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VOLUME IX

HIGHWAY SURVEYING AND SETTING OUT

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HIGHWAY SURVEYING AND SETTING OUT

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PREFACE

Many excellent text-books are available dealing with the general principles of surveying, but the increasing importance of highway engineering has created a demand for a book dealing specially with the practical application of these principles to road surveys.

The present volume is, therefore, confined to those branches of the subject which are concerned with the preparation of survey plans for the improvement of existing roads or construction on a new location.

Throughout the book an attempt has been made to emphasise the practical side of the subject, with occasional detailed notes on points of technique which may appear obvious but which are, nevertheless, often ignored, particularly by beginners.

Mathematical matter has been introduced only where essential for explaining the purpose of various methods of fieldwork procedure, or for obtaining the necessary data for setting out curves or plotting traverses.

The more advanced mathematical treatment of such points as errors and their rectification has been omitted as being of little practical use to surveyors engaged on highway surveys under the conditions found in this country. Field astronomy has been omitted for the same reason.

The author was privileged to receive his early instruction in the theory and practice of surveying from the late Professor M. T. M. Ormsby, formerly of University College, London, and would like to pay a tribute to his sound and clear teaching. The author's gratitude is also due to Robert Wood, B.A., formerly Mathematical Master at Watford Grammar School, for endowing him with a basic knowledge of geometry and trigonometry which has been of constant use in surveying work.

Thanks are also due to Asst. Professor M. K. Rice-Oxley, A.C.G.I., M.I.C.E., of the Civil Engineering Dept., Imperial College of Science and Technology, for very kindly reading the MS. and making valuable suggestions and corrections; to the Director-General of the Ordnance Survey for providing particulars of bench marks and giving permission for the reproduction of illustrations of the old and new types and also the map showing the National Grid; to Major H. E. Aldington, M.I.C.E., Chief Engineer, Ministry of Transport, for permission to quote his comments on the subject of transition curves and data from the Ministry's Memorandum No. 575; to J. J. Leeming, A.M.I.C.E., County Surveyor of Dorset, and A. N. Black, Oxford University, for permission to include particulars of their experiments relating to curve design; to Messrs. Aitken and Boyd, A.M.M.I.C.E., for permission to quote from their paper on vertical curves; to Messrs. McGraw Hill for permission to quote the correction table for spirals from Hickerson's "Highway Surveying"; and to Messrs. Cooke, Troughton, and Simms and Messrs. E. R. Watts for their courtesy in loaning photographs of typical instruments for illustrations.

B. G. MANTON.

GERRARD'S CROSS,
BUCKS.

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CHAPTER I

INTRODUCTION

The work which a highway engineer is called upon to carry out may range from the construction of a new road across a continent to the improvement of a dangerous corner, but in every instance the working drawings are based upon survey plans and sections, and the manner in which the preliminary surveying and levelling is conducted has, therefore, a vital influence on the design of the projected work, on the production of quantities and estimates and, finally, on the actual construction itself.

Incorrect or incomplete details on the original plan or sections may escape undetected until the work is in progress, when some unforeseen difficulty may arise in consequence, and in some cases costly amendments to the design may be required.

Inadequate levelling inevitably results in errors in the calculation of earthwork quantities, and on more than one occasion expensive litigation over disputed figures could have been obviated had the original levelling been carried out in a proper manner.

It will be appreciated, therefore, that a heavy responsibility rests upon the individual who is making a survey or taking levels for a projected scheme, although this work is frequently entrusted to junior assistants.

Accuracy combined with reasonable speed and economy are the two ideals to be aimed at, and the term " accuracy " implies something more than the taking of correct measurements—an accurate plan is one that includes correctly every possible item of detail (or " topography," as it is termed) reproducible to the scale of the drawing, just as an accurate section shows every point at which there is a perceptible change of gradient. In other words, measurements must not only be correct, but they must also be adequate in number.

Relationship between Fieldwork and Plotting. The close relationship between the fieldwork, the subsequent work in the drawing office, and the ultimate purpose of the plans and sections must be constantly borne in mind.

The degree of accuracy in the field measurements of topographical details, for example, should be suited to the scale of the drawing for which these measurements are required, and rough preliminary levelling does not call for the same precision as the setting out of levels for the foundations of a steel-framed structure.

An experienced draughtsman can scale off measurements to about 0·01 of an inch and the actual lengths which this dimension represents on drawings of various scales are tabulated below :

Scale	Equivalent Fractional Representation	Used for	Length in feet represented by 0·01 inch
6 inches to 1 mile	$\frac{1}{10560}$	Ordnance Survey Maps	8·8
25·344 inches to 1 mile	$\frac{1}{2500}$	*ditto*	2·08
200 feet to 1 inch	$\frac{1}{2400}$	Key Plans of long road schemes	2
41·66 feet to 1 inch	$\frac{1}{500}$	Detailed Plans of road schemes	0·4
40 feet to 1 inch	$\frac{1}{480}$	*ditto*	0·4
20 feet to 1 inch	$\frac{1}{240}$	*ditto*	0·2
10 feet to 1 inch	$\frac{1}{120}$	Common vertical scale for Sections	0·1
4 feet to 1 inch	$\frac{1}{48}$	Working Plans and Sections showing full constructional details	0·04

Accuracy depends not only on technique, but also on the equipment used for the work. It is useless, for instance, to take measurements to a fraction of an inch with a linen tape which may, itself, be incorrect by an inch or two in the first 10 feet, as is usually the case with linen tapes after a little wear.

Neat and clear booking of the field notes is a matter of vital importance. Slovenly and untidy notes are usually accompanied by incorrect and inadequate data, and the standard of the booking should be sufficiently good to enable the plotting to be carried out with ease and certainty by a draughtsman who is completely unfamiliar with the site of the survey.

Various routine methods of checking field measurements and calculations are adopted in every branch of surveying. If correctly and faithfully applied, such checks eliminate the risk of undetected errors and should never be omitted.

Survey Methods. The wide variation in the extent of highway engineering schemes results in the adoption of different methods of surveying as a preliminary step in the production of the required drawings, and three distinct types of survey are carried out : (1) the chain survey, (2) the traverse, and (3) minor triangulation, supplemented by chain surveys and traverses.

In all three cases we set out on the ground a framework of lines which can be reproduced on paper and from this framework we take subsidiary measurements to fix the detail.

Chain surveys are based entirely on linear measurements and are only employed for schemes of limited extent, such as the improvement of a corner or a road intersection.

Traverses consist of a framework in which we measure both the lengths of the lines and the angles between them. They are admirably suited to a long strip or ribbon survey such as that required for a new by-pass, and are also useful as a method of surveying built-in blocks for city street improvements.

A minor triangulation is employed where the extent of the survey is a large area, such as an entire urban district as distinct from a long ribbon, and is therefore useful for a comprehensive planning scheme. Its basic principle is the establishment of a network of triangles covering the area, built up from an accurately measured base-line by means of angular measurements only. The corners of the triangles form control stations whose positions are known with a high degree of precision and chain surveys and traverses are run between these stations to locate the topography.

The various operations necessary for preparing the plan are almost invariably accompanied by "levelling," and here again the fieldwork methods have to be suited to the type of scheme which is under consideration.

A small improvement at a corner, for instance, may only necessitate the indication on the plan of the heights of the ground at a few vital points. This is termed "spot levelling".

At the same time, the plan for a new by-pass would be of very limited use without the corresponding "sections"—the longitudinal section showing the undulations of the ground and the levels of the proposed road along the entire centre line, supplemented by cross-sections taken, in general, at right-angles to the centre line at regular intervals in order to show how the land slopes in a sidelong direction.

The plan of a large area derived from a minor triangulation is, again, of limited utility unless there is some indication thereon of the heights of the ground. This may be done in two ways: by a large number of spot-levels, or, better, by contours, and, in either case, an elaborate scheme of levelling will be required.

This brief outline may serve to give some idea of the scope of surveying and levelling as practised by the highway engineer and we may now proceed to consider the subject in greater detail.

CHAPTER II

CHAIN SURVEYING

It has already been mentioned that chain surveys are based entirely on linear measurements, and it therefore follows that the survey lines which constitute the framework must be arranged to form triangles in order that the network may be plotted entirely from the lengths of the sides.

The ordinary geometrical construction involving intersecting arcs is used for plotting the triangles and this method becomes inaccurate if he arcs give an indefinite meeting-point, as they do in an elongated triangle with a short base, such as that shown in fig. 2.1.

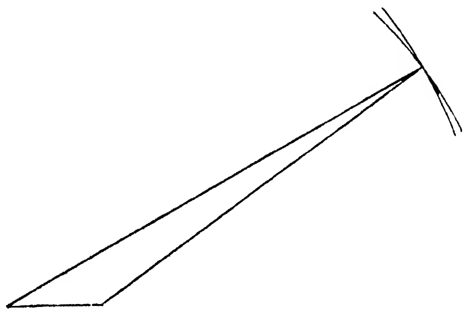


Fig. 2.1.—Intersecting Arcs in Plotting a Chain Survey.

For this reason, among others, the chain survey is not well adapted to road surveys of any magnitude since these, of necessity, take the form of long ribbons and if no angular measurements are taken,

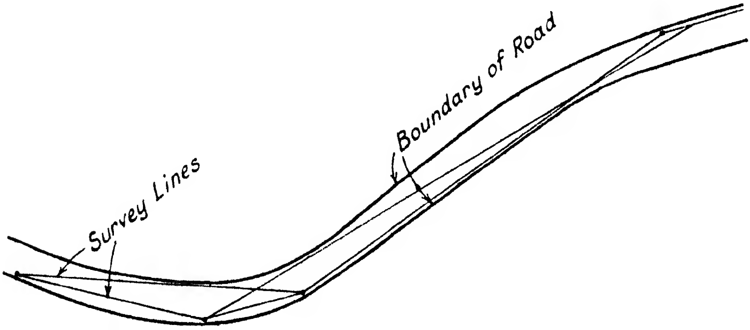


Fig. 2.2.—Chain Survey of a Winding Road showing "ill-conditioned" Triangles.

the framework of survey lines will consist of a series of straights connected by inadequate tie-lines, giving elongated and, therefore, badly shaped triangles, as shown in fig. 2.2, the whole arrangement being ineffective and inconvenient.

For surveys of this kind the only effective method is the traverse, described in Chapter V.

The chain survey, however, is useful for obtaining the plan of a small area, such as a road junction with its immediate vicinity. In such cases it is usually possible to arrange the lines to give well-shaped triangles, forming an adequately tied network which can be plotted accurately; and as plans of this nature are very frequently required for minor improvement schemes, the chain survey will be discussed at some length.

Furthermore, that portion of the fieldwork concerned with the fixing of the topographical details is common to both chain surveys and traverses, and will therefore receive full consideration in this chapter only.

Equipment.

Chains. The lengths of the lines forming the framework of the survey are determined by means of chains, bands, or steel tapes. Surveyors' chains, or "land chains" as they are sometimes termed, consist of a number of sections of straight steel wire, properly hardened and tempered, looped round at the ends and connected together by three oval rings. Each end of the chain is provided with a brass hand-grip, as shown in fig. 2.3, and the full length of the chain is

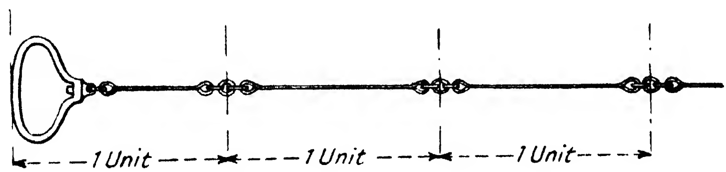


FIG. 2.3.—Portion of Surveyor's Chain.

given by the overall dimension to the outside of these hand-grips. In the case of the Engineer's chain this is 100 feet and in the case of the Gunter's chain 66 feet. It may be mentioned, in passing, that the latter was introduced by Edmund Gunter (1581–1626), a well-known mathematician and astronomer, of Elizabethan days.

The wire of which the sections are made is usually 8, 10, or 12 gauge, giving diameters of 0·16, 0·128, or 0·104 inches respectively and in the more expensive chains the joints are brazed. Cheaper varieties are obtainable in which iron wire is used in place of steel, but when used continuously under rough conditions they are naturally less satisfactory.

In the Engineer's chain the length of the sections, measured from the middle point of one central connecting ring to the middle point of the next, is 1 foot, and in the Gunter's chain 1 link, or 7·92 inches. From the outside of the hand grip to the central connecting ring between the two sections immediately adjacent, is, again, either 1 foot or 1 link, as the case may be.

Intermediate lengths along the chain are given by specially shaped brass tags hung to the central connecting rings at intervals of 10 feet, or 10 links, as shown diagrammatically in fig. 2.4.

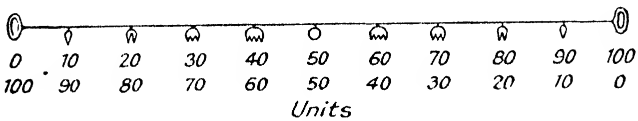


Fig. 2.4.—System of Tags on Surveyor's Chain.

It will be noticed that the tag with a single point is used at both 10 and 90 feet (or links), a tag with two points at 20 and 80, a tag with three points at 30 and 70, and a tag with four points at 40 and 60. The 50-foot and 50-link marks are circular tags. This arrangement makes the chain reversible, end for end, but care must be exercised when measurements are being taken near the centre, to make sure that the 60 tag is not mistaken for the 40, or the 70 for the 30.

When not in use, the chain is folded into a kind of sheaf, by starting at the middle point and doubling over the sections in pairs. Each pair, formed by folding together two individual links, is laid across the previous pair symmetrically but slightly askew, thus keeping a well-defined waist at the centre about which the chain is strapped. When correctly folded, the hand-grips and all the brass tags should be at the same end and when again required for use, the chain is thrown out, holding both hand-grips. It should then open out without tangles into a perfect loop.

The chain can scarcely be called a precise measuring instrument, and owing to the large number of joints at which wear takes place and the liability of the straight sections to become bent, it is a common occurrence for a chain, after a few months of rough treatment, to be an inch or so wrong in length. Errors due to this cause in chaining a line are, of course, cumulative.

For this reason the length of the chain should be checked, at intervals, against a standard length. An excellent method of setting up such a standard is to embed two brass plugs, of about three-quarters of an inch diameter, in concrete, and to scribe on the surface of the brass the necessary lines to indicate the length. The top of the concrete and the surface of the plugs must be flush with the ground which, of course, must be perfectly level between the marks.

As an alternative, two lines may be chiselled to give the standard length on the top face of a kerb, or two brass plugs, as before, may be inserted therein. A municipal or county council would find such a standard very useful if established at the surveyor's headquarters.

The standard length may be obtained from a brand-new chain or, preferably, a brand-new steel band or tape and the measurement should be carried out at 60° F., or thereabouts, since this temperature is usually recognised by instrument makers as that at which chains and steel bands and tapes should give their correct length.

The length of a chain may be adjusted by taking out or inserting connecting rings between the sections.

Highway engineers use the 100-foot chain almost invariably, rather than the Gunter's, but the latter continues to be used by the Ordnance Survey Department and also by railway engineers. It should be noted that 10 Gunter's chains are equal to 1 furlong or 80 Gunter's chains to 1 mile. Furthermore, 10 square chains are equal to 1 acre and for this reason the Gunter's chain is useful in surveying land.

The Engineer's chain is, of course, heavier and bulkier than a 66-foot chain of the same gauge and the sections, being longer, are rather more likely to become bent, but in measuring a long line it requires re-setting for length and alignment considerably fewer times and the chances of error are therefore reduced and time is saved.

The principal disadvantage of the Gunter's chain, however, is the confusion of units which it involves. Even though the survey lines are measured in chains and links, the measurements for fixing the topography are usually taken in feet and inches. Again, when the survey is plotted, the scale adopted would be a certain number of chains or links to 1 inch, but such details as the dimensions of buildings, the lengths of boundaries, or the widths of roads are always expressed in feet. These dimensions must either be scaled off in links and converted arithmetically, or a specially calibrated scale must be used which gives readings in feet from a drawing plotted on a basis of Gunter's chains.

Suppose, for example, the scale is 1 chain (i.e. 66 feet) to an inch. The appropriate scale will be graduated along one edge in inches and tenths, giving chains and the nearest ten links by direct reading, the nearest link being obtained by estimation. The other edge of the scale will be divided in such a way that a length of 1 inch represents 66 units. To divide each inch into 66 parts would result in very close spacing of the graduations and it is customary, therefore, to space the sub-divisions at intervals of one thirty-third of an inch and to indicate by the numbering that these sub-divisions represent distances of 2 feet. Accurate estimation to the nearest foot is easily possible, but the use of a double scale of this kind is clumsy and confusing.

These " surveyor's scales ", as they are called, are used, however, by land agents and estate and agricultural officials who still employ the Gunter's chain of 66 feet.

Tapes and Bands. For the subsidiary measurements, and " offsets ", from the survey lines to the various details in the topography, a linen tape may be used in ordinary work, if a high degree of accuracy is not required. Linen tapes, although they are generally interwoven with fine wire, are somewhat unreliable, varying in length according to the humidity of the atmosphere and easily stretching. They should on no account be wound up when wet or dirty, but should be carefully dried and cleaned first.

For more accurate work, involving precisely fixed detail, such as complicated railway tracks and buildings, plotted to a large scale, neither the chain nor the linen tape should be used, but a steel band, known as a " band chain," should be employed for the survey lines and a steel tape for the remaining measurements.

Band chains are usually half an inch or five-eighths of an inch wide and are provided with brass hand-grips like a chain, winding on to a frame or holder when not in use. They are made in a variety of different forms, corresponding to both the Engineer's and the Gunter's chains. Some are of blued steel, with small brass studs at every foot (or link) and larger studs with numerals or dots at every 10 feet (or links).

Others are of bright steel, with etched graduations, usually reading in feet, inches, and eighths on one side and links on the reverse side.

Steel tapes are made to wind into leather or bakelite cases, exactly like linen tapes. A steel tape five-eighths of an inch wide and 100 feet long is rather heavy and clumsy in use, but lighter sections, down to an eighth of an inch in width, may be obtained. These are handier, but somewhat fragile.

Stainless steel tapes are marketed, but this material is not entirely homogeneous, and such tapes require even more careful handling than the ordinary varieties of plain carbon steel.

Steel tapes must never be allowed to get into a loop, nor to stand on edge on the ground, otherwise fractures and kinks will be inevitable and, needless to say, these tapes should be carefully cleaned and oiled after use.

An occasional break is bound to occur, however, but repairs may be carried out quite easily by means of the outfits supplied by instrument makers, using either a riveted joint or a soldered sleeve.

Ranging Rods and other Equipment. Ranging rods, also known as "poles" or "pickets", are made in various lengths, from 6 to 12 feet; they are fitted with pointed iron shoes and marked off in multi-coloured bands, each 1 foot in width. Although the standard arrangement of colours appears to be black, white and red, a better arrangement is red and white only. The disadvantage of the standard type is not experienced in chain surveying, but is mentioned in connection with theodolite work in Chapter V.

When taking very long sights, particularly against a dark background, it is a great help to have a small red and white flag, or failing this, a handkerchief attached to the top of the rod.

In chaining the survey lines it is customary to use arrows or "pins" to mark the end of each complete chain length. These are made of No. 8 gauge steel wire, pointed at one end and looped into a ring at the other, with an overall length of about 15 inches. Arrows are usually kept in sets of 10 and since they are easily lost, especially in long grass, pieces of brightly coloured braid should be tied to the rings.

Arrangement of Lines in a Chain Survey. The first step in a chain survey consists in the location of the station points which should be so arranged that the lines joining them conform to the following conditions:

(1) The lines must run clear of any obstacles which might be a hindrance to chaining and must also pass close to important detail, so that the measurements connecting the latter to the framework of the survey are short.

(2) The lines should form triangles which are as nearly as possible equilateral in shape, to ensure open intersections of the arcs when plotting.

(3) The lines must build up into a figure which can be plotted from their lengths alone.

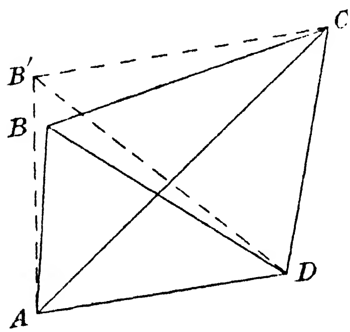


Fig. 2.5.—Method of checking a Quadrilateral by measuring both Diagonals.

(4) There must be a sufficient number of lines to provide check dimensions to every part of the framework. The importance of this may be understood from the following simple example (see fig. 2.5):

The quadrilateral *ABCD* cannot be plotted unless it is sub-divided into two triangles by measuring a diagonal, such as *AC*. Suppose, however, that *AB* is incorrectly measured, its length being booked too long. The figure will, of course, still plot, but the point *B* will be incorrectly located on the plan in some such position as *B′*, all the other lengths remaining unaltered.

If, now, the second diagonal, *BD*, is measured in the field, its length will not check with the plotted length *B′D* and the error in the measurement of *AB* will be revealed.

(5) The number of lines should be kept to a minimum, provided the previous conditions are satisfied.

Although each individual survey requires its own appropriate treatment, the following general rules may be found useful:

(1) It is usually possible to run one or two long lines across the area forming, as it were, " principal axes " from which well-shaped triangles can be built up.

(2) It is inconvenient to cross a busy road with the chain at a very oblique, or " flat ", angle, as the interruptions due to traffic are obviously more pronounced when measuring a long distance diagonally across the carriageway, than when taking a shorter line straight across.

(3) It is often quicker, in the long run, to establish a small subsidiary framework to locate distant topography, instead of taking long offsets and tie measurements (see page 17).

(4) The area covered by the survey should be carried well beyond the extent of any projected works. It is a very risky proceeding to extend the kerb lines, fences, and similar details even a short distance on an inadequate plan without verifying the facts on the site.

The consequences may be awkward and expensive if a project is based, even partially, on guesswork where details on the survey plan are concerned.

Marking the Station Points. Ranging rods are used for temporarily indicating the positions of the stations, and as soon as their locations are settled they are usually marked by wooden pegs where the ground permits of their being driven.

If the public have access to the site of the survey the pegs should be driven flush with the ground, but on private property stouter and longer pegs may be used with their tops left protruding, and the latter may be painted red or white to render them conspicuous.

Flush-driven pegs rapidly become discoloured, particularly in wet weather, and are very difficult to find after a short lapse of time. For this reason two or three guide measurements should always be taken from each peg to any permanent landmarks near at hand, such as fence posts, trees, lamp standards, or the edges of roads and paths. In moorland, or similar country, where such marks may be non-existent, a station peg driven flush in the close vicinity of a road or public footpath may be located from two or three guide pegs from which measurements are taken. The latter are left protruding, but are placed in such positions that they are not likely to be disturbed.

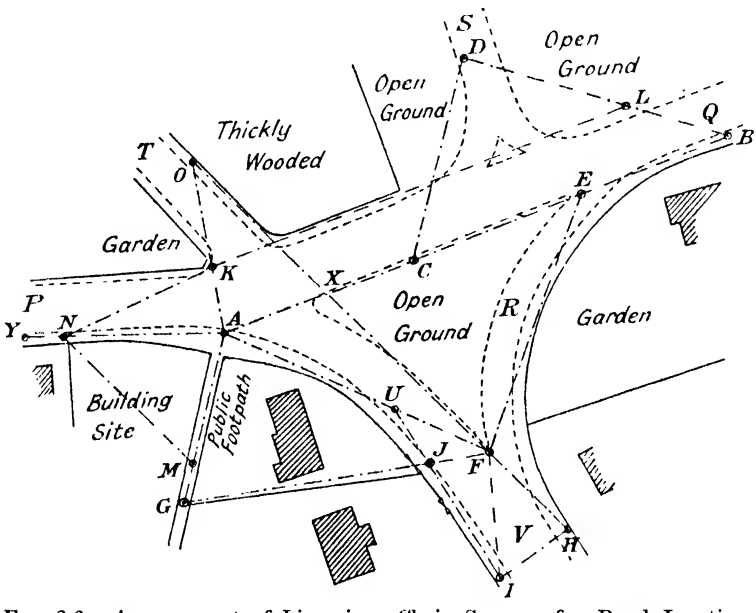


FIG. 2.6.—Arrangement of Lines in a Chain Survey of a Road Junction.

On metalled roads station points can usually be marked by large nails hammered in between the stones and on paved footpaths, or concrete, a small mark may be chiselled.

If frequent references to the stations are likely to be made over a lengthy period wooden pegs may not be sufficiently permanent for important points and short lengths of iron gas barrel embedded in concrete will be found satisfactory.

The position of each station relative to permanent points or guide pegs is sketched in the field book with the accompanying dimensions, the station itself being identified by a letter. The general arrangement of the survey lines can then be drawn, indicating a few prominent features in the topography in their approximate positions and adding a north point if its direction is known.

The lines are identified by their terminal letters, e.g. " Line *AB* ", and there appears to be no advantage in numbering them, although this method is sometimes advocated.

The arrangement of lines for a small chain survey for an improvement scheme at a simple road junction is shown in fig. 2.6.

Example of a small Chain Survey. This is a plan of an actual road junction, in which the full lines represent boundary fences and hedges and the dotted lines kerbs or the edges of turfed common land, marked " Open Ground " (fig. 2.6). There is continuous traffic, including 'buses and coaches from *P* to *Q*, fairly heavy traffic along road *V* and very occasional traffic along roads *R*, *S* and *T*.

The dot-and-dash lines represent the framework of a chain survey. In establishing the survey lines the first step is to set out one or more long straights which will form useful lines from the point of view of topography.

One such line is *AB* and another is *HO*. Two well-shaped triangles, *AEF* and *BCD*, are based on the line *AB*, and although portions of the lines *AF*, *CD* and *BD* cross busy roads more or less obliquely, the parts actually in the carriageway need not be used for taking offsets.

Thus, we should measure the full length *AF*, but an intermediate station is established at the edge of the carriageway at *U* from which the line *UI* runs and measurements to the curved boundary and kerb are taken from lines *AU* and *UI*. Once the length *UF* has been obtained, therefore, this part of the line *AF* is no longer required.

Advantage is taken of the fact that station *F* is visible from a point *G* on the public footpath to establish the triangle *AFG*. This is not only useful for tying the framework together, but also for providing strategic lines for fixing topography, particularly of the corner house which would form a vital feature in the consideration of any improvement scheme at the junction.

The lines *NK* and *FI*, which cross busy roads obliquely, are used as ties only and the hindrance of passing traffic is thus reduced to a minimum. Such lines should not be used for taking offsets.

This example illustrates an inherent drawback of chain surveys when applied to the normal configuration of a highway, as well as the difficulty of avoiding badly shaped triangles, such as *NKA* and *AOX*, where the survey is restricted by boundary fences.

Owing to the almost incessant traffic along roads *P* and *Q* the additional line *KL* was inserted to avoid taking offsets across the carriageway from the line *AB*. This line also provides a useful check on the triangles *AOX* and *BCD*.

The line *AN* is produced to *Y* for the purpose of completing the topography, but the distance *NY* is short compared with *AN*. In general, it is not good practice to prolong lines in this way unless the extension is very short; if it is unavoidable the end point must be tied to another station to provide a check. In a road survey this often results in a badly shaped triangle.

Ample opportunities exist for the measurement of further tie-lines for checking purposes, such as *FD*, *JH*, *DE*, *KC* and *UC*.

It will be noticed that only two lines, *GJF* and *MN*, cross private property, but access to gardens would, of course, be necessary in order to measure the offsets and tie-lines for locating the various buildings on the plan.

The representative of a highway authority has no legal right to enter private property for the purpose of taking measurements, but if permission to enter is asked for with tact and courtesy, and a letter of authority from the county or municipal council or other official body is shown to the householder, it will very rarely happen that reasonable access is refused.

Measurement of the Lines. For work involving much detail of a type which can be fixed accurately, such as buildings, tramway tracks or bridge structures, which are to be plotted to a scale of 40 feet to an inch or larger, the measurement of the survey lines should be carried out preferably with a steel band or tape. The latter should also be used for locating topographical detail.

For preliminary surveys plotted to a small scale, or rural surveys in which hedges, grass verges and similar features form the bulk of the topography, a chain for the measurement of the survey lines and a linen tape for the detailed measurements will provide sufficient accuracy.

Two matters of importance must be borne in mind when measuring lines: (1) careful adjustment of the chain or tape for alignment and distance and (2) careful recording of each successive chain length.

At first glance, nothing could be simpler than measuring the distance between two points separated by a few hundred feet. In actual practice, however, if a dozen inexperienced surveying assistants carried out this measurement independently, scarcely two would agree to an inch and it is highly probable that at least one of the twelve would give an answer 100 feet in error through omitting to keep a strict tally of the successive chainages.

Alignment is carried out by sighting along the edges of three ranging rods, one placed at the rear end of the chain or band, one at the front end and the third at the station at the forward end of the line. Attempts at alignment using less than three rods is certain to be inaccurate.

In measuring long lines a regular routine should be adopted and this assists with regard to both speed and accuracy. The normal procedure is as follows :

The surveyor, or responsible person, in charge of the booking, stands at the rear of the chain, with the assistant, or " chainman ", at the front end, the latter having a complete set of ten arrows when commencing the measurement.

The rear man places a rod at the starting-point and " lines in " the rod held by the front man at a distance slightly less than a full chain length, by bringing the edges of these two rods into coincidence with the third, placed at the distant station.

Definite prearranged signals must be given for such instructions as " move left ", " move right ", " hold the rod vertically " and " correct position ".

Having fixed the alignment, the rear man holds his end of the chain at the zero point and the front man straightens it by sending a gentle undulation along it combined with a slight pull, drawing it past the point of the ranging rod which has been carefully kept at its correct position. He then holds an arrow vertically against the end of the hand-grip of the chain and the rear man checks the alignment of the arrow relative to the distant rod. If correct, the arrow is inserted at the end of the 100-foot chainage, or if the ground is too hard to permit of this, two distinct scratches or chalk marks are made, one to indicate the direction of the line and the other to indicate the distance. The arrow is then laid at the side of the mark to act as a tally in the manner described later.

The front man then moves forward another chain length and the process is repeated, using the back of the arrow as the new zero.

Booking the Chainage. On his arrival at the point just previously marked out the rear man books the appropriate chainage and checks it by counting the arrows. The number of arrows in possession of the rear man, including the one which is marking the back end of the chain, indicates the number of chain lengths up to that point. If the line is greater in length than 1,000 feet, the front man will have no arrow left with which to mark 1,100 feet and the rear man hands over the complete set of 10 when the 1,000 and 1,100-feet chainages have been located. If, however, the arrows become distributed indiscriminately they lose their value as tallies.

The simplest form of booking consists of a single column running across the pages of the field book in the manner shown in fig. 2.7, the chainage distances being entered in this column, commencing at the foot and working steadily forward, page after page.

A convention which renders the booking of distances rather clearer is the separation of the hundreds from the tens and units by a plus sign, thus :

2 + 73	indicates	273 feet
119 + 48	,,	11,948 feet
0 + 04·5	,,	4 feet 6 inches.

This is helpful in the case of very long lines, or in the continuous booking of distances over several miles such as occurs in extensive traverses.

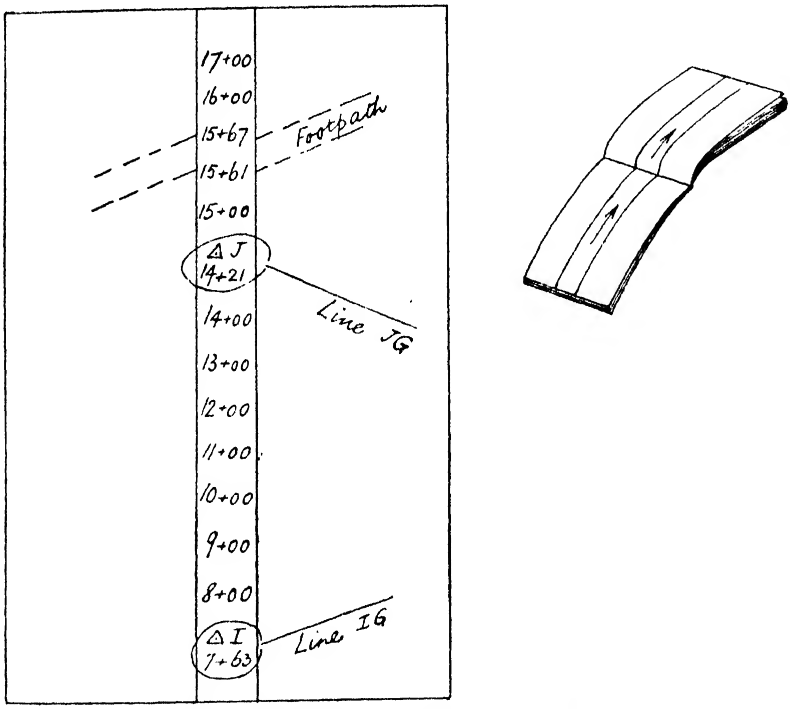


Fig. 2.7.—Method of booking Chainage.

Incidentally, if this convention is used when working with the Gunter's chain, the figures to the left of the plus sign indicate the chains and the figures to the right the links, the above examples reading respectively :

2 chains 73 links, 119 chains 48 links and $4\frac{1}{2}$ links.

While booking the chainage it is worth while indicating the approximate positions at which paths, streams, and other topographical details cross the line, or approach very closely. This may be done quite rapidly without taking any measurements and acts as a useful check, assisting in the location of errors should discrepancies appear as the work proceeds.

It is important to indicate the position of any other survey lines which may intersect at the terminal pegs and the chainman must keep a sharp look-out for intermediate pegs along the line which mark junctions with other lines. The chainage at such pegs must be accurately determined and booked with the appropriate station letter obtained from the sketch showing the general arrangement of the lines.

A typical fragment of chainage booking is shown in fig. 2.7. Note the method of indicating the footpath crossing the chain.

It is advisable to measure the survey lines first, before proceeding to fix the topography. In carrying out the latter portion of the work, the chain or steel band is left in position while the subsidiary measurements are taken and is liable to be slightly displaced by passing traffic or pedestrians. There is thus a tendency for errors to be introduced which do not occur in making rapid successive measurements along the line.

It is also possible to check the accuracy of the measurements by plotting the framework as soon as the complete dimensions are available.

Location of Topographical Details. The various topographical details such as boundary fences, hedges, buildings, trees and other items are located on the plan by subsidiary measurements taken at known distances along the survey lines.

These subsidiary measurements are of two distinct kinds: "Offsets", which are set out at right-angles to the chain line, and "ties", measured obliquely to the chain-line. The latter are intended for check purposes, or for locating detail which cannot be located accurately or conveniently by offsets.

In the case of an offset, if the right-angle is set out inaccurately in the field, the position of the point to be located will be displaced from its true position in a direction parallel to the survey line when it is plotted and its distance from the survey line will also be incorrect, although the extent of the latter error will be much smaller than that of the former.

The amount of displacement, in both cases, depends on the length of the offset and the angular error. In fig. 2.8, for example, if the angle α at which the offset AB is taken is 88° instead of 90° and the offset is 10 feet long, AC is $10.\sin.\ 2°$, or 0·35 feet, i.e. rather more than 4 inches. For an offset 100 feet long, however, the displacement AC becomes 3 feet 6 inches.

The true length of the offset is $AB.\cos\ 2°$, i.e. 9·99 feet if AB is 10 feet and 99·9 feet if AB is 100 feet.

It is thus the sideways displacement which is a serious matter and the importance of keeping offsets as short as possible will be readily appreciated.

A definite limit to the length of offsets is sometimes quoted, but this depends on a number of factors such as the scale to which the drawing is to be plotted, the skill of the surveyor in judging right-angles and the nature of the topography.

If, however, an item such as a boundary runs away from the survey line to such an extent that offsets become greater than the length of the tape, it will often be quicker and more accurate to set out a small subsidiary framework close to the boundary and take short offsets therefrom within the range of the tape, thus obviating the awkward process of re-aligning the tape and extending the measurement. Both offsets and ties are measured whenever possible by running the tape along the ground, the ring or zero end being held at the point whose position is to be fixed and the required distance being read off at the point of contact of the tape and the chain.

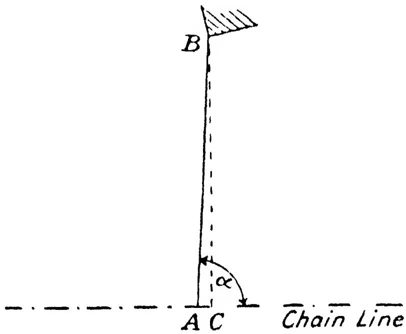


FIG. 2.8.—Effect of Error in judging a Perpendicular Offset.

If, as frequently happens, the measurement cannot be obtained without raising the tape off the ground, a ranging rod must be held vertically at the correct point on the chain and the tape must be stretched horizontally between the rod and the distant object.

For short offsets a ranging rod serves as a very convenient measure and when obtaining the distance to inaccessible points, such as the centre of a hedge or the wall of a building separated from the chain line by a low fence, the ring of the tape may be hooked on to the point of the rod and the latter held against the particular feature in the topography.

Methods of Setting Out Right-angles

Although experienced surveyors can judge a right-angle with considerable accuracy without instrumental or other aid, there are several simple methods of setting out perpendiculars the adoption of which will enhance the accuracy of the work and enable long offsets to be taken with more confidence. The simplest method is applicable only to the case in which a perpendicular is required from a fixed point to the chain line, but this is the normal condition which exists when taking offsets. It consists merely in swinging the tape about the fixed point as centre, the ring being held at that position, and noting the minimum distance from the point to the chain, as shown in fig. 2.9.

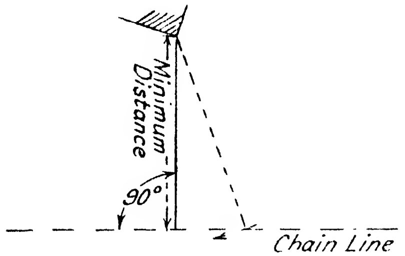


FIG. 2.9.—Method of obtaining a Perpendicular Offset from a Distant Point.

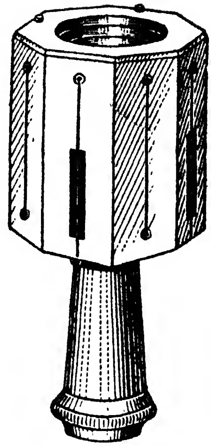


FIG. 2.10.—Octagonal Type of Cross-staff.

The Cross Staff. This simple instrument may be used for setting out a perpendicular from a given point on the chain line and is illustrated in fig. 2.10. It consists of a hollow octagonal box, about $2\frac{1}{2}$ or 3 inches in height and 2 to $2\frac{1}{2}$ inches across the flats, with sighting slits and wires in the vertical faces, so arranged to give lines of sight at right-angles to one another and also at angles of 45°. It is usually screwed to a threaded collar attached to the top of a 5-foot ranging rod.

The Optical Square. This is a more effective instrument than the cross staff and it obviates the necessity, inseparable from the latter, of focussing the eye more or less simultaneously on a cross wire at extremely close range, a slit a little farther away and a ranging rod at a considerably greater distance.

The optical square consists of a circular brass case about 2 inches in diameter and half an inch deep containing two small mirrors mounted on edge so that their planes are at 45° to one another. Fig. 2.11 represents a diagrammatic plan of this arrangement, F and G being the mirrors. The rim of the case is provided with two large apertures, C and D, and a small sighting hole, E.

If it is assumed that the instrument is located immediately above the point H at which the lines BK and AH intersect, and if ranging rods are placed at A and B and the optical square is held with the apertures and sighting hole in the positions shown, the rod at B will be visible through the hole E by direct vision over the top of mirror G and an image of the rod at A will appear in the mirror G, after reflection from the mirror F, provided that the lines BK and AH are nearly at right angles. If they are exactly at right-angles, the image of the rod at A will appear vertically under the rod at B and this condition is shown in the diagram.

To set out a right-angle at any point on a survey line, such as H, therefore, we place the optical square vertically above the point, usually mounting it upon a short ranging rod, and keeping it horizontal. Then sight directly through the instrument to a ranging rod placed at the forward end of the line, B. Another rod is now placed as nearly as possible on the perpendicular and moved slowly until its reflected

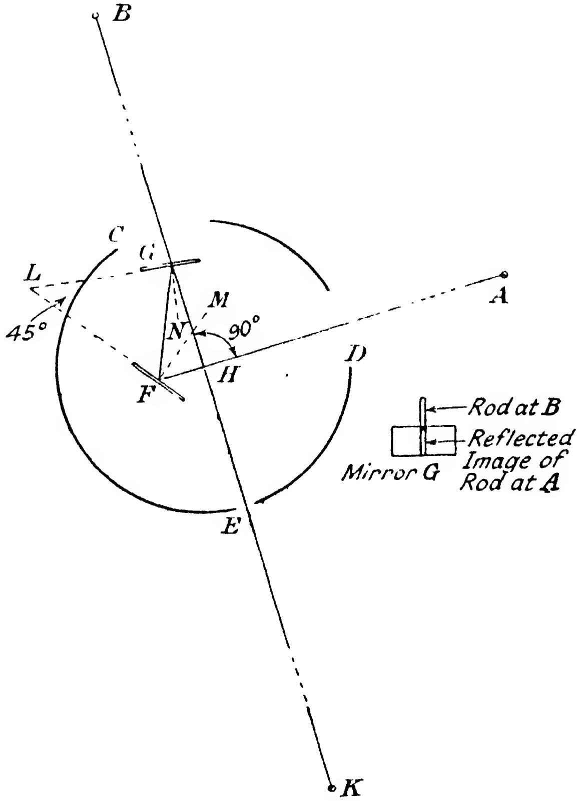


FIG. 2.11.—Principle of the Optical Square.

image falls exactly in the same vertical line with the rod at B. This gives the position A which fixes the perpendicular AH.

GEOMETRICAL PROOF

Let the planes of the mirrors intersect at L and let FM and GN be the normals to the mirrors. Then $\angle FLG = 45°$.

$$\begin{aligned}\angle AHG &= \angle HFG + \angle FGH \\ &= 2\ \angle MFG + 2\ \angle FGN \\ &= 2(90° - \angle LFG) + 2(90° - \angle LGF) \\ &= 360° - 2(180° - \angle FLG) \\ &= 360° - 2(180° - 45°) \\ &= 90°.\end{aligned}$$

Testing and Adjustment of the Optical Square. The more expensive optical squares are provided with a means of adjustment to enable the angle between the planes of the mirrors to be set exactly at 45°. This is effected by rotating one mirror through a small angle by means of a diminutive box-spanner, provided with the instrument, and engaging with a square-headed nut protruding through the case. To test the instrument, four ranging rods, A, A', B, B', are very carefully set out in perfect alignment as shown in fig. 2.12, the distance AB' being about 200 feet.

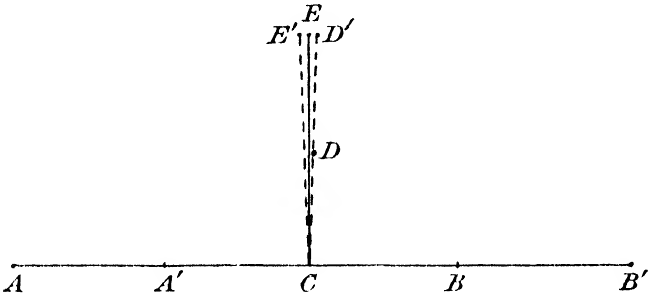


FIG. 2.12.—Test for Adjustment of Optical Square.

The optical square is held horizontally on a short ranging rod placed at an intermediate point, C, about half-way between A and B' and directly in line with B and B' which are viewed directly through the instrument.

While viewing either pair of rods, say B and B', and keeping truly aligned, set out two more rods, D and D', so that their images are in coincidence with one another and fall vertically in line with B and B'.

Then, theoretically, CDD' will be perpendicular to AB'.

Now turn the instrument so that D and D' are seen in coincidence by direct vision and A and A' by reflection. If CDD' is accurately perpendicular to AB', the two rods A and A' will fall in the same vertical line as D and D'.

If the instrument is wrongly adjusted, however, this condition will not be obtained. In this case an additional rod is placed at E', keeping the distances CE' and CD' approximately equal and setting the rod exactly in alignment with the coincident images of A and A'. Bisect the distance $D'E'$ at E and reset the rod at this point. Next turn the adjustable mirror very slightly until the rod E falls into perfect vertical alignment with the coincident images of the rods A and A'. Finally check the new setting by viewing the rod at E in conjunction with the rods at B and B'.

Setting out Right-angles Geometrically. The perpendicular distance from a given point to a survey line may be obtained quite easily by describing an arc with the tape, about the point as centre, to cut the chain line in two places, noting the chainages at the points of intersection, bisecting the distance between them and taking the measurement from the mid-point to the distant point, as indicated in fig. 2.13.

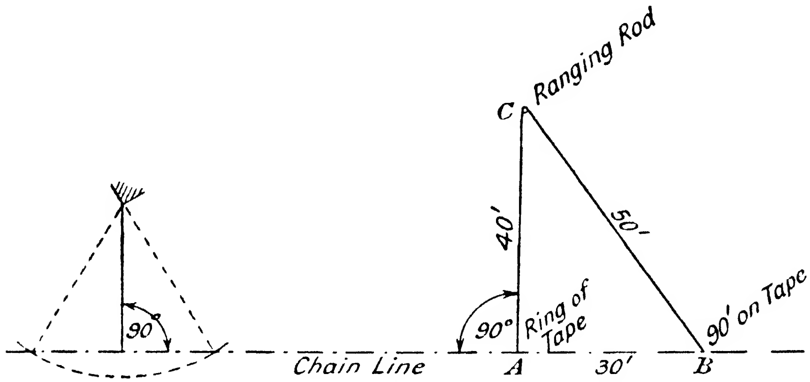


FIG. 2.13.—Geometrical Method and "3, 4, 5" Rule for setting out Right-angles.

Another geometrical method is based upon the fact that a triangle whose sides have lengths in the ratio 3 : 4 : 5 is right-angled (5 : 12 : 13 is another, but less suitable, ratio). Hence, if a right-angle to a chain line is required at *A*, in fig. 2.13, we may measure 30 feet along the line, giving the point *B*, then hold the ring of the tape at *A*, place a ranging rod at the 40-foot mark, *C*, and bring the 90-foot mark back to the point B, making *CB* 50 feet long. When both portions of the tape are pulled tight, the position of the rod *C* will give the perpendicular.

The process should be repeated, measuring 30 feet on the opposite side of *A* to *B*, or on the opposite side of the chain line.

Ties and Check Measurements. When important and definitely fixed detail such as a building is encountered, it is not sufficient to depend upon offsets alone to locate the corners, but oblique ties, or check measurements must be taken, in addition. Thus, in fig. 2.14, corners *A*,*B*,*C*,*D* and *E* can be plotted from the offsets 1, 2, 3, 4 and 5, but the ties 6, 7, 8 and 9 are necessary as checks, in conjunction with the dimensions *AB*, *BG*, *GC*, etc., of the building itself.

It is a convenience in fieldwork and simplifies booking if ties are run from the same points on the chain line as offsets, as at *K* and *L*, but this procedure should not be adopted unless a good "fix" is obtained, i.e. the tie should run at roughly 45° to the chain line.

It is useful when booking the topographical details of a building, to indicate the points at which the walls, if produced, would intersect the chain. Thus, the wall AF, produced, cuts the chain line at M and the wall HE at N. These points are determined by holding a

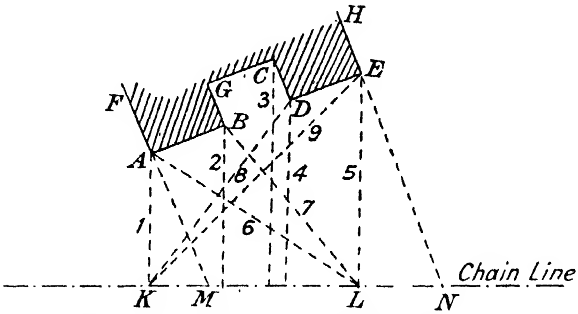


FIG. 2.14.—Method of "trying-in" a Building from the Chain Line.

ranging rod on the chain line and sighting it in alignment with the walls.

An elaboration of this principle will sometimes enable an inaccessible building to be located from three, or more, survey lines in the manner indicated in fig. 2.15.

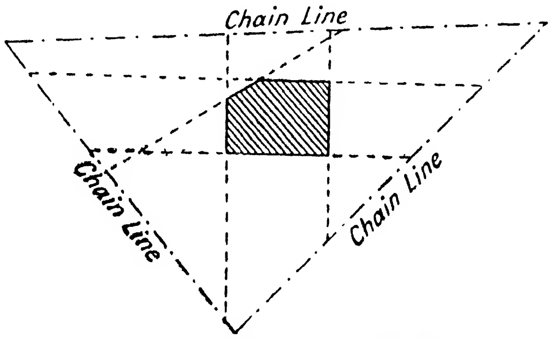


FIG. 2.15.—Method of "sighting-in" an Inaccessible Building.

Booking Topographical Notes. Efficient booking of topographical notes calls for the ability to sketch neatly, quickly, and in correct proportion. Some surveying assistants find it difficult to acquire the necessary skill in this respect, but it is largely a matter of practice.

Untidy and ill-kept notes should not be tolerated—they cause delays, uncertainty and inaccuracy in plotting; and an efficient surveyor can keep clear and neatly sketched notes just as quickly as the inefficient man with his indecipherable scrawl and badly drawn diagrams.

The usual form of ruling for the field notes is identical with that recommended for booking the chainages of the lines, i.e. a single column running centrally up and down the page. Distances along the survey line are, as before, entered in this column, commencing at the foot and running continuously page after page.

Details right and left of the chain are sketched in their correct relative positions, although not necessarily to scale, and offsets are written opposite the appropriate chainage, close to the points to which they are taken, without using dimension lines.

The following hints on booking topography may be helpful :

(1) Stations are indicated either by a small circle with a dot, or by a small triangle with a dot, together with the reference letter and a ring should be drawn round the station letter and its chainage to render this quite distinct in the chainage column.

(2) Do not attempt to keep to the same approximate scale throughout the work, but where detail is complicated, expand the sketch to give ample room for dimensions and where there is very little detail use a smaller scale. Note particularly the correct method of sketching an item such as a footpath, or a fence-line, which crosses the chain, shown in fig. 2.7, and again in fig. 2.16.

(3) When making the sketch, stand over the chain line, facing the direction in which the chainage is running, so that the angles at which fence-lines, walls of buildings, kerbs and other linear details occur are seen in their correct perspective in relationship to the direction of the chain line and on the correct side of it.

It is, of course, often necessary to inspect distant detail at close quarters to verify doubtful features, but the bulk of the sketch should be made from the chain line.

(4) Do not sketch too far ahead, but keep the drawing just in advance of the measurements. Do not permit the sketch on one side of the column to get too far ahead of that on the other side, otherwise it may be difficult to fit in the chainages which refer to the topographical details on the latter side.

(5) Do not use dimension lines for offsets, but figure in the distance close to the point to which it is taken and enter the corresponding chainage in the centre column exactly opposite. Dimension lines are necessary, however, in the case of ties.

Great care must be taken, when booking a series of measurements from the same chainage point, to make quite clear whether these are running dimensions or separate measurements. Thus, in fig. 2.16(*a*), the booking shows the location of the edge of a footpath, edges of a ditch, and a fence-line by running dimensions—the normal procedure in the field. Fig. 2.16(*b*) shows the same detail booked as separate measurements.

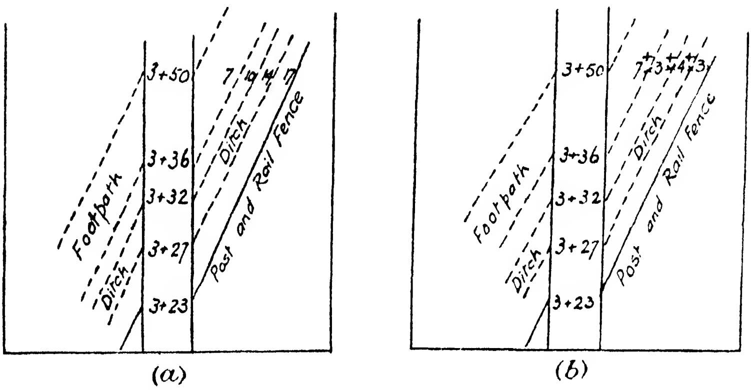


FIG. 2.16.—Methods of booking Topographical Detail.

(6) It frequently happens that some such complicated detail as a large building of irregular shape has one or more corners fixed by offsets and ties from the chain line. In this case the plan of the entire building should not be cramped into the limited space available at the side of the chainage column, but a sketch should be made showing only the corners fixed directly from the chain. A separate block plan can then be drawn, occupying an entire page of the field book, if necessary, with the same corners clearly referenced, as shown in fig. 2.17.

Incidentally, it may be mentioned here that if a large building has a comparatively short frontage facing the chain line and a considerable depth, it cannot be accurately fixed by offsets and ties from the front only and it is essential to tie the corners at the back of the building even if this necessitates running another survey line to do so.

(7) Any other survey lines running into a station should be indicated at roughly the correct angles.

The usual type of field book in which the pages measure about 6 inches by 4 inches is somewhat small for complicated topography. A sensibly sized field book was adopted some time ago by the Port of London Authority for dock surveys. These books, measuring about 11 by 8 inches with pages of stout paper ruled in small squares, were very convenient for sketching complex detail.

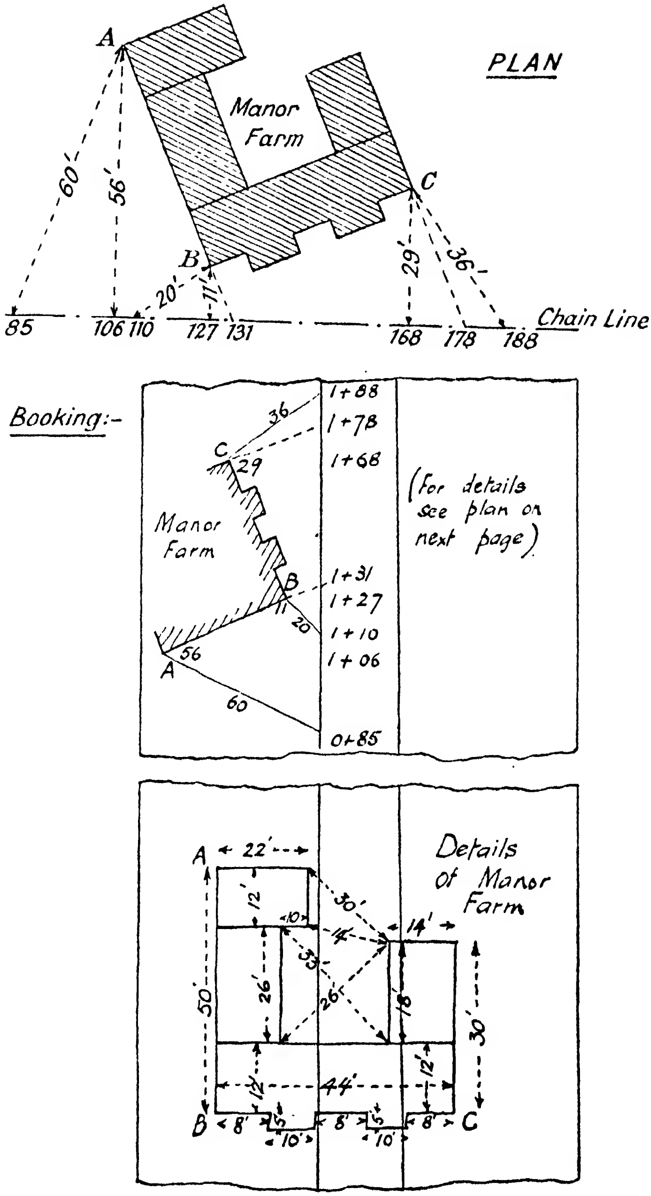


Fig. 2.17.—Method of booking Complicated Detail.

Data provided by a Plan. The fullest possible information concerning topography should be included on the plan and particulars which, at first sight, might be considered to be immaterial often prove to be vitally important.

The following are some of the items which should be noted at the time of taking topographical measurements :

Type of land or mode of cultivation, under such headings as " Arable ", " Pasture ", " Rough Pasture ", " Furze ", " Orchard ", etc.

Varieties of timber, when trees are sufficiently large to be of value, e.g. " Oak ", " Ash ", etc., also whether deciduous or evergreen.

Descriptions of all fences, e.g. " Post and Rail ", " Concrete Post and Wire ", etc. Fences are shown as a full line on the plan and a written description should be added. Brick or stone walls are sometimes shown by a double line and should also be described on the drawing.

The direction of flow of rivers and streams should be noted, and a road plan must show the position of every lamp post, telephone and electricity pole, Post Office manhole, fire hydrant, water valve, gas valve, electricity manhole, sewer manhole and lamphole, milestone, guide-post and Ordnance Bench Mark.

The importance of the latter item will be discussed in the chapter on levelling.

Whenever possible it is advisable to adopt the conventional signs used on the large-scale Ordnance maps.

Problems in Chain Surveying

I. Chaining on Sloping Ground. All topographical measurements in a chain survey or a traverse should be horizontal, since a plan depicts the horizontal projection of the various topographical details.

The error introduced by taking measurements along the surface of sloping ground depends upon the length and the degree of the slope, and unless this error is sufficiently large to be visible to the scale of the drawing it may be neglected. For instance, if we consider a slope of 1 in 10, meaning 1 foot vertical to 10 feet horizontal, the length of the hypotenuse corresponding to a 10-foot base is $\sqrt{101}$, or 10·05 feet, very nearly. This results in a discrepancy between the sloping and horizontal measurements amounting to nearly 6 inches in every 100 feet. Considering, next, a slope of 1 in 4, the length of the hypotenuse corresponding to a 4-foot base is $\sqrt{17}$, or 4·12 feet and the discrepancy, in this case, between the sloping and horizontal measurements is about 3 feet in every 100.

If the scale of the plan is 40 feet to an inch and we assume that a skilful draughtsman can plot to 0·01 of an inch, the difference between the inclined and horizontal measurements would just be visible in the case of a gradient of 1 in 10, 100 feet in length.

In the same way, if the plan is to be drawn to the scale of 200 feet to an inch, the difference between the inclined and horizontal measurements would just be visible in the case of a slope of 1 in 4, 67 feet long.

Provided the slope is not extremely steep or lengthy, the most practical method of obtaining the projected horizontal length is known as "chaining in steps", the slope being measured in short horizontal stages.

It is easier to carry out this operation working downhill and the procedure is as follows :

A heavy chain cannot be stretched out horizontally for a greater length than 10 feet, or thereabouts, without introducing an error due to sag which might well be more serious than that involved in accepting the sloping measurement. A light steel tape should be used in preference, but even in this case 25 or 30 feet will be found the maximum workable length. With a light tape a strong side wind will make accurate measurement somewhat difficult and in very windy weather a chain is better.

A set of specially distinct arrows must be kept for marking off the short lengths. Thus, if the ordinary arrows, used for marking the full chain lengths have red braid attached, the special arrows should be distinguished by a different colour and the red-braided arrows only used at the hundred-foot chainage points.

The setting of the tape or chain in a horizontal position cannot be judged by viewing it from the side, owing to the deceptive effect of the sloping ground, and the simplest method of obtaining the horizontal distances is the following : A plumb-line is held against the 10- or 20-foot mark on the tape or chain which is then swung in a vertical plane about its upper end until the plumb-line indicates that the maximum length is spanned, corresponding to the horizontal position, as shown in fig. 2.18. A special arrow is then inserted below the plumb-bob and the process is repeated.

On very steep slopes the length of the horizontal measurement will necessarily be very short since the chain or tape would otherwise be too high at its forward end and on long slopes the method becomes tedious and the repetition of many short lengths introduces errors.

An alternative method may then be adopted in which the chaining is carried along the surface of the ground and some form of clinometer is used for determining the angle of slope so that the necessary corrections can be applied.

Several forms of clinometer are available, the two most commonly applied being the small Watkins' instrument and the Abney level.

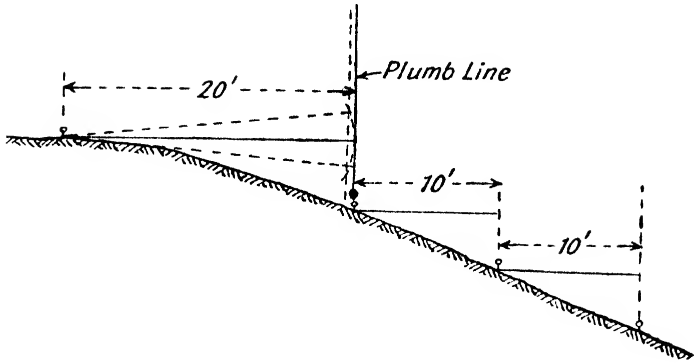


FIG. 2.18.—Chaining "in steps" down a Slope.

The Watkins' Clinometer. This instrument (fig. 2.19) consists of a pendulum, *AB*, pivoted about its middle point, *C*, and carrying a circular frame, *D*, to which is attached a curved scale, *E*, of white celluloid, graduated in degrees on its inner or concave surface. The pendulum and its attachments are enclosed in a circular case, *K*, in the rim of which are two sighting apertures, *F* and *G*, located at opposite ends of a diameter. An index mark inscribed on white celluloid is attached to the inside of the case alongside the aperture *F*

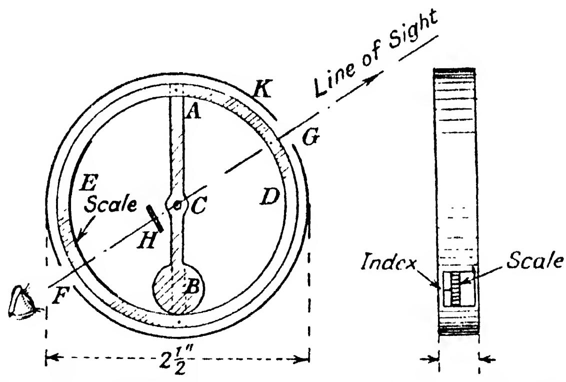


FIG. 2.19.—Working Principle of the Watkins' Clinometer.

and both this mark and the scale are reflected in the small mirror, *H*, placed to one side of the line of sight through *F* and *G*. The scale reading is thus visible while an observation is being taken to a distant point.

The instrument is held at a known height above the ground and a sight is taken to a rod or staff on which the same height is clearly indicated. The pendulum, which is normally clamped by a spring-loaded friction device, is then released and takes up a vertical position, carrying the scale with it, and the reflected reading of the latter against the index mark gives the angle of inclination of the line of sight.

For correct results, the ground should have a uniform gradient between the observer and the distant rod.

The Abney Level. This instrument (fig. 2.20) consists of a sighting tube, *AB*, about 6 inches long, on the top of which is a small spirit

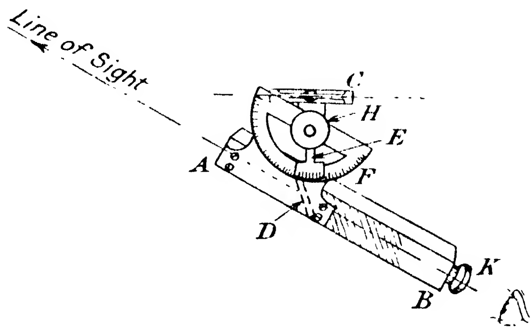


FIG. 2.20.—The Abney Level.

level, *C*, mounted on a pivot. The underside of the spirit-level casing is cut away and there is an opening in the top of the sighting tube immediately beneath. Below this opening and extending half-way across the interior of the sighting tube is a small mirror, *D*, inclined at an angle of 45° to the longitudinal axis of the instrument. This permits a reflected image of the bubble to be seen through the eye aperture, *K*. A short arm, *E*, carrying a vernier scale at its lower end, is attached to the spirit level in a direction at right-angles to the axis of the bubble tube and rotates with it when the small wheel, *H*, is turned, the vernier travelling over a graduated arc, *F*, rigidly mounted on the side of the sighting tube.

The instrument is held at a known height above the ground, preferably supported on a short ranging rod, and the line of sight is directed to a distant rod, or staff, on which the same height is clearly marked. The spirit level is then rotated until the bubble is seen to be central and the scale reading is noted. The vernier enables readings to be taken to the nearest 5 minutes.

Corrections for Slope. If a line of length l is measured along a uniform slope inclined at an angle α to the horizontal, the correct length for plotting will be $l.\cos\alpha$. If α is known corrections may either be applied to the booked chainage in the drawing office, or corrections may be applied in the field and the chainage adjusted in the manner described below which enables the booked lengths to be plotted without revision.

Let AB in fig. 2.21 represent the slope and let AC be 100 feet. Then the projected length AD would be the correct plotting distance for AC. If a length of 100 feet were plotted to scale, the point on the ground corresponding to this length would be F, where AE is 100 feet and EF is the perpendicular.

$$AF = 100.\sec \alpha = FC + 100, \text{ i.e. } FC = 100(\sec \alpha - 1).$$

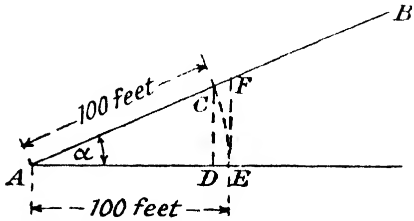


Fig. 2.21.—Comparison of Inclined and Horizontal Measurements.

The additional length FC must therefore be added to the measured length of 100 feet and the chain drawn forward to F to commence the next 100-foot length. The distance AF is then booked as 100 feet.

The accompanying table gives these additive corrections per hundred feet for various angles of slope and also the sloping distances which differ from their horizontal projections by one foot and the horizontal projected lengths of 100 feet measured on the slope.

Angle of Slope (Degrees)	Approx. Equivalent Gradient	Additive Correction to Slope Measurement per 100 feet (feet)	Length of Slope which differs by 1 foot from its Horizontal Projection (feet)	Horizontal Projected Length of 100 feet measured on Slope
1 . . .	1 in 57·2	0·02	6666·7	99·98
2 . . .	1 in 28·6	0·06	1639·3	99·94
5 . . .	1 in 11·4	0·38	262·4	99·62
10 . . .	1 in 5·7	1·54	65·8	98·48
15 . . .	1 in 3·7	3·53	29·3	96·59
20 . . .	1 in 2·75	6·42	16·6	93·97
25 . . .	1 in 2·1	10·34	10·7	90·63
30 . . .	1 in 1·7	15·47	7·4	86·60
35 . . .	1 in 1·4	22·08	5·5	81·92
40 . . .	1 in 1·2	30·54	4·3	76·60
45 . . .	1 in 1	41·42	3·4	70·71

Sloping Ground. Additional Methods. In cases where a survey line is carried across a railway cutting, embankment, or similar obstacle where the banks may slope as steeply as 45°, it is probably best to take linear measurements in conjunction with levels. This will enable an accurate section to be drawn from which horizontal dimensions may be derived. At such a location, which will usually be the site for a proposed bridge, a detailed and accurate survey is very important.

Difficulties due to undulating and uneven ground may be overcome very effectively by the tacheometric methods described in Chapter VI.

II. Ranging a Line over High Ground. It is sometimes necessary to range a line over high ground which renders the terminal stations invisible from one another so that direct sighting from end to end is impracticable with the usual ranging which are 6 or 8 feet long.

This difficulty may often be overcome by tying two rods together or by sighting to an extended levelling staff, which will give a height of 14 feet if the standard type is employed, but even if these methods are not applicable, the line can always be ranged in provided that both ends are visible from some part of the elevated area.

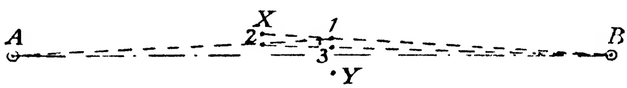


Fig. 2.22.—Lining in Intermediate Points over High Ground.

This condition of affairs is illustrated in plan in fig. 2.22. Ranging rods are placed at the ends A and B of the line and the surveyor, with an assistant, proceeds to the high ground, each with a further ranging rod. They place themselves as nearly in the correct position as possible, occupying some such points as X and Y and it is essential that A should be visible from Y and B from X. The man at X then places the man at Y in line with B, shifting him to 1. The man at 1 then places the man at X in line with A, shifting him to 2. The man at 2 then moves the man at 1 to a point 3 in line with B, and so on, until no further movement is required by either man. They will then be correctly on the line AB.

III. Chaining across a River or Lake. It happens occasionally that a survey line has to be carried across a river or lake too wide to be crossed by one span of the chain or band and even were this procedure possible, an accurate measurement could not be obtained with the full length of a chain, band or tape suspended across the obstacle unless a pull of known magnitude were applied and a calculated correction made for the sag which is always present to a greater or less extent.

Provided sufficient space is available, it is possible to obtain the required measurement by setting out on the ground a pair of triangles which are equal in every respect and in which the inaccessible dimension forms one side in the one and appears as an accessible length in the other.

There are many ways of carrying out this process and a general case is dealt with below:

Let AB in fig. 2.23, represent a portion of the survey line crossing the river or lake. Set out a length AD roughly perpendicular to the survey line. This may conveniently be 100 feet. The middle point, C, of AD is marked on the ground by a ranging rod and a point E is chosen on the survey line so that the length EA is less than AB. The reason for this stipulation will be evident later. The distance ED is measured and bisected at F and the distance AF is measured and produced to G making AG twice AF. For checking purposes it should be seen that DG is equal to EA and GE to DA.

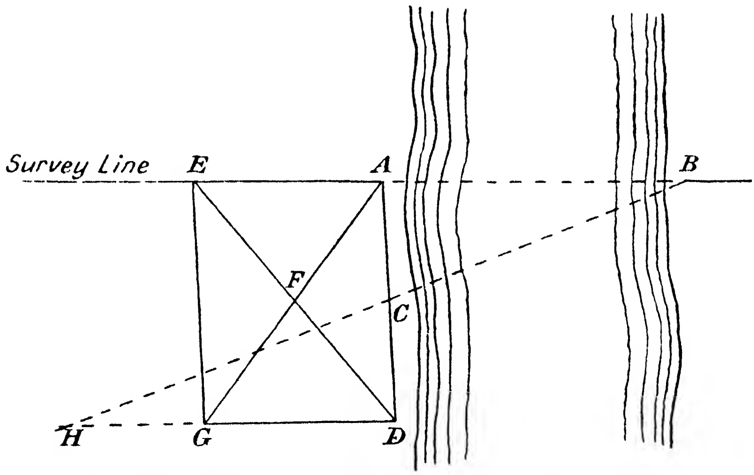


FIG. 2.23.—Method of continuing Chainage over a River.

The surveyor, carrying a ranging rod, then walks along the line DG produced, keeping himself in alignment with rods placed at D and G until he also comes into alignment with rods located at C and B. This will occur at the point H and the length HD will then be equal to the required length AB.

By making EA less than AB it is ensured that H will fall outside DG and this facilitates the dual lining-in of G with D and C with B.

GEOMETRICAL PROOF

Since $AF = FG$ and $EF = FD$, $EADG$ is a parallelogram and therefore HD is parallel to AB.

Thus, in the triangles ABC and HCD we have:

$AC = DC$, $\angle ACB = \angle HCD$, and $\angle ABC = \angle CHD$.

Therefore the triangles are equal in every respect and hence $AB = HD$.

Chaining across a small Lake or Pond. A small lake or a pond may often be dealt with by the simplified method illustrated in fig. 2.24, where AB is again the portion of the survey line crossing the water. The lines AC and CB are set out to clear the edge of the pond and the lengths of both lines are measured. AC is then produced its own length to E and BC is produced its own length to D. As a check it should be seen that AD and BE are equal in length and the distance AB will, of course, be given by DE.

Both the foregoing methods should be repeated by entirely re-arranging the lines and re-determining the inaccessible lengths.

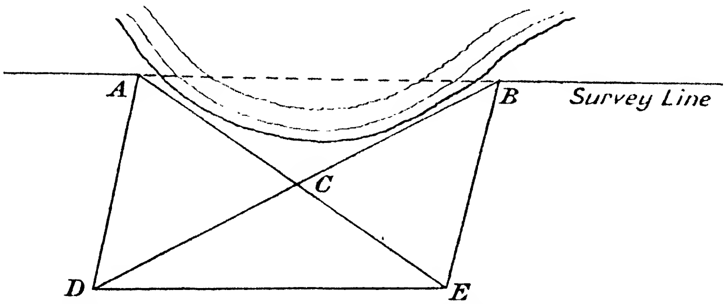


FIG. 2.24.—Method of continuing Chainage past an obstruction which permits Visual Alignment, e.g. a Pond.

Methods involving the use of a theodolite are quicker and better and it frequently happens that there is not a sufficient area of fairly level ground available to enable the above geometrical constructions to be carried out, in which case a method requiring the measurement of angles or, alternatively, a tacheometric method must be adopted.

Angular measurements are discussed in Chapter V and tacheometric surveying in Chapter VI.

IV. Producing a Line past an Obstacle. It occasionally happens that a survey line requires to be produced beyond an obstacle which prevents the sighting through of the alignment as well as the direct measurement of the distance. Such an obstacle would always be avoided in choosing the position of survey lines if it is at all possible to do so, but the difficulty can be overcome by a process of " squaring off ".

This is illustrated in fig. 2.25, in which the survey line is obstructed by a building beyond the point A and it is required to extend the line on the far side of the obstruction.

To do this, we set off lines AC and ED, equal in length and perpendicular to the chain line, making them sufficiently long to clear the obstacle. The length AE should be much longer than the distance AB, since the former length, repeated as CD, is used for giving a direction parallel to the survey line and is produced forward to give the re-alignment on the far side of the obstacle. The producing of a line is avoided whenever possible, but it is sometimes necessary, as in the present instance, *DC* having to be extended to *FG*, and the process is likely to be less inaccurate when the length of the portion produced is short in comparison with the original length.

Having set out the points *C* and *D*, the diagonals *CE* and *DA* are measured as checks and *CD* should equal *AE*. The points *F* and *G* are then located by lining-in with ranging rods and the lengths *FB* and *GH* are set off equal to *AC* and *ED* and perpendicular to *DG*. The direction *BH* then gives the continuation of the survey line and the chainage of *B* is known from the dimensions of the figure.

The diagonals *BG* and *FH* are measured as checks and the whole process should be repeated by setting out a similar arrangement of lines on the other side of the survey line.

Great care must be taken to keep strict tally of the chainage

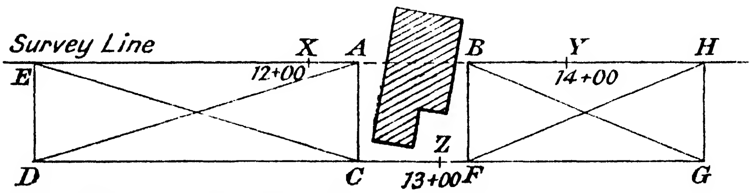


FIG. 2.25.—Method of continuing Chainage past an Obstruction which does not permit Visual Alignment.

whenever the regular routine of chaining is broken and here, again, a set of arrows with distinctive braid of a different colour from that on the standard set will prove useful.

It is also better to add lengths automatically by means of the chain or tape whenever possible, in the manner indicated in the following example :

> Reverting to fig. 2.25, suppose that *X* represents the 1,200 chainage point and that *XA* is 37 feet and *CF* 84 feet. An arrow with red braid could be inserted at *X* and one with blue braid at *A* and another at *C*. The 37-foot mark would then be placed at *C* and the 1,300 chainage would fall at *Z*, 63 feet beyond. This point would be marked with a red arrow, thus keeping correct tally of the 100-foot lengths. Since the length of *CF* is 84 feet, the chainage of *F* will be $1300 + (84 - 63)$, i.e. 1321, which will be given automatically by the chain. The 21-foot mark on the chain is then set at *B* and the chain is extended, thus locating the 1,400 chainage at *Y*, 79 feet beyond *B*, where an arrow with red braid is inserted and the chainage continued in the ordinary way.

CHAPTER III

LEVELLING

General Principles. If soundings were taken at various points in a shallow lake by the simple process of dipping a graduated rod into the water until it touched the bottom, the depths of water thus recorded would give an indication of the relative levels at the corresponding points on the floor of the lake. The horizontal water surface provides a " datum plane " from which the vertical measurements are taken.

In the process of " levelling " an axial line passing longitudinally through the telescope of the surveyor's level, and defined in the manner described on page 52, is brought to a horizontal position by adjusting the instrument and if the telescope is directed to a vertical measuring rod held at a number of points, the successive positions of this axial line will all lie in a horizontal datum plane corresponding to the surface of the lake. The telescope is provided with a hair line coincident with this horizontal plane, thus enabling relative heights to be obtained from the readings on the measuring rod.

Ordnance Datum. Bench Marks. For many purposes we merely require to know the extent by which two or more points differ in elevation and we are not at all concerned with the relationship between the elevations of these points and the elevations of other points beyond the immediate vicinity.

On the other hand, when an extensive road scheme is under consideration, the levels of the projected new works will be influenced by the levels of existing roads, railways and other topographical features which may be considerable distances apart, and in order to make comparisons between their elevations it is obviously necessary to have some common basis, or " datum ", from which all our levels can be measured.

