PRACTICAL BUILDING MECHANICS

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By

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THIRD IMPRESSION

LONDON

CHAPMAN & HALL LTD.

37 ESSEX STREET W.C.2

1950

First published . . . 1945
Second Impression . . 1946
Third Impression . . 1950

CATALOGUE No. 268/4

Printed and bound in Great Britain by Butler & Tanner Ltd., Frome and London

AUTHOR'S PREFACE

Many years of teaching and association among Builders and Engineers have convinced me that there is a definite demand for a book where the principles of Building Mechanics are treated in simple and general terms.

It is primarily intended to introduce building mechanics in such a way that the student may readily visualize the essentials of the subject without the necessity of an advanced knowledge of mathematics.

Whatever small measure of success in the engineering field the writer has achieved, he owes very largely to the keen and very able tutors under whom he studied at evening classes.

Students who are attending classes for the ordinary National Certificate, "Strength of Materials" or "Building Mechanics" may find this book helpful.

It is designed to make certain principles clear and to encourage students to stick to their studies. If that object is achieved, the Author will be well satisfied.

I would like to take this opportunity of expressing my thanks to the editor and proprietor of the *Illustrated Carpenter and Builder* for their permission to incorporate in this volume many of my articles which have appeared in their excellent journal, and also to place on record my appreciation of the help and assistance given to me at all times by my publishers, Messrs. Chapman & Hall Ltd. Finally, I wish to thank Mr. Colin Peter, A.M.I.M.E., and Mr. E. Walters Page, who have assisted me in reading and revising proofs.

My thanks are due to Mr. G. R. Lloyd Jones for pointing out a number of slips in the original printing, and for making a number of valuable suggestions for improving the book.

N. T.

LONDON.

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CHAPTER I

TECHNICAL TERMS

Bolts. A bolt may be described as a metal pin or rod with a head at one end and a thread or nut at the other. There are a great many different forms and shapes, and usually bolts are considered as a temporary fixing. However, although rivets are nearly always used for making connections on steel-framed buildings more than two storeys high, there are quite a number of single-storey constructions, such as garages or sheds, where bolts can be satisfactorily used for fixing the various parts together. Even in larger constructions there is no great objection to the use of bolts provided the holes are drilled and the bolts are turned and fitted. A point directly arises, that where turned and fitted bolts are used the cost may be as much or more than putting in rivets. Whatever may be said about the theory of using black bolts, there is no doubt that many scores of single-storey buildings have been made with bolted connections and have proved satisfactory in service.

The most common type of bolt used in constructional steelwork and steel bridges is shown in Fig. 1 (*a*). This is known as X O X,

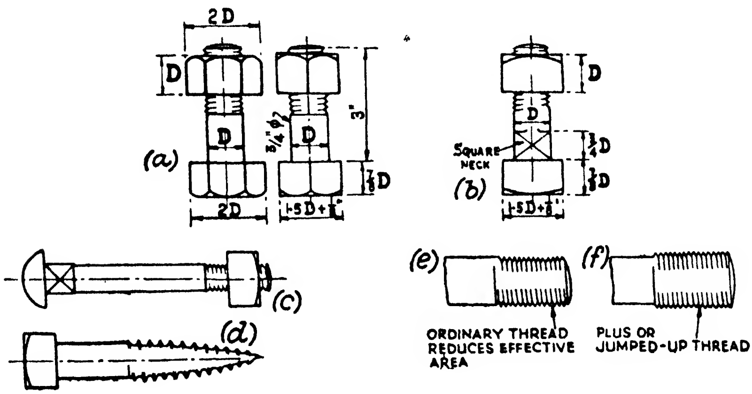


FIG. 1 (*a–f*).

which means hexagonal head, round shank, and hexagonal nut. The length of the bolt is the distance from the inside of the bolt head to the end of the shank, and a bolt such as shown in Fig. 1 (*a*) would be denoted " bolt X O X, ¾ in. diameter by 3 in. long ".

Quality of Steel. The British Standard Specification No. 15 gives the quality of steel from which mild steel bolts are made. Large quantities are made black, that is, with only the thread of the bolt and the thread of the nut machined. All the rest of the bolt is unmachined. In machine-tool work and for important constructional work when bolts are used, turned and fitted bolts are required. In these cases sometimes all the bolt and nut are machined, while in other cases the shank or rod part is machined and the bolt head and nut are left black; there are, however, cases where the use of black bolts is not allowed.

Fig. 1 (*b*) shows a bolt with a square head and square nut, and to prevent the bolt from rotating while the nut is being screwed on, a part of the shank is also made square.

Two types of bolts used in timber constructions are shown in Fig. 1 (*c*) and (*d*). Fig. 1 (*c*) has a cup head and square shank, and Fig. 1 (*d*) is the well-known coach screw.

Jumped-up threads are illustrated in Fig. 1 (*f*). Clearly the ordinary thread, as shown in Fig. 1 (*e*), has a less area at the bottom of the thread than the area of the bolt shank. In the case of the plus-thread or jumped-up thread, the bar is enlarged so that the area at the bottom of the thread is at least equal to the area of the bar itself. These jumped-up threads are often used in tie rods.

Rivets. A rivet is a permanent fastening. It is described by its diameter and its length, the length being the distance from the inside of the head to the end of the shank. A typical cup-head rivet before being closed is shown in the drawing. Before closing the rivet is heated so that it is easy to form. It will contract when cooled, and will tend to draw the plates connected closer together. The closing is done either by hand riveting,

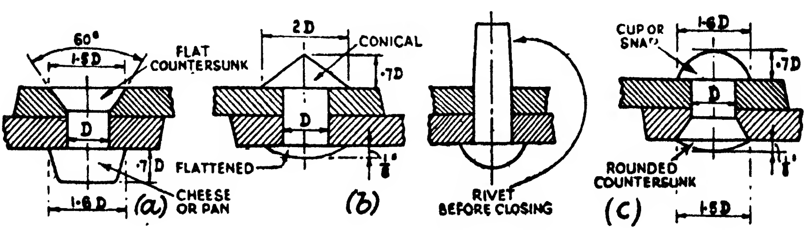


FIG. 2.

hydraulic riveting or a compressed air machine. The grip of the rivet is the distance between the finished heads of a snap or cup rivet. In Figs. 2 (*a*), (*b*) and (*c*) the various types and proportions of the more common types of rivets are shown. Probably the one most generally used is the cup or snap rivet, in which the head of the rivet is cup shape and the closed end is also cup shape. It is to be clearly understood that although only six different types are shown, it is quite possible to have a rivet with a cup at both ends, or a rivet countersunk at both ends, or a rivet pan shaped at both ends, and so on.

In structural work most rivets are $\frac{5}{8}$ in., $\frac{3}{4}$ in. or $\frac{7}{8}$ in. diameter.

Rivets formed in the shop or yard are likely to be better made than rivets made in the field or on the site. Pneumatic riveters are now very much used, both in the shops and during erection, but it is generally accepted that site riveting is not as good as the work done in the structural shops. For this reason it is common practice to allow one or two more rivets where the work will be done on the site than is strictly necessary by calculations, and in ordinary steel-frame building practice the diameter of the rivets which are to be put in on the site is generally not more than $\frac{3}{4}$ in.

Rolled Steel Sections.—Some of the principal sections used in building constructional work are shown in Figs. 3 and 4. The student will be well advised to notice carefully the manner in which these various sections are referred to in practice. All these sections are produced by passing white or red-hot solid bars through rolls, and they are called rolled steel sections.

The H Beam. The rolled steel joist shown in Fig. 3 (*a*) is the most common form of steel beam. It is sometimes called an I beam and sometimes an H beam. The two thick parts are

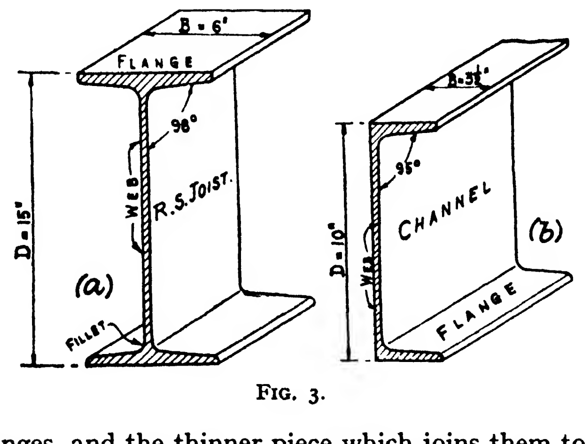


Fig. 3.

called flanges, and the thinner piece which joins them together is the web. The round part between the web and the flange is known as a fillet. The largest British section of joist is 24 in. deep, 7½ in. across the flange, and weighs 95 lb. per foot of length, while the smallest is 3 in. by 1½ in. and weighs only 4 lb. per foot. When specifying for British rolled steel joists it is only necessary to refer to three things, the total depth, the width of the flange and the weight per foot, so that if a beam of the section shown in Fig. 3 (*a*) is required anywhere in Britain, it would be sufficient to denote it as 15 in. by 6 in. by 45 lb. per foot.

The channel section shown in Fig. 3 (*b*) is denoted in the same way. In this case it will be 10 in. by 3½ in. by 24·46 lb. per foot.

An angle section is shown in Fig. 4 (*a*). This section is very commonly used for roof-truss members and bracings. It is

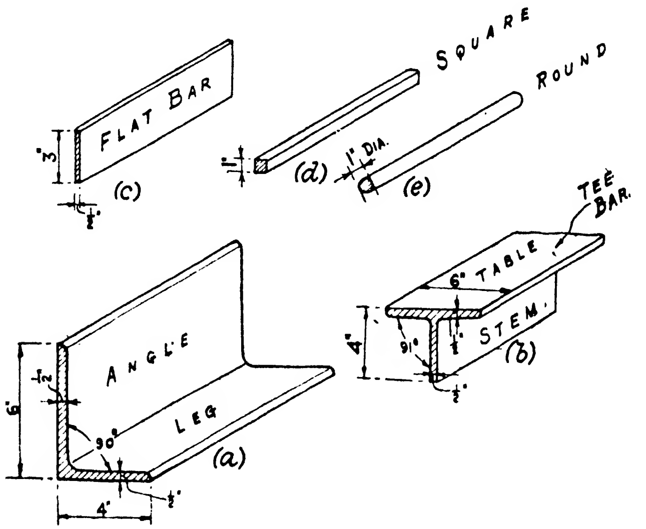


FIG. 4.

denoted by the dimension of each leg and the thickness of steel. The weight is not used. The angle shown would be 6 in. by 4 in. by ½ in.

In Fig. 4 (*b*) a tee-section is shown. Obviously it gets its name from its shape. It is very *important* that the student should remember that the first dimension given is the width of the table or flange. The correct indication for the tee shown would be 6 in. by 4 in. by ½ in. It would be wrong to mark this section 4 in. × 6 in. × ½ in.

Two dimensions only are required for the flat bar, the width and the thickness, while the square bar is denoted by the measurement of one side, and the round bar by the diameter.

Riveted Joints. The design of riveted joints will be dealt with later, but attention is now drawn to two important methods of failure. It will be noticed in Fig. 5 (*a*) and (*b*) that the rivet is in single shear—that is to say, that if the two plates pull apart as shown in Fig. 5 (*b*), the rivet will only have been cut or broken across one section.

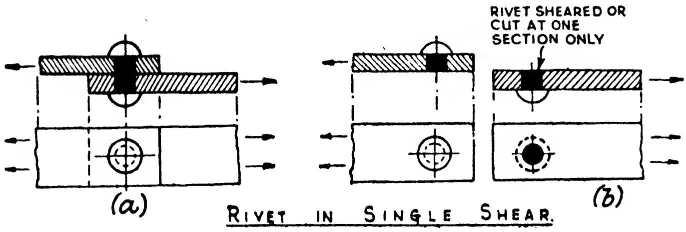


FIG. 5.

In Fig. 6 (*b*) the middle plate has been pulled out, and in order to do this the rivet itself must have been cut or sheared across at two sections. It is therefore not difficult to see that as far as the strength of the rivet is concerned, it will take twice as much pull to cause failure as shown in Fig. 6 (*b*) as it takes to cause failure as by the method shown in Fig. 5 (*b*).

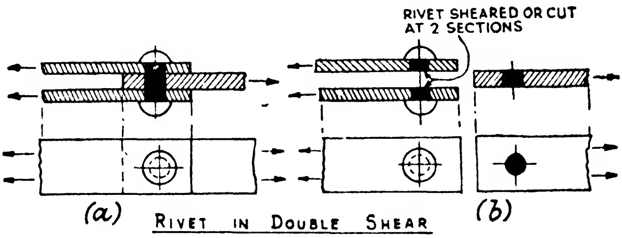


FIG. 6.

Technical Terms. Frequently the non-technical man uses words which to him mean something quite different from what they would convey to a technically trained man, and for this reason some of the terms commonly used will be explained here more fully than is possible by a sentence or two giving vague definitions.

Stress. Stress may be described as (1) The internal distributed force which resists a change in shape and size of a body subjected to external forces; (2) the state of affairs existing between the particles of a body transmitting a load or force.

To try and make this clear, consider two wood blocks, *A* and *B*, each 3 in. square and 8 in. long, as shown in Fig. 7 (*a*). A slab of plasticine or soft putty is placed between these two blocks. If a weight of 56 lb. is placed on the top of the blocks as shown in Fig. 7 (*b*), then (if the weight of the wood itself is neglected) it is easy to see that the load of 56 lb. will be transmitted to the slab of soft putty or plasticine, which would not be strong enough to resist the pressure and would be squashed and spread out as shown. If the wood blocks were built of a series of slabs all the same size as the plasticine slab, there is a tendency to squeeze each little block out of shape. The only thing which prevents this being done in the case of a wood block is the internal strength of the wood itself, and it is this internal resistance of the wood itself which is called " stress ". If the wood was not strong enough to

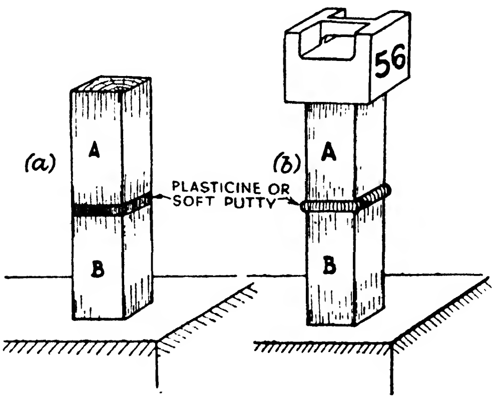


FIG. 7.

resist or balance the external load or force, the block would change in shape and crush.

The timber block is 3 in. square, and if the weight of this is neglected, then the total stress at any horizontal section will be 56 lb. Obviously there is much less likelihood of the block breaking than if it is made 2 in. square, although the total load and total stress remain constant at 56 lb. What is really altered is the unit of unital stress. This is the intensity of stress per unit of area, and when the load is evenly distributed over the area the unit stress at any point is the total stress divided by the area. With a block 2 in. square the area at any horizontal section will be 4 sq. in. With one 3 in. square, the area will be 9 sq. in.

It is very necessary to make clear whether the stress is the total stress or unit stress, and in the case of unit stress the figures should always be expressed as pounds per square inch, hundredweights per square inch, tons per square foot, etc. It is not sufficient to ive a figure of so many pounds or so many tons without stating whether it is per square inch or per square foot.

As a formula:

$$\text{Unit stress} = \frac{\text{Load}}{\text{Area}} \quad . \quad . \quad . \quad (1)$$

Problems. The load on a short timber post 2 in. square is 720 lb. What is the stress in pounds per square inch?

The total stress is the total load, which is 720 lb.

$$\text{Unit stress} = \frac{\text{Load}}{\text{Area}} = \frac{720}{2 \times 2} = 180 \text{ lb. per square inch.}$$

A short timber post 3 in. square carries a load of 720 lb. What is the intensity of stress?

$$\text{Intensity of stress} = \frac{720}{3 \times 3} = 80 \text{ lb. per square inch.}$$

The question of what is a long and what is a short timber post is determined by the relationship of the length to one side. This question will be dealt with more fully later, but the student can assume that if the length is less than 10 times the side of a wood post, it can be considered as a short post.

Compressive Stress or Compression. In the case we have just considered, the weight on the post tries to shorten it, and the stress is therefore compressive. In building and engineering work there are many examples of parts or members which are subjected to these compressive stresses. Look, for instance, at

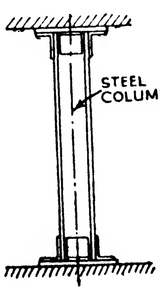


Fig. 8.

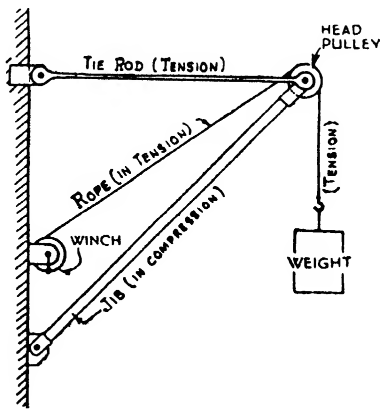


Fig. 9.

the crane illustrated in Fig. 9. A wire rope from a winch drum passes round a head pulley and suspends a weight. It is not difficult to see that the inclined member, which is called the jib, is in compression, that is, it tends to shorten in length. A steel column, as shown in Fig. 8, is another example of compression member.

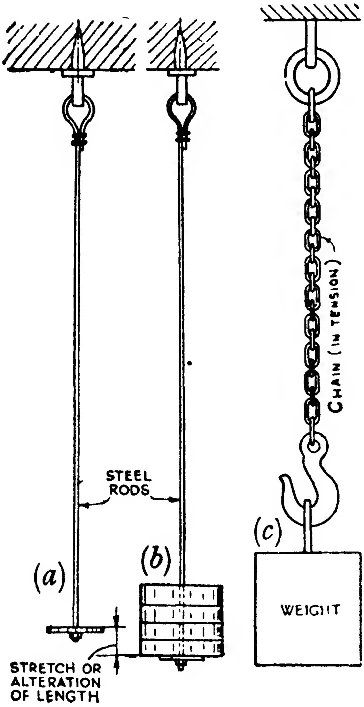


FIG. 10.

Tensile Stress or Tension. A tension or tensile stress has exactly the opposite effect to a compression or compressive stress. Three examples of tension are shown in the diagram Fig. 10 (*c*) shows a chain carrying a weight. Obviously the chain tends to lengthen and not shorten. Similarly the horizontal tie-bar in the crane illustration would stretch if made of rubber, and it requires very little imagination to realize that the rope on the crane is also in a state of tension.

Mild streel is strong both to resist tensile and compressive stresses. On the average it will break at about 30 tons for each square inch of area.

PROBLEM. If a tie-rod or tension member is $1\frac{1}{2}$ in. diameter, what pull or force would probably break it ?

It should be noticed in this case that no length is mentioned. The area of any cross-section would be the area of a $1\frac{1}{2}$-in. diameter bar.

$$= D \times D \times \frac{\pi}{4} = \frac{3}{2} \times \frac{3}{2} \times \frac{22}{28} = 1{\cdot}76 \text{ sq. in.}$$

If it takes 30 tons to break 1 sq. in., then it will take 30 tons $\times$ 1·76 = 52·8 tons to break a tie-rod.

Shear Stress. When one part of a body tends to slide past another it is said to be in shear. In Fig. 11 (*a*) is illustrated a

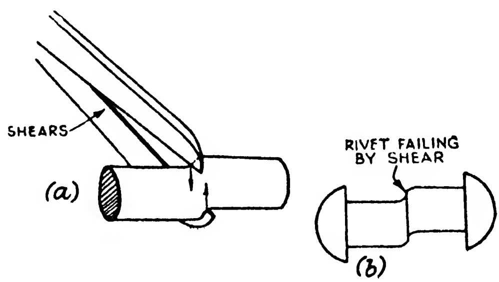


FIG. 11.

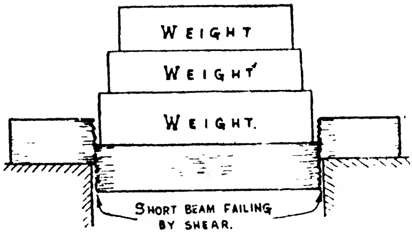


FIG. 12.

plasticine bar being cut by a pair of shears. Obviously, when the bar has been cut into two pieces, one piece will slide or shear past the other. In the case of a steel rivet connecting two plates together, this shearing tendency is clearly shown, and if the rivet is not strong enough it would break by shearing, as shown in Fig. 11 (*b*).

In Fig. 12 we see a short wood beam broken by shear. In actual practice beams are subject to bending stresses as well as to shearing stresses, but the illustration clearly shows the tendency of one part to slide past another.

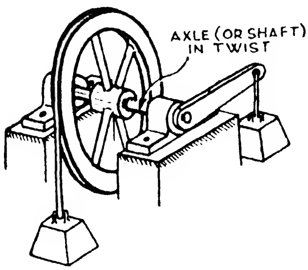


FIG. 13.

Torsion or Twisting Stress. This is illustrated in Fig. 13. A weight attached to the end of a lever is holding the weight supported by the rope passing over the pulley. The shaft or axle is supported by two bearings quite close to the pulley, and the shaft itself is in torsional or twisting stress. If, for instance, the shaft is considered as being made up of a number of small discs, it is easy to see that each disc would try to slide around the one next to it.

Strain. Strain has been described as the deformation in a body due to the application of load or stress. This deformation under a tensile force or pull is elongation or stretch, and under a compressive force or push is shortening. The student should be very careful not to mix up the words stress and strain. Sometimes the word strain is used when stress is meant. *Strain is the alteration of shape* as the result of a stress.

Refer now to Fig. 14 (*a*), (*b*) and (*c*). In Fig. 14 (*a*) an india-rubber band is shown; Fig. 14 (*b*) shows the same band being stretched by a weight attached at its lower end; while in Fig. 14 (*c*) a second load has been applied. Although the stretch of the rubber is related to the strain, it is not correct to say that the stretch is the strain.

We will try to make this clear. Suppose the distance between pins *A* and *B* in Fig. 14 (*a*) was originally 3 ft., and after a weight

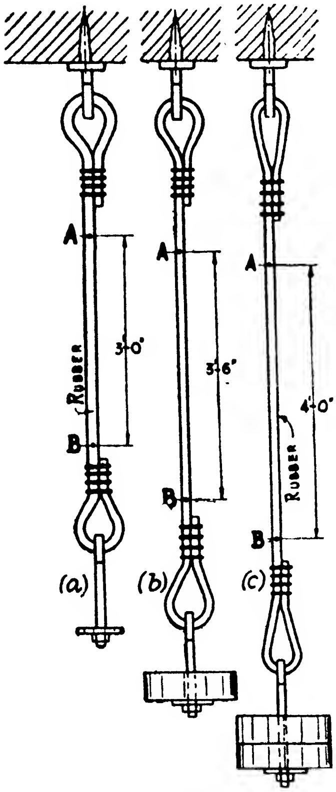


FIG. 14.

had been put on the collar the distance had increased to 3 ft. 6 in. Then the alteration of length is clearly 6 in. This would be the stretch, but it would not be the strain. Actually the strain would be the alteration of length of stretch divided by the original length. As a formula it could be written—

$$\text{Strain} = \frac{\text{Alteration of length}}{\text{Original length}} \quad . \quad . \quad (2)$$

In the case we are considering—

$$\text{Strain} = \frac{6 \text{ in.}}{3 \text{ ft.}} = \frac{6 \text{ in.}}{36 \text{ in.}} = \tfrac{1}{6}.$$

It should be noticed that this one-sixth is the relationship between the alteration of length and the original length. It is not a dimension—that is to say, it is not one-sixth of an inch, or one-sixth of a foot, but is one-sixth of the original length. The importance of clearly understanding this will be seen if we consider Fig. 14 (*c*). Here the rubber has been stretched so that the distance from *A* to *B* is 4 ft. The stretch is therefore 1 ft. as compared with the original unstretched length (Fig. 14 (*a*)).

$$\text{Strain} = \frac{1 \text{ ft.}}{3 \text{ ft.}} = \tfrac{1}{3}.$$

Students with an elementary knowledge of mathematics will see that by using this formula the actual change of length can be found if the strain and the original length are known.

$$\text{Change of length} = \text{Strain} \times \text{Original length} \quad . \quad (3)$$

$$\text{Original length} = \frac{\text{Change of length}}{\text{Strain}} \quad . \quad . \quad . \quad (4)$$

PROBLEMS. **If the strain on a loaded steel rod (in Fig. 5) is $\frac{1}{100}$, and the original length of the rod before loading was 50 in., what is the change of length ?**

$$\textbf{Change of length} = \frac{1}{100} \times 50 = \tfrac{1}{2} \textbf{ in.}$$

Similarly it is easy to find the original length if we are told that the stretch is $\frac{1}{2}$ in. and the strain is $\frac{1}{100}$.

$$\text{Original length} = \frac{\frac{1}{2}}{\frac{1}{100}} = \frac{1}{2} \times 100 = 50 \text{ in.}$$

The loaded rod shown in Fig. 10 (*b*) is in a state of tension and will definitely stretch, although, of course, not nearly as much as if the rod was made of rubber. This question is very important, and will be dealt with at length later. At this stage we will only say that steel and nearly all other metals have definite elastic properties, and if they are put under a tensile stress they will stretch, and if the load is removed the bar will return to its original length. Similarly, when a steel column in a building is loaded, it definitely shortens in length, and if the load is removed the column will again stretch out to its original length.

CHAPTER 2

STRESS. STRAIN. ELASTICITY

All metals are to some extent elastic, which means in other words that if a tensile or pulling force is applied to them they stretch, and when the load or force is removed they return to their original length. In like manner, if a compressive load or force is put upon them they shorten, and return to their original length if and when the load is removed. The amount of shortening or lengthening is in direct proportion to the load.

Hooke's Law. This elastic property of metals breaks down long before the breaking strength of the material is reached. The fact that a bar lengthened or shortened in direct proportion to the load placed on it was discovered by Dr. Hooke, and is known as Hooke's law. The elastic limit can be described as the maximum stress per square inch which can be applied without causing any appreciable set or alteration in length.

Fig. 15 (*a*) shows the results of a test which was made to find out the elastic properties of a mild steel rod, and the student is advised to consider this test carefully, as it will probably enable him more clearly to understand what has just been said about the elastic properties of metals.

In the test a mild steel rod $\frac{1}{4}$ in. diameter and about 11 ft. long was suspended from a very stiff beam and gauge points 120 in. apart were marked on the suspended rod.

In a previous chapter it was shown that

$$\text{Stress} = \frac{\text{Load}}{\text{Area}}.$$

The area of a rod $\frac{1}{4}$ in. in diameter is 0·049 sq. in., which is practically $\frac{1}{20}$ of a square inch. It is therefore easy to see that if a load of 1 cwt. be put on such a rod the stress per square inch will be

$$\text{Stress} = \frac{1 \text{ cwt.}}{\text{Area}} = 1 \text{ cwt.} \div \frac{1}{20}$$
$$= \text{very nearly } 1 \text{ ton per square inch.}$$

In other words, if a pull of 1 cwt. is exerted on a bar $\frac{1}{4}$ in. diameter, it will have the same effect as a pull of 1 ton on a bar having an area of 1 sq. in.

An Experiment.—Now let us return to the experiment shown in Fig. 15 (*a*). A load of 5 cwt. was steadily applied to

:he rod, and the length between the gauge points are found to neasure 120·045 in. Therefore, since the gauge length before .oading was 120 in., the load of 5 cwt. had caused a stretch of

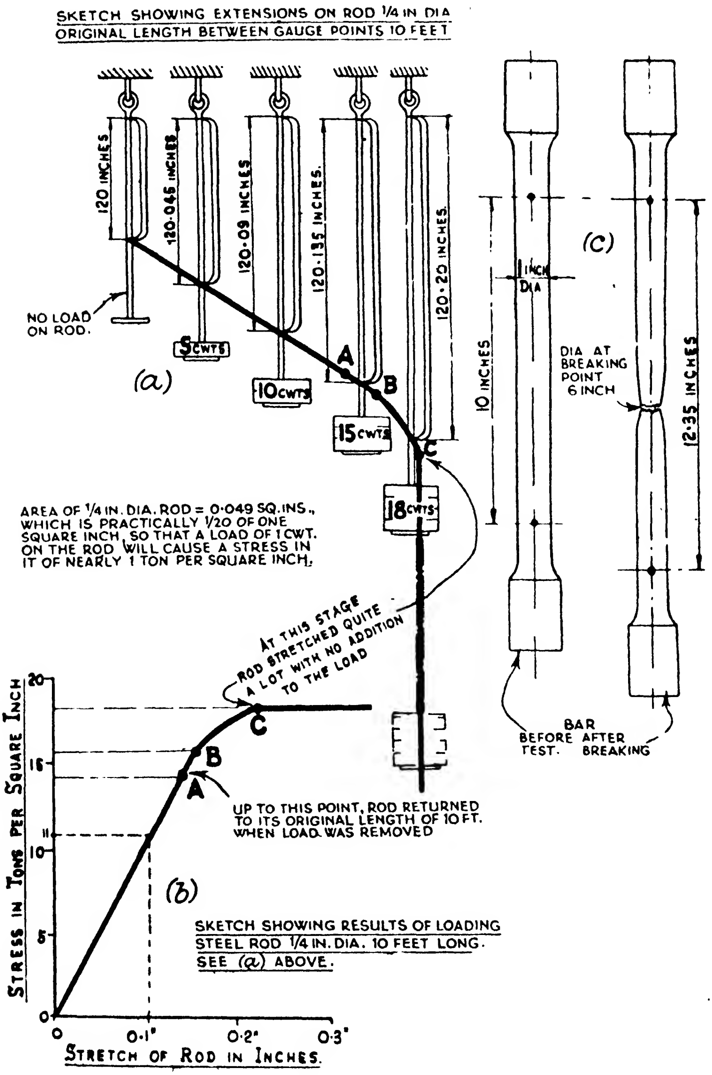


FIG. 15.

0·045 in. The load was removed and the rod quickly returned to its original length. A load of 10 cwt. steadily applied caused a stretch of 0·09 in. Again the rod returned to its original length when the load was removed. From 10 cwt. upwards additional loads of 1 cwt. were applied. The rod stretched in proportion to the load, and when 14 cwt. had been applied the rod returned to its original length after this 14 cwt. load had been removed. This shows that up to this point the bar was perfectly elastic. (As the load applied was 14 cwt., the stress was approximately 14 tons per square inch.)

The student should pay particular attention to the behaviour of the rod. A load of 15 cwt. produced a stretch of 0·135 in. Notice that the stretch for each 5 cwt. has been 0·045 in. The stretch up to this point is still proportional to the load, but when the 15 cwt. load was removed the bar did not completely return to its original length of 120 in. It has therefore lost some of its elastic properties, and the true elastic limit has been passed, although the stress and strain are still proportional.

Yield Point. A load of 16 cwt. was now applied to the rod, and it was found that the stretch was still very nearly proportional, but with a load of 17 cwt. the extension was not proportional, and a load of 18 cwt. stretched the bar so that the gauge length was 120·20 in. It is clear to see that for each 5 cwt. of load up to 15 cwt. the rod had stretched 0·045 in. For the 3 cwt. addition after the elastic limit had been passed the stretch was 0·065 in., showing clearly that the rod is now stretching out of proportion to the load applied.

As near as could be measured the proportionality of extension to load held good until the point *B* was reached, that is, until a weight of 16 cwt. had been applied to the rod. A load of 18¼ cwt. stretched the bar to 123 in. before the experiment was stopped. All these results are shown in Fig. 15 (*a*). The point *A* indicates the elastic limit, point *B* the limit of proportionality, and the point *C* the yield point.

Although it is very necessary that the student should understand the difference between these three points, in actual building practice the yield point is often the only one used, and it is generally assumed that the elastic limit and the yield point are about the same.

Fig. 15 (*b*) shows the same curve as Fig. 15 (*a*). The points *A*, *B* and *C* are indicated, and this curve will help the student to understand Fig. 16.

In the experiment which has just been described, a long rod was used so that the extensions could be measured by students, and the experiment was stopped after the yield point had been reached. In practical testing short test specimens are broken in machines specially made for this purpose.

In using a short test bar and a testing machine, the results would be as shown in Fig. 16. Here the bar is made of mild steel, machined to 1 in. diameter in the centre part. Gauge or centre punch marks are made 10 in. apart. Fig. 15 (*c*) shows what the bar would look like before and after the test. It follows that on such a short length as 10 in. the extensions must be very small, and these would be measured by a special instrument called an extensometer, by which it is possible to measure changes of length as small as $\frac{1}{5000}$ part of an inch.

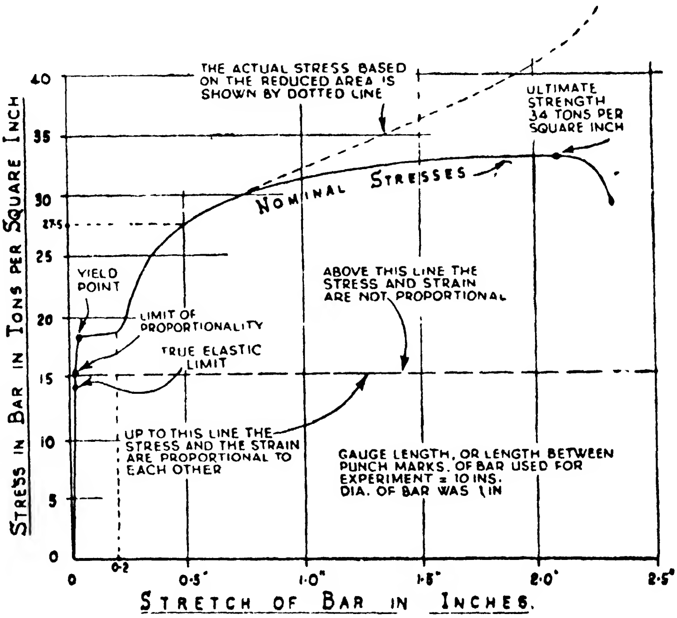


FIG. 16.

The results of the tests follow on exactly the same lines as for the previous experiment. The position of the true elastic limit is found to be at nearly 14 tons per square inch; the limit of proportionality is reached at a stress of about 15 tons per square inch, and the yield point when the stress in the bar is 18 tons per square inch. It will be seen that when the stress on the bar is about 18 tons per square inch, the bar continues to stretch until the gauge points are 10·2 in. apart. The steel then seems to recover strength, and the stretch is only 0·5 in. when the stress per square inch reaches 27·5 tons.

Increasing loads now stretch the bar rapidly, because it is now necking or reducing in diameter. The test is continued until the bar breaks. In this case the ultimate strength was 34 tons per square inch. At this stage the bar has stretched so much and the neck is so small that if some part of the load is taken off the stretching still continues, and the break will take place at a lower stress per square inch than the ultimate. Strictly speaking, the full line indicates the nominal stresses, and does not indicate the actual stresses, as the bar is reducing in area, the nominal stresses being taken on the original area of the bar. These actual stresses taken on the reduced area are indicated by a dotted line. *In practical testing no account is taken of the reduced area, and the breaking strength is based on the original dimensions of the test bar.*

Strength of Materials. In the table the average strength of materials are indicated, and the student is advised to remember

AVERAGE STRENGTHS OF MATERIALS

Material	Ultimate Strength			Safe Stress for Dead Load			Factors of Safety			Modulus of Elasticity
	Compression	Tension	Shear	Compression	Tension	Shear	Dead Load	Variable Load	Shocks	
	Tons per Square Inch			Tons per Square Inch						Tons per Sq In.
Mild Steel	30	30	20	7½	7½	5	4	6	10	13,400
Wrought Iron	18	24	20	4½	6	5	4	6	10	12,500
Cast Iron	40	10	12	7	1½	2	6	10	20	8,000
	Lbs per Square Inch			Lbs per Square Inch						Lbs per Sq In.
Oak	7,500	10,000		1,250	1,600		6	8	15	1,300,000
Pitch Pine	6,000	7,000		1,000	1,200		6	8	15	1,500,000
Douglas Fir	5,000	6,000		800	1,000		6	8	15	1,600,000

that they are average strengths and that the actual strengths of materials vary quite a lot. For instance, the average ultimate strength of mild steel in compression and tension is given at 30 tons per square inch. Actually the strength of mild steel may vary between 28 and 35 tons per square inch, but the figure of 30 is about the average. All students will be aware that the strength of timber varies over very wide limits, depending on whether it is a good timber, and whether it is green or seasoned.

In constructional work it would be *courting disaster* to load a member up to its ultimate strength, or above the elastic limit, as a permanent set in the member would probably cause distortion of the structure it forms part of, and eventually the collapse of the structure.

For this reason a member is only stressed to a fraction of the ultimate strength, and it is this divisor that is termed the factor of safety. As a formula it would be written:

$$\text{Safe working stress} = \frac{\text{Ultimate strength}}{\text{Factor of safety}} \quad . \quad . \quad (5)$$

In the table the ultimate strength of steel is given as 30 tons per square inch, but a safe working stress of $7\frac{1}{2}$ tons per square inch is the maximum that can be set up in a member. In this case, therefore,

$$\text{Factor of safety} = \frac{\text{Ultimate strength}}{\text{Safe working stress}} = \frac{30}{7\frac{1}{2}} = 4.$$

The safe stresses which can be used when the load is a dead load are shown in the table. However, if the load is a variable one or takes the form of a series of shocks on the member, then the factor of safety must be increased, as a body is more liable to failure under fluctuating loads and shocks than under a dead load. Suitable average factors of safety are shown in the table for variable and shock loads and the safe working stress for these types of loads will be obtained by dividing the ultimate strength of the material by the factor of safety given.

The last column on the table gives the modulus of elasticity of materials. It is often termed Young's modulus, and is denoted by the letter E.

$$\text{Modulus of elasticity} = E = \frac{\text{Stress per square inch}}{\text{Strain}}$$

If it were possible to stretch a bar originally 10 in. long to 20 in. (that is, twice its original length) without breaking it, the unit stress which would do this would be the modulus or measure of elasticity.

The modulus of elasticity of the steel used for the experiment shown in Fig. 15 (a) can be found from the results obtained. Up to B, the limit of proportionality, stress and strain are proportional, and the modulus of elasticity equation will hold good. In Fig. 15 (b) it will be seen that the rod had stretched 0·1 in. when the stress of the rod was 11 tons per square inch.

We have previously shown that

$$\text{Strain} = \frac{\text{Stretch}}{\text{Original length}}$$

$$\text{Therefore strain} = \frac{0{\cdot}1 \text{ in.}}{120 \text{ in.}} = 0{\cdot}000833$$

It has also been shown that

$$\text{Modulus of elasticity} = \frac{\text{Stress}}{\text{Strain}}$$

from which

$$\text{Modulus of elasticity} = \frac{11 \text{ tons}}{0{\cdot}000833}$$

$$= \frac{110{,}000}{8{\cdot}33} \doteqdot 13{,}200 \text{ tons per square inch.}$$

For structural steel the modulus of elasticity is usually taken at 30,000,000 lb. (13,400 tons) per square inch, but it should be remembered that the actual modulus of elasticity of mild steel varies between 29,000,000 lb. and 31,000,000 per square inch.

It is of course, not possible to stretch a steel bar to twice its length, as it would break long before this extension was reached, but the importance of the modulus of elasticity will be clear if the student has understood the elastic properties of steel, and the fact that, until the elastic limit is reached, the amount of stretch or elongation of length is in direct proportion to the load on the bar.

It should, therefore, not be difficult to see that if a load of 30,000,000 lb. per square inch is required to stretch a bar to twice its length, a stress of 30,000 lb. ($13\frac{1}{2}$ tons) per square inch would stretch the bar $\frac{1}{1000}$th part of its length, so that if a bar of 1 sq. in. area and 100 in. long is pulled by a force of 30,000 lb., the bar will stretch $\frac{1}{10}$ in.

Similarly, if the same bar was bedded in a concrete column, and pushed with a load of 30,000 lb. it would shorten $\frac{1}{10}$ in. It has already been made clear by the experiments which have been described that the elastic limit is not very much more than 30,000 lb. ($13\frac{1}{2}$ tons) per square inch, and therefore the actual alteration of length in beams or columns in a bridge or building is not much, because the actual stress which can be allowed must be less than the stress which would destroy the elastic properties of the steel.

Various problems will now be worked out so as to make clear how the various terms, such as stress, strain, modulus of elasticity, factor of safety, and ultimate strength are made use of in actual practice.

Problem 1. A flat steel bar 3 in. by $\frac{1}{2}$ in. in section and 10 ft. long is used as a tie-bar in a roof truss. It is subjected to a pull of 11 tons at the ends, and the modulus of elasticity for the steel is 13,400 tons per square inch. Find how much the bar stretches. If the ultimate strength of the steel is 30 tons per square inch, and a factor of safety of 4 is to be used, find if the bar is stressed above the safe working stress.

Answer.

$$E = \frac{\text{Stress}}{\text{Strain}} = 13{,}400 \text{ tons per square inch.}$$

from which $\quad \text{Strain} = \dfrac{\text{Stress}}{13{,}400}$

Also $\quad \text{Stress} = \dfrac{\text{Load}}{\text{Area}} = \dfrac{11 \text{ tons}}{3 \text{ in.} \times \frac{1}{2} \text{ in.}} = 7{\cdot}33$ tons per square inch.

Therefore $\quad \text{Strain} = \dfrac{7{\cdot}33}{13{,}400} = 0{\cdot}000547$

so that $\quad \text{Strain} = \dfrac{\text{Change of length}}{\text{Original length}} = 0{\cdot}000547$

from which

$$\begin{aligned}\text{Change of length} &= \text{Original length} \times 0{\cdot}000547\\ &= 120 \text{ in.} \times 0{\cdot}000547\\ &= 0{\cdot}0656 \text{ in.}\end{aligned}$$

If the ultimate strength of the steel is 30 tons per square inch, and a factor of safety of 4 is used, then

$$\begin{aligned}\text{Safe working stress} &= \frac{\text{Ultimate strength}}{\text{Factor of safety}}\\ &= \frac{30}{4} = 7{\cdot}5 \text{ tons per square inch}\end{aligned}$$

Actual stress in bar as found previously

$$= 7{\cdot}33 \text{ tons per square inch.}$$

Therefore the actual stress in the bar is less than the allowable stress, and the member is strong enough.

PROBLEM 2. A timber post 12 in. square and 10 ft. long carries a load of 40 tons. If E for the timber is 1 500,000 lb. per square inch, how much would the post shorten ?

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{40 \text{ tons} \times 2{,}240}{12 \text{ in.} \times 12 \text{ in.}} = 622 \text{ lb. per square inch}$$

therefore $\quad E = 1{,}500{,}000 = \dfrac{622}{\text{Strain}}$

from which $\quad \text{Strain} = \dfrac{622}{1{,}500{,}000} = 0{\cdot}000415$

$$\text{Strain} = \frac{\text{Change of length}}{\text{Original length}} = \frac{\text{Change of length}}{120 \text{ in.}}$$

from which

$$\text{Change of length} = \text{Strain} \times 120 \text{ in.} = 0{\cdot}00415 \times 120 = 0{\cdot}0498 \text{ in.}$$

Thus the column would shorten about $\frac{1}{20}$ in.

PROBLEM 3. A concrete column is 12 in. square and 10 ft. long. When loaded with 40 tons it shortens 0·0375 in. Find the modulus of elasticity of the concrete.

$$E = \frac{\text{Stress}}{\text{Strain}}$$

$$\text{Unit stress} = \frac{\text{Load}}{\text{Area}} = \frac{40 \text{ tons} \times 2{,}240}{12 \text{ in.} \times 12 \text{ in.}}$$

$$= 622 \text{ lb. per square inch}$$

$$\text{Strain} = \frac{\text{Change of length}}{\text{Original length}} = \frac{0{\cdot}0375}{120} = 0{\cdot}000312$$

Therefore
$$E = \frac{622 \text{ lb. per square inch}}{0{\cdot}000312}$$

$$= \text{say, } 2{,}000{,}000 \text{ lb. per square inch.}$$

CHAPTER 3

STRESSES ON OBLIQUE PLANES

Metals are elastic, and the elastic properties of mild steel, as shown by experiments, were dealt with in the preceding chapter and such terms as factor of safety, modulus of elasticity, ultimate strength, and safe stresses for various materials have been explained. The simple formula that the stress per unit area can be found by dividing the total load by the total area concerned is fundamental.

Up to the present stresses have only been considered on a plane or cut which was perpendicular to, or at right angles to, the axis of the bar or column. The load was also considered to be axial or concentric. In other words, the load was considered as acting down the centre of the bar, and the unit stress was found on the area cutting straight across the bar.

Stresses on Oblique Planes.—When materials are tested in compression they generally fail along a surface which is not at right angles to the vertical axis. In other words, they seem to slide away. A piece of cast iron loaded so that it fails under pressure would probably break in a manner somewhat similar to that shown in Fig. 17 (*a*). Results of many tests show that

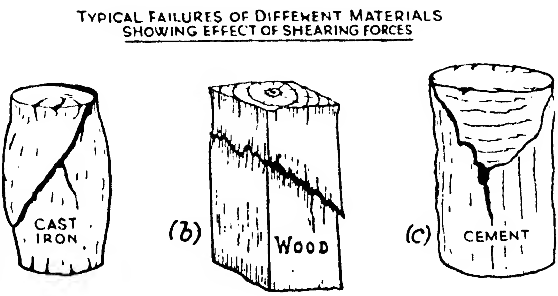


FIG. 17.

a timber block loaded until it breaks in compression would probably look like Fig. 17 (*b*). A piece of concrete might fail in the manner shown in Fig. 17 (*c*).

The student will immediately notice that these materials are relatively brittle, and that the actual line where the failure takes place is not straight across the specimen, but on an inclined or oblique plane. It is therefore necessary for us to give some

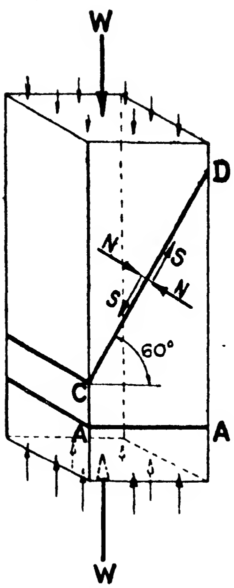


FIG. 18.

consideration to the stresses which occur on a plane which is not at right angles to the load.

Consider a bar in compression. The conditions are as shown in Fig. 18. We have already seen that on a cut or plane AA the stress per square inch would be

$$\text{Stress} = \frac{\text{Load}}{\text{Area}} = \frac{W}{A}$$

where W is the total load.

A is the cross-sectional area on line AA. If $W = 27$ tons and $A = 9$ sq. in., then Stress per square inch on $AA = \frac{27}{9} =$ 3 tons per square inch.

The line CD represents an inclined or oblique plane inclined at 60 deg. to AA, and we shall consider the stresses on this plane due to the load W. At this stage we introduce a little elementary trigonometry so as to make clear the terms used and the method of applying them. Fig. 19 (*a*) represents a right-angled triangle, and the lengths of the different sides are, of course, AC, BC, and AB, which are respectively the hypotenuse, perpendicular, and base lines. It is well known that there are rules for finding the lengths of the various sides when one side and the angles are known. Three of the most common terms in trigonometry are sine, cosine, and tangent.

$$\sin\theta = \frac{BC}{AC} \qquad \cos\theta = \frac{AB}{AC} \qquad \tan\theta = \frac{BC}{AB}.$$

A very helpful method for remembering these formula is the first letters of the words of the following sentence : " Peter's horse brings home Peter's bread. Notice

$$\frac{P}{H} = \sin \qquad \frac{B}{H} = \cos \qquad \frac{P}{B} = \tan.$$

One angle remains fixed at 90 deg., then if we know one of the other two angles it is quite an easy matter to find the third, since there are 180 deg. in every triangle.

We now return to considering the stresses on the oblique plane (Fig. 18). The length AA is known and the angle called θ

is also known, therefore we can easily find the length of CD by using the simple rules just mentioned.

$$CD = \frac{AA}{\cos \theta}.$$

Reference to any table of trigonometrical ratios will show that cos 60 deg. is 0·5. Therefore if length AA is 3 in.,

$$\text{then the length } CD \text{ will be } \frac{3 \text{ in.}}{0{\cdot}5} = 6 \text{ in.}$$

A little consideration of Figs. 17 (*a*), (*b*) and (*c*) will show that failure occurs by the two pieces sliding one over the other, and this is, of course, a shear, as was explained in an earlier chapter. Therefore in some way or the other a vertical push or load has been changed to an inclined force. Students who have an elementary knowledge of forces and their resolution, or who have done graphic statics, will know that one force can have two components, or that two forces can be substituted for or in place of one force. There may be some who have either forgotten or have not understood this point, and we shall try to make it perfectly clear by a simple example.

Fig. 19 (*b*) represents a block of stone. The lines and arrows marked M represent men pushing in the direction shown, each exerting the same force as the other. If sufficient force was used the block of stone would move in the direction shown by the arrow marked W. It is obvious that if, instead of the men pushing at the back of the stone as shown by the arrows, they were pulling on it by means of a rope as shown by W, the same effect would be gained (and with less effort). The point I am trying to make is that the stone could be moved forward 12 in. either by a pull on a single rope in the direction of arrow W, or it could be moved 12 in. forward by men pushing at the corners. In technical language we therefore say that one force can be substituted in place of two others, or we can resolve a single force into two other forces, which would have the same effect.

The load or force W in Fig. 18 will now be split up into two other forces, one marked S and one marked N. S is the compression or force of the load W which tries to make the top part T slide past the bottom part B, as shown in Fig. 19 (*c*). N is the normal force, or force at 90 deg., to the plane CD, which tries to squeeze or crush the two pieces together, as shown in Fig. 19 (*d*). If the material is weak in shear, the shearing forces which are shown separately in Fig. 19 (*c*) are those which will cause

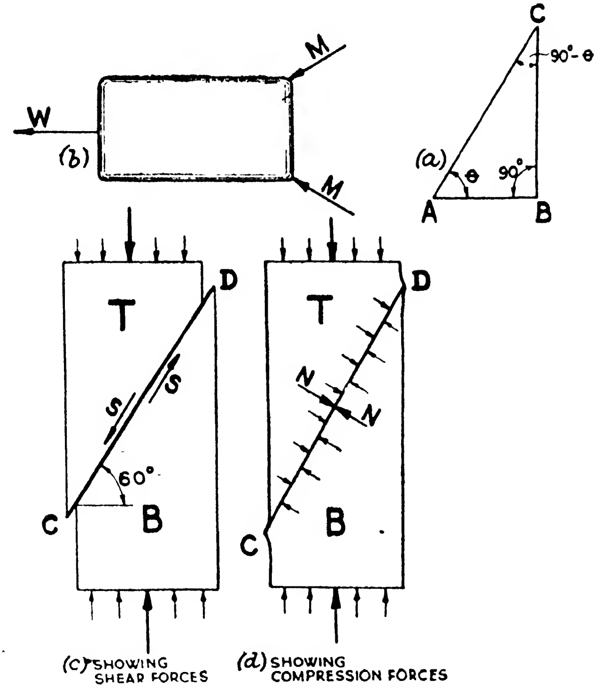


FIG. 19.

failure, and this explains why the two pieces break off in the oblique or inclined plane, as shown in Fig. 17.

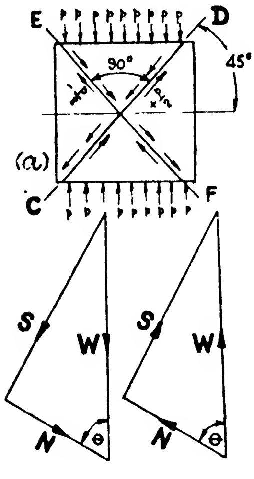
Normal and Shear Stresses. The forces which act at right angles or normal to the plane, Fig. 19 (*d*) are called normal forces. The forces which act along or parallel to or tangential to the plane (Fig. 19 (*c*)) are called shear forces.

Our next problem is to find the amounts of these normal and shear forces. We know the direction and the amount of the force *W*. We also know the direction, but not the amount of forces *N* and *S*. The amount of these forces can be found by drawing as shown in Fig. 20 (*b*). *W* is drawn to scale, and the two other forces are known by direction, so that they can only intersect at one point. The length of *N* and *S*, if scaled off to the same scale as *W*, will show the amount of the forces. It is also simple trigonometry that as *S* is parallel to plane *CD* and *N* is normal to it, the angle between *N* and *S* must be 90 deg. The other two angles can be found by inspection. Then it is not difficult to see that $N = W \times \cos\theta$ and $S = W \times \sin\theta$.

N is the total normal force on the plane *CD* and *S* is the total shear force on *CD*.

The stress due to shear can now be easily found, because we have the total shear load, and the total area on which it acts will be the length of CD multiplied by the thickness of the block. We have considered the thickness as being 3 in. It has been previously shown that

$$\text{Length of } CD = \frac{\text{Length of } AA}{\cos\theta}$$



(b) SHOWING TRIANGLE OF FORCES ON INCLINED PLANE C.D.

FIG. 20.

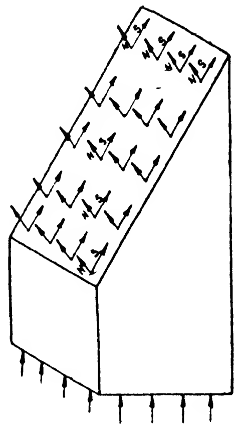


FIG. 21.

Cos θ for 60 deg. is 0·5, so that length CD is twice the length AA. The through dimension remains the same, so that the new area on the plane CD will be 6 in. $\times$ 3 in. $=$ 18 sq. in. Fig. 21 shows this.

Then total normal load $N = W \times \cos\theta = W \times 0{\cdot}5$

$$= 27 \times 0{\cdot}5 = 13{\cdot}5 \text{ tons.}$$

Total shear force $S = W \times \sin\theta$

$$= W \times 0{\cdot}866$$
$$= 27 \times 0{\cdot}866 = 23{\cdot}4 \text{ tons.}$$

$$\text{Normal stress per square inch} = \frac{N}{18} = \frac{13{\cdot}5}{18}$$

$$= 0{\cdot}75 \text{ per square inch.}$$

$$\text{Shear stress per square inch} = \frac{S}{18} = \frac{23{\cdot}4}{18}$$

$$= 1{\cdot}30 \text{ tons per square inch.}$$

The explanation which has just been given in very simple language is generally written in text-books as follows:

Normal intensity of stress per square inch on

$$\text{inclined plane } CD = \frac{W}{A}\cos^2\theta.$$

Tangential or shear stress per square inch on inclined plane $CD = \frac{W}{A} \times \sin\theta\cos\theta$. This can also be written as shear stress

$$= \frac{W}{2A}\sin 2\theta.$$

Let $P = \frac{W}{A}$ = stress per square inch in plane AA,

then

$$\text{shear stress} = \frac{P}{2}\sin 2\theta.$$

The maximum value of shear stress occurs when $\theta = 45$ deg.

Then $2\theta = 90$ deg. and $\sin 2\theta = +1$

Therefore maximum shear stress occurs on a plane inclined at 45 deg. to the horizontal and

$$= \frac{P}{2} \times 1 = \frac{P}{2}.$$

From which it will be seen that the maximum shear stress produced in a body due to a stress being applied at its ends and acting down the centre of the body is equal to one-half this stress P.

Fig. 20 (*a*) shows this plane of maximum shear. Consider now another plane EF inclined at 90 deg. to the plane of maximum shear. This plane will be inclined at 45 deg. + 90 deg = 135 deg. to the horizontal.

$$\text{Shear stress} = \frac{P}{2}\sin 2\theta.$$

In this case $\theta = 135$ deg. $2\theta = 270$ deg. $\sin 2\theta = -1$.

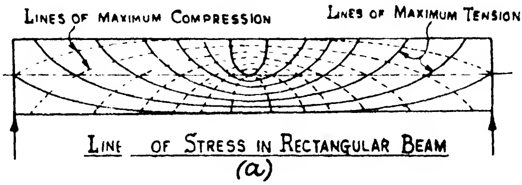
Therefore shear along plane $EF = \frac{P}{2} \times -1 = \frac{P}{2}$.

Thus the shear occurring along a plane at 90 deg. to the principal plane is also a maximum. This shows that the maximum shear stress on one plane is accompanied by an equal shear stress acting on a plane perpendicular to it.

In the first case the shear is positive, and in the second case negative, which shows that they are acting in opposite directions, see Fig. 20 (*a*).

A piece of cast iron may have a breaking strength of 40 tons per square inch in compression and 12 tons in shear. It is 1 in. square. If it is loaded with 30 tons then a compressive stress is set up = 30 tons per square inch, which is only three-quarters of the breaking strength of the material in compression. The shear stress on a plane inclined at 45 deg. to the axis of the bar $= \frac{30}{2} = 15$ tons per square inch, which is more than the ultimate strength of the material. Therefore the bar will fail by shearing in a manner similar to that shown in Fig. 17 (*a*).

Fig. 22 (*a*) shows the lines of stress in a simply supported beam uniformly loaded.



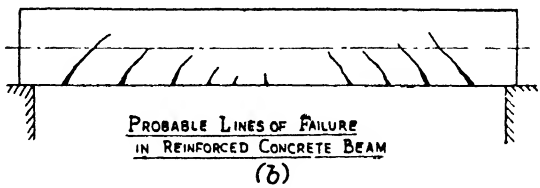


FIG. 22.

In a concrete beam, if failure occurs, it is probable it will be along the wavy lines shown inclined in Fig. 22 (*b*).

This shows the tensile stresses produced by the shear forces. These stresses will be further explained later when we shall deal with the design of wood, steel and reinforced concrete beams.

Bolts and rivets are the principal means used for fastening together steel plates, angles and beams, although welding is now being much more used than formerly. In timber construction bolts are generally used. They are also used for temporary steel-work. They have the advantage when compared with rivets that they can be taken out if the nut is unscrewed, whereas the rivet is a permanent fastening, which when once made can only be taken out by knocking off the head. Well-formed rivets fill completely the holes, whereas if ordinary black bolts are used the holes are not completely filled and therefore there is not the same strength nor rigidity in a joint secured with black bolts as there is in one fastened with well-made rivets. Turned bolts which are made to fit tight into drilled holes are as good as rivets, but they have the disadvantage of being more expensive.

Types of Rivets. Several types of rivets and the proportions of the heads were given in Chapter I. Of these, the snap or cuphead is the most common, but countersunk heads are also much used. The grip of the rivet is the distance between the finished heads, or in other words the total thickness of the plates which have to be joined together. Where there are three or four plates it is difficult to get the holes exactly opposite, and in practice, there is a certain amount of zig-zagging. It is therefore considered good practice that the maximum grip should not be more than four or five times the diameter of the rivet.

The distance from the centre of one rivet to the centre of the next is called the pitch, and should never be less than three times the diameter of the rivet. In general constructional work rivets of $\frac{5}{8}$ in., $\frac{3}{4}$ in., or $\frac{7}{8}$ in. diameter are most commonly used, but in light sheet steel tanks and hoods, rivets of $\frac{1}{4}$ in. diameter are very commonly encountered.

Riveting which is done in the steel shops or steel yard is known as shop riveting, while that done on the site is called site riveting or field riveting. In both these spheres pneumatic or compressed-air tools are now chiefly used. There is still a good deal of hand riveting, and in the shops hydraulic machines are still largely used on heavy work.

Size of Rivets. The student will very soon find that one of his difficulties is to get a formula giving a suitable diameter of rivet. Unfortunately, in text-books one frequently finds words like these: "According to Unwin's formula the diameter of the rivet should be 1 in. This is larger than is suitable, and for practical reasons we shall use rivets $\frac{7}{8}$ in. diameter." In general it can be said that $\frac{7}{8}$-in. diameter rivets are about the largest which can be made by hand riveting, and the student will not be far wrong if he assumes that this is about the largest size of rivets which should be used when dealing with problems which he is likely to get in examinations.

The general routine of hand riveting is to heat the shank to a red heat, and to hold the formed head against the underside of the plates by a dolly or die which is carried at the end of a long steel (or wood) handle. At a point a few inches from the rivet there is a fulcrum provided either by a hook bolt or chains, and it is easy to see that by pressing on the long end of the handle a tremendous upward force can be exerted on the head of the rivet. The red-hot shank is then hammered down either by hand hammers or by a compressed-air machine, and during this process the red-hot metal fills the drilled or punched holes. The newly formed heads are then finished off by snaps or cutters. A sketch showing the position of the rivet before the closing is done was shown in Chapter 1.

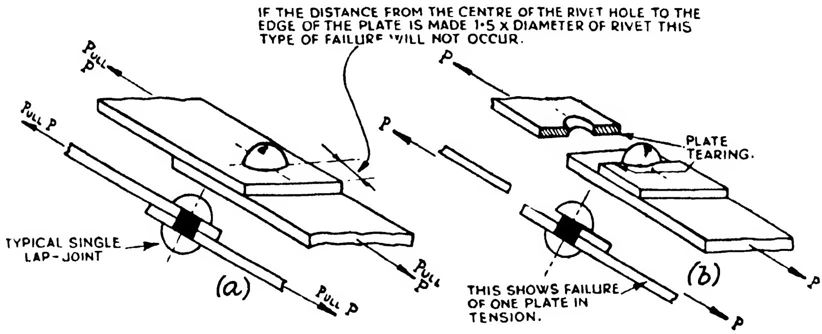


FIG. 23.

Different Joints. *Types of Joints.* Pages 29 to 32 show various types of joints and the methods by which they are likely to fail if badly designed are shown. Fig. 23 (*a*) shows a typical single lap-joing. It is easy to see that this takes its name from the fact that one plate is placed over or laps over another plate. This sort of joint is easy to make, but has certain disadvantages. As shown in Fig. 24, the forces or pulls or tensions tend to bend the plate so that the forces act in one straight line. This causes the rivets to bend and may result in the rivet heads breaking off.

A single-cover butt joint is shown in Fig. 25 (*a*), and has the same disadvantages as the lap joint. It gets its name from the fact that the main two plates are butted together, and on one side only there is a strip or single cover strap.

The question of single shear has already been briefly referred to, and it will be considered more fully later, but in the mean-

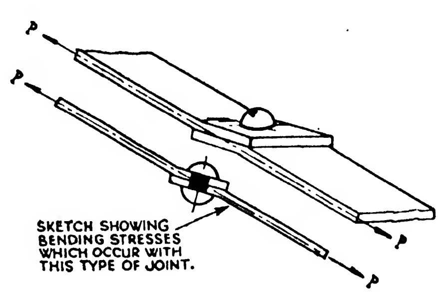


FIG. 24.

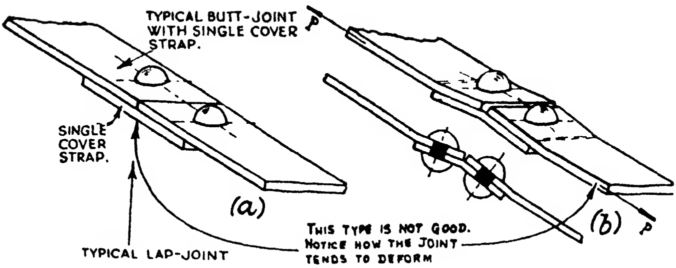


FIG. 25.

time the student will notice that in both the cases just mentioned the rivets are in single shear. A much better joint than either the lap joint or the butt joint with single strap is the butt joint with double cover straps, as shown in Fig. 26. In this case there is no tendency to bend the plates, and the rivets are in double shear.

