ENGINEERING ECONOMY

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Preface

ENGINEERING IS ESSENTIALLY a means of obtaining desired results with economy. The engineer's approach to the solution of problems has been so successful that his field of activity has broadened to the extent that his success now depends as much upon his ability to cope with economic aspects as with physical aspects of situations.

Throughout this book the fact is stressed that the purpose of engineering economy is to arrive at decisions of what action to take in the present that will prove fruitful in the future. The subject is presented as an aid to judgment, analyses being directed to evaluating the future economic effect of factors individually and in combination.

Young engineers tend to disregard economic factors and are often appalled at the necessity for making decisions in situations where reliance must in large measure be placed upon estimates and judgments. To be most successful they must learn to make sound use of both.

One purpose of this book is to aid the engineering student to extend to economy the application of factual analyses in which he is proficient.

A second purpose is to develop an attitude that will cause him to approach engineering problems with regard for their economic as well as their physical implications.

Much emphasis is given in this book to the fact that the forerunner of engineering application is economic feasibility. The engineer who aspires to a creative rather than a responsive role will find proficiency in economic analyses a necessity. A similar proficiency is of paramount importance to the increasingly large number of engineers who eventually become engaged in the activities of management. This trend in employment is recognized by a quantitative economic approach to some aspects of management in Chapter 17, Economy and Utilization of Personnel.

The fact that the engineer must satisfy human wants as they are rather than as the engineer may wish them to be is discussed at some length.

In general *Engineering Economy* aims at the extension of quantitative thinking to situations embracing engineering and economic factors. Numerous approaches and methods of analyses are presented, most of which are illustrated by concrete examples. Methods and concepts are in many cases further illustrated in graphic form. Many problems for solution are included as a means of giving the student practice in making analyses and of fixing concepts and principals in his mind.

A new method for designating interest factors which is designed to conserve the instructor's and student's effort in making computations is presented on pages 45 and 72.

The author is grateful to many individuals and industrial firms who have been generous in discussing ideas and presenting illustrative situations. Thanks are extended to Mr. James C. Phelps, Mrs. R. J. Dwyer, and Mrs. Alin M. Waddill for competent assistance in preparing the manuscript.

H. G. T.

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CHAPTER

1

Engineering Economy and Its Functions

THE WORLD WE LIVE IN has two aspects; one is physical, and the other is social. Both impinge on each individual and are of concern to him. The individual himself is both physical and social. Physically he is made up of atoms, molecules, forces, and energy. Socially his gregariousness almost always causes him to be dependent upon other people.

The physical world consists of matter, forces, and energy in a myriad of forms. These elements are manifested in earth, sea, sky, and living things. They constitute the environment in which man lives. From the physical world as it exists, man must wrest his physical wants. In this he has been quite successful, for he has learned many of nature's secrets, though many more yet remain to be solved. The physical world is characterized by the immutable laws of nature, which govern the interaction of its elements.

The social world consists of the interactions of the people who inhabit it. From their associations patterns of conduct have developed. These have been crystallized into custom. Thus social life is governed in part by what has happened in the past. People also think about the future, and so their action may also be influenced by what they think will happen in the future. Thus the social world is characterized by the fact that it is governed by past and present thought, by its history and its conceivable future.

In this world things are as they are. Wishful, unrealistic thinking will not change them. Changes can only be brought about by operating on conditions as they exist. A serious bar to the attainment of desired ends in this world is lack of knowledge. Charles F. Kettering of General Motors research fame has stated, "We have only begun to knock a few chips from the great quarry of knowledge. We know almost nothing about anything."

Chester I. Barnard has advanced the important concept that ours is a world of the unknown and the unknowable. The fact that much more is unknown than known is obscured by the emphasis that is placed on what is known. Barnard believes that it is important to understand "what constitutes rational behavior toward the unknown and the unknowable." He states further, "I believe it is desirable in teaching to give courageous emphasis to the practical fact that we often have to act without sufficient knowledge and that there is much that is literally unknowable."[1]

Engineering in Modern Civilization

Engineering is utilitarian; its purpose is to aid people in satisfying their wants. This is clearly implied in the widely accepted definition of engineering: "Engineering is the art of organizing men and of directing and controlling the forces and materials of nature for the benefit of the human race."

Modern civilization rests upon engineering. Large, integrated states could not exist without printing, telephony, and telegraphy for communicating information to coordinate the actions of their people. Far fewer people could be supported in the world if present methods of transportation for distributing goods were abandoned, making it necessary for each community to be self-sufficient.

One of the most important factors in increasing production of foodstuffs in the world is mechanization. Agriculturists have bred more desirable animals and plants and developed methods of production that have materially increased yields of foodstuffs per unit of ground area. The engineers' contribution to the production of foodstuffs has been the substitution of mechanical power for that of humans or animals and the mechanization of implements. Mechanization has increased the production of foodstuffs per man-hour of effort manyfold.

The production of clothing has been mechanized to the extent that "hand made" goods are a novelty to be proclaimed. The methods whereby modern man provides himself with shelter have also been extensively mechanized.

Though the engineer takes comparatively little part in the affairs

[1] Chester I. Barnard, "Education for Executives," *Journal of Business of the University of Chicago*, Vol. XVIII, No. 4, Oct. 1945, pp. 175 ff.

of governing bodies, a large portion of governmental expenditures is made for items embodying engineering.

Perhaps there are some who would deny that engineering is directly cultural, but few will deny that its application has been instrumental in providing the leisure time for pursuing and enjoying culture. Through the development of the printing processes, radio, and rapid transportation, engineering has provided the means for both cultural and economic improvement of the individual. The satisfaction of almost all human wants, physical, cultural, and spiritual, is in some way related to engineering.

Engineering Economy

The complex equations used in design and operation of engineering works are patently not an end in themselves but are a means for satisfying human wants. Engineering has two aspects. One aspect concerns itself with materials and physical forces of nature; the other is related to satisfying people's wants. Thus engineering is closely associated with the subject of economics. This relationship is recognized in such a definition of engineering as the following: "Engineering is the conscious application of science to the problem of economic production."[2] The term *engineering economy* embraces the consideration of human desires and the cost of satisfying them.

Economy, the attaining of an end at low cost in terms of human effort, has always been associated with engineering. During much of human history the limiting factor has been predominantly physical. Thus a great invention, the wheel, awaited invention, not because it was useless or costly, but because the mind of man could not synthesize it earlier. But, with the development of science, things have become physically possible that people are interested in not at all or only slightly. Costs have become important and may become a dominant factor. Thus a new type of lawnmower, cigarette lighter, or excavating machine may be perfectly feasible from the physical standpoint but enjoy limited use because of its first cost or cost of operation. Likewise, customs and aesthetics are often dominant factors in economic success. Aesthetics, the sense of beauty, is undoubtedly the consideration that has led to the "streamlining"

[2] C. F. Harding and D. T. Canfield *Business Administration for Engineers* (New York: McGraw-Hill Book Co., Inc., 1937), p. 3.

of kitchen appliances in an effort to increase their appeal to customers.

Customs, as they affect people's wants, may also have important consequences. Some years ago, a prominent automobile manufacturer met a voiced demand for streamlining by producing an automobile design streamlined, from a scientific standpoint, more fully than any that had been offered to the public previously. Commercially the design was unsuccessful, probably because the design was too great a departure from previous designs to which people were accustomed, and because the demand for streamlining had been misinterpreted. The clamor proved to have been for aesthetic streamlining instead of for streamlining to reduce air drag.

In spite of an ever-increasing technology for overcoming physical limitations, engineers are increasingly concerned with nonphysical considerations, which determine, in the final analysis, the worth of their work. As pertains to a tool for the examination of engineering proposals, the term *engineering economy* means an attitude toward human factors and a body of knowledge, techniques, and practices of analysis and synthesis for evaluating the worth of physical objects and services in relation to their cost. One function of engineering economy is to predict the worth and cost of the results of engineering —much in the same way that one aspect of engineering is to predict the material, size, and proportions of a structural element to resist a certain force.

The importance of considering nonphysical aspects of enterprise has perhaps been obscured by the tremendous emphasis now given the physical sciences. That other considerations are of importance has been acknowledged by an executive of General Motors Corporation, an organization that is a leader in automotive research. Alfred P. Sloan, Jr., said "It is just as important to apply research to all fundamental activities of a business as it is to the technical phases of an enterprise."

Responsive and Creative Engineering

There are those, and some are engineers, who feel that the engineer should restrict himself to the consideration of the physical and leave the humanistic and the economic aspects of engineering to others; some would not even consider these aspects as coming under engineering. This viewpoint may arise in part because those who take pleasure in revealing and applying the well-ordered certainties of nature find it difficult to adjust their thinking to consider the vagaries of people.

Now that economic factors are the strategic consideration in many, if not most, engineering activities, engineering practice may be either responsive or creative. If the engineer takes the attitude that he should restrict himself to the physical, he is likely to find that the initiative for the application of engineering has passed to those who will consider the economic factors. Thus others propose and determine the feasibility of undertakings from a social or profit standpoint and call upon the engineer only for solution of physical details.

The engineer who acts in a responsive manner fails to accept responsibility. Although his position leaves him relatively free from criticism, he gains his freedom at the expense of applying engineering in ways and to situations over which he has little control, and at the expense of recognition and prestige. In many ways he is more a technician than a professional man. A technician has been defined by Sir Richard Livingston as a man who understands everything about his job except its ultimate purpose and its place in the order of the universe. Responsive engineering is a distinct hindrance to the development of the engineering profession.

The creative engineer, on the other hand, not only seeks to overcome physical limitations but also initiates, proposes, and accepts responsibility for the success of projects involving human and economic factors. The general acceptance of the viewpoint of creative engineering would extend the usefulness of engineering and might be expected to avoid many misapplications. Engineers in any capacity have an opportunity to consider human and economic factors in their work. The machine designer may design machines that can be operated with less fatigue. The highway builder may build for greater durability and safety. The engineer can use his influence to see that public works are planned, built, and operated in conformity with good engineering practice. An opinion frequently voiced by engineers is that the cost of selling products is excessive. At least a partial remedy for this situation is for engineers to assume the responsibility for creating products with built-in sales appeal.

An engineer with a creative outlook might conceive of reclaiming a land area and proceed to demonstrate the worth of his proposal by securing the necessary capital, doing the necessary engineering and construction work, and finally placing the reclaimed area in the hands of farmers for the profit of both the farmers and himself. In face of a similar opportunity an engineer with a responsive outlook would prefer to wait until promoters who had seen the promise in prospect for an undertaking employed him to do the engineering work.

Many engineers are now chosen for executive positions in business and industrial concerns. Those who aspire to such positions must recognize that creativeness is an important requirement.

Engineering is an emerging profession. People have great confidence in the integrity and ability of engineers. Perhaps nothing will enhance the prestige of the individual engineer and the profession of engineering more than the acceptance of a creative role.

The Functions of Engineering Economy

The functions of engineering economy will be discussed under the following headings:

Determination of objectives
Determination of strategic factors and means
Evaluation of engineering alternatives
Interpretation of the economic significance of engineering proposals
Assistance in decision-making

Determination of objectives. One important function of engineering economy is to seek new objectives for engineering application—to learn what people want that can be supplied by engineering. In the field of invention, success does not necessarily attend the construction of a new device; rather, it is dependent on how people judge the invention will satisfy their needs. Thus, market surveys seek to learn what people want. Motor car companies make surveys to learn what mechanical, style, and comfort features people want in cars. Highway officials make traffic counts to learn what construction programs will be of greatest use. Only after what is wanted has been determined come considerations of technical feasibility and cost.

The things that people want may be the result of logical consideration of facts but more often are the result of emotional drives. There seems to be no logical reason why one prefers a certain make of car, a certain type of work, or a certain style and color of clothes. The bare necessities needed to maintain physical existence, in terms of calories of nourishment, clothing, and shelter needed to cope with the elements, are limited and may be determined fairly accurately. But the wants that stem from emotional drives seem to be unlimited.

The aspect of engineering economy that seeks to learn of human wants requires not only a general knowledge of people, but also an understanding and appreciation of psychology, economics, government, literature and the fine arts, and the other fields related to the understanding of human nature.

Determination of strategic factors and means. After an objective has been determined, the next step is to devise means for reaching it.

The factors that stand in the way of attaining objectives are known as *limiting factors*. An important purpose of analysis is the determination of the limiting factors standing in the way of accomplishing an end. Once the limiting factors have been determined they are examined to see which ones may be operated upon with success. Suppose that a person is manufacturing and selling a product to which the following data apply:

Unit manufacturing cost	$.30
Unit selling expense	$.20
Unit sale price	$.80
Annual sales in units	5,000

The annual net income of the enterprise is [$.80 − ($.30 + $.20)] × 5,000, or $1,500. Each of the four factors—unit manufacturing cost, unit selling expense, unit sale price, and annual sales in units—may in turn be considered to be the limiting factor limiting the annual net income to $1,500. A favorable change in any one of the four factors could result in increased income. But it is likely that it is feasible to strive for a favorable change in only one or two of the factors. Rarely is it possible to circumvent all limiting factors. A limiting factor that can be circumvented is known as a *strategic factor*. The determination of strategic factors is important, for it makes it possible to concentrate corrective effort on those areas in which success is attainable.

In the above situation the strategic factor may be the unit manufacturing cost. Can unit manufacturing costs be cut by better engineering, more alert purchasing of materials, or better labor relations? A first step in answering this question is to conceive of a means that is believed to be able to operate on the limiting factor with success. This may require inventive ability, or the ability to put known things together in new combinations, and is distinctly creative in character. The means that will achieve the desired end may consist of a procedure, a technical process, or a mechanical, organizational, or administrative improvement.

It is apparent from even the above simple example that strategic factors limiting the success of a venture may be circumvented by operating on engineering, human, and economic aspects individually and jointly. One function of engineering economy is to enable the engineer to recognize strategic limiting factors and to operate on them successfully.

The determination of the means for attaining an objective is necessarily always subordinate to the determination of the objective. Ordinarily those who limit themselves to the consideration of the means alone will find themselves in a position subordinate to those who have the imagination, courage, and ability to gain acceptance of objectives. Limiting factors may often be circumvented in several ways; these ways must then be evaluated to determine which will be most successful in terms of economy. If the means devised come within the engineering field, they may conveniently be termed engineering alternatives.

Evaluation of engineering alternatives. It is usually possible to accomplish a desired result by several methods, each of which is feasible from the standpoint of the physical aspects of engineering. The most desirable of the several methods is then the one that can be performed at the least expense. The evaluation of engineering alternatives in terms of comparative cost is a function of engineering economy.

A wide range of factors may be considered in determining the worth of engineering methods. If investments are required, the time value of money must be taken into account. Labor costs are usually a factor and may involve wage payment plans. The consideration of material costs may lead to the study of market conditions and purchasing plans. Risks of a physical or an economic nature must often be evaluated. Income will usually be a factor and may involve such human considerations as market demand.

Though engineering alternatives are most often evaluated to determine which is most desirable economically, exploratory evaluations are also made to determine whether *any* engineering method that can be devised to reach a goal can be profitable.

Interpretation of the economic significance of engineering proposals. An important function of engineering economy is to interpret the economic significance of engineering works to people unacquainted with engineering technology. The average owner of an automobile cannot judge its worth on the basis of its engineering features. Such things as compression ratio, displacement, and fuel-air ratio have no meaning for him. The value of the car must be interpreted to him in terms of cost, comfort, and satisfaction if he is to understand. This necessity for interpretation, true of all engineering works, is perhaps of particular importance in regard to public works.

Engineers are becoming increasingly aware of the fact that many sound ventures fail because those who might have benefited by them did not understand their significance. Engineers can render a great service to their communities, the nation, and their profession by taking the initiative in pointing out the economic significance of engineering works to the public.

Assistance in decision-making. Engineering is concerned with action to be taken in the future. It is concerned with deciding what to do. An important function of engineering economy is to improve the certainty of decision with respect to the economy of engineering work. Correct decisions can offset many operating handicaps and shortcomings. On the other hand, incorrect decisions may and often do hamper all subsequent action. No matter how expertly a bad decision is carried out, results will be at best unimpressive and may even be disastrous.

Consider for example, a community in which two competing filling stations are located. One of these is managed by a likable, painstaking, and energetic man and the other by a person of just the reverse qualifications. In spite of these differences, the first man is finding it difficult to meet his bills, while the latter prospers. This apparent injustice hinges on one decision of each man. The likable chap made the disastrous mistake of selecting a station in a poor location, while the second man selected a fortunate location. These men may remain "behind the eight ball" and "on easy street" respectively for the balance of their lives, all because of one decision made by each. Since making correct decisions is so important, anything that will serve to improve one's ability to decide correctly should prove worth while. A study of engineering economy should result in an improved ability to make correct economic decisions in engineering.

To make a decision is merely to select a course of action from several. A correct decision is the selection of that course of action which will result in an outcome more desirable than would have resulted from the selection of any other of the several. Decision rests on the possibility of choice, that is, on the fact that there are alternatives from which to choose. The engineer with initiative proceeds on the thought that there is a most desirable solution, an alternative more satisfactory than any other, if he can only find it. The loss due to the omission of fruitful possibilities is undoubtedly very great.

The logical determination and evaluation of alternatives in tangible terms provided by engineering economy studies improve certainty of decision. After evaluation, making the selection that is most fruitful from an economic standpoint is a matter of choosing the alternative that is least expensive in terms of dollars and cents.

Why Economic Analyses Should Be Made by the Engineer

A user of a service is primarily interested in its utility and its cost. Thus, if he considers a design for a machine, for instance, his first concern will be: Will it pay? Or, in other words, what is the relationship between the worth of the prospective machine and its prospective cost? The person who lacks an understanding of engineering technology may find it difficult or even impossible to grasp the technical aspects of the machine sufficiently to arrive at its economic desirability. The uncertainty so engendered may easily cause loss of confidence and a decision to drop further consideration. If nontechnical people are to have full confidence in engineering proposals, the proposals must usually be analyzed to bring out their economic significance.

Being accustomed to the use of facts and being proficient in computation, the engineer should accept the responsibility for making an economic interpretation of his work. It is much easier for the engineer to master the fundamental concepts and economic techniques necessary to bridge the gap between technical and economic aspects of engineering than it is for the person who is not technically trained to acquire the necessary technical background.

The more general acceptance by engineers of responsibility for seeing that their proposals are both technically and economically sound, and for interpreting their plans in terms of utility and cost, may be expected to promote confidence in their work and enhance the worth of their services.

QUESTIONS

1. What is the ultimate purpose of engineering?

2. Give your reason why engineers take comparatively little part in the affairs of the governing bodies of societies, even though a large portion of governmental expenditures are made for items that embody engineering.

3. Define *engineering economy.*

4. Why should engineers be concerned with the culture and customs of people?

5. Contrast the meaning of the terms *responsive engineering* and *creative engineering.*

6. Summarize the functions of engineering economy.

7. What is (a) a limiting factor, (b) a strategic factor?

8. Why should the engineer consider it his responsibility to interpret the economic significance of his proposals?

9. Why should economic analyses of engineering activities be made by the engineer?

CHAPTER

2

Present Economy

THERE ARE MANY SITUATIONS in which interest charges are not a factor or are so slight as to be a negligible consideration in evaluating alternative methods for attaining an end or providing a service. These situations occur when the alternative methods to be compared will provide identical results and when the expenditures for them will occur during substantially the same short period of time considered to be the present.

Since the results of the alternatives to be compared will be identical and the expenditures for them occur at the same time, the immediate cost of each alternative is a measure of its comparative economy. The terms *present economy* and *immediate economy* mean a condition wherein the worth of a proposal can be evaluated on the basis of present or immediate costs. A number of illustrative examples will be used to explain selections in present economy.

Selection of Material

A designer has a choice of specifying either aluminum or grey iron castings for an intricate housing for an instrument to be permanently mounted in a power plant. He has ascertained that either metal will serve equally well. The aluminum casting will weigh .8 pound and the grey iron casting 2.2 pounds. His analysis of the cost of providing each type of casting follows:

	Grey Iron Casting	*Aluminum Casting*
Cost of casting delivered to factory.....	$ .76	$ .92
Cost of machining.....................	.63	.52
Total cost of finished housing..........	$1.39	$1.44

On the basis that either material will serve equally well and provide an identical service, the grey iron casting will be selected because its immediate or present cost is the lowest.

In the above example, in which it is specified that the instrument is to be mounted on a power plant wall, differences in weight were not considered to be a factor. However, for an instrument for airplane service the difference in weight might be the deciding economic factor. In airplane service the lighter instrument casting would have an economic value in lessened fuel consumption or greater pay load and would thus have greater utility. It is obvious that for airplane service the two alternatives in the example would not provide identical services.

In determinations of economy, care must always be exercised to see that alternatives provide identical services. "Sales appeal" of one type of casting over the other may easily render their worths nonidentical. Even for power plant use it might have been recognized that the lighter metal had "sales appeal" over the heavier metal.

If concrete value can be given to a quality, this difference in service can be taken into account to render the services of the two alternatives identical. For instance, the quality of lightness might have service value to the manufacturer in that it would reduce the delivery cost of his product to the consumer. If this had applied in the example of the instrument casting, the two types of castings could have been placed on an identical service basis for comparison as follows:

	Grey Iron Casting	*Aluminum Casting*
Cost of casting delivered to factory	$ .76	$ .92
Cost of machining	.63	.52
Average additional cost to deliver grey-iron-casting-equipped instruments to customer	.26	.00
	$1.65	$1.44

Economy of Method

The same end result may often be attained by two or more different methods. For instance, parts made to the same specifications on a machine lathe or an automatic lathe have equal objective or functional value. However, we should not lose sight of the fact that the method by which an article is made often affects people emotionally so that they impute value to an article because of the method by which it was made. Thus, hand-sewn leather goods frequently command a higher price than those which are machine-sewn, though they possess no greater functional value. It is in recognition of this fact that such terms as hand-sewn, hand-polished, and hand-set are used in sales literature.

Comparing the economic desirability of methods is only significant when their end results are of equal value. Both functional and intangible values must be considered.

A builder of homes who is operating in a small town is building a house for a client who specifies a full basement. The basement is to have outside dimensions of 30 ft. × 32 ft. and will require excavation to a depth of 6 ft. below grade. The plans for the basement walls specify 12-in. concrete blocks; and, since the location is high and under drainage is provided, no waterproofing will be required. Thus, the excavation may be as close to exact size as practical for the method used.

A survey of equipment and labor available reveals that the contractor can have a local man do the excavating with a team and scraper for $12 per eight-hour day; or, he can rent a power shovel from a neighboring town for $75 per day plus a $100 moving charge. In either case some handwork will be necessary to trim corners and to level at the rate of $4.50 per cubic yard. Either method of excavation will be equally satisfactory to the client, since the end result will have equal functional value regardless of the method used.

The contractor analyzes the costs associated with the two methods as follows.

Team scraper method. The team with scraper can remove an average of 2.5 cubic feet per load and will require approximately 5 minutes per trip to load and unload. To enable teams to move in and out, a slope must be maintained which leaves 4 cubic yards to be removed by hand shoveling. Trimming of corners will require removal of an additional 10 cubic yards by hand.

Total cubic yards to be removed, $\frac{30 \times 32 \times 6}{27}$	214
Cubic yards removable by team scraper	200
Cubic yards to be removed by hand	14
Team cost, $\frac{200 \text{ cu. yds.} \times 27 \text{ cu. ft. per cu. yd.}}{2.5 \text{ cu. ft. per load} \times 12 \text{ loads per hr.}} \times \$1.50$ per hr.	$270
Hand excavation 14 cu. yds. @ $4.50	63
Total	$333

Power shovel method. The power shovel can excavate the basement in two days, leaving 8 yards to be completed by hand.

Moving cost	$100
Rental charge, 2 days @ $75.00	150
Hand excavation 8 × $4.50	36
Total	$286

Based on the above calculations the contractor would hire the power shovel to do the excavation since its use results in the lower net cost to him.

Later on in the construction of the residence, the contractor finds that he is unable to secure the services of a painter to hand-paint the exterior as specified in the contract. He asks the client to accept spray painting in return for a reduction of the contract price by $30. Although agreeing that the functional result of the two methods will be equal, the client refuses to accept the spray painting and the reduction in cost to him. The reason for his refusal is that he is on a bowling team on which there is a painter.

It should be noted that the client was concerned only with the final results in considering methods of excavation, but that while agreeing that either method of painting resulted in equal functional value to him, he held the intangible consideration of offending a teammate to be in excess of the $30 he might have saved.

Many decisions are made on the basis of intangible considerations. It is defensible to make such decisions when they are made knowingly, in full knowledge of the cost and other considerations.

It should be the aim of economy studies to assign concrete values to as many of the factors involved as possible, in order to minimize the consideration of intangible factors in making an economic decision.

Economy of Location

There are many situations in which geographical location is a factor in economy. Situations in which location is important over a short period of time, say a year or less, come properly under the heading of present or immediate economy. Small differences of location, as for example the location of part bins and tools in a work place, may have considerable effect upon economy and are often given very detailed consideration by motion study analysts.

It is not uncommon for concerns to set up temporary service facilities in an area in which they have unusual temporary activities. They are then confronted with determining the economy of setting up such facilities and the economy of various locations within the area.

A firm having a contract to build a dam requiring 300,000 cubic yards of gravel found two feasible sources whose characteristics are summarized as follows:

	Source A	*Source B*
Distance, pit to dam site	1.8 miles	.6 mile
Cost of gravel per cubic yard at pit	...	$.08
Purchase price of pit	$7,200	...
Road construction necessary	$2,400	None
Overburden to be removed at $.24 per cubic yard	...	60,000 cu. yds.
Hauling cost per cubic yd. per mile as bid by hauling contractor	$.072	$.084

In order to make a selection on the basis of economy, the cost of securing the required gravel from either source should be determined by the following analysis:

Cost of 300,000 Cubic Yards of Gravel, Source A

Purchase price of pit	$ 7,200
Road construction	2,400
Hauling cost, 300,000 cu. yd. × 1.8 miles @ $.072 per cu. yd. per mile	38,900
	$48,500

Cost of 300,000 Cubic Yards of Gravel, Source B

Cost of gravel at pit, 300,000 yds. @ $.08	$24,000
Removal of overburden, 60,000 yds. @ $.24	14,400
Hauling cost, 300,000 cu. yds. × .6 mile @ $.084 per cubic yard per mile	15,100
	$53,500

The cost of making analyses similar to the one covering the situation above is often a small percentage of the saving that may result from a correct decision. A cursory analysis or "hunch" cannot often be defended for decision-making on the basis of its lesser cost.

Economy of Proficiency

Value of proficiency is not necessarily directly proportional to degree of proficiency. Thus, a baseball player whose batting average is .346 will ordinarily command more than twice as much salary as one whose average is .173, if they are equal in other respects. In some activities the compensation of those with extremely high proficiencies is extremely high, but those with ordinary abilities can find no market for their services. Consider acting; the motion picture industries pay fabulous salaries for those whose box office pull is high when they undoubtedly could cast all their productions with volunteer actors of fair ability without cost.

The cost of human effort is a considerable portion of the total cost of carrying on nearly all business activities. People are employed with the idea of earning a profit on the skills they possess. In general, the higher a person's proficiency in any skill, be it manual dexterity, creativeness, leadership, inventiveness, or physical ability, the greater his value to his employer. Consider a machine operation for which a workman is paid $1 per hour for producing 10 pieces per hour on a machine whose rate is $3 per hour. (The machine rate embraces all costs incident to operating the machine.)

In this example the cost per piece is equal to

$$\frac{\$1 + \$3}{10} = \$.40$$

If a second worker of less proficiency completes 9 pieces per hour, his relative worth to his employer (as compared with that of the first workman) is calculated as follows:

Let W equal the hourly pay of the second workman to result in a cost per piece of $.40. Thus,

$$\frac{W + \$3}{9} = \$.40$$

and

$$W = \$.60$$

It will be noted in this example that a 10 per cent reduction in the ability of the workman to turn out pieces results in his services being worth 60 per cent as much per hour to his employer as the services of the first workman, all other things being equal.

If the second workman also received $1 per hour, the cost per piece of the 9 pieces he turns out in an hour will be as follows:

$$\text{Cost per piece} = \frac{\$1 + \$3}{9} = \$.444$$

It is conceivable that the resulting cost per piece may be so high that the employer cannot profit if he must pay the second workman $1 per hour.

The comparative worth of a third workman, who produces only 7 pieces per hour, is calculated as follows:

Let W equal the hourly pay of the third workman to result in a cost per piece of $.40.

$$\frac{W + \$3}{7} = \$.40$$

$$W = -\$.20$$

The negative result is illustrative of the fact that there are levels below which proficiencies have little or no economic value.

Thus, the fixing of wage rates by minimum wage laws or other artificial limitations may result in unemployment for persons of low output and increased demand for persons of high proficiency.

The fact that a legal minimum wage establishes a standard of employability is usually overlooked. It is obvious, however, that when the community says that a worker may not be paid less than a given amount, the community is in effect saying that anyone who is not worth that amount shall be regarded as unemployable. Unfortunately passage of minimum wage legislation has not been coupled with provision for support of the persons who are made unemployable by the legal minimum.[1]

Economy and Preparation

Nearly any activity requires some preparation prior to its performance. After the activity has been performed, it is frequently necessary to expend further effort in restoration—cleaning the machine, returning tools to the cabinet, and so forth. These two items are frequently termed *set-up* and *break-up*.

A student about to do an assigned plate in mechanical drawing must assemble his books and instruments, go to the drafting laboratory, arrange his books and instruments upon the drafting table, and study the assignment prior to performing any actual work upon the plate. After he has completed the plate and handed it to his instructor, he will gather up books and instruments and return to his room. It may be noted that set-up and break-up, for convenience lumped

[1] Sumner H. Slichter, *Basic Criteria Used in Wage Negotiations* (Chicago: The Chicago Association of Commerce and Industry, 1947), p. 11.

together as preparation in the example, may represent a large proportion of the total effort. It may also be noted that the preparation is constant for a considerable range of time devoted to the actual drafting in a single period.

In a certain office where mimeographed sheets were collated, it had been the practice to collate each day. Observation of the collating job revealed that it required an average of 24 minutes of preparation time for the girl employed at this task each time it was performed. The actual collating was done at the average rate of 50 sheets per minute. This situation is presented diagrammatically in Fig. 1.

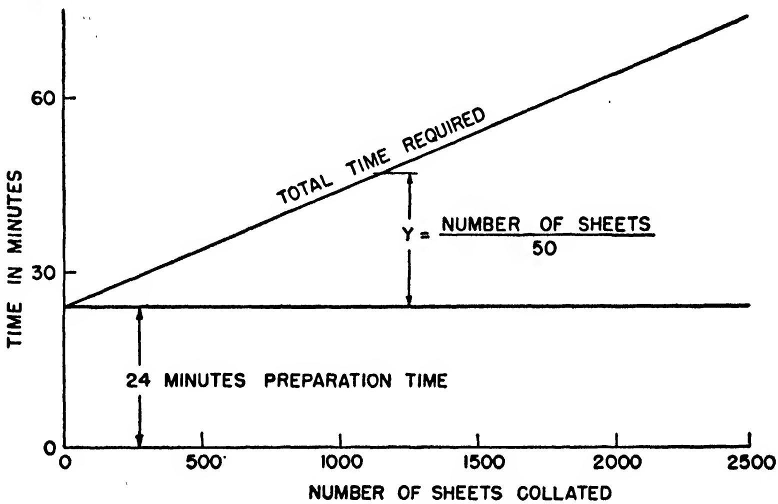


FIG. 1. Preparation and actual collating time required in collating.

Further observation revealed that an average of 500 sheets were collated per day and that this work could as well accumulate and be done weekly. The girl's hourly pay rate was \$.80.

When the collating is done daily, the cost per thousand sheets equals

$$\frac{24 \text{ min.} + 10 \text{ min.}}{.5 \text{ M sheets}} \times \frac{\$.80}{60 \text{ min.}} = \$.91$$

when the collating is done weekly (5 days), the cost per thousand sheets equals

$$\frac{24 \text{ min.} + 50 \text{ min.}}{2.5 \text{ M sheets}} \times \frac{\$.80}{60 \text{ min.}} = \$.39$$

The decision to collate weekly would thus result in a cost reduction of

$$\frac{\$.91 - \$.39}{\$.91} = 57\%$$

A great many situations both in present and future economy conform to the pattern of the situation illustrated in the example.

Economy of Tool Maintenance

Through the ages man has developed many tools to increase his output and lighten his burden. Such tools as shovels, axes, saws, screw drivers, and chisels, though relatively inexpensive, increase the output of a person manyfold.

Many common tools are the results of centuries of use and experimentation and have reached a high state of perfection. Yet it appears that too little attention has been given to the consideration of the effectiveness of tools as an aid in reducing effort. In a famous study in 1898 at the Bethlehem Steel Works, Frederick W. Taylor investigated shoveling, an activity that composed much of the work of 600 men. Taylor found a great variety of shovels in use. Shovelers were lifting shovel loads ranging from 3.5 pounds (of rice coal) to 38 pounds (of ore). Taylor set out to determine what shovel load permitted a man to move the maximum amount of material per day. After an extended study, he found that a shovel designed to hold 21.5 pounds was most effective.

All too frequently tools actually found in the hands of workmen are in very poor condition. This is often the result of indifference or a false sense of economy on the part of workmen or supervisors.

In a study of yard maintenance, the author investigated the loss of effectiveness caused by normal wear in a number of common tools. One of these was a pick used to break up hard soil. Since a pick is a penetrating tool, its penetration with a fixed blow was taken to be a measure of effectiveness. Workmen were given picks in new condition and the penetrations made by the picks as they dulled were observed under uniform conditions.

The results given in Fig. 2 are typical of those observed.

The penetration decreased rather rapidly at first and less later on, as indicated by the curve. For convenience in discussion, only the three points A, B, and C will be considered, and it will be assumed that curves AB and BC are straight lines.

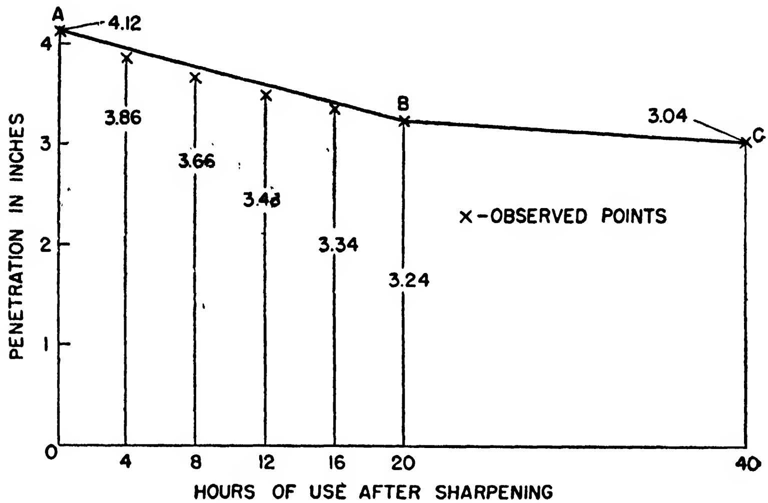


FIG. 2. Decrease in penetration of a pick as a result of use.

The average lost penetration during the first 20 hours of use was

$$(4.12 \text{ in.} - 3.24 \text{ in.}) \div 2 = .44 \text{ in.}$$

During the next 20 hours of use the average lost penetration was

$$[(4.12 \text{ in.} - 3.24 \text{ in.}) + (4.12 \text{ in.} - 3.04 \text{ in.})] \div 2 = .98 \text{ in.}$$

The average loss of penetration during the 40-hour period under consideration was

$$(.44 \text{ in.} + .98 \text{ in.}) \div 2 = .71 \text{ in.}$$

The average percentage loss of effectiveness during the first 20 hours was

$$\frac{.44 \text{ in.}}{4.12 \text{ in.}} = 10.7\%$$

The average percentage loss of effectiveness during the 40-hour period was

$$\frac{.71 \text{ in.}}{4.12 \text{ in.}} = 17.2\%$$

The estimated cost of sharpening a pick was $.62. The wage rate for pick work was $1.25 per hour.

Plan A. Sharpen pick at 20-hour intervals.

Costs Incident to Dulling of Pick for a 40-Hour Period

Cost of sharpening pick, 2 × $.62	$1.24
Cost of loss in effectiveness, 40 hrs. × $1.25 × 10.7%	5.35
Total	$6.59

Plan B. Sharpen pick at 40-hour intervals.

Costs Incident to Dulling of Pick for a 40-Hour Period

Cost of sharpening pick, 1 × $.62	$ .62
Cost of loss in effectiveness, 40 hrs. × $1.25 × 17.2%	8.60
Total	$9.22

The adoption of Plan A in place of Plan B would result in a saving of $9.22 − $6.59 = $2.63 per 40 man-hours of pick use. This amounts to $131 per 50-week man-year of pick labor.

TABLE 1. Relationship of Frequency of Sharpening a Pick and Cost

Sharpening Interval in Hours	Number of Sharpenings Per 40-Hour Period	Cost of Sharpening @ $.62 Per Time = X	Average Loss of Penetration in Inches	Average Per Cent Loss of Effectiveness	Cost of Loss of Effectiveness Per 40-Hour Period @ $1.25 Per Hour = Y	Total Cost for 40-Hour Period Incident to Dulling of Pick = $X + Y$
4	10	6.20	.13	3.2	1.60	7.80
8	5	3.10	.23	5.6	2.80	5.90
12	3.3	2.06	.32	7.8	3.90	5.96
16	2.5	1.55	.39	9.5	4.75	6.30
20	2	1.24	.44	10.7	5.35	6.59

Indications are that the pick should be sharpened even more frequently.

Costs incident to dulling of pick for a 40-hour period when pick is sharpened at various intervals appear in Table 1.

Almost everyone has seen situations in small tool use where an analysis similar to the above can be made.

Where unit labor costs are high and the tools used are relatively inexpensive, it is rarely economical to use tools that are in other than the best condition. Thus, it often happens that those who feel that they can least afford to maintain good tools are those who can least afford not to do so.

Economy of Crew Size

Much work, particularly maintenance work, is performed by crews in industry.

Where a large variety of nonrepetitive work that precludes preplanning is performed, the number of men in a crew is an important factor from the standpoint of the crew's effectiveness.

One reason for loss in effectiveness, which increases with crew size, is the time and cost associated with going to and from the job. This aspect will be illustrated with an absurd example. Assume that a maintenance job involving 60 man-minutes of work is to be performed at such a distance that it will require 10 minutes to go to the job and 10 minutes to return from the job. Assume further that the work to be performed can be done effectively by either a one-man or a 60-man crew. Labor cost is taken at $1.20 per man-hour, or $.02 per man-minute. A comparison of the over-all effectiveness of each crew is revealed by the following analyses:

One-Man Crew

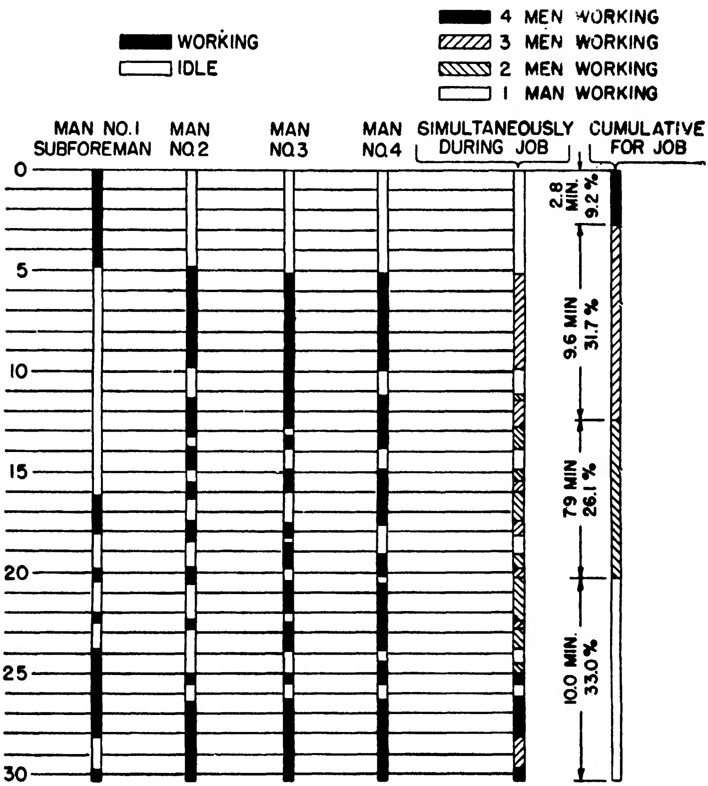
Activity	*Minutes Required in Performance*	*Man-Minutes*
Walk to job	10	10
Perform job	60	60
Walk from job	10	10
Total man-minutes		80

Total cost, 80 man-min. @ $.02 = $1.60

SIXTY-MAN CREW

Activity	*Minutes Required in Performance*	*Man-Minutes*
Walk to job	10	600
Perform job	1	60
Walk from job	10	600
Total man-minutes		1260

Total cost, 1260 man-min. @ $.02 = $25.20



[FIG. 3. An analysis of enforced idleness of members of a four-man crew. Reproduced from H. G. Thuesen and M. R. Lohmann, "Job Design," Oklahoma A. and M. College, Engineering Experiment Station, *Publication No. 66*, January 1948.]

The difference between $25.20 and $1.60, or $23.60, is a measure of the ineffectiveness resulting from travel time attributable to crew size.

In many cases the work is of such a nature or the place in which it must be performed is so limited that all crew members cannot work at one time. The idleness enforced on crew members results in loss of effectiveness.

On the activity chart of a four-man crew doing a maintenance job, shown in Fig. 3, the enforced idleness of each crew member has been recorded as it occurred. The chart also shows the number of men who were working at any given time, and that, during the 30.3 minutes required in the performance of the job, the number of minutes one, two, three, and four men were working simultaneously was respectively 10.0, 7.9, 9.6, and 2.8 minutes.

Taking labor costs of \$.02 per man-minute, the performance on the job with a four-man crew may be summarized as follows:

Total man-minutes consumed, 4×30.3	121
Total cost, 121 man-min. $\times$ \$.02	\$2.42

Cost of job—three-man crew. Since three or less men were working during $10 + 7.9 + 9.6$ minutes, this portion of the job could have been done in 27.5 minutes by three men. The portion of the job that required 2.8 minutes for four men could have been performed in $2.8 \times 4/3$ or 3.7 minutes by three men.

Total man-minutes consumed, $[10 + 7.9 + 9.6 + (2.8 \times 4/3)] \times 3$	93.7
Total cost, 93.7 man-min. $\times$ \$.02	\$1.87
Saving over cost with four-man crew, $\dfrac{2.42 - 1.87}{2.42}$	22.7%

Cost of job—two-man crew

Total man-minutes consumed, $[10.0 + 7.9 + (9.6 \times 3/2) + (2.8 \times 4/2)] \times 2$	75.8
Total cost, 75.8 man-min. $\times$ \$.02	\$1.52
Saving over cost with four-man crew, $\dfrac{2.42 - 1.52}{2.42}$	37.2%

Cost of job—one-man crew

Total man-minutes consumed, $[10.0 + (7.9 \times 2/1) + (9.6 \times 3/1) + (2.8 \times 4/1)]$	65.8
Total cost, 65.8 man-min. $\times$ \$.02	\$1.32
Saving over cost with four-man crew, $\dfrac{2.42 - 1.32}{2.42}$	45.5%

It is recognized that realization of the savings indicated would depend in large measure upon personal factors. The argument could be raised that enforced idleness would serve as a needed rest period; or the point could be made that the fewer the men in the crew, the less the time devoted to visiting.

It should also be borne in mind that crews may be too small in number for economy. In general, a crew should consist of the number of men needed to do that part of a job which requires the greatest number of men simultaneously. If crews use equipment, equipment costs should be taken into consideration in arriving at an economical crew size.

Economy of Vigilance and Error

Some years ago a large manufacturing concern placed on its employee bulletin boards an illustrated poster stating that 14 per cent of its production cost was due to errors. The first reaction that one might have to such a statement is that a reduction in errors would result in reduced production costs. This certainly would be true if the cost of eliminating errors would be less than the cost arising from the errors that could be eliminated. Since the concern in question was recognized for its efficient operation, it is probable that errors would not be reduced substantially unless added vigilance were exercised. It is reasonable to assume that the added vigilance would not be obtained without cost. Thus, if the errors eliminated do not result in a reduction in costs equal to the cost of the added vigilance, the latter is not an economy.

The question of costs arising from errors and the vigilance necessary to keep errors at an economical level in a manufacturing establishment is very complex and difficult of concrete analysis. But it is possible to deduce that there is a level of error, or for that matter of vigilance, which will result in greatest economy. Consider Fig. 4.

Curve *A-B* represents the cost in dollars arising from error as production costs due to error increase. Curve *C-D* represents cost of vigilance and is given its trend on the basis of the following consideration. It is reasonable to assume that the cost of vigilance necessary to eliminate all errors will approach an infinite amount. On the other hand, it is reasonable to assume that some things would be performed correctly without vigilance, even though only by chance. Thus, it is reasonable to assume that the curve *C-D* will approach zero cost when production cost due to error approaches 100 per cent.

The optimum relation between errors and vigilance exists when the sum of their costs is at a minimum; this relation may be determined from curve *E-F*, produced by adding curves *A-B* and *C-D*.

Proofreading will serve to illustrate further the concept expressed in Fig. 4. In the first reading of the copy for a book, for instance, a proofreader may be expected to find a majority of the errors. During each subsequent rereading he may be expected to find fewer and fewer errors. But it is not likely that he will detect all errors in a reasonable number of rereadings.

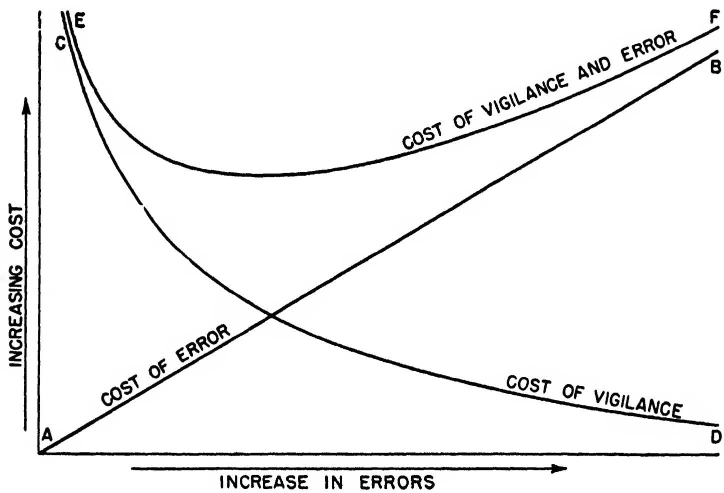


Fig. 4. Economy of vigilance and error.

In some operations the relationship between vigilance and error can be established. One such operation is the percentage inspection of parts, where only a percentage, determined on the basis of economy, of the parts in a lot are inspected. This is done on the premise that the consequences of passing some defective parts will be less than the cost of the additional inspection necessary to inspect all.

Industry seeks to reduce errors in many ways. One way is to reduce the possibility for error; this is done, for example, in equipping machine tools with interlocking controls to make them "foolproof."

Another way is to make vigilance easier or less costly. Fewer drivers run out of gasoline since the dash gasoline gauge replaced the rear-tank mounted gauge. Mechanical and electrical inspection devices are being used increasingly to reduce the cost of vigilance and, in turn, the cost of errors.

One aspect of personnel selection is the elimination of the error-prone and the accident-prone. It is common knowledge among safety engineers that a majority of accidents can be eliminated by eliminating a small minority of accident-prone persons. Employment techniques are also used to select persons for all manner of inspection work who have a higher-than-average proficiency in detecting errors.

Economy of Initiative

There are many things that are highly profitable to the person who initiates them and much less profitable to those who follow. The person who initiated the idea of a chocolate coating for a piece of ice cream on a stick is reputed to have made a fortune. The government recognizes the merit of originality and through patent and copyright laws extends special privileges to those who present new ideas.

Air conditioning in theaters has become commonplace. The theater owner who was first in a city to make an installation generally profited handsomely, drawing customers because of the novelty as well as because of the comfort that air conditioning could provide. Increased sales during the first summer frequently were sufficient to more than pay for the installation. Attendance at the favored theater increased largely at the expense of loss of attendance in competing theaters.

As competitors became aware of what was happening, they too installed air conditioning. In many cases these later installations served only to recapture attendance, that is, to offset a loss occasioned by the competing installations. On the whole, air conditioning has been an important factor in stabilizing the entire motion picture industry by minimizing seasonal variation in theater attendance.

Economy Inherent in Design

Design affords many opportunities for economy. In fact, the end sought in design is economy. Design, no matter how poorly done, is predicated on the thought that the effort devoted to it will be outweighed by the result.

To design is to project and evaluate ideas for attaining an objective. Suppose that a person employed in a machine design department has been assigned the task of designing a machine for a special purpose. His first step will be to project, literally to invent, new combinations of materials, machine elements, forces, and kinematic motions that it is believed may meet the purposes of the machine. His next step will be to evaluate these combinations.

There may be many bases of evaluation: certainty of operation, consequences of failure, operator attention required, safety of operator, rate of operation, power required, maintenance required, floor space required, and so forth. On the whole, these bases of evaluation all relate to economy in one way or another. In fact, economy is inherent in design. Consider a roller bearing in conjunction with a plain brass bearing. The friction induced by a turning load on a roller bearing is less than that of a plain bearing, by virtue of the fact that the rolling friction inherent in the former is less than the sliding friction of the latter. Where friction is economically undesirable, the roller bearing is inherently more economical than the plain bearing.

A decision to accept a design is a decision to assume the advantages and disadvantages associated with it, and to discard other designs that might have been chosen. Examples will be presented to bring out several aspects of economy as it relates to design.

For many years a company had marketed a tar soap in a package carrying a picture of an elderly Negro's face enveloped in lather printed in brown ink. Later a new package was adopted which had a motif of pine trees in green and white and referred to the "woodsy" fragrance of the soap. Sales increased substantially. In this example it is clear that the design of the new package embodied more sales appeal than that of the old and consequently had added economic value.

In the design of jigs and fixtures for carrying on manufacturing, the designer may become too engrossed in the mechanical features and give insufficient attention to over-all economy.

Assume that a jig is to be designed for adjusting 100,000 assemblies for a special production order. Two designs are presented and are evaluated as follows:

	Jig A	*Jig B*
Life, estimated	100,000 pieces	100,000 pieces
Cost	$200.00	$200.00
Hourly rate of class of labor required	$.76	$.94
Operator hourly output rate	62	54

Cost of Adjusting the 100,000 Assemblies with Jig A

Labor cost, $\dfrac{100{,}000 \text{ pcs.} \times \$.76 \text{ per hour}}{62 \text{ pcs. per hour}}$	$1226.00
Cost of Jig *A*	200.00
Total cost with Jig *A*	$1426.00

Cost of Adjusting the 100,000 Assemblies with Jig B

Labor cost, $\dfrac{100{,}000 \text{ pcs.} \times \$.94 \text{ per hour}}{54 \text{ pcs. per hour}}$	$1740.00
Cost of Jig *B*	200.00
Total cost with Jig *B*	$1940.00

The net advantage of selecting Jig *A* over selecting Jig *B* is $1940 − $1426 = $514. This advantage of $514 is inherent in the design of Jig *A*. Its characteristics were such that it could be operated with greater rapidity and that it required a lesser degree of skill from the operator.

In industrial literature may be found many examples in plant layout in which economy of operation has been improved, primarily, by rearrangement of equipment formerly used. In such cases it is clear that the advantage of the new layout is inherent in the arrangement.

The economy inherent in a design may be due to many factors. For instance, in rectangular beams, strength in the resistance of bending moments varies directly as the width and as the square of the depth in accordance with the expression $bd^2/6$, where b is the width of the beam in inches and d is the depth of the beam in inches.

In the design of a certain building it was found that 180 2 in. × 8 in. × 16 ft. rafters placed upon 16-inch centers are just sufficient to meet the contemplated load. It was also found that the sheathing to span rafters placed on 24-inch centers was just adequate for strength and stiffness. Consequently, this sheathing will be stronger and stiffer than necessary for a span of 16 inches. The excess is termed "overdesign." Overdesign has no functional value and may be disadvantageous because of its extra weight, bulk, or the like.

A new design is contemplated using 2 in. × 10 in. cross-section rafters. On the principle that the bending moment resisted by a rafter is in accordance with $bd^2/6$, the ratio of the load-carrying capacity of the 2 in. × 10 in. rafter to that of the 2 in. × 8 in. rafter is

$$\frac{\dfrac{2 \times 10^2}{6}}{\dfrac{2 \times 8^2}{6}} = \frac{10^2}{8^2} = \frac{100}{64} = 1.56$$

As far as load-carrying capacity is concerned, the 2 in. × 10 in. rafters may be spaced 16 in. × 1.56 or 25 inches. To conform to practical standards the rafters will be spaced on 24-inch centers. An analysis for comparing the two designs follows:

Rafters 2 in. × 8 in. × 16 ft. Long on 16-inch Centers

Number of rafters required	180
Overdesign of rafters	0%
Overdesign of sheathing, $\dfrac{24 - 16 \text{ in.}}{24 \text{ in.}}$	33%
Cost of rafters @ \$.10 per board foot, $\dfrac{180 \text{ rafters} \times 2 \text{ in.} \times 8 \text{ in.} \times 16 \text{ ft.}}{12 \text{ in.}} \times \$.10$	\$384.00

Rafters 2 in. × 10 in. × 16 ft. Long on 24-inch Centers

Number of rafters required, $\dfrac{180 \times 16 \text{ in.}}{24 \text{ in.}}$	120
Overdesign of rafters, $\dfrac{25 \text{ in.} - 24 \text{ in.}}{24 \text{ in.}}$	4%
Overdesign of sheathing	0%
Cost of rafters @ \$.10 per board foot, $\dfrac{120 \text{ rafters} \times 2 \text{ in.} \times 10 \text{ in.} \times 16 \text{ ft.}}{12 \text{ in.}} \times \$.10$	\$320.00

Not only is the design with 2 in. × 10 in. rafters adequate, but it is also cheaper by $64 plus whatever savings result from having to handle less pieces.

From a review of the foregoing it may be concluded that the function of a designer is to devise ways and means that are, when all things are considered, inherently most economical.

PROBLEMS

1. The allowable stresses for long-leaf yellow pine and tamarack are 1,200 and 1,000 pounds per square inch respectively. Yellow pine costs $100 and tamarack costs \$60 per thousand board feet. Calculate the ratio of the cost of a cantilever beam made of yellow pine to the cost of a beam made of tamarack, if the width, length, and load are the same for each beam. The load-carrying capacity for a rectangular cantilever beam is given by the formula $Pl = Sbd^2/6$, where P = load in pounds; l = length in inches; S = allowable stress in pounds per square inch; b = width of beam in inches; and d = depth of beam in inches.

2. Worker A receives \$1.20 per hour. He produces 80 items per hour of which 2 per cent are rejected. Worker B produces 90 items per hour of which 4 per cent are rejected. The hourly cost of the machines used by both A and B is \$2.40 and each rejected item results in a loss of \$.06 for material scrapped.

(a) What is A's cost per acceptable piece?

(b) What hourly wage should B receive if his cost per acceptable piece is to be equal to that of A?

3. What would have been the results of parts (a) and (b) of problem No. 2 if the hourly machine rate had been \$.80 and all other conditions had remained the same?

4. A homeowner who is painting his home after office hours finds that it requires an average of 20 minutes for him to change clothes and get equipment ready and 20 minutes to clean up when he stops painting each day. If there are 1,600 square feet of surface to be painted and his rate of painting is 45 square feet per hour, what will be the total number of hours required to complete the job if he devotes 1, 2, or 3 hours per day to the job?

5. In a study of shoveling it was found that the load per shovelful and the number of shovelfuls handled per minute vary with the condition of the shovel as shown in the accompanying table.

Period of hours after sharpening	0–8	8–16	16–24	24–32	32–40
Average load in pounds over given period	3.23	2.87	2.65	2.51	2.42
Average shovelfuls per minute over period	6.72	6.08	5.66	5.30	5.04

(a) If the cost of sharpening a shovel is \$.28 and the changes of both load and pace are due entirely to the dulling of the shovel, what reduction in cost in terms of dollars per 100 pounds of earth moved will result if shovels are sharpened at intervals of 16 hours instead of 40 hours? The shoveler's wage is \$1.00 per hour.

(b) What is the optimum interval of sharpening, assuming shovels can be sharpened only at the end of an 8-hour interval?

6. The hourly cost of operating an impregnating retort is \$26.40, exclusive of labor. The impregnating period (closed to open time) is 46 minutes. The unloading and loading period (open to closed time) when 1, 2, 3, 4, and 5 crew members are used is 36, 16, 11.2, 9.0, and 8.4 minutes respectively. Crew members will be idle during the impregnating period.

(a) One lead man is required regardless of crew size. If his wage rate is \$1.40 per hour and that of the other crew members is \$1.10 per hour, what crew size will result in a minimum cost per retort load?

(b) What is the most economical crew size if the time and wages of the crew during the impregnating period are charged to other jobs?

(c) What per cent wage incentive is justified for a 10 per cent reduction in open to closed time if conditions remain as given in Problem 6(a) and if a 3-man crew is used?

7. Observations of a 3-man crew engaged in a repetitive maintenance job in a refinery reveals that limitation of work space left two men idle 34 per cent of the time and one man idle 42 per cent of the time. All the men in the crew worked effectively 24 per cent of the time. Except for enforced idleness the crew size has no bearing upon the effectiveness of the efforts of individual crew members. Each crew member receives $1.56 per hour. The hourly rate of the equipment used is $1.40 and is independent of crew size. Determine the crew size that will result in a minimum cost for a maintenance job that can be performed in 4 hours by a 3-man crew.

8. The stress in a beam of rectangular cross section carrying a uniform load is given by the expression $S = 6L^2w/8bd^2$, where S = stress in beam in pounds per square inch; L = length of the beam in inches; w = uniform load on the beam in pounds per inch of length; b = width of the beam in inches; d = depth of the beam in inches.

An area 10 feet by 14 feet, surrounded by load-bearing walls, is to be spanned by floor joists spaced 2 feet apart. What is the comparative cost of the lumber required for beams of 10-foot and 14-foot span lengths, if the width of the beams is the same regardless of span length and the cost of lumber is $110 per thousand board feet?

9. A ¼-inch hole must be made in a number of pieces made of ⅛-inch steel. This may be done by laying out the position of the holes with dividers and a center punch and then drilling. If this method is employed, a workman whose wage rate is $1.36 per hour can complete 3 pieces per minute. An alternate method is to make a drill jig at a cost of $18 for use in drilling the holes. If the drill jig is used, a workman whose wage rate is $1.10 per hour can complete 14 pieces per minute.

(a) If all other costs of the two methods are considered equal, what will be the comparative costs of producing 2,500 pieces?

(b) What will be the comparative costs of producing 3,000 pieces?

(c) For what total production will the jig pay for itself?

CHAPTER

3

Interest and Interest Formulas

IN THE BUSINESS WORLD the term interest is used to designate a rental for the use of money. Fundamentally the rental paid for the use of a house, for instance, is essentially the same as interest paid for the use of money. Consider a person, Mr. Brown, who has arrived at a new location to go into business. He desires a house to shelter his family. If he is fortunate on his arrival at the new location, he may find a suitable dwelling that is for rent. After negotiations with the owner, Mr. Green, it is agreed that Mr. Brown is to pay the owner a rental of $80 at the end of each month for a period of one year and that Mr. Green shall let Mr. Brown have the use of the house for one year, at the end of which he is to relinquish it to Mr. Green.

Mr. Brown may also need funds, say $2,400, to establish himself in business. He therefore seeks out a person, Mr. Funds, who has money to rent, or, in the language of business, money to loan. After suitable negotiation, it is agreed that Mr. Brown is to pay Mr. Funds 8 per cent of $2,400 or $192 at the end of one year, and that, in return for this payment, Mr. Funds shall let Mr. Brown have $2,400 to use for one year, at the end of which the latter shall return or pay back $2,400 to Mr. Funds. Thus the total payment to be made to Mr. Funds at the end of the year will be $2,400 + $192, or $2,592.

Charging a rental for the use of money is a practice of long standing. The ethics and economics of interest has been a subject of discussion for philosophers, theologians, statesmen, and economists through the ages. The student interested in these aspects will find extensive literature for study, but the charging of interest for the use of money is a practical fact that confronts the person who makes studies in engineering economy. To provide the student with an understanding of the thinking of practical men in business, interest will be considered in turn from the standpoint of the lender and that of the borrower.

Interest Rate from Lender's Viewpoint

A person who has a sum of money has several alternatives.

1. He may exchange his sum of money for goods and services that will give him personal satisfaction, such as food, jewelry, clothing, medical care, and charity.

2. He may exchange his sum of money for productive goods or instruments, such as a stock of goods for resale, mining property for resale or development, or a lathe to facilitate production of goods, and thereby eventually recover a greater sum of money than he originally invested.

3. He may hoard his sum of money or let it lie idle without increase or decrease, either for the satisfaction of gloating over it, or in awaiting an opportunity for its subsequent use, or as reserve for use in an emergency.

4. He may lend his sum of money, asking only that the original sum be returned to him at some future date.

5. He may lend his sum of money on the condition that the lender will repay the sum at a future date and periodically pay him interest on the sum until it is repaid.

If he has decided on the last alternative, he must consider a number of things in deciding on the interest rate, of which the following are perhaps the most important:

(1) What is the probability that the borrower will not repay the loan? The lender may base his answer to this question on several things, such as the integrity of the borrower, his wealth, his potential earnings in the future, and the possibility for gain in the purpose for which the loan is to be used. For example, if in considering a loan for a year there is one chance in 50 that the loan will not be repaid, the lender will be justified in making a charge of $2 for each $100 or 2 per cent of the sum to compensate him for the risk of loss.

(2) What charge is necessary to compensate him for the effort of investigating the lender, drawing up the loan agreement, transferring the funds to the lender, and collecting the loan? If the sum of the loan is $1,000 for a period of a year and he values his effort

at $10, he is justified in charging $1 per $100 or one per cent of the sum to compensate him for his effort.

(3) What net amount of interest does he consider will compensate him for being deprived of electing other alternatives for disposing of his money? Assume that $3 per hundred or 3 per cent is considered adequate.

On the basis of the above reasoning, the interest rate arrived at will be 2 per cent plus 1 per cent plus 3 per cent, or 6 per cent.

An interest rate may be thought of, for convenience, as being made up of percentages for (1) risk of loss, (2) administrative expenses, and (3) pure gain or interest.

The Interest Rate from Borrower's Viewpoint

In many, if not most, cases the alternatives open to the borrower for the use of borrowed funds are limited by the lender, who may grant the loan only on condition that it be used for a specific purpose. Except as limited by the conditions of a loan, the borrower has open to him essentially the same alternatives for the use of money as a person who has ownership of money, but the borrower is faced with the necessity of repaying the amount borrowed and the interest on it in accordance with the conditions of the loan agreement or suffering the consequences. The consequences may be loss of reputation, seizure of property or of other moneys, or the placing of a lien on his future earnings. Organized society provides many pressures, legal and social, to induce a borrower to repay a loan. Default may have serious and even disastrous consequences to the borrower.

The prospective borrower's viewpoint on the rate of interest will be influenced by the use he intends to make of funds he may borrow. If he borrows the funds for personal use, the interest rate he is willing to pay will be a measure of the amount he is willing to pay for the privilege of having satisfactions immediately instead of in the future.

If funds are borrowed to finance operations expected to result in a gain, the interest to be paid must be less than the expected gain. An example of this is the common practice of banks and similar enterprises of borrowing funds to lend to others. In this case it is evident that the amount paid out as interest, plus risks incurred, plus administrative expenses must be less than the interest received on the money reloaned, if the practice is to be profitable. A borrower may be expected to seek to borrow funds at the lowest interest rate possible.

The Earning Power of Money

Funds borrowed for the prospect of gain are commonly exchanged for goods, services, or instruments of production. This leads us to the consideration of the earning power of money that may make it profitable to borrow money. Consider the following example.

Mr. Digg manually digs ditches for draining land. For this he is paid \$.06 per linear foot and averages 200 linear feet per day. Weather conditions limit this kind of work to 120 days per year. Thus, he has an income of \$12 per day worked or \$1,440 per year.

An advertisement brings to his attention a power ditcher that can be purchased for \$1,200. He buys the ditcher after borrowing \$1,200 at 8 per cent interest. The machine will dig an average of 800 linear feet per day. By reducing the price to \$.05 per linear foot he can get sufficient work to keep the machine busy when the weather will permit.

At the end of the year the ditching machine is abandoned because it is worn out. A summary of the venture follows.

Mr. Digg's Receipts and Disbursements
for the Year during which Ditcher Was Used

Receipts		
Amount of loan	\$1,200	
Payment for ditches dug, 120 days × 800 lin. ft. × \$.05	\$4,800	\$6,000
Disbursements		
Purchase of ditcher	\$1,200	
Fuel and repairs for machine	700	
Interest on loan, \$1,200 × .08	96	
Repayment of loan	1,200	\$3,196
Receipts less disbursements		\$2,804

For Mr. Digg, \$2,804 represents an increase in net earnings for the year with the ditcher over the previous year of \$2,804 − \$1,440 = \$1,364. Thus, the borrowed funds have made it possible for Mr. Digg to increase his earnings by \$1,364.

The above example is an illustration of what is commonly spoken of as the "earning power of money." It should be noted that the money borrowed was converted into an instrument of production. It was the instrument of production, the ditcher, which enabled Mr. Digg to increase his earnings. If Mr. Digg had held the money throughout the year it could have earned him nothing; also, if he had exchanged it for an instrument of production that turned out to be unprofitable, he might have lost money. Indirectly money has earning power when exchanged for profitable instruments of production.

Simple Interest

The rental rate for a sum of money is usually expressed as the per cent of the sum that is to be paid for the use of the sum for a period of one year. Interest rates are also quoted for periods other than one year, known as interest periods. In order to simplify the following discussion, consideration of interest rates for periods of other than one year will be deferred until later.

In simple interest the interest to be paid on repayment of a loan is proportional to the length of time the principal sum has been borrowed. A simple interest loan may be made for any period of time. Interest and principal become due only at the end of the period of the loan. It is common practice to consider a year as composed of twelve 30-day months in determining the length of time money has been borrowed.

Suppose that Mr. A borrowed $1,000 from Mr. B at simple interest at a rate of 6 per cent per annum. At the end of one year Mr. A would owe Mr. B $.06 \times \$1,000 = \$60$ for interest, plus the principal sum of $1,000.

On a loan of $100 at an interest rate of 7% per annum for the period February 1 to April 20 (80 days), the interest due on April 20 along with the principal sum of $100 would be $.07 \times \$100 \times (80 \div 360) = \$1.55$. Loans are rarely made at simple interest for periods of more than one year.

Compound Interest

In compound interest the interest that has been earned at the end of an interest period becomes due at that time, even though the loan extends for more than one interest period. For example, the payments on a loan of $1,000 at 6 per cent interest per annum for a period of 4 years would be as shown in Table 2.

It is also a common practice to increase a loan by an amount equal to the interest due at the end of each year. In this case no yearly interest payments are required and interest is said to be compounded. On this basis the pertinent facts relative to a loan of $1,000 at 6 per cent interest compounded annually for a period of 4 years will be as shown in Table 3.

TABLE 2. An Illustration of the Application of Compound Interest when Interest Is Paid Annually

Year No.	Amount Owed at Beginning Year	Interest To Be Paid at End of Year	Amount Owed at End of Year	Amount To Be Paid by Borrower to Lender at End of Year
1	$1,000	$60	$1,060	$ 60
2	1,000	60	1,060	60
3	1,000	60	1,060	60
4	1,000	60	1,060	1,060

TABLE 3. An Illustration of the Application of Compound Interest when Interest Is Permitted To Compound

Year No.	Amount Owed at Beginning of Year (A)	Interest To Be Added to Loan at End of Year (B)	Amount Owed at End of Year (A + B)	Amount To Be Paid by Borrower to Lender at End of Year
1	$1,000	$1,000 × .06 = $60	$1,000 (1.06) = $1,060.00	$00.00
2	1,060	1,060 × .06 = 63.60	1,000 $(1.06)^2$ = 1,123.60	00.00
3	1,123.60	1,123.60 × .06 = 67.42	1,000 $(1.06)^3$ = 1,191.02	00.00
4	1,191.02	1,191.02 × .06 = 71.46	1,000 $(1.06)^4$ = 1,262.48	1,262.48

Where the interest earned each year is added to the amount of the loan, as in the above example, it is said to be *compounded annually.* The final amount of a sum on which interest has been compounded is known as the *compound amount.*

FORMULAS FOR THE CALCULATIONS OF PROBLEMS INVOLVING INTEREST

General Terms Used in Interest Formulas

Let i = the interest rate for a given interest period
Let n = the number of interest periods
Let P = the principal sum at a time regarded to be the present

Let S = a sum, n interest periods hence, (1) equal to the compound amount of a principal sum P at an interest rate i, or (2) equal to the sum of the compound amounts of payments, R, in a series

Let R = a single payment in a series of n equal payments made at the end of each interest period

Single-Payment Compound-Amount Factor

When interest is permitted to compound, as in the example above, the interest is added to the principal at the end of each interest period. Results of substituting general terms in place of numerical

TABLE 4. Development of Single-Payment Compound-Amount Factor

Interest Period Number	Amount at Beginning of Interest Period	Interest Earned during Interest Period	Compound Amount at End of Year
1	P	Pi	$P + Pi = P(1 + i)$
2	$P(1 + i)$	$P(1 + i)i$	$P(1 + i) + P(1 + i)i = P(1 + i)^2$
3	$P(1 + i)^2$	$P(1 + i)^2i$	$P(1 + i)^2 + P(1 + i)^2i = P(1 + i)^3$
n	$P(1 + i)^{n-1}$	$P(1 + i)^{n-1}i$	$P(1 + i)^{n-1} + P(1 + i)^{n-1}i = P(1 + i)^n = S$

values in Table 3 are shown in Table 4. $(1 + i)^n$ is known as the Single-Payment Compound-Amount Factor.

EXAMPLE: If \$200 is invested at 5 per cent interest on January 1, 1950, what will be its compound amount January 1, 1960?

SOLUTION: Let S = the compound amount. Then

$$S = \$200(1 + .05)^{10}$$
$$= \$200 \times 1.629^{1}$$
$$= \$325.80$$

Single-Payment Present-Worth Factor

From the development of the Single-Payment Compound-Amount Factor,

$$P(1 + i)^n = S$$

[1] Values of this and similar factors to be developed are given in Interest Tables 47 to 57.

Solving for P,

$$S \times \frac{1}{(1+i)^n} = P$$

$\frac{1}{(1+i)^n}$ is known as the Single-Payment Present-Worth Factor.

EXAMPLE: What, on January 1, 1952, is the present worth of a $200 payment to be made on January 1, 1962, if the interest rate is 5 per cent?

SOLUTION: Let P = the present worth. Then

$$P = \$200 \times \frac{1}{(1+.05)^{10}}$$

$$= \$200 \times .6139 = \$122.78$$

It should be noted that the Single-Payment Compound-Amount Factor and the Single-Payment Present-Worth Factor are reciprocals.

Equal-Payment-Series Compound-Amount Factor

In many business situations and in many engineering economy problems, a series of equal payments occurring at the end of succeeding interest periods is encountered. The sum of the compound amounts of the several payments at the time the final payment of the series is made may be calculated with the aid of single-payment compound-amount factors.

Consider the calculation of the compound amount of a series of five $100 payments made at one year intervals at 6 per cent interest as illustrated in Table 5.

TABLE 5. Compound Amount of a Series of Annual Payments

End of Year	Amount of Year-End Payment, R	Compound Amount Factor	Compound Amount of Each Year-End Payment at End of Fifth Year	Compound Amount of Series, S
1	$100 ×	$(1.06)^4$ =		
2	$100 ×	$(1.06)^3$ =		
3	$100 ×	$(1.06)^2$ =		
4	$100 ×	$(1.06)^1$ =		
5	$100 ×	1 =	→ $100 + $106 + $113 + $119 + $126	= $564

It is apparent that the method illustrated will be cumbersome for calculating the compound amount of an extensive series; therefore it is desirable to develop a simpler solution for this type of calculation. Using the previously defined terms, i, n, S, and R, and referring to Table 5, it may be deduced readily that

$$(1)\quad S = R \times 1 + R(1 + i) \ldots + R(1 + i)^{n-2} + R(1 + i)^{n-1}$$

Multiplying this equation by $(1 + i)$ results in

$$(2)\quad S(1 + i) = R(1 + i) + R(1 + i)^2 \ldots + R(1 + i)^{n-1} + R(1 + i)^n$$

Subtracting (1) from (2) results in

$$S(1 + i) - S = -R + R(1 + i)^n$$

$$S = R\left[\frac{(1 + i)^n - 1}{i}\right]$$

$\dfrac{(1 + i)^n - 1}{i}$ is known as the Equal-Payment-Series Compound-Amount Factor.

EXAMPLE: What is the compound amount of a series of seven \$100 year-end payments at 5 per cent interest, seven years hence?

SOLUTION: Let S = the compound amount of the series. Then

$$S = \$100 \times \frac{(1 + .05)^7 - 1}{.05}$$

$$= \$100 \times 8.142$$

$$= \$814.20$$

Equal-Payment-Series Sinking-Fund Factor

From the development of the Equal-Payment-Series Compound-Amount Factor,

$$S = R\left[\frac{(1 + i)^n - 1}{i}\right]$$

Solving for R,

$$R = S\left[\frac{i}{(1 + i)^n - 1}\right]$$

$\frac{i}{(1+i)^n - 1}$ is known as the Equal-Payment-Series Sinking-Fund Factor.

EXAMPLE: If it is desired to accumulate $700 by making a series of twelve equal year-end deposits at 5 per cent interest, what must be the amount of each payment?

SOLUTION: Let R = the year-end deposit. Then

$$R = \$700 \times \frac{.05}{(1+.05)^{12} - 1}$$

$$R = \$700 \times .06283$$

$$= \$43.98$$

Equal-Payment-Series Capital-Recovery Factor

Substituting $P(1+i)^n$ for S in the relationship

$$S\left[\frac{i}{(1+i)^n - 1}\right] = R$$

results in

$$P\left[\frac{i(1+i)^n}{(1+i)^n - 1}\right] = R$$

$\frac{i(1+i)^n}{(1+i)^n - 1}$ is known as the Equal-Payment-Series Capital-Recovery Factor.

EXAMPLE: $1,000 invested at 5 per cent will provide for 8 equal year-end payments of what amount?

SOLUTION: Let R = the amount of each payment. Then

$$R = \$1{,}000 \times \frac{(1+.05)^8 \times .05}{(1+.05)^8 - 1}$$

$$R = \$1{,}000 \times .15472$$

$$= \$154.72$$

That the Equal-Payment-Series Capital-Recovery Factor is equal to the Sinking-Fund Factor plus i may be demonstrated as follows:

$$\frac{i(1+i)^n}{(1+i)^n-1} = \frac{i(1+i)^n}{(1+i)^n-1} - i + i$$
$$= \frac{i(1+i)^n - i(1+i)^n + i}{(1+i)^n-1} + i$$
$$= \frac{i}{(1+i)^n-1} + i$$

And, conversely,

$$\frac{i(1+i)^n}{(1+i)^n-1} - i = \frac{i}{(1+i)^n-1}$$

Equal-Payment-Series Present-Worth Factor

$$P\left[\frac{i(1+i)^n}{(1+i)^n-1}\right] = R$$

Solving for P,

$$P = R\left[\frac{(1+i)^n-1}{i(1+i)^n}\right]$$

$\dfrac{(1+i)^n-1}{i(1+i)^n}$ is known as the Equal-Payment-Series Present-Worth Factor.

EXAMPLE: What is the present worth of a series of 6 equal annual payments of \$100 if the interest rate is 5 per cent and the first payment of the series is made one year from now?

SOLUTION: Let P = the present worth. Then

$$P = \$100 \times \frac{(1+.05)^6-1}{(1+.05)^6 \times .05}$$
$$P = \$100 \times 5.076$$
$$= \$507.60$$

Summary of Factors

Single-Payment Compound-Amount Factor	$(1+i)^n$
Single-Payment Present-Worth Factor	$\dfrac{1}{(1+i)^n}$

Equal-Payment-Series Compound-Amount Factor	$\frac{(1+i)^n - 1}{i}$
Equal-Payment-Series Sinking-Fund Factor	$\frac{i}{(1+i)^n - 1}$
Equal-Payment-Series Present-Worth Factor	$\frac{(1+i)^n - 1}{i(1+i)^n}$
Equal-Payment-Series Capital-Recovery Factor	$\frac{i(1+i)^n}{(1+i)^n - 1}$

The numerical values of the above factors for the most frequently used values of i and n appear in the Interest Tables 47 to 57.

Designation of Factors

Since problems will be solved primarily by use of the tables, it will be convenient to have a simple way to designate the source of the values taken from the table. The designation used in this text is as follows:

Single-Payment Compound-Amount Factor	SP *i-n* (xxxxx)
Single-Payment Present-Worth Factor	PS *i-n* (xxxxx)
Equal-Payment-Series Compound-Amount Factor	SR *i-n* (xxxxx)
Equal-Payment-Series Sinking-Fund Factor	RS *i-n* (xxxxx)
Equal-Payment-Series Present-Worth Factor	PR *i-n* (xxxxx)
Equal-Payment-Series Capital-Recovery Factor	RP *i-n* (xxxxx)

In solving a problem such as *Find the present worth of $100 eight years hence at 6 per cent interest,* the solution will be stated, "The present worth of $100 eight years hence at 6 per cent interest is

PS 6–8
$100 × (.6274) = $62.74.

Two important advantages of this scheme for designating factors are: (1) the equations for solving problems may be set up prior to looking up any values of factors from the tables and inserting them in the parenthesis, and (2) the source and the identity of values taken from the tables are maintained throughout the solution. See page 72.

TABLE 6. Schematic Illustration of Use of Factors

Time	Single Payment: Use of Compound-Amount Factor	Single Payment: Use of Present-Worth Factor	Equal Payment Series: Use of Compound-Amount Factor	Equal Payment Series: Use of Sinking-Fund Factor	Equal Payment Series: Use of Present-Worth Factor	Equal Payment Series: Use of Capital-Recovery Factor
Present	P	P	—	—	P	P —
End of Year 1			R	R	R	R
End of Year 2			R	R	R	R
End of Year 3			R	R	R	R
End of Year $n-1$			R	R	R	R
End of Year n	S	S	R S	S R	R	R
	$S = P(\text{xxxxx})$ SP i-n	$P = S(\text{xxxxx})$ PS i-n	$S = R(\text{xxxxx})$ SR i-n	$R = S(\text{xxxxx})$ RS i-n	$P = R(\text{xxxxx})$ PR i-n	$R = P(\text{xxxxx})$ RP i-n

Schematic Illustration of Use of Factors

If the factors are to be used with facility, the purpose of each must be visualized so that they readily come to mind as needed. The schematic arrangement of the factors in Table 6 should be helpful in this connection.

In Table 6 five important points are to be noted in the use of the factors.

1. The end of one year is the beginning of the next year.

2. P is at the beginning of a year at a time regarded as being the present.

3. S is at the end of the nth year from a time regarded as being the present.

4. An R occurs at the *end* of each year of the period under consideration. When P and R are involved, the first R of the series occurs one year after P. When S and R are involved, the last R of the series occurs at the same time as S.

5. In the solution of problems, the quantities P, S, and R must be set up to conform with the pattern applicable to the factors used.

Interest Periods Other Than One Year

For simplicity of explanation the discussion of interest to this point has embraced interest periods of only one year. Interest rates are usually quoted on an annual basis known as the *nominal interest rate*. However, loan agreements frequently specify that interest shall be paid more frequently, say each half year, each quarter year, or each month. Such agreements result in interest periods of one-half year, one-quarter year, or one-twelfth year, and the compounding of interest twice, four times, or twelve times a year respectively.

Effective Interest Rates

The effect of this more frequent compounding is an *effective interest rate* higher than the nominal interest rate. Consider a nominal interest rate of 6 per cent with an interest period of one-half year. The true rate is in reality 3 per cent for one-half year. The compound amount of \$1 for one year at 3 per cent for one-half year is equal to $\$1\,(1.03)^2 = \$1.0609$. The effective interest rate is equal to $1.0609 - 1.0000 = .0609$ or 6.09 per cent.

If $i =$ the nominal interest rate, and $c =$ the number of interest periods per year, then the effective annual interest rate $= \left(1 + \frac{i}{c}\right)^c - 1.$

The effective interest rates of a nominal interest rate of 6 per cent compounded two, four, and twelve times per year are respectively .0609, .0612, and .0617, or 6.09 per cent, 6.12 per cent, and 6.17 per cent.

For some calculations where continuous functions are desirable, interest may be considered to be compounded an infinite number of times per year, that is, continuously. When interest is compounded continuously the expression for effective interest rate may be derived as follows:

$$\left(1 + \frac{i}{c}\right)^c - 1 = \left[\left(1 + \frac{i}{c}\right)^{c/i}\right]^i - 1$$

$$\lim_{c \doteq \infty} \left(1 + \frac{i}{c}\right)^{c/i} = e$$

and

$$\left[\left(1 + \frac{i}{c}\right)^{c/i}\right]^i - 1 = e^i - 1$$

and $e^i - 1$ is the effective annual interest rate.

The effective annual interest rate of a nominal rate of 6 per cent compounded continuously is

$$\begin{aligned} e^{.06} - 1 &= 2.718^{.06} - 1 \\ &= 1.0619 - 1 \\ &= .0619 \end{aligned}$$

An interest rate that if compounded continuously will be equivalent to a nominal interest rate may be found as follows. Suppose that it is desired to find an interest rate i which, if compounded continuously, will be equivalent to a nominal interest rate of 6 per cent per annum.

$$\begin{aligned} e^i - 1 &= .06 \\ e^i &= 1.06 \\ i(\log e) &= \log 1.06 \\ i &= \frac{\log 1.06}{\log e} \\ i &= \frac{.0253}{.4343} = .0583 \end{aligned}$$

Use of Interest Tables for Interest Periods Other than One Year

Interest Tables on pages 481 to 491 are set up on the basis of an interest rate for an interest period. For a nominal interest rate of 6 per cent compounded annually, for instance, the interest rate is 6 per cent for an interest period of one year and the 6 per cent table should be used for calculations. For a nominal interest rate of 6 per cent compounded semiannually, the interest rate is 3 per cent for an interest period of one-half year and the 3 per cent table should be used. The number of interest periods will be double the number of years involved.

For example, a loan of $1,000 for a period of 2 years at 6 per cent interest compounded quarterly would require eight interest payments of (.06 ÷ 4) × $1,000 = $15 at three-month intervals.

In most practical engineering economy studies, nominal interest rates and interest periods of one year are adequate; they should be used unless there are reasons to do otherwise.

Interpolation on Interest Tables

Within the range of the tables, values of factors for all interest rates and years between those given may be obtained by interpolation. The errors introduced in so doing are slight and are of negligible significance in practical problems. See Table 7.

TABLE 7. Per Cent Error in 5½ Per Cent Factors Obtained by Interpolating 5 Per Cent and 6 Per Cent Factors

Factor	Years					
	5	10	20	30	40	60
Single-Payment Compound-Amount Factor	+.022	+.101	+.428	+.980	+1.76	+3.99
Equal-Payment-Series Sinking-Fund Factor	+.005	+.010	+.089	+.324	+.687	+1.95
Equal-Payment-Series Capital-Recovery Factor	+.004	+.014	+.041	+.065	+.080	+.079

Payments Occurring during Interest Periods

To use the several interest factors that have been developed, it is necessary that the disbursements conform to the pattern for which the factors are applicable. See Table 6.

In the analysis of engineering economy problems, disbursements made to initiate an alternative are considered to take place at the beginning of the period embraced by the alternative. Disbursements occurring during interest periods are usually assumed to occur at the end of the interest period in which they occur.

PROBLEMS

1. How long will a \$3,000 note bearing 5 per cent simple interest have to run to amount to \$3,525?

2. If \$1,500 earns \$37.50 in six months, what is the annual rate of interest?

3. A man loans \$500 at 6 per cent simple interest for four years. At the end of this time he invests the entire amount (principal plus interest) at 5 per cent compounded annually for 10 years. How much will he have at the end of the 14-year period?

4. Compare the interest earned by \$100 for 10 years at 6 per cent simple interest with that earned by the same amount for 10 years at 6 per cent compounded annually.

5. Rewrite the formula given for the Single-Payment Compound-Amount Factor to apply to the compounding of interest at the end of each period, where p represents the number of compounding periods per year, y represents the number of years, and j represents the nominal annual rate of interest. Use P as the present sum and S as the compound amount and express S in terms of P, j, p, and y.

6. Set up equations with the proper designation of factors for the following situations:

   (a) Find the present value V of a compound amount A at y years hence, if interest is at the rate u compounded annually.

   (b) Find the compound amount B at the end of x years, if T dollars are invested now at a rate of interest w compounded annually.

   (c) To what value C will U dollars accumulate in z years at a rate of interest v compounded annually?

   (d) What value W can be paid now for the promise of D dollars c years hence, if interest at a rate t compounded annually is desired?

7. Develop a formula for finding the accumulated amount S at the end of n interest periods which will result from a series of beginning-of-period payments each equal to B, if the latter are placed in a sinking fund for which the interest rate per period is i, compounded each period.

8. Set up the equations for the following situations using the proper designation of factors.

   (a) What amount W will be accumulated at the end of x years if Y dollars are deposited at the end of each year into a fund that yields a rate of interest v compounded annually?

   (b) What sum A can be paid now for the promise of an income of B dollars every year end for the next c years, if a rate of interest d is used?

(c) What uniform year end payment M is necessary to assure an amount N at a time o years in the future, if an interest rate of p is received?

(d) What year end income T can be received over a period of r years, if an amount L is deposited now at a rate of interest h?

9. How would you determine a desired *equal-payment-series present-worth factor* if you had only

(a) a table of single-payment present-worth factors?
(b) a table of equal-payment-series capital-recovery factors?
(c) a table of equal-payment-series sinking-fund factors?
(d) a table of equal-payment-series compound-amount factors?
(e) a table of single-payment compound-amount factors?

10. What effective annual interest rate corresponds to the following situations?

(a) Nominal interest rate of 7 per cent compounded quarterly.
(b) Nominal interest rate of 4 per cent compounded monthly.
(c) Nominal interest rate of 8 per cent compounded quarterly.
(d) Nominal interest rate of 8 per cent compounded every 2 years.

11. (a) An effective interest rate of 6 per cent is desired. What nominal rate should be expressed if compounding is to be quarterly?

(b) What nominal interest rate is required to give an effective rate of 8 per cent if compounding is semiannually?

12. The Square Deal Loan Company offers the following terms: ½ per cent interest per week compounded weekly. What is the effective annual interest rate?

13. (a) What is the equivalent annual interest rate if a nominal rate of 8 per cent is compounded continuously?

(b) If an equivalent annual interest rate of 8 per cent is desired, what must the nominal rate be if compounding is continuous?

14. Using proper factor designations, set up the equations for the following situations:

(a) What amount L can a building construction company afford to pay for a crane? The service life of the crane is expected to be m years and will result in a saving of Q dollars every 3 months. A nominal interest rate x compounded quarterly is required on all investments.

(b) What sum O must a man deposit every six months if he desires to have an amount M available for the purchase of a home u years from now? Interest obtainable is at a nominal rate y compounded semiannually.

(c) What amount E must be accumulated at the end of v years on a present investment B, if a nominal interest rate x compounded annually is desired?

15. Using proper factor designations, set up the equations for the following situations:

(a) What value T can be paid for a home x years from now, if Y dollars are deposited into a sinking fund annually. A nominal rate of interest u compounded annually can be obtained.

(b) A loan can be repaid by a single payment of N dollars g years hence, or by a single payment of F dollars now. Show how to find F if N is known and if a nominal interest rate of c compounded quarterly is used.

(c) What semiannual return V would be required from a machine costing X dollars, if the service life of the machine is estimated to be n years and a nominal interest rate r compounded seimannually is desired?

16. From the interest tables given in the text determine the value of the following factors by interpolation:

(a) The Single-Payment Compound-Amount Factor for 10 periods at 5½ per cent interest.

(b) The Equal-Payment-Series Sinking-Fund Factor for 42 periods at 6 per cent interest.

(c) The Equal-Payment-Series Present-Worth Factor for 12 periods at 3¼ per cent interest.

(d) The Equal-Payment-Series Compound-Amount Factor for 39 periods at 8 per cent interest.

17. From the interest tables in the text determine the following value of the factors by interpolation:

(a) Single-Payment Present-Worth Factor for 37 periods at 5½ per cent interest.

(b) The Equal-Payment-Series Capital-Recovery Factor for 48 periods at 6¼ per cent interest.

CHAPTER

4

Calculations of Equivalence Involving Interest

THE WORTH OF ENGINEERING DESIGNS and plans of operation is measured in economic terms. A first step in evaluation is to translate engineering features into concrete expenditures and receipts. Many engineering proposals involve money disbursements and receipts over a period of time. Thus the time value of money in the form of interest becomes an important consideration in economy studies.

The methods for determining the economic desirability of other than short-run engineering proposals require calculations involving interest. For this reason it is desirable that the application of the interest factors that have been developed be so well understood that their use becomes a routine operation incidental to proper analysis. The subject of engineering economy will be not only more enjoyable and profitable, but also easier if calculations involving interest are mastered at the outset. The purpose of this chapter is to define the terms used and to illustrate by example the applications of the several factors used in interest calculations.

Meaning of Equivalence

The purpose of many calculations in engineering economy is to place prospective receipts and disbursements of two or more proposals on an equivalent basis for comparison.

Two things are said to be equivalent when they have the same effect. For instance, the torques produced by applying forces of 100 pounds and 200 pounds 2 feet and 1 foot respectively from the fulcrum of a lever are equivalent. This equivalence can be expressed as follows:

$$100 \text{ lb.} \times 2 \text{ ft.} = 200 \text{ lb.} \times 1 \text{ ft.}$$

$$200 \text{ lb.-ft.} = 200 \text{ lb.-ft.}$$

Another concept is that two things are equivalent when they have the same value in exchange. For instance, 4 tons of Material A at \$18 per ton are equivalent to 12 tons of Material B at \$6 per ton, since

$$4 \text{ tons} \times \frac{\$18}{\text{ton}} = 12 \text{ tons} \times \frac{\$6}{\text{ton}}$$

$$\$72 = \$72$$

Similarly, \$384.64 on June 6, 1950 is equivalent to \$526.40 on June 6, 1958, for an interest rate of 4 per cent per annum.

This is so because persons who consider 4 per cent a satisfactory rate of interest would be willing to pay \$384.64 on June 6, 1950 to receive \$526.40 on June 6, 1958, or vice versa.

The above equivalence may be expressed thus:

$$\$384.64 = \frac{1}{(1 + .04)^8} \times \$526.40$$

$$\$384.64 = \overset{\text{PS 4-8}}{(.7307)} \times \$526.40$$

$$\$384.64 = \$384.64$$

The above equivalence might also have been expressed thus:

$$\$384.64 \times (1 + .04)^8 = \$526.40$$

$$\$384.64 \times \overset{\text{SP 4-8}}{(1.369)} = \$526.40$$

$$\$526.40 = \$526.40$$

Three factors are involved in the equivalence of sums of money. These are (1) the amount of the sums, (2) the time of occurrence of the sums, and (3) the interest rate.

The interest factors that have been developed embody consideration of time and the interest rate. They constitute a convenient way for taking the time value of money into consideration in placing sums of money occurring at different times upon an equivalent basis.

The Basis for Comparison Is Equivalence

If two or more situations are to be compared, their characteristics must be placed on an equivalent basis. Which is worth more, 4 ounces of Item A or 1800 grains of Item A? This question cannot be answered until the two amounts are placed upon an equivalent basis.

By use of proper conversion factors the question becomes: Which is worth more, 1750 grains of Item A or 1800 grains of Item A? This question can be answered.

The relative economy of the several alternatives may not be apparent even though the payments to be made for the service under each alternative may be known. Consider the following example.

Electric current is desired at an isolated location by the Sun Company. It is estimated that 80,000 kw.-hr. of current will be needed each year during the next ten years. A power company makes two proposals for supplying the needed current.

In one proposal the power company agrees to build the necessary connecting transmission equipment and supply the 80,000 kw.-hr. of power at a rate of \$.05 per kw.-hr. This alternative will result in an annual power bill of \$4,000.

In the second proposal the power company agrees to supply the 80,000 kw.-hr. of power at a rate of \$.02 per kw.-hr. This will result in an annual bill of only \$1,600, but the Sun Company is to provide the connecting transmission equipment. Sun Company engineers estimate that the transmission equipment will cost \$18,000 to install, that its maintenance during the ten years will be negligible, and that at the end of the ten years it will be possible to sell the used equipment for \$6,000. The Sun Company considers an interest rate of 8 per cent adequate in making a comparison of these alternatives.

Identical service will be provided by either Proposal No. 1 or Proposal No. 2. The question to be answered is: Which proposal will supply 80,000 kw.-hr. of energy per year for a period of 10 years at least cost?

The patterns of disbursements in prospect for these proposals are given in Table 8.

It is not apparent from a cursory examination of the disbursements of the two proposals which is financially the most desirable. The disbursements of the two proposals can be placed on a comparable basis by calculating their present worths or by calculating the equivalent annual cost of each.

The present worths of the disbursements for Proposal No. 1 and Proposal No. 2 are respectively \$26,840 and \$25,957. This means in effect that 10 years of service will cost the equivalent of a present expenditure of \$26,840 if Proposal No. 1 is selected and \$25,957 if Proposal No. 2 is selected. These figures are directly comparable.

Thus the desired service may be provided at least cost by Proposal No. 2.

Calculations made on the basis of equivalent annual cost result in equivalent annual costs for Proposal No. 1 and Proposal No. 2 of $4,000 and $3,866 respectively.

TABLE 8. Pattern of Disbursements of Proposal No. 1 and Proposal No. 2

End of Year No.	Disbursements Proposal No. 1	Disbursements Proposal No. 2
0	$ 000	$18,000
1	4,000	1,600
2	4,000	1,600
3	4,000	1,600
4	4,000	1,600
5	4,000	1,600
6	4,000	1,600
7	4,000	1,600
8	4,000	1,600
9	4,000	1,600
10	4,000	1,600–6,000*

* A receipt, *i.e.* a negative disbursement.

This means that each year's service will cost the equivalent of $4,000 for Proposal No. 1 and $3,866 for Proposal No. 2. Again, these figures are directly comparable.

Single-Payment Compound-Amount Factor Calculations

SP i-n

This factor is $(1 + i)^n$ and is designated by (xxxxx).

The term *compound amount* is used here to designate a sum S, at a given time in the future, that is equivalent to a principal sum P at a given earlier time when time value of money is considered.

In Fig. 5, $295.40 on March 1, 1954 is equivalent to $200 on March 1, 1946 for an interest rate of 5 per cent compounded annually. Stated in another way, $295.40 on March 1, 1954 is the compound amount of $200 on March 1, 1946.

Note that the principal sum P occurs at the beginning of an interest period and that the compound amount S occurs at the end of an interest period.

Solution for S when P, i, and n are known. The solution for finding the compound amount on March 1, 1954 that is equivalent to a principal sum P equal to \$200 on March 1, 1946 for an interest rate of 5 per cent compounded annually follows:

$$n = 1954 - 1946 = 8$$

Then

$$S \text{ (compound amount)} = \$200\overset{\text{SP } 5\text{-}8}{(\text{xxxxx})}$$

Substituting the correct figure from the interest table,

$$S = \$200\overset{\text{SP } 5\text{-}8}{(1.477)}$$

$$= \$295.40$$

Solution for i when P, S, and n are known. If the principal sum P, its compound amount S, and the number of interest periods n are known, the interest rate i may be determined by interpolation of the interest tables or algebraically. Let

$$P = \$300$$
$$S = \$525$$
$$n = 9$$
$$i = ?$$

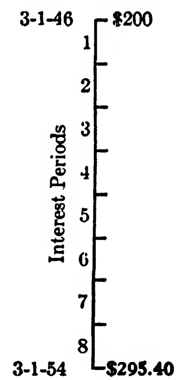


FIG 5. Use of single-payment compound amount factor.

The interest rate i may be solved for by interpolation of the interest tables in the following manner:

$$S = P \times \overset{\text{SP } i\text{-}n}{(\text{xxxxx})}$$

$$\$525 = 300\overset{\text{SP } i\text{-}9}{(\text{xxxxx})}$$

$$\overset{\text{SP } i\text{-}9}{(\text{xxxxx})} = \frac{525}{300}$$

$$\overset{\text{SP } i\text{-}9}{(1.750)} = 1.750$$

A search of the interest tables reveals that 1.750 falls between the Single-Payment Compound-Amount Factors in the 6 per cent and 7 per cent tables for $n = 9$. From the 6 per cent table

$$\overset{\text{SP } 6\text{-}9}{(1.689)}$$

From the 7 per cent table

SP 7-9
(1.838)

By proportion (See Fig. 6),

$$i = \left[6 + 1 \times \frac{1.750 - 1.689}{1.838 - 1.689}\right] \text{per cent}$$

$$= \left[6 + \frac{.061}{.149}\right] \text{per cent}$$

$$= 6.41 \text{ per cent}$$

The interest rate i might have been solved for as follows:

$$S = P\,(1 + i)^n$$

$$\$525 = \$300\,(1 + i)^9$$

$$(1 + i)^9 = \frac{525}{300}$$

$$i = \sqrt[9]{1.750} - 1$$

$$\log 1.750 = .24304$$

$$\log \sqrt[9]{1.750} = .02700$$

$$\sqrt[9]{1.750} = 1.0641$$

and

$$i = 1.0641 - 1$$

$$i = .0641, \text{ or } 6.41 \text{ per cent}$$

Solutions with logarithms as illustrated above are usually more time-consuming than solutions by interpolation. This is particularly true when any of the four equal-payment-series factors are involved, since these require quite cumbersome solutions. For use in connection with engineering economy studies, which always embrace estimates, the error introduced by the interpolation method will rarely be of significance. This method is accordingly recommended.

Solution for *n* when *P*, *S*, and *i* are known. If the principal sum P, its compound amount S, and the interest rate i are known, the number of interest periods n may be determined by interpolation of the interest tables or algebraically. Let

$$P = \$400$$
$$S = \$704.40$$
$$i = .07$$
$$n = ?$$

The number of interest periods n may be solved for by interpolation of the interest tables in the following manner:

$$S = P \times \overset{\text{SP } i\text{-}n}{(\text{xxxxx})}$$

$$\$704.40 = \$400\overset{\text{SP } 7\text{-}n}{(\text{xxxxx})}$$

$$\overset{\text{SP } 7\text{-}n}{(\text{xxxxx})} = \frac{\$704.40}{\$400}$$

$$\overset{\text{SP } 7\text{-}n}{(1.761)} = 1.761$$

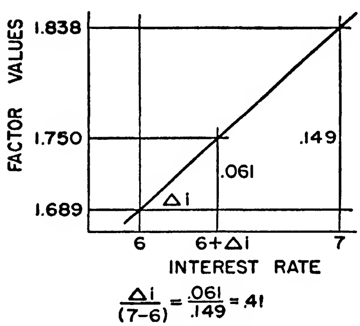


FIG. 6. Interpolation for i.

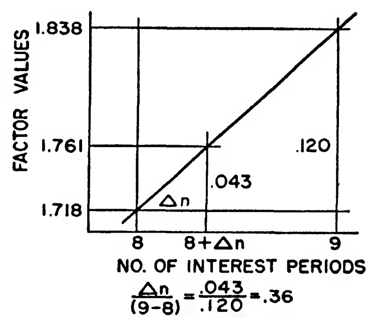


FIG. 7. Interpolation for n.

A search of the 7 per cent interest tables reveals that 1.761 falls between the Single-Payment Compound-Amount Factors for $n = 8$ and 9. For $n = 8$

$$\overset{\text{SP } 7\text{-}8}{(1.718)}$$

For $n = 9$

$$\overset{\text{SP } 7\text{-}9}{(1.838)}$$

By proportion (See Fig. 7),

$$n = \left[8 + 1 \times \frac{1.761 - 1.718}{1.838 - 1.718}\right] \text{years}$$

$$= \left[8 + \frac{.043}{.120}\right] \text{years}$$

$$= 8.36 \text{ years}$$

The number of interest periods n might have been solved for with logarithms as follows:

$$S = P \times (1 + i)^n$$

$$\$704.40 = \$400\,(1 + .07)^n$$

$$(1.07)^n = \frac{\$704.40}{\$400}$$

$$n \log 1.07 = \log \frac{704.40}{400}$$

$$n = \frac{\log 704.40 - \log 400}{\log 1.07}$$

$$n = \frac{2.84782 - 2.60206}{.02938}$$

$$n = 8.37 \text{ years}$$

Single-Payment Present-Worth Factor Calculations

This factor is $\frac{1}{(1 + i)^n}$ and is designated by $\overset{\text{PS } i\text{-}n}{(\text{xxxxx})}$.

The term *present worth* is used here to designate a sum P, at a given time regarded as being the present, which is equivalent to a second sum S, at a given future time when time value of money is considered.

In Fig. 8, \$198.80 on August 1, 1956 (a time regarded as being the present) is equivalent to \$400 on August 1, 1968 for an interest rate of 6 per cent compounded annually. Stated in other words, \$198.80 as of August 1, 1956 is the *present worth* of \$400 as of August 1, 1968.

Again note that the principal sum P occurs at the beginning of an interest period and that the compound amount S occurs at the end of an interest period.

Solution for P when S, i, and n are known. The method of finding the present worth P, on August 1, 1956, of a sum S equal to

\$400 on August 1, 1968, for an interest rate of 6 per cent compounded annually follows:

$$n = 1968 - 1956 = 12$$

Then

$$P \text{ (present worth)} = \$400\overset{\text{PS 6-12}}{(\text{xxxxx})}$$

From the interest table $\overset{\text{PS 6-12}}{(.4970\ )}$, and

$$P = \$400\overset{\text{PS 6-12}}{(.4970\ )}$$
$$P = \$198.80$$

Solution for *i* when *P*, *S*, and *n* are known. If a future sum *S*, its present worth *P*, and the number of interest periods *n* are known, the interest rate *i* may be determined in a manner similar to that previously illustrated under the heading *Single-Payment Compound-Amount Factor Calculations* or as follows.

Let *i*, the nominal interest rate, be compounded semi-annually and let 6 years intervene between the two sums, *P* and *S*, and:

$$S = \$400 \qquad j = \frac{i}{2}$$
$$P = \$294.80$$
$$n = 2 \times 6 = 12 \qquad i = ?$$

Then

$$P = S \times \overset{\text{PS } j\text{-12}}{(\text{xxxxx})}$$

$$\$294.80 = \$400\ \overset{\text{PS } j\text{-12}}{(\text{xxxxx})}$$

$$\overset{\text{PS } j\text{-12}}{(\text{xxxxx})} = \frac{294.80}{400}$$

$$\overset{\text{PS } j\text{-12}}{(.7370\ )} = .7370$$

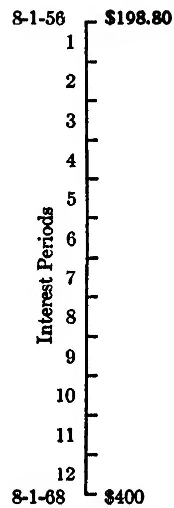


FIG. 8. Use of single-payment present-worth factor.

A search of the interest tables reveals that .7370 falls between the Single-Payment Present-Worth Factors in the 2 per cent and 3 per cent tables for *n* = 12. From the 2 per cent table

$$\overset{\text{PS 2-12}}{(.7885\ )}$$

From the 3 per cent table $\overset{\text{PS 3-12}}{(.7014\ )}$

By proportion,

$$j = \left[2 + 1 \times \frac{.7370 - .7885}{.7014 - .7885}\right] \text{ per cent}$$

$$= \left[2 + \frac{-.0515}{-.0871}\right] \text{ per cent}$$

$$= 2.59 \text{ per cent}$$

and

$$i = 2 \times 2.59 \text{ per cent}$$

$$i = 5.18 \text{ per cent}$$

Solution for *n* when *P*, *S*, and *i* are known. If a future sum S, its present worth P, and the interest rate i are known, the number of interest periods n may be determined in the manner illustrated under the heading *Single-Payment Compound-Amount Factor Calculations* or as follows. Let

$$S = \$800$$
$$P = \$491.36$$
$$i = .04 \text{ compounded annually}$$
$$n = ?$$

$$P = S \times \overset{\text{PS } i\text{-}n}{(\text{xxxxx})}$$

$$\$491.36 = \$800\overset{\text{PS 4-}n}{(\text{xxxxx})}$$

$$\overset{\text{PS 4-}n}{(\text{xxxxx})} = \frac{491.36}{800}$$

$$\overset{\text{PS 4-}n}{(.6142)} = .6142$$

A search of the 4 per cent interest tables reveals that .6142 falls between the Single-Payment Present-Worth Factors for $n = 12$ and 13. For $n = 12$

$$\overset{\text{PS 4-12}}{(.6246\ )}$$

For $n = 13$ $\overset{\text{PS 4-13}}{(.6006\ )}$

By proportion,

$$n = \left[12 + 1 \times \frac{.6142 - .6246}{.6006 - .6246}\right] \text{ years}$$

$$n = \left[12 + \frac{-.0104}{-.0240}\right] \text{ years}$$

$$n = 12.43 \text{ years}$$

Solution with logarithms for i and n where the Single-Payment Present-Worth Factor is involved is similar to that explained under the heading *Single-Payment Compound-Amount Factor Calculations.*

Equal-Payment-Series Compound-Amount Factor Calculations

This factor is $\frac{(1 + i)^n - 1}{i}$ and is designated by $\overset{\text{SR } i\text{-}n}{(\text{xxxxx})}$.

The term *compound amount* is used here to designate a future sum S at a given time in the future, which is equivalent to a series of payments R, occurring at the end of successive interest periods such that the last R concurs with S.

In Fig. 9, \$335.76 on February 2, 1958 is equivalent to a series of seven \$40 year-end payments as shown for an interest rate of 6 per cent compounded annually.

Stated in another way, \$335.76 on February 2, 1958 is the *compound amount* of a series of \$40 year-end payments between February 2, 1951 and February 2, 1958 at 6 per cent interest compounded annually.

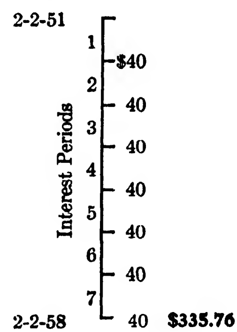


FIG. 9. Use of equal-payment series compound-amount factor.

Among people familiar with interest calculations it is often sufficient to say that \$335.76 is the compound amount of a series of seven \$40 year-end payments.

It should be noted that the R payments occur only at the end of interest periods; that the first R payment occurs after an elapse of one interest period from the initial date of the total period under consideration; and that the final R payment and the compound amount S both concur with the final date of the total period under consideration. The implications of the above statement should be mastered.

Solution for *S* when *R*, *i*, and *n* are known. The solution for finding the compound amount, on February 2, 1958, of a series of seven $40 year-end payments whose final payment occurs simultaneously with the compound amount being determined, for an interest rate of 6 per cent compounded annually, follows:

$$n = 1958 - 1951 = 7$$

Then

$$S \text{ (compound amount)} = \$40\overset{\text{SR 6-7}}{(\text{xxxxx})}$$

From the interest table $\overset{\text{SR 6-7}}{(8.394)}$, and

$$S = \$40\overset{\text{SR 6-7}}{(8.394)}$$
$$S = \$335.76$$

Solution for *i* when *R*, *S*, and *n* are known. Let

$$R = \$100$$
$$S = \$441.10$$
$$n = 4$$
$$i = ?$$

The interest rate i may be solved for by interpolation of the interest tables in the following manner:

$$S = \text{R} \times \overset{\text{SR } i\text{-}n}{(\text{xxxxx})}$$

$$\$441.10 = \$100 \times \overset{\text{SR } i\text{-4}}{(\text{xxxxx})}$$

$$\overset{\text{SR } i\text{-4}}{(\text{xxxxx})} = \frac{441.10}{100}$$

$$\overset{\text{SR } i\text{-4}}{(4.411)} = 4.411$$

This value falls between the Equal-Payment-Series Compound-Amount Factors in the 6 per cent and 7 per cent tables for $n = 4$. From the 6 per cent table

$$\overset{\text{SR 6-4}}{(4.375)}$$

From the 7 per cent table SR 7-4 (4.440)

By proportion,

$$i = \left[6 + 1 \times \frac{4.411 - 4.375}{4.440 - 4.375}\right] \text{ per cent}$$

$$i = \left[6 + \frac{.036}{.065}\right] \text{ per cent}$$

$$i = 6.55 \text{ per cent}$$

There is no simple direct way for solving for the interest rate i algebraically. Where interpolation from tables does not suffice, or if tables are not available, i may be determined by a series of trial solutions based on estimated values of i. Interpolation between results of trial solutions makes it possible to approach accurate results. For best results, seven-place logarithms should be used.

Solution for n when R, S, and i are known. Let

$$R = \$200$$
$$S = \$3847.26$$
$$i = .04$$
$$n = ?$$

The number of interest periods n may be solved for by interpolation of the interest tables as follows:

$$S = R \times \underset{(\text{xxxxx})}{\text{SR } i\text{-}n}$$

$$\$3847.26 = \$200 \times \underset{(\text{xxxxx})}{\text{SR 4-}n}$$

$$\underset{(\text{xxxxx})}{\text{SR 4-}n} = \frac{3847.26}{200}$$

$$\underset{(19.2363)}{\text{SR 4-}n} = 19.2363$$

This value falls between the Equal-Payment-Series Compound-Amount Factor for $n = 14$ and $n = 15$ in the 4 per cent table. For $n = 14$

SR 4-14
(18.292)

For $n = 15$ SR 4-15 (20.024)

By proportion,

$$n = \left[14 + 1 \times \frac{19.236 - 18.292}{20.024 - 18.292}\right] \text{ years}$$

$$n = \left[14 + \frac{.944}{1.732}\right] \text{ years}$$

$$n = 14.55 \text{ years}$$

The number of interest periods n might have been obtained as follows:

$$S = R \times \frac{(1 + i)^n - 1}{i}$$

$$3847.26 = 200 \times \frac{(1.04)^n - 1}{.04}$$

$$(1.04)^n = 1.7695$$

$$n(\log 1.04) = \log 1.7695$$

$$n = \frac{\log 1.7695}{\log 1.04}$$

$$= \frac{.24785}{.01703}$$

$$= 14.55 \text{ years}$$

Equal-Payment-Series Sinking-Fund Factor Calculations

This factor is $\dfrac{i}{(1 + i)^n - 1}$ and is designated by $\overset{\text{RS } i\text{-}n}{(\text{xxxxx})}$.

This factor is used to determine the amount R of each payment of a series of payments, occurring at the end of successive interest periods, which are equivalent to a future sum S.

In Fig. 10, payments of $58.81 at the end of each interest period from March 6, 1951 to March 6, 1957 are equivalent to $400 on March 6, 1957 for an interest rate of 5 per cent compounded annually.

This may also be stated as: Six year-end payments of $58.81 are required to accumulate a sinking fund of \$400 in 6 years when the interest rate is 5 per cent compounded annually.