In British practice this is known as " Ordnance Datum ", or " Mean Sea Level " which until 1929 was an arbitrary level based on a series of tidal observations taken at Liverpool. As the result of further observations by the Ordnance Survey Department, a new datum has been established at Newlyn.

Ordnance surveyors have levelled the entire country and have established innumerable points at which the height above mean sea level is accurately known. These points are known as " Ordnance Bench Marks " and are of three main types :

(*a*) fundamental,
(*b*) flush bracket,
(*c*) ordinary cut mark.

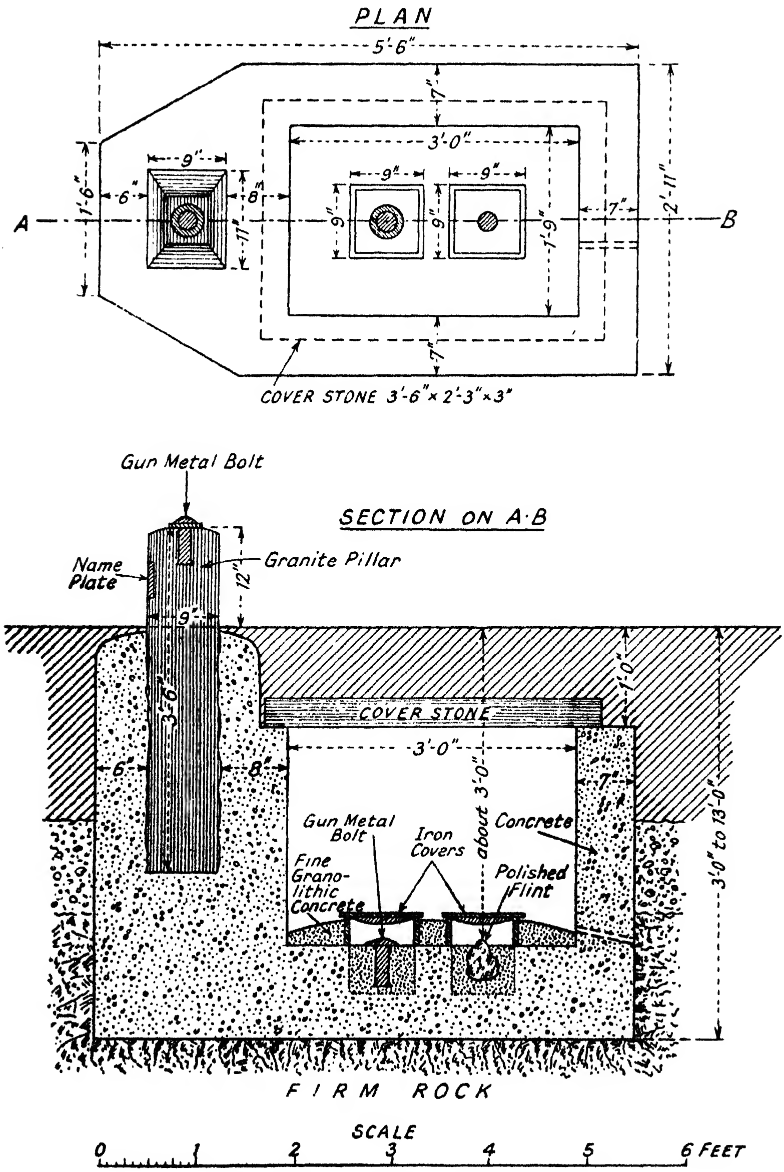


Fig. 3.1.—Fundamental Bench Mark.

Fundamental bench marks are constructed at specially selected sites, approximately 30 miles apart, in the primary or geodetic levelling network. Each has two reference points placed in a covered concrete pit and an external reference point for public use consisting of a gunmetal bolt inserted in the top of a granite pillar, 1 foot high. A bench mark of this type is illustrated in fig. 3.1.

Flush brackets are cemented into the face of walls or buildings at intervals of about 1 mile along the primary lines of levelling in the Ordnance network and at certain additional points. When using flush bracket bench marks a detachable auxiliary bracket is required for supporting the staff, as shown in fig. 3.2.

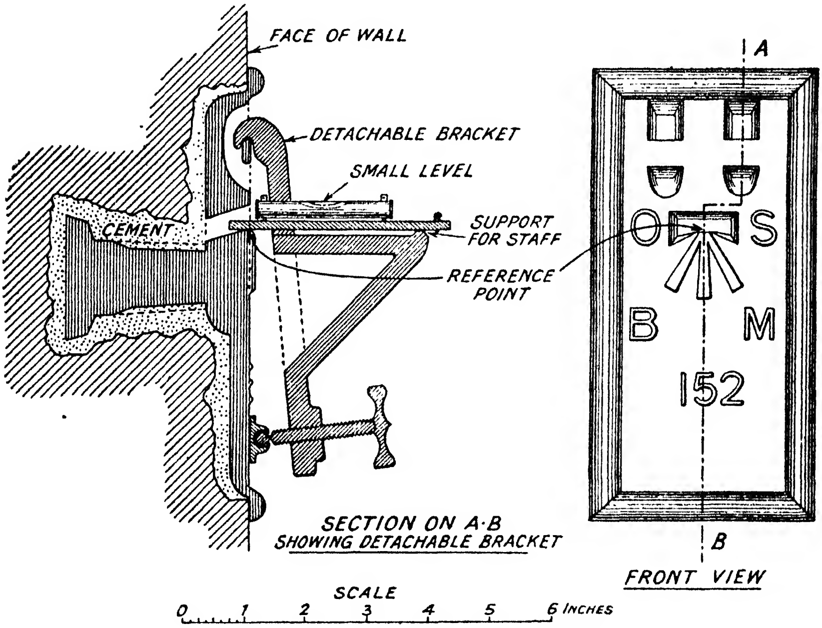


Fig. 3.2.—Flush Bracket Bench Mark.

The ordinary cut marks consist of a broad arrow with a horizontal top bar inscribed on the walls of buildings, parapets of bridges, mile-stones, boundary stones, gate-posts, and, should a more permanent location be unavailable in the vicinity, on fully grown trees. A mark of this kind is shown in fig. 3.3.

Bench marks are occasionally located on horizontal surfaces, such as the top of the coping on a dwarf wall, and they are then indicated by a distinctive dot or rivet head, accompanied by the same type of broad arrow as the vertical marks, but without the top bar. Brass bolts, about 2 inches in diameter, are sometimes inserted when no suitable site exists for fixing a flush bracket.

When using a bench mark of the ordinary cut type the base of the staff should be held at exactly the same level as the centre of the horizontal line at the top of the broad arrow. It is difficult to do this with precision, but it will be found helpful to place the end of a stiff steel rule, or similar thin rigid object, in the groove and rest the staff on this. It is scarcely an exaggeration to say that two men are required if a staff is to be held on a vertical cut bench mark with accuracy and certainty.

Greater accuracy is obtainable with the flush-bracket type. It should be noted that bench marks on such objects as gate-posts, signposts and milestones are subject to disturbance and marks in more permanent positions, such as the walls of brick or masonry buildings, should be used in preference, whenever possible.

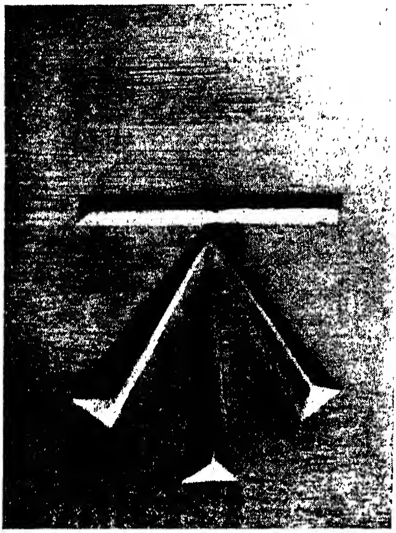


Fig. 3.3.—Ordinary Cut Bench-Mark on a Vertical Wall. Reproduced by permission of the Director-General, Ordnance Survey.

The position of every bench mark with the corresponding elevation above Ordnance Datum, or the "Reduced Level" as it is termed, is shown on Ordnance maps drawn to the scales of 25·344 inches and 6 inches to the mile.

Levels based on the old Liverpool datum are given in feet to one place of decimals, but the establishment of the Newlyn datum has necessitated the re-levelling of the country and this has altered the bench-mark levels by varying amounts, the maximum difference between old and new values being about 2 feet.

The new levels, expressed in feet to two places of decimals, are given on Ordnance maps revised after April 1929 and it is clearly stated on such maps that the levels are based on the Newlyn datum. Certain maps, however, published before April 1929, show levels based on this datum to one place of decimals only.

In most cases, it is immaterial which bench-mark values are used, provided they are used consistently, i.e. the levelling throughout the entire survey must be based either on the old or the new datum.

It happens very frequently in levelling that operations start from a bench mark and levels are carried along an existing road, or along the centre line of a projected road, checking on to any other bench marks which are encountered en route.

Levelling Instruments

The instruments in common use for producing the horizontal axial line referred to at the commencement of this chapter may be classified in three groups:

(1) Dumpy levels.
(2) " Quick-set " or tilting levels.
(3) Self-checking levels.

In the first group the telescope is rigidly attached to a tapered vertical shaft supported by a central bearing in a three- or four-armed underframe and is brought into a horizontal position by adjusting the latter by means of the levelling screws which raise or lower these radial arms.

In the second group the telescope is set on a hinge at or near the middle point of its length and may be tilted in a vertical plane, and thus brought into a horizontal position, by an adjusting screw fitted beneath the rear end.

In the third group the telescope can be tilted by an adjusting screw located at its rear end, as before, and it can also be rotated about its longitudinal axis.

Dumpy Levels. This type has been the standard levelling instrument, with but few modifications, for a considerable period and it is only of recent years that it has been superseded to some extent by the " quick-set " type.

The general construction of a dumpy level may be understood from the diagrammatic sketch given in fig. 3.4.

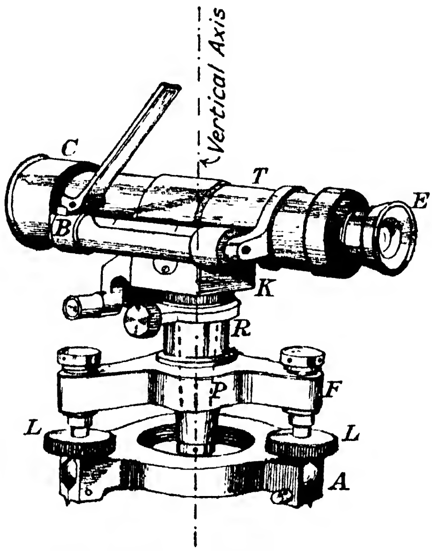
T is the telescope, equipped with a lens system, *O*, at the front end, known as the " object glass " ; a sun or rain cap, *C* ; a bubble tube, *B*, containing a spirit level ; a diaphragm, *D*, mounted inside the telescope at the rear end ; an eye-piece, *E* ; and a focussing adjustment operated by a milled-edged wheel, *M*.

The bubble tube is sometimes attached to the side of the telescope instead of to the top, and the eye-piece is both detachable and adjustable in length.

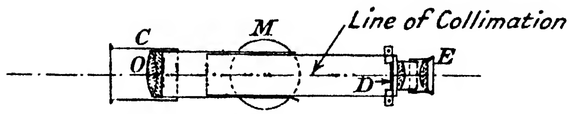
When correctly adjusted, the object glass produces an inverted image of the distant staff in the plane of the diaphragm and the eye-piece contains a lens system which enlarges this image and renders the staff reading clearly visible at distances up to 400 feet or more. The inversion of the image causes no difficulty.

The telescope is rigidly mounted on a bracket, *K*, which has a tapered spindle, *P*, projecting downwards into a tapered opening in the central boss of the underframe, or levelling base, *F*. It is secured thereto by a washer and set-screw. The weight of the telescope is carried by the collar, *R*. The spindle and the tapered opening are fitted with extreme precision so that the telescope may be rotated smoothly and freely without a trace of shake.

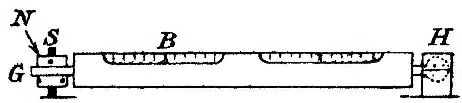
The underframe, or levelling base, F, may have either three or four arms which carry the equivalent number of levelling screws



A Three-Screw Dumpy Level



Diagrammatic Section through Telescope (External Focussing Type)



Method of attaching Bubble Tube to Telescope

FIG. 3.4.

L, threaded through their extremities, the level being called a "three-screw" or a "four-screw" instrument accordingly.

In the case of a three-screw instrument, the bases of the levelling screws are spherical and fit into cups formed in the base plate, A, which is tapped centrally to fit the screwed head of the tripod.

The construction of the base of a four-screw instrument is shown diagrammatically in fig. 3.5.

The underframe, *F*, has a central projection, *M*, formed somewhat like a mushroom head, which fits inside a dome-shaped boss, *D*, integral with the base plate, *A*. The spigot, *P*, is accurately fitted in a tapered opening passing right through the centre of the underframe, or levelling base, to the underside of the mushroom head where it is secured by a set-screw and washer, *W*.

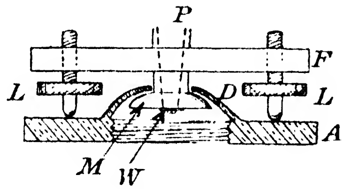


FIG. 3.5.—Section through Levelling Base of Four-screw Dumpy Level.

The levelling screws, *L*, merely rest upon the base plate, *A*, without being recessed. The effect of tightening the levelling screws is to pull the mushroom head, *M*, tight against the inner surface of the dome, *D*, so that the upper part of the instrument is rigidly held. If the levelling screws are slackened, the reverse action takes place and the upper part of the instrument is then only connected loosely to the base. This procedure is adopted before packing the instrument in its case, the loosening of the mushroom bearing obviating any possibility of strain.

Three- and Four-screw Levels Compared. The four-screw level is becoming obsolete, but a certain number are still in use and the type is still marketed by one or two makers. The four-screw base and centre bearing give a very rigid setting, whereas the three-screw type possesses the disadvantage that slackness becomes apparent when the spherical ends of the levelling screws become worn and it is not always possible to take up this wear without fitting new screws with oversize ends.

Beginners, however, find four-screw instruments more difficult to level up than the three-screw type.

Some dumpy levels are fitted with a ball-and-socket joint beneath the telescope. This joint can be loosened or tightened by a locking ring and enables the telescope to be roughly levelled before using the levelling screws at the base for the final adjustment. The advantage of this device is not particularly marked, however, except for the fact that this freedom of movement permits the instrument to be set up on steeply sloping ground with the tripod head considerably tilted.

Quick-set Levels. (Tilting Telescope Levels). These instruments resemble the dumpy in so far as the telescope and its accessories are concerned, but the method of mounting the telescope is quite different, the underside being provided with a hinged joint, *H*, in fig. 3.6, which connects it to the member, *M*, corresponding to the rigid bracket of the dumpy. In some instruments the telescope is supported by a spring-loaded plunger located at the front end of the member *M*. A fine-threaded adjusting screw, *A*, passes through the rear end of the member *M* and, when manipulated, gives a tilting

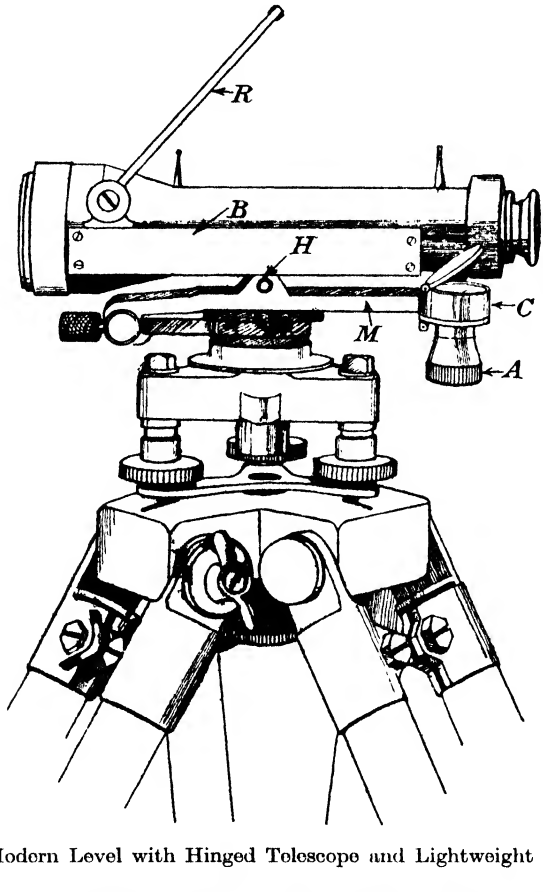


FIG. 3.6.—A Modern Level with Hinged Telescope and Lightweight Framed Tripod.

movement to the telescope axis. The member *M* also carries a small circular spirit level, C, and is attached to a three-screw levelling base by means of a tapered bearing of the usual type or, alternatively, a ball-and-socket joint may be substituted for the three-screw base.

The circular spirit level is used for rough preliminary levelling, this adjustment being carried out either by the foot screws or the ball joint, as the case may be. The accurate levelling, which must be carried out immediately before taking each reading, is performed by the fine-threaded screw, *A*, under the telescope, in conjunction with the sensitive spirit level, *B*. The latter is provided with a hinged mirror or an optical device (see page 57) for enabling the position of the bubble to be seen from the eye-piece end of the telescope.

It is stated by one well-known maker that a modern quick-set level can be taken out of its case, set up on its tripod, the circular bubble levelled by the ball-and-socket joint, the sensitive bubble adjusted and the staff reading taken within one minute ; if the instrument is carried from station to station on its tripod, the time occupied in making each observation is under 30 seconds. The normal time for setting up, levelling and reading with a standard-type dumpy is two to three minutes.

Quick-set levels are certainly more rapid in use and more accurate than a dumpy level in doubtful adjustment, but a good dumpy in correct adjustment, is a very efficient instrument and possesses the advantage that once it is set up properly there is no necessity for levelling the telescope before taking each reading. When a large number of readings are taken from one position of the level, this fact counteracts to some extent the advantages of the quick-set type.

Unfortunately, many dumpy instruments will not maintain their correct " permanent " adjustment during long periods of strenuous use and hence the increasing use of the newer forms.

Self-checking Levels. These may be considered to be an elaboration of the quick-set variety and are considerably more effective by virtue of the fact that the telescope can be rotated about its longitudinal axis so that two staff readings can be obtained simply and quickly at each staff position. Even should a discrepancy between these readings indicate that the instrument is not in perfect adjustment, the mean of the two will give the correct figure. Self-checking levels are rather more expensive than the other types mentioned, but are useful when a high degree of accuracy is required, as in setting out the levels for bridge foundations and similar precise work.

Diaphragms. The diaphragm is a vitally important part of the instrument and consists of a brass ring carrying the cross wires and mounted within the telescope near the eye-piece end, being retained in position either by two or four screws threaded through the telescope tube. These screws are often protected by a screwed cap. Some instruments possess the added refinement that the diaphragm itself is held in a separate diaphragm ring, or socket, by means of a bayonet joint, this ring being secured to the telescope tube in the manner described above.

The so-called " cross wires," or " cross hairs " are never actually wires or hairs, but may be either:

(1) spider's webs stretched across the brass ring and secured thereto by shellac varnish,

(2) fine lines engraved on a glass plate inserted in the brass ring, or

(3) fine needle points set in the inner circumference of the ring.

Fig. 3.7 indicates typical arrangements of cross wires. The web type gives the clearest line and does not collect dust or moisture films as the glass plate does, but is fragile and liable to be damaged if the eye-piece is withdrawn from the telescope in windy weather.

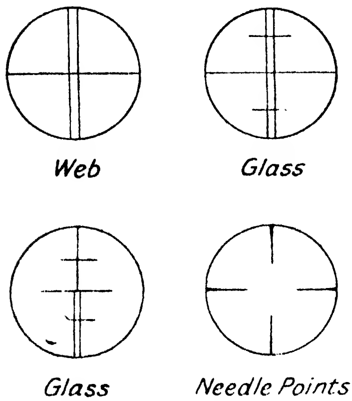


Fig. 3.7.—Types of Diaphragms.

It sometimes happens with a glass diaphragm that the lines are very difficult to see and they may be intensified by sprinkling a little powdered graphite from a pencil on the glass and wiping off the surplus with a silk handkerchief, or a camel-hair brush.

The needle-point type is durable and does not collect dust but will not give such a definite indication against the face of the staff as a fine line does. It will be noticed that the glass diaphragms have three horizontal cross-lines. The middle one is used for obtaining heights and the outer ones for obtaining distances (see Chapter VI). This arrangement is likely to cause errors through the accidental reading of one of the outer lines instead of the middle one when levelling and it is suggested that instrument makers might emphasise the middle line by adding a distinguishing mark such as a small circle or cross.

Desirable Features in a Level

(1) The tripod should be heavy, strong and rigid to avoid vibration of the instrument in windy weather. It is a mistake to attempt to save weight by using a light, flimsy tripod and the telescopic variety should be avoided. There should be some provision for taking up slack in the joints at the tripod head.

(2) Internal focussing by means of an additional diverging lens within the telescope is now almost universal. Focussing adjustment is obtained by moving this lens along the telescope axis. This method is better than the old-fashioned practice in which an inner sleeve of the telescope extended outwards, admitting dust and providing a source of wear. The focussing

screw should be accessible and in instruments of the non-rotating telescope type, should be preferably on the right-hand side of the telescope tube.

(3) The eye-piece should be adjustable for length by means of a threaded sleeve, rather than by the clumsy push-and-pull telescopic method. The purpose of this adjustment will be explained later.

(4) In instruments with a three-screw base there should be some provision for taking up slackness at the spherical bases of the levelling screws.

(5) The main bubble should be well protected and it is a convenience if it is fitted with a hinged mirror, or other device rendering the bubble visible from the eye-piece end of the telescope, thus obviating the necessity of walking round to the side of the instrument to inspect the bubble.

(6) A slow-motion, or tangent screw, should be fitted to enable the telescope to be accurately centred on the staff by moving it through a small arc in a horizontal plane.

Levels are classified in size by quoting the focal length of the telescope, e.g. 8-, 10- or 12-inch.

The Sopwith Staff. The Sopwith levelling staff is used almost universally in English practice and although alternative forms of graduation are introduced from time to time, the Sopwith pattern still retains its pre-eminent position.

The usual type reads in feet, tenths and hundredths and is derived from ordinary decimal graduations by a very simple device which renders the markings clearly visible through a good level up to about 400 feet, or rather less with the smaller and cheaper instruments. Some makers claim that their more expensive levels enable the staff to be read to 0·01 of a foot at a distance of 1,000 feet.

The Sopwith system of markings and their method of derivation are shown in fig. 3.8 which shows the first 2 feet of the staff with an enlarged drawing of the portion within the circle. Ordinary decimal sub-divisions are shown on the right and the lines are projected across to the enlarged part of the staff in the centre of the diagram. The spaces between alternate pairs of lines are filled in black, leaving the intermediate spaces white, hence the width of each black line and each white space is 0·01 of a foot.

The odd tenths of a foot are figured with black numerals one-tenth of a foot in depth and the top of the figure, in each case, indicates the number of tenths shown by the digit, the bottom of the numeral indicating one-tenth less, e.g. the top of a black 7 indicates 0·7 of a foot and the bottom 0·6 of a foot. The Roman numeral is used for " 5 ". Red numerals indicate the number of feet and here again the top of the figure gives the height, but these figures are not of any standard depth.

Small red numerals, such as the " 1 " in the diagram, are sometimes placed between the larger figures to indicate the number of feet in the reading when the staff is held close to the level and only a restricted view is obtainable which may not include a large red numeral, leaving the number of feet in doubt unless these small guide numbers are present.

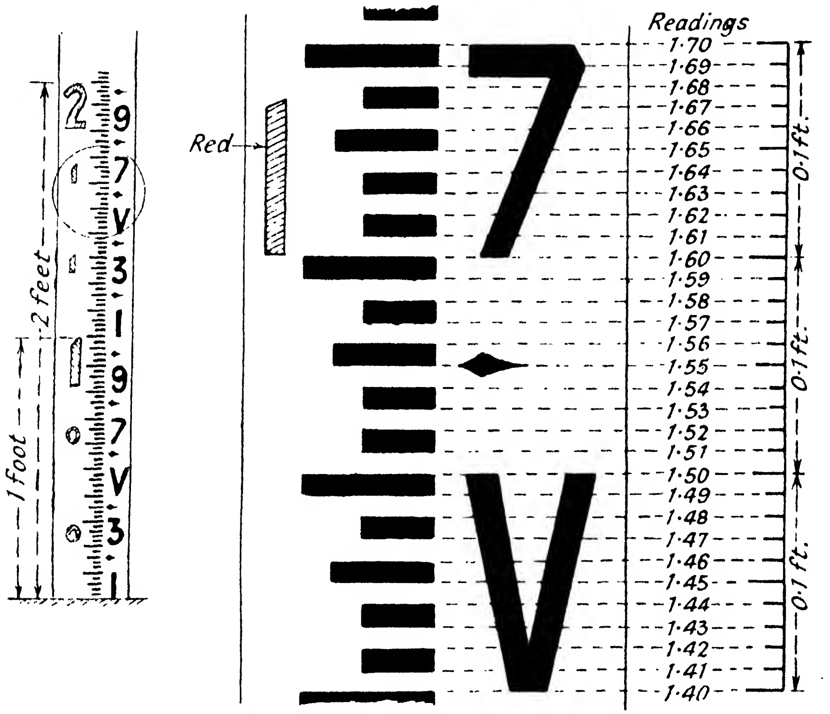


FIG. 3.8.—Details of Sopwith Levelling Staff.

Diamond-shaped marks indicate the five in the second place of decimals in the spaces unoccupied by the black numerals, i.e. at such readings as 0·15, 0·35, 0·55 and so on.

It is useful to remember that when the horizontal cross wire cuts a large red figure the number of feet in the reading is one less than the numeral intersected and when a black numeral is cut the number of tenths in the reading is, again, one less than that numeral.

It is possible to obtain staves graduated in feet and fiftieths instead of hundredths, but these are unusual.

Staves are constructed in mahogany, are either telescopic or folding and are supplied in heights varying from 10 to 18 feet when fully extended, 14 feet being the usual size. A telescopic 14-foot staff is made in three sections, the two lower ones having a hollow rectangular cross section and the top section being solid, the whole closing down to a height of slightly over 5 feet. The sliding sections are secured in position, when extended, by spring clips. These telescopic staves are not altogether satisfactory. The thin mahogany of which they are made is apt to split and the inner sections sometimes swell after exposure to rain, causing jamming. The small screws by which the brass shoe and collars are secured to the wood frequently work loose. The inner sections usually fit badly after a period of wear and tear and vibrate to and fro in a strong wind when fully extended. The graduations are usually painted direct on the wood or lithographed on varnished paper pasted to the face of the staff. In better-quality staves the graduations are cut by a dividing engine before painting.

For very accurate work special " precise " staves are used. These consist of an invar steel strip on which is an engine divided scale mounted in one 10-foot length of H-section mahogany.

Principles of Levelling.

I. Rise and Fall Method. Suppose the instrument is set up between two points, *A* and *B*, and adjusted to give a horizontal line of sight (see page 55), as shown in fig. 3.9. Let the staff be placed

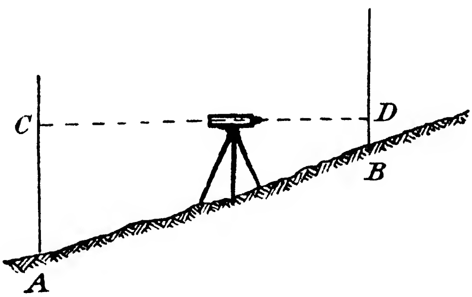


FIG. 3.9.—Principle of " Rise and Fall " Method of calculating Levels.

at *A* and let the horizontal line of sight intersect it at *C*. Then let the staff be placed at *B* and let the horizontal line of sight intersect it at *D*.

Then it will be apparent that the difference in level of the points *A* and *B* will be given by the difference in the staff readings.

It will be noticed that if *B* is higher than *A*, as in the diagram, the staff reading at *B* will be less than that at *A*, and vice versa.

The number of staff readings which can be taken from one setting-up of the level is limited, partly by the range of the telescope and also by the undulations of the ground. The level is normally set up at a height of about 5 feet, and when working with a 14-foot staff, this means that we can obtain a staff reading through the instrument up to a maximum rise of roughly 5 feet and down to a maximum fall of roughly 9 feet.

Booking of Readings. Staff readings are classified under three headings, thus :

(1) The first reading taken after setting up is called a " Back Sight ".

(2) The last reading taken before moving the level is called a " Fore Sight ".

(3) Any readings taken in between are called " Intermediate Sights ".

Thus, in fig. 3.9, if AC were the first reading and BD the last, AC would be a back sight and BD a fore sight. It must be pointed out, however, that " back " and " fore ", in this connection do not necessarily imply that the observer is looking in a backward or a forward direction. It is quite possible to have a fore sight in much the same direction as the back sight, indeed, it sometimes happens that a whole series of intermediates are taken on lines radiating in all directions from the instrument and the final fore sight is taken on the same point as the original back sight.

Change Points. When it becomes necessary to move the instrument to another position, the staff position at which the final reading was taken is termed a " Change point " and it should be located with care.

It should be well defined, so that it can be re-located with absolute certainty, should the necessity arise while the levelling is in progress, and it should be on firm ground so that the staff does not sink while the level is being moved and readjusted. When working along a road or pavement, the staff holder should make a chalk mark round the base of the staff at change points and when working over fields, commons or similar ground the change points should be chosen, if possible, on any convenient survey pegs which may exist in the vicinity, or on large stones firmly embedded in the ground. On marshy land the sinking of the staff at a change point may be prevented by interposing a small metal plate or a piece of board between its base and the soft earth.

Since the readings taken at the change points constitute vital links in the chain of levels as we proceed from one instrument station to the next, it is essential that the staff position should remain unaltered during the process of moving and re-setting the level.

In fig. 3.10, AB is a back sight ; CD, EF, and GH intermediates ; and IJ the fore sight at the change point, I. IK is the back sight taken to the same change point from the second position of the level ; LM an intermediate and NO a fore sight.

The final reading of a series of levels is always booked as a fore sight whether it is taken at a change point, or not.

Level Books. Readings are entered in tabular form and level books are obtainable with columns ruled in accordance with well-recognised practice, thus:

Back Sight	Int.	Fore Sight	Rise	Fall	Reduced Level (or Height above Base)	Distance	Remarks

These rulings occupy two pages in the book, the second page, which opens opposite the first, being usually devoted entirely to "Remarks", thus giving ample room for descriptive notes or sketches to illustrate the positions at which the levels were taken.

The method of working out the levels will be best understood from a numerical example and the following values have been assigned

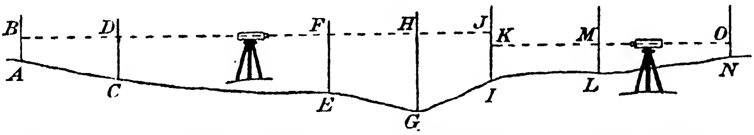


Fig. 3.10.—" Rise and Fall " Method of calculating Levels, illustrating Procedure at a Change Point.

to the points shown in fig. 3.10 and booked in the orthodox manner:

It will be noticed that the back sight, 2·31 stands alone, the first intermediate, 4·47, being written one line lower, followed line after line, by further intermediates and the fore sight, 5·07, at the change point, *I*.

The back sight, 3·94, from the second position of the level to the change point is written on the same line as the fore sight, 5·07, each line in the book representing one definite position of the staff.

Back Sight	Int.	Fore Sight	Remarks
2·31			Point *A*
	4·47		" *C*
	6·28		" *E*
	8·77		" *G*
3·94		5·07	" *I*. Change point
	3·02		" *L*
		0·14	" *N*

The second series of levels then follows, consisting of the intermediate, 3·02, and the fore sight 0·14.

The first reading booked is always a back sight standing alone and the last reading a fore sight standing alone and at change points we have a fore sight and a back sight written on the same line, the former being read and booked before moving the instrument.

The working out of these levels is carried out thus : In any series of readings taken from one position of the level, obtain the differences of the successive readings, taking the back sight and the first

(a)

Back Sight	Int.	Fore Sight	Rise	Fall	Reduced Level or Height above Base	Distance	Remarks
2·31					100·00		Peg A
	4·47			2·16	97·84		Peg C
	6·28			1·81	96·03		Peg E
	8·77			2·49	93·54		Peg G
3·94		5·07	3·70		97·24		Peg I Change point
	3·02		0·92		98·16		Peg L
		0·14	2·88		101·04		Peg N
6·25		5·21	7·50	6·46	101·04 100·00		
	diff =	1·04	diff =	1·04	diff. 1·04		

(b)

Back Sight	Int.	Fore Sight	Height of Instrument	Reduced Level or Height above Base	Distance	Remarks
2·31			102·31	100·00		Peg A
	4·47			97·84		Peg C
	6·28			96·03		Peg E
	8·77			93·54		Peg G
3·94		5·07	101·18	97·24		Peg I Change point
	3·02			98·16		Peg L
		0·14		101·04		Peg N
6·25		5·21		101·04 100·00		
	diff =	·04		diff. 1·04		

Fig. 3.11.—" Rise and Fall " and " Height of Instrument " Methods of calculating Levels.

intermediate, then proceeding line after line for the remaining intermediates, and finally considering the last intermediate and the fore sight.

This procedure is illustrated diagrammatically in fig. 3.11(*a*). If there are no intermediate readings, as sometimes happens, we take the differences between the back and fore sights read from *the same position of the instrument*, remembering that the back and fore sights written on the same line refer to the same *staff* position, *not* to the same *instrument* position.

The readings from successive instrument positions form independent groups which are connected at the change points.

Let us proceed to carry out the subtractions, remembering that if the second reading is less than the first, there is a rise from the first point to the second and if greater, a fall.

Thus, from A to C there is a fall the extent of which is

$$4{\cdot}47 - 2{\cdot}31, \text{ or } 2{\cdot}16 \text{ feet.}$$

This is entered in the "Fall" column on the line referring to point *C*. Similarly, there is a fall from *C* to *E* of 6·28 − 4·47, or 1·81 feet, and a fall from *E* to *G* of 8·77 − 6·28, or 2·49 feet.

The next pair of readings to be considered is the intermediate 8·77 and the fore sight 5·07, which indicate a rise from *G* to *I* of 8·77 − 5·07, or 3·70 feet. This completes the first group of readings and we now consider the back sight 3·94 and the intermediate 3·02 which form the first two readings in the second group. These indicate a rise from *I* to *L* of 3·94 − 3·02, or 0·92 feet.

Finally, there is a rise from *L* to *N* of 3·02 − 0·14, or 2·88 feet.

Reduced Level or Height above Base. The strict definition of the "Reduced Level" of any point is "the height of that point above Ordnance Datum, or mean sea level", but it sometimes happens that there is no Ordnance bench mark conveniently near the work from which we can base our levels and provided that we merely wish to know the relative elevations of a number of points, quite irrespective of their heights above sea level, we may take an assumed datum, or, in other words, we may assume a reduced level for the first point at which the original back sight is taken. If, for instance, we assume that the reduced level of *A* is 100·00, our datum is a horizontal line 100 feet below the point *A* and the levels of the other points relative to this datum are obtained by adding the rises and subtracting the falls, line by line, one after another.

Thus, level of	*C*	100·00 − 2·16 =	97·84
,, ,,	*E*	97·84 − 1·81 =	96·03
,, ,,	*G*	96·03 − 2·49 =	93·54
,, ,,	*I*	93·54 + 3·70 =	97·24
,, ,,	*L*	97·24 + 0·92 =	98·16
,, ,,	*N*	98·16 + 2·88 =	101·04

If reduced levels are required with reference to Ordnance datum it is, of course, necessary for the original back sight to be taken on an Ordnance bench mark, the level of which is obtained from a 25-inch or a 6-inch Ordnance map, or else on a point the level of which has been accurately determined from an Ordnance bench mark.

Arithmetical Check on Level Calculations. The complete entries in the level book for the readings in the above example are shown in fig. 3.11 and a simple arithmetical check is always applied to test the accuracy of the working.

The net difference in level between the first and last points is given by the difference in the sums of the back and fore sights or, equally, by the difference in the sums of the individual rises and falls. These three differences should therefore be identical.

In the foregoing example, sum of back sights = 6·25
sum of fore sights = 5·21
Difference = 1·04

Since the sum of the fore sights is less than the sum of the back sights, the net result is a rise from the first point to the last.

Again, sum of rises = 7·50
sum of falls = 6·46
Difference = 1·04 (rise)

Finally, the difference between the first and last reduced levels is also a rise of 1·04 from the first to the last.

When taking a large number of levels it is convenient to check the arithmetic in the level book page by page, since any error in the calculations, either in the rise and fall, or the reduced level columns will be carried forward. If such an error occurs on the first page and the readings cover several pages, the discovery of an arithmetical mistake when the checking is applied at the end to the booking as a whole, involves a lengthy revision of the entire work from the point at which the mistake occurred.

When checking each page individually, if the last reading on the page is an intermediate, it is re-written, in brackets, as a fore sight and its reduced level is calculated. The arithmetical check is then applied and, if correct, this artificial foresight, with its checked reduced level, is re-written as a back sight, again in brackets, at the top of the next page.

II. Height of Instrument, or Collimation Method of Calculating Levels. In this alternative method of calculating levels, the procedure in the field and the classification of the readings as back, intermediate, and fore sights are exactly the same as that already described for the Rise and Fall method. But in the Height of Instrument method the line of sight when taking a reading, more correctly known as the " line of collimation ", is used as a reference line from which all the vertical measurements are calculated direct.

The line of collimation may be defined as an imaginary line passing through the middle point of the horizontal cross wire and the optical centre of the object glass. If there are three horizontal cross wires, as is sometimes the case, the central one is taken for the purpose of this definition. The optical centre of the object glass may be defined as a point in the lens system through which rays of light pass without deviation.

Referring to fig. 3.12, which illustrates the same set of levels as those in fig. 3.11, the line of collimation for the first position of the level is BJ and for the second position KO. Taking any convenient datum line, the heights of the lines of collimation are, respectively,

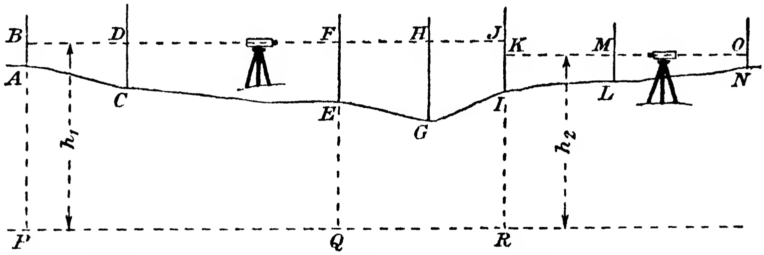


FIG. 3.12.—" Height of Instrument " Method of calculating Levels, illustrating Procedure at a Change Point.

h_1 and h_2 and this dimension is the " Height of Instrument ", abbreviated commonly to " H.I.". Let the vertical line through the point A be produced downwards to cut the datum line at P. Then the height of collimation $h_1 = BP = AB + AP$

$= \text{Back sight at } A + \text{reduced level of } A.$

Again, let EF be the staff reading at any intermediate point, E. Produce the vertical through E downwards to cut the datum line at Q. Then the reduced level of the point $E = EQ = FQ - EF = BP - EF$ = height of instrument − intermediate staff reading at E. Let I be the change point, as before, and IJ the fore sight from the first position of the level. Produce the vertical through I downwards to cut the datum line at R. Then reduced level of

$$I = RI = RJ - IJ = BP - IJ$$
$$= \text{height of instrument} - \text{fore sight at } I.$$

Let the level be moved to a second position and a back sight, IK, taken to the change point I. The level will now have a different collimation height, h_2, and this may be determined as follows : Height of instrument, or collimation, for second position of level $= RK = RI + IK$ = reduced level of change point + back sight to change point from new position of level.

Hence we have the following rules :

(1) Height of collimation = back sight + reduced level of point at which back sight is taken.

(2) Reduced level of any intermediate point = height of collimation − intermediate reading to that point.

(3) Reduced level of change point = height of collimation − fore sight to change point.

(4) New height of collimation after moving level = reduced level of change point + back sight to change point from new position of level.

Level books ruled in this form, as shown in the lower part of fig. 3.11, are obtainable as an alternative to "Rise and Fall" books and the method of calculation may be understood from the accompanying numerical example, using the same figures as before.

The height of collimation for the first position of the level is found by adding the first back sight to the reduced level of the point, in this case assumed to be 100·00,

i.e. H.I. $= 2{\cdot}31 + 100{\cdot}00 = 102{\cdot}31$ feet.

We now subtract the intermediates and the fore sight belonging to the first group of levels successively from this H.I. and thus obtain the corresponding reduced levels, i.e.:

Reduced Level of $C = 102{\cdot}31 - 4{\cdot}47 = 97{\cdot}84$ feet.
" " " $E = 102{\cdot}31 - 6{\cdot}28 = 96{\cdot}03$ feet.
" " " $G = 102{\cdot}31 - 8{\cdot}77 = 93{\cdot}54$ feet.
" " " $I = 102{\cdot}31 - 5{\cdot}07 = 97{\cdot}24$ feet.

The height of instrument for the second position of the level
= back sight to change point + reduced level of change point
$= 3{\cdot}94 + 97{\cdot}24 = 101{\cdot}18$ feet,
and proceeding as before:

Reduced level of $L = 101{\cdot}18 - 3{\cdot}02 = 98{\cdot}16$ feet.
and " " " $N = 101{\cdot}18 - 0{\cdot}14 = 101{\cdot}04$ feet.

Arithmetical Check. As the intermediates are subtracted independently from the height of collimation, any errors in these are not carried forward as they are in the Rise and Fall method and there is no quick direct check on these readings. Continuity occurs, however, at the change points and by summing the back and fore sights, obtaining the difference of the sums and comparing this with the difference in the first and last reduced levels we obtain a check on the change point levels and also on the calculation of the heights of collimation.

Thus, sum of back sights = 6·25 and sum of fore sights = 5·21, as before, giving a net rise of 1·04 feet which agrees with the difference in the reduced levels of the first and last points. This check may be carried out for each page when the levels cover several pages, by using the device already described in the Rise and Fall method.

It is possible to check the intermediates also in the following way:

Find the sum, in turn, of the group of intermediates corresponding to each particular height of collimation, e.g. for the first H.I. this sum will be: $4{\cdot}47 + 6{\cdot}28 + 8{\cdot}77$, or 19·52.

Find the sum of the reduced levels of each of these points, i.e. $97{\cdot}84 + 96{\cdot}03 + 93{\cdot}54$, or 287·41. Add the sum of the intermediates to the sum of their reduced levels, i.e. $19{\cdot}52 + 287{\cdot}41$, giving 306·93. This amount should be equal to the product of the height of collimation for the group multiplied by the number of intermediate readings in the group, i.e.

$$102{\cdot}31 \times 3, \text{ or } 306{\cdot}93$$

Rise and Fall and Height of Collimation Methods Compared. The simply applied arithmetical check covering *all* the readings is an advantage in the Rise and Fall method, but the Height of Collimation is more direct and therefore quicker. Is also more readily adaptable to contouring.

Temporary Adjustments of Levelling Instruments.

I. The Spirit Level. The spirit level consists of a glass tube, graduated on its upper surface (and, in certain cases, on its lower surface, in addition), containing alcohol, or petroleum ether, but leaving sufficient space to form an air bubble, or " bell ". Either a curved tube is used, or, in the better-quality instruments, the bore is machine ground to the shape of an elongated barrel, the cross-section at any point being circular. The upper and lower boundaries of the bore, in a longitudinal section, are thus circular arcs of large radius, from 30 to 1,000 feet, or more, and the air bubble naturally travels to the highest point of the arc. The glass tube is enclosed in a brass casing cut away to reveal the bubble and graduations.

Ordinarily the length of the air bubble diminishes if the volume of the spirit increases owing to a rise of temperature and vice versa, but spirit levels are obtainable in which the cross-section of the bore is elliptical and the bubble maintains a constant length throughout a wide range of temperatures.

Circular Levels are used for approximate levelling only and the upper surface is spherical. A small ring is inscribed at the centre of the circular glass to indicate when the bubble is correctly set.

(1) *Spirit Level Temporary Adjustment with a 3-screw Dumpy.* Place the telescope parallel to any pair of levelling screws and bring the spirit level bubble to the centre of its run by adjusting these two screws in the following way: Grip the screws between the fore-fingers and the thumbs of each hand and move thumbs either inwards or outwards, according to the indications of the bubble. This has the effect of turning the screws in opposite directions, thus altering the inclination of the levelling base and consequently of the telescope and bubble tube. If the levelling screws were both rotated in the same direction and by the same amount, the levelling base would be raised or lowered parallel to itself, without altering the inclination.

Next turn the telescope through 90° to bring one end vertically over the third levelling screw and bring the bubble to the centre of its run by manipulating this screw only.

Return the telescope to its original position, without reversing it end for end and re-adjust the bubble, if necessary. Return to the second position, again without reversing the telescope end for end and re-adjust, if necessary. When the bubble remains at the centre of its run for these two positions at right-angles, reverse it end for end and if the instrument is in correct "permanent" adjustment, so far as the spirit level is concerned, the bubble will remain central

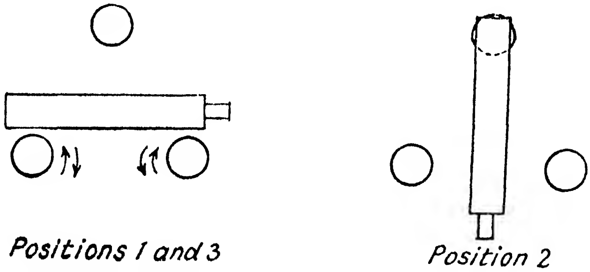


FIG. 3.13.—Method of Levelling a 3-screw Instrument.

after reversal and for all other positions of the telescope. Fig. 3.13 shows the procedure.

If it will not maintain its central position, or very nearly so, the "permanent" adjustment must be carried out in the manner described later.

As a temporary makeshift the bubble may be brought to the centre of its run by means of the levelling screws before taking each reading, but this is a tedious procedure and will not give accurate results unless the line of collimation is parallel to the bubble tube.

Another makeshift method consists in correcting *half* the maximum displacement of the bubble from its central position by means of the levelling screws. The bubble will usually maintain this position while the telescope is rotated, indicating that the axis of rotation is vertical, although the bubble tube is not horizontal. The accuracy of the work will then depend upon whether the line of collimation is perpendicular to the vertical axis. An explanation of this procedure is given on page 59.

(2) *Spirit Level Temporary Adjustment with a 4-screw Dumpy.* The method is exactly similar to that described for the three-screw instrument except that the telescope is placed vertically above one pair of levelling screws and then turned through 90° to bring it over the other pair, as shown in fig. 3.14.

It will be found with most four-screw instruments that it is impossible to adjust the spirit level unless the levelling screws are first tightened equally, thus making the plate through which they are threaded parallel to the base plate screwed to the tripod. The bubble is then roughly levelled by manipulating the tripod legs, leaving only a slight degree of adjustment to be carried out by means of the levelling screws.

It should be mentioned that it is a good practice with any level to set it up with the tripod legs so disposed that the bubble is roughly central in two positions at right-angles. This can be done quite easily after a little experience and saves time in the ultimate levelling, besides preventing undue strain and jamming of the levelling screws by forcing them into positions near the extremities of their threads.

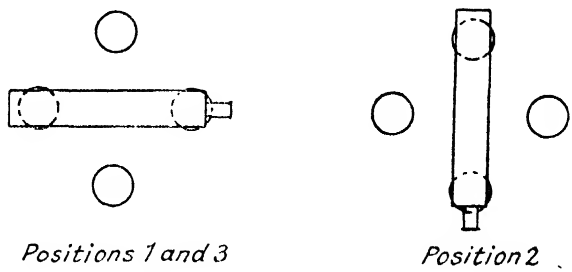


Fig. 3.14.—Method of Levelling a 4-screw Instrument.

(3) *Spirit Level Temporary Adjustment with Quick-set and Self-Checking Levels.* With the telescope pointing in any direction the instrument is approximately levelled, by manipulating either the ball and socket mounting or the levelling screws, and using the circular spirit level as a guide. The line of sight is then directed to the staff and the telescope clamped in this position, the accurate levelling being carried out by means of the fine adjusting screw under the telescope. The initial approximate levelling ensures that the final exact adjustment will be within the range of the accurate adjusting screw.

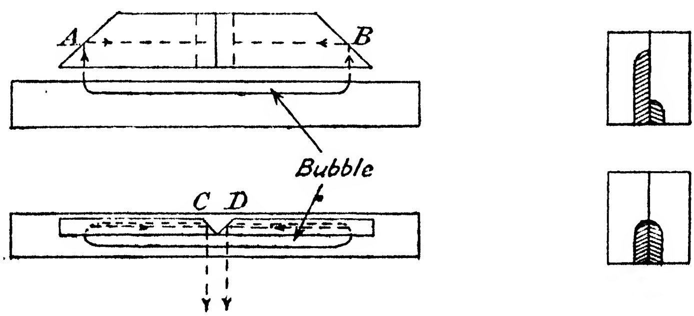


Fig. 3.15.—Optical Reflection of Bubble. Incorrect Adjustment in Upper View. Correct Adjustment in Lower View.

Many instruments of the "quick-set" type are fitted with a prismatic device which enables the bubble position to be seen from the eye-piece end of the telescope. The working principle of this device is shown in fig. 3.15, from which it will be seen that the images of the two ends of the bubble are obtained in juxta-position by reflection at *A*, *B*, *C* and *D*. By the addition of another totally reflecting prism the rays are deviated in a direction parallel to the telescope and thus rendered visible from the eye-piece end. The relative positions of the images of the bubble ends when the instrument is correctly and incorrectly levelled are shown at the side of the diagram.

II. Focussing and "Parallax". Though the focussing of the distant staff is an obvious and simple process, it is often forgotten that the eye-piece is adjustable and needs re-setting to suit differing powers of eye-sight. Hence this is usually a necessary adjustment if different observers are using the instrument or if the eye-sight of any particular observer varies, as it may do through fatigue.

The purpose of the eye-piece is to enlarge the small inverted image of the distant staff which the object glass produces within the telescope at or near the plane of the diaphragm, much in the same way that a photographic lens produces an inverted image which may be seen on a ground glass screen placed at the back of the camera. The usual magnification of the eye-piece is 25 to 30.

The first essential in taking an observation is to obtain the sharpest possible view of the cross wires, and this will be secured when the eye-piece is correctly adjusted. The procedure is of the "trial and error" description, the telescope being directed towards a light background, such as a well illuminated cloud or a white wall, and the length of the eye-piece varied by carefully manipulating its screwed or telescopic sleeve until the clearest image of the cross wires appears. Failing a suitable background, a sheet of white paper should be held in front of the object glass and tilted to throw reflected light into the telescope.

Having set the eye-piece, the observer can now proceed with the staff readings, varying only the telescope focus by means of the appropriate focussing screw which will usually accommodate any length of sight from 8 or 10 feet up to 300 or 400 feet, or more.

The correct focus is obtained when the image of the staff falls exactly in the plane of the cross wires and when this occurs no relative movement will be apparent between the image and the cross wires if the observer's eye is moved up and down and from side to side.

It is very important that the eye-piece setting and the subsequent focussing should be carefully carried out, since it is impossible to obtain a definite reading on the staff if the adjustment is imperfect and the cross-wire appears to travel over several graduations when the eye is raised and lowered—a state of affairs known as "parallax".

Permanent Adjustments of Levelling Instruments.

The so-called "permanent" adjustments must be re-examined from time to time, the period during which an instrument will remain in good "permanent" adjustment depending upon its design and quality and upon the manner in which it is treated.

It is easy to determine by simple tests whether any adjustments are needed, but it requires a certain amount of delicate mechanical skill to carry them out and rough handling with unsuitable appliances will do more harm than good.

Permanent Adjustments of the Dumpy Level

To give correct results throughout the full range of the telescope focus with sights of unequal lengths, two conditions must be satisfied :

(1) The axis about which the telescope and spirit level rotate must be truly vertical, and

(2) The line of collimation and the spirit level must be at right-angles to the vertical axis.

The effects of atmospheric refraction and the earth's curvature, dealt with on page 82, may be ignored for the present.

Vertical Axis and Spirit Level Setting. The first condition and the latter part of the second will be satisfied when the bubble remains at the centre of its run for all positions of the telescope.

The indication of the bubble are influenced by two factors : (1) the setting of the bubble tube relative to the telescope and, therefore, to the vertical axis, and (2) the adjustment of the levelling screws. The bubble tube is attached to the telescope in the manner shown in fig. 3.4. At one end, H, the tube is pivoted or hinged, and a projecting lug, G, at the other end, is secured to a stud, S, integral with the telescope barrel, by means of a pair of capstan-headed nuts, N. The inclination of the bubble tube can thus be varied by altering the position of the capstan-headed nuts. The adjustment is carried out as follows :

The instrument is levelled as carefully as possible and the telescope is reversed end for end, its position now being parallel to two foot screws. If the bubble will not maintain its central position, the extent of the deviation is noted from the graduations on the tube and the bubble is brought back towards the centre through *half* this number of graduations by slackening the capstan nuts on the supporting stud and re-adjusting them to suit. This must be done by means of a properly fitting tommy bar which engages accurately with the holes in the nuts and on no account should a makeshift instrument such as the point of a knife blade be employed. The adjustment should be carried a little too far with one nut so that the final tightening up of the second nut brings the bubble back to its pre-determined position.

The remaining half of the bubble deviation is then corrected by means of the levelling screws. It should be noted that this method of adjustment is based on the assumption that the displacement of the bubble is directly proportional to the angle of tilt and with an accurate spirit level this will be the case.

It is usually necessary to repeat the adjustment with the telescope at 90° to the previous position before a perfect setting of the bubble tube can be obtained.

The principle of the method may be understood from fig. 3.16. If the bubble tube, AB, is perpendicular to the axis of rotation, XX,

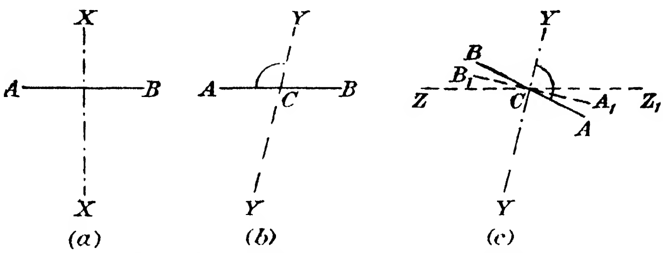


Fig. 3.16.—Principle of Spirit-level Test.

and if the latter is vertical, the bubble will always remain central (fig. 3.16(a)).

If, however, the axis of rotation is inclined as at YY in fig. 3.16(b), it will still be possible to bring the spirit level to a horizontal position by means of the levelling screws and the bubble will then be central for one given position of the telescope, but the tube will be inclined to the axis at an angle ACY.

If the telescope is now turned through 180°, the axis remains in the position YY, provided the levelling screws are not moved, and the bubble tube will still be inclined to it at an angle ACY, but on the opposite side, as shown in fig. 3.16(c). The bubble will then move towards the high end of the tube by n divisions, let us say.

Let ZCZ_1 be a horizontal line. Then the bubble deviation of n divisions corresponds to an angle of tilt ACZ_1 and if we reduce the deviation to $n/2$ we reduce this angle by half, the position of the bubble tube being now represented by the line B_1CA_1, which bisects the angles ACZ_1 and ZCB. Then

$$\angle ZCB_1 = \angle A_1CA \text{ and } \angle ZCY = \text{acute } \angle ACY.$$

Therefore $\angle B_1CY = \angle A_1CY$ and each is equal to 90°. In other words, the bubble tube is now at right-angles to the axis of rotation, although the latter is still not vertical and if the telescope be rotated the bubble will maintain a constant position, but it will not be central.

Finally, by correcting the remaining half of the original bubble deviation by means of the levelling screws we make the bubble tube horizontal while still maintaining its perpendicularity to the axis of rotation and the latter will therefore be vertical.

Collimation Test and Adjustment. Having made the axis of rotation vertical it is necessary to determine whether the line of collimation is perpendicular to the axis, i.e. truly horizontal.

The direction of the line of collimation is determined by the optical centre of the object glass and the centre of the middle horizontal cross wire. The former, although detachable, should never be disturbed and in the unlikely event of its requiring removal it must not be unscrewed from the instrument without first scribing a line on its rim and on the adjacent part of the telescope tube, so that it may be replaced exactly in its previous position. In many instruments it will be found that such a reference line has been provided by the makers.

The diaphragm carrying the cross wires may be raised or lowered quite easily, however, by manipulating the screws by which it is retained in the telescope tube, thus altering the inclination of the line of collimation, although this is an adjustment which should rarely be necessary.

Before making the collimation test it is as well to check the setting of the diaphragm by carefully levelling the instrument and taking a reading with the extreme end of the horizontal cross wire superimposed on a distant levelling staff. The telescope is then slowly turned to bring the opposite end of the cross wire on to the staff, during which process the reading should, of course, remain constant. If it does not do so the cross wire is not truly horizontal and the diaphragm bolts are slackened and the diaphragm rotated slightly until a constant reading is maintained. The bolts are then re-tightened.

The collimation test is carried out in the following way: The instrument is set up midway between two pegs, *A* and *B*, as shown in fig. 3.17, placed a convenient distance apart on fairly level ground, 200 feet being a suitable distance. It is levelled up carefully and staff readings are taken at the two pegs.

Let *FG* be a horizontal line through the centre of the instrument. This will represent the line of collimation, should the adjustment be correct. Then the correct staff readings would be *AF* and *GB* and their difference would give the difference in level between the two pegs. Suppose, however, that the line of collimation is not horizontal, but is inclined upward. Its respective positions when taking the staff readings at *A* and *B* may be represented by *HD* and *KE* and the readings will be AD and BE, the extent of the errors being FD and GE. But $\angle FHD = \angle GKE$ and $FH = KG$, hence the errors FD and GE are equal and therefore the difference in the readings AD and BE will give the correct difference in level of the pegs.

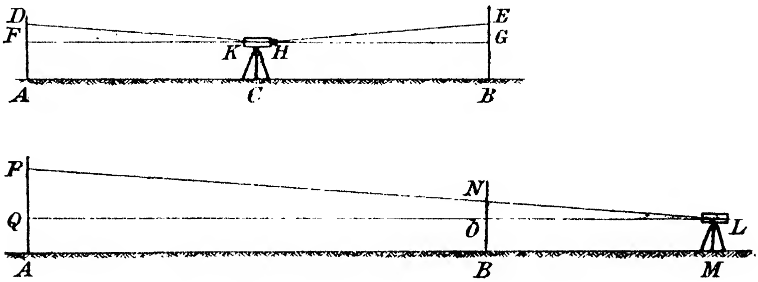


FIG. 3.17.—Principle of Collimation Test.

In general terms, the magnitude of the error in any staff reading produced by incorrect adjustment of the line of collimation is proportional to the distance between the staff and the level and if sights are made equal in length, errors are equalised and therefore cancel out on subtraction.

This fact may be usefully remembered and will be referred to again under the heading " Levelling Procedure ".

The level is now moved to the point M so that the lengths of the sights to the pegs will be markedly unequal—a suitable position will be 100 feet from B and 300 feet from A, as shown in the diagram. Let QOL be a horizontal line passing through the centre of the instrument and let PNL be the tilted line of collimation.

Then the correct readings to A and B, respectively, should have been QA and OB, in which case the difference between them would be equal to that previously obtained when the level was set up midway between A and B.

But the errors are now PQ and NO and if the distance AM is three times the distance BM, as in the diagram, PQ will be three times NO and the readings AP and BN, on subtraction, will not give the same result as that obtained previously for the difference in level of the pegs.

Hence the error in the line of collimation will be detected and it may be rectified in two ways :

1st Method. By raising or lowering the diaphragm until the correct difference of readings is obtained on the pegs.

As already mentioned, the diaphragm is secured to the telescope either by two or four bolts. In the latter case, the upper and lower bolts move the diaphragm vertically and the side bolts engage in slots in the diaphragm ring and prevent the diaphragm swivelling sideways. These latter must be slackened before the upper and lower bolts are adjusted. Usually the diaphragm bolts are capstan headed and a correctly fitted tommy bar must be used for their manipulation. The adjustment should be slightly overdone, before tightening up the last bolt, so that the latter procedure just takes up the excess and leaves the final setting correct.

Instead of a lengthy trial-and-error process, reading first on to one peg and then on to the other, the correct staff readings can be calculated in advance, provided that the lengths of the sights are measured and the working is simplified if these lengths are proportioned in an exact ratio, as in the example illustrated in fig. 3.17.

The staff can then be kept at one peg until the correct reading is obtained and then moved to the other peg for a check.

Example: Level at C. $AC = BC = 100$ feet.

Reading on peg A: 5·23

" " " B: 4·40

True rise from A to $B = 5{\cdot}23 - 4{\cdot}40 = 0{\cdot}83$ feet.

Level at M. $AM = 300$ feet. $BM = 100$ feet.

Reading on peg A: 5·51

" " " B: 4·62

Apparent rise from A to $B = 5{\cdot}51 - 4{\cdot}62 = 0{\cdot}89$ feet.

Let x = error per 100-foot length of sight.

Then correct reading at A, from $M = 5{\cdot}51 - 3x$

" " " B $= 4{\cdot}62 - x$

Hence $(5{\cdot}51 - 3x) - (4{\cdot}62 - x) = 0{\cdot}83$

i.e. $0{\cdot}89 - 2x = 0{\cdot}83$

whence $x = 0{\cdot}03$

Therefore, correct reading at A, from $M = 5{\cdot}51 - 0.09 = 5{\cdot}42$

" " " B $= 4{\cdot}62 - 0{\cdot}03 = 4{\cdot}59$

giving the true rise from A to B, as before 0·83

In the above example we have assumed that the line of collimation sloped upward. If this assumption had been wrongly made, the fact would have become apparent during the calculation, since the value of "x" would have been negative. The correct value of the staff readings could still have been obtained, however, by applying this value of x algebraically, as the following example will show:

Level at M. $AM = 300$ feet, $BM = 100$ feet.

True rise from A to $B = 0{\cdot}83$ feet.

Reading on peg A: 5·19

" " " B: 4·42

Again, let x = error per 100-foot length of sight.

Then, correct reading at $A = 5{\cdot}19 - 3x$
,, ,, ,, $B = 4{\cdot}42 - x$

Therefore $(5{\cdot}19 - 3x) - (4{\cdot}42 - x) = 0{\cdot}83$
or $0{\cdot}77 - 2x = 0{\cdot}83$
whence $x = -0{\cdot}03$

Hence, correct reading at $A = 5{\cdot}19 + 0{\cdot}09 = 5{\cdot}28$
,, ,, ,, $B = 4{\cdot}42 + 0{\cdot}03 = 4{\cdot}45$

giving the true rise from A to B, as before, 0·83.

It simplifies matters if pegs A and B are driven to exactly the same level, determined by obtaining identical staff readings on each when the level is set up midway between them.

2nd Method. If difficulty is experienced in keeping the bubble central the collimation test may be carried out as already described and the bubble brought to the centre of its run immediately prior to each observation.

If the collimation is imperfect the cross wire is then brought to the correct reading by altering the inclination of the telescope by means of the levelling-screws. This will, of course, throw the bubble out of centre, and after the correct difference of readings has been obtained on the pegs, the bubble tube is reset by means of the capstan nuts at one end to give a central position of the bubble.

We know then that the line of collimation and the spirit level are parallel.

Permanent Adjustment of Quick-set and Self-Checking Levels. It will be remembered that in quick-set and self-adjusting levels the telescope is hinged and has an adjusting screw at one end. In the case of these instruments it is only necessary to ensure that the line of collimation is parallel to the spirit level.

To ascertain whether this condition is satisfied with a quick-set level the same method of testing the line of collimation as that used for the dumpy may be adopted. To correct the adjustment, if found imperfect, the telescope is tilted to give the correct readings by means of the fine adjusting screw and the bubble is then brought to the centre of its run by the capstan nuts at one end.

With a self-checking level a very simple method of testing the collimation is available. The telescope in a level of this type may be rotated through 180° about its longitudinal axis, carrying the spirit level with it and the latter is so made as to function in either position of the telescope.

To carry out the test, a reading is taken to a distant staff with the bubble central. The telescope is then inverted about its longitudinal axis, re-levelled, if necessary, and the staff again read.

The two readings should be identical if the line of collimation is parallel to the spirit level, but if this is not the case one reading will be too high and the other too low by exactly equal amounts and the mean of the two will give the correct reading.

To correct any error which the test may reveal, the centre horizontal cross wire is set to the mean reading by means of the telescope adjusting screw. This will throw the bubble out of centre and the latter is then re-set by its capstan nuts.

Levelling Procedure

1. Flying Levels. Flying levels consist of back and fore sights only, without intermediates, and form the normal procedure in determining the difference in level between two points which are too far apart to permit of a direct comparison of their elevations by a back and fore sight from the same position of the level.

It frequently happens, for instance, that a number of points are established along the projected line of a new road and marked by stakes embedded in concrete, or some similar method, to give a certain degree of permanency. These points may be used as " temporary bench marks " (T.B.M's) for future levelling along the line of work and their levels require accurate determination and checking from Ordnance bench marks.

To do this, we start initially, with a back sight to the nearest Ordnance bench mark and proceed from point to point, using the longest range of the telescope which will give clear staff readings and keeping each pair of back and fore sights from the same position of the level equal in length, for the reason explained under the heading " Collimation Test and Adjustment " (page 62.)

It is sufficient to pace out these lengths. They need not be chained.

Every position of the staff, except the initial and final, will be a change point and must be chosen with due regard to the precautions mentioned under the heading " Change Points " (page 48.)

As the levelling proceeds, checks are taken to any other Ordnance bench marks in the vicinity, although it is more satisfactory to check the levels by working back to the original starting-point, as there is always the possibility that in the course of years the level of a bench mark may alter slightly with reference to its neighbours, through settlement.

This is particularly likely in districts where mining subsidences are common, or where the subsoil is unstable, as in parts of the Fen District.

Furthermore, the levels shown on the older Ordnance maps, based on the Liverpool datum, are only given to one place of decimals and therefore do not provide a good check on levels read to two places of decimals unless readings are carried back to the starting-point.

Since no linear measurements are taken along the line of route, the results of flying levels cannot be plotted in the form of a section.

2. Levelling for a Longitudinal Section. An essential part of the data required in the design of such highway schemes as the improvement of an existing road, the construction of an entirely new road, or the erection of a new bridge, is a longitudinal section taken through the area concerned, from end to end.

When dealing with an existing road it is customary to take levels along its centre-line at hundred-foot or closer intervals, with additional readings at any points at which marked changes of gradient are seen to occur, in order that the section, when plotted, will be properly representative of the facts. 20- or 25-foot intervals are often adopted.

When a survey is being carried along a road as a preliminary to a projected scheme, it is customary to mark the hundred-foot points derived from the chaining by pegs driven into the verges, or roadside banks, exactly opposite the end of each chain length on a rural highway, or by marks painted on kerb faces in urban areas, the appropriate chainage being indicated at each point, together with the offset distance to the chain line.

If this is done, the staff holder can readily locate himself at the correct positions when levels for the longitudinal section are being taken.

On a straight stretch of road, provided the gradient does not exceed about 1 in 60, seven or eight readings can often be obtained from one position of the level, if a 14-foot staff is used, but on steeper gradients change points must naturally occur more frequently. The instrument is set up to one side of the centre-line and not on the centre-line itself.

When the surveyor's experience is limited, there is invariably a tendency to move the level much too far uphill or downhill when changing position and much time is wasted in this way. The estimation by eye of the vertical rise or fall of a slope is apt to be very inaccurate since one's idea of a horizontal line is usually very far from correct when sighting against a slanting surface.

It is helpful when levelling to obtain the maximum illumination of the staff and, at the same time, prevent the ingress of direct sunshine into the telescope. Hence, when taking levels along a line running roughly east and west, the instrument should be set up, if possible, on the south side of the line. The instrument is normally set up with a telescope height of about 5 feet. Consequently, when using a 14-foot staff, there is a tendency for the back sights to be persistently longer than the fore sights when working uphill on a uniform gradient, since we are within the range of the staff for a vertical fall of about 9 feet, but for a vertical rise of only about 5 feet. The reverse takes place when working down the gradient.

Hence when levelling of this description is to be carried out, it is very important that the collimation of the instrument should be in good adjustment, or, alternatively, that the back and fore sights should be equalised in length. For this reason, the use of a 10-foot levelling staff is sometimes advocated, since the limiting readings, at the top and bottom of a staff of this length occur when the back and fore sights are approximately equidistant from the level, as shown in fig. 3.18.

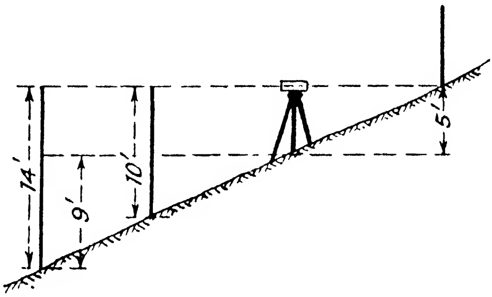


FIG. 3.18.—Comparative Range of 14-foot and 10-foot Staves, up and downhill.

The error introduced when working on a long slope, using the full range of a 14-foot staff and a level with a badly adjusted line of collimation, is, of course, cumulative.

In the case of an entirely new road, the line of route is first surveyed by means of a traverse, in a manner to be given full consideration later (see pages 101 *et seq.*) and the original survey lines may diverge considerably from the centre line of the projected road, but a longitudinal section is usually run along the traverse to give an approximate idea of the elevations of the ground *en route*.

Here again, hundred-foot pegs should be left in at the time of making the topographical survey and ground levels are determined at each peg and at any intermediate points where breaks of gradient occur. Levels of this kind are frequently carried to one place of decimals only, except at change points, which are carefully chosen and read to two places.

3. Levelling for Cross-sections. Cross-sections are necessary for presenting a more complete picture of the undulations of the ground than that provided by a longitudinal section alone. For such work as pipe lines where the excavation is of very limited width, a longitudinal section gives sufficient information of trench depths and volumes of excavation, but the overall width of a modern twin carriage-way road may be 120 feet or more and although its centre-line may run practically at the same level as the existing ground, if the latter has a sideways slope not only will excavation and filling be required, but the width of construction will be increased beyond the width of the highway proper by the extent of the cutting on one side and the embankment on the other.

The increase in constructional width in ground with a sidelong slope is even more marked when the centre-line of the new work is located at a level differing considerably from that of the existing ground, as shown in fig. 3.19, which represents a typical cross-section of a road in cutting.

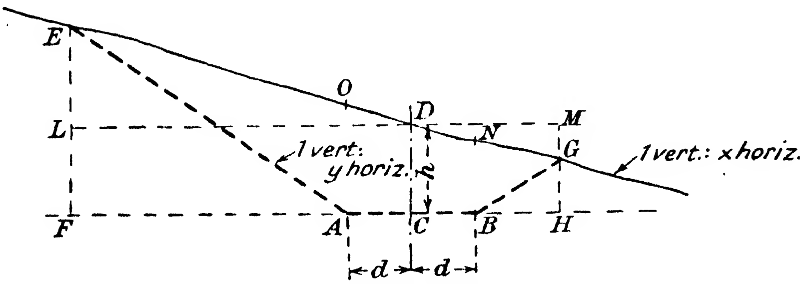


Fig. 3.19.—Cross-section on Sidelong Ground.

Let the road width, AB, be $2d$ feet, the centre point, C, being located h feet below the ground level, D.

Let the ground have an average side-slope of 1 vertical in x horizontal, as represented by EDG, and let the sides of the excavation, AE and BG, be sloped at y horizontal to 1 vertical. Let LDM be a horizontal line and EF and GH vertical lines. Then

$$FA = y.EF \text{ and } LD = x.EL$$

also $$LD = FA + d = y.EF + d = y(h + EL) + d = y(h + \frac{LD}{x}) + d$$

i.e. $$\frac{LD}{y} = h + \frac{LD}{x} + \frac{d}{y}, \text{ i.e. } LD\left(\frac{1}{y} - \frac{1}{x}\right) = h + \frac{d}{y}, \text{ i.e. } LD = \frac{h + \frac{d}{y}}{\left(\frac{1}{y} - \frac{1}{x}\right)}$$

Similarly, $$BH = y.GH \text{ and } DM = x.GM$$

Thus

$$DM = BH + d = y.GH + d = y(h - GM) + d = y\left(h - \frac{DM}{x}\right) + d$$

i.e.

$$\frac{DM}{y} = h - \frac{DM}{x} + \frac{d}{y}, \text{ i.e. } DM\left(\frac{1}{y} + \frac{1}{x}\right) = h + \frac{d}{y}, \text{ i.e. } DM = \frac{h + \frac{d}{y}}{\left(\frac{1}{y} + \frac{1}{x}\right)}$$

The difference in sign in the denominator of these expressions is due to the fact that the ground slopes upwards on one side of D and downwards on the other side.

To consider a numerical example we may assume the following data :

d = half width of road proper = 60 feet

h = 20 feet

x = 8 (sidelong gradient of ground : 1 in 8)

y = $1\frac{1}{2}$ (side slopes of excavation : $1\frac{1}{2}$ to 1)

On substitution in the above equations it will be found that LD, the overall constructional width on the high side of the centre-line is 111 feet and DM, the corresponding dimension on the low side of the line, 76 feet, making a total width of 187 feet between the tops of the cuttings.

This numerical example demonstrates the importance of carrying the levels well beyond the width of the road proper when obtaining data on ground which has a pronounced side slope. Suppose, for instance, that no levels were taken beyond the points N and O which correspond to the net width of the road. We should then know nothing definite about the slope of the ground from O to E or from N to G and in the absence of sufficient data, we could only assume that the ground line might be represented by a continuation of the line ODN. It might, in fact, slope quite differently and the assumed cross-section, in that case, would give a wrong idea of the area of excavation and also of the overall width of the cutting.

Location of Cross-sections. In new work, where accurately fixed traverse lines or projected centre-lines have already been marked on the ground by pegs, cross-section lines may be ranged out at right-angles to these by any of the methods described in the chapter on Chain Surveying (pages 17 *et seq.*)

Three or more ranging rods should be set out on each cross-section line so that the staff holder may be able to locate himself correctly by placing the staff in coincidence with two of the rods.

On portions of the centre-line which are curved, the cross-sections are judged to be as nearly as possible radial. It must be constantly borne in mind that cross-sections are ultimately used for estimations of the volume of cut and fill involved in a projected scheme, and that these volumetric calculations are based on the assumption that the cross-sections are perpendicular to the centre-line on straights and radial on curves. In undulating country it may be necessary to take cross-section levels at intervals spaced at closer intervals than 100 feet along the centre line.

Cross-sections in Hilly Country. Cross-sections on very undulating ground sometimes become a little complicated owing to the fact that one entire section cannot be completed from one setting-up of the level if the total rise or fall is beyond the range of the staff or the vertical range of the instrument. It is usually possible, however, to deal with either the low or high parts of several different sections from one and the same position of the level.

Thus, suppose that a part of a centre-line, XY, in fig. 3.20, runs through a valley in which the ground rises in the manner indicated by the arrows.

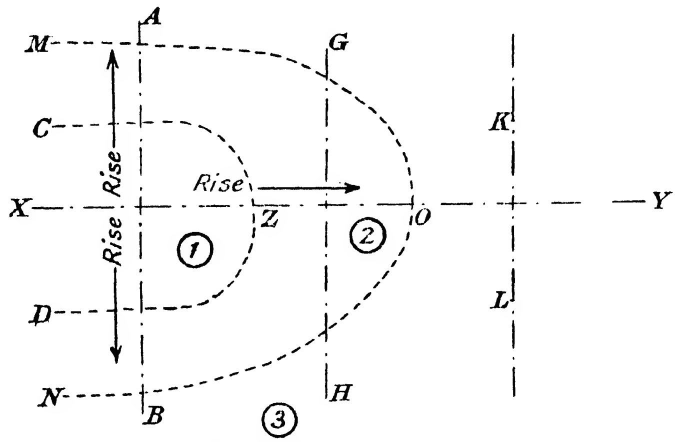


FIG. 3.20.—Range of Instrument in Cross-section Levelling.

If the level is set up in position 1 to read the lowest point X, the limit of the instrument in an uphill direction may be represented by the dotted line CZD, the high areas beyond this line being inaccessible. On moving the level uphill to position 2 so as to obtain a reading near the top of the staff when it is placed at the low point Z, the limiting range in an uphill direction may now be represented by the dotted line MON. From a position 3, however, on the hillside it may be possible to obtain readings to the higher parts of the first three sections, i.e. to such positions as A, G, K, B, H and L. This procedure is invariably quicker than completing one cross-section at a time, but such a disjointed set of levels naturally requires the greatest care in booking so that every reading is allocated to its correct section line.

Cross-sections. Fieldwork and Booking. Cross-section levelling may be carried out in the following way:

When the section line has been set out and marked by three or more ranging rods, the staff holder proceeds along it, carrying with him the ring end of the tape, while a second man is stationed on the centre-line at the chainage point with the remainder of the tape. The chainage figure at the centre-line point should be marked on a stake set at the side of the line opposite the point if it is not possible to indicate it on the centre-line peg itself.

