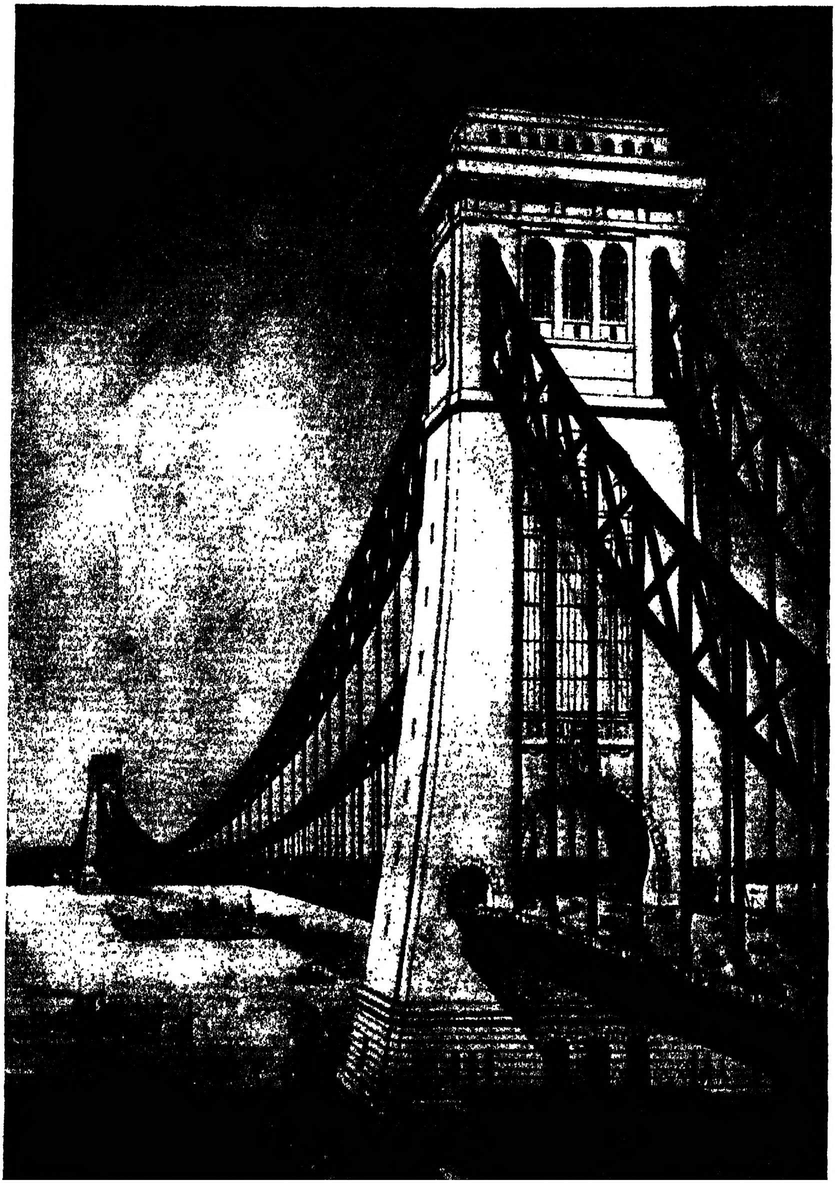
# A PRACTICAL TREATISE

ON

# SUSPENSION BRIDGES

THEIR DESIGN, CONSTRUCTION AND ERECTION



Proposed Hudson River Bridge, Perspective from New York Shore. Span 3240 Ft.
Design by G. Lindenthal, 1921.

(*Frontispiece*)

# A PRACTICAL TREATISE

ON

# SUSPENSION BRIDGES

## THEIR DESIGN, CONSTRUCTION AND ERECTION


BY

D. B. STEINMAN, A.M., C.E., Ph.D.

CONSULTING ENGINEER

*Member American Society of Civil Engineers; Member American Railway Engineering Association; Formerly Professor in Charge of Civil and Mechanical Engineering at the College of the City of New York*


WITH APPENDICES ON:

DESIGN CHARTS FOR SUSPENSION BRIDGES

THE FLORIANOPOLIS BRIDGE

THE OHIO RIVER SUSPENSION BRIDGE AT PORTSMOUTH

THE DEFLECTION THEORY

CHRONOLOGICAL TABLE OF SUSPENSION BRIDGES

*SECOND EDITION*


NEW YORK

JOHN WILEY & SONS, INC.

LONDON: CHAPMAN & HALL, LIMITED

SECOND EDITION
*Sixth Printing, June, 1957*

PRINTED IN THE UNITED STATES OF AMERICA

# PREFACE TO THE SECOND EDITION

During the few years since the first publication of this book, considerable progress has been recorded in the art of suspension-bridge design and erection. There have been new developments in materials, forms, methods, and principles of construction. In preparing this second edition, the author has endeavored to keep the book abreast of the latest developments in the art, by revisions in the body of the text as well as by additions of new material.

Among the portions of the text that have been rewritten or amplified are those bearing on the topics of wire cables *vs.* eye-bars, rocker towers, anchorage design, materials used in suspension bridges, economic and limiting proportions, comments on the deflection theory, formulas for temperature stresses, multiple-span bridges, proportioning cable bands, calculation of cable diameter, and time records of erection. Such typographical or mathematical errors as have been discovered in the original pages have been corrected, and some convenient new formulas have been added.

Following the original Appendix on Design Charts for Suspension Bridges, four new Appendices have been added. These, the author hopes, will substantially enhance the usefulness of the book for study and reference.

To bring to a focus, in the light of recent progress, the previous separate treatments of design theory, construction details, and erection methods, there have been added comprehensive illustrative accounts of the planning and execution of actual projects, from conception of design to completion of construction. For this purpose, two representative modern structures have been used: Appendix B, on the Florianopolis Bridge, covers the latest developments in the design and erection of *eyebar* suspension bridges, as represented by the largest executed example of that type. Appendix C, on the Ohio River Bridge at Portsmouth, presents an illustrated account of the details of design and construction of a modern *wire-cable* suspension bridge of ordinary span-length. In both cases, the explanatory descriptions of typical and novel features and methods, and the record of problems met and expedients developed, should be helpful to others who may be confronted with like problems in field and office.

Appendix D, on the Deflection Theory, is perhaps the most important contribution to this new edition. It presents the "more exact" method of computing stresses in suspension bridges, to supplement the common, approximate theory developed in Chapter I. In writing this section on the Deflection Theory, the author has aimed at maximum simplicity, conciseness, and practical convenience. The treatment is illustrated by a numerical example of the application to an actual design, - the Mount Hope Bridge. Finally, to supplement the quick design charts given in Appendix A based on the Elastic Theory, there is presented in Appendix D a new graphic chart by which the more exact stresses of the Deflection Theory may be expeditiously evaluated.

Appendix E presents a Chronological Table of Suspension Bridges, intended to facilitate reference and to aid historical review of the development of the wire-cable and eyebar types of construction. The author wishes to express his indebtedness to the many who have contributed data to help make this the most complete tabulation of suspension bridges thus far published.

In conclusion, the author wishes to state that this book has been a labor of love, prepared at considerable personal sacrifice, in pursuance of the principle that a man should give of his best to his profession. In proportion as this work may prove helpful to others,—fellow-engineers and students,—the author will feel repaid for his effort.

D. B. Steinman.

New York City
October 7, 1928.

## PREFACE TO THE FIRST EDITION

THIS book has been planned to supply the needs of practicing engineers who may have problems in estimating, designing or constructing suspension bridges, and of students who wish to prepare themselves for work in this field. The aim has been to produce an up-to-date, practical handbook on the subject, distinguished by simplicity of treatment and convenience of application.

In the first division, on Stresses in Suspension Bridges, the formulas have been corrected to conform to modern practice. and reduced to their simplest and most convenient form for direct application by the designing engineer. The formulas are supplemented by curves for their expeditious solution, and by alternative graphical methods for determining stresses.

The second division, on Types and Details of Construction, presents data and illustrations to assist the designing engineer in the selection of type of suspension bridge and in the determination of proportions, specifications, and details for the various elements of suspension construction.

The third division, on Typical Design Computations, gives numerical examples of suspension designs of different types worked out by methods that have proved most efficient in the author's practice. The designing engineer will find here the formulas to be used in each successive step of the design, and the practical methods of applying them, with tabulations, graphs and short-cuts.

The fourth division, on Erection of Suspension Bridges, describes and illustrates the successive stages in the erection of representative structures, from towers to trusses. The operations of stringing wire cables are presented in detail, with an outline of the computations for adjustment and control.

Methods of erecting eyebar chains and other types are also described and illustrated.

The Appendix presents a series of design charts, specially devised for this book, for the expeditious proportioning of suspension bridges. These charts give quickly and accurately the governing stresses throughout any span, saving the time and labor of applying the stress formulas otherwise required.

The author desires to express his indebtedness to his associate, Mr. Holton D. Robinson, for reviewing the manuscript on Erection; and to the Department of Plant and Structures of New York City for many courtesies extended.

D. B. STEINMAN.

NEW YORK CITY
August 1, 1922.

# CONTENTS

## CHAPTER I

## STRESSES IN SUSPENSION BRIDGES

### SECTION I.—THE CABLE



### SECTION II.—UNSTIFFENED SUSPENSION BRIDGES



### SECTION III.—STIFFENED SUSPENSION BRIDGES



### SECTION IV.—THREE-HINGED STIFFENING TRUSSES



### SECTION V.—TWO-HINGED STIFFENING TRUSSES

## CHAPTER III

## TYPICAL DESIGN COMPUTATIONS

### Example 1

*Calculations for Two-hinged Suspension Bridge with Straight Backstays*
*(Type 2F.)*



### Example 2

*Calculations for Two-hinged Suspension Bridge with Suspended Side Spans*
*(Type 2S.)*



### Example 3

*Calculations for Towers of Two-hinged Suspension Bridge*
*(Type 2S.)*



### Example 4

*Estimates of Cable and Wrapping*



### Example 5

*Analysis of Suspension Bridge with Continuous Stiffening Truss*

## APPENDIX C

### THE OHIO RIVER SUSPENSION BRIDGE AT PORTSMOUTH



## APPENDIX D

### THE DEFLECTION THEORY

## APPENDIX E

### CHRONOLOGICAL TABLE OF SUSPENSION BRIDGES

# A PRACTICAL TREATISE

ON

# SUSPENSION BRIDGES

THEIR DESIGN, CONSTRUCTION AND ERECTION

---

## CHAPTER I

## STRESSES IN SUSPENSION BRIDGES

### SECTION I.—THE CABLE

**1. Form of the Cable for Any Loading.**—If vertical loads are applied on a cable suspended between two points, it will assume a definite polygonal form determined by the relations between the loads (Fig. 1*a*).

The end reactions ($T_1$ and $T_2$) will be inclined and will have horizontal components $H$. Simple considerations of static equilibrium show that $H$ will be the same for both end reactions, and will also equal the horizontal component of the tension in the cable at any point. $H$ is called the horizontal tension of the cable

Let $M'$ denote the bending moment produced at any point of the span by the *vertical* loads and reactions, calculated as for a simple beam. Since $H$, the horizontal component of the end reaction, acts with a lever-arm $y$, the total moment at any point of the cable will be

$$M = M' - H \cdot y. \qquad (1)$$

This moment must be equal to zero if the cable is assumed to be flexible. Hence,

$$M' = H \cdot y, \qquad (2)$$

and

$$y=\frac{M'}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Equation (3) gives the ordinates to the cable curve for any loading, if the horizontal tension $H$ is known. Since $H$ is constant, the curve is simply *the bending moment diagram for the applied loads*, drawn to the proper scale. The scale for con-

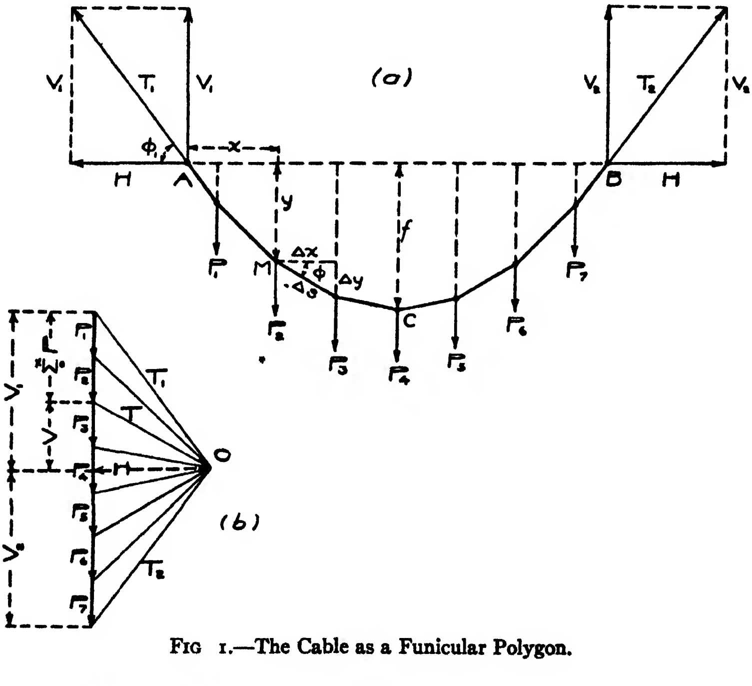



FIG 1.—The Cable as a Funicular Polygon.

structing this diagram is determined if the ordinate of any point of the curve, such as the lowest point, is given. If $f$ is the sag of the cable, or ordinate to the lowest point $C$, and if $M_c$ is the simple-beam bending moment at the same point, then $H$ is determined from Eq. (2) by

$$H=\frac{M_c}{f}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

To obtain the cable curve graphically, simply draw the equilibrium polygon for the applied loads, as indicated in Fig. 1

$(a, b)$. The pole distance $H$ must be found by trial or computation so as to make the polygon pass through the three specified points, $A$, $B$, and $C$. The tension $T$ at any point of the cable is given by the length of the corresponding ray of the pole diagram. $H$, the horizontal component of all cable tensions, is constant. By similar triangles, the figure yields

$$T = H \cdot \frac{\Delta s}{\Delta x} = H \cdot \sec \phi, \quad . \quad . \quad . \quad . \quad . \quad (5)$$

where $\phi$ is the inclination of the cable to the horizontal at any point. It should be noted that the tensions $T$ in the successive members of the polygon increase toward the points of support and attain their maximum values in the first and last members of the system.

If $V_1$ is the vertical component of the left end reaction, the vertical shear at any section $x$ of the span will be

$$V = V_1 - \sum_0^x P. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

This will also be the vertical component of the cable tension at the same point. By similar triangles,

$$V = H \cdot \frac{\Delta y}{\Delta x} = H \cdot \tan \phi. \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

(This relation is also obtained by differentiating both members of Eq. 2). Combining Eqs. (6) and (7), we may write

$$\frac{\Delta y}{\Delta x} = \frac{V_1 - \sum_0^x P}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If the loads are continuously distributed, the funicular polygon becomes a continuous curve. If $w$ is the load per horizontal linear unit at any point having the abscissa $x$, Eq. (8) becomes

$$\frac{dy}{dx} = \frac{V_1 - \int_0^x w \cdot dx}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

from which (by differentiation) we obtain the following as the differential equation of the equilibrium curve:

$$\frac{d^2y}{dx^2} = -\frac{w}{H}. \qquad (10)$$

For any given law of variation of the continuous load $w$, the integration of Eq. (10) will give the equation of the curve assumed by the cable.

**2. The Parabolic Cable.**—For a uniform distributed load, the bending moment diagram is a parabola. Consequently, by Eq. (3), if a cable carries a uniform load ($w$ per horizontal linear unit), the resulting equilibrium curve will be a parabola.

The maximum bending moment in a simple beam would be

$$M_c = \frac{wl^2}{8}.$$

Substituting this value in Eq. (4), the horizontal tension is determined:

$$H = \frac{wl^2}{8f}. \qquad (11)$$

To obtain the equation of the curve, integrate Eq. (10). With the origin of coordinates at the crown, the integration yields

$$y = \frac{wx^2}{2H}. \qquad (12)$$

Substituting the value of $H$ from Eq. (11), we obtain the equation of the parabola,

$$y = 4f\frac{x^2}{l^2}. \qquad (13)$$

If the origin of coordinates is taken at one of the supports (as $A$, Fig. 1), the equation becomes,

$$y = \frac{4fx}{l^2}(l - x). \qquad (14)$$

The maximum tension in the cable, occurring at either support, will be

$$T_1 = \sqrt{H^2 + (\tfrac{1}{2}wl)^2},$$

or, by Eq. (11),

$$T_1 = \frac{wl^2}{8f}\sqrt{1+16n^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where $n$ denotes the ratio of the sag $f$ to the span $l$:

$$n = \frac{f}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Equation (15) may also be derived from Eq. (5) by noting that the inclination of a parabolic cable at the support is given by,

$$\tan \phi_1 = \frac{4f}{l} = 4n. \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

To find the length of the cable, $L$, use the general formula,

$$L = 2\int_0^{\frac{l}{2}} \left[1 + \left(\frac{dy}{dx}\right)^2\right]^{1/2} \cdot dx. \quad . \quad . \quad . \quad . \quad (18)$$

Substituting the value of $\frac{dy}{dx}$ obtained from Eq. (13), we have,

$$L = 2\int_0^{\frac{l}{2}} \left[1 + \frac{64f^2x^2}{l^4}\right]^{1/2} \cdot dx, \quad . \quad . \quad . \quad . \quad (19)$$

which yields, upon integration,

$$L = \frac{l}{2}(1+16n^2)^{1/2} + \frac{l}{8n}\log_e\left[4n + (1+16n^2)^{1/2}\right]. \quad . \quad (20)$$

This formula gives the exact length of the parabola between two ends at equal elevation.*

For more expeditious solution, when a good table of hyperbolic functions is available, Eq. (20) may be written in the form

$$L = \frac{l}{16n}(2u + \sinh 2u), \quad . \quad . \quad . \quad . \quad (20')$$

where $u$ is defined by $\sinh u = 4n$.

An approximate formula for the length of curve may be obtained by expanding the binomial in Eq. (19) and then integrating. This gives,

$$L = l\left(1 + n\tfrac{8}{3}{}^2 - \tfrac{32}{5}n^4 + \dots\right), \quad . \quad . \quad . \quad (21)$$

* A convenient form of the exact formula, Eq. (20), is:

$$L = \frac{l}{8n}\left\{\tan\phi_1 \cdot \sec\phi_1 + \log_e(\tan\phi_1 + \sec\phi_1)\right\}.$$

where $n$ is defined by Eq. (16). For small values of the sag-ratio $n$, it will be sufficiently accurate to write,

$$L = l(1 + \tfrac{8}{3}n^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

for the length of a parabolic cable in terms of its chord $l$.

The following table gives the values of $L$ as computed by Eqs. (20) and (21), respectively.

| Sag-Ratio $\left(n = \frac{f}{l}\right)$ | Length-Ratio $= \frac{L}{l}$ | |
|---|---|---|
| | Exact (Eq. 20) | Approximate (Eq. 21) |
| 05 | 1 00663 | 1 00663 |
| 075 | 1 01480 | 1 01480 |
| .1 | 1 02606 | 1 02603 |
| 125 | 1 04023 | 1 04010 |
| 15 | 1 05712 | 1 05676 |
| 175 | 1 07652 | 1 07566 |
| 2 | 1 09823 | 1 09643 |

**3. Unsymmetrical Spans.**—If the two ends of a cable span are not at the same elevation, the ordinates $y$ should be measured vertically from the inclined closing chord $AB$ (Fig. 2). If that is done, all of the principles derived above will remain applicable, and Eqs. (1) to (14), inclusive, may be kept unchanged. For a load uniform along the horizontal, the curve will be a parabola, and its equation, referred to the origin $A$ and to the axis $AB$, will be as before,

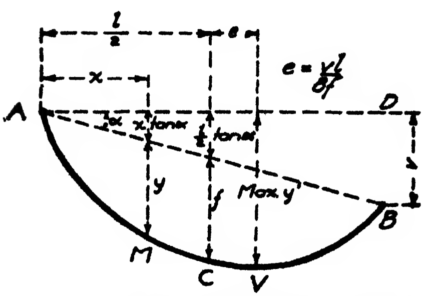



FIG. 2.—Unsymmetrical Parabolic Cable.

$$y = \frac{4fx}{l^2}(l - x). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

If it is desired to refer the curve to the horizontal line $AD$, with which the closing chord makes an angle $\alpha$, the equation becomes,

$$y' = y + x \cdot \tan \alpha = \frac{4fx}{l^2}(l - x) + x \cdot \tan \alpha. \quad . \quad . \quad . \quad (23)$$

To find the lowest point in the curve, located at $V$, a little to one side of the center, differentiate Eq. (23) and place the result equal to zero. Solving for $x$, we obtain,

$$x_v = \frac{l}{2}\left(1 + \frac{l}{4f} \cdot \tan \alpha\right). \quad . \quad . \quad . \quad . \quad . \quad (24)$$

To find the exact length of the curve, apply Eq. (20) to the segments $VA$ and $VB$ (Fig. 2), treating each of these segments as one-half of a complete parabola, and add the results.

An extreme case of the unsymmetrical parabolic curve occurs in the side-span cables of suspension bridges. Using the notation shown in Fig. 3, the equation of the curve may be written in the same way as Eq. (14),

$$y_1 = \frac{4f_1 x_1}{l_1^2}(l_1 - x_1). \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Here, again, $y_1$ and $f_1$ are measured vertically from the closing chord, and $x_1$ and $l_1$ are measured horizontally.

The true vertex of the curve or lowest point, $V$, will generally be found, by an equation similar to Eq. (24), to be outside point $D$ (Fig. 3). The exact length of curve will be $VA - VD$, or the difference between two semi-parabolas each of which may be calculated by Eq. (20).

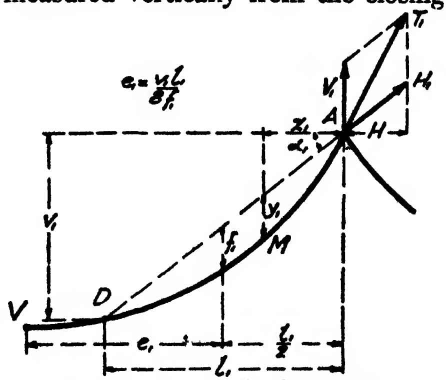



FIG. 3.—Parabolic Cable in Side Span.

An approximate value of the length may be obtained by taking the closing chord. $AD = l_1 \cdot \sec \alpha_1$, and adding the parabolic curvature correction as in Eq. (22). This will yield

$$L_1 = l_1\left(\sec \alpha_1 + \frac{8}{3} \frac{n_1^2}{\sec^3 \alpha_1}\right), \quad . \quad . \quad . \quad . \quad (26)$$

where

$$n_1 = \frac{f_1}{l_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

The cable tension in the side span acts in the line of the closing chord $AD$ (Fig. 3) and is designated by

$$H_1 = H \cdot \sec \alpha_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Since the lever arms $y_1$ are vertical, they must be multiplied by the horizontal component of $H_1$, or $H$, to obtain the bending moments produced by this force. Hence, as in Eq. (2), we have,

$$M' = H \cdot y_1, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

and, as in Eq. (11), we obtain,

$$H = \frac{w_1 l_1^2}{8 f_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

In order that the main and side spans may have equal values of $H$, by Eqs. (11) and (30), we have,

$$\frac{wl^2}{8f} = \frac{w_1 l_1^2}{8 f_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

Hence the necessary relation between the sags is

$$\frac{f_1}{f} = \frac{w_1 l_1^2}{wl^2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (32)$$

The stress at any point in the cable is given by Eq. (5), which may be rewritten as

$$T = H(1 + \tan^2 \phi)^{1/2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

At the center of the side span, where $x_1 = \frac{l_1}{2}$, the curve is parallel to the chord, and the inclination is equal to $\alpha_1$; hence, at that point

$$T=H(1+\tan^2 \alpha_1)^{1/2}. \quad . \quad . \quad . \quad . \quad (34)$$

At the support, where $x_1=0$, the inclination of the cable is given by

$$\tan \phi_1=\tan \alpha_1+\frac{4f_1}{l_1}, \quad . \quad . \quad . \quad . \quad . \quad (35)$$

and formula (33) yields

$$T_1=H\left[1+\left(\tan \alpha_1+\frac{4f_1}{l_1}\right)^2\right]^{1/2}, \quad . \quad . \quad . \quad (36)$$

which is the maximum stress in the cable.

**4. The Catenary.**—If the load $w$ is not constant per horizontal unit, but per unit length of the curve, as is the case where the load on the cable is due to its own weight, Eq. (10) takes the form,

$$\frac{d^2y}{dx^2}=-\frac{w\cdot\sec \phi}{H}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

Since $\tan \phi=\frac{dy}{dx}$, Eq. (37) may be written,

$$\frac{d^2y}{dx^2}=-\frac{w}{H}\left[1+\left(\frac{dy}{dx}\right)^2\right]^{1/2}. \quad . \quad . \quad . \quad . \quad (38)$$

Integrating this equation, taking the origin at the lowest point of the curve, we obtain the equation of the cable curve:

$$y=\frac{1}{2c}(e^{cx}+e^{-cx}-2), \quad . \quad . \quad . \quad . \quad . \quad . \quad (39)$$

where $c=\frac{w}{H}$.

This is the equation of a catenary; a cable under its own weight hangs in a catenary.

Replacing the exponential terms by hyperbolic functions, Eq. (39) may be written,

$$y=\frac{1}{c}(\cosh cx-1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (40)$$

To find the length of the catenary, substitute $\frac{dy}{dx}$ obtained from Eq. (39) in Eq. (18). This gives

$$L=2\int_0^{\frac{l}{2}}\tfrac{1}{2}(e^{cx}+e^{-cx})dx=\frac{1}{c}(e^{\frac{cl}{2}}-e^{-\frac{cl}{2}}). \quad . \quad . \quad (41)$$

Expressed in hyperbolic functions, Eq. (41) may be written,

$$L=\frac{2}{c}\sinh\frac{cl}{2}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42)$$

Equations (40) and (42) are useful in computations for the guide wires employed for the regulation of the strands in cable erection. If the length $L$ is known, Eq. (42) may be solved for the parameter $c$, by a method of successive approximations, and the ordinates may then be obtained from Eq. (40). For the expeditious solution of these equations, good tables of hyperbolic functions are required.

If the integration in Eq. (41) is performed between the limits 0 and $x$, and the value of $y$ substituted from Eq. (39), we obtain,

$$L=\frac{1}{c}\sqrt{2cy+c^2y^2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

as a formula for the length from the vertex to any point of the curve. Equation (43) may be used for unsymmetrical catenaries.

The stress at any point in the cable is again given by Eq. (5), or,

$$T=H\cdot\frac{ds}{dx}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

Since $H=\frac{w}{c}$, Eq. (44) may be written:

$$T=\frac{w}{c}\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{1}{2}}.$$

Substituting the value of $\frac{dy}{dx}$ derived from Eq. (39), we obtain,

$$T=\frac{w}{2c}(e^{cx}+e^{-cx}). \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

Replacing the exponential by hyperbolic functions, Eq. (45) becomes,

$$T = H \cdot \cosh cx. \quad \ldots \ldots \quad (46)$$

This tension will be a maximum at the ends of the span, where $x = \frac{l}{2}$, yielding,

$$T_1 = H \cdot \cosh \frac{cl}{2}. \quad \ldots \ldots \quad (47)$$

Comparing Eqs. (40) and (46), we find,

$$T = w\left(y + \frac{1}{c}\right) = wy + H \quad \ldots \quad (48)$$

At the span center, where $y = 0$, this gives $T = H$; and at the supports, where $y = f$, we obtain,

$$T_1 = wf + H. \quad \ldots \ldots \quad (49)$$

If the sag-ratio $\left(n = \frac{f}{l}\right)$ is small, all of the formulas for the catenary may be replaced, with sufficient accuracy, by the formulas for parabolic cables.

**5. Deformations of the Cable.**—As a result of elastic elongation, slipping in the saddles, or temperature changes, the length of cable between supports may alter by an amount $\Delta L$; as a result of tower deflection or saddle displacement, the span may alter by an amount $\Delta l$. Required to find the resulting changes in cable-sag, $\Delta f$.

For parabolic cables, the length is given with sufficient accuracy by Eq. (21). Partial differentiation of that equation with respect to $l$ and $f$, respectively, yields the two relations:

$$\Delta L = \tfrac{1}{15}(15 - 40n^2 + 288n^4) \cdot \Delta l, \quad \ldots \quad (50)$$

$$\Delta L = \tfrac{16}{15}n \cdot (5 - 24n^2) \cdot \Delta f. \quad \ldots \ldots \quad (51)$$

From Eqs. (50) and (51), there results,

$$\Delta f = -\frac{15 - 40n^2 + 288n^4}{16n(5 - 24n^2)} \cdot \Delta l. \quad \ldots \ldots \quad (52)$$

The required center deflections may be calculated by means of Eqs. (51) and (52) when $\Delta L$ and $\Delta l$ are known.

For a change in temperature of $t$ degrees, coefficient of expansion $\omega$, the change in cable-length will be,

$$\Delta L = \omega \cdot t \cdot L. \qquad \qquad (53)$$

For any loading which produces a horizontal tension $H$, the average stress in the cable will be, very closely,

$$\frac{L}{l} \cdot H,$$

and the elastic elongation will be,

$$\Delta L = \frac{L}{l} \cdot \frac{HL}{EA}, \qquad \qquad (54)$$

where $E$ is the coefficient of elasticity and $A$ is the area of cross-section of the cable.

Another expression for the elastic elongation is

$$\Delta L = \frac{H}{EA} \int_0^L \frac{ds^2}{dx} = \frac{Hl}{EA}\left(1 + \tfrac{16}{3}n^2\right). \qquad \qquad (55)$$

For a small change in the cable-sag $\Delta f$, the resulting change in the horizontal tension is obtained by differentiating Eq. (12):

$$\Delta H = -\frac{H}{f} \cdot \Delta f. \qquad \qquad (56)$$

From Eqs. (56), (51) and (52), may be found the deformations of the cable produced by any small change in the cable stresses.

## SECTION II.—UNSTIFFENED SUSPENSION BRIDGES

**6. Introduction.**—The unstiffened suspension bridge is not used for important structures. The usual form, as indicated in Fig. 4, consists of a cable passing over two towers and anchored by back-stays to a firm foundation. The roadway is suspended from the cable by means of hangers or suspenders. As there is no stiffening truss, the cable is free to assume the equilibrium curve of the applied loading.

**7. Stresses in the Cables and Towers.**—If built-up chains are used, as in the early suspension bridges, the cross-section may be varied in proportion to the stresses under maximum loading. In a wire cable, the cross-section is uniform throughout.

As the cable and hangers are light in comparison with the roadway, the combined weight of the three may be considered as uniformly distributed along the horizontal. Let this total dead load be $w$ pounds per lineal foot. The cable will then assume a parabolic curve; and all of the relations derived for a parabolic cable, represented by Eqs. (11) to (22), will apply.

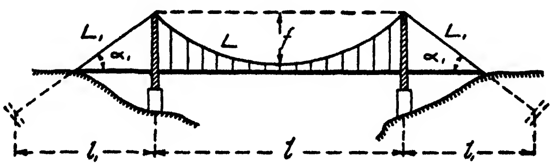



FIG 4.—Unstiffened Suspension Bridge.

The maximum dead-load stress in the cable, occurring at the towers, is given by Eq. (15):

$$T_w = \frac{wl^2}{8f}(1+16n^2)^{1/2}, \qquad (15)$$

where $n$ is the ratio of the sag $f$ to the span $l$.

Let there be a uniform live load of $p$ pounds per lineal foot. The maximum cable stress will evidently occur when the load covers the whole span, and will have a value,

$$T_p = \frac{pl^2}{8f}(1+16n^2)^{1/2}. \qquad (57)$$

Adding the values in Eqs. (15) and (57), we find the total stress in the cable at the towers:

$$T_{w+p} = \frac{(w+p)l^2}{8f}(1+16n^2)^{1/2}. \qquad (58)$$

If $\alpha_1$ is the inclination of the backstay to the horizontal (Fig. 4), the stress in the backstay will be:

$$T_1 = H \cdot \sec \alpha_1 = \frac{(w+p)l^2}{8f} \cdot \sec \alpha_1 \quad . \quad . \quad (59)$$

If cable and backstay have equal inclinations at the tower, their stresses, represented by Eqs. (58) and (59), will be equal.

The vertical reaction of the main cable at the tower is $(w+p)l/2$. If the backstay has the same inclination as the cable, it will also have the same vertical reaction; so that the total stress in the tower will be,

$$T = (w+p) \cdot l. \quad . \quad . \quad . \quad . \quad . \quad . \quad (60)$$

**8. Deformations under Central Loading.**—Under partial loading, the unstiffened cable will be distorted from its initial parabolic curve. It is required to find the deflections produced by the change of curve, disregarding for the present any stretching of the cable or any displacement of the saddles.

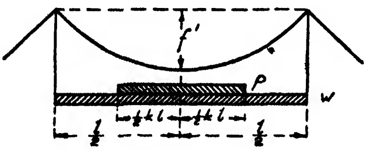



FIG. 5.—Loading for Maximum Vertical Deflection.

The maximum vertical deflection at the center of the cable will occur when a certain central portion of length $kl$ is covered with live load ($p$), in addition to the dead load ($w$) covering the whole span (Fig. 5). The sag of the distorted cable will be, by Eq. (3),

$$f' = \frac{wl^2}{8H} + \frac{pl^2}{8H} \cdot k(2-k). \quad . \quad . \quad . \quad . \quad (61)$$

Equating the expressions for the cable-length corresponding to the initial and distorted conditions, respectively, the lengths being obtained from the approximate equation (22), and introducing the symbol $q = p/w$, we obtain:

$$L = l(1 + \tfrac{8}{3}n^2) = l + \frac{w^2l^3}{24H^2}(1 + 3qk + 3q^2k^2 - qk^3 - 2q^2k^3). \quad (62)$$

Solving this equation for $H$, and substituting in Eq. (61), there results:

$$\frac{f'}{f}=\frac{1+2qk-qk^2}{(1+3qk+3q^2k^2-qk^3-2q^2k^3)^{1/2}}. \quad . \quad . \quad (63)$$

By differentiating this expression with respect to $k$, we obtain the following condition for a maximum value of $f'$:

$$k^4(1+2q)q+2k^3(1-q)q+3k^2(1-q)-4k+1=0. \quad . \quad (64)$$

Solving this equation for $k$ and substituting the result in Eq. (63), we obtain the following values for the maximum crown deflection $\Delta f=f'-f$:

| For $q=\frac{p}{w}=$ | 0 | $\frac{1}{8}$ | $\frac{1}{4}$ | $\frac{1}{2}$ | 1 | 2 | 3 |
|---|---|---|---|---|---|---|---|
| $k=$ | 1.0 | 0.64 | 0.30 | 0.28 | 0.25 | 0.23 | 0.21 |
| $\Delta f=$ | 0 | .013 | .022 | .028 | .045 | .067 | .080$f$ |

From this tabulation we may obtain the following empirical values, sufficiently accurate between the limits $q=\frac{p}{w}=\frac{1}{4}$ to 4:

$$\left.\begin{aligned} k&=0.20+\frac{0.05}{q} \\ \Delta f&=(0.007+0.046q-0.0075q^2)f \end{aligned}\right\} \quad . \quad . \quad . \quad (65)$$

**9. Deformations under Unsymmetrical Loading.**—The greatest distortion of the cable from symmetry, represented by the maximum horizontal displacement of the low point or vertex, will be produced by a continuous uniform load extending for some distance $kl$ from the end of the span. (Fig. 6.)

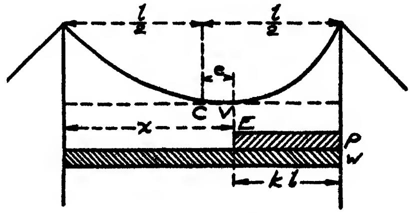


FIG. 6.—Maximum Horizontal Displacement of the Crown.

Applying the principle of Eq. (3), the lowest point of the cable curve is located by the condition $\frac{dM'}{dx}=0$; accordingly, with the notation of Fig. 6, so long as the crown $V$ is to the left of the head of the load ($E$),

$$\frac{dM'}{dx}=\frac{w}{2}(l-2x)+\tfrac{1}{2}pk^2l=0. \quad . \quad . \quad . \quad . \quad (66)$$

Inspection of this equation shows that $x$ will have its maximum value when $k$ has its maximum value; that is, when $kl=l-x$; in other words, the greatest lateral displacement occurs when the head of the moving load reaches the low point, $V$. Substituting this value in Eq. (66), we obtain:

$$k=-\frac{w}{p}+\sqrt{\frac{w}{p}+\frac{w^2}{p^2}}. \quad . \quad . \quad . \quad . \quad . \quad (67)$$

Hence the maximum deviation of the crown ($V$) from the center of the span ($c$), will be (Fig. 6),

$$\frac{e}{l}=\frac{1}{2}+\frac{w}{p}-\sqrt{\frac{w}{p}+\frac{w^2}{p^2}}. \quad . \quad . \quad . \quad . \quad (68)$$

The total sag of the cable is practically invariable for all ordinary values of $p/w$. Consequently, the uplift of the cable at the center of the span will amount to

$$\Delta f=\left(\frac{2e}{l+2e}\right)^2\cdot f. \quad . \quad . \quad . \quad . \quad . \quad (69)$$

We thus obtain the following values:

| For $\frac{p}{w}=$ | $\frac{1}{4}$ | $\frac{1}{3}$ | $\frac{1}{2}$ | 1 | $\frac{4}{3}$ | 2 | 3 | 4 |
|---|---|---|---|---|---|---|---|---|
| $e=$ | .028 | .036 | .051 | .086 | .105 | .134 | .167 | .191$l$ |
| $\Delta f=$ | .003 | .004 | .008 | .021 | .030 | .045 | .062 | .076$f$ |

**10. Deflections Due to Elongation of Cable.**—The total length of cable, including the backstays (Fig. 4), is, by Eq. (21),

$$L+2L_1=l(1+\tfrac{8}{3}n^2-\tfrac{32}{5}n^4)+2l_1\sec\alpha_1. \quad . \quad . \quad (70)$$

For a change in temperature of $t$ degrees, the total elongation of cable will be

$$\Delta L=\omega\cdot t(L+2L_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad (71)$$

For the elongation of the cable due to elastic strain, we may write, by Eq. (55),

$$\Delta L = \frac{H}{EA}[l(1 + \tfrac{16}{3}n^2) + 2l_1 \cdot \sec^2 \alpha_1]. \quad . \quad . \quad (72)$$

In addition there may be a contribution to $\Delta L$ from yielding of the anchorages.

If the cable is capable of slipping over the fixed saddles, the resulting deflection $\Delta f$ is obtained by substituting the above values of $\Delta L$ in Eq. (51).

If, however, a displacement of the saddles or a movement of the tops of the towers will occur before the cable will slip, any elongation of the backstays will alter the span ($l$), but not the length ($L$), of the cable in the main span. In that case, the combined effects of temperature and elastic strain will give:

$$\Delta L = \omega t L + \frac{Hl}{EA}(1 + \tfrac{16}{3}n^2), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

and

$$\Delta l = -2 \sec \alpha_1 \cdot \left( \omega t l_1 \cdot \sec \alpha_1 + \frac{Hl_1}{EA_1} \cdot \sec^2 \alpha_1 \right). \quad . \quad (74)$$

Substituting these values of $\Delta L$ and $\Delta l$ in Eqs. (51) and (52), respectively, we obtain the resulting deflection ($\Delta f$) of the main cable:

$$\Delta f = \frac{15}{16(5n - 24n^3)} \cdot \Delta L - \frac{15 - 40n^2 + 288n^4}{16(5n - 24n^3)} \cdot \Delta l. \quad . \quad (75)$$

If a displacement of the saddles ($\Delta l$) is accompanied by a slipping of the cable, so that the total length of the latter between anchorages (Fig. 4) remains unchanged, then the changes in length and span of the main cable must satisfy the relation

$$\Delta L = \Delta l \cdot \cos \alpha_1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (76)$$

Substituting these values in Eq. (75), the crown deflection becomes,

$$\Delta f = \frac{15 \cdot \cos \alpha_1 - (15 - 40n^2 + 288n^4)}{16(5n - 24n^3)} \cdot \Delta l. \quad . \quad . \quad (77)$$

## SECTION III.—STIFFENED SUSPENSION BRIDGES

**11. Introduction.**—In order to restrict the static distortions of the flexible cable discussed in the preceding pages, there is introduced a stiffening truss connected to the cable by hangers (Figs. 7, 15, 16). The side spans may likewise be suspended from the cable (Figs. 10, 11, 18), or they may be independently supported; in the latter case the backstays will be straight (Figs. 15, 16, 20). The main-span truss may be simply supported at the towers (Figs. 11, 16), or it may be built continuous with the side spans (Figs. 18, 20). A hinge may be introduced at the center of the stiffening truss in order to make the structure statically determinate (Fig. 8), or to reduce the degree of indeterminateness.

Another form of stiffened suspension bridge is the braced-chain type. This type does not make use of the straight stiffening truss suspended from a cable; instead, the suspension system itself is made rigid enough to resist distortion, being built in the form of an inverted arch (Figs. 21, 22, 23, 24).

For ease of designation, it will be convenient to adopt a symbolic classification of stiffened suspension bridges, based on the number of hinges in the main span of the truss, as tabulated on page 19.

In types $2F$ and $3F$, the side spans are not related to the main elements of the structure and may therefore be omitted from consideration. Hence these types are called "single-span bridges."

The suspension bridges with straight stiffening trusses will be analyzed first.

**12. Assumptions Used.**—In the theory that follows, we adopt the assumption that the truss is sufficiently stiff to render the deformations of the cable due to moving load practically negligible; in other words, we assume, as in all other rigid structures, that the lever arms of the applied forces are not altered by the deformations of the system. The resulting theory is sufficiently accurate for shorter spans and for those having comparatively deep stiffening trusses; any errors are on the side of safety.

- *Stiffened Suspension Bridges*
  - *Stiffening truss*
    - *Continous*
      - Side span free $=0F$ (Fig. 20)
      - Side span suspended $=0S$ (Fig. 18)
    - *One-hinged*
      - Side span free $=1F$
      - Side span suspended $=1S$
    - *Two-hinged*
      - Side span free $=2F$ (Fig. 16)
      - Side span suspended $=2S$ (Fig. 11)
    - *Three-hinged*
      - Side span free $=3F$ (Fig. 8)
      - Side span suspended $=3S$ (Fig. 26)
  - *Braced chain*
    - *Hingeless* $=0B$ (Fig. 24)
    - *One-hinged* $=1B$
    - *Two-hinged* $=2B$ (Fig. 23)
    - *Three-hinged* $=3B$ (Figs. 21, 22)

If the stiffening truss is not very stiff or if the span is long, the deflections of truss and cable may be too large to neglect. To provide for such cases, there has been developed a more exact method of calculation which is known as the *Deflection Theory* because it takes into account the deformations of the system. For a presentation of the Deflection Theory and its application, the reader is referred to Appendix D of this book.

The common theory developed in the following pages for the analysis of stiffened suspension bridges is known as the *Elastic Theory* because it is deduced from the simple considerations of elastic equilibrium of the system. It is based on the following five assumptions which are very near the actual conditions (except the fourth assumption, from which the variation may be of sufficient amount to require special consideration as is given by the more exact Deflection Theory):

1. The cable is supposed perfectly flexible, freely assuming the form of the equilibrium polygon of the suspender forces.

2. The truss is considered a beam, initially straight and

horizontal, of constant moment of inertia and tied to the cable throughout its length.

3. The dead load of truss and cable is assumed uniform per lineal unit, so that the initial curve of the cable is a parabola.

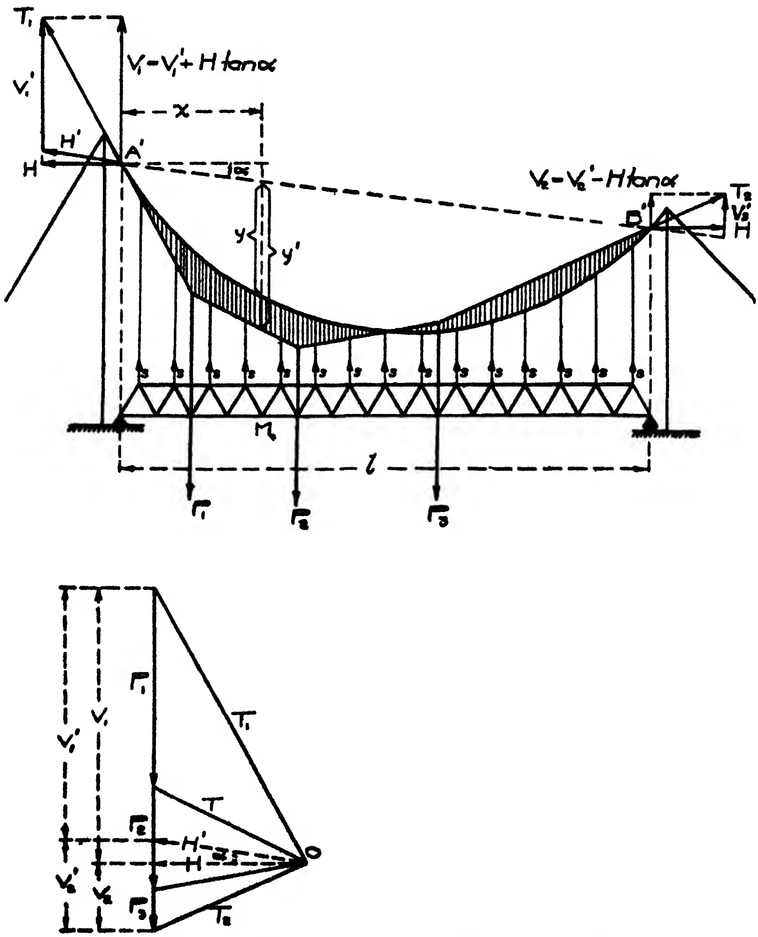



FIG. 7.—Forces Acting on the Stiffening Truss.

4. The form and ordinates of the cable curve are assumed to remain unaltered upon application of loading.

5. The dead load is carried wholly by the cable and causes

no stress in the stiffening truss. The truss is stressed only by live load and by changes of temperature.

The last assumption is based on erection adjustments, involving regulation of the hangers and riveting-up of the trusses when assumed conditions of dead load and temperature are realized.

**13. Fundamental Relations.**—Since the cable in the stiffened suspension bridge is assumed to be parabolic, the loads acting on it must always be uniform per horizontal unit of length. All of the relations established for a uniformly loaded cable (Eqs. (11) to (36), inclusive) will apply in this case.

If the panel points are uniformly spaced (horizontally), the suspender forces must be uniform throughout (Fig. 7). These suspender forces are loads acting downward on the cable, and upward on the stiffening truss. It is the function of the stiffening truss to take any live load that may be arbitrarily placed upon it and distribute it uniformly to the hangers.

The cable maintains equilibrium between the horizontal tension $H$ (resisted by the anchorages) and the downward acting suspender forces. If these suspender forces per horizontal linear unit are denoted by $s$, they are given by Eq. (11) as

$$s = H \cdot \frac{8f}{l^2}. \qquad (78)$$

The truss (Fig. 7) must remain in equilibrium under the arbitrarily applied loads acting downward and the uniformly distributed suspender forces acting upward. If we imagine the latter forces removed, then the bending moment $M'$ and the shear $V'$ at any section of the truss, distant $x$ from the left end, may be determined exactly as for an ordinary beam (simple or continuous according as the truss rests on two or more supports). This moment and shear would be produced if the cable did not exist and the entire load were carried by the truss alone. If $-M_s$ represents the bending moment of the suspender forces at the section considered, then the total moment in the stiffening truss will be

$$M = M' - M_s. \qquad (79)$$

Similarly, if $-V_s$ represents the shear produced by the suspender forces at the same section, the total shear in the stiffening truss will be

$$V=V'-V_s. \qquad (80)$$

Equations (79) and (80) are the fundamental formulas for determining the stresses in any stiffening truss. By these formulas, the stresses can be calculated for any given loading as soon as the value of $H$ is known.

The dead load is assumed to be exactly balanced by the initial suspender forces, so that it may be omitted from consideration in these equations.

In calculating $M'$ and $V'$ from the specified live load, and $M_s$ and $V_s$ from the uniform suspender loading given by Eq. (78), the condition of the stiffening truss as simple or continuous must be taken into account.

If the stiffening truss is a simple beam (hinged at the towers), by a familiar property of the funicular polygon, represented by Eq. (2),

$$M_s=H\cdot y, \qquad (81)$$

where $y$ is the ordinate to the cable curve measured from the straight line joining $A'$ and $B'$, the points of the cable directly above the ends of the truss (Fig. 7). Consequently, Eq. (79) may be written,

$$M=M'-H\cdot y. \qquad (82)$$

which is identical with equation (1). (In the unstiffened suspension bridge, $M=0$.)

If $\phi$ is the inclination of the cable at the section considered, the shear produced by the hanger forces is given by Eq. (7) as,

$$V_s=H\cdot\tan\phi. \qquad (83)$$

Consequently, Eq. (80) may be written

$$V=V'-H\cdot\tan\phi. \qquad (84)$$

If the two ends of the cable, $A'$ and $B'$, are at unequal elevations (Fig. 7), Eq. (84) must be corrected to the form,

$$V=V'-H\,(\tan\phi-\tan\alpha)=V'-H\cdot 4n\left(1-\frac{2x}{l}\right), \qquad (84')$$

where $\alpha$ is the inclination of the closing line $A'B'$ below the horizontal.

In Eqs. (82), (84) and (84′), the last term represents the relief of bending moment or shear by the cable tension $H$.

Representing $M'$ by the ordinates $y'$ of an equilibrium polygon or curve, constructed for the applied loading with a pole distance $=H$, Eq. (82) takes the form,

$$M=H(y'-y). \quad . \quad . \quad . \quad . \quad . \quad (85)$$

Hence the bending moment at any section of the stiffening truss is represented by the vertical intercept between the axis of the cable and the equilibrium polygon for the applied loads drawn through the points $A'B'$ (Fig. 7).

If the stiffening truss is continuous over several spans, the relations represented by Eqs. (81) to (85), inclusive, must be modified to take into account the continuity at the towers. The corresponding formulas will be developed in the section on continuous stiffening trusses (Section VI).

**14. Influence Lines.**—To facilitate the study and determination of suspension bridge stresses for various loadings, influence diagrams are most convenient.

The base for all influence diagrams is the $H$-curve or $H$-influence line. This is obtained by plotting the equations giving the values of $H$ for varying positions of a unit concentration. In the case of three-hinged suspension bridges, the $H$-influence line is a triangle (Figs. 8 and 9). In the case of two-hinged stiffening trusses, the $H$-lines (Figs. 11, 14) are similar to the deflection curves of simple beams under uniformly distributed load. In the case of continuous stiffening trusses, the $H$-line (Fig. 18) is similar to the deflection curve of a three-span continuous beam covered with uniform load in the suspended spans.

To obtain the influence diagrams for bending moments and shears, all that is necessary is to superimpose on the $H$-curve, as a base, appropriately scaled influence lines for moments and shears in straight beams.

The general expression for bending moments at any section (Eq. 82) may be written in the form,

$$M=y\left(\frac{M'}{y}-H\right), \quad . \quad . \quad . \quad . \quad . \quad (86)$$

(excepting that in the case of continuous stiffening trusses, $y$ is to be replaced by $y-ef$; see Eq. 212). For a moving concentration, $\frac{M'}{y}$ represents the moment influence line of a straight beam, simple or continuous as the case may be, constructed with the pole distance $y$. Hence the moment $M$ is proportional to the difference between the ordinates of this influence line and those of the $H$-influence line. If the two influence lines are superimposed (Figs. 8*b*, 11*b*, 11*c*, 18*b*), the intercepts between them will represent the desired bending moment $M$. In the case of stiffening trusses with hinges at the towers, $M'$ is the same as the simple-beam bending moment, and its influence line is familiarly obtained as a triangle whose altitude at the given section is,

$$\frac{M'}{y}=\frac{x(l-x)}{l \cdot y}. \quad . \quad . \quad . \quad . \quad . \quad (87)$$

For a parabolic cable, this reduces (by Eq. 14) to

$$\frac{M'}{y}=\frac{l}{4f}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (88)$$

Hence the $\frac{M'}{y}$ triangles for all sections will have the same altitude $\frac{l}{4f}$ (Figs. 8*b*, 11*b*). The corresponding altitude for sections in the side spans is $\frac{l_1}{4f_1}$ (Fig. 11*c*). The areas intercepted between the $H$-line and the $\frac{M'}{y}$ triangles, multiplied by $py$, give the maximum and minimum bending moments at the given section, $X$, of the stiffening truss. Areas below the $H$-line represent positive moments, and those above represent negative moments

(Figs. 8, 11, 18). Where the two superimposed lines intersect, we have a point $K$, which may be called the zero point, since a concentration placed at $K$ produces zero bending stress at $X$. $K$ is also called the critical point, since it determines the limit of loading for maximum positive or negative moment at $X$. Load to one side of $K$ yields plus bending, and load to the other side produces negative bending.

The shear at any section of the stiffening truss is given by Eq. (84), which may be written in the form,

$$V=\left(\frac{V'}{\tan\phi}-H\right)\cdot\tan\phi. \quad . \quad . \quad . \quad . \quad (89)$$

(If the two ends of the cable span are at different elevations, $\tan\phi$ in this equation is to be replaced by $\tan\phi-\tan\alpha$, where $\alpha$ is the inclination of the closing chord below the horizontal. See Eq. 84′). For any given section $X$, $\tan\phi$, the slope of the cable, is a constant and is given by,

$$\tan\phi=\frac{4f}{l}\left(1-\frac{2x}{l}\right)+\tan\alpha \quad . \quad . \quad . \quad . \quad (90)$$

The values assumed by the bracketed expression in Eq. (89) for different positions of a concentrated load may be represented as the difference between the ordinates of the $H$-line and those of the influence line for the shears $V'$, the latter being reduced in the ratio $\frac{1}{\tan\phi}$. The latter influence line is familiarly obtained by drawing the two parallel lines $as$ and $bt$ (Figs. 9$a$, 9$b$, 14$a$), their direction being fixed by the end intercepts

$$am=bn=\frac{1}{\tan\phi-\tan\alpha}=\frac{l}{4f\left(1-2\frac{x}{l}\right)}. \quad . \quad . \quad (91)$$

The vertices $s$ and $t$ lie on the vertical passing through the given section $X$. The maximum shears produced by a uniformly distributed load are determined by the areas included between the $H$ and $V'$ influence lines; all areas below the $H$-line are to be considered positive, and all above negative. These areas must be multiplied by $p \cdot \tan \phi$ (or by $p[\tan \phi - \tan \alpha]$) to obtain the greatest shear $V$ at the section; and $V$ must be multiplied by the secant of inclination to get the greatest stress in the web members cut by the section.

## SECTION IV.—THREE-HINGED STIFFENING TRUSSES

**15. Analysis.**—This is the only type of stiffened suspension bridge that is statically determinate (Types $3F$, $3S$, $3B$). The provision of the stiffening truss with a central hinge furnishes a condition which enables $H$ to be directly determined; viz., at the section through the hinge the moment $M$ must equal zero. Consequently, if the bending moment at the same section of a simple beam is denoted by $M'_0$, and if $f$ is the ordinate of the corresponding point of the cable, by Eq. (82),

$$H = \frac{M'_0}{f}. \qquad (92)$$

Hence the value of $H$ for any loading is equal to the simple-beam bending moment at the center hinge divided by the sag $f$. Accordingly, the cable will receive its maximum stress when the full span is covered with the live load $p$. In that case Eq. (92) yields

$$H = \frac{pl^2}{8f}, \qquad (93)$$

and, comparing this with Eq. (78), we see that

$$s = p. \qquad (94)$$

Hence, under full live load, the conditions are similar to those for dead load, the cable carrying all the load, the trusses having no stress. The bending moment at any section will be

$$\text{Total } M = 0. \qquad (94')$$

For a single load $P$ at a distance $kl$ from the near end of the span, the simple-beam moment at the center hinge will be

$$M'_0 = \frac{Pkl}{2}. \qquad (95)$$

Hence, the value of $H$, by Eq. (92), will be

$$H=\frac{Pkl}{2f}, \qquad (96)$$

This value of $H$ will be a maximum for $k=\frac{1}{2}$, yielding,

$$\text{Max. } H=\frac{Pl}{4f}. \qquad (97)$$

According to Eq. (92), the influence line for $H$ will be similar to the influence line for bending moment $M'_0$ at the center of a simple beam; hence it will be a triangle. It is defined by Eq. (96); and its maximum ordinate (at the center of the span) is given by Eq. (97) as $l/4f$. Figures 8*b* and 9*a* show the $H$-influence line constructed in this manner.

If the truss is uniformly loaded for a distance $kl$ from one end, the value of $H$ may be found by integrating Eq. (96) or directly from Eq. (92). We thus obtain:

$$\text{for } k<\tfrac{1}{2}, \quad H=\frac{pl^2}{4f}\cdot(k^2), \qquad (98)$$

$$\text{for } k>\tfrac{1}{2}, \quad H=\frac{pl^2}{8f}(4k-2k^2-1). \qquad (99)$$

For full load ($k=1$), Eq. (99) gives the maximum value of $H$:

$$H=\frac{pl^2}{8f}, \qquad (100)$$

which is identical with Eq. (93). Equations (98) to (100), inclusive, may also be obtained directly from the $H$-influence line (Figs. 8*b* and 9*a*).

For the half-span loaded, Eqs. (98) and (99) yield,

$$H=\frac{pl^2}{16f}, \qquad (101)$$

which is one-half of the value for full load. Substituting this value in Eq. (78), we find,

$$s=\tfrac{1}{2}p. \qquad (102)$$

One-half of the span is thus subjected to an unbalanced upward load, $s=\frac{1}{2}p$, per lineal foot, and the other half sustains an equal downward load, $p-s=\frac{1}{2}p$. Consequently there will be produced positive moments in the loaded half, and equal negative moments in the unloaded half, amounting to

$$M=\tfrac{1}{4}px\left(\frac{l}{2}-x\right); \quad . \quad . \quad . \quad . \quad . \quad (103)$$

and the maximum moments for this loading, occurring at the quarter points, $(x=\frac{1}{4}l,\ x=\frac{3}{4}l)$, will be,

$$\mathrm{M}=\pm\tfrac{1}{64}pl^2=\pm 0.01562pl^2. \quad . \quad . \quad . \quad (104)$$

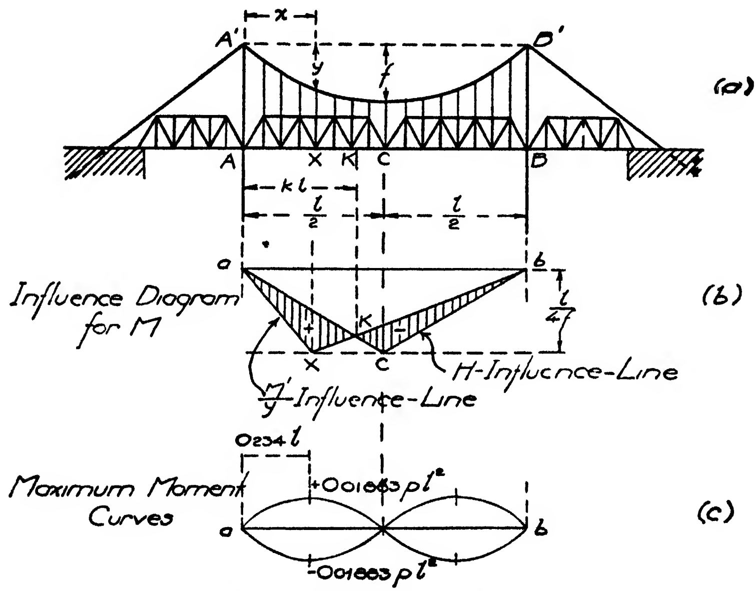



FIG. 8.—Three-hinged Stiffening Truss Moment Diagrams. (Type 3F).

**16. Moments in the Stiffening Truss.**—The influence diagrams for bending moments are constructed, in accordance with Eq. (86), by superimposing the $\dfrac{M'}{y}$ triangles upon the $H$-influence triangle. By Eq. (88), the $\dfrac{M'}{y}$ triangles for all sections have the same altitude $\dfrac{l}{4f}$; and, in the case of the three-hinged stiffening truss, this altitude is identical with that of the $H$-influence triangle.

The two triangles are shown superimposed in Fig. 8*b*. The shaded area between them is the influence diagram for bending moment at the section $X$.

For $x=\frac{l}{2}$, the two triangles would coincide. Hence the moment at the center hinge is zero for all conditions of loading, which agrees with the condition that the hinge can carry no bending.

For $x<\frac{l}{2}$, the two influence triangles intersect at a point $K$, a short distance to the left of the center. $K$ is the zero point or critical point. All load to the left of $K$ yields plus bending, and all load to the right produces negative bending.

Since the two superimposed triangles have the same base and equal altitudes, the plus and minus intercepted areas will be equal. Hence, if the whole span is loaded, the two areas will cancel each other, yielding zero moment as required by Eq. (94′).

If either of the shaded areas is multiplied by $py$, it will give the maximum value of the bending moment at $X$. The bending moments may also be obtained analytically from Eq. (82), as follows:

If the load covers a length $kl$ from one end of the span, the bending moment at any section $x<kl$, by Eqs. (82), (98) and (14), will be,

$$M=\tfrac{1}{2}pkl(2-3k)x-\tfrac{1}{2}px^2(1-2k^2). \quad . \quad . \quad (105)$$

Setting $\frac{dM}{dk}=0$ in this equation, we find that for maximum $M$,

$$k=\frac{l}{3l-2x}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (106)$$

This equation defines the distance $kl$ to the critical point $K$ (Fig. 8*b*). For this value of $k$, Eq. (105) gives the maximum value of $M$ for any value of $x$:

$$\text{Max. } M=\frac{px(l-x)(l-2x)}{2(3l-2x)}. \quad . \quad . \quad . \quad (107)$$

This value may also be obtained from the shaded areas in the influence diagram (Fig. 8*b*). Setting $\frac{dM}{dx}=0$ in the last equation, we find that the absolute maximum $M$ occurs at,

$$x=0.234l. \quad . \quad . \quad . \quad . \quad . \quad . \quad (108)$$

Substituting this value in Eqs. (107) and (106), we find that the absolute maximum value of $M$ is,

$$\text{Abs. Max. } M=+0.01883pl^2, \quad . \quad . \quad . \quad (109)$$

or about $\frac{1}{53}pl^2$, and that it occurs at $x=0.234l$, when $k=0.395$.

By loading the remainder of the span $(0.605l)$, we obtain the maximum negative moment at the same section. This will be numerically equal to the maximum positive moment, since their summation at any section must give zero according to Eq. (94′). Hence the absolute maximum negative moment will be,

$$\text{Abs. Min. } M=-0.01883pl^2. \quad . \quad . \quad . \quad (109')$$

After the maximum moments at the different sections along the span are evaluated from the influence lines, or from Eq. (107), they may be plotted in the form of curves, as shown in Fig. 8*c*. For the three-hinged stiffening truss, these maximum moment curves are symmetrical about the horizontal axis. They may be used as a guide for proportioning the chord sections of the stiffening truss.

**17. Shears in the Stiffening Truss.**—The shears produced in the stiffening truss by any loading are given by Eq. (84); but the maximum values at the different sections are most conveniently determined with the aid of influence lines (Fig. 9).

The influence line for $H$ is a triangle, with altitude $=\frac{l}{4f}$ at the center of the span. Upon this is superimposed the influence line for shears in a simple beam, reduced in the ratio $1 : \tan\phi$. The resulting influence diagram for shear $V$ at a given section $x<\frac{l}{4}$ is shown in Fig. 9*a*. There are two zero points or critical points: at $x$ and at $kl$. The portion of the left span between these two points must be loaded to produce maximum positive shear at the given section. From the geometry of the figure we find the position of the critical point $K$ to be given by,

$$k=\frac{1}{3-4\frac{x}{l}}. \qquad (110)$$

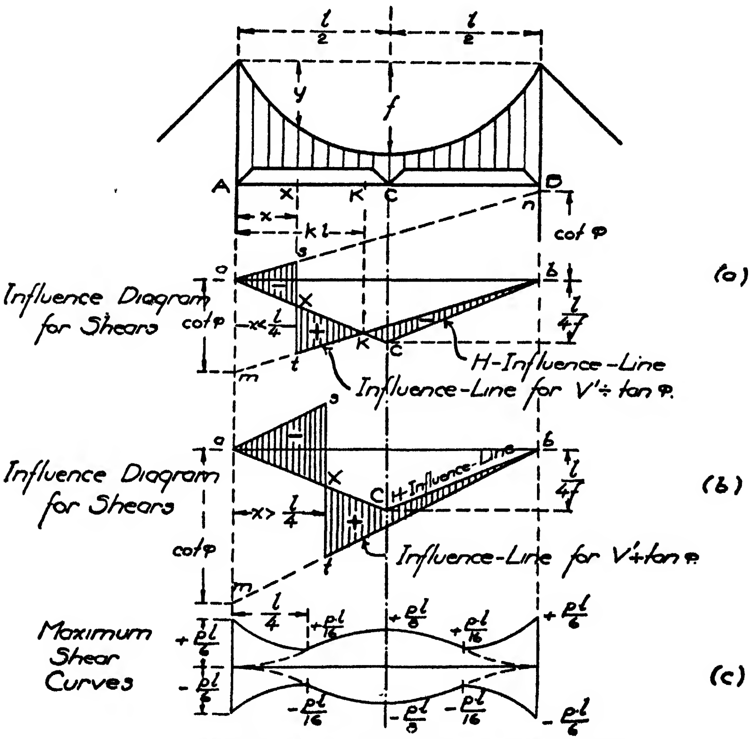



FIG. 9.—Shear Diagrams for Three-hinged Stiffening Truss. (Type 3F).

With the load covering the length from $x$ to $kl$, we find the maximum positive shear at $x$, either from the diagram or from Eq. (84), to be given by

$$\text{Max. } V=\frac{pl}{2}\cdot\frac{\left(1-3\frac{x}{l}+4\frac{x^2}{l^2}\right)^2}{3-4\frac{x}{l}}. \qquad (111)$$

When $x=0$, or for end-shear, Eq. (110) gives $k=\frac{1}{3}$, and Eq. (111) yields,

$$\text{Abs. Max. } V=\frac{pl}{6}. \qquad . \quad . \quad . \quad . \quad . \qquad (112)$$

When $x=\frac{1}{4}l$, we find $k=\frac{1}{2}$, and

$$\text{Max. } V=\frac{pl}{16}. \qquad . \quad . \quad . \quad . \quad . \qquad (113)$$

For $x>\frac{1}{4}l$, the influence diagram takes the form shown in Fig. 9*b*. There is only one zero point, namely at the section $X$. Loading all of the span beyond $X$, we find the maximum positive shear, either from the diagram or from Eq. (84), to be given by,

$$\text{Max. } V=\frac{px^2}{2l}\left(3-4\frac{x}{l}\right). \qquad . \quad . \quad . \quad . \qquad (114)$$

This has its greatest value for $x=\frac{1}{2}l$, or at the center, where it has the value,

$$\text{Max. } V=\frac{pl}{8}. \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (115)$$

Writing expressions for the maximum negative shears in the same manner, we obtain values identical with Eqs. (111) to (115), but with opposite sign. In other words, the plus and minus areas in any shear influence diagram are equal; their algebraic sum must be zero, since full span loading produces no stress in the stiffening truss. (See Eq. 94).

Figure 9*c* gives curves showing the variation of maximum positive and negative shears from end to end of the span. The curves are a guide for the proportioning of the web members of the stiffening truss. For the three-hinged truss, these curves are symmetrical about the horizontal axis.

If the two ends of the cable are at unequal elevations, the foregoing results for shear (Eqs. (110) to (115), inclusive) must be modified on account of the necessary substitution throughout the analysis of $(\tan\phi-\tan\alpha)$ for $\tan\phi$ as required by Eq. (84').

## SECTION V.—TWO-HINGED STIFFENING TRUSSES

**18. Determination of the Horizontal Tension** $H$.—In these bridge systems, the horizontal tension $H$ is statically indeterminate. The required equation for the determination of $H$ must therefore be deduced from the elastic deformations of the system

Imagine the cable to be cut at a section close to one of the anchorages. Then (with $H=0$), under the action of any loads applied on the bridge, the two cut ends would separate by some horizontal distance $\Delta$. If a unit horizontal force ($H=1$) be applied between the cut ends, it would pull them back toward each other a small distance $\delta$. The total horizontal tension $H$ required to bring the two ends together again would evidently be the ratio of these two imaginary displacements, or,

$$H=\frac{\Delta}{\delta} \quad . \quad (116)$$

Substituting for $\Delta$ and $\delta$ the general expressions for the displacement of a point

FIG. 10.—Philadelphia–Camden Bridge. Span 1750 Feet. (Type 2S)

Design by Delaware River Bridge Commission 1921 in an elastic system under the action of given forces, Eq. (116) becomes,

$$H=\frac{\Delta}{\delta}=-\frac{\int\frac{M'm}{EI}dx}{\int\frac{m^2}{EI}\cdot dx+\int\frac{u^2}{EA}\cdot ds}, \quad . \quad . \quad . \quad (117)$$

where $M'$ = bending moments (in the stiffening truss) under given loads, for $H=0$.

$m$ = bending moments (in the stiffening truss) with zero loading, for $H=1$.

$u$ = direct stresses (in the cable, towers and hangers) with zero loading, for $H=1$.

$I$ = moments of inertia (of the stiffening truss).

$A$ = areas of cross-section (of the cable, towers and hangers).

In the denominator of Eq. (117) there are two terms, since the system is made up of members, part of which are acted upon by bending moments, and part by direct, or axial, stresses. In the numerator, there is no axial stress term, since for the condition of loading producing $\Delta$, the cable tension $H=0$, and all of the axial stresses (in cables, towers and hangers) vanish.

Equation (117) is the most general form of the expression for $H$, and applies to any type of stiffened suspension bridge.

When there are no loads on the span, the bending moments in the two-hinged stiffening truss are, by Eq. (82):

$$M=-H\cdot y. \quad . \quad . \quad . \quad . \quad . \quad (118)$$

Hence we have, when $H=1$,

$$m=-y. \quad . \quad . \quad . \quad . \quad . \quad . \quad (119)$$

The stress at any section of the cable is given by Eq. (5), which, for $H=1$, reduces to,

$$u=\frac{ds}{dx}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (120)$$

Substituting Eqs. (119) and (120) in Eq. (117), we obtain the following fundamental equation for $H$ for two-hinged stiffening trusses (Types $2F$ and $2S$):

$$H=\frac{\Delta}{\delta}=\frac{\int\frac{M'y}{EI}dx}{\int\frac{y^2dx}{EI}+\int\frac{ds^3}{EAdx^2}}. \quad . \quad . \quad . \quad (121)$$

The integral in the numerator and the first integral in the denominator represent the contributions of the bending of the stiffening truss to $\Delta$ and $\delta$ respectively; the integrations extend over the full length of the stiffening truss suspended from the cable. The second integral in the denominator represents the contribution of the cable stretch to the value of $\delta$; the integration extends over the full length of cable between anchorages.

In the denominator of Eq. (121), the truss term contributes about 95 per cent, and the cable term only about 5 per cent of the total. Hence, certain approximations are permissible in evaluating the cable term.

Terms for the towers and hangers have been omitted, as they are negligible. (Their contribution to the denominator would be only a small decimal of 1 per cent.)

The terms in the denominator are independent of the loading and will now be evaluated. See Fig. 11$a$ for the notation employed. The symbols $l$, $x$, $y$, $f$, $\alpha$, $I$, $A$ have already been defined for the main span; and, adding a subscript, we have the corresponding symbols $l_1$, $x_1$, $y_1$, $f_1$, $\alpha_1$, $I_1$, $A_1$, for the side spans. In addition we have,

$l'$ = span of the cable, center to center of towers, which distance may be somewhat greater than the truss span $l$ (Fig. 11$a$);

$l_2$ = horizontal distance from tower to anchorage, which distance may be greater than the truss span $l_1$ (Fig. 11$a$).

Substituting for $y$ its values from Eqs. (14) and (25), the first integral in the denominator of Eq. (121), extending over main and side spans, becomes,

$$\int\frac{y^2dx}{EI}=\frac{1}{EI}\int_0^{l}y^2dx+2\cdot\frac{1}{EI}\int_0^{l_1}y_1^2dx=\frac{8}{15}\frac{f^2l}{EI}+2\left(\frac{8}{15}\frac{f_1^2l_1}{EI_1}\right). \quad (122)$$

The moments of inertia $I$ and $I_1$ are here assumed constant over the respective spans.