Solution for *R* when *S*, *i*, and *n* are known. The solution for finding the amount of annual sinking fund deposits R for the period March 6, 1951 to March 6, 1957, that are equivalent to a sinking fund S equal to \$400 on March 6, 1957 at 5 per cent compound interest follows:

$$n = 1957 - 1951 = 6$$

Then

$$R \text{ (sinking fund deposit)} = \$400 \times \overset{\text{RS } 5\text{-}6}{(\text{xxxxx})}$$

From the interest table $\overset{\text{RS } 5\text{-}6}{(.14702)}$, and

$$R = \$400 \times \overset{\text{RS } 5\text{-}6}{(.14702)}$$
$$R = \$58.81$$

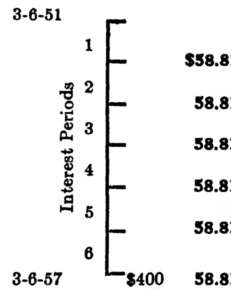


Fig. 10. Use of equal-payment series sinking-fund factor.

Solution for *i* and *n* when *S*, *R*, and *n* or *i* are known. See solutions under the heading *Equal-Payment-Series Compound-Amount Factor Calculations.*

Equal-Payment-Series Present-Worth Factor Calculations

This factor is $\frac{(1+i)^n - 1}{i(1+i)^n}$ and is designated by $\overset{\text{PR } i\text{-}n}{(\text{xxxxx})}$

The term *present worth* is used here to designate a sum P, at a given time taken to be the present, which is equivalent to a series of payments R, occurring at the end of successive interest periods following the time taken to be the present.

In Fig. 11, \$347.16 on June 7, 1968 is equivalent to a series of seven \$60 year-end payments as shown for an interest rate of 5 per cent compounded annually.

Stated in another way, \$347.16 on June 7, 1968 is the *present worth* of a series of seven \$60 year-end payments between June 7, 1968 and June 7, 1975 for an interest rate of 5 per cent compounded annually.

Among people familiar with interest calculations it is often sufficient to say that \$347.16 is the present worth of a series of seven \$60 year-end payments. It should be noted that an interest period intervenes between the present worth P and the first payment R.

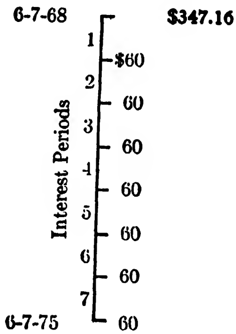


FIG. 11. Use of equal-payment series present-worth factor.

Solution for P when R, i, and n are known. The solution for finding the present worth P on June 7, 1968 which is equivalent to a series of seven \$60 year-end payments beginning at the end of the first interest period after June 7, 1968 for an interest rate of 5 per cent follows:

$$n = 1975 - 1968 = 7$$

Then

$$P \text{ (present worth)} = \$60 \times \overset{\text{PR } 5\text{-}7}{(\text{xxxxx})}$$

From the interest table $\overset{\text{PR } 5\text{-}7}{(5.786)}$, and

$$P = \$60 \times \overset{\text{PR } 5\text{-}7}{(5.786)}$$
$$P = \$347.16$$

Solution for i or n when P, R, and n or i are known. See schemes of solution suggested under the heading *Equal-Payment-Series Compound-Amount Factor Calculations.*

Equal-Payment-Series Capital-Recovery Factor Calculations

This factor is $\dfrac{i(1+i)^n}{(1+i)^n - 1}$ and is designated by $\overset{\text{RP } i\text{-}n}{(\text{qxxxx})}$.

This factor is used to determine the amount R of each payment of a series of payments occurring at the end of successive interest periods which are equivalent to a present sum P.

In Fig. 12, payments of \$71.22 at the end of each interest period during the period July 5, 1955 to July 5, 1960 are equivalent to \$300 on July 5, 1955 for an interest rate of 6 per cent compounded annually.

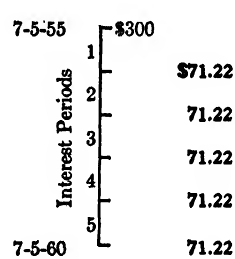


FIG. 12. Use of equal-payment series capital-recovery factor.

This might have been stated as: \$300 will provide an annuity of \$71.22 for a period of 5 years when the interest rate is 6 per cent.

Solution for R when P, i, and n are known. The solution for finding the amount R of annual year-end payments for the period July 5, 1955 to July 5, 1960 which are equivalent to a sum P of \$300 on July 5, 1955, for 6 per cent interest, follows:

$$n = 1960 - 1955 = 5$$

Then

$$R \text{ (annual payment)} = \$300 \times \overset{\text{RP 6-5}}{(\text{xxxxx})}$$

From the interest table $\overset{\text{RP 6-5}}{(.23740)}$, and

$$R = \$300 \times \overset{\text{RP 6-5}}{(.23740)}$$
$$R = \$71.22$$

Solution for i and n when P, R, and n or i are known. See schemes of solution suggested under the heading *Equal-Payment-Series Compound-Amount Factor Calculations.*

Equivalence Calculations Requiring Use of Several Factors

In Table 9 the left-hand column of sums represents a number of payments made at the times indicated. Suppose that it is desired to find the present worth of the payments at 6 per cent interest. The first step in the solution is to arrange the payments so that they conform with patterns for which the interest factors are applicable. This may be done in several ways. Two versions appear under the headings *Scheme A* and *Scheme B.*

TABLE 9. Schemes of Arrangement of Sums for Equivalence Calculations

Time and Amount of Payments			*Scheme A*		*Scheme B*
Beginning of 1954 or beginning of Year No.	1—$200	=	$200	=	$200
End of 1954 or end of Year No.	1— 50	=	50	=	50
1955 " " " " "	2— 50	=	50	=	50
1956 " " " " "	3— 50	=	50	=	50
1957 " " " " "	4— 50	=	50	=	50
1958 " " " " "	5— 450	=	50 + 400	=	50 + 400
1959 " " " " "	6— 50	=	50	=	50
1960 " " " " "	7— 50	=	50	=	50
1961 " " " " "	8— 120	=	120	=	50 + 70
1962 " " " " "	9— 120	=	120	=	50 + 70
1963 " " " " "	10— 120	=	120	=	50 + 70
1964 " " " " "	11— 120	=	120	=	50 + 70

On the basis of Scheme A the calculations to be made to find the present worth of the payments at the beginning of 1954 (or at the beginning of Year No. 1) at 6 per cent are the following.

The present worth of the $200 payment, $200 × 1..................$ 200.00
PS 6-5
The present worth of the $400 payment, $400(.7473).................. 298.92
PR 6-7
The present worth of the seven $50 payments, $50(5.582)............. 279.10
The present worth of the four $120 payments at the beginning of 1961
PR 6-4
(or at the beginning of Year No. 8) is equal to $120(3.465); and the
PR 6-4
present worth at the beginning of 1954 is equal to $120(3.465) ×
PS 6-7
(.6651)...$ 276.55

Total present worth.......................................$1054.57

If it had been desired to calculate the amount of equal year-end payments equivalent to the above series of payments, Scheme B might have been used. As a first step find the sum of the present worths of the $200, the $400, and the four $70 payments.

The present worth of the $200 payment, $200 × 1..................$ 200.00
PS 6-5
The present worth of the $400 payment, $400(.7473).................. 298.92
The present worth of the four $70 payments at the beginning of 1961
PR 6-4
is equal to $70(3.465) and their present worth at the beginning of
PR 6-4 PS 6-7
1954 is equal to $70(3.465) × (.6651)............................ 161.32

Total...$ 660.24

This total of $660.24 is equivalent to equal year-end payments for the
RP 6-11
period under consideration of $660.24(.12679).....................$ 83.71
Total amount of equal year-end payments equivalent to all the payments in the original statement equals $50 + $83.71................$ 133.71

Equal year-end payments of $133.71 are of course equivalent to the present worth of $1054.57 obtained above. This fact may be verified as follows:

RP 6-11
$1054.57(.12679) = $133.71

Schematic Representation for Calculation

Where a number of calculations of equivalence involving several interest factors are to be made, some difficulty may be experienced

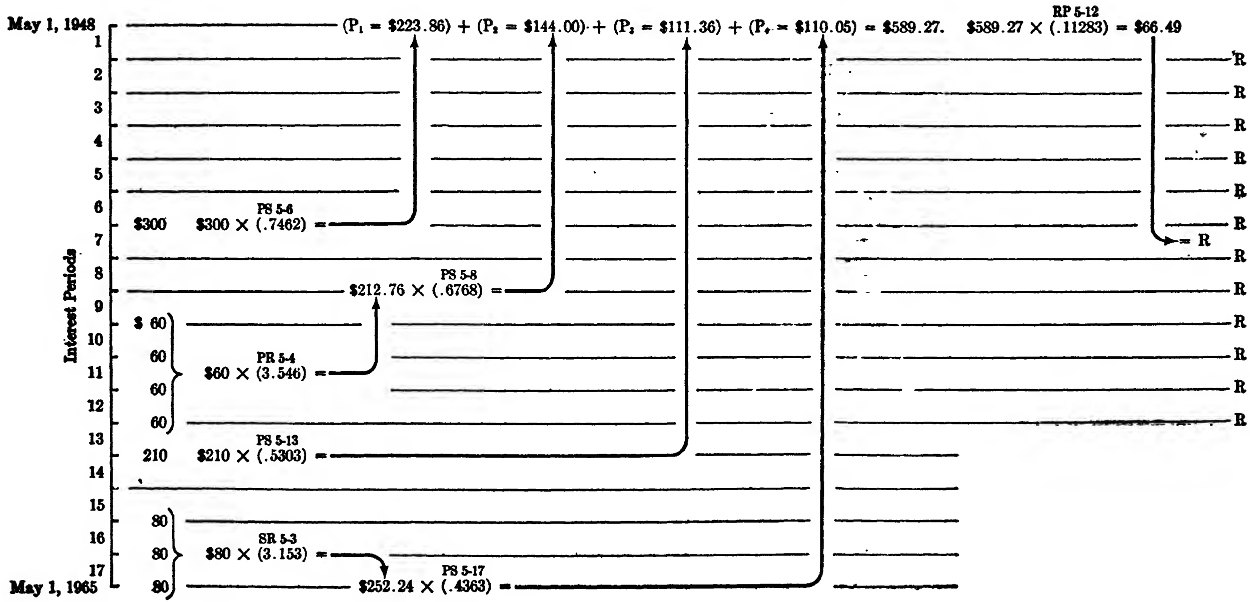


FIG. 13. Schematic illustration of solution of an equivalence problem.

in laying out a plan of attack. Also, until considerable experience has been gained with this type of calculations it may be difficult to keep track of the elapse of time. For complex problems, both the speed and the certainty of calculations can usually be improved by a schematic representation.

Suppose that it is desired to determine what series of 12 equal payments beginning May 1, 1949 is equivalent to the following described payments for an interest rate of 5 per cent: \$300 on May 1, 1954; \$60 on May 1, 1957, 1958, 1959, and 1960; \$210 on May 1, 1961; and \$80 on May 1, 1963, 1964, and 1965. See Fig. 13.

The plan of solution is to determine the sum of the present worths of the given payments as of May 1, 1948, and then to convert this sum to the 12 equal annual payments R, as required by the conditions of the problem as indicated on the diagram. In the formulas to be used, P payments occur at the beginning of interest periods and R payments occur at the end of interest periods. To satisfy this condition, the present worth of the several payments was calculated as of May 1, 1948, which is one interest period prior to the first R payment, set for May 1, 1949. Note also that one interest period intervenes between \$212.76 and the first \$60 payment. This is in accordance with the characteristics of the conversion formula used, which requires that the sum P occur one interest period prior to the first R payment. In the diagram, the \$252.24 as of May 1, 1965 represents the future worth of the three \$80 payments. Note that the \$252.24 amount concurs with the last \$80. This is in accordance with the characteristics of the formula used, which require that the sum S concur with the final R payment.

The sequence of calculations in the solution of this problem is clearly indicated in the diagram. The position of the arrowhead following each multiplication represents the position of the result with respect to time. The intermediate quantities \$212.76 and \$252.24 need not have been found. Much time may be saved if all calculations to be made in solving a problem are indicated prior to looking up factor values from the tables and making calculations. In the above example this might have been done as follows:

$$P_1 = \$300 \times \overset{\text{PS 5-6}}{(\qquad)} \qquad =$$

$$P_2 = \$\ 60 \times \overset{\text{PR 5-4}}{(\qquad)} \times \overset{\text{PS 5-8}}{(\qquad)} =$$

$$P_3 = \$210 \times \overset{\text{PS 5-13}}{(\qquad)} =$$

$$P_4 = \$\ 80 \times \overset{\text{SR 5-3}}{(\qquad)} \times \overset{\text{PS 5-17}}{(\qquad)} =$$

$$(P_1 + P_2 + P_3 + P_4) =$$

$$R = (\qquad) \times \overset{\text{RP 5-12}}{(\qquad)} =$$

Next, all factor values printed in italics are found and inserted in the parentheses. Calculations are then made as follows to obtain the results printed in bold face type:

$$P_1 = \$300 \times \overset{\text{PS 5-6}}{(\mathit{.7462})} = \mathbf{\$223.86}$$

$$P_2 = \$\ 60 \times \overset{\text{PR 5-4}}{(\mathit{3.546})} \times \overset{\text{PS 5-8}}{(\mathit{.6768})} = \mathbf{144.00}$$

$$P_3 = \$210 \times \overset{\text{PS 5-13}}{(\mathit{.5303})} = \mathbf{111.36}$$

$$P_4 = \$\ 80 \times \overset{\text{SR 5-3}}{(\mathit{3.153})} \times \overset{\text{PS 5-17}}{(\mathit{.4363})} = \mathbf{110.05}$$

$$(P_1 + P_2 + P_3 + P_4) = \mathbf{589.27}$$

$$R = \$589.27\overset{\text{RP 5-12}}{(\mathit{.11283})} = \mathbf{66.49}$$

Determining *i* When Two or More Interest Factors Are Involved

It is sometimes desired to determine the interest rate for which two series of payments will be equivalent. For example, at what rate of interest will the following two series of payments be equivalent?

	Series A	*Series B*
January 1, 1946	$1,300	
" " 1950		$800
" " 1952		600
" " 1953		600
" " 1954		600
" " 1955		600
" " 1960	2,000	

This problem may be solved by trial and interpolation. The first step is to guess at an interest rate. Guess is used advisedly; for, until sufficient experience has been gained to permit approximate calculations to be made to estimate an interest rate, it will usually be less time-consuming to guess at an interest rate to try at the outset.

Suppose that it has been decided to try 6 per cent and to make calculations on the basis of present worth as of January 1, 1946.

Try 6 per cent:

$1,300 × 1	$1,300.00
$2,000 × $\overset{\text{PS 6-14}}{(.4423)}$	884.60
Present worth Series A for 6 per cent	$2,184.60
$800 × $\overset{\text{PS 6-4}}{(.7921)}$	$ 633.68
$600 × $\overset{\text{PR 6-4}}{(3.465)}$ × $\overset{\text{PS 6-5}}{(.7473)}$	1,553.64
Present worth Series B for 6 per cent	$2,187.32

Try 7 per cent:

$1,300	$1,300.00
$2,000 × $\overset{\text{PS 7-14}}{(.3878)}$	775.60
Present worth Series A for 7 per cent	$2,075.60
$800 × $\overset{\text{PS 7-4}}{(.7629)}$	610.32
$600 × $\overset{\text{PR 7-4}}{(3.387)}$ × $\overset{\text{PS 7-5}}{(.7130)}$	1,448.96
Present worth Series B for 7 per cent	$2,059.28

Since the present worth of Series A is less than the present worth of Series B at 6 per cent and the present worth of Series A is greater than the present worth of Series B at 7 per cent, the two series will be equal for an interest rate between 6 and 7 per cent. The existing conditions may be summarized for interpolation as follows:

$$\begin{aligned}
&(\text{P.W. Series A for } 6\%) - (\text{P.W. Series B for } 6\%) &= -\$2.72\\
&(\text{P.W. Series A for } i) - (\text{P.W. Series B for } i) &= 0.00\\
&(\text{P.W. Series A for } 7\%) - (\text{P.W. Series B for } 7\%) &= \$16.32
\end{aligned}$$

By interpolation

$$i = \left[6 + 1 \times \frac{-2.72 - 0}{-2.72 - 16.32}\right] \text{ per cent}$$

$$= \left[6 + \frac{-2.72}{-19.04}\right] \text{ per cent}$$

$$= 6.14 \text{ per cent}$$

Calculation of Interest Rate on Bonds for Other Than Par Value

Calculations to determine an interest rate for which two money series are equivalent occur frequently in evaluating bonds and similar instruments.

A bond is an instrument setting forth the conditions under which money is loaned. It consists, usually, of a pledge of a borrower of funds to pay a stated amount or per cent of interest on a principal sum at stated intervals and to repay the principal sum at a stated time. Bonds are commonly written to cover amounts in multiples of $100 or $1,000. A typical $100 bond may embrace a promise to pay its holder $4, for example, one year after purchase and each succeeding year until the principal sum or par value of $100 is repaid on a designated date. Such a bond would be referred to as a 4 per cent bond, interest payable annually. Bonds may also provide for interest payments to be made semiannually or quarterly. Since pledges to pay as they are embodied in bonds have value, bonds are bought and sold. The price of a bond may range above or below its par or face value.

Suppose that on September 1, 1951, Mr. A purchased for $1,040 a $1,000 five per cent bond with interest payable annually and face amount due September 1, 1958. What will be the equivalent rate of interest earned on the $1,040 if all payments stipulated on the bond are met? A schematic arrangement of this situation is given in Table 10.

TABLE 10. Schematic Arrangement of Bond Receipts and Disbursements

Pay	$1,040	9-1-51
Receive	50	9-1-52
	50	9-1-53
	50	9-1-54
	50	9-1-55
	50	9-1-56
	50	9-1-57
	50 + 1,000	9-1-58

This problem may be solved on the basis that present worth of payments equals present worth of receipts at some interest rate.

This may be written:

$$\$1{,}040 = \$50\overset{\text{PR } i\text{-}7}{(\text{xxxxx})} + \$1{,}000\overset{\text{PS } i\text{-}7}{(\text{xxxxx})}$$

Solve for i by trial and error. The present worth of the amount to be paid for the bond at the time of payment is obviously \$1,040 for any interest rate.

Try $i = .04$. Then

$$\begin{aligned}\text{P.W. of Receipts} &= \$50\overset{\text{PR 4-7}}{(6.002)} + \$1{,}000\overset{\text{PS 4-7}}{(.7599)}\\ &= \$1{,}060\end{aligned}$$

Try $i = .05$. Then

$$\begin{aligned}\text{P.W. of Receipts} &= \$50\overset{\text{PR 5-7}}{(5.786)} + \$1{,}000\overset{\text{PS 5-7}}{(.7107)}\\ &= \$1{,}000\end{aligned}$$

(Note: At 5 per cent the present worth of receipts of a 5 per cent bond is its face value.)

The value of i sought falls between $i = .04$ and $i = .05$. Thus

For $i = .04$, P.W. of amount to be received = \$1,060
For $i = ?$, P.W. of amount to be received = 1,040
For $i = .05$, P.W. of amount to be received = 1,000

By interpolation for the desired value of i,

$$\begin{aligned} i &= \left[4 + 1 \times \frac{1040 - 1060}{1000 - 1060}\right] \text{ per cent}\\ &= \left[4 + \frac{-20}{-60}\right] \text{ per cent}\\ &= 4.33 \text{ per cent}\end{aligned}$$

Determining *n* When Two or More Interest Factors Are Involved

Sometimes it is desirable to calculate the number of interest periods for which two series of payments will be equivalent. For example, suppose that it is desired to compare the two money series, Series A and Series B, which are described as follows.

Series A consists of a single payment in the amount of $600 made January 1, 1950. Series B consists of annual payments of $100 each, beginning January 1, 1951 and continuing for n payments, and a receipt of $200 on the date the nth annual payment is made. Assume that it has been stipulated that an interest rate of 6 per cent is to be used. In tabular form, Series A and Series B can be described as follows:

	Series A	*Series B*
January 1, 1950	$600	
" " 1951		$100
" " 1952		100
" " 1950 + (n − 1)		100
" " 1950 + n		100 − $200

Suppose that it has been decided to try $n = 10$ and to make calculations on the basis of present worth as of January 1, 1950.

Try $n = 10$:

Present worth Series A, for any value of n..	$ 600.00
$100 × (7.360) [PR 6-10]	736.00
−$200 × (.5584) [PS 6-10]	−111.68
Present worth Series B, for n = 10.........	$ 624.32

Try $n = 9$:

$100 × (6.802) [PR 6-9]	$ 680.20
−$200 × (.5919) [PS 6-9]	−118.38
Present worth Series B, for n = 9..........	$ 561.82

It is obvious that the desired value of n lies between 9 and 10. The above results may be summarized for interpolation as follows:

(P.W. Series A for 9 years) − (P.W. Series B for 9 years) = $38.18
(P.W. Series A for n years) − (P.W. Series B for n years) = .00
(P.W. Series A for 10 years) − (P.W. Series B for 10 years) = −24.32

By interpolation,

$$n = \left[9 + 1 \times \frac{38.18 - 0}{38.18 - (-24.32)}\right] \text{years}$$

$$= \left[9 + \frac{38.18}{62.50}\right] \text{years}$$

$$= 9.61 \text{ years}$$

PROBLEMS

1. What amount will be accumulated by each of these present investments?
   (a) \$4,100 in 10 years at 6% compounded annually.
   (b) \$800 in 3 years at 4% compounded semiannually.
   (c) \$1,250 in 18 years at 4½% compounded annually.
   (d) \$15,000 in 38 years at 8% compounded annually.
   (e) \$1,720 in 6 years at 2% compounded quarterly.

2. What is the present worth of these future payments?
   (a) \$9,000 12 years from now with interest at 8% compounded annually.
   (b) \$9,000 12 years from now with interest at 8% compounded quarterly.
   (c) \$1,200 8 years from now with interest at 3½% compounded annually.
   (d) \$2,900 14 years from now with interest at 5% compounded annually.
   (e) \$14,000 7 years from now with interest at 4½% compounded annually.

3. (a) What rate of interest compounded annually must be received if an investment of \$7,500 made now will result in a receipt of \$9,000 5 years hence?
   (b) A college has a special research grant of \$12,000 to invest at the present time. What interest rate compounded semiannually must it receive on the investment if it needs \$15,000 for equipment 5 years from now?

4. (a) How many years must \$600 be left in a savings account paying 4% interest compounded semiannually to amount to \$1,000?
   (b) A man now has \$1,500. How many years will be required before he can purchase an \$1,800 automobile if the \$1,500 is invested at 4% compounded quarterly?
   (c) How many years does it take for money to double itself if interest is at the rate of 4% compounded annually?

5. What is the accumulated value of each of the following series of payments?
   (a) \$1,250 at end of each year for 9 years with interest at 5% compounded annually.
   (b) \$800 at the end of every 6 months for 5 years with interest at 6% compounded semiannually.
   (c) \$6,000 every six months for 16 years with interest at 7% compounded semiannually.

(d) \$450 every year for 42 years with interest at 4% compounded annually.

(e) \$1,700 every year for 8 years with interest at 7% compounded annually.

6. What is the present value of the following prospective future payments?

(a) \$700 a year for 14 years with interest at 5% compounded annually.

(b) \$1,000 semiannually for 11 years with interest at 6% compounded semiannually.

(c) \$1,400 a year for 7 years and \$4,000 at the end of 7 years with interest at 7% compounded annually.

(d) \$2,200 every six months for 10 years with interest at 7% compounded semiannually.

(e) \$800 every six months for 18 years with interest at 4% compounded semiannually.

7. What equal annual payments are necessary to repay the following present amounts if interest is compounded annually?

(a) \$7,000 in 12 years at 4%.
(b) \$12,000 in 27 years at 3%.
(c) \$9,000 in 13 years at 4½%.
(d) \$25,000 in 37 years at 5%.
(e) \$4,000 in 6 years at 8%.

8. What equal annual payments must be paid into a sinking fund to accumulate the following amounts if interest is compounded annually?

(a) \$12,000 in 15 years with interest at 5%.
(b) \$22,000 in 7 years with interest at 4½%.
(c) \$8,000 in 9 years with interest at 8%.
(d) \$14,000 in 41 years with interest at 4%.
(e) \$17,000 in 27 years with interest at 3%.

9. (a) What interest rate compounded annually is paid if \$515 a year for 8 years will repay an original loan of \$3,000?

(b) What interest rate compounded semiannually is received if 10 semiannual deposits of \$85 are made and at the end of 5 years \$1,000 is accumulated?

10. (a) An interest rate of 10% compounded annually is desired on an investment of \$8,600. How many years will be required to recover the capital with the desired interest if \$1,750 is received each year?

(b) A building is priced at \$37,000. If a down payment of \$14,000 is made and a payment of \$1,500 every six months thereafter is required, how many years will be necessary to pay for the building? Interest is charged at the rate of 6% compounded semiannually.

11. A mining property is offered for sale for \$57,000. On the basis of estimated production, an annual return of \$8,000 is foreseen for a period of 10 years. After 10 years the property will be worthless. What annual rate of return is in prospect?

12. (a) How many years will it take to accumulate $12,000 with which to buy a home, if $650 is deposited in a sinking fund every six months? Interest at 4% compounded semiannually is received on the sinking fund.

(b) A company sets aside $10,000 each year in a fund for plant expansion. If the fund draws 6% interest compounded annually, how long will it be before a new warehouse costing $75,000 can be built?

13. Flood damage to a certain area averages $7,000 annually. A dyke to restrain the water will cost $25,000 and involve annual maintenance of $200. With interest at 8% compounded annually, how many years will it take for the dyke to pay for itself?

14. With interest at 6% compounded annually, solve the following problems:

(a) How much will be required 7 years hence to repay an $800 loan made today?

(b) How much can be loaned now if $600 will be repaid at the end of 8 years?

(c) If a down payment of $6,000 is made on a house and $800 a year for the next 12 years is required, what was the price of the house?

(d) What payment 12 years from now is equivalent to a payment of $700 9 years from now?

15. Using interest at 4% compounded semiannually, solve the following problems:

(a) What lump-sum payment now can be made to prevent an expense of $250 every six months for the next 12 years?

(b) What semiannual deposit into a fund is required to total $5,000 in 10 years?

16. (a) On his sixth birth anniversary a boy is left an inheritance. The inheritance will be paid in a lump sum of $10,000 on his twenty-first birth anniversary. What is the present value of the inheritance, as of the boy's sixth birth anniversary if interest is 4% compounded annually?

(b) What should be the quarterly deposit in a savings bank paying interest of 2% compounded quarterly in order for the depositor to accumulate $12,000 in 7 years?

17. (a) How much must be paid monthly to repay $750 in 2 years at an annual nominal rate of 8.4% compounded monthly?

(b) What present loan can be repaid over the next 4 years with payments of $60 a month? Interest is at a nominal rate of 6% per annum compounded monthly.

18. A young couple have decided to make advance plans for financing their 2-year-old son's college education. Money can be deposited at 4% compounded annually. What annual deposit on each birth anniversary from the 3rd to the 17th inclusive must be made to provide $1,200 on each birth anniversary from the 18th to the 21st inclusive?

19. A city that was planning an addition to its water supply and distribution system contracted to supply water to a large industrial user for 10 years under the following conditions: The first five years of service were to be paid for in advance, and the last five years of service were to be paid for at a rate of $15,000 a year payable at the beginning of each year.

Two years after the system is in operation the city finds itself in need of funds and desires that the company pay off the entire contract so that the city can avoid a bond issue.

(a) If the city uses 5% interest compounded annually in calculating a fair receipt on the contract, what amount can they expect?

(b) If the company uses 3% interest compounded annually, how much can they afford to pay to be relieved of further payments under the contract?

20. Two plans of furnishing a given service involve the following expenses:

Year End	*Plan A*	*Plan B*
0	$1,000	$500
1	100	175
2	100	190
3	100	240
4	100–300 (operation for 4th year less salvage)	300–200

At what interest rate are the two plans equivalent?

21. A typewriter sales and repair agent, in an effort to increase his business, offers to lease a certain company new typewriters and furnish five years of maintenance for a fee of $220 per typewriter payable now. The company can purchase a new typewriter for $180 that will have a trade-in value of $36 at the end of the five-year period. Annual maintenance costs, if the company purchases the typewriter, will be $16 per year. At what interest rate compounded annually are the two plans equal?

22. A company let a contract for a group of 50-H.P. motors. The contract provides that the motors shall have an efficiency of 92% and are to cost the company $430 each. After the motors are completed, tests show that they have an efficiency of only 91%. The contractor agrees to sell them to the company for $370 each. Each motor will have a life of 12 years with no salvage value at that time, and all annual operational costs except that for electric energy will be the same for both motors. At what interest rate is this offer exactly equivalent to the original contract, provided that each motor will be operated 1,000 hours a year with energy costing 1.5 cents per kw-hr.? One H.P.-hr. equals .746 kw-hr.

23. A $1,000 4-per-cent bond is offered for sale for $950. If interest is payable annually and the bond will mature in 7 years, what interest rate will be received?

24. A $1,000 bond will mature in 10 years. The annual rate of interest is 3% payable semiannually. If the bond can be purchased for $960, what annual interest compounded semiannually will be received?

25. A bond is offered for sale for $1,040. Its face value is $1,000 and the interest is 4% payable annually. What rate of interest will be received if the bond matures 9 years hence?

26. A man desires to make an investment in bonds, provided he can realize 6% on his investment. How much can he afford to pay for a $1,000 bond that pays 5% interest annually and will mature 12 years hence?

27. How much can be paid for a $1,000 5-per-cent bond with interest paid semiannually, if the bond matures 8 years hence? Assume the purchaser will be satisfied with 4% interest compounded semiannually, since the bonds were issued by a very stable and solvent company.

28. A bond having a face value of $100 and paying 4% interest payable annually is offered for sale for $92.50. Can a purchaser who requires 5% on his investments afford to buy the bond if it matures 15 years hence? 8 years hence?

29. Two plans are offered that furnish the same service. Plan A involves the purchase of automatic equipment costing $9,700. Because this equipment is made specially for a particular service, the salvage value will be $150 regardless of the length of time it is used. Annual labor costs will be $2,000 a year. Plan B involves an annual labor cost of $4,500 a year. All other expenses and incomes under the two plans will be the same. If interest is at 6% compounded annually, how long must the equipment be used to pay for itself?

30. A chapter of a social fraternity is being organized and is in need of housing facilities. A local real estate man agrees to lease the fraternity a suitable house and pay all maintenance costs for $6,800 a year. The fraternity can purchase a building site for $9,500 and construct a house for $70,000. Annual maintenance, taxes, and insurance will cost the fraternity $1,500 if they own their house. With interest at 5%, how many years will it require before the new house would pay for itself, assuming that the building site will continue to be valued at $9,500 and that all other costs would be the same regardless of whether the house is leased or built?

31. Oil reservoir engineers estimate the annual production of an oil well for the next 12 years to be as follows:

Year End	*Annual Production in Barrels*
1	23,800
2	15,100
3	10,600
4	6,700
5	4,300
6	2,600
7	1,400
8	790
9	430
10	260
11	140
12	70

Assuming that the oil will sell for $2.20 per barrel during the 12-year perioa, what is the present worth of estimated future production, if interest is at 5% compounded annually?

CHAPTER

5

Depreciation and Depreciation Calculation

BROADLY SPEAKING, depreciation is the lessening in value of physical assets with the passage of time. Everyone has observed this process. No matter how the purchaser of a shiny new car may desire to keep it in new condition, it is only a matter of time until the paint becomes scarred and scratched, a fender becomes bent or worse, and the upholstery begins to show soil. Mechanically, the car also deteriorates: the brakes may fail to hold as well as when the car was new; it becomes necessary to shift on more and more hills; and gasoline and oil consumption go up. The car is doomed; it has started the inexorable march to the junk heap. Good care and maintenance may slacken the pace somewhat, but the end result is the same. It is characteristic of all physical assets, with the possible exception of land, to decrease in value with the passage of time.

Physical Assets Are Purchased for Profit

In the business world, with which engineers are concerned, one important objective is profit. Assets are purchased with this objective in mind. Such assets as machines and tools make it possible to get work done with less effort. It is often more profitable to invest money in tools than in labor, and in consequence huge sums are spent for superlative labor-saving machines in modern industry. One mark of an advanced civilization is a high investment per worker in production facilities. Although investments in tools of production may often be the source of handsome profit, they may also be lost in whole or part. A moment's reflection will reveal that a physical asset such as a bulldozer, lathe, or building has value only to the extent that the services it can provide can be utilized profitably. If great care is not exercised, the physical asset may have reached the junk heap before enough of its service has been utilized to repay even the original investment in the asset. For this reason, depreciation is a major factor in the economy of many enterprises.

Classifications of Depreciation

A common classification of types of depreciation sufficient for most purposes is:

1. Physical depreciation
2. Functional depreciation
3. Depletion
4. Depreciation and its reverse, appreciation, due to fluctuations in price levels
5. Accident

Physical depreciation. Depreciation resulting in physical impairment of an asset is known as *physical depreciation.* Physical depreciation manifests itself in such tangible ways as the wearing of particles of metal from a bearing and the rotting out of the roof of a building. This type of depreciation results in a lowered ability of a physical asset to render its intended service. Thus bearings in a lathe may become so worn that the lathe loses its usefulness. The causes usually considered to result in physical depreciation are (1) deterioration due to action of the elements and (2) wear and tear from use.

Functional depreciation. Functional depreciation results not from a deterioration in an asset's ability to serve its intended purpose but from a change in the demand for the services it can render.

The demand for the services of a machine may change because (1) it is more profitable to use a more efficient machine, (2) there is no longer work for the machine to do, or (3) the work to be done exceeds the capacity of the machine. Depreciation resulting from a change in need for the service of a machine or other asset is designated by the terms *obsolescence* and *inadequacy.* The meaning of the term *obsolescence* and its application in relation to depreciation will be illustrated by two examples.

Suppose that a manufacturer has a hand riveter in good physical condition, but he has found it more profitable to dispose of it and purchase an automatic riveter because of the reduction in riveting costs that the latter makes possible. The difference between the use value of the hand-operated riveter and the amount received for it on disposal represents a decrease in value due to the availability of the automatic riveter. This cause of loss in value is termed obsolescence. Literally the hand riveter had become *obsolete* as a result of an improvement in the art of riveting.

As a second example, consider the case of a manufacturer who has ceased producing a certain item and finds that he has machines in good operating condition which he no longer needs. If he disposes of them at a value less than their former use value to him, the difference will be termed depreciation, or loss resulting from obsolescence. The inference is that the machine has become obsolete or out of date as far as the user is concerned.

Inadequacy, a cause of functional depreciation, occurs when changes in demand for the services of an asset result in a demand beyond the scope of the asset. For example, a small electric generating plant has a single 500-kva generating unit whose capacity will soon be exceeded by the demand for current. Analyses show that it will be cheaper in the long run to replace the present unit with a 750-kva unit, which is estimated to meet the need for some time, than to supplement the present unit with a 250-kva unit.

In such a case the original 500-kva unit is said to be *inadequate* and to have been *superseded* by the larger unit. Any loss in the value of the unit below its use value is the result of inadequacy or supersession. The former term is preferred for designating this type of depreciation.

Rapid advances in technology and rapid growth and change in industrial output make obsolescence and inadequacy important causes of depreciation and, therefore, important factors in economy.

A disposition to replace machines when it becomes profitable to do so instead of when they are worn out has probably been an important factor in the rapid development of this nation. The sailing ship has given way to the steamship; in street transportation, the sequence of obsolescence has been horse cars, cable cars, electric cars, and automotive buses; steam locomotives are rapidly being replaced by internal-combustion-engine locomotives. In power generation, steam turbines have replaced steam engines. In manufacturing, improvements in the arts of processing have resulted in widespread obsolescence and inadequacy of equipment. Hardly has an advance in equipment been made before improvements render it obsolete.

Depletion. Depletion differs in theory from depreciation in that the latter is the result of use and the passage of time while depletion is the result of the *intentional piecemeal removal of an asset.* This term is used in reference to exhaustive activities and literally means *to empty.* When coal is removed from a mine the mine loses value through depletion. Other clear-cut examples of depletion are the removal of timber from a forest, stone from a quarry, and oil from an oil sand.

Fluctuation of price levels. A change of price levels is normal with the passage of time. Price levels may go up or down, and ordinarily these fluctuations may be expected to influence the dollar value of both new and used assets. This change in value is said to be due to the *fluctuation of price conditions.*

Accidents. Accidents may result in a very rapid loss of value. In general, accidents are not predictable and since the resulting loss may be high it has become a common business practice to insure against losses resulting from such accidents as fire, flood, windstorms, wrecks, collisions, and explosion. Insurable losses are not treated as depreciation. Minor damage caused by accidents is usually regarded as being due to actions of the elements or to wear and tear from use.

Summary of Depreciation Classifications and Causes

Deterioration due to action of the elements. Under this classification are such causes of deterioration as corrosion of pipe in soil, rotting of structural elements, chemical decomposition, and bacterial action, all of which are substantially independent of use.

Wear and tear from use. The use of assets subjects them to abrasion, shock, impact, chemical and bacterial action, and other forces that reduce their life. This sort of deterioration is occasioned primarily by use.

Obsolescence. Assets become obsolete because they are no longer needed or because another asset, the superiority of which makes it uneconomical to continue using the original asset, becomes available.

Inadequacy. An asset may be in excellent condition but unable to meet the volume of the demands placed upon it. This situation arises from changes in demand not contemplated when the asset is purchased.

Depletion. Depletion applies to exhaustion of an asset through piecemeal removal where replacement is not contemplated. Depletion applies to such assets as mines, petroleum pools, and timber holdings.

Fluctuations of market conditions. An asset may depreciate (suffer a loss in value) or appreciate (gain in value) as a result of changes in the general market value of like assets.

Accidents. Causes in this category, such as fire, flood, or accidental breakage, may result in very rapid depreciation. Only uninsurable losses of this nature are ordinarily treated as depreciation.

The Effect of Maintenance on Depreciation

Experience shows that even the best of care will not prevent deterioration with age. At best, maintenance will only retard the rate of depreciation.

Deferred Maintenance

Sometimes a more than normal rate of decline results from neglecting to make repairs as needed—failure, for example, to paint a building, repair the hearth of a furnace, straighten bent fenders on a truck, or regrind the valves of an engine. In appraisals such neglect of repairs is known as *deferred maintenance.* The amount of deferred maintenance for a given asset is determined by estimating the cost of putting the asset into a good, serviceable condition.

Depreciation Is Revealed by History

Depreciation cannot be known in advance with certainty. With experience some forms of physical depreciation can be approximated, but functional depreciation can rarely be determined objectively. How history reveals depreciation may be illustrated by example.

A lathe was purchased for $2,000 in 1938. In 1948 it was sold for $400 to make room for a replacement. The situation may be diagrammed as in Fig. 14. From the statement of the situation and from Fig. 14, it is known that the lathe had a first cost value of $2,000 and a disposal or salvage value of $400 at the end of a ten-year period. The lathe had depreciated $1,600 in value in a ten-year period. These values are a matter of history.

Capital Recovery with a Return

Capital assets such as trucks, buildings, and boilers are purchased in the belief that they will earn more than they cost. One part of the prospective earnings will be considered to be *capital recovery.* Capital invested in an asset is recovered in the form of income derived from the services rendered by the asset and from its sale as salvage. Suppose that a machine during its life provided services valued at $800 and that $200 was then received from its sale. The total of $1,000 would then be considered to be recovered capital. If the machine had cost $1,000, the capital invested in it would have been fully recovered. Capital invested in an asset is ordinarily recovered

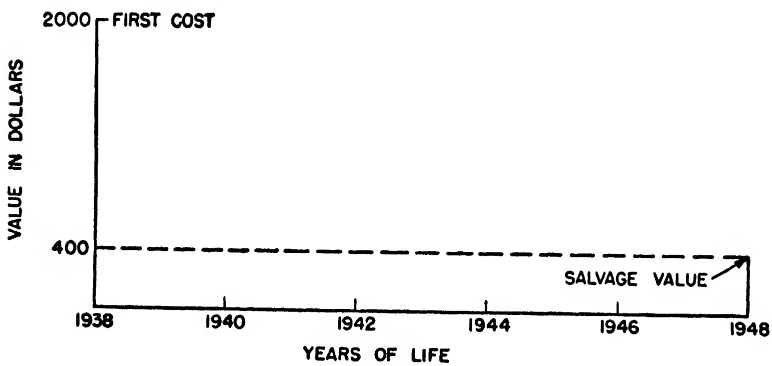


FIG. 14. Depreciation of an asset.

piecemeal. For instance, during the first, second, and third years in which the machine above was used, its services might have had value equal to $80, $140, and $100 respectively. Each of these sums represents the capital recovered during a particular year. The unrecovered balance of capital remaining in the asset at the end of the first, the second, and the third year is, respectively, equal to $920, $780, and $680.

A second part of the prospective earnings will be considered to be *return.* Money is presumed to have a time value, that is, it is considered to be capable of earning a return, usually spoken of as interest. An investment in an asset such as a machine is expected to result in income sufficient not only to recover the amount of the original investment but also to provide for a return on the diminishing unrecovered balance of capital remaining in the asset at any time during its life. This gives rise to the expression *capital recovery with a return.*

Depreciation Should Be Known in Advance

An asset such as a lathe is actually consumed in producing goods and thus its depreciation is a cost of production. As a cost of production it would be desirable to know the amount and pattern of an asset's depreciation at any time during the asset's life in order that proper charges could be made against products as they are produced. Unfortunately, the depreciation of an asset cannot be known with certainty until after the asset has been retired from service.

If the lathe in our example had produced 16,000 units of product each year for a total of 160,000 units, the cost for depreciation would be \$.01 per unit. This calculation is simple enough and can easily be made, assuming proper records have been kept, but it could not be made until *after* the lathe had been sold, ten years after its purchase.

In the usual business situation it is impractical to defer calculation of depreciation costs until after an asset has been disposed of at the end of its life. In fact, depreciation costs of an asset should be taken into account prior to the purchase of the asset, as one of the factors to be considered in arriving at the desirability of purchasing it.

Depreciation Rates Are Based on Estimates

Since information on depreciation is needed on a current basis for making decisions in the production of goods, it has become a practice to estimate the amount of depreciation an asset will suffer and the pattern in which this depreciation will occur.

Usually the first cost of an asset proposed for purchase can be known at the time the decision is made to purchase it. At the time of the decision to purchase, neither the prospective life of an asset nor its salvage value at the end of its life can be known with certainty. Since the future cannot be foretold, it is necessary to estimate both the life and the depreciated value of an asset. These estimates are usually based upon the experience with similar assets and the judgment of the estimator. Some methods in use for approximating life and salvage values of assets are presented later in this chapter.

The calculated depreciated value of an asset also known as book value, at any time during its life, is necessarily no more accurate than the estimates upon which it is based. The usual method is to assume that depreciation of an asset from its first cost to its salvage value takes place year by year in accordance with one of several mathematical formulas or curves. Several of these methods of depreciation are in general use.

Methods of Depreciation

Depreciation, on the basis of the three curves most commonly used, may be classified as:

Straight-Line Method of Depreciation
Sinking-Fund Method of Depreciation
Fixed-Percentage-on-Diminishing-Balance Method of Depreciation

The pertinent facts in establishing a depreciation rate by any method of depreciation are the first cost and the estimated life and salvage value at the end of the estimated life (see Fig. 14). The total depreciation is the difference between the original value and the estimated salvage value.

Straight-Line Method of Depreciation

In the straight-line method of depreciation it is assumed that depreciation is equal for each year of the life of the asset. Thus, if an asset has an original cost of $5,000, an estimated life of 5 years, and an estimated salvage value of $1,000, the total depreciation will be $4,000 and the depreciation each year will be $800. This situation is graphed in Fig. 15.

It should be borne in mind that it is necessary to establish the pattern of depreciation at the time the asset is purchased. Thus, the life and the salvage value are but estimates and, consequently, the annual depreciation arrived at is also an estimate. This means, in other words, that the annual expense for depreciation of the asset is estimated to be $800 per year. This is the *amount* which it is *estimated must be recovered in useful service each year* from the use of the asset. The fact that depreciation rests on estimates is disturbing to many. It should be realized that any decision relative to the future can be based only upon what is known in the present. Thus decisions relative to the future always must be based in some measure on judgment.

Recovery of asset investments. Money is usually invested in an asset for the purpose of gain or return. The first step in the attainment of this purpose is the recovery of the capital that has been lost through the depreciation of the asset (as it depreciates during its useful life). The benefits received from the use of the asset are a charge against the products it produced. The benefits received are recovered capital.

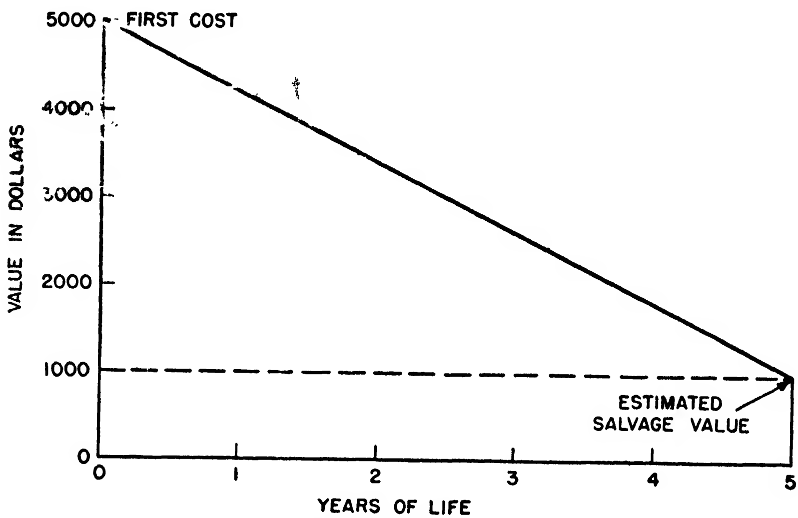


FIG. 15. Straight-line method of depreciation.

Return or interest on asset investments. The second step is to secure a gain or return (interest) on the unrecovered capital remaining in the asset each year. In the example above, the investment in the asset during the first year was \$5,000; during the second, \$4,200; during the third, \$3,600; and so forth. Assuming that the rate of return desired on the capital invested is 6 per cent, the return for the first year should be $\$5,000 \times .06 = \$300$; for the second year, $\$4200 \times .06 = \$252$; and so forth. The capital recovery and the return are illustrated in Table 11.

For simplicity of calculation and on the premise that the earning power of an asset is uniform throughout its life, it is common practice to calculate an average annual value to represent the interest on the unrecovered balance. This is done by finding the average of the

TABLE 11. An Example Showing Straight-Line Depreciation and Interest on Undepreciated Balances

	1st Year	*2nd Year*	*3rd Year*	*4th Year*	*5th Year*
Unrecovered or balance of capital at beginning of year designated.	\$5,000	\$4,200	\$3,400	\$2,600	\$1,800
Capital recovered during year designated (considered to be recovered at end of year designated)	800	800	800	800	800
Return or interest on unrecovered capital during year designated..	300	252	204	156	108
Sum of capital recovered and interest on unrecovered balance...	1,100	1,052	1,004	956	908

interest for the first and the last year. In the example, this is (\$300 + \$108) ÷ 2 = \$204. Based on this, the sum of capital recovered and average interest in the example will be (\$800 + \$204) = \$1,004.00 per year. It should be borne in mind that this average value only approximately represents the true situation.

Straight-line depreciation and average interest. A general expression for straight-line depreciation and average interest is derived in the following manner:

Let P = first cost of the asset
L = estimated salvage value at end of estimated life
n = estimated life of the asset in years
i = the rate of interest

(1) Depreciation in n years $= P - L$

(2) Depreciation per year $= \dfrac{P - L}{n}$

(3) Interest on Unrecovered Balance, first year $= (P - L)i + Li$

(4) Interest on Unrecovered Balance, nth year $= \left[\dfrac{P - L}{n}\right] i + Li$

(5) The average of (3) and (4) $= \left[(P - L) + \dfrac{P - L}{n}\right] \dfrac{i}{2} + Li$

$$= \left[(P - L)\,\frac{n + 1}{n}\right] \frac{i}{2} + Li$$

Consolidating (2) and (5) results in straight-line depreciation and average interest equal to

$$\frac{P-L}{n} + \left[(P-L)\frac{n+1}{n}\right]\frac{i}{2} + Li$$

or

$$(P-L)\left[\frac{1}{n} + \left(\frac{n+1}{n}\right)\frac{i}{2}\right] + Li$$

It should be borne in mind that this is only an approximate solution. It is fairly accurate for relatively low interest rates and for a relatively short life. See Fig. 16.

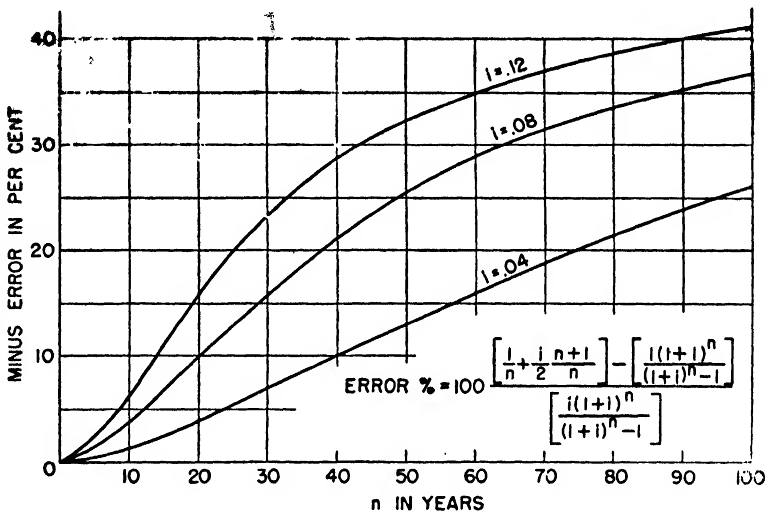


FIG. 16. Error of approximate capital-recovery factor.

Sinking-Fund Method of Depreciation

In the sinking-fund method of depreciation, one of a series of equal amounts is assumed to be deposited into a sinking fund at the end of each year of the asset's life. The sinking fund is ordinarily compounded annually. At the end of the estimated life of the asset, the amount accumulated in the sinking fund equals the amount of the estimated depreciation during the life of the asset.

The amount deposited into the sinking fund annually is equal to the estimated total depreciation times the sinking-fund factor corresponding to the estimated life and the interest rate taken.

Consider the following example:

First cost, $5,000
Estimated salvage value, $1,000
Estimated life, 5 years
Interest rate, 6 per cent

The amount to be deposited into the sinking fund for depreciation at the end of each year is

$$(\$5{,}000 - \$1{,}000)\overset{\text{RS 6-5}}{(.17740)} = \$709.60$$

The amount the asset is depreciated during any year is the sum of (1) the amount deposited into the sinking fund at the end of that year and (2) the amount of interest earned on the sum on deposit in the sinking fund during the year. Calculations are indicated in Table 12.

TABLE 12. An Example of Sinking-Fund Depreciation

Year No.	Depreciation or Capital Recovered during Year Designated			Total Depreciated to End of Year Designated	Undepreciated Balance at End of Year Designated
	Deposit	Interest Earned During Year	Total Depreciation		
0	$000.00	$000.00	$000.00	$ 000.00	$5,000.00
1	709.60		709.60	709.60	4,290.40
2	709.60	.06 × 709.60	752.18	1,461.78	3,538.22
3	709.60	.06 × 1,461.78	797.31	2,259.09	2,740.91
4	709.60	.06 × 2,259.09	845.15	3,104.24	1,895.76
5	709.60	.06 × 3,104.24	895.85	4,000.(09)	1,000.00

Capital recovery with a return is equal to the depreciation and the interest on the unrecovered balance of an asset. This is illustrated for the sinking-fund method of depreciation in Table 13.

It will be noticed that in the sinking-fund method of depreciation the sum of *the depreciation* and the *interest on the undepreciated balance* (see Column 6 of Table 13) is the same for each year.

The amount of this sum for the example above is equal to

$$(\$5{,}000 - \$1{,}000)\overset{\text{RS 6-5}}{(.17740)} + \$5{,}000 \times .06 = \$1{,}009.60$$

Capital recovery with a return. The solution for finding the sum of the annual sinking-fund depreciation and return on the unrecovered capital, that is, capital recovery with return, is given by

TABLE 13. An Example Showing Sinking-Fund Depreciation and Interest on Undepreciated Balance

Year No.	Depreciation or Capital Recovered during Year Designated			Interest on Undepreciated Balance at Beginning of Year (C)	Sum of Depreciation Plus Interest on Undepreciated Balance (B + C)	Sum of Interest on Sinking Fund and Interest on Undepreciated Balance (A + C)
	Deposited in Sinking Fund	Interest Earned on Amount in Sinking Fund at Beginning of Year (A)	Total (B)			
1	$709.60		$709.60	$5,000.00 × .06 = 300.00	$1,009.60	$300.00
2	709.60	.06 × $ 709.60 = 42.58	752.18	4,290.40 × .06 = 257.42	1,009.60	300.00
3	709.60	.06 × 1,461.78 = 87.71	797.31	3,538.22 × .06 = 212.29	1,009.60	300.00
4	709.60	.06 × 2,259.09 = 135.55	845.15	2,740.01 × .06 = 164.45	1,009.60	300.00
5	709.60	.06 × 3,104.24 = 186.25	895.85	1,895.76 × .06 = 113.75	1,009.60	300.00

$$\text{Capital Recovery with a Return} = (P - L)\overset{\text{RS } i\text{-}n}{(\text{xxxxx})} + Pi$$

or, if $\overset{\text{RS } i\text{-}n}{(\text{xxxxx})}$ is replaced by its equal $\overset{\text{RP } i\text{-}n}{(\text{xxxxx})} - i$, the result is

$$\text{Capital Recovery with a Return} = (P - L)\overset{\text{RP } i\text{-}n}{(\text{xxxxx})} + Li$$

Thus, while the calculation of depreciation by the sinking-fund method is somewhat laborious, the sum of the sinking-fund depreciation plus interest on the unrecovered balance is easily found. This sum is determined with mathematical accuracy by means of the capital-recovery factor. This fact and the convenience of the sinking-fund depreciation method for calculating capital recovery with a return make it a preferred method with many, particularly for theoretical analysis of business problems.

It will be noticed that depreciation by the sinking-fund method becomes progressively greater as the age of the asset increases. It is, therefore, less conservative than the straight-line method in the event that an asset is retired prior to the end of the estimated period of depreciation (see Fig. 17).

Two-Interest-Rate Capital Recovery with Return

On page 44 it was demonstrated that the capital-recovery factor

$$\frac{i(1+i)^n}{(1+i)^n - 1} = i + \frac{i}{(1+i)^n - 1}$$

If the latter expression is used to determine capital recovery with a return of an investment equal to P, we have

$$\text{Annual Capital Recovery with a Return} = Pi + P\frac{i}{(1+i)^n - 1}$$

Thus the annual capital recovery of an investment with a return is mathematically equivalent to the amount of the investment P times the interest rate i plus a sinking-fund deposit which, at an interest rate i, will amount to P in n years.

A modified form of this expression, known as Hoskold's formula, is in use for calculating capital recovery with a return in which two interest rates, i' and i, are applied as follows:

$$\text{Annual Capital Recovery with a Return} = Pi' + P\frac{i}{(1+i)^n - 1}$$

The interest rate i' is called the *stipulated rate.* It is the interest rate considered necessary to justify the risk in a particular investment. The interest rate i, referred to as the *sinking fund rate,* is the amount that can be obtained on conservative "risk-free" investments.

At first glance the expression may seem to provide for the high or stipulated rate of interest on the amount in the investment, as in an oil or mine property. Actually this is only true for the first year, for the amount actually remaining as an investment in the property diminishes year by year as the property is depleted or depreciates by the sinking-fund method.

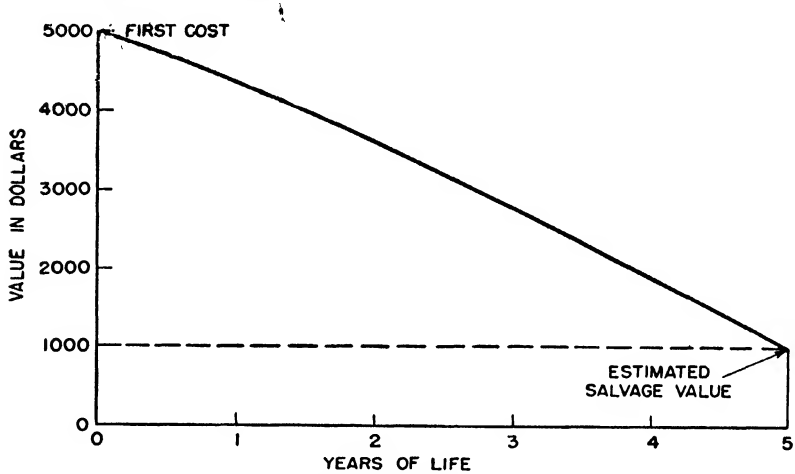


Fig. 17. Sinking-fund method of depreciation.

Example: What equal annual year-end payments received during the next ten years are equivalent to \$100,000 invested now, if the stipulated interest rate is 12 per cent and the sinking-fund rate is 3 per cent?

Solution: The amount of each equal payment,

$$
\begin{aligned}
R &= \$100{,}000\left[.12 + \frac{.03}{(1 + .03)^{10} - 1}\right] \\
&= \$100{,}000\ (.12 + .08723) \\
&= \$100{,}000\ (.20723) \\
&= \$20{,}723
\end{aligned}
$$

This plan of arriving at capital recovery with a return is used in evaluating depleting assets, particularly mine properties.

Corresponding to the above Two-Interest-Rate Equal-Payment-Series Capital-Recovery Factor is the Two-Interest-Rate Equal-Payment-Series Present-Worth Factor, which is equal to

$$\frac{1}{i' + \frac{i}{(1+i)^n - 1}}$$

Depreciation by the Method of Fixed Percentage on Diminishing Balance

By this method an asset is depreciated a fixed percentage of the undepreciated amount remaining at the end of each year. Consider the illustrative example:

First Cost, \$5,000
Life, 5 years
Salvage Value, \$1,000

By this method the depreciation during any year is equal to the undepreciated amount at the beginning of the same year times D, where $D \times 100$ equals the fixed percentage rate of depreciation. It follows that the undepreciated balance remaining at the end of any year equals the undepreciated balance at the beginning of the same year times $(1 - D)$.

In the example above the undepreciated balance at the end of the first, second, and third year, and so on, is \$5,000 $(1 - D)$, \$5,000 $(1 - D)^2$, and \$5,000 $(1 - D)^3$ respectively, and so forth, with \$5,000 $(1 - D)^5$ equal to \$1,000.

$$(1 - D) = \sqrt[5]{\frac{1{,}000}{5{,}000}} = .72478$$

and

$$D = 1 - \sqrt[5]{\frac{1{,}000}{5{,}000}} = 1 - .72478 = .27522$$

The depreciation during each year and the undepreciated balances are given in Table 14.

TABLE 14. An Example Showing Depreciation by the Method of Fixed Percentage on Diminishing Balance

Year No.	Depreciation during Year	Undepreciated Balance at End of Year
0	$ 000.00	$5,000.00
1	1,376.10	3,623.90
2	997.37	2,626.53
3	722.87	1,903.66
4	523.93	1,379.73
5	379.73	1,000.00

This method results in a relatively high depreciation during the first year and progressively diminishing depreciation during subsequent years. Therefore it is more conservative than the straight-line method in the event retirement takes place prior to the attainment of the contemplated service life (see Fig. 18).

The general solution for determining the value of D, the fixed percentage of depreciation, is given by

$$D = 1 - \sqrt[n]{\frac{L}{P}}$$

This method of depreciation is little used in the United States and has little application in economy studies because of the difficult calculations it requires.

Depreciation Based on Service Output

In some cases an asset is assumed to depreciate on the basis of service performed without regard to the duration of the asset's life.

Thus a trencher might be depreciated on the basis of the number of linear feet of pipeline trench dug. If the total amount of expected depreciation was equal to $11,000 minus $600, or $10,400, and it was estimated that the trencher would dig 1,500,000 linear feet of trench during its life, depreciation could be calculated as

$$\frac{\$10{,}400}{1500 \text{ M ft.}}$$

or $6.93 per M linear feet.

Depreciation is also calculated for some types of equipment on the basis of the number of hours of actual operation.

Capital Recovery with a Return Is Equivalent for All Methods of Depreciation

When retirement takes place at the age and salvage value for which depreciation and interest on the undepreciated balance of an asset were calculated, the sum of depreciation and interest on the undepreciated balance for all methods of depreciation will be equivalent. This fact may be proved as follows:

Let P = first cost of asset
L = salvage value of asset
n = estimated and actual life of asset
i = the interest rate
$(P - L) = A + B + \ldots + N$

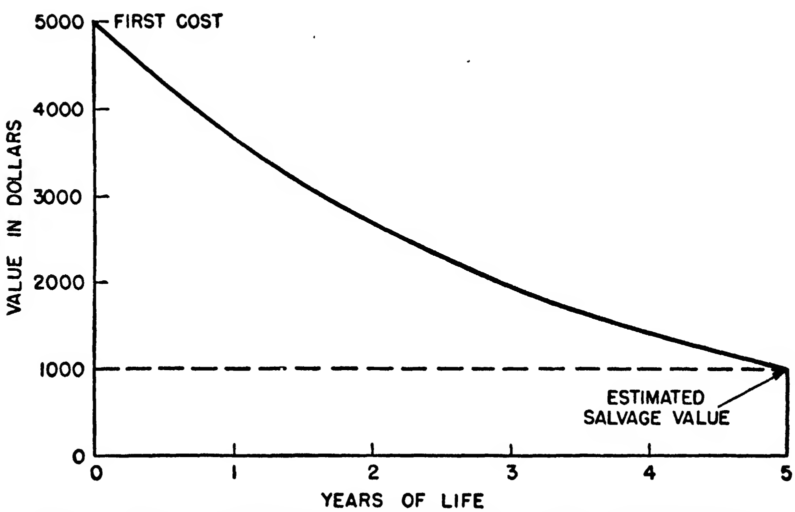


FIG. 18. Fixed percentage on diminishing-balance method of depreciation.

where A, B, $\ldots$, N are depreciation amounts for successive years. Then

End of Year No.	*Depreciation at End of Year*	*Interest on Undepreciated Balance at End of Year*
0	0	0
1	A	$Ai + Bi + \ldots + Ni + Li$
2	B	$Bi + \ldots + Ni + Li$
n	N	$Ni + Li$

Interest Li on the salvage value L will be equal for all methods of depreciation and need be given no further consideration.

$B + Bi$ as of the end of Year No. 2 is equivalent to

$$(B + Bi) \times \frac{1}{(1 + i)} = B$$

as of the end of Year No. 1. Addition of this amount, B, to Bi results in a total $(B + Bi)$ as of the end of Year No. 1. This sum is in turn equivalent to B as of the end of Year No. 0. By similar calculations, quantities involving symbols A to N inclusive will be found to have a worth of $A, B, \ldots N$ as of the end of Year No. 0 respectively. Since $(A + B + \ldots + N)$ equals $(P - L)$, and since $A, B, \ldots, N$ may be chosen to represent depreciation by any method, it may be concluded that the present worth of the depreciation calculated by any method plus the interest on the undepreciated balance is equal to the total depreciation at the beginning of the depreciation period.

In economy studies where estimated service life and salvage value are assumed to be realized, any depreciation method may be used. This equivalence permits the selection of method on the basis of convenience.

In the illustrative example on straight-line depreciation, an asset having a first cost of \$5,000 was depreciated to a salvage value of \$1,000 in a period of 5 years. The sum of *capital recovered* and *interest on unrecovered balance* was given as shown in the second column of the following table.

Year No.	*Sum of Capital Recovered and Interest on Unrecovered Balance*				
1	\$1,100	×	PS 6-1 (.9434)	=	\$1,037.74
2	1,052	×	PS 6-2 (.8900)	=	936.28
3	1,004	×	PS 6-3 (.8396)	=	842.96
4	956	×	PS 6-4 (.7921)	=	757.25
5	908	×	PS 6-5 (.7473)	=	678.55
Total Present Worth					\$4,252.78

The comparable figures calculated for sinking-fund depreciation are given in the second column of the following table.

Year No.	*Sum of Capital Recovered and Interest on Unrecovered Balance*				
1	$1,009.60	×	PS 6-1 (.9434)	=	$ 952.46
2	1,009.60	×	PS 6-2 (.8900)	=	898.54
3	1,009.60	×	PS 6-3 (.8396)	=	847.66
4	1,009.60	×	PS 6-4 (.7921)	=	799.70
5	1,009.60	×	PS 6-5 (.7473)	=	754.47
Total Present Worth					$4,252.83

The slight difference between the two resulting values, $4,252.78 and $4,252.83, results from using tables of too few places.

There are ordinarily only two real transactions in an asset's depreciation. These are its purchase and its sale as salvage. In the above example the asset was purchased for $5,000 and its salvage five years later was presumed to have a value of $1,000. The present worth of these two amounts as of the time of purchase follows.

Present worth of $5,000 disbursement at time of purchase of asset, $5000 × 1	$5,000
Present worth of $1,000 received from sale of salvage value (a receipt is a negative disbursement), $1,000 × PS 6-5 (.7473) . .	−747.30
Total present worth	$4,252.70

Compare with the two previous results.

The Straight-Line-Depreciation-Plus-Average-Interest Factor Is an Approximate Capital-Recovery Factor

The previously developed factor for calculating straight-line depreciation and average interest is

$$(P - L)\left[\frac{1}{n} + \left(\frac{n+1}{n}\right)\frac{i}{2}\right] + Li$$

Of this factor, the bracketed term

$$\left[\frac{1}{n} + \left(\frac{n+1}{n}\right)\frac{i}{2}\right]$$

in effect constitutes an approximation for $\left(\overset{\text{RP } i\text{-}n}{\text{xxxxx}}\right)$.

This may be inferred from the formula previously developed for calculating capital recovery with a return by the sinking-fund depreciation method,

$$(P - L)\left(\overset{\text{RP } i\text{-}n}{\text{xxxxx}}\right) + Li$$

This knowledge may be of value in understanding and appreciating the relationship of capital recovery with a return for straight-line and sinking-fund depreciation methods.

Service Life Predictions

In economy problems, patterns of the future depreciation of all manner of assets must be predicted. This is a difficult task. When a machine depreciates through use, a prediction must be made of the extent it will be used. If depreciation is caused by the elements, the rate at which deterioration progresses must be established. Even more difficult are the predictions that seek to determine when a machine will become obsolete because of new inventions and new needs, or inadequate owing to unlooked-for demand.

Much has been written concerning the service life of equipment. Compilations summarizing the depreciation of many types of equipment in many different situations are available.

Unfortunately such data are only of limited value as a basis for predicting the service life of a particular item of equipment. For the most part, the information that is available consists of tables giving the average life of various types of structures, machines, and so forth. These have been prepared by people of various degrees of competence and ability. In any event, they are largely based on judgments. One difficulty with such tables is that the conditions under which the facilities were used are not sufficiently described to enable application to be made to a particular situation.

In a well-documented table[1] of average service life of industrial

[1] Anson Marston and Thomas R. Agg, *Engineering Valuation* (New York: McGraw-Hill Book Company, Inc., 1936), pp. 497–514.

equipment listing the average life of more than 700 items, only approximately 100 of the examples given are based on actual mortality data.

Mortality data are very useful for purposes of adjudicating a fair cost for services. It may also be very useful in making decisions involving great numbers of units used under similar conditions, such as railroad ties, telephone poles, and electric light bulbs. However, where the future service life of a single unit is a factor in decision, particularly where obsolescence may intervene, mortality data may be of but limited use.

Mortality Curves

Where the service lives of a large number of like assets used under similar conditions are known, a number of useful curves can be drawn. Table 15 gives the mortality data of 30,009 wooden telegraph poles. For ease of understanding it may be supposed that all the poles were installed at one time. On the basis of this supposition, the "age interval" in Column 1 will also be the elapsed time since installation.

From the data in Table 15 several useful curves may be plotted (see Fig. 19). Examination of the mortality frequency curve shows that some poles do not survive as long and some poles survive much longer than the average of the group, but that the greatest rate of retirement centers around the average age. In order to maintain service, poles that are retired are immediately replaced. These replacements are subject to the same mortality frequency curve as the original group. Thus after a few years there are renewals of original poles, renewals of renewals, renewals of renewals of renewals, and so forth. A summation of these renewals is given by the renewals curve in Fig. 19.

The mortality survivor curve shows the number of original poles surviving at any service age. This curve is very similar to the survivorship curves for humans which are used in life insurance actuarial work.

The curve showing probable life of survivors is useful in predicting the life of those of the original groups that remain in use. Note that the probable life of the group at zero service age is equal to the average life of the group. The curves in Fig. 19 apply only to assets that have the same types of mortality frequency curve and have an average service life of 10.68 years.

To extend the use of this type of data Kurtz[3] has generalized its application by setting up a group of mathematical expressions repre-

TABLE 15. Compilation of the Mortality Data of 30,009 Wooden Telegraph Poles Treated with Coal Tar[2]

Age Interval, Years (1)	Units Retired during Age Interval, % (2)	Survivors at Beginning of Age Interval, % (3)	Service during Age Interval, %-years (4)	Remaining Service at Beginning of Age Interval, %-years (5)	Expectancy at Beginning of Age Interval, Years (6)	Probable Life at Beginning of Age Interval, Years (7)
0 - ½	0.00	100.00	50.00	1,067.88	10.68	10.68
½- 1½	0.35	100.00	99.82	1,017.88	10.18	10.68
1½- 2½	0.74	99.65	99.28	918.06	9.21	10.71
2½- 3½	1.45	98.91	98.19	818.78	8.28	10.78
3½- 4½	3.19	97.46	95.86	720.59	7.39	10.89
4½- 5½	2.96	94.27	92.79	624.73	6.63	11.13
5½- 6½	5.68	91.31	88.47	531.94	5.83	11.33
6½- 7½	6.11	85.63	82.58	443.47	5.18	11.68
7½- 8½	7.28	79.52	75.88	360.89	4.54	12.04
8½- 9½	9.63	72.24	67.42	285.01	3.95	12.45
9½-10½	10.37	62.61	57.43	217.59	3.48	12.98
10½-11½	10.33	52.24	47.07	160.16	3.07	13.57
11½-12½	9.54	41.91	37.14	113.09	2.70	14.20
12½-13½	9.06	32.37	27.84	75.95	2.34	14.84
13½-14½	7.26	23.31	19.68	48.11	2.06	15.56
14½-15½	6.44	16.05	12.83	28.43	1.77	16.27
15½-16½	3.77	9.61	7.73	15.60	1.62	17.12
16½-17½	3.01	5.84	4.33	7.87	1.35	17.85
17½-18½	1.84	2.83	1.91	3.54	1.25	18.75
18½-19½	0.50	0.99	0.74	1.63	1.65	20.15
19½-20½	0.12	0.49	0.43	0.89	1.82	21.32
20½-21½	0.13	0.37	0.31	0.46	1.24	21.74
21½-22½	0.21	0.24	0.13	0.15	0.63	22.13
22½-23½	0.03	0.03	0.02	0.02	0.67	23.17
23½-24½	0.00	0.00	0.00	0.00	0.00	23.50
Total.......	100.00	...	1,067.88	Av. life = 1,067.88 ÷ 100 = 10.68 yrs.		

sentative of mortality experience of a large number of items of property. These expressions facilitate mathematical analysis of problems related to mortality. They are particularly valuable for the valuation of property composed of original and renewal items or exclusively of

[2] Anson Marston and Thomas R. Agg, *Engineering Valuation* (New York: McGraw-Hill Book Company, Inc., 1936), p. 40.

[3] Edwin B. Kurtz, *The Science of Valuation and Depreciation* (New York: The Ronald Press, Inc., 1937).

renewal items, such as railroad ties on an old road bed. The Kurtz expressions also greatly facilitate the extension of analyses over periods in the future.

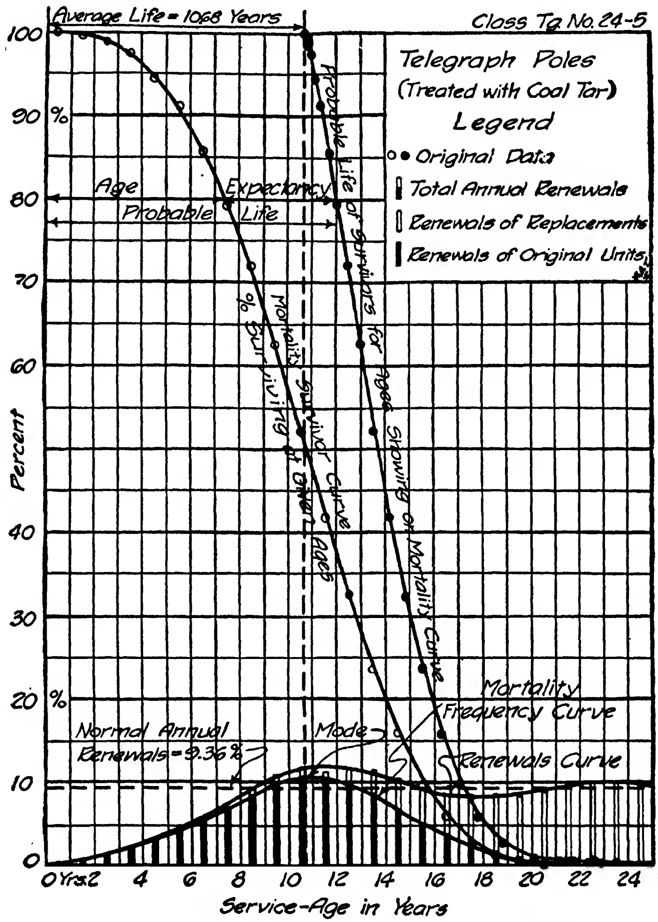


FIG. 19. Mortality curve of 30,009 wooden telegraph poles. [Reproduced by permission from Marston, Anson, and Agg, *Engineering Valuation*, McGraw-Hill, 1936.]

Reasons for Considering Depreciation

Depreciation is taken into account in business principally for the following reasons:

1. To recognize and account for the expenditure of capital assets in carrying on the activities of the enterprise.

2. To serve as a basis for dealing with agencies (usually governmental) external to an enterprise: relative to taxes, rate setting, franchises, contracts for goods and services, court cases, and the sale of property.

3. To serve as a basis for decision in matters of economy.

Recognizing and accounting for expenditure of capital assets. An understanding of depreciation is complicated by the fact that there are two aspects to be considered. One is the actual lessening in value of an asset with use and the passage of time, and the other is accounting for the lessening in value.

An asset such as a machine is a unit of capital. Such a unit of capital loses value over a period of time in which it is used in carrying on the productive activities of a business. This loss of value of an asset represents actual piecemeal expenditure of capital. For instance, a truck tire is a unit of capital. The particles of rubber that wear away with use are actually small physical units of capital expended in the intended service of the tire. In a like manner the wear of machine parts and the deterioration of structural elements are physical expenditures of capital. Expenditures of capital in this way are often difficult to observe and are usually difficult to evaluate in monetary terms, but they are nevertheless real.

One aim in accounting for depreciation is to have, continuously, a monetary measure of the value of an enterprise's unexpended physical capital, both collectively and by individual units such as specific machines. This aim can only be approximated with the accuracy with which the future life of the asset and the effect of deterioration can be estimated.

A second aim is to arrive at the physical expenditure of physical capital, in monetary terms, that has been occasioned by each unit of goods as it is produced. This aim is difficult of realization because the depreciation per unit of product depends upon the total depreciation and the total number of units that are processed, neither of which can be known until after the asset in question has ceased to be used.

In an enterprise physical capital in the form of machines, buildings, and the like is used in carrying on production activities. As machines wear out in productive activities, physical capital is converted to value in the product. Thus the capital that is lost in wear by machines is recovered in the product processed on them. This needs to be accounted for in order to determine production costs and to account for the recovered capital.

Depreciation is a cost of production. Depreciation is important because it is a cost of production. Moreover, it must be recognized as such; for, if it is not, profits will appear to be greater than they are by the amount of the depreciation that has taken place during a given period.

Consider the experience of Mr. Smith, who had been working for a salary of $2,400 per year. Through a plan for making a saving regularly he had accumulated $1,800. He decided that he could better his situation by purchasing a cab and going into business for himself, so he purchased a cab for $1,800 cash. Smith did not keep books on his business venture except to record the differences between his income and his business expenses at the end of each day. At the end of the first and second year his net income totaled $2,800 and $2,600 respectively. He felt quite well satisfied that he had gone into business for himself, and, on the strength of his apparent increase in income, increased his personal expenses. By the end of the second year repairs on the cab were becoming excessive, and Smith concluded that he must buy a new one. A new cab was priced to him at $1,800, or $1,400 and his old cab. At this point it dawned upon him that the $1,400 difference had actually been a business expense which he had failed to take into account. On reviewing his situation he realized that his income for the two years had been $2,800 plus $2,600 minus $1,400, or $4,000, an average of $2,000 per year. It then became clear to him that he should have estimated in advance the amount his cab would depreciate over the period he used it, and that he should have regarded the depreciation on the cab as a business expense to be deducted from his receipts in arriving at his net income.

There are many methods used in accounting for depreciation, but the essential feature of each is to record that portion of income which represents depreciation as a portion of an investment that is expended through use. How this portion of income is used is of no consequence if it is so recognized. Mr. Smith may well have spent the entire income from his business whether he recognized this or not, but knowing that he was spending his capital savings for living expenses he might not have been so pleased with his business perspicacity.

Two concepts of depreciation. There are two basic concepts in regard to depreciation. One of these is that "the fundamental basis of the exchange value of any specific item of property is the actual present worth, to the present owner and to the would-be purchaser, of the probable future services of the item during its probable future life in service."[4] In accordance with this concept an asset may be considered to depreciate by an amount each year equal to the net worth of the service provided. This is a view often presented in dealing with agencies outside of the enterprise in rate hearings and similar situations.

A second concept is to base depreciation on the cost of the asset. This is the concept most used where depreciation is a factor in determining the cost of production.

Depreciation and External Agencies

There are many situations in which depreciation is a factor in dealing with agencies external to an enterprise. Those most commonly encountered are governmental agencies.

The rate that private firms may charge for electric current, gas, transportation, communication, and similar services is usually determined by governmental regulatory bodies. Firms also negotiate with governmental agencies about franchises or concessions to render a service. In some cases governmental contracts for services and goods are the basis of their cost of production. In nearly all cases depreciation is a factor by virtue of being a cost of production and having a bearing on the net amount of capital on which a return is sought. As a consequence, many practices relative to depreciation have become established through custom, court action, and the opinions of recognized experts.

Much less frequently, dealings between private agencies are based on cost of production and capital invested. In a private situation, where cost of production and amount of capital are factors, the method by which these are to be taken into account is determined by agreement of parties concerned. Precedent of court cases need not be considered in such agreements.

Taxes and depreciation. Almost all enterprises must file income tax returns and so must deal with federal and sometimes state governmental tax agencies.

[4] Anson Marston and Thomas R. Agg, *Engineering Valuation* (New York: McGraw-Hill Book Company, Inc., 1936), p. 6.

Since the amount of taxes to be paid during any one year is dependent upon deductions made for depreciation, the latter is a matter of consideration for the Bureau of Internal Revenue of the United States Treasury Department and state taxing agencies.

The government does not prescribe the methods a concern must use for calculating depreciation in arriving at deductions to be made from income for tax purposes. The United States Supreme Court decisions make it clear that each situation is to be decided upon its merits and on the basis of testimony of experts in the field. However, directives are issued by governmental agencies as guides to the taxpayers in properly handling depreciation for tax purposes. In this connection the following statement from Bulletin F of the Bureau of Internal Revenue, a publication giving the probable service life and the depreciation rate of several thousand kinds of property, is of interest:[5]

Estimated Average Useful Lives and Depreciation Rates

A reasonable rate for depreciation is dependent not only on the prospective useful life of the property when acquired, but also on the particular conditions under which the property is used as reflected in the taxpayer's operating policy and the accounting policy followed with respect to repairs, maintenance replacements, charges to the capital asset account and to the depreciation reserve. If the useful life of the various assets shown hereafter could be determined precisely, which can not be done, there still could not be established standard rates of depreciation unless there existed standard methods of operation and of accounting from which there could be no deviation.

Being based on the usual experience of property owners, the probable useful lives shown herein for each kind or class of assets are predicated on a reasonable expense policy as to the cost of repairs and maintenance. Therefore, in the determination of the depreciation allowance in each case, due consideration should be given the maintenance and replacement policy of the taxpayer and the accounting practice regarding the same.

The estimates of useful life set forth herein are for new properties only. In applying them, consideration should be given to salvage values, to that portion of the service life already expired, and to that portion of the cost previously recovered or recoverable through prior depreciation deductions or other allowances.

It has been found that normal obsolescence is a very important factor in determining the useful life of property. The estimated useful lives shown herein include an allowance for normal obsolescence, but do not contain any provision for extraordinary obsolescence, such as is occasioned by revolu-

[5] "Income Tax Depreciation and Obsolescence, Estimated Useful Lives and Depreciation Rates" (Bureau of Internal Revenue; Revised, January 1942).

tionary inventions, abnormal growth or development, radical economic changes, or other unpredictable factors which may force the retirement or other disposition of property prior to the termination of its normal useful life.

Depreciation and Decisions in Economy

Economy studies are made for the primary purpose of arriving at a decision about what should be done in the future. While it is recognized that the future cannot be foretold with certainty, the fact remains that there can be no action without decisions relative to future events. Many decisions relative to economy involve predictions relative to depreciation.

The choice of the depreciation method is a decision as to the pattern of the decrease of the asset's value during its life. The calculated amount of decrease in value per year depends jointly upon the method of depreciation chosen and the estimated salvage value at the end of an estimated life. Once estimates have been made in regard to depreciation, they are acted upon as though they represent actual facts until another decision is made. When a new decision is made, it is based upon the facts as they exist at the time the decision is made. Thus, it often happens that depreciation rates that have been set up in the past are proved to be in error when a new economy study is made. Since economy studies must always be based on the facts as they exist at the time the studies are made, past estimates and predictions that have turned out to be in error must be corrected.

Accounting for Errors in Depreciation Rates

Errors in depreciation rates are sometimes incorrectly adjusted by charging for depreciation during the balance of the asset's life at a rate sufficient to compensate for the error in the original rate. Another erroneous practice is to add the difference of the book value and the actual value of an asset to the cost of a replacement. The effect of these practices is to embrace the errors of past depreciation in rates charged to the products made in the future.

As painful as it may be to admit error in setting up depreciation rates, the difference between book value and actual value should be admitted and charged to Profit and Loss or Surplus. This practice removes the influence of past errors and permits the accounts to reflect the facts accurately.

PROBLEMS

1. Mr. A, who purchased a pick-up truck in August 1940 for $1,100, was offered amounts for his truck in succeeding years as follows:

August 1941	$1,050	August 1946	$1,300
1942	1,300	1947	1,250
1943	1,400	1948	1,150
1944	1,375	1949	950
1945	1,350	1950	400

On the basis of the offers Mr. A received for his truck, plot the depreciation (and appreciation) and the undepreciated balance of the truck for each year of its life.

2. A certain asset has an initial cost of $2,200 and a salvage value of $200 at the end of a life of 10 years. The interest rate is 10%.

(a) Prepare a graph showing the relationship of the annual depreciation obtained by the straight-line, sinking-fund, and fixed-percentage-of-a-diminishing-balance methods for each year.

(b) Prepare a second graph showing the relationship of depreciated values of the asset obtained by the three methods for the end of each year.

(c) On a third graph plot the total depreciation of the asset at the end of each year as obtained by the three methods.

3. Calculate the depreciated value at the end of each year by the straight-line, sinking-fund, and fixed-percentage-of-a-diminishing-balance methods for an asset whose initial cost is $2,400 and where a salvage value is $400 at the end of 6 years. Use an interest rate of 6% and arrange results in tabular form.

4. A temporary water line will cost $4,200 and its salvage value will be $1,200 at the end of 4 years.

(a) Calculate the sum of the cost of depreciation and interest on the undepreciated balance for each year of the asset's life by the straight-line, sinking-fund, and fixed-percentage-of-a-diminishing-balance methods of depreciation, using an interest rate of 8%. Arrange results in tabular form.

(b) Convert each of the series of costs calculated in (a) to an equivalent annual cost for the four years.

5. An air compressor was purchased for $1,160. Its value at the beginning of succeeding years of its life, as determined by offers to buy it, were as follows: 2nd year, $1,240; 3rd year, $760; 4th year, $320. The interest rate is 8%.

(a) Determine the actual depreciation each year of the first three years.

(b) Determine the interest on the undepreciated balance for each of the first three years.

(c) Determine uniform year end amount for the three year period which is equivalent to the sum of depreciation and interest on the undepreciated balance as found in (a) and (b).

(d) Determine the annual cost of capital recovery with a return (sinking-fund depreciation plus interest on the undepreciated balance) for the three-year period, using only the initial cost of $1,160 and the value at the beginning of the fourth year of $320. Compare this result with that obtained in (c).

6. What is the "book value" of an asset at the end of 12 years, if the asset originally cost $1,800 and had an estimated salvage value of $400 at the end of an expected life of 20 years, by (a) straight-line depreciation, (b) 6% sinking-fund depreciation and (c) fixed-percentage-of-a-diminishing-balance depreciation?

7. A manufacturer purchased a machine lathe at an installed cost of $3,200. He estimated the service life of the machine at 10 years and its salvage value ten years hence at $800. Actually he disposed of the machine for $600 at the end of seven years.

(a) For an interest rate of 8% what was the estimated equivalent annual cost of capital recovery with a return?

(b) How much did he underestimate the equivalent annual cost of capital recovery with a return?

8. At what interest rate is the single-interest-rate capital-recovery factor equal to the capital-recovery factor whose stipulated rate is 12% and whose sinking-fund rate is 4%, if the period of recovery is 20 years? (Solve by interpolation.)

9. (a) Determine the value of the approximate capital-recovery factor based upon straight-line depreciation and average interest if the interest rate is 8% and the life of the asset under consideration is 10 years.

RP 8-10

(b) Compare the result in (a) with (xxxxx) and calculate the percentage of error of the approximate factor.

10. For convenience a contractor calculates capital recovery with a return on his equipment as the sum of straight-line depreciation plus interest on the *original* investment.

(a) On this basis what will be the annual cost of capital recovery with a return of an asset that is purchased for $4,600 and has a service life estimated as 10 years with a salvage value of $800 at that time? The interest rate is 8%.

(b) In what per cent error does the approximation result?

11. Mr. A purchased a car for $2,200. He maintained the car in excellent condition and sold it for $1,300 at the end of the third year of use. Mr. B purchased a similar car for $2,200 but negligence in care and maintenance of the car resulted in a resale value of only $600 at the end of 3 years' use. Mr. A spent $100, $175, and $200 for the first 3 years respectively for maintenance in excess of what was spent by Mr. B.

Assuming the service furnished by the two cars to be equal, what was the net equivalent annual value of Mr. A's extra care of his automobile? Use interest at 7%.

12. Mr. Brown purchased a small hotel in a town near a United States cavalry post for $100,000. At the time of the purchase Mr. Brown estimated that the net income from rental of the property would be $12,000 per year during the ensuing 15 years and that the property would be of negligible value 15 years hence. Mr. Brown received a rental of $13,000 for the hotel the first year it was in his possession. During the first year there was an increase in the personnel stationed at the cavalry post and Mr. Brown was offered $140,000 for the hotel at the end of that year but he declined to sell in view of an offer to rent the property for the second year for $22,000 which was accepted. Business for the hotel declined during the second year and the rental received for the third year was $16,000. Near the end of the third year the cavalry post was abandoned. Mr. Brown was unable to find a tenant for the hotel for the fourth year. At the end of the fourth year of ownership the hotel was sold for $53,000. To evaluate the above situation assume that rental payments are made at the end of the year and that these payments represent net income exclusive of interest and depreciation.

(a) What rate of return would result on the basis of Mr. Brown's original estimate?

(b) What rate of return did Mr. Brown actually receive on his investment?

(c) What rate of return would have been received if the hotel had been sold at the end of the first year?

13. A grain elevator was constructed in a small agricultural town at a cost of $75,000. The town was serviced by a spur railroad. The elevator owner estimated the life of the elevator to be 20 years, with no salvage value at the end of that time. Annual expenditures, exclusive of interest, were expected to be $12,000, an annual gross income was estimated at $22,500.

(a) What was the expected rate of return on the basis of the estimated period of depreciation?

(b) Unexpectedly the railroad discontinued service on the spur line after 6 years of operation. The elevator was sold at that time for $20,000. What rate of return was actually received on the investment?

14. A contractor can purchase a concrete mixer for $1,100. He estimates that it will have a salvage value of $300 6 years hence. He also estimates that the mixer will be used to produce 270 yards of concrete per year.

(a) Calculate the annual cost of capital recovery with a return if interest is at 6%.

(b) Calculate the cost of capital recovery with a return per yard of concrete produced with interest at 8%.

15. A mine estimated to produce a total of 32,200 ounces of gold is purchased for $280,000.

(a) Calculate the annual cost of depletion with interest on the undepleted balance, if the mine is exhausted at a uniform rate over an 8-year period. A stipulated interest rate of 15% and a sinking fund rate of 3% are required.

(b) Calculate the cost of capital recovery with a return per ounce of gold on the basis of the data given above.

16. The table below has been compiled from information obtained from studies of a certain type of automobile. Plot a mortality survivor curve, a mortality curve, and a renewals-of-original-units curve similar to those shown in Fig. 19.

Age Interval, Years	Units Retired During Age Interval, %	Survivors at Beginning of Age Interval, %	Service During Age Interval, %-years	Remaining In Service at Beginning of Age Interval, %-years	Expectancy at Beginning of Age Interval, Years	Probable Life at Beginning of Age Interval, Years
0 – ½	1	100	49.75	494.75	4.94	4.94
½–1½	4	99	97.00	445.00	4.50	5.00
1½–2½	7	95	91.50	348.00	3.66	5.16
2½–3½	11	88	82.50	256.50	2.91	5.41
3½–4½	16	77	69.00	174.00	2.26	5.76
4½–5½	22	61	49.50	105.00	1.72	6.22
5½–6½	17	39	30.50	54.50	1.40	6.90
6½–7½	12	22	16.00	24.00	1.09	7.59
7½–8½	7	10	6.50	8.00	.80	8.30
8½–9½	3	3	1.50	1.50	.50	9.00

CHAPTER

6

Some Concepts Useful in Economy Studies

IN DEALING WITH the physical aspects of nature, the engineer has a core of physical laws on which to base his reasoning. Such laws as Boyle's Law, Ohm's Law, and Newton's laws of motion permit reasoning to a conclusion with mathematical accuracy. These physical laws are supplemented by many formulas and known facts, all of which enable the engineer to come to conclusions about many physical phenomena which square with the facts within narrow limits. Many things are known with certainty about the physical world.

Much less, particularly of a quantitative nature, is known about the economic phenomena with which engineering economy is concerned. In engineering economy, thinking and reasoning are based on concepts, that is, ideas that the user finds helpful. Some of these ideas are the products of an individual's own ingenuity and experience. Many are very effective; others are false and must be guarded against by factual thinking. Later in this chapter a number of concepts are described and applications of them are given.

Economics Deals with Actions of People

The engineer is often confused by the lack of certainty associated with the economic aspect of engineering. It must be recognized that economic considerations embrace many of the subtleties and complexities characteristic of people.

> It is concerned with the relation of man to man and of man to nature, rather than with the physical, chemical, or astronomical law of nature per se. Principles of economics are built in part upon physical laws; but there would be no science of economics unless men themselves were present, for economics also rests in part on psychological laws.[1]

[1] Edison L. Bowers and R. Henry Rountree, *Economics for Engineers* (New York: McGraw-Hill Book Company, Inc., 1931), p. 3.

Economics deals with the behavior of people individually and collectively, particularly as their behavior relates to the satisfaction of their wants. The wants of people are motivated largely by emotional tensions and to a lesser extent by logical reasoning processes. A part of human wants can be satisfied by physical goods and services such as food, clothing, shelter, transportation, communication, entertainment, medical care, educational opportunities, and personal services; but man is rarely satisfied by physical things alone. In food, sufficient calories to meet his physical needs will rarely satisfy. He will want the food he eats to satisfy his energy needs and also his emotional needs. In consequence we find people concerned with the flavor of food, its consistency, the china and silverware with which it is served, the person or persons who serve it, the people in whose company it is eaten, and the "atmosphere" of the room in which it is served. The cost of the food is also a consideration. Similarly, there are many desires associated with clothing and shelter, in addition to those required merely to meet physical needs.

Vagaries of People Must Be Accepted

Anyone who has a part in satisfying human wants must accept the vagaries of people as a factor with which he must deal, even though he finds them inexplicable. Much or little progress has been made in discovering knowledge on which to base predictions of human reactions, depending upon one's viewpoint. Most of what has been learned appears to be more applicable to groups of people than to individuals. The idea that human reactions will someday be well enough understood to be predictable is accepted by many people; but in spite of the fact that this end has been the objective of the thinkers of the world since the beginning of time, it appears that progress in psychology has been meager compared to the rapid progress made in the physical sciences. But despite the fact that human reactions can often be neither predicted nor explained, they must be considered by those who are concerned with satisfying human wants, as are engineers.

Concepts Useful in Economy Studies

In the following paragraphs a number of ideas variously termed laws, principles, hypotheses, definitions, and concepts are presented. It will be observed that they are qualitative in nature and not necessarily universal in application. Concepts are crystallized thought. If they are thought out and in accordance with fact they may be very useful in suggesting solutions to problems or in stimulating new ideas and new possibilities for action.

Those that are given here are by no means all there are. Others will be found throughout this text. Anyone who wishes to improve his ability to arrive at sound decisions pertaining to engineering economy will find it profitable to give careful thought to the development of additional concepts to supplement those that will be presented.

Consumer and Producer Goods

Two classes of goods are recognized by economists; one class is consumer goods and the other is producer goods. Consumer goods are things and services that directly satisfy human wants, that are consumed or are directly used by persons. Examples of consumer goods are beefsteak, radios, houses, shoes, books, orchestras, barber services, and medical service. Producer goods also satisfy human wants but do not do so directly.

Broadly speaking, the ultimate end of all activities is to supply goods and services that people consume to satisfy their desires and needs. Some kinds of human wants are much more predictable than others. The demand for the food, clothing, and shelter that are needed for bare physical existence is much more stable and predictable than the demand for those items of consumer goods that satisfy man's emotional needs.

The amount of foodstuffs needed for bare existence is ascertainable within reasonable limits in terms of calories of energy, and the clothing and shelter needed may be fairly accurately approximated from data on climate. But once man is assured of physical existence he reaches out for satisfactions related to his being as a person rather than merely to his being as a physical organism.

The personal wants of people appear to be limited in scope and quantity only by the ability to acquire them. These wants are much more difficult of prediction than are the requirements of physical existence. The higher the standard of living, the higher the consumption of goods of this class. It is probable, in the more advanced nations of the world, that a greater portion of enterprise and more concern of the individual person are directed toward satisfying wants that go beyond bare physical existence than toward providing food, clothing, and shelter.

Producer Goods Are a Means to an End

Producer goods are, in the long run, a means to an end, namely that of producing goods and services for human consumption. Examples of this class of goods are such items as lathes, bulldozers, paper forms, boats, and railroad cars. These are an intermediate step in man's effort to supply his wants. They are not desired for themselves but because they may be instrumental in producing something that man can consume for the satisfaction it gives him.

Once the kind and amount of consumer goods to be produced has been determined, the kinds and amounts of facilities and producer goods necessary to produce them may be approached objectively. In this connection consider the satisfaction of the human want for harmonic sounds as in music. Suppose it has been decided that the desire for music can be met by 100,000 phonographic records. Then the organization of the artists, the technicians, and the equipment necessary to produce the records becomes predominantly objective in character. The amount of material that must be compounded and processed to form one record is calculable to a high degree of accuracy. If the concern has been making records for some time, it will know the various operations that are to be performed and the unit times for performing them. From these data, the kind and amount of producer service, the amount and kind of labor, and the number of various types of machines are determinable within rather narrow limits.

Thus, whereas the determination of the kinds and amounts of consumer goods needed at any one time may depend upon the most subjective of human considerations, the problems associated with their production are quite objective by comparison.

Engineering Is Utilitarian

As has previously been pointed out, engineering activities are rarely carried out for the satisfaction that may be derived from them directly. Engineering is, therefore, essentially a producer activity that comes into demand to satisfy human wants. But the engineer may also concern himself with determining what the wants of people are.

Thus extensive research has been done by engineers to learn what people deem to constitute comfort in automobiles and railway cars, air conditioning systems, and a great many other products in which the user's comfort is a paramount consideration.

With the continued development of science, the standard of living may be expected to improve, at least for a time, and further increase the demand for those things and services that minister to people's love of the comfortable and the beautiful. The fact that these aspects of human wants may be expected to engage the attention of engineers to an increasing extent is, in part, the basis of the widespread movement to broaden the humanistic content of engineering curricula. A knowledge of psychology, economics, sociology, and the fine arts is now recognized to be useful in nearly all fields of modern engineering and essential in some.

Law of Diminishing Returns

The term *law of diminishing returns* was originally used to designate the relation of input of fertilizers to land and the output of crops. It is a special application of the law of diminishing productivity, which may be stated as follows: *The amount of product obtained in a productive process varies with the way the agents of production are combined. If only one agent is varied, the product per unit of this agent may increase to a maximum amount, after which the product per unit may be expected to diminish but not necessarily proportionately.*

This concept may easily be illustrated by an example. A certain production line where an assembly operation is performed may be manned by a varying number of men. As the number of men on the line is increased from 15 to 19, production increases from 40.6 units per hour to 53.7 units per hour. The men are paid at the rate of $1.16 per hour. A number of relationships between output of product and input of labor are presented in tabular form in Table 16.

In this tabulation the output is given in terms of units of product and the agent of production is given in terms of man-hour input and dollar input per unit of product. It will be noticed that output per unit of input increases, reaches a maximum, and then diminishes.

The chief value of this concept in engineering economy situations is that it produces an awareness that output does not necessarily increase in a straight-line relationship with an increase in input of an agent of production. The solution of many problems in economy

TABLE 16. An Illustration of the Law of Diminishing Returns

Number of Men Employed on Assembly Line	Labor Input In Man-Hours Per Hour of Operation	Labor Input in Dollars Per Hour at Hourly Wage Rate of $1.16	Output in Number of Assemblies Per Hour	Units of Output Per Man-Hour of Input	Units of Output Per Dollar of Labor Input
15	15	$17.40	40.6	2.71	2.33
16	16	18.56	46.7	2.92	2.51
17	17	19.72	52.5	3.09	2.66
18	18	20.88	53.3	2.96	2.55
19	19	22.04	53.7	2.83	2.44

centers around adjusting the amounts of agents of productivity to produce maximum output per unit of input. This end is frequently expressed as an effort to find the input ratio that will result in least cost per unit of output. Such expressions as horsepower-hour output per pound of fuel, miles per gallon of fuel, sales per dollar of advertising, units of output per fatal accident, and defects per 100 units of product are associated with analyses in search of a maximum output per unit of input.

Value

The term *value*, like most other widely used terms, has a variety of meanings. In economics, value designates the worth that a person attaches to an object or a service. Thus the value of an object is not inherent in the object but is inherent in the regard that a person or people have for it. The subject of engineering valuation for instance is directed at assigning values to property in accordance with precepts that will be judged fair by the parties concerned. Value should not be confused with the cost or the price of an object in engineering economy studies. There may be little or no relation between the value of an article and the cost of providing it or the price that is demanded for it.

Utility

The general economic meaning of the term *utility* is *the power to satisfy human wants.* The utility that an object has for an individual is determined by him. Thus the utility of an object, like its value, is not inherent in the object itself but is inherent in the regard that a person has for it.

Utility and value in the sense here used are closely related. The utility that an object has for a person is the satisfaction he derives from it. Value is in the nature of an appraisal of utility in terms of media of exchange.

In ordinary circumstances a large variety of goods and services are available to an individual. The utility that available items may have in the mind of a prospective user may be expected to be such that his desire for them will range from abhorrence, through indifference, to intense desire. His evaluation of the utility of various items is not ordinarily constant but may be expected to change with time. Each person also possesses either goods or services that he may render. These have the utility for the person himself that he regards them to have. These same goods and possible services may also be desired by others, who may ascribe to them very different utilities. The possibility for exchange exists when each of two persons possesses utilities desired by the other. Some aspects of the economy of exchange will be discussed later in this chapter.

A person will consider two kinds of utility. One kind embraces the utility of goods and services that he intends to consume personally for the satisfaction he gets out of them. Thus it seems reasonable to believe that the utility a person attaches to goods and services that are consumed directly is at least in large measure a result of subjective nonlogical mental processes. That this is so may be inferred from the fact that sellers of consumer goods apparently find emotional appeals more effective in selling consumer goods than factual information. Early automobile advertising took the form of objective information related to design and performance, but more recent practice stresses such subjective aspects as comfort, beauty, prestige values, and the like.

An analysis of advertising and sales practices used in selling consumer goods will reveal that they appeal primarily to the senses rather than to reason, and perhaps rightly so. If the enjoyment of consumer goods stems almost exclusively from how one feels about them rather than what one reasons about them, it seems logical to make sales presentation on the basis on which customers ascribe utility.

Since the ultimate end of almost all engineering activity is consumer goods and services, the basis on which they are sold and evaluated by the consumer is a subject that might be given greater emphasis by engineers than it usually is.

The second kind of utility that an object or service may have for a person is its utility as a means to an end; this is the case with producer goods. Producer goods are not consumed for the satisfaction that can be directly derived from them but as a means of producing consumer goods.

It is inconceivable that a person would acquire such items as coal, steel bars, crude oil, or hay—none of which he has any intention of consuming directly—except as producer items. Although the utility of consumer goods is primarily determined subjectively, the utility of producer goods as a means to an end may be, and usually is, in large measure considered objectively. The value ascribed to producer goods is often quite objective. The energy, ash, and other contents of coal, for instance, can be determined very accurately and are the basis of evaluating the utility of the coal. The extent to which producer utility may be considered by logical processes is limited only by factual knowledge and the ability to reason.

Economy of Exchange

A person who possesses utilities of one kind may wish to exchange them for others. In an exchange he will seek to trade a utility he possesses for a utility that he values equally or more highly. If he is rational, he will never voluntarily exchange a utility for one he values less highly at the time of exchange.

Thus it appears that a person can never suffer a loss in an exchange. The illusion that one loses in an exchange stems from a change in one's regard for utilities with time. A person makes an exchange and on the basis of his evaluation at the time is well satisfied, is sure that he has suffered no loss. Later, on the basis of new information or because of a shift in outlook, he may re-evaluate the objects exchanged and wish he had done differently.

Both parties in an exchange may profit. Mutual benefit should be the objective of any exchange. The fact that both parties in an exchange may profit seems often not to be appreciated. That this is the case may be illustrated by an example. Suppose Mr. A purchased a small mine for the sum of $100,000. A few hours later Mr. B offers Mr. A $110,000 for the mine and Mr. A sells it to him.

It turns out that the mine contains 22,500 tons of ore. The ore is sold for $80 per ton and it costs $20 per ton to mine and deliver the ore to the place of sale. Both Mr. A and Mr. B borrow money for their business transactions at an interest rate of 5 per cent. If Mr. A had retained the mine, he would have removed the ore at the rate of 150 tons per year and the mining period would have extended over a period of 15 years. Mr. B was so situated that he removed the ore at the rate of 450 tons per year, exhausting the mine in a 5-year period. The present worth of the results of Mr. A's decision to sell the mine to Mr. B and Mr. B's decision to buy the mine and operate it as well as the result that would have been realized had Mr. A retained the mine and removed the ore over a 15-year period will be calculated for comparison.

SITUATION I

Mr. A purchases mine for $100,000 and immediately sells it for $110,000.

Present worth realized from sale of mine..............	$110,000
Cost of mine....................................	100,000
Net present worth of Mr. A's profit................	$ 10,000

SITUATION II

Mr. B purchases mine for $110,000 and depletes it in 5 years by removing ore at the rate of 450 tons per year.

Cost of mine....................................	$110,000
Annual income 450 tons at ($80 − $20)...$ 27,000	
Present worth of annual income, $27,000(4.329) (PR 5-5)........	116,880
Present worth of Mr. B's profit....................	$ 6,880

SITUATION III

Mr. A operates and depletes the mine in 15 years by removing ore at the rate of 150 tons per year.

Cost of mine....................................	$100,000
Annual income 150 tons at ($80 − $20)...$9,000	
Present worth of annual income, $9,000(10.380) (PR 5-15).......	93,420
Present worth of Mr. A's loss......................	$ 6,580

In accordance with the conditions of this example the exchange of the mine enabled Mr. A to gain $10,000 and Mr. B to enjoy a gain equivalent to $6,880 at the time the exchange was made. The purchase of the mine at $110,000 by Mr. B proved profitable to both parties. Moreover, had Mr. A operated the mine at the rate of 150 tons per year, as he first planned, he would have sustained a loss.

What was the strategic factor that enabled Mr. B to operate the mine at a profit when Mr. A would have sustained a loss had he operated it? The strategic factor was Mr. B's faster rate of operation of the mine; because of this factor, Mr. B's money remained invested in the mine a shorter time than Mr. A's would have been if he had operated it. The lesser interest on the invested funds enabled Mr. B to receive an income whose present-worth equivalent of $116,880 exceeds by $23,460 the present-worth equivalent of $93,420 in prospect for Mr. A.

Even if Mr. A had sold the mine for less than he paid for it, say for $97,000, his loss would have been less than if he had operated it according to his plan.

Each party in an exchange should seek to give something that has little utility for him but that will have great utility for the receiver. In this manner each exchange can result in the greatest gain for each party. Nearly everyone has been a party to such a favorable exchange. When a car becomes stuck in snow only a slight push may be required to dislodge it. The slight effort involved in the dislodging push may have very little utility for the person giving it, so little that he expects no more compensation than a friendly nod. On the other hand it might have very great utility for the person whose car was dislodged, so great that he may offer a substantial tip. The aim of much sales and other research is to find products, that not only will have great utility for the buyer but that can be supplied at a low cost—that is, have low utility for the seller. The difference between the utility that a specific service has for the buyer and the utility it has for the seller represents the profit or net benefit that is available for division between buyer and seller.

Customer Appeal

People purchase goods and services they know are available because such goods and services appeal to them. What is the nature or the qualities of an object that cause people to desire it? First there are the recognized tangible aspects—usefulness, durability, strength, portability, and the like. Secondly there are intangible appeals. A certain object may be purchased because of its history. Social pressure, as illustrated by the expression "keeping up with the Joneses," is important. The surroundings in which an object is sold may enable it to command a higher price. Accordingly some goods are sold in exclusive shops or may be seen only on appointment. Designers consider all manner of appeals in the hope of incorporating them into products: color, form, feel, taste, sound, and more recently, scent are considered.

If three brands A, B, and C of some common object such as a safety razor were offered for sale by mail without great promotional pressure and at the same price, it is almost certain that the sales of the brands would not be equal. Say that the campaign resulted in sales as follows:

Brand A sales during test period	120,000
Brand B sales during test period	80,000
Brand C sales during test period	40,000

It is quite likely that the relative sale could not be predicted in advance. Nor is it likely that the physical features that resulted in the disparity of sales could be deduced from the sales record unless inquiry were made of the customers. But that more A's than B's and more B's than C's were sold is a concrete fact, no matter how intangible the physical characteristics that resulted in the difference in sales may be. The economic success or failure of the product may rest upon the intangible features. Such considerations are thus of importance to engineers and others who design products to be used by consumers.

Persuasion in Exchange

It is not uncommon for an equipment salesman to call on a prospective customer, describe and explain a piece of equipment, state its price, offer it for sale, and have his offer rejected. This is concrete evidence that the machine does not possess sufficient utility at the moment to induce the prospective customer to buy it. In such a situation the salesman may be able to induce the prospect to listen to further sales talk, during which the prospect may decide to buy on the basis of the original offer. This is concrete evidence that the machine now possesses sufficient utility to induce the prospective customer to buy. Since there was no change in the machine or the price at which it was offered, there must have been a change in the customer's attitude or regard for the machine. The pertinent fact to grasp is that a proposition at first undesirable now has become desirable as a result of a change in the customer, not in the proposition.

What brought about the change? A number of reasons could be advanced. Usually it would be said that the salesman persuaded the customer to buy, in other words that the salesman induced the customer to believe something, namely that the machine had sufficient utility to warrant its purchase. There are many aspects to persuasion. It may amount only to calling attention to the availability of an item. A person cannot purchase an item he does not know exists. A part of a salesman's function is to call attention to the things he has to sell.

A second method of selling is to show objectively how an item has utility for a prospective customer. This may consist of presentation of facts and all manner of analyses to enable the prospective customer to arrive at the worth of the machine to him through the processes of reason.

Appeals are also made quite frequently to the emotions. Such appeals are rarely admitted by either party and so must, therefore, be made to appear factual and subject to reason. They take such forms as "The Jones Company purchased one last week," "As you can see, this machine has all the modern attachments," "Buy this machine and your progressiveness will be rewarded," and "I came to you first because I was sure that you (being a man of high intelligence, keen insight, progress, etc.) would appreciate having such a machine."

> Buying motives for industrial goods are both rational and emotional, but in the great majority of cases rational motives predominate. . . . Emotional motives cannot be disregarded. . . . New factories are sometimes built primarily because of the pride of possession rather than because of a well-considered need for the new plant. Banks and many retailers have often installed new fixtures and equipment because of their beauty rather than because of a carefully balanced consideration of their inherent utility or effect on business efficiency. New machines are sometimes installed because they are the latest thing on the market rather than from their practical use value.[2]

It should be recognized that even the most rational industrial buyer considers a number of factors that are not direct attributes of the item under consideration. Suggestive of these are dependability

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[2] Harold H. Maynard, Walter C. Weidler, and Theodore N. Beckman, *Principles of Marketing* (New York: The Ronald Press Company, Inc., 3rd Ed., 1939), pp. 290–291.

and reliability of the vendor, his labor policies, his position in the field under consideration, the personal relations between buyer and vendor, promptness and certainty of delivery, and availability of repair parts and service.

Whatever the approach, factual or emotional, persuasion consists of taking a person on an excursion into the future in an attempt to show and convince him what will happen if he acts in accordance with a proposal. The purpose of engineering economy analyses is to estimate, on as factual a basis as possible, what the economic consequences of a decision will be. It is therefore a useful technique in persuasion. With its wide use by both sellers and buyers many foolish decisions might be avoided. The morality of persuasion may be decried by some, but there seems to be no reason to decry it when it results in benefit to the buyer greater than he might otherwise have enjoyed. Sound persuasion is akin to leadership, which has been thus defined by Tead: "Leadership is the activity of influencing people to cooperate toward some goal which they come to find desirable."[3]

It is observable that persuasive ability is much in demand, is often of inestimable beneficial consequences to all concerned, and is usually richly rewarded. Persuasion as it applies to the sale of goods is of economic importance to industry. A manufacturer must dispose of the goods he produces. He can increase the salability of his products by building into them greater customer appeal in terms of greater usefulness, greater durability, or greater beauty, or he may elect to accompany his products to market with greater persuasive effort in the form of advertising and sales promotion. Either plan will require expenditure and both are subject to the law of diminishing returns. It is a study in economy to determine what levels of perfection of product and sales effort will be most profitable.

For example, let it be assumed that a new type of industrial truck has been developed. The manufacturer has the choice of spending say $2,000 on its manufacture and nothing on sales promotion or of spending say $500 on manufacture and $1,500 on sales promotion, or of spending for the two items in some other proportion.

