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principles of heat engineming

## Principles of

# HEAT ENGINEERING 

NEIL P. BAILEY<br>GLNLRAI. EI ECTIRIC (OMPANY<br>SCHENC(TADY, NI W YORK

Copirigitt, 1942
BY
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## PREFACE

Every engineer should be familiar with the accumulated experimental knowledge of heat and its uses. The principles which may be considered the minimum requirement for any engineer are rovered in this book. The information is presented in a framework of theory and is expressed quantitatively so that it may be useful for the solution of engineering problems.

The junior enginecring student who plans no additional formal study of heat power and the student who will take more advanced work in the senior year will not necessarily find their requirements vitally different. Both need a firm granp of principles and a basic knowledge of the most important applications.

Constant use has been made of the kinetic theory of gases and other qualitative concepts that aid in making the subject a logically connected science. Such explanations can have no quantitative applications, but this in no way detracts from their value for the purposes of thin book.

Every effort has been made to include up-to-date and reliable tables of the physical constants needed for the solution of elementary heat power problems and to illustrate the important applications with sample solutions. The limitation of the descriptive material to the functional aspects of heat power equipment reflects the conviction that the purely descriptive portion of a course is best presented by the instructor through the medium of the laboratory and current literature.

During the seven-year period in which this book developed I have been grateful for the suggestions and criticisms of Stephen Bencze, Colin Carmichael, William Schulte, Robert Burns, Edward Gamarekian, and Harold Juram. I also arknowledge my indebtedness to Dr. R. C. H. Heck for the gas tables published by him and for many stimulating discussions. I salute the patience of the engineering students of Rutgers University upon whom many ideas were tested.

Neil P. Bailey
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## SYMBOLS AND UNITS

A Area in square fect.
$a$ Area, usually square feet.
c Specific heat in Btu per pound per degree Fahrenheit.
$C$ Specific heat in Btu per mol per degree Fahrenhent.
$e$ Radiation emissivity, also efficiency.
$g$ Acceleration due to gravity, 322 ft . per sec. per sec.
$G$ Mass flow in pounds per second per square foot.
$h$ Enthalpy in Btu per pound.
$J$ Mechanical equivalent of heat, $77826 \mathrm{ft}-\mathrm{ll}$ ) per Btu.
$k$ Thermal conductivity in Btu per hour per square foot per degree Fahrenheit per foot.
$m$ Mass in slug.. (An olject that weighs 32.2 lb has a mass of 1 slug.)
$n$ Polytropic exponent.
$P$ Pressure in pounds per square foot.
$O$ Quantity of heat in Btu.
$q$ Rate of heat transfer in Btu per hour.
$r$ Radius in feet, also compression ratio.
$R$ Gas constant.
$s$ Entropy in Btu per pound per degree Rankme.
$t$ Temperature in degrees Fahrenheit, aloo time.
$T$ Absolute temperature in degrees Rankme (absolute Fahrenheit) unless otherwise noted.
$u$ Internal energy in Btu per pound.
$U$ Conductance in Btu per hour per square foot per degree Fahrenheit.
$v$ Velocity in fect per second.
$V$ Volume in cubic feet.
$W$ Weight in pounds.
$x$ Quality of a vapor mixture.
$\alpha$ (alpha) Angle in degrees.
$\beta$ (beta) Angle in degrees, also refrigeration coefficient of performance.
$\gamma$ (gamma) Adiabatic exponent.
$\boldsymbol{\epsilon}$ (epsilon) Base of natural logarithms, also energy ratio in the kinetic theory of gases.
$\mu$ (mu) Viscosity in slugs per second per foot.
$\rho$ (rho) Mass density in slugs per cubic foot.
$\omega$ (omega) Angular velocity in radians per second.

## CHAPTER 1

## BASIC CONCEPTS

1.1. Energy Concepts. The concept of heat is based on the everyday physical experience that its addition warms an object and its loss renults in cooling. After heat has been added to a body it may again be withdrawn by placing the body in contact with a cooler medium. This has led to the concept of a heated body possensing "thermal cnergy." The fact that the combustion of such a fuel as coal or ganoline will give off heat results in the idea that a fuel possenses "potential energy" that may be releaned under proper burning conditions.

Basic arientific concepts depend for their reality upon the physical senses. After all word, have been exhausted the idea of length or distance is associated with the visual difference between "here" and "there," and the concept of time is dependent upon the sense of what happens "now" and what happened "then." A foree is real only in terms of the muscular effort required to exert one, and temperature and heat are realized through the physical sense of "hot" and "cold."

The simplest energy concept is that of the mechanical work performed when a force act.s through a distance and the work done is measured by the product of force and the distance through which it acts. When work is done to lift a weight, experience dictates that this work can be recovered when the weight is again lowered. This has led to the concept that the object possesses energy or a capacity for action when in its elevated position. This energy or capacity for action is equal in magnitude to the work done in elevating it and is referred to as "potential energy."

A force must act on a body through a distance in order to accelcrate it and give it a velocity. A moving object is thought of as possessing "kinetic energy" equal in amount to the work done on it. This idea is borne out by the fact that by properly decelerating it, the object may be caused to perform this amount of work.

To be strictly consistent, the term "heat" should be reserved for the transfer of energy between objects at different temperatures, and never to the capacity of an object to store or give off heat. There is the same distinction between "heat" and "thermal energy" as there is between work and potential or kinetic energy.
1.2. The Principle of Conservation of Energy. The subjects of heat, electricity, chemistry, and mechanics have been developed independently over a long period of time and it has been customary to refer to thermal energy, electrical energy, chemical energy, and mechanical energy just as though they had nothing in common. Since about the year 1700 it has been known that in mechanical systems, kinetic energy of motion and potential energy of position are mutually convertible and quantitatively related by the amount of mechanical work represented.
Later, the heat absorbed and given off during chemical reactions was observed and the relationship between electrical and chemical energy was discovered. In the field of electricity the conversion of electrical energy into heat was measured and by 1850 quantitative measurements had been made on the conversion between mechanical and electrical energy. About this time the possibility of conversion between mechanical energy and thermal energy was appreciated, and the mechanical equivalent of heat was evaluated and stated in the form of the first law of thermodynamics.
Encouraged by these possibilities of energy conversion but still believing the different forms of energy to be unrelated phenomena, the principle of conservation of energy was stated. In its most general form this is but a statement of faith in the belief that in any system energy that disappears in one form must reappear in some other form. As the structure of matter in its different states is better understood and the nature of thermal, chemical, and electrical energy is appreciated, there is encouragement for the belief that only the concepts of kinetic and potential energy as defined in terms of mechanical work will be needed.

1-3. Energy Units. The common engineering unit of work is the foot-pound, which is the amount of work done in overcoming the resistance of 1 lb through a distance of 1 ft . The original definition of the British thermal unit ( Btu ) as the heat required to raise the temperature of one pound of water through one degree Fahrenheit was based on the idea of pure water being a standard substance of unit heat capacity. When the variation of the specific heat of water was recognized it became necessary to specify a standard degree as from 59 to 60 deg F and later the mean specific heat between 32 and 212 deg F came into general use. Back of the use of a water standard to define a Btu lay the experimental method of water calorimetry which then prevailed and yet persists in practical heat work.

Joule made the first accurate determination of the mechanical equivalent of heat and established its value as approximately 778 ft -lb to the Btu. The use of the water standard had the disadvantage that each supposed more accurate determination of the mechanical equivalent of
heat, usually designated by the symbol $J$, had the effect of changing the size of the heat unit. This, combined with the increasing use of electrical energy in precise calorimetry has caused the joule or watt-second to be adopted as the primary energy unit, and this in turn is defined in the mechanical system.

The International Steam Table Conference of 1929 adopted a system which defines the Btu as an energy unit 778.26 times as large as a footpound. The result is that instead of being a heat unit, the Btu is now actually an energy unit equal in size to $778.26 \mathrm{ft}-\mathrm{lb}$. A time may come when a single energy unit will be in common use which will eliminate the use of conversion factors, but at present it is necessary to develop skill in shifting from any one unit system to another.

The energy units in common use are the foot-pound, the British thermal unit and the kilowatt-hour. The corresponding units of power or rate of energy delivery are the foot-pound per second, the Btu per hour, and the kilowatt. The horsepower ( hp ) is a power unit defined as $550 \mathrm{ft}-\mathrm{lb}$ per sec and 1 hp is equivalent to 0.746 kw . This means that 1 hp is equal to $(550 \times 3600) / 778.26=2545 \mathrm{Btu}$ per hr and 1 kw is equal to $2545 / 0.746=3415$ Btu per hr. When converting from one set of units to another it helps to remember that a Btu is a much larger energy unit than a foot-pound and that a kilowatt is a larger power unit than a horsepower.
1.4. Matter and Energy. Although the experimentally determined thermal properties of matter are of primary importance in heat engineering, it is very helpful to associate each thermal operation with an explanation of what is taking place even when the present limitations of knowledge leave such a picture somewhat incomplete.

The capacity of physical material to store thermal energy is explained by the kinetic theory of matter, which assumes an atomic and molecular structure of matter and recognizes only two forms of energy, kinetic and potential. The concept of atomic and molecular structure comes from the realm of chemistry where it has been demonstrated that matter is composed of molecules and that all molecules of the same chemical substance are exactly similar.
These molecules and atoms possess kinetic energy by virtue of motion and potential energy by virtue of atomic and molecular forces of mutual attraction. Chemical energy is potential energy that is stored in the molecules by interatomic forces, and it is converted into kinetic energy only when chemical changes alter the atomic spacing and arrangement and these attractive forces act through a distance.

Mutual forces of attraction between molecules are large in solids and liquids where the molecular spacing is small. During the vaporization
of a liquid the molecular spacing is greatly increased and potential energy is stored as the rapidly diminishing attractive forces are overcome. In the vapor state the molecular forces are still significant, but in the gaseous form the molecular spacing is so great that attractive forces are negligible.

The type of molecular motion depends upon the physical state of matter. It is restricted in the solid phase, freer in the liquid state, and very free in the gaseous phase. It is in this gaseous state that thermal energy can be explained most simply in terms of molecular motion.
1.5. Solids. Modern research into the structure of solids has made it clear that in this state the mutual forees between molecules hold them in equilibrium in definite arrangements that are more highly organized in crystalline than in amorphous solids. Any attempt to compress the material and thus reduce molecular spacing is opposed by repulsive forces between molecules, and any attempt at dilation is opposed by attractive forees which come into action as the molecular separation tends to become larger than that for equilibrium. Density is dependent upon molecular spacing for equilibrium and strength upon the magnitude of the forces involved.

Thermal energy present in a solid is imagined as kinctic energy of vibration of atoms and molecules, each about its mean position of equilibrium. When heat is added this vibration increases in intensity and the temperature of the material is said to rise. Also, when mechanical work is expended upon a solid, as when one surface slides over another or when a wire is repeatedly bent, the resulting increase in vibrational energy is sufficient to account for the mechanical work done. Such vibration also functions as a readily imagined mechanism for the propagation or conduction of heat through solids when different levels of vibrational intensity or temperature exist.
1.6. Liquids. A solid heated to a certain temperature which is only slightly affected by imposed pressure and is widely different for various materials begins to liquefy, melt, or fuse. At this melting or fusion temperature a large amount of energy, known as the latent heat of fusion, must be supplied to break up the fixed structure of the molecules of the solid state. Solids that are strongly crystalline, such as ice and copper, have sharply defined melting temperatures, but amorphous solids, such as some earthy slags, may run through several hundred degrees between first softening and free fluidity.

In a liquid the mean molecular spacing is essentially the same as in a solid, but a fixed molecular arrangement no longer exists, and the molecules are visualized as moving among each other with very short free paths between collisions. This random arrangement results in balanced
attractive forces on any molecule by those surrounding it and no resultant forces exist to oppose a change in shape except at the surface where unbalanced forces exist in the top molecular layers, resulting in surface tension phenomena. Repulsive forces strongly resist compression and thermal energy is visualized as being stored as kinetic energy of translation and vibration.
1.7. Vaporization. All molecules of a liquid do not possess the same kinetic energy. At any instant there are a few molecules that by a series of favorable impacts have a very high energy level and others that have low velocities because of a series of unfavorable collisions. The majority of the molecular velocities are grouped about a mean or most probable value. This type of a "normal distribution" seems characteristic of all random occurrences in nature.

It is estimated that one out of each several hundred molecules possesses enough translational energy at any instant to overcome the attractive forces of the surface tension layer and escape into the space above the liquid. When such a molecule reaches the surface or collides with a surface molecule, a molecule is projected from the liquid. The attractive force decreases very rapidly as the molecule leaves the liquid and the kinetic energy of the leaving molecule also decreases as it does work to free itself. If this same molecule should chance to return into the field of force of the surface molecules of the liquid this potential energy would be returned to it as kinetic energy of motion as it is again drawn back into the liquid.

If the space above the liquid is enclosed this molecular exchange will come into equilibrium when as many molecules return to the liquid as leave it, when the energy of molecules leaving is balanced by that of those returning to the liquid and when the entering and leaving momentums are equal. For a given liquid temperature or mean kinetic energy level, the molecular density of the "vapor" molecules will always be the same, and the impacts of these molecules on the walls of the enclosure exert a pressure which is known as the saturation pressure corresponding to that liquid temperature.

For a higher liquid temperature the mean molecular kinetic energy will be higher and a larger fraction of the liquid molecules will have the velocity necessary to leave the liquid surface. The result is an increased density of vapor molecules above the liquid and the mean kinetic energy of these molecules will be higher than for the lower temperature. Since the pressure exerted by the impact of the vapor molecules on the enclosure is increased by both the molecular density and velocity, it would be-expected that the saturation pressure of a given liquid would increase rapidly with increased liquid temperature. This is borne out by the
curve of saturation pressure plotted against liquid temperature shown for water in Figure 1•1.

If the space above a liquid is not enclosed, as when water is exposed to the air, the high-velocity molecules that escape from the liquid surface are carried away by diffusion in the air and do not return to the liquid. The result of this continued loss of only high-speed molecules having kinetic energy far greater than the


Fig. 1.1. Vapor pressure of water. mean energy level of the liquid molecules is a reduction of the average energy level of the remaining molecules. This means that the liquid temperature will be reduced. If this temperature is to be held constant, heat must be supplied to compensate for the energy loss. This amount of heat, expressed as Btu added per pound of liquid vaporized, is known as the "latent heat of vaporization."

This latent heat is made up of two parts. First, and most important, it contains the energy necessary to overcome the attractive forces of the surface layer of the liquid and to free the molecules. In addition, it must also account for the work necessary for 1 lb to expand from the specific volume of the liquid to the volume that it occupies as a vapor. For water at 70 deg F this latent heat of vaporization is 1054.3 Btu per lb of vapor formed. Of this 996.1 Btu per lb represents the work done against molecular attraction and 58.2 Btu per lb is the energy required for the volume to increase from the liquid value of 0.01606 cu ft per lb to 867.9 cu ft per lb working against the saturation pressure of 0.3631 lb per sq in.
In a saturated vapor the molecular spacing is usually small enough that weak attractive forces between molecules are still present and the space occupied by the actual molecules themselves is an appreciable part of the total volume filled by the vapor. However, if no liquid is present and the vapor is heated above its saturation temperature or "superheated" and allowed to expand, the molecular spacing soon becomes great enough to reduce molecular attraction to a negligible value. Such a highly superheated vapor, existing at a temperature far above its saturation value, has much simpler properties than a saturated vapor and is usually referred to as a "gas."

## Sugaested Reading

Thermodynamics Princoples and Definitions
Moyer, Calderwood, and Potter, Elements of Engineering Thermodynamics. Pages 1-10
Historical Outline
Barnard, Ellenwood, and Hirshfeld, Heat Power Engineering Chapter II, Part I.

## CHAPTER 2

## CHARACTERISTICS OF GASES

2.1. Kinetic Theory of Gases. ${ }^{1,2 *}$ The physical properties of gases are determined by experiment on gas molecules in the aggregate, but these collective characteristics of gas molecules can be more readily understood and interpreted by considering the probable action of individual molecules. The kinetic theory of gases, first developed in 1855, explains the physical properties of gases and aids in understanding many thermal processes. It is applicable quantitatively only to simple monatomic gases such as argon and helium because it assumes no intermolecular attraction and ignores any spin or rotation of the molecules. In spite of such simplifying assumptions, however, the conclusions reached are qualitatively applicable to the more complex gases such as oxygen and carbon dioxide.
The basic concept of the kinetic theory is that of numberless identical gas molecules colliding and repelling each other in a completely random


Fig. 2.1. Molecular velocity distribution. manner. The condition of any one molecule varies continuously from one instant to another. It may at one moment have a very high velocity in one direction and at the next have a low velocity in another. There is overwhelming evidence ${ }^{1}$ that at any instant the molecular scalar velocities are distributed as shown in Figure $2 \cdot 1$ which is a distribution curve of the general type that is characteristic of random occurrences in nature.

The average of all the molecular velocities, $v_{\text {av. }}$, is greater than the most probable or most frequently occurring velocity, $v_{p}$, by the ratio $v_{u v} . / v_{p}=1.128$ because the few molecules that have very high velocities have an exaggerated effect on the average. The kinetic energy of any molecule depends on the square of its velocity with the result that the very high-speed ones have a much greater kinetic energy than the

[^0]slower ones. The net result is that the average kinetic energy per molecule is much greater than that of a molecule moving at the average velocity. The effective root-mean-square velocity, $v_{\text {eff }}$, is the one that must be used in calculating the kinetic energy of the molecules and the ratio of effective to most probable velocity is $v_{\text {eff }} / v_{p}=1.224$.
2.2. The Source of Gas Pressure. When a gas that has no flow velocity occupies a space that is all at the same temperature, it is known that the pressure exerted by it is the same in all directions and that the molecular density is constant throughout. This means that the amount of movement of the molecules is equal in the threc coordinate planes for otherwise the pressure would not be equal in all directions and the density would not remain constant throughout. A simple equivalent of this would be to imagine one-third of the molecules moving back and forth in each of the three planes.

When a molecule strikes a wall of the enclosure with a velocity $v$ and rebounds elastically, it would experience a change in velocity of $2 v$ in that direction and undergo a momentum change of $2 m v$, where $m$ is its mass. If this molecule were alone in the enclosure it would have to travel the length $l$ of the enclosure, rebound from the opposite wall and travel back a distance $l$ before it would again strike the original surface. This would require $2 l / v$ seconds, with the result that the average change in momentum per second experienced by one molecule colliding with one wall of the enclosure would be $\frac{2 m v}{2 l / v}=\frac{m v^{2}}{l}$.

Since the change in momentum per second is produced by the wall, the force on it due to one molecule would be $m v^{2} / l$. One third of the molecules or $n / 3$ may be considered as moving only in that plane, so the total force exerted on the surface is $\frac{n}{3} \frac{m v^{2}}{l}$. This total force is also equal to the unit pressure exerted $P$ times the surface area $A$ or

$$
P A=\frac{m n v^{2}}{3 l}
$$

Since $A l$ is gas volume $V$ and $v^{2}$ for all the molecules is $v_{\text {eff }}^{2}$, equation $2 \cdot 1$ becomes

$$
P=\frac{m n}{3 V} v_{\mathrm{eff}}^{2}
$$

This brings out the fact that, according to the kinetic theory, the pressure exerted by a gas depends on the density $m n / V$ and the molecular velocity.

By using values of pressure and density of gases from experimental data, it is possible to evaluate the molecular velocity as given by the kinetic theory and compare it with experimental information. At atmospheric pressure of $14.7 \times 144 \mathrm{lb}$ per sq ft, 1 lb of air, or a mass of $1 / 32.2$ slugs at a temperature of 70 deg F occupies a volume of 13.35 cu ft . The root-mean-square or effective value of the molecular velocity from equation $2 \cdot 2$ is

$$
v_{\mathrm{eff}}=\sqrt{\frac{3 P V}{m n}}=\sqrt{\frac{3 \times 14.7 \times 144 \times 13.35}{1 / 32.2}}=1655 \mathrm{ft} \mathrm{per} \mathrm{sec}
$$

As illustrated by Figure 2•1, the probable or most frequently occurring molecular velocity is $v_{p}=\frac{v_{\text {eff }}}{1.224}=\frac{1655}{1224}=1350 \mathrm{ft}$ per sec. It is significant that the velocity with which a pressure or sound wave travels in air at these same conditions is 1120 ft per sec for such waves are transmitted by molecular motion.

The kinetic theory of gases finds its greatest field of usefulness in furnishing a logical explanation of facts that are determined experimentally and in weaving such information together to make it into a logically connected subject rather than an accumulation of isolated facts.
2.3. Experimental Gas Equations. The volume that is occupied by a constant weight of any gas varies with both pressure and temperature and the experimental approach has been to study the variations independently. The relationship between pressure and volume that results when the temperature remains constant was formulated from experimental information by Boyle in 1660. The curve representing the effect of changes in pressure on the volume of a quantity of gas when the temperature is held constant was identified as the equilateral hyperbola shown in Figure 2-2. The equation that expresses this relation, which is known as Boyle's law, is

$$
P_{1} V_{1}=P_{2} V_{2}=P V=\mathrm{Constant}
$$

After many years of dispute it was finally established by Gay-Lussac in 1802 and by Charles in 1887 that, if the volume of a quantity of gas is held constant, the relationship between pressure and temperature is a straight line as shown in Figure $2 \cdot 3$. If this line is extended until it intersects the temperature axis the average for the permanent gases gives a value of 460 deg F below zero Fahrenheit as an origin.