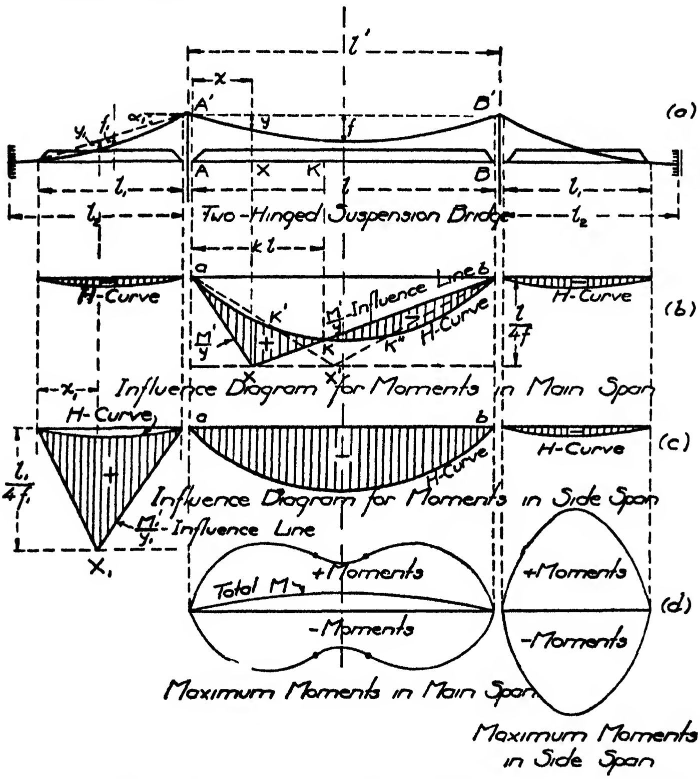



FIG. 11.—Moment Diagrams for Two-hinged Stiffening Truss. (Type 2S).

The second integral in the denominator of Eq. (121), with the aid of Eqs. (13) and (23), may be written,

$$\int\frac{ds^3}{EA\cdot dx^2}=\frac{2}{EA}\int_0^{\frac{l'}{2}}\left(1+64\frac{f^2x^2}{l^4}\right)^{3/2}dx$$
$$+\frac{2}{EA_1}\cdot\int_0^{l_2}\left[1+\left(\frac{4f_1}{l_1}-\frac{8f_1x_1}{l_1^2}+\tan\alpha_1\right)^2\right]^{3/2}\cdot dx_1. \quad (123)$$

The cable sections $A$ and $A_1$ are here considered to be uniform in the respective spans. Usually $A=A_1$. Expanding the binomials and integrating, we obtain, with sufficient accuracy,

$$\int \frac{ds^3}{EA\cdot dx^2}=\frac{l'}{E_cA}(1+8n^2)+\frac{2l_2}{E_cA_1}\cdot\sec^3\alpha_1(1+8n_1^2), \quad (124)^*$$

where $n$ and $n_1$ are the sag-ratios in main and side spans, respectively:

$$n=\frac{f}{l}, \quad n_1=\frac{f_1}{l_1}. \quad . \quad . \quad . \quad . \quad . \quad (124')$$

Setting the values given by (122) and (124) in Eq. (121), and multiplying through by $\frac{3EI}{f^2l}$, the formula for $H$ becomes,

$$H=\frac{\frac{3}{f^2l}\left[\int_0^l M'ydx+i\int_0^{l_1}M_1'y_1dx_1\right]}{\frac{8}{5}(1+2irv^2)+\frac{3I}{Af^2}\frac{E}{E_c}\frac{l'}{l}\cdot(1+8n^2)+\frac{6I\cdot E\cdot l_2}{A_1f^2\cdot E_c\cdot l}\cdot\sec^3\alpha_1(1+8n_1^2)}.. \quad (125)^*$$

where, for abbreviation,

$$i=\frac{I}{I_1}, \quad r=\frac{l_1}{l}, \quad v=\frac{f_1}{f}. \quad . \quad . \quad . \quad . \quad (126)$$

The elastic coefficient $E_c=E$, unless the cable is made of wire ropes. The denominator of Eq. (125), to be used for all suspension bridges of Type 2S, will henceforth be designated by $N$. It is a constant for any given structure.

The second term in the numerator represents the contribution of any loads in the side spans, and will vanish if the side spans are built independent of the backstays. In the latter case

---

* If the cable section is not constant but varies with the cable stress (as in eyebar chains), change $8n^2$ to $\frac{16}{3}n^2$, $8n_1^2$ to $\frac{16}{3}n_1^2$, and $\sec^3\alpha_1$ to $\sec^2\alpha_1$; using $A_0$ (cable section at crown) instead of $A$ and $A_1$.

the backstays will be straight (Type $2F$, Fig. 16), all terms containing $y_1, f_1, n_1$, or $v$ will vanish, and Eq. (125) reduces to

$$H=\frac{\frac{3}{f^2l}\int_0^l M'ydx}{\frac{8}{5}+\frac{E}{E_c}\cdot\frac{3Il'}{Af^2l}(1+8n^2)+\frac{6I}{A_1f^2}\cdot\frac{E}{E_c}\cdot\frac{l_2}{l}\cdot\sec^3\alpha_1}. \quad . \quad (127)^*$$

where $\alpha_1$ is the slope of the backstay.

**19. Values of *H* for Special Cases of Loading.**—In the preceding equations, the value of $M'$ depends upon the loading in the particular case. Expressing $M'$ as a function of $x$, using the value of $y$ given by Eq. (14), and performing the integration as indicated, we find, for a single load $P$ at a distance $kl$ from either end of the span,

$$\int M'ydx=\tfrac{1}{3}Pfl^2k(1-2k^2+k^3). \quad . \quad . \quad . \quad (128)$$

Hence, by Eq. (125), for a concentration in the main span, the value of the horizontal tension will be,

$$H=\frac{1}{N\cdot n}\cdot B(k)\cdot P, \quad . \quad . \quad . \quad . \quad (129)$$

where $N$ denotes the denominator of Eq. (125), and the function

$$B(k)=k(1-2k^2+k^3), \quad . \quad . \quad . \quad (129')$$

and may be obtained directly from Table I or from the graph in Fig. 12. The above value of $H$ is a maximum when the load $P$ is at the middle of the span; then $k=\frac{1}{2}$, and Eq. (129) yields,

$$\text{Max. } H=\frac{5}{16}\cdot\frac{1}{N\cdot n}\cdot P. \quad . \quad . \quad . \quad . \quad . \quad (130)$$

Similarly, for a concentration $P$ in either side span, at a distance $k_1l_1$ from either end,

$$H=\frac{1}{N\cdot n}\cdot ir^2v\cdot B(k_1)\cdot P, \quad . \quad . \quad . \quad . \quad (131)$$

* If the cable section is not constant but varies with the cable stress (as in eyebar chains), change $8n^2$ to $\frac{16}{3}n^2$, and $\sec^3\alpha_1$ to $\sec^2\alpha_1$; using $A_0$ (cable section at crown) instead of $A$ and $A_1$.

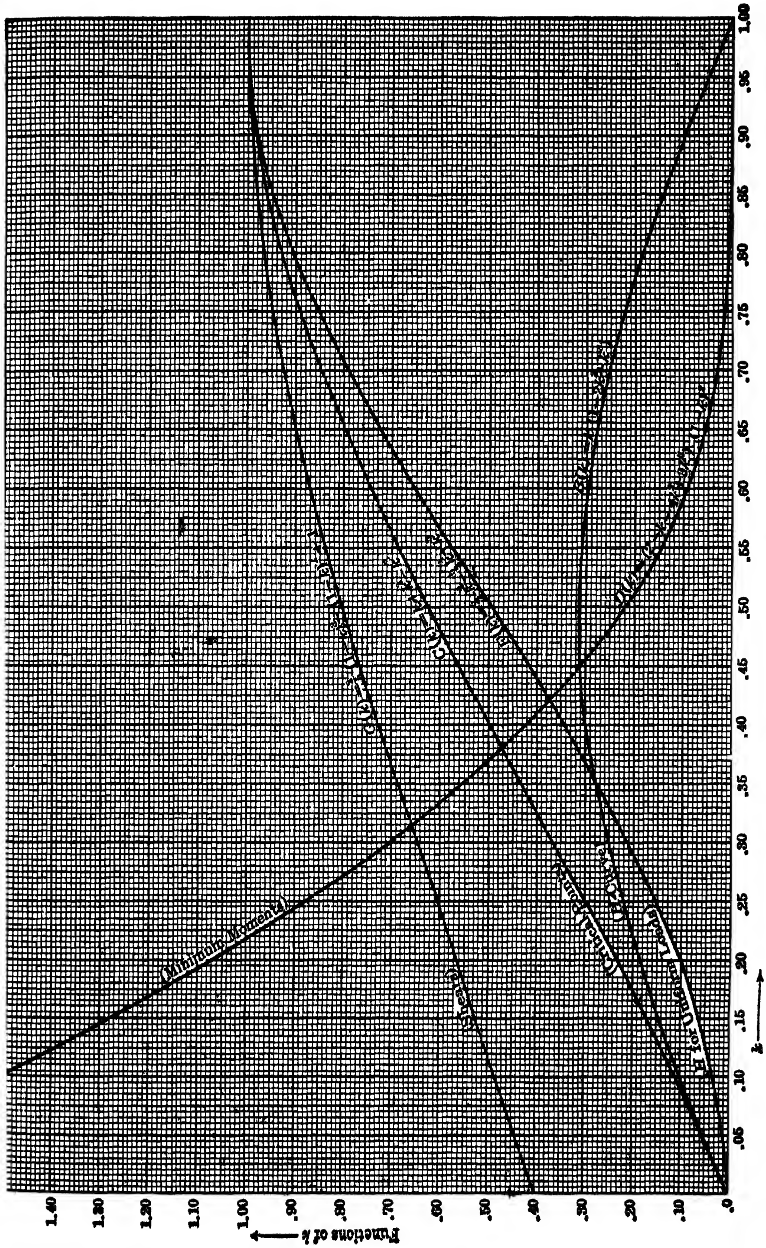



FIG. 12.—Graphs for the Solution of Suspension Bridge Formulas.

(See Table I, page 42).

where $B(k_1)$ is the same function as defined by Eq. (129′). This value of $H$ is a maximum when the load $P$ is at the middle of the side span; then $k_1=\frac{1}{2}$, and Eq. (131) yields,

$$\text{Max. } H=\frac{5}{16}\cdot\frac{1}{N\cdot n}\cdot ir^2v\cdot P. \quad . \quad . \quad . \quad (132)$$

By plotting Eqs. (129) and (131) for different values of $k$ and $k_1$, we obtain the $H$-curves or influence lines for $H$ (Figs. 11, 14). The maximum ordinates of these curves are given by Eqs. (130) and (132).

For a uniform load of $p$ pounds per foot, extending a distance $kl$ from either end of the main span, we find, by integrating the function $B(k)$ in Eq. (129′),

$$H=\frac{1}{5N\cdot n}\cdot F(k)\cdot pl, \quad . \quad . \quad . \quad . \quad (133)$$

where the function,

$$F(k)=\tfrac{5}{2}k^2-\tfrac{5}{2}k^4+k^5, \quad . \quad . \quad . \quad . \quad (133')$$

and may be obtained directly from Table I or from the graph in Fig. 12. For $k=1$, $F(k)=1$.

For similar conditions in either side span, we find for a loaded length $k_1l_1$,

$$H=\frac{1}{5Nn}\cdot ir^3v\cdot F(k_1)\cdot p_1l, \quad . \quad . \quad (134)$$

where $F(k_1)$ is the same function as defined by Eq. (133′).

The horizontal component of the cable tension will be a maximum when all spans are fully loaded, or when $k=1$ and $k_1=1$. Hence, by Eqs. (133) and (134),

$$\text{Total } H=\frac{1}{5N\cdot n}(1+2ir^3v)pl. \quad . \quad . \quad . \quad (135)$$

For a live load covering the central portion, $JK$, of the main span, from any section $x=jl$ to any other section $x=kl$, the application of Eq. (133) yields,

$$H=\frac{1}{5N\cdot n}[F(k)-F(j)]\cdot pl, \quad . \quad . \quad (136)$$

where $F(j)$ and $F(k)$ are the same function as defined by Eq. (133′).

The graph of $F(k)$ in Fig. 12 shows the proportional increase in the value of $H$ as a uniform load comes on and fills the main span (or either side span). The difference between the two ordinates for any sections, $J$ and $K$, multiplied by $\frac{pl}{5Nn}\left(\text{or by } \frac{p_1 l i r^3 v}{5Nn}\right)$, will give the value of $H$ for the corresponding partial loading $JK$.

For opposite loading conditions, that is, load covering both side spans and all of the main span with the exception of the central portion $JK$, we find the value of $H$ by subtracting the members of Eq. (136) from those of Eq. (135):

$$H=\frac{pl}{5N\cdot n}[1-F(k)+F(j)]+\frac{2p_1 l}{5Nn}\cdot ir^3 v. \quad . \quad . \quad (137)$$

**20. Moments in the Stiffening Truss.**—The bending moment at any section (main or side span) is given by Eq. (82),

$$M=M'-Hy, \quad M_1=M_1'-Hy_1. \quad . \quad . \quad . \quad (138)$$

If any span is free from load, the moments for that span are obtained by placing $M'$ (or $M_1'$) equal to zero, giving,

$$M=-Hy, \quad M_1=-Hy_1, \quad . \quad . \quad . \quad . \quad (139)$$

where $H$ is the cable tension produced by loads in the other spans, or by temperature.

With all three spans loaded, using the value of $H$ given by Eq. (135), Eq. (138) yields, for any section in the main span,

$$\text{Total } M=\tfrac{1}{2}px(l-x)\left[1-\frac{8}{5N}(1+2ir^3v)\right], \quad . \quad . \quad (140)$$

and, for any section in the side span,

$$\text{Total } M_1=\tfrac{1}{2}px_1(l_1-x_1)\left[1-\frac{8}{5N}(1+2ir^3v)\frac{v}{r^2}\right] \quad . \quad (141)$$

The influence diagrams for bending moment are constructed, in accordance with Eq. (86), by superimposing the influence triangle for $\frac{M'}{y}$ on the $H$-influence curve: The H-curve is

TABLE I

FUNCTIONS OCCURRING IN SUSPENSION BRIDGE FORMULAS

| | *H* Influence Line | Critical Points | Minimum Moments | *H* for Uniform Loads | Shears | |
|---|---|---|---|---|---|---|
| $k$ | $B(k)$ | $C(k)$ | $D(k)$ | $F(k)$ | $G(k)$ | $k$ |
| | $k(1-2k^2+k^3)$ | $k+k^2-k^3$ | $(2-k-4k^2+3k^3)(1-k)^2$ | $\frac{5}{2}k^2-\frac{5}{2}k^4+k^5$ | $\frac{2}{5}(1-k)^3-(1-k)^2+1$ | |
| 0 | 0 | 0 | 2.0 | 0 | 0.4 | 0 |
| .05 | .0498 | .0524 | 1.7511 | .0062 | .4404 | .05 |
| .1 | .0981 | .1090 | 1.5090 | .0248 | .4816 | .10 |
| .15 | .1438 | .1691 | 1.2790 | 0550 | .5232 | .15 |
| .2 | .1856 | .2320 | 1.0650 | .0963 | .5648 | .20 |
| .25 | .2227 | .2969 | .8704 | .1474 | .6062 | .25 |
| .3 | .2541 | .3630 | .6962 | .2072 | .6472 | .30 |
| .35 | .2793 | .4296 | .5445 | .2740 | .6874 | .35 |
| .4 | .2976 | .4960 | .4147 | .3462 | .7264 | .40 |
| .45 | .3088 | .5614 | .3065 | .4222 | .7640 | .45 |
| .5 | .3125 | .6250 | .2188 | 0.5 | .8000 | .50 |
| .55 | .3088 | .6861 | .1497 | .5778 | .8340 | .55 |
| .6 | .2976 | .7440 | .0973 | .6538 | .8656 | .60 |
| .65 | .2793 | .7979 | .0593 | .7260 | .8946 | .65 |
| .7 | .2541 | .8470 | .0332 | .7928 | .9208 | .70 |
| .75 | .2227 | .8906 | .0166 | .8526 | .9438 | .75 |
| .8 | .1856 | .9280 | .0070 | .9037 | .9632 | .80 |
| .85 | .1438 | .9584 | .0023 | .9450 | .9788 | .85 |
| .9 | .0981 | .9810 | .0005 | .9752 | .9904 | .90 |
| .95 | .0498 | .9951 | .0003 | .9938 | .9976 | .95 |
| 1.00 | 0 | 1.0 | 0 | 1.0 | 1.0 | 1.0 |

plotted with ordinates given by Eqs. (129) and (131); the $\frac{M'}{y}$ triangles have a constant height, $\frac{l}{4f}$ in the main span and $\frac{l_1}{4f_1}$ in the side spans. The resulting influence diagrams are shown in Figs. 11*b* and 11*c*. The intercepted areas, multiplied by $py$, give the desired bending moments; areas below the *H*-curve represent positive or maximum moments, and those above represent negative or minimum moments.

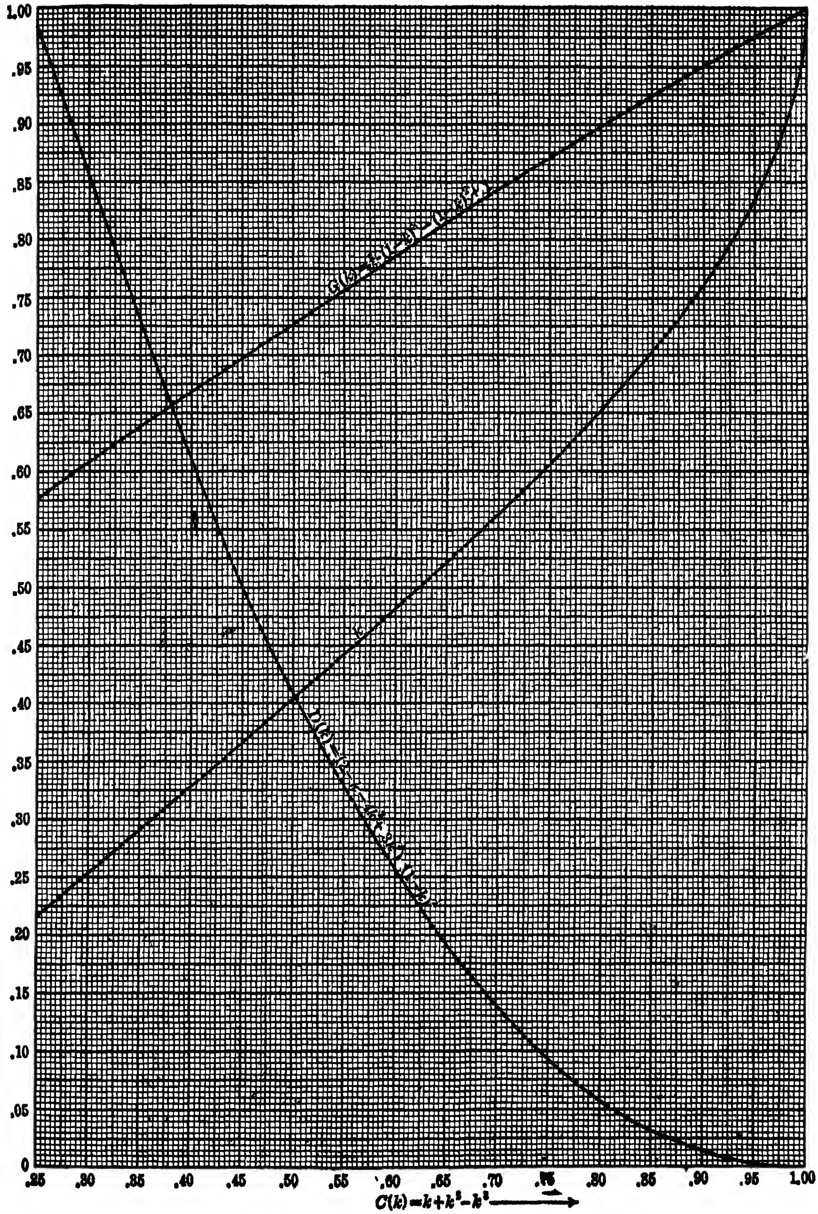



FIG. 13.—Graphs for the Solution of Suspension Bridge Formulas.
(Supplementary to Fig. 12).

For any section in the main span, there is a zero point or critical point $K$ (Fig. 11$b$), represented by the intersection of the superimposed influence lines. The distance $kl$ to this critical point is given by the relation,

$$C(k)=k+k^2-k^3=N\cdot n\cdot\frac{x}{y}=\frac{N}{4}\cdot\frac{l}{l-x}. \quad . \quad . \quad (142)$$

Values of the function $C(k)$ are listed in Table 1 and plotted in a graph in Figs. 12 and 13, to facilitate the solution of Eq. (142) for $k$.

The maximum negative moment at any section of the main span is obtained by loading the length $l-kl$ in that span and completely loading both side spans (Fig. 11$b$). Then, using the values of Eqs. (133) and (135), Eq. (138) yields,

$$\text{Min. } M=-\frac{2px(l-x)}{5N}[D(k)+4ir^3v], \quad . \quad . \quad (143)$$

where the function,

$$D(k)=(2-k-4k^2+3k^3)(1-k)^2, \quad . \quad . \quad (143')$$

and is given, for different values of $k$, by Table I and by the graph in Fig. 12 or 13. The value of $k$ or $C(k)$ obtained from Eq. (142) is to be used.

Equation (143) applies to all sections from $x=0$ to $x'=\frac{N}{4}\cdot l$. For the minimum moments at the sections near the center, from $x'$ to $l-x'$, it is necessary to bring on some load also from the left end of the span, as there are two critical points, $K'$ and $K''$, for these sections (see dotted diagram, Fig. 11$b$); so that the expression (143) for these moments must be corrected by replacing $D(k)$ by $D(k')+D(k'')$, where $k'$ is the value of $k$ (Eq. 142) corresponding to the given section $x$, and $k''$ is the value of $k$ corresponding to the symmetrically located section $(l-x)$.

The maximum positive moments are given by the relation,

$$\text{Max. } M=\text{Total } M-\text{Min. } M. \quad . \quad . \quad . \quad . \quad (144)$$

Subtracting the values given by Eq. (143) from those given by Eq. (140), we obtain,

$$\text{Max. } M = \tfrac{1}{2}px(l-x)\left[1 - \frac{8}{5N}[1 - \tfrac{1}{2}D(k)]\right]. \quad . \quad (144')$$

The loading corresponding to this moment is indicated in Fig. 11*b*; only a portion of the main span is loaded, the side spans being without load.

There are no critical points in the side spans. For the greatest negative moment at any section $x_1$ in one of the side spans, load the other two spans (Fig. 11*c*), giving,

$$\text{Min. } M_1 = -y_1 \cdot \frac{1 + ir^3v}{5Nn} \cdot pl. \quad . \quad . \quad . \quad . \quad . \quad . \quad (145)$$

Loading the span itself produces the greatest positive moments, which are obtained by the relation,

$$\text{Max. } M_1 = \text{Total } M_1 - \text{Min. } M_1. \quad . \quad . \quad . \quad . \quad . \quad (146)$$

Subtracting the values given by Eq. (145) from those given by Eq. (141), we obtain,

$$\text{Max. } M_1 = \frac{ry_1}{8n_1}\left(1 - \frac{8}{5N}irv^2\right) \cdot pl. \quad . \quad . \quad . \quad . \quad . \quad (146')$$

The maximum and minimum moments for the various sections of a stiffening truss (Type 2*S*), as calculated from Eqs. (143), (144), (145) and (146), are plotted in Fig. 11*d*, to serve as a guide in proportioning the chord members.

**21. Shears in the Stiffening Truss.**—With the three spans completely loaded, the shear at any section $x$ in the main span will be, by Eqs. (84), (90) and (135),

$$\text{Total } V = \tfrac{1}{2}p(l - 2x)\left[1 - \frac{8}{5N}(1 + 2ir^3v)\right], \quad . \quad . \quad (147)$$

and, in the side spans,

$$\text{Total } V_1 = \tfrac{1}{2}p(l_1 - 2x_1)\left[1 - \frac{8}{5N} \cdot \frac{v}{r^2} \cdot (1 + 2ir^3v)\right]. \quad (148)$$

The influence diagram for shear at any section is constructed according to Eq. (89), by superimposing on the $H$-curve (Eqs.

129 and 131) the influence lines for $\frac{V'}{\tan\phi}$. The latter will have end intercepts $=\cot\phi$, where $\phi$ is the slope of the cable at the given section. The resulting influence diagram is shown in Fig. 14*a*. The intercepted areas, multiplied by $p\cdot\tan\phi$, give the desired vertical shears $V$. Areas below the $H$-curve represent positive or maximum shears, and areas above represent negative or minimum shears.

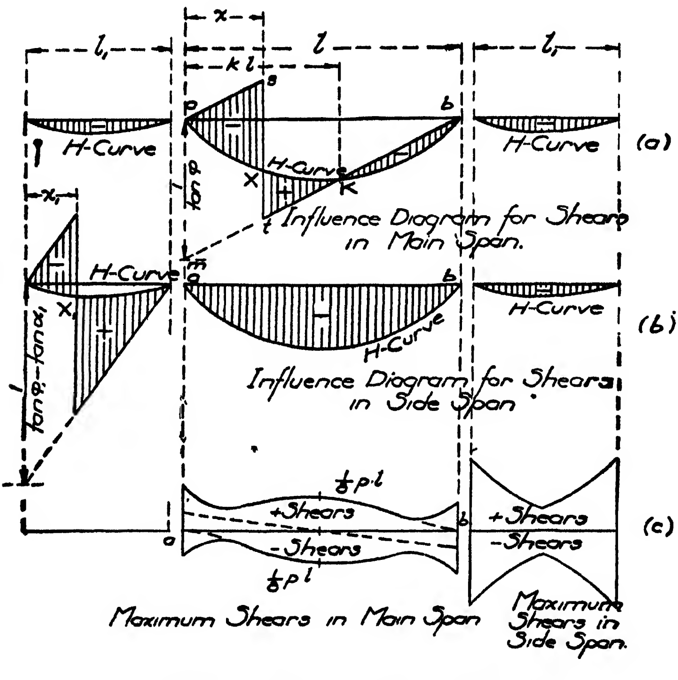



FIG. 14.—Shear Diagrams for Two-hinged Stiffening Truss. (Type 2S).

Loading the main span from the given section $X$ to the end of the span, we obtain the maximum positive shears by Eqs. (84), (90) and (133),

$$\text{Max. } V=\tfrac{1}{2}pl\left(1-\frac{x}{l}\right)^2\left[1-\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)\cdot G\left(\frac{x}{l}\right)\right], \quad . \quad (149)$$

where the function,

$$G\left(\frac{x}{l}\right)=\frac{2}{5}\left(1-\frac{x}{l}\right)^3-\left(1-\frac{x}{l}\right)^2+1, \quad . \quad . \quad . \quad . \quad . \qquad (149')$$

and is given by Table I and the graph in Fig. 12.

For the sections near the ends of the span, from $x=0$ to $x=\frac{l}{2}\left(1-\frac{N}{4}\right)$, the loads must not extend to the end of the span to produce the maximum positive shears, but must extend only to a point $K$ (Fig. 14*a*) whose abscissa $x=kl$ is determined by the following equation:

$$C(k)=k+k^2-k^3=\frac{N}{4}\cdot\frac{l}{l-2x}. \quad . \quad . \quad . \qquad (150)$$

For these sections, the positive shears given by Eq. (149) must be increased by an amount,

$$\text{Add. } V=\tfrac{1}{2}pl(1-k)^2\cdot\left[\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)\cdot G(k)-1\right], \quad . \qquad (151)$$

where the function,

$$G(k)=\tfrac{2}{5}(1-k)^3-(1-k)^2+1, \quad . \quad . \quad . \quad . \quad . \qquad (151')$$

and, like the same function in Eq. (149′), is given by Table I and the graph in Fig. 12 or 13.

Formula (150) for the critical section is solved in the same manner as Eq. (142) with the aid of Table I or the graph in Fig. 12 or 13.

There are no critical points for shear in the side spans. The influence diagram (Fig. 14*b*) shows the conditions of loading. For maximum shear at any section $x_1$, the load extends from the section to the tower, giving,

$$\text{Max. } V_1=\tfrac{1}{2}pl_1\left(1-\frac{x_1}{l_1}\right)^2\left[1-\frac{8}{N}\,irv^2\left(\frac{1}{2}-\frac{x_1}{l_1}\right)\cdot G\left(\frac{x_1}{l_1}\right)\right], \qquad (152)$$

where $G\left(\frac{x_1}{l_1}\right)$ is the same function as defined by Eqs. (149′) and (151′).

The maximum negative shears in main and side spans are given by the relations,

$$\text{Min. } V = \text{Total } V - \text{Max. } V, \quad . \quad . \quad . \quad (153)$$

and

$$\text{Min. } V_1 = \text{Total } V_1 - \text{Max. } V_1. \quad . \quad . \quad (153')$$

The maximum positive and negative shears for different sections of the main and side spans, as given by Eqs. (149), (152), (153) and (153′), are plotted for a typical suspension bridge, in Fig. 14*c*, to serve as a guide in proportioning the web members.

**22. Temperature Stresses.**—The relative horizontal deflection of one end of the cable due to a temperature change of $t$ degrees, with coefficient of expansion $\omega$, is

$$\Delta = \omega t\left[\int_0^l ds \cdot \frac{ds}{dx} + 2\int_0^{l_1} ds_1 \cdot \frac{ds_1}{dx_1}\right]. \quad . \quad . \quad . \quad (154)$$

Denoting the expression in the brackets by $L_t$, the expansion of the integrals yields, very closely,

$$L_t = l(\sec^2 \alpha + \tfrac{16}{3}n^2) + 2l_1(\sec^2 \alpha_1 + \tfrac{16}{3}n_1{}^2), \quad . \quad (154')$$

and Eq. (154) reduces to

$$\Delta = \omega t L_t. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (155)$$

Substituting this value for the numerator in Eqs. (116) to (125), we obtain,

$$H_t = -\frac{3EI\,\omega t L_t}{f^2 \cdot N \cdot l}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (156)$$

where $N$ denotes the denominator of Eq. (125) and $L_t$ is given by Eq. (154′).

The resulting bending moment at any section of the truss is given by,

$$M_t = -H_t \cdot y, \quad . \quad . \quad . \quad . \quad . \quad . \quad (157)$$

and the vertical shear by,

$$V_t = -H_t(\tan \phi - \tan \alpha), \quad . \quad . \quad (158)$$

where $\phi$ is the inclination of the cable at the given section, and $\alpha$ is the inclination of the cable chord (Eqs. 84′, 90).

**23. Deflections of the Stiffening Truss.**—For any specified loading, the deflections of the stiffening truss may be computed as the difference between the downward deflections produced by the applied loads and the upward deflections produced by the suspender forces, the stiffening truss being treated as a simple beam (for Types 2F and 2S). The suspender forces are equivalent to an upward-acting load, uniformly distributed over the entire span, and, by Eq. (78), amounting to,

$$s=\frac{8f}{l^2}\cdot H. \quad \dots\dots\dots \quad (78)$$

For a uniform load $p$ covering the main span, the resultant effective load acting on the stiffening truss will be, by Eqs. (78) and (135),

$$p-s=p\left(1-\frac{8}{5N}\right), \quad \dots\dots\dots \quad (159)$$

and the resulting deflection will be,

$$d=\frac{5}{384}\left(1-\frac{8}{5N}\right)\frac{pl^4}{EI}. \quad \dots\dots \quad (160)$$

In the general case, the applied loads will produce a deflection at a distance $x$ of,

$$d'=\frac{l-x}{l}\int_0^x\frac{M'}{EI}xdx+\frac{x}{l}\int_x^l\frac{M'}{EI}(l-x)\cdot dx. \quad \dots \quad (161)$$

The suspender forces, given by Eq. (78), will produce an upward deflection, at a distance $x$, of,

$$d''=\frac{1}{3EI}x(l^3-2lx^2+x^3)\frac{f}{l^2}\cdot H. \quad \dots \quad (162)$$

It should be noted that this deflection curve (Eq. 162) is similar to the $H$-influence curve given by Eq. (129). Using the function defined by Eq. (129'), Eq. (162) may be written,

$$d''=\frac{fl^2}{3EI}\cdot B\left(\frac{x}{l}\right)\cdot H. \quad \dots\dots \quad (162')$$

The resulting deflection of the truss at any point will then be obtained from Eqs. (161) and (162) as

$$d=d'-d''. \quad \dots\dots\dots \quad (163)$$

Equation (160), for a full-span load, may be derived directly from Eq. (163).

If merely the half-span is loaded with $p$ per unit length, then the deflection at the quarter-point will be, by Eqs. (161) and (162), in the loaded half,

$$d=\frac{1}{6144}\left(31-\frac{57}{2}\cdot\frac{8}{5N}\right)\cdot\frac{pl^4}{EI}, \quad . \quad . \quad . \quad . \qquad (164)$$

and, in the unloaded half,

$$d=-\frac{1}{6144}\left(\frac{57}{2}\cdot\frac{8}{5N}-26\right)\frac{pl^4}{EI}. \quad . \quad . \quad . \qquad (164')$$

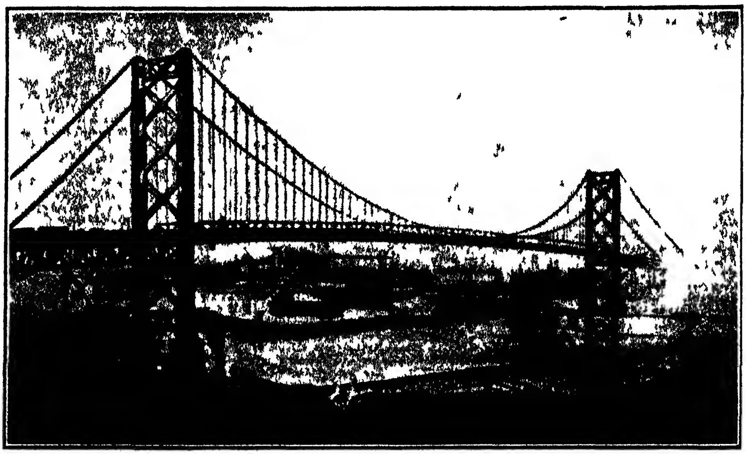


FIG. 15.—Detroit River Bridge (1929).
(The Ambassador Bridge.)
(Type $2F$.)

International highway bridge—Span 1850 ft
Two cables, 19-in dia, heat-treated wire
Two trusses, 22 ft deep, 59½ ft c to c, silicon steel

By Eq. (125), $N$ will always be greater than $\frac{8}{5}$. Substituting this minimum value in Eq. (164) or (164′), we obtain the upward or downward deflections at the quarter-points:

$$d=\frac{1}{2}\cdot\frac{5}{384}\cdot\frac{p}{EI}\cdot\left(\frac{l}{2}\right)^4. \quad . \quad . \quad . \quad . \quad . \quad . \qquad (165)$$

The deflections produced by temperature effects, or by a yielding of the anchorages, are given by Eq. (162′), upon substituting for $H$ the horizontal tension caused by the given influence. Substituting the expression from Eq. (156), we obtain,

$$d''=B\left(\frac{x}{l}\right)\cdot\frac{\Delta L}{N\cdot n}, \quad . \quad . \quad . \quad . \quad . \quad (166)$$

where the function $B\left(\frac{x}{l}\right)$ is defined by Eq. (129′) and is given by Table I and the graph in Fig. 12.

**24. Straight Backstays (Type 2*F*).**—If the stiffening truss is built independent of the cables in the side spans (Figs. 15, 16),

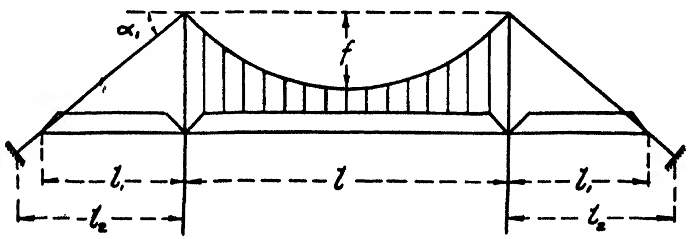



FIG. 16.—Two-hinged Stiffening Truss with Straight Backstays. (Type 2*F*).

the backstays will be straight and $f_1=0$. Consequently all terms containing $f_1$, $y_1$, $n_1=\frac{f_1}{l_1}$, or $v=\frac{f_1}{f}$ will vanish in Eqs. (125) to (166) inclusive.

The side spans will then act as simple beams, unaffected by any loads in the other spans; and the main-span and cable stresses will be unaffected by any loads in the side spans.

The denominator of the general expression for $H$ (Eq. 125) will then reduce to the denominator of Eq. (127):

$$N=\frac{8}{5}+\frac{3I}{Af^2}\frac{E}{E_c}\frac{l'}{l}(1+8n^2)+\frac{6I}{A_1f^2}\frac{E}{E_c}\frac{l_2}{l}\cdot\sec^3\alpha_1. \qquad (167)^*$$

Equations (131), (134), and (145), will vanish.

* See Footnote to Eq. 127.

The maximum value of $H$ will be produced by a uniform load $p$ covering the main span, and will be, by Eq. (135),

$$\text{Total } H=\frac{pl}{5N\cdot n}. \qquad (168)$$

The bending moment at any section $x$ of the main span will then be, by Eq. (140),

$$\text{Total } M=\tfrac{1}{2}px(l-x)\left(1-\frac{8}{5N}\right). \qquad (169)$$

The greatest negative bending moment will be, by Eq. (143),

$$\text{Min. } M=-\frac{2px(l-x)}{5N}\cdot D(k). \qquad (170)$$

The greatest positive moment is then given by Eq. (144'), or by,

$$\text{Max. } M=\text{Total } M-\text{Min. } M. \qquad (171)$$

In the side spans, there will be no negative moments. The greatest positive moments will be, by Eq. (141),

$$\text{Max. } M_1=\text{Total } M_1=\tfrac{1}{2}p_1x_1(l_1-x_1), \qquad (172)$$

exactly as in a simple beam.

With load covering the entire span, the shears in the main span will be, by Eq. (147),

$$\text{Total } V=\tfrac{1}{2}p(l-2x)\left(1-\frac{8}{5N}\right), \qquad (173)$$

and, in the side spans, by Eq. (148),

$$\text{Total } V_1=\tfrac{1}{2}p_1(l_1-2x_1). \qquad (174)$$

The maximum shears in the main span will be given by Eqs. (149), (150) and (151). In the side spans, the maximum shears will be, by Eq. (152),

$$\text{Max. } V_1=\tfrac{1}{2}p_1l_1\left(1-\frac{x_1}{l_1}\right)^2, \qquad (175)$$

exactly as in a simple beam.

The total length of cable will be, by Eq. (154),

$$L=l'(1+\tfrac{8}{3}n^2)+2l_2\cdot\sec\alpha_1, \qquad (176)$$

and the temperature stresses are then given by Eqs. (156), (157) and (158).

## SECTION VI.—HINGELESS STIFFENING TRUSSES

### (Types 0F and 0S)

**25. Fundamental Relations.**—Hingeless stiffening trusses (Figs. 17, 18) are continuous at the towers; hence there will be bending moments in the truss at the towers.

The moments and shears at any section in the stiffening truss will be the resultants of the values produced by the downward-acting loads ($M'$ and $V'$) and the upward-acting suspender

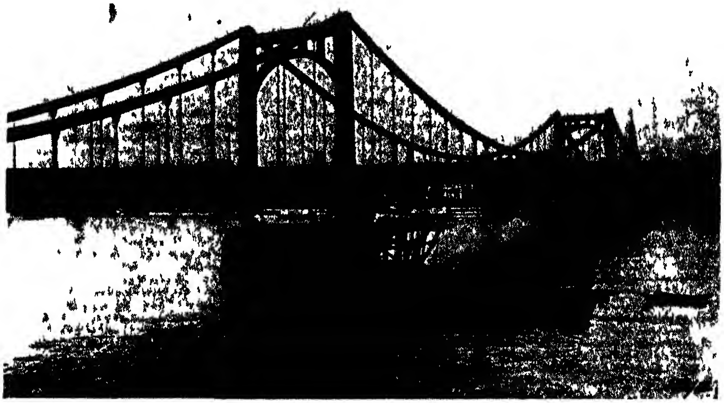


FIG. 17.—Suspension Bridge over the Rhine at Cologne. (Type 0S).

Continuous Stiffening Girder. Eyebar Chains. Self-anchored. Rocker Towers. Span 605 feet. Erected 1915.

forces ($M_s$ and $V_s$). Equations (78), (79) and (80) will apply; but the continuity of the truss must be taken into account in calculating the respective moments and shears.

If $M_1$ and $M_2$ are the bending moments at the towers produced by the downward loads on the stiffening truss, and if $M_0$ is the simple-beam bending moment at any section $x$, then the resultant bending moment due to the downward loads acting on the continuous truss will be,

$$M' = M_0 + \frac{l-x}{l} M_1 + \frac{x}{l} \cdot M_2, \quad . \quad . \quad . \quad (177)$$

in the main span, and,

$$M' = M_0 + \frac{x_1}{l_1} M_{1,2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (177')$$

in the side spans.

The upward-acting suspender forces will be uniform over each span. For any value of $H$, by Eq. (78), the upward pull will be $H \cdot \frac{8f}{l^2}$ per lineal foot in the main span and $H \cdot \frac{8f_1}{l_1^2}$ in the side spans. Then, by the Theorem of Three Moments for uniform load conditions, we find the moments at the towers (for symmetrical spans) to be,

$$-H \cdot m_1 = -H \cdot m_2 = -H \cdot (e \cdot f), \quad . \quad . \quad . \quad (178)$$

in which the coefficient of $f$ is a constant defined by,

$$e = \frac{2 + 2irv}{3 + 2ir}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (179)$$

where $i$, $r$, and $v$ are defined by Eq. (126).

The simple-beam bending moment produced by the suspender forces is given by Eq. (81) as $H \cdot y$. Adding the correction for the end moments at the towers (Eq. 178), we obtain the resultant suspender moments as,

$$M_s = H \cdot (y - e \cdot f), \quad . \quad . \quad . \quad . \quad . \quad . \quad (180)$$

for any section in the main span; and, for any section in the side-spans,

$$M_s = H \cdot \left(y_1 - \frac{x_1}{l_1} \cdot ef\right), \quad . \quad . \quad . \quad . \quad . \quad (181)$$

where $x_1$ is measured from the free end of the span, and $y_1$ is the vertical ordinate of the side cable below the connecting chord $D'A'$ (Fig. 18*a*).

Substituting (177), (177′), (180) and (181) in Eq. (79), we have, for bending moments in the main span, (Fig. 19),

$$M=M_0+\frac{l-x}{l}M_1+\frac{x}{l}\cdot M_2-H(y-ef), \quad . \quad . \quad (182)$$

and, for bending moments in the side span,

$$M=M_0+\frac{x_1}{l_1}\cdot M_{1,2}-H\left(y_1-\frac{x_1}{l_1}\cdot ef\right). \quad . \quad . \quad (183)$$

If any span is without load, $M_0$ for that span will vanish.

The shears produced by the downward-acting loads will be,

$$V'=V_0+\frac{M_2-M_1}{l} \quad . \quad . \quad . \quad . \quad . \quad (184)$$

in the main span, and

$$V'=V_0+\frac{M_1}{l_1}, \quad \text{or} \quad V'=V_0-\frac{M_2}{l_1}, \quad . \quad . \quad (185)$$

in the side spans. In these equations, $V_0$ denotes the simple-beam shears for the given loading.

The shears produced by the upward-acting suspender forces will be

$$V_s=H(\tan\phi-\tan\alpha) \quad . \quad . \quad . \quad . \quad . \quad (186)$$

in the main span, and

$$V_s=H\left(\tan\phi_1-\tan\alpha_1-\frac{ef}{l_1}\right) \quad . \quad . \quad . \quad (187)$$

in the side spans.

Substituting (184), (185), (186) and (187) in Eq. (80), we have, for resultant shears in the main span,

$$V=V_0+\frac{M_2-M_1}{l}-H\cdot(\tan\phi-\tan\alpha), \quad . \quad . \quad (188)$$

and, for resultant shears in the side spans,

$$V=V_0\pm\frac{M_{1,2}}{l_1}-H\cdot\left(\tan\phi_1-\tan\alpha_1-\frac{ef}{l_1}\right). \quad . \quad (189)$$

If any span is without load, $V_0$ for that span will vanish. If the two towers are of equal height, then, in the main span, $\alpha=0$.

**26. Moments at the Towers (Types 0$F$ and 0$S$).**—The values of the end-moments $M_1$ and $M_2$, used in Eqs. (177) to (189), may be determined, for any given loading, by the Theorem of Three Moments.

For a concentration $P$ in the main span, at a distance $kl$ from the left tower, we thus obtain,

$$M_1 = -Pl\cdot k(1-k)\frac{(3+2ir)(1-k)+2ir}{(3+2ir)(1+2ir)}, \quad . \quad (190)$$

$$M_2 = -Pl\cdot k(1-k)\frac{(3+2ir)k+2ir}{(3+2ir)(1+2ir)}. \quad . \quad . \quad (191)$$

The sum of these two end-moments will be,

$$M_1+M_2 = -\frac{3Pl\cdot k(1-k)}{3+2ir}, \quad . \quad . \quad . \quad . \quad . \quad (192)$$

and the difference will be,

$$M_1-M_2 = -Plk(1-k)\frac{1-2k}{1+2ir}. \quad . \quad . \quad . \quad . \quad (193)$$

For a concentration $P$ in the left side span, at a distance $kl_1$ from the outer end, the Theorem of Three Moments yields:

$$M_1 = -Pl\frac{2ir^2(1+ir)(k-k^3)}{(3+2ir)(1+2ir)}, \quad . \quad . \quad . \quad . \quad (194)$$

$$M_2 = +Pl\frac{ir^2(k-k^3)}{(3+2ir)(1+2ir)}. \quad . \quad . \quad . \quad . \quad (195)$$

For a uniform load covering the main span, we obtain,

$$M_1 = M_2 = -\frac{pl^2}{4(3+2ir)}. \quad . \quad . \quad . \quad . \quad . \quad (196)$$

For a uniform load covering the left side span, we obtain,

$$M_1 = -\frac{p_1l^2}{4}\,\frac{2ir^3(1+ir)}{(3+2ir)(1+2ir)}, \quad . \quad . \quad . \quad . \quad (197)$$

$$M_2 = +\frac{p_1l^2}{4}\,\frac{ir^3}{(3+2ir)(1+2ir)}. \quad . \quad . \quad . \quad . \quad (198)$$

For a uniform load covering all three spans, we obtain,

$$M_1 = M_2 = -\frac{pl^2}{4}\cdot\frac{1+ir^3}{3+2ir}. \quad . \quad . \quad . \quad . \quad . \quad (199)$$

**27. The Horizontal Tension** $H$.—The general formula (117) for the horizontal tension $H$ is applicable to the continuous stiffening truss (Types $0F$ and $0S$).

Equation (118), for bending moments produced by the suspender forces, is now replaced by the expressions (180) and (181), and Eq. (119) becomes,

$$m=-y+ef, \qquad (200)$$

for the main span, and

$$m=-y_1+\frac{x_1}{l_1}\cdot ef, \qquad (201)$$

for the side spans.

Substituting these values and integrating over all three spans, we obtain, as a substitute for Eq. (122),

$$\int\frac{m^2}{EI}dx=\frac{f^2l}{3EI}(\tfrac{8}{5}-4e+3e^2)+\frac{2f^2l_1}{3EI_1}(\tfrac{8}{5}v^2-2ev+e^2). \qquad (202)$$

Equations (120), (123), (124) and (124′) are retained unchanged. Collecting all the values and substituting in Eq. (117), we obtain the expression for $H$ in the continuous type of suspension bridge (in place of Eq. 125):

$$H=\frac{\frac{3}{f^2l}\left[\int_0^l M'(y-ef)dx+i\int_0^{l_1}M_1'\left(y_1-\frac{x_1}{l_1}ef\right)dx_1\right]}{\tfrac{8}{5}-4e+3e^2+2ir(\tfrac{8}{5}v^2+e^2-2ev)+\frac{E}{E_c}\frac{3Il'}{Af^2l}(1+8n^2)+\frac{E}{E_c}\frac{6I}{A_1f^2}\cdot\frac{l_2}{l}\cdot\sec^3\alpha_1(1+8n_1^2)}. \qquad (203)^*$$

The denominator of this expression is a constant for a given structure, and will henceforth be denoted by $N$. (If hinges are inserted at the towers, the coefficient of continuity $e$ will be zero, and Eq. [203] reduces to Eq. [125]).

**28. Values of $H$ for Special Cases of Loading.**—In the last equation (203), the value of the numerator depends upon the loading in any particular case. Expressing $M'$ as a function of $x$ (Eq. 177), substituting the value of $y$ given by Eq. (14), and

---

* See Footnote to Eq. (125).

performing the integration as indicated, we find, for a single load $P$ at a distance $kl$ from either end of the main span,

$$H=\frac{1}{Nn}[B(k)-\tfrac{3}{8}e(k-k^2)]\cdot P, \quad . \quad . \quad . \quad (204)$$

where $N$ denotes the denominator of Eq. (203), and the function $B(k)$ is defined by Eq. (129′) and is given by Table I and Fig. 12.

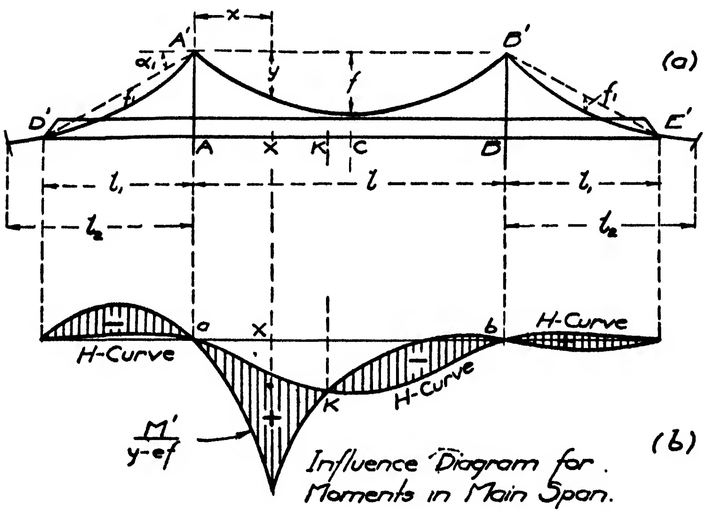



FIG. 18.—Moment Diagram for Continuous Stiffening Truss. (Type 0S).

Similarly, for a concentration $P_1$ in either side span, at a distance $k_1l_1$ from the free end, we obtain,

$$H=\frac{ir^2}{Nn}\left[v\cdot B(k_1)-\frac{e}{2}(k_1-k_1^3)\right]\cdot P_1. \quad . \quad (205)$$

Plotting Eqs. (204) and (205), we obtain the $H$-influence line, Fig. 18$b$.

If the main span is completely loaded, we obtain, by integrating Eq. (204),

$$H=\frac{1}{N\cdot n}\left(\frac{1}{5}-\frac{e}{4}\right)\cdot pl. \quad . \quad . \quad . \quad . \quad . \quad (206)$$

If both side spans are completely loaded, we obtain, by integrating Eq. (205),

$$H=\frac{2ir^3}{Nn}\left(\frac{v}{5}-\frac{e}{8}\right)\cdot p_1 l. \quad . \quad . \quad . \quad . \quad . \quad . \quad (207)$$

If the main span is loaded for a distance $kl$ from either tower, we obtain, from Eq. (204),

$$H=\frac{1}{5N\cdot n}\left[F(k)-\frac{5e}{4}(3-2k)k^2\right]\cdot pl, \quad . \quad . \quad (208)$$

where $F(k)$ is defined by Eq. (133′) and is given by Table I and Fig. 12.

If either side span is loaded for a distance $k_1 l_1$ from the free end, we obtain, from Eq. (205),

$$H=\frac{1}{5Nn}\cdot ir^3[v\cdot F(k_1)-\tfrac{5}{8}e(2-k_1^2)\cdot k_1^2]p_1 l, \quad . \quad (209)$$

where $F(k_1)$ is the same function as defined by Eq. (133′).

In the foregoing equations, $N$ represents the denominator of Eq. (203).

(If the stiffening truss is interrupted at the towers, the factor of continuity $e=0$, and the above formulas reduce to the corresponding equations [129] to [135] for the two-hinged stiffening truss.)

**29. Moments in the Stiffening Truss.**—With all three spans loaded, the bending moment at any section of the main span is given, very closely, by Eqs. (182) and (199), as,

$$\text{Total } M=\left(\tfrac{1}{2}p-H\cdot\frac{4f}{l^2}\right)x\cdot(l-x)-e(\tfrac{1}{8}pl^2-H\cdot f), \quad (210)$$

and, at any section of the side span distant $x_1$ from the free end by Eqs. (183) and (199), as,

$$\text{Total } M=\left(\tfrac{1}{2}p-H\cdot\frac{4f_1}{l_1^2}\right)x_1(l_1-x_1)-e(\tfrac{1}{8}pl^2-Hf)\frac{x_1}{l_1}, \quad (211)$$

where $e$ is defined by Eq. (179), and $H$ is given by the combination of Eqs. (206) and (207).

The moments for other loadings must be calculated by the general Eqs. (182) and (183), with the values of $H$ given by Eqs. (204) to (209), and the values of $M_1$ and $M_2$ given by Eqs. (190) to (199).

Influence lines for moments may be drawn as in the previous cases. For moments in the main span, Eq. (182) is written in the form,

$$M=\left[\frac{M_0+M_1\frac{l-x}{l}+M_2\cdot\frac{x}{l}}{y-ef}-H\right]\cdot(y-ef), \quad . \quad (212)$$

thus giving the bending moments as $(y-ef)$ times the intercepts obtained by superimposing the influence line for $\frac{M'}{y-ef}$ upon the influence line for $H$. This construction is indicated in Fig. 18*b*. For moments in the side spans, the corresponding influence line equation is obtained from Eq. (183):

$$M=\left[\frac{M_0+\frac{x_1}{l_1}\cdot M_{1,2}}{y_1-\frac{x_1}{l_1}\cdot ef}-H\right]\left(y_1-\frac{x_1}{l_1}\cdot ef\right). \quad . \quad . \quad (213)$$

For the continuous stiffening truss, the influence line method just outlined is not very convenient, as the $M'$ influence line (Fig. 18*b*) is a curve for which there is no simple, direct method of plotting.

A more convenient method is that of the Equilibrium Polygon constructed with pole-distance $H$, corresponding to Eq. (85) and Fig. 7. For the continuous stiffening truss, this construction is modified as follows (Fig. 19): At a distance $ef$ below the closing chord $A'B'$, a base line $AB$ is drawn, so that the cable ordinates measured from this base line will be $(y-ef)$ and will therefore represent $M_s$ (Eq. 180). The equilibrium polygon $A''MB''$ for any given loads is then constructed upon the same base line, with the same pole-distance $H$; the height $AA''$ represents $-M_1$, the height $BB''$ represents $-M_2$, and the polygon ordinates below $A''B''$ represent $M_0$; hence, by Eq. (177), the ordinates measured below the base line $AB$ represent $M'$. Then, by Eq. (79), the intercept between the cable curve and the superimposed equilibrium polygon, ***multiplied by H***, will give the resultant bending moment $M$ at any section.

For a single concentrated load $P$, the equilibrium polygon $A''MB''$ is a triangle, and the $M$ intercepts can easily be scaled or figured. By moving a unit load $P$ to successive panel points, we thus obtain a set of influence values of $M$ for all sections.

The corresponding construction in the side spans is also indicated in Fig. 19.

**30. Temperature Stresses.**—The horizontal tension produced by a rise in temperature of $t°$ is given by,

$$H_t = -\frac{3EI\omega t L_t}{f^2 \cdot N \cdot l}, \quad . \quad . \quad . \quad . \quad . \quad (214)$$

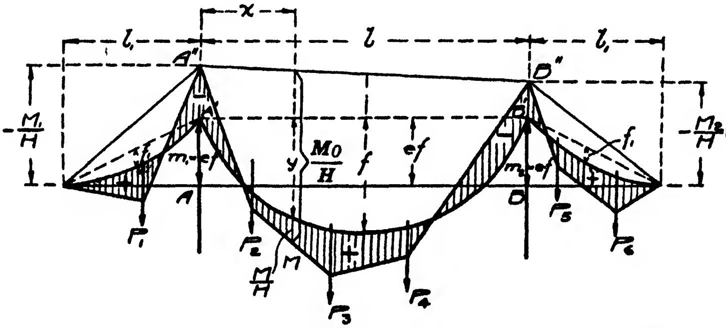



FIG. 19.—Equilibrium Polygon for Continuous Stiffening Truss. (Type OS).

where $N$ is the denominator of Eq. (203), and $L_t$ is given by Eq. (154′).

The resulting moments in the stiffening truss will be given, by Eqs. (180) and (181), as,

$$M_t = -H_t \cdot (y - ef), \quad . \quad . \quad . \quad . \quad . \quad . \quad (215)$$

for the main span, and,

$$M_t = -H_t\left(y_1 - \frac{x_1}{l_1} \cdot ef\right), \quad . \quad . \quad . \quad . \quad . \quad (216)$$

for the side spans.

The vertical shears are given by Eqs. (186) and (187) as,

$$V_t = -H_t(\tan \phi - \tan \alpha), \quad . \quad . \quad . \quad . \quad . \quad (217)$$

for the main span, and

$$V_t = -H_t\left(\tan\phi_1 - \tan\alpha_1 - \frac{ef}{l_1}\right), \quad . \quad . \quad . \quad (218)$$

for the side spans.

**31. Straight Backstays (Type 0*F*).**—If the stiffening truss in the side spans is built independent of the cable (Fig. 20), the backstays will be straight and $f_1 = 0$. Consequently, all terms containing $f_1$, $y_1$, $n_1 = \frac{f_1}{l_1}$, or $v = \frac{f_1}{f}$, will vanish in Eqs. (177) to (218), inclusive.

On account of the continuity of the trusses, however, each span will be affected by loads in the other spans.

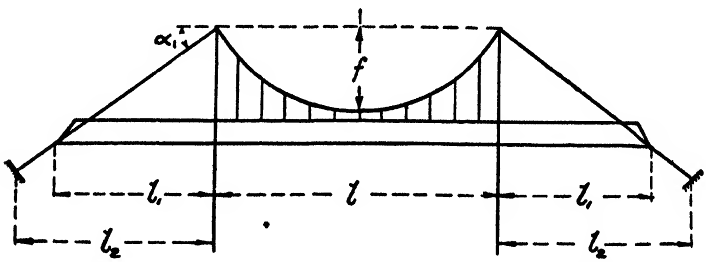



FIG. 20.—Continuous Stiffening Truss with Straight Backstays. (Type 0*F*).

The denominator of the expression for $H$, Eq. (203), will become,

$$N = \tfrac{8}{5} - 2e + \frac{E}{E_c}\frac{3I}{Af^2}\frac{l'}{l}(1+8n^2) + \frac{6I}{A_1 f^2}\frac{E}{E_c}\frac{l_2}{l}\cdot\sec^3\alpha_1, \qquad (219)^*$$

where $e$, the factor of continuity, now has the value,

$$e = \frac{2}{3+2ir}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (220)$$

Equation (183), for bending moments in the side spans, will become,

$$M = M_0 + \frac{x_1}{l_1}M_{1,2} + H\frac{x_1}{l_1}\cdot ef, \quad . \quad . \quad . \quad (221)$$

---

* See Footnote to Eq. (127).

and Eq. (189), for shears in the side spans, will become,

$$V = V_0 \pm \frac{M_{1,2}}{l_1} + H \cdot \frac{ef}{l_1}. \quad . \quad . \quad . \quad . \quad . \qquad (222)$$

For a concentration $P_1$ in either side span, Eq. (205) becomes,

$$H = -\frac{1}{2N \cdot n} \cdot ir^2 e(k_1 - k_1^3) \cdot P_1. \quad . \quad . \qquad (223)$$

For a uniform load covering both side spans, Eq. (207) becomes,

$$H = -\frac{ir^3 e}{4N \cdot n} \cdot p_1 l. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (224)$$

For a uniform load in either side span, covering a length $k_1 l_1$ from the free end, Eq. (209) becomes,

$$H = -\frac{ir^3 e}{8Nn}(2 - k_1^2) \cdot k_1^2 \cdot p_1 l. \quad . \quad . \quad . \quad . \quad . \qquad (225)$$

For a uniform load covering all three spans, Eq. (211), for the bending moments in the side spans, becomes

$$\text{Total } M = \tfrac{1}{2} p x_1 (l_1 - x_1) - \frac{e x_1}{l_1}(\tfrac{1}{8} p l^2 - Hf). \quad . \qquad (226)$$

Equation (216), for temperature moments in the side spans, becomes,

$$M_t = +H_t \cdot \frac{x_1 \cdot ef}{l_1}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad (227)$$

and Eq. (218), for the shears, becomes,

$$V_t = +H_t \cdot \frac{ef}{l_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (228)$$

## SECTION VII.—BRACED-CHAIN SUSPENSION BRIDGES

**32. Three-hinged Type** (3*B*).—The three-hinged type of braced-chain suspension bridge is statically determinate. The suspension system in the main span is simply an inverted three-hinged arch. The equilibrium polygon for any applied loading will always pass through the three hinges. The $H$-influence line for vertical loads reduces to a triangle whose altitude, if the crown-hinge is at the middle of the span and if the corresponding sag is denoted by $f$, is,

$$H=\frac{l}{4f}. \qquad (229)$$

The determination of the stresses is made, either analytically or graphically, exactly as for a three-hinged arch.

Figure 21 shows the single-span type, in which the backstays are straight (Type $3BF$). If the lower chord is made to coincide with the equilibrium polygon for dead load or full live load, the stresses in the top chord and the web members will be zero for such loading conditions. These members will then be stressed only by partial or non-uniform loading. Under partial loading,

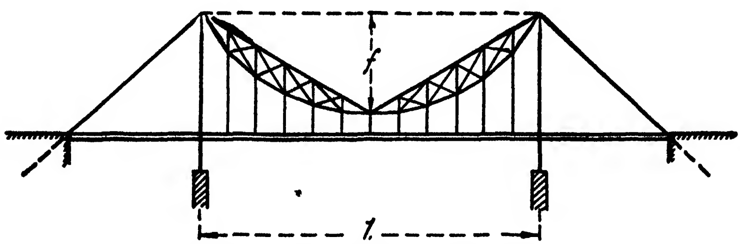



FIG. 21.—Three-hinged Braced Chain with Straight Backstays. (Type $3BF$).

the equilibrium polygon will be displaced from coincidence with the lower chord: where it passes between the two chords, both will be in tension; where it passes below the bottom chord, this member will be in tension and the top chord will be in compression. If the curve of the bottom chord is made such that the equilibrium polygon will fall near the center of the truss or between the two chords under all conditions of loading, the stresses in both chords will always be tension.

Figure 22 shows the three-hinged braced-chain type of suspension bridge provided with side spans (Type $3BS$). The stresses in the main span trusses are not affected by the presence of the side spans, and are found as outlined above. The stresses in the side spans are found as for simple truss spans of the same length, excepting that there must be added the stresses due to the top chord acting as a backstay for the main span. This top chord receives its greatest compression when the span in question is fully loaded, and its greatest tension when the main span is fully loaded.

Temperature stresses and deflection stresses in three-hinged structures are generally neglected.

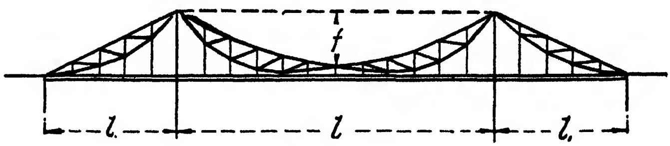



FIG. 22.—Three-hinged Braced Chain with Side Spans. (Type 3BS).

**33. Two-hinged Type (2*B*).**—This system (Fig. 23) is statically of single indetermination with reference to the external forces, so that the elastic deformations must be considered in determining the unknown reaction.

The structure is virtually a series of three inverted two-

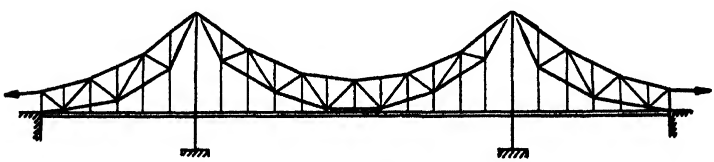


FIG. 23.—Two-hinged Braced Chain with Side Spans (Type 2BS).

hinged arch trusses, having a common horizontal tension $H$ resisted by the anchorage.

The value of $H$ may be determined by the same method as was used for writing Eqs. (116) and (117). In this case, the general equation for $H$ takes the form,

$$H=\frac{\Delta}{\delta}=-\frac{\sum\frac{Zul}{EA}}{\sum\frac{u^2l}{EA}}, \quad . \quad . \quad . \quad . \quad . \quad (230)$$

where $Z$ denotes the stresses in the members for any external loading when $H=0$ (i.e., when the system is cut at the anchorages); $u$ denotes the stresses produced under zero loading when $H=1$; $l$ denotes the lengths of the respective members and $A$ their cross-sections. The summations embrace all the members in the entire system between anchorages.

The stress in any member is given by adding to $Z$ the stress produced by $H$, or,

$$S=Z+H\cdot u. \quad \ldots \ldots \quad (231)$$

For a rise in temperature, the elastic elongations $\frac{Zl}{EA}$ are replaced by thermal elongations $\omega tl$, and Eq. (230) becomes,

$$H_t=\frac{\Delta}{\delta}=-\frac{\Sigma\omega tul}{\sum\frac{u^2l}{EA}}. \quad \ldots \ldots \quad (232)$$

For uniform temperature rise in all the members, Eq. (232) may be written,

$$H_t=-\frac{\omega tL}{\sum\frac{u^2l}{EA}}, \quad \ldots \ldots \quad (233)$$

where $L$ is the total horizontal length between anchorages.

Equations (230) to (233) may also be used for the ordinary types of suspension bridge with straight stiffening truss (Types 2*F* and 2*S*) if the summations are applied to the individual members of the stiffening truss and to the segments of the cable between hangers. (The hangers and towers may also be included.) This will give more accurate results than the ordinary method, as it takes into account the varying moments of inertia of the stiffening truss and any variations from parabolic form of cable.

A graphic method of determining $H$ is to find the vertical deflections at all the panel points produced by a unit horizontal force ($H=1$) applied at the ends of the system. The resulting deflection curve will be the influence line for $H$. If the ordinates of this curve are divided by the constant $\delta$ (the horizontal displacement of the ends of the system produced by the same force

$H=1$), they will give directly the values of $H$ produced by a unit vertical load moving over the spans.

**34. Hingeless Type** ($0B$).—This type of suspension bridge (Fig. 24) is threefold statically indeterminate, the redundant unknowns being the horizontal tension $H$ and the moments at the towers. Instead, the stresses in any three members, such as the members at the tops of the towers and one at the center of the main span, may be chosen as redundants. Let the stresses in the three redundant members under any given loading be denoted by $X_1$, $X_2$, $X_3$. When these three members are cut, the structure is a simple three-hinged arch; in this condition, let $Z$ denote the stresses produced by the external loads, and let $u_1$, $u_2$ and $u_3$ denote the stresses produced by applying internal

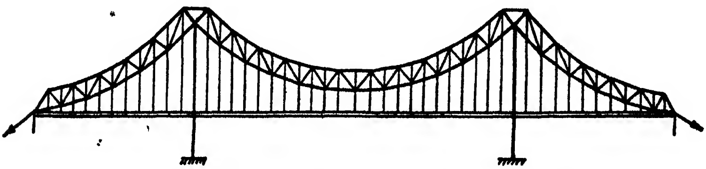


FIG. 24.—Hingeless Braced Chain Suspension Bridge. (Type $0B$.)

forces $X_1=1$, $X_2=1$, and $X_3=1$. Then, when the three redundants are restored, the stress in any member will be,

$$S=Z+X_1u_1+X_2u_2+X_3u_3. \quad . \quad . \quad . \quad (234)$$

The restoration of the redundant members must satisfy the three conditions,

$$\sum\frac{Su_1l}{EA}=0, \quad \sum\frac{Su_2l}{EA}=0, \quad \text{and} \quad \sum\frac{Su_3l}{EA}=0, \quad (235)$$

which, with the aid of Eq. (234), may be written:

$$\left.\begin{aligned}
\sum\frac{Zu_1l}{EA}+X_1\sum\frac{u_1^2l}{EA}+X_2\sum\frac{u_1u_2l}{EA}+X_3\sum\frac{u_1u_3l}{EA}&=0\\
\sum\frac{Zu_2l}{EA}+X_1\sum\frac{u_1u_2l}{EA}+X_2\sum\frac{u_2^2l}{EA}+X_3\sum\frac{u_2u_3l}{EA}&=0\\
\sum\frac{Zu_3l}{EA}+X_1\sum\frac{u_1u_3l}{EA}+X_2\sum\frac{u_2u_3l}{EA}+X_3\sum\frac{u_3^2l}{EA}&=0
\end{aligned}\right\}. \quad (236)$$

The redundant members are to be included in these summations.

The solution of these three simultaneous equations will yield the three unknowns $X_1$, $X_2$ and $X_3$, and their substitution in Eq. (234) will give the stresses throughout the structure.

## CHAPTER II

## TYPES AND DETAILS OF CONSTRUCTION

**1. Introduction.**—The economic utilization of the materials of construction demands that, as far as possible, the predominating stresses in any structure should be those for which the material is best adapted. The superior economy of steel in tension and the uncertainties involved in the design of large-sized compression members point emphatically to the conclusion that the material of long-span bridges, for economic designs, must be found to the greatest possible extent in tensile stress. This requirement is best fulfilled by the suspension-bridge type.

The superior economy of the suspension type for long-span bridges is due fundamentally to the following causes:

1. The very direct stress-paths from the points of loading to the points of support.
2. The predominance of tensile stress.
3. The highly increased ultimate resistance of steel in the form of cable wire.

For heavy railway bridges, the suspension bridge will be more economical than any other type for spans exceeding about 1500 feet. As the live load becomes lighter in proportion to the dead load, the suspension bridge becomes increasingly economical in comparison with other types. For light highway structures, the suspension type can be used with economic justification for spans as low as 400 feet.

Besides the economic considerations, the suspension bridge has many other points of superiority. It is light, aesthetic, graceful; it provides a roadway at low elevation, and it has a low center of wind pressure; it dispenses with falsework, and is easily constructed, using materials that are easily transported;

there is no danger of failure during erection; and after completion, it is the safest structure known to bridge engineers.

The principal carrying member is the cable, and this has a vast reserve of strength. In other structures, the failure of a

FIG. 25.—Brooklyn Bridge.
East River New York Span 1595½ feet Completed 1883.

single truss member will precipitate a collapse; in a suspension bridge, the rest of the structure will be unaffected. In the old Niagara Railway Suspension Bridge (built 1855), the chords of the stiffening truss were broken (due to overloading) and repaired repeatedly, without interrupting the railroad traffic.

| | | | | | | |
|---|---|---|---|---|---|---|
| STIFFENED SUSPENSION BRIDGES | Stiffening truss | Non-continuous | Center hinge (3-hinged) | Side spans free, | $3F$, | |
| | | | | Side spans suspended, | $3S$, | Brooklyn Br., N. Y. |
| | | | No center hinge (2-hinged) | Side spans free, | $2F$, | Williamsburg Br., N. Y. |
| | | | | Side spans suspended, | $2S$, | Manhattan Br., N. Y. |
| | | Continuous | Center hinge (1-hinged) | Side spans free, | $1F$, | Ohio River Br., Cincinnati |
| | | | | Side spans suspended, | $1S$, | Steubenville, Ohio |
| | | | No center hinge (0-hinged) | Side spans free, | $0F$, | Elizabeth Br., Budapest |
| | | | | Side spans suspended, | $0S$, | Rondout Br., Kingston |
| | Braced-chain $B$ | Three-hinged $3B$ | Parabolic lower chord $3BL$ | Side spans free, | $3BLF$, | Point Br., Pittsburgh |
| | | | | Side spans suspended, | $3BLS$, | |
| | | | Parabolic upper chord $3BU$ | Side spans free, | $3BUF$, | Youngstown, Ohio |
| | | | | Side spans suspended, | $3BUS$, | Grand Ave., St. Louis |
| | | | Parabolic center line $3BC$ | Side spans free, | $3BCF$, | |
| | | | | Side spans suspended, | $3BCS$, | 2nd Quebec Design |
| | | Two-hinged $2B$ | Parallel chords $2BP$ | Side spans free, | $2BPF$, | |
| | | | | Side spans suspended, | $2BPS$, | Seventh St., Pittsburgh |
| | | | Varying depth $2BV$ | Side spans free, | $2BVF$, | |
| | | | | Side spans suspended, | $2BVS$, | 1st Quebec Design |
| | | | Horizontal lower chord $2BH$ | Side spans free, | $2BHF$, | |
| | | | | Side spans suspended, | $2BHS$, | |
| | | Hingeless $0B$ | Parallel chords $0BP$ | Side spans free, | $0BPF$, | |
| | | | | Side spans suspended, | $0BPS$. | |

*Special Symbols:*. $E$=Eyebar Chains. $D$=Diagonal Stays.