It is interesting to note, in this connection, that the cost of some things that increase the sale of a product may be more than out-

[3] Ordway Tead, *The Art of Leadership* (New York: McGraw-Hill Book Company, Inc., 1934), p. 20.

weighed by the economies that result. Thus qualities that make a product more desired, or sales promotion, may have two beneficial effects. One of these is that it may be feasible to increase the price asked for the product and the other is that the volume of product sold may increase to the extent that a lower manufacturing cost per unit may result.

For example, a manufacturer may be making and marketing a product on which he is losing money. Assume that the product costs $.60 to make and $.40 to sell at the present volume. Suppose now it is decided to improve the quality of the product by using a better-quality material at a cost of $.10 per unit. Temporarily, the manufacturing cost increases to $.70, with the selling cost remaining at $.40. Later, however, it is quite possible that the improved quality will result in a greater volume of sales, which in turn will permit more economical methods of manufacture that will reduce the manufacturing cost to say $.50, even with the improved quality of material.

Specialization and Generalization

To specialize is to restrict a person or thing to a particular activity, place, time, or in general, to a particular situation. Specialization in this sense results when there is a division of labor, as for example, when operations in a process performed by a single person are divided among a number of persons. It is clear that machines can also be specialized in this sense, that is, specialized as to activity.

Persons or things can also be specialized as to use, place, or time. Thus a person whose assignments require that he report to work in St. Louis is specialized as to place. A passenger train that operates on a schedule is specialized as to time and place; so also is a person who works in one place from nine to five o'clock five days a week.

Specialization is of interest in relation to economy studies because it is often a means whereby the cost of accomplishing a given result can be reduced. In any design or planning effort the desirability of specialization should be considered as a routine matter. It is widely recognized that specialization, particularly as it relates to people, may result in improved performance. Of interest in this connection is a principle stated by Alford:

Principle of Division of Work, or Specialization of the Job. Subdividing work so that one or very few manual or mental operations can be assigned to a worker improves the quality and increases the quantity of output.[4]

Specialization usually requires pre-planning and specialized arrangements, the cost and maintenance of which may more than offset the advantage gained. The gains of a specialization of one kind may require specialization of another that is too costly. For instance, it might be desirable to have two men perform one each of two activities that must be performed one after the other, *unless* the new arrangement would also require the men to be specialized as to time. The latter specialization might easily result in one man's being idle while the other worked.

Specialization is subject to the law of diminishing return. It is apparent that it would not be economical to specialize to such an extent, for example, that one truck driver made only right turns and another made only left turns.

Generalization, the reverse of specialization, may also be a factor in economy. Thus in machine design it is often desirable to have one part serve several purposes; the crankcase of an engine, for instance, serves as a base for the engine, a container for oil, and a support for the crankshaft and cylinder. The term "all-around man" is evidence of generalization with respect to personnel.

Standardization and Simplification

A standard is a specification. Products designated as *standardized* conform to a previously accepted specification. Standardization, the conduct of activities in accordance with previously determined specifications, appears to be a modern development, particularly as it relates to the production of goods. Both products and the procedures by which they are made are subject to standardization.

Simplification is a name for the practice of examining a line of products for the purpose of eliminating useless variety in style, color, dimensions, size range, and the like. Its practice results in setting up what might be termed *most useful specifications* for a line of products. Thus simplification is closely related to standardization. Simplification may be thought of as being the practice of eliminating undesirable standards.

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[4] L. P. Alford, *Principles of Industrial Management for Engineers* (New York: The Ronald Press Company, Inc., 1940), p. 53.

There are a number of reasons, related to economy, why standardization has become a widespread practice. One factor is undoubtedly that human actions are characterized by variation and individualism whereas machine action is characterized by repetition of identical patterns. Thus it is relatively easy to build machines that operate and produce goods in accordance with predetermined specifications. As the burden of production has been shifted from human effort and skills to machines, it has been necessary, at least to the extent of machine processes, to standardize.

The chief drive to standardization is probably specialization. It is recognized as characteristic that the skill and speed of performance is increased and the effort of performance is decreased as a person repetitiously performs a specified task. It is also known that persons have greater aptitude for doing some things than others; more people may be found with one skill than several. It is rarity that places triple-threat men in demand in football.

Specialization is not economically practical unless it can be engaged in for some time. It is rarely profitable to build a machine for a total lifetime production of a single part. Nor would it be profitable to set up an assembly line of a number of specialized people to assemble a single unit of product.

Standardization is a method of increasing the number of units of one kind for a given total production. Suppose that a pottery has had an annual output of 1,000 unstandardized vases made in accordance with 40 different patterns, each in 5 different sizes. Thus the output embraces 200 different specifications and an average annual output of 5 units per specification. If it were decided to standardize on one vase of one size, the annual output per specification would be increased from 5 to 1,000 units. Thus it appears that the economy of standardization, lies in large measure, in the increased opportunity for specialization afforded by increased volume of one kind of product or activity.

Since standards ordinarily have extended usefulness in terms of time and volume, it is often economical to take great pains to perfect them. A standard is in reality a predetermined decision to act in accordance with a certain specification in certain situations. Even though a standard may be developed only after a considerable expenditure, it will probably result in a lower cost per decision than if each specification problem were decided as it arose without benefit of a standard. Suppose that a certain situation necessitating decision arises in a concern 1,000 times per year and that the cost of making each decision individually, owing to confusion, error, and loss of time, amounts to $1.00. A standard procedure that could reduce this cost to $.10 per decision, for instance, would justify an expenditure of $900 (or more, if applicable for more than one year).

One valuable feature of standards is that they greatly facilitate communication. Involved directions, for instance, may be given simply by pointing out the standards that should be applied. This is of tremendous value in coordinating the activities of large groups of people as in modern industry.

Pre-Planning

Pre-planning is the formulation, prior to beginning an undertaking, of more or less detailed plans for carrying it out. One important economy of this practice is that it permits specialization. People with unusual ability can be assigned the task of making plans covering a narrow scope of activities. Repeated experience with similar situations may also be expected to improve performance. Not only may people who are specialized in the task of planning be expected to produce plans that can be executed economically, but it is likely that the cost of the plans themselves will be less.

Planning is essentially a form of design and may be defined as being the projection and evaluation of ideas. In planning and designing, ways for accomplishing a contemplated end are literally invented and then evaluated on the basis of strength, fuel consumption, hazard economy, cost, and so forth. The conception of different ways to accomplish a desired end does not appear to be a logical process, but their evaluation is largely a matter of reasoning. One important aspect of planning is that it makes it possible to try out a great many ideas on paper more or less conclusively. The cost of such trials may be negligible in comparison with actual trials.

Consider the case of a manufacturer who has been asked to supply in quantity a certain chemical not now being produced. Where the art and science is well known, it is quite likely that he will be able to determine on paper and within narrow limits what material, labor, equipment, and related requirements will be necessary to make the product in several technically possible ways. Once he has learned the physical requirements, these can readily be converted into expenditures to determine the comparative economy of the several processes. Thus he may know the cost of producing the chemical without making costly trials of all the feasible methods.

Identification

Much effort is devoted to ascertaining what things are. After a thing has been identified, further effort is often expended in having the thing carry its identification along with it for subsequent ready reference. Identification refers to the establishment of certain qualities of a thing and may embrace a wide variety of attributes. Some of these that are related to physical qualities are chemical constituents, proportion of ingredients in a mix, history of processing (such as in the case of heat treatment of steel), and dimensions.

Inability to identify an object or a service may greatly impair its value. This may be illustrated by an amusing example. As a prank, a youngster removed the labels from the family's stock of canned goods. The prank proved to be a costly one, for to get a desired item for a meal it often became necessary to open and discard several cans of fruit or vegetables. In military activities, identification is considered of such importance that great precaution is exercised to see that the enemy cannot identify military units, equipment, installations, and roads.

Identification is an important factor in economy; many practices are based upon knowing the attributes of things. The activity of inspection is based upon the value of identification, the knowing that the attributes of the things being inspected come within prescribed limits. A failure to identify industrial stores properly may result in serious losses. It is not uncommon for extensive damage to be caused to elaborate cutting tools through the substitution of a material of inferior machinability in an automatic machining process. Airplanes may be rendered hazardous by inadvertent substitution of inferior fuel. Some chemical processes become dangerous through the inclusion of an undesired ingredient through misidentification. Though welding is far superior in many applications for joining materials, the strategic factor in rejecting it for some applications has been the difficulty of identifying inferior welds. Better methods of identifying the quality of welds have been the object of tremendous effort; welding has been extended as these have been developed. In the manufacture of pharmaceuticals, unusual precautions are taken to insure that each item manufactured is correctly identified by its label. Means for identification may be an object of investigation; more and more technical research is directed to this end.

The economic value of an identification depends upon (1) the cost of identification, (2) the consequences of misidentification, and (3) the cost related to making a decision when identification is lacking. Consider a large lot of castings, all of which are supposed to be annealed, which is suspected of including several unannealed castings. If the cost of inspecting the lot at the machine or elsewhere is $8.00, and the consequences of each attempt at machining an unannealed casting is $2.00 in broken tools, delay, and so forth, it is reasonable to suppose that the lot will be inspected. If the lot is inspected, the lack of identification will add $8.00 to the cost of the lot. If the lot is not inspected, the lack of identification will add $2.00 times the number of unannealed castings to the cost of the lot. In either case a decision must be made whether to spend $8.00 or take a chance that there will be less than 4 defective castings in the lot. The cost of making this decision also adds to the cost of this lot.

Identification is also important in relation to knowledge about nonphysical things. One of the most important assets that a concern may have is that large numbers of customers and prospective customers identify the concern as being a vendor of good products at reasonable prices. It is not enough that a concern makes a good product, if this fact remains unknown to potential buyers of such a product. Thus huge sums are spent upon establishing brand names as means whereby people can identify products.

Success in engineering consulting practice depends upon being known for sound counsel as well as upon the actual rendering of sound counsel. Most consultants realize this and take definite steps to identify themselves to people as being dependable. This they may do by rendering service that will elicit the favorable comment of those they serve, and by taking part in the activities of technical societies, writing for publication, and in other ways directing the attention of prospective clients to their capabilities.

The Relation of Objective and Performance to Success

Attention may be focused on doing very worth-while things or on doing things very well. Economic success depends to an extent on each.

It appears that greater emphasis is usually directed to doing things well than is directed to seeking to do things that have maximum promise. All too often the effort of each is not given consideration separately in contemplating undertakings. Views on this subject may be clarified by consideration of an example.

Let it be assumed that two prospectors are seeking ore veins to mine. After a short period of search Prospector A discovers a vein and immediately begins to remove the ore. Prospector A is both ingenious and industrious in his mining operations. Prospector B diligently continued his prospecting long after A was actively mining because he felt that the discovery of a potentially profitable ore body was paramount to success; his search was finally attended with the discovery of a rich vein after he had uncovered numerous poorer ones, including the one that A was now working. In the operation of his mine, B was only reasonably effective in both method and industry. Assume that this example is summarized by the following analysis.

PROSPECTOR A'S SITUATION

The vein was exhausted at the end of five years after the removal of 20 tons of ore.

Total gross income, 20 tons ore at $300 per ton		$ 6,000
Cost of prospecting to discover mine	$ 400	
Cost of supplies to operate mine at $30 per ton mined, 20 × $30	600	
Total costs		1,000
Net return to Prospector A for 5 years of effort		$ 5,000
Net return per year of effort		$ 1,000

PROSPECTOR B'S SITUATION

The vein was exhausted at the end of five years after the removal of 150 tons of ore.

Total gross income, 150 tons at $400 per ton		$60,000
Cost of prospecting to discover mine	$2,500	
Cost of supplies to operate mine at $50 per ton, 150 × $50	7,500	
Total costs		10,000
Net return to Prospector B for 5 years of effort		$50,000
Net return per year of effort		$10,000

Now suppose that A's mine had been operated by B and that, as a result of his less industrious and efficient operation, the supply cost per ton had risen to $60 and that 8 years were required to deplete the mine. B would then have had a net annual income of $550 for his effort.

Also suppose that B's mine had been operated by the more industrious and efficient A with the result that the supply cost per ton *dropped to $10 and the removal period to 4 years. The annual* return received by A would then have been $14,000.

Note that even when A's mine was efficiently operated it resulted in an annual income of only $1,000, whereas B's mine produced an annual income of $10,000 despite its inefficient operation.

On the basis of the above analysis, it appears warranted to draw the conclusion that the outcome of an undertaking is jointly dependent upon the potentialities of the undertaking itself and upon how well it is prosecuted. The potentialities of A's and B's mines in the above example were taken to be a total output of 20 and 150 tons, respectively, of ore of a certain grade located in a certain situation. If it is assumed for convenience that the income per ton could not be increased by action of the operators (which would ordinarily be reasonably true), $6,000 and $60,000 would represent the monetary income potentialities of the two mines, respectively.

A decision to select A's mine was a decision to undertake an enterprise that could not result in a total income greater than $6,000 regardless of subsequent action.

It may also be noted that there are costs associated with seeking out and deciding upon undertakings. The cost of seeking out desirable undertakings is a charge that must be deducted from the income potentialities of the undertakings that are decided upon. This limits the outlay that can be justified for search on the basis of economy. The measure of the net success of a venture may be thought of as being the difference between its potentialities for income and the sum of the outlay incurred in finding and deciding to undertake it and the outlay incurred in carrying it to completion.

The total success of an individual or an organization is the summation of the successes of all the ventures that he or it has undertaken. Also, the success of a major undertaking is the summation of the successes of the minor ventures of which it is constituted.

In Fig. 20 each vertical block represents the income potentialities of a venture, the outlay incurred in seeking it, the outlay incurred in prosecuting it, and the net gain of carrying it on. In this conceptual scheme the several quantities are considered to be measured in a single commensurable term, such as money.

From this figure it is apparent that the extent of the success of a venture depends upon its potentialities for income less the sum of the costs of finding it and carrying it on. It is also apparent that the success of an individual or an enterprise is the summation of the net success of the several ventures undertaken during a period of time.

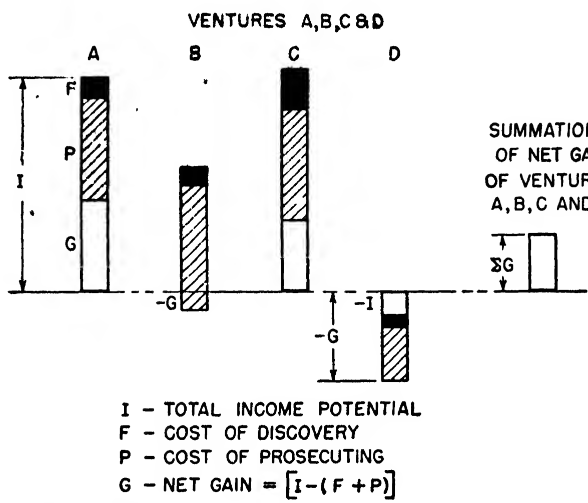

FIG. 20. Relation of income potential, cost of discovery, cost of prosecuting ventures, and success.

Qualitative and Quantitative Knowledge

Two types of knowledge will be encountered in economy studies. These are qualitative knowledge and quantitative knowledge.

Qualitative knowledge is knowledge of the attributes of the thing under consideration. It is descriptive and tells how a thing is constituted by naming the thing's distinctive characteristics. In the expression of qualitative knowledge such statements are used as: Repair cost of the truck will be low; Many units of this item will be sold this year; This method was cheaper than that method; The engine had much power and used little fuel. Such expressions cannot be precisely evaluated and have comparatively little value in economy studies.

Quantitative knowledge is knowledge of the amount or extent of the attributes of a thing in terms that are capable of being counted. It is information that is capable of being expressed in numbers. Representative of quantitative knowledge are such statements as: The repair cost of the truck amounted to \$358 per year; This year 7,288 units of that item were sold; This method costs \$.67 per piece but that method costs \$.78 per piece; The engine developed 88 horsepower and used .62 pound of fuel per horsepower-hour.

The eminent British scientist Lord Kelvin expressed his regard for quantitative knowledge in the following quotation:

> I often say that when you can not measure what you are speaking and express it in numbers your knowledge is of meager and unsatisfactory kind; it may be the beginning of knowledge, but you have scarcely in your thoughts advanced to the stage of science, whatever the matter may be.

It appears that qualitative knowledge is primarily evaluated by one's feeling for it and one's judgment. Different people are likely to vary widely in their interpretation of the significance of qualitative ideas. Quantitative knowledge, on the other hand, is quite precise and may be evaluated in large measure by the processes of reasoning. Also, most people may be expected to attach about the same significance to quantitative statements pertaining to fields with which they are familiar. This is an important characteristic because it facilitates communication of ideas.

A comparison of the effectiveness of the two kinds of knowledge may be gained from a consideration of two statements: (1) The engine had much power and used little fuel; and (2) The engine developed 88 horsepower and used .62 pound of fuel per hour at that load. Let each statement be considered in conjunction with the knowledge that a machine for which a prime mover is sought will require a turning torque of 420 foot-pounds at 1,000 R.P.M., that the cost of fuel in \$.02 per pound, and that

$$\text{H.P.} = \frac{\text{Torque} \times 2\pi \times \text{R.P.M.}}{33{,}000}$$

If the economy of using the engine as a prime mover for the machine was being considered, two questions would be pertinent. These are: (1) Is the engine powerful enough? and (2) What will be the fuel cost per hour? It will be noted that qualitative statements cannot be used to arrive at the desired answers.

The quantitative information can be used as follows.

$$\text{Required H.P.} = \frac{420 \times 2\pi \times 1{,}000}{33{,}000} = 80$$

This answers the first question. The second question can be answered fairly reliably by the following expression:

$$\text{Fuel cost per hour} = 80 \times .62 \times \$.02 = \$.99$$

One important aspect of quantitative data is that it can often be combined with other quantitative data by logical processes to produce new quantitative knowledge as was illustrated above. This is a very important characteristic in making economy studies and suggests that quantitative knowledge be exhausted before consideration of qualitative ideas.

However, it must be realized that need for action demands decision in many situations not fully covered by concrete facts. Then decision must often be based in part upon qualitative knowledge.

Action and Knowledge of Results Thereof

A great deal of time and energy is spent in acquiring information about an undertaking that has been completed. Thus motion pictures of last week's football game may be examined with great care. Business firms make quarterly or annual reports purporting to summarize their activities and to show their status at the end of these periods. The economic justification for acquiring knowledge of the past is that it will enable better decisions to be made in the future. It is a recognized psychological fact that a person cannot improve his skill by practice unless he learns the outcome of his actions. Thus a person could not improve his ability to throw darts if the lights on the target are extinguished from the time a dart leaves his hand until after it is removed from the target.

In a similar way, it is reasonable to believe that skill in making decisions must be dependent upon knowing the result of previous decisions. Since the chief purpose of information of past activity is to aid in performing activities more efficiently in the future, this purpose should be considered in relation to economy. Investigation often reveals that huge sums are spent in business for reports, records, audits and analyses. It often happens that much of such information is of little use. At the same time some items that would be of great value are not available. The benefits to be gained by the availability of information should always be considered in relation to the cost of collecting it.

Advantages and Disadvantages

Any contemplated action has its advantages and disadvantages. In considering alternative automobile routes between two points, nearly every one has said the equivalent of "Route A is shorter but rougher than the Route B," or the reverse, "Route B is longer but smoother than Route A."

A person who has enthusiasm for a proposal, particularly its originator, is likely to give undue favorable weight to its advantages and to minimize its disadvantages. But experience has taught most people to be skeptical of new things. The opposition to a new proposal may tend to stress its disadvantages and disregard its advantages.

Quite often a proposal is presented to replace a present practice that affects the working routine of a number of people. In such a situation the proposed method may encounter a great deal of opposition. Real and fancied disadvantages will be pointed out and the claimed advantages will not be admitted. The personal feelings with which people who will be affected by a proposal regard it may in themselves constitute very important advantages or disadvantages. Many meritorious proposals have been rejected or have failed after adoption because they had the disadvantage of the opposition of those who were affected by them. Enthusiasm for a measure, on the other hand, may be an advantage that results in indifferent proposals being adopted and being carried on with excellent results.

The point to realize is that every proposal has its advantages and disadvantages, whether these exist in the proposal itself or in people's regard for it. The sponsor of a proposal should school himself to consider advantages and disadvantages as impersonally as possible, because the economic advantage of a proposal depends upon its advantages and disadvantages in comparison with those of other alternatives. That concurrence of these opposing qualities has long been recognized is indicated by the following amusing quotation from Samuel Johnson: "There is now less flogging in our great schools than formerly,—but then, less is learned there; so that what the boys get at one end, they lose at the other."

Size and Economy

The cost of many items varies on the basis of size. Extremely large units and extremely small units may cost relatively more than sizes that come in between. One reason for this variation in unit cost is that the number produced is often greater in the sizes that lie between the extremes. Some products, such as valves, electric motors, lathes, and engines, are essentially identical except for size. Thus the same

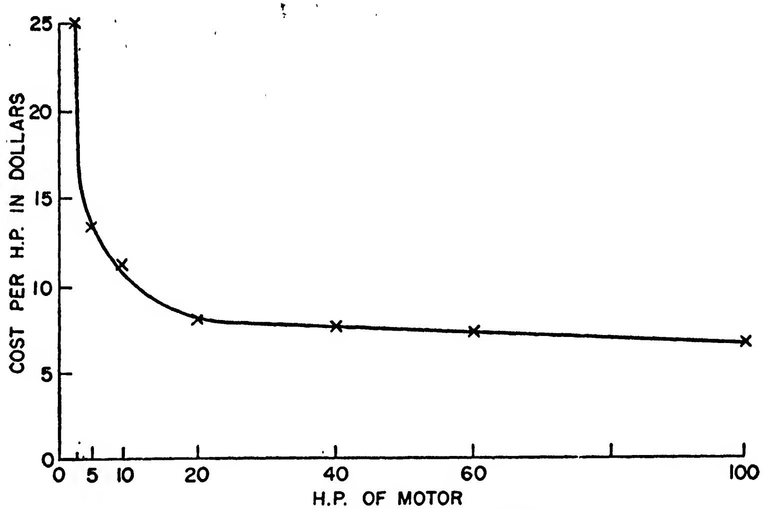

Fig. 21. Cost of electric motors per horsepower, based on a manufacturer's list price on squirrel-cage induction, 3-phase, 60-cycle, 220-volt, 1800-r.p.m. motors.

number of parts will have to be processed. Though larger parts may be expected to require longer time for processing than smaller parts, it is unlikely that a part twice the size of another will require twice the time to process.

Figure 21 shows the relationship between the cost per unit and the size of a certain make of electric motor. From this chart it may be seen that a motor capacity of 20 H.P. which could be provided for approximately $160 with a 20-H.P. motor would cost approximately $260 with four 5-H.P. units. It is clear that the latter installation could only be justified where it would permit offsetting advantages.

Similarly, there are many activities that cannot be carried on economically above or below a certain scale. It is estimated that the minimum establishment to compete successfully in the manufacture and sale of passenger automobiles would require a minimum investment of $100,000,000. On the other hand, certain hazardous processes in the manufacture of explosives are carried on in small isolated units as a matter of economy. There are many activities that are best carried on on a small scale. This fact is obscured in many cases because of an extreme regard for size by many people.

Theory, the Practical, Conjecture, and Hypotheses

The word *theory* means conjecture to some. Others consider theory to be an intellectual but unreal statement that does not square with the facts of the practical world. It seems desirable for engineers to reserve this term to designate statements of high validity, as suggested by the following definition:

> THEORY—*A statement of relationships between defined factors which is in accordance with fact.*

The term *practical* designates that which works out in practice. A thing that works out in practice is, of course, in accordance with fact. Therefore, practical knowledge is knowledge that is in accordance with fact though the underlying relationships may not be known.

Theoretical knowledge is practical knowledge with known underlying relationships. This means that theory is practical. Engineers should strive to establish the concept that theory is practical.

The term theory should never be used to designate conjecture or hypotheses. *Conjecture* is opinion inferred from inadequate evidence. A *hypothesis* is a supposition provisionally and tentatively adopted as an aid to investigation. The concept embraced by this term is very useful in making exploratory calculations in engineering economy. Certain assumptions are made in order to be able to calculate the outcome that would result if the assumptions were true.

PROBLEMS AND QUESTIONS

1. What is the justification for spending money to improve the appearance of producer goods such as machine tools (a) as viewed by the seller, (b) as viewed by the buyer?

2. (a) Illustrate the operation of the law of diminishing returns with a quantitative example.

(b) Which of the problems in the list following Chapter 2 illustrate the operation of the law of diminishing returns?

3. The *X* Manufacturing Company, located in City A, dispatches a delivery truck each day to City B, which is 150 miles distant. The truck is loaded on the trip to City B but returns to City A empty.

The *Y* Trucking Company has a contract to haul newspapers from City B to City A but the return trip is made without pay load. The *X* Manufacturing Company has offered to subcontract the hauling of the newspapers and has calculated that its cost for operating its truck is \$.11 per mile plus the cost of a driver at \$1.10 per hour. The *Y* Trucking Company has established its cost for the newspaper haul as \$.12 per mile plus \$1.20 per hour for the driver. Average time for a round trip for both companies is 9 hours.

It is estimated that the assumption of the newspaper haul by the *X* Manufacturing Company will cause an increase in the expenses equivalent to an increase in the round-trip mileage of 16 miles and an increase in the round trip time of 1 hour and 45 minutes.

(a) What is the minimum the *X* Manufacturing Company can charge for the newspaper haul?

(b) What is the maximum the *Y* Trucking Company can pay in subcontracting the newspaper haul?

4. Mr. A possesses a mine property estimated to contain 90,000 tons of coal. The mine is now leased to the B Coal Company, which pays Mr. A \$.60 royalty per ton of coal removed. Coal is removed at the rate of 15,000 tons per year. This rate is expected to continue until the mine is exhausted, at which time the mine property is estimated to be worth \$4,000. Mr. A now employs a checker whose duty is to measure the coal removed and to bill the B Coal Company for the royalty on the coal removed. The checker receives \$2,100 per year.

(a) If interest is at 8% compounded annually and taxes are neglected for how much can Mr. A afford to sell the property?

(b) If interest is at 6% compounded annually, how much can the B Coal Company afford to pay for the property?

(c) What is the most important factor causing the difference in the results obtained in (a) and (b)?

5. A certain household appliance has been sold through variety stores. There has been no change in design for many years and sales have stabilized at 1,200,000 units annually. The manufacturer knows that the average profit per unit is \$.063 at the present time. He is approached by an industrial designer who believes that an improvement in design can be made which will improve the "sales appeal" of the product. It is estimated that the improved design may increase the cost of manufacture by as much as \$.007 per unit. The profit per unit will probably be reduced by this amount, for it is not believed that it is feasible to increase the unit cost to vendors.

Development of an improved design will require extensive development work and market surveys. The industrial designer quotes a fee of $30,000 for undertaking the necessary development and market surveys. The proposal will be usable for 6 years and interest is taken at 10% compounded annually. How much must average annual sales be increased above the present level to warrant payment of the designer's fee?

6. A baker has been selling bread through local retailers without using advertising. His average annual sales have been $140,000. Annual cost of sales has averaged $128,000, leaving an average net profit of $12,000 a year. The advertising agent of a local newspaper suggests an advertising campaign requiring an expenditure of $140 per week for 10 weeks to bring the product to the attention of the customers and an expenditure of $20 per week thereafter.

It is estimated that average annual sales of the product will be increased by 20% and that unit cost of sales, exclusive of advertising, will be decreased by 2% owing to the estimated increase in volume. If these estimates are correct and interest is taken at 8% compounded annually, what will be the equivalent annual increase in profit for a period of 3 years?

7. A manufacturer has been awarded a contract to make 4,000 mechanisms for a timing device. Assembly of the device can be performed by highly skilled operators working individually. The assembly can also be performed by workmen of lesser skill working as a team, if they are provided with specialized assembly equipment and given special supervision.

It is estimated that the highly skilled workmen, whose wage rate per hour is $2.12, will each require an average of 2½ hours per assembly.

An assembly line that will permit specialization of labor can be set up for an initial cost of $6,200. With it the assembly can be performed by a team of four operators, whose hourly wage cost is $1.05 per hour, supervised by a lead man whose wage cost is $2.24 per hour. It is estimated that the average time required for the five-man crew to perform the assembly will be 36 minutes per unit. What loss or gain will result if the assembly equipment is installed and is scrapped as valueless at the termination of the contract?

8. In the manufacture of a line of equipment, a number of metal cylinders 3 inches long and of the following diameters were specified to be used in the indicated quantities:

Diameter in inches	¼	⅜	½	⅝	¾	⅞	1
Annual quantity	12,000	10,000	8,000	5,000	4,000	3,000	2,000

It is estimated that each size eliminated will result in an annual savings of $70 owing to decrease in set-up costs, record keeping, and so forth. It is proposed that the ⅜-in., ⅝-in., and ⅞-in. sizes be eliminated and that the next larger size be used in each case. The simplification of sizes will increase material costs. Determine the savings resulting from the elimination of each of the three sizes if the $70 estimate is correct. The metal weighs .29 pound per cubic inch and costs $.11 per pound.

9. A manufacturer makes 7,000,000 radio tubes per year. An assembly operation is performed on each tube for which a standard piece rate of $1.02

per hundred pieces is paid. The standard cycle time per piece is .624 minute. Overhead costs associated with the operation are estimated at $.36 per operator hour.

If the standard cycle time can be reduced by .01 of a minute and if one-half of the overhead cost per operator hour is saved for each hour by which total operator hours are reduced, what will be the maximum amount that can be spent to bring about the .01 of a minute reduction in cycle time? Assume that the improvement will be in effect for one year, that piece rates will be reduced in proportion to the reduction in cycle time, and that the average operator's time per piece is equal to the standard time per piece in either case.

10. List a number of situations in which the value of a product has been greatly impaired because an owner or a prospective buyer was uncertain of its qualities.

11. (a) Describe a number of situations in which success resulted primarily from excellence of execution.

(b) Describe a number of situations in which success resulted primarily from the selection of fruitful objectives.

12. Examine a number of advertisements of products in a technical magazine and list the qualitative and quantitative statements in each.

13. (a) Describe a number of situations in which correct actions were being taken but in which the actions were abandoned prematurely because it was not known that continuing them would lead to success.

(b) Describe a number of apparatuses or activities employed primarily for the purpose of revealing the results of industrial activities.

14. (a) List examples of products whose unit cost is in some measure inversely proportional to their capacity.

(b) List examples of products whose unit cost is in some measure directly proportional to their capacity.

CHAPTER

7

A Pattern for Engineering Economy Studies

ENGINEERING AND ECONOMICS jointly embrace many aspects of the physical and social world. Much progress has been made in securing and classifying information and in developing methods of analysis for attacking physical problems. This knowledge is predominantly quantitative. Much of the knowledge is based on physical laws, and the method of analysis is often delineated by a body of mathematical formulas. Even judgments as embodied in acceptable practices are often expressed in quantitative terms.

By contrast, economic analysis rests, to a large extent, on qualitative concepts. Methods of analysis scarcely have been suggested. Often those who are most successful in evaluating economic factors are unable to define their methods of analysis. Success in coping with economic factors seems to be intuitive in large measure. There are few accurately defined and standardized methods applicable to the analyses of economic situations in quantitative terms.

Patterns for Economy Studies

Grant,[1] a pioneer in the field of engineering economy, set up a pattern for economy studies in a sequence of four steps. The purpose of a pattern of study is to delineate a plan of analysis that will aid in arriving at sound conclusions. A good pattern of attack serves the purpose of pointing out the nature and the sequence of analysis that will be required in most situations. If followed, a sound pattern is effective in eliminating errors of omission for it focuses attention on the steps that should be taken. The purpose of this chapter will be to discuss additional factors pertinent to the evaluation of contem-

[1] Eugene L. Grant, *Principles of Engineering Economy* (New York: The Ronald Press Company, Inc., 1938), p. 25.

plated ventures and to develop a pattern for engineering economy studies.

The Aim of Private and Public Enterprise

Since the purpose of engineering economy is to increase the certainty of success of activities undertaken by enterprises, it will be desirable to examine the aim of such enterprises. The two most important types of enterprise in which engineering economy is applicable are private commercial concerns and governmental agencies.

The goals that prompt activity are many and varied. Perhaps the most easily understood and certainly the most precisely stated aim is profit, as this term applies to private commercial organizations. Profit is not only an aim of commercial organizations but a necessity for their survival.

Profit is a resultant of the worth of output and the cost of input. The achievement of the economic aim of private enterprise is measured in terms of profit, income, and cost expressed in monetary values. The development of a pattern for economy studies in this chapter will be related primarily to private enterprise.

The primary aims of the United States are "to provide for the common defense" and "to promote the general welfare." To discharge this aim, Congress is given the power in the Constitution of the United States "to lay and collect taxes for the common defense and the general welfare" on condition that funds so acquired are expended only in "consequence of appropriations made by law." It is clear that neither "the common defense" nor "the general welfare" can be wholly measured in terms of dollars. For this reason special consideration will be given to the application of economy studies to public activities in a separate chapter.

Major and Subordinate Objectives

Any enterprise has a major objective and subordinate objectives. For example, the ultimate aim of a contractor may be to earn a profit through activities associated with earth moving, such as digging pipeline trenches, excavating for buildings, or building earthen dams. Let earning a profit by digging pipeline trenches be considered to be a major objective designated by A. Objective A is an abstract aim that cannot be realized except by digging pipeline trenches in definite ways in definite places. An objective, a_1, subordinate to A, might be to earn a profit by digging a pipeline trench from Welltown to Pipetown via a certain route and in accordance with certain specifications. Realization of Objective A depends upon the realization of objectives a_1, a_2, a_3, and so on. Also each subordinate objective, as for instance a_1, has objectives subordinate to it. And each of these subordinate objectives has its subordinate objectives and so on until the final subordinate objective involves activities of very narrow scope.

An economy study may concern itself with the evaluation of contemplated activities associated with any level of objective. In one case a study may be made to decide whether or not a job involving the digging of 100 miles of trench should be undertaken. In another case the thing that is to be decided is whether it will be more profitable to dig a short length of trench where rocks are encountered with a machine or by men with shovels. How the next step may be performed with economy is a concern in any activity performed for a purpose.

The realization of any major objective is a summation of the realization of all its subordinate objectives.

Measures of Success

Any activity that is undertaken requires an input of thought, effort, force, material, and so forth for its performance. In a purposeful activity, an input of some value is surrendered in the hope of securing an output of greater value. The terms input and output as used here have the same meaning as when they are used to designate, for instance, the number of heat units that are supplied to an engine and the number of energy units it contributes for a defined purpose.

Success of a venture in terms of economy is determined by considering the relationship between the input and output of the venture. The profit of a venture is the difference between output and input. If the output consists of goods that are sold for dollars and input consists of materials and services bought for dollars, the resulting profit will be in terms of dollars and highly quantitative. The profit of a venture A is the summation of the profits of its subordinate parts a_1, a_2, and a_3. If P_A represents the profit of the major venture and p_{a_1}, p_{a_2}, and p_{a_3} represent the profits of the subordinate ventures, then

$$P_A = p_{a_1} + p_{a_2} + p_{a_3}$$

Let inputs and outputs of the above major and subordinate ventures be designated respectively by I_A, i_{a_1}, i_{a_2}, and i_{a_3} and by O_A, o_{a_1}, o_{a_2}, and o_{a_3}. Then

$$\begin{aligned} P_A &= O_A - I_A \\ &= (o_{a_1} - i_{a_1}) + (o_{a_2} - i_{a_2}) + (o_{a_3} - i_{a_3}) \end{aligned}$$

and also $$P_A = (o_{a_1} + o_{a_2} + o_{a_3}) - (i_{a_1} + i_{a_2} + i_{a_3})$$

As a measure, absolute profit, as expressed by $P_A = O_A - I_A$ and $p_{a_1} = o_{a_1} - i_{a_1}$, for example, has the advantage that the profit of major objectives may be obtained as a summation of the profits of subordinate objectives. It is frequently used.

Absolute profit as a measure has the disadvantage that it does not reveal the magnitude of the venture that resulted in the profit. In this connection, consider the following analysis embracing ventures *A* and *B*.

Venture	*Output*	*Input*	*Profit*
A	\$1,100	\$1,000	\$100
B	150	50	100

Measured in terms of absolute profit, ventures A and B are equal.

Another widely used measure of success is the ratio of output to input. This ratio will be recognized by engineers as an expression of efficiency. Following the scheme of symbols used above, the ratios of outputs to inputs may be expressed by R_A, r_{a_1}, r_{a_2}, and r_{a_3}. Then

$$R_A = \frac{O_A}{I_A}$$

and $$r_{a_1} = \frac{o_{a_1}}{i_{a_1}}, \text{ and so forth}$$

Also let R_B represent the ratio of output O_B and the input I_B of a venture *B*. Then the success of ventures *A* and *B* in the concrete example above may be stated as follows.

Venture	*Output*	*Input*	*Ratio R Output over Input*
A	\$1,100	\$1,000	1.1
B	150	50	3.0

Considering R as a measure of success, venture B is nearly three times as successful as venture A. The significance of the ratio of output over input is perhaps more clearly seen if it is considered to be the number of units of output per unit of input. Resources available for use in making a gain are usually limited. Thus it is desirable that each unit of resource be employed in ventures that will result in the greatest output per unit of input.

The ratio of output to input is a measure particularly useful for evaluating the comparative success of two or more ventures, a type of evaluation that is very important in engineering economy studies.

A disadvantage of output-over-input ratios as a measure is that the ratios that apply to subordinate parts of a venture cannot be added to determine the ratio that applies to a venture as a whole. If O_A, o_{a_1}, o_{a_2}, o_{a_3}, I_A, i_{a_1}, i_{a_2}, and i_{a_3} equal respectively \$260, \$80, \$60, \$120, \$200, \$60, \$40 and \$100, then

$$R_A = \frac{\$260}{\$200} = 1.30$$

$$r_{a_1} = \frac{\$80}{\$60} = 1.33$$

$$r_{a_2} = \frac{\$60}{\$40} = 1.50$$

$$r_{a_3} = \frac{\$120}{\$100} = 1.20$$

R_A is of course not equal to $r_{a_1} + r_{a_2} + r_{a_3}$.

In situations where it is useful to be aware of both the efficiency and the magnitude of the subordinate ventures that constitute a major venture, the following expression is applicable:

$$R_A = \frac{1}{I_A}\left[i_{a_1}r_{a_1} + i_{a_2}r_{a_2} + i_{a_3}r_{a_3}\right]$$

Input

Input will be considered as it applies to private commercial enterprises and government, and to the subordinate divisions and activities of each. Nearly all applications of engineering economy are found in such organizations.

The input of even a reasonably large commercial organization embraces a wide variety of items. A number of input items common to many organizations are presented in the following paragraphs.

Human service. A most important item of input is services of people for which wages are paid. In a commercial organization the total input of human services, as measured by the cost for a given period of time, is ordinarily reflected accurately by the amount of wage payments. Where involuntary service is secured, as in military services of governments, it will be clear that the input of human services is not necessarily reflected by amounts paid out as wages. In a commercial organization the input of human services may be classified under the following more or less well-defined headings:

Direct Labor	Management
Indirect Labor	Promotional Effort
Supervision	Investigation and Research

Of these, direct labor is the only human service input item whose amount is accurately known and whose identity is preserved until it becomes a part of output.

Input devoted to investigation and research is particularly hard to relate to particular units of output. Much research is conducted with no particular specified goal in mind and much of it results in no appreciable benefit that can be associated with a particular output. Expenditures for people for investigation and research may be made for some period of time before this type of service has a concrete effect upon output. Successful research of the past may continue to affect output for a long time in the future. The fact that expenditures for this type of service do not parallel in time the benefits provided makes it very difficult to relate them to an organization's output.

Expenditures for promotional effort have many of the characteristics that pertain to expenditures for investigation and research in the matter of relationship between time of input and effect on output.

Indirect labor, supervision, and management have characteristics falling between those of direct labor and investigation and research. The input of indirect labor and, to a lesser extent, that of supervision, parallel the output fairly closely in time, but their effects can ordinarily be identified only with broad classes of output items.

Management is associated with the operations of an organization as a whole. Its important function of seeking out desirable opportunities is similar in character to research. Input in the form of management effort is difficult to associate with output either in relation to time or to classes of product.

As difficult as it may be to associate certain inputs with a final measurable output, such as of products sold on the market, input can ordinarily be identified closely with intermediate ends that may or may not be measurable in concrete terms. For example, the cost of the input of human effort assigned to the safety engineering department, the legal department, the labor relations department, and the engineering department are reflected with high degree of accuracy by the payrolls of each. However, the worth of the output of the personnel assigned to these departments may and usually does defy even reasonably accurate measurement.

Where quantitative data on human service input is needed for economy studies, it is often necessary to use estimates, or, in other words, to evaluate input by judgment.

Material items. Many material items are acquired to attain the ends of both commercial and governmental enterprises. These may be classified for discussion under the following headings:

Direct Materials and Power	Equipment
Indirect Materials and Power	Land and Buildings

Inputs of direct material and power are directly allocated to final and measurable outputs. The measure of material items of input is their purchase price plus costs for purchasing, storage, and the like. This class of input is subject to reasonably accurate measurement and may be quite definitely related to final output, which in the case of commercial organizations is easily measurable.

Indirect material and power inputs are measurable in much the same way and with essentially the same accuracy as are direct material and power inputs. One of the important functions of accounting is to allocate this class of input in concrete terms to items of output or classes of output. On the whole, this may ordinarily be done with reasonable accuracy.

An input in the form of an item of equipment requires that an immediate expenditure be made, but its contribution to output takes place piecemeal over a period of time in the future which may vary from a short time to many years, depending upon the use life of the equipment. Inputs of equipment are accurately measurable and can often be accurately allocated to definite output items except in amount. The latter limitation is imposed by the fact that the number and kinds of output to which any equipment may contribute are often not known until years after many units of the product have been distributed. The function of depreciation procedures is to allocate equipment inputs to outputs.

Inputs of land and buildings are treated in essentially the same manner as inputs of equipment. They are somewhat more difficult to allocate to output because of their longer life and because a single item, such as a building, may contribute simultaneously to a great many output items. Allocation is made with the aid of depreciation and cost accounting techniques and practices.

Allocation of inputs of indirect materials, equipment, buildings, and land rests finally upon estimates or judgments. Although this fact is often obscured by the complexities and the necessary reliance upon accounting practices for day-to-day operations, it should not be lost sight of when economy studies are to be made.

Interest. Capital in the form of money is a very necessary input, although it must ordinarily be exchanged for producer goods in order for it to make a contribution to output. Interest on money used is usually considered to be a cost of production and so may be considered to be an input. Its allocation to output will necessarily be related to the allocation of human effort, services, material, and equipment in which money has been invested.

Pattern of input. Almost all activities require inputs of human service, physical equipment, and capital. Where these three classes of input are present, an analysis of input for an economy study may usually be summarized as follows:

INPUT ITEMS

Depreciation
- Equipment
- Buildings
- Land

Interest
- On investments in depreciable items

Other items
- Direct labor
- Direct materials and services
- Indirect labor
- Indirect material and services
- Interest on funds in nondepreciable items
- Taxes on property and service
- Taxes on profit

The first two main items, depreciation and interest, are the familiar items of capital recovery with return. In economy analyses the estimates of the amounts of these two items of input will ordinarily be based upon the techniques that have been developed to cope with depreciation.

Direct labor and direct material as given in the above classification are accurately known in cost and destination and therefore are matters of fact not requiring estimates.

The costs of inputs of indirect labor and indirect material are ordinarily accurately known, but the amounts of each that contribute to a specific output under consideration in an economy study are a matter of estimate. Accounting schemes for allocating these items are but orderly procedures based upon estimates. They are fairly accurate for input and output as wholes but may be grossly in error for specific items of output.

Output

The outputs of commercial organizations and governmental agencies are endless in variety. Commercial outputs are differentiated from governmental outputs by the fact that it is usually possible to evaluate the former accurately but not the latter. A commercial organization offers its products to the public. Each item of output is evaluated by its purchaser at the point of exchange. Thus the monetary values of past and present outputs of commercial concerns are accurately known item by item.

Since engineering economy is concerned with future economy, it is concerned with future output. Generally, information on two subjects is needed to come to a sound conclusion. One of these is the physical output that may be expected from a certain input. This is a matter for engineering analysis. The second is a measure of output that may be expressed in terms of monetary income.

Monetary income is dependent upon two factors: one is the volume of output, in other words the amount that will be sold; the other is the monetary value of the output per unit. The determination of each of these items for the future must of necessity be based upon estimates. Market surveys and similar techniques are widely used for estimating the future output of commercial concerns.

Internal intermediate outputs of commercial organizations are determined with great difficulty and are usually estimated as dictated by judgment. The value of an individual employee to a final output is rarely known with reasonable accuracy to either himself or his superior. Similarly, it is difficult to determine the value of the contributions of most intermediate activities to the final result.

The outputs of many governmental activities are distributed without regard to the amount of taxes paid by the recipient. When there is no evaluation at the point of exchange, it appears utterly impossible to evaluate many governmental activities. John Doe may recognize the desirability of the national military establishment, the U. S. Forest Service, or Public Health activities, but he will find it impossible to demonstrate their worth in monetary terms. However, some government outputs, particularly those which are localized, such as highways, drainage, irrigation, and power projects, may be fairly accurately evaluated in monetary terms by calculating the reduction in cost or the increase in income they result in for the user.

Choice Is between Alternatives

Except in extreme situations, there are usually several if not many things that are within the powers of an individual or an organization to do. But each choice imposes limitations of resources, time, and place. Thus, though it might be within the power of an individual to go to Place A and Place B, he cannot be at both places simultaneously. Also he may have the resources, time, talent, and desire to carry Activity A and Activity B to successful conclusions. But he may find that he cannot do both and that he is forced to make a choice between them.

Courses of action between which choice is contemplated are conveniently called alternatives. Both different ends and different methods are embraced by the term alternative. All proposed alternatives are not necessarily attainable. Alternatives are frequently proposed for analysis even though there seems to be little likelihood that they will prove feasible. This is done on the thought that it is better to consider many unprofitable alternatives than to overlook one that is profitable. Alternatives that are not considered cannot be adopted no matter how desirable they might prove to be. Consider the contractor who is trying to evaluate the comparative merit of Method A and Method B when Method C, of which he is not aware as a possibility, is far superior to either of the other methods.

An ever-present alternative of importance is to take no action at a given time. Thus, when Method A and Method B are being considered, choice lies between the adoption of Method A, the adoption of Method B, and the alternative of making no decision.

Conception of alternatives is a creative process. A complete, well-rounded, practical alternative rarely emerges full blown on consciousness. It begins rather as a hazy but interesting idea; only after analysis and synthesis is it formed into a definite proposal. In its final form an alternative should consist of a complete description of its objectives and the requirements in terms of outputs and inputs. Since this can rarely be done completely, it is often necessary to substitute estimates for knowledge of facts.

Success Is Comparative

The activities that may be undertaken by an individual or an organization are limited only by the natural limitations on man's ability to think of things to do. But the resources of both individuals and enterprises are limited. This gives rise to the desire to procure the greatest output for a given input or, in engineering terms, to operate at high efficiency. Thus the search is not merely for a fair, passable, or good opportunity for the employment of the limited resources, but for the best opportunity. The burden of much knowledge and much of man's conscious thought is to apply available resources to the best advantage. The criteria for judging the desirability of an alternative is its results in comparison with the results of other alternatives that may be undertaken.

All Alternatives Cannot Be Considered

While an objective of economy is to find the best opportunity, this objective can rarely be realized; to realize it completely would require that all possible alternatives of a situation be delineated for comparison with each other.

Alternatives are not outlined and evaluated without cost and the passage of time.

The attention of a superintendent was directed by a friend to a loss of heat from a bare pipe in his plant. The superintendent calculated the heat loss and found it to amount to $260 per month. A covering that would reduce the heat loss by $240 per month was offered by a salesman for $680. The superintendent thought he could secure the needed covering for less and actually accepted a bid for

$632 three weeks later. Thus the elapse of time in seeking a better alternative had resulted in a saving of $48 on the covering and a heat loss of $180 that could have been prevented by accepting the first offer for the covering.

Similarly the cost of considering alternatives will ordinarily force a choice before all are considered. Suppose that a concern was seeking a location for a plant in which to employ 20 women workers in manufacturing a line of leather novelties. Such a plant could be located in at least a thousand communities in the United States. Obviously the cost of investigating all the possibilities would be prohibitive. Further, suppose that the annual net income of the plant would be $10,000 at the first site considered. Suppose also that an annual income of $1,000 per year in the future is considered to justify a present investment of $5,000,[2] and that the cost of evaluating a plant site is $1,600. Thus each investigation of a site after the first should result in an increase in income of

$$\$1{,}000 \times \frac{\$1{,}600}{\$5{,}000} = \$320$$

on the average for each site investigated. If the potential annual income of the first site considered was $10,000, the annual income of the site selected to justify the cost of investigation of a number of sites would be as follows:

Number of Sites Investigated	1	2	3	4	6	10	20
Annual Income to Justify Investigation	$10,000	$10,320	$10,640	$10,960	$11,600	$12,880	$16,080

Ordinarily too few alternatives are probably considered, but the above analysis shows that there is a limit beyond which considerations cannot be justified. The number of alternatives to consider is in itself a matter of economy worthy of study.

It being recognized that all alternatives cannot be considered, the number of alternatives to be considered is limited on the basis of general assumptions. For instance, a fireman works on the assumption that it is his duty to minimize property loss by fire. When an alarm is sounded, his alternatives are limited to the methods by which

[2] This corresponds to a capital-recovery period of 10 years at 15 per cent.

$\$5{,}000 \times (\overset{\text{RP 15-10}}{.19925}) = \$996.25.$

he may minimize property loss by extinguishing the fire. The assumption under which he works precludes the necessity for him, for instance, to consider the alternative of letting a particular area be destroyed because mankind might be better served thereby.

Alternatives may be limited in a progressive series of assumptions. If the limiting assumptions are sound, no desirable alternatives will have been excluded. Skill and knowledge that will result in judgment that will enable consideration of none but desirable alternatives without excluding any that are desirable is an important function of intellect. Consider the following illustration:

> Suppose the problem to be to select a dam site for a storage reservoir. For simplicity, it will be assumed that the desideratum is to secure a specified volume of water storage at minimum cost, and that water storage above the specified amount will be of no value. Usually the real problem is not so simple. The cost can be estimated for each point along the river of building a dam with the required storage capacity. However, to make an accurate estimate, detailed studies would have to be made of the foundation conditions of each point. Then this huge array of cost estimates could be compiled and the dam site with the least cost selected.
>
> Actually, the engineer proceeds quite differently. By inspection of a topographic map, he immediately picks out a half-dozen "plausible" dam sites and forgets about the rest. He is sufficiently familiar with dam construction cost to know with fair degree of certainty that any other site he might choose would have a higher construction cost. Next, he makes an approximate estimate of dam costs for each of the plausible sites, assuming "normal" foundation conditions. Finally, he selects the most promising sites and makes careful foundation studies as a basis for final estimates.
>
> At each step in this process there is a chance that the dam site which really is most desirable will be eliminated without complete analysis.[3]

The degree of approximation that is allowable at each point in procedures similar to the one illustrated depends upon skill or judgment.

Individuals and organizations frequently limit the number of alternatives to be considered by assuming that the best opportunities for them lie within areas of activity. Thus a manufacturer of farm tractors would ordinarily be expected to reject with but cursory consideration an opportunity to manufacture cosmetics or to engage in the distribution of motion pictures.

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[3] Herbert A. Simon, *Administrative Behavior* (New York: The Macmillan Company, 1948), p. 99.

A Pattern for Engineering Economy Studies

The functions of engineering economy were stated in Chapter 1 to be:

Determination of objectives
Determination of strategic factors and means
Evaluation of engineering alternatives
Interpretation of the economic significance of engineering proposals
Assistance in decision-making

The aim of the next paragraphs will be to describe a series of steps to serve as a guide in making economy studies that will embrace the several functions.

1. Creative Step

(a) Seek opportunities not now known for profitable employment of resources.
(b) Seek means for circumventing factors that limit success of present activities.

2. Definitive Step

(a) Define alternatives to be compared.
(b) Enumerate prospective quantitative and qualitative outputs and inputs of each alternative.

3. Conversion Step

(a) Convert prospective outputs and inputs of each alternative to monetary receipts and disbursements at specified dates to the extent that this is possible.
(b) Place monetary receipts and disbursements of each alternative upon a basis that is comparable when time value of money is considered.
(c) Enumerate remaining irreducible outputs and inputs of each alternative.

4. Decision Step

(a) Weigh and compare quantitative and qualitative output and input of each alternative.
(b) Decide what action to take.

The Creative Step

Engineers, whether engaged in physical research or in general administrative activities, are concerned with the profitable employment of resources. When known opportunities fail to hold sufficient promise for the profitable employment of resources, more promising opportunities are sought. People with vision are those who accept the premise that better opportunities exist than are known to them. This view accompanied by initiative leads to exploratory activities aimed at finding the better opportunities. Exploration, research, investigation, and similar activities are creative. In such activities steps are taken into the unknown to find new possibilities, which may then be evaluated to determine if they are superior to those that are known.

Opportunities are not made; they are discovered. The possibility of flight in heavier-than-air machines, for example, has always existed and merely awaited discovery.

The universe may be considered to consist of a summation of factors, or, in other words, a summation of all that exists. If individual facts are designated by such symbols as a_1, a_2, and a_3, then the following representation applies:

$$\text{The Universe} = a_1 + a_2 + a_3 \ldots\ldots + a_n$$

Some of the facts of the universe are known. Most are unknown. Any situation embraces groups of facts of which some may be known and others unknown. The material out of which new opportunities for profit are to be fashioned are the facts as they exist.

Search for facts and new combinations of facts. Many successful ideas are merely new combinations of commonly known facts. The highly successful confection widely sold under many names and best described as a piece of chocolate-covered ice cream on a stick is the result of combining several simple known facts or ideas. The principal constituents are a stick, ice cream, and chocolate. It was known that people liked ice cream in conjunction with chocolate. Chocolate coverings for candies had been known for years. The stick had been previously used with candy in the "all-day sucker." Nevertheless, the exploiters of the resulting new combination of these ideas are reported to have made a fortune.

Some successful ideas are dependent upon the discovery of new facts. New facts may become known through research effort or by accident. Research is effort consciously directed to the learning of new facts. In pure research, facts are sought without regard for their specific usefulness, on the premise that a stockpile of knowledge will in some way contribute to man's welfare. Much of man's progress rests without doubt upon facts discovered from efforts to satisfy curiosity.

Both new facts and new combinations of facts may be consciously sought after. The creative aspect of engineering economy consists in finding new facts and new combinations of facts out of which may be fashioned opportunities to provide profitable service through the application of engineering.

Aside from the often quoted statement that "inspiration is ninety per cent perspiration," there are few guides to creativeness. It appears that both conscious application and inspiration may contribute to creativeness. Some people seem to be endowed with marked aptitudes for conceiving new and unusual ideas.

It may be presumed that a knowledge of facts in a field is a necessity for creativeness in that field. Thus, for example, it appears that a person who is proficient in the science of combustion and machine design is more likely to contrive a superior internal-combustion engine than a person who has little or no such knowledge. Also it appears that knowledge of costs and people's desires as well as of engineering is necessary to conceive of opportunities for profit that involve engineering.

Origin of parking meters. For an illustration of creative thinking in engineering economy consider the following example.

During 1931 a committee was appointed to work out ways and means for alleviating an existing parking situation in the business section of a midwestern city. Business people in the downtown area were complaining that their businesses were suffering because customers rarely could find space to park in the streets adjacent to the business houses. In spite of a one-hour-parking ordinance, surveys revealed that about eighty per cent of the parking space on streets was occupied by automobiles that remained parked for from four to eight hours. The solution seemed simple enough. Merely require the police to enforce the parking ordinance. But repeated campaigns to "tighten up" on parking had been neither effective nor popular. It was broadly hinted that most of those who were overparking were also those who would benefit by enforcement of the parking ordinance.

Hearings revealed that the businessmen favored enforcement but felt privileged to circumvent the ordinance as long as others were doing the same thing. The committee worked earnestly on the problem but developed no satisfactory solution.

Several weeks after the committee had ceased its deliberations, one of its members[4] was driving his car when the idea of a parking meter came to him, as he put it, "out of a clear sky." After he had elaborated on the basic idea, he proceeded to evaluate it.

Based on his observance of coin-operated machines for vending candy, cigarettes, stamps, and so forth, he concluded that a coin-operated signal device was feasible from the standpoint of engineering. Further consideration of the engineering aspect was therefore deferred.

Estimates of the income from such a device considered in relation to its "guessed-at cost" satisfied him that parking meters could be financially successful if their use were permitted.

He realized that if the collection of a fee through the use of a mechanical device stationed on the street as a means for regulating parking of automobiles were contrary to the constitution of the state or nation, the idea of a parking meter would have no profit possibilities. Regardless of how overwhelmingly the citizens of a city expressed a desire for an installation of parking meters, a single person could have their use banned by instituting a suit questioning their constitutionality. Thus success of the parking meters hinged on their legality. Accordingly the legality of the use of parking meters was investigated. To the extent that the matter could be decided without a test case it was the considered opinion of competent counsel that the use of parking meters would be legal.

With this assurance, the originator of the parking-meter idea turned his attention to the development of a suitable mechanism, the securing of patent protection, and the formation of a company to produce and distribute parking meters. This effort culminated in the installation of parking meters in Oklahoma City in June 1935. A decade later this means of controlling parking had become commonplace throughout the United States and in many foreign countries. The manufacture and distribution of parking meters is now an activity of considerable magnitude.

The above example shows the result of creative thinking in seeking

[4] Carl C. Magee, the first to develop and install parking meters.

out an opportunity for the profitable employment of resources involving engineering in a new field.

The idea of parking meters originated as a result of consideration of a problem that pointed to a need for a better way for controlling parking. The thoughts that led to the synthesis of the parking-meter idea probably cannot be traced, but it is quite likely that they involved two concepts. One of these is the concept of identification. Identification of overparked cars is the purpose of the parking meter. The second concept is related to persuasion. A police officer finds it advantageous to maintain friendly relations with people in his patrol area and is subject to much persuasive effort to grant privileges. The opportunity for persuasion is considerably minimized with parking meters, which identify violators and are ordinarily policed by special officers on a city-wide basis.

It is interesting to note that though the parking meter involves much careful engineering in design, manufacture, and distribution its legality was the factor of first concern. The income that might be derived from it was next considered; engineering feasibility was the third concern.

Faith backed by investment. As early as 1893 geologists of the Canadian Geological Survey investigated iron ore bodies in Labrador. From then until 1929, when an important discovery was made by a geologist of the Canadian Geological Survey, the vast area was studied sporadically. In 1937 an Indian brought a sample of rich iron ore to the geologist of a concern exploring the area for gold. One handicap to getting funds for exploring for iron ore bodies in this area was the realization that sufficient proved deposits would have to be found to justify an investment in the neighborhood of $100,000,000 for railroad facilities to bring it out. In 1942 a mining group became interested, but they found that an adequate exploration would cost several millions of dollars. The decision to spend this sum had to be on faith; for no matter how excellently exploration was done, it would have to be written off as a loss if deposits sufficient to justify facilities for bringing it out were not found. Decision was made to carry on exploration as rapidly as possible, in order to keep expenses at a minimum. The goal was to locate 300,000,000 tons of ore lying within 250 feet of the surface and having an iron content of 55 per cent. The attainment of this goal considered necessary to justify railroad facilities came in 1948.

In this example success was dependent upon a group of people having control of and a willingness to chance millions of dollars to learn if an opportunity for profit existed. A leader of the group is reported to have said, "Anybody who ever saw the country knew about the ore. What wasn't known was that people could be found who would find it fun to chance millions of dollars on faith. Such people are harder to find than iron." The faith that prompted support of the Canadian Geological Survey for a half a century was also a factor in success.

There appear to be many situations in which there must be great faith and the willingness and ability to make large investments of time, effort, and money in the hope of success.

Circumventing factors limiting success. So far discussion of the creative step has been confined to consideration of seeking new opportunities for profitable employment of resources. Attention will now be turned to a search for means for circumventing factors that limit success of present activities. Though this is not particularly different in character from the activity first discussed in the creative step, separate consideration is justified because of the more objective approach that is possible.

Where the aim is to improve the success of a given activity, search may be directed to means for circumventing factors that limit success. Factors that can be circumvented are known as strategic factors. Known strategic factors and those that seem most likely to be strategic factors should be selected for consideration.

The understanding that results from the delineation of limiting factors and their further consideration to arrive at the strategic factors often stimulates the conception of improvements.

There is obviously no point in operating upon some factors. Consider for example a situation in which an operator is hampered because he had difficulty in loading his machine with a heavy piece. Three factors are involved: the pull of gravity, the mass of the piece, and the strength of the man. Not much success would be expected from an attempt to lessen the pull of gravity. Nor is it likely that it is feasible to reduce the mass of the piece. A stronger man might be secured, but it seems more logical to consider overcoming the need for strength by devices to supplement the strength of the man. A consideration of the strength factors would thus lead to consideration of devices that might circumvent the limiting factor of strength.

It is possible that consideration of the factor of the piece's weight could have resulted in a reduction in weight by substitution of material, lighter sections, or a similar change. In this connection it may be noted that weight of pipe was considered a strategic factor in the cost of petroleum pipelines. This factor was circumvented by the use of high-strength steels made possible by the development of new welding processes.

Solutions result from search for limiting factors. In the drilling of oil wells with a rotary drill, a drill bit is rotated at the bottom of the hole by heavy members known as drill pipe joined end to end with screw joints. When it becomes necessary to replace the drill bit, successive joints of drill pipe are raised and unscrewed until the drill bit is reached. One alert person considered the joining method a strategic factor in drilling cost, for each length of drill pipe had to be turned a number of times in making and unmaking joints. After some consideration he devised the taper thread joint, which has saved millions of dollars in drilling oil wells. This thread becomes disengaged in a few turns, has all the advantages of straight threads and some in addition, and costs about the same as straight thread.

The creative step has been given much emphasis in the discussion above for it is believed to be of first importance in economy studies. It is directly related to the delineation and selection of objectives that are without doubt the most important functions of engineering economy and certainly the first steps toward success in any field of endeavor. Since the mental processes involved are in large measure nonlogical, this step must be approached with considerable alertness and curiosity and a willingness to consider new ideas and unconventional patterns of thought.

The Definitive Step

The definitive step consists of defining the alternatives that have originated in the creative step or which have been selected for comparison in some other way. In the first stage of the definitive step, the engineer's aim should be to delineate each alternative on the basis of its major and subordinate physical units and activities. The purpose of this stage is to insure that all factors of each alternative and no others will be considered in evaluating it.

The second stage of the definitive step consists of enumerating the prospective items of output and input of each alternative, in quantitative physical terms as far as possible and then in qualitative terms.[5] Though qualitative items cannot be expressed numerically they may often be of major importance. They should be listed carefully so that they may be considered in the final evaluation.

On completion of the definitive step there should be an enumeration giving all items of input and output and the time of their occurrence for each alternative.

The Conversion Step

In order that alternatives may be compared effectively, it is necessary that they be converted to a common measure. The common denominator usually chosen for economic comparison is value expressed in terms of money.

The first phase of the conversion step is to convert the prospective output and input items enumerated in the definitive step into receipts and disbursements at specified dates. This phase consists essentially of appraising the unit value of each item of output or input and determining their total amounts by computation. On completion, each alternative should be expressed in terms of definite amounts of receipts and disbursements occurring at specified dates in the future plus an enumeration of qualitative considerations that it has been impossible to reduce to money terms. For such items Grant[6] has used the term "irreducibles." This term seems to be superior to the term "intangibles" in this connection for many items described in qualitative terms are clearly known even though not expressible in numbers. The second phase of the conversion step consists of placing the receipts and disbursements of all alternative on a comparable basis, considering the time value of money. This involves calculation of equivalence. The interest rate to be used should have been determined as a part of the definitive step, where the use of funds to carry on a venture would be considered to be an input. Where money is an input item, it seems most logical to use cost of money for the interest rate in the calculations of equivalence. However, there are many who use the interest rate as a vehicle for expressing risk. This practice will be discussed at some length later. The

[5] For examples of quantitative and qualitative statements see page 137.

[6] Eugene L. Grant, *Principles of Engineering Economy* (New York: The Ronald Press Company, Inc., 1938), p. 21.

interest rate received from output items should be that actually received. Where calculations of equivalence involve depreciation, the method used should be based on the pattern of depreciation set forth in the definitive step.

A final phase in the conversion step is to enumerate all irreducibles that remain so that they may be considered in the final decision step.

Economic significance of engineering proposals. Control over the acceptance or rejection of an engineering proposal is exercised more often than not by persons who have not been concerned with the technical phases of the proposal. Also the persons who control acceptance are likely to lack understanding of technical matters.

A proposal should be explained in terms that will best interpret its significance to those who will control its acceptance. The aim of a presentation should be to take persons concerned with a proposal on an excursion into the future to experience what will happen if the proposal is accepted or rejected.

Suppose that a proposal for a new factory layout is to be presented. Since those who must decide if it should be adopted rarely have the time and background to go into and appreciate all the technical details involved, the significance of these details in terms of economic results must be made clear. Of interest to those in a position to decide will be such things as the present outlay required, capital-recovery period, flexibility of the layout in event of market changes, effect upon cost and quality of product, how employees will be affected, and difficulties of financing. Cost and other data should be broken down and presented so that attention may be easily focused upon pertinent aspects of the proposal. Diagrams, graphs, pictures, and even solid models should be used where these devices will contribute to understanding. The effort expended in developing a sound proposal is often lost through its rejection because of poor presentation.

The aim of economy studies is sound decision. Thus a first consideration should be to present a proposal in terms that will help those who must decide to understand the implications of the proposal to the fullest extent possible.

The Decision Step

In addition to the alternatives formally set up for evaluation, another alternative is always present. This is the alternative of making no decision on the formal alternative being considered. The decision not to decide may be a result of either active consideration or passive failure to act; it is usually motivated by the thought that there will be opportunities in the future which will prove more profitable than any known at present.

For example, if a venture to build a plant to process and sell fruit is under consideration, there frequently is only one formal alternative to consider. Those concerned will usually decide the issue on how favorably the prospective venture measures up to those generally open for the employment of resources. Thus the decision not to decide is clearly a decision based on a comparison with future, though perhaps unknown, alternatives.

Differences are bases for decision. On completion of the conversion step, quantitative and qualitative outputs and inputs of each alternative, so far as these are known, will form the basis for comparison and decision. Quantitative input may be deducted from quantitative output to obtain quantitative profit, or the ratio of quantitative profit to quantitative input may be found. Each of these measures is then supplemented by what qualitative consideration may have been enumerated.

Decision between alternatives should be made on the basis of their differences. Thus all identical factors can be canceled out for the comparison of any two or more alternatives at any step in an economy study. In this process great care must be exercised that factors canceled as being identical are actually of the same significance. Unless it is very clear that factors considered for cancellation are identical, it is best to carry them through the first stage of the decision step. This may entail a greater amount of computation and other paper work, but the slight added complexity and loss of time is ordinarily insignificant in comparison with the value of greater accuracy in decision.

It frequently occurs that alternatives to provide an identical output or service are under consideration. Then output need not be considered and decision is made on the basis of input.

Where all facts about alternatives are known in accurate quantitative terms, the relative merit of each alternative may be expressed in terms of a single number. Decision in this case is simple. Where all factors are known, evaluation may be made on the basis of reason.

Decision must also be made when the quantitative considerations are based on estimates that are subject to error and where quantitative knowledge must be used to fill gaps in knowledge. It must always be remembered that the final calculated amounts embody the errors of the estimated quantities.

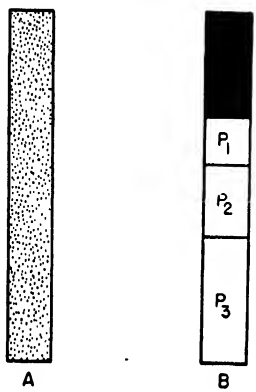
When facts are missing, judgment must be used. Where alternatives cannot be completely delineated in accurate quantitative terms, the choice is made on the basis of the judgment of one or more individuals. Judgment seeks to predict the outcome of a course of action. Few believe that any person is endowed with a sixth sense or intuition to enable him to peer into the future. Fortune tellers get rich on the "suckers" they serve rather than upon the soundness of their advice.

If judgment is not clairvoyant, the ability to predict the future rests entirely on a cause-and-effect relationship. The only basis for the prediction of the outcome of a course of action is the facts in existence at the time the prediction is made. A person's judgment rests, therefore, upon his knowledge of the facts involved and his ability to use these facts. When complete knowledge of all facts concerned and their relationships exists, reason can supplant judgment and predictions become a certainty. Judgment tends to be qualitative. Reason is both qualitative and quantitative. Judgment is at best an informal consideration and weighing of facts; at its worst it is merely wishful thinking. Judgment appears to be an informal process for considering information, past experience, and feeling in relation to a problem. In this connection, contrast aviators who fly "by the seat of their pants" with those who fly by instruments. No matter how sketchy factual knowledge of a situation may be, some sort of a conclusion can always be drawn in regard to it by judgment. This is an important advantage of judgment.

Since engineering economy is concerned with the future, there can never be complete factual knowledge about any proposed alternative. Judgment must always be exercised as a final step in all economy studies.

Figure as far as you can, then guess. An important purpose of economy study procedures is to marshal the facts so that reason may be used to the fullest extent in arriving at a decision. In this way judgment can be reserved for parts of situations where factual knowledge is absent. This idea is embraced in the statement "Figure as far as you can, then guess."

Where diligent search uncovers insufficient information to reason the outcome of a course of action, the problem is to render as accurate a decision as the lacking facts permit. In such situations there is a decided tendency, on the part of many, to make little logical use of the data that are available in coming to a conclusion, on the thought that, since some pretty rough estimating has to be done on some elements of the situation, the estimate might as well embrace the entire situation. But an alternative may usually be subdivided into parts, and the available data are often adequate for a complete or nearly complete evaluation of several of the parts (see Fig. 22). Thus the



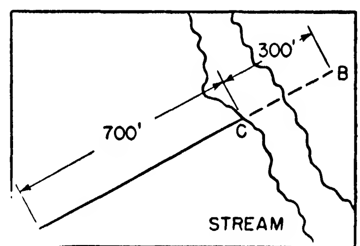


FIG. 22 (left). A schematic representation of an alternative before and after marshalling facts for effective use. (A) represents an alternative for which facts are available, if effectively used, to predict outcome with 70 per cent certainty. (B) represents the same alternative as in (A) after facts have been applied to parts P_1, P_2, and P_3, rendering 70 per cent of the whole predictable.

FIG. 23 (right). Judgment of a situation should be limited to those parts to which more accurate methods of determination cannot be applied.

outcome of a portion of the total situation may be predicted with considerable certainty, leaving a balance about which little is known.

In this connection consider the situation represented in Fig. 23, where the distance between *A* and *B* is to be determined by a man on side *A* and unable to cross the stream. The distance *A B* may be estimated in its entirety or *AC* may be measured and *C B* estimated. It is interesting to note that a 50 per cent error in an estimate of *CB* in Fig. 23 results in only a 15 per cent error in the conclusion, which is probably less than would have been the case if the distance *A B* had been estimated. Estimate or judgment should be reserved for those parts of the alternative for which a more accurate method of evaluation cannot be used.

The segregation of the known and unknown parts is in itself additional knowledge. Also, the unknown parts, when subdivided, frequently are recognized as being similar to parts previously encountered and thus become known.

QUESTIONS

1. Contrast the aim of private and public enterprise.

2. Show by an example of your selection that the attainment of a major objective is a summation of the attainment of subordinate objectives.

3. Explain two measures of success.

4. List items of input commonly encountered in commercial and industrial activities..

5. List items of disposable output of commercial and industrial organization.

6. As used in this text, what is the meaning of the term *alternative?*

7. (a) Give a concrete illustration of why all alternatives cannot be considered.

(b) What method is used to reduce the great bulk of possible alternatives to a number that is practical to consider?

8. Apply the 4 steps outlined as a pattern for an economy study to a concrete problem of your selection.

9. Describe the nature of the creative step in some detail.

10. (a) What is a strategic factor?

(b) Why should attention be directed toward strategic factors?

11. (a) Discuss the nature of judgment and contrast it with reason.

(b) Why is judgment applicable to more situations than reason?

12. Apply the "figure as far as you can, then guess" idea to a concrete situation of your choice.

CHAPTER

8

Treatment of Estimates in Economy Studies

Any decision relative to the future is fraught with uncertainty. As has been stated in poetic language, the plans of mice and men often go awry. The expression "nothing ventured, nothing gained" characterizes the motivating force behind undertaking a venture. The object of a great portion of man's conscious thinking is to seek out ventures with profit potentialities that are high in proportion to the risk involved. The logical bases for thinking about the future are cause-and-effect relationships as they apply to the future situation under consideration. The past may be examined with profit in the hope of uncovering cause-and-effect relationships. But care should be exercised to avoid the common error of thinking that events will take place in the future as they have in the past.

Recovery and Return on Investments

A person buys things to consume for the satisfaction he expects to get from them. This satisfaction is in terms of physical and mental experience and is the end of a transaction except as the satisfaction of the things consumed may be reproduced in memory.

Investments in goods not to be consumed for the mental or physical satisfactions to be obtained from them are made in the hope of deriving a profit. Something is ventured, that is, present satisfaction is foregone, in the hope that there may be greater satisfaction in the future. Thus an investment in an item of utility is hazarded in hope of recovering the amount originally invested plus a profit.

A portion of an investment in a physical asset is lost through physical or functional depreciation. If the loss resulting from depreciation is offset by an equal income from the use of the asset, that portion of the investment has been recovered. The actual undepreciated balance of the asset represented by its salvage value has not been lost and so need not be recovered. The salvage value of the asset is merely reconverted into another medium, usually money, by selling what remains of the asset.

In the recovery of an investment, attention is focused upon realizing an income to offset value lost through depreciation. And since depreciation is taken to be the difference between first cost and the amount realized from salvage, the prime problem in capital recovery is the securing of income equal to depreciation.

Return on Investments

After an investment has been recovered the next step in attaining capital recovery with a return is to secure a return or profit, usually thought of as interest on the sum invested. Profit is a prime objective in making an investment and the last to be realized, for until the investment has been recovered there has been no profit. Profit on investments is usually expressed as a rate of return, often spoken of as interest. The reason for expressing profits resulting from investments in terms of a rate is that a rate is more easily comprehended than a total. This is illustrated by an example. Mr. A invested $400 in 1944 and received $708.80 in return in 1950. His profit after recovery of his investment was $308.80. Mr. B. invested $1,100 in 1938 and received $2,213.20 in return in 1950. His profit after recovery of his investment was $1,113.20.

Which of these investments was the more desirable from the standpoint of the amount of investment and the duration of investment? The relative desirability of the two investments is not apparent from a statement of the profit derived from each. If the rates of return are calculated, it will be found that Mr. A received 10 per cent per year on his investment, whereas Mr. B received only 6 per cent per year.

Three Items Associated with Enterprise Involving Investments

It has been pointed out above that where the objective of making an investment is profit, consideration must be given to securing income sufficient to offset depreciation and a return on the sum invested in an asset. The income must also be sufficient to offset any expenses incurred as a result of the investment.

In most situations in which there is an investment in physical equipment there are expenses incident to operation of the equipment.

These embrace expenditures for such items as wages of labor, power, supplies, repairs, taxes, insurance, housing, heat, and light. The aggregate of expense for these items may be designated under the single general heading *operating expense.*

The three items to be provided for after an investment has been made in a physical asset are depreciation, return on investment, and operating expense.

These items can only be provided for out of income. If each undertaking is to be evaluated on its own merits, the income should be that derived from the undertaking for which the investment in the physical equipment was made. An investment in a venture is successful when

$$\text{INCOME} \left.\vphantom{\begin{matrix}a\\b\\c\end{matrix}}\right\} \text{EQUALS} \left\{\begin{matrix}\text{DEPRECIATION}\\ \text{RETURN ON INVESTMENT}\\ \text{OPERATING EXPENSE}\end{matrix}\right.$$

Operating expense is presumed to be current, that is, operations cannot be carried on without incurring operating expenses which, therefore, are the first to be met out of income.

Depreciation takes place currently with operations and the passage of time, but restitution need not be made until the physical equipment involved in the operation must be replaced.

A return on an investment is a resultant of income received and the sum of depreciation and operating expense.

Estimating the Total Outcome of a Decision

The outcome as a whole may be estimated. Thus, if the purchase of a steam shovel is under consideration, it might be directly estimated that the purchase of the steam shovel will result in an income equivalent to 8 per cent per year on the amount invested. This return will be a resultant of a number of prospective receipts and disbursements, which may be classified as income, depreciation, interest on invested funds, and operating expense. It is a rare intellect that can combine accurately four complex items to obtain their resultant without resort to paper and pencil, even when the items are clearly known. Thus for best results in estimating the final outcome of an undertaking, it will almost always be found advantageous to begin with detailed estimates of income and costs as they may be expected to originate in the future. The detailed estimates are then combined mathematically to obtain their results.

Prospective receipts and disbursements are a secondary result of prospective activities in a situation. Estimates of receipts and disbursements should therefore be based on the prospective activities. It follows that receipts and disbursements often arise from the same data. For example, if a steam shovel is used to excavate 10,000 cubic yards of earth, this activity may be expected to result in both income and expense. Thus an estimate that an asset, for example, a steam shovel, will be used to do certain work in the future is the basis for estimating prospective income, depreciation, interest on investment, and operating expense. It is important to recognize that prospective receipts and disbursements have a common origin in the prospective use of the asset under consideration.

Estimating Income

If there is a demand for a service, an income can be derived from supplying that demand. If the cost of supplying the service is less than the income received, a profit can be made. The first step toward a profit is an income. Thus, in estimating the desirability of a prospective undertaking, it seems logical to estimate income as a first step. As used here, income also embraces the possibility for making a saving. Estimates of income should be based upon objective information as far as possible.

Estimates of a result will usually be more accurate if they are based upon estimates of the factors having a bearing on the result than if the result is estimated directly. For example, in estimating the volume of a room, it will usually prove more accurate to estimate the several dimensions of the room and calculate the volume than to estimate the volume directly. Similar reasoning applies to estimates of economic factors in prospect.

If, for example, the problem under consideration is the saving that may result from replacing a hand operation by a machine operation, the amount of saving will depend upon the number of units processed in a given time and the saving per unit. The first step is to bring all possible information to bear upon estimating the number of units expected to be processed during each year of the future period to be considered. In this connection, use should be made of such information as the records of past sales, present sales trends, the product's relation to the building trades, general business activity, and anything else that may be useful in arriving at the most accurate estimate of future sales.

In estimating the savings per unit, all items of saving—direct labor, direct material, overhead items, storage, inspection, and any others—should be estimated separately and totaled in preference to estimating the total of the savings of the items directly. The total estimated saving is then determined as a product of the number of units processed and the saving per unit. If more than one product is to be processed on a machine, the savings for each product should be estimated as above and totaled.

Under some circumstances—for example, if the income is represented by the saving resulting from an improvement in a process for manufacturing a staple product made at a constant rate—an estimate of income is easily made. But estimating income for new products with reasonable accuracy may be very difficult. Extensive market surveys and even trial sales campaigns over experimental areas may be necessary to determine volume. When work is done on contract, as is the case with much construction work, for example, the necessity for estimating income is eliminated. Under these circumstances the income to be received is known in advance with certainty from the terms of the contract.

Estimating Depreciation

Depreciation of machines often parallels income. The period and extent of use of single-purpose machines are dependent upon the number of units to be processed, which will have been estimated in arriving at income. Where depreciation is dependent upon wear and tear, the extent of use will be the determining factor; experiences in the past with like or similar machines may be helpful. If weathering or other causative factors of depreciation associated with the passage of time are the determining factors in estimating depreciation, mortality tables may prove to be of aid.

A depreciation estimate should be considered to be a summation of four estimates covering (1) installed cost, (2) service life in years, (3) salvage value at the end of service life, and (4) pattern of depreciation during service life.

Little difficulty is ordinarily experienced in making a reasonable estimate of the installed cost of an asset. The pattern of depreciation is not ordinarily of great importance. This leaves estimates of service life and salvage value. Of these, the estimate of service life is most important and perhaps the most difficult to make. The longer the service life of an asset, the less is the significance of error in estimates of salvage value.

It is a common practice in industry to base depreciation on arbitrarily set length-of-service lives that are often much shorter than actual lives. This is usually done to express a policy of conservatism in regard to investment in equipment, particularly in industries where obsolescence is an important factor. As a basis of determining costs this practice seems unrealistic and unsound, for it assumes that an arbitrarily set payout period for a class of equipment is more accurate than an individual estimate of the life of a particular unit of equipment.

Estimating Interest

There are many viewpoints in regard to interest. A borrower may consider it as a rental he pays for the use of funds. He may also consider it to be an expense. The lender may consider it to be a profit received as the result of investing funds. An investor may also consider interest to be a vehicle for expressing risk. An investor holding this view will determine the interest rate he deems necessary to justify an investment on the basis of the risk of loss he believes to be involved.

In determining the desirability of an investment, it is believed to be sound to take the view that interest is an item of cost, along with depreciation and operating expense, at least during a part of the analysis. Profit is then the return in excess of that needed to offset depreciation, operating costs, and interest which is regarded to be a cost.

If interest is considered as an expense, the interest rate can be determined more or less objectively. A concern that borrows money for its operations may use the rate that it is paying for funds. A concern investing its own funds is justified in using the rate that it can receive for funds for purposes similar to the one under consideration. Considered as an item of expense, the interest rate for most industrial investments will be relatively small. Thus this view minimizes the effect of interest in estimates.

Additional points relative to the consideration of interest as an expense will be discussed later in this chapter.

Estimating Operating Expense

Operating expense, as the term is used here, is taken to embrace both direct and indirect costs pertinent to situations under consideration. Operating expense originates as expenditures for such items as fuel, water, electric current, materials, supplies, wages, taxes, insurance, and so forth.

Some of these items will be based on the same facts that determine income. For instance, the income derived from a welding machine will be based, at least in part, upon the number of hours that it will be used. The number of hours of use will also be a factor in estimating current consumption, repairs, and labor used. In general, operating expenses should be estimated item by item rather than as a whole.

The Basis of Comparison

The aim of estimates of income, depreciation, operating expense, and interest as an expense is, of course, to aid in arriving at a decision. Still considering interest as an expense, a venture will just break even when

$$\text{INCOME} \left.\right\} \text{EQUALS} \left\{ \begin{array}{l} \text{DEPRECIATION} \\ \text{INTEREST} \\ \text{OPERATING EXPENSE} \end{array} \right.$$

The above equation may be set up on the basis of either annual cost, present worth, or capitalized cost. For most industrial purposes an annual cost basis seems to be preferred and will be used in this discussion.

To place depreciation upon an annual basis any of the several methods of depreciation may be used. Selecting a method of depreciation amounts to making an estimate of the pattern in which an asset will actually depreciate.

The first cost of an asset may usually be known with a fair degree of accuracy prior to its purchase. Service life and salvage value at retirement must be estimated. Of these two, the former is usually the more difficult to estimate and nearly always of greater importance.

In calculating capital recovery with a return based on estimates, the estimated first cost, service life, and salvage value are treated as fact. Since capital recovery with a return for straight-line and sinking-fund depreciation are equivalent when an asset is retired at the end of the service life and at the salvage value for which it was calculated, either method may be used.

When studies are to be made for others, the method of depreciation selected should be the one that is most easily understood and most readily accepted by them. Where there are no considerations other than convenience, the sinking-fund method is preferable.

Estimates, Treatment of Estimates, and Decision

A situation will now be described to illustrate some aspects of estimating, treating estimated data, and arriving at a decision. This example will be used throughout the chapter.

The purchase of a machine for a certain operation, now performed in another manner, is considered likely to result in saving. It is known with almost absolute certainty that the machine will cost $1,000 installed.

Income estimate. From a study of available data and the result of judgment, it has been estimated that a total of 3,000 units of the product involved will be made during the next 6 years. The number to be made each year is not known; but, since it is believed that production will be fairly well distributed over the 6-year period, it is believed that the annual production should be taken as 500 units. A detailed consideration of materials used, time studies of the methods employed, wage rates, and the like has resulted in an estimated saving of $1.04 per unit, exclusive of the costs incident to the operation of the machine if the machine is used. Combining the estimated production and the estimated unit saving results in an estimated saving (income) of 500 × $1.04, or $520 per year.

Depreciation and interest estimate. The machine is a single-purpose machine and no use is seen for it except in processing the product under consideration. Its service life has been taken to be 6 years to coincide with the estimated production period of 6 years, although it is believed to have a wear life of 12 or more years. It is believed that the salvage value of the machine will be offset by the cost of removal at retirement. Thus the estimated net receipts at retirement will be zero. The pattern of depreciation is not known in detail and the sinking-fund method of depreciation is selected for its convenience.

Interest has been considered to be an expense and the rate of interest has been estimated at 5 per cent. The next step is to combine the estimates of first cost, service life, salvage value, and the patterns of depreciation and interest rate to determine the estimated annual capital recovery with a return for sinking-fund depreciation. For the first cost of $1,000, a service life of 6 years, a salvage value of zero, a sinking-fund pattern of depreciation and an interest rate of 5 per cent, the resultant estimate of annual depreciation and interest will be:

$$\text{Estimated Annual Capital Recovery and Return}$$
$$\text{(Depreciation and Interest)} = (\$1{,}000 - 0)\overset{\text{RP 5-6}}{(.19702)} + 0 \times .05$$
$$= \$197$$

Operation cost estimate. Operating costs will ordinarily be made up of several items such as fuel, maintenance, supplies, and labor. Consider for simplicity that the operating expense of the equipment in this example consists of items A, B, C, and D. Each of these items is estimated on the basis of the number of units of product that it is estimated are to be processed per year. Assume that these items have been estimated as follows:

Item A, $90 per year
Item B, $60 per year
Item C, $40 per year
Item D, $ 7 per year

Estimates of Items A, B, C, and D total $197 per year.

Estimates Are Most Objective Bases for Decision

The estimated income and cost items of the example may be summarized as follows:

Estimated Annual Capital Recovery and Return, $1,000(.19702) (RP 5-6)	$197
Estimated Annual Operating Cost, $90 + $60 + $40 + $7	197
Estimated Total Annual Cost	$394
Estimated Total Annual Income	520
Estimated Net Annual Profit for Venture	$126

The final statement means that the venture will result in a profit of \$126 per year for a period of 6 years if the several estimates prove to be accurate.

The resultant annual profit is itself an estimate, and experience teaches that the most certain characteristic of estimates is that they nearly always prove to be in error, sometimes in small degree and often in large degree. Once the best possible estimates have been made, however, whether they eventually prove to be good or bad, they remain the most objective basis on which to base decision. It should be realized that decision can never be an entirely objective process.

Conversion of Resultant Estimates to Other Bases

In the above example the resultant estimated annual profit may be converted by calculation into such other estimates as (1) present worth at some interest rate, (2) net saving per unit of product processed, (3) rate of return on investment, and (4) payout period for the equipment at some interest rate. These have been calculated as follows:

(1) Estimated present worth of profit = $\$126\overset{\text{PR } 5\text{-}6}{(5.076)} = \$640$.

(2) Net estimated saving per unit of product = \$126 ÷ 500 = \$.252.

(3) Rate of return on investment is determined as follows: Income annually applicable to capital recovery and return is

$$\$520 - \$197 = \$323$$

and

$$\$1{,}000\overset{\text{RP } i\text{-}6}{(\text{xxxxx})} = \$323$$

By interpolation i is approximated at 22.9 per cent. If 5 per cent interest is considered to be expense, the profit in terms of a rate of return will be 22.9 per cent less 5 per cent, or 17.9 per cent.

(4) Payout period at 5 per cent interest is determined as follows: Income annually applicable to capital recovery and return is

$$\$520 - \$197 = \$323$$

and

$$\$1{,}000\overset{\text{RP } 5\text{-}n}{(\text{xxxxx})} = \$323$$

By interpolation on the 5 per cent interest table, n is approximated at 3.5 years.

Allowance for Errors in Estimates

The effectiveness of the scientific approach in estimating in the physical field approximates certainty in many situations as, for instance, the pressure that a centrifugal pump will develop, the trajectory of a missile, the lift of an airfoil, the load induced on a structure by wind, and the outcome of combining a number of chemicals.

The success of the scientific approach, *i.e.*, dependence on a cause-and-effect relationship, in the physical field has carried over into other fields. The idea that the future can be predicted if sufficient knowledge is available is now generally accepted. Where this viewpoint prevails, great emphasis is placed on securing sufficient data and applying it carefully in arriving at estimates that are representative of actualities to the highest possible degree. The better the estimates, the less allowance need be made for error.

After the best possible estimates have been made, it is recognized that there will still be errors for which allowances must be made. At the outset it should be realized that allowances for errors do not make up for a deficiency of knowledge in the sense that allowances correct errors. Allowances are merely a means of eliminating some consequences of error at a cost.

In illustration of how allowances for errors in estimates function, consider an example from the engineering field. Let it be assumed that a scientific instrument is to hang from the center of a steel beam that has a horizontal span of 60 inches. It has been decided that the beam is to have a width of one inch; the depth is to be determined. The instrument has a value of $1,000 and an estimated weight of 800 pounds. Tests of several samples of steel from the melt from which the beam is to be made show the steel capable of resisting strains of 50,000 to 52,000 pounds per square inch. This steel costs $.10 per pound. The depth of the beam is determined by the use of the formula $PL/4 = Sbd^2/6$ as follows:

$$\frac{800 \times 60}{4} = \frac{50{,}000 \times 1 \times d^2}{6}$$

$$d = 1.2 \text{ in.}$$

The cost of the beam, 1 in. × 1.2 in. in cross section and 62 in. long (allowing 1 inch for support at each end), made of steel at $.10 per pound, is $2.11.

In the above example the nature of the length of beam required, its width, the strain-resisting properties of steel, and the formula by which the depth was calculated are such that they may be determined objectively, that is, estimated with a degree of accuracy approaching certainty. Thus a beam with a calculated depth of 1.2 inches and costing $2.11 is almost certain to be adequate to carry the 800 pounds.

But the consequences of a $1,000 loss in case of failure prompts caution, particularly since a stronger beam can be had for an additional cost equal to only a small fraction of the loss in case of failure. So possible errors for which allowances should be made are sought. The extent of the errors in the estimates of the length, breadth, and depth of the beam and in the strain resistance of steel is so small that no allowance will be made for them. But a question may arise in regard to the estimated load the beam is to carry. May not the estimated weight of the instrument be in error? Is it not possible that a load of greater than 800 pounds may be placed on the beam? A man might decide to cross the span by walking on the beam, for example, or the instrument might suddenly be dropped upon the beam.

Calculation reveals that an additional expenditure of $2.11 will provide a beam that will support four times as much as the original. Since this amount is relatively small in comparison to the loss that would be occasioned by failure of the beam, it is likely that a beam of greater depth than required to hold the instrument will be selected. If this is done, an allowance has been made for an error in the estimated load that the beam will be called upon to carry. By spending $2.11 it is hoped to eliminate the consequences that might have resulted from errors in the original estimate.

In this situation a large factor of safety can be introduced for a small cost. In fact, it might be cheaper to allow for a large error in the estimated load than to exercise much effort in trying to determine its amount.

As calculated above, a beam just adequate to carry a load of 800 pounds will weigh 21.1 pounds. If its depth is doubled to quadruple its load-carrying capacity, its weight will double and will be increased by 21.1 pounds. No cognizance was taken of this fact above. In some applications the increase in weight would be more important than an increase in its cost. It might prove prohibitive to allow for errors in the estimated load by introducing a factor of safety of four that results in increasing the weight of the beam by 21.1 pounds. On the basis of expected passenger miles, the worth of saving weight in a commercial plane has been estimated as high as several dollars per ounce. As a result, in aircraft design, great effort is made to determine accurately loads and the structural elements to resist them so that prohibitive allowances for error may be avoided.

In business situations where many elements to be estimated are of a nonphysical nature, the matter of allowances is more difficult. Ordinarily the degree of error that may exist in estimates is unknown and difficult to approximate. If too great allowances are made for possible error, many profitable ventures may be rejected. If too small allowances are made, unprofitable ventures may be undertaken.

This may be illustrated by an example. Suppose that chances to shoot at a certain rifle target cost S dollars each and that a reward of R dollars is provided if the upper half of the target is hit and a penalty of P dollars must be paid if the lower half of the target is hit. The sights of the guns to be used are known to be in error but the amount of error E is unknown. In the above, S represents the cost of seeking and evaluating an alternative, R the possible gains, P the possible losses, and E the error of estimates. Suppose further that the quantities S, R, P, and E vary according to no known pattern for each shot taken. Then the problem of holding the rifle to compensate for the error in the sights is comparable to that of compensating for errors in estimates in economy studies.

Risk

Risk may be defined as an exposure to loss or injury. In many situations the probability that certain events will happen in certain ways may be known. For instance, experience in the manufacture of a certain article had revealed that one piece was lost owing to breakage for every 132 acceptable pieces made. Analysis also revealed that the loss of a piece amounted to a money loss of \$86. Thus the probable loss per piece due to breakage amounted to \$86 ÷ 132 or \$.65 per piece. Stated in other words, the risk of breakage amounted to \$.65 per piece.

Where the risk is known it may be considered to be an input cost. This is often done by insuring against risk and entering the amount spent for insurance as an input cost.

Where the risk is not known it may be estimated in quantitative terms and added as an input cost, or it may be regarded as an irreducible factor to be considered in making decisions.

Risk as used here designates a loss that may be experienced by a happening in a situation under consideration. Action based on estimates that are in error may also result in loss, but in this case the loss results from *what is believed* about a situation. The difference in the implications of risk and error should be recognized if they are to be considered properly.

Allowing for Error in Estimates by High Interest Rates

It is a common policy in industrial concerns to require that a prospective undertaking be justified on the basis of a high interest rate, say 25 per cent. One basis for this practice is that there are so many opportunities which will result in a return of 25 per cent or more that those yielding less can be ignored. But since this is a much greater return than most concerns make on the average, the high rate of return represents an allowance for error. It is hoped that if undertaking of ventures is limited to those that promise a high rate of return, none or few will be undertaken that will result in a loss.

Returning to the example of the new machine (see p. 179 ff.), suppose that the estimated income and the estimated cost of carrying on the venture when the interest rate is taken at 25 per cent are as follows:

Estimated Annual Income (for six years)		$520
Estimated Annual Capital Recovery and Return, $1,000(.33882) [RP 25-6]	$339	
Estimated Annual Operating Cost	197	
Estimated Total Annual Operating Cost		536
Estimated Net Annual Profit of Venture		−$ 16

If the calculated loss based on the high interest rate is the deciding factor, the venture will not be undertaken. Though the estimates as given above, except for the interest rate of 25 per cent, might have been correct, the venture would have been rejected because of the arbitrary high interest rate taken, even though the resulting rate of return would be 22.9 per cent (see calculations on p. 181).

Suppose that the total operating costs had been estimated as above but that annual income had been estimated at $600. On the basis of a policy to accept ventures promising a return of 25 per cent on investment, the venture would be accepted. But if it turned out that the annual income was, say, only $150, the venture would result in loss regardless of the calculated income with the high interest rate. In other words, an allowance for error embodied in a high rate of return does not prevent a loss that stems from incorrect estimates if a venture is undertaken that will result in loss.

Allowing for Error in Estimates by Rapid Payout

The effect of allowing for error in estimates by rapid payout is essentially the same as that of using high interest rates for the same purpose. Let it be assumed that a concern has a policy that equipment purchases must be based upon a 3-year payout period when interest is taken at 5 per cent.

Returning to the example of past paragraphs, suppose that the estimated income and the estimated cost of carrying on the venture when a 3-year payout period is taken is as follows:

Estimated Annual Income (Estimated for six years but taken as being for three years to conform to policy)		$520
Estimated Annual Capital Recovery and Return, $1,000(.36721) (RP 5-3)	$367	
Estimated Annual Operating Cost	197	
Estimated Total Annual Operating Cost		564
Estimated Net Annual Profit for Venture		−$ 44

Thus the venture would not have been undertaken.

The Effect of Allowance on the Side of Conservatism

The effect of allowances on the conservative side is to reduce the probability of entering into some unprofitable venture at the expense of rejecting some profitable ventures.

The use of high interest rates or short payout periods as a basis for the statement of policy has the merits of clarity and conciseness. One objection to the use of either of these bases is that they are often misunderstood. They often come to be regarded as compensators for error and thus result in carelessly made estimates.

Use of Least Favorable, Fair, and Most Favorable Estimates

A plan for the treatment of estimates considered to have some merit is to make a least favorable estimate, a fair estimate, and a most favorable estimate of each situation.

The *fair estimate* is the estimate that appears most reasonable to the estimator after a diligent search for and a careful analysis of data. This estimate might also be termed the most likely estimate.

The *least favorable estimate* is the estimate that results when each item of data is given the least favorable interpretation that the estimator feels may reasonably be realized. The least favorable estimate is definitely not the very worst that could happen. This is a difficult estimate to make. Each element of each item should be considered independently in so far as this is possible. The least favorable estimate should definitely not be determined from the fair estimate by multiplying the latter by a factor.

The *most favorable estimate* is the estimate that results when each item of data is given the most favorable interpretation that the estimator feels may reasonably be realized. Comments similar to those made in reference to the least favorable estimate, but of reverse effect, apply to the most favorable estimate.

The use of the three estimates will be illustrated by application to the example of previous paragraphs.

Items Estimated	*Least Favorable Estimate*	*Fair Estimate*	*Most Favorable Estimate*
Annual Number of Units	300	500	700
Savings Per Unit	$.80	$1.04	$1.15
Annual Saving	$240	$520	$805
Period of Annual Savings, n	3	6	10
Capital Recovery and Return, $1,000(xxxxx) [RP 5-$n$]	$367	$197	$130
Operating Cost			
Item A	$60	$90	$120
Item B	60	60	70
Item C	50	40	60
Item D	20	7	2
Estimated Total of Capital Recovery, Return and Operating Items	$557	$394	$382
Estimated Net Annual Saving in Prospect for n years	−$317	$126	$423

An important feature of the least favorable, fair, and most favorable estimate plan of comparison is that it provides for bringing additional information to bear upon the situation under consideration. Additional information results from the estimator's analysis and judgment in answering two questions relative to each item. These questions are: "What is the least favorable value that this item may reasonably be expected to have?" and the reverse, "What is the most favorable value that this item may reasonably be expected to have?"

Judgment should be made item by item, for a summation of judgments can be expected to be more accurate than a single judgment of the whole. A second advantage of the three-estimate plan is that it reveals the consequences of deviations from the fair or most likely estimate. Even though the calculated consequences are themselves estimated, they show what is in prospect for different sets of conditions. It will be found that the small deviations in the direction of unfavorableness may have disastrous consequences in some situations. In others even a considerable deviation may not result in serious consequences.

The above results can be put on other bases for comparison. The present-worth basis has merit in this instance because of the variation in the number of years embraced by the above three estimates. The payout period and saving per unit of product can easily be calculated for other viewpoints.

Estimated Present Worth of Saving or Profit

Items Estimated	*Least Favorable Estimate*	*Fair Estimate*	*Most Favorable Estimate*
Net Annual Saving	−$317	$126	$423
Period of Annual Savings, n	3	6	10
Present Worth Factor, $\left(\text{xxxxx}\right)^{\text{PR } 5\text{-}n}$	2.723	5.076	7.722
Present Worth of Saving	−$863	$640	$3,270

Estimated Payout Period

Items Estimated	*Least Favorable Estimate*	*Fair Estimate*	*Most Favorable Estimate*
Annual Saving	$240	$520	$805
Annual Operating Cost, Items A, B, C, and D	190	197	252
Difference Applicable to Capital Recovery and Return at 5 Per Cent	50	323	553
Payout Period, Years	Infinite	3.5	2

Net Estimated Saving Per Unit of Product

Items Estimated	*Least Favorable Estimate*	*Fair Estimate*	*Most Favorable Estimate*
Net Annual Saving	−$317	$126	$423
Annual Number of Units	300	500	700
Net Saving Per Unit of Product	−$1.057	$.252	$.604

There are many who feel that it is an aid to judgment to have several bases on which to compare a single situation. Since the cost of making extra calculations is usually insignificant in comparison with the worth of even a small improvement in decision, the practice should be followed by all who feel they benefit from the additional information. But there are limits beyond which further calculations can serve no useful purpose. This occurs when calculations are made that are beyond the scope of the data used. In the example above, it would seem that no useful purpose would be served, for instance, by averaging the results calculated from the three estimates, least favorable, fair, and most favorable.

Use of Tentative Guesses

In evaluating some alternatives in which there are one or two obscure elements, it may be helpful to make a series of tentative guesses about the obscure elements.

The results obtained with different guesses are compared and the effect of the different guesses noted. The results of the guesses may reveal that no guess within the realm of reasonableness will result in the desirability of the alternative; on the other hand, it may be found that the missing element has little bearing on the alternative. Trends with the different guesses may be observed. If one of the guesses makes the alternative seem desirable, the probability that that guess is correct may be considered.

The systematic use of conjecture will nearly always result in a clearer understanding of an obscure situation and is particularly effective in guarding against a wholly disastrous decision.

Making the Decision

After a situation has been carefully analyzed and the possible outcomes have been evaluated as accurately as possible, a decision must be made. Even after all the data that can be brought to bear on a situation have been considered, some areas of uncertainty may be expected to remain. If a decision is to be made, these areas of uncertainty must be bridged by consideration of nonquantitative data or, in other words, by the evaluation of intangibles. Some call the type of evaluation involved in the consideration of intangibles *intuition;* others call it *hunch* or *judgment.*

Whatever it be called, it is inescapable that this type of thinking or, perhaps better, this type of feeling, must always be the final part in coming to a decision about the future. There is no other way if action is to be taken. There appears to be a marked difference in people's abilities to come to sound conclusions when some facts relative to a situation are missing. Perhaps much more attention should be devoted to developing sound judgment, for those who possess it are richly rewarded. But as effective as intuition, hunch, or judgment may sometimes be, this type of thinking should be reserved for those areas where facts on which to base decision are missing.

QUESTIONS

1. What is the primary reason for investing funds one has in an asset that it is known will eventually be rendered useless through use?

2. State as an equation the condition that must hold, if a venture requiring an investment in physical assets is to be successful.

3. Why are decisions related to the future based on estimates instead of upon the facts that will apply?

4. Explain why an estimate of a result will probably be more accurate if it is based upon estimates of the factors that have a bearing on the result than if the result is estimated directly.

5. Why is it logical to make estimates of the income of a prospective venture prior to estimating other factors having a bearing on the success of the venture?

6. What conditions exist when a venture just "breaks even"?

7. Explain why economy studies, no matter how carefully made, are merely estimates.

8. Explain how allowances for error in estimates may result in the rejection of profitable ventures.

9. Differentiate between risks and errors in estimates.

10. Discuss the effect of allowances for error through considering the interest rate to be higher than its cost or through considering the life of equipment to be less than its useful life.

11. Discuss the value to be gained from making least favorable, fair, and most favorable estimates of factors affecting the success of a venture.

12. How may tentative guesses be used in economy studies?

13. What is the nature of the process of decision?

14. Why is judgment or a similar process always necessary to come to a conclusion relative to an outcome in the future?

CHAPTER

9

First, Fixed, Variable, Increment, Differential, Marginal, and Sunk Costs

A NUMBER OF CLASSIFICATIONS OF COST have come into use that are helpful in understanding some aspects of the economy of an enterprise. These classifications are by no means universally used, nor do they mean the same thing to all people. Nevertheless, they provide concepts for considering many problems. The most important of these classifications of cost are:

First Cost	Differential Cost
Fixed Cost	Marginal Cost
Variable Cost	Sunk Cost
Increment Cost	

These will be defined and explained, and applications of the concepts of each will be made in turn.

First Cost

First cost is generally considered to embrace the cost of getting an activity started. In the illustration on page 19 it was pointed out that, in a certain collating task, experience had shown that 24 minutes, on the average, were spent in getting ready to do the required work. Here the 24 minutes' preparation time can be considered to be a first cost for the task of collating. On a larger scale, costs associated with the promotion and the development of a new bus route are illustrative of first cost. Initiating a new bus route will give rise to expense for such items as: investigation of schedules, passenger load, and cost of operation over different routes; legal fees for presenting petitions to franchising commissions; promotional work with chambers of commerce in the communities to be served; effort of acquiring needed facilities; and initial license fees. These costs will not be repeated, but are first costs associated with setting up the new bus route.

The chief advantage of this classification is, perhaps, that it calls attention to a group of costs associated with the beginning of something new that might not otherwise be given proper consideration. Many new enterprises fail in the early stages, not because the opportunity for profit was inadequate, but because adequate financing to take care of first costs is not provided.

Of the several classifications this one is least widely used; nevertheless it is a concept of very wide application to the undertaking of both small and large activities.

It should be realized that many otherwise profitable ventures cannot be undertaken for the reason that their associated first cost cannot safely be met. Many mines, for instance, have been abandoned from this cause only to be made profitable by further development later.

Fixed Costs

The term *fixed cost* has two common meanings. This term is used in business to designate such fixed charges as interest payments on bonds and amortization payments. It is also used to designate a group of expenses in a going enterprise whose total will remain relatively constant or fixed throughout the range of operational activity of the enterprise. The latter meaning of fixed costs is used in this discussion.

Fixed costs and variable costs make up the total cost of carrying on an activity. Because of their close association they will be discussed together after variable costs have been defined.

Variable Costs

Variable costs are those costs in an enterprise that vary in some relationship with a change in extent or character of the activities of an enterprise. In general usage, this term ordinarily gives greater emphasis to change in cost resulting from changes in extent of activity than to that resulting from changes in character of activity. An example of one such cost is the cost for material. Since the amount of material needed per unit of a product may be expected to remain constant, the material costs will vary in some proportion to the quantity of products produced.

Discussion of Fixed and Variable Costs

The ideas embraced by the terms "fixed costs" and "variable costs" are theoretical concepts. From a practical standpoint it is difficult to identify accurately all the costs of an actual business as being either fixed or variable. These concepts are nevertheless very useful, and accounting procedures ordinarily identify costs approximately in accordance with these two classifications.

As an aid to understanding fixed and variable costs, consider an enterprise whose activity consists of making and marketing doughnuts. Mr. A purchased for $1,400 a doughnut machine whose life is estimated at 8 years. He entered the following agreement with Mr. B, who operates a baking shop.

Mr. B is to keep the machine in his shop, furnish the electric current, and provide all the labor associated with the manufacture and sale of doughnuts; in return for his services he is to be paid $.11 per dozen doughnuts sold. Mr. A is to provide the ingredients, which are found to cost $.095 per dozen doughnuts. Mr. A considers an interest rate of 8 per cent necessary to justify investment in the machine.

The costs associated with this venture are:

	Fixed Costs (Annual)	*Variable Costs (Per Dozen)*
Capital recovery and return on equipment, $1,400 × $\overset{\text{RP 8-8}}{(.17401)}$	$244	
Insurance and taxes	34	
Repairs and maintenance	22	$.005
Material		.095
Labor, electricity, and space		.11
Total	$300	$.210

The difficulty of making a clear-cut separation between fixed and variable costs becomes apparent when attention is focused on the item for repair and maintenance in the above classification. In practice it is very difficult to distinguish between repairs that are a result of deterioration that takes place with the passage of time and those that result from the wear and tear of use. However, in theory the separation can be made as shown in this example and is in accord with the fact, with the exception perhaps, of the assumption that repairs from wear and tear will be in direct proportion to the number of doughnuts made. To be in accord with actualities, depreciation also should undoubtedly have been separated so that a part would appear as variable cost.

Let C_f equal annual fixed cost = \$300
C_v equal annual variable cost = $c \times n$
c equal variable cost per dozen doughnuts = \$.21
C_t equal total annual cost
n equal annual output of doughnuts in dozens
C_d equal cost of producing doughnuts per dozen
s equal sale price of doughnuts per dozen = \$.30
I equal income per year = $n \times s$

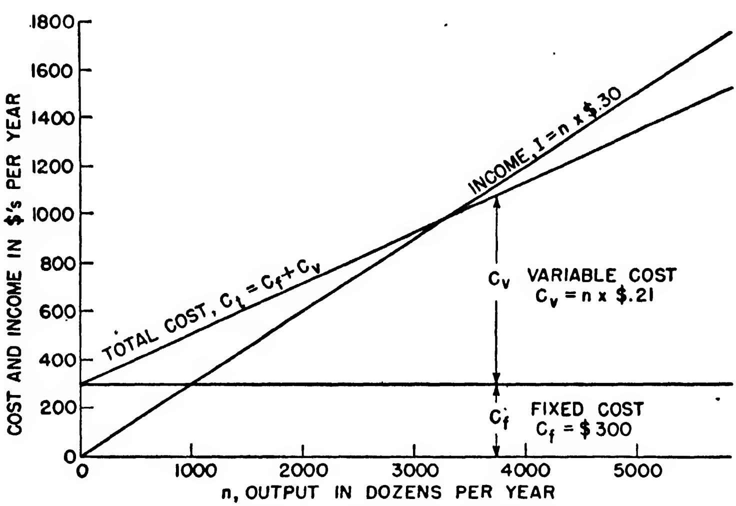


FIG. 24. Fixed cost, variable cost, and income per year.

Then the following expressions apply:

$$C_t = C_f + C_v$$
$$= C_f + (c \times n)$$

If C_t is plotted as n varies from 0 to 6,000, the result will be as shown in Fig. 24.

The cost of producing doughnuts will vary with the number made per year. The cost of production per dozen doughnuts is given by

$$C_d = \frac{C_f}{n} + c$$

Values of C_d, cost per dozen doughnuts, as n varies from 0 to 6,000 are given in Fig. 25.

It will be noted that fixed cost per dozen doughnuts may range to an infinite amount. Thus in determining unit costs, fixed cost has little meaning unless the number of units to which it applies is known.

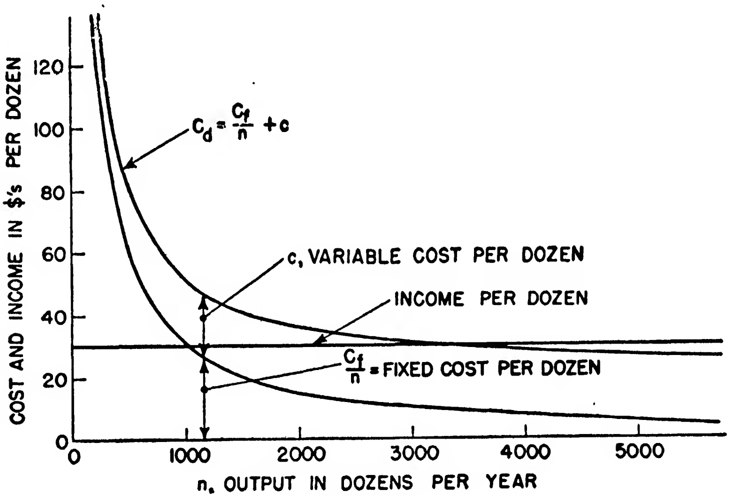


Fig. 25. Fixed cost, variable cost, and income per dozen.

The income of most enterprises is substantially proportional to the number of units of output sold. For the above example the annual income will be $I = ns$.

Thus the income per unit may easily be exceeded by the sum of the fixed cost and the variable cost per unit for low outputs. This is shown by the income curve in Fig. 24 and the income per unit curve in Fig. 25 based on a selling price of \$.30 per dozen doughnuts.

Fixed costs incurred for preparation for the future. Fixed costs arise from making preparation for the future. A machine is purchased now in order that labor costs may be reduced in the future.