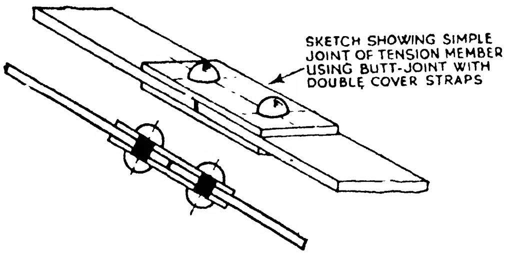


FIG. 26.

Failure of Riveted Joints. There are several ways by which a riveted joint may fail:

(1) By the plate tearing (Fig. 23 (*b*)).
(2) By the plate bursting at the end (Fig. 27 (*a*)).
(3) By the plate crushing (Fig. 27 (*b*)).
(4) By the rivet shearing (Fig. 28 (*a*)).

It is extremely unlikely that the plate will fail by bursting in front of the rivet, as shown in Fig. 27 (*a*), if the distance from the centre of the rivet to the edge of the plate is made $1{\cdot}5 \times$ diameter of the rivet, or, in other words, if the distance from the edge of the rivet to the edge of the plate is not less than the diameter of the rivet there is plenty of strength. We are therefore left with three probable causes of failure, and these will be considered in detail.

In the following, let

P = load applied on each end of plates
W = gross width of plate in inches
d = diameter of rivet and rivet hole in inches
t = thickness of plate in inches.

Failure of Plate in Tension. If the forces P are sufficiently big it is not difficult to see that the two plates might pull apart as shown in Fig. 23 (*b*). The area of the material which is broken is shown shaded in Fig. 29 (*b*). The amount of metal which has pulled apart is $2 \times B \times t$, but $2 \times B$ is the same as $W - d$, so that the area broken is $(W - d) \times t$.

Failure in Bearing or Compression. This method of failure is shown in Fig. 27 (*b*), and it may help the student to

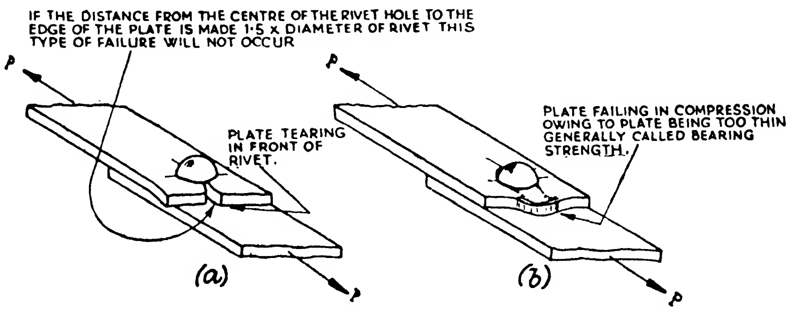


FIG. 27.

realize clearly the principles involved if he will consider one of the plates as being ½ in. thick, and the other, say, only ⅜ in. It is not difficult to visualize that if the plates were pulled apart the thinner plate might fail by compression or crushing of the plate in front of the rivet. This sort of failure is not likely to arise if the thickness of the plate is made somewhere near half the diameter of the rivet, but this is a practical hint only, and will not enable students to solve questions set in examinations or from the actual design point of view.

Effective Bearing Area.—Refer to Fig. 29 (*b*) and (*c*) and consider the forces pulling these plates apart and causing the top plate to bear against the rivet. Although theoretically it would touch on half the circumference of the rivet, the effective bearing area is only the diameter of the rivet multiplied by the thickness of the plate. I know only too well that many students will use half the circumference multiplied by the thickness of the plate as the bearing area. Bearing area however $= d \times t$.

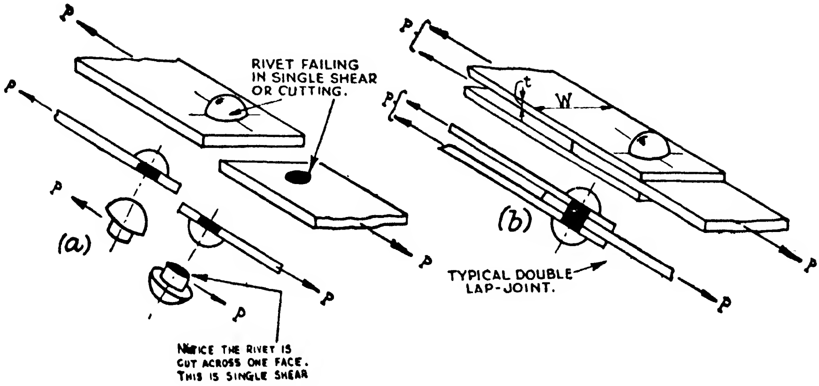


FIG. 28.

Shear strength. Fig. 28 (*a*) shows the joint failing by the rivet shearing or being cut in half. For the rivet to do this it must be cut right through its cross-section. The area of the cross-section of a rivet is that of a circle, and the area of a circle is

$$\pi \times r^2 \text{ or } \frac{\pi \times d^2}{4} \text{ where } \pi = 3{\cdot}14 \text{ approximately.}$$

$$r = \text{radius of rivet.} \quad d = \text{diameter of rivet.}$$

Before the joint can fail, therefore, the rivet must be cut through an area equal to $\frac{\pi \times d^2}{4}$. It will be noted that the rivet is cut in one place only in this case, and this is known as single shear. If the rivet had been cut as shown in Fig. 29 (*a*), it would have been cut at two sections, and this is known as double shear. A rivet in double shear is about twice as strong as one in single shear.

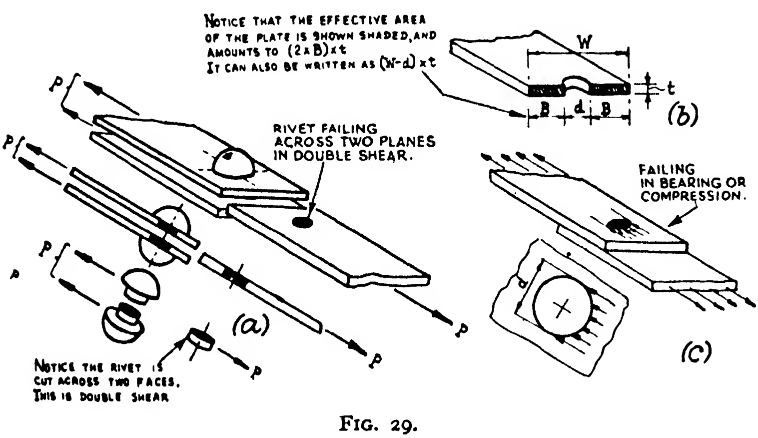


FIG. 29.

Breaking and Safe Stresses. In Chapter 1 we showed that it is not the ultimate or breaking strength of a material which is used when designing, but something considerably less. The relationship between the stress which is allowable and the breaking stress is called the factor of safety, and in riveted construction work a factor safety of 4 is generally used. If the crushing strength of steel is 46 tons per square inch, and a factor of safety of 4 is used, then the safe or working or permissible stress in crushing or bearing will be $46 \div 4 = 11\frac{1}{2}$ tons per square inch. In the table of strengths of materials (Chapter 2) it was shown that the breaking strength of mild steel in tension is about 30 tons per square inch. Again, if we use a factor of safety of 4, the permissible tensile stress will be

Ultimate tensile stress $\div$ Factor of safety

$$= 30 \div 4 = 7\tfrac{1}{2} \text{ tons per square inch.}$$

In like manner we find the permissible shear stress, given that the ultimate shear stress is about 20 tons per square inch. So that permissible shear stress will be $\frac{20}{4} = 5$ tons per square inch.

The student is warned against thinking these figures are absolutely correct. They are average figures, and nothing more. It should be realized, and will certainly be shown to students in technical classes, that the ultimate bearing strength of mild steel might be anything between 44 and 50 tons per square inch, the ultimate tensile strength anything between 28 and 33 tons per square inch, and the ultimate shear strength anything between 18 and 24 tons per square inch.

Similarly, the factor of safety of 4, which is very commonly used, is not the only factor of safety. It might be in some cases justifiable to use a factor of safety of only 3, while in other cases, where there is vibration, shock, or temperature variation, a factor of safety of 6 or 8 is not uncommon. It is for these reasons that the student will find in the various text-books and handbooks different values given for shearing and bearing strength of mild steel rivets.

Bearing Strength. The safe bearing or crushing strength is generally taken as 10 or 12 tons per square inch. Both these values are commonly used.

The safe tensile strength of mild steel plates is generally taken at $7\frac{1}{2}$ or 8 tons per square inch, and the safe shear strength of rivet steel is generally taken at 5 or 6 tons per square inch.

Let Ft be the safe tensile strength of plate $= 1$
Fb be the safe bearing strength of plate $= 1\frac{1}{2}$
Fs be the safe shear strength of rivet steel $= \frac{3}{4}$.

We shall now return to the various methods by which a riveted joint may fail, and calculate the amounts which would probably cause failure, and also the safe loads.

Consider case 1, the failure of plate in tension. The net area broken is $(W - d) \times t$. Let breaking strength of steel $= 30$ tons. The probable pull to cause failure would be $(W - d) \times t \times$ breaking strength, which equals $(W - d) \times t \times 30$ tons.

If Width of plate, $W = 3$ inches
Diameter of rivet hole, $d = \frac{3}{4}$ inch
Thickness of plate, $t = \frac{1}{2}$ inch

then probable pull which would cause failure will be

$$(3 - \tfrac{3}{4}) \times \tfrac{1}{2} \times 30 = 2\tfrac{1}{4} \times 15 = 33\tfrac{3}{4} \text{ tons.}$$

It has been shown that the permissible value would be very much less than this, in fact, it would only be about a quarter of this load.

We can calculate as follows:

Safe load so far as the strength of the plate in tension is concerned equals

$$\text{Safe load} = (W - d) \times t \times Ft.$$

If Ft is taken as 8 tons per square inch, then

$$\text{Safe load} = (3 - \tfrac{3}{4}) \times \tfrac{1}{2} \times 8 \text{ tons}$$
$$= 2\tfrac{1}{4} \times 4 = 9 \text{ tons.}$$

It will be clear that this is the maximum pull which could be safely resisted by the plate against failure, as shown in Fig. 23 (*b*).

Failure by Crushing. We shall now consider the possible failure by crushing or bearing (Fig. 27 (*b*)), and here, again, we can only find the approximate force which would cause bearing or crushing failure. This is found by the formula:

Breaking load $= d \times t \times$ crushing strength

If d is $\frac{3}{4}$ in. diameter
t is $\frac{1}{2}$ in. thick.

Crushing strength is 44 tons per square inch, then
Breaking load $= \frac{3}{4} \times \frac{1}{2} \times 44 = 16\frac{1}{2}$ tons.

Here, again, the student will clearly understand that though it would require a force of $16\frac{1}{2}$ tons to cause failure by the plate crushing, the permissible load would be only about a quarter of this amount. Then—

$$\text{Safe bearing stress} = d \times t \times Fb$$

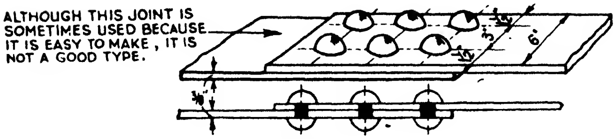


FIG. 30.

which equals in the case considered, taking Fb at 12 tons per square inch.

$$\text{Safe bearing stress} = \tfrac{3}{4} \times \tfrac{1}{2} \times 12 = 4\tfrac{1}{2} \text{ tons.}$$

This method failure is not very easy to visualize, but the fact that the rivet might pull apart is very easy to imagine.

The strength of one rivet in single shear is area $\times$ shearing strength. Let shearing strength of rivet steel $= 20$ tons per square inch, then

$$\text{Breaking load on rivet} = \frac{\pi}{4} \times d^2 \times 20$$

$$= \frac{3\cdot14}{4} \times \tfrac{3}{4} \times \tfrac{3}{4} \times 20 = \text{say } 8\cdot8 \text{ tons.}$$

Using a factor of safety of 4, the safe shear stress Fs of mild steel would be 5 tons per square inch, and the safe working load on one rivet $\frac{3}{4}$ in. diameter in single shear would be

$$\frac{3\cdot14}{4} \times d^2 \times Fs = \frac{3\cdot14}{4} \times 0\cdot75 \times 0\cdot75 \times 5 = 2\cdot2 \text{ tons.}$$

Collecting all these figures together, we find that a joint of the dimensions given would be safe as far as the tensile strength of the plate is concerned to resist a pull of 9 tons. It would be safe as far as bearing or crushing is concerned to resist a pull of $4\frac{1}{2}$ tons, while the rivet would be safe in shear if the pull on the plates was 2·2 tons.

Figs. 30 and 31 show a lap joint and a double cover butt joint.

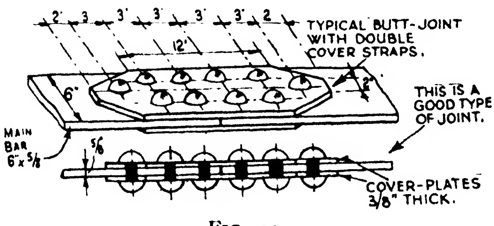


FIG. 31

CHAPTER 4

SIMPLE STRESSES AND STRAINS

We have already dealt with several technical terms and their meanings, and shall now proceed to show how these can be used.

If the student has clearly understood the difference between stress and strain, and the meaning of elastic limit and yield point, he will not find much difficulty in understanding why the permissible or working stress is only about one-quarter of the breaking stress. It is not very easy to realize why it is that if it will take a pull of 30 tons to break a bar, it is only considered safe for the same bar to be pulled to 7½ to 8 tons. There seems to be such an extraordinary difference in these values.

Fig. 32 shows three different bars of the same cross-sectional area but of a different shape. If these bars are made of mild steel

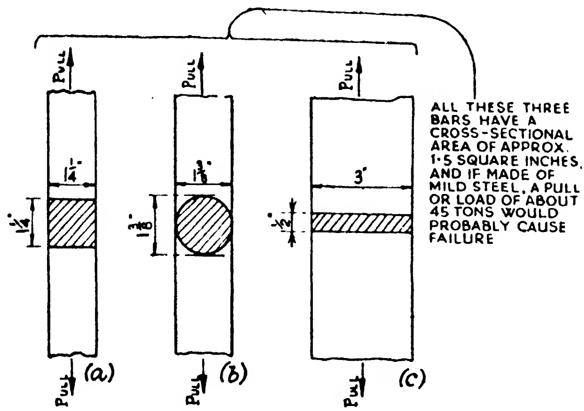


FIG. 32.

they will have a breaking strength of about 30 tons per square inch. The area of the square, round and rectangular bar is in each case about 1½ sq. in. It therefore follows that if we apply a load of about 45 tons on the end of each bar it will probably break. This shows that the strength of a bar in tension is in proportion to its cross-sectional area.

The following questions will help to make clear the points which have been dealt with so far with regard to the design of riveted joints.

QUESTION 1. What is the probable pull which would cause a mild steel bar 3 in. wide and ½ in. thick to break in tension?

Answer. This bar would be of the type shown in Fig. 32 (*c*). The table given on page 16 showed that the average breaking strength for mild steel in tension is 30 tons per square inch. The area of a bar 3 in. wide and ½ in. thick is 1½ sq. in., so that

$$\text{Breaking load} = \text{Area} \times \text{Breaking strength}$$
$$= 1{\cdot}5 \times 30 = 45 \text{ tons}$$

QUESTION 2. What is the safe load which could be carried by a mild steel bar 3 in. by ½ in. in section ?

Answer. Using a factor of safety of 4

$$\text{Working stress} = \frac{\text{Breaking stress}}{\text{Factor of safety}}$$

$$\text{Working stress} = \frac{30}{4} = 7\tfrac{1}{2} \text{ tons per square inch}$$

$$\text{Safe load} = \text{Area} \times \text{Allowable stress}$$
$$= 1{\cdot}5 \times 7{\cdot}5 = 11{\cdot}25 \text{ tons}$$

Notice that this is not tons per square inch, but the total load which can safely be carried by a mild steel bar of the size given.

QUESTION 3. What load would probably break a mild steel bar 3 in. wide and ½ in. thick, with a hole ¾ in. diameter drilled in the bar ?

Answer. Fig. 33 shows this bar, and it is easy to see that less load will be required to break the bar in this case than for the bar given in Question 1. The effective area is now reduced by the hole which has been drilled.

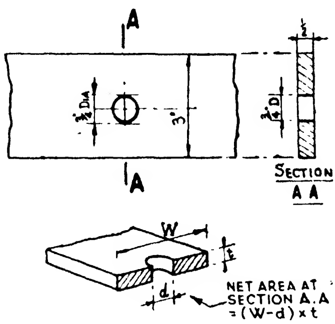


FIG. 33.

The amount of material which would have to be pulled apart or broken is called the net area, and, as explained in the preceding chapter, the net area is the total width less the diameter of the hole, multiplied by the thickness of the plate.

It can be written as

$$\text{Net area} = (W - d) \times t$$

where W is the width of the plate
d is the diameter of the rivet hole
t is the thickness of the plate

The probable force required to break the bar would be

$$\text{Breaking load} = \text{Net area} \times \text{Breaking strength}$$
$$= \text{Net area} \times 30 \text{ tons}$$
$$= (3 - \tfrac{3}{4}) \times \tfrac{1}{2} \text{ by } 30 \text{ tons}$$
$$= 2\tfrac{1}{4} \times \tfrac{1}{2} \times 30$$
$$= 1\tfrac{1}{8} \times 30 = 33{\cdot}75 \text{ tons}$$

QUESTION 4. What is the safe load or pull which a mild steel bar 3 in. wide by ½ in. thick, with a hole ¾ in. diameter drilled through the bar, can stand ?

Answer. Again using a factor of safety of 4—

$$\begin{aligned}\text{Safe load} &= \text{Net area} \times \text{Allowable stress}\\ &= 1\tfrac{1}{8} \times 7{\cdot}5 \text{ tons}\\ &= \text{nearly } 8\tfrac{1}{2} \text{ tons}\end{aligned}$$

In practice, the load or force which a bar must carry is often known from the design sheets, and it is necessary to fix up a bar of suitable size to resist this load.

QUESTION 5. What is the probable force which would cause a rivet $\frac{3}{4}$ in. diameter to fail in single shear?

Answer. The student will understand what shear is, and also the difference between single and double shear. The probable force which would cause a rivet or round bar to fail in shear is

$$\text{Shearing force} = \text{Area} \times \text{Shearing strength}$$

The area of a round bar is $\pi \times r^2$.

This can also be written as $\dfrac{\pi \times d^2}{4}$.

π is the relationship between the circumference and the diameter of a circle, and amounts to $3{\cdot}14$.

The area of a circle can also be written as $\dfrac{22}{28} \times d^2$.

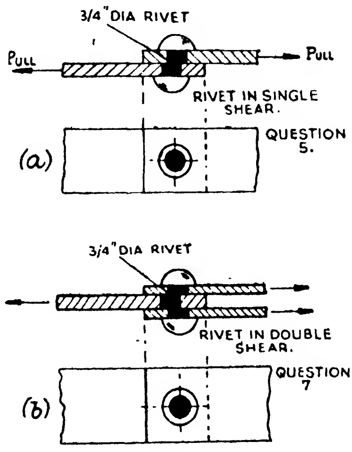


FIG. 34.

Fig. 34 (*a*) shows how in single shear the rivet is cut at one section only, therefore

$$\begin{aligned}\text{Shearing force} &= \text{Area} \times \text{Shearing strength}\\ &= \frac{3{\cdot}14}{4} \times \tfrac{3}{4} \times \tfrac{3}{4} \times 20 \text{ tons}\\ &= 0{\cdot}44 \times 20 \text{ tons}\\ &= 8{\cdot}8 \text{ tons}\end{aligned}$$

QUESTION 6. What is the safe strength of a $\frac{3}{4}$-in. diameter rivet in single shear?

Answer.

$$\text{Safe shearing strength} = \frac{\text{Breaking strength}}{\text{Factor of safety}}$$
$$= \frac{8\cdot8}{4} = 2\cdot2 \text{ tons}$$

QUESTION 7. What is the probable force which would cause a rivet ¾ in. diameter to fail in double shear ?

Answer. The term double shear means that the rivet will be cut across two faces, as shown in Fig. 34 (*b*), and it will therefore require twice as much force to do this as to cause the rivet to fail in single shear.

$$\text{Shearing force} = 2 \text{ (Area of rivet} \times \text{Shearing strength)}$$

$$\text{Area} = \frac{\pi}{4} \times d^2$$

Therefore

$$\text{Shearing force} = 2 \times \frac{3\cdot14}{4} \times \tfrac{3}{4} \times \tfrac{3}{4} \times 20$$
$$= 2 \times 0\cdot44 \times 20$$
$$= 0\cdot88 \times 20 = 17\cdot6 \text{ tons}$$

In practice, the resistance in double shear is often taken as only 1·75 times the value in single shear, but unless the local authorities limit the double shear to only 1·75 times the single shear value, there is no reason why the double shear value should not be taken as twice the single shear value.

QUESTION 8. What is the safe strength of a ¾ in. diameter rivet in double shear ?

Answer. This will be twice that of a rivet in single shear (Question 6)

$$= 2 \times 2\cdot2 = 4\cdot4 \text{ tons}$$

QUESTION 9. Two plates are connected by a lap joint, as shown in Fig. 35 (*a*). If the plates are ½ in. thick, what force would probably cause crushing in front of the rivet, if this was ¾ in diameter ?

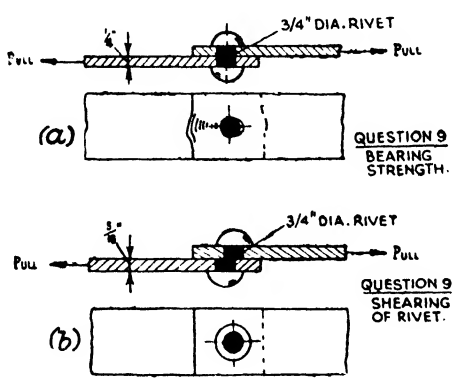


FIG. 35.

Answer. This is generally called the bearing strength, and

$$\text{Bearing force} = d \times t \times Fc$$

where d is the diameter of the rivet
t is the thickness of the plate
Fc is the crushing strength of the plate

$$\text{Force to cause failure} = \tfrac{3}{4} \times \tfrac{1}{4} \times 40 \text{ tons}$$

$$= \frac{3}{16} \times 40 = 7\tfrac{1}{2} \text{ tons}$$

We know that if a pull of $7\frac{1}{2}$ tons would cause failure that it is only safe to allow a pull of about a quarter of this amount. The allowable or safe resistance against bearing or crushing failure in the conditions just mentioned would be

$$\text{Safe load} = \frac{\text{Crushing strength}}{\text{Factor of safety}} = \frac{7\frac{1}{2}}{4} = 1\tfrac{7}{8} \text{ tons}$$

By similar reasoning the student will have no difficulty in finding that the safe crushing strength, if the plate is made $\frac{5}{16}$ in. thick, would be 2·3 tons, and we have already found that the allowable single shear on a $\frac{3}{4}$-in. rivet was 2·2 tons. Therefore if the plate is made $\frac{1}{4}$ in. thick the safe bearing force is 1·9 tons, which is less than the shear value, in which case the bearing strength is less than the shearing strength of the rivet. If, however, the plates are made $\frac{5}{16}$ in. thick, the safe bearing strength is 2·3 tons, which is greater than the single shear value of the rivet. From this it follows that when using $\frac{3}{4}$-in. diameter rivets in single shear, if the plates are made $\frac{5}{16}$ in thick, they are not likely to fail by bearing in front of the rivet, whereas if the plates are made $\frac{1}{4}$ in. thick there is more likelihood of the plate failing in bearing than of the rivet failing in shear.

QUESTION 10. The tension in a railway bridge tie-bar is 40 tons. If the bar is to be made $\frac{3}{4}$ in. thick, what is a suitable width for the bar, if the permissible working stress is 7 tons per square inch ?

Answer.

$$\text{Permissible load} = \text{Area} \times \text{Permissible stress}$$

$$\text{Area} = \frac{\text{Permissible load}}{\text{Permissible stress}}$$

$$\text{Area} = \frac{40 \text{ tons}}{7 \text{ tons per square inch}}$$

$$= 5{\cdot}7 \text{ sq. in.}$$

The area of a flat bar is Width × Thickness.

Therefore Width × Thickness = 5·7

And since the thickness is known, we can find the necessary width.

$$\text{Width} = \frac{5{\cdot}7}{\frac{3}{4}} = 5{\cdot}7 \times \frac{4}{3} = 7{\cdot}6 \text{ in.}$$

It should be carefully noted that this 7·6 in. is the actual net width of bar which is required, and if three holes on the same line are drilled in the bar, as shown in Fig. 36, and these holes are 1 in. diameter, it is clear that the effective or net width would be 7·6 less 3 in. or an effective width of 4·6 in. This is clearly not strong enough. In order to leave the bar with an effective width of 7·6 in. after the rivet holes had been taken out, it would be necessary to make the bar $10\frac{1}{2}$ in. wide in the first case. Three 1-in. holes taken out of this would give the required net area.

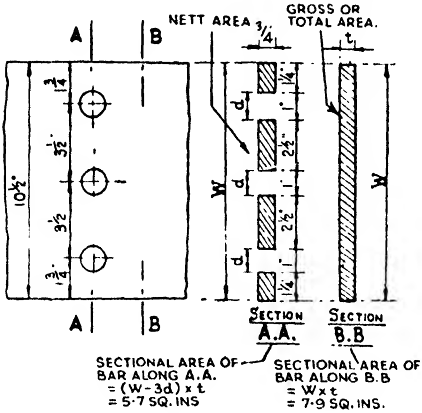


FIG. 36.

It will be noted that the plate is weakest along section *AA* where the cross-sectional area is least, while along section *BB* the plate is a good deal stronger. The rivets could be placed as shown in the drawing.

QUESTION 11. A tie-bar in a roof truss has a pull of 10 tons on it. Design a suitable butt joint with double cover straps, using rivets $\frac{3}{4}$ in. diameter.

Answer.

$$\text{Tension: Net area of bar required} = \frac{\text{Load}}{\text{Safe stress}}$$

Allowing a safe stress of $7\frac{1}{2}$ tons per square inch.

$$\text{Net area} = \frac{10 \text{ tons}}{7\frac{1}{2} \text{ tons}} = 1{\cdot}33 \text{ sq. in.}$$

Rivets in Shear: Since the joint is a butt joint with cover straps on each side, the rivets will be in double shear.

$$\text{Number of rivets required} = \frac{\text{Total load}}{\text{Strength of 1 rivet in double shear}}$$

Strength of one $\frac{3}{4}$-in. diameter rivet in double shear $= 4{\cdot}4$ tons.

$$\text{Number of rivets required} = \frac{10 \text{ tons}}{4{\cdot}4 \text{ tons}} = \text{use 3 rivets.}$$

The student should be careful to note that three rivets are required on each side of the joint, see Fig. 37 (*a*). This will be clear by looking at Fig. 37 (*b*). The pull on the main plate is 10 tons, and if the rivets were too weak the plate could be pulled away from the cover straps as shown, and to do this it is only necessary to shear the rivets on one side of the joint.

Bearing. We have now found the net area required and the shearing strength of a $\frac{3}{4}$-in. rivet in double shear, but we have not found the shape of the main member. The thickness will be such that the bearing strength is at least equal to the shearing strength of a rivet in double shear.

Fig. 37 (*c*) shows how the plate tends to bulge in front of each rivet if the plate is made too thin.

$$\text{Bearing strength} = d \times t \times Fb$$

where d is the diameter of the rivet

t is the thickness of the plate

Fb is the allowable bearing strength, say 10 tons per square inch.

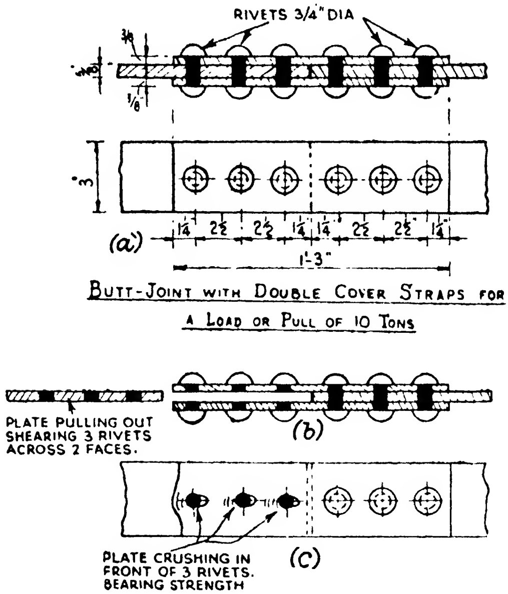


FIG. 37.

The total shear resistance of one rivet in double shear is $2 \times \text{Area} \times Fs$.

$$\text{Area} = \frac{22}{28} \times d^2$$

Fs = allowable shear stress = say, 5 tons per square inch.

If we make the total bearing strength equal to the total shear strength of one rivet in double shear, we get

$$d \times t \times 10 = 2 \times \frac{22}{28} \times d^2 \times 5$$

Divide each side by $10d$, and

$$t = \frac{22}{28}d$$

$$= 0{\cdot}8d.$$

So that it is safe to assume that if the plate thickness is made about 0·8 of the rivet diameter when the rivet is in double shear, the joint will be strong enough for bearing. Similarly, if the plate is made 0·4 × the diameter of the rivet, when the rivet is in single shear the bearing strength will be adequate.

It is therefore now possible to find a suitable thickness for the bar, using the formula just obtained.

In this case $t = 0{\cdot}8 \times \frac{3}{4} = 0{\cdot}6$, say $\frac{5}{8}$ in.

$$\text{Net area} = (W - d) \times t$$
$$1{\cdot}33 = (W - \tfrac{3}{4}) \times \tfrac{5}{8}$$
$$\frac{1{\cdot}33 \times 8}{5} = W - \tfrac{3}{4}$$
$$2{\cdot}13 + 0{\cdot}75 = W$$
$$W = 2{\cdot}88, \text{ say } 3 \text{ in.}$$

In practice, cover plates are generally made about five-eights the thickness of the main bar. In this case $\frac{3}{8}$ in. thick would do nicely.

Pitch of Rivets. The distance centre to centre of rivets in the same row is called the pitch, and for rivets $\frac{3}{4}$ in. diameter, $2\frac{1}{2}$ in. or 3 in. centres are very common. The pitch should never be less than three times the diameter of the rivet. The complete design is shown in Fig. 37 (*a*).

To sum up, it is near enough to assume that the single shear strength of a rivet $\frac{5}{8}$-in. diameter is $1\frac{1}{2}$ tons, in double shear 3 tons, while a $\frac{3}{4}$-in. rivet is good enough for $2\frac{1}{4}$ tons in single shear and $4\frac{1}{2}$ tons in double shear.

The table below gives the shear strengths of various diameters of rivets and the bearing strengths of plates using these rivets.

SHEARING AND BEARING VALUES OF RIVETS AND BOLTS.

Diameter of Rivet or Bolt Inches	Area of Rivet or Bolt Square Inches	Shearing Values at 5 tons per square inch		Bearing Values at 10 tons per Sq. Inch Thickness of Plate in Inches.			
		Single Shear	Double Shear	1/4	3/8	1/2	5/8
1/2	·196	·98	1·96	1.25	1.88		
5/8	·307	1·53	3·07	1.56	2.34		
3/4	·442	2·21	4·42	1.88	2.81	3·75	
7/8	·601	3·01	6.01	2.19	3.28	4.38	5.47

Welding. Notes on welding appear in Chapter 21, page 216.

CHAPTER 5

RIVETED JOINTS

We have already discussed the elementary principles of riveted joints, and showed that the point to be kept in mind when designing them is the strength of the rivets in shear, and strength of the main plate in tension, and the crushing or bearing strength of the plate in front of the rivet, should be kept as near equal as possible.

We shall now show a design for a butt joint with double cover straps suitable for connecting two flat bars together. We shall also show the importance of getting a correct arrangement of the rivets if the most economical joint is to be obtained. Students preparing for examinations in strength of materials, machine design, theory of structures, or for the examinations of the Institution of Structural Engineers or the Royal Institute of British Architects, should know something of the principles involved in finding the strength and efficiency of riveted joints.

In examination questions the size of the flat bar for which a joint is required is often given. In practice, the size of this bar must be found from the amount of stress or pull which the tension bar is to carry. The joint itself is required either at the end of the tension member to connect it with gusset plates, or alternatively it may be required near the centre of the member if the length is too long to permit one bar being used.

Example. A mild steel tension bar in a structure is 9 in. wide and $\frac{5}{8}$ in. thick. Using a safe tensile stress for the steel of 8 tons per square inch, a safe shearing strength for the rivets of 5 tons per square inch, and an allowable bearing or crushing strength of 10 tons per square inch, design a suitable butt joint, using double cover straps. What is the efficiency of the joint? Design two alternative butt joints which will withstand the same load.

Answer. There are many possible arrangements for the joint. Three are shown in design *A*, design *B* and design *C* (Fig, 38).

Design *A* is the one most commonly used, and we shall make our design on these lines. Later we shall show that this type is more economical than either design *B* or design *C*.

All the rivets are in double shear. The joint may fail in four ways:

(1) By all the rivets on one side of the joint shearing. Notice

that it is on one side of the joint, because if the rivets numbered 1 to 9 are not strong enough in shear the left-hand main bar would

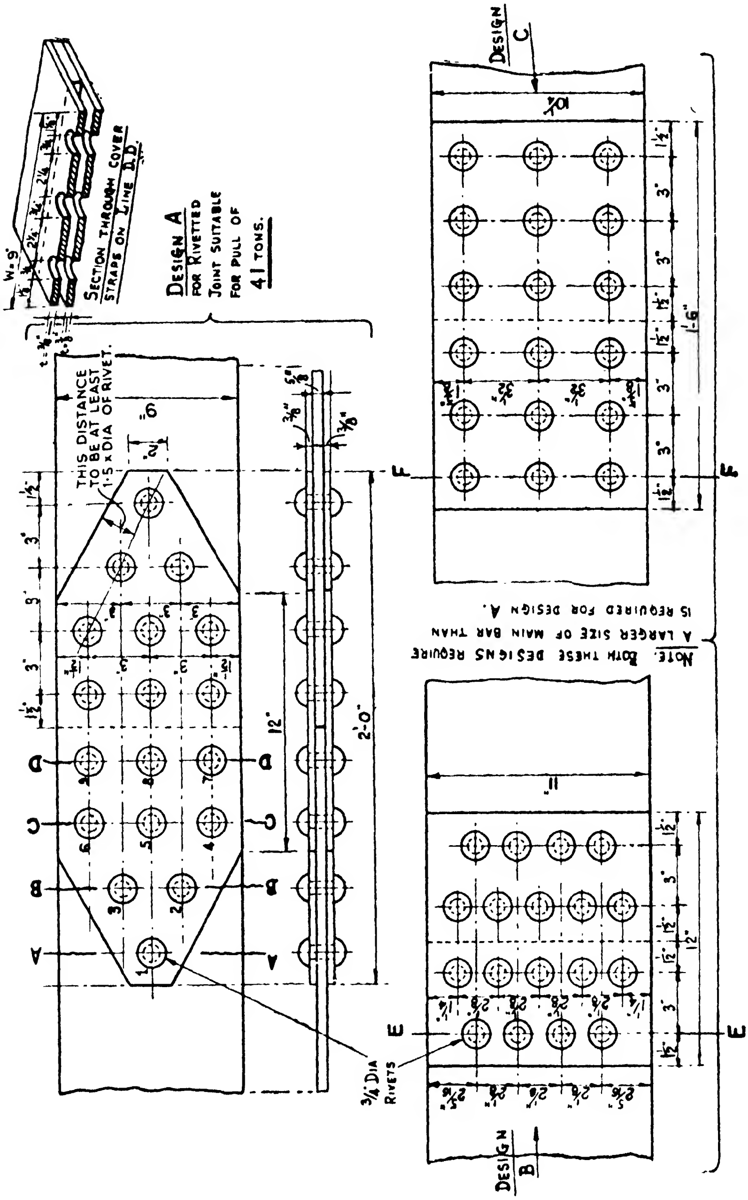


FIG. 38.—Designs A, B, and C

pull right out from between the cover plates and the joint would fail.

(2) The main bar may fail by tearing across any of the lines A B, C, or D.

(3) The joint may fail by crushing in front of the rivets if the bar or cover plates were too thin.

(4) The joint may fail by both the cover straps tearing across line DD. (In practice it will generally be found that if each of the cover straps be made five-eights the thickness of the main bar they will be satisfactory.

Number of Rivets. Before we can find the number of rivets required we must find the maximum force which the joint can be called upon safely to resist. This will depend upon the tearing strength of the main bar.

Tearing Strength of Main Bar. The tearing strength of the main bar is least across line AA. This will be proved shortly. At this point there is one rivet hole, so that across AA the main bar has a net sectional area $= (W - d) \times t$ (see Fig. 39 (a)).

Rivets either $\frac{3}{4}$ in. or $\frac{7}{8}$ in. diameter could be used. For this design we shall use rivets $\frac{3}{4}$ in. diameter.

Net section area of main bar across AA

$$\begin{aligned} &= (9 - \tfrac{3}{4}) \times \tfrac{5}{8} \text{ in.} \\ &= 8\tfrac{1}{4} \times \tfrac{5}{8} \text{ in.} \\ &= 5{\cdot}15 \text{ sq. in.} \end{aligned}$$

Safe tensile stress is 8 tons per square inch, so that

Safe load in tension on main bar

$$\begin{aligned} &= \text{Net area} \times \text{Safe stress} \\ &= 5{\cdot}15 \times 8 \text{ tons} \\ &= \text{say } 41 \text{ tons.} \end{aligned}$$

Therefore the main bar can safely resist a pull of 41 tons.

Number of Rivets Required. The number of rivets required will be governed either by shear or bearing. (This was explained fully in the preceding chapter.)

The strength of 1 rivet in double shear

$$\begin{aligned} &= 2 \times \text{Area of rivet} \times \text{Safe shear stress} \\ &= 2 \times \frac{\pi}{4} \times \tfrac{3}{4}^2 \times 5 \text{ tons} \\ &= 4{\cdot}42 \text{ tons.} \end{aligned}$$

The strength of $\frac{3}{4}$ in. rivet in $\frac{5}{8}$ in. thick plate in bearing

$$\begin{aligned} &= d \times t \times \text{Safe bearing stress} \\ &= \tfrac{3}{4} \times \tfrac{5}{8} \times 10 \text{ tons} \\ &= 4{\cdot}7 \text{ tons.} \end{aligned}$$

As the shear strength is lower than the bearing value the shear strength must be used in the design.

Number of rivets required in double shear

$$= \frac{\text{Total load}}{\text{Shear strength of 1 rivet}}$$

$$= \frac{41 \text{ tons}}{4{\cdot}42 \text{ tons}} = 9 \text{ rivets.}$$

The nine rivets will be arranged as shown in design *A* (Fig. 38).

Safe Loads. *Efficiency of Joint.* The efficiency of a joint is the relationship between the load the main bar could safely carry if there were no rivets in it, and the actual load the joint can safely carry. Consider the efficiency of the joint across line *AA*. A 9 in. × ⅝ in. solid bar could safely carry, allowing a safe tensile stress of 8 tons per square inch, a load of

$$9 \text{ in.} \times \tfrac{5}{8} \text{ in.} \times 8 \text{ tons} = 45 \text{ tons.}$$

At section *AA* the safe load which the main bar could carry

$$= (9 - \tfrac{3}{4}) \times \tfrac{5}{8} \times 8 \text{ tons} = 41 \text{ tons.}$$

Thus the efficiency of the joint at this section

$$= \frac{41}{45} \times 100 = 91 \text{ per cent.}$$

That is to say, the joint at this section is $\frac{91}{100}$ths of the strength of the main plate.

Now consider section *BB*. Here two rivets are taken out, as shown in Fig. 39 (*b*).

Thus the net area of main bar at this section

$$= (W - 2d) \times t$$
$$= (9 - 1\tfrac{1}{2}) \times \tfrac{5}{8}$$
$$= 7\tfrac{1}{2} \times \tfrac{5}{8} \text{ in.} = 4{\cdot}7 \text{ sq. in.}$$

Safe tearing strength of main member at *BB*

$$= 4{\cdot}7 \times 8 \text{ tons} = 37{\cdot}6 \text{ tons.}$$

It is important to notice, however, that before the bar can tear along *BB* the rivet No. 1 must be cut in double shear. Until this occurs failure by the bar tearing along *BB* cannot take place. The safe strength of a ¾-in. diameter rivet in double shear has been found to be 4·42 tons. We have just seen that the bar itself at section *BB* is safe to resist a pull of 37·6 tons. The strength of the end rivet in double shear (4·42 tons) must be added to this amount in order to find the total strength at this section.

Total safe load along *BB* = 37·6 + 4·42 = say 42 tons.

This shows that the joint on this line is stronger than along *AA*.

Section *CC* is shown in Fig. 39 (*c*). The main bar itself has a safe strength against tearing of

$$(W - 3d) \times t \times \text{Safe tensile stress}$$
$$= (9 - 2\tfrac{1}{4}) \times \tfrac{5}{8} \times 8 \text{ tons}$$
$$= 6\tfrac{3}{4} \times 5 = 33{\cdot}8 \text{ tons.}$$

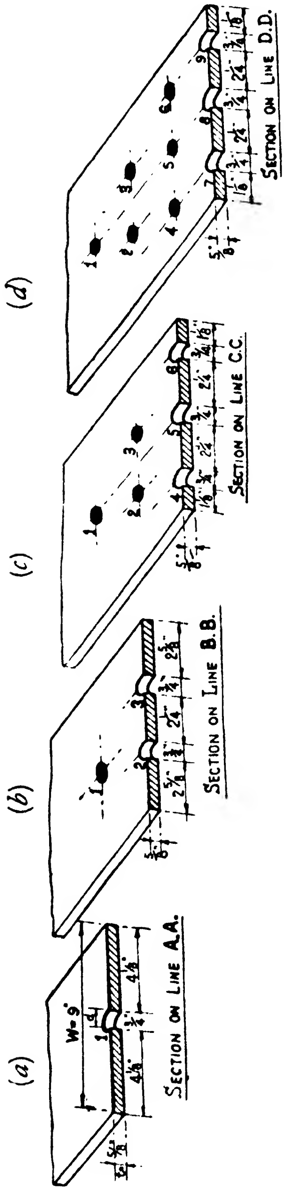


Fig. 39.

Before the joint can fail along *CC* the rivets 1, 2, and 3 must shear. The strength of three rivets in double shear

$$= 3 \times 4{\cdot}42 = 13{\cdot}26 \text{ tons.}$$

By similar reasoning it is easy to see that

Safe load on joint at *CC* =
$$33{\cdot}8 + 13{\cdot}2 = 47 \text{ tons.}$$

Since we have already shown that the safe strength of the main bar itself at any point outside the joint is only 45 tons, it is clear that the safe strength of the joint on the line *CC* is more than the actual bar itself, and therefore failure will not take place by the main bar tearing along this line.

Tensile Strength. At section *DD* there are again three rivet holes in the main bar (Fig. 39 (*d*)). Therefore along this section the bar will have a safe tensile strength of 33·8 tons (as shown in section *CC*). Before failure of the main bar can occur along this line the rivets 1, 2, 3, 4, 5, and 6 must all fail by shear, and the strength of six rivets in double shear $= 6 \times 4{\cdot}42 = 26{\cdot}5$ tons.

Safe load on main bar at *DD*

$$= 33{\cdot}8 + 26{\cdot}5 = 60{\cdot}3 \text{ tons.}$$

Failure will therefore not occur by the main plate tearing along this line.

Summarizing the results, we have

Safe load along section AA – 41 tons
,, ,, ,, BB – 42 ,,
,, ,, ,, CC – 47 ,,
,, ,, ,, DD – 60 ,,

It follows that the efficiency of the joint is determined by Section AA, and is 91 per cent.

The statement made previously that the main member is weakest along AA has thus been proved. This is an important point, because when we know this, it is possible to find the section of the main member if we know the load which the bar must carry. This is always the procedure in practice, because the force in the bar must first be found and then a suitable section to carry that force.

Tearing Strength of Cover Straps. The cover straps must be made sufficiently thick to ensure that they are as strong as the safe strength of the main bar along line AA. We have just seen that this is 41 tons. If the cover plates were made too thin, examination of design A will show that the whole joint might fail by the cover plates tearing along a line DD, as shown in Fig. 38. If the cover plates fail by tearing along this line the joint fails.

The strength of two cover plates along DD will be

$$\begin{aligned} &2 \times (W - 3d) \times t \times \text{Safe tensile stress} \\ &= 2 \times (9 - 2\tfrac{1}{4}) \times t \times 8 \text{ tons} \\ &= 2 \times 6\tfrac{3}{4} \times t \times 8 = 108t. \end{aligned}$$

The value of t must be such that the cover plates are safe for a tearing stress of 41 tons.

Therefore $\qquad 108t = 41$ tons

$$t = \frac{41}{108} = 0{\cdot}378 \text{ in.}$$

so that the thickness of each cover could be $\frac{3}{8}$ in.

We will now consider the alternative design B. The main member (shown in Fig. 38) will be weakest along the row of four rivets (line EE).

Although the net area of the bar along the line of the five rivets is less, it must be remembered that the row of four rivets must fail in double shear before failure of the joint can occur.

Net sectional area of bar required

$$= \frac{\text{Load}}{\text{Safe stress}} = \frac{41 \text{ tons}}{8} = \text{say, } 5 \text{ sq. in.}$$

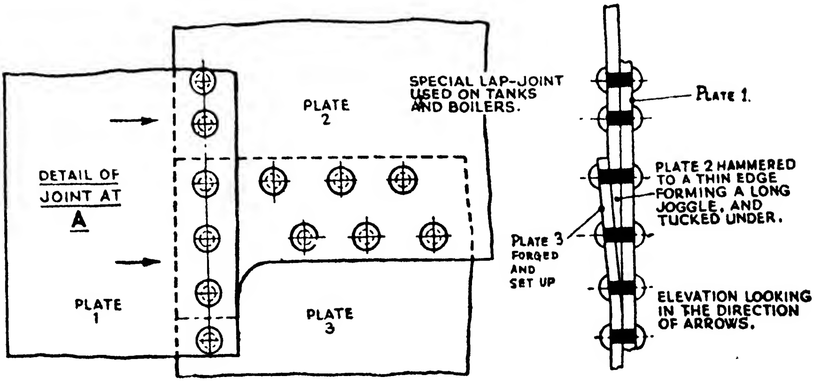


FIG. 40.

Along the row of 4 rivets,

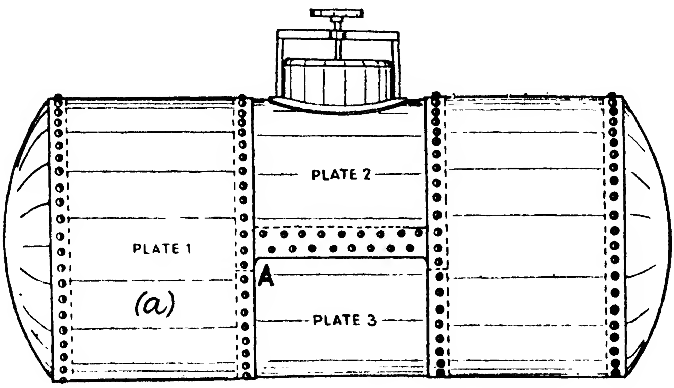
Net area $= (W - 4d) \times t = (W - 3 \text{ in.}) \times \frac{5}{8}$ in.

This must equal 5 sq. in., so that

$$(W - 3) \times \tfrac{5}{8} = 5 \text{ sq. in.}$$
$$W - 3 = 8$$
$$= 8 + 3 = 11 \text{ in.}$$

In this case the bar would need to be 11 in. $\times \frac{5}{8}$ in. as against 9 in. $\times \frac{5}{8}$ in. for design *A*. Some length would be saved in the cover straps, but this would not justify using design *B*.

Weakest Section. Consider design *C* (Fig. 38). Note that in this case, since there is the same number of rivets at any section throughout the main bar and at any section through the cover straps, the covers could be made half the thickness of the main bar



RIVETTED TANK FOR CARRYING LIQUIDS

FIG. 41.

in this case. The weakest section of the member will be along the outside row of three rivets, at FF.

$$\text{Net sectional area} = (W - 3d) \times t = 5 \text{ sq. in.}$$

$$(W - 2\tfrac{1}{4}) \times \tfrac{5}{8} = 5$$

$$W - 2\tfrac{1}{4} = \frac{5}{1} \times \frac{8}{5} =$$

$$W = 8 + 2\tfrac{1}{4} = 10\tfrac{1}{4} \text{ in.}$$

Again, in this case a wider plate than for design A is required, so that design C would not be as economical as design A.

In Fig. 40 a special type of lap joint is shown, and it will be noticed that one of the plates is hammered to a thin wedge shape. This type of joint is quite commonly used on large storage tanks and boilers. A type of tank used on the railways for carrying liquids is shown in Fig. 41.

CHAPTER 6

BASES FOR COLUMNS

In Chapter 5 we showed the importance of the arrangement of rivets, and also the complete design of a riveted joint for connecting mild steel flat bars together by using a butt joint, with double cover straps.

On page 54, the details of a riveted base for a steel column are shown. The chief principles involved in making such a design will be considered here.

The base plate is necessary in order to spread the load passing down the column over a sufficient area to ensure that the column does not sink into the ground or crush the support on which it rests. It is not difficult to imagine that if there was no base plate provided, and that if the rolled steel joist which forms the column shaft was set directly on a masonry or concrete block, the steel would sink into the pier and break up the bricks or concrete as shown in Fig. 42 (*a*).

Masonry and Concrete. The safe bearing capacity of masonry and concrete has been found by many years of experience and actual tests. It is quite safe to allow the following pressure on concrete foundations:

1 : 10 mass concrete	= 10 tons per square foot
1 : 8 ,, ,,	= 15 ,, ,,
1 : 6 ,, ,,	= 20 ,, ,,

For good concrete with a mixture by volume of one of cement, two of sand and four of aggregate, the bearing pressure between the steel base plate and the foundation block might be as high as 30 tons per square foot.

While on this subject of bearing pressure we may as well keep in mind that the safe bearing pressure on the ground below the concrete block is very much less than the safe bearing pressure between the steel base of the column and the top of the concrete block. It is easy to imagine that if a concrete foundation block 2 ft. square and 1 ft. thick was put on to new-made ground, a load of 10 tons would probably cause it to sink several inches into the ground, as shown in Fig. 42 (*b*). In order to ensure that the foundation block does not settle, it must be made with an area big enough to ensure that the load on each square foot is not more than the ground will carry.

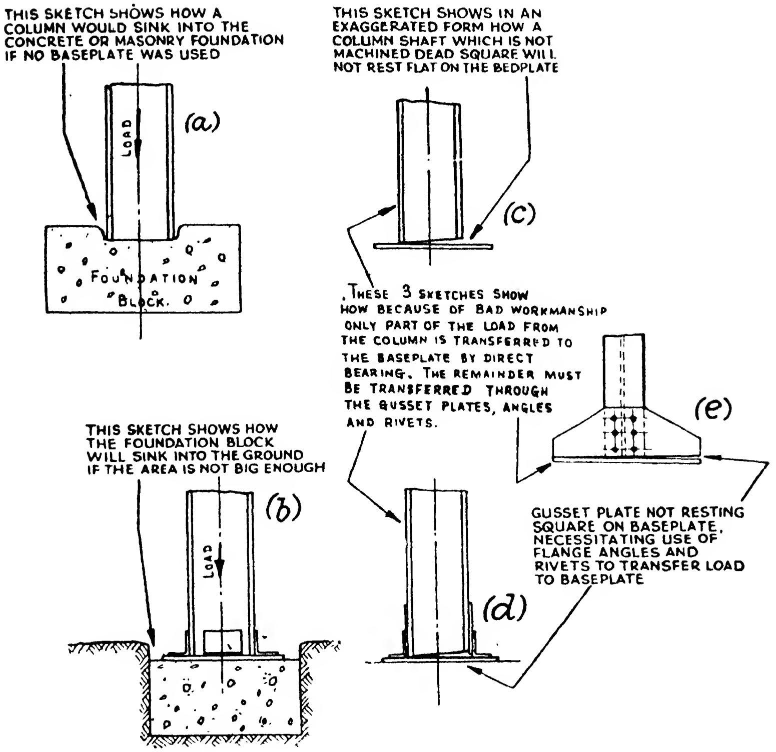


FIG. 42.

The following table gives values of what are generally considered to be about the maximum permissible loads on ground in tons per square foot :

Newly-made ground . . .	not more than ½ ton
Soft clay, wet or loose sand . .	,, ,, ,, 1 ton
Firm dry clay or ordinary good ground	,, ,, ,, 2 to 3 tons
Compact sand or gravel, very good ground	,, ,, ,, 4 tons

Where the foundation is hard, solid chalk or soft rock, a safe bearing pressure of anything up to 20 tons per square foot may be allowed.

QUESTION. What area of concrete block would be required under a steel column carrying a load of 80 tons, if the ground was good hard clay capable of safely supporting a load of 4 tons per square foot ?

Answer.

$$\text{Area of concrete block} = \frac{\text{Load}}{\text{Safe pressure}} = \frac{80}{4}$$
$$= 20 \text{ sq. ft.}$$

If the block were made square it would be about 4 ft. 9 in. each way, and if made rectangular, it could be about 5 ft. by 4 ft.

It has already been stated that the object of the steel plate under the column shaft is to spread the load, but the load can only be transmitted to the baseplate either by direct bearing of the shaft with the plate or by passing the load through other members connected by rivets or bolts.

With a base plate perfectly flat and the end of the column shaft machined, very few rivets will be required to make the connection. In actual construction, however, the end of the column shaft is often sawn off and not machined up square. As a result only three of the four corners of the bottom of the shaft may touch the baseplate in a manner similar to a table or chair when one of the four legs is shorter than the others (see Fig. 42 (*c*)). For this reason it is generally assumed that part of the load is transmitted direct to the baseplate from the column, the remaining being transmitted to the baseplate by means of rivets, side plates and angles.

Load on a Column. Where the load carried by a column is relatively small, say, less than 20 tons, it is usual to put in sufficient rivets to transfer the whole of the load to the baseplate. Where the column carries more than 50 tons, it is generally assumed that 40 or 50 per cent of the load is transferred to the baseplate by direct bearing, and that about 50 or 60 per cent is taken by the rivets. In other words, in designing a column base for a column carrying 100 tons, it can be assumed that 40 tons of the load is transferred to the baseplate by part of the column section resting on the baseplate, and that sufficient rivets must be provided to transmit the other 60 tons. The following design will make this clear.

QUESTION. Design a suitable riveted base for a column made of one R.S.J. 12 in. by 8 in. The load on the column is 80 tons. Assume the rivets take 60 per cent of the total load. Safe bearing pressure between steel baseplate and concrete foundation block equals 20 tons per square foot.

Answer. Baseplate :

$$\text{Required area of baseplate} = \frac{\text{Total load}}{\text{Safe pressure}} = \frac{80 \text{ tons}}{20 \text{ tons}} = 4 \text{ sq. ft.}$$

The baseplate should be kept as near square as possible, so that a baseplate 2 ft. by 2 ft. will be suitable. Baseplates are generally made between ½ in. and ⅞ in. thick. In this case use baseplate ⅞ in. thick.

Transmitted to Baseplate. Load transmitted direct to baseplate ; 40 per cent of 80 tons is transmitted by direct bearing to the baseplate.

$$= \frac{40}{100} \times 80 = 32 \text{ tons.}$$

Load to be transmitted to baseplate by rivets : 60 per cent of 80 tons.

$$= \frac{60}{100} \times 80 = 48 \text{ tons.}$$

If the base of the column does not touch the baseplate squarely (see Fig. 42 (*d*)), then this 48 tons must be transmitted through web angles, flange plates and flange angles to the base.

Cross-section area of 12 in. by 8 in. joist = 19·1 sq. in. Referring to Fig. 43, it will be seen that the area of the web of the column is 10 in. × 0·45 in. = 4·5 sq. in., and the area of the two flanges of the column is 2 × 8 in. × 0·9 in. = 14·4 sq. in.

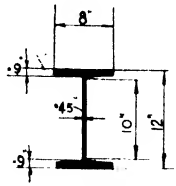


FIG. 43.

Over the total sectional area of the column of 19·1 sq. in. there is a total load of 48 tons, which has to be transferred to the baseplate by web angles, flange angles, flange plates, and rivets.

The web angles (see Fig. 44) should transmit their share of this

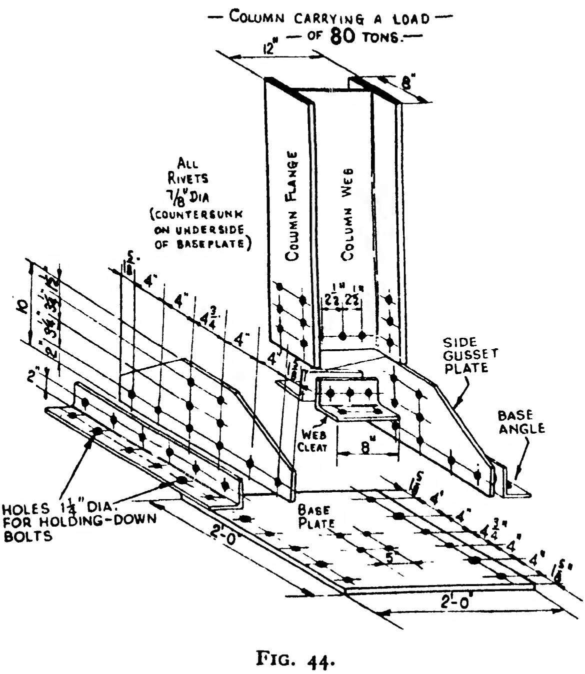


FIG. 44.

48 tons to the baseplate. The area of the web is 4·5 sq. in., so that the load transmitted to baseplate by web angles

$$= \frac{\text{Area of web of column}}{\text{Total area of column}} \times 48 \text{ tons} = \frac{4{\cdot}5}{19{\cdot}1} \times 48$$

$$= \text{roughly } 11{\cdot}5 \text{ tons.}$$

The remainder of the 48 tons (that is, $48 - 11{\cdot}5 = 36{\cdot}5$ tons) must be transmitted to the base by the flange plates and angles.

Reference to Fig. 42 (*e*) will show that if the bottom of the side plate (known as gusset plate) is not machined true the load will not be transmitted correctly to the baseplate, and to prevent this, flange angles are connected by rivets to the side plates, and these angles transfer the whole 36·5 tons from the plates to the baseplate.

Rivets. The web angles are connected to the column web by rivets, and the number of these depends on the strength of one rivet. Through the column web the rivets will be in double shear, and if we use $\frac{7}{8}$-in. diameter rivets the strength of one rivet in double shear = 6 tons. The strength in bearing of $\frac{7}{8}$-in. rivet in 0·45 in. thick column web

$$= \tfrac{7}{8} \text{ in.} \times 0{\cdot}45 \text{ in.} \times 10 \text{ tons} = 3{\cdot}94 \text{ tons.}$$

The bearing value being the lower, this will be used in the design.

Number of rivets required through web of column.

$$= \frac{\text{Load on web}}{\text{Strength of 1 rivet}} = \frac{11{\cdot}5 \text{ tons}}{3{\cdot}94} = 3 \text{ rivets.}$$

To prevent failure of the web angles themselves in bearing, they should be each made at least half the thickness of the column web. Two angles, $3\frac{1}{2}$ in. $\times$ $3\frac{1}{2}$ in. $\times$ $\frac{3}{8}$ in. will do nicely.

The flange plates or gusset plates are connected to the column flanges by a sufficient number of rivets to transmit 36·5 tons to the baseplate. This 36·5 tons load is the load down two flanges, so that the rivets in each flange will only take one half this amount, say, 18·25 tons.

With a flange of about $\frac{7}{8}$ in., rivets of $\frac{7}{8}$ in. diameter could be used.

Bearing Strength. In Chapter 4 the question of single shear, double shear, and the strength of rivets in shear and bearing have been fully dealt with, and the student will have no difficulty in checking up that the strength of a $\frac{7}{8}$-in. diameter rivet in single shear is 3 tons. The gusset plates should have a thickness to ensure that the bearing strength of the rivets is approximately the same as the single shear value of 3 tons.

Bearing strength $= D \times t \times 10$ tons.

Single shear strength of one rivet $= 3$ tons.

Therefore $\frac{7}{8}$ in. $\times t \times 10$ tons $= 3$ tons

from which $t = \dfrac{3 \times 8}{7 \times 10} = \dfrac{24}{70} = 0{\cdot}34$ in.

The flange plates can therefore theoretically be $\frac{3}{8}$ in. thick. In practice they would probably be made $\frac{7}{16}$ in. or $\frac{1}{2}$ in. thick.

Number of rivets required in each flange

$$= \frac{\text{Load on one flange}}{\text{Shear strength of one rivet}} = \frac{18{\cdot}25}{3} = 6 \text{ rivets.}$$

The position is that we have now found the number of rivets to fasten the side plates to the column, but the bottom edge of the side plate might not fit absolutely square on the baseplate, and it is therefore generally considered better to assume that the base angle rests on the baseplate, and that sufficient rivets are put in the vertical leg of the angle to take the total load coming down the side gusset plates. Two rivets will pass through the angle, side plate, and the column flange, so that we will require four more, and these are provided two on each side as shown on the drawing. Angles $3\frac{1}{2}$ in. $\times$ $3\frac{1}{2}$ in. $\times$ $\frac{3}{8}$ in. can be used for these flange angles. Fig. 45 (*a*) shows the complete base riveted up together.

It will be noticed that in this design it has been necessary to use flange angles and plates and web angles to make a satisfactory job. Nowadays it is becoming more and more

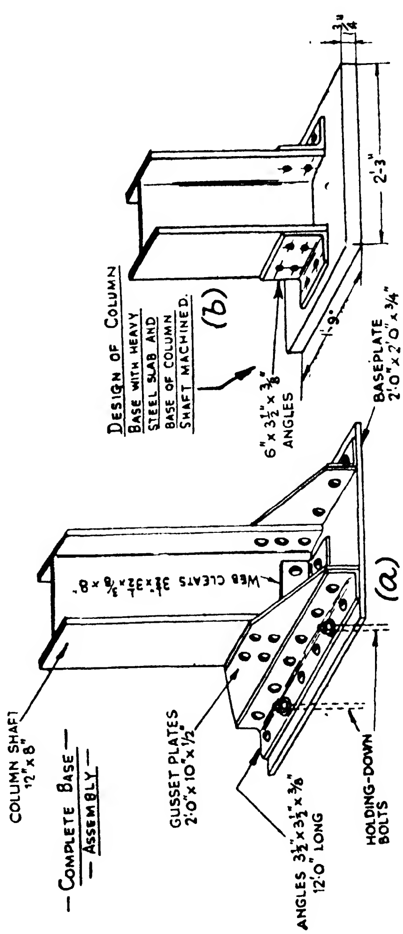


FIG. 45.

general practice to do away with these plates, and angles to a considerable extent by machining the end of the column so that the column fits exactly on to the baseplate.