The staff holder should be experienced in judging where breaks of gradient occur, so that he includes these points, rather than placing the staff at distances spaced at uniform intervals right and left of the centre-line.

If the ground slopes uphill, the man at the centre of the section should be provided with a ranging rod so that he can elevate the tape sufficiently to obtain a horizontal reading.

As soon as the instrument man has booked the staff reading, the man at the centre-line calls out (1) the centre-line chainage and (2) the distance right and left, as indicated by the tape.

The terms " right " and " left " must obviously be consistent with reference to a fixed direction along the centre-line, i.e. that in which the chainage is running.

Back Sight	Int	Fore Sight	Rise	Fall	Reduced Level	Distance	Remarks
6·30					231·10		Centre of existing road. Chainage 77+00
	6·55					10' L at 77	Pt A
	6 29					11 . .	. B
	6·31					14 . .	. C
	8·68					15 . .	. D (Ditch)
	6·30					16 . .	. E
	4·00					23 . .	. F (Hedge)
	3·87					40 . .	Field approx
	3·83					60 . .	level.
	6·56					10R at 77	Pt. G C. S. No 21 at 77+00.
	5·75					12 . .	. H (Hedge)
	5·07					17 . .	. K
	2·29					27 . .	. L
	1·04					40 . .	
	0·08					50 . .	Ground rising steadily beyond.

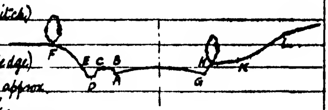


Fig. 3.21.—Booking of Cross-section Levels.

There are various methods of booking, based on the standard " Rise and Fall " or " Height of Collimation " rulings already described.

For instance, two further columns may be added beyond that headed " Distance " for booking the right and left measurements, and these are headed accordingly, keeping the original " Distance " column for centre-line chainages. Alternatively, we may use the distance column for both chainages along the centre-line and the right and left measurements, adding any descriptive notes which might provide useful information.

A sketch representing the cross-section is a very useful addition, reference letters being written thereon to correspond with the appropriate staff readings, distances and descriptive notes. An example is shown in fig. 3.21.

4. Setting out Batter Pegs or Slope Stakes. While dealing with cross-sections it will be opportune to consider the setting out of " batter pegs " or " slope stakes " which indicate the points on a cross-section where future cuttings and embankments will " run out ", i.e. points at which their slopes coincide with the existing ground levels, as at *E* and *G* in fig. 3.19. It may be noted that " batter

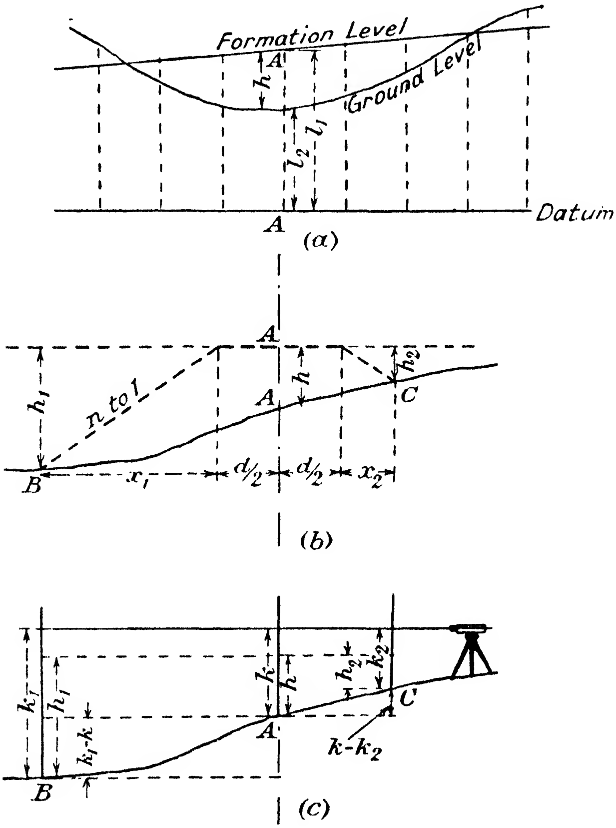


FIG. 3.22.—Cross-section Levelling. Determination of Overall Width of Fill.

pegs " are defined in the British Standard Glossary of Highway Engineering Terms as " pegs set out to mark the foot and top of an inclined surface to be formed ".

The first step in this process is to ascertain the height of the projected embankment or the depth of the projected cutting at the centre-line or, in other words, the " formation level " at that point. This may be defined as the level of the ground before the application of the actual constructional layers of the road itself, and is distinct from the finished surface level from which it may differ by as much as 18 inches.

The formation level at any point along the centre line is derived from the longitudinal section, of which a short typical length is shown in fig. 3.22(a).

Suppose at any particular point A along the centre line, the road is on embankment, the formation level being l_1 and the ground level l_2, giving the height of fill $l_1 - l_2$, or h.

Let the cross-section at this point be as shown in fig. 3.22(b).

Let the predetermined side-slopes of the embankments be n horizontal to 1 vertical and let these side-slopes run out at B and C. If the level at B is h_1 below the formation level and the level of C is h_2 below the formation level, and if x_1 and x_2 are the respective horizontal distances from the nearest edges of the formation to the toes of the embankment, the horizontal distance of B from the centre-line peg, $A = x_1 + \frac{d}{2} = n.h_1 + \frac{d}{2}$ and the horizontal distance of C from the centre-line peg, A, $= x_2 + \frac{d}{2} = n.h_2 + \frac{d}{2}$, where d is the formation width.

Suppose a level is set up in the position shown in fig. 3.22(c) and the reading on the centre-line peg $A = k$. Assuming that the points B and C were located correctly, let their respective staff readings be k_1 and k_2.

Then fall from A to $B = k_1 - k$. $\therefore h_1 = h + (k_1 - k)$

Similarly, rise from A to $C = k - k_2$. $\therefore h_2 = h - (k - k_2)$

Hence distance of B from centre-line $= x_1 + \frac{d}{2} = n.h_1 + \frac{d}{2}$

$$= n[h + (k_1 - k)] + \frac{d}{2}$$

and distance of C from centre-line $= x_2 + \frac{d}{2} = n.h_2 + \frac{d}{2}$

$$= n[h - (k - k_2)] + \frac{d}{2}$$

The points B and C could be scaled from the plotted cross-section and their positions set out on the ground, but it is often more convenient to set out these points direct from levels taken on the spot, and this can be done in the following way, provided that we know the formation levels at points along the centre-line and also the formation width and the inclination of the slopes of the proposed cuttings and embankments :

A levelling staff is placed at the centre-line peg and its reading noted. It is then moved up or down the slope, keeping on the cross-section line to trial positions for points B and C and the readings again taken. A simple calculation is then made to ascertain whether the distance of the staff from the centre-line, as measured on the ground, agrees with the distance derived from the above equations, based on the staff readings. If it does not, the staff is moved to another position and the process repeated.

It is sufficiently accurate if agreement to the nearest foot is obtained and levels in this kind of work are often read to one place of decimals only. This furnishes another example of co-ordination between the preliminary fieldwork and the ultimate object for which such fieldwork is being carried out—the tipping of an embankment or the excavation of a cutting is not a matter which can be controlled to a fraction of an inch and the setting out of batter pegs, though requiring a reasonable degree of accuracy, does not necessitate the same precision as, for example, the setting out of the bed stone levels for the girders of a bridge.

The fieldwork in setting out slope pegs is rendered more orderly and calculations are simplified if results are tabulated. The steps in the working are reduced by the introduction of an imaginary factor termed the "Grade Staff Reading", defined as the theoretical staff reading which would be obtained at the formation level.

Grade Staff Method of calculating Slope Peg Position. The grade staff reading, as defined above, may be either positive or negative as figs. 3.23 and 3.24 illustrate. In fig. 3.23, for example, the ground level is 100 and the formation level 107, so that the height of the fill is 7 feet. If a level were placed on the high ground so as to give a reading of 12·5 at the centre-line, the reading at the formation level

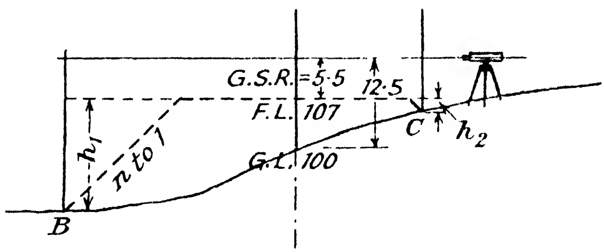


FIG. 3.23.

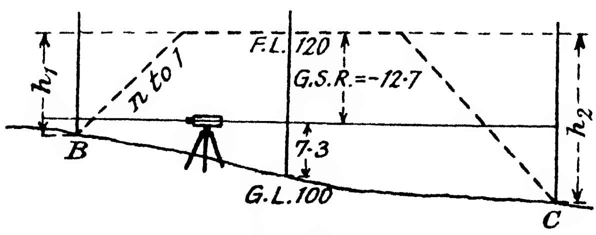


FIG. 3.24.

would be 12·5 − 7, or 5·5, which is, consequently, the grade staff reading for the section considered.

In fig. 3.24, however, the formation level is 120 and the ground level 100, the height of fill being 20 feet. If, now, the instrument were set up to read 7·3 at the centre-line, the grade staff reading will be negative, and is obtained, as before, by subtracting the height of fill from the staff reading at the centre-line, i.e. 7·3 − 20, or − 12·7.

It will be apparent from figs. 3.23 and 3.24 that the staff reading at any other point, e.g. *B* or *C*, minus the grade staff reading gives the depth of the latter point below formation level, i.e. our values of h_1 or h_2. The sign of the grade staff reading must, of course, be taken into account.

By multiplying these values by the slope ratio, n, we get our distances x_1 or x_2 and by adding half the road width $(d/2)$ to each we obtain the calculated distances of the points in question from the

Centre Line Chainage	C L Ground Level (g)	C L Formation Level (f)	C L Staff Reading (k)	Grade Staff Reading $s = g + k - f$	Staff Readings at Slope Peg Positions		h_1 $(k_1 - s)$	h_2 $(k_2 - s)$	Calculated		Measured	
					k_1	k_2			d_1 $(nh_1 + \frac{d}{2})$	d_2 $(nh_2 + \frac{d}{2})$	d_1	d_2

FIG. 3.25.—Location of Slope Stakes by the "Grade Staff" Method.

centre-line. These, finally, should agree with the measured distances and the fieldwork procedure consists, as before, in finding the position of points such as *B* and *C* by repeated trials until reasonable agreement is obtained.

The whole of this sequence of operations can be arranged concisely in tabular form in the manner indicated in fig. 3.25. The number of columns may appear formidable, but their use is far preferable to the random scribbling of calculations, staff readings and distances all over the field book, a proceeding which inevitably wastes time and causes mistakes.

The column headings are self-explanatory, with the exception of "Grade Staff Reading" which is given by Staff reading at centre line − Formation height or depth, the latter being considered negative. The first three columns are filled in from the longitudinal section before commencing the fieldwork and the formation height or depth is given by $f - g$.

5. Spot Levelling. This process consists in obtaining the heights of a number of points the positions of which are located by measurements so that their positions may be plotted on the plan of the area concerned.

It expedites both the fieldwork and the plotting if the points are arranged on some simple system of lines along which the staff holder can proceed, keeping his correct alignment, meanwhile, by sighting the levelling staff into coincidence with two ranging rods.

For instance, if fig. 3.26 represents an area of land on which spot levels are required, it will happen, very possibly, that the pegs used in making the topographical survey are still intact at the positions

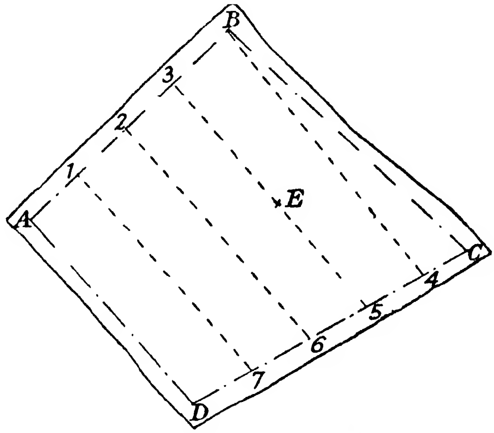


Fig. 3.26.—Arrangement of Lines for Spot Levelling.

A, *B*, *C* and *D*. If not, pegs must be placed in some such positions and sufficient measurements taken to enable them to be plotted accurately. By measuring off hundred-foot, or any other suitable lengths, along *AB* and *CD*, as shown by the numbered points, it is possible to obtain a well-distributed series of lines covering the area.

If three ranging rods are set up on any line, as at 3, *E* and 5, the staff holder can keep himself in alignment since he will always be in sight of two rods. He can also obtain his distance along the line by securing one end of a tape at the starting-point by means of an arrow, should another assistant be unavailable.

6. Contouring. This may be looked upon as a development of spot levelling and is sufficiently to justify a full consideration which is contained in the next chapter (pages 85 *et seq.*).

General Precautions in Levelling.

1. Change Points. The precautions in connection with these have already been mentioned (page 48).

2. Instrument Position. A firm site for the level position should be selected, if possible, and the tripod legs should be pushed well into the ground to give greater stability. If wing nuts are fitted at the top of the legs to facilitate folding, see that these are tightened after setting up the instrument.

When setting up on pavements, or smooth road surfaces set the points of the tripod legs in any convenient joints or small holes, or tie a cord round the base of the legs to give security from slipping.

In general, on undulating ground, set up as high as possible, i.e. in a position which will give as high a reading on the staff as possible when the latter is held at the lowest point to be levelled.

3. Swinging the Staff. It is essential that the staff should be held vertically both with reference to fore-and-aft and sideways directions. Any lack of verticality in the latter case can be detected immediately when sighting through the telescope by checking against the vertical cross wire and instructions given to the staff holder accordingly, but if the staff is tilted from the vertical either backwards or forwards, this fact is not visible when the reading is being taken.

To assist the staff holder in judging when the staff is vertical, some staves are fitted at the side with a plumb-line, the bob of which swings in a small ring, and others are provided with a small circular spirit

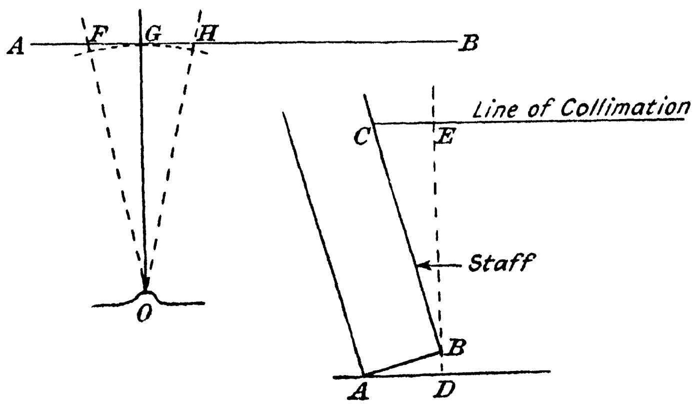


Fig. 3.27.—" Swaying " the Staff. The minimum reading only corresponds to the vertical staff position when the base is resting on a rounded surface.

level, located at the side or back, but these attachments, especially the latter, are rather liable to damage.

A simple method of ensuring that the staff is vertical in a fore-and-aft direction is termed " swinging " and consists in rocking the staff backwards and forwards while its base is supported on a rounded surface. The purpose of this procedure is illustrated in fig. 3.27, in which AB represents the horizontal line of sight from the level intersecting a vertical staff at G. If OF and OH are the positions of the staff when it is inclined backwards and forwards respectively, it will be apparent from the diagram that the staff readings at F and H are greater than that at G and the latter is the *minimum* reading obtainable.

If the staff be slowly rocked backwards and forwards, the horizontal cross wire will appear to travel up and down, coming to its lowest reading when the staff is held in the vertical position, OG, and this reading is, therefore, booked. Care must be taken, however, that the staff is not rocked about its rear edge, as this will have the effect of raising the front and consequently reducing the reading, in which case, for readings near the bottom of the staff, the minimum value may no longer correspond to the vertical position. Fig. 3.27 will make this clear, the reading BC being less than ED, the reading with the staff vertical.

The extent of the error due to the non-verticality of the staff in a fore-and-aft direction may be considerable, as the following example will show:

Suppose that the correct reading, OG, is 12·00 feet and that the staff is held 5° from the vertical, giving the reading OF.

Then $OF = OG. \sec. 5° = 12 \times 1{\cdot}004 = 12{\cdot}05$ feet very nearly. Hence the error would be nearly 0·05 of a foot and as the staff will read to 0·01 of a foot, the discrepancy is by no means insignificant. Furthermore, it is not unusual for an inexperienced or careless man to hold the staff fully 5° out of vertical without being conscious of the fact.

It will be evident that this error is of greater consequence when readings are being taken near the top of the staff.

4. Telescopic Staves. When using an extended telescopic staff, it is vitally important to make sure that the spring clips at the back which lock the sections in position are properly engaged and secure in their notches. The staff holder should be trained to stand immediately behind the staff with his hands flat against the sides.

5. Staff at Bench Mark. As already mentioned the standard type of Ordnance bench mark, inscribed on the vertical face of a wall, is not well adapted for locating the staff accurately at the correct height, since it offers no support and it is not an easy matter to hold up a heavy staff, particularly when fully extended, making perfectly certain, at the same time, that its base is kept exactly level with the centre of the horizontal V-shaped groove.

If possible, two men should be engaged on this job, one of whom inserts a steel rule, or a similar strip of thin metal such as the end of a knife blade, in the groove, to support the base of the staff at the correct level, while the second man holds the staff vertically.

6. Water Levels. The levels of streams and rivers naturally fluctuate to a greater or less extent, varying with the season, and when such levels are taken, the date should be noted and inscribed on the drawing, thus giving a rough guide as to whether the level is representative of summer or winter flow It is important to search for evidences of past floods in the vicinity of stream or river crossings and levels should be taken at any available flood marks.

7. Refacing Staves. A clean, bright staff enables the length of sight to be increased to a surprising extent, in addition to obviating uncertainties in the readings. If the face of a levelling staff becomes indistinct through scratches or grime, its clarity can be easily and cheaply restored by pasting over the old face paper strips on which the standard type of graduations and numerals are printed. These paper strips, known as " staff refills " are supplied by the principal instrument makers. The paper is subsequently sized and varnished.

Carrying Levels past an Obstruction. It sometimes happens, particularly in town surveying, that levels have to be transferred from one side to the other of some such obstruction as a high wall. This state of affairs is illustrated in fig. 3.20, and the difficulty may be overcome in the following way :

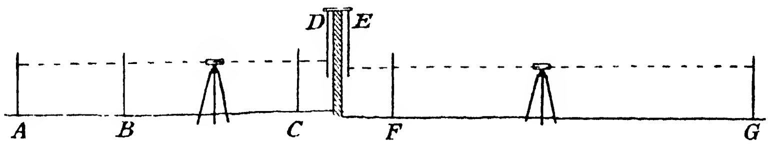


FIG. 3.28.—Continuation of Levels past an Obstruction.

A short batten is secured to the top of the wall and levelled carefully so as to be exactly horizontal ; the levels are taken at any required points, such as *A*, *B* and *C* on one side of the wall and the staff is then held upside down with its base touching the underside of the batten. The staff is then transferred to the other side of the obstruction and again held upside down in contact with the underside of the batten and a reading is taken from the new position of the level on the far side of the wall, after which the work proceeds normally, from point *F* onwards. The top of the wall is, in effect, a change point.

This method is adaptable to the " Height of Collimation " booking, if the following rules are observed :

The first reading to the inverted staff in position *D* is booked as a fore sight with a negative sign and is consequently *added* to the height of collimation, when the usual subtraction is carried out algebraically to obtain the reduced level of the change point.

The second reading to the inverted staff, i.e. in position *E*, is booked as a back sight from the second position of the level, but again with a negative sign and this must be added algebraically to the reduced level of the change point to give the new height of collimation which, in consequence of the minus sign, will be less than the level at the top of the wall.

Apart from these divergences, the levels are worked out in the ordinary way and the usual arithmetical check may be applied, adding the back sights and fore sights, but paying due regard to signs, thus :

Back Sight	Int. Sight	Fore Sight	Height of Collimation	Reduced Level	Distance	Remarks
3·99			181·24	177·25		Point *A*. R.L. 177·25
	4·00			177·24		,, *B*
	3·80			177·44		,, *C*
– 4·38		– 3·75	180·61	184·99		Points *D* and *E*. (Top of wall)
	3·90			176·71		Point *F*
		4·01		176·60		Point *G*

Sum of back sights: 3·99 – 4·38 = – 0·39
Sum of fore sights: – 3·75 + 4·01 = 0·26
Difference = – 0·65 ft.
indicating a fall from *A* to *G*.

First Reduced Level: 177·25
Last Reduced Level: 176·60
Difference: 0·65 feet fall from *A* to *G*.

In cases such as this which are somewhat out of the ordinary, it is always advisable to make a dimensioned sketch on the "Remarks" page.

Transferring Levels across a River. Reciprocal Levelling. When levels have to be transferred from one side of a wide river to the other, it usually happens that the readings consist of one very short sight and one very long one and, consequently, a slight error in the line of collimation of the instrument will produce a marked error in the result.

Thus, if in fig. 3.29 it is required to find the relative heights of the points *A* and *B*, staff readings would be taken to the two points from a level set up in the position indicated and, assuming that the line of collimation is slightly inclined, the error e_1 at *A* will be negligible, but the error e_2 at *B* may be considerable if the distance *AB* is several hundred feet.

If, however, we set up the level on the other side of the river, at the same distance beyond *B* as it was from *A*, the new staff reading at *B* will have the same error e_1 as the previous reading at *A* and the new reading at *A* will have the same error e_2 as the previous reading at *B*.

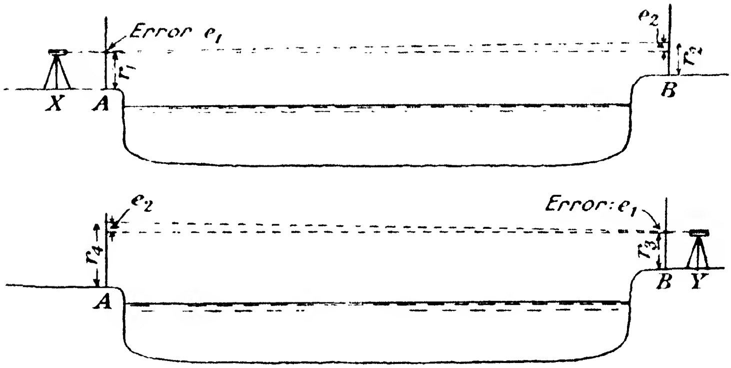


FIG. 3.29.—Reciprocal Levelling.

Let the readings from X to A and B be r_1 and r_2 respectively and let the readings from Y to A and B be r_4 and r_3, respectively. Then from the first pair of readings, the correct rise from A to B

$$= (r_1 - e_1) - (r_2 - e_2) = r_1 - r_2 - e_1 + e_2$$

and from the second pair of readings, the correct fall from B to A

$$= (r_4 - e_2) - (r_3 - e_1) = r_4 - r_3 + e_1 - e_2$$

Hence, by addition, twice correct difference in levels at A and B

$$= r_1 - r_2 + r_4 - r_3$$

or the correct difference of levels at A and B

$$= \frac{r_1 + r_4}{2} - \frac{r_2 + r_3}{2}.$$

i.e. the correct result, assuming refraction has not changed, is obtained by taking the difference between the mean of the readings to A and the mean of the readings to B.

A numerical example is appended:

Point	Reading from X	Reading from Y	Mean Reading
A	5·38	6·19	$\frac{11{\cdot}57}{2} = 5{\cdot}785$ feet
B	4·58	5·23	$\frac{9{\cdot}81}{2} = 4{\cdot}905$ feet

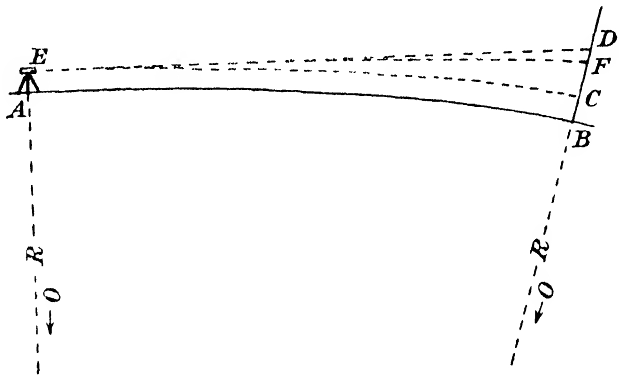
Mean Difference = True Rise from A to B = 0·880 feet

It will be noticed that the first pair of readings gave a rise from A to B of 0·80 feet and the second pair of readings a rise of 0·96, the mean of these results also giving the true rise of 0·88 feet.

This process of double reading is known as " Reciprocal Levelling " and should always be adopted when transferring levels directly across a river. It is a vitally important precaution when the levelling is a preliminary to future bridge construction.

The foregoing remarks refer mainly to levelling by the dumpy or quick-set type of instrument. When using a level of the self-checking type, described on page 43, a second value of the readings from each instrument station can be obtained immediately by inverting the telescope and their mean, if they differ, will automatically give the correct result ; but, nevertheless, reciprocal levelling could still be adopted with advantage as an additional check.

Curvature and Refraction. So far, we have considered a " level surface " to be a horizontal plane. Owing to the curvature of the



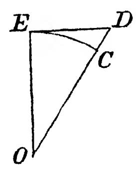


FIG. 3.30.—Effect of Curvature and Refraction in Levelling.

earth this is only true for a limited distance. A " horizontal line " is sometimes defined as a line perpendicular to the direction of a plumb-line and, neglecting the deflection due to the attraction of large mountain masses, the plumb-line points to the centre of the earth. If we now consider, as a rough-and-ready approximation, that the earth is a sphere of radius 4,000 miles, it will be apparent that a " level surface " passing through any point A in fig. 3.30 is spherical and that in a sectional view a " level line " passing through A is a circular arc.

If a level were set up at A, the line of collimation could be represented by a horizontal line ED, intersecting a staff held vertically at a distant point B having the same level as A, and the arc AB would represent a line of constant altitude.

Let EC be an arc concentric with AB. Then a staff reading BC would indicate that A and B had the same altitude, whereas the actual staff reading is BD.

Hence the error due to neglecting the curvature of the earth is DC.

Let O be the centre of the arc AB. Then

$$OE^2 + ED^2 = OD^2 = (OC + DC)^2 = (OE + DC)^2$$
$$= OE^2 + 2.OE.DC + DC^2$$
$$ED^2 = DC.(2OE + DC), \text{ or } DC = \frac{ED^2}{2OE + DC}$$

The term DC in the denominator will be very small compared with OE and may be neglected, so that we may write $DC = \dfrac{ED^2}{2.OE}$.

If we assume that the mean diameter of the earth is 8,000 miles,

$$\text{correction } DC, \text{ in feet} = (\text{length of sight in miles})^2 \times \frac{5280}{8000}$$
$$= 0{\cdot}66 \times (\text{length of sight in miles})^2$$

If the length of sight is 1 mile, the curvature correction is 0·66 feet, or nearly 8 inches.

Actually, for sights of this length it is incorrect to assume that the line of collimation is horizontal since atmospheric refraction causes the line to curve downwards to some such position as EF. The magnitude of the distance DF will vary with atmospheric conditions, but it is usually taken as one-seventh of the curvature correction DC.

It will be noticed that the effect of refraction partially compensates for the earth's curvature and the combined correction may be expressed (in feet) as

$$\tfrac{6}{7} \times 0{\cdot}66 \times (\text{length of sight in miles})^2$$
$$\text{or} \quad 0{\cdot}565 \times (\text{length of sight in miles})^2$$

Such long sights as we have been considering cannot, of course, be read with an ordinary staff and a special target staff is used in which a sliding disc, or target, with a clearly marked horizontal line, is moved up and down the graduated scale as directed by the instrument man by flag signals or heliograph messages until the horizontal index comes into coincidence with the central horizontal cross wire of the instrument.

A vernier is sometimes incorporated with the target so that readings may be taken to 0·001 of a foot. Fig. 3.31 shows a form of target known as a "Philadelphia rod".

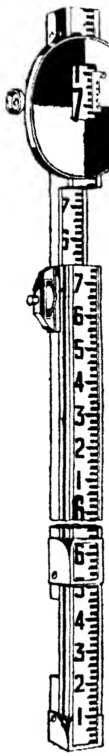


FIG. 3.31. — The "Philadelphia" Rod, with target. (Reproduced by courtesy of Messrs. E. R. Watts & Son.)

Instead of applying the calculated corrections for curvature and refraction it is preferable to arrange for their elimination by reciprocal levelling. Thus, in the example already given in fig. 3.29, the combined effect of curvature and refraction will give too small a rise from A to B when the level is at X and too large a fall from B to A when the level is at Y. Provided that the readings are taken under the same atmospheric conditions, so that refraction is unaltered, we may write, if c = combined correction for refraction and curvature :

From first pair of readings :

$$\text{Rise from } A \text{ to } B = r_1 - r_2 - e_1 + e_2 + c$$

From second pair of readings :

$$\text{Fall from } B \text{ to } A = r_4 - r_3 - e_2 + e_1 - c$$

By addition, twice difference in level between A and $B = r_1 - r_2 + r_4 - r_3$ or, difference in level corrected for collimation, curvature and refraction $\frac{r_1 + r_4}{2} - \frac{r_2 + r_3}{2}$, as before.

Accuracy of Levelling. In ordinary levelling, under favourable conditions, it may be reasonably expected that the errors will not exceed 0·1 feet per mile. Some authorities consider that the permissible error in this class of levelling may be expressed as $0{\cdot}1\sqrt{M}$ feet, where M is the distance levelled in miles.

Considerably greater accuracy is obtained in precise levelling.

CHAPTER IV

CONTOURS

We have seen that the project for a new highway involves the preparation of a plan, a longitudinal section and cross-sections.

To a certain extent the information provided by these three types of drawing can be combined in one—namely, a contoured plan, and although a plan of this kind will not provide a sufficiently precise method of determining accurate earthwork quantities, it is extremely useful in the preliminary consideration of possible routes for a new road, showing, as it does, not only the topographical details, but also the configuration of the hills and valleys, and, within limits, the elevation of the ground at any point.

The idea of a contour is very familiar and a " contour line " is defined in the British Standard Glossary of Highway Engineering Terms as " an imaginary line connecting a series of points on the surface of the ground which are all the same height above a prescribed datum ; also the line representing this on a map or plan ". The accuracy with which a contour depicts the actual conditions on the ground depends, consequently, upon the number and closeness of the points which give the contour its shape.

Thus, an approximate contour could be drawn through the three

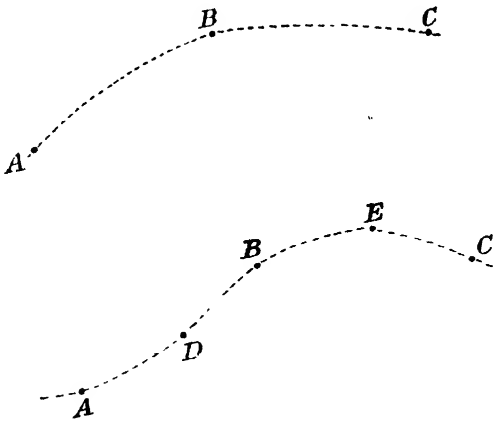


Fig. 4.1.—Variation in Contour Shape obtained by locating Additional Points.

points A, B and C in fig. 4.1, but if two additional points D and E, having the same level as A, B and C, were located in the field and plotted on the plan, the shape of the contour would become completely altered and would then give a much closer representation of the actual conditions at the site.

Herein lies the disadvantage of contours—unless they are based on a large number of points they become only approximate.

Contours are shown on six-inch-to-one-mile Ordnance maps, but not on the twenty-five-inch maps (1/2500 scale). One inch to the mile Ordnance maps are also available in contoured editions and on a road scheme of considerable magnitude these maps are very useful for preliminary location.

Application of Contours to Road Location.

1. Determination of Line of Limiting Gradient. Let us imagine that a road is to be constructed connecting two points *A* and *B* shown on the portion of a contoured plan reproduced in fig. 4.2.

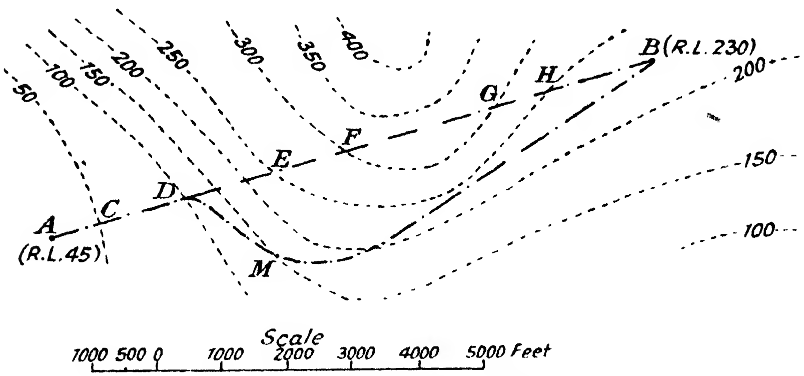


Fig. 4.2.—The Use of Contours in Road Location.

It will be apparent from the general run of the contours that a direct line from *A* to *B* on the ground surface will commence to rise at a comparatively easy gradient as far as *D* and will then climb steeply from *D* to *E*, where the close spacing of the contours indicates a sharp gradient, easing off somewhat from *E* to *F*, running fairly level across the flat " headland " of the hill from *F* to *G*, dropping suddenly down the hillside from *G* to *H*, and finally running by a gentle downgrade from *H* to *B*.

If, now, we stipulate that the limiting gradient of the proposed road is to be 1 in 20, this will mean that the horizontal distance along the route from one contour to the next must nowhere be less than twenty times the vertical interval between the contours, i.e. 20×50, or 1000 feet.

It will be seen from the scale of the drawing that the above stipulation is complied with throughout the route except between *D* and *E* and between G and H. If cutting and filling is to be avoided the line between these points must therefore be diverted from the direct route and slewed in a direction more in keeping with the general run of the contours in order to extend the horizontal distance between the successive 50-foot rises or falls, giving some such deviation as that shown at M, where the route runs round the promontory, instead of cutting directly across.

2. Derivation of Longitudinal Section from Contours. Continuing our consideration of the foregoing example, we could, of course, achieve the same result with regard to the grade limitation by using the direct line and carrying out heavy excavation.

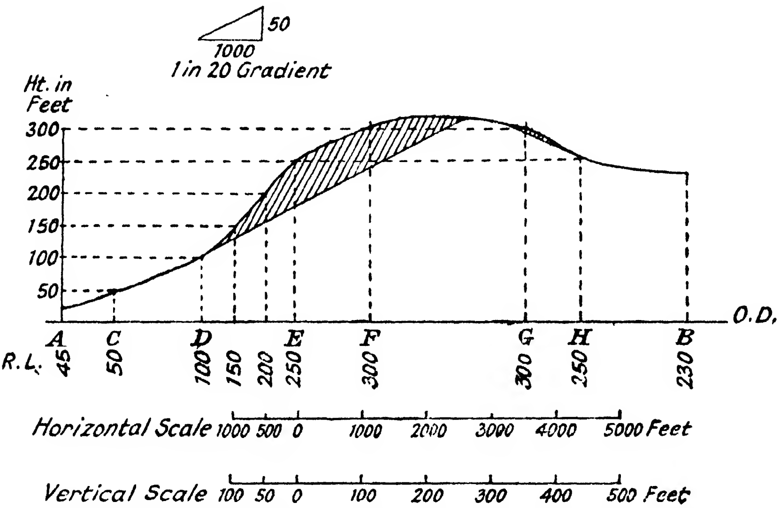


FIG. 4.3.—Reduction of Gradient by Excavation.

This possibility can be investigated quite simply by plotting an approximate longitudinal section along the line AB from the data provided by the contours, and this has been done in fig. 4.3.

It may be opportune to mention here that a longitudinal section, if plotted to the same scale horizontally and vertically does not give a sufficiently graphic representation of the rises and falls of the ground. This will be appreciated if a gradient of 1 in 4 (1 vertical to 4 horizontal) is drawn to its true or " natural " scale. The resulting slope appears, on paper, to be merely a gentle incline, whereas a grade of this magnitude is extremely steep when encountered in practice.

Hence, in the longitudinal section shown in fig. 4.3, the vertical scale has been exaggerated ten times by comparison with the horizontal scale.

3. Derivation of Cross-sections from Contours. It is, of course, possible to derive cross-sections from a contoured plan in much the same way as a longitudinal section.

Suppose, for instance, that *AB* in fig. 4.4, represents a part of the proposed centre-line for a new road and that a cross-section is required along the line *DE*.

The formation level on the centre-line at this point is found from the longitudinal section and the diagram illustrates how the cross-section could be obtained by direct projection, assuming that the formation level at the centre-line peg *C* is 150, the formation width is 50 feet, and the side slopes of the cutting are inclined at $1\frac{1}{2}$ horizontal

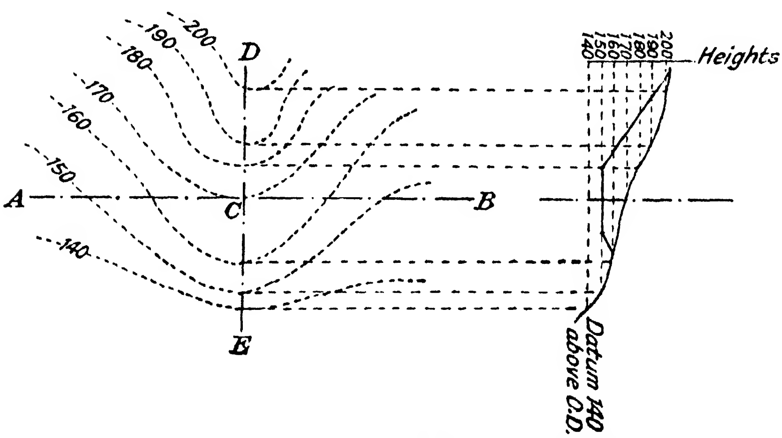


FIG. 4.4.—Derivation of Cross-section from Contours.

to 1 vertical. This is not the normal drawing office method but serves to show the relationship between the contoured plan and the cross-section.

Actually, cross-sections are usually plotted to the same scale horizontally and vertically and except for very deep excavations and very high embankments, they would become insignificantly small if projected directly from a contoured plan drawn to a small scale, such as 200 feet to an inch, often used for preliminary location work. An embankment 20 feet high, for example, would be represented on this scale by a vertical dimension of 0·1 inch.

Cross-sections, however, are only required at crucial points when the preliminary consideration of possible routes is in progress, i.e. at points where the centre-line cuts well into a hill or is carried high above a valley floor, so that the scale of the plan is sometimes sufficiently large to give a rough idea of the extent of excavation and filling at such points.

In this case, the points of intersection of the contours with the cross-section line, together with the position of the centre-line and formation width, as shown on the plan, may be marked off on the edge of a sheet of paper and transferred directly to a horizontal line used as a convenient datum. Verticals are drawn through the points so obtained and the corresponding heights are set off on these, thus giving the ground line. The formation width is then set off at its correct height symmetrically about the centre-line and the side slopes are drawn in to complete the cross-section.

By comparing the formation level, obtained from the longitudinal section, with the contour heights in the vicinity of the cross-section line, it is easy to judge whether the cross-section, if drawn to the same scale as the plan, will give a reasonable figure; if not, the distances of the contours, right and left of the centre-line, must be scaled from the plan and replotted to a larger scale.

Methods of Contouring.

There are two methods of producing a contoured plan:

(1) by direct location of the contours in the field and subsequent plotting of each contour point;

(2) by obtaining spot levels in the field, plotting them on the plan and drawing in the contours by interpolation.

1. Direct Location of Contours. The "Height of Collimation" method of computing levels provides a simple rule for predetermining the staff reading from any instrument position which will correspond to a given contour height.

We have already seen on page 53 that the reduced level at any intermediate point or change point is equal to the height of collimation minus the intermediate or fore sight reading.

In contouring, we assign values to the reduced levels, which then become the contour heights, and the above equation may therefore be written:

Contour height = height of collimation − staff reading at contour

or

Staff reading at contour = height of collimation − contour height.

We choose the contour heights at convenient round numbers and space them at vertical intervals of height which are suited to the locality, always attempting to visualise how the contour lines will appear on paper when plotted to the scale of the plan.

On steep slopes like 1 in 4, we should not attempt to locate contours at such a small height interval as 1 foot, for instance, if the scale of the plan is 200 feet to an inch, since the contour lines, on paper, would be separated by only 0·02 of an inch and could not be drawn. A height interval of 5 feet would be suitable in these circumstances.

On gentle slopes with a fairly smooth surface it would be quite feasible to adopt a height interval of 1 foot, but if the ground were very uneven, the surface irregularities might well give rise to false impressions and a larger height interval would be less influenced by these local undulations.

Method of Location using a Dumpy or similar Level. The first step in contouring is the arrangement of a system of lines on which the contours are to be located. These lines, obviously, must cut across the contours, or, in other words, run up and down the slopes and not follow a comparatively level line, roughly parallel to the contours.

Fortunately, it usually happens that the projected route for a new road tends to follow the latter direction whenever possible and the

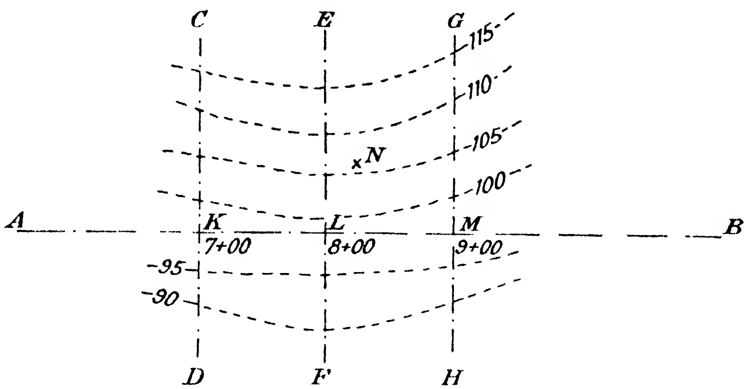


FIG. 4.5.—Arrangement of Survey Lines for locating Contours.

traverse lines used for surveying the route will naturally tend to assume much the same direction and thus serve admirably as centre-lines along which perpendiculars can be set off to intercept the contours.

Thus, in fig. 4.5, *AB* represents a portion of such a traverse line, *K*, *L* and *M* being chainage pegs which form convenient reference points at which to set out the perpendiculars *CD*, *EF* and *GH*. These should be located accurately by means of the cross staff, optical square, or some other method.

The reduced levels at the hundred-foot pegs along the line *AB* must be obtained in the ordinary way from the nearest Ordnance or temporary bench mark.

Suppose, then, that the section lines *CD, EF* and *GH* have been duly set out and marked by ranging rods, and that the level is set up at *N* and a reading of 12·31 obtained on a staff placed at *K*, where the reduced level is known to be 97·62. Then, the height of collimation is 97·62 + 12·31, or 109·93.

We read our original back sight to two places of decimals and our centre-line level will also be given to two places, but for the intermediate readings used in locating the contours one place of decimals is sufficient. Hence, for this purpose we consider the height of collimation to be 109·9.

At a change point, however, the fore sight must be read to two places and the corresponding reduced level worked out to the same degree of accuracy. The subsequent back sight and the new height of collimation are again booked to two places, reverting to one place for the intermediates only.

If the staff is 14 feet long, the lowest possible staff position visible from the instrument at its given setting will have a reduced level of 109·9 – 14·0, or 95·9. Similarly, a reading at the base of the staff would correspond to a reduced level of 109·9. Hence, assuming that contours are to be located at 5-foot intervals, those available from the given position of the level are at heights of 100 and 105, although the 110 contour could possibly be located with sufficient accuracy, in addition.

For the 100 contour, staff reading will be 109·9 – 100, or 9·9
,, ,, 105 ,, ,, ,, ,, ,, 109·9 – 105, or 4·9
,, ,, 110 ,, ,, ,, ,, ,, 109·9 – 110, or approximately zero.

The staff holder moves along the section lines, *CD*, *EF*, etc., until the required staff readings are obtained and the distance of the staff right or left of the centre line peg is then measured.

It is a great help to have a second man acting as assistant to the staff holder in taking these measurements.

It will be noticed that the above three contours could probably be located on several section lines from the assumed instrument position, *N*, four or five frequently being within the range of the level provided that trees or bushes do not obstruct the view. When the possibilities of one instrument position are exhausted, a change point is taken and the level moved to command a further series of heights.

Frequent checking back to the accurately levelled centre-line pegs is advisable, as this not only detects the presence of errors, but also tends to localise them.

Booking of Results. The procedure described above very much resembles levelling for cross-sections, referred to on pages 69 *et seq.* and the " Height of Collimation " level book may be readily adapted for booking the results, as shown by the following example which gives the data for some of the contours in fig. 4.5. The existing column in the book headed " Distance " is used for the centre-line chainage and two additional columns may be ruled on the " Remarks " page for the distances right and left to the contour positions.

Back Sight	Int.	Fore Sight	Height of Collimation	Reduced Level	Distance (on Centre-line)	Contour Distance	
						Right	Left
12·31			109·93	97·62	7 + 00	—	—
	9·9		(109·9)	100·0	,,	—	27
	4·9			105·0	,,	—	61
	0·0			109·9	,,	—	98
	9·9			100·0	8 + 00	—	12
	4·9			105·0	,,	—	47
	0·0			109·9	,,	—	79
	9·9			100·0	9 + 00	—	32
13·24		4·89	118·28	105·04	,,	—	68
	8·3		(118·3)	110·0	,,	—	109
	3·3			115·0	7 + 00	—	128
	8·3			110·0	8 + 00	—	79 (check)
	3·3			115·0	,,	—	117

Use of the Stadia Diaphragm in Contouring. In a previous reference (on page 43) to the diaphragms used in levelling instruments, the fact was mentioned that three horizontal cross wires are frequently used, the central line giving the staff reading for heights and the outer lines providing the data for obtaining the distance from the staff to the instrument.

The theoretical principles involved are discussed later in the chapter on "Tacheometry" (pages 159 *et seq.*), but it is opportune to remark at this stage that diaphragms of this type, known as "stadia", are very useful in contouring. Without adding appreciably to the time taken in obtaining the single staff reading necessary for determining the level and without complicating the work in the slightest, they obviate the necessity for measuring by tape the distance between each contour point and the centre-line.

All that is necessary in the field, after locating a contour by obtaining the predetermined reading on the central wire, is to book the readings of the two outer wires as well.

The difference between the outer readings multiplied by a simple factor, usually 100, and, in some cases, increased by a constant length, gives the distance between the staff and the level.

The multiplying factor and additive constant, if any, are usually inscribed by the maker on a label inside the instrument box, but if this information is not given, the necessary data can easily be obtained by making a simple test with measured distances, such as 100, 200 and 300 feet. If this method of determining the contour distances is adopted, it is, of course, essential that the level should be set up over a point whose position can be plotted on the plan.

We may start, for instance, by setting up over a centre-line peg, obtain our collimation height from a back sight taken to an adjoining peg and then proceed to locate contours on the section line passing through the instrument station.

When it becomes necessary to change the instrument position the level may be set up over a contour point previously located since the distance from the centre-line to this point will be known and further measurements may therefore be taken from it.

Some levels are provided with a plumb-line to facilitate setting up over a definite point.

Method of Booking using a Stadia Diaphragm. An orderly system of booking the stadia readings may be maintained by adding a few columns to the existing ruling in a " Height of Collimation " level book and a suggestion is given below, in which the bookings relate to the example previously discussed and illustrated in fig. 4.5.

The instrument, which is assumed to have a multiplying factor of 100 and no additive constant, was set up over the peg K, at chainage 7 + 00, and a back sight was taken to peg L, at chainage 8 + 00, reduced level 98·77.

The height of collimation is found, as before, to two places of decimals, but the intermediates are carried to one place only.

Additional columns to accommodate the stadia data are ruled on the " Remarks " page and it will be noticed that the outer cross wire readings are booked to two places of decimals. This is necessary because the difference between the readings is multiplied by 100 to obtain the corresponding distance.

The central cross wire reading should be the mean of the outers and it will be seen that this is so within the limits of the single decimal place in the case of the intermediates.

Back Sight	Int.	Fore Sight	Ht. of Coll'n.	Reduced Level	Distance (Centre-line)	Outer Cross Wires		Distance (Diff. × 100) Left	Distance (Diff. × 100) Right
Instrument station : K									
4·82			103·59 (103·6)	98·77	Stn. L	—	—	—	—
	3·6			100·00	7 + 00	3·74	3·47	27	—
	8·6			95·00	,,	8·75	8·46	—	29
	13·6			90·00	,,	13·89	13·30	—	59

Setting out Right-angles with the Level. Many levels of the modern " Quick set " type have a useful additional fitment in the form of a graduated ring located horizontally beneath the telescope. A clamp enables this ring to be held stationary at any particular setting relative to the vertical axis of rotation of the telescope, or, if released, the ring is free to rotate about this same axis.

The underside of the telescope carries a small index mark, or, in some cases, a vernier, which engages with the graduated circle and enables the angle described by the telescope to be measured when the latter is rotated. The accuracy of the angular measurement varies from the nearest degree in the cruder instruments to the nearest 5 minutes of arc in the more refined forms (see Chapter XI, page 268.)

A fitment of this type is provided in the modern level shown on page 237 (fig. 11.5). The eye piece for reading the horizontal circle will be seen to the right of and slightly lower than the main eye piece.

This attachment gives us a handy method of setting out the section lines for contour location, particularly when the stadia method is adopted and the instrument is set up over the centre-line peg. The method is as follows using the simple type mentioned above:

The telescope is directed along the centre-line and clamped and the graduated ring is rotated until the zero mark coincides with the telescope index or vernier zero. The ring is clamped at this setting and the telescope rotated until an angle of 90° is registered when it is again clamped and used for sighting in ranging rods on the section line.

Contouring by Hand Level. For more approximate contouring, which may often be quite good enough for the preliminary investigation of possible routes for a new road before a definite line is decided upon, a hand level may take the place of the more cumbersome dumpy, or similar, instrument.

Hand levels are made in various forms, but basically they all consist of a short sighting tube, usually without any optical aids for increasing the range of vision, the top of the tube having an aperture so that the indications of a small spirit level attached thereto may be reflected in an interior mirror fitted at an angle of 45° to the horizontal axis and visible through a small eye hole at one end. The horizontal position of the level is shown either by the symmetrical situation of the bubble about a central mark on the bubble tube or by the similar situation of the bubble with respect to a horizontal cross wire stretched across the front end of the sighting tube.

The use of the hand level, or "pocket reflecting level", as it is sometimes termed, simplifies the procedure in contouring very considerably and the general method described in the preceding pages is usually followed by first ranging out a series of section lines, running up and down the sloping ground in a direction perpendicular to a centre-line.

The hand level is normally used in conjunction with a simplified staff, of which there are several possible patterns, that shown in fig. 4.6

being eminently satisfactory. A staff of this type may be constructed of a single piece of well-seasoned deal, about ¾ inch thick and 3 inches wide, the usual height being 10 feet. The markings illustrated are painted in black on a white background, with red numerals.

The observer will find it convenient to steady the hand level against the side of a ranging rod which is held upside down, i.e. with its flat top on the ground and the level is placed at the mark on the rod which indicates a height of exactly 5 feet. By gently rocking the rod to and fro, a position will be found in which the reflected image of the bubble shows that the level is horizontal, and the cross wire then gives the staff reading.

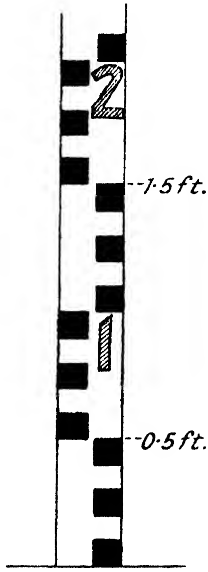


Fig. 4.6.—Contour Staff, reading to 0·1 feet.

The work is commenced by supporting the level horizontally in this manner above a point of known elevation, such as a centre-line peg. By adding 5 to the reduced level of the point we obtain the height of collimation and by subtracting the appropriate contour heights from the height of collimation we obtain the corresponding staff readings.

The following is a numerical example:

Reduced level of centre-line peg:	97·62
Height of level above point:	5·00
Height of Collimation:	102·62
say	102·6

Hence, staff reading 2·6 will give the 100 contour
" " " 7·6 " " " 95 "
and so on.

The staff holder, carrying the ring of the tape while the observer retains the case, proceeds along the section line until the required staff reading is obtained, when the distance is measured.

Assuming that contours are to be located at 5-foot intervals of height and that a 10-foot staff is being used, only the 100-foot contour on the uphill side of the centre-line can be obtained from the above position of the level; the staff holder remains at this point and the observer then proceeds along the section line until the level, held at a height of 5 feet reads exactly at the top of the staff. When this occurs the observer is at the next contour position, i.e. 5 feet higher than the staff position, and he then measures the distance from the previous point, thus locating the 105 contour.

The staff holder then moves forward until the observer, who has maintained his position meanwhile, obtains a reading at the bottom of the staff, indicating a further rise of 5 feet and thus locating the next contour.

This procedure, in so far as the numerical particulars are concerned, applies, of course, only in the case of 5-foot contours, a 10-foot staff and a height above ground of the hand level of 5 feet ; but the general principle applies equally to other height intervals and staff lengths and a closer contour interval will enable more contours to be located from one position of the level.

Booking Results. Hand Level Method. A simple method of booking is usually adopted with hand-level contouring by using an ordinary chain book with a centre column. The centre-line chainage is written in this column, accompanied by the reduced level of each point, and the contour heights and distances are written on the right- and left-hand sides of the centre column according to the side on which they occur in the field.

The following example, in which distances are entered immediately beneath the heights to which they refer, shows the booking for the contours in fig. 4.5 :

115 148	110 109	105 68	100 32	96·51 △ M 9 + 00	95 25	90 54
115 117	110 79	105 47	100 12	98·77 △ L 8 + 00	95 31	90 77
115 128	110 98	105 61	100 27	97·62 △ K 7 + 00	95 29	90 59

2. Contours from Spot Levels by Interpolation. We have seen that the object of spot levelling is to provide a number of points on a given area at each of which the reduced level is known and it is a simple matter to convert a spot levelled plan into a contoured plan by drawing in the contour lines to conform with the levels indicated at the points.

One method of doing this is to assume that the slope between each pair of spot levels is reasonably uniform and to locate any contours which may occur within the range of these levels by proportion. If the spot levels have been taken at well-chosen points, between which no sudden break of gradient exists, this assumption may not introduce any considerable errors in the location and configuration of the contours, but if the spot levels are inadequate in number and important points have been omitted, the contours will probably be very far from correct.

On the whole, unless the spot levelling has been carried out very thoroughly, the direct location of contours in the field, using a dumpy level, or similar instrument, is a more accurate method and from the point of view of quickness there is little to choose between the two procedures.

Simple mathematical interpolation may be illustrated by a numerical example :

Let the points A, B, C and D, in fig. 4.7, have the reduced levels indicated. Then it is evident that the 100-foot contour will occur between A and B, A and C and A and D, and assuming that the slopes

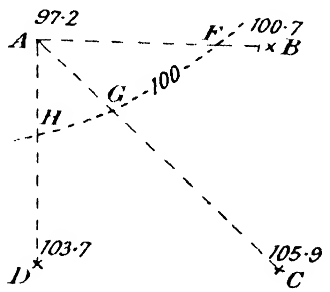


FIG. 4.7.—Contour interpolated from Spot Levels.

between these points are reasonably uniform, it may be located by proportion, thus :

Rise from A to B: 3·5 feet

„ „ A to 100 Contour: 2·8 feet

Hence, distance from A to 100 contour $= \frac{2\cdot 8}{3\cdot 5}$ of $AB = 0\cdot 8$ of AB

Rise from A to C: 8·7 feet

Hence, distance along AC to 100 Contour $= \frac{2\cdot 8}{8\cdot 7}$ of $AC = 0\cdot 32$ of AC

Rise from A to D: 6·5 feet

Hence, distance along AD to 100 Contour $= \frac{2\cdot 8}{6\cdot 5}$ of $AD = 0\cdot 43$ of AD

By setting off the lengths AF, AG and AH at these proportions of AB, AC and AD respectively, three points on the 100-foot contour are obtained.

Actually, it would be somewhat tedious to make these calculations for a large number of points and the contour lines are usually sketched in, the proportionate distances being judged by eye. An experienced cartographer, in fact, reviews the spot levels as a whole and does not consider them in individual pairs, thus breaking away from strictly mathematical proportioning and sketching the contours in accordance with the general trend which the spot levels indicate.

This has the effect, in certain circumstances which will be discussed later, of displacing the contours from the positions derived from purely numerical proportions.

Graphical Method of Interpolating Contours. Various graphical methods may be employed for locating contours from spot levels, one of the simplest, based on the assumption of uniform slopes between the points, being the following:

A sheet of tracing paper is ruled with parallel lines, spaced at any convenient constant interval and numbered in such a way that they represent the full range of heights of the spot levels to a scale which permits of estimation to a tenth of a foot.

Thus, supposing that the range of heights is from 200 to 240 feet, a suitable scale would be a tenth of an inch to each foot of height and the rulings would occupy a width of 4 inches numbered at every inch in the manner shown in fig. 4.8. The tracing paper is then placed over any two spot levels in such a way that the lines representing

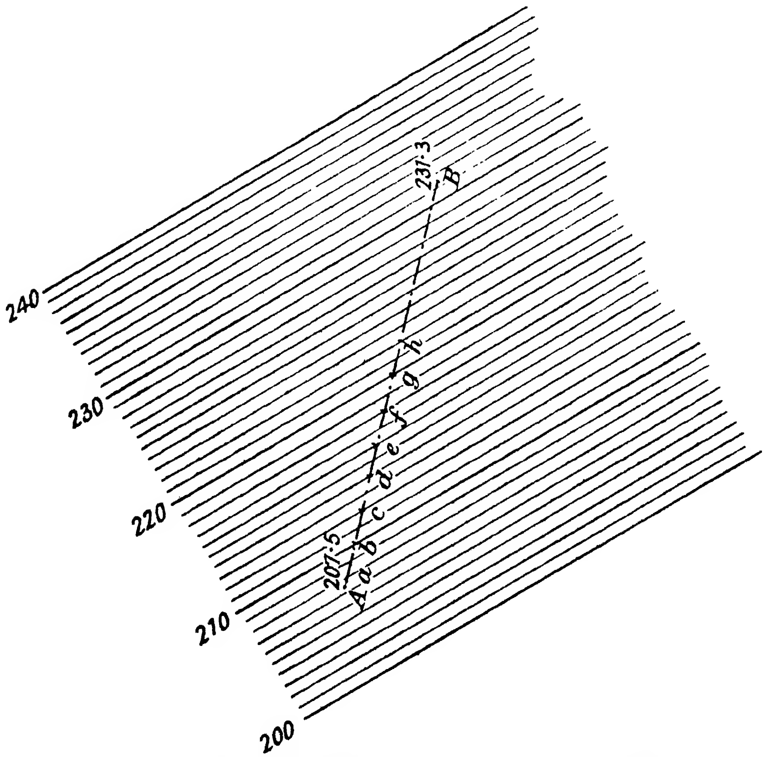


Fig. 4.8.—Graphical Method of Interpolating Contours.

their heights pass over the two points concerned. Any required contours can then be located by pricking through at the points where the appropriate parallel lines cross the line joining the spot levels. Thus, contours at even 2-foot intervals are located at the points *a*, *b*, *c*, etc., between *A* and *B*.

This provides a quick and handy method of contouring by proportion and is very frequently used.

"Adjustment" of Interpolated Contours. The methods of interpolation already described have their limitations and contours derived therefrom, even when based on adequate spot levels, are apt to be inaccurate where the sloping ground has a pronounced concavity or convexity, i.e. where there is a very definite change in the gradient associated with a rounded section between the two grades, as illustrated in fig. 4.9 where a gentle slope, *AB*, merges into a steep slope, *BC*.

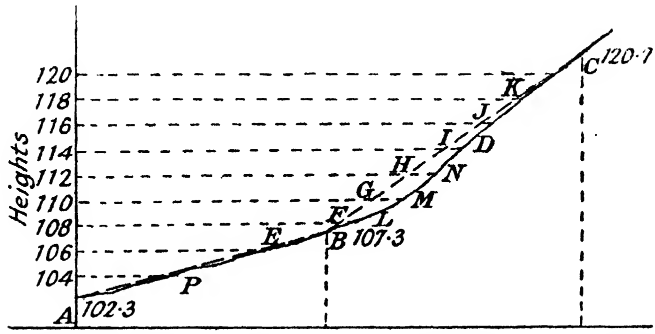


FIG. 4.9.—Comparison of Actual and Interpolated Contour Positions.

Let us suppose that spot levels are taken at *A*, *B* and *C*, and that they have the values shown in the diagram.

If we assume that the slopes between the successive points are uniform, the section of the ground becomes two straight lines, *AB* and *BC* and, by proportion, the contours at even 2-foot intervals would occur at the points *P* to *K*.

But except in the case of artificially battered banks, an abrupt change of gradient, such as that indicated by the junction of the two lines *AB* and *BC*, does not represent natural conditions and the point, *B*, in reality, will be somewhat indefinite in its situation. There will be a concavity in the neighbourhood of *B* and instead of a uniform gradient from that point to *C*, the slope will at first be more gentle than that indicated by the line *BC* and will then become steeper than the assumed uniform grade, the actual section being something like the curve *BDC*.

Hence the first few contours between *B* and *C* will be rather more widely spaced than those derived from simple proportioning, and those towards C will be rather more closely spaced, in conformity with the steeper slope in that vicinity.

In this way the points F, G and H, for instance, become displaced to the positions L, M and N.

A similar effect is produced at a convexity where a steep slope is rounded off into a gentle one.

Although approximate proportioning is an essential aid in fixing the general location of contours in relation to spot levels, it should be remembered, therefore, that when the latter indicate the presence of conditions such as those described above, a little adjustment of the contours by slight lateral displacement will give a closer representation of the actual facts.

This procedure can only be described as " intelligent guesswork ", but it is justifiable in the present instance since contours are inevitably an approximate representation of the levels of the ground and cannot give the precision of carefully surveyed sections.

Spot Levels by Plane-Tabling. In certain preliminary schemes it sometimes happens that contours are not required with the degree of accuracy obtainable by using a dumpy or quick-set level. In such cases they may be interpolated from spot levels derived by plane-tabling by means of a tacheometric alidade. This method is described on pages 179 *et seq.*

CHAPTER V

TRAVERSE SURVEYS

Application of Traverses. Methods of surveying which exclude the measurement of angles are only suitable for small surveys such as that of a road juction and its immediate surroundings, or of a short length of road sufficiently free from curves to permit of one survey line being used throughout.

When a plan is required of a longer stretch of road or of the proposed route for a new highway, the method adopted is the " traverse ", in which we measure a series of consecutive lines and also determine either the angles between them, or the angle which each makes with a fixed reference direction.

Thus the winding road shown in fig. 5.1, could be surveyed by the traverse *ABCDE*, in which the lines follow the general direction

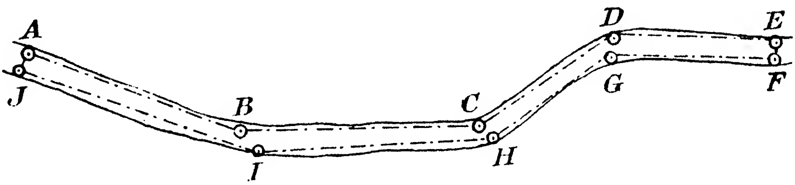


Fig. 5.1.—Traverse along a Winding Road.

of the road, but are kept to one side to avoid hindrance to and from traffic. Offsets would have to be taken across the road at intervals to fix the topography on the far side, but if the traffic was very heavy and there was considerable topographical detail on each side, a second traverse *FGHIJ* could be run, connected with the first at *EF* and *JA*, thus converting the first arrangement of lines, which forms an " open traverse " into a " closed traverse ". The latter, as will be seen later, forms a self-checking system which has certain advantages.

Again, when investigating the possible route for a new road the framework of the survey will consist of a series of lines forming an open traverse, following approximately the proposed line, but located where the local topography permits of clear sighting.

A further example of the traverse occurs in urban conditions where surveys for road improvement schemes involving the demolition of property are frequently required. In these cases it is often necessary to survey built-in blocks, bounded by narrow streets with no possible chance of arranging the survey lines to form the triangles which are essential in a chain survey.

A series of interconnected closed traverses, such as those shown in fig. 5.2, forms an excellent method of carrying out surveys of this type.

The measurement of the survey lines and also of the offsets and ties for fixing the topography is identical with the corresponding procedure in chain surveying, except that in a long traverse the chainage is generally run continuously, whereas in a chain survey

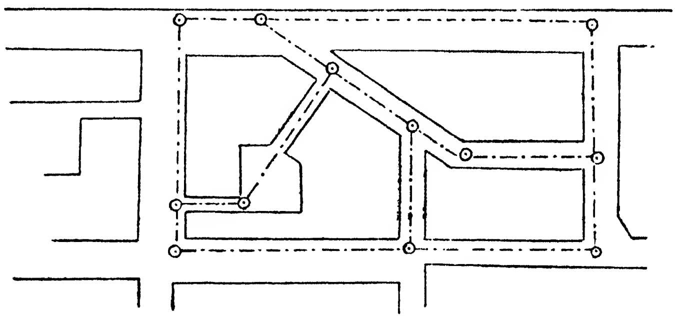


FIG. 5.2.—Traverses in Built-up Area.

and in small traverses the chainage of each line is measured separately. For this part of the work, therefore, the reader is referred to Chapter II on Chain Surveying, pages 4 *et seq.*

Measurement of Angles. The angles in a traverse may be measured in two ways:

(1) We may either measure the angles between the lines in accordance with a recognised routine, described later, deriving the so-called " Included Angles ", or

(2) We may measure the angle which every line makes, in turn, with a fixed reference direction, usually magnetic north, as given by the magnetic compass, in which case the angles so obtained are known as " bearings ".

The only practical instrument for accurately measuring traverse angles is the theodolite and it is important that its constructional details should be fully understood.

The Theodolite. The main features of a standard type of vernier instrument are shown diagrammatically in fig 5.3 (a). Fig. 5.3 (b) shows a similar instrument in detail and partly in section.

Referring to fig. 5.3 (a) the telescope, T, resembles that used for a level, but has two projecting shafts near the centre of its length which are supported in bearings each located at the apex of a triangular frame, A. These **A**-shaped supports are mounted on a circular plate, U, usually from 4 to 6 inches in diameter. This plate has a tapered spindle projecting downwards through a suitably tapered hole in the underframe, or levelling base, F, which is exactly similar to the three-screw base of a dumpy level. Beneath the circular

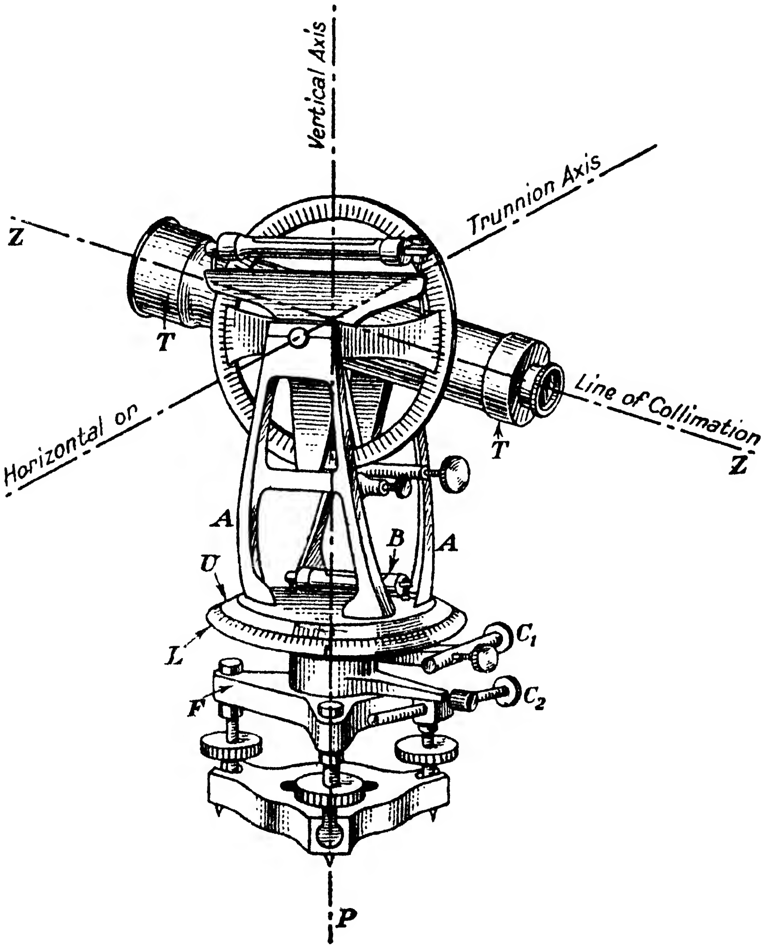


FIG. 5.3 (a).—Vernier Theodolite—Simplified Diagram.

plate, U, is another plate, L, of slightly larger diameter, which also has a bearing in the underframe concentric with that of the plate U. In fig. 5.3 (b) the A-frame is not attached directly to the upper plate, U, but to a cover plate, rigidly fastened to the upper plate and extending over both plates to exclude dust and damp.

These circular plates have bevelled edges and are fitted together with extreme precision so that the upper edge of the lower plate, *L*, and the lower edge of the upper plate, *U*, remain in perfect coincidence while the plates rotate freely, independently of one another. They may also be clamped together and rotated as one unit. The lower plate may be clamped to the body of the instrument.

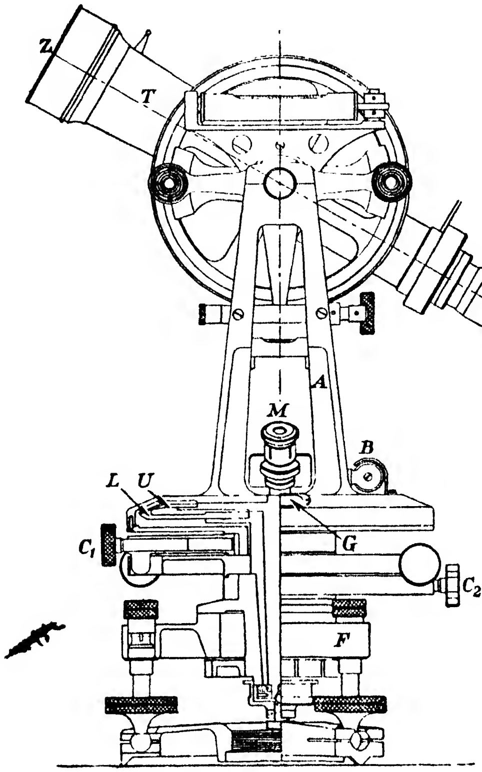


FIG. 5.3 (*b*).

The upper plate, *U*, carries on its periphery either two short vernier scales, or in the standard type of micrometer instrument two micrometer microscopes, located at the ends of a diameter at right-angles to the longitudinal axis of the telescope, being placed in this position for convenience in reading. In fig. 5.3 (*b*) the cover plate is provided with two glass-covered apertures immediately over the verniers. One of these is visible at *G*, with the magnifying eye-piece, *M*, above it.

The edge of the lower plate, *L*, is graduated clockwise completely round its circumference in degrees and fractions, the usual sub-divisions being halves or thirds for a vernier instrument, or sixths (i.e. 10 seconds) in a micrometer instrument.

The main circular scale and the vernier scales are usually of silver inset in the respective plates and are graduated automatically by a dividing engine. Makers of repute, realizing that the graduated circle forms a most vital part of the instrument, invariably ensure that their dividing engines give an extremely high standard of accuracy. Glass circles are used on certain modern instruments (see page 268).

In the case of vernier instruments, magnifying eye-pieces are fitted above each vernier to enable the fine graduations to be read easily. These eye-pieces are capable of slight lateral movement and can also be adjusted telescopically to give the correct focus and it is very important that they should be set vertically above the point of reading and accurately focussed to avoid parallax effects.

The principle of the vernier and the micrometer, as applied to the theodolite, will be discussed later.

The graduations on the circular scales are numbered at intervals, depending on the size and type of instrument, but increasing in a clockwise direction, with the zero marked " 360 ". An extension of the upper plate forms a dust-and-rainproof shield protecting the lower plate scale except in the vicinity of the verniers or micrometers.

Each plate is fitted with a clamp and a slow-motion, or " tangent " screw, the latter when turned producing a very slight rotation and thus enabling a very fine adjustment to be obtained. The tangent screws will not operate unless the appropriate clamp is slightly tightened.

The upper clamp, C_1, locks the upper plate to the lower and the lower clamp, C_2, locks the lower plate to the central boss of the underframe, i.e. to the body of the instrument. Four possible settings are thus obtainable :

(1) If the upper and lower clamps are both tightened, both plates are held stationary and the telescope cannot be rotated about the vertical axis of the instrument.

(2) If the upper clamp is released while the lower clamp remains tightened, the upper plate, carrying with it the verniers or micrometers, as the case may be, may be rotated while the lower plate remains stationary. The telescope also rotates about the vertical axis while the upper plate is turned and the reading of the horizontal circle changes while this movement is taking place.

(3) If the upper clamp is tightened and the lower clamp released the telescope may still be rotated about the vertical axis, but the two plates turn as one unit and consequently the reading remains constant.

(4) By releasing both clamps, both plates may be rotated independently and this setting has no practical application beyond the fact that it is advisable to provide complete freedom of movement in this way before packing the instrument in its box.

It may be mentioned here that this proceeding often requires a certain amount of knack. Straining and distortion of the instrument may easily result if force is applied.

In addition to the horizontal circles, a vertical circle is attached to the telescope and rotates with it as one unit when the latter is turned about its horizontal, or " trunnion " axis. This movement, when carried through 180°, or thereabouts, is called " transiting " the telescope and gives the names " transit theodolite " or " transit " which are sometimes applied to the instrument.

The arrangement for obtaining readings from the vertical circle is shown diagrammatically in fig. 5.4. An index arm, *I*, carrying two verniers or micrometer microscopes, has a central bearing concentric with the horizontal axis of the telescope and is also supported by the vertical arm, *J*, which is clamped to a cross piece or projection of the A-frame by the adjustable screws, *K*, known as the clip-screws. Alternatively the attachment may consist of an adjusting screw and a spring-loaded plunger. Movement of these screws alters the inclination of the centre-line of the index arm, the upper side of which is provided with a spirit level, *M*.

A second vertical arm, *N*, attached to the telescope, is clamped to the A-frame by a somewhat similar clipping arrangement incorporating a tangent screw which imparts slow motion to the telescope about its horizontal axis, tilting it up or down in a vertical plane. This latter arm is sometimes located on the same side of the telescope as the index arm and sometimes to the opposite side, as shown in fig. 5.4.

The telescope sometimes carries a spirit level, as in a dumpy level, though this is frequently omitted in modern instruments, but at least one spirit level is attached to the upper plate, as shown at *B* in figs. 5.3 and this is used when levelling up the instrument by means of the levelling screws located below the underframe, *F*.

Further attachments include a detachable plumb-line, *P*, adjustable in length and suspended exactly from the centre of the instrument and thus coincident, with its vertical axis.

The plumb-line is used for centring the instrument over a station and is apt to be deviated in windy weather, making accurate centring difficult. To obviate this, optical devices for centring are obtainable as a substitute.

A magnetic compass is another essential part of the equipment. On older instruments this is often circular in form and occupies the space on the upper plate between the telescope supports, but in modern theodolites the detachable trough-type is used and is either clamped to the side of the A-frame or secured in a grooved holder

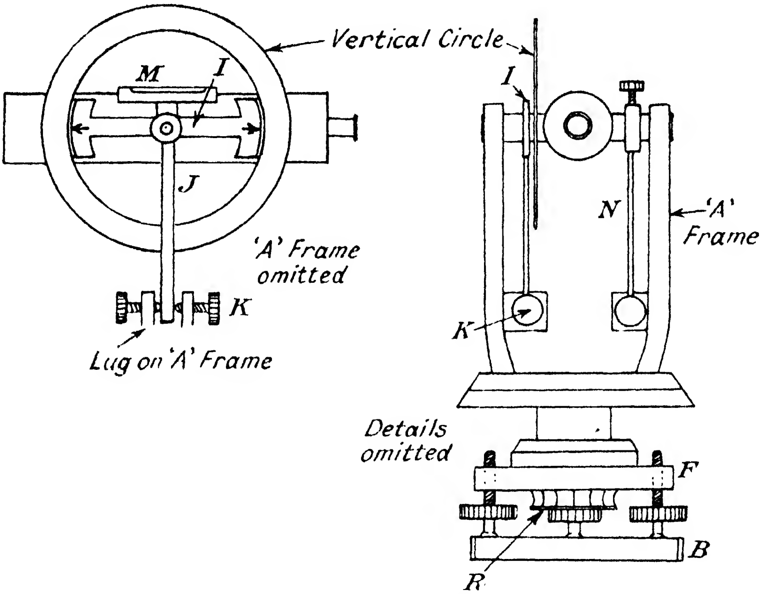


Fig. 5.4.—Attachment of Vertical Circle.

on the underside of the lower plate. In both cases, the axis of the trough is parallel to the longitudinal axis of the telescope.