Materials that may never be needed are purchased in large quantities and stored at much expense and with some risk in order that idleness of men and production facilities may be avoided. Research is carried on with no immediate benefit in prospect on the thought that it will pay in the long run.

The investments that give rise to fixed cost are made in the present in the hope that they will be recovered with a profit as a result of reductions in variable costs or of increases in income in the future. Fixed costs are a function of the original investment and the elapse of time.

Elapse of Time and Rate of Use Affect Fixed Costs

The elapse of time and rate of use of assets materially affect the fixed costs per annum and per unit of output. This may be illustrated by an example. Suppose that a machine can be purchased for $1,000 and that it will reach zero value either from the wear of processing 10,000 units of product or from 10 years of deterioration due to action of the elements. This machine has capacity sufficient to process 10,000 units per year. It is estimated that the annual cost of space used by the machine is $60. Interest is taken at 8 per cent.

A tabulation of the fixed costs per annum and per unit of output for this machine is given in Table 17.

TABLE 17. Fixed Costs Per Annum and Per Unit of Output of a Machine for Several Rates of Use

Annual output	10,000	5,000	2,000	1,000	500
Years of service, *n*	1	2	5	10	10
Capital Recovery Factor for *n* years	1.0800	.56077	.25046	.14903	.14903
Capital Recovery Factor × $1,000	$1,080	$561	$250	$149	$149
Charge for space	60	60	60	60	60
Total annual fixed cost	$1,140	$621	$310	$209	$209
Fixed cost per unit of output	$.114	$.124	$.155	$.209	$.418

At an annual output of 500 units the machine will have weathered to uselessness when only 5,000 units have been made.

If the entire 10,000 units of product had been made during the tenth year the cost per unit would be calculated as follows:

Compound amount of investment, \$1,000 × (2.159) [SP 8-10]	\$2,159
Compound amount of space charges, \$60 × (14.487) [SR 8-10]	869
Total	\$3,028
Cost per unit of output, \$3,028 ÷ 10,000	\$.3028

These analyses serve to show that fixed costs are a function of the amount of investment and the elapse of time. The fixed cost per unit output is markedly affected by the time element and warrants serious consideration as a means of cost reduction. It is also apparent that an unwise investment in an asset may be the source of high costs throughout its life. The problem of controlling fixed cost per unit of product is further complicated by depreciation due to obsolescence or inadequacy, which can result in the retirement of a machine before it either wears or weathers to scrap. This will be treated in the discussion of sunk costs.

Fixed costs arise from past decisions. Fixed costs are made up of such cost items as depreciation, maintenance, taxes, insurance, lease rentals, and interest on invested capital, sales programs, certain administrative expenses, and research. It will be observed that these arise from the decisions of the past and in general are not subject to rapid change. Volume of operational activity, on the other hand, may fluctuate widely and rapidly. As a result fixed costs per unit may easily get out of hand. It is probable that this is the cause of more business failures than any other, for few have the foresight or luck to make commitments in the present which will fit requirements of the future even reasonably well. Since fixed costs cannot be changed readily, consideration must be focused upon maintaining a satisfactory volume and character of activity.

In general, all costs such as direct labor, direct material, direct power, and the like, which can readily be allocated to each unit of product, are considered to constitute the variable costs and the balance of the costs of the enterprise are regarded as fixed costs.

In a practical situation, fixed costs are only relatively fixed and their total may be expected to increase somewhat with increased activity. The increase will probably not follow a smooth curve but will vary in accordance with the characteristics of the enterprise.

Consider a plant of several units that has been shut down or is operating at zero volume. No heat, light, janitor, and many other services will have been required. Many of these services must be reinstated if the plant is to operate at all, and if reinstated only on a minimum basis it is probable that these services will be adequate for quite a range of activity. Further increases in activity will require expenditures for other services that cannot be provided to just the extent needed. Thus what are termed "fixed costs" in business may be expected to increase in some stepped pattern with an increase in activity.

Variable expense may also be expected to increase in a stepped pattern. To increase production beyond a certain extent another machine may be added. Even though its full capacity may not be utilized, it may be necessary to employ a full crew of men to operate it. Also, an increase in productivity may be expected to result in the use of materials in greater quantities, and thus in their purchase at a lower cost per unit.

The practices followed in designating fixed and variable costs are usually at variance with the strict interpretation of these terms. Analyses in which they are a factor must recognize this fact or the results may be grossly misleading.

Increment, Differential, and Marginal Costs

The terms *increment cost, differential cost*, and *marginal cost* refer to essentially the same concept; the first term will be used here.

The term *marginal cost* refers specifically to an increment of output whose cost is barely covered by the return derived from it, in other words, a marginal cost is an increment cost that is on the borderline of attaining a contemplated result.

The term *increment* means *increase* and an increment cost means an increase in cost. Usually reference is made to an increase of cost in relation to some other factor, thus resulting in such expressions as increment cost per ton, increment cost per gallon, and increment cost per unit of production.

In Fig. 26, which is typical of cost-output curves, the increment cost of producing ten units between outputs of 60 and 70 units per year is shown. The increment output, Δ output $= 70 - 60 = 10$ units.

The increment cost, Δ cost, as read from the curve, is \$12. The average increment cost of the 10 units in question is $12/10 or $1.20 per unit.

The average cost of producing 60 units per year is $120/60 or $2 per unit. The average cost of producing 70 units per year is $132/70, or approximately $1.89 per unit.

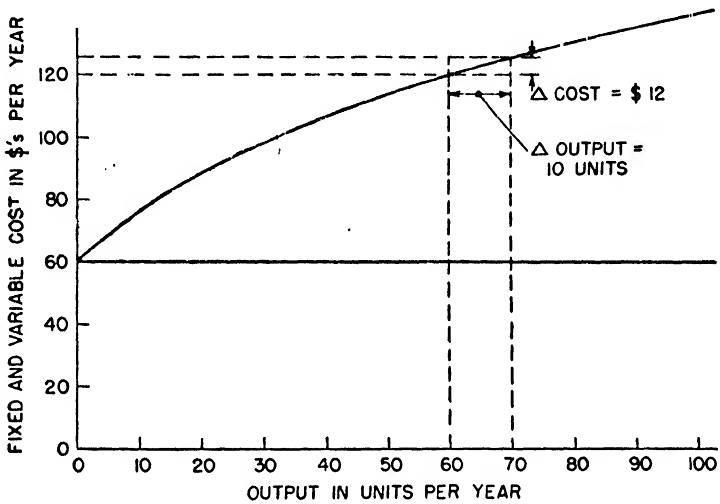


FIG. 26. Diagram showing increment cost.

The concept of increment costs is very useful in considering economy. Suppose, for instance, that in the above example 60 units per year are being sold at $2.20 per year when an offer of $1.50 per unit is made for 10 additional units. In view of the average cost of $2.00 and $1.89 for 60 and 70 units per year respectively, it might be concluded that acceptance of the offer would result in a loss, when in fact an average gain of $.30 per unit is in prospect.

Difficulty of Determining Increment Costs

In the above example a curve was given which enabled the increment cost to be precisely determined as $12 for ten units. As a practical matter in actual situations it is ordinarily difficult to determine increment cost. There is no general approach to the problem, but each case must be analyzed on the basis of the facts that apply to it at the time and the future period involved. Increment costs can be overestimated or underestimated, and either error may be costly. The overestimation of increment costs may obscure a profit possibility; on the other hand, if they are underestimated, an activity may be undertaken which will result in a loss. Thus accurate information is necessary if sound decisions are to be made.

Pattern for Considerations of Increment Costs

Where increment costs are to be considered, the question is: Will it be profitable to add a certain activity or subtract a certain activity from the total activities now in progress? To answer this question basic data are needed to give a true picture of the present situation in a physical sense. These data should take the form of such concrete terms as pounds of scrap produced, square feet of floor space used, number of hours that machine No. 16 operates, number of hours that machine No. 20 can operate before an overhaul will be needed, pounds of steam used, and so forth. Factual information on such items over the period that the contemplated increment activity will be in force forms the base line for evaluating the increment.

The next step is to determine, in similarly concrete terms, the changes that will take place as a result of undertaking the increment activity. The accuracy and completeness with which this can be done ranges all the way from the point where predictions are almost a certainty to the point where success of prediction is a matter of chance. In any event, the changes that the increment activity will bring about should be set down item by item. The plan above is analogous to the use of a free-body diagram in mechanics to arrive at the effect of a force.

Cost of increment activity. After the changes that will take place as the result of an increment activity have been isolated as far as possible, the next step is to convert the changes as found into monetary values. The point to recognize is that the monetary values that can be assigned to happenings in the future can at best be no more accurate than the data to which they apply. They may be less accurate. Even where it is known that an event will take place, it may be impossible to determine the expenditure it will entail.

Sources of Cost Data for Increment Analyses

The first and easiest method of arriving at costs is through judgment. This amounts to an informal consideration of the facts involved, or thought to be involved, in the future in light of facts, opinions, impressions, biases, and conjectures of the past. As faulty as judgment may sometimes be, the fact remains that it must necessarily be the basis for deciding on much activity for which there is insufficient objective knowledge. Judgments often must be made to undertake this or to drop that increment activity even though objective data are lacking, for the decision not to act is itself a decision relative to the question in hand.

Determination of costs by investigation and experimentation. There are a surprising number of situations in which costs that are unknown can be determined by investigation or experimentation. A relatively small sum spent experimentally may determine the result of a future activity under consideration. This is the idea behind the construction and operation of pilot plants. Not only are these plants effective in determining the feasibility of taking on a new product, but they also permit experimentation to arrive at improvements which, with a full-scale installation, it would not be feasible to work out.

In many cases information is available and needs merely to be brought to bear on the problem. Information is often available in handbooks and technical literature. Supervisors and operators of machines often have valuable concrete information. In other cases information is at hand and needs only to be used. In a recent conversion from the manufacture of aluminum to the production of steel pistons, no one took the trouble to determine the bulk of the shavings that would be produced as the result of the change. Thus provisions for removing them were inadequate and expensive changes had to be made after the manufacturing line had been installed. This large cost could have been avoided by calculating from the blueprints the amount of shavings that would be produced.

Increment costs based on cost accounts. The most prevalent data available on costs, and particularly unit costs, are those produced by accounting and cost accounting systems. Unfortunately such data are often not to be relied on in determining the effect of an increment activity.

It has been brought out, for instance, that accounting records give "book values" of equipment instead of the true depreciated balance. Cost accounting systems are compromises between the cost of getting accurate cost figures and the value of their accuracy in the operation of an enterprise.

Furthermore, cost data are based on estimates of future volume of activity in order that cost data may be predictive and serve as a basis for planning. Many bases for distributing burden rates such as direct labor dollars, direct labor hours, and machine hours are satisfactory from a practical standpoint and as an over-all guide but are inadequate for the determination of the specific costs needed for the analysis of increment activities.

A firm that was punching a circular disk from a square blank conceived the idea of altering the die so that four advertising novelties were punched out of the formerly wasted corners of the blank. After the punching, the burrs were removed and the pieces were flash plated. At first the product was given to salesmen without bothering to determine costs. As the demand grew, costs were determined on the basis of the scrap value of the actual weight of metal in each piece and the cost of burring and flash plating. No labor was charged to the novelty for the punching operation; this was considered to be carried by the disk. On this basis the increment cost was inordinately low and the company's salesmen not only gave the pieces to their customers freely but also offered a quantity for sale, basing their price upon an increment cost of $.0024 per piece. When the orders were totaled, the number required greatly exceeded the number that could be made in conjunction with the disks. When the pieces were made separately, where all costs had to be borne by the novelty, the cost was $.0208 per piece—over nine times as much as before—and resulted in a high percentage loss.

In this case the cost of production of $.0024 was an increment cost and so recognized by the firm, but it was only true for the number of pieces that could be punched with the disk. Cost accounting, ordinarily, could not be expected to direct attention to the fact that the cost of $.0024 only held good for a quantity proportional to the number of disks produced.

Dumping

Recognition of the increment principle sometimes makes it possible to take advantage of certain market situations by selling "below cost." This practice is suggested in the example on page 198 and is often engaged in during a period of depression. A plant that is operating below capacity can frequently get orders by pricing on the basis of increment costs, on which a profit can be made. The problem is to do so without disturbing the price structure of the company's other items. This may sometimes be accomplished by selling or dumping in a foreign market. Another practice that is quite common is to market essentially identical articles under different brand names. For example, a manufacturer of radio batteries marketed a product, identical except for labeling, at different prices through the same dealers. Customers who believed price was an indication of quality purchased the higher priced brand; the less affluent and perhaps more discriminating purchased the cheaper.

A midwestern city recently received three bids for a large-diameter concrete water line thirty miles long. Two bids were quite close in amount and a third was much lower. The disparity led to an investigation of the low bidder's competence. It developed that he had acquired a heavy back-hoe and other heavy equipment for a previous job. This equipment was heavier than ordinarily needed in the area but was ideally suited for the job in question. He correctly reasoned that the job provided an opportunity to get further use out of equipment that otherwise would have remained idle and he bid accordingly.

Though an understanding of increment costs may often make it possible to take advantage of a profitable situation not otherwise apparent, it is not feasible to make all decisions on this basis. Activities whose costs are determined on an increment basis should in general be limited to those that extend over a relatively short time. The advantages associated with increment activities are often due to unusual or unstable conditions. Thus their advantages may vanish.

Sunk Cost

The intent in making an investment in a capital item, such as a bulldozer, is to receive a net income, in the form of services, rentals, and receipts from sales, sufficient at least to equal the investment. When the net income received from an item equals or exceeds the amount invested in it, the investment is said to have been recovered. When the net income received is less than the amount invested, an unrecovered balance remains. This unrecovered balance is referred to as a *sunk cost.*

Thus the term *sunk cost* may be defined as the difference between the amount invested in an item, namely its cost, and the net worth of services and incomes resulting from the item. Sunk costs arise from decisions based on plans that are not realized. In an illustration of the concept of sunk costs, consider the following theoretical example in which interest will be ignored for simplicity. Mr. Black purchased for $2,800 a machine that he expected to have a service value of $1,000 per annum. When purchased, the machine was estimated to have a service life of 5 years, a salvage value of $200 at the end of its life, and an annual operating cost of $400 per year. On the basis of these estimates, the future years were pictured as shown in Table 18.

TABLE 18. Estimated Receipts and Disbursements

Receipts and Disbursements	*Year No. 0*	*Year No. 1*	*Year No. 2*	*Year No. 3*	*Year No. 4*	*Year No. 5*
Value of service rendered by machine during the year	—	$1,000	$1,000	$1,000	$1,000	$1,000
Amount received from salvage	—	—	—	—	—	200
Operating cost during year	—	400	400	400	400	400
Amount of investment recovered during year	—	600	600	600	600	800
Unearned balance of investment in machine at the end of year	$2,800	2,200	1,600	1,000	400	−400

On the basis of the estimates that had been made, the machine was expected to render services whose net value plus the $200 to be received from salvage was expected to be sufficient to recover the investment in the machine plus $400 in a 5-year period.

Actually Mr. Black's estimates had been in error. Instead of the machine's services having an annual value of $1,000, the services turned out to be worth only $700 per year. Operating costs totaled $600 instead of the estimated $400. Instead of having a service life of 5 years, the machine became so worn that it was sold for $300 at the end of three years. Thus the service value and the life of the machine had been overestimated and its salvage value had been underestimated.

The actual history of the investment is shown in Table 19.

TABLE 19. Actual Receipts and Disbursements

Receipts and Disbursements	*Year No. 0*	*Year No. 1*	*Year No. 2*	*Year No. 3*
Value of service rendered by machine during the year...........	—	$ 700	$ 700	$ 700
Amount received from salvage.....	—	—	—	300
Operating cost during year........	—	600	600	600
Amount of investment recovered during year....................	—	100	100	400
Unrecovered balance of investment in machine at the end of year...	$2,800	2,700	2,600	2,200

Thus on the basis of the facts pertinent to Mr. Black's venture, $2,200 of the investment was not recovered. This unrecovered capital or cost equal to $2,200 is a *sunk cost.*

In the above example the position was taken that an investment in an asset is recovered as a result of the worth of services and incomes received from the item. This is a correct view but a most difficult one to apply. It is usually impossible to determine the worth of the services rendered by a production unit. Suppose that a machine, labor to operate the machine, and a quantity of material were purchased and expended in one year, and that as a result of this action a person was able to earn $5,000 more than he might otherwise have done. Obviously the $5,000 income is attributable jointly to machine, material, and labor. The amount that is attributable to the machine alone cannot be determined objectively.

Worth of Services Based on Cost

Because of the difficulty of establishing the worth of services performed by physical property, the worth is estimated to be equal in value to the cost of providing it. A method commonly followed for making this estimate will be illustrated by example. Suppose that a machine that has been purchased for $5,000 has an expected salvage value of $1,000 at the end of a service life of 5 years.

From the above estimates the amount of capital investment to be recovered in 5 years is $5,000 − $1,000, or $4,000. Assume that straight-line depreciation is to be used. Then the amount of investment to be recovered per year is $4,000 ÷ 5 or $800 per year. This amount can only be recovered by the worth of the services provided by the machine. But, since it is all but impossible to determine the worth of a machine's services, the usual practice is to account for capital recovery by making a charge for a machine's service equal to an amount estimated necessary to recover the capital invested in it. In the above example, the amount estimated necessary to recover the capital invested in the machine is equal to the annual depreciation of $800 per annum for five years. If this amount can be charged to products processed on the machine, the capital invested in it will be considered to be recovered.

Thus capital invested in a machine is considered to be recovered to the extent that charges are made for the machine's services against the products that benefit thereby. The aim of most cost accounting procedures is to make such charges equal per unit of similar product and of such amount that the estimated depreciation of an asset will be recovered during its life.

Thus capital recovery as practiced consists of allocating the cost of depreciation of an asset to its output on some proportional basis. But the number of units that will be processed by a machine during its lifetime cannot be known prior to its retirement. For this reason it is common practice to estimate a *normal* annual output of a machine as a basis for allocating its depreciation costs to the product.

Depreciation based on normal activity. Assume it is estimated that a machine will process 1,000 units during a year of *normal* activity. Normal activity may be defined as the average activity expected over a number of years.

The charge to be made to the product for depreciation per unit is equal to the annual depreciation cost divided by normal output—in the above example, depreciation of $800 divided by 1,000 units of output, or $.80 per unit. This charge per unit is the estimated cost per unit arising from depreciation and it is assumed that this amount is the worth of the services of the machine per unit.

This practice is a practical approach made necessary by the fact that it is nearly impossible to determine the worth of services contributed by a machine to a product. Also it is more practical to make the necessary day-to-day decisions on the basis of present estimates than on the basis of a history of actual facts revealed in the future.

Below and above normal activity. On the basis of a charge of

$.80 per unit and an output of 1,000 units, the annual depreciation of $800 will be exactly recovered. But suppose that only 750 units were processed during a year; then only $600 will be recovered and $200 will remain unrecovered. On the theory that the machine will depreciate $800 per year even though not used fully and in order that the book value of the machine may be in accord with estimates made when it was purchased, some disposal must be made of the $200 unrecovered balance. A common practice is to charge the unrecovered amount to Profit and Loss as a loss incurred during the year's operations. This loss is in the nature of a sunk cost, since it represents unrecovered investment. It serves notice that the charge made per unit of product is less than the estimated depreciation cost of the machine per unit.

If the activity had been above normal and 1,250 units had been produced during the year, the charge to the product would have been $1,000, which is $200 in excess of the depreciation set up on the books for the machine. This excess may be entered as a profit in Profit and Loss and indicates to those in control that the charge made per unit of product is greater than the estimated depreciation cost per unit.

Error in Estimated Life and Salvage Value

For another viewpoint, consider a situation in which the estimated life and salvage value of an asset are in error. Assume that in the example above the machine was sold for $1,400 at the end of three years for reasons not pertinent to this discussion.

The history of the investment, assuming the production had been at the normal rate of 1,000 units per year, would appear as given in Table 20.

TABLE 20. Effect of Error in Estimated Life and Salvage Value of a Machine

Receipts and Disbursements	*Year No. 0*	*Year No. 1*	*Year No. 2*	*Year No. 3*
Estimated depreciation during year	—	$ 800	$ 800	$ 800
Amount considered to be received through charges to products made during year (A)........	—	800	800	800
Amount received from sale of salvage during year (B)...........	—	—	—	1,400
Unrecovered balance of investment at end of year, $5,000—(A + B)	$5,000	4,200	3,400	1,200

On the basis of this record the unrecovered balance, or sunk cost, is shown to be equal to $1,200. It should be remembered that this value is true only to the extent that the data on which it is based actually represent the facts of the case.

It is evident that the machine had not depreciated $800 per year, as shown by the record, but had depreciated $5,000 − $1,400 or $3,600 in three years—$1,200 per year on the average. The sunk cost is equal to the difference between the actual depreciation and the depreciation charged, in this case $3,600 − (3 × $800) = $1,200.

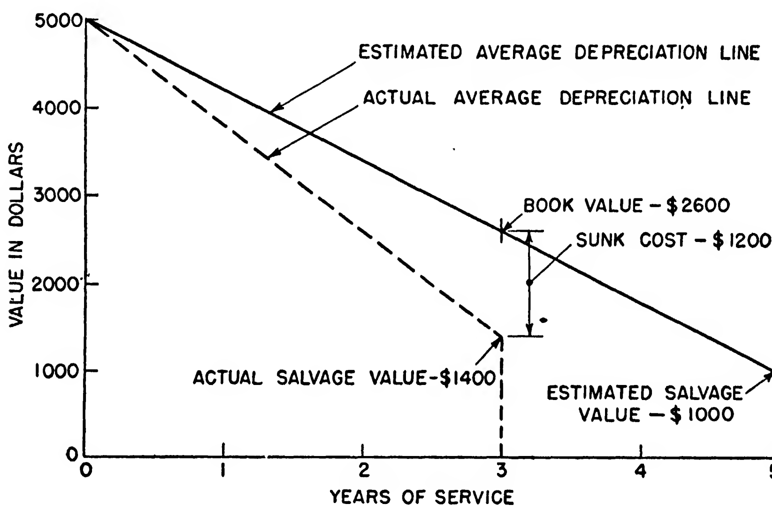


Fig. 27. Estimated and actual depreciation and sunk cost.

Stated another way, sunk cost, as usually determined by depreciation accounting, is equal to the estimated depreciated value or *book value* minus the realized salvage value of the asset, in this case $2,600 − $1,400 = $1,200. The financial facts relating to the machine are shown in Fig. 27.

In the above case an investment of $5,000 was made in a machine with the expectation that the total investment with a return would be recovered. The expectation was in error, $1,200 has been lost as a sunk cost, and nothing further can be done about it.

Face-Saving Sunk-Cost Practices

In spite of the fact that sunk costs cannot be recovered, a face-saving practice of charging the sunk cost of a machine to the cost of a replacing machine is often employed. This practice, human but unrealistic, will be illustrated by an example.

Three years ago Mr. S, who authorizes machine purchases in a manufacturing concern, was approached by Mr. F for authorization to purchase a machine. Mr. F pleaded his cause in glowing terms and with enthusiasm. He had many figures and arguments to prove that an investment in the machine he proposed would easily pay out. Mr. S was at first skeptical, but he also became enthusiastic about the purchase as the profits in prospect were calculated, and authorized the purchase. After three years Mr. F realized that the machine was not coming up to expectations and would have to be replaced, at a loss of $1,200.

Mr. F was well aware of the necessity of admitting this sunk cost when he went to Mr. S to get authorization for a replacement. He realized the difficulty of trying to establish confidence in his arguments for the replacement and at the same time admit an error in judgment that had resulted in a loss of $1,200. But he hit on the expendient of focusing attention on the proposed machine by emphasizing that the $1,200 loss could be added to the cost of the new machine and that the new machine had such possibilities for profit that it would pay out shortly, even though burdened with the unamortized balance of the previous machine. Such improper handling of sunk cost is merely deception designed to make it appear that an error in judgment has been corrected.

Calculations for Comparisons of Alternatives where Sunk Costs Are Involved

The following examples illustrate correct and fallacious methods of comparing alternatives.

Suppose that Machine A was purchased in 1946 for $2,200. It was estimated to have a life of 10 years and a salvage value of $200 at the end of its life. Its operating expense had been found to be $700 per year, and it appeared that the machine would serve satisfactorily for the balance of its estimated life. It had been depreciated by the straight-line method and its book value in 1950 was

\$2,200 − (4 × \$200) = \$1,400. In 1950 a salesman offered Machine B for \$2,600. Its life was estimated at 10 years and its salvage value at the end of its life was estimated to be \$300. Operating costs were estimated at \$400 per year.

The operation for which these machines are used will be carried on for many years in the future. Equipment investments are expected to justify 8 per cent interest, in accordance with the policy of the company concerned. The salesman offers to take the old machine in on trade for \$600. This appears low to the company, but the best offer received elsewhere is \$450. All estimates relative to both machines above have been carefully reviewed and are considered sound.

Base comparison on salvage value. If Machine A is traded in for \$600, a sunk cost of \$1,400 − \$600 or \$800 is revealed. Also, if Machine B is purchased, Machine A will be "sold" for \$600. The logical alternatives then are (1) to consider Machine A to have a value of \$600 and to continue with it for 6 years and (2) to purchase Machine B for \$2,600 and use it for 10 years.

CONTINUE WITH MACHINE A FOR 6 YEARS MORE
Cost Analysis in Terms of Annual Cost

Annual Capital Recovery and Return, (\$600 − \$200)(.21632 [RP 8-6]) + \$200 × .08	\$102.53
Annual Operating Cost	700.00
Total Annual Cost	\$802.53

DISPOSE OF MACHINE A, BUY MACHINE B AND USE FOR 10 YEARS
Cost Analysis in Terms of Annual Cost

Annual Capital Recovery and Return, (\$2,600 − \$300)(.14903 [RP 8-10]) + \$300 × .08	\$366.77
Annual Operating Cost	400.00
Total Annual Cost	\$766.77

If the latter alternative is adopted the annual saving in prospect for the next six years is equal to \$802.53 − \$766.77 = \$35.76. For the next four years after that time the amount of savings will be dependent upon the characteristics of the machine that might have been purchased six years from the present to replace Machine A. This is a question that cannot be answered, and in not answering it the assumption is made that costs will be the same during this period as with Machine B.

The "outsider" viewpoint. As an aid to taking a correct position in situations similar to that above, the situation may be viewed from the standpoint of a person who has need for the service that Machine A or Machine B will perform but owns neither. In attempts to purchase a machine, he finds that he can purchase Machine A for $600 and Machine B for $2,600. His analysis of which to buy will be identical to that above. The "outsider" viewpoint is helpful in many situations when there is a tendency for a person to be biased, as, for instance, when one alternative forces him to admit a sunk cost.

Calculation of comparative use value of old machine. A second method of comparison, which is particularly good for demonstrating the correctness of the above comparison to skeptical people, is to calculate the value of the machine to be replaced which will result in an annual cost equal to the annual cost of operation with the replacement. In this calculation, let X equal the present value of Machine A for which annual cost with Machine A equals annual cost with Machine B. Then

$$(X - \$200)\overset{\text{RP 8-6}}{(.21632)} + \$200 \times .08 + \$700$$
$$= (\$2{,}600 - \$300)\overset{\text{RP 8-10}}{(.14903)} + \$300 \times .08 + \$400$$

Solving for X results in

$$X = \$435$$

Machine A has a comparative use worth in comparison with Machine B of only $435. Thus it is obvious that it should be replaced if it can be disposed of for $600. Compare this result with that obtained in the previous section. Note that $600 − $435 = $165 is equivalent to $\$35.76 \times \overset{\text{PR 8-6}}{(4.623)} = \$165$.

Fallacy of adding sunk cost to a replacement. The fallacy of adding the sunk cost of an old machine to the cost of a replacement is demonstrated in the following example.

Machine C, purchased for $3,400 a year ago, had an estimated life of 6 years and a salvage value of $400. Its operating cost is $3,200 per year. At the end of the first year a salesman offers Machine D for $4,600. This machine has an estimated life of 5 years, a salvage value of $600, and, owing to improvements it embodies, an operating cost, as shown by trial, of only $2,200. The salesman offers to allow $1,400 for Machine C on the purchase price of Machine D.

At the end of the first year, the book value of Machine C on the basis of straight-line depreciation is equal to $2,900; thus the sunk cost is $1,500.

Annual cost with Machine C, on the basis of its present trade-in value and estimated salvage value 5 years hence is

Annual Capital Recovery and Return, ($1,400 − $400)(.25046) (RP 8-5) + $400 × .08	$ 282
Annual Operating Cost	3,200
Total	$3,482

Annual cost with Machine D, as incorrectly calculated when sunk cost of $1,500 of Machine C is added to the cost of Machine D, is

Annual Capital Recovery and Return, ($6,100 − $600)(.25046) (RP 8-5) + $600 × .08	$1,426
Annual Operating Cost	2,200
Total	$3,646

On the basis of this *incorrect* result, Machine C is continued for the next year on the erroneous belief that $3,646 less $3,482, or $164, is being saved annually.

Annual cost with Machine D as correctly calculated is

Annual Capital Recovery and Return, ($4,600 − $600)(.25046) (RP 8-5) + $600 × .08	$1,050
Annual Operating Cost	2,200
Total	$3,250

On this correct basis, purchase of Machine D should result in an annual saving of $3,482 less $3,250, or $232.

Comparison based on future receipts and disbursements. For a different viewpoint, the alternatives of continuing five years with Machine C and of replacing Machine C with Machine D and using the latter for the next five years can be compared on the basis of receipts and disbursements associated with them.

RECEIPTS AND DISBURSEMENTS IF MACHINE C IS RETAINED FOR THE NEXT 5 YEARS

Receipts and Disbursements	*Year No. 0*	*Year No. 1*	*Year No. 2*	*Year No. 3*	*Year No. 4*	*Year No. 5*
Disbursements (considered to come at end of year).		$3,200	$3,200	$3,200	$3,200	$3,200
Receipts at end of year...	—	—	—	—	—	$ 400

The present worth of the net cost of five years of service with Machine C is equal to

$$\overset{\text{PR 8-5}}{\$3{,}200(3.993)} - \overset{\text{PS 8-5}}{\$400(.6806)} = \$12{,}506$$

RECEIPTS AND DISBURSEMENTS IF MACHINE C IS SOLD AND MACHINE D IS PURCHASED AND USED FOR 5 YEARS

Receipts and Disbursements	*Year No. 0*	*Year No. 1*	*Year No. 2*	*Year No. 3*	*Year No. 4*	*Year No. 5*
Disbursements (considered to come at end of year)	$4,600	$2,200	$2,200	$2,200	$2,200	$2,200
Receipts at end of year..	$1,400	—	—	—	—	$ 600

The present worth of the net cost of five years of service if Machine C is sold and Machine D is purchased and used for 5 years is equal to

$$\$4{,}600 - \$1{,}400 + \overset{\text{PR 8-5}}{\$2{,}200(3.993)} - \overset{\text{PS 8-5}}{\$600(.6806)} = \$11{,}577$$

The net present worth of the advantage of accepting the latter alternative is $12,506 less $11,577, or $929. This is equivalent to

$$\$929 \times \overset{\text{RP 8-5}}{(.25046)} = \$232$$

and is equal to the annual saving resulting from the previous correct comparison made above.

If the incorrect calculation had been the one used, the decision would have been to retain Machine C on the thought that this action would result in an annual saving of $164. Actually this action would have resulted in annual loss of $232.

PROBLEMS

1. The fixed cost of a machine (depreciation, interest, space charges, maintenance, indirect labor and supervision, insurance, and taxes) is F dollars per year. The variable cost of operating the machine (power, supplies, and similar items, excluding direct labor) is V dollars per hour of operation. N

is the number of hours the machine is operated per year, A the annual total cost of operating the machine, C_h the hourly cost of operating the machine, t the time in hours to process one unit of product, M_p the machine cost of processing a unit of product, and n the number of units of product processed per year. In terms of these symbols, write expressions for (a) A, (b) C_h, (c) M_p.

2. In Problem 1, F = \$600 per year, t = .2 hour, V = \$.50 per hour, and n varies from 0 to 10,000 by increments of 1,000. Plot values of M_p as ordinates for various values of n plotted as abscissas.

3. Let W equal the hourly cost of direct labor necessary to operate the machine described in Problem 1. Using the symbols given in Problem 1, write the expression for C_p, the total of direct labor and machine cost per unit of product.

4. A certain machine costs \$32,000 new. Estimated life and salvage value are 8 years and \$6,000 respectively. Annual cost for space, supervision, interest, taxes, maintenance, and so forth is \$400. The hourly wage rate of the operator is \$1.12. The hourly operating cost of power, supplies, and so forth amounts to \$.84. The operating time per unit of product is 44 minutes. Assuming the machine is used only for a special product, calculate the cost per unit of processing of the product if (a) 100, (b) 500, (c) 1,000, (d) 2,000 units of product are made per year. Take interest at 6% and use sinking-fund depreciation.

5. Mr. A drives a car 60,000 miles per year. Mr. B drives a car 60,000 miles in 6 years. A car costs \$2,000 and has a trade-in value of \$1,200 at the end of one year and 60,000 miles and a trade-in value of \$400 at the end of 6 years and 60,000 miles. Fuel, oil, grease, and tires cost \$.02 per mile. Storage batteries cost \$17 each and have an average life of 2 years. Storage costs are \$6 per month and insurance costs \$48 per year. Each car is washed twice a month at a cost of \$1.50 per washing and is waxed four times a year at a cost of \$8 per waxing. To keep his car neat in appearance Mr. B deems it necessary to have it painted at the end of 3 years at a cost of \$100. Mechanical maintenance is estimated at \$4 per 1,000 miles for each car.

(a) With interest at 6%, determine the cost of operation per mile for each car.

(b) Mechanical maintenance was estimated for simplicity at \$4 per 1,000 miles for both Mr. A's car and Mr. B's car. Which car would be likely to have the higher maintenance? Why?

6. It will cost \$12,000 to build a special machine whose salvage value at the end of an estimated service life of 8,000 hours will be \$1,000. The cost of housing the machine is estimated at \$360 per year. Maintenance of the machine is estimated at \$.15 per hour and its power consumption is estimated at \$.32 per hour of operation. Labor costs per hour of operation are to be taken at \$1.56 and interest is 6%. It will require one hour to process a unit of the product to be made on the machine. Calculate the cost of processing a unit of product if (a), 8,000, (b) 4,000, (c) 2,000, and (d) 1,000 units are produced per year.

7. A truck owner who has a freight line estimates that it costs him $320 per month plus $.05 per mile to operate a truck. These costs include the drivers' salaries and central office expenses. The trucks are driven 1,000 to 2,400 miles per month. What is the cost of operating for the increment of 2,000 to 2,200 miles per month?

8. A manufacturing concern estimates that its expenses per year for different levels of operation would be as shown in the accompanying table.

Nature of Expense	Output, Units of Product					
	0*	10	20	30	40	50
Administration and sales expense	$4,900	$ 5,700	$ 6,200	$ 6,700	$ 7,100	$ 7,500
Direct labor and materials	0	2,500	4,600	6,400	8,100	9,800
Overhead charges	4,120	4,190	4,270	4,350	4,440	4,550
Total	$9,020	$12,390	$15,070	$17,450	$19,640	$21,850

* For a temporary period.

(a) What is the increment cost of maintaining the plant ready to operate (the increment cost of making 0 units of product)?

(b) What is the average increment cost per unit of manufacturing the first increment of 10 units of product per year?

(c) What is the average increment cost per unit of manufacturing the increment of 31 to 40 units per year?

(d) What is the average total cost per unit when manufacturing at the rate of 20 units per year?

(e) At a time when the rate of manufacture is 20 units per year a salesman reports that he can sell 10 additional units at $260 per unit without disturbing the market in which the company sells. Would it be profitable for the company to undertake the production of the 10 additional units?

9. A chemical company installed a piping system at a cost of $9,400 and based cost calculations on an estimate that it would serve for four years and that at the end of that time it would have no salvage value. At the end of 3 years it had deteriorated so badly that it was replaced. Depreciation was calculated by the straight-line method.

(a) What was the estimated annual depreciation?
(b) What was the actual annual depreciation?
(c) What was the sunk cost at the time of replacement?

10. Calculate Problem 9(c) using 10% sinking-fund depreciation.

11. Mr. Black purchased a plot of land on a highway for $3,000 and built a filling station at a cost of $11,000. He estimated that the land would remain constant in value and that the station would depreciate to $7,000 in 15 years. On the basis of these estimates net profits (after deduction of depreciation and interest on the basis of straight-line depreciation plus average interest) were calculated to be $3,600, $4,200, and $4,400 for the 1st, 2nd, and 3rd years of operation respectively. At the end of the third year a new highway

was built which virtually isolated Mr. Black's station. As a result Mr. Black sold the land and station for $8,000.

(a) What sunk cost loss did Mr. Black suffer?

(b) What was the actual net profit for the 1st, 2nd, and 3rd years of operation? Use straight-line depreciation plus average interest, taking interest at 6%.

12. Mr. Brown invented a rubber product and undertook the manufacture of it. Being short of funds he purchased a simple machine that required much hand labor. The machine was purchased for $4,400 and was estimated to have a service life of 5 years and negligible scrap value at the end of its life. Operating expense of the machine including labor but excluding depreciation and interest on the funds invested in the machine amounted to $2,600 per year. At the end of the first year he was urged to purchase for $6,200 a new machine which, as he was convinced by a demonstration, could produce the output of the present machine for an annual operating expense of $800, exclusive of depreciation and interest on funds invested in it. $1,600 would be allowed for the old machine on the purchase price of the new machine. For simplicity assume that the new machine will have negligible salvage value and a service life of 4 years, so that its service life will terminate with that estimated for the old machine. Take an interest rate of 8%.

(a) List Mr. Brown's receipts and disbursements during the next four years if he keeps the old machine; if he disposes of the old machine and buys the new machine. Compare the present worth of the two series of receipts and disbursements.

(b) Take the "outsider's viewpoint" and calculate the equivalent annual cost for each of the two alternatives.

(c) Calculate the use value of the old machine in comparison with that of the new machine.

(d) Should Mr. Brown purchase the new machine? Why?

13. Machine A was purchased four years ago for $1,200 and had an estimated life of 10 years and an estimated salvage value of $200. It has been depreciated by the straight-line method. Annual operating costs, exclusive of depreciation and interest on the investment in the machine, have averaged $1,200 and will probably continue at this rate. Machine B is under consideration as a replacement for Machine A. Machine B will cost $2,400 and has an estimated life and salvage value of 8 years and $400 respectively. Annual operating costs are estimated at $900. Machine A can be sold to a used machinery dealer for $300. The interest rate is 8%.

(a) What is the present book value of Machine A?

(b) What will be the sunk-cost loss if Machine A is sold for $300?

(c) If the salvage value for Machine A is now estimated to be $100 six years hence, should it be replaced now if capital recovery is based on sinking-fund depreciation?

(d) If the salvage value of Machine A is now estimated to be $100 six years hence, should it be replaced now if capital recovery is based on straight-line depreciation and average interest?

14. Machine X was purchased n years ago at a cost of m dollars. It had an estimated life of t years and an estimated salvage value of s dollars. Its annual operating cost exclusive of depreciation and interest is p dollars. It can be sold for j dollars and a replacement, Machine Y, can be purchased for M dollars. Machine Y has an estimated salvage value of S dollars L years hence. The interest rate is 10%. Calculate the maximum value of P, annual operating cost in dollars exclusive of depreciation and interest, for which it would be theoretically desirable to make the replacement. Capital recovery is to be calculated on the basis of sinking-fund depreciation. Assume that the original estimates of life and salvage value for Machine X are still believed to be good.

CHAPTER

10

Bases for Comparison of Alternatives

IN THE FIRST PHASE of the conversion step of an economy study, the prospective physical outputs and inputs of alternatives under consideration are converted to monetary receipts and disbursements at specified dates, insofar as this conversion is possible. On completion of this phase, each alternative has been converted to a series of receipts and disbursements of definite amount at a definite date plus an enumeration of irreducible considerations.

Since the amounts and dates of receipts and disbursements are given in quantitative terms, they may be converted mathematically to a number of bases for comparison.

Bases for Comparison of Alternatives

Any number of bases for comparison of alternatives can be set up, but most purposes can be served by the seven most commonly used. These are

- Equivalent Annual Amount
- Present-Worth Amount
- Capitalized Amount
- Rate of Return
- Service Life
- Minimum Cost Points
- Break-Even Cost Points

The first five of these bases will be illustrated and explained in a number of examples in this chapter. Minimum cost points and break-even cost points will be considered in a separate chapter.

Choice between Accepting and Not Accepting a Proposal

Often the decision on a proposal under consideration rests between putting or not putting it into effect. In this case the decision will be based on the relative merits of the proposal and other opportunities believed to exist, even though none of the latter have been crystallized into definite proposals.

Suppose that Mr. Brown is seeking a profitable outlet for his funds and for one reason or another wishes to consider building and operating an apartment house. Acting on this desire, he may seek possible locations and finally find one that interests him. Theoretically there are any number of alternative sites that could be considered. But practically it is often very difficult to find even one suitable site in a community for a specified purpose. After Mr. Brown has located the site, he will subject it to analyses of increasing thoroughness. The first of these analyses may be rather sketchy; but if on the basis of it the opportunity still appears promising, Mr. Brown will order more detailed analyses, which may embrace surveys by experts, preparation of architect's drawings and estimates, and bids on the cost of construction. Finally he must make the decision to build the apartment or not to build it.

When there is only one specified alternative, as in the above example, it should be evaluated in terms that can be used to compare its desirability with that of opportunities that are believed to exist but are unspecified. The following example will be used to illustrate several bases of evaluation when the decision to be made is to accept or reject a proposal and when there is no specific alternative proposed for comparison.

A person is considering the purchase of a portable Diesel 210 c.f.m. air compressor for renting to contractors who need such equipment temporarily. The receipts and disbursements associated with the purchase and subsequent rental of the air compressor are given below.

Description of Disbursements and Receipts	*Date*	*Disbursements*	*Receipts*
Cost of compressor	6-1-50	$5,000	
Rental received, 1st year	6-1-51		$2,400
Operating cost, 1st year (power, supplies, taxes, insurance, repairs, etc.)	6-1-51	1,000	
Rental received, 2nd year	6-1-52		2,400
Operating cost, 2nd year	6-1-52	1,000	
Overhaul, 2nd year	6-1-52	700	
Rental received, 3rd year	6-1-53		2,400
Operating cost, 3rd year	6-1-53	1,000	
Rental received, 4th year	6-1-54		2,400
Salvage sale of compressor	6-1-54		1,200
Operating cost, 4th year	6-1-54	1,000	

These disbursements and receipts tabulated above, with the exception of the purchase cost of the compressor, are estimates because all occur in the future. The life of the compressor has been estimated at four years. For convenience, disbursements and receipts occurring during the year are considered to come at the end of the year in which they occur. In this example the rental of $2,400 received during the first year is considered to occur at the close of May 31, 1951. The error introduced by this practice is insignificant in comparison with the usual errors in estimates, except for extremely high interest rates. It should be borne in mind that the end of one year may be considered to be the beginning of the next. The cost of money in this example is taken at 5 per cent. Also, June 1, 1950 will be considered to be the present, in other words the date as of which evaluation is to be made.

Present-worth evaluation. The present worths of disbursements and receipts as of June 1, 1950 at 5 per cent interest are first determined as follows:

Present worth of receipts as of 6-1-50 at 5%:		
$2,400 × (3.546) [PR 5-4]	$8,510	
$1,200 × (.8227) [PS 5-4]	987	
		$9,497
Present worth of disbursements as of 6-1-50 at 5%:		
$5,000 × 1	$5,000	
$1,000 × (.9524) [PS 5-1]	952	
($1,000 + $700) × (.9070) [PS 5-2]	1,542	
$1,000 × (.8638) [PS 5-3]	864	
$1,000 × (.8227) [PS 5-4]	823	9,181
Present worth of receipts less disbursements		$ 316

On the assumption that all estimates eventually prove to be correct, the significance of this analysis is that if $5,000 is invested in the compressor on June 1, 1950 the investor will receive a 5 per cent return on the money he has invested in the enterprise plus the equivalent of a receipt of $316 on June 1, 1950. Note that the amount invested in the enterprise begins at $5,000 and diminishes until the entire investment has been recovered.

To decide whether or not to purchase the compressor, the prospective purchaser should compare the gain in prospect from it against a feeling or opinion regarding nonspecified opportunities that appear to be in prospect or against the prospects from some specific alternative. In the latter case, as will be demonstrated later, the net present worth of receipts less disbursements for the latter specified alternative can be calculated for comparison.

Equivalent annual basis of evaluation. The equivalent annual difference between receipts and disbursements at 5 per cent interest may be calculated. Ordinarily the first step in this calculation is to find the present worths of receipts and disbursements; this was done for the present-worth evaluation above. Next these quantities are converted to an annual basis in the following manner:

Equivalent annual receipts, 6-1-50 to 6-1-54:	
$\$9,497 \times \overset{RP\ 5\text{-}4}{(.28201)}$	\$2,678
Equivalent annual disbursements, 6-1-50 to 6-1-54:	
$\$9,181 \times \overset{RP\ 5\text{-}4}{(.28201)}$	2,589
Equivalent annual receipts less disbursements	\$ 89

This result means that if \$5,000 is invested in the compressor on June 1, 1950, a 5 per cent return will be received on the amount invested in the compressor for the time it is invested plus the equivalent of receipts of \$89 on June 1, 1951, 1952, 1953, and 1954.

Capitalized basis of evaluation. The capitalized basis of evaluation is in favor in some quarters for consideration of long-term opportunities. Typical of such opportunities are highway and railway cuts, fills, embankments, water power developments, and other assets of long life. On a capitalized basis, the income and disbursements will be calculated as though they will continue in perpetuity or, in other words, forever. This basis of evaluation is not ordinarily used for assets of short life.

The capitalized basis of evaluation consists of finding a single amount in the present whose return at a given rate of interest will be equivalent to the net difference of receipts and disbursements if the given patterns of disbursements and receipts were repeated in perpetuity. This would take place in the example if a new compressor were purchased each fourth year in the future and if given disbursements and receipts were repeated during the life of each compressor.

Calculations for a capitalized evaluation may begin with the equivalent annual receipts and disbursements previously calculated to be \$2,678 and \$2,589 for each year-end of the first four years. To capitalize these amounts, it is only necessary to determine amounts in the present whose annual return at 5 per cent will be \$2,678 and \$2,589 respectively.

Capitalized receipts as of 6-1-50:	
\$2,678 ÷ .05	\$53,560
Capitalized disbursements as of 6-1-50:	
\$2589 ÷ .05	51,780
Capitalized receipts less capitalized disbursements	\$ 1,780

This result means that an investment of \$5,000 followed by renewals of compressors out of earnings each four years forever, provided that the patterns of receipts and disbursements of the first compressor are repeated, will have results equivalent to an investment of \$5,000 at 5 per cent interest forever plus a receipt of \$1,780 on June 1, 1950.

Rate-of-return evaluation. An opportunity can be evaluated on the basis of the rate of return that may be secured on the funds invested in it. This can be done by determining the rate of interest for which the present worth of disbursements and receipts will be equal. This may be done by trial-and-error calculations for different rates of interest and interpolation. The necessary calculation of present worths will follow the pattern illustrated previously under Present-Worth Evaluation.

Try 7%:

Present worth of receipts at 7%	\$9,044
Present worth of disbursements at 7%	8,999
Present worth of receipts less disbursements at 7%	\$ 45

Try 8%:

Present worth of receipts at 8%	\$8,831
Present worth of disbursements at 8%	8,912
Present worth of receipts less disbursements at 8%	−\$ 81

To find the desired value of i, remember that the present worth of receipts less present worth of disbursements equals 0 when the interest rate equals i. By interpolation:

$$i = \left[7 + 1 \times \frac{45 - 0}{45 - (-81)}\right]$$

$$= 7.36 \text{ per cent}$$

This result means that, if $5,000 is invested in the compressor on June 1, 1950, a return of 7.36 per cent will be received on the amount invested for the time it is invested.

Service life evaluation. The desirability of an opportunity involving investment of funds in equipment may be evaluated by determining the life of the equipment for which the present worth of receipts and disbursements at a given per cent of interest will be equal. In the example it may reasonably be assumed that if the estimated service life of the compressor were decreased by one year, receipts and disbursements would be decreased by $2,400 and $1,000 respectively. On this basis, the length of life that will make the present worths of receipts and disbursements equal is then determined by trial-and-error calculations.

Try a service life of 4 years. For a service life of 4 years previous calculations have resulted in:

Present worth of receipts as of 6-1-50 at 5%	$9,497
Present worth of disbursements as of 6-1-50 at 5%	9,181
Present worth of receipts less disbursements for life of 4 years	$ 316

Try a service life of 3 years. Under this condition the fourth-year receipts and disbursements are omitted:

Present worth of receipts at 5% as of 6-1-50:		
$2,400 × (2.723) [PR 5-3]		$6,535
$1,200 × (.8638) [PS 5-3]		1,037
		$7,572
Present worth of disbursements at 5% as of 6-1-50:		
$5,000 × 1	$5,000	
$1,000 × (.9524) [PS 5-1]	952	
($1,000 + $700) × (.9070) [PS 5-2]	1,542	
$1,000 × (.8638) [PS 5-3]	864	$8,358
Present worth of receipts less disbursements for life of 3 years		−$ 786

To find the desired value of n, recall that the present worth of receipts less disbursements for n is equal to 0. By interpolation

$$n = \left[3 + 1 \times \frac{-786 - 0}{-786 - (+316)}\right]$$

$$n = 3.29$$

This result means that the investment of \$5,000 in a compressor will yield 5 per cent on the basis of the estimated receipts and disbursements for a service life of 3.71 years for the compressor.

The several methods of evaluation illustrated above should be considered to be different ways of measuring. The best method to use is the one that is most easily interpreted by the persons concerned. In illustration of this point, consider expression of the capacity of a pump as 1,350 gallons per minute, 3.0 cubic feet per second, or 648,000 pounds per hour. Any of these would be meaningful to a person who was accustomed to their use.

It should be noted that present-worth amount, equivalent annual amount, and capitalized amount methods result in equal ratios as follows:

$$\frac{\text{Present Worth of Receipts}}{\text{Present Worth of Disbursements}} = \frac{\$9{,}497}{\$9{,}181} = \frac{\text{Equiv. Annual Receipts}}{\text{Equiv. Annual Disbursements}} = \frac{\$2{,}678}{\$2{,}589} = \frac{\text{Capitalized Receipts}}{\text{Capitalized Disbursements}} = \frac{\$58{,}560}{\$51{,}780}$$

It is also worth while to remember that the above ratios are the results of calculations as follows:

$$\text{Present-Worth Amount} \times \overset{\text{RP } i\text{-}n}{(\text{xxxxx})} = \text{Equivalent Annual Amount}$$

$$\text{Equivalent Annual Amount} \div i = \text{Capitalized Amount}$$

$$\text{Present-Worth Amount} \times \overset{\text{RP } i\text{-}n}{(\text{xxxxx})} \div i = \text{Capitalized Amount}$$

Of the several methods explained above for evaluating an opportunity where the choice is between accepting it or not accepting it, the rate-of-return method is probably meaningful to more people than any other. The service-life method is also readily understood and has merit particularly as a supplement to other methods.

The hazard of this opportunity is quite clearly revealed by the fact that a decrease of prospective service life from 4 to 3.71 years will reduce the rate of return to 5 per cent. The calculated rate of interest of 7.36 per cent shows that a very good rate of return is in prospect if things turn out as estimated.

The equivalent annual amount, present-worth amount, and capitalized amount methods are little used where the choice is between accepting or rejecting an opportunity.

Other bases of evaluation. It is often desirable to evaluate situations on bases other than those outlined above. Since the cost of calculation is ordinarily negligible, several bases of evaluation should be used if their use will improve the accuracy of decision.

A first concern in investment is to recover capital. For this reason evaluation might have been made on the basis of a 0 per cent interest rate. On this basis

Present worth of receipts	$10,800
Present worth of disbursements	9,700
Difference of receipts and disbursements for 0% interest	$ 1,100

Additional appreciation of a situation may often be gained by determining the annual income, the annual operating cost, or the receipts from salvage sale of equipment for which the present worths, for example, of receipts and disbursements will be equal.

The results of calculations suggested above follow. At 5 per cent interest, present worth of receipts and disbursements will be equal in the above example when

Annual receipts are reduced from $2,400 to $2,311

or

Annual operating costs are increased from $1,000 to $1,089

or

Salvage value is decreased from $1,200 to $816.

At 0 per cent interest, present worth of receipts and disbursements will be equal when

Annual receipts are reduced from $2,400 to $2,125

or

Annual operating costs are increased from $1,000 to $1,275 or

Salvage value is decreased from $1,200 to $100.

The above analyses reveal that a relatively small percentage decrease in annual receipts or increase in annual operating cost would result in the loss of capital in the venture.

Choice between Alternatives of Different Inputs and Outputs

In a search for opportunities for the profitable employment of resources, alternatives of different input and output may be found. For instance, a person may be seeking opportunities for the investment of $200,000. In his search he may find an apartment building to purchase for $200,000 or an opportunity to invest $150,000 in an enterprise being organized to manufacture a line of automobile accessories. Assuming that other alternatives have not been delineated, this situation results in at least three alternatives designated by letters as follows:

(A) Invest $200,000 apartment building.
(B) Invest $150,000 in new manufacturing enterprise and $50,000 in undelineated alternatives.
(C) Invest $200,000 in undelineated alternatives exclusive of A and B.

In this situation the choice is between A and B, A and C, and B and C.

Since Alternatives A and B may vary widely in the period of years they embrace, activities to be carried on, nature of equipment to be used, cost of money, and so forth, they are difficult to compare. Each should be evaluated independently in a manner similar to that illustrated with the compressor example. When this has been done comparison should be made between A and C, between B and C, and, finally, between A and B. This plan avoids the error of selecting one from two alternatives, as for instance from A and B, when neither may be as desirable as C.

Choice between Alternatives That Provide Services of Identical or Equal Value

A great many, if not most, problems in engineering economy involve the comparison of alternatives with outputs that are identical or equal in value. In such situations an aim in economy is to provide the desired service at least cost.

Suppose that it is desired to route a highway over a stream. The desired service or output is a passageway for traffic. This may be provided, for example, with a suspension bridge or a reinforced concrete bridge with many piers.

If both types of bridge will provide traffic-ways that are either *identical* or *equal in value,* their outputs will cancel and the choice between them can be made on the basis of cost.

Alternatives that provide identical services. Two alternatives will be considered to provide identical services when they meet needs that are the same in all significant particulars. Consider concrete mixtures. Where ability to carry a load is the only consideration, mixtures that have the same strength per unit area will be considered identical. But where abrasion, color, and imperviousness to moisture and quicksetting are of economic significance, mixtures will be considered to be identical only if they are identical in the qualities enumerated.

When alternatives will provide identical services, the services of one alternative will offset or cancel those of the other. Therefore the comparison of alternatives that provide identical services may be made on the basis of input or the cost of providing them.

Alternatives that provide services of equal value. In some cases alternatives will be found to provide services that are equal in value though not identical. Where outputs are equal in value, they can be canceled and the alternatives compared on the basis of their inputs.

Where alternatives have unequal outputs, monetary compensation may be made for inequalities in value of output so that comparison may be made on the basis of input.

Suppose that two insulating coverings, A and B, to reduce heat loss from a steam pipe, are being considered. Covering A will reduce the present heat loss by 90 per cent and Covering B will reduce it by 92 per cent. If the total annual cost of heat lost from the surface to be covered is \$600, the loss will be $\$600 \times (1 - .90) = \$60$ if A is applied and $\$600 \times (1 - .92) = \$48$ is B is applied.

It is clear that the value of the service or output of B is \$12 per annum greater than the output of A.

This difference in output may be compensated for by adding \$12 to the annual output and $12 to the annual input of A, thus placing A and B on an equal basis with regard to output.

In the above example, the service to be rendered might have been considered to be the reduction of the annual heat loss to zero. Then $60 and $48 might have been added to the annual cost of the inputs of A and B respectively to place the two alternatives on a common footing in regard to service.

Alternatives that provide service of equal unit value. In some cases alternatives have outputs that are identical or equal in value per unit of time. For example, an untreated telephone pole will give just as good service while it lasts as a pole treated with a preservation to lengthen its life. Suppose that an untreated pole will cost $16 and will last 8 years and that a treated pole will cost $21 and will last 14 years. If interest is neglected, the cost per year of service is $16 ÷ 8 = $2 for the untreated pole and $21 ÷ 14 = $1.50 for the treated pole. Here a comparison can be made on the basis of the cost per unit of output provided by the alternatives. The unit of output in either case is the service provided by a pole for a period of one year.

When a service is to be provided over a period of time by a series of renewable facilities, the outputs of alternative facilities per unit of time may be equal. If so, comparison of the alternatives can be made on the basis of the cost per unit of service.

Positive and Negative Outputs, Inputs, Receipts, and Disbursements

In the above example involving insulation it was proper to add $12 to both output and input of Covering A because a positive output is equivalent to a negative input. The reverse, that a positive input is equivalent to a negative output, is of course also true. Similarly a positive receipt is equivalent to a negative disbursement and vice versa, as has been suggested in previous illustrative problems.

Frequently proposals are made for improvements that will result in a saving. A saving may be considered to be a negative disbursement or expenditure. Thus a saving is equivalent to a receipt or an income.

Methods of Comparing Alternatives That Provide Services Which Are Identical or Equal in Value

The application of the five bases of comparison outlined earlier in the chapter will be illustrated below. Where only cost of input need be taken into consideration, these bases may be referred to as follows:

Equivalent Annual Cost
Present-Worth Cost
Capitalized Cost
Rate of Return
Service Life

Example of alternatives requiring approximately equivalent initial and future annual expenditures. Two alternative methods have been proposed for providing a certain service for 12 years.

Alternative A will require an immediate investment of $30,000 in a building that will depreciate to zero value in 12 years. The cost of upkeep, heat, light, taxes, and insurance has been estimated at $2,200 per annum.

Alternative B will require an immediate investment of $20,000 in a building that will depreciate to zero in 12 years. The cost of upkeep, heat, light, taxes, and insurance upon the building plus rental of additional storage space has been estimated at $3,200 per annum.

The interest rate to be used is 5 per cent.

PRESENT-WORTH COST COMPARISON

Alternative A	
Present worth of initial investment, $30,000 × 1	$30,000
Present worth of operating cost, $2,200 $(\overset{PR\ 5\text{-}12}{8.863})$	19,499
	$49,499
Alternative B	
Present worth of initial investment, $20,000 × 1	$20,000
Present worth of operating cost, $3,200 $(\overset{PR\ 5\text{-}12}{8.863})$	28,361
	$48,361

The sum of $49,499 for Alternative A can be thought of as the lump-sum present cost of twelve years of service for interest at 5 per cent. The comparison shows Alternative B to have an advantage of a present-worth cost of $1,138 over Alternative A.

Equivalent annual cost comparison. The equivalent annual cost can be obtained by multiplying the present-worth costs found above by the capital-recovery factor $(\overset{RP\ 5\text{-}12}{.11283})$. The result of this method is the sum of the sinking-fund depreciation, interest on unrecovered capital in the depreciating asset, and annual operating cost.

EQUIVALENT ANNUAL COST COMPARISON

Alternative A

Capital recovery with return on initial investment, $30,000 (.11283) [RP 5-12]	$3,385
Annual operating cost, $2,200 × 1	2,200
	$5,585

Alternative B

Capital recovery with return on initial investment, $20,000 (.11283) [RP 5-12]	$2,257
Annual operating cost, $3,200 × 1	3,200
	$5,457

$5,585 and $5,457 are interpreted as the annual year-end costs of providing the desired service with Alternative A and Alternative B, respectively.

On the basis of the above comparison, the equivalent annual cost of Alternative B is less than that of Alternative A by $128 per annum.

Capitalized cost comparison. Capitalized cost can most easily be obtained by dividing the equivalent annual cost by the interest rate. This method will be used to calculate the capitalized cost of Alternative A. Capitalized cost of Alternative B will be calculated in a way that gives a clearer insight into the significance of capitalized cost. It should be borne in mind that the capitalized cost of an alternative is the present sum that will pay for the service provided by the alternative forever. Capitalized cost may also be considered to be the present-worth cost of providing a service forever. Capitalized cost is most widely used for comparison of assets of long life, particularly if the annual cost associated with them is relatively low.

CAPITALIZED COST COMPARISON

Alernative A

Equivalent annual cost ÷ i, $5,585 ÷ .05	$111,700

Alternative B

Cost of initial asset..................................	$ 20,000
Capital cost of renewals, $20,000 $\overset{\text{RS 5-12}}{(.06283)}$ ÷ .05...........	25,140
Capital cost of annual operating cost, $3,200 ÷ .05........	64,000
	$109,140

In the calculation of capitalized cost of Alternative B, it is clear that $20,000 would have to be available to purchase the initial asset. To accumulate $20,000 to purchase a renewal at the end of each 12 year period,

$$\$20{,}000 \overset{\text{RS 5-12}}{(.06283)} = \$1{,}257$$

might be deposited at the end of each year in a sinking fund to compound at 5 per cent interest. A sum whose annual interest would be sufficient to provide for the annual deposit of $1,257 would be equal to $1,257 ÷ .05 = $25,140. An additional capital amount of $3,200 ÷ .05 = $64,000 will be needed to earn $3,200 each year to meet annual operating costs.

In the above example $111,700 may be interpreted as the present sum that will provide the service of Alternative A forever.

On the basis of the above comparison, the desired service can be had forever with Alternative B for a present cost $2,560 less than the present cost of perpetual service of Alternative A.

Rate-of-return comparison. Previous comparisons have shown that Alternative A is the less desirable choice for an interest rate of 5 per cent. Alternative A also requires a higher initial investment than Alternative B. On the basis of interest at 5 per cent there is no justification for making the larger investment required by Alternative A. However, at some interest rate less than 5 per cent Alternative A will be more economical than Alternative B. An insight into the situation can be gained by calculating the interest rate for which the two alternatives will have equal annual costs as follows:

At some interest rate i, the present-worth cost of Alternative A equals the present-worth cost of Alternative B. Then

$$\$30{,}000 + \$2{,}200 \overset{\text{PR } i\text{-12}}{(\text{xxxxx})} = \$20{,}000 + \$3{,}200 \overset{\text{PR } i\text{-12}}{(\text{xxxxx})}$$

$$\$1{,}000 \overset{\text{PR } i\text{-12}}{(\text{xxxxx})} = \$10{,}000$$

$$\overset{\text{PR } i\text{-}12}{(10.000)} = 10{,}000 \div 1{,}000$$

From the tables,

$$\overset{\text{PR 2-12}}{(10.575)} \quad \text{and} \quad \overset{\text{PR 3-12}}{(9.954\ )}$$

By interpolation

$$i = 2 + 1 \times \frac{10.575 - 10.00}{10.575 - 9.954}$$

$$i = 2 + \frac{.575}{.621}$$

$$i = 2.93 \text{ per cent}$$

Where funds are considered to earn less than 2.93 per cent interest, Alternative A will be most desirable.

Service life comparison. The service life of 12 years for Alternatives A and B is of necessity the result of estimates and may be in error. If the service for which the alternatives are under consideration were actually needed for some period other than 12 years, the advantage might pass from B to A. The service life for which the two alternatives will have equal annual costs may be calculated as follows.

For some service life of *n* years, the equivalent annual cost of Alternative A will equal the equivalent annual cost of Alternative B. Expressing this as an equation,

$$\$30{,}000 \overset{\text{RP 5-}n}{(\text{xxxxx})} + \$2{,}200 = \$20{,}000 \overset{\text{RP 5-}n}{(\text{xxxxx})} + \$3{,}200$$

$$\$10{,}000 \overset{\text{RP 5-}n}{(\text{xxxxx})} = \$1{,}000$$

$$\overset{\text{RP 5-}n}{(.10000)} = 1{,}000 \div 10{,}000$$

From the tables,

$$\overset{\text{RP 5-14}}{(.10102)} \quad \text{and} \quad \overset{\text{RP 5-15}}{(.09634)}$$

By interpolation

$$n = 14 + 1 \times \frac{.10102 - .10000}{.10102 - .09634}$$

$$n = 14 + \frac{.00102}{.00468}$$

$$n = 14.22 \text{ years}$$

This result means that if the desired service will be needed for more than 14.22 years, Alternative A will be the less costly alternative.

Another comparison based upon service life is sometimes pertinent. Suppose that the service for which the alternatives are being considered is of a continuing nature and will be provided through a series of renewals of buildings as they become unserviceable. Also suppose there is a feeling that Alternative A might have a service life longer than that of Alternative B. Then, assuming the service life of Alternative B to be 12 years, the service life of Alternative A for which the two alternatives will have equal annual costs may be calculated as follows:

Let the equivalent annual cost of Alternative A, based on a service life of n years, equal the equivalent annual cost of Alternative B. based on a service life of 12 years. Expressing this as an equation,

$$\$30{,}000 \overset{\text{RP 5-}n}{(\text{xxxxx})} + \$2{,}200 = \$20{,}000 \overset{\text{RP 5-12}}{(.11283)} + \$3{,}200$$

$$\$30{,}000 \overset{\text{RP 5-}n}{(\text{xxxxx})} = \$2{,}257 + \$3{,}200 - \$2{,}200 = \$3{,}257$$

$$\overset{\text{RP 5-}n}{(.10857)} = 3{,}257 \div 30{,}000 = .10857$$

From the table,

$$\overset{\text{RP 5-12}}{(.11283)} \quad \text{and} \quad \overset{\text{RP 5-13}}{(.10646)}$$

By interpolation

$$n = 12 + 1 \times \frac{.11283 - .10857}{.11283 - .10646}$$

$$n = 12 \times \frac{.00426}{.00637}$$

$$n = 12.67 \text{ years}$$

Comparison of Alternatives of High Initial Cost with Alternatives of High Annual Cost

A bank has determined that a central proofer is needed for balancing its accounts each day. Proofers may be purchased outright or may be acquired on a rental basis. Interest is taken at 6 per cent because this return can be realized on loans made by the bank.

Alternative C. The proofer can be purchased for $5,900 with the provision that the vendor will service the machine without charge for a period of 10 years. At the end of 10 years the vendor will allow $400 for the old machine on the purchase of new equipment.

Alternative D. A proofer can be rented for $66 per month or $792 per year for 10 years. The owner will service the machine and replace it when it becomes unserviceable.

Present-worth cost comparison

Alternative C, present-worth cost of 10 years of service:	
Present worth of first cost	$5,900
Present worth of receipts from salvage, $400 (.5584) [PS 6-10]	223
	$5,677
Alternative D, present-worth cost of 10 years of service:	
Present worth of rental, $66 × 12 (7.360) [PR 6-10]	$5,829
Advantage in favor of Alternative C for 6% interest	$ 152

Equivalent annual cost comparison

Alternative C, equivalent annual cost of 10 years of service:	
Sinking fund depreciation plus return,	
($5,900 − $400) (.13587) [RP 6-10] + $400 × .06	771
Alternative D, annual cost of 10 years of service:	
Annual rental $66 × 12	792
Advantage in favor of Alternative C for 6% interest	$ 21

Rate-of-return comparison. On the basis of the present-worth comparison given above, there is a positive advantage in favor of Alternative C of $152 when the interest rate is 6 per cent. The problem is to find the value of i for which the advantage in favor of Alternative C is equal to zero.

For 7% interest

Alternative C, present worth of 10 years of service,	
$5,900 − $400 (.5083) [PS 7-10]	$5,697
Alternative D, present worth of 10 years of service,	
$66 × 12 (7.024) [PR 7-10]	5,563
Advantage in favor of Alternative C for 7% interest	−$ 134

Let i equal the interest rate for which the advantage in favor of Alternative C is equal to zero. Then by interpolation

$$i = 6 + 1 \times \frac{152 - 0}{152 - (-134)}$$

$$i = 6 + \frac{152}{286}$$

$$i = 6.53 \text{ per cent}$$

Service life comparison. When an investment is made in a unit, such as the proofer under consideration in this problem, there is always a possibility that a more desirable way of meeting the need may become available. Thus the service life of the original unit may be terminated before the end of its estimated life. An advantage of renting a service is that it may be discontinued without penalty whenever a more desirable service presents itself.

For this reason it may be desirable to determine the service life for which the costs of Alternatives C and D will be equal. It will be assumed that the trade-in value of the purchased proofer will remain $400.

For some value of n, the present-worth cost of Alternative C will equal the present-worth cost of Alternative D. For 6 per cent

$$\$5{,}900 - \$400 \overset{\text{PS 6-}n}{(\text{xxxxx})} = \$66 \times 12 \overset{\text{PR 6-}n}{(\text{xxxxx})}$$

Try $n = 9$:

Alternative C, present-worth cost of 9 years of service,	
PS 6-9 $5,900 − $400 (.5919)	$5,663
Alternative D, present-worth cost of 9 years of service,	
PR 6-9 $66 × 12 (6.802)	5,387
Advantage in favor of Alternative C for $n = 9$	−$ 276

From previous calculations, the advantage in favor of Alternative C for $n = 10$ is equal to $152. By interpolation

$$n = 9 + 1 \times \frac{-276 - 0}{-276 - 152}$$

$$n = 9 + \frac{276}{429}$$

$$n = 9.65 \text{ years when } i \text{ is 6 per cent}$$

The above results show that a slight shortening of the service life causes the advantage to pass from Alternative C to Alternative D for an interest rate of 6 per cent.

Comparison of Alternatives of Different Service Lives

Poles for a certain telephone line that is expected to be carried on poles for many years can be provided as follows.

Alternative E. Use untreated poles whose average life is estimated at 12 years. These poles can be purchased for $15.75 each and can be installed at a cost of $1.75 per pole. Thus each pole in place will cost $17.50.

Alternative F. Buy poles suitable for treatment with a preservative. Treated poles are estimated to have a service life of 18 years. These poles will cost $15.25 each and treatment will cost $4.25 per pole; installation will cost $1.75 per pole as in Alternative E. Thus each pole in place will cost $21.25.

Present-worth comparison. Since the two alternatives provide for different periods of service, some method must be used to embrace equal periods of service in a present-worth comparison. This can be done by making the present-worth comparison on the basis of a service period of 36 years. The service period will require 3 poles of Alternative E, one purchased at the beginning of each 12-year period, and 2 poles of Alternative F, one purchased at the beginning of each 18-year period of the 36 years under consideration. Interest will be taken at 6 per cent.

Alternative E, present-worth cost of 36 years of service per pole	
Present worth of first pole, $17.50 × 1	$17.50
Present worth of second pole, $17.50 $(.4970)_{PS\ 6\text{-}12}$	8.70
Present worth of third pole, $17.50 $(.2470)_{PS\ 6\text{-}24}$	4.32
	$30.52
Alternative F, present worth cost of 36 years of service per pole	
Present worth of first pole, $21.25 × 1	$21.25
Present worth of second pole, $21.25 $(.3505)_{PS\ 6\text{-}18}$	7.44
	$28.69

These results show that 36 years of pole service can be had with Alternative F for a present-worth cost that is $1.83 less than that of

Alternative E. In view of the fact that the service period greatly exceeds the service life of either pole, an interpretation that might be more meaningful is that the prospective cost for pole service with Alternative F is ($28.69 ÷ $30.52) = .94 times or 94 per cent of the cost of the same service with Alternative E.

Equivalent annual cost comparison

Alternative E, equivalent annual cost per pole,	
$17.50 \times \overset{RP\ 6\text{-}12}{(.11928)}$	$2.09
Alternative F, equivalent annual cost per pole,	
$21.25 \times \overset{RP\ 6\text{-}18}{(.09236)}$	$1.96

These results show that the prospective annual cost of Alternative F is $.13 less per annum than that of Alternative E. It may also be useful to state the cost of service of Alternative F as a percentage of the cost of Alternative E. The annual cost of service with Alternative F will be ($1.96 ÷ $2.09) = .94 or 94 per cent of the cost with Alternative E.

Alternative E and Alternative F may also be compared by other methods previously illustrated.

Discussion of Present-Worth, Equivalent Annual, and Capitalized Amount Methods of Comparison

Each of these three old and established bases of comparison has its champions. These bases of comparison are mathematically mutually convertible in accordance with the following expression:

$$\text{Present Worth} \times \overset{RP\ i\text{-}n}{(\text{xxxxx})} = \text{Equivalent Annual Amount}$$
$$= i \times \text{Capitalized Amount}$$

Capitalized amount basis. The capitalized amount basis is best suited for evaluation and comparison of long-term alternatives with stable income and interest costs that are high in relation to subsequent costs. Suppose that a railway tunnel whose future upkeep will be negligible and which will reduce operating costs by $12,000 per year is under consideration. The capitalized cost method provides a ready means of determining how much may be invested in constructing the tunnel. For an interest rate of 6 per cent, the reduction in operating cost will justify investing

$$\$12{,}000 \div .06 = \$200{,}000$$

in the tunnel on the supposition that it will be used forever. For practical purposes, forever may be considered to be 50 years and over. In the above example the reduction of operating cost by $12,000 per year for 50 years will justify investing

$$\overset{\text{PR } 6\text{-}50}{\$12{,}000\ (15.762)} = \$189{,}144$$

in the tunnel. This amount differs slightly more than 5 per cent from $200,000.

An objection to the capitalized amount method is that its results are difficult to comprehend, particularly when applied to short-lived assets. In general this method is losing in favor and is now rarely applied to investments involving periods of less than 30 or 40 years.

Present-worth basis. The present-worth basis of evaluation is an outgrowth of valuation techniques and practices developed mainly since the turn of the century. The aim of valuation is ordinarily to determine a present worth of a property. The present worth of a property is often considered to be the present worth of the difference between future receipts and disbursements associated with it at a certain interest rate. The present-worth basis of comparison often results in amounts that are quite large in comparison with the amounts encountered in the alternatives being compared. A slight change in the interest rate may result in large changes in present-worth amounts. This method of comparison is somewhat cumbersome in comparing alternatives embracing different periods of time.

Equivalent annual amount basis. The development of this basis of comparison is associated with the development and general use of cost accounting, which necessitated consideration of costs on an annual basis and therefore consideration of depreciation as an annual cost. A distinct advantage of the equivalent annual amount basis of comparison is that it conforms to thought patterns that are used in accounting practices and are therefore familiar to most people in business. A second important advantage of this method is that the period under consideration is always a unit of time, namely one year. The amounts determined with it are actually a summation of receipts and disbursements per unit of time. Since equivalent annual amounts are based on a common unit, they are easier to comprehend and compare. Most people appear to live "by the year" and tend to think in terms of annual disbursements and receipts. Thus it seems logical to make economic comparisons on the basis of yearly periods. Because of inherent advantages and its general and growing use in activities associated with engineering, the equivalent annual amount basis of comparison should be favored over either the present-worth or the capitalized amount basis.

Rate-of-Return, Service Life, and Other Bases of Comparison

It is widely recognized that the best way to become familiar with the implications of a situation is to work with it. A keener insight into the factors involved and their relationships may often be obtained by comparing economic situations on several bases. Also, after common methods have been exhausted, it may often be profitable to make calculations to answer the question, "What would be the result if this happened?"

Rate-of-return comparison. This method is probably the best for comparing a concrete proposal with other opportunities believed to exist but not delineated. Rate of return is a universal measure of economic success. The meaning of rates of return is widely understood. Also the rates of return to be expected from different classes of opportunities are usually well established and generally known. Thus rates of return are norms representative of opportunities in general. These characteristics make this basis of comparison particularly well adapted to situations where the choice is between engaging in an opportunity or not engaging in it.

This basis of comparison also has merit as a supplement to other methods of making a choice between two concrete alternatives. Determination of the rate of return for which the alternatives will have equal cost is often revealing.

Service life comparison. Expressions like "This machine will pay for itself in less than three years" are common in industry and are indicative of the tendency to evaluate assets in terms of their service life. It is generally conceded that the longer the life, the greater the uncertainty. One merit of the service life comparison is that it specifically directs attention to the length of life embraced by an alternative. The hazard of an opportunity may often be pointedly revealed by calculating results for service lives less than that originally estimated. The service life method of evaluation is useful as a supplement to other methods of comparison.

Calculated Comparisons Embody Errors of Data Used

It must always be borne in mind that correctly calculated results are true only to the extent that the data used represent the actual facts. In reference to economy calculation, it is well to adopt the mental habit of thinking "This will be the result if things turn out in accordance with estimates." Calculations to determine the effect of errors in estimates on results will be helpful in developing a proper attitude toward the significance of calculated comparisons.

Also, it should be borne in mind that calculated results embrace only quantitative estimates. Though irreducible considerations are difficult to evaluate, they are no less worthy of careful consideration than quantitative aspects. Calculated results and irreducibles should serve jointly as material for judgment in deciding problems in engineering economy.

Methods of Depreciation

In economy studies calculations are made on the assumption that the estimated lives of assets will be realized. It would be absurd, for example, to estimate the life of an asset at 10 years and then make calculations on the basis of a shorter life, except to investigate the effect of deviations from the original estimate. When estimated service lives of assets are realized, depreciation plus interest on the undepreciated balance of an asset, *i.e.*, capital recovery with a return, is identical for all methods of depreciation. See page 100.

In present-worth calculations, a depreciation method need not be designated if the first cost of the asset is considered as a cost and the salvage value is considered as a receipt at the end of the asset's estimated life. The result is equivalent to calculations based on any depreciation method.

When comparisons are on an annual basis, the sinking-fund method of depreciation may be selected for its greater convenience, if there is no reason to the contrary. Since the straight-line method of depreciation is most widely used in this country, it is often desirable to present calculations based upon it. This will require calculation of the return on the undepreciated balance for each year, unless some approximation is used. The approximation of average interest on the undepreciated balance, as previously explained in connection with straight-line depreciation and average interest, is quite widely used and is reasonably accurate for periods under twenty years and for low interest rates.

Interest Rate, Service Life, and Risk as Elements of Cost

Economic activities undertaken for profit are entered into only after it is found that estimated receipts exceed estimated costs. When an activity has failed to yield a profit, it is because disbursements exceeded receipts. This may have resulted because some receipts were less than estimated, because some costs were greater than estimated, or from a combination of both varieties of incorrect estimates.

The usual items of cost of an activity are depreciation, interest, and operating expense. To estimate these costs, estimates are ordinarily made of the following detailed items:

Investment Cost
- A. First cost of asset

Depreciation Cost
- B. Salvage value of asset
- C. Life of asset

Interest Cost
- D. Interest rate

Operating Cost
- E. Direct labor
- F. Direct material
- G. Overhead, exclusive of depreciation, interest, and insurance.

Estimates of items A, B, and D are not ordinarily sources of error of important consequences. Items A and D are usually known within narrow limits. Item B, salvage value, can usually be estimated with reasonable accuracy when the prospective life of the asset is short. If the prospective life of the asset is long, an error in the estimate of its salvage value will usually be of minor importance. Item C, the life of the asset, and the amounts of items E, F, and G during the life of the asset are estimated with great difficulty and have important consequences in the success of a venture. Estimates of items C, E, F, and G are closely related to quantity of output and the total income to be received during the life of the activity in question.

But once items A to G above have been estimated on the best available information, there seems little point in making allowance for error; if an estimate is believed to be in error, it should be corrected. Furthermore, it seems unrealistic to make adjustments in the estimate of one item for suspected errors in the estimates of others, as is often done when the interest rate and service life are over- or under-estimated to compensate for errors in other items.

Risk associated with hazards as contrasted to errors associated with estimates can be entered, if known quantitatively, as a cost of operation. This is ordinarily done with fire and accident insurance premiums. If risk is not known in quantitative terms, it must be estimated and entered as an estimated cost of operation.

Income of an economic activity as measured in terms of receipts is equal to a summation of a number of units of products or services multiplied by the amount received for each. Thus the detailed items to be estimated before income can be estimated are the number of each type of service or product that will be made and the amount that will be received for each.

Effect of Composition of Alternatives, Interest Rate, and Service Life

When the amounts and patterns of occurrence of the costs of alternatives are similar, the interest rate and the service life used will have relatively little effect on the comparative advantage of the alternatives being considered.

Low-interest-rate estimates tend to favor alternatives whose costs originate predominantly as initial costs in contrast to alternatives whose predominant costs occur later in the period under consideration—those including such costs, for example, as annual costs.

Alternatives A and B described in a previous example have costs fairly similar in amount and pattern of distribution. The present worths of initial and annual costs of each of these alternatives are also comparable in amount. Alternatives C and D previously described have dissimilar patterns of costs. The costs of Alternative C are predominantly initial costs. Those of Alternative D are exclusively annual costs.

The effect of using different interest rates on the ratio of advantage of Alternative A to Alternative B and of Alternative C to Alternative D on the basis of present-worth calculation is given below:

Interest rate	4%	6%	8%
Present worth, Alt. A	\$50,647	\$48,445	\$46,579
Present worth, Alt. B	\$50,032	\$46,829	\$44,115
Ratio of advantage of Alt. A to Alt. B	1.010	1.035	1.056
Ratio change for interest rate change of 4 to 8%, in per cent:	$\frac{1.056 - 1.010}{1} = 4.6\%$		

Interest rate	4%	6%	8%
Present worth, Alt. C	\$5,630	\$5,677	\$5,715
Present worth, Alt. D	\$6,424	\$5,829	\$5,314
Ratio of advantage of Alt. C to Alt. D	.876	.974	1.078
Ratio change for interest rate change of 4 to 8%, in per cent.	$\frac{1.078 - .876}{1} = 20.2\%$		

Since the apparent ratio of advantage of alternatives is in some cases materially affected by the interest rate used in calculations, the interest rate should be chosen to represent the actual situation as nearly as possible.

When the costs for depreciation of alternatives are unequal, the alternative for which the depreciation cost is the lowest will be favored in a comparison by the use of relatively low service life estimates. Considerable error may be introduced in economy studies by the selection of fictitious service lives for comparison of alternatives whose costs for depreciation are dissimilar.

PROBLEMS

1. Mr. Green has an opportunity to engage in a venture for which the following disbursements and receipts are in prospect:

Year End	*Disbursements*	*Receipts*
0	\$4,000	0
1	600	800
2	600	1,800
3	400	2,200
4	200	1,600

Determine the desirability of the venture on the basis of:

(a) Present-worth amount with interest at 6%.

(b) Equivalent annual amount with interest at 6%.

(c) Capitalized amount if the above series is repeated indefinitely at 6%.

2. The present-worth amount of a series of payments extending over a period of 16 years at 6% is \$5,680.

(a) What is the equivalent annual amount of the series of payments?

(b) What investment, made at the beginning of the first year at 6% would make the series of payments and leave a balance equal to the original investment at the end of the 16th year?

3. The equivalent annual amount of a series of payments extending over a period of 12 years is $92.

(a) What is the present-worth amount of the series at 8%?

(b) What is the capitalized amount at 8% if the series repeats every 12 years for an infinitely long period?

4. Sam White earns a salary of $2,600 per year. What is the present worth of his salary if the interest rate is (a) 3%, (b) 4%, and (c) 6% and if he will earn the same salary for 30 years?

5. An investment results in an annual return of $2,000 a year. Determine the percentage of error that will result if capitalized amount is used instead of present worth to evaluate the returns for periods of investment of (a) 30 years, (b) 60 years, and (c) 90 years. Interest is 4%.

6. An investment of $20,000 is made at 6% in a venture that is expected to continue indefinitely.

(a) What annual payment is received?

(b) Assuming that the above investment was made in 6% bonds and that after 30 years the concern issuing the bonds became bankrupt, what is the loss expressed as of the time of the investment if there is no recovery of principal?

(c) Calculate the loss in part (b), assuming that the bankrupt company pays 50 cents on every dollar on outstanding bonds.

7. A manufacturer pays a patent royalty of $.62 per unit of a product he manufactures, payable at the end of each year. The patent will be in force for an additional 4 years. At present he manufactures 8,000 units of the product per year but it is estimated that output will be 10,000, 12,000, 14,000, and 16,000 in the four succeeding years. He is considering (a) asking the patent holder to terminate the present royalty contract in exchange for a single payment at present or (b) asking the patent holder to terminate the present contract in exchange for equal annual payments to be made at the beginning of each of the next four years. If 4% interest is used, what is (a) the present single payment and (b) the beginning-of-the-year payments that are equivalent to the royalty payments in prospect under the present agreement?

8. As usually quoted, the prepaid premium of insurance policies covering loss by fire and storm for a 3-year period is 2.5 times the premium for one year of coverage. What rate of interest does a purchaser receive on the additional present investment if he purchases a three-year policy now rather than three one-year policies at the beginning of succeeding years?

9. An investor has $40,000 to invest. He can invest the entire amount in Venture A, which is estimated to yield a return of 7%. Or he can invest $10,000 in Venture B, on which the estimated rate of return is 12%. If he undertakes Venture B, what return must be received on the balance of

$30,000 in order that he may receive a total return equal to that estimated for Venture A?

10. Assume that your prospective earnings will be $4,000, $5,000, $7,000, and $10,000 per year in the succeeding four decades of your life. What will be (a) the present worth, (b) the equivalent annual value, and (c) the capitalized amount of your earnings if interest is taken at 3%.

11. One factor considered by John Doe in deciding whether to attend college was the fact that his prospective earnings might be increased. He estimated that if he started to work upon completion of high school he could earn $2,400 per year for four years and this his prospective annual earnings would be $4,000, $5,000, $6,000, and $7,000 in succeeding decades thereafter. He estimated that if he went to college his earnings for the 4-year period would be negligible and that his expenses would be $1,200 per year greater than they would have been had he not gone. Assume that his prospective earnings after completion of college will be ($4,000 + X), ($5,000 + X), ($6,000 + X), and ($7,000 + X) per year in succeeding decades. Determine the value of X for which John Doe's education venture will just pay out if the interest rate is 3%.

12. A person can buy a household refrigerator for $260 cash. He is offered an identical refrigerator for 24 equal beginning-of-the-month payments of $13. What annual interest rate is charged by the vendor in extending credit?

13. A college graduate estimated that his education had cost the equivalent of $14,000, as of the date of graduation, considering his increased expenses and loss of earnings while in college. He estimated that his earnings during the first decade after leaving college would be no greater than if he had not gone to college. If, by virtue of his added preparation, $500, $1,000, and $2,000 additional per year is earned in succeeding decades, what is the rate of return realized on his $14,000 investment in education?

14. A person is offered a dwelling for $16,000. The down payment is to be $3,000 and the balance, plus 6% interest on unpaid balances, is to be paid in 12 equal year-end payments. The dwelling can also be purchased for $14,000 cash. At what interest rate are the two plans of purchase equivalent?

15. A bending fixture that will save $.06 operating cost per unit of product can be built for $600. Maintenance cost of the fixture is estimated at $30 per year. If 3,500 units are made per year, how long will it take the fixture to pay for itself with interest at 7%?

16. A temporary building whose salvage value at any time is estimated to be negligible can be built for $8,000. Annual rentals received less current operating costs are estimated at $1,260 per year.

(a) Evaluate the proposal by calculating the rate of return in prospect on the investment if the building is used for 8 years.

(b) Calculate what life of the building will result in a return of 10% on the investment.

17. James Roe estimates the cost of his college education at $12,000 as of the date of his graduation. He estimates that during the first five years after graduation his earnings will be the same as they would have been had he not had a college education. If after five years he enjoys additional earnings of $1,000 per year attributable to his investment in his education, how many years after graduation will it be before his investment will "pay for itself" at 4% interest?

18. Every year the stationery department of a large concern uses 1,200,000 sheets of paper with three holes drilled for binding and 250,000 sheets that have the corners rounded. At present the drilling and corner cutting is done by a commercial printing establishment at a cost of $.35 and $.30 per thousand sheets respectively.

Two alternatives are being considered. Alternative A consists of the purchase of a paper drill for $465 and Alternative B consists of the purchase of a combination paper drill and corner cutter for $650. Obviously the two alternatives do not provide equal service. The following data apply to the two machines:

	Drill	*Combined Drill and Cutter*
Life	12 years	12 years
Salvage value	$46.50	$55.00
Annual maintenance	5.00	6.00
Annual space charge	11.00	11.00
Annual labor to drill	32.00	36.00
Annual labor to cut corners		24.00
Interest rate	5%	5%

(a) Alter one or the other of the alternatives given above so that they may be compared on an equitable basis. Calculate the equivalent annual cost of each of the revised alternatives, using straight-line depreciation plus average interest.

(b) What other alternative or alternatives should be considered?

19. A certain service can be purchased for $100 per unit. The same service can be provided by the purchase at a cost of $90,000 of equipment to which the following estimates apply: Salvage value at the end of 6 years, $30,000; annual operating expense, $5,000 per year plus $20 per unit.

(a) What will be the rate of return on the investment if 250 units are made per year and if estimates prove to be correct?

(b) What will be the rate of return if 200 units are made per year?

(c) What will be the rate of return if 250 units are made per year, the useful life of the equipment is 5 years, and the salvage value is $30,000 at the end of the equipment's service life?

(d) Calculate the rate of return if the output is 250 units per year and the salvage value is $24,000 at end of 6 years.

(e) Calculate the rate of return if the output is 250 units per year and the variable cost is $25 per unit.

20. The heat loss of a bare steam pipe costs $206 per year. An insulation that will reduce the heat loss cost by 93 per cent can be installed for $116, and an insulation that will reduce the heat loss cost by 89 per cent can be installed for $60. The insulations require no additional expenses and will have no salvage value at the end of the pipe's estimated life of 8 years. Determine which of the two insulations is the more desirable if the interest rate is 10%.

21. A refinery can provide for water storage with a tank on a tower or a tank of equal capacity placed on a hill some distance from the refinery. The cost of installing the tank and tower is estimated at $82,000. The cost of installing the tank on the hill, including the extra length of service lines, is estimated at $60,000. The life of the two installations is estimated at 40 years, with negligible salvage value for either. The hill installation will require an additional investment of $6,000 in pumping equipment, whose life is estimated at 20 years with a salvage value of $500 at the end of that time. Annual cost of labor, electricity, repairs, and insurance incident to the pumping equipment is estimated at $500. The interest rate is 5%.

(a) Compare the two plans on the basis of equivalent annual cost, using sinking-fund depreciation.

(b) Compare the two plans on the basis of the present worth of 40 years of service.

(c) Compare the capitalized costs of the two plans.

22. Mr. Doe has been quoted a price of $11,000 on a house. *Plan I.* He can pay $1,000 on taking possession and the balance in equal payments over 10 years with 5% interest. The payments are to begin one year after taking possession. *Plan II.* He can purchase the house for $10,200 cash. He has $5,000 that he can invest and he can borrow $5,200 to be re-paid in 10 equal annual payments at 4%.
Determine the rate of interest for which the two plans are equal.

23. A copper roof and a wood shingle roof can be put on a certain railroad depot for $6,000 and $3,500 respectively. The life of the copper roof is estimated at 60 years (equal to the life of the building). The shingle roof is estimated to have a life of 15 years. Consider that neither roof will have any salvage value.

(a) Calculate the annual cost of each type of roof for an interest rate of 4%; for an interest rate of 10%.

(b) If the copper roof lasts the estimated 60 years, what must be the life of the shingle roof to make it equivalent to the copper roof if the interest rate is 4%? If the interest rate is 10%?

24. An investor can purchase a plot of ground at the present time for $30,000. If he receives no income from the land and taxes and other disbursements incident to ownership amount to $600 per year, what is the maximum number of years he can hold the land before disposing of it for $40,000 if he desires an 8% return on his investment?

25. A person purchased a number of lots for $3,200 on June 1, 1938 and gleefully reported that he had sold them on June 1, 1950 for double what he had paid for them. Annual taxes had been $50 and he had received no income during his period of ownership. What rate of interest had he made on his investment?

26. A warehouse and its contents are insured for $700,000. The present annual insurance premium is $.86 per hundred dollars of coverage. A sprinkler system with an estimated life of 20 years and no salvage value at the end of that time can be installed for $18,000. Annual operation and maintenance cost is estimated at $360. Taxes are .8 per cent of worth of warehouse, equipment and contents. If the sprinkler is installed, the premium rate will be reduced to $.38 per hundred dollars of coverage.

(a) Using straight-line depreciation plus average interest, determine the rate of return paid on the investment in the sprinkler system.

(b) How many years will be required for the sprinkler to pay for itself if the straight-line depreciation plus average interest method of calculation is used with an interest rate of 8%?

CHAPTER

11

Accounting, Cost Accounting, and Engineering Economy Studies

THE ACCOUNTING SYSTEMS OF ENTERPRISES are one of the most important sources of data for engineering economy studies. Also, the outcome of decisions based upon forecasts inherent in engineering economy studies will usually be evaluated in the future as their outcome is revealed by accounting records. For this reason it is desirable to examine the data that accounting systems provide in relationship to the requirements of economy studies.

Two classifications of accounting are generally recognized in industrial accounting. These are general accounting and cost accounting. Cost accounting, properly considered to be a branch of general accounting, is usually of greater importance in making engineering economy studies and evaluating the outcome of decisions based upon them than other phases of accounting.

General Accounting and Its Functions

Accounting consists of recording and summarizing transfers of assets and liabilities to determine the amount of various desired classifications of assets and liabilities. An important purpose of accounting is to provide summaries of the status of an enterprise or its subdivisions in terms of assets and liabilities from time to time as desired. Such summaries serve as a basis for judging the condition of the enterprise as a whole or of any of its subdivisions.

The *balance sheet* is a common form of a summary of assets and liabilities. It lists the assets and liabilities of an enterprise in terms of their monetary values as of a given date. The following balance sheet of the A.B.C. Company shows the principal items found on such records.

BALANCE SHEET

A.B.C. Company as of closing of business Dec. 31, 1949

Cash	$161,000	Notes payable	$ 22,000
Accounts receivable	7,000	Accounts payable	4,700
Raw materials	9,000	Accrued taxes	3,200
Work in process	17,000	Depreciation reserve	31,000
Finished goods	3,700	Capital stock	200,000
Land	11,000	Declared dividends	40,000
Factory building	82,000		
Equipment	34,000		$300,900
Prepaid services	1,300	Surplus	25,100
	$326,000		$326,000

In general each item on the balance sheet is itself a summary. For example, the item *raw materials* is a summary of the value of all items of raw material as revealed by actual inventory or by detailed records. Balance sheets are normally drawn up annually, quarterly, or monthly or at other regular intervals. The change of a company's condition during the interval between balance sheets may be determined by comparing successive balance sheets.

In order to have information relative to the change of conditions that has taken place during the interval between successive balance sheets, it is a usual accounting practice to draw up a *statement of income and expense.* This statement of income and expense is also known as a profit-and-loss statement. It is a summary in more or less detail, depending on the purpose it is to serve, of the source and amount of gross income, gross profit, cost of goods sold, selling expenses, and net profit from operations during the interval covered.

Balance sheets and similar accounting statements convey the appearance of absolute accuracy. Actually many items in accounting are based upon estimates. For example, values shown for accounts receivable usually involve estimates of uncollectible accounts. Also, the asset value of land, buildings, and equipment and the liability in terms of depreciation reserve are estimates in terms of book value, which may be at great variance with actual current values. Similarly, an examination of other items on a balance sheet will generally show that they embody estimates.

Cost Accounting

Cost accounting is a branch of accounting adapted to registering the costs for labor, material, and overhead item by item as a means of determining the cost of producing specific items or lots of items of products or services.

The detailed information about the costs of products and services provided by cost accounting has four principal purposes. These are:

(1) To determine the actual cost of products
(2) To serve as a basis for the control of expenditures
(3) To serve as a basis for pricing products
(4) To provide information on which to base operating decisions and policies.

The last-named purpose is of primary concern in engineering economy studies, for data provided by cost accounting is used in such studies as a basis for arriving at decisions and for evaluating their outcome.

Classifications of Cost

The costs that are incurred to produce and sell an item of product are commonly classified as illustrated in the following tabulation of costs for a specific item:

Direct material	$1.57		
Direct labor	2.06		
Factory overhead	1.90		
Factory cost		$5.53	
Administrative cost		.62	
Production cost			$6.15
Selling cost			1.16
Cost of sales			$7.31

Direct material. The material whose cost is directly charged to a product is termed *direct material.* Ordinarily, the costs of principal items of material required to make a product are charged to it as direct material costs. Charges for direct material are made to the product at the time the material is issued, through the use of forms and procedures designed for that purpose. The sum of charges for materials that accumulate against the product during its passage through the factory constitutes the total direct material cost.

In the manufacture of many products, small amounts of a number of items of material may be consumed which are not directly charged to the product. These items are charged to factory overhead, as will be explained later. They are not directly charged to the product on the premise that the advantage to be gained will not be enough to offset the increased cost of record keeping.

Though perhaps less subject to gross error than records of other elements of cost, records of direct material costs should not be used in engineering economy studies without being questioned. Their accuracy in regard to quantity and price of material should be ascertained. Also, their applicability to the situation being considered should be established before they are used.

Direct labor. *Direct labor* is labor whose cost is charged directly to the product. The source of this charge is time tickets or similar forms used to record the time and wages of workmen whose efforts are applied to a product during its journey through the factory. Unless the allocation of labor costs to products is very closely controlled, records of labor costs charged to specific products are likely to be in error.

As a result of either carelessness or a desire to conceal an undue amount of time spent on a job, some of the time applied to one job may be reported as being applied to another. Thus, direct-labor cost records should be carefully examined for accuracy and applicability to the situations under investigation before being used as data for engineering economy studies.

Various small amounts of labor may not be considered to warrant the record-keeping that is required to charge them as direct labor. Such items of labor became part of the factory overhead. The labor of personnel engaged in such activities as inspection, testing, or moving the product from machine to machine or in pickling, painting, or washing the product is often charged in this way.

Factory overhead. This classification of costs is also designated by such terms as factory expense, shop expense, burden, indirect costs, and on-cost. Factory overhead costs embrace all expenses incurred in factory production which are not directly charged to products as direct material or direct labor.

Factory overhead costs embrace costs of material and labor not charged directly to product and fixed costs. Fixed costs embrace charges for such things as taxes, insurance, interest, rental, depreciation and maintenance of buildings, furniture and equipment, and salaries of factory supervision, which are considered to be independent of volume of production.

Indirect material and labor costs embrace costs of all items of material and labor consumed in manufacture which are not charged to the product as direct material or direct labor.

Bases for allocation of factory overhead. There are several methods of allocating factory overhead charges to the product. Four of these methods will be explained and compared in the following pages. These four methods are

1. Direct-labor-cost method.
2. Direct-labor-hour method.
3. Direct-material-cost method.
4. Machine-rate method.

In each of these methods it is necessary to estimate the expected annual overhead charges and the expected normal activity of the factory or department under consideration. Estimates of normal overhead charges and normal activity are based upon calculations, past experience, and budgets for the coming year. Overhead charges are expressed in terms of dollars, and activity is expressed in terms of cost of direct labor, number of hours of direct labor, cost of direct material, or number of hours of operations by classes of machines. When normal overhead and normal activity have been estimated, rates for the allocation of overhead to the product by the methods under consideration are calculated as follows:

$$\text{Direct-labor-cost rate} = \frac{\text{Annual overhead in dollars}}{\text{Annual direct labor cost in dollars}}$$

The overhead charge for a product as determined by this rate will be equal to the rate times the direct labor cost of the product. The following rates are applied in a similar manner.

$$\text{Direct-labor-hour rate} = \frac{\text{Annual overhead in dollars}}{\text{Annual direct labor hours}}$$

$$\text{Direct-material-cost rate} = \frac{\text{Annual overhead in dollars}}{\text{Annual direct material cost in dollars}}$$

$$\text{Machine rate} = \frac{\text{Annual overhead in dollars allocated to a particular machine class}}{\text{Annual machine hours of same class}}$$

Factory cost. The *factory cost* of a product is the sum of direct material and direct labor costs, which are sometimes termed *prime costs,* and factory overhead.

Administrative costs. *Administrative costs* arise from expenditures for such items as salaries of executive, clerical, and technical personnel, office space, office supplies, depreciation of office equipment, travel, and fees for legal, technical, and auditing services that are necessary to direct the enterprise as a whole as distinct from its production and selling activities. Expenses so incurred are often recorded on the basis of the cost of carrying on subdivisions of administrative activities deemed necessary to take appropriate action to improve the effectiveness of administration. It is generally impractical to relate administrative costs directly to specific products. The usual practice is to allocate administrative costs to the product as a percentage of the product's factory cost. For example, if the annual administrative costs and factory costs of a concern are estimated at $10,000 and $100,000 respectively for a given year, 10 per cent will be added to the factory cost of products manufactured to absorb the administrative costs.

Production cost, selling cost, and cost of sales. *Production cost* of a product is the sum of its factory cost and its administrative cost.

Selling cost arises from expenditures incurred in disposing of the enterprise's products and services. This class of expense will include such items as salaries, commissions, office space, office supplies, rental and depreciation, operation of office equipment and automobiles, travel, market surveys, entertainment of customers, displays, and sales space.

Selling expenses may be allocated to various classes of products, sales territories, sales of individual salesmen, and so forth, as a means of improving the effectiveness of selling activities. In many cases it is considered adequate to allocate selling expense to products as a percentage of their production cost. For example, if the annual selling expense is estimated at $22,000 and the annual production cost is estimated at $110,000, 20 per cent will be added to the production cost of products to obtain the cost of sales.

The *cost of sales* is equal to the sum of the production cost and the selling cost.

Cost Accounting Is Based on Estimates

The more nearly a cost accounting procedure can provide cost data that are contemporary with the manufacture of a product, the more effectively it serves its purpose of providing data for the control of expenditures, the making of operating decisions, and the pricing of products.

Cost data obtained long after a product has been made and sold are practically valueless. But where long-lived assets such as buildings and equipment are used, the amount of depreciation that takes

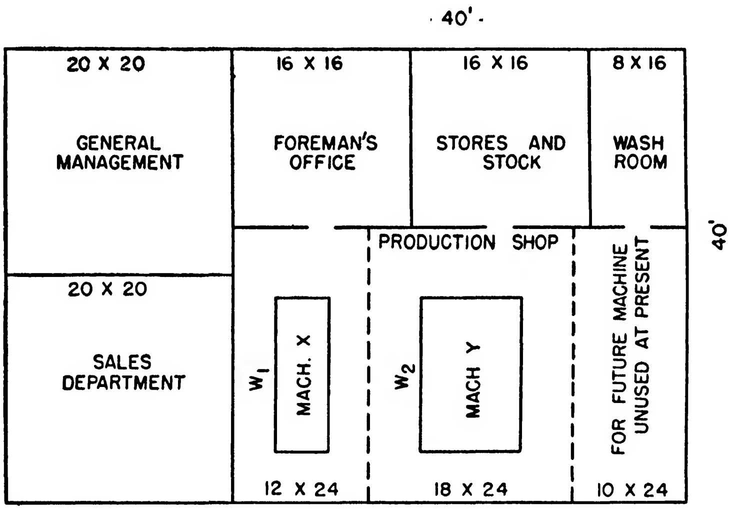


FIG. 28. Plan of Acme Company offices and factory.

place per unit of product cannot be determined until the assets are retired. For this reason cost accounting is usually based on estimates, in order that cost information may be more or less contemporary with the production to which it applies. This results in some loss in accuracy. Also, as has been indicated, accuracy is sacrificed by the use of arbitrary bases of allocation to simplify and reduce the cost of accounting procedures. Thus, costs shown by accounting should be recognized as approximations and subjected to analysis before being used in engineering economy studies.

An Example to Illustrate Distribution of Overhead by Several Methods

The Acme Company, a hypothetical small manufacturing concern, will be used to illustrate some aspects of cost accounting pertinent to the use of cost accounting data in engineering economy studies. The plan of the Acme Company's plant is shown in Fig. 28. Land for the plant cost $3,000 and the plant, constructed four years ago, cost $12,000. Two-thirds of each of these items, or $2,000 and $8,000, is attributed to the production department. Other data pertinent to the determination of rates for the allocation of overhead charges for the plant's fifth year of operation are given in the following paragraphs and tables.

Tabulation of Acme Company Production Facilities

Description	*Cost*	*Age*	*Depreciation Rate*	*Depreciation 5th Year*	*Present Value*
Land	$ 2,000	—	0	$ 0	$ 2,000
Building and equipment	8,000	4	4%	320	6,720
Factory furniture	400	4	10%	40	240
Small tools	400*	—	40%	160	400
Machine X	2,000	4	10%	200	1,200
Machine Y	8,000	4	10%	800	4,800
Stores and stock inventory	5,000	—	0	0	5,000
	$25,800			$1,520	$20,360

* Present value.

Estimated Factory Salaries and Wages during Fifth Year

Foreman F supervises factory operation	$3,000
Handyman H moves material, takes care of stock and stores, and does janitor work	1,600
Total indirect labor	$4,600
Workman W_1 operates Machine X, $1.80/hr. × 1,600 hrs.	$2,880
Workman W_2 operates Machine Y, $1.20/hr. × 1,600 hrs.	1,920
Total direct labor	$4,800

Estimated Supplies Used in Factory during Fifth Year

Office and general supplies	$ 200
Water (Est. as ¾ of bill for entire building)	60
Lighting current (Est. as ⅔ of bill for entire building)	150
Heating fuel (Est. as ⅔ of bill for entire building)	210
Electric power ($160 for Machine X and $300 for Machine Y)	460
Cutting oil, expendable tools, abrasive paper, etc. ($100 for Machine X and $280 for Machine Y)	380
	$1,460

In practice, allocations of supplies and power to Machine X and Machine Y would be based on records or estimates.

TABLE 21. Estimated Activity of Acme Company Factory during Fifth Year

Product	Estimated Output	Material Cost		Direct Labor Hours				Machine Hours			
				Workman W_1		Workman W_2		Machine X		Machine Y	
		Each	Total	Each	Total	Each	Total	Each	Total	Each	Total
L	1,000	$4.00	$ 4,000	1	1,000	—	—	1	1,000	—	—
M	1,400	4.00	5,600	—	—	1	1,400	—	—	1	1,400
N	800	4.00	3,200	.75	600	.25	200	.75	600	.25	200
			$12,800		1,600		1,600		1,600		1,600

Miscellaneous data relative to Acme Company

Annual taxes are estimated at 2 per cent of the cost of facilities.

Insurance is estimated at .5 per cent of the cost of facilities.

Interest on invested funds is to be taken as 6 per cent.

Annual maintenance of building and factory furniture is estimated at 3 per cent of original cost.

Annual maintenance of Machine X and Machine Y is estimated at 8 per cent of original cost.

Payroll, taxes, compensation insurance and the like are estimated at 9 per cent of the factory payroll.

Calculation of Acme Company factory overhead rates

On the basis of information and estimates given above and in Tables 21 and 22, overhead allocation rates are calculated as follows:

$$\text{Direct-labor-cost rate} = \frac{\text{Total factory overhead}}{\text{Total direct labor wages}} = \frac{\$11{,}334}{\$4{,}800} = 2.36$$

$$\text{Direct-labor-hour rate} = \frac{\text{Total factory overhead}}{\text{Total hours of direct labor}} = \frac{\$11{,}334}{3{,}200 \text{ hours}} = \$3.54 \text{ per hour}$$

$$\text{Direct-material-cost rate} = \frac{\text{Total factory overhead}}{\text{Total direct material cost}} = \frac{\$11{,}334}{\$12{,}800} = .89$$

TABLE 22. Estimated Factory Overhead of Acme Company during Fifth Year

A. *Overhead items equivalent to rent*		
Depreciation, insurance, maintenance on building, $8,000 × (.04 + .005 + .03)	$ 600	
Taxes on cost of building and land, $10,000 × .02	200	
Interest on present value of building and land, $8,720 × .06	523	
Water, light, and fuel for factory, $60 + $150 + $210	420	
Total		$ 1,743
B. *Miscellaneous items of overhead*		
Depreciation, taxes, insurance, and maintenance on factory furniture, $400 × (.10 + .02 + .005 + .03)	$ 62	
Interest on present value of factory furniture, $240 × .06	14	
Depreciation, taxes, insurance, and interest on present value of small tools, $400 × (.40 + .02 + .005 + .06)	194	
Taxes, insurance, and interest on stores and stock inventory, $5,000 × (.02 + .005 + .06)	425	
Office and general supplies	200	
Total		$ 895
C. *Indirect labor and labor overhead*		
Salaries of indirect labor of F and H, $3,000 + 1,600	$4,600	
Payroll taxes, ($3,000 + $1,600 + $2,880 + $1,920) × .09	846	
Total		$ 5,446
D. *Machine X overhead items*		
Depreciation, taxes, insurance, and maintenance on Machine X, $2,000 × (.10 + .02 + .005 + .08)	$ 410	
Interest on present value of Machine X, $1,200 × .06	72	
Supplies for Machine X	100	
Power for Machine X	160	
Total		$ 742
E. *Machine Y overhead items*		
Depreciation, taxes, insurance, and maintenance on Machine Y, $8,000 × (.10 + .02 + .005 + .08)	$1,640	
Interest on present value of Machine Y, $4,800 × .06	288	
Supplies for Machine Y	280	
Power for Machine Y	300	
Total		$ 2,508
Grand Total All Factory Overhead Items		$11,334

Further analysis must be made before machine rates can be established for Machine X and Machine Y. In establishing machine rates, as many items of overhead as possible are directly allocated to each machine before their identity is lost by being charged to an overhead account.

Consider Item A in Table 22. This item is equal to $1,743 and is equivalent to rent of the factory building, which has a floor area of 1,600 square feet.

Annual cost per sq. ft. of floor area, $1,743 ÷ 1,600	$1.09
Area directly occupied by Machine X, 12 × 24	288 sq. ft.
Area directly occupied by Machine Y, 18 × 24	432 sq. ft.
Space charge, Machine X, 288 × $1.09	$314
Space charge, Machine Y, 432 × $1.09	$471
Total space charged to Machines X and Y	$785
Balance of space cost to be allocated, $1,743 − $785	$958

This balance of Item A, together with Item B and Item C from Table 22, must be distributed to Machine X and Machine Y on some basis that is estimated to reflect actual conditions. In this example the sums of these items will be allocated equally to the two machines. One-half of the unallocated sum is equal to ½ ($958 + $895 + $5,446) = $3,649.

