficulty, using standard apparatus, has, however, been found in the " director " system. The general idea of the system is that the call dialled is received and stored in the director, and that subsequently impulses are sent out through a prearranged route. The trains of impulses (both in number of impulses and number of trains) sent out by the director may be quite different from those received, depending upon the existing layout of junction routes and exchanges. As soon as the required connection has been established, the director disconnects itself from the line and becomes available for another call.

The subscriber is provided with a dial on which the numbers (except 1 and 0) are associated with three code letters, and dials first the code (usually the first three letters of the exchange), and then the number of the subscriber required. The director itself responds to the last two of the code digits, stores the numerical digits, and routes the call by sending out trains of impulses to operate "code" switches or selectors. These trains depend only on the last two code digits, and may be "translated" in the director to a maximum number of six trains of ten impulses each, the number required depending upon the route and the number of code switches to be operated. After the call has been extended to the required exchange, the director sends out the numerical call without translation, to operate the "numerical" selectors in the required exchange in the usual way, and connection is made to the wanted subscriber. The director and its associated apparatus now release from the line and become available for another call.

The transition difficulties have been solved by means of "call indicator B position," at which incoming calls from an automatic to a manual exchange appear as illuminated figures on a ground glass screen. The director routes the call to a "coder," a group of relays and switches which translates the numerical impulses into reversals which operate three serially connected polarised relays. From the contacts of these a further group of relays is operated to produce the number display in front of the operator. The latter then connects the call in the usual way and the number display is restored.

The arrangements for connections to trunk lines are different. A subscriber wanting a trunk call asks for trunks, is put through to the trunk exchange on a "record" circuit, and gives details of the call he requires to a "record" operator. The latter enters the details on a ticket which is passed to the appropriate trunk operator at the switchboard either by hand or by pneumatic tube. When the required trunk is about to be free, the trunk operator calls the calling subscriber over a trunk junction, and passes the call over the trunk to the distant exchange, where the far operator calls the required subscriber.

The switchboard termination of a trunk is shown in Fig. 210. Calling over the trunk loop AB is by magneto round the loop to operate the line relay L. Current then flows from the earthed battery through the pilot relay P, extra jack springs, holding relay H, *l* operated to earth. H is operated and current now flows through the calling lamp CL through h_2. Also h_1 puts earth on the battery, so that even though the ring is only momentary and l breaks, H remains operated and CL lit. The clearing signal is obtained from the earthed 12-volt batteries in series with the bridging coils C_1 and C_2, acting to operate the clearing relay CR_1 in

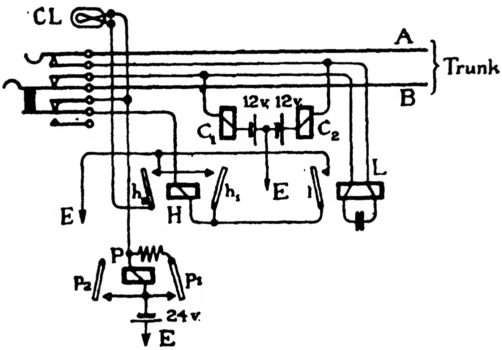


FIG. 210.—Trunk Exchange—Line Termination.

the cord circuit (Fig. 211). The black plug BP is used on the trunk side and the red plug RP on the junction side of the connections. Clearing from the junction side is obtained on CR_2 or CR_3, according to whether, depending on the type of exchange to which the junction is connected, the current is sent over one wire to earth or round the

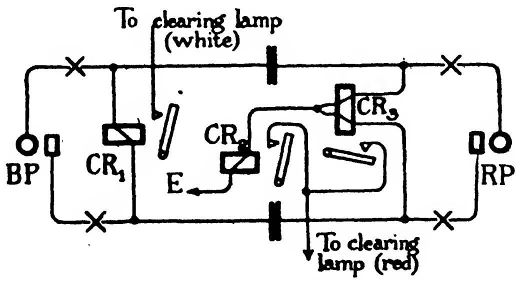


FIG. 211.—Trunk Exchange—Cord Circuit.

loop. The operator's ringing and speaking circuit is connected at the points marked with crosses.

The connection of trunk to trunk terminating at different points in the exchange is effected by means of transfer circuits running round the exchange, with signalling on a similar principle to that of Fig. 208.

The huge subject of switching has been but barely touched upon. The number of circuit diagrams to meet all the different conditions arising in practice is legion, and a detailed examination of all would fill volumes. But enough has been written to show that almost any condition can be met with the apparatus at the disposal of the circuit engineer, and the intervention of human agency in making and clearing connections can be reduced to a minimum.

REFERENCES

G.P.O.—"Telephone Diagrams." H.M. Stationery Office.
T. E. Herbert.—"Telephony."
J. Poole.—"The Practical Telephone Handbook."
T. H. Turney.—"The Principles of Relay Timing in Connection with Automatic Telephone Circuits." *J.I.E.E.*, Vol. LXVI., p. 341.
W. Atkins.—"Common Battery Simplified."
C. W. Brown.—"Automatic Telephony Simplified."
H. H. Harrison.—"Automatic Telephony."
T. F. Purves.—"The Post Office and Automatic Telephones." *J.I.E.E.*, Vol. LXIII., p. 617.
G. F. O'Dell.—"An Outline of the Trunking Aspect of Automatic Telephones." *J.I.E.E.*, Vol. LXV., p. 185.

PART III

WIRELESS TELEGRAPHY AND TELEPHONY

CHAPTER XI

ELECTROMAGNETIC WAVES

(55) Radiation of Energy

It is not difficult to understand that when a sounding body (such as a bell) vibrates, energy in the form of sound waves is radiated away from it, and that part of the total effective mechanical resistance of the vibrating body is due to this radiation. It has been seen in section (28) that the radiation is caused by the elasticity and inertia of the air, which determine the velocity of the waves produced.

It is far more difficult to understand exactly how, whenever an electric current changes in a circuit, energy in the form of electromagnetic waves is radiated away from the circuit. The mere statement that electric and magnetic forces produce strains in an all-pervading ether, that the ether has properties corresponding to elasticity and inertia, and that therefore electromagnetic waves can be propagated through the ether, hardly carries conviction.

Another point of view is perhaps more helpful.* Consider a circuit of inductance L and resistance R through which an alternating current $i = \mathrm{I} \sin \omega t$ is flowing. There will be, at any instant, a magnetic flux of Φ lines through the circuit, and if the current changes are indefinitely slow, to every value of the current i there is a flux Φ given by $\Phi = \mathrm{L}i$. But since, if some wave transmission is assumed, the flux lines must take a definite time to spread out from the wires of the circuit (or close in upon them) for every change of current, it is reasonable to suppose that the flux lags behind the current producing it in some such way as is expressed by :—

$$\Phi = \mathrm{LI} \sin (\omega t - \theta) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This being so, the electromotive force necessary to drive the current round the circuit is given by

* L. B. Turner, "Outline of Wireless."

$$\begin{aligned} e &= \mathrm{RI} + \frac{d\Phi}{dt} \\ &= \mathrm{RI} \sin \omega t + \omega \mathrm{LI} \cos (\omega t - \theta) \\ &= (\mathrm{R} + \omega \mathrm{L} \sin \theta)\mathrm{I} \sin \omega t + \omega \mathrm{LI} \cos \theta \cos \omega t \quad . \quad . \quad . \quad (2) \end{aligned}$$

The effective resistance is increased by $\omega\mathrm{L}\sin\theta$, and the total power absorbed by the circuit is

$$\frac{\mathrm{I}^2}{2}(\mathrm{R} + \omega\mathrm{L}\sin\theta) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Of this $\frac{\mathrm{I}^2}{2}\mathrm{R}$ is dissipated as heat in the circuit and $\frac{\mathrm{I}^2}{2} . \omega\mathrm{L}\sin\theta$ is radiated away from the circuit. $\omega\mathrm{L}\sin\theta$ may be called the "radiation resistance" R_r of the circuit, and :—

$$\mathrm{R}_r = \omega\mathrm{L}\sin\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

In the ordinary circuits of electrical engineering θ is so small that R_r, and therefore the power radiated, is quite negligible. To increase R_r, ω, L and θ must be increased. L is increased by enlarging the circuit or by winding on more turns ; θ, by providing the inductance by a few large open turns rather than by many close turns ; θ also will increase with the frequency. A vertical aerial wire is such a large open circuit, in which the field is very much spread out, and in which accordingly the radiation resistance is large with currents of high frequency.

Exactly similar considerations apply to the electric field.

Actual calculation has shown that for a single vertical aerial wire of height h cm. earthed at the foot

$$\mathrm{R}_r = 7{\cdot}1 \times 10^{-19} f^2 h^2 \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and for a vertical loop of T close turns of area S sq. cm.

$$\mathrm{R}_r = 7{\cdot}8 \times 10^{-38} f^4 \mathrm{S}^2 \mathrm{T}^2 \text{ ohms} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

showing a general agreement with the rough conclusion arrived at above.

Electromagnetic waves were first produced, and detected, from an electric circuit by Hertz, in 1888. The circuit in effect was a straight metal rod joining two metal plates, and so approximated closely to a "dipole," in which all the inductance of the circuit is contributed by the rod and all the capacity is contributed by the plates. A magnetic field is associated with currents flowing up and down the rod and an electric field with the varying charges on the plates. For a dipole of length $2h$ cm. the radiation resistance is

$$R_r = 3{\cdot}52 \times 10^{-18}h^2f^2 \text{ ohms} \quad . \quad . \quad . \quad . \quad (7)$$

The action is the same if a perfectly conducting plane is inserted half-way along the rod and perpendicular to it, and one half of the dipole removed; but now energy is radiated into only half the space, and the radiation resistance for the half dipole is accordingly

$$R_r = 1{\cdot}76 \times 10^{-18}h^2f^2 \text{ ohms} \quad . \quad . \quad . \quad . \quad (8)$$

An aerial system with an extensive system of horizontal wires at a height h above the ground, and a single earthed uplead approximates to the half dipole, as the earth approximates to a perfectly conducting plane, and the current in the lead to the "flat top" is approximately constant. But in the case of a single vertical earthed wire, the capacity to earth is distributed along the wire and the current falls off along the wire from a maximum value at the foot of the wire to zero at the top. The radiation resistance (by which the square of the current at the *foot* of the wire must be multiplied in order to obtain the power radiated) is accordingly less than that of the corresponding half dipole, as is seen by comparing formulæ (5) and (8). The height of the half dipole having the same radiation resistance as the actual aerial is known as the *effective height* of the aerial.

Thus the attachment to a vertical uplead of a large horizontal system of wires leads to an increase of the effective height of the aerial. It has, however, another object. At a given frequency and with a given aerial system, the power radiated can only be increased by increasing the current in the uplead, and consequently the quantity Q of electricity that must be periodically stored in the capacity (C) of the flat top. The voltage V that the system acquires is $V = Q/C$, and a limit to this is reached when brush discharge (at 50,000 to 200,000 volts) sets in. Thus the power radiated can only be increased if C is increased.

Instead of speaking of the frequency of currents in aerial and other circuits, the wireless engineer often speaks of the wavelength of the radiation they produce. The velocity of electromagnetic waves is the velocity of light, *i.e.*, 3×10^{10} cm. per second, and since frequency and wavelength are connected by the expression $f\lambda = c$, if λ is measured in metres,

$$f = \frac{3 \times 10^8}{\lambda} \qquad (9)$$

For example, a frequency of a million cycles per second corresponds to a wavelength of 300 metres.

If R is the total resistance of an aerial circuit (as measured at the foot of the uplead), the total power put into the aerial is I^2R, while the radiated power is I^2R_r. The aerial efficiency η is accordingly

$$\eta = R_r/R \qquad (10)$$

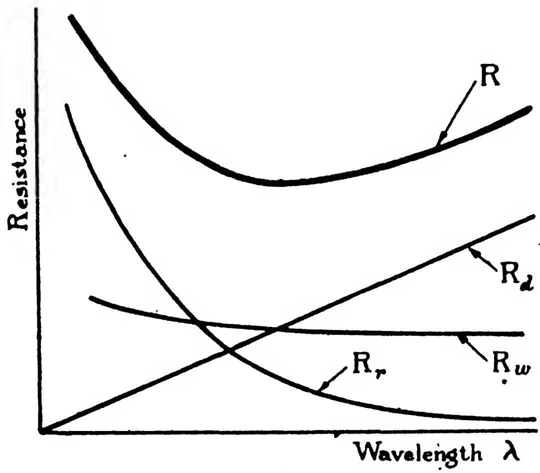


FIG. 212.—Resistance of Aerial.

The total resistance includes three main components; the wire resistance R_w (including effective resistance due to eddies in the earth and in surrounding objects), the resistance R_d taking account of the losses in surrounding dielectrics such as trees, buildings, grass, etc., and the useful radiation resistance. Of these, R_w varies inversely as the square root of the wavelength (directly as the square root of the frequency) R_d varies directly as the wavelength (inversely as the frequency) and R_r varies inversely as the square of the wavelength (directly as the frequency^2). Fig. 212 gives a general idea of these components plotted against the wavelength, the total resistance R showing a minimum at a particular wavelength.

The total resistance

$$R = R_w + R_d + R_r$$

can be written

$$R = \frac{a}{\lambda^{\frac{1}{2}}} + b\lambda + \frac{C}{\lambda^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

If the total resistance curve of Fig. 212 is found experimentally, and the constants of equation (11) are found to fit the curve, then the various components of the total resistance at any wavelength can be estimated, and the effective height of the aerial can be calculated from (8).

A considerable part of the resistance of the aerial is located at the earth connection, and usually a number of earth plates are provided connected by earth wires in the form of a fan under the aerial. But better results are obtained by an "earth screen," which is another aerial system carried on short masts 10 or 12 feet high, and extending throughout the length and breadth of the top of the aerial and a little beyond. The total resistance may in this way be reduced from, say, 5 ohms to 0·5 ohms without alteration of the radiation resistance (0·05 ohm), with a consequent increase of aerial efficiency from 1 per cent. to 10 per cent. These figures are for a particular aerial 100 feet high, working at a wavelength of 5,700 metres.

Generally speaking, transmitting aerials radiate more or less equally in all directions, a desirable condition in a broadcast station, but not in the case of two fixed stations which have to work with each other. Hertz's original experiments were made with wavelengths of the order of a metre, and it was quite practicable to construct parabolic reflectors to send the electromagnetic wave out as a beam, in the same way as light is projected in a beam from a search-lamp. But in the first application to telegraphy, Marconi, by using a single earthed vertical wire of 100 metres, increased the wavelength to 400 metres, and the construction of a reflector of sufficient size became impracticable. Further developments involved the use of greater and greater wavelengths to obtain sufficient range. But latest of all, in the "beam" stations, there is a return to far shorter wavelengths with directive transmission, the reflection being obtained not from a parabolic reflector, but by exciting a number of parallel aerials behind each of which (at something less than a quarter-wavelength distance) is arranged another aerial not separately excited. The wavelength is from 10 to 50 metres, and the radiation is very markedly directional. The frequency being very high, the radiation resistance is also very high, and for a given range much smaller powers are required in the aerial system than with the large long wavelength non-direction aerials.

(56) Electromagnetic Waves and their Propagation

The theory of electromagnetic waves rests on three fundamental laws and an assumption. The laws are the familiar ones of electromagnetism :—

(i.) *Gauss's Law.*—The total electric flux through a closed surface is equal to 4π times the total charge within the surface, and the total magnetic flux is equal to 4π times the total pole strength within the surface, *i.e.*, is zero, since positive and negative magnetic "charges" cannot be separated.

(ii.) *Ampère's Law.*—The work done in carrying a unit pole in a closed path round a current is equal to 4π times the current; or the line integral of the magnetic field round the closed path is 4π times the current in absolute electromagnetic units;

$$\int_o \mathrm{H}dl = 4\pi\mathrm{I} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

(iii.) *Faraday's Law.*—The electromotive force induced in a circuit is equal to minus the rate of change of magnetic flux through the circuit; or the line integral of the electric force round a circuit is equal to the rate of change of magnetic flux through the circuit;

$$\int_o \mathrm{E}dl = -\frac{d\Phi}{dt} = -\mu\frac{d\mathrm{HS}}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where μ is the permeability of the medium in which the circuit lies, S is the area of the circuit, E is the electric force, and H the magnetic force, both in absolute electromagnetic units.

(iv.) The assumption, made by Clerk Maxwell in 1865, is that a varying electric field in a dielectric is equivalent to an electric current. If D is the electric flux per square centimetre (called by Maxwell the displacement) then $\mathrm{D} = \kappa\mathrm{E}/4\pi$, where κ is the dielectric constant, and $\dfrac{d\mathrm{D}}{dt} = \dfrac{\kappa}{4\pi}\dfrac{d\mathrm{E}}{dt}$ is in its magnetic effect a current density.

Thus equation (1) is modified, if there is a possibility of "displacement currents," to

$$\int_a \mathrm{H}dl = 4\pi\left(\mathrm{I} + \frac{\kappa}{4\pi}\frac{d\mathrm{E}}{dt}\,.\,\mathrm{S}\right)$$

$$= 4\pi\,\mathrm{I} + \kappa\mathrm{S}\,.\,\frac{d\mathrm{E}}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

These statements take a different mathematical form when instead of isolated charges, poles, and circuits, an extended homogeneous isotropic medium is considered. Let J be the current density. Using rectangular co-ordinates Ox, Oy, Oz, the subscripts x, y, z indicating components in those directions, and measuring H in electromagnetic units and E and J in electrostatic units, the ratio of the first to the second being c, (2) and (3) become

$$\left.\begin{aligned}
-\frac{\mu}{c}\frac{\partial \mathrm{H}_x}{\partial t} &= \frac{\partial \mathrm{E}_z}{\partial y} - \frac{\partial \mathrm{E}_y}{\partial z}\\
-\frac{\mu}{c}\frac{\partial \mathrm{H}_y}{\partial t} &= \frac{\partial \mathrm{E}_x}{\partial z} - \frac{\partial \mathrm{E}_z}{\partial x}\\
-\frac{\mu}{c}\frac{\partial \mathrm{H}_z}{\partial t} &= \frac{\partial \mathrm{E}_y}{\partial x} - \frac{\partial \mathrm{E}_x}{\partial y}
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and

$$\left.\begin{aligned}
\frac{4\pi}{c}\mathrm{J}_x + \frac{\kappa}{c}\frac{\partial \mathrm{E}_x}{\partial t} &= \frac{\partial \mathrm{H}_z}{dy} - \frac{\partial \mathrm{H}_y}{\partial z}\\
\frac{4\pi}{c}\mathrm{J}_y + \frac{\kappa}{c}\frac{\partial \mathrm{E}_y}{\partial t} &= \frac{\partial \mathrm{H}_x}{\partial z} - \frac{\partial \mathrm{H}_z}{\partial x}\\
\frac{4\pi}{c}\mathrm{J}_z + \frac{\kappa}{c}\frac{\partial \mathrm{E}_z}{\partial t} &= \frac{\partial \mathrm{H}_y}{\partial x} - \frac{\partial \mathrm{H}_x}{\partial y}
\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (5)$$

while Gauss's Laws are written

$$\frac{\partial \mathrm{E}_x}{\partial x} + \frac{\partial \mathrm{E}_y}{\partial y} + \frac{\partial \mathrm{E}_z}{\partial z} = 4\pi q \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and

$$\frac{\partial \mathrm{H}_x}{\partial x} + \frac{\partial \mathrm{H}_y}{\partial y} + \frac{\partial \mathrm{H}_z}{\partial z} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where q is the volume density of electric charge. To these may be added

$$\mathrm{J} = \sigma\mathrm{E} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

by Ohm's Law, where σ is the specific conductivity in electrostatic units.

In the ether, and very nearly in air, J and q are zero, and (4), (5), (6) and (7) are then Maxwell's equations. In this case differentiation of the first equation of (5) with regard to t, substitution from (4) differentiated with regard to y and z to eliminate H_z and H_y, and using (6) leads to

$$\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} = \frac{\mu\kappa}{c^2} \cdot \frac{\partial^2 E_x}{\partial t^2} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

In the same way a similar equation is found for E_x and E_z, and for H_x, H_y and H_z.

These equations are those of a wave motion with velocity $c/\sqrt{\mu\kappa}$, which in the ether and very nearly in air, with $\mu = 1$ and $\kappa = 1$, is a velocity c. Now the ratio c of the electromagnetic to the electrostatic unit is 3×10^{10} cm./sec., and was known experimentally to be practically the same as the velocity of light. Hence Maxwell concluded that light is propagated by electromagnetic or ether waves.

In a plane wave the electric and magnetic forces are of constant value at any instant throughout the plane of the wave. If this plane is YOZ in Fig. 213, then the differential coefficients with regard to y and z are zero, and Maxwell's equations become

$$-\frac{\mu}{c}\frac{\partial H_x}{\partial t} = 0 \qquad -\frac{\mu}{c}\frac{\partial H_y}{\partial t} = -\frac{\partial E_z}{\partial x} \qquad -\frac{\mu}{c}\frac{\partial H_z}{\partial t} = \frac{\partial E_y}{\partial x} \quad . \quad . \quad (10)$$

$$\frac{\kappa}{c}\frac{\partial E_x}{\partial t} = 0 \qquad \frac{\kappa}{c}\frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial x} \qquad \frac{\kappa}{c}\frac{\partial E_z}{\partial t} = \frac{\partial H_y}{\partial x} \quad . \quad . \quad (11)$$

Since $\partial H_x/\partial t$ and $\partial E_x/\partial t$ are zero, H_x and E_x are either constant or zero, and must, therefore, be zero since constant quantities do not enter into the wave motion. It follows that the electric and magnetic forces lie wholly in the plane of the wave.

The direction of the electric force can be taken as parallel to OY without loss of generality. Then $E_z = 0$, and it follows from either (10) or (11) that $H_y = 0$. The only components not zero are thus E_y and H_z; the electric and magnetic forces are at right angles to each other.

Equations (10) and (11) are now reduced to

$$-\frac{\mu}{c}\frac{\partial H_z}{\partial t} = \frac{\partial E_y}{\partial x} \text{ and } \frac{\kappa}{c}\frac{\partial E_y}{\partial t} = -\frac{\partial H_z}{\partial x} \quad . \quad . \quad . \quad (12)$$

whence and
$$\left.\begin{aligned}\frac{\partial^2 E_y}{\partial x^2} &= \frac{\mu\kappa}{c^2}\frac{\partial^2 E_y}{\partial t^2}\\ \frac{\partial^2 H_z}{\partial x^2} &= \frac{\mu\kappa}{c^2}\frac{\partial^2 H_z}{\partial t^2}\end{aligned}\right\} \quad \ldots \ldots \quad (13)$$

equations identical in form with 29·04 and representing two waves travelling in opposite directions along the axis OX.

If the time variations of E_z and H_z are sinusoidal, E_y can be written $E_y\epsilon^{j\omega t}$ and (13) gives

$$\frac{\partial^2 E_y}{\partial x^2} = -\frac{\mu\kappa}{c^2}\omega^2 E_y \quad \ldots \ldots \quad (14)$$

with a vector solution for the wave travelling from left to right

$$E_y = A\epsilon^{j\omega\left(t - \frac{\sqrt{\mu\kappa}}{c}x\right)} \quad \ldots \ldots \quad (15)$$

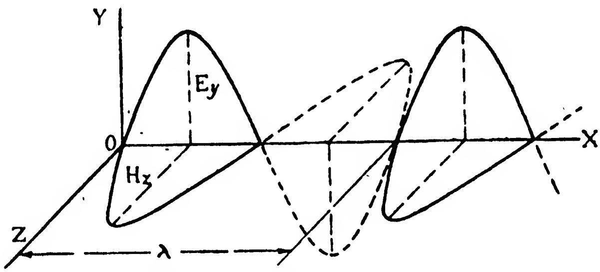


Fig. 213.—Plane Electromagnetic Wave.

Similarly
$$H_z = B\epsilon^{j\omega\left(t - \frac{\sqrt{\mu\kappa}}{c}x\right)} \quad \ldots \ldots \quad (16)$$
(Compare section 29, p. 115.)

The relation between the constants A and B is found from (12) by substitution from (15) and (16) to be

$$\frac{A}{B} = \sqrt{\frac{\mu}{\kappa}} \quad \ldots \ldots \quad (17)$$

which in space or in air, with $\mu = \kappa = 1$, shows that the electric force in electrostatic units is equal to the magnetic force in electromagnetic units. From (15) and (16) it is clear that the two forces are in time phase. The curves of Fig. 213 show the wave at a particular instant.

Fig. 213 would represent the wave close to the earth produced by currents in an aerial situated a distance equal to a large number of wavelengths to the left of O, if the earth were a perfectly conducting plane.

The general solution of equation (9) and the other similar equations involves much mathematics and cannot be attempted. In a spherical wave the various components of E and H depend upon the distance r from the origin only, and (9) can be transformed to

$$\frac{\partial^2(rE_x)}{\partial r^2} = \frac{\mu\kappa}{c^2}\frac{\partial^2(rE_x)}{\partial t^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

with similar equations for the other components. This is identical in form with the equation (29·17) for a spherical sound wave, and shows that rE_x is propagated unchanged. It follows in the same way as with the sound waves, that at great distances the electric and magnetic forces are inversely proportional to the distance from the source.

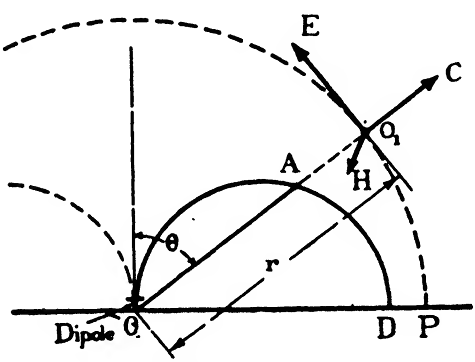


FIG. 214.—Wave from Half-Dipole.

The wave from a half-dipole (or ideal aerial) is, however, not spherical, but has, at large distances from the dipole, a polar distribution in the vertical plane given, as in Fig. 214, in every horizontal direction by a semicircle with a horizontal diameter. Thus OA represents either the electric or magnetic force at O_1, of the propagation in a direction inclined at an angle θ to the vertical, if OD represents the corresponding force at P on the same sphere. E and H at considerable distances are both tangential to the sphere and at right angles to each other and to the radius OO_1; H is horizontal and the direction of propagation is O_1C in continuation of OO_1.

The forces at P are given by

$$\left.\begin{aligned} E &= 4\pi\frac{Ih}{\lambda r} \\ H &= 4\pi\frac{Ih}{\lambda r} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where I is the dipole current in c.g.s. electromagnetic units, h is the height of the half-dipole in centimetres, and λ and r are also in centimetres.

If I is in amperes and h, λ and r in metres, the electric force in volts per metre is given by

$$E = 377 \frac{Ih}{\lambda r} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Close to the dipole the static forces of the dipole are combined with the forces of the wave, and the total field distribution is far more complicated. There are components depending inversely on the square and cube of r.

With a beam aerial system the propagation is ideally all in one horizontal direction, but is distributed in the vertical plane somewhat in the same way as in Fig. 214.

Three considerations arise in the actual transmission of electromagnetic waves which make the field strengths actually established differ from those given by equation (20). These are (i) the fact that the earth's surface is not a perfect conductor and so energy is absorbed in transmission, (ii) (for long-range transmission) the fact that the earth's surface is spherical, and since the waves are propagated in straight lines it is only by diffraction that they can curve round the earth; (iii) on the other hand, the waves are reflected downwards towards the earth from the upper atmosphere, so that the field strengths at long ranges may be far greater than those given by calculations based on a simple diffraction round the earth's surface.

Over short ranges the first effect only is operative. The magnetic field H_y induces currents in the earth at its surface in the direction OX, which involve an electric field E_x. In the earth $H_x = H_y = E_y = E_z = 0$, and, neglecting any displacement currents in the earth, equations (5) give

$$\frac{4\pi}{c} . J_x = \frac{\partial H_z}{\partial y} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

which by (8) is

$$\frac{4\pi}{c} \sigma E_x = \frac{\partial H_z}{\partial y} \quad . \quad . \quad . \quad . \quad . \quad (22)$$

and equations (4) give

$$-\frac{\mu}{c}\frac{\partial H_z}{\partial t} = -\frac{\partial E_x}{\partial y} \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Eliminating E_x from (22) and (23) gives

$$\frac{\partial^2 H_z}{\partial y^2} = \frac{4\pi\sigma\mu}{c^2}\frac{\partial H_z}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

for the propagation of the magnetic field into the earth.

If H_z varies sinusoidally (24) can be written

$$\frac{\partial^2 H_z}{\partial y^2} = j\,\frac{4\pi\sigma\mu}{c^2}\,\omega H_z \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

This is of the same form as equation 39.07, and the solution is (39.11)

$$H_z = A \cosh Py + B \sinh Py \quad . \quad . \quad . \quad . \quad (26)$$

with (by 39.09)

$$P = \sqrt{j\,\frac{4\pi\sigma\mu}{c^2}\,\omega} \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

whence (from 39.14)

$$\alpha = \beta = \frac{2\pi}{c}\sqrt{\sigma\mu f} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Calling the field at the surface H_o, and considering the earth of infinite extent, it follows as in the case of the infinitely long telephone line (39.13a) that

$$H_z = H_o\,\epsilon^{-Py} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

and from (21)

$$\begin{aligned} J_x &= \frac{c}{4\pi}(-P)H_o\,\epsilon^{-Py} \\ &= -\frac{c}{4\pi}\cdot\sqrt{2}\,\alpha\,\epsilon^{j\frac{\pi}{4}}\,H_o\,\epsilon^{-\alpha y}\,.\,\epsilon^{-j\alpha y} \\ &= -\frac{c}{2\sqrt{2}\,\pi}\,\alpha\,\epsilon^{-\alpha y}\,H_o\,\epsilon^{j\left(\frac{\pi}{4}-\alpha y\right)} \quad . \quad . \quad . \quad . \quad . \quad (30) \end{aligned}$$

and

$$\left|J_x\right| = \frac{c}{2\sqrt{2}\,\pi}\,\alpha\,\epsilon^{-\alpha y}\,H_o \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

The current amplitude in a thin slab 1 cm. wide (in the direction OZ) and dy cm. deep is $|J_x|\,dy$, and the resistance of 1 cm. length (in the direction OX) of this slab is $1/\sigma dy$. The heat losses in the slab are, therefore, $[\frac{1}{2} I^2R] = \frac{1}{2\sigma}\left|\dot{J}_x\right|^2 dy$, and the total losses for each square centimetre of surface are

$$\frac{1}{2\sigma}\int_0^\infty \left|J_x\right|^2 dy = \frac{1}{\sigma}\,\frac{c^2}{16\pi^2}\,a^2 H_o^2 \cdot \frac{1}{2a}$$

$$= \frac{c}{16\pi}\sqrt{\frac{\mu f}{\sigma}}\,H_o^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

The losses increase with the permeability and the frequency, but are smaller the greater the conductivity.

The total current flowing under each square centimetre of the surface is

$$\int_0^\infty J_x dy = \frac{c}{4\pi} H_o$$

by (21). If this current is supposed to occupy a depth t, so that the resistance encountered by the current is $1/\sigma t$, the heat losses are

$$\frac{1}{2} \cdot \frac{c^2}{16\pi^2} H_o^2 \cdot \frac{1}{\sigma t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

and equations (32) and (33) give

$$t = \frac{c}{2\pi}\sqrt{\frac{1}{\mu f \sigma}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

t is known as the equivalent depth of penetration of the current. A similar idea applies to the penetration of a high frequency current into a conductor, that is, to the "skin effect," and equation (34) enables the high frequency resistance of a conductor to be calculated, provided t is small compared with the dimensions of the cross-section of the conductor. For example, the equivalent penetration into a thick copper bar 1 cm. wide at a frequency of one million is found from (34) by putting $\mu = 1$, $f = 10^6$, $\sigma c^2 = 0{\cdot}58 \times 10^{-3} c^2$ ($0{\cdot}58 \times 10^{-3} = \sigma$ in electromagnetic units at 20° C., the σ in (34) is in electrostatic units), giving $t = 0{\cdot}05$ mm. Thus if the strip is 2 mm. thick, and is carrying a current at a frequency of a million, the effective cross-section is $2 \times 0{\cdot}05 = 0{\cdot}1$ mm., and the resistance to the current will be $2/0{\cdot}1 = 20$ times the resistance to direct currents.

The specific conductivity of the earth's surface varies enormously according to its nature; sea water, fresh water, damp, dry and very dry soil. In the last cases displacement currents in the soil must be taken into account. While not involving actual heat losses, the presence of these currents means that the energy radiated from an aerial is spread out over a larger space, and in consequence the field strength at the earth's surface is less.

The following table gives some idea of the effect of the nature of the surface on the relative field strength (Zenneck).

Nature of surface.	Distance from aerial for equal field strength (kilometres).
Perfect conductor . .	1,000
Sea water . . .	920
Fresh water or marsh .	700
Wet soil . . .	560
Damp soil . . .	270
Dry soil . . .	150
Very dry soil . . .	55

It is clear from the above considerations that the loss of field strength on these accounts can be taken account of by the introduction of an attenuating factor in equation (20), the factor depending upon the frequency or wavelength. From measurements over sea water, with wavelengths from 600 to 4,000 metres, Austen and Cohen found as the result of many experiments that the following formula

$$E = 377 \frac{Ih}{\lambda r} \epsilon^{-0 \cdot 0018 r \sqrt{\lambda}}$$

applied up to $r = 2000 \times 10^3$ metres.

The propagation of electromagnetic waves over large distances—even right round the earth—is a far more complicated problem and is still not completely understood. It seems certain, however, that there is a conducting layer in the upper atmosphere, which results more or less in the waves being confined between two conducting spheres, the earth and the so-called Heaviside layer. The conductivity of the latter is due to ionisation of the rarefied air by the sun's rays, and very considerable changes occur as between day and night, and as between summer and winter. For instance, the effective height of the layer for 400 metre waves which have travelled eighty miles increases steadily during the night from 90 kilometres to 130 kilometres, but other reflections have been found during the winter from a layer from 250 to 350 kilometres high, and it appears that the two reflections may exist simultaneously. The changes are most pronounced during sunset and sunrise.