Some form of centring adjustment is usually incorporated rendering it possible to move the instrument slightly in a lateral direction on the tripod head without altering the position of the tripod legs.

Desirable Features in a Theodolite. Many features are common to both theodolites and levels and the remarks on page 44 concerning the tripod, method of focussing the telescope, method of adjusting the eye-piece, provision of adjustment for the spherical ends of the levelling screws and protection of bubbles applies equally to both instruments.

Further points, however, include the following:

(1) *Unit Construction*

Some instruments are made with the telescope, vertical circle and index arm detachable from the A-frame and are packed with these parts dismantled, some of the larger instruments requiring two boxes. For general highway surveying, however, an instrument with a non-detachable telescope is preferable. This type packs into its box as one unit and in order that it may be retained therein securely, it is essential that adequate support should be given to the theodolite at suitable points, with very little clearance. For this reason it must always be replaced in the box exactly as intended by the makers and when using an unfamiliar instrument the position of the various parts relative to the pads and distance pieces should be noted carefully before it is unpacked. Simpler methods of packing are adopted in more modern instruments (see page 268).

The instrument may easily be strained if lifted out of its box and replaced therein carelessly. It should never be lifted by the telescope, the **A**-frame or the micrometer brackets, but by the underframe and base plate which screws on to the tripod.

(2) *Centring Device*

The device allowing slight lateral movement to the instrument on the tripod head may be located either (i) in the tripod head itself, (ii) above the tripod head, but below the levelling screws, or (iii) above the levelling screws. The third type is incorporated in the levelling base, or underframe, *F*, as shown in fig. 5.4 and is controlled by a large diameter locking ring, *R*, on releasing which the instrument may be moved about an inch in any direction from the centre. If the instrument has already been levelled, the underframe, *F*, will be horizontal and the lateral movement will not disturb the level to any extent, but with centring adjustments located below or within the base plate of the levelling screws, *B*, any lateral movement will be accompanied by a disturbance in level, unless the tripod head is set horizontally, and the necessary re-levelling, if at all extensive, will bring the plumb-bob off the reference mark.

Hence it is important to place the tripod head as nearly horizontal as possible with this type of centring device—a proceeding which is desirable, in any case.

Telescope Sights. External sights, both on the top and underside of the telescope are of great assistance when turning the instrument to bring a distant rod into the field of view.

Diaphragms. Glass diaphrams with stadia rulings are the most satisfactory. The latter should always be included to render the instrument suitable for tacheometric work.

Tacheometers. In discussing contouring by stadia methods with the level (pages 92 *et seq*) it was mentioned that an additive correction was not always involved in the calculation of the distance between the instrument and the staff. The elimination of the additive constant was formerly achieved by fitting within the telescope an additional lens defined by the term "anallactic".* In more modern instruments, however, an additional lens, provided with a means of longitudinal movement within the telescope, is commonly used for focussing. In

* Frequently spelt "anallatic" although the above spelling is now considered more correct.

such " internal focussing " instruments the telescope retains a constant length. It is difficult to combine an internal focussing lens with an " anallactic " lens, but the former may be so designed as to virtually eliminate the additive correction. The instrument may then be used for tacheometry, provided it has a suitable diaphragm, and a theodolite of this type is often called a " tacheometer ". This matter is referred to again on pages 163 and 164.

Clamps, Tangent and Adjusting Screws. These should be readily accessible and those for the upper and lower plates should be of distinctive forms so that they may be distinguished by a sense of touch, while the observer is looking through the telescope.

The telescope focussing screw should never be located on the top of the telescope, as it is on some instruments, since on transiting it will be on the underside and consequently very inaccessible. Some up-to-date instruments are fitted with a telescope focussing screw in the form of a large knurled ring at one end of the trunnion axis—an excellent position.

Micrometers and Vernier Eyepieces. It is desirable that these should be so located that the readings may be taken easily without the observer finding it necessary to remove his hat—a proceeding which is required with a large number of instruments.

Size and Accuracy. The classification of theodolites with regard to size is determined by the diameter of the graduated circle on the lower horizontal plate, the usual dimensions being 4, 4½, or 5 inches for vernier instruments and 5 or 6 inches for micrometer instruments. The former read to the nearest 30 or 20 seconds, although an accuracy down to single minutes only is sometimes adopted and is, indeed, sufficient for a great deal of work.

Micrometer instruments read directly to the nearest 10 or 5 seconds, with estimation to the nearest second, a degree of accuracy not required in road traverses, but called for in minor triangulation.

There has recently been a tendency to break away from the more or less standardized patterns to which theodolites have been constructed for many years, and completely re-designed instruments are now available which combine compact build and light weight with a greater accuracy that that obtainable with much heavier and bulkier instruments of the older type.

One of these is the " Tavistock ", designed by collaboration between the manufacturers and officials of the Ordnance Survey (see page 268). This is a micrometer instrument in which the horizontal circles have a mean diameter of only 3½ inches, but which reads directly to the nearest second and weighs only 13 pounds, as against 20 pounds which is the average weight of a standard 6-inch micrometer instrument reading directly to only 10 seconds.

For ordinary road traverses a vernier instrument reading to half-minutes or 20 seconds is quite satisfactory.

The Principle of the Vernier. The vernier consists of two scales reading edge to edge. One, which may be termed the main scale, is graduated in any required units and fractions, e.g. inches and tenths, and the other, which is the vernier proper, is graduated in such a way that n divisions thereon are equal in length either to $n - 1$ or $n + 1$ divisions on the main scale. The former ratio is more usual, i.e. in the above example, 10 divisions on the vernier would extend over nine-tenths of an inch.

The principle of the device may be understood by considering this simple example and assuming that the vernier scale, nine-tenths of an inch long, slides along the main scale, starting from a position where the two zero readings coincide as shown in fig. 5.5 (a).

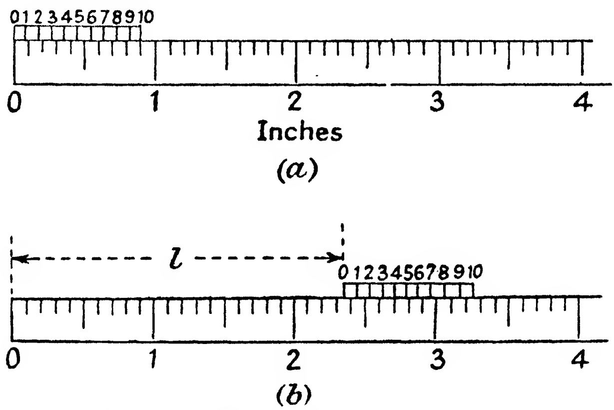


FIG. 5.5.—Simple Example of a Vernier.

It will be apparent in fig. 5.5 (a) that only two graduations of the vernier are in coincidence with graduations on the main scale, the 0 which coincides with the 0 on the main scale, and the 10 which coincides with the 0·9 inch on the main scale. All the other graduations are out of phase, commencing with the 1 on the vernier and the 0·1 on the main scale, which differ by 0·01 inch and the divergence increases by 0·01 at each subsequent pair of graduations.

Let the vernier be moved along to another position, as shown in fig. 5.5 (b). The zero of the vernier is now between 2·3 and 2·4 inches on the main scale and it is required to determine the length l through which the vernier has moved. It will be seen that only one of the vernier divisions coincides with a main scale graduation, i.e. the 4, and if we count back from this point to the zero of the vernier, we shall see that the 3 on the vernier is 0·01 inch out of phase with the corresponding main scale graduation, the 2 is 0·02 out of phase, the 1 is 0·03 out of phase and the 0 is 0·04 out of phase, i.e. the discrepancy between the division lines is increased by 0·01 inch at each vernier graduation, starting from the point of coincidence, and thus, if the vernier reading at this point was n, the displacement of the zero from the previous main scale reading would be n times 0·01 inch, or, in other words, the vernier reading at the point of coincidence gives the number of hundredths of an inch by which the sliding scale has been displaced beyond the previous graduation on the main scale.

Hence, the length l is 2·3 inches plus 0·04 of an inch, or 2·34 inches. It is very important to notice that the main scale graduation at the point of coincidence is entirely without significance and it will be observed from the above example that the vernier has enabled us to obtain a reading to a hundredth of an inch from a scale graduated to tenths only.

Application of the Vernier Principle to Theodolites. In applying the vernier principle to the theodolite, the main scale is represented by the graduated lower plate which reads in degrees and fractions, with the numbers increasing in a clockwise direction so that we count from right to left and not from left to right, as in the previous example.

The reading is obtained by first noting the graduation on the main scale immediately before the zero of the vernier and then adding to this the vernier reading at the point of coincidence.

An arrow marks the zero or reference mark on each vernier and one or two additional graduations are provided to the right of this arrow and to the left of the last numbered graduation at the opposite end of the scale. The object of these additional graduations is to assist in forming an accurate idea of the exact point of coincidence when it occurs near either end of the vernier. The displacement of the vernier division immediately to the right of the point of coincidence will be to the left of the nearest main scale graduation and the displacement of the vernier division immediately to the left of the point of coincidence will be to the right of the nearest main scale graduation. Hence the point of coincidence can be located most accurately by determining which two divisions give equal, but opposite, displacements and taking the division midway between them.

The method of graduating the main circle and the verniers depends on the sub-divisions of the former and on the fineness to which the latter are required to read. Typical types will now be discussed.

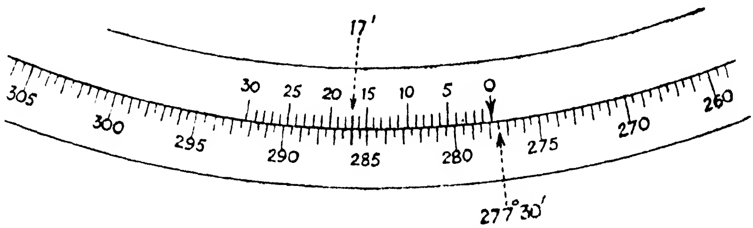
Single Minute Reading.

(1) *Degrees on main scale divided into halves* (Fig. 5.6.)

The vernier is numbered from 0 to 30 and has no sub-divisions. 30 vernier divisions equal 29 main scale divisions, hence, ignoring the additional graduations beyond the 0 and 30 numberings which would occur in practice but are not shown in the diagrams, the vernier extends over 29 × 30 minutes, or 14° 30′ on the main scale. The reading for the setting shown in fig. 5.6 is 277° 30′ from the main scale plus 17′ from the vernier, i.e. 277° 47′.

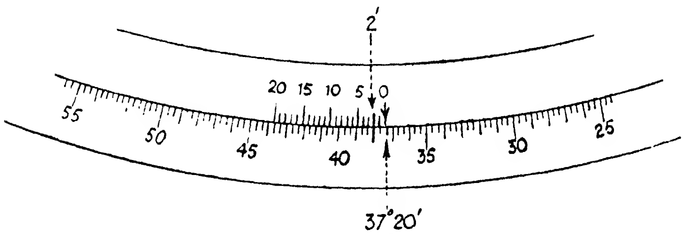
(2) *Degrees on main scale divided into thirds* (fig. 5.7.)

The vernier is numbered from 0 to 20 and has no sub-divisions.



Reading: 277° 47′

FIG. 5.6.



Reading: 37° 22′

FIG. 5.7.—Examples of Verniers.

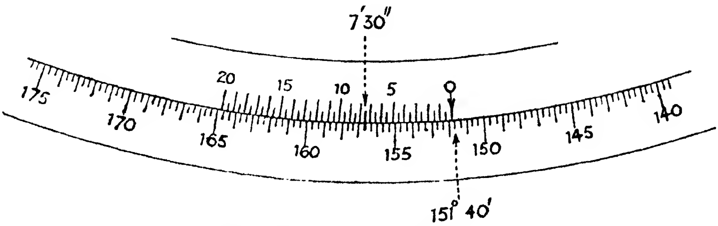
20 vernier divisions equal 19 main-scale divisions; 1 main scale division = 20 minutes. Hence, ignoring the extra graduations at the ends, the vernier extends over 19 × 20 minutes, or 6° 20′ of the main scale.

The reading for the setting shown in fig. 5.7 is 37° 20′ from the main scale plus 2′ from the vernier, making 37° 22′. By sub-dividing the vernier graduations into halves or thirds, the fineness of reading becomes half-minutes, or 20 seconds respectively, but the length of the vernier requires alteration, thus:

Half-minute Reading. (Fig. 5.8.) Main scale divided into degrees and thirds. Vernier graduations numbered 0 to 20 and sub-divided into halves. Total number of vernier divisions, ignoring the extra graduations at the ends, is 40 and this length is equal to 39 main-scale divisions each representing 20 minutes. Hence the vernier extends over 780′ or 13° of main scale.

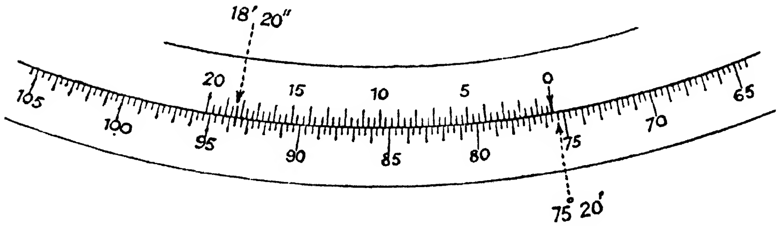
Twenty-second Reading (fig. 5.9.) Main scale divided into degrees and thirds. Vernier graduations numbered 0 to 20 and subdivided into thirds. Total number of vernier divisions, ignoring the extra graduations at the ends, is 60 and this length is equal to 59 main-scale divisions, each representing 20 minutes. Hence the vernier extends over 59 × 20 minutes, or 19° 40′ of the main scale.

Examples of half-minute and twenty-second verniers are shown



Reading: 151°47′30″

FIG. 5.8.



Reading: 75°38′20″

FIG. 5.9.—Examples of Verniers.

in figs. 5.8 and 5.9, the respective readings being 151° 47′ 30″ and 75° 38′ 20″.

It will be observed that when the main scale is graduated in half-degrees, the vernier divisions are numbered 0 to 30 and where the main scale is graduated in 20-minute intervals the vernier divisions are numbered 0 to 20.

To set the Vernier to Zero. Clamp the lower plate. Release the upper plate and rotate until the zero is set approximately correctly. Tighten the upper plate clamp and bring the reading exactly to zero by the upper plate tangent screw.

Instrumental Errors with Vernier Theodolites.

Although the better-class instruments are made with scrupulous care, the possibility of slight instrumental errors must not be overlooked and it is as well that their causes and the methods of correction should be understood.

Eccentricity of Horizontal Plates. The verniers should be located with their zeros diametrically opposite, consequently the readings at any setting should differ by exactly 180°.

In some instruments it will be found that this constant difference of 180° is not obtained, but the discrepancy may be one of two kinds :

(1) the difference between the vernier readings at various settings may be constant, though not exactly 180°, or

(2) the difference between the vernier readings at various settings may not even be constant.

In the first case, the error is due to the fact that the zeros of the verniers are not located exactly at the ends of a diameter, one having a slight, but constant difference of phase relative to the other.

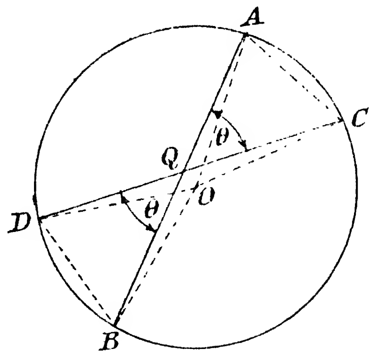


FIG. 5.10.—Eccentricity of Horizontal Plates.

This error may be eliminated by always reading the same vernier and either will give the correct value of the angle by taking the difference of two successive readings.

In the second case, however, a lack of uniformity in the differences may indicate that the upper and lower plates do not rotate about the same vertical axis, or, in other words, that the error is due to eccentricity.

Thus, in fig. 5.10, let O represent the centre of the lower plate carrying the graduated circle and let Q represent the centre of the upper plate carrying the verniers and the telescope. Then, if A and B are the positions of the verniers corresponding to a given position of the telescope and if the latter is turned through an angle θ, the verniers will also move through an angle θ about Q as centre, taking up the positions C and D respectively. Hence, the angles AQC equals the angle DQB, both equalling θ, but the angles indicated on the graduated circle, during this movement will be AOC and DOB, neither of which is equal to θ, the circle having been graduated about O as centre. But since the angles of a triangle sum to 180°,

$$\begin{aligned}
\theta &= 180^\circ - (QCA + QAC) \\
&= 180^\circ - (OCA - OCQ + OAC + OAQ) \\
&= [180^\circ - (OCA + OAC)] + OCQ - OAQ \\
&= AOC + OCQ - OAQ
\end{aligned}$$

also

$$\begin{aligned}
\theta &= 180^\circ - (QBD + QDB) \\
&= 180^\circ - (OBD - OBQ + ODB + ODQ) \\
&= [180^\circ - (OBD + ODB)] + OBQ - ODQ \\
&= DOB + OBQ - ODQ
\end{aligned}$$

Hence, by addition,

$$2\theta = AOC + DOB + OCQ - OAQ + OBQ - ODQ$$

But $$OCQ = ODQ \text{ and } OAQ = OBQ$$

Therefore $$\frac{AOC + DOB}{2} = \theta$$

Hence, by taking the mean of the two vernier readings, we obtain correctly the angle through which the telescope has turned and eliminate the eccentricity error.

Errors due to Graduation of Circle. Errors due to imperfect graduation of the circles are likely to be extremely small in any good instrument and in traverse work no special precautions need be taken to eliminate possible microscopic discrepancies due to this cause, but in the accurate reading of angles in a triangulation survey the possibility of error is dealt with in the manner described later (page 251).

Application of the Micrometer to the Theodolite

Although the majority of instruments used for road traverses are of the vernier type, micrometer theodolites are also employed and a description of their working principle is therefore necessary.

Instead of verniers, two micrometer microscopes are provided for reading the horizontal circle and two for the vertical circle, the former being mounted on the upper plate and rotating with it and the latter being mounted on the index arm.

The micrometer microscope consists of an objective and an eye-piece between which are located a pair of movable cross wires and a reference mark, the latter giving the reading in conjunction with the graduated scale on the lower plate of the theodolite.

The objective is a lens system at the end of the microscope nearest to the graduated circle and it forms an image which is viewed by the magnifying eye-piece, much in the same manner as the image formed by the object glass is viewed by the eye-piece in a level or theodolite telescope.

The objective is focussed on the graduations by moving the entire microscope tube in its mounting but this is a "permanent" setting and not a temporary adjustment. White porcelain reflectors are usually fitted to illuminate the circular scale and thus obtain a bright image.

Between the objective and the eye-piece is a sliding diaphragm, carrying two closely spaced parallel cross wires which may be moved sideways by turning the micrometer screw. These cross wires are accurately focussed by means of the eye-piece, exactly as the cross wires in a level or theodolite telescope, to eliminate parallax, and the pitch of the micrometer screw is so proportioned that one complete revolution moves the cross wires from one graduation on the circular scale to the next.

The movement of the micrometer screw is recorded by a graduated drum with a fixed index mark mounted at the side and the reference

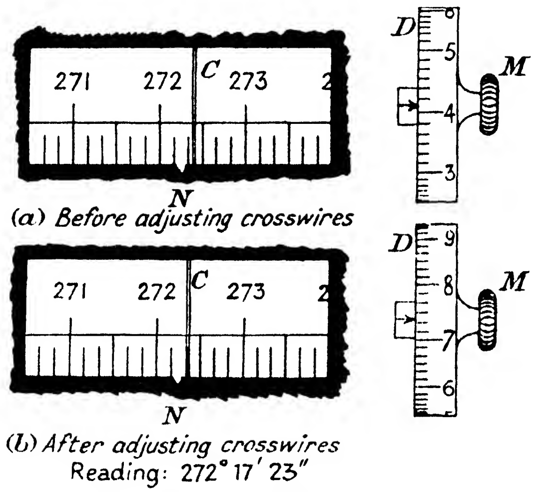


FIG. 5.11.—Principle of Micrometer Theodolite.

mark for giving the reading on the circular scale usually takes the form of a V-notch located in front of the plane of the cross wires which shows up clearly on a dark background against the edge of the bright graduated circle. This notch replaces the arrow-head which forms the reference mark on a vernier.

Fig. 5.11 represents diagrammatically the view of the graduated circle, the parallel cross wires, C, and the notch, N, as seen through the micrometer eye-piece, together with the micrometer screw, M, and the micrometer drum and index, D, which are mounted at the side.

The first part of the reading is obtained from the position of the notch with reference to the graduated circle and owing to the fact that an inverted image is obtained in the microscope, the numbered graduations appear to increase from left to right, instead of from right to left, as in the vernier instruments.

Thus, in the example illustrated, the notch is between 272° 10′ and 272° 20′. The micrometer drum has ten numbered divisions each sub-divided into six parts and one complete revolution of the drum carries the cross-wires from one graduation of the circular scale to the next, i.e. through 10 minutes. Hence the numbers on the drum represent minutes and a movement of the micrometer screw corresponding to one sub-division on the drum represents a movement of the cross wires of one-sixth of a minute or 10 seconds. It is quite possible to estimate a fractional part of a drum division, by eye, down to a tenth, i.e. to a single second.

When the notch *N* is exactly midway between the cross wires the micrometer drum, if properly adjusted, reads zero and to obtain the reading for any setting, the cross wires are moved from the notch to either of the circle graduations immediately adjacent, the division line on the circle occupying a position exactly midway between the two parallel wires.

The extent of this movement is recorded on the micrometer drum and is added to the reading given by the notch.

In the example shown, the micrometer reading after adjustment, is 7 minutes 23 seconds, the minutes and nearest 10 seconds being given by the drum graduation and the additional 3 seconds by estimation. Thus the total reading is 272° 17′ 23″.

Had the cross wires been moved to the circle graduation on the other side of the notch, the micrometer screw would have been rotated in the other direction, but would have arrived at the same recording, if in proper adjustment.

It is usual, in accurate work, to read on to both the graduations adjacent to the notch and take the mean of the micrometer readings.

To set the Circle Reading to zero with the Micrometer. Move the cross wires until the notch *N* is exactly midway between them. The reading on the drum should now be zero, but if it is not, grip the milled head of the screw *M* and rotate the drum independently until the zero reading is obtained.

This movement is permitted by the fact that the recording drum is only friction tight on its spindle.

Unclamp the upper plate and rotate it until the 360° reading is nearly reached, obtaining the final accurate setting, when the 360° graduation will be midway between the cross wires, by means of the upper clamp and tangent screw.

Temporary Adjustments of the Theodolite

1. Centring. The instrument must be set up exactly over a nail head or the centre of a pencilled cross on the top of the station peg, the tripod being placed so that the plumb bob is approximately in its correct position with the levelling base approximately horizontal. Hence an eye must be kept both on the centring and the level and it will be found easiest and quickest to adjust the tripod by moving two legs simultaneously, instead of attempting to alter the position of each leg separately.

The accurate centring can then be carried out by means of the sliding attachment common to most instruments which permits of some slight lateral movement of the theodolite on the tripod, but it is useless to centre precisely unless the spirit level on the upper plate indicates that this part of the instrument is practically horizontal, otherwise the levelling process will throw the plumb-line out of centre. The centring must be checked from two positions at right-angles to one another.

In windy weather the plumb-line and bob must be sheltered in some way or a pronounced deflection may be caused. The length of the cord must be adjusted so that the bob is just above the reference point. A means of adjustment is provided by threading the cord through two holes in a small disc, and forming a loop which can be varied in length by sliding the disc up and down. A button will serve quite well instead of the orthodox type of disc. An optical centring device is fitted to some instruments, obviating the use of a plumb-line.

2. Levelling. The process of levelling up the instrument is precisely similar to the procedure with a three-screw dumpy level. The plates are unclamped and the spirit level on the upper plate is placed parallel to any pair of levelling screws. These are rotated, moving the thumbs both in or both out, until the bubble is central. The upper plate is then turned through 90° bringing the level over the third foot-screw and the bubble centred by this screw alone, the telescope is then returned to its original position and a slight re-adjustment made, if necessary.

The upper plate is finally rotated through 180° and the bubble should remain central.

If there are two spirit levels of unequal length on the upper plate the longer should be used for this adjustment and if there is a marked deviation of the bubble from the mid-position on reversing end for end, a " permanent " adjustment is required.

It may be mentioned, however, that no perceptible error in reading a horizontal angle will be caused by a slight imperfection in levelling up the instrument, provided that the lines of sight are not inclined at widely different angles to the horizontal.

After levelling, the centring should be re-checked and both processes should be repeated, if necessary. Accurate centring is vitally important, especially with the short traverse lines which are often necessary when surveying a winding road.

3. Optical Adjustments. The adjustment of the eye-piece to give sharp definition of the cross-wires and eliminate parallax, and the focussing of the telescope on the distant rod are precisely similar to the corresponding adjustments when levelling, already described on page 58.

When sighting on to the rod, the telescope should be tilted by the vertical motion tangent screw until the base of the rod is bisected by the vertical cross wire or intersected symmetrically by both cross wires if the latter are of the diagonal type.

A rod painted with red and white bands only is preferable to one painted red, white and black, since the cross wires are invisible against a black background.

In a vernier instrument the magnifying eye-pieces for obtaining the circle readings must be adjusted laterally so as to be vertically over the appropriate graduations on the horizontal circle or exactly opposite the graduations on the vertical circle and the telescopic adjustment must be set to give the clearest possible view of the scales.

The corresponding adjustments in a micrometer instrument have already been mentioned on page 116.

"Permanent" Adjustment of the Theodolite

The conditions which should be satisfied in a well-adjusted theodolite may be summarised as follows:

(1) The plates should rotate in a horizontal plane about a common vertical axis. This is termed "Rotation in azimuth."

(2) The line of collimation of the telescope should be at right-angles to the horizontal or trunnion axis. (Lateral Collimation).

(3) The trunnion axis should be truly horizontal.

(4) The line of collimation should be horizontal when the vertical circle reads zero (or 90°, according to the method of numbering). The bubbles of the index arm and/or telescope spirit levels should then be central. (Vertical Collimation.)

It is very important that the permanent adjustments should be carried out in the order given below.

1. Plate Level or Vertical Axis Adjustment. Clamp the lower plate and leave the upper plate free. If there are two spirit levels of unequal length on the upper plate place the longer parallel to two levelling screws and bring the bubble to the centre of its run by these screws. Turn the upper plate through 90° and bring the bubble central by the third screw. Return to original position and re-adjust, if necessary. Turn upper plate through 180°, thus reversing spirit level end for end. If bubble moves from centre, correct half the displacement by the two levelling screws and half by the capstan nuts at the end of the bubble tube. Repeat in a direction at right-angles to first position, if necessary. If there is a level on the telescope or the index arm of the vertical circle this will be more sensitive than the plate level and the following method of adjustment may be used as an alternative to that given above: If the telescope level is to be used, first set the vertical circle to the reading corresponding to the horizontal position of the telescope, i.e. to 0° or 90° according to the method of graduating vertical circle. Level up the instrument as accurately as possible by the plate level and place the telescope or index arm level parallel to two levelling screws. If the bubble is not central correct it by the clip screws. Reverse the spirit level, end for end, by turning the upper plate through 180° and if the bubble does not remain central, correct half the displacement by the clip screws and half by the two levelling screws. Turn the upper plate through 90° and correct any displacement entirely by the levelling screws; then through 180° and half by each, as before. Repeat this procedure until adjustment is perfect.

If the plate levels are not central on completing the above adjustment correct them by the capstan nuts at the ends of the bubble tubes.

If the upper plate is now clamped and the lower plate released the bubbles should remain central when the plates are rotated. If they do not the instrument can only be rectified by the makers.

2. Lateral Collimation Test and Adjustment. In order to discover whether the line of collimation is at right-angles to the horizontal, or trunnion, axis, it is first necessary to ascertain whether the horizontal and vertical cross-wires are set truly in these directions when the instrument is carefully levelled.

This may be done by directing the telescope to some well-defined point, such as the intersection of two pencilled lines on a distant wall, and bringing one end of the horizontal cross wire exactly on to the reference mark. The upper plate tangent screw is then turned so that the mark appears to travel along the cross wire. If it does not maintain its position thereon correctly, the diaphragm must be rotated very slightly, after slackening the four bolts which hold it in place in the telescope tube, until the reference mark remains coincident with the cross wire across the entire field of view.

The diaphragm bolts are then re-tightened very carefully. The vertical cross wire may be checked by focussing it against a plumb-line, but if these two tests, in conjunction, show that the cross wires are not at right-angles to one another, the diaphragm must be changed, or, if of the web type, one or other of the webs must be removed and replaced more accurately. The method of carrying out this procedure is described on page 122.

The lateral collimation test is carried out as follows: The instrument is set up about midway between two levelling staves supported horizontally at such a height that they may be sighted by the telescope

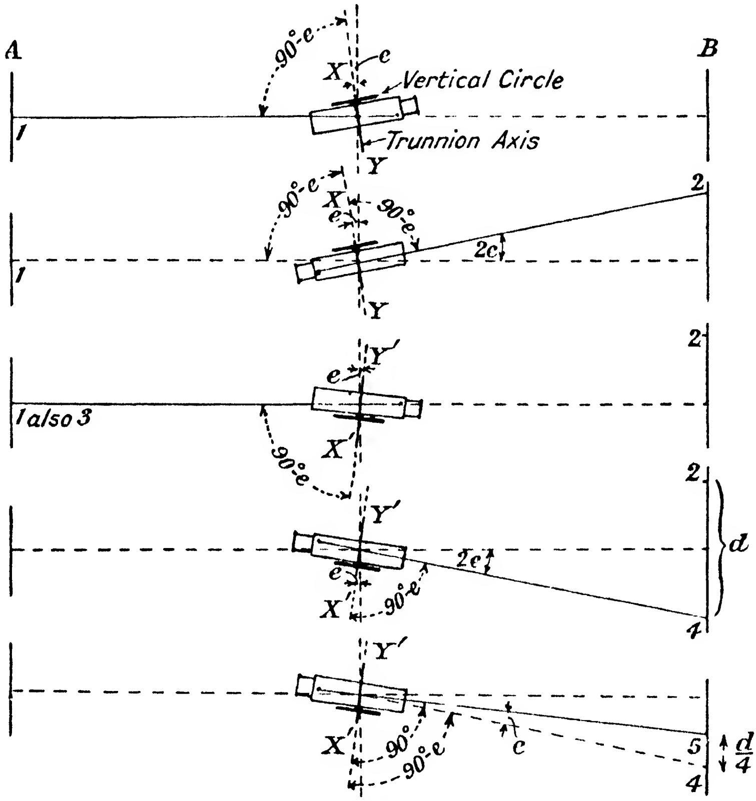


FIG. 5.12.—Test for Lateral Collimation Adjustment.

without requiring more than a very slight displacement of the latter from a horizontal position.

The general arrangement is shown in plan view in fig. 5.12, an error e in the line of collimation being assumed, or in other words, the line of collimation is inclined at 90°-e to the trunnion axis XY.

Both the theodolite and the staves are carefully levelled, the lower plate is clamped, the telescope is directed to one staff, e.g. A, the upper plate is clamped and the staff reading at point 1 noted, using the vertical cross wire.

The telescope is then transited, i.e. rotated about the trunnion axis, and the reading on staff B is noted at point 2. As the diagram shows, the angular difference between the second line of sight and the first produced backwards will now be $2e$, since the line of collimation will still be inclined at $90-e$ to XY which has remained stationary.

The upper plate is now released and the telescope turned about its vertical axis until the vertical cross wire again reaches the previous reading obtained on staff A, the accurate setting being obtained by the upper plate tangent screw. The trunnion axis will then take up the position $X'Y'$.

The upper plate is then clamped and the telescope again transited, giving a second reading on staff B, at point 4.

If the line of collimation had been perpendicular to the trunnion axis of the telescope, the two readings on staff B would have been identical, but assuming an initial angular error e, the effect of the foregoing procedure is to quadruple the error, thus making the angular difference between the two lines of sight to the staff B, $4e$. The difference between the readings gives a numerical value d to the collimation error and since the initial discrepancy is only one-fourth of this exaggerated value, the correction is applied by bringing back the line of sight one-quarter of d, to point 5.

This is done by careful manipulation of the adjusting screws of the diaphragm. After carrying out this adjustment, the test should be repeated and it is as well to make sure that the diaphragm has not been rotated inadvertently, thus disturbing the horizontal and vertical setting of the cross wires.

3. Trunnion Axis Test and Adjustment. In order to ascertain whether the trunnion axis is horizontal, a levelling staff is laid horizontally on the ground at such a distance that it can be read through the instrument and after carefully levelling by means of the plate level, the telescope is elevated to bring the central intersection of the cross wires into exact coincidence with a well-defined point at a high altitude, such as the spike of a lightning conductor.

The plates are clamped at this setting and the telescope is then tilted downwards and the reading given by the vertical cross wire on the staff is noted. If the trunnion axis is horizontal the telescope will have moved in a vertical plane during this procedure.

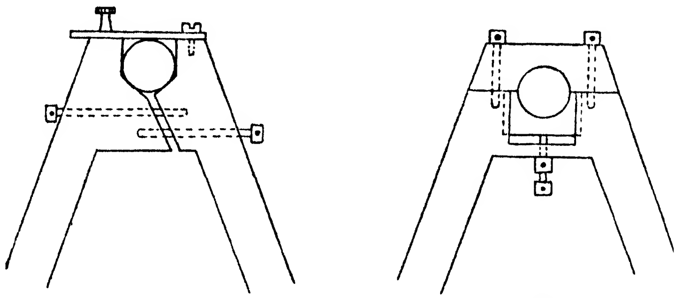
The upper plate is now unclamped and rotated through 180° and the telescope is transited and again centred on the elevated point. The upper plate is clamped at this setting and the telescope tilted downwards to obtain a second reading on the staff, which should be identical with the first if the telescope has again moved in a truly vertical plane.

If the trunnion axis is not horizontal the telescope will swing down to a different reading. The axis is adjusted by raising or lowering one end until the reading obtained on the staff is the mean of the two already noted.

The exact method of carrying out this adjustment will depend upon the construction of the instrument.

In some types, in which the telescope is detachable, the latter is supported in a recess at the top of the A-frame which can be opened or closed by manipulating two small capstan-headed bolts located at the side, thus lowering or raising the trunnion axis. Fig. 5.13 (*a*) shows this method of construction and is self-explanatory.

In instruments in which the telescope is not designed to be readily detachable, an adjusting bolt with a capstan-head is sometimes located beneath the trunnion axis bearing, as shown in fig. 5.13 (*b*) and any



(*a*) (*b*)

FIG. 5.13.—Trunnion Axis Mountings.

necessary adjustment of the axis can be made by slightly slackening the bolts which hold the bearing cap and turning this capstan-headed bolt, after releasing the lock-nut.

Instead of using levelling staves for the collimation and trunnion axis tests, reference marks may be made on two opposite walls for the former and on a sheet of paper or a board placed on the ground within the focussing range of the telescope for the latter. This procedure may be found preferable for instruments equipped with diagonal cross wires.

4. Vertical Collimation Adjustment. This adjustment is not required in traverse surveying since we are only concerned with horizontal angles in this type of survey, but it is mentioned here in order to make the details of theodolite adjustments complete.

The object aimed at is to ensure that the line of sight of the telescope is horizontal when the vertical circle reads zero and the bubble of the index arm spirit level is at the centre of its run.

The instrument is levelled up carefully, using the plate level and main levelling screws and the bubble of the index arm spirit level is brought to the centre of its run by means of the clip screws which, it will be remembered, alter the position of the verniers.

The telescope is now set by means of the vertical-motion tangent screw to give a reading of zero on the vertical circle and the reading of the central horizontal cross wire is taken on a levelling staff held vertically at a convenient distance, say 300 feet, away.

The upper plate is now unclamped and rotated through 180°, the telescope is transited and the bubble of the index arm spirit level again brought to the centre of its run by the clip screws, if necessary.

The vertical circle is again set to read zero by the vertical motion tangent screw and a second reading obtained on the staff. If it differs from the first reading, the mean of the two will correspond to a horizontal line of sight and the cross wire is brought to this reading by the vertical tangent screw. This will disturb the zero reading which is re-set by the index arm clip screws keeping the vertical circle and telescope clamped at the mean reading on the staff. This, in turn, will disturb the index arm bubble which is finally brought to the centre of its run by adjusting the capstan nuts at one end. If there is a telescope spirit level and its bubble is not central after making the above adjustment this bubble is corrected by the level tube capstan nuts, taking care that the cross wire remains at the mean reading on staff.

Reading with " Both Faces ". A theodolite is said to be reading with " right " or " left face " according as to whether the vertical circle is on the right or left side of the telescope when viewed from the eye-piece end. If the telescope is reversed end for end by rotation about the trunnion axis and the upper plate is rotated through 180° about the vertical axis the face of the instrument is reversed. This process is called " transiting and reversing face " and has the effect of automatically eliminating certain errors due to maladjustment of the instrument which will now be discussed in detail.

Except in the unlikely event of such errors being very large, their elimination by this method is far preferable to the tedious methods of permanent adjustment previously described.

Effect of Reversing Face on Vertical Angles. Assume that the line passing through the zeros of the vertical circle verniers is horizontal but that the line of collimation is inclined upwards at an angle α when the telescope is set at the zero reading with face left, as in fig. 5.14 (a). When the telescope is transited through 180° the line of collimation will be inclined downwards at an angle α, as in fig. 5.14 (b). On rotating the upper plate through 180° this inclination will be maintained, as in fig. 5.14 (c). Let P represent an elevated point at a true elevation β. If the telescope is directed to this point with face left, the angle turned through by the vertical circle and recorded by the vernier will be $\alpha - \beta$ and with face right the angle will be $\alpha + \beta$. The mean of the face left and face right readings gives the correct value β, thus eliminating the error in vertical collimation.

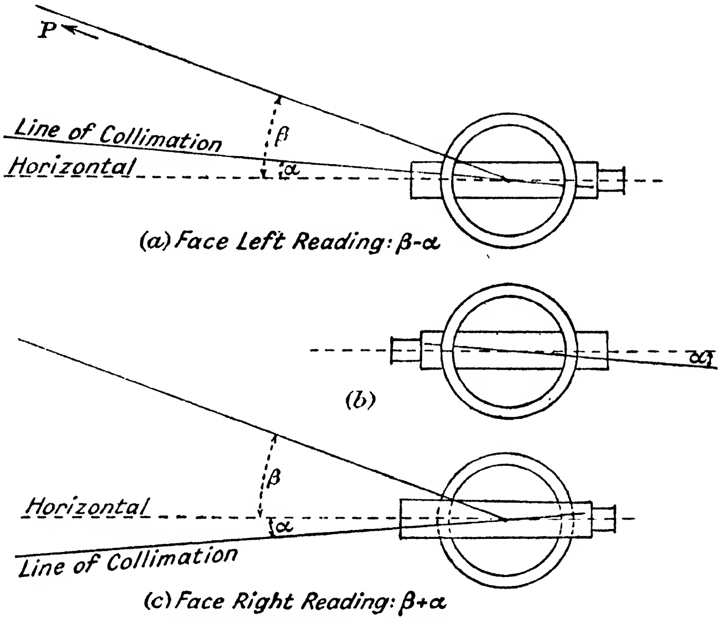


FIG. 5.14.—Effect of Reversing Face on Vertical Angles.

Effect of Reversing Face on Lateral Collimation Error. In reading the angle between two rods which are almost at the same level, the fact that the line of collimation is not at right-angles to the trunnion axis will not cause any measurable error in the angle, but if the rods are so located that the angle of elevation when sighting one of them is widely different to the angle of elevation when sighting the other, an appreciable error may be introduced, since the line of collimation, in rotating about the trunnion axis, will not describe a vertical plane, but a cone.

This may be visualised from fig. 5.15, in which T represents the theodolite, A is a rod to which an approximately horizontal sight is obtainable and B is a rod at a much lower level. Consider the vertical lines AA' and BB' passing through the rods and OO' passing through the plumb-line of the instrument. These three lines define two vertical planes $ATO'A'$ and $B'TO'B$, somewhat resembling the adjacent pages of an open book standing on edge and the correct value of the angle between the theodolite and the two rods is, of course, the horizontal angle θ between these planes. If oa and ob are horizontal lines in the planes $ATO'A'$ and $B'TO'B$, respectively, $\theta = \angle aob$. If the line of collimation TA be swung round without altering the tilt of the telescope to intersect the line BB' it will assume the position TC, the angle ATC being θ, for all practical purposes, but if the collimation is imperfectly adjusted laterally, in tilting the telescope downwards to view the rod B, the line of sight will not remain in the vertical plane $OB'BO'$, but will move outwards, either to D or to E, according to which side of the longitudinal axis of the telescope the collimation line deviates, the latter sweeping out the surface of a cone as the trunnion axis rotates instead of a vertical plane. Suppose it assumes the position D. The circle reading will remain at the value θ, but the cross wires will have to be moved

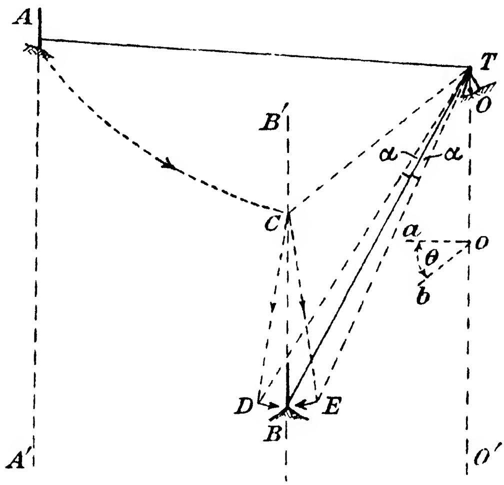


FIG. 5.15.—Effect of Reversing Face on Lateral Collimation Error.

counter-clockwise from D to B through an angle α, so that the recorded angle will be $\theta - \alpha$.

If, however, the telescope is transited and the upper horizontal plate swung round to repeat the reading of the angle with the reverse face, the displacement of the line of collimation will now be to the point E, with the circle still reading θ and in order to bring the cross wires on to the rod B, the telescope must be turned clockwise through an angle α, so that the reading will now be $\theta + \alpha$.

Hence, by taking the mean of the right and left face readings, we obtain the correct value of the angle.

Effect of Reversing Face on Trunnion Axis Error. If the trunnion axis is not truly horizontal, the line of collimation will not describe a vertical plane when the telescope is rotated and a similar effect will be produced to that illustrated in fig. 5.15, but the mean of the right and left face readings will again give the true value of the horizontal angle.

When the instrument is set up over a station, the process of reversing face and obtaining a second reading takes up very little additional time and when " included angles " are being measured it is well worth reading with both faces, thus eliminating the two errors mentioned above.

Included Angles. It has been mentioned already that " Included Angles " are measured according to a standardised routine and the purpose of keeping strictly to the recognised procedure will be clear from a consideration of the following example :

Let *AB*, *BC*, and *CD*, in fig. 5.16 represent three traverse lines. Then at each of the junction pegs, *B* and *C*, the angle between the

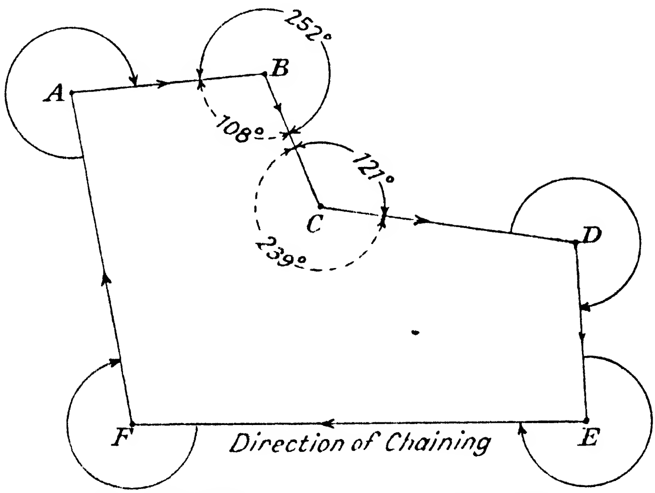


FIG. 5.16.—Measurement of " Included Angles ".

lines has two possible values. Thus, at *B*, the angle *ABC* may be either 252° or 108°, and at *C*, the angle *BCD* may be either 121° or 239°. If these angles were measured and plotted at random, complete chaos would ensue and it is obviously essential that the measurements should be carried out in a consistent manner and that the plotting should conform to the same system.

In practice, the angles are measured by a theodolite set up in turn at every junction point which becomes, for the time being, an " instrument station " and the conventional rule to be observed is the following :

Work along the traverse in the same direction as the chaining is running. Then at any instrument station we regard the previous point, from which the chainage has run, as the " back station " and the point ahead as the " forward station ". The angle must always be read in a clockwise direction from the back station to the forward and it is then termed an "Included Angle". Thus, the included angle at B is 252° and at C 121°, if the chainage runs as shown. The same rule may be stated in other words thus:

At any instrument station, face towards the forward station, i.e. look in the direction in which the chainage is running. Then the correct angle to measure is the one on the *left* hand side of the survey lines.

Included Angles in a Closed Traverse. Confusion sometimes arises between the included angles in a closed traverse and the interior angles of the figure. Actually, the included angles may be either the interior or the exterior angles of the figure according to the direction in which the chainage is run.

If the chaining is carried round the polygon in a clockwise direction, the included angles will be outside the figure and if the chaining is run counter-clockwise, the included angles will be inside, as an inspection of fig. 5.16 will show.

Bearings. The angle which a survey line makes with any reference direction is called a "bearing" and if the reference direction is magnetic north, as indicated by a compass, the angle is known as a "magnetic bearing". The bearing of a survey line with reference to north may be expressed in two ways:

(1) We may measure the angle in a clockwise direction from north round to the line, in which case we obtain the "whole circle bearing". A whole circle bearing may have any value between 0° and 360°, due east being 90°, due south 180°, and due west 270°; or

(2) We may measure the angle between the survey line and the north and south direction, proceeding either clockwise or counter-clockwise, from north or from south, but choosing the shortest way from the reference line to the survey line.

This gives the "reduced bearing" of the line. This, in itself, is valueless unless it is accompanied by the cardinal points to indicate whether the line lies between north and east, south and east, south and west, or north and west. A reduced bearing can never be greater than 90° and it provides a means of expressing a whole circle bearing in a form suitable for calculations involving sines and cosines, as will be seen later. Actually, the bearings measured in the field are always the whole circle and the angles so obtained are converted to the "reduced" form by a series of simple rules which will now be discussed.

Relationships between Whole Circle and Reduced Bearings. The survey lines in a traverse may be placed in four different groups, according to the value of their whole circle bearings and for each group a simple relationship exists between the whole circle bearing and the reduced bearing. The groups are bounded by the north–south and east–west directions and any survey line will fall within one of the four areas, conveniently visualised as " quadrants " and designated "north-east, south-east, south-west and north-west", shown in fig. 5.17. These four quadrants will now be considered in turn, *AB*, *CD*, *FG* and *HQ* representing the traverse lines.

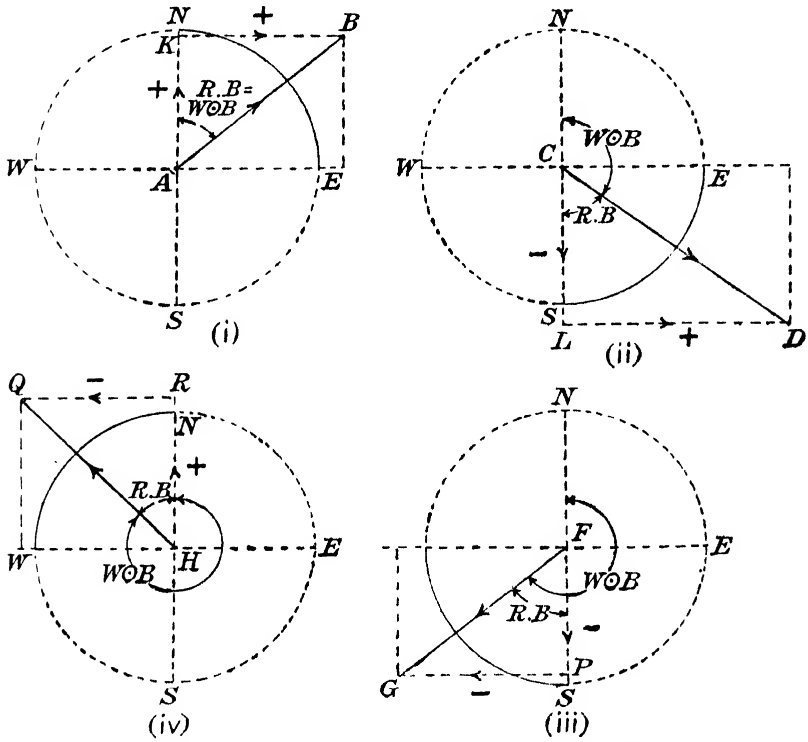


FIG. 5.17.—Whole Circle and Reduced Bearings.

(i) *North-east Quadrant.* The value of the whole circle bearing may range from 0° to 90° and the reduced bearing is measured clockwise from north. Hence, reduced bearing = whole circle bearing.

(ii) *South-east Quadrdnt.* The value of the whole circle bearing may range from 90° to 180° and the reduced bearing is measured counter-clockwise from south. Hence, reduced bearing = 180° − whole circle bearing.

(iii) *South-west Quadrant.* The value of the whole circle bearing may range from 180° to 270° and the reduced bearing is measured clockwise from south. Hence, reduced bearing = whole circle bearing − 180°.

(iv) *North-west Quadrant.* The value of the whole circle bearing may range from 270° to 360° and the reduced bearing is measured counter-clockwise from north. Hence, reduced bearing = 360° − whole circle bearing.

These relationships may be summarised thus :

Limiting Values of Whole Circle Bearing ($\theta°$)	Reduced Bearing and Cardinal Points
0° to 90°	N $\theta°$ E
90° to 180°	S (180° − $\theta°$) E
180° to 270°	S ($\theta°$ − 180°) W
270° to 360°	N (360° − $\theta°$) W

The above notation indicates that the reduced bearings in the first group are measured from north towards east, in the second group from south towards east, and so on. It must be emphasised that a reduced bearing without its cardinal points is meaningless, but a whole circle bearing, expressed merely as an angle, defines the position of a survey line completely.

Relationship between Whole Circle Bearings and Included Angles. It is possible to derive the whole circle bearings of all the lines of a traverse from the included angles, provided that the whole circle bearing of one line is known, by the application of the following simple rule :

If the included angle between two traverse lines is measured and the whole circle bearing of the first line is known, the whole circle bearing of the second line is found by adding together the included angle and the known whole circle bearing and subtracting 180° from the answer if the latter is greater than 180° or adding 180° to the answer if the latter is less than 180°.

In cases where the addition of the whole circle bearing and included angle results in an angle greater than 360°, subtract 360° before applying the rule.

The rule may be proved by simple geometry, thus :

Case I. Where the sum of the included angle and whole circle bearing is greater than 180°.

Let AB and BC, in fig. 5.18, be the two survey lines and N_1S_1 and N_2S_2 the parallel north–south lines at A and B, respectively.

Then $\qquad \angle N_1AB = \angle ABS_2$

and
$$\begin{aligned} \angle N_2BC &= \angle ABC - (180° - \angle ABS_2) \\ &= \angle ABC - (180° - \angle N_1AB) \\ &= \angle ABC + \angle N_1AB - 180° \end{aligned}$$

i.e. whole circle bearing of BC = included angle ABC + whole circle bearing of $AB°$ − 180°.

Case II. Where the sum of the included angle and whole circle bearing is less than 180°.

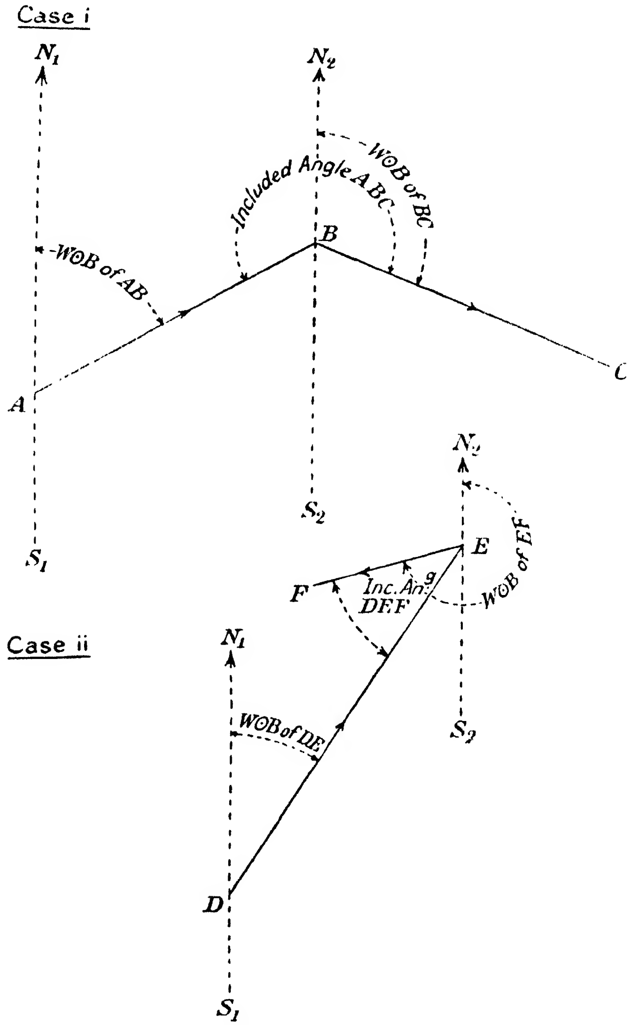


FIG. 5.18.—Whole Circle Bearings and Included Angles.

Let DE and EF be the survey lines and N_1S_1 and N_2S_2 the parallel north–south lines, as before.

Then, $\angle N_1DE = \angle DES_2$

and reflex angle $N_2EF = \angle DEF + \angle DES_2 + 180°$

$= \angle DEF + \angle N_1DE + 180°$

i.e. whole circle bearing of EF = included angle DEF + whole circle bearing of DE + 180°.

Plotting a Traverse by Departures and Latitudes. At first sight, the plotting of a traverse would appear to be a simple process involving the use of a protractor, but we have seen that a vernier theodolite will measure angles to the nearest half-minute or 20 seconds and a micrometer instrument to an even finer degree of accuracy, whereas the ordinary plain protractor, even in the 12-inch metal form, will not read closer than 15 minutes and an expensive vernier type only to the nearest minute.

Moreover, the setting out of angles by a protractor has the disadvantage that if the length of the line is greater than the radius of the instrument, the former must be produced beyond the point set out and any small error is thereby magnified.

In order to reproduce on paper the accuracy with which the theodolite measures angles in the field, a method of plotting is adopted which involves linear measurements only and the accuracy of the plotting can thus be increased by increasing the scale of the drawing.

The method is known as plotting by " departures and latitudes ", or, alternatively, as plotting by " co-ordinates " and is a simple application of the well-known mathematical device of locating a point by two distances measured parallel to two right-angled axes from a common origin.

Considering, again, the survey lines in the four quadrants shown in fig. 5.17, it will be apparent that each line can be located at its correct angle relative to the north–south line by two rectangular dimensions. Thus, AB is located by AK and KB, CD by CL and LD, FG by FP and PG, and HQ by HR and RQ.

The lengths KB, LD, PG and RQ run east or west and are called " departures " and the lengths AK, CL, FP and HR run north or south and are called " latitudes ".

These two terms may be sub-divided thus: the departures KB and LD are " eastings " and PG and RQ are " westings "; the latitudes AK and HR are " northings " and CL and FP are " southings ". In this country eastings and northings are considered positive and westings and southings negative. It will be noticed that the survey line, in each case, forms the hypotenuse of a right-angled triangle and we can obtain the following relationships:

$$KB = AB.\sin \angle KAB; \qquad AK = AB.\cos \angle KAB$$
$$LD = CD.\sin \angle LCD; \qquad CL = CD.\cos \angle LCD$$
$$PG = FG.\sin \angle GFP; \qquad FP = FG.\cos \angle GFP$$
$$RQ = HQ.\sin \angle QHR; \qquad HR = HQ.\cos \angle QHR.$$

In each case, the angle concerned is the reduced bearing and hence we derive the following general relationships:—

The departure of a line = Length of line × sin of reduced bearing

The latitude of a line = Length of line × cos of reduced bearing.

Both the terms on the right-hand side of the equation are obtained from the fieldwork and therefore the departure and latitude of each line can be calculated and the plotting carried out by setting off these linear dimensions without using a protractor.

Fieldwork of a Traverse Survey

I. Location of Stations

The location of the station points in a traverse differs somewhat from the usual practice in a chain survey. For instance, there is no longer the necessity to consider whether the lines will form well-shaped triangles or whether there are adequate ties. Instead, the stations must be so placed that the instrument can be set up over them conveniently, bearing in mind, at the same time, that the lines should pass close to important detail, to keep offsets short and should be as long as possible, provided they are within the clear range of the instrument for sighting purposes.

It would be manifestly absurd to set up the instrument in the middle of a busy road if a suitable station point could be fixed at the side and equally ridiculous to place the station peg so near to a wall or fence that the theodolite tripod could not possibly be set up over the reference mark. It is as well to make sure that a clear sight can be obtained to the base of the rod, whenever possible, as errors due to the rod being out of plumb may be considerable and are, of course, magnified if a sight is taken to the top.

It must be remembered that the lines will have to be chained and a clear run for the chain or steel band must be assured whenever possible.

There is sometimes a tendency to avoid an additional station by going too far ahead and fixing a point from which only the top of the rod is visible from adjacent stations, a grass bank or some other obstacle intervening. This is not good practice, either from the point of view of the theodolite work or the chaining, and the saving of an extra station does not compensate for the lack of accuracy which this procedure entails.

The marking of the station points and the necessity of taking measurements therefrom to fixed reference marks to facilitate re-location have already been mentioned in the chapter on chain surveying (pages 10 *et seq.*).

II. Chaining Lines and Fixing Topographical Detail

This procedure has been described in the chapter on chain surveying (pages 13 *et seq.*).

III. Measurement of Angles and Bearings

(1) *Measurement of the Whole Circle Bearing from Magnetic North.* It is customary to carry out this measurement for the first line of the

traverse, provided that one or other of its terminal points is situated where the magnetic needle will be reasonably free from the influence of iron or steel in the immediate vicinity. The girders of a steel bridge or iron railings, for example, would result in a deviation of the needle from its correct direction. If this condition cannot be fulfilled for the first line of the survey, the bearing of another line must be obtained.

The procedure is as follows:

Having set up, centred and levelled the instrument in the way described under "Temporary Adjustments", the compass is attached, if of the trough form, or if of the circular type permanently attached to the upper plate, the needle is released by unclamping and allowed to swing freely. The needle in the trough compass should not be released until the appliance is fixed in place on the theodolite, otherwise the pivot bearing may be damaged. The lower plate is clamped and the upper plate rotated until one vernier reads nearly zero (marked 360° on the scale) and the accurate setting to the zero graduation is carried out by the upper plate tangent screw. The upper plate is then clamped, the lower plate unclamped and the instrument rotated until the compass swings north and south. The final setting of the compass is carried out by the lower plate tangent screw which will not affect the zero reading.

The upper plate is then unclamped and the telescope directed, without transiting, to a ranging rod placed at the other end of the line, the accurate setting of the cross wires on the rod being accomplished by the upper plate tangent screw.

The same vernier as that originally set to zero will then give the whole circle bearing of the line, the procedure being indicated diagrammatically at $\triangle E$ in fig. 5.19.

The readings may be booked in the following way:

Instrument Station: E	Reading on Vernier A
Sighting magnetic north . . .	0° 0′ 0″
Sighting station F	75° 0′ 0″ = Whole Circle Bearing of EF.

Owing to the approximate nature of the compass indication on which this measurement is based, it is an unnecessary refinement to transit and reverse face.

Location of Distant Rod when Sighting. In reading traverse angles or bearings, the theodolite is centred accurately over a precise mark and is sighted on to a distant rod. This rod should be held vertically over the centre of the peg, or similar precise mark at the far station, and small tripod stands are available for this purpose. The importance of placing the rod exactly on the reference mark will be realised from the fact that a displacement of 1 inch at a distance of 500 feet will cause a measurable error when observing with a theodolite reading to half a minute or less.

If the length of the sight is 300 feet or less, a ranging rod is too thick, when viewed through the instrument, to permit of accurate centring of the cross wires and a chaining arrow forms a more suitable reference mark. A sheet of white paper may be placed behind it to give a good background and it may be inserted into the top of a wooden peg exactly at the intersection of the diagonal lines drawn thereon. For very accurate town surveys the " three tripod " system is sometimes used to reduce centring errors (see page 274).

(2) *Measurement of the Included Angle between Two Lines.* The instrument is levelled and centred at the junction station and rods are located accurately at the adjacent stations. The included angle, as already stated, is measured clockwise from the back station to the forward. We therefore clamp the lower plate at any random setting, turn the upper plate until the rod at the back station is intersected by the cross wires as low down as possible, completing the final adjustment with the upper plate tangent screw and vertical motion tangent screw and then read both verniers and note the face of the instrument. Next release the upper plate and, without transiting, direct the telescope to the forward station, again using the upper plate and vertical motion tangent screws to bring the cross wires on to the lowest part of the rod. Read both verniers.

This completes the procedure for one face, but it is advisable to repeat the measurement with the opposite face.

To do this, release the upper and lower plates, rotate the lower plate to another random position about 90° from its former position, and clamp it. Transit the telescope and reverse the face and bring the cross wires into coincidence with the rod at the back station, using the upper plate and vertical motion tangent screws as before. Read both verniers and note the face of the instrument. Release the upper plate and direct to the forward rod, using upper plate and vertical motion tangent screws and again read both verniers.

A refinement when reading with both faces consists in reversing the direction in which the fine setting of the cross wires on the reference mark is carried out, i.e. if they are moved laterally from right to left in one case, they should be moved from left to right after reversing face. This procedure is particularly useful in minimising the effects of light and shade on the distant rod which often produce a deceptive appearance.

It is important to notice that the lower plate remains clamped throughout the proceedings for each face, only being released at the time of reversing face to bring another part of the graduated circle into juxtaposition with the verniers. This gives a completely different set of angles, thus obviating the bias in reading which is likely to occur if the same angles are repeated and also distributing the readings over different parts of the circular scale, thereby reducing, to some extent, any small errors due to imperfect graduation which might possibly arise with a second-rate instrument.

It is also important to remember that the fine adjustment of the cross wires when sighting both the back and the forward rod is accomplished by the upper plate tangent screw, leaving the lower plate tangent screw undisturbed.

There are several methods of booking, but the following is as satisfactory as any:

Instrument Station	Pointing	Face	Vernier *A*	Vernier *B*	Mean of *A* and *B*	Included Angle	Mean Included Angle	Sketch or Remarks
F	*E*	Left	37° 43′ 00″	43′ 00″	37° 43′ 00″	257° 38′ 45″	257° 38′ 37″	
	G		295° 21′ 30″	22′ 00″	295° 21′ 45″			
	E	Right	198° 23′ 30″	23′ 30″	198° 23′ 30″	257° 38′ 30″		
	G		96° 02′ 00″	02′ 00″	96° 02′ 00″			

The following points should be noticed regarding the above example:

The readings are booked in the order in which they are taken, giving the degrees, minutes and seconds for vernier *A*, but the minutes and seconds only for vernier *B*. The fact the degrees differ by 180° on the two verniers is, however, verified when reading, but the mean of the degrees is not taken as this would give a meaningless result. In the column headed "Mean of *A* and *B*", the degrees of vernier *A* are entered together with the average of the minutes and seconds from both verniers.

Since the upper plate is turned clockwise from the first pointing to the second and the lower plate graduations increase in the same direction, the first reading will, in general, be less than the second and is subtracted from it to give the included angle, as shown with the left face readings above. It frequently happens, however, that in rotating the upper plate to sight the forward station, the vernier crosses the 360° graduation and arrives at a reading which is numerically less than the first. In such cases 360° is added to the second reading and the first subtracted from it, as before. This is shown in the right face readings in the above example, where the first reading is 198° 23′ 30″ and the second 96° 02′ 00″.

Here the value of the included angle is

$$96° \ 02' \ 00'' + 360° \ 00' \ 00'' - 198° \ 23' \ 30''$$
$$\text{i.e. } 456° \ 02' \ 00'' - 198° \ 23' \ 30''$$
$$\text{i.e. } 257° \ 38' \ 30''$$

The right and left face values of the included angle should not differ by more than some predetermined amount depending upon the type of instrument used and the accuracy required in the work. In ordinary road traverses, using an instrument reading to 30″, a discrepancy between right and left face readings of not more than 1 minute is usually accepted. When taking the mean of the right and left face angles, the nearest second is booked, since there is no justification for assuming an apparent accuracy down to half a second. It is probable, in fact, that the mean can only be relied upon to the nearest 15 seconds, or so, when using a vernier instrument reading to half-minutes, but this is amply sufficient for the ordinary road traverse.

(3) *Direct Reading of Bearings.* It has been mentioned that traverses are usually plotted from the departures and latitudes of the lines, derived from the lengths and the reduced bearings.

The included angles between the lines are not directly adaptable to the calculation of departures and latitudes. This necessitates the working out of, first, the whole circle bearings and then the reduced bearings for every line.

This arithmetical work may be avoided by the direct measurement of the whole circle bearing of each line in the field. It would be impossible to use the compass for each bearing in turn owing to the fluctuations which would probably occur in the direction of the needle due to the presence of variable forces of attraction. Thus, if one traverse station were located in the middle of an open field and the next were situated close to an iron railing the compass indications at the two stations would be far from parallel.

It is, of course, essential that the reference direction from which the bearings are measured should remain parallel to itself at every station along the traverse and a method of manipulating the theodolite is adopted which automatically ensures that the imaginary reference line is kept parallel to the direction of magnetic north as given by the compass at the first station. In fact, the compass may be discarded after making this preliminary determination and, if the correct routine is carried out, the bearings throughout the traverse will be measured from a direction parallel to this initial setting of the needle by an automatic orientation of the lower plate of the instrument.

Assuming that the whole circle bearing of the first line has been found in the manner already described, the upper plate is kept clamped and the lower plate released before leaving the first station. The instrument is now set up at the second station and sighted back to the first *without transiting the telescope* and with one or other of the verniers still reading the whole circle bearing of the first line. To do this, the upper plate must be kept clamped and the accurate sighting of the cross wires on the rod must be accomplished by the lower plate tangent screw. The procedure is illustrated in fig. 5.19.

Let the whole circle bearing of the first line, *EF*, be 75°, as recorded by vernier *A*. Remembering that the verniers are located on a line at right-angles to the telescope, the position of this reading on the lower plate will be as shown. On moving to station *F* and sighting back to station *E*, the 75° reading will assume the position indicated and if the upper plate were released and swung round until vernier *A*

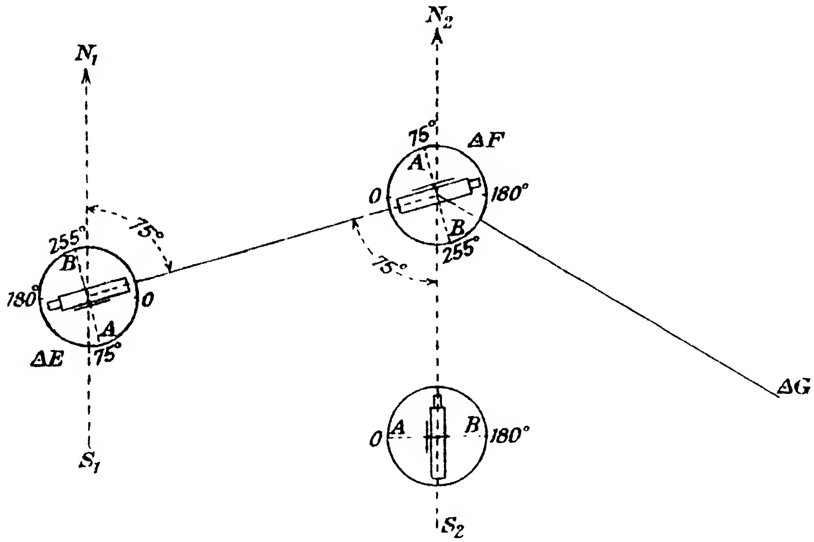


FIG. 5.19.—Direct Reading of Bearings.

arrived at the zero reading, it will be apparent that the telescope would then be pointing due south, i.e. it will again be parallel to the north–south line, but will be facing the wrong direction. In other words, the lower plate is now set with the graduations 180° out of phase with respect to the original compass direction, but since the verniers themselves differ by 180°, this setting of the lower plate will give the correct reading for an angle measured at station *F*, provided that vernier *B* is read instead of vernier *A*.

If the lower plate is kept clamped, therefore, and the upper plate released and rotated until the rod *G* is sighted, using the upper plate tangent screw for the fine setting, vernier *B* will give the whole circle bearing of the line *FG*. On repeating the procedure at station *G* to obtain the whole circle bearing of the line *GH*, vernier *A* must be read in order to get the correct angle.

It will be noticed that in the method described above, the theodolite automatically carries out the arithmetical rule for deriving the whole circle bearing from the included angle. Thus, the whole circle bearing of $EF = \angle N_1EF = \angle EFS_2$. After sighting back from F to E, the telescope describes the angle EFG, which, of course, is the included angle. The addition or subtraction of 180° is carried out by reading each vernier alternately.

The following points must be borne in mind when using this method :

(1) It is absolutely essential that the forward bearing should not alter when moving the instrument from one station to the next. If the theodolite is jerked in any way, the upper plate may move slightly, thus altering the reading. Before sighting back, therefore, it is important to check the vernier reading and reset it, if necessary. There is less likelihood of the upper plate slipping relatively to the lower, if the latter is unclamped, thus permitting the two plates to rotate as one unit.

(2) The telescope must not be transited between the back and forward readings, otherwise any collimation defect, if present, will be doubled.

(3) The accurate setting of the cross wires on the back station is carried out with the lower plate tangent screw and on the forward station by the upper plate tangent screw.

(4) Both verniers should be read at each station and the readings booked in the following simple way :

Use the central column in the chain book for the successive station letters, commencing at the bottom and working upwards, and write the angle given by vernier A on the left of this column and the angle given by vernier B on the right, placing the figures midway between the corresponding stations, as shown below :

	O	
$209^\circ\ 52'\ 20''$		$\underline{29^\circ\ 52'\ 20''}$
	N	
$\underline{196^\circ\ 44'\ 40''}$		$16^\circ\ 44'\ 40''$
	M	
$175^\circ\ 18'\ 00''$		$\underline{355^\circ\ 18'\ 00''}$
	L	
$\underline{223^\circ\ 31'\ 20''}$ correct		$43^\circ\ 31'\ 20''$
	K	
Vernier A		Vernier B

It is very important that the vernier giving the correct bearing of the first line should be clearly marked by underlining or some other method and alternate verniers will then give the correct bearings for the subsequent lines.

By spacing out the station letters, room is left for inserting additional values of the bearings if the work is repeated for the purpose of checking, as it often is.

The direct bearing method is quick and simple when once the routine becomes familiar, but it has the disadvantage that it is not readily adaptable to right and left face readings and errors are liable to escape notice. It is a useful method for an experienced surveyor when rapid work is necessary and it saves the time occupied in the calculation of the whole circle bearings from the included angles.

The included angle method is better, however, from the point of view of safety.

Calculation of Departures and Latitudes

The formulae

$$\text{Departure} = \text{Length of line} \times \sin \text{ of reduced bearing}$$
$$\text{Latitude} = \text{Length of line} \times \cos \text{ of reduced bearing}$$

may be re-written:

$$\text{Log departure} = \log \text{length} + \log \sin \text{reduced bearing}$$

and
$$\text{Log latitude} = \log \text{length} + \log \cos \text{reduced bearing.}$$

Five-figure logarithms are generally used for making these calculations and the data may be conveniently tabulated in the form given later.

If the traverse angles have been read as included angles, the first step is to convert these into whole circle bearings, which, in turn, are converted into reduced bearings, with their appropriate cardinal points, and it is advisable to sketch the traverse lines roughly with scale and protractor, as a preliminary, in order to obtain an idea of the general arrangement.

A small closed traverse will be taken as an example in which the data derived from the fieldwork is as follows:

Lengths of Lines

AB	907	feet
BC	249	,,
CD	348	,,
DE	523	,,
EA	444·8	,,

Included Angles

ABC	328° 41′ 20″
BCD	122° 57′ 00″
CDE	275° 26′ 40″
DEA	241° 06′ 13″
EAB	291° 48′ 47″

Whole Circle Bearing of *AB* = 77° 38′ 40″.

Calculation of Whole Circle Bearings of the remaining Lines

Whole Circle Bearing of *AB*		77	38	40
Included Angle *ABC*		328	41	20
		406	20	00
	less	360	00	00
		46	20	00
	add	180	00	00
Whole Circle Bearing of *BC*		226	20	00
Included Angle *BCD*		122	57	00
		349	17	00
	less	180	00	00
Whole Circle Bearing of *CD*		169	17	00
Included Angle *CDE*		275	26	40
		444	43	40
	less	360	00	00
		84	43	40
	add	180	00	00
Whole Circle Bearing of *DE*		264	43	40
Included Angle *DEA*		241	06	13
		505	49	53
	less	360	00	00
		145	49	53
	add	180	00	00
Whole Circle Bearing of *EA*		325	49	53
Check : Whole Circle Bearing of *EA*		325	49	53
Included Angle *EAB*		291	48	47
		617	38	40
	less	360	00	00
		257	38	40
	less	180	00	00
Whole Circle Bearing of *AB*		77	38	40

This agrees with the previous value.

Calculation of Reduced Bearings :

Line	Whole Circle Bearing	Reduced Bearing and Cardinal Points
AB . . .	77° 38′ 40″	N 77° 38′ 40″ E
BC . . .	226° 20′ 00″	226° 20′ 00″ − 180° = S 46° 20′ 00″ W
CD . . .	169° 17′ 00″	180° − 169° 17′ 00″ = S 10° 43′ 00″ E
DE . . .	264° 43′ 40″	264° 43′ 40″ − 180° = S 84° 43′ 40″ W
EA . . .	325° 49′ 53″	360° − 325° 49′ 53″ = N 34° 10′ 07″ W

The reduced bearings are in a suitable form for obtaining log sines and log cosines and the data is now arranged in tabular form, in the manner shown, entering the log sine, log length and log cosine in adjacent columns to facilitate the additions which give, respectively, the log departure and the log latitude. The departure column is sub-divided into east and west and the latitude column into north and south.

Tabular Form for Traverse Calculations

Line	Length (feet)	Included Angle	Whole Circle Bearing	Reduced Bearing (R.B.) and Cardinal Pts.	Log sin R.B.
		° ′ ″	° ′ ″	° ′ ″	
AB . .	907		77 38 40	N. 77 38 40 E.	$\bar{1}$·98982
		328 41 20			
BC . .	249		226 20 00	S. 46 20 00 W.	$\bar{1}$·85936
		122 57 00			
CD . .	348		169 17 00	S. 10 43 00 E.	$\bar{1}$·26940
		275 26 40			
DE . .	523		264 43 40	S. 84 43 40 W.	$\bar{1}$·99816
		241 06 13			
EA . .	444·8		325 49 53	N. 34 01 07 W.	$\bar{1}$·74945
		291 48 47			
AB . .			77 38 00		
		Σ1,260 00 00			

Note that the included angles are written midway between the lines to which they refer.

Log length	Log cos R.B.	Log Dep.	Log Latitude	Departure East	Departure West	Latitude North	Latitude South
2·95761	$\bar{1}$·33037	2·94743	2·28798	886·0		194·1	
2·39620	$\bar{1}$·83914	2·25556	2·23534		180·1		172·0
2·54158	$\bar{1}$·99236	1·81098	2·53394	64·7			342·0
2·71850	$\bar{2}$·96326	2·71666	1·68176		520·8		48·1
2·64817	$\bar{1}$·91771	2·39762	2·56588		249·8	368·0	
				950·7	950·7	562·1	562·1

(In practice this tabular form would be continuous across the page.)

Checks on a Closed Traverse

It will be seen from the above table that the included angles sum to exactly 1,260° and also that the eastings and westings both sum to 950·7 and the northings and southings both sum to 562·1.

It is obvious that when the survey lines form a polygon there will be a definite summation value which the included angles should satisfy, the total distance traversed east must be equal to the total distance traversed west and a similar condition must be satisfied by the northings and southings.

We thus have a very complete check both on the fieldwork and the calculations in the case of a closed traverse.

Check for Included Angles in a Closed Traverse. The check on the angles is the well-known geometrical rule : For a polygon of n sides, the sum of the interior angles is equal to $(2n - 4)$ right-angles and the sum of the exterior angles is $(2n + 4)$ right-angles.

In the example given, $n = 5$, and since the chainage runs clockwise, the included angles are external. Hence, the sum of the included angles $= (10 + 4) \times 90° = 1{,}260°$. But even with very careful fieldwork, an exact agreement of the measured angles with the theoretical total is never obtained, although the discrepancy should be small. Using an ordinary 4- or 5-inch vernier theodolite, the error at each angle should not much exceed $\pm$ 15 seconds and it is generally assumed that the total error is proportional to the square root of the number of sides in the traverse.