There are two main classes of suspension bridges: those with suspended stiffening truss (Figs. 25 to 36), and those with overhead braced-chain construction (Figs. 37 to 41). For purpose of reference, there is given here (page 71) a comprehensive system of classification of suspension bridges, with mnemonic type symbols and outstanding examples.

**2. Various Arrangements of Suspension Spans.**—The simplest form of suspension bridge is a single span (Type *2F* or *3F*)

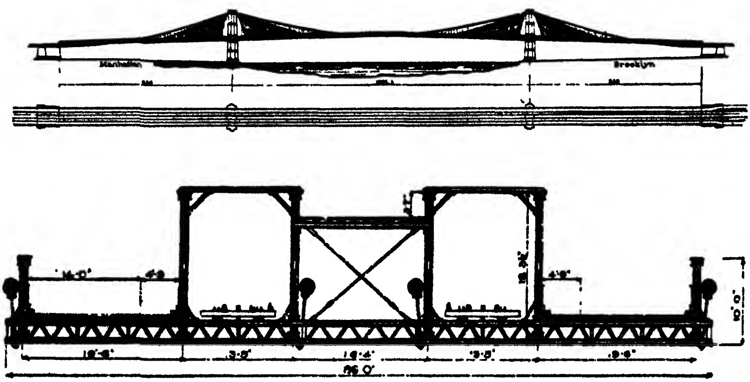


FIG. 26.—Brooklyn Bridge.
(Type *3SD*).
Elevation, Plan, and Cross-section.

with the cable carried past the towers as diagonal backstays (Figs. 27, 29). If side spans are added (Fig. 28), they are independent of the cable and of the main span. The single-span suspension bridge may be built either with or without a stiffening truss (Fig. 27).

The next form is the bridge having three suspended spans (Types *0S*, *1S*, *2S*, *3S*). In this form, stiffening trusses (or girders) are indispensable. Only two towers are required, and each side span is about one-half the length of the main span (Figs. 10, 17, 25, 30, 33, 35).

If the main span is provided with a center hinge (in addition to end hinges), the three-span structure becomes statically determinate (Type *3S*, Fig. 26). The side spans are suspended

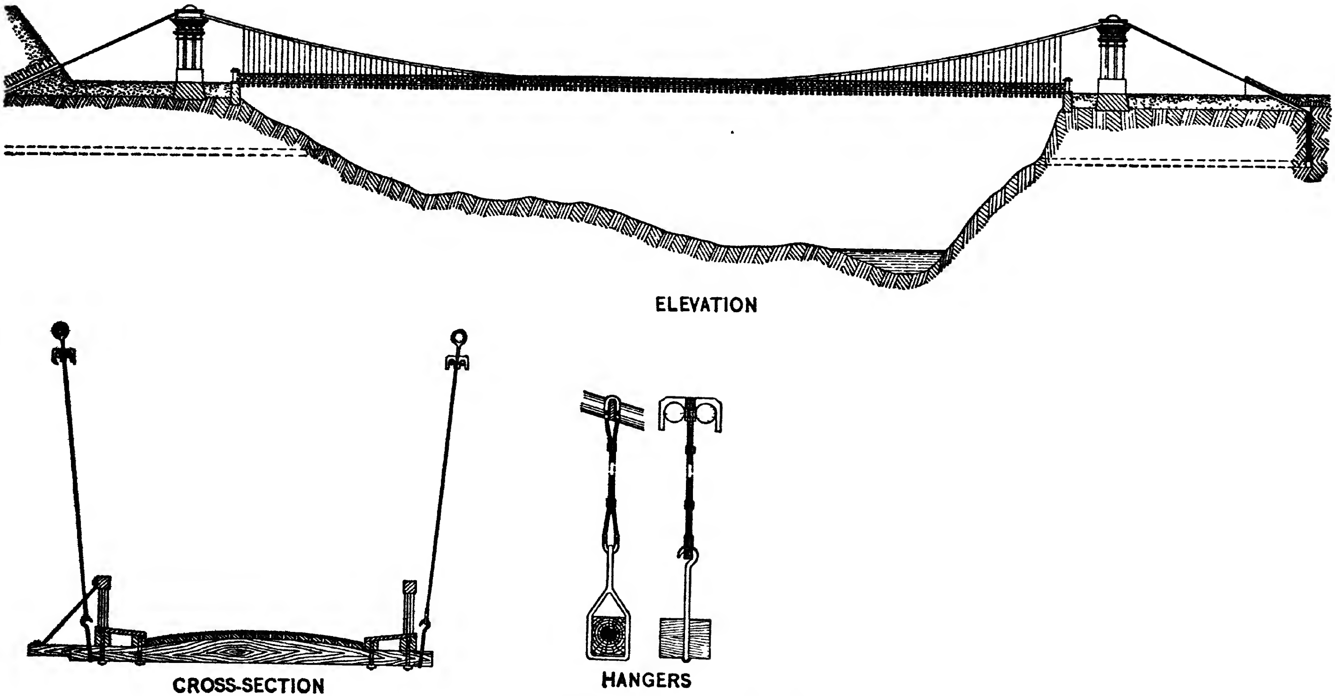



FIG. 27.—Freiburg Suspension Bridge (Switzerland).
Span 870 feet. Unstiffened. Built 1834. Two cables added 1881. Removed 1924.

from the cable, but carry their loads as simple beams without affecting the stresses in the cable or in the main span; on the other hand, any load in the main span or tension in the cable will produce relieving stresses in the side spans.

Multiple-span suspension construction, with more than two towers, is not efficient; the great economy of the suspension type is lost. As the number of spans increases, the value of the cable tension $H$ is proportionately reduced, and more of the load is thrown upon the stiffening trusses; the bending moments and the deflections are thus greatly increased. Examples of this type are the Lambeth Bridge, London, with three equal spans of 280 feet; and the former Seventh St. Bridge, Pittsburgh. To secure greater efficiency in multiple-span suspension designs, the tops of the towers may be connected together with ties, or rigid braced towers may be used. The aim is to minimize the tower-top deflections from unequal loading of the spans.

A suspension bridge of two spans with a single tower would not be economical. The tower would have to be twice the normal height to give the desired sag-ratio for the cables.

**3. Wire Cables *vs.* Eyebar Chains.**—One of the first questions to be decided in the design of a suspension bridge is the choice between a wire cable and a chain of eyebars (or flats) for the principal carrying member. The latter enables the bracing for the prevention of deformation under moving load to be incorporated in the suspension system; the other ordinarily requires a separate stiffening truss for the reduction of these deflections.

The earliest suspension bridges were built with chains. At first (1796) forged wrought-iron links were employed; then (1818) wrought-iron eyebars were introduced; and later (1828) open-hearth steel eyebars came into use. John A. Roebling established the use of parallel wire cables (about 1845). Following his time, wire cables were used in practically all suspension bridges. (Two notable exceptions were the Elizabeth Bridge at Budapest, Fig. 34, and the Rhine Bridge at Cologne, Fig. 17.) With the development of heat-treated eyebars and the adoption of this new material for the Florianopolis Bridge (1922–5), the economic

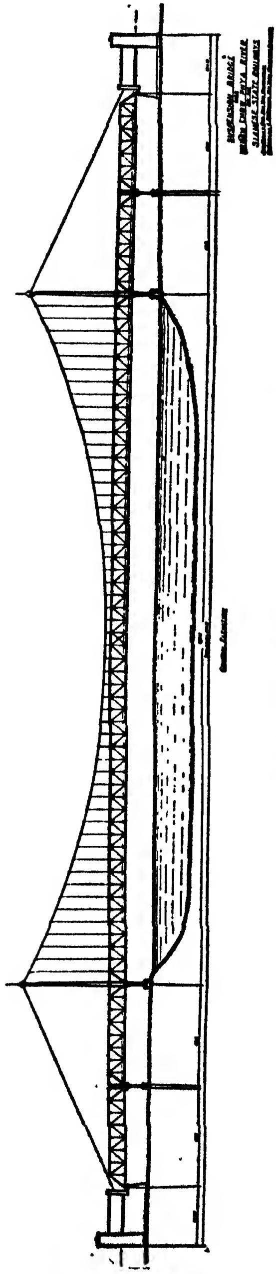


FIG. 28.—Suspension Bridge for the Siamese State Railways. (Type 2*FE.*)

Span 900 feet. Eyebar Chains. Design by D. B. Steinman and F. H. Frankland, 1921.

competition between wire cables and eyebars has been reopened. Heat-treated eyebars have since been adopted for three suspension bridges (of 430-foot and 442-foot spans) at Pittsburgh, and for two 700-foot spans (of the Florianopolis type) over the Ohio River. On the other hand, wire cables have since been adopted for the 1500-foot Hudson River span at Poughkeepsie, the 1850-foot span at Detroit (Fig 15), the Mount Hope Bridge of 1200-foot span in Rhode Island, the 700-foot and 689-foot spans at Portsmouth and Steubenville over the Ohio River, the 948-foot span at Grand' Mere, Quebec, and the 3500-foot Hudson River span at New York.

In the competitive bidding on the Hudson River Bridge at New York (1927), the wire design won by a margin of $2,000,000, indicating that the eyebars cannot compete with wire cables for extremely long spans The awards of the contracts on the Ohio River bridges at Point Pleasant and St Mary's indicate that high-tension heat-treated eyebars, at present (1928) relative prices and working stresses, are more economical than wire cables for spans as short as 700 feet It is difficult to predict the relative economy of the two materials for span lengths between these limits (700 feet and 3500 feet) without further bidding competitions Any conclusions are subject to revision with changes in relative working stresses and with fluctuations in unit prices.

The present accepted practice allows design stresses of 80,000 to 84,000 pounds per square inch in wire cables, and 45,000 to 50,000 pounds per square inch in high-tension heat-treated eyebars. The specified minimum ultimate strengths are 215,000 for galvanized wire and 105,000 for the eyebars, and the specified minimum yield points are 144,000 for the wire and 75,000 for the eyebars; these values indicate that, for a fair comparison, the respective allowable stresses for the two materials should be approximately in the ratio of 2 to 1.

The most recent development in connection with wire cables has been the introduction (in 1928) of heat-treated bridge wire. The heat-treatment raises the minimum yield point from 144,000 to 190,000, the ultimate strength from 215,000 to 220,000, and the modulus $E$ from $26\frac{1}{2}$ million to 30 million pounds per square inch. The heat-treated wire has already been adopted for the Mount Hope and Detroit Bridges.

Since eyebar chains are ordinarily 2 to 2½ times as heavy as the alternative wire cables, the respective unit prices for the two materials (delivered and erected) must be approximately in the inverse ratio for economic competition.

Aside from the economic comparison, the principal arguments in favor of wire construction are the following: Wire cables are self-supporting during erection, and all the problems involved have been worked out and successfully demonstrated in the longest suspension spans. The method of stringing to equal sags automatically secures uniform tensions. With the protection of wire wrapping, supplemented (in coastal locations) by the use of galvanized bridge wire, demonstrated assurance against any possibility of corrosion is secured. Wire cable bridges have shown long endurance and high reserve capacity for loadings greatly exceeding the original design loads. The smaller weight of wire cables makes them the logical material for very long spans. In addition, the following points are cited against eyebar construction: high secondary stresses in the bars, unequal stressing of the bars, difficulty of inspection and painting between the eyebar heads and in the pin-holes, and untried problems in the erection of eyebar chains for very long spans.

The principal advantages cited in favor of eyebar construction are: Where two designs are of equal cost, the heavier bridge is to be preferred as giving a more rigid structure. For railway bridges, the eyebar construction with its larger cable section and consequently greater rigidity is especially advantageous. Pin-connected eyebars permit speedier erection, especially in bridges which would require large-sized cables. Finally, the eyebar construction facilitates the adoption of the Florianopolis type or other type in which the cables and the stiffening system are combined in whole or in part.

In their design for the Sydney Harbor Bridge (1923), Robinson and Steinman developed plans and details for combining wire cables with a continuous stiffening truss of the Florianopolis principle. Thus far, however, there has been no opportunity of demonstrating the feasibility of integrally connecting the stiffening system to wire cables.

**4. Methods of Vertical Stiffening.**—On account of the deformations and undulations under moving load, unstiffened suspension bridges should not be used for ordinary spans.

If no stiffening truss is provided, the distortions and oscillations of the cable may be limited by using a small sag-ratio; by making the floor deep and continuous; or by employing a latticed railing as a stiffening construction (Figs. 27, 33).

Another method of stiffening the suspension bridge is by the introduction of diagonal stays between the tower and the roadway (Fig. 26). These, however, have the disadvantage of making the stress-action uncertain, and of becoming either overstressed or inoperative under changes of temperature; moreover, they introduce unbalanced stresses in the towers.

In recent French construction, diagonal stays are utilized, but the redundancy of members is more or less remedied by

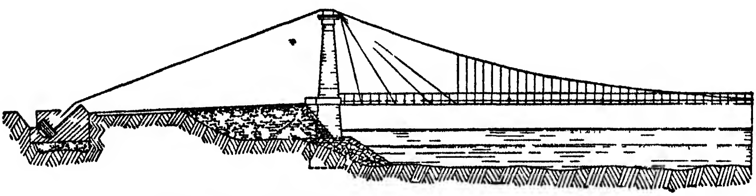


FIG. 29.—Suspension Bridge at Cannes–Ecluse.
(Type 2*FD*).
Over the Yonne River (France) Span 760 feet Built 1900 Wire Rope Cables. Diagonal Stays

omitting the suspenders near the towers (Fig. 29). The indeterminateness is thus relieved, and the cable stress is reduced. This arrangement may be used to advantage in the reconstruction of weak suspension bridges.

A different method of vertical stiffening, known as the Ordish-Lefeuvre System, dispenses with cable and suspenders; it consists of diagonal stays running from the tops of the towers and meeting at a number of points along the span, so as to provide a triangular suspension for each point. These diagonal stays are held straight by hangers from a light catenary cable overhead. This system was used for a bridge at Prague (1868) and for the Albert Bridge in London (1873). It proved to be uneconomical and unsatisfactory. A modified form, known as the Gisclard System, was devised for a bridge at Villefranche in 1907 and has since been copied for several other spans in France, despite its structural and aesthetic drawbacks.

Practically all modern suspension bridges are stiffened by means of a truss construction, either separate (Figs. 25–36) or incorporated in the cable system (Figs. 37–41). The different types of stiffening trusses and braced-chain designs will be discussed in separate sections.

**5. Methods of Lateral Stiffening.**—To give the structure lateral stiffness against wind forces, the most effective means is a complete system of lateral bracing. If this bracing is in the plane of the top or bottom chords of the stiffening truss, these chords may act as members of the lateral systems (Figs. 30, 36); otherwise, separate wind-chords must be provided (Figs. 38, 39, 40).

The wind bracing just described is sometimes supplemented by land-ties or wind-anchors, i.e., ropes connecting points on the roadway to the piers (Fig. 38) or to points on shore. A horizontal suspension system may thus be formed (Fig. 38).

Another device for securing lateral stiffness is by building the cables and suspenders in inclined planes (Figs. 27, 30). This "cradling" of the cables, however, does not appreciably increase the lateral stability of the structure if there is but one cable on each side. If two or more cables of different inclinations are provided on each side (Figs. 26, 32), lateral stability is secured, but at the sacrifice of equal division of cable stresses.

Cradled cables, even if they do not prevent lateral deflection, will help to bring the resulting oscillations more quickly to rest—an important desideratum in long spans.

**6. Comparison of Different Types of Stiffening Truss.**—As a result of a comparative estimate of different types of stiffened suspension bridges, the following relative weights of cable and truss (in main span) were obtained:

| Type | Relative Weight of Cable | Relative Weight of Truss | Relative Combined Weight |
|---|---|---|---|
| 0S | 99 | 103 | 102 |
| 1S | 111 | 107 | 109 |
| 2S | 100 | 100 | 100 |
| 2F | 101 | 89 | 95 |
| 3F | 103 | 82 | 92 |

The hingeless type (0*S*) (Fig. 17), gives the most rigid structure, as a result of the continuity of the stiffening truss over the towers. The deflections will be about $\frac{1}{4}$ less than those of a two-hinged stiffening truss of the same dimensions. This greater rigidity is secured at an expense of only 2 per cent increase in the cost of the structure.

The one-hinged type (1*S*) is the least desirable construction. It has the highest cable stresses and chord stresses of any type of stiffening truss. It will cost more than any other type; and the large variation in chord stresses, the abrupt reversals of shear, and the lack of rigidity are serious disadvantages.

The two-hinged types (2*S* and 2*F*) are widely used and are probably the most efficient types, all things considered. They are more economical than the continuous types, and are simpler to figure. They are far more rigid than type 3*F*. The hinges are located in the towers, where they are least objectionable.

Comparing types 2*S* and 2*F*, we find that leaving the side spans free (straight backstays, Type 2*F*) (Fig. 31) reduces the bending moments in the main span. The main-span truss weight is thus reduced by about 11 per cent, without sensibly affecting the cable weight. For lightness of truss, type 2*F* is exceeded only by the three-hinged suspension bridge. Type 2*F* is also more rigid than type 2*S*.

Suspending the side spans (Type 2*S*) (Figs. 30, 35) makes the cable more flexible, thus throwing more load on the stiffening truss. As a result, about 11 per cent is added to the weight of the truss in the main span, and the cable stress is slightly relieved. The increase in cost of the main span is generally more than offset, however, by the saving in the side spans as a result of their suspension. Without any addition to its weight, the cable relieves the side spans of their full dead load and nearly all of their live load. Type $2S$ will consequently be more economical than $2F$ or $3F$ unless the conditions at the site are favorable to cheap, independent approach spans (Fig. 31). Another advantage of suspended side spans is the dispensing with falsework for their erection (Fig. 50).

The three-hinged type ($3F$) is determinate for calculations. The addition of the center hinge slightly increases the cable stress, but effects a small reduction in weight of stiffening truss. This type is little used on account of its lack of rigidity and other disadvantages arising from the hinge at mid-span. Intermediate hinges are troublesome and expensive details, particularly in long spans; besides augmenting the deflections, they cause sudden reversals of shear under moving load, and constitute a point of weakness and wear in the structure. There is also a large waste of material in the minimum chord sections near the hinges which, in many cases, will offset the theoretical reduction in the weight of the stiffening truss. Furthermore, a center hinge conduces to a serious distortion of the cable from the ideal parabolic form, with a resulting overloading of some of the hangers. In the case of the Brooklyn Bridge (Fig. 25), the center hinge or slip joint has caused excessive bending stresses in the cable at that point, and the breaking of the adjacent suspenders; 120 suspenders near the hinge had to be replaced by larger rods.

**7. Types of Braced-Chain Bridges.**—A stiffening construction incorporated in the suspension system may be used instead of the straight stiffening truss at roadway level. The former construction, as a rule, involves the use of eyebar chains instead of wire cables (Figs. 37, 38, 40, 41).

A braced-chain suspension bridge is virtually an inverted arch in which the ends are capable of restricted horizontal movements. The stresses are the same as those in an arch, but with opposite signs; the principal stress is tension, instead of compression.

Braced-chain bridges may be classified as to the number of hinges ($0B$, Fig. 41; $2B$, Fig. 39; $3B$, Fig. 38); or as to outline of the suspension system (Parabolic Top Chord, Figs. 37, 39; Parabolic Bottom Chord, Fig. 38; Parabolic Center Line, Fig. 40; Parallel Chords, Fig. 41).

If the suspension system has a parabolic top chord and a straight bottom chord (Type $2BH$; Type $3BUH$, Fig. 37) it corresponds to a spandrel braced arch. The Lambeth Bridge, London, is an example. The top chord, like a cable, carries the entire dead load. If the live load is not too great in proportion, the top chord will never have its tensile stresses reversed; it may then be built as a flexible cable (Lambeth Bridge) or chain (Frankfort Bridge, Fig. 37). The bottom chord members suffer reversals of stress, hence they must be built as compression members. For erection, the diagonals should be omitted until all the dead load and one-half the live load are on the structure at mean temperature; this procedure will minimize the extreme stresses in bottom chord and web members. The advantage of making the bottom chord straight is to save hangers and extra wind chords.

To avoid having very long diagonals near the ends of the span, the bottom chord may be bent up toward the towers (Type $2BV$, Fig. 39). This construction has the advantage of maximum truss depth near the quarter-points where the bending moments are also a maximum. The main part of the lower chord remains at the roadway level, thereby saving hangers and extra wind chords over that length.

If the bottom chord is made parabolic, it becomes the principal carrying member. This outline is best adapted for three-hinged systems (Type $3BL$). A notable example is the former Point Bridge at Pittsburgh (Fig. 38). In this structure, the top chord consists of two straight segments, intersecting the bottom chord at ends and center. Since the bottom chord is the equilibrium curve for dead load, there are no dead-load stresses in the top chords or in the web members. The top chords must be made stiff members, as they are subject to reversals of stress. This form of suspension bridge (Type $3BL$) is statically determinate and easily figured. It avoids the use of long diagonals required in the spandrel braced types ($2BV$, Fig. 39; $2BH$; $3BUH$, Fig. 37), but it requires the addition of longitudinal and lateral stiffening in the roadway.

Instead of being straight lines (Fig. 38), the two top-chord segments may be curved. In a system proposed by Eads, they are made convex upward.

To avoid reversals of stress in the chord members, a form known as the Fidler Truss may be used. In this form (Type $3BC$), both chords are concave upward; and the line midway between top and bottom chords is made parabolic, so that the two chords will have equal tensions under dead load and uniform live load. An example of this form is Lindenthal's Second Quebec Design (Fig. 40). The outlines of the chords are obtained by superimposing the two equilibrium curves for total dead load plus live load covering each half of the span in turn.

For two-hinged systems (Type $2BF$) a crescent-shaped truss may be used. The top and bottom chains meet at common supports on the towers, where they are connected to single backstays. There are no examples of this type.

If the top and bottom chains are kept parallel, we have either Type $2BP$ or Type $0BP$ (Fig. 41), according as the truss bracing is interrupted or continuous at the tower. Both of these types are indeterminate, and may involve some uncertainty of stress distribution. Unless the tower and anchorage details are properly worked out, there is danger of one of the parallel chains becoming overstressed or inoperative. Examples of these types are Lindenthal's Seventh St. Bridge at Pittsburgh (Type $2BP$) and his Hudson River Bridge design (Type $0BP$, Fig. 41).

An important advantage of the braced-chain system of construction over the straight stiffening truss is the greater flexibility of outline, with the possibility of varying the truss depths for maximum efficiency. By having the greatest depth of bracing at the quarter points of the span, where the maximum moments occur, the stiffness of the bridge with a given expenditure of material is greatly increased; and by using a shallow depth along the middle third of the span, the temperature stresses are reduced.

The braced-chain construction (Types $2BV$, $2BH$, or $3BU$) saves one chord of the truss, as the cable itself forms the upper chord.

Advantages of the suspended stiffening truss (Figs. 25–36) are more graceful appearance, dispensing with extra wind chords, lower elevation of surfaces exposed to wind, less live-load effect on hangers and cables, simpler connections, easier and safer erection.

In addition, the braced-chain and suspended-truss types carry with them the respective advantages of eyebars and wire cables, unless the practically untried combination of overhead bracing with wire cables is adopted.

**8. Economic Proportions for Suspension Bridges.**—The general ratio of side spans to main span is about $\frac{1}{4}$ for straight backstays, and about $\frac{1}{2}$ for suspended side spans. Shorter ratios tend to make the stresses or sections in the backstays greater than in the main cable. The length of side span is also controlled by existing shore conditions, such as relative elevations and suitable anchorage sites.

The economic ratio of sag to span of the cable between towers is about $\frac{1}{9}$ if the backstays are straight and about $\frac{1}{8}$ if the side spans are suspended. (See the author's book "Suspension Bridges and Cantilevers.") For light highway and foot-bridges, the sag-ratio may be made as low as $\frac{1}{10}$ to $\frac{1}{12}$.

For efficient lateral stiffness the width, center to center of outer stiffening trusses or wind chords, should preferably not be less than about $\frac{1}{30}$th of the span.

The proper depth of stiffening truss is determined by the degree of rigidity desired. Reducing the depth diminishes the cost. For a railroad bridge of ordinary span-length, the truss depth (at the quarter points) should not be less than about $\frac{1}{75}$th of the span, or the deflection gradients will exceed 1 per cent. (See "Suspension Bridges and Cantilevers.") For highway bridges, the depth may be made as low as $\frac{1}{60}$th to $\frac{1}{70}$th of the span, for spans up to 1000 feet; $\frac{1}{70}$th to $\frac{1}{90}$th of the span, for spans up to 2000 feet; $\frac{1}{90}$th to $\frac{1}{150}$th of the span, for spans up to 3000 feet; and the stiffening trusses may be dispensed with for longer spans. The increasing ratio of dead-load to live-load reduces the need for extraneous stiffening.

Suspension spans up to 10,000 feet may be regarded as feasible. The bridge under construction (1927-1932) over the Hudson River at New York has a span of 3,500 feet, and the proposed bridge over the Narrows at New York will have a span exceeding 4500 feet.

**9. Arrangements of Cross-sections.**—The unit of suspension bridge design is the vertical suspension system, consisting of a vertical (or slightly inclined) plane.

Since the suspension systems are above the roadway, their number is limited; they seldom exceed two (Figs. 27, 30, 32, 37–41). In wide bridges having a number of roadways, four suspension systems may be provided (Figs. 26, 36).

The main carrying element in each suspension system may be a single cable (Figs. 26, 36), two cables side by side (Figs. 27, 32), two cables superimposed, or a group of cables or wire ropes (Figs. 27, 29, 30); or it may consist of a single chain of bars (Figs. 28, 37), two chains simply superimposed (Figs. 33, 34, 39), or two chains connected by web members to make a vertical stiffening system (Figs. 38, 40, 41).

There is generally one stiffening truss for each suspension system, and in the same plane; hence, there are ordinarily two (Figs. 30, 32, 37–41), and at most four stiffening trusses (Fig. 36). An exception is the Brooklyn Bridge (Fig. 26), having six stiffening trusses for four cables; this, however, has proved to be an unsatisfactory and inefficient arrangement.

Between the stiffening trusses are the roadways, generally on a single deck (Figs. 27, 30, 33, 34, 37–40). Sometimes two decks are provided, in order to provide the required number of traffic-ways (Figs. 26, 32, 36, 41). Where two decks are used, the railways are best placed below and the vehicular roadways above (Fig. 41). In the Williamsburg Bridge (Fig. 32), a transverse truss is employed to carry the inside floorbeam reactions to the two outside suspension systems.

The floorbeams either terminate at their connections to the outer suspension systems (Figs. 26, 27, 30, 37–40); or they extend beyond as cantilever brackets to carry outside sidewalks or roadways (Figs. 32, 36, 41). The latter arrangement saves floorbeam weight; reduces width of towers, piers and anchorages; and helps in the separation of different traffic-ways. In very long spans, the first arrangement (with all roadways inside) may be necessary in order to maintain the requisite width between trusses for lateral stiffness.

Where there are four suspension systems, the floorbeams may be made continuous for greater stiffness (Fig. 36); or they may be provided with hinges to eliminate the indeterminateness.

**10. Materials used in Suspension Bridges.**—The stiffening trusses are generally built of structural steel, but silicon steel or other alloy steels may be used; and for minor structures, timber trusses have been employed.

The cables are generally made of galvanized steel wires having an ultimate strength of 215,000 to 230,000 pounds per square inch, and an elastic limit as high as 144,000 pounds per square inch, or 190,000 in heat-treated wire.

The suspenders are generally galvanized steel ropes. These are manufactured in diameters ranging from 1 to $3\frac{1}{4}$ inches, and have a tested ultimate strength given by $80{,}000 \times (\text{diameter})^2$.

In smaller bridges (Figs. 29, 30), the cables may be made of galvanized steel ropes or rope strands instead of parallel wires.

The towers are generally built of structural steel (Figs. 30, 31, 35, 37, 38, 41); although stone (Figs. 25, 27, 29, 33), concrete, and timber have been used.

Cast steel is used for all castings, such as saddles (Figs. 32, 33), cable bands (Figs. 32, 36), strand shoes (Figs. 32, 36), anchorage knuckles (Figs. 32, 33), and anchor shoes (Figs. 33, 37, 38). Suspender sockets (Figs. 32, 36) are made by drop-forging and machining.

If eyebar chains are adopted instead of wire cables, special steels may advantageously be employed. The chains for the

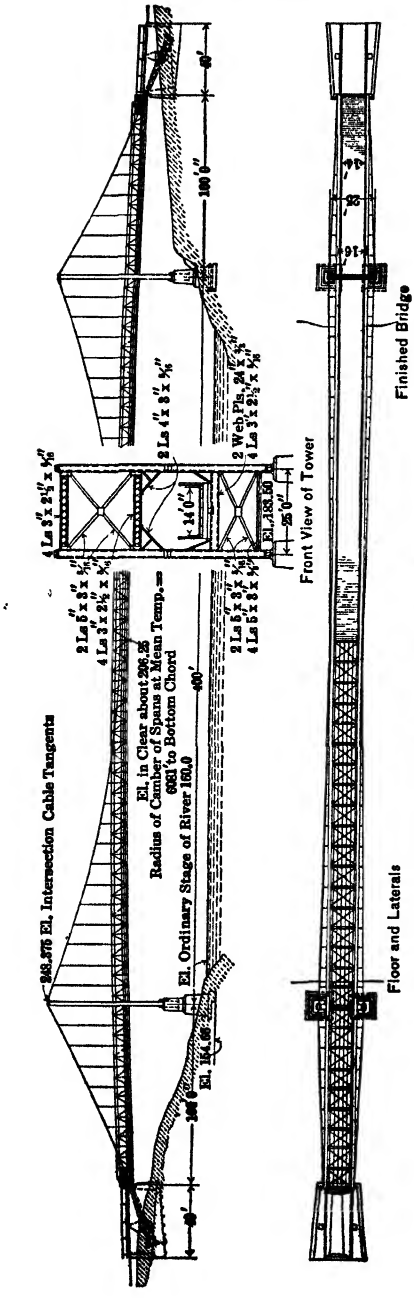



FIG. 30.—Suspension Bridge at Massena, N. Y.
(Type 2S.)

Span 400 feet. Rope strand cables. Designed and built by H. D. Robinson, 1910.

Elizabeth Bridge at Budapest (Fig. 34) were made of open-hearth steel with 70,000 ultimate strength. Nickel-steel eyebars with 100,000 ultimate strength were to be used for Lindenthal's Manhattan Bridge design (1902), and were adopted for the Rhine Bridge at Cologne, 1915 (Fig. 17). Since 1922, a more economical material has been available, in the form of high-tension heat-treated carbon-steel eyebars, with 75,000 elastic limit, 105,000 ultimate, and 5 per cent elongation. This material received its first application in the Florianopolis Bridge in Brazil (1114-foot span, 1922–6; Robinson and Steinman, Consulting Engineers), and has since been adopted in the 700-foot spans over the Ohio River at Point Pleasant (1928) and St. Mary's (1929). A mild grade of the heat-treated eyebars, having 50,000 elastic limit, 80,000 ultimate and 8 per cent elongation, was adopted for the three self-anchored suspension bridges over the Allegheny River at Pittsburgh (1926–1928).

**11. Wire Ropes.**—Calvanized steel ropes used for suspenders and for small bridge cables (Figs. 29, 30), are manufactured in diameters ranging from 1 to $3\frac{1}{4}$ inches. Each rope consists of 7 strands, each strand containing 7, 19, 37 or 61 wires; the wires are twisted into strands, in the opposite direction to the twist of the strands into rope, the angle of twist being about 18°. The weight of the rope in pounds per lineal foot is $1.68 \times (\text{diameter})^2$.

The strength of a twisted wire rope is less than the aggregate strength of the individual wires. The spiral wires are stressed about 4 or 5 per cent higher than the mean stress per square inch in the rope, and the center wire is stressed 15 per cent higher than the spiral wires. The tested ultimate strength of galvanized steel suspension bridge rope is given by $80{,}000 \times (\text{diameter})^2$.

When twisted wire ropes are used for cables, care must be observed, when applying the fundamental design formulas, to allow for the reduced elastic coefficient ($E$) of this material; it is only about $\frac{2}{3}$ of the value of $E$ for structural steel.

The coefficient of elasticity ($E$) of a single rope strand with an angle of twist of 18° is 85 per cent of $E$ for parallel wires, or about 24,000,000. The coefficient of elasticity ($E$) of a twisted wire rope composed of 7 or more strands is 85 per cent of $E$ for a single strand, or about 20,000,000.

Twisted wire ropes have a large initial stretch under load, on account of the spiral lay of the wires and strands. Consequently, at small loads, tests show a high rate of stretch yielding a modulus of elasticity ($E$) as low as 10,000,000. After the initial stretch has been taken up (at a unit stress of about 20,000 pounds per square inch), the rate of elongation is considerably reduced, yielding a value of 20,000,000 for the true elastic coefficient ($E$). The lower values of $E$ (10,000,000 to 15,000,000) are to be used in estimating the dead-load elongation of the cable (if composed of wire ropes), and the higher value (20,000,000) should be used in figuring live-load and temperature stresses.

On account of the high and variable elongations, including the influence of time, suspenders and cables made of wire ropes should preferably be provided with screw and nut adjustments to regulate their lengths to the assumed deflections and elevations.

Cables may be built either of twisted wire ropes or of parallel wires. For long or heavy spans, parallel wire construction is best adapted; for light bridges, the use of twisted wire ropes may be more convenient and expeditious.

In cables formed of twisted wire ropes, the individual ropes are limited to 250 to 300 wires each, so as to avoid excessive stiffness and difficulty of handling; consequently, large cable sections require several such ropes.

A multi-strand cable may be formed of twisted strands surrounding a straight central strand; or of parallel strands united at intervals by clamps. Twisted strands ensure a more even division of load, except that the central strand carries a little more than its share; but the resulting cable suffers greater elongation under load. Moreover, since a twisted-strand cable must be erected as a unit, it is limited in weight and section. Equal stressing of parallel strands is dependent upon the efficiency of the clamps or bands in gripping them. An advantage of the parallel construction with bolted clamps is the ease of correcting overstress in individual strands and of replacing damaged strands. Clamping systems have been designed for large groups of parallel ropes, to ensure unit stress action and to facilitate renewal of individual ropes; at the same time preserving ample spacing between the ropes to permit inspection and protection against rusting.

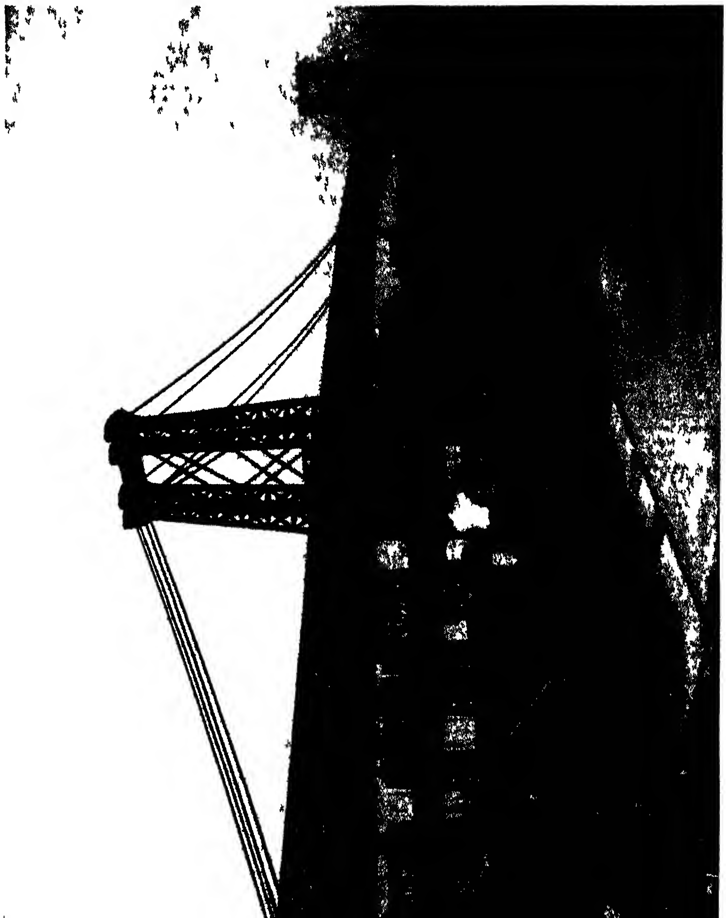


Fig. 31.—Williamsburg Bridge.
East River New York Span 1600 feet Completed 1903

A twisted wire cable of patent locked wire has been developed. In it the spiral wires have trapezoidal and $Z$-sections, locking together so as to leave practically no voids. The advantages are compactness, smooth outer surface, firm gripping of the individual wires, and sealing against the entrance of moisture. Cables of this construction, ready to erect, have been made in single strands up to 800 tons tensile strength, and in seven strands up to 1500 tons.

The application of twisted ropes on a large scale involves problems requiring further study; whereas parallel wire construction has had ample satisfactory demonstration in the largest existing suspension bridges.

**12. Parallel Wire Cables.**--Parallel wire cables have the advantages of maximum compactness, maximum uniformity of stress in all the wires, and the easiest and safest connection of the cable to the anchorage. Twisted wire ropes are used for shorter spans, up to 600 or 700 feet, to save time in erection. Parallel wires are applicable to spans of any length, and will cost somewhat less than twisted ropes of the same strength; they will not stretch as much as twisted ropes, and will therefore keep more of the load off the stiffening truss. The only disadvantage of parallel wire cables is that they consume several weeks or months in erection.

A common size of wire for cables is No. 6 (Roebling gauge) which is 0.192 inch diameter and weighs 0.0973 pound per foot before galvanizing; after galvanizing, the diameter is about 0.195 inch, and the weight is practically $\frac{1}{10}$ pound per foot. The breaking strength of this wire at 220,000 pounds per square inch is 6400 pounds; the elastic limit at 150,000 pounds per square inch is 4350 pounds; the working stress at 75,000 pounds per square inch is 2180 pounds per single wire. Other common sizes of wire for cables are No 7 (0.177 inch diameter) and No. 8 (0.162 inch diameter), recommended for shorter spans.

About 250 to 350 of these wires are treated as a single strand during erection. The cable consists of 7, 19, 37 or 61 of these strands. At the anchorages, the strands are looped around grooved shoes (Fig. 36) which are pin-connected to the anchorage eyebars (Fig. 32). For the rest of their length, the strands are compacted and bound to form a cylindrical cable of parallel wires.

The following table gives data on the wire cables of the East River suspension bridges:

| | Brooklyn (Fig. 25) | Williamsburg (Fig. 31) | Manhattan (Fig. 35) |
|---|---|---|---|
| Date | 1876–1883 | 1898–1903 | 1903–1909 |
| Main span | 1595.5 ft. | 1600 ft. | 1470 ft. |
| Cable-sag | 128 ft. | 177 ft. | 160 ft. |
| Total load, p. l. f. | 35,500 lb. | 75,000 lb. | 104,000 lb. |
| Number of cables | 4 | 4 | 4 |
| Strands per cable | 19 | 37 | 37 |
| Wires per strand | 278 | 208 | 256 |
| Wire diameter | .180 in. | .192 in. | .195 in. |
| Total cross-section | 533 sq. in. | 838 sq. in. | 1092 sq. in. |
| Cable diameter | $15\frac{3}{4}$ in. | $18\frac{5}{8}$ in. | $20\frac{3}{4}$ in. |
| Size of wrapping wire | .135 in. | ............ | .148 in. |
| Max. stress in cables | 47,500 lb./sq. in. | 50,300 lb./sq. in. | 73,000 lb./sq. in. |
| Ult. strength of cables | 160,000 lb./sq. in. | 200,000 lb./sq. in. | 210,000 lb./sq. in. |

For security against corrosion, the wire should be galvanized. The only drawback is a reduction of about 7 per cent in the strength of the wire per square inch of final gross section (4 per cent actual reduction in the strength of the wire, and 3 per cent increase in gross section.)

The splicing of individual wires was formerly effected by wrapping the overlapping ends with fine wire. A more efficient splice (giving 95 per cent efficiency) is made by mitering the ends, threading them and connecting with small sleeve-nuts (Figs. 32, 36). Both methods have the disadvantage of disturbing the uniformity of the cable section. To reduce the number of such splices, the lengths of the individual wires as manufactured have been increased to 3300 feet. In some French bridges, the ends of the wires, after beveling, were joined by brazing; but the heat reduces the strength of the wire at the splice.

Besides using galvanized wires, additional protection is secured by providing a tight and continuous wire wrapping around the cable. Soft, annealed, galvanized wire of No. 8 or No. 9 Roebling gauge is commonly used. The function of this wrapping is to exclude moisture, to protect the outer wires, and to hold the entire mass of wires so tightly as to prevent chafing and ensure united stress action.

No record can be found of any rusting of wire cables employing either or both of the above described methods of protection.

**13. Cradling of the Cables.**—In many older suspension bridges, the main span cables do not hang vertically but in planes inclined toward one another, the inclination ranging from 1 : 20 to as much as 1 : 6. The stiffening trusses, however, are kept vertical. Even in designs with overhead bracing, the suspension systems have been cradled with inclinations ranging from 1 : 20 to 1 : 16.

Cradling is employed principally because it is supposed to augment the lateral stiffness of the structure; however, the advantage in this respect over vertical cables is but slight. With an inclination of 1 : 10, the increased resistance to lateral displacement is only 1 per cent. Moreover, with cradled cables any lateral displacement is accompanied by a tilting of the suspended structure, resulting in secondary stresses which are difficult to evaluate.

Resistance to lateral displacement is more significantly improved in proportion as the cable-sag is reduced and as the weight of the suspended structure is increased.

If cradling is adopted, the cables should not be wrapped until they are pulled into the final inclined position; otherwise serious local stresses will be produced in the cable wires near the saddles.

If the cables are cradled, the saddle reactions on the towers will be correspondingly inclined unless the backstay cables are made divergent; the latter arrangement (used in the Williamsburg Bridge, Fig. 32) increases the necessary width of the anchorage.

Cradling will be effective in producing lateral stiffness if two cables of different inclination are provided on each side (Figs. 26 and 32). This arrangement has the disadvantage of throwing unequal load on the cables when the wind acts on the structure; load is added to the cables inclined in the direction of the wind, and those inclined in the opposite direction are relieved of load.

**14. Anchoring of the Cables.**—Parallel wire cables are anchored by ~~making the end of each~~ strand in the form of a sling. With the wrapping omitted at the end of the cable, the free wires loop around a half-round, flanged casting called a strand shoe

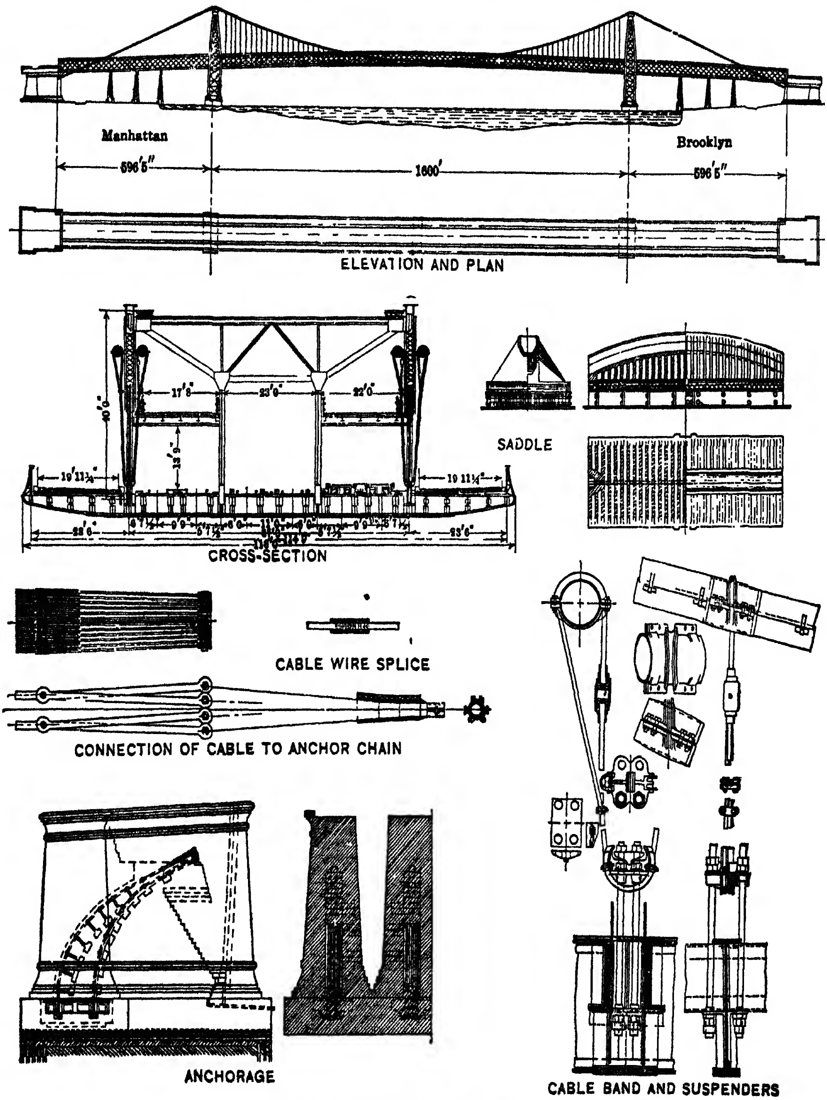



FIG. 32.—Williamsburg Bridge.
(Type 2*F*).

(Fig. 36), and then pass back into the strand. Large cables are divided for this purpose into 7, 19 or 37 strands; and such cables accordingly have 7, 19, or 37 strand shoes at each end. Steel pins pass through these shoes for connection to the anchorage eyebars (Fig. 32).

The strand shoes are grouped into a number of horizontal rows (generally 2 or 4), and the anchor chain divides into an equal number of branches to effect the connection (Fig. 32).

About 10 feet forward of the shoe, the two halves of a strand are combined into one; and all strands, before leaving the masonry, are squeezed into a round cable.

The shoes have slotted pin-holes which are provided with shim-blocks (Fig. 36) to permit regulation of the individual strands before combining into a cable.

Bending the wires around the shoe produces bending stresses exceeding the elastic limit; but the resulting stretching of the outer fibers redistributes the stress over the cross-section of the wire; with properly ductile steel wire, the strength at the loop is not materially impaired.

If the wire is very hard, or if the cable consists of wire ropes, a larger radius of curvature must be provided or some other form of connection must be used.

For wire ropes a larger shoe is used, with the end of the rope fastening into a socket after bending around the shoe.

The sling construction is avoided by setting the ends of the strands or ropes directly into steel sockets. After inserting the rope into the expanding bore of the socket, the wires are pried apart and spread with a point tool, and the intervening space filled with fusible metal (preferably molten zinc). Such socket connections are now made to develop the full strength of the rope. The sockets may be designed to bear directly against the under side of the anchor girder; or they may be threaded to receive the end of a rod which serves as a continuation of the strand.

**15. Construction of Chains.**—Chains may be constructed of horizontal flats piled together and spliced at intervals by means

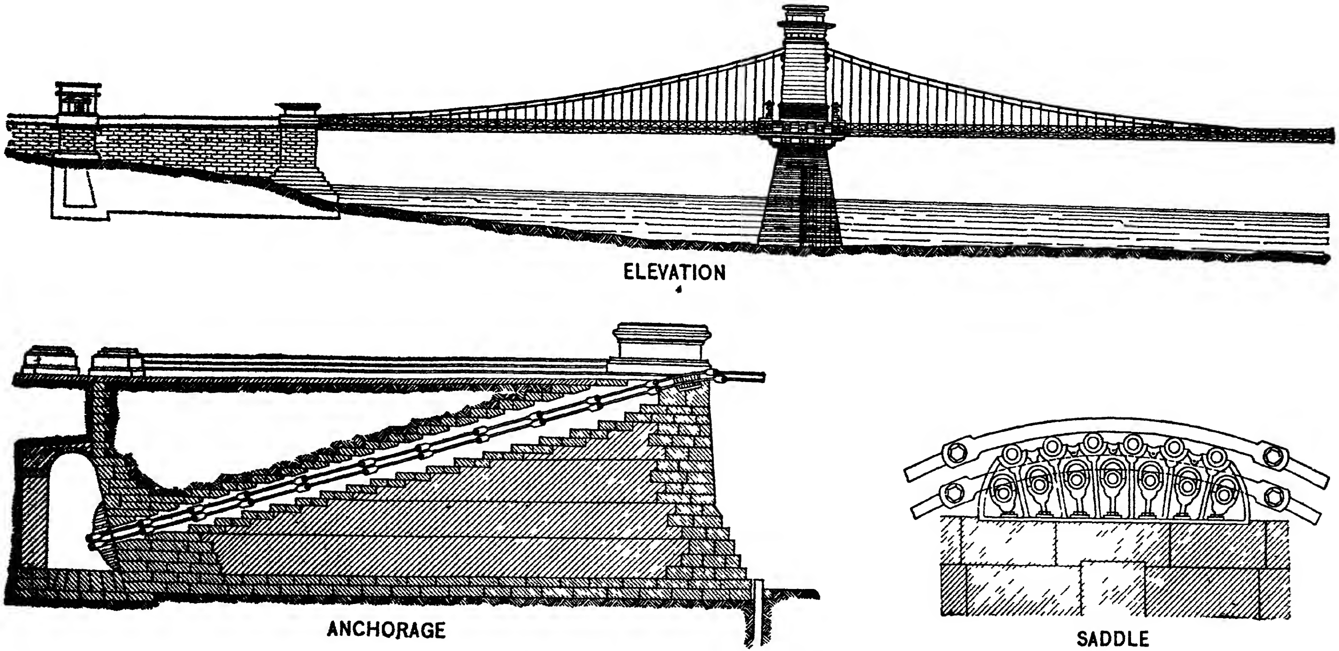



FIG. 33.—Clark's Bridge over the Danube at Budapest.

Span 663 feet  Eyebar Chains  Unstiffened  Completed 1845  Reconstructed 1914 of friction clamps with bolted flanges. Suspenders are bolted to these clamps. This laminated construction is subject to high secondary stresses from bending.

Chains may be constructed of closed links overlapping around connecting pins to which the hangers are attached (Fig. 39).

Chains may consist of eyebars or flats bored at their ends to receive pins (Figs. 33, 34, 38, 40). The pin holes may be bored oval or elongated in order to facilitate the erection of the eyebars. This procedure was adopted in the Florianopolis bridge.

In American practice (Figs. 38, 40), forged eyebars are used. In European practice (Figs. 17, 33, 34), the eyebars are made by welding or riveting, or by cutting from wide flats. The last is an extravagant procedure.

Where flats are used, the reduction of section by the pinholes may be largely made up by riveting pin-plates at the ends of the bars.

Chains composed of vertical flats riveted together have been proposed, but the secondary stresses from bending would be very high.

For long spans, the chains would have large cross-sections, requiring pins of excessive length.

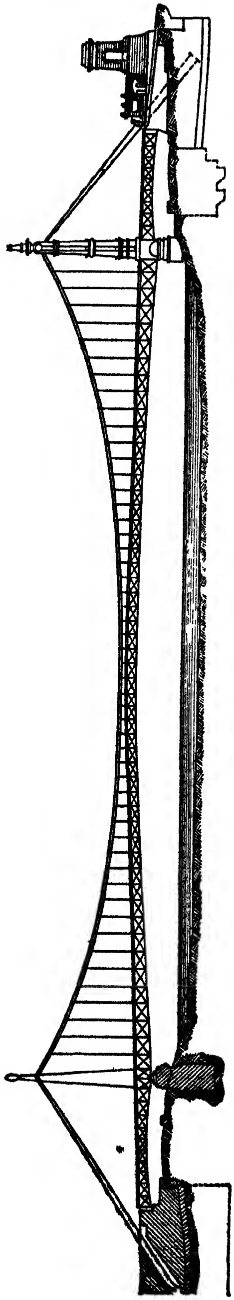


FIG. 34.—Elizabeth Bridge over the Danube at Budapest. (Type 0*FE*).
Span 951 feet. Eyebar Chains. Completed 1903.

This is circumvented by using two chains, either side by side or superimposed; in the latter case, if the panels are not too short, successive hangers are connected alternately to the upper and lower chains (Figs. 33, 34).

A disadvantage of chain construction is unequal division of stress in the individual bars between two pins. This may be caused by inaccuracies in length, differences in temperature, variations in elastic modulus, bending of the pins, and eccentric suspender loading. The unequal stressing of the eyebars is frequently apparent on superficial examination or upon comparing the ringing pitch under hammer blows. Actual measurements (by comparing deflections under lateral test loads) have revealed varying stresses in a single group of eyebars ranging from 40 to 200 per cent of the mean stress.

**16. Suspender Connections.—Cable Bands and Sockets.—** The attachment of the suspenders to the cable is generally made by means of cast steel collars called cable bands (Figs. 26, 32, 36). The cable band may be an open ring with flanged ends to receive a clamping and connecting bolt (Fig. 26). More generally it is made in two halves with flanges (Figs. 32, 36). The band must grip securely to prevent slipping. The inside of the band should be left rough to minimize the tendency to slip on the cable; and space should be left between the flanges for taking up any looseness of grip, when necessary. A cam-clamping device has been proposed for automatically increasing the grip as load is applied through the suspender.

If the hangers are of rigid section, they are bolted to vertical flanges cast integral with the cable band for this purpose.

If rope suspenders are used, the cable band is cast with a groove or saddle to receive the rope which passes over it (Figs. 32, 36). On account of the varying slope of the cable, the grooves in the cable bands are at varying angles, requiring a number of different patterns. To avoid this, the bearing flange of the grooves may be made curved in elevation.

If the cable is used as a chord of an overhead bracing system, the rigid web members connect to the cable bands; and the latter must be made long enough, with ample clamping bolts, to

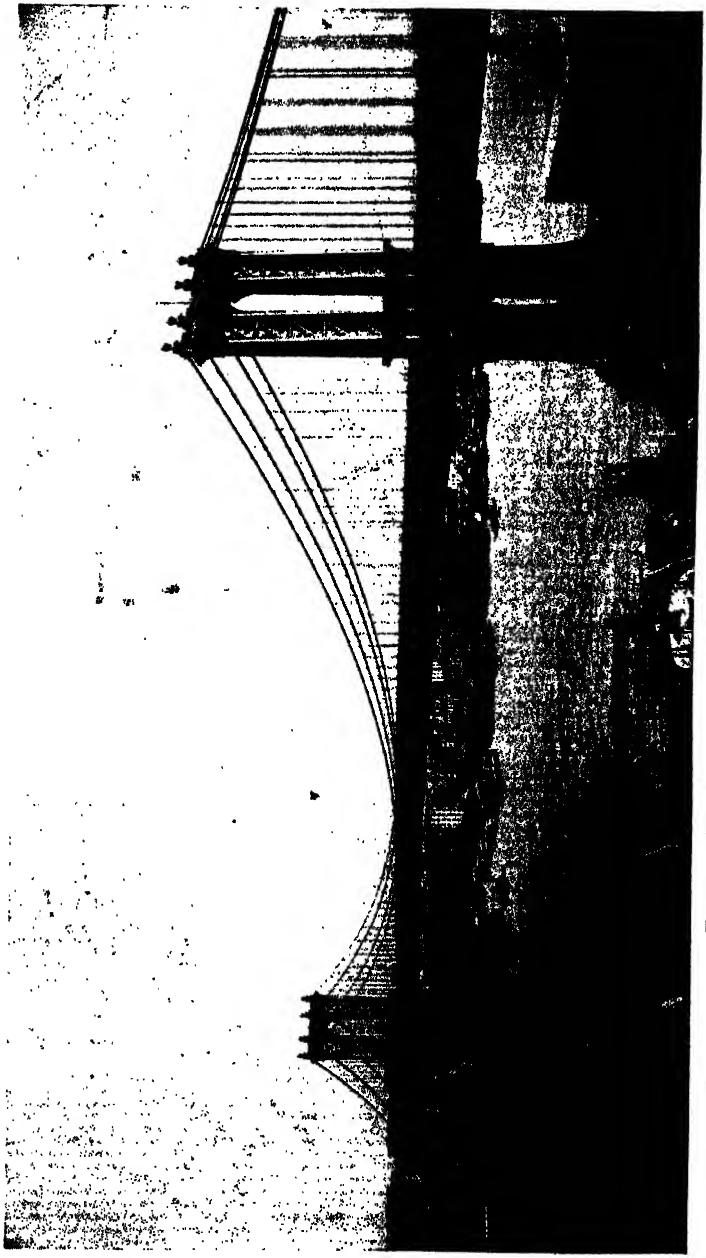


FIG. 35.—The Manhattan Bridge over the East River, New York.
Span 1470 feet. Completed, 1909.

develop the friction requisite to take up the chord increment of the web stresses. A tight layer of wire wrapping against the ends of the cable bands will add to their security against slipping.

The frictional grip attainable in a two-piece cable band is 50 per cent of the aggregate tension in the clamping bolts. If made of high-tensile steel of 65,000 elastic limit, the bolts may be stressed up to 25,000 pounds per square inch. On this basis may be calculated the number of bolts required to resist a given component parallel to the cable; a safety factor of 2 should be applied to the result.

If the cable consists of a cluster of wire ropes, soft metal fillers should be inserted within the band to improve the grip and to exclude moisture.

The cable band should be designed so as to prevent the admission of moisture to the cable. The flanges should be designed for excluding rain, and the joints all around should be securely calked. The band should preferably be undercut at both ends for the insertion of the first few turns of the wire wrapping (Fig. 36).

The free ends of the suspender ropes are secured in sockets made of high-grade steel drop-forgings (Figs. 32, 36). The end of the rope is inserted into the expanding shell of the socket, the ends of the wires are spread apart and the interstices are filled with molten metal (preferably zinc) which will not shrink appreciably on cooling. This fastening of the end of the rope is found, by test, to be unaffected by the ultimate loads causing failure of the rope.

Closed sockets terminate in a closed loop with which other links can be engaged. Open sockets terminate in two parallel eye-ends to receive a bolt or pin for connection to other structural parts. Threaded sockets (Figs. 32, 36) are cylindrical and are threaded on the outside to receive adjusting and holding nuts; these sockets may be passed through truss chords or girder flanges, with the nuts bearing up against the lower cover plates of these members (Fig. 36).

Sockets are furnished by the wire rope manufacturers, either loose or fastened to the ropes.

**17. Suspension of the Roadway.**—The suspenders may consist of wire ropes (Figs. 26–28, 30–32, 36); or of rods, bars or

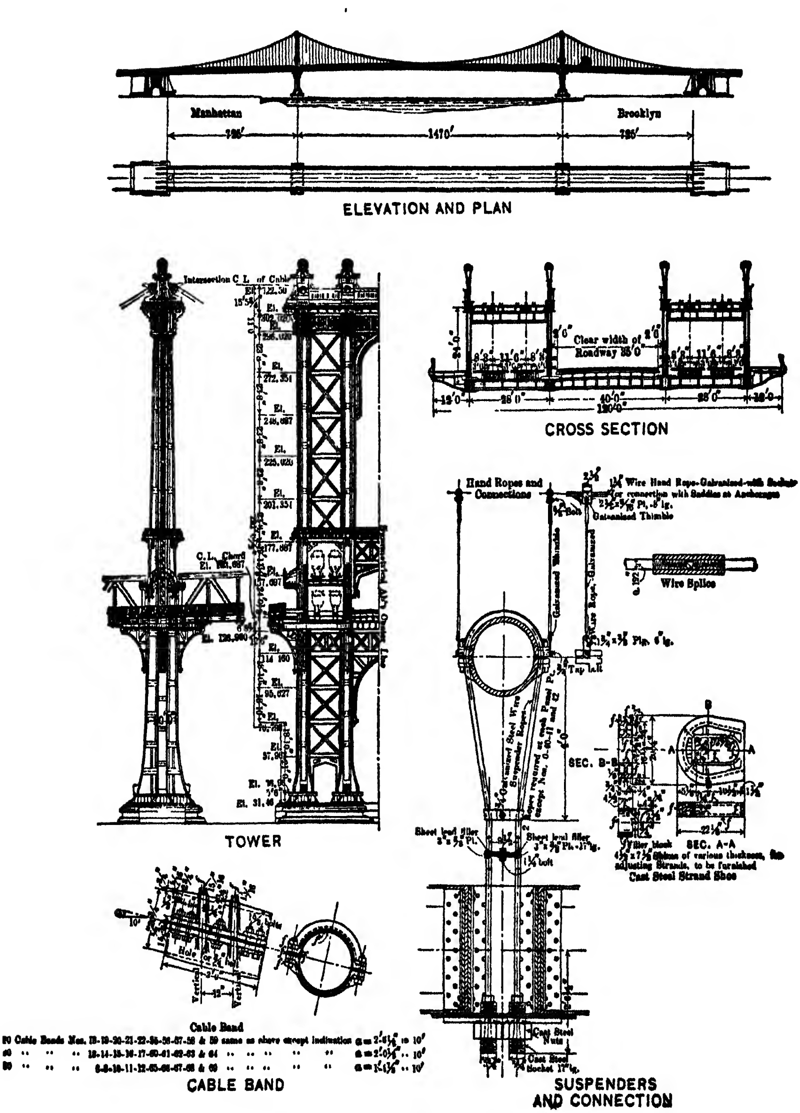



FIG. 36.—Manhattan Bridge.
(Type 2S).

rolled shapes (Figs. 29, 33, 34, 38–41). There may be one (Fig. 26), two (Fig. 30), or four suspenders (Figs. 32, 36) at a panel point.

If the hangers are made of rigid rods (instead of wire ropes), bending stresses due to lateral or longitudinal swaying of the bridge are avoided by inserting pin connections or links (Figs. 38, 39, 40).

Solid steel rods used for hangers generally have a high slenderness ratio and are subject to bending and to vibrations; to provide greater stiffness, tubular and built-up sections have been substituted (Figs. 34, 38, 41).

Where there are two or more suspenders at a panel point, the possibility of unequal division of load should be taken into consideration. Equalizers may be used to advantage (Fig. 32).

After passing around the cable band, the suspender may extend down as two separate ropes (Fig. 36); or the short end may be clamped to the main suspender, which then extends down as a single rope (Fig. 32).

The suspenders may connect directly to the floorbeams (Figs. 26, 27, 29, 38, 41), or to the top or bottom chord of the stiffening truss (Figs. 32, 36). In the latter case, the floorbeams frame into the chords or into the posts of the stiffening truss.

For connection to the floorbeams or chords, the suspenders may pass through and bear up against the lower cover plate with the aid of washers or special castings (Figs. 26, 32, 36); or they may loop around the floorbeams or chords either directly (Fig. 27) or with the aid of steel cross-pieces or yokes.

Connecting the suspenders to the top chord of the stiffening truss requires the entire length of the cable to be above the truss; this has aesthetic advantages (see Figs. 28, 34, 35), but it adds the depth of the truss to the required height of the towers. Lowering the cable saves height (Figs. 25–27, 29–33), but requires either lengthening of the floorbeams or spreading of the trusses or towers.

Another method of suspending the roadway is to loop the suspender rope under a small saddle casting from which there extend downward rigid rods terminating in holding nuts (Fig. 32) or steel flats bored to receive connecting pins.

Provision for adjustment of the hangers may be made by means of the holding nuts (Figs. 32, 36), or by means of sleeve nuts or turn buckles with right and left threads (such as shown in Fig. 32). Some engineers prefer to omit provision for adjustment, depending upon careful computation of required length before cutting the ropes and attaching the sockets.

**18. Construction of Stiffening Trusses.**—The function of the stiffening truss is to limit the deformations of the cable and to so distribute any concentrated, unsymmetrical, or non-uniform loads as to keep the suspender tensions in a constant proportion (or equal if the cable is parabolic). In other words, the stiffening truss is required to hold the cable (or chain) in its initial curve of equilibrium. This will limit the deflections of the structure, and will resist the setting up of vertical oscillations.

The first suspension bridge provided with a stiffening truss was the 820-foot railway span at Niagara, built by John A. Roebling in 1851–55. The Brooklyn Bridge (Fig. 26), completed in 1883 by Roebling's son, was built with 4 cables and 6 stiffening trusses, and, in addition, was provided with diagonal stays.

Until comparatively recent years, stiffening trusses were only roughly figured and were made of constant section. The scientific design of suspension bridges dates from about 1898.

Stiffening trusses are generally built with parallel chords (Figs. 25, 30, 32, 35); a small variation in depth is sometimes introduced (Fig. 34). To prevent an unsightly and otherwise undesirable sag under load, and to counteract the illusion of sag, a generous camber is usually provided (Fig. 31).

The web system may be of the single Warren (Figs. 30, 35) double intersection (Fig. 34), or latticed types (Fig. 31). The $K$-truss may also be applied to advantage. Instead of a truss, plate girders may be used; the Vierendeel girder or quadrangular truss (like the floorbeam in Fig. 41) has also been proposed.

To make the design statically determinate, a hinge at the center of the stiffening truss is necessary (Type $3F$ or $3S$, Fig. 26); but this construction has many drawbacks. In long spans, the angle change at the hinge would be so great as to cause serious bending stresses in the cable and overloading of adjacent suspenders. Moreover, the wind stresses must be transferred through the hinge, and the details become more difficult and costly.

Making the truss continuous past the towers (Types $0F$, $0S$, Fig. 34) yields more effective stiffening: either less material is required or the deflections are reduced; furthermore, the impact effects of moving loads entering the main span are reduced. On the other hand, continuity renders the structure indeterminate (in the third degree); inaccuracies in construction, settlement of supports, and unequal warming of the chords will affect the stresses adversely.

Introducing hinges in the continuous stiffening truss relieves the indeterminateness and the accompanying uncertainty of stress conditions. In the Williamsburg Bridge (Fig. 31) a hinge is placed in each side span, close to the tower; in a prize design for the Elizabeth Bridge at Budapest (Fig. 34), two hinges were located in the main span; in both cases, the resulting system is singly indeterminate. In the usual two-hinged construction (Types $2F$, $2S$, Figs. 15, 36), the truss is hinged or interrupted at the towers.

As in other indeterminate structures, all precautions must be observed in construction to avoid false erection stresses. If the suspenders are adjustable, a definite apportionment of dead load between cable and stiffening truss may be secured. The stiffening truss may be totally relieved of dead-load stress by adjusting it under full dead load and mean temperature to the exact form it had when assembled in the shop at the same temperature; or by omitting certain members until full dead load (at mean temperature) is on the structure. In any case, the joints should not be riveted until the dead load is on and all adjustments are made.

The stiffening truss may be made of any height, depending upon the degree of stiffness desired. With increasing depth, the stiffness is naturally augmented; but the cost of the structure is thereby increased. In the Williamsburg Bridge (Fig. 31), the truss depth was made $\frac{1}{40}$ of the span. Generally a shallower truss is preferred, for appearance as well as for economy. (See

Fig. 35.) The depth in the vicinity of the quarter-points of the span is most effective in controlling the critical deflections. Shallow depth at mid-span reduces stresses from temperature and from full-span loading. Shallow depth at ends and center, where the shears are highest, yields economy in the web members. A truss of variable depth, following the graph of maximum moments, would be the most efficient. This principle is utilized in designs of the Florianopolis type.

Bearings must be provided at the towers and abutments to take the positive reactions of the stiffening truss (even if continuous); and these points of the truss must also be anchored down to resist the uplift of negative reactions. At the expansion bearings, the anchorage must be so designed as to permit free horizontal movement; this may be accomplished either by the use of pins in slotted guide bearings, or by means of pin-connected rocker arms. One bearing of each truss should be fixed against horizontal movement, in order to resist longitudinal forces. (An exception was the Niagara Railway Suspension Bridge, where an automatic wedge device, for dividing the expansion equally between the two ends of the span, was provided.)

**19. Braced-Chain Construction.**—Overhead stiffening trusses may be regarded as inverted arches. A common form is the three-hinged truss with horizontal lower chord (Type *3BH*, Fig. 37), and these are designed similar to spandrel-braced arches. The chords and web members are built up of plates and angles with riveted panel points. The center hinge is designed to transmit the full value of $H$ for dead and live load, and the maximum vertical shear from live load. At the towers, both chords are supported on expansion plates; the bottom chord ends there, but the top chord passes over cast-steel saddles and continues toward the anchorage. The top chord is supported at the top of the towers either on rockers or on rollers (Fig. 37).

In the three-hinged type (*3BH*), the center hinge may be located either in the upper or in the lower chord. In the former case, the upper chord will carry all of the dead load and full-span live load; the lower chord and web members will be

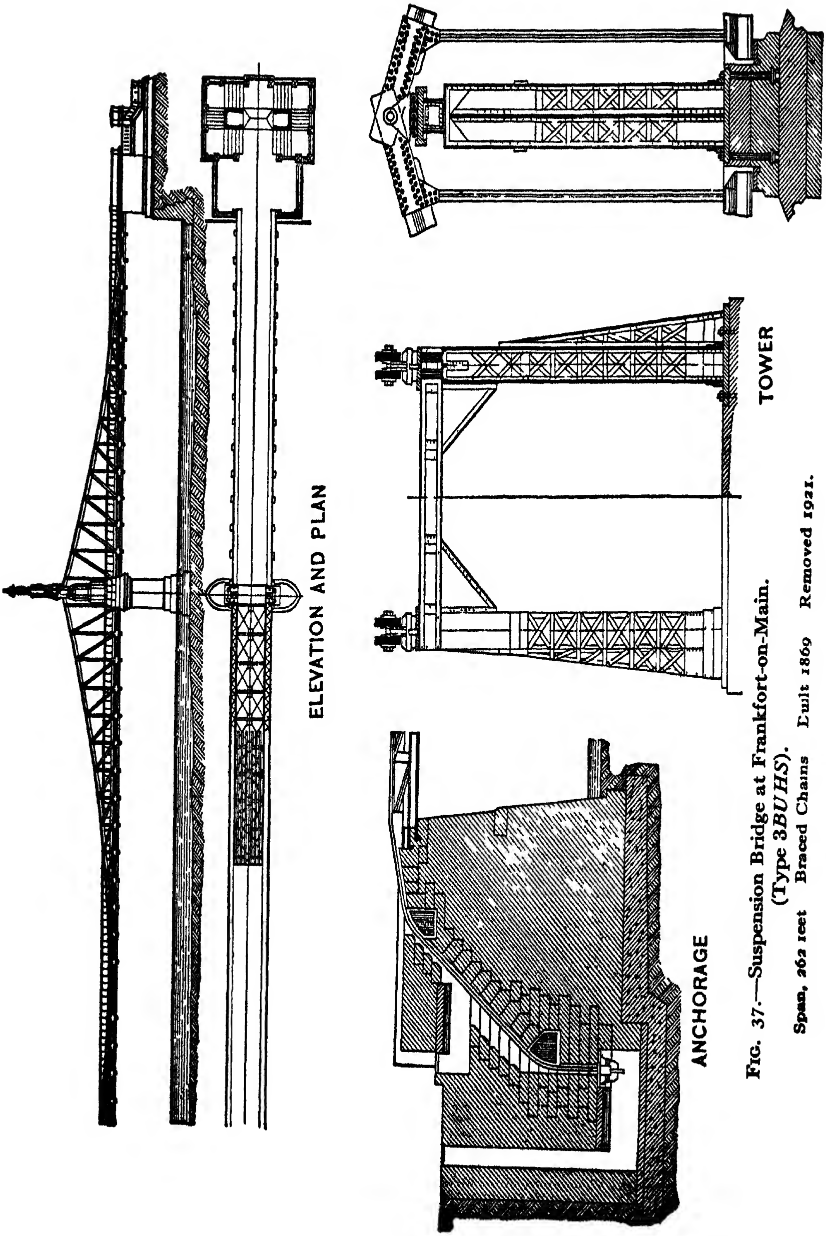



Fig. 37.—Suspension Bridge at Frankfort-on-Main.
(Type 3*BUHS*).
Span, 262 feet Braced Chains Built 1869 Removed 1921.

stressed only by partial loading. Accordingly, the upper chord will have a fairly uniform section, while the other members will be comparatively light. This arrangement has a disadvantage, however, in the necessary break in the floor system under the hinge; the stringers must be provided with expansion joints, and the wind bracing must be interrupted.

If the hinge is placed in the lower chord, false members are required, for the sake of appearance, to cover the interruption in the top chord. Furthermore, there results a large variation in the chord stresses: near mid-span, the top chord stresses become light and the bottom chord stresses become heavy; and the reverse occurs near the towers.