In general, therefore, the field strengths at any point on the earth's surface are the vector sum of those of a wave which has been propagated along the earth's surface and one which has been reflected from the upper atmosphere. With long wavelengths, say, 10,000 to 20,000 metres, the surface wave predominates, being propagated between the two conducting spheres, and the day and night variations of field strength are not marked. With short waves, say, 100 metres and shorter, the surface wave is almost completely absorbed in a short distance, and the field strengths observed are due to the reflected wave, and are generally greater when the whole of the path is in darkness; during the day the lower ionisation absorbs greater energy. The wave may, however, penetrate the lower ionised layer without great loss and be reflected at the upper layer. In this case there is little difference between day and night transmission. There is, moreover, the phenomenon of a "skip distance." Over a certain distance from the transmitting aerial the surface wave has been absorbed and the reflected wave has not yet reached the earth and practically no field can be observed. With waves of 30 metres, for instance, the "silent" zone may extend to 500 miles, and good field strengths extend from 500 to 1,500 miles. With intermediate wavelengths both surface and space waves contribute to the total field and interference may be caused. The upper layer is continuously varying, and with it the phase of the reflected wave. Thus the vector sum of the two waves varies, and the phenomenon of "fading" occurs.

(57) Reception of Energy.

Whenever electromagnetic waves encounter a conductor, oscillatory electromotive forces are set up and produce currents in the conductor of the same frequency as that of the wave, and in the same way that a circuit having a large radiation resistance will radiate a large amount of energy, one having a large radiation resistance will have induced in it a large electromotive force, and if tuned to the wave will receive a large amount of energy. The currents that flow in the receiving aerial produce a radiation which interferes with the oncoming wave to cause points of maximum and minimum field strength, but opposes and weakens the wave progressing beyond the receiving aerial.

For all ordinary purposes, therefore, the receiving circuit is an elevated aerial earthed at the foot, and if h cms. is the height of the up-lead and E volts/cm. the vertical electric field strength of the wave at the aerial, the electromotive force set up is Eh, and if R is the total resistance of the tuned aerial, the current produced is $I_r = Eh/R$. The total resistance is the sum of the radiation resistance R_r which is proportional to the square of h (see equation 55·05), the total loss resistance R_l, and a resistance R_a of the necessary apparatus for detecting the waves. The useful power received is then

$$\begin{aligned} W &= \frac{1}{2} I_r^2 R_a \\ &= \frac{1}{2} \frac{E^2 h^2 R_a}{R^2}. \\ &= A \frac{R_r R_a}{R^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1) \end{aligned}$$

where A is a constant.

This is a maximum when R_a, which is under control, is made equal to $\frac{1}{2}R$, and is then

$$W_m = A \frac{R_r}{2R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

By means of the triode valve, with certain limitations, the total resistance can be reduced to any desired extent, say, to R_r/m. Then the useful power is $Am/2$, and is independent of the height h of the aerial. It appears also from (2) that the effectiveness of any receiving aerial in delivering power to detecting apparatus depends only on the ratio R_r/R. Thus receiving aerials need not be very high or extensive; and may even be replaced by coils of wire of comparatively small dimensions (the loop or frame aerial).

Frame aerials have the advantage of directional properties. It will be clear on consideration that if the plane of the frame is at right angles to the direction of the wave, equal electromotive forces of equal phase will be induced in each vertical side, and no current will flow. Alternatively, considering the magnetic field of the wave, no flux will cut the coil, and, therefore, no electromotive force be produced. On the other hand, when the plane of the frame is in the plane of the wave, the difference of phase of the voltages in the coil sides, or alternatively the magnetic flux cutting the coil, is a maximum, and maximum electromotive force is produced.

Let the plane of the frame be inclined at an angle θ to the direction of propagation of the wave, and let the width of the frame be b and the height h, and the distances of the two sides be r_1 and r_2 from the distant source of the waves. The wave is considered to be plane. If E is the amplitude of the electric force at the frame, the instantaneous e.m.fs. in each wire of the two sides of the coil may be

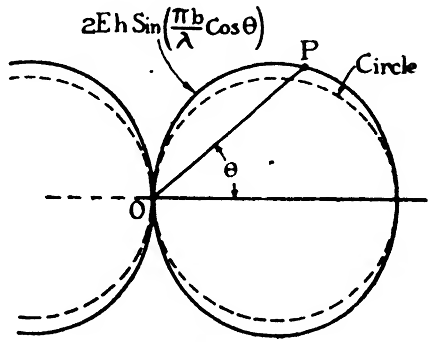


FIG. 215.—Voltage in Frame Aerial.

written (from 56.15) as $\mathrm{E}h\cos\omega\left(t-\frac{r_1}{c}\right)$ and $\mathrm{E}h\cos\omega\left(t-\frac{r_2}{c}\right)$ respectively, and the net e.m.f. per turn as

$$\mathrm{V}=\mathrm{E}h\left\{\cos\omega\left(t-\frac{r_2}{c}\right)-\cos\omega\left(t-\frac{r_1}{c}\right)\right\} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$=2\mathrm{E}h\sin\frac{\omega}{2c}\left(r_2-r_1\right)\sin\omega\left(t-\frac{r_1+r_2}{2c}\right)$$

with an amplitude

$$\mathrm{V}=2\mathrm{E}h\sin\frac{\omega}{2c}\left(r_2-r_1\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$=2\mathrm{E}h\sin\left(\frac{\omega b}{2c}\cos\theta\right)$$

$$=2\mathrm{E}h\sin\left(\frac{\pi b}{\lambda}\cos\theta\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

since $\lambda f=\lambda\omega/2\pi=c$.

The shape of polar curve given by this equation is shown in Fig. 215, which gives, therefore, the received voltage as the frame is rotated through various angles θ from the direction of the wave. If b is small compared with λ so that the angle $\left(\frac{\pi d}{\lambda}\cos\theta\right)$ is small, the voltage is approximately

$$V = 2\pi\frac{\mathrm{E}hb}{\lambda}\cos\theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and the polar curve is a pair of tangent circles.

As showing that consideration of either the electric or magnetic fields of the wave must lead to identical results, let the amplitude of the magnetic field at the frame be H, and the instantaneous value $\mathrm{H}\cos\omega\left(t-\frac{r}{c}\right)$. The total instantaneous flux through the frame is

$$\begin{aligned}\Phi &= h\int_{r_1}^{r_2}\mathrm{H}\cos\omega\left(t-\frac{r}{c}\right)dr \\ &= -h\mathrm{H}\frac{c}{\omega}\left[\sin\omega\left(t-\frac{r_2}{c}\right)-\sin\omega\left(t-\frac{r_1}{c}\right)\right]\end{aligned}$$

and the induced e.m.f. per turn is

$$\begin{aligned}v &= -\frac{d\Phi}{dt} \\ &= h\mathrm{H}c\left[\cos\omega\left(t-\frac{r_2}{c}\right)-\cos\omega\left(t-\frac{r_1}{c}\right)\right] \quad . \quad . \quad . \quad (7)\end{aligned}$$

This expression is very similar to (3) and leads to an amplitude

$$V = 2\mathrm{H}hc\sin\left(\frac{\pi b}{\lambda}\cos\theta\right) \quad . \quad . \quad . \quad . \quad . \quad (8)$$

which is seen to be identical with (5), when E and H are measured in the same system of units, and remembering (56·15, 56·16, and 56·17) that in a plane wave E in electrostatic units is equal to H in electromagnetic units.

This directional property is the basis of methods of direction and position finding by wireless. If a ship, for instance, radiates waves which are picked up, at two suitably placed stations, on frame aerials which can be rotated to positions giving maximum signal strength, or better, as is clear from the polar diagram, to positions of minimum strength, the intersection of lines drawn on a map at the angles thus determined will give the position of the ship. Because of its importance a large amount of work has been done on the subject and there is already an extensive literature. Apart from instrument errors, which can readily be kept below 1 per cent., errors arise from the fact that the waves may not be propagated in perfectly straight lines from the sending to the receiving station, and from the peculiar effects due to the wave reflected from the Heaviside layer. When the waves pass obliquely from sea to land they suffer refraction and apparent change of direction, and the same effect is produced by other aerials, trees, telegraph wires and any other conductors in the vicinity of the receiving station. On board ship, for instance, the masts and superstructure produce a certain error. In the main these errors are constant and may be allowed for by constructing a suitable calibration chart, and as a result sufficiently accurate bearings can be given up to, say, 100 miles, that is, at distances where they are most required as an aid to marine or aerial navigation.

Over longer distances, and especially during sunset and sunrise and at night, very large errors may occur, and it may not be possible to obtain a minimum signal strength angle at all. These effects are ascribed to the Heaviside layer. The reflected wave has its field components at angles to those of the direct wave, and the angle varies considerably. It may even vary continuously, so that a rotating field (similar to that in an induction motor) is formed, and in that case obviously no balance is possible. Where a balance is obtained, the direction found is the apparent direction of the combination of the direct and the reflected waves, and this may differ considerably from the true direction.

One of the greatest of the unsolved problems of wireless telegraphy is the elimination of disturbances due to the effects on the aerial of naturally produced electromagnetic waves, known variously as atmospherics, strays or X's. These waves are supposed to originate in electric discharges in the atmosphere such as lightning flashes; they are very powerful, and can travel over great distances. They vary in intensity at different points on the earth's surface, and are generally most severe in the tropics. They vary also with the time of the year, being generally worse in summer than in winter.

The interference caused is greater on long wavelengths than on very short ones. In many cases atmospherics appear to come mostly from one direction, so that relief can be obtained by placing the receiving station at a point where directional reception which favours the desired signal does the reverse to the atmospherics.

REFERENCES.

L. B. Turner.—" Outline of Wireless."
G. W. Pierce.—" Electric Oscillations and Electric Waves."
L. S. Palmer.—" Wireless Principles and Practice."

CHAPTER XII

HIGH FREQUENCY CIRCUITS

(58) Capacity, Inductance, and Resistance.

It is clear from the considerations of the previous chapter that systems of telegraphy and telephony depending upon electromagnetic waves in space must involve high frequency currents in transmitting and receiving apparatus. With the exception of the radiating property of extended circuits such as aerials already noticed, there is no fundamental difference in the theory of such circuits as compared with the theory of power circuits at 50 cycles a second, or voice frequency circuits of, say, 2,000 cycles a second, but the quantities involved are enormously different. Reactances are usually far greater than resistances, and in order to obtain sufficient currents from the electromotive forces available circuits are usually resonant, that is, the inductive reactance is made to counter[illegible]t the capacitative reactance (see section 31).

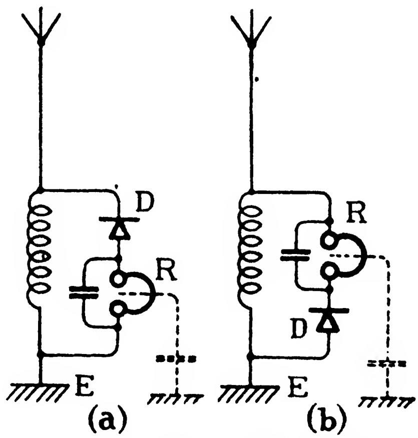


Fig. 216.—Effect of Stray Capacity.

The importance of small capacities and inductances, whether self or mutual, cannot be over-emphasised. A capacity of only 10 micro-microfarads at a frequency of 3×10^6 cycles per second (a wavelength of 100 metres) has an impedance ($1/\omega C$) of only about 5,000 ohms, and 5 volts would drive a milliampere through it. At the same frequency an inductance of 10 microhenries has the very considerable impedance (ωL) of about 200 ohms.

Stray capacities and inductances, which would be quite unnoticed in low frequency circuits, may thus profoundly modify the behaviour of high frequency circuits. Of special importance are stray capacities to earth. If care is not taken, currents will flow through the stray capacities to earth instead of through essential apparatus. A common example is the very simple receiving circuit of Fig. 216, in which (*a*) shows the right way of connecting the detector or rectifier D and the telephone receivers R, and (*b*) the wrong way. The receivers when strapped to the head have a large capacity to earth, and in (*b*) currents produced by the electromotive forces, set up in the aerial by the electromagnetic wave, flow to earth through this stray capacity (shown dotted) instead of passing through and being rectified by the detector. In (*a*) the stray capacity produces no ill effect; it is merely in parallel with the necessary condenser across the receiver and all the high frequency current flows through the detector.

Inductances and capacities at high frequencies differ as a rule inappreciably from the values measured at low frequencies, although the wire-to-wire capacity or self capacity of an inductance coil must be taken into account. But the reverse is true of resistances, owing to the "skin effect" already noticed in section 56. With large conductors formula 56.34 for the equivalent depth of penetration enables the high frequency resistance of a straight wire to be calculated, but this no longer applies when the penetration is a considerable fraction of the cross-sectional dimensions. Some idea of the importance of the skin effect is given by the ratios of high frequency resistance (R′) to direct current resistance (R) for copper wires in the following table :—

Frequency.	R^1/R.	
	Dia. of wire 2·0 mm.	Dia. of wire 0·2 mm.
10^3	1·000	1·000
10^4	1·144	1·000
10^5	8·67	1·000
10^6	84·2	1·144
10^7	839·	8·67

Evidently there is no great gain in increasing the diameter of the wire beyond a certain size; tubes will be equally effective as large-diameter wires; tinned copper is not desirable, as the current is mostly carried by the poorly conducting tin, but copper-covered steel wires and even galvanised steel wires may be useful where great strength is required as well as good conductivity (aerial wires, for instance). Lower resistances for a given weight of copper may be obtained by using stranded wire, in which a large number of small-gauge insulated wires are braided together so that each wire lies as often on the outside of the bundle as on the inside.

When a wire is bent round to form a coil a further increase in effective resistance to high frequency currents takes place, as instead of the current distribution being uniform round the surface of the wire, the current is now mostly carried on the side of the wire which lies on the inside of the coil. An accurate estimate of the high-frequency wire resistance of a coil involves very complicated mathematics, and the results are generally less than the actual effective resistance as measured, owing to losses in the dielectric between the turns. Stranded wire may be employed with advantage on the longer wavelengths, but on short wavelengths a solid conductor is better.*

(59) Resonance and Tuning.

The variation of the rotating vector representing the current in the circuit of Fig. 217 when an electromotive force of varying frequency is impressed in the circuit has already been studied in section 31. It was found that the vector locus is a circle, and that the current amplitude plotted against the frequency gives a "resonance curve," the maximum current occurring when $\omega L = 1/\omega C$, and that the sharpness of the resonance curve depends upon the decay factor $R/2L$. When the electromotive force is derived from aerial currents producing electromagnetic waves, or from the currents produced by electromagnetic waves, and the values of L and C are adjusted so that $\omega L = 1/\omega C$, the circuit is said to be "tuned" to the wave. If the circuit contains some alternating current indicating or measuring device, such as a thermojunction and galvanometer, and the condenser is continu-

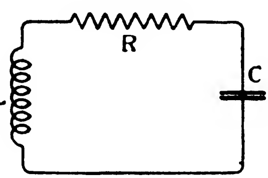


FIG. 217.—Oscillatory Circuit.

* C. L. Fortescue, J.I.E.E., Vol. 61, p. 933.

ously variable and the circuit is tuned by adjusting the condenser until the current is a maximum, the circuit can be used to measure (from a knowledge of L and C) the frequency of the currents and hence the wavelength of the wave. The arrangement is then known as a wavemeter.

Instead, however, of calculating the wavelength from L and C the circuit is best calibrated by reference to high frequency currents of accurately known frequency. If the values of λ^2 are plotted against C a straight line passing through the origin should be obtained, since

$$f = \frac{c}{\lambda} = \frac{1}{2\pi\sqrt{CL}}$$

and hence

$$C = \frac{\lambda^2}{4\pi^2 c^2 L} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

But, as shown in Fig. 218, although a straight line is obtained in such a calibration, it does not pass through the origin, but through a point L, and if continued backwards cuts the C axis in a point O′. This is due to the wire-to-wire or self capacity of the coil, which is equivalent to a single capacity of value $C_0 = OO'$ shunting the coil, and therefore in parallel with C. If C were made zero, the circuit would still resonate, to a wavelength $= \sqrt{OL}$.

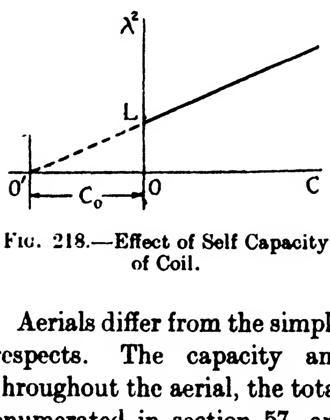


FIG. 218.—Effect of Self Capacity of Coil.

Aerials differ from the simple circuit of Fig. 217 in some important respects. The capacity and the inductance are distributed throughout the aerial, the total resistance includes the various parts enumerated in section 57, and the current varies throughout the aerial, being a maximum at the foot where the aerial is earthed and zero at the extreme ends. Any aerial is, however, resonant or tuned to a particular frequency or wavelength, and round about this frequency a simple equivalent circuit such as that of Fig. 217 can be found which will have the same current as that at the foot of the aerial with the same applied e.m.f. The frequency referred to is the fundamental resonance frequency. But aerials, in common with all other systems with distributed inductance and capacity (or mass and stiffness), have multiple resonances, and for each resonance an equivalent circuit can be found.

Aerials almost invariably have an inductance coil inserted at the foot for the purpose of introducing an electromotive force to produce aerial currents in the case of a transmitting aerial, or of producing electromotive forces from the aerial currents in receiving circuits in the case of a receiving aerial (Fig. 219 (*a*)). In the latter case the inductance is frequently shunted by a condenser for the purpose of tuning to the desired wavelength (Fig. 219 (*b*)). This "loading" of the aerial modifies the current and voltage distribution in the aerial, and makes the overtones inharmonic, but equivalent circuits can still be found, with the additional condition that small variations of the inductance or capacity in the equivalent circuit must make the same alteration of the resonant frequency as the alterations of L or C make to the resonant frequency of the aerial.

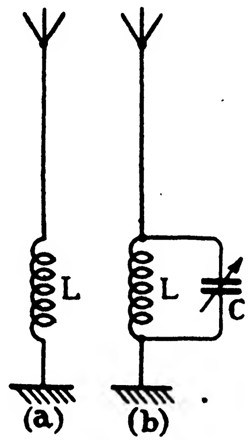


FIG. 219.—Aerial Tuning.

The calculation of the resonant frequencies (or natural wavelengths) of an aerial is usually very complicated, but some insight into the matter is obtained by considering the simple case of a single horizontal wire aerial with an uplead, in which an electromotive force is introduced, at one end. If l is the length of the horizontal wire, and C and L its capacity and inductance per unit length, and if the capacity and inductance of the uplead are neglected, then the current in the uplead is obtained by putting $x = 0$ in equation 39.30

as
$$I_s = \frac{V_s}{Z_0} \tanh Pl \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where V_s is the voltage in the uplead and

$$Z_0 = \sqrt{\frac{L}{C}}$$

and
$$P = j\omega\sqrt{LC}$$

from 39.10 and 39.09, on neglecting R and G in comparison with ωL and ωC respectively, which is justified for the present purpose at the large values of ω concerned.

Thus the impedance of the aerial from the foot of the uplead is

$$Z = \frac{V_s}{I_s} = Z_0 \coth Pl$$

$$= \sqrt{\frac{L}{C}} \coth (j\omega\sqrt{LC})l$$

$$= -j\sqrt{\frac{L}{C}} \cot \omega\sqrt{LC}l$$

i.e., the reactance X is

$$X = -\sqrt{\frac{L}{C}} \cot \omega\sqrt{LC}l \quad . \quad . \quad . \quad . \quad (3)$$

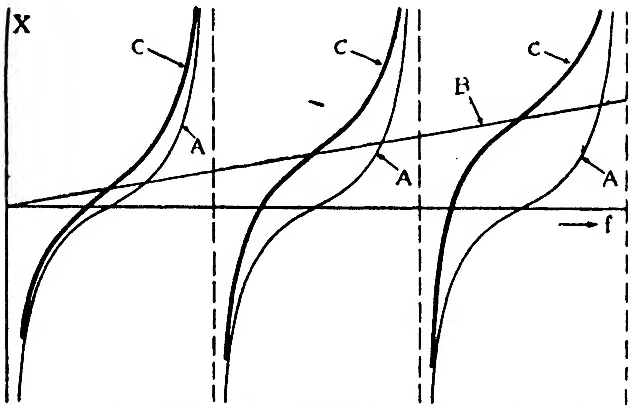


FIG. 220.—Reactance-Frequency Curves of Aerial.

This is plotted as the curves A in Fig. 220. Resonance occurs when the reactance is zero, that is, when

$$\omega\sqrt{LC}l = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \text{ etc.,}$$

or when

$$f = \frac{m}{4\sqrt{LC}l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where m has the values 1, 3, 5, etc.

Since $c = 1/\sqrt{LC}$ = the velocity of light in the case of an open wire when resistance and leakance are neglected (see equation 39.24 and the following paragraphs), and since $\lambda = c/f$, equation (4) can be rewritten

$$\lambda = \frac{4l}{m} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and this gives the natural wavelengths of the aerial. For the fundamental, the natural wavelength is thus four times the length of the aerial. This is, however, only a rough approximation, as the effect of the vertical portion of the aerial has been neglected. Similar considerations apply to a single vertical aerial wire, but here the capacity to earth varies along the length. However, equation (5) even in this case gives a rough approximation to the natural wavelengths.

When the aerial is loaded by an inductance L_l at the foot, the reactance ωL_l must be added to the reactance given by (3). In Fig. 220 the line B represents ωL_l, and the firm curves C the total reactance found by adding curves A and B. This is seen to be zero at lower frequencies (greater wavelengths) than when the aerial is unloaded, and the frequencies are no longer harmonic, although at the higher modes the frequency intervals tend to become the same. The effect of loading by a condenser, or by a shunted condenser and inductance, can be found in a similar manner.

For the fundamental wavelength the graphical method can be avoided and a sufficiently good approximate formula obtained. For small values of θ, cot θ can be expanded in two terms as

$$\cot\theta = \frac{1}{\theta} - \frac{\theta}{3} \qquad \text{. . . . . . . (6)}$$

Applying this to equation (3) the reactance becomes

$$\mathrm{X} = -\sqrt{\frac{\mathrm{L}}{\mathrm{C}}}\left(\frac{1}{\omega\sqrt{\mathrm{LC}}l} - \frac{\omega\sqrt{\mathrm{LC}}l}{3}\right)$$

$$= -\frac{1}{\omega\mathrm{C}l} + \frac{\omega\mathrm{L}l}{3} \qquad \text{. . . . . (7)}$$

which is the reactance of a capacity Cl and an inductance Ll/3 in series. With the aerial loaded with an inductance L_l the reactance is

$$\mathrm{X} = -\frac{1}{\omega\mathrm{C}l} + \frac{\omega\mathrm{L}l}{3} + \omega\mathrm{L}_l$$

and equating this to zero gives

$$\omega = \frac{1}{\sqrt{\left(\frac{\mathrm{L}l}{3} + \mathrm{L}_l\right)\mathrm{C}l}} \qquad \text{. . . . . . (8)}$$

whence

$$\lambda = 1884\sqrt{\left(\frac{Ll}{3} + L_l\right)Cl} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

where λ is in metres, Ll and L_l in microhenries, and Cl in microfarads.

An idea of the distribution of current and potential in a vertical wire aerial can in a similar manner be obtained from equations 39.29 and 39.30. The curves given for the fundamental and first and second higher modes of vibration are drawn in Fig. 221 as *a*, *b* and *c*.

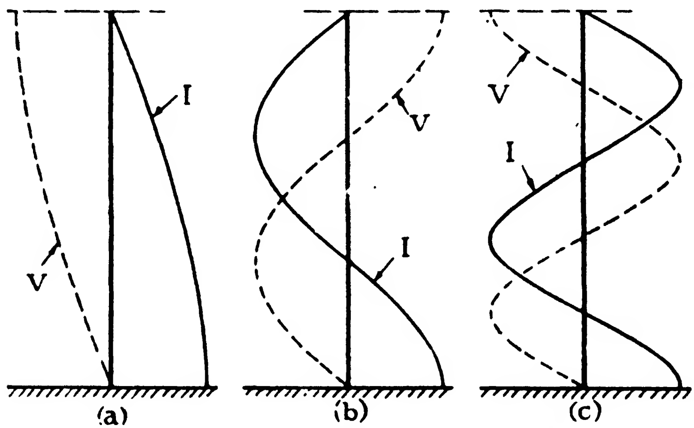


FIG. 221.—**Distribution of Current and Potential along Aerial. (*a*) Fundamental, (*b*) First overtone, (*c*) Second overtone.**

The contribution of each element of length of the aerial to the total field at any point is proportional to the product of the current in the element and the length of the element, and due regard must be paid to the sign of the current. This results, in the higher modes of vibration, in a polar distribution showing only a small power radiated upwards at an angle to the earth in comparison with the horizontal radiation. This is a desirable condition for broadcasting station aerials, where long ranges are not desired owing to the probable interference with other stations, and the most stable reception is to be obtained from the surface wave. But it is undesirable in the case of very long range beam stations, where the upward radiation is required, and in which, owing to the shortness of the wave, the aerials must necessarily oscillate at a higher mode. The difficulty has been overcome by C. M. Franklin by the insertion of loading or phase changing coils as indicated in Fig. 222, in which also the effect on the current distribution is shown. The current, though varying in magnitude, is in the same direction throughout the aerial. The radiation from the coils, in which the current is flowing in the reverse direction, is negligible.

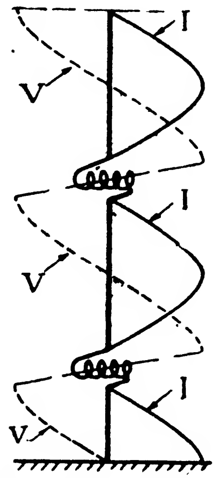


FIG. 222. — Effect of Insertion of Phasing Coils.

The effects on the current distribution of a loading coil at the base of the aerial and of a capacity at the top can similarly be studied by the methods of section 39. But the approximation is very rough. Besides the variation of capacity to earth along the length of the aerial, there is also the capacity between one element of the aerial and another, which is of importance when the elements are at different potentials, and which will be of increasing importance the higher the mode of vibration.

(60) Coupled Circuits.

Two circuits, one of which may be the equivalent circuit of an aerial, coupled together are frequently met with in "wireless." In Fig. 223 the coupling is by mutual inductance M between the coils of the two circuits, $L_1C_1R_1$ and $L_2C_2R_2$ are the constants of the two circuits respectively, and an electromotive force V cos ωt is supposed to be introduced into the first or primary circuit. It is required to find the currents I_1 and I_2 in the primary and secondary circuits respectively.

After transients have disappeared, the following vector relationships may be written down :—

$$V = \left\{ R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right) \right\} I_1 + j\omega M I_2 \quad . \quad . \quad (1)$$

$$0 = \left\{ R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2}\right) \right\} I_2 + j\omega M I_1 \quad . \quad . \quad (2)$$

Writing Z_1 and Z_2 for the series impedances of the two circuits, *i.e.*, writing

$$\left.\begin{aligned} Z_1 &= R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right) \\ Z_2 &= R_2 + j\left(\omega L_2 - \frac{1}{\omega C_2}\right) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

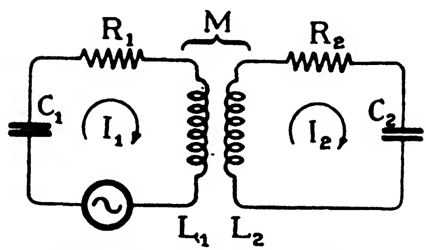


FIG. 223.—Oscillatory Circuits Magnetically Coupled.

from (2)

$$I_2 = -\frac{j\omega M I_1}{Z_2}$$

which inserted in (1) gives

$$V = \left(Z_1 + \frac{\omega^2 M^2}{Z_2}\right) I_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Also from (2)

$$I_1 = -\frac{Z_2}{j\omega M} I_1$$

which in (1) gives

$$V = -\frac{1}{j\omega M}(\omega^2 M^2 + Z_1 Z_2) I_2 \quad . \quad . \quad . \quad (5)$$

Equations (4) and (5) give the primary and secondary currents for an impressed primary e.m.f. in terms of the circuit constants. Interest generally lies chiefly in the secondary current, which can be written

$$I_2 = \frac{V}{Z''} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where

$$Z'' = \frac{j}{\omega M}(\omega^2 M^2 + Z_1 Z_2) \quad . \quad . \quad . \quad (7)$$

is an impedance determining the secondary current for a given primary e.m.f.

In the case of a single circuit, $I = E/Z$ and the locus of Z with ω varied is a straight line, which inverted gives a circular locus for the current (section 31). When the two circuits of Fig. 223 are tuned, that is, when $L_1C_1 = L_2C_2 = 1/\omega_0^2$, say, the locus of Z'' is a parabola. For, writing X_1 and X_2 for the reactances of the two circuits, from (3)

$$\left.\begin{aligned} X_1 &= \omega L_1 - \frac{1}{\omega C_1} = \omega L_1\left(1 - \frac{\omega_0^2}{\omega^2}\right) \\ \text{and}\quad X_2 &= \omega L_2 - \frac{1}{\omega C_2} = \omega L_2\left(1 - \frac{\omega_0^2}{\omega^2}\right) \end{aligned}\right\} \quad . \quad . \quad . \quad (8)$$

and

$$\begin{aligned} Z_1Z_2 &= (R_1 + jX_1)(R_2 + jX_2) \\ &= R_1R_2 - X_1X_2 + j(R_1X_2 + R_2X_1) \\ &= R_1R_2 - \omega^2 L_1L_2\left(1 - \frac{\omega_0^2}{\omega^2}\right)^2 + \\ &\quad j\omega(R_1L_2 + R_2L_1)\left(1 - \frac{\omega_0^2}{\omega^2}\right) \quad . \quad . \quad . \quad . \quad . \quad (9) \end{aligned}$$

In Cartesian co-ordinates the locus of Z_1Z_2 is accordingly given by

$$x = R_1R_2 - \omega^2 L_1L_2\left(1 - \frac{\omega_0^2}{\omega^2}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad (10)$$

$$y = \omega(R_1L_2 + R_2L_1)\left(1 - \frac{\omega_0^2}{\omega^2}\right). \quad . \quad . \quad . \quad . \quad (11)$$

whence

$$y^2 = \frac{(R_1L_2 + R_2L_1)^2}{L_1L_2}(R_1R_2 - x) \quad . \quad . \quad . \quad . \quad . \quad (12)$$

This is the equation of a parabola (Fig. 224) with intercepts on the x axis $Ov = R_1R_2$, and on the y axis $Oc = Ob =$

$$(R_1L_2 + R_2L_1)\sqrt{\frac{R_1R_2}{L_1L_2}}.$$

When the circuits are identical ($R_1 = R_2$ and $L_1 = L_2$) $Oc = Ob = 2R_1^2$ and $Ov = R_1^2$; O is at the focus of the parabola.

If P is any point on the parabola OP is the complex quantity Z_1Z_2 at that point. The frequency at the point is determined by either (10) or (11). With ω very small, y has a large negative value and P is situated far to the left on the lower arm. As ω increases P moves to the right along the lower arm until v is reached at $\omega = \omega_0$ when $y = 0$. With further increase of ω, y is positive and of increasing value, and P moves to the left along the upper arm of the parabola. The parabola is accordingly described in a counterclockwise direction with increasing ω, as indicated by the arrows.

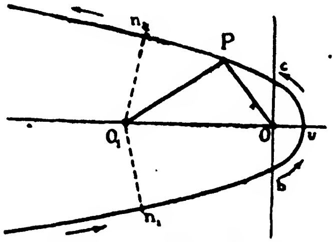


FIG. 224.—Locus of Z″.

As the next step in the construction of the locus of Z″ from equation (7), ω^2M^2 has to be added to every ray OP. This can be done by marking OO_1 to the left from O along the x axis and making $OO_1 = \omega^2M^2$. O_1P is then equal to the vector sum of O_1O and OP, *i.e.*, to $\omega^2M^2 + Z_1Z_2$. Actually the distance OO_1 is different for every point P, but with most wireless circuits the coupling M is small enough and the resistances are small enough for the whole of the resonance phenomenon to be passed through with such a small percentage change in ω, that ω^2M^2 is sensibly constant and O_1 nearly a fixed point. Finally, to multiply by $j/\omega M$ the whole diagram is rotated counterclockwise through a right angle, or Ob taken as the new reference axis, and the scale altered by dividing by ωM, again considered constant.

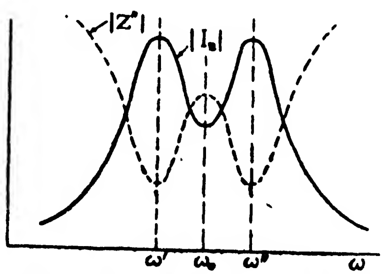


FIG. 225.—Double Hump Resonance Curve of Secondary Current.

In general three normals O_1n_1, O_1v and O_1n_2 can be drawn from O_1 to the parabola; at n_1 and n_2 the impedance is a minimum and at v a maximum. The shape of the impedance $|Z''| - \omega$ curve is accordingly as shown by the dotted line in Fig. 225 and that of the $|I_2| - \omega$ curve (the reciprocal of the $|Z''| - \omega$ curve) by the full line. The current resonance curve shows a double hump. The greater the coupling (*i.e.*, the greater M) the further O_1 is to the left, and the greater the difference of frequency between n_1 and n_2; the humps of the resonance curve are further apart and the trough between them deeper. On the other hand, if the coupling is very small the three normals may coincide in Ov, and only a single hump resonance curve is obtained.

The lengths of the normals from a point on the axis of the parabola $y^2 = px$ distant a from the vertex are given by

$$\sqrt{p\left(a - \frac{p}{4}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

and, of course, a. The three normals will coincide when $\sqrt{p\left(a - \frac{p}{4}\right)}$ $= a$, *i.e.*, when $a = p/2$. Hence the double hump will just begin to appear when

$$\omega^2 M^2 + R_1 R_2 = \frac{(R_1 L_2 + R_2 L_1)^2}{2 L_1 L_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

since $a = \omega^2 M^2 + R_1 R_2$ by construction and
$p = (R_1 L_2 + R_2 L_1)^2 / L_1 L_2$ by (12).