No actual gas could be cooled to such a low temperature without becoming a solid, but the temperature of minus 460 deg F represents a
theoretical temperature at which molecular activity would be absent if a gas acted at low temperature as it does at higher ones. The remarkable growth in the crystal size in metals and other phenomena that have been observed when this temperature is approached, makes it appear that minus 460 deg F actually has physical significance and may not be just a mathematical concept. Temperature measured from this origin of minus 460 deg F is known as "absolute temperature."


Fig. 2.2. $P-V$ curve for 1 lb of arr at 70 deg F .


Fig. $23 \quad P$ - $T$ rurve for 1 lb of arr in 1 cu ft volume.

The gas properties formulated by Gay-Lussac and Charles may be expressed mathematically for constant volume as

$$
\begin{equation*}
\frac{P}{t+460}=\frac{P}{T}=\mathrm{Constan} \mathrm{t} \tag{2.4}
\end{equation*}
$$

where $t$ equals degrees Fahrenheit and $T$ equals degrees absolute Fahrenheit, which is also known as degrees Rankine or deg R.

A more general gas equation may be developed from the observations of Boyle and Charles by thinking of the change of a quantity of a gas from a pressure $P_{1}$, volume $V_{1}$, and absolute temperature $T_{1}$ to conditions $P_{2}, V_{2}$, and $T_{2}$ as being made in two steps. If first the temperature remains constant at $T_{1}$ and the volume changes to $V_{2}$, the resulting pressure $P^{\prime}$ is given by

$$
P_{1} V_{1}=P^{\prime} V_{2} \quad \text { or } \quad P^{\prime}=\frac{P_{1} V_{1}}{V_{2}}
$$

If the volume is then held constant at $V_{2}$ and the pressure changes from $P^{\prime}$ to $P_{2}$ while the temperature changes from $T_{1}$ to $T_{2}$

$$
\frac{P^{\prime}}{P_{2}}=\frac{T_{1}}{T_{2}} \quad \text { or } \quad P^{\prime}=\frac{P_{2} T_{1}}{T_{2}}
$$

By combining equations $2 \cdot 5$ and $2 \cdot 6$,

$$
\begin{equation*}
\frac{P_{1} V_{1}}{V_{2}}=\frac{T_{1} P_{2}}{T_{2}} \text { or } \frac{P_{1} V_{1}}{T_{1}}=\frac{P_{2} V_{2}}{T_{2}}=\text { Constant }=\frac{P V}{T} \tag{2.7}
\end{equation*}
$$

which is a very convenient relationship that follows from the conclusions of Boyle and Charles.
$\mathbf{2 - 4}$. The Significance of Temperature. When two gases unite chemically, the volumes entering into the reaction are in a simple relation such as 1 to 2 and 2 to 4 . Since the theory of chemical reactions assumes that an integral number of molecules of each substance unite to form the new molecule, Avagadro, in the year 1811, proposed the hypothesis that at the same temperature and pressure equal volumes of all gases contain the same number of molecules. The entire structure of molecular theory that has since been crected on this hypothesis has so well fitted experimental observations that his hypothesis is now looked upon as a scientific principle. From this other conclusions become inevitable. By re-arranging equation 2.2 and by dividing both sides by the absolute temperature $T$, it may be combined with equation $2 \cdot 7$ to give

$$
\frac{P V}{T}=\frac{1}{3} m n \frac{v_{\mathrm{eff}}^{2}}{T}=\mathrm{Constant}
$$

The experimentally determined gas equations that are summarized by equation $2 \cdot 7$ indicate that for a given weight of gas, $P V / T$ is a constant. If this conclusion and Avagadro's hypothesis that the number $n$ of molecules in a given volume of gas at the same temperature and pressure is constant, are applied in equation $2 \cdot 8$, it must be concluded that $m v_{\text {eff }}^{2} / T$ is a constant.

This means that the absolute temperature $T$ of a gas is proportional to the translational kinetic energy per molecule. For a light gas with a small mass $m$ per molecule, the translational velocity $v$ must be greater than it is for a heavy gas with a large mass $m$ per molecule. As indicated by Figure 2•1, this docs not mean that the kinetic energy of translation of each individual molecule of a gas is the same at all times. It is a statistical conclusion relating the mean kinetic energy of translation of all the molecules. Also, it should not be extended to refer to the total kinetic energy per molecule. Any molecule possesses kinetic energy of translation and at the same time kinetic energy of molecular spin and atomic vibration. This spin and vibrational energy varies from a negligible amount in monatomic gases such as helium and argon to a significant quantity in such complicated molecules as carbon dioxide.
2.5. The General Gas Equation. The obvious impracticability of actually counting the number of molecules in a volume of gas or of weighing single molecules early led chemists to adopt the concept of molecular weights. By observing the relative weights of the elements that entered into chemical reactions, a relative scale of molecular weights based on an oxygen molecular weight of 32 or atomic weight of 16 was erected. Oxygen was chosen because it reacts directly with so many different substances. It may thus be said that the number of molecules $n$ of a quantity of gas is proportional to the weight $W$ divided by the molecular weight. This, combined with the conclusion that $m v_{\text {eff }}^{2}$ is proportional to the absolute temperature $T$ makes it possible to write equation $2 \cdot 2$ as

$$
P V=\text { Constant } \times \frac{W}{\text { mol. wt. }} \times T
$$

The best average value of the constant in equation 2.9 has been experimentally established as 1544 when the pressure $P$ is measured in pounds per square foot absolute, the volume $V$ in cubic feet, and the weight of gas $W$ is in pounds with the temperature $T$ measured in degrees absolute Fahrenheit or degrees Rankine. The ratio $1544 / \mathrm{mol}$. wt. is usually known as the gas constant $R$ and the general gas equation is customarily written

$$
P V=W R T
$$

This equation, relating pressure, volume, temperature, and weight of gas, applies accurately to any gas under ordinary conditions.

Since 1544 is only a mean value for all gases, more exact values of $R$ are given for common gases in Table 2•1.

## Example

The volume in cubic feet of 2 lb of oxygen at 80 deg F and 50 lb per sq in . gage pressure may be calculated as

$$
V=\frac{W R T}{P}=\frac{2 \times \frac{1544}{32} \times(460+80)}{(50+14.7) \times 144}=5.78 \mathrm{cu} \mathrm{ft}
$$

It is very important to remember to change temperatures to absolute values and to change values of pressure to absolute pressure in pounds per square foot. In practice, pressures are usually stated as pounds per square inch gage and it is first necessary to change this to an absolute pressure by adding to it the atmospheric pressure in pounds per square inch, and then to convert this absolute pressure to pounds per square foot by multiplying it by 144 . This is the most common set of mistakes made in handling the general gas equation.

## TABLE $2 \cdot 1$

Gas Constants

| Gas | Molec- <br> ular <br> Symbol | Atoms <br> per <br> Molecule | Exact <br> Molec- <br> ular <br> Weight |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Exact $R$ | Approxi- <br> mate <br> Molec- <br> ular <br> Weight | Approxi- <br> mate <br> $R$ <br> 1544 |  |  |  |  |
| Helium | He | 1 | 4.00 | 386.00 | 4.0 | 386.00 |
| Argon | Ar | 1 | 39.9 | 38.70 | 40.0 | 38.6 |
| Hydrogen | $\mathrm{H}_{2}$ | 2 | 2.016 | 76586 | 2.0 | 772.0 |
| Oxygen | $\mathrm{O}_{2}$ | 2 | 32.0 | 48.25 | 32.0 | 48.3 |
| Nitrogen | $\mathrm{N}_{2}$ | 2 | 2808 | 54.99 | 28.0 | 55.2 |
| Carbon monoxide | $\mathrm{CO}_{\mathrm{O}}$ | 2 | 28.0 | 55.14 | 28.0 | 55.2 |
| Carbon dioxide | $\mathrm{CO}_{2}$ | 3 | 44.0 | 35.09 | 44.0 | 35.1 |
| Methane | $\mathrm{CH}_{4}$ | 5 | 1603 | 96.31 | 16.0 | 96.5 |
| Ethylene | $\mathrm{C}_{2} \mathrm{H}_{4}$ | 6 | 28.03 | 55.08 | 28.0 | 55.2 |
| Water vapor | $\mathrm{H}_{2} \mathrm{O}$ | 3 | 18016 | 85.60 | 18.0 | 85.7 |
| Air | A | A | equiv. |  | equiv. |  |
|  | mixture | mixture | 28.95 | 53.4 | 29.0 | 53.3 |

2.6. The Mol of Gas. (Pronounced "mole.") Avagadro's hypothesis that under the same condifions of pressure and temperature the number of molecules in a given volume is the same for all gases has resulted in the use of a unit of gas measurement known as a mol. Since the molecular weight of a gas is a number that is proportional to the weight of the gas molecules, it follows that there are the same number of molecules in 32 lb of oxygen as in 28 lb of nitrogen or 44 lb of carbon dioxide. Since the same number of molecules occupy a volume that is the same for all gases, the volume occupied by 32 lb of oxygen is the same as that for 28 lb of nitrogen and 44 lb of carbon dioxide, provided temperatures and pressures are the same.

A mol of gas is, then, a number of pounds of the gas equal to its molecular weight, and the advantage of the mol concept is that it is a weight of the gas that has the same volume for all gases. This mol volume may be found at any temperature and pressure from the general gas equation $2 \cdot 9$.

$$
P V=1544 \times \frac{W}{\text { mol. wt. }} \times T
$$

Since $W$ for a mol is equal to the molecular weight, the volume $V$ of a mol becomes

$$
V=\frac{1544 T}{n}
$$

The volume of any gas at atmospheric pressure of 14.7 lb per sq in. absolute and 68 deg F is

$$
V=\frac{1544(460+68)}{14.7 \times 144}=385 \mathrm{cu} \mathrm{ft}
$$

This mol volume varies with both pressure and temperature, but at any given conditions, equation $2 \cdot 12$ indicates that it is independent of the kind of gas. This mol concept is very useful when working with gas mixtures and also in combustion problems.

For example, if it is known that air is made up of approximately $0.23 \mathrm{O}_{2}$ and $077 \mathrm{~N}_{2}$ by weight, it would follow that in 100 lb of air there are $\frac{23}{32}=0.72 \mathrm{mols}$ of $\mathrm{O}_{2}$ and $\frac{77}{28}=2.75 \mathrm{mols}$ of $\mathrm{N}_{2}$. Since the mol is also a volume unit, this would mean that air is approximately $\frac{0.72}{0.72+2.75} \approx 0.21 \mathrm{O}_{2}$ and $\frac{2.75}{0.72+2.75} \approx 0.79 \mathrm{~N}_{2}$ by volume.

## REFERENCES

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Moyer, Calderwood, and Potter, Elements of Engineering Thermodynamacs Chapter II.

## CHAPTER 3

## THERMAL ENERGY

3.1. Forms of Energy. The principle of conservation of energy may be stated in many general ways, but before it can be a useful tool for solving thermal problems, it must be expressed quantitatively in terms of the properties of gases and vapors. The most general case that is commonly of interest is that of a gas or vapor that is confined under pressure as it flows along with varying velocity, pressure, volume, and temperature as heat is added to it or conducted away.

Heat may be added by such things as the burning of a fuel-air mixture in an internal-combustion engine or the transfer of heat from the hot furnace gases to the steam in the tubes of a steam generator. Heat may be lost in many ways such as the heat conducted from the hot gases of an engine cylinder to the cooler jacket water.

The internal energy of a gas or vapor is altered when the temperature changes with the accompanying change in the kinctic energy of the molecules. It may also vary when a change in volume alters the potential energy of the molecules by changing the molecular spacing in the presence of the mutual attractive forces between molecules.

Since gases and vapors are compressible, any change in volume results in expansion or compression work being done as it is when the gases in an internal-combustion engine undergo an increase in pressure and an accompanying decrease in volume.

Flow work is involved only when a volume of fluid is moved from a region at one pressure to one at a different pressure. Flow work may be present with both incompressible and compressible fluids, whereas the previously mentioned compression work is associated only with gases and vapors which undergo an appreciable volume change when the pressure is changed.

When the flow velocity of a gas or vapor is changed, the resulting change in kinetic energy of flow must be considered as it must when steam passes through the nozzles of a turbine or when air passes through the impeller of a centrifugal blower. Acceleration work is involved whenever there is a change in the velocity of mass flow, and this should not be confused with the random velocities of the individual molecules.

It is useful to state the principle of conservation of energy in the form of an equation which is often referred to as the general flow equation or as the general energy equation.

Heat added $=$ Increase in internal energy + Expansion work

$$
+ \text { Flow work }+ \text { Accelcration work }
$$

This equation is simply a statement of the known fart that if heat is added to a quantity of gas or vapor it may increase the internal energy, do expansion work or flow work, cause an increase in the kinetic energy of mass flow, or any combination of these. These changes have been stated in the positive sense but each change can occur in the reverse sense, which would cause it to be negative in equation $3 \cdot 1$. However, before quantitative problems can be solved, each of these energy components must be capable of numerical evaluation.
3.2. Internal Energy Changes. Changes in the internal energy of vapors are made up of three components, molecular kinetic energy of translation which is proportional to the absolute temperature 7 ', kinetic energy of molecular spin and vibration which varies with both molecular structure and temperature, and potential energy resulting from mutual attractive forces between molecules and depending on molecular spacing. This means that the total internal energy of a vapor such as steam or ammonia near saturation depends upon both temperature and pressure, and for such a situation it is not practical to attempt to use a mathematical equation to express it. The result is that the internal energy $u$ is listed in "vapor tables" for different conditions of temperature and pressure.

In highly superheated vapors or gases the molecular spacing is very great and forces of mutual attraction are negligible. Consequently, changes in molecular density of a gas do not affect the internal potential energy appreciably, with the result that in the gascous region, changes in internal energy depend only on temperature variations. This is a conclusion that was reached experimentally by Joule.

The experimental method of evaluating changes in the internal energy of a gas is to determine the amount of heat that must be added to produce a given temperature rise when the gas is heated at constant volume. For such a condition all the heat added goes to change the internal energy, since the expansion work, flow work, and acceleration work changes are zero. Any change in internal energy for a gas can be expressed as

$$
d u=c_{v} d T
$$

where $d u=$ change in internal energy in Btu per pound of gas.
$d T=$ temperature change in degrees Rankine.
$c_{v}=$ specific heat at constant volume in Btu per pound per degree Rankine. Values are given in Table $3 \cdot 1$.

The fact that equation $3 \cdot 2$ expresses the internal energy change in terms of the specific heat at constant volume $c_{v}$ should not be interpreted as meaning that it can be used only when a constant volume change is involved. It gives the change in internal energy of a gas for any kind of an operation and the use of the symbol $c_{v}$ refers only to the experimental method used in evaluating it.
3.3. Expansion and Compression Work. When a gas or vapor expands or is compressed as in the cylinder illustrated by Figure $3 \cdot 1$, the


Fig. 3.1. Cylinder with movable piston.


Fig. 3.2. Pressure-volume expansion work area.
total force acting against the piston face is $P A$, where $P$ is the unit absolute pressure in the cylinder and $A$ is the area of the piston. When the piston moves through a distance $d s$ the expansion or compression work involved is PAds. Since Ads represents the change in volume $d V$, the expansion work in foot-pound units is $P d V$. This may be represented graphically as the work area under the expansion curve as shown in Figure 3-2.

If it is desired to express this expansion work in Btu units to correspond to those of the internal energy $d u$, this may be done by dividing by $J$.

$$
\text { Increment of expansion or compression work }=\frac{1}{J} P d V
$$

3.4. Flow Work. Flow work should be associated with the transportation of a fluid from one region to another. For example, it is involved in an engine cylinder when gases are entering or leaving the cylinder, but not while all the valves are closed during the expansion or
compression strokes. It may be expressed in a usable form by considering a quantity of fluid as it is being forced into a region of higher pressure as illustrated in Figure 3•3.

When the surface (1) has been moved to (2) a volume of fluid $A S$ will go from a region at pressure $P$ to one at a pressure $P+d P$. The work done by the force $F$ will be $A S d P$ and since $A S$ is the volume $V$ that has been moved, the flow work done is $V d P$. When it is expressed in Btu,

$$
\text { Flow work }=\frac{1}{J} V d P
$$



Fig. 3.3. Flow work.
3.5. Acceleration Work. When a flowing gas or vapor is being accelerated to a higher flow velocity, it must be acted upon by a force $F=\frac{W}{g} a$, where $W$ is the pounds of material involved and $a$ is the acceleration. If the velocity is to be increased an amount $d v$ in a distance $d s$, the acceleration work is $F d s=\frac{W}{g} a d s$. But since $a=\frac{d v}{d t}$ and $v=\frac{d s}{d t}$, $a d s$ may be replaced by $v d v$. Thus, when expressed in Btu,

$$
\text { Acceleration work }=\frac{1}{J} \frac{W}{g} v d v
$$

To be consistent in units, velocity $v$ must, of course, be in feet per second.
3.6. The General Energy Equation. Equation $3 \cdot 1$ may now be written in its usual form for 1 lb of gas or vapor in Btu units, by combining it with equations $3 \cdot 2,3 \cdot 3,3 \cdot 4$, and $3 \cdot 5$ as

$$
d Q=d u+\frac{1}{J} P d V+\frac{1}{J} V d P+\frac{1}{J g} v d v
$$

where $d Q$ is the amount of heat added in Btu per pound. This is a general equation for either gases or vapors, but for gases, since the internal energy is a simple function of the temperature only, it may be written

$$
d Q=c_{v} d T+\frac{1}{J} P d V+\frac{1}{J} V d P+\frac{1}{J g} v d v
$$

This equation covers the general case, but in specific cases, any of the terms may be absent or negligible. When no heat is added or removed, $d Q$ is zero or if the flow velocity $v$ is constant, the kinetic energy change is zero.

During the compression and expansion strokes in engines and pumps both the flow work and acceleration work terms are not present, and the general equation reduces to a widely used form,

$$
d Q=c_{v} d T+\frac{1}{J} P d V
$$

This simplified energy equation can be integrated for any case only when the relationship between $c_{v}$ and $T$ is known and when the connection between $P$ and $V$ can be expressed. One simple but important case is that of heating a gas at constant pressure as is commonly done in furnaces and air heaters. If the temperature rise is not too great, the specific heat at constant volume $c_{v}$ may be considered approximately constant even though it varies considerably at higher temperatures.