The hinge may be either of the ordinary pin type or of the plate type. In the latter case, the chord section is concentrated into wide horizontal plates to connect the two halves of the span; and the vertical shears at the point are transmitted by means of vertical spring plates.

The false members in the interrupted chord may be connected with friction bolts in slotted holes, so that the resulting friction may act as a brake against oscillations.

To eliminate bending moments in the stiffening truss at the tower, the end member of the lower chord may be suspended from the saddle; or it may simply rest on an expansion bearing at the tower, with the end vertical omitted from the truss. Another arrangement is to make the tower integral with the main span truss, the tower being pivoted at the base, and the side span having only a hinged connection at the top of the tower.

The use of a wire cable for the top chord had an illustration in the Lambeth Bridge, London; but the details did not constitute an example worth copying. The development of a satisfactory detail for the panel point connections is necessary before the overhead or integral bracing system can be used in conjunction with wire cables. A solution by the use of a cable composed of pin-connected wire links (Fig. 39) was proposed by Lindenthal in 1899. A detail for the direct connection of web members to a continuous parallel wire cable was developed and proposed by Robinson and Steinman in their design for the Sydney Harbor Bridge in 1923.

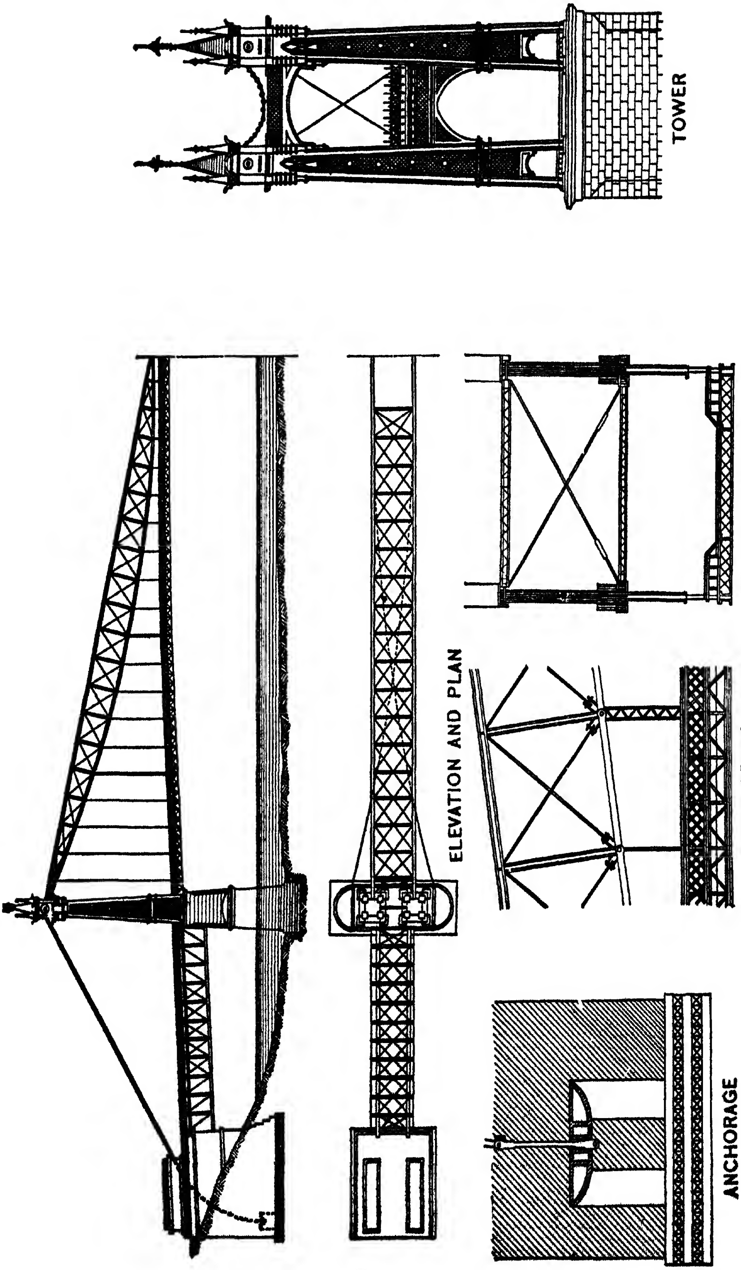



FIG. 38.—Point Bridge at Pittsburgh. (Type 3*BLF*.)
Span, 800 feet. Braced Chain. Erected 1877. Removed 1927.

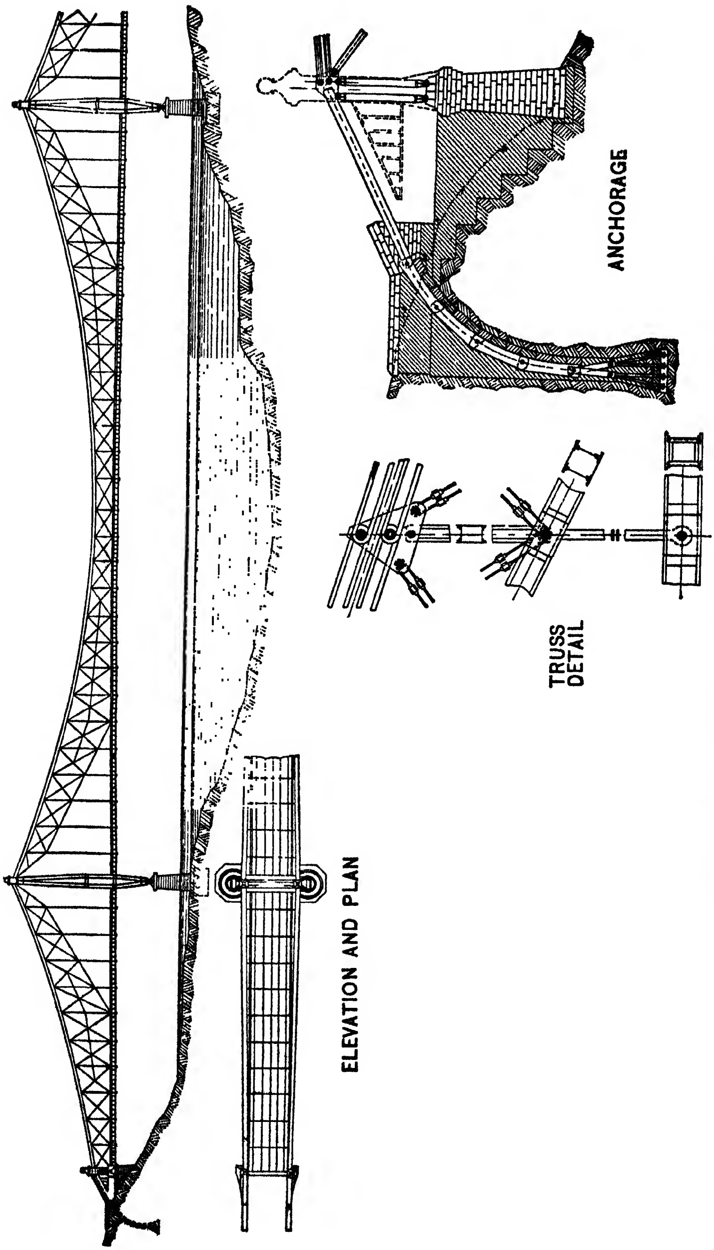



FIG. 39.—Lindenthal's First Design for the Quebec Bridge. 1899. (Type 2*BVS*).
Span, 1801 feet. Chain of pin-connected wire links.

When the braced-chain (or braced-cable) system is used, the web members should preferably not be connected until full dead load and half live load are on the structure at mean temperature. There will thus be accomplished a reduction in extreme stresses in the web members and lower chords; the maximum tension will be equal to the maximum compression in each member, and this stress will be only the arithmetic mean of the extreme stresses that would be produced without this precaution. However, if the design specifications prescribe stringent allowances for alternating stresses, the reduction in sections by this device will not be material.

The truss depth at the crown (Type *3B*) should preferably be between 0.15 and 0.3 of the sag of the chain. Sufficient depth must be provided to take care of the shearing stresses and to prevent undue flexibility in the central portion of the span; but excessive depth, besides increasing the metal required, impairs the desired graceful appearance of the suspension construction. If the hinge is omitted, any increase in crown-depth serves to augment the temperature stresses.

The form of braced-chain construction (Type *2BV*) proposed by Lindenthal for his first Quebec design (Fig. 39), and for the Manhattan Bridge, has many advantages. The bottom chord is bent up toward the towers, so as to obviate the necessity for long web members. The requirement of extra wind chords at the ends of the span is not, comparatively, an important objection.

Suspension trusses with bracing above the principal chain (Type *3BL*) are exemplified in the Point Bridge at Pittsburgh Fig. 38). In this system the stiffening chords are straight, and the bottom chord is parabolic. The top chord members have to resist both tension and compression.

A system having many advantages is the Fidler Truss (Type *3BC*), exemplified in Lindenthal's second Quebec design (Fig. 40), in a Tiber bridge at Rome, and in the Tower Bridge at London. In this system, consisting of two crescent-shaped half-arches, both chords are curved, the bottom chord having a sharper curvature than the equilibrium curve for full load.

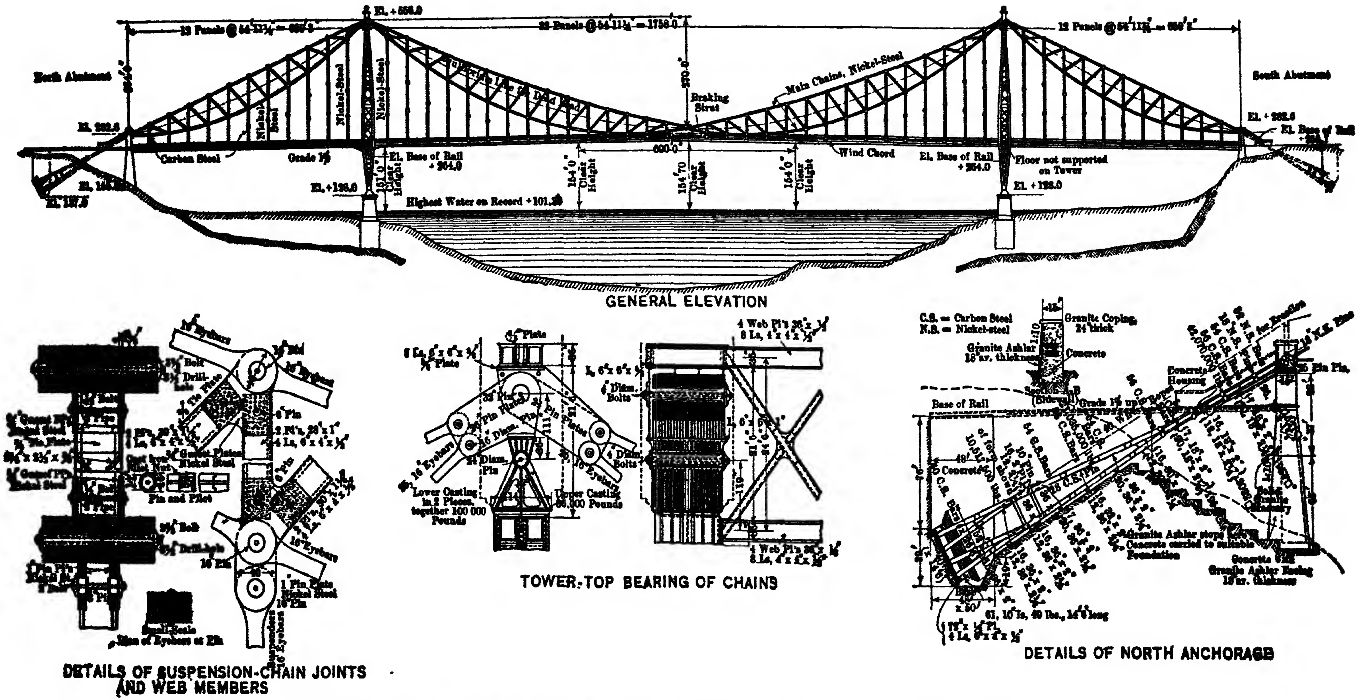



FIG. 40.—Lindenthal's Second Design for the Quebec Bridge (1910). (Type 3BCS)

Span, 1758 feet. Pidler Truss.

The parabolic curve passes midway between the two chords, so that they are about equally stressed. With this system it is possible to avoid compressive stresses in both chords.

In the foregoing systems (Types 3*BL*, 3*BC*), it should be noted that the suspended floor system must be interrupted, or else of negligible moment of inertia, under the center hinge. It is more important here than in three-hinged arches, on account of the greater crown deflections in suspension systems.

The systems using parallel chains connected with web-bracing have been little used on account of the difficulties in stress analysis. If each chord has its own backstay (Type 0*BP*), the system is threefold statically indeterminate. If the top chord is interrupted at the towers (Type 2*BP*), the indeterminateness is reduced. It would be more effective, in such case, to bring the two chords together in crescent form instead of using parallel chords. In Lindenthal's Seventh St. Bridge at Pittsburgh and in his first design for the North River Bridge, the bottom chord rested directly on bearings on the top of the towers, and the top chord was connected thereto by a double quadrangular linkwork equivalent to a hinge; this made the system singly indeterminate (aside from the redundancy of web members). When parallel chains are used in the side spans, the bottom chord may be connected to the anchorage; or both chords may be brought together at a common pin for connection to the anchor chain.

The latest example of parallel chain construction is Lindenthal's new design for the Hudson River Bridge (Fig. 41). The main span is 3240 feet, and the bridge is continuous at the towers (Type 0*BP*).

**20. Wind and Sway Bracing.**—To take care of transverse wind pressure and lateral forces from moving train loads, and to carry these forces to the piers, systems of lateral and sway bracing are required.

A system of wind bracing must be provided in the plane of the roadway, since the principal horizontal forces originate there. Such bracing system is obtained by inserting diagonals between the floorbeams, so as to form a horizontal truss; using

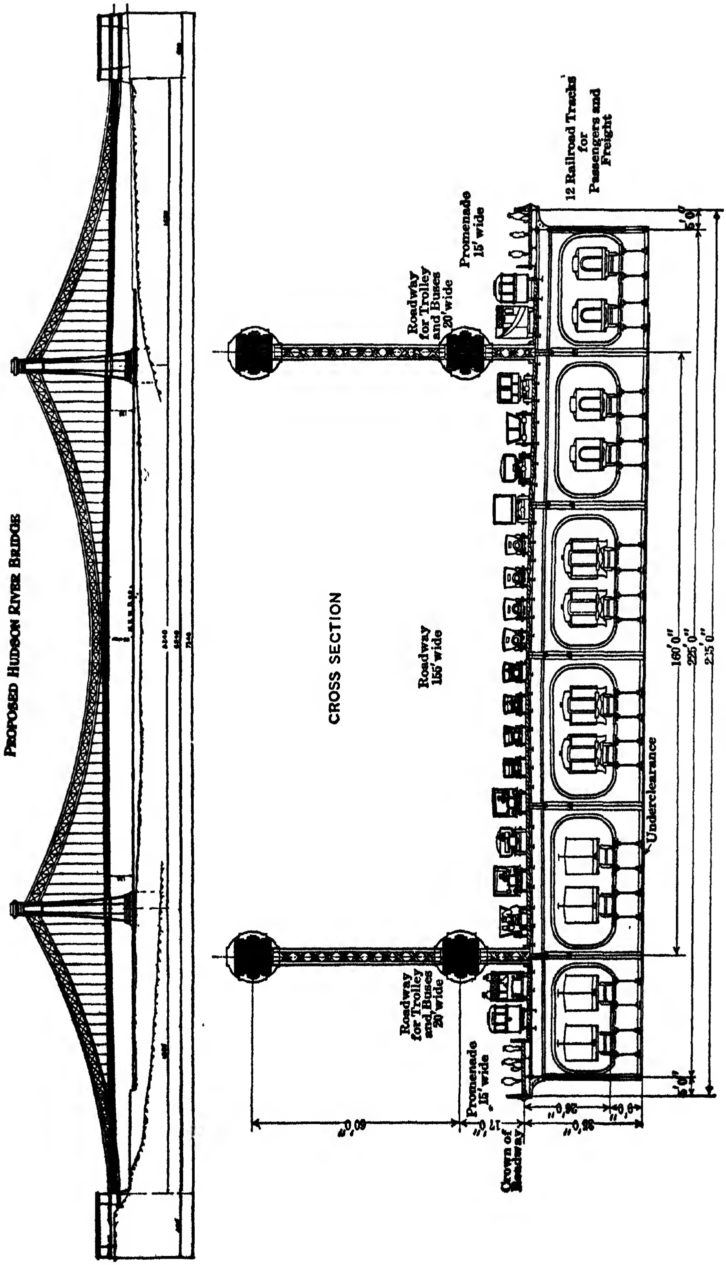



FIG. 41.—Proposed Hudson River Bridge. (Type 0BPS).
Span, 3240 feet. Parallel Chains. Design by G. Lindenthal, 1921

for chords either the bottom chords of the main stiffening truss (Figs. 26, 30, 32, 36, 37) or else adding extra longitudinals called wind chords (Figs. 38–41, 56).

If the stiffening truss is high enough to afford necessary clearances, a bracing system in the plane of the top chord may be added, giving a closed cross-section to the structure (Figs. 26, 32, 36, 43).

If the roadway is elevated, vertical sway frames of cross-bracing may be introduced between the trusses (Fig. 26); or the floorbeams may be built of deep latticed construction. In such case, a single plane of horizontal wind bracing will suffice.

A novel method of transverse bracing is introduced in the design for the Hudson River Bridge (Fig. 41) in the form of deep floorbeams (32 feet high) of quadrangular construction (Vierendeel Girders); the rectangular openings are used as passageways for the railway tracks.

Bearings must be provided at the towers to take the horizontal reactions of the wind truss without hindering longitudinal expansion. Vertical pins bearing in slotted guides may be used for this purpose.

The cables or chains present so small a surface to the wind as to require no wind bracing; or at most they may be connected together by horizontal ties.

On account of the inherent stable equilibrium of the suspension system, the wind bracing is to a certain extent relieved of its duty.

Braced-chain systems are provided with a single wind truss below the roadway, using either the lower chords of the main truss (Type *3BH*, Fig. 37) or special wind chords (Types *2BV*, *3BC*, etc., Figs. 38, 39, 40, 41). In addition, transverse sway frames are located at intervals between the two suspension trusses in the planes of verticals or diagonals, so far as clearance requirements permit (Fig. 38). In comparison with other types of bridges, but little material is needed for this sway bracing since, in the first place, the low position of the center of gravity makes the suspension truss stable without bracing, and, in the second place, there are no top chord compression members (in most of these types) to be braced against buckling. It is essential, however, to provide properly designed rigid portal and sway bracing between the legs of the main towers (Figs. 37, 38).

A profusion of overhead bracing, besides being structurally unnecessary, will impair the graceful appearance sought in suspension constructions.

A center hinge in the stiffening truss introduces complications in the design of the wind-bracing system. If the hinge is, as usual, in the top chord, the wind bracing must follow the two central diagonals to make connection at the hinge; these central diagonals then act as wind-chord members, and their sections must be increased accordingly. If the top chord hinge lies above the roadway, the cross-bracing in the two central panels connecting with the hinge has to be omitted. This produces a point of weakness in the horizontal bracing system, and should be avoided in long spans either by omitting the hinge or by locating the hinge in the bottom chord.

Early suspension bridges that were found to have excessive lateral deflections and oscillations were stiffened by means of wind cables (wire ropes) placed in a horizontal plane under the roadway; these ropes were connected to the floorbeams and were anchored to the piers, so as to form a horizontal suspension system of cable and stiffening truss. To take care of wind in both directions, a double system of wind cables must be used, and their sag-ratio should be made as large as possible (Fig. 38). For greater stiffness, straight auxiliary cables have sometimes been added.

The efficacy of the above-described system of wind cables is doubtful, since it is ordinarily impossible to give the ropes proper initial tension; consequently they do not commence to take stress until the horizontal deflection of the structure exceeds a certain amount. For this reason, wind cables have not been relied upon for modern designs, but rigid wind trusses have been adopted instead, to take care of the wind pressures.

**21. Towers.**—For purposes of discussion, the tower may be considered as composed of two parts: the substructure or pier; and the tower proper, extending above the roadway and supporting the cables or chains. The pier does not involve any special features differentiating it from ordinary bridge piers.

The tower must be designed so as not to obstruct the roadways. It is therefore composed of a column or tower leg for each suspension system (Figs. 30, 35–38). For lateral stability, the tower legs are braced together by means of cross-girders and cross-bracing (Figs. 30, 35) or by arched portals (Figs. 37, 38). The sway and portal bracing are necessary to brace the tower columns against buckling, to take care of lateral components from cradled cables or chains, and to carry wind stresses down to the piers.

The design of the tower depends upon the material employed. This is either masonry (Figs. 25, 27, 29, 33) or, more commonly steel (Figs. 15, 17, 28, 30–32, 34–41), and occasionally timber. If masonry is used, the tower may consist of shafts springing from a common base beneath the roadway and connected together at the top with gothic arches (Fig. 25); or, for smaller spans, the tower may consist of two separate tapering shafts or obelisks.

To meet architectural requirements and to express resistance to transverse forces, the outline of the tower should taper toward the top. This also conforms to structural requirements.

The tower legs must be designed as columns to withstand the vertical reaction of the cables; also as cantilevers to resist the unbalanced horizontal tension. The latter will depend upon the saddle design (fixed or movable), the temperature and loading conditions, and the difference in inclination of main and side-span cable tangents at the saddle. Forces due to wind pressure on the cables, towers, and trusses must also be provided for.

The application of steel to suspension-bridge towers offers many advantages. The lower cost permits a greater height in order to secure a more favorable sag-ratio. The thermal expansion of the steel tower balances that of the suspenders, so as to eliminate serious temperature stresses which would otherwise arise in indeterminate types ($2F$, $2S$, $0F$, $0S$).

Steel tower columns (Figs. 15, 17, 30, 36–39) are made up of plates and angles to form either open or closed cross-sections;

the sides may be either latticed or closed with cover plates. The relative dimensions are governed by the usual specifications for the design of compression members, particularly in respect to limiting unsupported widths of web plates. The cross-section enlarges toward the base, or outside stiffening webs are added; and the base must be anchored to resist the horizontal forces (Fig. 37).

For high towers, the individual legs may be made of braced-tower construction, each leg consisting of four columns spreading apart toward the base and connected with lacing or cross-bracing (Fig. 31).

Rocker towers, pin-bearing at the base, afford the most economical and scientific design for bridges of longer span. They eliminate the stresses from unbalanced horizontal forces without requiring movable saddle construction. The most notable examples actually constructed are the Elizabeth Bridge at Budapest (Fig. 34) and the Cologne Bridge (Fig. 17). (See also Figs. 15, 28, 39, 40.)

If rocker towers are adopted, they must be secured against overturning during erection. This may be accomplished by temporary connections to the adjoining truss structure, by wedging or bracing at the base, or by guying the upper portions of the tower.

For foot bridges and bridges of small span, the simplest tower construction employs a stiffened plate for each leg, the two legs being braced together and carrying steel castings at the top to hold the cables.

If timber is used, each cable support consists of four battered posts with framed bracing, the two legs thus formed being connected at the top with cross-bracing.

**22. Saddles and Knuckles.**—The cables are generally continuous over saddles on top of the towers.

Designs have been made with the main cables terminating at the towers, with a special connection at the top of the towers to the backstay cables (e.g., Morison's North River design). The advantages claimed were shorter cable strands to handle in erection, elimination of stresses due to bending of the cable over the towers, and the possibility of increasing the section of the backstays to permit steeper inclination. The latter advantages, however, can be secured with continuous cables by employing certain design features.

If the stress in the backstay, as a result of steeper inclination, is much greater than in the main cables, auxiliary strands may be incorporated in the backstay to increase its section; and provision should be made for the connection of these auxiliary strands to the saddle. (An example is the Rondout Bridge at Kingston, N. Y., Fig. 56.)

Cable saddles are generally made of cast steel (Fig. 32). They are either bolted to the top of the tower (Fig. 36) or provided with rollers (Figs. 32, 33, 37).

Where fixed saddles are used (Fig. 36), the resultant unbalanced horizontal forces must be calculated and provided for in the design of the tower, unless the tower is made of rocker type (Figs. 15, 17, 34, 39, 40). If the saddles are movable, the eccentricity of the vertical reaction under various loading conditions must be provided for.

The simplest but least satisfactory construction, used in some smaller bridges, consists of fixed saddles over which the chain or cable is permitted to slide. In early cable bridges, the wrapping was discontinued near the towers, and the wires were spread out to a flat section to pass over the saddle casting; this is objectionable as it gives access to moisture. It is preferable to give the saddle a cross-section conforming to the cable section; to reduce wear from the rubbing, the cable may be protected by a lead sleeve. On account of the friction, this arrangement does not eliminate the unbalanced horizontal pull on the top of the tower. On the whole, this construction, or any sliding saddle arrangement, is not to be recommended.

Another saddle arrangement consists of steel pulleys, free to rotate, over which the cable passes (Fig. 27). A similar arrangement used with chains consists of a fixed roller nest over which curved eyebars slide (Fig. 33); the resulting bending stresses, however, are objectionable.

The best arrangement to permit horizontal movement on top of the tower consists of a roller support for the saddle (Figs. 32, 37). In modern designs, the rollers are of equal height between two plane surfaces. The construction comprises a bed plate, a nest of rollers connected by distance bars, and the superimposed saddle casting. In the saddle rests the cable (Fig. 32) which is held from sliding by friction or clamping. Instead of a saddle, the movable part may consist of a casting to which the chains are pin-connected (Fig. 37). The resultant of the tensions of the cables or chains should pass through the middle of the roller nest to give an even distribution of stress. The friction of the rollers is so small that the obliquity of the resultant reaction is negligible.

Instead of circular rollers, segmental rollers (rockers) may be used, so as to furnish a greater diameter and thereby reduce friction and roller-bearing stress. Segmental rollers, however, must be secured against excessive motion liable to cause overturning.

Rollers (Figs. 32, 33, 37) serve to reduce the bending stresses on the towers due to unbalanced horizontal cable pull resulting from temperature and special loading conditions. On the other hand, they add expense, increase erection complications, give trouble in maintenance, and merely substitute eccentric vertical loading for unbalanced horizontal pull. On the whole, fixed saddles provide a simpler and safer construction.

Another saddle design consists of a rocker, pin-connected at either upper or lower end to the tower and carrying the cable or chain at the other end. The rocker-hanger or pendulum type has been used only in earlier bridges; the main objection to it is the increased height of tower required. The rocker-post type (Fig. 39) has pin-connection to cable or chain at the upper end, and has a pin or cylindrical bearing at the lower end.

Short rocker posts should not be used for long spans; after such posts assume an inclined position under temperature variation, the return to normal position is seriously resisted by the necessity of raising the point of cable support through a vertical height.

The tower itself may be made to serve as a rocker post for its full height by providing hinge action at the base. This construction was used in the Elizabeth Bridge at Budapest, 1903 (Fig. 34); in the Rhine Bridge at Cologne, 1915 (Fig. 17); in the Florianopolis Bridge in Brazil, 1925; in the Portsmouth (Ohio) Bridge, 1927, and three subsequent bridges (1928) over the Ohio River; and in three suspension bridges at Pittsburgh (1926–1928). Instead of using pins at the lower end (Figs. 39, 40), the hinge-action is more efficiently secured by providing the tower leg with a segmental base (Fig. 17). In very large bridges, a concave nest of rollers may be used.

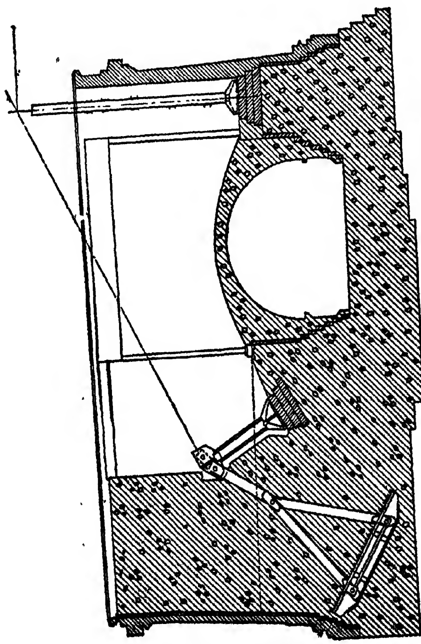
Knuckles are provided in the anchorages at all points where the backstays or anchor chains alter their direction (Figs. 29, 32, 33, 36–40, 42). They are similar in function to tower saddles, and should be designed to permit any movement due to thermal or elastic elongation of the anchor chain.

Sliding bearings (Figs. 37, 39) are commonly used at knuckle joints, and may be considered suitable where the directional change is small. In the Rondout Bridge at Kingston, N. Y. (Fig. 56), the design consists of vertical pin plates supporting the knuckle pin and sliding on steel plates anchored in the masonry.

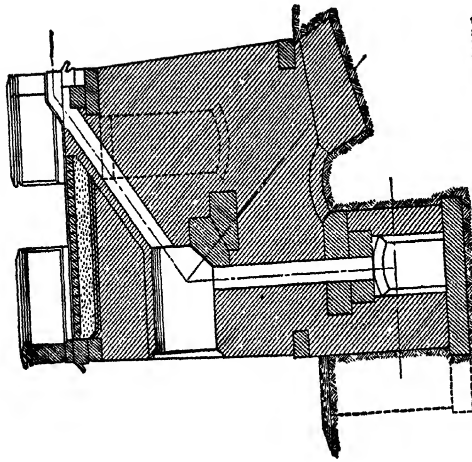
Roller bearings (Figs. 33, 36, 40) give a better design for the anchorage knuckles. A cable may be carried in a saddle casting resting on rollers (Fig. 36); and chain eyebars may be either directly supported on rollers (Fig. 33) or may be pin-connected to a casting carried on rollers (Fig. 40). The plane of the rollers should be normal to the bisector of the angle of the chain or cable.

Rocker supports (Figs. 32, 39, 42) are also used for anchorage knuckles. The change in direction may be accomplished at one main rocker strut (Fig. 42), or may be distributed over a large number of small rocker knuckles (Fig. 32). The direction of the rocker should preferably coincide with the bisector of the angle of the chain or cable.

**23. Anchorages.**—The safety of a suspension bridge depends upon the security of the anchorages. Consequently, in any new design, the anchorages should receive thorough study and their construction should be carefully supervised; and, after



LINDENTHAL'S MANHATTAN BRIDGE DESIGN (1902)



DESIGN FOR FOOTBRIDGE AT GOTHA (GERMANY)

FIG. 42.—Designs for Anchorages.

completion, the condition of the anchorage should receive watchful attention. Accessibility for inspection and maintenance should be considered in the design.

In rare cases it is possible to anchor the cables in natural rock (Figs. 27, 39, 40). The shaft or tunnel which is to contain the anchor chain must then be driven to such depth as to reach and penetrate rock that is perfectly sound, proof against weathering and of sufficient thickness to afford the necessary anchorage.

In most cases, the anchorage requires a masonry construction which resists the cable pull by friction on its base or by the resisting pressure of the abutting earth (Fig. 34).

Masonry anchorages may be imbedded below ground level, with backstays connecting them to the nearest towers (Fig. 29); or they may constitute the end abutments of the side spans (Figs. 26, 28, 30, 32–38, 41). The latter arrangement generally requires bending or curving the line of the cable or chain from its initial inclination to a more vertical direction, in order to secure the necessary depth of anchor plate without excessive length of anchorage (Figs. 32, 33, 36–38, 42). In addition to stability against sliding, such anchorage must also be designed for stability against tilting or overturning. Furthermore, the applied forces must be followed through the masonry and the resulting normal and shearing stresses at all sections and joints must be provided for. The extreme pressures on the base should also be investigated, to make sure that they do not exceed the allowable load on the foundation material (Fig. 44). Foundations on piles (Figs. 32, 36) should be avoided, as they give insufficient security against displacement of the anchorage; if such foundations are unavoidable, an ample proportion of batter piles should be provided.

The anchor chains go through the masonry and are fastened at their ends to anchor plates or to reaction girders. In larger structures, anchor tunnels may be left in the masonry, affording access for inspection of the anchor chains (Figs. 33, 34, 37, 42).

As a rule, each cable or chain is separately anchored; in rare cases, the two cables have been connected in the anchorage so as to form a loop around a body of masonry or rock.

The anchor chains are commonly secured by means of an anchor plate which is either a single casting or built up of cast sections; the last link is passed through this anchor plate and is fastened behind it with a special pin or bolt (Figs. 33, 37, 38). The anchor plate is stiffened against bending by means of perpendicular webs or ribs (Figs. 33, 38); it must have a bearing surface large enough to transfer and distribute the pressure to a sufficient area and mass of masonry.

Instead of cast-steel anchor plates, anchor girders built up of rolled sections have been used in more recent designs (Figs. 32, 39, 40, 42). The chains are pin-connected to the webs of these girders, and the latter transmit the reaction to grillages or castings bearing against the masonry.

Provision for adjustment of backstay length may be made in the anchorage, if not elsewhere. Adjustment may be secured through the use of wedges in the connection behind the anchor plate (Fig. 37); or by means of a threaded connection between the strand sockets and round rods passing through the anchor plate.

The anchor plates (Figs. 33, 37, 38) are designed like any other masonry plates. The area is determined by the allowable bearing pressure on the concrete or stone, and the section of the plate and stiffening ribs are determined by the shearing and bending stresses. The holding bolts or wedges must also be proportioned for shearing and bending stresses; for greater bending strength, the bolts may be made of oval rather than circular section.

A connection of cable to anchor chain is illustrated in Fig. 32.

In small bridges, the cable is often anchored directly, by passing the wire slings around anchored bolts or around anchor blocks. This method can be used only with parallel wire strands; and, if the diameter of the sling is too small, excessive bending stresses will arise in the wire.

At each change in direction of the cable or chain in the anchorage, a bearing is required. This may consist of a casting with rounded surface over which the cable or chain may slide

(Figs. 37, 39), or of a flat plate on which the eyebar heads rest. A knuckle casting with bearing for the eyebar pin is preferable to one on which the eyebar heads bear. If the change in direction of the anchor chain is considerable, roller bearings (Figs. 33, 36, 40) or rockers (Figs. 32, 42) must be provided.

In general, aside from greater simplicity, a straight anchor chain (Figs. 30, 34) is preferable to a curved or bent chain (Figs. 29, 32, 33, 36–38, 42), as the latter results in greater lengthening of the cable from compression or settlement of the masonry. Space limitations, however, frequently make this arrangement unavoidable.

If it is desired to leave the anchorage steel accessible, shafts or tunnels must be provided in the masonry, large enough for a man to pass through (Figs. 27, 29, 33, 34, 37, 42); a clearance of 2 to 3 feet is necessary. Near the lower end, the shaft generally becomes constricted in order to reduce the required size of the anchor plate; consequently the end of the chain is not fully accessible to inspection. For the examination of the anchor plate and fastenings, vaulted chambers or horizontal passageways (about 3 to 4 feet wide and 5 to 6 feet high) are provided behind the anchor plate (Figs. 33, 37, 38, 42). These chambers may lead to the sides of the anchorage, where they are sealed by doors (Figs. 33. 42), or they may be reached through horizontal tunnels (Fig. 27). Inclined shafts (Figs. 27, 33, 37, 42) may be roofed with stepped slabs, flat slabs or an arched vault. Rain and dirt must be excluded, and the points of emergence of the cables or chains should be protected accordingly.

Instead of leaving open passageways for inspection and painting, the opposite course may be adopted and the anchorage completely sealed against the entrance of air or water (Figs. 39, 40, 42). In such designs, the anchorage steel is imbedded in concrete, or surrounded with waterproofing material, so as to exclude air and moisture and thereby prevent oxidation. The shafts receiving the anchor chains or cables are made as narrow as possible, and are subsequently filled with concrete, cement mortar, asphaltic cement or other waterproofing substance.

The anchorage may be built of stone masonry or of concrete.

The use of reinforced concrete gives maximum flexibility in design.

The forces acting on the anchorage as a whole are the cable pull, the weight of the masonry and any superimposed reactions, and the pressure or resistance of abutting earth. For the study of the internal stresses, the outside cable pull is replaced by the reactions of the anchor plates and the knuckle castings. By graphic composition of these various applied forces, the lines of pressure in the masonry are determined. The resultant of all the external forces, including the weight of the anchorage, must intersect the base within the limits necessary to prevent uplift at the heel (Fig. 44); and the inclination of the resultant from the normal must not exceed the angle of friction. If it proves impracticable to secure this stability against sliding with a horizontal foundation, the base may be sloped (Fig. 42) or stepped to increase the sliding resistance. Stepping the base is not effective save on hard foundations. In soft ground, requiring piles, the pile caps should be imbedded in masonry; and the piles should preferably be battered in the direction of the resultant pressure.

Granite or other stone blocks should preferably be used to take the direct pressure of the anchor plate and knuckle castings (Figs. 32, 33, 37, 42). Extending forward from the anchor plate, cut-stone blocks may be laid in arch formation, following the curving line of resultant pressure and with joints normal thereto. The rest of the anchorage, serving only to provide weight, may be built of rubble masonry or brickwork in horizontal courses; or of rubble concrete.

Special designs of anchorages are frequently necessitated by physical conditions and economic limitations.

Anchorages may be designed as large reinforced-concrete boxes filled with rock or compacted sand as an economical means of producing mass resistance. Such use of sand-filled chambers, to furnish weight while saving concrete, has proved effective in the anchorages for the suspension bridges at Massena, N. Y., Portsmouth, Ohio, and Mount Hope Bay, Rhode Island.

In the Philadelphia-Camden bridge (Fig. 10), the front portion of the anchorage contains an inclined steel rocker bent which supports the two cables at the point where they change direction for straight-line attachment to the anchor chains in the heel of the anchorage.

In the case of the Portsmouth Bridge over the Ohio River, the topography and existing railroad tracks interfered with the customary location of the anchorages at the ends of the suspension structure; rocker bents were therefore provided to support the cables at those points, and the anchorages were placed 150 feet away. In the Mount Hope Bridge in Rhode Island, rocker cable-bents were similarly used, with the anchorages located 220 feet away; this resulted in a large saving in cost since it reduced the necessary height of the anchorages and permitted them to be placed in more favorable locations.

Where the anchorage can be imbedded in solid rock, a large volume of anchorage concrete or masonry is saved. Foundations on rock should be stepped or sloped to increase resistance to sliding. Where rock foundation is not available, piles (preferably battered) may be used to provide bearing and resistance. In extreme cases, caissons sunk to rock or hard pan may be used to provide the necessary foundation for the anchorage.

For anchorages on rock foundation, additional security against uplift and sliding may be obtained by setting and grouting steel rods into the rock under the heel of the anchorage, to serve as dowels. This expedient was adopted for the suspension bridges at Portsmouth, Ohio, and Grand'Mere, Quebec. On a clay foundation, inclined piles might similarly be used as dowels under the heel of the anchorage.

In general, the form and mass distribution of an anchorage are governed by the requirement of concentrating the weight toward the heel, so as to counteract the overturning moment from cable pull.

# CHAPTER III

## TYPICAL DESIGN COMPUTATIONS

(NOTE.—All references are to Chapter I, "Stresses in Suspension Bridges")

---

### EXAMPLE 1

### Calculations for Two-hinged Suspension Bridge with Straight Backstays (Type 2*F*)

**1. Dimensions.**—The following dimensions are given:

$l$ = Main Span = 50 panels at 22.5 ft. = 1125 ft. $(l' = l)$.

$l_2$ = Side Span = 281.25 ft. $= \frac{l}{4}$.

$f$ = Cable-sag in main span = 112.5 ft. $\left(n = \frac{f}{l} = \frac{1}{10}\right)$.

$f_1$ = Cable-sag in side spans = 0. (Straight backstays).

$d$ = Depth of Truss = 22.5 ft.

Mean Chord Sections (gross):

Top = 94 sq. in. Bottom = 161 sq. in.

$I$ = Mean Moment of Inertia of stiffening truss in main span $= 94(14.2)^2 + 161(8.3)^2 = 30{,}000$ in.$^2$ ft.$^2$ (1 truss).

Width, center to center of trusses or cables = 34.5 ft.

$A$ = Cable Section = 84 sq. in per cable. $(A_1 = A)$.

$\tan\alpha$ = Slope of Cable Chord in main span = 0.

$\tan\alpha_1$ = Slope of Cable Chord in side span $= 4\frac{f}{l} = 4n = 0.4$.

**2. Stresses in Cable.**—Given:

$w$ = Dead Load per cable (including cable)
= 2650 lb. per lin. ft.

$p'$ = Live Load per cable = 850 lb. per lin. ft.
$t$ = Assumed Temperature Variation = $\pm 60°$ F.
($E\omega t$ = 11,720 lb. per sq. in.)

All values are given per cable.

For Dead Load, by Eq. (11), the horizontal component of cable stress is,

$$H=\frac{wl^2}{8f}=\frac{10}{8}wl=3730 \text{ kips.} \quad (1 \text{ kip}=1000 \text{ lb.})$$

For Live Load, by Eq. (167), the denominator of the formula for $H$ is,

$$N=\frac{8}{5}+\frac{3I}{Af^2}\cdot\frac{l'}{l}\cdot(1+8n^2)+\frac{6I}{A_1f^2}\cdot\frac{l_2}{l}\cdot\sec^3\alpha_1$$
$$=\tfrac{8}{5}+3(.0283)(1+.08)+6(.0283)\tfrac{1}{4}(1.08)^3=1.745.$$

By Eq. (168), the live-load tension will be,

$$H=\frac{p'l}{5Nn}=\frac{2}{N}\cdot p'l=1.146\,p'l=1095 \text{ kips.}$$

The total length of cable between anchorages is given by Eq. (176):

$$\frac{L}{l}=(1+\tfrac{8}{3}n^2)+2\cdot\frac{l_2}{l}\cdot\sec\alpha_1=(1+.027)+\tfrac{2}{4}(1.08)=1.567.$$

Then, for temperature, by Eq. (156),

$$H_t=-\frac{3EI\omega tL}{f^2Nl}=\mp\frac{3(30{,}000)}{(112.5)^2}\cdot\frac{1.567}{1.745}(11{,}720)=\mp 75 \text{ kips.}$$

Adding the values found for $H$:

| | |
|---|---|
| D. L. | 3730 kips |
| L. L. | 1095 |
| Temp. | 75 |

we obtain, Total $H$ = 4900 kips per cable.

The maximum tension in the cable is, by Eq. (5),

$$T_1=H\cdot\sec\alpha_1=H(1.08)=5300 \text{ kips.}$$

At 65,000 lb. per sq. in., the cable section required is $5300\div 65$ = 82 sq. in. (Section provided = 84 sq. in.)

**3. Moments in Stiffening Truss.**—Given:

Live Load $= p = 1600$ lb. per lin. ft. per truss.

(All values given and calculated are per truss.)

With the main span completely loaded, the bending moment at any section $x$ is given by Eq. (169):

$$\text{Total } M = \tfrac{1}{2}px(l-x)\left(1 - \frac{8}{5N}\right) = \tfrac{1}{2}px(l-x)(.085).$$

In other words, only 8.5 per cent of the full-span live load is carried by the truss. Accordingly, at the center,

$$\text{Total } M = .085\frac{pl^2}{8} = +21{,}500 \text{ ft. kips per truss.}$$

At other points, the values of $M$ are proportional to the ordinates of a parabola. They are obtained as follows:

| Section $\left(\frac{x}{l}\right)$ | Parabolic Coefficient | Total $M$ (ft. kips) |
|---|---|---|
| 0 | $4 \times 0 \times 1 = 0$ | 0 |
| 0.1 | $4 \times .1 \times .9 = .36$ | + 7,800 |
| 0.2 | $4 \times .2 \times .8 = .64$ | +13,800 |
| 0.3 | $4 \times .3 \times .7 = .84$ | +18,100 |
| 0.4 | $4 \times .4 \times .6 = .96$ | +20,600 |
| 0.45 | $4 \times .45 \times .55 = .99$ | +21,300 |
| 0.5 | $4 \times .5 \times .5 = 1.00$ | +21,500 |

For maximum and minimum moments, the critical points are found by solving Eq. (142):

$$C(k) = N \cdot n \cdot \frac{x}{y} = .1745\frac{x}{y}.$$

The values of the minimum moments are then given by Eq. (170):

$$\text{Min. } M = -\frac{2px(l-x)}{5N} \cdot D(k) = -\frac{4}{5N-8} \cdot (\text{Total } M) \cdot D(k)$$
$$= -5.52(\text{Total } M) \cdot D(k).$$

The tabulations in Table I or the graphs in Fig. 12 are used in solving the values of $C(k)$ for $k$ and then finding the corresponding values of $D(k)$. The following tabulation shows the successive steps:

| $\frac{x}{l}$ | $\frac{y}{l}$ | $\frac{x}{y}$ | $C(k)$ | $k$ | $D(k)$ | Min. $M$ (ft. kips) |
|---|---|---|---|---|---|---|
| 0 | 0 | (2.50) | (.436) | (.355) | (.531) | 0 |
| .1 | .036 | 2.78 | .485 | .392 | .437 | −18,800 |
| .2 | .064 | 3.12 | .544 | .437 | .334 | −25,400 |
| .3 | .084 | 3.57 | .623 | .497 | .224 | −22,400 |
| .4 | .096 | 4.17 | .728 | .584 | .113 | −12,800 |
| .45 | .099 | 4.55 | .795 | .647 | .061+ 001 | − 7,300 |
| .5 | .100 | 5.00 | .872 | .729 | .024+ 024 | − 5,700 |
| .55 | .099 | 5.55 | .970 | .876 | .001+.061 | − 7,300 |

(By using Fig. 13, $D(k)$ may be found directly from the values of $C(k)$.)

For all sections between $x=\frac{N}{4}\cdot l=.436l$ and the symmetrical point $x=.564l$, a correction is made in the above table for the second critical point, as explained under Eq. (143′).

To find the maximum moments, apply Eq. (144):

$$\text{Max. } M=\text{Total } M-\text{Min. } M.$$

Dividing the maximum and minimum moments by the truss depth ($d=22.5$ ft.), we obtain the respective chord stresses as follows:

| Section $\frac{x}{l}$ | Maximum $M$ (ft. kips) | Chord Stresses (kips) Maximum | Chord Stresses (kips) Minimum |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0.1 | +26,600 | ∓1180 | ± 840 |
| 0.2 | +39,200 | ∓1740 | ±1130 |
| 0.3 | +40,500 | ∓1800 | ±1000 |
| 0.4 | +33,400 | ∓1490 | ± 570 |
| 0.45 | +28,600 | ∓1270 | ± 320 |
| 0.5 | +27,200 | ∓1210 | ± 250 |

In this tabulation of the stresses, the upper signs refer to the top chord and the lower signs to the bottom chord. Dividing the above values by the specified unit stresses in tension and compression, respectively, we obtain the required net and gross sections of the chord members. For the bottom chords, these sections must be increased to provide for the wind stresses, computed as indicated below. In addition, the temperature stresses must be taken into consideration.

The moments produced by temperature variation are given by Eq. (157):

$$M_t = -H_t \cdot y.$$

As found above, $H_t = \mp 75$ kips for a temperature variation of $\pm 60°$ F. At the center,

$$M_t = \pm(75 \text{ kips} \times 112.5 \text{ ft.}) = \pm 8450 \text{ ft. kips.}$$

At other sections, the moments are proportional to the parabolic ordinates $y$:

| Section | Parabolic Coefficient | $M_t$ |
|---|---|---|
| $\frac{x}{l}=0$ | 0 | 0 |
| .1 | .36 | ±3040 |
| .2 | .64 | ±5400 |
| .3 | .84 | ±7100 |
| .4 | .96 | ±8100 |
| .5 | 1.00 | ±8450 |

(The upper signs pertain to a rise in temperature, the lower signs to a fall.)

The resulting stresses are to be combined with the live-load stresses previously found, in whatever manner the specifications may prescribe. For the sections $\frac{x}{l} = 0$ to 0.4, the temperature moments amount to less than 25 per cent of the live-load Max. $M$, in which case, according to some specifications, the temperature stresses may be ignored.

**4. Shears in Stiffening Truss.**—

$$(p = 1600 \text{ lb. per lin. ft., } \tfrac{1}{2}pl = 900 \text{ kips.})$$

With the main span fully loaded, the shears at the various sections are given by Eq. (173):

$$\text{Total } V = \tfrac{1}{2}p(l-2x)\left(1-\frac{8}{5N}\right) = \tfrac{1}{2}pl\left(1-\frac{2x}{l}\right)(.085) = 76.5\left(1-\frac{2x}{l}\right).$$

| Section | $\left(1-\frac{2x}{l}\right)$ | Total $V$ |
|---|---|---|
| $\frac{x}{l}=0$ | 1 | 76 kips |
| .1 | .8 | 61 |
| .2 | .6 | 46 |
| .3 | .4 | 31 |
| .4 | .2 | 15 |
| .5 | 0 | 0 |

The maximum shears are given by Eq. (149):

$$\text{Max. } V = \tfrac{1}{2}pl\left(1-\frac{x}{l}\right)^2\left[1-\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)\cdot G\left(\frac{x}{l}\right)\right].$$

The values of $G\left(\frac{x}{l}\right)$ are taken from Table I, and the shears are obtained as follows:

| Section | $\left(1-\frac{x}{l}\right)^2$ | $\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)$ | $G\left(\frac{x}{l}\right)$ | [ ] | Maximum $V$ |
|---|---|---|---|---|---|
| $\frac{x}{l}=0$ | 1 | 2.29 | .400 | .084 | + 76+219 |
| .1 | .81 | 1.84 | .482 | .114 | + 83+110 |
| .2 | .64 | 1.38 | .565 | .220 | +131+ 28 |
| .3 | .49 | 0.92 | .647 | .405 | +179 |
| .4 | .36 | 0.46 | .726 | .666 | +216 |
| .5 | .25 | 0 | .800 | 1.000 | + 225 kips |

For all sections $x < \frac{l}{2}\left(1 - \frac{N}{4}\right) = .282l$, the loading for maximum shear extends from the given section $x$ to a critical point $kl$ defined by Eq. (150):

$$C(k) = \frac{N}{4} \cdot \frac{l}{l - 2x} = \frac{.436}{1 - \frac{2x}{l}}.$$

The values $C(k)$ are solved for $k$ with the aid of Fig. 13.

| Section | $1 - \frac{2x}{l}$ | $C(k)$ | $k$ |
|---|---|---|---|
| 0 | 1 | 436 | 355 |
| .1 | 8 | .545 | 438 |
| .2 | .6 | .730 | .588 |

For these sections, a correction is to be added to the values of Max. $V$ found above. This additional shear is given by Eq. (151):

$$\text{Add. } V = \tfrac{1}{2}pl(1-k)^2\left[\frac{8}{N}\left(\frac{1}{2} - \frac{x}{l}\right) \cdot G(k) - 1\right].$$

| Section | $k$ | $(1-k)^2$ | $\frac{8}{N}\left(\frac{1}{2} - \frac{x}{l}\right)$ | $G(k)$ | $[\ ]$ | Add. $V$ |
|---|---|---|---|---|---|---|
| $\frac{x}{l} = 0$ | .355 | .416 | 2.29 | .691 | .585 | +219 kips |
| .1 | .438 | .316 | 1.84 | .755 | .389 | +110 |
| .2 | .588 | .170 | 1.38 | .858 | .185 | + 28 |

(By using Fig. 13, $G(k)$ may be found directly from the values of $C(k)$.)

The minimum shears are then given by Eq. (153):

$$\text{Min. } V = \text{Total } V - \text{Max. } V.$$

| Section | Total $V$ | Maximum $V$ | Minimum $V$. |
|---|---|---|---|
| $\frac{x}{l}=0$ | +76 kips | +295 kips | −219 kips |
| .1 | +61 | +193 | −132 |
| .2 | +46 | +159 | −113 |
| .3 | +31 | +179 | −148 |
| .4 | +15 | +216 | −201 |
| .5 | 0 | +225 | −225 |

The temperature shears are given by Eq. (158):

$$V_t = -H_t \cdot (\tan \phi - \tan \alpha).$$

In this case, $\tan \alpha = 0$, and $H_t = \mp 75$ kips. At the ends,

$$\tan \phi = \frac{4f}{l} = 4n = 0.40,$$

and the slope ($\tan \phi$) diminishes uniformly toward the crown.

| Section | $\tan \phi$ | $V_t$ |
|---|---|---|
| $\frac{x}{l}=0$ | 0.40 | ±30 kips |
| .1 | .32 | ±24 |
| .2 | .24 | ±18 |
| .3 | .16 | ±12 |
| .4 | .08 | ± 6 |
| .5 | 0 | 0 |

These shears are to be combined with the maximum and minimum live-load shears, as may be required by the specifications.

**5. Wind Stresses in Bottom Chords.**—

(Assumed wind load $= p = 400$ lb. per lin. ft.)

If the lateral bracing is in the plane of the bottom chords, these chords act as the chords of a wind truss. The applied wind pressure $p$ is partly counteracted by a force of restitution $r$ due to the horizontal displacement of the weight of the stiffening truss $w$. The resulting reduction in the effective horizontal load is given with sufficient accuracy by the formula,

$$\frac{r}{p}=\frac{.013\frac{wl^4}{vEI}}{1+.013\frac{wl^4}{vEI}}.$$

(For the derivation of this formula, see Steinman's "Suspension Bridges and Cantilevers," D. Van Nostrand Co., 1913, page 76.) In this case, $w$ = total dead load (both trusses) = 5300 lb. per lin. ft.; $v$ = vertical height from cable chord to center of gravity of the dead load = 135 ft.; $I$ = moment of inertia of wind truss = $\frac{1}{2}(161)\times(34.5)^2=96{,}000$ in.²ft.² Substituting these values, we obtain,

$$\frac{r}{p}=\frac{.284}{1+.284}=0.22.$$

Hence the force of restitution $r$ (due to the obliquity of suspension after horizontal deflection) amounts, in this case, to 22 per cent of the applied wind load ($p$) at the center of the span. The force $r$ diminishes to zero at the ends of the span, and the equivalent uniform value of $r$ may be taken as $\frac{5}{8}$ of the mid-span value. The resultant horizontal load on the span is,

$$p-\tfrac{5}{8}r=400-\tfrac{5}{8}(88)=327 \text{ lb. per lin. ft.}$$

Treating this value as a uniform load, the bending moment at the center is,

$$M_w=\frac{327l^2}{8}=51{,}900 \text{ ft. kips.}$$

Dividing by the truss width 34.5 ft., we obtain the chord stress $=\pm 1500$ kips at mid-span. The wind stresses at other sections will be proportional to parabolic ordinates; being zero at the ends of the span.

The shears in the lateral system may also be calculated for the resultant uniform load of 327 lb. per lin. ft.

## EXAMPLE 2

## Calculations for Two-hinged Suspension Bridge with Suspended Side Spans (Type 2S)

**1. Dimensions.**—The following dimensions are given:

$l$ = Main Span = 1080 ft. $(l' = l)$.

$l_1$ = Side Span = 360 ft. $r = \frac{l_1}{l} = \frac{1}{3}$.

$l_2$ = Distance, tower to anchorage = 400 ft.

$f$ = Cable-sag in main span = 108 ft. $n = \frac{f}{l} = \frac{1}{10}$.

$f_1$ = Cable-sag in side spans = 12 ft. $n_1 = \frac{f_1}{l_1} = \frac{1}{30}$. $v = \frac{f_1}{f} = \frac{1}{9}$

$d$ = Depth of Stiffening Truss = 22.5 ft.

Mean Chord Section (gross):

Main Span, Top = 83, Bottom = 137 sq. in.

Side Spans, Top = 52, Bottom = 52 sq. in.

$I$, (Main Span) $= 83(14)^2 + 137(8.5)^2 = 26{,}200$ in.² ft.²

$I_1$, (Side Spans) $= 2 \times 52(11.25)^2 = 13{,}100$ in.² ft.²

$$i = \frac{I}{I_1} = 2.0.$$

Width, center to center of trusses or cables = 42.5 ft.

$A$ = Cable Section = 78 sq. in. per cable. $(A_1 = A)$.

$\tan \alpha$ = Slope of Cable Chord in Main Span = 0.

$\tan \alpha_1$ = Slope of Cable Chord in Side Span $= 4(n - n_1) = .267$.

$\sec \alpha_1 = 1.034$.

**2. Stresses in Cable.**—(All values given per cable).

Given:

$w$ = Dead Load (including cable) = 2385 lb. per lin. ft.

$p'$ = Live Load = 860 lb. per lin. ft.

$t$ = Temperature Variation = ±60° F. $(E\omega t = 11{,}720$ lb. per sq. in.).

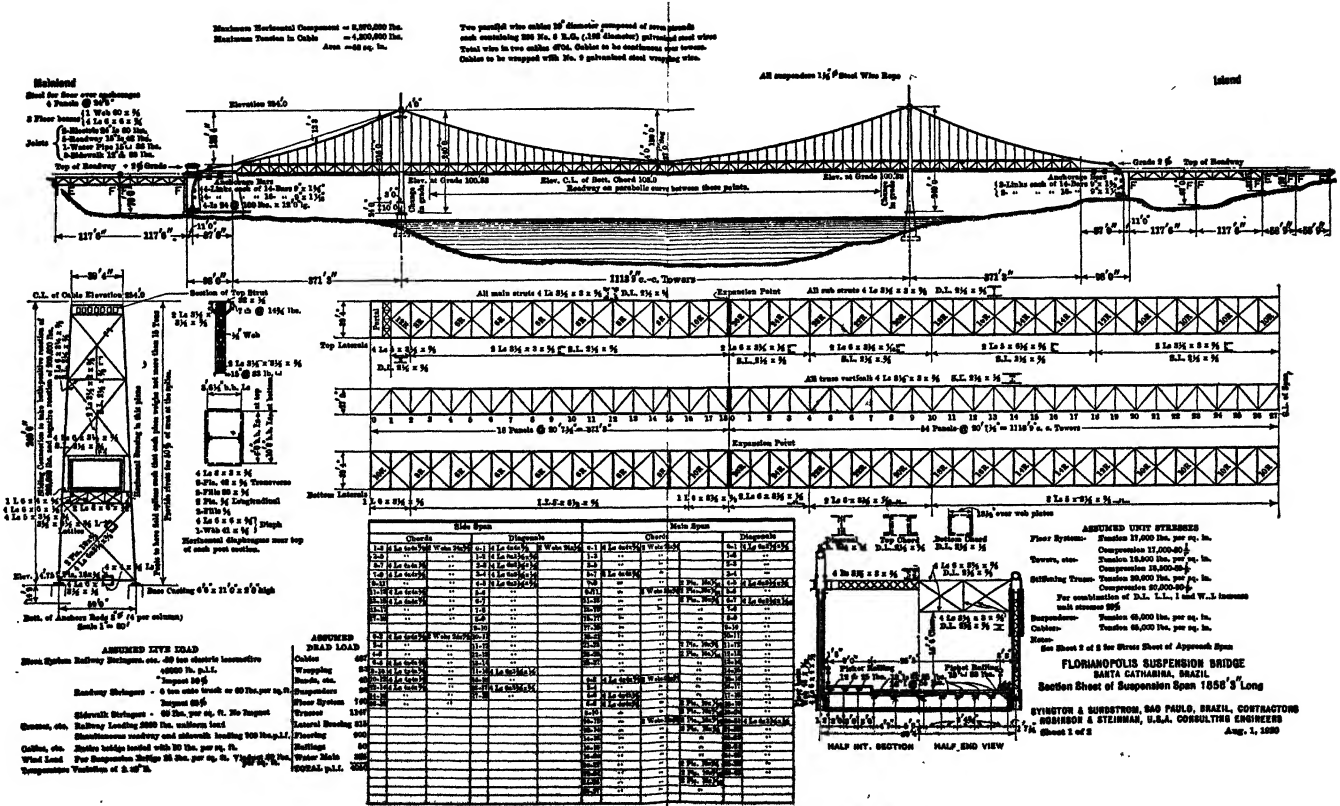



Fig. 43.—Suspension Bridge at Florianopolis, Brazil. (Type 2S)
Span, 1113 feet 9 inches. Design by H. D. Robinson and D. B. Steinman, 1926.

(To face page 134.)

For Dead Load, by Eq. (11), the horizontal component of cable stress is,

$$H=\frac{wl^2}{8f}=\frac{10}{8}wl=3220 \text{ kips.} \quad (1 \text{ kip}=1000 \text{ lb.})$$

For Live Load, by Eq. (125), the denominator of the $H$-equation is,

$$N=\tfrac{8}{5}(1+2irv^2)+\frac{3I}{Af^2}\cdot\frac{l'}{l}(1+8n^2)+\frac{6I}{A_1f^2}\cdot\frac{l_2}{l}\cdot\sec^3\alpha_1(1+8n_1^2)$$

$$=1.626+.093+.071=1.790.$$

By Eq. (135), the horizontal tension produced by live load, covering all three spans, will be,

$$H=\frac{1}{5Nn}(1+2ir^3v)p'l=\frac{2}{1.790}(1.0164)(930)=1050 \text{ kips.}$$

The total length of cable between anchorages is given by Eq. (154):

$$\frac{L}{l}=\left(1+\frac{8}{3}n^2\right)+2\frac{l_2}{l}\left(\sec\alpha_1+\frac{8}{3}\frac{n_1^2}{\sec^3\alpha_1}\right)=1.027+.767=1.794.$$

Then, for temperature, by Eq. (156),

$$H_t=-\frac{3EI\omega tL}{f^2Nl}=\frac{3(11720)(26200)(1.794)}{(108)^2\times 1.790}=\mp 80 \text{ kips.}$$

Adding the values found for $H$:

| | |
|---|---|
| D. L. | 3220 kips |
| L. L. | 1050 |
| Temp. | 80 |

we obtain, Total $H=4350$ kips per cable.

The maximum tension in the cable is, by Eq. (5),

$$T_1=H\cdot\sec\phi_1=H(1.08)=4700 \text{ kips.}$$

At 60,000 lb. per sq. in., the cable section required is:

$$4700\div 60=78 \text{ sq. in. per cable.}$$

**3. Moments in Stiffening Truss—Main Span.—**

Given:

$$\text{Live Load} = p = 1600 \text{ lb. per lin. ft.}$$

(All values given and calculated are per truss.)

With the three spans completely loaded, the bending moment at any section $x$ of the main span is given by Eq. (140):

$$\text{Total } M = \tfrac{1}{2}px(l-x)\left[1 - \frac{8}{5N}(1+2ir^3v)\right] = \tfrac{1}{2}px(l-x)[.091].$$

Hence only 9.1 per cent of the full live load is carried by the stiffening truss. Accordingly, at the center,

$$\text{Total } M = .091\,\frac{pl^2}{8} = +21.200 \text{ ft. kips.}$$

At other points, the values of $M$ are proportional to the ordinates of a parabola. They are obtained as follows:

| Section | Parabolic Coefficient | Total $M$ |
|---|---|---|
| $\frac{x}{l}=0$ | $4\times 0 \times 1 = 0$ | 0 |
| 0.1 | $4\times.1 \times .9 = .36$ | + 7,600 |
| 0.2 | $4\times.2 \times .8 = .64$ | +13,600 |
| 0.3 | $4\times.3 \times .7 = .84$ | +17,800 |
| 0.4 | $4\times.4 \times .6 = .96$ | +20,400 |
| 0.45 | $4\times.45\times .55 = .99$ | +21,000 |
| 0.5 | $4\times.5 \times .5 = 1.00$ | +21,200 ft. kips |

For maximum and minimum moments, the critical points are found by solving Eq. (142):

$$C(k) = N\cdot n\cdot\frac{x}{y} = 0.179\frac{x}{y},$$

with the aid of Table I or Fig. 12 or 13:

| $\frac{x}{l}$ | $\frac{y}{l}$ | $\frac{x}{y}$ | $C(k)$ | $k$ | $D(k)$ |
|---|---|---|---|---|---|
| 0 | 0 | (2.50) | (.448) | (.364) | (.508) |
| .1 | .036 | 2.78 | .498 | .402 | .411 |
| .2 | .064 | 3.12 | .559 | .448 | .310 |
| .3 | .084 | 3.57 | .640 | .512 | .202 |
| .4 | .096 | 4.17 | .747 | .603 | .095 |
| .45 | .099 | 4.55 | .815 | .667 | .050+.000 |
| .5 | .100 | 5.00 | .895 | .755 | .016+.016 |
| .55 | .099 | 5.55 | .995 | .950 | .000+.050 |

The values of $D(k)$, found from Table I or Fig. 13, are recorded in the above tabulation. For all sections from $x=\frac{N}{4}\cdot l=.447l$ to $x=.553l$, there are double values of $D(k)$, as explained under Eq. (143′).

The values of the minimum moments are then given by Eq. (143):

$$\text{Min. } M=-\frac{2px(l-x)}{5N}[D(k)+4ir^3v]$$

$$=-417{,}000\frac{x}{l}\left(1-\frac{x}{l}\right)[D(k)+.033],$$

and the maximum moments are then given by Eq. (144):

$$\text{Max. } M=\text{Total } M-\text{Min. } M.$$

| Section | $\frac{x}{l}\left(1-\frac{x}{l}\right)$ | [ ] | Minimum $M$ | Total $M$ | Maximum $M$ |
|---|---|---|---|---|---|
| $\frac{x}{l}=0$ | 0 | (.541) | 0 | 0 | 0 |
| .1 | .09 | .444 | −16,700 | + 7,600 | +24,300 |
| .2 | .16 | .343 | −22,900 | +13,600 | +36,500 |
| .3 | .21 | .235 | −20,600 | +17,800 | +38,400 |
| .4 | .24 | .128 | −12,800 | +20,400 | +33,200 |
| .45 | .248 | .083 | − 8,600 | +21,000 | +29,600 |
| .5 | .25 | .065 | − 6,800 | +21,200 | +28,000 ft. kips |

Dividing the maximum and minimum moments by the truss depth ($d=22.5$), we obtain the respective chord stresses. Adding the temperature and wind stresses found as in Example 1 and dividing by the specified unit stresses, we obtain the required chord sections.

**4. Bending Moments in Side Spans.**—

($p=1600$ lb. per lin. ft. per truss).

With all three spans completely loaded, the bending moment at any section $x_1$ of either side span is given by Eq. (141):

$$\text{Total } M_1=\tfrac{1}{2}px_1(l_1-x_1)\left[1-\frac{8}{5N}(1+2ir^3v)\frac{v}{r^2}\right]=\tfrac{1}{2}px_1(l_1-x_1)[.091].$$

Accordingly, at the center,

$$\text{Total } M_1=.091\,\frac{pl_1^2}{8}=+2300 \text{ ft. kips.}$$

There are no critical points for moments in the side spans. The minimum moments are given by Eq. (145):

$$\text{Min. } M_1=-y_1\cdot\frac{1+ir^3v}{5Nn}\cdot pl=-y_1(1.124)pl.$$

Accordingly, at the center,

$$\text{Min. } M_1=-12(1.124)(1730)=-23{,}400 \text{ ft. kips.}$$

The maximum moments are given by Eq. (146):

$$\text{Max. } M_1=\text{Total } M_1-\text{Min. } M_1.$$

Accordingly, at the center,

$$\text{Max. } M_1=+2300+23{,}400=+25{,}700 \text{ ft. kips.}$$

At other sections, the moments are proportional to the ordinates of a parabola:

| Section | Parabolic Coefficient | Total $M_1$ | Minimum $M_1$ | Maximum $M_1$ |
|---|---|---|---|---|
| $\frac{x_1}{l_1}=0$ | 0 | 0 | 0 | 0 |
| .1 | .36 | + 800 | − 8,400 | + 9,200 |
| .2 | .64 | +1500 | −15,000 | +16,500 |
| .3 | .84 | +1900 | −19,600 | +21,500 |
| .4 | .96 | +2200 | −22,400 | +24,600 |
| .5 | 1 | +2300 | −23,400 | +25,700 ft. kips |

**5. Shears in Stiffening Truss—Main Span.—**

($p=1600$ lb. per lin. ft.)

With the three spans completely loaded, the shear at any section $x$ of the main span is given by Eq. (147):

$$\text{Total } V=\tfrac{1}{2}p(l-2x)\left[1-\frac{8}{5N}(1+2ir^3v)\right]=\tfrac{1}{2}p(l-2x)[.091].$$

The shears will be the same as would be produced by loading the span with 9.1 per cent of the actual load, or with .091 $pl=157$ kips.

Total $V=157\left(\frac{1}{2}-\frac{x}{l}\right)$.

| Section | $\frac{1}{2}-\frac{x}{l}$ | Total $V$ |
|---|---|---|
| $\frac{x}{l}=0$ | 0.5 | +79 kips |
| .1 | 0.4 | +63 |
| .2 | 0.3 | +47 |
| .3 | 0.2 | +31 |
| .4 | 0.1 | +16 |
| .5 | 0 | 0 |

The maximum shears are given by Eq. (149):

$$\text{Max. } V=\tfrac{1}{2}pl\left(1-\frac{x}{l}\right)^2\left[1-\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)\cdot G\left(\frac{x}{l}\right)\right],$$

where the values of $G\left(\frac{x}{l}\right)$ are taken from Table I or Fig. 12. The shears are obtained as follows: ($\frac{1}{2}pl=864$ kips).

| Section | $\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)$ | $G\left(\frac{x}{l}\right)$ | $[\;\;]$ | $\left(1-\frac{x}{l}\right)^2$ | Maximum $V$ |
|---|---|---|---|---|---|
| $\frac{x}{l}=0$ | 2.23 | .400 | .107 | 1 | + 92+194 |
| .1 | 1.79 | .482 | .136 | .81 | + 95+ 96 |
| .2 | 1.34 | .565 | .243 | .64 | +134+ 22 |
| .3 | 0.89 | .647 | .424 | .49 | +179 |
| .4 | .45 | .726 | 673 | .36 | +210 |
| .5 | 0 | .800 | 1.000 | .25 | +216 kips |

For all sections, $x<\frac{l}{2}\left(1-\frac{N}{4}\right)=.277l$, the loading for maximum shear extends from the given section $x$ to a critical point $kl$ defined by Eq. (150):

$$C(k)=\frac{N}{4}\cdot\frac{l}{l-2x}=\frac{.446}{1-\frac{2x}{l}}.$$

The values $C(k)$ are solved for $k$ and $G(k)$ with the aid of Fig. 13:

| Section | $1-\frac{2x}{l}$ | $C(k)$ | $k$ |
|---|---|---|---|
| 0 | .1 | .446 | .362 |
| .1 | .8 | .558 | .448 |
| .2 | .6 | .744 | .600 |

For these sections, a correction is to be added to the values of Max. $V$ found above. This additional shear is given by Eq. (151):

$$\text{Add. } V=\tfrac{1}{2}pl(1-k)^2\left[\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)\cdot G(k)-1\right].$$

| Section | $k$ | $(1-k)^2$ | $\frac{8}{N}\left(\frac{1}{2}-\frac{x}{l}\right)$ | $G(k)$ | [ ] | Add. $V$ |
|---|---|---|---|---|---|---|
| $\frac{x}{l}=0$ | .362 | .407 | 2 23 | .696 | .552 | +194 kips |
| .1 | .448 | .305 | 1.79 | .762 | .365 | + 96 |
| .2 | .600 | .160 | 1.34 | .866 | 160 | + 22 |

The minimum shears are then given by Eq. (153):

$$\text{Min. } V=\text{Total } V-\text{Max. } V.$$

| Section | Total $V$ | Maximum $V$ | Minimum $V$ |
|---|---|---|---|
| $\frac{x}{l}=0$ | +79 | +286 kips | −207 kips |
| .1 | +63 | +191 | −128 |
| .2 | +47 | +156 | −109 |
| .3 | +31 | +179 | −148 |
| .4 | +16 | +210 | −194 |
| .5 | 0 | +216 | −216 |

**6. Shears in Side Spans.—**

($p=1600$ lb. per lin. ft., $l_1=360$ ft.)