Writing the decay factors $a_1 = R_1/2L_1$ and $a_2 = R_2/2L_2$, (14) becomes

$$\begin{aligned} \omega^2 M^2 &= 2L_1 L_2 (a_1 + a_2)^2 - R_1 R_2 \\ &= 2L_1 L_2 \{(a_1 + a_2)^2 - 2a_1 a_2\} \\ &= \frac{1}{2} R_1 R_2 \left(\frac{a_1}{a_2} + \frac{a_2}{a_1}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15) \end{aligned}$$

When the decay factors of the two circuits are the same this critical coupling becomes

$$\omega^2 M^2 = R_1 R_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

The size of the impedance Z'' at the normals $O_1 n_1$ and $O_1 n_2$ is by (13) and (7), and introducing a_1 and a_2 as in obtaining (15),

$$\begin{aligned} |Z_m''|^2 &= \frac{1}{\omega^2 M^2} [4L_1 L_2 (a_1 + a_2)^2 \{\omega^2 M^2 + R_1 R_2 - L_1 L_2 (a_1 + a_2)^2\}] \\ &= \frac{1}{\omega^2 M^2} [4L_1 L_2 (a_1 + a_2)^2 \{\omega^2 M^2 - L_1 L_2 (a_2 - a_1)^2\}] \quad . \quad (17) \end{aligned}$$

If the decay factors are the same, and approximately if $L_1 L_2 (a_2 - a_1)^2$ is small compared with $\omega^2 M^2$, this reduces to

$$|Z_m''| = 2\sqrt{L_1 L_2}(a_1 + a_2) \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

showing that in this case increasing the coupling beyond the critical value produces no alteration in the secondary current maxima.

At critical coupling given by (15), (17) becomes

$$|Z_m''|^2 = \frac{2L_1L_2(a_1 + a_2)^4}{a_1^2 + a_2^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and this gives the maximum value of the current with "sufficient" coupling to produce the double hump. An increase of coupling results in a decrease of current maximum, and a decrease of coupling in an increase. With "insufficient" coupling, that is, with couplings below the critical value, the minimum impedance is found along the axis and its size is

$$|Z_m''| = \frac{\omega^2M^2 + R_1R_2}{\omega M} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Differentiating with regard to ωM and equating to zero to find the condition for a minimum gives

$$\omega^2M^2 = R_1R_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

and this is the coupling which in any case gives the maximum possible value of the secondary current. The value is $E/2\sqrt{R_1R_2}$ from (6), (2) and (21).

The frequencies for which the current is a maximum can be found as follows. At the normals the value of x is given by

$$-x = O_1O - p/2$$

(the sub-normal of a parabola $y^2 = px$ is $p/2$) which equated to the value given by (10) gives

$$\omega^2M^2 - \frac{(R_1L_2 + R_2L_1)^2}{2L_1L_2} = -R_1R_2 + \omega^2L_1L_2\left(1 - \frac{\omega_0^2}{\omega^2}\right)^2 \quad (22)$$

which by simple reduction becomes

$$\omega^4(1 - \tau^2) + 2\omega^2(a_1^2 + a_2^2 - \omega_0^2) + \omega_0^4 = 0 \quad . \quad . \quad . \quad (23)$$

where τ^2 is written for M^2/L_1L_2. τ is known as the *coupling coefficient*, and has a maximum value of unity, in the ideal case of no magnetic leakage between the two coils.

In the second term a_1^2 and a_2^2 are generally small compared with ω_0^2 and can be neglected. Equation (23) then becomes

$$\omega^4(1 - \tau^2) - 2\omega_0^2\omega^2 + \omega_0^4 = 0$$

with solutions

$$\omega^2 = \frac{1}{1 \pm \tau}\,\omega_0^2.$$

Hence the angular frequencies ω' and ω'' of the humps are given by

$$\left.\begin{aligned}\omega' &= \frac{1}{1+\tau}\,\omega_0 \\ \text{and} \qquad \omega'' &= \frac{1}{1-\tau}\,\omega_0\end{aligned}\right\} \qquad (24)$$

when, as is usually the case, the effect of the resistance on the hump frequencies can be neglected.

The primary current may be dealt with in a similar manner. Equation (4) may be written

$$I_1 = \frac{E}{Z'} \qquad (25)$$

where

$$Z' = Z_1 + \frac{\omega^2 M^2}{Z_2} \qquad (26)$$

With zero coupling ($M = 0$) the locus of Z' is a straight line. As the coupling is increased the line bulges at the axis, and develops a

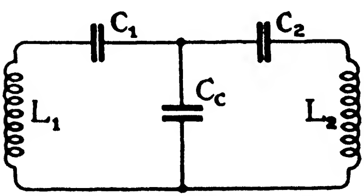


FIG. 226.—Coupling by Common Condenser.

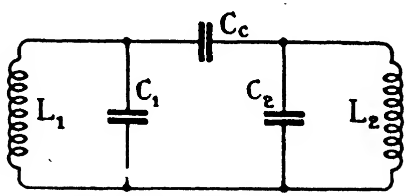


FIG. 227.—Electric Coupling.

loop as the coupling is still further increased. The family of curves with various couplings is the cissoid family. Here, as with the secondary current, a double minimum impedance appears with sufficient coupling, corresponding to a double hump primary resonance curve.*

There are other methods of coupling circuits. There may, for instance, be an impedance common to the two circuits. In Fig. 226 this impedance is shown as a condenser, but it may be an inductance or a resistance or both. Or there may be an impedance connecting the two circuits but not common to either as shown in Fig. 227. Here the coupling impedance is a condenser, and this is an important case owing to the fact that this type of coupling almost invariably exists, even though not intentionally, owing to stray capacities.

* E. Mallett, Proc. Royal Soc. A., Vol. 117, p. 331.

In addition, the inductances L_1 and L_2 of Fig. 227 may be coupled by a mutual inductance M, and this represents what is actually obtained in practice when only the simple coupling of Fig. 223 is intended.

In all these cases Z″ can be obtained from a parabola $y^2 = p(-x)$ by drawing rays to the parabola from a pole $x_p y_p$ and dividing each ray by a complex quantity A. The distribution of ω round the parabola is found from

$$y = 4L_1L_2(a_1 + a_2)(\omega - \omega_0 + \omega_a)$$

and in all cases

$$p = 4L_1L_2(a_1 + a_2)^2$$

When the circuits are tuned x_p, y_p, A and ω_a are as follows .—

(i.) Magnetic coupling

$$-x_p = R_1R_2 + \omega^2M^2$$
$$y_p = 0$$
$$A = -j\omega M$$
$$\omega_a = 0$$

This is the case considered above (Fig. 223). If the coupling is by a common inductance l (Fig. 226), l replaces M in the above expressions and Z_1 and Z_2 refer to the whole circuits composed of $L_1\ C_1\ l$ and $L_2\ C_2\ l$ with their total resistances, and the reactances of these circuits must vanish for the same angular frequency ω_0.

(ii.) Coupling by common condenser C_c (Fig. 226)

$$-x_p = R_1R_2 + \frac{1}{\omega^2C_c^2}$$
$$y_p = 0$$
$$A = -j/\omega C_c$$
$$\omega_a = 0.$$

Here again Z_1 and Z_2 must comprise the whole circuits $L_1C_1C_c$ and $L_2C_2C_c$.

(iii,) Coupling by series condenser C_c between circuits (Fig. 227).

$$-x_p = R_1R_2 + \frac{1}{4}\omega^2C_c^2L_1L_2(1/C_1 + 1/C_2)^2$$

$$-y_p = \omega C_c\left\{\frac{1}{2}(L_1R_2 - L_2R_1)(1/C_1 - 1/C_2)\right\}$$

$$A = -jC_c/\omega C_1C_2$$

$$\omega_a = \frac{1}{4}\omega C_c(1/C_1 + 1/C_2)$$

(iv.) Combined magnetic and electric coupling (cases (i.) and (iii.) combined).

$$-x_p = R_1R_2 + \omega^2M^2(1 + C_c/C_1 + C_c/C_2) + 2MC_c/C_1C_2 + \frac{1}{4}\omega^2C_c^2L_1L_2(1/C_1 + 1/C_2)^2$$

$$-y_p = \omega C_c \frac{1}{2}(L_1R_2 - L_2R_1)(1/C_1 - 1/C_2)$$

$$A = -j\omega M - jC_c/\omega C_1C_2$$

$$\omega_a = \frac{1}{4}\omega C_c(1/C_1 + 1/C_2)$$

It is seen that in the first case

$$-x_p - R_1R_2 = -A^2$$

and that this condition is also fulfilled in the second case. This may be taken as one criterion for an exact parallel existing between any type of coupling and magnetic coupling. Another essential is $y_p = 0$, which means equal current humps.

These conditions are seen not to be fulfilled in general in cases (iii.) and (iv.), but in case (iii.) if C_1 is not very different from C_2 and/or the decay factors are nearly the same,

$$y_p \doteqdot 0 \text{ and } -x_p - R_1R_2 \doteqdot C_c^2/\omega^2C_1^2C_2^2 = -A^2$$

And if in case (iv.), in addition to C_1 and C_2 and/or a_1 and a_2 being not very different, $C_c(1/C_1 + 1/C_2)$ can be neglected compared with unity,

$$-x_p - R_1R_2 \doteqdot (\omega M + C_c/\omega C_1C_2)^2 = -A^2$$

With these conditions understood, and they are conditions very often fulfilled in wireless circuits, it may be said that all types of coupling are exactly similar in their effects to coupling by mutual inductance, and that all expressions for the case of mutual inductance can be converted to expressions for other types by replacing ωM by $1/\omega C_c$ in case (ii.), by $C_c/\omega C_1C_2$ in case (iii.) and by $(\omega M + C_c/\omega C_1C_2)$ in case (iv.), with the additional provision in the last two cases that the ω distribution must be altered by ω_a. It may be noted also that in the last case the sign of M is of importance, and the combined coupling may be either greater or less than by the two separately.

The graphical method can be extended to cases where there are three or more coupled identical circuits,* the mathematical aspect of which has already been dealt with in section 40.

(61) **Transients in Oscillatory Circuits.**

(i.) The battery of voltage V in the circuit of Fig. 228 charges the condenser C to a voltage V through the inductance L of resistance R. When the key K is depressed the condenser discharges through the inductance and the equation of the instantaneous voltages round the circuit leads to

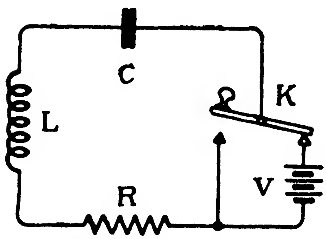


FIG. 228.—Circuit for Oscillatory Charge and Discharge of Condenser.

$$L\frac{di}{dt} + Ri + \frac{q}{C} = 0 \quad . \quad . \quad (1)$$

where q is the instantaneous charge on the condenser. Differentiating (1) gives

$$L\frac{d^2i}{dt} + R\frac{di}{dt} + \frac{1}{C}i = 0 \quad . \quad (2)$$

For a solution try $\quad i = A\epsilon^{mt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$

where A and m are constants to be determined. Then

$$\frac{di}{dt} = Am\epsilon^{mt}$$

and

$$\frac{d^2i}{dt^2} = Am^2\epsilon^{mt}$$

and substitution in (2) gives

$$LAm^2\epsilon^{mt} + RAm\epsilon^{mt} + \frac{1}{C}A\epsilon^{mt} = 0$$

or

$$Lm^2 + Rm + \frac{1}{C} = 0$$

whence, say,

$$\left.\begin{aligned} m &= -\frac{R}{2L} \pm \sqrt{\frac{R^2}{4L^2} - \frac{1}{LC}} \\ &= -a \pm \beta \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Thus the complete solution (from (3) and (4)) can be written

$$i = A_1\epsilon^{m_1t} + A_2\epsilon^{m_2t} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

* E. Mallett, Chains of Resonant Circuits, J.I.E.E., Vol. 66, p. 968.

where $\quad m_1 = -\alpha + \beta$
and $\quad m_2 = -\alpha - \beta$ $\quad \Big\} \quad \ldots \ldots \ldots$ (6)

and A_1 and A_2 are constants to be determined.

At the instant when the key is depressed $t = 0$ and $i = 0$ and from (5)

$$0 = A_1 + A_2 \quad \ldots \ldots \ldots \quad (7)$$

Also the initial charge in the condenser is CV, and this must be $-\int idt$ with $t = 0$. From (5)

$$\int idt = \frac{A_1}{m_1}\epsilon^{m_1 t} + \frac{A_2}{m_2}\epsilon^{m_2 t}$$

and when $t = 0$ this gives with (7) and (6)

$$CV = -A_1\left(\frac{1}{m_1} - \frac{1}{m_2}\right)$$

$$= A_1 \frac{2\beta}{\alpha^2 - \beta^2}$$

whence, using (4) and (7),

$$A_1 = -A_2 = \frac{CV}{2\beta} \cdot \frac{1}{LC}$$

and the solution (5) becomes

$$i = \frac{V}{2\beta L}\epsilon^{-\alpha t}\left\{\epsilon^{\beta t} - \epsilon^{-\beta t}\right\}$$

$$= \frac{V}{\beta L}\epsilon^{-\alpha t}\sinh \beta t \quad \ldots \ldots \ldots \quad (8)$$

If β is real the discharge current is unidirectional. But if β is imaginary, that is, if

$$\frac{1}{LC} > \frac{R^2}{4L^2} \quad \ldots \ldots \ldots \quad (9)$$

then, writing

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \quad \ldots \ldots \ldots \quad (10)$$

ω will be real and equal to $j\beta$, and (8) becomes

$$i = \frac{V}{\omega L}\epsilon^{-\alpha t}\sin \omega t \quad \ldots \ldots \ldots \quad (11)$$

The discharge is now oscillatory, but with logarithmically decreasing amplitude, as indicated in Fig. 229.

Intermediately, with $1/LC = R^2/4L^2$, β is zero, $\sinh \beta t/\beta$ is t, and (8) becomes

$$i = \frac{Vt}{L}\epsilon^{-\alpha t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The discharge is now known as critically damped.

In wireless circuits the condition (9) almost invariably holds, and the discharge current is oscillatory and of the "damped wave"

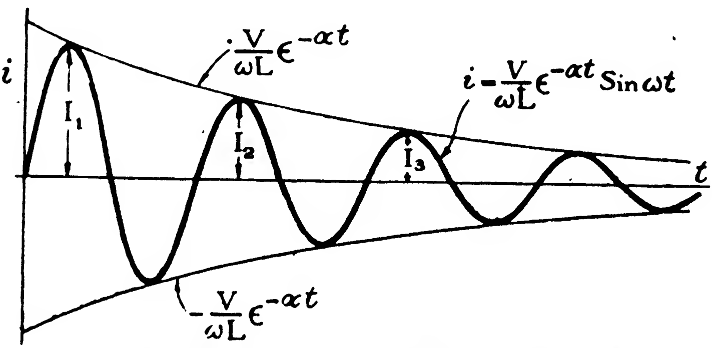


FIG. 229.—Discharge of Condenser through Inductance.

form shown in Fig. 229. $R^2/4L^2$ is, moreover, generally so much less than $1/LC$ that the frequency of the free oscillations given by (10) is very nearly the resonant frequency of the circuit given by $\omega = 1/\sqrt{LC}$.

The dying away or damping of the oscillations is determined (equation (11)) by the decay factor $\alpha = R/2L$. The time interval between successive maxima is from (10) and (11),

$$T = \frac{2\pi}{\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

a formula first given by Lord Kelvin.

If t is chosen in (11) to give the first current maximum, successive maxima are obtained when t is increased by T and ωt by 2π. The current maxima are therefore given by

$$I_1 = \frac{V}{\omega L}\epsilon^{-at}\sin \omega t$$

$$I_2 = \frac{V}{\omega L}\epsilon^{-a(t+T)}\sin(\omega t + 2\pi)$$

$$I_3 = \frac{V}{\omega L}\epsilon^{-a(t+2T)}\sin(\omega t + 4\pi)$$

and so on, whence the ratio of one maximum to the next is found as

$$\frac{I_1}{I_2} = \frac{I_2}{I_3} = \frac{I_3}{I_4} = \ldots = \epsilon^{aT} = \epsilon^{\delta} \quad \ldots\ldots \quad (14)$$

δ as defined by equation (14) is known as the *logarithmic decrement* of the circuit, the name arising from the fact (from (14)) that

$$\log I_1 - \log I_2 = \log I_2 - \log I_3 = \ldots = \delta \quad \ldots \quad (15)$$

Evidently from (14)

$$\delta = aT = \frac{a}{f} \quad \ldots\ldots \quad (16)$$

and approximately

$$\delta = \frac{R}{2L}2\pi\sqrt{LC}$$

$$= \pi R\sqrt{\frac{C}{L}} \quad \ldots\ldots \quad (17)$$

When the key of Fig. 228 is released the condenser is charged through the inductance and capacity, and equations (8), (11) and (12) with a negative sign give the charging currents under the various circuit conditions. This follows in an exactly similar manner.

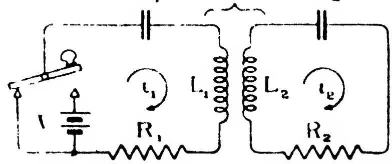


FIG. 230.—Transients in coupled circuits.

(ii.) If the circuit of Fig. 228 is coupled with another oscillatory circuit, say, as in Fig. 230, by mutual inductance M between the two coils, the equations connecting the instantaneous currents $i_1 i_2$, the instantaneous charges $q_1 q_2$ on the condensers and the circuit constants (denoted by subscripts 1 and 2 for the primary and secondary respectively) are

$$\left.\begin{aligned} L_1\frac{di_1}{dt} + M\frac{di_2}{dt} + R_1 i_1 + \frac{q_1}{C_1} = V \\ L_2\frac{di_2}{dt} + M\frac{di_1}{dt} + R_2 i_2 + \frac{q_2}{C_2} = 0 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \ (18)$$

In the special case in which the mutual inductance is so small that the currents in the secondary produce no appreciable reaction on the primary (as, for instance, when the secondary represents a receiving aerial and the primary a transmitting aerial), the secondary current is given by

$$L_2\frac{di_2}{dt} + R_2 i_2 + \frac{q_2}{C_2} = -M\frac{di_1}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \ (19)$$

and the primary current by (11)

$$i_1 = \frac{V}{\omega_1 L_1}\epsilon^{-a_1 t}\sin\omega_1 t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \ (20)$$

whence

$$L_2\frac{di_2}{dt} + R_2 i_2 + \frac{1}{C_2}\int i_2 dt = -\frac{MV}{\omega_1 L_1}\frac{d}{dt}(\epsilon^{-a_1 t}\sin\omega_1 t) \quad . \quad . \ (21)$$

Now $\qquad \dfrac{d}{dt}(\epsilon^{-a_1 t}\sin\omega_1 t) = -a_1\epsilon^{-a_1 t}\sin\omega_1 t + \omega_1\epsilon^{-a_1 t}\cos\omega_1 t$

In wireless circuits the decay factor a is usually very small compared with ω and the term containing a_1 can be ignored, hence (21) becomes on differentiating very nearly :—

$$L_2\frac{d^2 i_2}{dt^2} + R_2\frac{di_2}{dt} + \frac{1}{C_2}i_2 = \frac{\omega_1 MV}{L_1}\epsilon^{-a_1 t}\sin\omega_1 t \quad . \quad . \quad . \ (22)$$

One solution is given by equating the left-hand side to zero. This is from (11) of the form

$$i_2 = A\epsilon^{-a_2 t}\sin\omega_2 t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \ (23)$$

in which $\sin\omega_2 t$ can be written as the imaginary part of $\epsilon^{j\omega_2 t}$, giving

$$i_2 = A\epsilon^{(-a_2 + j\omega_2)t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \ (24)$$

The simplest and most important case is that in which the circuits are tuned so that $\omega_1 = \omega_2 = \omega = 1/L_1C_1 = 1/L_2C_2$ very nearly. Then another solution can be obtained by the substitution

$$i_2 = B\epsilon^{(-a_2 + j\omega)t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \ (25)$$

which gives

$$L_2B(-a_1+j\omega)^2 + R_2B(-a_1+j\omega) + \frac{B}{C_2} = \frac{\omega MV}{L_1}$$

which is very nearly

$$L_2B(-2j\omega a_1 - \omega^2) + j\omega R_2B + \frac{B}{C_2} = \frac{\omega MV}{L_1}$$

and since $L_2\omega^2 = 1/C_2$,

$$B = \frac{\omega MV}{L_1(-2j\omega L_2a_1 + j\omega R_2)}$$

$$= \frac{jMV}{2L_1L_2(a_1-a_2)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

and (25) becomes

$$i_2 = \frac{jMV}{2L_1L_2(a_1-a_2)} \cdot \epsilon^{(-a_1+j\omega)t} \quad . \quad . \quad . \quad . \quad (27)$$

The complete solution of (22) is found as the imaginary part of the sum of the solutions (24) and (27), *i.e.*,

$$i_2 = A\epsilon^{(-a_1+j\omega)t} + \frac{jMV}{2L_1L_2(a_1-a_2)}\epsilon^{(-a_1+j\omega)t}$$

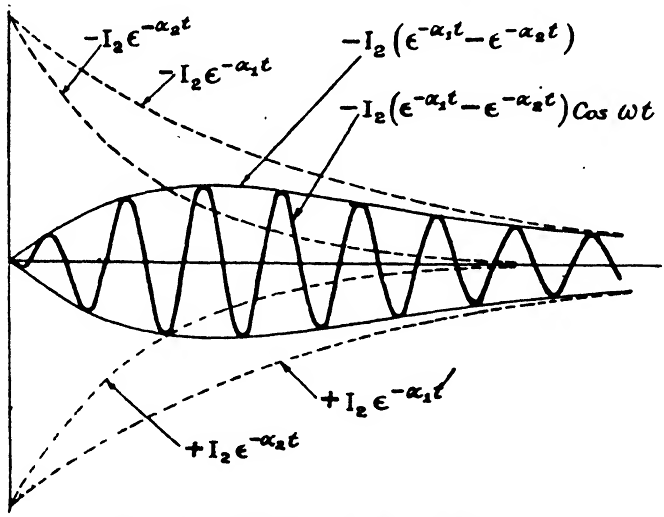


FIG. 231.—Secondary Current—loose coupling.

In order that i_2 may be zero when $t = 0$,

$$A = -\frac{jMV}{2L_1L_2(a_1-a_2)}$$

and

$$i_2 = \frac{MV}{2L_1L_2(a_1 - a_2)} \left\{ \epsilon^{-a_1 t} - \epsilon^{-a_2 t} \right\} j\epsilon^{j\omega t}$$

and the imaginary part is

$$i_2 = \frac{MV}{2L_1L_2(a_1 - a_2)} \left\{ \epsilon^{-a_1 t} - \epsilon^{-a_2 t} \right\} \cos \omega t \quad . \quad . \ (28)$$

The wave form of the current is shown in Fig. 231. The envelope is given by $\mp I_2 \left\{ \epsilon^{-a_1 t} - \epsilon^{-a_2 t} \right\}$, where $I_2 = MV/2L_1L_2(a_1 - a_2)$, and the method of constructing the curve is clear from the figure.

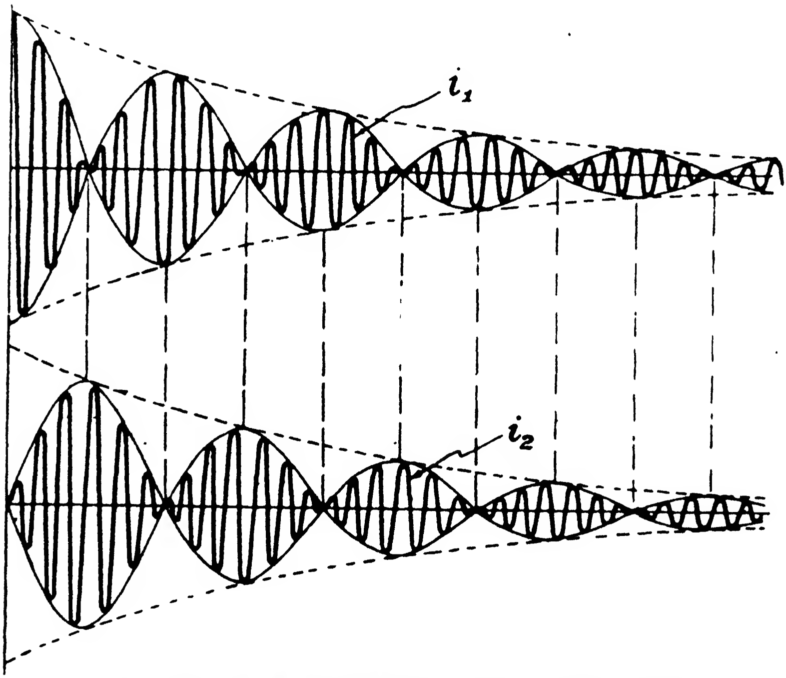


FIG. 232.—Primary and Secondary Currents—tight coupling.

When the decay factors of the two circuits are the same $a_1 = a_2$ and the envelope cannot be evaluated directly from (28). In this case (see Appendix 10) the equation becomes

$$i_2 = -\frac{MV}{2L_1L_2} t\epsilon^{-at} \cos \omega t \quad . \quad . \quad . \quad . \quad . \ (29)$$

If the coupling between the two circuits is large enough to make the effect of the secondary currents on the primary circuit appreciable, the solution of equation (18) is far more difficult and the whole phenomenon far more complicated.

The currents which flow are now of two frequencies, given very nearly by equation 60.24, and these currents by their interference with each other produce wave forms of the type shown in Fig. 232. The available energy surges backwards and forwards between the two circuits.

(iii.) The transients occurring when the electromotive force applied to the circuit is alternating involve in general far more complicated mathematics. Currents of two frequencies flow initially in the circuit, one frequency that of the applied electromotive force and the other the natural frequency of the circuit, and these currents by their superposition produce maxima and minima in the envelope of the current curve. Finally, of course, the natural frequency currents die away and the currents are determined by the expressions of section (31).

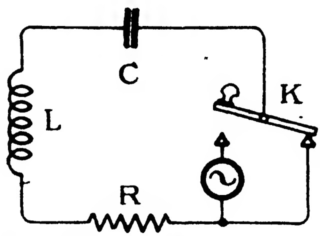


FIG. 233.—Transients on Application of Alternating Voltage.

The equation for the voltages in the circuit of Fig. 233, when the key is depressed, if the alternating e.m.f. is $V \sin \omega t$, is :—

$$Ri + L\frac{di}{dt} + \frac{1}{C}\int idt = V \sin \omega t \quad . \quad . \quad . \quad . \quad (30)$$

The case of most interest is that in which the circuit is adjusted so that its natural frequency is the same as the frequency of the applied e.m.f., *i.e.*, equation (10) applies, or very nearly $\omega = 1/\sqrt{LC}$. In that case the impedance of the circuit is very nearly R, and writing the solution of (30) as the sum of the steady state and transient solutions (from (31·11), and (5) and (6)) gives

$$i = \frac{V}{R}\sin \omega t + \epsilon^{-\alpha t}(A_1\epsilon^{j\omega t} + A_2\epsilon^{-j\omega t}) \quad . \quad . \quad . \quad . \quad (31)$$

If the switch is closed when $t = 0$, $i = 0$ when $t = 0$, which necessitates $A_2 = - A_1$ and (31) becomes

$$i = \frac{V}{R}\sin \omega t + \epsilon^{-\alpha t}\, 2jA_1 \sin \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

If the switch is closed at time $t = t_1$ (which allows the voltage to have any value $V \sin \omega t_1$, at the instant of closing the switch) $i = 0$ when $t = t_1$, and (32) gives

$$0 = \frac{V}{R} \sin \omega t_1 + \epsilon^{-\alpha t_1} . 2jA_1 \sin \omega t_1$$

whence

$$2jA_1 = -\frac{V}{R} . \epsilon^{\alpha t_1}$$

and the solution becomes

$$i = \frac{V}{R} \{1 - \epsilon^{-\alpha(t-t_1)}\} \sin \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

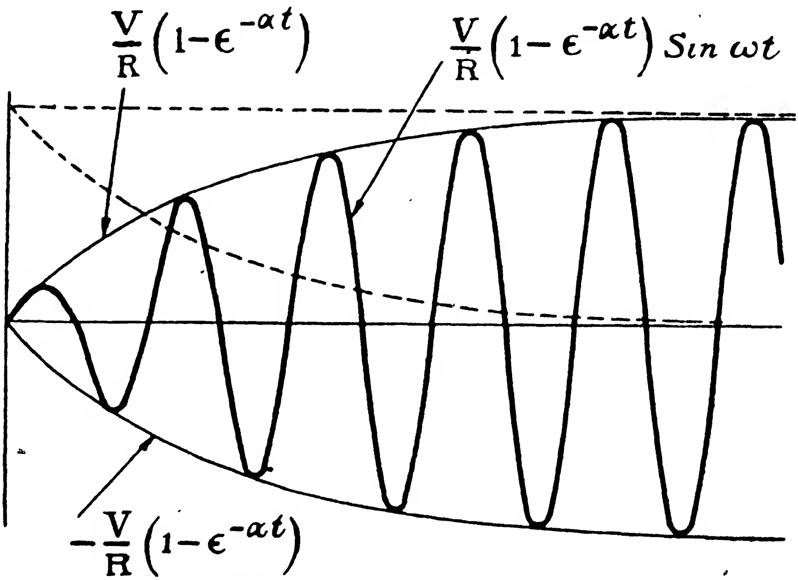


FIG. 234.—Transient in oscillatory circuit.

This curve is plotted in Fig. 234 for the case of $t_1 = 0$, that is when the switch is closed at the instant when the voltage is zero. The curve lies between the envelope formed by the curves $\pm \frac{V}{R}(1 - \epsilon^{-\alpha t})$ and the envelope is of the same shape whatever the value of t_1.

On releasing the key the current dies away in a similar manner, the equation of the envelope being

$$i = \pm \frac{V}{R} \epsilon^{-at} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34)$$

Equations (33) and (34) should be compared with equations (3) and (4) of section 13, and Fig. 234 with Fig. 38. The similarity of shape of the envelope of Fig. 234 with the curves of Fig. 38 is clear, and since (as is shown in section 68) the rectified or actual signal current that is available for working telegraph instruments depends upon the envelope of the curve, the effect of the transients on the speed of signalling in wireless telegraphy depends upon quite similar considerations as in the direct current case.

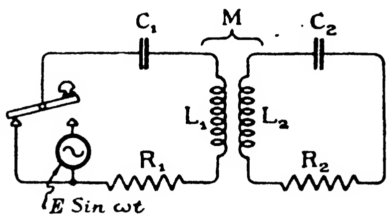


Fig. 235.—A.C. Transients in coupled circuits.

(iv.) If the circuit of Fig. 233 is loosely coupled with a second tuned circuit as in Fig. 235, the electromotive force induced in the secondary is

$$M \frac{di_1}{dt} = \frac{\omega ME}{R_1} \left\{1 - \epsilon^{-a_1 t}\right\} \cos \omega t \quad . \quad . \quad . \quad . \quad . \quad (35)$$

on neglecting a in comparison with ω and taking $t_1 = 0$, which simplifies the mathematics somewhat, but does not affect the shape of the envelope.

Writing
$$\frac{\omega ME}{R_1} = V \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (36)$$

the electromotive force in the secondary is

$$V \cos \omega t - V\epsilon^{-a_1 t} \cos \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

The first term produces a current

$$\frac{V}{R_2} \left\{1 - \epsilon^{-a_2 t}\right\} \cos \omega t. \quad . \quad . \quad . \quad . \quad . \quad (38)$$

by (33), and the second term a current

$$\frac{V}{2L_2(a_1 - a_2)} \left\{\epsilon^{-a_2 t} - \epsilon^{-a_1 t}\right\} \cos \omega t \quad . \quad . \quad . \quad . \quad . \quad (39)$$

by (28), noting that in the case of (28) the applied voltage is by (21)

$$\left(-\frac{MV}{L_1} \epsilon^{-a_1 t} \cos \omega_1 t\right).$$

The secondary current is thus the sum of (38) and (39), *i.e.*,

$$i_2 = V\left\{\frac{1}{R_2} + \frac{1}{2L_2(a_1 - a_2)}\epsilon^{-a_1 t} - \frac{a_1}{R_2(a_1 - a_2)}\epsilon^{-a_2 t}\right\}\cos\omega t \quad . \quad (40)$$

The form of the secondary current is as shown in Fig. 236, which also indicates how the envelope is found from equation (40). The transient is of longer duration than in the case of the single circuit

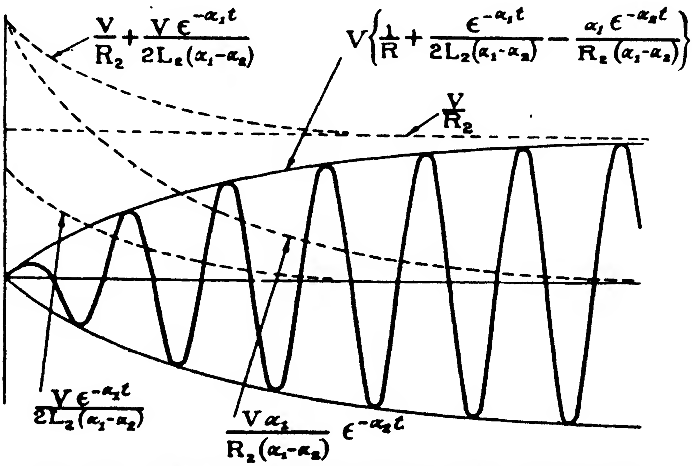


Fig. 236.—Transient in Secondary of two tuned coupled circuits.

(Figs. 233 and 234), but the general shape of the envelope is similar, and also the effect on the speed of signalling.

REFERENCES FOR FURTHER READING.

S. Butterworth.—" H.F. Resistance of Coils." *Phil. Trans.*, Vol. CCXXIIA., p. 57.

C. L. Fortescue.—" H.F. Resistance of Coils." *J.I.E.E.*, Vol. LXI., p. 933.

P. P. Eckersley, T. L. Eckersley and H. L. Kirke.—" The Design of Transmitting Aerials for Broadcasting Stations." *J.I.E.E.*, Vol. LXVII., p. 507.

E. Mallett.—" A Vector Loci Method of Treating Coupled Circuits." *Proc. Royal Society A.*, Vol. CXVII., p. 331.

G. W. Pierce.—" Electric Oscillations and Electric Waves."

CHAPTER XIII

PRODUCTION OF TRANSMITTING CURRENTS

(62) **Valve Methods.**

From what has been written in the previous chapters, it is clear that communication by means of electromagnetic waves, which are not guided by wires, involves at the transmitting station the production in resonant aerials of large currents of high frequency, their keying in the Morse or other codes in the case of telegraphy, and their modulation in accordance with the speech wave in the case of telephony.