In general terms, if the traverse has n sides and the allowable error per station is e, the allowable summation error is $e\sqrt{n}$. Thus, allowing 15 seconds at each station, the permissible summation error for the above five-sided traverse would be $15\sqrt{5}$ seconds, or 34 seconds.

Check for Direct Bearings in a Closed Traverse. It is usual, when obtaining direct bearings in a closed traverse to work round the figure and re-determine the bearing of the first line, i.e. after finding the bearing of EA in the above example, the instrument, still clamped at this bearing, is set up again at A and the bearing of the line AB found again. This forms a limited check and would reveal any slipping of the plates, but owing to the purely automatic action of the theodolite in adding each included angle to the previous bearing, there is nothing to prevent a bearing being wrongly read and booked at any of the intermediate points in spite of the second reading of the bearing of AB giving excellent agreement with its previous value.

Check on Departures and Latitudes in a Closed Traverse. Just as the sum of the included angles cannot be expected to give a perfect check, the departures and latitudes will not usually sum to an exact closure. The accuracy of the angular measurements is usually greater than that of the linear measurements and the permissible closing error is frequently taken as 1/1000 of the perimeter of the polygon for a traverse on variable ground, using a chain for measuring the lines, or 1/2000 for a traverse on similar ground, using a steel band. On smooth level ground better figures than these are possible.

The degree of accuracy required in any kind of survey work, however, is mainly determined by the purpose for which it is being carried out and the scale to which it will be plotted. A closing error too small to be visible to the scale of the drawing can be ignored.

It should be remembered that it is possible to obtain a very small closing error even if the traverse lines have been measured with an inaccurate chain or band, provided the same chain or band has been used throughout.

Correction of Closed Traverses

1. Correction of Included Angles

Provided the summation is sufficiently close to the theoretical amount, the error is distributed over the angles in the manner shown in the following example in which the error in the observed angles is 50 seconds.

	Angles as read			Correction	Corrected Angles		
	°	′	″	″	°	′	″
ABC	328	41	10	10	328	41	20
BCD	122	56	50	10	122	57	00
CDE	275	26	30	10	275	26	40
DEA	241	06	03	10	241	06	13
EAB	291	48	37	10	291	48	47
	1,259	59	10		1,260	00	00

This correction, with the observed and adjusted angles, should be entered in the tabular form for the traverse calculations, adding the appropriate columns where required. Observed whole circle bearings, corrected in the manner shown below, should be entered in the tabular form in the same way.

2. Correction of Bearings

The correction of any observed whole circle bearing is carried forward to the bearings which follow in the manner shown below :

Line	Observed Whole Circle Bearing			Correction	Corrected Whole Circle Bearing		
	°	′	″	″	°	′	″
EF	28	16	30	—	28	16	30
FG	117	22	00	− 10	117	21	50
GH	183	53	30	− 20	183	53	10
HK	79	03	30	− 30	79	03	00
KL	217	41	00	− 40	217	40	20
LM	172	18	00	− 50	172	17	10
ME	94	07	30	− 01′ 00″	94	06	30
EF	28	17	30	− 01′ 00″	28	16	30
error :		01′	00″				

In the above example the error of 1 minute has been distributed among six angles by subtracting 10 seconds successively and carrying forward the previous correction each time.

3. Correction of Latitudes and Departures

(i) *Methods involving no Change in Angles.* Since the angular measurements are likely to be more accurate than the lengths of the sides, and provided that the angles check well, it is logical to obtain a perfect closure of the figure by altering the lengths of the lines, but not the angles between them. This may be done graphically, or by a combination of graphical and arithmetical methods.

Let fig. 5.20 represent a traverse $ABCDEFA'$, in which AA' is the closing error, greatly exaggerated for diagrammatic purposes. In practice, AA' will be small, otherwise the fieldwork data would

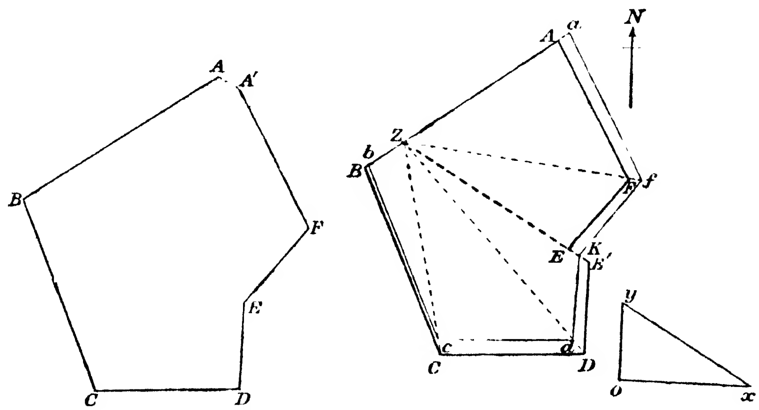


FIG. 5.20.—Adjustment of Closing Error in a Traverse by " Axis of Correction " Method.

be rejected. It can, however, be represented to an enlarged scale by setting off the errors in departure and latitude, ox and oy, respectively, when yx will be the closing error in magnitude and direction. The direction of the errors in departure and latitude must be carefully noted when plotting this enlarged diagram. Thus, in the example, the southings are too large by the amount yo and the eastings by ox.

In the figure, the plotting was commenced at A and terminated at A', but the closing error will be of the same magnitude and will be parallel to AA' at whatever station the plotting is commenced since it depends entirely on the differences in the total east and west departures and the total north and south latitudes. Thus, if the plotting had been started at E, the resulting figure would be $EFABCDE'$

and EE' will be equal and parallel to AA'. There will be one or more stations in the traverse through which a line drawn parallel to the closing error will divide the area into two parts roughly equal in extent and in the given example it is apparent that it should be drawn through E. Let the line so drawn, intersect AB in Z. This line, $E'Z$, is then called the " axis of correction ".

Let K be the middle point of EE'. Then, if we accept K as a compromise closing-point for the traverse, and no alteration is to be made in the angles, it will be seen from the figure that lines on one side of the axis must be increased in length, while those on the opposite side must be decreased in length, assuming that the errors are to be distributed round the figure and not applied wholly to two lines. It will be logical to alter the lengths of the lines according to some rule of proportion and the proportionate increase in the lengths of the lines on the north-east side of the axis will be equal to the ratio $EK : ZE$. Similarly, the proportionate decrease in the lengths of the lines on the south-west side of the axis will be equal to the ratio $KE' : ZE'$. Note that the line BA, intersected by the axis, must be considered in two parts, BZ and ZA. But $EK = KE' =$ half the closing error and ZE and ZE' will be nearly equal, both being large compared with EE'. We may, in fact, for all practical purposes consider that ZE and ZE' are both equal to the axial length, giving us the simple rule that a traverse line of length l must be increased or decreased by an amount

$$l \times \frac{\text{half closing error}}{\text{axial length}}.$$

The axial length is scaled off the outline drawing of the traverse plotted with the closing error ignored and the closing error is scaled off the enlarged diagram yox. Sufficient data is thus obtainable for re-plotting the traverse with the lines of corrected length drawn parallel to their original directions. Alternatively, this may be done graphically by joining ZC, ZD and ZF and constructing the figure $KfabcdK$ in the manner shown.

If, however, the length of a line is altered, its departure and latitude will be altered in the same proportion. Hence, if a trial figure be plotted and the correction factor

$$\frac{\text{Half closing error}}{\text{axial length}}$$

determined, this factor may be used for correcting the departures and latitudes which should then give perfect agreement in the summation check. The traverse can then be re-plotted from the corrected latitudes and departures. A worked example is given later. The magnitude of the correction factor is inversely proportional to the axial length and to give the minimum change to the dimensions of the traverse, the axial length should therefore be as long as possible.

(ii) *Bowditch's Method of Correcting a Closed Traverse.* This method is based on the rule:

Correction to the departure or latitude of any line

$$= \text{Total error in departure or latitude} \times \frac{\text{length of line}}{\text{perimeter of traverse}}$$

Due attention must be paid to the algebraic sign when applying the calculated correction. Thus, if the eastings and southings are too large, as in the worked example given later, the corrections must be subtracted from east departures and south latitudes and added to west departures and north latitudes.

(iii) *Ordinary Method of Correcting a Closed Traverse.* A frequent way of correcting a closed traverse is based on the rules:

Correction to any east departure

$$= \text{half total error in departures} \times \frac{\text{that departure}}{\text{sum of eastings}}$$

Correction to any west departure

$$= \text{half total error in departures} \times \frac{\text{that departure}}{\text{sum of westings}}$$

and

Correction to any north latitude

$$= \text{half closing error in latitudes} \times \frac{\text{that latitude}}{\text{sum of northings}}$$

Correction to any south latitude

$$= \text{half closing error in latitudes} \times \frac{\text{that latitude}}{\text{sum of southings}}$$

Again, the corrections must be applied with due regard to sign, so that the summations of the eastings and westings are equal after adjustment and also the summations of the northings and southings. Both of the above methods of correction distort the figure, the angles becoming changed from their original values in order to obtain a closed figure. This appears to be illogical if the summation of the angles, as read, checked well, but both methods are in general use and give a working compromise which is satisfactory for most of the purposes for which a traverse is required and they have the advantage that there is no need to plot a trial figure as there is in the "axis of correction" method.

Worked Example. All three methods have been applied for correcting a traverse in which the departures and latitudes, after adjustment of the angles, were as follows:

Line	Length (feet)	Departures East	Departures West	Latitudes North	Latitudes South
AB	525		85·6		518·1
BC	718		567·6		439·5
CD	633		632·8	14·4	
DE	540		196·2	503·1	
EF	302		75·7	292·3	
FG	814	813·5		22·7	
GA	776	746·0		122·1	
Summations :	4,308	1,559·5	1,557·9	954·6	957·6
Closing Errors : Departures : 1·6 feet ; Latitudes : 3·0 feet					

The extent of the corrections resulting from each method will be seen from the following summary. Discrepancies of 0·01 foot will be noticed, but for practical purposes these may be ignored.

Line	Uncorrected				Corrections by Method (i)					
	Departure		Latitude		Dep.	Lat.	Corrected			
							Departure		Latitude	
	E.	W.	N.	S.			E.	W.	N.	S.
AB . .		85·6		518·1	− 0·17	− 1·04		85·43		517·06
BC . .		567·6		439·5	− 1·13	− 0·88		566·47		438·62
CD . .		632·8	14·4		+ 1·04	+ 0·02		633·84	14·42	
DE . .		196·2	503·1		+ 0·32	+ 0·83		196·52	503·93	
EF . .		75·7	292·3		+ 0·12	+ 0·48		75·82	292·78	
FZ . .	(470·0)		(13·1)		+ 0·77	+ 0·02	470·77		13·12	
	813·5		22·7							
ZG . .	(343·5)		(9·6)		− 0·69	− 0·02	342·81		9·58	
GA . .	746·0		122·1		− 1·49	− 0·24	744·51		121·86	
							1,558·09	1,558·08	955·69	955·68

Corrections by Method (ii)						Corrections by Method (iii)					
Dep.	Lat.	Corrected				Dep.	Lat.	Corrected			
		Departure		Latitude				Departure		Latitude	
		E.	W.	N.	S.			E.	W.	N.	S.
+ 0·20	− 0·37		85·80		517·73	+ 0·04	− 0·81		85·64		517·29
+ 0·27	− 0·50		567·87		439·00	+ 0·29	− 0·69		567·89		438·81
+ 0·23	+ 0·44		633·03	14·84		+ 0·32	+ 0·02		633·12	14·42	
+ 0·20	+ 0·38		196·40	503·48		+ 0·10	+ 0·79		196·30	503·80	
+ 0·11	+ 0·21		75·81	292·51		+ 0·04	+ 0·46		75·74	292·76	
− 0·30	+ 0·57	813·20		23·27		− 0·42	+ 0·04	813·08		22·74	
− 0·29	+ 0·54	745·71		122·64		− 0·38	+ 0·19	745·62		122·29	
		1,558·91	1,558·91	956·74	956·73			1,558·70	1,558·69	956·10	956·10

(i) *Axis of Correction Method*

If the traverse be plotted in the manner shown in fig. 5.21, starting at station C, the axis of correction roughly bisects the figure. This plotting should be on the largest convenient scale since the accuracy of the method depends on the careful measurement of the axial line, CZ, and the portions into which it sub-divides the opposite side

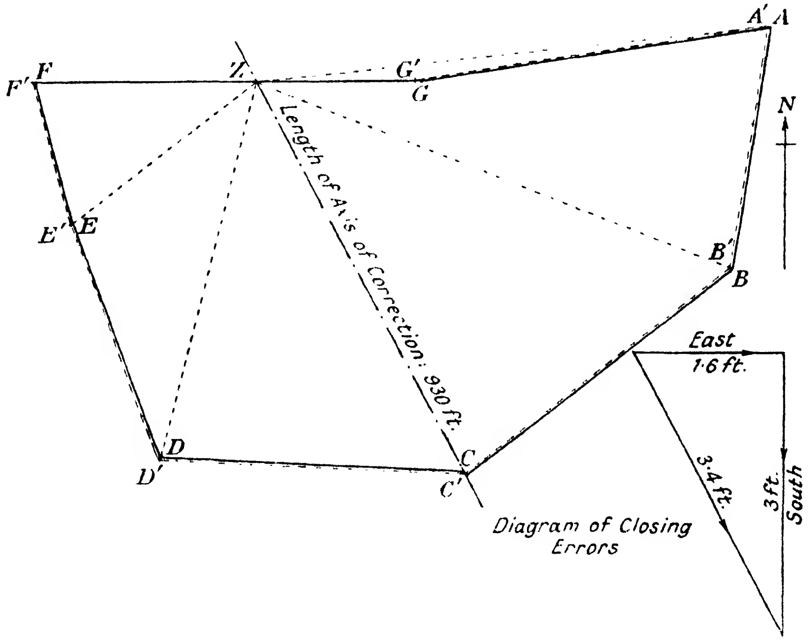


FIG. 5.21.—Example of " Axis of Correction " Method applied to a Traverse.

(FZ and ZG). A large scale must also be used for plotting the closing errors. In the example, the length of the axis of correction is 930 feet, the resultant closing error is 3·4 feet, FZ is 470 feet, and ZG is 344 feet. The perimeter of the figure is divided by the axis in the ratio 0·45 : 0·55, the left-hand part being the smaller. Hence, the correction factors are : For left-hand part : $\dfrac{0{\cdot}45 \times 3{\cdot}4}{930} = 0{\cdot}00165$ and for right-hand part : $\dfrac{0{\cdot}55 \times 3{\cdot}4}{930} = 0{\cdot}002$. The lengths, departures and latitudes on the left side of the axis will be increased and on the right side decreased.

(ii) *Bowditch's Method*

Length of perimeter : 4,288 feet.

Correction to departure of AB : $\frac{525}{4288} \times 1{\cdot}6 = 0{\cdot}20$ feet.

Correction to latitude of AB : $\frac{525}{4288} \times 3{\cdot}0 = 0{\cdot}37$ feet.

and so.

(iii) *Ordinary Method*

Correction to westing AB : $85{\cdot}6 \times \frac{0{\cdot}8}{1557{\cdot}9} = 0{\cdot}04$ feet.

Correction to southing AB : $518{\cdot}1 \times \frac{1{\cdot}5}{957{\cdot}6} = 0{\cdot}81$ feet.

and so on.

The complete corrections by all three methods are given in the tables on p. 148 (see fig. 5.21).

It is customary, when using the tabular form shown on page 142, to show the calculated departures and latitudes before correction, adding columns for the amounts to be added or subtracted from the calculated figures and finally giving the adjusted values so obtained which should sum to a perfect closure.

Checking Road Traverses. The usual road survey, except in certain urban clearance schemes, does not take the form of a closed traverse, but is almost invariably an open traverse, often of considerable length. The very satisfactory property of the closed traverse in giving such excellent opportunity for checking need not be lost, however, in every instance.

It frequently happens, for example, that the proposed line for a by-pass runs roughly parallel to existing roads to which cross connections are available at intervals, as shown in fig. 5.22. The proposed route is, of course, surveyed fully from the main traverse lines *ABCD*, etc., but the framework only of a subsidiary traverse can be carried along the existing roads and connected to the main traverse where convenient as indicated by the dotted lines in the figure, thus forming a self-checking network. Only the lengths of the lines and the included angles or bearings are measured in the subsidiary traverse and this data can be obtained without adding much to the period occupied in making the main survey, since it is the location of the topographical details which takes up so much time in a long survey, not the measurement of the lines and angles.

Again, if the traverse forms part of a triangulation survey, the terminal stations of the traverse should be chosen at triangulation stations. These points are accurately fixed and the distance between them can be computed with precision, so that the traverse becomes, in effect, a closed network with the line joining the triangulation stations as the closing link.

If it is not possible to adopt either of these methods, the reliability of an open traverse may be enhanced by first measuring the lines, without locating topography and, in open country, leaving pegs at every 100 feet. The topography is then located, necessitating, of

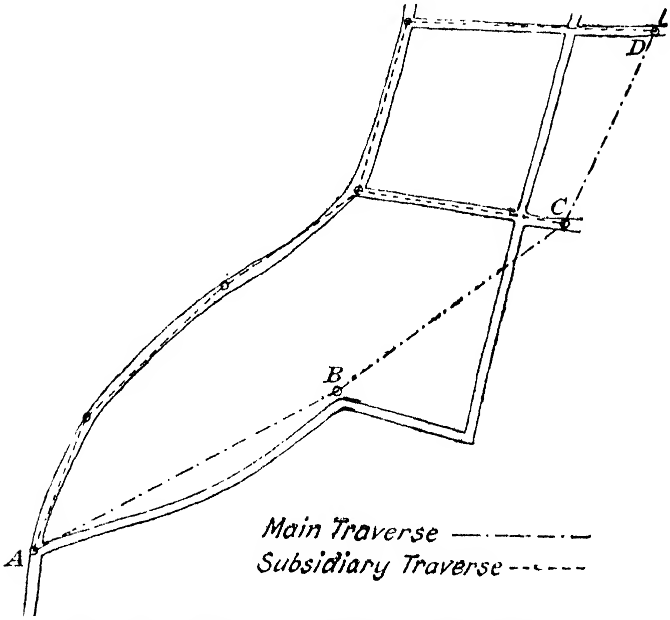


FIG. 5.22.—Method of checking a Road Traverse.

course, the re-chaining of the lines and a check is thereby obtained for every 100-foot length. The included angles should be measured, reversing face, and the right and left face readings will act as a check on one another.

In British practice, the 25-inch Ordnance maps provide sufficient check to reveal any glaring errors. The terminal traverse stations and selected intermediate points can be located on the appropriate sheet to within about 2 feet and the traverse lines are plotted to the 25-inch scale (1/2500) on tracing paper and placed over the map.

Plotting by Co-ordinates. If a traverse is plotted by setting off successive departures and latitudes, errors are carried forward, but if the departures and latitudes are summed algebraically and the points located by measuring the total departures and latitudes from the common origin of two right-angles axes, this carrying forward of errors is avoided.

Considering, once again, the example given on page 142, the departures and latitudes are repeated in the first four columns below :

Line	Departure		Latitude		Total Departure	Total Latitude	Station	Co-ords. of Stns.	
	East	West	North	South				East	North
AB. .	886·0		194·1		0	0	*A*	0	400·0
					886·0	194·1	*B*	886·0	594·1
BC .		180·1		172·0					
					705·9	22·1	*C*	705·9	422·1
CD .	64·7			342·0					
					770·6	− 319·9	*D*	770·6	80·1
DE. .		520·8		48·1					
					249·8	− 368·0	*E*	249·8	32·0
EA. .		249·8	368·0						
					0	0	*A*	0	400·0

It is conventional to consider north and east as positive and west and south as negative. The successive departures and latitudes to the station points have been summed algebraically in accordance with this convention and the results entered under the headings " Total Departure " and " Total Latitude ", placing the figures midway between the lines on which the invidual departures and latitudes are written to indicate that the total departure up to *B* is 886·0 and total latitude up to *B* 194·1 and so on.

It will be apparent from the figures that *A* is the most westerly point of the traverse since the departures are all positive, *B* is the most northerly point as shown by the maximum positive latitude and *E* the most southerly point with the maximum negative latitude. By considering the departure and latitude at *A* to be zero, this point becomes, of course, the origin of the rectangular axes from which the other departures and latitudes are measured, but it is usual to locate these axes so that all departures and latitudes measured therefrom are positive. This may be done by assigning an assumed departure and latitude to the starting-point of the traverse which will have the effect of moving the axes so that the entire traverse lies to the east and north of them. The departures and latitudes of the stations calculated on this new basis are termed their " co-ordinates " and in the above example the co-ordinates of *A* may be taken as 0 for departures and 400 for latitudes. The co-ordinates of the other stations will then have the values tabulated and it will be noticed that no negative quantities appear. The plotting of a traverse from co-ordinates is rendered more accurate if the paper is first ruled into squares to some convenient scale.

Traverse Tables. Tables can be obtained which give the departure and latitude for any combination of distance and reduced bearing and these will be found very useful when extensive traverses are being plotted. Particulars of such tables are given in the bibliography at the end of the volume.

Magnetic and True North. It has been stated that the bearings of traverse lines are measured from magnetic north as a reference direction. This differs from the direction of true north by an amount which varies from year to year and from place to place. There is even a slight fluctuation in the reading of the magnetic needle during each 24-hour period and completely irregular variations occur from time to time.

It is customary, in ordinary surveying practice, to consider the difference between magnetic and true north as constant for any given year at any given locality and this difference is termed the magnetic declination, or " deviation ".

The phenomena associated with terrestial magnetism are described in any good text-book on magnetism and electricity and are of considerable scientific interest. If lines are drawn on a map connecting points which have the same declination, they will be found to follow well defined trends, much as contours do, and they are termed " isogonic lines " or " isogons ". Annual charts showing isogonic lines for the entire world are published by the Admiralty, and the Ordnance Survey publishes a map of England and Wales to a scale of 1 : 1,000,000 showing isogonic lines and also the observed deviations at a large number of points.

The former run roughly parallel to a line passing through Newcastle-on-Tyne and Lyme Regis, i.e. slightly west of south, although irregularities occur in many localities, especially, of course, where magnetic rocks are present at or near the surface.

There is a general tendency in the British Isles for the deviation to increase as one travels in a westerly direction, the rate of increase being, very approximately, 40 seconds west per mile. Thus the average values during 1933 were 10° 42′ W. at the South Foreland and 13° 48′ W. at Falmouth.

The declination in the British Isles is decreasing with time at a rate which itself shows a tendency to decrease and the following figures will give some idea of the variation which has occurred at Greenwich during several centuries :

Year	Declination
1580	11° 15′ East
1659	0° 0′ (Magnetic and true north coincident)
1823	24° 30′ West
1928	14° 0′ West (Rate of decrease 12′ per annum)
1944	10° 08′ West (Rate of decrease 10′ per annum)
1947	9° 36′ West (inferred)

The magnetic deviation is shown on Ordnance Survey 1-inch and Bartholomew ½-inch maps and if shown on a plan should always be accompanied by the date. Much interesting information regarding terrestial magnetism, including the current value of the declination at Greenwich, will be found in *Whitaker's Almanac.* True north is, of course, the recognised reference direction for a plan and this may be derived approximately from the magnetic north in two ways:

(1) If the latter is used as the reference direction for plotting the traverse, the north point may be drawn on the plan by setting out a line parallel to the north–south axis of the co-ordinates and then measuring off the declination angle to the east of this line to give the direction of true north.

(2) The declination may be subtracted from every whole circle bearing before calculating the reduced bearing, departure, latitude and co-ordinates. The traverse will then be orientated directly to true north. If included angles have been obtained in the field together with the whole circle bearing of the first line, the latter may be converted into the bearing from true north before calculating the subsequent whole circle bearings.

The Prismatic Compass. This is a handy instrument for obtaining the direction of magnetic north and it may also be used for measuring, very roughly, the magnetic bearing of the lines in a traverse.

Fig. 5.23 represents a typical prismatic compass in perspective and plan. The magnetic needle, N, is supported by a jewelled bearing, B, on a central pivot and forms the diameter of a light aluminium ring, R, graduated in degrees and half-degrees. The whole is enclosed in a glass lidded brass case, 3 to $4\frac{1}{2}$ inches in diameter, on the rim of which are located a 45° prism, P, and a sighting indicator, I, diametrically opposite each other. Both these attachments fold down when not in use and the sighting indicator automatically clamps the needle when folded on to the glass lid and releases it when erected into the observing position. The line of sight is defined by a notch in the upper edge of the brass case in which the prism, P, is fitted and by a vertical horsehair, H, stretched across the frame, I. The effect of the 45° prism is to render visible the graduations on the circular scale when the observer's eye is placed horizontally behind the vertical face of the prism, in which position the line of sight can be directed simultaneously towards a distant rod by aligning it with the notch and the sighting hair.

The prism forms an enlarged mirror image of the small section of the graduated ring immediately beneath it and for this reason the numerals are inscribed on the ring backwards (but not upside down), starting from zero at the south end of the needle and increasing in a clockwise direction.

This arrangement provides for a zero reading when the notch and the sighting hair are aligned with the freely swinging needle and automatically gives the whole circle bearing from magnetic north when the line of sight is directed elsewhere.

To use the instrument, it is usually placed on the top of a short ranging rod at a convenient height for sighting and the rod is placed over the station peg. The needle and scale are allowed to float, this movement being damped, if necessary, by a small plunger located beneath the sighting indicator, *I*. The needle will then assume the direction of magnetic north, more or less correctly according to the intensity of any local magnetic attraction in the immediate vicinity. The notch and sighting hair are directed to a rod at the forward station and the prism will then be vertically over the graduation which gives the whole circle bearing of the line. In the figure this is 55°.

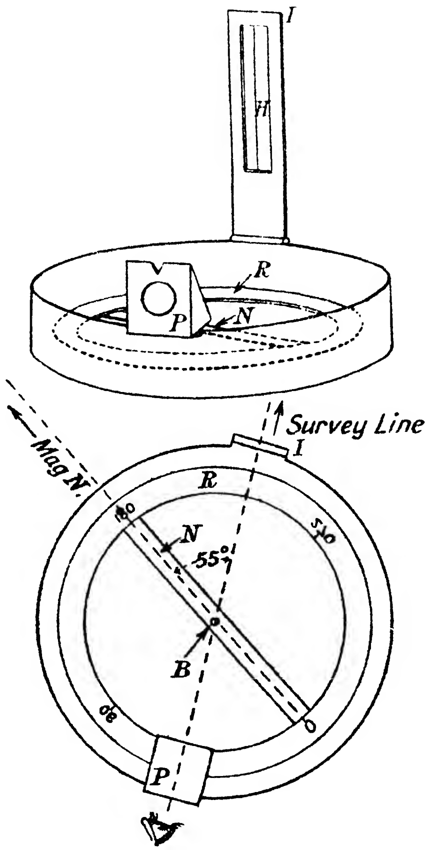


FIG. 5.23.—Prismatic Compass.

When determining the whole circle bearings of a number of successive lines, it is usual to read both the forward and back bearings, i.e. we obtain the bearing of the line *AB*, for example, from station *A* and the bearing of *BA* from station *B*. These two bearings should differ, of course, by exactly 180°. If there is a marked discrepancy in the forward and back bearings on two successive lines, when reading from their junction station, the presence of local magnetic attraction in the vicinity of that station may be assumed and the readings from this point ignored.

Thus, in the following series of readings, the forward and back bearings give differences reasonably near to 180° with the exception of the two bearings read from station *C*. It will be seen that the back bearing from *C* to *B* is 4° less than the forward bearing from *B* to *C* plus 180° and the forward bearing from *C* to *D* is 4° 15′ less than the back bearing from *D* to *C* minus 180°. We are entitled to assume, therefore, that local attraction has, in all probability, produced these effects at station C, particularly as the extent of the divergence is

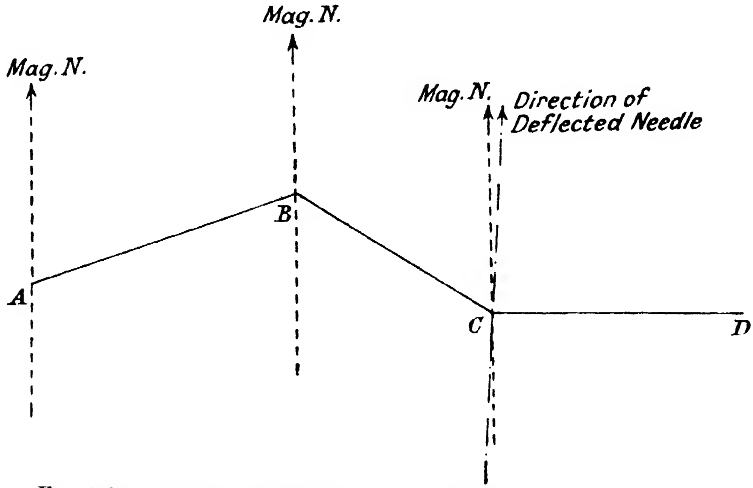


FIG. 5.24.—Magnetic Disturbance in a Prismatic Compass Traverse.

about the same for both lines meeting there and it may be deduced from the figures that the needle has been deflected towards the east, as shown in fig. 5.24.

Line	Forward Bearing	Back Bearing	Difference
AB . .	72° 15′	(BA) 252° 00′	179° 45′
BC . .	122° 30′	(CB) 298° 30′	176° 00′ (− 4° discrepancy)
CD . .	87° 00′	(DC) 271° 15′	184° 15′ (+ 4° 15′ ,,)
DE . .	137° 00′	(ED) 317° 00′	180° 00′
EF . .	89° 30′	(FE) 269° 00′	180° 30′

The "scale and protractor" method is used for plotting a prismatic compass traverse. The graphical "axis of correction" method may be used for adjusting a closing error in a closed traverse of this type.

Approximate Measurement of Angles using a Box Sextant. The box sextant, shown diagrammatically in fig. 5.25, is an elaboration of the optical square, described in Chapter II, page 18, but instead of the stationary mirrors in the latter instrument, one mirror, M_2 known as the "horizon glass", is fixed in position and silvered on its lower half only, while the other, M_1, known as the "index glass", may be rotated about a vertical axis and is entirely silvered. Both are enclosed in a brass case about 3 inches in diameter and the movable mirror is rotated by a milled-edged disc attached to a spindle which protrudes through the top of the case. The angle of rotation is recorded by a vernier arm, directly connected to the mirror, and reading on a graduated silver arc on the upper surface of the box. The arc is usually divided into degrees and halves and the vernier enables angles to be read to one minute, a hinged magnifying glass being provided to enable the fine graduations to be seen more easily. The rim of the box is cut away at F and G and has a sighting aperture at E. When the vernier reads 0° the mirrors will be parallel if the instrument is correctly adjusted.

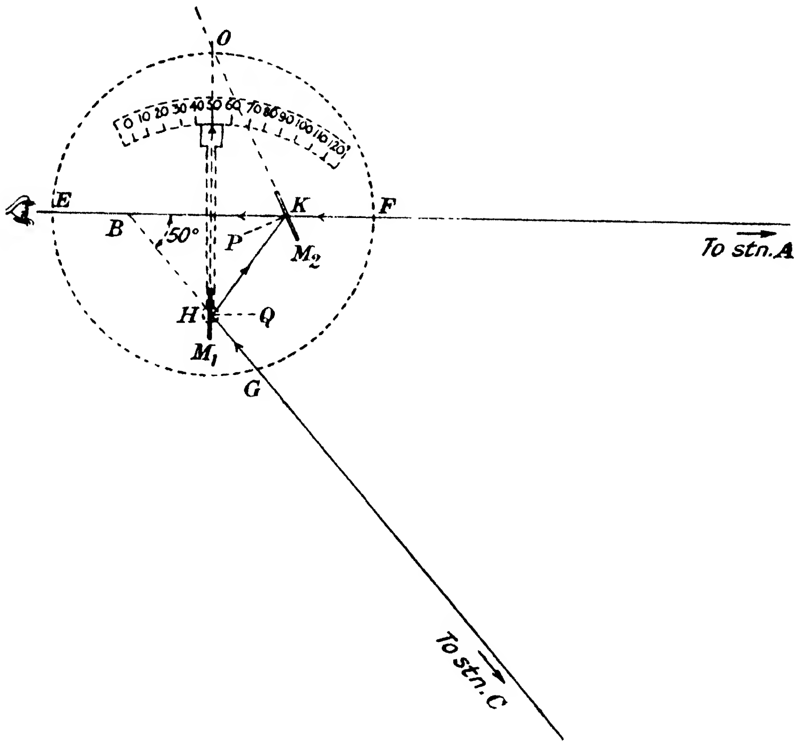


Fig. 5.25.—Principle of the Box Sextant.

To measure an angle ABC, the instrument is mounted on a short ranging rod placed at station B and turned until a direct view of the rod at A is obtained through the unsilvered portion of mirror M_2 and the opening G is opposite the general direction of the line BC. The milled-edged disc is rotated until the image of the rod at C is seen in the mirror M_2 immediately beneath the rod at A and the angle is then read off the graduated arc by means of the vernier. The principle of the instrument is based on the fact that the angle between the mirrors for any setting giving coincidence between one rod and the image of another is half the angle subtended by the rods at the instrument.

This may be proved by the following simple geometry: Let a ray from C be reflected from mirror M_1 (index glass) at H and from mirror M_2 (horizontal glass) at K, passing thence to the sighting aperture, E; let HQ and KP be the normals to the mirrors and let the planes of the mirrors intersect at O.

Then

$$\angle ABC = 180° - (\angle BHK + \angle HKB) = 2.\angle KHQ - 2.\angle HKP,$$

by the laws of reflection, i.e.

$$\begin{aligned} ABC &= 2.(90° - \angle OHK) - 2.(\angle OKH - 90°) \\ &= 2.[180° - (\angle OHK + \angle OKH)] \\ &= 2.\angle KOH \end{aligned}$$

For this reason the numbering on the graduated arc is double the actual value of the subtended angles, i.e. a movement of the vernier arm corresponding to n degrees reads $2n$ on the scale and hence the latter gives the value of the angle between the observed lines and not the angle between the mirrors. Strictly speaking, the centre of rotation of mirror M_1 should be vertically above the station.

Most box sextants will only read up to 120° and an angle greater than this must be divided into two or more parts by additional ranging rods and the parts measured separately.

The test for zero error should be made from time to time by sighting on to a distant rod, after setting the vernier to zero, and ascertaining whether the reflected image of the rod falls in the same vertical line as the rod itself, seen by direct vision. If not, the fixed mirror can be rotated very slightly to give perfect coincidence by means of the special key supplied with the instrument which engages with a small square headed bolt protruding through the side of the case. Movement of this bolt is transmitted to the frame in which the fixed mirror is mounted.

The box sextant should only be used on fairly level ground and angles measured under these circumstances will be far more accurate than the whole circle bearings obtained by means of a prismatic compass, but both instruments are only intended for approximate work and cannot be regarded as really practical appliances. The box sextant might conceivably be of use, however, for determining the approximate value of the angles in a small survey where the arrangement of the lines in well shaped triangles is not possible and when a theodolite is not available.

A theodolite in correct adjustment will give the true value of an angle, i.e. the horizontal projection, whatever may be the inclination of the telescope while sighting, but a box-sextant will not do so unless held horizontally.

CHAPTER VI

TACHEOMETRY

Theoretical Principles. The utility of the tacheometric method of measurement in connection with contouring has already been mentioned in Chapter IV, pages 92 *et seq.* It was stated that the readings of the outer cross wires on the staff determine the distance between the staff and the instrument, but the principle on which the method is based has not yet been explained.

It may be rendered clear by considering the paths taken by selected rays of light in travelling to the stadia diaphragm from the distant staff, taking advantage of two rules which govern the transmission of light through the object glass of the instrument. These rules may be stated thus:

(1) A ray travelling parallel to the optical axis of the telescope will be deflected through the principal focus of the object glass, this point being located outside the instrument at a distance of a foot, or so, from the lens; and

(2) A ray passing through the optical centre of the object glass will be transmitted without deviation.

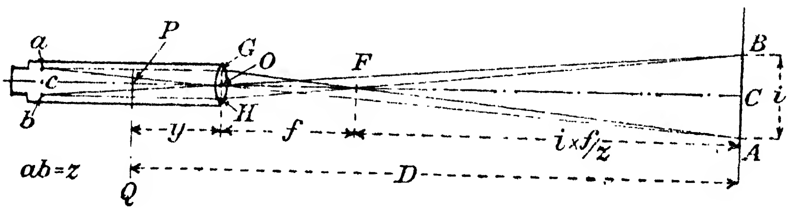


FIG. 6.1.—The Principle of Tacheometry. Horizontal Sight.

To simplify matters, these rules will first be applied to the case of a horizontal sight, illustrated in fig. 6.1. Let a, b and c represent the three horizontal stadia wires, O the optical centre and F the principal focus of the object glass and A, B, C, points on a staff intersected by the stadia wires. Consider the two rays aOA and bOB, passing through the top and bottom cross wires respectively and the optical centre O. These rays will not be deviated.

Consider the rays aG and bH, parallel to the optical axis cO, and neglect the refraction in the object glass. These will be deviated through the principal focus F at a constant angle, irrespective of the focussing adjustment of the telescope and their paths are represented by $aGFA$ and $bHFB$. The intersection of two rays defines a point on an image and hence, when the instrument is correctly focussed, the image of the point B on the staff will be formed at b and the image of A at a, or, in other words, an inverted image of the portion AB of the staff will be formed at ab, giving the familiar appearance of the cross wires cutting the staff at the readings A, B and C.

Incidentally, this diagram of the light rays offers a simple explanation of the fact that levels and theodolites produce inverted images, unless a special form of " erecting eye-piece " is fitted to the instrument.

The difference between the staff readings at A and B is termed the " Staff Intercept " and since the angle of divergence of the rays at the principal focus, F, is constant, the staff intercept will be proportional to the distance FC between the focal point and the staff.

To a very close approximation, we may consider GFO and FCA to be similar triangles, from which we obtain the relationship

$$\frac{CA}{GO} = \frac{FC}{OF}$$

But $GO = ac$. Hence

$$\frac{CA}{ca} = \frac{FC}{OF} \text{ or } FC = OF \times \frac{CA}{ac}$$

Also $CA = \frac{1}{2}.AB$ and $ac = \frac{1}{2}.ab$

Therefore,

$$FC = OF \times \frac{AB}{ab} = \text{staff intercept} \times \frac{\text{focal length of object glass}}{\text{spacing of outer crosswires}}$$

Let staff intercept, or difference in readings of outer cross wires, $= i$; focal length of object glass $= f$, and spacing of outer crosswires $= z$.

Then $$FC = i.\frac{f}{z} = i \times \text{a constant } k$$

By proportioning the spacing of the cross wires in relation to the focal length of the object glass the constant k may be given any convenient numerical value and 100 is usually adopted by the instrument makers.

The distance FC, however, is not the length required. For surveying purposes we wish to know the distance from the staff to the vertical axis of the instrument, PQ, from which the plumb bob is suspended and centred over the station peg. This distance is made up of three parts : the length FC plus the focal length of the object glass, f, plus the distance from the object glass to the vertical axis, y. Furthermore, in an instrument with internal focussing, $f + y$ is a constant length, which may be conveniently called k'. In an instrument with external focussing the variation in the value of $f + y$ will be very small. Hence, if D is the distance between the staff and the vertical axis of the instrument:

$$D = FC + f + y = i \times k + k'$$

k is termed the multiplying constant and k' the additive constant.

Case of Inclined Sight. With a theodolite, as distinct from a level, tacheometric measurements may be carried out with an inclined

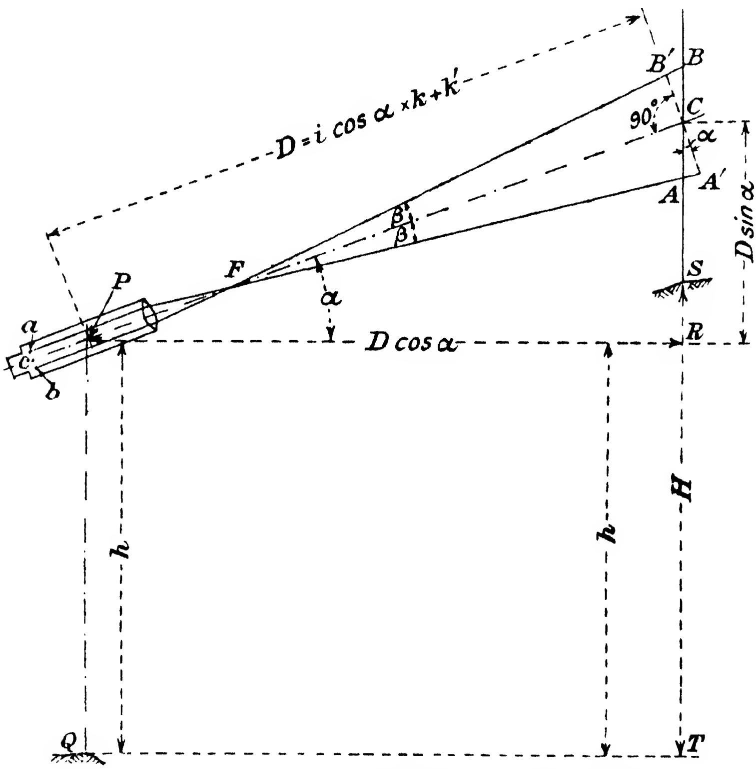


Fig. 6.2.—The Principle of Tacheometry. Inclined Sights.

line of sight, thus enormously increasing the scope of the instrument. The basic principle of the process remains unchanged, but certain amendments must be made to the horizontal sight formula already derived.

Let fig. 6.2 represent the conditions when the telescope of the theodolite is inclined at an angle of elevation α, the staff being held vertically, and the letters in the diagram corresponding to those in fig. 6.1. Very occasionally surveyors prefer to hold the staff normal to the line of sight, but this method need not be considered. The staff intercept, AB, as read, must first be converted into an equivalent intercept, $A'B'$, perpendicular to the optical axis. Although not strictly correct from a mathematical point of view, it will be a very close approximation to the actual conditions if we write $B'C = BC.\cos B'CB = BC.\cos \alpha$. This assumes that $\angle CB'B$ is 90° which, of course, is not mathematically true. Assuming that the multiplying constant of the instrument is 100, however, as it usually is, tan $B'FC$ will be very nearly $\frac{1}{200}$ or 0·005, so that $\angle B'FC$ is only about 17 minutes. Consequently $\angle CB'B$ is 90° 17′ instead of 90° and for all practical purposes no error will be introduced by neglecting this small difference.

This calculation, however, involves the further approximation that $BC = CA$ and the closeness of this assumption to the actual facts depends upon the angle of elevation of the telescope. If we call $\angle BFC$, or $\angle AFC$, β, it will be seen from fig. 6.2 that $\angle B'BC = 90° - \beta - \alpha$, $\angle CAA' = 180° - [(90° - \beta) + \alpha]$, or $90° + \beta - \alpha$, $\angle BB'C = 90° + \beta$ and $\angle CA'A = 90° - \beta$.

Hence, by the sine formula for a triangle:

$$\frac{AC}{A'C} = \frac{\sin(90° - \beta)}{\sin(90° + \beta - \alpha)}$$

and

$$\frac{BC}{B'C} = \frac{\sin(90° + \beta)}{\sin(90° - \beta - \alpha)}$$

But $A'C = B'C$, hence

$$\frac{AC}{BC} = \frac{\dfrac{\sin(90° - \beta)}{\sin(90° + \beta - \alpha)}}{\dfrac{\sin(90° + \beta)}{\sin(90° - \beta - \alpha)}} = \frac{\sin(90° - \beta - \alpha)}{\sin(90° + \beta - \alpha)}$$

We have seen that for a multiplying constant of 100, β will be about 17 minutes and except in very exceptional circumstances α, the angle at which the telescope is inclined, will not exceed 45°. Substituting these values for β and α in the above expression, the ratio $\frac{AC}{BC}$ becomes 0·990. As the value of α decreases, this ratio approaches more nearly to unity, thus when $\alpha = 10°$, $\frac{AC}{BC}$ becomes 0·998.

Accepting this assumption, therefore, we may write

$$A'B' = AB.\cos \alpha$$

Then, as for a horizontal sight,

$$FC = k.A'B' \text{ and } PC = k.A'B' + k'$$

Hence $$PC = k.AB.\cos \alpha + k'. = i.k.\cos \alpha + k'$$

The horizontal distance, D, between the staff and the vertical axis of the theodolite is PR and from the figure,

$$PR = PC.\cos\alpha, \text{ i.e. } D = i.k.\cos^2\alpha + k'.\cos\alpha,$$

where i is the staff intercept and k and k' are constants for the instrument.

Determination of Heights. It is also possible to determine tacheometrically the difference in level between the instrument station and the point at which the staff is held, thus :

Let S be the base of the staff and Q the ground level at the instrument. Then, if QT is a horizontal line and if h is the height of the trunnion axis of the theodolite, the difference in level, H, between S and Q is given by

$$\begin{aligned}
& TR + RS \\
&= h + RS \\
&= h + CR - CS \\
&= h + PC.\sin\alpha - \text{reading of middle cross wire} \\
&= h + i.k.\cos\alpha.\sin\alpha + k'.\sin\alpha - \text{reading of middle cross wire} \\
&= i.k.\cos\alpha.\sin\alpha + k'.\sin\alpha + \text{height of trunnion axis} - \text{reading} \\
&\qquad \text{of middle cross wire.} \\
&= i.k.\frac{\sin 2\alpha}{2} + k' \sin\alpha + \text{ht. of instrument} - \text{reading of middle} \\
&\qquad \text{cross wire.}
\end{aligned}$$

The angle will be positive or negative according as to whether the telescope is pointed upwards or downwards, and due regard must be paid to algebraic signs in the above formula. The summation, if positive, indicates a rise from the instrument station to the staff position and if negative, a fall.

It must be emphasised that the staff should be held in a truly vertical position when readings are being taken with inclined sights, but the procedure of swinging, described in Chapter III, page 77, is no longer applicable when the line of collimation is not horizontal. A plumb-line or spirit level attached to the staff will consequently be found of value and some such method of ensuring verticality should undoubtedly be adopted.

Elimination of Additive Constant. The insertion of a correctly proportioned lens between the object glass and the eyepiece enables the additive constant, k', to be obviated and greatly simplifies the formulae for distances and heights. This additional fitment is called an " anallactic lens " and the advantage of eliminating the additive constant outweighs the disadvantage of a slight loss of optical power which always accompanies the addition of a further lens to the telescope. The principle of the anallactic lens may be understood from the diagram of rays given in fig. 6.3. L represents the anallactic lens which occupies a fixed position on the telescope axis at a constant distance from the object glass O and F' is its principal focus. The cross wires are a, b and c, and considering, again, the rays passing through a and b in a direction parallel to the optical axis, these will be deviated after passing the lens L and will pass through the focal point F', travelling thence, at a constant angle of deviation to the object glass, O. The object glass will refract the rays in the manner shown and they will give a staff intercept AB. Since the angle of deviation of the rays at F' is constant, the angle of emergence of the rays from the object glass will also be constant. Let the rays be produced backwards to intersect the optical axis at P and let the constant angle BPC be θ. Then $PC = BC.\cot\theta = \frac{1}{2}.AB.\cot\theta$. Hence, if the point P could be made to coincide with the centre of the instrument, i.e. with the intersection of the vertical and longitudinal axes, PC would become the required distance D and we could write :

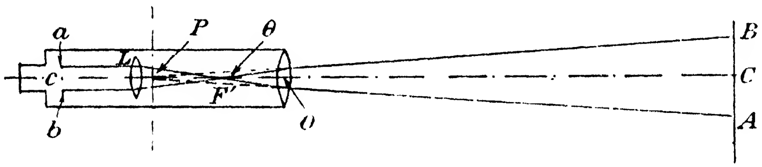


FIG. 6.3.—The Principle of the Anallactic Lens.

$D = AB.\frac{1}{2}\cot\theta = AB \times$ a constant. By correctly proportioning the various parts of the instrument, the makers are able to equate $\frac{1}{2}\cot\theta$ to a convenient round number usually 100, thus giving the simple result that the distance from the staff to the instrument is the staff intercept multiplied by a constant. The additive correction is reduced to zero and the formulae previously derived may be expressed as follows in the case of an instrument fitted with an anallactic lens :

With a horizontal sight : $D = i.k$.

With the telescope inclined at an angle α, $D = i.k.\cos^2\alpha$ and $H = i.k.\cos\alpha.\sin\alpha +$ height of trunnion axis $-$ reading of middle cross wire. In order to maintain a constant distance between the anallactic lens and the object glass, the focussing in older tacheometric instruments is often accomplished by adjusting the eye-piece end of the telescope where a short length can be racked in or out without disturbing the position of the anallactic lens. It is difficult to combine the anallactic lens with the internal focussing lens now commonly adopted in levels and theodolites, but instruments can now be obtained in which internal focussing is retained and the additive constant is so small that it can safely be neglected. Two prominent makers, for example, produce instruments in which the additive correction is only half an inch for sighting distances over 40 or 50 feet.

Correction to Distance with Inclined Sights. The correction factor for obtaining the horizontal distance from the staff to the instrument when reading with an inclined telescope is the square of the cosine of the angle of inclination. For small angles this correction may be neglected and the following table gives values of $\cos^2 \alpha$ up to an angle of 7°. In deciding the limit to be placed on the angle α beyond which corrections must be applied, the length of the sight and the scale of the drawing must be taken into account.

α in degrees	$\cos^2 \alpha$
1	0·9997
2	0·9987
3	0·9973
4	0·9951
5	0·9924
6	0·9891
7	0·9852

The Utility of Tacheometry in Road Surveys. In rough and hilly country, particularly where most of the topographical detail is somewhat indefinite in character, the adoption of tacheometric methods may be recommended, both for locating topography and for obtaining approximate spot levels. It is a very useful method in the case of preliminary surveys for projected new roads in the type of country where linear measurements with chain and tape can only be carried out with difficulty and, in such circumstances, is quite as accurate as chaining, but it would scarcely be used in comparatively flat areas containing a great deal of definite topography, such as buildings which require accurate location.

One reason for drawing this distinction lies in the fact that topographical detail obtained by tacheometry is fixed on the plan by scaling distances along a series of lines which radiate from each station point at various angles from a reference direction and these lines are normally set out by a protractor. This introduces a certain amount of approximation which will usually be permissible in the case of indefinite detail such as hedges, rough tracks, ridges or banks of streams, but would not be suitable in the case of more accurately fixed topography. As mentioned on previous occasions, however, due consideration in all matters of accuracy must be paid to the scale and the purpose of the drawing.

Tacheometric methods may also be employed for locating points difficult of access for taping, such as the remote bank of a river to which a tape cannot be taken, or the upper edge of a quarry from which horizontal linear dimensions cannot be obtained. In difficult circumstances such as these, tacheometry is very useful since the location and level of any point can be determined provided that the staff-holder is able to reach it.

Fieldwork

I. Checking the Constants of the Instrument. Set up the theodolite on a level site and measure distances of 100, 200 and 300 feet from the plumb bob with an accurate chain or steel band. Take the readings of the outer cross wires, with the telescope horizontal, at each of these distances and read the central horizontal cross wire as a check. Let the respective staff intercepts be i_1, i_2 and i_3.

Then
$$100 = i_1.k + k'$$
$$200 = i_2.k + k'$$
$$300 = i_3.k + k'$$

from which equations the constants k and k' can be obtained. It is usual for the instrument maker to give this information on a label inside the lid of the instrument box, but a check may sometimes be desirable.

II. Location of Stations. It sometimes happens that tacheometry is used for locating the topographical detail on a traverse

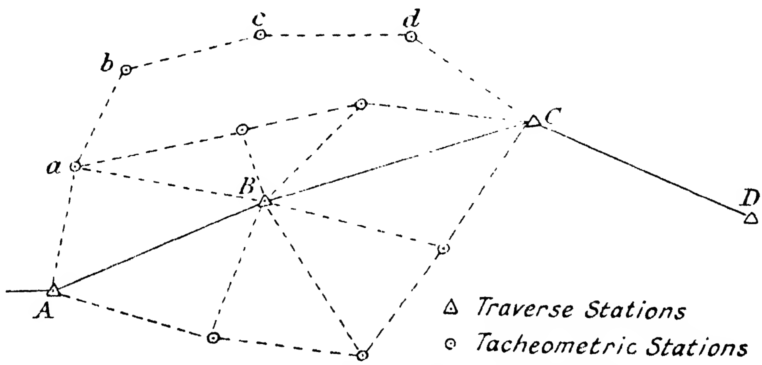


FIG. 6.4.—Arrangement of Tacheometric Stations.

instead of the chain and tape method, in which case the traverse stations may be used for tacheometric stations. The limiting length of sight with a 4- or 5-inch theodolite, when viewing the staff, is about 500 feet for accurate reading, although longer sights are frequently taken. If the distances between the traverse stations are to be determined tacheometrically and the lengths of the lines exceed 500 feet, or so, intermediate stations should be located, but whenever possible a tacheometric survey should be based on a series of main stations accurately fixed, either by normal traversing or by triangulation. The tacheometric observations can then be arranged in circuits and errors can then be localised. Intermediate tacheometric stations should be so placed that two other stations are visible therefrom, thus building up a system of triangles or subsidiary closed traverses and tying-in the work for checking purposes as shown in fig. 6.4.

III. Taking the Readings. Having centred and levelled the instrument, the vernier on the vertical circle should be set to zero in the manner described on page 123, since the angle of elevation must be observed for every sighting and except when reading to station points, these angles are taken on one face only.

One vernier on the horizontal circle is now set to zero, noting the particular vernier concerned, the upper plate is clamped, the lower plate released and the telescope pointed to another station to give a reference direction. If the tacheometric instrument station is a traverse point, the reference direction may be conveniently taken to another station on the traverse, but the procedure will depend, to some extent, on whether the traverse lines have been chained or whether their lengths are to be found tacheometrically. In the first case a long line can be used as the reference direction since it will only be necessary to sight on to a ranging rod at the distant point, but in the second case the other station must be near enough to enable a staff reading to be obtained with certainty.

The procedure will also differ slightly according to whether the traverse bearings or included angles have been obtained separately or whether they are to be obtained at the same time as the tacheometric measurements. In the latter case, the whole circle bearing of the first line should be determined and magnetic north used as the reference direction at the first station. On moving to the next station, the reference direction is taken to the station previously used, the horizontal vernier being set to zero and the telescope directed back to this point.

The lower plate is clamped at this setting, the upper plate released and the telescope directed to the staff held at various points to define the topographical details or to locate spot-levels. The staff holder should move round from point to point in a progressive circuit, without retracing his steps, thus saving time, and among the series of observations radiating from the instrument in this way, readings must be taken to the next station ahead. Unlike the readings taken only for locating topography or spot levels, however, the observations to station points must be taken with both faces. Readings will include the horizontal angle, the vertical angle of the telescope and the readings of the three cross wires on the staff. This information will give the angles for locating the rays meeting at the instrument station and their lengths can be readily calculated.

If intermediate stations are required, the lines joining them may form a small closed traverse with the main traverse lines as the closing links, but to make the checking effective the length of the main traverse lines must be chained if they are too long to be determined accurately by tacheometry. In fig. 6.4, *Aab* . . . *C* is a small subsidiary traverse of this kind and the whole circle bearing of Aa must be determined, together with the included angles BAa, Aab, abc, . . . dCB.

In the survey of a restricted area which can be covered from one instrument station, it is usual to take magnetic north as the reference direction, setting the instrument with the compass north and south and one horizontal vernier reading zero. Having oriented the instrument, the lower plate must be kept clamped and in all cases it is advisable to check back to the reference point from time to time to see that the zero direction has not deviated. In tacheometric work a great deal depends upon the intelligence of the staff holder. When locating an irregular boundary, for instance, it is essential that the staff should be held at all the necessary points for delineating its shape correctly and in obtaining spot levels it is equally important that the staff holder should locate the points where marked changes of gradient occur, in addition to well-defined high and low places. The staff positions are often at much greater distances from the observer than the usual length of the offsets in chain surveying and it is not easy for the man at the instrument to distinguish items in the topography or undulations in the ground at these long distances. It is therefore advisable to inspect the area before commencing to take readings, noting the important points and instructing the staff holder accordingly. This procedure is also of assistance in preparing the sketch which always accompanies the numerical data. Although the latter will automatically enable the points to be plotted, it is essential to have an accurately detailed sketch with full descriptive notes to enable the plan to be drawn satisfactorily.

It effects a considerable saving of time if two staff holders are employed, the instrument being directed to one while the other is moving to the next position. This procedure is very effective when points are to be observed on both sides of a river or a railway cutting. In this connection it may be mentioned that a simplified form of staff, such as that illustrated in fig. 4.6 is sometimes used for tacheometry, instead of the standard Sopwith pattern. The open graduations enable a reading to be taken at a greater distance but, of course, do not permit of accuracy in reading to a hundredth of a foot. For many jobs this will not be good enough if the multiplying constant is 100. Full use is made of the vertical movement of the telescope and readings should be taken as low down the staff as possible to minimise the errors due to the latter not being vertical. Small adjustments of the vertical motion may be made at each reading to bring either the lower cross wire or the middle cross wire on to an exact number of feet on the staff. In the former case this facilitates subtraction in calculating the staff intercept and in the latter case it is easy to check whether the middle reading is the mean of the outer readings, although it must be remembered that this rule cannot be strictly applied when the telescope is inclined at a large angle. Opinion is divided as to which procedure is the better, but on fairly level ground the vertical movement of the telescope required to bring either the lower or the middle cross wire on to a whole number of feet on the staff is usually so small that it will not involve a correction for the distance calculation, i.e. $\cos^2 \alpha$ may be neglected as it approaches so closely to unity.

Reading the Vertical Circle. The "zero" of the vertical circle previously referred to (see Chapter V), means, of course, the

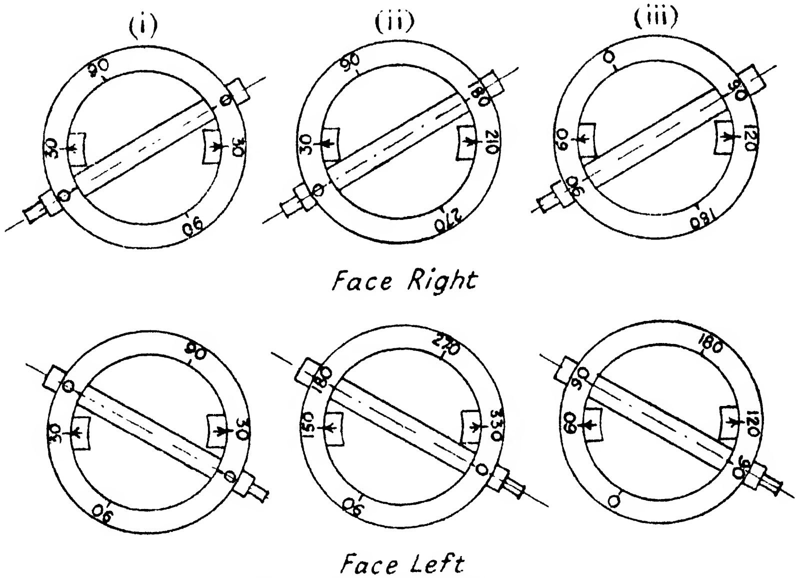


FIG. 6.5.—Methods of Numbering Vertical Circles.

datum reading when the telescope is horizontal, but, numerically, this is not always 0° 0′ 0″. Although the method of numbering the graduations on the horizontal circle is obvious, this is by no means the case with the vertical circle and fig. 6.5 shows three methods in common use. Type (i) gives the angle of elevation or depression directly on both verniers and with either face and is the most popular form. In type (ii), if readings are taken on the vernier originally set to 0°, with face right, an angle of elevation is obtained directly and an angle of depression is obtained by subtracting the vernier reading from 360°. With face left these conditions are reversed. This makes a clear distinction between angles of elevation and depression when the telescope is only slightly inclined from the horizontal. Type (iii), in which the numerical zero is at the top of the circle when the telescope is horizontal on one face, reads the " zenith distance ", i.e. the angle measure downwards from the zenith to the line of collimation. The angle of elevation or depression is then obtained automatically on one vernier with the correct sign by subtracting the zenith distance from 90°, e.g., if the reading of the vertical circle on this vernier is 75° 23′, the angle of elevation is 90° − 75° 23′, or 14° 37′, or if the reading of the vertical circle is 110° 38′, the angle of elevation becomes 90° − 110° 38′, or − 20° 38′, i.e. it is actually an angle of depression. The same vernier conforms to this rule for either face.

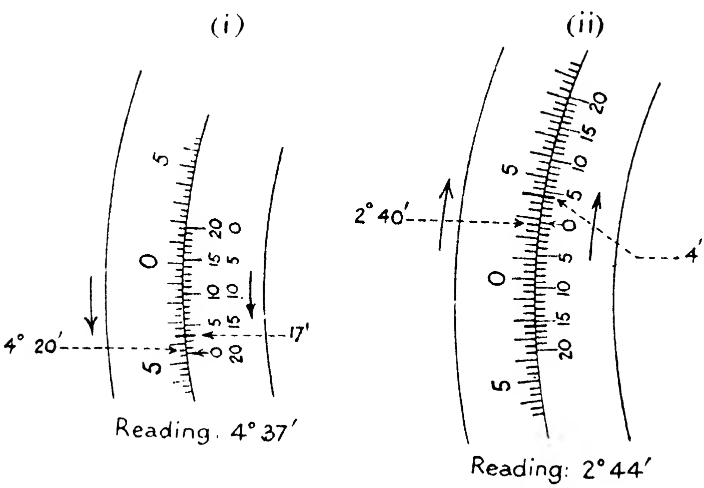


Fig. 6.6.—Vertical Circle Verniers.

Although identical in principle with the verniers on the horizontal circle, the vertical circle verniers are of the so-called " double " type to suit the case where the graduations on the main scale are numbered in two directions from a common zero. Double verniers may be of two kinds, both of which are shown in fig. 6.6. In type (i) the position of the arrow head relative to the main scale gives the first part of the reading. In the example, the graduations are degrees and thirds and the arrow is between 4° 20′ and 4° 40′, running downwards from the zero. The vernier graduations are numbered 0 to 20 in two directions, thus giving readings to single minutes, and the reading at the point of coincidence is taken from the numbering which runs in the same direction as the reading on the main scale. In the example shown, therefore, the numbering on the vernier must also run downwards and at the point of coincidence the vernier reading is consequently 17′, making the complete value of the angle 4° 37′.

In type (ii) the graduations shown are of the same magnitude as those in the previous example, but the arrow head of the vernier is located centrally and the numbering of the graduations runs up and down to correspond with the main scale. The vernier reading at the point of coincidence is taken from the upper or lower part of the scale, according to whether the arrow head is to the upper or lower side of the zero on the main scale. Thus the reading in the example shown is $2° 40'$ on the main scale, upwards from the zero, plus 4 minutes from the upper vernier scale, making the value of the angle $2° 44'$. A second point of coincidence occurs on the wrong half of the vernier and care must be taken not to read this point by mistake. In fig. 6.6 it will be seen that there is coincidence at the 16-minute reading on the *lower* half of the vernier, for example. This reading is ignored although it gives the correct angle if *subtracted* from the main scale reading *beyond* the zero of the vernier ($3° - 16' = 2° 44'$).

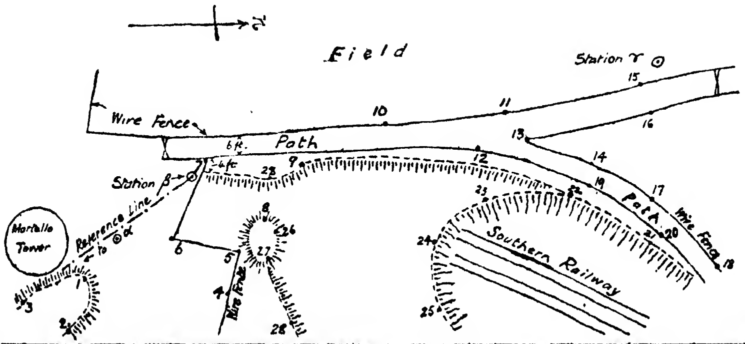
Booking Tacheometric Data. The information to be booked is as follows :

(1) The name of the instrument station.
(2) The reference direction to which the zero on the horizontal plate is directed.
(3) The identification number or letter for each staff position with the horizontal angle between the reference direction and line of collimation to the staff position.
(4) The readings of the three cross wires.
(5) The height of the trunnion axis of the instrument which may be read directly from a staff placed against the theodolite and held vertically, or by taping.
(6) The reduced level of the instrument station, generally obtained from accurate levelling carried out previously.

The numerical data must be supplemented by a sketch showing the station points and staff positions, the latter being numbered or lettered to correspond with the booked figures.

The clarity of the sketch is just as important in tacheometry as it is in topographical location by chain and tape and it should be borne in mind that an apparently comprehensive set of readings covering a large area often looks somewhat sparse when plotted. Small dimensions such as the widths of roads and paths, the distances between gate-posts, and the measurements of buildings should be obtained with a tape to supplement the tacheometric readings and the staff holder can carry out these measurements quite frequently single-handed, giving the results to the instrument man for booking on the sketch. The staff may, of course, be used for obtaining such dimensions instead of a tape. Additional descriptive notes in the " Remarks " column of the field book should always be made if the information is likely to prove of use either in plotting, or in the designing of future works, and an efficient surveyor is always on the look-out for such information and should not grudge the time occupied in entering the notes.

Specially ruled tacheometric field books can be obtained, but the method of ruling and the headings of the columns are not standardised. An excellent form of booking, devised by the late Professor Ormsby,



Station and Zero.	Point.	Bearing.	Readings.	Generating Number.	Zenith Distance.	Angle of Elevation.	Difference of Level.	Theodolite and Middle Reading.	Difference.	Rise.	Fall.	Reduced Level.	Distance.	Remarks.
		° ′ ″			° ′ ″	° ′ ″	ft						ft.	
β α = 0								4·75				259 90		
	1	356 9 30	{2·38 3·64	126	96 42 30	−6 42 30	−14 62	3 0	+1·75		12·87	247·03	124·3	
	2	336. 31. 30	{3·29 4·71	142	93 40 0	−3 40 0	−9·06	4·0	+0·75		8·31	251·59	141·5	
	3	359 37.0	{1·03 2·97	194	93 35 0	−3 35 0	−12 10	2·0	+2 75		9·35	250·55	193·3	
	4	309.20.0	{5·60 6·40	80	81 57 0	+08 03 0	+11·09	6·0	−1·25	9·84		269·74	78 6	
	5	276.53.0	{1·72 2·26	54	77 05 0	+12 55 0	+11·76	2·0	+2 75	14·61		274·41	51·3	
	6	336.07.0	{1·93 2·07	14	90 07 30	−0 07 30	−1·81	2·0	+2·75	0·94		260·84	13 8	
	7	178.19.0	{2·90 3·10	20	90 22 0	−0 22 0	−0·12	3·0	+1·75	1·63		261·53	10·0	
	8	247.51.0	{3·61 4·38	77	78 42 30	+11 17 30	+14·79	4·0	+0·75	15·64		275·44	74·0	
·	9	230.29.30	{5·58 6·42	84	79 29 0	+10 31 0	+15 08	6 0	−1 25	13·83		273·73	80·1	
	10	201.11.0	{5·53 7·47	194	78 24 0	+11 36 0	+38·22	6·0	−1·25	36 97		296·87	186·2	

FIG. 6.7.—Field Notes and Sketch for part of an actual Tacheometric Survey.

of University College, London, is shown in fig. 6.7, which is reproduced from some actual fieldwork notes. The instrument station is entered in the first column and the first reading will be taken to a reference point, usually another station with the horizontal circle reading zero. If magnetic north is used as the reference direction " Mag. N " may be entered in the second column with the corresponding bearing 0° 0′ 0″. If a survey station is used as the reference point its identification letter is entered in the first column under the instrument station. The outer cross wire readings are entered in column 4 and their difference gives the staff intercept, i. The " Generating Number " is the quantity $i.k + k'$, where k is the multiplying constant and k' the additive constant, the latter being zero if the instrument has an anallactic lens. The generating number is so called because it gives the distance from the instrument to the staff and the difference in level between the instrument station and the staff position when the appropriate factors are applied.

The heading " Zenith Distance " is for use with a vertical circle having the 0° reading at the top and graduated downwards on both sides to 180° at the bottom. The angle of elevation will then be 90°— the zenith distance and will be positive or negative accordingly and must be booked with due regard to sign. The " difference in level " is really the vertical difference in height between the trunnion axis of the instrument and the point on the staff at the middle hair reading. Referring again to fig. 6.2, page 161, it is the dimension CR, and is given by the formula $i.k.\sin\alpha.\cos\alpha + k'.\sin\alpha$, or for an instrument with an anallactic lens, $i.k.\sin\alpha.\cos\alpha$, or $i.k.\frac{1}{2}\sin 2\alpha$. In this case the " difference in level " is the generating number $\times \sin\alpha.\cos\alpha$. The height of instrument (h in fig. 6.2) is the height of the trunnion axis above the ground and is entered opposite the corresponding station letter, at the top of the column, followed beneath by the middle cross wire readings for the various staff positions. The next column, headed " Difference ", is intended for the difference between the height of instrument and middle cross wire readings at the various staff positions and may be positive or negative. It may obviously be made zero by setting the middle cross wire at a staff reading h. The " Difference of level " and " Difference " are summed algebraically for each point, in conformity with the rule, quoted on page 163:

Rise or fall from station to staff position

$= i.k.\cos\alpha.\sin\alpha + k'.\sin\alpha +$ height of instrument

— reading of middle cross wire.

$K'.\sin\alpha$ is, of course, zero with an anallactic lens. The algebraic summation of the " Difference of level " and the " Difference ", if positive, indicates a rise and, if negative, a fall from the instrument station to the staff position. The rise and fall columns are filled in accordingly and the reduced level of each point calculated in the usual way, starting from the known reduced level of the instrument station. The distance from the instrument station to the staff is given by the formula $i.k.\cos^2\alpha + k'.\cos\alpha$, or, with an anallactic lens, $i.k.\cos^2\alpha$, or, in other words, the generating number $\times \cos^2\alpha$. The instrument used for the fieldwork shown in fig. 6.7 was fitted with an anallactic lens and had a multiplying constant of 100. The generating number is therefore the staff intercept $\times$ 100. In this example the numerical results are only given for the first ten points observed from station β.

Effect of Inclined Staff. The importance of holding the staff vertical when reading with inclined sights has already been emphasised and the error introduced by tilting the staff may be estimated approximately in the following way.

In fig. 6.8, let AB represent a vertical staff, BC the staff tilted at an angle θ to the vertical and O the instrument. If OAC is a sight inclined at an angle α to the horizontal, the correct reading would be

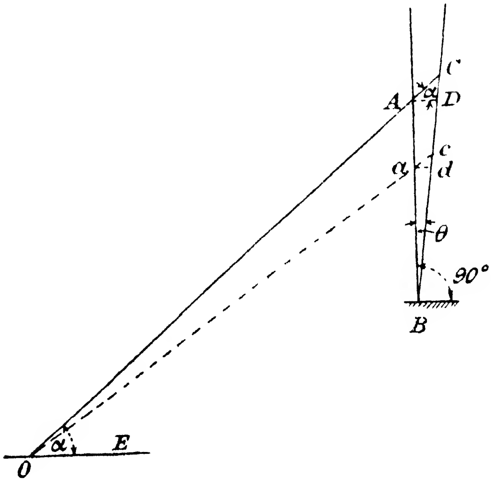


FIG. 6.8.—Error due to Inclined Staff in Tacheometry.

BA and the incorrect reading BC. The error may be approximately represented by CD, where AD is drawn perpendicular to AB.

Then $\qquad \angle DAC = \angle EOC = \alpha$

and $\qquad \angle ACD = 180° - (\theta + 90° + \alpha) = 90° - (\theta + \alpha)$.

Therefore $\qquad \dfrac{CD}{AD} = \dfrac{\sin \alpha}{\sin 90° - (\theta + \alpha)} = \dfrac{\sin \alpha}{\cos (\theta + \alpha)}$.

But $\qquad AD = AB . \tan \theta$.

Therefore $\qquad CD = AB . \tan \theta . \dfrac{\sin \alpha}{\cos (\theta + \alpha)}$.

If θ is very small compared with α, $CD = AB . \tan \theta . \tan \alpha$, nearly. If we consider another sight Oac, inclined very nearly at the same angle α, such as would be the case with the sights from the outer cross wires of the tacheometer, we may write approximately:

$$cd = aB . \tan \theta . \tan \alpha.$$

The correct staff intercept is Aa and the incorrect staff intercept Cc. Hence the error in the staff intercept is $Cc - Aa$, which is equal to $Cc - Dd$, or $Cc - (Cc - CD + cd)$, i.e. $CD - cd$, or its equivalent $(AB - aB).\tan\theta.\tan\alpha$ or $Aa.\tan\theta.\tan\alpha$.

This error, though small when θ and α are small, becomes of some importance when θ and α are larger, as the following table shows:

VALUES OF $\tan\theta.\tan\alpha$

θ degrees	α degrees			
	1	3	10	45
1 . . .	0·0003	0·0009	0·0030	0·0175
2 . . .	0·0006	0·0018	0·0061	0·0349
5 . . .	0·0015	0·0046	0·0154	0·0875

Plotting a Tacheometric Survey. The instrument stations, acting as control points, must be plotted from co-ordinates derived from the whole circle bearings and the lengths of the lines joining them, but the remaining points are located on the drawing by scale and protractor. Tacheometric tables, charts and slide rules are obtainable to expedite the calculations of distances and levels, and ingenious tacheometric protractors, which combine the usual angular graduations with a linear scale, are supplied by instrument makers at reasonable prices.

Time-saving aids should always be adopted if much tacheometric plotting is to be carried out, thus taking the fullest advantage of the rapidity with which the tacheometric data can be obtained in the field.

The Tacheometric Plane Table. In plane table surveying the plan is drawn in the field. This procedure has several obvious advantages. Details in the topography are not so likely to be missed as they are when measurements are taken by the ordinary chain and tape method for subsequent plotting in the drawing office and doubtful points can be corrected immediately on the ground.

It often happens that the draughtsman responsible for plotting a survey from field notes has never seen the actual site and unless the notes are booked clearly and are very comprehensive, uncertainties inevitably occur and their interpretation is frequently left to guesswork—a thoroughly bad practice.

In plane table work, with a suitable instrument, an experienced surveyor can produce an accurate and fully detailed plan much more quickly than by chain and tape methods followed by the usual plotting.

The necessary equipment consists of a drawing board mounted on a tripod fitted with a head which enables the board to be levelled, rotated, and clamped in any position, together with an " alidade " which rests on the board and forms the measuring and observing instrument. The tripod head should preferably include a slow-motion device which enables the rotary movement of the table to be carried out with precision.

Simple forms of alidade are obtainable but the only practical type consists of a tacheometric telescope mounted on a vertical support, the base of which forms a straightedge parallel to the longitudinal axis of the telescope. The latter can be tilted in a vertical plane or set horizontally, exactly like a theodolite telescope and its angle of inclination is recorded on a graduated arc in conjunction with a vernier, the arc, of course, moving with the telescope.

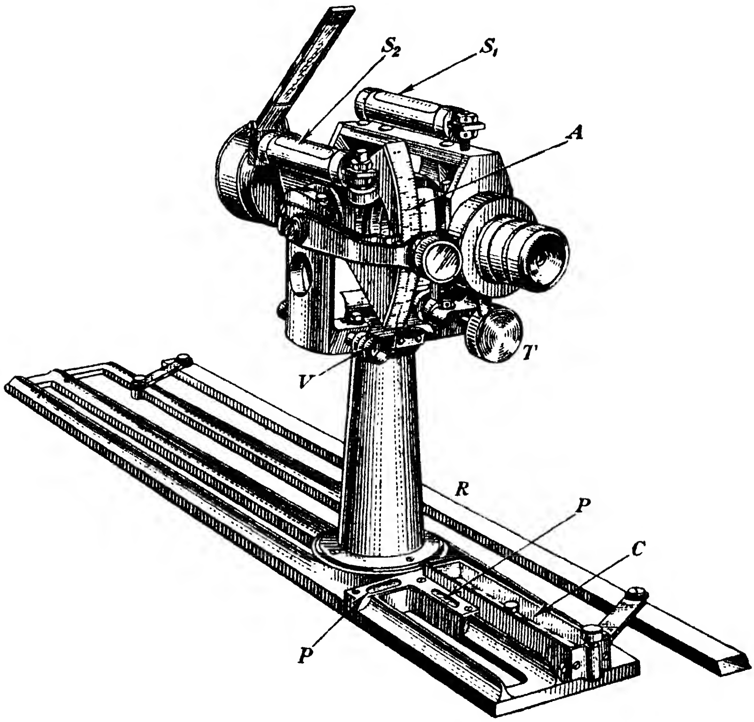


FIG. 6.9 (*a*).—Alidade.