It is clear that the same area of baseplate will be needed for the same pressure per square foot to be kept between the solid slab and the concrete block. Examination of Fig. 45 (*b*) will show that there are no side gusset plates to stiffen up the baseplate, so that this needs to be a good deal thicker than is required for the design shown in Fig. 45 (*a*). There is, however, a good deal less cutting and riveting to do, and for columns carrying heavy loads a type of base (Fig. 45 (*b*)) is more economical than the design shown in Fig. 45 (*a*).

Stresses in Thin Boiler Shells. In Chapter 5 an example of a riveted lap joint suitable for tanks and boilers was given. If this figure is referred to it will be seen that while there is only one row of rivets round the circumference of the shell, there are two rows horizontally along the shell. The following explanation of the stresses acting in these thin shells will make the reason for this type of riveting clear.

Cylindrical Tanks. Fig. 46 shows a transverse section of a cylindrical tank. If we fill this tank with steam or a liquid

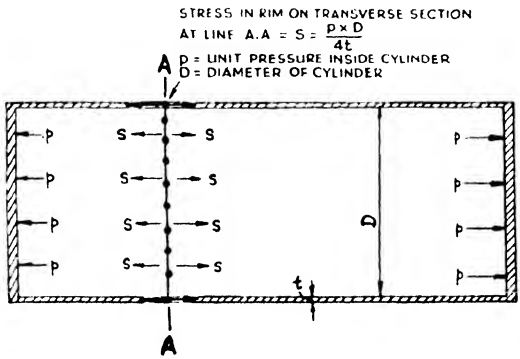


FIG. 46.

under pressure, then a time will come when the shell will tend to expand and finally crack. The steam will push against each end of the shell, as shown in Fig. 46. If p is the pressure per square inch of the steam, it follows that as the area of the end of the cylinder will be $\frac{\pi \times D^2}{4}$, that the total pressure will be

$$p \times \frac{\pi}{4} \times D^2.$$

Such a pressure on each end of the cylinder will tend to stretch it and to cause failure along some line such as AA. It follows, if failure is not to occur, that some forces such as S must act in the opposite direction to p, and that the total pressure on the end of the cylinder must be balanced by the total stress acting in the opposite direction around the rim of the cylinder.

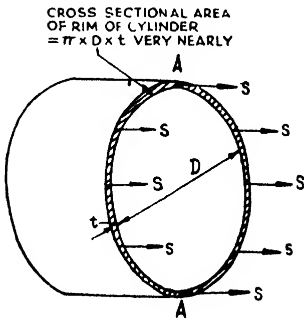


FIG. 47.

Fig. 47 shows these stresses S. The sectional area of the thin rim of a cylinder is very nearly equal to circumference $\times$ t. The circumference $= \pi \times D$, from which it follows that average total stress in rim $= \pi \times D \times t \times S$, where S is the tensile stress per square inch in the cylinder. As the pressure on end of cylinder is balanced by total stress in rim, then

$$\pi \times D \times t \times S = p \times \frac{\pi}{4} \times D^2$$

$$S = \frac{p \times \frac{\pi}{4} \times D^2}{\pi \times D \times t} = \frac{\frac{1}{4}p \times D}{t} = \frac{p \times D}{4t}.$$

This is called the longitudinal tension.

Now consider Fig. 48 (a). As well as pressing on the ends of the cylinder, the steam will try to increase the diameter of the cylinder by pressing out radially. Imagine that the cylinder was half full of ice, as shown in Fig. 48 (b), then the same conditions obtain as for Fig. 48 (a), as regards the part of the cylinder above the centre line.

For equilibrium the pressure on the part of the cylinder above

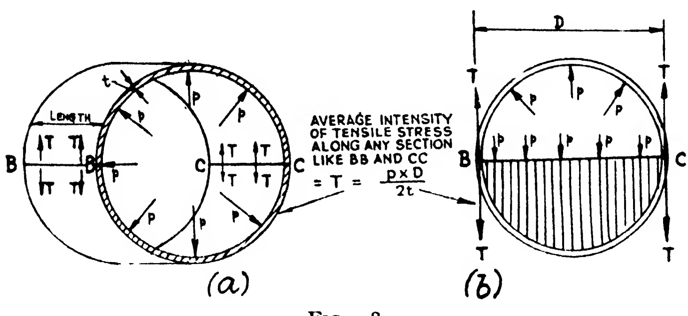


FIG. 48.

the centre line BC must be balanced by that below the centre line, or else the cylinder would move its position.

The pressure acting down on the ice on a length of cylinder equal to unity (that is – 1) $= p \times D \times 1$.

If the lower half of the cylinder is not to break off from the whole cylinder, stresses T must act on both sides of the rim along BB and CC, and these must balance the internal pressure.

Upward tensile force $= T \times$ Area of rim. Where T is the tensile stress per square inch in the rim.

There are two sections along which T is acting, that is, BB and CC, so that area of rim on a length equal to unity $= 2t \times 1$. Upward tensile force $= T \times 2t \times 1$.

For equilibrium,

$$T \times 2t \times 1 = p \times D \times 1$$

from which

$$T = \frac{p \times D \times 1}{2t \times 1} = \frac{p \times D}{2t}.$$

This is termed the circumferential tension. It will be observed that the longitudinal tension S is half the circumferential tension T. For this reason more rivets are required in the longitudinal joints which resist the circumferential tension than are provided in the joint running around the shell which resists the longitudinal tension. This will make clear the reason for forming the joints as shown in Figs. 49 and 50.

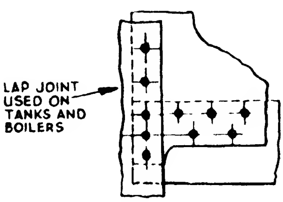


FIG. 49.

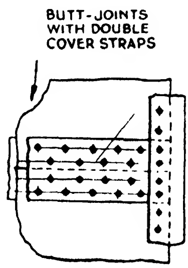


FIG. 50.

CHAPTER 7

CENTRE OF GRAVITY

We shall shortly be coming to the designing of beams, when the term neutral axis will be constantly cropping up. In order to find the neutral axis it is necessary to be able to find the centre of gravity, hence we will now deal with the centre of gravity of various shapes and sections and the various methods of finding them.

Centre of Gravity. Various methods can be used to find the centre of gravity of a section—

(1) by suspension (see Fig. 52 (*a*) and (*b*))
(2) by balancing on a knife edge (Fig. 54)
(3) by the principle of moments (Figs. 56, 58 (*a*) and (*b*))
(4) by graphic statics (Figs. 55, 57 (*a*) and (*b*)).

For such shapes as rectangles and squares it is easy to see where the centre of gravity lies. For instance, Fig. 51 shows a square plate, and the intersection of the diagonal lines drawn from the corners of the square will show the position of the centre of gravity or centroid.

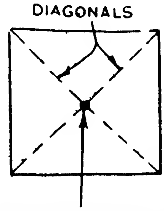


FIG. 51.

Where the figure is irregular in shape the position of the centroid, or centre of gravity, is not so easy to obtain. One method would be to make use of the well-known fact that if a thin sheet of metal or cardboard is freely suspended it will hang so that the centre of gravity is vertically below the point of suspension.

Fig. 52 (*a*) shows an example of this.

A flat-bottomed rail section cut out of a piece of thin sheet steel is shown suspended by a piece of string. Through the same hole in the rail another piece of string is placed to which a small weight is attached to keep the string hanging vertically. The web of the rail is marked where the string passes over it. Somewhere on this line lies the centre of gravity.

The rail is now suspended from another hole, as shown in Fig. 52 (*b*), and again the web is marked where the string passes over it. We shall now have two lines on the web, and where these intersect is the centre of gravity of the whole section.

In this particular case it is easy to see that the centre of gravity will certainly be on the centre line marked *AA*, as the

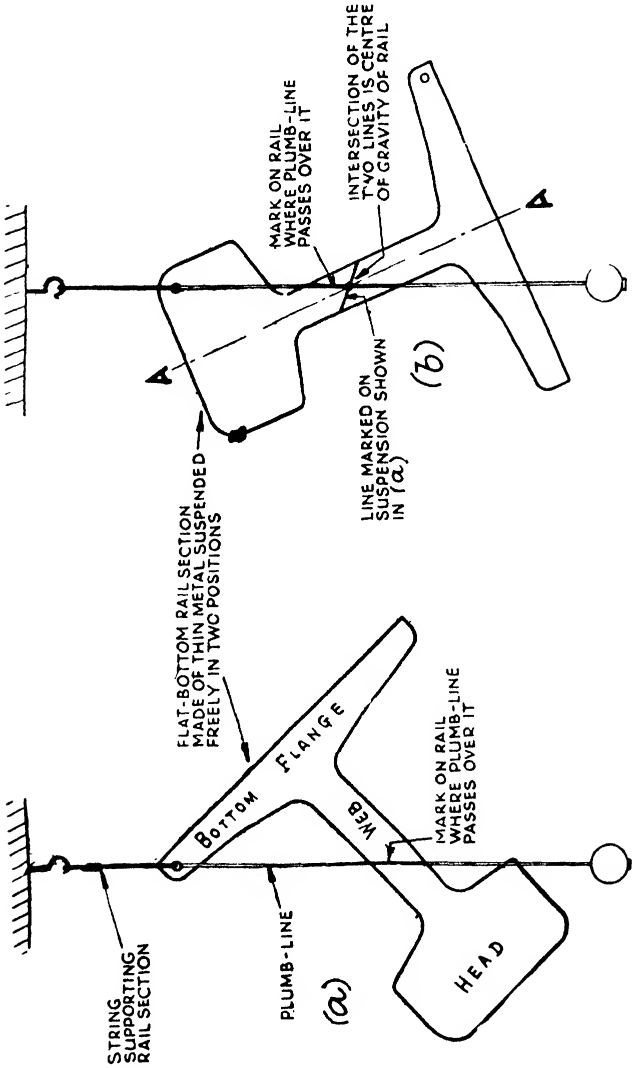


Fig. 52.

section is balanced on each side of this line, but if there is any doubt on this point the section could be suspended from another point. There would then be three lines all intersecting at one point. The hole from which the suspension is made should be kept as small as possible.

Centre of Gravity by Moments. The moment of an area with respect to any axis is the same as the sum of the moments of the small parts of the area. The moment of each little piece is its own area multiplied by the distance from itself to the axis or line considered.

Consider a figure such as shown in Fig. 53. If it is assumed to be made up of a large number of little pieces of area dA located at distances h from the axis XX, then the position of the centre of gravity of the complete figure can be determined by the following equation:

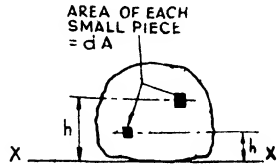


FIG. 53.

Let A be the total area of the figure

H be the distance of the centre of gravity of the whole figure from the axis XX.

Then $$A X H = \Sigma h \times dA.$$

In other words, the total area multiplied by the distance of its centre of gravity from the axis XX is equal to the sum of all the small areas multiplied by the distance of their respective centres of gravity from the same axis XX.

This will probably be clearer if we consider the example shown in Fig. 54. The section is split up into three rectangular pieces marked D, B, and C. The centre of gravity of each of these rectangles can be found at the intersection of the diagonals, and will, of course, be in the centre of each rectangle. We now take each separate area and multiply it by the distance of its own centre of gravity from the base line. Now add the results obtained as follows:

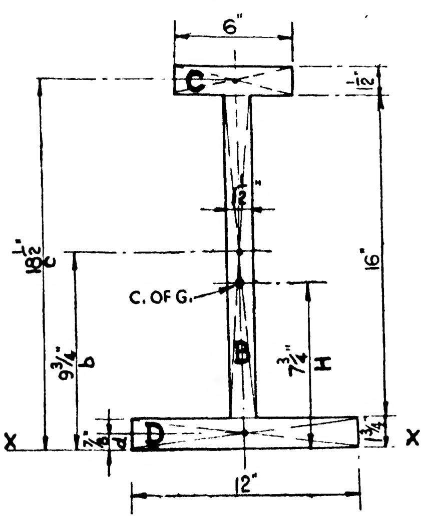


FIG. 54.

$$(D \times d) + (B \times b) + (C \times c)$$

and all this is equal to the total area multiplied by the distance of the centre of gravity of the total section from the base line. The total area of the section will be $D + B + C$.

$$\text{Area of } D = 12 \times 1\tfrac{3}{4} = 21 \text{ sq. in.}$$
$$B = 16 \times 1\tfrac{1}{2} = 24 \quad ,,$$
$$C = 6 \times 1\tfrac{1}{2} = 9 \quad ,,$$
$$\text{Total area } A = D + B + C = 54 \quad ,,$$

The moments of the various rectangles will be

$$(21 \times \tfrac{7}{8}) + (24 \times 9\tfrac{3}{4}) + (9 \times 18\tfrac{1}{2}).$$

These three added together are the sum total of all the moments of each separate area multiplied by the distance of its own centre of gravity from the base line XX, and we have already said that this is equal to the total area multiplied by the distance of the centre of gravity of the complete section from the line XX.

Therefore

$$A \times H = (21 \times \tfrac{7}{8}) + (24 \times 9\tfrac{3}{4}) + (9 \times 18\tfrac{1}{2}).$$
$$H = \frac{(21 \times \tfrac{7}{8}) + (24 \times 9\tfrac{3}{4}) + (9 \times 18\tfrac{1}{2})}{54}$$
$$= 7{\cdot}75 \text{ in.}$$

Thus the distance of the centre of gravity from the base line is $7\tfrac{3}{4}$ in., and since the total depth of the beam is $19\tfrac{1}{4}$ in., the distance of the centre of gravity from the top flange is $11\tfrac{1}{2}$ in.

The same position would have been found if the moments had been taken about the top flange. In this case the result would have shown that the distance of the centre of gravity from the top flange would be $11\tfrac{1}{2}$ in.

Alternate Method. *Centre of Gravity by Graphic Statics.* An alternative to the method described for finding the centre of gravity is by graphic statics. Consider Fig. 55. The centre of gravity for this section has been calculated. To find its centre of gravity graphically proceed as follows: Divide the figure up into rectangles C, B, and D. The place where the diagonals cross will be the centre of gravity.

Calculate the area of each rectangle and set these off to some suitable scale. One inch on the line can be made to represent 10 sq. in. of area. Select a pole O somewhere below this line and join up the points as shown. The exact position of this point O is not important as the same result would be obtained wherever the point O was chosen, but experience shows that it is wise to choose a point approximately in the position shown.

From the centres of gravity of the three rectangles project horizontal lines out to the right of the figure. From some point along the line projected from the centre of gravity of rectangle *C* draw line *b* parallel to line *b* in the polar diagram, finishing where it meets the line projected from the centre of gravity of rectangle *B*. From here draw line *c* parallel to *c* in the polar diagram, stopping where it reaches the line projected from the centre of gravity of rectangle *D*.

Now draw a line *d* parallel to *d* in the polar diagram, and another line *a* parallel to line *a* in the polar diagram from the

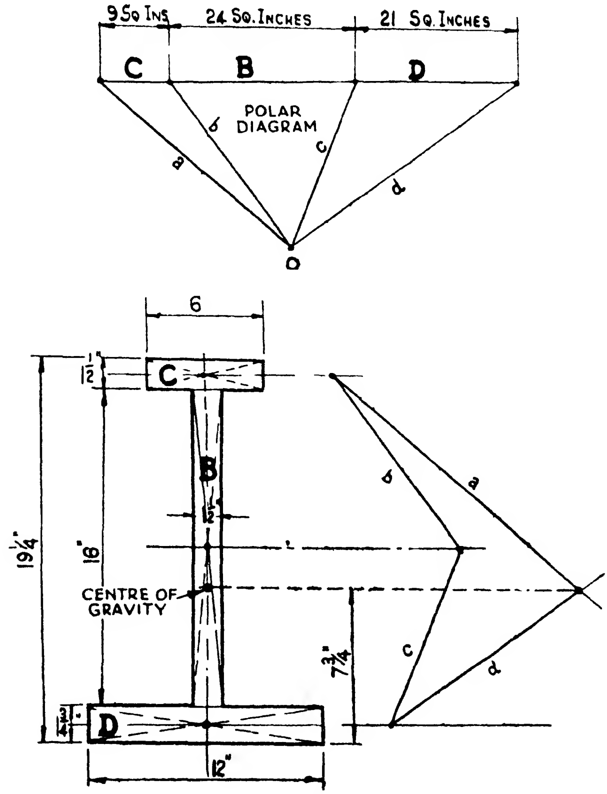


Fig. 55.

upper junction point. These lines *a* and *d* cross as shown in Fig. 55. A horizontal line projected from this point will pass through the centre of gravity of the complete section. As the figure is symmetrical about the vertical axis it follows that the centre of gravity will pass down the centre of the web.

If we cut out of a piece of sheet metal a shape of the dimensions shown in Fig. 55 and drill a small hole through the point marked as the centre of gravity, the whole piece would remain

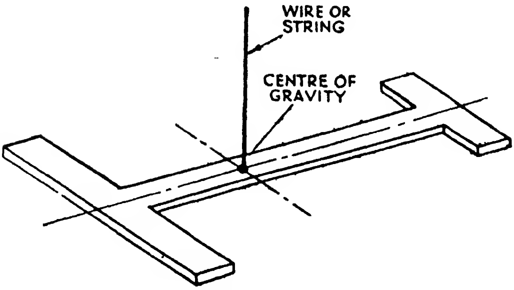


FIG. 56.

hanging horizontally as shown in Fig. 56 if suspended from a piece of string or wire. Again, if a wire was put through the hole so that it fitted through loosely, and the ends of the wire were supported, the whole piece of plate could be rotated, and it would not stop at any particular position.

Fig. 57 (*a*) shows the method of finding graphically the centre of gravity of a section such as is used for retaining walls, dams, etc. To find the centre of gravity divide *AB* into two and *CD* into two equal parts. Join *EF*. From *B* draw a horizontal line equal in length to *CD*, and from *C* a horizontal line equal in length to *AB*. Join *G* to *H*. Where *GH* and *EF* intersect is the position of the centre of gravity of the body.

The centre of gravity of any four-sided figure can be found as follows : Divide each side of the body into three equal parts as shown in Fig. 57 (*b*), and draw lines through the points nearest to each corner. The four points so drawn will form a parallelogram, and if diagonals are drawn from the corners of this parallelogram their intersection point will show the centre of gravity

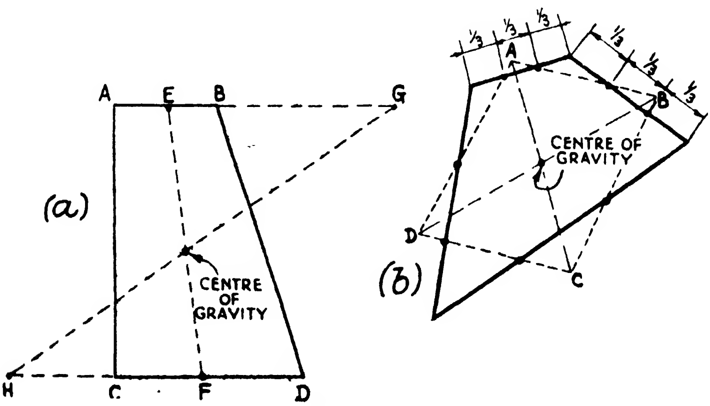


FIG. 57.

of the original figure. This method would be equally applicable for the wall section shown in Fig. 57 (*a*).

Problem. (1) The flange of a girder is made up of two angles 4 in. × 4 in. × ½ in. and two flange plates 14 in. × 1½ in. thick (Fig. 58 (*a*)). Find the position of the centre of gravity of the whole section.

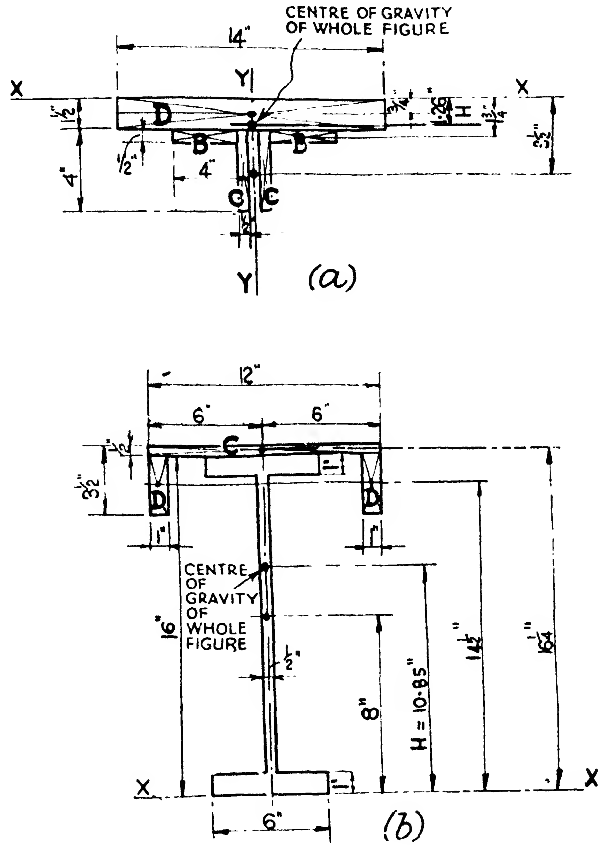


Fig. 58.

Answer.—The centre of gravity can be found by the method of suspension as shown in Figs. 52 (*a*) and (*b*), or by the graphical method shown in Fig. 55, or by the system of moments. We shall employ the system of moments to find the solution, and the students should check the result by one of the other methods.

Then $A \times H = \Sigma dA \times h.$

Divide the flange into rectangles *D*, *B*, and *C*.

$$\text{Area of } D = 14 \text{ in.} \times 1\tfrac{1}{2} = 21 \text{ sq. in.}$$
$$B = 2 \times 3\tfrac{1}{2} \text{ in.} \times \tfrac{1}{2} \text{ in.} = 3\tfrac{1}{2} \text{ sq. in.}$$
$$C = 2 \times 4 \text{ in.} \times \tfrac{1}{2} \text{ in.} = 4 \text{ sq. in.}$$
$$\text{Total area of section} = 21 + 3\tfrac{1}{2} + 4 = 28\tfrac{1}{2} \text{ sq. in.}$$

Then using the outside edge of the flange plates as our axis XX,

$$28\tfrac{1}{2} \times H = (21 \times \tfrac{3}{4}) + (3\tfrac{1}{2} \times 1\tfrac{3}{4}) + (4 \times 3\tfrac{1}{2})$$

$$H = \frac{15\tfrac{3}{4} + 6 + 14}{28\tfrac{1}{2}} = \frac{35\tfrac{3}{4}}{28\tfrac{1}{2}} = 1\cdot26 \text{ in.}$$

The flange is symmetrical about axis YY, so that the centre of gravity occurs on axis YY in one direction and 1·26 in. from the outside of the flange plates in the other direction.

PROBLEM. (2) A runaway girder if made of a rolled steel channel 12 in. × 3½ in. set on a 16 in. × 6 in. joist (Fig. 58 (*b*)). Find the position of the centre of gravity of the whole section.

Answer. The rolled steel joist 16 in. × 6 in. is a symmetrical figure, and its centre of gravity will occur along the centre of the web and half-way up.

Taking axis XX about the lower edge of the girder, proceed as follows: Divide the channel into pieces C and D.

$$\text{Area of rolled steel joist} = (2 \text{ in.} \times 6 \text{ in.} \times 1 \text{ in.}) + (14 \text{ in.} \times \tfrac{1}{2} \text{ in.}) = 19 \text{ sq. in.}$$

$$\text{Area of piece } C = 12 \text{ in.} \times \tfrac{1}{2} \text{ in.} = 6 \text{ sq. in.}$$

$$\text{Area of piece } D = 2 \times 3 \text{ in.} \times 1 \text{ in.} = 6 \text{ sq. in.}$$

$$\text{Total area of girder} = 19 + 6 + 6 = 31 \text{ sq. in.}$$

Then

$$31 \times H = (19 \times 8) + (6 \times 14\tfrac{1}{2}) + (6 \times 16\tfrac{1}{4})$$

$$H = \frac{152 + 87 + 97\tfrac{1}{2}}{31} = \frac{336\tfrac{1}{2}}{31} = 10\cdot85 \text{ in.}$$

The centre of gravity of the girder will occur down the web of the joist and at a distance 10·85 in. from the bottom flange.

The student will now have a fair idea of what is meant by such terms as stress, strain, modulus of elasticity, working stress, factor of safety, ultimate strength, single and double shear, dead and live loads, centre of gravity and bearing pressures.

CHAPTER 8

MOMENTS AND REACTIONS

It is necessary to understand clearly what the term bending moment really means before we can design beams. The type of scale shown in Fig. 59 was formerly used for weighing coal. The platform carrrying the weight W is suspended at a point 3 ft. from the pivot, while the load marked P is suspended at a point 1 ft. from the pivot, which is called the fulcrum. Clearly the load P in the box would have to be considerably more than the weight W on the table to keep the beam horizontal. Actually the load P would be three times the weight W.

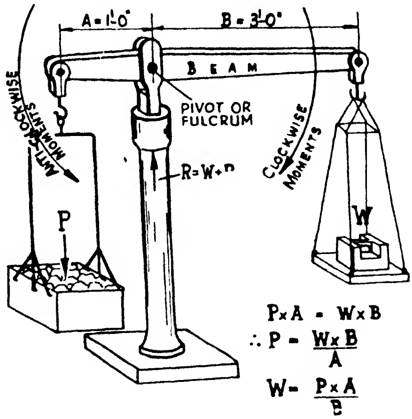


FIG. 59.

Equal "Moments". The moment of the weight W about the pivot will be the weight multiplied by the distance at which it acts. In this case the distance is marked B and is 3 ft. The moment of the load P will be its weight times the distance at which it acts from the pivot; in this case A is 1 ft. If the beam is to remain horizontal these two moments must equal each other.

The moment of the load P is $P \times A$ (Force $\times$ Distance)

The moment of the load W is . $W \times B$ (Force $\times$ Distance)

and if these moments balance each other it follows that

$$(P \times A) = (W \times B), \text{ from which}$$

$$P = \frac{W \times B}{A} \text{ and } W = \frac{P \times A}{B}.$$

Considering the fulcrum point as the centre of a clock face, the weight W tries to turn the beam in the same direction as the hands of the clock, and it is therefore called a clockwise moment. The load P tries to turn the beam in a contrary direction, and is called an anti-clockwise or counter-clockwise moment.

A moment is the product of a force multiplied by a distance (the distance is called the arm) and a moment can only be balanced by another moment acting in the opposite direction. Where the moment causes bending, it is called a bending moment. (Sometimes the moment causes twisting, as in the case of a crankshaft. In this case it is called twisting or turning moment.)

The centre post and pivot of the scale shown in Fig. 59 will have to support the combined loads P and W. This force is known as the reaction. In the case of Fig. 63 (*a*) and (*c*), each wall will carry half the total load. These reactions are indicated as R_1 and R_2.

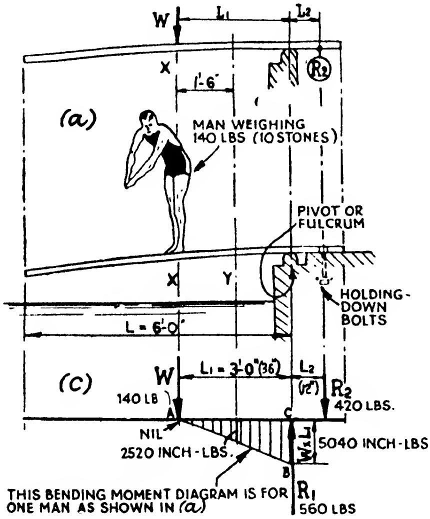


FIG. 60 (*a*) and (*c*).

Fig. 60 (*a*) shows a diving-board on which is a man weighing 140 lb. His position is 3 ft. from the point where the diving-board is supported. Notice that this is the fulcrum point, and that the holding-down bolt, which is marked R_2, is really a balancing force. Looking at the three arrows in Fig. 60 (*c*), marked W, R_1 and R_2, and imagining these to be a balance, with R_1 as the centre post, W as the weight, and R_2 as the load P, then we can write

$$W \times L_1 \times R_2 \times L_2.$$

The moment of W is $W \times L$. This is the bending moment at the support, and its amount will be

$$\text{Bending moment at support} = W \times L_1$$
$$= 140 \text{ lb.} \times 3 \text{ ft.} = 420 \text{ ft.-lb.}$$

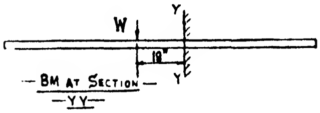
It is very important to be quite clear in what units the bending moment is taken. It would be quite correct to say

Bending moment of support = 10 stones × 36 in. = 360 in.-stones.

It would be equally correct to say

$$\text{B.M.} = 140 \text{ lb.} \times 36 \text{ in.} = 5{,}040 \text{ in.-lb.}$$

Although the figures are very different, the bending moment is the same. This shows the importance of stating the units.



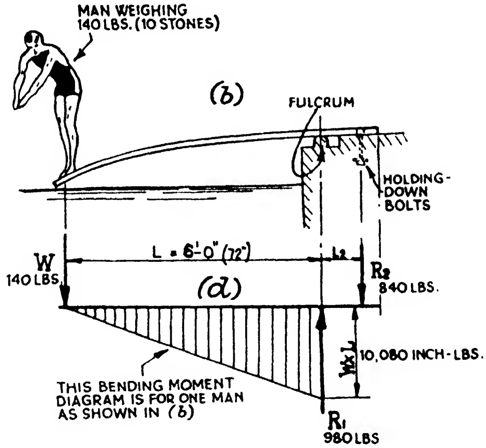


FIG. 60 (*b*) and (*d*).

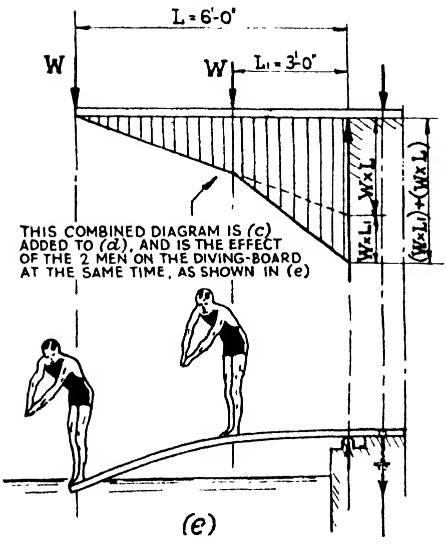


FIG. 60 (*e*).

In Fig. 60 (*c*) on the reaction line marked R_1, mark off a distance below the beam to represent the bending moment at the support to some scale. Just as on a motoring map 1 in. can represent a mile, or a $\frac{1}{4}$ in. can represent a mile, so on these diagrams the scale used is the one most convenient.

If we choose a scale of, say, 1 in. represents 20,000 in.-lb., then a line $\frac{1}{4}$ in. long will represent 5,000 in.-lb. and a line about this length will represent the bending moment at the support or on the line R_1. To find the bending moment at any other point on the beam, proceed as follows.

It is not difficult to see that if the man was standing directly over the place where the plank is supported, there would be no bending moment, because at that point we have a weight acting at no distance. Therefore B.M. at point $X = W \times O = O$.

A Common Snag. Consider section Y at a point 18 in. from the man. The author, after many years of teaching experience, is well aware that the clear understanding of bending moments and the position at which bending moment occurs is where many students become rather shaky. If we consider a bending moment at any section, all the forces or loads on one side can be completely ignored, and it is a good idea to imagine the beam built into a wall up to the section which is being considered. If this is done there is no difficulty in seeing that the only force is W, which acts at a distance of 18 in.

$$\text{B.M. at section } YY = 140 \times 18 = 2{,}520 \text{ in.-lb.}$$

A line representing this bending moment to scale will be exactly half as long as the line representing the bending moment at R. There is no bending moment under the head W. If the student will take the trouble to work out the bending moment on every 6 in. from the line R_1 and mark these to scale, then if the points are joined together he will get a triangle such as is shown in ABC, Fig. 60 (*c*), and this is called the bending-moment diagram. In this case of a cantilever the maximum bending moment occurs at the support. It will be shown later that in the case of beams loaded as shown in Fig. 63 (*a*) and (*c*), the maximum bending moment occurs at the centre of the beam.

Fig. 60 (*b*) shows that if the man walks to the end of the spring-board, which is 6 ft. from the point of support, the plank will bend a lot more, and the bending moment will also increase.

The bending moment at the support will now be

$$W \times L = 140 \text{ lb.} \times 72 \text{ in.} = 10{,}080 \text{ in.-lb.}$$

By exactly the same methods as we used previously, the bending moment at any point along the beam can be calculated, and the bending moment diagram drawn as shown in Fig. 60 (*d*).

Reactions. These have already been briefly mentioned. If the holding-down bolts shown in Fig. 60 (*a*) and (*b*) are taken out, the right-hand ends of the planks would fly up into the air. We can find the amount of force in these bolts in order to keep the right-hand end of the plank down.

To keep the beam balanced the sum of the clockwise moments about the support R_1 must equal the sum of the anti-clockwise moments. We have two moments around the pivot or turning-point. The weight of the man W acts at a distance of 3 ft. in the case of Fig. 60 (*a*), and the force in the holding down bolts R_1, acts at a distance of 1 ft. from the pivot. These two moments are equal and therefore

$$W \times L_1 = R_2 \times L_2$$

$$140 \text{ lb.} \times 36 \text{ in.} = \text{R}_2 \times 12 \text{ in.}$$

Therefore $$R_2 = \frac{140 \times 36}{12} = 420 \text{ lb.}$$

We have therefore a force R_2 of 420 lb. and a load W of 140 lb., so that the upward force or reaction on the point on which the plank turns is $W + R_2 = 560$ lb.

An Interesting Problem.—In the case of Fig. 60 (*b*) the man has moved along the plank, and therefore the holding-down bolts will have to resist a greater moment. If the reader will work out the figures, it will be found that R_2 is 840 lb. and R_1 is 980 lb. These results are very interesting, because although the man's weight does not vary, the actual amount of upward reaction R_1 varies with the man's position on the board. If the man stands directly over the point of support there is no bending moment at all, and the reaction R_1 will then be only 140 lb.

Combined Loads. With two men on the diving-board, as shown in Fig. 60 (*e*), there will clearly be more acting sagging or deflection of the board, and the bending moment at the support will be found by adding together the bending moments shown in Fig. 60 (*c*) and (*d*). The effect of the man at the end of the plank is shown in the top triangle, and the effect of the man in the middle of the board is shown in the second triangle, which is added to it. In text-book language, the bending moment at any given point is described as "The sum of all the moments of all the forces on either side of the point considered." The simple problems and explanations which have just been given will make this definition clear.

Before we can deal with the practical design of cantilever beams it is necessary to see what effect this bending moment actually has. Fig. 61 (*a*) shows a beam in which a notch is cut from the outside to the middle. If this beam is fixed into a wall at one end and a weight is placed at the other end of the beam, it will bend, and the notch will open on the top side. If the beam is placed the other way up, as shown in Fig. 61 (*c*), the notch will close. This shows us that the top layer or skin of a canti-

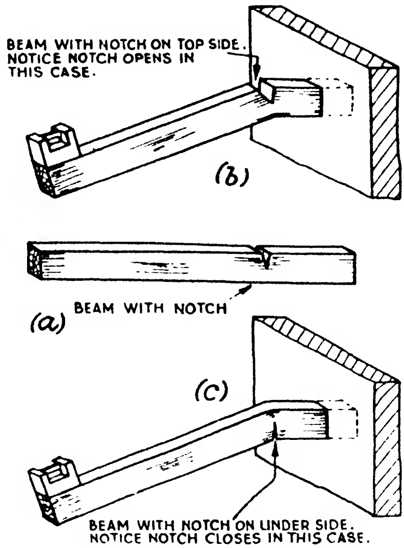


Fig. 61.

lever beam is stretched when the beam is loaded, and that the fibres or skin on the underside of the beam is compressed or squeezed.

Where a beam is supported at both ends and loaded so that it bends, the stresses are exactly the opposite. This is shown in Fig. 62 (*a*), (*b*) and (*c*). In this case the wood fibres on the top side of the beam are closed or squeezed by the action of bending, while those on the underside are stretched or opened. The reader can very easily check this by bending a piece of rubber between the fingers and making an indentation with the thumb on one side. The rubber will very quickly open. If a small notch is made in one side of the rubber, and it is bent the other way, the notch closes.

The student will remember that in an earlier chapter we dealt at considerable length with the question of elasticity of materials. It was shown that if a body is pulled it is in tension and stretches, while if it is pushed it is in compression and shortens. These points will be more fully dealt with later on when we come to beam design.

One of the things which it is not easy to see is that a wall or support pushes upwards. To illustrate this, consider Fig. 62 (*d*). Spring balances placed between the beam and the support will register the amount of each reaction. The balances are shown with the pan on top of the wall, and since the hand has turned round there must be an upward force to make it do this.

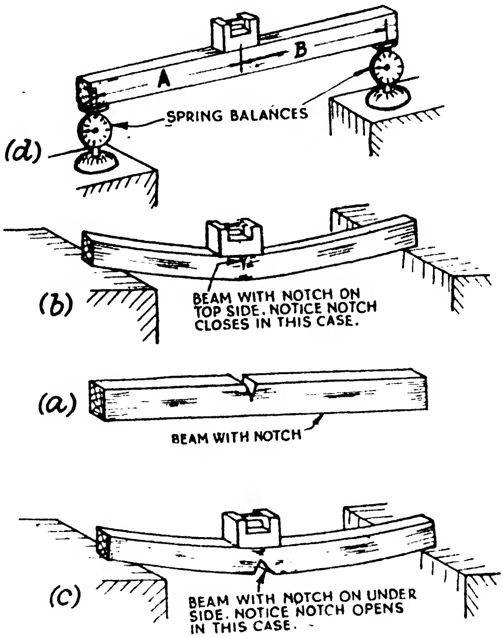


FIG. 62.

Elastic Properties. The student will readily understand that a wall prevents the beam from falling, but it is not so easy to visualize that the wall pushes up. This pushing up force is due to the effect of the downward load on the wall, which compresses the wall and therefore shortens it. (Read again Chapter 1.) The wall has elastic properties and tries to get back to its original height. It is this actual pushing-up force which is technically called a reaction.

The position of the load on the beam greatly affects the bending moment and the amount the beam bends. In Fig. 63 (*a*) a plank spans between two walls and is loaded with bags of cement. In this case the load is said to be uniformly distributed over the whole length (often written U.D.L.). This loading would cause the plank to bend as shown.

In Fig. 63 (*c*) the same bags are placed on the same plank, but in this case they are placed one on top of each other at the centre of the beam. Every student knows perfectly well that this would cause the beam to bend more than with the bags laid out as shown in Fig. 63 (*a*). This system of loading is called concentrated loading, or a point load at the centre of the span. Strictly speaking, very few loadings act at a point, but in practice it is often considered that they do. Fig. 63 (*b*) and (*d*) show the method generally used in technical journals and text-books for indicating the loadings shown in Fig. 63 (*a*) and (*c*).

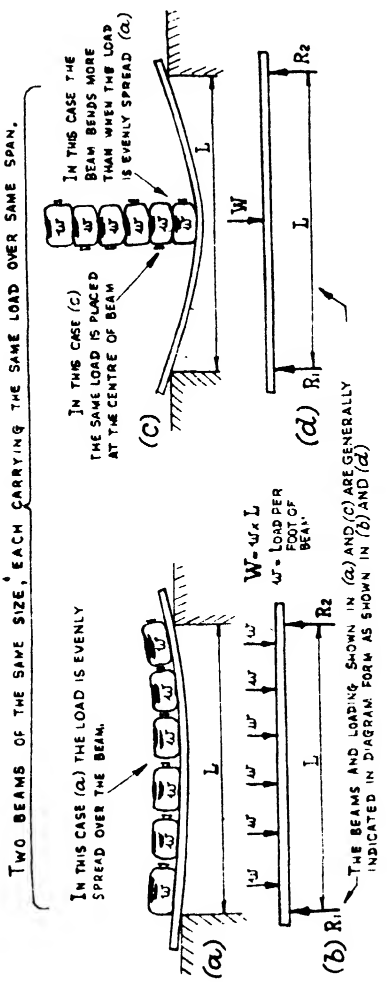


Fig. 63.

We will now proceed to consider the case of a beam which is supported on a wall at both ends and carries a load distributed over its whole length. A case such as this occurs where a beam spans over a doorway or large window-frame and carries a brick wall. A beam loaded in this manner is shown in Fig. 64.

Arching Effect. We shall show later that where the brick wall is higher than the span of the beam, there is a certain arching effect which should be taken into account, and this makes it possible to use a lighter beam than if the total weight of the wall was considered as being supported by the beam. At this stage, however, and with a wall only about half the height of the span between the supporting walls, it will be correct to consider all the weight as being carried on the steel beam.

Weight of Wall. Assuming brickwork to weigh 120 lb. per cubic foot, then the total weight of the wall will be

$$12 \text{ ft.} \times 7 \text{ ft.} \times \tfrac{3}{4} \text{ ft.} \times 120 \text{ lb.} = 7{,}560 \text{ lb.}$$

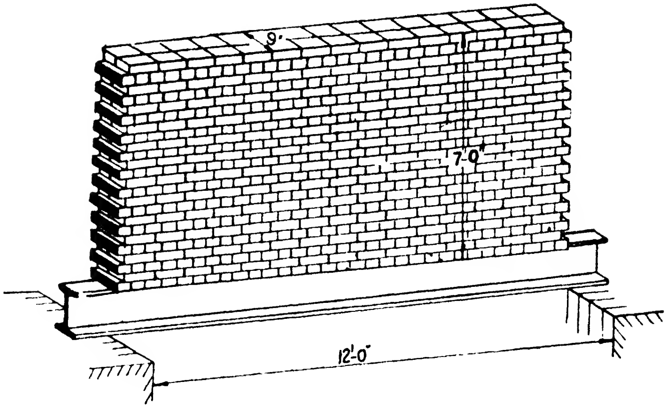


FIG. 64.

The beam spans over 12 ft., so that each foot of the beam will carry $\frac{7{,}560}{12} = 630$ lb.

In addition to the weight of the wall, the beam must, of course, carry its own weight. We do not yet know how much the beam will weigh, but we can make some allowance. If we assume that the total load on each foot run of the beam including its own weight is $\frac{3}{10}$th tons (672 lb.), we shall then have added to the weight of the wall 42 lb. to cover for the weight of each foot of the steel beam. This may be on the top side, but we shall assume the beam to carry a uniformly distributed load of 0·3 tons on each foot.

In Fig. 65 (*a*) this loading is represented by 12 small rectangles, each 1 ft. long and weighing 0·3 ton. It was shown in Chapter 7 that the weight of a body acts through its centre of gravity, and that the centre of gravity of a rectangle acts down its middle. We can assume, then, that each 0·3 ton weight acts down the centre of the rectangle as shown in Fig. 65 (*a*).

Reaction. The total weight which the walls have to support will be the weight of the brick and the weight of the beam. Each wall will carry half the total load, as the loading is symmetrical.

Therefore the reaction of the left-hand wall will be

$$R_1 = \frac{\text{Total load}}{2} = \frac{0{\cdot}3 \text{ tons} \times 12}{2} = 1{\cdot}8 \text{ tons}$$

$$R_2 = R_1 = 1{\cdot}8 \text{ tons.}$$

In this case the reactions are easy to find by examination only. In many cases it is necessary to calculate them by using the principle of moments. This will be dealt with when we come to deal with more advanced problems of bending moments.

We shall now work out fully the bending moment which occurs at each of the sections marked, *AA*, *BB*, *CC*, etc., in Fig. 65 (*a*). The student will remember that a bending moment

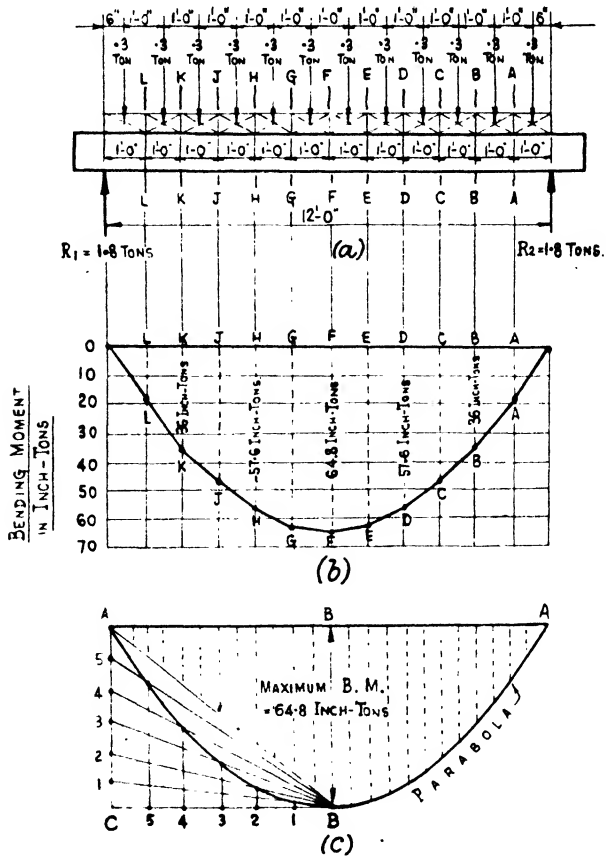


Fig. 65.

has been described as " The algebraic sum of all the moments of all the forces on either side of the point considered."

That word " algebraic " has certainly caused a lot of anxiety to many. If one has 2*s*. 6*d*. in one pocket and 1*s*. in another pocket, while at the same time he owes 1*s*. 6*d*. to his friend, then the real amount of money to his credit is the algebraic sum of these, that is 2*s*. 6*d*. + 1*s*. – 1*s*. 6*d*. His credit is therefore 2*s*. The sum of 2*s*. 6*d*., 1*s*. and 1*s*. 6*d*. is 5*s*., and clearly his credit is not 5*s*. The algebraic sum really means that we take away from the credit the amount that he is owing.

When dealing with bending moments the algebraic sum means the difference between the moments clockwise and the moments anti-clockwise. That is, we add together all the clockwise moments, then we add together all the anti-clockwise moments; the difference between these two totals is the bending moment at the section considered.

Consider the section at AA, Fig. 65 (*a*). In order not to get confused with the loads on the left-hand side of the section, make a little sketch (Fig. 66 (*a*)) and show only the forces on one side of the section we are considering. Students are strongly advised to cultivate the habit of making sketches on the lines shown, for most of the mistakes which occur in finding a bending moment arise from the fact that a sketch of the conditions has not been made.

At section AA there are only two forces to the right of the section. One force is a load of 0·3 ton acting at the centre of gravity of the foot-long rectangle, that is, at a distance 6 in. from the wall, and this is a clockwise moment. The other is the upward force of the reaction, which is 1·8 tons, acting at a distance of 12 in. from the section considered. This is anti-clockwise, and the difference between the moments of these two forces will be the actual bending moment at the section AA.

$$\text{B.M. at } AA = (1{\cdot}8T \times 12 \text{ in.}) - (0{\cdot}3T \times 6 \text{ in.})$$
$$= 21{\cdot}6 - 1{\cdot}8 = 19{\cdot}8 \text{ in.-tons.}$$

Bending Moments. The bending moments at the various sections, BB, CC, DD, EE, and FF, have been fully worked out, and are shown on the drawing. To summarize these we find the bending moments as follows:

Section	AA	19·8 in.-tons
,,	BB	36·0 ,,
,,	CC	48·6 ,,
,,	DD	57·6 ,,
,,	EE	63·0 ,,
,,	FF	64·8 ,,

With regard to the bending moments at sections LL, KK, JJ, HH, and GG, it is not necessary to make a separate calculation, as a little consideration will show. For instance, the bending moment at LL will be exactly the same as at AA. The conditions are shown in Fig. 66 (*a*). Similarly at HH the bending moment will be exactly as was calculated for *DD*. Fig. 66 (*f*) shows the loading in this case.

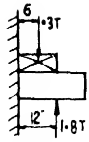
Earlier it was stated that the bending moment was the algebraic sum of all the moments of all the forces on either side of the point considered. In order to make this clear, consider the section *EE*. Fig. 66 (*g*) shows all the forces which act on the right side of the section, and the bending moment is shown as 63 in.-tons. Fig. 66 (*h*) shows the same section, but considering the forces on the left side of the section. The calculation shows that the bending moment for this case is also 63 in.-tons. It will be found in any and every case that the bending moment can be calculated on either side of the section considered and the same result will be obtained.

Cantilever Beam. *Bending-Moment Diagram.* In Chapter 8 the bending-moment diagram for a cantilever beam was shown. In Fig. 65 (*b*) a diagram has been drawn to represent the bending-moments which have now been found by calculation. The length of line on section *FF*, which represents 64·8 in.-tons, will depend on the scale chosen. One inch might represent 50 in.-tons or 1 in. might represent 25 in.-tons. The scale is chosen as a matter of convenience. What is important is that the scale used for section *FF* must be used for all other sections, such as *DD*, *BB*, etc. The student will therefore see that the saucer shape may be very shallow or very deep, according to the scale chosen.

Accurate Bending-Moment Diagram. Fig. 65 (*b*) showed the bending-moment diagram plotted out from the results obtained by calculating the bending moment at various sections of the beam. It will be remembered that we assumed that the loading on the beam was divided up into 1-ft. lengths, each foot carrying a load of 0·3 ton. If, however, we had divided the beam into 4-in. lengths, each length would carry 1 ton. The same loading would apply, but the number of sections would be increased, and the bending-moment diagram would be made up of a greater number of straight lines than is shown in Fig. 65 (*b*).

If we still further reduced the lengths of the rectangles in Fig. 65 (*a*) a time would come when the bending-moment diagram, instead of being made up of a number of short straight lines, would become a curve as shown in Fig. 65 (*c*). It can be proved that this curve is a parabola. If the maximum bending moment is known we can draw a parabola and scale off the bending moment at any distance along the beam.

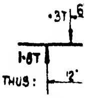
The method of constructing a parabola for a bending-moment

(a)


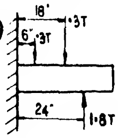
SECTION A.A.

B.M. AT SECTION A.A. = (1·8T × 12") − (·3T × 6")
= 21·6 − 1·8 = 19·8 INCH-TONS.

NOTE THAT BENDING MOMENT AT SECTION L.L WILL ALSO BE 19·8 INCH-TONS, AS THE LOADING AND DISTANCES ARE MERELY REVERSED THUS:

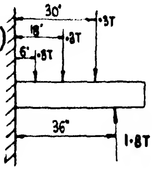


(c)

(b)


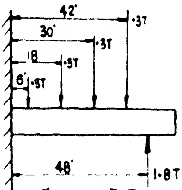
SECTION B.B.

B.M. AT SECTION B.B = (1·8T × 24") − (·3T × 18") − (·3T × 6")
= 43·2 − 5·4 − 1·8 = 36·0 INCH-TONS.

(d)


SECTION C.C.

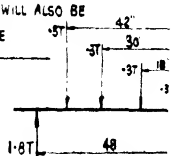
B.M. AT SECTION C.C. = (1·8T × 36") − (·3T × 30") − (·3T × 18") − (·3T × 6")
= 64·8 − 9 − 5·4 − 1·8 = 48·6 INCH-TONS

(e)


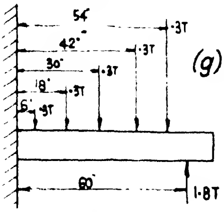
SECTION D.D.

B.M. AT SECTION D.D. = (1·8T × 48") − (·3T × 42") − (·3T × 30") − (·3T × 18") − (·3T
= 86·4 − 12·6 − 9 − 5·4 − 1·8 = 57·6 INCH-TONS.

NOTE THAT THE BENDING MOMENT AT SECTION H.H WILL ALSO BE 57·6 INCH-TONS, AS THE LOADS AND DISTANCES ARE MERELY REVERSED THUS



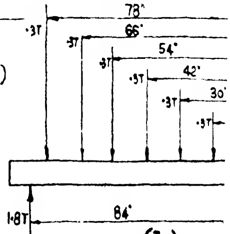
(f)



(g)
SECTION E.E.

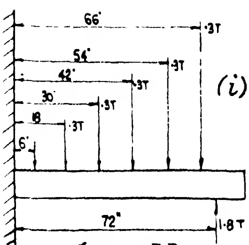
B.M. AT SECTION E.E = (1·8T × 60") − (·3T × 54") − (·3T × 42") − (·3T × 30") − (·3T × 18")
= 108 − 16·2 − 12·6 − 9 − 5·4 − 1·8 = 63·0 INCH-TONS

THE SAME RESULT WILL BE OBTAINED WHICHEVER END OF THE BEAM THE MOMENT ARE TAKEN ABOUT. IF THE LEFTHAND END OF THE BEAM IS USED THEN THE FOR SECTION E.E WILL BE AS SHOWN



(h)

THEN B.M. AT SECTION E.E = (1·8T × 84")
− (·3T × 78") − (·3T × 66") − (·3T × 54") − (·3T × 42")
− (·3T × 30") − (·3T × 18") − (·3T × 6")
= 151·2 − 23·4 − 19·8 − 16·2 − 12·6 − 9 − 5·4 − 1·8
= 63·0 INCH-TONS, WHICH IS THE SAME RESULT AS WAS OBTAINED USING THE RIGHTHAND END OF BEAM



(i)
SECTION F.F.

B.M. AT SECTION F.F = (1·8T × 72") − (·3T × 66") − (·3T × 54") − (·3T × 42") − (·3T × 30")
− (·3T × 6") = 129·6 − 19·6 − 16·2 − 12·6 − 9 − 5·4 − 1·8 = 64·8 INCH-TONS

FIG. 66.

diagram is as follows. Draw a horizontal line AA, see Fig. 65 (c), equal in length to the span of beam. Divide AA into two equal lengths, AB and BA. Draw a vertical line BB equal to scale to a maximum bending moment at the centre of beam. From B draw BC parallel to AA, and from A draw AC parallel to BB. Divide AC into a number of equal lengths, such as 1, 2, 3, 4, and 5. Divide BC into the same number of equal lengths, 1, 2 3, 4, and 5. Join points 1, 2, 3, 4, and 5 on AC with point B. From point 1 on BC project vertically upwards until the line running from 1 on AC to B is reached. This junction joint is a point on the parabola. From 2 on BC project vertically up till the line joining 2 on AC to B is reached. This is another point on the parabola.

Proceed exactly the same for the other points, and finally, a set of points as shown in Fig. 65 (c) will be obtained. If these points are joined by a curve, then the result is the left half of a parabola. The right half of the curve can be drawn in like manner, and finally the bending moment diagram will be obtained from which the bending moment at any point along the beam can be reached.

In practice the bending moment at the various sections of a beam which is loaded uniformly need not be calculated out fully as we have done. Only the maximum bending moment at the centre of the beam is required. If a parabola is drawn the bending moments at any other section can be scaled off.

The bending moment at the centre of a beam, uniformly loaded as shown in Fig. 65 (a), can be found directly if we assume that all the load on the right of section FF is one rectangle instead of six as shown.

Centre of Gravity. The centre of gravity (Centroids, Chapter 7) of a rectangle 6 ft. long would occur at 3 ft. from FF. At this point would be concentrated the load on the right half of the beam $= 0{\cdot}3$ tons $\times$ 6 ft. $= 1{\cdot}8$ tons. Therefore we will have a clockwise moment of $1{\cdot}8$ tons $\times$ 36 in. due to this force.

We shall also have an anti-clockwise moment due to the reaction of the wall R of $1{\cdot}8$ tons $\times$ 72 in. Therefore bending moment at $FF = (1{\cdot}8t \times 72 \text{ in.}) - (1{\cdot}8t \times 36 \text{ in.})$

$$= 129{\cdot}8 - 64{\cdot}8 = 64{\cdot}8 \text{ in.-tons.}$$

This figure agrees with the previously calculated. If we call the span of beam $= L = 144$ in., $W =$ total weight carried by beam $= w \times L = W = 3{\cdot}6$ tons, then $1{\cdot}8$ tons $= \frac{1}{2}W$ and

72 in. $= \frac{1}{2}L$, while 36 in. $= \frac{1}{4}L$. Therefore bending moment at $FF = (\frac{1}{2}W \times \frac{1}{2}L) - (\frac{1}{2}W \times \frac{1}{4}L) = \frac{1}{4}WL - \frac{1}{8}WL = \frac{1}{8}WL = \frac{WL}{8}$.

This is the maximum bending moment on a beam simply supported at the ends and carrying a uniformly distributed load.

You have already been shown how to find the maximum bending moment for a beam supported at the ends and carrying a uniformly distributed load, and also how to construct the bending-moment diagram for this loading. It will be remembered that we found that the bending-moment curve took the form of a parabola, and that if we knew the value of the maximum bending moment on the beam, the bending moment at any other part of the span would be in direct proportion to it.

The beam and loading considered and the bending-moment diagram constructed are shown again in Fig. 67 (*a*) and (*b*). The maximum bending moment at the centre of the beam was found to be 64·8 in.-tons. To refresh the student's memory, the easiest way to find this moment will be again shown. Consider Fig. 67 (*c*), which shows the left half of the beam. The bending moment at any section (in this case at the centre of the beam) is equal to the algebraic sum of all the moments on one side of the section. We have a clockwise moment at the centre of span which amounts to reaction from wall $\times$ 72 in.

Clockwise moment $=$ 1·8 tons $\times$ 72 in. $=$ 129·6 in.-tons.

The load on half the span will produce an anti-clockwise moment. Each foot of beam carries a load of 0·3 tons; half the span is 6 ft., so that the total load on half-span will be 1·8 tons: This load will act through its centre of gravity, which is 36 in. from the centre of span.

Anti-clockwise moment $=$ 1·8 tons $\times$ 36 in. $=$ 64·8 in.-tons.

The bending moment is the difference between all the clockwise moments and all the anti-clockwise moments on either side of the point considered. (In this case we have only one clockwise moment and one anti-clockwise moment, the point considered is the centre of the beam, and we are taking moments to the left side, as shown in Fig. 67 (*c*)).

Sum of clockwise moments $=$ 129·6 in.-tons.
Sum of anti-clockwise moments $=$ 64·8 in.-tons.
Difference in bending moment $=$ 64·8 in.-tons.

Therefore the maximum bending moment is 64·8 in.-tons, as shown in Fig. 67 (*b*), and the B.M. at any other point in the span can be obtained by scaling from the diagram.

It was shown in Chapter 8 (by means of a pair of scales) that one moment can only be balanced by another moment. (It will be remembered that the definition of a moment is—*a force multiplied by an arm or distance.*) Having now a clear idea of what a bending moment is, we will proceed to deal with another kind of moment called " resisting moment ".

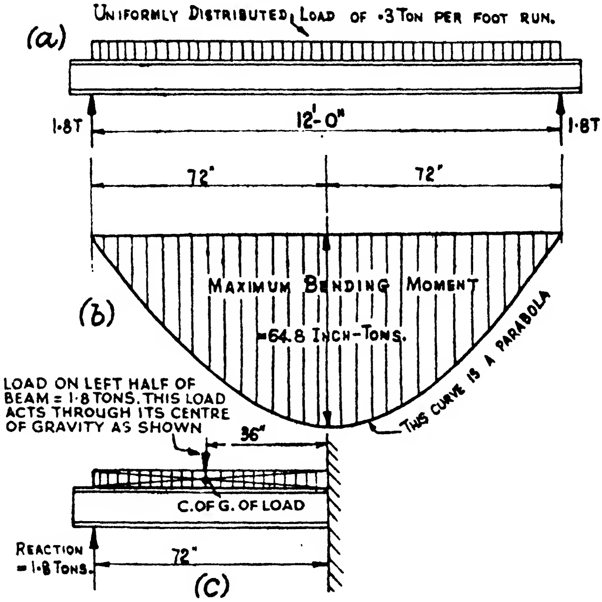


FIG. 67.

A resisting moment is the strength of a beam to resist bending moment. The bending moment is the effect on the beam caused by the load or weight which the beam carries. The resisting moment is the strength of the beam itself, and if it is more than the bending moment the beam will not fail.

Fig. 68 (*a*) shows an ordinary rolled-steel beam before loading. Fig. 68 (*b*) shows the same beam after loading The bending has been greatly exaggerated in order to show clearly that the top flange shortens when the beam is bent and the bottom flange lengthens. It has already been shown that all metals have elastic properties and that a compression force causes shortening, while a tension or pulling force causes lengthening. The top flange is said to be in compression and the bottom flange to be in tension.

The Neutral Axis. Assuming that the top flange shortens by the same amount as the bottom flange lengthens, there will be no change of length along a line half-way between the top and bottom flanges. Along this line the steel has no stress due to bending, and for this reason it is called the neutral axis, since it is not in tension or compression.

Resisting Moment. If the beam is hinged at the centre as shown in Fig. 68 (*c*), and the top and bottom flanges cut away, we shall be able to see more clearly what happens when the

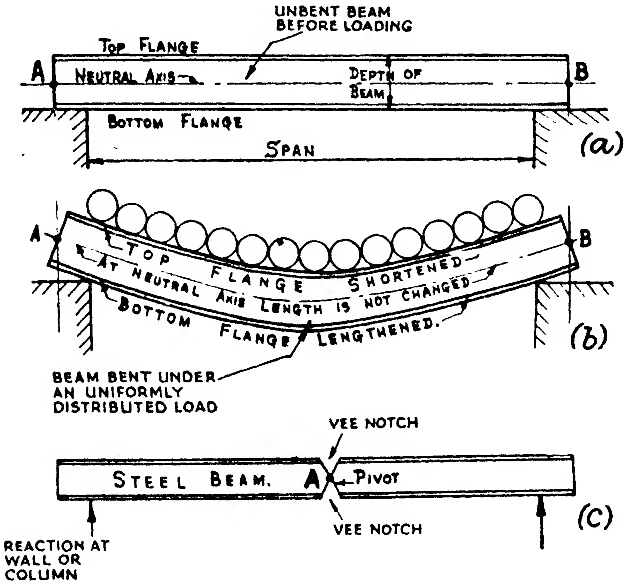


FIG. 68.

beam bends. Fig. 69 (*a*) shows an enlarged view of the hinge and the beam before bending.

A block of rubber is placed in the vee-notch opposite the top flange. A strip of rubber is bolted to the bottom flange. If the beam is bent it will turn around the hinge pin and the top block of rubber will be compressed or squeezed, while the bottom strip will be stretched. This is shown in Fig. 69 (*b*). In an actual beam there is, of course, no rubber, and although it is not so easy to see the actual shortening and lengthening of the steel in the top and bottom flanges, it does nevertheless take place.

Fig. 69 (*c*) shows the forces set up in the beam flanges in order to resist the bending moment.