High frequency currents may be produced by spark, by valve, by alternator and by arc. Of these the spark method produces a series of trains of damped oscillations (one train is drawn in Fig. 231), but the other methods can produce continuous oscillations of constant amplitude. The spark method will probably remain owing to the simplicity of the apparatus required, but the valve is gradually replacing the alternator and the arc. One of the greatest difficulties in the design and construction of valves to deal with large powers is to arrange for the dissipation of the heat generated at the anode. The early glass containers were poor conductors of heat, and silica instead of glass gave a much higher power for the same size of valve. But the greatest advance has been made by arranging that the anode, in the form of a copper cylinder, itself forms part of the containing vessel, the remainder being of glass sealed to the copper cylinder in a special manner. The copper anode can be water or oil cooled, and the power of the valve increased very greatly.

One way of finding the conditions necessary for the production of high frequency currents by means of a triode valve has already been given in section 48. The more usual analysis makes the same assumptions, that is, that the straight part of the valve characteristics is used and that no grid current flows. In Fig. 237 let i_1 i_2 and i_a be the instantaneous currents (alternating components) through the inductance L of resistance R, the capacity C and the valve in order.

Then

$$i_a = i_1 + i_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and by 45·08

$$R_a i_a = \mu v_g + v_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and

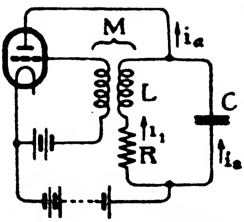


FIG. 237.—Valve Oscillator.

$$v_a = -Ri_1 - L\frac{di_1}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$v_g = M\frac{di_1}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$i_2 = -C\frac{dv_a}{dt}$$

$$= CR\frac{di_1}{dt} + CL\frac{d^2i_1}{dt^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Substitution of (1), (3), (4) and (5) in (2) gives

$$R_a\left(i_1 + CR\frac{di_1}{dt} + CL\frac{d^2i_1}{dt^2}\right) = \mu M\frac{di_1}{dt} - Ri_1 - L\frac{di_1}{dt}$$

i.e.,

$$L\frac{d^2i_1}{dt^2} + \left\{R + \frac{1}{CR_a}\left(L - \mu M\right)\right\}\frac{di_1}{dt} + \frac{1}{C}\cdot\left(1 + \frac{R}{R_a}\right)i_1 = 0 \quad . \quad (6)$$

Comparing this equation with the corresponding equation (61·02) for the current in a simple oscillatory circuit, it is seen that the effective resistance has been reduced by $(\mu M - L)/CR_a$, and the effective capacity reduced by CR/R_a very nearly. The solution of (6) is by analogy with 61.11.

$$i_1 = \frac{V}{\omega L}\epsilon^{-at}\sin\omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where

$$a = \frac{R}{2L} + \frac{1}{2LCR_a}(L - \mu M)$$

$$= \frac{R}{2L} + \frac{1}{2CR_a} - \frac{\mu M}{2LCR_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and

$$\omega = \sqrt{\frac{1}{LC}\left(1 + \frac{R}{R_a}\right)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

very nearly.

The current would be given by (7) if the condenser C were charged initially to a voltage V, and if a were positive the current wave would be of the form shown in Fig. 229. If at any instant a were made zero, the current would persist indefinitely with the amplitude it had at that instant, while if a were negative the current amplitude would rise in value indefinitely. Actually in the valve circuit the switching on of the anode battery or of the filament battery causes a small initial oscillation, and initially a is made negative by choosing a suitable value for the mutual inductance M. The current amplitude therefore increases in value with time, but as the alternating voltages in the valve carry the working point on the characteristic curves on to the curved portions, the mean internal resistance R_a of the valve is increased, and eventually a stable condition is reached in which a is zero. The rise of current to the final study amplitude is somewhat similar to that of Fig. 234, and keying to start and stop the oscillations will involve similar transients.

The condition for maintenance of the oscillations is thus $a = 0$, or from (8)

$$M = \frac{1}{\mu}(R_a RC + L) \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

in agreement with equation 48.07, while (9) for the frequency of the oscillations agrees with 48.08.

In drawing an approximate vector diagram of the oscillations (Fig. 179), the resistance of the oscillatory circuit was neglected, but, as was mentioned, the alteration of the angles on this account is only very small in all practical cases. The effect of the resistance will, however, be included in finding the path of the working point on the valve characteristic curves.

Writing $\qquad i_1 = I_1 \sin \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$

from (3) . $\qquad v_a = - RI_1 \sin \omega t - \omega LI_1 \cos \omega t \quad . \quad . \quad (12)$

from (4) $\qquad v_g = \omega MI_1 \cos \omega t \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$

and from (2) and (12)

$$i_a = \frac{1}{R_a}(\mu v_g - RI_1 \sin \omega t - \omega LI_1 \cos \omega t) \quad . \quad . \quad . \quad . \quad (14)$$

From (13) $\qquad \cos \omega t = \dfrac{v_g}{\omega MI_1}$

whence $\qquad \sin \omega t = \sqrt{1 - \dfrac{v_g^2}{\omega^2 M^2 I_1^2}} \quad . \quad . \quad . \quad . \quad . \quad (15)$

Using (15), time may be eliminated from (14), giving

$$i_a = \frac{1}{R_a}\left(\mu v_g - RI_1\sqrt{1 - \frac{v_g^2}{\omega^2 M^2 I_1^2}} - \omega L I_1 \frac{v_g}{\omega M I_1}\right)$$

$$= -\frac{L - \mu M}{R_a M} v_g - \frac{R}{R_a} I_1 \sqrt{1 - \frac{v_g^2}{\omega^2 M^2 I_1^2}}$$

whence

$$\left(i_a + \frac{L - \mu M}{R_a M} v_g\right)^2 = \left(\frac{R}{R_a}\right)^2 I_1^2 - \frac{R^2 v_g^2}{\omega^2 M^2 R_a^2}$$

or

$$\frac{v_g^2}{\omega^2 M^2} + \frac{R_a^2}{R^2}\left(i_a + \frac{L - \mu M}{R_a M} v_g\right)^2 = I_1^2 \quad . \quad . \quad . \quad (16)$$

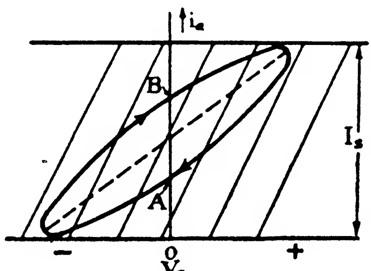


FIG. 238.—Oscillation Ellipse.

This is the equation of an ellipse, drawn on the ideal i_a/v_g characteristics in Fig. 238. The points A and B of intersection with the i_a axis are given, putting $v_g = 0$ in (16), by

$$i_a = \pm \frac{R}{R_a} I_1 \quad . \quad . \quad . \quad . \quad (17)$$

If R is put equal to zero, (16) gives

$$i_a + \frac{L - \mu M}{R_a M} v_g = 0.$$

This is the equation of the dotted straight line (the major axis of the ellipse). The slope of the line is

$$\frac{di_a}{dv_g} = \frac{1}{R_a}\left(\mu - \frac{L}{M}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and is necessarily less than the slope μ/R_a of the characteristics.

The ellipse is described in a clockwise direction, as is clear from (13) and (14). If for instance $\omega t = \pi/2$, $\cos \omega t = 0$, $\sin \omega t = 1$, $v_g = 0$ and $i_a = - RI_1/R_a$, giving the point A. At a later time, with ωt between $\pi/2$ and $3\pi/2$, $\cos \omega t$ and therefore v_g are negative. When $\omega t = 3\pi/2$, $\cos \omega t = 0$, $\sin \omega t = -1$, $v_g = 0$ and $i_a = + RI_1/R_a$, giving the point B.

In section 48 it was shown that maximum power output was obtained, with the symmetrical adjustment considered, when the ratio of L to RC was made equal to R_a, and it was mentioned that an equivalent adjustment was by means of the anode tap arrangement. This is shown in Fig. 239. The anode is connected to an intermediate point T on the inductance AB. Let the voltage across AT be b times the voltage across AB. Then neglecting resistances in comparison with reactances, and assuming that at resonance $i_1 = -i_2$, the effective impedance of the part of the coil AT is $j\omega b\mathrm{L}$, and of the circuit TBCA is $\left\{ j(1-b)\,\omega\mathrm{L} - j\dfrac{1}{\omega\mathrm{C}} \right\}$.*

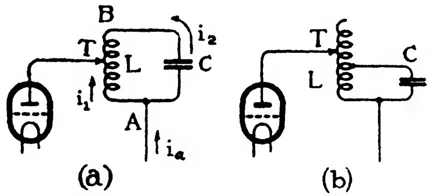


FIG. 239.—Anode Tap.

Hence the parallel impedance between A and T is

$$Z_p = \frac{(j\omega b\mathrm{L})\left\{ j(1-b)\,\omega\mathrm{L} - j\dfrac{1}{\omega\mathrm{C}} \right\}}{Z}$$

where Z is the series impedance of the whole circuit.

Hence at resonance

$$Z_p = \frac{b^2\mathrm{L}}{\mathrm{CR}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

and this must be made equal to R_a for maximum power output, *i.e.*,

$$b = \sqrt{\frac{\mathrm{CRR}_a}{\mathrm{L}}}.$$

If b is greater than unity, the coil should be extended as indicated at (b) Fig. 239.

With the anode tap arrangement at resonance (from 46.05)

$$I_a = \frac{\mu V_g}{R_a + \dfrac{b^2\mathrm{L}}{\mathrm{CR}}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

and approximately

$$I_1 = -\frac{j\omega b\mathrm{L}}{\mathrm{R}} I_a$$

* See Appendix 11 for a more detailed treatment.

and
$$\begin{aligned} V_g &= j\omega M I_1 \\ &= \frac{b\omega^2 ML}{R} I_a \\ &= \frac{bM}{RC} I_a \quad \ldots \ldots \ldots \quad (21) \end{aligned}$$

For oscillations to persist, (20) and (21) must hold simultaneously, hence

$$\frac{\mu}{R_a + \dfrac{b^2 L}{CR}} = \frac{RC}{bM}$$

or
$$M = \frac{1}{\mu}\left\{\frac{R_a RC}{b} + bL\right\} \quad \ldots \ldots \ldots \quad (22)$$

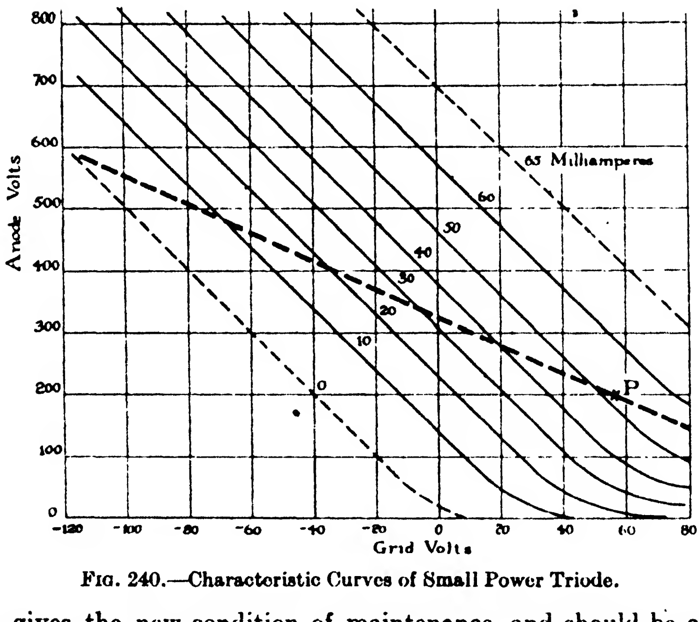


Fig. 240.—Characteristic Curves of Small Power Triode.

(22) gives the new condition of maintenance, and should be compared with (10).

Without the anode tap, maximum power is obtained when $L/RC = R_a$, which makes the two terms on the right-hand side of (10) each equal to L, and $M = \dfrac{2L}{\mu}$.

With the anode tap, condition (20) makes the two terms on the right-hand side of (22) each equal to bL, and $M = \frac{2bL}{\mu} = \frac{2}{\mu}\sqrt{R_aRCL}$.

This symmetrical arrangement of the working point on the valve characteristics gives, as has been seen, the maximum power output and the purest possible sine wave, but the efficiency (neglecting the power taken to heat the filament) is only 50 per cent. While this is of little consequence in the case of the small powers required for line telegraphy and telephony and for measurement purposes, it becomes quite a serious matter when the generation of the large powers required in wireless transmitting stations is concerned.

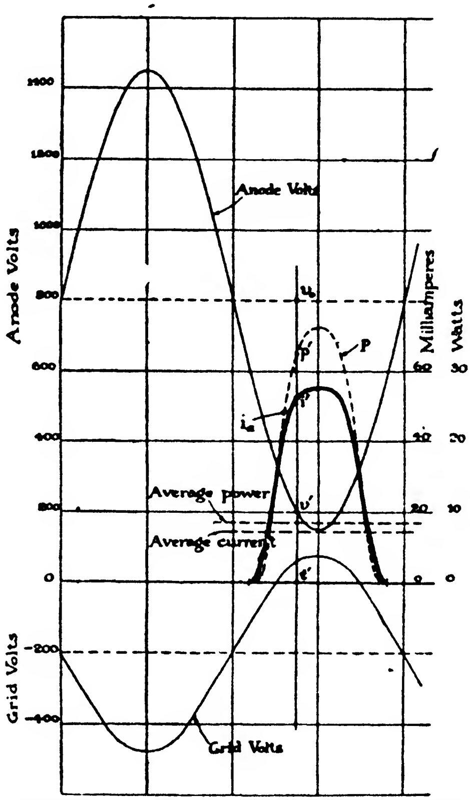


FIG. 241.—Potential and Current Fluctuations in Valve Oscillator.

By making the grid more negative, the mean anode current, and therefore the power wasted in the valve, is reduced, and while the power output to the oscillatory circuit is also reduced, the reduction is not as great proportionally, so that the efficiency may be greater than 50 per cent., although the total output is less than the maximum possible. An example will make this clear. Fig. 240 gives characteristic V_a/V_g curves (corresponding to the dotted lines of Fig. 164) for a small power valve. To the left the curves are straight lines which may be extended indefinitely, and the slope of which is $-\mu$. The curves are drawn for different values of I_a extending from 0 to the saturation value I_s (65 mA), and are contained within the dotted lines for these extreme cases. For points in the V_a/V_g plane to the right of the band of curves the current is I_s, while for points to the left the current is zero.

Suppose that the valve is to be used to obtain high-frequency oscillations by means of the circuit of Fig. 237, that the available direct current supply for the anode is at 800 volts, and that the grid bias is $-$ 200 volts. Suppose further, that the circuit is arranged so that the minimum anode voltage is 150 volts, and the maximum positive grid voltage is 80 volts, and assume that the grid and anode voltage fluctuations are sine shaped and exactly opposite in phase. Then the instantaneous anode and grid voltages can be drawn as in Fig. 241, and the instantaneous anode current i_a can be drawn by reference to Fig. 240. For instance, at the time t', the anode voltage is 200 and the grid voltage is 58, which gives the point P in Fig. 240, from which the anode current is read off as 52 mA. The points P thus found lie on the straight dotted line which is called the oscillation line, and corresponds to the dotted line of Fig. 238. $t'i'$ in Fig. 241 is made equal to 52 and in this way the complete i_a curve is drawn. It is seen that anode current flows during less than half the cycle. By integrating the curve and dividing by the time of the cycle, the mean anode current is found to be 14·4 mA, whence the average power supplied by the anode battery is $800 \times 14{\cdot}4 \times 10^{-3} = 11{\cdot}5$ watts.

At the instant t', the anode voltage is $t'v'$ and the supply voltage is $t'v_o$. The voltage $v'v_o$ (= 600 volts), therefore represents the voltage dropped in the oscillatory circuit, and since the current is $t'i' = 52$ mA, the instantaneous power in the oscillatory circuit is $600 \times 52 \times 10^{-3} = 31{\cdot}2$ watts. This is plotted as $t'p'$, and in the same way other points on the curve of instantaneous power p in the oscillatory circuit are found, and from the curve the average power is found to be 8·5 watts.

Thus of 11·5 watts supplied to the oscillator, 8·5 watts are absorbed in the oscillatory circuit; the "anode efficiency" is thus 8·5/11·5, or 74 per cent. The remaining 3·0 watts appear as heat at the anode.

Suppose further that the oscillations are required for a radiation of 4,000 metres wavelength, and that the resistance R of the oscillatory circuit (Fig. 237) may be taken as 15 ohms. The current in the oscillatory circuit is found from $I^2R = W$ to be $I = \sqrt{8{\cdot}5/15} = 0{\cdot}75$ amps. rms. The amplitude of the alternating component of the anode voltage is 650 volts, and this must be very nearly ωLI, where I is the amplitude of the oscillatory current $= \sqrt{2} \times 0{\cdot}75 = 1{\cdot}06$ ampere

$$\omega = 2\pi \frac{c}{\lambda} = 2\pi \times \frac{3 \times 10^8}{4{,}000} = 0{\cdot}47 \times 10^6.$$

Hence $$L = \frac{650}{0{\cdot}47 \times 10^6 \times 1{\cdot}06} = 1{,}300 \text{ microhenries.}$$

Similarly the amplitude of the grid swing = 280 volts, and

$$M = \frac{280}{\omega I} = \frac{280}{0{\cdot}47 \times 10^6 \times 1{\cdot}06} = 560 \text{ microhenries.}$$

Finally C is given by

$$C = \frac{1}{\omega^2 L} = \frac{1}{0{\cdot}47^2 \times 10^{12} \times 1300 \times 10^{-6}} = 3500\ \mu\mu\text{F}.$$

and the design of the circuit is complete.

If the adjustment of the circuit were for maximum power (anode efficiency 50 per cent.), the grid bias necessary to give a mean anode current of half the saturation value (*i.e.*, 32·5 mA) with the same anode battery would be — 95 volts, the total power supplied would be $32{\cdot}5 \times 10^{-3} \times 800 = 26$ watts, and the power in the oscillatory circuit would be 13 watts.

The anode efficiency could in this way be increased indefinitely, but the power output would become smaller and smaller, and the power used in heating the filament of more and more importance in determining the overall efficiency. It is usual therefore to work with an anode efficiency of about 75 per cent.

In the example above, if the grid bias were — 200 when the anode voltage is first switched on, no anode current would flow, and the oscillations could not commence. The bias must be much less (conveniently zero) in order to start the oscillations, and be gradually reduced to — 200 when the oscillations have commenced.

The large negative bias required for the grid may be provided by a direct current generator in a large transmitting station, or it may be provided automatically by the grid current which flows during the part of the cycle which makes the grid potential positive. The necessary arrangement is shown in Fig. 242 as a high resistance R_g shunted by a condenser, in series with the grid coil between filament and coil. Electrons flow from filament to grid within the valve and from grid through grid coil and shunted resistance in the external circuit, resulting in a lowering of the mean grid potential with regard to the filament. Before oscillations commence, therefore, the mean grid potential is zero, but as oscillations build up the mean grid potential gradually becomes more and more negative until a stable value depending upon the resistance R_g and the grid current characteristic of the valve is reached.

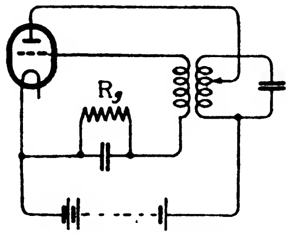


FIG. 242.—Arrangement for giving Negative Grid Bias.

The direct current high voltage supply required for large power valves is several thousand volts, and is obtained by valve rectification from a low-frequency power supply. The arrangement for a single phase supply is drawn in Fig. 243. The valves are two electrode valves (diodes), and the filaments are heated through a transformer T_2 from the same low-frequency supply S that provides the high-tension voltage to the valves V_1 and V_2 through the transformer T_1. C_1, C_2 and L_1 L_2 are condensers and iron-cored chokes for smoothing out the ripple on the rectified current from the valves. The action will be understood by reference to Fig. 163,

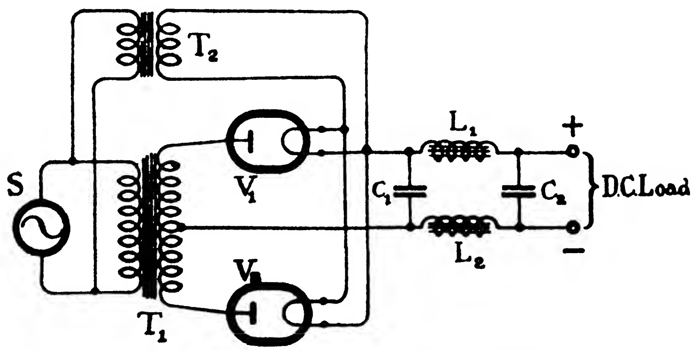


FIG. 243.—Anode Supply from A.C. Source.

which can be taken to show the current through one valve during one set of half cycles. During the alternate half cycles similar

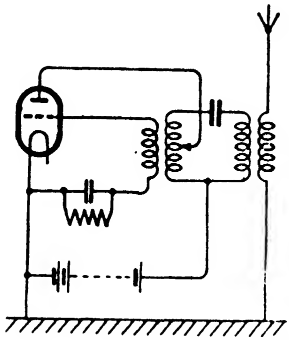


FIG. 244.—Coupling to Aerial.

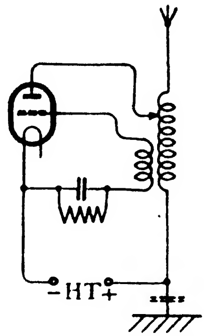


FIG. 245.—Aerial as Oscillatory Circuit.

currents flow through the other valve in the same direction through the load. Several diodes may be connected in parallel to obtain the necessary power.

A three-phase supply may also be used with double valve rectification in each phase. The problem of eliminating the ripple is then considerably simplified.

The valve oscillator may be coupled to the aerial as in Fig. 244, or the aerial itself may be used as the oscillatory circuit as in Fig. 245, and there is only one oscillatory circuit and one mutual inductance to adjust instead of two, and power is not wasted in an intermediate oscillatory circuit. A serious disadvantage of the arrangement of Fig. 245, however, is that the filament battery is at a potential below earth equal to the potential of the anode supply. If this is avoided by inserting a condenser (shown dotted) at the foot of the aerial and earthing the negative terminal of the filament battery, the whole of the aerial is at a high potential above earth. These disadvantages are avoided in the

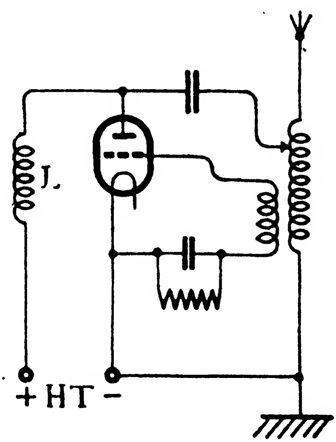


FIG. 246.—Choke Feed.

arrangement of Fig. 246. The direct current to the valve is supplied through a choke L, which effectively prevents high-frequency fluctuations passing through the supply. The condenser

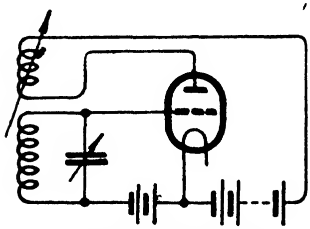


Fig. 247.—Tuned Grid Oscillator.

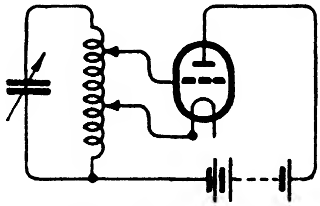


Fig. 248.—Hartley Circuit for Valve Oscillator.

C allows the high frequency currents to pass, but stops direct currents; the filament battery and also the foot of the aerial are at earth potential.

There are other forms of valve circuit suitable for producing oscillations. Fig. 247 shows an arrangement called the "tuned grid" oscillator in which the oscillatory circuit is connected to the grid, while in Fig. 248 there is only one coil, the tapping points to the grid and anode being on opposite sides of the tapping point to the filament. This is known as the "Hartley Circuit."

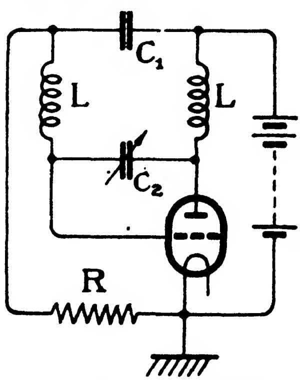


Fig. 249.—Short Wave Oscillator.

In Fig. 249 the oscillatory circuit is connected between anode and grid. C_1 is a comparatively large condenser to prevent the anode battery sending current through the high resistance R to earth; the actual tuning capacity is C_2 in parallel with the anode grid electrode capacity. This circuit is used for generating very high frequency currents for short waves; for very short waves the capacity C_2 may be omitted altogether (the tuning capacity being the anode-grid capacity) and the coils L may be replaced by a rectangular circuit of a single turn.

In order to stabilise the frequency of the oscillations as much as possible, it is becoming more and more the practice to "separately excite " the grids of the power valves; that is, instead of the power valves being the actual oscillatory valves, the oscillations are produced initially by quite small power oscillators, and the voltages obtained amplified, usually in several stages, to give the necessary grid swing to the power valves. The initial oscillation may be produced from a quartz crystal, or even the low frequency oscillations of a tuning fork may be used, by purposely distorting the wave-form in an amplifier circuit, selecting a harmonic by means of a circuit tuned to it and amplifying again, and repeating the process until the required frequency and voltage are obtained.

The quartz crystal is of increasing importance in wireless telegraphy, both as an oscillator and as a frequency standard. Its value depends upon the piezo-electric effect in quartz crystals, upon the extreme constancy of its mechanical properties, and upon the fact that it has a far smaller decay factor than can be obtained from an electrical resonant circuit at wireless frequencies. A rectangular block is cut from a large crystal, with the long axis of the block at right angles to the optical axis of the crystal. If two electrodes are applied to opposite sides of the block, and a voltage is established between them, a force tending to extend the block will be experienced. Conversely, if the block is extended, a voltage will appear at the electrodes.

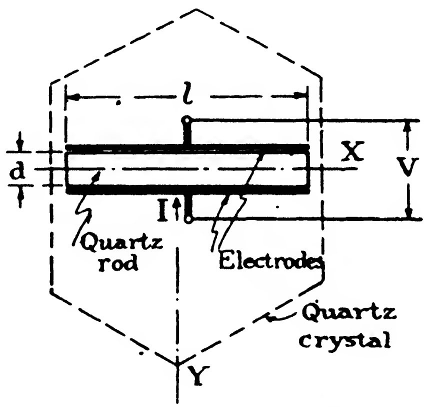


FIG. 250.—Quartz Crystal Resonator.

If the length of the block or rectangular rod is l, its depth d and breadth b, and if a voltage V is established between the electrodes, the electric field strength is V/d, the stress produced along the X axis (Fig. 250) is $\epsilon V/d$ (where ϵ is the piezo-electric constant), and the force is the stress multiplied by the area, *i.e.*,

$$f = \frac{\epsilon V}{d} \times db = \epsilon bV \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

If the voltage is alternating, the rod will vibrate longitudinally if it is free to do so, and the equation of motion of the vibrations, where x is the extension of the rod, is

$$m\frac{d^2x}{dt^2}+a\frac{dx}{dt}+sx=\epsilon b\mathrm{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where m, r and s are the equivalent mass, mechanical resistance, and stiffness of a simple system replacing the rod (compare Fig. 90 and equation 31·04). Writing $\epsilon b = \mathrm{B}$, the solution (for steady state vibrations) of (24) when the alternations of V are sinusoidal is

$$\frac{dx}{dt}=\frac{\mathrm{BV}}{z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

where z is the mechanical impedance of the equivalent system. (Compare 31·08, where U is written for dx/dt, and 34·05.)

The current through the crystal is made up of two parts, (i.) that due to the normal electric displacement D, which, if κ is the dielectric constant of quartz, is given by

$$\mathrm{D}=\frac{\kappa\mathrm{V}}{4\pi d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

and (ii.) the displacement due to the converse piezo-electric effect, which can be written

$$\mathrm{D}_p=\frac{\epsilon}{l}\,.\,x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The total current is given by the area multiplied by the rate of change of displacement, *i.e.*,

$$\begin{aligned}\mathrm{I}&=bl\frac{d}{dt}(\mathrm{D}+\mathrm{D}_p)\\&=\kappa\frac{bl}{4\pi d}\frac{d\mathrm{V}}{dt}+b\epsilon\frac{dx}{dt}\\&=\mathrm{C}_1\frac{d\mathrm{V}}{dt}+\mathrm{B}\frac{dx}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)\end{aligned}$$

where C_1 is the capacity of the crystal between the electrodes in the absence of vibrations. $\frac{d\mathrm{V}}{dt}$ can be written $j\omega\mathrm{V}$, and $\frac{dx}{dt}$ by (25) is BV/z, hence (28) becomes

$$\mathrm{I}=\left(j\omega\mathrm{C}_1+\frac{\mathrm{B}^2}{z}\right)\mathrm{V} \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

This equation should be compared with the telephone receiver (or electrically vibrated tuning fork) equation, 34·09.

$\frac{B^2}{z}$ could be called the motional admittance of the crystal. Its locus with varying ω is a circle.

There are several different circuit arrangements by which a valve can be used to maintain oscillations in a quartz crystal. One is drawn in Fig. 251. L_1, L_2 and L_3 are large inductances having

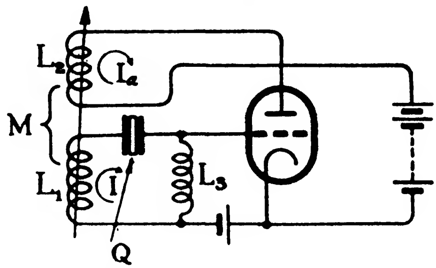


FIG. 251.—Valve Oscillator controlled by Quartz Crystal.

natural frequencies higher than that of the crystal Q. If I is the current in the grid circuit, L_1 Q L_3, and I_a the anode current, then

$$I = \frac{-j\omega M I_a}{j\omega(L_1 + L_3) + \dfrac{1}{j\omega C_1 + \dfrac{B^2}{z}}}$$

by (29). The impedance $j\omega(L_1 + L_3)$ is small compared with the crystal impedance, and hence approximately

$$I = -j\omega M\left(j\omega C_1 + \frac{B^2}{z}\right) I_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

and the grid voltage V_g is

$$\begin{aligned} V_g &= j\omega L_3 I \\ &= \omega^2 L_3 M \left(j\omega C_1 + \frac{B^2}{z}\right) I_a \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31) \end{aligned}$$

On the other hand, if the voltage $j\omega MI$ induced in the anode circuit from the grid circuit is neglected in comparison with μV_g, the anode current can be written

$$I_a = \frac{\mu V_g}{R_a + j\omega L_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

and in order that (31) and (32) may hold simultaneously,

$$\mu\omega^2 L_3 M \left(j\omega C_1 + \frac{B^2}{z}\right) = (R_a + j\omega L_2) \quad . \quad . \quad . \quad . \quad (33)$$

is obtained as the condition of maintenance corresponding to 48.13. If each side of (33) is multiplied by I_a, it is seen to state the condition that μV_g must be equal to $-V_a$.

Equation (33) is drawn vectorially in Fig. 252. At (*a*) OO_1 is made equal to $j\omega C_1 \times \mu\omega^2 L_3 M$, and the circle is drawn on the horizontal diameter $O_1R = \frac{B^2}{r} \times \mu\omega^2 L_3 M$. At (*b*), $OA = R_a$ and $AB = \omega L_2$. (33) states that OP in (*a*) must be equal and parallel with OB in (*b*), and this will occur with suitable choice of the circuit constants very nearly indeed when P lies on R; that is, the frequency of the oscillations is practically that of the quartz crystal, and this can be made (within limits) any desired high frequency simply by a choice of the length of the crystal.

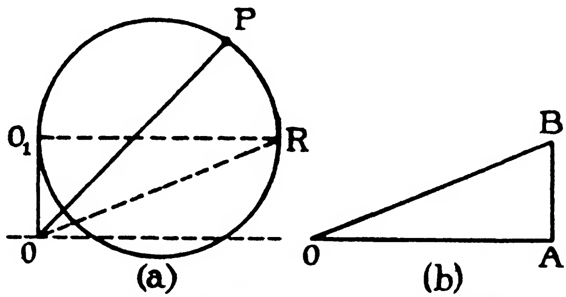


FIG. 252.—Condition of Maintenance of Quartz Crystal Oscillator.

(63) Keying and Modulation.

Keying for the purpose of signalling by the Morse code presents no difficulty in the valve circuits described in the previous section. The key or relay contact is placed in the grid circuit of the oscillating valve, or where the valve is separately excited, in the oscillator valve or one of the small power amplifier valve grid circuits, and used so that in the spacing position the grid is made so negative that no anode current flows and the oscillations, and hence the aerial currents, cease. Merely breaking the grid leak (R_g in Fig. 242) will have this effect, since then the negative electrons which reach the grid can no longer flow away to the filament, but accumulate and give the grid a large negative potential, but it is sometimes arranged that the grid is given a definite negative potential from an independent source. The power in the grid circuit that the key must interrupt is very small, and large powers in the aerial can be controlled with practically no sparking.

Modulation of the amplitude of the aerial currents for the purpose of telephony is a somewhat more difficult problem. The grid-type method of modulation described in section 49 is not generally used where large powers are involved. Instead, the valve oscillator circuits are so adjusted that the amplitude of the oscillations produced is limited by the anode supply voltage. Any increase

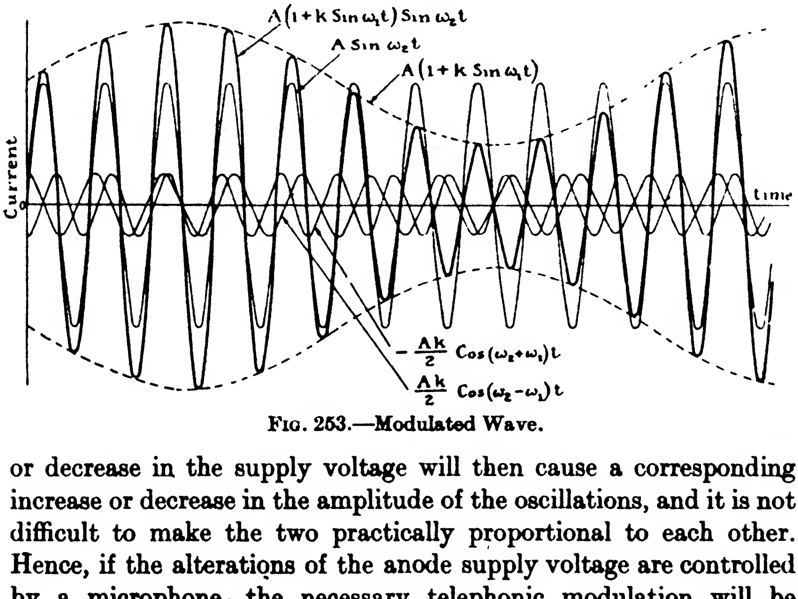


FIG. 253.—Modulated Wave.

or decrease in the supply voltage will then cause a corresponding increase or decrease in the amplitude of the oscillations, and it is not difficult to make the two practically proportional to each other. Hence, if the alterations of the anode supply voltage are controlled by a microphone, the necessary telephonic modulation will be achieved. If $\omega_1/2\pi$ is the oscillator frequency, and $\omega_2/2\pi$ one of the telephonic frequencies, then the expression for the modulated current can be written

$$I = I_0 (1 + k \sin \omega_2 t) \sin \omega_1 t \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where $I_0 \sin \omega_1 t$ is the unmodulated high frequency current, and k is a coefficient of modulation, essentially less than unity, and depending upon the ratio between the anode voltage produced by the microphone to the steady anode voltage supply.