The accompanying drawing (fig. 6.9 (*a*)) illustrates a typical alidade of this type in which the following features will be noticed :

A reversible and detachable spirit level, S_1, which can be placed on the telescope or used to check the levelling of the drawing board ; a second spirit level, S_2, attached to the bracket of the vernier working in conjunction with the graduated arc, A ; a tangent screw, T, actuating the tilting movement of the telescope ; a tangent screw, V, which alters the setting of the index bubble, S_2 and, simultaneously, the position of the vernier; a trough compass, C; small spirit levels, P, P, on the base of the alidade; a parallel ruler, R, which increases the range of the straightedge. As will be seen later, the process of plane tabling involves the drawing of rays along, or parallel to, the straightedge through fixed points on the plan and the parallel rule allows the alidade to be slightly offset from the point.

A stadia diaphragm is provided and the telescope so designed with an internal focussing lens as to give a simple multiplying constant, almost invariably 100, and a negligible additive correction.

If the instrument is required for obtaining spot-levels in addition to locating topography, it is essential that some reducing device should be incorporated to obviate the necessity of working out the factor $i.k.\sin\alpha.\cos\alpha$ for every reading, and in hilly country, necessitating inclined sights of more than 5°, or so, some device will be needed for obtaining rapidly and easily the $\cos^2\alpha$ correction for distances.

Both these requirements are admirably satisfied by the Beaman Arc.

The Beaman Arc. This device only involves the addition of two further sets of graduations to the standard graduated arc. The latter gives the angle of tilt of the telescope, usually to the nearest minute.

The complete Beaman arc is illustrated in fig. 6.9 (b). The right-hand scale, Z, is an ordinary arc, graduated in degrees and halves, which, in conjunction with the vernier, V, gives a reading to minutes. The centre set of graduations gives the unit percentage corrections for the reduction of distances to the horizontal, or, in other words, the whole number values of n in the equation

$$100 - 100.\cos^2\alpha = n.$$

The approximate values of α corresponding to values of n from 1 to 10 are tabulated below:

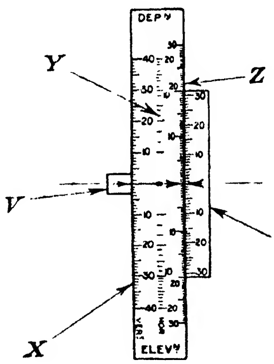


FIG. 6.9 (b).—Principle of the Beaman Arc.

% Correction (n)	Angle of Tilt to Nearest Minute (α)	% Correction (n)	Angle of Tilt to Nearest Minute (α)
1	5° 45′	6	14° 11′
2	8° 6′	7	15° 20′
3	9° 59′	8	16° 25′
4	11° 32′	9	17° 28′
5	12° 55′	10	18° 26′

The left-hand scale, X, is calibrated in whole number values of $k.\sin\alpha.\cos\alpha$. With the customary multiplier of 100, this factor becomes 50.sin 2α and the following values of α reduce this expression to simple whole numbers:

α	50.sin 2α	α	50.sin 2α
0° 34′ 30″	1	2° 17′ 30″	4
1° 9′ 0″	2	2° 52′ 30″	5
1° 43′ 30″	3	3° 27′ 0″	6

These numbers appear on scale X opposite the corresponding angles on scale Z.

The centre and left-hand scales are read against the index mark, V.

To use the Beaman arc the telescope is focussed on any convenient part of the distant staff and the arc bubble levelled by the appropriate tangent screw. The telescope is then adjusted by its vertical motion tangent screw until the left-hand scale arrives at the nearest graduation to give a whole-number reading at the index line and the staff readings are taken for the three horizontal cross wires. Suppose, for example, that the multiplying factor is set at 3 and the stadia interval is then 3·42, simple multiplication gives the difference in height of the middle cross wire reading and the telescope pivot as 10·26 feet. If the middle reading is 5·07, the difference in height between the ground at the staff position and the telescope pivot is, of course, 10·26 − 5·07, or 5·19 feet.

We now require the height of the telescope pivot above the ground at the plane table station. This is obtained by direct measurement with a tape or the levelling staff. Suppose it is 4·22 feet Then the difference in height between the ground at the staff position and the ground at the plane table station is 5·19 + 4·22, or 9·41 feet. Usually the reduced level of the latter has been determined previously by ordinary levelling and hence the reduced level at the staff position can be found immediately.

This addition and subtraction may be eliminated by setting the middle cross wire to give the same staff reading as the height of the telescope pivot. This is done by the telescope tangent screw but, as a result, one will generally obtain a multiplying factor on scale X which is no longer a whole number, thus losing one of the great advantages of the Beaman arc.

Permanent Adjustment of the Alidade. The object of this adjustment is to ensure that a horizontal line of collimation will be obtained when the vernier of the graduated arc reads zero and the bubble of the index spirit level is central. If a detachable spirit level is provided for placing on the telescope, the bubble of this should also be central.

The procedure is as follows:

Drive two pegs into the ground, 200 feet apart, and set up the plane table midway between them. Direct the telescope to a levelling staff held vertically on one peg and bring the index bubble central by its tangent screw. Adjust the telescope tangent screw until the arc reads zero and then read the staff.

Turn the alidade round, lifting it bodily off the board, and repeating the previous procedure, obtain the staff reading on the second peg.

The difference in the readings will give the true difference in peg levels in spite of any possible collimation error.

If the instrument is equipped with a detachable telescope level proceed as follows :

Set up the plane table in line with the pegs, about 50 feet from one and 250 feet from the other. Place the level on the telescope and make the bubble central by the telescope tangent screw. Reverse the level end for end, and, if the bubble does not remain central, correct half the divergence by the telescope tangent screw and half by the capstan nuts on the bubble tube.

Obtain the staff readings on the two pegs, keeping the telescope bubble central by slight adjustment of the telescope tangent screw, if necessary. If the difference in readings is the same as before, the collimation is correct. If not, the diaphragm must be raised or lowered by very careful manipulation of the diaphragm capstan bolts until the same difference in readings is obtained.

The vernier is then set to zero by means of its tangent screw and if the index bubble is not central, it is corrected entirely by the bubble tube capstan nuts.

The test should then be repeated.

If no detachable spirit level is provided, the plane table is set up in the position described above and the staff readings are obtained on the two pegs with the index bubble central and the vernier reading zero each time.

If the difference in readings is not the same as before, the telescope is tilted up or down, as required, until the difference gives the correct value of the relative peg levels.

The vernier is then set to zero by its tangent screw and the index bubble brought to the centre of its run, if necessary, by the bubble tube capstan nuts. The adjustment is then checked by repeating the test.

Method of Using the Plane Table. The table is set up over a peg, the position of which is marked on the drawing, and the alidade is directed to a staff held at any required position. The direction of the straightedge then gives the direction of the line of collimation, and the stadia readings, with the necessary corrections, give (1) the distance from the instrument to the staff, and (2) the difference in ground levels at the instrument station and the staff position.

A ray is drawn along the straightedge, or parallel ruler, the distance to the staff is scaled off and the reduced level at the staff position written on the drawing. We thus have the means of producing a contoured plan and with a good instrument, skilfully used, the method is quite accurate enough for detailed plans drawn to as large a scale as 40 feet to an inch, or 1/500, so far as the topography is concerned. Heights cannot be determined with the same degree of accuracy as by normal levelling but plane table methods are sufficiently accurate for preliminary plans, on a scale of 100 feet to an inch, or for smaller scales, contoured at vertical intervals not closer than 5 feet.

In a road traverse the plane table may be used advantageously for filling in topography and obtaining approximate spot levels. For this purpose it should be set up initially over a traverse station, or a chainage peg on one of the traverse lines. At the commencement of the work the table is set up with the board placed most suitably in relation to the area to be surveyed. If this lies entirely to the east of the peg, for instance, the peg position may be located on the extreme westerly side of the drawing board. The peg position is transferred to the board by means of the plumbing fork shown in fig. 6.10. The orientation of the table is then fixed by directing the alidade to one or more stations, or chainage pegs, and locating these points on the plan by scaling off their distances along the appropriate rays, keeping the table clamped.

The height of the telescope pivot is measured, as already described, and the direction of magnetic north is indicated by ruling along the edge of the trough compass when the latter is correctly set.

Rays are then taken to important points in the topography and to useful spot levels and the staff holder, as in ordinary tacheometry, should exercise sound judgment as to where the staff should be held. An undulating boundary hedge, or stream, for instance, requires a ray at every point where there is a change of direction, while a straight fence only needs rays at widely spaced intervals. A Sopwith staff, reading to hundredths of a foot, should be used.

Having completed the work at the first station, the table is moved to another point. This may be another traverse station, or a chainage peg, or it may be a plane table station which has been already located and plotted at the first setting-up.

The second, and subsequent, set-ups involve a more complicated procedure than the first since we have to reconcile the position of the peg at the new station with the plotted position of that peg on the drawing. Moreover, the table must be correctly oriented, i.e. placed in such a way that rays passing through the plotted position of the new station to other points already fixed on the plan will, if produced, pass through the actual points on the ground.

Centring the Plane Table. On moving the plane table to a new station it is centred as nearly as possible over the peg by the plumbing fork, the pointer of the latter being held at the plotted position of the peg, and it is oriented approximately by the compass (fig. 6.10). The alidade is then laid with its straightedge along a ray to a distant point and the table is rotated until the vertical cross wire intersects a ranging rod at the point concerned. This rotation may displace the plumb-bob some way from the peg beneath the table and if the displacement on the ground is measurable on the scale of the drawing, the table must be re-centred and re-oriented.

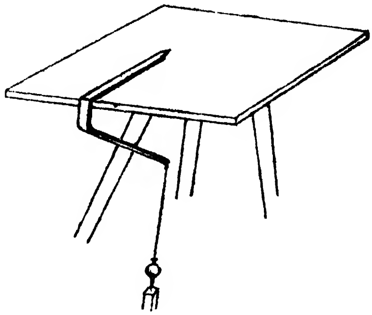


FIG. 6.10.—Use of Plumbing Fork with Plane Table.

If not, it may be clamped and the orientation checked by sighting to other well-defined points whose positions have already been plotted. Unless these fall in very close alignment with their plotted positions the centring and orientation must be improved.

The ideal to be aimed at is, of course, to set the table with correct orientation and the plotted position of its station peg vertically above the peg itself, but slight errors in centring may not be sufficiently serious to warrant correction.

Thus, in fig. 6.11, let O represent the true position of the plane table station peg and o its plotted position. Then the lines OA and

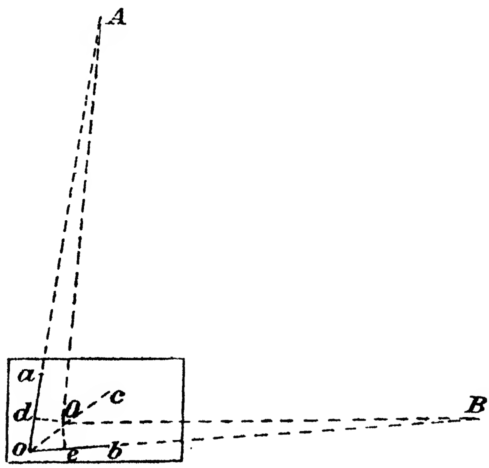


FIG. 6.11.—Centring Error with Plane Table.

OB on the ground will be shown on the plan as oa and ob and $\angle AOB$ will be represented by $\angle aob$. Produce oO to c. Then

$$\angle AOc = \angle oAO + \angle AoO$$

and
$$\angle cOB = \angle OBo + \angle BoO.$$

$$\therefore\ \angle AOB = \angle aob + \angle oAO + \angle OBo$$

i.e. the error in $\angle aob = \angle oAO + \angle OBo$.

Draw Od and Oe perpendicular to oA and oB, respectively.

Then
$$\angle oAO = \sin^{-1}\frac{Od}{OA} \text{ and } \angle OBo = \sin^{-1}\frac{Oe}{OB}.$$

The angular error will obviously increase as the distances OA and OB decrease. Suppose that OA and OB are each 20 feet long and that Od and Oe are each 3 inches long. The angular error is then 1° 26′. If the scale of the plan is 1/500, the plotted lengths oa and ob will only be 0·04 feet, or 0·48 inches, and the relative displacement of the points a and b, due to this angular error will be not more than about 0·012 inches, an amount which it is hardly possible to scale off.

If OA and OB are each 50 feet long and Od and Oe are, again, each 3 inches, the angular error becomes 34 minutes and the relative displacement of a and b, assuming, as before, a scale of 1/500, is again not more than about 0·012 inches and, therefore, negligible.

If OA and OB are each 500 feet long, Od and Oe each 3 inches, and the scale the same as before, the angular error is less than 4 minutes and the total displacement of the plotted points 0·014 inches.

The centring error, Oo, will be rather more than 3 inches in the above example and the effect on the relative positions of the two plotted points a and b is seen to be insignificant.

Re-section. The Three-point Problem. Although it is standard practice to locate and plot the position of a new plane table station before leaving the previous station, it is occasionally necessary, as the work proceeds, to set the table up in a new position which has not previously been fixed on the plan. The problem then arises as to how this station is to be plotted in its correct position relative to other points on the drawing. It is usually possible to adopt three other well-spaced points, already plotted on the plan, as reference points and the question then becomes the "Three-point Problem", the procedure for its solution being known as "Re-section".

It occurs in other branches of surveying and there are several methods of solution, but the simplest is the following:

Drive in a peg, or mark the position of the new station O, in some other way on the ground and place a sheet of tracing paper on the drawing board. Having set up the table, transfer the peg position to the tracing paper by means of the plumbing fork, giving the point o', let us say. Direct the alidade, in turn, to the three reference points,

A, B and C, marked by ranging rods and draw the corresponding rays $o'a'$, $o'b'$ and $o'c'$ respectively. Now adjust the tracing paper over the plan so that these three rays pass through the corresponding plotted points a, b and c. Prick through the point o', giving the point o on the plan which represents the station O.

Adjust the centring and orientation to suit the position of o thus obtained.

Graphical Solution of the Three-point Problem. Let the table be set up over the point O, the position of which is required on the plan. Let the reference points A, B and C be represented on the drawing by a, b and c, as shown in fig. 6.12. Place the alidade straightedge along ba, with a towards A, and turn the table into position 1, in which A is sighted in alignment with ba. Clamp the table, place the straightedge at b and turn the alidade until C is sighted thus obtaining the line xbx, which is ruled in.

Rotate the table to position 2, with b towards B and the straightedge along ab sighting B. Clamp the table, place the straightedge at a and turn the alidade to sight C, thus obtaining the line yay, which is ruled in. Let xbx and yay intersect at z. Join zc, place the alidade along this line and rotate the table until C is sighted, as shown in position 3.

Clamp the table in this position, which is the correct orientation. Place the straightedge at a and turn the alidade to sight A, producing the ray backwards to intersect zc at o. Place the straightedge at b and turn the alidade to sight B. This ray, if produced backwards, should intersect zc in the same point, o, as the previous ray ao; o then fixes the position of the station O.

It should be noticed that if the points a, b, c and o are concyclic, o cannot be located as there is an infinite number of solutions to the problem. If the four points are almost concyclic it is difficult to obtain an accurate solution. The reference points A, B and C should therefore be chosen to avoid this possibility.

Advantages of the Plane Table in Highway Surveys. The plane table is little used in British practice, partly because there appears to be a mistaken impression that it is unsuitable for accurate work, and partly because it is, of course, a fine weather instrument.

It is possible to continue urgent work with a chain and tape in a drizzle which would effectively stop plane tabling, but, on the other hand, the proportion of fine days in many parts of Britain is sufficiently high to render the inclusion of a tacheometric plane table in the surveying equipment of a highway authority well worth while.

In addition to possessing the advantages enumerated on page 175 plane tabling is extremely useful in fixing topography inaccessible for taping, such as the far bank of a river, the boundary of a quarry face

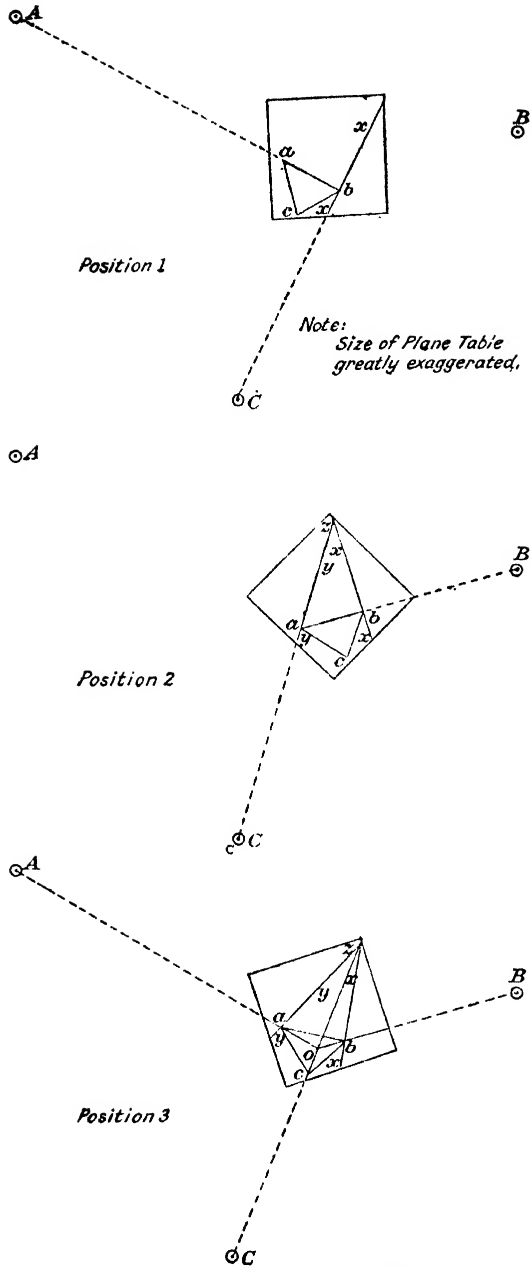


Fig. 6.12.—Re-section with Plane Table.

separated by a deep declivity from the chain line, or fences and hedges surrounding fields of growing crops.

If the staff holder can get to any point, however awkward, and if the instrument man can obtain the staff readings with the middle cross wire and one outer cross wire, the point can be plotted and its level calculated.

Where difficult topography occurs, the plane table can follow the traverse lines of a road survey and where extra stations are required to one side or other of the traverse, these are readily located and plotted in advance, being tied in thoroughly to traverse stations or chainage pegs.

A highway survey resembles a ribbon and it is rarely necessary for the plane table stations to be established at positions remote from the accurately located pegs and reference marks which occur at frequent intervals along the main framework of the survey.

The plane table sheets should be plotted to the same scale as the finished drawing. They can then be traced and transferred direct to the latter. Hence the necessity for including plenty of check points, such as traverse stations and chainage peg positions, on the plane table drawings.

CHAPTER VII

SETTING OUT CIRCULAR CURVES

Types of Curves used in Highway Construction. The centre-line of a projected highway is first located in plan as a series of straights which subsequently form the tangents to connecting curves. The type of curve used may be (1) a circular arc, (2) a " compound " curve consisting of a central circular arc " eased " into the tangents by the insertion of circular arcs of larger radius at the beginning and end of the curve, (3) a central circular arc " eased " into the tangents by the insertion of curves having a variable radius. The latter are known as " transition " curves. Their radius is infinity at their junction with the tangents and decreases gradually until the junction point with the central circular arc is reached where the radii of the two curves become identical. A fourth possibility is the wholly transitional curve which is useful in certain circumstances.

The longitudinal section of a projected road is first drawn with the gradients and any level stretches shown as a series of straight lines. In actual practice the sharp intersections thus formed at changes of gradient are smoothed out by means of " vertical curves ", for which purpose the parabola is commonly used and is quite satisfactory.

Both transition curves in plan and vertical parabolic curves are dealt with in subsequent chapters, the present chapter being devoted to circular arcs.

Circular Arcs in Plan. General Conditions. In practice the positions of the straight lines forming the tangents to the arc will have been fixed on the ground, although their intersection, or " apex ", may not be accessible. The radius will have been determined from a consideration of local topography, the requirements of traffic, and cost, the usual procedure being to adopt the largest radius that is economically possible.

Small curves may be set out by chain and tape methods, without carrying out any angular measurements, but for quick and accurate setting out, a theodolite is necessary, points on the curve being located by a combination of angles and lengths.

Setting out a Circular Arc without a Theodolite. It will be assumed that the tangents have been located by pegs and that their intersection is accessible. In fig. 7.1, *AB* and *AC* are the tangents and *R* is the pre-determined radius. It should be noted that the centre of the curve, O, is not used in the setting out procedure, but is shown in the figure for explanatory purposes.

The first step in any method of curve ranging is to fix the points X and Y at which the curve diverges from and re-joins the straights. These are known as " tangent points " and are frequently lettered on plans " $T.P._1$" and " $T.P._2$", or, alternatively, as " $P.C.$" and " $P.T.$", meaning respectively, " point of curve " and " point of tangent " and referring to the starting and finishing points, in that order. The initials " $B.C.$" and " $E.C.$" are also used meaning " beginning " and " end " of curve, respectively.

If the intersection, A, is accessible, X and Y are fixed by calculating the " tangent length ", AX or AY. This may be done in the following

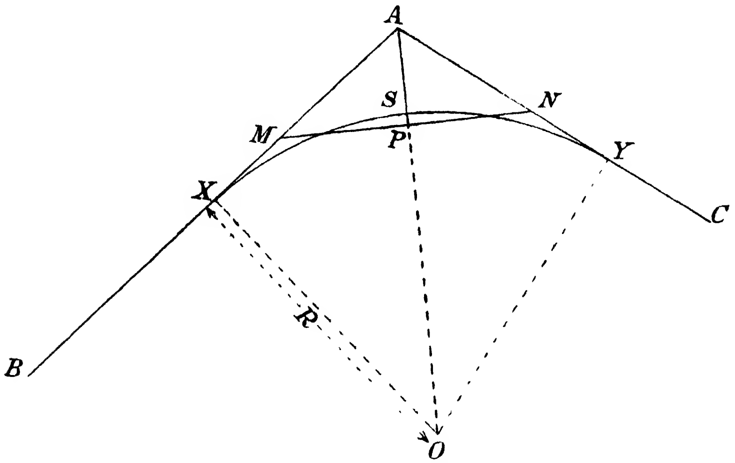


FIG. 7.1.—Location of Tangent Points for a Circular Arc without using a Theodolite.

way: Measure two equal lengths, AM and AN, along the tangents. The full length of the 100-foot chain will often be convenient for this purpose. Measure the distance MN and bisect it at P. Measure AP. Now AP, if produced, would pass through O and it is easily seen that AMP and AXO are similar right-angled triangles. Hence $\frac{AX}{XO} = \frac{AP}{PM}$

or $AX = XO \times \frac{AP}{PM} = R \times \frac{AP}{PM}$. All the dimensions on the right-hand side of this equation are known, thus giving the tangent length which is set out along each straight, fixing the points X and Y.

If A is inaccessible, the tangent length may readily be calculated from angular measurements, but for small curves, such as would be set out by chain and tape methods, the usual procedure with an inaccessible apex is to fix the tangent points by measurements from well-defined points in the surrounding topography scaled from the site plan. Having fixed the tangent points, the curve may be set out in various ways and two methods will be considered.

Method I.—By Offsets from Chords. Figs. 7.2 and 7.3 illustrate the procedure. Measure XY, known as the " long chord ", and bisect it at Z. Let $XZ = l$. We require to know the offset SZ which will fix the point S on the arc.

Let $SZ = x$.

Then $ZO = R - x$ and $XZ^2 + ZO^2 = XO^2$, i.e. $l^2 + (R - x)^2 = R^2$, whence $$x = R - \sqrt{R^2 - l^2}.$$

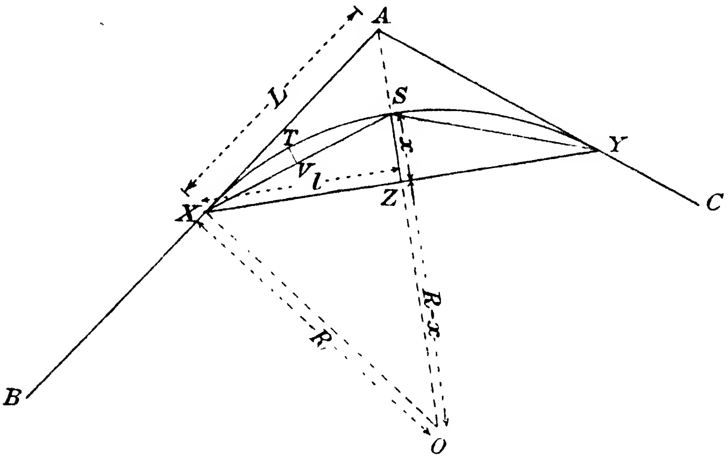


FIG. 7.2.—Setting out a Circular Arc by offsets from Chords.

The offset SZ can thus be computed and measured off, care being taken to ensure that it is perpendicular to XY. The next step in the work consists in measuring XS and YS, thus checking the position of S. The chords XS and YS are then bisected, if the check is satisfactory and perpendicular offsets from their mid-points, such as VT in fig. 7.2, are set out after computing their length exactly as for ZS.

Alternatively, the lengths of further perpendicular offsets from the chord XY, such as PQ in fig. 7.3 may be calculated as follows :

Let $QZ = y$ and draw PM perpendicular to OS.

Then $OM^2 = OP^2 - PM^2 = R^2 - y^2$, i.e. $OM = \sqrt{R^2 - y^2}$.

But $OM = MZ + ZO = PQ + \sqrt{R^2 - l^2}$.

Hence, the offset length, $PQ = \sqrt{R^2 - y^2} - \sqrt{R^2 - l^2}$.

Method II.—By Extended Chords. Fig. 7.4 illustrates the procedure. Choose a chord length, l, not greater than one-tenth of the radius. (The chord length is exaggerated in the diagram for the sake of clearness.) Consider the location of the first chord, XZ. Let VZ be the offset to Z measured perpendicular to the tangent XA. Bisect XZ at S. Then XOS and XVZ are similar right-angled triangles since $\angle VXS = \angle XOS$. Therefore $\frac{VZ}{XZ} = \frac{XS}{XO}$, i.e. $VZ = XZ \times \frac{XS}{XO} = \frac{l^2}{2R}$.

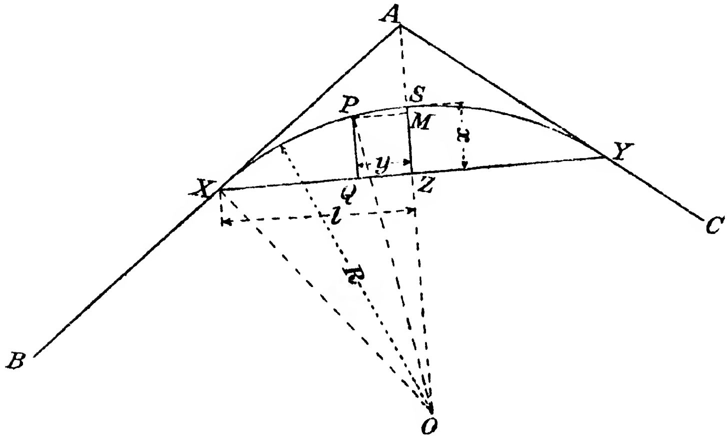


FIG. 7.3.—Setting out a Circular Arc by offsets from Long Chord.

If, now, a chain, or, preferably, a steel tape is stretched from X to give the chord length, l, and swung about X as centre until the offset

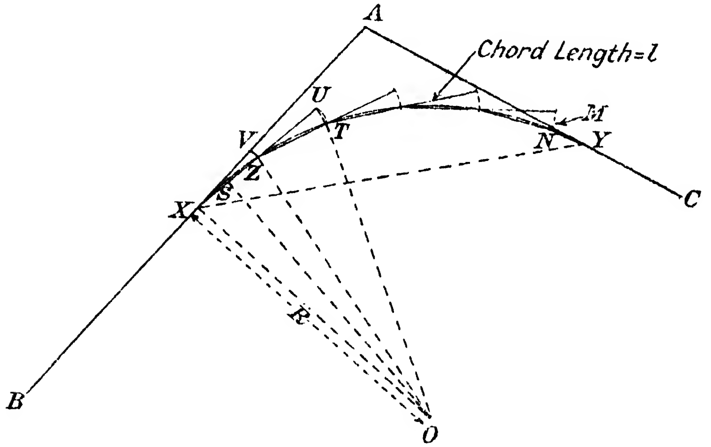


FIG. 7.4.—Setting out a Circular Arc from Extended Chords.

VZ, measured perpendicular to XA, either by a second tape or a 5-foot rod, is equal to $\frac{l^2}{2R}$, the point Z will be correctly located, thus fixing the first point on the curve.

Let the chord XZ be produced its own length to U and consider the location of the second point, T, on the curve. $ZT = ZU = l$. Now the isosceles triangles ZUT and ZTO are similar. Thus

$$\frac{UT}{ZT} = \frac{ZT}{ZO}, \text{ or } UT = \frac{ZT^2}{ZO} = \frac{l^2}{R}$$

Hence if the extended chord length ZU is swung about Z as centre through a distance $\frac{l^2}{R}$, T will be located on the curve. The distance UT may be conveniently measured with a 5-foot rod or a small steel tape. The chord ZT is then produced its own length and swung about the point T through a distance $\frac{l^2}{R}$ to fix the next point on the curve. This process is repeated until we arrive at a point N distant less than the chord length, l, from Y. The distance NY is measured and also the offset NM perpendicular to the tangent AY. If the setting out is accurate NM should equal $\frac{NY^2}{2R}$.

It will be noted that the extended chords are swung through a distance equal to twice the initial offset measured perpendicular to the tangent. This method requires very careful work to obtain accurate results. The process of producing the chords their own length is apt to introduce errors and method II is not so effective as either of the alternatives in method I, but the latter involves more arithmetic.

Setting out Circular Arcs by Theodolite. The most satisfactory method of setting out a circular arc is known as the " Tangential Angle Method " and involves the use of a theodolite. Fig. 7.5 (i) and (ii) illustrates the principle on which this method is based.

As before, AB and AC are the straights, or tangents, meeting at the " intersection point ", or " apex ", A located on the ground by pegs. We will consider the case, first, when A is accessible. Imagine the tangent BA produced to A'. Then $\angle A'AC$ is termed the " Deflection Angle ", the " Deviation Angle ", or, sometimes, the " Intersection Angle ", for which the symbols Δ, or " I " are usually employed. " Deviation Angle " is, perhaps, the best term. " Deflection Angle " is apt to be confused with " Tangential Angle " and " Intersection Angle " is indefinite. The angle BAC is sometimes called the " Apex Angle ".

Let O be the centre of the curve in fig. 7.5 (ii), X and Y the tangent points, and R the radius. Then $\angle XOY = \angle A'AC = \Delta$ and $\angle XOA = \frac{\Delta}{2}$.

Hence the tangent length, XA or $YA = R.\tan\frac{\Delta}{2}$.

The deviation angle, Δ, is measured carefully, reading with both faces of the instrument. The tangent length is then calculated and

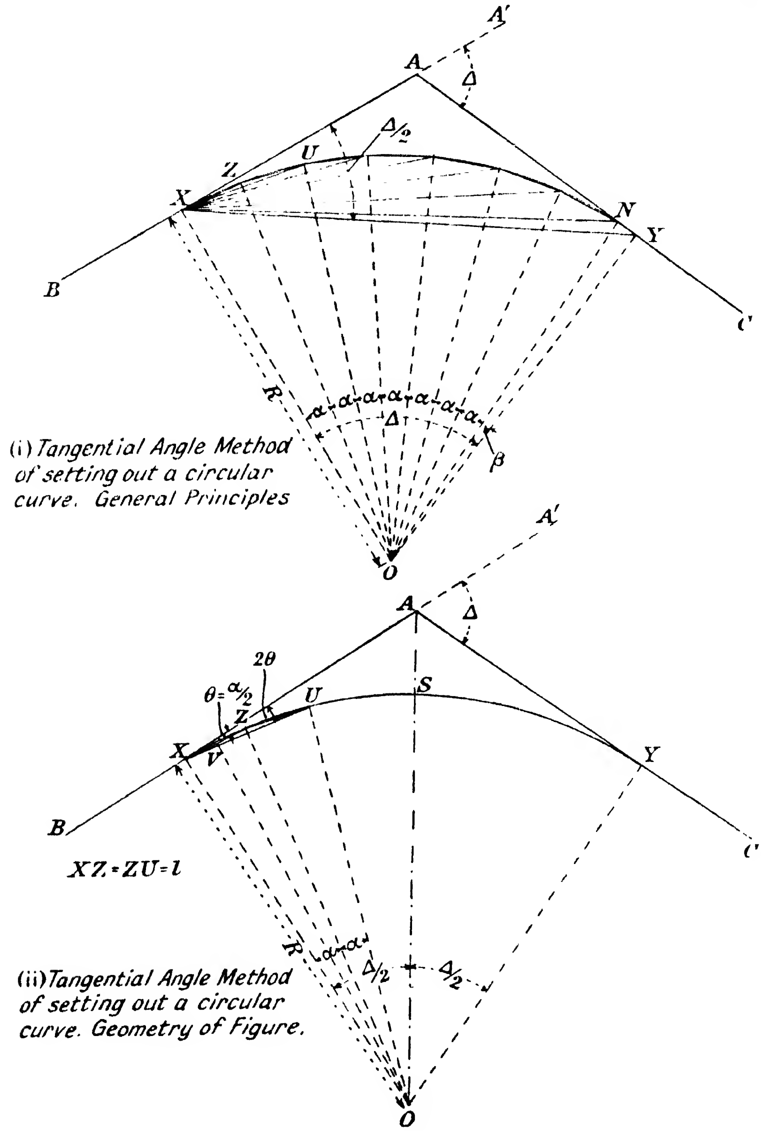


FIG. 7.5.

pegs inserted at X and Y, at the measured distances, using the theodolite for giving the alignment.

The theodolite is now moved to X, the tangent point from which the curve diverges in a right-hand direction. The reason for choosing a right-hand divergence will be apparent later. The curve is set out as a series of chords and the chord length should be sufficiently short to ensure that there is only a negligible difference between the lengths of the chord and the arc. The former should never exceed one-twentieth of the radius.

Let XZ be the first chord, of length l, and bisect XZ at V. Then $\angle AXZ$ is termed the "first tangential angle".

It will be apparent that

$$\angle AXZ = \angle XOV = \sin^{-1}\frac{XV}{XO} = \sin^{-1}\frac{\text{Half chord length}}{\text{Radius}}.$$

The value of the first tangential angle, θ, can thus be computed.

Suppose, now, that U is the second point on the curve, the chord length ZU being, again, l. Let the angle subtended by ZU at the centre be α. The angle subtended by XZ will also be α and the total angle subtended by the curve up to U will be 2α. Hence the tangential angle $AXU = \frac{1}{2}\ XOU = \alpha = 2\theta$. Thus, for every additional chord length l, the tangential angle is increased by θ.

Let us now consider the practical application of this elementary geometry. The theodolite having been set up at X, one vernier is set to zero, the upper plate clamped and the lower plate unclamped. The telescope is then directed to a rod at A, using the lower-plate tangent screw for the final accurate setting and the lower plate is clamped. The upper plate is now released and the vernier previously set at zero is set to read the first tangential angle, using the upper-plate tangent screw for the accurate adjustment. A chainman with the chord length stretched on a chain, or, preferably, a steel tape, is then directed by the instrument man until the end of the chord falls into correct alignment with the vertical cross wire. The preliminary lining-in may be effected by a ranging rod, but at short distances a finer sighting mark is required, such as a chaining arrow skewered through a sheet of white paper.

This procedure will locate the first point, Z, on the curve, which is then pegged. The second tangential angle is now set on the theodolite and the second chord length measured out and placed in correct alignment. It should be noticed that the tangential angles radiate from the tangent point, whereas the chord lengths are measured successively round the curve. It is advisable to check back from time to time to the rod at A to ensure that the zero setting of the theodolite has not shifted.

The procedure is continued until we ultimately arrive at a point distant less than the chord length, l, from the second tangent point, Y. The final chord length, NY in fig. 7.5 (i), can be predetermined thus :

The total angle XOY will be made up of a number of angles α, each corresponding to a chord length l, plus a smaller angle, β, i.e.

$$\Delta = n\alpha + \beta, \text{ or } \beta = \Delta - n\alpha.$$

and, by proportion, length of final chord $= l \times \left(\frac{\Delta - n\alpha}{\alpha}\right)$.

$$\text{The total length of the curve} = nl + l\left(\frac{\Delta - n\alpha}{\alpha}\right) = l \times \frac{\Delta}{\alpha}.$$

With the theodolite at X reading the final tangential angle, the vertical cross wire should accurately bisect a ranging rod placed at the tangent point Y. Furthermore, the calculated length of the final chord should agree exactly with the measured length.

Chainage Pegs on the Curve. It is customary in setting out road curves to continue the chainage from the initial tangent point and insert pegs at the exact 100-foot chainage points. It will generally happen that the chainage at the initial tangent point is an odd amount, and, when using 100-foot chords, a short chord will, therefore, be necessary at the start of the curve in order that the first point on the curve may fall at an exact 100-foot chainage. We shall, thus, usually require a short chord at the start and finish of the curve and the tangential angles for these chords are calculated by the rule already given :

$$\text{tangential angle} = \sin^{-1} \frac{\text{Half chord length}}{\text{Radius}}$$

The intermediate chords will be of uniform length and equal increments in the tangential angle will occur for each additional intermediate chord. If there are n such chords, the tangential angle for each being θ, and if the tangential angles for the first and last short chords are γ and δ, we have the relationship $n\theta + \gamma + \delta = \frac{\Delta}{2}$.

This equation provides a check on the calculation. A further check consists in comparing the length of the curve calculated as a series of chords with the true length of the arc calculated from the equation

$$\text{length of arc} = \frac{\Delta}{360} \times 2\pi R$$

These two lengths should agree very closely.

The method of calculating the tangential angles for a specific curve will now be considered :

Worked Example

Preliminary Data : Deviation Angle : 14° 38′ 00″ (Δ).
Radius : 3,200 feet (R).
Chainage at Intersection Point : 269 + 84.
Theodolite reads to 30 seconds.

First calculate the Tangent Length.

$$\text{Tangent Length} = R \tan \frac{\Delta}{2} = 3200 . \tan 7^\circ\ 19' = 410{\cdot}88 \text{ feet.}$$

For all practical purposes this may be called 411 feet. Chainage of Initial Tangent Point (X) $= (269 + 84) - (4 + 11) = 265 + 73$.
Hence length of first chord $= 100 - 73 = 27$ feet.
The first point on the curve will thus be at chainage $266 + 00$.

We may use intermediate chords 100 feet long since this length complies with the rule that the chord length must not be greater than one-twentieth of the radius.

$$\text{Tangential Angle for a chord 100 feet long} = \sin^{-1} \frac{\text{Half Chord Length}}{\text{Radius}}$$

$$= \sin^{-1} \frac{50}{3200} = 53'\ 43''$$

and the angle subtended by this chord at the centre

$$= 2 \times 53'\ 43'' = 1^\circ\ 47'\ 26''.$$

$$\text{Length of Curve, measured as chords} = \frac{14^\circ\ 38'\ 00''}{1^\circ\ 47'\ 26''} \times 100 \text{ feet}$$

$$= \frac{14{\cdot}633}{1{\cdot}791} \times 100$$

$= 817$ feet, to the nearest foot.

As a check, calculate the true length of the arc, thus :

$$\text{Length of arc} = \frac{14{\cdot}633}{360} \times 2\pi \times 3200,$$

which, to the nearest foot, is also 817 feet. Hence,

Chainage at end of curve, Y, $= (265 + 73) + (8 + 17) = 273 + 90$

The last chainage peg on the curve will, therefore, be at $273 + 00$ and the length of the final chord will be 90 feet.

We must now calculate the tangential angles for the initial chord, 27 feet long, and the final chord, 90 feet long.

$$\text{Tangential Angle for 27-foot chord } \sin^{-1} \frac{13{\cdot}5}{3200} = 14'\ 30''$$

$$\text{Tangential Angle for 90-foot chord } \sin^{-1} \frac{45}{3200} = 48'\ 21''$$

The tangential angles must now be checked for summation to $\Delta/2$, there being seven intermediate 100-foot chords in addition to the short chords at the start and finish.
Thus,

the total tangential angle $= 14'\ 30'' + 7 \times 53'\ 43'' + 48'\ 21''$ giving $7^\circ\ 18'\ 52''$, instead of $7^\circ\ 19'\ 00''$. This error of 8 seconds is due to approximation in the derivation of the tangential angles from their sines and is sufficiently small to be neglected for practical purposes.

We are now in a position to tabulate the tangential angles, together with the corresponding chord distances. The successive summation of the increments in the angles gives the readings which must be set on the theodolite as the chord measurements proceed. The readings on the graduated circle of the instrument increase in a clockwise direction, hence with a curve which diverges in a right-handed way and consequently has a clockwise increase in its tangential angles, the theodolite gives direct readings without the necessity for any subtractions.

The tabulated data for setting out the circular curve under consideration is given below:

Chainage	Tangential Angle								Tangl. Angle to nearest 30″		
	°	′	″	′	″	°	′	″	°	′	″
Point X ($P.C.$) 265 + 73											
266 + 00		14	30				14	30		14	30
267 + 00		14	30	+ 53	43	= 1	08	13	1	08	00
268 + 00	1	08	13	+ 53	43	= 2	01	56	2	02	00
269 + 00	2	01	56	+ 53	43	= 2	55	39	2	55	30
270 + 00	2	55	39	+ 53	43	= 3	49	22	3	49	30
271 + 00	3	49	22	+ 53	43	= 4	43	05	4	43	00
272 + 00	4	43	05	+ 53	43	= 5	36	48	5	37	00
273 + 00	5	36	48	+ 53	43	= 6	30	31	6	30	30
Point Y ($P.T.$) 273 + 90	6	30	31	+ 48	21	= 7	18	52	7	19	00

$$\text{Check: } \frac{I}{2} = 7^\circ\ 19'\ 00''.$$

Degree of Curvature. In American practice circular curves are specified as having a particular "degree of curvature" instead of a radius of so many feet, or chains. The degree of curvature is generally defined as the number of degrees in the angle subtended at the centre of the arc by a chord 100 feet long. Thus, in the worked example given previously, the angle subtended at the centre by a chord 100 feet long has a value 1° 47′ 26″ which is, therefore, the degree of curvature corresponding to a radius of 3,200 feet. The larger the radius, the smaller will be the degree of curvature.

Usually, when using the degree system, curves are chosen having a simple round number as their degree of curvature. This system has the advantage that the tangential angle for a chord 100 feet long is known immediately, being half the specified degree of curvature. Tangential angles for chords of other lengths can be worked out by proportion.

An alternative definition is sometimes used in which an *arc* 100 feet long is substituted for a *chord* of that length. For large radii curves the difference is insignificant.

The relationship between the degree of curvature (arc definition), $D°$, and the radius, R, is given by $R \times \frac{D°}{57 \cdot 3} = 100$, or $R = \frac{5730}{D°}$, approx. Thus, a 3-degree curve (arc definition) has a radius of 1,910 feet.

Use of Gunter's Chain. It should be mentioned here that, in British practice, the radii of railway and tramway curves are almost invariably expressed in Gunter's chains. Thus, a "10-chain" tramway curve has a radius of 660 feet. The hundred-foot or "engineers'" chain is used in highway work.

Location of Intersection Point (or Apex). There are usually two or more chainage pegs on each tangent, but the intersection point may not be marked. To locate the latter a theodolite is set up over a peg well back on one tangent and aligned on a rod or arrow placed at a peg on the tangent between the instrument station and the intersection point. A rod is then set out on this alignment *beyond* the intersection point. The same process is repeated for the other tangent. Wires are now stretched between each instrument peg and the corresponding distant peg, thus delineating on the ground the line of each tangent. The point at which the wires cross is, of course, the intersection point, or apex, of the straights.

Case of an Inaccessible Apex. This is illustrated in fig. 7.6. Suppose that the straights, or tangents, are defined by the lines AB

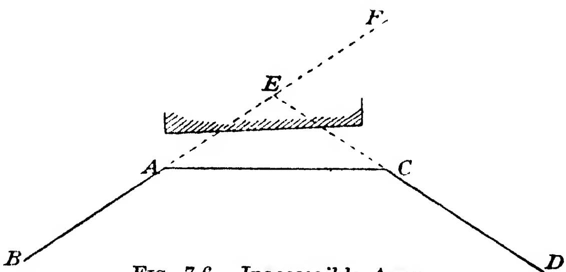


FIG. 7.6.—Inaccessible Apex.

and CD, located on the site by pegs, those at A and C being mutually visible. The intersection point, E, is inaccessible.

Measure the distance AC and, with a theodolite, measure the angles BAC and ACD, reading with both faces of the instrument. Then

$$\begin{aligned}\text{the deviation angle, } \Delta, &= \angle FEC = \angle EAC + \angle ECA \\ &= 180° - \angle BAC + 180° - \angle ACD \\ &= 360° - (\angle BAC + \angle ACD).\end{aligned}$$

Knowing $\triangle$ and R, the tangent length can be calculated. The lengths EA and EC can be found from the sine formula:

$$\frac{EA}{\sin ECA} = \frac{EC}{\sin EAC} = \frac{AC}{\sin AEC}$$

Hence the tangent points X and Y can be set out by measurements from A and C and the curve data calculated exactly as in the worked example. In setting out the tangential angles, however, we cannot sight on to a rod at the intersection point with a zero reading set on the theodolite and there will probably be only a short length of tangent available beyond the point X. If so, an arrow skewered through a sheet of white paper should be used for the reference mark instead of a ranging rod at the point A. It may sometimes be possible, however, to extend the tangent beyond the point A and thus secure a more suitable distance for fixing the zero direction of the theodolite. In this case the instrument is sighted back along the tangent to B and the telescope then transited and a reference mark made as far from A as possible. The face of the instrument is then reversed and the process repeated, thus obtaining a second reference mark which will coincide with the first if the instrument is in good adjustment. If coincidence is not obtained a mean position between the two reference marks will locate correctly the extension of the tangent.

Case of an Intervening Obstruction. Fig. 7.7 illustrates this problem. Suppose that the points Z and V have been set out on the curve, but that the next point, U, cannot be seen from X owing to an intervening obstruction.

Let TVS be the tangent at V and let the tangential angle for the point V be θ_2. Produce the ray XV to Q. Then $\angle QVS = \angle TVX = \angle TXV = \theta_2$. If the theodolite be moved to V, the verniers set to read zero and 180° and the telescope sighted back to X with the upper plate clamped at these readings, releasing the upper plate and rotating it through 180° would result in the telescope pointing in the direction VQ. If the tangential angle θ_2 were then set out using the opposite vernier to that previously reading zero, the telescope would be pointing along the line VS, i.e. along the tangent at V. If the tangential angle $AXU = \theta_3$, the tangential angle measured from V for setting out the point U is $\angle SVU$, or $\dfrac{\angle VOU}{2}$, which is $\theta_3 - \theta_2$ since $\angle VOU = 2\theta_3 - 2\theta_2$.

Hence, if the tabulated tangential angle θ_3 is now set on the theodolite, the telescope will be pointing in the required direction VU and the setting out can proceed normally, using the tabulated tangential angles in their original order with no alterations. (The chord length is exaggerated in the figure for the sake of clearness.)

When setting out a sharp curve in a deep cutting it may be necessary to shift the theodolite repeatedly and the initial tangent point may become invisible. In this case it is only necessary to sight back to *any* previous station with one vernier reading the tabulated tangential angle corresponding to that station. The upper plate is clamped to the lower during this process and the lower plate tangent screw is used for obtaining an accurate setting on the back station. This is

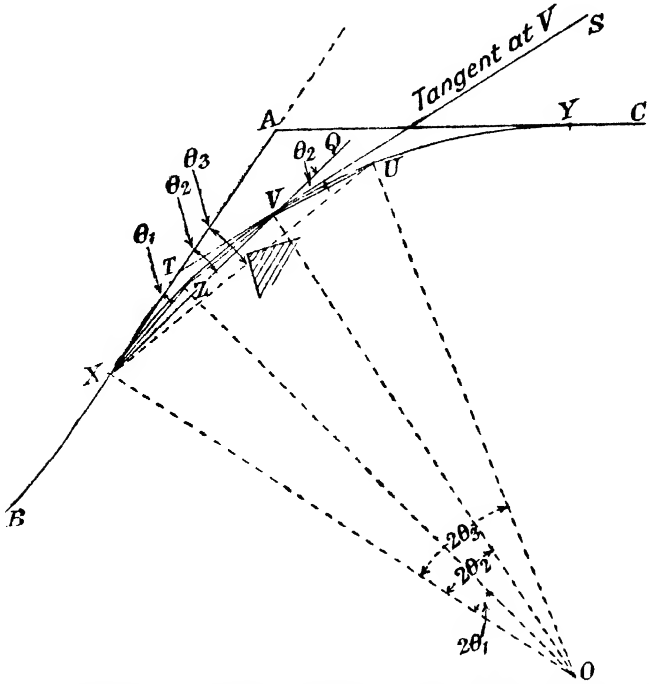


FIG. 7.7.—Obstruction preventing setting out of Circular Curve from initial Tangent Point.

equivalent to sighting back to the start of the curve with one vernier reading zero. The upper plate is now released and turned until the next forward tangential angle is obtained on the opposite vernier to that used when sighting back. The correctness of this procedure may be proved quite easily from the geometry of fig. 7.7.

External Distance. The distance between the intersection point of the tangents and the nearest point of the curve, i.e. AS in figs. 7.1, 7.2, 7.3 and 7.5, is known as the " External Distance ". It is often a limiting dimension in deciding the radius of a curve, e.g. in cases where buildings have to be avoided, and for a circular arc of radius R and tangent length L its value is obviously $\sqrt{R^2 + L^2} - R$.

If Δ is the deviation angle, the external distance is $R(\sec \Delta/2 - 1)$.

CHAPTER VIII

SUPERELEVATION AND TRANSITION CURVES

Superelevation. Preliminary Theory. When the centre-line of a road is curved, the normal cambered cross-section is unsuitable unless the radius of the curve is very large, or the speed of vehicles using the road is strictly limited.

Newton's First Law of Motion, given in his " Principia ", published in 1686, states that " Every body continues in its state of rest, or of uniform motion in a straight line, except in so far as it be compelled by external impressed force to change that state." From the elementary principles of dynamics it can be shown that a body moving with a velocity v round a curve of radius r has an acceleration of $\frac{v^2}{r}$ towards the centre of the curve. The force required to produce this acceleration is the product of the acceleration and the mass of the body, or, if W is the weight of the body, the force is $\frac{Wv^2}{gr}$. When the front wheels of a motor vehicle are turned to negotiate a curve the forward driving force has a tangential and a radial component, the latter acting towards the centre of the curve and producing the radial acceleration.

But by Newton's Third Law, to every action there is an equal and opposite reaction and a force of $\frac{Wv^2}{gr}$, acting outwards from the centre of the curve is exerted on the vehicle. This is termed the " centrifugal force " and its effects are well known. It tends to push the vehicle away from the centre of the curve and may thus produce skidding and it also has a turning moment about the points of contact of the outer wheels and the road surface which may result in overturning. The latter tendency is very pronounced in the case of a motor-cycle, with a light sidecar attached, when driven too fast round a left-hand bend. The sidecar wheel lifts from the road surface and the equilibrium of the vehicle is seriously disturbed. In similar circumstances a car may not overturn, but the body heels over on the springs, away from the centre of the curve, movable objects in the vehicle are displaced in the same direction, and the occupants feel the effect of the outward force.

It is necessary, however, to consider the combined effect of the centrifugal force and the weight of the vehicle, since these two forces may act in conjunction, or in opposition, according to the direction of the cross-fall of the road. Fig. 8.1 represents cross-sections of a curved road, (*a*) showing the normal camber improperly applied,

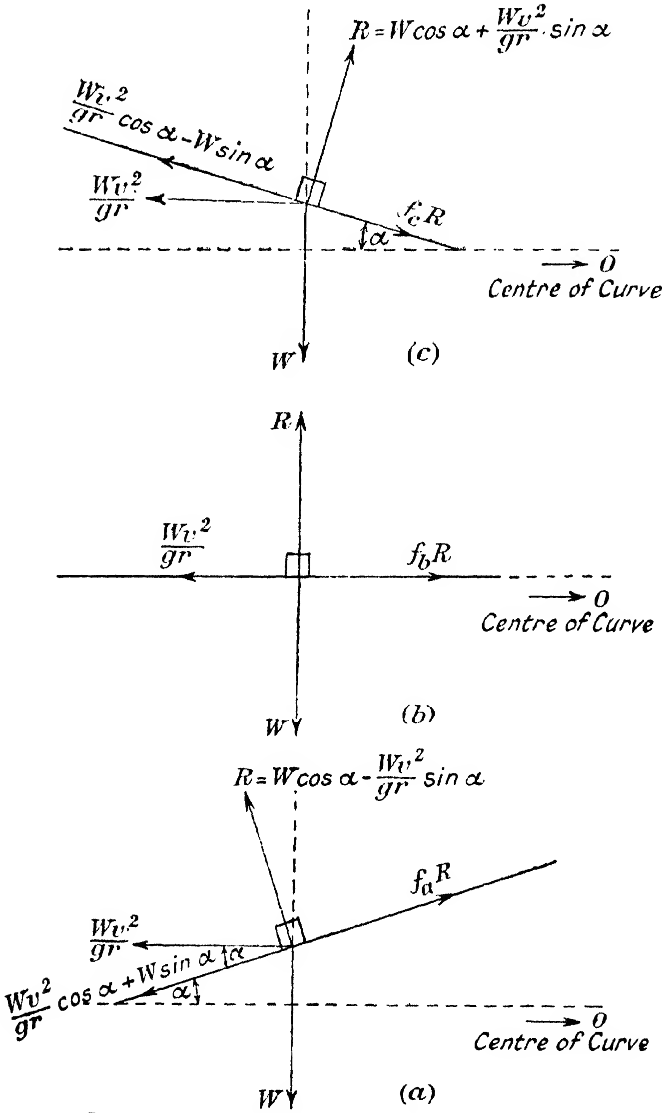


FIG. 8.1.—Forces on a Vehicle on a curved Path.

(*b*) showing a horizontal surface, and (*c*) showing the correct application of superelevation. O is the centre of the curve in each case.

Resolving forces parallel to the road surface, we have in case (*a*), if α is the inclination of the cross-fall to the horizontal, the outward lateral force is $\frac{Wv^2}{gr}\cos\alpha + W.\sin\alpha$, i.e. the weight and centrifugal force act together.

In case (*b*) the weight has no lateral component and the outward lateral force is $\frac{Wv^2}{gr}$.

In case (*c*) the outward lateral force is $\frac{Wv^2}{gr}\cos\alpha - W.\sin\alpha$, i.e. the weight of the vehicle reduces the effect of the centrifugal force. In this case if $\frac{Wv^2}{gr}\cos\alpha$ is greater than $W.\sin\alpha$, the resultant will act outwards and if $\frac{Wv^2}{gr}\cos\alpha$ is less than $W.\sin\alpha$ the resultant will act inwards, towards the centre of the curve. For a given value of r there will be a certain value of v which satisfies the equation $\frac{Wv^2}{gr}\cos\alpha = W.\sin\alpha$, or $v^2 = g.r.\tan\alpha$. In this case no lateral force is applied to the vehicle and Leeming[1] has termed the speed, under these circumstances, the " hands off " speed. In general, however, a lateral force will be exerted and to avoid skidding it must be counteracted by the lateral adhesion of the road surface.

Lateral Equilibrium of a Vehicle on a Curve. The lateral resisting force is the product of a frictional co-efficient "f", let us say, and a reaction, R, normal to the plane of the road. The factor "f" is not the same as the " sideways force co-efficient " obtained when a wheel is actually skidding. The value of the latter may vary from 0·1, or less, on an ice-covered road to unity on a good " non-skid " surface, a fair average value being 0·7. When a vehicle is travelling round a curve only sufficient lateral adhesion is called into play to balance the resultant lateral force and it is considered that "f" should not exceed 0·15. If higher values occur, as they sometimes do, excessive speed is indicated and extravagant tyre wear is caused, even if no actual danger is involved.

Referring, again, to fig. 8.1, let f_a, f_b and f_c be the respective values of f corresponding to given values of v and r. Then, equating the resisting force to the applied lateral force resolved along the plane of the road, we have in case (*a*):

$$f_a.R = f_a.W\cos\alpha = \frac{Wv^2}{gr}\cos\alpha + W.\sin\alpha, \text{ or } f_a = \frac{v^2}{gr} + \tan\alpha$$

neglecting the slight effect of the centrifugal force on the reaction normal to the plane of the road, represented by $\frac{Wv^2}{gr}\sin\alpha$.

In case (*b*) we have, simply,

$$f_b.R = f_b.W = \frac{Wv^2}{gr}, \text{ or } f = \frac{v^2}{gr},$$

and in case (*c*), considering only speeds in excess of the "hands off" speed, we have

$$f_c.R = f_c.W \cos\alpha = \frac{Wv^2}{gr}\cos\alpha - W.\sin\alpha, \text{ i.e. } f_c = \frac{v^2}{gr} - \tan\alpha.$$

Tan α is the superelevation slope, i, and we may write :

$$\text{for case } (a): \ f_a = \frac{v^2}{gr} + i, \qquad \text{for case } (b): \ f_b = \frac{v^2}{gr},$$

$$\text{for case } (c): \ f_c = \frac{v^2}{gr} - i.$$

f_c is obviously smaller than f_a or f_b. Alternatively, if we assign a safe limit to f, say f_s, and consider v to be the variable, we have for case (*c*) : $\frac{v^2}{gr} = f_s + i$, giving a higher value of v than cases (*a*) or (*b*).

If v is small and i is large in case (*c*), f may become negative. This indicates that the superelevation more than compensates the centrifugal force and the lateral adhesion then acts up the slope, thus counteracting the tendency of the vehicle to slide down.

Authorities differ as to the maximum permissible value of i. The steepest cross-fall specified in Memorandum No. 575 on "The Construction and Lay-out of Roads", published by the Ministry of Transport, is 1 in $14\frac{1}{2}$, in which case $i = 0{\cdot}07$. A maximum of 1 in 10 has been proposed by other authorities, but is regarded in this country as too steep. If, however, this figure is accepted, together with a maximum of 0·15 for f, we have by substitution, for case (*c*) : $\frac{v^2}{gr} = 0{\cdot}10 + 0{\cdot}15 = 0{\cdot}25$. The quantity $\frac{v^2}{gr}$ is sometimes termed the "Centrifugal Ratio" and the above equation is a mathematical interpretation of the rule, sometimes quoted, that the "centrifugal ratio" should not exceed $\frac{1}{4}$. In actual practice this value is frequently exceeded by a considerable amount.

In the limiting case of 0·25 for $\frac{v^2}{gr}$ and 0·10 for i, it is apparent superelevation is contributing 0·4 of the total effect resisting centrifugal force and it has been suggested that this proportion is a satisfactory figure.

Reaction Normal to the Road Surface. The equilibrium equations given on page 201 are only approximately true, since the effect of the centrifugal force on the normal reaction has been omitted therein to give simplified expressions which are sufficiently accurate for practical purposes. Referring to fig. 8.1 (*a*), in which the superelevation is applied in the wrong direction, the centrifugal force has a component $\frac{Wv^2}{gr}.\sin\alpha$ normal to the road surface acting in the opposite direction to the normal reaction $W.\cos\alpha$. This has the effect of slightly reducing the " adhesive weight ", i.e. the component of the weight normal to the road surface which, when multiplied by the co-efficient of lateral adhesion, gives the lateral adhesive force.

Thus in case (*a*), the resultant normal reaction $= W.\cos\alpha - \frac{Wv^2}{gr}.\sin\alpha$. In case (*b*), however, the normal component of the centrifugal force acts in the same direction as the normal component of the weight, or, in other words, the " adhesive weight " is slightly increased by the centrifugal force when superelevation is correctly applied, the resultant normal reaction then being $W.\cos\alpha + \frac{Wv^2}{gr}.\sin\alpha$.

Recommended Superelevation. The following table of recommended cross-falls for curves of various radii is extracted from the Ministry of Transport Memorandum No. 575 :

Radius (in feet)	Superelevation	Radius (in feet)	Superelevation
1,200, or less	1 in 14½	4,000	1 in 38
1,400	1 in 17	4,500	1 in 39
1,600	1 in 19	5,000	1 in 40
1,800	1 in 22	5,000 to 7,500	1 in 40 to 1 in 48 according to nature of material.
2,000	1 in 24		
2,500	1 in 30		
3,000	1 in 36	over 7,500	normal camber or crossfall retained.
3,500	1 in 37		

Assuming a coefficient of lateral adhesion of 0·15, the recommended superelevation will safely accomodate 30 miles per hour on a radius of 300 feet, 62 miles per hour on a radius of 1,200 feet, or 115 miles per hour on a radius of 5,000 feet.

For roads in built-up areas, subject to a speed limit of 30 m.p.h. superelevation should be provided, if practicable, at the following rates :

For curves of 800 feet radius, or over	1 in 40
,, ,, ,, 600 ,, ,,	1 in 30
,, ,, ,, 500 ,, ,,	1 in 25

Adverse camber should always be eliminated.

Application of Superelevation. Transition Curves. We have seen that a superelevated cross-section should be introduced on curves other than those of very large radii, but if the configuration of the centre-line changes abruptly from a straight line to a circular arc of less than 7,500 feet radius, we are confronted with the difficulty that the straight portion of the road will normally have the ordinary camber, while the arc ought, theoretically, to have a superelevated crossfall. As a practical compromise we may either introduce a change of cross-section on the straight, gradually eliminating the camber on the appropriate half and then raising the edge until the specified superelevation is reached at the tangent point, or we may continue the camber up to the tangent point and gradually change the cross-section on the curve, or we may eliminate the camber without applying any superelevation on the straight and gradually increase the superelevation to its full value on the curve. Opinions differ as to which is the best compromise, but the only correct way of introducing superelevation is by means of a transition curve, which, as mentioned in Chapter VII, has a variable radius, gradually decreasing from infinity at the tangent point to some predetermined minimum. If, as usually happens, the transition connects the tangent with a circular arc, the minimum radius of the transition is, of course, that of the arc. It is questionable whether the camber should be eliminated from the appropriate half of the road before or after the tangent point. American practice appears to favour the first method but the Ministry of Transport Memorandum No. 575 recommends that "the elimination of adverse camber and the attainment of the required superelevation should be obtained by raising the outer above the inner channel at a uniform rate along the length of the transition, from the end of the straight to the commencement of the circular curve, where the superelevation should reach its maximum."

Types of Transition Curve. To provide a uniform increase in superelevation the radius of a transition curve should decrease uniformly with the distance along the curve from the tangent point. This requirement is exactly satisfied in the simple spiral, although other curves may be used for transition purposes. The cubic parabola is largely employed in railway practice where deviation angles on the main running lines are invariably small, but it cannot be used with large deviation angles. The lemniscate is suitable for road curves and it conforms very closely to the requirement that its radius should vary inversely with the length of the curve, but it possesses no marked advantages over the spiral. The latter has been adopted as standard in the United States, Canada, Australia, and elsewhere, its mathematical treatment is straightforward and simple, and it may easily be set out by equal chord lengths and tangential angles, exactly like a circular arc.

Further Advantages of Transition Curves. A driver approaching a circular arc at too fast a speed in relationship to the radius will automatically describe a larger radius than that of the arc by pulling over towards the centre of the road, or even encroaching on the wrong side in the manner shown in fig. 8.2. The introduction of transition curves at each end of the circular arc gives more time for modifying speed and direction to suit the changing curvature, but does not completely eliminate the possibility of cutting the curve. It must be remembered that for a given deviation angle and external distance there is only one possible radius for a plain circular arc, and if transitions are introduced the radius of the central circular arc will necessarily be sharper than that of the plain circular curve. Consequently a reckless driver may still find himself unable to conform properly to the lay-out of the road as the radius diminishes and will be forced away from his correct side in order to describe a larger radius to suit his excessive speed. The longer the transition the greater the reduction in radius and the greater the tendency for a bad driver to " cut in ".

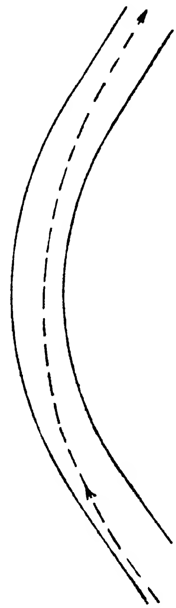


FIG. 8.2.—Path of a badly driven Car on a Curve.

If, for example, we consider a deviation angle of 60° and an external distance limited to 150 feet, a circular arc to fit these conditions will have a radius of 969 feet. The insertion of 100-foot transitions at each end would reduce this radius by about 5 feet—not a serious amount. If the length of each transition were increased to 200 feet, however, the radius of the central circular arc would be reduced to about 955 feet, while an all-transitional curve to suit the given conditions would have a minimum radius of only 710 feet and the middle portion of the curve would thus be considerably sharper than the plain circular arc. It will, therefore, be apparent that the application of transition curves needs careful judgment, as too long a curve will defeat its own object from the point of view of safety.

Transition curves have some aesthetic value in obviating the ugly kink and apparent bulge often seen when kerbing is set out as a circular arc changing abruptly into a straight line.

The Spiral Transition Curve. Fig. 8.3 represents a curve of this type and is lettered in the customary manner. *TS* is the tangent point, or the tangent-spiral junction, and *SC* is the point at which it merges into the circular arc, or the spiral-circular arc junction.

Similarly, CS is the junction of the circular arc and the second spiral, and ST is the second tangent point, considering the curve to be right-handed. The angle subtended by the spiral, or the "spiral angle", is θ_s. It should be noted that the spiral angle subtended up to any point along the transition curve may be defined as the angle between

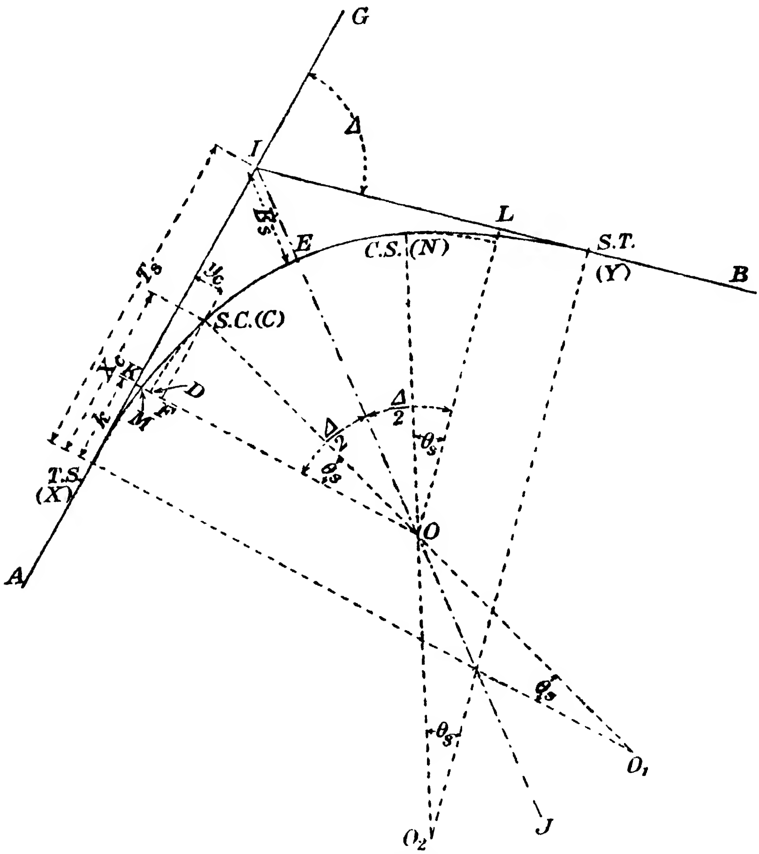


FIG. 8.3.—Circular Arc with Spirals at Entrance and Exit.

the perpendicular to the tangent at TS (or ST) and the perpendicular to the tangent at the point considered. The total spiral angle of the first spiral is subtended at the point O_1 which does not fall on the centre-line IJ of a symmetrical arrangement of two equal transitions and a central arc, but lies on an extension of the line passing through O, the centre of the arc, and the point $S.C.$ Similarly, the total spiral angle of the second spiral is subtended at O_2 which lies on the line passing through O and $C.S.$ To avoid complication in the lettering, $T.S$ in fig. 8.3 will be called "X" and $S.C$ will be called

" C ". I is the intersection point of the tangents, IA and IB, and if AI is produced to G, $\angle GIB$ is the deviation angle, $\triangle$. If IO cuts the circular arc at E, IE is termed the " external distance " (E_s) and IX is the tangent length (T_s). Draw OK and OL perpendicular to the tangents IA and IB respectively and let the point $C.S.$ be called " N " and the point $S.T.$ " Y ". Then $\angle KOC = \angle LON = \theta_s$. But $\angle KOL = \triangle$. Hence the angle CON subtended by the circular arc is $\triangle - 2\theta_s$.

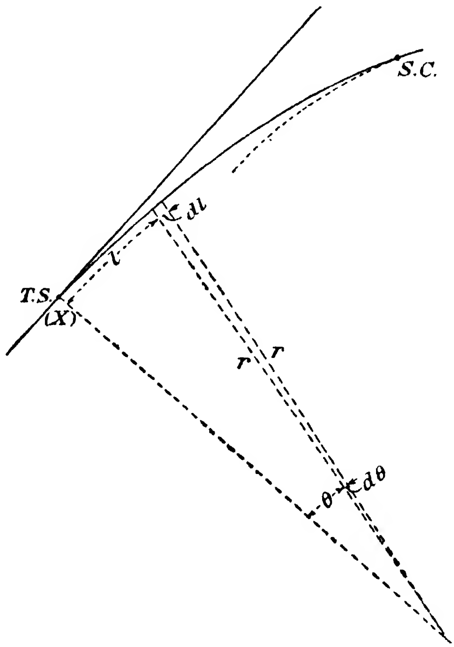


FIG. 8.4.—Derivation of Formula for Spiral Angle.

The spiral angle, θ_s, may be readily expressed in terms of the length of the spiral, L_s, and the radius of the circular arc, R_c. Consider, in fig. 8.4, a small element of the spiral of length dl at a mean distance of l from the tangent point X and let the angle subtended by this short length of curve be $d\theta$. It may be assumed that the radius remains constant over this very short length. Let this radius be r. Then the short length of curve may be treated as a circular arc and we may write $dl = r.d\theta$, the angle being expressed in radians.

It is the basic property of the spiral that the radius is inversely proportional to the distance along the curve from the tangent point. This relationship may be expressed by the equation $r = \frac{K}{l}$, where K is a constant. But when $r = R_c$, $l = L_s$.

Therefore

$$K = r.l = R_c.L_s. \text{ or } r = \frac{R_c.L_s.}{l}$$

Hence $$dl = \frac{R_c.L_s}{l}.d\theta, \text{ or } l.dl = R_c.L_s.d\theta$$

By integration, $$\frac{l^2}{2} = R_c.L_s.\theta,$$

or θ, the spiral angle up to the point considered, $= \dfrac{l^2}{2.R_cL_s}$

When $l = L_s$, $\theta = \theta_s$,

i.e. spiral angle, $\theta_s = \dfrac{L_s}{2.R_c}$ in radians $= \dfrac{L_s}{2.R_c} \times 57{\cdot}3°$

This is a very important equation from which the main dimensions of the curve may be deduced.

We can see, immediately, for example, that

$$\frac{\theta}{\theta_s} = \frac{\dfrac{l^2}{2.R_cL_s}}{\dfrac{L_s}{2.R_c}} = \frac{l^2}{L_s{}^2},$$

or the spiral angles of various points along the curve are proportional to the squares of the distances of the points from the tangent point.

In order to make the necessary calculations for setting out the curve it is necessary to obtain expressions for the co-ordinates of the terminal point, C, of the spiral, measured parallel and perpendicular to the tangent.

Let these be x_c and y_c, respectively.

It can be shown that $x_c = L_s\left[1 - \dfrac{\theta_s{}^2}{10} + \dfrac{\theta_s{}^4}{216} - \text{smaller terms}\right]$

and $y_c = L_s\left[\dfrac{\theta_s}{3} - \dfrac{\theta_s{}^3}{42} + \dfrac{\theta_s{}^5}{1320} - \text{smaller terms}\right]$

The derivation of these expressions is given in Appendix I, page 273. Only the first two terms in the brackets need be considered and, in some cases, all terms after the first may be neglected, but it must be remembered that θ_s is expressed in radians.

Referring again to fig. 8.3, let the circular arc be produced backwards to cut OK at D and draw FC perpendicular to OK. Then KD is known as the " shift ". It will be seen that a circular arc of slightly larger radius than R_c could be fitted between the points K and L as tangent points, with O as centre. If this circular arc were then "spiralised", or smoothed into the tangents by transition curves, its radius would be slightly decreased and it would be moved inwards to enable the transitions to be introduced. Hence the name "shift" which represents the extent of this movement.

In the accepted nomenclature, the shift is represented by "p" and, from the figure, it will be seen that

$$KD + DO = KF + FO$$

or $$p + R_c = y_c + R_c.\cos\theta_s$$

i.e. $$p = y_c + R_c(\cos\theta_s - 1)$$
$$= y_c - R_c(1 - \cos\theta_s)$$

An approximate, but simpler, expression for the shift may be obtained thus: The sine or cosine of an angle may be represented by a series of gradually diminishing terms, the accuracy of the value depending on the number of terms considered. Cos θ may be calculated from the series

$$1 - \frac{\theta^2}{1.2} + \frac{\theta^4}{1.2.3.4} - \frac{\theta^6}{1.2.3.4.5.6} \cdots$$

θ in the above terms being the radian measure of the angle. An approximate value of cos θ will be given by $1 - \frac{\theta^2}{2}$.

But $$y_c = \frac{L_s\theta_s}{3} \text{ approximately and } \theta_s = \frac{L_s}{2.R_c}$$

Hence $$p = \frac{L_s^2}{6.R_c} - R_c\left(1 - 1 + \frac{L_s^2}{8.R_c^2}\right) = \frac{L_s^2}{24R_c}$$

This is a very useful approximate formula.

Let the spiral intersect OK at M. Then M will be very nearly the mid-point of the spiral and also the mid-point of the shift KD. The distance XK is usually represented by the symbol "k" and is very closely $\frac{L_s}{2}$. Actually $k = x_c - R_c.\sin\theta_s$.

The tangent length, IX or IY, usually written "T_s" may be expressed in several ways. For instance:

$$IX = XK + KI = XK + OK \tan\frac{\Delta}{2}$$

or $$T_s = k + (R_c + p)\tan\frac{\Delta}{2}$$

The external distance, E_s, or IE may be found from the expression

$$\frac{IE + EO}{OK} = \sec\frac{\Delta}{2}, \text{ or } E_s = (R_c + p)\sec\frac{\Delta}{2} - R_c$$

The foregoing expressions give the values of the leading dimensions of the spiral based on the values of R_c and Δ, both of which form the basic data for the design of the curve, and θ_s which is readily calculated when the spiral length has been chosen.

It may be convenient to summarise the formulae thus far obtained :

$$\text{Spiral angle, } \theta_s = \frac{L_s}{2.R_c} \text{ (radians)}$$

$$\text{Co-ords. of } S.C \begin{cases} x_c = L_s\left(1 - \dfrac{\theta_s^{\,2}}{10}\right) \\ y_c = L_s\left(\dfrac{\theta_s}{3} - \dfrac{\theta_s^{\,3}}{42}\right) \end{cases}$$

$$\text{Shift, } \quad p = y_c - R_c(1 - \cos\theta_s), \text{ or approx. } \frac{L_s^{\,2}}{24R_c}$$

$$k = x_c - R_c.\sin\theta_s, \text{ approx. } \frac{L_s}{2}$$

$$\text{Tangent length, } \quad T_s = k + (R_c + p)\tan\frac{\Delta}{2}$$

$$\text{External distance, } E_s = (R_c + p)\sec\frac{\Delta}{2} - R_c.$$

Setting out a Spiral. When Δ, L_s and R_c are known, the tangent length can be calculated and the points *T.S.* and *S.T.* set out. The spirals may then be located either by offsets from the tangents or by tangential angles. In the offset method, values of x and y are calculated for given values of l and θ, using the formulae for co-ordinates in conjunction with the relationship $\theta = \dfrac{l^2}{2R_cL_s}$.

The theorectical principle of the more usual tangential angle method is as follows : From fig. 8.5, if ϕ is the tangential angle for any point of the curve whose co-ordinates are x and y and spiral angle θ,

$$\tan\phi = \frac{y}{x} = \frac{l\left(\dfrac{\theta}{3}\right)}{l} = \frac{\theta}{3} \text{ radians, approximately.}$$

Hence, assuming that the tangent is equal to the radian measure, $\phi = \dfrac{\theta}{3}$. This is an approximation which is only justifiable up to a limiting value of θ, depending upon the accuracy with which the curve is to be set out. A better expression is : $\phi = \dfrac{\theta}{3} - c$, where c is a correction.