We shall now consider the forces in the two pieces of flange which have been cut away and replaced by a piece of rubber. In the actual beam the piece of top flange which is shown in Fig. 70 (*a*) can be considered as a short column, of width *W* and

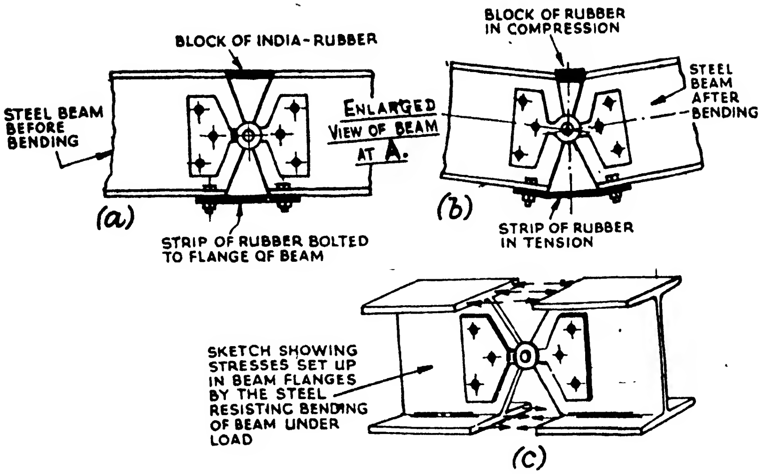


FIG. 69.

thickness T. If W is 4 in. and the thickness T is $\frac{1}{2}$ in., the area is 2 sq. in. The breaking strength of mild steel will be around 30 tons per square inch, so that the force or load to crush this piece of steel would be 60 tons.

Tensile Stresses. A short length of bottom flange is shown in Fig. 70 (*b*). In this case the force is tensile or trying to pull the bar apart. (These tensile or pulling stresses have already been dealt with.) The cross-sectional area is the same as for the top flange, that is 4 in. × $\frac{1}{2}$ in. or 2 sq. in., and if the tensile

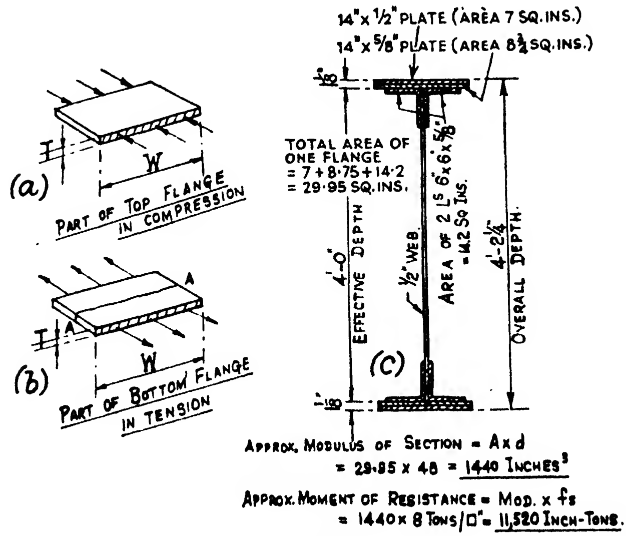


FIG. 70.

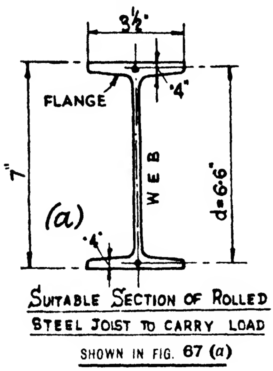
strength of the steel is 30 tons per square inch, it would take a pull of 60 tons to break this flange along a line AA.

Now look at Fig. 71 (*b*), and a little study will show that the strength moment of the top flange will be 60 tons multiplied by its effective arm, which is $\frac{d}{2}$, while the strength moment of the bottom flange will also be 60 tons multiplied by its effective arm, which is also $\frac{d}{2}$.

Total resisting strength of the beam is therefore

$$\left(60 \text{ tons} \times \frac{d}{2}\right) + \left(60 \text{ tons} \times \frac{d}{2}\right)$$

which equals 60 tons $\times$ d



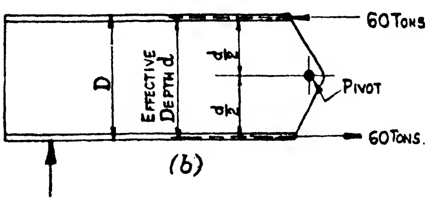


FIG. 71.

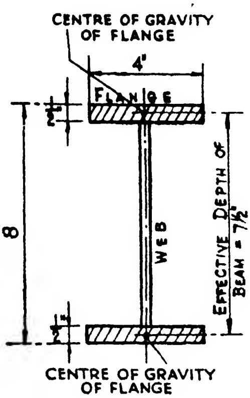


FIG. 72.

where d is the effective depth of the beam, or the distance between the centres of gravity of the forces in the two flanges. It is *not* the total depth of the beam.

If the beam is 8 in. deep the effective depth d will be about $7\frac{1}{2}$ in. (see Fig. 72).

The resisting moment will then be

$$60 \text{ tons} \times 7\tfrac{1}{2} \text{ in.} = 450 \text{ in.-tons.}$$

What this really means to say is that if we had a beam 8 in. deep with flanges 4 in. wide by $\frac{1}{2}$ in. thick, which was made of mild steel with a breaking strength of 30 tons per square inch, then if the loading on the beam was such that the resulting bending moment amounted to 450 in.-tons, the beam would be just about at breaking-point.

In practice, of course, beams must be strong enough to be safe, and for this reason the bending moment which a beam will safely carry should not be more than a quarter of the bending moment which would break the beam. This matter was dealt with at length in an earlier section under the heading of "Factor of Safety".

To return to our steel beam 8 in. deep and 4 in. wide, with flanges $\frac{1}{2}$ in. thick, if we use a factor of safety of 4, the safe stress in the flanges will only be $7\frac{1}{2}$ tons per square inch, and the safe resisting moment will therefore be a quarter of 450 in.-tons, or $112\frac{1}{2}$ in.-tons.

Therefore a rolled steel joint 8 in. × 4 in. × $\frac{1}{2}$ in. would be safe if the maximum bending moment did not exceed 110 in.-tons.

Resisting Moment of R.S.J.—

Let A be the area of one flange in square inches
d be the effective depth of beam in inches
f_s be allowable stress in tons per square inch.

Then resisting moment of R.S.J. $= A \times d \times f_s$.

In practice, the allowable stress is often taken at 8 tons per square inch.

TYPICAL PROBLEM. *Design of R.S.J.* Find a suitable section of R.S.J. to carry a uniformly distributed load of 0·3 ton per foot run over a span of 12 ft. The weight of the beam itself has been included in the uniformly distributed load (see Fig. 67 (*a*)). The allowable stress per square inch is to be 8 tons.

Answer. The maximum bending moment can be shown to be 64·8 in.-tons, and the diagram is shown in Fig. 67 (*b*). In order to avoid excessive deflection, the depth of a steel beam is not to be less than $\frac{1}{2}$ in. for each foot of span. (In bridges and plate girders the depth of the girder is often made about one-tenth of the span.) In this case the beam must be at least $\frac{1}{24}$th of the span, or $\frac{144}{24}$, say, 6 in.

The safe resisting moment of the beam must be at least equal to the maximum bending moment.

$$\text{Therefore Maximum B.M.} = \text{Resisting moment}$$
$$\text{Resisting moment} = A \times d \times f_s.$$

$f_s = 8$ tons per square inch, and if the effective depth is assumed as 6 in., we can find the area of the flange.

$$64{\cdot}8 = A \times 6 \times 8$$
$$64{\cdot}8 = 48A$$
$$\frac{64{\cdot}8}{48} = A = 1{\cdot}35 \text{ sq. in.}$$

If the width of the flange is 3 in., the thickness required will be

$$\frac{1{\cdot}35}{3} = 0{\cdot}45 \text{ in}$$

If the flange is 4 in. wide, the thickness required will be

$$\frac{1\cdot35}{4} = 0\cdot34 \text{ in.}$$

Reference to any steel-maker's lists will show that one of the British Standard sections is a 7-in. × 3½ in. R.S.J., and that the flange thickness is 0·4 in. Our previous calculations seem to show that this section will be about right, but we shall now check it. The effective depth will be 6·6 in.

Therefore $$64\cdot8 = A \times 6\cdot6 \times 8.$$

The width of the flange in this case is 3½ in. and the thickness 0·4 in. The resisting moment of a 7-in. × 3½-in. R.S.J. is therefore

$$(3\tfrac{1}{2} \times 0\cdot4) \times 6\cdot6 \times 8 = 73\cdot9 \text{ in.-tons.}$$

This is greater than the bending moment, and this beam would be a suitable section to use. (Fig. 71 (*a*) shows this section.)

The next smaller section is a 6-in. × 3-in., with flanges ⅜ in. thick, and the student should test to make sure that this section is not strong enough.

Plate Girders. Where the span is too large or the load too heavy to permit a rolled-steel section to be used, a built-up girder of the type shown in Fig. 70 (*c*) may be used. This is called a plate girder, because the web is made of a rolled-steel plate. The flanges are made of plates and angles. The effective depth of such a girder will be approximately the distance between the centre of gravity of the two flanges. In Chapter 7 the method of finding the centre of gravity for such a type of flange was shown.

Plate Girders. The approximate resisting moment of a plate girder will be : Effective area of one flange (shown in section lines) × Allowance stress × Effective depth of the girder.

In other words—

$$\text{Approx. R.M.} = A \times f_s \times d$$

where R.M. is the resisting moment of the beam

A is the area of one flange

f_s the safe stress per square inch.

If the area is in square inches, the depth in inches, and the safe stress in tons per square inch, then the resisting moment will be in inch-tons. If the effective depth is taken in feet the resisting moment will be in foot-tons. The student should be very careful to see that the bending moment units and the resisting moment units are the same. A more accurate method of finding the resisting moment of a built-up section will be given later.

Question. What is the maximum bending moment which could be safely resisted by a plate girder with a cross section as shown in Fig. 70 (*c*) ? Take f_s = 8 tons per square inch.

Answer. The girder consists of two flanges connected by a web plate. Each flange consists of

One plate 14 in. × $\frac{1}{2}$ in.	Area =	7 sq. in.
One plate 14 in. × $\frac{5}{8}$ in.	Area =	8·75 sq. in.
2 angles 6 in. × 6 in. × $\frac{5}{8}$ in.	Area =	14·20 sq. in.
Total area of one flange . . .		29·95 sq. in.

$$\text{R.M.} = A \times d \times f_s$$

where A is the area of one flange = 29·95 sq. in.

d is the effective depth of the beam, or the distance between the centres of gravities of the flanges, which in this case will be 48 in.

$$f_s = 8 \text{ tons per square inch.}$$

$$\text{R.M.} = 29{\cdot}95 \times 48 \times 8 = 11{,}520 \text{ in.-tons.}$$

Therefore, if the section has a safe resisting moment of 11,520 in.-tons, it can safely resist a bending moment of the same amount = 11,520 in.-tons.

CHAPTER 9

RESISTING MOMENTS

We have found when dealing with a rolled-steel joist that

$$\text{Resisting moment} = A \times d \times f_s$$

where A is the area of one flange of the joint

d is the effective depth of the joist, i.e. approximately the distance between the centres of gravity of the flanges

f_s is the safe stress per square inch in the steel.

This expression is easily applied to rolled-steel joists which are made with two thick flanges, assumed to take all the bending stresses. The web which joins them is assumed to take none of these bending stresses. However, when it is desired to find the resisting moment of a rectangular section, such as a timber beam, the equation given above does not apply.

Fig. 73 shows a timber beam section 4 in. wide and 8 in. deep. Fig. 74 (*a*) shows an elevation of the beam. If the beam was made up of eight planks of equal length, securely strapped together, it would bend as shown in Fig. 74 (*b*) when loaded. Notice that the upper planks have shortened and the lower planks have lengthened. The diagram is, of course, considerably exaggerated so that this shortening and lengthening may be seen. In practice the beam does not bend nearly as much as shown, but the shortening and lengthening does actually take place when the beam is bent.

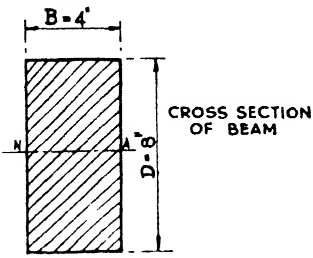


FIG. 73.

Half-way between the top and bottom faces there would be a length where the plank would remain unchanged. It is therefore easy to see that the wood on the top face shortens more than any other and the wood on the bottom face lengthens more than any other. The shortening and lengthening of any particular fibre will depend on its distance from the neutral axis.

To make this point clear, consider Fig. 75 (*a*). This is an enlarged view of the centre of the beam which has been notched top and bottom and hinged along the neutral lamina or neutral axis. At the faces of the beam the notches are 1 in. wide, reducing to zero at the neutral axis or centre of gravity of the section. Under the

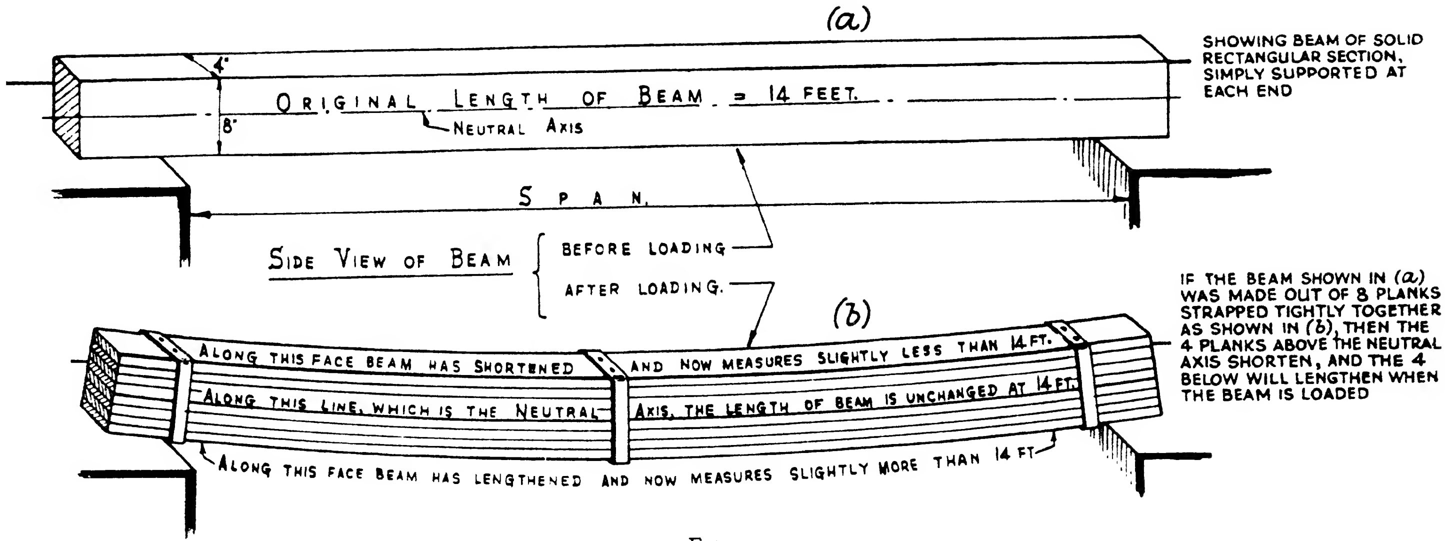

(a)
SHOWING BEAM OF SOLID RECTANGULAR SECTION, SIMPLY SUPPORTED AT EACH END
4"
8"
ORIGINAL LENGTH OF BEAM = 14 FEET.
NEUTRAL AXIS
SPAN.
SIDE VIEW OF BEAM
BEFORE LOADING
AFTER LOADING.
(b)
IF THE BEAM SHOWN IN (a) WAS MADE OUT OF 8 PLANKS STRAPPED TIGHTLY TOGETHER AS SHOWN IN (b), THEN THE 4 PLANKS ABOVE THE NEUTRAL AXIS SHORTEN, AND THE 4 BELOW WILL LENGTHEN WHEN THE BEAM IS LOADED
ALONG THIS FACE BEAM HAS SHORTENED AND NOW MEASURES SLIGHTLY LESS THAN 14 FT.
ALONG THIS LINE, WHICH IS THE NEUTRAL AXIS, THE LENGTH OF BEAM IS UNCHANGED AT 14 FT.
ALONG THIS FACE BEAM HAS LENGTHENED AND NOW MEASURES SLIGHTLY MORE THAN 14 FT.

FIG. 74

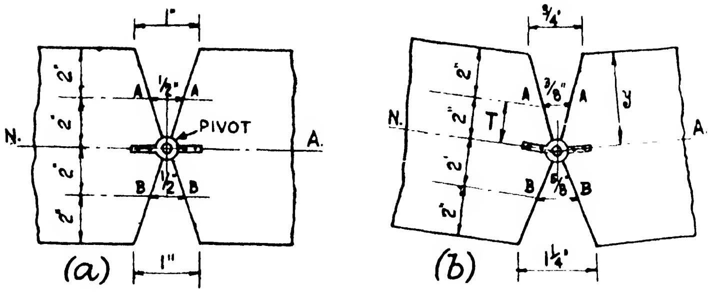


FIG. 75.

load the beam is bent as shown in Fig. 75 (*b*). Then the top notch closes while the lower notch opens. If the lower notch now measures $1\frac{1}{4}$ in. it will have stretched $\frac{1}{4}$ in. and the upper notch will have decreased the same amount, and will now measure $\frac{3}{4}$ in.

Section Modulus. At any other point between the neutral axis and the outside face of the beam the stretch or contraction is in direct proportion to the distance from the outside fibres. For instance, at AA, which is half-way between the outer face and the neutral axis, the contraction will be half of $\frac{1}{4}$ in. or $\frac{1}{8}$ in. Similarly, at BB the stretch will be $\frac{1}{8}$ in. Then the fibres of the beam itself will stretch and contract in exactly the same manner.

Then we can write

$$\frac{e}{y} = \frac{S}{T}$$

where e is the alteration in length of the fibres at the outside face of the beam

y is the distance from the neutral axis to the outside fibres

T is the distance from the neutral axis to the part considered

S is the alteration of length of the fibres at distance T from the neutral axis.

Earlier we showed that stress and strain are proportional within the elastic limit and we had a formula which read—

$$\text{Modulus of elasticity } E = \frac{\text{Stress}}{\text{Strain}}$$

or

$$E = \frac{\text{Stress} \times \text{Original length}}{\text{Alteration of length}}.$$

The original length at the outer face and at the neutral axis will be identical when the beam is unbent, and the value of E will be constant for any given material.

Then as Stress $= \dfrac{E \times \text{Alteration of length}}{\text{Original length}}$

we can call $\dfrac{E}{\text{Original length}} = \text{a constant} = K$

then $\quad\text{Stress} = \dfrac{\text{Alteration of length}}{K}$

or $\quad\text{Alteration of length} = \text{Stress} \times K.$

The stress in the outside fibres of the beam being f, then we can immediately find the stress of any point between the NA and the outside face of the beam by direct proportion.

Then $\quad S = K \times \text{stress at point } T \text{ distant from } NA$

$\quad e = K \times \text{stress at point } y \text{ distant from } NA$

$\quad = K \times f.$

As $\dfrac{e}{y} = \dfrac{S}{T}$ then $S = \dfrac{e \times T}{y}$

and substituting the values of S and e in this equation we have

$$K \times \text{stress at point } T \text{ distant from } NA = \frac{K \times f \times T}{y}$$

and stress at point T distant from $\quad NA = f \times \dfrac{T}{y}$

where f is the stress at the outside fibres of section
T is the distance from NA to point considered
y is the distance from NA to outside fibres of the beam.

From this formula it will be seen that f is a maximum at the outside fibres and is zero on the neutral axis.

It is therefore clear that the intensity of stress may be illustrated as shown in Fig. 76 (*a*) and (*b*). Fig. 76 (*a*) shows the stresses due to the bending moment in the beam, while Fig. 76 (*b*) shows the resisting stresses set up in the material acting in the opposite direction to and resisting these bending stresses. We have shown that the stress is proportional to the distance of the fibres from the neutral axis, and we can now proceed to find the moment of resistance of the rectangular section.

A clear understanding of this feature is absolutely essential if the student is to really get a grip on this all-important subject of the strength of a beam. He must have no doubts that in a rectangular beam there is a lot of material which is useless so far as the strength to resist bending moment is concerned. Suppose the loading of the beam produces a stress of 1,000 lb. on each square inch of material along the outside face EE (see Fig. 76 (*c*)).

The same load and the same bending moment can only produce a stress of half this amount, or 500 lb. per square inch, along the layers at *FF*. It produces no stress at all along the fibres at the centre of the beam, that is, along the neutral axis.

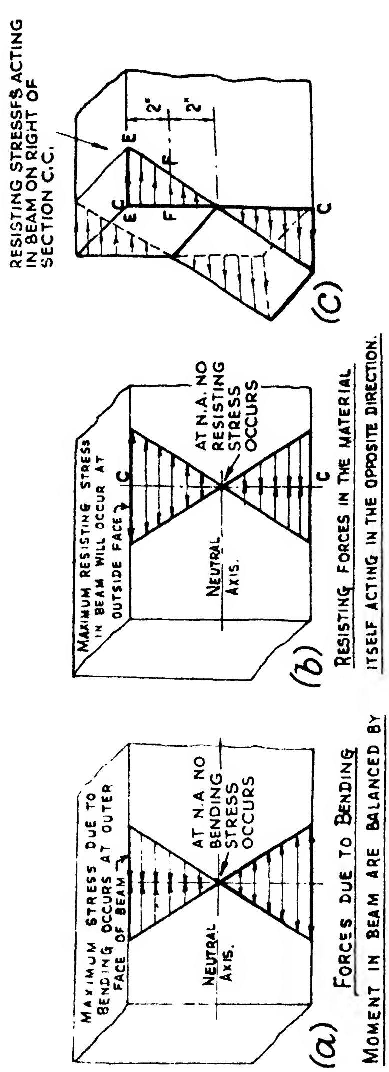


FIG. 76.

It therefore follows that if we could make a beam as shown in Fig. 77, then each layer of the beam would have the same stress to it per square inch. Each little piece of timber would do the same amount of work as each other piece. There would be no waste of material. In other words, the parts shown shaded in Fig. 77 are useful, while the parts shown not shaded are waste material so far as the strength to resist bending is concerned. Of course, for practical reasons it is not possible to make beams of this section, and there is also the question of shear to be considered. (This question of shear will be dealt with later.)

The effective strength of a rectangular beam is therefore the shaded part shown in Fig. 77. This really forms the two flanges of a beam and resists the bending moment. In the case of a rolled-steel beam, the effective lever arm was shown to be the distance between the centres of gravities of the forces in the two flanges. In this case the effective lever arm is exactly the same, that is, the distance between the two centres of gravity. Remember that in this figure each little piece of material is equally stressed. Therefore the total strength of the top half of the beam to resist compression will be :—

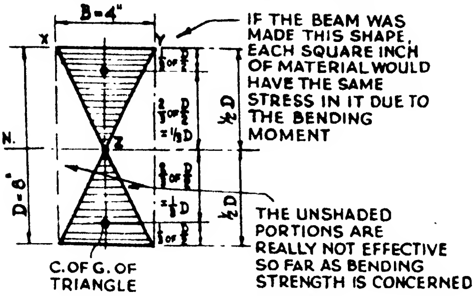


FIG. 77.

$$\text{Area of triangle } XYZ \times Fc = \frac{B}{2} \times \frac{D}{2} \times Fc$$

where $$\text{Area } XYZ = \frac{B}{2} \times \frac{D}{2}$$

and Fc is the compressive strength of the material.

Total strength of the bottom triangle will be $\frac{B}{2} \times \frac{D}{2} \times Ft$, where Ft is the tensile strength of the material.

Notice that this is exactly the same as we showed was the case for a steel beam. What we have now obtained is really the strength of the tension side and the compression side. The compression side acts with a lever arm of $\frac{2}{3} \times \frac{D}{2}$ and the tension side also acts with a lever arm of $\frac{2}{3} \times \frac{D}{2}$.

Total strength to resist tension is therefore

$$\frac{B}{2} \times \frac{D}{2} \times Ft \times \frac{2}{3} \times \frac{D}{2}.$$

This can be reduced to $\frac{BD^2}{12} \times Ft$.

Total strength to resist compression is therefore

$$\frac{B}{2} \times \frac{D}{2} \times Fc \times \frac{2}{3} \times \frac{D}{2}$$

which can be reduced to $\frac{BD^2}{12} \times Fc$.

Total strength of the beam is the tensile strength added to the compressive strength

$$= \left(\frac{BD^2}{12} \times Ft\right) + \left(\frac{BD^2}{12} \times Fc\right).$$

If the beam is assumed to be approximately as strong in compression as it is in tension, then

Total strength of the beam to resist bending moment will be

$$2\left(\frac{BD^2}{12} \times f\right) = \frac{BD^2}{6} \times f$$

where f is the safe strength of the material against bending stresses. This is called the resisting moment or moment of resistance of a rectangular beam.

There are two distinct parts in this formula: $\frac{BD^2}{6}$ is the modulus or shape, measure of strength, and depends only on the size and shape of the beam. The strength of the material is denoted by f. It is obvious that if two solid rectangular beams are both made 8 in. deep and 4 in. wide, and one was made of timber while the other was mild steel, that the steel beam will safely carry more load than the timber beam. In both cases the modulus of shape measure is the same, that is $\frac{BD^2}{6}$, but in the case of timber the safe stress would probably be about ½ ton per square inch, while in the case of the mild steel beam the safe stress f would probably be about 8 tons per square inch, so that the mild steel beam would be about 16 times as strong as the timber beam.

The terms bending moment, section modulus (usually denoted by Z), and moment of resistance have now been fully explained, and in future chapters we shall use them with the greatest confidence.

A Timber Beam. It will be remembered that in Chapter 8 we designed a steel beam suitable for carrying a bending moment of 64·8 in.-tons. We shall now proceed to design a timber beam for the same conditions of loading.

QUESTION. Design a suitable beam of northern pine to carry the load shown in Fig. 78 (*a*). Take f, the safe stress of the timber at 1,000 lb. per square inch.

Answer. The maximum bending moment for a beam simply supported at each end and carrying a uniformly distributed load $= \frac{W \times L}{8}$

where W is the total load on the beam

$$= 3{\cdot}6 \text{ tons or } 8{,}064 \text{ lb.}$$

$$L \text{ is the span} = 12 \text{ ft.} = 144 \text{ in.}$$

$$\text{Max. B.M.} = \frac{8{,}064 \text{ lb.} \times 144 \text{ in.}}{8} = 145{,}152 \text{ in.-lb.}$$

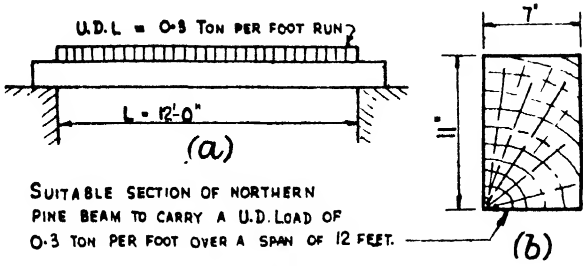


FIG. 78.

If the resisting moment of the beam is equal to or greater than the bending moment the beam will not fail.

$$\text{Resisting moment} = \text{Bending moment}$$

$$\text{Resisting moment} = \frac{B \times D^2}{6} \times f$$

$$= \frac{B \times D^2}{6} \times 1{,}000 \text{ lb.}$$

Then

$$\frac{B \times D^2}{6} \times 1{,}000 = 145{,}152$$

$$B \times D^2 = \frac{145{,}152 \times 6}{1{,}000} = 870.$$

Try a beam 7 in. wide, then

$$7 \times D^2 = 870$$

$$D^2 = \frac{870}{7} = 124$$

$$D = \sqrt{124} = \text{say, } 11 \text{ in.}$$

This section will be suitable, and we could use a beam of northern pine 7 in. broad and 11 in. deep.

It is a good plan to make the breadth of a timber beam between half and two-thirds of the depth.

It will be well to note that the figure of 145,152 in.-lbs. for the bending moment need not be taken as accurate. The loading is never known to exact limits, and even distribution is almost impossible. In practice the figure would be quite accurate enough at 145,000 in.-lbs. Working to figures of extreme accuracy are not necessary and a waste of time.

CHAPTER 10

BENDING MOMENTS

The student cannot hope to understand bending moments and the design of beams except by actually working out many problems for himself. It is quite a different thing to read an article and say, " Yes, I understand that," from being able to do it in an examination room or to reason it out without the aid of a text-book. Accordingly he is strongly advised to work out for himself many problems, not only on the question of bending moments, but on each of the various subjects which have already been covered and will be dealt with in this book.

Various types of loading for beams are shown on pages 101 and 105. These are a little more complicated than the problems we have dealt with so far. In each case keep in mind that the beam is considered as being simply supported at the ends not built-in or fixed.

Cantilever with Point Load at Free End. This type is shown in Fig. 79.

Taking moments about R_2

$$W \times L = R_1 \times A$$

from which $$R_1 = \frac{W \times L}{A}.$$

Total load on support $= W + R_1 = W + \dfrac{WL}{A} = R_2.$

To find the bending moment at any point on the span, calculate the algebraic sum of the moments on either side of the section considered. For instance, to find bending moment of R_1 take moments to left of section, and

$$\text{B.M. at } R_1 = W \times \tfrac{1}{2}L.$$

At a distance half-way along the span B.M. $= \frac{1}{2}L = \frac{1}{2}WL$, while at distance quarter-way along the span from W, B.M. $= W \times \frac{1}{4}L = \frac{1}{4}WL$. At the load W the bending moment would be zero.

If we construct a bending-moment diagram from these results, a figure such as shown in Fig. 79 will be obtained. The maximum B.M. will occur at the support R_1 and will equal WL(WL means $W \times L$). The portion of the bending-moment diagram shown dotted is not usually drawn in practice.

QUESTION 1. A cantilever beam carries a load of 3 tons at the end of a 10-ft. span. Calculate the maximum bending moment on the beam.

Answer.

$$\begin{aligned}\text{Maximum bending moment} &= W \times L \\ &= 3 \text{ tons} \times 10 \text{ ft.} \\ &= 30 \text{ ft.-tons} \\ &= 360 \text{ in.-tons.}\end{aligned}$$

Cantilever Carrying Uniformly Distributed Load Over Entire Span. This is shown in Fig. 80. The load on the beam is w per foot run, so that total load on beam is wL, which will also be called W. From the theorem of centre of gravity it will be remembered that a load acts through its centre of gravity, which in this case will be half-way along the span of it. The load acting through this point will be wL or W.

Then $$W \times \frac{L}{2} = R_1 \times A$$

from which $$R_1 = \frac{WL}{2A}.$$

The support has to carry the load W and also R_1, so that

$$R_2 = W + R_1 = W + \frac{WL}{2A}.$$

The bending moment at any point along the beam can be found by calculating the algebraic sum of the moments acting on either side of the section considered. Taking moments to the left of R_2 we have

$$\text{B.M. at } R_2 = W \times \tfrac{1}{2}L = \frac{WL}{2}.$$

The bending moment at midspan can be found from the fact that if we take moments to the left of midspan we have a load of $\frac{1}{2}W$ acting through its own centre of gravity, that is, half-way along half the span, or at $\frac{1}{4}L$.

$$\text{B.M. at midspan} = \tfrac{1}{2}W \times \tfrac{1}{4}L = \frac{WL}{8}.$$

The student should calculate the bending moment at other places along the beam and plot the bending-moment diagram from these. Finally, from the values obtained a bending-moment diagram similar to that shown in Fig. 80 can be drawn, and it will be found that the shape of the curve is a parabola. The dotted shape of the bending-moment diagram to the right of R_1 is not generally drawn in practice. It will be noted that the maximum bending moment again occurs at the support R_2 and that its value is

$$\frac{WL}{2} \text{ or as } W = wL \text{ maximum B.M.} = \frac{wl^2}{2}.$$

QUESTION 2. A cantilever beam carries a load of $\frac{1}{2}$ ton per foot over a span of 12 ft. Calculate the maximum bending moment on the beam.

Answer.

$$\begin{aligned} \text{Maximum bending moment} &= W \times \tfrac{1}{2}L \\ &= \frac{w \times L^2}{2} \\ &= \frac{\tfrac{1}{2}\text{ ton} \times 12\text{ ft.} \times 12\text{ ft.}}{2} \\ &= 36\text{ ft.-tons} \\ &= 432\text{ in.-tons.} \end{aligned}$$

Beam Simply Supported at Each End and Carrying Point Load at Midspan. This is shown in Fig. 81. As the load W is located at midspan, each support will take one-half of this load or $\frac{1}{2}W$. Therefore

$$R = \tfrac{1}{2}W \text{ and } R_2 = \tfrac{1}{2}W.$$

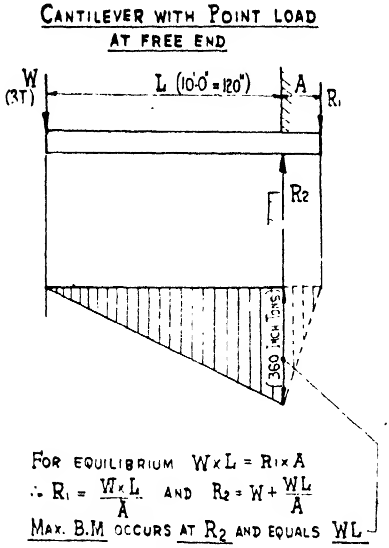


FIG. 79.

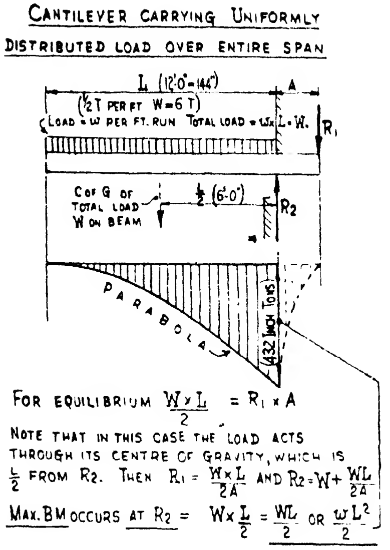


FIG. 80.

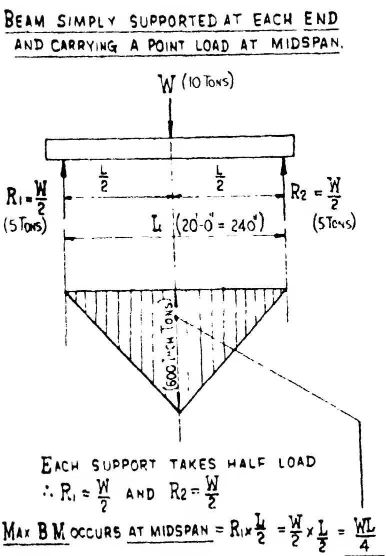


FIG. 81.

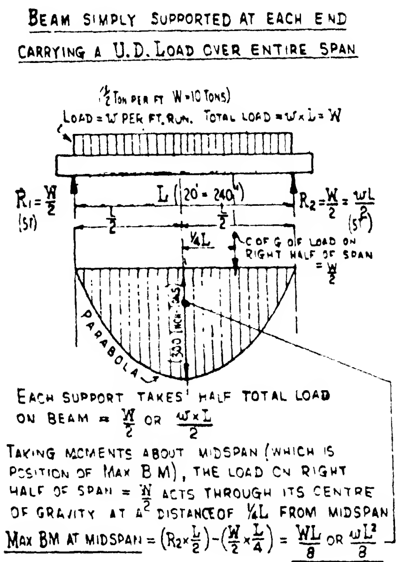


FIG. 82.

Taking moments to the left of midspan we have

$$\text{B.M. at midspan} = R_1 \times \tfrac{1}{2}L = \tfrac{1}{2}W \times \tfrac{1}{2}L = \tfrac{1}{4}WL \text{ or } \frac{WL}{4}.$$

At a distance $\frac{1}{4}L$ to left of load W we have, taking moments to the left of this section

$$\text{B.M. at quarter span} = R_1 \times \tfrac{1}{4}L = \tfrac{1}{2}W \times \tfrac{1}{4}L = \frac{WL}{8}.$$

Further calculations should be made at various points along the span, and finally from the values obtained a bending-moment diagram similar to that shown in Fig. 81 can be drawn. Note that bending moment at the supports is zero, while the maximum bending moment, which occurs at midspan, is $\frac{WL}{4}$.

QUESTION 3. A beam 20 ft. span carries a point load of 10 tons at its centre. Find the maximum bending moment on the beam.

Answer.

Maximum bending moment at centre of beam

$$= \frac{WL}{4}$$
$$= \frac{10T \times 20}{4}$$
$$= 50 \text{ ft.-tons}$$
$$= 600 \text{ in.-tons.}$$

Beam Simply Supported at Each End and Carrying a Uniformly Distributed Load Over Entire Span. This type of loading is shown in Fig. 81. The beam carries a load of w per foot run, or a total load of $w \times L$, which is also called W. The load being symmetrical on the beam, each support will take half the total load or $\frac{1}{2}W$ each. Take moments to the right of midspan to find the bending moment at this point. The load on the right half of the beam will be half total load $= \frac{1}{2}W$. This $\frac{1}{2}W$ load will act through its centre of gravity, which will be at a point half-way between midspan and support R_2, or at a distance $\frac{1}{4}L$ from midspan. Now take moments.

$$\text{We have anti-clockwise moment} = R_2 \times \tfrac{1}{2}L$$
$$\text{and clockwise moment} = \tfrac{1}{2}W \times \tfrac{1}{4}L.$$

If we take the difference between these two moments we find the bending moment at midspan.

Bending moment at midspan

$$= (R_2 \times \tfrac{1}{2}L) - (\tfrac{1}{2}W \times \tfrac{1}{4}L)$$
$$= (\tfrac{1}{2}W \times \tfrac{1}{2}L) - (\tfrac{1}{2}W \times \tfrac{1}{4}L)$$
$$= (\tfrac{1}{4}WL) - (\tfrac{1}{8}WL).$$

A quarter minus one-eighth equals one-eighth, so that

$$\text{B.M. at midspan} = \frac{WL}{8}, \text{ or as } W = w \times L$$

$$\text{B.M.} = \frac{wL^2}{8}.$$

The bending moment at any other point along the span can be found in like manner. The bending moment at a point $\frac{1}{4}L$ to right of midspan can be calculated as follows. Take moment to right of point considered. We have a load of $\frac{1}{4}W$ (the load one-quarter of span) acting through its centre of gravity at a distance $\frac{1}{8}L$ from the section being considered. We have therefore

$$\text{Anti-clockwise moment} = R_2 \times \tfrac{1}{4}L$$
$$\text{Clockwise moment} = \tfrac{1}{4}W \times \tfrac{1}{8}L$$

$$\text{B.M. at quarter span} = (R_2 \times \tfrac{1}{4}L) - (\tfrac{1}{4}W \times \tfrac{1}{8}L)$$
$$= (\tfrac{1}{2}W \times \tfrac{1}{4}L) - (\tfrac{1}{4}W \times \tfrac{1}{8}L)$$
$$= \left(\frac{W \times L}{8}\right) - \left(\frac{W - L}{32}\right)$$
$$= \frac{3WL}{32}.$$

The bending moment at any other point along the beam can be found in a similar manner and finally from the results so obtained a bending-moment diagram as shown in Fig. 82 can be constructed. It will then be found that the maximum bending moment occurs at midspan and equals $\frac{WL}{8}$ or $\frac{wL^2}{8}$. Note that the bending moment is only a half of that obtained with a point load at midspan as shown in Fig. 81. A lot of work can be saved from the fact that the curve of this bending-moment diagram is a parabola, and as the maximum value of the bending moment is known, the moment at any other part of span can be obtained by drawing a parabola, using this maximum value as the apex value. The method of constructing a parabola was dealt with in Chapter 8.

QUESTION 4. A beam 20 ft. span carries a uniformly distributed load of ½ ton per foot run over its extreme length. Find the maximum bending moment on the beam.

Answer.

Maximum bending moment, at centre of beam

$$= \frac{WL}{8}$$
$$= \frac{wL^2}{8}$$
$$= \frac{\tfrac{1}{2}T \times 20 \text{ ft.} \times 20 \text{ ft.}}{8}$$
$$= 25 \text{ ft.-tons}$$
$$= 300 \text{ in.-tons.}$$

Beam Simply Supported at Each End and Carrying Two Point Loads. The first things to calculate in this case (see Fig. 83) are the reactions R_1 and R_2. If distances ABC are unequal, the loading will not be symmetrical over the span, so that R_1 and R_2 will probably have different values.

The bending moment over the support of any beam simply supported at its ends is zero. If we take moment at about R_1, algebraic sum of the clockwise and anti-clockwise moments to the right of this point must be zero.

We have clockwise moments of $(W_1 \times A) + (W_2 \times D)$ and an anti-clockwise moment of $R_2 \times L$.

Then $(W_1 \times A) + (W_2 \times D) - (R_2 \times L) = 0$

or $(W_1 \times A) + (W_2 \times D) = R_2 \times L$, and

$$R_2 = \frac{(W_1 \times A) + (W_2 \times D)}{L}$$

R_1 will be the difference between the total load on the beam and R_2 or $R_1 = (W_1 + W_2) - R_2$.

The bending moment at any section can be found by the usual manner of taking moments about any side of the section considered, and if we take moments to the left of W_1 we have

$$\text{B.M. at } W_1 = R_1 \times A,$$

and taking moments to the right of W_2 we have

$$\text{B.M. at } W_2 = R_2 \times C.$$

If other sections are taken, and the bending moments obtained for these, a bending-moment diagram as shown in Fig. 83 can be drawn. The maximum value of the bending moment will be found to occur under either W_1 or W_2, depending on the values of these.

QUESTION 5. A beam 15 ft. span carries two point loads, one of 3 tons at a distance of 3 ft. from the left-hand end of the beam, and one of 5 tons at 8 ft. from the right-hand end of the beam. Find the amount and position of the maximum bending moment.

Answer. Referring to Fig. 83, then

$$\begin{aligned} W_1 &= 3 \text{ tons} \\ W_2 &= 5 \text{ tons} \\ A &= 3 \text{ feet} \\ B &= 4 \text{ feet} \\ C &= 8 \text{ ft.} \\ D &= 7 \text{ ft.} \\ L &= 15 \text{ ft.} \end{aligned}$$

Then

$$\begin{aligned} R_2 \times L &= (W_1 \times A) + (W_2 \times D) \\ R_2 \times 15 &= (3T \times 3) + (5T \times 7) \\ R_2 &= \frac{9 + 35}{15} = \frac{44}{15} = 2{\cdot}93 \text{ tons}, \\ R_1 &= \text{total load on beam} - R_2 \\ &= 5 + 3 - 2{\cdot}93 = 5{\cdot}07 \text{ tons} \end{aligned}$$

Maximum bending moment occurs under either

$$W_1 \text{ or } W_2.$$

Under W_1, B.M. $= R_1 \times A = 5{\cdot}07 \times 3 = 15{\cdot}21$ ft.-tons.
Under W_2, B.M. $= R_2 \times C = 2{\cdot}93 \times 8 = 23{\cdot}44$ ft.-tons.

The maximum bending moment therefore occurs under the 5-ton load and equals 23·44 ft.-tons or 282 in.-tons.

Beam Simply Supported at Each End and Carrying Two Point Loads and a Uniformly Distributed Load Over Entire Span. This type of loading is shown in Fig. 84. It is a combination of the loadings shown in Figs. 82 and 83, and the bending moment at any section will therefore be a combination of those shown in these figures. The reactions will also be a combination of those obtained for these loadings.

$$R_2 = \frac{W}{2} + \frac{(W_1 \times A) + (W_2 \times D)}{L}.$$

$$R_1 = W + W_1 + W_2 - R_2.$$

The position of the maximum bending moment is not so easy to see in this case. The maximum bending moment for U.D. Loads is at mid-span, and for point loads under one of the loads, but when combined, the position of the maximum bending moment will be somewhere between midspan and the position for the point loads.

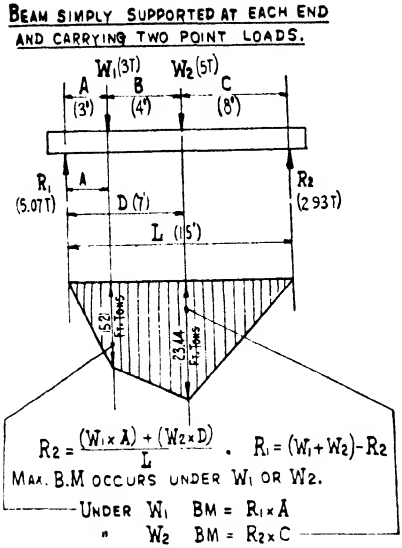


FIG. 83.

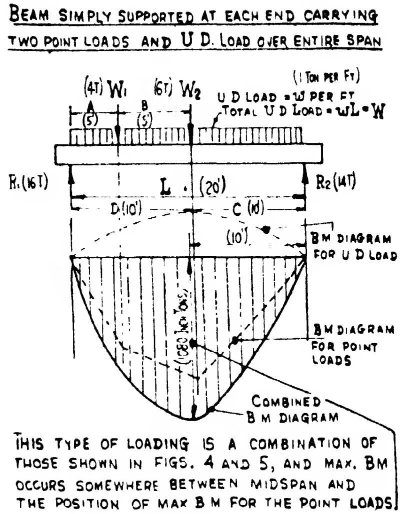


FIG. 84.

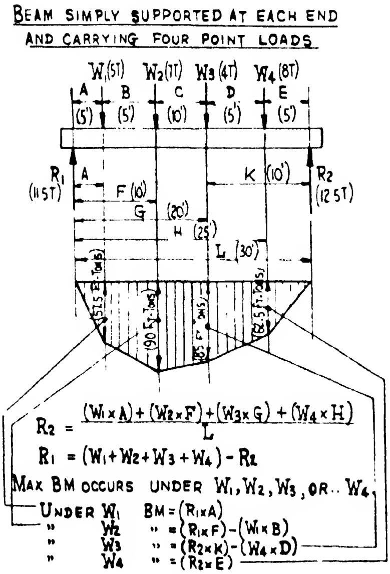


FIG. 85.

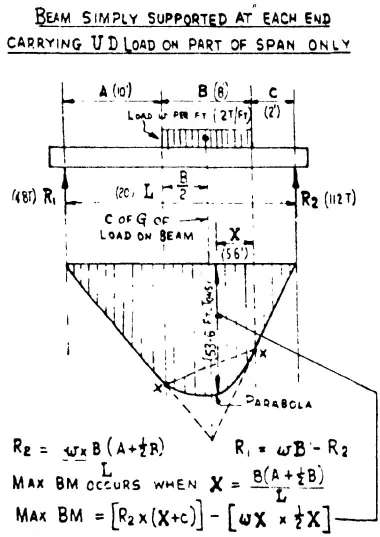


FIG. 86.

An easy way to find the position of the maximum B.M. is as follows: Starting from one support, note the value of the reaction, which is a force acting upwards. Travel along the beam adding together all loads acting downwards until a point is reached where the sum of the downwards loads is as near as possible equal to the reaction. This is the position of minimum shear and maximum bending moment.

Consider in Fig. 84 that $R_1 = 16$ tons, $R_2 = 14$ tons, W_1 and W_2 each equal 5 tons, $A = 3$ ft., $B = 2$ ft., and $C = 5$ ft., while $L = 10$ ft. U.D.L. on beam = 2 tons per foot. Starting from R_1, we have an upward force due to the reaction of 16 tons. For every foot we move along to the right of R_1, there is a downward load due to the uniformly distributed load of 2 tons, so that at 3 ft. from R_1, or at W_1, the U.D.L. = 3 ft. × 2 tons = 6 tons. Also at W_1 there is a downward load of 5 tons, making a total down load of $6 + 5 = 11$ tons.

The difference between this figure and the reaction $R_1 = 16 - 11 = 5$ tons. To balance upward and downward loads we therefore need a downward load of 5 tons. Moving along to the right of W_1 the U.D.L. increases at 2 tons per foot, so that at 2 ft. 6 in. from W_1 the U.D.L. will have increased $2{\cdot}5 \times 2 = 5$ tons. This section, therefore, is where upward and downward forces balance, and where the maximum bending moment occurs.

Checking to the right of this reaction we have a U.D.L. of 4 ft. 6 in. × 2 tons = 9 tons, and a point load $W_1 = 5$ tons, making a total of 14 tons, which agree with the reaction R_2. The maximum bending moment therefore occurs at 4 ft. 6 in. from R_2, and it is now only necessary to take moments on one side of this point to calculate the value of the maximum bending moment.

QUESTION 6. A beam 20 ft. span carries two point loads, one of 4 tons at 5 ft. from the left-hand end, and one of 6 tons at 10 ft. from the right-hand end, and also a uniformly load of 1 ton per foot over its entire length. Find the position and amount of the maximum bending moment.

Answer. The supports will each take one-half of the uniformly distributed load and part of the point loads.

Then

$$\begin{aligned} W_1 &= 4 \text{ tons} \\ W_2 &= 6 \quad ,, \\ A &= 5 \text{ ft.} \\ B &= 5 \quad ,, \\ C &= 10 \quad ,, \\ D &= 10 \quad ,, \\ L &= 20 \quad ,, \end{aligned}$$

Then $R_2 \times L = (W_1 \times A) + (W_2 \times D) + (W \times \frac{1}{2}D)$

$$R_2 = \frac{(4 \times 5) + (6 \times 10) + (20 \times 10)}{20}$$

$$= \frac{20 + 60 + 200}{20} = \frac{280}{20} = 14 \text{ tons.}$$

R_1 takes the remainder of the load $= 4 + 6 + 20 - 14 = 16$ tons.

It is now necessary to find the position of the maximum bending moment before this can be calculated. This will occur where the sum of the downward loads is equal, or as near equal as possible, to the reaction.

Starting from $R_1 = 16$ tons acting upwards, proceed along the beam. From R_1 to W_1 there is a 5-ft. length of the uniformly distributed load of 1 ton per foot, that is, a total load of $5 \times 1 = 5$ tons. At W_1 the point load is 4 tons, making a total downward load up to this point of $5 + 4 = 9$ tons.

Proceeding still further along the beam, there is a 5-ft. length of the U.D.L. between W_1 and W_2, that is, a total load of $5 \times 1 = 5$ tons. Total downward load just before W_2 is reached $= 9 + 5 = 14$ tons. $R_1 = 16$ tons, so that reaction still exceeds downward loads by 2 tons. At W_2, however, the point load of 6 tons will bring the total downward loads to 20 tons, thus exceeding R_1 by $20 - 16 = 4$ tons. At this section the sum of downward loads is as near equal as possible to the reaction, and the maximum bending moment therefore occurs under load W_2. Taking moments to the right of W_2 we have

$$\text{B.M.} = (R_2 \times C) - (w \times C \times \tfrac{1}{2}C).$$

The $\frac{1}{2}C$ comes from the fact that the load on length C will act through its centre of gravity at a point $\frac{1}{2}C$ from W_2.

Then maximum B.M. under W_2

$$\begin{aligned} &= (14 \text{ tons} \times 10) - (1 \times 10 \times 5). \\ &= 140 - 50 = 90 \text{ ft.-tons} \\ &= 1{,}080 \text{ in.-tons.} \end{aligned}$$

Beam Simply Supported at Each End and Carrying Four Point Loads. This loading is shown in Fig. 85. It will be solved in the same manner as in Fig. 83.

B.M. at R_1 will be zero, so that

$$R_2 \times L = (W_1 \times A) + (W_2 \times F) + (W_3 \times G) + (W_4 \times H)$$

$$R_2 = \frac{(W_1 \times A) + (W_2 \times F) + (W_3 \times G) + (W_4 \times H)}{L}$$

R_1 will be the difference between the total load on the beam and R_2.

$$R_1 = (W_1 + W_2 + W_3 + W_4) - R_2.$$

The maximum bending moment will occur under any one of the loads, depending on their values. Taking moments at the various sections, then

$$\begin{aligned} \text{Under } W_1 \text{ B.M.} &= (R_1 \times A) \\ W_2 \text{ B.M.} &= (R_2 \times F) - (W_1 \times B) \\ W_3 \text{ B.M.} &= (R_2 \times K) - (W_4 \times D) \\ W_4 \text{ B.M.} &= (R_2 \times E). \end{aligned}$$

The bending moment at each of these sections should be calculated and a bending moment diagram as in Fig. 85 can then be constructed. The results of the calculations will show under which load the maximum bending moment occurs.

QUESTION 7. A beam 30 ft. span carries four point loads situated at 5 ft., 10 ft., 20 ft., and 25 ft., respectively, from the left-hand end. Find the position and amount of the maximum bending moment if the loads are 5 tons, 7 tons, 4 tons, and 8 tons, respectively.

Answer.

$$\begin{aligned} R_2 \times 30 \text{ ft.} = (5T \times 5 \text{ ft.}) + (7T \times 10 \text{ ft.}) + (4T \times 20 \text{ ft.}) \\ + (8T \times 25 \text{ ft.}) \end{aligned}$$

$$R_2 = \frac{25 + 70 + 80 + 200}{30} = \frac{375}{30} = 12{\cdot}5 \text{ tons}$$

$$R_1 = \text{Total load} - R_2 = 5 + 7 + 4 + 8 - 12{\cdot}5 = 11{\cdot}5 \text{ tons}$$

The maximum bending moment will occur under one of the loads.

Under W_1 B.M. $= R_1 \times 5$ ft. $= 11{\cdot}5T \times 5$ ft. $= 57{\cdot}5$ ft.-tons.
Under W_2 B.M. $= (R_1 \times 10 \text{ ft.}) - (5T \times 5 \text{ ft.})$
$= (11{\cdot}5T \times 10 \text{ ft.}) - 25$ ft. $= 90{\cdot}0$.
Under W_3 B.M. $= (R_2 \times 10 \text{ ft.}) - (8T \times 5 \text{ ft.})$
$= (12{\cdot}5T \times 10 \text{ ft.}) - 40$ ft. $= 85{\cdot}0$.
Under W_4 B.M. $= R_2 \times 5$ ft. $= 12{\cdot}5T \times 5$ ft. $= 62{\cdot}5$.

The maximum bending moment therefore occurs under the 7 tons load and its amount is 90 ft.-tons, or 1,080 in.-tons.

Beam Simply Supported at Each End and Carrying a Uniformly Distributed Load on Part of Span only. For this loading see Fig. 86. It is a type of loading half-way between a point load, as shown in Fig. 81 and a U.D.L. as shown in Fig. 82. The beam carries a load w per foot run over a length B on a beam which is L long. If it is assumed that all the load acts through its centre of gravity, in the manner of a point load, at a distance $\frac{1}{2}B$ from the end of the load, then the total load on the beam will be wB, and taking moments about R we can find the values of the reactions.

$$R_2 \times L = wB \times (A + \tfrac{1}{2}B) \text{ and } R_2 = \frac{wB(A + \frac{1}{2}B)}{L}.$$

R_1 will be the difference between the total load on the beam wB and the reaction R_2, therefore

$$R_1 = wB - R_2.$$

At the right-hand edge of the load, taking moments about this point.
B.M. at right edge of load $= R_2 \times C$.
Left $= R_1 \times A$.

These two figures will give us the points XX on the bending-moment diagram. The bending moment varies from zero at the supports, and will be represented on the diagram by straight lines as far as XX. Although for the purposes of calculating the reactions we assumed the load to act through its centre of gravity, it is a uniformly distributed one, and the bending moment diagram for a U.D.L. has been found to take the form of a parabola. Therefore the part of the diagram between XX will not be a straight line as shown dotted, but a parabola as shown in full, the base line of the parabola being the dotted straight line XX.

The position of the maximum bending moment can be found easily, as it will occur at the point where downward loads equal the reaction. Starting from R_2, then the maximum bending moment occurs at some distance X from the right-hand end of load. On this length X there will be a downward load of wX. This will equal the value of R_2 at the place of maximum bending moment.

Then

$$wX = R_2$$

$$wX = \frac{wB(A + \frac{1}{2}B)}{L}$$

$$X = \frac{wB(A + \frac{1}{2}B)}{wL} = \frac{B(A + \frac{1}{2}B)}{L}.$$

CHAPTER 11

PLATE GIRDERS

The question of what safe stresses should be allowed will depend to some extent on the nature of the loading. The questions of live loads, dead loads, and factor of safety have already been covered.

Span of Girder. For the purpose of this particular design it will be considered that the plate girder has a 60-ft. span and is in a steel-framed building, the load from the floors being transmitted to the plate girder by smaller size floor beams, generally known as secondary beams. These will be assumed effectively to stiffen the top flange against side buckling, and for this reason the full stress of 8 tons per square inch will be allowed. Actually, since we shall make the compression flange of the same area as the tension flange, the actual stress on the compression flange will be something less than 8 tons (actually it will be a little over 7 tons per square inch).

Loading. The girder will carry two point loads of 30 tons each. These loads come down steel columns and are located at 10 ft. on each side of the centre line. In addition, the girder will carry a uniformly distributed load of 4 tons per foot run. This load includes the weight on the floor, the weight of the floor itself, and the weight of the plate girder and any concrete casing which may be used. The plate girder is carried from steel columns.

Reactions. Total load carried by the plate girder will be :

$$\begin{aligned} \text{Point loads, } 2 \times 30 &= 60 \text{ tons} \\ \text{Distributed load, } 4 \times 60 &= 240 \text{ tons} \\ \hline \text{Total load} &= \underline{\underline{300 \text{ tons}}} \end{aligned}$$

All the loads are symmetrical about the centre line of the girder, so that the reaction at each end will be one-half the total load, therefore the rivets in the end angles which connect the plate girder to the columns will have to be strong enough to deal with a shear of 150 tons. The general layout of the loading is shown in Fig. 87 (*a*).

Depth of Girder. For economical construction the depth of a plate girder is generally made between one-tenth and one-twelfth of the span where the head room permits. In this case we shall assume a depth of web plate equal to one-twelfth of the span, so that the distance outside to outside of the flange angles will be 60 ft. $\div$ 12 = 60 in., and for preliminary calculations this will be considered as the effective depth of the beam.

Bending Moment Due to Point Loads Only:

Bending moment at one 30-ton load

$$= 30 \text{ tons} \times 20 \text{ ft.} = 600 \text{ ft.-tons.}$$

Bending moment at centre

$$= 30 \text{ tons} \times 30 \text{ ft.} - (30 \text{ tons} \times 10 \text{ ft.})$$

Bending moment at centre = 900 − 300 = 600 ft.-tons.

The bending-moment diagram for these point loads is shown in Fig. 87 (*b*), points *A, B, C, D, E, F, G,* and *H.*

Bending Moment Due to Distributed Load Only.

$$\text{Bending moment at centre} = \frac{\text{Total load} \times \text{Span}}{8}$$

$$\text{Bending moment of centre} = \frac{4 \times 60 \times 60}{8} = 1{,}800 \text{ ft.-tons.}$$

The bending-moment diagram for the uniformly distributed load will be a parabola. This is shown in Fig. 87 (*b*) by the points *A, H, G, F, E, L, K,* and *J.*

To form the complete bending moment diagram add the bending-moment diagram for the point loads to the bending-moment diagram for the uniformly distributed load, as shown in the drawing. A little thought will make it clear that the bending-moment diagram due to all the loads on the plate girder is as shown in *A, H, F, E, O, N,* and *M.*

Maximum Bending Moment. This occurs at the centre of the beam, and amounts to

$$1{,}800 \text{ ft.-tons} + 600 \text{ ft.-tons} = 2{,}400 \text{ ft.-tons.}$$

This can also be written as 28,800 in.-tons.

Flange area required at centre of span

$$= \frac{\text{Bending moment in inch-tons}}{\text{Effective depth in inches} \times \text{Safe stress}}$$

$$= \frac{28{,}800}{60 \times 8} = 60 \text{ sq. in.}$$

The bottom flange is in tension and will be weakened by rivet holes. The top flange is in compression and will not be weakened by rivet holes, since the spaces will be filled up by rivets. Strictly speaking, therefore, the tension flange should have more steel provided than the compression flange. In practice they are often made the same.

Some designers assume that one-sixth of the gross area of the web can be considered as part of the flange to resist bending moment. Others use one-eighth of the gross area of the web. Many designers assume that none of the web is available to resist bending stresses, and that the flanges, which consist of the flange

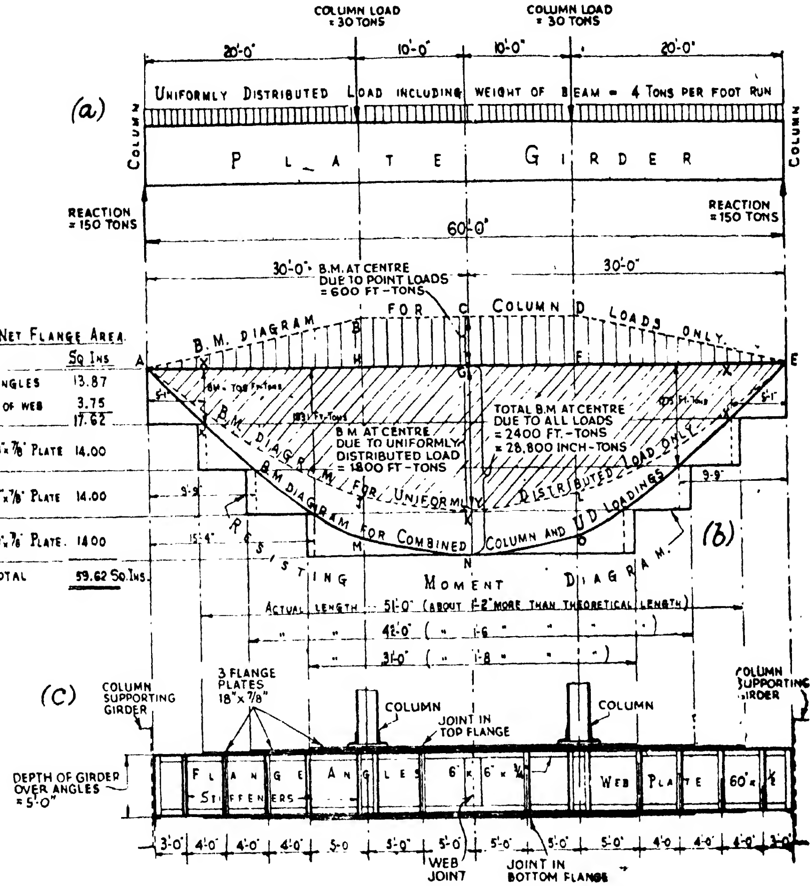


FIG. 87.

angles and flange plates, should be sufficiently strong to take all the bending moment without any assistance at all from the web.

Area of Tension Flange. In this case we shall assume that one-eighth of the web can be considered as part of the flange. We shall also consider the effective area of the tension flange as being the gross area less the area of the rivet holes.

Although not strictly necessary, we shall make the compression flange the same area as the bottom or tension flange.