With the three spans completely loaded, the shear at any section $x_1$ in the side spans will be, by Eq. (148),

$$\text{Total } V_1=\tfrac{1}{2}p(l_1-2x_1)\left[1-\frac{8}{5N}\,\frac{v}{r^2}\,(1+2ir^3v)\right]=pl_1\left(\frac{1}{2}-\frac{x_1}{l_1}\right)[.091].$$

Since $l_1=\frac{1}{3}l$, these shears will be $\frac{1}{3}$ of the corresponding values in the main span.

| Section | Total $V_1$ |
|---|---|
| $\frac{x_1}{l_1}=0$ | +26 kips |
| .1 | +21 |
| .2 | +16 |
| .3 | +10 |
| .4 | + 5 |
| .5 | 0 |

There are no critical points for shear in the side spans. The maximum shear at any section $x_1$ is given by Eq. (152):

$$\text{Max. } V_1 = \tfrac{1}{2} p l_1 \left(1 - \frac{x_1}{l_1}\right)^2 \left[1 - \frac{8}{N} irv^2 \left(\frac{1}{2} - \frac{x_1}{l_1}\right) \cdot G\left(\frac{x_1}{l_1}\right)\right].$$

| Section | $\frac{8}{N}(irv^2)\left(\frac{1}{2}-\frac{x_1}{l_1}\right)$ | $G\left(\frac{x_1}{l_1}\right)$ | $[\quad]$ | $\left(1-\frac{x_1}{l_1}\right)^2$ | Maximum $V_1$ |
|---|---|---|---|---|---|
| $\frac{x_1}{l_1}=0$ | .0183 | .400 | .993 | 1 | +286 kips |
| .1 | .0147 | .482 | .993 | .81 | +232 |
| .2 | .0110 | .565 | .994 | .64 | +183 |
| .3 | .0073 | .647 | .995 | .49 | +140 |
| .4 | .0037 | .726 | .997 | .36 | +103 |
| .5 | 0 | .800 | 1.000 | .25 | + 72 |

The minimum shears in the side spans are given by Eq. (153′):

$$\text{Min. } V_1 = \text{Total } V_1 - \text{Max. } V_1.$$

| Section | Minimum $V_1$ |
|---|---|
| $\frac{x_1}{l_1}=0$ | −260 kips |
| .1 | −211 |
| .2 | −167 |
| .3 | −130 |
| .4 | − 98 |
| .5 | − 72 |

**7. Temperature Stresses.—**

$$(H_t = \mp 80 \text{ kips}).$$

The stresses in the main span from temperature variation are figured exactly as in Example 1 (Type 2*F*), using Eqs. (157) and (158):

$$M_t = -H_t \cdot y,$$

$$V_t = -H_t(\tan\phi - \tan\alpha). \quad (\text{Here, } \tan\alpha = 0).$$

The temperature moments in the side spans are given by the formula:

$$M_t = -H_t \cdot y_1,$$

and will therefore be $v(=\frac{1}{8})$ times the corresponding main-span values.

| Section | Parabolic Coefficient | $M_t$ |
|---|---|---|
| $\frac{x_1}{l_1}=0$ | 0 | 0 |
| .1 | .36 | $\pm 350$ |
| .2 | .64 | $\pm 610$ |
| .3 | .84 | $\pm 810$ |
| .4 | .96 | $\pm 920$ |
| .5 | 1 | $\pm 960$ ft. kips |

The temperature shears in the side spans are given by the formula:

$$V_t = -H_t(\tan\phi_1 - \tan\alpha_1). \quad (\text{Here, } \tan\alpha_1 = .267.)$$

They will be $\frac{n_1}{n}(=\frac{1}{3})$ times the corresponding main-span values.

| Section | $\tan\phi_1 - \tan\alpha_1$ | $V_t$ |
|---|---|---|
| $\frac{x_1}{l_1}=0$ | $4n_1 = .133$ | $\pm 11$ kips |
| .1 | .106 | $\pm 8$ |
| .2 | .080 | $\pm 6$ |
| .3 | .053 | $\pm 4$ |
| .4 | .027 | $\pm 2$ |
| .5 | 0 | 0 |

**8. Wind Stresses.**—The wind stresses in the bottom chords and lateral bracing are calculated exactly as in Example 1 (Type 2*F*).

The assumed wind load ($=p=400$ lb. per lin. ft.) is reduced by the fractional amount, at span center,

$$\frac{r}{p}=\frac{.013\dfrac{wl^4}{vEI}}{1+.013\dfrac{wl^4}{vEI}}=\frac{.173}{1+.173}=0.147.$$

($w=4770$ lb. per lin. ft.; $v=130$ ft.; $I=124{,}000$ in.$^2$ ft.$^2$)

Since the equivalent uniform value of $r$ is $\frac{5}{6}$ of the mid-span value, the resultant horizontal load on the span is,

$$p-\tfrac{5}{6}r=400-\tfrac{5}{6}(59)=351 \text{ lb. per lin. ft.}$$

Treating this value as a uniform load, the bending moment at the center is,

$$M_w=\frac{351l^2}{8}=\pm 51{,}000 \text{ ft. kips.}$$

Dividing by the truss width, 42.5 ft., we obtain the chord stress $=\pm 1200$ kips at mid-span. The wind stresses at other sections will be proportional to parabolic ordinates.

The end shears in the lateral system will be:

$$V_w=\frac{351l}{2}=\pm 190 \text{ kips.}$$

In the side spans, unless they exceed 1000 feet in length, the reduction in effective wind pressure may be neglected. (In this example, $\frac{r}{p}$ would amount to only 1 per cent.) Hence the moments and shears are calculated for the full specified wind load of 400 lb. per lin. ft., acting on simple spans 360 ft. in length.

### EXAMPLE 3

### Calculations for Towers of Two-hinged Suspension Bridge (Type 2S)

**1. Dimensions.**—The bridge is the same as in Example 2.

Each tower consists of two columns of box section, stiffened with internal diaphragms, and rigidly tied together with transverse bracing in a vertical plane. Each tower column is 225 feet high and is made of a double box section, 42.5 inches wide. The other dimension ($d$), parallel to the stiffening truss, is 4 feet at the top, increasing to 9 feet at the base. The walls are $1\frac{1}{4}$ inches thick (made up of $\frac{5}{8}$-inch plates and corner angles) and the vertical transverse diaphragm is $\frac{5}{8}$-inch thick. Splices are provided at such intervals as to keep the individual sections within specified limitations of length or weight for shipment. Horizontal diaphragms are provided at splices and, in general, at 10-foot intervals.

The tower columns are battered so as to clear the trusses. They are 42.5 feet center to center at the top and 53.5 feet center to center at the base.

**2. Movement of Top of Tower.**—The towers are assumed fixed at the base and the cable saddles immovable with respect to the tower.

The maximum fiber stress in the tower columns will occur when the live load covers the main span and the farther side span at maximum temperature. Under this condition of loading, the top of the tower will be deflected toward the main span, as a result of the following deformations:

1. The upward deflection ($\Delta f_1$) at the center of the unloaded side span.

2. The elongation of the cable between the anchorage and the tower, due to the elastic strain produced by the applied loads.

3. The elongation of the cable due to thermal expansion.

These deformations are computed as follows:

(Live Load $= p' = 860$ lb. per lin. ft. $H = 1040$ kips.)

1. The upward deflection $\Delta f_1$ is found by considering the unloaded side span as a simple beam subjected to an upward loading equal to the live-load suspender tensions (Eq. 78):

$$s = \frac{8f}{l^2} \cdot H = \frac{8}{10} \cdot \frac{1040}{1080} = 770 \text{ lb. per lin. ft. per truss,}$$

$$\Delta f_1 = \frac{5}{384} \frac{s l_1^4}{E I_1} = 0.428 \text{ ft.}$$

2. The elastic elongation of the cable in the side span is, by Eq. (55),

$$\Delta L_1=\frac{Hl_2}{EA}(1+\tfrac{16}{3}n_1^2+\tan^2\alpha_1)=.178(1.077)=.192 \text{ ft.}$$

3. The temperature expansion of the cable in the side span is, by Eqs. (53) and (26),

$$\Delta L_1=\omega t l_2\left(\sec\alpha_1+\frac{8}{3}\frac{n_1^2}{\sec^3\alpha_1}\right)=.156(1.037)=0.162 \text{ ft.}$$

We also have (from Eq. 26):

$$\frac{\Delta L_1}{\Delta l_1}=\cos\alpha_1+\frac{8}{3}n_1^2(2\cos^3\alpha_1-3\cos^5\alpha_1)=0.964,$$

$$\frac{\Delta L_1}{\Delta f_1}=\frac{16}{3}\frac{n_1}{\sec^3\alpha_1}=0.160.$$

The deflection of the top of the tower is then given by,

$$y_0=\Delta l_1=\frac{\Delta l_1}{\Delta L_1}\cdot\frac{\Delta L_1}{\Delta f_1}\cdot\Delta f_1+\frac{\Delta l_1}{\Delta L_1}\cdot\Sigma(\Delta L_1).$$

Substituting the values just calculated, we obtain the maximum tower deflection:

$$y_0=\frac{.160}{.964}(.428)+\frac{1}{.964}(.192+.162)=0.439 \text{ ft.}$$

**3. Forces Acting on Tower.**—Considering this deflection as produced by an unbalanced horizontal force $P$ applied at the top of the tower, this force may be calculated, if the sectional dimensions of the tower are known, by the formula,

$$y_0=\frac{P}{E}\cdot\Sigma\left(\frac{x^2}{I}\cdot\Delta x\right).$$

In the present case, we find $\Sigma\frac{x^2}{I}\cdot\Delta x=1740$. Hence,

$$P=y_0\cdot\frac{E}{1740}=17{,}200y_0=7550 \text{ lb. per column.}$$

The other loads acting on the tower are the vertical reaction ($V$) at the saddles, and the end-shears ($V_1$) at the point of support of the stiffening truss. The saddle reaction is given by the formula:

$$V=2H\cdot\tan\phi=2\times4340\times0.4=+3470 \text{ kips per column.}$$

The truss reaction, with all spans loaded and maximum temperature rise, is,

$$V_1 = (42+32)+(14+11) = +99 \text{ kips per column.}$$

With one side span unloaded, as assumed above,

$$V_1 = (45+32)+(11-140) = -52 \text{ kips per column.}$$

The inaccuracy introduced by neglecting this uplift, $V_1$, will be on the side of safety; therefore the column need be figured only for the horizontal load $P$ and the vertical load $V$.

At any section $x$ of the tower, the horizontal deflection ($y$) from the initial vertical position of the axis is given with sufficient accuracy by the equation for the elastic curve of the cantilever:

$$y = y_0\left[1 - \frac{3}{2}\left(\frac{x}{h}\right) + \frac{1}{2}\left(\frac{x}{h}\right)^3\right].$$

**4. Calculation of Stresses.**—The resulting extreme fiber stresses at any section of the tower will be:

$$\text{Combined Stress} = \frac{V}{A} + \frac{Pxc}{I} + \frac{V(y_0-y)c}{I}.$$

The computations may be arranged as follows, the stresses being figured for convenience at 25-foot intervals:

| Joint No. | $x$ ft. | $y_0-y$ ft. | $d=2c$ ft. | $A$ sq. in. | $I$ in.² ft.² | $\frac{x^2}{I}$ | $\frac{V}{A}$ | $\frac{Pxc}{I}$ | $\frac{V(y_0-y)c}{I}$ | Combined Stresses lb./sq. in. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 4.0 | 280 | 560 | 0 | 12,400 | 0 | 0 | 12,400 |
| 1 | 25 | .068 | 4.5 | 295 | 730 | .86 | 11,800 | 500 | 700 | 13,000 |
| 2 | 50 | .134 | 5.0 | 310 | 940 | 2.66 | 11,200 | 900 | 1100 | 13,200 |
| 3 | 75 | .197 | 5.5 | 325 | 1170 | 6.40 | 10,700 | 1200 | 1600 | 13,500 |
| 4 | 100 | .254 | 6.0 | 340 | 1440 | 6.94 | 10,200 | 1500 | 1800 | 13,500 |
| 5 | 125 | .305 | 6.5 | 355 | 1740 | 8.98 | 9,800 | 1600 | 2000 | 13,400 |
| 6 | 150 | .348 | 7.0 | 370 | 2080 | 10.80 | 9,400 | 1800 | 2000 | 13,200 |
| 7 | 175 | .380 | 7.5 | 385 | 2460 | 12.42 | 9,000 | 1900 | 2000 | 12,900 |
| 8 | 200 | .400 | 8.0 | 400 | 2880 | 13.88 | 8,700 | 1900 | 1900 | 12,500 |
| 9 | 225 | .408 | 9.0 | 430 | 3850 | 13.20 | 8,100 | 1800 | 1700 | 11,600 |
| | | | | | | $\Sigma = 69.54$ | | | | |

**5. Wind Stresses.**—To the above tower stresses produced by live load and temperature, must be added the stresses due to wind loads.

The truss wind load of 400 lb. per lin. ft. produces a horizontal reaction at each tower of,

$$360\frac{l}{2}+400\frac{l_1}{2}=266 \text{ kips.}$$

This acts at Joint No. 4, ($x=100$).

The deflection of the stiffening truss under wind load produces a horizontal reaction at the top of each tower of $40\frac{l}{2}$; and the wind on the surface of the cables produces an addition to this reaction amounting to $10\left(\frac{l}{4}+\frac{l_1}{2}\right)$; hence the total reaction at the tower top $=26$ kips.

The wind acting directly on the tower is assumed at 25 lb. per sq. ft. of vertical elevation. This produces, at each joint, an equivalent concentrated load of $25\times(25d)$.

| Joint No. | $x$ ft. | $d$ ft. | Wind Load kips | Shear kips | Moment ft. kips | Column Dist. ft. | $A$ sq. in. | Stress from W. L. lb./sq. in. | Stress from L. L.+Temp. lb./sq. in. | Total Stress lb./sq. in. |
|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 4 | 26+1 | 0 | 0 | 42.5 | 280 | 0 | 12,400 | 12,400 |
| 1 | 25 | 4.5 | 3 | − 27 | − 675 | 43.5 | 295 | 100 | 13,000 | 13,100 |
| 2 | 50 | 5 | 3 | − 30 | − 1,425 | 44.5 | 310 | 100 | 13,200 | 13,300 |
| 3 | 75 | 5.5 | 3 | − 33 | − 2,250 | 46.5 | 325 | 100 | 13,500 | 13,600 |
| 4 | 100 | 6 | 266+4 | − 36 | − 3,150 | 48.5 | 340 | 200 | 13,500 | 13,700 |
| 5 | 125 | 6.5 | 4 | −306 | −10,800 | 49.5 | 355 | 600 | 13,400 | 14,000 |
| 6 | 150 | 7 | 4 | −310 | −18,550 | 50.5 | 370 | 1000 | 13,200 | 14,200 |
| 7 | 175 | 7.5 | 5 | −314 | −26,400 | 51.5 | 385 | 1300 | 12,900 | 14,200 |
| 8 | 200 | 8 | 5 | −319 | −34,375 | 52.5 | 400 | 1600 | 12,500 | 14,100 |
| 9 | 225 | 9 | 3 | −324 | −42,475 | 53.5 | 430 | 1800 | 11,600 | 13,400 |

The bending moments, divided by the column distance, give the column stresses, and these divided by the areas give the unit stresses from wind load.

The transverse bracing is proportioned to resist the shears tabulated above.

## EXAMPLE 4

### Estimates of Cable and Wrapping

**1. Calculation of Cable Wire.**—Given a suspension bridge in which a cable section of 70 sq. in. is to be provided. To find the material required for cables and wrapping. Other data as in Example 2.

The total length of each cable is given by Eq. (154):

$$L=l(1+\tfrac{8}{3}n^2)+2l_1\left(\sec\alpha_1+\frac{8}{3}\frac{n_1^2}{\sec^3\alpha_1}\right)$$

$$=1080(1.027)+720(1.034+.003)=1110+746=1856 \text{ ft.}$$

To this must be added 43 ft. of cable at each end, between end of truss span and anchorage eyebars (scaled from drawing); hence,

$$\text{Total } L=1856+86=1942 \text{ ft. per cable.}$$

No. 6 galvanized cable wire will be used $=0.195$ in. diameter $=$ .030 sq. in. area. Each cable consists of 7 strands of 336 wires each $=2352$ wires at .030 sq. in. $=70.5$ sq. in.

Weight of No. 6 galvanized wire $=0.1$ lb. per ft.

Total cable wire $=2\times 2352$ wires at 1942 ft. $=9,150,000$ lin. ft.

Total weight of cable wire $=9,150,000$ ft. at 0.1 lb. $=915,000$ lb.

**2. Calculation of Cable Diameter.**—The diameter $D$ of a cable, composed of $n$ wires of diameter $d$, is given by $D=K\cdot\sqrt{n}\cdot d$ where $K$ is the void constant. In practice, $K$ varies between 1.09 and 1.12, depending on the compacting. Using $K=1.11$, a cable of 2352 wires of 0.195 in. diameter will have a diameter of 10.5 inches. (Adding the thickness of wrapping, the finished cable will be 10.8 inches in diameter.)

**3. Calculation of Wrapping Wire.**—The wrapping consists of No. 9 galvanized wrapping wire (soft, annealed), weighing .06 lb. per ft. Deducting lengths of cable bands, etc., there will be 3250 ft. of cable to be wrapped. Since the wrapping wire is 0.15 in. diameter, it will make 80 turns per lin. ft. The diameter of the cable is 10.5 inches, hence the length of each turn will be 2.8 ft.

Length of wrapping wire = 80 turns at 2.8 ft.
= 224 ft. per lin. ft. of cable.

Weight of wrapping wire = 224 ft. at .06 lb.
= 13.44 lb. per lin. ft. of cable.

Total wrapping wire = 3250 ft. of cable at 13.44 lb.
= 44,000 lb.

**4. Estimate of Rope Strand Cables.**—Instead of building the cable of individual wires, manufactured rope strands may be used. In the case at hand, with a factor of safety of 3, there would be required sixty-one $1\frac{5}{8}$-inch strands per cable. These galvanized steel ropes weigh 4.34 lb. per ft.; hence the total weight in the cables would be,

$$2 \times 1942 \text{ ft.} \times 61 \text{ strands at } 4.34 \text{ lb} = 1{,}030{,}000 \text{ lb.}$$

The diameter of the resulting cable would be $7 \times 1\frac{5}{8}$-in. = 11.4 in., plus the wrapping. (If rope strands are used, it should be remembered that their modulus of elasticity $E$ is only about 20,000,000, as compared with 27,000,000 for parallel wire cables.)

## EXAMPLE 5

## Analysis of Suspension Bridge with Continuous Stiffening Truss (Type 0S)

(See Chap. I.. Pages 53 to 63.)

**1. Dimensions.**—The following dimensions are given:

$$l = \text{Main Span} = 40 \text{ panels at } 17'\ 7\tfrac{1}{2}'' = 705 \text{ ft.} \quad (l' = l).$$

$$l_2 = l_1 = \text{Side Span} = 10 \text{ panels at } 17'\ 7\tfrac{1}{2}'' = 176.25 \text{ ft.} \quad \left(r = \frac{l_1}{l} = \frac{1}{4}\right).$$

$$f = \text{Cable sag in Main Span} = 74.285 \text{ ft.} \quad \left(n = \frac{f}{l} = \frac{1}{9.5}\right).$$

$f_1$ = Cable-sag in Side Spans = 4.65 ft.

$$\left(n_1 = \frac{f_1}{l_1} = \frac{1}{38}\right). \quad \left(v = \frac{f_1}{f} = \frac{1}{16}\right).$$

$d$ = Depth of Truss = 15.′0 at towers, 10.′833 at center, 11.′346 at ends.

Width, center to center of trusses or cables = 27 ft.

$I$ = Mean Moment of Inertia in main span = 1642 in.$^2$ ft.$^2$ (per truss).

$I_1$ = Mean Moment of Inertia in side spans = 2278.

$$\left(i = \frac{I}{I_1} = 0.72\right).$$

$A$ = Cable Section in main span = 7 strands of 282 wires at 0.192 in. diameter = 57.2 sq. in. per cable.

$A_1$ = Cable Section in side spans = $A$, + 2 strands of 76 wires = 61.6 sq. in. per cable.

tan $\alpha$ = Slope of Cable Chord in main span = .026.

tan $\alpha_1$ = Slope of Cable Chord in side spans = 0.5.

$$e = \text{Coefficient of Continuity} = \frac{2+2irv}{3+2ir} = 0.602$$

**2. Stresses in Cables.**—(All values per cable).

For dead load ($w$ = 2850 lb. per lin. ft. per cable), the horizontal tension is given by Eq. (11):

$$H = \frac{wl^2}{8f} = \frac{9.5}{8}wl = 2380 \text{ kips per cable.}$$

The denominator for other values of $H$ is given by Eq. (203):

$$N = \tfrac{8}{5} - 4e + 3e^2 + 2ir(\tfrac{8}{5}v^2 + e^2 - 2ev)$$
$$+ \frac{3I}{Af^2}\cdot\frac{l'}{l}(1+8n^2) + \frac{6I}{A_1f^2}\cdot\frac{l_2}{l}\cdot\sec^3\alpha_1(1+8n_1^2) = 0.413.$$

With the live load ($p$ = 750 lb. per lin. ft. per cable) covering the main span, Eq. (206) gives,

$$H = \frac{1}{Nn}\left(\frac{1}{5} - \frac{e}{4}\right)\cdot pl = \frac{.470}{N}pl = 600 \text{ kips per cable.}$$

With the live load ($p_1 = 750$ lb. per lin. ft. per cable) covering both side spans, Eq. (207) gives,

$$H = \frac{2ir^3}{N \cdot n}\left(\frac{v}{5} - \frac{e}{8}\right) \cdot p_1 l = -\frac{.0134}{N} \cdot p_1 l = -17 \text{ kips.}$$

The temperature factor is given by Eq. (154′):

$$\frac{L_t}{l} = (1 + \tfrac{16}{3}n^2) + 2\frac{l_1}{l}(\sec^2 \alpha_1 + \tfrac{16}{3}n_1^2) = 1.687.$$

Substituting this value in Eq. (214), we obtain the cable tension produced by temperature variation ($t = \pm 60°$ F., $E\omega t = 11{,}720$):

$$H_t = -\frac{3EI\,\omega t L_t}{f^2 \cdot N \cdot l} = \mp 43 \text{ kips per cable.}$$

Combining the values for dead load, main-span live load, and fall in temperature, we obtain,

$$\text{Max. } H = 3023 \text{ kips per cable.}$$

In the main span, the maximum slope of the cable is $\tan \phi = \tan \alpha + 4n = .447$; $\sec \phi = 1.096$. For this slope, the stress in the cable is, by Eq. (5),

$$\text{Max. } T = H \cdot \sec \phi = 3314 \text{ kips.}$$

At 60,000 lb. per sq. in., the cable section required is $3314 \div 60 = 55.2$ sq. in. (The section provided is $A = 57.2$ sq. in.)

In the side spans, the maximum slope of the cable is $\tan \phi_1 = \tan \alpha_1 + 4n_1 = .605$; $\sec \phi_1 = 1.17$. For this slope, the stress in the cable is, by Eq. (5),

$$\text{Max. } T_1 = H \cdot \sec \phi_1 = 3538 \text{ kips.}$$

At 60,000 lb. per sq. in., the cable section required is $3538 \div 60 = 58.8$ sq. in. (The section provided is $A_1 = 61.6$ sq. in.)

**3. Influence Line for *H*.**—For a concentration $P$ traversing the main span, the values of $H$ are given by Eq. (204):

$$H = \frac{1}{N \cdot n}[B(k) - \tfrac{5}{2}e(k - k^2)] \cdot P.$$

Taking the values of $B(k)$ from Table I or Fig. 12, we obtain the following main-span influence ordinates for $H$:

| Load Position | Ordinate $\frac{H}{P}$ |
|---|---|
| $k=0$ | 0 |
| 0.1 | .386 |
| 0.2 | .945 |
| 0.3 | 1.484 |
| 0.4 | 1.861 |
| 0.5 | 1.994 |

For a concentration $P_1$ traversing either side span, the values of $H$ are given by Eq. (205):

$$H=\frac{ir^2}{N\cdot n}\left[v\cdot B(k_1)-\frac{e}{2}(k_1-k_1^3)\right]\cdot P_1.$$

where $k_1l_1$ is measured from the free end of the span. Substituting the values of $B(k_1)$, we obtain the following side-span influence ordinates for $H$:

| Load Position | Ordinate $\frac{H}{P_1}$ |
|---|---|
| $k_1=0$ | 0 |
| 0.2 | −.048 |
| 0.4 | −.085 |
| 0.6 | −.100 |
| 0.8 | −.078 |
| 1.0 | 0 |

**4. Bending Moments in Main Span.**—The bending moments will be obtained by the method of unit loads applied at successive panel points, using Eq. (182):

$$M=\left(M_0+\frac{l-x}{l}\cdot M_1+\frac{x}{l}\cdot M_2\right)-H(y-ef)=M'-H(y-ef).$$

In this case, $ef=44.7$ ft., and the values of $(y-ef)$ are as follows:

| Panel Point | $\frac{x}{l}$ | $y-ef$ |
|---|---|---|
| No. 20 | 0 | −44.7 ft. |
| 16 | .1 | −18.0 |
| 12 | .2 | + 2.8 |
| 8 | .3 | +17.6 |
| 4 | .4 | +26.5 |
| 0 | .5 | +29.7 |

NOTE.—In this bridge, the panel points were numbered consecutively in both directions, starting with No. 0 at the middle of the span; No. 20 is at the towers, and No. 30 at the free ends.

The value of $H$ is a constant for each load position and is taken from the influence table figured above.

For each load position, the moments $M_1$ and $M_2$ at the towers are given by Eqs. (190) and (191):

$$M_1 = -Pl\cdot k(1-k)\frac{(3+2ir)(1-k)+2ir}{(3+2ir)(1+2ir)}$$

$$= -[55.5+518(1-k)]P\cdot k(1-k).$$

$$M_2 = -Plk(1-k)\frac{(3+2ir)k+2ir}{(3+2ir)(1+2ir)}$$

$$= -[55.5+518k]P\cdot k(1-k).$$

The bending moment $M'$ at the section carrying the load is,

$$M' = M_0 + M_1(1-k) + M_2k = Pk(1-k)l + M_1(1-k) + M_2k.$$

Using the three above equations, we obtain the following controlling values of $M'$ for a unit load $P=1$.

| Load Position | $M_1$ at Near Tower | $M'$ at Load | $M_2$ at Far Tower |
|---|---|---|---|
| $k=0$ | 0 | 0 | 0 |
| .1 | −47.0 | +20.2 | − 9.7 |
| .2 | −75.3 | +47.5 | −25.5 |
| .3 | −87.9 | +73.2 | −44.3 |
| .4 | −88.0 | +91.2 | −63.1 |
| .5 | −78.7 | +97.6 | −78.7 |

These values give the three vertices of the equilibrium triangle; and, for each load position, the values of $M'$ for other sections may be tabulated by straight-line interpolation. Subtracting from each value of $M'$ the corresponding value of $H(y-ef)$, we obtain the unit-load bending moments $M$. A typical tabulation for this computation is as follows:

UNIT LOAD AT PANEL POINT 12. ($k=0.2$). ($H=.945$)

| Panel Point | $M'$ | $H(y-ef)$ | $M$ |
|---|---|---|---|
| No. 20 | −75.3* | −42.3 | −33.0 |
| 16 | −13.8 | −17.0 | + 3.2 |
| 12 | +47.5* | + 2.6 | +44.9 |
| 8 | +38.4 | +16.7 | +21.7 |
| 4 | +29.3 | +25.1 | + 4.2 |
| 0 | +20.2 | +28.1 | − 7.9 |
| 4 | +11.0 | +25.1 | −14.1 |
| 8 | + 1.9 | +16.7 | −14.8 |
| 12 | − 7.2 | + 2.6 | − 9.8 |
| 16 | −16.3 | −17.0 | + 0.7 |
| 20 | −25.5* | −42.3 | +16.8 |

With the left side-span completely loaded (unit load at each panel point), Eqs. (197), (198) and (207) give:

$$M_1=-\frac{p_1l^2}{4}\cdot\frac{2ir^3(1+ir)}{(3+2ir)(1+2ir)}=-.001453p_1l^2=-41.0$$

$$M_2=+\frac{p_1l^2}{4}\cdot\frac{ir^3}{(3+2ir)(1+2ir)}=+.000616p_1l^2=+17.4.$$

$$H=\frac{ir^3}{Nn}\left(\frac{v}{5}-\frac{e}{8}\right)\cdot p_1l=-.0017\frac{p_1l^2}{f}\qquad=-\ 0.645.$$

The resulting bending moments in the main span will be, by Eq. (182),

$$M=\frac{l-x}{l}M_1+\frac{x}{l}\cdot M_2-H(y-ef),$$

and are obtained by a tabulation similar to the one above.

The influence values of $M$ obtained in the series of tabulations just described may be summarized as follows (only every fourth panel point shown here):

INFLUENCE VALUES OF $M$ FOR UNIT LOADS

| Load Position | $M$ at Panel Point | | | | | |
|---|---|---|---|---|---|---|
| | 20 | 16 | 12 | 8 | 4 | 0 |
| Panel point No. 20..... | 0 | 0 | 0 | 0 | 0 | 0 |
| 16...... | − 29.7 | + 27.1 | + 15.8 | + 6.7 | − 0.1 | − 4.6 |
| 12...... | − 33.0 | + 3.2 | + 44.9 | + 21.7 | + 4.2 | − 7.9 |
| 8..... | − 21.5 | − 7.5 | + 15.4 | + 47.1 | + 17.1 | − 4.4 |
| 4...... | − 4.8 | − 9.7 | − 3.6 | + 13.6 | + 41.8 | + 10.2 |
| 0...... | + 10.5 | − 7.5 | − 13.7 | − 8.1 | + 9.4 | + 38.4 |
| 4...... | + 20.1 | − 3.9 | − 16.8 | − 18.8 | − 9.6 | + 10.2 |
| 8..... | + 22.0 | − 0.8 | − 14.8 | − 20.1 | − 16.5 | − 4.4 |
| 12...... | + 16.8 | + 0.7 | − 9.9 | − 14.8 | − 14.1 | − 7.9 |
| 16...... | + 7.6 | + 0.6 | − 4.1 | − 6.5 | − 6.7 | − 4.6 |
| 20..... | 0 | 0 | 0 | 0 | 0 | 0 |
| Left side span......... | − 69.3 | − 46.4 | − 27.2 | − 11.9 | − 0.4 | + 7.4 |
| Right side span...... | − 11.6 | − 0.2 | + 7.4 | + 11.2 | + 11.1 | + 7.4 |
| Maximum $M$.......... | +308.1 | +108.7 | +292.0 | +344.0 | +277.3 | +223.7 |
| Minimum $M$...... .. | −450.7 | −163.7 | −278.5 | −282.3 | −186.2 | −129.1 |

Max. $M$ is the summation of all positive influence values, and Min. $M$ is the summation of all negative influence values.

These results are multiplied by the panel load $\left(P=\frac{pl}{40}=13.22\right.$ kips) to obtain the bending moments in foot-kips; and the latter values are divided by the truss depths at the respective panel points to obtain the chord stresses in kips.

The temperature moments are given by Eq. (215):

$$M_t=-H_t(y-ef),$$

where $H_t=\mp 43$ kips; the values of $(y-ef)$ have been tabulated above. These moments are combined with the live-load moments as the specifications may prescribe.

**5. Shears in Main Span.**—The shears in the main span are given by Eq. (188):

$$V=V_0+\frac{M_2-M_1}{l}-H(\tan\phi-\tan\alpha)=V'-H(\tan\phi-\tan\alpha).$$

The method of unit loads will be used. The values of $H$, $M_2$ and $M_1$ have been calculated above for different load positions.

| Load Position | $V_0$ | $\frac{M_2-M_1}{l}$ | $V'$ (to left of load) | $V'$ (to right of load) | $H$ |
|---|---|---|---|---|---|
| Panel point No. 20... | 1.0 | 0 | +1.0 | 0 | 0 |
| 16.. | .9 | .053 | + .953 | −.047 | .386 |
| 12... | .8 | .071 | + .871 | −.129 | .945 |
| 8... | .7 | .062 | + .762 | −.238 | 1.484 |
| 4... | .6 | .035 | + .635 | −.365 | 1.861 |
| 0.. | .5 | 0 | + .500 | −.500 | 1.994 |

The value of $(\tan\phi - \tan\alpha)$ decreases uniformly from $4n = .42$ at the left tower (Panel Pt. No. 20) to 0 at the center (Panel Pt. No. 0) and to $-4n = -.42$ at the right tower (Panel Pt. No. 20).

Substituting these values in the above formula for $V$, an influence table for shears is constructed, similar to the preceding influence table for moments. (Only every fourth panel point shown here):

INFLUENCE VALUES OF $V$ FOR UNIT LOADS

| Load Position | $V$ at Panel Point | | | | | |
|---|---|---|---|---|---|---|
| | 20 | 16 | 12 | 8 | 4 | 0 |
| Panel point No. 20..... | +1.00 | 0 | 0 | 0 | 0 | 0 |
| 16...... | + .79 | − .18 | − .15 | − .11 | − .08 | − .05 |
| 12...... | + .47 | + .55 | − .37 | − .29 | − .21 | − .13 |
| 8..... | + .14 | + .26 | + .39 | − .49 | − .36 | − .24 |
| 4..... | − .15 | + .01 | + .16 | + .32 | − .52 | − .37 |
| 0..... | − .34 | − .17 | 0 | + .16 | + .33 | ± .50 |
| 4..... | − .42 | − .26 | − .11 | + .05 | + .21 | + .37 |
| 8..... | − .39 | − .26 | − .14 | − .01 | + .11 | + .24 |
| 12...... | − .27 | − .19 | − .11 | − .03 | + .05 | + .13 |
| 16...... | − .12 | − .08 | − .05 | − .02 | + .02 | + .05 |
| 20...... | 0 | 0 | 0 | 0 | 0 | 0 |
| Left side span.......... | + .35 | + .30 | + .24 | + .19 | + .13 | + .08 |
| Right side span......... | + .19 | + .14 | + .08 | + .03 | − .03 | − .08 |
| Maximum $V$........... | +7.56 | +5.76 | +4.07 | +3.58 | +4.17 | +4.42 |
| Minimum $V$........... | −6.68 | −4.34 | −3.12 | −3.00 | −3.90 | −4.42 |

Max. $V$ is the summation of all positive influence values, and Min. $V$ is the summation of all negative influence values. These results are multiplied by the panel load $\left(P=\frac{pl}{40}=13.22 \text{ kips}\right)$ to obtain the vertical shears in kips.

The temperature shears are given by Eq. (217):

$$V_t=-H_t(\tan\phi-\tan\alpha).$$

The shears are then multiplied by the respective secants of inclination, to obtain the stresses in the web members of the stiffening truss.

**6. Bending Moments in Side Spans.**—The bending moments in the side spans are obtained by the method of unit loads, using Eq. (183):

$$M=M_0+\frac{x_1}{l_1}M_1-H\left(y_1-\frac{x_1}{l_1}\cdot ef\right)=M'-Hy'.$$

For loads in the main span, $M_0=0$, and the values of $M_1$ and $M_2$ are the same as calculated above. For the far side span completely loaded, $M_0=0$, and the value of $M_1$ is the same as the value of $M_2$ calculated above. For unit loads ($P=1$) in the given side span, the moment $M_1$ is given by Eq. (194):

$$M_1=-Pl\frac{2ir^2(1+ir)(k_1-k_1^3)}{(3+2ir)(1+2ir)}=-16.4P(k_1-k_1^3),$$

and

$$M_0=Pl_1(k_1-k_1^2).$$

The values of $H$ will be the same as calculated above. Accordingly, we have the following values for a unit load ($P=1$) traversing the side span.

| Load Position | $k_1$ | $M'$ | $H$ |
|---|---|---|---|
| Panel point No. 20...... | 1.0 | 0 | 0 |
| 22...... | .8 | +24.4 | −.078 |
| 24...... | .6 | +38.5 | −.100 |
| 26...... | .4 | +40.1 | −.085 |
| 28...... | .2 | +27.6 | −.048 |
| 30...... | 0 | 0 | 0 |

The values of $y' = y_1 - \frac{x_1}{l_1} \cdot ef$ are as follows:

| Panel Point: | 20 | 22 | 24 | 26 | 28 | 30 |
|---|---|---|---|---|---|---|
| $\frac{x_1}{l_1} =$ | 1.0 | .8 | .6 | .4 | .2 | 0 |
| $y' =$ | −44.7 | −32.8 | −22.4 | −13.4 | −6.0 | 0 |

Substituting the various values in the equation:

$$M = M' - H \cdot y',$$

we obtain the following influence table for side-span moments (only every second panel point shown here):

INFLUENCE VALUES OF $M$ FOR UNIT LOADS

| Load Position | $M$ at Panel Point | | | | | | |
|---|---|---|---|---|---|---|---|
| | 20 | 21 | 22 | 24 | 26 | 28 | 30 |
| Panel point No. 20 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | − 8.2 | + 6.9 | + 21.9 | + 16.6 | + 11.2 | + 5.7 | 0 |
| 24 | − 10.8 | + 1.1 | + 12.8 | + 36.3 | + 24.3 | + 12.2 | 0 |
| 26 | − 9.3 | − 1.2 | + 6.9 | + 23.0 | + 39.0 | + 19.5 | 0 |
| 28 | − 5.3 | − 1.2 | + 3.0 | + 11.1 | + 19.3 | + 27.3 | 0 |
| 30 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Maximum $M$ | 0 | + 23 | + 88 | +173 | +187 | +126 | 0 |
| Minimum $M$ | − 69 | − 5 | 0 | 0 | 0 | 0 | 0 |
| Far side span | − 12 | − 10 | − 7 | − 4 | − 2 | 0 | 0 |
| Main span | +308 | +231 | +167 | + 76 | + 25 | + 4 | 0 |
| | −370 | −363 | −352 | −316 | −252 | −154 | 0 |
| Total Maximum $M$ | +308 | +254 | +254 | +249 | +212 | +130 | 0 |
| Total Minimum $M$ | −451 | −378 | −359 | −320 | −254 | −154 | 0 |

The above results are to be multiplied by the panel load $\left(P = \frac{pl}{40} = \frac{pl_1}{10} = 13.22 \text{ kips}\right)$ to obtain the maximum and minimum bending moments in the side span.

The temperature moments are calculated by Eq. (216):

$$M_t = -H_t\left(y_1 - \frac{x_1}{l_1}\cdot ef\right) = -H_t \cdot y',$$

where $H_t = \mp 47$ kips.

**7. Shears in Side Spans.**—The left side-span shears are calculated by Eq. (189):

$$V = \left(V_0 + \frac{M_1}{l_1}\right) - H\left(\tan\phi_1 - \tan\alpha_1 - \frac{ef}{l_1}\right) = V' - K\cdot H.$$

At the tower (Panel Point 20), $K = -4n_1 - \frac{ef}{l_1} = -.105 - .254 = -.359$; and the value of $K$ diminishes uniformly to $K = +4n_1 - \frac{ef}{l_1} = +.105 - .254 = -.149$ at the free end (Panel Point 30).

Substituting in the above formula the known values of $H$, $M_1$ and $K$, we obtain the following table of influence ordinates for side-span shears (only every second panel point shown here):

INFLUENCE VALUES OF $V$ FOR UNIT LOADS

| Load Position | V at Panel Point | | | | | |
|---|---|---|---|---|---|---|
| | 20 | 22 | 24 | 26 | 28 | 30 |
| Panel point No. 20 | 0 | 0 | 0 | 0 | 0 | 0 |
| 22 | −.86 | −.85 | +.15 | +.16 | +.16 | +.16 |
| 24 | −.67 | −.67 | −.66 | +.34 | +.35 | +.35 |
| 26 | −.46 | −.46 | −.45 | −.45 | +.55 | +.56 |
| 28 | −.24 | −.23 | −.23 | −.23 | −.23 | +.78 |
| 30 | 0 | 0 | 0 | 0 | 0 | +1.00 |
| Maximum $V$ | 0 | +0.2 | +0.8 | +1.8 | +3.3 | +4.2 |
| Minimum $V$ | −5.0 | −4.0 | −2.4 | −1.1 | −0.3 | 0 |
| Far side span | −0.1 | −0.1 | −0.1 | −0.1 | 0 | 0 |
| Main span | +5.4 | +3.8 | +2.4 | +1.2 | +0.4 | 0 |
| | −0.9 | −1.3 | −1.8 | −2.5 | −3.6 | −5.1 |
| Total Maximum $V$ | +5.4 | +4.0 | +3.2 | +3.0 | +3.6 | +4.2 |
| Total Minimum $V$ | −6.0 | −5.4 | −4.2 | −3.7 | −3.9 | −5.1 |

These results are to be multiplied by the panel load ($P = 13.22$ kips) to obtain the maximum and minimum shears in kips.

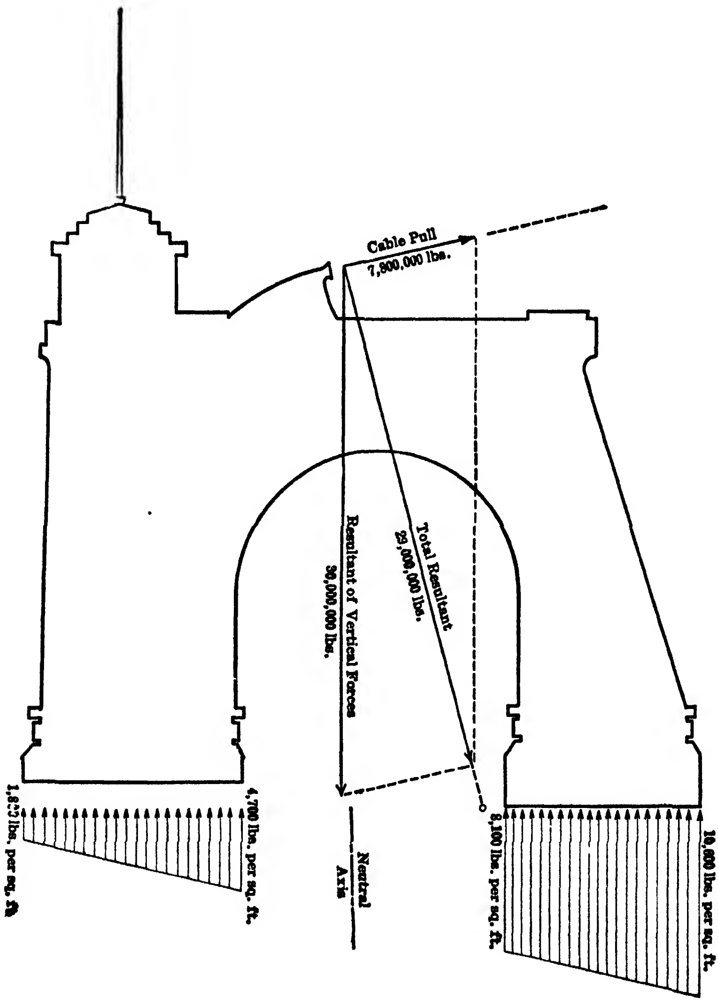



FIG. 44.—Design of Anchorage.

For the right side span, the shears will be the same, with the signs changed.

The temperature shears are given by Eq. (218):

$$V_t = -H_t \cdot \left(\tan \phi_1 - \tan \alpha_1 - \frac{ef}{l_1}\right) = -K \cdot H_t.$$

The shears are then multiplied by the respective secants of inclination, to obtain the stresses in the web members.

### EXAMPLE 6

### Design of Anchorage

**1. Stability against Sliding.**—The outline of a design for a reinforced concrete anchorage is shown in Fig. 44.

The principal forces acting are the cable pull, and the weight of the anchorage (including any superimposed loads). In the case at hand, the cable pull $= H \cdot \sec \phi = 7800$ kips. The weight of the anchorage and the superimposed loads is 30,000 kips. This weight is represented in the diagram as a vertical force drawn through the center of gravity of the anchorage and applied loads. By a parallelogram of forces, the total resultant is found, amounting to 29,000 kips. If its inclination from the vertical is less than the angle of friction, the anchorage is safe against failure by sliding.

**2. Stability against Tilting.**—The resultant is prolonged to intersection with the plane of the base, and its vertical component ($V = 28{,}000$ kips) is considered as an eccentric load applied at the point of intersection. The toe and heel pressures are given by,

$$p = \frac{V}{A} \pm \frac{Vec}{I},$$

where $A$ is the area of the base (sq. ft.), $I$ is its moment of inertia about the neutral axis (ft.$^4$), $e$ is the distance (ft.) of the resultant $V$ from the neutral axis, and $c$ is the distance (ft.) from the neutral axis to the respective extreme fiber. We thus obtain, for the case at hand, a toe pressure of 10.6 kips per sq. ft. and a heel pressure of 1.8 kips per sq. ft. The allowable foundation pressure was 6 tons per sq. ft., so the anchorage figures safe against settlement or overturning.

## CHAPTER IV

## ERECTION OF SUSPENSION BRIDGES

**1. Introduction.**—The erection of suspension bridges is comparatively simple, and is free from dangers attending other types of long span construction.

The normal order of erection is: substructure, towers and anchorages, footbridges, cables, suspenders, stiffening truss and floor system, roadways, cable wrapping.

The cables are the only members requiring specialized knowledge for their erection. The other elements of the bridge, for the most part, are erected in accordance with the usual field methods for the corresponding elements of other structures.

**2. Erection of the Towers.**—The erection of the towers may proceed simultaneously with the construction of the anchorages.

In the case of the Manhattan Bridge, the tower (Fig. 45) consists of four columns supported on cast-steel pedestals resting on base plates set directly on the masonry pier. The sections of the pedestals (weighing up to 40 tons) were delivered by lighters and lifted by their derricks to the pier-tops; they were rolled into position on cast-steel balls placed on the bed plate, and then jacked up to release the balls.

The tower columns were erected by the use of ingenious derrick platforms (one for each pair of columns) adapted to travel vertically up the tower as the erection proceeded (Fig. 45). Each platform (21 feet by 34 feet) projected out from the face of the tower on the shore side and was supported by two bracket-struts below. The tipping moment was resisted by two pairs of rollers or wheels, one at each column, engaging vertical edges of the projecting middle portion of the column, the upper wheel being on the river side and the lower wheel on the shore side. The vertical support was furnished by hooks engaging the projecting gusset plates of the bracing system. A stiff-leg derrick with 45-foot steel boom was mounted at the middle of the inner side of the platform, being braced back to the outer corners of the platform. With this derrick the sections of the tower (weighing up to 62 tons) were lifted from the top of the pier and set in place, the material having been transferred from scows to the pier by floating derricks. When a full section had been added to the tower, blocks were fastened to the top and falls

FIG. 45.—Manhattan Bridge. Erection of Towers.

(See Fig. 35, page 97).

attached to the derrick platform by which it then lifted itself to the next level.

For purposes of handling and erection, each column was divided by transverse and longitudinal field splice joints into sections of convenient size. The transverse joints were 12 feet to $27\frac{1}{2}$ feet apart, and were staggered to break joint. Where the three longitudinal sections changed to two, shim plates were used to level off. The riveting of the field splices (with 1-inch rivets) was kept several sections back of the erection work in order to give opportunity for the transverse joints to come to full bearing.

Each tower column was finished with a cap section (52 tons) upon which was set the saddle (15 tons).

In addition to the two traveling derricks, the following equipment was required for the erection of each tower: two hoisting engines on the pier; one stiff-leg derrick (10-tons, 60-foot boom) on the pier between the tower legs, used in the assembly of the traveling derricks; two large storage scows moored to the pier, supplying the respective traveling derricks; a power plant on shore with two 50-H.P. horizontal boilers, a steam turbine blower for forced draft, and an air compressor; 30 pneumatic riveting hammers; 6 pneumatic forges.

The force at each tower consisted of a hundred men, including six riveting gangs. Riveting scaffolds were erected around the tower for field riveting, and were provided with stairs and safety railings. The erection record was 2000 tons of steel at one tower in sixteen working days.

Figure 46 shows the completed tower, 282 feet high above the masonry, and weighing 12,500,000 pounds.

For the Manhattan tower of the Williamsburg Bridge, a stationary derrick on the approach falsework was used to erect the steel up to roadway level; the erection was then completed by two stiff-leg derricks mounted on a timber tower built up on the cross-girder between the two tower legs. (The completed tower is shown in Fig. 57.)

For smaller bridges, the towers may be erected by gin-pole or by stationary derrick alongside. For the suspension bridge at Kingston, N. Y. (H. D. Robinson, Chief Engineer), a guyed derrick with 95-foot steel boom was set up on a square timber tower 80 feet high, for the erection of each steel tower; the same derricks later erected the adjoining panels of the stiffening truss (Fig. 56).

**3. Stringing the Footbridge Cables.**—The first step in cable erection consists in establishing a connection between the two banks. Various methods have been used since prehistoric times, when the first thread was fastened to an arrow and shot across from bank to bank. In building the Niagara bridge, a kite was used to take the first string across the gorge; at other places, a light rope is drawn across with a rowboat.

In the erection of the Brooklyn Bridge, a $\frac{3}{4}$-inch wire rope was first laid across the East River by means of a tugboat and scow, and then raised to position. With another line taken over in the same manner, an endless rope was made, and this was used for hauling over the remaining traveler ropes and an

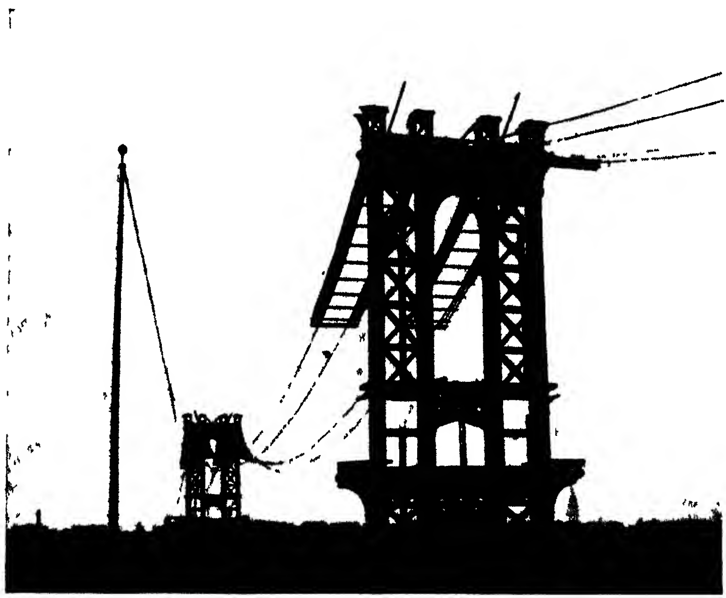


FIG. 46.—Manhattan Bridge. Erection of Footbridges.

auxiliary $1\frac{3}{4}$-inch carrier rope; the latter served to carry the load of the footbridge cables and cradle cables ($2\frac{3}{8}$ and $2\frac{5}{8}$ inches diameter) when these were hauled across the river.

For the Manhattan Bridge, sixteen $1\frac{3}{4}$-inch wire ropes were swung between the towers in four groups of four (Fig. 46). One group (to make a single footbridge cable) was taken across at a time. The four reels were mounted on a scow brought alongside one of the towers, *A*. The end of each rope was unreeled and hauled up over a temporary cast-iron roller saddle mounted on top of the tower, and thence carried back to the anchorage *A*, where it was made fast. Then the scow was towed across the river, laying the ropes along the bottom, to the opposite tower *B*. The remainder of each rope was then unreeled and coiled on the deck of the scow. Then, while all river traffic was stopped for a few minutes, the free end of each rope was hauled up by a line to the top of Tower *B*, over a roller saddle and thence to the Anchorage *B*; the middle of the rope, or bight, rose out of the water during this operation, and came up to the desired position. After being made fast at the Anchorage *B*, the ropes were socketed and drawn up to the precise deflection desired, as determined by levels. Each group of four ropes then formed a temporary cable for the support of the footbridges (Fig. 46).

The footbridge ropes for the Williamsburg Bridge were strung in the same manner as for the Manhattan Bridge, except that three were laid, instead of four, at each trip of the flatboat across the river.

**4. Erection of Footbridges.**—The next step is the construction, for each cable, of a footbridge or working platform which permits the wires to be observed and regulated throughout their length and greatly facilitates the entire work on the cables.

For the Manhattan Bridge (Fig. 46), traveling cages, hanging from the footbridge cables as a track, were used by the men placing the double cantilever floorbeams. These floorbeams, 35½ feet long and spaced 21 feet apart, were supported on the footbridge cables in pairs, and were secured to the upper side of these cables by U-bolt clamps. Upon the outer portions of the floorbeams were dapped the stringers, three lines on each side, and on these were spiked the floorboards, spaced 1½ inches apart in the clear. In this manner, four platforms were constructed, 8 feet wide, placed concentric with the main cables and 30 inches (clear) below them. The platforms were provided with wire-rope handrails. Passage from one platform to another was provided only at the towers and anchorages, and at mid-span. Each platform carried nine small towers called "hauling towers" (Fig. 47), about 250 feet apart, to support the sheaves of the carrying and hauling ropes used for placing the strand wires. The platforms were braced and guyed underneath by backstays from each tower, and by inverted storm cables connected to them at intervals of 54 feet. The entire construction was of light wooden plank (maximum size $3 \times 12$) and all connections were thoroughly bolted with washer bearings. All woodwork had been previously cut to length, framed, bored and marked, and hoisted to the tops of the towers. The floorbeams were slipped down on the cables toward the center of the span

FIG. 47.—Manhattan Bridge. Footbridges and Sheave Towers.

and toward the anchorages, set by the men in the traveling cages, and maintained in position by the stringers dapped to them. Then the sheave towers and handrails were erected on the platforms, practically completing the falsework (Fig. 47).

The temporary platforms for the Williamsburg Bridge are shown in Fig. 57. Two footbridges were used, 67 feet center to center, connected by transverse bridges every 160 feet.

For the Brooklyn Bridge, Fig. 25, the timber staging consisted of one longitudinal footbridge and five transverse platforms, called "cradles," from which the wires were handled and regulated during cable-spinning.

**5. Parallel Wire Cables.**—Smaller spans have been built with ready-made parallel wire cables, served with wire wrapping at short intervals; but the individual wires in such cables lack freedom to adjust themselves to the necessary curvature of suspension, so that objectionable stress conditions arise. For these reasons it has become general practice to use the method, introduced by Roebling, of spinning the desired number of parallel wires in place and then combining them into a cable. The cable is pressed into cylindrical form and wound with continuous wire wrapping. This wrapping, together with the tight cable bands to which the suspenders are attached, serves to create enough friction pressure between the wires to ensure united stress action.

Guide wires are used as a means of adjusting the individual wires to equal length. Slight differences in length, if distributed over the entire span, will be immaterial. To avoid the excess in length of the longer wires from accumulating at a single point, the wire wrapping should be started at a considerable number of points distributed along the cable; and a large cable should first be bound into smaller temporary strands by serving with wire at intervals.

The length of the guide wire must be accurately computed, so that the resulting cable shall have the desired sag (assumed in the design) after the bridge is completed. Length corrections must be made for any cradling of the cables, variation from mean temperature, the curve of the cable saddle, and the elastic elongation due to the suspended load.

**6. Initial Erection Adjustments.**—Special computations have to be made for the location of the guide wires, for setting the saddles on top of the towers, and for the length of the strand legs.

When the desired final position of the cable, under full dead load, is known, its length is carefully computed, including the main span parabola between points of tangency at the saddles, the short curved portions in the saddles, and the side-span parabolas or tangents from point of tangency at the saddle to the center of shoe pin at the anchorage. Applying corrections for elastic elongation (due to suspended load) and for difference of temperature from the assumed mean, the length of unloaded cable is determined. This gives the length of the guide wire between the same points.

Assuming no slipping of the strands in the saddles, the initial position of the saddles is computed so as to balance tensions (or values of $y$) between the main and side span catenaries. This gives the distance the saddles must be set back (toward shore) from their final position on the tops of the towers.

Since the strands will be spun about 2 feet above their final position in the tower saddles, the initial position of the strand shoes will be a short distance forward of their final position. This distance is carefully computed and gives the required length of the "strand legs" (Fig. 48). The distance may also be determined or checked by actual trial with the guide wire.

Taking into consideration the previously calculated and corrected total length of cable between strand shoes, the initial raised position of the strands above the tower saddles, and the length of strand legs shifting the initial position of the strand shoes, the ordinates of the initial catenaries in main and side spans are carefully computed. These ordinates are used for setting the guide wires with the aid of level and transit stationed at towers and anchorages.

For the Cumberland River footbridge (540-foot span, see page 184), the saddles on the two towers were set back about 5 inches toward shore from center of tower. This distance was figured from backstay elongation and tower shortening due to dead load plus one-half live load, so that the center of the tower would bisect the movement of the shoe (on rollers) for live load at mean temperature. Allowing for displacement of saddles and cable stretch, the no-load cable-sag was made 38½ feet in order that the sag in final position under full live load should be 45 feet.

In the case of the Brooklyn Bridge, the strands were spun about 57 feet above their final position at mid-span, the purpose, as stated, being to avoid interference with regulation and to increase the tension as an initial strength-test of the individual wires. In consequence, the strand leg had to be designed so as to hold the shoe 12 feet *behind* the anchor pin. After the strand was finished, the shoes were let forward into their final places and, at the same time, the strand was lowered from the rollers on top of the saddle into the saddle, which double operation caused the vertex to sink into correct position as previously calculated.

For the Williamsburg Bridge, the strands were spun 15 feet above their final position, requiring the shoes to be initially set *back* of the anchor pin, as in the Brooklyn Bridge.

For the Manhattan and Kingston Bridges (H. D. Robinson, Engineer-in-charge), the strands were spun parallel to (and slightly above) their final position. In these cases, the strand leg held the shoe a short distance *in front* of the anchor pin; and the shoe had to be pulled back that distance when the strands were lowered into the saddle (Fig. 49).

In the case of the Kingston Bridge (705-foot span, Fig. 56), instead of setting the saddles back on the towers, the tops of the towers were tipped back toward the shore a distance of 6 inches, by means of temporary backstays, the anchor bolts at the toe being loosened ½ inch to permit the tilting. The cables were erected with the towers and the attached saddles in this position. As the steelwork in the main span was erected, the backstays gradually elongated until the towers returned to their final vertical position.

The initial erection adjustments for the Brooklyn, Williamsburg, Manhattan and Kingston Bridges are summarized and compared in the following table:

Initial Position of Cable Strands
(With Reference to Final Position)

| | Height Above Crown | Height Above Saddle | Distance Saddle Set Back | Distance Shoe Set Forward |
|---|---|---|---|---|
| Brooklyn | 57 ft. | 2.1 ft. | 0.1 ft. | −12 ft. |
| Williamsburg | 15 | 2 | 2.75 | − 3 |
| Manhattan | 2 | 2 | 0 | + 1.83 |
| Kingston | 1.25 | 1.25 | 0.5 | + 0.25 |

**7. Spinning of Cables.**—The operation of cable-spinning requires an endless wire rope or "traveling rope" (Fig. 48) suspended across the river and driven back and forth by machinery

FIG. 48—Strand Shoes and Traveling Sheaves Ready for Cable Spinning (Manhattan Bridge.)

for the purpose of drawing the individual wires for the cable from one anchorage to the other. There is also suspended a "guide wire" which is established by computations and regulated by instrumental observations so as to give the desired deflection of the cable wires.

Large reels, upon which the wires are wound, are placed at the ends of the bridge alongside the anchor chains (Fig. 48). The free end of a wire is fastened around a grooved casting of horseshoe outline called a "shoe" (Fig. 48), and the loop or bight, thus formed is hung around a light grooved wheel (Fig. 48) which is fastened to the traveling rope. The traveling rope with its attached wheel, moving toward the other end of the bridge, thus draws two parts of the wire simultaneously across from one anchorage to the other; one of these parts, having its end fixed to the shoe, is called the "standing wire"; while the other, having its end on the reel, is called the "running wire" and moves forward with twice the speed of the traveling rope. Arriving at the other end, the wire loop is taken off the wheel and laid around the shoe at that end. The two parts of the wire are then adjusted so as to be accurately parallel to the guide wire, the operation of adjustment being controlled by signals from men stationed along the footbridge. The wire is then temporarily secured around the shoe, and a new loop hung on the traveling wheel for its second trip. After two or three hundred wires have thus been drawn across the river and accurately set, they are tied together at intervals to form a cable strand.

For the Manhattan Bridge, the wires (drawn in 3000-foot lengths) were spliced to make a continuous length of 80,000 feet (4 tons) wound on a wooden reel (Fig. 48). These reels were 48 inches in diameter (at bottom of groove) and 26 inches long, and were provided with brake drums. On each anchorage were set eight reel stands, each with a capacity of four reels.

The equipment used for cable-spinning consisted of an endless $\frac{3}{4}$-inch steel traveling rope passing around a 6-foot horizontal sheave at each anchorage; machinery for operating the endless rope; devices for removing and adjusting the wires and strands; apparatus for compacting and wrapping the cables; hoisting machinery and power plant.

Attached to the endless rope ("traveling rope") at two equidistant points, were deeply-grooved 4-foot carrier sheaves ("traveling wheels") in goose-neck frames (Fig. 48). These frames were held securely in a vertical plane, and were designed with clearance to ride over the supporting sheaves.

The "strand shoe" was held 22 inches in front of final position by a special steel construction called a "strand leg" (Fig. 48) attached to the pin between two anchorage eyebars.

The bight of wire was placed around the traveling wheel and

FIG. 49.—Manhattan Bridge. Anchoring a Completed Strand.

pulled across. As each part of the wire became dead, it was taken by an automatic Buffalo grip at the tower and, with a 4-part handtackle of manila rope, adjusted to the guide wire. It required about seven minutes for a trip across from anchorage to anchorage (3223 feet). Only ten field splices were required to a strand (256 wires). After the strand was completed, the wires were compacted with curved-jaw tongs and fastened (or "seized") with a few turns of wire, every 10 feet. Then, with a "strand-bridle" attached to a 35-ton hydraulic jack (Fig. 49), the shoe was pulled toward the shore, releasing the strand leg and the eyebar pin. The strand shoe was then revolved 90° to a vertical position (Fig. 49), and pulled back to position on the eyebar pin.

The strand was then lifted from the temporary sheaves in which it was laid at the anchorages and the towers, and lowered into the permanent saddles; a 20-ton chain hoist and steel "balance beam" were used for this operation. The strand was then adjusted to the exact deflection desired, by means of shims in the strand shoe.

After the seven center strands were completed, they were bunched together with powerful squeezers to make a cylinder about 9¼ inches in diameter, secured with wire "seizings" at intervals. Then the remaining strands were completed, and compacted in two successive layers around the core, the interstices being filled with petrolatum. A hydraulic compacting machine was used for this squeezing, and temporary clamps applied.

Then the cable was coated with red-lead paste, and the permanent cable bands and suspenders were attached.

After the stiffening trusses and floor were suspended, the spaces between the cable bands were covered with wire wrapping.

The spinning of these cables took six days for a strand (256 wires); but four strands in each cable were strung simultaneously. The four cables (each consisting of thirty-seven strands or 9472 wires) were completed in four months. The work of compressing and binding the cables and attaching the suspender clamps and ropes took two or three months more, but the erection of the suspended trusses proceeded at the same time.

As soon as the strands were completed, the footbridges were hung to the main cables to be later used for the work of cable wrapping. The temporary footbridge cables were cut up for use as suspenders.

For the Williamsburg Bridge (Fig. 57), the wire was supplied on 7-foot wooden reels carrying 90,000 feet (9000 pounds) per reel. An engine on the New York side operated the driving wheels around which two endless ropes passed. Two carrier sheaves on each endless rope traveled back and forth, carrying two bights across (for two strands) on the forward trip, and two bights (for two adjacent strands) on the return trip. In this manner each endless rope was laying four strands at the rate of fifty wires in each strand in ten hours.

Eight reels of wire were required for each strand. When the end of a coil was reached, it was held in a vise and connected to a wire from a fresh reel, by screwing up a sleeve nut over the screw-threaded ends (which were formed by a special machine to roll the threads).

As the wire was laid, it was adjusted to conform to the catenary of the guide wire, in order to secure uniform tension in the wires of the finished cable.

The carrier wheels moved 400 feet per minute. There were three men at each anchorage to handle the reels, make splices, adjust the wire and take the bights off and on the carrier wheels. As the carrier wheel passed each tower, three men on the top of the tower clamped handtackle to the wire and pulled up until the wire was adjusted exactly parallel to the guide wire, as signaled by men distributed along the footbridge (three men on each side span and seven men on the main span). These men clamped the wire to the strand after adjustment. After the standing wire was adjusted, the running wire was regulated in the same manner, but in the reverse order. A total of twenty-five men were thus required to handle the wire as it was laid.

As soon as the strand was completed, the shoe was drawn clear of its support by a 25-ton ratchet jack anchored to the masonry. Then the shoe was twisted by hand with a long bar and thus revolved 90° into a vertical plane and allowed to slip back towards the tower, thus lowering the strand in the middle of the main span. The shoe was then permanently connected to the end pin of the anchor-chain eyebars. Shims back of the pin in the slotted pin-hole of the strand shoe provided adjustment for the strand length; each $\frac{1}{8}$-inch shim corresponded to a vertical movement of about 1 inch at mid-span.

When the inner strands were completed, their ties were removed and they were made into one strand to avoid trouble in handling them after they were surrounded by the remaining strands.

**8. Compacting the Cables.**—Each cable consists of 3, 7, 19 or 37 strands, depending upon its size, and these have to be compacted to make a cylindrical cable.

For the Manhattan Bridge, the temporary seizings around the strands were removed and the cable was compacted by hydraulic squeezers. Sixteen duplicate squeezers were used, each consisting of a hinged collar with a hydraulic jack of 6-inch stroke opposite the hinge. A hydraulic hand-pressure pump was used to produce a pressure of 5000 pounds per square inch or a total force of 43,000 pounds on the squeezer piston. Seizing (12 turns of No. 8 wire) was applied close to the squeezer, which was then moved 2 feet forward to repeat the operation. With two men operating each squeezer, the four cables were compacted in a few weeks.

**9. Placing Cable Bands and Suspenders.**—After the cables are compacted (with wire seizing at short intervals to hold them), the cable bands are placed at the panel points.

For the Manhattan Bridge, the cable bands (Fig. 55) consist of split cast-steel sleeves, 3 feet long, with ten $1\frac{5}{8}$-inch bolts through the longitudinal flanges. The upper half has two semicircular grooves, 12 inches apart, for holding the suspender ropes. The bolts were screwed up tight by means of socket wrenches with 4-foot handles, operated by two or three men each.

The $1\frac{3}{4}$-inch suspender ropes, made by cutting up the temporary footbridge cables, were fitted with cast-steel sockets $5\frac{1}{4}$ inches in diameter by 17 inches long. These sockets were threaded on the outside to receive a cast-steel nut $5\frac{1}{4}$ inches thick. The ends of the rope were served; and the wires beyond the serving were spread, cleaned in dilute acid, washed in water and dried with a painter's torch. The end of the rope was then passed through the socket (which had been carefully cleaned of sand and scale), the wires were spread to fill the covered portion, and melted spelter (heated to a very thin consistency) was poured in, filling all the interstices. Some of the finished ropes were tested, and showed an ultimate strength of 287,000 to 290,000 pounds, with the rope breaking 4 to 8 feet from the socket, there was no sign of injury at the socket, thread or nut.

The suspenders were then placed in position around the cable bands, with their lower ends ready to engage the bottom chords of the stiffening trusses (Fig 50).

FIG 50—Manhattan Bridge. Erection of Lower Chords and Floor System.

**10. Erection of Trusses and Floor System.**—The suspension from the cables permits the steelwork to be erected without falsework. In planning the program of erection there must be considered the method of connection to the suspenders, clearances for travelers, and the reach of the booms. In addition, the scheme should aim to balance the dead-load distribution along the span, so as to minimize the distortion of the cables during erection.

In the Manhattan Bridge, the truss is supported at each panel point by four parts of $1\frac{3}{4}$-inch steel rope suspenders (Fig. 50) with their bights engaging the main cables and having, at the lower end, nut bearings on horizontal plates across the bottom flanges of the lower chord.

All members were shipped separately, the chord members in two-panel-length pieces weighing 26,000 to 30,000 pounds each.

The erection proceeded at four points simultaneously, working in both directions from each tower (Fig. 50). Traveler der-

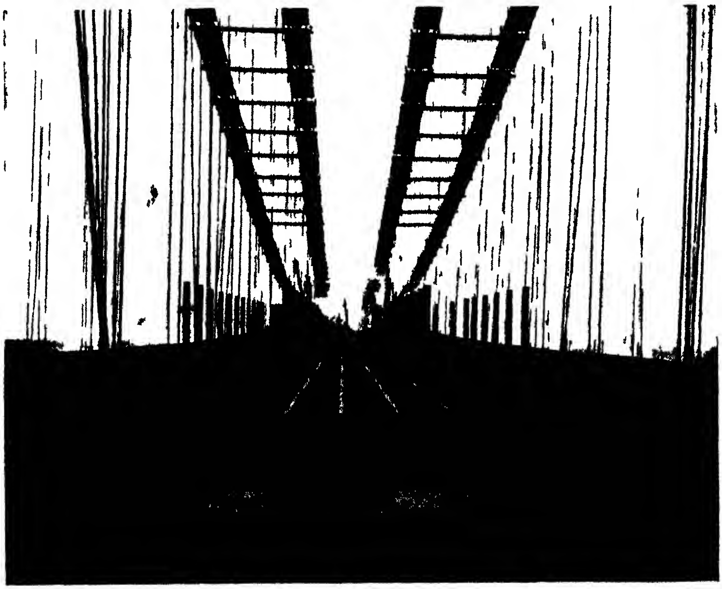


FIG. 51.—Manhattan Bridge. Erection of Verticals.

ricks of 25-ton capacity were used, with 34-foot mast and 50-foot boom (covering two panels in advance) and provided with bull-wheel. At each point of erection there were two of these large derricks, also one jinnywink derrick with 30-foot boom and 7-ton capacity. In addition to these twelve movable derricks, there were four stationary steel-boom derricks at the towers.

Starting at the towers, the lower chords and floor system were assembled two panels in advance of the travelers, making temporary connections to the suspenders, until the anchorages and mid-span were reached. Then the travelers returned to the towers to commence their second trip.

The material was hoisted by the tower derricks and loaded on service cars which delivered it to the traveler derricks The service cars ran on the permanent track between the inner and outer trusses, the cars were hauled away from the tower by cables operated by hoisting engines on the tower, and returned empty, by gravity, on the grade furnished by the camber

FIG 52—Manhattan Bridge Erection of Diagonals

On the first trip, the lower chords, lower deck and verticals were erected (Fig. 51); on the second trip, the truss diagonals were erected (Figs 52, 53), and on the return (Fig 54), the upper deck and transverse bracing were put up, thus completing the structure.

On the first trip, temporary suspender connections were made to the lower chord at alternate panel points so as to miss the upper chord splices. After the return of the travelers, permanent connection and adjustment of the suspenders at the other points were made. The temporary suspender connections were removed before the top chords were erected, and were connected again (permanently) after the top chords were in place.

A force of three hundred men was employed on this work, and their record was 300 tons of steel erected in a day. There were about 1,000,000 field rivets in the three spans. The bridge was formally opened ten months after the floor hanging commenced.

Where the side spans are not suspended from the cables,

FIG. 53.—Manhattan Bridge. View before Erection of Top Chords.

falsework is generally required. In the Kingston Suspension Bridge, Fig. 56, the side spans (although suspended) were erected on light falsework, as time was thereby saved.

The first few panels of the main span are generally erected by the stationary derricks at the tower, as far as their booms can reach. Additional panels may be erected by drifting or outhauling from the cable; or by the use of "runners," that is, block and falls suspended from the advance cable band and operated by the hoisting engine at the tower. At Kingston, the latter method was adopted, dispensing with the use of travelers for the main portion of the span.

**11. Final Erection Adjustments.**—The equilibrium polygon is computed for the dead load acting on the cable, and levels taken at a number of points on the cable should check these ordinates. The elevations and camber of the roadway are also

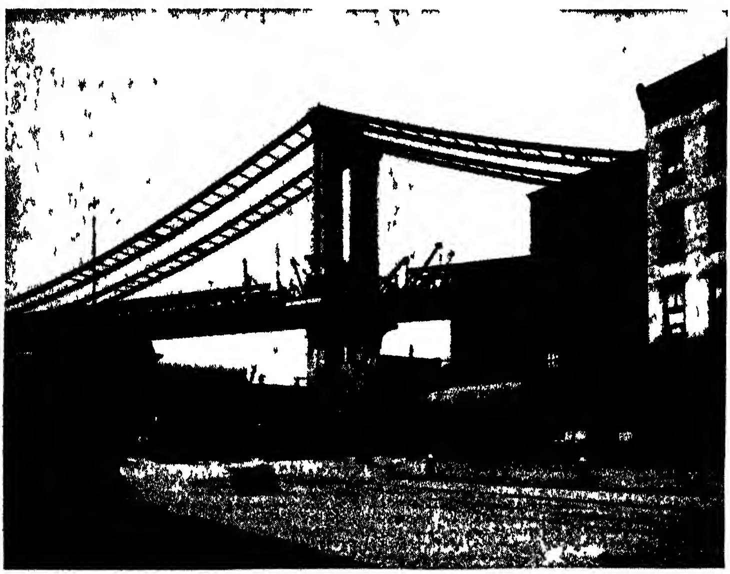


FIG. 54.—Manhattan Bridge. Erection of Top Chords.

checked with levels and corrected, where necessary, by adjusting the lengths of the suspenders.