Machine X Overhead Charges and Machine Rate

Item D (Table 22)	$ 742
Space charge as calculated above	314
One-half of unallocated balance of Item A, Item B, and Item C, ½ ($958 + $895 + $5,446)	3,649
Total	$4,705

$$\text{Machine rate (Machine X)} = \frac{\text{Overhead allocated to Machine X}}{\text{Estimated annual hours of operation}}$$

$$= \frac{\$4{,}705}{1{,}600 \text{ hrs.}} = \$2.94 \text{ per hour}$$

Machine Y Overhead Charges and Machine Rate

Item E (Table 22)	$2,508
Space charge as calculated above	471
One-half of unallocated balance of Item A, Item B, and Item C, ½ ($958 + $895 + $5,446)	3,649
Total	$6,628

$$\text{Machine rate (Machine Y)} = \frac{\$6{,}628}{1600 \text{ hrs.}} = \$4.14 \text{ per hour}$$

COST OF PRODUCTS L, M, AND N AS DETERMINED BY FOUR METHODS OF ALLOCATING OVERHEAD

The Factory Cost of Product L

Direct-labor-cost method	
Direct material	$ 4.00
Direct labor, 1 hr. × $1.80/hr.	1.80
Overhead, $1.80 × 2.36	4.25
	$10.05
Direct-labor-hour method	
Direct material	$ 4.00
Direct labor, 1 hr. × $1.80/hr.	1.80
Overhead, 1 hr. × $3.54/hr.	3.54
	$ 9.34
Direct-material-cost method	
Direct material	$ 4.00
Direct labor, 1 hr. × $1.80/hr.	1.80
Overhead, $4.00 × .89	3.56
	$ 9.36
Machine-rate method	
Direct material	$ 4.00
Direct labor, 1 hr. × $1.80/hr.	1.80
Overhead, 1 hr. × $2.94/hr.	2.94
	$ 8.74

The Factory Cost of Product M

Direct-labor-cost method	
Direct material	$ 4.00
Direct labor, 1 hr. × $1.20/hr.	1.20
Overhead, $1.20 × 2.36	2.83
	$ 8.03
Direct-labor-hour method	
Direct material	$ 4.00
Direct labor, 1 hr. × $1.20/hr.	1.20
Overhead, 1 hr. × $3.54/hr.	3.54
	$ 8.74
Direct-material-cost method	
Direct material	$ 4.00
Direct labor, 1 hr. × $1.20/hr.	1.20
Overhead, $4.00 × .89	3.56
	$ 8.76
Machine-rate method	
Direct material	$ 4.00
Direct labor, 1 hr. × $1.20/hr.	1.20
Overhead, 1 hr. × $4.14	4.14
	$ 9.34

The Factory Cost of Product N

Direct-labor-cost method	
Direct material	$ 4.00
Direct labor, .75 hr. × $1.80/hr. + .25 hr. × $1.20/hr.	1.65
Overhead, $1.65 × 2.36	3.89
	$ 9.54
Direct-labor-hour method	
Direct material	$ 4.00
Direct labor (calculated as above)	1.65
Overhead, (.75 hr. + .25 hr.) × $3.54/hr.	3.54
	$ 9.19
Direct-material-cost method	
Direct material	$ 4.00
Direct labor	1.65
Overhead, $4.00 × .89	3.56
	$ 9.21
Machine-rate method	
Direct material	$ 4.00
Direct labor	1.65
Overhead, .75 hr. × $2.94 hr. + .25 hr. × $4.14/hr.	3.24
	$ 8.89

Cost of sales of the Acme Company. The cost of sales is obtained by adding administrative and selling costs to the factory cost. Continuing with the example of the Acme Company, suppose that after careful analysis of expenditures annual administrative costs have been estimated at $4,600 and annual selling costs at $9,300.

Annual direct material costs, direct labor costs, and factory overhead costs for the Acme Company have been estimated previously as $12,800, $4,800, and $11,334 respectively.

The annual estimated cost of sales of the Acme Company for the fifth year may be summarized as follows:

Estimated annual direct material cost	$12,800		
Estimated annual direct labor cost	4,800		
Estimated annual factory overhead cost	11,334		
Estimated annual factory cost		$28,934	
Estimated annual administrative cost		4,600	
Estimated annual production cost			$33,534
Estimated annual selling cost			9,300
Estimated annual cost of sales			$42,834

Rates for allocation of administrative cost and selling cost to products. It will be observed that

$$\frac{\text{Est. Annual Admin. Cost}}{\text{Est. Annual Factory Cost}} = \frac{\$4,600}{\$28,934} = .159 \text{ or } 15.9\%$$

and

$$\frac{\text{Est. Annual Selling Cost}}{\text{Est. Annual Production Cost}} = \frac{\$9,300}{\$33,534} = .277 \text{ or } 27.7\%$$

The cost of sales per unit of product—for example Product L, whose factory cost by the direct-labor-cost method of allocating factory overhead has previously been calculated to be $10.05—is obtained as follows:

Unit Cost of Product L

Factory cost	$10.05	
Administrative cost, $10.05 × .159	1.60	
Production cost		$11.65
Selling cost, $11.65 × .277		3.23
Cost of sales		$14.88

Evaluation of Cost Accounting Data

An examination of the factory costs obtained for Product L in the Acme Company example with the four methods of factory overhead employed shows considerable variation.

Each of the three costs that make up the total is subject to error. Direct material costs may be incorrect because of such errors as pricing, charging a product with more material than is actually used, and the use of approximate methods generally. Similarly, and for much the same reasons, direct labor costs as charged will usually be in error to some extent. However, with reasonably good control and accounting procedures, direct material costs and direct labor costs are generally fairly reliable.

If attention is directed to factory overhead costs allocated to Product L, it will be observed that amounts allocated by the several methods range from $4.25 to $2.94. The ratio of these amounts is equal to approximately 1.5. The fact that the amounts shown for the several methods differ is proof that at least three of the methods of allocations are in error. It is probable that all are in error.

In actual practice the item of factory overhead is a summation of a great number and variety of costs. It is therefore not surprising that the use of a single simple method will not allocate factory overhead costs to specific products with precision. Though a particular method may be generally quite satisfactory, gross errors may result in some specific situations.

For example, consider the direct-labor-hour method as it applies to Product L and Product M. The assumption is that $3.54 will be incurred for each hour of direct labor, regardless of equipment used. Thus, the amount of overhead allocated to Product L and Product M is identical, even though it is apparent that the cost of operating the respective machines on which they are processed is quite different. Note that the machine rates of Machine X and Machine Y are $2.94 and $4.14 respectively.

If the direct-material-cost method is used, it is clear that overhead allocations to products will be dependent upon the unit price as well as upon the quantity of materials used. Suppose, for example, that in the manufacture of tables a certain model may be made from either pine or mahogany. Processing might be identical but the amount charged for overhead might be several times as much for mahogany as for pine because of the difference in the unit price of the materials used.

Effect of changes in extent of activity. In the determination of rates for the allocation of overhead, the activity of the Acme Company for the fifth year was estimated in terms of products L, M, and N. This estimate served as a basis for determining annual material cost, annual direct labor cost, annual direct labor hours, and machine hours. These items then became the denominator of the several allocation rates.

The numerator of the allocation rates was the estimated factory overhead, totaling $11,334. This numerator quantity will remain relatively constant for changes in activity, as an examination of the items of which it is composed will reveal.

For this reason the several rates for allocating factory overhead will vary in some generally inverse proportion with activity. Thus, if the actual activity is less than the estimated activity, the overhead rate charged will be less than the amount necessary to absorb the estimated total overhead. The reverse is also true. When the under or overabsorbed balance of overhead becomes known at the end of the year, it is usually charged to Profit and Loss, or Surplus.

In engineering economy analyses, the effect of activity on overhead charges and overhead rates is an important consideration. The total overhead charges of the Acme Company would remain relatively constant over a range of activity represented, for example, by 800 to 2,000 hours of activity for each of the two machines. Thus, after the total overhead has been allocated, the increment cost of producing additional units of product will consist of direct material and direct labor costs.

Cost data must be pertinent. The need for exercising care to see that cost accounting data are pertinent to situations under consideration will be illustrated by two examples.

It is a common error to infer that a reduction in labor costs will result in a proportionate decrease in overhead costs, particularly if overhead is allocated on a labor cost basis. In one instance a company was manufacturing an oil field specialty. An analysis revealed costs as follows:

Direct labor	$2.18
Direct material	.84
Factory overhead, $2.18 × 2.56	5.58
	$8.60

The factory cost of $8.60 was slightly less than was being received for the item in question. The first suggestion was to cease making the article. But after further analysis it became clear that the overhead of $5.58 would not be saved if the item was discontinued. The burden rate used was based on heavy equipment required for most of the work in the department and on hourly earnings of workmen who averaged $1.56 per hour. For the job in question only a light drill press and hand tools were used. Little actual reduction in cost would have resulted from not using them in the manufacture of the article.

It has previously been calculated that items of overhead of the Acme Company that are equivalent to rent are equal to $1.09 per square foot of factory floor space. Figure 28 reveals a currently unused 10 ft. × 24 ft. space for future machines. The annual rental cost equivalent of this space is 10 ft. × 24 ft. × $1.09/ft.^{2} = $262.

This item need not be included as an item of cost attributable to a machine that may be purchased to occupy the space in question, since no actual additional cost will arise should the space be occupied. The $262 item has been entered as an overhead charge to be allocated to products made on the basis of one of several overhead rates. The addition of a new machine will probably result in changes in the overhead allocation rate used, but it will not result in a change in the overhead item equivalent to rent.

Average costs are inadequate for specific analysis. An important function of cost accounting, if not the primary one, is to provide data for decisions relative to the reduction of production costs and the increase of profit from sales. Errors in cost data that are believed to be accurate may lead to costly errors in decisions. Cost data that give true average values and are adequate for over-all analyses may be inadequate for specific detailed analyses. Thus, cost data must be carefully scrutinized and their accuracy established before they can be used with confidence in engineering economy studies.

In Table 23 actual and erroneous cost data relative to the cost of three products have been tabulated. The actual production costs of

TABLE 23. Illustration of How Unknown Errors in Production Costs of Products May Result in Errors in Decisions

Product	Direct Labor and Material Costs	Overhead Costs, Actual	Overhead Costs, Believed To Be	Production Cost, Actual	Production Cost, Believed To Be
A	$6.50	$2.50	$3.50	$9.00	$10.00
B	7.00	3.00	3.00	10.00	10.00
C	7.50	3.50	2.50	11.00	10.00
Average	7.00	3.00	3.00	10.00	10.00

products A, B, and C are respectively $9.00, $10.00, and $11.00, but owing to unknown errors in overhead costs the production costs of products A, B, and C are believed to be equal to $10.00 for each.

It should be noted that even though the average of a number of items of cost may be correct, there is no assurance that individual items are not in error. For this reason the accuracy of each item of cost should be ascertained before the item is used in an economic analysis.

If, for example, the selling price of the products is based upon their believed production cost, Product A will be overpriced and Product C will be underpriced. Buyers may be expected to shun Product A and to buy large quantities of Product C. This may lead to a serious unexplained loss of profit. Average values of cost data are of little value in making decisions relative to specific products.

PROBLEMS AND QUESTIONS

1. Summarize the functions of cost accounting and engineering economy analyses.

2. What precautions should be exercised in using cost accounting data in economy studies?

3. What is the difference in viewpoint in respect to time between a balance sheet and an economy study?

4. What relationship between accuracy and cost determines the extent of expenditure that can be justified for the maintenance of a cost accounting system?

5. Name the classifications of cost whose total is the cost of sales.

6. Define each of the classifications of cost whose total is factory cost.

7. Name and define several bases for the distribution of overhead costs.

8. In electrolytic processes, the cost of current is often considered to be a direct cost of production, but the cost of current used in driving a lathe is usually considered as part of overhead or an indirect cost. What is the justification for this variation in practice?

9. (a) The manufacturing costs of Products M and N are believed to be $10 per unit. On the basis of this cost and a desired profit of 10 per cent, the selling price is set at $11 per unit. What is the apparent profit of 500 units of Product M and 1,500 units of Product N are sold?

(b) If the actual manufacturing costs of Products M and N are $9 and $11 respectively, what actual profit is made using the selling price and quantities sold given in part (a)?

(c) If the actual cost of manufacture is 10% greater for each item than given in part (b), what profit is made using the selling price and quantities sold given in part (a)?

10. A small factor is subdivided into three departments for accounting purposes. The following table gives the pertinent information concerning the departments for a certain year.

Department	*Direct Labor Hours*	*Cost of Direct Labor*	*Direct Material Cost*
A	16,840	$13,472	$ 7,300
B	11,330	$13,596	$ 3,650
C	6,520	$ 6,520	$10,300

Overhead items to be distributed to the departments are: rent, $4,800; power, $1,920; superintendent's salary, $4,500; miscellaneous expenses, $1,520.

Distribute the total of these items to Departments A, B and C on the bases of direct labor hours, direct labor cost, and direct material cost and compare the results.

11. A small shop manufactures four products A, B, C, and D whose annual production and sales are 1,000, 500, 2,000, and 10,000 pieces respectively. Shop overhead expenses including supervision, rent, utilities, repairs, depreciation, and taxes total $86,500 per year. The shop uses only one type of machine, engine lathes, and the owner believes that the overhead varies with machine hours. Time-keeping records indicate that the total machine hours spent on each unit of each product are as follows: A, 11 hours; B, 6 hours; C, 8 hours; D, 2 hours.

Use the machine-hour method of apportioning overhead charges and calculate the amount of overhead that should be charged to *each unit* of *each product* so that the total overhead will be absorbed.

CHAPTER

12

Break-Even and Minimum Cost Points

IN MANY SITUATIONS encountered in industry, the costs related to one or more methods of accomplishing specified ends are affected by a number of variables. Where a common variable affects the costs of two methods, it is often desirable to know the value of the common variable that will result in equal or *break-even* cost.

Similarly, if the costs associated with a venture are affected by a variable that can be defined, it may be desirable to determine the value of the variable for which the cost of the venture will be at a minimum.

An understanding of break-even and minimum cost points is valuable both for their direct applications and for a clearer appreciation of cost trends.

Break-Even Point

The break-even point of two variable situations is the value or point at which they become equal as the result of the effects of a common variable.

If the costs of each of two situations under consideration can be expressed in terms of a common variable, a break-even point exists. The cost equations involved will be of the form

$$C_M = f_1(x) \qquad \text{and} \qquad C_N = f_2(x)$$

where

C_M = a specified cost such as annual cost, total cost, cost per day, or cost per piece, applicable to a situation designated M.

C_N = a specified cost applicable to a situation designated N and comparable to that under consideration in situation M.

x = a variable affecting C_M and C_N.

To solve for the value of x for which C_M and C_N are equal, let

$$C_M = C_N$$

Then
$$f_1(x) = f_2(x)$$

which may be solved for x.

The variable x may pertain to a great variety of factors. Some of these are extent of operation per year, expected life, expected period of operation, size of job to be undertaken, wage rates, interest rate, probability of error or damage, cost of materials, cost of power, and rate of production and sales.

Break-even point, example of mathematical solution. Consider the following example. A 20 hp. unit is needed to drive a pump to remove water from a railway tunnel. The number of hours that the power unit will operate per year is dependent upon weather conditions and is thus uncertain. The power unit will be used for four years. Two plans for supplying the power are under consideration.

Plan A calls for the construction of a power line and the purchase of an electric motor, at a total cost of $1,400. The salvage value of this equipment at the end of the four-year period under consideration is estimated at $200. The cost for current per hour of operation is estimated at $.84. The equipment is automatic and will require no attendant. Maintenance is estimated at $120 per year. Interest is to be taken at 10 per cent. Let

A = equivalent annual cost of Plan A.
D = the equivalent annual cost of depreciation and interest
$$\overset{\text{RP 10-4}}{=} (\$1{,}400 - \$200)(.31547) + \$200 \times .10 = \$399.$$
M = annual maintenance = $120.
C = cost of current per hour of operation = $.84.
N = the number of hours of operation per year.

Then
$$A = D + M + NC$$

Plan B calls for the purchase of a gasoline motor at a cost of $550. The motor will be scrapped at the end of the four-year period. The cost of fuel and oil per hour of operation is estimated at $.42. The hourly wage of the operator who tends the engine when it runs is $.80. Maintenance is estimated at $.15 per hour of operation. Let

B = equivalent annual cost of Plan B.
D' = the equivalent annual cost of depreciation and interest
RP 10-4
= \$550(.31547) = \$174.
H = hourly cost of fuel and oil, operator attention, and maintenance = \$.42 + \$.80 + \$.15 = \$1.37.
N = the number of hours of operation per year.

Then
$$B = D' + NH$$

There is a value of N for which the costs of Plan A and Plan B will be equal. This value of N may be found as follows: Let $A = B$. Then

$$D + M + NC = D' + NH$$

and
$$N = \frac{D' - (D + M)}{C - H}$$

Substituting the values given above for the several terms,

$$N = \frac{\$174 - (\$399 + \$120)}{\$.84 \text{ per hr.} - \$1.37 \text{ per hr.}} = 651 \text{ hrs.}$$

The value of A or B for $N = 651$ hours is

$$A = B$$
$$D + M + NC = D' + NH$$
$$\$399 + \$120 + 651 \times \$.84 = \$174 + 651 \times \$1.37$$
$$\$1066 = \$1066$$

For the given conditions, annual costs of the two alternatives are calculated to be equal for 651 hours of usage per year. If usage amounts to less than 651 hours per year, selection of the gasoline motor is suggested; for a number of hours greater than 651 the electric motor promises to be the more economical selection. A graphical representation of the above example is given in Fig. 29.

The consideration of a break-even cost point is pertinent when a service may be provided by two alternatives, one of which has higher fixed costs and lower variable costs than the other.

The difference in equivalent annual cost between the two alternatives given above for any number of hours of operation, as for example 100 hours, may be calculated as follows: Let

$$S_{100} = A - B$$
$$= D + M + NC - (D' + NH)$$

Substituting the values for the several quantities given above,

$$S_{100} = \$399 + \$120 + (100 \times \$.84) - \$174 - (100 \times \$1.37)$$
$$= \$292$$

Break-even point, graphical solution. There are many situations in which setting up equations to represent the cost patterns of two alternatives is either too difficult or too time-consuming to be a feasible approach for the determination of a break-even point. In situations for which the cost patterns of two alternatives can be

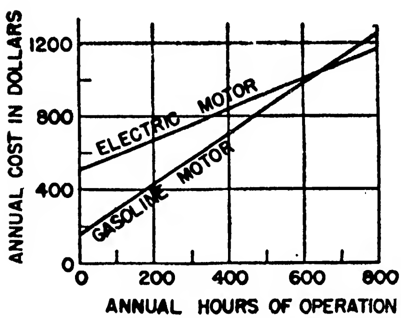


FIG. 29. Graph of an algebraic solution for break-even point of an electric motor and a gasoline motor installation.

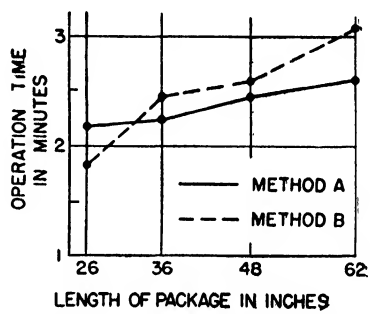


FIG. 30. Graphical solution for break-even point for two methods of packing a product in cartons 24 in. in girth and of several lengths.

established by determining a number of points of the pattern by calculation or experiment, the break-even point may be determined graphically.

Consider the following example. Two methods, A and B, are under consideration for packing different lengths of a window display material in paper cartons of 24 inches in girth and 26, 36, 48, and 62 inches in length, respectively. The average times required to pack a number of cartons of the given lengths by Method A and Method B were found by time studies. The results are given in Fig. 30. From Fig. 30 the length for which the two methods will break even may be approximated at 32 inches.

The concept embraced by the break-even point is widely used in consideration of the relationship of sales income and production costs.

as they are affected by the variable rate of sales and production. The use of the break-even chart in this connection is discussed in some detail in Chapter 15.

Minimum Cost Point, Mathematical Solution

A summation of two or more components that are modified differently by a common variable may have a minimum value for a certain value of the variable. The value of the variable for which the value of the summation is a minimum is termed a minimum point. When the quantities under consideration are costs, the minimum point is designated a minimum cost point.

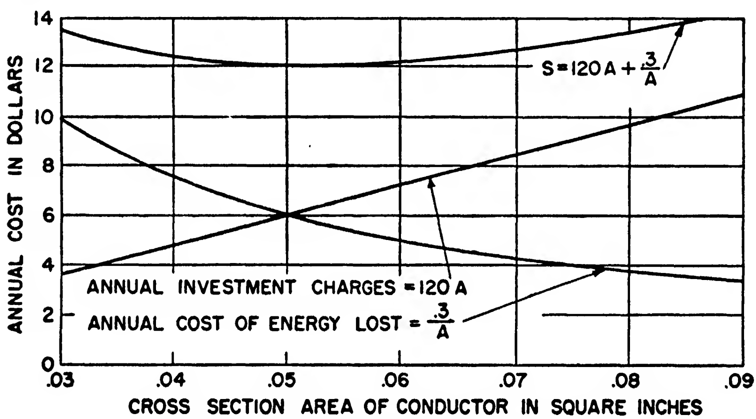


FIG. 31. Graphical exposition of Kelvin's Law.

There are many situations in which costs are the summation of components that are affected differently by a common variable. When such costs can be expressed in terms of a common variable, the minimum cost can be calculated. Lord Kelvin recognized that this condition applied to the economics of electric current flow in a conductor and in 1881 announced his findings, now known as Kelvin's Law.

Kelvin's Law. The gist of Kelvin's Law is that the greatest economy (least cost) occurs when a conductor is of such cross-sectional area that the annual cost of energy lost (I^2R loss) and the annual investment charges for the conductor are equal (see Fig. 31). Kelvin's Law, predicated on the supposition that the cost of the conductor is proportional to the area of its cross section, may be explained mathematically as follows: Let

A = cross-sectional area of the conductor in square inches.
M = annual investment charges on the conductor in dollars.
N = annual cost of energy lost owing to resistance of conductor, in dollars.
R = electrical resistance of the conductor in ohms.
S = $M + N$, total annual cost in dollars.

The annual investment charges for the conductor will be in proportion to its weight, which is in turn proportional to A, its cross-sectional area. Thus

$$M = K_1A$$

where K_1 is a constant.

The annual cost of energy lost is proportional to R and R is inversely proportional to A. Thus

$$N = \frac{K_2}{A}$$

where K_2 is a constant. Then

$$S = K_1A + \frac{K_2}{A}$$

If the length of the conductor, the unit cost of copper, the interest rate, and similar charges are such that $K_1 = 120$ and the resistance of the conductor, resistance losses of energy, and unit cost of energy are such that $K_2 = .3$,

$$S = 120\,A + \frac{.3}{A}$$

By observation it is clear that the minimum annual cost occurs for the value of A for which $M = 120A$ and $N = .3/A$ are equal.

The value of A for which S will be of minimum value may be found by differentiating the expression for S with respect to A, setting the derivative equal to zero, and solving for A as follows:

$$\frac{dS}{dA} = 120 - \frac{.3}{A^2} = 0$$

$$\text{A} = \sqrt{\frac{.3}{120}} = .05 \text{ sq. in.}$$

Though Kelvin's Law does not have a broad, direct application in economy problems, it does bring out the very useful and widely applicable concept that costs are often a summation of opposing components. Thus the decrease of one component is accompanied by an increase in the other. This situation is encountered in a great variety of problems in engineering economy. If costs can be expressed mathematically, the minimum cost points may be found mathematically as above. Generally costs are not readily expressed mathematically. In such cases a graphical solution will be most desirable.

Economic purchase order quantity. The mathematical solution of minimum cost points will be further illustrated by an example in which the aim is to determine the amount of a raw material to order per order for minimum cost.

When a decision has been made to manufacture a certain article, it becomes necessary to procure materials. Consider the case of a manufacturer who produces a certain article at a uniform rate throughout the year. He may meet the requirement that material be on hand when needed by purchasing a year's supply at the beginning of the year, or by purchasing a day's supply at the beginning of each day. But neither of these plans may be the most economical method of providing the needed material; consequently, the frequency of purchasing or, to put it another way, the quantity to be purchased each time should be investigated.

Assume that a manufacturer uses a certain material at a uniform rate throughout the year. If he meets his requirements for material by ordering several times per year, he incurs the cost incident to ordering (clerical cost of requisitioning, ordering, paying the vendor, paying transportation, checking invoices, entering the items received on the store's record, and so forth) several times per year, but since relatively small quantities are received, the cost of storing them and the charges on the money invested in them (interest, insurance, taxes, and the like) will be relatively small. Conversely, if he orders but once a year, the cost incident to ordering will occur but once, but the large quantity received at one time will give rise to relatively large costs for storage and for interest, insurance, and taxes.

If his decision is to be based on economy, he must determine what quantity to order in order to minimize the cost of ordering, storage, interest, insurance, taxes, and so forth. Let

Y = all costs incident to providing the year's needs (purchase price, ordering cost, storage cost, interest, insurance, taxes, and so on).
N = the number of units of the material under consideration needed during the year.
C = purchase price per unit.
X = the number of lots ordered per year = N/n.
n = the quantity ordered per lot = N/X.
D = order cost per lot (cost of requisitioning, ordering, paying vendor, paying transportation, checking invoice, entering the item received on stores record, and so on).
I = interest, insurance, taxes, and depreciation in per cent per year on the average of funds invested in the supply.
T = cost of storage per unit per year.

Then
$$Y = NC + XD + \frac{NCI}{2X} + \frac{NT}{X}$$

In the equation above,

NC = purchase price for the year's supply.
XD = cost of ordering a year's supply.
$\frac{NCI}{2X}$ = interest, insurance, taxes, and depreciation on average investment in items on hand during the year.

The quantity $NCI/2X$ is obtained as follows:

If a year's supply of N pieces is purchased at the rate of C dollars per unit in one lot, the initial investment at the beginning of the year will be NC dollars. See solid lines, Fig. 32.

If the N pieces are used at a uniform rate during a period of a year, the investment at the end of the year will be zero. The average investment will be

$$\left(\frac{NC + 0}{2}\right) = \frac{NC}{2}$$

If two lots of $N/2$ had been purchased, the initial investment would have been $NC/2$, and the average investment would have been

$$\left(\frac{NC}{2} + 0\right) \div 2 = \frac{NC}{4}$$

See dotted lines, Figure 32.

When X lots are purchased per year, there are N/X pieces per lot requiring an initial investment of NC/X and an average investment of $NC/2X$. This investment times the rate I is equal to $NCI/2X$.

$\frac{NT}{X}$ = {cost of storing the supply during the year. (It is assumed that storage space provided for a unit of the supply in question cannot be used for any other purpose.)

The quantity NT/X is obtained as follows: When storage can be used only for a particular kind of item, it is clear that sufficient storage must be provided for the largest lot that will be received. Thus if the unit cost per year for storage is T, and the material is received in a lot of N pieces (a year's supply), the storage cost will be NT per year.

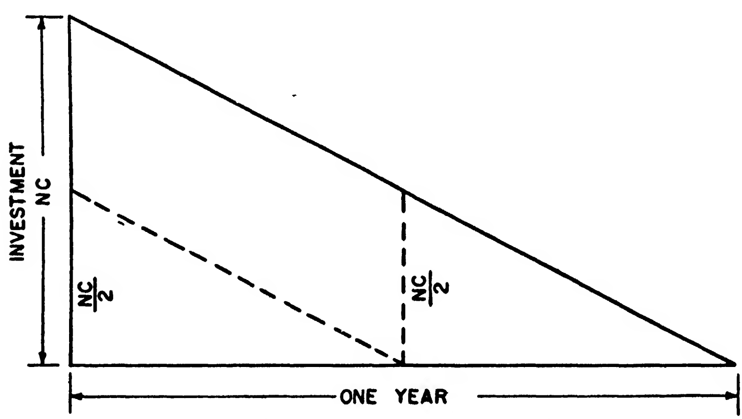


FIG. 32. Pattern of investment when one and two lots are purchased per year.

If the year's supply is received in X lots, storage must be provided for N/X, and the storage cost will be NT/X.

If the nature of the storage required is such that it can and is used for other purposes when not used for the material under consideration, reasoning similar to that presented in regard to average investment applies, and the storage cost will be $NT/2X$.

To find the value of X lots per year for minimum cost, the equation

$$Y = NC + XD + \frac{NCI}{2X} + \frac{NT}{X}$$

is differentiated with respect to X and the result is set equal to 0 and then solved for X.

$$\frac{dY}{dX} = 0 + D - \frac{NCI}{2X^2} - \frac{NT}{X^2} = 0$$

$$X = \sqrt{\frac{NCI + 2NT}{2D}}$$

and

$$n = \frac{N}{X}$$

Illustration of use of economic purchase order quantity equations. Consider the following example:

N =	2,000 units	D =	\$10.00
C =	\$1.00	X =	number of lots per year
I =	.10	n =	number of units per lot
T =	\$.10	Y =	yearly cost

Solving for X in the equation, $X = \sqrt{\frac{NCI + 2NT}{2D}}$

$$X = \sqrt{\frac{2{,}000 \times \$1.00 \times .10 + 2 \times 2{,}000 \times \$.10}{2 \times \$10.00}}$$

= 5.5 or say 6 lots per year corresponding to lots of 333 units.

In order to determine the difference in cost resulting from ordering different numbers of lots, different values of X are substituted in the equation

$$Y = NC + XD + \frac{NCI}{2X} + \frac{NT}{X}$$

For X = 1 and n = 2,000; $Y = 2{,}000 \times 1 + 1 \times 10 + \frac{2{,}000 \times 1 \times .10}{2} + \frac{2{,}000 \times .10}{1}$

Y = 2,000 + 10 + 100 + 200 = \$2,310

For X = 2 and n = 1,000; Y = 2,000 + 20 + 50 + 100 = \$2,170

For X = 3 and n = 667; Y = 2,000 + 30 + 33.3 + 66.7 = \$2,130

For X = 4 and n = 500; Y = 2,000 + 40 + 25 + 50 = \$2,115

For X = 5.5 and n = 364; Y = 2,000 + 55 + 18.18 + 36.36 = \$2,109.54

For X = 6 and n = 333; Y = 2,000 + 60 + 16.7 + 33.3 = \$2,110

For X = 8 and n = 250; Y = 2,000 + 80 + 12.5 + 25 = \$2,117.50

For X = 12 and n = 167; Y = 2,000 + 120 + 8.3 + 16.7 = \$2,145

The results of the above solutions are presented graphically in Fig. 33. It should be noted that costs increase more rapidly as the lot size increases than as the lot size decreases from the most economical lot size. This indicates that the trend should be toward small lot sizes, particularly if it seems likely that costs associated with depreciation, obsolescence, and storage may exceed the figures given in the equation.

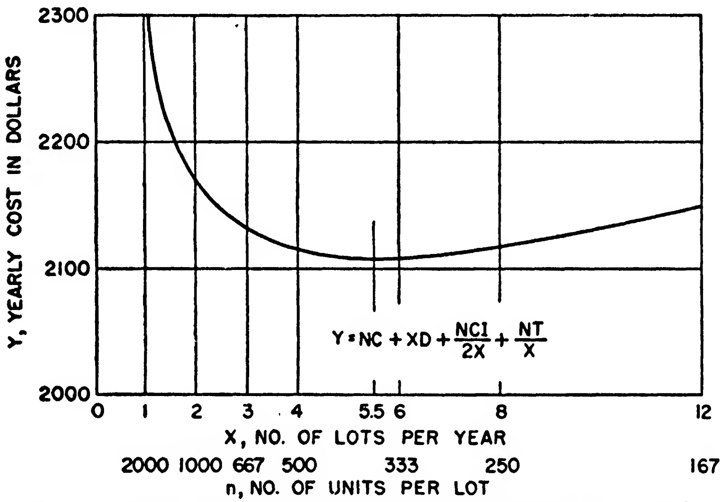


FIG. 33. Yearly cost for purchase of different numbers of lots of a certain material per year.

Since the purchase price, $NC = \$2,000$, is a constant, it is obvious that each of the above results less \$2,000 represents the cost of the items on which a decision as to the number of lots per year has a bearing. Subtracting the \$2,000 from the tabulated values above, the results are:

Number of Lots Per Year	*Number of Pieces Per Lot*	*Costs Incident to Ordering and Storing*
1	2,000	\$310
2	1,000	170
3	667	130
4	500	115
5.5	364	109.54
6	333	110
8	250	117
12	167	145

Note that the decision to order in lots of 333 six times per year rather than in a single lot of 2,000 results in a saving of $200 per year.

Minimum cost point, graphical solution. There are many situations for which minimum cost points are sought but for which it is very difficult to set up equations that truly characterize the existing cost relationships. Since finding minimum cost points from cost equations involves differentiation, equations that only approximate true situations may result in grossly misleading results.

For most situations and particularly for those that are nonrepetitive, it is simpler to calculate the costs that result when different values of a variable affecting the situation are used. The costs so obtained may then be plotted to establish the minimum cost points as well as cost trends.

The graphical method for finding minimum cost points will be illustrated in the following example. The manufacturer of a pharmaceutical product plans to use an evaporative process. In this process one or several evaporators in multiple may be used. It is known that variable operating costs, of which the chief item is the cost of steam, will be approximately inversely proportional to the number of evaporators used in the installation. Fixed costs will be approximately in proportion to the number of evaporators used. Trial had demonstrated that a rather complex equation would be required to express accurately the cost relationships that existed, so a graphical solution was sought.

Estimates were made of the variable and the fixed operation costs that would be obtained if one, two, three, or four evaporators were used. Results from equipment manufacturers, experimental data, and a knowledge of the process formed the basis for estimating the costs. The following is a summary of the estimates based on 200 days' operation per year.

Costs	Number of Evaporators Used			
	1	*2*	*3*	*4*
Fixed costs (depreciation, insurance, taxes, space, etc.)	$ 860	$1,680	$2,350	$3,030
Variable costs (steam, labor, maintenance, etc.)	7,850	4,560	3,610	3,190
Total annual cost	$8,710	$6,240	$5,960	$6,220

The plots of these several costs are shown as curves 1, 2 and (1 + 2) in Fig. 34.

It is clear that, on the basis of 200 days' operation per year, the lowest annual cost of $5,960 results when three evaporators are used.

In a similar manner costs were estimated for several periods of operation other than 200 days per year in order to note the effect of alterations in this variable. The results of estimates for 100 days of use per year appear as Curves 1, 3 and (1 + 3) in Fig. 34. For this

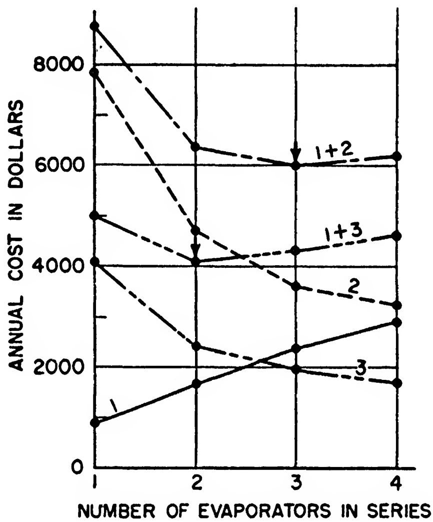


FIG. 34. Graphical determination of minimum-cost points for two levels of operation of an evaporative process in the manufacture of a pharmaceutical product.

Curve No. 1—Annual fixed costs of installations.
Curve No. 2—Variable cost for 200 days of operation per year.
Curve No. (1 + 2)—Total cost for 200 days of operation per year.
Curve No. 3—Variable cost for 100 days of operation per year.
Curve No. (1 + 3)—Total cost for 100 days of operation per year.

activity two evaporators result in a minimum annual cost equal to $4,060.

In the above graphical solution two variables, the number of evaporators and the number of days' use per year, have been taken into consideration. In addition to being simpler to apply in many complex situations, the graphical determination of minimum costs has the advantage of being more readily understood than mathematical solutions.

PROBLEMS

1. An oil company may furnish a car for use of pumpers for visiting the oil wells they tend or the company may pay the pumper for the use of his car at a rate of $.11 per mile for this purpose. The following estimated data apply to company furnished cars. A car costs $1,800 and has a life of 4 years and a trade-in value of $700 at the end of that time. Monthly storage cost for the car is $3 and the cost of fuel, tires, and maintenance is $.028 per mile. What annual mileage must a pumper travel by car for the cost of the two methods of providing transportation to be equal if the interest rate is 8%?

2. Two methods of exploiting an ore deposit are under consideration. Method A will recover 82% of the ore in the deposit and will cost $3.30 per ton of ore removed. Method B will recover 89% of the ore in the deposit and will cost $3.60 per ton of ore removed. For what value of recovered ore will the net return be equal for the two methods of removal?

3. A man purchased a car for $2,260, of which $60 represented the cost of an overdrive estimated to increase the mileage obtained per gallon of gasoline from 15.0 to 16.5 during the period that it was used. When the owner sold the car for $1,100 three years later, he was of the opinion that the overdrive did not affect the amount he received. He estimated that the overdrive was used during 40% of the total miles he had driven the car. The average fuel cost was $.26 per gallon. The owner considered the cost of money to be 6%.

(a) If the car was driven 10,000 miles per year, did the overdrive pay for itself in fuel savings?

(b) For what annual mileage would the overdrive just pay for itself?

4. A coffee concern has a weigher for filling cans with coffee. Because of its lack of sensitivity the weigher must be set so that the average filled can contains 16¼ ounces in order to insure that no cans contain less than 16 ounces of coffee. The present weigher was purchased 10 years ago at a price of $1,500 and is expected to last an additional 10 years. A new weigher is being considered whose sensitivity is such that the average "overage" per pound can is ⅛ ounce. This weigher will cost $2,500. $300 will be allowed for the present weigher on the purchase price of the new weigher. The salvage value of either weigher 10 years from now is considered to be negligible and their operating costs are estimated to be equal. The value of the coffee is $.28 per pound. What is the minimum number of one-pound packages packed annually for which the purchase of the new weigher is justified, if the interest rate is 6%?

5. A contractor can purchase a heavy-duty truck with a 12-cubic-yard dump body for $13,000. Its estimated life is 7 years and its estimated salvage value is $2,000. Maintenance is estimated at $1,100 per year. Daily operating expenses are estimated at $23, including the cost of the driver. The contractor can hire a similar unit and its driver for $53 per day.[1] If the interest

[1] Based on data in "Contractor Equipment Ownership Expense" book of the Associated General Contractors of America, Inc.

is taken at 8%, how many days per year must the services of a dump truck be required to justify the purchase of a truck?

6. The cost of constructing a hotel, including materials, labor, taxes, and carrying charges during construction and other miscellaneous items, is $980,000. The lots on which the hotel is located cost $300,000. Furniture and furnishings cost $150,000. Working capital of 30 days' gross income at 100 per cent capacity is required. The investment in the furniture and furnishings should be recovered in 7 years and the investment in the hotel should be recovered in 25 years. The land on which the hotel is built is considered not to depreciate in value.

If the hotel operates at 100% capacity, the gross annual income will be $1,200 per day for 365 days. The hotel has fixed operating expenses (exclusive of capital recovery and interest) amounting to $115,000 a year and a variable operating cost of $78,000 for 100 per cent capacity. Assume that the variable cost varies directly with the level of operation. If interest is taken at 8% compounded annually, at what per cent of capacity must the hotel operate to break even?

7. Two brands of a protective coating are being considered. Brand A costs $4.50 per gallon. Past experience with Brand A has revealed that it will cover 350 square feet of surface per gallon, will give satisfactory service for 3 years, and can be applied by a workman at the rate of 70 square feet per hour. Brand B, which costs $7.60 per gallon, is estimated to cover 400 square feet of surface per gallon and can be applied at the rate of 80 square feet per hour. The wage rate of the workman is $1.92 per hour. If an interest rate of 10% is used, how long should Brand B last to provide service at equal cost with that provided by Brand A?

8. Pumpers are hired to visit each well in an oil field at regular intervals throughout the 24 hours of each day. The purpose of these visits may be classified under the following headings:

(1) To service, adjust, and make minor repairs on equipment and to take care of tank batteries.

(2) To detect "down" wells (wells that are idle because of some mechanical failure) and report them so that they may be put into service again with the least loss of oil production.

The second classification of purpose is satisfied by a visit to the well for either purpose.

Loss of production chargeable to the pumper function is the value of the oil that could have been produced between the time a well goes down and the time the pumper detects the down well and reports it by telephone. In this situation it is desirable to determine the most economical number of visits to make to each well at regular intervals in 24 hours. Let

P = the net value in dollars of the oil normally produced by a well in 24 hours.
M = number of visits per well per 24 hours to satisfy Purpose No. 1.
K = cost of each visit for Purpose No. 1.

N = sum of the number of visits per well per 24 hours to satisfy Purposes No. 1 and No. 2.
C = cost of each visit for Purpose No. 2.
T = average number of days between downs per well in a field.
S = sum of the probable daily average cost of lost production and the cost of N visits to the well.

Assume that when a well fails the failure occurs midway in point of time between visits.

(a) What is the value of S if P = \$400, $M = 2$, K = \$.40, $N = 8$, C = \$.20, and T = 40 days?

(b) Write an equation for S in terms of P, M, K, N, C, and T.

(c) Plot the value of S for the data given in part (a) as N varies from 2 to 10.

(d) Differentiate S with respect to N in the equation obtained in part (b). Set the result of the differentiation equal to zero and solve for N to find the value of N for which S will be a minimum.

(e) With the result found in part (d), calculate the value of N for minimum cost, using values given in part (a).

9. Taylor found that metal is removed at minimum cost if feeds and speeds are such that a lathe tool will endure for T hours between changes for sharpening where $T = 7t$. In this expression, t multiplied by the sum of the lathe operator's and the lathe's hourly cost is equal to all costs incident to a single change of tools. The costs incident to a change of tools includes the time of the lathe operator and the lathe during a change of tools, and all costs related to sharpening the tool.

Calculate T in hours when the lathe's rate is \$1.00 per hour. The lathe operator receives \$1.20 per hour. The lathe is stopped 6 minutes to change tools. Time consumed by the tool grinder and tool grinder operator at rates \$.80 and \$1.60 per hour, respectively, is 9 minutes per regrind. The tool is purchased ready for use for \$2.50 and may be reground 12 times before scrapping.

Note. As a matter of interest it may be noted that the above expression is based on Frederick Taylor's famous metal cutting experiments which revealed that the life of a lathe tool is inversely proportional to the cutting speed to the eighth power.

10. A manufacturing firm uses 3,000 sheets of insulating material per year at a constant rate. The average delivered cost per sheet is \$2.20. The firm estimates that it costs \$25 to place and receive an order. Interest taxes, insurance, and losses on this material are estimated at 12% of the average value on hand per year. The storage cost per sheet is estimated at \$.25 per year in general warehouse space that is also used to store other materials.

(a) Calculate the number of lots to order per year for least total cost.

(b) What will be the costs and the differences of costs of providing a year's needs by ordering 2, 6, or 12 lots per year?

11. Ethyl acetate is made from acetic acid and ethyl alcohol. The relationship of input and output is given by the expression

$$\frac{Z^2}{(1.47\,X - Z)(1.91\,Y - Z)} = 3.91$$

where X = pounds of acetic acid, Y = pounds of ethyl alcohol, and Z = pounds of ethyl acetate.

(a) Determine the output of ethyl acetate per pound of acetic acid, where the ratio of acetic acid to ethyl alcohol is 2, 1.5, 1, ⅔, and ½ and graph the result.

(b) Graph the cost of material per pound of ethyl acetate for each of the ratios given in part (a), and determine the ratio for which the material cost per pound of ethyl acetate is a minimum if acetic acid costs $.08 per pound and ethyl alcohol costs $.06 per pound.

(c) Calculate the ratio in part (b), taking the cost of acetic acid to be $.04 per pound.

12. The daily electrical load transmitted by a conductor in a generating station is considered to be represented by 800, 1,800, and 3,200 amperes for 12, 6, and 6 hours per day respectively for 365 days per year.

The following data apply to the conductor: length of conductor, 140 feet; installed cost, $160 + $.18 per pound of copper; estimated life, 20 years; salvage value, $.08 per pound of copper. Electrical resistance of a copper conductor 140 feet long and of 1 square inch cross section is .0011435 ohm, and the electrical resistance is inversely proportional to the area of the cross section. The energy loss in kw-hrs. in a conductor due to resistance is equal to $I^2R \times$ no. of hours $\div$ 1,000, where I is the current flow in amperes and R is the resistance of the conductor in ohms. Copper weighs 555 pounds per cubic foot. The energy lost is valued at $.007 per kw-hr.; taxes, insurance, and maintenance are negligible; the interest rate is 6%. Use sinking-fund depreciation in calculating capital recovery.

(a) Plot the total annual cost, including capital recovery with a return and lost energy cost, for conductors of 4-, 5-, 6-, 7-, and 8-square-inch cross sections to find the optimum cross section.

(b) Set up an equation for annual cost C in terms of the cross section A. Differentiate C with respect to A, set the result equal to zero, and solve for the optimum cross section.

13. The loss of heat through frame construction walls made up of $\frac{7}{16}$-inch clap boards, paper, ¾-inch sheathing, studdings, wooden lath, and plaster is .23 Btu per hour per square foot of wall area per degree Fahrenheit temperature difference. If a certain insulating material is placed between the walls, the heat loss in Btu per hour per degree temperature difference per square foot of wall area is taken as being equal to

$$\frac{1}{.23} + \frac{t}{.27}$$

where t is the thickness of the insulation in inches. The in-place cost of insulation 1, 2, and 3 inches thick is \$.075, \$.104, and \$.134 per square foot respectively. The dwelling under consideration has 1,600 square feet of outside wall area. The dwelling is heated by a gas furnace with an efficiency of 50 per cent. The heating value of the natural gas used is 1,000 Btu per cubic foot and costs \$.70 per 1,000 cubic feet. The mean outside temperature during a 150-day heating period is 40 degrees Fahrenheit and the desired inside temperature is 70 degrees Fahrenheit for 24 hours per day.

What thickness of insulation, if any, should be used if the interest rate is 5% and if the resale value of the house 12 years hence is enhanced \$150 by having insulated walls, regardless of the thickness of insulation used?

CHAPTER

13

Evaluation of Proposals for New Activities

IN A GOING CONCERN situations often arise where action must be taken. Consider the situation of a power company one of whose boilers has failed. Ordinarily the question is not "Shall action be taken?" but "What action shall be taken?" In other words, circumstances force some action to be taken.

Usually, when a new activity is under consideration, it may be rejected or accepted. Thus the first question to be answered is "Shall the proposed new activity be undertaken?" An affirmative answer to this question requires an answer to "What action shall be taken?" as above.

When a proposal to undertake an activity includes methods for implementing it, the question "Shall the proposed new activity be undertaken?" is often answered in the affirmative without proper consideration because of enthusiasm for answering the second, "What action shall be taken?" The first question relates to the selection of a desirable objective and the second question relates to its attainment. When economy is a consideration, the aim is first to select a potentially profitable venture, and second to develop it effectively.

For this discussion a new activity will be considered to be a proposed new opportunity for profit whose acceptance or rejection is substantially independent of previous commitments. A new activity may be either independent or an extension of some present undertaking. Examples of the latter are the opening of a branch store by an existing firm or the addition of a new line of products by a manufacturing concern.

Some Advantages of a New Activity

Certain advantages favor a new activity. Since it is independent of previous commitments, it may be implemented in large measure on the basis of present and future considerations. Little consideration need be taken of the past except to profit by previous mistakes. Contrast this with the situation of railroads, whose future development is for all practical purposes limited by the present standard distance between rails. This gauge is not the best for either economy or comfort for higher-speed trains, but to change it is not feasible. Similarly, a bar to advances in communication is the necessity for integrating new developments into the existing system of equipment.

A new activity may enjoy a tremendous temporary advantage from being first in the field. Its launching is analogous to the surprise attack in war, where traditionally a small force overwhelms a much larger force. Where the new enterprise is based upon a patented device, initiative of discovery results in the advantage of monopoly to a greater or lesser extent. But even without patents the discovery and the vigorous prosecution of an economic opportunity often result in a tremendous initial gain before rivals can come into the field.

Embarking upon a new activity has an element of adventure associated with it and usually requires more creative thinking than the expansion of an old activity. Also new activities result in employment of new personnel or in a change of activity of existing personnel. These factors often create enthusiasm and in turn superlative accomplishment of both individuals and groups.

Some Disadvantages of a New Activity

A serious disadvantage faced in launching many new enterprises is that they often require an input of money and effort for long periods of time prior to realization of income from them. Many new ventures of ultimate promise fail because funds and enthusiasm are exhausted before adequate returns can be established. Unless strong financial backing without returns for long periods of time is available, many otherwise profitable undertakings are not feasible. Inadequate financing is recognized as one of the most important causes of failure of new enterprises.

Some new enterprises can be entered into on a small scale where income is substantially current with input and then be expanded gradually. Many of our present large firms became established in this way and the opportunity for modest beginning and gradual growth still exists, apparently unimpaired. But there are many activities that cannot be entered successfully without tremendous initial investment; automobile and steel manufacture are two examples.

In any pioneering activity lack of experience often results in the arising of unforeseen contingencies for which provision has not been made. Lack of experience or ability in regard to financing, organization, marketing, production, and other important activities is an important cause of failure in otherwise promising opportunities. Thus in launching a new enterprise competent personnel is of great importance. Incompetence of management ranks along with inadequate financing as a cause of failure of new enterprises.

Many new enterprises fail because the opportunities they are designed to exploit are inadequate for success. Although the opportunities for profit should be ascertained prior to deciding upon a proposal if possible, many opportunities cannot be evaluated even reasonably well except by trial. When this is the case, the uncertainty of the enterprise should be realized so that steps may be taken to meet the outcome, whatever it may be. If the outlook for the new activity seems promising, immediate steps should be taken to exploit the opportunity with vigor. If it seems unpromising, steps should be taken to withdraw from the venture with a minimum loss of money and effort. The development of mining properties and oil fields, theatrical presentations, and marketing of new products, particularly those that are dependent upon people's whims, are examples of ventures that are often impossible to evaluate except by trial.

Unforeseen Hazards of New Enterprises

Experience leads to the conclusion that, despite the most careful planning, a new enterprise will be more subject to unforeseen hazards or contingencies than one that is well established. Furthermore, the new enterprise usually has very limited reserves to meet even temporary reversals.

Demands for new products are subject to both seasonal and long-time fluctuations. A new enterprise that begins on a long-time decline of demand for its product may fail from this cause. A new enterprise entering a seasonal market may lose much of its first year's income through any cause that may delay completion of its plant in time to take advantage of the active period for its product.

A new enterprise usually needs considerable credit during the period of rapid expansion before it has time to establish itself favorably with sources of credit. It is very vulnerable to any tightening of credit in this period. Even if its need for credit may arise from a more rapid expansion of sales than expected, it may fail from inability to secure funds to bridge the period between expenditures for the manufacture of its product and receipts from its sales.

Established firms do not ordinarily welcome a new competitor into the field with enthusiasm. Thus a new firm often finds active and concerted efforts directed toward discrediting it and its products. This can be very damaging and difficult to offset.

A most important factor in the success of a new venture is the enthusiasm and capabilities of its leaders. These men are subject to the hazards of sickness and accident. Impairment of health or death of a capable leader can easily cause a new enterprise to fail. This hazard may be offset in some measure by carrying life and accident insurance upon principal officers. But payments from insurance often do not compensate for loss of leadership that is essential to success.

Analysis Should Begin with Income

In a free-enterprise economy, success of a new enterprise is usually more dependent upon market limitations than upon the ability to produce. Thus the problem is to organize production to conform to the market in respect to a product, its quantity, and price. Eidmann has said that ". . . analyses should begin with the customer and be developed toward the supply of raw materials and the production end of the business rather than in reverse order."[1] This statement is particularly pertinent in the consideration of a new activity. Through experimentation, research, and analysis the quality, price, and quantity that can be produced and sold most profitably should be determined.

The new enterprise may have to limit itself to lines or markets on which a quick return can be realized. It may also have to skim the market by selling only where sales can be made with minimum promotion. These restrictions will tend to limit sales.

After income estimates have been made, they should be next translated into certain products to be manufactured in certain quantities at certain times at a certain cost. Once production schedules have been determined, production facilities to meet them can be worked out on the basis of economy.

[1] Frank L. Eidmann, *Economic Control of Engineering and Manufacturing* (New York: McGraw-Hill Book Company, Inc., 1931), p. 323.

Income Lags behind Expense

The income from many new activities lags behind the expenses associated with them for a considerable period of time. In some instances this lag, in terms of time and amount of funds needed, is known with considerable accuracy. For example, contracts for buildings, bridges, and large units of equipment usually stipulate a schedule of payments to be made based upon progress of the work.

Where income is derived from sales of a new product, income often expands gradually over a long period of time before it equals expenses. Such a situation imposes special problems in the selection of equipment.

Equipment Selection for Expanding Activities

In the early stages of the enterprise, when production is low, it will usually prove to be economical to purchase equipment whose fixed costs are low. In the latter stages, when sales are approaching the ultimate level, high fixed cost equipment permitting low variable production costs may be most economical. In this connection consider the following example:

It is estimated that annual sales of a new product will begin at 1,000 units the first year and increase by increments of 1,000 units per year until 4,000 units are sold during the fourth and subsequent years. Two proposals for equipment to manufacture the product are under consideration.

Proposal A embraces equipment requiring an investment of approximately $10,000. Annual fixed cost with this equipment is calculated to be $2,000 and the variable cost per unit of product will be $.90. The life of the equipment is estimated at 4 years.

Proposal B embraces equipment requiring an investment of approximately $20,000. Fixed cost of this equipment is estimated at $3,800 per year and variable cost per unit of product will be $.30. The life of this equipment is also estimated at 4 years.

On the basis of the ultimate annual production of 4,000 units, cost per unit will be as follows:

PROPOSAL A

Annual cost for 4,000 units of product, $2,000 + (4,000 × $.90)	$5,600.00
Cost per unit, $5,600 ÷ 4,000	1.40

PROPOSAL B

Annual cost for 4,000 units of product, $3,800 + (4,000 × $.30)	$5,000.00
Cost per unit, $5,000 ÷ 4,000	1.25

On the basis of the ultimate rate of production, Proposal B is superior to Proposal A. On the basis of the total production during the life of the equipment, the following analyses apply:

PROPOSAL A

Year of Life	*No. of Units Made*	*Fixed Cost*	*Variable Cost*
1	1,000	$2,000	1,000 × $.90 = $ 900
2	2,000	2,000	2,000 × $.90 = 1,800
3	3,000	2,000	3,000 × $.90 = 2,700
4	4,000	2,000	4,000 × $.90 = 3,600
	10,000	$8,000	$9,000

Cost per unit = ($8,000 + $9,000) ÷ 10,000 = $1.70

PROPOSAL B

Year of Life	*No. of Units Made*	*Fixed Cost*	*Variable Cost*
1	1,000	$3,800	1,000 × $.30 = $ 300
2	2,000	3,800	2,000 × $.30 = 600
3	3,000	3,800	3,000 × $.30 = 900
4	4,000	3,800	4,000 × $.30 = 1,200
	10,000	$15,200	$3,000

Cost per unit = ($15,200 + $3,000) ÷ 10,000 = $1.82

The calculated advantage of Proposal A over Proposal B would have been increased by considering the time value of money. But perhaps even more important than the difference in cost per unit, particularly for a new enterprise that must conserve its funds or where there is considerable doubt that production schedules will be reached, is the lesser investment required by Proposal A.

Single-Purpose vs. General-Purpose Facilities

At rates of operation approaching capacity, single-purpose machines may be expected to produce more economically than general-purpose machines. Specialized facilities usually result in higher fixed costs and lower variable costs than general-purpose facilities. Consequently specialized equipment is generally advantageous for high volumes of work and general-purpose equipment is advantageous for low volumes of work.

Single-purpose machines particularly are subject to obsolescence owing to change in design or demand for the product on which they are specialized. Soundness of analyses involving specialized facilities rests heavily upon the accuracy of estimates of the volume of work to be performed by them.

Future Prices in Economy Studies

Prices of specific items and price levels in general fluctuate with the passage of time. Figure 35 shows the trend of construction costs over a period of nearly a century. This chart shows that price levels may

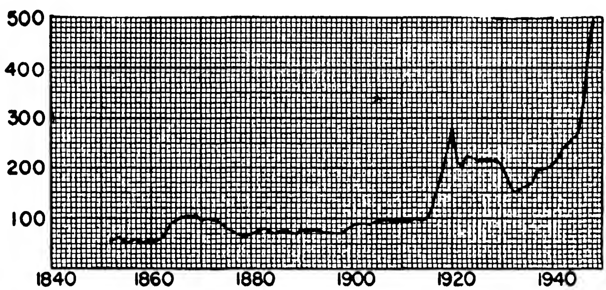


FIG. 35. The American Appraisal Company construction cost index on which 1913 = 100. This index is based on the construction materials and labor required for four representative types of frame, brick, concrete, and steel buildings (exclusive of building fixtures) averaged for thirty representative cities. It gives no recognition to exceptional overtime labor. [Reproduced by permission of the American Appraisal Company, Milwaukee, Wis.]

fluctuate widely and rapidly. If future prices could be known, it would be simple to take them into consideration in economy studies. But no matter how thoroughly price trends of the past are known, prices in the future can only be estimated. The processes of buying and selling involve estimates of future prices. The estimates of buyers and sellers are based upon what they think will happen in the future. For example, a person who possesses something to sell will base its present price upon what he expects he can get for it in the future less what he thinks it will cost him to possess it until the time of sale is reached. Thus present prices of most items of commerce represent a composite of future-price estimates of many buyers and sellers.

Unless there is good reason to the contrary, the composite estimate of many buyers and sellers as embodied in present prices should be used in economy studies.

How Long To Plan for

How long to plan for is a question that confronts all who plan for the future. The period to be covered by a plan should be based upon three considerations and the relationships between them. These considerations are:

(1) The income period of the proposed activity.
(2) The life of the facilities involved.
(3) The payout period of the facilities involved.

It is clear that the income period of an activity must at least be equal to the payout period of a facility that is to be used. It is also clear that life of a facility must at least equal the payout period at the rate of income received.

In some activities the income period may be relatively long in comparison to either the life or the payout period of equipment involved; a cab driver, for example, may operate a taxi all his life but get a new cab each year. There are also activities whose income period is nearly equal to the payout period of the facilities involved.

Though a concrete answer cannot be given to the question of how long to plan for, it can be said that the minimum period should not be less than the payout period of the facilities involved. The maximum period to be considered is the income period of an activity.

Economy of Present Expenditures To Defer Decision

In a new activity where success embodies elements of uncertainty, there is often a disposition to hold capital equipment investment to a minimum until outcomes become clearer, even though this plan may result in higher production costs. Such action amounts to a decision to incur higher costs temporarily in order to reserve the privilege of making a second decision when the situation becomes clearer.

The accuracy of estimates with respect to events in the future is at least to some extent inversely proportional to the span of time between the estimate and the event. It is often sound to incur expense for the privilege of deferring decision. This premise may be illustrated by the following example.

A firm has need for warehouse space in a new business territory it is about to enter. It is estimated that the equivalent annual cost of providing a warehouse to meet estimated needs through ownership will amount to $5,000 per year. Warehouse space can be leased for $8,000 for the first year, with an option to renew the lease. In spite of the higher cost, it is desired to lease for the first year until the situation becomes clearer. It is considered possible that during the year (1) the firm will withdraw from the territory, or (2) warehouse needs will be either lesser or greater than estimated. In other words it is considered prudent to spend $3,000 during the first year in order to be free to consider the warehouse problem one year hence.

When To Buy and When To Manufacture

In some cases the ability to market overshadows the ability to produce, to the extent that a new enterprise may begin as a sales organization dependent upon others for its product. The practice of most firms lies between this extreme and the extreme in which a product is completely produced and sold by one firm. The question of when to buy and when to manufacture is particularly pertinent in regard to new activities. In the manufacture of a product certain parts may require fixed investment out of proportion to the product as a whole. For instance, parts of a product may require specialized machines or they may require facilities that can be used only for a small part of their capacities. Such parts may often be purchased for less than the cost of producing them in a new activity, particularly at low levels of operation. The economic advantage of this practice can easily be determined by getting bids from manufacturers for parts and comparing them with estimates of the cost of producing them under the conditions that would apply if their manufacture were to be undertaken.

It is often justifiable to buy parts at a cost considerably higher than that of manufacturing them if so doing will reduce investment for equipment. Also, in a new venture initial designs of products are often materially changed or even abandoned, with a resulting high loss in equipment investment.

Selection of a Location for a Plant

The selection of a location for a plant is a long-time commitment. A new enterprise may be hampered throughout its life by an unfavorable location. Once a plant has been built, the expense and disruption of activities necessary to move it to a more favorable location is generally so great as to be impractical, even though failure may result from the unfavorable characteristics of the original location. Therefore, the search for and evaluation of plant sites justify very careful consideration.

When profit is the measure of success, the best location is the one in which production and marketing effort will result in the greatest profit. Location may affect the cost at which raw materials are gathered, the cost of production, the cost of marketing, and the volume of product that can be sold.

The basic items involved in evaluating the economy of a location are related to:

Market	Labor
Raw material	Laws and taxation
Transportation	Miscellaneous considerations
Fuel and power	

Evaluation of a location begins with research to determine the volume of sales and income promised by the given location. Then research is directed along the lines suggested by the factors listed above to gain data with which to calculate the cost of gathering raw material, processing it, and delivering finished products to the consumer in the volume that can be sold. Evaluation of plant locations consists essentially of operating the enterprise under consideration "on paper" at each location studied. The results of this approach in evaluating three locations for a small glassware plant requiring an investment of approximately $180,000 are given in Table 24. The values given in this table represent a summation of costs and income based on detailed analyses of the operation of the plant "on paper."

Consider the competitive advantage of a plant at Location C over one at Location B. A plant at Location C has an advantage in net income of over 4 per cent by virtue of its location to offset price reductions and operating and selling efficiency of a competing plant at Location B.

Figure 36 shows the result of an analysis, based upon rail freight rates, raw material costs, and manufacturing costs, of the locations of plants to manufacture and market ethylene glycol. If it is assumed that there is sufficient market adjacent to Philadelphia, Kansas

TABLE 24. Comparative Evaluation of Three Locations for a Proposed Glassware Plant

	Location A	Location B	Location C
Market			
Annual sales....................(A)	$260,000	$260,000	$260,000
Selling expense................(B)	44,000	43,000	46,000
Net income from sales..........	$216,000	$217,000	$214,000
Production Costs			
Supplies and raw materials........	$ 69,000	$ 71,000	$ 62,000
Transportation—in and out........	27,000	26,000	23,000
Fuel, power, and water............	13,000	17,000	18,000
Wages and salaries...............	64,000	63,000	61,000
Miscellaneous items..............	8,000	8,000	10,000
Fixed costs other than interest.....	11,000	11,000	11,000
Net Production Costs........(C)	$192,000	$196,000	$185,000
Selling plus Production Costs			
B + C.....................(D)	236,000	239,000	231,000
Annual Profit, A − D...........(E)	24,000	21,000	29,000
Rate of Return. E ÷ $180.000.......	13.3%	11.7%	16.1%

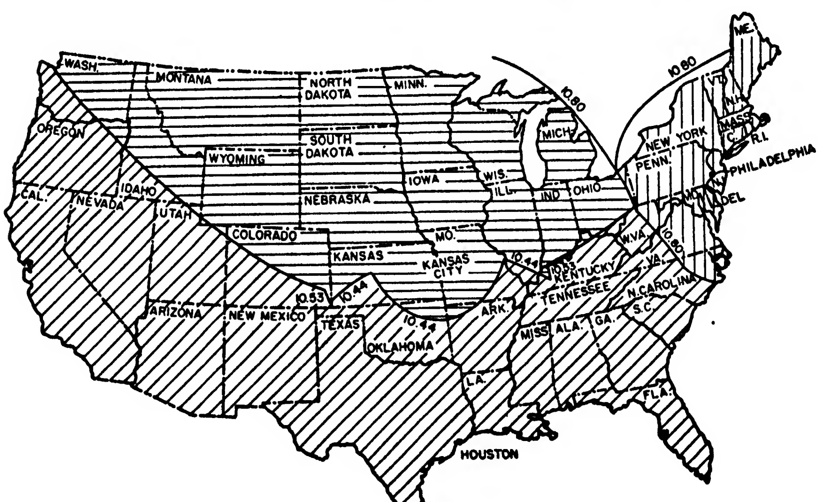


FIG. 36. Approximate non-competitive marketing areas of ethylene glycol for plants at Houston, Kansas City, and Philadelphia. Based on rail freight rates and raw material costs. Figures shown at boundaries of areas are approximate manufacturing cost in cents per pound. [Reproduced through the courtesy of the publisher from Sittenfield, Marcus, "The Economics of Petroleum Chemical Plant Locations," *Chemical Engineering Progress*, Vol. 45, No. 5, p. 322, May 1949, published by the American Institute of Chemical Engineers.]

City, and Houston to absorb the output of a minimum economically-sized plant at each of these cities, then plants should be designed to serve the areas shown.

Levels of Development

Since a new development is not hampered by past commitments, the extent or scope of its development is open for consideration. This permits optimum development, all things considered. There are two basic considerations in the development of an opportunity, the limitations of the opportunity itself and the limitations of the means for exploiting it.

Often only the limitations of the opportunity are considered. In this case it is assumed that the means, usually expressed in terms of funds, will be sufficient to provide a fair rate of return from the enterprise.

Limitations Imposed by Ability to Finance

Often the levels to which an opportunity can be developed are dependent upon the availability of funds. Industrial history records many cases in which the benefits of excellent opportunities were lost to their discoverers because they could not finance their development to fruition.

Many opportunities can be profitable only when prosecuted above certain levels. If funds cannot be obtained to reach the necessary level of development, the opportunity may have to be abandoned. In many cases a person or a firm controls an opportunity but has insufficient funds to exploit it to the best advantages. In such situations, sufficient funds for the optimum level of development from a profit standpoint can be acquired only by relinquishing control of the opportunity. Such a situation gives rise to three alternatives: to abandon the opportunity, to develop it at less than the optimum level, to sell it to others more able to exploit it. The last-mentioned alternative is chosen, for example, by inventors who sell their patents.

The determination of levels of development not limited by funds is often made for the purpose of determining the amount of funds needed to exploit an opportunity and the rate of return to be expected. The results of such analyses are of value in making the most profitable use of the funds that are available or that can be made available. In the following paragraphs a number of analyses, involving different levels of development with respect to several factors, will be made without consideration of limitations imposed by availability of funds.

Levels of investment for a rental property. A new enterprise must decide to what extent it should develop its opportunity. For example, it must decide the volume of sales on which to base its activities. If the sales volume is set too high, the sales effort required may have to be so great that profits disappear. On the other hand, sales volumes can be set so low that economical processing is impossible.

The effect of levels of development may be simply illustrated by an example in real estate development. Mr. A has a 40-year lease on a building site in an area of neighborhood stores; on this site he wishes to construct a building for rental. He intends to consider the feasibility of buildings of one, two, three, or four stories. The first story will be planned for shops, the upper stories for professional offices. Since Mr. A does not own the land, the proposed building will have no salvage value for him on termination of the lease 40 years hence. Based on research and estimate, the investment necessary and the net income to be expected by each of the levels of development are given in Table 25.

TABLE 25. Analysis of Return for Several Levels of Development of a Rental Property

	No. of Stories			
	1	2	3	4
Investment in property......(A)	$105,000	$160,000	$208,000	$253,000
Increment investment per story....................(B)	105,000	55,000	48,000	45,000
Net income (after depreciation but before interest)...(C)	8,000	15,000	20,000	23,000
Increment net income per story ....(D)	8,000	7,000	5,000	3,000
Rate of return based on A and C	7.61%	9.38%	9.61%	9.09%
Rate of return on increment investment per story based on B and D....................	7.61%	12.73%	10.42%	6.67%

In this example Mr. A must choose one of four alternatives—the building of 1, 2, 3, and 4 stories—which will result in a return on invested funds equivalent to an interest rate of 7.61, 9.38, 9.61, and

9.09 per cent respectively. If he has the funds to invest and considers 8 per cent an adequate return on his funds, it appears that he would be justified in building four stories.

If the investment is considered in increments, the rate of return for the investment in each story may be found. The increment analysis reveals that only a two- or three-story building can be justified if a rate of return of 8 per cent is desired.

The above example showing to what extent to develop rental property is representative of a situation that confronts new enterprises in general. It is important that a new enterprise be planned for the level of sales that will prove most desirable in relation to investment and profit. As sales are converted into production programs and equipment, levels of investment should be considered item by item. Levels of investment may be based on several considerations. Some considerations pertinent to the selection of equipment are capacity, durability, efficiency, flexibility, and provision for expansion.

Levels of durability or life span. It is often possible to construct different buildings and machines or their elements so that they will render essentially identical service except for maintenance and frequency of replacement. Consider the following example from the chemical industry.

A piping system to carry a corrosive liquid in a proposed chemical plant could be built either from steel or from a more corrosive-resistant alloy. The steel pipe had an estimated life of 4 years and the estimated life of the alloy pipe was 10 years. A comparison of the two systems on an annual cost basis follows:

EQUIVALENT ANNUAL COSTS

Steel Piping	
Capital recovery and return on installed cost, \$23,000 × (.28201) RP 5-4	\$6,486
Estimated average maintenance cost	1,300
	\$7,786
Alloy Piping	
Capital recovery and return on installed cost, \$48,000 × (.12950) RP 5-10	\$6,216
Estimated average maintenance cost	600
	\$6,816
Net annual difference in favor of alloy piping	\$ 970

The rate of interest for which the equivalent annual costs for the two types of systems are equal is approximately 10.8 per cent. This is well above the rate of interest of 5 per cent which the new enterprise was paying on funds it was borrowing. However, the cheaper steel piping was selected because the officials of the new enterprise felt that the additional $25,000 required for alloy piping could not be justified in the face of the many opportunities to invest their limited funds to better advantage in other needed units of equipment. In fact, a policy had been adopted to limit purchases to equipment that would net a return of 25 per cent or more. For an interest rate of 25 per cent the following comparison applies to the two types of piping systems:

Equivalent Annual Costs

Steel Piping

Capital recovery and return on installed cost, $23,000 × (.42344) [RP 25-4]	$ 9,739
Estimated average maintenance cost........................	1,300
	$11,039

Alloy Piping

Capital recovery and return on installed cost, $48,000 × (.28007) [RP 25-10]	$13,443
Estimated average maintenance cost........................	600
	$14,043
Net annual difference in favor of steel piping...............	$ 3,004

This difference of $3,004 can be considered to be the equivalent annual loss that would result if the more durable piping system were installed, assuming opportunities existed for investing all available funds in equipment that would net a return of 25 per cent.

Levels of efficiency. Equipment may be considered to have two efficiencies, physical efficiency and economic efficiency. There may be little relationship between these efficiencies. When the profit motive is the prime consideration, the latter efficiency must prevail.

Consider the thickness of insulation which is economically most desirable for a proposed cold storage room. The purpose of insulation is to decrease infiltration of heat; the greater the thickness of insulation, the less the heat inflow. Also, the greater the thickness, the greater the cost. Thus there is a level of thickness of insulation beyond which there is no gain in economic efficiency. In illustration of this fact consider the following example.

The owner of a refrigeration plant is planning to build a temporary storage room and wishes to determine the most economical thickness of cork board to use to insulate the walls. The storage room will have a use life of 2 years. Used cork board in place is estimated to cost \$110 per thousand square feet for each inch of thickness and to have a salvage value of 30 per cent of the original cost at the end of 2 years.

The average difference in temperature outside and inside the storage room is estimated at 80 degrees F. The heat transfer through the cork board will be in accordance with the formula: Heat transfer in Btu. per square foot of exposed surface per degree F temperature difference = .26/*t*, where *t* is the thickness of the cork board in inches. Thus the rate of heat inflow per thousand square feet of exposed surface for the conditions given above will be

$$\text{Heat inflow, Btu. per hr.} = 1{,}000 \text{ sq. ft} \times 80 \text{ degrees F} \times .26/t$$

Accounts of the plant reveal that the energy and fixed costs are such that the cost of removing heat at the rate of one Btu. per hour over a period of one year amounts to \$.036. Interest is to be taken at 6 per cent. Insurance, taxes, and maintenance are considered negligible. On the basis of the above conditions, the desirability of various thicknesses of cork board may be calculated on the basis of a thousand square feet of wall surface as follows:

COST PER 1,000 SQUARE FEET OF WALL SURFACE FOR 2-IN. CORK BOARD

Equivalent annual cost of cork board,	
RP 6-2	
(\$220 − .3 × \$220) (.54544) + \$220 × .3 × .06...	\$ 88
Annual cost of refrigeration	
1,000 square feet × 80 deg. F × $\frac{.26}{2}$ × \$.036............	374
Total equivalent annual cost........................	\$462

In a similar manner costs are calculated for other thicknesses; a tabulation of the results appears in Table 26.

TABLE 26. Summary of Cost for Different Thicknesses of Insulation

Thickness of Cork Board	Depreciation and Interest	Cost of Refrigeration	Total Annual Cost
2″	\$ 88	\$374	\$462
3″	132	249	381
4″	176	187	363
5″	220	150	370
6″	264	125	389

On the basis of the conditions given, 4 inches of cork board will be most desirable. Note that the difference in annual cost between 2 and 3 is much larger than the difference between 3 and 4 inches of cork board. Cork board 4 inches thick was selected.

Initial or Future Provision of Facilities for Expansion of Activities

At its outset a new enterprise operates at a zero level of activity. From this level its activity expands if the enterprise is successful. Ordinarily the activity of a new enterprise may be expected to grow slowly but at an increasing rate in the early stages of its existence. Later growth may be expected to take place at a decreasing rate, which ultimately approaches zero as activities stabilize at an upper level. A pattern of growth characteristic of many enterprises appears in Fig. 37.

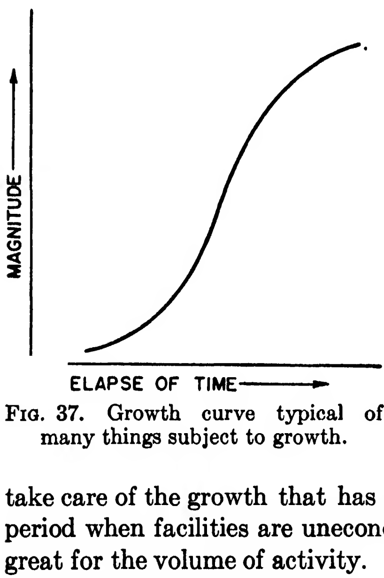


FIG. 37. Growth curve typical of many things subject to growth.

The expansion of the activity of an enterprise poses many problems in economy. One of these is related to the capacity of facilities. Facilities for an expanding activity, like clothes for a growing boy, must be purchased too large in order that they may be large enough at a later date to take care of the growth that has taken place. Thus there will be a period when facilities are uneconomical because their capacity is too great for the volume of activity. Rarely is it feasible to add facilities in increments that closely parallel increments of growth. Thus equipment investments are an economic compromise between what is most desirable at present and what is necessary for the future.

The problem of planning for expansion involves determining how much it is economical to spend now to reduce an expenditure for expansion in the future.

Economy of providing for future needs. In illustration consider the following example. An engineer who is planning a home owns a car but expects to own two cars sometime in the future. He is confronted with a decision whether to build a single or a two-car garage now. Inquiry reveals that he can have a single garage built for $810 and a double garage built for $1,440. If he has a single garage constructed now, it can be rebuilt into a double garage any time in the future. To do so will require demolition of one wall and part of the roof of the original garage. It is estimated that the enlargement will cost $1,000 on the basis of present prices. On the basis that extra garage space has no value until the owner acquires a second car 6 years hence, that maintenance cost, insurance, and taxes will amount to 3 per cent of first cost per annum, that present building costs will prevail, and that the additional funds necessary can be had for 5 per cent, the situation can be analyzed as follows:

Plan A. Build Single Garage Now and Enlarge Later

Present-worth cost of building single garage...............	$ 810
Present worth of 6 years maintenance, insurance, and taxes, $810 × .03 (5.076) [PR 5-6].................................	123
Present-worth cost of enlargement, $1,000 × (.7462) [PS 5-6].......	746
	$1,679

Plan B. Build Double Garage Now

Present-worth cost of building double garage..............	$1,440
Present worth of 6 years maintenance, insurance, and taxes, $1,440 × .03 (5.076) [PR 5-6]................................	219
	$1,659

On the basis of the data used, Plan B is slightly advantageous. If a double garage were needed before an elapse of 6 years time, it would be still more advantageous. High interest rates and maintenance costs would tend to favor Plan A.

Effect of load characteristics of equipment on economy. Most production facilities, whether buildings or equipment, may be operated with greatest economy at a rate corresponding to their normal capacity. It follows that facilities are usually inefficient and costly to operate when utilized at a relatively low rate of output in terms of normal capacity. For example, if a production department is housed in a space considerably larger than is needed for its efficient operation, a number of losses may be expected to arise. The fixed costs such as taxes, insurance, interest, and maintenance will be higher than necessary. Heat and light and, perhaps, janitor service may have to be provided for the unused space. Similarly the unneeded space may increase the cost of material handling and the difficulties of supervision.

The losses incident to operating facilities at low rates in terms of capacity are very clearly evident in electrical machines. For this reason an electrical machine will be used in illustrating the effect of load characteristics of equipment on economy, particularly as it relates to expanding activities.

A mining company that is slowly expanding its operations is in need of direct current for an electrolytic process. During the next year it expects to need an average of 12 kw of direct-current energy for 2,000 hours. Planned expansion for the mine is expected to increase the rate at which direct current is needed by 2 kw per year until 30 kw are needed. At this point the rate that direct current is needed will remain constant. Energy will be needed 2,000 hours per year regardless of the rate it is used.

Investigation reveals that the need for direct current can best be provided by an AC-DC motor-generator set. These sets can be purchased in a variety of capacities. Two plans are considered.

Plan A. Purchase a 20-kw set now and use for five years, by which time the demand will have reached 20 kw, then purchase a 30-kw set.

Efficiency-load curves provided by the vendor show that the 20-kw set has efficiencies of 48, 69, 78, and 76 per cent at ¼, ½, ¾, and full load respectively. The 30-kw set is slightly more efficient at its ¼, ½, ¾ and full load. The purchase rate of AC current is $.02 per kw-hr. Interest is taken at 10 per cent. Taxes, insurance

Analysis of Plan A

Present-worth cost of current used:

Year	Output Rate DC Current in kw (A)	Efficiency at DC Output Rate (B)	Input Rate AC Current, A ÷ B (C)	Annual Power Bill C × $.02 × 2,000 (D)	Present Worth of Annual Power Bill D × (xxxx) PS 10-*n* (E)
1	12	74	16.2	$ 648	$589
2	14	77	18.2	728	602
3	16	78	20.5	820	616
4	18	78	23.1	924	631
5	20	76	26.3	1,052	653

Total present worth of power bill for 5 years (ΣE in table)	$3,091
Present-worth cost of 20-kw set installed	$1,230
Present worth of 20-kw set trade-in receipt, $800(.6209) (PS 10-5)	−497
Present worth of 30-kw set, $1,580(.6209) (PS 10-5)	981
Present-worth cost of providing 5 years of service with 20-kw set and purchasing 30-kw set 5 years hence so that the latter will be available for the future	$4,805

maintenance, and operating costs will be neglected in the interest of simplicity. The 20-kw set will cost $1,230 installed. It is estimated that $800 will be allowed for the 20-kw set on the installed purchase price of $1,580 of the 30-kw set 5 years hence.

Plan B. Purchase 30-kw set now.

ANALYSIS OF PLAN B

Present-worth cost of current used:

Year	Output Rate DC Current in kw (A)	Efficiency at DC Output Rate (B)	Input Rate AC Current A ÷ B (C)	Annual Power Bill C × $.02 × 2,000 hrs. (D)	Present Worth of Annual Power Bill PS 10-n D × (xxxxx) (E)
1	12	65	18.5	$ 740	$673
2	14	70	20.0	800	661
3	16	74	21.6	864	649
4	18	77	23.4	936	639
5	20	79	25.3	1,012	628

Total present worth of power bill for 5 years (ΣE in table)	$3,250
Present-worth cost of 30-kw set installed	$1,580
Present-worth cost of providing 5 years of service with 30-kw set and having it available for the future	$4,830

Power costs have not been taken into account beyond the first 5-year period in the above analysis because they will be equal in subsequent years, since a 30-kw set will be used regardless of whether Plan A or B is adopted.

Also the above analysis embodies the assumption that a 30 kw unit that has been used 5 years is equivalent in use value to a new 30 kw set. This is substantially true because of the long life of electrical equipment.

Steps to Cope with Hazards

There are a number of common hazards such as fire, flood, theft, and arson to which enterprises are subject. These are unpredictable with regard to time of occurrence and the extent of the loss they cause. Thus planning to meet such hazards is difficult and is given special consideration.

Hazards may originate either from the action of the elements or from the action of men. Common causes of loss arising from action of the elements are earthquake, flood, wind, hail, lightning, and spontaneous combustion. Since these manifestations are ordinarily not considered to be in the control of man, their visitations are considered to occur on the basis of chance.

Man constitutes a hazard to enterprise by virtue of such criminal derelictions as deceit, theft, arson, riot, and other acts contrary to law, and such noncriminal acts as mistakes and accidents.

Certain recognizable hazards may be met in several ways. A common method is to insure against loss. Also, loss from hazards may often be prevented or minimized by precautionary measures taken in the construction or operation of facilities. Such precautionary steps are not taken without cost and may be expected to result in higher initial investments or in increased operating expense.

Insurance. Insurance consists essentially of spreading the financial burden of losses that may occur to individuals of a group over all members of the group with respect to certain hazards and in accordance with a certain plan.

For example, assume that the 1,000 houses in a community have an aggregate value of $8,000,000 and that the annual total loss due to fire in the future will average $20,000 per year. Loss due to fire would thus be equivalent to $2.50 per $1,000 of property per year. Individuals suffering loss could be compensated for their entire loss by assessing all house owners $2.50 per $1,000 of property each owned.

It should be noted that insurance as illustrated in the above example does not reduce losses; it merely spreads them. Thus insurance is "substitution of certain for uncertain loss."[2] A person in the above example who owned a $12,000 house would substitute an annual certain loss of

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[2] S. S. Huebner, *Property Insurance* (New York: D. Appleton-Century Company, 1938), p. 6.

$$\frac{\$12,000}{\$1,000} \times \$2.50 = \$30$$

in the form of an insurance premium for an uncertain loss that might be as great as $12,000.

The economy of insurance lies in the fact that the value of freedom from the necessity of being continually prepared to meet a loss that is unpredictable in respect to time and amount outweighs the cost of insurance.

In practice, the amount charged for insurance exceeds the aggregate of losses sustained by the insured, because of administrative expenses and profit to those engaged in insurance activities. These two items compose about 40 per cent of fire insurance premiums. The balance represents loss.

Fire insurance practices recognize that the type of construction, the nature of the use and contents of a building, and the fire protection and hazards of the community in which it is located affect the probability and extent of loss. One scheme takes these factors into account by establishing rates on the basis of a defined standard building in a defined standard city. The rate applicable to a standard building in a standard city is about .25 per cent of the value of the property insured. Deviations from the established standards result in higher rates.

Insurance is available to spread the loss of a great many hazards arising from the elements or from acts of man.

Precautionary measures to minimize the effect of hazards. The effect of future hazards may often be substantially reduced by precautionary measures that increase the initial cost of facilities. For example, the loss that might be sustained by the explosion of a boiler is minimized by making it capable of sustaining pressures several times the pressure it is expected to withstand and by adding such appurtenances as pressure gauges, safety valves, and soft plugs. These precautions increase the initial costs but, unlike insurance, they may and often do serve to reduce the loss from hazards.

On the basis of economy, initial investments for precautionary measures may not exceed the present worth of the losses that they eliminate. Losses that may be sustained in the future and the amount that they may be reduced by specific precautionary measures are approached through consideration of probabilities. Probabilities, as they relate to loss that may result from specific causes, are determined by scientific analyses, past experience, estimates, or a combination of these. The aim is to establish the probability of a specific event's occurring during a given period of time.

For example, if the probability of temperatures below freezing can be established, steps that are economically feasible to prevent frost damage may be calculated. A truck gardener has a number of glass-covered but unheated propagating beds. If temperatures drop below 20 degrees F for a period of 6 hours during the propagating season, he estimates that he will sustain a loss of $7,200. An examination of weather records reveals that damaging conditions may be expected to occur at 12-year intervals. This is equivalent to stating that his annual probable loss will be equal to $7,200 ÷ 12 = $600. A precautionary measure such as the installation of electric heating elements that will eliminate prospective frost damage entirely may be expected to prove profitable if all annual costs associated with it are less than $600.

Levels of precautionary measures. It is not always practical to eliminate entirely the chances of damage from a specific probable occurrence. Completely fireproof buildings, power lines that will withstand hurricanes, and practices that will prevent all theft of property are rarely feasible. Thus the usual problem is to find the extent of precautionary measures that it is practical to take. Precautionary measures may either reduce the probability of occurrence of loss or reduce the probable extent of loss. Overhead sprinkler fire protection systems are designed to do the latter. They do not prevent the incidence of fires, but they limit their extent.

The following example is an illustration of the economy of precautionary measures to reduce the probability of occurrence of loss.

A power line to be extended to a mining property is expected to be in use for 20 years. On the basis of expected wind and ice loads, the power company's engineers have designed lines that can be built for $1,530, $1,630, $1,750, $1,900, and $2,080; the probabilities of failure of these lines in terms of interval of occurrence are estimated at 9, 14, 30, 52, and 86 years respectively. A failure is expected to result in a loss equal to 40 per cent of the original cost of the line. General maintenance, insurance, taxes, and so forth amount to 4 per cent of first cost and an interest rate of 6 per cent is considered adequate. The salvage value of lines 20 years hence is taken at zero.

PROBABLE ANNUAL COST OF $1,530 LINE

	RP 6-20
Capital Recovery and Return, $1,530 × (.08718)	$133
Maintenance, taxes, and insurance, $1,530 × .04	61
Probable cost of failure, $1,530 × .40 × 1/9	68
Total probable annual cost	$262

Results for other lines have been calculated in a similar manner and appear in Table 27.

TABLE 27. Summary of Probable Annual Costs of Power Lines of Different Designs

Cost	Probable Interval of Failure, in Years	Probable Annual Cost of Failure	Capital Recovery	Maintenance, Taxes, and so forth	Total Annual Cost
$1,530	9	$68	$133	$61	$262
1,630	14	47	142	65	254
1,750	30	23	152	70	245
1,900	52	15	166	76	257
2,080	86	10	181	83	274

On the basis of the estimates used, it is probable that the $1,750 line should be installed.

Methods of Financing and Their Effect on Return on Investment

Funds may be acquired in exchange for ownership. Ownership may be extended by taking in a partner who invests funds in the enterprise or by selling shares of a corporation's stock. A partner or stockholder is a part owner of an enterprise and as such may have a voice in management and be entitled to a share of the earnings. There is no obligation to pay him when profit has not been made.

Funds may also be acquired by borrowing. When funds are borrowed either directly or through the sale of bonds, the borrower agrees to pay interest at a given rate on the principal amount at stated intervals and to repay the principal amount on a stipulated date. The pledge to pay interest on borrowed funds and to repay the principal amount when it comes due must be honored whether or not the borrower's operations have been profitable. Thus loans represent fixed obligations. Failure to meet loan obligations may result in serious embarrassment, extending to loss of control and ownership under some circumstances.

Most enterprises use both ownership and borrowed funds. This leads to consideration of the proportion of each of these types of funds on which to operate. For example, consider an enterprise that can borrow funds at the rate of 5% interest and that requires funds totaling $100,000.

The enterprise may elect to acquire the entire amount as ownership funds or it may elect to borrow varying amounts of the required total. The effect of several ratios of ownership funds to borrowed funds upon the rate of return on the ownership funds is shown for several assumed annual earnings before interest payments in Table 28.

TABLE 28. The Effect of Ratio of Ownership to Borrowed Funds and Annual Earnings before Interest upon the Per Cent Return on Ownership Funds

Per cent of ownership funds..........		100%	80%	60%	40%
Per cent of borrowed funds..........		0%	20%	40%	60%
Amount of ownership funds..........		$100,000	$80,000	$60,000	$40,000
Amount of borrowed funds...........		0	$20,000	$40,000	$60,000
Interest on borrowed funds at 5%....		0	$ 1,000	$ 2,000	$ 3,000
Return on ownership funds for annual earning before interest of	$10,000	10.0%	11.2%	13.3%	17.5%
	8,000	8.0	8.7	10.0	12.5
	6,000	6.0	6.2	6.7	7.5
	4,000	4.0	3.7	3.3	2.5
	2,000	2.0	1.2	0.0	*
	1,000	1.0	0.0	*	*
	0	0.0	*	*	*

* Earnings insufficient to meet interest on borrowed funds.

It will be noticed that the return on ownership funds for earnings above $5,000, before interest, increases as the proportion of borrowed funds increases. Contrariwise, the return on ownership funds for earnings less than $5,000, before interest, decreases as the proportion of borrowed funds increases. The advantage of using a high proportion of borrowed funds is offset by the fact that if earnings are insufficient to pay the interest on the borrowed funds the owners may lose their control of and their equity in the enterprise.

Table 28 does not reflect the fact that the interest rate is usually less when a small percentage of funds is borrowed than it is when a large percentage is borrowed.

Economic Design Prior To Engineering Design

Since engineering design is a means to an economic end, it seems logical to precede engineering or physical design of a structure or machine with what may be termed economic design.

Suppose that a person has a piece of city property in a location suitable for the construction of a furniture store, an apartment house, or a factory. It seems logical to explore these possibilities thoroughly from the economic standpoint before undertaking the design of the building to be erected. Suppose investigation reveals that an apartment house of relatively small living units appears most profitable. This amounts to saying that the economic design favors an apartment house of small living units. In general, detailed physical design should be deferred until thorough analysis has been made to establish the characteristics a structure or a machine must have to be economically feasible.

There are many examples of structures, machines, and developments that were first designed from the physical standpoint and built, only to be revealed by later economy analysis to have little economic merit. Losses due to this cause are undoubtedly of great magnitude.

Often an economic approach to design will reveal physical characteristics that a machine or structure should have to be economically sound. Consider, for example, the design of a power tubing tong to be used in unscrewing and screwing up joints of tubing in the repair of oil wells. The tubing used varies in nominal diameter from $1^1/_2$ inches to 3 inches and extends to the bottom of the well. In case of failure the tubing is lifted and stacked in the derrick. The heaviest work in this operation and the subsequent operation of lowering the tubing into the well after repairs have been made is the unscrewing and screwing up of tubing joints. This is usually done manually and usually requires three men. Fig. 38 presents an analysis to uncover characteristics of a power tubing tong that will be economically successful. The work performed in pulling rods and tubing and replacing them is shown adjacent to the time scale A-W.

The chart brings out the fact that items *f* and *g*, unscrewing and screwing up tubing, represent just a small portion of the total time required for the total job, yet these are the only items for which a power tubing tong might be advantageous.

The set-up and break-up time for the tong appears as item *W*.

The right-hand portion of the chart gives the time required to perform each item of work under varying conditions and the total cost for the job.

	Power Tong Used?	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
	Number of Men in Well Servicing Crew	5	5	5	5	5	4	4	4	4
	% Screwing Time Reduction Due to Tong	—	0	0	25	25	0	0	25	25
	Set-up Time for Tong in Man Minutes	—	40	10	40	10	40	10	40	10
A	Time to drive to and from well	24	24	24	24	24	24	24	24	24
C	Time to set up and break up for rods and tubing $= \frac{300}{\text{no. of men in crew}}$ (time for tong not included)	60	60	60	60	60	75	75	75	75
D	Time to repair pump, etc.	10	10	10	10	10	10	10	10	10
E	Time to pull rods, less unscrewing time	38	38	38	38	38	38	38	38	38
e	Time to unscrew rods	11	11	11	11	11	11	11	11	11
F	Time to pull tubing, less unscrewing time	160	160	160	160	160	160	160	160	160
f	Time to unscrew tubing	48	48	48	36	36	48	48	36	36
G	Time to run tubing, less screwing time	55	55	55	55	55	55	55	55	55
g	Time to screw up tubing	72	72	72	54	54	72	72	54	54
H	Time to run rods, less unscrewing time	35	35	35	35	35	35	35	35	35
h	Time to screw up rods	9	9	9	9	9	9	9	9	9
W	Time to set up and break up tong $= \frac{W}{\text{no. of men}}$	—	8	2	8	2	10	2.5	10	2.5
	Total Job Time in Minutes	522	530	524	500	494	547	539.5	517	509.5
	Crew and Unit Cost per Minute	1333	.1333	.1333	.1333	.1333	.1167	.1167	.1167	.1167
	Cost for Job in Dollars	69.5	70.1	69.8	66.7	65.9	63.7	62.9	60.3	59.4

FIG. 38

Economic analysis to uncover characteristics of a power tubing tong necessary for its economic success. [Reproduced from H. G. Thuesen and M. R. Lohmann, "Job Design," Oklahoma A. and M. College Engineering Experiment Station, *Publication No. 66*, January 1948.]

The analysis has been planned to bring out the effect on economy of a tubing tong that (a) does not reduce the time for items *f* and *g*; (b) reduces the time for items *f* and *g* by 25 per cent; (c) requires a 40-man-minute set-up and break-up; (d) requires a 10-man-minute set-up and break-up; and (e) reduces the number of men required in the well-servicing crew.

A comparison of the costs indicates that characteristics which will permit the reduction of the number of men in the well-servicing crew are of greatest importance. A reduction of one crew member will reduce the cost of the job by about $7.00 or roughly 10 per cent. A 25 per cent reduction of time required for items *f* and *g* will reduce the cost of the job by about $3.50 or 5 per cent. The cost for set-up and break-up is about $.027 per man-minute or approximately $.27 for a 10-minute and $1.08 for a 40-minute set-up and break-up period.

From the above analysis several desirable characteristics of the tubing tong may be deduced. It should undoubtedly reduce the numbers of crew members necessary. Since it should reduce unscrewing and screwing-up time, its control mechanism should be such that the tong can engage the tubing, do its work, and disengage the tubing with dispatch. This in turn indicates the necessity for a hydraulic or electric control system, and, since its set-up time should be relatively short, it should be light enough in weight to be carried by one or two men.

The knowledge of these and other characteristics necessary to economic success often provides helpful insight into mechanical design.

PROBLEMS

1. Two locations A and B are being considered for a brick plant. Analysis reveals the information given in the following table:

	Location A	*Location B*
Capital cost per 1,000 brick	$ 2.17	$ 1.56
Material cost per 1,000 brick	2.92	3.36
Labor cost per 1,000 brick	6.73	5.28
Distribution cost per 1,000 brick	6.35	6.41
Total cost per 1,000 brick	$18.17	$16.61

Calculate the ratio of the profit per 1,000 brick produced at location A to the profit per 1,000 brick produced at location B for selling prices of $17.50, $18.50, $19.50, and $20.50 per 1,000 brick.

2. A short conveyor costing \$150 and having a life of one year is placed between Machines A and B to eliminate the necessity of hand trucking in tote boxes. The services of a trucker at a rate of \$.90 per hour is eliminated to the extent of 20 minutes daily. The machine rate of Machine A is \$1.50 per hour and the operator receives \$1.12 per hour. The convenience of the new equipment makes it possible to reduce the cycle time on the operation done by Machine A from 1.7 to 1.6 minutes. The conveyor will be used 8 hours a day during 300 days of the year. Determine the rate of return on the investment.

3. A machine not equipped with a brake "coasts" 15 seconds after the power is turned off upon the completion of each piece, thus preventing the removal of the piece from the machine. The time per piece exclusive of this stopping time is 1 minute and 45 seconds. The machine is used to make 40,000 pieces per year. The operator receives \$1.34 an hour and the machine rate is \$1.05 per hour.

How much can be paid for a brake that reduces stopping time to 3 seconds, if it will be used for 6 years and have no salvage value? Repairs and maintenance for the brake are estimated at \$20 per year. The interest rate is 8%.

4. You are about to purchase a truck tank for delivering a volatile chemical. You have two alternatives. One is to purchase a tank 10 ft. × 5 ft. × 3 ft. made of steel ⅛ inch thick, the other is to purchase a tank 10 ft. × W ft. × 3 ft. made of aluminum 3/16 inch thick. Steel weighs 490 pounds and aluminum 160 pounds per cubic foot. The chemical weighs 6.7 lbs. per gallon (7.48 gal. = 1 cu. ft.). The steel and aluminum tanks will cost \$.14 and \$.60 per pound respectively and are made in any dimensions desired. The scrap value of the steel and aluminum tanks will be \$.01 and \$.16 per pound respectively.

Assume that it costs \$.16 per mile plus the cost of the tanks to operate the truck for an average mileage of 50,000 miles per year and that the tanks will be scrapped at the end of 5 years. Also assume that a full load is carried over .5 of the mileage traveled.

(a) For what value of W will the weights of the filled tanks be equal?
(b) Give the capacity of each tank in gallons.
(c) Give the weight of the tanks and load.
(d) Calculate the first cost of each tank.
(e) Calculate the scrap value of each tank.
(f) If \$.0004 is received per gallon-mile for the hauling of the chemical and if interest is at 6%, which tank should be used?

5. A manufacturing concern will need 260,000 square feet of warehouse area to replace space formerly leased and will need an additional 80,000 square feet of warehouse area when a present lease for the latter amount of space expires 8 years hence. Two alternatives are being considered. One is to build a warehouse with 260,000 square feet now at a cost of \$2.60 per square foot and add 80,000 square feet 8 years later at an estimated cost of \$18,000 plus \$2.60 per square foot.

The other alternative is to build a warehouse having an area of 340,000 square feet at a cost of $2.60 per square foot. The annual rental paid on the 80,000 square feet is $22,000 per year. This space can be sublet for the next 8 years for $16,000 per year. It is estimated that handling costs would be reduced by $4,000 per year by virtue of having a single warehouse.

A warehouse built at present as well as the proposed 80,000-square-foot addition is expected to be retired 25 years hence and have a salvage value of $.30 per square foot at that time. The cost of funds is considered to be 6% and annual maintenance is estimated at $.05 per square foot on all space owned. Annual taxes and insurance are estimated at $.03 per square foot for the buildings owned and $.03 per square foot for contents in storage.

Should a warehouse of 260,000 square feet or one of 340,000 square feet be built now, on the basis of equivalent annual cost?

6. In a hydroelectric development under consideration the question to be decided is the height of dam to be built. The function of the dam is to create a head of water. Because of the width of the proposed dam site at different elevations, heights of the dam under consideration are 173 feet, 194 feet, and 211 feet; costs for these heights are estimated at $1,860,000, $2,320,000, and $3,020,000 respectively. The capacity of the power plant is based on the minimum flow of the stream of 990 cubic feet per second. This flow will develop $(h \times 990 \times 62.4) \div (550 \times .75)$ horsepower where h equals the height of the dam in feet. A horsepower-year is valued at $31. The cost of the power plant, including building and equipment, is estimated at $180,000 for the building and $29 per hp of capacity for the equipment.

To be conservative, the useful life of the dam and buildings is estimated at 40 years with no salvage value. Life of the power equipment is also estimated at 40 years with no salvage value. Annual maintenance, insurance and taxes on the dam and buildings are estimated at 2.8 per cent of first cost. Annual maintenance, insurance, and taxes on the equipment are estimated at 4.7 per cent of first cost. Operation costs are estimated at $38,000 per year for each of the alternatives. Determine the rate of return for each height of dam and the rate of return on the added investment for each added height. Which height dam should be built if 10% is required on all investments?

7. A company is about to build a processing plant that will require 5,000,000 kw-hr. of electrical energy per year. The maximum rate of consumption will be 1,400 kw.

The electric power can be purchased from a power firm for $.013 per kw-hr., providing the company will construct the required 25 miles of transmission line and furnish necessary transformers. A high-tension transmission line will cost $9,500 per mile and the necessary transformers will cost $22,000. The life of these assets will be 30 years, after which they will have a salvage value of $12,000.

There is a possibility that the company can construct and operate its own power plant more cheaply than current can be purchased. The company's plant will be located on a river that will furnish adequate water for condensing purposes for a steam plant or for cooling if a Diesel plant is installed. A good location is also available for a hydrogenerating plant 36 miles from the company's plant site. Historical data on the river flow reveal that the flow has been sufficient for the past 10 years to maintain a constant load of 1,800 kw. Costs involved in the construction and maintenance of the three types of plants are given in the following table:

	Steam	*Diesel*	*Hydro*
Total investment per kw of generating capacity	$210.00	$260.00	$310.00
Transmission equipment	0	0	$364,000
Fuel required in pounds per kw-hr.	1.85	.68	0
Cost of fuel	$7.50/ton	$.0092/lb.	0
Operating, management and maintenance (annual)	$27,000	$19,000	$15,000
Taxes and insurance (% of invested value)	2.0	2.0	2.0
Life of plant and equipment (years)	20	16	30
Salvage at end of life, (% of original cost)	10	12	5

Assume an interest rate of 6% compounded annually, and determine the cost of the four alternate methods of supplying the required power.

8. Mr. Smith has $60,000 invested in his business and has no indebtedness. He wishes to expand his activities and will need $50,000 to do so. He can borrow $50,000 at 6% interest or he can sell Mr. Black a ¼ interest in the business for $20,000 and borrow $30,000 at 5% interest.

(a) What minimum earnings before interest are necessary to meet interest payments on borrowed funds for each method of financing?

(b) What will be the rate of return on the $60,000 that Mr. Smith has invested if the annual net earnings of the expanded business amount to $7,700 before interest for each method of financing?

(c) Calculate the rate of return in part (b) if the annual earnings before interest are $3,800.

9. It is desired to purchase a number of 25-hp, three-phase motors with reduction gears. Bids have been received as follows:

Make	*Guaranteed Full Load Efficiency*	*Price*	*Estimated Life*
A	87 per cent	$520	10 years
B	86 per cent	$360	10 years

Maintenance and operation costs of the machines are estimated to be equal except for the power consumption. Annual taxes and insurance combined amount to 1.5 per cent of first cost. If current cost is at the rate of $.018 per kw-hr. and the motors are operated at full load 1,400 hours a year, what is the total operation cost per hour for each motor, assuming that neither motor will have a salvage value at the end of its estimated life? The purchaser requires a rate of return of 8% on all investments. 1 hp = .746 kw.

10. A motor of 100 hp output is required. Estimated data for two motors under consideration are tabulated below:

	Motor A	*Motor B*
Cost	$3,600	$3,000
Life	12 yrs.	12 yrs.
Salvage value	0	0
Efficiency ½ load	85%	83%
Efficiency ¾ load	90%	89%
Efficiency full load	89%	88%
Hours use per year at ½ load	800	800
Hours use per year at ¾ load	1,000	1,000
Hours use per year at full load	600	600

Current cost per kw-hr. is $.03, and $.03(.746) = $.02238 per horsepower-hour input. Annual maintenance, taxes, and insurance amount to 1.4% of the original cost and interest is at 5%.

(a) What will be the equivalent annual cost for each motor if capital recovery with a return is based upon sinking-fund depreciation?

(b) What will be the return on the additional amount invested if Motor A is selected?

11. A small manufacturing concern is buying parts and assembling them to produce a product invented by its principal owner. Sales are being made at the rate of 8,000 units per year. The purchase cost of parts for the product delivered to the shop is $.87 per unit of product. The inventor urges the purchase of tools costing $1,260 to produce the product. He estimates that material cost and labor cost per part including overhead will be $.42 and $.24 respectively if the company manufactures the parts, and that the tools will soon pay for themselves. Others in the company urge that the purchase of tools be deferred for one year so that the trend of sales may be more clearly revealed, and their view prevails.

At the end of the year 6,800 units had been sold; sales were stabilized at 6,000 units per year and appeared likely to continue at the later rate. It was therefore decided to purchase the necessary tools and manufacture the product. This was done at a cost of $1,080. Material and labor costs including overhead were found to be the same as had previously been estimated. The useful life of the tools is 4 years with no salvage value and the interest rate is taken at 10%. Make and state assumptions you believe to be reasonable and calculate the cost of deferring the decision to buy the tools as of the time they were purchased.

12. A bridge and approaches are being planned to cross a stream of erratic flow. The greater the capacity of the bridge, the less the danger that it will be destroyed by flood. It has been determined that bridges and approaches of sufficient capacity to withstand flows whose probability of being exceeded in any one year is .1, .05, .025, .0125, and .00625 will cost $142,000, $154,000, $170,000, $196,000, and $220,000 respectively; will require annual maintenance amounting to $4,600, $4,900, $5,400, $6,500, and $7,200 respectively; and will suffer damage of $122,000, $133,000, $145,000, $170,000, and $190,000 respectively if subjected to flows exceeding their capacity. Assume the life of the structures will be 40 years with no salvage value. The interest rate is 4%.

Calculate annual costs including probable damage for each of the five proposed plans and determine which will result in a minimum annual cost.

13. A single-purpose machine can be purchased for $4,600 and will have a salvage value of $800 at the end of a useful life of 8 years. It can produce certain units of product at a rate of 8 per hour with an operator whose wage rate is $1.12 per hour. Space and maintenance charges on the machine are estimated at $400 per year and other charges exclusive of depreciation and interest are estimated at $.30 per hour of operation. The part in question can be made on a general-purpose machine at the rate of 3 per hour with an operator whose wage rate is $1.56 per hour. Space and maintenance charges on this machine are estimated at $600 per year and other charges exclusive of interest and depreciation are estimated at $.20 per hour of operation. It is estimated that this machine will be used 800 hours per year on parts other than the one in question. This machine costs $2,200 and has an estimated salvage value of $200 in 10 years.

(a) What is the cost per unit for each alternative if 2,600 units are made per year and the interest rate is 5%?

(b) What will be the cost per unit of product for each alternative if the manufacture of the product will be discontinued 5 years hence, the special purpose machine will be sold for $1,600 at the end of a life of 5 years, and the idle time of the general-purpose machine caused by discontinuing the product in question will be used for other work?

14. You have made a survey of the market possibilities for bread in Towns A, B, and C and you believe you can sell in these towns, 2,000, 4,000, and 7,000 loaves of bread per day, 6 days per week, respectively. You are considering two possibilities: (1) building a plant of sufficient capacity at C to take care of Towns A, B, and C, or (2) building a plant in each of the three towns. Town C is between Towns A and B and is 20 miles from Town A and 30 miles from Town B. Base labor rates in Town A, B, and C are respectively $1.05, $1.20, and $1.35 per hour.

Table A gives experience on labor costs for various-sized plants. Table B gives experience on overhead costs for various-sized plants applicable to plants in Towns A, B, and C. Trucking costs amount to $.12 per truck-mile, including the driver's salary. Eleven hundred loaves constitute a load. If a central plant at C is decided upon, terminal space must be rented at Towns A and B at costs of $80 and $120 per month respectively.

TABLE A

Comparison of labor cost per loaf to capacity of plant on the basis of labor at $1.00 per hour. Interpolate for desired capacities.

Capacity in loaves per day	1,000	2,000	5,000	10,000	15,000
Labor cost in cents per loaf	3.4	2.8	2.4	2.0	1.8

TABLE B

Comparison of overhead cost per loaf to plant capacity in loaves per day.

Capacity in loaves per day	1,000	2,000	5,000	10,000	15,000
Overhead cost per loaf in cents	3.2	2.6	2.2	2.0	1.9

Find the average cost per loaf and the total annual cost in each of the two possibilities.

CHAPTER

14

Evaluation of Replacements to Continue Activities

EQUIPMENT does not last forever; therefore there must be replacement of equipment in a going concern. There are two basic reasons for replacement of equipment. One of these is that it ceases to function physically for one reason or another. Pipe systems corrode, roofs weather, and machine elements wear or break to the extent that they must be replaced if the purpose they serve is to continue to be served. The question to be answered is then, "Is it desirable to continue the service provided by the equipment that has failed?" Equipment failure provides an opportune time to consider a change of objective. If it is decided that the service of a failed machine should be continued, the next step is to determine how best to replace the machine.

A second basic reason for replacing a facility is that it does not provide a needed service as economically as a replacement apparently would. Whereas the failure of a machine forces consideration of its replacement, a machine that is uneconomical in comparison with another that is available may go unobserved for long periods of time. In 1945 it was estimated that 800,000 machine tools in American industry were from 15 to 30 years old and uneconomical by virtue of obsolescence.[1]

Replacement Should Be Based on Economy

Where profit is a motive, replacement should be based on economy of future operation. Though production facilities are inanimate and should be considered to be means to an end, namely production

[1] Joseph L. Trecker, in an address on "The Economics of Machine Tool Replacement" before the Society for the Advancement of Management, Milwaukee, Wisconsin, March 8, 1945.

at lowest cost, there is ample evidence that motives other than economy often enter in to thinking concerned with their use and replacement.

The idea that replacement should occur when it is most economical rather than when an item of equipment is worn out is contrary to the fundamental concepts of thrift of many people. Also, old machines are often venerated like old friends. People tend to feel secure with familiar old equipment and to be skeptical in regard to change, even though they may profess a progressive outlook. Replacement of equipment requires a shift of enthusiasm. When a person initiates a proposal for new equipment, he ordinarily must generate considerable enthusiasm to overcome inertia standing in the way of its acceptance. Later, enthusiasm may have to be transferred to a replacement. This is difficult to do, particularly if one must confess to having been overenthusiastic about the equipment originally proposed.

Part of the reluctance to replace physically satisfactory but economically inferior units of equipment has roots in the fact that the import of a decision to replace is much greater than that of a decision to continue with the old. A decision to replace is a commitment for the life of the replacing equipment. But a decision to continue with the old is usually only a deferment of a decision to replace that may be reviewed at any time when the situation seems clearer. Also a decision to continue with old equipment that results in a loss will usually result in less censure than a decision to replace it with new equipment that results in an equal loss.

Replacement Is a Method of Conservation

The economy of scrapping a functionally efficient unit of productive equipment lies in the conservation of effort, energy, material, and time resulting from its replacement. The unused remaining utility of an old unit is sacrificed in favor of savings in prospect with a replacement. Consider, by way of illustration, a shingle roof. Even a roof that has many leaks will have some utility as a protection against the weather and may have many sound shingles in it. The remaining utility could be made use of by continual repair. But the excess of labor and materials required to make a series of small repairs over the labor and materials required for a complete replacement may exceed the utility remaining in the roof. If so, labor and materials can be conserved by a decision to replace the roof.

Causes of Replacement

The main considerations leading to replacement may be classified under the following headings:

(1) Inadequacy
(2) Obsolescence
(3) Excessive Maintenance
(4) Declining Efficiency

An asset may be replaced for any one of the above reasons, but not infrequently two or more are involved. For example, a person who does oil field hauling has an old truck and is considering replacing it. He has noticed that frequent repairs are needed, fuel consumption is increasing, and oil consumption is excessive; several jobs have been lost because his truck was of insufficient capacity; and in addition he feels that improvements on new models place his present truck at a disadvantage. In this example all four of the reasons classified above are jointly being considered.

In the following paragraphs an approach to replacement for each of the four reasons acting singly will be considered first; then more general methods will be explained.

Replacement Due to Inadequacy

An item of equipment that is inadequate is insufficient in some capacity to perform services required of it, even though it may be in excellent operating condition. For example, a company has a boring mill that is used almost exclusively to face and bore pulleys. At the time it was purchased, three years ago, the largest pulley contemplated was 54 inches in diameter. Since then there appears to have been a change in the practices of customers; orders are being received for pulleys up to 72 inches in diameter and these orders seem to be on the increase.

Orders for pulleys between 54 and 72 inches are subcontracted to another concern. Not only is this costly but it occasions delays that are detrimental to the reputation of the company. The factor entering into consideration of replacement in this example is inadequacy. Though the present boring mill is up to date, efficient, and in excellent condition, consideration of its replacement is being forced by the need for a boring mill of greater capacity.

Where there is inadequacy, a usable piece of equipment, often in excellent condition, is on hand. Often, as in the case of the boring mill, the desired increased capacity can only be had by purchasing a new unit of equipment of the desired capacity.

In many cases, such as with pumps, motors, generators, and fans, the increased capacity desired can be had by purchasing a unit to supplement the present machine, should this alternative prove more desirable than purchasing a new unit of the desired capacity.

The method of comparing alternatives where inadequacy is the principal factor will be presented in the following example.

A year after a 10 H.P. motor has been purchased to drive a belt coal conveyor, it is decided to double the length of the belt. The new belt requires 20 H.P. The needed power can be supplied either by adding a second 10 H.P. motor or by replacing the present motor with a 20 H.P. motor.

The present motor cost $420 installed and has a full load efficiency of 88 per cent. An identical motor can now be purchased and installed for $440. A 20 H.P. motor having an efficiency of 90 per cent can be purchased and installed for $780. The present 10 H.P. motor will be accepted as $270 on the purchase price of the 20 H.P. motor. Current costs $.02 per kw-hr., and the conveyor system is expected to be in operation 2,000 hours per year.