Area of web plate = Depth × Thickness

$$\text{Area required} = \frac{\text{Max. shear}}{\text{Safe shear stress}} = \frac{\text{Max. reaction}}{\text{Safe shear stress}}.$$

Allowing a safe shear stress of 5 tons per square inch:

$$\text{Area required} = \frac{150}{5} = 30 \text{ sq. in.}$$

Depth of web is 60 in., so that required thickness

$$= \frac{30}{60} = \frac{1}{2} \text{ in.}$$

Tension Flange Area. It has been shown that the net effective flange area required is about 60 sq. in. With one-eighth of the web available, the net flange area consisting of the flange angles and flange plates, will be:

$$60 - (\tfrac{1}{8} \times 30) = 60 - 3{\cdot}75 = 56{\cdot}25 \text{ sq. in.}$$

Assuming the flange angles should be at least one-fourth of this, net area of the flange angles required will be:

$$\frac{56{\cdot}25}{4} = \text{about } 14 \text{ sq. in.}$$

The largest British standard section of angle which is commonly made is $6 \times 6 \times \frac{3}{4}$ in.

		Sq. in.
Area of 2 angles, $6 \times 6 \times \frac{3}{4}$	=	16·87
Two holes, 1 in. dia. from each angle $4 \times (1 \times \frac{3}{4})$	=	3·00
Net area of two angles (16·87 − 3·00)	=	13·87
Net area of flange angles and cover plates required	=	56·25
Net area of flange angles	=	13·87
Net area of flange plates (56·25 − 13·87)	=	42·38

$$\text{Assume width of flange plates as } \frac{\text{Span}}{40}$$

$$= \frac{60 \times 12}{40} = 18 \text{ in.}$$

Since it is necessary to deduct the rivet holes from the flange plates, the gross sectional area would have to be more than 42·38.

		Sq. in.
Area of 3 plates, 18 in. × $\frac{7}{8}$ in.	=	47·25
Area of 2 rivet holes, 1 in. dia. $= 2 \times 2\frac{5}{8} \times 1$	=	5·25
Net area of flange plates	=	42·00

This section seems about right for the tension flange, and as stated before, although the compression flange could, strictly speaking, be somewhat less, the areas will be kept the same.

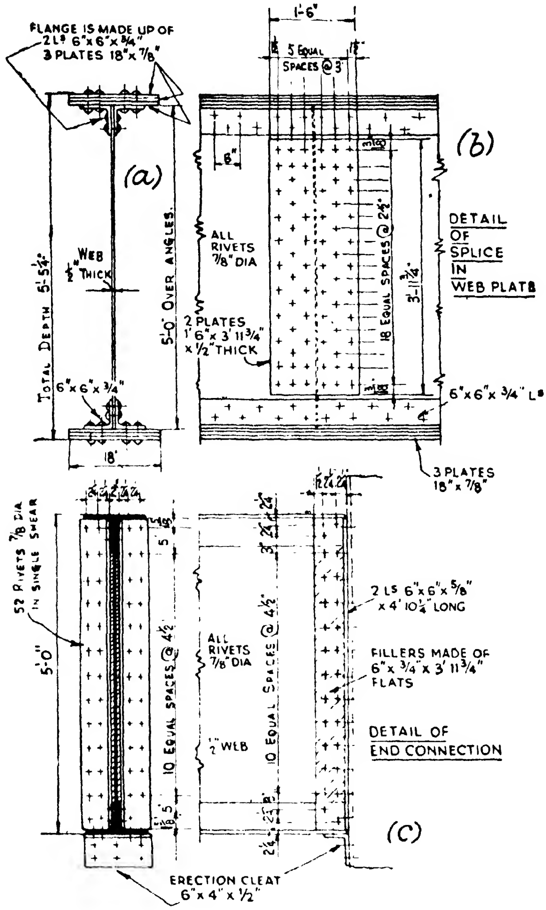


FIG. 88.

The cross-section of the girder at the centre line has therefore been found to be :

Web plate, 5 ft. deep and $\frac{1}{2}$ in. thick.
Each flange composed of

3 plates, 18 in. wide $\times$ $\frac{7}{8}$ in. thick
2 angles, 6 in. $\times$ 6 in. $\times$ $\frac{3}{4}$ in.

This section is shown in Fig. 88 (*a*).

Although the section just found by approximate methods is not, strictly speaking, accurate, it is a method very often used in practical design.

Bending Moment. The section of the girder which has now been found is the material required to resist the maximum bending moment, which, as shown in Fig. 87 (*b*), is in the centre of the girder. Examination of the bending-moment diagram shows that at other places along the beam the bending moment reduces, and consequently less material is required in the flanges. For instance, it is easy to see that if the beam is kept the same depth and the bending moment is reduced from 28,800 in.-tons to 14,400 in.-tons, that the net area of one flange will require to be only one-half of the 60 sq. in. necessary at the centre of the girder. Therefore there is no need to carry all the plates the full length of the girder, as this would be waste of material.

We have allowed one-eighth of the web as being considered part of the flange area. The web plate section is not altered and the flange angles run the full 60 ft. span. The net area of two angles 6 in. × 6 in. × $\frac{3}{4}$ in. and one-eighth the web is 17·62 sq. in. The net area of the total flange is 59·62 sq. in. Assuming the effective depth is not changed, the bending moment which the angles and part of the web will resist (without any flange plates) will be

$$\frac{17{\cdot}62}{59{\cdot}62} \times 2{,}400 \text{ ft.-tons} = 709 \text{ ft.-tons.}$$

Therefore, until the bending moment exceeds this amount, no flange plates are required.

Increased Strength. When one flange plate is added, the strength of the girder is increased so that it is capable of resisting a bending moment of

$$\frac{31{\cdot}62}{59{\cdot}62} \times 2{,}400 \text{ ft.-tons} = 1{,}275 \text{ ft.-tons.}$$

This section is therefore sufficient until the bending moment exceeds this amount. By scaling the bending-moment diagram the position at which these bending moments occur can be found. For instance, the position when the maximum bending-moment is 709 ft.-tons is shown at *XX* in Fig. 87 (*b*), and the horizontal distance to this point from the end of the girder is found to be 5 ft. 1 in. The total length of the girder is 60 ft. and 5 ft. 1 in. off each end indicates that the theoretical length of the flange plate nearest the angles will be 60 ft. − 10 ft. 2 in = 49 ft. 10 in. Generally, in order to ensure development of rivet strength, the plates are actually made between 1 ft. and 2 ft. longer than this theoretical length. The dotted lines shown in Fig. 87 (*b*) are the theoretical length of the flange plates and the full lines the actual lengths.

Length of Flange Plates by Calculation. Instead of drawing the bending-moment diagram, the lengths of the flange plates could be found by calculation as shown below. At any point between the reaction and the column load of 30 tons the bending moment on the beam

$$= (\text{Reaction} \times P) - (\text{U.D.L. on length } P \times \tfrac{1}{2}P)$$

where P is the distance from the support to the position being considered. The U.D.L. on length P will be 4 tons $\times P = 4P$. This load will act through its centre of gravity, that is, at a position $\frac{1}{2}P$ from the position being considered.

$$\text{B.M.} = (150 \text{ tons} \times P \text{ ft.}) - (4P \times \tfrac{1}{2}P)$$
$$= 150P - 2P^2 \text{ ft.-tons.}$$

The net flange area is made up as follows:

	Sq. in.
Net area of angles and one-eighth of web .	17·62
,, ,, 1 flange plate, 18 in. × $\frac{7}{8}$ in. .	14·00
,, ,, 1 ,, ,, ,, ,, .	14·00
,, ,, 1 ,, ,, ,, ,, .	14·00
Total net flange area	59·62

If the total area of 59·62 sq. in. will resist a bending moment of 2,400 ft.-tons, it follows that

		Ft.-tons.
Angles and web plate only will resist	$\frac{17\cdot62}{59\cdot62} \times 2{,}400 =$	709
Angles, web plate, and one flange plate will resist	$\frac{31\cdot62}{59\cdot62} \times 2{,}400 =$	1,275
Angles, web plate, and two flange plates will resist	$\frac{45\cdot62}{59\cdot62} \times 2{,}400 =$	1,831

It we equate each of these resisting moments to the bending-moment formula we can find the value of P.

Taking first the angles and one-eighth of the web plate we have

$$150P - 2P^2 = 709$$
$$75P - P^2 = 354\cdot5$$
$$P^2 - 75P^2 = -354\cdot5$$
$$P^2 - 75P + \left(\frac{75}{2}\right)^2 = -354\cdot5 + \left(\frac{75}{2}\right)^2.$$

Take the square root of each side, then

$$P - 37\cdot5 = \sqrt{-354\cdot5 + 1{,}406} = \sqrt{1{,}051\cdot5} \pm 32\cdot41$$
$$P = 37\cdot5 - 32\cdot41 = 5\cdot09 \text{ ft.} = 5 \text{ ft. } 1 \text{ in.}$$

Then at a distance of 5 ft. 1 in. from each end of the girder the bending moment equals the resisting moment of the flange angles and one-eighth of the web, so that at this point a flange plate must be added to take care of the increasing bending moment.

Length of plate theoretically $= 60$ ft.
$- (2 \times 5$ ft. 1 in.)
$= 60$ ft. $- 10$ ft. 2 in. $= 49$ ft. 10 in.

Taking now the angles, web plate, and the one flange plate, we have

$$150P - 2P^2 = 1{,}275$$
$$75P - P^2 = 637\cdot5$$
$$P^2 - 75P = -637\cdot5$$
$$P - 37\cdot5 = \sqrt{-637\cdot5 + 1{,}406} = \sqrt{768\cdot5} = \pm 27\cdot71$$
$$P = 37\cdot5 - 27\cdot71 = 9\cdot79 \text{ ft.} = 9 \text{ ft. } 9 \text{ in.}$$

Theoretical length of second flange plate

$= 60$ ft. $- (2 \times 9$ ft. 9 in.) $= 40$ ft. 6 in.

Actual length of second flange plate

$= 40$ ft. 6 in. $+ 1$ ft. 6 in. $= 42$ ft.

Lastly we have the angles, web plate and two flange plates, which will resist a maximum bending moment of 1,831 ft.-tons.

$$150P - 2P^2 = 1{,}831$$
$$75P - P^2 = 915\cdot5$$
$$P - 37\cdot5 = \sqrt{-915\cdot5 + 1{,}406} = \sqrt{490\cdot5} = \pm 22\cdot13$$
$$P = 37\cdot5 - 22\cdot13 = 15\cdot37 \text{ ft.} = 15 \text{ ft. } 4 \text{ in.}$$

Length of outer flange plate would therefore theoretically need to be 60 ft. $- (2 \times 15$ ft. 4 in.) $= 29$ ft. 4 in.

Actual length of outside flange plate will be made

29 ft. 4 in. $+ 1$ ft. 8 in. $= 31$ ft.

It will be found that these lengths of plates agree with those found by the graphical method.

Number of Rivets.—Fig 88 (c) shows a detail of the end connection of the girder to the supporting column. Remembering that the rivets passing through the web of the girder will be in double shear, and those passing through the column will be in single shear, the student should check up that the number of rivets shown are sufficient to make a sound connection between girder and column.

Although it is possible to get a web plate 60 ft. long, it is very probable that owing to the difficulty of handling the plate, a joint would be made somewhere near the centre of the girder. Sometimes the flange and web joints are made at the same place, but this is not considered very good practice, and at Fig. 88 (*b*) the detail of a suitable web joint is shown.

CHAPTER 12

SHEAR STRESSES

In Chapter 1 the nature of shear and its definition were given. In addition to the bending action which takes place in a beam, there is also a shearing action. For instance, consider the beam shown in Fig. 89 (*a*). This beam is of rectangular section, and is simply supported at each end.

If this beam was made out of 13 separate small blocks, as shown in Fig. 89 (*b*), then when the beam is loaded the small blocks will try to slide past each other vertically as shown. This is known as vertical shear. If each small block *K* to *L*

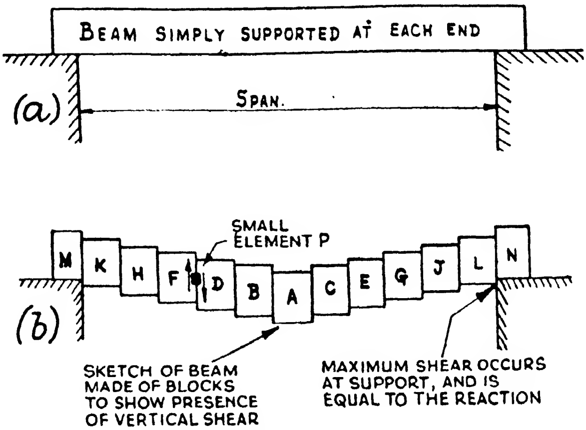


FIG. 89.

carried a load of 1 ton, then block *A* would carry 1 ton, and if block *A* was not to fall out of the beam, the two side faces of this block must each exert an upward force or reaction of ½ ton.

If the blocks were glued together along their side faces, then the glue between *A* and *B* and between *A* and *C* must be strong enough to resist a shear of ½ ton. As each block, *A*, *B*, and *C*, has a load of one ton on it, then blocks *A*, *B*, *C* carry a load of 3 tons, so that the shear on faces *BD* and *CE* will each be one-half of this, that is 1½ tons. Then if blocks *A*, *B*, and *C* are not to fall out by failure along *BD* and *CE*, the glue along these two faces must resist a shear of 1½ tons.

If we continue this procedure until we reach the supports, then the 11 blocks *A* to *K* and *A* to *L* will be supported by the glue along faces MK and LN. Total load on these 11 blocks will be 11 tons and each of faces MK and LN will take one-half of this 11 tons, or $5\frac{1}{2}$ tons. Therefore the glue along faces MK and LN will have to be strong enough to resist a shear of $5\frac{1}{2}$ tons.

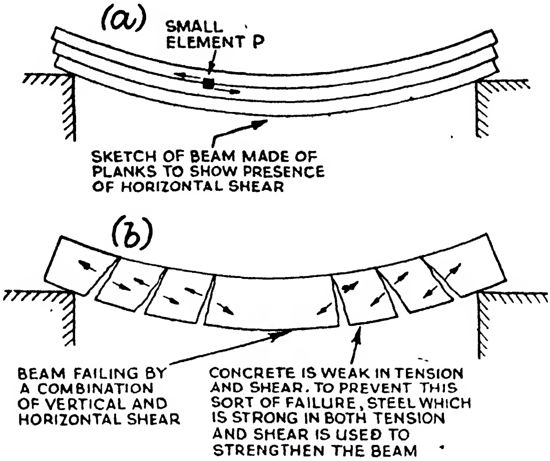


FIG. 90.

It is obvious that, even if the beam is not made of separate blocks, when it is loaded this same sliding or shearing action will tend to take place, and that the timber at the supports of the beam must be strong enough to resist a shear force of $5\frac{1}{2}$ tons. Note, also, that the shear at the support is equal in value to the reaction.

Next we come to horizontal shear. Let the beams be made of three planks as shown in Fig. 90 (*a*), then, upon bending, the three planks tend to slide one over the other. If the three planks were glued together, the glue would have to be strong enough to prevent this sliding action.

Both horizontal and vertical shear take place in a beam, and the resultant failure would be as shown in Fig. 90 (*b*). A small element, P, taken from

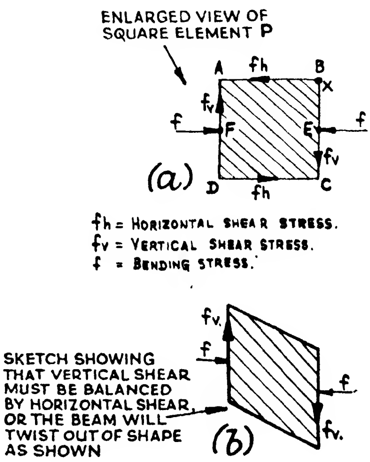


FIG. 91.

the top half of the beam and enlarged, would appear as shown in Fig. 91 (*a*). If extremely small, then we have horizontal shear acting along the top and bottom faces of the element $= f_h$. Also we have vertical shear acting along the side faces $= f_v$. Then we have the compressive stress due to bending acting normally to the side faces $= f$.

From the theory of moments, it is known that if a body is to be in equilibrium the sum of the moments about any point must be zero. Taking moments about point X we have—

Clockwise moments $= (f_v \times 0) + (f_v \times AB) + (f \times BE)$.
Anti-clockwise moments $= (f_h \times 0) + (f \times AF) + (f_h \times BC)$.

Summing these we obtain

$$(f_v \times AB) + (f \times BE) = (f \times AF) + (f_h \times BC).$$

But $BE = AF$, so that

$$(f_v \times AB) + (f \times AF) = (f \times AF) + (f_h \times BC)$$
$$f_v \times AB = f_h \times BC.$$

But $AB = BC$ so that $f_v = f_h$.

For the element to be in equilibrium the vertical shear must equal the horizontal shear. In a beam vertical and horizontal shear are equal. If this was not so the section would twist out of shape, as shown in Fig. 92 (*b*).

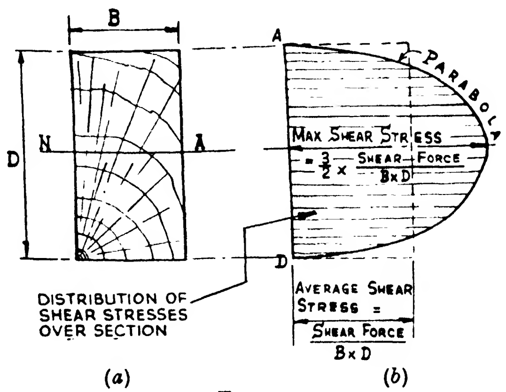


FIG. 92.

Shear Stresses. It will be remembered that when dealing with bending moments we assumed that the bending-moment stresses were taken care of by the flange of a steel beam. In like manner the shear stresses are taken care of by the web. In a rectangular timber section, the section must take care of both bending and shear stresses. In a steel plate girder the combination of horizontal and vertical shear produces diagonal stresses in the web, which tend to make this buckle in one direction and to tear in the opposite direction. For this reason it is often necessary to put in stiffeners as shown in the plate girder in Chapter 11.

Distribution of Shear Stress. The shearing stress across a beam is not uniformly distributed. For instance, in a timber beam 4 in. wide and 8 in. deep, and assuming the shear at one of the supports (that is, the maximum shear) is 3,200 lb., then

$$\text{Average shear stress per square inch} = \frac{\text{Total shear}}{\text{Area}}$$

$$= \frac{3{,}200 \text{ lb.}}{8 \times 4} = 100 \text{ lb.}$$

Reference to Fig. 92 (*a*) and (*b*) shows how the shear stress is distributed over the section. It varies from a maximum at the neutral axis to zero at the upper and lower faces of the beam. (Note that this is exactly the opposite to the bending stress, which varies from zero at the neutral axis to a maximum at the outside faces.) The distribution of the stress, however, takes the form of a parabola as shown. The total shear force in the beam is represented by the area shown shaded. If the length AD represents area of beam $= B \times D$, and the horizontal shade lines, the stress at various sections of the beam, total area of shaded portion will represent $B \times D \times$ Average stress = Shear force in beam

The area under a parabolic curve

$$= \text{Base} \times \frac{2}{3} \text{ Height} = B \times D \times \text{Average shear stress.}$$

From this it will be seen that at the neutral axis the shear stress will be

$$\text{Average stress} \div \frac{2}{3} = \text{Average stress} \times \frac{3}{2}.$$

Therefore

$$\text{Maximum shear stress} = \frac{3}{2} \times \frac{\text{Shear force}}{B \times D}$$

$$\text{Maximum shear stress} = \frac{3}{2} \times \frac{3{,}200}{4 \times 8} = 150 \text{ lb. per square inch.}$$

The strength of the timber in shear must take care of this stress of 150 lb. per square inch, and not the 100 lb. per square inch average stress.

Shearing Force Diagrams. As well as for finding the value of the maximum shearing force in a beam, shear force diagrams are also very useful for finding the position to the maximum bending moment. Maximum bending moment occurs at the position of minimum shear. Minimum shear can occur at more than one place; for instance, it can be zero at one point and very nearly zero at another. In such cases it is necessary to calculate the bending moment at each point in order to find which is the maximum value.

Various types of beam and loadings are dealt with below, and the accompanying examples will help to make clear the method of finding maximum and minimum shear.

Cantilever with Point Load at Free End. This is shown in Fig. 93. When dealing with bending-moment diagrams, the first

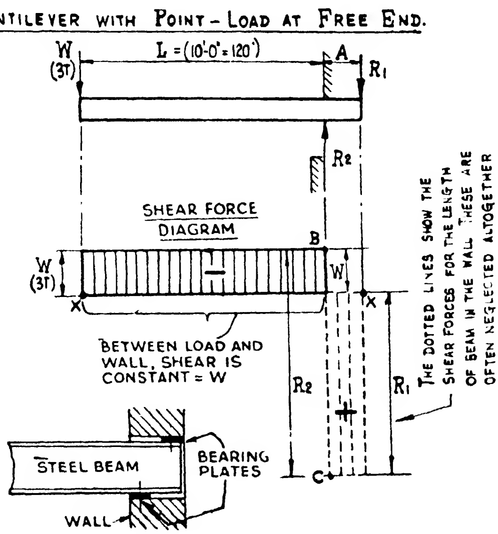


FIG. 93.

thing we had to calculate was the reactions, this applies to shear-force diagrams also. Then remembering that the algebraic sum of the moments about any section must be equal to zero for the beam to be in equilibrium, we have

$$W \times L = R_1 \times A, \text{ from which } R_1 = \frac{W \times L}{A}.$$

The reactions of a beam must total the same as the loads, so that

$$R_2 = W + R_1 = W + \frac{W \times L}{A}.$$

The method of constructing a shear-force diagram is as follows. Starting from one end of the beam, in this case from the right-hand end, take note of all upward reactions and downward loads coming on the beam. In Fig. 93 we have a downward force of R_1. Choosing some suitable scale, say, 1 in. to represent 2 tons, draw a vertical line equal in length to R_1 immediately below R_1. This is the shear acting at R_1. As there are no other loads until we reach R_2, the shear does not alter and is the same on the shear diagram. Therefore we draw a horizontal line until point C is reached, which is directly under R_2.

At this point we have an upward force due to the reaction R_2. An upward vertical line representing this reaction runs upwards from C. It finishes at B. Between B and the load W no other loads occur, so that the shear is constant between these two points. At W there is a downward force which is represented by a downward vertical line equal to scale to the value of W. If the diagram has been correctly drawn, a horizontal line XX will joint the starting and finishing points of our diagram. Then the shear at any point on the span can be scaled off the diagram.

Note that one part of the diagram is above the line XX and one part below. This is because the shear changes from positive to negative at the support R_2. It is usual when dealing with cantilevers to deal only with the part of the beam outside of the wall, and the part of the shear diagram shown dotted is not usually drawn.

QUESTION 1. A cantilever carries a load of 3 tons at the end of a 10-ft. span. Find the maximum shear occurring on the part of the beam outside the wall.

Answer. The shear force remains constant between the support and the load is equal in value to the load = 3 tons. Maximum shear = 3 tons.

Cantilever Carrying Uniformly Distributed Load Over Entire Span. This is shown in Fig. 94. It will be remembered that a uniformly distributed load can be assumed to act through its centre of gravity, so that

$$W \times \tfrac{1}{2}L = R_1 \times A$$

$$R_1 = \frac{W \times L}{2A}.$$

Reaction = Total of loads on beam.

$$R_2 = \frac{W \times L}{2A} + W.$$

To draw the shear-force diagram, start from the right-hand end of the beam. Directly under the force R_1 draw a line equal to this. Shear is constant until point C is reached, and then comes the upward reaction R_2, represented by the line CB. Now every foot we travel along the beam we have the U.D.L. of w acting downwards. It follows that the shear decreases accordingly. At the free end of the beam the shear will

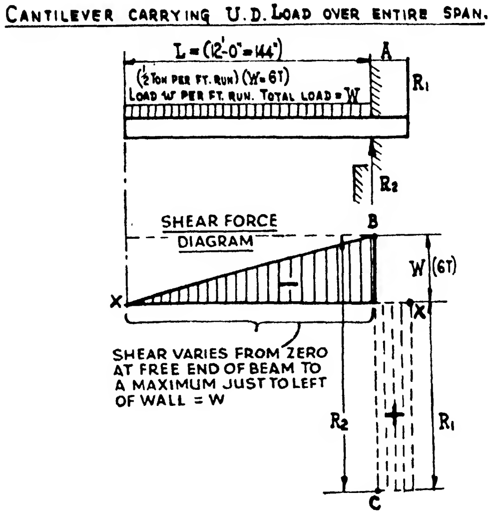


FIG. 94.

be zero, so the decrease of shear due to U.D.L. between support and end of beam will be $w \times L = W$. At point X the shear line will be a distance equal to W below point B. Then a horizontal line XX will join starting and finishing points of the shear diagram. The dotted part of the diagram is not usually drawn in practice. From this diagram it will be seen that outside the support the shear varies from a maximum of W to zero.

QUESTION 2. A cantilever beam carries a load of $\frac{1}{2}$ ton per foot over a span of 12 ft. Find the value and position of the maximum shear.

Answer. Maximum shear occurs just to left of support

$$= W \times L = W = 6 \text{ tons.}$$

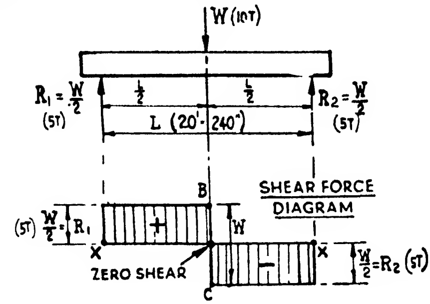
Note that shear changes from positive to negative at R_2, so that the maximum bending moment occurs here.

Beam Simply Supported at Each End and Carrying Point Load at Midspan. This is shown at Fig. 95. As the load is located at the centre of span each reaction will take half total load $= \frac{1}{2}W$ each. Starting at the left side of the beam from point X we have an upward force of $\frac{1}{2}W$ from the reaction R_1. The shear is then constant till the load W is reached, so that a horizontal line is drawn on the diagram from R_1 to W. At midspan we have a downward load of W, represented on the diagram by the length BC. Shear is then again constant until R_2 is reached, where there is an upward force of $\frac{1}{2}W$. A line XX joining starting and finishing points completes the shear-force diagram.

The shear at any point on the span can now be scaled off. Note that the shear changes from positive to negative at midspan. Midspan is the point of minimum shear and position of maximum bending. Maximum shear occurs between the support and midspan, and is equal to the reaction in value.

QUESTION 3. A beam 20 ft. span carries a point load of 10 tons at its centre. Find the maximum shear and position of minimum shear.

BEAM SIMPLY SUPPORTED AT ENDS AND CARRYING POINT-LOAD AT MIDSPAN



SHEAR IS CONSTANT BETWEEN SUPPORT AND LOAD AT MIDSPAN, WHERE SHEAR CHANGES FROM POSITIVE TO NEGATIVE.
AT MIDSPAN SHEAR IS ZERO.

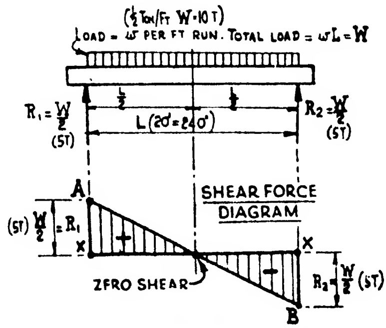
MAXIMUM SHEAR = $\frac{W}{2}$ = REACTION.

FIG. 95.

Answer. Maximum shear occurs at support (also at any point between support and midspan) = Reaction = $\frac{1}{2}W = 5$ tons. Minimum shear occurs at midspan.

Beam Simply Supported at Each End and Carrying U.D.L. over Entire Span. This is shown in Fig. 96. Starting from the left-hand side of the beam, draw upwards line equal to R_1. As the load is uniformly distributed, then for every foot we move along the beam we have a downward force of w. Total down force on $\frac{1}{2}$ span $= \frac{1}{2}WL = \frac{1}{2}W$. Therefore at midspan the shear will have decreased from $\frac{1}{2}W$, its value at R_1, to zero. Continuing along the beam, the total downward force due to the U.D.L. will equal $w \times L = W$. Therefore point B on the diagram should be a distance equal in value to W below point A. At

BEAM SIMPLY SUPPORTED AT EACH END, CARRYING U.D.LOAD OVER ENTIRE SPAN



SHEAR VARIES FROM A MAXIMUM OF $\frac{W}{2}$ AT SUPPORT TO ZERO AT MIDSPAN.

MAXIMUM SHEAR = $\frac{W}{2}$ = REACTION.

FIG. 96.

R_2 we have an upward force of $\frac{1}{2}W$. A horizontal line XX joining starting and finishing points completes our diagram. Then it will be seen that the maximum shear force occurs at the support and is equal in value to the reaction. The minimum shear occurs at midspan, which will also be the point of maximum bending moment.

In Practice

QUESTION 4. A beam 20 ft. span carries a uniformly distributed load of $\frac{1}{2}$ ton per foot run over its entire length. Find the value and position of the maximum shear and maximum bending moment.

Answer. Maximum shear occurs at the support

$$= \tfrac{1}{2}wL = \tfrac{1}{2} \times \tfrac{1}{2} \times 20 = 5 \text{ tons.}$$

Maximum bending occurs at position of minimum shear = at midspan.

Beam Simply Supported at Each End and Carrying Two Point Loads. This is shown in Fig. 97. First it is necessary to find the reactions.

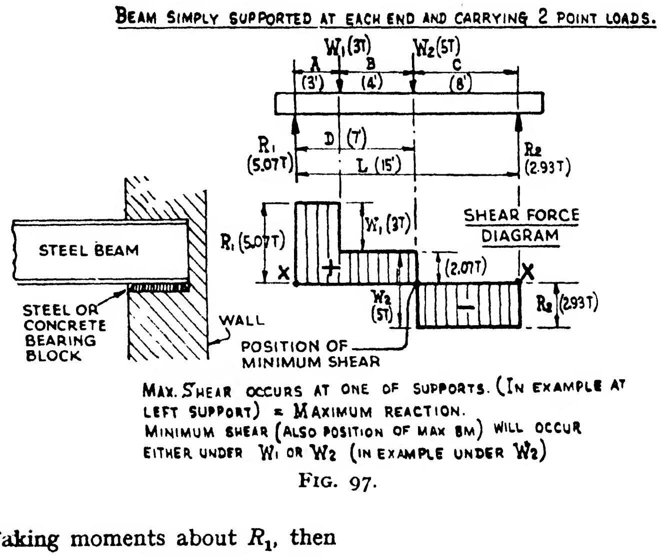


FIG. 97.

Taking moments about R_1, then

$$R_2 \times L = (W_1 \times A) + (W_2 \times D)$$

from which
$$R_2 = \frac{(W_1 \times A) + (W_2 \times D)}{L}$$

$$R_1 = W_1 + W_2 - R_2.$$

Starting from the left-hand support, commence the shear diagram by drawing upward line equal to R_1. Shear remains constant until W_1 is reached, so that a horizontal line is drawn from R_1 to W_1. At W_1 there is a downward load of W_1, and the shear is decreased on the diagram by this amount. Shear now remains constant until W_2 is reached, where load W_2 acts downwards. Shear is again constant until R_2 is reached, where there is an upward force equal in value to the reaction. Line XX joining the starting and finishing points will complete the diagram. The maximum shear occurs at one of the supports, whichever has the greater reaction. Minimum shear will occur under one of the loads.

QUESTION 5. A beam 15 ft. span carried two point loads, one of 3 tons at a distance of 3 ft. from the left-hand end of the beam, and one of 5 tons at 8 ft. from the right-hand end of beam. Find the value and position of maximum shear and position of maximum bending moment.

Answer. Taking moments about R_1, then

$$R_2 \times 15 \text{ ft.} = (3T \times 3 \text{ ft.}) + (5T \times 7 \text{ ft.})$$

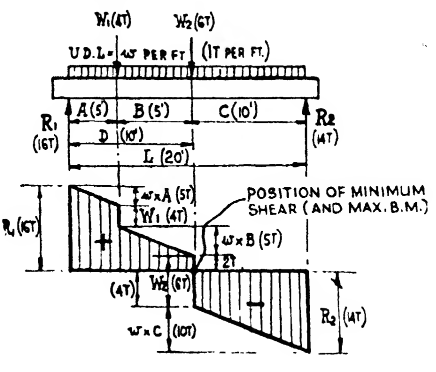
$$R_2 = \frac{9 + 35}{15} = 2{\cdot}93 \text{ tons.}$$

$$R_1 = 3T + 5T - 2{\cdot}93T = 5{\cdot}07 \text{ tons.}$$

Maximum shear = Value of greatest reaction = 5·07 tons. Therefore maximum shear occurs at left-hand support = 5·07 tons. To the right of W_1 this reaction is reduced by $W_1(3T)$ to $5{\cdot}07 - 3 = 2{\cdot}07$ tons. To the right of W_2 shear is equal to $R_2 = 2{\cdot}93$ tons. The part of the diagram above XX is positive and that below the line is negative, so minimum shear and maximum bending moment occurs under W_2.

Beam Simply Supported at Each End and carrying Two Point Loads and a U.D.L. over Entire Span. This is shown in Fig. 98. It is a combination of the types of loading shown in Figs. 96 and 97. The first step is to find the reactions. Having done so, begin the shear

BEAM SUPPORTED EACH END, CARRYING 2 POINT LOADS AND U.D. LOAD.



IN THIS TYPE OF LOADING MAXIMUM SHEAR OCCURS AT ONE OF THE SUPPORTS AND WILL EQUAL THE VALUE OF MAXIMUM REACTION.

FIG. 98.

diagram by drawing upward line R_1, equal in value to this reaction. From R_1 till W_1 is reached the shear is decreased by the U.D.L. acting downwards. Just as W_1 is reached the U.D.L. will have decreased the shear by $w \times A$, as shown by the diagonal line. At W_1 the shear is further decreased by the point load $- W_1$. From W_1 to W_2 is reached the U.D.L. decreases the shear by $w \times B$. A diagonal line from W_1 to W_2 represents this on the diagram. At W_2 the point load acts downwards. From W_2 to R_2 the U.D.L. produces a downward load starting at zero at W_2 and increasing to $w \times C$ at the support. At R_2 we have an upward force of R_2.

A horizontal line starting and finishing points will complete the diagram. Then the maximum shear will occur at the support which has the maximum reaction. The position of minimum shear will depend on the values of the loads, but can be found easily if the method described above is followed.

QUESTION 6. A beam 20 ft. span carries two point loads of 4 tons at a distance of 5 ft. from the left-hand end of the beam, and of 6 tons at 10 ft. from the right-hand end, and also a uniformly distributed load of 1 ton per foot over its entire length. Find the value and position of maximum shear and position of maximum bending.

Answer. First calculate the reactions. Take moments about R_1.

$$R_2 \times 20 \text{ ft.} = (4T \times 5 \text{ ft.}) + (6T \times 10 \text{ ft.}) + (1T \times 20 \text{ ft.} \times 10 \text{ ft.})$$

$$R_2 = \frac{20 + 60 + 200}{20} = \frac{280}{20} = 14 \text{ tons}$$

$$R_2 = \text{Total load} - R_2 = 4T + 6T + 20T - 14T = 16 \text{ tons.}$$

Maximum shear occurs at maximum reaction, that is, at left-hand support, and its value is 16 tons.

From R_1 to W_1 shear decreases by 5 ft. $\times$ 1 ton = 5 tons. At W_1 it is further decreased by a point load of 4 tons. Between W_1 and W_2 it is further decreased by 5 ft. $\times$ 1 ton = 5 tons. Just to the left of W_2 shear $= R_1 - (5T + 4T + 5T) = 16T - 14T = 2$ tons. W_2 provides a downward load of 6 tons, so that just to right of W_2 there is a shear of -4 tons.

Minimum shear occurs under W_2, and this will be the position at which the maximum bending moment occurs.

Beam Simply Supported at Each End and Carrying Four Point Loads. This is shown in Fig. 99. The shear diagram can be drawn in exactly the same manner as for Fig. 97, and the maximum shear occurs at one of the supports, whichever has the greater reaction. Minimum shear will occur under one of the loads.

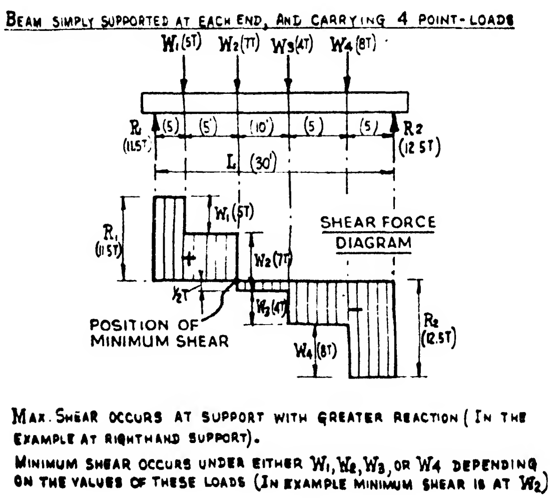


FIG. 99.

QUESTION 7. A beam 30 ft. span carries four point loads situated at 5 ft., 10 ft., 20 ft., and 25 ft., respectively, from the left-hand end. Find the position and value of maximum shear and position of maximum bending if the loads are 5 tons, 7 tons, 4 tons, and 8 tons, respectively.

Answer. Take moments about R_1, then

$$R_2 \times 30\text{ ft.} = (5T \times 5\text{ ft.}) + (7T \times 10\text{ ft}) + (4T \times 20\text{ ft.}) + (8T \times 25\text{ ft.})$$

$$R_2 = \frac{25 + 70 + 80 + 200}{30} = \frac{375}{30} = 12{\cdot}5 \text{ tons}$$

$$R_1 = \text{Total load} - R_2 = 24T - 12{\cdot}5T = 11{\cdot}5 \text{ tons.}$$

Maximum shear occurs at R_2, the right-hand support $= 12{\cdot}5$ tons. Reference to Fig. 15 shows that minimum shear occurs under W_2.

Beam Simply Supported at Each End and Carrying a U.D.L. over Part of Span only. This is shown in Fig. 100. First calculate the value of the reactions. For this purpose it can be assumed that the load acts through its centre of gravity. At R_1 there is an upward force equal to the reaction. From R_1 to the point where the load commences the shear is constant.

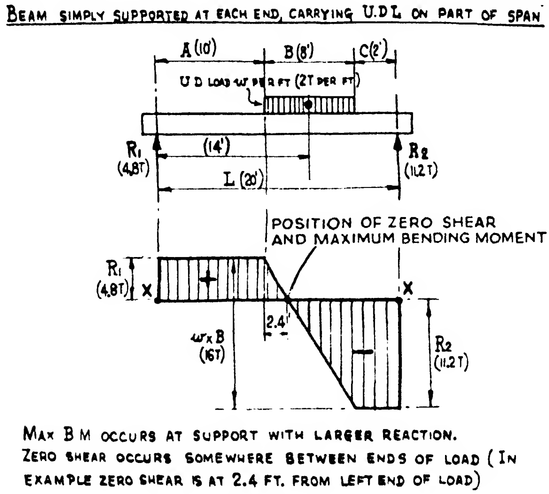


FIG. 100.

From the start to finish of the uniformly distributed load there is a downward force of w per foot run. The diagonal line in Fig. 16 shows the effect of this load. From the finish of the load to R_2 the shear is again constant, and represented by a horizontal line. At R_2 we have an upward force due to this reaction. Horizontal line XX joining starting and finishing points of the diagram complete it.

The part of the diagram above the line XX is positive and that below is negative. Maximum shear occurs at whichever support has the maximum reaction, while minimum shear will occur somewhere between the ends of the uniformly distributed load.

QUESTION 8. A beam 20 ft. span carries a uniformly distributed load of 2 tons per foot over 8 ft. of its length, the left-hand end of the load being at 10 ft. from the left-hand support. Find the position and magnitude of maximum shear and position of minimum shear.

Answer. Assuming that the load acts through its centre of gravity, then we have a total load of 2 tons × 8 ft. = 16 tons acting on a distance of 14 ft. from R_1.

Taking moments about R_1 we have

$$R_2 \times 20 \text{ ft.} = 16T \times 14 \text{ ft.}$$

$$R_2 = \frac{16 \times 14}{20} = 11{\cdot}2 \text{ tons}$$

$$R_2 = \text{Total load} - R_2 = 16T - 11{\cdot}2T = 4{\cdot}8 \text{ tons.}$$

Maximum shear occurs at the right-hand support R_2, and equals 11·2 tons. At the left-hand support R_1 = 4·8 tons. This shear remains constant until the left-hand end of the load is reached. It then decreases at the rate of 2 tons per foot, so that at 2·4 ft. from the left end of load the shear has decreased 2·4 × $2T$ = 4·8 tons, and its value is now zero. Therefore minimum shear occurs at a point 2·4 ft. from left end of load or 10 + 2·4 = 12·4 ft. from R_1.

The student should make up problems for himself on shear forces, and work them out, for by so doing he will soon become conversant with the methods employed in shear calculations.

In some cases the shear forces acting in a beam control the size of it. For instance, the following problem shows how necessary it is to check up and see that a beam is strong enough against failure in shear.

QUESTION 9. A beam is required to span over 8 ft. and to carry a uniformly distributed load of 1½ tons per foot run. Find a suitable section using a safe horizontal shear stress of 120 lb. per square inch, and a safe bending stress of 1,000 lb. per square inch.

Answer.

$$\text{Max. B.M.} = \frac{w \times L^2}{8} = \frac{1\tfrac{1}{2} \text{ tons} \times 8 \text{ ft.} \times 8 \text{ in.}}{8} = 12 \text{ ft.-tons}$$

$$= 12 \times 2{\cdot}240 \times 12 = 322{\cdot}500 \text{ in.-lb.}$$

Resisting moment of section = Max. B.M. = 322·500.

$$\text{Resisting moment of section} = \frac{B \times D^2}{6} \times \text{Safe bending stress.}$$

Therefore

$$\frac{B \times D^2}{6} \times 1{,}000 = 322{,}500$$

$$BD^2 = \frac{322{,}500 \times 6}{1{,}000} = 1{,}935.$$

If B, the breadth of the beam, is made two-thirds of the depth D, then

$$\tfrac{2}{3}D \times D^2 = 1{,}935$$

$$D^3 = 1{,}935 \times \tfrac{3}{2} = 2{,}900$$

$$D = \sqrt[3]{2{,}900} = \text{say, } 14 \text{ in.}$$

$$B = \tfrac{2}{3}D = \tfrac{2}{3} \times 14 = \text{say, } 10 \text{ in.}$$

A beam 10 in. wide and 14 in. deep is suitable as regards bending. We shall now check up for shear strength.

$$\text{Reaction} = \frac{\text{Load}}{2} = \text{Maximum shear} = \frac{8 \times 1\frac{1}{2}}{2} = 6 \text{ tons} = 13{,}440 \text{ lb.}$$

$$\begin{aligned} \text{Maximum shear stress} &= \frac{3}{2} \times \frac{\text{Total shear}}{B \times D} \\ &= \frac{3}{2} \times \frac{13{,}440}{10 \times 14} \\ &= 144 \text{ lb. per square inch.} \end{aligned}$$

Since the shear stress must not exceed 120 lb. per square inch, it is quite clear that the beam must be made of a larger section than 14 in. × 10 in., otherwise it is too weak in shear.

Maximum shear stress = 120 lb.

$$= \frac{3}{2} \times \frac{\text{Total shear}}{B \times D} = \frac{3}{2} \times \frac{13{,}440}{B \times D}$$

$$B \times D - \frac{3 \times 13{,}440}{2 \times 120} = 168.$$

If B is made 10 in., then

$$D = \frac{168}{10} = \text{say, } 17 \text{ in.}$$

Therefore, although a beam 14 in. deep and 10 in. wide would have been large enough to resist the stresses due to bending, it is necessary to use a beam 17 in. deep × 10 in. wide, because of the shear stresses. (This does not often arise in practice.)

CHAPTER 13

REINFORCED CONCRETE

We have previously dealt with the design of steel beams, both of the rolled joist type and the built-up type, and with timber beams. Beams can also be made of concrete, suitably reinforced with longitudinal bars of steel, which take the tensile stresses. Concrete by itself is strong in compression, being capable of safely carrying a stress of from 600 lb. to 800 lb. per square inch, depending on the quality of the mix.

Various types of mix, and their approximate compressive strengths, are given in a table on page 137. The figures 1–2–4 mean that the concrete is made of a mix of one part of cement, two parts of sand, and four parts of aggregate or broken stones. Similarly, a mix of 1–1½–3 means one part of cement to one and a half parts of sand, to three parts of aggregate.

In tension, concrete is very weak, and as any loaded beam will have compression stresses at one face and tensile stresses on the opposite face, the tensile side of the beam must be strengthened. This is done by running bars of steel along the beam near to the tension face. These steel rods can be stressed to from 16,000 lb. (about 7 tons) to 18,000 lb. (about 8 tons) per square inch with safety.

Central Load. In a beam simply supported at each end and loaded at the centre, the bottom face of the beam is in tension. The tensile stresses will vary from a maximum at the outside face to zero at the neutral axis. Therefore we need to have steel reinforcing near the lower face of the beam to take this tension. If the beam was a cantilever, then the upper face would be in tension and the lower face in compression. In this case we need the bars near the upper face of the beam.

It is generally assumed that the concrete resists compressive stresses only. The tensile strength of the concrete is small and is neglected, sufficient steel being put in to resist all tensile stresses.

Fig. 101 shows a water tank carried on rolled steel joists which rest on two concrete beams A. Assuming these to be simply supported at each end, the part of the beam above the neutral axis is in compression, while that below is in tension.

Strength of Reinforced Concrete Beams. When dealing with the elasticity of materials we found that materials stretch at different rates. For instance, a piece of rubber would stretch more under a load than a similar piece of mild steel would under the same load. We found (see Chapter 2) that within the elastic limit

$$E = \frac{\text{Stress}}{\text{Strain}}$$

where E is the modulus of elasticity of the material

$$\text{Strain is } \frac{\text{Alteration of length}}{\text{Original length}},$$

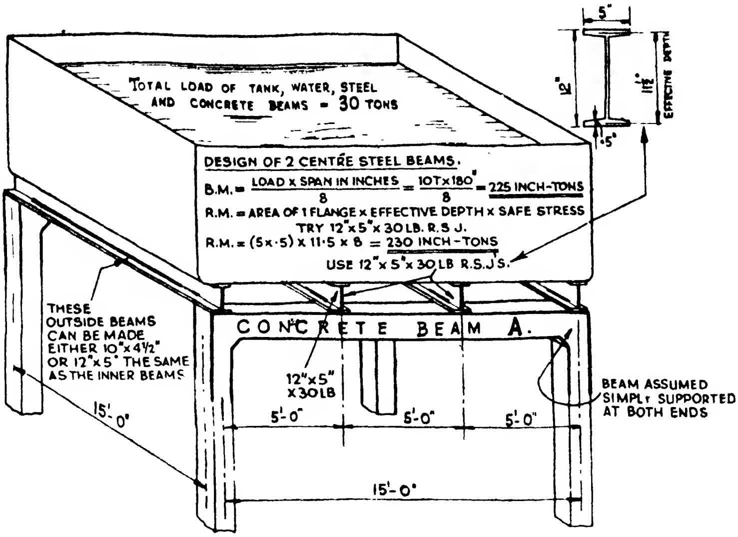


FIG. 101.

Concrete will stretch about 15 times as much as mild steel under a given load (within its elastic limit).

Then we can say that

$$\frac{E_s}{E_c} = 15 = m$$

where E_s is the modulus of elasticity of the steel

E_c is the modulus of elasticity of the concrete

m is the ratio between modulus of elasticity of steel and modulus of elasticity of concrete, and is called the modular ratio.

When a steel bar is embedded in solid concrete, the steel bar cannot stretch with the concrete round it stretching the same amount, and vice versa. Within the elastic limit, stress is proportional to strain, and on bodies of the same original length stress is proportional to stretch.

When Concrete Stretches. If a steel bar 8 ft. 6 in. long is stressed to 16,000 lb. per square inch of cross-sectional area it may stretch, say, $\frac{1}{20}$ in. Therefore the concrete surrounding that bar must stretch $\frac{1}{20}$ in. Under any given load the concrete by itself would stretch 15 times as much as the steel, but it can only now stretch as much as the steel, then the stress in the concrete can only be $\frac{1}{15}$th of that in the steel. If the steel is stressed to 16,000 lb. per square inch, the concrete immediately surrounding the steel will be stressed

$$\frac{16{,}000}{m} = \frac{16{,}000}{15} = 1{,}066{\cdot}6 \text{ lb. per square inch.}$$

Generally, 600 lb. per square inch is the safe stress allowed on concrete in compression. (750 lb. is sometimes used.)

The student is already aware that the maximum compressive stress in a beam occurs at the outside face, and gets less towards the neutral axis, where it becomes zero. From the neutral axis to the other face of the beam the stress increases, but instead of being compressive, it is now tensile. This was fully explained in Chapter 8. The stress distribution due to bending is shown in Fig. 102 (*a*). Notice that triangles *ABC* and *CDE* are similar.

It follows that $\dfrac{AC}{AE} = \dfrac{AB}{AB + DE}$

therefore $\dfrac{AC}{AE} = \dfrac{600}{600 + 1{,}066{\cdot}6} = 0{\cdot}36.$

But *AE* is the distance from the top of the beam to the reinforcing bars. We call this distance *d*, the effective depth of the beam, and the reader should be careful to remember it is not the total depth of the beam. The distance *AC* from the top of the beam to the neutral axis is denoted usually as ***n***.

$$\text{Therefore } \frac{n}{d} = 0{\cdot}36 \text{ or } n = 0{\cdot}36d.$$

This remains to say that the neutral axis is located at a distance of 0·36 of the effective depth below the top face of the beam. The stress in the concrete above the neutral axis varies from a maximum of 600 lb. at the outside fibres to zero at the neutral axis, so that the average stress will be

$$\text{Average stress} = \frac{600 + 0}{2} = 300 \text{ lb. per square inch.}$$

The cross-sectional area of the beam above the neutral axis

$$= B \times n$$
$$= B \times 0{\cdot}36d$$

where *B* is the breadth of the beam.

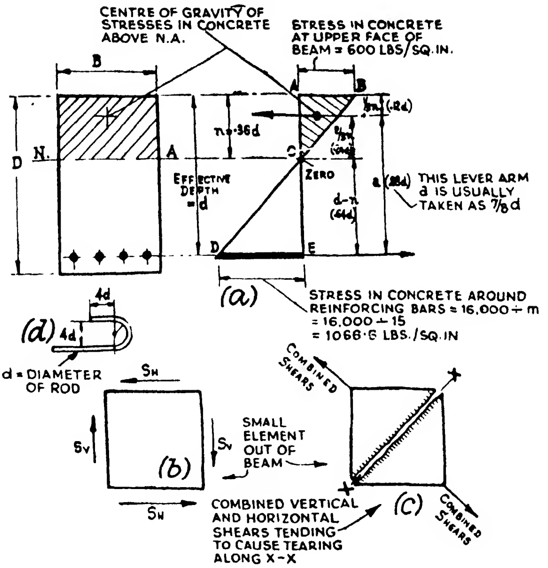


FIG. 102.

In the part of the beam above the neutral axis, we have therefore a total compressive force acting of

Compressive strength = Average stress × Area of compression side of beam

$$= 300 \times B \times 0{\cdot}36d \text{ lb.}$$

Now consider the tension side of the beam. If it is assumed that the concrete on this side takes none of the tensile stresses, the force acting will depend on the steel area.

$$\text{Tensile strength} = A_s \times 16{,}000 \text{ lb.}$$

where A_s is the cross-sectional area of the horizontal reinforcing bars.

In Chapters 8 and 9 we showed that the moment of resistance of a beam was

Moment of resistance = Force in one flange × Distance between the centres of gravity of the forces in the two flanges.

If we consider that the concrete above the neutral axis is one flange of a beam, and the steel bars below the neutral axis are the other flange, then we can apply the above formula.

The stress distribution in the concrete above the neutral axis is that of a triangle, so that it can be assumed that the force acts through the centre of this triangle, which will be at a distance of one-third the height of the triangle from the base line AB, that is, at a distance of $\frac{1}{3}n$ from the top face of the beam. The force in the steel reinforcing bars can be assumed to act through their centre.

The distance from the centre of gravity of the compressive force to the neutral axis $= n - \frac{1}{3}n = \frac{2}{3}n$, which in our case will be $\frac{2}{3}$ of $0{\cdot}36d = 0{\cdot}24d$. The distance from neutral axis to centre of steel bars is the difference between d and n, in our case $d - 0{\cdot}36d = 0{\cdot}64d$. Then the distance from centre of compressive force to centre of gravity of tensile force $= 0{\cdot}24d + 0{\cdot}64d = 0{\cdot}88d$. This is known as the lever arm of the beam, and generally it is taken as $\frac{7}{8} \times d$.

If the steel is stressed to 16,000 lb. per square inch, and the concrete to 600 lb. per square inch, then

$$\begin{aligned}\text{Resisting moment of concrete} &= \text{Compressive force} \times \text{Lever arm} \\ &= 300 \times B \times 0{\cdot}36d \times 0{\cdot}88d \\ &= 95Bd^2 \text{ in.-lb.}\end{aligned}$$

If we equate this to the actual bending moment on the beam we can find the breadth and depth of a suitable beam to withstand this bending moment.

$$\begin{aligned}\text{Resisting moment of steel} &= \text{Tensile force} \times \text{Lever arm} \\ &= A_s \times 16{,}000 \times 0{\cdot}88d \\ &= 14{,}080\ A_s \times d.\end{aligned}$$

Therefore, if we know the bending moment on a beam and a suitable depth d, we can calculate the necessary cross-sectional area of reinforcing bars required to take care of the tensile stresses in the beam.

Equating these formulae we can find the area of steel required.

$$14{,}080 \times A_s \times d = 95 \times B \times d^2$$

$$A_s = \frac{95 \times B \times d^2}{14{,}080 \times d} = 0{\cdot}00675Bd.$$

Let p = the economic percentage of reinforcement in the beam with regard to the cross-sectional area $B \times d$, then

$$p = \frac{0{\cdot}00675 \times B \times d}{B \times d} \times 100 = 0{\cdot}675 \text{ per cent.}$$

This means that if we had a concrete beam 10 in. wide and with an effective depth of 20 in., giving an area of 200 sq. in., the percentage of steel would be 0·675 per cent, and the area of steel would be 0·675 per cent of area of beam

$$= \frac{0{\cdot}675}{100} \times 200 = 1{\cdot}35 \text{ sq. in.}$$

By varying the allowable stresses in steel and concrete, the position of the neutral axis will be altered, and this will alter the values of the effective lever arm (denoted by a) and the value of the resisting moments. The table with Fig. 103 gives values for n, a, p, and the resisting moment of concrete, using various safe stresses for the steel and concrete.

Shear Stresses. In addition to the bending stresses, there are also horizontal and vertical shear stresses in the beam. The effective cross-sectional area of concrete which resists these shear stresses is breadth of beam × lever arm a. The proof of this is as follows.

Fig. 103 (a) shows the bending and shear stresses on a small length of a beam of the type we are considering. C and C_1 denote

FACTORS FOR REINFORCED CONCRETE BEAM DESIGN

SAFE STRESS IN STEEL LBS/SQ IN	SAFE COMPRESSION IN CONCRETE LBS/SQ IN.	SAFE SHEAR STRESS IN CONCRETE LBS/SQ IN	VALUE OF n	VALUE OF m	VALUE OF a	ECONOMIC PERCENT. OF STEEL	R.M. OF CONCRETE	TYPE OF CONCRETE MIX :-
16,000	600	60	·36d	15	·88d	·675 %	95 Bd²	1 : 2 : 4.
	700	70	·40d	15	·87d	·875 %	121 Bd²	1 : 2 : 4.
	800	80	·43d	15	·86d	1.075 %	147 Bd²	1 : 1½ : 3.
18,000	600	60	·33d	15	·89d	.555 %	89 Bd²	1 : 2 : 4.
	700	70	·37d	15	·88d	.720 %	113 Bd²	1 : 2 : 4.
	800	80	·40d	15	·87d	.890 %	139 Bd²	1 : 1½ : 3.

THE RESISTING MOMENT OF THE STEEL IS EQUAL TO THE RESISTING MOMENT OF CONCRETE

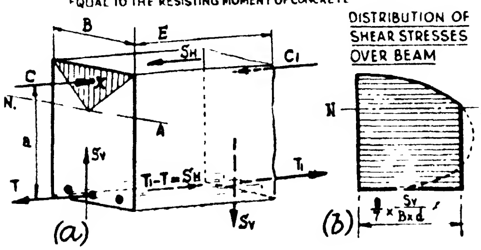


FIG. 103.

the compressive forces acting in the compression side of the beam, and T and T_1 the tensile forces acting in the steel reinforcing bars It should be noted that as the bending moment varies along the beam, then if the beam is the same depth all the way along, then the stresses due to bending will vary along the span.

S_v and S_h are the shear stresses in the beam. Then the horizontal shear acting just above the reinforcing bars will be $T_1 - T$. This shear acts on a horizontal area of length of small piece of beam × breadth of beam $= E \times B$. Then the unit shear stress at any place between the neutral axis and the steel

$$= \frac{T_1 - T}{E \times B}.$$

Above the neutral axis the shear will diminish to zero at the outside face.

If this piece of beam is to be in equilibrium, then the sum of the moments about any point will equal zero. Take moments about X.

Then $$(T_1 - T) \times a = S_v \times E$$

or $$T_1 - T = \frac{S_v \times E}{a}$$

where a is the lever arm of the beam.

Substitute this in the first formula, and

$$\text{Unit shear stress} = \frac{S_v}{a \times B}$$

or

Shear force = Unit shear stress × Breadth of beam × Lever arm.

The effective area of the beam to resist shear is therefore breadth of beam multiplied by lever arm.

The lever arm is approximately $\frac{7}{8}d$, so that

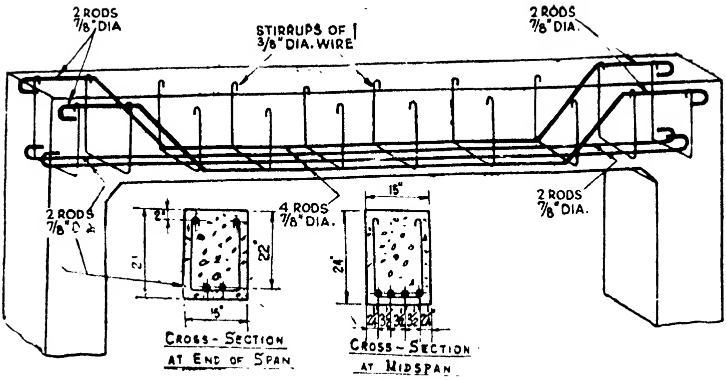
$$\text{Unit shear stress} = \frac{S_v}{\frac{7}{8}d \times B} = \frac{8}{7} \times \frac{S_v}{B \times d}$$

where S_v is the shear force of the section considered
d is the effective depth of the beam
B is the breadth of the beam.

A small element of the beam shown in Fig. 105 (*b*) is enlarged in Fig. 102 (*b*). Here we have the horizontal and vertical shearing forces acting on it. These forces combined will tend to crack the concrete along line XX (Fig. 102 (*c*)). Unless the concrete itself is strong enough to resist this tension, steel reinforcing must be provided, otherwise the beam will crack along this sloping plane, due to shear stresses (see Fig. 18, Chapter 3).

To prevent this failure occurring, vertical steel stirrups are inserted in the beam as shown in Fig. 104. These bars act in the same manner as the stiffeners in plate girders. The main reinforcing bars are turned up at about 45 deg. and they help to resist failure due to the diagonal tension caused by shear.

Design of Concrete Beams under Water Tank. We now proceed to design the concrete beams A shown in Fig. 101. As the load, including the weight of tank, water, steel, and concrete beams, is 30 tons, each beam A will carry one-half of this, that is, 15 tons. This load will be transmitted to the concrete beam by rolled steel joists, which are placed at 5-ft. centres. The two centre joists will each carry 10 tons, and the two outer joists 5 tons. Each concrete beam takes half of these loads, as shown in



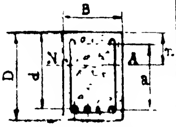
	NOTE. f_c = STRESS PER SQUARE INCH IN CONCRETE, f_s = " " " " STEEL.	ACCURATE FORMULA	APPROX. FORMULA.
	DISTANCE FROM TOP OF BEAM TO NEUTRAL AXIS = n =	$\left[\frac{m \times f_c}{f_s + (f_c \times m)}\right] d$	$\frac{3}{8} d$
	LEVER ARM . . . . = a =	$d - \frac{1}{3}n$	$\frac{7}{8} d$
	RESISTING MOMENT OF STEEL =	$f_s \times .01p \times a \times Bd.$	$f_s \times A_s \times \frac{7}{8} d$
	RESISTING MOMENT OF CONCRETE	$\frac{1}{2} f_c \times B \times n \times a$	$\frac{1}{6} Bd^2 \times f_c$
COVER OF CONCRETE OUTSIDE BARS = 1½ IN MINIMUM	AREA OF REINFORCING STEEL = A_s =	$\frac{R.M}{f_s \times a} = \frac{B.M}{f_s \times a}$	$\frac{BM}{f_s \times \frac{7}{8} d}$
	PERCENTAGE OF STEEL = p =	$\frac{A_s \times 100}{B \times d} = \frac{BM \times 100}{f_s \times a \times B \times d}$	$20 \times \frac{f_c}{f_s}$

FIG. 104.

Fig. 105 (*a*). As the load is symmetrically distributed over the beam, each concrete post will exert an upward reaction of $7\frac{1}{2}$ tons.

Note that at the supports we have an upward force of $7\frac{1}{2}$ tons and a downward load of $2\frac{1}{2}$ tons. The $2\frac{1}{2}$ tons load in the end steel beams will pass straight through the concrete beam and down the concrete post. We can therefore consider the loading on the concrete beam to be as shown in Fig. 105 (*b*).

The maximum bending moment occurs under the 5-ton loads. Taking moments about the left 5-ton load (point X) we have

$$\text{Max. B.M.} = \text{Reaction} \times 5 \text{ ft.} = 5T \times 5 \text{ ft.}$$

$$= 25 \text{ ft.-tons} = 25 \times 2{\cdot}240 \times 12 = 672{,}000 \text{ in.-lb.}$$

The complete bending moment diagram is shown shaded in Fig. 105 (*c*). We have to design a beam strong enough to provide a resisting moment equal to (or greater than) this bending moment.

Where compressive stress in concrete = 600 lb. per square inch, and tensile stress in steel = 16,000 lb. per square inch,

$$95Bd^2 = \text{resisting moment of concrete}$$
$$= \text{bending moment}$$
$$= 672{,}000$$

$$Bd^2 = \frac{672{,}000}{95} = 7{\cdot}070.$$

The depth of a beam should not be less than $\frac{1}{20}$th of the span, and the breadth is usually between $\frac{1}{2}$ and $\frac{3}{4}$ of the depth. Try a beam 15 in. wide, then

$$15d^2 = 7{,}070$$
$$d^2 = \frac{7{,}070}{15} = 475$$
$$d = \sqrt{475} = \text{say, } 22 \text{ in.}$$

Allowing 2 in. extra for covering the bars, the total depth D of the beam will be 24 in.