In completing the stiffening truss, the closing chord members should be inserted after all the dead load is on the structure, the connecting holes at one end being drilled in the field.

If the closure of the stiffening truss has to be made before full dead load is on the structure, or at other than mean temperature, the vertical deflections are computed for these variations from assumed normal conditions and the suspenders adjusted accordingly, before connecting the closing members.

In adjusting the suspenders, the center hanger is shortened or lengthened the calculated amount, and the other hangers are corrected by amounts varying as the ordinates to a parabola.

If the trusses are assembled on the ground before erection, the exact camber ordinates can be measured and reproduced (by suspender adjustment), so as to secure zero stress under full dead load at mean temperature.

An ideal method of checking the final adjustments is by means of an extensometer, which should check zero stresses throughout the stiffening truss when normal conditions are attained, or calculated stresses for any variation from assumed normal conditions.

Instead of adjusting to zero stress for full dead load, it would be more scientific and somewhat more economical to adjust for zero stress at dead load plus one-half live load.

**12. Cable Wrapping.**—Close wire wrapping has proved to be the most effective protection for cables.

For the Manhattan Bridge, No. 9 galvanized soft-steel wire (0.148-inch diameter) was used. This was rapidly wound around the cable by a very simple and ingenious self-propelling machine operated by an electric motor. This machine, designed by H. D. Robinson, is illustrated in Fig. 55.

In advance of the machine, the temporary seizings are carefully removed and the cable painted with a stiff coat of red-lead paste. The end of the wrapping wire is fastened in a hole in the groove at the end of the cable band. The machine, carrying the wire on two bobbins or spools, travels around the cable and applies the wire under a constant tension. The machine presses the wire against the preceding coil and at the same time pushes itself along. The rate is about 18 feet per hour.

The machine weighs 1000 pounds and is operated by a 1½-H.P. motor at a speed of 13 R.P.M. It is handled by a force of six men.

(The small hand-operated device, which was superseded by the motor-driven machine, is seen at the extreme right in Fig. 55. It was used to complete the wrapping close to the cable bands.)

For the Williamsburg Bridge, wire wrapping was not used; instead, the cables were covered with a preservative coating of oil and graphite, then wrapped spirally with three layers of waterproof duck, and finally enclosed in a thin steel-plate shell made in two semi-cylindrical portions with overlapping joints and locked fastenings. This protection has proved inadequate to keep out moisture and prevent rust, and it has recently (1917–1921) been replaced by wire wrapping applied with Robinson's machine.

FIG. 55.—Manhattan Bridge. Cable Wrapping Machine.

**13. Erection of Wire Rope Cables.**—The individual wire ropes composing a cable of this type may be towed across the river in the same manner as the temporary footbridge ropes of a parallel wire cable; or they may be strung across by means of a single working cable stretched from tower to tower.

The latter method was used for a footbridge of 540-foot span built in 1919 over the Cumberland River by the American Bridge Co. Each cable consisted of seven ropes of $1\frac{7}{8}$-inch diameter.

A working cable of 1-inch wire rope was first stretched across between the towers for each of the main cables The main ropes were unwound from the reels back of one tower. One end of a rope was lifted to the top of the tower and hauled across the river to the top of the opposite tower, the rope being supported from the 1-inch working cable by blocks attached at intervals of about 60 feet, thus preventing too much sag. The

FIG 56—Erection of Rondout Creek Bridge at Kingston, N Y, 1921
Type OS
Span 705 feet

rope was then lowered to approximately correct position, and the sockets attached to the tower shoes The remaining ropes were then stretched in the same manner, and all were then adjusted by nuts at the ends until they touched a level straight-edge held on the fixed line of sag determined by a transit in the tower. The cable clamps and suspenders were then placed by men on a movable working platform hung from the cables, beginning in the center and working toward each end. A "boatswain chair" was used to carry out men, materials and tools. The floor system was also erected by men on the working platform, in this case working from both ends toward the center. The platform was then removed, and the trusses were erected from the ends toward the center by workmen on the floor system, using the two working cables (shifted to the center of the bridge) as a trolley cable for transporting the truss sections to position. When the top lateral bracing, railings, and wood floor were added, the structure was completed. A total

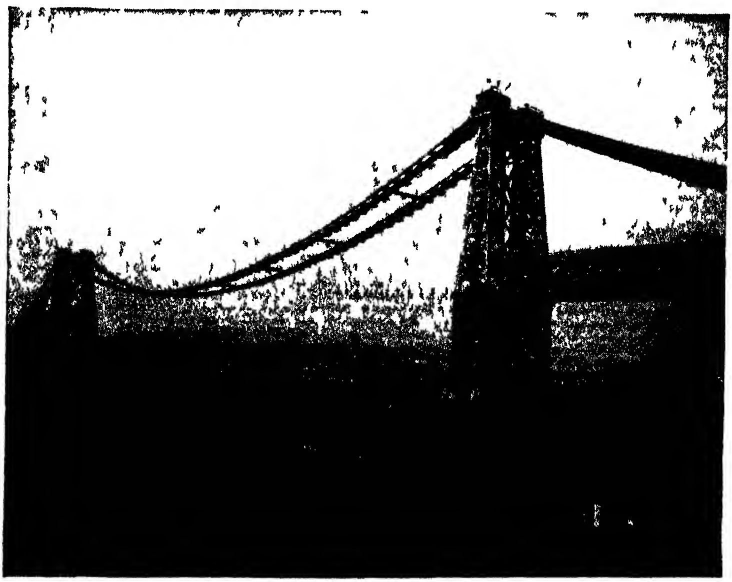


FIG. 57.—Footbridges for Erection of Williamsburg Bridge.

(See Fig 31, page 88)

of 205 tons of structural steel and 45 tons of cables were thus erected in a period of twelve weeks.

**14. Erection of Eyebar Chain Bridges.**—Chain suspension bridges have, as a rule, been erected upon falsework.

The falsework used for the erection of the Elizabeth Bridge at Budapest (1902, Span 951 feet) is shown in Fig. 58. The falsework consisted of huge scaffoldings built on piles and protected from floating ice by ice breakers. Four openings of 160 feet were left for vessels; these openings were spanned by temporary timber bridges floated into place on pontoons. After the falsework was completed, the main chains were erected in twelve weeks. The falsework was then taken down and the steelwork completed.

At a crossing like the East River or the Hudson River, the use of such falsework would be out of the question. A comparison of the cumbersome construction employed for the Elizabeth Bridge (Fig. 58) with the comparatively insignificant scaffolding required for the Williamsburg Bridge (Fig. 57), is an argument for wire cable *vs.* eyebar bridges.

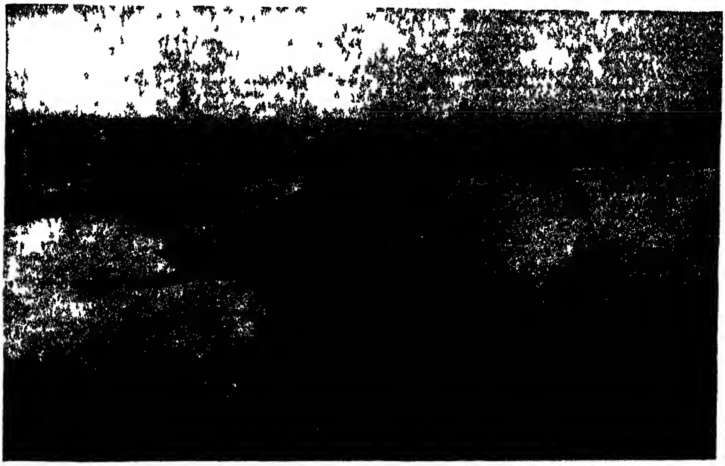


FIG. 58.—Falsework for the Elizabeth Bridge (Eyebar Chains).

(See Fig 34, page 95)

A different scheme, eliminating heavy falsework, was used for the Clifton Bridge (1864, Span 702 feet). Under each set of three chains, a suspension footbridge was constructed, using wire ropes. Above this staging, another rope was suspended to carry the trolley frames for transporting the links. The chains were commenced simultaneously at the two anchor plates, the lowest of the three chains being put in first. Commencing at the anchorage, there were inserted the whole number of links, namely 12, then 11, 10, 9, 8, and so on until the chain was diminished to 1 link; then the chain was continued with 1 and 2 links, alternately, until the two halves met at mid-span. The suspended footbridge was strong enough to carry the weight of this chain (consisting of 1 and 2 links, alternately) until the center connection was made; the chain was then made to take its own weight by removing the blocking under it. The next operation was to add the remaining links of the chain on the pins already in place. The process was repeated for the upper chains, and then the roadway was suspended.

The Cologne Suspension Bridge (1915, Span 605 feet, Fig. 17), was the first large bridge to be built hingeless. (The Kingston Bridge, 1921, was the second.) It is of the self-anchored type, the stiffening girder taking up the horizontal tension; and the towers are hinged at the base. Nickel steel was used for the chains, the eyebars being of the European type, that is, of flat plates (36 to 59 inches wide) bored for 12-inch pin-holes near the ends. The erection of the chains and stiffening girders proceeded simultaneously on special staging, and was so conducted that the girders were completed first. The girders were made three-hinged during erection and then changed to hingeless by riveting on splice plates.

The procedure was as follows: Falsework was built for the side spans and a traveler was assembled at each end. The side-span girders and deck were erected on the falsework, and the staging built up (on the girders) for the land chains, the traveler moving forward from the anchorage to the tower during this operation. The traveler then moved out on cantilever false-work spans in the main opening, erecting the stiffening girders and the staging for the chains from the tower to mid-span. The erection of the chains followed closely upon the erection of the girders. When the stiffening girders were completed and the suspension chains connected to the ends (with 24-inch pins), every third hanger was coupled up. The staging carrying the chains was then removed, and the remaining hangers were connected and adjusted by means of their turnbuckles to bring the pin points in the chains into their correct positions. The splices in the webs and flanges of the stiffening girders at the three hinge-points were then riveted up, thus completing the erection.

For Lindenthal's Quebec Design (1910, Span 1758 feet, Fig. 40), the following scheme of erection was proposed: The side spans were to be erected on steel falsework—first the floor system, then the eyebars and pins of the lower chord chain, then the verticals and upper chain eyebars, leaving the pins projecting out to receive the diagonals and remaining eyebars after the main span chains were erected and self-supporting. The towers were to be riveted up in place and temporarily anchored to the steel staging which, in turn, was to be anchored to the abutment. The first sets of eyebars (one and two alternating per panel) of the chains of the middle span were to be erected from temporary wire rope cables, each consisting of forty steel wire ropes of $2\frac{3}{4}$-inch diameter. Then the remaining eyebars and gusset plates were to be pushed on to the pins until the chains were completed. Thereafter the verticals and diagonals were slipped in place, and the suspenders and floor system of the middle span erected.

**15. Time Required for Erection.**—The time schedule for the Manhattan Bridge (1470-foot span, Fig. 35) was as follows:

| | | |
|---|---|---|
| First substructure contracts let | | 1901 |
| Pier foundations commenced | May, | 1901 |
| Work commenced on final (revised) design | March, | 1904 |
| Steel towers commenced | July, | 1907 |
| Steel towers completed (12,500 tons) | July, | 1908 |
| Temporary cables strung | June 15–20, | 1908 |
| Footbridges constructed | July 7–13, | 1908 |
| Spinning of main cables commenced (4 cables) | Aug. 10, | 1908 |
| Last wire strung (37,888 wires) | Dec. 10, | 1908 |
| Erection of suspended steel commenced | Feb. 23, | 1909 |
| Erection of suspended steel completed (24,000 tons) | June 1, | 1909 |
| Approaches completed and bridge formally opened | Dec. 31, | 1909 |

The steel erection, amounting to 42,000 tons of steel between anchorages and including towers, cables, trusses, and decks, was accomplished in two and a half years.

The Kingston Suspension Bridge (705-foot span, Fig. 56) was completed in one year (1920–1921).

The bridge contains 1430 tons of structural steel and 300 tons of cables.

The 400-foot-span suspension bridge at Massena, New York, (Fig. 30; H. D. Robinson, Consulting Engineer), containing 400 tons of steel, was erected complete in six months.

Progress in the art is recorded by the following comparative time-records for the cable construction on various large suspension bridges:

| | Brooklyn | Williams-burg | Manhattan | Bear Mountain | Phila-delphia |
|---|---|---|---|---|---|
| Number of cables | 4 | 4 | 4 | 2 | 2 |
| Cable diameter | $15\frac{3}{4}''$ | $18\frac{5}{8}''$ | $20\frac{3}{4}''$ | $18\frac{3}{4}''$ | 30″ |
| Wires per cable | 5296 | 7696 | 9472 | 7252 | 18,666′ |
| Wire diameter | 0.180″ | 0.192″ | .0195″ | 0.195″ | 0.195″ |
| Total wire (tons) | 3500 | 4500 | 6300 | 1900 | 6500 |
| Construction of foot-walks (months) | 10 | 7 | 4 | $1\frac{1}{3}$ | $3\frac{1}{3}$ |
| Spinning of cables (months) | 21 | 7 | 4 | $2\frac{1}{2}$ | 5 |
| Maximum tons wire spun in a day | $19\frac{1}{2}$ | 75 | 130 | 76 | 100 |

There have also been improvements in the erection of the structural steel of suspension bridges. In the case of the Ohio River Bridge at Portsmouth, Ohio (Robinson and Steinman, Consulting Engineers, 1927), the entire stiffening trusses and floor system of the 700-foot main span and 350-foot side spans were erected in a little over two weeks.

# APPENDIX A

## DESIGN CHARTS FOR SUSPENSION BRIDGES

INTRODUCTION.—To expedite the proportioning or checking of suspension bridges, the author has devised the three charts which are presented in this Appendix. These charts give directly the maximum and minimum moments and shears in the stiffening truss, throughout the main and side spans. The charts are constructed for the usual form of construction, parabolic cable with two-hinged stiffening truss; and they cover both types:

Type 2*F*—Free Side Spans (Straight Backstays).

Type 2*S*—Suspended Side Spans (Curved Backstays).

To use the charts, it is simply necessary to calculate $N$, which is a constant for any given structure. This constant $N$ is defined by Eq. (125) or (167), Chapter I; the formulas for $N$ are also reproduced on the charts. In these formulas:

$I$ = moment of inertia of the truss, main span;
$I_1$ = moment of inertia of the truss, side span;
$A$ = area of cable section, main span;
$A_1$ = area of cable section, side span;
$E$ = coefficient of elasticity for truss;
$E_c$ = coefficient of elasticity for cable;
$f$ = cable sag, main span;
$f_1$ = cable sag, side span;
$l$ = main span of cable (c. to c. of towers);
$l'$ = main span of truss (c. to c. of bearings);
$l_1$ = side span of truss (c. to c. of bearings);
$l_2$ = side span of cable (tower to anchorage);
$\alpha_1$ = inclination of cable chord in side span.

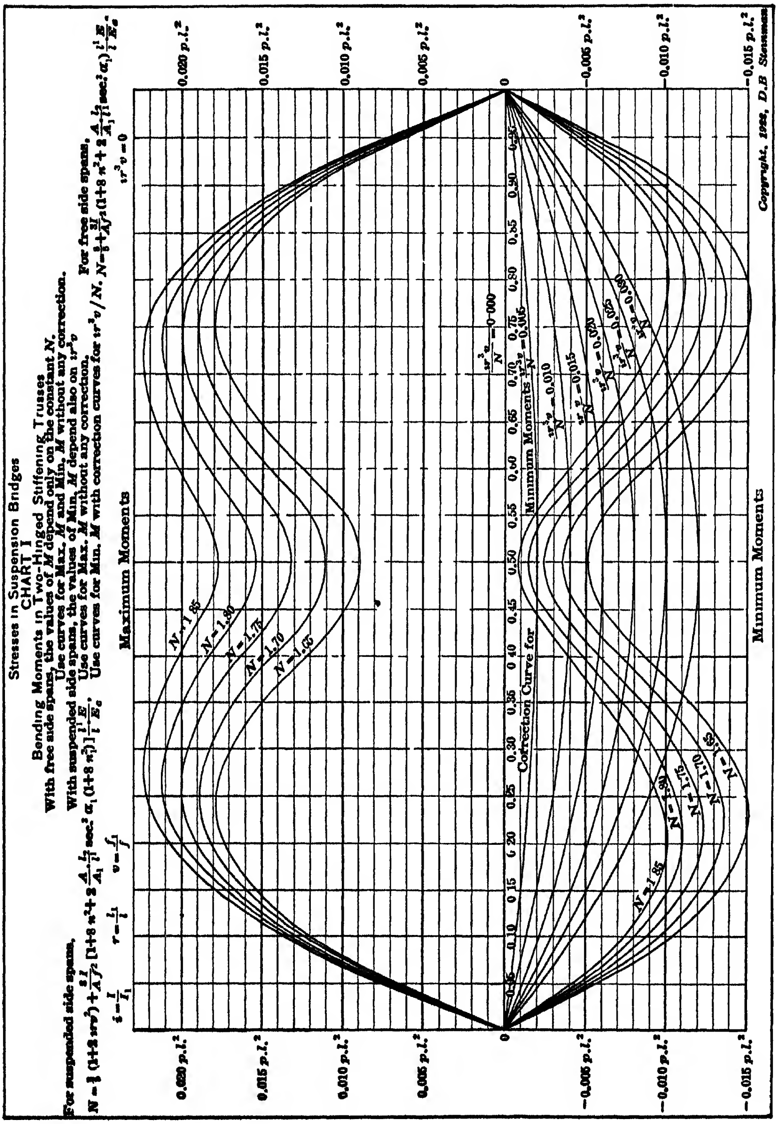



CHART I.—Bending Moments in Main Span.

The value of $N$ is usually about 1.70 for the case of free side spans (Type $2F$), and about 1.80 for the case of suspended side spans (Type $2S$).

For the case of suspended side spans (Type $2S$) it is also necessary to figure the ratio-product $ir^3v$, where

$$i=\frac{I}{I_1},\quad r=\frac{l_1}{l},\quad v=\frac{f_1}{f}.$$

This ratio-product is also a constant for any given structure. (It is equal to zero when the backstays are straight, Type $2F$. For Type $2S$ we may usually assume $i=1$, and $v=r^2$, so that $ir^3v=r^5$, approximately.)

With the values of the two constants $N$ and $ir^3v$ known, the maximum and minimum moments and shears for all points in main and side spans may be read directly from the charts, thus dispensing with the usual laborious computations.

**Chart I.—Bending Moments in Main Span.**—This chart gives the governing bending moments throughout the main span. The upper curves (for different values of $N$) give the maximum bending moments, and the lower curves (for different values of $N$) give the minimum bending moments. No correction is required except for minimum moments in the case of suspended side spans (Type $2S$). The corrections for this case are given by the parabolic curves plotted below the axis $\left(\text{for different values of } \frac{ir^3v}{N}\right)$. These corrections, like the minimum moments, are negative in sign, and the two should therefore be added arithmetically. (These corrections represent the effect of load covering both side spans.)

The values of Total $M$ (for full loading of all spans) may be obtained, if desired, by arithmetically subtracting the corrected Min. $M$ from Max. $M$. Total $M$ for load covering the main span alone may be obtained by subtracting the uncorrected Min. $M$ from Max. $M$. The resulting values, in either case, would be represented by parabolas above the axis.

**Chart II.—Shears in Main Span.**—This chart gives the governing shears throughout the main span. The upper curves (for

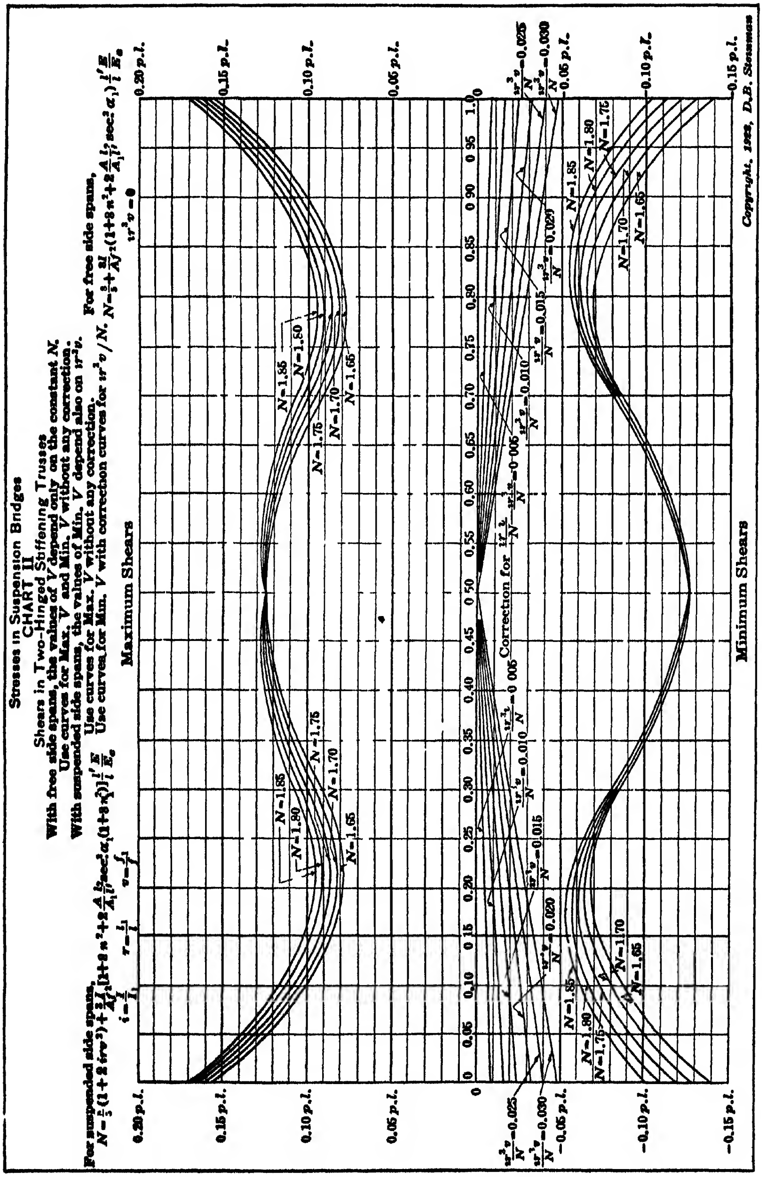



CHART II.—Shears in Main Span.

different values of $N$) give the maximum shears, and the lower curves (for different values of $N$) give the minimum shears. No correction is required except for minimum shears in the case of suspended side spans (Type 2$S$). The corrections for this case are given by the straight lines plotted below the axis $\left(\text{for different values of } \frac{ir^3v}{N}\right)$. These corrections are of the same algebraic sign as the minimum shears, and the two should therefore be added arithmetically. (These corrections represent the effect of load covering both side spans.)

On this chart, the plus sign indicates a shear upward on the outer side and downward on the inner side of a section; the minus sign indicates a shear in the opposite direction.

The values of Total $V$ (for full loading of all spans) may be obtained, if desired, by arithmetically subtracting the corrected Min. $V$ from Max. $V$. Total $V$ for load covering the main span alone may be obtained by subtracting the uncorrected Min. $V$ from Max. $V$. The resulting values, in either case, would be represented by radiating straight lines above the axis.

**Chart III.—Moments and Shears in Side Spans.**—This chart gives the governing stresses throughout a side span.

In the left-hand diagram, the upper parabolic curves $\left(\text{for different values of } \frac{ir^3v}{N}\right)$ give the maximum bending moments. (These curves represent the effect of load covering the given side span.) The lower parabolic curves $\left(\text{for different values of } \frac{1+ir^3v}{N}\right)$ give the minimum bending moments. (These curves represent the effect of load covering the two other spans.) The values of Total $M$ (for load covering all three spans) may be obtained, if desired, by arithmetically subtracting Min. $M$ from Max. $M$. The resulting values would be represented by flat parabolas above the axis.

In the right-hand diagram, the upper curves $\left(\text{for different values of } \frac{ir^3v}{N}\right)$ give the maximum shears. (These curves represent the effect of load covering the given side-span.) The

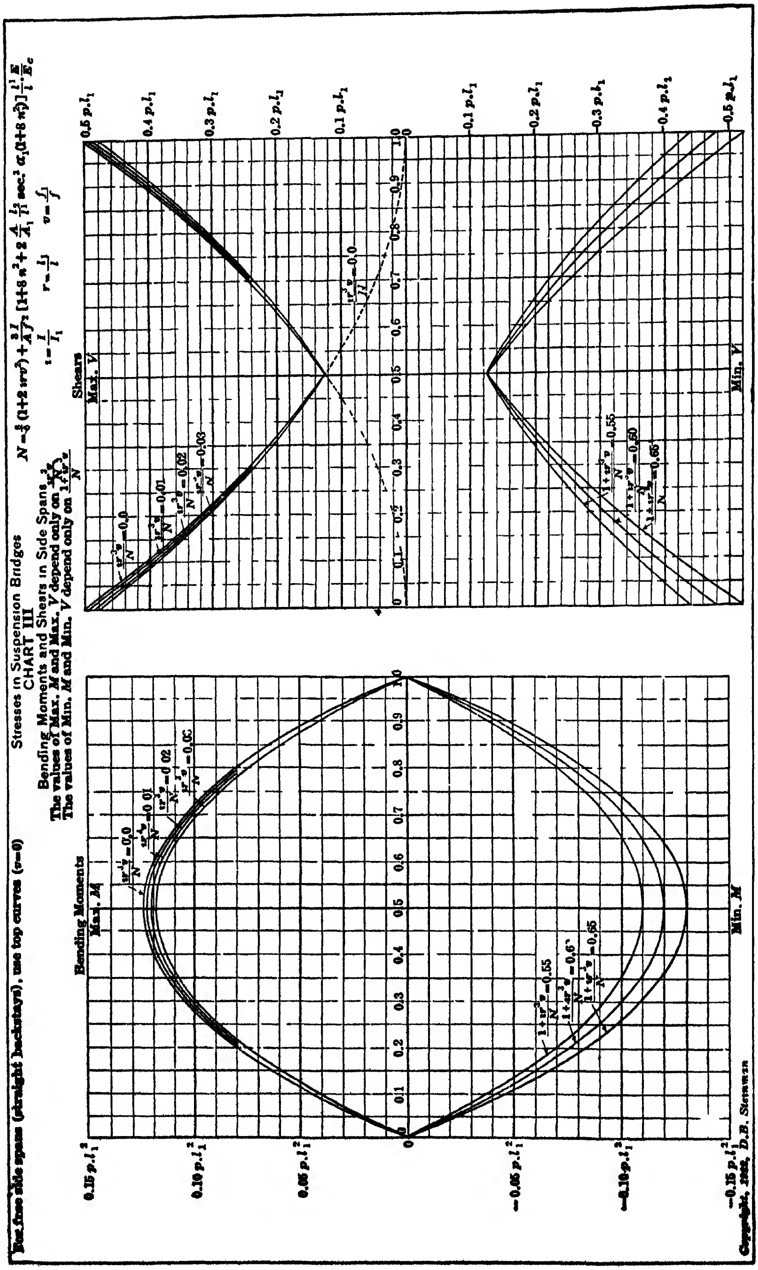



CHART III.—Moments and Shears in Side Spans.

lower curves $\left(\text{for different values of } \frac{1+ir^3v}{N}\right)$ give the minimum shears. (These curves represent the effect of load covering the two other spans.) The values of Total $V$ (for load covering all three spans) may be obtained, if desired, by arithmetically subtracting Min. $V$ from Max. $V$. The resulting values would be represented by radiating straight lines above the axis.

In this diagram, the plus sign indicates a shear upward on the outer side and downward on the inner side of a section; the minus sign indicates a shear in the opposite direction.

Chart III can also be used for a side span not suspended from the backstays (Type $2F$), or for any independent simple span. The maximum bending moments produced by uniform load are given by the top curve in the left-hand diagram; the minimum bending moments are zero. The maximum shears produced by uniform load are given by the top curve in the right-hand diagram; the minimum shears are given by the dotted continuation curve in the same diagram.

In constructing the graphs for the side spans in Chart III, it has been assumed for simplicity that $v/r^2=1$. This ratio is generally between 1.00 and 1.04, but in extreme cases may be as high as 1.10. If $v/r^2$ is greater than 1.00, the values of Min. $M$ and Min. $V$ given by the chart should be multiplied by this ratio.

Where locomotive loadings with axle-concentrations are specified, the equivalent uniform loads are to be used for $p$ in these charts.

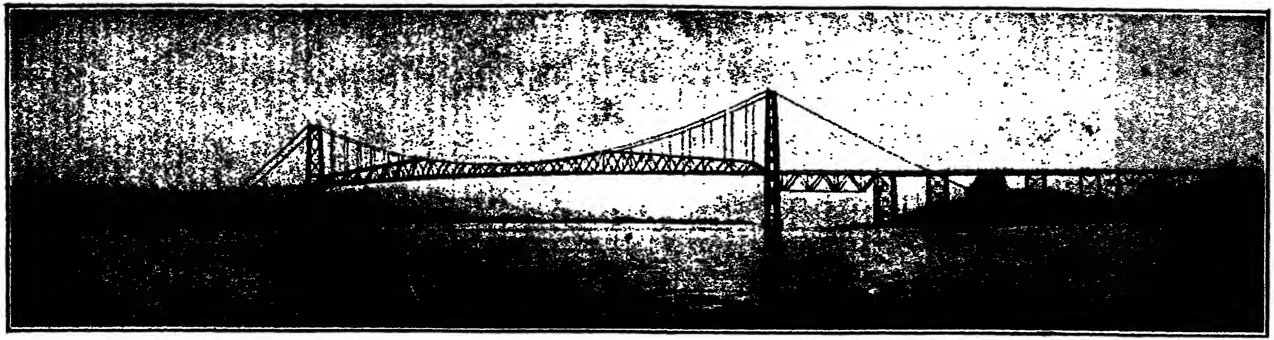


FIG. B1.—Florianopolis Bridge (1922–1926).

(Longest Eyebar Suspension Span in World.)

Span 1114 ft. Cables of Heat-treated Eyebars.
New Form of Stiffening Truss. Rocker Towers.
Trusses, 22½ to 45 ft. Deep, 33½ ft. c. to c.
Built for Highway, Railway and Aqueduct.
Designed by Robinson and Steinman, Consulting Engineers.

# APPENDIX B

## THE FLORIANOPOLIS BRIDGE

**1. Introduction.**—The Florianopolis Suspension Bridge, completed 1926, with a main span of 1113 ft. 9 in., is the longest span bridge in South America and the longest eyebar suspension span in the world. The bridge was constructed for the Brazilian state of Santa Catharina, and spans the waters of a strait of the Atlantic Ocean. It was built to carry a highway, electric railway, and water-supply main to Florianopolis, the island capital of that state.

The structure is of interest to bridge engineers as the first executed example of a new form of suspension stiffening construction, whereby greatly increased rigidity and economy of material are secured simultaneously. The distinctive feature of this construction is the utilization of the cable to replace a part of the top chord of the stiffening truss, and the consequent change from the conventional parallel-chord truss to a stiffening truss of more effective outline.

Another departure from customary practice is the use of rocker towers, yielding advantages in economy of material. The Florianopolis Bridge is the first large suspension bridge in the Americas to be built with rocker towers.

The bridge is of further interest to engineers as the first application of an important new structural material. Instead of wire for the cables, eyebars are used; and these are made of the newly developed, high-tension, heat-treated carbon steel, having a yield point exceeding 75,000 lb. per sq. in., and intended to be used with a working stress of 50,000 lb. per sq. in. The bid price for this type reduced the total cost to the lowest estimated cost with wire cables.

Originally the suspension type, of conventional design, was adopted for this bridge in economic competition with cantilever designs. Subsequent modifications yielded further economies in favor of the suspension type.

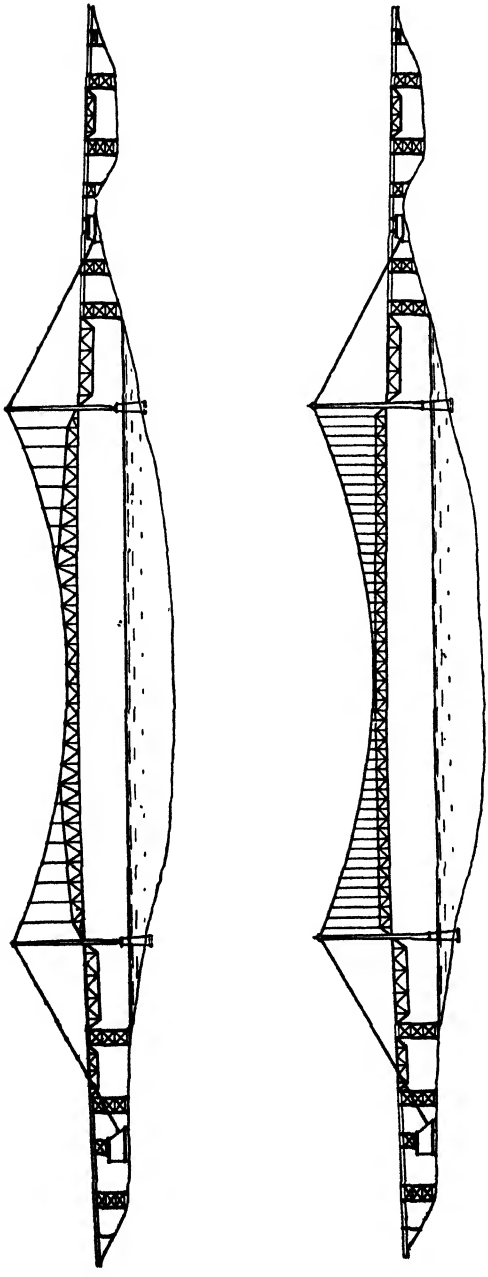
In the foundation work for the main piers, novel construction methods were devised to overcome unusual difficulties. The concrete anchorages are of special design, U-form in plan, for maximum efficiency. One anchorage is founded on rock, the other on piles.

Finally, an entirely new method was developed for the erection of the eyebar cable and suspension span stiffening trusses, using an overhead trolley, thus eliminating wooden falsework and working platforms.

**2. New Type of Stiffening Truss.**—The form of stiffening construction adopted for the Florianopolis Bridge is an innovation. A comparison (Fig. B2) of the adopted design with the conventional layout which it superseded shows the distinctive characteristics of the new type.

In the central portion of the conventional form of suspension construction, the cable, sustaining the dominant tensile stress, closely follows the upper chord of the stiffening truss, which has compression for its governing stress. Such juxtaposition of two principal members carrying opposing stresses represents a waste of material or, rather, a neglected opportunity for economizing. By combining the two opposing structural elements, one member is made to take the place of two; the result is a subtraction of stresses instead of an addition of sections. This effects a partial neutralization of the maximum tension in the middle portion of the cable, and the omission of a corresponding portion of the upper chord of the truss.

This utilization of the cable as the upper chord of the stiffening truss should preferably be limited to the central half of the span. To extend this construction to the ends of the span would not be economical, because the saving of the top chord in the outer quarters would be offset by the increase in the length of the web members in a region where the stiffening truss has its maximum shears. Moreover, beyond the quarter-points there would be an addition instead of a subtraction of stresses, since the condition of loading that produces maximum tension in those



PREVIOUS DESIGN

FIG. B2.—Adopted Design for the Florianopolis Bridge.

Compared with Previous (Parallel-Chord) Design.

top chords is one that produces nearly maximum tension in the cable.

Another neglected opportunity for increasing economy and efficiency in the conventional form of suspension construction is in the use of parallel-chord stiffening trusses. For maximum economy the truss should have a profile conforming to the variation of maximum bending moments along the span, a principle that is recognized in designing other structures, as simple trusses, cantilevers, continuous trusses, and arches. Since the economic depth at any section is a function of the governing bending moment, a truss should have its greatest depth at the points of greatest bending moment, and should be made shallow where the bending moments are comparatively small.

In a suspension-bridge stiffening truss, the greatest bending moments occur near the quarter-points of the span; consequently, the economic profile of a stiffening truss is one having maximum depth near the quarter-points and minimum depth at mid-span and at the ends. This conclusion is strengthened by the fact that the shears in a stiffening truss are a minimum near the quarter-points and attain maximum values at the middle and ends of the span. Thus, a truss profile with maximum depth near the quarter-points also gives economy in web members since it provides the shallowest depth in the regions where the web stresses are greatest. Such a profile yields the additional advantage of greater uniformity of required chord sections throughout the span, as a wide range of variation usually involves a waste of material in those chord members requiring minimum sections.

Another consideration governing truss depth is that of efficiency in reducing deflections. The most serious deflection of a stiffening truss, as measured by the resulting deflection gradients, is produced under the condition of live load covering approximately one-half the span. Half-span loading produces a downward deflection of the loaded segment and a smaller upward deflection of the unloaded segment, with a maximum deflection gradient at the loaded end. The magnitude of this deformation depends on the truss depth at and near the quarter-points. Calculations show that to limit the deflection gradient to 1 per cent, a truss depth of about $\frac{1}{45}$ of the span is required. A parallel-chord stiffening truss of such depth (as illustrated by the Williamsburg Bridge, New York) would render the structure unsightly. To secure the requisite stiffness without resorting to a stiffening truss of clumsy proportions, it is necessary to depart from the parallel-chord type and to adopt an outline providing the extreme depth only where it is needed, namely, in the vicinity of the quarter-points of the span.

From the foregoing considerations, the two logical means for improving conventional suspension design for increased economy and efficiency are:

1. Utilization of the cable to replace a portion of the top chord of the stiffening truss (preferably limited to the middle half of the span).
2. Variation of the truss profile to give maximum depth near the quarter-points of the span.

Fortunately, compliance with the first of these requirements automatically helps with the second. The result is the form of suspension construction adopted for the Florianopolis Bridge.

**3. The Revision of Design.**—The bridge had already been designed along conventional lines, when the decision to substitute eyebar cables for wire cables prompted consideration of the revised truss design and facilitated its application. In the parallel-chord design, the stiffening truss was 25 ft. deep throughout; the revised layout, utilizing part of the chain as the top chord, provides a truss with depth varying from 22.5 to 42.5 ft.

In the first plans for the new design, the upper chord in the outer quarters of the span was made curved so as to produce an effect of symmetry about the quarter-points. Straight chords were substituted, however, at the decision of the purchaser, in order to minimize fabrication costs. Minimum cost was the outstanding requirement governing the entire design. The question of relative appearance of straight and curved chords in the outer quarters of the span is a matter of individual preference.

Although the revision from the conventional design was prompted and facilitated by the adoption of eyebar cables for the principal suspension elements, the new form of stiffening construction can also be used in conjunction with wire cables. Approved details of the necessary connections between truss members and wire cables have been developed by Robinson and Steinman for designs of this type.

**4. Economy of the Adopted Design.**—Comparative cost estimates of the two designs shown in Fig. B2 demonstrate a material saving in favor of the adopted design. In addition to the major elements of economy inherent in the salient features of the new form of construction, a number of incidental savings arise from the change in design:

1. Saving the material represented by the middle half of the top chord of each stiffening truss.

2. A general saving in the remaining chord material resulting from the use of an economic truss profile conforming to the variation of bending moments along the span; and, in particular, a material reduction in the maximum chord sections near the quarter-points.

3. A saving in details and in minimum sections resulting from the greater uniformity of required chord sections throughout the span.

4. A saving in web material on account of the reduced truss depth in the regions of maximum shear.

5. The omission, in the middle half of the span, of the subverticals previously required to shorten the compression chord members, now replaced by tension members.

6. The omission of the intermediate top laterals previously required to stay these shortened compression chord members in a horizontal plane.

7. As a result of these savings, a reduction of about one-third in the total weight of the stiffening truss, and a consequent further saving in all parts affected by the dead load of the truss.

8. A saving in cable sections resulting from the reduced dead load of the truss and from the consequent reduced dead load of the cable.

9. The omission of the suspenders in the middle half of the span and a reduction in length of the remaining suspenders.

10. A combined saving in the towers resulting in part from

the reduction (6.5 ft. in the case of the Florianopolis Bridge) in the total height in consequence of the reduced distance between the cable and lower chord at the center.

11. A material saving in the anchorages resulting from the reduced dead load of the truss and cable and the reduced elevation of the back-stays.

In the case of the Florianopolis Bridge, much of this economy was not capitalized but was turned back into the structure in the form of a general reduction of unit stresses to provide a greater margin of safety for future load increases. Thus, the design stress for the stiffening truss was lowered from 20,000 to 18,500 lb. per sq. in. (actually 14,500 lb. as calculated by the exact method); and the unit stress in the cable was reduced from 50,000 to 46,500 lb. per sq. in.

Designs have been proposed, in the past, in which the cable would be utilized as the top chord of an overhead bracing system. G. Lindenthal advocated, for the Manhattan Bridge and for the Quebec Bridge, eyebar cable designs with bracing systems having the maximum depth near the quarter-points of the span. (See Fig. 39, page 107.) In those designs, however, the stiffening system would have to leave the roadway level to follow the line of the cable in the outer quarters of the span.

The design of the Florianopolis Bridge secures the desired advantages while retaining a stiffening truss at the roadway level from tower to tower. The use of an overhead trussing system (departing from the roadway level) would necessitate separate wind chords for lateral stiffening.

**5. Gain in Rigidity.**—Although the governing consideration in the change of design was that of economy, the change yielded greatly increased rigidity (300 per cent) as an incidental advantage.

According to the deflection graphs calculated for the adopted design, the maximum deflection under full-span loading is only 1.88 ft., or $\frac{1}{592}$nd of the span; and the maximum deflection under half-span loading is only 1.36 ft., or $\frac{1}{820}$th of the span, with uplift in the unloaded half practically eliminated. The actual deflections will be about 30 per cent less than these calculated values, since the deflection calculations were based on the elastic, or "approximate," method and since no allowance was made for the stiffening effect of details. These deflections are approximately one-fourth of the corresponding values for the previous (conventional) design. As the Florianopolis Bridge is designed to carry a railway as its principal element of live load, this reduction of the governing deflections is of practical significance.

An increase in rigidity of about 35 per cent may be attributed to the substitution of eyebars for wire cables; the remaining 265 per cent increase is the direct consequence of the new form of stiffening construction.

The following elements of the new design contribute to this increase in rigidity:

1. The revised truss profile is more efficient in resisting deflections, since it provides a greater average depth, with maximum depth in the regions of greatest bending moment.

2. The depth at the quarter-points has been made nearly twice as great as in the previous design, and the stiffness in the vicinity of the quarter-points is the principal factor in determining the rigidity of a suspension bridge under the critical condition of half-span loading.

3. The functioning of the full section of the cable as the top chord of the stiffening truss in the middle half of the span greatly increases the moment of inertia in that part of the span.

4. The fact that the live load introduces tension in the middle half of the top chord (by virtue of its forming part of the cable) further reduces mid-span deflections.

As a result of these various factors, the change of design yields greater stiffness with less material in the structure. In approximate figures, the design is four times as rigid with only two-thirds as much material in the stiffening truss. Thus, greater efficiency has been secured through a more scientific design of the suspension stiffening system.

In addition to the marked increase in vertical rigidity yielded by the new design, the lateral stiffness is improved by the large cable sections functioning as wind chords in the middle half of the span; and ideal longitudinal rigidity is secured by the direct connection of the truss to the cables. Longitudinal or braking forces are carried directly into the cable.

The net result of the change in design as applied to the Florianopolis Bridge is a reduction in cost (through actual saving in material), an increase in safety and longevity (through lowered unit stresses), and an increase in efficiency (as measured by resistance to deflections).

**6. Heat-Treated Eyebars.**—In the Florianopolis Bridge a new material, in the form of high-tension, heat-treated, carbon-steel eyebars, found its first application. This material, intended to be used with a working stress of 50,000 lb. per sq. in., was developed through experimental research by the American Bridge Company. It is furnished under guaranty of minimum elastic limit of 75,000 and minimum ultimate strength of 105,000 lb. per sq. in., and minimum elongation of 5 per cent in 18 ft.

Heat treatment of steel has been known and used for some time, but it is only since 1915 that its application has been made to structural steel bridges. Carbon steel is the material used, but the amount of carbon is higher than in the ordinary structural grade with an ultimate strength of 55,000 to 65,000 lb. per sq. in.

After the steel is manufactured and rolled in the eyebar sizes, the eyebars are upset in the usual manner, except that for the Florianopolis Bridge the heads were made $\frac{1}{8}$ in. thicker than the body of the bar. The bars are then placed in the heat-treating or annealing furnaces and subjected to temperatures necessary to produce elastic limits and ultimate strengths of the desired amounts. After quenching, the bars are reheated and then cooled slowly. Each bar is treated separately.

**7. Use of Rocker Towers.**—The Florianopolis Bridge is the first large American suspension bridge built with rocker towers. The only large bridges previously built with this feature are the Elizabeth Bridge at Budapest, Hungary (1903), and the bridge over the Rhine at Cologne, Germany (1915).

The rocker type offers the most economical and scientific design for suspension bridge towers. It eliminates the bending stresses from unbalanced cable pull, thereby yielding a saving in tower material, and it obviates the difficulties of the necessary erection operation of pulling back the tops of the towers prior to stringing the cables.

In the case of the Florianopolis Bridge, the change was made from fixed bases after comparative estimates showed a net saving of about 20 per cent in the weight of the towers in favor of the rocker type. There is a substantial reduction in the main sections by the elimination of the bending stresses. Another important advantage is the elimination of the bending stress from the piers, permitting their size and reinforcement to be reduced.

**8. Details of the Tower Design.**—As shown in Fig. B3, the towers are approximately 230 ft. high. The legs are battered, from a top width of 33 ft. 6 in. to a width of 55 ft. 6 in. at the base. The width at the top corresponds to the spacing, center to center, of trusses, the cables hanging in vertical planes. This form of tower design, introduced into suspension bridge construction by H. D. Robinson, has for its chief advantage the facility of running the truss and roadway construction through the tower portal without interference from the tower legs; an additional advantage is the increased transverse stability, of particular importance for narrow bridges.

The two legs of the Florianopolis tower design are braced together with rigid diagonal and transverse bracing members, the latter including a transverse distributing girder at the tower top, a transverse reaction girder supporting the stiffening truss and approach truss bearings, and a transverse portal girder immediately above the stiffening truss portal.

The reaction girder has a box section and supports the roller expansion bearings of the approach span and the pin-connected rocker supports of the main-span stiffening truss. These rockers take care of the vertical reactions (both positive and negative) while permitting the necessary longitudinal expansion movements.

Each tower leg or column is made up of a double-box section (Fig. B3), having a maximum longitudinal width of 8 ft. and a constant transverse width of 3 ft. 6½ in. The 8-ft. width tapers to 4 ft. 6 in. at the top, and to 5 ft. 0 in. at the base. Transverse

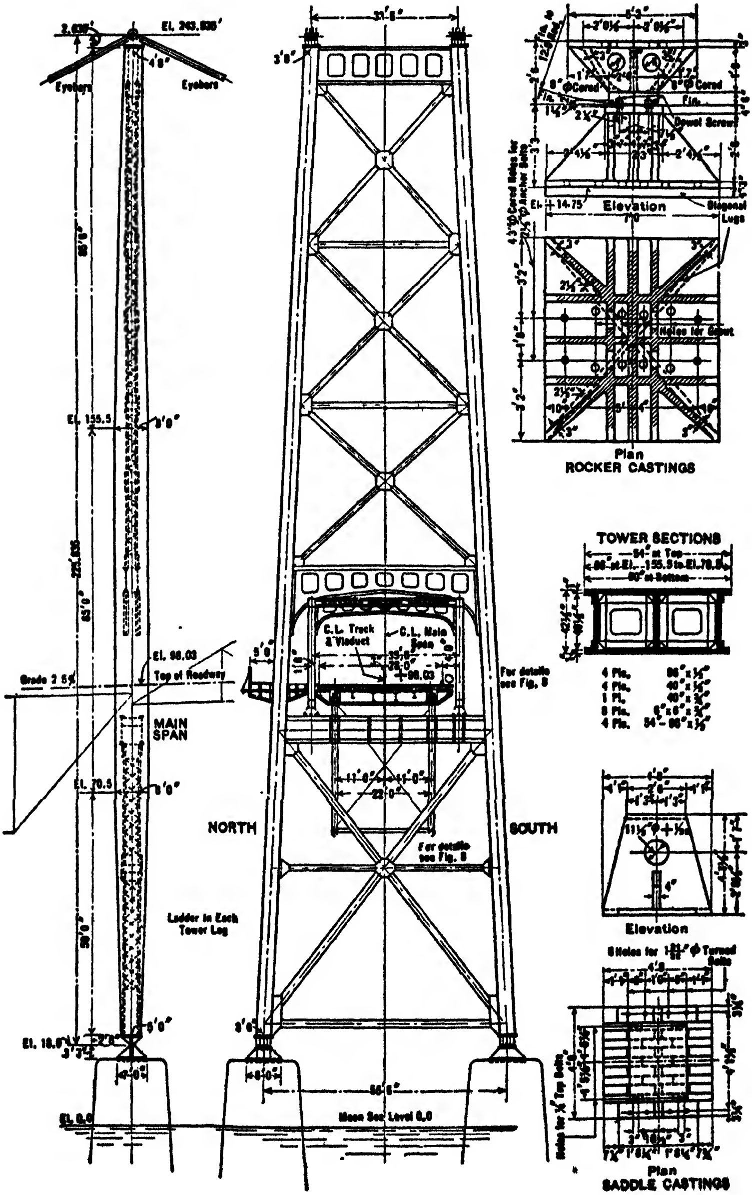



Fig. B3.—Details of Towers, Florianopolis Bridge.

stiffening diaphragms are provided at intervals, two in each column section.

The tower is designed for a maximum horizontal component of 3,860,000 lb. in each cable, resulting in a maximum vertical reaction of 3,790,000 lb. per column; this is supplemented by the vertical reactions of the stiffening truss (240,000 lb.) and approach span (300,000 lb.), which reactions, however, affect only the lower sections of the tower. In addition, the tower top is subjected to a maximum lateral pull of 75,000 lb. per tower from wind forces transmitted through the cables; and, at the lower level, the tower receives the lateral reactions (190,000 lb.) from the trusses. All these forces are taken into account in proportioning the tower columns and the transverse bracing. Unbalanced cable pull at the top of the towers, which would otherwise materially affect the column sections, is eliminated by the rocker feature.

The tower column has a maximum cross-section of 370 sq. in., tapering to a minimum of 285.5 sq. in. at the top and 297.5 sq. in. at the base. This variation in cross-section (and in moment of inertia), similar to that provided in derrick booms, is calculated to take care of the varying flexural stress produced by the long-column action.

The rocker base details are shown in Fig. B3. The pedestal or base casting, which rests on the concrete piers, is finished to a plane top surface, 27 by 45 in. The rocker casting, which is affixed to the column base, is finished on its lower or bearing surface to a radius of 12 ft. The line of contact is 45 in. long. For security against any possibility of creeping displacement, four screw dowels, 3 in. in diameter, are provided (Fig. B3). The rocking of the upper casting on the lower was tested in the shop with the dowels temporarily in place. In the bottom face of the pedestal casting are two full-length diagonal lugs which engage corresponding grooves in the masonry to prevent any possible sliding of the casting. The maximum vertical reaction on each rocker bearing is 2420 tons. Details of the saddle casting are also shown in Fig. B3.

**9. Design and Construction of the Anchorages.**—The design of the Island anchorage is shown in Fig. B4. Both anchorages are U-form in plan, for maximum efficiency.

The east anchorage (Fig. B4) is on rock. The anchor cables and reaction girders are embedded in concrete in two stepped

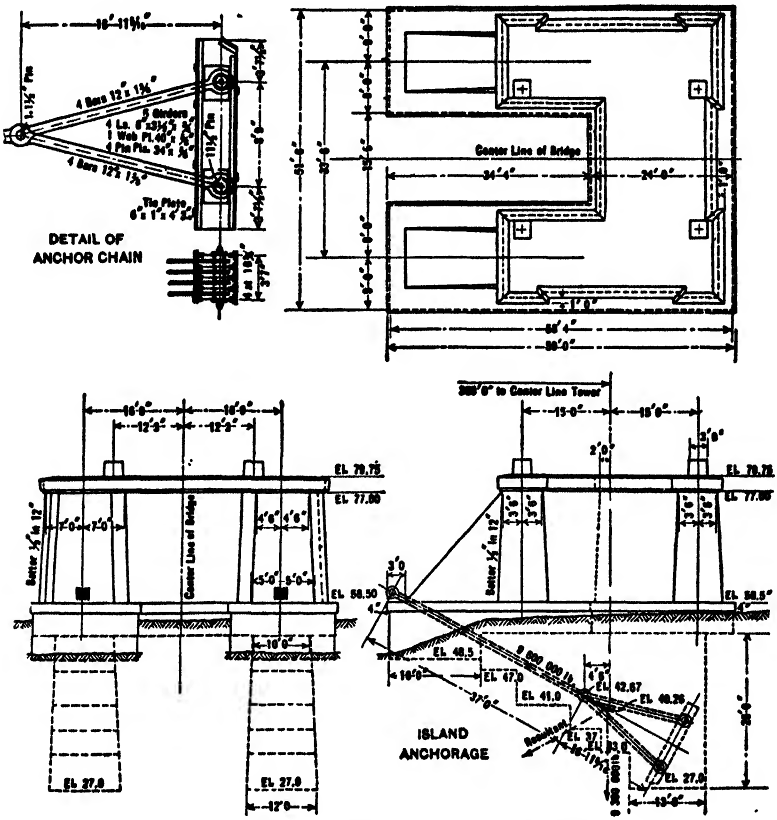



FIG. B4.—Anchorage of Florianopolis Bridge.
(Island Anchorage on Rock Foundation.)

trenches excavated in the solid rock; the sides of these trenches were given a negative batter to increase the vertical resistance. On this construction is superimposed the buttressed concrete anchorage structure.

The west anchorage is on lower ground on the mainland. The excavation did not reveal rock as anticipated, and a pile foundation had to be used; about 25 per cent of the piles (located under the forward portion of the anchorage) are battered in the direction of the resultant pressure.

Each anchor cable, consisting of heat-treated eyebars, divides into two branches for connection to the anchor girders (Fig. B4). Each anchor girder, 16 ft. long, consists of five built-up girders reinforced with pin-plates and connected together by tie-plates. At the Y-point of each anchor chain, a built-up pin seat was provided to hold the pin in accurate position during the placing of the concrete around the anchor girder and the connecting eyebars; these temporary pin seats were burned off after the girders and the lower eyebars were securely concreted. Above these points, the anchor cables were boxed in during the completion of the concreting in order to prevent adhesion of the concrete before full dead-load strain was in the cables. When the anchorage erection was completed, the boxing was removed; the tunnels were left around each anchor cable until the steel superstructure was erected and the full dead-load stress was in the cables. Then the eyebars were covered with a protective coating of minwax and the tunnels filled with concrete.

**10. Construction of the Main Piers.**—The four main piers of the Florianopolis Bridge are cylindrical concrete shafts, 16 ft. in diameter, with coping 17 ft. in diameter. The base of each pier is 30 ft. square. The unusually small size of the pier shafts was made possible by the adoption of rocker towers, eliminating the pier bending stresses due to tower flexure. A separate pier cylinder is provided under each tower leg.

Construction was commenced in the spring of 1923.

An open square cofferdam of steel sheet piles was driven for each of the four main pier foundations. The rock lay at depths from 30 to 60 ft. below water.

The entire substructure work, including the concrete piers and anchorages, was completed in June, 1924.

**11. Cross-section of the Bridge.**—The Florianopolis Bridge was specified to carry a 28-ft. roadway, a meter-gauge electric railway, a 24-in. water main, and a 9-ft. sidewalk. The arrangement of cross-section finally adopted is shown in Fig. B5. The sidewalk is carried on an outside bracket along the north truss; and the water main is located just inside the south truss, to help equalize the loads. The railway track is near the middle of the roadway, the trackway being covered with planking to provide a continuous surface. This planking is so detailed as to facilitate fitting the steel rails whenever the railway connections are completed.

On account of the inherent stability of the suspension con-

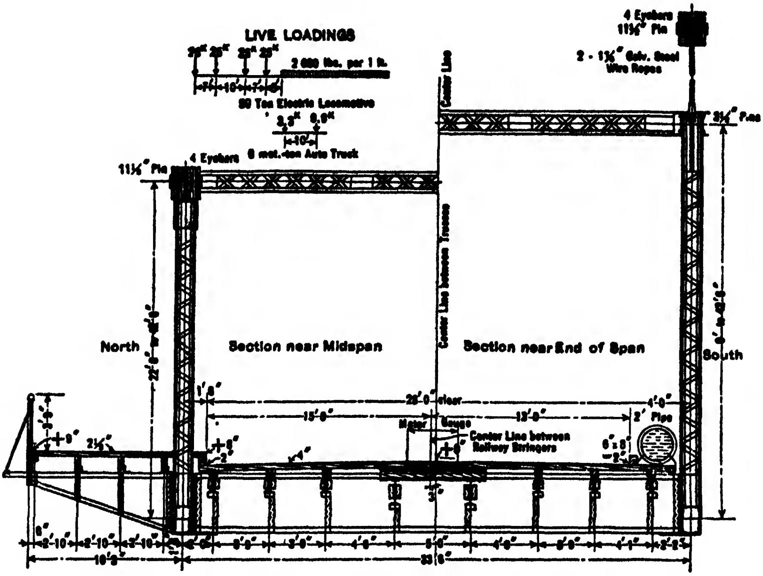



FIG. B5.—Cross-section of Main Span, Florianopolis Bridge.

struction, sway-bracing is unnecessary. Adequate systems of lateral bracing have been provided.

**12. Design Loads.**—The dead load used in the design of the main span totaled 4370 lb. per lin. ft.

The live load was taken at 2000 lb. per lin. ft., plus 10 per cent for impact, or a total of 2200 lb. per lin. ft., for the design of the stiffening trusses. For the design of the cables, which require full-span loading (and extreme temperature) for maximum stress, the live load was taken at 1850 lb. per lin. ft.

The floor was proportioned for the concentrated moving loads.

**13. Stresses by Method of Elastic Weights.**—The stresses in the main span of the Florianopolis Bridge were first calculated by the method of "elastic weights."

The first step is to calculate the stresses, $u$, produced in all the members of the unloaded structure by $H = 1$. For each truss member, that stress is given by $u = \frac{y}{r}$, in which $r$ equals the lever arm of the member about its center of moments, and $y$ equals the vertical ordinate representing the respective effective lever arm of $H$.

The ordinate $y$ is always measured along the vertical through the center of moments, and it is always measured from the closing chord of the cable. For the cable chord members, $y$ is measured to the actual center of moments; for all other chord members, $y$ is measured to the point of the chain directly above the actual center of moments; for the diagonals, $y$ is measured from the closing chord of the cable to the prolongation of the cable member above the diagonal.

The strains, $\Delta s$, producible by the stresses, $u$, in the individual members are given by $\Delta s = \frac{us}{EA}$, in which $s$ and $A$ are the length and gross section of the member.

The next step is to calculate, for each truss member, the elastic weight, $w$, given by,

$$w = \frac{\Delta s}{r} = \frac{us}{EAr} = \frac{ys}{EAr^2}.$$

Each of these weights $w$ is considered as applied at the center of moments of the respective member, except that, in the case of a diagonal, the elastic weight, $w$, is resolved into two parallel opposing components, $P$ and $Q$, applied at the respective ends of the diagonal:

$$P = -q\frac{w}{a}; \quad \text{and} \quad Q = p\frac{w}{a};$$

in which $p$ and $q$, differing by the panel length, $a$, are the respective distances of $P$ and $Q$ from $w$.

These elastic weights, $w$, for the chord members, and the component elastic weights, $P$ and $Q$, for the diagonals, are combined and treated as applied simultaneously on the span. The resulting moment diagram or equilibrium polygon is the "elastic curve." It is the influence line for $H$, if all ordinates are divided by a constant $N$, given by $N = \Sigma(u\Delta s)$, in which the summation extends over all members of the structure that are affected by $H$, including anchorage steel, towers, back-stays, cables, suspenders, and truss members.

On the elastic curve, or $H$-curve, constructed as described, the simple-span straight-line influence diagrams for the various truss members (drawn to corresponding scale) are superimposed, and the intercepted areas are the required influence areas for the respective members.

**14. Stresses by the Deflection Theory.**—Following the calculation of the stiffening truss stresses by the Method of Elastic Weights, a re-calculation was made by the "More Exact" Method, or Deflection Theory. The methods of analysis that do not make correction for the deformed configuration of the suspension system are only approximate; the resulting values of the stresses are too high, satisfying safety but not economy. The application of the Deflection Theory to the Florianopolis Bridge yielded a material reduction in the values of the stresses previously calculated by the less exact method. Thus, for the quarter-points of the span, the Deflection Theory showed a maximum bending moment of 24,367,000 ft. lb. (with load extending from one end over 0.450 of the span), as compared with a bending moment of 30,300,000 ft. lb. obtained by the approximate method—a reduction of 20 per cent. For the middle or half-point of the span, the corresponding moments were 16,929,000 ft. lb. (with load covering the middle 0.460 of the span), as compared with 26,800,000 ft. lb. obtained by the approximate method, a reduction of 37 per cent.

In more flexible suspension spans, greater reductions of calculated stress are found by application of the "more exact" method. In the Philadelphia–Camden Bridge, these were 34 and 38 per cent, respectively. In the Mount Hope Bridge, the reductions were 50 and 35 per cent, respectively.

The final values of the maximum stresses in the Florianopolis stiffening truss, by the "more exact" theory, are about 14,500 lb. per sq. in. for the assumed loading. Since a working stress of 18,500 lb., or even 20,000 lb., is amply safe for stiffening truss members, the Florianopolis Bridge has a capacity for concentrated loads considerably in excess of that specified.

**15. Design of Eyebars and Pins.**—The suspension span was designed so that the entire dead load was carried by the eyebar cables. This dead load was not uniform either along the cable or along the horizontal so that the pins connecting the eyebars lay neither in a catenary nor in a parabola. The position of the eyebar cable in space was an equilibrium polygon passing through three fixed points, the top of each tower and a point at the center of the span having a sag of 120 ft.

Each panel-length of the cable consists of 4 similar eyebars of 12-in. depth and of width varying from 2 to $1\frac{13}{16}$ in.

For connecting the 12-in. eyebars of the cables, pins $11\frac{1}{2}$ in. in diameter were used; consequently, with eyebar heads the same thickness as the body of the bar, the bearing pressure on these pins would have exceeded the working tension in the eyebars. To reduce this high unit bearing pressure the heads were made $\frac{1}{8}$ in. thicker than the remainder of the bars. To resist the high unit stress, the pins were made of special heat-treated steel with a yield point ranging between 60,000 and 65,000 lb. per sq. in., and a tensile strength ranging between 100,000 and 105,000 lb. per sq. in. A few of the pins are of chrome-nickel steel having the same range of strength.

To facilitate the entry of the pins during erection, a novel detail in the form of oval pin-holes was developed. The hole is made somewhat elongated axially and enlarged on the inside, to provide more clearance for the insertion of the pin or for slipping the bar over the pin, while retaining a close fit in the segment in contact. The outer or bearing semicircle exceeds the diameter of the pin by only 0.005 in., while the inner semicircle is bored to the diameter of the pin plus $\frac{1}{32}$ in., and the two centers are separated $\frac{1}{8}$ in. along the axis of the eyebar. The unusually close fit thus secured along the bearing surface reduces the secondary stresses in the eyebar head.

**16. Method of Erecting Cables.**—To make the erection of the eyebars as simple and cheap as possible, it was decided to hang them by hand-operated chain hoists which in turn were suspended from flexible ropes; then, after all the eyebars and pins were placed and supported by the ropes, to swing the eyebar cables by playing out the chain hoists.

To save as much erection material as possible, flexible hoisting ropes 1 in. in diameter were used, so that after having served their purpose in the erection of the two eyebar cables they could be cut up and used for ordinary hoisting rope. The grade selected had an approximate ultimate strength of 90,000 lb. per rope; a factor of safety of three was used for erection. At 32,000 lb. per rope, twenty-four 1-in. ropes were required.

During the erection of the eyebars, the main tower columns were locked into the 185-ft. approach spans. To insure that the horizontal components of the stresses in the erection rope would be taken entirely by the anchorages, temporary I-beam grillages were provided, bearing on rollers which in turn rested on top of the column castings.

It was necessary to ascertain either the modulus of elasticity of the ropes or their elongation under a stress of 32,000 lb. per rope. The lengths of the erection ropes, under full load, had then to be shortened by the amount of this elongation so that when they supported all the eyebars, the sag at the center would not exceed 115 ft. (Ordinarily, the rope used for suspension bridge cables, consisting of 6 strands with a wire center, has a modulus of elasticity of from 12,000,000 to 20,000,000, depending upon the lay of the rope and the stress it carries. Ordinary flexible hoisting rope, consisting of 6 strands with a hemp center, has a modulus of elasticity of from 5,000,000 to 10,000,000.)

The method adopted for the erection of the eyebars was to attach them and their pins to the lower ends of the hand-operated chain hoists, the upper ends of which were held by clamps around the ropes. The load at each panel point was 19,000 lb., so that hoists with capacities of 20,000 lb. were selected. When all bars were suspended, the chain hoists could be played out and the eyebars lowered from a sag of 115 ft. until they were carrying their own weight under a sag of 116 ft. As the loads on the hoists were released, the erection ropes were relieved of their stress, and shortened to a length corresponding to their own weight (but not to their original length because of the permanent set they had acquired). The sag meanwhile decreased from 115 ft. until it reached an amount dependent on the new length of the ropes. This decrease had to be determined by advance tests in order to provide the proper amount of chain for the operation of the chain hoists. The greatest vertical movement was of course at the center of the span, the amount decreasing toward the towers at which points there was none. The center of the ropes was 6 ft. above the center of the main pins on top of the towers. The chain hoists were originally set so that the eyebar pins would be a uniform distance of 6 ft. below the ropes when the eyebars were first lifted into position.

As this method of erection of eyebars was a new departure in bridge construction, it was decided to make a series of advance tests on the actual ropes used.

The ropes were to be used twice in the field during the erection of the bars. They were first to be put up over the north columns. Supporting its own weight, each rope would be stressed to about 3000 lb. The eyebars for the north cable were then to be supported from the ropes, the average stress per rope under this condition being about 30,000 lb. The north cable would then be swung and the ropes again would support their own weight only, under a stress of 3000 lb. per rope. Next the ropes were to be transferred to the tops of the south columns and the operation repeated.

To approximate erection conditions as far as possible, each of the test pieces was stressed twice to an amount exceeding 30,000 lb.

Readings of the elongations at each increment were plotted, making it possible to compute the moduli of elasticity for any stresses. From these moduli could be determined the proper lengths for the ropes and the movements that would take place during the erection of the cables.

From the tests, the average modulus of elasticity of the ropes was found to be 8,000,000 when stressed the first time to 28,000 lb., and 8,300,000 when stressed the first time to 32,000 lb.; and practically the same values of $E$ were found for the second stressing to the same loads.

**17. Erection of Ropes, Trolleys, Clamps and Chain Hoists.**—The steel towers were erected by February 1, 1925. The beam grillages, temporary rollers, and rope shoes were then placed on top of the two north columns. All the eyebars from the anchorages up to the viaduct floor level had been placed previously, and the temporary eyebars and the rope girders on the north side of the bridge were erected during the first week of February. A continuous ⅞-in. rope was then run across the main channel, lifted to the top of each tower, and placed on the top transverse tower strut adjacent to the north eyebar shoes.

Twelve of the twenty-four reels containing the erection ropes had been transferred from the Continent to the Island Side and, on February 10, 1925, everything was ready for the placing of the pilot rope. The free end of this pilot rope was unwound from its reel on the Continent Side and lifted to the floor level of the viaduct. It was then fastened to the continuous ⅞-in. rope, hauled up to the top of the tower, across the strait, and over the top of the Island tower, using the hoisting engine on the Island viaduct. The two ends were then wound one and one-half times around the rope girders and clamped into position.

The centers of the rope shoes had been placed 2½ ft. back from the center of the eyebar shoes on the tops of the main columns in accordance with the calculations. The pilot rope was then lifted into the grooves on the rope shoes. An observation showed the sag to be between 86 and 87 ft., or within 1 ft. of the calculated distance. During the next three days the remaining twenty-three erection ropes were placed in a similar manner and fastened to the rope girders so that the sags were the same as the sag of the pilot rope. The rope ends were fastened together in pairs, the free end of one rope after one and one-half complete turns around the rope girder being clamped to its adjacent neighbor by six rope clamps.

After the main-span and back-stay trolleys had been placed in position on the ropes, the main-span trolley was moved across from the Island to the Continent Side, placing the clamps which bound the twenty-four ropes into one cable unit. These clamps were placed at variable distances along the erection ropes in such positions that the chain hoists would hang approximately vertical when all the eyebars were supported by the ropes. After the twenty-six clamps were in position, the main-span trolley moved back to the Island side attaching the twenty-six Harrington chain hoists to the clamps. These two operations took about a week. In the meantime, the back-stay trolleys had been placing the clamps on the Continent and Island back-stay portions of the erection ropes, starting at the top of the towers and working down toward the rope girders.

**18. Erection of the Eyebar Cables.**—By February 23 all the clamps and chain hoists were in place and everything was ready for the erection of the eyebars. To eliminate the driving of pins up in the air, the two inner eyebars of alternate panels were assembled in the yard and the two pins inserted in the holes. In this manner all pins except those at the tops of the tower columns were inserted in the material yard.

The eyebars were loaded on a lighter which was towed out to position under the ropes and anchored. The main-span trolley moved to a position over the lighter, picked up the two inner eyebars with pins for Panel 24–26, Island Side, raised them up into position at the proper chain hoists where they were fastened to the lower ends of the chain hoists approximately 6 ft. below the erection ropes. The trolley then lifted the eyebars, 24–26, Continent Side, into position. Fig. B6 shows the trolley lifting the second pair of eyebars, the first pair having already been attached to the chain hoists.

At the start the eyebars were grouped toward the center of the span. The object was to put as much weight as possible on the central part of the main-span erection ropes so as to bring the rope shoes from 2½ ft. back of the center of the columns to a position over the center of the columns as quickly as possible. By the time twenty-six eyebars had been supported on the erection ropes, the rope shoes had moved from a position 2½ ft. away from the column center to 1 ft. from the column center. It was then felt safe to commence the erection of the eyebars on the two back-stays.

When all the eyebars had been placed, the temporary rope shoes had moved from the original position of 2 ft. 6 in. back of the saddle-casting to a position practically over the center of the saddle. The entire erection of the 156 bars took two weeks.

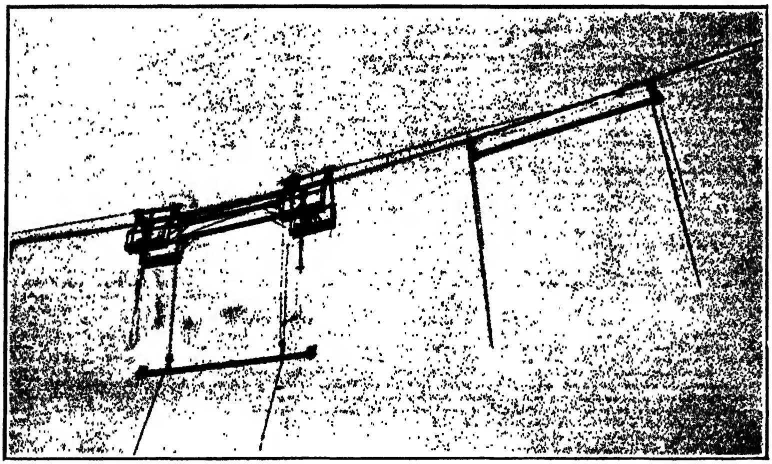


Fig. B6.—Trolley Lifting Pair of Eyebars, Florianopolis Bridge.

By March 7, all the eyebars were erected, the entire eyebar cable hanging from the erection ropes. The day being clear and calm, it was decided to swing the eyebar cable that afternoon. The actual sag at that time when the erection ropes carried all the eyebars was 113 instead of 115 ft., the difference being partly due to not placing the gusset-plates and hangers as was originally contemplated when the calculations were made. The eyebars had been slipped over the pins without difficulty, due to the elongated pin-holes and also to the use of short pilot nuts on each pin.

Men took positions on the eyebars at the chain hoists and at 2:30 P.M. the signal was given for the men to slacken off the chain hoists; in a few minutes the normal 6-ft. gap between the erection ropes and eyebars began to increase. Some of the men operated the chain hoists faster than others and to keep the eyebar cable in a smooth curve it was necessary to have them wait until the slower ones could catch up. By 3:15 all the chain hoists were slack, showing that the eyebar cable was swinging free of the ropes under its own weight and the erection ropes were carrying their own weight under a sag of between 99 and 100 ft. instead of the calculated sag of 99 ft. The entire operation of swinging took 45 min. and was a complete success.

