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List of Abbreviations

RDG(s):	rectangularly dualizable graph(s)
MRDG(s):	maximal rectangularly dualizable graph(s)
PTG(s):	plane triangulated graph(s)
EPTG(s):	extended plane triangulated graph(s)
CIP(s):	corner implying path
BNG:	block neighborhood graph
T-structure:	turnable structure
4-cycle:	cycle of length 4

List of Symbols

G:	RDG
R:	rectangular dual
v_i:	vertex of a graph
V:	vertex set of a graph
E:	edge set of a graph
R_i:	component rectangle
E(G):	extended rectangular dual of G
C:	cycle of a graph
W_n:	wheel graph
M:	MRDG
$N(v_i)$:	neighborhood of v_i
P:	path
$d(v_k)$:	degree of v_k

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