Economical percentage of steel = 0·675 per cent.

$$\text{Economical area of steel} = \frac{0{\cdot}675}{100} \times B \times d$$
$$= \frac{0{\cdot}675}{100} \times 15 \times 22 = 2{\cdot}2 \text{ sq. in.}$$

A $\frac{7}{8}$-in. diameter round bar has an area of 0·6 sq. in. Four of these will give an area of 2·4 sq. in., which is slightly more than the required area of 2·2 sq. in. These four bars will be arranged as shown in Figs. 104 and 105 (*d*).

Plate Girders. It will be remembered when dealing with plate girders that the flange plates were cut short to suit the bending moment. In a concrete beam the full number of reinforcing bars is not required to run the full length of the beam, as the bending moment decreases towards the end of the span. Instead of cutting some of the bars short it is usual to turn them up at an angle of about 45 deg. and continue them up to the top of the beam. By doing this the inclined part of the rods will take care of diagonal tension due to shear in the beam. We can in this case turn up two of the $\frac{7}{8}$-in. bars shown in Fig. 104.

The maximum bending moment on the beam occurs at one of the 5-ton loads, and is 672,000 in.-lb. (see Fig. 105 (*c*)). It requires 2·2 sq. in. of reinforcing to resist this. Four $\frac{7}{8}$-in. diameter bars have an area of 2·4 sq. in., and two have an area of 1·2 sq. in.; 2·2 sq. in. of steel will resist 672,000 in.-lb., so that 1·2 sq. in. will resist $\frac{1{\cdot}2}{2{\cdot}2} \times 672{,}000 = 367{,}000$ in.-lb. Therefore, when the bending moment falls below 367,000 in.-lb., it is only necessary to have two reinforcing bars near the tension face of the beam. The bending moment varies directly from zero at the support to 672,000 at the 5-ton load, and their distance is 5 ft. or 60 in. Therefore the bending moment will be 367,000 in.-lb. at a distance of $\frac{367{,}000}{672{,}000} \times 60$ in. = say at 33 in. from the support.

We can turn up two of the reinforcing bars as shown in Figs. 104 and 105 (*d*). As the inclined length of steel will be assumed to take tensile stresses, then its resisting moment will have to

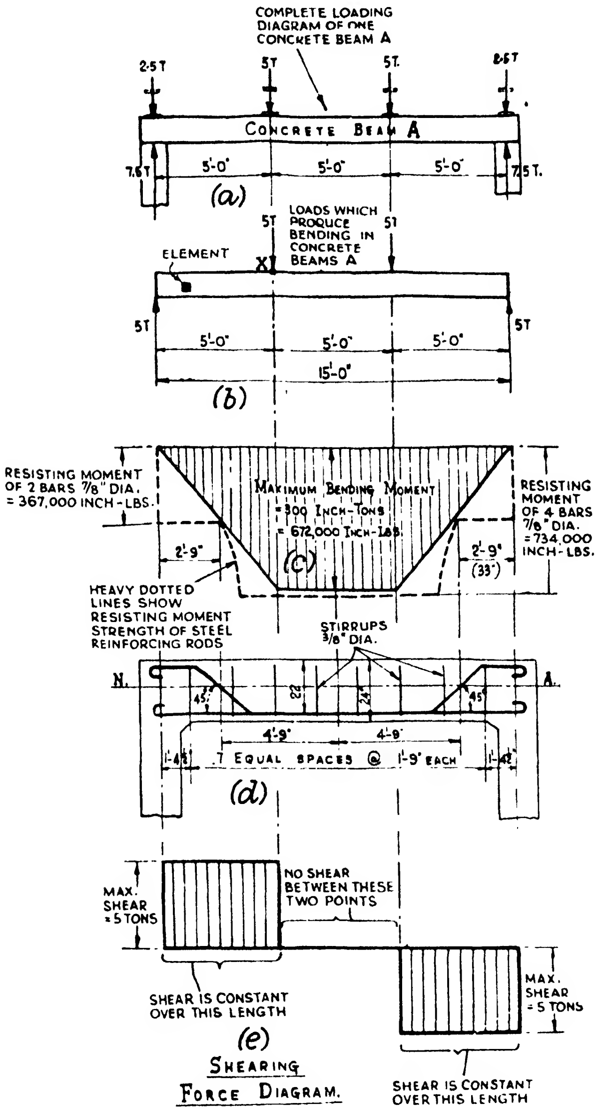


FIG. 105.

be taken into account until the neutral axis is passed. This is shown in the resisting moment diagram (see Fig. 105 (*c*)).

Fig. 105 (*e*) shows the shearing-force diagram. Between the loads, there is no shear on the beam, and it is constant at 5 tons from support to the load.

It is necessary to check up to see if the safe unit shear stress of about 60 lb. per square inch in this case has been exceeded, and if any stirrups are required.

$$\text{Max. shear stress} = \frac{8}{7} \times \frac{\text{Max. shear}}{B \times d}$$

$$= \frac{8}{7} \times \frac{5T \times 2{,}240}{15 \times 22} = 39 \text{ lb. per square inch.}$$

There, theoretically, no stirrups are required to resist shear. In practice, however, it may be advisable to put these in to stop the main reinforcing bars from sagging, and the spacing between these stirrups should not exceed a distance equal to the effective depth of the beam. Fig. 105 (*d*) shows a suitable spacing of about 21 in. The diameter of the stirrups will be made $\frac{3}{8}$ in. The ends of the main reinforcing bars should be hooked to prevent the bars from pulling out of the concrete. The general proportions of these hooks is given in Fig. 102 (*d*).

Design of Steel Beam. In Fig. 101 the tank is shown resting on steel joists. We found that the two outer joists carried a load of 5 tons each, the two outer ones 10 tons each. The depth of steel joists should be at least $\frac{1}{24}$th of the span or $\frac{1}{24}$th of

$$15 \text{ ft.} = \frac{15 \times 2}{24} = 7\tfrac{1}{2} \text{ in.}$$

The breadth should be at least $\frac{1}{40}$th of the span, or

$$\frac{15 \times 12}{40} = 4\tfrac{1}{2} \text{ in.}$$

Therefore we require beams at least $4\frac{1}{2}$ in. wide and $7\frac{1}{2}$ in. deep.

Inner Joists.

$$\text{Max. B.M.} = \frac{W \times L}{8} = \frac{10T \times 15 \times 12}{8} = 225 \text{ in.-tons.}$$

Resisting moment of joist = Area of one flange
$\times$ Effective depth $\times$ Safe stress
= Modulus of section $\times$ Safe stress.

Try a 12-in. $\times$ 5-in. $\times$ 30-lb. R.S.J. with a flange thickness of $\frac{1}{2}$ in.

Take safe stress at 8 tons per square inch.

Resisting moment of 12-in. $\times$ 5-in. R.S.J.

$$= 5 \text{ in.} \times \tfrac{1}{2} \text{ in.} \times 11\tfrac{1}{2} \text{ in.} \times 8T$$
$$= 28{\cdot}75 \times 8$$
$$= 230 \text{ in.-tons.}$$

As the maximum bending moment is 225 in.-tons this section is suitable for the inner-joists.

Outer Joists. The smallest standard section with depth and breadth dimensions of over $7\frac{1}{2}$ in. $\times$ $4\frac{1}{2}$ in. is the 10-in. $\times$ $4\frac{1}{2}$-in. $\times$ 25-lb. R.S.J. with a flange thickness of $\frac{1}{2}$ in. Try this section for the outer joists.

Maximum bending moment will be half that on the inner joists or

$$\frac{225}{2} = 112{\cdot}5 \text{ in.-tons.}$$

A 10-in $\times$ $4\frac{1}{2}$-in. joist will have a section modulus of $4\frac{1}{2}$ in. $\times$ $\frac{1}{2}$ in. $\times$ $9\frac{1}{2}$ in. $= 21{\cdot}4$ in.^{3} units.

$$\text{Resisting moment of 10-in.} \times 4\tfrac{1}{2}\text{-in. joist} = \text{Section modulus} \times f_s$$
$$= 21{\cdot}4 \times 8 = 171 \text{ in.-tons.}$$

This section is strong enough for the outside beams. A pad could be made on the concrete frame so as to make the top side of all four beams level.

Note.—The reinforced concrete beam has been assumed to be only freely supported at each end, so as to show the design of a simple beam. In reinforced concrete frames the beam ends are fixed and the posts and beams are designed as a complete frame. It would, of course, be possible to make the concrete beams and the concrete posts separate from each other, in which case the conditions assumed for the problem would be correct.

ROOF TRUSSES

The built-up frames which carry the covering over buildings are called roof trusses, or roof principles. Fig. 106 shows a

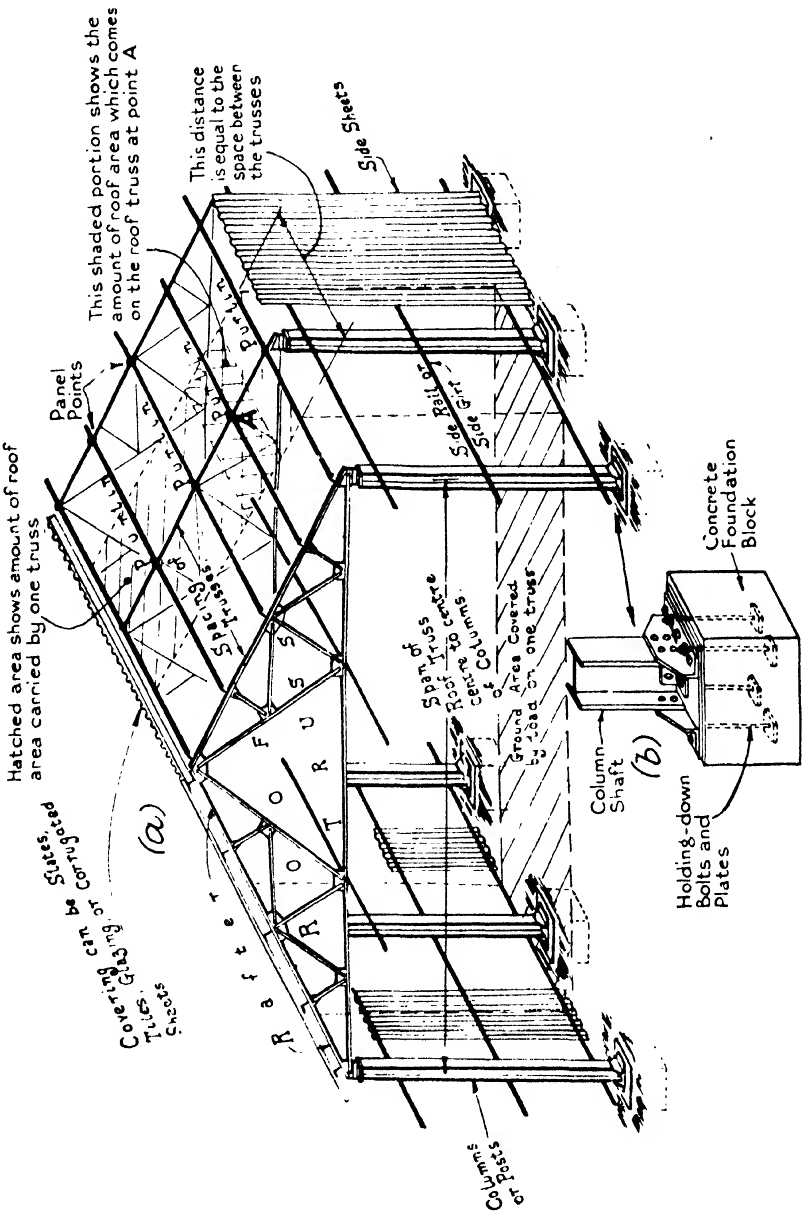


FIG. 106.

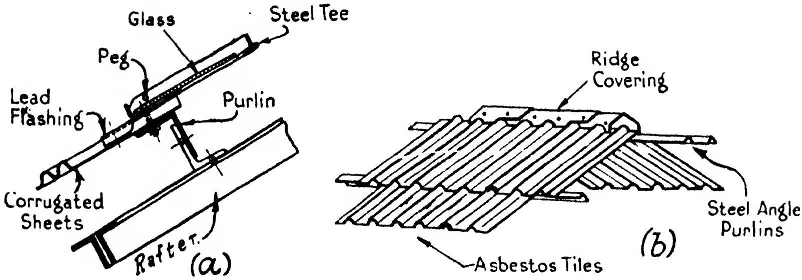


FIG. 107.

general view of a steel shed, and the roof truss shown is a very common type where the span is between 40 ft. and 60 ft. This truss is known as the Fink type, and is one of the commonest forms of steel truss.

The covering for these roof trusses can be of various materials, either slates or boards, slates on laths, tiles, or corrugated sheets made of asbestos or steel. Purlins between these coverings and the roof trusses run lengthways of the building, and are made of wood, rolled steel angles, channels, or joists.

Fig. 107 (*a*) shows a detail quite commonly used where glazing covers part of the roof and corrugated sheets the remainder. The purlins transmit the load to the roof rafters. On each rafter there is a small angle called a cleat, to which the ends of the purlins are connected. Various forms of these cleats are shown in Figs. 110, 111, and 112.

The glazing is carried on steel tee-bars generally $1\frac{1}{2}$ in. by $1\frac{1}{2}$ in. by $\frac{1}{4}$ in., and to prevent the glazing from sliding off the end, each tee-bar is slightly turned up. A hard-wood peg is generally put in the vertical leg of this tee astragal to keep the glazing from jumping up. The joint is made tight by lead flashing, which runs along the underside of the glazing and along the top side of the corrugated sheets. When used in chemical

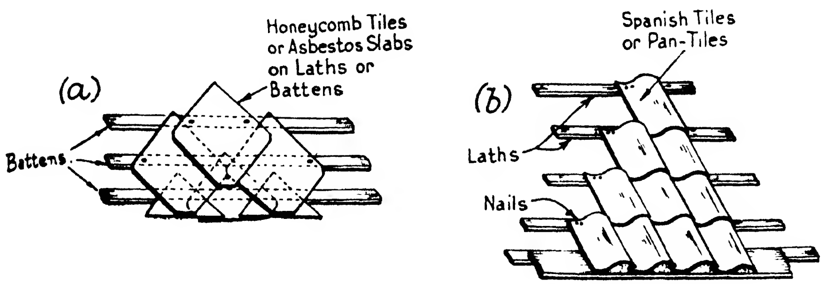


FIG. 108.

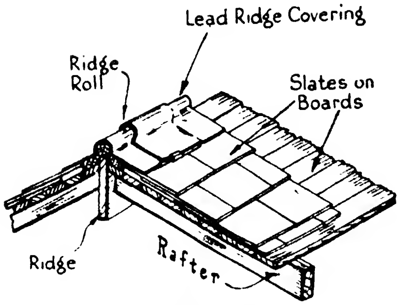


FIG. 109.

or metallurgical plants where there would be a good deal of corrosion, various brands of specially protected corrugated sheets are used.

Fig. 108 (*b*) shows a type of roof covering which is now being largely used on house construction and cinema buildings. These Spanish or pan tiles are made in various colours and have a very good appearance. The tiles can, of course, be laid on boards instead of laths if a better job is desired.

In Figs. 107 (*b*), 108 (*a*), 109, 110 and 111 are shown other forms of covering which call for no special comment.

Fig. 112 shows a picture view of a good detail for a steel roof shoe and the method of making the connection to the top of a steel column.

Loads on Roof Trusses. These fall into two general categories: (1) dead loads, and (2) live loads.

The dead loads consist of the roof covering—slates, sheets, tiles, etc.—the purlins, and the roof truss itself. The live load consists of the wind. In countries where there is a heavy snow load, this must also be taken into account, but in English practice it is often assumed that if a wind load is allowed for the snow load can be neglected.

In industrial construction, runway-beams are often fastened to the underside of the roof truss, along which run pulley blocks and tackle for lifting weights. These must also be considered

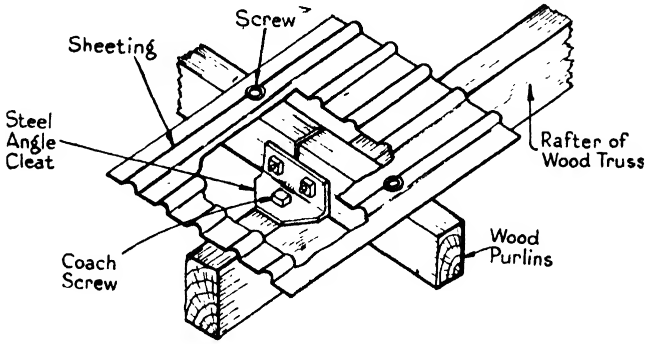


FIG. 110.

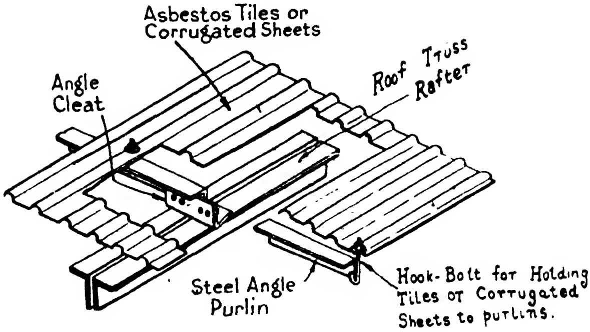


FIG. 111.

as live loads, and it is very important that their effect should not be forgotten when considering the strength of steel roofs.

Two Methods. There are two methods of considering dead loads: (1) The load per square foot of ground area covered: (2) The load per square foot of inclined surface.

The ground area covered is the width of the building multiplied by its length. Thus, if the distance between the side walls is 40 ft., and the distance between the end walls 80 ft., the ground area covered is 40 ft. by 80 ft., which amounts to 3,200 sq. ft.

Although the length of the roof covering in this case would still remain as 80 ft., the roof surface would be considerably more, as there are two sloping sides. The length of each sloping side may be 25 ft., in which case the total area of the covering would be 2 by 25 by 80, which amounts to 4,000 sq. ft. This makes clear the difference between area of the roof covering, and the ground area thus covered.

Wherever possible, the actual weights of the roof coverings should be used, but if actual weights are not available, the table

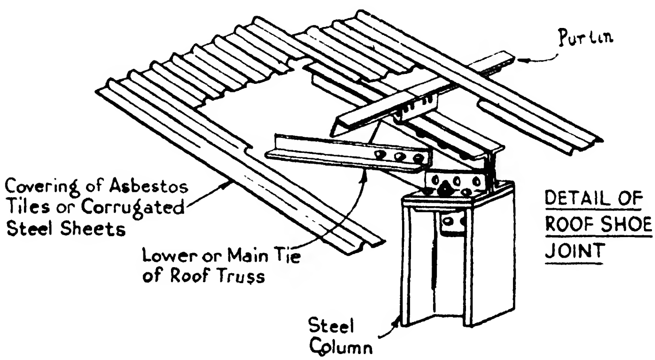


FIG. 112.

given below can be used, to give a general idea of what roof coverings weigh. In each case, the weight is per square foot of roof covering.

TABLE I

Material.	Approx. Weight in lb. per square foot
Galvanized corrugated Sheets	3
Slates on timber laths	9
Slates on boards	11
Tiles	10
Glazing	6
Steel Purlins	3

The table below will give a guide to the approximate weights of steel roof trusses, but it must be remembered that the table is only intended to serve as a guide, and that the reader should make careful notes of the actual weights of roof trusses whenever he can.

TABLE II

APPROXIMATE WEIGHT OF STEEL ROOF TRUSSES

Span of Truss in feet.	Centre of Trusses, in feet	Approx. Weight in cwt.
35	10	8
40	10	10
45	12	12
50	12	16
55	15	20
60	15	22

Generally no allowance is made for snow load in England, because it is assumed that when a high wind is blowing no snow will remain on the roof.

Although the speed of wind seldom exceeds 60 m.p.h., it sometimes reaches 80 or 90 m.p.h. when a gale is blowing. Many experiments have been made to find the pressure of the air which would result on a vertical surface in terms of speed or velocity. It may appear that if the wind is blowing against a vertical bill-boarding at a speed of 40 m.p.h., it will be pushing with half as much pressure as would result if the wind became a gale of 80 m.p.h. This is not so. The pressure varies as some constant multiplied by the velocity squared, and can be written :

$$P = C \times V^2$$

where P is the pressure of the air on each square foot of vertical surface, due to a horizontal wind pressure.

C is a constant (some authorities say this should be 0·0032 and others 0·004. The average seems to be 0·0032).

V is the velocity of the wind in miles per hour.

Example. What would be the pressure per square foot on a vertical surface resulting from a wind blowing at a speed of 90 m.p.h.?

Answer. If constant is 0·0032, then

$$P = 0{\cdot}0032 \times 90 \times 90 = 26 \text{ lb. per square foot.}$$

If constant is 0·004, then

$$P = 0{\cdot}004 \times 90 \times 90 = 32 \text{ lb. per square foot.}$$

In many cases roofs are designed to resist a horizontal wind pressure of 30 lb. per square foot.

Generally, wind is assumed to blow horizontally, and as the sides of the roof are nearly always inclined, we must consider the effect of this.

Assume a piece of timber 5 ft. square is hinged at one end (Fig. 113) and laid flat on the ground, then the wind would blow right over this, and have very little effect on it. If it is raised,

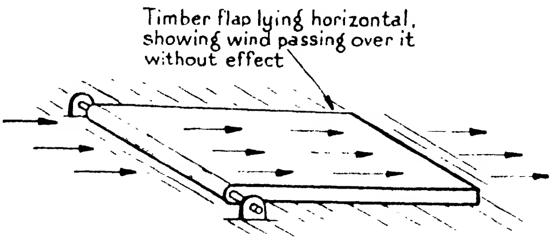


Fig. 113.

as shown in Fig. 114 (*a*), and the wind blows against it, it will certainly fall down flat again unless it is propped up. A newspaper held along the top side of the timber would show that part of the wind was sliding up the timber. It is also easy to see that part of the wind tries to force the board down to the ground, and is only prevented from doing so by the inclined prop. We see the wind has two separate and distinct forces, one parallel to the board and the other perpendicular or at right angles to the board. This last force, which passes right down the inclined post, is called the normal component of the wind force.

Many experiments have been carried out to find how much of the wind force slides up the incline, and how much tries to push at right angles to the timber. Most of the formulae for finding the normal wind pressure are complicated. If the horizontal wind pressure is around 30 lb. per square foot, there is a simple formula which gives satisfactory results. The formula is:

$$\text{Normal wind pressure} = \frac{2 \times A}{\ }$$

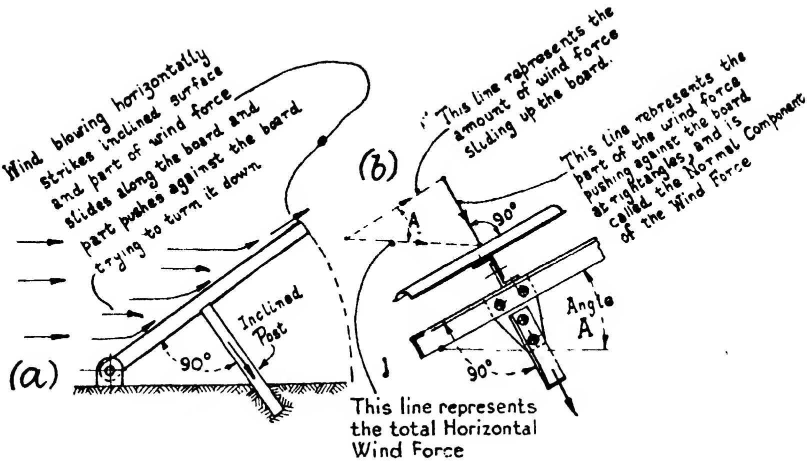


FIG. 114.

where A is the angle which the rafter of the roof truss makes with the horizontal. The following table gives the approximate normal wind pressure when the horizontal wind pressure is 30 lb. per square foot.

Angle A, degrees	Approx. Normal Wind Pressure per square foot of Roof Area
20	13
25	17
30	20
35	23
40	30

Pitch or Slope of Roof. This is given in two forms: (1) The ratio of the rise of the roof to the span: (2) As an angle of inclination between the horizontal and the rafter.

There are two very common slopes for steel roof trusses, and probably at least 80 per cent of steel roof trusses are made to one or other of these. They are:

$$(a)\ \text{Rise} = \frac{\text{Span}}{4}.$$

$$(b)\ \text{Inclination in degrees} = 30.$$

In house construction and other special construction, such as cinemas or church roofs, the angle of slope is often made 45 degrees.

Combined Dead and Wind Loads. We have shown that a roof truss must be strong enough to carry (1) Dead load of the covering, purlins, and its own weight: (2) The effect of the wind load.

It is shown in Figs. 113 to 114 how the horizontal wind pressure can be split up into two forces, one of which slides up the sloping face of the roof, and the other pushes normal or at right angles to the roof.

"Normal Force." To proceed a step further, we can consider that the proportion of the wind which slides up the roof is of no material importance, and that we are left with the force which acts normal on to the roof, as shown in Fig. 115 (*a*).

This force has two effects; one to try and push the board horizontally, and the other to push it down vertically. Fig. 115 (*b*) shows the effect of the horizontal force which pushes the roller along the ground, and Fig. 115 (*c*) shows the vertical force which pushes the roller vertically into a pit or cavity. These forces are generally represented on drawings by lines, as shown in Fig. 115 (*d*). The horizontal force illustrated in Fig. 115 (*b*) is called the horizontal component of the normal wind pressure, and the vertical force represented in Fig. 115 (*c*) is called the vertical component of the normal wind pressure.

The horizontal wind force pushing against the side of the building or shed, and the

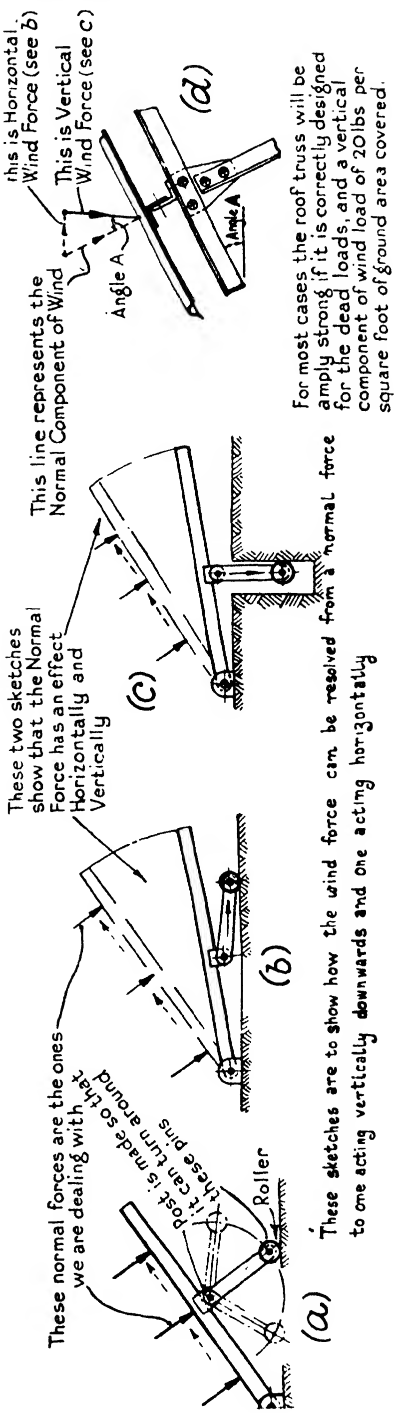


FIG. 115.

horizontal component of the wind force on the roof, try to push the building over, and the columns themselves and the fixing at the base must be strong enough to resist this wind force. The vertical component of the wind force on the roof tries to buckle up or distort the members of the framework called the roof truss.

Generally, when designing new trusses, the dead load of the covering and the normal wind load only are used, but it is possible to design a roof truss which will be quite satisfactory by considering the weight of the covering, purlins, roof truss, and the vertical component of the wind force only.

A perfectly satisfactory design will be obtained if it is assumed that the total of all loads amounts to 40 lb. per square foot of ground area covered. This 40 lb. might roughly be split up into 20 lb. for the roof, covering, and purlins, and 20 lb. for the vertical component of the wind pressure.

Fig. 106 shows how each roof truss in the centre part of the building carries a roof area of the length of the two rafters multiplied by the spacing between the trusses. The ground area covered by this area of roofing is the span of the truss multiplied by the spacing between the trusses.

QUESTION. What is the roof area carried by a roof truss 60 ft. span if the spacing between the trusses is 15 ft.? What is the ground area covered? The trusses have rafters inclined to 30 deg. to the horizontal.

Answer.

$$\text{Length of one rafter} = \frac{\tfrac{1}{2}\text{ span of truss}}{\text{cosine of angle}} = \frac{30\text{ ft.}}{\cos 30\text{ deg.}} = \frac{30\text{ ft.}}{0{\cdot}866} = 34\text{ ft. }7\tfrac{1}{2}\text{ in.}$$

$$\text{Roof area} = 2 \times \text{Length of rafter} \times \text{Spacing between trusses} = 2 \times 34\text{ ft. }7\tfrac{1}{2}\text{ in.} \times 15\text{ ft.} = 1{,}040\text{ sq. ft.}$$

$$\text{Ground area covered} = \text{Span of truss} \times \text{spacing between trusses} = 60\text{ ft.} \times 15\text{ ft.} = 900\text{ sq. ft.}$$

QUESTION. If the horizontal wind pressure on a 60-ft. span with trusses spaced at 15-ft. centres is 30 lb. per square foot, what is the total vertical wind load on one truss causing distortion of its members? The rafters of the truss make an angle of 30 deg. to the horizontal.

Answer. From the table previously given a horizontal wind pressure of 30 lb. per square foot will produce a normal wind load of 20 lb. per square foot of roof area covered. Only the vertical part of this normal wind load is assumed to cause distortion in the truss members. Consideration of Fig. 115 (*d*) will show that the angle between the normal wind load and the vertical component is the same as the angle the rafter makes to the horizontal, in this case 30 deg. We therefore say

Vertical wind component = Normal wind pressure $\times$ cosine of angle A
$= 20$ lb. $\times \cos 30$ deg.
$= 20 \times 0{\cdot}866 = 17{\cdot}3$ lb. per square foot of roof area covered.

In the previous question we found that roof area $-$ 1,040 sq. ft. From which

Total vertical wind load on one truss

$$= \text{Roof area} \times \text{Vertical wind pressure}$$
$$= 1{,}040 \times 17{\cdot}3$$
$$= 18{,}000 \text{ lb.}$$

We previously found that the ground area covered by the load on one truss was 900 sq. ft. Therefore we can say

Vertical wind pressure per square foot of ground area covered

$$= \frac{\text{Total vertical wind load on truss}}{\text{Ground area covered}}$$
$$= \frac{18{,}000 \text{ lb.}}{900} = 20 \text{ lb.}$$

This is the same figure as the normal wind force per square foot of roof area covered, and it is a good deal easier to work on ground area than roof area covered. To find the vertical wind load on a truss all we have to do is to multiply the vertical wind pressure per square foot of ground area covered (which is exactly the same figure as the normal wind pressure per square foot of roof area covered and can be obtained from the table given previously) by the ground area covered.

Roof Coverings. Note, however, that the weight of roof coverings given are per square toot of roof area, and if we work in square feet of ground area these weights must be increased accordingly. Then

Weight of roof covering per square foot of ground area covered = Weight of roof covering per square foot of roof area $\div \cos A$.

where A is the angle the rafter of the truss makes with the horizontal.

QUESTION. A truss has rafters inclined at 30 deg. to the horizontal. The covering weighs 9 lb. per square foot of roof area. What is the weight of the roof covering per square foot of ground covered ?

Answer.

Weight of roof covering per square foot of ground area covered

$$= \frac{9}{\cos 30 \text{ deg.}} = \frac{9}{0{\cdot}866} = \text{say, } 10\tfrac{1}{2} \text{ lb.}$$

The weight due to dead loads and wind on a truss are transmitted to the truss by means of purlins which are connected to the trusses by cleats. In Fig. 106 there are ten purlins and each truss has nine panel points where the roof load is transmitted to the truss. The shaded area shows that all the panel points except the shoes take one-eighth of the roof load, while the shoe panel points take one-sixteenth of the total load.

QUESTION. What is the vertical load at point A in Fig. 106, assuming the trusses as in the previous questions, and that the truss is to be designed for a total vertical load of 40 lb. per square foot of ground area covered?

Answer.

Ground area covered by load on A

$$= \tfrac{1}{8} \times 60 \text{ ft.} \times 15 \text{ ft.} = 112\tfrac{1}{2} \text{ sq. ft.}$$

Total vertical load on A = 40 lb. $\times$ $112\frac{1}{2}$ sq. ft. = 4,500 lb.

All the other panels of the truss will have to carry 4,500 lb. except the shoes, which carry one-half of this, that is, 2,250 lb.

The above questions and answers will give the reader a good idea of how to calculate the loads coming on roof trusses, and in the next chapter we shall show how to calculate the stresses in the various members of a roof truss, and how to draw the stress diagram.

CHAPTER 15

STRESSES IN ROOF TRUSSES

We already know how the horizontal wind load acting on a roof truss could be resolved into a vertical load (see Fig. 115 (*b*) and (*c*) on page 151). So far as the roof truss is concerned, a perfectly satisfactory design can be obtained by considering all the loads as being vertical, and if the truss is designed to carry a vertical load of 40 lb. per square foot of ground area covered, the sections which are so obtained will be quite satisfactory.

Three Main Types. The attention of the student is at this point drawn to the fact that there are about three main divisions into which steel roof trusses can be placed:

(1) Very light roof trusses, suitable for covering hayricks, or for conditions where failure would not have serious effects. In this case the tension members would be made of round bars or flat bars.

(2) The general type of design for garages and industrial buildings. In these cases all the members are usually made of angles, so that bars which are normally in tension will be capable of resisting some compression should these arise. (This condition may easily arise due to the suction on the leeward side, or by someone hanging a pulley block to some member of the roof truss and lifting a motor or heavy machinery. This might cause not only increased stresses in some of the members, but might even change the stresses in a bar which is normally in tension to stresses in compression.) These terms, stress, tension and compression, have already been explained in earlier chapters.

(3) The heavy type, where the truss is located over a chemical plant where the dusts and acids are corrosive. In such a case the members would be made somewhat thicker than theoretically required, to allow for some corrosion.

The author knows of two roof trusses, both of the same span, and with the same arrangement of members, where the weight of one stress is very nearly twice the weight of the other. In one case the truss was designed by a firm in what is known as the "light construction" trade, while the heavy truss was designed by plant engineers who had had some experience of corrosion, and were also well aware that whether a roof truss is designed for it or not, there is always a danger of someone lifting weights by slinging chains round the horizontal members and attaching thereto a pulley block.

These points are not likely to arise in examination questions, but they are mentioned as practical notes in the hope that they will help to explain to students and designers some of the reasons why there is such a variation in the weights of roof trusses.

Design of Roof Trusses, 55 *ft. Span. Spaced at* 12-*ft. Centres.* Fig. 116 (*a*) shows the roof area and ground area covered by

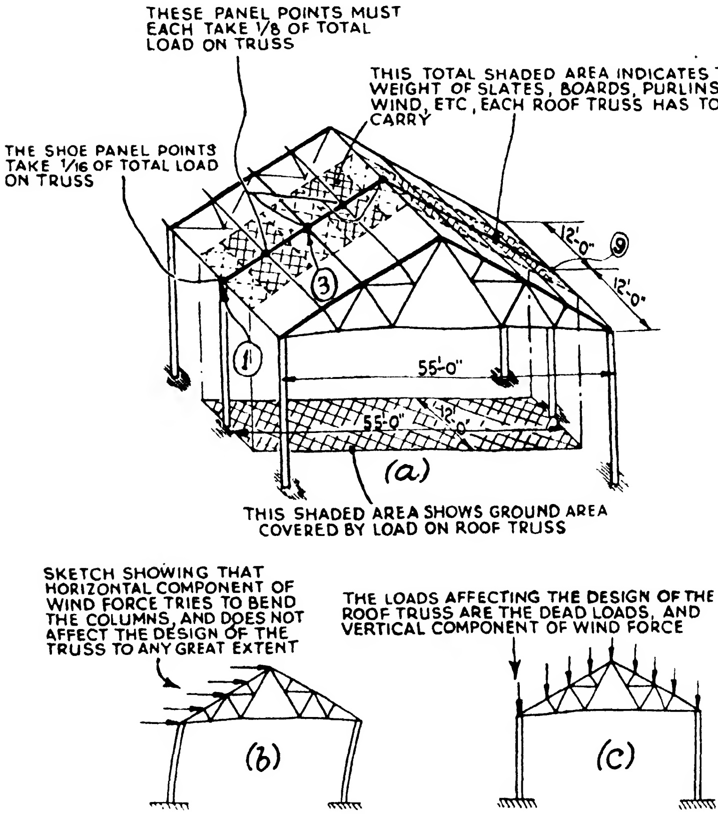


FIG. 116.

each roof truss. (Obviously the two roof trusses at the ends of the building only take half the loads of the intermediate trusses.)

An allowance of 20 lb. per square foot of ground covered is very ample for the roof-covering purlins and roof truss. A further 20 lb. to allow for the vertical component of the wind load is also sufficient.

Fig. 117 shows in diagrammatical form the shape and arrangement of the members suitable for a roof truss of 55 ft. span. In this case the purlins are assumed to be at the panel points. A panel is the part of the roof truss rafter between the struts. The panel points are therefore located at 1, 2, 3, 4, etc.

QUESTION. What is the load carried at joint 1 and at joint 3?

Answer. The ground area covered by the load on one truss 55 ft. × 12 ft. which amounts to 660 sq. ft. A little study of Fig. 116 (*a*) will show that the loads at points 1 and 9 will be one-half as much as the loads at the other panel points.

We have therefore seven full-load panel points located at 2, 3, 4, etc., and two half-load panel points located at 1 and 9. Therefore, if we divide the total weight on the ground area covered by 8, we shall get the load coming on each of the points 2, 3, 4, etc., and if we again divide the figure so obtained by 2, we shall get the loads at the points 1 and 9.

Total load on roof truss = Span × Spacing × Load per square foot of ground area covered

$$= 55 \text{ ft.} \times 12 \text{ ft.} \times 40 \text{ lb.}$$
$$= 26{,}400 \text{ lb.} = \text{say, } 12 \text{ tons.}$$

$$\text{Load of panel point } 3 = \frac{\text{Total load}}{8}$$

$$= \frac{12 \text{ tons}}{8} = 1\tfrac{1}{2} \text{ tons.}$$

$$\text{Load at panel point } 1 = \frac{\text{Load at panel point } 3}{2}$$

$$= \frac{1\tfrac{1}{2} \text{ tons}}{2} = \tfrac{3}{4} \text{ ton.}$$

These forces are represented by vertical lines in Fig. 117, and the amount at each panel point is shown. Since all the load is vertical and symmetrical about the centre line, the load down each wall, or the reaction at each wall, will also be vertical, and will be each equal to half the total load, that is, 6 tons.

Lettering each Space. We now proceed to letter each space between two forces (see Fig. 117) in accordance with Bow's notation. Thus *B* denotes the space between point 1 and 2, *C* the space between 2 and 3, and so on. Each space in the roof truss itself also carries a letter. Each member or load is known by the letters on each side of it. For instance, the part of the roof rafter between 2 and 3 is known as member *CN* or *NC*.

The next stage is to draw the load line, which is marked *a* to *k* (Fig. 118). This line represents to some suitable scale the loads coming on the roof truss. All these loads are vertical in this case, so that our load line will also be vertical. Then *ab* on the load line represents *AB* (at point 1) = $\frac{3}{4}$ ton. The line *bc* represents load *BC* = $1\frac{1}{2}$ tons. The lines *cd*, *de*, *ef*, *fg*, *gh*, and *hj*, each represent a load of $1\frac{1}{2}$ tons, which is the load at points

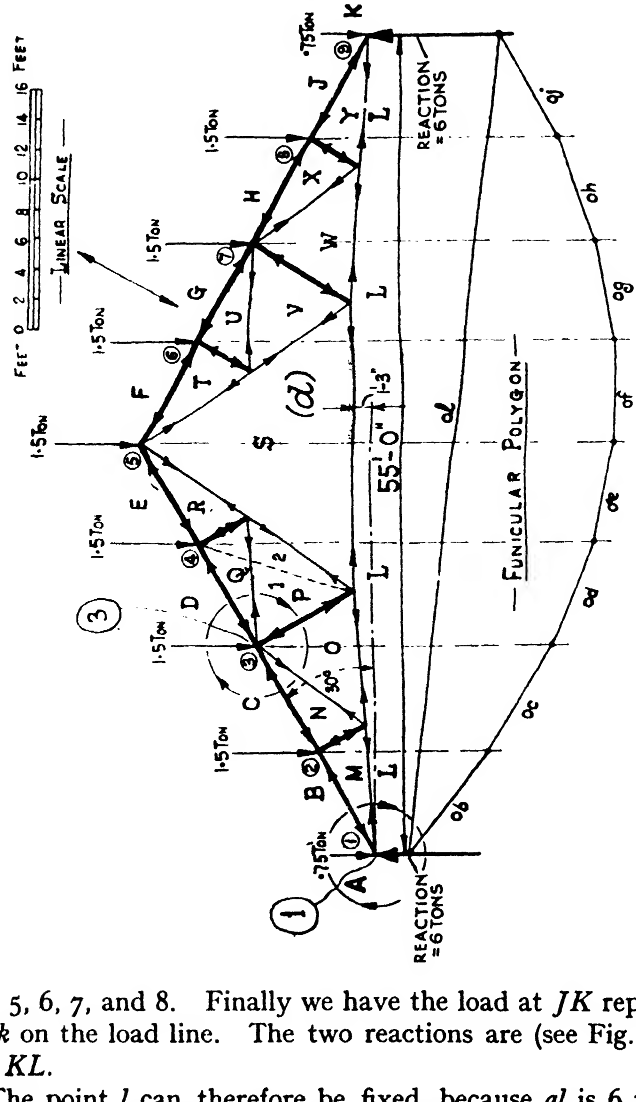


Fig. 117.

3, 4, 5, 6, 7, and 8. Finally we have the load at *JK* represented by *jk* on the load line. The two reactions are (see Fig. 117) *AL* and *KL*.

The point *l* can therefore be fixed, because *al* is 6 tons and *kl* 6 tons. In this case it is not necessary to draw either the polar diagram or the funicular polygon, but it has been drawn because in many cases the point l, which gives the two reactions, is more easily found by this means. We shall see in a later chapter that the reactions may be different both in amount and direction.

To find point l by graphical means, choose any pole O. It is perfectly immaterial where the point O is chosen. Three other places are indicated in Fig. 118, and the result would have been exactly the same if the pole O had been taken in any of these positions. Generally it is a good thing to place the pole so that the outside lines of the diagram form something near to an isosceles triangle. Joint points b, c, d, etc., to the pole O. This completes the polar diagram.

Graphical Methods. From any point on the reaction line AL (Fig. 117) draw a line ob parallel to ob in the polar diagram (Fig. 118). This line runs between the reaction line and the projection of the load BC. From this intersection point draw oc parallel to oc in the polar diagram, then od, oe, etc., until oj finally cuts the reaction line LK.

Join this point with the starting-point on reaction line AL by line ol; refer to the polar diagram (Fig. 118), and from the pole O draw line ol parallel to ol in the funicular polygon. Where the line intersects the load line a to k is the point l. Then la is the left-hand reaction and lk is the right-hand reaction. If these distances are scaled they will each be found to be 6 tons, and this shows how the reactions can be obtained by graphical methods.

Stress Diagram. We can now proceed to find the stresses in the members of the roof truss by drawing, or, as it is technically called, by graphic statics. Using the load line as a base, we can construct a figure, the sides of which to scale will represent the forces acting in the various members of the roof truss. From point b on the load line draw bm parallel to rafter BM on the frame diagram (Fig. 117). From l draw line lm parallel to member LM. Where these two lines intersect we get the point m. If the length bm is measured to the same scale as was used for the load line to a to k it will give the stress in the member BM. Similarly the length of line lm gives the stress or force in the roof truss member LM. These figures are shown in the table on page 161.

The stress in member BM is 11·9 tons, and the stress LM is 10·3 tons. Now from point c draw a line cn parallel to CN in Fig. 117, and from point m (which has already been determined) draw mn parallel to member MN. The intersection of these lines gives point *n*. By scaling, the stresses in *MN* and *CN* can be found to be 1·3 tons and 11·2 tons respectively. From *n* draw line *no* until it intersects the line *lm*, this intersection point being *o*. Then the stress in member *LO* will be found to be 8·8 tons and that in member *NO* 1·4 tons. From *d* draw a line *dq* parallel to *DQ* and a line *er* parallel to *ER*.

At this stage we find difficulty as we cannot locate either point *q* or point *r*. In order to locate point *r* we substitute another member into the roof truss. Looking at Fig. 117, we take out members *QP* and *QR* and substitute in their place the member (1) (2) shown dotted. Point *o* has already been found, and we can therefore draw line *o* (1) parallel to member *O* (1). This line strikes the line from *d* at point (1). Then *d* (1) represents the force in member *D* (1). From point (1) draw line (1) (2) parallel to member (1) (2) until it meets the line *e* (2) drawn parallel to member *E* (2). This locates points (1) and (2).

We have now found the stress in member *E* (2), and this stress will be the same whether members *PQ* and *QR* are in position or whether it is made with *PQ* and *QR* removed and the bar (1) (2) substituted in their place. Point (2) is therefore also point *r*.

Having located this point, we now figuratively take out the bar (1) (2) and return to the original framework with the members *PQ* and *QR*. The length *er* is known and scales 9·7 tons. From *r* draw *rq* until it intersects line *dq* at *q*. The stress in bar *DQ* is found to be 10·4 tons. The stress in member *QR* scales 1·3 tons. The lines *rs* and *ls*, drawn respectively parallel to members *RS* and *LS*, show the stresses in these to be 4·9 tons and 5·6 tons. Then, by drawing *op* parallel to *OP* and then drawing *qp*, we find the stresses in *OP* and *QP* are 2·6 and 1·4 tons respectively.

It will be noted that the stresses in the members shown in the table are the total load or force in the member, and not the unit stress or stress per square inch. The difference between these two terms has already been fully explained.

Nature of Stress. By this term we mean whether the force tries to shorten the bar, in which case it is in compression, or whether it tries to lengthen the bar, in which case it is in tension. In Fig. 117 the members which are in compression, that is, the rafters and struts, are shown in thick lines, and the members that are in tension, or the ties, are shown in thin lines. The method of finding whether a member is in tension or compression will now be explained fully.

STRESSES IN MEMBERS OF ROOF TRUSS.			
MEMBERS		COMPRESSION TONS.	TENSION TONS.
B-M AND J-Y	RAFTER.	11.9	
C-N AND H-X	"	11.2	
D-Q AND G-U	"	10.4	
E-R AND F-T	"	9.7	
L-M AND L-Y	LOWER TIE		10.3
L-O AND L-W	"		8.8
L-S ---------	"		5.6
M-N AND X-Y	STRUT	1.3	
N-O AND W-X	TIE		1.4
O-P AND V-W	MAIN STRUT	2.6	
P-Q AND U-V	TIE		1.4
P-S AND S-V	"		3.4
Q-R AND T-U	STRUT	1.3	
R-S AND S-T	TIE		4.9

Examining Forces. Consider point 1. Here we have only two members, *BM* and *ML*. Imagine the centre of the joint as being the centre of a clock face, and proceed to examine the forces in a clockwise direction. It makes no difference which force is taken first, so long as they are considered in a clockwise direction. If we start with the load *AB* in Fig. 117 we see that this load is acting downwards. On the load line in Fig. 118 put an arrow on length *ab* also acting down. Returning to Fig. 117, our next force acts in the member *BM*. In Fig. 118 we put an arrow pointing from *b* to *m*. Now return to Fig. 117 and make an arrow near to the joint pointing in the same direction as that on the stress diagram. Still proceeding in a clockwise direction, our next member is *ML*. On Fig. 118 we make an arrow pointing from *m* to *l*, and returning to Fig. 117 we mark an arrow pointing in the same direction.

The direction of the arrows which result in Fig. 117 shows whether the members are in tension or compression. If the arrow points towards the joint, then the member which carries this arrow is in compression. If the arrow points away from the joint the member is in tension. The arrow on member *BM* points towards the joint we are considering, so that member *BM* is in compression. The arrow on *ML* points away from the joint we are considering, so that *ML* is in tension.

Each joint can be dealt with separately, and we will now proceed to joint number 3 to show a further application of the

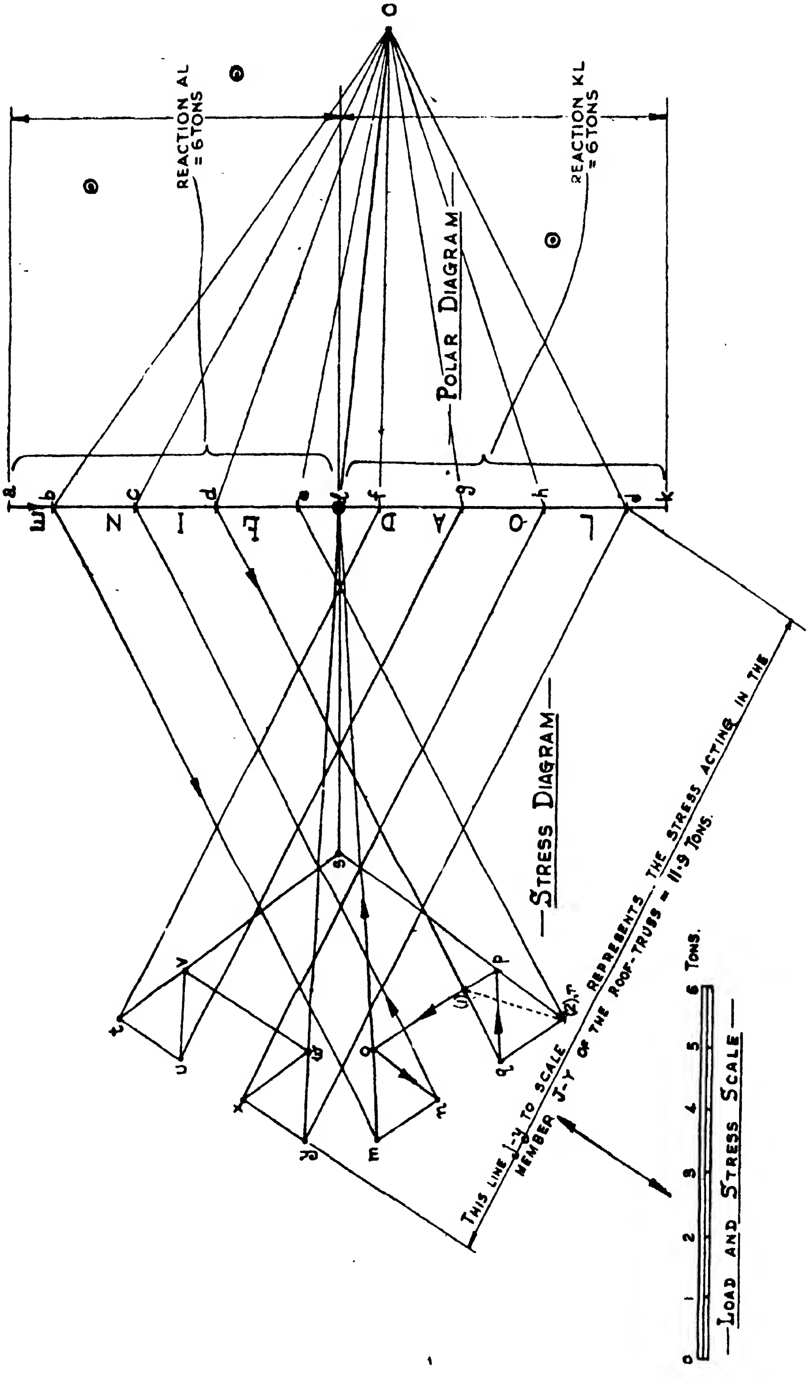

REACTION AL = 6 TONS
REACTION KL = 6 TONS
POLAR DIAGRAM
STRESS DIAGRAM
THIS LINE J-Y TO SCALE REPRESENTS THE STRESS ACTING IN THE MEMBER J-Y OF THE ROOF-TRUSS = 11·9 TONS.
LOAD AND STRESS SCALE
0 1 2 3 4 5 6 TONS.

FIG. 118.

method. To show that it makes no difference at which member we start, so long as we work around the joint in a clockwise direction, we will begin at member *PO* in Fig. 117. An arrow pointing from *p* to *o* is made on the stress diagram, and an arrow pointing in the same direction is transferred back to joint *3* in Fig. 117. The next member is *ON*. Again in Fig. 118 make an arrow pointing from *o* to *n* and another arrow in the same direction in Fig. 117. Proceed in like manner for the member *NC*, the load *CD* and members *DQ* and *QP*, and the nature of the force in each of these members will be found. The arrows for all the joints and all the members are shown in Fig. 117, and the arrows for the two joints we have just described are shown in Fig. 118.

The stresses in the various members of a roof truss, such as shown in Fig. 117, where the loading is considered to be vertical, can also be easily found by calculation or by means of the tables giving coefficients of stresses which are to be found in many of the excellent hand-books issued by the makers of rolled-steel sections. In the tables referred to the lower tie of the truss is considered as being horizontal for all its length. If the lower chord is raised, as shown in Fig. 117, the effective depth of the roof truss will be lessened and the stresses in the rafters increased, but generally it will be found that this increase of stress is not sufficient to call for a larger size of member. Raising the roof tie slightly at the centre improves the appearance of a roof truss to a considerable extent.

The table which follows gives the coefficients for roof trusses with a slope of 30 deg. and having the lower member horizontal. The coefficients can be used for any span of roof truss providing the arrangement of the various members is the same as shown in Fig. 117 and also provided that the slope of the rafter is 30 deg. If the slope of the roof is changed, the coefficients cannot be used, and the stresses in the members will be different. For instance, if the arrangement of members is kept the same and the slope of the rafter is less than 30 deg., that is to say, the roof is made flatter, then the stresses in the members will be increased. The stress in any member of a truss is to be found by multiplying the load on each internal joint by the coefficient given in the table. (In this case the load on each internal joint is 1·5 tons.)

COEFFICIENT OF STRESSES FOR TYPE OF TRUSS SHOWN IN FIG. 4.

SLOPE 30°.

Members.	Coefficient.	Stress.
BM and *JY*	+ 7·00	+ 10·5 tons
CN and *HX*	+ 6·50	+ 9·75 tons
DQ and *GU*	+ 6·00	+ 9·00 tons
ER and *FT*	+ 5·50	+ 8·25 tons
LM and *LY*	− 6·06	− 9·09 tons
LO and *LW*	− 5·20	− 7·8 tons
LS	− 3·46	− 5·19 tons
MN and *XY*	+ 0·87	− 1·3 tons
NO and *WX*	− 0·87	− 1·3 tons
OP and *VW*	+ 1·73	+ 2·6 tons
PQ and *UV*	− 0·87	− 1·3 tons
PS and *SV*	− 1·73	− 2·6 tons
QR and *TU*	+ 0·87	+ 1·3 tons
RS and *ST*	− 2·60	− 3·9 tons

In the above table + denotes that the stress is a compressive one, while − denotes it is tensile.

The total stress or force in every member of the roof truss, and the nature of these stresses, has now been found, and now we will proceed to find the size of the bars for resisting these stresses.

CHAPTER 16

DESIGN OF ROOF TRUSSES

We found by graphical means the stresses acting in the members of a roof truss of 55 ft. span. Fig. 119 (*a*) on the folding plate shows this roof truss and also indicates the stresses acting in each member. We will now proceed to consider the design of suitable steel sections for these members.

The stresses shown are of two kinds, one compression and the other tension. The tension members, or ties as they are called, are relatively easy to design, and the method to be followed is exactly that shown in Chapter 5, which dealt with riveted joints. It was found that the safe load which could be put to a plate, flat, or angle which was in tension, equals

Safe load = Net sectional area × Safe stress per square inch.

To connect the ends of the ties to the rafters or other members, gusset plates are used (see Fig. 119 (*d*), (*e*), (*f*) and (*g*)). Rivet holes made in a tie reduce the net area of the section and also its load-carrying capacity. In most of the sections used in roof truss construction only one rivet hole is taken out across any cross-section, so that generally (but not always, see Chapter 5):

Net sectional area of tie = Gross area − Area of one rivet hole.

In roof truss design a safe stress in tension of 7 tons per square inch can be used for ordinary construction, while for light construction up to 8 tons can be adopted.

The design of compression members, or struts, in a roof truss provides a different problem. If these struts are long compared with their cross-section, they will tend to bend when a compressive load is put on them. At this stage it is therefore necessary to consider how the strength of the strut is affected by the bending.

Columns, struts and compression members in general may be divided into three classes:

(1) Short pieces, where failure would be due to direct compression only.

(2) Relatively short columns where failure would be due to a combination of compression and banding.

(3) Long columns, where failure is chiefly due to bending.

In Chapter 3 we showed the various kinds of failure which would result if short blocks of brittle and ductile materials were loaded to destruction.

The great majority of struts in roof trusses and columns in buildings can be classified as those where failure would result in a combination of compression and bending.

Fig. 119 (*b*) shows a solid steel column 4 in. diameter and 12 ft. high. If a 4-in. diameter steel bar 12 in. long was loaded to destruction, it might easily take a load of 350 tons to cause failure. It is not difficult to see that a very much smaller load would cause failure of a steel column made of 4-in. diameter metal, if the length was 12 ft. instead of 12 in. Long before this load was reached the column would have bent as shown (greatly exaggerated) by the dotted lines, and the column would fail.

Fig. 119 (*b*) shows a column of exactly the same height as that shown in Fig. 119 (*c*), but made of hollow steel. The section shown has the same area of metal as the solid steel column. In both cases the area is approximately 12½ sq. in. It does not require a knowledge of mathematics or mechanics to see that the column shown in Fig. 119 (*b*) would bend more easily than the hollow column shown in Fig. 119 (*c*). In technical language we say that " the hollow steel column is stiffer than the solid steel column," or, in other words, it is less slender. This slenderness ratio determines very largely the safe load which a column will carry.

$$\text{Slenderness ratio} = \frac{L}{R}$$

where L is the length of the column in inches.
R is the radius of gyration in inches.

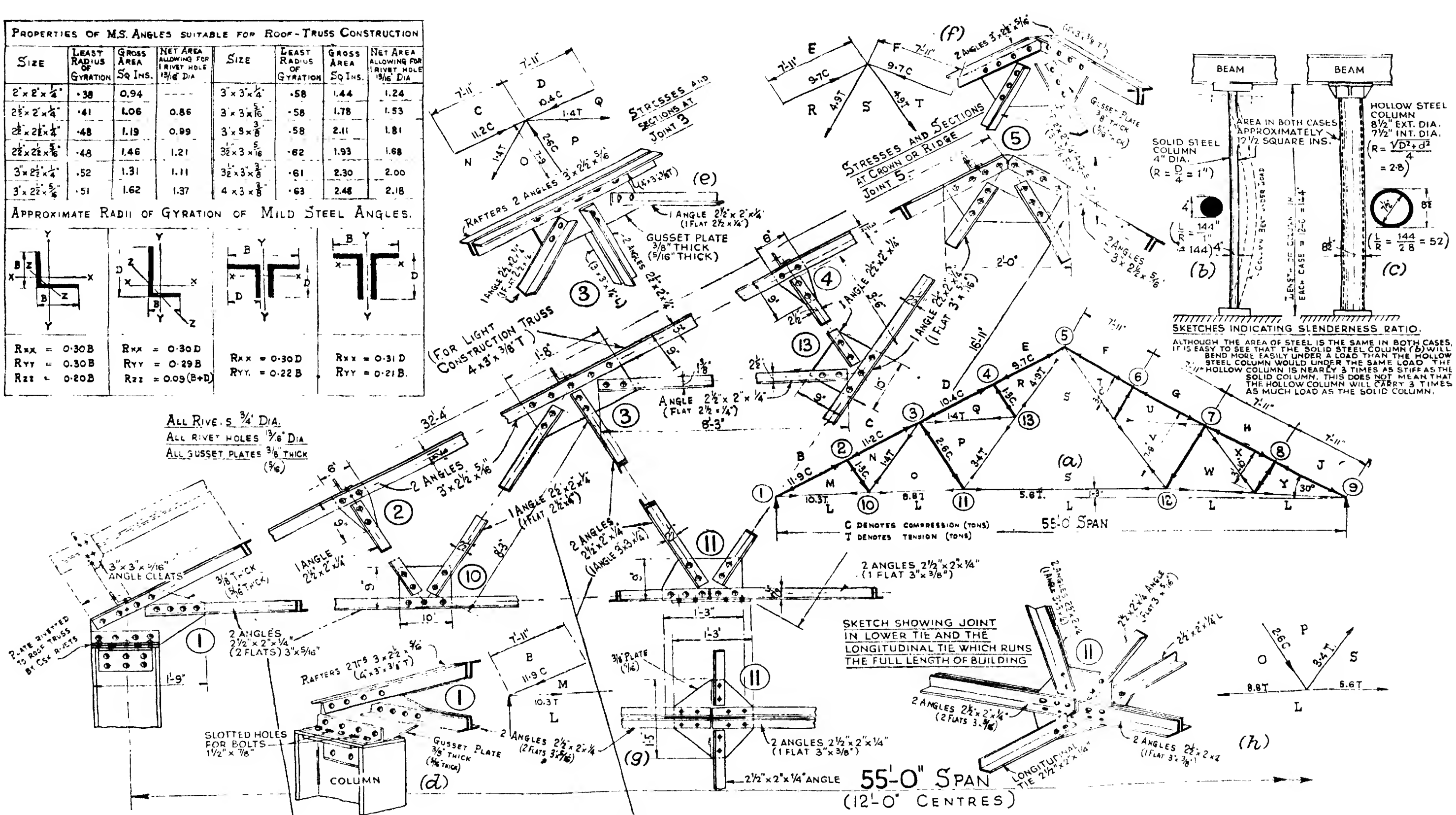
We shall have more to say about this term radius of gyration later, but in the meantime it can be stated that the radius of gyration is the square root of the moment of inertia divided by the area of the section.

It is written

$$R = \sqrt{\frac{I}{A}}.$$

If the length and the area of a column or strut remain constant, the strength will be increased as the radius of gyration gets bigger.

The formulæ used for finding the radius of gyration of a solid section and of a hollow section are shown in Fig. 119 (*b*) and (*c*) and in a table facing are shown formulæ for finding the approximate radii of gyration of various angles such as are commonly used in roof construction.



Properties of M.S. Angles suitable for Roof-Truss Construction

Size	Least Radius of Gyration	Gross Area Sq Ins.	Net Area allowing for 1 rivet hole 13/16" Dia	Size	Least Radius of Gyration	Gross Area Sq Ins.	Net Area allowing for 1 rivet hole 13/16" Dia
2"×2"×1/4"	·38	0.94	----	3"×3"×1/4"	·58	1.44	1.24
2½"×2"×1/4"	·41	1.06	0.86	3"×3"×5/16"	·58	1.78	1.53
2½"×2½"×1/4"	·48	1.19	0.99	3"×3"×3/8"	·58	2.11	1.81
2½"×2½"×5/16"	·48	1.46	1.21	3½"×3"×5/16"	·62	1.93	1.68
3"×2½"×1/4"	·52	1.31	1.11	3½"×3"×3/8"	·61	2.30	2.00
3"×2½"×5/16"	·51	1.62	1.37	4×3×3/8"	·63	2.48	2.18

Fig. 119 (*a–h*).