As the next operation, the main-span trolley, starting from the Continent tower, moved across the ropes removing the chain hoists and clamps and placing the rope hangers and steel hangers. In the meantime the back-stay trolleys were removing the clamps and eyebar supports from the back-stay parts of the ropes. These operations took about a week. The main-span and back-stay trolleys were taken down and the twenty-four ropes were moved from the north columns to the top struts of the towers.

The rope shoes, rollers, and I-beam grillages were transferred to the tops of the south columns, and the rope girders and temporary eyebars were transferred to the south sides of the viaducts. The rope ends were then fastened to the rope girders in a manner similar to that used for the north cable, and the ropes were lifted into the grooves on the rope shoes on top of the tower columns. The three trolleys were then re-erected on the ropes and the rope clamps and chain hoists placed. These several operations took about two weeks.

On March 26, erection of the south eyebar cable was commenced. The method used here was the same as that used for the north cable and the entire 156 eyebars were placed in one week, just one-half the time that it took for the north cable.

By April 2 all the eyebars were hanging from the erection ropes. At 3:30 P.M. the signal was given for the men at the hoists to slacken off; by 4:00 P.M. all the hoists were loose and the south eyebar cable was self-supporting. Clamps and chain hoists were then removed; rope hangers and steel hangers were erected; and the three rope trolleys were taken down. These operations took about four days.

**19. Erection of Trusses.**—The first two truss panels at each end of the bridge were erected by jinniwinks standing on the viaducts, the remainder of the main span being erected by the overhead trolley method.

Two erection ropes were lifted from the rope shoe and placed on supports on top of the transverse struts adjacent to the cast-steel shoes, and a small trolley was mounted to run on these ropes.

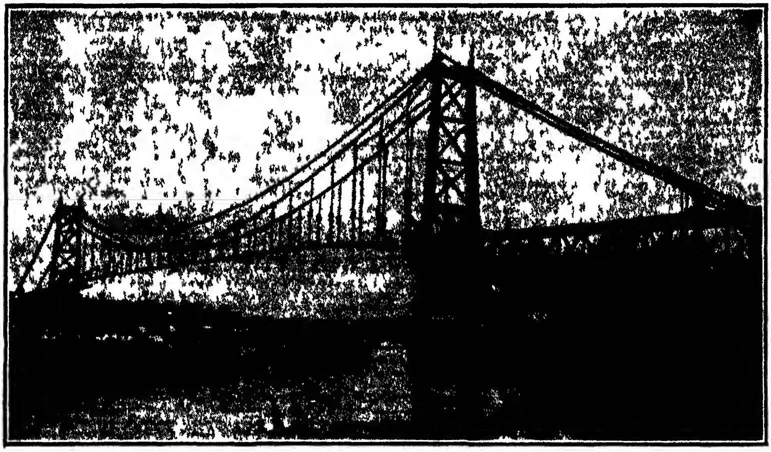


FIG. B7 —Erection of Stiffening Truss, Florianopolis Bridge.

The south bottom chords were the first truss members erected by the overhead method. These were placed starting from the Continent and working toward the Island. When the Continent half of the bottom chords was suspended from the hangers the weight distorted the eyebar cable. As the remainder of the south bottom chord was placed the eyebar cable gradually came back to its symmetrical form (Fig. B7); and when the last section of bottom chord was placed there was a gap of about 6 in., due to the camber being high under the partial dead load. The south truss diagonals were then erected, using the same trolley. The bottom chords were erected in 14 hours and the diagonals in an equal period.

Another trolley was then erected on two more of the erection ropes placed adjacent to the cast-steel shoes on the north columns and the north bottom chords and diagonals were erected in a similar manner. The bottom chords of this truss were placed in 10 hours and the diagonals also in 10 hours.

The north and south top chords were then erected, first on the Continent Side and then on the Island Side. This operation took about 18 hours. Erection of the floor-beams, stringers, and bottom laterals then took place, followed by the top laterals, top struts, and portals. The total time consumed in the main-span erection was 45 working days for the erection of about 1000 members.

**20. Placing Temporary Counterweight and Drilling Holes in Web Members.**—When all the steel was erected the total dead load was only 3000 lb. per lin. ft., so that the sag had not reached 120 ft. Under this condition it was impossible to connect the web members and top chords because the fabricated lengths of these members were based on the geometric lengths they would occupy under a sag of 120 ft.

The delivery date of the floor lumber was uncertain so that some temporary load had to be placed upon the main span to bring the eyebar cable to the correct dead-load position, in order that the holes in the blank gusset-plates could be drilled and the diagonals outside Panel Points 16 could be connected under zero stress. As there was considerable sand on the beach along the Continent shore it was decided to make use of this material. A sand load equivalent to 1400 lb. per lin. ft. of bridge was placed on the steel floor system, and this brought the diagonals from Points 0 to 16 at each end of the bridge to such position that connections could be made with practically no drifting. The 1728 holes in the bottom-chord gusset-plates between Points 16 were then drilled and the diagonals connected under this condition. One of the erectors set a record by drilling 240 holes (in ½-in. material) in 1 day of 8 working hours.

**21. Riveting and Painting.**—There were more than 50,000 field rivets to be driven on the main span and this work was completed during July, 1925. The temporary sand counterweight was then removed and the structure given two coats of field paint.

The last contingent of the erection organization arrived in the United States on August 31, so that a few days less than fourteen months elapsed from the time that the first section of field forces left the country until the last section returned; the actual time of field erection of main span and viaducts was just about one year.

**22. Total Weights, Quantities, and Costs.**—The weight of steel in the Florianopolis Bridge, including the approaches, is approximately 4400 tons, made up as follows:

| | | Tons |
|---|---|---|
| Cables: | Eyebars and pins | 780 |
| Main Span: | Trusses and bracing | 840 |
| | Floor system | 420 |
| Main Towers: | Columns and bracing | 830 |
| | Castings | 90 |
| Anchorages: | Eyebars and girders | 110 |
| Approaches: | Spans (including floor and bracing) | 960 |
| | Towers and bracing | 290 |
| Miscellaneous: | Railings, etc | 80 |
| Total | | 4400 |

The total quantity of concrete in the anchorages and piers is approximately 14,500 cu. yd., made up as follows:

| | Cu. Yd. |
|---|---|
| Island anchorage | 3,500 |
| Continent anchorage | 6,000 |
| Piers and abutments | 5,000 |
| Total | 14,500 |

The total cost of the Florianopolis Bridge, including the contractor's profit, as represented by the amount of the general contract, was upward of $1,400,000. (The exact amount is difficult to state on account of the fluctuating value of the

Brazilian currency at the time.) Of this total amount, the cost of the superstructure, as represented by the amount of that sub-contract, was approximately one-half.

**23. Conclusions.**—The successful design, fabrication, transportation, and erection of the Florianopolis Bridge has demonstrated the fact that suspension bridges with eyebar cables are practicable and that falsework of the ordinary kind, or, on the other hand, elaborate staging to support the eyebars during erection, is unnecessary.

A particularly gratifying feature of the bridge from an engineering standpoint was the manner in which the actual movements both of the erection ropes and of the permanent steel towers and main span agreed with the designed movements. It was possible from the calculations to predict the movements or positions under various conditions of erection with a very small error.

The placing of the wooden floor was done by the general contractors, Byington and Sundstrom, and completed during March, 1926. The official opening of the bridge took place on May 3, 1926.

The entire field operations covering a period of one year from August 1, 1924 to August 1, 1925, were successfully accomplished, 6000 miles from home, without the loss of a single human life. Furthermore, the actual cost of the entire steel superstructure was within the estimate. The Florianopolis type of suspension bridge, therefore, has shown itself to be a practical, a safe, and an economical type of structure.

The economic advantage of the Florianopolis type of suspension bridge has received commercial recognition and confirmation in the proposal and adoption of this form of construction in two successive bidding competitions. These are the 700-ft. spans over the Ohio River at Point Pleasant, W. Va., and St. Marys, W. Va., respectively. In each case, the Florianopolis type was proposed by the American Bridge Company as an alternative bid, at a price sufficiently below the tenders on the conventional suspension designs to win the respective competitions. The bids showed that, for a span length of 700 ft. and for the relative unit stresses specified, heat-treated eyebars are more economical than wire cables at present prices; and that the combination of the eyebars with the Florianopolis type offers a further saving in the cost of the structure.

For further particulars relating to the design and construction of this bridge, the reader is referred to the paper "The Eyebar Cable Suspension Bridge at Florianopolis, Brazil" by D. B. Steinman and Wm. G. Grove, in the *Transactions* of the American Society of Civil Engineers (Vol. 92, p. 266, 1928).

# APPENDIX C

## THE OHIO RIVER SUSPENSION BRIDGE AT PORTSMOUTH

**1. Introduction.**—The "General U. S. Grant Bridge" (Fig. C1) between Portsmouth, Ohio, and Fullerton, Ky., completed 1927, is of interest because of a number of unusual features.

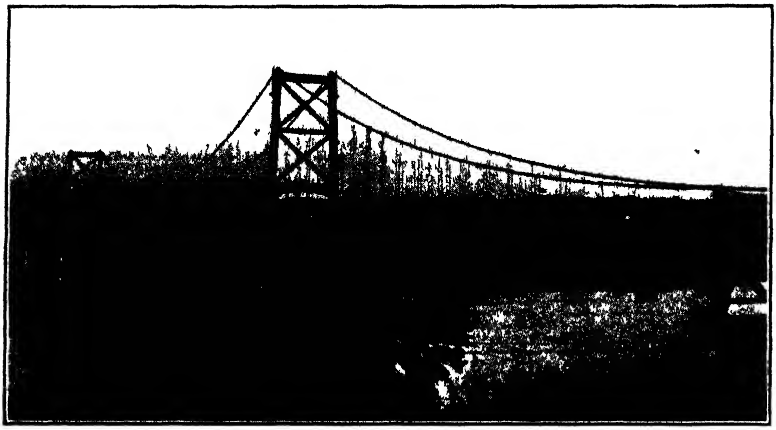


FIG C1—General U S Grant Bridge (1927) over Ohio River at Portsmouth, Ohio
Span 700 ft 2 Wire Cables Rocker Towers
Continuous Trusses 14 ft Deep Spaced 31½ ft
Designed by Robinson and Steinman Consulting Engineers

In general design, it departs from conventional practice in the use of a continuous stiffening truss, rocker towers, rocker bents for supporting the cables, and sand-filled anchorages. In details, it embodies novel features in the expansion connections, in the adoption of forged cable bands, in the splicing of stringers for continuity, and in the railing design. In construction, the method of spinning and placing the cables and the method of erecting the stiffening trusses were unusual.

The modern period of suspension bridge construction in Ohio River territory commences with the adoption of this type for the Portsmouth bridge. Following two decades of almost exclusive adherence to the cantilever, the pendulum has swung back to the suspension type and practically all of the new Ohio River spans begun or projected since 1927 are designed as suspension bridges. Some of these are patterned after the Portsmouth bridge. (In the case of the Point Pleasant and St. Marys bridges, a departure was introduced by the adoption of the Florianopolis type employing eyebar chains, following the receipt of an alternative proposal for such design in the bidding; these will therefore be the second and third suspension bridges to be built of this type and to employ high-strength heat-treated eyebars, the Florianopolis Bridge being the first.)

**2. Competition of Types.**—The Portsmouth bridge was undertaken as a toll bridge project, and the official design was a cantilever bridge with a main span of 700 ft. The Dravo Contracting Company, which was interested in the bridge both as part owner and as prospective contractors, retained Robinson and Steinman to prepare an alternative design of the suspension type for the purpose of bidding. Three bids were received on the cantilever design, but the bid on the suspension design was 10 per cent under the lowest bid for the cantilever. The suspension design was accordingly adopted.

In this instance, the suspension type was handicapped by unfavorable local conditions. The topography and the existing railroad tracks interfered with the customary location of the anchorages at the ends of the suspension structure, without a material increase in the length of the main structure. The problem was solved partly by lengthening the side spans from 275 ft. to 350 ft., and partly by placing the anchorages 150 ft. away from the ends of the side spans. This necessitated lengthening the cables and carrying them an idle distance of 150 ft. at each end, and providing cable-supporting bents that would otherwise have been unnecessary.

Despite these handicaps there remained sufficient inherent economy in the suspension design to make it successful in the

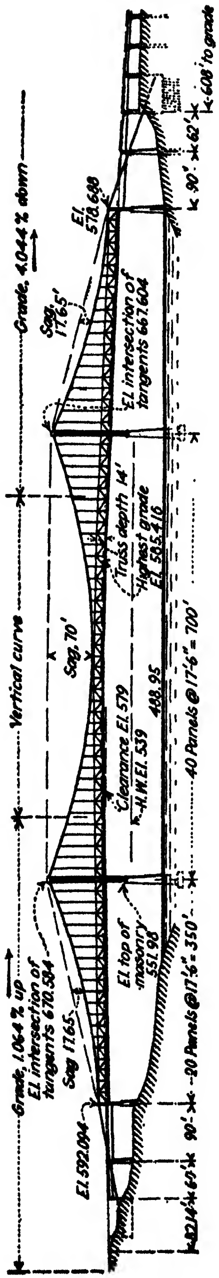


FIG. C2.—General Elevation of Portsmouth Bridge.
(Note Extension of Cables to Anchorages.)

competition. This result has served to correct an impression that the suspension type cannot compete with the cantilever for spans as small as 700 ft. Where local conditions especially unfavorable to the economy of the suspension type do not obtain, it can compete with the cantilever for much smaller spans. Comparative estimates in fact show the suspension type to be economical for spans as small as 400 ft.

**3. Principal Dimensions.** — As built, the Portsmouth bridge (Fig. C2) has a main span of 700 ft. and two side spans of 350 ft. each. The main cable sag is 70 ft. or 1 : 10 of the span; the side-span sag was made 17.65 ft. in order to equilibrate the cable under full dead load. The stiffening truss was made 14 ft. deep, or 1 : 50 of the span length, determined by the degree of rigidity desired.

To provide a clear roadway of 28 ft. the spacing of the trusses was made $31\frac{1}{2}$ ft. (See Fig. C3.) The cables are hung in the truss planes. The width of the towers increases from this saddle spacing at the top to 40 ft. $9\frac{7}{8}$ in. at the base, corresponding to an adopted batter of $\frac{1}{2}$ in. per foot for the tower legs. This form of tower, with main legs battered, was introduced into suspension bridge practice by H. D. Robinson, having been recommended by him in 1911 for the proposed Rhine bridge at Cologne and first applied by him in 1919 in the design of the Rondout Bridge at Kingston, N. Y., completed 1922, the object being to provide clearance for roadway and stiffening truss without interference

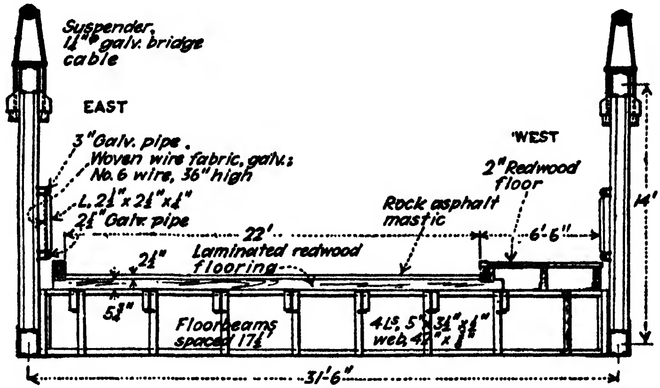



FIG. C3.—Portsmouth Bridge. Cross-section of Suspended Spans.
(Design provides for future widening of roadway to 28 ft. and future sidewalk on outside brackets.)

by the tower columns, incidentally securing the advantage of increased transverse stability.

**4. Continuous Stiffening Truss.**—The Portsmouth bridge is the second American suspension bridge built with a continuous stiffening truss; the first application was made in the design of the Rondout Bridge. The continuous truss was adopted because it offers greater efficiency than the conventional two-hinged type. This increased efficiency may be in the form of superior economy, or superior rigidity, or both, depending upon the proportions adopted. Comparative designs show that the continuous type is more rigid for the same total quantity of steel, and more economical for the same specified degree of rigidity. It offers incidental advantages in greater efficiency of the continuous lateral truss, in improved and simplified supporting details at the towers, and in reduced variation between minimum and maximum sections.

In the Portsmouth stiffening truss, the variation of chord sections is from a minimum of 24.3 sq. in. (at the $L_0$ end of the side spans, and near the ⅛th points of the main span) to a maximum of 33.8 sq. in. (at the towers and at mid-span). Only two intermediate sectional areas were required. All chord sections were made up of two 15-in. channels (34, 40, 45, or 50-lb.) with top cover plate $12 \times \frac{3}{8}$ in. Instead of bottom lacing, $9 \times \frac{3}{8}$ in. batten plates at 3-ft. spacing were used. All diagonals are made up of two 10-in. 20-lb. channels, tied together with batten plates, except in the panels adjoining the tower, where 12-in. 25-lb. channels were necessary.

**5. Rocker Towers.**—The Portsmouth bridge is one of the first suspension bridges to be built in the United States with towers of the rocker type. (The Florianopolis Bridge, also designed by Robinson and Steinman, was an earlier application of rocker suspension towers.) This type was adopted because it is believed to be the most scientific and most economical. It yields a substantial saving in main material of the tower (by eliminating the bending stresses caused by tower deflections), in the pedestals at the base, and in substructure; and it minimizes the difficulty and expense of pulling the towers back toward the shores during the erection of the cables. The main pier shafts, only 8 ft. in diameter under coping, could not have served for towers of the fixed-base type.

The Portsmouth towers (Fig. C4), like those of the Florianopolis Bridge, rock on line bearings (33 in. long) between upper and lower steel castings. The upper casting, secured to the tower leg, has its bottom face finished to a convex surface of 10-ft. radius. The lower casting, secured to the masonry, has its top finished to a plane surface. This line bearing has to carry a reaction of 1000 tons per tower leg. Flanges hold the castings in proper relative position, and the lower casting has diagonal ribs set in grooves in the pier concrete for security against displacement, supplementing the anchor bolts.

To hold the towers during erection, the batten plates at the base of each column were extended and formed into wing-brackets; under these, small steel struts were wedged up on the pier masonry, while each bracket was tied to the pier by three $2\frac{1}{4}$ in. hog-rods. After the cables were in place the towers were freed at the base by burning off the temporary extensions of the base batten plates.

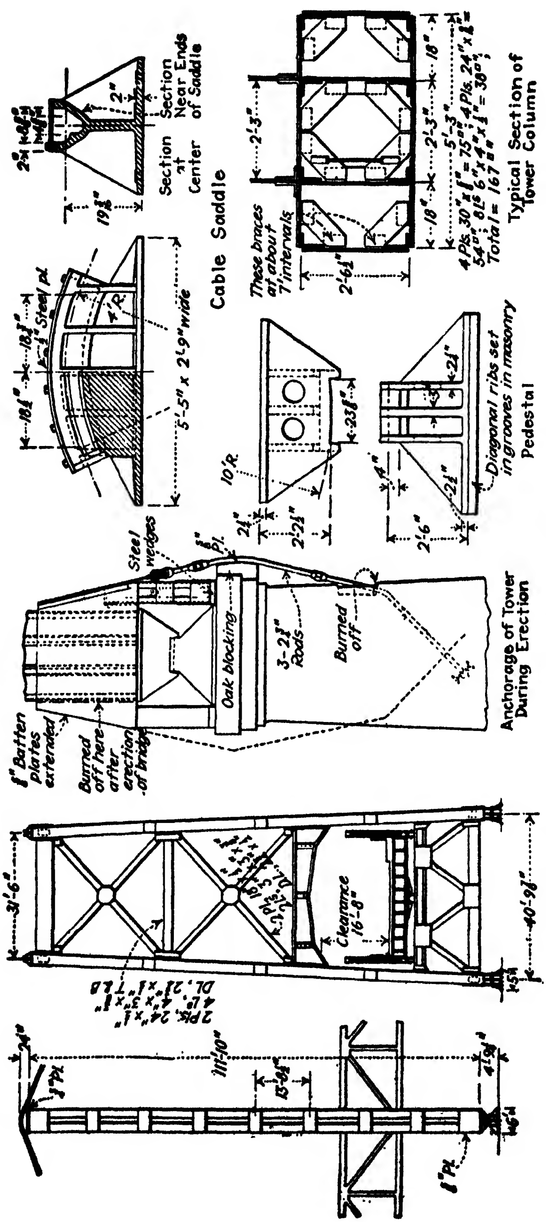



FIG. C4.—Portsmouth Bridge. Rocker Towers and Bearings.
Tower Stayed During Erection by Struts and Ties to Pier.

Each tower leg is made up of two closed box sections, tied together at intervals with batten or splice plates. This section is exceptionally efficient, yielding a radius of gyration of 23 in. Manholes and ladders are provided to give access to the interior of the columns; and the section is stiffened with interior diaphragm bracing at suitable intervals.

The basic unit stress of 18,000 lb. per sq. in., reduced by column formula and increased 25 per cent for inclusion of wind stress, yields a permissible unit stress of 17,700 lb. per sq. in. for the tower columns. The actual stress (11,700 from load, 1000 from wind and 4600 from portal flexure) is 17,300 lb. per sq. in.

At each end of the suspension structure (see Fig. C2) is a rocker bent to support the cables at the point where they bend downward on their way to the anchorage 150 ft. away.

In addition to sustaining the cable reactions, the rocker bent has to support the reversible end-reactions of the continuous stiffening truss, and the end-reactions of the 90-ft. approach girder span. Rocking action is necessitated by the thermal and elastic elongation of the cables between anchorage and top of bent. The difference of cable tension on either side, corresponding to the difference of cable inclination, necessitates provision for fixing the cables against sliding in their saddles at the top of the bent. The rocking of the tower, combined with the expansion of the main structure and of the approach span, necessitated sliding or rocking details for the respective end bearings.

The legs of the rocker bent are vertical, about 32 ft. high; they rest on pier bases and carry cable saddles at their tops. To permit the end of the stiffening truss to pass through the column, the cover plates are interrupted; sliding pin bearings transfer the truss reaction directly into the column. The approach girders are seated on a transverse truss connecting the two columns below the bridge deck.

**6. Design of Cables.**—For the cables of the Portsmouth bridge, bright wire was adopted instead of the more customary galvanized wire, on account of the faith of the designing engineers in the adequacy of the protection afforded by modern cable wrapping. The bright wire yields advantages of lower cost and greater strength. The cables of the Williamsburg Bridge, made of bright wire, are in an excellent state of preservation. When Roebling's Niagara railway suspension bridge was taken down (after a half-century of severe service under increasing locomotive loading), the bright wire composing the cables was found free from rust. Except possibly for coastal locations exposed to salt air, the author would unhesitatingly recommend the use of ungalvanized wire for bridge cables; with the proviso, of course, that the outside of the cables be suitably protected with tight wire wrapping.

When the design was prepared, the conventional method of spinning-in-the-air was contemplated for the Portsmouth cables (though later another method of construction was adopted). In order to avoid difficulty in the air-stringing arising from the stiffness of the wires in so short a span, such as was encountered in the cable stringing for the 705-ft. span of the Rondout Bridge, it was decided to adopt smaller wire, No. 8 of 0.162-in. diameter, in place of the usual No. 6 wire of 0.192-in. diameter. Three strands, each containing 486 wires, made up the required 30 sq. in. per cable. Each strand terminates in strand-shoes which are pin-connected to the anchorage eyebars. After the three strands were squeezed together (this was done by three-plunger hydraulic squeezing machines, Fig. C5, devised by H. D. Robinson) the cable had a diameter of $7\frac{1}{8}$ in. under the wrapping wire. The cables were wrapped with No. 9 soft annealed double-galvanized steel wrapping wire, by an electrically operated cable-wrapping machine, also the invention of Dr. Robinson.

The wire for the Portsmouth cables was specified to have a minimum yield point of 140,000 and a minimum ultimate strength of 220,000 lb. per sq. in.; the tested strength averaged over 230,000 lb. per sq. in. The adopted working stress was 70,000 lb. per sq. in. for the initial condition of the bridge (22-ft. roadway and inside sidewalk), and 80,000 for the possible future condition of the bridge (28-ft. roadway and outside sidewalk). In this connection, the author wishes to record his conviction that suspension bridge designers have been too conservative in the past in fixing wire cable working stresses, in comparison with working values permitted for other materials, and that stresses as high as 100,000 lb. per sq. in. or even higher may safely be used for this unexcelled bridge material.

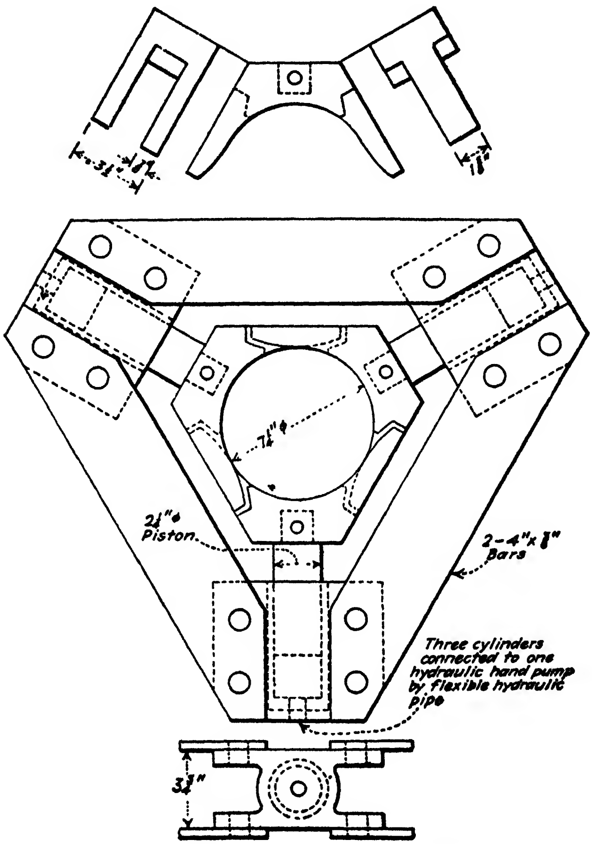



Fig. C5.—Three-plunger Cable Squeezer.
Actuated by Hand-operated Hydraulic Pump.

The maximum horizontal tension per cable, for the ultimate bridge loading is 2,158,000 lb. (1,530,000 dead load, 548,000 live load, and 80,000 lb. temperature), and the maximum inclined tension 2,380,000 lb., which is slightly under 80,000 lb. per sq. in.

The suspender at each panel point consists of two parts of

$1\frac{1}{8}$-in. diameter double-galvanized steel bridge cable, stranded 6 by 7, with a steel wire rope center. This suspender rope weighs approximately 2.57 lb. per lin. ft., and has a guaranteed minimum breaking strength of 62 tons. The calculated maximum panel concentration (including impact) is 79,400 lb., so that a factor of safety exceeding 3 is provided by these suspenders.

Departing from conventional precedent, the suspenders of the Portsmouth bridge were made *non-adjustable*. It was deemed preferable to depend upon scientific pre-calculation of correct suspender lengths and careful measurement in cutting and socketing, than to place reliance upon variations of field adjustment. Non-adjustable suspenders are a protection against future accidental or misguided disturbances from correct adjustment.

The attachment of the suspender ropes is made by means of their button sockets bearing under angle lugs riveted to the truss posts near their tops.

**7. Design of Anchorages.**—The outstanding novel feature in the Portsmouth anchorages (Fig. C6) is their design as large reinforced-concrete boxes filled with compacted sand as an economical means of producing mass resistance; this use of sand-filled chambers yields a material saving of concrete. The only previous application of such a feature was in the suspension bridge at Massena, N. Y., 400-ft. span, designed and built by H. D. Robinson in 1910.

The south or Kentucky anchorage, which also serves as end abutment, is founded on hard blue shale; this was stepped off, with top slopes of 1 to 6, to increase the resistance to cable pull; in addition, about two hundred $\frac{7}{8}$-in. rods, 12 ft. long, were driven into the rock at 45 degrees under the heel of the anchorage, to serve as dowels for greater security. The anchorage weight comprises 3800 tons of concrete and 1600 tons of sand; the latter contribution represents a saving of 800 cu. yd. of concrete.

The Ohio anchorage, similar in general principle of design but differing slightly in shape for local reasons, has caisson and pile foundations as rock did not occur near the surface. Three caissons were sunk by open dredging; one under each corner of the toe through 45 ft. of clay to hardpan, and one under the full length of the heel through 27 ft. of clay to a bed of sand and

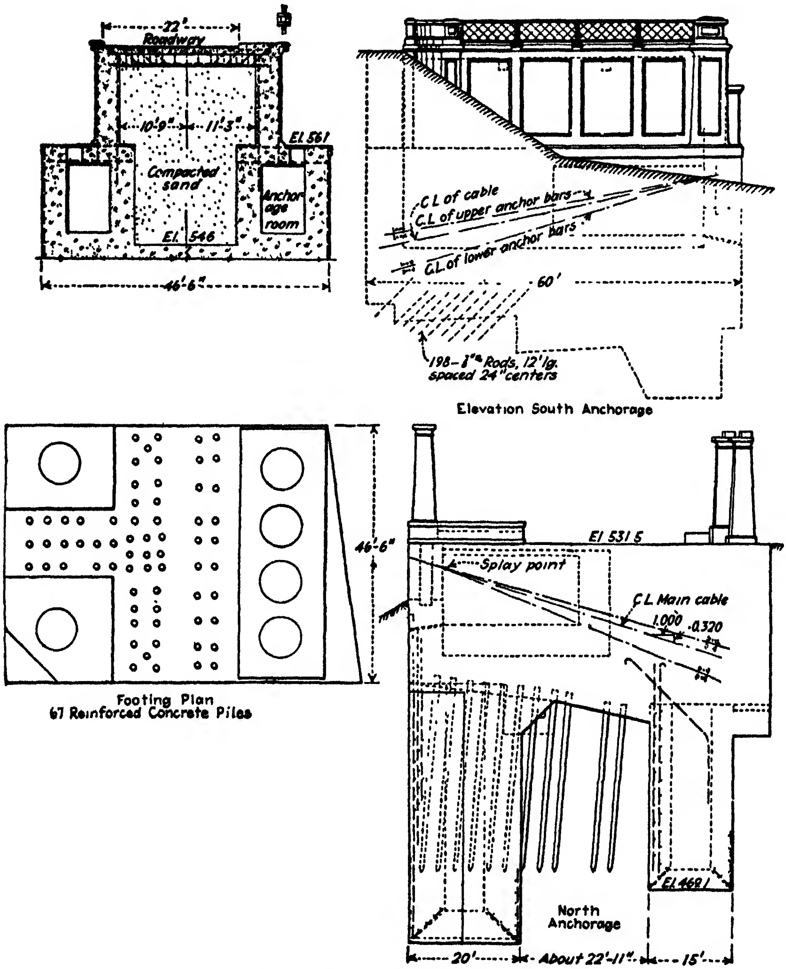



FIG. C6.—Anchorage Construction. Portsmouth Bridge.
South Anchorage Foundations on Soft Rock.
North Anchorage Foundations on Concrete Piles and Caissons.

gravel. In the T-shaped area between these caissons, 67 reinforced-concrete piles were driven, on an average batter of 1 in. per foot.

**8. Roadway Design.**—The Portsmouth bridge is designed and proportioned for an ultimate three-lane roadway, 28 ft. wide between curbs, and a future 6-ft. sidewalk on outside brackets. Pending the development of increased traffic demands, a part of the roadway space is utilized for an inside sidewalk, 6½ ft. wide. (See Fig. C3.)

The roadway floor is of asphalt on a laminated redwood base. The redwood planking, set on edge across the stringers, is alternately 5¼ and 5¾ in. high, so as to yield a more effective bond with the asphalt wearing coat; the latter is of rock asphalt mastic and has a minimum thickness of 2½ in. above the higher planks.

This floor construction of asphalt on redwood base is lighter (though not cheaper) than a reinforced-concrete floor, resulting in a saving of material in the supporting structure. It was adopted for the suspension design of the Portsmouth bridge in the bidding competition because the same floor construction was proposed in the official cantilever design. If a concrete floor had been adopted for both competing designs, the suspension design would have enjoyed an easier victory; for the suspension type is less affected in cost by dead load than any other bridge type. (The stiffening truss sections are not increased, but actually somewhat reduced, by an augmentation of the design dead load.)

**9. Loads and Stresses.**—The floor system of the Portsmouth bridge was designed for a maximum load of a 20-ton truck at any point, or for a normal load of three 15-ton trucks abreast. Impact of 37½ per cent was added in proportioning the stringers, and of 30 per cent in proportioning the floorbeams.

For the design of the stiffening trusses, cables, towers and anchorages, the assumed live load was a uniform load of 1400 lb. per lineal foot of bridge plus a superimposed concentrated load of 42,000 lb. at any point. The uniform load represents three lanes of motor vehicles averaging 50 lb. per sq. ft., and the concentration represents the excess weight of three 15-ton trucks abreast at any point in this stream of traffic.

The estimated dead load (per cable) used in the stress calculations for the final condition of the structure (with 28-ft.

roadway and outside sidewalk) was 1748 lb. per lin. ft. (flooring and sidewalk 674 lb., floor steel and railings 551 lb., trusses and bracing 387 lb., cables and suspenders 136 lb.). These figures are for the east cable; the dead loads for the west cable are somewhat less in consequence of the unsymmetrical cross-section of the bridge.

The design loading also included a moving wind load of 30 lb. per sq. ft. on $1\frac{1}{2}$ times the vertical projection of the structure, and a temperature variation of $\pm 60$ deg. F.

**10. Unit Stresses.**—The structural steel used is medium carbon steel of minimum elastic limit 36,000 lb. per sq. in. and ultimate strength 60,000 to 70,000. A basic unit stress of 18,000 lb. per sq. in., reduced for compression, was adopted for the towers, floor system and approach spans, and 24,000 for the stiffening truss (with column reduction and a limiting value of 21,000 for compression). The higher basic stress for the stiffening truss is justified by the fact that the truss is not essential for the safety of the structure against collapse.

In accordance with the usual practice, it was specified that the basic unit stresses for proportioning the towers and stiffening trusses may be increased 25 per cent when wind and temperature stresses are included, provided the required section is not thereby reduced. Accordingly, the maximum total unit stress in the stiffening truss (for contingent future maximum loading) is 30,000 lb. per sq. in. This leaves a safety margin of 6000 lb. per sq. in. below the minimum elastic limit of the steel used.

**11. Some Special Details.**—The continuity of the stiffening truss and its support against reversible vertical and lateral reactions on the rocking towers and cable-bents gave rise to special problems in devising efficient bearing connections; these connections had to take care of the reversible reactions while permitting the necessary relative longitudinal movements due to combined span expansion and tower rocking. The connection detail adopted consisted of sliding pins, rotating in annular bushings in one of the members to be connected and sliding between bronze bearings in the other member.

For the connection of the truss to the towers, the pins turn in the truss gusset and slide in a casting attached to the tower girder. Each pin is 6 in. square for a length of 11 in., to slide between top and bottom phosphor bronze bushings in a rectangular slot in a steel casting bolted to the reaction girder of the tower; the ends of the pin are turned to 6-in. diameter, to turn in phosphor bronze annular bushings set in annealed steel castings bolted to the projecting truss gussets at $L_{20}$. The effective length of the slot is $14\frac{1}{2}$ in., providing amply for combined truss expansion and tower movement.

To take care of the lateral wind reaction at the main tower, a thrust bracket is built up from the tower cross-girder, to engage the faces of two built-up brackets on the underside of the floorbeam. The central thrust bracket has its two bearing sides faced with 1-in. steel plates, planed to smooth finish; the two engaging thrust brackets have their bearing ends faced with 2-in. steel plates, planed to convex cylindrical surface. This detail permits relative longitudinal movement of truss and tower, while providing effective line-bearing for taking the lateral wind reactions from the continuous bottom lateral truss system into the main towers.

The cable bands are of novel design, developed for ease of fabrication and economy. While the upper or suspender-saddle portion is an annealed steel casting, the lower half is a steel plate, $10 \times \frac{3}{8} \times 11$ in., hot-pressed to the desired contour; this pressed steel plate is less expensive than the usual casting detail. Two $1\frac{1}{2}$-in. U-bolts, going around under this curved plate, pass up through the cast cable band top. The suspender groove in the top casting is flared at the sides, to permit all the cable band castings to be built with a single pattern. Small keeper plates, $2 \times \frac{3}{8}$ in., are tap-bolted over the side suspender grooves, after the rope is in place, as a safeguard against accidental displacement of the suspender.

Departing from conventional practice, the stringers are spliced together with top tie-plates over the floorbeams. The resulting effect of partial continuity increases the efficiency of the stringers and relieves the connection of certain objectionable secondary stresses.

A wire fencing design was adopted for the railings of the Portsmouth bridge. Top and bottom rails of $2\frac{1}{2}$-in. galvanized pipe, spaced 37 in., are U-bolted (with spacing blocks) to the truss posts and diagonals; between these pipe rails are mounted panels of woven fabric of No. 6 galvanized wire, 2-in. mesh; these panels are two per truss panel, and are framed with $1\frac{1}{2}$-in. galvanized channels for stiffening and attachment. This design, while comparatively light and inexpensive, has an advantage of superior resilience in comparison with conventional bridge railings.

**12. Construction of the Cables.**—The cables of the Portsmouth bridge were not constructed by the conventional method of stringing-in-the-air with the aid of temporary footbridges, but by stringing the individual strands on the ground. Complete sets of erection drawings were prepared by Robinson and Steinman for the two methods of cable construction; the contractor selected the ground-stringing method. Since, for small cables, the footbridge method involves a high plant cost per pound of cable material, the ground-stringing method may be more economical under favorable conditions. But the principal advantage offered by the ground-stringing method is the possibility of constructing the cable strands before the towers and anchorages are completed, thereby saving time in the completion of the bridge. At Portsmouth, the cable strands were completed on the ground before the steel towers were ready to receive them.

The method of ground-stringing, revived for the Portsmouth bridge, was the prevailing practice for the Ohio River bridges built by Laub and his school about a generation ago; whence it is sometimes called "the Ohio River method." One objection to it is the lack of conformation of the strand, as built, to the curvature over the saddles; this tends to produce an excess of tension in the upper wires of the strand. To meet this objection, Robinson and Steinman introduced into the strand-stringing plan means for producing pre-calculated initial bends or curvature in the strand corresponding to the computed difference of length between the top and bottom wires; for this purpose, the strands during stringing were inclined downwards for a distance near each end to the temporary anchorages of the strand shoes.

For the stringing of the strands, an existing railroad siding on the Kentucky shore was extended to provide the necessary length of 1800 ft. A gasoline track-car was used, running back and fourth on this track, to lay the wires in a sheet-metal lined V-trough on either side. The track-car carried the 26-in. coils of wire on swifts, which were controlled when necessary by hand-operated brakes; thence inclined guides guided the wire to the troughs. Below the level of the track, at either end of the 1800-ft. length, an anchorage of concrete and steel rails was provided to hold the strand shoes at pre-calculated elevation, inclination and distance. The individual wires were stretched, in adjustment around the strand shoes, to a spring-balance pull pre-calculated for varying temperatures. Two strands (486 wires each) were strung simultaneously; temporary wire seizings were placed at short intervals around each completed strand. When the two strands were completed, they were shifted to one side along the track; and the operation was repeated for the second and third pairs of strands.

After the six strands were completed, and the towers were ready to receive them, they were taken individually across the river and raised to position. Each strand was guided around a quarter-turn to the shore, and pulled across the river on barges; the strand shoes at the two ends were secured to their permanent anchorages, and the strands were then hoisted up to their position in the tower saddles by means of lines from gallows frames mounted on the tower tops.

The use of footbridges was completely dispensed with. Erection cages suspended from trolleys running on the cables were employed for the operations of squeezing the cables and placing the cable bands and suspenders.

**13. Spiraling of Strand.**—In the ground-stringing of the strands a mishap occurred which was temporarily upsetting but which was corrected without serious consequences. When the ground-stringing of the first pair of strands was completed, one of these strands was released from its temporary anchorage at one end by prying the strand shoe from its fastening; immediately upon this sudden release of tension, the end of the strand, for a length of about 50 ft., sprang into a spiral coil, the attached strand shoe turning over and over on the ground with the twisting of the strand. Somehow, it appears, the wires had worked around in the strand into a condition where their individual tendencies to coil produced a similar summation effect for their aggregate.

For the moment it appeared that the strand would have to be given up as lost. Various measures were considered, including salvaging the wire by unstringing the first pair of strands, and mechanically or heat-straightening all wire before continuing with the ground-stringing procedure. It was finally decided to make an attempt to unspiral the coiled strand, and this was successfully accomplished with far less difficulty than was anticipated and without any injury to wires or strand. With a few precautions, including the use of a snub-line to keep the strand under tension and the use of a "monkey-tail" to keep the strand shoe from turning over, it was found possible to handle the strands without a recurrence of the spiraling. The remaining four strands were strung in the same manner as the first pair, without resorting to the use of straightened wire; and all six strands were subsequently erected without loss or too serious trouble in handling in spite of their lively nature.

Thus the mishap fortunately had no serious consequences. The author has since learned that similar curling up of strands occurred during the cable stringing for one of the Ohio River bridges of a generation ago, and that in that instance one or two strands actually had to be given up as a complete loss. Had that incident been reported in the published accounts, the near-recurrence at Portsmouth could have been anticipated and avoided.

The safest precaution in the future, when the ground-stringing method is adopted, is to use wire that is either mechanically or heat-straightened so that all tendency to coil may be avoided. Unstraightened wire was purchased for the Portsmouth bridge before the substitution of ground-stringing for air-stringing was contemplated.

**14. Erection of Trusses.**—The stiffening trusses were shop-riveted and shipped in units of two panels each. These units were raised from the barges by means of a derrick boat, and were connected directly to the suspenders ready to receive them (Fig. C7). When all of the truss units were thus suspended, they were pinned and bolted together, ready for final field riveting.

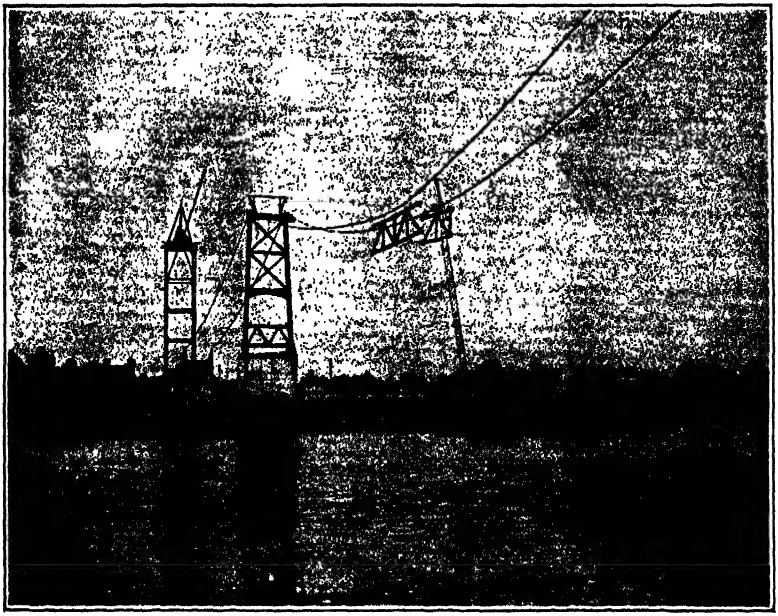


FIG. C7.—Method of Erection of Trusses. Portsmouth Bridge.
Hanging First Sections of Stiffening Trusses in Main Span.

This method of truss erection proved rapid and cheap. The entire stiffening trusses and floor system were erected in a little more than two weeks.

To facilitate the field connections between the truss units, the gusset plate on the inside of the truss was shop-riveted to one truss unit, and the gusset plate on the outside of the truss was shop-riveted to the other truss unit. The top and bottom splice plates were similarly handled. In this manner, the necessity of entering connections was avoided.

# APPENDIX D

## THE DEFLECTION THEORY

**1. Introduction.**—The common or approximate theory for the stress analysis of stiffened suspension bridges, presented in Chapter I of this book, is known as the *Elastic Theory*. A more exact method of analysis, which takes into account the deformed configuration of the structure, is known as the *Deflection Theory*.

The values of the bending moments and shears yielded by the Elastic Theory are too high, satisfying safety but not economy. The method is expeditious and therefore convenient for preliminary designs and estimates. It is generally sufficiently accurate for short spans and for designs having deep rigid stiffening systems that limit the deflections to small amounts.

In designs with long spans, shallow trusses, or high dead load, the results of the approximate method become too wasteful, and the Deflection Theory should therefore be applied. (As a time-saving device, the stresses may be found by the more expeditious Elastic Theory and then corrected by proper coefficients representing the reduction by the more exact theory.)

**2. History of the Deflection Theory.**—The Deflection Theory or " More Exact Theory " for the analysis of stiffened suspension bridges was originated by J. Melan and was first published by him in 1888 in the second edition of his classic work " Theorie der eisernen Bogenbrücken und der Hängebrücken." It was republished in 1906 in his third edition, which was translated in 1909 by D. B. Steinman and published in the latter's English translation " Theory of Arches and Suspension Bridges " in 1913. In Professor Melan's fourth edition, published in 1925, the Deflection Theory appears again, in amplified form. All of the basic formulas were developed by Professor Melan and were included in his original presentation.

The first application of the Deflection Theory in practice was in the computations of the Manhattan Bridge by L. S. Moisseiff and in the checking of the stresses by F. E. Turneaure in 1909. This work included the amplification of the working formulas for application to suspended side spans. A similar amplification of the theory for bridges with suspended side spans was made independently by D. B. Steinman in 1909.

The Deflection Theory was subsequently applied in the design computations of the Philadelphia–Camden Bridge, the Florianopolis Bridge, the Mount Hope Bridge, the Grand' Mere Bridge, and other suspension structures. The utilization of the theory became more common with the growing realization of the amount of the resultant saving, especially in longer spans and shallower trusses.

A different form of the Deflection Theory, using converging trigonometric series and involving a refinement over the usual form, was published in 1928 by S. Timoshenko. The refinement consists in eliminating the assumption of uniform suspender loading in the derivation of the $H$-equation. The resulting difference in the value of $H$ is found, however, to be small and practically negligible.

In the course of the professional practice of Robinson and Steinman, as successive bridge designs were calculated by the Deflection Theory, data were gradually accumulated on the percentages of reduction from the corresponding stresses by the approximate theory for different lengths and proportions of spans. These data soon indicated that there was some law of relationship between the amount of reduction and the flexibility of the span. The plotted records were used to indicate the reduction percentages for use in approximate estimates and preliminary designs on new bridge projects. In 1927 the author suggested to A. H. Baker, as a thesis problem, the development, from the office data and from further computations, of graphs that would give the correction percentages closely as functions of the span constants. After a year of computations and comparisons, Mr. Baker developed the graphs reproduced in Fig. D5 and described in the accompanying text. These graphs afford a short-cut method for figuring the " more exact " stresses, with little error, directly from the results of the more expeditious Elastic Theory.

**3. Fundamental Assumptions of the Deflection Theory.**—The Deflection Theory for the analysis of stiffened suspension bridges is based on the following assumptions:

1. The initial curve of the cable is a parabola. (In practice, the greatest ordinate deviation from a true parabola is seldom as large as $\frac{1}{2}$ per cent.)

2. The initial dead load ($w$) is carried by the cable (producing the initial horizontal tension $H_w$) without causing stress in the stiffening truss.

Unlike the Elastic Theory, however, the Deflection Theory does not assume that the ordinates of the cable curve remain unaltered upon application of loading. In other words, the alteration of the lever arms of the cable forces is taken into account. This change in cable ordinates or lever arms makes the initial cable tension $H_w$ significant.

The theory that follows is applicable to two-hinged suspension bridges either with or without suspended side spans. It is not applicable, without modification, to suspension bridges having continuous (hingeless) stiffening trusses.

**4. Fundamental Equations of the Deflection Theory.**—If the deflections of the span are neglected, the general expression for the bending moment at any point of the span is given by the basic formula of the Elastic Theory (Eq. 82, page 22):

$$M = M' - Hy,$$

where $M'$ denotes the simple-span bending moment and $Hy$ represents the relieving moment due to the cable tension.

In consequence of the deflections $\eta$, the bending moments are relieved by an additional amount $(H_w + H)\eta$ and the expression for $M$ becomes:

$$M = M' - Hy - (H_w + H)\eta. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

This is the basic equation of the Deflection Theory.

Neglecting the elongation of the suspenders, the truss at any point will have the same deflection $\eta$ as the cable at that point.

By the common theory of flexure applied to the truss,

$$\frac{d^2\eta}{dx^2}=-\frac{M}{EI}.$$

Substituting Eq. (1), and introducing the symbol

$$c^2=\frac{H_w+H}{EI}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

we obtain:

$$\frac{d^2\eta}{dx^2}=c^2\eta-\frac{c^2}{H_w+H}(M'-Hy).$$

The solution of this differential equation yields the general formula for deflections or equation of the deflection curve:

$$\eta=\frac{H}{H_w+H}\left[C_1e^{cx}+C_2e^{-cx}+\left(\frac{M'}{H}-y\right)-\frac{1}{c^2}\left(\frac{p}{H}-\frac{8f}{l^2}\right)\right], \quad . \quad (3)$$

where $p$ is the uniform applied loading producing $M'$ and $H$. (The terms in the last parentheses are the second derivatives of the terms in the preceding parentheses. The student may verify Eq. (3) by differentiating it twice and substituting back in the preceding equation.)

Substituting Eq. (3) in Eq. (1), the general formula for $M$ or equation of the $M$-curve is obtained:

$$M=-H\left[C_1e^{cx}+C_2e^{-cx}-\frac{1}{c^2}\left(\frac{p}{H}-\frac{8f}{l^2}\right)\right]. \quad . \quad . \quad . \quad (4)$$

From Eq. (4) we observe that the moment $M$ is not simply proportional to the load $p$ that produces it. Eq. (1) also shows that the value of $M$ is affected by the dead-load stress $H_w$ in the cable before the application of the live load. *In the Deflection Theory, influence lines cannot be used.* Stresses producible by a combination of loadings cannot be found by adding algebraically the respective stresses producible by the component loadings.

The general formula for shears $V$ or equation of the $V$-curve is obtained by differentiating Eq. (4), which gives:

$$V=\frac{dM}{dx}=-Hc[C_1e^{cx}-C_2e^{-cx}]. \quad . \quad . \quad . \quad (5)$$

The constants of integration $C_1$ and $C_2$ in the foregoing equations for $\eta$, $M$, and $V$ appear also in the general equation (Eq. 6) for $H$. They are determined for a given structure from the conditions of loading, as illustrated in Art. 6. Values of $C_1$ and $C_2$ for special cases of loading are tabulated in Art. 9.

Differentiating Eq. (5), we obtain an expression for the live-load per unit of length actually carried by the stiffening truss:

$$p-(s_1-s_0)=-\frac{d^2M}{dx^2}=-\frac{dV}{dx}=Hc^2[C_1e^{cx}+C_2e^{-cx}]$$

where $s_0$ and $s_1$ are the initial and final values, respectively, of the suspender loading per unit length of the span at any point $x$. This equation shows that the suspender loading $s_1$ is no longer constant, as in the Elastic Theory, but becomes a variable in the Deflection Theory.

**5. Derivation of the Basic Equation for $H$.**—The horizontal cable tension $H$ due to live load, temperature change, and supplementary dead load (following the condition represented by the initial tension $H_w$), may be evaluated as follows:

The total virtual work ($W_1$) done in the vertical displacements $\eta$ of the suspender loads ($s_1$) plus the cable weight ($g$) must equal the total virtual work ($W_2$) done by the cable tension ($H_w+H$) in stretching the cable. These work quantities $W_1$ and $W_2$ are expressed as the integrated products of the forces and their respective displacements, as follows, using the symbol $\Sigma$ to denote the summation of similar expressions for all the spans:

$$W_1=\Sigma\int_0^l(s_1+g)\eta dx=\Sigma\frac{8f}{l^2}(H_w+H)\int_0^l\eta dx \quad \text{(approximately)};$$

$$W_2=\Sigma(H_w+H)\left[\frac{H}{E_cA_0}\int_0^l\frac{A_0}{A}\cdot\frac{ds^3}{dx^2}+\omega t\int_0^l\frac{A_0}{A}\cdot\frac{ds^2}{dx}\right]$$

$$=(H_w+H)\frac{H}{E_cA_0}\cdot L_s+(H_w+H)\omega tL_t;$$

where

$$L_s=\Sigma\int_0^l\frac{A_0}{A}\cdot\frac{ds^3}{dx^2}, \quad \text{and} \quad L_t=\Sigma\int_0^l\frac{A_0}{A}\cdot\frac{ds^2}{dx}.$$

In these expressions, $A$ denotes the cable section at any point and $A_0$ denotes the cable section at mid-span.

For a parabolic wire cable, having uniform $A$, we may write with sufficient accuracy:

$$L_s = \Sigma l(\sec^3 \alpha + 8n^2), \quad L_t = \Sigma l\left(\sec^2 \alpha + \frac{16}{3}n^2\right)$$

where $n = \frac{f}{l}$, and $\alpha$ is the inclination of the closing chord in any span.

Similarly, for a parabolic eyebar cable, assuming $A$ varying with the slope secant $\frac{ds}{dx}$, we may write with sufficient accuracy:

$$L_s = \Sigma l\left(\sec^2 \alpha + \frac{16}{3}n^2\right), \quad L_t = \Sigma l\left(\sec \alpha + \frac{8}{3}n^2\right).$$

Equating the expressions given above for $W_1$ and $W_2$, we obtain the work equation:

$$\Sigma \frac{8f}{l^2}\int_0^l \eta dx = \frac{H}{E_c A_0} L_s + \omega t L_t.$$

Substituting the expression for $\eta$ from Eq. (3), the work equation may be written:

$$\Sigma K \int_0^l \left[C_1 e^{cx} + C_2 e^{-cx} + \left(\frac{M'}{H} - y\right) - \frac{1}{c^2}\left(\frac{p}{H} - \frac{8f}{l^2}\right)\right] dx = \frac{c^2 l^2}{8f}\left(\frac{IL_s}{A_0}\cdot\frac{E}{E_c} + \frac{EI\omega t L_t}{H}\right).$$

Solving for $H$, we obtain the basic $H$-equation:

$$H = \frac{\Sigma K \int_0^l \left(M' - \frac{p}{c^2}\right) dx - rc^2 EI\omega t L_t}{\Sigma K\left[-\int_0^l (C_1 e^{cx} + C_2 e^{-cx}) dx + \frac{2}{3}fl - \frac{l}{rc^2}\right] + rc^2 \cdot \frac{IL_s}{A_0}\cdot\frac{E}{E_c}}, \quad (6)$$

where $E$ is the modulus of elasticity of the truss material and $E_c$ that of the cable; and $r$ is the parameter of the cable parabola defined by:

$$r = \frac{l^2}{8f}.$$

The summations $\Sigma$ in Eq. (6) embrace the corresponding expressions for all the spans. The coefficient $K$ occurring in the summations denotes the ratio of $\frac{f}{l^2}$ for any span to $\frac{f}{l^2}$ of the main span; hence $K=1$ for the main span; also $K_1=1$ for the side spans if they have the same $\frac{f}{l^2}$ as the main span. (Generally the ratio $K_1$ for suspended side spans is between 1.00 and 1.05, representing the ratio of side-span weight to main-span weight per unit of length. For "unloaded" or "straight" backstays, $K_1=0$ and all side-span terms vanish.) In any case:

$$K=1, \quad \text{and} \quad K_1=\frac{f_1/l_1^2}{f/l^2}=\frac{r}{r_1}.$$

It should be noted that two approximations are involved in the foregoing derivation of the $H$-equation (Eq. 6). In writing the transformed expression for $W_1$, it is assumed that the suspender and cable loading $(s_1+g)$ is uniformly distributed over the span; this is contrary to the actual condition as demonstrated in Art. 4. (In the Deflection Theory formulas presented by Timoshenko, this assumption is avoided; the effect on the resulting stresses is, however, found to be practically negligible.) The second approximation consists in writing the original cable sag $f$ instead of the augmented cable sag $(f+\eta)$ in the expression for $W_1$. This affects only the terms containing $L_s$ and $L_t$ in the $H$-formula (Eq. 6), and the effect of this approximation on the value of $H$ does not, in extreme cases, exceed 1 per cent.

**6. Evaluation of the Integration Constants.**—The constants of integration $C_1$ and $C_2$, appearing in the basic Equations (3), (4), (5), and (6), must be determined for each different condition of loading. For each segment of the span having a constant value of $p$ and of $I$, there is a pair of values for $C_1$ and $C_2$.

In the treatment that follows it will be assumed, as is usually done for the sake of simplicity, that the moment of inertia $I$ (or $I_1$) is constant throughout the length of any span under consideration, although it may have different respective values for the three different spans. The error of ignoring the variation of $I$ within a span is found to be practically negligible and on the side of safety. (For greater accuracy, instead of the average value of $I$ for any span, the value of the equivalent uniform $I$ should preferably be used in the computations; this equivalent uniform $I$ may be determined by figuring equal deflections under governing loadings.)

In this article, the integration constants are evaluated for three general cases, covering the division of a span into *one*, *two*, and *three* differently loaded segments, respectively. From the general formulas thus derived, special formulas may be written for a large variety of loading conditions, including all of the loading conditions that arise in the usual design computations (Cases I to VIII, Art. 9).

*One Loading Segment.*—For the case of the main span fully loaded with a uniform applied load $p$, and assuming constant moment of inertia $I$, the quantities $C_1$ and $C_2$ are obtained from the two known conditions that for $x=0$ and $x=1$ in Eq. (4), $M=0$. Substituting these values and solving the resulting two independent equations, we find:

$$\left.\begin{aligned} C_1 &= \frac{1}{c^2(1+e^{cl})}\left(\frac{p}{H}-\frac{1}{r}\right) \\ C_2 &= C_1 e^{cl} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

For a side span fully loaded, identical expressions are similarly obtained, except that $c$, $l$, and $r$ are replaced by $c_1$, $l_1$, and $r_1$. (These values are recorded in Case V, Art. 9, and, for $p=0$, in Case VI.)

*Two Loading Segments.*—For the case of partial loading of the main span (Case I, Art. 9), with a uniform load $p$ per unit length extending a distance $k$ from the left end of the span, the constants $C_1$ and $C_2$ for the loaded segment ($k$) and the constants $C_3$ and $C_4$ for the unloaded segment ($m=l-k$) are obtained from the four known conditions that the moment and shear at the right end $B$ of the loaded segment must be equal respectively to those at the left end $B$ of the unloaded segment, and that $M=0$ at each end of the span. Substituting these relations in

Eqs. (4) and (5), and solving the resulting four independent equations, we find:

$$\left.\begin{aligned} C_1&=\frac{p}{2Hc^2}\cdot\frac{(e^{cm}+e^{-cm}-2e^{-cl})}{(e^{cl}-e^{-cl})}-\frac{1}{rc^2(1+e^{cl})}\\ C_2&=-C_1+\frac{1}{c^2}\left(\frac{p}{H}-\frac{1}{r}\right)\end{aligned}\right\} \quad . \quad . \quad . \quad (8)$$

$$\left.\begin{aligned} C_3&=\frac{pe^{-cl}}{2Hc^2}\cdot\frac{(e^{ck}+e^{-ck}-2)}{(e^{cl}-e^{-cl})}-\frac{1}{rc^2(1+e^{cl})}\\ C_4&=-C_3e^{2cl}-\frac{e^{cl}}{rc^2}\end{aligned}\right\} \quad . \quad . \quad . \quad (9)$$

The values of the integration constants given by Eqs. (8) for the loaded segment are recorded in Case I, Art. 9.

To calculate the deflection, moment or shear at any point in the loaded segment of the span, the foregoing values of $C_1$ and $C_2$ must be substituted in Eqs. (3), (4), or (5), respectively. To calculate the corresponding values in the unloaded segment, the values of $C_3$ and $C_4$ replace the general constants $C_1$ and $C_2$ in the same basic equations; and, since the segment is unloaded, the value of $p$ is taken as zero in Eq. (3) or (4), though not in Eqs. (9).

As a check upon the foregoing formulas, Eqs. (8) for the loaded segment may be reduced, by substituting $m=0$, to Eqs. (7) for the span fully loaded.

Upon substituting for $c$, $l$, and $r$ in Eqs. (8) and (9), the corresponding side-span quantities $c_1$, $l_1$, and $r_1$, the same expressions also apply to the case of partial loading of the side spans. Eqs. (8) thus yield the constants for Cases VII and VIII, Art. 9.

For a span fully unloaded, the constants may be obtained by substituting $p=0$ in either Eqs. (7) or (8), or $k=0$ in Eqs. (9). This yields the constants recorded in Case VI, Art. 9.

*Three Loading Segments.*—For other loading conditions, the integration constants are determined by a procedure similar to that followed in the cases represented by Eqs. (7), (8) and (9). If a span is divided into *three* segments ($k+m+z=l$) having different uniform loads $j$, $p$, and $q$, respectively, the three corresponding pairs of integration constants are obtained from the six known conditions that $M$ and $V$ at the right end of the first segment must be equal to $M$ and $V$ at the left end of the second segment, that the same two equalities also hold at the junction of the second and third segments, and that $M=0$ at each end of the span. Upon substituting these relations in Eqs. (4) and (5), the solution of the resulting six independent equations yields the following values of the three pairs of integration constants:

$$\left.\begin{aligned}
C_1 &= -\frac{1}{rc^2(1+e^{cl})}+\\
&\frac{1}{2Hc^2}\left\{\frac{(j-p)[e^{c(l-k)}+e^{-c(l-k)}]+(p-q)(e^{cz}+e^{-cz})-2je^{-cl}+2q}{(e^{cl}-e^{-cl})}\right\}\\
C_2 &= -C_1+\frac{1}{c^2}\left(\frac{j}{H}-\frac{1}{r}\right).
\end{aligned}\right\} \tag{10}$$

$$\left.\begin{aligned}
C_3 &= -\frac{1}{rc^2(1+e^{cl})}+\\
&\frac{1}{2Hc^2}\left\{\frac{(j-p)(e^{ck}+e^{-ck})e^{-cl}+(p-q)(e^{cz}+e^{-cz})-2je^{-cl}+2q}{(e^{cl}-e^{-cl})}\right\}\\
C_4 &= -C_3-\frac{1}{2Hc^2}[(j-p)(e^{ck}+e^{-ck})-2j]-\frac{1}{rc^2}.
\end{aligned}\right\} \tag{11}$$

$$\left.\begin{aligned}
C_5 &= -\frac{1}{rc^2(1+e^{cl})}+\\
&\frac{1}{2Hc^2}\left\{\frac{(j-p)(e^{ck}+e^{-ck})e^{-cl}+(p-q)[e^{-cz}+e^{-c(2l-z)}]-2je^{-cl}+2q}{(e^{cl}-e^{-cl})}\right\}\\
C_6 &= -C_5e^{2cl}+\frac{e^{cl}}{c^2}\left(\frac{q}{H}-\frac{1}{r}\right).
\end{aligned}\right\} \tag{12}$$

The pair of integration constants given by Eqs. (11) for the middle segment ($m$), when the uniform loads ($j$ and $q$) in the adjoining segments are made zero, are recorded in Case III, Art. 9. The pair of integration constants given by Eqs. (11) for the middle segment $m$, when the uniform load in that segment is made zero and the adjoining segments ($k$ and $z$) are loaded with $p$, are recorded in Case IV, Art. 9. As a check: for $k=0$, the constants of Case III reduce to those of Case I; and, for $k=0$, the constants of Case IV reduce to those of Case II.

The three-segment loading condition represented by Eqs. (10) to (12) may be regarded as a general case from which the integration constants for the more usual loading conditions may be evaluated by simple substitution. Thus, Eqs. (7) may be written:

From Eqs. (10), by substituting
$k=l,\quad m=0,\quad z=0,\quad p=0,\quad q=0,\quad j=p;$
from Eqs. (11), by substituting
$k=0,\quad m=l,\quad z=0,\quad j=0,\quad q=0;$
or from Eqs. (12), by substituting
$k=0,\quad m=0,\quad z=l,\quad j=0,\quad p=0,\quad q=p.$

Eqs. (8) may be written:
From Eqs. (10), by substituting
$z=0,\quad p=0,\quad q=0,\quad j=p;$
or from Eqs. (11), by substituting
$k=0,\quad m=k,\quad z=m,\quad j=0,\quad q=0.$

And Eqs. (9) may be written:
From Eqs. (11), by substituting
$z=0,\quad p=0,\quad q=0,\quad j=p;$
or from Eqs. (12), by substituting
$k=0,\quad m=k,\quad z=m,\quad j=0,\quad q=0.$

In Art. 9 are tabulated, with their respective loading diagrams, the working formulas for eight cases of loading that require to be considered in actual design computations. The integration constants there recorded for those eight loading conditions may be written from the more general Eqs. (10) to (12) by simple substitution, as follows:

Case I.—From Eqs. (10), substituting
$z=0,\quad p=0,\quad q=0,\ j=p;$
or from Eqs. (11), substituting
$k=0,\ m=k,\quad z=m,\quad q=0,\quad j=0.$

Case II.—From Eqs. (10), substituting
$z=0,\quad j=0,\quad q=0;$
or from Eqs. (11), substituting
$k=0,\ m=k,\quad z=m,\ j=0,\ p=0,\ q=p.$

Case III.—From Eqs. (11), substituting

$$j=0, \quad q=0.$$

Case IV.—From Eqs. (11), substituting

$$p=0, \quad j=p, \quad q=p.$$

Case V.—From Eqs. (10), substituting

$$k=l, m=0, \; z=0, \; p=0, \; q=0, \quad j=p; \; l_1, \; c_1, \; r_1;$$

or from Eqs. (11), substituting

$$m=l, \; k=0, \; z=0, \; j=0, \quad q=0; \; l_1, \; c_1, \; r_1.$$

Case VI.—From Eqs. (10), substituting

$$k=l, \quad m=0, \quad z=0, \; j=p=q=0; \; l_1, \; c_1, \; r_1;$$

or from Eqs. (12), substituting

$$z=l, \; k=0, \; m=0, \; j=p=q=0; \; l_1, \; c_1, \; r_1.$$

Case VII.—From Eqs. (10), substituting

$$z=0, \quad p=0, \quad q=0, \; j=p; \; c_1, \; l_1, \; r_1.$$

Case VIII.—From Eqs. (11), substituting

$$k=0, \quad z=k, \quad q=0, \; j=0; \; c_1, \; l_1, \; r_1.$$

The expressions for the constants $C$ for side-span conditions (Cases V to VIII) are identical with those for corresponding main-span conditions, except that the side-span dimensions replace the main-span dimensions, and $c_1$ (calculated from $I_1$) replaces $c$. With the same simple changes in notation, Eqs. (3), (4), and (5) for $\eta$, $M$, and $V$, respectively, are also applicable to the side spans.

It should be noted that unsymmetrically loaded spans are not reversible left to right without altering the values of the integration constants (unless the origin of $x$ is also reversed). That is because the integration constants occur in Eqs. (3), (4), and (5), in which $x$ is assumed measured from the left end of the span and which represent the unsymmetrical curves of $\eta$, $M$, and $V$, respectively. It is for this reason, for instance, that the values of the integration constants given by Eqs. (9) for a right-hand unloaded segment cannot be applied to Case II (Art. 9) representing a left-hand unloaded segment. Eqs. (9) would properly be applicable to the unloaded segments of Cases I, VII, and

VIII; they would also be applicable to Case II if $x$ were measured from the other end of the span.

It may also be noted that (in non-continuous stiffening trusses, to which this analysis is confined) the values of the integration constants $C$, for any loading condition in a span, are unaffected by the loading conditions in the other spans.

**7. Derivation of Working Formulas for *H*.**—The basic equation for $H$, Eq. (6), may be simplified, for any particular loading condition, by substituting detailed expressions for the terms that depend on the loading, transferring some of the terms containing $H$, and re-solving for $H$.

For the case of partial loading, with a uniform load $p$ covering the left segment $k$ $(=l-m)$ of the main span (Case I, Art. 9), the simplification of the general equation for $H$ is as follows:

For this loading condition, the summation term in the numerator of Eq. (6) takes the following form upon substituting the respective expressions for $M$ and integrating for the two segments of the span:

$$\int_0^l \left(M' - \frac{p}{c^2}\right)dx = pk\left[\frac{k}{12}(3l-2k) - \frac{1}{c^2}\right]. \quad . \quad . \quad (13)$$

If the side spans have no load, the corresponding side-span terms are zero.

The integration summation term in the denominator of Eq. (6) may be written in the form:

$$-\Sigma K\int_0^l (C_1e^{cx}+C_2e^{-cx})\,dx = -\int_0^k (C_1e^{cx}+C_2e^{-cx})dx$$
$$-\int_k^l (C_3e^{cx}+C_4e^{-cx})dx - 2K_1\int_0^{l_1}(B_1e^{c_1x}+B_2e^{-c_1x})dx.$$

Substituting for the main-span constants $C_1$, $C_2$, $C_3$, $C_4$, the values given by Eqs. (8) and (9), and for the side-span constants $B_1$ and $B_2$ values similar to those given by Eqs. (7) except that $p$ is zero for the unloaded side spans, the foregoing summation reduces, upon integration, to the form:

$$-\Sigma K\int_0^l (C_1e^{cx}+C_2e^{-cx})dx=$$

$$\frac{1}{c^3(e^{cl}-e^{-cl})}\left\{\frac{p}{H}[2-e^{cl}-e^{-cl}-e^{ck}-e^{-ck}+e^{cm}+e^{-cm}]+\frac{2}{r}(e^{cl}+e^{-cl}-2)\right\}$$
$$+\frac{4K_1}{r_1c_1^3}\cdot\frac{(e^{c_1l_1}-1)}{(e^{c_1l_1}+1)}\cdot \quad . \quad . \quad . \quad . \quad . \quad (14)$$

Substituting the expressions given by Eqs. (13) and (14) for the summations in Eq. (6), and solving for $H$, we finally obtain:

$$H=\frac{\left\{\begin{array}{l} pk\left[\frac{k}{12}(3l-2k)-\frac{1}{c^2}\right]-\frac{p}{c^3(e^{cl}-e^{-cl})}\cdot \\ \qquad [2-e^{cl}-e^{-cl}-e^{ck}-e^{-ck}+e^{cm}+e^{-cm}]-rc^2EI\omega tL_t\end{array}\right\}}{D} \quad (15)$$

where $D$, the denominator of the $H$-formula, is given by:

$$D=\Sigma K\left[\frac{2}{rc^3}\cdot\frac{(e^{cl}-1)}{(e^{cl}+1)}+\tfrac{2}{3}fl-\frac{l}{rc^2}\right]+rc^2\frac{I}{A_0}\cdot\frac{E}{E_c}\cdot L_s. \quad (16)$$

Eq. (15) for $H$ applies only to the special loading condition of uniform load $p$ covering the segment $k$ of the main span from *either* end, with no loading in the side spans, and with rise in temperature.

Eq. (16) for the denominator $D$ is found, however, to remain unchanged for all other loading conditions; it contains no terms involving the load intensity $p$, the load length $k$, nor the temperature change $t$. Eq. (16) is therefore the expression for the denominator of the $H$-formula for any condition of loading. It should be noted, however, that $D$ is not a constant; although the *expression* for $D$ remains unchanged, it contains the variable $c$ which depends upon $H$, and therefore the *value* of $D$ varies with the loading.

The last term in Eq. (15), representing temperature effect, has the minus sign for a rise in temperature above normal, and must be reversed in sign (to plus) for a drop in temperature below normal. In other words, $H$ is diminished by a rise in temperature and augmented by a drop in temperature.

The values of $D$ are dependent upon $H$, because of the quantities $c$ (and $c_1$ for the side spans) that appear in Eq. (16). The calculations for a given structure are facilitated by calculating in advance the values of $D$ for varying values of $H$, and tabulating or plotting the results for reference in the subsequent computations. (See Fig. D2.)

The formula for $H$ for any loading or combination of loadings may be written in the general form:

$$H=\frac{N}{D}, \qquad \ldots \ldots \ldots \quad (17)$$

where the *expression* for the denominator $D$ as given by Eq. (16) is the same for all loadings, but the expression for the numerator $N$ varies with the loading condition.

From the expression for $N$ in Eq. (15), the $N$-expressions for the various loading conditions represented by Cases I to VIII (Art. 9) are easily written.

For partial loading of the main span from either end, the expression for $N$ is given by the numerator in Eq. (15). This yields the value of $H$ in Case I.

For partial loading of either side span from either end, the same express'on from Eq. (15) may be used for $N$ after substituting the side-span terms $K_1$, $l_1$, and $c_1$. This yields the value of $H$ in Case VII.