Equation (1) can be written

$$I = I_o \sin \omega_1 t + \frac{1}{2} kI_o \cos (\omega_1 - \omega_2)t - \frac{1}{2} kI_o \cos (\omega_1 + \omega_2)t \quad . (2)$$

showing the carrier wave, and the lower and upper side frequencies, which, when the whole range of frequencies comprised in telephony are considered, become the lower and upper side bands already

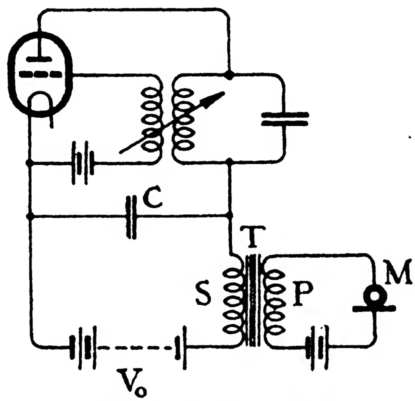


FIG. 254.—Telephone Modulation.

defined in section 49. Fig. 253 is a numerical example from (2) with k equal to 50 per cent. and ω_2 ten times ω_1.

The manner in which such a modulated current could be produced is shown by the circuit diagram of figure 254. The anode supply voltage is at any instant the sum of the battery voltage V_o and the voltage on the secondary S of the transformer T, the primary P of which contains the microphone and a battery. The high frequency variations of anode current find a path of low impedance through the condenser C; the low frequency variations of anode supply voltage cause corresponding variations of the amplitude of the high frequency currents in the oscillatory circuit.

The power that such a circuit could give would be very limited, since the microphone circuit must supply or absorb the differences in power in the oscillatory circuit and valve from the mean as determined by the anode battery V_o. Increase of power can be obtained either by high frequency amplification of the modulated currents in the oscillatory circuit, or by low frequency amplification between the microphone and the transformer T.

The latter is the more usual arrangement, and by a slight modification becomes the *anode choke* method of control illustrated in Fig. 255. Here the same anode supply V_0 delivers anode current to the oscillating valve V_1 and to the control valve V_2, in each case through the large iron-cored choke L. The current taken by the control valve depends upon the voltage on its grid, and this in turn, through the transformer T, depends upon the sound wave reaching the microphone M. The greater the current taken by V_2, the greater the voltage drop in the choke L, the smaller the voltage available for the oscillating valve V_1, and in consequence the smaller the amplitude of the high frequency currents in the oscillatory circuit O.

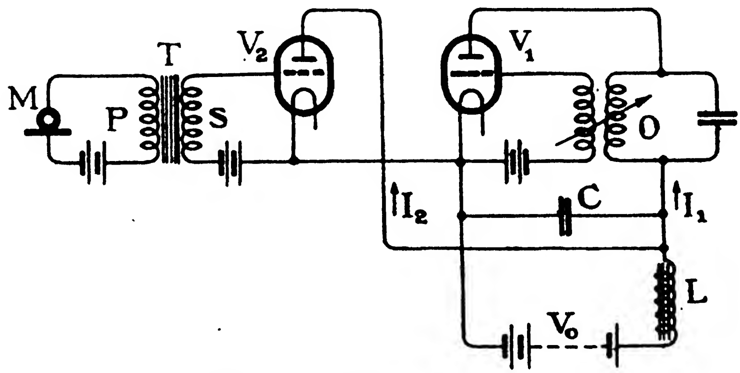


FIG. 255.—" Anode Choke " Modulation.

Let the low-frequency anode current fluctuations in the oscillating and control valves respectively be I_1 and I_2 (I_1 is the mean anode current with regard to the high frequency), so that the current fluctuation through the choke is $(I_1 + I_2)$ and the voltage fluctuation across it is $j\omega L\ (I_1 + I_2)$ on neglecting the resistance of the choke. If the anode resistances of the valves are R_1 and R_2 then

$$I_1 = -j\omega L\,(I_1 + I_2)/R_1 \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

whence

$$I_2 = -\left(1 + \frac{jR_1}{\omega L}\right) I_1 \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

Also

$$I_2 = \frac{1}{R_2}\left\{-j\omega L\,(I_1 + I_2) + \mu V_g\right\} \quad . \quad . \quad . \qquad (5)$$

where μ is the amplification factor of the control valve and V_g the voltage applied to its grid.

Adding (3) and (5) gives

$$I_1 + I_2 = -j\omega L\left(\frac{1}{R_1} + \frac{1}{R_2}\right)(I_1 + I_2) + \frac{1}{R_2}\mu V_g$$

whence

$$I_1 + I_2 = \frac{1}{R_2}\frac{\mu V_g}{1 + j\omega L\left(\frac{1}{R_1} + \frac{1}{R_2}\right)} \quad . \quad . \quad . \quad . \quad . \quad (6)$$

Multiplying each side of (6) by $j\omega L$ and dividing by V_g gives

$$\frac{j\omega L(I_1 + I_2)}{V_g} = \frac{j\omega L\mu}{R_2\left\{1 + j\omega L\left(\frac{1}{R_1} + \frac{1}{R_2}\right)\right\}} \qquad (7)$$

The left-hand side is clearly the ratio of the voltage fluctuation at the anode of the oscillating valve to the voltage applied to the grid of the control valve, and this from the right-hand side is seen to be greatest when $\omega L >> R_1$ or R_2, and is then :

$$\frac{\mu}{\frac{R_2}{R_1} + 1}$$

This again is greater the larger R_1 is compared with R_2; the limiting value is μ.

With $\omega L >> R_1$ (4) gives

$$I_2 = -I_1.$$

The total anode current supplied by the battery is constant; the current fluctuations of the two valves are of equal magnitude and of opposite phase. The power that the control valve must handle is therefore of the same order as the power given by the oscillating valve, and the valves must be of similar rating.

If the microphone in Fig. 255 is replaced by a low frequency oscillator (say, 1,000 cycles), a sinusoidally modulated wave is obtained which can be used for telegraphy. The system is known as "tonic train," and has the advantage of simpler reception than in the case of telegraphy by unmodulated wave, called "continuous wave." The sinusoidally modulated wave may be produced by a single valve circuit as shown in Fig. 256. The high frequency oscillatory circuit is L_1C_1, which is coupled to the grid by mutual inductance between L_1 and L_o. C_2 and C_3 are large condensers which offer only small impedance to the high frequency currents, while L_2 and L_3 are large inductances which offer high impedances to the high frequency currents. L_2C_2 (with L_1) is the low frequency oscillatory circuit, L_2 being coupled with L_3 to give the necessary feed back. The grid is biased so that the valve is working on the square law part of the characteristics, and modulation takes place as in Fig. 183.

The advantages of the tonic train system of telegraphy can be obtained in a simpler manner by omitting the smoothing condensers and inductances (C_1C_2 and L_1 L_2 in Fig. 243) from the rectifying circuit supplying current to the anode of the oscillating valve from the alternating current supply. The envelope of the high frequency current is then composed approximately of a succession of half sine waves, two to each cycle of the A.C. supply (see

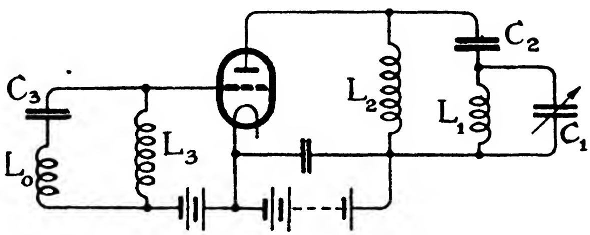


FIG. 256.—Tonic Train Circuit.

Fig. 273). If the latter is at a frequency of 500 cycles per second a note of 1,000 cycles per second is accordingly heard in reception. This system is known as interrupted continuous wave (I.C.W.)

(64) **Spark Methods.**

The spark method of obtaining the high frequency currents necessary for the production of electromagnetic waves was the method employed by Hertz in his original experiments, and by Marconi in his first application to practical telegraphy, and it is still extensively used for short-range transmission between ships and between ships and shore stations on account of the simplicity of the apparatus required.

When the key of Fig. 228 is depressed the current of Fig. 229 results, and the frequency of the alternations can be made as high as is desired by suitable choice of the inductance and capacity. When the key is released the condenser charges up again and further currents flow, similar in wave form to that of Fig. 229. If the key is continuously rocked, a series of trains of damped oscillations is produced, and if the frequency of the rocking is an audible frequency, the trains on rectification will produce an audible note in a telephone receiver. The energy stored by the condenser is $\frac{1}{2}CV^2$, and this is the energy that is converted into high frequency oscillations in each train, and high voltages are necessary in order to obtain the energy required.

A spark gap functions as an automatic key. In the circuit of Fig. 257 the oscillatory circuit is CLRG, while R_0 is a high resistance which makes the charging of the condenser C from the battery V non-oscillatory or a-periodic (*i.e.*, according to equation (61·08)). When the key K is depressed the condenser C gradually charges. When the potential across the spark gap G reaches a certain value, a spark passes owing to cumulative ionisation of the air between the balls of the gap, the resistance of the gap becomes very low, and an oscillatory discharge of the condenser round the circuit LCRG takes place. When the current falls to a sufficiently low value the ionisation is no longer maintained, the gap again becomes insulating, and the condenser is again charged from the battery until the voltage is sufficient to cause a breakdown of the insulation of the gap again, and so on. While the key is depressed trains of damped oscillations flow in the oscillatory circuit. The resistance R_0 prevents the formation of a continuous arc across the gap.

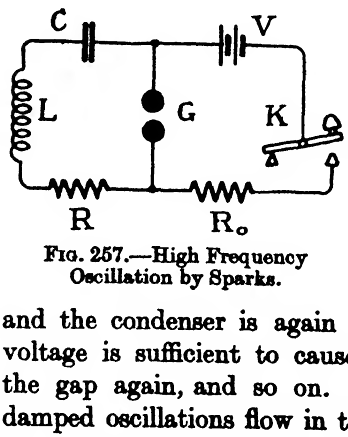


FIG. 257.—High Frequency Oscillation by Sparks.

The voltage required (say, 10,000 volts) is provided in practice either by a Rhumkoff coil, for small power sets, or by an alternator and step-up transformer for larger power sets. In the first case the spark gap is set so that its breakdown potential is just below that of the potential given at the secondary of the coil at break, and there is a spark and consequent train of oscillations at each vibration of the contact breaker. The note heard at the receiver therefore has the frequency of the contact breaker. The adjustment is similar in the second case if the alternator frequency is from 300 to 500 cycles per second. A spark passes just before each voltage maximum (positive or negative), and the frequency of the note heard is twice the frequency of the alternator. If, however, the alternator frequency is 50 cycles per second, the spark gap is set to break down at a voltage considerably less than the maximum, and several sparks pass during each half cycle. The spark frequency is now not nearly so definite and the sound heard has more the character of a buzz than of a musical note.

An outline diagram of connections of a spark transmitting set is given in Fig. 258. The high voltage is supplied by the alternator A through the transformer T. Keying involves interrupting the whole power supply, and specially designed keys are necessary. The low frequency choke L_0 is to give some measure of tuning to the alternator frequency on the low voltage side of the transformer when the spark gap is " open " ; when the spark gap is " closed " the circuit

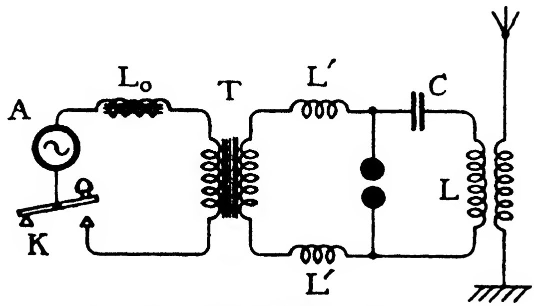


Fig. 258.—Spark Transmitting Circuit.

is no longer tuned, smaller currents flow, and the danger of the formation of a continuous arc is minimised. The high frequency chokes L′L′ are for the purpose of protecting the secondary winding of the transformer against high frequency currents set up by reaction from the aerial when the gap is " open." Such currents could flow through the self capacity of the secondary winding and produce high voltages which might damage the insulation of the winding.

Considerable heat is produced at the spark gap, and cooling of the electrodes is a problem of some importance. Cooling vanes are generally provided, and sometimes an air blast. High-power sets are usually arranged with one electrode rotating, a disc carrying the same number of electrodes as there are poles on the alternator (or rotary converter) being mounted on the alternator shaft. Two fixed electrodes diametrically opposite each other are so arranged that rotating electrodes come opposite them at the instants of maximum voltage, and a spark passes through the two gaps in series. In this system, the "synchronous spark," the cooling is provided by the rotation, and there is no danger of continuous arcing.

Another problem arises in coupling the oscillatory circuit to an aerial. The arrangement is now theoretically that of Fig. 230. If the coupling is loose, the rise of oscillating current in the aerial is slow, as in Fig. 231, and most of the available energy is dissipated in the oscillatory circuit instead of in the aerial. If the coupling is tight the energy surges backwards and forwards as in Fig. 232, more of the available energy is dissipated in the aerial, but the radiation is of two wavelengths.

In the "quenched spark" system the gap becomes non-conducting the first time the envelope of the high frequency current becomes zero, and the resulting currents are accordingly as shown in Fig. 259. The advantages of tight coupling are obtained without the corresponding disadvantages. The quenching of the spark in this manner is achieved by the special form of gap illustrated in Fig. 260. The gap is actually a large number in series of small gaps 0·2 mm. in length of extensive surface, and the electrodes are solid copper discs (sometimes silver plated) with large copper cooling fins F. The annular grooves G are to prevent the spark passing across the mica surface. Cooling and therefore de-ionisation is thus very rapid,

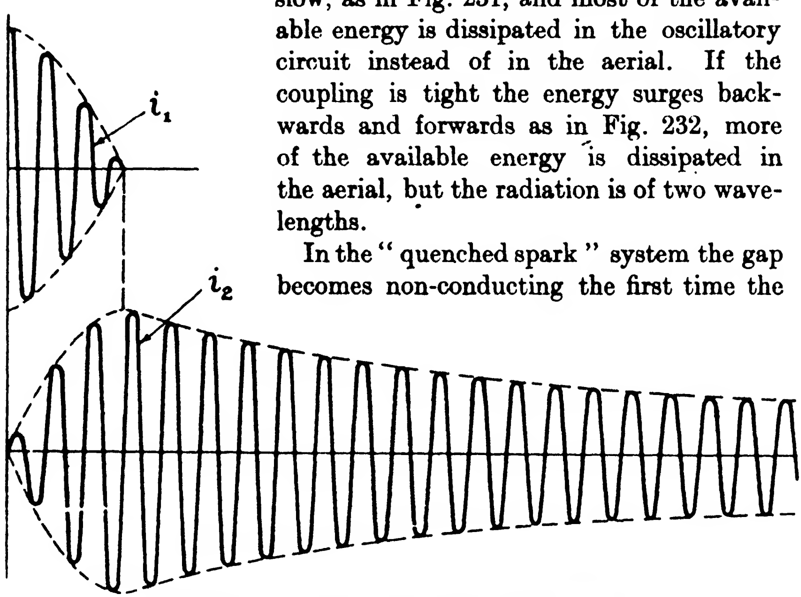


FIG. 259.—Currents in Quenched Spark System.

and the effect is enhanced by the fact that the air between the electrodes is enclosed, since sudden temperature rises increase the air pressure, and with it the sparking potential.

The quenched spark system should not be confused with the synchronous spark. The duration of a single train of high frequency currents is of the order 1/10,000th of a second, while the number of trains per second may be 1,000. Thus the time that the electrodes in the synchronous spark arrangement are opposite each other is comparatively so long as to produce no quenching. The chief function of the rotating electrodes is to prevent arcing, and the

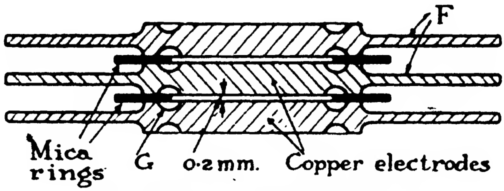


FIG. 260.—Quenched Spark Gap.

quenched gap arrangement of Fig. 260 may be used in series with the rotating gap to obtain the advantages of both systems.

(65) **Other Methods.**

The discovery that a direct current arc between carbon electrodes could be used to produce alternating currents was made by W. Duddell in 1900. A series condenser C and inductance L of total resistance R is shunted across the arc as shown in Fig. 261, and the arc is fed from a direct current source of voltage V through a choke L_0 and resistance R_0 which maintain the total current supply I sensibly constant. If the instantaneous current into the arc is i_a and that into the shunting oscillatory circuit is i_c, and the voltage across the arc is v_a, then

$$i_a + i_c = I \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and, by adding all the voltages round the circuit ARLC,

$$L\frac{di_c}{dt} + Ri_c + \frac{1}{C}\int i_c dt = v_a \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Now it is characteristic of an arc that as the current in it is increased, the voltage across it is decreased; and for very slow changes an equation of the form

$$v_a = A + \frac{B}{i_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where A and B are constants, can be found to connect the two.

Putting (1) in (3) gives

$$v_a = A + \frac{B}{I - i_c} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and putting (4) in (2) and differentiating gives

$$L\frac{d^2 i_c}{dt^2} + R\frac{di_c}{dt} + \frac{1}{C}i_c = \frac{B}{(I - i_c)^2} \cdot \frac{di_c}{dt}$$

which, while the oscillatory current i_c is small compared with the supply current I, is

$$L\frac{d^2 i_c}{dt^2} + \left(R - \frac{B}{I^2}\right)\frac{di_c}{dt} + \frac{1}{C}i_c = 0 \quad . \quad . \quad . \quad . \quad (5)$$

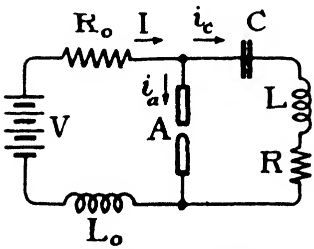


FIG. 261.—Arc Oscillator.

an equation of the same form as 62.06, and from which similar conclusions can be drawn. If $R = B/I^2$ sustained oscillations of constant amplitude are produced. If $R < B/I^2$ the oscillatory current grows in amplitude until i_c can no longer be neglected in comparison with I. On the assumption made I is the arc current i_a of equation (3), from which

$$\frac{dv_a}{di_a} = -\frac{B}{i_a^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

dv_a/di_a is the resistance of the arc to current changes, and this is seen by (6) to be negative; it is generally stated that the arc has a negative resistance, and the condition for oscillations to be maintained is that the resistance of the oscillatory circuit shall not be numerically greater than the negative resistance of the arc. The frequency of the oscillations is from (5) given by $\omega = 1/\sqrt{LC}$.

The static characteristic of equation (3) depends upon changes of ionisation within the arc, and if the current changes are rapid there is not sufficient time for these ionisation changes to be effected; the negative resistance will therefore be less and less as the frequency is increased. The Duddell arc gave comparatively small power, and the highest frequency obtained was only about 10,000 cycles per second; but three modifications introduced by Poulsen led to the use of the arc in the largest power wireless stations. These are (i.) the arc is enclosed in a water-cooled chamber filled with hydrogen, coal gas or alcohol; (ii.) the anode is made of a hollow copper rod through which water is continuously circulated; (iii.) a powerful magnetic field is maintained across the arc itself, tending always to blow the arc out. The complete theory of the Poulsen arc is complicated and still somewhat obscure.* It is clear, however, that the hydrogen gas and the copper anode increase the negative resistance, and that the arc is actually extinguished every cycle. The maintenance of the oscillations is by a shock every cycle, as in the case of the valve oscillator with large negative grid bias, instead of by continuous sinusoidal maintenance.

To avoid re-striking the arc every time a dot or dash is sent, the Morse code is sent on two wavelengths, one for marking and one for spacing, and the key shorts part of the inductance of the oscillatory circuit for this purpose. This is a distinct disadvantage of the arc method, both on account of the fact that power is being consumed during the whole time signals are being sent, and on account of each station requiring the allocation of two separate wavelengths.

The alternator would at first sight hardly be considered as a possible generator of high frequency oscillations, partly owing to the mechanical difficulties of providing a sufficiently high peripheral speed, and partly owing to the large eddy current losses in the iron cores to be expected at the high frequencies. However, by adopting an inductor type of alternator, in which both field and alternator winding are carried on the stator, the necessary flux variations being produced by teeth on the rotor, and using very finely laminated iron for the stator core, machines have been constructed to give frequencies up to 100,000 cycles per second. In later and higher power machines the frequency is from 8,000 to 10,000 cycles per second, and the frequency is raised either by independent static frequency multiplying transformers (Joly) or by an ingenious arrangement of resonant circuits on the rotor and stator of the machine itself (Goldschmidt).†

Several high-power long wavelength transmitting stations are run

* P. O. Pedersen, *Proc. I.R.E.*, Vol. 5, p. 255.

† M. Latour, "Radio Frequency Alternators," *Proc. I.R.E.*, June, 1920.

by alternator and many by arc, but it seems unlikely that any further stations of either type will be installed ; the advantages of the valve are too many.

REFERENCES FOR FURTHER READING.

C. L. FORTESCUE.—" The Three-Electrode Thermionic Valve as Alternating Current Generator." *Engineering*, October 10th and October 17th, 1919.

C. L. FORTESCUE.—" Power Rectifier." *Phys. Soc.*, Vol. XXXI., p. 319, and Vol. XXXIX., p. 313.

R. V. HANSFORD and H. FAULKNER.—" Design Details of a High-Power Radio-Telegraphic Transmitter using Thermionic Valves." *J.I.E.E.*, Vol. LXV., p. 297.

C. F. ELWELL.—" The Holweck Demountable Type Valve." *Proc. Wireless Section I.E.E.*, Vol. II., p. 155.

H. MORRIS AIREY, G. SHEARING and H. G. HUGHES.—" Silica Valves in Wireless Telegraphy." *Proc. Wireless Section I.E.E.*, Vol. II., p. 157.

W. J. PICKEN.—" Cooled Anode Valves, and Lives of Transmitting Valves." *Proc. Wireless Section I.E.E.*, Vol. II., p. 162.

W. G. CADY.—" The Piezo-Electric Resonator." *Proc. I.R.E.*, Vol. X., p. 83.

D. W. DYE.—" Piezo-Electric Quartz Resonator and Equivalent Electric Circuit." *Proc. Phys. Soc.*, Vol. XXXVIII., p. 399.

G. W. N. COBBOLD and A. E. UNDERDOWN.—" Some Practical Applications of Quartz Resonators." *Proc. W. S., I.E.E.*, Vol. III., p. 151.

CHAPTER XIV

RECEPTION

(66) Aerial Circuits.

A RECEIVING aerial is very usually tuned to the frequency of the signal that it is desired to receive (section 59), and the sharpness of the tuning depends upon the decay factor of the aerial circuit. There will at any instant be many electromagnetic waves of different frequencies causing voltages in the aerial, and any of these producing appreciable currents will cause interference with the desired signal, the resonance curve of Fig. 262 is the curve of aerial current at different values of ω for the same field strength. Thus if I_0 is the current produced by the wave of angular frequency ω_0 to which the aerial is tuned, I_1 is the current produced by a wave of the same field strength but angular frequency ω_1.

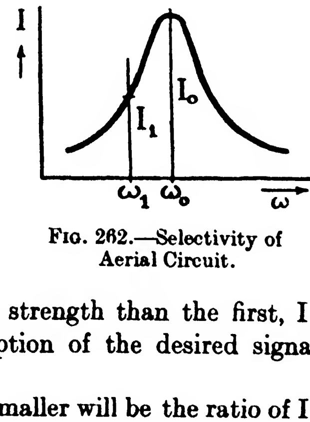


FIG. 262.—Selectivity of Aerial Circuit.

If the second wave is of much greater strength than the first, I_1 may be greater than I_0, and the reception of the desired signal rendered difficult if not impossible.

The sharper the resonance curve the smaller will be the ratio of I_1 to I_0. Upon this sharpness of resonance depends the "selectivity" of the circuit; the smaller the decay factor the greater the selectivity.

The decay factor of the aerial circuit may be reduced by means of retroactive amplification; power is "fed back" from the anode to the grid circuit, but not to a sufficient extent to cause valve-maintained oscillations. This may be done by any of the methods described in section 62. In the arrangement of Fig. 237 the oscillatory circuit would represent the aerial, and the effective decay factor would be reduced by

$$\frac{\mu M}{2LCR_a} - \frac{1}{2CR_a}$$

(Equation 62.08).

A more usual arrangement is that of Fig. 263, in which the oscillatory circuit (representing the aerial) is connected to the grid, and back coupling is by mutual inductance M between the oscillatory circuit coil L_1 and the small coil L_2 in the anode circuit. Let V be the voltage produced in the coil L_1 by the electromagnetic wave, C_1 the condenser of the oscillatory circuit, I_1 the current in the oscillatory circuit and I_a the anode current (alternating component). Then the grid voltage is

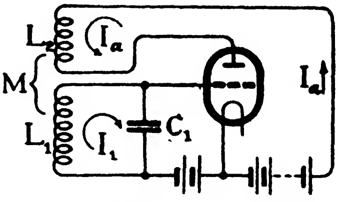


FIG. 263.—Retroactive Amplification.

$$V_g = -\frac{j}{\omega C_1} I_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the anode current is

$$I_a = \frac{\mu V_g + j\omega M I_1}{R_a + j\omega L_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Neglecting $j\omega L_2$ in comparison with R_a, (1) and (2) give

$$I_a R_a + j\left(\frac{\mu}{\omega C_1} - \omega M\right) I_1 = 0 \quad . \quad . \quad . \quad . \quad (3)$$

Also, from a consideration of the voltages acting in the oscillatory circuit,

$$V + j\omega M I_a = I_1 Z_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where Z_1 is the series impedance of the oscillatory circuit, *i.e.*,

$$Z_1 = R_1 + j\left(\omega L_1 - \frac{1}{\omega C_1}\right) \quad . \quad . \quad . \quad . \quad (5)$$

Elimination of I_a from (3) and (4) gives

$$I_1 = \frac{V}{Z_1 - \frac{\mu M}{R_a C_1} + \frac{\omega^2 M_2^2}{R_a}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

The effective impedance of the circuit is now

$$Z_1 - \frac{\mu M}{R_a C_1} + \frac{M^2}{R_a C_1 L_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

(writing $\omega^2 = 1/L_1C_1$) instead of Z_1. The last term is usually small compared with the second, since M/L_1 is usually small compared with μ, and the result of the feed-back is a reduction of effective resistance by $\mu M/R_a C_1$, or a reduction of decay factor by $\mu M/2R_a C_1 L_1$.

In this manner the selectivity of the aerial could be increased indefinitely, the only limit being the stability of the valve. (If the retroaction is pushed too far, a small increase of μ or reduction of R_a will cause oscillations to commence.)

But another limit is set by the increase of the transient period, during which the oscillations build up, which necessarily accompanies the reduction of the effective resistance (see equation 6 (1.33) and Fig. 234), with a consequent lowering of the speed of signalling attainable. Hence the selectivity must not be made too great, and it follows that in order that a reasonable speed of signalling with reasonable freedom from interference may be attained, the wavelengths allocated to different stations must not be too close together.

Apart altogether from transients, the selectivity in telephony must not be too great, as the whole of the side bands must be received without undue distortion in amplitude. If the decay factor is very small, and the aerial is tuned to the carrier wave (ω_o in Fig. 262), and if ω_1 is one of the side frequencies, then that side frequency will suffer reduction in amplitude compared with the carrier wave in the ratio of I_1/I_o. Thus if a simple aerial is used the decay factor must be sufficiently large for the whole of the side bands to be received near the peak of the resonance curve. With the shorter wavelengths this is quite practicable, but on long wavelengths it is better to use a chain of resonant circuits or a filter having a band pass characteristic in order to minimise distortion and at the same time avoid interference. One side band may be completely eliminated at the transmitting station in order to reduce the width of the band of frequencies required.

(67) **High Frequency Amplification.**

The amplification by means of triode valves of the small high frequency voltages received in the aerial is the same in principle as the amplification of low frequency voltages considered in section 46, but special attention must be paid to capacity effects in the resistances of Fig. 170 and the transformers of Fig. 171, and the likelihood of feed-back through the anode-grid capacity causing oscillations (Appendix 9) is considerably increased.

If the resistance R_1 in Fig. 170 is 20,000 ohms, and the stray capacity across it is equivalent to a capacity of 10 $\mu\mu$F, then at a wavelength of 300 metres ($\omega = 2\pi \times 10^6$) the impedance of the capacity is 16,000 ohms, and the effective impedance of the resistance is only 12,500 ohms instead of 20,000 ohms. Ordinary wire-wound resistance bobbins are not suitable on account of their large capacities, and other devices such as a platinum film grid or a carbonised cellulose thread must be resorted to. Care must, moreover, be taken to minimise all small shunting capacities in the wiring and any switches; even the capacity between anode and filament may have an appreciable effect at the shorter wavelengths. Generally speaking, the resistance coupled amplifier is quite effective on long wavelengths, but its efficiency falls off considerably as the wavelength is reduced.

The transformers T_1 and T_2 of Fig. 171 are air cored for high frequency amplification instead of iron cored, and consist of closely coupled inductances formed of single layer solenoids one within the other, or of turns of wire in alternate grooves cut in an ebonite cylinder. It is impossible to obtain sufficient reactance to match the valve resistance without obtaining so much self capacity that at a particular frequency the coils are in parallel resonance. When this is so (see section 48) the impedance at resonance ($= L/CR$) and near resonance is large and the amplification accordingly satisfactory but at all other frequencies the impedance is small and the amplification also small. It is usual to wind sufficient wire on the coils to obtain parallel resonance at a frequency corresponding to the wavelength the amplifier has most generally to receive.

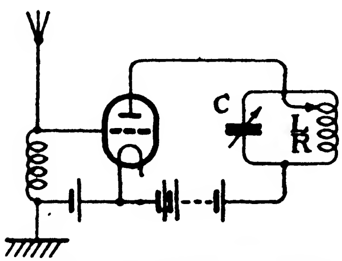


Fig. 264.—Tuned Anode Amplifier.

Or the inductance may be made smaller, and resonance obtained by shunting it with a variable air condenser as in Fig. 264. This constitutes the "tuned anode" amplifier. The circumstances controlling the best position of the anode tap are the same as those examined in section 62. The amplified voltage may be fed to the next stage either through a condenser and grid leak or through another coil, which may also be tuned, coupled with that of the anode circuit. If C_1L_1 and R refer to the oscillatory circuit, Z is the series impedance of the circuit and b the fraction of the inductance tapped, the parallel impedance of the oscillatory circuit is very nearly

$$Z_p = b^2 \frac{L}{CZ} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

the anode current is

$$I_a = \frac{\mu V_g}{R_a + b^2 \frac{L}{CZ}}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

the current in the oscillatory circuit is very nearly

$$I = \frac{j\omega L b}{Z} I_a, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and the amplified voltage available across the condenser (or whole inductance) is

$$V = -\frac{j}{\omega C} I$$

$$= \frac{bL}{CZ} I_a \qquad \text{(by (3))}$$

$$= \frac{\mu V_g}{\frac{CZ}{bL} R_a + b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

(by (2))

The amplified voltage is a maximum when the denominator in (4) is a minimum, that is, when

$$\frac{d}{db}\left(\frac{CZ}{bL} R_a + b\right) = 0$$

or

$$b^2 = \frac{CZ}{L} R_a$$

and when the circuit is tuned, so that $Z = R$, the best value of b is given from

$$b^2 = \frac{CRR_a}{L} \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The amplified voltage may be written from (4) as

$$V = \mu V_g \frac{\frac{bL}{CR_a}}{Z + \frac{b^2 L}{CR_a}}$$

$$= \mu V_g \frac{\frac{bL}{CR_a}}{R + \frac{b^2 L}{CR_a} + j\left(\omega L - \frac{1}{\omega C}\right)} \quad . \quad . \quad . \quad (6)$$

a form which makes clear the increase of effective resistance and decay factor of the oscillatory circuit due to the shunting effect of the valve. The best value for b from (5) evidently causes the effective resistance and decay factor to be doubled by the valve.

As an example, consider the reception of a signal on a wavelength of 3,780 metres with $C = 1{,}000\ \mu\mu F$, $L = 4{,}000\ \mu H$, $R = 10$ ohms and $R_a = 25{,}000$ ohms. The best value for b is found from (5) to be 1/4, and the voltage amplification from (6) is 2μ. The decay factor is $R/L = 2{,}500$.

If, as is very frequently the case, the anode tap scheme had not been utilised, the anode being connected to the end of the coil, b in the above expression would be 1, the voltage amplification would be $\mu/1{\cdot}06$, and the decay factor

$$\frac{R}{2L} + \frac{1}{2CR_a} = 1{,}250 + 20{,}000$$
$$= 21{,}250.$$

The voltage amplification at resonance is only half, and the decay factor has been increased nearly ten times, with a corresponding loss of selectivity.

The tendency of high frequency amplifiers to burst into sustained oscillations is generally present unless definitely counteracted in some way. Biasing the grid to a positive potential and so damping the input circuit by the grid currents which then flow, or lowering the anode or filament potentials to give reduced amplification are devices often adopted, but they are not to be recommended, especially for telephony, owing to the distortion that ensues.