The following table gives values of c in minutes for various values of θ in degrees and is extracted from Hickerson's " Highway Surveying and Planning ", published in the United States by Messrs. McGraw Hill:

θ in Degrees	c in Minutes	θ in Degrees	c in Minutes
20	0·4	49	6·2
25	0·8	50	6·6
30	1·3	51	7·0
35	2·1	52	7·5
40	3·2	53	7·9
41	3·4	54	8·4
42	3·7	55	8·8
43	3·9	56	9·3
44	4·2	57	9·8
45	4·5	58	10·4
46	4·9	59	10·9
47	5·3	60	11·4
48	5·8	61	12·0

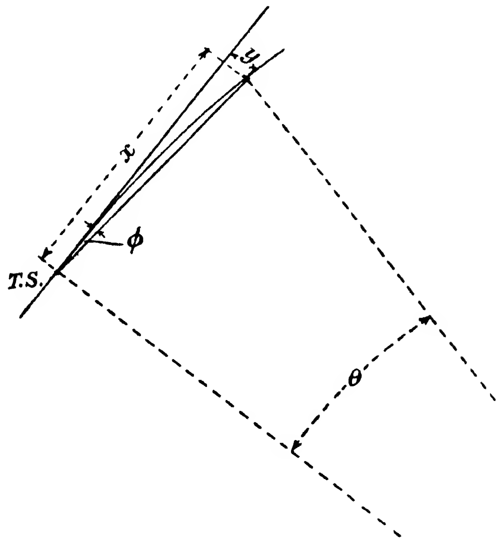


Fig. 8.5.—Derivation of Formula for Tangential Angle for a Point on Spiral.

Hence, for given values of l and R_c, we can calculate the value of θ corresponding to any distance l along the curve and thus obtain the tangential angle, ϕ, which, in conjunction with the distance, locates the point in question. The tangential angles are set out from the tangent points, $T.S.$ and $S.T.$, and the distances are measured as successive chords, the chord length being chosen so that it agrees closely with the equivalent curve length. The setting out is thus a similar procedure to the tangential angle method for circular arcs.

In a great many cases the correction, c, nay be neglected. For values of θ up to 20° it is less than 30 seconds and most road engineers would be satisfied if the tangential angles in curve ranging were set out correct to the nearest half-minute. In the case of a wholly transitional curve of a symmetrical type, $2\theta_s = \triangle$, and, if the above limit of accuracy be accepted, uncorrected tangential angles can be used for deviation angles up to 40° with a curve of this type.

Much larger deviations could be accommodated without correcting the tangential angles by inserting a circular arc between the transitions. This, it may be added, is the normal and correct practice. It should be pointed out that in the lemniscate the tangential angle is exactly one-third of the angle subtended by the curve up to any point, but the radius does not diminish at a strictly uniform rate and, on the whole, the spiral is the simpler curve for calculations.

Transition Curve Design. The procedure of "designing" a combination of circular arc and symmetrical transitions is briefly as follows :

The curve is first drawn on the plan as a circular arc of the largest practicable radius. Governing conditions may be the existence of a building which cannot be demolished and which must, therefore, be given adequate clearance, or, in the case of a road to be constructed round the shoulder of a hill, an economic limit to the depth of cutting involved. The extent of the latter may be found, roughly, by inspection from a contoured plan if an approximate longitudinal section has been prepared first, and the formation levels provisionally settled.

If the plain circular curve is now combined with reasonably short spirals at entrance and exit, the radius will be slightly reduced and the tangent length and external distance increased. A reasonable value for R_c may be deduced from the plan, the value of $\triangle$ will have been measured in the field, and the remaining essential dimension is the transition length, L_s. The complete data for setting out the curve can be calculated quite simply from R_c, L_s, and $\triangle$, and the suitability of the lay-out with regard to the surrounding topography can be judged from a knowledge of the shift, the tangent length, and the external distance.

The method of determining the transition length, however, is a somewhat controversial matter and will be discussed in some detail.

Determination of Transition Length.

1. Shortt's Method. This method was originally applied by W. H. Shortt [2] to railway curves and, according to Royal-Dawson [3], is equally applicable to road curves. It is based on a limiting value of the rate of gain of radial acceleration. If a vehicle travels at a uniform speed, V, over a transition curve of length L having a minimum radius R, the maximum value of the radial acceleration will be $\frac{V^2}{R}$ and this will be acquired in a time $\frac{L}{V}$. If V, R and L are in the appropriate units, the rate of gain of radial acceleration will be $\frac{V^2}{R} \div \frac{L}{V}$, or $\frac{V^3}{R.L}$ feet per second per second per second.

According to Shortt [2] the maximum rate at which the increase in radial acceleration will pass unnoticed by a railway passenger is 1 foot per second per second per second and Royal-Dawson [3] has accepted this figure as a criterion for the design of highway transitions. Conditions are not identical with those on a railway, however, and doubt has been cast on the validity of the specified value when applied to roads. Nevertheless, Shortt's principle has been widely used, although much uncertainty exists as to the correct figure for the limiting value of $\frac{V^3}{R.L}$. The following table gives some of the values specified by highway authorities abroad :

Authority	Max. Value of $\frac{V^3}{R.L}$ in ft./sec.^{3}
U.S. Bureau of Public Roads	2
Country Roads Board, Victoria, Australia :	
For speeds less than 50 m.p.h.	2
For speeds of 50 m.p.h. and over	1·536
Oregon Highway Commission	3

No reference is made to Shortt's method of determining transition length in the Ministry of Transport Memorandum No. 575.

Recent experiments by J. J. Leeming [4] and A. N. Black, in which autographic accelerometer records were taken while vehicles were travelling round a variety of curves, indicate that the rate of gain of radial acceleration does not appear to have a decisive influence on driving procedure. A further reference is made to these experiments on page 215. The expression $\frac{V^3}{R.L}$ takes no account of superelevation and, as we have seen, the centrifugal reaction is partly neutralised by suitable banking.

If the value of unity is accepted as the maximum permissible value of $\frac{V^3}{R.L}$ in ft./sec., an unduly long transition frequently results and the central circular arc becomes so short as to be impracticable. Hence Royal-Dawson advocates the use of the wholly transitional curve. As already mentioned on page 205, this type of curve is not altogether suitable for general road use and may, in fact, encourage drivers to encroach on the wrong side of the road in order to traverse a larger radius than that afforded by the transition in the neighbourhood of its mid-point.

2. Rate of Application of Superelevation. In American practice the normal camber on the tangent is removed gradually on the appropriate side of the road when approaching a curve so that the outer half is horizontal at the tangent point as shown in fig. 8.6. In British practice, however, the camber is usually continued up to the tangent point. The outer edge is then gradually raised until the maximum superelevation is obtained at the junction of the tran-

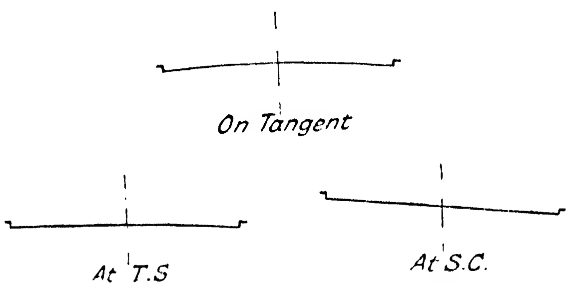


FIG. 8.6.—Change of Cross-section of Road.

sition and the circular arc. The inner edge of the road is kept at the normal level as a rule, the entire superelevation being applied on the outer edge. As a possible alternative, half the superelevation could be obtained by raising the outer edge and half by lowering the inner edge, but this method causes an ugly sag in the inner kerb line and is not recommended. In all cases the changes of gradient must be smoothed out by vertical curves (see pages 224 *et seq*). There must obviously be a limit to the rate at which the outer edge is raised since banking applied too suddenly would result in considerable discomfort to the occupants of fast vehicles and might well be highly dangerous. Barnett, in the handbook [5] on transition spirals published by the U.S. Bureau of Roads, stipulates that superelevation should not be applied at a steeper longitudinal gradient than 1 in 200. With a maximum crossfall of 1 in $14\frac{1}{2}$ and a 22-foot carriageway, this rule gives a minimum transition length of rather more than 300 feet, if the inner channel levels are unaltered and the superelevation applied entirely to the outer edge. In some situations this transition length would be difficult to fit in and slightly steeper rates may be necessary unless the required transition length is halved by raising the outer edge of the carriageway and lowering the inner edge by equal amounts.

3. Leeming and Black's Experiments. Drivers of road vehicles habitually describe their own transition curves. In doing so, encroachment on the opposite lane occurs at times, indicating too high a speed for the lay-out of the road, but a careful driver will describe a curve within the limits of his own traffic lane. Consequently, if the curves described under normal driving conditions can be analysed, it is possible that a practical basis for transition curve design may be obtained. This would enable curves to be designed to suit normal driving methods, instead of basing curve design on theoretical principles which may, or may not, be correct, and expecting drivers to conform to the configuration so produced.

As already mentioned, Leeming [4] and Black have investigated driving procedure by examining accelerometer diagrams automatically recorded while vehicles were travelling round curves. Among the conclusions which can be drawn from this investigation, the following is of considerable interest and reduces the determination of transition length to a matter of extreme simplicity: There is a strongly marked tendency for drivers to make a definite proportion of the curve transitional, the overall mean being about two-thirds, the entrance transition usually being slightly longer than the exit. It is, therefore, suggested, as a practical rule, that the length of each transition should be made one-third of the total length of the curve.

Now we have seen, on page 208, that for the simple spiral,

$$\theta_s = \frac{L_s}{2.R_c} \text{ or } L_s = 2R_c.\theta_s.$$

The total length of curve $= 2.L_s + R_c(\triangle - 2.\theta_s)$ where $\triangle$ and θ_s are in radians.

Hence $$L_s = 2.R_c.\theta_s = \frac{4.R_c\theta_s + R_c(\triangle - 2.\theta_s)}{3}$$

i.e. $$6R_c\theta_s = 4R_c\theta_s + R_c\triangle - 2R_c\theta_s,$$

$$\therefore 4\theta_s = \triangle,$$

or $$\theta_s = \frac{\triangle}{4}.$$

The spiral angle is thus known immediately and with a knowledge of R_c the whole of the remaining curve data can be readily obtained.

4. Empirical Rules used in Quebec. The Ministère de la Voirie of the Province of Quebec has standardised the circular arc with spiral transitions for road curves in which the degree of curvature of the arc is 1° 30′, or more, i.e. in which the radius is 3,819·7 feet, or less. For larger radii no transitions are used. The transition length is given by the empirical rules summarised in the following table:

Degree of Curvature of Arc ($D°$)	Equivalent Radius (feet)	Transition Length (feet)
1° 30′ to 4°	3,819·7 to 1,432·4	100.D (150 to 400)
4° 30′ to 6°	1,273·2 to 954·9	60.D (270 to 360)
6° 30′ to 9°	881·5 to 636·6	40.D (260 to 360)
10° to 14° . . .	573·0 to 409·3	20.D (200 to 280)

Setting out Points Invisible from Tangent Point. If the entire spiral from *T.S.* to *S.C.*, (or *S.C.S.* in fig. 8.9) cannot be set out from the initial tangent point, the theodolite must be moved forward to a point previously established on the curve, but the

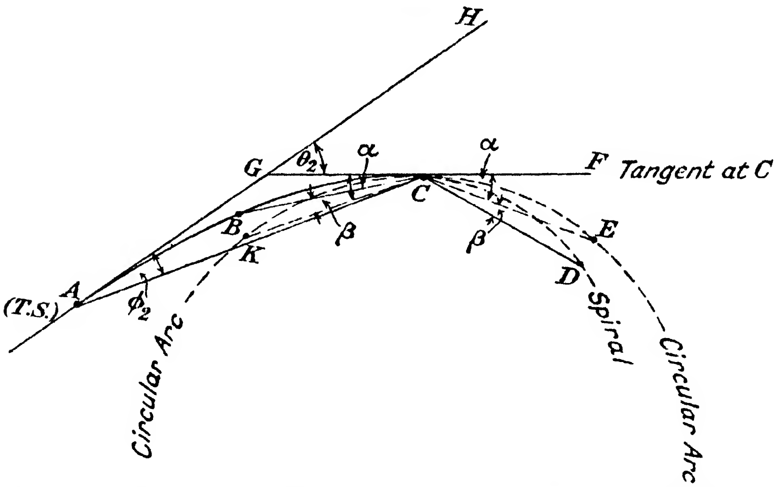


FIG. 8.7.—Continuation of Setting out of Spiral from an Intermediate Point.

subsequent procedure differs from that adopted when a similar necessity arises with a circular arc.

The angles required for setting out the spiral beyond the established point must be specially calculated and this may be done very simply by using the principle of "Equivalent Deviations" which may be expressed as follows: Both a straight line and a circular arc have constant curvature and the curvature of a spiral is changing at a constant rate. Consequently, if at any intermediate point along a spiral we describe a circular arc of the same radius as that of the spiral at that point, the spiral will diverge from the arc at the same rate as it does from the initial tangent.

Thus, in fig. 8.7, A,B,C are three consecutive points on a spiral, A being the tangent point ($T.S.$) and AH the tangent. Let $AB = BC = l$ and suppose that C is the last point visible from A so that the theodolite must be shifted from A to C for further setting out. Let D represent the next point ahead on the spiral and let CE be a circular arc of the same radius, r, as that of the spiral at C, CE being the uniform chord length, l. Let GCF be the tangent to the arc at C. Then the total tangential angle, FCD, consists of two parts, α and β, where α is the tangential angle FCE for setting out a circular arc of radius r with chord length l and β (or $\angle ECD$) is the same tangential angle as that required for setting out a spiral length l from the initial tangent point A.

i.e. $$\alpha = \sin^{-1}\frac{l}{2.r} \quad \text{(cf. page 192)}$$

and $$\beta = \frac{\theta}{3} - C$$

where $$\theta = \frac{l^2}{2R_cL_s} \quad \text{(cf. pages 207 and 208)}.$$

Hence, if the direction of the tangent FC could be located, the setting out of the point D and subsequent points on the spiral could be accomplished.

Now exterior $\angle HGC = \theta_2 =$ spiral angle up to the point C

$\therefore \angle GCA = \theta_2 - \phi_2$ where ϕ_2 is the tangential angle for the point C.

If the theodolite at C were sighted back to A with one vernier reading $360° - (\theta_2 - \phi_2)$ and the lower plate were clamped and the upper plate rotated in azimuth until a zero reading was obtained on the *same* vernier, the telescope would be pointing along the tangent at C in the direction CG. If the upper plate were then rotated further until the *opposite* vernier gave a zero reading, the telescope would be pointing along the tangent at C in the direction CF. A further rotation through the angle $\alpha + \beta$ would fix the direction of the line CD, thus enabling D to be located. Other methods of locating the direction of the tangent can be devised but this method has the advantage that transiting is obviated.

It should be noted that the "Back Angle", GCB, is $\alpha - \beta$, K being the corresponding point on the circular arc and $\angle BCK$ being β and $\angle GCK$ being α. Hence the spiral may be continued from any intermediate point by sighting back to the previous point with one vernier reading $360° - (\alpha - \beta)$ and then proceeding in the manner described above. This method is adopted when the original tangent point, $T.S.$, is invisible from intermediate points on the curve.

Numerical Example of a Calculation for a Circular Arc with Symmetrical Spirals

Preliminary Data :

Deviation angle, $\triangle = 34° \ 35'$.
Chainage of intersection : 182 + 59 feet.
Radius of central circular arc, R_c = 1,000 feet.
Suggested length of each spiral = L_s = 400 feet.
Recommended superelevation in M.O.T. Memorandum 575 for 1,000 feet radius : 1 in 14½.
Carriageway width : 22 feet on tangent, 24 feet on circular arc.
Max. superelevation : 1·65 feet.

The length of 400 feet is ample for the application of this superelevation, even allowing for 0·2 foot adverse camber at *T.S.*

Determination of length of Circular Arc :

$$\text{Spiral angle, } \theta_s = \frac{L_s}{2.R_c} = \frac{400}{2000} = 0{\cdot}2 \text{ radians}$$
$$= 11° \ 27' \ 33''$$

$$\text{Angle subtended by circular arc} = \triangle - 2\theta_s$$
$$= 34° \ 35' - 22° \ 55' \ 06''$$
$$= 11° \ 39' \ 54'' = 11{\cdot}665°$$

$$\text{Length of arc} = \frac{2.\pi.1000}{360} \times 11{\cdot}665 \text{ feet}$$
$$= 204 \text{ feet.}$$

This gives a combination of circular arc and spirals which is badly proportioned, the arc being too short. A spiral length of 300 feet is about the minimum which would suffice for the application of the full superelevation and the proportions of the curve will now be investigated assuming L_s = 300 feet.

$$\theta_s = \frac{300}{2000} = 0{\cdot}15 \text{ radians} = 8° \ 35' \ 40''$$

$$\text{Angle subtended by circular arc} = 34° \ 35' - 17° \ 11' \ 20''$$
$$= 17° \ 23' \ 40''$$

from which length of circular arc = 304 feet, to nearest foot. This gives a more satisfactory proportion. The external distance and tangent length should now be calculated and checked with the limitations imposed by surrounding topography.

First calculate the x and y co-ordinates of *S.C.*, thus :

$$x_c = L_s(1 - \frac{\theta_s^2}{10} + \ . \ . \ . \ ) = 300\left(1 - \frac{0{\cdot}15^2}{10}\right) = 299{\cdot}3 \text{ feet}$$

(The third term $L_s \frac{\theta^4}{216}$ gives only 0·0007 feet which is obviously negligible.)

$$y_c = L_s\left(\frac{\theta_s}{3} - \frac{\theta_s^3}{42} + \ldots\right) = 300\left(\frac{0{\cdot}15}{3} - \frac{0{\cdot}15^3}{42}\right) = 15{\cdot}00 \text{ feet}$$

Shift, $p = y_c - R_c(1 - \cos\theta_s) = 15{\cdot}00 - 1000(1 - \cos 8^\circ\ 35'\ 40'')$
$= 3{\cdot}8$ feet

$k = x_c - R_c.\sin\theta_s = 299{\cdot}3 - 1000.\sin 8^\circ\ 35'\ 40''$

$= 149{\cdot}9$ feet $\left(\text{very nearly } \frac{L}{2}\right)$

Tangent length, $T_s = (R_c + p).\tan\frac{\Delta}{2} + k$

$= 1003{\cdot}8.\tan 17^\circ\ 17'\ 30'' + 149{\cdot}9$
$= 462{\cdot}4$ feet.

External distance, $E_c = (R_c + p).\sec\frac{\Delta}{2} - R_c$

$= 1003{\cdot}8.\sec 17^\circ\ 17'\ 30'' - 1000$
$= 51{\cdot}3$ feet.

The above formulae need not be memorised as the expressions can be derived immediately from fig. 8.3.

Should the tangent length and external distance conform to practical requirements, the tangential angles are now calculated. Two methods of setting out may be adopted, either the spiral may be divided into a convenient number of equal chord lengths, usually ten, and the 100-foot chainage pegs for the continuous centre-line chainage interpolated afterwards in the field, or the initial chord length may be chosen to locate the first peg on the spiral at some convenient round number of feet by adding the requisite length to the chainage of the tangent point, *T.S.* The first peg on the spiral may, or may not, be a 100-foot chainage point, but such points will be located directly as they occur along the spiral by an appropriate choice of chord length.

Method I.—Calculation of Tangential Angles for a 10*-point Spiral.* In this case the chord length, l is $\frac{L_s}{10}$ and the calculation is extremely simple since the successive spiral angles are found by multiplying the first spiral angle by 2^2, 3^2, 4^2, and so on and the tangential angles may be taken as one-third of the corresponding spiral angles without correction up to a value of 20° for the latter.

The tangential angles are tabulated below:

Chainage (feet)	Dist. from *T.S.* l feet	$\theta = \dfrac{l^2}{2R_cL_s}$ Radians	$\phi = \dfrac{\theta}{3} - C$ Radians	ϕ °	′	″
177 + 96·6	*T.S.*	—	—	0	0	0
178 + 26·6	30	0·0015	0·0005	0	1	43
178 + 56·6	60	0·006	0·002	0	6	52
178 + 86·6	90	0·0135	0·0045	0	15	28
179 + 16·6	120	0·024	0·008	0	27	30
179 + 46·6	150	0·0375	0·0125	0	42	58
179 + 76·6	180	0·054	0·018	1	1	53
180 + 06·6	210	0·0735	0·0245	1	24	13
180 + 36·6	240	0·096	0·032	1	50	0
180 + 66·6	270	0·1215	0·0405	2	19	14
180 + 96·6 (*S.C.*)	300	0·15	0·05	2	51	53

Method II.—Tangential Angles for setting out 100-foot Chainage Pegs.

		foot
Chainage of Intersection	. . . .	182 + 59
Tangent Length	. . . . . .	4 + 62·4
Chainage of *T.S.*	. . . . . .	177 + 96·6

In this case a chord of only 3·4 feet would locate the 178 + 00 peg if measured from *T.S.* and the tangential angle would be too small to be measurable. The remainder of the spiral could be set out by 50-foot chords and the computation of the tangential angles is tabulated below:

Chainage (feet)	Dist. from *T.S.* l foot	$\theta = \dfrac{l^2}{2R_cL_s}$ Radians	$\phi = \dfrac{\theta}{3} - C$ Radians	ϕ °	′	″
177 + 96·6	*T.S.*	—	—	0	0	0
178 + 00	3·4	—	—	0	0	0
178 + 50	53·4	0·00475	0·00158	0	5	26
179 + 00	103·4	0·01782	0·00594	0	20	25
179 + 50	153·4	0·03922	0·01307	0	44	56
180 + 00	203·4	0·06895	0·02298	1	19	0
180 + 50	253·4	0·10702	0·03567	2	2	38
180 + 96·6 (*S.C.*)	300·0	0·15000	0·05000	2	51	53

It will be apparent that the computation is much more cumbersome in this case than for the 10-point Spiral.

Calculation for the Circular Arc

The tangential angles are calculated in the ordinary way (see page 194) and the chord length should be not more than one-twentieth of the radius, i.e. for $R_c = 1000$ feet, chord length should be not more than 50 feet.

Chainage of $S.C.$ = 180 + 96·6

First chord to locate peg at 181 + 00 = 3·4 feet

First tangential angle $= \sin^{-1} \frac{3.4}{2000} = 0{\cdot}00170 = \sin\ 0°\ 6'\ 1''$

Tangential angle for 50-foot chord $= \sin^{-1} \frac{50}{2000} = 0{\cdot}025$

$= \sin\ 1°\ 25'\ 58''.$

Length of curve measured by chords = 303·5 feet.

Chainage of C.S. = 184 + 00·1

Final chord length = 0·5 feet. Corresponding increment in tangential angle negligible.

Complete tangential angles are tabulated below :

Chainage (feet)	Chord Lengths (feet)	Increment in Tangential Angle	Total Tangl. Angle
180 + 96·6	$S.C.$	—	0° 0′ 0″
181 + 00	3·4	0° 5′ 52″	0° 5′ 52″
181 + 50	50	1° 25′ 58″	1° 31′ 50″
182 + 00	50	1° 25′ 58″	2° 57′ 48″
182 + 50	50	1° 25′ 58″	4° 23′ 46″
183 + 00	50	1° 25′ 58″	5° 49′ 44″
183 + 50	50	1° 25′ 58″	7° 15′ 42″
184 + 00·1 ($C.S.$)	50·1	1° 26′ 08″	8° 41′ 50″

The total tangential angle checks with the correct value, i.e. half the angle subtended by the arc at the centre.

Setting out the Central Circular Arc

When a circular arc is combined with spirals at entrance and exit, the arc is set out from the point $S.C.$ by the ordinary method given on page 192. Theoretically the direction of the tangent at $S.C.$ should be located and the telescope of the theodolite directed along this line with one vernier reading zero. In practice the tangent direction is not actually located, but the instrument is manipulated in such a way that this condition is, in effect, fulfilled. The procedure may be understood from fig. 8.8, in which A is the spiral tangent point ($T.S.$) and B is the junction of the spiral and the circular arc ($S.C.$) DE is the tangent at B, AF is the tangent at A, and GH is perpendicular to AF. Then $\angle FDE = \angle GHB = \theta_s$. $\angle DAB$ is the final tangential angle for the spiral, ϕ_s, and $\angle FDE = \angle DAB + \angle DBA$. Therefore $\angle DBA = \theta_s - \phi_s$. Hence, if the theodolite at B were directed back to A with one vernier reading $360° - (\theta_s - \phi_s)$, the lower plate clamped and the upper plate released and rotated until the *same* vernier reads 360°, the telescope would now be pointing along the tangent ED in the direction BD. If the upper plate were now rotated until the *opposite* vernier read 360°, the telescope would now be pointing along the tangent ED in the direction BE, which is the setting required. On further clockwise movement of the upper plate to the calculated vernier reading for the first tangential angle

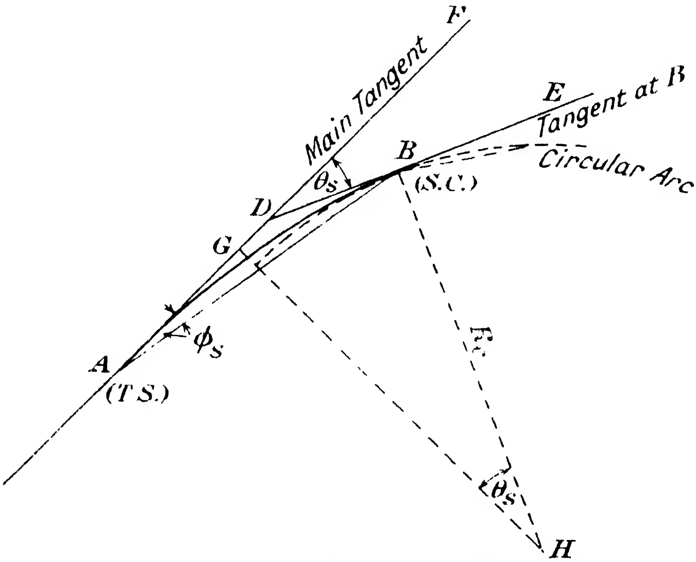


FIG. 8.8.—Location of Tangent for Central Circular Arc.

for the circular arc, the direction of the first chord will be correctly located. Thus the procedure consists in sighting back to the initial tangent point of the spiral with one vernier reading $360° - (\theta_s - \phi_s)$ and then setting the successive tangential angles for the arc on the opposite vernier.

When θ_s is not greater than 20°, ϕ_s is, of course, $\dfrac{\theta_s}{3}$.

Should the point A be invisible from B, the theodolite is directed back to an intermediate point on the spiral distant x from B with one vernier reading $360° - (\alpha - \beta)$, where α is the tangential angle for the circular arc using a chord length x, and β is the tangential angle for a spiral length x, measured from the initial tangent point, A (see page 216 and fig. 8.7). The tangential angles for the arc are then set out on the opposite vernier, as before.

It should be noticed that an excellent check on the setting out is afforded by measuring the external distance, E_s, from the intersection point of the main tangents to the middle point of the circular arc. This applies equally well to the middle point of a wholly transitional curve.

Second Spiral. This is set out from the tangent point *S.T.* using the same angles as those computed for the first spiral, the vernier readings being obtained by subtraction from 360° as the curve is set out left handed. The final tangential angle should check on *C.S.*

Wholly Transitional Curves. With certain combinations of deviation angle and minimum radius it is impossible to obtain an

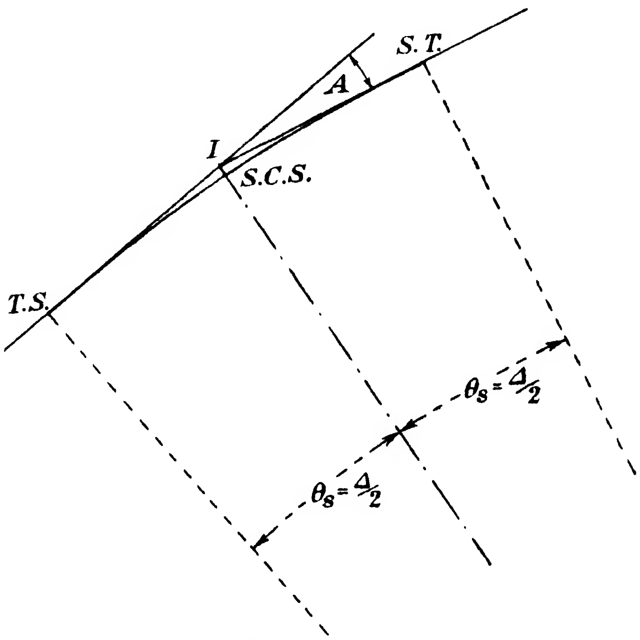


FIG. 8.9.—Wholly Transitional Curve.

adequate length of transition in conjunction with a reasonable length of circular arc. In such cases a wholly transitional curve may be used. The spiral angle, θ_s, is then half the deviation angle, Δ, and if the minimum radius, R_c, has been settled the spiral length is determined automatically from the expression $L_s = 2 \,.\, R_c \,.\, \theta_s = R_c . \Delta$. The calculation of the tangential angles is based on the theory already given on pages 219 to 221 and the points *S.C.* and *C.S.* coincide on the bisector of the apex angle at *S.C.S.* (fig. 8.9).

It will be noticed that for a given minimum radius the length of the entire transition curve is twice the length of a circular arc of the same radius. Again, for a given tangent length the radius of a plain circular arc will be about double the minimum radius of the transition, but the transition will give a shorter external distance, i.e. it will fit more closely into the angle between the tangents. This is often an advantage when a projected curve is hemmed in by buildings or when it is desired to save excavation when carrying a curve round the shoulder of a hill.

A numerical example is given below:

Deviation angle: 12° 38′ (Δ)

Minimum radius: 800 feet (R_c)

$$\theta_s = \frac{\Delta}{2} = 6° \ 19' = 0{\cdot}1102 \text{ radians} = \frac{L_s}{2 \, . \, R_c}$$

Therefore $L_s = 2 \times 800 \times 0{\cdot}1102 = 176{\cdot}3$ feet

The external distance, in this case, is 6·48 feet and the tangent length very nearly 177 feet.

A circular arc of 1,600 feet radius could be used with this same tangent length, but the external distance would then be very nearly 10 feet.

Transition Curve Tables. Tables are obtainable giving the necessary data for setting out transition curves of various types based on a variety of preliminary assumptions including the controversial rule regarding a limiting value of the rate of gain of radial acceleration. In most cases a certain amount of interpolation is necessary before these tables can be applied to specific cases and in the author's opinion the method of deriving the necessary fieldwork data for spiral transitions is so simple that tables are hardly required, particularly if the curve is set out as a ten-point spiral in the manner indicated on page 219. A list of some of the better-known tables is given, however, in the bibliography on page 281.

Vertical Curves

Vertical curves are necessary to smooth out the road profile at changes of grade. Various curves have been suggested for this purpose, including the circular arc, simple parabola, cubic parabola and lemniscate. The gradients normally encountered on main roads are rarely steeper than about 1 in 10 and with slopes of this steepness, or less, the grade lines which form the tangents to the curve give the same conditions as horizontal tangents with a very small deflection angle and all four types of curve mentioned above would practically coincide when used under these conditions. Preference may therefore be given to the curve which involves the simplest calculations and this is undoubtedly the simple parabola with the equation $y = mx^2$.

Incidentally, the slope of this curve changes at a uniform rate and this appears to be an appropriate quality for a vertical transition curve. This fact is expressed, mathematically, by the first differential of the equation to the curve : $\dfrac{dy}{dx} = 2x$.

It is usual to adopt various conventions and assumptions in vertical curve calculations and these are summarised below :

(1) The grades are expressed as percentages, positive for an ascent from left to right and negative for a descent from left to right in the longitudinal section. Thus, a 1 in 50 up gradient meeting a 1 in 33 down gradient, as shown to an exaggerated scale in fig. 8.10, gives a combination of 2% and − 3%.

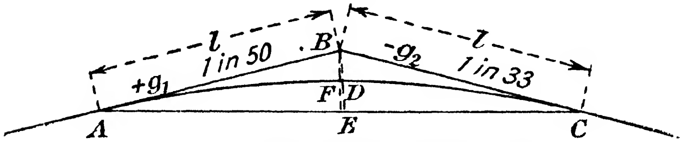


FIG. 8.10.—Symmetrical Summit Curve.

The algebraic difference of grade is 2 − (− 3), or 5%. This quantity, calculated with due regard to signs and usually given the symbol " g ", is a basic factor in vertical curve computations.

(2) A curve with equal tangents, AB and BC in fig. 8.10, is termed " symmetrical " and it is assumed that the length of the curve ADC is equal to the length of the chord AEC and that both are equal to twice the tangent length. If the chord is bisected at E, BE cutting the curve at D, $BD = DE$, from the properties of the parabola.

(3) It is assumed that the vertical dimension BF is equal to the slanting dimension BD. The latter is usually given the symbol " e ".

The above assumptions are justifiable for practical setting-out purposes owing to the comparative flatness of normal grade lines. This would be apparent if the grades were plotted to the same scale vertically and horizontally.

Symmetrical Curve. Calculation of Vertical Offset (e) from Intersection Point of Grades

A summit is shown in fig. 8.10, and a valley in fig. 8.11. In both cases length of curve $= L$, length of tangent $= l$, and $L = 2l$. Consider,

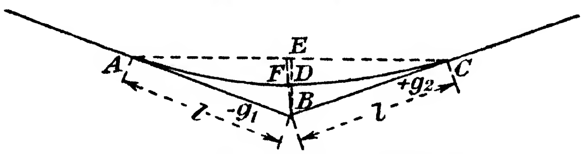


FIG. 8.11.—Symmetrical Valley Curve.

first, the summit and let the reduced level of A, for simplicity, be 0·00. E is the middle point of AC. Then

$$\text{the reduced level of } B = \frac{l}{100}g_1$$

$$\text{reduced level of } C = \frac{l}{100}g_1 - \frac{l}{100}g_2$$

$$\text{reduced level of } E = \frac{l}{200}(g_1 - g_2)$$

$$\therefore BE = \frac{l}{100}g_1 - \frac{l}{200}(g_1 - g_2) = \frac{l}{200}(g_1 + g_2)$$

$$\text{and } e = \frac{BE}{2} = \frac{l}{400}(g_1 + g_2) = \frac{L}{800}(g_1 + g_2) = \frac{L}{800}(g_1 - (-g_2))$$

Similarly for the valley:

$$\text{reduced level of } A = 0{\cdot}00,$$

$$\text{reduced level of } B = -\frac{l}{100}g_1$$

$$\text{reduced level of } C = -\frac{l}{100}g_1 + \frac{l}{100}g_2$$

$$\text{reduced level of } E = \frac{l}{200}(-g_1 + g_2)$$

$$\therefore EB = \frac{l}{200}(-g_1 + g_2) - (-\frac{l}{100}g_1) = \frac{l}{200}(+g_1 + g_2)$$

$$\text{and } e = \frac{l}{400}(+g_1 + g_2) = \frac{L}{800}(+g_1 + g_2) = -\frac{L}{800}(-g_1 - g_2)$$

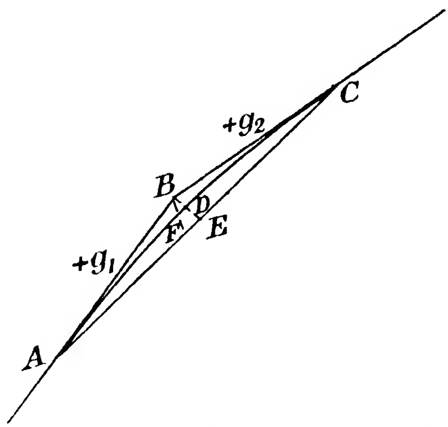


FIG. 8.12.—Curve connecting two Gradients of same Sign.

The expressions $(g_1 - (-g_2))$ and $(-g_1 - g_2)$ are, respectively, the algebraic differences in grade.

Finally, if we consider two upward grades of g_1 and g_2 (fig. 8.12) it can readily be deduced from the figure that $e = \frac{L}{800}(g_1 - g_2)$. A similar expression may be deduced for two downward grades $-g_1$ and $-g_2$. Hence, in all cases, $e = \frac{L}{800} \times$ (algebraic difference in grades). The algebraic difference in grades may be represented conveniently by "G". Then $e = \frac{LG}{800}$.

Calculation of "e" for Unsymmetrical Curves. In some cases, particularly in built-up surroundings, existing levels form the controlling factors in the configuration of a vertical curve and it may not be possible t› adopt a symmetrical curve, i.e. the tangent lengths cannot be made equal. The dimension "e", however, may be readily calculated in the following way:

Let the tangent lengths be l_1 and l_2 and the gradients g_1 and $-g_2$, as shown in fig. 8.13.

As before, let reduced level of $A = 0{\cdot}00$

Then reduced level of $B = \frac{g_1 l_1}{100}$

reduced level of $C = \frac{g_1 l_1}{100} - \frac{g_2 l_2}{100}$

With the small inclinations of grade lines encountered in practice we may assume that $AC = l_1 + l_2$ and $AE = l_1$. Let AFG be a horizontal line.

Then $CG = \frac{g_1 l_1}{100} - \frac{g_2 l_2}{100}$

also $\frac{FE}{CG} = \frac{l_1}{l_1 + l_2}$, or $FE = \frac{l_1}{l_1 + l_2}\left(\frac{g_1 l_1}{100} - \frac{g_2 l_2}{100}\right)$

and $BE = \frac{g_1 l_1}{100} - \frac{l_1}{l_1 + l_2}\left(\frac{g_1 l_1}{100} - \frac{g_2 l_2}{100}\right)$

therefore $e = \frac{1}{2}BE = \frac{g_1 l_1}{200} - \frac{l_1}{200(l_1 + l_2)}(g_1 l_1 - g_2 l_2)$

i.e. $e = \frac{g_1 l_1^2 + g_1 l_1 l_2 - g_1 l_1^2 + g_2 l_1 l_2}{200(l_1 + l_2)}$

$$= \frac{l_1 l_2}{200(l_1 + l_2)}(g_1 + g_2) = \frac{l_1 l_2}{200(l_1 + l_2)}(g_1 - (-g_2))$$

The expression $(g_1 - (-g_2))$ is, again, the algebraic difference in gradients represented by G. If an unsymmetrical valley curve is considered it will be found that we may again write $e = \frac{l_1 l_2}{200(l_1 + l_2)}.G$.

The same expression also holds good if l_2 is greater than l_1 although a somewhat different figure is obtained. Also, when $l_1 = l_2 = \frac{L}{2}$

$e = \frac{L}{800}.G$, as before, for a symmetrical curve.

Highest or Lowest Point on Curve. Except when the gradients and tangent lengths are equal the highest point of a summit curve and the lowest point of a valley curve will not occur at the middle point. The distance from the tangent point to the highest or lowest point, as the case may be, and its level may be determined in the following way:

An unsymmetrical curve, ADC, with unequal tangents AB and BC, is shown in fig. 8.13, the gradients being $+g_1$ and $-g_2$ and the tangent lengths l_1 and l_2. X is the highest point on the curve at a

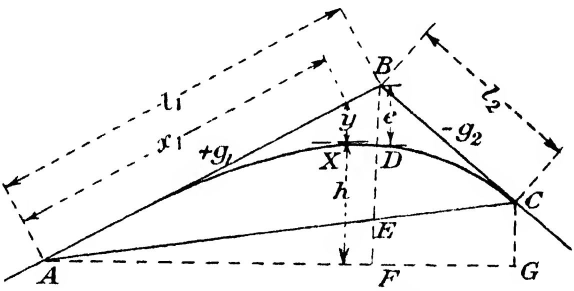


FIG. 8.13.—Unsymmetrical *Vertical* Curve.

distance x_1 from the tangent point, A, and at a vertical distance y below the tangent. The elevation of X above A is h. As before, it is assumed that the distance x_1 is the same whether measured along the curve or the tangent.

From the property of the parabola:

$$\frac{y}{e} = \frac{x_1^2}{l_1^2}, \text{ or } y = \frac{x_1^2}{l_1^2}.e$$

and

$$h = \frac{g_1 x_1}{100} - \frac{x_1^2}{l_1^2}.e$$

For a maximum value of h,

$$\frac{dh}{dx_1} = 0,$$

i.e. differentiating,

$$\frac{g_1}{100} - \frac{2x_1 e}{l_1^2} = 0$$

or

$$x_1 = \frac{g_1 l_1^2}{200e}.$$

If x_1, calculated in this way, is greater than l_1, the point X is on the other part of the curve and its distance must be determined from the other tangent point, B, using the expression

$$x_2 = \frac{g_2 l_2^2}{200e}$$

For a symmetrical curve, $l_1 = l_2 = \frac{L}{2}$ and $e = \frac{LG}{800}$ where G is the algebraic difference in gradients, or in fig. 8.13, $g_1 - (-g_2)$. Hence, for symmetrical curves:

$$x_1 = \frac{g_1 L}{G},$$

similarly, measuring from the opposite tangent point

$$x_2 = \frac{g_2 L}{G}$$

with equal gradients, $x_1 = x_2 = \frac{L}{2}$.

Length of Vertical Curves. The minimum length of a summit curve can be derived from any specified sighting distance over the brow of the hill, assuming, of course, that the gradients are known, but there is no definite rule for calculating the minimum length of a valley curve. In railway practice a limiting rate of change of gradient is sometimes stipulated, but the dynamics of road vehicles on vertical curves do not appear to have been fully investigated. Aitken and Boyd [6] suggest that vertical curves may be treated as circular arcs and the radial acceleration, given by the formula $\frac{V^2}{R}$, limited to 2·5 feet per second per second. For a vehicle travelling at 60 miles per hour the minimum radius of the curve is then about 3,000 feet. In practice, the length of a valley curve is frequently dependent upon the amount of material available for the fill.

Sighting Distance on Summit Curves. It is obviously necessary that vehicles approaching each other or overtaking when near the brow of a hill should have adequate sighting distance. Ministry of Transport Memorandum No. 575 recommends that single carriageway roads should have a minimum sighting distance of 1,000 feet at summits, or 500 feet with dual carriageways, the line of sight being measured at a height of 3 feet 9 inches above the road surface. In American practice the minimum sighting distance for two-lane trunk highways is 800 feet at summits, the height of the line of sight above the road surface being taken as 5 feet. If we consider two vehicles equidistant from the highest point of the curve, the requisite sighting distance will be obtained in some cases while the vehicles are on the curve and in other cases while the vehicles are on the tangents. Fig. 8.14 (a) and (b) illustrate the two cases, in each of which D is the sighting distance, L the length of the curve and h the height of the line of sight. "e", as before, is the vertical ordinate from the intersection of the gradients to the curve and has the value $\frac{L.G}{800}$ for a symmetrical curve.

Case I. L greater than D (Fig. 8.14 (*a*)).

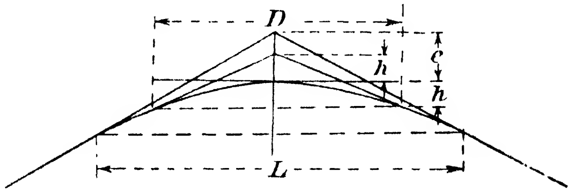


FIG. 8.14 (*a*).—Sighting Distance over Summit. L > D.

From the basic property of the parabola:

$$\frac{L^2}{D^2} = \frac{e}{h} \text{ or } L = \frac{D^2}{h} \cdot \frac{G}{800}$$

for a symmetrical curve. Putting $D = 1{,}000$ feet and $h = 3{\cdot}75$ feet, this formula becomes

$$L = 333{\cdot}3G.$$

Case II. D greater than L (Fig. 8·14 (*b*)).

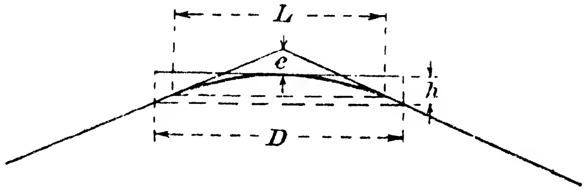


FIG. 8.14 (*b*).—Sighting Distance over Summit. D > L.

From similar triangles:

$$\frac{L}{D} = \frac{2e}{e + h} = \frac{2LG}{LG + 800h}$$

for a symmetrical curve, from which

$$L = 2D - \frac{800h}{G}$$

Putting $D = 1{,}000$ feet and $h = 3{\cdot}75$ feet, this formula becomes

$$L = 2000 - \frac{3000}{G}$$

It will be apparent from these two formulae that when G, the algebraic difference in gradients, is 3%, the sighting distance exactly equals the length of the curve. If G is greater than 3% the formula for case I must be applied and if G is less than 3% the formula for case II must be applied. For very slight gradient differences, when $G = 1{\cdot}5\%$, or less, an infinitely short vertical curve would give the required sighting distance of 1,000 feet. For a sighting distance of 500 feet, the corresponding expressions for a symmetrical vertical curve are:

(1) when L is greater than D: $L = 83{\cdot}3G$ (for G greater than 6%)

(2) when D is greater than L: $L = 1000 - \dfrac{3000}{G}$ (for G less than 6%)

An infinitely short curve would give a sighting distance of 500 feet when G is 3% or less.

Memorandum No. 575 of the Ministry of Transport gives no specific rules for the lengths of vertical curves, but states that "all changes of gradient should be gradually effected by means of vertical curves of ample length".

Vertical Curve Calculations. In setting out vertical curves we require the reduced level of a series of pegs usually placed at intervals of 50 or 100 feet along the centre-line of the road to give the correct configuration of the surface. The values of the gradients forming the tangents may be obtained from the longitudinal section of the road and in the case of summit curves where the algebraic difference of the gradients is sufficiently large, it is necessary to check the sighting distance. The economic aspect must also be considered. The greater the length of a summit curve cut through the top of a hill, the greater the excavation involved and the greater the length of a valley curve built on filling, the greater the volume of material required.

In addition to the peg levels, it is usually necessary to know the level of the highest or lowest point for a summit or valley curve respectively.

If a tangent with a gradient of $+g_1\%$ and length l_1 is divided into a number of equal parts, each x feet long, the levels of the successive points along the tangent will be $\dfrac{g_1x}{100}, \dfrac{2g_1x}{100}, \dfrac{3g_1x}{100}$, and so on. From the basic property of the parabola the successive vertical distances from the tangent to the curve will be y, $4y$, $9y$ and so on, where y is obtained from the relationship

$$\frac{y}{e} = \frac{x^2}{l_1{}^2}$$

Hence the heights of successive points along the curve above the level of the tangent point will be

$$\frac{g_1x}{100} - y,\ \frac{2g_1x}{100} - 4y,\ \frac{3g_1x}{100} - 9y,$$

and so on.

If x is 100 feet and the curve is symmetrical, so that $l_1 = \dfrac{L}{2}$, the height of the first point on the curve above the level of the tangent point becomes

$$g_1 - \frac{40,000LG}{L^2 800} = g_1 - 50\frac{G}{L}.$$

Similarly, the height of the second point above the level of the tangent point becomes $2g_1 - 200\frac{G}{L}$, and so on. There is, however, a very simple relationship between the levels of the successive equidistant points along the curve.

Consider, first, a summit curve. On the ascending side the rate of increase of height of successive points gradually diminishes. If the

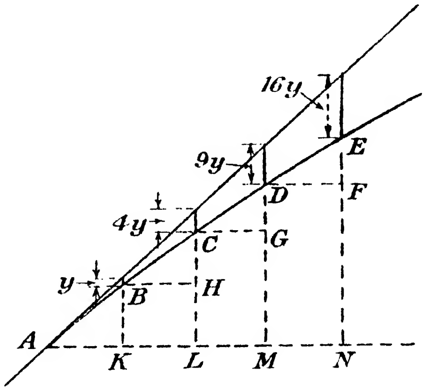


FIG. 8.15.—Levels of Points on Vertical Curve.

points are equidistant and y, as before, is the vertical distance between the first point on the curve and the tangent, it will be shown that the differences of height of successive pairs of points always diminishes by $2y$. On the descending side there is an equivalent increase in the successive falls. This rule may be readily proved from fig. 8·15.

For simplicity, assume level of A is zero, and let AN be a horizontal line. Then

$$BK = \frac{g_1 x}{100} - y$$

$$CL = \frac{2g_1 x}{100} - 4y$$

$$DM = \frac{3g_1 x}{100} - 9y$$

$$EN = \frac{4g_1 x}{100} - 16y$$

and so on. Also

$$CH = CL - BK = \frac{g_1 x}{100} - 3y$$

i.e. $BK - CH = 2y \quad \therefore CH = BK - 2y.$

Again, $DG = DM - CL = \dfrac{g_1 x}{100} - 5y$

i.e. $CH - DG = 2y \quad \therefore DG = CH - 2y.$

Similarly, $EF = EN - DM = \dfrac{g_1 x}{100} - 7y$

i.e. $DG - EF = 2y \quad \therefore EF = DG - 2y,$

and so on.

This fact provides a simple method of writing down the reduced levels of all the pegs after the first, as the following numerical example demonstrates :

Numerical Example of a Symmetrical Summit Curve.

Preliminary Data :

Gradients : g_1 2·5% (1 in 40 up)
g_2 – 1·5% (1 in 67 down)

Algebraic difference in gradients, per cent = 4 (G).
Total length of curve = 600 feet (L).
Pegs to be set out at 50-foot intervals.

$$e = \frac{LG}{800} = 3 \text{ feet}$$

Minimum sighting distance : 500 feet (dual carriageways), requiring a curve length of only 250 feet when $G = 4$, from the formula

$$L = 1000 - \frac{3000}{G}.$$

The above values of L and G give a sighting distance of 675 feet, from the formula

$$\frac{L}{D} = \frac{2e}{e + h}, \text{ applicable when } D \text{ is greater than } L.$$

Reduced level of intersection of gradients (B in fig. 8.16) : 349·72
Reduced level of tangent point A = 349·72 – 7·5 = 342·22
Reduced level of tangent point C = 349·72 – 4·5 = 345·22

$$\frac{y}{e} = \frac{50^2}{300^2}, \text{ i.e. } y = \frac{3}{36} = 0{\cdot}0833 \text{ and } 2y = 0{\cdot}1667.$$

Height of point no. 1 above A = 1·25 – 0·0833 = 1·1667.

Peg levels are only required to a hundredth of a foot, but to obtain a proper check it is necessary to work to closer accuracy owing to the cumulative effect of approximation in the value of $2y$. The results can be converted to the nearest two places of decimals subsequently.

We have seen that peg no. 1 will be 1·1667 feet higher than peg A; peg no. 2 will be $1{\cdot}1667 - 2y$, or 1·0000 feet higher than peg no. 1; peg no. 3 will be $1{\cdot}0000 - 2y$, or 0·8333 feet higher than peg no. 2; and so on. We can therefore write down the successive reduced levels thus:

		Nearest Two Decimals
Reduced level of peg A	342·22	342·22
	1·1667	
Reduced level of peg no. 1 . . .	343·3867	343·39
1·1667 − 0·1667 =	1·0000	
Reduced level of peg no. 2 . . .	344·3867	344·39
1·0000 − 0·1667 =	0·8333	
Reduced level of peg no. 3 . . .	345·2200	345·22
0·8333 − 0·1666 =	0·6667	
Reduced level of peg no. 4 . . .	345·8867	345·89
0·6667 − 0·1667 =	0·5000	
Reduced level of peg no. 5 . . .	346·3867	346·39
0·5000 − 0·1667 =	0·3333	
Reduced level of peg no. 6 . . .	346·7200	346·72

Check: Reduced level of peg no. 6 = Reduced level of peg $B - e$
$= 349{\cdot}72 - 3{\cdot}00 = 346{\cdot}72$.

This procedure is continued for the second portion of the curve, treating the additions and subtractions algebraically, thus:

		Nearest Two Decimals
Reduced level of peg no. 6 . . .	346·7200	
0·3333 − 0·1666 =	0·1667	
Reduced level of peg no. 7 . . .	346·8867	346·89
0·1667 − 0·1667 =	0·0000	
Reduced level of peg no. 8 . . .	346·8867	346·89
0·0000 − 0·1667 =	− 0·1667	
Reduced level of peg no. 9 . . .	346·7200	346·72
− 0·1666 − 0·1666 =	− 0·3333	
Reduced level of peg no. 10 . . .	346·3867	346·39
− 0·3333 − 0·1667 =	− 0·5000	
Reduced level of peg no. 11 . . .	345·8867	345·89
− 0·5000 − 0·1667 =	− 0·6667	
Reduced level of peg C	345·2200	345·22

This agrees with the previously calculated reduced level of peg C.

Unsymmetrical Curve. In the case of an unsymmetrical curve the above procedure requires modification. The curve is divided into the two portions defined by the respective tangent lengths and separate calculations are made for the vertical distances y and y' between the tangents and the curve at the first points on each branch.

This will be made clearer by considering a numerical example and the calculations for the peg levels for an unsymmetrical valley curve are given below:

Numerical Example of an Unsymmetrical Valley Curve

Preliminary Data :

Gradients : $g_1 = -3\%$
$g_2 = 2{\cdot}5\%$

Algebraic difference of gradients : $5{\cdot}5\%$ (G).

Tangent lengths : $l_1 = 200$ feet
$l_2 = 300$ feet

Reduced level of intersection of gradients : 394·77 (B in fig. 8.17).

Pegs required at 50-foot intervals.

$$e = \frac{l_1 l_2}{200(l_1 + l_2)} . G = \frac{200 \times 300 \times 5{\cdot}5}{200 \times 500} = 3{\cdot}3$$

$$y = 3{\cdot}3 \times \frac{50}{200} \times \frac{50}{200} = 0{\cdot}20625. \qquad 2y = 0{\cdot}4125$$

On the descending side of the curve the differences in successive falls will diminish each time by $2y$, but as the curve is unsymmetrical, this constant diminution will only apply for the length l_1, from A to C.

Reduced level of peg A (Tangent Point) $= 394{\cdot}77 + 6.00 = 400{\cdot}77$

Reduced level of peg no. 1 $= 400{\cdot}77 - 1{\cdot}50 + 0{\cdot}2062$
$= 400{\cdot}77 - 1{\cdot}2938$

Peg levels may thus be written down as follows :

		Nearest Two Decimals
Reduced level of peg A	400·77	400·77
	− 1·2938	
Reduced level of peg no. 1 . . .	399·4762	399·48
$-1{\cdot}2938 + 2y = -1{\cdot}2938 + 0{\cdot}4125 =$	− 0·8813	
Reduced level of peg no. 2 . . .	398·5949	398·59
$-0{\cdot}8813 + 0{\cdot}4125 =$	− 0·4688	
Reduced level of peg no. 3 . . .	398·1261	398·13
$-0{\cdot}4688 + 0{\cdot}4125 =$	− 0·0563	
Reduced level of peg no. 4 . . .	398·0698	398·07

Check : Reduced level of peg no. 4 = reduced level of peg $B + e$
$= 394{\cdot}77 + 3{\cdot}3 = 398{\cdot}07$.

For the second portion of the curve we start at the second tangent point, D, calculate y', the vertical distance between the tangent and the first point on the curve from D, and proceed as before.

$$y' = e \times \frac{50 \times 50}{300 \times 300} = \frac{3{\cdot}3}{36} = 0{\cdot}09167. \qquad 2y' = 0{\cdot}18334$$

Reduced level of peg no. 9
= reduced level of peg $D - 1{\cdot}25 + 0{\cdot}0917$
= reduced level of peg $D - 1{\cdot}1583$

Reduced level of peg D = reduced level of peg $B + 7{\cdot}5$
$= 394{\cdot}77 + 7{\cdot}5 = 402{\cdot}27$.

Peg levels may be written down thus :

		Nearest Two Decimals
Reduced level of peg D	402·27	402·27
	− 1·1583	
Reduced level of peg no. 9 . . .	401·1117	401·11
− 1·1583 + 0·1833 =	− 0·9750	
Reduced level of peg no. 8 . . .	400·1367	400·14
− 0·9750 + 0·1833 =	− 0·7917	
Reduced level of peg no. 7 . . .	399·3450	399·34
− 0·7917 + 0·1833 =	− 0·6084	
Reduced level of peg no. 6 . . .	398·7366	398·74
− 0·6084 + 0·1833 =	− 0·4251	
Reduced level of peg no. 5 . . .	398·3115	398·31
− 0·4251 + 0·1833 =	− 0·2418	
Reduced level of peg no. 4 . . .	398·0697	398·07

This agrees with the value previously determined.

Highest and Lowest Points. Considering, again, the summit curve in the foregoing example, the position and reduced level of the

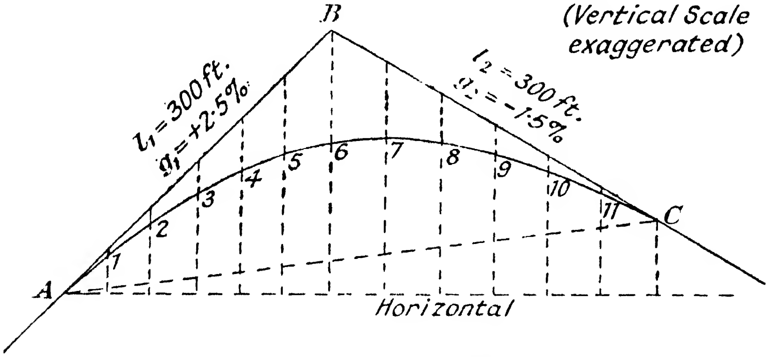


FIG. 8.16.—Example of Symmetrical Summit Curve.

highest point may be found thus : It cannot be discovered from inspection whether the highest point occurs before or after the mid-point, starting from A (fig. 8·16) and we will assume, first, that it occurs before the mid-point is reached. Applying the equation given on page 229 for symmetrical curves, we have :

$$x_1 = \frac{g_1 . L}{G} = \frac{2·5 \times 600}{4} = \mathbf{375} \text{ feet.}$$

This is greater than half the total curve length and we must, therefore, apply the alternative equation, giving the distance to the highest point from the tangent point C. We then have :

$$x_2 = \frac{g_2 . L}{G} = \frac{1·5 \times 600}{4} = 225 \text{ feet.}$$

The reduced level of the highest point

$$= \frac{225}{100} \times 1{\cdot}5 - e\left(\frac{225}{300}\right)^2 + 345{\cdot}22$$

$$= 3{\cdot}375 - 3 \times 0{\cdot}5625 + 345{\cdot}22$$

$$= 346{\cdot}9075,$$

or 346·91, to two decimal places.

In the second example, in which we have an unsymmetrical valley

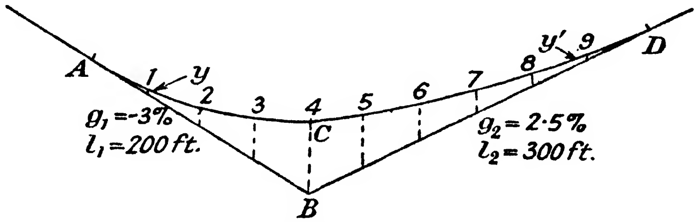


FIG. 8.17.—Example of Unsymmetrical Valley Curve.

curve (fig. 8.17) the equations giving the lowest point are $x_1 = \dfrac{g_1 . l_1^2}{200 . e}$ or $x_2 = \dfrac{g_2 . l_2^2}{200 . e}$ and, here again, we have to apply the method of trial and error. Assuming, first, that the lowest point occurs within the first l_1 feet of the curve, measuring from A, we have

$$x_1 = \frac{3 \times 200 \times 200}{200 \times 3{\cdot}3} = 181{\cdot}82 \text{ feet.}$$

The assumption is, therefore, correct.

The reduced level of the lowest point is

$$400{\cdot}77 - \frac{3 \times 181{\cdot}82}{100} + 3{\cdot}3 \times \left(\frac{181{\cdot}82}{200}\right)^2$$

$$= 398{\cdot}0427,$$

or 398·04 to two decimal places.

Widening of Carriageway on Curves. Long vehicles require additional width on curves to avoid encroachment on traffic lanes adjacent to their own. Ministry of Transport Memorandum No. 575 gives the following increases for curves of various radii :—

Radius of Curve	Additional Width per Traffic Lane
Less than 1,500 feet	1 foot
,, ,, 1,000 ,, ...	1 foot 6 inches
,, ,, 500 ,, ...	2 feet

The gradual application of these added widths can only be carried out properly on a transition curve. Memorandum No. 575 states that " such added width should be obtained by widening the carriageway at a uniform rate along the length of the transition curve. In the improvement of existing curves the widening should generally be made on the inside of the curve."

It is interesting to note that if a curve consists of a circular arc without transitions at entrance and exit and if the widening is commenced at the tangent point and gradually increased, the curve on the inner edge of the carriageway becomes a spiral, the radius decreasing uniformly with the distance along the curve, up to the point at which the full widening is obtained.

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CHAPTER IX

HIGHWAY SURVEYS AND SETTING OUT—GENERAL REVIEW OF PRACTICE

It has already been mentioned that a simple chain survey may suffice for preparing the necessary drawings for a small improvement scheme, such as the alteration of a dangerous corner, while a traverse serves excellently for schemes involving the widening of an existing road or for the construction of a short by-pass.

On a project of any considerable size, however, the survey work may be divided into four stages: (1) the reconnaissance survey, (2) the preliminary survey, (3) the location survey, and (4) the construction survey. In schemes of great importance involving a vast expenditure, parliamentary sanction may have to be obtained and plans must be presented to a parliamentary committee for this purpose. Such plans must comply with various regulations, or " Standing Orders ", as they are called, regarding scales, etc., and the survey work required for their preparation very much resembles that of a preliminary survey.

Reconnaissance Surveys. In Great Britian we are fortunate in having excellent large-scale Ordnance maps on which a possible route for a projected road can be chosen provisionally. At this stage the route would be mapped as a broad band, rather than a line, to permit of deviations and alternative loops. Maps to the scale of 6 inches to 1 mile are generally used for this purpose, the sheets being mounted to form a continuous roll if the scheme covers more than one map. The route is then followed on the ground, noting any new details not shown on the map, such as buildings erected since the last revision.

Other details to be noted are geological features, presence of marshy ground, suitable points for the crossings of streams and rivers, possibility of obtaining local material for constructional purposes, visible flood marks and any fact, however trivial, which may have a bearing on the scheme. From this field inspection it will be possible to narrow down the route to an approximate line which, in places, will almost certainly fork into two or more alternatives each appearing to have its own advantages. Sometimes two or more alternative routes may require consideration over the entire distance.

The use of aerial surveying for reconnaissance purposes is certain to become very much more common in the immediate future, even in this well-mapped country. In a country unprovided with large-scale maps it will doubtless be considered absolutely essential as a first step in projecting new highways. When accompanied by suitable ground control aerial photography is an ideal method of obtaining topographical maps, even of scales as large as 1/2500, and contours can be derived by the same method with sufficient accuracy for reconnaissance. Aerial survey has been greatly developed during the recent war and has become a highly specialised procedure, both in respect of the actual photography and the plotting. References are given in the bibliography, page 281, to text-books dealing with the subject.

Preliminary Surveys. The routes chosen in the reconnaissance survey are next investigated in detail, particular attention being paid to levelling. This work constitutes the " preliminary survey " which consists of a traverse following as closely as possible the mapped courses. Control points are first set out by transferring scaled dimensions from the 6-inch map to the ground at places where the projected line runs near to well-defined landmarks. The lines are measured carefully, preferably with a steel tape, an accuracy of 1 in 1,000 being usually specified, and the angles between the lines are measured, using both faces of the theodolite. Alternatively, the " direct-bearing " method may be used, in which case the bearing of each line will require an independent check. The accuracy of the angular measurements will depend partly on the type of theodolite but results should be dependable to the nearest minute as an outside limit and should preferably be accurate to a finer limit than this. The lines are pegged at 100-foot intervals and the ground level is obtained at every peg. Temporary bench marks are established at frequent intervals throughout the survey, the distances between them being not greater than about 1,000 feet and their levels being checked and rechecked until they are known with certainty to within 0·01 foot. The ground levels at the pegs need only be correct to 0·1 foot if they are merely to be used for contouring with a hand-level. Contouring is a very important part of the work and it is essential that the contoured area should amply cover the extent of the future construction, allowing for possible deviations and for the additional widths required in cuttings and on embankments. Particular care must also be taken where curves occur, it being remembered that the traverse lines may diverge considerably from the centre-line of the road at such places. The traverse forming the basis of the preliminary survey is plotted from departures and latitudes on a " grid ", i.e. a system of squares drawn with great accuracy, the side of each square representing a convenient round number of feet to the scale of the drawing and the orientation of the sides coinciding with the east–west and north–south directions respectively. The scales used for preliminary plans may range from 500 feet to an inch in open country with little or no topography to 100 feet to an inch where topography is more complicated. The scale of the plan, considered in conjunction with the steepness of the slopes encountered along the route, will serve as a guide for settling the vertical interval for the contours. Five feet is the usual interval in gently undulating country and 10 or 20 feet on steep hillsides.

A convenient scale for the preliminary plan is 1/2500, or roughly 208 feet to an inch, the scale of the " 25-inch " Ordnance maps. The adoption of this scale simplifies checking from the Ordnance sheets and the preliminary survey is sometimes plotted direct on the map, the topographical details being checked on the ground and revised where necessary.

During the course of the fieldwork the names of the tenants and landowners must be obtained and recorded for each property along the route.

Trial lines for the proposed road are superimposed on the contoured plan and longitudinal and cross sections are drawn from which earthwork quantities are computed and compared for each alternative route. Formation levels will be tied at certain points, such as the intersections with existing roads, railways and waterways, and the limiting gradient will be suited to the particular purpose for which the road is being designed. Thus, for important trunk roads in gently undulating country the steepest gradient would be 1 in 30, in compliance with Memorandum No. 575 of the Ministry of Transport. Curve radii, with or without the introduction of transitions, would be decided upon at this stage, and the choice of route, in any scheme of importance, would be influenced to some extent by data derived from a soil survey.

After due consideration has been given to all the various factors involved, a final line, representing the centre line of the proposed road, is drawn on the preliminary plan. In the case of dual carriageway roads, two centre-lines will be required.

The Location Survey. The location survey consists of two parts—the setting out of the line as closely as possible to that shown on the preliminary plan, and the detailed surveying and levelling of the line, as set out. The actual working drawings, together with an accurate longitudinal section and cross-sections, will be prepared from the location survey and quantities and estimates will be derived therefrom.

The line will first be set out as a series of straights, their location being fixed by dimensions from well-defined topographical features, such as the corners of boundary fences, walls and buildings, and also from the station pegs of the preliminary survey, all of which will have been carefully referenced so that they may be re-located in cases where they have been accidentally or deliberately removed. An adequate number of tie and check measurements must be taken to ensure that the station pegs in the location survey correspond in position to the intersection points of the straights shown on the preliminary plan and, in order to prevent confusion, the tops of the pegs in the location survey should be painted a distinctive colour. Having fixed and referenced the stations, the line is surveyed as a traverse and pegs are inserted at 100-foot intervals. On long lines, where sighting is difficult, these intermediate pegs should be set out by a theodolite. It must be borne in mind that the scale of the working drawings will be either 1/500, or 40 feet to an inch, and may be even larger at specially important places, such as bridge sites. Measurements will, therefore, require a higher degree of accuracy than that required for the preliminary survey. The distances between the stations should be accurate to 1 in 3,000 on reasonably good ground and the accuracy of the offsets and ties fixing topographical detail should correspond with the limit imposed by the draughtsman's skill, having regard to the scale of the plan and the nature of the topography. A good draughtsman can plot to 0·01 of an inch and if the scale of the drawing is 40 feet to an inch, well-defined detail, such as brick walls and wooden fences, should, therefore, be fixed to 0·4 of a foot, or less. A steel tape is necessary to ensure that dimensions are measured to this accuracy.

Every item of topographical detail must be included; the types of boundaries must be described, e.g. "post and rail fence", "wire fence", "hedge", etc.; the varieties of trees and approximate girth should be noted if these are likely to need removal; all manholes, hydrants, valve-boxes, post office cable indicators, and similar items must be carefully marked and described. Minor works in connection with such features will be included in the specification and must be allowed for in the estimate. It is, therefore, highly important that the smallest item should be included on the working plan. The deviation angles between the successive straight courses along the centre-line will be obtained automatically from the traverse angles. The minimum radii and transition lengths, in conjunction with the deviation angles, provide the essential data for the curve calculations and the curves are set out, usually by the tangential angle method. Special pegs are used to mark the tangent points and the junctions of spirals and circular arcs and the chainage is adjusted so that pegs at exact 100-foot intervals run continuously right through the line. The ground level is obtained at each peg and levels are also taken for cross-sections running through these points, or at closer intervals where the ground undulates sharply. The cross-sections must extend right and left of the line sufficiently far to cover the future work. Although intermediate levels on rough ground cannot be ascertained with certainty to closer limits than 0·05 of a foot or so, great care should be taken to pick reliable change points and the levelling must be checked repeatedly at temporary bench marks or pegs at which the level is accurately known. The error on checking should never exceed 0·01 of a foot. Although cross-sections derived from contours are sufficiently accurate for preliminary earthwork estimates, cross-sections plotted from levels obtained by a dumpy or quick-set instrument are necessary for more precise quantities.

The computation of earthwork quantities is dealt with in Chapter X, page 257.

The Construction Survey. The construction Survey is carried out by the resident engineer and his staff. The centre-line pegs set out in the location survey are first checked to ensure that the proprosed road is located in accordance with the plan and any missing pegs are replaced. New pegs are then set out, left and right of the centre-line, at a sufficient distance to be clear of the future works. If possible a constant offset distance is used in fixing these pegs and both chainage and offset should be marked on each. The same pegs are sometimes used as level pegs, the difference between the reduced level of the top of the peg and the formation or finished surface level being marked thereon, or recorded. Separate pegs, however, are frequently used for levels, in which case their tops are painted a distinctive colour to distinguish them from the chainage pegs. The slope stakes, or " batter pegs " will also be set out to mark the tops of the cuttings and the toes of the embankments. The location of these pegs may be obtained by scaled dimensions from cross-sections, or, preferably, by the " grade-staff " method, described on page 74.

All this work, of course, must be carried out ahead of the construction, but constant checking of line and level is carried out while the construction is in progress. It frequently happens that slight modifications of the centre-line, or levels, of the road appear advantageous as the work proceeds and sometimes such alterations are rendered necessary by unforeseen circumstances. In such cases small surveys are carried out and the appropriate plans and sections prepared. A very important part of the constructional survey is the setting out of bridges and culverts.

Every setting-out job has its own peculiar problems and it is difficult to lay down general rules applicable to every case. There are, however, one or two points which may be mentioned.

Since pegs will inevitably be disturbed during construction, or even before, if the public has access to the site, it is important to fix a number of master " pegs " well clear of the construction limits and less likely to be disturbed than the ordinary wooden variety.

Thus a short length of steel rod may be set flush with the upper surface of a concrete surround and scribed with a fine reference mark. The positions of these master "pegs", or control points, must be accurately located on a large-scale plan and the positions of other pegs are fixed from them by linear and angular measurements. It will then be possible to relocate the position of any missing or doubtful peg.

The ordinary surveying methods and principles which have been discussed already are applied to the setting-out of works. Thus, a length which cannot be measured by direct taping owing to obstructions can often be computed by a simple triangulation from two measured angles and a carefully measured base; a straight line may be extended by means of a theodolite, taking care to read on both faces if it is necessary to transit the telescope; the corner points of bridge abutments and wing-walls may be set out by offsets measured at right-angles to a main base-line; and so on.

It is, of course, essential to apply checks to every measurement, either by repeating it, or, preferably, by using an alternative method.

Setting out includes the establishment of marks at predetermined levels, but this work is merely a development of ordinary levelling and scarcely calls for a detailed description.

In order to give some idea of the work involved in setting out a fairly complicated road job, an actual example has been chosen and the setting out is described below.

The works included the re-alignment of the Oxford–Banbury road at Kidlington, Oxon, the construction of bridges over the Great Western Railway and the Oxford and Birmingham Canal and a diversion of the latter. The following description of the setting out of the railway bridge has been contributed by Mr. I. Kursbatt, B.Sc., M.I.C.E., formerly Bridge Engineer, Oxfordshire County Council, with the permission of Mr. G. T. Bennett, O.B.E., B.Sc., M.I.C.E., County Surveyor. The engineering features of the work have been described in the *Journal of the Institution of Civil Engineers.*[1]

Setting-Out of Bridge over Railway at Kidlington, Oxon. The survey of the site was plotted to a scale of 66 feet to an inch and a realignment of the carriageway with a 60-foot formation width superimposed as shown in fig. 9.1. For setting out the works this scale was not considered sufficiently accurate. A further plan to a scale of one-sixteenth of an inch to a foot was made and the positions of the new bridges fixed thereon. It will be noticed that the canal had to be diverted to enable a bridge to be built with a reasonable angle of skew. The latter was fixed at 45°.

From the larger plan it was possible to locate and fix on the site the tangent points of the curves and the intersection points of the tangents. Tangent point A of the centre-line of the north curve was fixed by measurement 5 feet from the existing kerb-line. A cup-headed stackpipe nail was driven into the road to locate it and a point marked on it with a centre pop. Intersection point, B, of this curve was marked with a 3-inch by 3-inch wooden peg encased in concrete, a 1-inch nail being used to mark the exact point. Intersection point C and tangent point D of the south curve were similarly located by measurements from existing topography.

An 8-inch micrometer theodolite and a 100-foot steel tape were used throughout.

The theodolite was set up over point B to read the intersection angle of the north curve and to give the alignment for measuring the length BC. Point C was obscured by the existing railway bridge parapet girder. A tripod was erected over this point to support a 12-foot ranging rod, carefully plumbed and centred. A sight from C to B was not possible without an elaborate beacon which would have been unsuitable for the site. Several intermediate stations, K, L and N were established on the line BC to facilitate its measurement. Points K and L were located on either side of the canal, and as the distance KL was greater than 100 feet, it was triangulated with a further point O. Point N was a centre-pop mark on the steel railway bridge parapet. Owing to the steep approach from the canal bank at point L to the top of the existing railway bridge parapet, a further triangulation was made with the point M. The actual site measurements of lengths and angles were as follows:

AB = 298 feet $5\frac{1}{2}$ inches, BK = 186 feet $0\frac{1}{4}$ inches,
KO = 56 feet 6 inches, LM = 191 feet $6\frac{7}{8}$ inches,
OL = 185 feet $3\frac{1}{4}$ inches, NC = 138 feet $6\frac{3}{4}$ inches.

$\angle OKL = 77^\circ\ 43'\ 40''$, $\angle KLO = 17^\circ\ 20'\ 40''$,
$\angle KOL = 84^\circ\ 55'\ 40''$ (by subtraction)
$\angle MLN = 46^\circ\ 27'\ 20''$, $\angle LMN = 102^\circ\ 25'\ 20''$.

The length LN averaged from several measurements plumbed up the slope was 361 feet $9\frac{5}{8}$ inches.

CD = 364 feet $3\frac{5}{8}$ inches and $\angle DCB = 166^\circ\ 11'\ 04''$.

At intersection angles six readings were taken, three on each face of the instrument, and averaged. Triangulation angles were taken as the average of two readings, one on each face of the instrument. The following were the calculations made:

$$\frac{KL}{OL} = \frac{\sin 84^\circ\ 55'\ 40''}{\sin 77^\circ\ 43'\ 40''}$$

$$\therefore\ KL = 185{\cdot}27 \times \frac{\sin 84^\circ\ 55'\ 40''}{\sin 77^\circ\ 43'\ 40''} = 188{\cdot}89 \text{ feet}$$

$$\frac{KL}{KO} = \frac{\sin 84^\circ\ 55'\ 40''}{\sin 17^\circ\ 20'\ 40''}$$

$$\therefore\ KL = 56{\cdot}5 \times \frac{\sin 84^\circ\ 55'\ 40''}{\sin 17^\circ\ 20'\ 40''} = 188{\cdot}79 \text{ feet}$$

Average length $KL = \frac{1}{2}(188{\cdot}89 \text{ feet} + 188{\cdot}79 \text{ feet})$
$= 188{\cdot}84$ feet $= 188$ feet $10\frac{1}{8}$ inches.

$$\frac{LN}{LM} = \frac{\sin 102^\circ\ 25'\ 20''}{\sin\ 31^\circ\ 07'\ 20''}$$

$$\therefore\ LN = 191{\cdot}57 \times \frac{\sin 102^\circ\ 25'\ 20''}{\sin\ 31^\circ\ 07'\ 20''} = 361 \text{ feet } 11\tfrac{1}{2} \text{ inches}$$

Average length LN from computation and measurement $=$ $\frac{1}{2}$(361 feet $11\frac{1}{2}$ inches + 361 feet $9\frac{5}{8}$ inches) = 361 feet $10\frac{1}{2}$ inches.

The total length BC is made up as follows:

	ft.	ins.
BK	186	$0\frac{1}{4}$
KL	188	$10\frac{1}{8}$
LN	361	$10\frac{1}{2}$
NC	138	$6\frac{3}{4}$
	875	$3\frac{5}{8}$

BL = 186 feet $0\frac{1}{4}$ inches + 188 feet $10\frac{1}{8}$ inches = 374 feet $10\frac{3}{8}$ inches.

Distance of L from tangent point of north curve
= (374 feet $10\frac{3}{8}$ inches − 298 feet $5\frac{1}{2}$ inches) = 76 feet $4\frac{7}{8}$ inches.