Maintenance and operating costs other than for current of each 10 H.P. motor are estimated at $35 per year and for the 20 H.P motor at $50 per year. Taxes and insurance are taken as 1 per cent of the purchase price. Interest will be at the rate of 6 per cent. The service lives of the new motors in the present application are taken as 10 years, with a salvage value of 20 per cent of their original cost at that time. The present motor will be considered to have a total life of 11 years.

This is done for simplicity and on the basis that it is an approximation that will introduce little practical error into the analysis. It is likely that all the motors will outlast the period of service they will have in the application under consideration. More refined methods for taking into consideration overlapping of the service periods of equipment will be introduced later.

On the basis of the data presented above, the following analysis of the two alternatives is made.

Alternative A. Purchase 20 H.P. Motor for $780, Dispose of Present Motor for $270

Annual Cost

Capital recovery and return, ($780 − $156)(.13587) + $156 × .06 (RP 6-10)	$ 94.22
Current cost, $\frac{20\text{ H.P.}}{.90\text{ Eff.}} \times \frac{.746\text{ kw}}{\text{H.P.}} \times \frac{\$.02}{\text{kw-hr.}} \times 2{,}000$ hrs..	663.11
Maintenance and operating cost..............................	50.00
Taxes and insurance, $780 × .01	7.80
Total equivalent annual cost..............................	$815.13

Alternative B. Retain Present Motor, Purchase Additional 10 H.P. Motor for $440

Annual Cost

Present 10 H.P. motor:	
Capital recovery and return, ($270 − $84)(.13587) + $84 × .06 (RP 6-10)	$ 30.31
Current cost $\frac{10}{.88} \times .746 \times \$.02 \times 2{,}000$......................	339.08
Maintenance and operating cost..............................	35.00
Taxes and insurance, $420 × .01..............................	4.20
New 10 H.P. motor:	
Capital recovery and return, ($440 − $88)(.13587) + $88 × .06.. (RP 6-10)	53.11
Current cost, $\frac{10}{.88} \times .746 \times \$.02 \times 2{,}000$......................	339.08
Maintenance and operating cost..............................	35.00
Taxes and insurance, $440 × .01..............................	4.40
Total equivalent annual cost..............................	$840.18

On the basis of the above analysis, the advantage of replacing the 10 H.P. motor over supplementing it is equivalent to $840.18 less $815.13 or $25.05 per year.

Owing to the incorrect decision to purchase a 10 H.P. motor a year ago, a sunk cost equal to its present book value less $270 has been sustained. This sunk cost has been revealed, rather than caused, by the present analysis. Since economy studies are concerned with the future, this passed sunk cost must not enter into an analysis embracing the future. Methods for correctly considering sunk costs were developed in Chapter 9 and may be profitably reviewed.

The trade-in value of $270 was taken as the present value of the original 10 H.P. motor because if it is replaced $270 will be received for it. Thus this value is germane in the above comparative analysis.

The annual charge for taxes and insurance was based on the original cost, because taxes and insurance charges are usually based upon book values; but for simplicity no reduction was made in these items to correspond to the expected decline in book values

The alternative to supplement the present motor will require an investment of $270 in the present motor plus $440 in a new 10 H.P. motor or a total investment of $710. The second alternative can be implemented by an investment of $780.

The fact that $270 can be realized from the sale of a capital asset and applied upon the purchase price of the 20 H.P. motor does not reduce the expenditure necessary to acquire the motor or the amount invested in it. Thus the above analysis reveals that an additional investment of $780 − $710 = $70 will result in a return of 6 per cent on the additional investment plus $25.05 per annum.

Replacement Due to Obsolescence

Tools of production are continually being improved. Often the rate of improvement is so great that it is an economy to scrap a unit that is in good operating condition and replace it with an improved unit. In some cases the activity for which a machine has been used declines to the point where it is an advantage to replace it with a smaller unit. In either case replacement amounts to disposing of the remaining utility of a present unit in order to replace it with a more effective unit. If less is received for the remaining utility than the amount of the remaining unrecovered balance of an asset, a sunk cost will be revealed. Obsolescence of equipment is characterized by changes external to the equipment itself.

A specialty manufacturer produces a hose coupling consisting of two parts. The parts are machined on a turret lathe purchased 13 years previously for $3,700 including installation. It has been depreciated on the basis of a 15-year life and a salvage value of $250 and now has a book value of $710 in the company's accounts.

A salesman proposes that the old machine be replaced by a new turret lathe whose installed cost is $6,800.

The production time per 100 sets of parts is as follows:

Part	*Present Machine*	*New Machine*
Connector	2.92 hours	1.98 hours
Swivel	1.84 hours	1.14 hours
Total	4.76 hours	3.12 hours

The company's sales of the hose couplings average 40,000 units per year and are expected to continue at near this level. Machine operators are paid \$1.42 per hour. The old and the proposed machine require equal floor space. The proposed machine will use power at a greater rate than the present one, but since it will be used fewer hours, the difference in cost is not considered worth figuring. This is also considered true of general overhead items. Interest is to be taken at 8 per cent. The salesman for the new machine has found a small shop that will purchase the old machine for \$900. The prospective buyer estimates the life of the new machine at 10 years and its salvage value at 10 per cent of its installed cost of \$6,800. (The old turret lathe is estimated to be physically adequate for 10 more years and then to have a salvage value of \$250.)

To evaluate the desirability of replacement, the prospective buyer analyzes the situation on the basis of straight-line depreciation and average interest as follows:

Turret Lathe Analysis

Present Machine—Equivalent Annual Cost

Capital recovery and return, $(\$900 - \$250)\left(\frac{1}{10} + \frac{.08}{2} \times \frac{10+1}{10}\right) +$ $\$250 \times .08$	\$ 114
Direct labor, $(4.76 \div 100) \times 40{,}000 \times \$1.42$ per hour	2,704
Total	\$2,818

Proposed Machine—Equivalent Annual Cost

Capital recovery and return, $(\$6{,}800 - \$680)\left(\frac{1}{10} + \frac{.08}{2} \times \frac{10+1}{10}\right)$ $+ \$680 \times .08$	\$ 936
Direct labor, $(3.12 \div 100) \times 40{,}000 \times \$1.42$ per hour	1,772
Total	\$2,708
Annual amount in favor of new machine	\$ 110

The prospective buyer does not feel that the annual saving is sufficient to warrant replacement. However, to get another viewpoint on the matter he calculates the payout period at the present rate of production in the following manner:

Assume that the annual cost with the present machine is equal to the annual cost with the new machine. Then

$$\$2{,}818 = (\$6{,}800 - \$680)\left(\frac{1}{n} + \frac{.08}{2} \times \frac{n+1}{n}\right) + \$680 \times .08 + \$1{,}772$$

Solving for n $$n = 8.5 \text{ years}$$

Because of the comparatively low annual saving in prospect and the relatively long payout period, the prospective buyer decides to defer replacement to a later date.

This decision may be interpreted as a decision to incur additional cost of \$110 per year in order to be in a position to be free to accept a more advantageous alternative that may become available in future.

In the above situation the old machine could be sold for more than its book value. This is not unusual in a period of inflation. The amount that will be realized for an old machine if replacement takes place is the correct one to use in a comparative analysis. It should be noted that the new machine will be used $(3.12 \div 100) \times 40{,}000 = 1{,}248$ hours per year. No cognizance is taken of the fact that it is available for use many more hours per year; the unused capacity is of no value until used. Since, however, the overcapacity is potentially of value and may prove a safeguard against inadequacy, it should be considered an irreducible in favor of the new machine.

Replacement because of Excessive Maintenance

Unlike the "one hoss shay" of literary fame, a machine rarely has all its elements wear out at one time. Experience shows that it is economical to repair many types of productive assets in order to maintain and extend their usefulness. Some repairs are of a current nature and minor in extent. Others are periodic and extensive. Examples of the latter class are the overhaul of an automobile engine and the complete replacement of a roof.

An extensive periodic repair is not usually contemplated until it becomes necessary to extend the life of the unit of equipment in question. Usually, for example, an engine is not overhauled until its failure to provide acceptable service has occurred or is believed to be imminent. Thus the cost of an extensive periodic repair may be considered to be an expenditure to purchase additional service by extending the life of a unit of equipment.

Before an expenditure for major repairs is made to extend the service life of a machine or structure, analyses should be made to determine if the needed service might be more economically provided by other alternatives.

In this connection consider the following situation. The main roadway through an oil refinery, six-tenths of a mile long and 20 feet wide and made of concrete, is badly in need of repair to continue it in service. The maintenance department of the refinery estimates that repairs which will extend the life of the roadway for three years can be made for $4,600. A contractor has offered to replace the present roadway with a type of pavement estimated to have a life of 20 years for $18,400.

Current maintenance cost on the repaired pavement is estimated to average $160 per year and that on the replacing pavement is estimated to average $80 per year. Other items are considered to be equal or negligible. The company has no difficulty in borrowing funds at 4%, so this rate of interest is used in the comparison. The salvage value of the present pavement is considered to be nil if it is replaced.

Annual-Cost Comparison of Pavement Repair and Replacement

Repair Pavement to Extend Its Life 3 Years	
RP 4-3	
Capital recovery and return, $4,600(.36035)	$1,657
Average annual repair cost	160
Total	$1,817
Replace with Pavement with Estimated Life of 20 Years	
RP 4-20	
Capital recovery and return, $18,400(.07358)	$1,354
Average annual repair cost	80
Total	$1,434
Annual advantage of replacing over repairing	$ 383

In some classes of equipment current repairs increase with age. Maintenance may be slight at first but increases at an accelerating rate with the elapse of time. Thus a point is ultimately reached where it is more economical to replace than to continue maintenance. To illustrate the economy of this situation consider the following example.

Example of replacement due to excessive maintenance. A piping system in a chemical plant was installed at a cost of $32,000. This system deteriorated by corrosion until it was replaced at the end of six years. The salvage value of the system was nil. Maintenance records show that maintenance costs in the past have been as given in Column B of Table 29.

TABLE 29. Analysis of Maintenance Costs of a Chemical Piping System

(A) Year	(B) Cost of Maintenance for Year	(C) Sum of Maintenance Cost to End of Year, Σ(B)	(D) Cost of n Years of Service, $32,000 + C	(E) Average Annual Cost of Service to End of Year, D ÷ A
1	$ 1,260	$ 1,260	$33,260	$33,260
2	3,570	4,830	36,830	18,415
3	6,480	11,310	43,310	14,437
4	9,840	21,150	53,150	13,287
5	14,230	35,380	67,380	13,476
6	19,820	55,200	87,200	14,533

It may be noted that annual cost of maintenance increases with elapse of time. This is typical of many classes of equipment and may be the primary reason for replacement. Total expenditures for repairs are given to the end of any year in Column C. The sum of the maintenance costs given in Column C and the original cost of the equipment is equal to the cost of providing the number of years' service designated in Column A.

The piping system could have been scrapped at the end of any year. Column E gives the average annual cost of service that would have resulted from scrapping the system at the end of any year. Thus if the system had been scrapped at the end of the first year, the cost for a year of service would have been $33,600. If it had been scrapped at the end of the second year, the two years of service would have cost $36,830, as given in Column D, and the average annual cost would have been $18,415.

The least average annual cost, $13,287, occurs for a four-year life.

If interest had been considered and equivalent annual costs had been used, the quantities in Column E would have been somewhat larger than those given. But the general pattern would have been much the same. Though the lowest annual cost in the above example occurs for a four-year life, it does not necessarily follow that greatest economy would have resulted from scrapping the system after four years of service. The economy of replacement depends upon a number of additional factors such as the need for services of a piping system in the future, changes in levels of maintenance cost, and the characteristics and cost of a replacement. A decision to replace the present equipment should be based on an analysis of costs in prospect with the present equipment and with a possible replacement.

Replacement Due to Declining Efficiency

Efficiency of equipment often declines with usage and age. A gasoline engine usually reaches its maximum efficiency after a short run-in period after which its efficiency declines as cylinder walls, pistons, piston rings, and carburetors wear and the ignition system deteriorates.

When loss of efficiency is due to the malfunctioning of only a few parts of a whole machine, it is often economical to replace them periodically and in this way maintain a high level of efficiency over a long life.

There are a number of facilities that decline in efficiency with use and age but which it is not feasible to repair. Pipes that carry hot water, for example, often fill with scale. As their internal diameter decreases, the amount of energy required to force a given quantity of water through them increases. Pipe lines often decline in efficiency as carriers of fluid or gas because of increasing loss by leakage due to external or internal corrosion with age. When it is not economical to restore efficiency by maintenance, the entire system should be replaced at intervals on the basis of economy. Consider the following example. The buckets on a conveyor are subject to wear that reduces the capacity of the conveyor in accordance with the data given in Table 30. As the capacity of the buckets becomes smaller, it is necessary to run the conveyor for longer periods of time, thus increasing operating costs. When the buckets are in new condition, the desired annual quantity of material can be handled in 1,200 hours of operation. Wear and tear on the balance of the conveyor.

power, labor, and delay awaiting loading of associated equipment is estimated at $6.40 per hour of operation. Buckets can be replaced for $960. Interest will be neglected for simplicity.

TABLE 30. Illustration of Effect of Decreased Efficiency of Equipment on Operating Costs

(A) Year No.	(B) Efficiency at Beginning of Year	(C) Average Efficiency during Year	(D) Annual Hours of Operation, 1200 ÷ C	(E) Annual Cost of Operation Exclusive of Replacement of Buckets, D × $6.40	(F) Sum of Operation Costs to End of Year, Σ(E)	(G) Average Annual Cost of Service to End of Year, ($960 + F) ÷ A
1	1.00	.97	1,237	$7,917	$ 7,917	$8,877
2	.94	.91	1,319	8,442	16,359	8,659
3	.88	.86	1,395	8,928	25,287	8,749
4	.84	.82	1,463	9,363	34,650	8,903
5	.80					

The above example is typical of many kinds of equipment whose efficiency declines progressively when it is not feasible to arrest the decline with maintenance. In the example the least cost of operation occurs when efficiency is permitted to decline to 88 per cent, corresponding to a life of two years before replacement takes place.

Though least cost of operation occurs for a life of two years in the above example, this is not conclusive evidence that least cost of operation will result from a policy to replace buckets at two-year intervals unless the replacing buckets will duplicate the buckets being replaced in first and subsequent costs. But determination of a least-cost life for a unit of equipment and casual consideration of subsequent replacement is often sufficient and as far as it is practical to go in many situations.

Replacement Due to a Combination of Causes

In many cases a combination of causes rather than a single cause leads to consideration of replacement. As a unit of equipment ages, its efficiency may be expected to decline and its need for maintenance to increase. More efficient units of equipment will be developed.

Moreover, it frequently happens that changes in activities result in a unit being either too large or too small for greatest economy.

But whatever the causes or combination of causes that lead to consideration of replacement, analysis and decision must be based upon estimates of what will occur in the future. These estimates will relate to:

(1) The service required in the future.
(2) Cost of providing the desired service:
    (a) Costs associated with ownership of the facilities.
    (b) Costs associated with the operation of the facilities.

Cost of Installation and Removal

When a new unit of equipment is purchased, a number of additional expenses beyond its purchase price may be incurred to put the unit into operation. Such expense items may embrace freight, cartage, construction of foundations, special connection of wiring and piping, guard rails, and personal services required during a period of run-in or adjustment. Expenses for such of the above items as apply are first-cost items and for all practical purposes represent an investment in a unit of equipment under consideration. For this reason all first-cost items necessary to put a unit of equipment into operation should be depreciated as part of the total original investment in the unit.

When a unit of equipment is replaced, its removal may entail considerable expense. Some of the more frequently encountered items of removal expense are dismantling, removal of foundations, haulage, closing off water and electrical connections, and replacing floors or other structural elements. The sum of such costs should be deducted from the amount received for the old unit to arrive at its net salvage value. It is clear that this may make the net salvage value a negative quantity.

In many cases facilities can be abandoned in place if their salvage value is less than the cost of removing them. Examples are pipelines, bridges, and roadways.

If the net salvage value is less than the book value at the time of retirement, the difference represents a sunk cost.

When the net salvage value of an asset is less than zero, it is mathematically correct to treat it as a negative quantity in capital-recovery calculations, although this practice may be a source of confusion to some. The annual capital recovery with a return for sinking-fund depreciation at 6 per cent of an asset whose life is 10 years, whose first cost is \$800, and whose net salvage value is a negative \$200 may be written:

$$\text{Capital recovery and return} = [\$800 - (-\$200)](\overset{\text{RP 6-10}}{.13587}) + (-\$200) \times .06$$
$$= \$123.87$$

Recurring Similar Replacements

In some situations a sequence of similar replacements will be made to continue a service. Common examples of this are replacement of light bulbs in an electric sign, roofs on a building, buckets in a conveyor, pipe systems, tires on motor vehicles, and motor vehicles themselves.

In situations involving recurring replacements, it is desirable to know what length of life of replacements will result in least cost. Least-cost life, often designated *economic life*, can be calculated if the characteristics of future replacements and the conditions under which they will be used are known.

The economic-life concept embraces the idea that there is a length of life for which the cost of operation of a facility will be at a minimum. Economic life is a goal toward which to strive in making replacements. Where the history of a succession of replacements is repeated and this fact is known, the economic life of a facility can be determined. This condition has prevailed in regard to automotive vehicles during periods of stable economy.

The fact that a ready market exists for used motor vehicles makes it possible to establish their worth for any length of service life. For this reason, a light automotive truck will be used to illustrate analyses to determine economic life of replacement. Assume that such a truck was owned and operated by Messrs. A, B, C, D, and E during successive one-year periods, and that the service received from the truck by each owner was identical. The truck, originally purchased by Mr. A for \$2,200, was sold for \$270 by Mr. E to Mr. F, who dismantled it. Interest has been taken at 10 per cent. An economic history of the truck's ownership is shown in Table 31.

TABLE 31. Economic History of a Light Automotive Truck

First-Year Costs While Owned by Mr. A	
Depreciation (purchased new for $2,200, sold to Mr. B for $1,500)	$ 700
Interest on investment during first year, $2,200 × .10	220
Operating expense (fuel, tires, repairs, license, etc.)	1,360
Net cost of first year of service	$2,280
Second-Year Costs While Owned by Mr. B	
Depreciation (purchased for $1,500, sold to Mr. C for $1,010)	$ 490
Interest on investment during second year, $1,500 × .10	150
Operating expense	1,500
Net cost of second year of service	$2,140
Third-Year Costs While Owned by Mr. C	
Depreciation (purchased for $1,010, sold to Mr. D for $660)	$ 350
Interest on investment during third year, $1,010 × .10	101
Operating expense	1,640
Net cost of third year of service	$2,091
Fourth-Year Costs While Owned by Mr. D	
Depreciation (purchased for $660, sold to Mr. E for $430)	$ 230
Interest on investlent during fourth year, $660 × .10	66
Operating expense	2,070
Net cost of fourth year of service	$2,366
Fifth-Year Costs While Owned by Mr. E	
Depreciation (purchased for $430, sold to Mr. F for $270)	$ 160
Interest on investment during fifth year, $430 × .10	43
Operating expense	2,260
Net cost of fifth year of service	$2,463

The above history serves to establish the cost of successive years of service at $2,280, $2,140, $2,091, $2,366, and $2,463. These will be considered to be year-end costs. Suppose now that Mr. A had retained ownership for 1, 2, 3, 4, or 5 years and that his annual costs had been as established above. What would have been the equivalent annual cost of operation if he had kept the truck 1, 2, 3, 4, or 5 years? The answer may be found by finding the sum of the present worths of the annual costs for 1, 2, 3, 4, and 5 years and then reconverting these to equivalent annual cost for 1, 2, 3, 4, and 5 years respectively.

The calculations necessary to answer the above question are given in Tables 32 and 33.

TABLE 32. Present Worth of Operating Cost of Successive Years of a Light Truck as of the Beginning of Its First Year

Present worth of 1st-year operating cost, \$2,280(.9091) [PS 10-1]	\$2,073
Present worth of 2nd-year operating cost, \$2,140(.8264) [PS 10-2]	1,768
Present worth of 3rd-year operating cost, \$2,091(.7513) [PS 10-3]	1,571
Present worth of 4th-year operating cost, \$2,366(.6830) [PS 10-4]	1,616
Present worth of 5th-year operating cost, \$2,463(.6209) [PS 10-5]	1,529

TABLE 33. Equivalent Annual Cost of First *n* Years of Service of a Light Automotive Truck

Present worth of operating cost, first year \$2,073	
Equivalent annual cost for one year, \$2,073(1.1000) [RP 10-1]	\$2,280
Present worth of operating cost, first two years = \$2,073 + \$1,768 = \$3,841	
Equivalent annual cost of first two years of service, \$3,841(.57619) [RP 10-2]	\$2,213
Present worth of operating cost, first three years = \$2,073 + \$1,768 + \$1,571 = \$5,412	
Equivalent annual cost of first 3 years of service, \$5,412(.40211) [RP 10-3]	\$2,176
Present worth of operating cost, first four years = \$2,073 + \$1,768 + \$1,571 + \$1,616 = \$7,028	
Equivalent annual cost of first 4 years of service, \$7,028(.31547) [RP 10-4]	\$2,217
Present worth of operating cost, first five years = \$2,073 + \$1,768 + \$1,571 + \$1,616 + \$1,529 = \$8,557	
Equivalent annual cost of first 5 years of service, \$8,557(.26380) [RP 10-5]	\$2,258

The costs associated with operation of the truck as given in Tables 31 and 33 have been plotted in Figure 39.

When interest is taken into consideration, the calculations necessary to determine equivalent annual costs for several lengths of service lives are rather time-consuming. Where the cost of interest is relatively small or fairly uniform throughout the life of an asset or the life of the asset is relatively short, interest may be neglected without significant error in calculating least-cost life. If interest had been neglected in calculating equivalent annual costs from the annual costs given in Table 31, the equivalent annual cost for 1, 2, 3, 4, and 5 years of operation would have been found to be $2,280, $2,210, $2,170, $2,219, and $2,268 respectively.

It should be remembered that the above annual operating costs have been revealed as history. The fact that the lowest equivalent annual cost would occur if replacement of the truck was made at the end of its third year did not become known until the truck was retired from service.

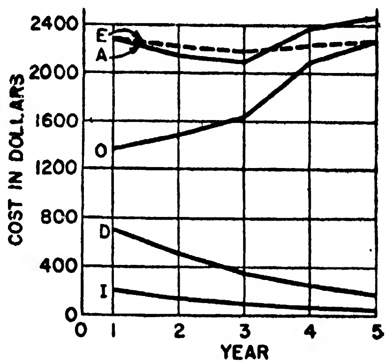


Fig. 39. Economic history of a light automotive truck. Curves *I*, *D*, *O*, and *A* respectively represent interest, depreciation, operating cost, and total cost for years designated—from Table 31. Curve *E* represents equivalent annual cost to end of year designated—from Table 33.

The determination of economic life has practical applications when cost patterns of successive replacements are similar. This is most likely to occur with short-lived facilities and in situations where there are a number of similar units whose service characteristics can be averaged.

When the cost patterns of a unit of equipment and its replacement have not been established by experience that can be depended upon to be repeated, cost patterns must of necessity be based upon estimates.

A practical approach to replacement at the end of a unit's economic life is to consider replacement at the end of each year. If this had been done with the truck in the illustrative example, its prospective yearly cost for each year of its remaining life would have been estimated at the end of its first year. These estimates would then be used for comparison with the estimated costs of a replacement to determine if replacement would be desirable at the end of the first year. This procedure should have been repeated at the end of each year until replacement took place.

If the costs associated with an asset continue to rise during the balance of its life, it should be replaced when its costs for the next year will exceed the equivalent annual cost of the prospective replacement. In Table 31 the costs for the 4th and 5th year of the truck life are given as $2,366 and $2,463. The truck should be replaced at the end of the third year if the lowest equivalent annual cost in prospect with a replacement is less than $2,366 and at the end of the fourth year if a replacement's lowest equivalent annual cost will be less than $2,463.

Comparison on the Basis of a Selected Study Period

It is usually presumed that each event in history is dependent upon those that came before. Thus in theory it is necessary, for accurate comparison of a pair of alternatives, to consider the entire future or a period from the present to a point in the future when the effect of both alternatives will be identical. It is rarely feasible to consider all links in the chain of events in the future. Also it is frequently impossible to be able to discern a point in the future at which the selection of one of a pair of alternatives in the present will have the same effect as the selection of the other.

In the example involving electric motors (under the heading *Replacement Due to Inadequacy*) earlier in the chapter, a common point in the future for each of two alternatives was brought about by assuming that no significant error would be introduced by taking the life of the original motor to be 11 years so that its life might terminate at the same time as that of the other motors under consideration.

In the paragraphs that follow, a more general method for placing alternatives on a comparative basis involving the selection of more or less arbitrary study periods will be illustrated. With this method, comparison of alternatives is made on the basis of costs and income that occur during a selected period in the future. The effect of values occurring after the selected study period is eliminated by suitable calculations. Study periods for all alternatives in a given comparison should be equal.

Consider the following example. A centrifuge whose capacity is 2,000 pounds per hour was purchased 5 years ago. Centrifuging capacity of 3,000 pounds per hour is needed now and is estimated to be required for many years hence.

Estimates have been made relative to centrifuges as follows:

	First Cost	*Life*	*Salvage Value at Age Given*	*Annual Operating Cost*
First, or present 2,000-pound unit	...	8	\$450—5 yrs.	\$1,050
Second 2,000-pound unit	\$1,600	8	\$600—5 yrs.	1,050
First 1,000-pound unit	1,000	8	\$625—3 yrs.	530
First 3,000-pound unit	2,200	8	1,375—3 yrs.	1,520
All units	...	8	0—8 yrs.	...

The desired capacity may be provided by the addition of a 1,000-pound unit now and an alternating succession of 2,000-pound and 1,000-pound unit 3, 8, and 11 years hence, and so forth. The desired capacity may also be provided by the purchase of a 3,000-pound unit now or at the time of retirement of a 2,000-pound or 1,000-pound unit 3, or 8, or 11 years hence, and so forth. See Fig. 40.

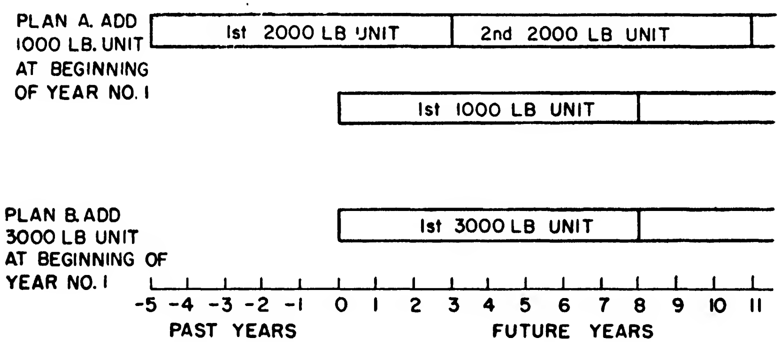


FIG. 0. Graphical representation of two plans for selecting study periods.

The alternatives are to provide 3,000-pound capacity for 3, or 8, or 11 years and so forth, (1) with 1,000-pound and 2,000-pound units, or (2) with a 3,000-pound unit. Comparisons of these alternatives will be made with study periods of 3 and 8 years to illustrate the method. An interest rate of 5 per cent is to be used.

Analysis on the basis of a 3-year study period. If a 3,000-pound unit is installed now, the present unit will be sold as salvage for \$450. If the first 1,000-pound unit is installed now its salvage value as scrap three years hence must be used at that time in comparing the desirability of adding the second 2,000-pound unit or installing a 3,000-pound unit. The estimated salvage value of a 1,000-pound unit three years old is \$625.

ADD 1,000-POUND UNIT NOW AND REPLACE WITH 3,000-POUND UNIT 3 YEARS HENCE

Capital recovery and return, first 2,000-pound unit, $450(.36721) (RP 5-3)	$ 165
Annual operating cost, first 2,000-pound unit	1,050
Capital recovery and return, first 1,000-pound unit, ($1,000 − $625)(.36721) (RP 5-3) + $625 × .05	169
Annual operating cost, first 1,000-pound unit	530
Equivalent annual cost	$1,914

REPLACE WITH 3,000-POUND UNIT NOW

Capital recovery and return, 3,000-pound unit, ($2,200−$1,375)(.36721) (RP 5-3) + $1,375 × .05	$ 372
Annual operating cost, 3000-pound unit	1,520
Equivalent annual cost	$1,892

Note that the effect of the values of the use life of all the units extending beyond the study period, and represented by their estimated salvage values, is eliminated to place the two alternatives on a comparable basis.

On the basis of the above results it is quite likely that the decision would be made in favor of installing the 1,000-pound unit now on the basis of irreducibles. At an estimated cost of $22 per year, an investment of $2,200 − $1,000 = $1,200 can be deferred for three years.

Analysis on the basis of an 8-year study period. If the first 1,000-pound unit is installed at present and a second 2,000-pound unit is installed three years hence, three years of service will remain in the latter unit 8 years hence.

ADD FIRST 1,000-POUND UNIT NOW AND SECOND 2,000-POUND UNIT 3 YEARS HENCE

Present worth of salvage value of first 2,000-pound unit	$ 450
Present worth of first 1,000-pound unit	1,000
Present worth of first cost of second 2,000-pound unit, $1,600(.8638) (PS 5-3)	1,382
Present worth of salvage of second 2,000-pound unit, $600(.6768) (PS 5-8)	−406
Total net present worth	$2,426
Equivalent annual cost, $2,426(.15472) (RP 5-8)	$ 375
Annual operating cost, $530 + $1,050	1,580
Equivalent annual cost	$1,955

REPLACE WITH 3,000-POUND UNIT NOW

	RP 5-8	
Capital recovery and return, 3,000-pound unit, $2,200(.15472)		$ 340
Annual operating cost, 3,000-pound unit		1,520
Equivalent annual cost		$1,860

On the basis of the estimates for the 8-year study period, the advantage in favor of installing the 3,000-pound unit now is $95 per year. Note that the calculated advantage of installing the 3,000-pound unit at present is different when an 8-year instead of a 3-year study period is used.

In general, the longer the study period, the more significant the results. But the longer the study period, the more likely estimates are to be in error. Thus the selection of a study period must be based on estimate and judgment.

Planning for Growth

Many activities are expected to expand with the passage of time. Others are expected to decline. Most cities in the United States whose populations are over 10,000 people are expected to increase in population. As a result, the systems for providing light and power, water, sewage disposal, transportation, fire protection, and the like will become inadequate periodically. Thus planning for growth consists essentially of planning to take care of contemplated replacement due to inadequacy.

The first step in planning for growth is to estimate the pattern of growth in prospect. In some cases there is little information and in other cases there is a wealth of information on which to base estimates of future growth; in either instance the end result is no more than an estimate.

Once an estimate of a future pattern of growth has been made, it is possible to conceive of alternative plans for meeting the pattern of growth as estimated. After the plans have been delineated, they should be compared for economy. One aspect of the comparison will involve inadequacy, which can be examined by methods that have been illustrated. Making a correct decision where inadequacy is a factor will often involve the use of a study period.

Excess Capacity in Comparisons

An existing facility is often replaced with a facility of greater capacity. For example, a turret lathe and its contemplated replacement may be of the same nominal size but the latter may be able to produce more per unit of time. The additional capacity of the replacement has no value unless it is profitably used. If the additional capacity will be used, this fact should be taken into consideration in an analysis of the economy of replacement. In this connection consider the turret lathe analysis on page 324.

The present machine will be used for

$$4.76\left(\frac{40{,}000}{100}\right) = 1{,}904$$

hours per year and the proposed machine can do the same amount of work in

$$3.12\left(\frac{40{,}000}{100}\right) = 1{,}248$$

hours per year. Thus the proposed machine will be available for other work for 1,904 − 1,248 = 656 hours per year more than the present machine. Suppose that new work can be procured for the proposed machine to do during part or all of these excess hours and that this new work will result in a net profit, attributable to the machine, of $260 per year. Then this amount should be entered as a net receipt in favor of the proposed machine in an analysis as follows.

Proposed Machine—Equivalent Annual Cost

Capital Recovery and Return, (6,800 − 680) $\left[\frac{1}{10} + \frac{.08}{2} \times \frac{10+1}{10}\right] + 680 \times .08$	$ 936
Direct Labor, (3.12 ÷ 100) × 40,000 × $1.42 per hour	1,772
Net receipts from utilization of excess capacity over that of present machine	−260
	$2,448

Net saving that may result from utilization of excess capacity in doing work formerly done less efficiently by present methods or machines should likewise be credited to the advantage of the proposed machine.

Approximate Methods and Formula

The methods for comparison of alternatives that have been presented in this text, with the exception of those embracing straight-line depreciation and average interest, will give mathematically correct results if the estimates used with them reflect actual conditions. If properly used the straight-line depreciation and average interest method of capital recovery with return will introduce only slight error.

Since estimates relative to the future often are grossly in error, there are many who see little point to using refined methods in making comparisons. Many approximate methods are used in practice. In this connection the following quotation from a brochure on the replacement of machine tools is of interest.

> In every replacement, certain elements of cost should be considered. We find some difference of opinion among machine tool users in the details of the analysis. This is not the place for a discussion of accounting principles. Each plant will adhere to its own traditional methods. It may be contended, for instance, that the amount of capital invested in the old machine and not yet recovered by depreciation is evidence of a past error in judgment or of unrealistic rates imposed by a government agency which should not be allowed to influence the present decision. But many plant managers take it into consideration.[2]

Some of the methods used are based upon correct premises and give significant results to those who understand the approximations involved. Others are based upon incorrect premises, whim, or wishful thinking and may be grossly misleading. In many cases incorrect methods of comparison expressed as mathematical formulas have been given wide circulation in management literature and have been used without discrimination. Such formulas should be avoided until they have been proved adequate for the situations to which they are to be applied.

When accurate methods of analysis are even slightly surer than approximate methods, the slight additional time and labor they require will be fully justified. If approximate methods are demanded because of custom or for their simplicity, the person charged with making analyses should determine the error that may be introduced by them.

PROBLEMS

1. An industrial establishment secures its water supply from a system of wells. They are using a 6-inch, single-stage centrifugal pump that is in

[2] From *The World's Best Investment.* a brochure of the National Machine Tool Builders Association, Cleveland, Ohio.

good condition. The pump was purchased 3 years ago for $1,350 and has a present book value of $1,005, having been depreciated on the basis of an expected life of 10 years. However, owing to improvements of design, the demand for a pump of this type is so small that its present sale value is only $500. It is anticipated that the pump will have a trade-in value of $200 seven years from now. An improved pump of the same type can now be purchased for $1,700 and will have an estimated life of 10 years with a trade-in value of $200 at the end of that time.

The pumping demand is 225 cubic feet per minute against an average head of 200 feet. The old pump has an efficiency of 75 per cent when furnishing the above demand. The new pump has an efficiency of 81 per cent in furnishing the same demand. Power costs $.02 per H.P.-hr. and either pump must operate 2,400 hours per year. Do the improvements made in design justify the purchase of a new pump if interest of 6% is required?

2. A suburban area has been annexed by a city, which henceforth is to provide water service for the area. The prospective growth of the suburban area has been charted and, on the basis of the charts and of trends in water consumption, it is estimated that water consumption will reach 200,000, 400,000, and 600,000 cubic feet per day 15, 24, and 40 years hence respectively. The estimated cost of a connecting main of capacities in the order named above is $42,000, $52,000, and $72,000. Mains of the material to be used are estimated to have a life of 60 years. The costs given above are estimated to be the same regardless of when the mains are installed. On the basis of a minimum equivalent expenditure during the next 40 years and an interest rate of 4%, compare the four possible plans of providing the needed service.

3. Four years ago an ore-crushing unit was installed at a mine at a cost of $81,000. Annual operating costs for this unit are $3,540, exclusive of charges for interest and depreciation. This unit was estimated to have a useful life of 10 years and this estimate still appears to be substantially correct. The amount of ore to be handled is to be doubled and is expected to continue at this higher rate for at least 20 years. A unit that will handle the same amount of ore and have the same annual operating cost as the one now in service can be installed for $75,000. A unit with double the capacity of the one now in use can be installed for $112,000. Its life is estimated at 10 years and its annual operating costs are estimated at $4,936. The present realizable value of the unit now in use is $26,000. All units under consideration will have an estimated salvage value at retirement age of 12 per cent of the original cost. The interest rate is 10%. Compare the two possibilities of providing the required service on the basis of equivalent annual cost over a study period of 6 years, taking cognizance of any unused service remaining in units at that time.

4. A stationery manufacturer rents 12,000 square feet of storage space with a 10-foot ceiling height for the storage of boxed paper. The rental rate is $.28 per square foot per year. At present the paper is stacked to a depth of 6 feet. The purchase of a fork truck is being considered. It is estimated that use of the fork truck will result in a saving of 800 man-hours per year and that it will be feasible to stack paper to a depth of 9 feet. Each man employed receives $.98 per hour. The fork truck will cost $2,600 and its annual operating cost, exclusive of depreciation and interest, is estimated at $460 per year. Assuming that storage-space rental will be reduced in proportion to the space used, how long will it take the lift truck to pay for itself if a return of 6% is required to justify investments?

5. A special-purpose machine was installed 8 years ago at a cost of $4,800. The following table shows a record of its annual operating cost, maintenance cost, book value, and scrap value. If interest is neglected, what is the average annual cost for each service life in years? For what service life was the average annual cost at a minimum?

Year of Service	*Operation Cost for Year*	*Maintenance Cost for Year*	*Book Value at End of Year*	*Scrap or Salvage Value at End of Year*
1	$5,140	$ 30	$4,300	$200
2	5,170	38	3,800	200
3	5,215	54	3,300	200
4	5,263	82	2,800	200
5	5,240	108	2,300	200
6	5,370	196	1,800	200
7	5,565	316	1,300	200
8	5,782	575	800	200

6. A manufacturer is considering the purchase of an automatic lathe to replace one of two turret lathes. The latter were purchased 12 years ago at a cost of $3,400 and were depreciated on the books to a value of $400 two years ago. At present they can be sold for $700 each. The automatic lathe will cost $8,600. Other data that apply follow:

	Turret Lathes	*Automatic Lathe*
Annual output, Part A	40,000 each	80,000
Annual use other than on Part A	400 hrs. each	0
Production, units of Part A per hour	34 each	82
Labor	1 man each at $1.80 per hour	1 man at $1.70 per hour
Estimated annual maintenance	$80 each	$120
Power cost per hour of operation	$.06 each	$.11
Taxes and insurance, 1.6% of	Present value	Original cost
Space charges per year	$24 each	$32

The turret lathes are in good mechanical condition and may be expected to serve an additional 10 years before maintenance rises appreciably. Their salvage value will probably never drop below $300 each. If one of the turret lathes is replaced by the automatic lathe, it is assumed that the automatic lathe will be used to produce the entire 80,000 units of Part A and that the remaining turret lathe will be used 800 hours per year. If the interest rate to be used is 8%, how many years will be required for the automatic lathe to pay for itself?

7. A contractor is now using wheelbarrows that have steel bodies and steel-tired wheels. A salesman suggests that the contractor will make money by scrapping the present wheelbarrows and buying pneumatic-tired wheelbarrows fitted with aluminum bodies of improved design. The salesman claims that experience has shown that men using the improved equipment will not only carry greater loads but will increase their pace.

The present equipment was purchased 3 years ago at a cost of $14.00 per unit and is estimated to have a life of 2 additional years. The new equipment will cost $32.50 per unit. A payroll clerk reveals that the wheelbarrows have been used an average of about 340 hours per year with a labor cost of $1.12 per hour and will probably continue to be used at the same rate in the future. The present equipment can be disposed of at $3.50 per unit; if it is kept an additional 2 years, it will have a salvage value of $1.00.

What per cent increase in output per hour per unit will justify investment in the new units for an interest rate of 10%, a life of 5 years, and a salvage value of $5.00?

8. Mr. A has a gasoline-driven pump unit which he uses for pumping irrigation water from a lake. The unit is used 6 hours per day 120 days per year at full load capacity and delivers 450 cubic feet per minute against a head of 18 feet. Fuel consumption of the unit is at the rate of 4.4 gallons per hour and costs $.22 per gallon.

An equipment dealer offers to sell a similar unit and states that the fuel consumption will be not more than 3.5 gallons per hour. Mr. A states that he will purchase the unit for an installed cost of $720 if, after a month's trial, it demonstrates that its fuel consumption will conform to the claims of the dealer. At the end of the trial period it is found that fuel consumption has been 3.58 gallons per hour and Mr. A declines to purchase the unit. The dealer then offers to sell the unit at a 20 per cent reduction in price and will accept the old unit as $60 in payment for the new unit. Mr. A estimates that the old unit will be usable for two more years and then will be scrapped for no salvage value. Mr. A estimates that the new unit will have a life of 5 years and no salvage value. Mr. A pays 6% interest on all funds he borrows.

(a) On the basis of equivalent annual costs based on the above information, should Mr. A accept the dealer's offer at the discounted price?

(b) What discount on the quoted price of the new unit will just compensate for the difference in claimed and actual fuel consumptions, on the assumption that the latter will continue throughout the expected life of the unit?

9. Two years ago a centrifugal pump driven by a direct-connected induction motor was purchased to meet a need for a flow of 2,000 gallons of water per minute for an industrial process. The unit cost $2,100 and had an estimated salvage value of $300 at the end of 6 years of use. It consumed electric power at the rate of 43 kw and was used 16 hours per day for 300 days a year. The process for which the water is required has been changed and in the future a flow of only 800 gallons per minute is needed 3 hours per day for 300 days a year. At the decreased flow both pump and motor are relatively inefficient and the current consumption is 35 kw.

A new unit of a capacity conforming to future needs will cost $1,180. This new unit will have an estimated life of 8 years with an estimated salvage value of $180 at the end of that time. Its current consumption will be 19 kw. The present unit has a book value of $1,100 and can be sold for $600. The original estimates of useful life and salvage value are still believed to be reliable. Insurance, taxes, and maintenance are estimated at 6 per cent of the original cost for both units. The cost of power is $.015 per kw-hr.

(a) Should the old unit be retained or should the new unit be purchased, if interest is at 8%?

(b) At what number of hours per day for 300 days a year with a requirement of 800 gallons of water per minute will the future equivalent annual cost of the two units be equal, using 8% interest?

10. Assume that the date is January 1 of the year 19X1. The Ajax Company, a manufacturer of heavy sheet metal products, carries on its operation in a leased building for which the annual rental is $23,000 payable in advance on January 1 of each year. The present lease runs until December 31, 19X5. The owner of the building requests termination of the lease on December 31, 19X1, and on that date the owner will pay the Ajax Company $5,000 if it complies with the request.

If the lease is not terminated on December 31, 19X1, it may be presumed that it will be terminated December 31, 19X5.

The Ajax Company owns a building site and has plans for a building that can be constructed in one year. It is estimated that the desired building will cost $320,000 whether it is completed by January 1, 19X2 or by January 1, 19X6.

If the Ajax Company elects to continue in the leased quarters, it will have expenditures for temporary facilities in the amounts indicated during the following years: 19X2, $20,000; 19X3, $17,000; and 19X4, $12,000. The temporary facilities will have no salvage value.

It is estimated that operating costs, exclusive of interest and depreciation, will be $8,000 per year less in the new building than in the present quarters for comparable levels of operations. Annual taxes, insurance, and maintenance will be 5% of the first cost of the new building. Its estimated life is 30 years with no salvage value. The interest rate is 8%.

(a) Determine the present worths of the two plans, as of the end of the present year, for a study period of 30 years beginning one year hence, taking cognizance of the prospective difference in the lengths of service provided. (Both plans will have identical costs for the present year.)

(b) Find the present worths of the two plans, as of the end of the present year, taking no cognizance of the prospective difference in lengths of service provided by the two plans.

(c) The company considers that being able to defer the decision to build for four years has a present worth of $10,000. On the basis of this fact and the results of part (a), which plan should be adopted?

11. Fourteen years ago a 1,200 kw steam electric plant was constructed at a cost of $220 per kw. Annual operating expenses have been $31,000 to produce the annual demand of 5,400,000 kw-hrs. Estimates are that the annual operating and annual demand for current will be the same in the future. The original estimate of a 20-year life with a 5 per cent salvage value at that time is still expected to be correct.

Officials are contemplating the replacement of the old steam plant with a new Diesel plant. The old plant can be sold for $75,000, while the new Diesel plant will cost $245 per kw to construct. The Diesel plant will have a life of 25 years with a salvage value of 10% at the end of that time and will cost $23,000 annually to operate. Taxes and insurance will be 2.3 per cent of the first cost of either plant. Determine whether the officials are financially justified in replacing the old steam plant if the interest rate is 5%.

12. Four years ago a hydroelectric plant was built to use a continuous flow of 11 cubic feet of water per second with an absolute head of 860 feet. An 18-inch pipe line in the system cost $92,000 for pipe, installation, and right of way. The loss of head due to flow friction in the pipe is 81 feet. The power company has acquired additional water rights and will have 22 cubic feet per second in the future. Three plans, designated as A, B, and C, are under consideration for utilizing the augmented flow.

Plan A. Use the present pipe line. This will entail no additional expense but will result in a total loss of head due to friction of 346 feet, owing to the increased velocity of the water.

Plan B. Add a second 18-inch pipeline at a cost of $68,000. The loss of head for this line will be 81 feet.

Plan C. Install a 26-inch pipeline at a cost of $91,000 and remove the old line, for which a net of $3,800 can be realized. The loss of head due to friction for the 26-inch line will be 63 feet.

The energy of the water delivered to the power plant is valued at $64 per horsepower-year where H.P. $= h \times F \times 62.4 \div 550$. In this equation h is the net head in feet and F is the flow in cubic feet per second.

Insurance and taxes amount to 2 per cent of first cost. Operating and maintenance costs are considered to be equal for all three plans. Use interest at 7% and assume that all lines, including the one now in use, will be retired in 30 years with no salvage value. Determine comparable equivalent annual costs of the three alternatives.

13. Mr. B purchased Machine X 4 years ago for $2,200 and estimated the machine would have a useful life of 10 years with a salvage value of $200 at the end of that time. Installation cost of the machine was $400 and annual operating costs of the machine have been $1,200 per year.

An equipment salesman has suggested that Machine X be replaced by Machine Y in the interest of economy. He offers to sell Machine Y for $3,600 and will allow $600 for Machine X as a part payment if Mr. B desires. Mr. B estimates that Machine Y will have a salvage value of $600 at the end of a service life of 8 years and that annual operating costs will be $875 throughout its service life. Mr. B also estimates that Machine *X* will continue to have an annual operating cost of $1,200. The original estimates of service life and salvage value for Machine *X* are still believed to be the best that can be made. A used machinery dealer has offered to pay $750 for Machine *X*. The purpose served by either machine is expected to continue for many years and the interest rate is taken to be 6% compounded annually.

(a) Determine the annual costs of retaining Machine *X* and of buying Machine *Y*, using straight-line depreciation and average interest.

(b) Determine the annual costs of retaining Machine *X* and of buying Machine *Y*, using sinking-fund depreciation with a return.

CHAPTER

15

Economy of Operations

IN A PRIVATE BUSINESS ENTERPRISE the objective sought is usually profit. Even if profit is not the primary objective, it is usually necessary that there be a profit for the enterprise to survive.

In a sense profit cannot be sought directly. Profits are an economic resultant of activities performed. Consider the person who is manufacturing and selling an article. If his economic outlay for all items associated with the production and sale per article is less than the income he receives, he has made a profit. Profit is the excess of income over expenditures for an activity or a series or group of activities. Viewed from this standpoint, profit is a measure of the economic success of activities.

Profit Is a Resultant of Income and Outlay

It is apparent that profit is the resultant of two components, one of which is the economy associated with the performance of an activity and the other the economy associated with income from the activity.

It is obvious that some activities have greater profit potentialities than others. In fact some activities can only result in loss. When profit is a consideration, it is important that activities be selected for their profit potentialities. This is a first step and an important one, since once the decision to perform an activity has been made, the success of subsequent acts has been limited.

The second step, after an activity has been selected, is to perform the activity with economy; to obtain the desired performance with the least possible outlay.

The idea of profit as a resultant of the economies of income and outlay associated with an activity is presented diagrammatically in Fig. 41.

The Aim of Operations Is Economy of Performance

A great many activities of an enterprise may be classified under the heading of operations. As used here "operations" refers to the more or less routine activities performed in gaining chosen objectives of an enterprise. Representative of these in a manufacturing establishment are many of the day-to-day, month-to-month, and year-to-year activities of people whose duties are associated with purchasing, inspection, production, sales, engineering, safety engineering, industrial relations, line management and supervision, and so forth.

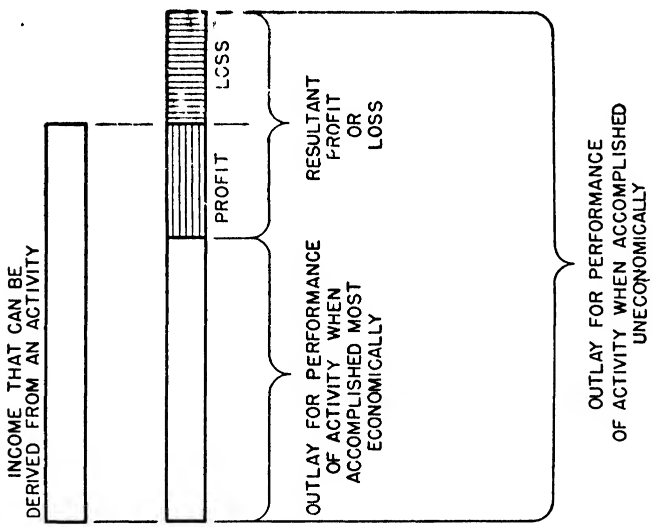


FIG. 41. Profit or loss is a resultant of the income of an activity and the economy of its accomplishment.

In operations, the stress is on the performance of given activities with economy. The aim is to get the work done with the least outlay. Not much attention is given to the desirability of the activity in question, *i.e.* to its income possibilities, for it is presumed that this has been considered in arriving at the decision to perform the activity.

Consider the example of a production department that has been directed to manufacture 600 motor shafts. The production department's problem is to manufacture the shafts at least cost; in general, the department will not question the economic desirability of making the shafts. It is concerned with the economy of activities to be performed in attaining an end rather than the economy of the end itself.

Economy of operations deals with the economical attainment of results through the selection of economical methods and their effective performance. Analyses illustrative of a variety of approaches to this problem follow.

Economic-Purchase-Order Quantities for Quantity Price Discounts

It is common practice to give discounts on purchase prices based on quantities ordered at one time. The approach to the determination of economic-purchase-order quantities when quantity price discounts are given is similar to that developed in the illustrative example on economic-purchase-order quantities in Chapter 12. The modification of this approach brought about by price discounts will be illustrated in an example. Let

Y = all costs incident to providing the year's needs (purchase price, ordering cost, storage cost, interest, insurance, taxes, and so forth).
N = the number of units of the material under consideration needed during a year.
C = the purchase price per unit in dollars.
X = the number of lots ordered per year, N/n.
n = the quantity ordered per lot, N/X.
D = the order cost per lot (clerical cost of requisitioning, ordering, paying vendor, paying transportation, checking invoice, entering the items received on store's record, and so forth).
I = interest, insurance, taxes, and depreciation in percent per year on average funds invested in the supply.
T = the cost of storage per unit per year.

Assume that the following price schedule has been quoted for a certain material and that it is desired to determine the most economical purchase-order quantity.

1 to 49 units........	$1.00 per unit
50 to 399 units........	.90 per unit
400 to 999 units........	.85 per unit
1,000 and more units........	.83 per unit

Assume that this material is used at a uniform rate and that the value of the terms defined above are as follows.

N = 4,000 units
D = \$20 per order
I = .10
T = \$.20 (Storage will be used for other purposes when not used for material under consideration)

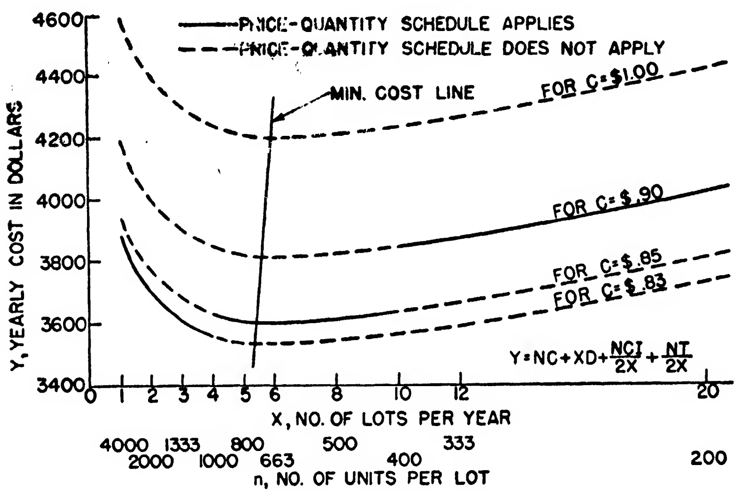


FIG. 42. Economic number of lots with price-quantity schedule.

The general equation for the cost per year is in this case

$$Y = NC + XD + \frac{NCI}{2X} + \frac{NT}{2X}$$

The values of Y (cost per year) for X (number of lots per year, ranging from 1 to 20), and for C as given in the price schedule above are graphed in Fig. 42.

When C as given in the price schedule is substituted in the equation

$$X = \sqrt{\frac{NCI + NT}{2D}}$$

the following values are obtained:

$$\begin{aligned} \text{For } C &= 1.00,\ X = 5.47 \text{ and } n = 732 \text{ units} \\ C &= .90,\ X = 5.39 \text{ and } n = 743 \text{ units} \\ C &= .85,\ X = 5.33 \text{ and } n = 750 \text{ units} \\ C &= .83,\ X = 5.31 \text{ and } n = 753 \text{ units} \end{aligned}$$

Since the values of X and n are substantially the same for all reasonable values of C, only one calculation need have been made, for in practice the values would be rounded to either 5 or 6 lots per year corresponding to lots of 800 or 667. Let it be assumed that 5 lots of 800 per year will be ordered.

Lots of this size come within the price schedule for \$.85 per unit; consequently Y (cost per year) will be equal to \$3,614. Next it must be ascertained if purchase in the smallest lot available at the next lowest price of \$.83 per unit will result in a cost smaller than \$3,614. At \$.83 the smallest lot that may be purchased is 1,000 units. This lot size will result in the purchase of 4 lots per year at \$.83 per unit; hence Y (cost per year) will equal \$3,541. Therefore for minimum cost the material should be ordered in four lots per year of 1,000 units each.

In the solution of this type of problem, where a price schedule based on quantity purchased is in effect, the following steps should be taken to determine the economic lot size.

1. Solve for the number of lots per year X and the number of pieces per lot n for minimum cost for one or more values of C in the price schedule.
2. Calculate the annual cost Y, using the value of X obtained above and the value of C applicable to the lot size n.
3. Calculate the annual cost Y, using the smallest lot size available at the next lower price.
4. Compare the results obtained in Steps 2 and 3 above and take the number of lots corresponding to the lesser cost.

Economic-Production-Order Quantities

Economic-production-order quantities are determined in a manner similar to that employed in determining economic-purchase-order quantities. The difference in method is brought about by the fact that a purchased lot is received at one time while a production order accumulates as it is made. Let

Y = all costs incident to supplying the year's needs (production cost, set-up cost, storage cost, interest, insurance, taxes, and so forth).

N = the number of units of the item under consideration needed during the year (assumed to be used at a uniform rate throughout the year).

C = the production cost per unit including cost of material but excluding set-up cost.

X = the number of lots manufactured per year.

n = the quantity manufactured per lot.

S = the set-up cost per lot (cost of requisitioning, set-up of machines, entering the items received on store's record, and so forth).

P = the rate in units per year at which items under consideration can be made.

I = interest, insurance, taxes, and depreciation in per cent per year on the average of funds invested in the supply.

T = the cost of storage per unit per year.

Then

$$Y = NC + XS + (P - N)\frac{NIC}{2PX} + (P - N)\frac{NT}{PX}$$

In the equation above

NC = production cost for a year's supply exclusive of set-up costs.

XS = set-up cost incurred in producing a year's supply.

$(P - N)\dfrac{NIC}{2PX}$ = interest, insurance, taxes, and depreciation on average investment in items on hand during the year.

When pieces are added to storage at the rate of P units per year and are taken from storage at N units per year, the net rate of accumulation is $(P - N)$ units per year.

The time required to make N pieces at the rate of P pieces per year is N/P years. If N pieces are made in a single lot, the maximum accumulation in storage will be $(P - N)\ N/P$. Since no pieces will be in storage at the end of the year, the average number in storage will be

$$\left[(P - N)\frac{N}{P} + 0\right] \div 2 = (P - N)\frac{N}{2P}$$

See Fig. 43. If X lots are produced per year, the average number in storage will be $(P - N)\ (N/2PX)$, and the investment charges will be $(P - N)\ (NIC/2PX)$.

$(P - N)\dfrac{NT}{PX}$ = cost of storing the supply during the year. (It is assumed that storage space provided for a unit of supply in question cannot be used for any other purpose. If the storage space is used for other purposes as the material in question is vacated, the expression should be $(P - N)\ (NT/2PX)$.

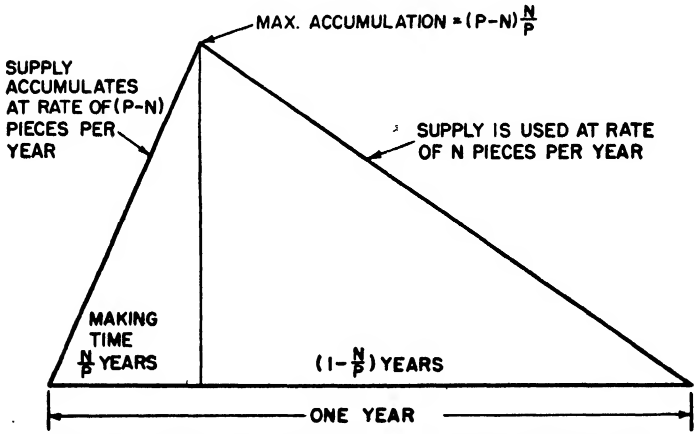


FIG. 43. Pattern of investment when one lot is made per year.

Reference to Fig. 43 shows that the maximum number of pieces in storage where one lot is made per year is $(P - N)\ N/P$. If X lots are made per year, the maximum number of pieces in storage will be $(P - N)\ N/PX$. The cost per year for storage is then $(P - N)\ (NT/PX)$ or $(P - N)\ (NT/2PX)$, depending on whether the storage space can not or can be used for other purposes.

For convenience in solution, the equation may be written

$$Y = NC + XS + \frac{(P - N)N(CI + 2T)}{2PX}$$

Differentiating with respect to X, setting the result equal to zero, and solving for X results in

$$X = \sqrt{\frac{(P - N)N(CI + 2T)}{2PS}}$$

Bearing in mind that the number of pieces per lot $n = N/X$, consider the following example:

N, the number of units needed per year = 400 pieces
C, production cost per unit exclusive of set-up cost = \$8.00
S, set-up cost per lot = \$40.00
P, production rate in units per year = 3,000 pieces
I, per unit on investment for interest, taxes, and so forth = .12
T, cost of storage per unit per year (in this case storage used for the material in question can be used for no other purpose) = \$1.10

To find the economic lot size and number of lots per year, substitute the given values in the equation obtained above.

$$X = \sqrt{\frac{(P - N)N(CI + 2T)}{2PS}}$$

$$X = \sqrt{\frac{(3{,}000 - 400)\ 400\ (8 \times .12 + 2 \times 1.10)}{2 \times 3{,}000 \times 40}}$$

$$X = 3.7, \text{ say } 4, \text{ lots per year}$$

and $$n = \frac{N}{X} = \frac{400}{4} = 100 \text{ pieces per lot}$$

The annual cost for four lots per year is \$3,497. The cost for one and 12 lots per year is respectively \$3,788 and \$3,725. Thus a decision to make one or 12 lots per year would result in a loss of \$291 and \$228 respectively.

Economical Size of Repair Crews

There are many situations in industry in which economy depends upon a balance between losses associated with machine failures and the costs incident to preventative or repair activities undertaken to minimize such losses.

In situations where effort is directed to the prevention of machine failures and the repair of machine failures, several economies are usually involved. Illustrative of such economies are the economy of the preventative activities, the economy of the repair activities, the economy associated with the loss of use of a machine, and, finally, the economy of the situation as a whole.

Suppose that the failure of a machine will result in a loss of $2.00 per hour during the period that it is inoperative or "down" for repairs. Past experience shows that a typical repair for this machine can be made by one man in 9 hours, by two men in 5 hours, by three men in 4 hours, by four men in 3.75 hours, and by five men in 4 hours. The wage rate of repair men is taken to be $1.00 per hour.

A tabulation of the costs incident to the failure of the machine is given in Table 34.

TABLE 34. Repair Costs for Different Numbers of Men in a Repair Crew

Number of men in repair crew	1	2	3	4	5
Hours required to make repair	9	5	4	3¾	4
Man-hours required to make repair	9	10	12	15	20
Labor cost of making repair at $1.00 per hour	$ 9	$10	$12	$15	$20
Down-time cost at $2.00 per hour for machine	$18	$10	$ 8	$ 7.5	$ 8
Resultant of labor cost of making repair and down-time loss of machine	$27	$20	$20	$22.5	$28

The economy associated with the loss of use of the machine suggests that the repair crew should be the size that will result in minimum down time. A person concerned only with the down-time loss of a machine would select a four-man crew, which would hold the down-time loss to $7.50 per repair.

The economy of the repair activity points to a crew of one man so that the repair may be made most efficiently and at least cost. A person primarily interested in the efficient utilization of repair men and low repair costs would have his men work singly.

Usually it is the over-all economy that is desired. In this case the greatest over-all economy occurs when crews of two or three men are used; the use of either would result in a total cost of $20.00.

This example brings out the fact that there are two aspects to be considered in a repair program. One of these is to make the needed repairs at least cost; the other is to minimize losses associated with the down time of machinery.

Economical Number of Repair Crews

Similar to the determination of economical size of repair crews illustrated above is the determination of the economical number of repair crews. Failures of equipment do not occur with regularity. Failures occur more or less by chance, and the amount of repair work needed during any short period may be expected to vary greatly both above and below the average amount needed over a long period of time. If the number of repair crews provided is just sufficient to take care of the average amount of repair work needed, there will always be a backlog of repair work waiting to be done and excessive down time of units that fail may result. Down time is equal to the sum of the time required for making repairs plus the time spent waiting for repair work to begin. If down time is costly, it may be wise to maintain repair crews in excess of those needed for the average repair load, even though this leads to some idleness.

An approach for determining the economical number of repair crews will be illustrated by an example from the petroleum industry.

In petroleum production, heavy equipment is used to pump oil to the surface. This equipment fails from a variety of causes necessitating its being removed from the well for repairs. The repairs are made by well-serving crews of three to five men who are equipped with heavy, portable equipment to pull the pumping equipment from the well.

Between the time the pumping equipment fails and the time it is repaired the well is idle. This results in a loss, known as *lost production,* equal to the amount of oil that would have been produced if there had been no failure of equipment. In some cases lost production may be only production that is deferred to a later date. In other cases lost production is partially lost because of drainage to competitor's wells. This loss may be quite large. Two days' down time of a well producing 200 barrels of oil per day at $2.20 per barrel, for instance, when drainage to competitor's well is judged to be half of lost production, results in a loss of $440.

The down time of a well is made up of the time that the well is idle before the repair crew gets to it and the time it is idle while repair is in process. Since the rate at which repairs can be made is substantially controlled by the repair equipment, reduction in loss is brought about by reducing the time a well is idle awaiting repair crews. This is done at the expense of having excess repair crews. Thus the problem is to balance the number of repair crews with the lost production associated with delays of repairs to wells. Consider the following data adapted from an actual situation.

N, number of oil wells in field = 30
T, average interval at which individual wells fail = 15 days
R, average actual time for company crew to repair well failure = 12 hours
U, hourly cost for company-operated repair unit and repair crew = $6.90 per hour
r, average actual time for contract crew to repair well failure = 16 hours
u, hourly cost for contract repair unit and repair crew = $9.00 per hour
L, lost production per well when "down" = $84 per day

Wells are operated 24 hours per day, seven days per week.

Under the present situation analysis has revealed that operation of one repair unit 24 hours per day, seven days per week can keep up with the repair of failures in the long run. However, serious losses are arising from delays in getting to wells after notification of their failure. Delay is due to the chance bunching of well failures. For instance, one period during which no wells fail may be followed by a period of an above-average number of failures. Since repairs cannot be made before failures occur, it is clear that crews sufficient to take care of the average number of failures will have a backlog of failures awaiting them most of the time.

The first step in an analysis to determine the most economical number of repair crews is to determine the number of wells that may be expected to fail during each and every day of a period in the future. The number of failures to expect in the future may be estimated on the basis of past records or by mathematical analysis. The former method is used in the example because it is more revealing.

In the solution of this example the pattern of well failures of a previous 30-day period selected at random will be considered to be representative of future periods. A 30-day period will be taken in the interest of simplicity, even though experience has shown that longer periods are advantageous.

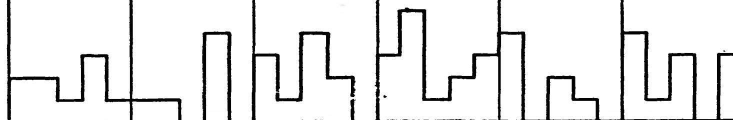
Since the company's unit can repair two wells per day when operating three eight-hour shifts per day, unrepaired wells will be carried over one day each day that the number of failures plus the carry-over from the previous day exceeds two.

Line A of Fig. 44 shows the number of wells that failed during the

FIG. 44

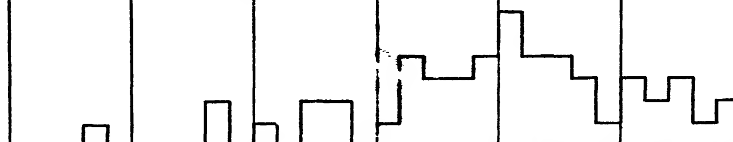
Pattern of oil-well failure and two repair methods.

SITUATION A: Company unit, only used



Day of Period	1				5					10					15					20					25					30
A. Number of wells failing each day.	2	2	1	3	1	1	1	0	4	0	3	1	4	2	0	3	5	1	2	3	4	0	2	1	0	4	1	3	0	3
B. Number of wells repaired each day (two can be repaired each day by company's unit in three 8-hr. shifts).	2	2	1	2	2	1	1	0	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
C. Unrepaired wells carried over 1 day.	0	0	0	1	0	0	0	0	2	0	1	0	2	2	0	1	4	3	3	4	6	4	4	3	1	3	2	3	1	2

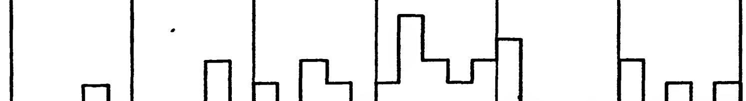
Total for month 57 (row A)



Total carry-over: 52 well days (row C)

SITUATION B: Contract unit used.

Day of Period	1				5					10					15					20					25					30
D. Number of wells failing each day.	2	2	1	3	1	1	1	0	4	0	3	1	4	2	0	3	5	1	2	3	4	0	2	1	0	4	1	3	0	3
E. Number of wells repaired each day (two can be repaired each day by company's unit in three 8-hr shifts and three can be repaired each day indicated by (^{x}) when contract unit is added).	2	2	1	2	2	1	1	0	2	2^x	2	2	2	3^x	1	2	2	3^x	3^x	2	3^x	3^x	2	1	0	2	3^x	2	1	2
F. Unrepaired wells carried over 1 day.	0	0	0	1	0	0	0	0	2	0	1	0	2	1	0	1	4	2	1	2	3	0	0	0	0	2	0	1	0	1



Total carry-over: 24 well days (row F)

30-day period selected at random. The number of wells failing during any one day ranged between 0 and 5 and their total was 57. The number of wells repaired each day with the company's unit and crews is given in Line B of Fig. 44. During the 30-day period 57 wells failed and 55 were repaired. Thus in spite of periodic backlogs there was idle time of the unit and crews on days numbered 3, 6, 7, and 8 sufficient to repair 5 wells. The carry-over of unrepaired wells totals 52 well-days and appears in Line C of Fig. 44.

A summary of operation costs on the present basis of operation (one unit operating 24 hours per day) follows.

Value of lost production due to carry-over of unrepaired wells, 52 well-days × $84	$4,368
Cost of company's unit and crews, $6.90 per hour × 24 hours × 30 days	4,968
Total cost incident to well failure and repair during 30-day period	$9,336

Analysis of the pattern of occurrence of unrepaired wells carried over reveals that, once a backlog has accumulated, wells may remain unrepaired and unproductive for a long period. As a remedy the supervisor in charge considers the feasibility of hiring an additional unit and a crew whenever a backlog of unrepaired wells has accumulated. He finds that a repair unit and crew can be hired on short notice when needed for $9.00 per hour. Because of greater travel distance and other reasons, a hired unit and crew has been found to require an average of 16 hours to repair a well.

The supervisor wishes to determine the effect of a policy of hiring an additional unit and crew for a 16-hour period on days when there is a carry-over of two or more unrepaired wells from the previous day. He assumes that the hired unit and crew will be paid for a minimum of 16 hours each time it is asked to report, whether or not it is used.

If this policy had been in effect during the 30-day period under consideration, the additional unit and repair crew would have been hired for 16 hours on days numbered 10, 14, 18, 19, 21, 22, and 26; the total number of wells repaired during each day would have been as given in Line E of Fig. 44; and the total carry-over totaling 24 well-days would have accumulated in accordance with the pattern given in Line F of Fig. 44. From Line E of Fig. 44 it can be determined that there was idle time of units and crew members during days numbered 3, 6, 7, 8, 10, 15, 24, 25, and 29 sufficient for them to have repaired 11 well failures.

The following costs are expected on the basis of the above analysis.

Value of lost production due to carry-over of unrepaired wells, 24 well-days × $84	$2,016
Cost of company's unit and crews, $6.90 per hour × 24 hours × 30 days	4,968
Cost of hired unit and crew, $9.00 per hour × 16 hours × 7 days	1,008
Total cost incident to well failures and repair for 30-day period when hired unit and crew is used	$7,992

Thus a decision to institute the policy stated above may be expected to result in a new reduction in costs incident to well failures of $9,336 − $7,992 = $1,344 per month.

Machine Interference[1]

Machine interference occurs when one or more machines are nonproductive because, having shut down in need of attention, they stand idle while the operator or operators are attending to other machines. For example, when one operator attends to two or more semi-automatic machines that have irregular shutdown times, machine idleness caused by interference must be tolerated because the running cycles of these machines cannot be coordinated.

The effects of machine interference are particularly apparent in the textile industry, where each operator usually attends to several semi-automatic machines. For a given work assignment, there may be times when all machines are producing and the operator is merely waiting for something to happen. There are other occasions, however, when many of the machines will have chanced to shut down simultaneously. In most cases, there is little or nothing that the operator can do to control unpredictable chance shutdowns of the machines and the consequent interference idleness.

The problem of evaluating machine interference losses is important to the engineer in two respects. First, machine interference loss must be known quantitatively before the economic number of semi-automatic machines to assign to one operator can be known. Second, the operator's actual productive efficiency cannot be determined for wage incentive purposes until the loss of production occasioned by machine interference has been properly evaluated.

[1] Adapted from W. D. Jones, "Mathematical and Experimental Calculations of Machine Interference Time," *The Research Engineer*, Georgia Institute of Technology, January 1949.

Countless hours have been spent by time-study men in attempts to evaluate machine interference by use of stop watches. However, the unpredictable, variable shutdowns of the many machines involved in a given assignment make the job of recording interference time an almost impossible task. Moreover, the question always arises whether the period of timing was of sufficient length to be representative of the actual conditions of operation.

Since machine interference is unpredictable, spasmodic, and variable in magnitude, accurate predetermination of interference mathematically requires some means of measurement that will take into account these uncertain characteristics. It has been found that one of the laws of probability serves this purpose very well; this law states, in effect, that when an event is based on chance as pertaining to each of several participants acting together, the various possible combinations of occurrence of that event will be distributed according to the terms of a binomial expansion. For example, suppose it is desired to determine the probabilities of each of the possible combinations of occurrence of the ace when rolling three dice together, *i.e.* the chance of all three dice turning up aces, the chance of rolling two aces, one ace, and a non-ace, and so forth. Substituting in the equation

$$(d + r)^n = d^n + nd^{n-1}r + \frac{n(n-1)d^{n-2}r^2}{1(2)} + \ldots + r^n$$

$(d + r)^n$ becomes $(\frac{1}{6} + \frac{5}{6})^3$, where d is the probability of occurrence of the ace for each participant for each roll, r is the probability of non-occurrence of the ace, and n is the number of participants. Expanding $(\frac{1}{6} + \frac{5}{6})^3$, the following distribution of probabilities is obtained:

Number of Aces Showing	*Coefficient Sums*		*(d)*		*(r)*	*Probability*
3	1	×	$(1/6)^3$	×		1/216
2	3	×	$(1/6)^2$	×	$(5/6)^1$	15/216
1	3	×	$(1/6)^1$	×	$(5/6)^2$	75/216
0	1			×	$(5/6)^3$	125/216

Here it can be seen that on the average, during 216 rolls of three dice, three aces would turn up once, two aces and a non-ace would occur 15 times, one ace and two non-aces would show 75 times, and three non-aces would turn up 125 times.

The expansion of $(d + r)^n$ as employed in this dice problem is also valid for evaluating interference when one operator tends several semi-automatic machines having either the same or different degrees of servicing-time requirements. The problem of computing interference for a given number of machines is handled in the same manner as the dice problem, except that interference waiting time for each probability must be factored in. For example, assume that three machines chance to shut down during the same interval of time. While one machine is being attended to by the operator, the other two must wait. The interference idleness inherent in the probability of three machines being down at the same time would therefore be that probability multiplied by the two consequent waits.

The total interference idleness for a given group of machines is secured by adding the products of the various shut-down probabilities and consequent number of waits. This total interference is then divided by the number of machines in the group to secure the average interference idleness per machine.

Example of interference calculations. A brief description of the application of this method to a simple problem now seems in order. To make this presentation clear, the problem selected involves the determination of interference after a period of operation; normally, the interference allowance would be predetermined and included in the piece rate.

In this example, assume that one operator attends to six semi-automatic machines. A count of the production at the end of the day shows that the average down time (nonproducing time) per machine was 20 per cent of the time the machines were operated. What then, was the average percentage interference per machine for the period of operation? Also, what was the average percentage servicing time per machine? Substituting, $(d + r)^n$ becomes $(1/5 + 4/5)^6$ when

d = probability of occurrence of down time = 1/5
r = probability of nonoccurrence of down time = 4/5
n = the number of machines = 6

Number of Machines Down Together	*Coefficient Sums*		*(d)*		*(r)*		*Waits*	*Interference*
6	1	×	$(1/5)^6$			×	5	5/15,625
5	6	×	$(1/5)^5$	×	$(4/5)^1$	×	4	96/15,625
4	15	×	$(1/5)^4$	×	$(4/5)^2$	×	3	720/15,625
3	20	×	$(1/5)^3$	×	$(4/5)^3$	×	2	2,560/15,625
2	15	×	$(1/5)^2$	×	$(4/5)^4$	×	1	3,840/15,625
1	6	×	$(1/5)^1$	×	$(4/5)^5$	×	0	
0	1				$(4/5)^6$	×	0	

Total Interference = 7,221/15,625
= .462

Since there are six machines in the group, the average interference per machine would be .462 ÷ 6 or 7.7 per cent. Since downtime D is equal to average servicing time plus average interference time, the average servicing time per machine would be 20.0 − 7.7 = 12.3 per cent.

It is interesting to note that, since total down time D is equal to regular servicing time plus average interference time, interference *causes* interference. Interference caused by regular servicing time would result when, for instance, three machines out of a group shut down at the same time. Two machines must wait while the other is being serviced. Later, moreover, while the operator services one of the remaining two idle machines, the other must continue to wait. This waiting time of the third machine would be attributed to the prior interference of the second machine, and it would therefore be a case of interference causing interference. Meanwhile, if any of the other machines in the group chances to shut down while the operator is engaged in servicing the three machines described above, there would be another case of interference causing interference.

Tables or charts can be compiled showing the relationships between interference, servicing time, and down time for a group of any number of machines. This can be done by assuming various percentages of down time, proceeding with calculations as illustrated above, and then recording or plotting the results.

In some cases the servicing time of a machine operated separately is the only known quantity. The interference that will take place in a bank of any number of similar machines can be determined from charts or tables compiled as mentioned above, even though total down time is the basis of the original calculations.

Determination of economical number of machines per operator. When a certain machine typical of a bank of machines is tended individually and no interference can take place, the average observed down time of the machine is 15 per cent of the over-all operating time.

It is desired to calculate the number of these machines that should be tended by one operator for greatest economy when the hourly cost of power, depreciation, and so forth of each machine is $1.00 and the operator's hourly wage is $1.00. Let

S = servicing time per machine in per cent of over-all operating time, assuming each machine is individually tended.

I = average interference time per machine in per cent of over-all operating time. The average interference time per machine for groups of 3, 4, 5, and 6 machines, each having an S of 15 per cent, as determined from tables described above, is 3, 5, 8, and 11 per cent, respectively.
D = average down time per machine in per cent of total operating time.
P = average productive time per machine (or the efficiency) in percentage of over-all operating time.
L = hourly pay for the operator's labor.
M = hourly machine rate (power, depreciation, and so forth).
n = number of machines tended by one operator.

When one operator tends one machine, the average productive time P of the machine is $100\% - D$. When one operator tends several machines, however, interference I becomes a factor in the operating disposition of each machine and D therefore becomes $I + S(100\% - I)$. In multiple machine operation then,

$$P = 100\% - I - S(100\% - I).$$

It therefore follows that the "equivalent" number of 100 per cent productive machines (an imaginary term) when one operator tends n machines is nP. Now, through the use of the formulas just developed, the economical number of machines for the stated conditions will be determined.

n	S	I	P	nP	nM	L	$nM + L$	$(nM + L)/nP$
3	15%	3%	82.5%	2.48	$3.00	$1.00	$4.00	$1.61
4	15%	5%	80.7%	3.23	4.00	1.00	5.00	1.55
5	15%	8%	78.2%	3.91	5.00	1.00	6.00	1.53
6	15%	11%	75.7%	4.54	6.00	1.00	7.00	1.54

Here it can be seen that an assignment of 5 machines would result in the lowest cost per equivalent machine (*i.e.* the lowest unit cost) for the conditions of this problem.

Economic Loading of Equipment

The load that it is considered proper to impose on units of equipment is often indicated by their makers. Such indicated loads are often not the optimum for economy.

Loading of equipment is stated in such terms as tons of dead load, tons of live load, pounds per square inch of pressure, volts, amperes, and revolutions per minute. The life of an asset is usually dependent in some measure upon the load imposed upon it. When this is the case, the load that results in least cost of operation will also result in a length of life that may be designated as the economic life of the asset. Thus in effect economic loading may be expressed in terms of economic life as is done with incandescent lamps.

In some cases, particularly with short-lived assets, the economic load may be determined by experiment. If the effect of various loadings upon the life, maintenance, and other operating costs of a unit of equipment are known or can be estimated with reasonable accuracy, it is practical to determine the load that will result in the greatest economy.

Consider the following example. A concern has a bending machine that produces one piece for each revolution it makes. It is now being operated at the rate of 150 rpm and produces 18.6 pounds of product per hour. The machine costs $6,150 and is estimated to have an operating life of 10,000 hours. Direct wages plus labor overhead of the operation amount to $1.37 per hour. Average present maintenance and power are $.072 and $.064 per hour respectively. Output of product and cost of power used per hour are estimated to be directly proportional to the rpm of the machine. Since centrifugal forces on machine parts increase with the square of the speed, it is estimated that maintenance costs will increase in proportion to the square of the speed, and that the useful life of the machine will be inversely proportional to the square of the speed. On the basis of these assumptions Table 35 was constructed.

TABLE 35. Relation of Costs to Operating Speed of a Machine

(A) Operating Speed in R.P.M.	(B) Estimated Life in Hrs. $10{,}000 \times (150)^2/A^2$	(C) Average Output in Pounds Per Hr. $18.6 \times A/150$	(D) Labor Cost Per Hour	(E) Average Maint. Cost Per Hr. $\$.072 \times A^2/(150)^2$	(F) Power Cost Per Hr. $\$.064 \times A/150$	(G) Average Depr. Cost Per Hr. $\$6{,}150 \div B$	(H) Total Cost of Op. Per Hr. D + E + F + G	(I) Cost Per Pound H ÷ C
150	10,000	18.6	$1.37	$.072	$.064	$ .615	$2.121	$.114
200	5,630	24.8	1.37	.128	.085	1.092	2.675	.108
250	3,600	31.0	1.37	.200	.107	1.708	3.385	.109

On the basis of the estimated use, it would be desirable to increase the speed of the machine to 200 rpm. The expected saving per pound of product would be

$$(\$.114 - \$.108) \div \$.114 = 5.3 \text{ per cent}$$

This is a worth-while saving, particularly since it shortens the capital-recovery period and so lessens the possibilities of losses from obsolescence and inadequacy.

Economy of Load Distribution between Machines

In many cases two or more machines are available for the same kind of production. Thus two or more boilers may be available to produce steam or two or more turbine generators may be on hand to produce power. If the total load is less than the combined capacity of the machines that are available, the economy of operation will, in some measure, depend upon the portion of the total load that is carried by each machine.

A simple method for determining load distribution between two machines for minimum input will be developed by means of an illustrative example. Suppose that two Diesel-engine-driven d-c generators, one of 1,000 kw capacity and the other of 500 kw capacity, are available. The efficiencies for different outputs and the corresponding inputs for each of these machines are given below in tabular form.

Output in kw	Machine A. 1,000 kw Output		Machine B. 500 kw Output	
	Efficiency in Per Cent	Input in kw	Efficiency in Per Cent	Input in kw
100	15.1	663	20.7	483
200	22.0	909	27.9	719
300	26.3	1,141	31.8	943
400	29.5	1,356	32.5	1,231
500	31.2	1,603	32.0	1,563
600	32.3	1,858		
700	32.5	2,154		
800	32.6	2,454		
900	32.4	2,778		
1,000	32.3	3,096		

Suppose that the total load to be met at a certain time is 1,200 kw. This load may be distributed in several ways with the result shown in the table below.

Output in kw			Input in kw		
Machine A	Machine B	Total	Machine A	Machine B	Total
1,000	200	1,200	3,096	719	3,815
900	300	1,200	2,778	943	3,721
800	400	1,200	2,454	1,231	3,685
700	500	1,200	2,154	1,563	3,717

It will be noticed that the desired output of 1,200 kw can be produced with least input when the larger unit carries 800 kw and the smaller 400 kw.

In the above example input can readily be converted to units other than kw. Suppose that it is desired to express input in terms of dollars. Let it be assumed that fuel oil having an energy content of 18,800 Btu weighs 7.48 pounds per gallon and costs $.084 per gallon delivered. One kw-hr. is equivalent to 3,410 Btu. From these data the cost per kw-hr. is calculated to be

$$\frac{\$.084 \times 3{,}410}{7.48 \times 18{,}800} = \$.002037$$

At this rate of cost for fuel, 1,200 kw-hr. output of energy can be produced for 3,685 × $.002037 = $7.51 if the loads on the two machines are 800 kw and 400 kw respectively. If the load distribution had been 1,000 kw and 200 kw on the machines, the cost would have been $7.78, an increase over the better method of operation of $.27 or $.27/$7.51 = 3.7 per cent. This is a rather high rate of saving considering the ease with which it is made. The saving is a result of a correct decision based on knowledge of load distribution.

This method for determining load distribution is applicable to all types of machines and to entire plants as well. Its greatest usefulness is perhaps in relation to the application of power machines and machines having similar efficiency characteristics. When more than three machines are involved, the process illustrated above becomes cumbersome and should be replaced by more refined methods based on incremental rates.[2]

Manufacturing to a Variable Demand

The demand for many manufactured goods is seasonal. The varia-

[2] Max J. Steinberg and Theodore Smith, *Economy Loading of Power Plants and Electric Systems* (New York: John Wiley & Sons, Inc., 1943), p. 3.

tion in demand may reflect the effects on living of seasonal climatic changes such as in temperature, hours of sunshine, and rainfall. Heavier clothing, for example, will be purchased for the colder periods of the year. Some items vary in demand during the year because of social customs. Thus the custom of using fireworks on July Fourth in this country results in the concentration of fireworks sales in a short period of the year.

The seasonal-goods manufacturer may make his product at a relatively low rate throughout the year and store it until it is needed. Or he may acquire sufficient facilities to manufacture the product at a rate equal to the demand during the period in which it is sold.

The disadvantage of the first plan is that storage cost is relatively high, and the disadvantage of the second method is that a relatively large equipment investment will be required. The most desirable plan may be a compromise of the above two plans.

The method of solution of this and similar situations will be illustrated by an example. Let it be assumed that 36,000 units of a product are sold during a four-month period each year as follows:

Month of Year	*Number Sold*
9th	2,000
10th	10,000
11th	15,000
12th	9,000

One machine can make 36,000 units of this product during a year. The fixed charges on the required machine for such items as interest, taxes, insurance, space to house machine, and depreciation due to causes exclusive of usage amounts to $2,000 per year. The cost for depreciation due exclusively to usage, power, supplies, maintenance, and so forth amounts to $.25 per hour or $40 per month of operation on the basis of a 160-hour month.

Although one machine can meet the demand for the product, the accumulation of finished products throughout the year will result in considerable expense for storage. The expense for storage can be reduced by using more machines to shorten the period required to make the year's needs.

The costs associated with plans of production based on using one, two, and three machines will be determined. For an output of 36,000 units per machine, 12, 6, and 4 months respectively will be required to manufacture the annual output of 36,000 units with one, two, and three machines. The number sold each month and the number in storage at the end of each month during the year are given in the accompanying table and are shown graphically in Fig. 45.

Month Number	Sales during Month	Number in Storage at End of Month for		
		1 Machine	2 Machines	3 Machines
1		3,000		
2		6,000		
3		9,000		
4		12,000		
5		15,000		
6		18,000		
7		21,000	6,000	0
8		24,000	12,000	0
9	2,000	25,000	16,000	7,000
10	10,000	18,000	12,000	6,000
11	15,000	6,000	3,000	0
12	9,000	0	0	0

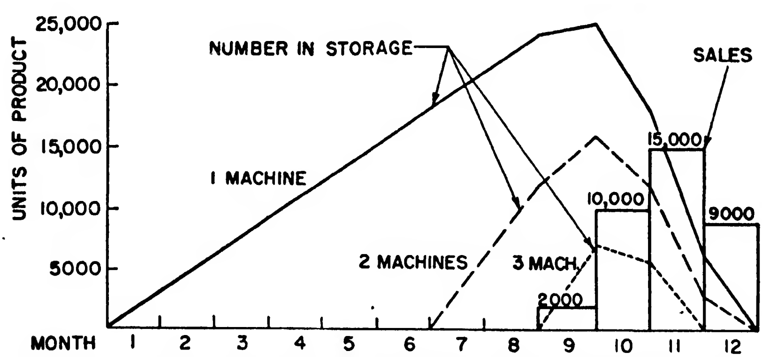


FIG. 45. Product in storage in manufacturing to seasonal demand with one, two, and three machines.

The average number in storage for one, two, and three machines is 13,083, 4,083, and 1,083 respectively. These results are obtained by adding the average of the number of units of product in storage at the beginning and end of each month in the year for each plan and dividing the resulting sum by 12. The product is valued at $3.00 per unit. The sum of interest, taxes, and insurance is taken as 10 per cent of the unit cost and storage costs as $.20 per unit per year of storage.

On the basis of the above data the cost with one, two, and three machines will be as follows.

ONE MACHINE

Fixed charges on machine, 1 × $2,000	$2,000
Variable charge on machine, 12 × $40	480
Interest, taxes, and insurance, 13,083 × $3 × .10	3,925
Storage cost, 13,083 × $.20	2,617
Total cost with one machine	$9,022

TWO MACHINES

Fixed charges on machines, 2 × $2,000	$4,000
Variable charge on machines, 6 × 2 × $40	480
Interest, taxes, and insurance, 4,083 × $3 × .10	1,225
Storage cost, 4,083 × $.20	817
Total cost with two machines	$6,522

THREE MACHINES

Fixed charges on machines, 3 × $2,000	$6,000
Variable charge on machines, 4 × 3 × $40	480
Interest, taxes, and insurance, 1,083 × $3 × .10	325
Storage cost, 1,083 × $.20	217
Total cost with three machines	$7,022

On the basis of the above analysis, a saving of $500 per year would result from using two machines in place of three and a saving of $2,500 would result from using two machines in place of one. It should be realized that there may be considerable hazard in the use of any of the three plans. If only one machine is used, there is a possibility that the product may become outmoded before the sales period or that sales may not be up to expectations. If three machines are used, there is a large investment that may not be recovered before the machines are rendered obsolete or inadequate.

General solutions have been developed to solve many problems that arise from seasonal demand for products.[3] These solutions are very useful where situations to which they are adapted arise frequently.

Economy of Jigs and Fixtures

Jigs and fixtures are adjuncts to machines. They may serve sev-

[3] See Paul N. Lehoczky, *Quantitative Management in Industry* (Columbus, Ohio: Harold L. Hedrick, 1938), pp. 214–271.

eral purposes. One purpose is to hold work in a desired geometric relationship with the cutting tools of the machine with which they are used. Fixtures are distinguished from jigs by the fact that they are usually more or less rigidly attached to the machine with which they are used, while jigs are usually not attached. Because of their similarity the term "fixture" will be used to include both jigs and fixtures in this discussion.

Not only may fixtures reduce the skill and strength required of the operator, but they may permit performance of work which is beyond the skill and strength of an operator. Both of these aspects have economic value. Attainment of results beyond human capacities is a characteristic of a civilization resting on mechanization. For example, fixtures permit the production of articles with a uniformity beyond human capacity. The economic value of this feature is beyond calculation for on it rests interchangeability of parts, the basis of mass manufacture. It is so commonplace for fixtures and like devices to attain perfection beyond that attainable by unaided human effort that this perfection is taken for granted.

Once a new level of desirability in attainment has been reached, attention is subsequently directed to gain this new level of desirability with least cost. Thus a fixture under consideration for a particular purpose may be evaluated on the basis of:

1. Savings it will effect in the use of material. It may reduce the amount or the quality of material needed, or both.

2. Savings resulting from a reduction in the degree of skill and extent of human labor required.

3. Savings resulting from increases in output of the machine with which it is used.

Offsetting such savings are the costs incident to the fixtures themselves, such as depreciation, interest on investment, taxes, insurance, supplies, repairs, and power.

Evaluation of Economy of Fixtures

Several general expressions will be developed for evaluating a fixture under consideration in comparison with another fixture or with a situation where no fixture is used. The expressions to be developed will also serve to demonstrate the use of equations for economic comparison. To illustrate the method of comparison, development will be in greater detail than is usually deemed necessary, in accordance with the idea that abridgment is best made after an expression has been fully developed. The following terms and their accompanying values will be used in an illustrative example.

N_2, annual number of pieces to be made = 8,000
I, decimal per cent allowance on investment for interest = .08
X, decimal per cent allowance on investment for taxes and insurance = .02

TERMS RELATING TO FIXTURE A

M, cost of machine used per hour = \$2.00
W, cost of operator per hour = \$.96
t, time per piece to process = .05 hour
T, cost of material per piece = \$.42
C_F, cost of fixture = \$1,200
F_F, annual fixed cost of fixture, $C_F(D + I + X)$
F_V, variable cost of fixture per hour = \$.20
N_1, life of fixture in terms of number of pieces made = 24,000
L, life of fixture (as estimated in years on the basis of the number of units it is expected to process—for a life of N_1 pieces life will be N_1/N_2 years) = 3 years
D, decimal per cent allowance for annual depreciation of fixture $= 1/L = N_2/N_1$
U, cost per set-up = \$16
N_3, number of pieces made per set-up = 2,000
C_P, cost per piece
S_P, savings per piece resulting from use of Fixture A

TERMS RELATING TO SITUATION B (NO FIXTURE USED) OR FIXTURE B

M', cost of machine per hour = \$2.40
W', cost of operator per hour = \$1.18
t', time per piece to process = .07 hour
T', cost of material per piece = \$.44
C_F', cost of fixture (none used) = 0
F_F', annual fixed cost of fixture, $C_F'(D + I + X)$ (none used) = 0
F_V', variable cost of fixture per hour (none used) = 0
N_1', life of fixture in terms of numbers of pieces made (none used) = 24,000
D', decimal percent allowance for annual depreciation of fixture, N_2/N_1' (none used) = 0

U', cost per set-up (none required) = 0
N_3', number of pieces made per set-up = 2,000
C_P', cost per piece

It should be noted that $(D + I)$, in the expression $(D + I + X)$, is an approximate capital recovery factor based on straight-line depreciation plus interest on the original investment. This is conservative and adequate for most fixtures whose life is relatively short. Where greater accuracy is desired, $(D + I)$ may be replaced by the exact capital-recovery factor or by

$$D + I\,\frac{N_1 + N_2}{2N_1}$$

for straight-line depreciation and average interest.

The term F_V embraces costs for supplies, power, repairs, and the like. The costs for these items are usually not large and they may often be neglected without introducing a significant error.

Since fixtures are usually used as adjuncts to machines, the machines with which they are used must be taken into consideration. A general expression for the cost of a piece when a fixture and a machine are used jointly is given by

$$(1) \quad C_P = T + Mt + Wt + \frac{U}{N_3} + \frac{C_F}{N_2}\left(\frac{N_2}{N_1} + I + X\right) + F_V t$$

For the data given for Fixture A the solution of this equation to arrive at the cost per piece is

$$C_P = \$.42 + \$2 \times .05 + \$.96 \times .05 + \frac{\$16}{2{,}000} + \frac{\$1{,}200}{8{,}000} \times \left(\frac{8{,}000}{24{,}000} + .08 + .02\right) + \$.20 \times .05 = \$.651$$

For Situation B (no fixture used)

$$(2) \quad C_P' = T' + M't' + W't' + \frac{U'}{N_3'} + \frac{C_F'}{N_2}\left(\frac{N_2}{N_1'} + I + X\right) + F_V't'$$

The net saving per piece resulting from use of the fixture is given by

$$(3) \quad S_P = C_P' - C_P$$

$$(4)\quad S_P = (T' - T) + (M' + W' + F_V')t' - (M + W + F_V)t + \left(\frac{U'}{N_3'} - \frac{U}{N_3}\right) + \left(\frac{C_F'}{N_1'} - \frac{C_F}{N_1}\right) + \frac{C_F' - C_F}{N_2}\left(I + X\right)$$

For the data given, the solution of this expression is given by

$$S_p = (\$.44 - \$.42) + (\$2.40 + \$1.18 + 0).07 - (\$2 + \$.96 + \$.20).05 + \left(\frac{0}{2{,}000} - \frac{\$16}{2{,}000}\right) + \left(\frac{0}{24{,}000} - \frac{\$1{,}200}{24{,}000}\right) + \frac{0 - \$1{,}200}{8{,}000}\left(.08 + .02\right)$$

The net annual savings from use of fixture is equal to

$$S_P \times N_2 = \$.040 \times 8{,}000 = \$320$$

Setting the expressions for C_P and C_P' equal to each other and solving for the break-even value of C_F results in

$$(5)\quad C_F = \left[(T' - T) + (M' + W' + F_V')t' - (M + W + F_V)t + \left(\frac{U'}{N_3'} - \frac{U}{N_3}\right) + \frac{C_F'}{N_2}\left(\frac{N_2}{N_1'} + I + X\right)\right] \div \frac{1}{N_2}\left(\frac{N_2}{N_1} + I + X\right)$$

For the data given the solution is

$$C_F = \left[(\$.44 - \$.42) + (\$2.40 + \$1.18 + 0).07 - (\$2 + \$.96 + \$.20).05 + \left(\frac{0}{2{,}000} - \frac{\$16}{2{,}000}\right) + \frac{0}{8{,}000}(0)\right] \div \frac{1}{8{,}000}\left(\frac{8{,}000}{24{,}000} + .08 + .02\right) = \$1{,}932$$

Substituting L for N_1/N_2, setting the expressions C_P and C_P' equal to each other, and solving for the break-even value of L results in

$$(6)\quad L = \frac{C_F}{N_2} \div \left[(T' - T) + (M' + W' + F_V')t' - (M + W + F_V)t + \left(\frac{U'}{N_3'} - \frac{U}{N_3}\right) + \frac{C_F'}{N_1'} + \left(\frac{C_F' - C_F}{N_2}\right)(I + X)\right]$$

For 8,000 pieces per year the solution of the expression results in

$$L = \frac{\$1{,}200}{8{,}000} \div \Big[(\$.44 - \$.42) + (\$2.40 + \$1.18 + 0).07 - (\$2 + \$.96 + \$.20).05 + \left(\frac{0}{2{,}000} - \frac{\$16}{2{,}000}\right) + \frac{0}{24{,}000} + \left(\frac{0 - \$1{,}200}{8{,}000}\right)(.08 + .02)\Big] = 1.7 \text{ years}$$

The above formulas are applicable to a great variety of situations. They provide rather completely for the consideration of items that may have a bearing on economy. For general application the rather large number of terms is probably an advantage in reducing the possibility of omitting significant items. Items that have little or no bearing may be omitted consciously and readily.

For special applications the formulas may be abridged to conform to the situations for which they are used. For example, the general formula for the cost per piece C_P may be abridged to

$$(7) \quad C_P = T + Mt + Wt + \frac{U}{N_3} + C_F\left(\frac{N_2}{N_1} + I + X\right)$$

when $F_V = 0$.

The general formula for the break-even cost C_F of the fixture may be abridged to

$$(8) \quad C_F = [C_F' + (M + W)(t' - t)] \div \frac{1}{N_2}\left(\frac{N_2}{N_1} + I + X\right)$$

when $W = W'$; $M = M'$; $T = T'$; $F_V = F_V'$; $N_1 = N_1'$; $U \doteq U' = 0$ or when $U = U'$ and $N_3 = N_3'$.

Such abridged formulas should not be used unless analyses show them to be adequate for the purpose for which they are applied.

Processing Rates of Material a Factor in Selection

In cases where two or more materials may serve a purpose equally well from a functional standpoint, the relationship of their cost and the cost of processing them may be the factor that determines which is chosen. Brass, for example, is often found to be less costly for parts than cold rolled steel because it may be machined at a higher rate, in spite of its greater weight per unit volume and greater cost per pound. Aluminum, which is easily machinable and in addition has a low specific weight, is being used in increasing amounts as a replacement for steel, cast iron, and other metals whose cost per pound is considerably less than that of aluminum. Because of the ease with which they can be processed, plastics have proved to be an economy in many applications as a replacement for materials of less cost per pound. The same can be said of die castings.

In some cases the decision to substitute one material for another will result in an entirely different sequence of processing. For instance, a change from grey iron to zinc alloy castings will require marked change in equipment.

To determine the comparative economic desirability of two materials, it is necessary to make a detailed study of the costs that arise when each is used. In some cases this may involve the economy of disposing of present equipment and acquiring of new equipment.

However, there are numerous instances in which a change of material will result in no change in the sequence and type of processes involved, as, for instance, a change from steel to brass rods for turned parts. When such situations are encountered, a formula may be developed to aid in the selection of the most economical material to use.

The development of a formula for the selection of steel or brass on the basis of economy when the method of processing is essentially identical for both metals will now be considered. In the development of formulas of this type there is less likelihood of omission and error if rather detailed consideration is given to the costs that make up the total. Also it is best to preserve the identity of items far into the development. Abridgment to fit special situations should be withheld until general equations have been developed, so that the full implication of contemplated abridgments may readily be ascertained. Abridged formulas, of which there are many in industrial literature, should not be used unless their adequacy for the situation to which they are to be applied has been established.

The cost of processing a part is made up of costs for material, preparation, use of machines, and labor. For purposes of comparison the cost per piece may be taken to be

$$C = T + u + (M + W)t$$

where

C = the cost per piece in dollars.
T = net cost for material per piece.
u = set-up cost in dollars per piece.
M = the cost of processing machine used per hour.
W = the cost of machine operator per hour.
t = time to process in hours per piece.

and

$$T = G[V_R P_R - (V_R - V_F)P_S]$$

where

G = specific weight of material in pounds per cubic inch.
V_R = volume of raw material needed in cubic inches.
V_F = volume per piece of finished piece in cubic inches.
P_R = cost of raw material in dollars per pound.
P_S = value of scrap in dollars per pound.

On the basis of the above terms a general expression for cost per piece is

$$C = G[V_R P_R - (V_R - V_F)P_S] + (M + W)t + u$$

From this equation will be developed an equation that will give the savings resulting from the use of brass over steel. Using subscripts S and B respectively for the terms that apply to steel and brass,

$$C_S = G_S[V_R P_{RS} - (V_R - V_F)P_{SS}] + (M + W)t_S + u_S$$

and $$C_B = G_B[V_R P_{RB} - (V_R - V_F)P_{SB}] + (M + W)t_B + u_B$$

Let S_B = saving per piece of brass over steel in dollars per piece, and let $u_S = u_B$. Then

$$\begin{aligned} S_B &= C_S - C_B \\ &= G_S[V_R P_{RS} - (V_R - V_F)P_{SS}] - G_B[V_R P_{RB} - (V_R - V_F)P_{SB}] \\ &\qquad + (M + W)(t_S - t_B) + (u_S - u_B) \\ &= V_R(G_S P_{RS} - G_B P_{RB}) - (V_R - V_F)(G_S P_{SS} - G_B P_{SB}) \\ &\qquad + (M + W)(t_S - t_B) \end{aligned}$$

G_S and G_B may be replaced by their numerical values of .283 and .309 pound per cubic inch respectively and

$$G_B = \frac{.309}{.283} \times G_S = 1.09G_S$$

$$S_B = V_R(.283P_{RS} - .309P_{RB}) - (V_R - V_F)(.283P_{SS} - .309P_{SB}) + (M + W)(t_S - t_B)$$

If the value of S_B turns out to be a negative value, the result is interpreted as the loss per piece that would be occasioned by the use of brass over steel.

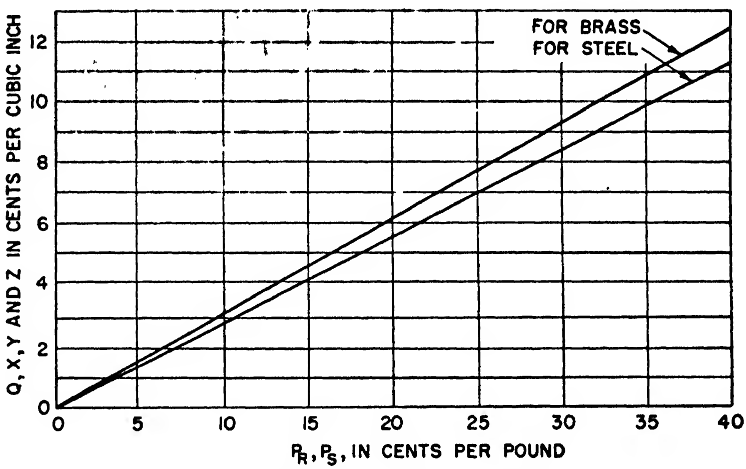


FIG. 46. Scheme of a graph for converting cost of brass and steel from cents per pound to cents per cubic inch.

The use of the above equation is further simplified by the use of the graph on the plan of Fig. 46 after the following terms have been substituted in the above equation:

Q for $.283P_{RS}$ Y for $.283P_{SS}$
X for $.309P_{RB}$ Z for $.309P_{SB}$
V_S for $(V_R - V_F)$ where V_S equals volume of scrap in cubic inches per piece

The equation then becomes

$$S_B = V_R(Q - X) - V_S(Y - Z) + (M + W)(t_S - t_B)$$

Consider the following example:

V_R, volume of raw material per piece = .221 cu. in.
V_F, volume of finished material per piece = .097 cu. in.
V_S, volume of scrap per piece = .221 − .097 = .124 cu. in.
P_{RS}, cost of steel, per pound = \$.09
V_{SS}, value of steel scrap, per pound = \$.02
P_{RB}, cost of brass, per pound = \$.21
P_{SB}, value of brass scrap, per pound = \$.10
M, machine rate per hour = \$1.60
W, labor cost per hour = \$.90

$$t_S\text{, time per steel piece in hours} = \frac{.52 \text{ min.}}{60} = .0087 \text{ hour}$$

$$t_B\text{, time per brass piece in hours} = \frac{.34 \text{ min.}}{60} = .0057 \text{ hour}$$

From a large scale graph on the plan of Fig. 46 the following values are obtained.

$$\begin{aligned} Q &= \$.09 \times .283 = \$.0254 \\ X &= \$.21 \times .309 = \$.0649 \\ Y &= \$.02 \times .283 = \$.0057 \\ Z &= \$.10 \times .309 = \$.0309 \end{aligned}$$

Substitution of the above values in the equation results in

$$\begin{aligned} S_B &= .221(\$.0254 - \$.0649) - .124(\$.0057 - \$.0309) \\ &\qquad + (\$1.60 + \$.90)(.0087 - .0057) \\ &= -\$.00873 + \$.00312 + \$.0075 \\ &= \$.00189 \text{ saving per piece or } \$1.89 \text{ per thousand pieces} \end{aligned}$$

Control

A clear concept of the term *control* as it relates to such things as costs, budgets, material, production, and quality is helpful in economic analyses. To control is to confine between limits, to make an action or thing conform to a plan. A car that is being driven along the highway is in control as long as the driver can make it go where he has planned.

The nature of the problem of control may be observed by trying to retrace freehand a line that has been drawn with the aid of a ruler. As long as the pencil is on the ruled line it is in control and no corrective action is required or can be taken. The pencil can never be controlled absolutely, since it must deviate from the ruled line before the fact that it is out of control can be observed and corrective action can be taken.

Since absolute control is impossible, practical plans provide for deviation from perfection by setting up limits of acceptable variability. Action confined between these established limits is considered to be

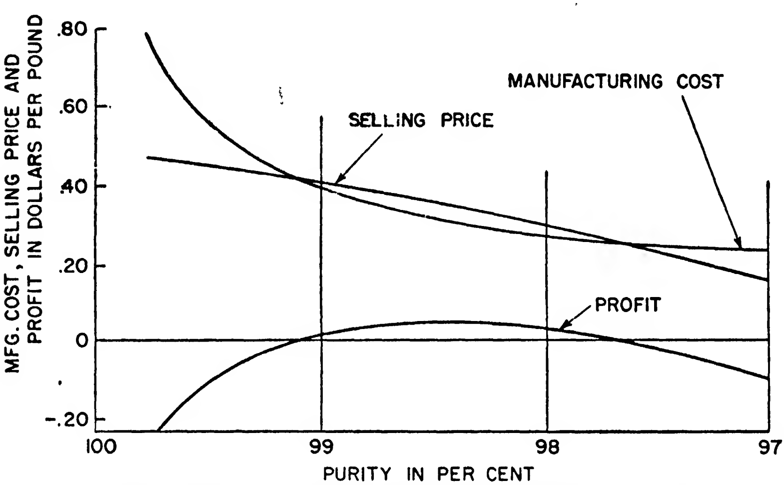


Fig. 47. Relationship of manufacturing cost, selling price, and profit to the quality of a chemical product.

in control. As a general thing, the wider the limits, the easier it is to maintain control, and the lesser the worth of the action. The balance between the cost of maintaining control and its worth is an important consideration in economy.

This relationship is illustrated by Fig. 47, which gives the production cost, sale price, and profit per pound of a chemical as its purity varies. In this example the cost of control is a part of the cost of production. Thus production should be controlled to the percentage of purity that will result in maximum difference between sale price and cost of production. The degree of control to be exercised is an economic consideration in nearly all utilitarian activities.

Statistical quality control. To maintain or control quality within desired limits involves three factors. First is the specification that establishes the plan or the limits of variability of the product.

Second is the production of the product in accordance with the specifications. Third is inspection to determine if the product conforms to the specifications.

Methods of analysis involving the use of statistics for considering these three factors are embraced by the general term *statistical quality control.* The statistical approach to quality control, generally credited to Dr. W. A. Shewhart, was applied in the Western Electric Company as early as 1924. But the application of this technique did not become widespread until during World War II. The methods of statistical quality control are essentially approaches to economy.

Statistical analysis has proved to be an important tool for reducing the cost of inspection. This approach is used extensively to determine what per cent of a lot of product may be inspected with greatest economy. It is usually found that inspection of less than the total lot is sufficient; the percentage is often quite small, ten per cent or less. The efficacy of inspecting a small sample may be superior to that of inspecting the entire lot in some cases. Rutherford[4] states: "It has been established that frequently the human error on large-volume, repetitive inspection work more than offsets any gain of quality through complete inspection."

Statistical analysis has important applications in relation to the production function. Some variability of a product that is dependent upon the methods and facilities used to make it will always be present. If the product resulting from a process does not meet specifications, the reasons for failure may often be determined by statistical analysis. In some cases statistical analysis may be used to detect changes in the variabilities of a product, changes that will foretell, even before any of the product fails to meet specifications, that some defective units of product are in prospect unless corrective action is taken. Thus corrective steps can be taken before a loss in incurred.

In some cases statistical analysis may demonstrate that a productive process is inadequate to meet the specifications that have been set up for a product. In such a case either the process must be changed or an analysis must be made to determine if the specifications may be made less difficult to meet.

Statistical quality control is an important modern tool of production. The subject is too extensive to be more than introduced in

[4] John G. Rutherford, *Quality Control in Industry* (New York: Pitman Publishing Corporation, 1948), p. 103.

this text, but it is one that merits consideration by all concerned with production and inspection.

PROBLEMS

1. The profit on a product selling for $8.20 is 10 per cent of the selling price. What percentage increase in production cost will reduce the profit by 60 per cent.

2. A product is selling for $18.75. The production cost of the product is $17.90.

(a) What per cent of the selling price represents profit?

(b) What per cent reduction in production cost will increase the margin of profit by 60 per cent?

3. A machine has been developed to produce a special glass product. The machine costs $6,000 and is estimated to have a useful life of 6 years and no salvage value 6 years hence. The output of the machine is 4 units per hour and it is estimated that 1,600 acceptable units will be made per year. The operator's hourly wage rate is $1.50 and the variable cost of operating the machine is $1.10 per hour. The material used amounts to $5.40 per unit. Five per cent of the units produced by the machine are defective. Defects are not detectable until after completion of the unit. The interest rate to be used is 10%.

(a) Determine the cost per nondefective unit produced.

(b) It has been suggested that the machine be rebuilt. If the percentage of defective units can be reduced to 2 per cent, what expenditure for rebuilding is justified, assuming there will be no change in the operator's rate, cost of material per unit, variable operating cost of the machine or the life of the machine?

4. A manufacturer uses 800 bags of a raw drug ingredient at a uniform rate throughout the year. The following prices are quoted:

1 to 8 bags	$2.80 each
9 to 39 bags	2.58
40 to 199 bags	2.43
200 bags and up	2.37

Analyses show the average cost of placing and receiving an order of this class to be $18. The storage cost per bag, in general storage that can be used for other purposes, is estimated at $.70 per bag per year. Interest, taxes, and insurance are estimated at 16 per cent of the average inventory in storage. How many lots and how many bags per lot should be ordered per year for greatest economy?

5. It is desired to determine the most economical lot size for the manufacture of 1,000 units of a part that is used at a uniform rate throughout the

year. The set-up cost per lot is \$40. Three of the parts can be made in an hour and the plant operates 1,880 hours per year. The manufacturing cost is \$5.20 per unit. Interest, insurance, and taxes are estimated at 12% on average inventory. The parts are stored in a general warehouse and the storage cost is estimated at \$.80 per unit per year. Calculate the most economical number of lots and the number of units to be made per lot.