The general gas equation may be written for one pound as

$$
P V=R T
$$

For the special case of a constant pressure change it may be differentiated to give

$$
P d V=R d T
$$

If this is combined with equation 3.8 the result is

$$
d Q=c_{v} d T+\frac{R}{J} d T=\left(c_{v}+\frac{R}{J}\right) d T
$$

Since the definition of the specific heat at constant pressure $c_{p}$ is the heat required to raise the temperature of one pound of gas or vapor through one degree Fahrenheit at constant pressure, equation $3 \cdot 11$ may also be written

$$
d Q=c_{p} d T=\left(c_{v}+\frac{R}{J}\right) d T
$$

This means that

$$
c_{p}=c_{v}+\frac{R}{J}
$$

The heat added to a gas during a constant-pressure change with no flow or acceleration work involved must go to change the internal energy and to do external expansion work. This means that an amount of external expansion work $R / J$ Btu per pound of gas is done when it is heated through one degree Fahrenheit at constant pressure. At first thought it might seem that a one-degree rise should result in more external expansion work at a high pressure than at a low one. However, it must be remembered that a greater change in volume will occur when the heat-

TABLE $3 \cdot 1$
Gas Specific Heats
(Pound Basis)

|  |  | $0^{\circ} \mathrm{F}$ | $200^{\circ} \mathrm{F}$ | $400^{\circ} \mathrm{F}$ | $600^{\circ} \mathrm{F}$ | $800^{\circ} \mathrm{F}$ | $1000^{\circ} \mathrm{F}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \text { Argon (Ar) } \\ & \text { Mol. wt. }=40 \\ & R=39.6 \end{aligned}$ | $c_{v}$ | 0.076 | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ |
|  | $c_{p}$ | 0.127 | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ |
|  |  | 1.67 | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ |  |
| Helium ( He ) Mol. wt. $=4$ $h=386.6$ | $r_{0}$ | 0.754 | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ |
|  | $c_{p}$ | 1.25 | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ |
|  | ${ }^{\prime}{ }_{p} / r_{2}$ | 1.66 | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ | $\rightarrow$ |
| $\begin{aligned} & \text { Oxygen }\left(\mathrm{O}_{2}\right) \\ & \text { Mol. wt. }=32 \\ & R=48.2 \end{aligned}$ | ${ }^{\prime}$ | 0.156 | 0.161 | 0.170 | 0.178 | 0.184 | 0.190 |
|  | $r_{p}$ | 0.218 | 0.223 | 0.232 | 0.240 | 0.246 | 0.252 |
|  | $c_{p} / c_{v}$ | 1.10 | 1.39 | 1.37 | 1.35 | 1.34 | 1.33 |
| $\begin{aligned} & \text { Nitrogen }\left(\mathrm{N}_{2}\right) \\ & \text { Mol. wt. }=28 \\ & R=55.3 \end{aligned}$ | $c_{v}$ | 0.175 | 0.177 | 0.179 | 0.187 | 0.195 | 0.202 |
|  | $r_{p}$ | 0.246 | 0.248 | 0.250 | 0.258 | 0.266 | 0.273 |
|  | $c_{p} / c_{v}$ | 1.40 | 1.40 | 1.39 | 1.38 | 1.37 | 1.35 |
| $\begin{aligned} & \text { Carbon monoxide (CO) } \\ & \text { Mol. wt. }=28 \\ & R=55.3 \end{aligned}$ | $c_{v}$ | 0.175 | 0.177 | 0.182 | 0.190 | 0.198 | 0.204 |
|  | $c_{p}$ | 0.246 | 0.248 | 0.253 | 0.261 | 0) 269 | 0.275 |
|  | $c_{p / 1} /{ }_{1}$ | 1.40 | 1.40 | 1.39 | 1.37 | 1.36 | 1.35 |
| $\begin{aligned} & \text { Hydrogen }\left(\mathrm{H}_{2}\right) \\ & \text { Mol. wt. }=2 \\ & R=762 \end{aligned}$ | $c_{v}$ | 2.48 | 2.48 | 2.49 | 2.52 | 2.54 | 2.58 |
|  | $c_{p}$ | 3.46 | 3.46 | 3.47 | 3.50 | 3.52 | 3.56 |
|  | $c_{p} / c_{v}$ | 1.40 | 1.40 | 1.40 | 1.39 | 1.39 | 1.38 |
| $\begin{aligned} & \text { Carbon dioxide }\left(\mathrm{CO}_{2}\right) \\ & \text { Mol. wt. }=44 \\ & R=35.0 \end{aligned}$ | $c_{v}$ | 0.152 | 0.169 | 0.194 | 0.211 | 0.225 | 0.236 |
|  | $c_{p}$ | 0.197 | 0.214 | 0.239 | 0.256 | 0.270 | 0.281 |
|  | $r_{p} / c_{v}$ | 1.30 | 1.27 | 1.23 | 1.21 | 1.20 | 1.19 |
| $\begin{aligned} & \text { Methane }\left(\mathrm{CH}_{4}\right) \\ & \text { Mol. wt. }=16 \\ & R=96.5 \end{aligned}$ | $c_{v}$ | 0.371 | 0.462 | 0.555 | 0.647 | 0.739 | 0.831 |
|  | $c_{p}$ | 0.495 | 0.586 | 0.679 | 0.771 | 0.863 | 0.955 |
|  | $c_{p} / c_{v}$ | 1.33 | 1.27 | 1.23 | 1.19 | 1.17 | 1.15 |
| Ethylene $\left(\mathrm{C}_{2} \mathrm{H}_{4}\right)$ <br> Mol. wt. $=28$ <br> $R=55.1$ | $c_{v}$ | 0.273 | 0.357 | 0.441 | 0.526 |  |  |
|  | $c_{p}$ | 0.344 | 0.428 | 0.512 | 0.597 |  |  |
|  | $c_{p} / c_{v}$ | 1.26 | 1.20 | 1.16 | 1.13 |  |  |
| $\begin{aligned} & \text { Ethane }\left(\mathrm{C}_{2} \mathrm{H}_{6}\right) \\ & \text { Mol. wt. }=30 \\ & R=51.5 \end{aligned}$ | $c_{v}$ | 0.319 | 0.428 | 0.536 | 0.645 |  |  |
|  | $c_{p}$ | 0.384 | 0.494 | 0.602 | 0.711 |  |  |
|  | $c_{p} / c_{v}$ | 1.20 | 1.15 | 1.12 | 1.10 |  |  |
| $\begin{aligned} & \text { Octane }\left(\mathrm{C}_{8} \mathrm{H}_{18}\right) \\ & \text { Mol. wt. }=114 \\ & R=13.55 \end{aligned}$ | $c_{v}$ | 0.165 | 0.271 | 0.376 | 0.482 |  |  |
|  | ${ }^{\prime}{ }_{p}$ | 0.312 | 0.418 | 0.523 | 0.629 |  |  |
|  | $c_{p} / c_{v}$ | 1.89 | 1.54 | 1.39 | 1.30 |  |  |
| Air (a mixture) Mol. wt. $=29$ $R=53.4$ | $c_{v}$ | 0.169 | 0.172 | 0.176 | 0.184 | 0.191 | 0.198 |
|  | $c_{p}$ | 0.238 | 0.241 | 0.245 | 0.253 | 0.260 | 0.267 |
|  | $c_{p} / c_{v}$ | 1.40 | 1.40 | 1.39 | 1.38 | 1.36 | 1.35 |

ing is done at a low pressure, and thus result in the same amount of external work. Values of $c_{p}$ and $c_{v}$ for common gases are listed in Table $3 \cdot 1$.
3.7. Enthalpy. In many engineering problems connected with the flow of vapors and gases it is very convenient to have the sum of the internal energy, external expansion work, and flow work evaluated as a unit, and this quantity is known as the enthalpy of the gas or vapor and is designated as $h$.

Thus,

$$
d h=d u+\frac{1}{J} P d V+\frac{1}{J} V d P
$$

With $d h$ so defined, the general energy equation becomes

$$
d Q=d h+\frac{1}{J g} v d v
$$

The expression for the enthalpy change of a vapor or gas can be simplified by recognizing that

$$
\frac{1}{J} P d V+\frac{1}{J} V d P=\frac{1}{J} d(P V)
$$

This reduces equation $3 \cdot 14$ to

$$
d h=d u+\frac{1}{J} d(P V)
$$

The change in enthalpy between any two conditions (1) and (2) may be expressed as

$$
\int_{h_{1}}^{h_{2}} d h=\int_{u_{1}}^{u_{2}} d u+\frac{1}{J} \int_{P_{1} V_{1}}^{P_{2} V_{2}} d(P V)
$$

or

$$
h_{2}-h_{1}=u_{2}-u_{1}+\frac{1}{J} P_{2} V_{2}-\frac{1}{J} P_{1} V_{1}
$$

This results in the general definition of enthalpy as

$$
h=u+\frac{1}{J} P V
$$

An enthalpy change represents a convenient grouping of internal energy, expansion work, and flow work terms in one numerical quantity, with the result that attempts to assign a single simple physical meaning to the concept of enthalpy is extremely difficult.

Because the internal energy of a vapor depends on both temperature and pressure, values of enthalpy for vapors are evaluated from experimental data and organized in vapor tables. In some tables the internal energy is also listed but wherever the volume of the vapor is known, equation $3 \cdot 20$ may be used to calculate values of $u$.

For gases and highly superheated vapors for which the general gas equation applies, changes in enthalpy may be evaluated from equation $3 \cdot 17$. The internal energy change $d u$ may be replaced by $c_{v} d T$, and $P V$ may be evaluated from the general gas equation as $R T$, since one pound of gas is being considered. This results in

$$
d h=c_{v} d T+\frac{R}{J} d T=\left(c_{v}+\begin{array}{l}
h \\
J
\end{array}\right) d T=c_{p} d T
$$

In the lower temperature range where $c_{v}$ and $c_{p}$ may be approximated as constant, it is a very simple thing to evaluate an enthalpy change from equation $3 \cdot 21$, but at higher temperatures where the specific heats of gases vary, the integration becomes tedious, since the equations relating $c_{p}$ and $T$ become quite involved. For that reason there is an increasing use of tables of gas properties that are similar to vapor tables.
3.8. Mol Specific Heats. The specific heat at constant volume $c_{v}$, and that at constant pressure $c_{p}$ have been used in Btu per pound per degree Fahrenheit, and for many applications this is the most convenient system of units. For other purposes, however, it is more useful to express specific hcats in the mol unit of Btu per nol per degree Fahrenheit. When this is done they are designated by capital $C$ instead of lower-case $c$, which refers to the pound unit. This means that

$$
C_{v}=\text { Mol. wt. } \times c_{v} \text { and } C_{p}=\text { Mol. wt. } \times c_{p}
$$

The significance of the mol specific heat may be brought out by combining equation $2 \cdot 2$ with equation $2 \cdot 10$ to give

$$
P V=\frac{m n}{3} v_{\text {eff. }}^{2}=W R T
$$

For one mol of gas $W=$ mol. wt. and $m n=$ the mass per mol or mol. wt. $/ \mathrm{g}$. Using the value of $R$ equals $1544 / \mathrm{mol}$. wt.,

$$
\frac{\text { Mol. wt. }}{g} \frac{v_{\text {eff. }}^{2}}{3}=\frac{1544}{J} T
$$

The total kinetic energy of translation of the molecules in a mol of gas is

$$
\frac{\text { Mol. wt. }}{g} \frac{v_{\text {eff. }}^{2}}{2}=\frac{3}{2} \times \frac{1544}{J} T=2.985 T
$$

This indicates that the change in kinetic energy of translation of gas molecules is 2.985 Btu per mol per deg F for all gases.

For simple monatomic gases such as helium and argon, the single atom molecule has most of its mass concentrated in the heavy central nucleus so the kinetic energy of spin and vibration should be small and, as in all gases, the potential energy changes due to molecular attraction are


Fig. 3.4. Specific heats of gases.
negligible. This would indicate that the value of $C_{v}$ for monatomic gases should be very close to the 2.985 Biu per deg F per mol suggested by equation $3 \cdot 25$. The fact that the experimental value of $C_{v}$ for helium and argon is 3.03 Btu per deg F per mol is good evidence of the validity of the concepts of the kinetic theory of gases.

For diatomic gases and others with more complex molecules, internal kinetic energy is not only stored in the translational kinetic energy of the molecules, but also in their spin and vibration. Spin and vibration account for much of the total internal energy, and their importance increases at higher temperatures. This may be illustrated by the properties of carbon dioxide $\left(\mathrm{CO}_{2}\right)$ which has a $C_{v}$ of 6.90 Btu per deg F per mol at 500 deg R and this increases to 11.425 at 2000 deg R . Since the
kinetic energy of molecular translation accounts for 2.985 Btu per deg F per mol, spin and vibration account for $6.90-2.985=3.915$ Btu per deg F per mol at 500 deg R and for $11.425-2.985=8.44$ Btu per $\operatorname{deg} \mathrm{F}$ per mol at 2000 deg R. This accounts for the constant specific heat of monatomic gases, for the moderate increase at higher temperature for the diatomic gases and the marked increase for carbon dioxide and water vapor as shown by Figure 3.4.

Only values of $C_{p}$ are shown by Figure 3.4 because by using equation $3 \cdot 13, C_{v}$ may be readily calculated from the relation

$$
C_{p}=\text { Mol. wt. }\left(c_{v}+\frac{R}{J}\right)=C_{v}+\frac{\text { mol. wt. } \times 1544}{\text { mol. wt. } \times J}=C_{v}+1.985
$$

This means that the external expansion work done while heating a gas at constant pressure is 1.985 Btu per deg F per mol and that this is the same for all gases and all pressures and temperatures. Thus, when a mol of $\mathrm{CO}_{2}$ is heated one degree at constant pressure near 500 deg R , the translational kinetic energy of the molecules accounts for 2.985 Btu , molecular spin and vibration for 3.915 Btu , and external expansion work for 1.985 Btu . Of this thermal energy that is added, only 1.985 Btu is involved in the external manifestation of expansion work and 6.90 Btu is stored in the random and uncontrolled motion of the molecules.
3.9. Entropy. If it could be possible to handle cach gas or vapor molecule individually, thermal energy could be readily caused to do useful mechanical work. However, with earh molecule moving, spinning, and vibrating in a completely random manner, such individual control is impossible and the method of converting thermal energy into mechanical work is more indirect and the resulting transformation is not complete. The only practical method so far devised for converting thermal energy into mechanical work is to take advantage of the pressure resulting from molecular impacts and to cause it to do work as a gas or vapor expands behind a moving piston or to produce a high-velocity jet as it expands through a nozzle.

When heat is added to a high-pressure gas or vapor, and work is done during expansion, the density of the gas decreases and its volume increases. The result is that work is being done by a medium that increases in volume as it does work and some of the energy added goes only to obtain an increased space for the gas or vapor to occupy. This can be illustrated by the cylinder and piston of Figure $3 \cdot 5 a$ and the corresponding pressure-volume diagram of Figure $3 \cdot 5 b$.

When the piston moves a distance $d s$ the total work done by the expanding gas is PAds where $A$ is the piston area. However, it must be recog-
nized that an amount of work $P_{o} A d s$ has been done against the external pressure ( $P_{o}$ ) and the useful work delivered to the piston rod of the engine is $F d s=\left(P-P_{o}\right) A d s$. This means that the work done against the external pressure represents an internal energy decrease in the


Fig. 3-5. Useful expansion work.
expanding gas in the cylinder that did no useful mechanical work on the piston rod of the engine.

The instantaneous efficiency of such an operation is

$$
\begin{align*}
& \text { Efficiency }=\frac{\text { Useful mechanical work done }}{\underbrace{}_{\text {Total work done }}} \\
& \text { Efficiency }=\frac{\left(P-P_{o}\right) A d s}{P_{o} A d s}=\frac{P-P_{o}}{P} \tag{3•27}
\end{align*}
$$

At first thought, it might seem possible to eliminate $P_{o} A d s=P_{o} d V$ by sealing the space on the external side of the piston and pumping out all the gas in that space, thus reducing the external pressure $P_{o}$ to zero. However, after the expansion is completed the cylinder would be full of spent gas or vapor that would have to be forced out of the cylinder against the external pressure $P_{o}$ before a new working stroke could be started. Consequently, such a scheme would not eliminate the work $P_{o} d V$.

Under certain conditions this nonproductive work may be reduced somewhat by using a closed-cycle engine. Such an engine would use the same charge of gas in the cylinder at all times and work would be performed by alternately heating and cooling this gas. There is no commercial engine that operates in just this manner, but it does represent the most efficient cycle that has so far been conceived for converting thermal energy into mechanical work.

Equation $3 \cdot 27$ indicates that the highest possible conversion efficiency would be attained by having the pressure during expansion as high as possible and that during the return stroke as low as possible.

This is illustrated by Figure $3 \cdot 6$ which indicates the conditions in the cylinder during the expansion stroke when the gas temperature is $T$ and its volume is $V$ and during the compression or return stroke for the same gas volume in the cylinder when the temperature is held at a $T_{0}$ which would naturally be that of the coolest temperature available, such as that of the water of a river. Since for the same volume of gas $\mathrm{V}^{\mathrm{V}}$, pressures are proportional to the tem-


Fig. 3 6. Closed-cy cle efficiency. peratures, the expression for the maximum efficiency for a closed cycle becomes

$$
\text { Efficiency }=\frac{P-P_{o}}{P}=\frac{\frac{W R T}{V}-\frac{W R T_{0}}{V}}{\frac{W R T}{V}}=\frac{T-T_{0}}{T}
$$

This adds a very important concept to the previously stated first law of thermodynamics, which, in conformity with the principle of conservation of energy, states that thermal energy and mechanical energy are mutually convertible in the ratio of $778.26 \mathrm{ft}-\mathrm{lb}$ of mechanical work per Btu of thermal energy. To this statement must now be appended the proviso, known as the second law of thermodynamics, that this transformation cannot be accomplished with equal ease in the two directions. Mechanical energy, being capable of complete control, can be readily and completely converted to thermal energy. Thermal energy, being in the form of random molecular motion that cannot be completely controlled can be converted to useful mechanical work only by an expansion process and the conversion is always incomplete. It is still true that when oneBtu of thermal energy is converted to mochanical work, it will produce 778.26 ft -lb of work. The significant point is that only a part of each Btu of thermal energy can be converted by any known method, and this fraction becomes greater as the temperature is increased.

Stated mathematically, when $d Q$ Btu are added to a gas at an absolute temperature $T$, the maximum portion of this that can be caused to do
useful mechanical work is $d Q\left(\frac{T-T_{0}}{T}\right)$ where $T_{0}$ is the lowest temperature available for cooling purposes. An amount $T_{0} d Q / T$ is unavazlable for doing useful mechanical work, and this is the result of using an expansive medium whose volume insists on increasing as work is done.

Whenever a conversion of thermal to mechanical energy is involved or contemplated, it is not sufficient to indicate only the quantity of heat added to a gas or vapor. It is just as important to report some measure of its degree of diepersion which will indicate its state of avazlabulity for conversion into mechanical work. For a given cooling temperature $T_{0}$, any operation that results in an increase in the value of $\int \frac{d Q}{T}$ means that there is an increase in unavailable energy. The ratio $d Q / T$ is such a uヶeful measure of the condition at which thermal energy exists, that it has been designated by the term cntropy, which is defined by

$$
\text { Change in cntropy }=d s=\frac{d Q}{T}
$$

The concept of a change in entropy, which represents the amount of heat added to a fluid, divided by the absolute temperature at which it is added, has, for the sake of simplicity, been developed for the permanent gases, but it applies also to thermal energy in any medium. Its significance may be realized when it is remembered again that thermal energy stored in the random motion of molecules can be caused to do useful mechanical work only by virtue of the expansion of a gas or vapor. There is an immense amount of thermal energy in the atmosphere about the earth, but with everything at the same temperature or energy level, expansive action is impossible and all this thermal energy is unavailable. It is only when a quantity of gas or vapor exists at a higher energy level than that of its surroundings that thermal energy is available to do mechanical work.

Since the addition of heat to a finite quantity of a gas results in a temperature rise, the entropy change must be evaluated as an integral. When the heating is at constant volume, the heat added becomes

$$
\int_{1}^{2} d s_{v}=\int_{1}^{2} \frac{d Q}{T}=\int_{T_{1}}^{T_{2}} \frac{W c_{v} d T}{T}
$$

Similarly for constant-pressure heating,

$$
\int_{1}^{2} d s_{p}=\int_{1}^{2} \frac{d Q}{T}=\int_{T_{1}}^{T_{2}} \frac{W c_{p} d T}{T}
$$

Whenever the temperature change is in a range where the specific heat $c_{p}$ or $c_{v}$ may be considered as approximately constant the integration of entropy change is simple, but it becomes more involved when the specific heat must be treated as a variable.

## Example of Entropy Change and Avallable Energy

If the exhaust gases of an internal-combustion engine leave at atmospheric pressure and at a temperature of 800 deg F what maximum portion of this thermal energy is available to do useful work if the lowest available temperature is 80 deg F ?
If the exhaust gases are considered to have essentially the same specific heat as nitrogen, the average value of $c_{p}$ from Table 31 is 0.25 Btu per lb per deg R . To heat one pound from 80 deg F to 800 deg F at atmospheric pressure would require the addition of

$$
\int_{1}^{2} d()=\int_{1}^{2} r_{p} d T=c_{p}\left(T_{2}-T_{1}\right)=0.25 \times 720=180 \text { Btu per lb }
$$

The change in entropy during such a heating process in

$$
\int_{1}^{2} d s=\int_{T_{1}}^{T_{2}}{\underset{\sim}{p}}^{c_{p}} d \underline{T}=c_{\mu} \log \frac{T_{1}}{T_{2}}=0.25 \log _{\epsilon} \frac{1260}{540}=0.211
$$

The unavailable portion of the $180 \mathrm{~J} 3 t u$ is $T_{0} \Delta s=(460+80) \times 0.211=114$ Btu per lb , leaving $180-114=66 \mathrm{l3tu}$ per lb as the maximum portion that could possibly be converted to useful mechanical work.

## Suggested Reading

Energy Equatıons, Enthalpy, and Specific Heats of Gases
Severns and Degler, Steam and Gas Power. Pages 11-30.

## The Energy Laws of Thermodynamics

Emswiler, Thermodynamics. Chapter XVI.

## Energy Equatzons

Sparks, Theory of Mechanical Refrigeration. Pages 5-15.

## CHAPTER 4

## PROPERTIES OF VAPORS AND GASES

4.1. Characteristics of Vapors. The essential differences between vapors and gases previously discussed can probably be best illustrated on a pressure-volume diagram. In Figure $1 \cdot 1$ the relationship between the temperature of a liquid and the vapor pressure exerted by its molecules was illustrated. The corresponding pressurc-volume history of a pound of liquid as it is heated is shown by the saturated llquid line on the $P-V$ diagram of Figure 4•1.

As the liquid temperature is raised the vapor pressure increases as shown by Figure 1.2, but a very slight increase in liquid volume occurs. This means that most of the heat that is added is stored as internal energy of the liquid and very little energy goes to do external work of expansion.

The history of one pound of the vapor is somewhat different and is shown by the saturated vapor line of Figure $4 \cdot 1$. At low temperatures the vapor molecules are widely separated and this results in a very large volume per pound of vapor and a low pressure. As the temperature and pressure are increased the vapor density increases and the volume of a pound of vapor becomes less.

At high pressures the saturated vapor line proves to be a continuation of the saturated liquid line. This point, known as the critical pressure with a corresponding critical temperature represents a condition where all the molecules possess sufficient energy to overcome the molecular forces of attraction and it is impossible for a true liquid to exist. At this critical pressure and temperature liquid and vapor volumes are equal. The critical temperatures and pressures of a few important substances are given in Table 4•1.

TABLE $4 \cdot 1$

| Critical Temperatures and Pressures |  |  |
| :--- | :---: | :---: |
|  | Critical Temperature | Critical Pressure in |
| Vapor | in Deg F | Lb per Sq In. Abs |
| Water $\left(\mathrm{H}_{2} \mathrm{O}\right)$ | 706 | 3226 |
| Ammonia $\left(\mathrm{NH}_{3}\right)$ | 271 | 1657 |
| Sulphur dioxide $\left(\mathrm{SO}_{2}\right)$ | 311 | 1160 |
| Carbon dioxide $\left(\mathrm{CO}_{2}\right)$ | 88 | 1070 |
|  | 30 |  |

If a gas remains at a temperature greater than the critical value, as $T_{4}$ in Figure $4 \cdot 1$, it cannot be liquefied by merely increasing the pressure. This fact is of great commercial importance in the liquefying of gas. An example is the recovery of gasoline by liquefying natural gases in some oil fields. There are some natural gases for which ordinary atmospheric temperatures would lie between $T_{3}$ and $T_{4}$ (Figure 4•1). In such cases, gasoline recovery is impossible unless the gas is refrigerated to some lower temperature such as $T_{2}$ during the compression.