The centre point of the new canal bridge was located from the large-scale plan as 310 feet from B. This was checked on site by setting up the theodolite over this point and turning the 45° skew angle of the bridge in line with the west section of the existing canal.

Centre point of canal bridge from tangent point of north curve
= 310 feet 0 inches − 298 feet $5\frac{1}{2}$ inches = 11 feet $6\frac{1}{2}$ inches.

Distance between L and centre point of canal bridge
= 76 feet $4\frac{7}{8}$ inches − 11 feet $6\frac{1}{2}$ inches = 64 feet $10\frac{3}{8}$ inches.

Square span of bridge = 27 feet 0 inches.

From L to south abutment face
= 64 feet $10\frac{3}{8}$ inches − 13 feet 6 inches sec 45°
= 45 feet $9\frac{1}{2}$ inches.

$LC = LN + NC$ = 361 feet $10\frac{1}{2}$ inches + 138 feet $6\frac{3}{4}$ inches
= 500 feet $5\frac{1}{4}$ inches.

Distance from L to tangent point of south curve
$= LC - CD$ = 500 feet $5\frac{1}{4}$ inches − 364 feet $3\frac{5}{8}$ inches
= 136 feet $1\frac{5}{8}$ inches.

Length of straight between reverse curves $= BC - (AB + CD)$
= 875 feet $5\frac{3}{8}$ inches − (298 feet $5\frac{1}{2}$ inches + 364 feet $5\frac{3}{8}$ inches)
= 212 feet $6\frac{1}{2}$ inches.

Before the detailed designs of the new railway bridge could be made the correct skew angle had to be measured on the site. The minimum clearance between the up line and the abutment face was specified as 5 feet 6 inches and the overall square clearance 49 feet 6 inches. The new bridge, superimposing the existing one, was on the south road curve and the latter had to be set out as a preliminary means of ensuring the correct measurement of the skew angle. The tangent distance *CD* and the intersection angle, ϕ, at *C* were taken from the large-scale plan ($\frac{1}{16}$th inch to 1 foot) as 364 feet and 13° 51′ 20″, respectively, the intersection angle being computed by continuing the line *BC* for 300 feet and taking a square offset to *CD*. The latter measured 74 feet, hence $\tan \phi = \frac{74}{300}$ giving the value of ϕ.

The angle was read subsequently on the site and found to be 13° 48′ 56″. The discrepancy of 2′ 24″ produced only very minute errors in the setting out.

The radius of the south curve was estimated in the following way: Tangent length, T, = 364 feet. Intersection angle, ϕ, = 13° 51′ 20″.

If radius $= R$, $L = R.\tan \frac{\phi}{2}$ or $R = T.\cot \frac{\phi}{2} = 2{,}996$ feet.

$$\text{The tangential angle for a 50-foot chord} = \sin^{-1}\frac{\text{half chord}}{\text{radius}}$$

$$= \sin^{-1}\frac{25}{2996}$$

$$= 28{\cdot}69 \text{ minutes.}$$

Alternatively, tangential angle = half angle subtended at centre by chord $= \frac{1}{2}\frac{50}{2996}$ radians $= 1718{\cdot}9 \times \frac{50}{2996}$ minutes $= 28{\cdot}69$ minutes.

A length of 400 feet of this curve was set out by the tangential angle method with the theodolite stationed over the tangent point *D*, the last peg arriving on the verge just short of the brick parapet of the existing railway bridge. It was then necessary to transfer the curve to the bottom of the embankment 30 feet away to mark the west side of the formation. By setting up the theodolite over peg 400 on the curve, sighting back to *D* and turning an angle of 90° − 8 × 28·69 minutes or 86° 10′ 30″, a peg *V* was set out 30 feet west of the centre-line, on a radius to the curve. Forward readings to the curve could then be continued by sighting back to peg 400 and turning 90° to obtain the direction of the tangent at *V*. Allowance was made for the shortened chords, the reduction in length being $\frac{30}{2996} \times 50$ feet, or approximately 6 inches per 50-foot chord.

The running edge of the up line which was nearest Oxford was straight and was taken as the datum for the parallel faces of the abutments of the new bridge. Two mason's lines were set up parallel to the track 5 feet 6 inches from the rail nearest Oxford and 44 feet on the opposite side, thus giving the required overall square clearance of 49 feet 6 inches. The points P and Q at which the curve 30 feet west of the centre-line of the new road intersected these lines were fixed by sighting and the chord between the two points was taken as the west spandril face of the bridge.

The skew angle was then read from both sides of the track and

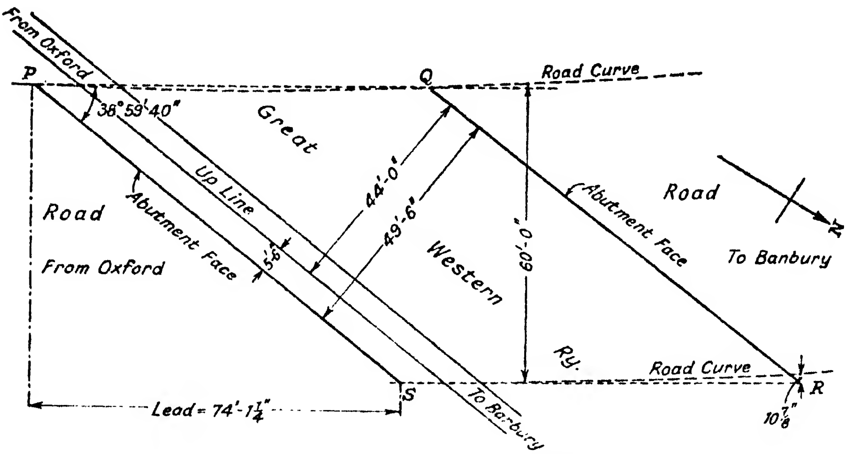


FIG. 9.2.—Setting-out Abutments of Bridge over Railway, Kidlington, Oxon.

averaged. The angles read were 38° 59′ 20″ and 141° 0′ 0″, giving a skew angle of 51° 0′ 20″ by subtraction of the mean from 90°.

Fig. 9.2 shows the outline layout of the abutment and spandril faces. Points P and Q being fixed, it was desirable to establish R and S 60 feet away and parallel to PQ. A direct sight along PS and QR was not possible owing to the obstruction of the wing walls of the old bridge. It was therefore decided to solve the triangle PQS and take a direct reading from point Q.

Thus, distance PQ = 49 feet 6 inches × cosec 38° 59′ 40″
= 78·6657 feet
(measured on site as 78 feet 8 inches)

PS = 60 feet × cosec 38° 59′ 40″ = 95·352 feet

$PS - PQ$ = 95·352 − 78·666 feet = 16·686 feet

$PS + PQ$ = 95·352 + 78·666 feet = 174·018 feet

$$\text{Tan } \tfrac{1}{2}(P\hat{Q}S - Q\hat{S}P) = \frac{16\cdot686}{174\cdot018} \cot \tfrac{1}{2}(38^\circ\ 59'\ 40'')$$

$$\text{From which } \tfrac{1}{2}(P\hat{Q}S - Q\hat{S}P) = 15^\circ\ 09'\ 14''$$

$$P\hat{Q}S - Q\hat{S}P = 30^\circ\ 18'\ 28''$$

$$P\hat{Q}S + Q\hat{S}P = 141^\circ\ 0'\ 20''$$

$$2.P\hat{Q}S = 171^\circ\ 18'\ 48''$$

$$P\hat{Q}S = 85^\circ\ 39'\ 24''$$

also
$$\frac{QS}{\sin 38^\circ\ 59'\ 40''} = \frac{PS}{\sin 85^\circ\ 39'\ 24''} = \frac{95{\cdot}352}{\sin 85^\circ\ 39'\ 24''}$$

Hence
$$QS = 95{\cdot}352 \times \frac{\sin 38^\circ\ 59'\ 40''}{\sin 85^\circ\ 39'\ 24''} = 60{\cdot}173 \text{ feet.}$$

For the final setting out, the south curve was set out from its north tangent point. Referring to fig. 9.1, this would be 511 feet 0 inches from B. It was also more convenient to set the curve 30 feet east of the centre-line, thus locating the east side of the formation, due allowance having been made for the increased length of the chord for the same angular deflection. On arriving at point R there was a deviation of $11\frac{1}{8}$ inches in an easterly direction and at S $\frac{3}{8}$ inch and the difference is explained by the "lead" on the skew. The value of this is $95{\cdot}352.\cos 38^\circ\ 59'\ 40'' = 74$ feet $1\frac{1}{4}$ inches. Therefore the offset from the curve should have been $\dfrac{(\text{chord length})^2}{2 \times \text{radius}}$ i.e. $\dfrac{74{\cdot}10^2}{2 \times 3026} = 0{\cdot}91$ feet $= 10\frac{7}{8}$ inches, the final error being $\frac{1}{4}$ inch. The errors of $\frac{3}{8}$ inch and $\frac{1}{4}$ inch on the curve were ignored.

From the plan measurements, taken on site, of the relation between the old and new bridges, all the necessary information was obtained at the design stage to scheme for the maintenance of traffic during the progress of the works.

Fig. 9.3 is a plan of the footings, the broken lines indicating the position of eight portal frame girders eventually to be erected on it. A base line was set out east of the spandril face, parallel to the abutment face and 6 feet behind it. A subsidiary line was set out 3 feet from the wing wall footing. A link between these lines, parallel to the spandril face at a known distance from it and from points P, Q, R and S, was sufficient to set out the terminal points of the footings from the given dimensions.

The work described was carried out under the direction of Mr. G. T. Bennett, O.B.E., B.Sc., M.I.C.E., whom the writer wishes to thank for permission to publish the details.

Triangulation Surveys. Although highway surveys do not normally require triangulation methods, the survey for a proposed road may sometimes form part of a larger survey based on a triangulation framework. Such a framework is frequently used when a survey is required of a comparatively large area, such as an entire borough, or urban district, where considerable new development has taken place and the large-scale Ordnance maps are out of date. The sides of the triangles of which the framework is formed are not used for obtaining topographical detail, but the points at which the sides intersect are known as "triangulation", or "trigonometrical", stations. The relative positions of these stations can be fixed with great accuracy and they serve as control points between which chain surveys and traverses are run for picking up the detail. These subsidiary surveys are thus checked and errors localised. The triangulation stations may also be used as control points in aerial surveying.

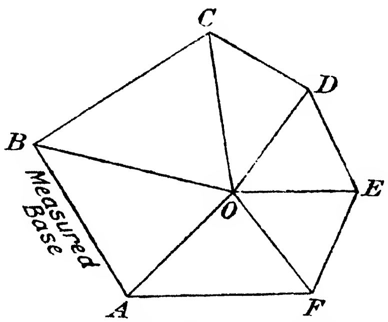


FIG. 9.4.—Principle of Triangulation.

The basic principle of triangulation is the well-known trigonometrical formula $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, where a, b, c are the sides and A, B, C are the angles of any triangle. In a network of triangles, such as ABO, BCO, CDO, etc., in fig. 9.4, if the length of one side, such as AB, is known and all the angles are measured, the lengths of all the other sides may be calculated and the network plotted. It is a comparatively easy matter to carry out the angular measurements with considerable accuracy and if the length of one side is determined with great care, the distances between all the triangulation points can be found very accurately also. The side chosen for measurement is known as the "base line". The same principle is used in mapping an entire country, in which case micrometer theodolites reading to single seconds would be used, the base line, and one or more check bases, would be measured with somewhat elaborate apparatus and the curvature of the earth's surface would be taken into account. For considerably smaller areas, such as those within the boundaries of a town of moderate size, less accuracy is permissible in both angular and lineal measurements and the curvature of the earth's surface is ignored. Some authorities state that areas up to 20 square miles may be treated as plane surfaces for mapping purposes, but in American practice areas as large as 100 square miles are regarded as plane.

Triangulation Stations. The positions of the triangulation stations are carefully chosen so that the triangles are "well conditioned", i.e. as nearly equilateral in shape as possible and obvious conditions as to visibility must be satisfied which will be apparent from fig. 9.8. As the lengths of the lines will be much greater than those in a traverse or chain survey special methods are adopted for marking them for sighting purposes. The simplest device is a tall pole, suitably painted and provided with a flag. Guys may be attached for keeping it vertical. More elaborate beacons and targets may be used for more difficult sights.

Measurement of Angles. In a small triangulation, a theodolite reading to 20 seconds is usually sufficient and the angles are commonly read by the method of "repetition". Suppose an angle BAC is to be measured by a theodolite set up at A. The instrument is set to read zero by clamping the lower plate and adjusting with the upper-plate tangent screw. The upper plate is clamped, the lower plate, or "outer axis", released and the telescope directed to B, the fine adjustment of the cross wires being made by the lower-plate tangent screw. The zero should be checked. The upper plate, or "inner axis", is released and the telescope directed to C, the fine adjustment of the cross wires being made by the upper-plate tangent screw. The upper plate is clamped, the lower plate released and the telescope directed back to B, the fine adjustment of the cross wires being made by the lower-plate tangent screw. The upper plate is released and the telescope directed again to C, the fine adjustment of the cross wires being again made by the upper-plate tangent screw. The instrument would now read twice the angle BAC. The process is repeated five times, usually, thus giving six times the angle, and the face of the instrument is changed between the third and fourth readings. The appropriate multiple of 360° must be added to the final reading to allow for the number of complete turns described during the summation of the angles. If, for example, the angle was 63°, the instrument would read 18° after five repetitions, the total angle turned through being rather more than 360°. Hence the final value of the angle would be booked as 378° which, when divided by the number of readings, gives 63°. The instrument need only be read after making the last repetition. The whole procedure is carried out a second time, measuring the angle in the opposite direction, i.e. from C to B.

Base Line Measurement. In a small triangulation the length of the lines will not be very great, varying, probably, between half a mile and 2 miles, and it will usually be possible to locate one line, at least, of the framework on fairly flat ground free of obstructions so that this line can be used as the measured base. It is advisable to measure a second line as a check base unless the framework is very simple. It may be impossible, however, to use a line of the main framework as a base, and the only available site for an accurate lineal measurement may only give sufficient room for a line much shorter than the average length of the triangulation lines. In this case the base is extended by triangulating in some such way as that shown in fig. 9.5, in which the length AE of the main framework is computed from the measured base XY and the angles in the subsidiary network by the sine formula. The subsidiary stations M and N are so located that the triangles are reasonably well shaped. This method is very frequently used in large surveys where the length of the base is necessarily short in comparison with the other lines of the triangulation net.

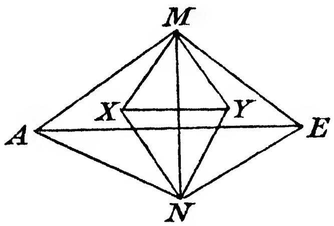


FIG. 9.5.—Method of Extending a Base Line.

The base line in a small survey may be measured with sufficient accuracy by means of quite simple equipment. Methods differ in detail but the general procedure consists in making a series of consecutive measurements with a standardised tape suspended over tripods or supporting posts and subjected to a known pull. The temperature of the tape must be ascertained as closely as possible during the proceedings. A special baseline tape should be used, preferably of invar. On these tapes the terminal markings are some inches from the ends and rings are provided for attachment to the straining device. A nominal length of 100 feet is convenient and the makers will supply a certificate stating the temperature and pull at which the tape gives its true length by comparison with a standard. The simplest way of applying the pull is to couple the tape by swivelling clips to a pair of sleeves which slide over stout ranging rods with a spring balance inserted between one end of the tape and one of the rods. This device is shown diagrammatically in fig. 9.6. The temperature of the tape during measurement is found approximately by clipping three base-line thermometers to it with their bulbs in contact with the metal. Wooden stakes about 3 feet long and 3 inches square can be used for supports. Metal bands should be nailed round the top to protect the wood during driving and zinc plates are tacked to the upper surface on which the measured lengths and the direction of the alignment are scribed. The stakes marking the ends of the base are set out first, having due regard to visibility to and from other stations, and intermediate stakes are then set out. Their alignment is fixed by theodolite and their distances apart are fixed by an approximate measurement with the tape, using the same pull as that to be used for the final measurement. The zinc plates are then attached and the accurate measurements are then made, a fine mark such as a surveying arrow being used for the alignment and the prescribed pull being applied steadily. The thermometers are attached with as little delay as possible and their readings taken. The work should preferably be carried out in still, dull weather as wide variations in temperature will occur in sunny weather with variable light breezes. If the base-line tape has no fine sub-divisions the total length of the base should be a multiple of a nominal 100 feet, or 50 feet if the middle point on the tape is marked.

The outward measurement having been completed, a return measurement is made and, provided there are no great fluctuations of temperature, the return distances will not be widely different to the

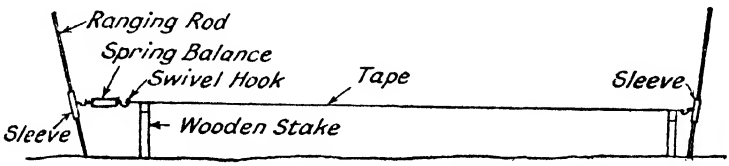


FIG. 9.6.—Simple Method of Base Measurement.

outward distances. The distance between the original zero and the final mark for the return measurement is scaled off with a finely divided steel rule. The nominal outward and return lengths are now amended by the application of a series of corrections and the two corrected lengths are compared with the measured difference between the zero and the final mark on the first stake. One of these corrections necessitates the determination of the levels of the tops of the stakes.

Base-line Corrections. Suppose that the tape gives a standard length L at a temperature $T°$ when subjected to a pull of P lb. In some cases, P is zero. T is frequently 62° F.

(1) *Temperature Correction.* If the mean temperature during the measurement of a nominal length L is $t°$ and the coefficient of lineal expansion of the tape is α, the correction is $L.\alpha.(t-T)$.

α for steel tapes is 0·0000065 per °F. and for invar it varies between 0·0000003 and 0·0000005 per °F. If t is greater than T, the tape will be too long and the correction is positive, and vice versa.

(2) *Pull Correction.* This is derived from the ordinary expression for elastic extension :

$$\frac{\text{stress}}{\text{strain}} = \text{Young's Modulus } (E) = \frac{\dfrac{\text{pull}}{\text{area}}}{\dfrac{\text{extension}}{\text{length}}}$$

E may be taken as 30×10^6 lb. per square inch and the cross-sectional area (a) of the tape must be calculated from micrometered measurements. If the pull used during the measurement is p, the correction for the elastic extension per length L is $\frac{(p - P)L}{aE}$. If p is greater than P this correction is added to the nominal length, L.

(3) *Sag Correction.* The tape actually hangs in the form of a catenary, but to simplify the derivation of a correction formula, it may be assumed to hang as a parabola without introducing any appreciable error. If W is the weight of a span L of the tape, the sag correction is $\frac{L}{24}\left(\frac{P}{W}\right)^2$. W and P must, of course, be expressed in the same units. The effect of sag is to shorten the tape and the correction is negative in consequence.

(4) *Slope Correction.* The tops of the stakes will not all be at the same level and the base line will therefore consist of a series of sloping measurements which require reduction to a horizontal line. The

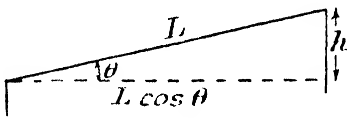


FIG. 9.7.—Slope Correction in Base Measurement.

relative heights of the stakes are found by levelling. Suppose that the difference in height of two successive stakes is h, as in fig. 9.7. Then if L is the nominal sloping distance, the horizontal distance is

$$L.\cos\theta = L.\frac{\sqrt{L^2 - h^2}}{L}$$

$$= (L^2 - h^2)^{\frac{1}{2}} = L.\left(1 - \frac{h^2}{L^2}\right)^{\frac{1}{2}}$$

which by expansion becomes

$$L\left(1 - \frac{1}{2}.\frac{h^2}{L^2} \ldots\right)$$

Neglecting further terms, the slope correction is $\frac{h^2}{2.L}$. Further refinements are introduced in a large geodetic survey, but the above corrections suffice in a small triangulation.

Equations of Condition. Many triangulation frameworks consist wholly, or in part, of lines arranged in the manner shown in fig. 9.8 (a) or (b). In one case we have a central station, X, and in the other a common vertex, X, outside the figure. These arrangements give rise to three equations relating to the angles known as the " equations of condition " by which the theodolite work can be closely checked.

The first two equations are obvious :

(1) the angles of each triangle must sum to 180°,

(2) the angles round the central station, or common vertex, must sum to 360°.

The third equation necessitates a convention regarding the naming of the base angles of the triangles. In fig. 9.8 (*a*), if an observer stands on any line of the perimeter and faces the central station the base angles can be classified immediately as " left " or " right-hand ". This rule applies equally to the triangles AXE, BXA and EXD in fig. 9.8 (*b*), the observer, in this case, facing the common vertex.

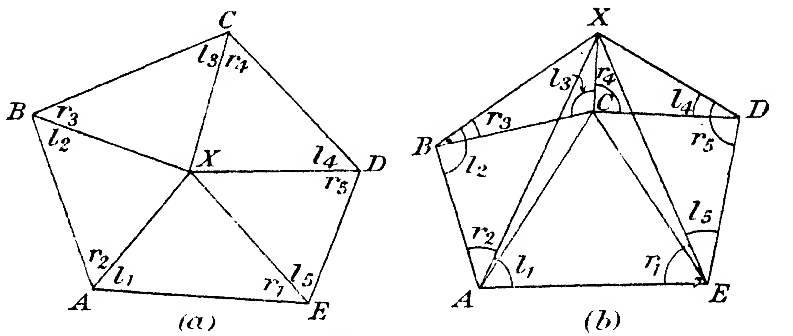


FIG. 9.8.—Triangulation Networks.

The triangles BXC and CXD, however, must be viewed from the lines BX and DX, in which case the angles XCB and XDC will be regarded as left-hand angles and the angles XBC and XCD as right-hand angles. In both diagrams, we then have :

$$\frac{\sin l_1}{\sin r_1} = \frac{EX}{AX}, \quad \frac{\sin l_2}{\sin r_2} = \frac{AX}{BX}, \quad \frac{\sin l_3}{\sin r_3} = \frac{BX}{CX}, \quad \frac{\sin l_4}{\sin r_4} = \frac{CX}{DX}, \quad \frac{\sin l_5}{\sin r_5} = \frac{DX}{EX}.$$

If these expressions are multipled together we obtain the result

$$\frac{\sin l_1 \times \sin l_2 \times \sin l_3 \times \sin l_4 \times \sin l_5}{\sin r_1 \times \sin r_2 \times \sin r_3 \times \sin r_4 \times \sin r_5} = 1,$$

and by conversion to logs we obtain the third equation :

(3) the sum of the sines of the left-hand angles must equal the sum of the sines of the right-hand angles.

Adjustment of Triangulation Angles. As in all other forms of practical fieldwork small errors are certain to occur in the observed triangulation angles and perfect agreement with all three equations of condition is never obtained. Before plotting, the angles are adjusted to give very close if not quite exact agreement and in a small survey this is usually done by trial and error. Certain rules, however, should be borne in mind.

(1) The sines of small angles increase at a greater rate as the angles increase than the sines of large angles. Hence, when adjusting the log.sine summation, a correction of a given magnitude will produce a smaller alteration in small angles than in large. On the other hand, when correcting the summation of the angles, an alteration of given magnitude will cause less alteration to the log.sine if applied to a large angle than if applied to a small one. If, therefore, the summation of the angles checks reasonably, but the summation of the log.sines does not, corrections should be applied to the smaller angles. If, however, the summation of the log.sines checks fairly well, but the summation of the angles does not, corrections should be applied to the larger angles.

(2) If the centre angles, or the angles at a common vertex, check well in the summation to 360° but two adjacent triangles have summation errors of approximately the same magnitude, but opposite in sign, it is highly probable that compensating errors have occurred in two adjacent centre angles, due to some such cause as a displaced signal.

In large surveys the adjustment of the angles is carried out by complicated mathematical methods. This subject, however, is outside the scope of highway surveying and for further information the reader is referred to appropriate text-books, some of which are listed in the bibliography, page 281.

Reference

1. I. Kursbatt, "The Kidlington Bridges". *Journal Inst. C.E.*, **13**, No. 3, January, 1940.

CHAPTER X

EARTHWORK QUANTITIES

The cross-sections show the extent of the excavation in cuttings and the filling for embankments at intervals spaced equally along the centre-line. In general, the shape of the sections through a cutting will be roughly similar to one another. This similarity will also occur with successive sections through an embankment, but will cease where the construction changes from wholly excavation, or wholly filling, to a mixture of cut and fill necessary, sometimes, on a hillside. Let us first consider the case of simple excavation or filling.

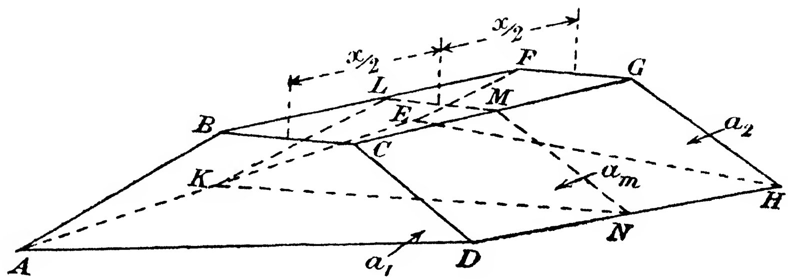


FIG. 10.1.—Estimation of Earthwork Quantities.

A part of an embankment is shown in fig. 10.1, *ABCD* and *EFGH* being two cross-sections distant x feet apart. For simplicity the ground lines *AD* and *EH* will be assumed straight. Two methods are in common use for determining the volume in such a case as this, and both will now be considered.

Trapezoidal Formula. The simplest procedure is the "trapezoidal" or "average end area" method which merely involves the estimation of each of the cross-sectional areas, the calculation of their mean and the multiplication of the mean area by the distance apart of the sections. Thus, if the area *ABCD* is a_1 and the area *EFGH* is a_2, the volume of the bank is taken as $\frac{a_1 + a_2}{2}.x$.

For a series of successive cross-sections distant x apart with areas a_1, a_2, a_3, etc., to a_n, the volume enclosed between the first and last sections is given by the expression

$$V = \frac{a_1 + a_2}{2}.x + \frac{a_2 + a_3}{2}.x + \frac{a_3 + a_4}{2}.x + \ldots + \frac{a_{n-1} + a_n}{2}.x$$

$$= x.\left(\frac{a_1 + a_n}{2} + a_2 + a_3 + a_4 + \ldots + a_{n-1}\right)$$

This " trapezoidal " formula gives only an approximation of the true volume, but in the opinion of many engineers it is sufficiently accurate for practical purposes since the somewhat indefinite factors of swelling and shrinkage which occur during the constructional processes render precise mathematical calculations of little value.

If, however, a closer approximation to the true volume is required, it may be obtained with little extra work by means of the " prismoidal " formula which is an adaptation of the well-known Simpson's Rule for obtaining the area of a figure with an irregular boundary.

Prismoidal Formula. Referring, again, to fig. 10·1, consider a section *KLMN*, midway between the two original sections, and let the area *KLMN* be a_m, which will not, in general, be the mean of the areas a_1 and a_2. Then, according to the prismoidal rule, the volume of the bank is $\frac{x}{6}(a_1 + 4a_m + a_2)$. For a series of successive cross-sections with areas a_1, a_2, a_3, etc., to a_n, *where n is an odd number*, the volume enclosed between the first and last sections is given by

$$V = \frac{d}{3}(a_1 + 4a_2 + 2a_3 + 4a_4 + 2a_5 + \dots + 2a_{n-2} + 4a_{n-1} + a_n)$$

where d is the constant distance between the sections. It is important to notice that the prismoidal formula can only be applied to an odd number of cross-sections. In the case of the three sections in fig. 10.1, $d = \frac{x}{2}$. Further, if it is assumed that the mid area, a_m, is the mean of the areas a_1 and a_2, the prismoidal rule gives the same expression as the trapezoidal rule, thus :

$$V = \frac{x}{6}.\left[a_1 + 4\left(\frac{a_1 + a_2}{2}\right) + a_2\right] = \left(\frac{a_1 + a_2}{2}\right)x.$$

Derivation of the Prismoidal Rule. Let the three areas a_1, a_m, and a_2 be represented by three ordinates *PU*, *QT* and *RS*, in fig. 10.2, *UT* and *TS* representing the distance $\frac{x}{2}$, or d. The area *UPQRS* can then be considered to represent the volume of the embankment and the points *P*, *Q* and *R* can be joined by a smooth curve, indicating a gradual change in cross-sectional area which is the normal condition in practice. The basic assumption in Simpson's Rule is that points such as *P*, *Q* and *R* lie on a parabolic arc. On this assumption, the area $UPQRS$ = trapezium $UPRS$ + parabolic segment PQR

$$= 2d\left(\frac{a_1 + a_2}{2}\right) + \frac{2}{3}\text{ rectangle } PVYR$$

$$= 2d\left(\frac{a_1 + a_2}{2}\right) + \frac{2}{3}.2d.QZ$$

$$= 2d\left(\frac{a_1 + a_2}{2}\right) + \frac{2}{3}.2d.\left(a_m - \frac{a_1 + a_2}{2}\right)$$

$$= d\left(a_1 + a_2 + \frac{4}{3}a_m - \frac{2}{3}a_1 - \frac{2}{3}a_2\right)$$

$$= \frac{d}{3}(a_1 + 4a_m + a_2).$$

In the foregoing, the proportions of a_1, a_m and a_2 are such that the

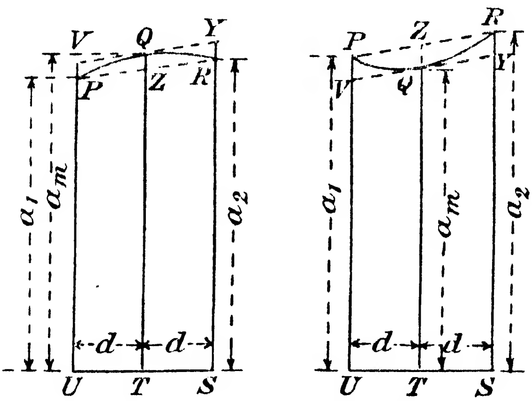


FIG. 10.2. FIG. 10.3.
Derivation of Prismoidal Rule.

parabolic arc is convex when viewed from above. With different values of the ordinates, a concave arc may be obtained, but the enclosed area will still be given by the same formula. Thus, in fig. 10.3

area $UPQRS$ = trapezium $UPRS$ − parabolic segment PQR

$$= 2d\left(\frac{a_1 + a_2}{2}\right) - \frac{2}{3}.2d.\left(\frac{a_1 + a_2}{2} - a_m\right)$$

$$= d\left(a_1 + a_2 - \frac{2}{3}a_1 - \frac{2}{3}a_2 + \frac{4}{3}a_m\right)$$

$$= \frac{d}{3}(a_1 + 4a_m + a_2), \text{ as before.}$$

If we have a continuous series of cross-sections, they would be numbered 1, 2, 3, etc., up to n, n being an odd number, and if the corresponding areas are a_1, a_2, a_3, etc., the volume up to no. 3 would be

$$\frac{d}{3}(a_1 + 4a_2 + a_3).$$

From no. 3 to no. 5 the volume would be $\frac{d}{3}(a_3 + 4a_4 + a_5)$, and so on. Hence, the total volume would be

$$\frac{d}{3}(a_1 + 4a_2 + 2a_3 + 4a_4 + 2a_5 + \ldots + 2a_{n-2} + 4a_{n-1} + a_n).$$

Expressed in words, the formula may be remembered thus: The volume equals $\frac{d}{3}$ multiplied by the sum of the first and last ordinates, plus four times the even ordinates and twice the odd. The fact that "four" and "even" are both four-letter words is often quoted as a useful mnemonic.

If, in figs. 10.2 and 10.3, P and R are joined by a straight line we obtain a graphical representation of the trapezoidal rule.

Volumes Tapering to Zero. At places where cuttings and

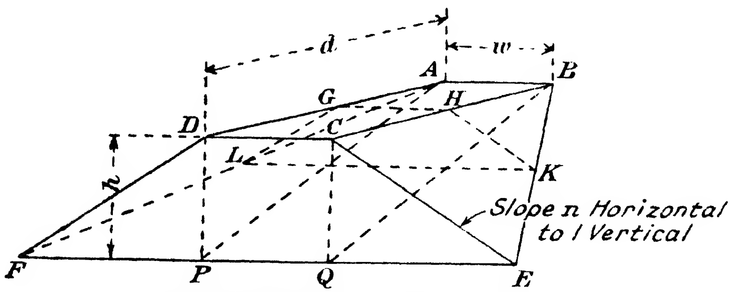


FIG. 10.4.—Embankment converging to a Level Section.

embankments run out to the natural ground level, a cross-section is reached of which the area is zero and the final portion of the excavation or filling assumes a shape which approximates to a wedge. This is illustrated in fig. 10.4 which represents a simplified case where the natural ground is assumed to be horizontal in a direction at right-angles to the centre-line. The formation width is w and the side slopes are n horizontal to 1 vertical. The area of the cross-section $CDEF$, distant d from AB, is $hw + nh^2$, where h is the height of the formation above the ground. By the trapezoidal rule the volume of the filling would be

$$d\left(\frac{hw + nh^2 + 0}{2}\right) = d\left(\frac{hw}{2} + \frac{nh^2}{2}\right)$$

By the prismoidal rule a midway section, $GHKL$, must be considered.

The area $GHKL = \frac{hw}{2} + \frac{nh^2}{4}$ and the volume of the filling would be

$$\frac{d}{6}\left(hw + nh^2 + 0 + 4\left(\frac{hw}{2} + \frac{nh^2}{4}\right)\right)$$
$$= \frac{d}{6}(3hw + 2nh^2)$$
$$= d\left(\frac{hw}{2} + \frac{nh^2}{3}\right)$$

This expression gives the correct value of the volume which is made up of the wedge $ABCDPQ$ and two tetrahedrons, $DPFA$ and $CQEB$, i.e. by ordinary mensuration, volume

$$= d\left(\frac{hw}{2} + 2.\frac{1}{3}.\frac{nh^2}{2}\right) = d\left(\frac{hw}{2} + \frac{nh^2}{3}\right)$$

The trapezoidal formula is noticeably inaccurate in such cases.

Complicated Sections. In fig. 10·1 where the sections are wholly filling, the solid figure thus formed approximates to a "prismoid" which may be defined as a solid whose ends are parallel, but not necessarily similar, figures each having the same number of sides, while the remaining faces are trapeziums. If the ends are similar, the prismoid becomes the frustum of a pyramid and a prismoid with rectangular ends is the frustum of a wedge.

It frequently happens that simple excavation or filling changes into sections which are a combination of cut and fill. When this occurs it is usually possible to divide the solid contained between certain cross-sections into portions which approximate to geometrical shapes such as the prism, pyramid and wedge and then apply the ordinary rules of mensuration.

Cross-sections on a Curved Centre-line. Where the centre-line is curved the usual practice is to take cross-sections in directions which are judged as nearly as possible to be radial. According to the theorem of Guldinus (also attributed to Pappus) if a plane area rotates about an axis in its own plane, but outside the area, the volume of the solid thus generated is equal to the product of the area and the length of the path described by the centre of gravity of the area. This theorem has a direct bearing on the computation of volumes from cross-sections on a curved centre-line, but it is scarcely of practical application except in special circumstances, which will now be considered.

Two consecutive cross-sections on a curved centre-line are shown in fig. 10.5. If their respective areas are a_1 and a_2, and the eccentricities of their centroids are e_1 and e_2, R being the radius of curvature of the centre-line and d the distance between the sections measured along the centre-line, by an adaptation of the theorem of Guldinus, the volume between the sections may be expressed as $\frac{a_1 + a_2}{2} \times$ distance between centroids, measured as an arc. The angle subtended at the centre of the curve $= \frac{d}{R} = \theta$ radians. The mean eccentricity $= \frac{e_1 + e_2}{2}$ and the distance between the centroids is approximately $\left(R + \frac{e_1 + e_2}{2}\right)\theta$, or $d\left(1 + \frac{e_1 + e_2}{2R}\right)$. Hence, the volume is given approximately by $\frac{a_1 + a_2}{2} \times d\left(1 + \frac{e_1 + e_2}{2R}\right)$, the extent of the correction for the curved centre-line being $\left(\frac{e_1 + e_2}{2R}\right)d$ added, in this case, to the distance d since the centroids are displaced to the side

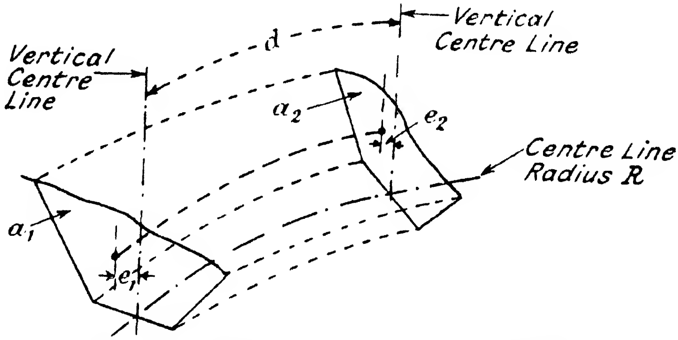


FIG. 10.5.—Cross-sections on a curved Centre-line.

of the centre-line remote from the centre of the curve. For displacements in the opposite direction, the eccentricities would be treated as negative.

A numerical example will serve to illustrate the effect of assuming the distance between the sections to be the centre-line distance instead of the distance between the centroids. Suppose the radius of curvature of the centre-line is 5,000 feet and the distance d is 100 feet. For an error of 1% in the distance between the sections

$$100\left(1 + \frac{e_1 + e_2}{2 \times 5{,}000}\right) = 101$$

or

$$\frac{e_1 + e_2}{2} = 0{\cdot}01 \times 5{,}000 = 50 \text{ feet.}$$

Hence a mean eccentricity of the centroids of 50 feet would be necessary to give a 1% error. The shape of cross-section which would give an eccentricity of this extent, even with a formation width of 100 feet, is not ordinarily met with in practice, except in the case of sections which are partly cut and partly fill. It must be remembered, however, that the magnitude of the correction is inversely proportional to R and with sharp radius curves a correspondingly smaller eccentricity would result in a 1% distance error if the curvature of the centre-line is ignored.

Determination of Cross-sectional Areas. In practice, cross-sections are usually spaced at 100-foot intervals. If a cutting or bank contains an even number the prismoidal formula may be used for finding the volume as far as the last section but one, the remaining volume being computed separately, or, if rather less accuracy can be

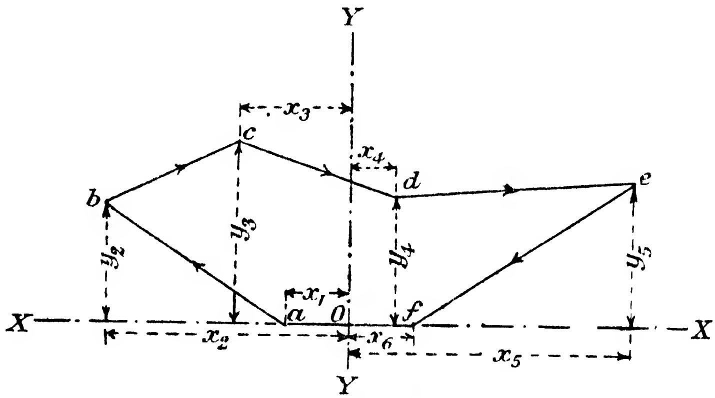


FIG. 10.6.—Computation of Cross-sectional Area from Co-ordinates.

accepted, the whole volume may be calculated by the trapezoidal formula. The cross-sectional areas are determined either by planimeter, or by dividing the area into triangles and trapeziums. If the ground line of the section is irregular it is replaced by "give-and-take" lines, thus making the figure rectilineal. The area of a figure of this kind may be calculated by the same method as that frequently used for a closed traverse. Suppose that the polygon *abcdef* in fig. 10.6 represents a cross-section in excavation, *af* being the formation and *bcde* the ground line. The co-ordinates of the corners of the figure are measured from the axis XX, coincident with the formation line, and the axis YY, the perpendicular through the mid-point of the formation. We may regard the XX axis as an east–west line and the YY axis as a north–south line, the y ordinates being northings. A circuit of the figure is now made in a clockwise direction. The sum of the y co-ordinates of the ends of each line is multiplied by the easting or westing of the line, an easting being considered positive and a westing negative, and these products are summed algebraically. The area is given by half the sum.

In the example,

$$\text{the cross-sectional area} = \tfrac{1}{2}[-(0+y_2)(x_2-x_1)+(y_2+y_3)(x_2-x_3) + (y_3+y_4)(x_3+x_4)+(y_4+y_5)(x_5-x_4)-(y_5+0)(x_5-x_6)]$$

The co-ordinates of the points a, b, c, d, e, indicated in the figure, may be derived directly from fieldwork data obtained from the cross-section levelling and the setting out of slope stakes by the "grade-staff" method. Consequently the above method of calculating the cross-sectional area renders a scale drawing unnecessary.

Mass-Haulage Diagrams. In projects involving heavy earthworks it is customary to prepare mass-haulage diagrams. These consist of graphs showing the algebraic summation of excavation and filling as one proceeds along the centre-line. They are plotted beneath the longitudinal section, using the same scale for horizontal distances and their characteristics and uses can best be understood from an example. Fig. 10.7 represents part of the longitudinal section of a proposed road with a mass-haulage diagram beneath. Starting from the point A the volumes of excavation between successive cross-sections are computed and plotted as an additive curve. Thus the ordinate xx' represents, to scale, the total amount of excavation up to the point X. The maximum ordinate, bb, will occur vertically beneath B and after this point the volume of excavated material will steadily decrease as it becomes used up in filling between B and C. The curve therefore drops until it reaches the datum line at d. Here the net amount of excavation and filling is zero, and if a vertical is drawn from d to intersect the formation line at D, it will be apparent that the excavated material between A and B will just balance the filling between B and D. Proceeding beyond D the volume of filling increases and the negative ordinates, indicating fill, lengthen until c is reached, corresponding to the point C on the longitudinal section at which the second cutting starts. The volume of filling now becomes steadily reduced as the excavation increases between C and E, at which point another peak occurs in the mass-haulage diagram. In the example shown, the mass-haul curve does not reach the datum line at e, the negative ordinate at e representing an excess of filling. In addition to the datum line, any other horizontal line such as fhk will intercept loops of the mass-haul curve which give a balance of cut and fill. If verticals are projected to the longitudinal section from the points f, g, h and k on the mass-haul curve, it will be seen that the filling from F to C is balanced by the excavation from C to G, the filling from E to H by the excavation from G to E, and the filling from L to H by the excavation from L to K. It is also possible to derive from the diagram the following rule : If the mass-haul curve is above the datum line, excavated material must be hauled from left to right and if the mass-haul curve is below the datum line, excavated material

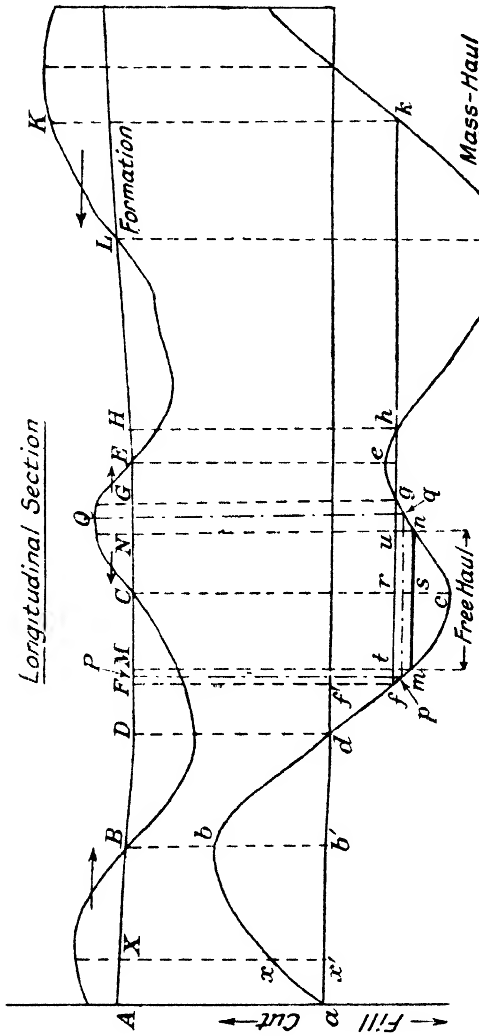


Fig. 10.7.—Longitudinal Section and Mass-Haulage Diagrams.

must be hauled from right to left. This presupposes, of course, that excavation is plotted as positive ordinates.

This rule also applies to any other base line, such as *fghk*, but the balance of excavation and filling will occur between different points along the centre-line depending on the position of the base line. The contract price for excavation usually includes haulage within a stated distance known as the "freehaul". Haulage over distances in excess of the freehaul is charged for at a cost which involves both the volume of the material and the length of "overhaul". The mass-haul curve is particularly useful for determining both these factors and thus arriving at a reasonable figure for the overhaul charges. For this purpose the work is dealt with in sections and the loop of the mass-haul curve intercepted by the portion of the base line *fg* will be taken as an example. The scaled length of the freehaul distance is set off horizontally so that it just fits the curve. Let *mn* be this distance. Referring to the corresponding points *M* and *N* on the longitudinal section, the excavated material between *N* and *C* will be hauled without extra charge to form the bank between *C* and *M*. The remainder of the material within the balanced loop, however, will be subject to overhaul charges and it will be necessary to estimate the volume of this material and the average distance it is hauled, in excess of the freehaul distance. The total volume of filling between *F* and *C* is represented by the ordinate *cr*, which also represents the balancing volume of excavation between *C* and *G*. Similarly, the freehaul volume is represented by *cs*. The volume subject to overhaul charges is therefore represented by *rs* (or *tm*, or *un*). The average overhaul distance is usually accepted as the distance between the centroids of the volumes of cut and fill concerned, less the freehaul. If horizontal lines are drawn through the middle points of the ordinates *tm* and *un* to intersect the mass-haul curve at *p* and *q*, the centroids of these volumes will lie on verticals through *p* and *q* projected up to the longitudinal section at *P* and *Q*. Hence the overhaul distance is $pq - mn$.

It will be noticed in fig. 10·7, that an unbalanced volume of filling represented by *ff'*, occurs between *D* and *F*. This would necessitate the haulage of material from outside the section of the project under consideration unless a borrow pit were available conveniently close to the point in question. In general, it is preferable to have an excess of excavation and this would be indicated by a similar gap in the mass-haul curve above the datum or base line instead of below. The haulage of material from a distant source, or from a borrow pit, is always avoided, if possible, by adjusting the formation levels to give a better balance and the mass-haul curve obviously enables a much closer balance to be obtained than the longitudinal section alone.

Bulking and Shrinkage. Excavation is measured as the nett volume of the opening formed but the loosened material expands or "bulks" and if dumped without compaction the volume obtained would be greater than the volume of the excavation. When placed as filling and properly consolidated the final volume is frequently less than the original volume of the undisturbed material. These facts must be taken into account when computing earthwork quantities and the reader is referred to the handbooks listed in the bibliography, page 281, for information regarding correction factors for various kinds of soil. Improved methods of compaction are now being adopted and a minimum density of the consolidated material is sometimes stipulated in earthwork specifications. A comparison of this density with that of the undisturbed soil before excavation would probably give a more accurate factor than that obtained from tables.

When computing volumes of filling in the mass-haul curve allowance must be made for the shrinkage due to compaction and also the fact that the topsoil from an excavation should never be used for filling but should be set aside for subsequent spreading on the slopes of the cuttings and embankments.

Earthwork Tables. Tables are available which facilitate the calculation of earthwork quantities and some of the better known are listed in the bibliography, page 281.

Correction for Shape of Subgrade. So far the formation line in the cross-sections has been assumed to be a horizontal straight line, but actually it follows a more complicated outline, such as that shown

FIG. 10.8.—Assumed and Actual Cross-section.

in fig. 10·8. The shaded area may not be very large, but when multiplied by several miles it will add up to an appreciable yardage. If the earthwork quantities have been computed on the assumption that *ab*, in fig. 10.8, was the formation line, the computed volume will be too small in excavation and too large in filling. The area enclosed between the assumed and actual formation may be found from a large-scale section and the correction per unit length of centre-line may be readily calculated.

CHAPTER XI

DEVELOPMENTS IN INSTRUMENT DESIGN

The design of British surveying instruments advanced considerably between the war of 1914–18 and the recent World War. This was due, to some extent, to the great strides made in the quality of British optical products. Improved lenses made it possible to adopt smaller telescopes without losing any efficiency and it has been found that graduated circles of glass need only be about half the diameter of metal circles of corresponding accuracy. Modern instruments are consequently more compact and lighter than older models. Another noteworthy change in design has been the incorporation of prismatic devices which render it unnecessary for the surveyor to move round to the side of the instrument to check the spirit-level adjustment or take reading of angles from the horizontal and vertical circles.

Focussing by means of a lens within the telescope is now common practice and, as already mentioned in Chapter VI, page 164, such lenses are usually so proportioned that the instrument has a tacheometric multiplier of 100 and a negligible additive correction. Internal focussing has several advantages over the older method in which a sleeved telescope of variable length is used.

Greatly improved instrument cases, mainly of metal, have been introduced enabling both levels and theodolites to be packed quickly and simply without any risk of strain or need of partial dismantling.

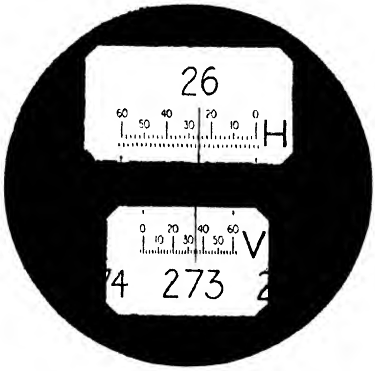
Levels. The difference in principle between the dumpy and the newer levels of the " quick-set ", or tilting telescope, type has been discussed in Chapter II, pages 41 *et seq.* Many instruments now have an additional eye-piece close to the main eye-piece for observing the bubble adjustment, the two ends of the bubble being resolved optically so as to appear in juxta-position in the manner explained on page 58. Levels can be obtained with a horizontal circle reading direct to 5 or 10 minutes of arc by means of a microscope with a graticule, or fine scale superimposed on the image of the circular scale. This reading device is described later. A graduated circle of this type possesses obvious advantages for contouring and also for setting out right-angles in cross-section levelling.

Theodolites. In 1926 a conference was held at Tavistock between officers of the Ordnance Survey and other Government departments and the principal instrument makers as a result of which the " Tavistock " theodolite was produced. A full description of this interesting instrument has been given in the technical press [1] and only a brief reference will be made to it here since it gives a standard of accuracy which is beyond the requirements of the highway engineer. Both horizontal and vertical circles are graduated on glass, the former being only 3·5 inches and the latter 2·75 inches in diameter, yet a direct reading is obtainable to a second of arc on each. The instrument weighs only 13 lb. A system of prisms and lenses enables the readings from both sides of the circle, 180° apart, to be read simultaneously and automatically meaned without moving from the normal observing position at the eye-piece end of the telescope. Certain features of the "Tavistock" theodolite, however, have been adopted in simpler modern instruments which are admirably suited to highway surveys, one of the most important being the use of glass for the graduated circles. The division lines are inscribed on the under-surface which is silvered and protected from dust and damp. Daylight, or artificial light, directed into the instrument by a reflector, passes through the glass and is reflected back to the observer's eye through a reading microscope in which there is some device for obtaining a finer value of the angle than that given by the graduated circles. The latter are usually divided only into single degrees. The image viewed through the microscope is brilliantly luminous and in bright sunlight it may be necessary to tone down the light by a tinted glass plate which may be interposed in the eye-piece.

Optical methods of centring are introduced in some of the more expensive instruments, obviating the use of a plumb-line.

Reading Systems. Both horizontal and vertical circles are usually viewed through the same eye-piece placed conveniently near to the telescope eye-piece. The images of both circles are formed at the same focal plane and the finer reading is obtained from a separate scale. Two simple methods of reading will be discussed, the "optical scale" method and the "optical micrometer" method.

Optical Scales. The circular scale on the lower plate is graduated at intervals of one degree, each of which is numbered. The fine scale may be graduated in various ways, a typical example being shown in fig. 11.1. For the horizontal circle there are two sets of graduations, the upper being 30 in number, each representing 2 minutes. The total length of the 30 graduations corresponds exactly with the space between two consecutive degree marks on the circle. A second set of graduations appears immediately below the first, 31 in number, and displaced laterally so that any given line in the upper scale falls midway between two lines in the lower scale. It follows that the upper scale gives the even and the lower scale the odd numbers of minutes. The fine scales, or graticules, are superimposed on the image of the degree readings and if one of the latter graduations is located exactly midway between a line on the upper and a line on the lower scale a fine illuminated gap is visible on each side of the degree line. This setting gives a reading of 30 seconds and it is possible to estimate to 15 seconds. This method of reading is more definite and satisfactory than the vernier, which is liable to marked inaccuracy unless the small magnifying eye-pieces are carefully placed immediately above the point of coincidence of the scales.



Horizontal Circle Reading: 26° 25′ 30″.
Vertical Circle Reading: 273° 35′.

FIG. 11.1.—Field of View of Optical Scales
(Cooke, Troughton & Simms).

The vertical circle is of minor importance in general engineering surveys where no astronomical work is contemplated and in the example shown in fig. 11.1 the fine scale for vertical readings consists of graduations spaced at 2-minute intervals, giving a reading to single minutes if exactly bisected by a degree mark, with a possibility of estimation to 30 seconds.

Optical Micrometers. In the optical micrometer there is either a fixed index or a pair of parallel cross wires and the image of the

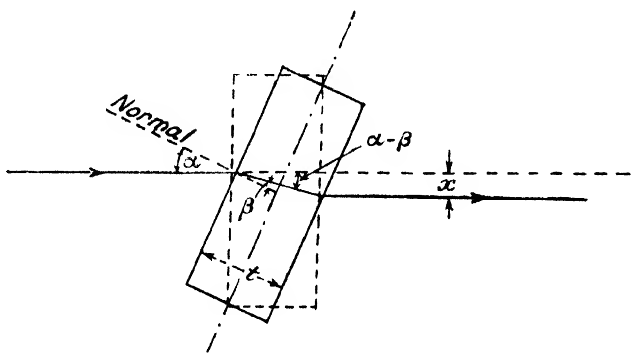


FIG. 11.2.—Principle of Parallel Plate Micrometer.

circle graduation, indicating the reading to the nearest degree, is displaced sufficiently to bring it into coincidence with the index or, alternatively, to bisect the space between the parallel cross wires. The extent of the displacement is recorded on a fine scale reading in minutes and fractions of a minute against another index or cross wire.

The displacement of the circle reading may be produced by passing the light from the graduated circle through the parallel sides of a pivoted glass block. If a light ray falls normally on a glass plate no deviation occurs, but if it falls at an angle it is deviated towards the normal on entering the glass and is deviated back again on emergence by an equal amount, thus continuing parallel to its original direction. The displacement of the ray is proportional to the thickness of the plate, the refractive index of the glass and the angle of incidence between the ray and the normal to the surface of the plate. In fig. 11.2 let α be the angle of incidence, t the thickness of the plate, μ the refactive index and x the displacement of the ray. On entering the plate the ray will be refracted towards the normal and if β is the angle between the normal and the refracted ray, $\dfrac{\sin\alpha}{\sin\beta} = \mu$. From the figure:

$$\frac{x}{\dfrac{t}{\cos\beta}} = \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos.\alpha.\sin\beta.$$

$$\therefore \quad x = t.(\sin\alpha - \cos\alpha.\tan\beta)$$

But $\sin\beta = \dfrac{\sin\alpha}{\mu}$, or $\sin^2\beta = 1 - \cos^2\beta = \dfrac{\sin^2\alpha}{\mu^2}$

$$\therefore \cos\beta = \sqrt{1 - \frac{\sin^2\alpha}{\mu^2}} \text{ and } \tan\beta = \frac{\dfrac{\sin\alpha}{\mu}}{\dfrac{\sqrt{\mu^2 - \sin^2\alpha}}{\mu}} = \frac{\sin\alpha}{\sqrt{\mu^2 - \sin^2\alpha}}$$

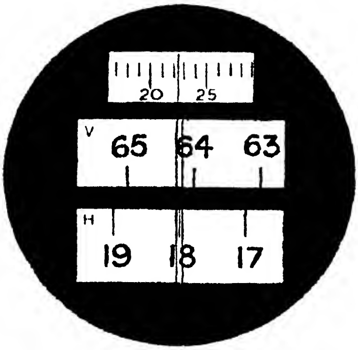
$$\text{Hence } x = t\left(\sin\alpha - \cos\alpha.\frac{\sin\alpha}{\sqrt{\mu^2 - \sin^2\alpha}}\right)$$

$$= t.\sin\alpha\left(1 - \frac{\sqrt{1 - \sin^2\alpha}}{\sqrt{\mu^2 - \sin^2\alpha}}\right)$$

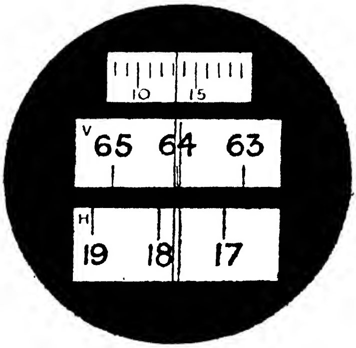
If α is small, $\sin\alpha = \alpha$ and $\sin^2\alpha$ may be neglected. We may then write $x = t.\alpha\left(1 - \dfrac{1}{\mu}\right)$, where α is in radians.

The parallel plate in the theodolite is tilted by manipulating the milled head of an adjusting screw until the appropriate degree graduation is brought to the correct position with reference to the index or cross wires and a specially calibrated scale records the angular movement of the plate, the complete range of the scale measuring the tilt which produces a displacement in the circle readings of one degree.

Two typical fields of view in optical micrometer instruments are shown in figs. 11.3 and 11.4. It will be noticed that one fine scale serves both horizontal and vertical circles. In fig. 11.3 direct readings are obtainable to single minutes, and owing to the openness of the scale it is possible to estimate to 0·1 of a minute. In fig. 11.4 a direct



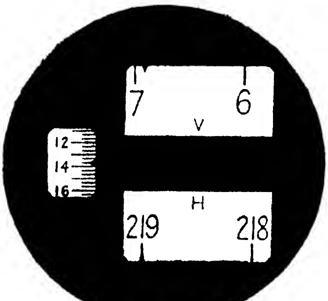
Horizontal Circle Reading: 18° 22′ 30″.



Vertical Circle Reading: 64° 13′ 20″.

FIG. 11.3.—Field of View of Optical Micrometer (E. R. Watts).

reading is obtainable to 20 seconds and it is possible to estimate to 10 seconds, or less.



Horizontal Circle Reading: 219° 13′ 40″.
FIG. 11.4.—Field of View of Optical Micrometer
(Cooke, Troughton & Simms).

Surveying instruments, if made by a firm of repute and handled with reasonable care, have a long life. Consequently there are still a large number of old pattern instruments giving good service and many of these will continue to do so for some time to come. The substitution of newer types, however, will occur gradually and it is important that the main features in their design should be understood, although the older standard patterns cannot yet be considered obsolete. Improvements are being introduced continually and the makers' literature should be studied by those who wish to be fully informed regarding progress in design.

Typical Modern Instruments

Typical modern British instruments are illustrated in figs. 11.5 to 11.8. Fig. 11.5 shows a level of recent design manufactured by Messrs. E. R. Watts & Son. It is a compact instrument of the tilting telescope type with internal focussing. The bubble adjustment is viewed, after prismatic reflection, through the eye-piece B, alongside the telescope, the bubble appearing divided and the method of end coincidence being used, as explained on page 57. A graduated circle of silvered glass enables horizontal angles to be measured and set out—a useful attachment for cross-sectioning and contouring. The circle reading is seen through the eye-piece E, a direct reading to the nearest 10 minutes being obtainable with a possibility of estimating to the nearest 2 minutes. All the observations may thus be made without the surveyor moving from his position at the back of the instrument.

Fig. 11.6 illustrates a "Microptic" theodolite also made by Messrs. E. R. Watts. In this instrument the readings of both the horizontal and vertical circles are transmitted by reflection to the same micrometer eyepiece, this being visible in the photograph at the side of the right-hand telescope support. The field of view in this eyepiece is somewhat similar to that shown in fig. 11.3. By rotating the milled-edged wheel seen immediately beneath the micrometer eyepiece the images of the degree graduations on both horizontal and vertical circles are displaced laterally and the method of reading consists in bringing the nearest degree graduation on either circle, as may be required, exactly midway between the parallel cross wires by adjusting this wheel. The extent of the displacement is registered by a single cross wire on a small scale reading direct to 20 seconds with easy estimation to 5 seconds, and the reading of this scale is added to the number of degrees.

The focussing of the telescope is carried out by rotating a knurled ring which encircles the telescope barrel near the eye-piece end and effects the necessary movement of an internal lens.

Fig. 11.7 illustrates an "Optical Scale" theodolite made by Messrs. Cooke, Troughton & Simms. In this instrument the images of both the horizontal and vertical circle graduations are formed simultaneously at a common focal plane on which is superimposed a fine reading scale. The readings are viewed through a single eye-piece seen in the photograph on the left-hand side of the instrument and the field of view is shown in fig. 11.1. The milled-edged wheel visible in fig. 11.7 at the right-hand end of the trunnion axis transmits motion to an internal lens for focussing the telescope.

Fig. 11.8 illustrates a Cooke, Troughton & Simms "Optical Micrometer" theodolite. In this instrument the image of the appropriate degree graduation is displaced by means of a tilting parallel plate to bring it into coincidence with a fixed index. The extent of this movement is a measure of the minutes and seconds which must be added to the number of degrees and it is recorded on a fine scale reading to the nearest 20 seconds. The degrees on both horizontal and vertical circles and the fine scale are viewed through one eye-piece located immediately to the left of the telescope eye-piece and a typical field of view is shown in fig. 11.4.

The diameter of the horizontal glass circles on most of these modern instruments is usually only 3 inches, or thereabouts, and that of the vertical circles rather less, in contrast to the customary 4 or 5 inches on an ordinary vernier instrument or 6 to 8 inches on a micrometer instrument of the older type.

Stadia diaphragms are invariably fitted, giving a multiplier of 100 with a negligible additive constant and an accuracy of 1 in 300 can normally be expected in the determination of distances tacheometrically. Many of the newer types of theodolites are adaptable to optical plumbing.

Equipment for the "Three Tripod" System for Traversing etc.

The "Three Tripod" system of traversing and triangulation has been introduced with a view to greatly reducing, or practically eliminating, centring errors. As the name implies, three tripods are used. These are identical and are fitted with centring and levelling heads in which the lateral adjustment is situated above the levelling screws so that centring adjustments do not seriously disturb the level. These tripod heads form the supports for a theodolite body minus the usual three-screw base, for a pair of distinctive targets on which a fine setting may be made with the vertical cross wire of the theodolite, and for an optical plumbing attachment, all three being mutually interchangeable and attached to the tripod heads by a bayonet joint. The centring heads are first plumbed over fine reference marks, the targets are then mounted at the back and forward stations and the theodolite at the intermediate. The interchangeable units are then moved as required as the survey proceeds.

The system has obvious advantages for any work where a high degree of accuracy is needed, such as surveys or setting-out at bridge sites, or town surveys for large-scale plans.

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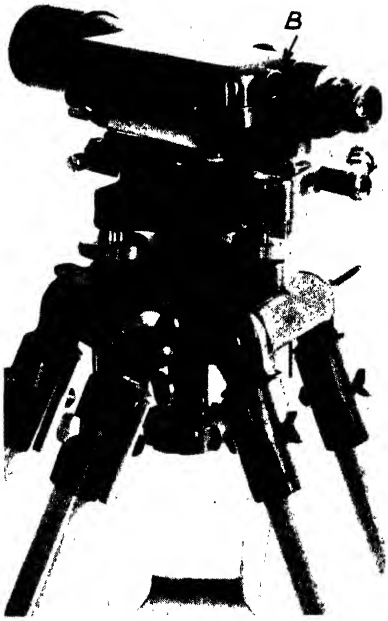

Fig. 11.5. A Modern Level (E. R. Watts & Son).

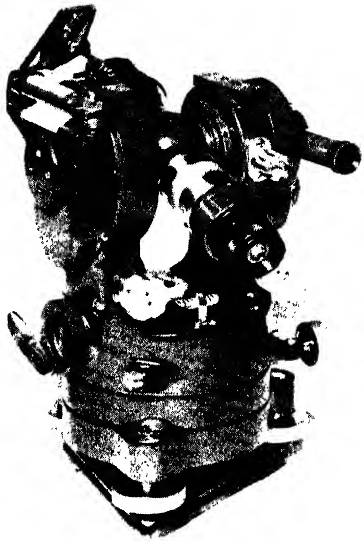

Fig. 11.6.—A "Microptic" Theodolite (E. R. Watts & Son).

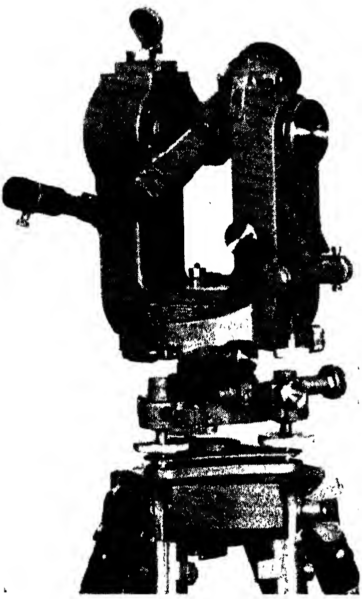

Fig. 11.7.—An "Optical Scale" Theodolite (Cooke, Troughton & Simms).

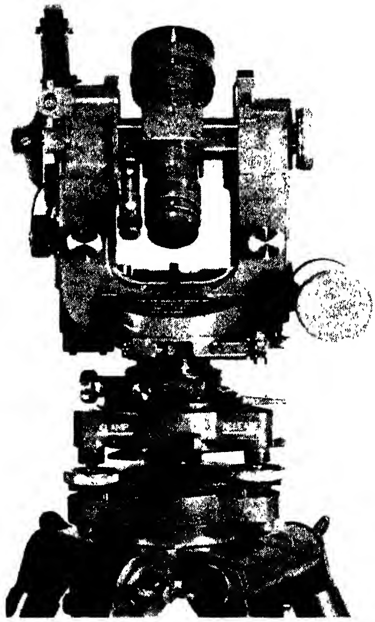

Fig. 11.8.—An "Optical Micrometer" Theodolite (Cooke, Troughton & Simms).

TYPICAL MODERN INSTRUMENTS

APPENDIX I

DERIVATION OF CO-ORDINATES FOR ANY POINT ON A SPIRAL

Consider a small element of the spiral, pq in fig. A1.1, of length dl, the spiral angle up to this element being θ. Draw qr parallel to and pr perpen-

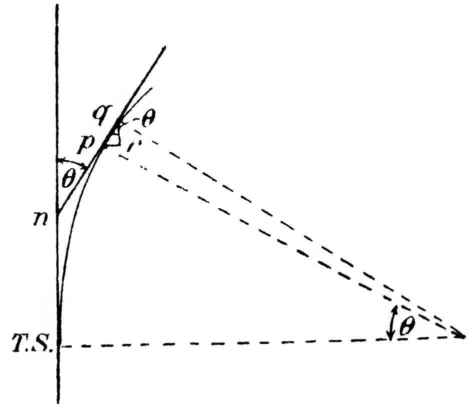

FIG. A1.1.—Derivation of Co-ordanates of a point on a Spiral.

dicular to the tangent at $T.S.$ Then $qr = dx$ and $pr = dy$. In the limit the small element may be considered to coincide with the tangent nq, i.e. $\angle pqr = \theta$. Then $dx = dl.\cos\theta$, and $\theta = \dfrac{l^2}{2R_c l_s}$ or $l = \theta^{\frac{1}{2}}.(2R_c l_s)^{\frac{1}{2}}$

$$\therefore \frac{d\theta}{dl} = \frac{2l}{2R_c l_s} \text{ or } dl = \frac{d\theta . 2R_c l_s}{2l}.$$

$$x = \int dl \cos\theta = \int \cos\theta . \frac{d\theta . 2R_c l_s}{2l}$$

$$= \frac{2R_c l_s}{2}\int\left(1 - \frac{\theta_2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \ldots\right)\frac{d\theta}{\theta^{\frac{1}{2}}(2R_c l_s)^{\frac{1}{2}}}$$

$$= \frac{2R_c l_s}{2(2R_c l_s)^{\frac{1}{2}}}\int\left(\theta^{-\frac{1}{2}} - \frac{\theta^{\frac{3}{2}}}{2!} + \frac{\theta^{\frac{7}{2}}}{4!} - \frac{\theta^{\frac{11}{2}}}{6!} + \ldots\right)d\theta.$$

$$= \frac{(2R_c l_s)^{\frac{1}{2}}}{2}\left[\frac{\theta^{\frac{1}{2}}}{\frac{1}{2}} - \frac{\theta^{\frac{5}{2}}}{\frac{5}{2}.2!} + \frac{\theta^{\frac{9}{2}}}{\frac{9}{2}.4!} - \frac{\theta^{\frac{13}{2}}}{\frac{13}{2}.6!} + \ldots\right]$$

$$= \frac{(2R_c l_s)^{\frac{1}{2}}\theta^{\frac{1}{2}}}{2}\left[2 - \frac{2\theta^2}{5.2.1} + \frac{2\theta^4}{9.4.3.2.1} - \frac{2\theta^6}{13.6.5.4.3.2.1} + \ldots\right]$$

$$= l\left[1 - \frac{\theta^2}{10} + \frac{\theta^4}{216} - \frac{\theta^6}{9360} + \ldots\right]$$

Similarly, $dy = dl \,.\, \sin\theta$

$$y = \int dl \,.\, \sin\theta = \int \sin\theta \,.\, \frac{d\theta \,.\, 2R_c l_s}{2l}$$

$$= \frac{2R_c l_s}{2} \int \left(\theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \ldots\right) \frac{d\theta}{\theta^{\frac{1}{2}}(R_c l_s)^{\frac{1}{2}}}$$

$$= \frac{2R_c l_s}{2(2R_c l_s)^{\frac{1}{2}}} \int \left(\theta^{\frac{1}{2}} - \frac{\theta^{\frac{5}{2}}}{3!} + \frac{\theta^{\frac{9}{2}}}{5!} - \frac{\theta^{\frac{13}{2}}}{7!} + \ldots\right) d\theta$$

$$= \frac{2R_c l_s}{2(2R_c l_s)^{\frac{1}{2}}} \left[\frac{\theta^{\frac{3}{2}}}{\frac{3}{2}} - \frac{\theta^{\frac{7}{2}}}{\frac{7}{2}.3.2.1} + \frac{\theta^{\frac{11}{2}}}{\frac{11}{2}.5.4.3.2.1} - \frac{\theta^{\frac{15}{2}}}{\frac{15}{2}.7.6.5.4.3.2.1} + \ldots\right]$$

$$= \frac{(2R_c l_s)^{\frac{1}{2}}\theta^{\frac{1}{2}}}{2} \left[\frac{2\theta}{3} - \frac{2.\theta^3}{42} + \frac{2\theta^5}{1320} - \frac{2\theta^7}{75600} + \ldots\right]$$

$$= l\left[\frac{\theta}{3} - \frac{\theta^3}{42} + \frac{\theta^5}{1320} - \frac{\theta^7}{75600} + \ldots\right]$$

APPENDIX II

ORDNANCE MAPS

British highway engineers and surveyors are constantly using Ordnance maps, the most useful varieties being those to the scales of 25·344 inches and 6 inches to the mile. These are conveniently referred to as "25-inch" and "6-inch" maps, respectively. Maps to the scale of 1 inch to the mile are used to a lesser extent and in the case of certain large cities and towns Ordnance plans are available to the scale of 1/1250, i.e. double the scale of the 25-inch maps which is expressed by the representative fraction 1/2500.

6-inch and 25-inch Maps. As these maps are so widely used it may be useful to give brief particulars of the numbering system adopted for their

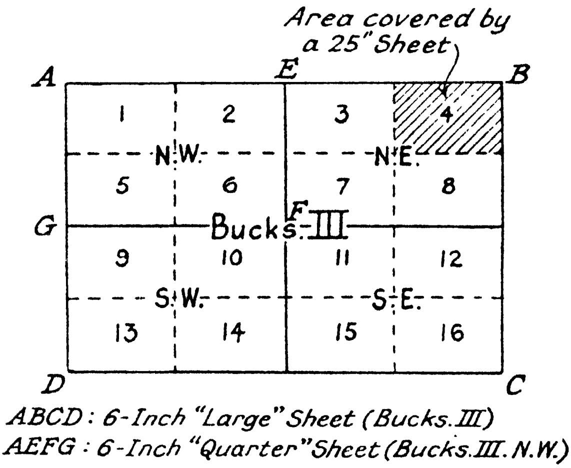

Fig. A2.1.—Numbering System for 6-inch and 25-inch Ordnance Maps.

identification. Each county, or, in some cases, a group of counties, is covered by the so-called "large" 6-inch sheets, each of which is identified by the county name and a Roman numeral. These sheets measure 36 inches by 24 inches and therefore cover 24 square miles of country. Each "large" 6-inch sheet is divided into four "quarter" sheets, distinguished by the appropriate cardinal points, N.W., N.E., S.E. or S.W.

The extent of country covered by a 25-inch map is one-quarter of that shown on a 6-inch "quarter"-sheet sheet, i.e. $1\frac{1}{2}$ square miles. There are thus sixteen 25-inch sheets covering the same area as that shown on a 6-inch "large" sheet and the sub-divisions of the latter corresponding to the 25-inch sheets are numbered from 1 to 16 in Arabic numerals as shown in fig. A2.1. A 25-inch map is consequently identified by (1) the county name and Roman numeral of the large 6-inch sheet in which its area is included, and (2) the Arabic numeral indicating the particular sub-division of the 6-inch sheet. The 25-inch map covering the shaded area in fig. A2.1 would thus be numbered " Bucks. III, 4 ".

Each 1/1250 town plan covers an area formed by dividing a 25-inch map into quarters and is identified by adding the appropriate cardinal points, N.W., N.E., S.E. or S.W., to the number of the 25-inch sheet, thus indicating the particular quarter.

The National Grid. A new system of co-ordinates, known as the " National Grid ", has been introduced for Ordnance maps of all scales and a new series of 25-inch maps (1/2500 scale) is to be published using the new system as a basis. A further series of maps to the scale of 1/25,000, or 2·534 inches to a mile, is also to be published on the same basic system. The origin of the co-ordinate axes is taken slightly to the south-west of

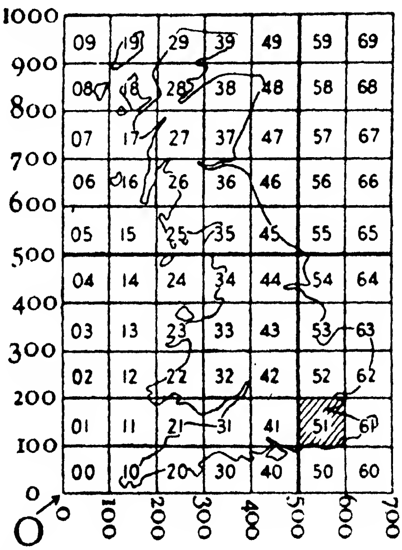

FIG. A2.2.—The National Grid. Numbering System for 100-km. Squares. (Reproduced by permission of the Director General, Ordnance Surveys.)

Land's End, from which any point in Great Britain can be located by an easting and a northing. The primary lines of the grid are spaced at intervals of 100 kilometres in each direction and the squares thus formed are numbered in the manner shown in fig. A2.2, each number being the co-ordinates of the south-west corner of the appropriate square, expressed in hundreds of kilometres with the first digit as the easting and the second the northing. To express the co-ordinates of a point to the nearest metre would generally involve the use of two letters and twelve numerals. Thus, a point within the London area, situated in the shaded square in fig A2.2, might be defined as E 538932, N 177061. This is too complicated for general use and the following modifications are introduced to simplify the reference :

(1) It is the accepted convention that the easting shall be given first and the northing second. This dispenses with the letters E and N.

(2) If, as frequently happens, the point need not be located more closely than the nearest 100 metres, the last two figures in both easting and northing may be deleted.

(3) If the number of the 100-kilometre square is known, the first number in both easting and northing may be omitted.

Hence the shortened reference to the point in the example becomes "389770" which is known as the "Normal National Grid Reference". If the number of the 100-kilometre square is included the reference becomes 51/389770, which is known as the "Full National Grid Reference". If it is sufficient to locate a point to the nearest kilometre, we can use a "Four-figure Reference", e.g. "3877", or a "Full Four Figure Reference", including the number of the 100-kilometre square, e.g. 51/3877.

National Grid References on 1-inch Maps. On 1-inch maps the grid lines are spaced and numbered at kilometre intervals. Consequently, the large numerals on the margin of these maps give "Four-Figure" co-ordinates. It is important to note that each kilometre square on the 1-inch map will cover the same area as one of the new series of 25-inch maps. Each of these 25-inch maps will therefore be identified by the "Full Four-Figure Reference" of its south-west corner. Every 1-inch map based on the National Grid will serve as an index to the new 25-inch maps covering the same area.

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