Facing page 166.

Safe Stress. If the steel column or strut were very short, it would be possible to allow a safe working stress of about 7 tons per square inch. To make this clear, suppose we had a piece of mild steel 4 in. diameter and 12 in. long, and we used this as a post or column, then it would safely carry a load of 7 tons on each square inch of area.

The area of a 4-in. diameter round bar is—

$$\text{Area} = \frac{22}{28} \times 4 \times 4 = \text{approximately } 12\tfrac{1}{2} \text{ sq. in.}$$

Therefore the safe load on this short steel post would be

$$\begin{aligned}\text{Safe load} &= \text{Area} \times \text{Safe stress} \\ &= 12\tfrac{1}{2} \times 7 = \text{say, } 87 \text{ tons.}\end{aligned}$$

If the column was 12 ft. long, it would bend much easier than the short column, and therefore the allowable stress on the steel must be reduced. We must use a working stress of something less than 7 tons per square inch. How much less than 7 tons is determined by the slenderness ratio

$$\frac{L}{R} \text{ of the column or strut.}$$

In designing roof trusses we can use a very simple formula which will give this safe working stress. It is

$$\text{Safe stress per square inch} = 7 - \frac{L}{30R} \text{ tons.}$$

QUESTION. What is the allowable working stress per square inch on a mild steel strut, which has a length of 6 ft. (72 in.) and a radius of gyration of 1 in.?

Answer. Safe stress per square inch

$$\begin{aligned}&= 7 - \frac{72}{30 \times 1} \\ &= 7 - 2{\cdot}4 \\ &= 4{\cdot}6 \text{ tons per square inch.}\end{aligned}$$

We have already seen that the total safe load will be obtained by multiplying the safe stress per square inch by the area of the section in square inches.

It is advisable, except in light work, to make the rafters and long struts of a roof truss of two angles with their long legs back to back. The most common sizes of angles for roof truss construction are shown in the table with Fig. 119.

For roof trusses up to 30 ft. span either $\frac{1}{2}$-in. or $\frac{5}{8}$-in. diameter rivets are generally used, while for spans above 30 ft., $\frac{3}{4}$-in. and $\frac{7}{8}$-in. diameter rivets are common. To provide for proper riveting when using these $\frac{3}{4}$-in. and $\frac{7}{8}$-in. rivets, the smallest size of angle that should be used is the $2\frac{1}{2}$ in. by 2 in. × $\frac{3}{8}$ in. where the rivets pass through the $2\frac{1}{2}$-in. leg. Also the smallest size of flat should be $2\frac{1}{2}$ in. × $\frac{1}{4}$ in.

We will now proceed to design the members of the 55-ft. roof truss shown in Fig. 119.

In this case, assume we require a sturdy construction roof truss, and to allow for reversals of stress due to wind pressure, the struts will all be made of two angles. The lower tie of the truss will also be made of two angles.

Quite apart from this design we will also calculate suitable sizes of the members for a "light construction" truss of similar span, using the same stresses.

Design of Compression Members. Rafters BM and JY (Fig. 119 (b) and (d)) have a compressive load of 11·9 tons and a length of 7 ft. 11 in. or 95 in.

It is a good guide for roof truss rafters to make the $\frac{L}{R}$ about 100.

In no case should $\frac{L}{R}$ exceed 160.

Then,

$$\text{Safe stress per square inch} = 7 - \frac{L}{30R} \text{ tons.}$$

If $\frac{L}{R}$ is to be 100, then

$$\text{Safe stress per square inch} = 7 - \frac{100}{30}$$

$$= 7 - 3{\cdot}33 = 3{\cdot}67 \text{ tons.}$$

Actual stress in member = 11·9 tons, so that

$$\text{Approximate sectional area required} = \frac{\text{Load}}{\text{Safe stress}}$$

$$= \frac{11{\cdot}9}{3{\cdot}67} = 3{\cdot}2 \text{ sq. in.}$$

If the rafter is made of two angles, then the sectional area of each angle will need to be

$$\frac{3{\cdot}2}{2} = 1{\cdot}6 \text{ sq. in.}$$

Reference to the table of angle sizes shows that two angles 3 in. × $2\frac{1}{2}$ in. × 5·16 in. thick will give a total gross area of 2 × 1·62 = 3·24 sq. in. Try this size of member.

$$\text{Safe stress per square inch} = 7 - \frac{L}{30R} \text{ tons.}$$

L is the length of the member = 95 in.

R is the least radius of gyration of the section.

There are two radii of gyration, one about axis XX and one about YY. The table of approximate radii gyration given shows that $RXX = 0{\cdot}31D$ and $RYY = 0{\cdot}21B$. In our case D, the depth of the angle leg = 3 in. To connect the rafters to the struts and ties, gusset plates are required, and it is convenient to have these plates inserted between the two angles as shown in the details. For an ordinary construction roof truss these gussets can be $\frac{3}{8}$ in. thick, while for light construction their thickness can be reduced to $\frac{5}{16}$ in.

The dimension B will amount to $2\frac{1}{2}$ in. + $2\frac{1}{2}$ + $\frac{3}{8}$ in. = 5·37 in.

Then $RXX = 0{\cdot}31D = 0{\cdot}31 \times 3 = 0{\cdot}93$ in.

$RYY = 0{\cdot}21B = 0{\cdot}21 \times 5{\cdot}37 = 1{\cdot}2$ in.

The least radius of gyration is therefore 0·93 in. and this must be used in the design. (A little consideration will show that if the other radius of 1·2 in. was used in the design, we would obtain a larger safe working stress and a larger safe load than if 0·93 in. was used, so that we would be putting a greater load on the rafter than it could safely bear on axis XX.)

$$\begin{aligned}\text{Safe stress per square inch} &= 7 - \frac{95}{30 \times 0{\cdot}93} \\ &= 7 - \frac{95}{27{\cdot}9} \\ &= 7 - 3{\cdot}4 = 3{\cdot}6 \text{ tons.}\end{aligned}$$

$$\begin{aligned}\text{Safe load} &= \text{Safe stress} \times \text{Sectional area} \\ &= 3{\cdot}6 \text{ tons} \times 3{\cdot}24 \text{ sq. in.} \\ &= 11{\cdot}9 \text{ tons.} \\ \text{Actual load} &= 11{\cdot}9 \text{ tons.}\end{aligned}$$

Therefore this section of two angles 3 in. × $2\frac{1}{2}$ in. × $\frac{5}{16}$ in., with the 3-in. legs vertical, is strong enough and will be adopted.

In the light construction trade or for temporary construction the safe stress per square inch might be taken as

$$8 - \frac{L}{30R}$$

Assuming a slendering ratio of 150, then the least radius of gyration will have to be more than $\frac{95}{150} = 0{\cdot}64$ in. A rolled steel tee 4 in. × 3 in. × $\frac{3}{8}$ in., has a least radius of gyration of 0·86 in. Using the formula given above, the safe stress will be:

$$8 - \frac{95}{30 \times 0{\cdot}86}$$
$$= 8 - 3{\cdot}68 = 4{\cdot}32 \text{ tons per square inch.}$$

The cross-section area of this tee is 2·5 sq. in., so that

$$\text{Safe load} = \text{Safe stress} \times \text{Section area}$$
$$= 4{\cdot}32 \times 2{\cdot}5 = 10{\cdot}8 \text{ tons.}$$

This section would be a little bit on the small side for members *BM*, *JY*, *CN*, and *HX*, but it would be adequate for the rafters *DQ*, *GU*, *ER*, and *FT*. As the vertical loading of 40 lb. per square foot is ample to take care of any conditions likely to arise and as the maximum stress which would result in members *BM* and *JY* if the 4 in. × 3 in. × $\frac{3}{8}$ in. tee was used would be less than 6 tons per square inch, this section would probably be used.

The compressive stress in rafters *CN*, *HX*, *DQ*, *ER*, and *FT* (Fig. 119 (*a*)) varies from 9·7 tons to 11·2 tons, so that theoretically we could use a smaller size of angle than that just adopted for rafter members *BM* and *JY*. For practical reasons the rafters of a roof truss are made of one continuous length, so that in this case two angles 3 in. × $2\frac{1}{2}$ in. × $\frac{5}{16}$ in., will be used for all rafters on the sturdy construction truss. Washers or spacers should be put in the space between the angle legs at intervals. In the light construction truss all the rafter members will be made of the 4 in. × 3 in. × $\frac{3}{8}$ in. tee section.

Main struts *OP* and *VW* (Fig. 119 (*a*) and (*e*)) have a compressive stress of 2·6 tons, and are 7 ft. 9 in., or 93 in. long. With such a small load it is evident that only a small cross-sectional area is required, and in this case it is a question of keeping the slenderness ratio below 160 and of the minimum practical size of angle that can be used.

To Prevent Buckling. To prevent buckling out of shape it is preferable to use two angles back to back for these main struts in ordinary construction. Try the smallest practical size, that is, two angles $2\frac{1}{2}$ in. × 2 in. × $\frac{1}{4}$ in. with the $2\frac{1}{2}$ in. legs back to back. Two such angles have a total area of 2 × 1·06 = 2·12 sq. in. Then $D = 2{\cdot}5$ in., and $B = 2$ in. + 2 in. + $\frac{3}{8}$ in. = 4·37 in.

$$R_{XX} = 0{\cdot}31D = 0{\cdot}31 \times 2{\cdot}5 = 0{\cdot}78 \text{ in.}$$
$$R_{YY} = 0{\cdot}21B = 0{\cdot}21 \times 4{\cdot}37 = 0{\cdot}91 \text{ in.}$$
$$\text{Least radius of gyration} = 0{\cdot}78 \text{ in.}$$

Therefore $\dfrac{L}{R} = \dfrac{93}{0{\cdot}78} = 119$, which is permissible.

$$\text{Safe stress per square inch} = 7 - \frac{L}{30R}$$
$$= 7 - \frac{119}{30}$$
$$= 7 - 3{\cdot}97 = 3{\cdot}03 \text{ tons.}$$

$$\text{Safe load} = \text{Safe stress} \times \text{Sectional area}$$
$$= 3\cdot03 \times 2\cdot12 = 6\cdot4 \text{ tons.}$$
$$\text{Actual load} = 2\cdot6 \text{ tons.}$$

For ordinary construction it is not advisable to reduce this size of section.

For light construction there is no reason why a single angle should not be used. If $\frac{L}{R}$ is not to exceed 160, then $R = \frac{93}{160}$ $= 0\cdot58$ in. The least size of angle which gives a radius of gyration larger than this is the 3 in. × 3 in. × ¼ in., which has a least radius of 0·58 in. It has a gross sectional area of 1·44 sq. in.

$$\text{Safe stress per square inch} = 8 - \frac{93}{30 \times 0\cdot58}$$
$$= 8 - 5\cdot35$$
$$= 2\cdot65 \text{ tons per square inch.}$$
$$\text{Safe load} = 2\cdot65 \times 1\cdot44 = 3\cdot82 \text{ tons.}$$
$$\text{Actual load} = 2\cdot6 \text{ tons.}$$

This 3-in. × 3-in. × ¼-in. angle can therefore be used.

Struts MN, XY, QR, and TU are generally known as the small struts, and the load they carry is so small that again it is a question of limiting the section so that its slenderness ratio does not exceed 160. Try the small practical size of angle for general construction, that is, the 2½ in. × 2 in. × ¼ in. Area $= 1\cdot06$ sq. in. Least radius of gyration of simple angle with unequal legs $= 0\cdot09\ (B + D) = 0\cdot09 \times 4\cdot5 = 0\cdot4$ in.

$$\frac{L}{R} = \frac{46}{0\cdot4} = 115.$$

$$\text{Safe stress per square inch} = 7 - \frac{L}{30R}$$
$$= 7 - \frac{117}{30}$$
$$= 7 - 3\cdot83 = 3\cdot17 \text{ tons.}$$
$$\text{Safe load} = \text{Safe stress} \times \text{Sectional area}$$
$$= 3\cdot17 \text{ tons} \times 1\cdot06 = 3\cdot33 \text{ tons.}$$
$$\text{Actual load} = 1\cdot3 \text{ tons.}$$

In ordinary construction and in light construction this would be the minimum practical size to use for these small struts.

Design of Tension Members. Lower ties, LM and LY, have a load of 10·3 tons. In ordinary construction either one or two angles can be used. Two angles have the advantage of being stiffer, and we will adopt these. In light construction flats will be used. Where the angle connects to the gusset plate there will be one rivet hole in each angle. The tie is therefore weakest across this section.

$$\text{Required net section} = \frac{\text{Load}}{\text{Safe stress}}$$

$$= \frac{10 \cdot 3}{7} = 1 \cdot 5 \text{ sq. in.}$$

Try two angles $2\frac{1}{2}$ in. $\times$ 2 in. $\times$ $\frac{1}{4}$ in. having a total gross area of $2 \times 1 \cdot 06 = 2 \cdot 1$ sq. in.

Net section = Gross area of angles − Area of two rivet holes. (It should be noted that there is one rivet hole in each angle.) Area of two rivet holes = 2 × Diameter of hole × Thickness of angle leg. For $\frac{3}{4}$-in. diameter rivets the holes will be $\frac{1}{16}$ in. larger, that is, $\frac{13}{16}$ in. diameter.

Area of two rivet holes $= 2 \times \frac{13}{16}$ in. $\times \frac{1}{4}$ in. $= 0 \cdot 4$ sq. in.

Net sectional area $= 2 \cdot 1 - 0 \cdot 4 = 1 \cdot 7$ sq. in.

Required net sectional area $= 1 \cdot 5$ sq. in.

Two angles $2\frac{1}{2}$ in. $\times$ 2 in. $\times$ $\frac{1}{4}$ in. will therefore be suitable. (In light construction work two flats 3 in. $\times$ $\frac{5}{16}$ in. thick could be used.)

Members of *LO* and *LW* are also part of the lower tie, and carry a load of 8·8 tons. For practical purposes they will be made a continuation of members *LM* and *LY*, respectively, and will be made of two angles $2\frac{1}{2}$ in. $\times$ 2 in. $\times$ $\frac{1}{4}$ in. (In light construction they would be two flats 3 in. $\times$ $\frac{5}{16}$ in.)

Member *LS* is the horizontal part of the lower tie, and is made separate from the ties just considered. It carries a load of 5·6 tons.

$$\text{Required net section} = \frac{\text{Load}}{\text{Safe stress}}$$

$$= \frac{5 \cdot 6}{7} = 0 \cdot 8 \text{ sq. in.}$$

Two angles $2\frac{1}{2}$ in. $\times$ 2 in. $\times$ $\frac{1}{4}$ in., having a net area of 1·7 in. could be used if a stiff job is wanted, or one angle $2\frac{1}{2}$ in. $\times$ 2 in. $\times$ $\frac{1}{4}$ in., having a net area of 0·86 sq. in., would be suitable. (For light construction a 3-in. $\times$ $\frac{3}{8}$-in. flat would be suitable.)

Diagonal ties *RS* and *ST* have a load of 4·9 tons. Try one angle $2\frac{1}{2}$ in. $\times$ 2 in. $\times$ $\frac{1}{4}$ in. having a net area of 0·86 sq. in.

$$\text{Net area required} = \frac{\text{Load}}{\text{Safe stress}}$$

$$= \frac{4 \cdot 9}{7} = 0 \cdot 7 \text{ sq. in.}$$

The size of angle is therefore suitable for ordinary construction. (For light construction a 3 in. × $\frac{5}{16}$ in. plat can be adopted.)

Though ties *PS* and *SV* have a less load than *RS* and *ST*, it is practical to make *PS* and *RS* of one length of member and also *SV* and *ST*. In ordinary construction these members will be one angle $2\frac{1}{2}$ in. × 2 in. × $\frac{1}{4}$ in. (In light construction one flat 3 in. × $\frac{5}{16}$ in. would probably be used.)

Ties *NO*, *WX*, *PQ*, and *UV* all have a tension of 1·4 tons.

$$\text{Required net section} = \frac{1 \cdot 4}{7} = 0 \cdot 2 \text{ sq. in.}$$

In ordinary construction the smallest practical size of angle is the $2\frac{1}{2}$ in. × 2 × $\frac{1}{4}$ in., having an area of 0·86 sq. in., and this must be adopted. (In light construction, if we use $\frac{3}{4}$-in. diameter rivets, the least size of flat that can be used is the $2\frac{1}{2}$ in. × $\frac{1}{4}$ in.)

Design of Joints. The design of joints for a roof truss follows exactly on the same lines adopted for ordinary riveted joints.

It will be noticed in the details of the truss shown that the members are so placed that lines drawn through the centres of the rivets meet at one point for each joint. This should always be so as far as possible, because the stress diagram we constructed assumed that the various members of the truss intersected at points.

We will now design some of the joints necessary for a general construction truss, the members for which we have just found.

The forces acting in the members around crown joint (5) are shown in Fig. 119 (*f*). We used $\frac{3}{4}$-in. diameter rivets for the general construction truss and $\frac{3}{8}$-in. gussets.

Members *ER* and *FT* are the rafters of the truss and are composed of two angles 3 in. × $2\frac{1}{2}$ in. × $\frac{5}{16}$ in. One angle passes each side of the gusset, so that the rivets which have to pass through two angle legs and the gusset will be in double shear. From the table given in Chapter 4 of this book, it will be found that the double shear value of $\frac{3}{4}$-in. rivet is 4·4 tons.

Failure of a joint can also occur by bearing. Bearing value of $\frac{3}{4}$-in. rivet in $\frac{3}{8}$-in. thick gusset = 2·8 tons. (Note that the joint will not fail by crushing of the angle legs as there are two of these each $\frac{5}{16}$ in. thick, making a total thickness of $\frac{5}{8}$ in. to be crushed.) The joint will be weaker in bearing than in shear, as the bearing value is lower than the shear.

Number of rivets required to connect each of members *ER* and *FT* to gusset plate

$$= \frac{\text{Load}}{\text{Bearing strength of 1 rivet}}$$

$$= \frac{9{\cdot}7}{2{\cdot}8} = \text{use 4 rivets.}$$

Members *RS* and *ST* are single angles $\frac{1}{4}$ in. thick and the rivets are in single shear. Single shear value for $\frac{3}{4}$-in. diameter rivet $= 2{\cdot}2$ tons.

Bearing value of $\frac{3}{4}$-in. rivet in $\frac{1}{4}$-in. angle leg $= 1{\cdot}9$ tons. Bearing again is the lower value, and must be used in the design.

Number of rivets required to connect each of members *RD* and *ST* to gusset plate

$$= \frac{\text{Load}}{\text{Bearing strength of 1 rivet}}$$

$$= \frac{4{\cdot}9 \text{ tons}}{1{\cdot}9 \text{ tons}} = \text{use 3 rivets.}$$

The complete detail for this joint is shown in Fig. 119 (*f*). When designing these joints the gusset plate itself should be made with as few cuts as possible, and if possible should be square or rectangular in shape.

Main Strut Joints (3 *and* 7). The forces acting in the members which converge on joint 3 are shown in Fig. 119 (*e*).

Members *NO* and *PQ* are single $2\frac{1}{2}$-in. $\times$ 2-in. $\times$ $\frac{1}{4}$-in. angles, so that the rivets are in single shear.

Single shear value of $\frac{3}{4}$-in. rivet $= 2{\cdot}2$ tons.

Bearing value of $\frac{3}{4}$-in. rivet in $\frac{1}{4}$-in. angle $= 1{\cdot}9$ tons.

Bearing governs the design.

Number of rivets required in each of members *NO* and *PQ* to connect them to gusset plate

$$= \frac{\text{Load}}{\text{Bearing strength of 1 rivet}}$$

$$= \frac{1{\cdot}4 \text{ tons}}{1{\cdot}9 \text{ tons}} = 0{\cdot}8.$$

It is not considered good practice in a truss which is of such a large span and which is to be a permanent structure to use less than two rivets to connect any member to a gusset plate. In this case two rivets should be used. (In light construction, one rivet can be used.)

Member *OP* is formed of two angles $2\frac{1}{2}$ in. $\times$ 2 in. $\times$ $\frac{1}{4}$ in., and the rivets are in double shear.

Again bearing will govern the design.

Number of rivets required

$$= \frac{\text{Load}}{\text{Bearing strength of 1 rivet}}$$

$$= \frac{2 \cdot 6}{2 \cdot 8} = \text{use 2 rivets.}$$

Members *CN* and *DQ* are continuous, so that the net load coming on the rivets will be the difference between the two opposing forces, that is, 11·2 − 10·4 = 0·8 ton. Although this calls for one rivet it is necessary to make a slightly stiff joint between gusset plate and rafter, and to do this five rivets are used, as shown in Fig. 119 (*e*).

Shoe Joints (1 *and* 9). The load coming on joint 1 is shown in Fig. 119 (*d*).

Member *BM*: This is of two angles 3 in. × $2\frac{1}{2}$ in. × $\frac{5}{16}$ in., and the rivets are in double shear. Bearing again governs the design.

Number of rivets required

$$= \frac{\text{Load}}{\text{Bearing strength of 1 rivet}}$$

$$= \frac{11 \cdot 9}{2 \cdot 8} = \text{use 4 rivets.}$$

Member *LM* is made of two angles $\frac{1}{4}$ in. thick and the rivets are in double shear, from which it follows that bearing governs the design.

Number of rivets required

$$= \frac{\text{Load}}{\text{Bearing strength of 1 rivet}}$$

$$= \frac{10 \cdot 3}{2 \cdot 8} = \text{use 4 rivets.}$$

The complete shoe joint is shown in Fig. 119 (*d*).

Joints in Lower Tie (11 *and* 12). It is common practice to carry the longitudinal wind bracing (which prevents distortion of the building by wind blowing on its ends) from these two joints. These bracings are shown in Fig. 119 (*g*) and (*h*).

Member *OP* is of two angles $\frac{1}{4}$ in. thick, and the rivets being in double shear the bearing value governs the design.

Number of rivets required

$$= \frac{\text{Load}}{\text{Bearing strength of 1 rivet}}$$

$$= \frac{2 \cdot 6}{2 \cdot 8} = \text{use 2 rivets.}$$

Member *PS* is a single ¼ in. thick angle. Single shear value of ¾-in. diameter rivet = 2·2 tons. Bearing value of ¾-in. rivet in ¼-in. angle = 1·9 tons. Bearing governs.

Number of rivets required

$$= \frac{\text{Load}}{\text{Bearing strength of 1 rivet}}$$

$$= \frac{3\cdot4}{1\cdot9} = \text{use 3 rivets.}$$

Member *LO* has a pull of 8·8 tons. Rivets are in double shear, so that bearing value governs the design.

Number of rivets required

$$= \frac{8\cdot8 \text{ tons}}{2\cdot8 \text{ tons}} = \text{use 3 rivets.}$$

Member *LS*. Again bearing governs.

Number of rivets required

$$= \frac{5\cdot6}{2\cdot8} = \text{use 2 rivets.}$$

The complete detail of this joint, complete with wind bracing, is shown in Fig. 119 (*h*). Note that the rivets connecting the wind bracing gusset to the main truss are staggered, so that at any cross-section the lower tie angles are only weakened by one rivet hole.

The remainder of the joints for the truss are detailed out on the drawing. The student should try his hand at checking up that the number of rivets shown is correct and he should also attempt to design suitable joints for a light construction truss, using the members shown in brackets on the drawing.

CHAPTER 17

WIND PRESSURE ON ROOFS

We have dealt with the method of finding the stresses in, and suitable sections for, a particular type of roof truss of 55 ft. span. In our design we assumed that the truss carried a vertical load which took care of the weight of the roof covering, the purlins and the truss itself, and also the wind pressure. Although this method gives very good results, in general design it is usual to design assuming the truss to carry a vertical load due to the weight of covering purlins and truss, and in addition a wind pressure acting normal or at right angles to the roof rafter.

Roof trusses may be pitched, flat, arched or domed, depending on the shape of the building and purpose for which it is required. In ordinary house construction, and for small buildings, roofs are usually made of timber. In most houses the trusses are of the pitched type, with the rafters sloped at 45 deg. to the horizontal. By having such a steep pitch it is possible to provide more headroom under the truss, and also to build an attic in the truss itself. Timber is also very often employed on temporary construction.

In some countries where steel sections are difficult to obtain large-span timber trusses are frequently used, but on the whole a steel truss is better, because of the greater stress-resisting properties and longer life of steel. Sometimes timber is used for the compression members of a truss while the tension members are made of steel flat bars or round bars. These trusses are known as the composite type. Roofs of reinforced concrete are sometimes used, and can be sloped, flat, or curved, as desired.

For sloped roofs the quarter rise truss is commonly used when the covering is corrugated sheeting, and the 30 deg. truss when covered with slates or tiles.

Spacing of Trusses. The most economical spacing is the one which gives the least amount of material in the columns, roof trusses, and purlins. It is not difficult to see that if the spacing between adjacent roof trusses is increased each truss will carry a larger amount of roof covering, and therefore will have to support a bigger load. On the other hand, although the members of the roof trusses require bigger members, and are therefore heavier, the proportion of the increase is less than the increased ground area covered.

For instance, consider the roof trusses are 40 ft. span and the centre of one truss to the centre of the next is 10 ft., then the ground area covered will be $40 \times 10 = 400$ sq. ft. for each truss. If the spacing is increased to 15 ft., the ground area covered will be 600 sq. ft., or 50 per cent more. The weight of the trusses spaced at 10-ft. centres would probably be about 11 cwt., and the trusses spaced at 15-ft. centres 12 cwt. It is easy to see that so far as the roof trusses are concerned the wider the spacing between the trusses the more economical it is. On the other hand, we have now to consider the purlins. If the spacing between the trusses is increased, the purlins have to carry over a bigger span, and in this case the proportionate increase in the weight of the purlins as compared with the ground area covered is greater. From this it will be seen that the roof truss spacing which is most economical is the one which gives the least amount of steel for both roof trusses and the purlins. In practice the centres of trusses for spans of between 25 ft. and 35 ft. are generally made between 8 ft. and 10 ft. For trusses between 35 ft. and 45 ft. span the centres may be 10 ft. to 12 ft. and where the span is between 45 ft. and 60 ft. the centres are generally between 12 ft. 6 in. and 15 ft.

In cinema and similar buildings the ceiling is often suspended from the roof trusses, and the weight should, of course, be taken into account when finding the stresses in the roof truss members.

Although roof truss with members designed to resist a vertical dead load of 40 lb. per square foot of ground area covered would be quite strong enough for general conditions, it is usual practice to resolve the wind from a horizontal pressure to a normal one.

Fig. 120 (*a*) shows a line diagram of a roof truss 55 ft. span. The space between every pair of forces is lettered *A*. *B*, *C*, *D*, etc. Before we can find the stresses in the various members it is necessary to know the loads the roof must carry. They can be split up into:

(*a*) *Dead loads*, consisting of the weight of roof covering, weight of purlin, and weight of the roof truss itself.

The length of each sloping rafter is 31·6 ft., and the distance between one roof truss and the next is 12 ft., so that the total roof area carried by one roof principle will be

Total roof area $= 2 \times 31{\cdot}6 \times 12 = 758$ sq. ft.

Still dealing with dead loads only, it is safe to assume that the vertical dead load per square foot of roof area, including the weight of slates, boards, purlins and the roof truss itself, will be about 18 lb., so that

Total dead load on one truss will be

$$758 \text{ sq. ft.} \times 18 \text{ lb.} = 13{,}650 \text{ lb.}$$

This load is carried by the purlins, which are located at panel points of the rafters marked 1, 2, 3, 4, etc.

The load on panel points, 2, 3, 4, 5, 6, 7, and 8 will be equal, and the load at the supports or shoes will be half the others.

The vertical dead loads at points 2, 3, 4, 5, 6, 7, and 8 will therefore be $\frac{13{,}650}{8} = 1{,}705$ lb. $= 0{\cdot}76$ tons, while the vertical dead load at points 1 and 9 will be 0·38 ton.

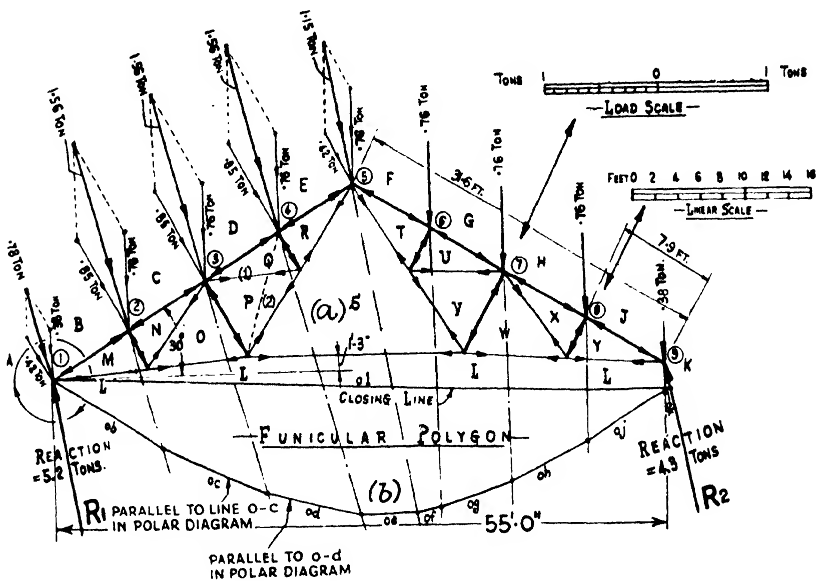


Fig. 120.

(*b*) *Wind Load.* In addition to these vertical dead loads we have a wind load, which is generally assumed to act horizontally and to have a minimum force of 30 lb. per square foot. After considering Chapter 16, the student will have little difficulty in seeing that if the horizontal pressure is 30 lb., then the normal component of this pressure against a rafter inclined at 30 deg., will be about 20 lb. per square foot of roof area.

Wind Pressure. This wind pressure can only act on one side of the roof at one time, but since at some time or the other it will act at the other side of the roof, the members must be designed strong enough to take the maximum stresses which result in the bars. Frequently in practice only half the stress diagram is drawn, sufficient to give the stresses in the members on the side of the truss where the wind is assumed to be acting. The amount of the wind pressure at each panel point is shown in Fig. 120 (*a*), and by the elementary principles of the parallelogram of forces, the vertical load and the normal wind pressure are resolved into a third force as shown. This amounts to 1·56 tons at the intermediate points, 1·15 tons at the apex or ridge, and 0·78 ton at the seat or shoe.

If the student has any difficulty in realizing the amount of area which is supported at each panel point of the roof truss, he should refer to Fig. 116 of Chapter 15.

Reaction to Loads. Having now found the loads on the roof principle, we proceed to find the direction and amount of the reactions. Reaction means the upward force that each wall or column must exert, and this will, of course, be equal to the amount of the downward push at the places where the roof truss rests on the wall or columns. It is easy to see that if the loads are all vertical, the reactions will also be vertical, but since in our case some of the loads are inclined, they will try to push and bend over the walls or columns (see Fig. 116 (*b*), Chapter 15).

The reactions, therefore, with the loading as shown in Fig. 120 (*a*), cannot be vertical. Sometimes one of the roof seats is put on a sliding plate or on rollers, in which case the reaction at that end of the truss will be vertical, and the other shoe will have to take all the horizontal push. In general practice it is safe to consider that both walls or both columns take a part of the horizontal push, and that the direction of the reactions is parallel. This has been assumed in the roof truss under consideration, and in order to find the amount and direction of the reactions we proceed as follows. It is as well to notice that, although the lines R_1 and R_2 are shown in Fig. 120 (*a*), we do not know at present their amount or direction.

Referring to Fig. 121, start at a convenient point *a*, and draw the load line marked *a* to *k* to some suitable scale, *ab* will be parallel to the line marked 0·78 tons at the joint No. 1 in Fig. 1, *bc* will be parallel to the line marked 1·56 tons, shown at joint 2. Proceed with *cd*, *de*, and *ef*. Note that *ef* is parallel to the line marked 1·15 tons at joint 5. From *f* to *k* the lines are vertical. Now join point *k* with point *a*, and this line represents the amount of and the direction of the two reactions marked R_1 and R_2.

As yet we do not know the amount of each reaction. To find these we proceed as follows:

Choose a point which is marked *O*. The position of this point is quite immaterial, but it is generally wise to take it in such a position that the lines *a–o* and *k–o* form angles between 30 deg. and 45 deg. to the horizontal. Now join the points *ao*, *bo*, *co*, *do*, etc., and complete the polar diagram shown in Fig. 121.

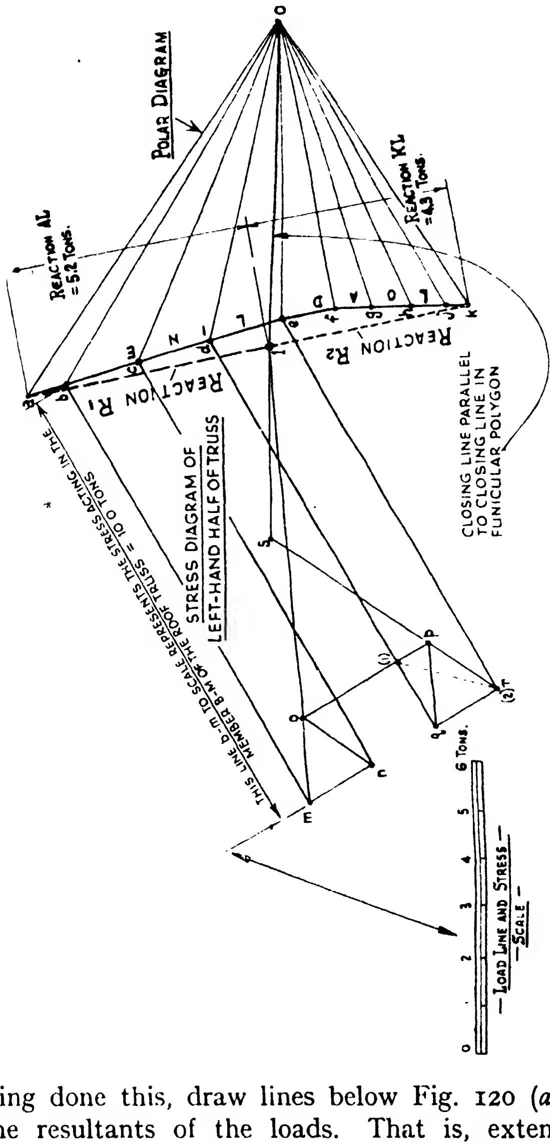


FIG. 121.

Having done this, draw lines below Fig. 120 (*a*) projecting from the resultants of the loads. That is, extend the lines marked 1·56 tons, 0·75 ton, etc. Starting at the joint 1, draw a line *ob* in Fig. 120 (*b*) parallel to *bo* in Fig. 121 until it strikes the projection from the line marked 1·56 tons in joint 2. In like manner proceed to draw *oc*, *od*, etc., and finally join the starting-point with the finishing-point, by a line *ol*, which is marked closing line in Fig. 120 (*b*). From the pole *O* in the polar diagram in Fig 121, draw a line parallel to this closing line until it intersects the reaction line at point *l*.

By measuring to scale we can find the amount of the two reactions. In this case the left-hand reaction R_1 is indicated by line *al* and amounts to 5·2 tons. The right-hand reaction marked R_2 is indicated by the line *kl* and amounts to 4·3 tons. These reactions are now drawn in on Fig. 120 (*a*) and marked R_1 and R_2.

[The method of drawing the stress diagram was fully explained in Chapter 15, to which the students should refer.] To find the stresses in *BM* and *LM*, draw a line from *b* on the load line (Fig. 121) parallel to *BM* in the frame diagram (Fig. 120 (*a*)). From *l* draw parallel to *LM*, and where these two lines intersect gives the point *m*. By scaling the lengths of the lines *bm* and *lm*, the stresses on the two members *BM* and *LM* can be found to be 10 tons and 9·4 tons, respectively. These stresses are shown in the table. As the student proceeds to draw the stress diagram he will find difficulty arising when he tries to find the points *q*, *r*, *p*, or *s*.

In order to get over this difficulty it is assumed that the internal members of the roof truss are altered. The member marked (1) (2) shown in dotted lines is put into the truss, and the members marked *PQ* and *QR* are taken out. When this is done we draw a line in the stress diagram *d*–(1) and the line *O*–(1). Where these lines intersect gives the point (1). From *e* on the load line draw a line parallel to *E*2, and from point (1) draw a dotted line parallel to the substituted member (1) (2), and where these two lines intersect gives the point (2).

At this stage the substituted member can be dispensed with, and the two original members in the roof truss can be put back into position. The point (2) now becomes point *r*, and the points *q*, *p*, and *s* can be found without difficulty. The stresses in all the members of the left-hand half of the truss can now be found by scaling from the stress diagram. If the wind was blowing from the right-hand side instead of the left side, the stresses in the members of the other half of the truss would be the same. That is, the stress in *JY* would be the same as the stress in *BM*.

Tension or Compression. To find whether a member of the roof truss is in tension or compression, proceed on the lines laid out in Chapter 15.

The sizes of the members can now be found without much difficulty by proceeding exactly on the same lines as was shown for the complete design given in Chapter 16. The number of rivets in each joint can also be found in like manner, and once

LOADS CARRIED BY ROOF-TRUSS PER SQ.FT. OF GROUND AREA COVERED,

SLATES AND BOARDS	13 LBS
PURLINS	3 "
ROOF TRUSS	4 "
TOTAL VERTICAL DEAD LOAD PER SQ.FT. OF GROUND AREA COVERED --	20 LBS
VERTICAL WIND PRESSURE PER SQ FT OF GROUND AREA COVERED --	20 "
TOTAL VERTICAL LOAD PER SQ. FT. OF GROUND AREA COVERED -	40 LBS

LOADS AT PANEL-POINTS 2,3,4,5,6,7 AND 8

EACH POINT TAKES $\frac{1}{8}$ OF TOTAL LOAD ON ROOF $= \frac{1}{8} \times (55' \times 12' \times 40 \text{ LBS}) = 3{,}300$ LBS $=$ 1·5 TONS

LOADS AT SHOES 1 AND 9

EACH POINT TAKES $\frac{1}{16}$ OF TOTAL LOAD ON ROOF $= \frac{1}{16} \times (55' \times 12' \times 40 \text{ LBS}) = 1{,}650$ LBS $=$ 0·75 TON.

STRESSES IN MEMBERS OF ROOF TRUSS.

MEMBERS		COMPRESSION TONS.	TENSION TONS.
B-M AND J-Y	RAFTER.	11·9	
C-N AND H-X	"	11.2	
D-Q AND G-U	"	10.4	
E-R AND F-T	"	9.7	
L-M AND L-Y	LOWER TIE		10.3
L-O AND L-W	"		8.8
L-S ---------	"		5.6
M-N AND X-Y	STRUT	1.3	
N-O AND W-X	TIE		1.4
O-P AND V-W	MAIN STRUT	2.6	
P-Q AND U-V	TIE		1.4
P-S AND S-Y	"		3.4
Q-R AND T-U	STRUT	1.3	
R-S AND S-T	TIE		4.9

more the student is advised that, although it may seem easy to understand the principles involved when reading about it, the best possible way to understand what he is doing is to work out for himself several examples.

Fig. 122 shows a suitable detail for the shoe joint, indicating one method of building the wall up around the shoe.

During the course of this book we have dealt with the various technical terms generally used in connection with the strength of materials and building construction problems. Such terms as stress, strain, modulus of elasticity, bearing moment, shearing forces, and double and single shear have been explained. Designs for simple wood, steel, and reinforced concrete beams have been shown, also methods of finding the stresses in and sizes for members of roof trusses have been dealt with.

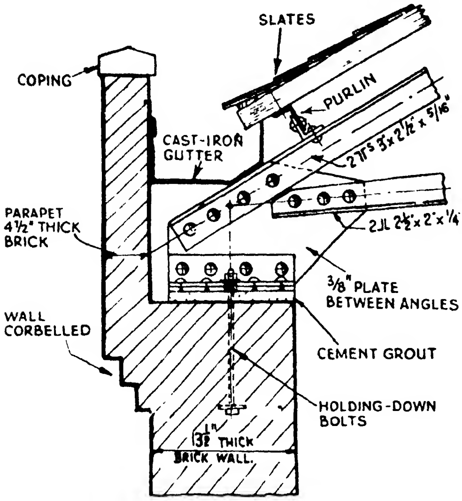


Fig. 122.

We show the layout and sections of steelwork for a building (see pages 185 and 186) is made of very light construction, with the main columns formed of two channels each 6 in. by 3 in. The roof trusses are 38 ft. span and are spaced at 13-ft. centres. It will be noticed that below the shoe of the roof truss there is a diagonal member running between the column and the horizontal tie of the roof truss. This diagonal is called a knee-brace, and its purpose is to stiffen up the column by making its effective length shorter than it would be if the knee-brace was not put in.

The walls are made of $4\frac{1}{2}$ in. brick set in cement mortar. They fit nicely between the flanges of the 6 in. columns. In

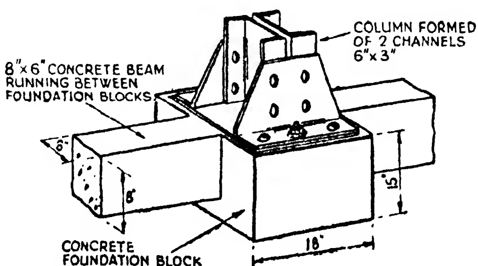


Fig. 123.

order to keep the size of the various panels small enough, various vertical members are put in between the main columns. Look at the drawing marked "elevation of building" and notice that along the full length, about half-way up the column, there is

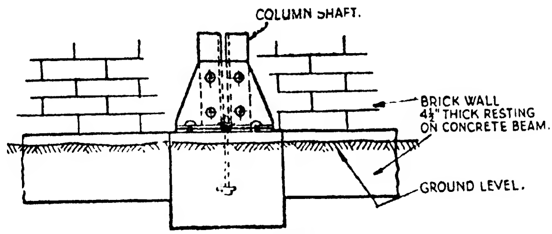


FIG. 124.

a 6-in. by 3-in. joist. Between this horizontal joist and the ground line there are various vertical members which are put in for door, window, and wall framings. Between the horizontal 6-in. by 3-in. side girts and the top of the columns, the panels would be 13 ft. by 9 ft. if the vertical 6-in. × 3-in. R.S.J. were not put in half-way between the columns; this joist reduces the size of the panel to 6½ ft. by 9 ft.

The brick panels in the end walls are also about the same size. Figs. 123 and 124 show the details at the base of the columns. Between the foundation blocks on which the main columns rest there is a concrete beam which serves two purposes. First, it is useful as a tie to connect the column foundation blocks together, a method frequently used where the ground is poor, and secondly, it serves as a base on which the 4½-in. walls can be built. A steel channel is sometimes run along the top of this continuous concrete beam and the wall is built into this.

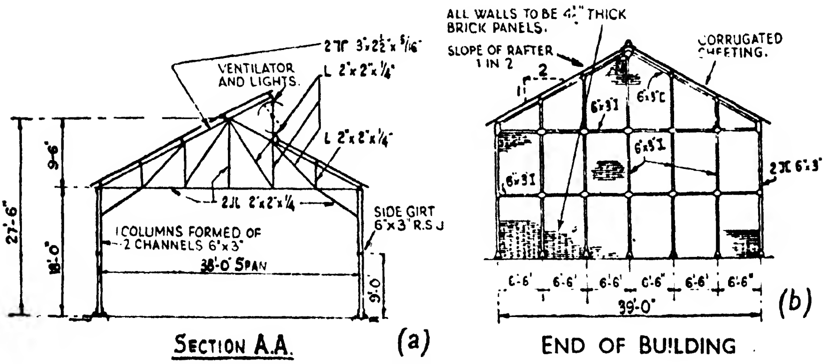


FIG. 125.

Light in the lower half of the building is provided by the windows, as shown in the elevation, and some light and ventilation is provided in the roof by using windows which swivel round a centre pin. These are shown in section *AA*, Fig. 125 (*a*).

The roof covering is corrugated sheets carried on purlins. These purlins span over a distance of 13 ft., and are made of a rolled steel joist 5 in. deep by 2½ in. wide.

To prevent the roof twisting and buckling due to wind pressure, diagonal bracings are put in each of the end bays, as shown in Fig. 126. These bracings are in the same plane as the rafters or the sloping sides of the trusses.

An enlarged view of the roof truss shoe, where it sits on top of the steel columns, is shown in Fig. 127. It will be noticed that a steel channel runs the full length of the building between the

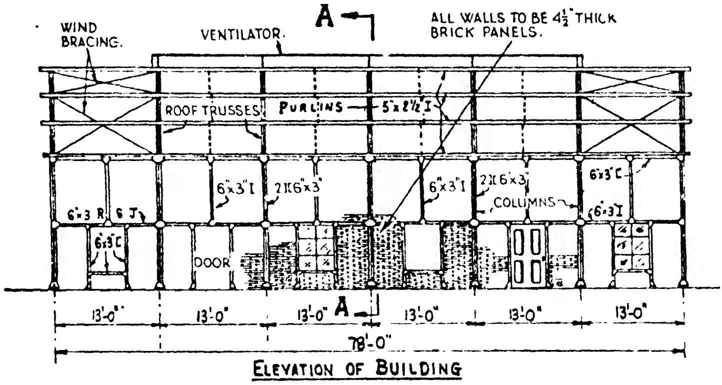


FIG. 126.

columns, and that the top of the wall is built into this channel, which also serves as stiffener between the tops of the steel columns.

The joists which run horizontally round the building split the wall height into two lengths of 9 ft. each, and this also serves as a stiffener between the columns.

The Roof. In this particular design the 6 in. by 3 in. side girts are connected to the columns by means of plates connected to their flanges. Frequently in this type of construction the horizontal beams are connected to the columns by means of angles bolted to the webs only.

As the roof trusses are spaced at 13 ft. centres, and the purlins are 5 in. deep and 2½ in. wide, round bars called sag rods are put in half-way between each pair of roof trusses. These can be made about $\frac{5}{8}$-in. diameter. They are shown dotted in Fig. 126.

Although in the sketch the doors are shown of wood, there is, of course, no reason why they should not be made of steel if so desired. If there are any rail tracks to run into the building, a doorway can easily be made in the two ends of the building, and a sliding door fitted. Along the vertical faces of the saw tooth, swivelling sash windows are shown. These can be made of standard wood frames pivoted at the centre, or they can be made so as to slide horizontally one past the other. If ventilation only is required, and not so much light, louvres could be used, or short steel plates could be substituted in the window frames instead of glass.

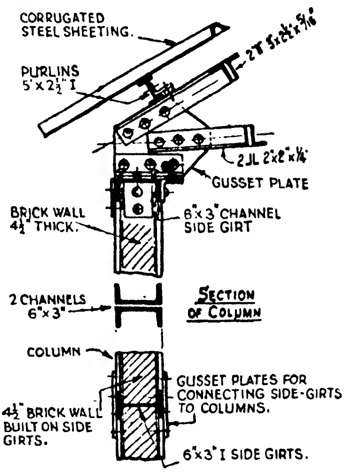


Fig. 127.

The roof covering could be of galvanized corrugated sheets about 18 gauge, which weigh about $2\frac{1}{2}$ cwt. for each 100 sq. ft. of area. At the end of the building, instead of using the standard form of roof truss, it will be noticed that the 6×3 in. joist columns are taken up to the rafters, which are made from a 6×3 in. channel with the two flanges pointing downwards. The wall runs up between these two flanges. It will be noticed that only two sections of angles are used for the roof trusses, the internal members being made from angles 2 in. $\times$ 2 in. $\times$ $\frac{1}{4}$ in. thick. The rafters are made from two angles 3 in. $\times$ $2\frac{1}{2}$ in. $\times$ $\frac{5}{16}$ in.

CHAPTER 18

RETAINING WALLS

As the name implies, a retaining wall is erected for the purpose of retaining, or holding back, either earth or water. Formerly most walls were made of the heavy gravity type, whereby the weight of the wall itself was used to resist the overturning force of the earth behind the wall.

Where dry stone walls of the type shown in Fig. 128 (*a*) are used, the top two or three layers are frequently laid in lime or weak cement mortar. Generally the width of the wall base is about one-third of the height.

Heavy Gravity Wall. A type of heavy gravity wall is shown in Fig. 128 (*b*). This construction may be either in cut

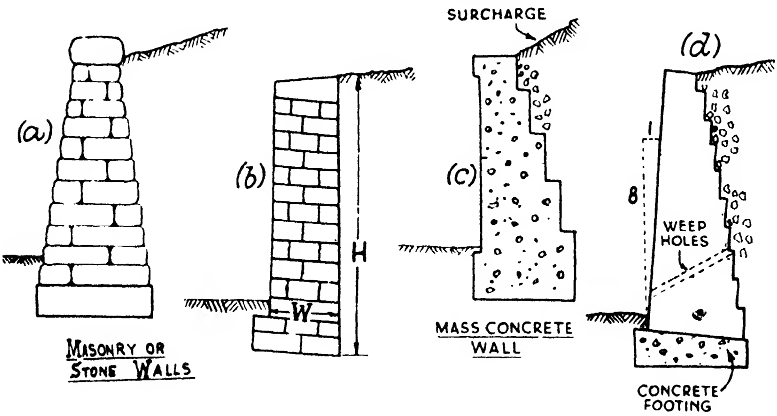


FIG. 128.

stone or brick. Trautwine, in his well-known *Engineers' Pocket Book*, gives various proportions for the thickness of such a wall. He says that when the backing is deposited loosely, or when it is dumped from lorries, the thickness of the wall, if made of cut stone or first-class large-ranged rubble set in mortar, should be 0·35 of the total vertical height of the wall. If made of brick, the thickness at the base should be 0·5 of the total height. When backing is consolidated in horizontal layers, the thickness of the wall may be somewhat reduced.

Another type of heavy gravity wall made of solid concrete is shown in Fig. 128 (*c*). As a general rule, the width of the base slab for such a wall is generally made about four-tenths of the total height. The front wall is generally slightly battered for the sake of appearance, the batter varying between about ½ in. in 12 and 1½ in. in 12.

The front face of the wall shown in Fig. 128 (*d*) is made with a definite grade or slope towards the ground. This form of construction is obviously very strong to resist overturning. Usually the slope or batter of such a wall is made not more than about 1 in 8, otherwise there is a danger of rainwater settling into the joists of the front face. A batter of 1 in 8 would, of course, mean that if the wall is 8 ft. high above the ground level, the top of the wall would be 1 ft. farther back than the base.

Many contractors are quite accustomed to building these various types of mass or gravity retaining walls, and know the thicknesses and proportions either by such rule-of-thumb methods as have just been given, or very often as a result of experience or by means of the practised eye.

In recent times there has been a distinct tendency towards building retaining walls in reinforced concrete. Considerably less material is used with this type of construction, as will easily be seen by looking at Fig. 129 (*a*). In this type of construction the weight of the earth itself rests on the bottom slab or footing, and this tends to prevent the wall overturning.

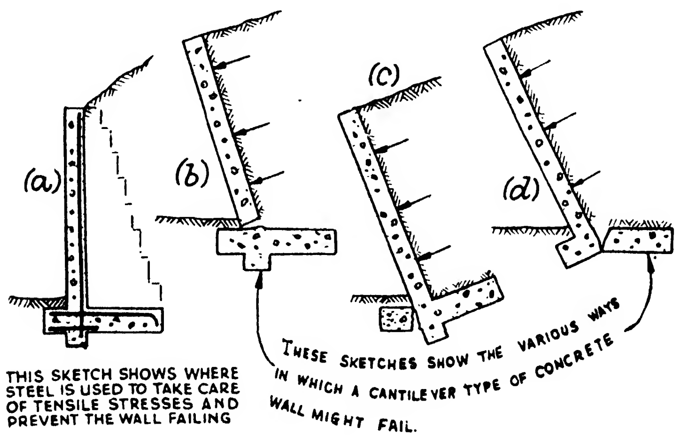


FIG. 129.

Before such a wall can be designed, it is necessary to know the pressure which is bearing against the wall tending to cause overturning. This overturning force depends upon the class of earth behind the wall, whether the ground is likely to become water-logged, and whether there is vibration through the ground caused by, say, traffic on a road or railway. We will show later on how to calculate the actual force pushing against a wall, but for the time being it will suffice if we get a thorough grasp of the various methods by which a retaining wall is likely to fail.

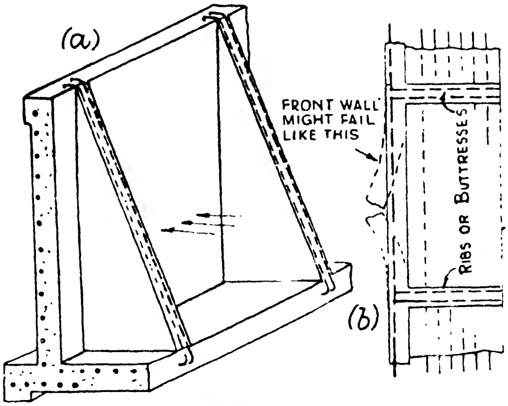
The type shown in Fig. 129 (*a*) is known as a cantilever wall, because the vertical side acts like a cantilever. It can be considered as fixed at the base, and the top of the wall as free, so that if we consider part of the wall 1 ft. long, the vertical face will really be a beam 1 ft. wide. This beam carries a weight which is the pressure of the earth behind the wall, and this pressure tends to bend the vertical face of the wall, so that if it is not properly designed, the wall may fail, as shown in Fig. 129 (*b*).

The base slab or footing may also fail at either of the two points shown in Fig. 129 (*c*) and (*d*), and in order to obviate this steel rods are put into the wall.

It is a well-known fact that concrete is very strong in compression and very weak in tension. It will be remembered that when dealing with reinforced concrete beams, steel bars are put in to resist the tensile or pulling forces, and in precisely the same way steel bars are put into reinforced concrete walls where there are tensile stresses, so that the concrete is really used to resist compression forces and steel is used to resist the tensile forces. A typical method of reinforcing and strengthening a cantilever type of wall is shown in Fig. 129 (*a*).

Reinforced Concrete. Another type of reinforced concrete wall is shown in Fig. 130 (*a*). This type is called a buttress wall, and is much used where walls are more than about 15 ft. high. The buttresses, or ribs, are generally spaced between 9-ft. and 12-ft. centres. The idea of these buttresses, or ribs, is to stiffen the wall by causing it to act as a slab which is supported along three of its edges. The vertical wall between two ribs is supported along three of the four edges, the two vertical edges being tied to the ribs by the buttresses. The bottom edge is tied to the footing slab, and only the top edge is more or less free. Sometimes a slab is run between the buttresses along the top edge of the vertical slab, in order to stiffen this edge. It will be noticed that if a wall of this type failed, it would probably do so by the vertical slab breaking down the middle and opening out, and the student should clearly see the difference between this type of wall and the cantilever wall shown in Fig. 129 (*a*).

In the buttress type of wall, the front or vertical slab can be considered as a series of beams, each supported at both ends



BUTTRESS-TYPE OF REINFORCED CONCRETE RETAINING WALL

FIG. 130.

by the buttresses, and loaded uniformly by the earth pressing behind the vertical slab. Although the various beams can be considered as uniformly loaded, this does not, of course, mean that the bottom 12 in. of the vertical carries the same pressure as the top 12 in. Obviously this is not so, as there is hardly any pressure against the vertical slab along the top, but there is a very considerable pressure against the vertical slab lower down near to the base; as a matter of fact, the pressure is continually increasing from the minimum at the top of the wall to a maximum at the base slab.

If a retaining wall is vertical and has to hold back water pressure, the water will press equally in all directions, and can be considered as acting normal, or at right angles, to the surface of the wall. In this case the pressure on the wall face would be directly proportional to the depth of the wall.

Water Pressure. One cubic foot of water weighs about $62\frac{1}{2}$ lb. At the surface of the water there will be no pressure, and at a given depth h below surface the pressure would be $62\frac{1}{2} \times h$. The centre of this pressure would be at one-third the height of the water, and the total amount of the pressure would be $0{\cdot}5 \times 62{\cdot}5 \times h^2$. This is generally written as

$$P = \frac{W \times h^2}{2}.$$

Where W is the weight of water per cubic foot
h is the depth of the water from the surface.
P is the total pressure against 1 ft. length of wall.

QUESTION 1. A dam wall is 20 ft. high. What is the pressure at the base of the wall and what is the total overturning pressure, and where does it act?

Answer. **The pressure on a square foot any depth from the top of the water will be $w \times h$.**

In this case h is the height of the wall, assuming the dam to be full, that is, 20 ft., w is the weight of 1 cu. ft. of water, say, $62\frac{1}{2}$ lb. The pressure of the water at the top of the wall will be nil, since h is nil. The pressure at the base of the wall will be $62 \cdot 5 \times 20 = 1{,}250$ lb. The pressure half-way up the wall will be $62 \cdot 5 \times 10 = 625$ lb.

It is easy to see that the pressure depends directly on the depth of the water. Therefore the pressure can be represented by a triangle in which the depth of the water is h, the pressure at the surface of the water nil, and gradually increasing all the way down until a pressure of $w \times h$ is reached at the base of the wall.

The total amount of pressure on the wall is the sum total of all the varying pressures and will be the area of the pressure triangle. This area is base x half height, or

$$wh \times \frac{h}{2}, \text{ which can be written as } \tfrac{1}{2}wh^2.$$

In the case we are considering the total pressure against the wall will be $\frac{1}{2} \times 62 \cdot 5 \times 20 \times 20 = 12{,}500$ lb. This is the total pressure of 1 ft. length of wall 20 ft. high. The pressure of water acts at right angles to or normal to the face of the wall, so that if the wall is vertical the pressure acts horizontally, and the centre of pressure below the surface of the fluid is $\frac{2}{3}h$, or, as it is generally considered, $\frac{1}{3}$ the height of the wall from the base, that is, at

$$\frac{20}{3} = 6 \text{ ft. } 8 \text{ in.}$$

In the case of a wall, which is to hold back ground, the conditions are quite different, for the ground does not act as a fluid. Consider some newly excavated ground, which has a face nearly vertical, as shown in Fig. 131 (*a*). If this ground were left exposed to the weather, sooner or later the ground would break away and take up a slope something like that shown in Fig. 131 (*b*).

This slope would remain more or less permanent, and the angle or inclination would depend upon the class of earth; for instance, if it were hard chalk, the face would probably remain pretty nearly vertical, but if it were moist vegetable earth, or

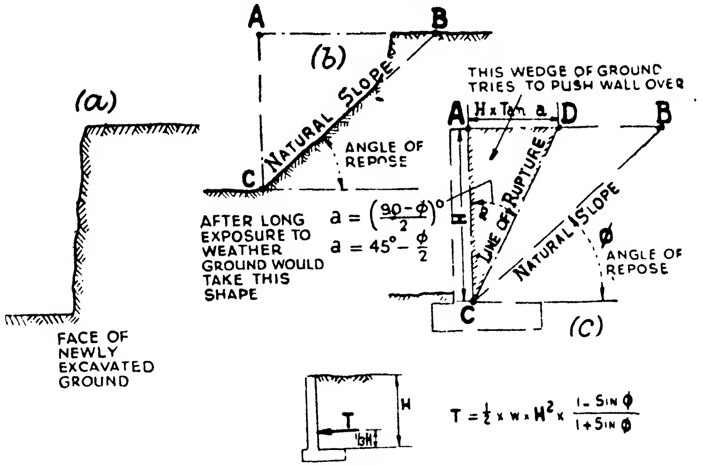


FIG. 131.

sand, the final slope would be very much flatter. This final slope is known as the natural slope, and the angle which this makes to a horizontal line is called the angle of repose. What the angle of repose is for various materials can only be found by actual experiment, or after having had considerable experience, but in general the following table will give a good idea of what could be expected.

Class of Earth.	Angle of Repose in Degrees.
Sand	25 to 35
Dry vegetable earth	30 to 35
Moist vegetable earth	40 to 45
Gravel with sand	25 to 30
Clean gravel	40 to 50

Although the natural slope of the ground is the final slope the ground would take after weathering, it must not be thought a weight of earth made up of a triangle *ABC* actually pushes against the wall. This point deserves some thought. If we consider ground which is weathered to its natural slope, and we filled on this ground a wedge of earth such as shown in *ABC*, all this earth would not slide down the surface marked *BC*. What would probably happen is that about half the earth, such as is shown by the shaded triangle *ADC* (Fig. 131 (*c*)), would try to slide down the face *DC*, and it is this amount of earth which the retaining wall has to hold back. The line *DC* is shown as the line of rupture, and this line is generally made so as to cut in half the angle between the natural slope of the ground and the vertical face of the wall. There are many different theories as to how much pressure is exercised against a wall; the particular one we have just considered is called the "wedge" theory.

Thrust against Wall. It can be said that the thrust or push against the wall when the earth is level behind the wall takes place when the line of rupture is such that the angle

$$ACD = \tfrac{1}{2}\ (90 \text{ deg.} - \phi).$$

Now the weight of a wedge of ground ACD can be found as follows:

$W = \frac{1}{2} \times w \times H^2 \times \tan$ angle

where w = Weight of 1 cub. ft. of earth

H is the height AC

W is the weight of a wedge ACD 1 ft. in length. The angle by the line of rupture is $\frac{1}{2}\ (90° - \phi)$

$$= \left(45° - \frac{\phi}{2}\right).$$

The maximum horizontal thrust produced by the shape ADC will be equal to the weight of this prism × tangent of the angle a, from which we get maximum thrust against the wall due to the weight of earth will be:

$$T = \tfrac{1}{2}wH^2 \tan^2 \frac{90 - \phi}{2}.$$

This can be rewritten in another form:

$$T = \tfrac{1}{2}wH^2 \frac{1 - \sin\phi}{1 + \sin\phi}.$$

This is the form used by Rankine for finding the thrust tending to push over a wall which has a straight face, and where the ground is level.

QUESTION 2. A retaining wall with a vertical face 20 ft. high has to hold back vegetable earth. The ground is level and the natural slope is such that the angle of repose is 30 deg. Find the maximum horizontal thrust tending to overturn the wall. Where does this overturning pressure act?

Answer.

$$\text{Thrust} = \tfrac{1}{2}wH^2 \tan^2\left(\frac{90 - \phi}{2}\right)$$

where w is the weight of 1 cu. ft. of earth, say 90 lb.

H is the height of wall = 20 ft.

ϕ – angle of repose = 30°

$$T = \tfrac{1}{2} \times 90 \times 20 \times 20 \times \tan^2\left(\frac{90 - 30}{2}\right)$$

$$= 18{,}000 \times \tan^2 30°.$$

The tan of 30° = 0·5774.

$$T = 18{,}000 \times 0{\cdot}5774 \times 0{\cdot}5774 = 6{,}000 \text{ lb.}$$

This is the total pressure on 1 ft. length of wall 20 ft. high. This pressure acts at a distance of $\frac{1}{3}H$ from Base = 6 ft. 8 in., and its direction is horizontal.