6. Take the number of oil wells failing each day (Line A, Fig. 44) in reverse. Use data relative to the performance of repair crews as given in Fig. 44.

(a) Determine the number of unrepaired wells carried over one day when only the company's unit is used.

(b) Determine the number of unrepaired wells carried over one day when a contract repair unit is used each day that two or more unrepaired wells are carried over from the previous day.

(c) Use the values of N, T, R, U, r, u, and L given in the illustrative example in the text and calculate the total cost incident to well failure and repair for part (a) and part (b) for the 30-day period.

7. A company has 24 identical machines that are operated 24 hours per day, 360 days per year. On the average each machine breaks down at intervals of 6 working days. The time required to repair each machine is 8 man-hours.

Repairmen receive \$1.00 per hour and work 8 hours per day. A loss of \$16 is sustained for each day that a machine is "carried over" unrepaired.

(a) Determine the number of machines that may be expected by chance to go "down" during each day of a month by using 24 dice to represent the 24 machines. Let the downs for each day be represented by the aces that come up for each throw of 24 dice.

(b) Assuming that all downs as found in part (a) above occur at 12 midnight, determine the most economical number of regular repair men to employ, if no extra repair men or overtime wages are to be allowed and regular repair men are not used except to repair the machines in question. Assume that the sequence for the 30-day month will be repeated 12 times during the year.

8. For a certain group of machines, a repair crew requires one day to repair a machine that breaks down. A loss of \$80 per day occurs each day a machine is carried over unrepaired. Each crew costs \$90 per day. Pattern of machine failures on succeeding days of a 30-day period is: 7, 5, 8, 4, 6, 6, 6, 7, 2, 9, 6, 8, 7, 6, 6, 9, 7, 5, 5, 3, 5, 4, 8, 7, 6, 6, 5, 6, 5, 6. Assume all breakdowns occur at the first of each day.

(a) Determine the idle time of crews for the given pattern of failures when 6, 7, or 8 crews are used.

(b) Determine losses occasioned by the carry-over of unrepaired machines.

(c) What is the optimum number of crews to employ?

9. The output of wells in an oil field ranges from less than 10 to more than 1,000 barrels per day per wall. When wells go down, they are repaired by a crew of men and a portable winch known as a "unit." Oil that could have been produced while a well is awaiting and undergoing repairs is known as lost production. Frequently a large well goes down when a crew and unit have partially completed repairs on a small well.

The question then arises, should repairs to the small well be completed before beginning the repairs of the large well or should the repairs on the small well be interrupted until the crew can be transferred to and complete the repairs on the large well? If the latter course is followed, the unit used on the small well will be left in its set-up position and a spare unit that is available will be driven to the large well. Thus only the crew will be transferred from the small well to the large well and the number of set-ups will remain one per well repair for either decision. Under average conditions, it has been found that interrupting repair of a small well to repair a large well results in a loss of one hour of the crew's time. The crew must be paid for this hour though nothing tangible is accomplished.

It is desired to set up equations for calculating the savings that will result from taking a crew away from a small well before completing its repair in order to get to the large well with least possible delay. Wells and repair crews operate 24 hours per day. Let

- S = savings in dollars that will result from interrupting repair of a small well to repair a large well.
- U = cost per unit per hour in dollars = \$3. This charge applies only when the unit is on the road or actually in use on a job.
- K = cost for crew per hour in dollars = \$5.
- D = delay, in hours, in getting back to small well after repairing large well.
- F = hours required to finish repairs in process on small well. This item is known by the person making decision.
- P_a = net production of small well in dollars per hour.
- P_b = net production of large well in dollars per hour.
- T = hours to repair a large well.

(a) Write the general equation for S.

(b) Determine savings S under the following conditions: $D = 1$ hr.; $F = 5$ hrs.; $P_a = \$4$; $P_b = \$15$; $T = 16$ hrs.

10. In the operation of a certain type of equipment, repairs average \$6 each and the cost of a shutdown for emergency repairs entails a loss of \$14 in addition. In the past, there has been no routine periodic inspection of the equipment. It is believed that periodic inspections would result in a worthwhile saving.

In order to get data on which to determine the frequency of periodic inspections for greatest economy, five groups of an equal number of machines A, B, C, D, and E were inspected 0, 1, 2, 3, and 6 times yer week respectively. The cost per inspection was found to be \$1.25. The plant operates 6 days per week, 50 weeks per year. The results of a year's run are shown in Table A.

TABLE A

Group	Inspections Per Week	Number of Emergency Repairs	Number of Non-emergency Repairs
A	0	31	27
B	1	20	36
C	2	13	41
D	3	9	43
E	6	3	47

What number of inspections should be made per week and what will be the total yearly cost for inspection, loss due to emergency repairs, and cost of repairs for this number of inspections per week?

11. If tended individually, a certain machine would be nonproductive 12 per cent of the time for servicing. Determine the number of these machines that should be tended by one operator for greatest economy when the operator's hourly wage is $.90 and the hourly cost of power, depreciation, and so forth for each machine is $.60. The average interference per machine for groups of 4, 5, 6, 7, and 8 machines, each requiring 12 per cent servicing time on an individual attention basis, is 3, 4, 6, 8, and 11 per cent, respectively.

12. The machine and handling time of Operation *A* on Machine *M* is 1.3 and .4 minutes respectively. The machine and handling time of Operation *B* on Machine *N* is .6 and .3 minute respectively. The hourly cost of operating machines *M* and *N* is $1.20 and $.70 respectively. The operators of both machines receive $1.08 per hour. Determine the cost of Operations *A* and *B*.

The economy of letting one operator handle both machines is being considered.

(a) Remembering that the handling times cannot overlap or be interrupted when started, determine the percentage of idle time of Machine *N* if Machine *M* is kept busy all of the time because of its higher rate.

(b) Determine the savings on each operation if the operator is paid $1.20 for operating both machines, of which $.65 is charged to *M* and $.55 to *N*.

(c) Determine the percentage of the total time that the operator of two machines will be working.

13. Electric lamps rated at 200 watts and 110, 115, or 120 volts can be purchased for $.30 each. When the lamps are placed in a 115-volt circuit, the following data apply:

	110 V Lamp	115 V Lamp	120 V Lamp
Average watts input	209.1	195.2	182.8
Average lumens output per watt of input	18.30	16.84	15.48
Average life of lamp in hours	420	750	1,340

Energy costs $.04 per 1,000 watt-hours. Determine the cost per million lumen hours for each lamp.

14. The normal operating speed of wire weaving looms is 300 R.P.M.; at this speed the looms have a life expectancy of 5 years, an annual maintenance cost of $60, and an annual power cost of $70. The machines cost $1,200 and have a scrap value of $100. Annual space charges are estimated at $170 per machine. The operator's rate is $1.20 per hour and each operator runs four machines regardless of speed of the looms.

Experimental work on the operation of these machines has resulted in data as follows:

R.P.M.	*Life, Years*	*Annual Maintenance Cost, Dollars*	*Annual Power Cost, Dollars*
250	7.2	40	60
300	5	60	70
350	3.7	80	80
400	2.8	100	90
450	2.2	120	100

Summarize the items of cost in a table and determine the most economical R.P.M. at which to run the machines. Assume that a loom operating at a normal operating speed of 300 R.P.M. will produce 1,200 units of product per year (each year consists of 40 hours a week for 50 weeks) and that production is directly proportional to the speed of the loom. Compute capital recovery with a return on the basis of the straight-line depreciation and average interest with interest at 7%.

15. An underground deposit has been estimated to contain 56,000 tons of recoverable sulphur, for which the market price at the mine is $15.50 per ton. The deposit will be recovered through wells by dissolution in hot water. Forty-five hundred tons per year can be recovered from each well. The time required to exhaust the deposit is inversely proportional to the number of wells. The company makes the following estimates of expenses for 1, 2, 3, and 4 wells.

No. of Wells	1	2	3	4
Investment	$47,000	$85,000	$123,000	$161,000
Variable Cost Per Ton	$2.72	$2.72	$2.72	$2.72
Years to Mine	12	6	4	3
Tons Per Year	4,500	9,000	13,500	18,000

The property is valued at $400,000 and will have no value after the sulphur deposit is exhausted. Assume that all equipment must be paid for by this mining operation. What number of wells will result in the highest rate of return on the investment?

16. The manufacturer of a gift article finds that his average sales are at the rate of 10,000 units per month each month of the year, except September and October when the rate of sales is 40,000 units per month. This product is made on single-purpose machines that have an initial cost of $12,000 each, an estimated life of 6 years, and no salvage value. Fixed charges other than interest and depreciation on each of these machines amount to $400 and the variable cost amounts to $.42 per hour of operation. Machine operators are paid \$1.20 per hour. Each machine normally produces 25 units of product per hour. The plant operates 160 hours per month. The material entering the product is received as used and costs \$2.80 per unit of product. Charges for interest, taxes, and insurance on the average inventory of finished goods are estimated to be 9% of the variable cost of manufacture and materials. Charges for storage of inventory amounts to \$.10 per unit per year of average inventory. The firm uses an interest rate of 8% on its investment in facilities in cost calculations. How many machines should be used for minimum cost?

17. The following data apply for the evaluation of the economy of a proposed fixture in terms of symbols used in the text:

$$N_1 = 120{,}000 \quad N_2 = 30{,}000 \quad I = .10 \quad X = .02$$
$$M = \$2.80 \quad W = \$1.20 \quad t = .032 \text{ hr.} \quad T = \$.24$$
$$C_F = \$920 \quad F_V = \$.03 \quad U = \$12 \quad N_3 = 2{,}500$$

Data applying to the alternative situation, in which no fixture is used follow:

$$M' = \$2.40 \quad W' = \$1.56 \quad t' = .044$$
$$T' = \$.24 \quad U' = \$2 \quad N_3' = 4{,}000$$

(a) Determine the net saving per piece if the proposed fixture is made.

(b) Determine the life at which the fixture will pay for itself.

18. The volume of the raw material required for a metal part is .123 cubic inch. Its finished volume is .064 cubic inch. The machining time per piece is .246 minute for steel and .144 minute for brass. The cost of the specified steel is \$.12 per pound and the value of steel scrap is negligible. The cost of the specified brass is \$.23 per pound and the value of brass scrap is \$.08 per pound. The hourly cost of the required machine and operator is \$3.20. Determine the comparative costs per piece for steel and brass parts.

19. A manufacturer of heavy earth-moving equipment estimates that rolling resistance of different qualities of roadbeds as follows:

Unstabilized, rutted, dirt roadway; soft under travel.....	150 lbs. per ton
Maintained, firm, smooth roadway; dirt surfacing flexing slightly under load..................................	65 lbs. per ton
Maintained, hard-surfaced roadway; little penetration under load..	40 lbs. per ton

The outputs of earth haulers over a 5,000-foot roadway for various values of rolling resistance are as follows:

Rolling resistance of roadway, lbs. per ton.....	150	65	40
Output, cubic yards per hour.................	30	60	80

Twelve thousand cubic yards of earth are to be moved; the hauler and driver cost \$9 per hour; and a roadway with a rolling resistance of 150 pounds per ton costs \$260. What expenditure is justified for the construction of a roadway whose rolling resistance is (a) 65 and (b) 40 pounds per ton?

CHAPTER

16

Capacity, Loading, Production and Distribution Costs, Income, and Profits

IN NEARLY ALL ACTIVITIES there are two types of input or costs. These are fixed costs and variable costs. True fixed costs are independent of volume or quantity of output.

The concept of fixed cost has a wide application. For example, certain losses in the operation of an automobile engine are in some measure independent of its output of power. Among its fixed costs, in terms of energy, are the power to drive the fan, the valve mechanism, and the oil and fuel pumps. Almost any task involves preparation independent of its extent. Thus to paint a small area may require as much effort for the cleaning of a brush as to paint a large area. Similarly the operation of a business involves fixed costs that are independent of volume of output.

Variable costs are those that in some manner are proportional to output. For example, the consumption of fuel of an engine may be expected to be proportional to its output of power and the amount of paint used may be expected to be proportional to the area painted in some degree. In businesses and industries also there are costs that are in some proportion to the output.

Capacity, Loading, and Economy

A particular facility of a given capacity will have, in regard to fixed and variable costs, certain characteristics that are not easily altered. Therefore its economy per unit of output will be determined by the relationship of its capacity to its load or output.

Since fixed costs per unit of output will be inversely proportional to output, the fixed cost per unit will decrease as output increases; and since variable costs per unit of output will be in some proportion to output, it follows that there is some load of a facility at which the total unit cost of output may be at a minimum. See page 195 for a graphical representation of fixed and variable costs per unit of output. The aim of such economic analysis is directed toward maintaining a balance between capacity and loading that will produce the optimum over-all result.

Load Factor, Capacity Factor, and Diversity Factor Concepts

Several concepts useful in the consideration of the capacity, loading, and economy have been defined as ratios. These ratios or factors, though developed particularly for the consideration of relationships in the electrical industry, embody concepts generally useful.

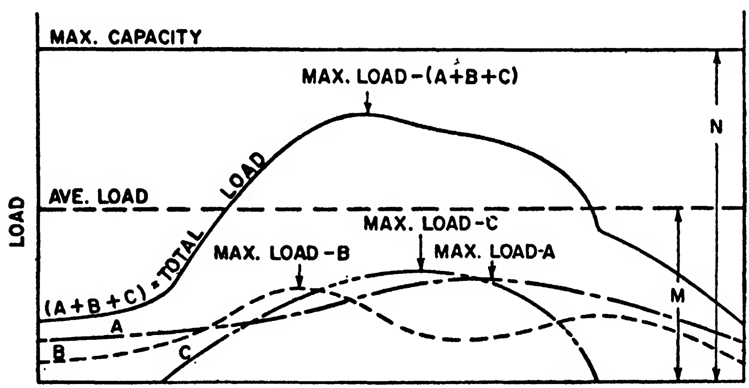


FIG. 48. Capacity and loading of a facility.

In Fig. 48 the maximum capacity, the average load, and the total load over a period of time of a facility are represented graphically. As shown in Fig. 48 the total load is the summation of loads A, B, and C.

Loads A, B, and C may be taken to represent, for example, the output of three products of a factory in dollars during a year, the output of three gas wells in a field in million cubic feet of gas per month, or the output of an electrical generating station to three classes of customers in kw-hrs. of energy during a 24-hour period.

Load factor. By definition *load factor* is the ratio of the average load to the maximum load. In terms of the symbols used in Fig. 48

$$\text{Load factor} = \frac{\text{M}}{\text{Max. load (A + B + C)}}$$

The load factor may be equal to unity or less. In Fig. 48 it is approximately equal to .64. The load factor is one measure of load fluctuation with time.

Capacity factor. *Capacity factor* is defined as the ratio of the average load to the maximum capacity. In terms of the symbols in Fig. 48

$$\text{Capacity factor} = \frac{\text{M}}{\text{N}}$$

The capacity factor of a facility may be equal to unity or less. In Fig. 48 its value is approximately equal to .56. The capacity factor is one measure of the extent of utilization of a facility. This means that usage of the facility aggregates little more than half of that theoretically possible. True fixed cost per unit of output will be a minimum when the capacity factor is equal to unity.

Diversity factor. By definition *diversity factor* is the ratio of the sum of the maximums of sub-loads to the maximum of the total load. In terms of the symbols in Figure 48

$$\text{Diversity factor} = \frac{\text{Max. load A + Max. load B + Max. load C}}{\text{Max. load (A + B + C)}}$$

The diversity factor of a facility may be equal to unity or more. In the figure its value is approximately equal to 1.14. The diversity factor is a measure of how effectively individual peak loads are disposed. A diversity factor equal to unity would indicate that the maximums of sub-loads coincide. This factor appears most frequently in literature of the electrical industry, but it is clear that it is applicable to other activities.

For example, manufacturers, though rarely using the term, take cognizance of diversity when they seek products that will be in maximum demand at different seasons of the year. The theater owner who offers a matinee for a lesser price than an evening show is attempting to improve his diversity factor. One purpose of having cylinders of a multicylinder engine fire at intervals instead of simultaneously is to improve the diversity factor so that such elements of the engine as its crankshaft, carburetor, exhaust manifold, and clutch need not be sufficient to meet loads that would be imposed if all cylinders fired at once.

Application and terms of expression of load, capacity, and diversity factors. The concepts embraced by the terms load factor, capacity factor, and diversity factor are of interest to the industrialist. A proper balance between load, capacity, and diversity of activities with respect to time is an important consideration in attaining profitable operation. In the electrical industry, loads and capacity are usually expressed in terms of units of product. But the usual practice, when a large variety of products are made in one factory, is to express loads and capacity in terms of dollars of product made and sold.

Ordinarily two inputs of costs are recognized. One of these is production cost and the other is distribution cost. Output is expressed as the amount of products sold or as income in terms of dollars.

Relationship of Capacity, Loading, and Cost of Electric Services

In the regulation of public utilities, the commission or court having jurisdiction may approve an evaluation of properties on which the utility is permitted to make a reasonable earning. The approved evaluation is known as a "rate base." If, for example, a rate base of $10,000,000 and a rate of return of 8 per cent have been allowed, a utility may establish rate schedules that will result in earnings of $\$10,000,000 \times .08 = \$800,000$ per annum.

Utilities ordinarily attempt to set up rate schedules that conform more or less closely to the cost of providing the service. A method for doing this will be illustrated by an example.

In this example all costs of providing electric service for one year will be considered to be classifiable on a simplified basis as follows:

1. *Capacity costs.* Costs under this heading consist of interest, depreciation, taxes, insurance, labor, and supplies necessary to maintain generating and distribution facilities in operable conditions. Capacity costs may be considered to be fixed for a given system under consideration.

2. *Energy costs.* Costs under this heading consist of items such as fuel, labor, and supplies which are incurred in the generation of current. Total energy costs are variable costs and vary in relation

to the amount of energy sold. They also represent the increment cost of energy generated and distributed.

3. *Customer cost.* Costs under this heading consist of such items as the metering of current, the rendering of customer service, and the billing and collection of service charges. Total customer costs vary in proportion to the number of customers served.

For the year and the utility under consideration in this example costs were as follows:

Capacity cost (max. demand of 36,300 kw). .	$1,120,000	or $30.85/kw
Energy cost (98,000,000 kw-hr.).	1,215,000	or $.0124/kw-hr.
Customer cost (48,400 customers)	390,000	or $8.05 each
Total	$2,725,000	

The plant has three classes of customers, which are designated Group D, Group E, and Group F. The cost of providing each of these groups of customers will be calculated on the basis of a rate equation first proposed by Henry L. Doherty in 1900 as follows:

$$\text{Cost of Service} = Ax + By + C$$

where

Ax = demand charge for the period under consideration.
By = energy charge for the period under consideration.
C = customer cost for the period under consideration (the aggregate for each customer group in this example).
A = demand rate in dollars per kw or other stated units.
x = demand during period under consideration.
B = energy rate in dollars per kw-hr. or other stated units.
y = number of kw-hrs. consumed during a period under consideration.

The demand A in this example will be determined by the *peak-responsibility method.* In accordance with this method, the demand of each of the three classes of customers will be taken to be that of each class at the time that the maximum demand on the system occurs. This corresponds to the ordinates of Curves A, B, and C in Fig. 48 at the point where Curve (A + B + C) is a maximum.

This method assumes that capacity costs are chargeable to customer groups in proportion to the amount each contributes to the need for plant capacity. During the year under consideration the maximum demand was 36,300 kw.

The load on the utility is summarized as follows:

Customer Group	*Demand in kw*	*Energy Consumed in kw-hr.*	*Number of Customers*
Group D	11,500	48,000,000	2,200
Group E	15,400	18,000,000	8,800
Group F	9,400	32,000,000	37,400
Total	36,300	98,000,000	48,400

On the basis of the unit costs previously calculated, the annual cost of serving Group D is as follows:

$$\text{Group D Cost} = \$30.85 \times 11{,}500 + \$.0124 \times 48{,}000{,}000 + \$8.05 \times 2{,}200$$
$$= \$967{,}700$$

A summary of the costs of serving each group follows:

Customer Group	*Demand Cost*	*Energy Cost*	*Customer Cost*	*Total Cost*	*Avg. Cost Per kw-hr.*
Group D	$355,000	$595,000	$17,700	$967,700	$.0202
Group E	475,000	223,000	70,900	768,900	.0427
Group F	290,000	397,000	301,400	988,400	.0309
Total	$1,120,000	$1,215,000	$390,000	$2,725,000	$.0278

For greater refinement in determining costs, allocation is often based on a greater number of customer groups or on a division of the customer groups as given into sub-groups.

Power Factor

The term *power factor* applies to alternating current and may be defined as the ratio of the output of power in watts to the product of volts and amperes. The maximum attainable value of the power factor is unity. Resistance losses in an electric circuit for a given power output are inversely proportional to the square of the power factor.

The power factor of power supplied to a customer depends upon the characteristics of the equipment in which the power is consumed. The power factor may be improved by the customer in various ways. Since low power factors tend to increase a utility's capacity costs and power losses, the power factor is considered in rate schedules.

Rate Schedules for Electric Service

A rate schedule based on the Doherty rate equation will have three parts and will be designated as a three-part rate. In practice, customer charges may be merged with capacity charges to produce a simpler, two-part rate. Also demand or energy charges or both may be quoted on the basis of increments.

For even greater simplicity, both customer and capacity charges may be merged with energy charges to produce a one-part rate.

Two-part schedule. The following is an example of a two-part schedule applicable to large industrial users.

a. *Rate.*

Primary Charge

First 100 kw of billing demand $1.80 per kw per month
Next 400 kw of billing demand $1.40 per kw per month
Next 500 kw of billing demand $1.20 per kw per month
Excess kw of billing demand $1.10 per kw per month

Secondary Charge

First 200,000 kw-hr. per month @ 0.7¢ per kw-hr.
Next 800,000 kw-hr. per month @ 0.55¢ per kw-hr.
Excess kw-hr. per month @ 0.4¢ per kw-hr.

b. *Minimum bill.* The minimum monthly bill shall be the amount computed under the above schedule based on 250 hours' use of the billing demand, but in no case less than 25,000 kilowatt-hours.

c. *Determination of maximum demand.* The customer's maximum demand shall be the maximum rate at which energy is used for any period of 30 consecutive minutes of the month for which the bill is rendered as shown by the company's maximum-demand meter, but shall not be considered as less than 100 kilowatts.

d. *Power factor clause.* The standard power factor upon which this rate is based is a monthly average power factor of 80 per cent.

When the average monthly power factor is more than 75 per cent and less than 85 per cent, no power factor adjustment in billing demand shall be made. Whenever the average monthly power factor is 75 per cent or less or 85 per cent or more, the monthly billed demand shall equal the measured maximum demand multiplied by 80 and divided by the monthly average power factor expressed in per cent.

One-part schedule. The following is an example of a one-part schedule applicable to residential users.

a. *Rate.* Where connected load in motors with individual capacities of 1 horsepower or over does not exceed 3 horsepower:

First 18 kw-hr. or less per month for $1.00
Next 22 kw-hr. per month @ 3.5¢ per kw-hr.
Next 100 kw-hr. per month @ 3.0¢ per kw-hr.
Excess kw-hr. per month @ 2.0¢ per kw-hr.

Relationship of Production and Distribution Costs, Income, and Profit of Industrial Enterprise

There are two aspects of an industrial enterprise. One consists of assembling labor, facilities, and material for the production of goods or services. The other consists of the distribution of the goods or services that have been produced. The success of an enterprise depends upon its ability to carry on these activities to the end that there may be a net difference between receipts for goods and services sold and the input necessary to produce and distribute them. In private enterprise the usual aim is to maximize net revenue or profit. Profit is a resultant of three factors and may be expressed as follows:

$$\text{Profit} = \text{Receipts for goods and services} - (\text{Cost of production} + \text{Cost of distribution})$$

For simplicity of discussion in this chapter, cost of distribution will be considered to be a summation of an enterprise's expenditures to influence the sale of its products and services. Such items as advertising, sales administration, salesmen's salaries, and expenditures for packaging and decoration of products done primarily for sales appeal will be included in cost of distribution.

Cost of production will be considered to be the summation of all costs of the enterprise not included in cost of distribution. Cost or production will embrace expenditure for labor, materials, power, equipment, insurance, taxes, interest, and the delivery of the product to the customer.

Expenditures for some items may be of the nature of both distribution and production cost. For example, cost of decorating a package is related to distribution and the basic package is related to production.

In this chapter attention will be directed to the consideration of some aspects and relationships of markets, production, distribution, and profit.

Distribution Is of Concern to the Engineer

The engineer is prone to consider production as his field and leave distribution to others. But both distribution and production are of concern to the engineer. For example, there is not much point in designing an engine that is highly efficient and producing it at a low cost unless it is sold and used. There are few products with which engineers are concerned that "sell themselves." Ordinarily, much attention must be given to incorporate sales appeal into products and to accompany them to market with considerable sales effort, if a profit is to be realized. Rightly or wrongly, the profit derived from some products depends more upon the effectiveness of distribution than upon effectiveness of design and production. It is significant that the cost of selling many articles of commerce, particularly those sold to the ultimate consumer, often outweighs the cost of production. Of each dollar spent by the consumer, it is estimated that \$.41 represents production costs and \$.59 represents distribution costs.[1]

Because of the important effects of distribution on the profits of a firm, the activity of distribution concerns the engineer whether or not he directly engages in the activity. It should be realized that distribution is a primary function, for there is no point to production unless the product can be distributed and put to use.

The Nature of Distribution

A buyer will purchase an article when he has money available and when he believes that the article has equal or greater utility for him than the amount required to purchase it. Conversely, a seller will sell an article he has when he believes that the amount of money to be received for the article has greater utility than the article has for him. Thus, an exchange will not be effected unless at the time of exchange both parties believe that they will benefit. Exchanges are made when they are thought to result in mutual benefit. This is

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[1] *Does Distribution Cost Too Much?* Twentieth Century Fund, New York, 1939.

possible because the objects of exchange are not valued equally by the parties to the exchange.

Suppose that A has a house he values at $10,000 and that B has $14,000, which he values equal to A's house. Then A may be expected to sell his house if he can receive $10,000 or more for it and B may be expected to buy the house if he can buy it for $14,000 or less. This situation may be represented as in Fig. 49.

The price at which exchange in the above situation will take place is dependent upon many factors. For instance, if prices have risen very rapidly and A is ignorant of the current values of property and timid, he may have found it difficult to name a price as high as $10,000. If this amount is offered to him, he is likely to accept.

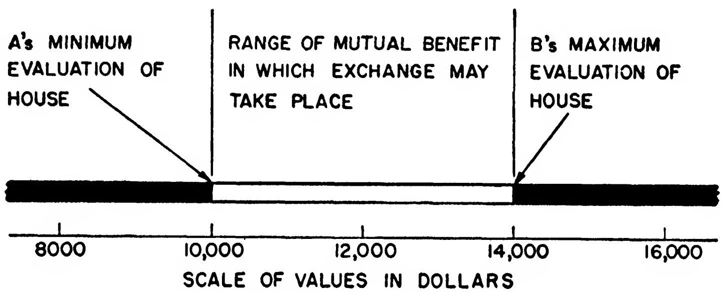

FIG. 49. Illustration of the range of mutual benefit in exchange.

On the other hand, if B is very anxious to get the house, he may "talk when he should be listening" and offer $14,000 and the exchange will take place at this amount.

The factors that may determine a price within the range of mutual benefit at which exchange will take place are infinite in variety. They may be either subjective or objective. A person seeking to sell may be expected to make two evaluations: the minimum amount he will accept and the maximum amount a prospective buyer can be induced to pay by persuasion. The latter estimate may be based upon mere conjecture or upon a detailed analysis of buyers' subjective and objective situations. In bargaining it is usually advantageous to obscure one's situation. Thus, sellers will ordinarily refrain from revealing the costs of the things they are seeking to sell or from referring a buyer to a competitor who is willing to sell at a lower price.

When fixed prices are adhered to by sellers or buyers, bargaining is limited to acceptance or rejection of the fixed price.

The Contribution of Sales Effort

The contribution of sales effort to the value of products to consumers is related to time and place utilities. The time and place at which products are available may greatly affect the value or utility they have for a consumer.

Consider the following example in illustration of time utility. A manufacturing concern has the opportunity to buy electric current at the rate of \$.015 per kw-hr during the period 8 A.M. to 5 P.M. and at the rate of \$.009 per kw-hr. during the period 12:01 A.M. to 8 A.M. The fact that the firm continues to buy current at the higher price is evidence that it considers current delivered during the daytime period to have a time utility of at least \$.006 per kw-hr. in excess of that delivered in the early morning hours.

An illustration of place utility is the case of a person who is willing to exchange a house in one town for a house of lesser value in another town, when the latter town has compensating advantages in location.

Demand, Supply, and Price

In general, demand (*i.e.* the number of buyers) for a particular article will increase as prices decrease. When a decrease in price will result in an increased demand, demand is said to be *elastic.* The elasticity of demand may range from *perfectly elastic* to *perfectly inelastic.* When demand is perfectly elastic, an infinitesimal decrease in price will result in an infinite increase in demand. When demand is perfectly inelastic, it will remain constant regardless of price changes.

Ordinarily stable commodities such as wheat, sugar, coal, and crude oil are quite elastic in demand. In other words, a small decrease in price will result in a great increase in the number of units of these items purchased. The demand for such items as punch presses, lathes, typewriters, shoes, bicycles, neckties, false teeth, and toupées may be expected to be relatively inelastic. A marked decrease in price of these items will result in only a slight increase in the number sold.

When an increase in price will call forth a greater number of offers to sell, the supply is said to be elastic. When a supply is perfectly elastic, an infinitesimal increase in price will call forth an infinitely greater supply. When perfectly inelastic, supply will remain constant regardless of price changes.

Mathematical expression of elasticity of demand and supply. Elasticity of demand may be expressed mathematically as follows. Let

E_D = elasticity of demand.
P_1 = a unit price corresponding to a demand equal to D_1 units of product (see Fig. 50).
P_2 = a unit price corresponding to a demand equal to D_2 units of product.

Then
$$E_D \times \frac{P_1 - P_2}{P_2} = \frac{D_1 - D_2}{D_1}$$

and
$$E_D = \frac{(D_1 - D_2)P_2}{(P_1 - P_2)D_1}$$

When $E_D = -\infty$, demand is said to be perfectly elastic. When $E_D = 0$, demand is said to be perfectly inelastic.

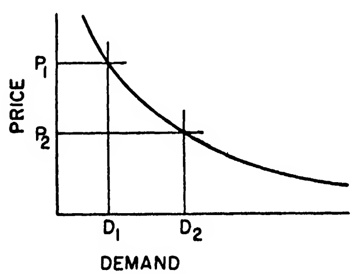

FIG. 50. Elasticity of demand.

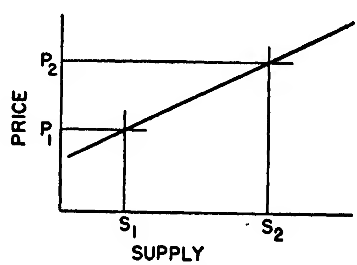

FIG. 51. Elasticity of supply.

Elasticity of supply may be expressed mathematically as follows. Let

E_S = elasticity of supply.
P_1 = a unit price corresponding to a demand equal to S_1 units of product (see Fig. 51).
P_2 = a unit price corresponding to a demand equal to S_2 units of product.

Then
$$E_S \times \frac{P_1 - P_2}{P_2} = \frac{S_1 - S_2}{S_1}$$

and
$$E_S = \frac{(S_1 - S_2)P_2}{(P_1 - P_2)S_1}$$

When $E_S = \infty$, supply is said to be perfectly elastic. When $E_S = 0$, supply is said to be perfectly inelastic.

Price Is Determined by Supply and Demand

In free enterprise, prices of goods and services are ultimately determined by supply and demand. Where there is elasticity in either supply or demand of a commodity or both, its price-demand curve

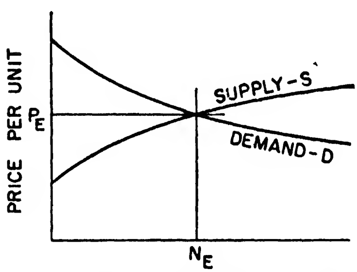

FIG. 52. Relationship of supply, demand, price, and the number of units exchanged.

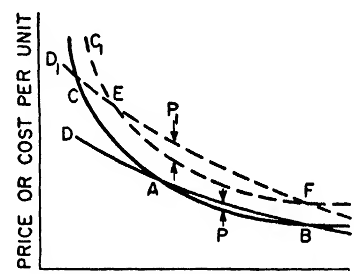

FIG. 53. Effect of sales effort on demand, cost of production, and profit.

and its price-supply curve must of necessity intersect. The intersection of these curves determines the price and the quantity exchanged. The quantity exchanged is equal to both quantity of supply and quantity of demand. Figure 52 represents a general price, supply, and demand relationship.

In this figure the price at which exchange will take place is P_E and the number that will be exchanged is N_E as determined by the intersection of the price-demand curve D and the price-supply curve S.

More pertinent to the management of a firm is the price-supply-demand relationships that apply to it. Unless a firm is a dominant factor with respect to either the total supply or the total demand for a certain commodity, it is subject to the prices determined by supply and demand in general. Thus it must purchase its raw materials at the market and sell its product at the market, except as it can bring influence to bear which will permit it to do otherwise.

If a firm is to realize a profit, it must sell its output for more than it costs to produce and distribute it. In doing this, a firm is confronted with a situation like that shown in Fig. 53. In this figure, D represents a price-demand curve for the firm's output of a single product when no sales effort is applied, and C represents the firm's cost-output curve for the same product. Outputs between those corresponding to the intersections of the two curves at A and B can be sold at a profit. Profit per unit is represented by P.

If the cost-output curve C for the product had failed to intersect the price-demand curve D, there would be no level of output at which the firm could operate at a profit. The firm might remedy the situation in several ways. One way is to lower the cost of production of the product by securing needed raw materials and facilities at lower prices and by improving the methods of operation. Ordinarily most cost reduction will have to be brought about by improving methods of application of materials and facilities rather than by acquiring materials at a lower cost. This is so because the firm is subject to general price-supply curves over which it has very little control.

A second way to realize a profit in the situation under consideration is to apply sales effort in the hope of developing a more favorable price-demand curve. The more favorable curve sought is a new curve applying to a new product. The new product is the old product plus persuasion. The cost of the new product is the old cost of production plus the new added sales promotion cost. The situation is represented in Fig. 53 by the dotted price-demand curve D_1 and the dotted cost-output curve C_1. Profit per unit under the new situation is indicated by P_1. It should not be inferred that the addition of sales effort will always result in increasing unit profit. It is clear from the figure that sales effort must result in a greater increase in the unit sale price than in the unit cost of production to be efficacious.

A third way in which a firm operating at a loss may realize a profit is to abandon a present product in favor of one for which a more favorable price-demand and cost-of-production relationship exists.

Flexibility of facilities (as contrasted to specialization), which will enable changes of product to take advantage of shifting economic conditions, is often a more important factor in the earning of profit than high production efficiency in the manufacture of specialized products.

Sales Effort, Profit, and Demand

Ordinarily a more favorable price-demand curve may be obtained for a product by applying sales effort in its distribution. Sales effort is applied primarily for the benefit of the vendor, but the consumer may also be benefited.

Consumers are primarily benefited by the informative aspects of sales promotion. Learning of a product not previously known to exist may prove to be of distinct advantage to a consumer. Likewise information relative to the specifications, characteristics, and application of a product may enable the consumer to make a wiser choice or more efficient use of products. A product plus such information undoubtedly has greater value to the consumer and warrants a higher price than the product alone. As a result, many technically trained people are employed to provide information on the application of products as a means of enhancing their value to the consumer. But information must be understood and acted upon to be efficacious. A vendor who receives an inadequate response from his efforts to bring knowledge of his products to customers often finds it advantageous to shift the object of his appeals from the customer's intelligence to the customer's gullibility and his method from facts to propaganda.

Some noninformative aspects of sales promotion may increase the value of a product for a consumer. For example, a consumer may feel that his prestige is enhanced by the use of a product endowed by advertising with an aura of distinction. Similarly, sales effort may bolster a consumer's confidence and induce him to feel more secure while using the product or service. The value of feeling secure is often the most important value that a product or service can have for a consumer and should not be minimized.

The purpose of much sales effort is to gain customer preference for a product over that of an equal or superior product of a competitor. There appears to be little if any advantage to the consumer from expenditure for this purpose, but its cost must ultimately be borne by him. When the aim of sales promotion is to gain preference for a certain product over similar products of competitors, a technique called differentiation is often employed.

In differentiation, a product that is similar to a number of competitive products is made to appear different or distinctive. This may be done by giving it a name or an unusual finish, by incorporating an identifying symbol in the product, or by other inconsequential variations readily apparent and distinguishable by a prospective customer. Products of competitors tend to become similar by incorporation of features most favored by consumers. Differentiation of product is practiced to confine the effect of sales effort on demand to the product of the particular firm employing it.

One form of sales effort is that of industry sales promotion. The cost of this is borne by trade associations representative of particular industries. Its aim is to increase the demand for certain classes of products or services. Such efforts are often competitive, as is indicated by advertisements aiming to encourage shipment by air and by rail, for example, or to build with wood or with concrete.

From the standpoint of an individual firm, its sales effort is justified when it will create a greater demand for its product, which in turn will result in greater profits to be realized. Greater profits may result from the sale of a product at a higher price or in greater volume or both. Often a greater volume may make possible a reduction in the price of a product per unit, while at the same time increasing the firm's total profit. This may happen because the advantages of mass production come into play.

But whatever the objectives of sales efforts, the methods by which they are conducted are a matter of concern. The measure of efficiency of sales efforts is the same as that of production methods, namely, output over input.

Competition

Competition is a relative condition. Perfect competition, which probably never exists, may be defined as a situation in which (1) an identical product is (2) available from a great many vendors in (3) a market placing no restriction upon the entry of new vendors. The essential feature of perfect competition is that the supply and demand of a product in question cannot be influenced significantly by any action a vendor can take.

Monopoly

A perfect monopoly exists when (1) a unique product is (2) available from a single vendor and (3) entry of all other possible vendors is prevented. A firm having a monopoly on a product can control the supply of it that is offered for sale. It can also indirectly control demand for its product through the price it names and the sales effort it applies. The advantage of monopoly to a firm lies in the fact that it can select the combination of supply, price, and promotional effort that will yield the greatest profit without interference of competitors. Monopoly prices are not necessarily higher than competitive prices; they can be and often are lower.

The government recognizes the possibility for lower prices through monopoly when it grants franchises for the operation of monopolies, as it does with such public utilities as railroad, power and light service, telephone and telegraph, and water companies. In these cases monopoly is granted on the premise that service can and will be rendered to consumers at less cost and that prices will be determined by regulatory bodies in place of competition.

Monopoly is so advantageous from the standpoint of profit that it is a common practice to direct effort actively toward attaining this end. Sometimes effort is directed to reducing the number of vendors by merger or pressure and by preventing entry of new vendors. Also the effect of monopoly may in some measure be obtained by acting in concert on such matters as the number of units each firm is to make and the price at which each unit is to be sold. Such restrictive efforts are not always successful and the benefits to be obtained from them are frequently not equal to their cost.

The most common method of establishing a degree of monopoly is by offering a *unique product*. Such a product can be obtained by secret formulas, patents, or unusual skills, but it is most often created by promotional effort directed at the differentiation of a product that is essentially identical to those offered by competitors.

Probably the simplest method for giving uniqueness to a product is to announce that it is being sold at the "friendly store with the purple front" or something of the sort. Promotional effort that creates "uniqueness" of product by virtue of the place where it is offered for sale often enables the product to command a higher price than similar products can be sold for next door.

Monopoly through uniqueness of product can be created to such an extent that its vendor becomes independent of the general competitive situation applying to similar products. It should be recognized that creation of monopoly through uniqueness is profitable only to the extent that benefits outweigh costs.

Oligopoly

An oligopoly exists when there are so few suppliers that the action of one vendor may affect the course to be followed by other vendors. Thus, in considering a contemplated course of action, a vendor in an oligopoly must consider not only what effect it will have on his situation but also what action it may cause other vendors to take.

Suppose there are three motion picture theaters in a town, none of which have air conditioning. If the manager of one is considering installation of an air conditioning system, he must consider the consequences of his action; it may result in the installation of air conditioning systems in one or both of the other theaters. Firms that are continually confronted with evaluating their actions in terms of their effect on consequent action of others in an oligopoly may be expected to develop viewpoints and adopt policies similar in effect to those of a monopoly.

Illustrations of Relationship of Production and Selling Costs, Income, and Profit

The consideration previously given to the pattern of production costs in Chapter 11 should be reviewed at this point. The relationship of production volumes, production costs, and production cost patterns to income, distribution costs, distribution cost patterns, and profit will be discussed and illustrated in succeeding paragraphs. For simplicity, illustrations and discussion will be based on examples in which a firm produces only a single product. The rate of production will be taken in terms of the number of units produced per year.

In Table 36 cost data are given relative to a hypothetical plant which has a maximum capacity of 10 units of product per year and rates of production ranging from 0 to 10 units per year. These data and those presented in Tables 37 and 38 will be used as a framework for an analysis of relationships between production costs, distribution cost, income, and profit. The pattern of the data presented is typical of many firms except that it has been chosen to accentuate the factors to be discussed.

TABLE 36. Relationship of Cost of Production, Net Income from Sales, Profit, and Number of Units Made and Sold Per Year

(1) Annual Output, Number of Units	(2) Annual Total Production Cost A, Fig. 54	(3) Annual Fixed Production Cost B, Fig. 54	(4) Annual Variable Production Cost C, Fig. 54	(5) Average Production Cost Per Unit D, Fig. 54	(6) Average Fixed Cost Per Unit	(7) Average Variable Cost Per Unit E, Fig. 54	(8) Increment Production Cost Per Unit F, Fig. 54
0	$200	$200	$ 0	$ ∞	$ ∞	$ 0	
1	300	200	100	300.00	200.00	100.00	$100
2	381	200	181	190.50	100.00	90.50	81
3	450	200	250	150.00	66.67	83.30	69
4	511	200	311	127.75	50.00	77.75	61
5	568	200	368	113.60	40.00	73.60	57
6	623	200	423	103.83	33.33	70.50	55
7	679	200	479	97.00	28.57	68.43	56
8	740	200	540	92.50	25.00	67.50	61
9	814	200	614	90.44	22.22	68.22	74
10	924	200	724	92.40	20.00	72.40	110

The annual total production cost of the hypothetical plant as its rate of production varies from 0 to 10 units per year is given in Column (2) of Table 36. It should be specifically noted that no distribution costs are embraced in these data.

Annual fixed cost is given as $200 per year in Column (3). The annual fixed cost in this example should be considered to be the annual cost of maintaining the plant whose capacity is 10 units per year in operating condition at a production rate of 0 units per year. The fixed cost of a plant is analogous to the standby costs of a steam boiler whose fire is banked but which is ready to furnish steam on short notice.

The difference between the annual total production cost in Column (2) and the fixed production cost in Column (3) constitutes the annual variable cost and appears in Column (4). Average unit production, fixed and variable costs appear in Columns (5), (6), and (7) respectively.

The increment production costs per unit given in Column (8) were obtained by dividing the differences between successive values of annual total production cost as given in Column (2) by the corresponding difference between successive values of annual output given in Column (1). In this case the divisor will be equal to unity but it should be understood that an increment of production of several units may be convenient in some analyses.

The values in all columns of Table 36 except those in Column (6) and the values of all columns of Tables 37 and 38 have been plotted with respect to the annual output in number of units in Figs. 54, 55, and 56, and have been keyed for easy identification.

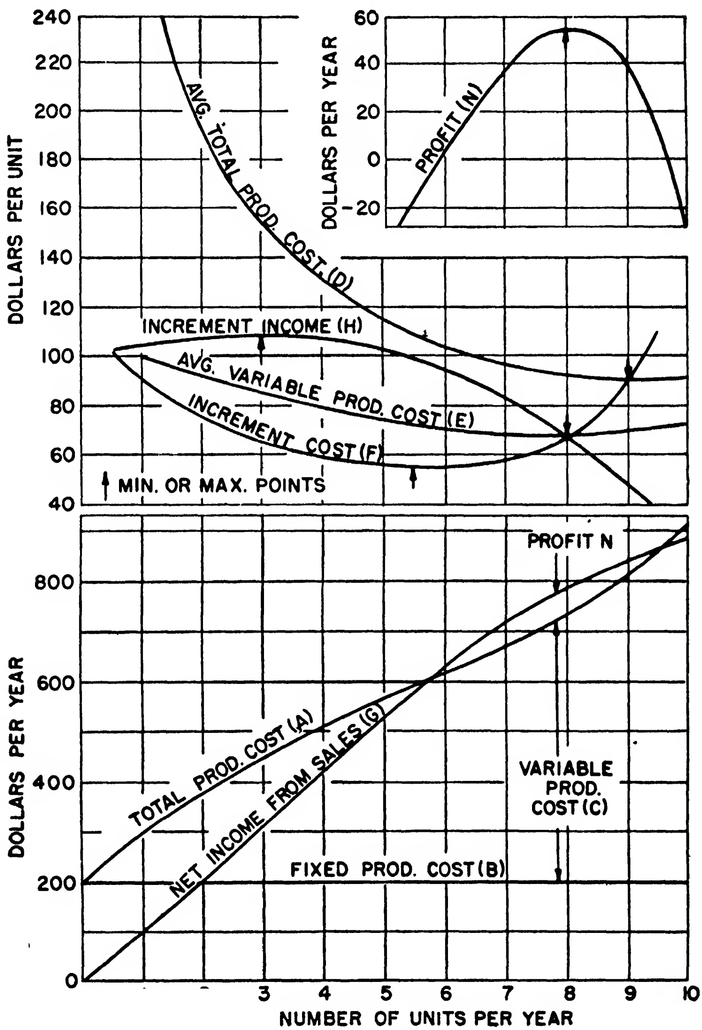

FIG. 54. Relationship of cost of production, net income from sales, profit, and number of units made and sold per year.

Consider Curve (A) in Fig. 54. Total production cost curves of actual business take a great variety of forms. Ordinarily, however, a producing unit will produce at minimum average cost at a rate of production between zero output and its maximum rate of output. Thus, the average unit cost of production will decrease with an increase in the rate of production from zero until a minimum average cost of production is reached and then the average cost per unit will rise.

The average total production cost is given by Curve (D) in Fig. 54. It will be noted that it reaches a minimum at a rate of 9 units per year. The average variable production cost is given in Curve (E); its minimum occurs at 8 units per year. Curves (D) and (E) should be considered in relation to Column (6) of Table 36. It should be noted that fixed cost per unit is inversely proportional to the number of units per year. Reduction of fixed cost per unit is an important factor tending toward lower average cost with increases in rates of production.

Increment production costs are given in Curve (F); their minimum value is reached at 6 units per year. The form of this curve shows that the increment cost of production per unit declines until 6 units per year are produced and then rises.

An interesting fact to observe is that the increment cost Curve (F) intersects the average total production cost Curve (D) and the average variable production cost Curve (E) at their minimum points.

The increment cost Curve (F) is a measure of the slope of Curve (A). It may be noted that the slope of Curve (A) decreases until 6 units per year is reached and then increases. The same could have been said of a variable production cost Curve (C) if one had been plotted.

Further consideration will be given to the curves of Fig. 54 after income from sales of product and the cost of distribution have been explained.

Pattern of income and cost of distribution. When demand is not perfectly elastic or competition is imperfect, or when both of these conditions exist, it is often profitable to increase the quantity of products that can be sold or the price at which a given quantity of products can be sold through the application of sales effort. Sales effort may embrace decorative packaging, advertising of various forms, and personal persuasion in the form of instruction, service, entertainment, and the like.

Gross annual income from sales is the total income received from customers as payment for products.

In Columns (2) and (3) of Table 37, gross annual income from sales

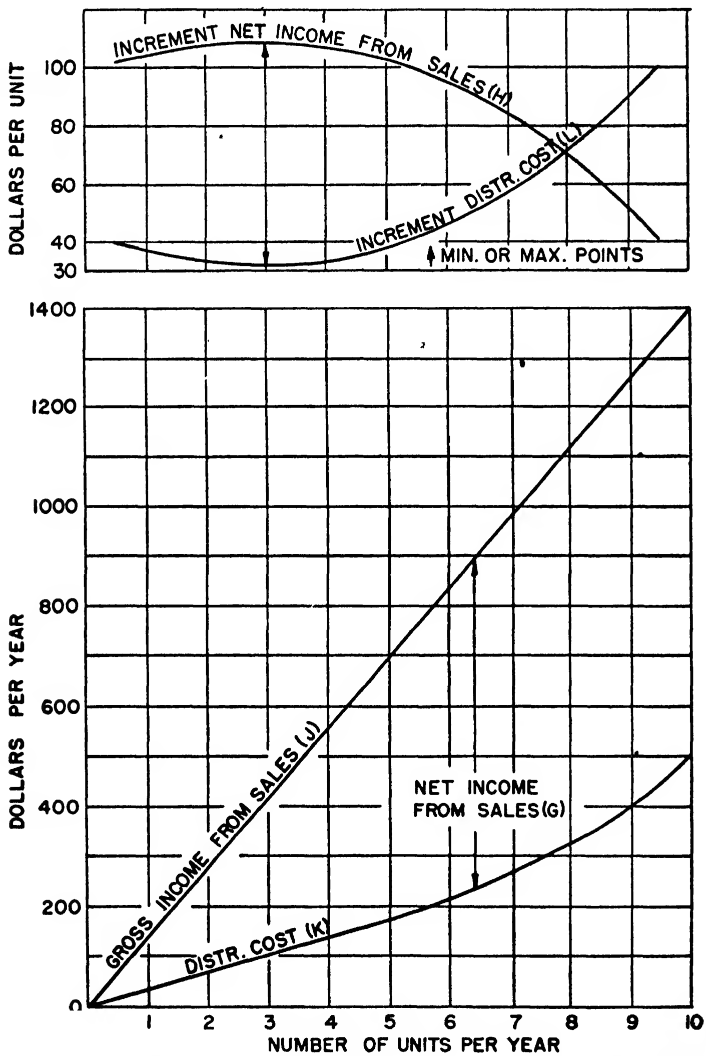

FIG. 55. Relationship of income from sales, net income from sales, distribution cost, and number of units sold per year.

and the annual cost of distribution are given for various rates of sale of product of the firm for which cost-of-production data were given in Table 36 and Fig. 54.

The difference between the gross annual income from sales and the annual cost of distribution is equal to the annual net income from sales given in Column (4). Increment distribution costs per unit are given in Column (5) and increment net income from sales per unit appears in Column (6). The values of each of these columns have been plotted with respect to annual output in Fig. 55.

The gross annual income from sales, Curve (J) in Fig. 55, is a straight line and is typical of situations in which products are sold at a fixed price. The annual increment distribution cost, Curve (L), first falls slightly until a minimum is reached and then rises. This is typical of situations in which sales effort is relatively inefficient at low levels and where sales resistance increases with increased number of units sold.

TABLE 37. Relationship of Gross Income from Sales, Net Income from Sales, Distribution Cost, and Number of Units Sold Per Year

(1) Annual Output, Number of Units	(2) Annual Gross Income from Sales J, Fig. 55	(3) Annual Distribution Cost K, Fig. 55	(4) Annual Net Income from Sales G, Fig. 55	(5) Increment Distribution Cost Per Unit L, Fig. 55	(6) Increment Net Annual Income from Sales Per Unit H, Fig. 55
0	$ 0	$ 0	$ 0		
1	140	38	102	$ 38	$102
2	280	72	208	34	106
3	420	104	316	32	108
4	560	136	424	32	108
5	700	170	530	34	106
6	840	211	629	41	99
7	980	261	719	50	90
8	1,120	325	795	64	76
9	1,260	405	855	80	60
10	1,400	505	895	100	40

A pattern of distribution cost, made up of fixed and variable costs similar to those of production, might have been taken. Ordinarily, fixed distribution costs are a minor consideration; they have been omitted from this analysis in the interest of simplicity.

A representation of the net income from sales is given as (G) in

Fig. 55. The positive difference between net income from sales and the total cost of production, Curve (A) in Fig. 54, represents profit. Profit is shown as (N) in Fig. 54; it should be noted that the maximum profit, or minimum loss if no profit is made, occurs at a rate of produc-

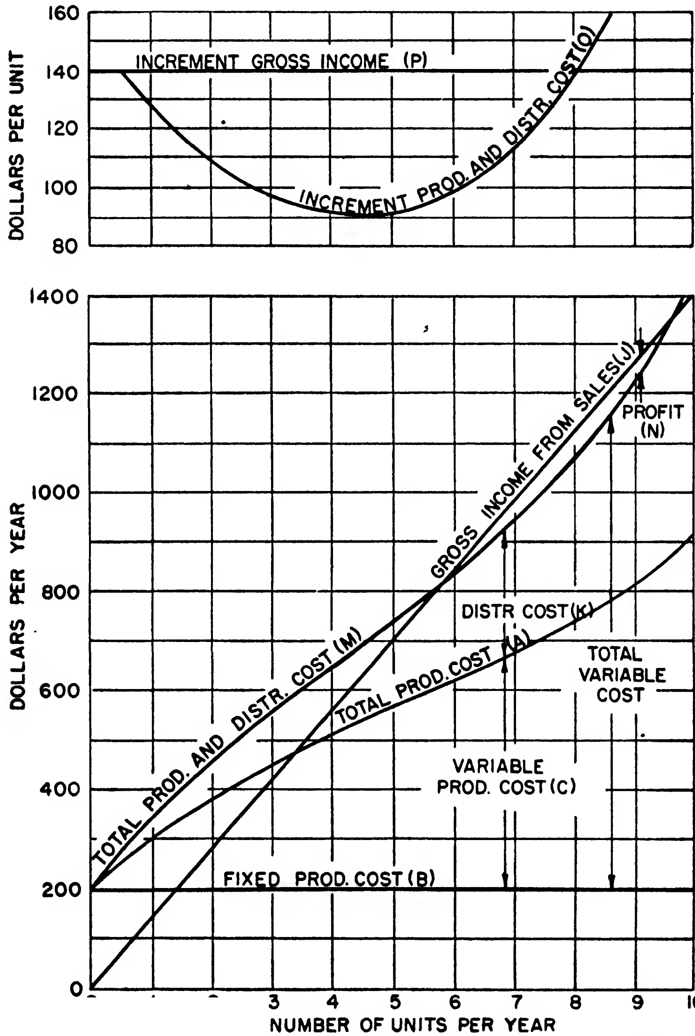

Fig. 56. Relationship of gross income, total production and distribution cost, profit, and number of units per year.

tion at which the increment income curve (H) intersects the increment cost Curve (F), or at 8 units per year. From Column (8), Table 36, and Column (6), Table 37, it may be observed that the increment production cost and increment net income for a ninth unit are $74 and $60 respectively. Thus, a loss of $14 profit would be sustained if the activity were increased to 9 units. When the profit motive governs, there is no point in producing beyond the point where the increment cost of the next unit will exceed the increment income from it.

If the sales effort in the above example is resulting in sales of 9 or 10 units per year, a number of steps might be taken. The sales effort could be reduced, causing sales to drop. The price could be increased, causing sales to decrease and the income per unit to increase. The plant could be expanded or other changes could be made to alter the pattern of production costs. Consideration of any of the above steps would require a new analysis embracing the altered factors.

Consolidation of production and distribution cost. It is common practice to consolidate production and distribution cost for analysis of operations. To illustrate this practice an analysis of the above example will be made in this manner. The method is illustrated in Table 38, whose values have been plotted as several curves in Fig. 56.

TABLE 38. Relationship of Gross Income, Production Cost, Distribution Cost, Profit, and Number of Units Made and Sold Per Year

Annual Output, Number of Units	Annual Total Production Cost A, Fig. 56	Annual Distribution Cost K, Fig. 56	Annual Total of Production and Distribution Cost M, Fig. 56	Annual Gross Income from Sales J, Fig. 56	Annual Profit N, Figs. 54 and 56	Increment Total of Production and Distribution Cost O, Fig. 56
0	$200	$ 0	$ 200	$ 0	− $200	$138
1	300	38	338	140	− 198	115
2	381	72	453	280	− 173	101
3	450	104	554	420	− 134	93
4	511	136	647	560	− 87	91
5	568	170	738	700	− 38	96
6	623	211	834	840	6	106
7	679	261	940	980	40	125
8	740	325	1,065	1,120	55	154
9	814	405	1,219	1,260	41	210
10	924	505	1,429	1,400	− 29	

Profit in this case will be equal to the difference between gross income (J) and total production and distribution cost (M). The point of maximum profit will occur when the increment gross income Curve (P) is intersected by a rising increment production and distribution cost Curve (O). This occurs at a rate of 8 units per year. It should be noted that if production reaches 10 units, a loss of $29 will be sustained from the year's operation.

Break-Even Chart Analysis

If a firm's income and variable costs are assumed to be proportional to the quantity of products made and sold, analysis of their relationships to profit is greatly simplified. If this is done, the patterns of income and costs of Fig. 56, for example, can be replaced by a diagram such as that shown in Fig. 57.

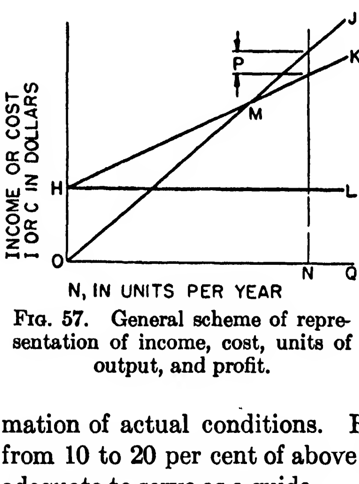

Fig. 57. General scheme of representation of income, cost, units of output, and profit.

In this figure the maximum capacity of a plant is represented by the abscissa *OQ*. Fixed production costs are represented by the ordinate *OH* and the line *HL*. The sum of variable production and distribution costs is represented by the line *HK*, and income from sales is represented by the line *OJ*. It should be recognized that this representation is only an approximation of actual conditions. However, over a range of output of from 10 to 20 per cent of above and below normal output, results are adequate to serve as a guide.

Accuracy over the range to be considered can be improved by having the line *HK* representing variable cost tangent to an actual variable cost curve at the mid-point of the range of output to be considered. The intersection of the tangent with *OH* will thus represent the fixed cost to be used. This refinement will not be used further in this discussion because it is not generally used.

Analyses similar to that shown in Fig. 57 may be made mathematically or by diagrams drawn accurately to a large scale. The mathematical method is relatively simple and more convenient to use when this type of analysis is needed frequently.

In Fig. 57 let

N = number of units of product made and sold per year.
R = the amount received per unit of product in dollars. R is equal to the slope of OJ.
I = RN, the annual income from sales in dollars. $I = RN$ is the equation of line OJ.
F = fixed cost in dollars per year, represented by OH and HL.
V = variable cost per unit of product. V is equal to the slope of HK.
C = the sum of fixed and variable cost of N units of product, $F + VN$. $C = F + VN$ is the equation of line HK.
P = annual profit in dollars per year. $P = I - C$. Negative values of P represent loss.
M = break-even point. At this point $P = 0$.
Q = capacity of plant in units per year.

Note. Where a firm produces a variety of items, it may be convenient to let N represent per cent capacity, R the amount received per per cent of output, and V the variable cost per per cent of output.

Calculation of "break-even" point. Income will be equal to costs at the point of intersection of OJ and HK. At this point $I = C$ and $RN = F + VN$. Solving for N,

$$N = \frac{F}{R - V}$$

Thus the abscissa is equal to $F/(R - V)$.

If $F/(R - V)$ is substituted for N in $I = RN$ or $C = F + VN$, the ordinate of the break-even point may be found. The value of the ordinate in terms of dollars of income or cost will be

$$I = R\frac{F}{R - V} \qquad \text{and} \qquad C = F + \frac{VF}{R - V}$$

EXAMPLE. $R = \$11$, $F = \$4{,}000$, and $V = \$5$; find the break-even point.

$$N = \frac{F}{R - V}$$

$$N = \frac{\$4{,}000}{\$11 - \$5}$$

$$N = 667 \text{ units per year}$$

and

$$I = C = RN$$
$$I = C = \$11 \times 667 = \$7{,}337$$

Calculation for P for any value of N:

$$P = I - C$$
$$= RN - (F + VN)$$
$$= (R - V)N - F$$

EXAMPLE. $I = \$11$, $F = \$4{,}000$, $V = \$5$, and $N = 800$; find the annual profit P.

$$P = (R - V)N - F$$
$$= (\$11 - \$5)800 - \$4{,}000$$
$$= \$6 \times 800 - \$4{,}000$$
$$= \$800$$

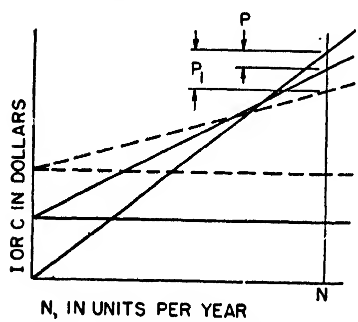

FIG. 58. Effect on profit of an increase in fixed cost that results in reduced variable cost.

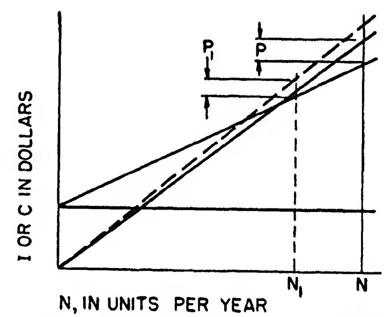

FIG. 59. Effect on profit of a proposed change in price and volume.

Figures 58, 59, and 60 show applications of the break-even chart for evaluating the effect of proposals. Proposals are indicated with dotted lines and corresponding symbols with primes. These applications are obvious from the figures except for the application in Figure 60, which will be explained in the next section.

Differentiated market or dumping. Under some circumstances it is possible for a firm to sell essentially similar products at different prices, without any relation between the number sold at one price and the number sold at the other price. For instance, some of a firm's output of light bulbs might be sold in the United States at a price different from that at which they are sold in another country to which they have been exported. Markets may often be differentiated and in large measure kept independent by the simple expedient of selling the same product through different trade channels under different brand names. Suppose that a firm is making and selling N units of product per year at a loss, as illustrated in Fig. 60. If now the market can be differentiated, it may be possible to sell enough additional units at a lower price to make a profit. The practice outlined above is often referred to as *dumping*.

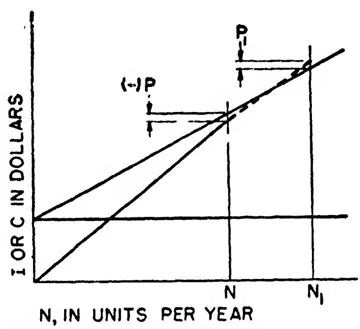

FIG. 60. Effect on profit of a proposed addition of a differentiated market.

EXAMPLE. R = \$11, F = \$4,000, V = \$5, N = 600, R_1 = \$8, and N_1 = 800. The loss sustained with sales of 600 at a price R of \$11 is equal to

$$\begin{aligned} P &= (600 \times \$11) - (\$4{,}000 + 600 \times \$5) \\ &= \$6{,}600 - \$7{,}000 \\ &= -\ \$400 \end{aligned}$$

If differentiation will permit the sale of an additional ($N_1 - N$) or 800 − 600 = 200 units at \$8 per unit, then the profit will be

$$\begin{aligned} P_1 &= (600 \times \$11 + 200 \times \$8) - (\$4{,}000 + 800 \times \$5) \\ &= (\$6{,}600 + \$1{,}600) - (\$4{,}000 + \$4{,}000) \\ &= \$8{,}200 - \$8{,}000 \\ &= \$200 \end{aligned}$$

Profit can be increased or losses decreased if an additional number of units can be sold at a price that exceeds the variable cost by any amount.

For some purposes it is convenient to have the equation of the new income line. If extended, the line will intersect the I-axis at $N(R - R_1)$ or, for the example above, 600 ($11 − $8) = $1,800. Its slope is $R_1 = 8$. The equation of the new income line is therefore

$$I = 8N + 1800$$

Effect of Price on Profit

Profits are the combined result of effort to produce and to distribute. The results and the costs of production and distribution activities must be considered in relation to their joint effect. Either one is meaningless when considered independently of the other. One aspect of this joint relationship will be illustrated by an example.

A firm had been marketing a specialized product for a number of years. On the basis of experience, sales research and estimate curves were drawn showing the relationship between price, cost of sales effort, and the number of units sold. Data taken and values calculated from these curves are given in Table 39.

TABLE 39. Relationship of Selling Price, Sales, Sales Effort, and Income

	Selling Price Per Unit								
	P_1 = $90			P_2 = $100			P_3 = $110		
N, Number of Units Sold	I, Income = $N \times P_1$	S, Sales Effort Cost	G, Net Income from Sales, $S = I - S$	I, Income = $N \times P_2$	S, Sales Effort Cost	G, Net Income from Sales, $S = I - S$	I, Income = $N \times P_3$	S, Sales Effort Cost	G, Net Income from Sales, $S = (I - S)$
10	$ 900	$ 40	$ 860	$ 1,000	$ 70	$ 930	$ 1,100	$ 90	$1,010
20	1,800	130	1,670	2,000	200	1,800	2,200	220	1,980
30	2,700	240	2,460	3,000	380	2,620	3,300	420	2,880
40	3,600	400	3,200	4,000	640	3,360	4,400	730	3,670
50	4,500	630	3,870	5,000	1,000	4,000	5,500	1,260	4,240
60	5,400	970	4,430	6,000	1,500	4,500	6,600	2,090	4,510
70	6,300	1,530	4,770	7,000	2,130	4,870	7,700	3,070	4,630
80	7,200	2,170	5,030	8,000	2,890	5,110	8,800	4,140	4,660
90	8,100	2,940	5,160	9,000	3,770	5,230	9,900	5,320	4,580
100	9,000	3,770	5,230	10,000	4,770	5,230	11,000	6,550	4,450
110	9,900	4,650	5,250	11,000	5,900	5,100	12,100	7,900	4,200
120	10,800	5,600	5,200	12,000	7,150	4,850	13,200	9,300	3,900

From this table curves representing the relationship of net income from sales, $I - S$, and the number of units sold, N, were superimposed on a cost-of-production chart, Fig. 61. The cost of production is composed of a fixed cost of $3,000 and a variable cost of $20 per unit. For the conditions given, the maximum profit will be approximately $510 for sales of approximately 80 units at a price of $100. Corresponding values for selling prices of $90 and $110 are $430 and $310 for sales of 80 and 60 units, respectively. These results illustrate the fact that sales effort, price, and cost of production must be considered jointly.

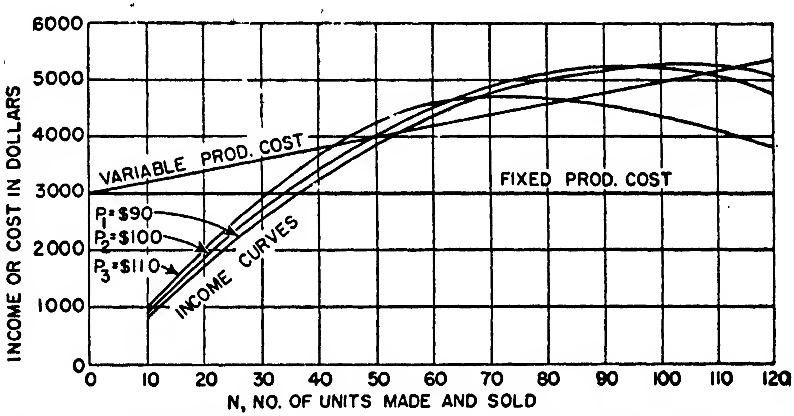

FIG. 61. Relationship of net income from sales for different sale prices, cost of production, and profit.

Combined Effect of Quality and Sales Effort

To increase the sale of a product at a given price, its quality may be improved or sales effort may be increased. The aim of much attention is to find what combination of these two factors will result in the greatest profit. To illustrate the thinking in this connection consider the following illustration.

It is desired to determine the optimum combination of quality and sales effort for a certain product whose variable cost for manufacture and sales effort is $10 per unit. Five qualities of this product as measured by the variable cost of manufacture are to be marketed. The variable cost of manufacture for qualities or grades designated Q_0, Q_1, Q_2, Q_3, and Q_4 is respectively $6, $7, $8, $9, and $10. For grades Q_1, Q_2, Q_3, and Q_4 the amount for added quality above that of Q_0 is then \$1, \$2, \$3, and \$4 respectively.

In marketing the product under consideration, sales effort costing \$4, \$3, \$2, \$1, and \$0 will be applied to grades Q_0, Q_1, Q_2, Q_3, and Q_4 respectively. Thus the total variable cost of any grade is constant at \$10 per unit.

In Fig. 62 the several combinations of added quality and sales effort are plotted against sales. In general it may be said that the combination that enables the greatest number of units to be sold is the best.

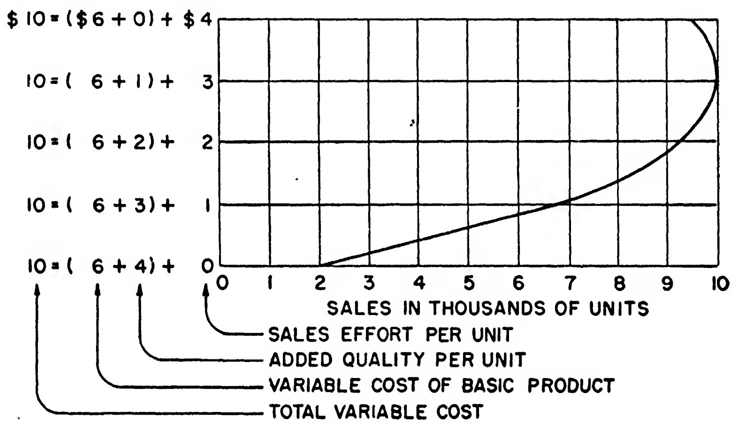

FIG. 62. Relationship of quality of product, sales effort, and sales.

In practice it is rarely possible to determine the most profitable combination of quality and sales effort; nevertheless, this is an end toward which most firms strive. The higher the ability of buyers to recognize quality, the higher the ratio of quality to sales effort may be expected to be.

Combined Effect of Price, Quality, and Sales Effort

In the previous section the effect of quality and sales effort was considered. It is clear that the effect of a number of different sales prices may be considered in relation to combinations of quality and sales effort. This is done in Table 40. The product under consideration is made in four grades G_0, G_1, G_2, and G_3 whose costs of production are respectively \$10, \$11, \$12, and \$13. Sales effort ranging from \$0 to \$4 by \$1 increments will be applied to each of the four grades named above.

The selling price of the product to be considered is equal to the sum of the manufacturing cost and the cost of the sales effort applied to the product plus 10 per cent. Thus when sales effort to the extent of \$3 is applied to grade G_2, the selling price will equal $\$12 + \$3 + (\$12 + \$3) \times .1 = \$16.50$.

A firm wishing to consider the effect of different combinations of price, quality, and sales effort might proceed as follows. By actual trial, sales research, or estimate, a curve is drawn for each grade showing the number of units sold when different amounts of sales effort are applied per unit of product. Information from these curves is then set up in an arrangement similar to that shown in Table 40. The advantage of this arrangement is that the effect of all combinations of price, quality, and sales effort is readily observed. For most purposes this form will be found simpler to construct and more useful than a plotting of the data and calculated quantities on three axes.

It should be recognized that it is practically impossible to secure sufficient facts to determine the optimum combination in the above example. Nevertheless, a combination believed to be the best will be selected in one way or another.

The usual way is to select a combination by estimate of the situation as a whole. The above method confines estimates to simpler relationships and may be expected to contribute to understanding of the problem and to the soundness of conclusions. The arrangement of Table 40 is applicable to a variety of situations in which the combined effect of three factors is considered.

Level of Sales Effort

Some sales effort must be applied to dispose of nearly all products, particularly those that are purchased by the ultimate consumer. The level of sales effort to apply for optimum results is an important consideration in economy.

Level of sales effort applied to a product or a group of products may be expressed in terms of the amount of advertising purchased, the number of dealer outlets, the number of calls by salesmen, and so forth, or a combination of these.

TABLE 40. Determination of Most Profitable Combination of Price, Quality, and Sales Effort Per Unit of Product

Cost of Sales Effort Per Unit in Dollars				
4	15.40 14.00 1.40 8.80 12.32	16.50 15.00 1.50 9.20 13.80	17.60 16.00 1.60 8.60 13.76	18.70 17.00 1.70 7.90 13.43
3	14.30 13.00 1.30 9.20 11.96	15.40 14.00 1.40 10.00 14.00	16.50 15.00 1.50 8.40 12.60	17.60 16.00 1.60 7.60 12.60
2	13.20 12.00 1.20 9.10 10.92	14.30 13.00 1.30 9.20 11.96	15.40 14.00 1.40 7.50 10.50	16.50 15.00 1.50 6.70 10.05
1	12.10 11.00 1.10 7.50 8.25	13.20 12.00 1.20 6.80 8.16	14.30 13.00 1.30 5.30 6.89	15.40 14.00 1.40 4.60 6.44
0	11.00 10.00 1.00 .90 .90	12.10 11.00 1.10 1.60 1.76	13.20 12.00 1.20 1.40 1.68	14.30 13.00 1.30 .60 .78
Grade of Product	G_0	G_1	G_2	G_3
Total Cost Per Unit in Dollars	10	11	12	13
Cost of Added Quality Per Unit in Dollars	0	1	2	3
Cost of Basic Product Per Unit in Dollars	10	10	10	10

KEY TO QUANTITIES IN BOXES

x x x x x	Selling Price Per Unit in Dollars
x x x x x	Cost of Product Plus Cost of Sales Effort in Dollars
x x x x x	Profit Per Unit in Dollars
x x x x x	Number of Units Sold
x x x x x	Total Profit in Dollars

In general there is a level that produces more favorable results than any other. This optimum level is continually being sought for through research, experimentation, and the exercise of reason and opinion.

Table 41 illustrates the effect of different levels of sales effort in the disposal of a truck accessory.

TABLE 41. Effect of Levels of Sales Effort

Number of Trucks Per Salesman	Number of Units Sold Per Salesman Per Year	Cost of Sales Effort Per Unit
30,000	1,240	$11.45
40,000	1,570	9.04
50,000	1,740	8.16
60,000	1,760	8.07
70,000	1,690	8.40
80,000	1,580	8.99
90,000	1,410	10.07

The first column gives the number of trucks in each salesman's territory. The second column gives values taken from a smooth curve drawn to represent average number of units sold per salesman per year. In the third column the cost of sales effort per unit of product is given. These values are found by dividing $14,200 by the number of sales per year. $14,200 represents the average cost per salesman per year and is independent of the number of trucks per salesman.

Where only the cost of sales effort is pertinent, one salesman per 60,000 trucks is most economical. The relationship of total sales to output and plant capacity must also be considered. Thus, if the plant's normal capacity cannot be sold by assigning one salesman to each 60,000 trucks, greater profit may result from decreasing the number of trucks per salesman, even though sales cost per unit of product is increased thereby.

PROBLEMS

1. A bakery makes white, whole wheat, and raisin bread. It has an oven with a capacity of 3,000 loaves per day. During the first week of June, the number of loaves baked was as follows:

	White	*Whole Wheat*	*Raisin*	*Total*
Sunday	1,200	500	100	1,800
Monday	1,050	550	140	1,740
Tuesday	950	600	70	1,620
Wednesday	900	500	80	1,480
Thursday	1,200	400	100	1,700
Friday	1,400	500	80	1,980
Saturday	0	0	0	0
Total				10,320

For the week under consideration what was (a) the load factor, (b) the capacity factor, and (c) the diversity factor?

2. A person who pays for electric energy in accordance with the one-part rate schedule given in the text was billed $5.97 for energy used in June. How many kilowatt-hours were used?

3. A residence owner who is billed for electricity in accordance with the one-part schedule given in the text was billed $4.20 for energy used in October.

(a) What was the average cost per kw-hr.?

(b) What would have been his bill if he had had an appliance that consumed an additional 68 kw-hr.?

(c) What would have been the increment cost for energy for the appliance?

(d) What would have been the cost per kw-hr. of the increment of energy used by the appliance?

(e) What would have been the average cost per kw-hr. of all energy consumed?

4. Three motors A, B, and C of 5, 5, and 10 horsepower capacities respectively are used to drive the equipment in a small woodworking shop. At full load each motor consumes energy at the rate of one kw per horsepower. The owner of the shop pays for energy in accordance with the two-part schedule given in the text in which the minimum bill clause and the 100 kw minimum demand clause has been deleted. To minimize demand charges he has switches arranged so that Motors A and B cannot be operated when Motor C is operating. This switch arrangement was inconvenient and was recognized to cause a loss of effectiveness of the workmen resulting in a loss of $.50 per hour for each hour Motor C was operated. During September Motors A, B, and C operated at full load for 80, 120, and 10 hours respectively. Motors A and B were operated simultaneously for 20 hours. The power factor was always between 75 per cent and 80 per cent.

(a) What was the total power bill for September?

(b) What would have been the total power bill if the three motors had been operated simultaneously for 30 minutes or more?

(c) With no change in the usage of Motors A and B, what is the maximum number of hours of usage of Motor C for which the switch arrangement can be justified?

5. A manufacturing concern uses 280,000 kw-hrs. of energy per month. The maximum 30-minute demand is 1,100 kw and the average power factor is .59. The low power factor is due in part to a 300-kw induction motor which drives a d-c generator. If this induction motor is replaced by a synchronous motor, which may be caused to take a leading current by overexcitation, the power factor of the total load may be raised to .76. Assuming that energy consumed and demand remain as given above and neglecting consideration of minor losses incident to the change in power factor, determine the monthly power bill for each alternative on the basis of the two-part rate schedule given in the text.

6. A small foundry and machine shop makes ornamental brass products. It requires electric power to drive machine tools and to operate a coreless induction furnace. An analysis of power bills reveals that the average power consumption is 1,600 kw-hrs. of energy during a typical month of 22 eight-hour working days and that the peak load during a typical month is 67 kw. The peak load occurs during a one-hour period between 3 and 4 P.M. each working day during which the induction furnace is in operation. This furnace requires a peak input of 46 kw and an average input of 36 kw during the hour it is in operation each day. It is estimated that the peak load of the plant would average 26 kw if the induction furnace were supplied with electric energy by a generator driven by an oil engine. A suitable second-hand engine and generator can be purchased and installed for $1,100. It is estimated that the engine will consume 5.5 gallons of fuel oil and one quart of lubricating oil per hour of operation. Maintenance is estimated at $.18 per hour of operation. One hour of labor costing $1.18 per hour in excess of that required at present will be needed if the oil engine-generator unit is purchased. Fuel oil costs $.12 per gallon and lubricating oil costs $.60 per gallon. Taxes and insurance amount to 1.8% of the cost of the equipment. Interest is taken at 6% and the oil engine-generator unit has an estimated life of 4 years and no salvage value. The power bill is based on the two-part schedule given in the text, but in which the power factor clause and the minimum bill clause are waived.

(a) What will be the electric power bill if operation of the plant continues as it is now?

(b) What will be the electric power bill if the energy for the furnace is supplied by the oil engine-generator unit?

(c) What annual savings will result from installing the oil engine-generator unit, if any?

7. In a typical month a power customer has a maximum demand of 1,200 kw and the power factor averages 69 per cent.

(a) If a static condenser that will raise the power factor to 84 per cent is installed, how much will the monthly bill be reduced if demand charges are calculated on the basis of the two-part schedule given in the text, assuming no changes in current consumption?

(b) If it is estimated that the life of the static condenser will be 12 years, salvage value will be zero, and annual maintenance, taxes, insurance, and so forth will be $260 per year, what purchase price is justified if interest is taken at 6% compounded annually?

8. A jobber had a quantity of one-inch pipe he wished to dispose of quickly. An advertisement offering to sell the pipe at $.28 per linear foot resulted in sales of 160 linear feet of the pipe. An advertisement the following day offering the pipe at $.25 per linear foot resulted in sales of 1,620 linear feet. What was the elasticity in demand?

9. An analysis of an enterprise and the market in which its product is sold results in the following data:

Level of Operation, Units of Product	Production Cost, Dollars	Net Income from Sales, Dollars	Level of Operation, Units of Product	Production Cost, Dollars	Net Income from Sales, Dollars
0	10,900	0	700	41,700	43,000
100	17,600	8,000	800	44,800	46,800
200	23,000	15,400	900	48,000	50,000
300	27,500	22,000	1,000	51,300	52,700
400	31,500	28,000	1,100	54,800	54,800
500	35,100	33,500	1,200	58,500	56,300
600	38,500	38,500			

(a) Determine the profit for each level of operation.
(b) Plot production cost, net income from sales, profit, average increment production cost per unit, and average increment net income from sales per unit for each level of operation.

10. The product of an enterprise has a fixed selling price of $62.00. An analysis of production and sales costs and the market in which the product is sold has produced the following results:

Level of Operation, Units of Product	Total Production and Selling Cost, Thousand Dollars	Level of Operation, Units of Product	Total Production and Selling Cost, Thousand Dollars
0	13.2	600	35.0
100	17.9	700	40.4
200	21.4	800	47.1
300	24.6	900	55.6
400	27.2	1,000	65.4
500	30.6		

(a) Determine the profit for each level of operation.
(b) Plot production and selling cost, income from sales, profit, average increment production and selling cost per unit, and average increment income per unit, for each level of operation.

11. The capacity of a plant is 1,600 units of product per year. Sales are at the rate of 1,200 units per year and the income per unit is $720. Variable costs are $416 per unit and fixed costs of the plant equal $365,000. A plan for replacing many units of present equipment which it is hoped will reduce variable costs is under consideration. It is estimated that fixed costs

will be increased by $58,000 per year and that variable costs will be reduced $56 per unit. (a) What will be the effect of the proposed plan on profit if estimates are realized?

A second plan under consideration is to increase sales effort by $22 per unit and increase the sales price to $730 per unit. If sales are increased by 15 per cent by this plan, what will be its effect on profit if it is adopted (b) in conjunction with the first plan? (c) Without putting the first plan into effect?

12. A concessionaire at football games has found that the amount of sales made during a game per thousand prospects varies with the ratio of the number of salesmen to the number of prospects approximately as shown in the following table.

Number of Prospects Per Salesman	*Amount of Sales Per 1,000 Prospects*
200	$56.00
300	53.20
400	50.00
500	46.60
600	39.20
700	30.40

(a) The salesmen are paid $3.00 for the afternoon. The profit on sales before the salesmen are paid is 50 per cent. What number of prospects should be assigned per salesman and what is the concessionaire's net profit per 1,000 prospects?

(b) Recalculate part (a), taking the profit before paying the salesmen to be 40 per cent.

(c) Recalculate part (a), assuming that the salesmen are paid $4.00.

13. The annual fixed production and selling cost of a company producing a certain product is estimated at $40,000. The variable cost is estimated at $65 per unit. Dissatisfied with the small margin of profit, the company is studying a plan to improve their product and increase their selling effort. The plan of improvement requires an additional annual fixed expenditure of $10,000 on production and selling. Also an additional $25 per unit will be spent to improve the product from the standpoints of quality and customer appeal. On the basis of a market survey, estimated sales, with and without the improvements, at various selling prices are given below:

Unit Selling Price	*Annual Sales Without Improvements*	*Annual Sales With Improvements*
$350	—	55
300	20	180
250	90	360
200	225	610
150	460	920
125	680	—
110	1,010	—

Plot the total production and selling cost of each plan, and plot the total income from each plan for the various selling prices against outputs as abscissa. Determine whether or not the plan for improvements should be adopted and the production and selling price that will result in the maximum profit.

14. A company has priced its product at $1.00 per pound, but is operating at a loss. Sales at the price of $1.00 per pound total 850,000 pounds per year. The company's fixed cost of manufacture and selling is $480,000 per year and the variable cost is $.46 per pound. It appears, from information obtained by a market survey, that price reductions of $.05, $.10, $.15, and $.20 per pound from the present selling price will result in total annual sales of 1,030,000 pounds, 1,190,000 pounds, 1,360,000 pounds, and 1,480,000 pounds per year respectively.

(a) Calculate the annual profit that will result from each of the selling prices given, assuming variable cost per unit will be the same for all productions.

(b) Determine graphically the annual profit that will result from each of the selling prices by drawing a break-even type chart.

15. The cost of manufacturing a certain article has been found by analyses to vary as shown in Table A and the market for the same article is estimated to vary as shown in Table B.

TABLE A

Yearly Quantity	*Unit Cost*
1,000	2.25
2,000	1.95
3,000	1.75
4,000	1.50
5,000	1.25
6,000	1.10
7,000	.95
8,000	.90
9,000	.85
10,000	.82

TABLE B

Factory Price	*Yearly Quantity*
3.00	1,200
2.50	2,400
2.00	3,700
1.50	5,300
1.25	6,500
1.00	8,000
.90	10,000

Graph each table and determine the maximum return possible and the level of output that will produce it. *Note.* Draw straight lines between plotted points and interpolate if necessary.

16. A cold storage locker plant (including lockers, insulation, and so forth) is constructed at a cost of $27,510. The land on which the plant is located is worth $2,000 and will not decrease in value. The investment in the building should be recovered in 20 years. The refrigeration machinery and other necessary equipment cost $8,700. This investment should be recovered in 6 years. The operation of the plant requires a working capital of $1,200. The fixed operating cost of salaries, advertising and so forth is $8,000 per year. Determine the profit or loss for each degree of operation given in the table below. Interest is at 7%.

Per Cent of Capacity	*Gross Annual Income*	*Variable Annual Expenses*
40	$ 9,600	$1,300
60	14,400	2,300
80	19,400	2,800
100	24,000	3,200

17. A certain firm has the capacity to produce 650,000 units of product per year. At present it is operating at 62 per cent capacity. The firm's annual income is $416,000. Annual fixed costs are $192,000 and the variable costs are equal to $.356 per unit of product.

(a) What is the firm's annual profit or loss?

(b) At what volume of sales does the firm break even?

(c) What will be the profit or loss at 70, 80, and 90 per cent capacity on the basis of constant income per unit and constant variable cost per unit?

18. A company is operating at capacity for one shift per day. Annual sales are 3,600 units per year and the income per unit is $200. Fixed costs are $280,000 and total variable costs are $440,000 per year at the present rate of operation. If output can be increased, 400 additional units can be sold at $200 per unit during the coming year. These additional units can be produced through overtime operation at the expense of a 20 per cent increase in the unit variable cost of the additional units. The elasticity of demand for the product in question is believed to be such that an increase in selling price to $208 will result in curtailing demand to 3,600 units. For greatest profit in the coming year should output be increased or should selling price be increased?

19. A manufacturing company owns two plants, A and B, that produce an identical product. The capacity of Plant A is 60,000 units annually while that of Plant B is 80,000 units. The annual fixed cost of Plant A is $260,000 per year and the variable cost is $3.20 per unit. The corresponding values for Plant B are $280,000 and $3.90 per unit. At present Plant A is being operated at 35 per cent of capacity and Plant B is being operated at 40 per cent of capacity.

(a) What are the unit costs of production of Plant A and Plant B?

(b) What is the total cost and the average cost of the total output of both plants?

(c) What would be the total cost to the company and the unit cost if all production were transferred to Plant A?

(d) What would be the total cost to the company and the unit cost if all production were transferred to Plant B?

20. A plant with a capacity of 100,000 units of product per year has constant costs of $225,000 per year and a variable cost per unit of $1.50. There is a standard home market for 50,000 units at $6.50 per unit and a foreign market for 50,000 units at $3.50 per unit. What profit will result if (a) only the standard home market demand is met, and (b) both standard home market and foreign market demand is met?

CHAPTER

17

Economy and Utilization of Personnel

An important ingredient in production and distribution of goods and services is human effort. Wages and salaries constitute a large portion of the total input of most organized endeavors. Charles F. Kettering has said, "American industry is cultivating ideas and men as its richest investment in the future." This statement may be interpreted to mean that human effort, mental and physical, is the most important factor in industrial progress.

When it is realized that the finished product of one organization is the raw material of another, and that a particular material may pass through several organizations in the manufacture of a product, it becomes apparent that human effort is a major cost of production. How to improve the effectiveness of human labor has been the subject of much study and is a major consideration of industrialists during the present era.

Improvements that result in less labor being used per unit of output of product are considered to be labor-saving.

How Labor Is Saved

It appears that individuals as well as organizations are characteristically motivated to obtain the things they desire with the least expenditure of effort. The means by which individuals and organizations seek to save labor may be classified as follows:[1]

Labor-saving equipment
Labor-saving materials
Labor-saving methods
Labor-saving organization

[1] H. G. Thuesen and M. R. Lohmann, "Job Design," *Publication No. 66*, Engineering Experiment Station, Oklahoma A. and M. College, 1948.

Many aspects of these means are covered in such courses of study as machine design, thermodynamics, chemistry of materials, motion and time study, and industrial organization and management. The purpose of this chapter will be to present additional ideas and approaches related to economy and the utilization of personnel. This will be done through the use of illustrative examples.

Labor-Saving Equipment

Much equipment is characterized by its labor-saving and time-saving features. These features increase in importance as cost of labor rises. When the labor saving is known or can be estimated reasonably well, methods of analysis previously presented may be used to determine the desirability of a unit of labor-saving equipment.

But there are many situations in which the labor saving that may result from a proposed method may not be known. In such cases it may be very revealing to calculate the labor saving that would have to result to justify the equipment in question.

An industrial firm employed an average of 64 janitors at an annual cost of approximately $110,000. The average hourly rate was $.92 per hour. Push brooms used in the past had cost $2.60 each. A new broom being considered appeared to be much superior to those previously purchased, but its price was $4.95. Though the superintendent agreed to the superiority of the latter broom, he did not feel that it was "twice as good," which he felt that it should be to justify the higher price. At the salesman's suggestion the superintendent estimated that janitors spent 40 per cent of their time using a push broom. Then the salesman made the following calculation, based on a broom life of one year and 2,000 working hours per janitor per year.

Let E equal the per cent increase in effectiveness necessary to justify the purchase of the superior broom.

Then

$$\$4.95 - \$2.60 = (.40 \times 2{,}000 \times \$.92)E$$

Solving for E,

$$E = .003 \text{ or } .3 \text{ per cent.}$$

Note that in the above calculation only the estimate of the percentage of time that janitors used brooms was subject to much error. For an estimate of 10 per cent the value of E would have been 1.2 per cent. These results can be interpreted to mean that if the higher-priced broom were only slightly more effective it would justify the higher price. Also, for the estimated utilization of 40 per cent, each per cent increase in effectiveness of sweeping above .3 per cent should result in an annual labor saving in the plant of \$110,000 $\times$ .40 $\times$ .01 = \$440.

When the increase effectiveness of a unit of equipment is known but the amount it will be used per year is unknown, the amount of the latter to justify the equipment may be calculated.

The employees in an office have requested their office manager to purchase a collating device. The device permits the employee to sit while working, thus making the job less fatiguing. The superior admits that the device reduces collating time to "about half" but states that not enough collating is done to justify an outlay of the purchase price of \$44. On the basis of service life of 5 years, an employee rate of \$.86 per hour, and a 50 per cent time saving, an employee calculated N, the number of hours' use per year to justify the purchase of the device, as follows.

$$N \times .50 \times \$.86 = \$44 \left[\frac{1}{5} + \frac{.05}{2}\left(\frac{1+5}{5}\right)\right]$$

$$N = 24 \text{ hours}$$

When the request was supplemented with the result of the above calculations, it was clear to the office manager that the device would easily pay for itself.

The shift in viewpoint provided by these approaches is often of very real advantage in improving the quality of judgment. In the first example above judgment was shifted from whether a superior broom would justify a higher price to whether it would result in a labor saving of from .3 to 1.2 per cent. These approaches are applicable to both minor and major items of equipment.

When a labor saving is in prospect, a common question is what will be done with the time saved. Sometimes time saved is wasted; other times it is used to improve quality. Occasionally the time saved would have "broken the camel's back," in other words, would have necessitated the employment of additional help. The utilization of time saved is a function of supervision.

Labor-Saving Materials

The approach to evaluation of labor-saving materials is frequently a trial with a sample. When more elaborate methods of comparison are needed, the patterns suggested in relation to labor-saving equipment and the general approaches developed in earlier chapters will serve.

Ordinarily the greatest difficulty is not evaluation but awareness of labor-saving materials, for their advantage is usually quite apparent.

A nurseryman was irked by the necessity for spending large sums to remove ties from grafts a few days after they were made. As a result, he had ties made from a material that deteriorated from the desired number of days' exposure to sunlight. The ties were much more expensive than ordinary cord, but as a labor-saving means they were an economy.

Examples of other labor-saving materials are quick-drying lacquers, ready-mixed paints, degreasers, stationery with carbon paper attached, use of identifying colors, and chemical weed killers. In general, materials that minimize repair and replacement save labor. A longer-lasting paint, for example, reduced the amount of labor for painting.

Labor-Saving Methods

Labor-saving methods are the outgrowth of the work of Frederick W. Taylor and Frank B. Gilbreth. Most proposed labor-saving methods can be tried either with no equipment changes or with makeshift devices to investigate their potentialities; ordinarily their evaluation is not difficult. The limiting factors to economies from labor-saving methods are the conception of and development of an improved method and the difficulty of getting the improved method adopted. Economy studies showing the comparative costs of the old and new methods are often effective in directing attention to the desirability of the new method and overcoming resistance to it.

Labor-Saving Organization

Since people are gregarious, it is probable that there will always be organizations, if only for their social implications.

But there are economic reasons for organizations as well. People work in groups because they can accomplish more that way than they can singly. Thus, organizations are means for increasing the output per unit of individual effort beyond what would result if individuals worked by themselves.

Suppose that a tank is to be riveted. One person stands on the inside to "buck" the rivets while they are driven by a man on the outside with a pneumatic hammer. The output of a single man on the operation might be nil; it would certainly be much less than that of two men working together.

It is conceivable that an apparatus might be constructed which will permit one man to perform the riveting operation unaided. But in this task, as in so many others, it is more economical to overcome the limitations of man by an organization of men than by equipment.

Organizations are means for overcoming limitations of people through cooperative effort.

Essentials of the Cooperative Action of Organizations

Barnard[2] states that the essentials of cooperative action and therefore the essentials of organization are:

1. The ability of the members of the organization to communicate with each other.
2. A common purpose acceptable to the members of the organization.
3. The willingness of the members to serve in the cooperative effort.

For convenience let the activities of managing, supervising, or exercising directive control over others be called leadership activities. Such activities will be performed by A, B_1, and C_1, for example, in Fig. 63a. Also, let people be called leaders while they are performing leadership activities. For the purpose of discussion, nonleadership duties of persons in leadership positions will be neglected. Leadership activities will be considered to be fundamentally similar, regardless of who performs them.

Thus defined, the leadership activities of B_1 in Fig. 63a will be

[2] Chester I. Barnard, *The Functions of the Executive* (Cambridge, Mass.: Harvard University Press, 1938), p. 82.

considered in relation to the essentials of cooperation action enumerated above.

Leadership Activities

With the above essentials of organization in mind, consider the activities performed by a leader in any formal organization. For convenience let Fig. 63a represent any formal organization. The leadership activities of B_1 may be enumerated and classified as follows:

I. Activities Related to Communication
   (a) B_1 communicates with his superior A, and his subordinates C_1 and C_2.

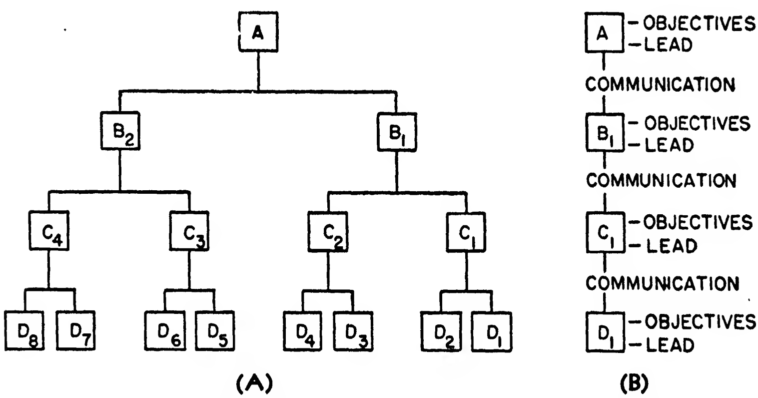

FIG. 63. (A) Chart of a line organizational structure. (B) Schematic representation of leadership activities on several organizational levels.

II. Activities Related to *Common Purpose, i.e.*, Objectives
   (a) B_1 receives objectives from A which he, B_1, is to accomplish with the men under his control, C_1 and C_2 and their subordinates D_1, D_2, D_3, and D_4.
   (b) B_1 sets up objectives (subordinate to the objectives he receives from his superior, A) for each of his subordinates, C_1 and C_2, to accomplish.

III. Activities Related to Willingness of Members of the Organization to Serve in Cooperative Action
   (a) B_1 incites the willingness of his subordinates to engage in the cooperative action necessary to accomplish

objectives, *i.e.*, B_1 exercises leadership. He does this by offers and payment of incentives and by persuasion.

In addition to the activities directly related to the essentials of cooperative action or organization, two leadership activities warrant special emphasis because of their importance. These two activities are:

A. Activities Related to Organizing
   (1) B_1 "organizes" his subordinates and means (equipment, material, and so forth, *i.e.* establishes the relationships between his subordinates, the work to be performed, the time the work is to be performed, and the means with which the work is to be performed to accomplish the objectives in cooperative effort. (These activities are elements of activities related to inciting willingness of subordinates to cooperate.)

B. Activities Related to Making Decisions
   (1) B_1 makes decisions as required by his superior, A, as requested by his subordinates, C_1 and C_2, and as necessary in the performance of all his other activities such as organizing the subordinates, means, and work under his control.

The above enumeration of leadership activities may be summarized for convenience in the following key words: Communication, Objectives, Leadership, Organization, and Decision.

Since leadership activities are fundamentally the same on all organizational levels, the graphical representative in Fig. 63b may be drawn to show the relationship between leadership activities on the several levels of organization.

Communication. If Fig. 63b is accepted as being representative of the relationships existing between leadership activities on different levels, it is apparent that the organization is tied together from top to bottom and from bottom to top by communication. Leaders are centers of communication and communication is one of the most important of their functions. It is through communication that the purpose of the organization is made known and through which willingness to cooperate is engendered and made possible.

Objectives. The setting of objectives is important. If objectives to be attained are not defined, they cannot be attained, any more than an unknown destination can be reached. The setting of sound objectives for the organization to attain is of first importance because unsound objectives, even if well executed, must of necessity be unfruitful.

Leadership. The leader must lead. Leadership has been defined as the activity of influencing people to cooperate toward some goal which they come to desire.[8] The leader may influence people by persuading them or he may secure cooperation by offering such incentives as material rewards, prestige opportunity for enlarged participation, penalties, and the like. Leadership is essential in the securing of willingness to cooperate.

Organization. The act of organizing consists of establishing relationships between persons, the work that they are to perform, the time the work is to be done, and the means to be used in performing it. Organizing is planning. It makes cooperative action possible. Organization is an important factor in the successful accomplishment of objectives and therefore an important leadership function. Careful study has established many aspects of organizing as relatively concrete; as a result most business activities are fairly well organized.

Decision. Decision is the selecting of a course of action from two or more courses of action. However difficult it is to render sound decisions, it is apparent that a leader's ability to do so is of great value to his organization.

Decision probably warrants much greater attention than it ordinarily receives. It is observable that many leaders are burdened by the sheer volume of the decisions they must make. A leader must, of course, decide on questions posed by his superior. But in regard to questions posed by subordinates, the leader has the choice of burdening himself with decision or of arming his subordinates with information and well-defined policies, so that he can be spared the burden of making routine decisions.

The Nature of Output at Different Organizational Levels

Observation leads to the conclusion that most of the concrete

[8] Ordway Tead, *The Art of Leadership* (New York: McGraw-Hill Book Co., Inc., 1935), p. 20.

aspects of products and services produced by an organization are the result of the activities of those on its lowest level. For example, it is not the orchestra leader, the sales manager, or the foundry superintendent who makes the music, the sale, or the castings. These are made by the men on the lowest level of organization.

This fact in no way minimizes the importance of the functions performed by those of the upper levels of organization. The function of much if not most of the activities of the upper hierarchy of organization is to facilitate the work of those on the lower levels.[4] This is particularly clear in regard to the work of the methods engineer. His work has no significance except as it enables those on the lowest levels to make a product with less effort. Though not as apparent, this seems equally true of the functions of design, production control, stores control, industrial relations, and cost accounting.

Persons in organization levels above the lowest are for the most part concerned with the determination of objectives and the means for attaining them. The higher the level, the greater the emphasis on general objectives, and the less the emphasis on means. Thus, top levels are concerned predominantly with objectives and the lower levels with the means for accomplishing objectives.

The exception to the rule that concrete aspects of production are performed by persons on the lowest organizational levels is some times found in procurement. Negotiation for funds, the purchase of principal items of raw material and equipment, and bargaining for labor are often carried on by persons on high organizational levels. Also, important sales are often made on high levels.

The results of upper-level activities to facilitate the work of those on lower levels are communicated to the point of use in the form of policies, specifications and directions and are effective only to the extent that they can be communicated.

Communication, a Necessity for Organized Effort

To enable a person to coordinate his actions in a cooperative effort with others, it is necessary that he specialize his efforts in time, place,

[4] "I suspect that at least nine-tenths of all organization activity is on the responsibility, the authority, and the specifications of those who make the last contributions, who apply personal energies to the final concrete objectives." Chester I. Barnard, *The Functions of the Executive* (Cambridge, Mass.: The Harvard University Press, 1938), p. 232.

and kind to conform to the needs of the concerted effort of the group. It is apparent that he can know what to do next only by information communicated to him in one way or another.

The element that binds organizations together is communication. The structure of organization is largely dictated by the necessities of communication. Some organizational precepts concerning channels of communication are: (1) they should be known; (2) they should lead to every member of an organization; and (3) they should be as short and direct as possible.

These objectives are met by the line type of organization, which is the core of the organizational structure of nearly all business and industrial concerns. A typical line organization of four levels and a span control of two appears in Fig. 63a. In Fig. 63b a channel of communication, A–B_1–C_1–D_1, is depicted.

Span of Control and Number of Organizational Levels

In considering organizational structure from the standpoint of economy, the first aspect to note is the relationship of span of control and the number of organizational levels. When all levels are complete and the span of control is constant throughout the organization, the relationship between the number of levels, the span of control, and the total number of persons in an organization may be established. Let

N = the total number of persons in an organization.
S = the span of control.
L = the number of levels of organization.

Then

$$N = 1 + S + S^2 + \dots S^{L-1}$$

This result is shown graphically for a number of spans of control and levels of organization in Fig. 64. From this graph it may be found, for example, that 10,000 people can be organized with a span of control of 6 or 20 and that the resulting number of levels would be 6 and 4 respectively.

It is recognized that the difficulty of communication is increased by the number of levels. In an organization of a given size the number of levels can be decreased by increasing the span of control. But there are economical limits to the span of control. Thus, one problem in the design of an organization's structure is to determine the most economical compromise between number of organization levels and span of control.

Because of the difficulty of measuring the effect of either the number of organizational levels or the span of control, the thinking about them has been predominantly in qualitative terms. The fact that effective span of control varies widely for different persons and classes of work further complicates the problem.

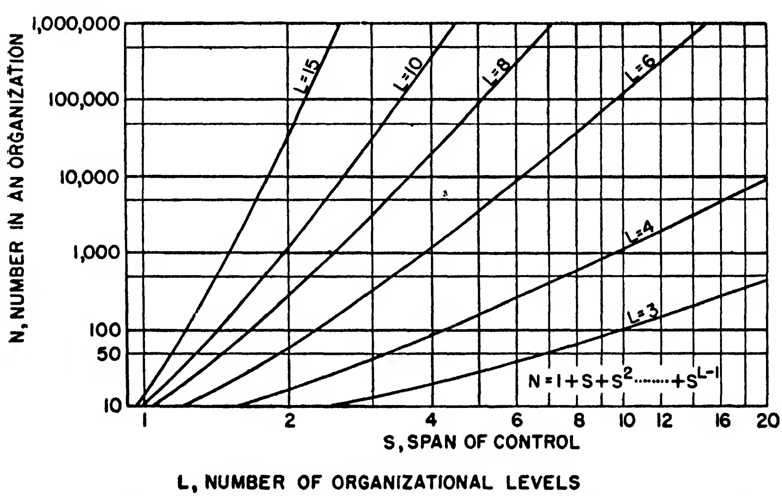

FIG. 64. Relationship of span of control, number of organizational levels, and the number in an organization.

But quantitative analyses can be made of some aspects that will be helpful in improving judgments in regard to the number of organizational levels and the span of control.

Relationship of Span of Control and Economy

It has been pointed out that the primary function of leadership is to facilitate the accomplishment of subordinates. Assuming that this is true, one question that may be considered is the ratio of input of leadership to that of those on the lowest level of an organization through whose efforts the concrete aspects of production are accomplished.

On the basis of the development in the previous section

$$N = 1 + S + S^2 \ldots S^{L-1}$$

Of N employees, $1 + S + S^2 \ldots S^{L-2}$ are on levels of organization above the lowest. The number on the lowest level equals S^{L-1}. The ratio r_L of these two groups is expressed by

$$r_L = \frac{1 + S + S^2 \ldots S^{L-2}}{S^{L-1}}$$

As the number of levels L increases, the ratio r_L approaches $1/(S - 1)$. For example, for a span of control S of six, the following values are obtained.

Number of Levels L	*Number in Upper Levels* $1 + S \ldots + S^{L-2}$	*Number in Lowest Level* S^{L-1}	*Ratio* r_L
1	0	1	0
2	1	6	0.167
3	7	36	0.194
4	43	216	0.199
5	259	1,296	⅕ nearly

Leader and subordinate relationships. There are a number of relationships that a leader must take into consideration in dealing with his subordinates. These are known as direct single, direct group, and cross relationships. Consider a leader, *s*, who has two subordinates, *a* and *b*. The span of control S is equal to two. The leader has direct single relationships with *a* and *b* when he deals with them individually. He has a group relationship with them when he deals with *a* and *b* together. Between *a* and *b* the cross relationship of *a* to *b* and *b* to *a* can exist.

The numbers of these three types of relationships are given on a minimum basis in Table 42 and Table 43.

The success of a leader depends in large measure upon his effectiveness in giving proper consideration to the relationships that are pertinent to effective direction and employment of his subordinates. It will be noted that the number of relationships that a leader must consider increases more rapidly than the span of control. Many students of leadership hold that the span of control should be limited to from 3 to 6 or 8 persons, depending upon the nature of work, because for a greater number of subordinates the number of relationships exceeds that with which a leader can cope efficiently. Certainly the number of relationships confronting a leader are a quantitative aspect of leadership.

TABLE 42. Organizational Relationship of a Leader to His Subordinates on a Minimum Basis

(*s* represents the leader and *a*, *b*, *c*, . . ., *S* represent subordinates.)

Span of Control		Number on Minimum Basis
	X, Direct Single Relationships	
1	*s* to *a*	1
2	*s* to *a* and *b*	2
3	*s* to *a*, *b*, and *c*	3
4	*s* to *a*, *b*, *c*, and *d*	4
S	*s* to *a*, *b*, *c*, . . . and *S*	*S*
	Y, Direct Group Relationships	
1		0
2	*s* to *ab*	1
3	*s* to *abc*, *ab*, *ac*, and *bc*	4
4	*s* to *abcd*, *abc*, *abd*, *acd*, *bcd*, *ab*, *ac*, *ad*, *bc*, *bd*, and *cd*	11
S	*s* to (*ab* . . . *S*) . . . etc.	$(2^s - S - 1)$
	Z, Cross Relationships	
1		0
2	*ab*	1
3	*ab*, *ac*, and *bc*	3
4	*ab*, *ac*, *ad*, *bc*, *bd*, and *cd**	6
S	*ab*, *ac*, *ad*, *ae*, . . . etc.	$\frac{S}{2}(S-1)$

* Others that could be considered are *a* to *bcd*, *b* to *acd*, *c* to *abd*, *d* to *abc*, *a* to *bc*, *a* to *bd*, *a* to *cd*, *b* to *ac*, etc., and their reverse.

$$\text{Let } Q = X + Y + Z = S + 2^s - S - 1 + \frac{S}{2}(S - 1).$$

Ratio of leadership to subordinate input. The number of subordinates assigned to a leader for the accomplishment of a given task may be varied. Such variation may be thought of as a variation in the ratio of leader to subordinate input on a man-to-man basis. Compensation may be considered and the ratio of leader to subordinate input may be obtained in terms of cost.

A function of leader activities is to utilize the effort of subordinates in such a manner that their output will be greater than it would be without leadership. Thus, the amount by which the output of unsupervised workers must be increased by leadership to justify an expenditure for leadership can be calculated. Let

S = number of workers per leader, *i.e.* span of control.
W = average daily wage per worker.
W_1 = average daily wage per leader.
P = ratio of leader's to subordinate's wage rate $= W_1 \div W$.

TABLE 43. Summary of Organizational Relationships of a Leader to His Subordinates on a Minimum Basis (See Table 42)

Span of Control, S	X	Y	Z	Q
1	1	0	0	1
2	2	1	1	4
3	3	4	3	10
4	4	11	6	21
5	5	26	10	41
6	6	57	15	78
7	7	120	21	148
8	8	247	28	283
9	9	502	36	547
10	10	1,013	45	1,068
11	11	2,036	55	2,102
12	12	4,083	66	4,161
13	13	8,178	78	8,269
14	14	16,369	91	16,474
15	15	32,752	105	32,872
16	16	65,519	120	65,655
17	17	131,054	136	131,207
18	18	262,125	153	262,296
19	19	524,268	171	524,458
20	20	1,048,555	190	1,048,765

U = average daily output per unsupervised worker.
R = ratio of daily output of supervised subordinate to daily output of unsupervised worker.
RU = average daily output per supervised subordinate.
T = leader's time per worker per 480-minute day.

The daily wage cost per supervised group of S subordinates and one leader is equal to $SW + W_1 = SW + PW$.

The daily output per supervised group of S subordinates and one supervisor is equal to SRU, and the cost per unit output for a supervised group of S subordinates and one supervisor is equal to $(SW + PW)/SRU$.

Let cost per unit output for a supervised group of S subordinates and one leader equal the cost per unit output of unsupervised workers; then

$$\frac{SW + PW}{SRU} = \frac{W}{U}$$

Solving for R,

$$R = 1 + \frac{P}{S}$$

Also

$$T = \frac{480}{S}$$

The values of R and T for values of S from 1 to 20 and for values of P from 1 to 3 may be read from the graph in Fig. 65.