A better scheme is to counteract the feed-back through the anode-grid capacity by an oppositely directed feed-back of equal magnitude. This can be done in Fig. 263 by reversing the connection to the coil L_2, and in Fig. 264 by including in the anode circuit a small coil coupled with the aerial coil in the same sense. It is necessary, however, to make a careful adjustment of the mutual for every change of the wavelength received. This adjustment is avoided in another scheme, known as the "neutrodyne," illustrated in Fig. 265. The coils LL′ are arranged as an autotransformer with a one-to-one ratio, and the capacity C′ has the same value as the anode grid capacity C_v. As far as the alternating currents are concerned, the bridge (b) is seen to be equivalent to the circuit (a), and the voltage fluctuations across AB (the output) cause no voltage fluctuations across GO (the input). Once the adjustment of L′ and C′ is made, it holds for all frequencies.

In the push-pull amplifier (Fig. 169) the additional coil L′ is not required, balance being obtained as in Fig. 266 by small condensers

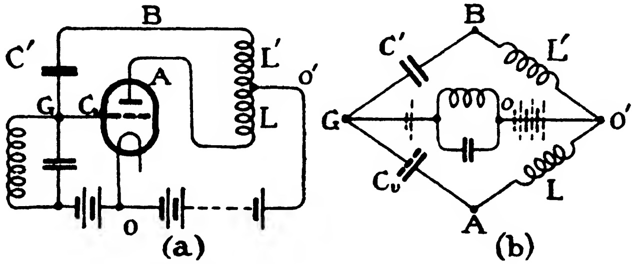


FIG. 265.—Neutrodyne Amplifier.

connected across from the anode of each valve to the grid of the other. C_1 and C_2 are the anode-grid capacities of the two valves, and C'_1 and C'_2 the balancing capacities. The equivalent bridge is drawn at (*b*), and it is seen that when $C_1 = C'_1$ and $C_2 = C'_2$ no voltage across the input $L'_1L'_2$ is caused by a voltage across the output L_1L_2. This arrangement is used in short wave working, both in reception

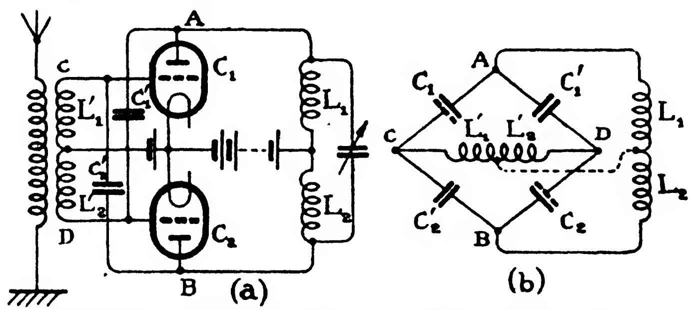


FIG. 266.—Push-pull Amplifier with Balancing Condensers.

and in power amplification from the small master oscillator at the transmitting station.

Another method of avoiding retroaction depends upon a modification to the valve itself; a second grid is interposed between the anode and the working grid, and this second grid is connected to a point at earth potential as far as the high frequency voltages are concerned (usually to a tapping on the anode battery). This second or screening grid thus reduces the effective capacity between the anode and the input grid to a very small value indeed, with a corresponding reduction in the feed-back.

(68) Rectification.

Any conducting device having a non-linear relation between the voltage applied across it and the current flowing through it can be used as a rectifier or detector of high frequency currents. If the potential is alternating (of mean value zero), the mean current will not be zero, and can be used to operate receiving apparatus which would not be affected by the high frequency currents. The simplest device that is used for this purpose is a "crystal detector," consisting of a contact between certain crystals or between a crystal and a metal. Fig. 267 shows the characteristic curves of two such crystals; (*a*) that of a combination of carborundum (SiC) and steel, (*b*) that of zincite (ZuO) and chalcopyrites ($CuFeS_2$), generally called a "perikon" crystal. The diode valve is another such device (see Fig. 162), and the triode valve if worked on the lower bend of the characteristic (Figs. 164 and 165) or on the upper bend near saturation (Fig. 164). The grid current characteristic (the dotted line of Fig. 165) can also be used for rectification.

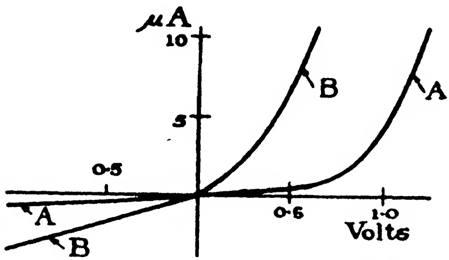


FIG. 267.—Crystal characteristics. A. Carborundum. B. Perikon.

The characteristic curve in any of these cases can be expressed by the equation

$$I_o = f(V_o) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

If i is the change of current due to a small change v of potential

$$I_o + i = f(V_o + v) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and on expansion by Taylor's theorem

$$I_o + i = f(V_o) + v\frac{d}{dV_o}f(V_o) + \frac{v^2}{2!}\frac{d^2}{dV_o^2}f(V_o) + \frac{v^3}{3!}\frac{d^3}{dV_o^3}f(V_o) + \; . \quad (3)$$

whence using (1), and replacing v by $V \sin \omega t$

$$i = V\sin\omega t\frac{d}{dV_o}f(V_o) + \frac{V^2}{2!}\sin^2\omega t\frac{d^2}{dV_o^2}f(V_o) + \; . \quad . \quad . \quad (4)$$

The average values of sin ωt, sin $^3\omega t$, etc., are zero, and the average value of sin $^2\omega t$ is 1/2. The useful rectified current I_r is the mean value of i; and if the fourth and higher differentials are disregarded as being usually small, then

$$I_r = \frac{V^2}{4} \cdot \frac{d^2 I_o}{dV_o^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Evidently the rectified current is proportional to the square of the voltage applied to the detector, and the detector will give poor results with very weak signals. Hence the importance of amplifying weak signals before rectification. Equation (5) also shows that the rectified current is proportional to the curvature of the characteristic, and a biasing voltage should be used to bring the working point to the point of maximum curvature. In the case of the carborundum detector this voltage should be about 0·8 volt, but in the case of the perikon detector it is unnecessary to have any bias. The biasing voltage is usually applied by the potentiometer arrangement in Fig. 268, which shows a very simple receiving set. D is the crystal detector and R a pair of telephone receivers. The condenser C provides a path of low impedance for the alternating currents, while the rectified current flows through the receivers.

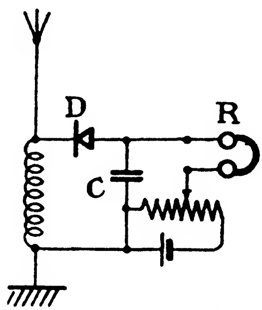


FIG. 268.—Crystal receiver.

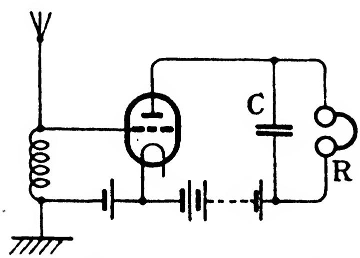


FIG. 269.—Receiver with anode-bend rectification.

The corresponding arrangement for detection on the anode bend of a triode valve is drawn in Fig. 269. The theory in this case has been written in section 46. The impedance to the high frequency currents offered by the condenser C is negligible, so that as far as these are concerned, R in equation 46·17 is zero. The impedance to the rectified current (low frequency changes) is that of the telephone receivers Z_r, and this must replace R in equation 46.18 In order to obtain the maximum possible power in the telephone receivers their impedance should match that of the valve, and high resistance phones are necessary, or if the ordinary low-resistance phones are used, a step-down transformer should be inserted between the anode circuit and the phones.

The use of grid current rectification leads to a more sensitive detector for weak signals than the anode bend method, but with strong signals it leads to distortion and appreciable damping of the receiving circuit. The arrangement is shown in Fig. 270. The condenser C_g is interposed between the grid and the receiving circuit, and is large enough to offer only small impedance to the high frequency currents, but small enough to reduce transients in conjunction with R_g to small dimensions. Voltage fluctuations accordingly reach the grid practically unchanged. R_g is a high resistance, of the order 1 to 5 megohms, known as the grid leak. (It makes very little difference if R_g is connected across the condenser instead of between condenser and filament; the arrangement is then the same as in Fig. 242.) Rectification of the grid current causes a rectified current to flow (in the usual convention) from filament to grid through R_g, with a consequent lowering of the mean grid potential with regard to the filament. This change of grid potential produces a corresponding change in anode current, which is used to operate the receiving apparatus. Usually no biasing battery is necessary to bring the initial grid potential to a suitable point on the grid current characteristic.

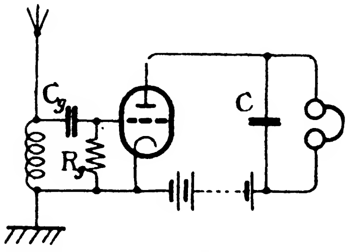


FIG. 270.—Receiver with grid-current rectification.

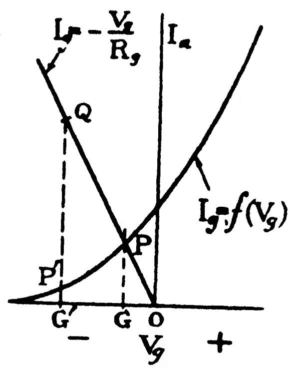


FIG. 271.—Grid-current rectification.

In Fig. 271 two curves of grid current plotted against grid potential are drawn; the first is

$$I_g = f(V_g) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and is the current through the valve ; the second is the straight line

$$I_g = -\frac{V_g}{R_g} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

and is the current through the grid leak.

In the absence of an incoming signal the intersection of these two curves in P evidently determines the grid current and potential, which, dropping PG perpendicular to the axis, are read off as GP and OG respectively. If an incoming signal of known amplitude causes an alternation of the grid potential, the corresponding rectified grid current I_r can be estimated from the grid characteristic, and a vertical QP′G′ can be drawn such that the intercept QP′ between the two curves is I_r. Then the total mean grid current in the presence of the signal is G′Q, the total mean grid potential is OG′, and the change of mean grid potential due to the signal is GG′. It is this change which determines the change of mean anode current through the telephone receiver. For small signals this change is, as in the case of the anode bend detector, proportional to the square of the incoming voltage, but the actual change in the grid leak case may be as much as thirty times that with the anode bend.

The sensitivity of both the anode bend and grid leak detectors may be considerably improved by using the same valve to give retro-active amplification, by including in the anode circuit a coil coupled in the correct sense to the aerial coil, as shown for the grid leak case in Fig. 272. In this diagram the aerial circuit is shown as being tuned by a variable condenser shunting the loading inductance. This is a very usual and convenient arrangement.

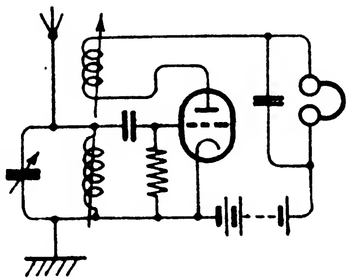


FIG. 272.—Rectification with retro-active amplification.

The current through the telephone receivers in all these cases depends upon the envelope of the high frequency wave, and is proportional at any instant to the square of the amplitude of the envelope at that instant. The envelopes in the various cases are summarised in Fig. 273. A, shows the continuous wave system (CW.), B, the interrupted continuous wave system (I.C.W.), C, the tonic train system (T.T.), D, telephony and E, spark. The total time of the diagrams is only a small fraction of one dot. I.C.W., T.T., or spark signals in the Morse code will accordingly be heard as notes or buzzes of the modulation or spark frequency, of short duration for the dot and long duration for the dash. Telephony will also be heard direct, but

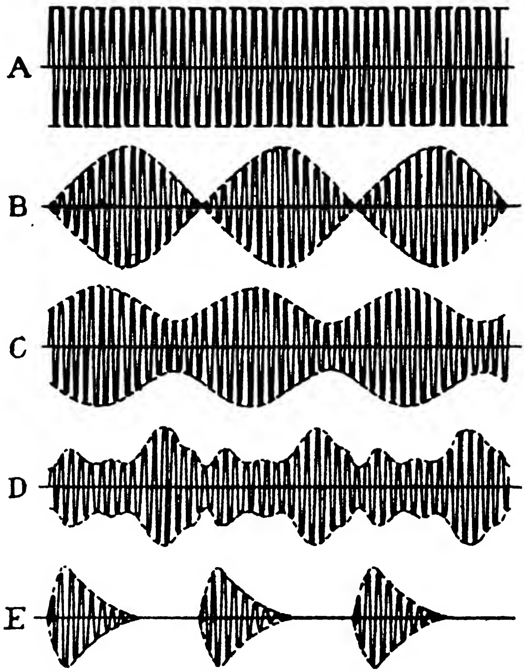


Fig. 273.—A. Continuous wave. B. Interrupted Continuous Wave. C. Tonic Train. D. Speech. E. Spark.

with some distortion owing to the square law of the rectifier. As has been seen (equation 63·01) the modulated wave can be written

$$A(1 + k \sin \omega_2 t) \sin \omega_1 t \quad . \quad . \quad . \quad . \quad . \quad (8)$$

where ω_2 is one of the low frequencies and ω_1 the high frequency. The rectified current is proportional to the mean value (with respect to the high frequency) of the square of this, that is, to

$$\frac{A^2}{2}(1 + k \sin \omega_2 t)^2$$

$$= \frac{A^2}{2}(1 + 2k \sin \omega_2 t + k^2 \sin^2 \omega_2 t)$$

$$= \frac{A^2}{2} + A^2k \sin \omega_2 t + \frac{A^2k^2}{4} - \frac{A^2k^2}{4} \cos 2\omega_2 t \quad . \quad . \quad . \quad (9)$$

Harmonic distortion thus ensues, and as is shown by considering two or more low frequencies, sum and difference tones are introduced, the amplitudes of which are in the ratio $k/4$ to the amplitudes of the fundamental frequencies transmitted. The modulation coefficient k at the transmitting station must not be made too great.

If one side band is eliminated (by band-pass filters) at the transmitting station, there is no such distortion. The received potential is now proportional to

$$A \sin \omega_1 t + \frac{Ak}{2} \cos (\omega_1 + \omega_2)t \quad . \quad . \quad . \quad . \quad (10)$$

which squared gives

$$A^2 \sin^2 \omega_1 t + \frac{A^2k^2}{4} \cos^2(\omega_1 + \omega_2)t + A^2k \sin \omega_1 t \cos(\omega_1 + \omega_2)t$$

$$= A^2 \sin^2 \omega_1 t + \frac{A^2k^2}{4} \cos^2(\omega_1 + \omega_2)t + A^2k \sin \omega_1 t \cos \omega_1 t \cos \omega_2 t$$

$$- A^2k \sin^2 \omega_1 t \cos \omega_2 t \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

The rectified current is proportional to the mean value of this, that is, to

$$\frac{A^2}{2} + \frac{A^2k^2}{8} - \frac{A^2k}{2} \cos \omega_2 t \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and the only fluctuation is that of the required frequency ω_2.

If the transmitted signals are by Morse code from continuous wave, the dots and dashes will cause alterations of anode current lasting as long as the dots and dashes. These may, after suitable amplification, be used to actuate a telegraphy relay; as, for instance, in Fig. 182, but reception by telephone will not be possible by the circuit arrangements considered, as the only sounds heard in the receiver will be clicks at the commencing and end of the dots and dashes. The telephone receiver may, however, be used in this case by use of the heterodyne principle described in the next section.

(69) Heterodyne Reception.

Continuous wave signals are made to give audible notes in the telephone receivers by means similar to those by which beats are produced in acoustics. An additional e.m.f. of frequency $\omega_2/2\pi$ is introduced into the receiving circuit. This produces currents which beat with those of the frequency $\omega_1/2\pi$ of the continuous wave, causing the total current envelope to rise and fall in amplitude at a frequency equal to the difference of the two frequencies. On rectifying this current, a note is heard in the receivers if it has been arranged that the frequency difference is an audible note.

The formation of beats is shown in Fig. 274. Analytically, if the

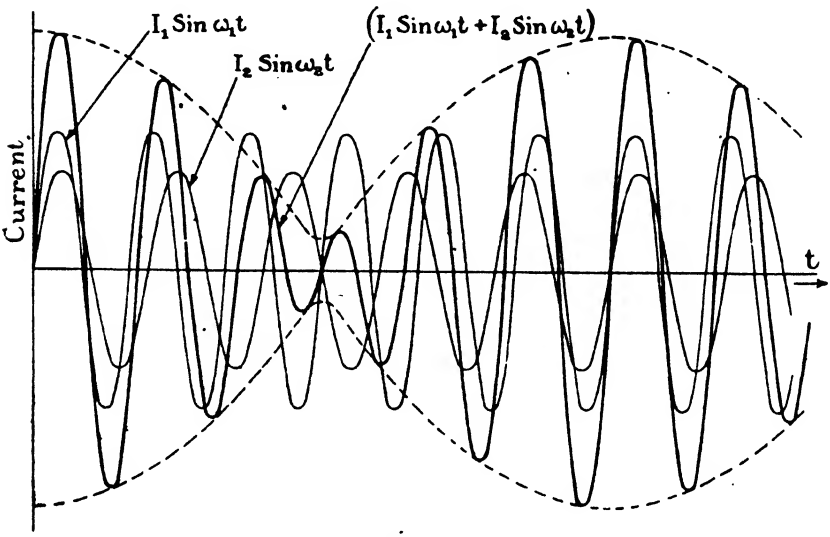


FIG. 274.—Heterodyne principle.

current amplitudes are I_1 and I_2 respectively, the total current can be written

$$I = I_1 \sin \omega_1 t + I_2 \sin \omega_2 t \quad . \quad . \quad . \quad . \quad (1)$$

and

$$\begin{aligned} I^2 &= I_1^2 \sin^2 \omega_1 t + I_2^2 \sin^2 \omega_2 t + 2I_1 I_2 \sin \omega_1 t \sin \omega_2 t \\ &= I_1^2 \sin^2 \omega_1 t + I_2^2 \sin^2 \omega_2 t + I_1 I_2 \cos (\omega_1 - \omega_2) t \\ &\qquad - I_1 I_2 \cos (\omega_1 + \omega_2) t \quad . \quad . \quad . \quad (2) \end{aligned}$$

The rectified current is proportional to the mean of this with regard to high frequencies, that is, to

$$\frac{I_1^2}{2} + \frac{I_2^2}{2} - \frac{I_1 I_2}{2} + I_1 I_2 \cos (\omega_1 - \omega_2) t \quad . \quad . \quad . \quad (3)$$

The first three terms simply produce inaudible changes of anode current, but the fourth term gives an audible note of frequency $(\omega_1 - \omega_2)/2\pi$ in the telephone receiver as long as the incoming signal persists.

Two important points should be noted. (i.) The effective rectified current is directly proportional to the strength of the incoming signal instead of to its square, and hence heterodyne reception is far more sensitive for weak signals than direct reception. (ii.) The rectified current is proportional also to the strength of the local oscillation, and weak signals can be increased by increasing this, within the limits imposed by the valve characteristics.

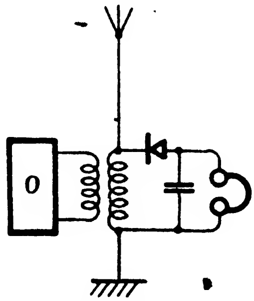


FIG. 275.—Simple Heterodyne Receiver.

The valve oscillator forms a ready means of providing the necessary local oscillation, and heterodyne reception can be carried out very simply, as in Fig. 275, by coupling a valve oscillator with the aerial coil of a crystal receiving set. The same may be done, of course, with the circuits of Figs. 269 and 270.

If in Fig. 272 the circuit is distuned a little from the wave frequency, and the reaction increased until the valve oscillates, the local oscillation is produced by the detecting valve and no separate oscillator is required. This arrangement is known as "auto-heterodyne." As the condenser is varied, and with it the frequency of the local oscillations, the beat note should also vary from a high value through zero to a high value as the local frequency passes through the signal frequency, as is indicated by the straight lines AC_0B in Fig. 276. (The lines are straight, as the capacity changes are very small compared with the whole capacity, so that the frequency changes of the oscillations are practically proportional to the condenser changes.) Actually, however, the curve traced is AC_1C_2B. Between the settings C_1 and C_2 of the condenser no note is heard at all. The incoming signal has pulled the local oscillator

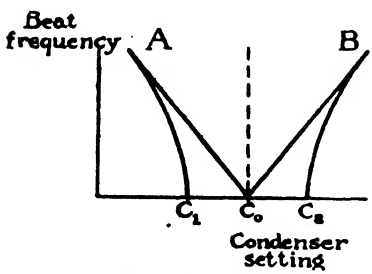


FIG. 276.—Silent Interval.

into step with itself. The range C_1C_2, over which this happens, is greater the stronger the incoming signal.

The heterodyne principle may be used in receiving comparatively short wave telephony (such as broadcast) to avoid the difficulties attending high frequency amplification and to obtain the increased sensitivity of heterodyne reception. The local oscillation differs in frequency from the carrier wave by a fairly large amount, say, 100,000 cycles, which makes the beats quite inaudible, but gives a modulated 100,000 cycle current instead of the original modulated million-cycle current. This is then amplified by tuned 100,000-cycle amplifiers, and finally rectified to give the speech currents. This arrangement is known as "super-heterodyne," and is also used in very short wave telegraph reception, in which case the heterodyning may be in two stages.

(70) **Low Frequency Amplification.**

When the signals have been rectified, or heterodyned and rectified, to give audible sounds, either the Morse code or telephony, in the telephone receivers, the amplitude of these audible sounds may be increased by low frequency amplification in the various manners described in section 46. The telephone receivers of the diagrams of the last section are replaced by the primary winding of a suitable "intervalve" transformer, the secondary winding of which is connected across the grid and filament of the low frequency amplifying valve, the output of which can be connected as may be required to telephone receivers, loud speaker, or land line for extending the wireless system to the land-line system.

REFERENCES FOR FURTHER READING

L. B. Turner.—"The Relations Between Damping and Speed in Wireless Reception." *J.I.E.E.*, Vol. LXII., p. 192.

C. L. Fortescue.—The Design of Multi-stage Amplifiers." *J.I.E.E.*, Vol. LVIII., January, 1920.

E. B. Moullin and L. B. Turner.—"The Thermionic Triode as Rectifier." *J.I.E.E.*, Vol. LX., p. 706.

APPENDIX

Appendix 1.—Mathematical Formulæ.

(i.) *Trigonometrical*

$$\left.\begin{aligned}\cos^2\theta + \sin^2\theta &= 1\\ \cosh^2\theta - \sinh^2\theta &= 1\end{aligned}\right\}$$

$$\left.\begin{aligned}\sin(\theta\pm\phi) &= \sin\theta\cos\phi \pm \cos\theta\sin\phi\\ \sinh(\theta\pm\phi) &= \sinh\theta\cosh\phi \pm \cosh\theta\sinh\phi\end{aligned}\right\}$$

$$\left.\begin{aligned}\cos(\theta\pm\phi) &= \cos\theta\cos\phi \mp \sin\theta\sin\phi\\ \cosh(\theta\pm\phi) &= \cosh\theta\cosh\phi \pm \sinh\theta\sinh\phi\end{aligned}\right\}$$

$$\sinh(\theta\pm j\phi) = \sinh\theta\cos\phi \pm j\cosh\theta\sin\phi$$

$$\cosh(\theta\pm j\phi) = \cosh\theta\cos\phi \pm j\sinh\theta\sin\phi$$

$$\left.\begin{aligned}\sin 2\theta &= 2\sin\theta\cos\theta\\ \sinh 2\theta &= 2\sinh\theta\cosh\theta\end{aligned}\right\}$$

$$\left.\begin{aligned}\cos 2\theta &= \cos^2\theta - \sin^2\theta\\ &= 1 - 2\sin^2\theta\\ &= 2\cos^2\theta - 1\\ \cosh 2\theta &= \cosh^2\theta + \sinh^2\theta\\ &= 1 + 2\sinh^2\theta\\ &= 2\cosh^2\theta - 1\end{aligned}\right\}$$

$$\left.\begin{aligned}\sin\theta \pm \sin\phi &= 2\sin\frac{\theta\pm\phi}{2}\cos\frac{\theta\mp\phi}{2}\\ \sinh\theta \pm \sinh\phi &= 2\sinh\frac{\theta\pm\phi}{2}\cosh\frac{\theta\mp\phi}{2}\end{aligned}\right\}$$

$$\left.\begin{aligned}\cos\theta + \cos\phi &= 2\cos\frac{\theta+\phi}{2}\cos\frac{\theta-\phi}{2}\\ \cosh\theta + \cosh\phi &= 2\cosh\frac{\theta+\phi}{2}\cosh\frac{\theta-\phi}{2}\end{aligned}\right\}$$

$$\left.\begin{aligned}\cos\theta - \cos\phi &= -2\sin\frac{\theta+\phi}{2}\sin\frac{\theta-\phi}{2}\\ \cosh\theta - \cosh\phi &= 2\sinh\frac{\theta+\phi}{2}\sinh\frac{\theta-\phi}{2}\end{aligned}\right\}$$

$$\left.\begin{aligned}\sin\theta\cos\phi &= \frac{1}{2}\{\sin(\theta+\phi)+\sin(\theta-\phi)\}\\ \sinh\theta\cosh\phi &= \frac{1}{2}\{\sinh(\theta+\phi)+\sinh(\theta-\phi)\}\end{aligned}\right\}$$

$$\left.\begin{aligned}\cos\theta\cos\phi &= \frac{1}{2}\{\cos(\theta+\phi)+\cos(\theta-\phi)\}\\ \cosh\theta\cosh\phi &= \frac{1}{2}\{\cosh(\theta+\phi)+\cosh(\theta-\phi)\}\end{aligned}\right\}$$

$$\left.\begin{aligned}\sin\theta\sin\phi &= \frac{1}{2}\{\cos(\theta-\phi)-\cos(\theta+\phi)\}\\ \sinh\theta\sinh\phi &= \frac{1}{2}\{\cosh(\theta+\phi)-\cosh(\theta-\phi)\}\end{aligned}\right\}$$

$$\left.\begin{aligned}a\sin\theta+b\cos\theta &= \sqrt{a^2+b^2}\sin\left(\theta+\tan^{-1}\frac{b}{a}\right)\\ &= \sqrt{a^2+b^2}\cos\left(\theta-\tan^{-1}\frac{a}{b}\right)\\ a\sinh\theta+b\cosh\theta &= \sqrt{a^2-b^2}\sinh\left(\theta+\tanh^{-1}\frac{b}{a}\right)\\ &= \sqrt{b^2-a^2}\cosh\left(\theta+\tanh^{-1}\frac{a}{b}\right)\end{aligned}\right\}$$

$$\epsilon = 1+1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\ \ldots \quad (= 2{\cdot}7183\ ..)$$

$$\epsilon^x = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\ \ldots$$

$$\left.\begin{aligned}\sin\theta &= \frac{\epsilon^{j\theta}-\epsilon^{-j\theta}}{2j}\\ \sinh\theta &= \frac{\epsilon^{\theta}-\epsilon^{-\theta}}{2}\end{aligned}\right\}$$

$$\left.\begin{aligned}\cos\theta &= \frac{\epsilon^{j\theta}+\epsilon^{-j\theta}}{2}\\ \cosh\theta &= \frac{\epsilon^{\theta}+\epsilon^{-\theta}}{2}\end{aligned}\right\}$$

$$\cosh\theta\pm\sinh\theta = \epsilon^{\pm\theta}$$

$$\left.\begin{aligned}\sin\theta &= \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \quad . \quad . \quad . \\ \sinh\theta &= \theta + \frac{\theta^3}{3!} + \frac{\theta^5}{5!} + \quad . \quad . \quad .\end{aligned}\right\}$$

$$\left.\begin{aligned}\cos\theta &= 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \quad . \quad . \quad . \\ \cosh\theta &= 1 + \frac{\theta^2}{2!} + \frac{\theta^4}{4!} + \quad . \quad . \quad .\end{aligned}\right\}$$

$$\frac{d\epsilon^x}{dx} = \epsilon^x$$

$$\left.\begin{aligned}\frac{d\sin\theta}{d\theta} &= \cos\theta \qquad \frac{d\cos\theta}{d\theta} = -\sin\theta \\ \frac{d\sinh\theta}{d\theta} &= \cosh\theta \qquad \frac{d\cosh\theta}{d\theta} = \sinh\theta.\end{aligned}\right\}$$

(ii.) *Geometric Progression.*

The sum s of n terms of a geometric progression with common ratio r and first term a is

$$s = a\frac{1 - r^n}{1 - r}$$

If r is between $+1$ and -1, and the number of terms is ∞, since r^n is then 0,

$$s = \frac{a}{1 - r}$$

(iii.) *Binomial Theorem*

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \quad . \quad . \quad .$$

If n is a positive integer, this is true for all values of x.

If n is fractional or negative, the series holds if x is less than unity (positive or negative). In particular, if x is much less than unity,

$$(1 + x)^n = 1 + nx$$

very nearly.

(iv.) *Taylor's Theorem*

$$f(x + h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) +$$

where
$$f'(x) = \frac{d}{dx} f(x)$$
$$f''(x) = \frac{d^2}{dx^2} f(x), \text{ etc.}$$

(v.) *Fourier's Theorem.*

Fourier's theorem has been stated in section 30 (pp. 123 and 124) for a quantity which varies periodically with time. But, with certain restrictions, any portion of any single valued curve can be represented by a portion of the graph of a Fourier series. That is,

$$y = f(x) \quad \ldots \quad (1)$$

can be written

$$y = A_o + \sum_{m=1}^{\infty} (A_m \sin mx + B_m \cos mx) \quad \ldots \quad (2)$$

where

$$A_o = \frac{1}{2\pi} \int_o^{2\pi} f(x) dx \quad \ldots \quad (3)$$

$$A_m = \frac{1}{\pi} \int_o^{2\pi} f(x) \sin mx \, dx \quad \ldots \quad (4)$$

$$B_m = \frac{1}{\pi} \int_o^{2\pi} f(x) \cos mx \, dx \quad \ldots \quad (5)$$

The particular analysis required (in Appendix 4) is that of

$$y = 1 - \frac{x}{l} \quad \ldots \quad (6)$$

for values of x between o and l.

Suppose $x = 2l$ is chosen as the fundamental period of the Fourier series, that is, $2l$ represents 2π in the expansion, and (2) can be written

$$y = A_o + A_1 \sin \frac{\pi x}{l} + A_2 \sin \frac{2\pi x}{l} + \ldots + A_m \sin \frac{m\pi x}{l} + \ldots$$
$$+ B_1 \cos \frac{\pi x}{l} + B_2 \cos \frac{2\pi x}{l} + \ldots + B_m \cos \frac{m\pi x}{l} + \quad . \quad (7)$$

The average value of y over the period $2l$ is zero, hence $A_o = 0$. From (4)

$$A_m = \frac{1}{l} \int_o^{2l} \left(1 - \frac{x}{l}\right) \sin \frac{m\pi x}{l} . dx$$
$$= \frac{1}{l} \int_o^{2l} \sin \frac{m\pi x}{l} dx - \frac{1}{l^2} \int_o^{2l} x \sin \frac{m\pi x}{l} dx \quad . \quad . \quad (8)$$

The value of the first integral of (8) is evidently zero. The second can be evaluated from the formula for integration by parts, viz :—

$$\int u dv = uv - \int v du \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Since $\quad \dfrac{d}{dx}\left(\cos\dfrac{m\pi x}{l}\right) = -\dfrac{m\pi}{l}.\sin\dfrac{m\pi x}{l},$

$$\int x \sin\frac{m\pi x}{l} dx = -\int x \frac{l}{m\pi} d\left(\cos\frac{m\pi x}{l}\right).$$

$$= -\frac{l}{m\pi}\int x\, d\left(\cos\frac{m\pi x}{l}\right)$$

$$= -\frac{l}{m\pi}\left\{x \cos\frac{m\pi x}{l} - \int \cos\frac{m\pi x}{l} dx\right\} \quad . \quad . \quad \text{by } (9)$$

$$= -\frac{l}{m\pi}\left\{x \cos\frac{m\pi x}{l} - \frac{l}{m\pi}\sin\frac{m\pi x}{l}\right\} \quad . \quad . \quad . \quad . \quad (10)$$

Taken between the limits $2l$ and 0, $x \cos\dfrac{m\pi x}{l} = 2l$ and $\sin\dfrac{m\pi x}{l} = 0$,

hence (10) becomes $-\dfrac{2l^2}{m\pi}$, and (8)

$$A_m = \frac{2}{m\pi} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

In a similar manner it can be shown from (5) that $B_m = 0$. (In the expression corresponding to (10) the sin and cos terms are interchanged, and between the limits both $x \sin\dfrac{m\pi x}{l}$ and $\cos\dfrac{m\pi x}{l}$ vanish.) It is clear also from the requirements of symmetry that the cosine terms in the Fourier expansion must vanish.

Thus the required series is

$$y = 1 - \frac{x}{l} = \frac{2}{\pi}\sin\frac{\pi x}{l} + \frac{2}{2\pi}\sin\frac{2\pi x}{l} + \frac{2}{3\pi}\sin\frac{3\pi x}{l} + \ldots$$

$$= \frac{2}{\pi}\sum_{m=1}^{\infty}\frac{1}{m}\sin\frac{m\pi x}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The sum of the first four terms is drawn in Fig. 277. A shows the line $y = 1 - \dfrac{x}{l}$ and B the series (12) with m values up to 4 found by adding the sine curves a, b, c and d.