QUESTION 3. We shall now check up to see if the Rankine formula gives the same results for the question 2.

$$T = \frac{1}{2}wH^2\frac{1 - \sin\phi}{1 + \sin\phi}$$

$$= \frac{1}{2} \times 90 \times 20 \times 20\frac{1 - \sin 30^\circ}{1 + \sin 30^\circ}.$$

The sin of 30°

$$= 0{\cdot}5$$

$$T = 18{,}000 \times \frac{1 - \frac{1}{2}}{1 + \frac{1}{2}}$$

$$= 18{,}000 \times \frac{\frac{1}{2}}{1\frac{1}{2}}$$

$$= 18{,}000 \times \frac{1}{3} = 6{,}000 \text{ lb.}$$

CHAPTER 19

REINFORCED CONCRETE WALL

About 150 years ago Coulomb developed a theory that the pressure trying to overturn a retaining wall was caused by a wedge of earth. This wedge, as we showed in the previous chapter, is very much less than the wedge formed with one side the face of the wall and the other side the natural slope of the ground; as a matter of fact, it can be shown that the maximum thrust against a wall which has a vertical face and where the surface of the ground is horizontal and level with the top of the wall, occurs when the wedge is considered as having an inclined side which bisects the angle between the wall face and the natural slope of the ground. The sloping line of the wedge is known as the line of rupture.

Let us try and form a clear picture of the wedge force which is causing a horizontal thrust against the wall. Looking at Fig. 132 (*a*), we can see that if the retaining wall was removed, some of the ground would immediately slip down. We also know that there is friction between the various surfaces, and that this friction is equal to a normal pressure multiplied by some constant which is known as the coefficient of friction.

BC is the natural slope of the ground, and the amount of friction between this surface of the solid ground and the underside of the surface *BC* on the wedge, would be just sufficient to prevent the wedge *ABC* from sliding forward in one solid mass, and therefore, provided the wedge-shaped ground *ABC* did not break up, there would be no horizontal thrust on the wall at all. What would happen in practice is that the first movement of ground after the wall has been removed would be that a slip would take place along what is known as a line of rupture and it can be proved the maximum horizontal thrust on the wall occurs when this line of rupture bisects, or divides equally in two, the angle formed between the back of the wall and the natural slope of the ground.

Horizontal Thrust. We can now find graphically the amount of horizontal thrust and the overturning moment which results from this horizontal thrust. Consider the back of the wall to be vertical, and the ground surface horizontal, and level with the top of the wall. Fig. 132 (*a*) represents these conditions, and the angle of slope in this case is assumed to be 30 deg. The line *BO* shows the plane of rupture, and the wedge of earth is *ABO*. This wedge tries to slide vertically downwards through its centre of gravity *G*.

Draw a line *ED* through *G*, representing to scale the weight of a prism of earth formed by the triangle *ABC*, and 1 ft. in length. Where this vertical strikes the line *OB*, draw a line *DH* normal, or at right angles to *OB*. (If the surface *OB* was perfectly smooth and without friction, the reaction would be normal or at right angles to line *BO*, that is, in the direction of the line *DH*. As there is friction, the reaction acts along the line *FD*, which is drawn so that the angle *FDH* is the same as the angle of repose of the natural ground.) Draw a line *DF*, and from *E* draw a horizontal line to find the point *F*. *FE*

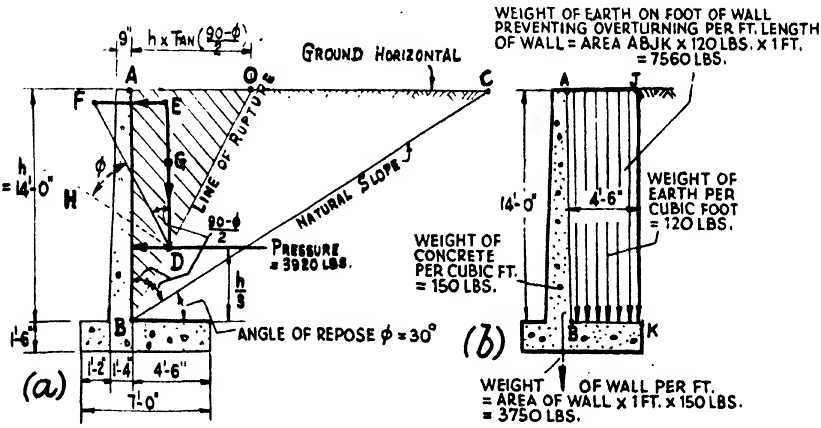


FIG. 132.

measured to the same scale as the vertical line *ED* will give the horizontal component or thrust which tries to overturn the wall. This force acts at a distance of $\frac{1}{3}h$ from the top of base.

The weight of the wedge *ABO* 1 ft. long is easily found. Since the line *BO* bisects the angle $90 - \phi$ at *B*, it follows that the angle *ABO* must be half $90 - \phi$. The area of the triangle *ABO* is easily seen to be

$$\frac{\text{Length } AO \times \text{Length } AB}{2}.$$

Since the surface of the ground is horizontal, the angle at *A* is 90 deg., and the length *AO* will be the tangent of the angle at *B* multiplied by the length *AB*, but the length $AB = h$, the height of the wall, and the angle at $B = \frac{1}{2}(90 - \phi)$.

We can now find the weight of the prism formed by a wedge of earth ABO, which is 1 ft. long (consider 1 ft. run of wall).

$$\text{Area of prism} = \frac{h}{2} \times \left(h \tan \frac{90 - \phi}{2}\right)$$
$$= \frac{h^2}{2} \times \tan \frac{90 - \phi}{2}.$$

Volume of a prism 1 ft. long will be area $AOB \times 1$, and the weight of the prism will be volume $\times\ w$ (w being the weight of 1 cub. ft. of earth).

$$\text{This equals } \frac{wh^2}{2} \times \tan \frac{90 - \phi}{2}.$$

This is the weight which is represented in the drawing by the line ED. Now the line EF represents the horizontal thrust, and the length of line EF can easily be found by multiplying length ED by the tangent of the angle at D. This angle is $\frac{90 - \phi}{2}$, from which we see that

$$EF = ED \times \tan \frac{90 - \phi}{2}$$

$$\text{but } ED = \frac{wh^2}{2} \times \tan \frac{90 - \phi}{2}.$$

$$\therefore\ EF = \left(\frac{wh^2}{2} \times \tan \frac{90 - \phi}{2}\right) \times \tan \frac{90 - \phi}{2}$$

$$\therefore\ EF = \frac{wh^2}{2} \times \tan^2 \frac{90 - \phi}{2}.$$

This can be written as

$$T = \tfrac{1}{2} wh^2 \tan^2 \left(45 - \frac{\phi}{2}\right)$$

where T = the maximum thrust on the wall with a vertical face, and with the ground horizontal and level with the top of the wall

w is the weight of 1 cu. ft. of earth

h is the weight of the wall in feet

ϕ is the angle of repose, or the angle formed by the natural slope of the ground.

For this particular case, Rankine by quite different methods arrives at exactly the same result, but the Rankine formula is generally written in another form—

$$T = \tfrac{1}{2} wh^2 \frac{1 - \sin \phi}{1 + \sin \phi}$$

The value $$\frac{1 - \sin\phi}{1 + \sin\phi}$$

is the trigonometrical reduction of $\tan^2\left(45 - \frac{\phi}{2}\right)$.

The thrust or pressure acts normal or at right angles to the back of the wall, and at a height of $\frac{2}{3}h$ below the surface, or $\frac{1}{3}h$ up from the base.

The student should be careful to remember that the formula which has just been given applies only where the ground is horizontal and at a level with the top of the retaining wall.

Design of Retaining Wall. Having now found the formula which gives the thrust or force tending to overturn a wall, we shall proceed to show you its use by making a complete design of a cantilever type of retaining wall.

PROBLEM. Design a reinforced concrete retaining wall of the cantilever type for the following conditions: Wall, 14 ft. high; angle of repose, 30 deg.; weight of earth, 120 lb. per cubic foot; ground horizontal and level with the top of wall.

Answer. Generally the base, or foot, is made about half the height of the wall, and the toe, or footing in front of the wall, is usually less than the heel, or part of the footing behind the wall. We can therefore start with rough overall dimensions as shown in Fig. 132 (*a*). We showed in the previous chapter that the vertical part of the wall acts like a cantilever, and if the previous Chapters dealing with bending moments on beams have been followed, there will be no difficulty in appreciating this point.

There are two main forces to be considered. One is the horizontal thrust trying to push the wall over (see Fig. 133 (*b*)), while the other is the downward weight of the wall itself and the weight of the earth which rests on the heel of the wall (see Figs. 132 (*b*) and 133 (*a*)). By taking moments about the lower corner of the toe, we can find the centre of gravity of the vertical loads (refer Chapter 7).

The weight of the reinforced concrete wall will be taken at 150 lb. a cubic foot, and the weight of earth is given in the problem as 120 lb. a cubic foot. Reference to Fig. 133 (*a*) will show that the centre of gravity of all the downward loads acts at a distance of 4 ft. from the toe of the wall. This force or load is what keeps the wall from overturning. In like manner, the horizontal thrust against the wall produces a moment which, of course, is a force acting at a distance, and this moment tries to overturn the wall. In our case the overturning moment amounts to the thrust × distance of centre of pressure above base.

$$\text{Thrust} = \tfrac{1}{2}wh^2\frac{1 - \sin\phi}{1 + \sin\phi}$$

and the distance at which this acts is $\frac{h}{3} + t$

where t is the thickness of base slab,

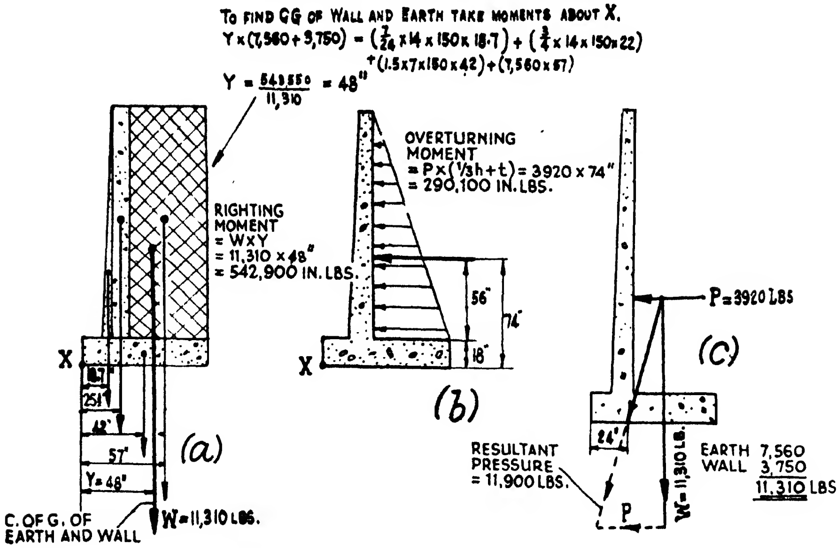


FIG. 133.

$$\text{Overturning moment} = \left(\tfrac{1}{2}wh^2\frac{1 - \sin\phi}{1 + \sin\phi}\right) \times \left(\frac{h}{3} + t\right)$$

$$h = 14 \text{ ft.}$$
$$t = 1{\cdot}5 \text{ ft.}$$
$$w = 120 \text{ lb. a cubic foot}$$
$$\sin\phi = \sin 30 \text{ deg.} = 0{\cdot}5.$$

$$\text{Overturning moment} = \left(\frac{1}{2} \times \frac{120}{1} \times 14^2 \times \frac{0{\cdot}5}{1{\cdot}5}\right) \times \left(\frac{14}{3} + 1{\cdot}5\right).$$

$$\text{Overturning moment} = 3{,}920 \times 6{\cdot}16$$
$$= 24{,}190 \text{ ft.-lb.}$$
$$= 290{,}100 \text{ in.-lb.}$$

The moment of the downward forces preventing this overturning amount to—

Total downward load × Distance of *CG* from toe

$$= 11{,}310 \text{ lb.} \times 4 \text{ ft.}$$
$$= 45{,}240 \text{ ft.-lb.} = 542{,}900 \text{ in.-lb.}$$

These figures show that the wall will not overturn. Graphically the resultant of the downward forces and the horizontal thrust is shown in Fig. 133 (*c*), and this resultant intersects the base slope at a distance of 2 ft. from the toe.

Design of Vertical Wall.—Consider 1 ft. run of wall, and then imagine the wall to be a cantilever beam 1 ft. wide, 14 ft. long, and loaded in the manner shown in Fig. 134. The load varies from nothing at the top of the wall, which is the free end of the beam, to a maximum at the base of the wall, which is the point of the support. The total area of this pressure

diagram will be $h \times \frac{W}{2}$, and this must equal the total thrust on the wall, which is—

$$\text{Thrust} = \tfrac{1}{2}wh^2\frac{1 - \sin\phi}{1 + \sin\phi}$$

$$\text{Thrust} = \tfrac{1}{2} \times 120 \times 14^2 \times \frac{0{\cdot}5}{1{\cdot}5}$$

$$\text{Thrust} = 60 \times 196 \times \tfrac{1}{3}$$

$$\text{Thrust} = 3{,}920 \text{ lb.}$$

$$3{,}920 \text{ lb.} = h \times \frac{W}{2}$$

h is 14 ft.

$$\frac{W}{2} = \frac{3{,}920}{14} = 280 \text{ lb.}$$

$$W = 560 \text{ lb.}$$

The bending moment at the foot of the wall can now easily be found by taking the total area of the pressure diagram, which amounts to the total horizontal thrust, or 3,920 lb. Since the load is not uniformly distributed, it can be considered it will act through the centre of gravity of this pressure, which is $\tfrac{1}{3}h$ from the base, and the total bending moment will be—

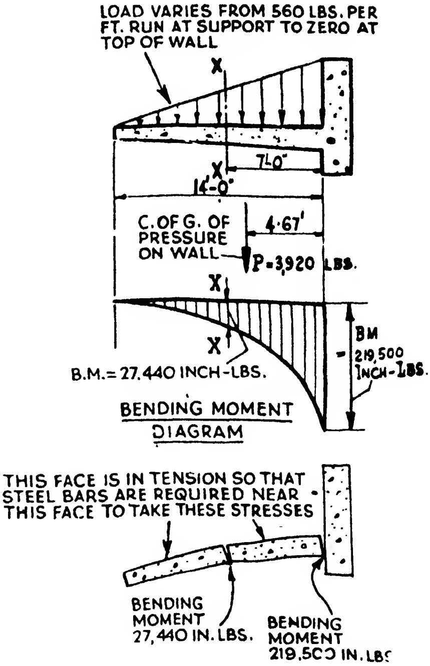


FIG. 134.

$$3{,}920 \text{ lb.} \times \tfrac{1}{3}h = 3{,}920 \times \frac{14}{3} \text{ ft.-lb.}$$

This equals $3{,}920 \times \frac{14}{3} \times 12 = 219{,}500$ in.-lb.

It is easy to see that the bending moment on the beam will get less the farther away we get from the point of support, and therefore we shall require to use less steel.

Bending Moment. Consider the bending moment a half-way up the wall, that is, at a height of 7 ft. above the base slab. Here the triangle of pressure or load will vary from nothing at the free end to 280 lb. at the section we are considering. The total thrust, therefore, on the top 7 ft. of the wall will be—

$$7 \text{ ft.} \times \frac{280}{2} = 980 \text{ lb.}$$

This will act at a distance of $\frac{7}{3} = 2\tfrac{1}{3}$ ft., from which it is easy to see that the bending moment at section xx will be

$$980 \text{ lb.} \times \frac{7}{3} \times 12 = 27{,}440 \text{ in. lb.}$$

Steel Reinforcement Required.—In Chapter 13 we showed that if the steel had a safe stress of 16,000 lb. a square inch, and the concrete a safe stress of 600 lb. a square inch, the resistance strength of the concrete would be the same as the resistance strength of the steel, if the amount of steel used was

$$0{\cdot}00675 \times B \times d$$

where B is the breadth of the beam
d is the effective depth of the beam.

It has been shown in Chapter 13 that the resisting moment, if the area of steel is $0{\cdot}00675 \times B \times d$, amounts to—

$$RM = 95 \times B \times d^2.$$

In this case we are considering 12 in. run of wall, and this is the breadth of the beam. We can therefore find the effective depth as follows:

RM must be equal to, or greater than, the B.M.

$$RM = 95 \times B \times d^2$$

$$\text{If } RM = \text{B.M., we get—}$$

$$d^2 = \frac{\text{B.M.}}{95 \times B}$$

$$d = \sqrt{\frac{\text{B.M.}}{95 \times B}}$$

$$d = \sqrt{\frac{219{,}500}{95 \times 12}}$$

$$= \sqrt{194}. \quad \text{Say 14 in.}$$

$$\text{Area of steel} = 0{\cdot}00675 \times 12 \times 14$$

$$= 1{\cdot}13 \text{ sq. in.}$$

A bar $\frac{7}{8}$ in. diameter has an area of 0·6 sq. in. Therefore two bars $\frac{7}{8}$ in. diameter will be sufficient to give the required area of steel, and we shall use for the reinforcing at the base of the wall $\frac{7}{8}$ in. diameter bars spaced at 6-in. centres. (See Fig. 145.)

Half-way up the wall the bending moment has been shown to be 27,440 in.-lbs. The thickness of the wall at this position would be $12\frac{1}{2}$ in., and the effective depth of the beam, which is the distance from the front of the wall and the centre of the steel would be $10\frac{1}{2}$ in. Obviously there is no point at all in carrying all the steel to the top of the wall, and very often half the bars are stopped half-way up the wall.

In the case of walls 20 ft. or more in height, three changes of steel area may be made, but for walls 14 ft. high, perhaps two changes would be sufficient. Cutting out half the steel will mean that every alternate bar will be cut off so that the bars will now be spaced at 12-in. centres instead of 6-in. centres. The following check will show how this affects the strength of the wall: Remember that we really have a beam which is 12 in. wide, the effective depth is $10\frac{1}{2}$ in., and the amount of steel reinforcing is one $\frac{7}{8}$ in. diameter bar, which has an area of 0·6 sq. in. Position of the neutral axis can be found by the formula

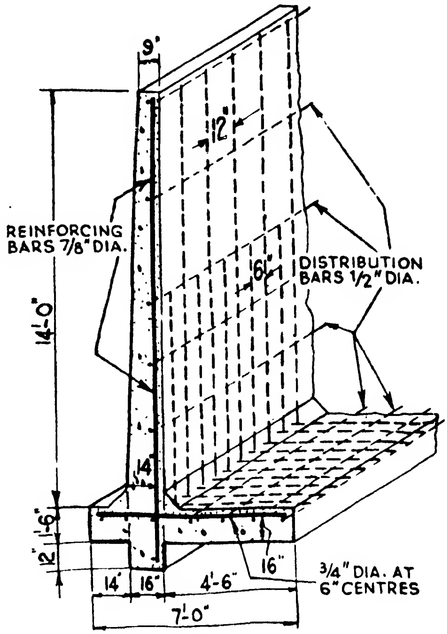


FIG. 135.

$$n = \sqrt{2pm + (pm)^2} - pm$$

where m is the modular ratio = 15

p is the ratio of steel to concrete

n is ratio between distance of neutral axis from front wall to effective depth

These terms were explained in Chapter 13.

$$p = \frac{0{\cdot}6}{12 \times 10{\cdot}5} = \frac{0{\cdot}6}{106} = 0{\cdot}005$$

$$\therefore\ n = \sqrt{2 \times 0{\cdot}005 \times 15 + (0{\cdot}005 \times 15)^2} - 0{\cdot}005 \times 15$$

$$n = 0{\cdot}320$$

$$a = 1 - \frac{1}{3}n = 0{\cdot}89$$

where a is ratio of lever arm to effective depth of beam.

The strength of the concrete to resist bending—

$$M_c = \tfrac{1}{2}f_c \times n \times ja(Bd^2)$$
$$f_c = 600 \text{ lb.}$$

$$\therefore\ M_c = \tfrac{1}{2} \times 600 \times 0{\cdot}32 \times 0{\cdot}89 \times (12 \times 10{\cdot}5^2)$$
$$M_c = 113{,}000 \text{ in.-lb.}$$

Strength of the steel—

$$M_s = p \times f_s \times j(Bd^2)$$
$$= 0{\cdot}005 \times 16{,}000 \times 0{\cdot}89 \times 12 \times 10{\cdot}5^2$$
$$M_s = 94{,}200 \text{ in.-lb.}$$

The lesser of these two figures determines the strength of the wall.

Since we have proved that the bending moment at this point is only 27,440 in.-lb., the wall will be amply strong if $\frac{7}{8}$-in. bars spaced at 12-in. centres are used from half-way up the wall to the top of the wall.

Design of Base Slab. Fig. 136 (*a*) shows the upward pressure which the ground must exert because of the vertical component of the resultant. This vertical component is 11,310 lb., as shown in Fig. 133 (*c*), and acts at a distance of 2 ft. from the front edge of the wall.

The vertical loading and the downward pressure which results therefrom is shown in Fig. 136 (*b*). The total net pressure on the

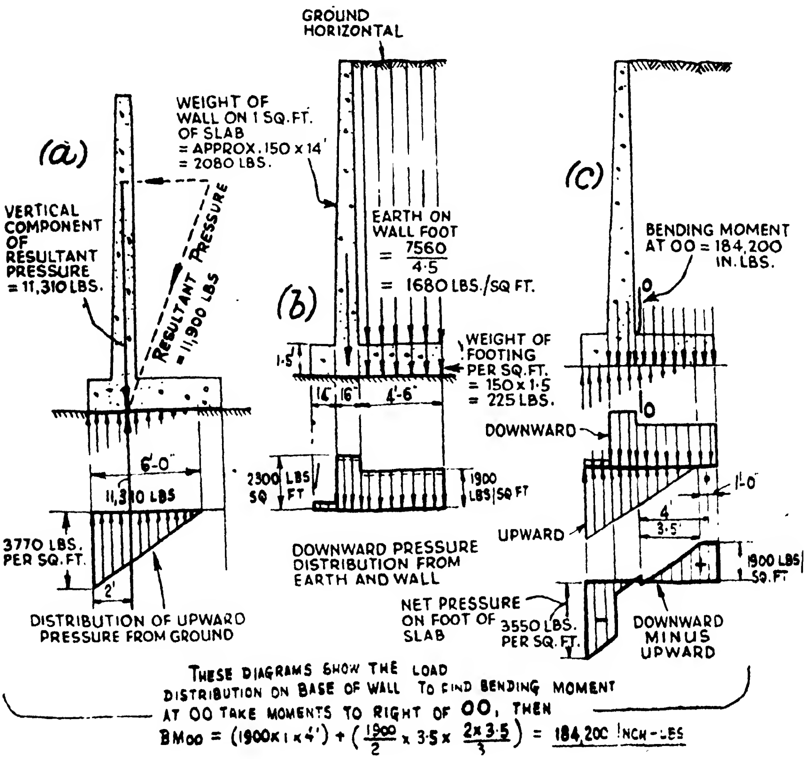


FIG. 136.

base slab is the difference between the downward pressure of the wall and the earth and the upward pressure of the ground beneath the wall. The critical point is at section *oo* in Fig. 136 (*c*), and the calculations shown on the drawing prove that the bending moment at this section is 184,200 in.-lb.

Two Cases. There are now cases open to us. We can either assume an economical percentage of steel and design for a thickness of concrete in exactly the same way as we found the required thickness of the vertical wall, or we can assume a thickness of concrete and find the required amount of steel to resist the bending moment. We shall assume the concrete footing to be 1 ft. 6 in. thick. It will be near enough to assume that the effective lever arm or distance between centres of gravity of the steel area and the concrete area are seven-eighths of the effective depth of the beam. The effective depth is the distance from the face of the concrete to the centre of steel, so that if the steel is placed 2 in. below the top face of the concrete slab, the effective depth of the beam will be 16 in. The effective lever arm would be approximately seven-eighths of this, or 14 in.

Let A equal the area of steel required in 1 ft. of length.

$$\text{Then } A = \frac{\text{B.M.}}{14 \times 16{,}000} = \frac{184{,}000}{14 \times 16{,}000} = 0{\cdot}825.$$

A $\frac{3}{4}$-in. diameter bar has an area of 0·44 sq. in., and a $\frac{7}{8}$-in. diameter bar an area of 0·6 sq. in. You can therefore use either $\frac{3}{4}$-in. bars spaced at 6-in. centres, or $\frac{7}{8}$-in. bars spaced at 8-in. centres. Since the vertical bars are spaced at 6-in. centres, we shall use $\frac{3}{4}$-in. bars at the same spacing for the base slab. The bars can be arranged as shown in Fig. 135.

CHAPTER 20

FOUNDATIONS AND FOOTINGS

We shall now consider various kinds of building foundations and footings. For footings under brick walls, as shown in Fig. 137 (*a*), the concrete bed should be at least 4½ in. wider than the footing of the wall on each side and generally not less than 9 in. thick. For heavy buildings, if there is any doubt as to the reliability of the sub-soil, trial holes should be dug and the underlying strata examined.

Where buildings are of a timber construction and in ground which is water-logged, the footing is sometimes made of the type shown in Fig. 137 (*b*). This sort of ground is frequently met alongside rivers and canals.

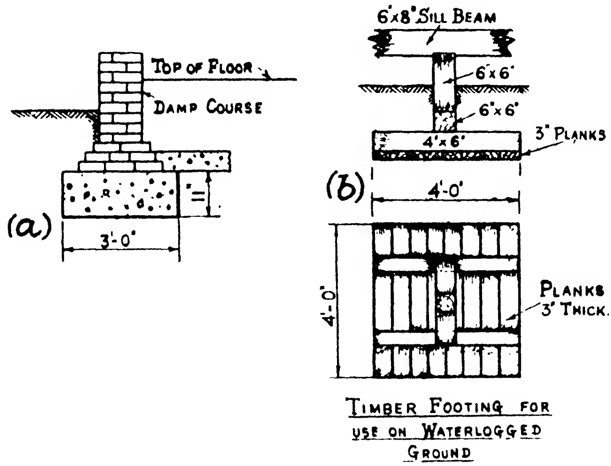


FIG. 137 (*a* and *b*).

Where the ground is soft to a great depth, a wide trench filled with concrete is sometimes used to distribute the weight of wall over the sufficiently large area. Frequently it is necessary to drive wooden piles into the ground to increase the load-carrying capacity of the concrete footing. In soft ground, the supporting power of the wood pile depends more on the skin friction than anything else. If the ground is patchy and has alternative soft and hard layers, it may be necessary to put a cast iron point on to the end of the piles as shown in Fig. 138. To prevent the

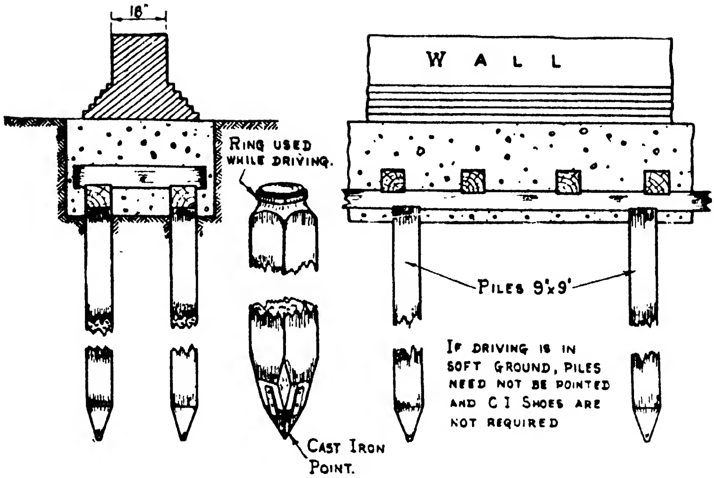


FIG. 138.

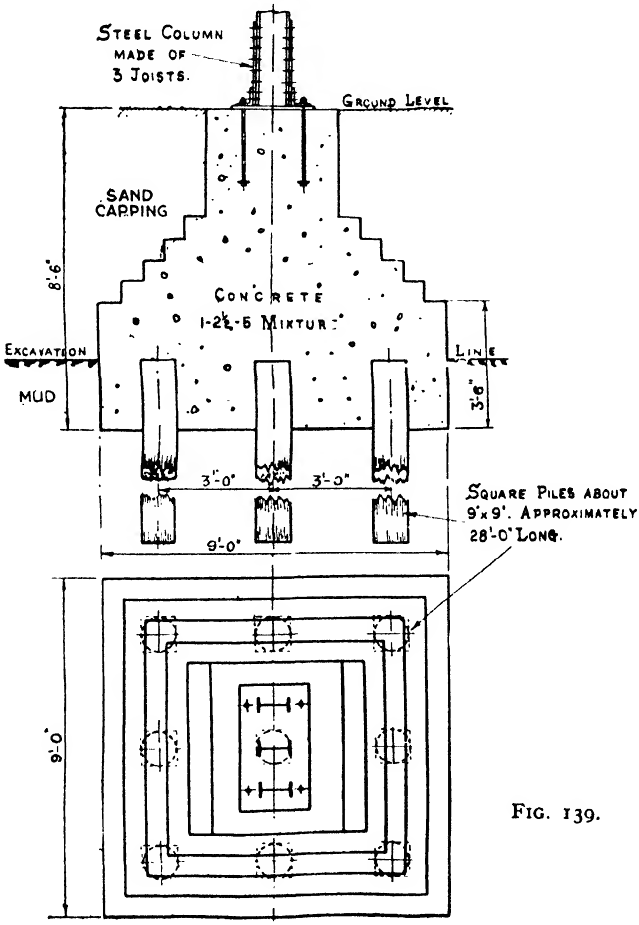


FIG. 139.

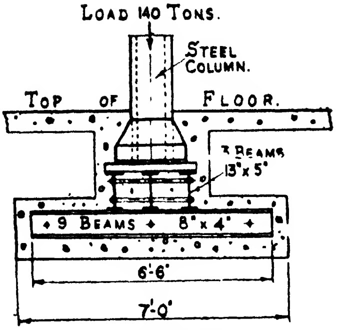


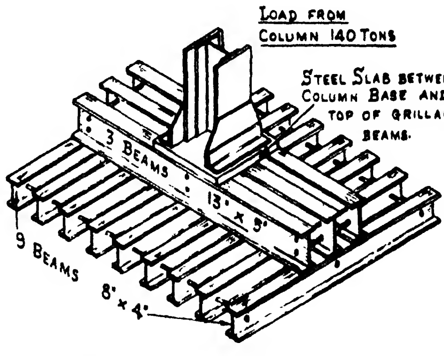
Fig. 140.

head of the pile spreading when it is being driven, a steel ring is sometimes used.

As an alternative to the type of timber footing shown in Fig. 137 (*b*), two layers of little planks are sometimes used for carrying walls in soft ground.

In buildings where the steel columns carry heavy loads and where the ground is too soft to carry the necessary weight, a foundation block of plain concrete carried on heavy square or round wooden piles, are made with pointed ends as shown in Fig. 139. The concrete in this case is unreinforced. In modern practice concrete piles are frequently used instead of timber. In order to spead the load over a sufficiently large area to prevent the footing sinking into the ground, foundations of the types shown in Figs. 140 and 141 are sometimes used. It will be seen that, in the steel column itself, there is a heavy steel slab and immediately under this a set of rough steel beams which are in turn carried on a lower set of small beams. Gas pipes are often used as separators. Each layer of beams is called a tier so that the grillage which is shown is known as two-tier grillage.

We now draw attention to the foundations shown in Fig. 142 (*a*). This type has been frequently used where there is heavy frost or where there is good ground at some feet below the surface. A square concrete block of sufficient area to keep the bearing pressure within reasonable limits is used as a base, and on this a plain concrete or masonry board is built on which the steel column is carried. Two very common types of foundations for steel columns are shown in Figs. 142 (*b*) and 142 (*c*). In recent years the reinforced concrete has come very much into the picture on building



Type of Steel Beam Grillage Footing under Heavy Loads

Fig. 141.

construction work. In Fig. 143 (*a*) we see the footing with a sloping top, and in Fig. 143 (*b*) a similar kind of footing with a flat top. For practical reasons the flat top is preferred. Steel bars running both ways are placed under the concrete to prevent failure which might otherwise take place, as shown in Fig. 144. Sometimes the plain concrete footings of the type shown in Fig. 145 are used; here the foundation block is stepped in order to save concrete. Fig. 146 shows timber footing.

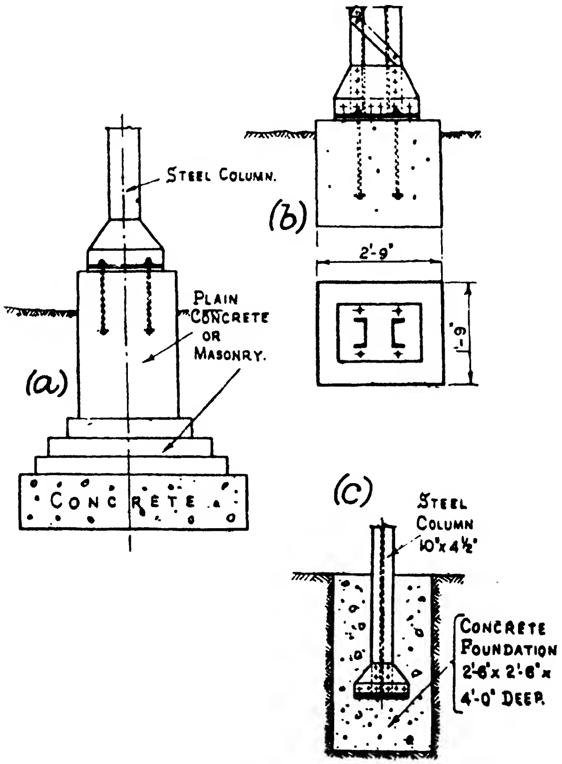
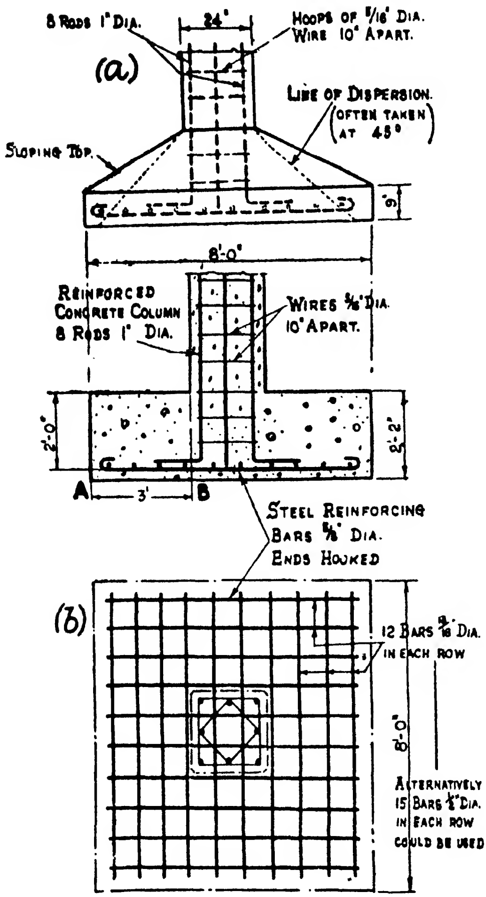


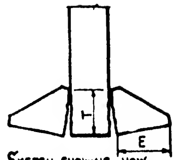
FIG. 142.

Concrete Mix. Concrete is a mixture of cement, sand and aggregate. The most common mix is 1 : 2 : 4. This has already been explained. The practical builder will not be very long in finding that this mix of concrete is not watertight. Concrete roofs are frequently a source of trouble unless they are made watertight either by some form of asphalt or other special waterproofing compound. In some cases trouble has been overcome by heating the flat concrete surface and applying hot paraffin



Plan of Footing

FIG. 143.



Sketch showing how Concrete footing may fail

FIG. 144.

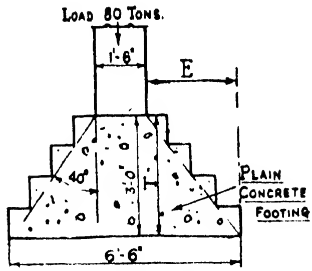
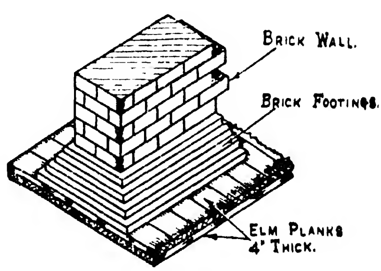


FIG. 145.



Timber Footing for Carrying Walls on poor waterlogged Ground

FIG. 146.

wax. The melted wax settles into the concrete and makes the roof watertight. There are quite a number of proprietary articles to be applied as a varnish which are used to make concrete watertight. Sometimes it is desirable to have a large mass, as in the case of the concrete foundation blocks in which case weaker mix than 1 : 2 : 4 is satisfactorily used. In other cases it is desirable that columns and floor beams should be as small as possible, in which case a richer mix can be used. A mix of 1 : 1½ : 3 is much stronger than a 1 : 2 : 4 mix and is almost watertight.

Chains. There is a rhyme easy to remember which gives within very reasonable limits the safe lifting capacity of chains. It runs like this:

"Doubling the square of the chain in eighths gives a safe load in hundredweights."

Suppose we wish to find the safe load which an iron chain made of round bar ⅜ in. thick will safely lift. We proceed as follows. First find the number of eighths in the chain metal. In this case it is obviously three. To square it, we multiply this three by itself, so that three times three = 9. We then double this nine and find eighteen, which is the safe load the chain will lift in hundredweights. By similar reasoning, the safe lifting capacity of a chain made from ½-in. iron would be the number of eighths, in this case it is four, four times four = 16. This doubled gives thirty-two, which is the safe lifting capacity of chain. If the chain is in a particularly good condition and the load to be lifted a dead load, it is safe to say that the chain can be used to lift one-quarter more than the amount which results from the above simple rule. If, on the other hand, the chain has not been annealed for some time, and if the crane driver is likely to lift the load with a shock, then the safe lifting capacity of the chain should be taken as about one-quarter less than the figure which results from the rule given above.

Wood Posts. Although steel has very largely superseded timber in this country, there is still a good deal of timber used on chemical plants, where acid would corrode steel. In the United States, Scandinavia, and South America wood construction is still used on a very large scale.

Wood is not so reliable as steel, and its strength depends on the kind of wood, quality, and dryness. Unseasoned timber is much weaker than dry timber (timber is called dry when it has not more than about 15 per cent of moisture. Unseasoned wood frequently contains 40 to 50 per cent of moisture).

As the strength of wood varies within very wide limits, it is useless to attempt the same closeness in design as can be done in steel construction.

Radius of Gyration. The approximate radius of gyration for various sections is shown in Fig. 147.

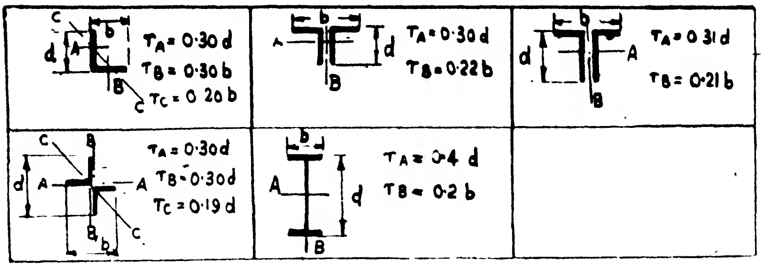


FIG. 147.

Eccentric Loading on Pier. A pier is loaded as shown in Fig. 148. It is 4 ft. high. Find the intensities of stress on

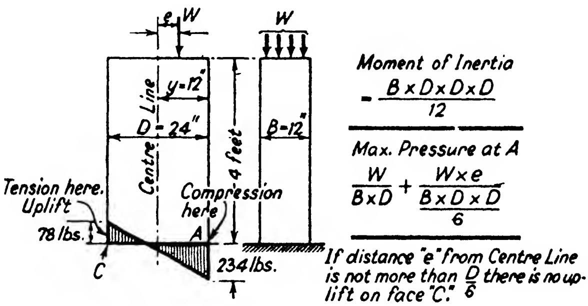


FIG. 148.

the faces of the pier nearest and farthest away from the load. What is the maximum eccentricity of load if no tension is to occur in pillar.

$$\text{Stress} = \frac{W}{A} \pm \frac{W \times e \times y}{I}$$

$$I \text{ for a rectangle} = \frac{B \times D^3}{12} = \frac{12 \times 24^3}{12} = 13{,}820 \text{ in.}^4$$

$A = B \times D = 12 \times 24 = 288$ sq. in.
$y = 12$ in.
$e = 8$ in.

Then stress $= \dfrac{10 \times 2{,}240}{288} \pm \dfrac{10 \times 2{,}240 \times 8 \times 12}{13{,}820}$

$$= 78 \pm 156 = +234 \text{ or } -78.$$

Tension on side furthest from load . . 78 *lbs.*
Compression on side nearest load . . 234 *lbs.*

If no tension is to occur, then

$$\text{Stress} = \frac{W}{A} - \frac{W \times e \times y}{I} = 0$$

$$= 78 - 19{\cdot}5e = 0, \text{ from which } e = \frac{78}{19{\cdot}5} = 4 \text{ in.}$$

Therefore maximum eccentricity for no tension to occur = 4 in.

CHAPTER 21

WOOD AND CAST-IRON COLUMNS. WELDING

Cast-Iron Columns. There is so much variation in the compressive strength of different cast irons that it is almost impossible to get a formula for the safe stress per square inch which can be considered accurate.

In many practical cases the end-fixing of cast-iron columns can be considered as being something between (one end fixed and one hinged) and (both ends fixed).

Any of the following formulas are accurate enough for cast-iron columns design in ordinary circumstances:

(1) Safe stress per square inch $= 9{,}000 - \dfrac{40L}{R}$ . . . New York.

(2) Safe stress per square inch $= 10{,}000 - \dfrac{60L}{R}$ . . . Chicago.

(3) Safe stress per square inch $= \dfrac{5}{1 + \dfrac{1}{5{,}000}\left(\dfrac{L}{R}\right)^2}$ . . . Rankine-Gordon.

(4) Safe stress per square inch $= 5{\cdot}5 - \dfrac{L}{20R}$ . . . London.

where L is length of column in inches.

R is least radius of gyration in inches.

Round hollow columns are the most common form of columns, but square and I section are also used because of the difficulty of making connections to round columns. The I section has also some advantage over either hollow round or hollow square because no core is required and brickwork can be built up to it in the same manner as is done with steel columns. The length of a cast-iron column should not exceed 15 times the least side, although sometimes 20 times the diameter is allowed. The ratio of $\dfrac{L}{R}$ should not exceed 70; with hollow round or hollow square columns great care is necessary to see that the thickness of metal does not vary owing to the movement of the core.

Calculate the safe load which can be carried by a round cast-iron column 10 in. outside diameter, 8 in. inside diameter

12 ft. long. The radius of gyration will not vary whichever formula is used, nor will the area of the section.

$$D = \frac{10 + 8}{2} = 9 \text{ in.}$$

For Round Hollow Columns. Approximate radius of gyration

$$= 0{\cdot}35D$$
$$= 0{\cdot}35 \text{ in.} \times 9 \text{ in.} = 3{\cdot}15 \text{ in.}$$

D is mean diam. = hole + thickness of metal.

$$\text{Correct Radius of Gyration} = \sqrt{\frac{\text{Moment of inertia}}{\text{Area}}}.$$

Moment of Inertia for Hollow Cylinder

$$= \frac{\pi(D^4 - d^4)}{64} = \frac{22}{7} \times \frac{10^4 - 8^4}{64} = \frac{22}{7}$$
$$\times \frac{(10 \times 10 \times 10 \times 10) - (8 \times 8 \times 8 \times 8)}{64}$$
$$= \frac{22}{7} \times \frac{10{,}000 - 4{,}096}{64} = \frac{22}{7} \times \frac{5{,}904}{64} = \frac{4{,}059}{14} = 289 \text{ in.}^4$$

$$\text{Area} = \frac{\pi(D^2 - d^2)}{4} = \frac{22}{7} \times \frac{10^2 - 8^2}{4} = \frac{22}{7} \times \frac{36}{4} = 28{\cdot}2 \text{ sq. in.}$$

$$\text{Radius of gyration} = \sqrt{\frac{I}{A}} = \sqrt{\frac{289}{28{\cdot}2}} = \sqrt{10{\cdot}2} = 3{\cdot}2 \text{ in.}$$

$$\frac{L}{R} = \frac{12 \times 12}{3{\cdot}2} = 45.$$

By formula (1). Safe stress per square inch $= 9{,}000 - \dfrac{40L}{R}$

$$= 9{,}000 - \frac{40 \times (12 \times 12)}{3{\cdot}2}$$
$$= 9{,}000 - \frac{5{,}760}{3{\cdot}2}$$
$$= 9{,}000 - 1{,}800$$
$$= 7{,}200 \text{ lb.} = 3{\cdot}21 \text{ tons per square inch.}$$

By formula (2). Safe stress per square inch $= 10{,}000 - \dfrac{60L}{R}$

$$= 10{,}000 - \frac{60 \times (12 \times 12)}{3{\cdot}2}$$
$$= 10{,}000 - \frac{8{,}640}{3{\cdot}2}$$
$$= 10{,}000 - 2{,}700$$
$$= 7{,}300 \text{ lb.} = 3{\cdot}26 \text{ tons per square inch.}$$

By formula (3). Safe stress per square inch $= \dfrac{5}{1 + \dfrac{1}{5{,}000}\left(\dfrac{L}{R}\right)^2}$

$$= \frac{5}{1 + \dfrac{1}{5{,}000}\left(\dfrac{12 \times 12}{3 \cdot 2}\right)^2}$$

$$= \frac{5}{1 + \dfrac{1}{5{,}000}(2{,}025)}$$

$$= \frac{5}{1 + 0 \cdot 405}$$

$$= 3 \cdot 56 \text{ tons per square inch.}$$

By formula (4). Safe stress per square inch $= 5 \cdot 5 - \dfrac{L}{20R}$

$$= 5 \cdot 5 - \frac{12 \times 12}{20 \times 3 \cdot 2}$$

$$= 5 \cdot 5 - \frac{144}{64}$$

$$= 5 \cdot 5 - 2 \cdot 25 = 3 \cdot 25 \text{ tons per square inch.}$$

Safe load = Safe stress per square inch = Area.
Safe load = by formula (1) = 3·21 × 28·2 = 90 tons.
Safe load = by formula (2) = 3·26 × 28·2 = 91 tons.
Safe load = by formula (3) = 3·56 × 28·2 = 100 tons.
Safe load = by formula (4) = 3·25 × 28·2 = 91 tons.

Note.—$R = \sqrt{\dfrac{I}{A}} = \sqrt{\dfrac{\dfrac{\pi}{64}(D^4 - d^4)}{\dfrac{\pi}{4}(D^2 - d^2)}}$

$$= \sqrt{\frac{1}{16}(D^2 + d^2)} = \frac{1}{4}\sqrt{D^2 + d^2}$$

Welding. An alternative method of uniting plates and pieces of metal together instead of using rivets is by welding. If two pieces of wrought iron or mild steel are heated together to a white heat and hammered or pressed together, they become permanently united by fusion, and this process is called welding.

The oldest form is that done by blacksmiths over an anvil. Two bars of wrought iron are easily welded together, but the join so made, although reliable, is not as strong as the original material, so that if the parts were put in a testing machine and pulled apart until they break, it is probable that the pieces would fail at the weld itself.

Thermit Welding. Thermit is the trade name for the mixture of aluminium and oxide. When ignited, the powder fuses at a very high temperature—probably somewhere in the region of 3,000° centigrade. The process is sometimes described as a casting process as special crucible methods are necessary. The parts which are to be united are placed in a mould with a crucible which contains the thermit over the mould. Once the thermit is ignited the metal runs into the mould at such a high temperature that it melts the parent metal of the two parts which

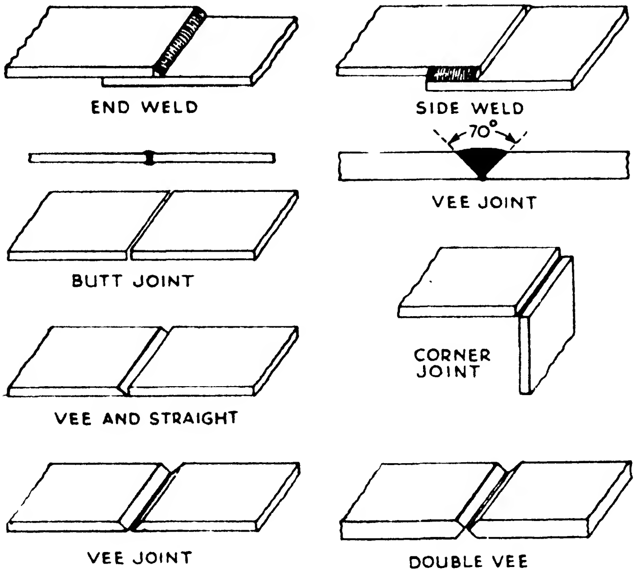


FIG. 149.

are to be united and a solid mass is formed which takes the shape of the mould. Heavy bridge bearings, repairs to ships' parts and repairs to tram rails and train rails have all been successfully done by the thermit welding.

Gas Welding. By mixing oxygen and acetylene or oxygen and hydrogen, or oxygen and coal gas, and igniting them under pressure, a very hot flame can be made. The compressed gases are usually stored in cylinders and usually the proportion is about 1·7 parts of oxygen to 1 part of acetylene. Various valves and gauges are provided so that the correct pressure can be adjusted and usually the oxygen is compressed to about 120 atmospheres, which has the very high pressure of 1,800 lb. per square inch (Fig. 150). The gases from both cylinders are connected to a nozzle and the flame is played on to the surface of the parts to be welded. The heat from the flame melts the parent metal and additional metal is laid on by melting rods in the flame.

Around engineering shops and shipyards oxy-acetylene is much used, both for welding parts together and also for cutting up plates and for cutting out special shapes of material.

Welding of Cast Iron. It is generally considered that cast iron is not weldable, and it is true that it cannot be welded by

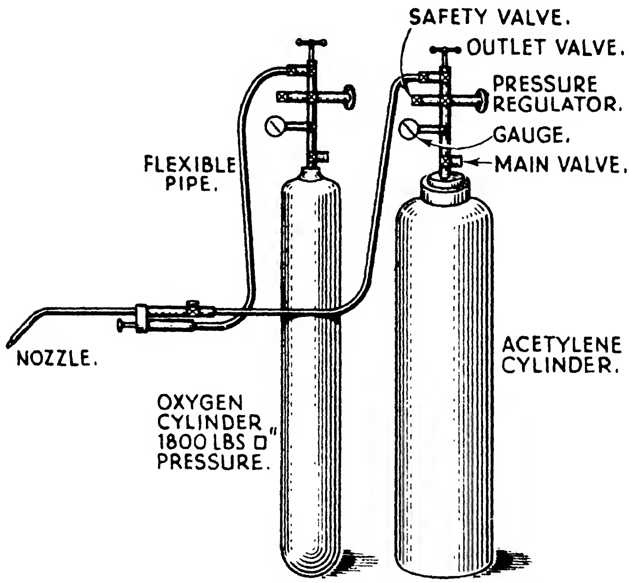


FIG. 150.

the same methods as the blacksmith uses for welding two wrought-iron rods together. Nevertheless there is no doubt that cast iron can be very satisfactorily welded together by using bronze for the electrode. Cast iron which has been clean cracked and where there is no very large stress, has been successfully welded by using oxy-acetylene and cast-iron electrodes. If this method is to be successful it is necessary that the melted metal at the crack shall be continuously scraped away by the cast-iron electrode. As the method is not much used, it need not be further described. Most of the difficulties met when welding cast iron can be overcome by using suitable bronze rods for welding. The great advantage of using bronze is that it has a relatively low melting point, and the bronze itself being very ductile forms a sort of shock absorber.

A further good point is that the bronze itself has a higher tensile strength value than cast iron, so that a good bronze weld can be made on quite heavy cast-iron parts which are subject to considerable stress.

Before the actual welding is commenced, the parts to be welded together should be cleaned, and any rust, scale or grease carefully removed. The parts to be welded should then be bevelled off so as to form an angle of approximately 70° in the vee. Bronze metal from a bronze electrode will flow into a smaller vee than the metal from a steel electrode. Before the actual welding takes place the parts to be joined should be heated to a dull red heat by means of a blow-lamp. A good flux should always be used and the surfaces will flux before the actual welded material has run into the vee. If the metal is heated too much the metal from the bronze rods will form into balls and run off. Like every other sort of welding, practice is necessary to get good results.

It is possible to weld cast iron by using electric arc welding and nickel or monel metal electrode. When this is done it is better to have the electric current with reverse polarity. That is to say, that the part to be welded should have negative current from the generator and the electrode itself should have positive current. In heavy welding repairs it is quite a common practice to screw steel studs into the mass of the cast iron and leave the ends projecting so that a better hold is made between the parent metal and the welded metal forming the vee.

Resistance Welding. In the case of gas welding using oxy-acetylene or electric arc welding, there is no actual pressure applied. The welding is done by adding molten metal to the surfaces to be joined.

Resistance welding is done by passing a low voltage electric current through the metal; a welding heat is created and the joint is made good by pressure or forging action—special machines are used for this purpose. Resistance welding can be either spot, butt, or along a seam.

The size of the weld zone in spot welding depends on the intensity of the current, the time the current is applied and the mechanical pressure applied to the plates through the electrodes. Some of the advantages which spot welding has are that the use of rivets is not necessary, working costs are low, there is no drilling or punching of holes and the parts are quickly assembled.

For continuous weld along the seams of thin sheets can be economically and quickly done by these special machines using resistance welding.

Electric Arc Welding. This is by far the most common method of welding used in connection with railway bridges, buildings or steel drums. During the past twenty years enormous advance has been made in the technique of welding, and to-day it is used on submarines, fighting tanks, boiler drums under heavy steam pressure, and all kinds of structural steelwork. The principle is simple. An electrode is a metal conductor which when heated to melting-point deposits metal on to other surfaces which have been brought to a molten state. The electric generator has one connection made to the parent material and

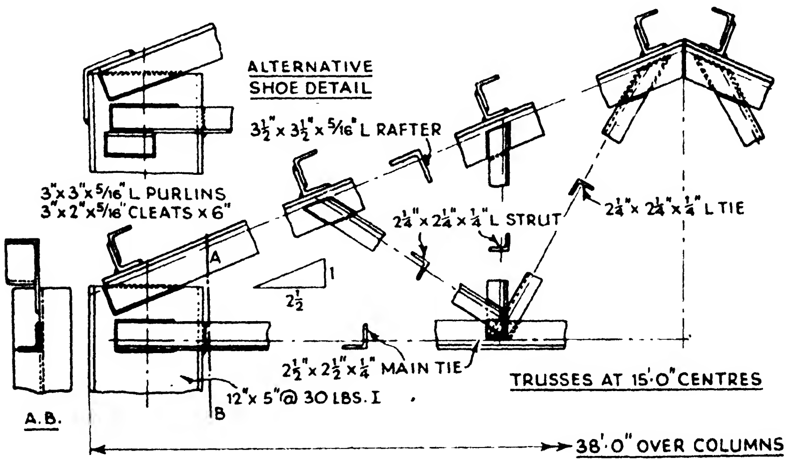


FIG. 151.

the other connection is made to the electrode. By this means an electric arc is formed and temperatures are generated sufficiently high to melt the parent material—that is to say, the material which is really the two parts to be moulded together.

The electrodes are sometimes made of bare steel wire ; sometimes they are coated. Plates to be joined are generally specially shaped as shown in the sketch. When the join is made along the end of a plate it is called the End Weld, and when it is made along the side, it is called a Side Weld. Fig. 151 shows a welded roof truss, and it is interesting to compare this with the riveted one of about the same span.

Strength of Timber Posts. Many complicated formulae for finding the safe stress in a timber post are available, but there is a very simple one which gives very satisfactory results. It is adopted by the American Railway Engineering Association.

$$p = S\left(1 - \frac{L}{60d}\right)$$

where p is the maximum allowable stress in lb. per square inch
S is the maximum allowable compression parallel to the grain for short blocks in lb. per square inch (see table below)
L is the length of column in inches
d is the (least) side of the column section.

VALUES OF S FOR TIMBER

Spruce, deal, Norway pine	800 lb.
Good oak or pitch pine	1,100 lb.
Good Douglas fir	1,100 lb.

The formula given above applies only to columns with both ends flat.

EXAMPLE.—Design a square spruce column to carry a load of 45,000 lb. (20 tons) for a 12-ft. length.

Try a 8 in. × 8 in.

$$p = 800\left(1 - \frac{L}{60d}\right)$$

$$= 800\left(1 - \frac{12 \times 12}{60 \times 8}\right)$$

$= 800(1 - 0{\cdot}3) = 800 \times 0{\cdot}7 = 560$ lb. per square inch allowable.

$\frac{P}{A} = \frac{45{,}000}{8 \times 8} = \frac{45{,}000}{64} = 703$ lb. per square inch actual.

Therefore an 8 in. × 8 in. column is not strong enough.

Try a 9 in. × 9 in.

$$p = 800\left(1 - \frac{12 \times 12}{60 \times 9}\right)$$

$= 800(1 - 0{\cdot}267) = 800 \times 0{\cdot}733 = 586$ lb. per square inch allowable.

$\frac{P}{A} = \frac{45{,}000}{9 \times 9} = \frac{45{,}000}{81} = 555$ lb. per square inch actual.

A 9 in. × 9 in. column is suitable.

Actually the safe strength is dependent on:

(1) Whether wood is sound, free from knots, shakes and cracks.

(2) Whether wood is to be exposed to water, sea, acids, heat.

(3) Whether wood is dry or wet (seasoned). Dry is less than 15% moisture.

(4) Class of wood.

(5) Condition of end-fixing, and whether load is live or dead.

(6) Slenderness ratio $\frac{L}{d}$.

L is the length or height of post in inches, and d is least side.

Where $\frac{L}{d}$ is *more than* 10, the following table gives good average values:

$\frac{L}{d}$	Safe Stress in lb. per square inch.	
	Spruce, Norway Pine and Fir.	Pitch Pine and Oak.
	lb. per square inch.	lb. per square inch.
10	650	850
12	620	800
15	580	750
20	510	650
25	420	570
30	350	500

Many authorities *limit* $\frac{L}{d}$ *to not more than* 30.

Example. What size of pitch pine post would be required to carry a load of 7,400 lb.; the length of the post is 134 in.?

Assuming 4 in. × 4 in. post:

One formula gives:

$$\text{Safe working stress} = 1{,}000 - \left(15 \times \frac{L}{d}\right)$$

$$\text{Safe working stress} = 1{,}000 - \left(15 \times \frac{134}{4}\right)$$

$= 1{,}000 - (15 \times 34) = 1{,}000 - 510 = 490$ lb. per square inch.

Area of 4 in. × 4 in. = 16 sq. in.

Safe load = Safe stress × Area = 490 × 16 = 7,840 lb.

If the $\frac{L}{d}$ is not to exceed 30 the *least side would have to be* $\frac{134}{30} = 4\frac{1}{2}$ *inches*, so that the post would be $4\frac{1}{2}$ in. or 5 in. square.

By formula (page 221) assuming 4 in. × 4 in. post:

$$\text{Safe working stress} = 1{,}000\left(1 - \frac{L}{60d}\right) = 1{,}000\left(1 - \frac{134}{60 \times 4}\right)$$

$= 1{,}000(1 - 0{\cdot}558) = 1{,}000 - 558 = 442$ lb. per square inch.

Area of 4 in. × 4 in. = 16 sq. in.

Safe load = Safe stress × Area = 442 × 16 = 7,072 lb.

For rough reinforced concrete beam design it is sufficiently correct to use the following approximations:

$$n = \tfrac{3}{8} \text{ roughly} \quad ; \quad a = \tfrac{7}{8} \text{ roughly}$$

$$M_s = f_s \times A \times \tfrac{7}{8} \times d \;; \quad M_c = \tfrac{1}{6} \times f_c \times b \times d^2$$

$$f_s = \frac{M}{\tfrac{7}{8} \times d \times A} \quad ; \quad f_c = \frac{6 \times M}{b \times d^2}$$

$$\text{Steel ratio } p = \frac{A}{b \times d} = \frac{3}{16} \times \frac{f_c}{f_s}$$

$$\text{Area of steel} = A = \frac{M}{\tfrac{7}{8} \times d \times f_s}$$

$$b \times d^2 = \frac{6 \times M}{f_c}.$$

For f_s 16,000 and f_c 600 . . . $M = 95 \times b \times d^2$.
For f_s 16,000 and f_c 650 . . . $M = 107 \times b \times d^2$
For f_s 20,000 and f_c 750 . . . $M = 120 \times b \times d^2$.

b = Breadth of beam in inches
d = Depth to steel in inches
M = Bending moment, inch-lb.
f_s = Stress in steel, lb. per square inch
f_c = Stress in concrete, lb. per square inch
A = Area of steel
a = Lever arm
n = Top to neutral axis

$$n = \frac{E_s}{E_c} = \frac{30{,}000{,}000}{2{,}000{,}000} = 15$$

E_s = Modulus of elasticity for steel
E_c = Modulus of elasticity for concrete.

Flitched Beams. Beams made of two flitches of timber with a steel plate between them and securely bolted together will carry the loads shown in Table I.

TABLE I

Section.	No. and Size of Timbers.	Safe Load evenly distributed on spans of:			
		10 ft.	12 ft.	14 ft.	16 ft.
2 timbers, with one steel flitch in centre of beam plate, ½ in. thick in all cases	2— 7 in. × 3 in.	70 cwt.	60 cwt.	50 cwt.	—
	2— 9 in. × 3 in.	120 ,,	90 ,,	85 ,,	70 cwt.
	2—11 in. × 3 in.	170 ,,	140 ,,	120 ,,	110 ,,
	2—12 in. × 6 in.	310 ,,	250 ,,	210 ,,	190 ,,

Beams made of three flitches of timber with two steel plates will carry the loads shown in Table II.

TABLE II

Section.	No. and Size of Timbers.	Safe Load evenly distributed on spans of:			
		10 ft.	12 ft.	16 ft.	20 ft.
3 timbers, and 2 steel plates. Steel plates ½ in. thick.	3— 9 in. × 4 in.	220 cwt.	190 cwt.	150 cwt.	110 cwt.
	3—12 in. × 4 in.	400 ,,	340 ,,	250 ,,	200 ,,
	3—12 in. × 6 in.	520 ,,	400 ,,	320 ,,	250 ,,
	3—14 in. × 4 in.	550 ,,	480 ,,	350 ,,	300 ,,

WEIGHTS OF MATERIALS (APPROXIMATE)

Material.	Weight, lb. per cubic foot.	Material.	Weight, lb. per cubic foot.
British oak	55	Cast iron	450
Pitch pine	45	Lead	710
Spruce	30	Copper	540
Larch	35	Dry clay	65
Elm	40	Clay and gravel	100
Ash	45	Moist earth, packed	100
Concrete	120 to 140	Sandstone	140
Brickwork	120 to 140	Granite	165
Steel	490	Marble	170
Wrought iron	480	Slate	160

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