Specific volume - cu ft per lb.
Fig. 4.1. Pressure-volume chart.
With the aid of Figure 4•1, the changes from liquid to vapor and to gas may be traced. Starting with liquid at the saturation pressure and temperature $A$, if heat is added and the pressure is held constant, the liquid begins to evaporate or vaporize at that temperature with a corresponding increase in volume. At point $B$ it is saturated vapor and between points $A$ and $B$ it is a mixture of liquid and vapor. This constantpressure evaporation represented by $A-B$ occurs at constant temperature because the temperature cannot rise above the saturation value until all the liquid present is evaporated.

When a fractional part $x$ of a pound of liquid has been converted to vapor, the mixture of liquid and vapor has a quality $x$ which is defined as the fractional part by weight of a liquid-vapor mixture that is in the vapor form. When enough heat has been added to vaporize all the pound
of liquid (point $B$, Figure 4•1) the quality $x$ is unity and it is referred to as dry saturated vapor. If more heat is added at constant pressure, since there is no more liquid to evaporate, the temperature will rise and the volume will increase. The energy added goes to increase the internal energy and do external expansion work. In the region to the right of the saturated vapor line (Figure 4.1), the vapor is superheated because it exists at a temperature higher than the saturation temperature for that pressure.

If the vapor is superheated further, a point $C$ will be reached where it begins to act as a gas because the molecules are widely separated and possess so much kinetic energy that the forces of attraction between molecules have a negligible effect. The equation of the constant-temperature curve $T_{4}$ is the same as for the permanent gases, $P V=$ constant. The water vapor in the atmosphere and in the products of combustion of fuels usually exists at such low pressures and densities that it may be dealt with as a gas in all practical cases.
However, in the saturation and low superheat regions corresponding to the steam conditions in power plants and the refrigerant pressures and temperatures in vapor refrigerators, a constant-temperature curve is


Fig. 4-2. Specific heat of steam at constant pressure. quite irregular and discontinuous as illustrated by $T_{1}$ and $T_{2}$, Figure $4 \cdot 1$. In this mixture region the concept of specific heat cannot be used because as long as liquid is vaporizing, heat is being added with no increase in temperature, and the specific heat would have to be conceived as being infinite.

The dependence of the specific heat of superheated vapors on both temperature and pressure, mentioned in article $3 \cdot 6$, is illustrated by
Figure 4.2. Low-pressure vapors that are highly superheated may be dealt with as gases, however, with fair accuracy. For steam near atmospheric pressure and at temperatures between 300 and $400 \mathrm{deg} F$, if values of the gas constant $R$ equals 85.5 and $c_{p}$ equals 0.47 Btu per lb per deg F are used, the results calculated from gas equations are satisfactory. At pressures around 1 lb per sq in. and temperatures near 100 deg F , values of $R$ equals 85.6 and $c_{p}$ equals 0.44 result in accurate predictions. However, at any higher pressures and near saturation temperatures, the properties of steam and other vapors are listed in vapor tables instead of using the very elaborate equations necessary to express them.
4.2. Vapor Tables. ${ }^{1,2,3}$ There are many ways in which the physical properties of vapor might be expressed and tabulated, but the method most universally used is associated with constant-pressure changes. When water evaporates in a steam boiler or into the atmosphere and when a refrigerant such as ammonia evaporates in the expansion coil of a refrigerator it is at constant pressure. When a vapor condenses in a condenser or when steam is superheated in a steam generator it is a con-stant-pressure process. These facts have resulted in the use of tables and charts of the properties of vapors based upon constant-pressure heating.

It was pointed out in article 1.7 that for any given pressure there is one unique equilibrium temperature at which a liquid will boil, and this is known as the saturation temperature. In saturated vapor tables these values of saturation temperature are listed for each pressure, these pressures usually being in pounds per square inch absolute. The values in such tables are given for one pound of liquid, vapor, or mixture. The subscript $f$ refers to the liquid or fluid condition, the subscript $g$ to vapor or gas, and $f g$ to the transition from liquid to saturated vapor. Thus, in such tables $V_{f}$ refers to the volume in cubic feet of one pound of liquid at the saturation temperature corresponding to that pressurc. The symbol $V_{f g}$ is the increase in volume during vaporization and $V_{g}$ is the volume of one pound of saturated vapor. For a mixture of quality $x$, the volume in cubic feet per pound of mixture would be calculated as

$$
V_{\text {mixture }}=V_{f}+x V_{f g} \text { or }=(1-x) V_{f}+x V_{g}
$$

The values of enthalpy $h$ that are listed in vapor tables are defined in terms of the general energy equation $3 \cdot 6$, the definition of enthalpy change, equation $3 \cdot 14$, and the definition of enthalpy, equation $3 \cdot 20$.

$$
\begin{gather*}
d Q=d u+{ }_{J}^{1} P d V+\frac{1}{J} V d P+\frac{1}{J g} v d v \\
d h=d u+{ }_{J}^{1} P d V+\frac{1}{J} V d P \\
h=u+\frac{1}{J} P V
\end{gather*}
$$

The most logical base or zero of reference for the measurement of enthalpy would be absolute zero of temperature, for at this point both the internal energy and enthalpy are theoretically zero. However, since the low-temperature range is of little interest in most engineering work, an arbitrary zero of temperature is used as a base above which internal energy is measured.

In the vapor tables for steam and other vapors used in power generation, internal energy is measured above 32 deg F. ${ }^{1,2}$ For refrigerants ${ }^{3}$ that operate at much lower temperatures, such a base as 32 deg F would result in the use of negative values of internal energy, which would be confusing. The result is that for refrigerants such as ammonia and sulphur dioxide the base of -40 deg F is arbitrarily used as the zero of reference for the measurement of internal energy. In any case, this base is not too important, since most problems involve only changes in internal energy.
The $\frac{1}{J} P V$ term of enthalpy $h$ equation $3 \cdot 20$, is, as shown by equation $3 \cdot 14$, the sum of the flow or pumping work and the expansion work involved in the generation of the vapor and to attempt to measure this above its value at some arbitrary zero would involve complications. As a result, the $\frac{1}{J} P V$ portion of the enthalpy is evaluated for the total volume per pound $V$ at that condition, and the total or absolute pressure $P$ which in reality puts it on an absolute zero basis instead of the arbitrary zero of 32 deg F or -40 deg F used for the internal energy. This theoretically inconsistent convention of evaluating the two components of the enthalpy concept on two different bases is done for practical reasons to simplify its use.

Vapor tables list values of enthalpy of saturated liquid $h_{f}$, enthalpy increase during vaporization ( $h_{f g}$ ), and enthalpy of saturated vapor $\left(h_{g}\right)$. For a mixture of liquid and vapor having a quality $x$ the enthalpy per pound of mixture is

$$
h_{\text {mixture }}=h_{f}+x h_{f g} \quad \text { or }=(1-x) h_{f}+x h_{g}
$$

Some vapor tables also list tables of the corresponding values of internal energy $u$, but in any case, the internal energy may be calculated for liquid, mixture, or vapor from equation $3 \cdot 4$ as

$$
u=h-\frac{1}{J} P V
$$

## Example

For saturated steam at $P=100 \mathrm{lb}$ per sq in. absolute, $V_{g}=4.432 \mathrm{cu} \mathrm{ft}$ per lb and $h_{g}=1187.2 \mathrm{Btu}$ per lb

$$
u_{g}=h_{g}-\frac{1}{J} P V_{g}=1187.2-\frac{100 \times 144 \times 4.432}{778.26}=1105.2 \text { Btu per lb }
$$

The evaluation of entropy as defined in equation 3.28 by $d s=\frac{d Q}{T}$ requires the use of the general energy equation (3.6) and an appreciation of the fact that there can be no change in entropy without the addition or extraction of heat $Q$. It is possible for a gas or vapor to do work or have work done on it and thus experience a change in enthalpy and internal energy with no change in the unavailable energy or entropy. This, in fact, is the condition sought when, for example, steam is expanding and doing work in an engine cylinder or when a refrigerant is being compressed in a refrigeration unit. Since vapor tables are for static conditions, there is no acceleration work involved and any flow work $\frac{1}{J} V d P$ such as that of pumping feed water into a boiler, is assumed to be an enthalpy change that results from the performance of mechanical work and is not the result of the addition of heat.

As used in vapor tables, entropy changes are defined by

$$
d s=\int \frac{d Q}{T}=\int \frac{d h-\frac{V d P}{J}}{T}
$$

The evaluation of this integral requires the formulation of the properties of liquids and vapors from experimental data which becomes quite involved, and this is the reason for the use of vapor tables. Values of entropy are referred to the same temperature base, 32 deg F or -40 deg $F$ that is used for enthalpy.

Tables list values of entropy of liquid $s_{f}$, entropy change during vaporization $s_{f g}$ and entropy of saturated vapor $s_{g}$. As in the case of enthalpy, the entropy of a mixture of vapor and liquid of quality $x$ is

$$
s_{\text {mixturv }}=s_{f}+x s_{f g}=(1-x) s_{f}+x s_{g}
$$

Since the vaporization takes place at constant pressure and at constant temperature $T_{\text {sat }}$, the entropy change during vaporization can be calculated from equation 4.4 as

$$
s_{f g}=\int_{h_{f}}^{h_{g}} \frac{d h}{T_{\text {sat. }}}=\frac{h_{f g}}{T_{\text {sat. }}}
$$

In the superheat region, to the right of the saturated vapor line in Figure $4 \cdot 1$, the vapor tables list values of specific volume, enthalpy, and entropy of superheated vapor at various temperatures for each pressure. The temperatures are reported as the total temperature and also as degrees of superheat which is the difference between the total temperature
and the saturation value at that pressure. Changes in enthalpy during a constant-pressure superheating for which $d P=0$ are given from equation $3 \cdot 14$ as,

$$
\begin{equation*}
\int_{P=\text { const. }} d h=\int_{P=\text { const. }} d u+\frac{1}{J} P d V \tag{4•7}
\end{equation*}
$$

In the same manner entropy changes in this region may be evaluated from equation $4 \cdot 4$. When the propertics of superheated vapors are required at values of temperature and pressure that fall between the increments used in the tables, a two-way interpolation is required, but such tables are so constructed that straight-line interpolation gives values that are sufficiently accurate for most purposes.
4.3. Vapor Calculations. When making any calculations for vapors or gases, it is vitally important to keep the physical meaning of equation $3 \cdot 6$ constantly in mind, and to be certain what energy changes are involved in any given problem. For example, the acceleration work is ignored in practically all problems except the formation of jets in nozzles. The changes in velocity of gases and vapors that do occur in engines and compressors represent amounts of energy that are quite small when compared with the other energy changes usually involved, the chief exception being that of low-pressure blowers which deliver most of the energy in the form of increased gas or vapor velocity.

The flow work term of equation $3 \cdot 6$ is present only when a fluid is actually moved from one region to another, as when a new charge of steam flows from a boiler to an engine or during the suction stroke in a compressor. The integral of $V d P / J$ may sometimes have a finite mathematical value where no flow work is actually involved. An example of this is that of heating a quantity of vapor or gas to a higher pressure and temperature with no change in volume. For this situation, $\int_{P_{1}}^{P_{2}} \frac{V d P}{J}$ would have a value of $\frac{V}{J}\left(P_{2}-P_{1}\right)$, but this would have no meaning in this problem since no flow work is involved. It is always necessary to look at the actual physical problem involved and to avoid permitting it to become merely a mathematical equation.

There are many engineering problems that are solved by the use of vapor tables, but a few illustrations will serve to illustrate the general pattern of procedure. In general, the initial conditions of the liquid or vapor are known and enough is known or must be assumed about the process involved to determine the final conditions from the tables. It is helpful to remember that once the pressure and quality or superheat of a vapor are known, the specific volume, internal energy, enthalpy, and
entropy are uniquely determined, regardless of the manner in which the vapor arrived at that condition. On the other hand, expansion work, flow work, acceleration work, and heat added or withdrawn are determined by the particular operation involved. At all times the energy equation must be satisfied or,

$$
\int_{1}^{2} d Q=\int_{1}^{2} d u+\frac{1}{J} \int_{1}^{2} P d V+\frac{1}{J} \int_{1}^{2} V d P+\frac{1}{J g} \int_{1}^{2} v d v
$$

## Example of Constant-Pressure Heating

Consider wet steam that is 0.10 moisture or has a quality $x=0.9$, leaving the briler of a steam generator at 300 lb per sq in. absolute and being heated at constant pressure in the superheater tubes to a final temperature of 600 deg F . From steam tables, ${ }^{1}$ the initial tenperature of the steam is 417.33 and the initial volume of the steam is

$$
\begin{aligned}
& V_{1}=\left(1-x_{1}\right) V_{f 1}+x_{1} V_{g 1}=0.1 \times 0.01890+0.9 \times 1.5433=1.391 \text { cu ft per lb } \\
& \left.h_{1}=h_{f 1}+x_{1} h_{f g 1}=393.83+0.9 \times 809.0=1121.9 \text { Btu per } 1 \mathrm{l}\right)
\end{aligned}
$$

The initial internal energy may be found in a similar manner from tables listing it or

$$
u_{1}=h_{1}-\frac{1}{J} P_{1} V_{1}=1121.91-\frac{144 \times 300 \times 1.391}{778}=1044.4 \mathrm{Btu} \text { per } \mathrm{lb}
$$

The initial entropy is

$$
s_{1}=s_{f 1}+x_{1} s_{f 61}=0.5879+0.9 \times 0.9225=1.4182 \text { Btu per lb per deg F }
$$

From the superheat tables at 300 lb per sq in. and a temperature of 600 deg F , $V_{2}=2.005 \mathrm{cu} \mathrm{ft}$ per $\mathrm{lb}, h_{2}=1314.7 \mathrm{Btu}$ per lb , and $s_{2}=1.6268 \mathrm{Btu}$ per lb per $\operatorname{deg} F$. The internal energy

$$
u_{2}=h_{2}-\frac{1}{J} P_{2} V_{2}=1314.7-\frac{300 \times 144 \times 2.005}{778}=1203.7 \text { Btu per } \mathrm{lb}
$$

For this particular case, the acceleration work or change in kinetic energy may be ignored because the flow velocity in the superheater tubes does not represent much energy. In this connection, it is of interest to note that when one Btu goes into the kinetic energy of one pound of fluid, it will produce a velocity given by

$$
\int d Q=\int_{0}^{v} \frac{v d v}{J g} \quad \text { or } \quad 1=\frac{v^{2}}{2 J g}
$$

or

$$
v=\sqrt{2 J g}=224 \mathrm{ft} \text { per sec }
$$

Since this is assumed to be a constant-pressure heating, it is implied that pressure losses in the superheater are negligible, so the flow work is neglected. This reduces equation $4 \cdot 8$ for $P_{1}=P_{2}$ to

$$
{ }_{1} \Delta Q_{2}=u_{2}-u_{1}+\frac{1}{J} P_{2} V_{2}-\frac{1}{J} P_{1} V_{1}
$$

This may be recognized as $h_{2}-h_{1}$, so the heat that must be added is $1314.7-1121.9=192.8 \mathrm{Btu}$ per lb . The change in internal energy $u_{2}-u_{1}=1203.7-1144.4=159.3$ Btu per lb. This means that the expansion work done during this heating process is $192.8-159.2$ or 33.5 Btu per lb.

If this pound of vapor and liquid had been heated at constant pressure in an engine cylinder, this expansion work would have been done directly on the engine piston. Since it was done in a steam generator, the expansion work went to push another pound of steam, previously generated, into the engine or turbine that loads the boiler, so the effect is precisely the same.

Since the entropy change involved was $s_{2}-s_{1}=1.6268-1.4182=$ 0.2086 , for a lowest available temperature of 70 deg F or $T_{0}=530 \mathrm{deg}$ R , the increase in unavailable energy is $\Delta s T_{0}=0.2086 \times 530=110.8$ Btu per lb . This means that even with a heat engine having the highest conceivable efficiency, only $192.8-T_{0} \Delta s=192.8-110.8=82.0 \mathrm{Btu}$ per lb or $\frac{82.0}{192.8}=0.426$ of the total amount of heat added could ever be converted into useful mechanical work.

## Example of Throttling

The familiar occurrence of a gas or vapor at a high pressure which is allowed to rush through a valve or other opening into a region at a lower pressure is known as throttling, when the high velocity of the vapor as it passes through the opening is allowed to revert to thermal energy by a process of turbulent mixing in the low-pressure region. If this process is assumed to occur quickly enough to preclude the possibility of any heat transfer, and if the initial and final kinetic energy are negligible, the energy equation for such conditions could be written

$$
0=d u+\frac{1}{J} P d V+\frac{1}{J} V d P
$$

This is equivalent to saying that there is no change in enthalpy or that $h_{1}=h_{2}$. Such would be true of dry saturated steam at 100 lb per sq in . absolute pressure being throttled to 15 lb per sq in . absolute for use in a heating system. From steam tables, $v_{1}=4.43 \mathrm{cuft}$ per $\mathrm{lb}, h_{1}=1187.2 \mathrm{Btu}$ per lb and $s_{1}=1.6026$

Btu per lb per deg. From the superheat tables for $h_{1}=h_{2}$, the final temperature would be 289 deg F . Also $v_{2}=29.43 \mathrm{cu} \mathrm{ft} \mathrm{per} \mathrm{lb}$ and $s_{2}=1.8066$. This represents a drop in temperature from 327.81 deg F to 289 deg F with an increase in volume from 4.43 to 29.43 cu ft per lb .

The really significant thing about such a throttling process is that even though no heat has been conducted away, there is a great increase in the dispersion or scattering of the original energy that is reflected in the entropy increase of $1.8066-1.6026=0.2040$. For a lowest available temperature of 530 deg R , this throttling process has increased the amount of thermal energy that cannot, by any known means, be converted into useful mechanical work by an amount $T_{0} \Delta s=108 \mathrm{Btu}$ per lb or $84,000 \mathrm{ft}-\mathrm{lb}$ per lb of vapor.

4-4. Low-Temperature Gas Calculations. Evaluating the properties of the permanent gases at low temperatures is a simpler procedure than it is to do the same thing for vapors. The phenomenon of vaporization is not present; pressure volume and temperature are simply related by the general gas equation; the specific heats are independent of pressure; and at temperatures below 500 deg F , the variations in specific heats are not so great but that the various terms of the energy equation may be evaluated with fair accuracy by assuming these specific heats as constant. These facts greatly simplify calculations for gas compressors, blowers, and allied low-temperature applications and it is for use in such problems that the values of $c_{p}, c_{v}, c_{p} / c_{v}$, and gas constant $R$ are included in Table 3.1. These data are from the same source as Figure 3.4 and cover the gases usually involved in engineering problems.

Gases are heated at constant volume in internal-combustion engines and at constant pressure in furnaces, air heaters, ctc. For simple con-stant-volume and constant-pressure heating and cooling problems, the acceleration and flow work terms of the general energy equation $3 \cdot 6$ are usually not present and the other terms are readily integrated and evaluated when $c_{v}$ and $c_{p}$ are considered constant. Entropy changes during such heating and cooling operations are evaluated from

$$
\begin{align*}
\Delta s_{p=\text { const. }} & =\int_{T_{1}}^{T_{2}} \frac{c_{p} d T}{T}=c_{p} \log _{\epsilon} \frac{T_{2}}{T_{1}}=2.3 c_{p} \log _{10} \frac{T_{2}}{T_{1}} \\
\Delta s_{v=\text { const. }} & =\int_{T_{1}}^{T_{2}} \frac{c_{v} d T}{T}=c_{v} \log _{\epsilon} \frac{T_{2}}{T_{1}}=2.3 c_{v} \log _{10} \frac{T_{2}}{T_{1}}
\end{align*}
$$

When a gas is compressed in the cylinder of a reciprocating compressor it is usually cooled to prevent excessive temperature rise and to reduce the amount of compression work required. Completely successful cooling would be represented by a constant-temperature operation and no cooling whatever would correspond to the case where no heat is removed
or added or $d Q=0$ in the energy equation. The constant-temperature condition is known as an isothermal change and when no heat is added or removed, the change is known as adrabatic.