(vi.) *Heaviside's Expansion Theorem*

This theorem is of value in finding direct current transients. It can be expressed by the following formula :—

$$i = \frac{E}{\phi(0)} + E \sum_m \frac{\epsilon^{y_m t}}{y_m \phi'(y_m)} \quad . \quad . \quad . \quad . \quad . \quad (13)$$

The periodic solution for the current produced in the circuit by

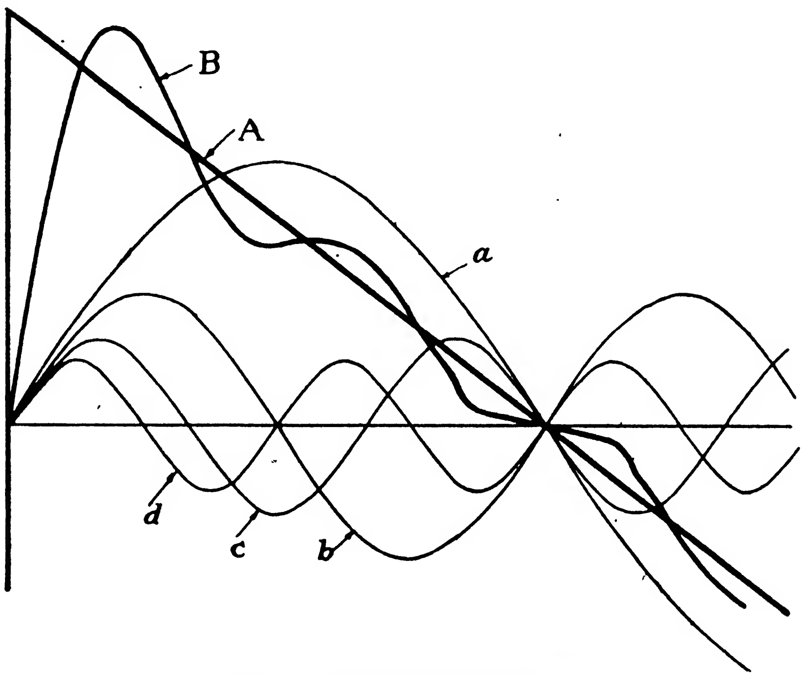


FIG. 277.—Illustrating Fourier's Series.

an alternating e.m.f. $E\epsilon^{j\omega t}$ instead of the steady e.m.f. E is first found in the vector form

$$I = \frac{E\epsilon^{j\omega t}}{Z}$$

In the impedance Z, y is written for $j\omega$, and Z is written $\phi(y)$. $\phi(0)$ in (1) is the value of $\phi(y)$ when y is put equal to 0. y_m is one of the m roots of the equation

$$\phi(y) = 0$$

$\phi'(y_m)$ is $\dfrac{d}{dy}\phi(y)$ evaluated with values of y equal to y_m.

As a simple example consider the rise of current in an inductive circuit (Fig. 37). The periodic solution is

$$I = \frac{E\,\epsilon^{j\omega t}}{R + j\omega L}$$

whence $$\phi(y) = R + yL \qquad \text{(i.)}$$
and $$\phi(0) = R \qquad \text{(ii.)}$$

There is only one root of the equation

$$\phi(y) = R + yL = 0$$

which is $$y = -\frac{R}{L} \qquad \text{(iii.)}$$

$$\frac{d\phi(y)}{dy} = L$$

and $$y\frac{d\phi(y)}{dy} = yL$$

and with $y = -\frac{R}{L}$, $$y\frac{d\phi(y)}{dy} = -R \qquad \text{(iv.)}$$

Substitution of (i.), (ii.), (iii.) and (iv.) in (13) gives

$$i = \frac{E}{R} + E\frac{\epsilon^{-\frac{R}{L}t}}{-R}$$

$$= \frac{E}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right)$$

in agreement with 13.03.

Appendix 2.—The Shunted Condenser or Maxwell Earth.

In Section 14 the following equations were obtained for the currents in the network of Fig. 278 :—

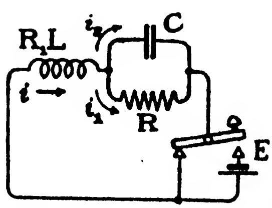


Fig. 278.—Rending Condenser Circuit.

$$i = i_1 + i_2 \qquad (1)$$

$$R_1 i + L\frac{di}{dt} + Ri_1 = E \qquad (2)$$

$$\frac{1}{C}\int i_2 dt = Ri_1 \qquad (3)$$

and $$RLC\frac{di^2}{dt^2} + (R_1RC + L)\frac{di}{dt} + (R_1 + R)i = E \qquad (4)$$

(Equations 14.07, 14.08, 14.09 and 14.13.)

One solution is evidently

$$i = \frac{E}{R_1 + R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

Others are to be obtained by equating the left-hand side of (4) to 0.

Doing this and trying the substitution

$$i = A\epsilon^{mt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

yields $\quad RLCm^2 + (R_1RC + L)\,m + (R_1 + R) = 0$

whence

$$m = -\left(\frac{R_1}{2L} + \frac{1}{2RC}\right) \pm \sqrt{\left(\frac{R_1}{2L} + \frac{1}{2RC}\right)^2 - \frac{R_1 + R}{RLC}}$$

$$= -\alpha \pm \beta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where $\quad \alpha = \frac{R_1}{2L} + \frac{1}{2RC} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$

and $\quad \beta = \sqrt{\left(\frac{R_1}{2L} - \frac{1}{2RC}\right)^2 - \frac{1}{LC}} \quad . \quad . \quad . \quad . \quad . \quad (9)$

The complete solution, using (5), (6) and (7), can therefore be written

$$i = A_1\epsilon^{(-\alpha+\beta)t} + A_2\epsilon^{(-\alpha-\beta)t} + \frac{E}{R_1 + R} \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where α and β are determined by (8) and (9), and the constants A_1 and A_2 have to be found from the terminal conditions.

When $t = 0$, $i = 0$; hence

$$A_1 + A_2 + \frac{E}{R_1 + R} = 0 \quad . \quad . \quad . \quad . \quad (11)$$

Also when $t = 0$, $Q = 0$, where Q is the charge on the condenser.

$$Q = \int i_2 dt = CRi_1 \qquad \text{by (3)}$$

$$= CE - CR_1 i - CL\frac{di}{dt} \qquad \text{using (2)}$$

Hence, substituting from (10) and putting $t = 0$,

$$0 = E - R_1\left(A_1 + A_2 + \frac{E}{R_1 + R}\right) - L\{A_1(-\alpha + \beta) + A_2(-\alpha - \beta)\}$$

$$= E\left\{1 - \frac{R_1}{R_1 + R}\right\} - A_1\{L(-\alpha + \beta) + R_1\} - A_2\{L(-\alpha - \beta) + R_1\} \quad . \quad . \quad . \quad (12)$$

(11) and (12) determine A_1 and A_2.

Substituting the value of A_2 from (11) in (12) gives

$$A_1\{L(-\alpha+\beta)+R_1\}+\left\{-A_1-\frac{E}{R_1+R}\right\}\{L(-\alpha-\beta)+R_1\}$$
$$=E\frac{R}{R_1+R}$$

or
$$A_1=\frac{E}{2\beta L}\cdot\frac{R_1+R+L(-\alpha-\beta)}{R_1+R} \quad . \quad . \quad . \quad . \quad (13)$$

Similarly
$$A_2=-\frac{E}{2\beta L}\frac{R_1+R+L(-\alpha+\beta)}{R_1+R} \quad . \quad . \quad . \quad . \quad (14)$$

Hence, putting (13) and (14) into (10), the solution for i is found as

$$i=\frac{E}{R_1+R}\left\{1+\frac{R_1+R+L(-\alpha-\beta)}{2\beta L}\epsilon^{(-\alpha+\beta)t}\right.$$
$$\left.-\frac{R_1+R+L(-\alpha+\beta)}{2\beta L}\epsilon^{(-\alpha-\beta)t}\right\} \quad . \quad (15)$$

$$=\frac{E}{R_1+R}\left\{1-\epsilon^{-\alpha t}\left(\cosh\beta t-\frac{R_1+R-\alpha L}{\beta L}\sinh\beta t\right)\right\} \quad . \quad (16)$$

If β is imaginary the current is oscillatory, as in Fig. 43, and the solution, by writing $\omega=j\beta$, becomes

$$i=\frac{E}{R_1+R}\left\{1-\epsilon^{-\alpha t}\left(\cos\omega t-\frac{R_1+R-\alpha L}{\omega L}\sin\omega t\right)\right\} \quad . \quad (17)$$

Note that if $R=\infty$, (17) reduces to

$$i=\frac{E}{\omega L}\epsilon^{-\alpha t}\sin\omega t,\ \text{with}$$
$$\alpha=\frac{R_1}{2L}\quad\text{and }\omega=\sqrt{\frac{1}{LC}-\frac{R_1^2}{4L^2}}$$

the equation for the oscillatory current in a circuit of series inductance and capacity. Compare equation 61.11.

If $C=0$, the reduction is a little more difficult. From (9)

$$\beta=\left(\frac{R_1}{2L}-\frac{1}{2RC}\right)\sqrt{1-\frac{1}{LC\left(\frac{R_1}{2L}-\frac{1}{2RC}\right)^2}}$$
$$=\left(\frac{R_1}{2L}-\frac{1}{2RC}\right)\sqrt{1-\frac{1}{\frac{L}{4R^2C}\left(1-\frac{RR_1C}{L}\right)^2}}$$

$$= \left(\frac{R_1}{2L} - \frac{1}{2RC}\right)\sqrt{1 - \frac{4R^2C}{L}} \quad \text{very nearly as C approaches 0.}$$

$$= \left(\frac{R_1}{2L} - \frac{1}{2RC}\right)\left(1 - \frac{2R^2C}{L}\right) \quad \text{very nearly by the Binomial}$$

Theorem

$$= \frac{R_1}{2L} + \frac{R}{L} - \frac{1}{2RC} \quad \text{very nearly.}$$

$$\alpha = \frac{R_1}{2L} + \frac{1}{2RC}$$

$$\therefore \alpha + \beta = \frac{R_1 + R}{L}$$

$$\alpha - \beta = \frac{R}{L} + \frac{1}{RC} = \infty \qquad \text{with } C = 0$$

$$\frac{\alpha}{\beta} = -1 \text{ with } C = 0$$

and $\beta = -\infty$ with $C = 0$.

Substitution in (15) gives

$$i = \frac{E}{R_1 + R}\left\{1 - \epsilon^{-\frac{R_1 + R}{L}t}\right\}.$$

the rise of current in an inductance L through a resistance $R_1 + R$. Compare equation 13.03.

If $C = \infty$,

$$\alpha = \frac{R_1}{2L}, \qquad \beta = \frac{R_1}{2L}$$

and (15) becomes

$$i = \frac{E}{R_1 + R}\left\{1 + \frac{R}{R_1} - \frac{R_1 + R}{R_1}\epsilon^{-\frac{R_1}{L}t}\right\}$$

$$= \frac{E}{R_1}\left\{1 - \epsilon^{-\frac{R_1}{L}t}\right\}$$

as it should, since the resistance R is shorted by the infinite capacity.

When $R = 0$ the capacity is shorted. Proceeding as in the case of $C = 0$,

$$\beta = \frac{R_1}{2L} - \frac{1}{2RC}$$

$$a = \frac{R_1}{2L} + \frac{1}{2RC}$$

$$a + \beta = \frac{R_1}{L} \quad a - \beta = \infty \quad \frac{a}{\beta} = -1 \text{ and } \beta = -\infty$$

$\therefore$ From (15) $$i = \frac{E}{R_1}\left\{1 - \epsilon^{-\frac{R_1}{L}t}\right\}$$

Appendix 3.—Artificial Submarine Cable of Two T-links.

The circuital equations Section 15 (Fig. 279) are

$$\left.\begin{aligned} E &= Ri_1 + \frac{1}{C}\int i_1 dt - \frac{1}{C}\int i_2 dt & \text{(i.)} \\ 0 &= 2Ri_2 + \frac{2}{C}\int i_2 dt - \frac{1}{C}\int i_1 dt - \frac{1}{C}\int i_3 dt & \text{(ii.)} \\ 0 &= Ri_3 + \frac{1}{C}\int i_3 dt - \frac{1}{C}\int i_2 dt & \text{(iii.)} \end{aligned}\right\} \quad . \quad . \quad (1)$$

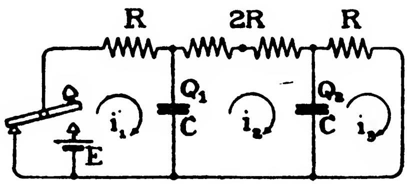


FIG. 279.—Artificial Submarine Cable of 2 T-links.

These differentiated give

$$\left.\begin{aligned} 0 &= R\frac{di_1}{dt} + \frac{1}{C}i_1 - \frac{1}{C}i_2 & \text{(i.)} \\ 0 &= 2R\frac{di_2}{dt} + \frac{2}{C}i_2 - \frac{1}{C}i_1 - \frac{1}{C}i_3 & \text{(ii.)} \\ 0 &= R\frac{di_3}{dt} + \frac{1}{C}i_3 - \frac{1}{C}i_2 & \text{(iii.)} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

From 2 (iii.)

$$i_2 = RC\frac{di_3}{dt} + i_3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

cc

Substituting in 2 (ii.) gives

$$i_1 = 2R^2C^2\frac{d^2i_3}{dt^2} + 4RC\frac{di_3}{dt} + i_3 \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Substitution of (3) and (4) in (2) (i.) and integration gives

$$\frac{d^2i_3}{dt^2} + \frac{3}{RC}\frac{di_3}{dt} + \frac{2}{R^2C^2} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

As a solution of (5) try $i_3 = A\epsilon^{mt}$, leading on substitution to

$$m^2 + \frac{3}{RC}m + \frac{2}{R^2C^2} = 0$$

whence

$$m = -\frac{3}{2RC} \pm \sqrt{\frac{9}{4R^2C^2} - \frac{2}{R^2C^2}}$$

$$= -\frac{1}{RC} \text{ or } -\frac{2}{RC}.$$

A further solution is obviously $i_3 = \frac{E}{4R}$, the final current. The complete solution may therefore be written

$$i_3 = A_1\epsilon^{-\frac{1}{RC}t} + A_2\epsilon^{-\frac{2}{RC}t} + \frac{E}{4R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

When $t = 0, i_3 = 0$

$$\therefore \quad 0 = A_1 + A_2 + \frac{E}{4R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Also when $t = 0$, $i_2 = 0$.

From (3) and (6), putting $t = 0$,

$$A_2 = \frac{E}{4R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

and then from (7)

$$A_1 = -\frac{E}{2R} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Hence, finally,

$$i_3 = \frac{E}{4R}\cdot\left\{1 - 2\epsilon^{-\frac{1}{RC}t} + \epsilon^{-\frac{2}{RC}t}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

From (3) using (10)

$$i_2 = \frac{E}{4R}\left\{1 - \epsilon^{-\frac{2}{RC}t}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and from (2) (ii.) using (10) and (11)

$$i_1 = \frac{E}{4R}\left\{1 + 2\epsilon^{-\frac{1}{RC}t} + \epsilon^{-\frac{2}{RC}t}\right\} \quad . \quad . \quad . \quad . \quad (12)$$

The solutions (10), (11) and (12) may be checked by using them to find the final charges Q_1 and Q_2 in the first and second condensers.

Since the final voltage across the first condenser is $\frac{3}{4}E$, and across the second condenser is $\frac{1}{4}E$, $Q_1 = \frac{3}{4}EC$ and $Q_2 = \frac{1}{4}EC$.

From (11) and (12),

$$Q_1 = \int_0^\infty (i_1 - i_2)\,dt = \frac{E}{4R}\int_0^\infty \left\{2\,\epsilon^{-\frac{t}{RC}} + 2\,\epsilon^{-\frac{2}{RC}t}\right\}dt$$

$$= \frac{E}{4R}\left[-2\,RC\epsilon^{-\frac{t}{RC}} - RC\epsilon^{-\frac{2}{RC}t}\right]_0^\infty$$

$$= \frac{3}{4}EC.$$

From (10) and (11),

$$Q_2 = \int_0^\infty (i_2 - i_3)\,dt = \frac{E}{4R}\int_0^\infty (2\epsilon^{-\frac{1}{RC}t} - 2\epsilon^{-\frac{2}{RC}t})\,dt$$

$$= \frac{E}{4R}\left[-2RC\epsilon^{-\frac{1}{RC}t} + RC\epsilon^{-\frac{2}{RC}t}\right]_0^\infty$$

$$= \frac{1}{4}EC.$$

Appendix 4.—The Kelvin Arrival Curve.

Equations (1), (2), (3) and (4) in section 16 are as follows :—

$$\frac{\partial v}{\partial x} = -Ri \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\frac{\partial i}{\partial x} = -C\frac{\partial v}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$\frac{\partial^2 v}{\partial x^2} = CR\frac{\partial v}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{\partial^2 i}{\partial x^2} = CR\frac{\partial i}{\partial t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

These give relationships which must hold at any instant and at any point in the cable in the case of a cable having only resistance and capacity.

It is further assumed that the resistance of the apparatus at the receiving end is negligible. Thus if a voltage E is maintained at the sending end, the voltage at the receiving end is zero, and the voltage at any intermediate point distance x from the sending end, will, after the steady state has been reached, be given by

$$V = E\left(1 - \frac{x}{l}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where l is the length of the cable, an expression which may be expanded into a Fourier series (see Appendix 1, equation 12) as

$$V = \frac{2E}{\pi} \sum_{m=1}^{m=\infty} \frac{1}{m} \sin \frac{m\pi x}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

During the time in which this voltage distribution is built up, or during the time in which it decays, the voltage at any point must obey the relationship of equation (3).

Consider the expression

$$v = \frac{2E}{m\pi} \epsilon^{-\frac{m^2\pi^2}{CRl^2}t} \sin \frac{m\pi x}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

from which

$$\frac{\partial^2 v}{\partial x^2} = -2E\frac{m\pi}{l^2} \epsilon^{-\frac{m^2\pi^2}{CRl^2}t} \sin \frac{m\pi x}{l}$$

and

$$\frac{\partial v}{\partial t} = -2E\frac{m\pi}{CRl^2} \epsilon^{-\frac{m^2\pi^2}{CRl^2}t} \sin \frac{m\pi x}{l};$$

i.e.,

$$\frac{\partial^2 v}{\partial x^2} = CR\frac{\partial v}{\partial t}$$

The expression (7) therefore satisfies (3); so also does an infinite series with m varying from 1 to ∞. Moreover, with $t = 0$, this series is identical with (6), and with $t = \infty$ the series has the value zero.

The series

$$v = \frac{2E}{\pi} \sum_{m=1}^{m=\infty} \frac{1}{m} \epsilon^{-\frac{m^2\pi^2}{CRl^2}t} \sin \frac{m\pi x}{l} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

therefore gives the voltage at any point x of the cable at a time t after the sending end voltage E has been removed (the sending end earthed).

It follows that the voltage at any time t at any point x after the *application* of a potential E is given by $(E - v)$ or by

$$v = \frac{2E}{\pi}\left[\sum_{m=1}^{m=\infty} \frac{1}{m} \sin \frac{m\pi x}{l} - \sum_{m=1}^{m=\infty} \frac{1}{m} \epsilon^{-\frac{m^2\pi^2}{CRl^2}t} \sin \frac{m\pi x}{l}\right] \quad . \quad (9)$$

and this is the required solution.

The corresponding current is to be obtained as $i = -\frac{1}{R}\frac{\partial v}{\partial x}$ (from (1)), giving

$$i = \frac{E}{Rl}\left[1 + 2\sum_{m=1}^{m=\infty} \epsilon^{-\frac{m^2\pi^2}{CRl^2}t} . \cos \frac{m\pi x}{l}\right] \quad . \quad . \quad . \quad (10)$$

(using (5)).

At the receiving end $x = l$ and

$$i_r = \frac{E}{Rl}\left[1 + 2\sum_{m=1}^{m=\infty} \epsilon^{-\frac{m^2\pi^2}{CRl^2}t} \cos m\pi\right]$$

and at the sending end $x = 0$ and

$$i_s = \frac{V_s}{Rl}\left[1 + 2\sum_{m=1}^{m=\infty} \epsilon^{-\frac{m^2\pi^2}{CRl^2}t}\right]$$

which are the solutions 16.05 and 16.06 which were to be found.

Appendix 5.—Arrival Curves by Heaviside's Expansion Theorem.

(i.) *Including Resistance of Receiving Apparatus*

Consider the case of a submarine cable of resistance R ohms and capacity C farads per mile, and resistance of terminal apparatus R_r ohms. It is required to find the current at the receiving end, using equation (13), Appendix 1.

The steady state periodic solution is from 39.40

$$I_r = \frac{E_s}{R_r \cosh Pl + Z_o \sinh Pl} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where, from 39.09, putting $L = O$ and $G = O$,

$$P = \sqrt{j\omega CR}$$

and, from 39.10

$$Z_o = \sqrt{\frac{R}{j\omega C}}$$

Thus $\quad \phi(y) = R_r \cosh l\sqrt{yCR} + \sqrt{\frac{R}{yC}} \sinh l\sqrt{yCR} \quad . \quad . \quad . \quad (2)$

Write
$$jx = l\sqrt{yCR} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

so that
$$y = -\frac{x^2}{CRl^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Then the equation $\phi(y) = 0$, of which the roots are required, becomes

$$R_r \cos x + \frac{Rl}{x} \sin x = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

or
$$\tan x = -\frac{R_r}{Rl} x \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$\frac{d\phi(y)}{dy} = \Big\{ R_r l\sqrt{CR} \sinh l\sqrt{yCR}$$
$$+ \frac{Rl}{\sqrt{y}} \cosh l\sqrt{yCR} - \frac{1}{y}\sqrt{\frac{R}{C}} \sinh l\sqrt{yCR} \Big\} \frac{1}{2\sqrt{y}}$$

and
$$y\phi'(y) = \frac{1}{2} R_r l\sqrt{CRy} \sinh l\sqrt{yCR} + \frac{Rl}{2} \cosh l\sqrt{yCR}$$
$$- \frac{1}{2}\sqrt{\frac{R}{Cy}} \sinh l\sqrt{yCR}$$
$$= -\frac{1}{2} R_r x \sin x + \frac{Rl}{2} \cos x - \frac{Rl}{2x} \sin x \quad . \quad . \quad . \quad (7)$$

and on substituting from (6)

$$= \frac{Rl}{2} \Big\{ \frac{\sin^2 x}{\cos x} + \cos x - \frac{\sin x}{x} \Big\}$$
$$= \frac{Rl}{2} \Big\{ \frac{1}{\cos x} - \frac{\sin x}{x} \Big\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

$$\phi(0) = R_r + Rl \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Hence the arrival current is given by

$$i_r = \frac{E}{Rl + R_r} + \Sigma_x \frac{2E\epsilon^{-\frac{x^2}{CRl^2}t}}{Rl\left(\frac{1}{\cos x} - \frac{\sin x}{x}\right)} \quad . \quad . \quad . \quad . \quad (10)$$

where the various x values are found from (6). When $R_r = 0$, the x values are found from $\tan x = 0$, *i.e.*, $x = m\pi$, and (10) becomes

$$i_r = \frac{E}{Rl} + \Sigma_m \frac{2E\epsilon^{-\frac{\pi^2 m^2}{CRl^2}t}}{Rl} \cos m\pi$$

which is Kelvin's solution.

(ii.) *Double Block*

Let (Fig. 60) the capacity of each blocking condenser be K, and neglect the resistance of the instrument.

Then the periodic solution from 39.40 is

$$I_r = \frac{E_s}{-\frac{2j}{\omega K}\cosh\sqrt{j\omega CR}\,l + \left(\sqrt{\frac{R}{j\omega C}} - \frac{1}{\omega^2 K^2}\sqrt{\frac{j\omega C}{R}}\right)\sinh\sqrt{j\omega CR}\,l} \qquad (11)$$

and hence

$$\phi(y) = \frac{2}{yK}\cosh\sqrt{CRy}\,l + \left(\sqrt{\frac{R}{yC}} + \frac{1}{y^2K^2}\sqrt{\frac{yC}{R}}\right)\sinh\sqrt{CRy}\,l$$

$$= -\frac{2CRl^2}{x^2K}\cos x + \left(\frac{Rl}{x} - \frac{1}{K^2}\cdot\frac{C^2Rl^3}{x^3}\right)\sin x \quad . \quad . \quad . \quad (12)$$

on writing $jx = l\sqrt{CRy}$.

$\phi(y) = 0$ when

$$\tan x = \frac{2Cl}{Kx - \frac{C^2l^2}{Kx}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

$$y\phi'(y) = y\frac{d\phi(y)}{dx}\cdot\frac{dx}{dy} = \frac{d\phi(y)}{dx}\cdot y\cdot\frac{1}{2}\frac{l\sqrt{CR}}{j\sqrt{y}}$$

$$= \frac{x}{2}\frac{d\phi(y)}{dx}.$$

Hence from (12)

$$y\phi'(y) = \frac{x}{2}\left[\frac{4CRl^2}{x^3K}\cos x + \frac{2CRl^2}{x^2K}\sin x + \left(\frac{Rl}{x} - \frac{1}{K^2}\cdot\frac{C^2Rl^3}{x^3}\right)\cos x\right.$$
$$\left. + \left(-\frac{Rl}{x^2} + \frac{3}{K^2}\frac{C^2Rl^3}{x^4}\right)\sin x\right] \quad . \quad . \quad . \quad . \quad (14)$$

$$= \frac{Rl}{2}\left[\left(\frac{4CL}{Kx^2} + 1 - \frac{C^2l^2}{K^2x^2}\right)\cos x + \left(\frac{2Cl}{Kx} - \frac{1}{x} + \frac{3C^2l^2}{K^2x^3}\right)\sin x\right]$$

$$= \frac{Rl}{2}\left[\left(\frac{4Cl}{Kx^2} + 1 - \frac{C^2l^2}{K^2x^2}\right)\cos x\right.$$
$$\left. + \left\{\frac{2Cl}{Kx} + \frac{2}{x} - \frac{3}{x}\left(1 - \frac{C^2l^2}{K^2x^2}\right)\right\}\sin x\right].$$

Now from (13) $\quad 1 - \dfrac{C^2l^2}{K^2x^2} = \dfrac{2Cl}{Kx}\cdot\dfrac{\cos x}{\sin x}$

Hence

$$y\phi'(y) = \frac{Rl}{2}\left[\frac{4Cl}{Kx^2}\cos x + \frac{2Cl}{Kx}\,\frac{\cos^2 x}{\sin x} + \frac{2Cl}{Kx}\sin x + \frac{2\sin x}{x} - \frac{6Cl}{Kx^2}\cos x\right]$$

$$= Rl\left[\frac{\sin x}{x} + \frac{Cl}{Kx}\left(\frac{1}{\sin x} - \frac{\cos x}{x}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Also
$$\phi(0) = \infty \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Hence the arrival current is given by

$$i_r = \sum_x \frac{E\epsilon^{-\frac{x^2}{CRl^2}t}}{Rl\left[\frac{\sin x}{x} + \frac{Cl}{Kx}\left(\frac{1}{\sin x} - \frac{\cos x}{x}\right)\right]} \quad . \quad . \quad . \quad . \quad (17)$$

where the x values are the multiple roots of (13). The x values are best found graphically in each numerical case and i_r found as the sum if the series (17). In this way Malcolm calculated the figures given below in Table II. for a cable of total resistance = 4,975 ohms and total capacity 875 μF, with blocking condensers each equal to $Cl/10 = 87{\cdot}5$ μF. The table can be used for any other cable with blocking condensers each equal to $Cl/10$ by multiplying the times by $\frac{CRl^2}{4{\cdot}351}$ and the currents by $\frac{4{,}975}{Rl}$.

Tables of ut/f(ut) and Arrival Current with Double Block Condensers

TABLE I.—$ut/f(ut)$

ut	$f(ut)$	ut	$f(ut)$
0·1	0·000	1·1	0·179
0·2	0·000	1·2	0·207
0·23	0·000	1·3	0·233
0·3	0·001	1·4	0·257
0·4	0·006	1·5	0·279
0·5	0·018	2·0	0·365
0·6	0·037	2·5	0·418
0·7	0·062	3·0	0·450
0·8	0·091	4·0	0·482
0·9	0·121	5·0	0·493
1·0	0·150	10·0	0·500

TABLE 11.—*Arrival Current, in microamps per volt, for a cable of 4,975 ohms total resistance and 875 μF total capacity with double block condensers each equal to 87·5 μF.*

Time.	Current.	Time.	Current.	Time.	Current.	Time.	Current.
0·10	0·04	0·90	5·02	1·70	1·47	3·00	0·19
0·20	1·29	1·00	4·36	1·80	1·26	3·50	0·08
0·30	4·16	1·10	3·76	1·90	1·07	4·06	0·03
0·40	6·21	1·20	3·23	2·00	0·92	—	—
0·50	6·97	1·30	2·77	2·20	0·67	—	—
0·60	6·89	1·40	2·36	2·40	0·49	—	—
0·70	6·38	1·50	2·02	2·60	0·35	—	—
0·80	5·72	1·60	1·73	2·80	0·26	—	—

Appendix 6.—Equality of Duplex Meshes.

To find the values of the resistances r, s_1, s_2 and the condensers K_1 and K_2 of Fig. 52 in order that the current i_1 into the mesh may be at every instant the same as the current i_1 into the mesh of Fig. 46.

Writing the operator $d/dt = D$, $\int dt = D^{-1}$, etc., the circuital equations for the mesh of Fig. 46 (equation 15.11) become

$$\left.\begin{aligned} E &= Ri_1 + \frac{1}{C}i_1D^{-1} - \frac{1}{C}i_2D^{-1} \\ 0 &= 2Ri_2 + \frac{2}{C}i_2D^{-1} - \frac{i_1}{C}D^{-1} - \frac{i_3}{C}D^{-1} \\ 0 &= Ri_3 + \frac{1}{C}i_3D^{-1} - \frac{1}{C}i_2D^{-1} \end{aligned}\right\}$$

From the third $\qquad i_3 = \dfrac{i_2}{RCD + 1}$

which used in the second gives

$$i_2 = \frac{i_1}{2RCD + 2 - \dfrac{1}{RCD + 1}}$$

and this in the first gives

$$E = i_1 \left\{ R + \frac{1}{CD} - \frac{1}{2RC^2D^2 + 2CD - \dfrac{CD}{RCD + 1}} \right\}$$

$$= i_1 \left\{ \frac{2R^3C^2D^2 + 6K^2CD + 4R}{2R^2C^2D^2 + 4RCD + 1} \right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

The circuital equations for the mesh of Fig. 52 are

$$\left.\begin{aligned} E &= ri_1 - ri_2 \\ 0 &= (r + s_1)i_2 - ri_1 + \frac{1}{K_1}\int i_2 dt - \frac{1}{K_1}\int i_3 dt \\ 0 &= \left(\frac{1}{K_1} + \frac{1}{K_2}\right)\int i_3 dt - \frac{1}{K_1}\int i_2 dt + s_2 i_3 \end{aligned}\right\}$$

which on the introduction of the operators D and D^{-1} become

$$\left.\begin{aligned} E &= ri_1 - ri_2 \\ 0 &= \left(r + s_1 + \frac{D^{-1}}{K_1}\right) i_2 - ri_1 - \frac{D^{-1}}{K_1} i_3 \\ 0 &= \left(\frac{D^{-1}}{K_1} + \frac{D^{-1}}{K_2} + s_2\right) i_3 - \frac{D^{-1}}{K_1} i_2 \end{aligned}\right\}$$

From the third

$$i_3 = \frac{1}{1 + \dfrac{K_1}{K_2} + s_2 K_1 D} i_2$$

which, used to eliminate i_3 from the second, gives

$$0 = \left(r + s_1 + \frac{1}{K_1 D} - \frac{1}{K_1 D + \dfrac{K_1^2}{K_2} D + s_1 K_1^2 D^2}\right) i_2 - ri_1$$

and substituting i_2 from this in the first gives

$$E = \left\{ r - \frac{r^2}{r + s_1 + \dfrac{1}{K_1 D} - \dfrac{1}{K_1 D + \dfrac{K_1^2}{K_2} D + s_2 K_1^2 D^2}} \right\} i_1$$

which on rearrangement becomes

$$E = \frac{rs_1s_2K_1K_2D^2 + \{rs_1(K_1 + K_2) + rs_2K_2\}D + r}{(r + s_1)s_2K_1K_2D^2 + \{(r + s_1)(K_1 + K_2) + s_2K_2\}D + 1} i_1$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

For this to be identical with (1) the numerators and the denominators must be separately equal and the coefficients of D and D^2 must be separately equal.

From the numerators it follows at once that

$$r = 4R \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

and also that

$$rs_1s_2K_1K_2 = 2R^3C^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$rs_1(K_1 + K_2) + rs_2K_2 = 6K^2C \quad . \quad . \quad . \quad . \quad (5)$$

and from the denominators

$$(r + s_1)s_2K_1K_2 = 2R^2C^2 \quad . \quad . \quad . \quad . \quad (6)$$

$$(r + s_1)(K_1 + K_2) + s_2K_2 = 4RC \quad . \quad . \quad . \quad (7)$$

Dividing (6) by (4) gives

$$\left.\begin{aligned} \frac{r + s_1}{rs_1} = \frac{1}{R} \quad \text{whence } s_1 &= \frac{4}{3}R \\ \text{or using (3),} \qquad s_1 &= \frac{r}{3} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (8)$$

(5) divided by (7) gives, using (3) and (8),

$$15\, s_2K_2 = 4r(K_1 + K_2) \quad . \quad . \quad . \quad . \quad . \quad (9)$$

From (7); using (3) and (8)

$$\frac{4}{3}r(K_1 + K_2) + s_2K_2 = rC \quad . \quad . \quad . \quad . \quad (10)$$

and from (9) and (10)

$$15\, C = 24(K_1 + K_2) \quad . \quad . \quad . \quad . \quad . \quad (11)$$

From (6), using (3), (8), (9) and (10),

$$K_1 = \frac{9}{16}C \quad . \quad . \quad . \quad . \quad . \quad (12)$$

and from (11)

$$K_2 = \frac{1}{16}C \quad . \quad . \quad . \quad . \quad (13)$$

Finally, from (9), using (11), (12) and (13),

$$s_2 = \frac{8}{3}r = \frac{32}{3}R \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Appendix 7.—Use of Kennelly's Tables and Charts.

The tables* of most general use in telephone line calculations are Tables VII., VIII. and IX., giving hyperbolic sines, cosines and tangents of $(x + i\underline{q})$ in the form $(u + iv)$, and Tables X., XI. and XII., giving hyperbolic sines, cosines and tangents of $(x + i\underline{q})$ in the form $r/\underline{\gamma}$. x extends from 0 to 3·95 and q from 0 to 2. i has the same significance as j. q is the imaginary "quadranted," that is, divided by $\pi/2$.

Tables VII., VIII. and IX. are arranged as shown by the following extract from Table VII. of hyperbolic sines.

q	$x = 0{\cdot}25$		$x = 0{\cdot}3$	
0	0·25261	0·00	0·30452	0·00
0·05	0·25183	0·08092	0·30358	0·08202
0·1	0·24950	0·16135	0·30077	0·16353
0·15	0·24563	0·24078	0·29611	0·24403
0·2	0·24025	0·31872	0·28962	0·32303

Suppose that it is desired to find the value of $x + iy$. y is first divided by $\pi/2$ to find q. q is now divided by 4, and the remainder only considered. If the remainder is less than 2, the table can be entered with this figure as q. If the remainder is greater than 2, 2 is subtracted from it, the table entered with the figure obtained as q, and the value of $u + iv$ found is multiplied by (-1).