Suppose that the question under consideration is whether 6 or 8 is the better span of control on the basis of which to organize a function, and that the ratio of leader's to subordinate's wage considered desirable is 1.2. From Fig. 65 the value of R for a span of 6 is 1.20 and for a span of 8 is 1.15 when $P = 1.2$. To justify a span of 6 as against a span of 8, one would have to be satisfied that the output per subordinate would be increased to the extent that R would be increased from 1.15 to 1.20 or by

$$\frac{1.20 - 1.15}{1.15} = .043 \text{ or } 4.3 \text{ per cent}$$

Supervision per worker can be increased from 60 to 80 minutes per subordinate per day or

$$\frac{80 - 60}{60} = .33 \text{ or } 33 \text{ per cent}$$

Reference to Table 43 reveals that a reduction of the span of control from 8 to 6 will reduce the number of organizational relationships from 283 to 78 or

$$\frac{283 - 78}{283} = .72 \text{ or } 72 \text{ per cent}$$

The question of whether to use a span of control of 6 or of 8 has been reduced to a decision whether a span of control of 6, which permits 33 per cent more time for leader activities per subordinate and which reduces the number of organizational relationships the leader must consider by 72 per cent, will result in an increase in output per subordinate of 4.3 per cent. It should be recalled that supervisory activities are associated with the key words Communication, Objectives, and Leadership (see p. 435). Also it should be realized that an increase in time per subordinate for leadership activities will not

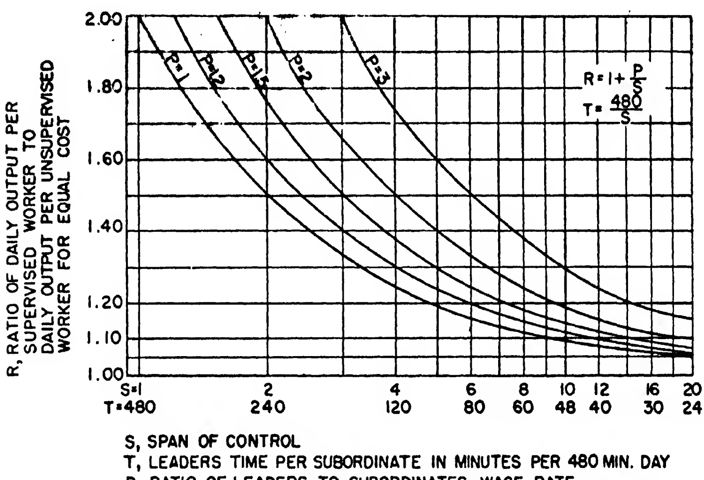

FIG. 65. Relationship of supervision, span of control, and worker output.

result in an increase of output per subordinate unless it can be and is utilized efficiently.

Selection of Personnel

Viewed objectively, subordinates are merely their leader's means for reaching ends. Of the several productive factors used in industry, none has such a variety of characteristics as personnel. Characteristics of both body and mind are of concern. One individual may accomplish several times as much of a task requiring physical strength, dexterity, or acuteness of vision as another. The range of mental proficiencies for given tasks is even wider.

The physical and mental endowments of people appear to be subject only to limited change by training and leadership. For this reason it is important that personnel be selected that have inherent characteristics as nearly compatible as possible with the work they are to perform and the position they are to occupy. Contrasted with the meticulous care with which materials are selected, the selection of personnel in most concerns is given but superficial attention.

The range of human capacities. Wechsler[5] shows that the range of proficiency of individuals to perform most physical and mental activities varies in the ratio of about two to one when the rare exceptions are excluded from consideration.

In an experiment conducted by T. R. Turnbull, 500 persons were instructed in the method of tossing thirty-two $\frac{3}{8}$ in. × $\frac{3}{8}$ in. × 2 in. blocks, pre-positioned in 4 rows on a work table into a 2 in. × 4 in. hole, $4\frac{1}{2}$ inches from the edge of the table. Each person was asked to perform the task as rapidly as possible. The results of the experiment are shown in Fig. 66.

The frequency distribution curve in Fig. 66 is typical of those for many physical or mental tasks. The ratio of the poorest to the best performance time is $.60 \div .28 = 2.14$. It is apparent that the cost of labor input per unit of output might be materially reduced by selecting only employees capable of better than average performance. In many cases superior employees receive little or no greater compensation than less capable employees. This is particularly true where performance is measured with difficulty. Superior managerial ability, for example, may go unrecognized and therefore unrewarded for long periods of time because it cannot be measured. But even where superior ability is proportionally compensated for, savings may still result (see example, page 16).

Superior ability that remains unused is not an advantage and may often be a disadvantage because of the resulting frustration and dissatisfaction of the employee. Generally speaking, the work assignment should require the workman's highest skill.

For many tasks tests can be devised for selection of prospective employees with superior ability. Representative of these are a

[5] David Wechsler, *The Range of Human Capacities* (Baltimore, Maryland: The Williams & Wilkins Company, 1935), p. 73.

battery of three tests for the selection of mail distributors for the Postal Service.[6] It was found, from extensive experimental trials, that of those making the highest 25 per cent of scores on these tests, over 93 per cent would be above average in proficiency. The economic desirability of having most employees above average in ability for the task under consideration (even though compensation be in proportion to ability, which it rarely is) can hardly be overestimated.

In this relatively new field, results of the installation of a program of selection practices are most difficult to estimate. Since the input

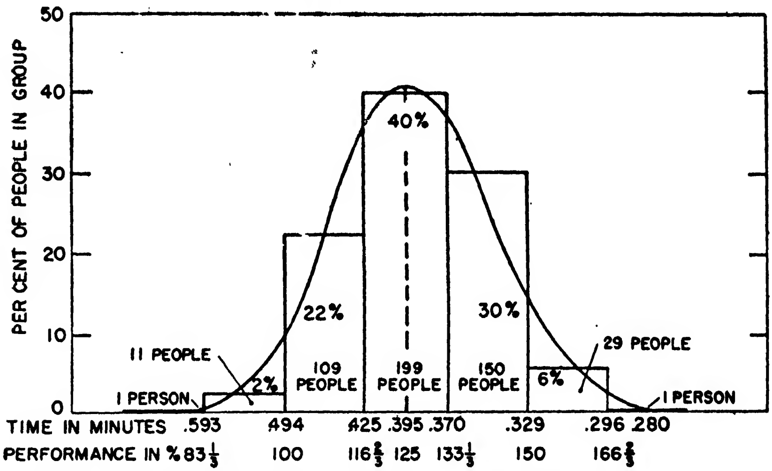

FIG. 66. Frequency distribution of time taken by 500 people to perform block-tossing operation based on .395 min. = 125 per cent performance. [Based on data, taken with permission of the publisher, from Table XXII and Fig. 243 from Barnes, Ralph M., *Motion and Time Study*, pages 355 and 359. New York: John Wiley and Sons, Inc., 1948.]

cost of a program can usually be estimated with reasonable accuracy, it may be helpful in arriving at a decision to calculate the benefits necessary to justify a contemplated selection program.

Suppose that a certain plant employs 40 operators. The average output is 28 pieces per hour, the average spoilage is 1.7 per cent, and each rejected piece results in a loss of $.60. Records reveal that average spoilage for the best 20 operators is 1.1 per cent, whereas that for the poorest 20 operators is 2.3 per cent. A consultant agrees

[6] Morris S. Viteles, *Industrial Psychology* (New York: W. W. Norton & Company, Inc., 1932), p. 317.

to prepare a set of tests and administer them for a year for $1,800. Administration subsequent to the first year is expected to cost $3.00 per operator hired or $140 per year based upon the estimated turnover.

Let P equal the per cent reduction in spoilage during first year to justify expenditure of $1,800. Then

$$P \times 40 \text{ operators} \times 28 \text{ pieces per hr.} \times 2{,}000 \text{ hrs. per year} \times \$.60 \text{ per reject} = \$1{,}800$$

$$P = .00134 \text{ or } .134 \text{ per cent}$$

This percentage reduction in spoilage seems attainable in view of the fact that a program that will reject applicants of less than average ability in relation to spoilage will result in an average reduction of 1.7 − 1.1 or .6 per cent. Moreover, it is reasonable to expect other benefits from the program.

By tending to place people in work for which they are best fitted, sound selection is generally beneficial. Even those who are rejected will generally be benefited; for, if continually rejected for work for which they are unfitted, they must eventually find a job that will permit them to work at their highest skill.

The Economy of Incentives

A person will accept and continue to work in a situation he believes to hold a net advantage for him in comparison with other opportunities that he knows to be open to him. The advantages of a situation may be considered to be positive incentives and the disadvantages may be considered to be negative incentives. People are induced to act on the basis of net incentives. Incentives are personal. A thing prized highly by one person may be regarded with indifference, or even with disgust, by another.

The following are representative of the incentives considered to be effective in inducing people to contribute their services to industrial activities.

1. Material incentives
2. Nonmaterial incentives
   (a) Social considerations
   (b) Opportunity for advancement
   (c) Opportunity for enlarged participation
   (d) Opportunity for creativeness

Much of the effort of supervisors and managers is directed toward providing maximum net incentives for minimum cost.

Material incentives. Material incentives offered to induce people to work in free enterprise are wages; wage supplements such as contributions to insurance, savings, and housing programs; and the physical condition of the plant and its locale. Up to the point where it is sufficient to provide the basic necessities of life material compensation is one of the strongest incentives. But even wages must be paid out in certain ways for greatest effectiveness. Recognition of this fact at one time led to some very complex incentive wage plans. But, "Today with our more advanced techniques and greater experience, the trend is to keep the plan itself as simple as possible."[7] Merit rating and job evaluation are other techniques that are being used to secure greater output for an input of wages.

Though material incentives as represented by compensation are emphasized, they have limited effectiveness, particularly after the individual receiving them is provided with physical necessities. This may be inferred from the fact that it is generally recognized that such valuable qualities as devotion, dependability, loyalty, initiative, industry, and honesty can be purchased for money in only a small degree. High rates of absenteeism often accompany high rates of compensation. Productivity per man does not necessarily increase with an increase in wages, and it has been known to fall. Ordinarily, doubling of wages cannot be expected to incite people to work twice as hard, if for no other reason than their physical limitations.

The overemphasis on material incentives stems, no doubt, in some measure from the sales effort, most readily discernible in modern advertising, directed at inculcating a desire for material things. The latter is a means for increasing output so that advantage may be taken of the economies of mass production. It has also resulted in demands for higher wages for a given service, or in other words, has resulted in a decrease in the value of wages as an incentive.

There are a number of material incentives that do not result in enlarged employee earnings. These are embodied in the objective plant conditions and involve such things as cleanliness, quality of buildings and equipment, maintenance, lighting, heating, ventilation,

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[7] J. K. Louden, *Wage Incentives* (New York: John Wiley & Sons, Inc., 1944), p. 7.

rest rooms, provision of parking space, and recreational facilities. These incentives seem to have more or less universal appeal and are evaluated in comparison with the conditions existing in other plants. They can be applied on an impersonal basis. Good physical conditions are often an important factor in inducing a favorable response, particularly in conjunction with other incentives of more personal nature, but alone their effectiveness appears to be very limited.

Nonmaterial incentives. There seems to be a wide individual divergence in the effectiveness of nonmaterial incentives. Some people are greatly influenced by the companionship afforded by fellow workers. Others can be induced to perform superlatively by a knowledge that such performance will result in praise or prestige. An important incentive for ambitious people is the opportunity a situation affords for advancement. The opportunity for enlarged participation in the affairs of the organization with which they are associated is a very strong incentive for many. For some this takes the form of being able to have a hand in directing the activities of the group; others will consider being "in the know" a reward for faithful service.

One of the strongest incentives is the opportunity for self-expression through creativeness in work that is in accord with natural aptitudes of the person concerned. This incentive is not feasible in work requiring strict obedience to specifications, policies, and superiors.

Leaders should strive to organize their subordinates in relation to the work to be done, so that each worker may receive the greatest satisfaction in doing that which contributes most toward reaching the objectives to be accomplished. When the work itself provides an incentive for doing it in whole or in part, there is true economy.

Nonmaterial incentives are not provided without cost, and they can easily be more costly for the beneficial motivation they provide than material incentives. They must, to a large extent, be applied upon a personal basis, for their effectiveness depends upon the characteristics of the persons to whom they are offered. For example, the prestige of a private office might be a great incentive to some employees, but not to others.

A peculiarity of nonmaterial incentives is that they often affect others beside the person to whom they are directed. Thus, giving one employee a private office might prove to be a negative incentive to several others and result in a net loss. On the other hand, if all employees of a group were given private offices, the beneficial motivation might be very slight.

Because of their personal nature, the successful and extensive offering of nonmaterial incentives depends upon the selection of personnel who are easily pleased and upon leaders who have an aptitude for distributing incentives to their subordinates in such a way that each feels he has received favorable individual attention. The selection of employees who are easily favorably motivated is an economy, since such employees require less input of leader attention.

The cost and effectiveness of nonmaterial incentives rests almost entirely upon the personal attributes of leaders. Nonmaterial incentives that are offered to a subordinate are controlled in large measure by leaders. Thus, the selection of leaders is an important factor in economy, though it appears not to receive the attention it warrants. Further improvement of leadership may result from training. The leader is also a subordinate. His proper motivation may be expected to enhance the value of his service.

Relation of Personal Qualities to Economy

The value of a person to an organization may be considered to be the difference between the worth of his output and the cost of the input that he requires. The output of a person in an organization consists of what he does that contributes (1) directly to the attainment of the organization's objectives, and (2) to the establishment and maintenance of a system of cooperative effort.

Examples of the former contributions are such outputs as negotiating a loan on which to base plant expansion, closing a sale for 600 units of product with XYZ Company, designing a new water pump, planning a transformer production layout, turning out 800 motor frame castings, typing a letter to John Doe and Company, helping an electrician install a motor. The value of such contributions depends upon their volume, quality, and timeliness. They are usually concrete and easily observable and are generally quite well understood.

Most things are accomplished in cooperation with others. This necessitates the coordination of several persons' activities in such a way that their joint effort is specialized in a system of cooperation in a way that will accomplish the desired result. Three conditions essential to the existence of systems of cooperation were stated on page 434. An individual may contribute to the establishment and the maintenance of any of these essentials. Persons who do so are integrating influences. Those who tend to destroy the essential conditions are disruptive influences. Maintenance of effective cooperation is so important in many situations that a member of an organization may be valued primarily for his integrating influence rather than for his concrete output. On the other hand, some star performers, whether on athletic teams or in industrial organizations, are dropped because their disruptive influence outweighs their concrete performance.

A variety of inputs are associated with the utilization of personal service. Among the more tangible of these are wages and allied compensation, cost of recruitment, and the maintenance of comfort facilities such as heat, light, furniture, and rest rooms. An important additional input is what may be conveniently called leadership attention. Leadership attention to a subordinate takes the form of directions, instructions, encouragement, discipline, transfer to more suitable work, and compensation for a shortcoming in ability or performance.

It is observable that there is a great difference in the amount of leadership attention required to secure satisfactory performance from individuals. Some individuals require so little leadership that they need only be given general directions. Some have shortcomings requiring so much leadership attention that the cost of this input exceeds the value of their output.

Direct and Indirect Wage Costs

There are usually a number of indirect wage costs associated with the employment of a workman in addition to the amount of wages paid to him per hour of active service. These indirect costs arise from granting vacations with pay, workman's compensation insurance, contributions to life insurance and retirement programs, unemployment compensation, and similar benefits and requirements. In illustration of these costs, the direct and indirect costs in effect for pumpers of 15 years' service in an Oklahoma petroleum company in 1949 are given in Table 44.

It is clear that the indirect wage cost in Table 44 is directly in proportion to direct wage payments. Thus an improvement that would result in a release of a pumper for 1,000 hours, for example, would result in a saving in labor cost of 120.2 per cent of 1,000 hours times $1.765 per hour or $2,122. In economy studies improvements that reduce direct wage costs should be credited with the indirect wage costs that apply.

TABLE 44. Direct and Indirect Costs in Effect for Pumpers (a labor classification) of 15 Years' Service in an Oklahoma Petroleum Company in 1949

HOURS	
Scheduled hours per year, 52 weeks × 40 hours	2,080 hrs.
Vacation with pay, 3 weeks × 40 hours	−120 hrs.
Holidays with pay, 6 day × 8 hours	−48 hrs.
Net working hours per year	1,912 hrs.
Direct-Wage Base Rate	$1.765 per hour
Indirect-Wage Rate	
COSTS	
Vacation and holiday pay, 168 hours × $1.765	$296.52
Workman's compensation (4.25 per cent, Oklahoma)	156.03
General American Insurance	21.60
Social security (1 per cent)	36.71
Unemployment compensation (.8 per cent, Oklahoma)	29.37
Retirement plan (based on employee aged 40)	93.64
Unavoidable absence (1.33 per cent average for 1948)	48.83
	$682.70
Indirect wage rate, $682.70 ÷ 1,912	$.357 per hour
Indirect wage cost in per cent of base rate, $.357 ÷ $1.765	20.2 per cent
Total Cost Per Working Hour	
Base rate of $1.765 plus indirect cost of $.357	$2.122 per hour

PROBLEMS AND QUESTIONS

1. Give two or three examples of labor-saving machines, labor-saving materials, labor-saving methods, and labor-saving organizations.

2. Two departmental heads in a manufacturing concern, whose activities must be closely coordinated, each receive $10,000 a year for a year consisting of 40 hours a week and 50 weeks. Since communication between them is essential for coordinating their activities, they must spend some time in conference for this purpose.

Construct a set of axes: Let *cost of communication* and *value of communication* be represented on the ordinate axis and *per cent of time spent in communication* from 0 to 100 per cent be represented as the abscissa. Plot the cost of communication, value of communication, and value less cost of communication on the basis of what you believe to be reasonable.

3. The annual expenditure for labor, material, and fixed cost of a factory employing a manager and 5 immediate subordinates totals $2,300,000. The salary of the manager is $18,000 per year and the subordinates receive an annual salary of $12,000 each. The manager holds a weekly conference with his subordinates as an aid in securing coordination and believes strongly that the conferences are well worth the two hours spent each week. The plant operates 40 hours a week for 50 weeks a year.

(a) On the basis of the salaries of the people involved, what is the weekly cost of the conference?

(b) What perçentage increase in the effectiveness of the total operation of the factory, attributable to the conferences, is necessary to justify the weekly meetings?

4. Foreman A directs the work of 12 men. He plans his work "as he goes" during the eight-hour work period.

Foreman B also directs the work of 12 men but he makes it a point to spend 2 hours each day in planning the work to be done in order to supervise more effectively. This reduces the time he can be in active direction of his men. What percentage more effective must his active supervision be, per unit of time, than that of Foreman A to compensate for the time spent in planning?

5. An executive receives an annual salary of $11,600 and his secretary receives a salary of $3,200 per year. The executive can perform a certain task in 3 hours. If he delegates the task to his secretary it will require ¾ hour for him to instruct the secretary how to perform the task and ½ hour to check the results. In addition, it will require the secretary 4 hours to accomplish the task, owing to her unfamiliarity with the work to be done. Considering salary cost only, what is the cost of performing the task by each method if a year is considered to consist of 1,800 hours?

6. A foreman receiving a salary of $4,200 per year now supervises 18 men whose average salary is $3,200 per year. The foreman is assisted by a clerk who is chiefly engaged in paper work and whose salary is $2,600 per year. The foreman's superintendent suggests that an assistant foreman be employed as a means of increasing the effectiveness of the men through closer supervision. The foreman opposes the suggestion on the grounds that it would "take as much time to supervise the assistant foreman as it does to supervise 5 men." If an assistant foreman is employed, his salary will be $3,600 per year.

(a) What average per cent increase in the output of the men is necessary to justify the employment of the assistant foreman?

(b) Assuming that it takes the foreman .5 hour per 8-hour day to supervise the clerk and that his statement quoted above is correct, what percentage incease in the average time spent in supervision per day per man can be made if the assistant foreman is employed?

7. A group of 60 employees whose average salary is $3,000 per year is directed by 5 foremen whose average salary is $4,000 per year. It is suggested that the number of foremen be increased to 6.

(a) What per cent increase in average output per man must result for equal cost of the two plans?

(b) If the suggested plan is adopted, what will be the percentage increase in average time that can be spent in supervision per worker and the percentage decrease in the number of organizational relationships with which each foreman will have to cope?

8. A foreman is in charge of a maintenance crew of eight men and takes great pride in the amount of work he personally performs on the job. Observation shows that the accomplishment of the men is impaired by lack of direction as a result of the foreman's active participation in the work to be done. The foreman receives $1.80 and his men $1.50 per hour. If the foreman does one-third as much work as the average of his men would do if they were properly directed, what loss in the effectiveness of the crew will just be compensated for by the actual work performed by the foreman?

9. Mr. A, Mr. B, and eight other men are supervised by the same foreman. The foreman states that "Mr. A requires twice as much and Mr. B requires half as much of my time as the average of my men." Records show that Mr. A's output averages 8 units and Mr. B's output averages 7 units of work per day. The foreman receives $325 per month and Mr. A receives $260 per month. On the basis of equal cost per unit, what monthly salary is justified for Mr. B considering 22 working days per month?

10. A company employs a number of young engineers. At the present time five engineers are assigned to each office. They feel that distractions caused by crowded conditions reduce their productivity greatly. Determine the per cent of increase in productivity per man over the present productivity per man necessary to justify having only 4, 3, 2, and 1 man in each office if the following data apply.

Size of each office	16 × 20 feet
Annual salary of each engineer	$4,200
Cost of building per square foot	$6
Estimated life of building	25 years
Estimated salvage value of building	$0
Annual taxes, insurance, and maintenance based on first cost	4%
Janitor service, heating and illumination, etc., per square foot per year	$.36
Interest rate	5%

11. A bank has 18 employees. Through payment of a prepaid premium of $1,702.35, the bank receives coverage for three years of losses due to dishonesty and fraud of employees up to $100,000 per employee per year. If the interest rate is taken at 6% compounded annually, and if 60 per cent of the premium represents the insurance company's operating costs and profit, what is the average annual cost of potential dishonesty per employee covered by the premium?

12. In a certain firm 60 employees are engaged in identical activities. The average output of the group as a whole is 46.4 units per hour. The average output of the less productive half is 40.2 satisfactory and 1.4 unsatisfactory units per hour and the average of the more productive half is 52.6 satisfactory and .8 unsatisfactory units per hour. The employees work on a straight piecework plan and receive $2.25 per hundred satisfactory units. The firm sustains a loss of $.08 for each unsatisfactory unit. One machine is required for each employee. Each machine has an annual fixed cost of $320 and a variable cost of $.16 per hour. Supervision and other overhead costs are estimated at $560 per employee per year. The average employee works 1,900 hours per year. How much could be paid annually for a selection, training, and transfer program which would result in raising the average productivity of the entire group to 52.6 satisfactory and .8 unsatisfactory units per hour.

13. The retirement plan in operation for the employees of a certain state operates as follows. Four per cent of the employee's salary is deducted and placed in a retirement fund to accumulate at 3% interest compounded annually. At the age of 65 years the employee has available in a fund to his credit the above amount plus an equivalent amount that has been contributed by the state.

What will be the amount available for retirement payments if an employee has received annual salaries of $3,000, $4,000, $5,000, and $6,000 for age periods 25 to 35, 35 to 45, 45 to 55, and 55 to 65 respectively?

14. An employee has a total retirement credit of $11,000 when he retires at the age of 65 years. He may elect to receive this credit as a lump sum or to receive year-end annuity payments. If his life expectancy at the age of 65 is 11 years, what will be the annual payment he will receive for this period if the unpaid balance remains in a fund at 3% interest compounded annually?

15. A college student has been solicited by a salesman representing a nationally known life insurance company. The salesman offers four $1,000 nonparticipating policies whose premium rates and surrender value based on the student's age of 20 years were as follows:

	Type of Policy			
	20-Year Term	*Ordinary Life*	*20-Payment Life*	*20-Year Endowment*
Annual premium	$8.60	$14.63	$26.20	$47.02
Surrender value at age 40	0	$229.00	$503.00	$1,000

What will be the equivalent annual cost of insurance for each policy if all premiums are paid when due (at the first of each year), if the insured terminates policies at the age of 40 and receives payment of the surrender values, and if the interest rate is (a) 2%, (b) 3%, and (c) 4%?

16. From the data given in Problem 15 determine the rate of interest at which the 20-year term policy and the ordinary life policy will result in equal costs to a policyholder.

17. In a certain process .15 pound of solder is theoretically required per unit of product. Under the piecework rate now in effect, workers receive $5.40 per hundred units for the soldering operation, and at this rate .24

pound of solder is used per unit. The overhead cost per worker is $.30 per hour and the cost of solder is $1.10 per pound. The job has been studied by the methods department and it is believed that a reduction in cost can be had at the expense of hourly output per worker in accordance with the following table:

	Excess Solder Per 100 Units, Pounds	*Average Hourly Output Per Worker, Units*
Plan A (Present)	9.0	20
Plan B	6.0	15
Plan C	3.6	10

(a) Determine the piece rate per hundred units that workers should be paid for Plans B and C in order for their earnings to be the same as they are under under Plan A.

(b) Determine the cost to the company of excess solder, overhead, and wages per hundred units for each of the plans, using the rates of pay determined in part (a).

(c) Determine the piece rates per hundred units for Plans B and C that will make the total cost of each the same as that of Plan A.

(d) The company feels it is fair to share equally the savings brought about by the adoption of Plan B or C with its employees. What will be the total cost to the company per 100 units and the earnings per hour of employees for each plan?

18. A firm has been organized to manufacture 1,200,000 units of a single product per year. The principal equipment needed to make the product is special machines costing $6,200 each and having a life of 12,000 hours with no salvage value. If the plant operates 40 hours per week 50 weeks per year, 12 machines will be required. The number of machines needed to meet the demand for the product will be in inverse proportion to the number of hours the plant operates per week. The variable cost of operating a machine, excluding labor, is $1.50 per hour. The material cost per unit of product is $.03. Direct labor will be $.04 per unit of product when the plant is operated 40 hours per week for 50 weeks per year.

Two plans are being considered: Plan A—purchase 12 machines and operate the plant 2,000 hours per year. Plan B—purchase 4 machines and operate the plant 6,000 hours per year with 3 shifts of workmen. With Plan A it is estimated that supervision will cost $9,000 per year and that fixed costs, exclusive of supervision and interest and depreciation for the 12 machines, will be $12,000 per year. With Plan B fixed costs, exclusive of supervision and interest and depreciation on the 4 machines, will be only $8,000 because less building space will be required.

Plan B has been rejected because it is believed that operation in three shifts will result in increased cost of supervision and direct labor which will more than offset the advantages of a smaller investment in equipment and building space. In the analysis of these plans it can be assumed that supervision and direct labor costs will be increased by X per cent by the difficulties of three-shift operation.

Calculate the value of X for Plans A and B to "break even."

CHAPTER

18

The Evaluation of Public Activities

AS A BASIS FOR EVALUATING PUBLIC ACTIVITIES it is necessary to have an understanding of the characteristics of the governmental agencies that sponsor them. For simplicity of discussion, the term *activity* will be taken to mean a project or undertaking of any nature.

A national government is a super-organization to which all agencies of the government and all organizations in a nation, including lesser political subdivisions such as states, counties, cities, townships, and school districts as well as private organizations and individuals, are subordinate. In some of its aspects the United States may be likened to a huge corporation. Its one hundred fifty million citizens play a role similar to that of stockholders. Each, if he chooses, may have a voice in the election of the policy-making group; the latter, the Congress of the United States, may be likened to a board of directors.

In the United States the lesser political subdivisions, such as states, counties, cities, and school districts, carry on their functions in much the same way as does the United States, for in them each citizen may have a voice in determining their policy. Each of these lesser political subdivisions has certain freedom of action, though each is in turn subordinate to its superior organization. The subdivisions of the government are delineated for the most part as continuous geographical areas that are easily recognized.

The Function of Government

In the United States it is a basic tenet that the purpose of government is to serve the people who constitute its citizens. The chief aims of the United States as stated in its constitution are the *national* *defense* and the *general welfare* of its citizens. For convenience in discussion these aims may be considered to be embraced by the single term *general welfare.* This simply stated aim is, however, very complex. To discharge it perfectly requires that the desires of each citizen be fulfilled to the greatest extent possible and in equal degree with those of every other citizen.

Since the general welfare is the aim of the United States, the super-organization to which the lesser political subdivisions are subordinate, it follows that the latter's aims must conform to the same general objective regardless of what other specific aims they may have.

The Nature of Public Activities

The government of the United States and its several subdivisions engage in innumerable activities all predicated upon the thesis of promotion of the general welfare. So numerous are the services available to individual citizens, associations, and private enterprises that books are required to catalog them.[1]

Governmental activities may be classified under the headings, Protection, Enlightenment and Cultural Enhancement, and Economic Benefits. A few common activities representative of life in the United States are given below.

PROTECTION

- The military establishment, state and national
- Police forces of a political subdivision
- The system of jurisprudence
- Flood control
- Mine rescue activities
- Lighthouses and other navigational aids
- Pension payments
- Sanitation and health services

ENLIGHTENMENT AND CULTURAL ENHANCEMENT

- The public school system
- Municipal, state, and national colleges
- Library of Congress and other publicly supported libraries

[1] Archie Robertson, *The Government at Your Service* (Boston: Houghton Mifflin Company, 1939).

Publicly supported research
Postal service
Recreational facilities, such as national parks, trailways, ski courses, golf courses, tennis courts, bathing beaches, fish hatcheries, game preserves

Economic Benefits

Postal service
Harbors and canals
Power development
Irrigation development
Flood control
Research and informational services such as those provided by the Department of Commerce, the Bureau of Standards, the Bureau of Mines, the Bureau of Fisheries, the National Forest Service, and the Department of Agriculture
Regulatory bodies like the Corporation Commission and similar commissions of most states, the Interstate Commerce Commission, and the Federal Trade Commission

In the above incomplete list it will be recognized that there is much overlapping in classification. For example, the educational system is considered by many to contribute to the protection, the enlightenment, and the economic benefit of people. Consideration of the purposes of governmental activities as suggested by the above classification is necessary in considering the pertinency of economic analysis to public activities.

Following World War II, approximately one-fourth of the national income was collected as taxes and expended for governmental activities. In point of expenditure, protection, particularly as represented by the military establishment, is of greatest importance. Other leading activities, as judged by the expenditure they require, are the public educational system, maintenance and construction of the highway system, and old-age benefits.

Engineering a Factor in Most Public Activities

Engineering is a factor in nearly all public activities because of the physical facilities these activities require. Most activities will at least have to be housed in some type of structure. The success of many activities is largely dependent upon the science and art of engineering.

Our system of highways embodies much engineering. The curves and grades of highways must be laid out in relation to the physical requirements of the vehicles that are to use them. Plans for their construction must encompass the physical characteristics of the soil on which they are to be built and the materials of which they are made. Engineering also comes into play in the design and usage of equipment required for highway construction.

The military might of nations is now largely expressed in terms of their industrial potential. Modern military establishments, if effective, are highly mechanized. Such military weapons as the tank and its gyroscopic gun stabilizer, proximity fuses, and atomic bombs are in large measure products of the application of engineering abilities.

A large percentage of expenditures on all governmental levels involve engineering directly or indirectly; many engineers find an outlet for their talents as employees or consultants of governmental agencies. Thus, public activities are a concern of all engineers as citizens and of many engineers as an outlet for their talents.

The Basis for Judging Desirability of Public Activities

Since each citizen may have a voice, if he will exercise it, in a government, the objectives of the government stem from the people. For this reason the objectives taken by the government must be presumed to express the objectives necessary for attainment of the general welfare of the citizenry as perfectly as they can be expressed. This must be so, for there is no superior authority to decide the issue.

Thus, when the United States declares war, it must be presumed that this act is taken in the interest of the general welfare. Similarly, when a state votes paving bonds, it also must be presumed to be in the interest of the general welfare of its citizens. The same reasoning applies to all activities undertaken by any political subdivision; for, if an opposite view is taken, it is necessary to assume that people collectively act contrary to their wishes.

Broadly speaking, the final measure of the desirability of an activity of any governmental unit is the judgment of the people in that unit. The exception to this is when a subordinate unit attempts an activity whose objective is contrary to that of a superior unit, in which case the final measure of desirability will rest in part in the people of the superior unit. Also, it must be clear that governmental activities are evaluated by a summation of judgments of individual citizens whose basis for judgment has been the general welfare as each sees it. The objectives of most governmental activities appear to be primarily social in nature, although economic considerations are often a factor. Public activities are proposed, implemented, and judged by the same group, namely the people of the governmental unit concerned.

The situation of the private enterprise is quite different. Those in control of private enterprise propose and implement services to be offered to the public, which judges whether the services are worth their cost. To survive, a private business organization must, at least, balance its income and costs; thus profit is of necessity a primary objective. For the same reason a private enterprise is rarely able to consider social objectives except to the extent that they improve its competitive position.

Thus, profit must be a first consideration in evaluating an activity of private enterprise. But when a public activity is being considered, the question to be decided is: Will it result in the greatest possible enhancement of the general welfare (in terms of economic, social, cultural, and other satisfactions) as judged by the people in the governmental unit concerned?

The General-Welfare Objective as Seen by the Individual Citizen

In the previous section it was stated that public activities are evaluated by a summation of judgments of individual citizens, each of whose basis for judgment has been the general welfare as he sees it. Each citizen is the product of his unique heredity and environment; his home, cultural patterns, education, and aspirations differ from those of his neighbor. Because of this and the additional fact that human viewpoints are rarely logically determined, it is rare for large groups of citizens to see eye to eye on the desirability of proposed public activities.

The father of a family of several active children may be expected to see more point to expenditures for school and recreational facilities than to expenditures for a street-widening program planned to enhance the value of downtown property. It is not difficult for a person to extol the value of aviation to his community if a proposed airport will increase the value of his property or if he expects to receive the contract to build it. Many public activities have no doubt been strongly supported by a few persons primarily because they would profit handsomely thereby.

But it is incorrect to conclude that activities are supported only by those who see in them opportunity for economic gain. For example, schools and recreational facilities for youth are often strongly supported by people who have no children. Many public activities are directed to the conservation of national resources for the benefit of future generations. Public projects are undertaken in periods of severe unemployment to alleviate suffering, even though this end might be accomplished more cheaply with a dole.

It is clear that the benefits of public activities are very complex. Some that are of great general benefit may spell ruin for some persons and vice versa. Lack of knowledge of the long-run effect of proposed activities is probably the most serious obstacle in the way of the selection of those activities that can contribute most to the general welfare.

The Financing of Public Activities

Funds to finance public activities are obtained through the assessment of various types of taxes and charges for services. Governmental receipts are derived chiefly from income, property, and excise taxes and duties on imports. Considerable income on some governmental levels is derived from fees collected for services. Examples of such incomes on the national level are incomes from postal services, and on the city level, incomes from supplying water service and from levies on property owners for sidewalks, pavements, and sewers adjacent to their property.

Two basic philosophies in the United States greatly influence the collection of funds and their expenditure by governmental subdivisions. These are collection of taxes on the premise of *ability to pay* and the expenditure of funds on the basis of *equalizing opportunity* of citizens. Application of the ability-to-pay viewpoint is clearly demonstrated in our income and property tax schedules. The equalization-of-opportunity philosophy is apparent in federal assistance to lesser subdivisions to help them provide improved educational and health programs, highway systems, old-age assistance, and the like. This philosophy is also manifested in the federal farm subsidy program.

Relationship of Payments for and Benefits Received from Public Activities by an Individual Citizen

Because of the two basic tenets of taxation on the basis of ability to pay and expenditure of tax funds on the basis of equalization of opportunity, there often is little relationship between the benefits that an individual receives and the amount he pays for public activities. This is in large measure true of such major activities as government itself, military and police protection, the highway system, and most educational activities.

There are, however, some public services that are paid for at the point of exchange in much the same way as occurs with services provided by private interests. This is common practice in the sale of city water, irrigation water, and electric current by public agencies. In such cases, the benefits received by an individual may be expected to be fairly closely related to the payments he makes for them.

Illustration of distribution of paving costs on the basis of benefit to users.[2] Let it be assumed that a certain state is contemplating a highway development embracing the construction of 8,000,000 square yards of pavement. Vehicle registration in the state is as follows:

Passenger cars	400,000
Light trucks	80,000
Medium trucks	15,000
Heavy trucks	5,000

The characteristics and pavements necessary to carry the above vehicles are taken as follows:

Class of Vehicle	*Pavement Thickness*	*Cost Per Sq. Yard*	*Increment Cost*
Passenger cars	5.5 inches	$1.80	$14,400,000
Light trucks	6.0 inches	2.10	2,400,000
Medium trucks	6.5 inches	2.30	1,600,000
Heavy trucks	7.0 inches	2.50	1,600,000

On the assumption that paving costs should be distributed on the basis of the number of vehicles in each class and the increment costs

[2] Adapted from an illustrative problem in Harry Tucker and Marc C. Leager, *Highway Economics* (Scranton, Pa.: International Textbook Company, 1942), pp. 221–222.

of paving required for each class of vehicle, the following analysis applies:

Allocation of Increment Per Vehicle	*Passenger Cars*	*Light Trucks*	*Medium Trucks*	*Heavy Trucks*
$14,400,000 ÷ 500,000	$28.80	$28.80	$28.80	$28.80
2,400,000 ÷ 100,000		24.00	24.00	24.00
1,600,000 ÷ 20,000			80.00	80.00
1,600,000 ÷ 5,000				320.00
Total	$28.80	$52.80	$132.80	$452.80

If it is desired to collect taxes on the basis of the cost of service, a suitable tax plan must be devised. This may be done by assessing a fuel tax and a vehicle license tax of proper amounts.

Pertinency of Economic Analysis of Public Activities

It must be recognized that the standards by which a private business, which must have profits to survive, evaluates its activities are markedly different from those that apply in the evaluation of public activities. Broadly speaking, private activities are evaluated in terms of profit and public activities are evaluated in terms of the general welfare. This is well illustrated by the following narrative.

On December 9, 1948, an Army airplane was forced to land with 7 Army personnel on the Greeland ice cap. Fortunately, no one was seriously hurt. Both the Army and the Navy made attempts to rescue the flyers. Expense was not spared, and the lives of additional personnel were risked. Three futile rescue attempts resulted in the marooning of five additional men on the ice cap and the loss of one rescue airplane and two gliders. After rescue of all the men had been brought about on December 28, 1948, one newspaper story itemized the cost of the rescue as follows:

B-17 airplane equipped for rescue........................	$300,000
Two G.S. 15A gliders....................................	44,000
Fuel, installation of special equipment and other items on the *Saipan*, Navy vessel..............................	50,000
Total...	$394,000

The story pointed out that the cost of rescue was much greater than the total given because many costs associated with the rescue had not been itemized. The story closed with the following significant paragraph:

"Cost of rescue is seldom tabulated by the armed services, spokesmen said. They explained that the rescues must be attempted regardless of expense; no one keeps close tab on the dollars and cents expended."[3]

No criticism of the huge expenditure of funds in the rescue was voiced by the press and none would have been voiced in regard to the expense had the rescue attempts failed. In the minds of the people, economic analysis of the rescue activity was not pertinent.

Analyses of economy are applicable to government activities only when dollars-and-cents economy is pertinent.

In this connection consider the postal service. This agency usually operates at a deficit. This deficit could be ascribed to a number of causes, such as that rates charged to transport certain classes of mail such as newspapers, magazines, postal cards, and air mail are lower than cost and that service in many sparsely settled areas is unprofitable.

Postal rates and the extent of postal service are based upon the premise of the general welfare. The people say in effect "charge these low rates and maintain service to the remotest sections of our land; you may tax us to do so if necessary." Obviously, economic measures in the ordinary sense are not pertinent in this connection.

But economic measures are pertinent in comparing the cost with which a given governmental activity is carried on in comparison with the cost of carrying on an identical or comparable private activity.

In general, it may be said that the general welfare as collectively and effectively expressed by the people concerned is the primary basis for evaluating public activities and that economy in terms of cost, income, and profit is a secondary basis.

Applicability of Engineering Economy Studies to Public Activities

In Chapter 7 a pattern for engineering economy studies was described as embracing the following steps:

(1) Creative Step
(2) Definitive Step

[3] *Oklahoma City Times*, December 29, 1948, p. 17.

(3) Conversion Step
(4) Decision Step

The creative step directs analysis toward (a) the discovery of opportunities for the profitable employment of resources or (b) the discovery of means to circumvent factors limiting the profit of present activities.

Proposals of public activities relate to the employment of public resources for the general welfare. The primary objective of public activities is the general welfare and not profit in the usual sense. Therefore, the creative step as defined is not applicable. This is easily illustrated.

Suppose that a municipality has under consideration two projects, one a swimming pool and the other a library. The municipality has resources for one or the other, but not both. The selection can not be made on the basis of profit, since no profit is in prospect for either venture. The selection must be made on the basis of which will contribute most to the general welfare as expressed by the citizens of the community, perhaps by a vote. There is no superior basis for evaluating the contribution of each alternative to the general welfare.

Because profit is the objective of the creative step as outlined for an engineering economy study, it is directly applicable to public activities only to the extent that it suggests objective search for activities of high merit in terms of the general welfare.

Engineering Economy Studies as Aids to Selection of Public Activities

It should be noted that evaluations of public activities in terms of the general welfare encompass both the benefits to be received from and the cost of the proposed activity. No matter how subjective an evaluation of the contribution of an activity to the general welfare may be, its cost may often be determined quite objectively. It may be fairly simple to determine the immediate and subsequent costs in prospect if either the swimming pool or the library in the above example is constructed. A knowledge of the costs in prospect for benefits to be gained may be expected to result in sounder selection of public activities.

The definitive, conversion, and decision steps as outlined for an engineering economy study are directly applicable to the analysis of the costs associated with public activities.

The analyst who wishes to serve the public well will take particular pains with the conversion step. Suppose that a proposal to install equipment to soften the water in a community is to be decided by a vote of the taxpayers. Since the average taxpayer has little understanding of technical matters, a detailed analysis of the hardness of the water and the chemical treatment necessary to soften it will help him little to understand the proposal. What he will be interested in is how it will affect him from day to day and what it will cost. Benefits at the point of use should be explained. For example, it might be shown that soft water will not stain fixtures, will permit better results in laundering and shampooing, will be easier on the hands in dishwashing, and will reduce expenditures for soap, water tanks, and plumbing repair.

Costs might be broken down as an expenditure per family per day, for the total of the necessary bond issue will ordinarily be meaningless. Still better results might be had by using diagrams, cartoons, and even window displays to visualize benefits and costs.

Probably the most serious obstacle in the way of wise choice of public activities is lack of accurate information in terms that are meaningful to the people concerned.

Consideration of Interest in Evaluation of Public Activities

The purpose of economic analysis of prospective public activities is to select and carry out activities that will result in the greatest satisfaction to the people concerned at the least cost.

Public activities financed through taxation require payments of funds from citizens. Broadly speaking, the funds expended for public activities should result in benefits comparable with those which the same funds would bring if expended in private ventures. It is almost universal for individuals to demand interest or its equivalent as an inducement to invest their private funds. To maintain public and private expenditures on a comparative basis, it seems logical to consider interest in economic evaluation of public activities that are financed through taxation.

Some public activities are financed in whole or in part through the sale of services or products. Examples of such activities are power developments, irrigation projects, housing projects, and toll bridges. Many such services could be carried on by private companies and are in general competitive with private enterprise. Again, since private enterprise must of necessity consider interest, it seems logical to consider interest in the economic analysis of public activities that compete in any way with private enterprise in order that both may be on a comparable basis.

Expenditures for capital goods are made on the promise that they will ultimately result in more consumer goods than can be had for a present equal expenditure. Interest represents the expected difference. Not to consider interest in the evaluation of public activities is equivalent to considering a future benefit equal to a present similar benefit. This appears to be contrary to human nature.

The interest rate to use in an economy study of a public activity is a matter of judgment. The rate used should not be less than that paid for funds borrowed for the activity. In many cases, particularly where the activity is comparable or competitive with private activities the rate used should be comparable with that used in private evaluations.

Consideration of Taxes in Evaluation of Public Activities

Many public activities result in loss of taxes through the removal of property from tax rolls or by other means, as, for example, the exemption from sales taxes.

In a nation where free enterprise is a fundamental philosophy, the basis for comparison of the cost of carrying on activities is the cost for which they can be carried on by well-managed private enterprises. Therefore, it seems logical to take taxes into consideration in economy analysis, particularly where the activities are competitive with private enterprise.

In a report of the Special Committee on Government of 1937 which was accepted by the Board of Directors of the Chamber of Commerce of the United States, the following statement appears:

> Should an emergency require that the government engage in any form of activity in competition with private enterprise, the true costs should be ascertained in accordance with standard business accounting practice and be made public. Full allowance should be made in costs for the loss to the federal government in taxes and for taxes of which the state and local government is deprived.[4]

[4] *Policy Declarations, 1948,* Chamber of Commerce of the United States, Washington, D. C.

Competitive Bidding, a Check on Costs of Public Activities

The point has been made that one basis for the evaluation of public activities is the cost for which they can be carried on by private enterprise. One common method for determining the cost of constructing and operating a public activity is to call for competitive bids from private organizations. The lowest bid received, after allowances for the bidder's ability to discharge the terms of his contract, is then a measure of cost. Except in unusual circumstances, there seems no justification for a governmental agency to undertake the construction and operation of a project unless it can do so for less than would be charged by private enterprise.

On the other hand, there are times when governmental agencies carry on activities as a means of determining if costs of private enterprise are fair.

It should be borne in mind that many of the activities of governmental units are of such a nature or size or of such uncertainty that they are beyond the scope of private enterprise. Such activities are not measurable through competitive bidding.

Economic Cost of Personal Injuries or Death

Knowledge of the economic losses associated with accidental injury and death are needed to evaluate the extent of precautionary measures that may be taken by both private and public agencies to minimize the number and severity of accidents. In spite of the most careful precautions, personal injuries or death due to accidents will occur.

Personal accidents are the source of pain, grief, and inconvenience to the victim and his dependents, his employer, and the nation. Pain, grief, and inconvenience appear impossible of evaluation in economic terms. But the economic losses associated with personal accidents have been the subject of much inquiry and considerable information is available for use.

The National Safety Council has advanced the following figures for evaluating the cost of traffic accidents:

For each death	$11,500
For each personal injury	425
For each property damage	125

These figures include wage loss, medical expense, overhead, cost of insurance, and property damage. On the basis of 150 property damages and 35 disabling accidents per fatality, each death is taken to involve a cost of $45,000.

The use of accident cost figures is illustrated in the following example. Three persons have been killed and 12 persons have been injured during a 10-year period at a railway crossing. During the same period, there were 22 accidents that resulted in property damage only. A tabulation of the cost of accidents on the basis of the National Safety Council figures follows:

For 3 fatalities, 3 × $11,500	$34,500
For 12 injuries, 12 × $425	5,100
For 22 property damage accidents, 22 × $125	2,750
Total for 10-year period	$42,350
Average annual accident cost	$ 4,235

On the basis of this annual cost, what investment in an overpass that will eliminate accidents is justified if interest is taken at 4 per cent and maintenance is estimated at 3 per cent of the cost of the structure and if its life is estimated at 40 years? Let C equal the cost of the structure. Then

$$\$4{,}235 = C \times \overset{\text{RP 4-40}}{(.05052)} + C \times .03$$

$$C = \frac{\$4{,}235}{.08052}$$

$$C = \$52{,}600$$

Evaluation of injuries and death in terms of prospective earnings of victims. When a person has suffered a partially or totally disabling accident, his prospective future earnings may be expected to be decreased; death, of course, would eliminate future earnings entirely. The money loss of earnings occasioned by disability, accidents, or death may be expressed as the present worth of the loss of the victim's prospective earnings at the time of the accident. The present worth of the prospective earnings of an individual depends upon his future annual earnings, his expected years of life, the rate of interest used in calculating present worth, and other factors. Table 45 gives the present net and gross future earnings of persons corresponding to specified annual earnings at stated ages after deductions of income tax. Net earnings are equal to gross earnings less the cost of living of the individual in question. According to the table, for example, the present value of the net and gross prospective earnings of an individual 35 years of age whose annual

TABLE 45. Present Value of Net and Gross Future Earnings Corresponding to Specified Annual Earnings* at Stated Ages—Average Mortality, 2½ Per Cent Interest[5]—Net Value Given in Italics, Gross Value Given in Bold-Face Type

Age	Annual Earnings at Stated Age*						
	$1,500	$2,000	$2,500	$3,000	$4,000	$6,000	$8,000
	$	$	$	$	$	$	$
25	*21,300*	*32,400*	*42,800*				
	35,500	**51,200**	**66,000**				
30	*15,400*	*23,800*	*32,300*	*41,000*			
	26,400	**38,500**	**50,600**	**63,000**			
35	*12,700*	*19,700*	*26,600*	*33,400*	*50,100*		
	22,700	**32,900**	**43,000**	**53,200**	**75,700**		
40	*10,200*	*15,700*	*21,300*	*26,800*	*39,300*	*87,100*	
	19,500	**28,100**	**36,600**	**45,300**	**63,300**	**126,700**	
45	*7,600*	*11,800*	*16,000*	*20,300*	*30,400*	*58,900*	*82,800*
	16,300	**23,500**	**30,700**	**38,200**	**53,500**	**92,500**	**124,600**
50	*5,200*	*8,100*	*11,000*	*14,000*	*21,600*	*39,400*	*58,000*
	13,300	**19,000**	**24,800**	**31,100**	**43,400**	**68,900**	**94,900**
55	*3,000*	*4,500*	*6,100*	*7,800*	*12,800*	*24,600*	*36,500*
	10,000	**14,400**	**18,900**	**23,800**	**32,900**	**51,000**	**68,800**
60	*800*	*800*	*800*	*900*	*3,300*	*9,300*	*15,300*
	6,400	**9,400**	**12,400**	**15,500**	**21,300**	**32,600**	**43,600**

* After deduction of income tax.
Note. Present value of gross earnings less the present worth of the future cost of living of the wage earner is equal to the present net value of earnings.

earning, at that age, is $3,000, are $33,400 and $53,200 respectively. The net value may be used to evaluate loss due to fatal accidents or death. The gross value may be used to evaluate loss due to disabling accidents.

Table 46 gives a schedule of permanent partial disabilities in percentages of total disability. In this table, for example, the loss of an eye by a worker 30 years old is shown to impair his earning power by

[5] Table 45 is an abridgement of Tables 52 and 55 in Louis I. Dublin, Alfred J. Lotka and Mortimer Spiegelman, *The Money Value of Man*, Revised Edition (New York: The Ronald Press Company, Inc., 1946), pp. 194, 195, 200, 201.

20 per cent. If his earnings had been at the rate of $2,500 per year, the present value of his loss in earnings may be calculated to be .20 × $50,600 = $10,120. The $50,600 figure is taken from Table 45.

Tables 45 and 46 are general guides for arriving at loss of earnings resulting from injury and death. For application to specific cases, values given should be modified to be in accord with conditions that apply. For example, the loss of a hand by a professional pianist may be expected to cause greater disablement than that shown in Table 46. Also, an individual life expectancy that is less than normal should be taken into consideration in applying the values given in Table 45.

TABLE 46. Permanent Partial Disability Schedule Stated in Percentages of Total Disability. Applicable to Ordinary Manual Workers[6]

Injury	Age, in Years				
	20 and Under	30	40	50	60
Loss of					
Arm at shoulder	40	50	55	65	80
Arm at or above elbow	34	42.5	47	55	68
Hand at or above wrist	26.5	33.5	36.5	43.5	53.5
Thumb	8	10	11	13	16
Index finger	4	5	5.5	6.5	8
Middle finger	3	4	4.5	5	6.5
Ring finger	2.5	3	3	4	5
Little finger	2.5	3	3	4	5
Leg at hip	40	50	55	65	80
Leg at or above knee	34	42.5	47	55	68
Foot at or above ankle	20	25	27.5	32.5	40
Great toe	3	4	4.5	5	6.5
Other toe	1	1	1	1	1.5
Eye	16	20	22	26	32
Hearing, one ear	4	5	5.5	6.5	8
Hearing, both ears	26.5	33.5	36.5	43.5	53.5

PROBLEMS AND QUESTIONS

1. A small city owns its electrical power plant and water plant. Income from the power plant is sufficient to pay for all the operating expenses of the city, including a deficit normally encountered each year from the sale

[6] Table 46 is an abridgement from Table 26 in Louis I. Dublin, Alfred J. Lotka and Mortimer Spiegelman, *The Money Value of Man*, Revised Edition (New York: The Ronald Press Company, Inc., 1946), p. 106.

of water. The city prides itself on being "tax-free." Discuss this situation in relationship to the "ability to pay" and "equalization of opportunity" concepts of taxation.

2. Many municipally-owned water systems receive insufficient income to equal the cost of operation and the deficit balance is taken from revenue received through property and other taxes. Is this evidence that water systems are inefficiently operated? Explain.

3. Name and contrast the criteria of evaluation for private and public enterprises or activities.

4. Give several reasons why interest should be considered in evaluating prospective investments in capital goods by governmental subdivisions even though they will be paid directly from income derived from tax levies.

5. Give reasons why loss of taxes, through removal from tax rolls of property acquired for public activities, should be considered in evaluating public activities.

6. The death rate per 100,000,000 miles of automobile travel was approximately 17 in 1930 and 8 in 1948. Calculate the cost of traffic hazards per vehicle mile for 1930 and 1948 on the basis of the National Safety Council evaluation given in the text.

What expenditure is justified per year per mile of a highway that has a traffic flow of 4,000 vehicles per day to reduce traffic hazards by 20 per cent on the basis of the 1948 death rate given above?

7. A state whose population is 3,000,000 has 4 state-supported colleges and universities with a total enrollment of 28,000 students and an annual budget for instructional programs approximating $7,000,000 per year.

(a) Most students enrolled in the colleges state that prospective increased earnings is an important reason for attending. What percentage of the state's population do you believe desire to support the program of higher education through taxation for this reason?

(b) Suppose that you are enrolled as a student in an undergraduate course of instruction in one of the state colleges. A legislator with whom you are speaking states that he finds it contrary to public interest to tax the entire population of the state to support the educational program in order that a relatively small number of students may benefit through increased future earnings. State several reasons that you can defend why he should support the appropriation bills for the colleges and universities.

8. Mr. A and Mr. B each invested $14,000 in a venture in 1949 which resulted in a net income of 7% on the investment before income taxes. Mr. A's and Mr. B's incomes from other sources were $4,000 and $40,000 respectively. State and federal income taxes on Mr. A's total taxable earnings for 1949 amounted to $1,007. Total income taxes he would have paid would have been $759 if there had been no return on his $14,000 investment. State and federal income taxes on Mr. B's total taxable earnings for 1949 amounted to $19,218.40. The amount of income taxes he would have paid would have been $18,551.20 if there had been no return on his $14,000 investment.

(a) What net return in per cent after income taxes did Mr. A and Mr. B each receive on their investment of $14,000?

(b) Discuss the relative desirability of a $14,000 investment in tax-exempt bonds paying 2.5% interest for both Mr. A and Mr. B.

9. In a certain city the fire insurance premium rate is $.28 per $100 of the insured amount. There are approximately 3,600 dwellings of an average valuation of $6,200 in the city. It is estimated that the insured amount represents 45 per cent of the value of the dwellings. The city commissioners have been advised that the fire insurance premium rate will be reduced to $.18 per $100 if the following improvements are made to reduce fire losses.

Improvement	*Cost*	*Life*
Increase capacity of trunk water lines from pumping station	$23,000	40 years
Increase capacity of pumps	$ 4,600	20 years
Add supply tank to increase pressure in remote sections of town	$32,000	40 years
Purchase additional fire truck and related equipment	$ 3,700	20 years
Add two firemen	$ 4,800 per year	

Operation costs of added improvements are expected to be offset by decreased pumping cost brought about by the enlargement of trunk water lines. The city is in a position to increase its bonded indebtedness and can sell sinking-fund bonds bearing 3% interest at par. Should the above improvements be made on the basis of prospective savings in insurance premiums paid by the home owners?

10. The main line of a railway runs through the business district of a city of 200,000 population. The business people of the city complain that the railway should be relocated to eliminate the smoke, noise, nuisance, and interruption of traffic that results from the passage of trains. The topography of the city is such that overpasses are not practical. The railway company agrees to relocate on a new route satisfactory to the city if the city pays $8,500,000. The right of way, which will revert to the city, has an estimated sale value of $4,300,000. Enhancement of property values due to relocation is estimated to result in an increase of city tax receipts of $62,000 annually. No concrete value has been placed on the advantage of smoke and noise elimination.

Interference with traffic was a factor at the intersections of 14 streets with the railway. The traffic flow averaged 5,800 vehicles per 24 hours per intersection. On the average 140 vehicles per intersection were held up 1.4 minutes in every 24-hour period by the passage of 21 trains. Vehicles not stopped by trains lost an average of 4.3 seconds each because of the roughness of the intersection or the caution of the drivers, which prompted a reduction of speed. Time losses were evaluated at $1.20 per hour per vehicle.

For purposes of evaluating the desirability of relocation of the rail route, tax receipts and time losses of vehicles are considered to continue at the present rate forever. The project, if undertaken, will be financed by bonds bearing 2.75% interest.

Find the capitalized amounts of the cost and the benefits of the relocation project to determine if the city should undertake it.

11. Many years ago a road was built between two sections of a city separated by a swamp. This road skirted the swamp. At first many people of the city considered the road unneeded and predicted that it would be little used because most people traveled by electric railway. With the passage of time the road was improved and some years ago was surfaced with two-lane pavement. Throughout its life the amount of traffic handled by the road has increased and now peak traffic exceeds 2,000 vehicles per hour per lane. With this traffic flow the observed average speed of traffic is reduced to 17 miles per hour. A survey has revealed that the weighted average speed throughout the day is 24 miles per hour. City officials are being pressed to "do something about it" and two proposals are being considered.

Proposal M provides for extension of the present road to accommodate four lanes of traffic through addition of pavement and reconditioning of the present pavement. The present road is 3.46 miles in length and the total cost of proposal M is $160,000. Annual maintenance is estimated at $4,000.

Proposal N provides for abandonment of the present road and construction of a 4-lane highway across the swamp 2.68 miles in length at a cost of $122,000 per mile. Annual maintenance is estimated at $11,000.

Based on a survey of population growth and trends in traffic counts, engineers estimate a peak traffic flow of 1,600 vehicles per hour per lane will be reached in twenty years and that the average total traffic flow during the next 20 years will be 26,000 vehicles per day. They estimate average speed of traffic when flow is 1,600 cars per hour per lane will be not less than 28 miles per hour and that the weighted average speed of cars throughout the next 20 years will be 36 miles per hour.

A survey of the traffic reveals the following data pertinent to the evaluation of the two proposals:

	Per Cent of Total Traffic	*Operation Cost Per Mile*	*Driver Cost Per Hour*
Passenger car, pleasure......	36	$.060	0
Passenger car and light truck, commercial..............	55	.064	1.40
Medium truck..............	7	.126	1.40
Heavy truck...............	2	.192	1.60

If Proposal N is selected, it is estimated that the development of property adjacent to the highway will result in an average net increase of tax receipts of $4,200 per year during the next 20 years. The city will finance either of the proposals through the sale of 3% sinking-fund bonds.

Compare proposals M and N on the basis of equivalent annual costs.

References

Alford, L. P., *Principles of Industrial Management.* New York: The Ronald Press Company, Inc., 1940.

Barnard, Chester I., *The Functions of the Executive.* Cambridge, Massachusetts: Harvard University Press, 1938.

Boulding, Kenneth E., *Economic Analysis.* New York: Harper and Brothers, 1941.

Bowers, Edison L., and Rountree, R. Henry, *Economics for Engineers.* New York: McGraw-Hill Book Company, Inc., 1938.

Bryant, J. M., and Herrman, R. R., *Elements of Utility Rate Determination.* New York: McGraw-Hill Book Company, Inc., 1940.

Bullinger, Clarence E., *Engineering Economic Analysis.* New York: McGraw-Hill Book Company, Inc., 1942.

Canfield, D. T., and Bowman, J. A., *Business, Legal, and Ethical Phases of Engineering.* New York: McGraw-Hill Book Company, Inc., 1948.

Dublin, Louis I., Lotka, Alfred J., and Speigelman, Mortimer, *The Money Value of Man.* New York: The Ronald Press Company, Inc., 1946.

Dublin, Louis I., and Lotka, Alfred J., *Length of Life, A Study of the Life Table.* New York: The Ronald Press Company, Inc., 1936.

Eidmann, Frank L., *Economic Control of Engineering and Manufacturing.* New York: McGraw-Hill Book Company, Inc., 1931.

Eigelberner, J., *The Investigation of Business Problems.* New York: McGraw-Hill Book Company, Inc., 1926.

Gillette, Halbert P., and Black, John C., *Road and Street Construction Methods and Costs.* Chicago: Gillette Publishing Company, 1940.

Gorham, R. C., *Power Economics for Engineering Students.* Pittsburgh: Pittsburgh Printing Company, 1939.

Grant, Eugene L., *Principles of Engineering Economy.* New York: The Ronald Press Company, Inc., 1938.

Grant, Eugene L., *Statistical Quality Control.* New York: McGraw-Hill Book Company, Inc., 1946.

Harding, C. Francis, and Canfield, Donald T., *Business Administration for Engineers.* New York: McGraw-Hill Book Company, Inc., 1937.

Heckert, J. Brooks, *Business Budgeting and Control.* New York: The Ronald Press Company, Inc., 1946.

Heckert, J. Brooks, *The Analysis and Control of Distribution Costs.* New York: The Ronald Press Company, Inc., 1940.

Hempel, Edward A., *The Economics of Chemical Industries.* New York: John Wiley and Sons, Inc., 1939.

Holmes, W. Gerald, *Plant Locations.* New York: McGraw-Hill Book Company, Inc., 1930.

Huebner, S. S., *Property Insurance.* New York: D. Appleton-Century Company, Inc., 1938.

Justine, Joel D., and Mervine, William G., *Power Supply Economics.* New York: John Wiley & Sons, Inc., 1934.

Kennedy, Clifford W., *Quality Control Methods.* New York: Prentice-Hall, Inc., 1948.

Kurtz, Edwin B., *The Science of Valuation and Depreciation.* New York: The Ronald Press Company, Inc., 1937.

Lehoczky, Paul N., *Quantitative Management in Industry.* Columbus, Ohio: Harold L. Hedrick, 1938.

Lester, Bernard, *Marketing Industrial Equipment.* New York: McGraw-Hill Book Company, Inc., 1935.

Lester, Bernard, *Sales Engineering.* New York: John Wiley & Sons, Inc., 1940.

Louden, J. K., *Wage Incentives.* New York: John Wiley & Sons, Inc., 1944.

Marston, Anson, and Agg, Thomas R., *Engineering Valuations.* New York: McGraw-Hill Book Company, Inc., 1936.

Maynard, Harold N., Weidler, Walter C., and Beckman, Theodore N., *Principles of Marketing.* New York: The Ronald Press Company, Inc., 1932.

Rautenstrauch, Walter, and Villers, Raymond, *The Economics of Industrial Management.* New York: Funk and Wagnalls Company, 1949.

Robertson, Archie, *The Government at Your Service.* Boston: Houghton Mifflin Company, 1939.

Rutherford, John G., *Quality Control in Industry.* New York: Pitman Publishing Company, 1948.

Simon, Herbert A., *Administrative Behavior.* New York: The Macmillan Company, 1948.

Steinberg, Max J., and Smith, Theodore N., *Economy Loading of Power Plants and Electric Systems.* New York: John Wiley & Sons, Inc., 1943.

Tucker, Harry, and Leager, Marc C., *Highway Economics.* Scranton, Pennsylvania: International Textbook Company, 1942.

Tyler, Chaplin, *Chemical Engineering Economics.* New York: McGraw-Hill Book Company, Inc., 1938.

Viteles, Morris S., *Industrial Psychology.* New York: W. W. Norton & Company, Inc., 1932.

Wilson, G. Lloyd, Herring, James M., and Eutsler, Roland B., *Public Utility Industries.* New York: McGraw-Hill Book Company, Inc., 1936.

Woods, Baldwin M., and DeGarmo, Paul E., *Introduction to Engineering Economy.* New York: The Macmillan Company, 1943.

Interest Tables

TABLE 47. ½% Compound Interest Factors

n	Single Payment: Compound-Amount Factor — Given P To find S $(1+i)^n$ — SP i-n (xxxxx)	Single Payment: Present-Worth Factor — Given S To find P $\frac{1}{(1+i)^n}$ — PS i-n (xxxxx)	Equal Payment Series: Compound-Amount Factor — Given R To find S $\frac{(1+i)^n-1}{i}$ — SR i-n (xxxxx)	Equal Payment Series: Sinking-Fund Factor — Given S To find R $\frac{i}{(1+i)^n-1}$ — RS i-n (xxxxx)	Equal Payment Series: Present-Worth Factor — Given R To find P $\frac{(1+i)^n-1}{i(1+i)^n}$ — PR i-n (xxxxx)	Equal Payment Series: Capital-Recovery Factor — Given P To find R $\frac{i(1+i)^n}{(1+i)^n-1}$ — RP i-n (xxxxx)	n
1	1.005	0.9950	1.000	1.00000	0.995	1.00500	1
2	1.010	0.9901	2.005	0.49875	1.985	0.50375	2
3	1.015	0.9851	3.015	0.33167	2.970	0.33667	3
4	1.020	0.9802	4.030	0.24813	3.950	0.25313	4
5	1.025	0.9754	5.050	0.19801	4.926	0.20301	5
6	1.030	0.9705	[illegible].076	0.16460	5.896	0.16960	6
7	[illegible]	0.9657	7.106	0.14073	6.862	0.14573	7
8	1.041	0.9609	8.141	0.12283	7.823	0.12783	8
9	1.046	0.9561	9.182	0.10891	8.779	0.11391	9
10	1.051	0.9513	10.228	0.09777	9.730	0.10277	10
11	1.056	0.9466	11.279	0.08866	10.677	0.09366	11
12	1.062	0.9419	12.336	0.08107	11.619	0.08607	12
13	1.067	0.9372	13.397	0.07464	12.556	0.07964	13
14	1.072	0.9326	14.464	0.06914	13.489	0.07414	14
15	1.078	0.9279	15.537	0.06436	14.417	0.06936	15
16	1.083	0.9233	16.614	0.06019	15.340	0.06519	16
17	1.088	0.9187	17.697	0.05651	16.259	0.06151	17
18	1.094	0.9141	18.786	0.05323	17.173	0.05823	18
19	1.099	0.9096	19.880	0.05030	18.082	0.05530	19
20	1.105	0.9051	20.979	0.04767	18.987	0.05267	20
21	1.110	0.9006	22.084	0.04528	19.888	0.05028	21
22	1.116	0.8961	23.194	0.04311	20.784	0.04811	22
23	1.122	0.8916	24.310	0.04113	21.676	0.04613	23
24	1.127	0.8872	25.432	0.03932	22.563	0.04432	24
25	1.133	0.8828	26.559	0.03765	23.446	0.04265	25
26	1.138	0.8784	27.692	0.03611	24.324	0.04111	26
27	1.144	0.8740	28.830	0.03469	25.198	0.03969	27
28	1.150	0.8697	29.975	0.03336	26.068	0.03836	28
29	1.156	0.8653	31.124	0.03213	26.933	0.03713	29
30	1.161	0.8610	32.280	0.03098	27.794	0.03598	30
31	1.167	0.8567	33.441	0.02990	28.651	0.03490	31
32	1.173	0.8525	34.609	0.02889	29.503	0.03389	32
33	1.179	0.8482	35.782	0.02795	30.352	0.03295	33
34	1.185	0.8440	36.961	0.02706	31.196	0.03206	34
35	1.191	0.8398	38.145	0.02622	32.035	0.03122	35
40	1.221	0.8191	44.159	0.02265	36.172	0.02765	40
45	1.252	0.7990	50.324	0.01987	40.207	0.02487	45
50	1.283	0.7793	56.645	0.01765	44.143	0.02265	50
55	1.316	0.7601	63.126	0.01584	47.981	0.02084	55
60	1.349	0.7414	69.770	0.01433	51.726	0.01933	60
65	1.383	0.7231	76.582	0.01306	55.377	0.01806	65
70	1.418	0.7053	83.566	0.01197	58.939	0.01697	70
75	1.454	0.6879	90.727	0.01102	62.414	0.01602	75
80	1.490	0.6710	98.068	0.01020	65.802	0.01520	80
85	1.528	0.6545	105.594	0.00947	69.108	0.01447	85
90	1.567	0.6383	113.311	0.00883	72.331	0.01383	90
95	1.606	0.6226	121.222	0.00825	75.476	0.01325	95
100	1.647	0.6073	129.334	0.00773	78.543	0.01273	100

TABLE 48. 1% Compound Interest Factors

n	Single Payment: Compound-Amount Factor	Single Payment: Present-Worth Factor	Equal Payment Series: Compound-Amount Factor	Equal Payment Series: Sinking-Fund Factor	Equal Payment Series: Present-Worth Factor	Equal Payment Series: Capital-Recovery Factor	n
	Given P To find S $(1+i)^n$	Given S To find P $\frac{1}{(1+i)^n}$	Given R To find S $\frac{(1+i)^n-1}{i}$	Given S To find R $\frac{i}{(1+i)^n-1}$	Given R To find P $\frac{(1+i)^n-1}{i(1+i)^n}$	Given P To find R $\frac{i(1+i)^n}{(1+i)^n-1}$	
	SP *i-n* (xxxxx)	PS *i-n* (xxxxx)	SR *i-n* (xxxxx)	RS *i-n* (xxxxx)	PR *i-n* (xxxxx)	RP *i-n* (xxxxx)	
1	1.010	0.9901	1.000	1.00000	0.990	1.01000	1
2	1.020	0.9803	2.010	0.49751	1.970	0.50751	2
3	1.030	0.9706	3.030	0.33002	2.941	0.34002	3
4	1.041	0.9610	4.060	0.24628	3.902	0.25628	4
5	1.051	0.9515	5.101	0.19604	4.853	0.20604	5
6	1.062	0.9420	6.152	0.16255	5.795	0.17255	6
7	1.072	0.9327	7.214	0.13863	6.728	0.14863	7
8	1.083	0.9235	8.286	0.12069	7.652	0.13069	8
9	1.094	0.9143	9.369	0.10674	8.566	0.11674	9
10	1.105	0.9053	10.462	0.09558	9.471	0.10558	10
11	1.116	0.8963	11.567	0.08645	10.368	0.09645	11
12	1.127	0.8874	12.683	0.07885	11.255	0.08885	12
13	1.138	0.8787	13.809	0.07241	12.134	0.08241	13
14	1.149	0.8700	14.947	0.06690	13.004	0.07690	14
15	1.161	0.8613	16.097	0.06212	13.865	0.07212	15
16	1.173	0.8528	17.258	0.05794	14.718	0.06794	16
17	1.184	0.8444	18.430	0.05426	15.562	0.06426	17
18	1.196	0.8360	19.615	0.05098	16.398	0.06098	18
19	1.208	0.8277	20.811	0.04805	17.226	0.05805	19
20	1.220	0.8195	22.019	0.04542	18.046	0.05542	20
21	1.232	0.8114	23.239	0.04303	18.857	0.05303	21
22	1.245	0.8034	24.472	0.04086	19.660	0.05086	22
23	1.257	0.7954	25.716	0.03889	20.456	0.04889	23
24	1.270	0.7876	26.973	0.03707	21.243	0.04707	24
25	1.282	0.7798	28.243	0.03541	22.023	0.04541	25
26	1.295	0.7720	29.526	0.03387	22.795	0.04387	26
27	1.308	0.7644	30.821	0.03245	23.560	0.04245	27
28	1.321	0.7568	32.129	0.03112	24.316	0.04112	28
29	1.335	0.7493	33.450	0.02990	25.066	0.03990	29
30	1.348	0.7419	34.785	0.02875	25.808	0.03875	30
31	1.361	0.7346	36.133	0.02768	26.542	0.03768	31
32	1.375	0.7273	37.494	0.02667	27.270	0.03667	32
33	1.389	0.7201	38.869	0.02573	27.990	0.03573	33
34	1.403	0.7130	40.258	0.02484	28.703	0.03484	34
35	1.417	0.7059	41.660	0.02400	29.409	0.03400	35
40	1.489	0.6717	48.886	0.02046	32.835	0.03046	40
45	1.565	0.6391	56.481	0.01771	36.095	0.02771	45
50	1.645	0.6080	64.463	0.01551	39.196	0.02551	50
55	1.729	0.5785	72.852	0.01373	42.147	0.02373	55
60	1.817	0.5504	81.670	0.01224	44.955	0.02224	60
65	1.909	0.5237	90.937	0.01100	47.627	0.02100	65
70	2.007	0.4983	100.676	0.00993	50.169	0.01993	70
75	2.109	0.4741	110.913	0.00902	52.587	0.01902	75
80	2.217	0.4511	121.672	0.00822	54.888	0.01822	80
85	2.330	0.4292	132.979	0.00752	57.078	0.01752	85
90	2.449	0.4084	144.863	0.00690	59.161	0.01690	90
95	2.574	0.3886	157.354	0.00636	61.143	0.01636	95
100	2.705	0.3697	170.481	0.00587	63.029	0.01587	100

TABLE 49. 2% Compound Interest Factors

n	Single Payment		Equal Payment Series				n
	Compound-Amount Factor	Present-Worth Factor	Compound-Amount Factor	Sinking-Fund Factor	Present-Worth Factor	Capital-Recovery Factor	
	Given P To find S $(1+i)^n$	Given S To find P $\frac{1}{(1+i)^n}$	Given R To find S $\frac{(1+i)^n-1}{i}$	Given S To find R $\frac{i}{(1+i)^n-1}$	Given R To find P $\frac{(1+i)^n-1}{i(1+i)^n}$	Given P To find R $\frac{i(1+i)^n}{(1+i)^n-1}$	
	SP i-n (xxxxx)	PS i-n (xxxxx)	SR i-n (xxxxx)	RS i-n (xxxxx)	PR i-n (xxxxx)	RP i-n (xxxxx)	
1	1.020	0.9804	1.000	1.00000	0.980	1.02000	1
2	1.040	0.9612	2.020	0.49505	1.942	0.51505	2
3	1.061	0.9423	3.060	0.32675	2.884	0.34675	3
4	1.082	0.9238	4.122	0.24262	3.808	0.26262	4
5	1.104	0.9057	5.204	0.19216	4.713	0.21216	5
6	1.126	0.8880	6.308	0.15853	5.601	0.17853	6
7	1.149	0.8706	7.434	0.13451	6.472	0.15451	7
8	1.172	0.8535	8.583	0.11651	7.325	0.13651	8
9	1.195	0.8368	9.755	0.10252	8.162	0.12252	9
10	1.219	0.8203	10.950	0.09133	8.983	0.11133	10
11	1.243	0.8043	12.169	0.08218	9.787	0.10218	11
12	1.268	0.7885	13.412	0.07456	10.575	0.09456	12
13	1.294	0.7730	14.680	0.06812	11.348	0.08812	13
14	1.319	0.7579	15.974	0.06260	12.106	0.08260	14
15	1.346	0.7430	17.293	0.05783	12.849	0.07783	15
16	1.373	0.7284	18.639	0.05365	13.578	0.07365	16
17	1.400	0.7142	20.012	0.04997	14.292	0.06997	17
18	1.428	0.7002	21.412	0.04670	14.992	0.06670	18
19	1.457	0.6864	22.841	0.04378	15.678	0.06378	19
20	1.486	0.6730	24.297	0.04116	16.351	0.06116	20
21	1.516	0.6598	25.783	0.03878	17.011	0.05878	21
22	1.546	0.6468	27.299	0.03663	17.658	0.05663	22
23	1.577	0.6342	28.845	0.03467	18.292	0.05467	23
24	1.608	0.6217	30.422	0.03287	18.914	0.05287	24
25	1.641	0.6095	32.030	0.03122	19.523	0.05122	25
26	1.673	0.5976	33.671	0.02970	20.121	0.04970	26
27	1.707	0.5859	35.344	0.02829	20.707	0.04829	27
28	1.741	0.5744	37.051	0.02699	21.281	0.04699	28
29	1.776	0.5631	38.792	0.02578	21.844	0.04578	29
30	1.811	0.5521	40.568	0.02465	22.396	0.04465	30
31	1.848	0.5412	42.379	0.02360	22.938	0.04360	31
32	1.885	0.5306	44.227	0.02261	23.468	0.04261	32
33	1.922	0.5202	46.112	0.02169	23.989	0.04169	33
34	1.961	0.5100	48.034	0.02082	24.499	0.04082	34
35	2.000	0.5000	49.994	0.02000	24.999	0.04000	35
40	2.208	0.4529	60.402	0.01656	27.355	0.03656	40
45	2.438	0.4102	71.893	0.01391	29.490	0.03391	45
50	2.692	0.3715	84.579	0.01182	31.424	0.03182	50
55	2.972	0.3365	98.587	0.01014	33.175	0.03014	55
60	3.281	0.3048	114.052	0.00877	34.761	0.02877	60
65	3.623	0.2761	131.126	0.00763	36.197	0.02763	65
70	4.000	0.2500	149.978	0.00667	37.499	0.02667	70
75	4.416	0.2265	170.792	0.00586	38.677	0.02586	75
80	4.875	0.2051	193.772	0.00516	39.745	0.02516	80
85	5.383	0.1858	219.144	0.00456	40.711	0.02456	85
90	5.943	0.1683	247.157	0.00405	41.587	0.02405	90
95	6.562	0.1524	278.085	0.00360	42.380	0.02360	95
100	7.245	0.1380	312.232	0.00320	43.098	0.02320	100

TABLE 50. 3% Compound Interest Factors

n	Single Payment: Compound-Amount Factor	Single Payment: Present-Worth Factor	Equal Payment Series: Compound-Amount Factor	Equal Payment Series: Sinking-Fund Factor	Equal Payment Series: Present-Worth Factor	Equal Payment Series: Capital-Recovery Factor	n
	Given P To find S $(1+i)^n$	Given S To find P $\frac{1}{(1+i)^n}$	Given R To find S $\frac{(1+i)^n-1}{i}$	Given S To find R $\frac{i}{(1+i)^n-1}$	Given R To find P $\frac{(1+i)^n-1}{i(1+i)^n}$	Given P To find R $\frac{i(1+i)^n}{(1+i)^n-1}$	
	SP i-n (xxxxx)	PS i-n (xxxxx)	SR i-n (xxxxx)	RS i-n (xxxxx)	PR i-n (xxxxx)	RP i-n (xxxxx)	
1	1.030	0.9709	1.000	1.00000	0.971	1.03000	1
2	1.061	0.9426	2.030	0.49261	1.913	0.52261	2
3	1.093	0.9151	3.091	0.32353	2.829	0.35353	3
4	1.126	0.8885	4.184	0.23903	3.717	0.26903	4
5	1.159	0.8626	5.309	0.18835	4.580	0.21835	5
6	1.194	0.8375	6.468	0.15460	5.417	0.18460	6
7	1.230	0.8131	7.662	0.13051	6.230	0.16051	7
8	1.267	0.7894	8.892	0.11246	7.020	0.14246	8
9	1.305	0.7664	10.159	0.09843	7.786	0.12843	9
10	1.344	0.7441	11.464	0.08723	8.530	0.11723	10
11	1.384	0.7224	12.808	0.07808	9.253	0.10808	11
12	1.426	0.7014	14.192	0.07046	9.954	0.10046	12
13	1.469	0.6810	15.618	0.06403	10.635	0.09403	13
14	1.513	0.6611	17.086	0.05853	11.296	0.08853	14
15	1.558	0.6419	18.599	0.05377	11.938	0.08377	15
16	1.605	0.6232	20.157	0.04961	12.561	0.07961	16
17	1.653	0.6050	21.762	0.04595	13.166	0.07595	17
18	1.702	0.5874	23.414	0.04271	13.754	0.07271	18
19	1.754	0.5703	25.117	0.03981	14.324	0.06981	19
20	1.806	0.5537	26.870	0.03722	14.877	0.06722	20
21	1.860	0.5375	28.676	0.03487	15.415	0.06487	21
22	1.916	0.5219	30.537	0.03275	15.937	0.06275	22
23	1.974	0.5067	32.453	0.03081	16.444	0.06081	23
24	2.033	0.4919	34.426	0.02905	16.936	0.05905	24
25	2.094	0.4776	36.459	0.02743	17.413	0.05743	25
26	2.157	0.4637	38.553	0.02594	17.877	0.05594	26
27	2.221	0.4502	40.710	0.02456	18.327	0.05456	27
28	2.288	0.4371	42.931	0.02329	18.764	0.05329	28
29	2.357	0.4243	45.219	0.02211	19.188	0.05211	29
30	2.427	0.4120	47.575	0.02102	19.600	0.05102	30
31	2.500	0.4000	50.003	0.02000	20.000	0.05000	31
32	2.575	0.3883	52.503	0.01905	20.389	0.04905	32
33	2.652	0.3770	55.078	0.01816	20.7[illegible]6	0.04816	33
34	2.732	0.3660	57.730	0.01732	21.132	0.04732	34
35	2.814	0.3554	60.462	0.01654	21.487	0.04654	35
40	3.262	0.3066	75.401	0.01326	23.115	0.04326	40
45	3.782	0.2644	92.720	0.01079	24.519	0.04079	45
50	4.384	0.2281	112.797	0.00887	25.730	0.03887	50
55	5.082	0.1968	136.072	0.00735	26.774	0.03735	55
60	5.892	0.1697	163.053	0.00613	27.676	0.03613	60
65	6.830	0.1464	194.333	0.00515	28.453	0.03515	65
70	7.918	0.1263	230.594	0.00434	29.123	0.03434	70
75	9.179	0.1089	272.631	0.00367	29.702	0.03367	75
80	10.641	0.0940	321.363	0.00311	30.201	0.03311	80
85	12.336	0.0811	377.857	0.00265	30.631	0.03265	85
90	14.300	0.0699	443.349	0.00226	31.002	0.03226	90
95	16.578	0.0603	519.272	0.00193	31.323	0.03193	95
100	19.219	0.0520	607.288	0.00165	31.599	0.03165	100

TABLE 51. 4% Compound Interest Factors

n	Single Payment: Compound-Amount Factor — Given P To find S $(1+i)^n$ — SP i-n (xxxxx)	Single Payment: Present-Worth Factor — Given S To find P $\frac{1}{(1+i)^n}$ — PS i-n (xxxxx)	Equal Payment Series: Compound-Amount Factor — Given R To find S $\frac{(1+i)^n-1}{i}$ — SR i-n (xxxxx)	Equal Payment Series: Sinking-Fund Factor — Given S To find R $\frac{i}{(1+i)^n-1}$ — RS i-n (xxxxx)	Equal Payment Series: Present-Worth Factor — Given R To find P $\frac{(1+i)^n-1}{i(1+i)^n}$ — PR i-n (xxxxx)	Equal Payment Series: Capital-Recovery Factor — Given P To find R $\frac{i(1+i)^n}{(1+i)^n-1}$ — RP i-n (xxxxx)	n
1	1.040	0.9615	1.000	1.00000	0.962	1.04000	1
2	1.082	0.9246	2.040	0.49020	1.886	0.53020	2
3	1.125	0.8890	3.122	0.32035	2.775	0.36035	3
4	1.170	0.8548	4.246	0.23549	3.630	0.27549	4
5	1.217	0.8219	5.416	0.18463	4.452	0.22463	5
6	1.265	0.7903	6.633	0.15076	5.242	0.19076	6
7	1.316	0.7599	7.898	0.12661	6.002	0.16661	7
8	1.369	0.7307	9.214	0.10853	6.733	0.14853	8
9	1.423	0.7026	10.583	0.09449	7.435	0.13449	9
10	1.480	0.6756	12.006	0.08329	8.111	0.12329	10
11	1.539	0.6496	13.486	0.07415	8.760	0.11415	11
12	1.601	0.6246	15.026	0.06655	9.385	0.10655	12
13	1.665	0.6006	16.627	0.06014	9.986	0.10014	13
14	1.732	0.5775	18.292	0.05467	10.563	0.09467	14
15	1.801	0.5553	20.024	0.04994	11.118	0.08994	15
16	1.873	0.5339	21.825	0.04582	11.652	0.08582	16
17	1.948	0.5134	23.698	0.04220	12.166	0.08220	17
18	2.026	0.4936	25.645	0.03899	12.659	0.07899	18
19	2.107	0.4746	27.671	0.03614	13.134	0.07614	19
20	2.191	0.4564	29.778	0.03358	13.590	0.07358	20
21	2.279	0.4388	31.969	0.03128	14.029	0.07128	21
22	2.370	0.4220	34.248	0.02920	14.451	0.06920	22
23	2.465	0.4057	36.618	0.02731	14.857	0.06731	23
24	2.563	0.3901	39.083	0.02559	15.247	0.06559	24
25	2.666	0.3751	41.646	0.02401	15.622	0.06401	25
26	2.772	0.3607	44.312	0.02257	15.983	0.06257	26
27	2.883	0.3468	47.084	0.02124	16.330	0.06124	27
28	2.999	0.3335	49.968	0.02001	16.663	0.06001	28
29	3.119	0.3207	52.966	0.01888	16.984	0.05888	29
30	3.243	0.3083	56.085	0.01783	17.292	0.05783	30
31	3.373	0.2965	59.328	0.01686	17.588	0.05686	31
32	3.508	0.2851	62.701	0.01595	17.874	0.05595	32
33	3.648	0.2741	66.210	0.01510	18.148	0.05510	33
34	3.794	0.2636	69.858	0.01431	18.411	0.05431	34
35	3.946	0.2534	73.652	0.01358	18.665	0.05358	35
40	4.801	0.2083	95.026	0.01052	19.793	0.05052	40
45	5.841	0.1712	121.029	0.00826	20.720	0.04826	45
50	7.107	0.1407	152.667	0.00655	21.482	0.04655	50
55	8.646	0.1157	191.159	0.00523	22.109	0.04523	55
60	10.520	0.0951	237.991	0.00420	22.623	0.04420	60
65	12.799	0.0781	294.968	0.00339	23.047	0.04339	65
70	15.572	0.0642	364.290	0.00275	23.395	0.04275	70
75	18.945	0.0528	448.631	0.00223	23.680	0.04223	75
80	23.050	0.0434	551.245	0.00181	23.915	0.04181	80
85	28.044	0.0357	676.090	0.00148	24.109	0.04148	85
90	34.119	0.0293	827.983	0.00121	24.267	0.04121	90
95	41.511	0.0241	1012.785	0.00099	24.398	0.04099	95
100	50.505	0.0198	1237.624	0.00081	24.505	0.04081	100

TABLE 52. 5% Compound Interest Factors

	Single Payment		Equal Payment Series				
	Compound-Amount Factor	Present-Worth Factor	Compound-Amount Factor	Sinking-Fund Factor	Present-Worth Factor	Capital-Recovery Factor	
n	Given P To find S $(1+i)^n$	Given S To find P $\frac{1}{(1+i)^n}$	Given R To find S $\frac{(1+i)^n-1}{i}$	Given S To find R $\frac{i}{(1+i)^n-1}$	Given R To find P $\frac{(1+i)^n-1}{i(1+i)^n}$	Given P To find R $\frac{i(1+i)^n}{(1+i)^n-1}$	n
	SP i-n (xxxxx)	PS i-n (xxxxx)	SR i-n (xxxxx)	RS i-n (xxxxx)	PR i-n (xxxxx)	RP i-n (xxxxx)	
1	1.050	0.9524	1.000	1.00000	0.952	1.05000	1
2	1.103	0.9070	2.050	0.48780	1.859	0.53780	2
3	1.158	0.8638	3.153	0.31721	2.723	0.36721	3
4	1.216	0.8227	4.310	0.23201	3.546	0.28201	4
5	1.276	0.7835	5.526	0.18097	4.329	0.23097	5
6	1.340	0.7462	6.802	0.14702	5.076	0.19702	6
7	1.407	0.7107	8.142	0.12282	5.786	0.17282	7
8	1.477	0.6768	9.549	0.10472	6.463	0.15472	8
9	1.551	0.6446	11.027	0.09069	7.108	0.14069	9
10	1.629	0.6139	12.578	0.07950	7.722	0.12950	10
11	1.710	0.5847	14.207	0.07039	8.306	0.12039	11
12	1.796	0.5568	15.917	0.06283	8.863	0.11283	12
13	1.886	0.5303	17.713	0.05646	9.394	0.10646	13
14	1.980	0.5051	19.599	0.05102	9.899	0.10102	14
15	2.079	0.4810	21.579	0.04634	10.380	0.09634	15
16	2.183	0.4581	23.657	0.04227	10.838	0.09227	16
17	2.292	0.4363	25.840	0.03870	11.274	0.08870	17
18	2.407	0.4155	28.132	0.03555	11.690	0.08555	18
19	2.527	0.3957	30.539	0.03275	12.085	0.08275	19
20	2.653	0.3769	33.066	0.03024	12.462	0.08024	20
21	2.786	0.3589	35.719	0.02800	12.821	0.07800	21
22	2.925	0.3418	38.505	0.02597	13.163	0.07597	22
23	3.072	0.3256	41.430	0.02414	13.489	0.07414	23
24	3.225	0.3101	44.502	0.02247	13.799	0.07247	24
25	3.386	0.2953	47.727	0.02095	14.094	0.07095	25
26	3.556	0.2812	51.113	0.01956	14.375	0.06956	26
27	3.733	0.2678	54.669	0.01829	14.643	0.06829	27
28	3.920	0.2551	58.403	0.01712	14.898	0.06712	28
29	4.116	0.2429	62.323	0.01605	15.141	0.06605	29
30	4.322	0.2314	66.439	0.01505	15.372	0.06505	30
31	4.538	0.2204	70.761	0.01413	15.593	0.06413	31
32	4.765	0.2099	75.299	0.01328	15.803	0.06328	32
33	5.003	0.1999	80.064	0.01249	16.003	0.06249	33
34	5.253	0.1904	85.067	0.01176	16.193	0.06176	34
35	5.516	0.1813	90.320	0.01107	16.374	0.06107	35
40	7.040	0.1420	120.800	0.00828	17.159	0.05828	40
45	8.985	0.1113	159.700	0.00626	17.774	0.05626	45
50	11.467	0.0872	209.348	0.00478	18.256	0.05478	50
55	14.636	0.0683	272.713	0.00367	18.633	0.05367	55
60	18.679	0.0535	353.584	0.00283	18.929	0.05283	60
65	23.840	0.0419	456.798	0.00219	19.161	0.05219	65
70	30.426	0.0329	588.529	0.00170	19.343	0.05170	70
75	38.833	0.0258	756.654	0.00132	19.485	0.05132	75
80	49.561	0.0202	971.229	0.00103	19.596	0.05103	80
85	63.254	0.0158	1245.087	0.00080	19.684	0.05080	85
90	80.730	0.0124	1594.607	0.00063	19.752	0.05063	90
95	103.035	0.0097	2040.694	0.00049	19.806	0.05049	95
100	131.501	0.0076	2610.025	0.00038	19.848	0.05038	100

TABLE 53. 6% Compound Interest Factors

	Single Payment		Equal Payment Series				
n	Compound-Amount Factor	Present-Worth Factor	Compound-Amount Factor	Sinking-Fund Factor	Present-Worth Factor	Capital-Recovery Factor	n
	Given P To find S $(1+i)^n$	Given S To find P $\frac{1}{(1+i)^n}$	Given R To find S $\frac{(1+i)^n-1}{i}$	Given S To find R $\frac{i}{(1+i)^n-1}$	Given R To find P $\frac{(1+i)^n-1}{i(1+i)^n}$	Given P To find R $\frac{i(1+i)^n}{(1+i)^n-1}$	
	SP i-n (xxxxx)	PS i-n (xxxxx)	SR i-n (xxxxx)	RS i-n (xxxxx)	PR i-n (xxxxx)	RP i-n (xxxxx)	
1	1.060	0.9434	1.000	1.00000	0.943	1.06000	1
2	1.124	0.8900	2.060	0.48544	1.833	0.54544	2
3	1.191	0.8396	3.184	0.31411	2.673	0.37411	3
4	1.262	0.7921	4.375	0.22859	3.465	0.28859	4
5	1.338	0.7473	5.637	0.17740	4.212	0.23740	5
6	1.419	0.7050	6.975	0.14336	4.917	0.20336	6
7	1.504	0.6651	8.394	0.11914	5.582	0.17914	7
8	1.594	0.6274	9.897	0.10104	6.210	0.16104	8
9	1.689	0.5919	11.491	0.08702	6.802	0.14702	9
10	1.791	0.5584	13.181	0.07587	7.360	0.13587	10
11	1.898	0.5268	14.972	0.06679	7.887	0.12679	11
12	2.012	0.4970	16.870	0.05928	8.384	0.11928	12
13	2.133	0.4688	18.882	0.05296	8.853	0.11296	13
14	2.261	0.4423	21.015	0.04758	9.295	0.10758	14
15	2.397	0.4173	23.276	0.04296	9.712	0.10296	15
16	2.540	0.3936	25.673	0.03895	10.106	0.09895	16
17	2.693	0.3714	28.213	0.03544	10.477	0.09544	17
18	2.854	0.3503	30.906	0.03236	10.828	0.09236	18
19	3.026	0.3305	33.760	0.02962	11.158	0.08962	19
20	3.207	0.3118	36.786	0.02718	11.470	0.08718	20
21	3.400	0.2942	39.993	0.02500	11.764	0.08500	21
22	3.604	0.2775	43.392	0.02305	12.042	0.08305	22
23	3.820	0.2618	46.996	0.02128	12.303	0.08128	23
24	4.049	0.2470	50.816	0.01968	12.550	0.07968	24
25	4.292	0.2330	54.865	0.01823	12.783	0.07823	25
26	4.549	0.2198	59.156	0.01690	13.003	0.07690	26
27	4.822	0.2074	63.706	0.01570	13.211	0.07570	27
28	5.112	0.1956	68.528	0.01459	13.406	0.07459	28
29	5.418	0.1846	73.640	0.01358	13.591	0.07358	29
30	5.743	0.1741	79.058	0.01265	13.765	0.07265	30
31	6.088	0.1643	84.802	0.01179	13.929	0.07179	31
32	6.453	0.1550	90.890	0.01100	14.084	0.07100	32
33	6.841	0.1462	97.343	0.01027	14.230	0.07027	33
34	7.251	0.1379	104.184	0.00960	14.368	0.06960	34
35	7.686	0.1301	111.435	0.00897	14.498	0.06897	35
40	10.286	0.0972	154.762	0.00646	15.046	0.06646	40
45	13.765	0.0727	212.744	0.00470	15.456	0.06470	45
50	18.420	0.0543	290.336	0.00344	15.762	0.06344	50
55	24.650	0.0406	394.172	0.00254	15.991	0.06254	55
60	32.988	0.0303	533.128	0.00188	16.161	0.06188	60
65	44.145	0.0227	719.083	0.00139	16.289	0.06139	65
70	59.076	0.0169	967.932	0.00103	16.385	0.06103	70
75	79.057	0.0126	1300.949	0.00077	16.456	0.06077	75
80	105.796	0.0095	1746.600	0.00057	16.509	0.06057	80
85	141.579	0.0071	2342.982	0.00043	16.549	0.06043	85
90	189.465	0.0053	3141.075	0.00032	16.579	0.06032	90
95	253.546	0.0039	4209.104	0.00024	16.601	0.06024	95
100	339.302	0.0029	5638.368	0.00018	16.618	0.06018	100

TABLE 54. 7% Compound Interest Factors

n	Single Payment		Equal Payment Series				*n*
	Compound-Amount Factor	Present-Worth Factor	Compound-Amount Factor	Sinking-Fund Factor	Present-Worth Factor	Capital-Recovery Factor	
	Given P To find S $(1+i)^n$	Given S To find P $\frac{1}{(1+i)^n}$	Given R To find S $\frac{(1+i)^n-1}{i}$	Given S To find R $\frac{i}{(1+i)^n-1}$	Given R To find P $\frac{(1+i)^n-1}{i(1+i)^n}$	Given P To find R $\frac{i(1+i)^n}{(1+i)^n-1}$	
	SP *i-n* (xxxxx)	PS *i-n* (xxxxx)	SR *i-n* (xxxxx)	RS *i-n* (xxxxx)	PR *i-n* (xxxxx)	RP *i-n* (xxxxx)	
1	1.070	0.9346	1.000	1.00000	0.935	1.07000	1
2	1.145	0.8734	2.070	0.48309	1.808	0.55309	2
3	1.225	0.8163	3.215	0.31105	2.624	0.38105	3
4	1.311	0.7629	4.440	0.22523	3.387	0.29523	4
5	1.403	0.7130	5.751	0.17389	4.100	0.24389	5
6	1.501	0.6663	7.153	0.13980	4.767	0.20980	6
7	1.606	0.6227	8.654	0.11555	5.389	0.18555	7
8	1.718	0.5820	10.260	0.09747	5.971	0.16747	8
9	1.838	0.5439	11.978	0.08349	6.515	0.15349	9
10	1.967	0.5083	13.816	0.07238	7.024	0.14238	10
11	2.105	0.4751	15.784	0.06336	7.499	0.13336	11
12	2.252	0.4440	17.888	0.05590	7.943	0.12590	12
13	2.410	0.4150	20.141	0.04965	8.358	0.11965	13
14	2.579	0.3878	22.550	0.04434	8.745	0.11434	14
15	2.759	0.3624	25.129	0.03979	9.108	0.10979	15
16	2.952	0.3387	27.888	0.03586	9.447	0.10586	16
17	3.159	0.3166	30.840	0.03243	9.763	0.10243	17
18	3.380	0.2959	33.999	0.02941	10.059	0.09941	18
19	3.617	0.2765	37.379	0.02675	10.336	0.09675	19
20	3.870	0.2584	40.995	0.02439	10.594	0.09439	20
21	4.141	0.2415	44.865	0.02229	10.836	0.09229	21
22	4.430	0.2257	49.006	0.02041	11.061	0.09041	22
23	4.741	0.2109	53.436	0.01871	11.272	0.08871	23
24	5.072	0.1971	58.177	0.01719	11.469	0.08719	24
25	5.427	0.1842	63.249	0.01581	11.654	0.08581	25
26	5.807	0.1722	68.676	0.01456	11.826	0.08456	26
27	6.214	0.1609	74.484	0.01343	11.987	0.08343	27
28	6.649	0.1504	80.698	0.01239	12.137	0.08239	28
29	7.114	0.1406	87.347	0.01145	12.278	0.08145	29
30	7.612	0.1314	94.461	0.01059	12.409	0.08059	30
31	8.145	0.1228	102.073	0.00980	12.532	0.07980	31
32	8.715	0.1147	110.218	0.00907	12.647	0.07907	32
33	9.325	0.1072	118.933	0.00841	12.754	0.07841	33
34	9.978	0.1002	128.259	0.00780	12.854	0.07780	34
35	10.677	0.0937	138.237	0.00723	12.948	0.07723	35
40	14.974	0.0668	199.635	0.00501	13.332	0.07501	40
45	21.002	0.0476	285.749	0.00350	13.606	0.07350	45
50	29.457	0.0339	406.529	0.00246	13.801	0.07246	50
55	41.315	0.0242	575.929	0.00174	13.940	0.07174	55
60	57.946	0.0173	813.520	0.00123	14.039	0.07123	60
65	81.273	0.0123	1146.755	0.00087	14.110	0.07087	65
70	113.989	0.0088	1614.134	0.00062	14.160	0.07062	70
75	159.876	0.0063	2269.657	0.00044	14.196	0.07044	75
80	224.234	0.0045	3189.063	0.00031	14.222	0.07031	80
85	314.500	0.0032	4478.576	0.00022	14.240	0.07022	85
90	441.103	0.0023	6287.185	0.00016	14.253	0.07016	90
95	618.670	0.0016	8823.854	0.00011	14.263	0.07011	95
100	867.716	0.0012	12381.662	0.00008	14.269	0.07008	100

TABLE 55. 8% Compound Interest Factors

n	Single Payment: Compound-Amount Factor — Given P To find S $(1+i)^n$ — SP i-n (xxxxx)	Single Payment: Present-Worth Factor — Given S To find P $\frac{1}{(1+i)^n}$ — PS i-n (xxxxx)	Equal Payment Series: Compound-Amount Factor — Given R To find S $\frac{(1+i)^n-1}{i}$ — SR i-n (xxxxx)	Equal Payment Series: Sinking-Fund Factor — Given S To find R $\frac{i}{(1+i)^n-1}$ — RS i-n (xxxxx)	Equal Payment Series: Present-Worth Factor — Given R To find P $\frac{(1+i)^n-1}{i(1+i)^n}$ — PR i-n (xxxxx)	Equal Payment Series: Capital-Recovery Factor — Given P To find R $\frac{i(1+i)^n}{(1+i)^n-1}$ — RP i-n (xxxxx)	n
1	1.080	0.9259	1.000	1.00000	0.926	1.08000	1
2	1.166	0.8573	2.080	0.48077	1.783	0.56077	2
3	1.260	0.7938	3.246	0.30803	2.577	0.38803	3
4	1.360	0.7350	4.506	0.22192	3.312	0.30192	4
5	1.469	0.6806	5.867	0.17046	3.993	0.25046	5
6	1.587	0.6302	7.336	0.13632	4.623	0.21632	6
7	1.714	0.5835	8.923	0.11207	5.206	0.19207	7
8	1.851	0.5403	10.637	0.09401	5.747	0.17401	8
9	1.999	0.5002	12.488	0.08008	6.247	0.16008	9
10	2.159	0.4632	14.487	0.06903	6.710	0.14903	10
11	2.332	0.4289	16.645	0.06008	7.139	0.14008	11
12	2.518	0.3971	18.977	0.05270	7.536	0.13270	12
13	2.720	0.3677	21.495	0.04652	7.904	0.12652	13
14	2.937	0.3405	24.215	0.04130	8.244	0.12130	14
15	3.172	0.3152	27.152	0.03683	8.559	0.11683	15
16	3.426	0.2919	30.324	0.03298	8.851	0.11298	16
17	3.700	0.2703	33.750	0.02963	9.122	0.10963	17
18	3.996	0.2502	37.450	0.02670	9.372	0.10670	18
19	4.316	0.2317	41.446	0.02413	9.604	0.10413	19
20	4.661	0.2145	45.762	0.02185	9.818	0.10185	20
21	5.034	0.1987	50.423	0.01983	10.017	0.09983	21
22	5.437	0.1839	55.457	0.01803	10.201	0.09803	22
23	5.871	0.1703	60.893	0.01642	10.371	0.09642	23
24	6.341	0.1577	66.765	0.01498	10.529	0.09498	24
25	6.848	0.1460	73.106	0.01368	10.675	0.09368	25
26	7.396	0.1352	79.954	0.01251	10.810	0.09251	26
27	7.988	0.1252	87.351	0.01145	10.935	0.09145	27
28	8.627	0.1159	95.339	0.01049	11.051	0.09049	28
29	9.317	0.1073	103.966	0.00962	11.158	0.08962	29
30	10.063	0.0994	113.283	0.00883	11.258	0.08883	30
31	10.868	0.0920	123.346	0.00811	11.350	0.08811	31
32	11.737	0.0852	134.214	0.00745	11.435	0.08745	32
33	12.676	0.0789	145.951	0.00685	11.514	0.08685	33
34	13.690	0.0730	158.627	0.00630	11.587	0.08630	34
35	14.785	0.0676	172.317	0.00580	11.655	0.08580	35
40	21.725	0.0460	259.057	0.00386	11.925	0.08386	40
45	31.920	0.0313	386.506	0.00259	12.108	0.08259	45
50	46.902	0.0213	573.770	0.00174	12.233	0.08174	50
55	68.914	0.0145	848.923	0.00118	12.319	0.08118	55
60	101.257	0.0099	1253.213	0.00080	12.377	0.09080	60
65	148.780	0.0067	1847.248	0.00054	12.416	0.08054	65
70	218.606	0.0046	2720.080	0.00037	12.443	0.08037	70
75	321.205	0.0031	4002.557	0.00025	12.461	0.08025	75
80	471.955	0.0021	5886.935	0.00017	12.474	0.08017	80
85	693.456	0.0014	8655.706	0.00012	12.482	0.08012	85
90	1018.915	0.0010	12723.939	0.00008	12.488	0.08008	90
95	1497.121	0.0007	18701.507	0.00005	12.492	0.08005	95
100	2199.761	0.0005	27484.516	0.00004	12.494	0.08004	100

TABLE 56. 10% Compound Interest Factors

	Single Payment		Equal Payment Series				
	Compound-Amount Factor	Present-Worth Factor	Compound-Amount Factor	Sinking-Fund Factor	Present-Worth Factor	Capital-Recovery Factor	
n	Given P To find S $(1+i)^n$	Given S To find P $\frac{1}{(1+i)^n}$	Given R To find S $\frac{(1+i)^n-1}{i}$	Given S To find R $\frac{i}{(1+i)^n-1}$	Given R To find P $\frac{(1+i)^n-1}{i(1+i)^n}$	Given P To find R $\frac{i(1+i)^n}{(1+i)^n-1}$	n
	SP i-n (xxxxx)	PS i-n (xxxxx)	SR i-n (xxxxx)	RS i-n (xxxxx)	PR i-n (xxxxx)	RP i-n (xxxxx)	
1	1.100	0.9091	1.000	1.00000	0.909	1.10000	1
2	1.210	0.8264	2.100	0.47619	1.736	0.57619	2
3	1.331	0.7513	3.310	0.30211	2.487	0.40211	3
4	1.464	0.6830	4.641	0.21547	3.170	0.31547	4
5	1.611	0.6209	6.105	0.16380	3.791	0.26380	5
6	1.772	0.5645	7.716	0.12961	4.355	0.22961	6
7	1.949	0.5132	9.487	0.10541	4.868	0.20541	7
8	2.144	0.4665	11.436	0.08744	5.335	0.18744	8
9	2.358	0.4241	13.579	0.07364	5.759	0.17364	9
10	2.594	0.3855	15.937	0.06275	6.144	0.16275	10
11	2.853	0.3505	18.531	0.05396	6.495	0.15396	11
12	3.138	0.3186	21.384	0.04676	6.814	0.14676	12
13	3.452	0.2897	24.523	0.04078	7.103	0.14078	13
14	3.797	0.2633	27.975	0.03575	7.367	0.13575	14
15	4.177	0.2394	31.772	0.03147	7.606	0.13147	15
16	4.595	0.2176	35.950	0.02782	7.824	0.12782	16
17	5.054	0.1978	40.545	0.02466	8.022	0.12466	17
18	5.560	0.1799	45.599	0.02193	8.201	0.12193	18
19	6.116	0.1635	51.159	0.01955	8.365	0.11955	19
20	6.727	0.1486	57.275	0.01746	8.514	0.11746	20
21	7.400	0.1351	64.002	0.01562	8.649	0.11562	21
22	8.140	0.1228	71.403	0.01401	8.772	0.11401	22
23	8.954	0.1117	79.543	0.01257	8.883	0.11257	23
24	9.850	0.1015	88.497	0.01130	8.985	0.11130	24
25	10.835	0.0923	98.347	0.01017	9.077	0.11017	25
26	11.918	0.0839	109.182	0.00916	9.161	0.10916	26
27	13.110	0.0763	121.100	0.00826	9.237	0.10826	27
28	14.421	0.0693	134.210	0.00745	9.307	0.10745	28
29	15.863	0.0630	148.631	0.00673	9.370	0.10673	29
30	17.449	0.0573	164.494	0.00608	9.427	0.10608	30
31	19.194	0.0521	181.943	0.00550	9.479	0.10550	31
32	21.114	0.0474	201.138	0.00497	9.526	0.10497	32
33	23.225	0.0431	222.252	0.00450	9.569	0.10450	33
34	25.548	0.0391	245.477	0.00407	9.609	0.10407	34
35	28.102	0.0356	271.024	0.00369	9.644	0.10369	35
40	45.259	0.0221	442.593	0.00226	9.779	0.10226	40
45	72.890	0.0137	718.905	0.00139	9.863	0.10139	45
50	117.391	0.0085	1163.909	0.00086	9.915	0.10086	50
55	189.059	0.0053	1880.591	0.00053	9.947	0.10053	55
60	304.482	0.0033	3034.816	0.00033	9.967	0.10033	60
65	490.371	0.0020	4893.707	0.00020	9.980	0.10020	65
70	789.747	0.0013	7887.470	0.00013	9.987	0.10013	70
75	1271.895	0.0008	12708.954	0.00008	9.992	0.10008	75
80	2048.400	0.0005	20474.002	0.00005	9.995	0.10005	80
85	3298.969	0.0003	32979.690	0.00003	9.997	0.10003	85
90	5313.023	0.0002	53120.226	0.00002	9.998	0.10002	90
95	8556.676	0.0001	85556.760	0.00001	9.999	0.10001	95
100	13780.612	0.0001	137796.123	0.00001	9.999	0.10001	100

TABLE 57. Capital-Recovery Factors for Interest Rates from 6% to 50%

$\frac{i(1+i)^n}{(1+i)^n - 1}$

Given P, To Find R

RP i-n
(xxxxx)

n	i = 6%	8%	10%	12%	15%	20%	25%	30%	40%	50%	n
1	1.06000	1.08000	1.10000	1.12000	1.15000	1.20000	1.25000	1.30000	1.40000	1.50000	1
2	0.54544	0.56077	0.57619	0.59170	0.61512	0.65455	0.69444	0.73478	0.81667	0.90000	2
3	0.37411	0.38803	0.40211	0.41635	0.43798	0.47473	0.51230	0.55063	0.62936	0.71053	3
4	0.28859	0.3[illegible]192	0.31547	0.32923	0.35027	0.38629	0.42344	0.46163	0.54077	0.62308	4
5	0.23740	0.2[illegible]046	0.26380	0.27741	0.29832	0.33438	0.37184	0.41058	0.49136	0.57582	5
6	0.20336	0.21632	0.22961	0.24[illegible]	0.26424	0.30071	0.33882	0.37840	0.46126	0.54812	6
7	0.17914	0.19207	0.20541	[illegible]	0.240[illegible]	0.27742	0.31634	0.35687	9.44192	0.53108	7
8	0.16104	0.17401	0.18744	0.2013[illegible]	[illegible]	0.26061	0.30040	0.34191	0.42804	0.52030	8
9	0.14702	[illegible].16008	0.17364	0.18768	0.20957	0.24808	0.28876	0.33123	0.42034	0.51335	9
10	0.13587	0.14903	0.16275	0.17698	0.19925	0.23852	0.28007	0.32346	0.41432	0.50823	10
11	0.12679	0.14008	0.15396	0.16842	0.19107	0.23110	0.27349	0.31773	0.41013	0.50585	11
12	0.11928	0.13270	0.14676	0.16144	0.18448	0.22526	0.26845	0.31345	0.40718	0.50388	12
13	0.11296	0.12652	0.14078	0.15568	0.17911	0.22062	0.26454	0.31024	0.40510	0.50258	13
14	0.10758	0.12130	0.13575	0.15087	0.17469	0.21689	0.26150	0.30782	0.40363	0.50172	14
15	0.10296	0.11683	0.13147	0.14682	0.17102	0.21388	0.25912	0.30598	0.40259	0.50114	15
16	0.09895	0.11298	0.12782	0.14339	0.16795	0.21144	0.25724	0.30458	0.40185	0.50076	16
17	0.09544	0.10963	0.12466	0.14046	0.16537	0.20944	0.25576	0.30351	0.40132	0.50051	17
18	0.09236	0.10670	0.12193	0.13794	0.16319	0.20781	0.25459	0.30269	0.40094	0.50034	18
19	0.08962	0.10413	0.11955	0.13576	0.16134	0.20646	0.25366	0.30206	0.40067	0.50023	19
20	0.08718	0.10185	0.11746	0.13388	0.15976	0.20536	0.25292	0.30159	0.40048	0.50016	20
25	0.07823	0.09368	0.11017	0.12750	0.15470	0.20212	0.25095	0.30043	0.40009	0.50002	25
30	0.07265	0.08883	0.10608	0.12414	0.15230	0.20085	0.25031	0.30011	0.40002	0.50000	30
40	0.06646	0.08386	0.10226	0.12130	0.15056	0.20014	0.25003	0.30008	0.40001	0.50000	40
50	0.06344	0.08174	0.10086	0.12042	0.15014	0.20002	0.25000	0.30001	0.40000	0.50000	50
100	0.06018	0.08004	0.10001	0.12000	0.15000	0.20000	0.25000	0.30000	0.40000	0.50000	100
∞	0.06000	0.08000	0.10000	0.12000	0.15000	0.20000	0.25000	0.30000	0.40000	0.50000	∞

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