For an operation that is performed in the cylinder of a compressor or engine the flow work and acceleration work terms of the energy equation


Fig. 4 3. Isothermal compression work. are not present and for an isothermal change where $d T$ is zero, the energy equation becomes

$$
d Q=\frac{1}{J} P d V
$$

For such a compression the total work done is represented in Figure $4 \cdot 3$ by the area $a 12 b$. Since the value of the compression work term integral depends upon the type of compression, its integration may be accomplished by replacing $P$ by its equivalent value in terms of $V$. Since $P V=$ $P_{1} V_{1}=P_{2} V_{2}$ for a constant-temperature casc

$$
P=\frac{P_{1} V_{1}}{V}=\frac{P_{2} V_{2}}{V}
$$

Combining equations $4 \cdot 13$ and $4 \cdot 14$ results in

$$
\int_{1}^{2} d Q=\frac{1}{J} \int_{V_{1}}^{V_{2}} P d V=\frac{P_{1} V_{1}}{J} \int_{V_{1}}^{V_{2}} \frac{d V}{V} \text { or } \frac{P_{2} V_{2}}{J} \int_{V_{1}}^{V_{2}} \frac{d V}{V}
$$

The integration of equation $4 \cdot 15$ gives

$$
{ }_{1} \Delta Q_{2}=\frac{P_{1} V_{1}}{J} \log _{\epsilon} \frac{V_{2}}{V_{1}}
$$

Since for a compression stroke, $V_{2}$ is smaller than $V_{1}$, the work term comes out negative. This is consistent with the signs of the energy equation in which work done by the gas is designated as a positive term, and since this work was done on the gas it should be represented by a negative quantity. In the same way, ${ }_{1} \Delta Q_{2}$ coming out as a negative quantity indicates that heat was withdrawn. Since $P_{1} V_{1}$ equals $W R T_{1}$ and for an isothermal change $P_{1} V_{1}$ equals $P_{2} V_{2}$, equation $4 \cdot 16$ may also be written

$$
{ }_{1} \Delta Q_{2}=\frac{W R T_{1}}{J} \log _{\epsilon} \frac{P_{1}}{P_{2}}
$$

when it is more convenient to have it expressed in terms of pressure and temperature.

When the compression is adiabatic $d Q=0$ as before, and the flow and acceleration work terms are zero. This reduces the energy equation to

$$
0=W c_{v} d T+\frac{1}{J} P d V
$$

Here all external compression work goes to increase the internal energy and for an expansion, the work done by the gas is at the expense of the internal energy. For an adiabatic compression from a temperature $T_{1}$ to $T_{2}$, the internal energy increase is

$$
\int_{T_{1}}^{T_{2}} W c_{v} d t \text { or } \quad\left[W c_{v}\left(T_{2}-T_{1}\right)\right]
$$

and the work term is [ $-W c_{v}\left(T_{2}-T_{1}\right)$ ].
The characteristics of an adiabatic compression may be established by integrating equation $4 \cdot 18$, but before that can be accomplished the three variables of equation $4 \cdot 18$ must be reduced to two by substituting from the differentiated general gas equation, $P V=W R T$. Differentiating this,

$$
P d V+V d P=W R d T
$$

Eliminating $d T$ between equations $4 \cdot 18$ and $4 \cdot 19$ and separating variables

$$
\begin{gather*}
\frac{c_{v}}{R} P d V+\frac{c_{v}}{R} V d P+\frac{1}{J} P d V=0 \\
\left(\frac{c_{v}}{R}+\frac{1}{J}\right) \frac{d V}{V}=-\frac{c_{v}}{R} \frac{d P}{P}
\end{gather*}
$$

Integrating this for a compression between $P_{1}, V_{1}$ and $P_{2}, V_{2}$.

$$
\left(\frac{c_{v}}{R}+\frac{1}{J}\right) \log _{\epsilon} \frac{V_{2}}{V_{1}}=-\frac{c_{v}}{R} \log _{\epsilon} \frac{P_{2}}{P_{1}}=\frac{c_{v}}{R} \log _{\epsilon} \frac{P_{1}}{P_{2}}
$$

or

$$
\left(\frac{V_{2}}{V_{1}}\right)^{\left(c_{v}+\frac{R}{J}\right) / c_{v}}=\frac{P_{1}}{P_{2}} \quad \text { or } \quad P_{1} V_{1}^{c_{1} / c_{v}}=P_{2} V_{2}^{c_{L} / c_{v}}
$$

Since $\left(c_{v}+\frac{R}{J}\right) / c_{v}=c_{p} / c_{v}$ and this ratio is called $\gamma$, the relationship between pressure and volume during an adiabatic change is

$$
P_{1} V_{1}^{\gamma}=P_{2} V_{2}^{\gamma}=P V^{\gamma}=\mathrm{Constant}
$$

Some problems are more conveniently solved in terms of pressure and
temperature, and the corresponding $P, T$ equation may be had by eliminating $V$ between equation $4 \cdot 22$ and the general gas equation

$$
\begin{gather*}
\frac{P_{1} V_{1}}{T_{1}}=\frac{P_{2} V_{2}}{T_{2}}=\frac{P V}{T} \\
\frac{V_{2}}{V_{1}}=\frac{P_{1} T_{2}}{P_{2}}=\left(\frac{P_{1}}{T_{1}}\right)^{1 / \gamma} \text { or } \frac{T_{2}}{T_{1}}=\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma-1) / \gamma}
\end{gather*}
$$

There are other low-temperature gas problems that are involved in engineering calculations, but the previously discussed constant-volume, constant-pressure, isothermal, and adiabatic changes cover the majority of applications and illustrate the method of attack.

## Examples of Isothermal and Adiabatic Changes

The use of gas equations and a comparison of isothermal and adiabatic changes may be illustrated by the compression of one pound of air from atmospheric pressure and 70 deg F to a final pressure of 100 lb per sq in . gage. The initial volume is

$$
V=\frac{W R T_{1}}{P_{1}}=\frac{1 \times 53.3 \times 530}{14.7 \times 144}=13.35 \mathrm{cu} \mathrm{ft} \mathrm{per} \mathrm{lb}
$$

The final temperature at the end of an isothermal compression is, of course, 70 deg F and at the end of an adiabatic compression it is found from equation $4 \cdot 23$ as

$$
T_{2}=T_{1}\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma-1) / \gamma}=530\left(\frac{114.7}{14.7}\right)^{(1.4-1) / 1.4}=955 \operatorname{deg} \mathrm{R} \quad \text { or }{ }^{\circ} 495 \operatorname{deg} \mathrm{~F}
$$

For an isothermal compression the heat extracted, which is also equal to the compression work done, is

$$
\begin{aligned}
{ }_{1} \Delta Q_{2} & ={ }_{1} \text { work }_{2}=\frac{W R T_{1}}{J} \log _{e} \frac{P_{1}}{P_{2}}=-\frac{W R T_{1}}{J} \log _{e} \frac{P_{2}}{P_{1}} \\
& =-\frac{1 \times 53.3 \times 530}{778} \times \log _{e} \frac{114.7}{14.7}=-74.5 \text { Btu per lb }
\end{aligned}
$$

or

$$
-74.5 \times 778=-57,900 \mathrm{ft}-\mathrm{lb} \text { per } \mathrm{lb}
$$

The minus sign indicates heat extracted and work done on the gas. For an adiabatic compression since the work done on the gas is stored in the form of increased internal energy,

$$
\begin{aligned}
\text { Compression work } & =-\int_{T_{1}}^{T_{2}} W c_{v} d T=-W c_{v}\left(T_{2}-T_{1}\right) \\
& =-1 \times 0.172 \times(955-460) \\
& =-85.2 \mathrm{Btu} \text { per } \mathrm{lb}
\end{aligned}
$$

4.5. Gas Tables ${ }^{4}$ and Their Use. The rapid increase in the specific heats of gases at higher temperatures shown in Figure 3.4 makes calculations based on the assumption that they are constant entirely too inaccurate for even rough calculations. Owing to the form of these specific heat-temperature curves, the equations expressing them are too involved for everyday use. For these reasons tables that give the properties of gases at higher temperatures have been prepared ${ }^{4}$ for use in the high-temperature problems involved in internal-combustion engines, furnace gases, and gas turbines.

The problem of preparing tables is similar to that of preparing superheated vapor tables, with the exception that for gases $c_{v}$ and $c_{p}$ are not dependent upon pressure, but are functions of temperature only. There is no need for listing gas volumes, since they may be readily calculated from the general gas equation. The concept of internal energy as $u=\int c_{v} d T$ is the same for gases as for vapors, but in gas tables it is advantageous to modify the vapor concept of enthalpy as discussed in article 4.2. There enthalpy was defined $h=u+\frac{P V}{J}$ with $u$ being measured from an arbitrary temperature base and $P V / J$ being an absolute or total quantity.

For a gas, since for one pound $P V=R T$ it is far better to measure both terms from the same base. Consequently, in the gas tables that use $540 \operatorname{deg} \mathrm{R}$ or $80 \operatorname{deg} \mathrm{~F}$ as a base, gas enthalpy is defined by

$$
h=\int_{540}^{T}\left(c_{v}+\frac{R}{J}\right) d T=\int_{540}^{T} c_{p} d T
$$

and

$$
u=\int_{540}^{T} c_{v} d T
$$

Since entropy is defined as

$$
s=\int \frac{d Q}{T}=\int \frac{c_{v} d T+\frac{1}{J} P d V}{T}
$$

it is definitely dependent upon the manner in which the temperature is increased, whether at constant pressure, constant volume, or otherwise. This would mean that a general entropy table would list values at each pressure and temperature combination just as is done in superheated vapor tables.

This complication is avoided in gas tables by taking advantage of the fact that the entropy of a gas is determined when any two of the three
variables, pressure, temperature, and volume, are specified. When a problem is being solved in terms of temperature and volume, the components illustrated by Figure 4.4 are most convenient and when the variables temperature and pressure are known the components of Figure 4.5 are best.

The entropy change from conditions 1 to 2 as shown in Figure 4.4 may be broken up into the algebraic sum of the change at constant temperature $T_{1}$ to the volume $V_{2}$ at $2^{\prime}$ plus the change along the constant volume from $T_{1}$ to $T_{2}$ represented by $2^{\prime}-2$. An alternate method is to take


Fig. 4.4. Temperature-volume components of entropy change.


Fig. 4.5. Temperature-pressure components of entropy change.
the algebraic sum of the constant-volume entropy change from 1 to $1^{\prime}$ and the constant-temperature entropy change from $V_{1}$ to $V_{2}$ along the constant-temperature $1^{\prime}-2$, which is, of course, the exact equivalent of the first method.
If the variables, temperature and pressure, are being used the method indicated by Figure 4.5 is convenient. Then the entropy change at constant temperature from $P_{1}$ to $P_{2}$ would be added algebraically to the entropy change at constant pressure from $T_{1}$ to $T_{2}$.

In gas tables it is necessary to provide values of entropy changes at constant temperature, constant volume, and at constant pressure. These are evaluated from the entropy equation $4 \cdot 26$. At constant temperature $d T$ is zero, so equation $4 \cdot 26$ becomes

$$
\Delta s_{T}=\int \frac{1}{J} \frac{P d V}{T}
$$

Since $\frac{P}{T}=\frac{R}{V}$ for 1 lb ,

$$
\Delta s_{T}=\int_{V_{1}}^{V_{2}} \frac{R}{J} \frac{d V}{V}=\frac{R}{J} \log _{e} \frac{V_{2}}{V_{1}}
$$

Since for constant temperature $P_{1} V_{1}=P_{2} V_{2}$, this may also be written as

$$
\Delta s_{T}=\frac{R}{J} \log _{\epsilon} \frac{P_{1}}{P_{2}}
$$

Since entropy changes at constant temperature are so simple to calculate, no tables of values are required. As a result the following gas tables include values of

$$
u=\int_{540}^{T} c_{v} d T \quad h=\int_{540}^{T} c_{p} d T \quad s_{v}=\int_{540}^{T} \frac{c_{v} d T}{T}
$$

and of $s_{p}=\int_{540}^{T} c_{p} d T$ which have been calculated ${ }^{4}$ from experimental data.

All are based on a zero value of 80 deg F or 540 deg R , and Table $4 \cdot 2$ for air assumes a weight composition of $\mathrm{O}_{2}=0.23 ; \mathrm{N}_{2}=0.756 ; \mathrm{Ar}=0.014$ for air. Table 4.8 for water vapor is based on the $c_{p}$ values at zero pressure shown in Figure $4 \cdot 2$ and can be used only for very low-pressure calculations.

When the properties of gas mixtures are needed they can be calculated from Tables 4.2 to 4.8 because all the propertics are additive. For example, $h$ at 800 deg R for a mixture of one part by weight $\mathrm{N}_{2}$ and two parts CO would be given by

$$
\frac{1 \times h \mathrm{~N}_{2}+2 \times h \mathrm{CO}}{3} \text { or } \frac{1 \times 64.8+2 \times 65.1}{3}=65.0 \mathrm{Btu} \text { per } \mathrm{lb}
$$

In internal-combustion engine calculations, it is necessary to deal with the products of combustion. Since the properties of such a gas mixture do not vary radically with composition, Table 4.9 has been included for such work. It is accurate for the theoretical combustion of a fuel that is 0.85 carbon and 0.15 hydrogen by weight burned with 20 per cent excess air, and the values given may be used with fair accuracy for most mixtures met in practice. This particular table is for a final gas composition by weight of $\mathrm{CO}_{2}=0.1634, \mathrm{H}_{2} \mathrm{O}=0.0708, \mathrm{~N}_{2}=0.7295$, and $\mathrm{O}_{2}=0.0363$ for which the gas constant $R=53.6$.

Once the basis upon which these gas tables have been calculated is understood, their use is simple and direct. For example, if 1 lb of combustion mixture (Table 4.9) in an engine cylinder at 1000 deg R absorbs 500 Btu per lb during a constant-volume burning, all this added heat goes to increase the internal energy $u$. The final $u$ is $u_{1000}+500=$
$88.0+500=588$ Btu per lb . Interpolating between 3100 and 3200 $\operatorname{deg} R$ in Table 4.9 gives the final temperature as 3126 deg R. From the $s_{v}$ tables the entropy change may be calculated, and the increase in unavailable energy for any lowest available temperature $T_{0}$ may be found from $T_{0} \Delta s_{v}$. An exactly similar procedure would be followed for a constant-pressure heating, using $h$ and $s_{p}$ instead of $u$ and $s_{v}$.

Another example of the use of the gas tables would be that of 1 lb of combustion mixture (Table 4.9) at 3500 deg R that is expanded without turbulence or heat transfer until its final volume $V_{2}$ is six times its initial volume $V_{1}$. Since for this type of adiabatic expansion $d Q=0$, it is also a constant entropy change, with temperature and volume as working variables (Figure 4.4). The initial entropy $s_{v 1}=0.4085$ and the entropy increase during the constant-temperature expansion to the final volume is, from equation $4 \cdot 28$,

$$
\Delta s_{T}=\frac{R}{J} \log _{\epsilon} \frac{V_{2}}{V_{1}}=\frac{53.8}{778} \times 2.3 \log _{10} \frac{6}{1}=0.1240
$$

Since the final total entropy is $s_{v 2}+\Delta s_{T}$ and this must be equal to the initial value of $s_{v 1}=0.4085$, it follows that for a constant entropy expansion

$$
s_{v 2}=s_{v 1}-\Delta s_{T}=0.4085-0.1240=0.2845
$$

Locating this value of $s_{v 2}$ in Table 4.9 gives a final temperature of 2146 deg R and a final internal energy $u_{2}=341.9 \mathrm{Btu}$ per lb. The initial internal energy is $u_{1}=686.2$ and the decrease $u_{1}-u_{2}=686.2-$ $341.9=344.3$ Btu per lb. Since, for an adiabatic or constant entropy change the external expansion work done by the mixture is equal to the decrease in internal energy, the total expansion work performed is also 344.3 Btu per lb of combustion mixture.

TABLE 4.2
The Pound of Air

| T ${ }^{\circ} \mathrm{K}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ | T ${ }^{\circ} \mathrm{R}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 570 | 7.1 | 5.1 | . 0130 | . 0093 | 2600 | 545.8 | 404.7 | . 4058 | . 2981 |
| 600 | 14.4 | 10.3 | . 0254 | . 0182 | 2700 | 574.7 | 426.7 | . 4167 | . 3064 |
| 650 | 26.4 | 18.9 | . 0445 | . 0318 | 2800 | 603.7 | 448.9 | . 4273 | . 3145 |
| 700 | 38.5 | 27.6 | . 0624 | . 0446 | 2900 | 632.8 | 471.1 | . 4375 | . 3223 |
| 750 | 50.6 | 36.3 | . 0792 | . 0567 | 3000 | 662.1 | 493.5 | . 4474 | . 3299 |
| 800 | 62.8 | 45.0 | . 0949 | . 0680 | 3100 | 691.5 | 516.0 | . 4570 | . 3373 |
| 850 | 75.0 | 53.8 | . 1097 | . 0786 | 3200 | 720.9 | 538.7 | . 4663 | . 3444 |
| 900 | 87.3 | 62.6 | . 1237 | . 0887 | 3300 | 750.5 | 561.5 | . 4754 | . 3514 |
| 950 | 99.6 | 71.5 | . 1370 | . 0983 | 3400 | 780.2 | 584.3 | . 4843 | . 3582 |
| 1000 | 112.0 | 80.5 | . 1497 | . 1075 | 3500 | 810.0 | 607.2 | . 4930 | . 3649 |
| 1050 | 124.5 | 89.6 | . 1619 | . 1163 | 3600 | 839.8 | 630.2 | . 5014 | . 3714 |
| 1100 | 137.1 | 98.7 | . 1736 | . 1248 | 3700 | 869.7 | 653.2 | . 5096 | . 3777 |
| 1150 | 149.7 | 107.9 | . 1848 | . 1330 | 3800 | 899.7 | 676.4 | . 5175 | . 3839 |
| 1200. | 162.4 | 117.2 | . 1957 | . 1409 | 3900 | 929.8 | 699.6 | . 5253 | . 3899 |
| 1250 | 175.2 | 126.6 | . 2061 | . 1486 | 4000 | 960.0 | 722.9 | . 5329 | . 3958 |
| 1300 | 188.1 | 136.0 | . 2162 | . 1560 | 4100 | 990.2 | 746.3 | . 5404 | . 4016 |
| 1350 | 201.1 | 145.5 | . 2260 | . 1632 | 4200 | 1020.5 | 769.7 | . 5477 | . 4072 |
| 1400 | 214.1 | 155.1 | . 2355 | . 1702 | 4300 | 1050.8 | 793.2 | . 5549 | . 4127 |
| 1450 | 227.2 | 164.8 | . 2447 | . 1770 | 4400 | 1081.2 | 816.7 | . 5619 | . 4182 |
| 1500 | 240.3 | 174.5 | . 2536 | . 1836 | 4500 | 1111.7 | 840.3 | . 5687 | . 4235 |
| 1600 | 266.8 | 194.2 | . 2707 | . 1963 | 4600 | 1142.2 | 864.0 | . 5754 | . 4287 |
| 1700 | 293.7 | 214.2 | . 2870 | . 2085 | 4700 | 1172.8 | 887.7 | . 5820 | . 4338 |
| 1800 | 320.9 | 234.5 | . 3026 | . 2201 | 4800 | 1203.4 | 911.5 | . 5884 | . 4387 |
| 1900 | 348.4 | 255.1 | . 3174 | . 2312 | 4900 | 1234.1 | 935.3 | . 5947 | . 4436 |
| 2000 | 376.0 | 275.9 | . 3316 | . 2419 | 5000 | 1264.8 | 959.2 | . 6009 | . 4484 |
| 2100 | 403.8 | 297.0 | . 3452 | . 2521 | 5100 | 1295.6 | 983.1 | . 6070 | . 4532 |
| 2200 | 431.8 | 318.2 | . 3582 | . 2620 | 5200 | 1326.4 | 1007.1 | . 6130 | . 4578 |
| 2300 | 460.0 | 339.6 | . 3708 | . 2715 | 5300 | 1357.2 | 1031.1 | . 6189 | . 4624 |
| 2400 | 488.4 | 361.1 | . 3829 | . 2806 | 5400 | 1388.1 | 1055.1 | . 6247 | . 4669 |
| 2500 | 517.0 | 382.8 | . 3945 | . 2895 | 5500 | 1419.1 | 1079.2 | . 6304 | . 4714 |