(i.) Let $x = 0{\cdot}25$ and $y = 0{\cdot}2355$

$$q = \frac{0{\cdot}2355}{\pi/2} = 0{\cdot}15$$

This is less than 2, so

$$\sinh(0{\cdot}25 + i\,0{\cdot}2355) = \sinh(0{\cdot}25 + i\,\underline{0{\cdot}15})$$

The horizontal line under the 0·15 is to show that it is a quadranted number.

From the table it is found that

$$\sinh(0{\cdot}25 + i\,\underline{0{\cdot}15}) = 0{\cdot}24563 + i\,0{\cdot}24078$$

* "Tables of Complex Hyberbolic and Circular Functions," A. E. Kennelly.

(ii.) Let $x = 0{\cdot}25$ and $y = 6{\cdot}52$

$$q = \frac{6{\cdot}52}{\pi/2} = 4{\cdot}15$$

Dividing q by 4, the remainder is 0·15. Hence

$$\begin{aligned} \sinh(0{\cdot}25 + i\,6{\cdot}52) &= \sinh(0{\cdot}25 + i\,\underline{4{\cdot}15}) \\ &= \sinh(0{\cdot}25 + i\,\underline{0{\cdot}15}) \\ &= 0{\cdot}24563 + i\,0{\cdot}24078 \end{aligned}$$

(iii.) Let $x = 0{\cdot}25$ and $y = 3{\cdot}378$

$$q = \frac{3{\cdot}378}{\pi/2} = 2{\cdot}15$$

q is between 2 and 4, hence 2 is subtracted and the result from the table multiplied by (-1). Thus

$$\begin{aligned} \sinh(0{\cdot}25 + i\,3{\cdot}378) &= \sinh(0{\cdot}25 + \underline{2{\cdot}15}) \\ &= -\sinh(0{\cdot}25 + \underline{0{\cdot}15}) \\ &= -0{\cdot}24563 - i\,0{\cdot}24078. \end{aligned}$$

Conversely, in finding the complex quantity whose hyperbolic sine is a given complex, any multiple of 4 may be added to the value of q found, and finally the imaginary is de-quadranted by multiplying by $\pi/2$.

Thus $\sinh^{-1}(0{\cdot}29611 + i\,0{\cdot}24403)$

$$= 0{\cdot}3 + i\,\underline{0{\cdot}15} = 0{\cdot}3 + i\,2{\cdot}355$$

or $$= 0{\cdot}3 + i\,\underline{4{\cdot}15} = 0{\cdot}3 + i\,6{\cdot}52$$

or $$= 0{\cdot}3 + i\,\underline{8{\cdot}15} = 0{\cdot}3 + i\,12{\cdot}8$$

and so on.

A negative sign is to be placed in front of figures appearing in the tables in heavy type.

Similar rules apply for the use of Tables X., XI. and XII., but the hyperbolic sine, cosine or tangent is entered in the form $r\underline{/\gamma}$, γ being in degrees.

The reason for these rules will be understood from the following considerations.

Putting $\cosh(x + iy)$ in its exponential form,

$$\cosh(x + iy) = \frac{\epsilon^{x+iy} + \epsilon^{-(x+iy)}}{2}$$

$$= \frac{\epsilon^{x}}{2}\,.\,\epsilon^{iy} + \frac{\epsilon^{-x}}{2}\,.\,\epsilon^{-iy} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

In Fig. 280 the first term is OA, found by rotating a length $\epsilon^x/2$ counter-clockwise through an angle y radians, the second term is OB, found by rotating a length $\epsilon^{-x}/2$ clockwise through an angle y radians, and the vector sum is OP, which is by (1) cosh $(x + iy)$,

$$\sinh (x + iy) = \frac{\epsilon^{x+iy} - \epsilon^{-(x+iy)}}{2}$$

$$= \frac{\epsilon^x}{2} \epsilon^{iy} - \frac{\epsilon^{-x}}{2} \epsilon^{-iy} \quad . \quad . \quad . \quad . \quad . \quad (2)$$

The sinh of $(x + iy)$ is therefore found by multiplying OB by -1 to give OB′, and adding OA and OB′ vectorially to give OQ.

If y is increased by 2π, or by any multiple of 2π, the same complex quantities OP and OQ represent the cosh and sinh respectively. If y is increased by π, or by π plus any multiple of 2π, the cosh and sinh are represented by OQ′ and OP′ respectively, drawn equal to and in the opposite direction to OQ and OP.

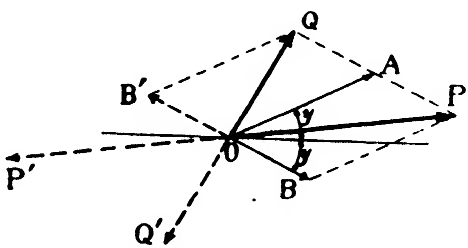


FIG. 280.—Sinh and Cosh of Complex Quantity.

If any complex quantity $a + ib$ is transferred to a new complex plane, keeping a the same in each, but making the imaginary $\dfrac{b}{\pi/2}$ in the new plane, the angle of the complex is $\tan^{-1} b/a$, which is measured by radians in the old plane, but in the new plane it is $\tan^{-1} b/\dfrac{a\pi}{2}$, and is measured in quadrants. Thus a right angle in the old plane measures $\pi/2$ radians, but in the new plane it measures 1 quadrant. The angles of rotation in (1) and (2) become in the new plane ϵ^{iq} and ϵ^{-iq} quadrants instead of ϵ^{iy} and ϵ^{-iy} radians, where $q = y \div \pi/2$, and the sinhs and coshs have the same value when q is increased by 4 quadrants, and have the same numerical value, but with a minus sign affixed when q is increased by 2 quadrants.

This quadranting of the imaginary thus shortens the tables by taking advantage of the periodic property of the hyperbolic functions of complex quantities.

Interpolation from the tables is difficult, but is easy from specially constructed charts.*

(i.) The locus of $\cosh(x + iy)$—or, to be more accurate, the locus of the extremity of the line representing the complex quantity $\cosh(x + iy)$—when y is varied but x remains constant, is an ellipse. For

$$\cosh(x + iy) = \cosh x \cos y + i \sinh x \sin y \quad . \quad (3)$$

which expressed in Cartesian co-ordinates, in which u is written for the abissa and v for the ordinate (instead of the more usual x and y), gives

$$u = \cosh x \cos y \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

$$v = \sinh x \sin y \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

By squaring (4) and (5), and dividing $(4)^2$ by $\cosh^2 x$ and $(5)^2$ by $\sinh^2 x$ and adding, the variable y is eliminated, and the equation obtained, viz :—

$$\frac{u^2}{\cosh^2 x} + \frac{v^2}{\sinh^2 x} = 1 \quad . \quad . \quad . \quad . \quad (6)$$

is the equation of an ellipse, whose horizontal major axis is $2 \cosh x$ and vertical minor axis is $2 \sinh x$.

(ii.) If x is varied but y remains constant the corresponding locus is a hyperbola. For subtracting (5) squared divided by $\sin^2 y$ from (4) squared divided by $\cos^2 y$ gives

$$\frac{u^2}{\cos^2 y} - \frac{v^2}{\sin^2 y} = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

Ellipses for values of x equal to 0·5 and 1·0, and hyperbolæ for values of y equal to 0·314, 0·784 and 1·256, are drawn in Fig. 281. The y values correspond to quadranted (q) values of $\underline{0{\cdot}2}$, $\underline{0{\cdot}5}$ and $\underline{0{\cdot}8}$. F_1F_2 are the foci of the curves. Limiting hyperbolæ are the lines F_1U and F_2U' ($q = 0$) and $V'OV$ ($q = \underline{1{\cdot}0}$).

Each hyperbola crosses each ellipse at four points, but the quadrant in which u and v are to be read is the same as that in which the complex $(x + iy)$ lies. This is clear from the construction of Fig. 280 or from (4) and (5). The q values affixed to the hyperbolæ differ for this reason in the four quadrants, being 0 (F_1U), $\underline{0{\cdot}2}$, $\underline{0{\cdot}5}$, $\underline{0{\cdot}8}$ (first quadrant), $\underline{1{\cdot}0}$ (OV), $\underline{1{\cdot}2}$, $\underline{1{\cdot}5}$, $\underline{1{\cdot}8}$ (second quadrant), $\underline{2{\cdot}0}$ (F_2U'), $\underline{2{\cdot}2}$, $\underline{2{\cdot}5}$, $\underline{2{\cdot}8}$ (third quadrant), $\underline{3{\cdot}0}$ (OV′) and $\underline{3{\cdot}2}$, $\underline{3{\cdot}5}$, $\underline{3{\cdot}8}$ (fourth quadrant), and $4{\cdot}0 = 0{\cdot}0$ (OV). In Kennelly's charts the

* A. E. Kennelly, "Chart Atlas of Complex Hyberbolic Functions."

whole numbers are omitted, to avoid confusion when the chart is used for sinhs.

It is a simple matter to read off values of coshs from the diagram. For instance,

$$\cosh(1{\cdot}0 + i\,\underline{0{\cdot}8}) = 0{\cdot}48 + i\,1{\cdot}12 \qquad \text{(point } p_1)$$
$$\cosh(0{\cdot}5 + i\,\underline{1{\cdot}5}) = -\,0{\cdot}80 + i\,0{\cdot}37 \qquad \text{(point } p_2)$$
$$\cosh(1{\cdot}0 + i\,\underline{3{\cdot}8}) = 1{\cdot}47 - i\,0{\cdot}36 \qquad \text{(point } p_3)$$

Actually all these values could have been read from one quadrant of

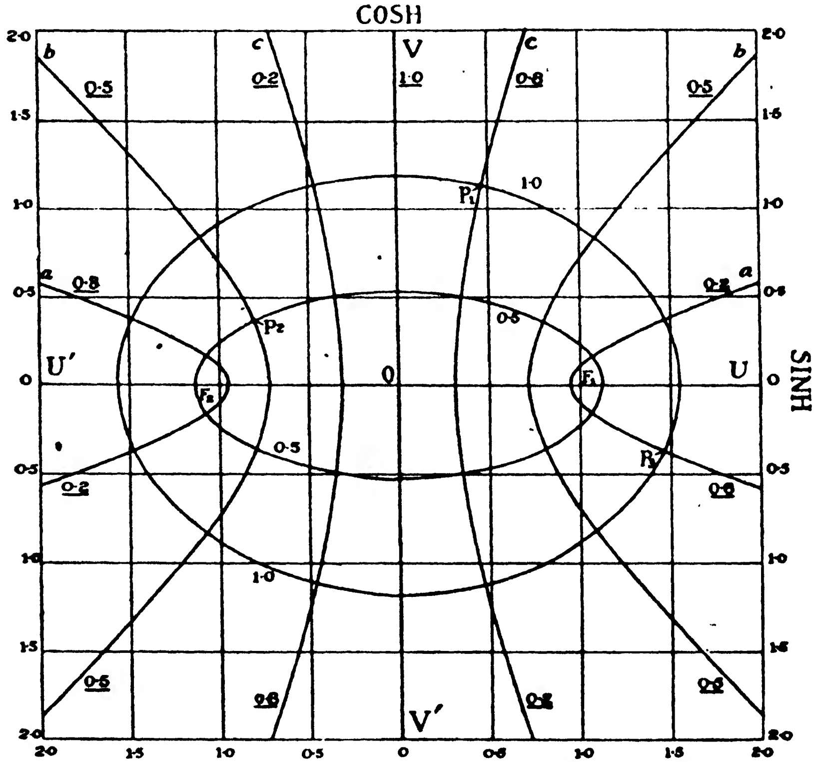


FIG. 281.—Loci of Cosh and Sinh of complex quantities.

the figure by suitable choice of q value and affixing suitable signs to u and v, and in Kennelly's charts only two quadrants are drawn.

The same diagram can equally well be used for finding sinhs.

(iii.) Proceeding as before,

$$\sinh(x + iy) = \sinh x \cos y + i \cosh x \sin y \tag{8}$$

which in Cartesian co-ordinates (OU, OV) gives

$$u = \sinh x \cos y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

$$v = \cosh x \sin y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Eliminating y as before gives

$$\frac{u^2}{\sinh^2 x} + \frac{v^2}{\cosh^2 x} = 1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

and eliminating x gives

$$-\frac{u^2}{\cos^2 y} + \frac{v^2}{\sin^2 y} = 1 \quad . \quad . \quad . \quad . \quad . \quad (12)$$

(11) and (12) are identical with (6) and (7) if u and v are interchanged. Hence if the diagram of Fig. 281 is turned through a right angle counter-clockwise, so that OV′ is read as the u axis and OU as the v axis, sinhs can be read off the diagram, the integer of q being 0 in the V′U quadrant, 1 in the UV quadrant, 2 in the VU′ quadrant and 3 in the U′V′ quadrant. Thus

$$\sinh(1{\cdot}0 + i\,\underline{0{\cdot}8}) = 0{\cdot}36 + i\,1{\cdot}47 \qquad \text{(point } p_3\text{)}$$

$$\sinh(1{\cdot}0 + i\,\underline{1{\cdot}8}) = -1{\cdot}12 + i\,0{\cdot}48 \qquad \text{(point } p_1\text{)}$$

$$\sinh(0{\cdot}5 + i\,\underline{2{\cdot}5}) = -0{\cdot}37 - i\,0{\cdot}80 \qquad \text{(point } p_2\text{)}$$

Appendix 8.—Star-mesh Transformation.

Calculations in complicated networks are frequently facilitated by the use of a star-mesh transformation, first given by Kennelly * for a three-ray star, and extended to a four-ray star by Kupfmüller,† and to a star of any number of rays by Rosen.‡

Let OA, OB, OC . . . represent a star in any complicated network, and let the admittances of the rays OA, OB, OC . . . be a, b, c, etc. (Generally, in the case of an a.c. network, a, b, c, etc., will be complex quantities.) The theorem states that the star can be replaced by a mesh network eliminating O, without altering the currents and potentials in the network external to the star, by giving the admittances between the points AB, AC, BC, etc., the values

$$Y_{AB} = \frac{ab}{\Sigma a}, \; Y_{AC} = \frac{ac}{\Sigma a}, \; Y_{BC} = \frac{bc}{\Sigma a}, \text{ etc.}$$

where $\Sigma a = a + b + c + \ldots$

* A. E. Kennelly, *Elect. World*, 1899, Vol. 34, p. 431.
† K. Kupfmüller, *Archiv. für Electrotecknik*, 1923, Vol. 12, p. 160.
‡ A. Rosen, *J.I.E.E.*, 1924, Vol. 62, p. 916.

If there are n rays in the star, there will be $\frac{1}{2}n(n-1)$ rays in the mesh.

(i.) As an example,* let the figures in circles in Fig. 282 (*a*) be the conductances in mhos of the various branches of the network ABCDE, and let it be required to find what current a 4-volt battery applied across E and D will supply to the network.

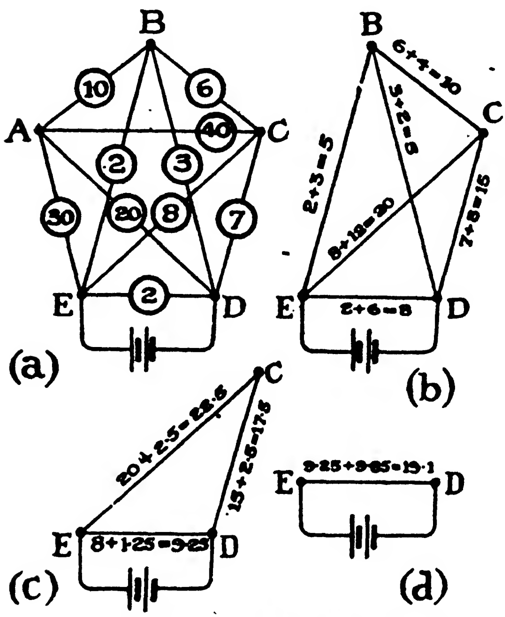


Fig. 282.—Star-mesh Transformation.

(*a*) Replace the star AB, AC, AD, AE, and so eliminate A. $\Sigma(a) = 10 + 40 + 20 + 30 = 100$ mhos. Hence $Y_{BC} = 10 \times 40/100 = 4$, $Y_{CD} = 40 \times 20/100 = 8$, $Y_{DE} = 20 \times 30/100 = 6$, $Y_{BE} = 10 \times 30/100 = 3$, $Y_{BD} = 10 \times 20/100 = 2$, and $Y_{DE} = 20 \times 30/100 = 6$, all in mhos. These conductances are added to the existing conductances to give the totals shown in Fig. 282 (*b*).

(*b*) The point B is next eliminated in the same way. $\Sigma(a) = 10 + 5 + 5 = 20$. $Y_{CD} = 10 \times 5/20 = 2{\cdot}5$, $Y_{DE} = 5 \times 5/20 = 1{\cdot}25$ and $Y_{CE} = 10 \times 5/20 = 2{\cdot}5$. The total conductances are then as shown in Fig. 282 (*c*).

(*c*) Finally, the point C is eliminated. $\Sigma(a) = 22{\cdot}5 + 17{\cdot}5 = 40$, and $Y_{ED} = 22{\cdot}5 \times 17{\cdot}5/40 = 9{\cdot}85$. The total conductances between E and D (Fig. 282 (*d*)) is thus 19·1 mhos, and the battery supplies $4 \times 19{\cdot}1 = 76{\cdot}4$ amperes.

(ii.) The theorem can be applied † to obtain the relations among

* From Rosen's paper.

† A. Rosen, "Interference between Circuits in Continuously Loaded Telephone Cables," *J.I.E.E.*, Vol. 66, p. 165.

the various quad capacities that must hold, in order that cross-talk and overhearing may be avoided (see p. 213). Fig. 283 shows the various capacities involved. Eliminating the point E, the quantities shown in Fig. 284 result. (Capacities can be added, since the admittances of the condensers are proportional to the capacity values.)

In the figure

$$\Delta = a + b + c + d \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

(Δ is the same as Σa above).

The capacities between AD, AC, BC and BD are all nearly equal; hence if a voltage is established across AB, there will be practically

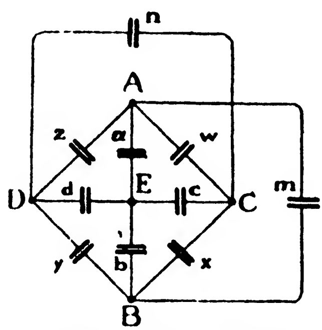


FIG. 283.—Quad capacities.

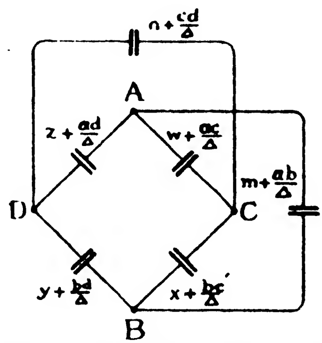


FIG. 284.—E eliminated from Fig. 283.

no voltage across DC if the difference between the capacities AC and AD is the same as the difference between the capacities CB and DB, *i.e.*, if

$$\left(w + \frac{ac}{\Delta}\right) - \left(z + \frac{ad}{\Delta}\right) = \left(x + \frac{bc}{\Delta}\right) - \left(y + \frac{bd}{\Delta}\right)$$

i.e., $$(w - z) + \frac{a}{\Delta}(c - d) = (x - y) + \frac{b}{\Delta}(c - d) \quad . \quad . \quad (2)$$

or $$(w - x) + \frac{c}{\Delta}(a - b) = (z - y) + \frac{d}{\Delta}(a - b) \quad . \quad . \quad (3)$$

Equations (2) or (3) thus express the requirement for no cross-talk between the two side circuits.

To find the conditions for no overhearing between phantom and side circuits the network may be redrawn as in Fig. 285, where G represents the admittance of the transformers (compare Fig. 154). Voltage is applied across P and Q, and listening is on A and B or C and D, and *vice versa*. Considering A and B, the potential across

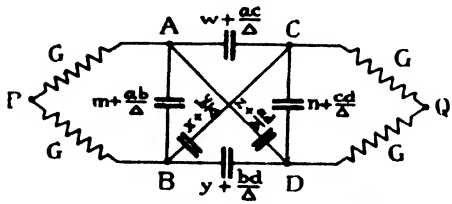


FIG. 285.—Showing phantom connections.

C and D is small for small out of balances, and the admittances between C and D may be neglected. Write

$$\left.\begin{aligned} W &= j\omega\left(w + \frac{ac}{\Delta}\right) \\ X &= j\omega\left(x + \frac{bc}{\Delta}\right) \\ Y &= j\omega\left(y + \frac{bd}{\Delta}\right) \\ \text{and}\qquad Z &= j\omega\left(z + \frac{ad}{\Delta}\right) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

Eliminating C as the star point of the star CA, CB, CQ gives

$$\frac{WG}{W + X + G} \text{ for the admittance of AQ}$$

and

$$\frac{XG}{W + X + G} \text{ for the admittance of BQ}$$

Eliminating D as the star point of the star DB, DA, DG gives

$$\frac{ZG}{Y + Z + G} \text{ for the admittance of AQ}$$

and

$$\frac{YG}{Y + Z + G} \text{ for the admittance of BQ}$$

Thus in Fig. 286 the total admittances of AQ and BQ are

$$\frac{WG}{W + X + G} + \frac{ZG}{Y + Z + G} \quad . \quad . \quad . \quad . \quad (5)$$

and
$$\frac{XG}{W+X+G}+\frac{YG}{Y+Z+G} \quad . \quad . \quad . \quad . \quad (6)$$

respectively, and for no overhearing these must be equal, *i.e.*,

$$\frac{(W-X)\,G}{W+X+G}=\frac{(Y-Z)\,G}{Y+Z+G} \quad . \quad . \quad . \quad . \quad (7)$$

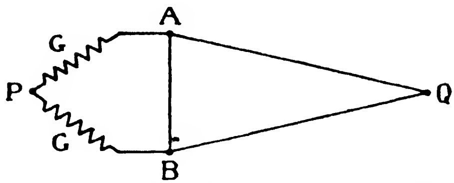


FIG. 286.—C and D eliminated from Fig. 285.

The denominators on each side are nearly equal, hence the condition may be written

$$W-X=Y-Z$$

or, from equation (4),

$$\left(\omega+\frac{ac}{\Delta}\right)-\left(x+\frac{bc}{\Delta}\right)=\left(y+\frac{bd}{\Delta}\right)-\left(z+\frac{ad}{\Delta}\right)$$

i.e.,
$$(w-x)+\frac{c}{\Delta}\cdot(a-b)=(y-z)+\frac{d}{\Delta}\cdot(b-a) \quad . \quad . \quad (8)$$

The same equation results from a consideration of the overhearing between PQ and CD.

In order that there may be no cross-talk and overhearing (2), (3), and (8) must all hold, and this necessitates

$$\left.\begin{aligned}(w-x)+\frac{c}{\Delta}(a-b)&=0\\(z-y)+\frac{d}{\Delta}(a-b)&=0\\(w-z)+\frac{a}{\Delta}(c-d)&=0\\(x-y)+\frac{b}{\Delta}(c-d)&=0\end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

These equations all hold if

$$(w-x)=(z-y)=(w-z)=(x-y)=(a-b)=(c-d)=0 \quad . \quad . \quad (10)$$

Appendix 9.—Valve Oscillations by Back-coupling through Grid-anode Capacity.

In Fig. 287 C is the grid-anode capacity, Z_g the impedance (resistance R_g, inductance L_g) of the transformer winding connected across grid and filament, Z_1 is the series impedance of C and Z_g, Z_2 is the impedance (resistance R_2, inductance L_2) of the transformer winding connected in the anode circuit, I_a is the anode current, and I_1 and I_2 the currents through the impedances Z_1 and Z_2 respectively (I_1, I_2 and I_a are rotating vectors). If a voltage E sin ωt is applied to the grid, the anode current from 46.05 is

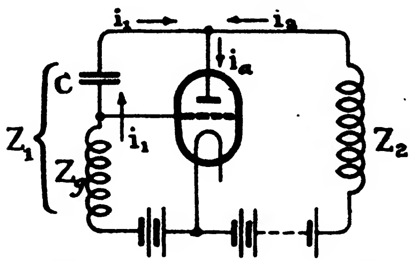


FIG. 287.—Back-coupling through Grid-anode capacity.

$$I_a = \frac{\mu E}{R_a + \dfrac{Z_1 Z_2}{Z_1 + Z_2}} \quad . \quad (1)$$

Since $Z_1Z_2/(Z_1 + Z_2)$ is the impedance of Z_1 and Z_2 in parallel, which is the effective " load " impedance.

The drop of potential in the valve is I_aR_a, hence the potential driving current through Z_1 and Z_2 is $(\mu E - I_aR_a)$. Hence

$$\begin{aligned} I_1 &= \frac{\mu E - I_a R_a}{Z_1} \\ &= \frac{\mu E}{Z_1}\left\{1 - \frac{R_a}{R_a + \dfrac{Z_1Z_2}{Z_1+Z_2}}\right\} \\ &= \frac{\mu E}{Z_1}\left(\frac{Z_1Z_2}{R_a(Z_1+Z_2)+Z_1Z_2}\right) \\ &= \frac{\mu E}{Z_1 + \dfrac{R_a}{Z_2}(Z_1+Z_2)} \quad . \; . \; . \; . \; . \; . \; . \quad (2) \end{aligned}$$

Similarly

$$I_2 = \frac{\mu E}{Z_2 + \dfrac{R_a}{Z_1}(Z_1+Z_2)} \quad . \; . \; . \; . \; . \; . \; . \quad (3)$$

Note also

$$\frac{I_1}{I_2} = \frac{Z_2}{Z_1} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

Now the current I_1 through Z_g produces a grid potential V_g' given by

$$V_g' = -I_1 Z_g$$
$$= -Z_g \cdot \frac{Z_2}{Z_1} I_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

The total grid potential is thus

$$V_g = E + V_g' = E - Z_g \frac{Z_2}{Z_1} I_2 \quad . \quad . \quad . \quad . \quad (6)$$

and this must be used instead of E in (3) to find the amplified current I_2. Hence

$$I_2 \left\{ Z_2 + \frac{R_a}{Z_1}(Z_1 + Z_2) \right\} = \mu \left\{ E - Z_g \frac{Z_2}{Z_1} I_2 \right\}$$

or
$$I_2 \left\{ Z_2 + \frac{(R_a + \mu Z_g) Z_2}{Z_1} + R_a \right\} = \mu E \quad . \quad . \quad . \quad . \quad (7)$$

If the expression in brackets is zero, oscillations can persist in the absence of E.

This condition becomes (on multiplying by Z_1)

$$Z_2(Z_1 + R_a + \mu Z_g) + Z_1 R_a = 0 \quad . \quad . \quad . \quad . \quad (8)$$

i.e.,
$$(R_2 + j\omega L_2)\left(R_g + j\omega L_g - \frac{j}{\omega C} + R_a + \mu R_g + j\omega\mu L_g\right)$$
$$+ \left(R_g + j\omega L_g - \frac{j}{\omega C}\right) R_a = 0 \quad . \quad . \quad . \quad (9)$$

Equating imaginaries gives

$$j\omega L_2(R_g + R_a + \mu R_g)$$
$$+ R_2\left(j\omega L - \frac{j}{\omega C} + j\omega\mu L_g\right) + R_a\left(j\omega L_g - \frac{j}{\omega C}\right) = 0$$
$$j\omega L_2 \left\{ R_a + (\mu + 1) R_g \right\} + j\omega L_g \left\{ (1 + \mu) R_2 + R_a \right\}$$
$$- \frac{j}{\omega C}(R_a + R_2) = 0$$

which on dividing by R_a and neglecting R_2/R_a and R_g/R_a as small, even when multiplied by $(\mu + 1)$, compared with unity, gives

$$\omega L_2 + \omega L_g - \frac{1}{\omega C} = 0$$

or
$$\omega = \sqrt{\frac{1}{(L_2 + L_g) C}} \qquad (10)$$

The oscillations take place round the circuit Z_1 and Z_2 in series.

Equating reals in (9) gives

$$R_2 \{ R_a + (\mu + 1) R_g \} - \omega^2 (1 + \mu) L_2 L_g + \frac{L_2}{C} + R_a R_g = 0$$

$$i.e., \; R_a (R_2 + R_g) + R_2 R_g (\mu + 1) - (1 + \mu) \frac{L_2 L_g}{(L_2 + L_g) C} + \frac{L_2}{C} = 0$$

using (10).

Multiplying throughout by $(L_2 + L_g)C$, and neglecting $R_2 R_g (\mu + 1)$ in comparison with $R_a(R_2 + R_g)$ gives

$$R_a (R_2 + R_g) (L_2 + L_g) C + L_2 (L_2 + L_g) - L_2 L_g = \mu L_2 L_g$$

whence
$$\frac{R_a (R_2 + R_g) (L_2 + L_g) C}{L_2} + L_2 = \mu L_g \quad . \quad . \quad . \quad (11)$$

is given as the condition of maintenance.

Appendix 10.

To find the value of
$$\frac{\epsilon^{-a_1 t} - \epsilon^{-a_2 t}}{a_1 - a_2}$$

when $a_1 = a_2$.

(See section 61, equation (28).)

$$\frac{\epsilon^{-x} - \epsilon^{-y}}{x - y} = \frac{1}{x - y} \left\{ \begin{array}{l} 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \ldots \\ -1 + y - \frac{y^2}{2!} + \frac{y^3}{3!} - \frac{y^4}{4!} + \ldots \end{array} \right\}$$

$$= \frac{1}{(x - y)} \left\{ -(x - y) + \frac{x^2 - y^2}{2!} - \frac{x^3 - y^3}{3!} + \frac{x^4 - y^4}{4!} - \ldots \right\}$$

$$= - \left\{ 1 - \frac{x + y}{2!} + \frac{x^2 + xy + y^2}{3!} - \frac{x^3 + x^2 y + x y^2 + y^3}{4!} + \ldots \right\}$$

and on putting $x = y$

$$= - \left\{ 1 - \frac{x}{1} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots \right\}$$

$$= - \epsilon^{-x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Now
$$\frac{\epsilon^{-a_1 t} - \epsilon^{-a_2 t}}{a_1 - a_2} = \frac{\epsilon^{-a_1 t} - \epsilon^{-a_2 t}}{a_1 t - a_2 t} \cdot t$$

and by (1) when $a_1 t = a_2 t = at$, say,

$$= -t\epsilon^{-at} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Thus equation (61.29) is proved from equation (61.28).

Appendix 11.—Parallel impedance with anode tap.

Let $r_1\, l_1$, $r_2\, l_2$ be the resistance and inductance of AT and TCA respectively (Fig. 239) (r_2 includes the condenser resistance) and m the mutual inductance between the two parts of the coil. Let I_a, I_1 and I_2 be the anode current, and the currents through AT and ACT in order, and V the voltage across AT.

Then
$$I_a = I_1 + I_2$$
$$V = I_1(r_1 + j\omega l_1) - j\omega m I_2$$
$$= I_2\left(r_2 + j\omega l_2 - \frac{j}{\omega C}\right) - j\omega m I_1$$

whence
$$I_1(r_1 + j\omega l_1 + j\omega m) = I_2(r_2 + j\omega l_2 - \frac{j}{\omega C} + j\omega m)$$

Write
$$z_1 = (r_1 + j\omega l_1 + j\omega m)$$
$$z_2 = \left(r_2 + j\omega l_2 - \frac{j}{\omega C} + j\omega m\right)$$

Then
$$I_1 z_1 = I_2 z_2 \text{ and } I_1 = I_a - I_2 = I_a - \frac{z_1}{z_2} I_1$$

$$\therefore \qquad I_1 = \frac{I_a}{1 + \dfrac{z_1}{z_2}}, \qquad I_2 = \frac{I_a}{1 + \dfrac{z_2}{z_1}}$$

Now
$$V = I_1(r_1 + j\omega l_1) - I_2(j\omega m)$$
$$= I_a\left(\frac{r_1 + j\omega l_1}{1 + \dfrac{z_1}{z_2}}\right) - I_a\left(\frac{j\omega m}{1 + \dfrac{z_2}{z_1}}\right)$$

and
$$Z_p = \frac{V}{I_a} = \frac{(r_1 + j\omega l_1)z_2 - j\omega m z_1}{z_1 + z_2} \quad . \quad . \quad . \quad . \quad (1)$$

Now $z_1 + z_2$ is the series impedance Z of the whole oscillatory circuit,

and
$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right),$$

if $R = r_1 + r_2$ and since $l_1 + 2m + l_2 = L$.

Neglecting the resistances in the numerator in comparison with the reactances,

$$Z_p = \frac{j\omega l_1\left(j\omega l_2 - \dfrac{j}{\omega C} + j\omega m\right) - j\omega m(j\omega l_1 + j\omega m)}{Z}$$

$$= \frac{\dfrac{l_1}{C} - \omega^2 l_1 l_2 + \omega^2 m^2}{Z} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is seen that $I_1 = -I_2$ if $z_1 = -z_2$, *i.e.*, if

$$r_1 + j\omega l_1 + j\omega m = -r_2 - j\omega l_2 + \frac{j}{\omega C} - j\omega m.$$

Again neglecting the resistances, this is true if

$$j\omega l_1 + j\omega l_2 + 2j\omega m - \frac{j}{\omega C} = 0$$

i.e.,

$$\omega^2 = \frac{1}{LC} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

which holds at resonance and very nearly round about resonance.

So that what has been assumed is that nearly the same current flows in AT as in TCA, or the oscillatory current is large compared with the anode current.

The ratio of the voltage across AT to that across AB has been defined as b. That is, with the approximations made

$$b = \frac{j\omega l_1 + j\omega m}{j\omega l_1 + j\omega l_2 + 2j\omega m} = \frac{l_1 + m}{l_1 + l_2 + 2m}$$

Introducing this into equation (2)

$$Z_p = \frac{\dfrac{l_1}{C} - \omega^2 l_1 l_2 + \omega^2 m^2}{Z} \qquad \text{and using (3)}$$

$$= \frac{l_1 L - l_1 l_2 + m^2}{CZL} = \frac{l_1(l_1 + l_2 + 2m) - l_1 l_2 + m_2}{CZL}$$

$$= \frac{l_1^2 + 2ml_1 + m^2}{CZL} = \frac{(l_1 + m)^2}{L^2} \cdot \frac{L}{CZ}$$

$$= b^2 \frac{L}{CZ} \qquad \text{as given in 62.19.}$$

INDEX

INDEX