For full loading of the main span, substitute $l$ for $k$ and o for $m$ in the $N$-expression given by Eq. (15) for partial loading. Omitting the temperature term, this yields:

$$N=D\cdot H=pl\left(\frac{l^2}{12}-\frac{1}{c^2}\right)+\frac{2p}{c^3}\frac{(e^{cl}-1)}{(e^{cl}+1)}, \qquad \ldots \quad (18)$$

for the main span fully loaded.

Similarly substituting $l_1$ for $k$ and o for $m$ in the $N$-expression of Case VII for partial loading of the side span, we obtain the $N$-expression of Case V for full loading of the side span. The same expression for $N$ in Case V is also obtained by substituting the side-span terms $K_1$, $l_1$, and $c_1$ in Eq. (18) for full loading in the main span.

By adding the respective $N$-expressions for the main span and a side span fully loaded (Eq. 18 and Case V), we obtain the $N$-expression for the two spans loaded as given in Case VI.

Doubling the side-span terms in this $N$-expression of Case VI, we obtain the following expression for the condition of full loading of all three spans:

$$N = D \cdot H = pl\left(\frac{l^2}{12} - \frac{1}{c^2}\right) + 2K_1pl_1\left(\frac{l_1^2}{12} - \frac{1}{c_1^2}\right) + \frac{2p}{c^3}\frac{(e^{cl}-1)}{(e^{cl}+1)} + \frac{4K_1p}{c_1^3}\frac{(e^{c_1l_1}-1)}{(e^{c_1l_1}+1)} \quad (19)$$

for the three spans fully loaded.

Subtracting from $N$ of Eq. (18) for the main span fully loaded the $N$-expressions given by Eq. (15) for partial loading from each end, we obtain $N$ for the case of partial loading of a segment near the middle of the span. This is given in Case III.

Subtracting from $N$ for all three spans loaded (Eq. 19) the $N$-expression for the loading of a segment of any span (Cases I, III, or VII), we obtain $N$ for the complementary case of the segment unloaded with the remainder of the structure fully loaded. This gives the expressions used in Cases II, IV and VIII. (Note that the eight Cases in Art. 9 are tabulated in complementary pairs.)

For the effect of a rise or fall in temperature, the value of $N$ is expressed by the last term in Eq. (15), namely

$$N = D \cdot H_t = \mp rc^2 EI \omega t L_t. \quad . \quad . \quad . \quad . \quad . \quad (20)$$

Adding this term (with the proper sign) to the live-load terms for the foregoing loading conditions, all of the expressions for $H$ in Cases I to VIII, inclusive, of Art. 9 are obtained.

The methods used above for deducing all other working formulas for $H$ from the expression given by Eq. (15) for one loading condition have consisted of the simple processes of substitution, addition and subtraction. We have made use of the convenient fact that the expression for $H$ (though not its value) is additive for combinations of loadings.

It is also of interest to note that unsymmetrically loaded spans are reversible (left to right) without affecting the value of $H$ (whereas the integration constants $C$ are altered by such reversal). That is because the directional variable $x$ does not occur in the working formulas for $H$.

**8. Loading Conditions for Maximum Moments and Shears.**—A study of the fundamental Eq. (1) of the Deflection Theory will indicate the selection of loading conditions for maximum and minimum moments at any point.

For maximum positive moment at any point, it is apparent from Eq. (1) that the loading must be such as to make $M$ as large as possible while keeping $H$ small. Hence a length of span embracing the given point must be covered with live load, with no load on the rest of the structure; and maximum temperature must be assumed in order to reduce the value of $H$. (These conditions govern in Cases I, III, and V, Art. 9.)

For maximum negative moment at any point, the reverse conditions must be satisfied. Hence a length of span embracing the given point must be unloaded, and the rest of the structure covered with live load at minimum temperature. (These conditions govern in Cases II, IV, and VI, Art. 9.)

The foregoing load placements for maximum and minimum values of $M$ are also indicated by the moment influence diagrams represented in Fig. 11, page 36.

The shear at any point is given by the general expression (Eq. 84, page 22):

$$V = V' - H \tan \phi.$$

From this equation it is apparent that $V'$ must be large and $H$ small for maximum positive shear, and the reverse for maximum negative shear at any point. Hence similar loading conditions govern for shears as for moments, except that the partial loading stops at the given section where the maximum shear is sought. (See also the shear influence diagrams represented in Fig. 14, page 46.) The loading conditions thus indicated for maximum positive shears are covered by Cases I, III, V, and VII (Art. 9); and those for maximum negative shears by Cases II, IV, VI, and VIII.

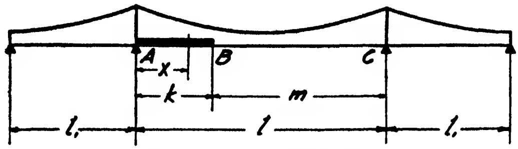
The load-lengths for maximum positive and negative moments (and for shears in the outer parts of the main span) have to be found by trial in the Deflection Theory. The load-lengths determined by the Elastic Theory may be used as a guide for the trial values to be substituted in the more exact theory. Generally three trial values of the load-length suffice to determine the maximum value of the function sought.

**9. Values of *H* and *C* for Special Cases of Loading.**—For the various loading conditions that are useful in design, formulas for $H$ with the corresponding values of $C_1$ and $C_2$ have been worked out (as outlined in Arts. 6 and 7) and are tabulated below. Eight cases are given, covering all loading conditions of practical importance; they are presented in complementary pairs. The odd numbered cases (with highest temperature) are for maximum *positive* moments, shears, or deflections; the even numbered cases (with lowest temperature) are for maximum negative moments, shears, or deflections. The first four cases are for *main-span* moments, shears, and deflections; the last four cases are for *side-span* moments, shears, and deflections.

Case I.—Main Span, partially loaded from left end.
Side Spans, unloaded.
Temperature, highest.

For: Maximum *positive moments* (and downward deflections) in *main span* from left end to near center.
Maximum *positive shears* in *main span* at left end and (with span reversed) near center.

CASE I




$$H=\frac{pk\left[\frac{k}{12}(3l-2k)-\frac{1}{c^2}\right]-\frac{p}{c^3(e^{cl}-e^{-cl})}\left[2-e^{cl}-e^{-cl}-e^{ck}-e^{-ck}+e^{cm}+e^{-cm}\right]-rc^2EI\omega tL_t}{D}$$

*In Segment A-B:*

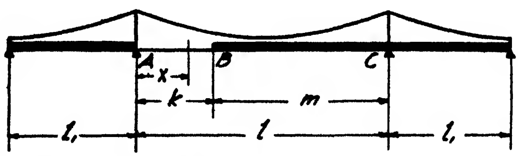
$$C_1=\frac{p}{2Hc^2}\cdot\frac{(e^{cm}+e^{-cm}-2e^{-cl})}{(e^{cl}-e^{-cl})}-\frac{1}{rc^2(1+e^{cl})}$$

$$C_2=-C_1+\frac{1}{c^2}\left(\frac{p}{H}-\frac{1}{r}\right)$$

Case II.—Main Span, partially loaded from right end.
Side Spans, fully loaded.
Temperature, lowest.

For: Maximum *negative moments* (and upward deflections) in *main span* from left end to near center.
Maximum *negative shears* in *main span* at left end and (with span reversed) near center.

CASE II




$$H=\frac{pl\left(\frac{l^2}{12}-\frac{1}{c^2}\right)+2Kpl_1\left(\frac{l_1^2}{12}-\frac{1}{c_1^2}\right)+\frac{2p(e^{cl}-1)}{c^3(e^{cl}+1)}+\frac{4Kp(e^{c_1l_1}-1)}{c_1^3(e^{c_1l_1}+1)}-[\text{Numerator Case I}]}{D}$$

*In Segment A-B:*

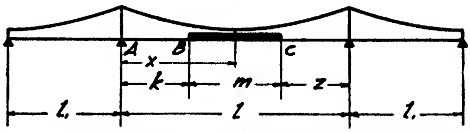
$$C_1=-\frac{p}{2Hc^2}\cdot\frac{(e^{cm}+e^{-cm}-2)}{(e^{cl}-e^{-cl})}-\frac{1}{rc^2(1+e^{cl})}$$

$$C_2=-C_1-\frac{1}{rc^2}$$

CASE III.—Main Span, partially loaded near center.
Side Spans, unloaded.
Temperature, highest.

For: Maximum *positive moments* (and downward deflections) in *main span* near center.
Maximum *positive shears* in *main span* from left end to near center.

CASE III




$$H=\frac{pm\left[\frac{k(l-k)+z(l-z)}{4}+\frac{m^2}{12}-\frac{1}{c^2}\right]-\frac{p}{c^3(e^{cl}-e^{-cl})}\left[e^{ck}+e^{-ck}+e^{cz}+e^{-cz}-e^{c(l-k)}-e^{-c(l-k)}-e^{c(l-z)}-e^{-c(l-z)}\right]-rc^2EI\omega tL_t}{D}$$

*In Segment B-C:*

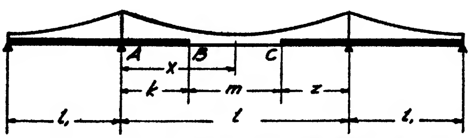
$$C_1=\frac{p}{2Hc^2}\frac{\left[e^{cz}+e^{-cz}-e^{-cl}(e^{ck}+e^{-ck})\right]}{(e^{cl}-e^{-cl})}-\frac{1}{rc^2(1+e^{cl})}$$

$$C_2=-C_1+\frac{p}{2Hc^2}(e^{ck}-e^{-ck})-\frac{1}{rc^2}$$

CASE IV.—Main Span, partially loaded from each end.
Side Spans, fully loaded.
Temperature, lowest.

For: Maximum *negative moments* (and upward deflections) in *main span* near center.
Maximum *negative shears* in *main span* from left end to near center.

CASE IV




$$H=\frac{pl\left(\frac{l^2}{12}-\frac{1}{c^2}\right)+2Kpl_1\left(\frac{l_1^2}{12}-\frac{1}{c_1^2}\right)+\frac{2p(e^{cl}-1)}{c^3(e^{cl}+1)}+\frac{4Kp(e^{c_1l_1}-1)}{c_1^3(e^{c_1l_1}+1)}-[\text{Numerator Case III}]}{D}$$

*In Segment B-C:*

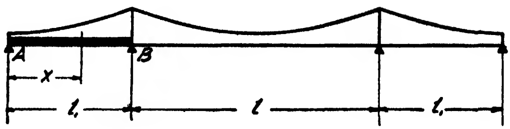
$$C_1=\frac{p}{2Hc^2}\frac{\left[e^{-cl}(e^{ck}+e^{-ck})-e^{cz}-e^{-cz}\right]}{(e^{cl}-e^{-cl})}+\frac{1}{c^2(1+e^{cl})}\left(\frac{p}{H}-\frac{1}{r}\right)$$

$$C_2=-C_1-\frac{p}{2Hc^2}(e^{ck}+e^{-ck})+\frac{1}{c^2}\left(\frac{p}{H}-\frac{1}{r}\right)$$

Case V.—One Side Span, fully loaded.
Main Span and other Side Span, unloaded.
Temperature, highest.

For: Maximum *positive moments* (and downward deflections) in the (loaded) *side span.*
Maximum *positive shear* at left end of (loaded) *side span.*

CASE V




$$H = \frac{Kpl_1\left(\frac{l_1^2}{12}-\frac{1}{c_1^2}\right)+\frac{2Kp(e^{c_1l_1}-1)}{c_1^3(e^{c_1l_1}+1)} - rc^2EI\omega tL_t}{D}$$

In Segment A-B:

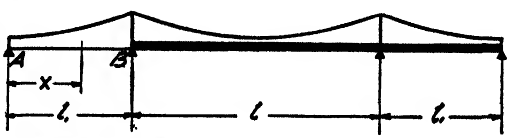
$$C_1 = \frac{1}{c_1^2(1+e^{c_1l_1})}\left(\frac{p}{H}-\frac{1}{r_1}\right)$$

$$C_2 = C_1e^{c_1l_1}$$

Case VI.—Main Span and one Side Span, fully loaded.
Other Side Span, unloaded.
Temperature, lowest.

For: Maximum *negative moments* (and upward deflections) in the (unloaded) *side span.*
Maximum *negative shear* at left end of (unloaded) *side span.*

CASE VI




$$H = \frac{pl\left(\frac{l^2}{12}-\frac{1}{c^2}\right)+Kpl_1\left(\frac{l_1^2}{12}-\frac{1}{c_1^2}\right)+\frac{2p(e^{cl}-1)}{c^3(e^{cl}+1)}+\frac{2Kp(e^{c_1l_1}-1)}{c_1^3(e^{c_1l_1}+1)} + rc^2EI\omega tL_t}{D}$$

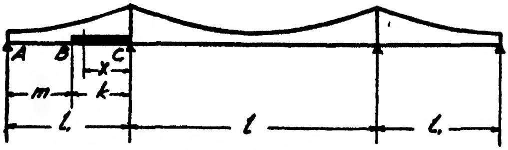
In Segment A-B:

$$C_1 = -\frac{1}{rc_1^2(1+e^{c_1l_1})}$$

$$C_2 = C_1e^{c_1l_1}$$

CASE VII.—Left Side Span, partially loaded from tower end.
Main Span and Right Side Span, unloaded.
Temperature, highest.
For: Maximum *positive shears* in the left *side span.*

CASE VII




$$H=\frac{Kpk\left[\frac{k}{12}(3L_1-2k)-\frac{1}{c_1^2}\right]-\frac{Kp}{c_1^3(e^{c_1L_1}-e^{-c_1L_1})}\left[2-e^{c_1L_1}-e^{-c_1L_1}-e^{c_1k}-e^{-c_1k}+e^{c_1m}+e^{-c_1m}\right]-rc^2EI\omega t L_t}{D}$$

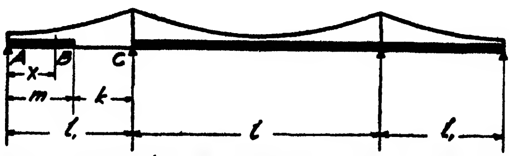
*In Segment B-C:*

$$C_1=\frac{p}{2Hc_1^2}\frac{(e^{c_1m}+e^{-c_1m}-2e^{-c_1L_1})}{(e^{c_1L_1}-e^{-c_1L_1})}-\frac{1}{r_1c_1^2(1+e^{c_1L_1})}$$

$$C_2=-C_1+\frac{1}{c_1^2}\left(\frac{p}{H}-\frac{1}{r_1}\right)$$

CASE VIII.—Left Side Span, partially loaded from anchorage end.
Main Span and Right Side Span, fully loaded.
Temperature, lowest.
For: Maximum *negative shears* in the left *side span.*

CASE VIII




$$H=\frac{pl\left(\frac{l^2}{12}-\frac{1}{c^2}\right)+2Kpl_1\left(\frac{l_1^2}{12}-\frac{1}{c_1^2}\right)+\frac{2p}{c^3}\frac{(e^{cl}-1)}{(e^{cl}+1)}+\frac{4Kp}{c_1^3}\frac{(e^{c_1L_1}-1)}{(e^{c_1L_1}+1)}-[\text{Numerator Case VII}]}{D}$$

*In Segment A-B:*

$$C_1=\frac{p}{2Hc_1^2}\frac{(e^{c_1k}+e^{-c_1k}-2e^{-c_1L_1})}{(e^{c_1L_1}-e^{-c_1L_1})}-\frac{1}{r_1c_1^2(1+e^{c_1L_1})}$$

$$C_2=-C_1+\frac{1}{c_1^2}\left(\frac{p}{H}-\frac{1}{r_1}\right)$$

**10. Maximum Deflections.**—The maximum deflection at the center of the main span is generally produced by full load covering the span, with the side spans unloaded, at highest temperature. This loading condition is represented by Case I with $k=l$, or by Case III with $m=l$. The respective values of $C_1$ and $C_2$ reduce in either case to those given by Eqs. (7) for main span fully loaded.

Substituting the special values of $C_1$, $C_2$, $x$, $M'$, and $y$, the general deflection formula Eq. (3) reduces to:

$$\text{Max. } \eta=\frac{H}{H_w+H}\left[2C_1e^{\frac{cl}{2}}-f+\frac{pl^2}{8H}-\frac{1}{c^2}\left(\frac{p}{H}-\frac{1}{r}\right)\right] \quad . \quad . \quad (21)$$

as the expression for the maximum mid-span deflection. For $C_1$, use the value given by Eq. (7). For $H$, use the value for main span fully loaded at highest temperature, namely (by Eq. 18):

$$H=\frac{pl\left(\frac{l^2}{12}-\frac{1}{c^2}\right)+\frac{2p}{c^3}\cdot\frac{(e^{cl}-1)}{(e^{cl}+1)}-rc^2EI\omega tL_t}{D}. \quad . \quad . \quad (22)$$

For maximum deflection at the center of the side span, use the values of $H$, $C_1$ and $C_2$ given by Case V in the general deflection formula Eq. (3). The expression for the maximum deflection will be the same as Eq. (21) with the side-span terms substituted.

In bridges with very shallow stiffening trusses, a portion of the main span at each end may have to be unloaded in order to obtain maximum deflection at mid-span. Under such condition, loading Case III (Art. 9) must be applied, and the central loaded length $m$ must be found by trial to yield the maximum value of $\eta$.

**11. Effect of Temperature Variation with No Load on Spans.**—Upon substituting $p=0$, the various formulas previously given for $H$ (Eqs. 6, 15, and Cases I to VIII) reduce to Eq. (20), or

$$H_t=\mp\frac{rc^2EI\omega tL_t}{D}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Eqs. (7) for the integration constants reduce to:

$$C_1=-\frac{1}{rc^2(1+e^{cl})}; \quad C_2=C_1e^{cl}. \quad . \quad . \quad . \quad (24)$$

The general formula for moments (Eq. 4) reduces to:

$$M_t = -H_t\left(C_1 e^{cx} + C_2 e^{-cx} + \frac{1}{rc^2}\right). \quad . \quad . \quad (25)$$

The moment at mid-span $\left(x=\frac{l}{2}\right)$ will then be:

$$\text{Max. } M_t = -H_t\left(2C_1 e^{\frac{cl}{2}} + \frac{1}{rc^2}\right). \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The temperature deflection at mid-span will be, by Eq. (21),

$$\text{Max. } \eta_t = \frac{H_t}{H_w + H_t}\left[2C_1 e^{\frac{cl}{2}} - f + \frac{1}{rc^2}\right]. \quad . \quad . \quad . \quad (27)$$

The corresponding expressions for either side span may be written by simply substituting the subscript terms $c_1$, $l_1$, $f_1$, $r_1$ for $c$, $l$, $f$, $r$, in the foregoing Eqs. (24) to (27).

**12. Typical Computations by the Deflection Theory.**—The following outline of the design calculations for the Mount Hope Bridge will serve to illustrate the application of the Deflection Theory.

The Mount Hope Bridge is a two-hinged suspension bridge with suspended side spans. The main span is 1200 ft. center to center of towers, with two side spans 504 ft. long. The layout of the structure is shown in Fig. D1.

*General Data—Calculation of Constants.*—The following dimensional constants are given:

Main Span: $l$ = 62 panels = 1188.33, $f$ = 118.795, $n$ = .099968

Side Span: $l_1$ = 26 panels = 498.33, $f_1$ = 20.891, $n_1$ = .041922

$\sec \alpha_1$ = 1.042362 $r = r_1$ = 1485.9 $K_1$ = 1.00 $A$ = 73.92 in.²

The following loading constants are given (all values per cable):

Dead load: $w$ = 2650 lb./ft. Live load: $p$ = 750 lb./ft.

$$H_w = \frac{wl^2}{8f} = 3940 \text{ kips.}$$

Temperature: $t = \pm 60°$ F., $E$ = 29,000,000, $\omega$ = .0000065

$E\omega t$ = 11,310 lb. per sq. in.

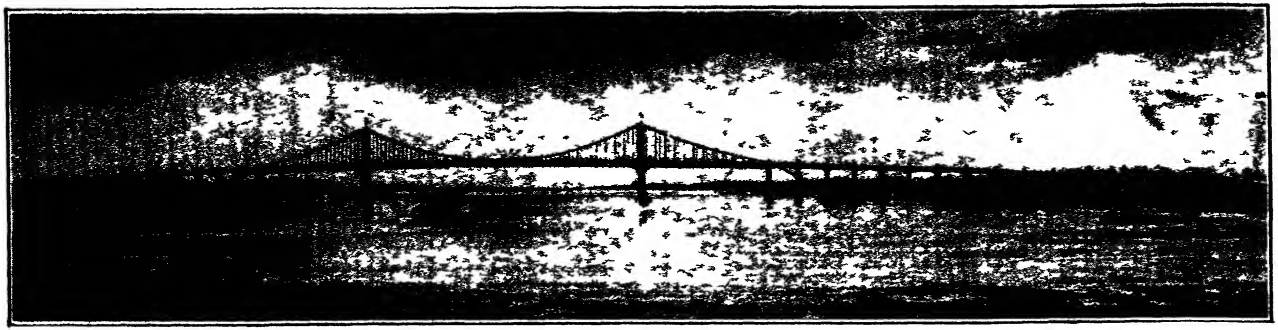


FIG. D1.—Mount Hope Bridge, Rhode Island (1929).

Main Span 1200 ft  Cables of Heat treated Wire
Trusses of Silicon Steel, 18 ft deep 34 ft center to center
Rocker Bents Supporting Cables at Ends of Structure
Designed by Robinson and Steinman, Consulting Engineers

By the approximate formulas in Art. 5,

$$L_t = 2996 \text{ ft.} \qquad L_s = 3133 \text{ ft.}$$

(By exact integration, $L_s = 3138$).

The truss constants are as follows:

Main Span: $I = 4259$ in.$^2$ ft.$^2$; $EI = 123{,}511{,}000$ ft.$^2$ kips

Side Span: $I_1 = 4152$ in.$^2$ ft.$^2$; $EI_1 = 120{,}408{,}000$ ft.$^2$ kips.

*Calculation of Values of D.*—The values of $D$, the denominator of the formulas for $H$, were calculated by Eq. (16) for different values of $H$ from $-100$ to $+1200$, varying by intervals of 100. For these computations, systematic tabular arrangements are used, covering the step-by-step numerical operations. In condensed form, the principal tabulated values are:

| $H$ (kips) | $c$ | $c_1$ | $e^{cl}$ | $e^{c_1l_1}$ | $D$ |
|---|---|---|---|---|---|
| −100 | .005576 | 005647 | 754 | 16.68 | 90,590 |
| 0 | .005648 | .005720 | 822 | 17.30 | 91,256 |
| 200 | .005790 | .005864 | 973 | 18.58 | 92,540 |
| 400 | .005928 | .006004 | 1146 | 19.92 | 93,765 |
| 600 | .006063 | .006140 | 1346 | 21.33 | 94,937 |
| 800 | .006195 | .006274 | 1574 | 22.80 | 96,062 |
| 1000 | .006324 | .006405 | 1836 | 24.34 | 97,144 |
| 1200 | .006451 | .006534 | 2134 | 25.94 | 98,187 |

For convenience of reference and interpolation, the values of $D$ are plotted in a graph, Fig. D2.

*Values of H for Advancing Uniform Load.*—The values of $H$, $C_1$ and $C_2$, for different lengths of a continuous advancing uniform load ($p = 750$) on the main span, with no load on the side spans and at highest temperature ($t = +60°$ F.), are calculated by the formulas of Case I, Art. 9, using an appropriate tabular form of computation. The successive load-lengths vary by 0.1 of the span. Trial values of $H$ are assumed and the operations are repeated once or twice until the resulting values of $H$ check the assumed values. The values of $D$ are taken from the

*D*-Graph, Fig. D2. The principal values and results, in abbreviated form, are:

| Load $k/l$ | Trial $H$ | $D$ (From Fig. D2) | Calculated $H$ | $C_1$ | $C_2$ |
|---|---|---|---|---|---|
| 0 | − 74 | 90,760 | − 74.0 | − .0278 | − 21.47 |
| 0.2 | + 17 | 91,360 | + 16.9 | +179.3 | +1177 |
| 0.4 | +245 | 92,830 | +245.0 | + 2.820 | + 67.66 |
| 0.6 | +519 | 94,510 | +518.6 | + 0.262 | + 21.12 |
| 0.8 | +741 | 95,730 | +741.4 | + .0287 | + 8.944 |
| 1.0 | +829 | 96,230 | +828.8 | + .0037 | + 6.004 |

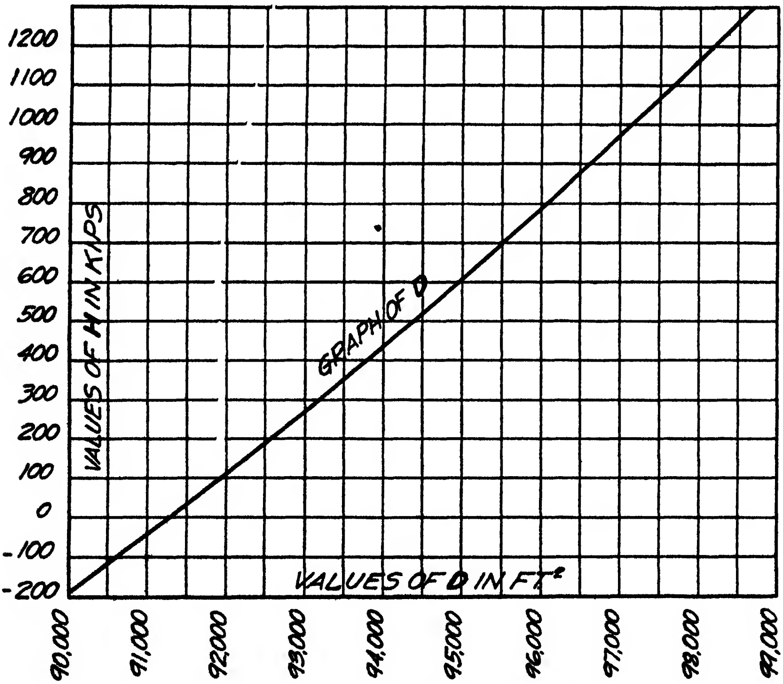



FIG. D2.—Graph for Determination of *D*.
(Mount Hope Bridge.)

For convenient reference and interpolation, these calculated values of *H* for advancing uniform load (at highest temperature) are plotted in a graph (Fig. D3) with the load-lengths as abscissas.

*Positive Moments in Main Span.*—The maximum positive moments for the points $\frac{x}{l}=0.1$, 0.2, 0.3, 0.4, are calculated by Eq. (4) for the load condition of Case I, Art. 9. For each section $x$, $M$ was computed for three or four different trial load-lengths

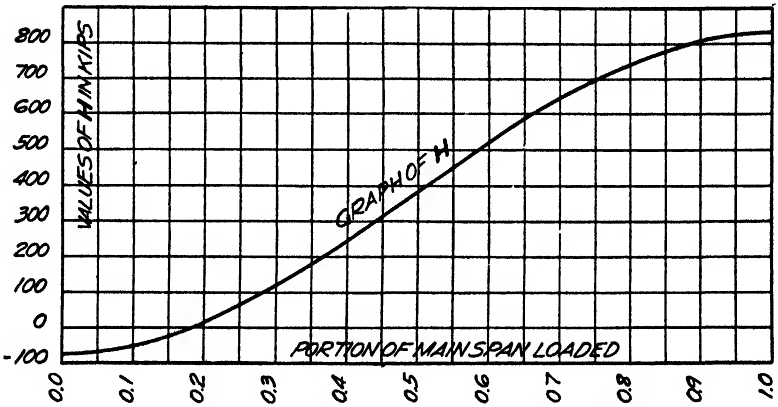



FIG. D3.—Graph for $H$ for Advancing Uniform Load on Main Span. (Mount Hope Bridge.)

$k$, until the maximum value of $M$ for the section was determined. The value of $H$ for each load-length is taken from the $H$-graph, Fig. D3. The tabulation of governing values, in condensed form, is as follows:

| Section: $x/l$ | 0.1 | 0.2 | 0.3 | 0.4 |
|---|---|---|---|---|
| Load: $k/l$ (assumed) | .325 | .375 | .450 | .550 |
| $H$ (from Fig. D3) | 150 | 213 | 312 | 450 |
| $c$ (by Eq. 2) | .00575 | .00580 | .00587 | .00596 |
| $e^{cl}$ | 933 | 983 | 1067 | 1193 |
| $C_1$ (by formula, Case I) | 8.16 | 3.94 | 1.50 | 0.46 |
| $C_2$ (by formula, Case I) | 122.5 | 81.0 | 48.8 | 27.5 |
| $e^{cx}$ | 1.98 | 3.96 | 8.10 | 17.01 |
| Max. $M$ (by Eq. 4) | +7901 | +10391 | +10026 | +8315 |

Different values of $k$ were tried for each section; the above values of $k$ gave the maximum results for $M$.

*Maximum M at Mid-Span.*—The load condition producing maximum positive moment at mid-span is given by Case III, Art. 9, with a uniform load of length $m$ symmetrical about the center line of the main span, no loads in the side spans, and highest temperature ($t = +60°$ F.). Different trial values of $m$ are used until the maximum value of $M$ is determined. For each value of $M$, successive trial values of $H$ are assumed until the calculated value of $H$ checks the trial value. The final values, in condensed form, are:

| | |
|---|---|
| $m$ (assumed) | .350 |
| $k = z$ | .325 |
| $H$ (assumed trial value) | 395 kips |
| $c$ (by Eq. 2) | .005924 |
| $e^{cl}$ | 1141.5 |
| $e^{ck}$ | 9.856 |
| $D$ (by Fig. D2) | 93,739 |
| $H$ (by formula, Case III) | 395.5 |
| $C_1$ (by formula, Case III) | 0.2189 |
| $C_2$ (by formula, Case III) | 249.9 |
| $x/l$ (for mid-span) | 0.5 |
| $e^{cx}$ | 33.786 |
| $p$ (live load per cable) | 750 |
| Max. $M$ (by Eq. 4) | +7954 ft. kips |

In the other trials, $m = .375$ yielded $M = 7898$, and $m = .340$ yielded $M = 7953$. Hence the value for $m = .350$ was recorded as the maximum. (A closer value of $m$ for maximum $M$ is estimated by graphic interpolation.)

*Maximum Positive Moments in Side Span.*—The load condition producing maximum positive moments in the side span is given by Case V, Art. 9, with this side span fully loaded and other spans unloaded at highest temperature. Two or three successive trial values of $H$ are assumed until the calculated value checks the assumed value. The final values, in condensed form, for maximum $M$ at the center of the side span, are:

| | |
|---|---|
| $H$ (trial value) | −36.5 |
| $c_1$ (by Eq. 2) | .005694 |
| $e^{c_1 l_1}$ | 17.071 |
| $D$ (by Fig. D2) | 91,024 |
| $H$ (by formula, Case V) | −36.7 |
| $C_1$ (by formula, Case V) | −36.222 |

| | |
|---|---|
| $C_2$ (by formula, Case V) ...................... | $-618.36$ |
| $x/l_1$ (for mid-span) ........................... | $0.5$ |
| $e^{C_1 x}$ .................................... | $4.1317$ |
| $M$ (by Eq. 4) ................................ | $+12,967$ ft. kips |

*Maximum Negative Moments.*—For computing the maximum negative moments in the main span, the load conditions and formulas of Cases II and IV are employed. Different load-lengths are tried until the greatest value of $-M$ is found for each section. Successive trial values of $H$ are used until the calculated $H$ checks the assumed $H$.

Case VI is used, with successive trial values of $H$, for calculating the maximum negative moments in the side spans.

The procedures are similar to those for positive moments, and the corresponding tabulations are not repeated here.

*Maximum Shears.*—For calculating maximum shears, the loading conditions and formulas of the following Cases are used:

| | |
|---|---|
| Positive $V$ in Main Span ............ | Cases I and III |
| Negative $V$ in Main Span ............ | Cases II and IV |
| Positive $V$ in Side Span ............ | Cases V and VII |
| Negative $V$ in Side Span ............ | Cases VI and VIII |

The values of $H$, $C_1$ and $C_2$ are computed by the same method of tabulation as for moments; the load-lengths, however, except in Cases I and III, are definite in figuring shears and need not be determined by trial. From the values of $H$, $C_1$ and $C_2$, the shears are given by Eq. (5).

*Comparison of Results with Approximate Theory.*—To indicate the saving in truss sections resulting from the use of the more exact theory, the maximum positive bending moments as calculated for the Mount Hope Bridge by the Elastic and Deflection Theories respectively have been plotted in Fig. D4. The comparative moment curves are drawn for both main span and side span; and below those curves, the percentage differences (figured as a reduction correction from the approximate values) are also plotted. It will be observed that the reductions range from 35 to 50 per cent, with an average value of about 45 per cent, in the spans of this structure.

This comparison (Fig. D4) indicates that the application of the Deflection Theory to the design of the Mount Hope Bridge has resulted in a saving of approximately 45 per cent of the steel weight of the stiffening trusses.

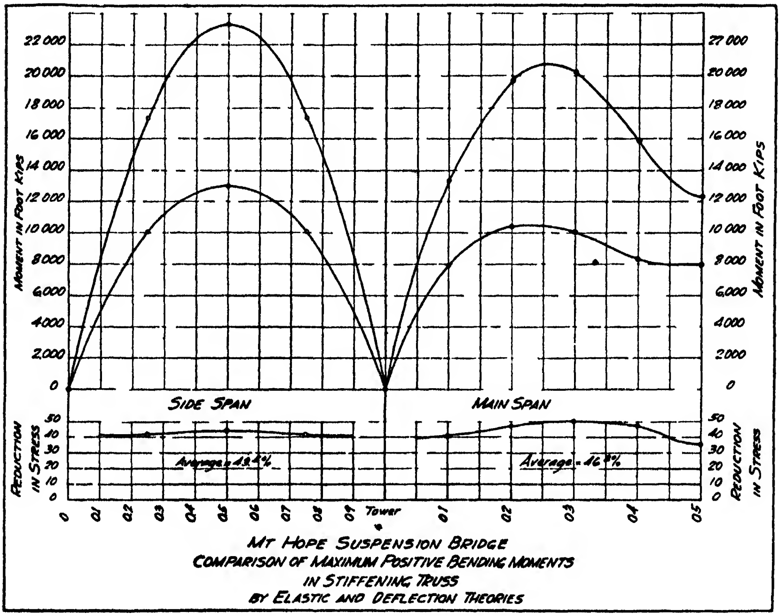



FIG D4 —Comparison of Maximum Bending Moments by Elastic and Deflection Theories
Mount Hope Bridge
Higher Moment Graph for Elastic Theory
Lower Moment Graph for Deflection Theory.
Percentage Reductions Plotted below the Moment Graphs

**13. Charts for Deflection Theory Stresses.**—In Appendix A of this book, the author has presented graphic charts giving directly the maximum and minimum moments and shears throughout the main and side spans of suspension bridges as figured by the Elastic Theory. If those charts are supplemented by graphs giving directly the percentage corrections from the Elastic Theory to the Deflection Theory, the combination will be adapted to the expeditious estimating of the more exact stresses in suspension bridges.

Occasions frequently arise in practice requiring an expeditious estimate of stresses and sections without going into the tedious computations of the Deflection Theory. For such purposes, as in preliminary designs and estimates, it is time-saving to figure the stresses by the approximate theory (or to take them from the charts of Appendix A) and then to apply simple percentage corrections that will reduce the stresses to those of the more exact theory.

A study of the basic formula (Eq. 1), representing the point of departure of the Deflection Theory from the Elastic Theory, indicates that the resulting stress-reductions will depend principally on the magnitude of the dead load and on the flexibility of the structure, since $H_w$ and $\eta$ are the new terms entering the theory. The stress reductions by the Deflection Theory will increase with the deflections and with $H_w$, and will therefore increase with the span length ($l$) and with the dead load ($w$), while decreasing with the truss stiffness ($EI$) and with the cable sag ($f$). (The effect of varying stiffness in a single structure was illustrated in the case of the Florianopolis Bridge, where the reduction of moments by the Deflection Theory was 37 per cent at mid-span, where the truss was shallow, and only 20 per cent at the quarter-points where the truss was deep.)

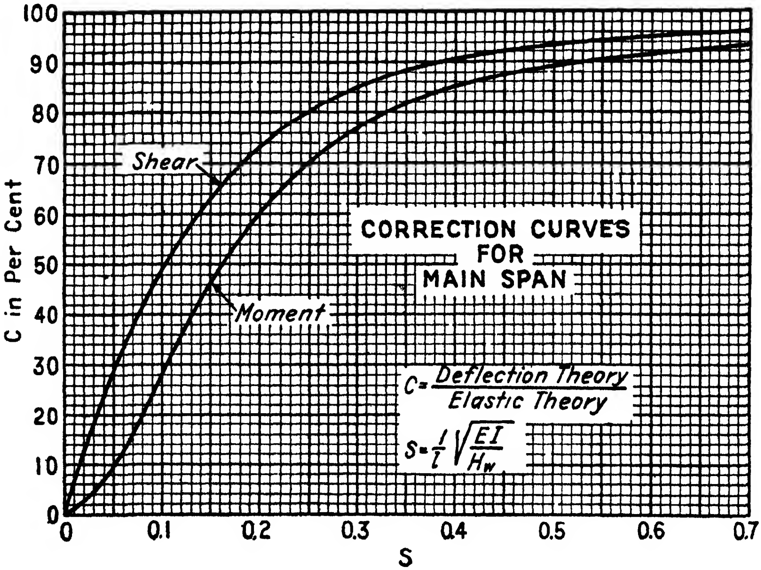
A single expression combining the foregoing factors, which is found to have significance in Deflection Theory analysis, is:

$$S = \frac{1}{l}\sqrt{\frac{EI}{H_w}} = \frac{1}{l^2}\sqrt{\frac{8fEI}{w}}. \qquad (28)$$

This function $S$, for convenience of reference, is called the *stiffness-factor*. The records of past design calculations and the results of new comparisons show that this factor $S$ governs the percentage relation between the results of the two theories.

Curves giving the percentage corrections for varying values of the stiffness factor $S$ are presented in Fig. D5.* The values of $S$ are plotted as abscissas, and the ordinates are the corrections $C$ representing the percentage ratio of stresses by the Deflection Theory to those by the Elastic Theory. The upper graphs are

* These curves were prepared by A. H. Baker while a student in the Graduate School at Rensselaer Polytechnic Institute in 1928, as the completion of studies commenced under the author's guidance in the office of Robinson and Steinman.



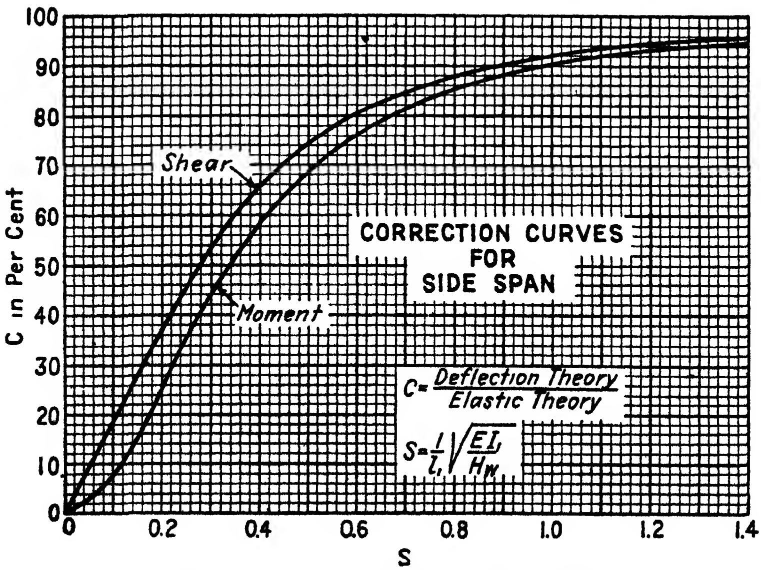




FIG. D5.—Chart for Estimating Deflection Theory Stresses from the Stresses Determined by the Elastic Theory.

for maximum moments and shears in the main span and the lower graphs are for maximum moments and shears in the side spans. For the side-span corrections, the stiffness factor $S$ is figured from the side-span dimensions, $l_1$ and $I_1$.

To illustrate the use of the correction curves, they are applied as follows to the case of the Mount Hope Bridge (Fig. D1). The stiffness factors are, by Eq. (28):

$$S = \frac{1}{l}\sqrt{\frac{EI}{H_w}} = \frac{1}{1188.3}\sqrt{\frac{123{,}511{,}000}{3940}} = 0.149$$

$$S_1 = \frac{1}{l_1}\sqrt{\frac{EI_1}{H_w}} = \frac{1}{498.3}\sqrt{\frac{120{,}408{,}000}{3940}} = 0.351.$$

For $S=0.149$, the upper chart yields $C=0.45$ for main-span moments, and $C=0.63$ for main-span shears, representing reductions of 55 per cent and 37 per cent, respectively, from the main-span moments and shears given by the approximate theory. For $S_1=0.351$, the lower chart yields $C=0.51$ for side-span moments, and $C=0.60$ for side-span shears, representing reductions of 49 and 40 per cent, respectively, from the side-span moments and shears given by the approximate theory.

A comparison of the foregoing factors given by the chart with those determined for the Mount Hope Bridge by actual computations (for live load, without temperature) is as follows:

The maximum bending moment at the quarter-points of the main span as calculated by the Deflection Theory is 9094 ft. kips, and by the Elastic Theory 19,189, yielding a ratio $C=0.47$. The chart yields, for main-span moments, $C=0.453$; adding the small supplementary corrections described below, for $l_1/l=0.42$ and for $w/p=3.53$, the chart factor becomes $C=0.46$, slightly less than the calculated ratio.

The computed maximum shear at the end of the main span is 90.47 kips by the Deflection Theory and 140.8 kips by the Elastic Theory, yielding a ratio $C=0.64$. The chart yields, for main-span shears, $C=0.63$, which becomes $C=0.64$ upon adding the small supplementary corrections, thus checking the calculated ratio.

The computed maximum bending moment at the center of the side span is 12,022 ft. kips by the Deflection Theory and 22,986 by the Elastic Theory, yielding a ratio $C=0.52$. The chart yields, for side-span moments, $C=0.51$, which becomes $C=0.52$ upon adding the small supplementary corrections, thus checking the calculated ratio.

The computed maximum end-shear in the side spans is 112.45 kips by the Deflection Theory and 184.5 kips by the Elastic Theory, yielding a ratio $C=0.61$. The chart yields $C=0.60$, which becomes $C=0.61$ upon adding the small supplementary corrections, thus checking the calculated ratio.

The comparisons represented by Fig. D4 for the Mount Hope Bridge cannot be checked directly by the graphs of Fig. D5, since the former include temperature stresses and the latter do not.

As a further numerical illustration of the use of the correction charts, they are applied as follows to the case of the Grand' Mere Bridge in Quebec (Span 948.5 ft.; straight backstays; Robinson and Steinman, Consulting Engineers): The stiffness factor is:

$$S=\frac{1}{l}\sqrt{\frac{EI}{H_w}}=\frac{1}{948.5}\sqrt{\frac{29,000\times 2150}{1430}}=0.22.$$

For $S=0.22$, the upper chart yields $C=64.5$ per cent for main-span moments. (Actual computation of the moments by the Deflection and Elastic Theories yields a ratio of $C=64.8$ per cent, which is almost identical with the value given by the graph.) This indicates a reduction of 35.5 per cent (or 35.2 per cent as actually computed) in the maximum moments by using the Deflection Theory. For maximum shears, the upper chart yields $C=76.5$ per cent, indicating a reduction of 23.5 per cent by using the Deflection Theory.

The correction curves given in Fig. D5 are for live-load stresses only. The values of temperature moments and shears can be determined quite easily by the Deflection Theory (Art. 11), and may be added to the live-load stresses with very little error (less than 1 per cent for ordinary conditions).

The correction graphs for the main span were computed for maximum positive moments at the quarter-points and maximum positive shears at the ends of the span. The graphs for the side span were computed for maximum positive moments at the center and maximum positive shears at the ends of the span. For other points along the main or side span, the values of $C$ vary somewhat (see Fig. D4); but the corrections given by the graphs will generally be very near the average for the entire span.

The calculation of the curves was based on the following assumed proportions: side span one-half the main span, sag-ratio one-tenth, moment of inertia of stiffening truss constant throughout main span and throughout side spans, cable design stress 80,000 lb. per sq. in., modulus of elasticity 29,000,000 for both cable and stiffening truss, and a dead-load live-load ratio of three. For other proportions, free backstays, etc., small supplementary corrections come into play. These have not been incorporated in Fig. D5, in order to preserve simplicity and since the combination of the supplementary corrections rarely amounts to more than one or two per cent and does not amount to more than five per cent in extreme cases.

The amounts of the supplementary corrections may be estimated by interpolation from the following determinations:

For the case of free side spans, increase $C$ by $2\frac{1}{2}$ per cent of its value.

For sag-ratio $= .12$ (or $.08$) instead of $.10$, decrease (or increase) $C$ by 2 per cent of its value.

For cable stress $= 120,000$ (or $40,000$) instead of $80,000$, increase (or decrease) $C$ by $\frac{1}{2}$ per cent of its value.

For $I/I_1 = 0.75$ instead of $1.00$, increase $C$ by $1\frac{1}{2}$ per cent of its value.

For $l_1/l = 0.25$ instead of $0.50$, increase $C$ by 2 per cent of its value.

For load-ratio $w/p = 5$ instead of 3, add 1 per cent to $C$; for $w/p = 2$, subtract 1 per cent from $C$; for $w/p = 1\frac{1}{2}$, subtract 2 per cent from $C$.

The reciprocal of the stiffness factor $S$ may be termed the *flexibility factor*. Its value is given by:

$$\frac{1}{S} = l\sqrt{\frac{H_w}{EI}} = l^2\sqrt{\frac{w}{8fEI}}. \qquad (29)$$

Roughly, the percentage reductions from the approximate theory are proportional to this flexibility factor $\frac{1}{S}$. Accordingly, the magnitude of the reductions increases with the span-length $l$ and the dead load $w$, while diminishing with the stiffness $EI$ and the sag $f$.

This confirms the observation that the error of the approximate method becomes increasingly significant with long spans, heavy dead load, and shallow trusses, to which may be added flat cable sag. The greatest influence (according to Eq. 29) is that of the span length, and this fact indicates the especial importance of using the Deflection Theory in long-span designs.

# APPENDIX E

## CHRONOLOGICAL TABLE OF SUSPENSION BRIDGES

**Introduction.**—The following table of suspension bridges is presented to furnish a condensed historical review of the progress in the art, also to facilitate reference to particular structures.

The tabulated record indicates the outstanding longevity of bridges of the suspension type, with the exception of a few early structures that were bui t in apparent ignorance of engineering principles. The table lists over a half-dozen suspension bridges that have passed the century mark, over a score that have remained in use more than 75 years, and some three-score that have carried their traffic more than 50 years. This record is unequaled by any other type of bridge structure in iron or steel.

The early bridges, as recorded in the table, were almost exclusively of chain construction. Subsequently the art of building wire cables was developed, and the more modern structures are predominantly of the cable type.

In order to facilitate reference in the table, the more interesting or important structures are listed in heavy type.

Appreciating the difficulty of making such tabulation complete and accurate from available records, the author will welcome communications from readers who have information to amplify or correct this table for revision in any subsequent edition.

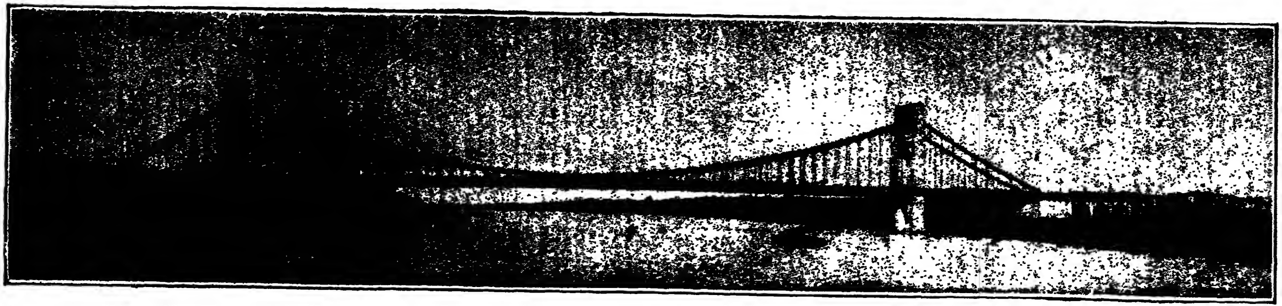


Fig. E1.—Hudson River Bridge (1932).
(The Longest-Span Bridge in the World.)
(Type 2S)

Span 3500 ft., 4 Wire Cables, 36-in. Diameter.
Trusses, Nickel Steel, 27 ft. deep, 106 ft. c. to c.
Steel Towers, to be Encased in Granite-faced Concrete.
Designed by O. H. Ammann, Chief Engineer of Bridges, The Port of New York Authority; and Cass Gilbert, Architect.

## CHRONOLOGICAL TABLE OF SUSPENSION BRIDGES

| Completion Date | Bridge | Location | Span, Feet | Type | Life, Years | Age, 1929, Years |
|---|---|---|---|---|---|---|
| 1741 | River Tees | England | 70 | Chain | 61 | |
| 1785 | Lahn | Weilburg, Germany | 98 | Chain | | |
| 1796 | Uniontown | Pennsylvania | 72 | Chain | | |
| 1803 | Wynch | Tees R., England | ...... | Chain | 105 | |
| 1807 | Potomac R. | Washington, D. C. | 130 | Chain | 33 | |
| 1809 | Schuylkill Falls | Philadelphia | 153 | Chain | 2 | |
| **1809** | **Newburyport** | **Massachusetts** | **244** | **Chain** | **100** | |
| 1811 | Northampton | Pennsylvania | 100 | Chain | | |
| 1815 | Allentown | Pennsylvania | 230 | Chain | 14 | |
| 1816 | Schuylkill Falls | Philadelphia | 408 | Cable | 1 | |
| 1816 | Galashiel | England | 112 | Cable | | |
| 1817 | King's Meadow | Tees R., England | 110 | Cable | | |
| 1818 | Dryburg Abbey | Tweed, England | 260 | Chain | | |
| **1819** | **Tweed** | **Berwick, England** | **449** | **Chain** | ..... | **110** |
| 1823 | Fosse | Geneva, Switzerland | 132 | Cable | | |
| 1823 | Bourbon Is. | France | 132 | Chain | | |
| 1824 | Fontanka | St. Petersburg | 120 | Chain | 82 | |
| 1824 | Strassnitz | Germany | 98 | Chain | 76 | |
| 1824 | Footbridge | Nürnberg, Germany | 112 | Chain | ..... | 105 |
| 1824 | Nienburg | Germany | 256 | Chain | 1 | |
| **1824** | **Tain-Tournon** | **Rhône, France** | **280** | **Chain** | ..... | **105** |
| 1825 | Irwell | Manchester | 138 | Chain | 6 | |
| 1825 | Sophia | Vienna | 232 | Chain | 47 | |
| **1826** | **Menai** | **Bangor, Wales** | **580** | **Chain** | ..... | **103** |
| **1826** | **Conway** | **North Wales** | **327** | **Chain** | ..... | **103** |
| 1826 | Egyptian | St. Petersburg | 180 | Chain | 79 | |
| 1826 | Invalides | Seine, Paris | 558 | Chain | 2 | |
| 1827 | Newburyport | Massachusetts | 160 | Chain | | |
| **1827** | **Hammersmith** | **London** | **400** | **Chain** | **60** | |
| 1827 | Eger | Saaz, Bohemia | 210 | Chain | 69 | |
| 1827 | Arcole | Seine, Paris | 131 | Chain | 27 | |
| 1828 | Beaucaire | Rhône, France | 394 | Cable | ..... | 101 |
| 1828 | Dordogne | Argentat, France | 344 | Cable | | |
| **1828** | **Karl** | **Vienna** | **312** | **Chain** | **48** | |
| 1828 | Rudolfs | Vienna | 110 | Chain | 62 | |
| 1828 | Saône | Lyon, France | 335 | Cable | 60 | |
| 1828 | Rhône | Valence, France | 384 | Cable | | |
| 1829 | Regnitz | Bamberg, Germany | 211 | Chain | 60 | |
| 1829 | Montrose | Scotland | 432 | Chain | ..... | 100 |
| 1829 | Invalides | Seine, Paris | 230 | Chain | 24 | |

CHRONOLOGICAL TABLE OF SUSPENSION BRIDGES—*Continued*

| Completion Date | Bridge | Location | Span, Feet | Type | Life, Years | Age, 1929, Years |
|---|---|---|---|---|---|---|
| 1829 | Serrieres | Rhône, France | 332 | Cable | | |
| 1830 | River Tees | England | 281 | Chain | 11 | |
| 1830 | Schikaneder | Vienna | 85 | Chain | 60 | |
| 1831 | Garonne | Langon, France | 262 | Chain | | |
| 1832 | Bercy | Seine, Paris | 148 | Chain | 32 | |
| 1832 | Bry-sur-Marne | France | 250 | Cable | 39 | |
| 1833 | Louis Philip | Seine, Paris | 231 | Cable | 27 | |
| **1834** | **Freiburg (Saane)** | **Switzerland** | **870** | **Cable** | **90** | |
| 1835 | Garonne | Areole, France | ..... | Chain | | |
| 1836 | Ellbogen | Eger, Bohemia | 222 | Chain | .... | 93 |
| 1836 | Roche Bernard | Vilaine, France | 650 | Cable | 30 | |
| 1837 | Lions' Bridge | Berlin | 57 | Cable | | |
| 1837 | Damiette | Seine, Paris | ..... | Cable | 11 | |
| 1837 | Constantine | Seine, Paris | 328 | Cable | 38 | |
| 1838 | Maine | Angers, France | 344 | Cable | 12 | |
| 1839 | Caille | Annecy, France | 635 | Cable | 44 | |
| 1839 | Weser | Hamlin, Germany | 312 | Chain | 50 | |
| 1839 | Dordogne | Cubzac, France | 360 | Cable | 44 | |
| 1840 | Railway Bridge | Saône R., France | 137 | Cable | 4 | |
| **1840** | **Gotteron** | **Freiburg, Switzerland** | **746** | **Cable** | | **89** |
| 1840 | Jaromer | Elbe R., Bohemia | ..... | Chain | 49 | |
| 1840 | Lezardrieux | Trieux, France | 500 | Cable | 84 | |
| 1841 | Charente | Rochefort, France | 295 | Cable | | |
| 1842 | Moldau | Prague | 435 | Chain | 57 | |
| 1842 | Schuylkill R. | Philadelphia | 358 | Cable | 32 | |
| 1842 | City Bridge | Seine, Paris | 207 | Cable | 19 | |
| **1842** | **Douro** | **Oporto, Portugal** | **557** | **Cable** | **44** | |
| 1843 | Maas | Seraing, Belgium | 345 | Chain | ..... | 86 |
| 1844 | Podebrady | Elbe R., Bohemia | 330 | Chain | ..... | 85 |
| **1844** | **Mülheim** | **Ruhr, Germany** | **320** | **Chain** | **63** | |
| **1845** | **Lancz** | **Budapest** | **663** | **Chain** | | **84** |
| 1845 | Hungerford | London | 676 | Chain | 15 | |
| 1845 | Karl Franz | Graz, Austria | 214 | Chain | | |
| 1845 | Neckar | Mannheim, Germany | 282 | Chain | 46 | |
| 1845 | St. Pierre | Toulouse, France | 295 | Cable | | |
| 1845 | Aqueduct | Allegheny R., Pittsburgh | 162 | Cable | 16 | |
| 1846 | Strakonice | Elbe R., Bohemia | 137 | Chain | | |
| 1846 | Garonne | Verdun, France | 500 | Cable | ..... | 83 |
| 1847 | Footbridge | Niagara R. | 770 | Cable | 7 | |
| 1847 | St. Christophe | Lorient, France | 604 | Cable | ..... | 82 |

CHRONOLOGICAL TABLE OF SUSPENSION BRIDGES—*Continued*

| Completion Date | Bridge | Location | Span, Feet | Type | Life, Years | Age, 1929, Years |
|---|---|---|---|---|---|---|
| 1847 | Smithfield St. | Pittsburgh | 188 | Cable | 35 | |
| 1848 | Guyandot | West Virginia | 450 | Cable | | |
| **1848** | **Wheeling** | **West Virginia** | **1010** | **Cable** | ..... | **81** |
| 1848 | Franz | Vienna | 274 | Chain | ..... | 81 |
| 1849 | Napoleon | Lyon, France | ...... | Cable | ..... | 80 |
| 1849 | Midi | Lyon, France | 398 | Cable | ..... | 80 |
| 1850 | Ostrawitza | Ostrau, Bohemia | 216 | Chain | 36 | |
| 1850 | Fairmont | West Virginia | ...... | Cable | 40 | |
| 1850 | Niagara | Lewiston, N. Y. | 1040 | Cable | 11 | |
| 1852 | Elk River | Charleston, W. Va. | 478 | Cable | 52 | |
| 1852 | St. John | New Brunswick | 628 | Cable | 63 | |
| 1853 | Fonda | Tribes Hill, N. Y. | 556 | Cable | ..... | 76 |
| 1853 | Welikaja | Ostrow, Poland | ...... | Chain | ..... | 76 |
| 1853 | Dnieper | Kief, Ukraine | 440 | Chain | ..... | 76 |
| 1854 | Aare | Switzerland | 330 | Chain | ..... | 75 |
| **1854** | **Railway Bridge** | **Niagara R.** | **821** | **Cable** | **43** | |
| 1855 | Morgantown | West Virginia | 608 | Cable | 53 | |
| 1855 | Tetschen | Elbe R., Bohemia | 373 | Chain | ..... | 74 |
| 1855 | Minneapolis | Mississippi R. | 620 | Cable | 20 | |
| 1857 | Victoria | Chelsea, London | 333 | Chain | ..... | 72 |
| 1857 | Allegheny R. | Pittsburgh | 344 | Cable | | |
| 1857 | Aare | Berne, Switzerland | 200 | Chain | ..... | 72 |
| **1860** | **Railway Bridge** | **Vienna** | **255** | **Chain** | **24** | |
| 1860 | Port Gibson | Mississippi | 150 | Chain | 68 | |
| 1862 | Auburn-Coloma | California | 258 | Cable | ..... | 67 |
| 1863 | Lambeth | London | 280 | Cable | ..... | 66 |
| **1864** | **Clifton** | **Bristol, England** | **702** | **Chain** | **59** | |
| 1864 | Aspern | Vienna | 200 | Chain | 59 | |
| 1864 | Weser | Porta, Germany | 200 | Chain | ..... | 65 |
| **1867** | **Ohio R.** | **Cincinnati, Ohio** | **1057** | **Cable** | ..... | **62** |
| **1868** | **Franz Joseph** | **Moldau, Prague** | **482** | **Chain** | ..... | **61** |
| 1869 | Footbridge | Moldau, Prague | 315 | Chain | ..... | 60 |
| **1869** | **Frankfort** | **Germany** | **262** | **Chain** | **52** | |
| 1869 | Passau | Germany | 246 | Cable | ..... | 60 |
| **1869** | **Clifton** | **Niagara Falls** | **1268** | **Cable** | **29** | |
| 1870 | Waco | Texas | 470 | Cable | ..... | 59 |
| 1870 | Singapore | Straits Settlements | 200 | Chain | ..... | 59 |
| 1870 | Connecticut R. | Turners Falls, Mass. | 452 | Cable | ..... | 59 |
| 1870 | Equinunk | Lordville, N. Y. | 345 | Chain | ..... | 59 |
| 1871 | Warren | Pennsylvania | 470 | Cable | 48 | |

CHRONOLOGICAL TABLE OF SUSPENSION BRIDGES—*Continued*

| Completion Date | Bridge | Location | Span, Feet | Type | Life, Years | Age, 1929, Years |
|---|---|---|---|---|---|---|
| 1872 | Gotha | Germany | 160 | Chain | ..... | 57 |
| 1873 | Albert | London | 400 | Chain | ..... | 56 |
| 1873 | Augarten | Vienna | 202 | Chain | ..... | 56 |
| 1875 | Aare | Brugg, Switzerland | 169 | Cable | ..... | 54 |
| 1877 | Oil City, Pa. | Allegheny R. | 500 | Cable | ..... | 52 |
| **1877** | **Point** | **Pittsburgh** | **800** | **Chain** | **50** | |
| 1877 | Minneapolis | Mississippi R. | 675 | Cable | 13 | |
| 1877 | Inverness | Scotland | 173 | Cable | ..... | 52 |
| 1879 | St. Ilpize | France | 232 | Cable | ..... | 50 |
| **1883** | **Brooklyn** | **New York** | **1595** | **Cable** | ..... | **46** |
| 1884 | Lamothe | Brioude, France | 377 | Cable | ..... | 45 |
| 1884 | Elk River | Charleston, W. Va. | 273 | Cable | 42 | |
| **1884** | **Seventh St.** | **Pittsburgh** | **330** | **Chain** | **42** | |
| 1884 | Windsor Locks | Connecticut R. | 500 | Cable | ..... | 45 |
| 1888 | Avignon | France | 282 | Cable | ..... | 41 |
| 1888 | Saône R. | Lyon, France | 261 | Cable | ..... | 41 |
| 1888 | Canyon | Ecuador | 275 | Cable | ..... | 41 |
| 1889 | Richmond | Indiana | 150 | Cable | ..... | 40 |
| 1889 | Hammersmith | London | ...... | Chain | ..... | 40 |
| 1889 | Tiber | Rome | ...... | Chain | ..... | 40 |
| 1889 | Valley Junction | Whitewater R. | 498 | Cable | ..... | 40 |
| 1890 | Grand Ave. | St. Louis | 400 | Chain | ..... | 39 |
| 1890 | Kellams | Hawkins, N. Y. | 380 | Cable | ..... | 39 |
| 1891 | Voulte | Ardèche, France | 590 | Cable | ..... | 38 |
| 1891 | North Sydney | Australia | 500 | Cable | ..... | 38 |
| 1893 | Loschwitz | Germany | 481 | Chain | ..... | 36 |
| 1893 | Est River | Reunion Is. | 475 | Cable | ..... | 36 |
| **1894** | **Cauca R.** | **Colombia, S. A.** | **940** | **Cable** | ..... | **35** |
| 1894 | Mill Creek Park | Youngstown, Ohio | 90 | Chain | ..... | 35 |
| 1895 | Tower | London | 302 | Chain | ..... | 34 |
| 1896 | E. Liverpool, Ohio | Ohio R. | 705 | Cable | ..... | 33 |
| **1896** | **Rochester, Pa.** | **Ohio R.** | **800** | **Cable** | ..... | **33** |
| 1898 | Langenargen | Germany | 236 | Cable | ..... | 31 |
| 1898 | Lackawaxen | Minisink, N. Y. | 135 | Cable | ..... | 31 |
| 1899 | Lewiston | Niagara R. | 800 | Cable | ..... | 30 |
| 1899 | Mühlentor | Lübeck, Germany | 147 | Chain | ..... | 30 |
| **1900** | **Miampimi** | **Mexico** | **1030** | **Cable** | ..... | **29** |
| 1900 | Easton, Pa | Lehigh R. | 279 | Cable | ..... | 29 |
| 1900 | Cannes Ecluse | France | 760 | Cable | ..... | 29 |
| 1901 | Aramon | France | 902 | Cable | ..... | 28 |

CHRONOLOGICAL TABLE OF SUSPENSION BRIDGES—*Continued*

| Completion Date | Bridge | Location | Span, Feet | Type | Life, Years | Age, 1929, Years |
|---|---|---|---|---|---|---|
| 1902 | Vernaison | France | 764 | Cable | ..... | 27 |
| **1903** | **Elizabeth** | **Budapest** | **951** | **Chain** | ..... | **26** |
| 1903 | Caperton | West Virginia | 510 | Cable | ..... | 26 |
| **1903** | **Williamsburg** | **New York** | **1600** | **Cable** | ..... | **26** |
| 1904 | Ticonic | Waterville, Me. | 400 | Cable | ..... | 25 |
| 1904 | Bonhomme | Blavet R., France | 525 | Cable | ..... | 25 |
| 1904 | Steubenville, Ohio | Ohio R. | 700 | Cable | ..... | 25 |
| 1904 | .................. | Bolivia | 131 | Cable | ..... | 25 |
| 1905 | Tuscumbia | Osage R., Missouri | 627 | Cable | ..... | 24 |
| 1905 | E. Liverpool, Ohio | Ohio R. | 750 | Cable | ..... | 24 |
| 1905 | Cowlitz R. | Kelso, Wash. | 300 | Cable | 18 | |
| 1905 | Borsig | Berlin | 167 | Chain | ..... | 24 |
| **1906** | **Villefranche** | **France** | **512** | **Cable** | ..... | **23** |
| 1908 | Jalapa | Mexico | 184 | Cable | ..... | 21 |
| 1909 | .................. | Costa Rica, C. A. | 208 | Cable | ..... | 20 |
| 1909 | Panama Canal | Empire, C. Z. | 600 | Cable | ..... | |
| 1909 | Newburyport | Massachusetts | 244 | Cable | ..... | 20 |
| **1909** | **Manhattan** | **New York** | **1470** | **Cable** | ..... | **20** |
| 1910 | Breslau | Germany | 415 | Chain | ..... | 19 |
| 1910 | Massena | New York | 400 | Cable | ..... | 19 |
| 1911 | Inn R. | Brail, Switzerland | 550 | Cable | ..... | 18 |
| 1911 | Linn Creek | Osage R., Missouri | 525 | Cable | ..... | 18 |
| 1913 | .................. | Guatemala, C. A. | 180 | Cable | ..... | 16 |
| 1913 | Brilliant | British Columbia | 331 | Cable | ..... | 16 |
| 1914 | 98th Meridian | Byers, Texas | 568 | Cable | ..... | 15 |
| **1915** | **Cologne** | **Rhine R.** | **605** | **Chain** | ..... | **14** |
| 1915 | Muskingum R. | Dresden, Ohio | 450 | Chain | ..... | 14 |
| 1915 | Wenatchee R. | Chiwaukum, Wash. | 190 | Cable | ..... | 14 |
| **1916** | **Parkersburg** | **West Virginia** | **775** | **Cable** | ..... | **13** |
| 1916 | .................. | Colombia, S. A. | 390 | Cable | ..... | 13 |
| 1916 | .................. | Colombia, S. A. | 66 | Cable | ..... | 13 |
| 1917 | Terrall | Ringold, Texas | 450 | Cable | ..... | 12 |
| 1917 | Rio Chiriqui | Panama | 410 | Cable | ..... | 12 |
| 1918 | .................. | Chile | 328 | Cable | ..... | 11 |
| 1919 | Beebee, Wash. | Columbia River | 632 | Cable | ..... | 10 |
| 1919 | Manawatu R. | New Zealand | ...... | Cable | ..... | 10 |
| 1919 | Cumberland R. | Nashville, Tenn. | 540 | Cable | ..... | 10 |
| 1919 | Hansen | Twin Falls, Idaho | ...... | Cable | ..... | 10 |
| 1920 | .................. | Jamaica, W. I. | 280 | Cable | ..... | 9 |
| 1920 | .................. | Colombia, S. A. | 339 | Cable | ..... | 9 |

CHRONOLOGICAL TABLE OF SUSPENSION BRIDGES—*Continued*

| Completion Date | Bridge | Location | Span, Feet | Type | Life, Years | Age, 1929 |
|---|---|---|---|---|---|---|
| 1921 | .................. | Peru | 137 | Cable | ..... | 8 |
| 1921 | .................. | Venezuela | 210 | Cable | ..... | 8 |
| 1921 | Bridgeport | Oklahoma | 600 | Cable | ..... | 8 |
| **1922** | **Rondout** | **Kingston, N. Y.** | **705** | **Cable** | ..... | **7** |
| 1922 | Alum Creek | Columbus, Ohio | ...... | Cable | ..... | 7 |
| 1922 | Ada–Kanawa | Oklahoma | 400 | Cable | ..... | 7 |
| 1923 | .................. | Colombia, S. A. | 230 | Cable | .... | 6 |
| 1923 | Hamm-Lipp Canal | Westphalia | 181 | Chain | ..... | 6 |
| 1924 | Lezardrieux | Trieux, France | 367 | Cable | ..... | 5 |
| **1924** | **Bear Mountain** | **Hudson River** | **1632** | **Cable** | .... | **5** |
| 1924 | .................. | Ecuador | 210 | Cable | ..... | 5 |
| 1924 | Nocona | Red River, Texas | 700 | Cable | ..... | 5 |
| 1924 | .................. | Panama | 250 | Cable | .... | 5 |
| 1925 | Railway & Highway | Costa Rica, C. A. | 250 | Cable | ..... | 4 |
| 1925 | Railway & Highway | Costa Rica, C. A. | 150 | Cable | .... | 4 |
| 1925 | Luzancy | Marne, France | 180 | Chain | .... | 4 |
| **1926** | **Florianopolis** | **Brazil** | **1114** | **Chain** | .... | **3** |
| **1926** | **Philadelphia** | **Delaware R.** | **1750** | **Cable** | .... | **3** |
| **1926** | **Seventh St.** | **Pittsburgh** | **442** | **Chain** | .... | **3** |
| 1926 | .................. | Colombia, S. A. | 623 | Cable | .... | 3 |
| 1926 | .................. | Colombia, S. A. | 156 | Cable | .... | 3 |
| 1927 | .................. | Venezuela | 180 | Cable | .... | 2 |
| 1927 | Bryan–Fannin | Bonham, Texas | 400 | Cable | .... | 2 |
| 1927 | Airline | St. Jo, Texas | 700 | Cable | .... | 2 |
| 1927 | Georgia–Florida | Donaldsonville, Ga. | 600 | Cable | .... | 2 |
| 1927 | .................. | Colombia, S. A. | 417 | Cable | .... | 2 |
| 1927 | Vaux-sous-Laon | Aisne, France | 115 | Chain | .... | 2 |
| 1927 | Mediacanoa | Colombia, S. A. | 380 | Cable | .... | 2 |
| **1927** | **Portsmouth, Ohio** | **Ohio R.** | **700** | **Cable** | .... | **2** |
| 1927 | Humboldthafen | Berlin | 315 | Chain | .... | 2 |
| 1927 | Ninth St. | Pittsburgh | 430 | Chain | .... | 2 |
| 1927 | Montjean | Loire, France | 302 | Cable | .... | 2 |
| 1927 | Girard-Arnodin | Alfortville, Paris | ...... | Cable | .... | 2 |
| **1928** | **Point Pleasant, Ohio** | **Ohio R.** | **700** | **Chain** | .... | **1** |
| 1928 | Steubenville, Ohio | Ohio R. | 689 | Cable | .... | 1 |
| 1928 | Sowells Bluff | Durant, Oklahoma | 500 | Cable | .... | 1 |
| 1928 | Sumida River | Tokyo, Japan | ...... | Chain | ..... | 1 |
| 1928 | .................. | Panama | 380 | Cable | ..... | 1 |
| 1928 | Roma | Rio Grande, Texas | 630 | Cable | ..... | 1 |
| 1928 | Hidalgo | Rio Grande, Texas | 350 | Cable | ..... | 1 |

CHRONOLOGICAL TABLE OF SUSPENSION BRIDGES—*Concluded*

| Completion Date | Bridge | Location | Span, Feet | Type | Life, Years | Age, 1929, Years |
|---|---|---|---|---|---|---|
| 1928 | Zapata | Rio Grande, Texas | ...... | Cable | ..... | 1 |
| 1928 | Mercedes | Rio Grande, Texas | ...... | Cable | ..... | 1 |
| 1928 | Des Arc | White River, Arkansas | 650 | Cable | ..... | 1 |
| 1928 | Anacaro | Colombia, S. A. | 417 | Cable | ..... | 1 |
| 1928 | Sixth St. | Pittsburgh | 430 | Chain | ..... | 1 |
| 1929 | St. Marys, Ohio | Ohio R. | 700 | Chain | | |
| 1929 | Bernardo Arango | Colombia, S. A. | 623 | Cable | | |
| **1929** | **Mount Hope** | **Rhode Island** | **1200** | **Cable** | | |
| **1929** | **Detroit** | **U. S. and Canada** | **1850** | **Cable** | | |
| **1929** | **Grand' Mere** | **Quebec** | **948** | **Cable** | | |
| **1930** | **Mid-Hudson** | **Poughkeepsie, N. Y.** | **1500** | **Cable** | | |
| **1932** | **Hudson River** | **New York** | **3500** | **Cable** | | |
| **a** | **Narrows** | **N. Y. Harbor** | **4500** | **Cable** | | |

a Proposed.

# INDEX

Numbers refer to pages. Illustrations are indicated by an asterisk (*) after page number. For definitions of symbols, consult pages listed under the index word *Notation.*

D



E

SEVEN
DAY
BOOK