TABLE 4.3
The Pound of Nitrogen

| T ${ }^{\circ} \mathrm{K}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ | T ${ }^{\circ} \mathrm{R}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 570 | 7.4 | 5.3 | . 0134 | . 0096 | 2600 | 559.8 | 413.8 | . 4165 | . 3051 |
| 600 | 14.9 | 10.7 | . 0263 | . 0189 | 2700 | 589.4 | 436.4 | . 4277 | . 3136 |
| 650 | 27.3 | 19.6 | . 0460 | . 0328 | 2800 | 619.2 | 459.1 | . 4385 | . 3219 |
| 700 | 39.8 | 28.5 | . 0645 | . 0460 | 2900 | 649.2 | 481.9 | . 4490 | . 3299 |
| 750 | 52.3 | 37.4 | . 0818 | . 0585 | 3000 | 679.3 | 504.9 | . 4592 | . 3377 |
| 800 | 64.8 | 46.4 | . 0980 | . 0702 | 3100 | 709.5 | 528.0 | . 4691 | . 3453 |
| 850 | 77.4 | 55.4 | . 1132 | . 0811 | 3200 | 739.8 | 551.3 | . 4787 | . 3526 |
| 900 | 90.0 | 64.5 | . 1276 | . 0914 | 3300 | 770.2 | 574.6 | 4881 | . 3598 |
| 950 | 102.7 | 73.6 | . 1412 | . 1012 | 3400 | 800.7 | 598.1 | . 4972 | . 3668 |
| 1000 | 115.4 | 82.8 | . 1542 | . 1106 | 3500 | 831.3 | 621.6 | . 5061 | . 3736 |
| 1050 | 128.2 | 92.0 | . 1667 | . 1196 | 3600 | 862.0 | 645.2 | . 5148 | . 3803 |
| 1100 | 141.0 | 101.3 | . 1787 | . 1283 | 3700 | 892.8 | 668.8 | . 5232 | . 3868 |
| 1150 | 153.9 | 110.7 | . 1902 | . 1367 | 3800 | 923.6 | 692.6 | . 5314 | . 3931 |
| 1200 | 166.9 | 120.2 | . 2013 | . 1447 | 3900 | 954.5 | 716.4 | . 5394 | . 3993 |
| 1250 | 180.0 | 129.7 | . 2120 | . 1525 | 4000 | 985.5 | 740.3 | . 5472 | . 4053 |
| 1300 | 193.2 | 139.3 | . 2223 | . 1600 | 4100 | 1016.5 | 764.2 | . 5549 | . 4112 |
| 1350 | 206.4 | 149.0 | . 2323 | . 1673 | 4200 | 1047.6 | 788.2 | . 5624 | . 4170 |
| 1400 | 219.7 | 158.8 | . 2419 | . 1744 | 4300 | 1078.8 | 812.3 | . 5697 | . 4227 |
| 1450 | 233.0 | 168.6 | . 2513 | . 1813 | 4400 | 1110.0 | 836.4 | . 5769 | . 4283 |
| 1500 | 246.5 | 178.5 | . 2604 | . 1880 | 4500 | 1141.2 | 860.6 | . 5839 | . 4337 |
| 1600 | 273.6 | 198.6 | . 2780 | . 2010 | 4600 | 1172.5 | 884.8 | . 5908 | . 4390 |
| 1700 | 301.1 | 218.9 | . 2947 | . 2134 | 4700 | 1203.8 | 909.1 | . 5975 | . 4442 |
| 1800 | 328.9 | 239.6 | . 3106 | . 2252 | 4800 | 1235.2 | 933.4 | . 6041 | . 4493 |
| 1900 | 357.0 | 260.6 | . 3258 | . 2366 | 4900 | 1266.7 | 957.7 | . 6106 | . 4543 |
| 2000 | 385.4 | 281.9 | . 3403 | . 2475 | 5000 | 1298.2 | 982.1 | . 6169 | . 4592 |
| 2100 | 414.0 | 303.4 | . 3542 | . 2580 | 5100 | 1329.7 | 1006.5 | . 6232 | . 4641 |
| 2200 | 442.8 | 325.1 | . 3676 | . 2681 | 5200 | 1361.2 | 1031.0 | . 6293 | . 4688 |
| 2300 | 471.7 | 347.0 | . 3805 | . 2778 | 5300 | 1392.8 | 1055.5 | . 6353 | . 4735 |
| 2400 | 500.9 | 369.1 | . 3929 | . 2872 | 5400 | 1424.4 | 1080.0 | . 6413 | . 4781 |
| 2500 | 530.3 | 391.3 | . 4049 | . 2963 | 5500 | 1456.0 | 1104.5 | . 6471 | . 4826 |

TABLE 4.4
The Pound of Oxygen

| T ${ }^{\circ} \mathrm{R}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ | T ${ }^{\circ} \mathrm{R}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 570 | 6.6 | 4.7 | . 0118 | . 0085 | 2600 | 518.1 | 390.3 | . 3838 | . 2863 |
| 600 | 13.2 | 9.5 | . 0232 | . 0167 | 2700 | 545.4 | 411.4 | . 3941 | . 2943 |
| 650 | 24.3 | 17.5 | . 0409 | . 0294 | 2800 | 572.8 | 432.6 | . 4041 | . 3020 |
| 700 | 35.5 | 25.6 | . 0575 | . 0414 | 2900 | 600.3 | 453.9 | . 4137 | . 3094 |
| 750 | 46.8 | 33.8 | . 0730 | . 0527 | 3000 | 627.8 | 475.3 | . 4231 | . 3167 |
| 800 | 58.1 | 42.0 | . 0877 | . 0633 | 3100 | 655.5 | 496.7 | . 4321 | . 3237 |
| 850 | 69.5 | 50.3 | . 1016 | . 0734 | 3200 | 683.3 | 518.3 | . 4409 | . 3306 |
| 900 | 81.1 | 58.8 | . 1148 | . 0831 | 3300 | 711.2 | 540.0 | . 4495 | . 3373 |
| 950 | 92.8 | 67.4 | . 1273 | . 0923 | 3400 | 739.2 | 561.8 | . 4579 | . 3438 |
| 1000 | 104.5 | 67.0 | . 1394 | . 1012 | 3500 | 767.3 | 583.7 | . 4660 | . 3501 |
| 1050 | 116.4 | 84.7 | . 1510 | . 1098 | 3600 | 795.5 | 605.6 | . 4739 | . 3563 |
| 1100 | 128.4 | 93.6 | . 1622 | . 1180 | 3700 | 823.8 | 627.7 | . 4817 | . 3623 |
| 1150 | 140.5 | 102.6 | . 1729 | . 1260 | 3800 | 852.1 | 649.9 | . 4893 | . 3682 |
| 1200 | 152.7 | 111.7 | . 1833 | . 1337 | 3900 | 880.6 | 672.2 | . 4967 | . 3740 |
| 1250 | 165.0 | 120.9 | . 1933 | . 1412 | 4000 | 909.1 | 694.5 | . 5039 | . 3797 |
| 1300 | 177.3 | 130.1 | . 2030 | . 1485 | 4100 | 937.8 | 716.9 | . 5109 | . 3852 |
| 1350 | 189.7 | 139.5 | . 2124 | . 1555 | 4200 | 966.5 | 739.5 | . 5178 | . 3906 |
| 1400 | 202.2 | 148.9 | . 2215 | . 1624 | 4300 | 995.3 | 762.1 | . 5246 | . 3959 |
| 1450 | 214.8 | 158.4 | . 2303 | . 1691 | 4400 | 1024.2 | 784.8 | . 5313 | . 4012 |
| 1500 | 227.5 | 167.9 | . 2389 | . 1756 | 4500 | 1053.2 | 807.5 | . 5378 | . 4063 |
| 1600 | 253.0 | 187.2 | . 2554 | . 1880 | 4600 | 1082.2 | 830.4 | . 5442 | . 4113 |
| 1700 | 278.7 | 206.7 | . 2710 | . 1998 | 4700 | 1111.4 | 853.3 | . 5505 | . 4162 |
| 1800 | 304.6 | 226.5 | . 2858 | . 2111 | 4800 | 1140.7 | 876.4 | . 5566 | . 4211 |
| 1900 | 330.8 | 246.5 | . 2999 | . 2219 | 4900 | 1170.0 | 899.6 | . 5626 | . 4258 |
| 2000 | 357.2 | 266.7 | . 3134 | . 2322 | 5000 | 1199.4 | 922.8 | . 5685 | . 4305 |
| 2100 | 383.7 | 287.0 | . 3264 | . 2421 | 5100 | 1228.9 | 946.1 | . 5744 | . 4351 |
| 2200 | 410.3 | 307.4 | . 3388 | . 2516 | 5200 | 1258.4 | 969.4 | . 5802 | . 4396 |
| 2300 | 437.1 | 327.9 | . 3507 | . 2608 | 5300 | 1288.1 | 992.8 | . 5858 | . 4441 |
| 2400 | 464.0 | 348.6 | . 3621 | . 2696 | 5400 | 1317.8 | 1016.3 | . 5914 | . 4485 |
| 2500 | 491.0 | 369.4 | . 3732 | . 2781 | 5500 | 1347.5 | 1039.8 | . 5969 | . 4529 |

TABLE 4.5
The Pound of Hydrogen

| T ${ }^{\circ} \mathrm{R}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ | T ${ }^{\circ} \mathrm{R}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 570 | 103 | 73 | 0.185 | 0.131 | 2600 | 7350 | 5321 | 5.554 | 3.996 |
| 600 | 206 | 147 | 0.361 | 0.258 | 2700 | 7732 | 5604 | 5.688 | 4.103 |
| 650 | 378 | 270 | 0.637 | 0.454 | 2800 | 8117 | 5891 | 5.827 | 4.207 |
| 700 | 551 | 393 | 0.892 | 0.637 | 2900 | 8504 | 6180 | 5.963 | 4.308 |
| 750 | 724 | 517 | 1.131 | 0.808 | 3000 | 8894 | 6472 | 6.095 | 4.407 |
| 800 | 896 | 640 | 1.355 | 0.968 | 3100 | 9286 | 6766 | 6.224 | 4.503 |
| 850 | 1069 | 764 | 1.565 | 1.118 | 3200 | 9681 | 7062 | 6.349 | 4.597 |
| 900 | 1243 | 888 | 1.763 | 1.260 | 3300 | 10079 | 7361 | 6.472 | 4.689 |
| 950 | 1416 | 1013 | 1.951 | 1.395 | 3400 | 10479 | 7663 | 6.591 | 4.780 |
| 1000 | 1590 | 1137 | 2.129 | 1.523 | 3500 | 10881 | 7967 | 6.708 | 4.868 |
| 1050 | 1764 | 1261 | 2.299 | 1.644 | 3600 | 11286 | 8273 | 6.822 | 4.954 |
| 1100 | 1937 | 1386 | 2.461 | 1.760 | 3700 | 11693 | 8582 | 6.934 | 5.038 |
| 1150 | 2111 | 1511 | 2.615 | 1.870 | 3800 | 12102 | 8893 | 7.043 | 5.121 |
| 1200 | 2286 | 1636 | 2.763 | 1.976 | 3900 | 12514 | 9206 | 7.149 | 5.202 |
| 1250 | 2460 | 1761 | 2.905 | 2.078 | 4000 | 12927 | 9521 | 7.253 | 5.282 |
| 1300 | 2635 | 1887 | 3.042 | 2.176 | 4100 | 13342 | 9837 | 7.356 | 5.360 |
| 1350 | 2810 | 2013 | 3.173 | 2.271 | 4200 | 13759 | 10155 | 7.457 | 5.437 |
| 1400 | 2985 | 2139 | 3.300 | 2.362 | 4300 | 14178 | 10476 | 7.556 | 5.513 |
| 1450 | 3161 | 2265 | 3.423 | 2.450 | 4400 | 14598 | 10798 | 7.652 | 5.587 |
| 1500 | 3337 | 2392 | 3.542 | 2.536 | 4500 | 15020 | 11122 | 7.747 | 5.660 |
| 1600 | 3689 | 2646 | 3.771 | 2.701 | 4600 | 15444 | 11447 | 7.840 | 5.731 |
| 1700 | 4044 | 2902 | 3.986 | 2.857 | 4700 | 15870 | 11774 | 7.932 | 5.802 |
| 1800 | 4401 | 3160 | 4.190 | 3.005 | 4800 | 16297 | 12103 | 8.002 | 5.871 |
| 1900 | 4761 | 3421 | 4.385 | 3.146 | 4900 | 16726 | 12434 | 8.110 | 5.939 |
| 2000 | 5123 | 3685 | 4.570 | 3.281 | 5000 | 17156 | 12765 | 8.197 | 6.006 |
| 2100 | 5488 | 3952 | 4.747 | 3.410 | 5100 | 17588 | 13098 | 8.283 | 6.072 |
| 2200 | 5855 | 4221 | 4.918 | 3.535 | 5200 | 18021 | 13432 | 8.367 | 6.137 |
| 2300 | 6225 | 4493 | 5.083 | 3.656 | 5300 | 18455 | 13768 | 8.449 | 6.201 |
| 2400 | 6597 | 4766 | 5.242 | 3.773 | 5400 | 18890 | 14105 | 8.531 | 6.264 |
| 2500 | 6972 | 5042 | 5.395 | 3.886 | 5500 | 19326 | 14443 | 8.611 | 6.326 |

TABLE 4.6
The Pound of Carbon Monoxide

| T ${ }^{\circ} \mathrm{R}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ | T ${ }^{\circ} \mathrm{R}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 570 | 7.5 | 5.3 | . 0134 | . 0096 | 2600 | 566.2 | 420.2 | . 4206 | . 3092 |
| 600 | 15.0 | 10.7 | . 0262 | . 0189 | 2700 | 596.2 | 443.1 | . 4319 | . 3179 |
| 650 | 27.5 | 19.6 | . 0460 | . 0329 | 2800 | 626.4 | 466.1 | . 4429 | . 3263 |
| 700 | 40.0 | 28.6 | . 0646 | . 0462 | 2900 | 656.6 | 489.3 | . 4535 | . 3344 |
| 750 | 52.5 | 37.6 | . 0820 | . 0587 | 3000 | 687.0 | 512.6 | . 4638 | . 3423 |
| 800 | 65.1 | 46.6 | . 0982 | . 0703 | 3100 | 717.5 | 536.1 | . 4738 | . 3499 |
| 850 | 77.7 | 55.6 | . 1135 | . 0813 | 3200 | 7481 | 559.6 | . 4835 | . 3573 |
| 900 | 90.3 | 64.7 | . 1279 | . 0917 | 3300 | 778.8 | 583.2 | . 4929 | . 3646 |
| 950 | 103.0 | 73.9 | . 1417 | . 1016 | 3400 | 809.6 | 606.8 | . 5021 | . 3717 |
| 1000 | 115.8 | 83.2 | . 1548 | . 1111 | 3500 | 840.4 | 630.6 | . 5111 | . 3786 |
| 1050 | 128.7 | 92.6 | . 1674 | . 1203 | 3600 | 871.3 | 654.4 | . 5198 | . 3853 |
| 1100 | 141.7 | 102.1 | . 1796 | . 1292 | 3700 | 902.3 | 678.3 | . 5283 | . 3919 |
| 1150 | 154.8 | 111.6 | . 1913 | . 1377 | 3800 | 933.3 | 702.2 | . 5366 | . 3983 |
| 1200 | 168.0 | 121.2 | . 2025 | . 1459 | 3900 | 964.4 | 726.2 | . 5447 | . 4045 |
| 1250 | 181.3 | 130.9 | . 2133 | . 1538 | 4000 | 995.6 | 750.3 | . 5526 | . 4106 |
| 1300 | 194.6 | 140.7 | . 2237 | . 1614 | 4100 | 1026.8 | 774.5 | . 5603 | . 4166 |
| 1350 | 208.0 | 150.6 | . 2338 | . 1688 | 4200 | 1058.1 | 798.7 | . 5678 | . 4224 |
| 1400 | 221.5 | 160.6 | . 2436 | . 1761 | 4300 | 1089.4 | 823.0 | . 5752 | . 4281 |
| 1450 | 235.1 | 170.6 | . 2532 | . 1832 | 4400 | 1120.8 | 847.3 | . 5824 | . 4337 |
| 1500 | 248.8 | 180.7 | . 2625 | . 1901 | 4500 | 1152.3 | 871.6 | . 5895 | . 4392 |
| 1600 | 276.4 | 201.2 | . 2803 | . 2033 | 4600 | 1183.8 | 896.0 | . 5964 | . 4446 |
| 1700 | 304.3 | 222.0 | . 2972 | . 2159 | 4700 | 1215.3 | 920.4 | . 6032 | . 4498 |
| 1800 | 332.4 | 243.1 | . 3134 | . 2280 | 4800 | 1246.8 | 944.9 | . 6098 | . 4549 |
| 1900 | 360.9 | 264.5 | . 3288 | . 2396 | 4900 | 1278.4 | 969.4 | . 6163 | . 4599 |
| 2000 | 389.6 | 286.1 | . 3435 | . 2507 | 5000 | 1310.0 | 993.9 | . 6227 | . 4649 |
| 2100 | 418.6 | 307.9 | . 3576 | . 2613 | 5100 | 1341.7 | 1018.5 | . 6290 | . 4698 |
| 2200 | 447.7 | 330.0 | . 3712 | . 2716 | 5200 | 1373.4 | 1043.1 | . 6351 | . 4746 |
| 2300 | 477.1 | 352.3 | . 3842 | . 2815 | 5300 | 1405.2 | 1067.8 | . 6412 | . 4793 |
| 2400 | 506.7 | 374.8 | . 3968 | . 2910 | 5400 | 1436.9 | 1092.4 | . 6472 | . 4840 |
| 2500 | 536.4 | 397.4 | . 4089 | . 3002 | 5500 | 1468.6 | 1117.1 | . 6531 | . 4886 |

TABLE 4.7
Tue Pound of Carbon Dioxide

| T ${ }^{\circ} \mathrm{R}$ | $h$ | $u$ | $s_{p}$ | $s v$ | T ${ }^{\circ} \mathbf{R}$ | $h$ | $u$ | $s_{p}$ | $\because v$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 570 | 6.1 | 4.8 | . 0111 | . 0087 | 2600 | 572.1 | 479.2 | . 4135 | . 3426 |
| 600 | 12.4 | 9.7 | . 0218 | . 0171 | 2700 | 603.9 | 506.4 | . 4255 | . 3528 |
| 650 | 23.1 | 18.1 | . 0389 | . 0305 | 2800 | 635.8 | 533.8 | . 4371 | . 3628 |
| 700 | 34.1 | 26.8 | .0552 | . 0435 | 2900 | (667.8 | 561.4 | . 4483 | . 3725 |
| 750 | 45.4 | 35.8 | . 0708 | . 0559 | 3000 | 700.0 | 589.1 | . 4592 | . 3819 |
| 800 | 56.9 | 45.1 | . 0857 | . 0679 | 3100 | 732.4 | 616.9 | . 4698 | . 3910 |
| 850 | 68.7 | 54.6 | . 0999 | . 0794 | 3200 | 764.9 | 644.9 | . 4801 | . 3999 |
| 900 | 80.7 | 64.4 | . 1136 | . 0906 | 3300 | 797.5 | 673.0 | . 4902 | . 4085 |
| 950 | 92.9 | 74.4 | . 1268 | . 1014 | 3400 | 830.2 | 701.2 | . 5000 | . 4169 |
| 1000 | 105.4 | 84.6 | . 1396 | . 1118 | 3500 | 863.1 | 729.5 | . 5095 | . 4252 |
| 1050 | 118.1 | 95.0 | . 1520 | . 1220 | 3600 | 896.0 | 757.9 | . 5188 | . 4332 |
| 1100 | 130.9 | 105.6 | . 1639 | . 1318 | 3700 | 929.0 | 786.4 | . 5278 | . 4410 |
| 1150 | 143.9 | 116.4 | . 1755 | . 1414 | 3800 | 962.1 | 815.0 | . 5366 | . 4486 |
| 1200 | 157.1 | 127.3 | . 1868 | . 1508 | 3900 | 995.3 | 843.7 | . 5152 | . 4560 |
| 1250 | 170.5 | 138.4 | . 1977 | . 1599 | 4000 | 1028.5 | 872.4 | . 5536 | . 4633 |
| 1300 | 184.0 | 149.7 | . 2083 | . 1687 | 4100 | 1061.8 | 901.2 | . 5618 | . 4704 |
| 1350 | 197.7 | 161.1 | . 2186 | . 1773 | 4200 | 1095.2 | 930.1 | . 5698 | . 4774 |
| 1400 | 211.5 | 172.7 | . 2287 | . 1857 | 4300 | 1128.7 | 959.1 | . 5777 | . 4842 |
| 1450 | 225.4 | 184.4 | . 2385 | . 1939 | 4400 | 1162.2 | 988.1 | . 5854 | . 4909 |
| 1500 | 239.5 | 196.2 | . 2479 | . 2019 | 4500 | 1195.8 | 1017.1 | . 5930 | . 4974 |
| 1600 | 268.0 | 220.2 | . 2664 | . 2174 | 4600 | 1229.4 | 1046.2 | . 6004 | . 5038 |
| 1700 | 297.0 | 244.7 | . 2840 | . 2323 | 4700 | 1263.1 | 1075.4 | . 6077 | . 5101 |
| 1800 | 326.4 | 269.6 | . 3008 | . 2465 | 4800 | 1296.9 | 1104.7 | . 6148 | . 5162 |
| 1900 | 356.1 | 294.8 | . 3168 | . 2601 | 4900 | 1330.7 | 1134.0 | . 6218 | . 5223 |
| 2000 | 386.2 | 320.4 | . 3322 | . 2732 | 5000 | 1364.6 | 1163.4 | . 6286 | . 5282 |
| 2100 | 416.6 | 346.2 | . 3470 | . 2858 | 5100 | 1398.6 | 1192.9 | . 6353 | . 5340 |
| 2200 | 447.2 | 372.3 | . 3613 | . 2980 | 5200 | 1432.6 | 1222.4 | . 6419 | . 5398 |
| 2300 | 478.1 | 398.7 | . 3751 | . 3097 | 5300 | 1466.6 | 1251.9 | . 6484 | . 5454 |
| 2400 | 509.2 | 425.3 | . 3883 | . 3210 | 5400 | 1500.7 | 1281.4 | . 6548 | . 5509 |
| 2500 | 540.5 | 452.1 | . 4011 | . 3320 | 5500 | 1534.8 | 1311.0 | . 6611 | . 5564 |

TABLE 4.8
The Pound of Steam

| T ${ }^{\circ} \mathrm{R}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ | T ${ }^{\circ} \mathrm{R}$ | $h$ | 14 | $s_{p}$ | $s_{v}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 570 | 13.3 | 10.0 | . 0239 | . 0181 | 2600 | 1083.5 | 856.5 | 0.7916 | 0.6184 |
| 600 | 26.7 | 20.1 | . 0468 | . 0352 | 2700 | 1145.3 | 907.3 | 0.8149 | 0.6376 |
| 650 | 49.2 | 37.0 | . 0828 | . 0623 | 2800 | 1207.8 | 958.8 | 0.8377 | 0.6564 |
| 700 | 71.8 | 54.1 | . 1163 | . 0877 | 2900 | 1271.1 | 1011.0 | 0.8600 | 0.6748 |
| 750 | 94.5 | 71.3 | . 1476 | . 1115 | 3000 | 1335.0 | 1063.9 | 0.8817 | 0.6928 |
| 800 | 117.4 | 88.7 | . 1771 | . 1339 | 3100 | 1399.6 | 1117.5 | 0.9029 | 0.7104 |
| 850 | 140.4 | 106.2 | . 2050 | . 1551 | 3200 | 1464.8 | 1171.7 | 0.9236 | 0.7275 |
| 900 | 163.6 | 123.9 | . 2315 | . 1752 | 3300 | 1530.5 | 1226.4 | 0.9438 | 0.7443 |
| 950 | 186.9 | 141.8 | . 2567 | . 1945 | 3400 | 1596.7 | 1281.6 | 0.9636 | 0.7608 |
| 1000 | 210.5 | 159.9 | . 2809 | . 2130 | 3500 | 1663.5 | 1337.3 | 0.9829 | 0.7770 |
| 1050 | 234.3 | 178.2 | . 3042 | . 2309 | 3600 | 1730.8 | 1393.6 | 1.0019 | 0.7929 |
| 1100 | 258.4 | 196.7 | . 3266 | . 2482 | 3700 | 1798.5 | 1450.4 | 1.0205 | 0.8085 |
| 1150 | 282.7 | 215.4 | . 3481 | . 2649 | 3800 | 1866.8 | 1507.6 | 1.0387 | 0.8238 |
| 1200 | 307.2 | 234.4 | . 3689 | . 2810 | 3900 | 1935.5 | 1565.3 | 1.0566 | 0.8388 |
| 1250 | 331.9 | 253.6 | . 3891 | . 2966 | 4000 | 2004.6 | 1623.4 | 1.0741 | 0.8535 |
| 1300 | 356.8 | 273.0 | . 4086 | . 3118 | 4100 | 2074.1 | 1681.9 | 1.0913 | 0.8679 |
| 1350 | 381.9 | 292.6 | . 4276 | . 3266 | 4200 | 2143.9 | 1740.7 | 1.1081 | 0.8821 |
| 1400 | 407.2 | 312.4 | . 4460 | . 3410 | 4300 | 2214.1 | 1799.8 | 1.1247 | 0.8960 |
| 1450 | 432.7 | 332.4 | . 4640 | . 3551 | 4400 | 2284.6 | 1859.3 | 1.1409 | 0.9097 |
| 1500 | 458.4 | 352.6 | .$^{.4815}$ | . 3689 | 4500 | 2355.4 | 1919.1 | 1.1568 | 0.9232 |
| 1600 | 510.5 | 393.8 | . 5151 | . 3955 | 4600 | 2426.6 | 1979.2 | 1.1725 | 0.9365 |
| 1700 | 563.7 | 435.9 | . 5473 | . 4210 | 4700 | 2498.1 | 2039.7 | 1.1879 | 0.9496 |
| 1800 | 617.9 | 479.1 | . 5783 | . 4456 | 4800 | 2569.9 | 2100.5 | 1.2031 | 0.9624 |
| 1900 | 673.1 | 523.2 | . 6081 | . 4694 | 4900 | 2642.0 | 2161.6 | 1.2180 | 0.9750 |
| 2000 | 729.2 | 568.3 | . 6369 | . 4926 | 5000 | 2714.3 | 2222.9 | 1.2326 | 0.9874 |
| 2100 | 786.2 | 614.2 | . 6647 | . 5151 | 5100 | 2786.9 | 2284.5 | 1.2470 | 0.9997 |
| 2200 | 844.0 | 661.0 | . 6917 | . 5369 | 5200 | 2859.7 | 2346.3 | 1.2612 | 1.0117 |
| 2300 | 902.6 | 708.7 | . 7178 | . 5581 | 5300 | 2932.7 | 2408.3 | 1.2751 | 1.0235 |
| 2400 | 962.1 | 757.2 | . 7431 | . 5787 | 5400 | 3005.9 | 2470.4 | 1.2888 | 1.0351 |
| 2500 | 1022.4 | 806.5 | . 7677 | . 5988 | 5500 | 3079.3 | 2532.8 | 1.3024 | 1.0466 |

TABLE 4.9
The Pound of Combustion Mixture

| T ${ }^{\circ} \mathrm{R}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ | T ${ }^{\circ} \mathrm{R}$ | $h$ | $u$ | $s_{p}$ | $s_{v}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 570 | 7.6 | 5.6 | . 0137 | . 0100 | 2600 | 595.5 | 453.5 | . 4400 | . 3317 |
| 600 | 15.2 | 11.1 | . 0268 | . 0196 | 2700 | 627.6 | 478.7 | . 4521 | . 3412 |
| 650 | 28.0 | 20.5 | . 0471 | . 0344 | 2800 | 659.9 | 504.1 | . 4639 | . 3504 |
| 700 | 40.9 | 29.9 | . 06662 | . 0483 | 2900 | 692.4 | 529.7 | . 4753 | . 3594 |
| 750 | 53.8 | 39.3 | . 0841 | . 0614 | 3000 | 725.0 | 555.4 | . 4863 | . 3681 |
| 800 | 66.8 | 48.9 | . 1009 | . 0738 | 3100 | 757.8 | 581.3 | . 4971 | . 3766 |
| 850 | 79.9 | 58.5 | . 1168 | . 0855 | 3200 | 790.7 | 607.3 | . 5075 | . 3848 |
| 900 | 93.1 | 68.3 | . 1318 | . 0966 | 3300 | 823.7 | 633.5 | . 5177 | . 3929 |
| 950 | 106.4 | 78.1 | . 1461 | . 1072 | 3400 | 856.9 | 659.8 | . 5276 | . 4008 |
| 1000 | 119.7 | 88.0 | . 1598 | . 1173 | 3500 | 890.2 | 686.2 | . 5373 | . 4085 |
| 1050 | 133.2 | 98.0 | . 1730 | . 1271 | 3600 | 923.7 | 712.8 | . 5467 | . 4160 |
| 1100 | 146.8 | 108.2 | . 1856 | . 1366 | 3700 | 957.3 | 739.4 | . 5559 | . 4233 |
| 1150 | 160.4 | 118.4 | . 1978 | . 1457 | 3800 | 990.9 | 766.2 | . 5649 | . 4304 |
| 1200 | 174.2 | 128.7 | . 2095 | . 1545 | 3900 | 1024.7 | 793.1 | . 5737 | . 4374 |
| 1250 | 188.1 | 139.2 | . 2208 | . 1630 | 4000 | 1058.4 | 820.0 | . 5822 | . 4442 |
| 1300 | 202.1 | 149.7 | . 2318 | . 1713 | 4100 | 1092.4 | 847.0 | . 5906 | . 4508 |
| 1350 | 216.2 | 160.3 | . 2424 | . 1793 | 4200 | 1126.4 | 874.1 | . 5988 | . 4574 |
| 1400 | 230.3 | 171.0 | . 2527 | . 1871 | 4300 | 1160.5 | 901.3 | . 6068 | . 4638 |
| 1450 | 244.6 | 181.8 | . 2628 | . 1947 | 4400 | 1194.7 | 928.6 | . 6147 | . 4700 |
| 1500 | 258.9 | 192.7 | . 2725 | . 2020 | 4500 | 1228.9 | 955.9 | . 6224 | . 4762 |
| 1600 | 287.9 | 214.9 | . 2912 | . 2164 | 4600 | 1263.1 | 993.3 | . 6299 | . 4823 |
| 1700 | 317.3 | 237.4 | . 3090 | . 2300 | 4700 | 1297.5 | 1010.8 | . 6373 | . 4882 |
| 1800 | 347.1 | 260.2 | . 3260 | . 2430 | 4800 | 1331.9 | 1038.3 | . 6445 | . 4940 |
| 1900 | 377.2 | 283.4 | . 3423 | . 2556 | 4900 | 1366.4 | 1065.9 | . 6516 | . 4996 |
| 2000 | 407.6 | 307.0 | . 3579 | . 2677 | 5000 | 1401.0 | 1093.6 | . 6586 | . 5052 |
| 2100 | 438.3 | 330.8 | . 3729 | . 2793 | 5100 | 1435.6 | 1121.3 | . 6655 | . 5107 |
| 2200 | 469.3 | 354.9 | . 3873 | . 2895 | 5200 | 1470.3 | 1149.1 | . 6722 | . 5161 |
| 2300 | 500.5 | 379.2 | . 4012 | . 3013 | 5300 | 1505.0 | 1176.9 | . 6788 | . 5214 |
| 2400 | 531.9 | 403.7 | . 4146 | . 3118 | 5400 | 1539.7 | 1204.7 | . 6853 | . 5266 |
| 2500 | 563.6 | 428.5 | . 4275 | . 3219 | 5500 | 1574.5 | 1332.6 | . 6917 | . 5317 |

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2. Sheldon, "Properties of Mercury Vapor," Bulletin of the General Electric Company.
3. "Tables of Thermodynamic Properties of Ammonia," Bureau of Standards Circular 142.
4. Heck, The New Specific Heats.

Mechanical Engineering, January, 1940, and February, 1941.
The gas tables included in this section were calculated by Dr. R. C. H. Heck, Professor Emeritus of Mechanical Engmeering, Rutgers University.

## Suggested Reading

Properties of Vapors
Chaig and Anderson, Steam Power and Internal Combustion Engines. Pages 32-40. Moyer, Calderwoon, and Potter, Elements of Engineering Thermodynamics. Pages 81-88.
Barnard, Ellenwood, and Mirshfeld, Heat Power Engineering. Part I, Chapter XV.

Steam
Norris and Therkelsen, Heat Power. Chapter XI.
Variable Specific Heats of Gases, based on 1924 data now replaced by spectroscopic data of 1938.
Barnard, Ellenwood, and Hirshfeld, Heat Power Engineering. Pages 79-83.

## CHAPTER 5

## GAS AND VAPOR MIXTURES

5.1. Theory of Gas Mixtures. In the permanent gases the molecules have dimensions so small in comparison with the intermolecular distances that they may be regarded as points. This picture of a gas led Dalton to conclude that if two or more gases that do not react chemically are mixed together in the same space, they do not interfere with each other. This conclusion that each gas acts precisely as if it alone filled the space, and that the resulting pressure is the sum of all the partial pressures of the individual gases is amply verified by experiment. It applies accurately to the permanent gases and to all but the heavicst vapors.

To find the pressure exerted by a gas mixture it is only necessary to find the partial pressure each constituent would exert if it filled the same space at the same temperature and then to sum up all the partial pressures.

From the hypothesis of Avogadro that equal volumes of gas at the same conditions have the same number of molecules and the concept of a mol developed in article $2 \cdot 6$ a useful conclusion may be reached. From the mol form of the general gas equation,

$$
P=\frac{1544 T}{V} \frac{W}{\text { Mol. wt. }}
$$

it may be seen that when several gases fill the same volume $V$ at the same temperature $T$, the partial pressure exerted by any constituent varies as its $W / \mathrm{mol}$. wt. or as the number of mols of that gas present.

This may be illustrated by considering air at atmospheric pressure of 14.7 lb per sq in . Since air is essentially a mixture of 0.23 oxygen and 0.77 nitrogen by weight, one pound of air contains $\frac{0.23}{32}=0.0072 \mathrm{mols}$ of $\mathrm{O}_{2}$ and $\frac{0.77}{28}=0.0275 \mathrm{mols}$ of $\mathrm{N}_{2}$. In one mol of air there would then be $\frac{0.0072}{0.0275+0.0072}=0.21$ (approx.) mols of $\mathrm{O}_{2}$ and $\frac{0.0275}{0.0275+0.0072}=0.79$ (approx.) mols of $\mathrm{N}_{2}$. This would mean that $0.21 \times 14.7=3.1 \mathrm{lb}$ per
sq in . pressure is exerted by the oxygen and $0.79 \times 14.7=11.6 \mathrm{lb}$ per sq in. by the nitrogen.

As a general thing gas mixtures are analyzed on a volumetric basis. If a gas mixture has a volumetric analysis of $\mathrm{CO}_{2}=0.12$ and $\mathrm{N}_{2}=0.88$, it would mean that a mol of the mixture would contain 0.12 mols or $0.12 \times 44=5.28 \mathrm{lb}$ of $\mathrm{CO}_{2}$ and 0.88 mols or $0.88 \times 28=24.64 \mathrm{lb}$ of $\mathrm{N}_{2}$. This would correspond to a weight composition of

$$
\frac{5.28}{5.28+24.64}=0.177 \mathrm{O}_{2} \text { and } \frac{24.64}{5.28+24.64}=0.823 \mathrm{~N}_{2}
$$

It is often desirable to simplify the calculations for a gas mixture by considering it to be a single gas with appropriate constants. For example, this scheme has been used for air in all the tables of gas properties so far listed. The equivalent values for these constants may be found from the general gas equation. For a mixture of three gases occupying a volume $V$ at a temperature $T$,

$$
P_{1}=\frac{W_{1} R_{1} T}{V} \quad P_{2}=\frac{W_{2} R_{2} T}{V} \quad P=\frac{W_{3} R_{3} T}{V}
$$

The total pressure is the sum of the partial pressures, so

$$
\Sigma P=\frac{T}{V}\left(W_{1} R_{1}+W_{2} R_{2}+W_{3} R_{3}\right)
$$

Multiplying through this equation by the total weight $\Sigma W$ puts it in the form of the general gas equation,

$$
\Sigma P \times V=\Sigma W\left(\frac{W_{1} R_{1}+W_{2} R_{2}+W_{3} R_{3}}{\Sigma W}\right) T \cdots
$$

It then follows that the equivalent $R$ for the gas mixture is

$$
R=\frac{W_{1} R_{1}+W_{2} R_{2}+W_{3} R_{3} \cdots}{W_{1}+W_{2}+W_{3}+\cdots}
$$

Equation $5 \cdot 3$ is a convenient form when the weight composition of the gas mixture is known. When a volume composition is known it is more convenient to sum up the total weight of a mol of mixture and use $R=1544 / \mathrm{mol}$. wt.

The gas mixture of $\mathrm{CO}_{2}=0.12$ and $\mathrm{N}_{2}=0.88$ by volume, previously referred to, would have a mol weight of $0.12 \times 4.4+0.88 \times 28=29.92$ lb and the corresponding value of the gas constant would be $R=$ $1544 / \mathrm{mol} . \mathrm{wt} .=1544 / 29.92=51.6$.

The equivalent specific heat of such a mixture would be, in a similar manner,

$$
\begin{equation*}
c_{v}=\frac{W_{1} c_{v 1}+W_{2} c_{v 2}+W_{3} c_{v 3} \cdots}{W_{1}+W_{2}+W_{3} \cdots} \tag{5•4}
\end{equation*}
$$

If the volumetric analysis is known, it is often convenient to calculate the mol specific heat and divide it by the cquivalent molecular weight. If $V_{1}, V_{2}, \cdots$, represent the fractions of each gas by volume, the mol specific heat is

$$
C_{p}=V_{1} C_{p 1}+V_{2} C_{p 2} \cdots
$$

5.2. Mixing Processes. Many gas mixtures such as air and natural gas occur as such, but there are also many engineering processes where gases are mixed together. When such a mixing of gases at different initial conditions occurs, it is necessary to be able to anticipate the final conditions at which the mixture will exist.

The simplest example of mixing is that of two quantities of gas existing at the same initial temperature and pressure that have the same total pressure and the original temperature after mixing. The initial volumes would be $V_{1}=W_{1} R_{1} T / P$ and $V_{2}=W_{2} R_{2} T / P$. Since $T$ is to remain constant and together they exert the initial pressure $P$, they would have to occupy a volume $V_{1}+V_{2}$ equal to the sum of their initial volumes. This is the simplest kind of diffusion of one gas into another.

Slightly more involved constant-pressure mixing occurs when an incoming charge of new air at atmospheric pressure and at about 150 to 200 deg F mixes with the hot products of combustion in the clearance volume of a Diesel engine. At the time the intake valve opens to admit the new air charge the products of combustion fill the clearance space at atmospheric pressure and at a temperature as high or higher than 1000 $\operatorname{deg} \mathrm{F}$. The problems involved in such a situation as this are to predict the final temperature of the mixture and the amount of new air drawn into a cylinder of known volume and clearance space.

Since such a mixing of $W_{1}$ pounds of gas at a lower temperature $T_{1}$ with $W_{2}$ pounds at a higher temperature $T_{2}$ is at constant pressure, the final temperature $T_{m}$ of the mixture would be given by

$$
W_{1} c_{p 1}\left(T_{m}-T_{1}\right)=W_{2} c_{p 2}\left(T_{2}-T_{m}\right)
$$

The total volume occupied by the mixture at a pressure $P$ would be

$$
\begin{equation*}
\text { Total volume }=\frac{W_{1} R_{1} T_{m}}{P}+\frac{W_{2} R_{2} T_{m}}{P} \tag{5•7}
\end{equation*}
$$

In such a problem it is usually possible to calculate the weight of clear-
ance gases $W_{2}$ from initial conditions, and since the final total volume would be known, the weight of air $W_{1}$ could be calculated.

This analysis assumes that no heat is conducted to the mixture from the cylinder wall during the mixing time and that the temperatures involved are low enough to permit the use of the assumption that $c_{p}$ is constant. A high-temperature problem would have to be solved by seeking in the gas tables for a final temperature such that

$$
W_{1} \Delta h_{1}=W_{2} \Delta h_{2}
$$

and this would be a trial-and-error or a graphical solution.
5.3. Vapor Mixture. The principles of mixtures of articles $5 \cdot 1$ and $5 \cdot 2$ and the gas equations may be used to investigate gas mixtures under any conditions met in practice. However, when there are vapors in the mixture, as is usually true, a compression or cooling may result in vapor condensation or a heating or expansion may cause vapor evaporation. In such circumstances the methods of calculation must be modified.

As long as the vapor is superheated and at a low pressure, the partial pressure exerted by it may be calculated from the amount of vapor present, just as if it were a gas. However, should the temperature of the mixture drop below the saturation temperature corresponding to this pressure, condensation of part of the vapor will occur. This condensation will continue until the partial pressure as calculated from the mols remaining in the vapor form corresponds to the pressure that the liquid is capable of exerting at that temperature as found from vapor tables. This temperature at which condensation of the vapor content of the mixture starts is known as its dew point temperature, and the mixture in that condition is known as saturated with vapor.

The dew point of atmospheric pressure air that contains 0.01 lb of water vapor per pound of dry air may be found as follows. One pound of dry air with an equivalent molecular weight of 29 contains $\frac{1}{29}=0.0345$ mols of air, and 0.01 lb of water vapor represents $\frac{0.01}{18}=0.00056 \mathrm{mols}$. This is $\frac{0.00056}{0.0345+0.00056}=0.016 \mathrm{mols}$ of water vapor per mol of mixture and the partial pressure exerted by the water vapor is $0.016 \times 14.7=$ 0.237 lb per sq in. From steam tables, a pressure of 0.237 lb per sq in. corresponds to a saturation temperature or dew point of 58 deg F , which is the lowest temperature to which this moist air can be cooled without condensation.

If it is cooled to 40 deg F the maximum partial pressure that the water vapor can exert is 0.1217 lb per sq in . as found in steam tables. The
volume filled by 1 lb of vapor at this condition is $V_{g}=2445 \mathrm{cu} \mathrm{ft}$ according to the vapor tables. If the volume is calculated from the gas equation using $R=85.7$ it is

$$
V_{g}=\frac{W R T}{P}=\frac{1 \times 85.7 \times(40+460)}{0.1217 \times 144}=2450 \mathrm{cu} \mathrm{ft}
$$

which illustrates how closely low-pressure vapors conform to the gas equations. The dry air that occupies this same space of 2445 cu ft exerts a partial pressure of $14.70-0.1217=14.58 \mathrm{lb}$ per sq in . and has a weight of

$$
W=\frac{P V}{R T}=\frac{14.58 \times 144 \times 2445}{53.3 \times(40+460)}=193 \mathrm{lb}
$$

This means that only $\frac{1}{193}=0.0052 \mathrm{lb}$ of water per pound of dry air remains in the form of saturated vapor at 40 deg F , and $0.01-0.0052=$ 0.0048 lb of moisture per pound of dry air was condensed as the mixture cooled from 58 to 40 deg F.

The mist of gasoline seen in the venturi throat of an internal-combustion engine carburetor or the water mist in fogs or clouds are mixtures of vapor and finely divided liquid


Fig. 5•1. Vapor pressure of liquid fuels. in air. Since the vapors are colorless the mist or cloud represents atomized liquid in suspension. The moisture in the air or in the products of combustion when fuels are burned normally exists as superheated vapor and its presence is apparent only when cooled below its dew point temperature.

In actual practice, the successful formation of saturated vapor-gas mixtures requires a well atomized spray of liquid, thorough mixing, and enough elapsed time to complete the vaporization process. Although this is not feasible in many applications, in this text equilibrium conditions are assumed to be established.

The factors involved in the formation of a saturated air-fuel vapor mixture for an engine or furnace may be illustrated by considering a saturated mixture that is 10 lb of dry air per pound of ethyl alcohol vapor (Figure $5 \cdot 1$ and Table 5•1). Since the molecular weight of $\mathrm{C}_{2} \mathrm{H}_{6} \mathrm{O}$ is 46 and the equivalent value for air is 29 , this would correspond
to a mixture that has $\frac{1}{46}=0.0217 \mathrm{mols}$ of alcohol vapor for each $\frac{10}{29}$ or 0.345 mols of air. If it exists at atmospheric pressure, the vapor pressure of the alcohol would be $\frac{0.0217}{0.0217+0.345} \times 14.7=0.87 \mathrm{lb}$ per sq in. From Figure $5 \cdot 1$, the temperature corresponding to this pressure is 68 $\operatorname{deg} \mathrm{F}$ which is the lowest temperature at which 1 lb of alcohol vapor can exist in 10 lb of dry air.

An even more important problem is that of finding the initial temperature at which the air must be supplied if this mixture is to be formed without external heating. This means that enough heat must be given up by the air and liquid fuel to furnish the latent heat of vaporization of the fuel (Table $5 \cdot 1$ ). If it is known that the liquid fuel is supplied at

TABLE 5•1

| Fuel | Average Latent Ifeat of <br> Vaporization <br> (Btu per lb ) | Specific Heat of <br> liquid Fuel <br> (Btu per lb per deg F) |
| :--- | :---: | :---: |
| Gasoline | $135-145$ | 0.58 |
| Ethyl alcohol |  |  |
| Kerosene | 410 | 0.50 |
| $105-110$ | 0.50 |  |

80 deg F , and the initial air temperature is represented by $t_{a}$ degrees Fahrenheit, it is possible to write a heat balance equation for a constantpressurc process,
wt. fuel $\times$ sp. heat fuel $\times(80-68)+$ wt. air $\times c_{p}$ air $\left(t_{a}-68\right)$
$=w t$. fuel $\times$ latent heat of vaporization of fucl [5.9]
or

$$
\begin{gathered}
1 \times 0.50 \times(80-68)+10 \times 0.24 \times\left(t_{a}-68\right)=1 \times 410 \\
t_{a}=278 \operatorname{deg} \mathrm{~F}
\end{gathered}
$$

If it should not be feasible to heat the incoming air to 278 deg F, it would then be necessary to supply external heat, the amount of which could be predicted for any set of conditions, by writing a similar heatbalance equation.
5.4. Atmospheric Moisture. Many times in engineering practice it is necessary to determine the moisture content of air by experimental methods. An obvious solution would be to cool a sample to its dew point and from this temperature calculate the moisture content. However, this involves practical difficulties because, if there are no dust
particles or other nuclei to promote the formation of water drops, it is possible to cool the mixture well below its dew point with no condensation. This is an unstable condition known as supersaturation and a disturbance will usually cause precipitation of moisture.

The experimental difficultios involved in an accurate dew point temperature determination have led to the use of other methods for determining the moisture content of air. If water is available in unsaturated air, evaporation will continue until the partial pressure of the vapor is equal to the vapor pressure of the liquid. If no external heat is supplied the latent heat for vaporizing moisture must be supplied by the air and its initial superheated moisture and its temperature will drop. The equilibrium temperature that results from a constant heat or adrabatic saturation may be used to calculate the initial moisture content of the air.
The usual experimental method used to determine this adiabatic saturation temperature depends upon a wet bulb thermometer which is usually a mercury-glass thermometer with a wet gauze wick over the bulb. If the air is blown over the bulb or the thermometer is swung vigorously through the air as a sling psychrometer, its reading will drop an amount that depends upon the rate of moisture cvaporation. Each portion of air that comes into contact with the wet bulb picks up vapor but must supply the heat for its evaporation. The wet bulb temperature that results closely approaches the theoretical adiabatic saturation temperature.

The adiabatic saturation process is illustrated schematically by Figure $5 \cdot 2$ and it is very important not to think of the wet bulb temperature $T_{w}$ and the amount of moisture $\Delta W$ that is evaporated as existing in the air before it is "processed." The dry bulb reading is the temperature of the one pound of dry air and its accompanying $W$ pounds of superheated vapor that are taken as a sample to determine the value of $W$.

This sample may be thought of as being placed in an insulated enclosure where water at the wet bulb temperature $T_{d}$ is supplied. This temperature must be used for the water supply because it corresponds to the situation present in the wick of the wet bulb thermometer. The air being tested is circulated through this wick until a minimum thermometer reading is found, which means that the remaining water in the wick must also be at that temperature. With this value of wet bulb temperature known it is possible to solve for the initial moisture content of the air sample.

If $T_{d}$ represents air or dry bulb temperature, $T_{w}$ the wet bulb temperature, $W$ the initial pound of moisture per pound of dry air, $\Delta W$ the pound of moisture evaporated into each pound of dry air that comes into contact with the wet wick, and $h_{f g}$ is the latent heat of vaporization at the wet, bulb temperature, a heat balance equation may be written for each pound of dry air that contacts the wet bulb. The Btu used for the evaporation of $\Delta W$ pounds of moisture is $\Delta W h_{f g}$. The Btu given up by the $W$ pounds of superheated water vapor is $W c_{p}\left(T_{d}-T_{w}\right)=$ $0.44 W\left(T_{d}-T_{w}\right)$ and that from the pound of dry air having a $c_{p}=0.24$ is $0.24\left(T_{d}-T_{w}\right)$,

$$
\Delta W h_{f g}=0.44 W\left(T_{d}-T_{w}\right)+0.24\left(T_{d}-T_{w}\right)
$$

The amount of vapor, $W+\Delta W$, necessary to saturate 1 lb of dry air at the wet bulb temperature $T_{w}$ may be found as illustrated in article $5 \cdot 3$ and with this known, $W$, the initial pounds of moisture per pound of dry air may be calculated from equation $5 \cdot 10$. This may be illustrated by using (for convenience) 58 deg F air ( $T_{d}=58$ ) having a wet bulb reading ( $T_{w}=40$ ) deg F . As calculated in article $5 \cdot 3, W+\Delta W$ for saturation of air at the wet bulb temperature of 40 deg F is 0.0052 lb of vapor per pound of dry air and from steam tables $h_{f g}$ at 40 deg F is 1069.1 Btu per lb.

From cquation 5•10

$$
[\Delta W=0.0052-W] 1069.1=0.44 W(58-40)+0.24(58-40)
$$

which gives $W=0.00115 \mathrm{lb}$ of moisture per pound of dry air as the moisture content.

The ratio of the pounds of vapor per pound of dry air to the pounds necessary to saturate it at the same temperature $T_{d}$ is defined as the humidity. In this case, since 0.01 lb is necessary for saturation at 58 $\operatorname{deg} \mathrm{F}$ (article $5 \cdot 3$ ), the humidity is $0.00115 / 0.01=0.115$, which is usually spoken of as 11.5 per cent humidity.

The various bases of zero enthalpy used in vapor and gas tables have been previously discussed but yet another base for the enthalpy of an air-vapor mixture is in common use in the air conditioning industry. The enthalpy of the air is measured from zero degrees Fahrenheit and that of the vapor includes only the latent heat of vaporization. It is assumed that the moisture for vaporization is always furnished at the wet bulb temperature and that is quite accurate for commercial humidifiers and wet bulb thermometers.

Any attempt to calculate the enthalpy of an unsaturated air-vapor mixture directly would involve the complications of superheated vapor. It is convenient that the heat content of a pound of dry air and its vapor
is the same as that of a pound of dry air and its saturated vapor after adiabatic saturation. It is only necessary to calculate the enthalpy after adiabatic saturation is completed.

In the case just discussed the enthalpy of one pound of air and the 0.0052 lb of saturated vapor at 40 deg F wet bulb temperature, is the same as that of one pound of air and the 0.00115 lb of superhcated vapor at the initial condition of 58 deg F . The enthalpy of the air is $1 \times 0.24 \times$ $40=9.6 \mathrm{Btu}$ and the enthalpy of vaporization of 0.0052 lb of saturated vapor at 40 deg F is $0.0052 \times 1069.1=5.56 \mathrm{Btu}$. This gives a total enthalpy of $9.6+5.56=15.15$ Btu per lb of dry air.


Fig. 5-3. Psychrometric chart.
5.5. Psychrometric Charts. The method of calculating humidity, dew point temperature, enthalpy, etc., just outlined is too slow and tedious for practical work and this has resulted in the construction of
"psychrometric" charts that yield graphical solutions that are sufficiently accurate for most work. A widely used form of chart now current is shown in skelcton form by Figure $5 \cdot 3$.

The primary coordinates of this chart are dry bulb temperature in degrees Fahrenheit and moisture content in grains per pound of dry air ( 7000 grains $=1 \mathrm{lb}$ ). For 100 per cent humidity or saturation and for other percentages of humidity, the corresponding moisture content is calculated, and these lines are plotted as shown. Since at saturation the dry bulb, wet bulb, and dew point temperatures are identical, the projection of dry bulb temperatures locates the wet bulb and dew point values on the saturation curve.

A constant dew point temperature corresponds to a constant moisture content, which is the reason that constant dew point lines are horizontal on the chart. The calculation of moisture content at constant wet bulb temperature and varying dry bulb temperatures, results in the constant wet bulb lines that slope downward to the right.

Since the enthalpy or total heat depends only upon the wet bulb temperature, it is possible to calculate the nomogram of total heat that is at the left of the saturation line. The solution of a problem where the wet bulb and dry bulb temperatures are known is indicated by the arrows on Figure 5.3. If the original data are in terms of other variables, the order of the solution may be changed to fit. There are many special problems that can be solved quickly, but the really important thing is to understand thoroughly the basis of humidity calculations given in article $5 \cdot 4$.

Sugaemted Reading
Mixtures of Gaseous Substances
Emswiler, Thermodynamics. Chapter XIV.
Dry and Wet Bulb Temperatures
Severns, Heating, Ventilatıng, and Air Condıtioning Fundamentals. Pages 20-24.
Moisture in the Atmosphere
Barnard, Ellenwood, and Hirsiffeld, Heat Power Engineering. Pages 308-310.

## CHAPTER 6

## COMBUSTION OF FUELS

6.1. The Combustion Process. In solid carbon the atoms are in equilibrium in a lattice arrangement under the influence of mutual forces of attraction and repulsion, and a large quantity of potential energy is stored in such a system. Gas molecules also possess potential or chemical energy stored in a similar manner. Carbon or hydrogen molecules may be mixed with the oxygen of the air at ordinary temperatures without being affected; however, at high temperatures, the impact of high-velocity molecular collisions completely upsets the structure of the fuel and oxygen molecules.

In the new molecules that are formed, the re-arranged atoms possess a greatly reduced amount of potential energy and the potential energy released appears as increased molecular kinetic energy which is known as the heat of combustion. The ignition temperature at which combustion is sustained varies widely with the kind of fuel and the burning conditions, but for hydrogen and gaseous fuels with air, it may be thought of as being near 1000 deg F . Coal and coke do not glow below 850 deg F but under favorable conditions slow burning may be maintained at lower temperatures.

The rate of burning and heat release when coal is burned in a furnace is determined largely by the rate at which fuel and air are supplied and the rate at which heat is transferred to the surroundings. In mixtures of gaseous or vapor fuels with air in engine cylinders, the combustion normally occurs as a flame front traveling across the cylinder. This happens because the temperature is adequate for ignition only at the flame front, and is too low in the unburned portion. Should the temperature of the unburned mixture exceed a critical value, spontaneous combustion or detonation occurs as the entire mass ignites with explosive violence.

When fuel oil is sprayed into an engine cylinder or is burned in an oilfired furnace, as well as when finely pulverized coal burns as it is blown into a furnace, combustion is largely a surface action. The drops of oil and particles of coal are extremely large in comparison with the individual moiecules involved in the combustion of gaseous fuels and the
problem is to get the particle or drop of fuel in contact with enough oxygen molecules to complete its combustion. Consequently the particle or drop size and the degree of gas turbulence that sweeps the inactive gas layer away from the surface are the factors which govern the burning time and ignition temperature.

The final combustion temperature reached usually depends upon the heat of combustion released, the weight of products of combustion formed, the rate of combustion, and the rate of heat loss to the surroundings. When the heat losses from the flame are not large dissociation or chemical equilbbrium may govern the final combustion temperature. Since molecular impacts are credited with breaking up fuel and oxygen molecules it is not difficult to imagine some of the collisions of the molecules of the products of combustion as being violent enough to shake them apart. It is necessary to visualize combustion reactions as taking place in both directions. At lower temperatures many oxygen and fuel molecules are uniting, and a very few of the resulting molecules are being dissociated. The dissociation increases with increased temperature until at a temperature, usually between 4000 and 6000 deg F, equilibrium is reached. This temperature varies with the fuel and the burning conditions, but once it is reached, no further release of heat of combustion can occur.
6.2. Composition of Fuels. Solid Fuels. The most important solid fuel is coal which, during past ages, has been formed from wood and other vegetable materials, largely cellulose $\left(\mathrm{C}_{6} \mathrm{H}_{10} \mathrm{O}_{5}\right)$, water, organic salts, resinous materials, and ash. Chemically dry coal is composed of carbon, hydrogen, oxygen, sulphur, nitrogen, and ash. Commercial coal contains free moisture and a wet analysis includes this, together with the other components, and is usually on a weight basis. The usual complete chemical analysis is on a dry basis and gives the total amounts of the above constituents without regard to the form in which they occur in the coal.

Besides the free moisture, coal also contains water of crystallization which is not removed by drying and consequently is not included in the moisture content reported in the analysis. This water of crystallization is the source of all the oxygen and a part of the hydrogen of the chemical analysis. For this reason the heat of combustion of the part of the hydrogen that comes from this water is not available.

Since the full chemical analysis of a coal sample is expensive, coal is more commonly identified by its proximate (not approximate) analysis. Such an analysis is on a weight basis and specifies the moisture, volatile material, fixed carbon, and ash contents. The moisture content is found by oven drying a sample, and the volatile part is made up of the water
of crystallization, absorbed nitrogen, and the hydrocarbon compounds that are driven off when the sample is heated briefly in the absence of air. The ash is the silica and the inert material that remains after combustion has been completed, and the fixed carbon is assumed to account for the remainder. The moisture content of commercial coal is usually less than 4 per cent, the ash is between 2 and 10 per cent, and the various classifications of coal on a moisture


Fig. 6.1. Classifications of coal. and ash-free or combustible basis are shown in Figure 6.1.
Liquid Fuels. Regardless of its source, petroleum has a chemical composition by weight ranging from 0.83 to 0.87 carbon, 0.11 to 0.14 hydrogen, and small amounts of sulphur and nitrogen. It is made up of many complex hydrocarbons and is separated into gasoline, kerosene, fuel oil, and lubricating oil by a proces of fractional distillation. Gasoline is also produced from the heavicr components by a method of destructive heating called cracking, and also by the hydrogenation process of heating heavier hydrocarbons in the presence of hydrogen to reduce them to lighter hydrocarbons. Approximately 0.40 of crude oil goes to gasoline, 0.40 to fuel oil, and less than 0.1 each to kerosene and lubricating oil. Benzene ( $\mathrm{C}_{6} \mathrm{H}_{6}$ ) which is a coal distillate and ethyl alcohol $\left(\mathrm{C}_{2} \mathrm{H}_{6} \mathrm{O}\right)$ which is made by the fermentation and distillation of grains, vegetables, or molasses are potential gasoline substitutes.

Gaseous Fuels. The gaseous fuels of commercial importance are probably led by natural gas, the chicf combustible parts of which are usually methane $\left(\mathrm{CH}_{4}\right)$ and ethane $\left(\mathrm{C}_{2} \mathrm{H}_{6}\right)$. Producer gas, which is made by the incomplete combustion of coal contains CO and $\mathrm{H}_{2}$ as important combustibles. The various types of illuminating gas that are usually made by the distillation of coal or petroleum are largely $\mathrm{H}_{2}$ and $\mathrm{CH}_{4}$ with some CO. By-product gas, which includes both coke and blast furnace gases, are made up chiefly of $\mathrm{H}_{2}, \mathrm{CO}$, and $\mathrm{CH}_{4}$.

TABLE 61

## Reference Table of Combustion Elements

| Substance | Formula | Molecular Weight |
| :--- | :---: | :---: |
| Hydrogen | $\mathrm{H}_{2}$ | 2 |
| Oxygen | $\mathrm{O}_{2}$ | 32 |
| Nitrogen | $\mathrm{N}_{2}$ | 28 |
| Carbon | C | 12 |
| Sulphur | S | 32 |

TABLE 6.2
Reference Table of Fuel Elements

| Substance | Formula | Substance | Formula |
| :--- | :--- | :--- | :---: |
| Carbon (sold) | C | Ethane (gas) | $\mathrm{C}_{2} \mathrm{H}_{6}$ |
| Carbon monoxide (gas) | CO | Benzene (lquid) | $\mathrm{C}_{6} \mathrm{H}_{6}$ |
| Sulphur (sold) | S | Hexane (hquid) | $\mathrm{C}_{6} \mathrm{H}_{14}$ |
| Hydrogen (gas) | $\mathrm{H}_{2}$ | Octane (hquid) | $\mathrm{C}_{8} \mathrm{H}_{18}$ |
| Methane (gas) | $\mathrm{CH}_{4}$ | Methyl alcohol (hquid) | $\mathrm{CH}_{4} \mathrm{O}$ |
| Acetylene (gas) | $\mathrm{C}_{2} \mathrm{H}_{2}$ | Ethyl alcohol (liquid) | $\mathrm{C}_{2} \mathrm{H}_{6} \mathrm{O}$ |
| Ethylene (gas) | $\mathrm{C}_{2} \mathrm{H}_{4}$ |  |  |

6.3. Combustion Reactions. The combustion or oxidation of a fuel may be studied by using the chemical reaction equation which lists the elements or mixtures undergoing reaction on the left-hand side of the equation and the final products of the reaction on the right-hand side, with the number of molecules of each element balanced on the two sides. Such a scheme does not reflect any of the intermediate steps that may be involved in the reaction but it does make it possible to calculate the quantities of each clement involved. Thus for hydrogen,

$$
2 \mathrm{H}_{2}+\mathrm{O}_{2}=2 \mathrm{H}_{2} \mathrm{O}
$$

This equation may be read: 2 molecules of $\mathrm{H}_{2}+1$ molecule of $\mathrm{O}_{2}=2$ molecules of $\mathrm{H}_{2} \mathrm{O}$, or,

$$
2 \text { mols } \mathrm{H}_{2}+1 \mathrm{~mol} \mathrm{O}_{2}=2 \text { mols } \mathrm{H}_{2} \mathrm{O}
$$

Since, at the same temperature and pressure a mol of any gas occupies the same volume,

$$
2 \mathrm{cuft} \mathrm{H}_{2}+1 \mathrm{cuft} \mathrm{O}_{2}=2 \mathrm{cuft} \mathrm{H}_{2} \mathrm{O}
$$

As air is 0.79 nitrogen and 0.21 oxygen by volume, each cu ft of oxygen is accompanied by

$$
\frac{1}{0.21} \times 0.79=3.76 \mathrm{cu} \mathrm{ft} \text { of nitrogen }
$$

The complete volume combustion equation, assuming water vapor to act as a gas is
2 cuft $\mathrm{H}_{2}+1$ cuft $\mathrm{O}_{2}+3.76$ cu ft $\mathrm{N}_{2}=2$ cuft $\mathrm{H}_{2} \mathrm{O}$ +3.76 cu ft $\mathrm{N}_{2} \quad[6.4]$

On the weight basis, since each pound of $\mathrm{O}_{2}$ is accompanied by $0.77 / 0.23$ $=3.35 \mathrm{lb} \mathrm{N}_{2}$, equation 6.4 becomes
$4 \mathrm{lb} \mathrm{H}_{2}+32 \mathrm{lb} \mathrm{O}_{2}+105.2 \mathrm{lb} \mathrm{N}_{2}=36 \mathrm{lb} \mathrm{H}_{2} \mathrm{O}+105.2 \mathrm{lb} \mathrm{N}_{2} \quad[6 \cdot 5]$

In a similar manner, the complete combustion of carbon is represented by

$$
1 \mathrm{~mol} \mathrm{C}+1 \mathrm{~mol} \mathrm{O}_{2}=1 \mathrm{~mol} \mathrm{CO} 2
$$

or

$$
\begin{equation*}
12 \mathrm{lb} \mathrm{C}+32 \mathrm{lb} \mathrm{O}_{2}=44 \mathrm{lb} \mathrm{CO}_{2} \tag{6.7}
\end{equation*}
$$

The incomplete combustion of carbon when insufficient oxygen is supplied is given by

$$
2 \mathrm{~mol} \mathrm{C}+1 \mathrm{~mol} \mathrm{O}_{2}=2 \mathrm{~mol} \mathrm{CO}
$$

or

$$
24 \mathrm{lb} \mathrm{C}+32 \mathrm{lb} \mathrm{O}_{2}=56 \mathrm{lb} \mathrm{CO}
$$

The nitrogen that accompanies the oxygen when air is used may be included as it was for the combustion of hydrogen.

Most fuels, especially petroleum products, contain compounds of hydrogen and carbon or hydrocarbons. They range from the simpler gases like methane $\left(\mathrm{CH}_{4}\right)$ and ethylene $\left(\mathrm{C}_{2} \mathrm{H}_{4}\right)$ to the great variety of complex hydrocarbons that constitute gasoline and fuel oil. In combustion calculations gasoline is often arbitrarily represented as octane $\left(\mathrm{C}_{8} \mathrm{H}_{18}\right)$ simply because that fuel has about the same ratio between hydrogen and carbon. In addition to hydrogen and carbon some fuels such as wood, coal, methyl, alcohol $\left(\mathrm{CH}_{4} \mathrm{O}\right)$, and ethyl alcohol $\left(\mathrm{C}_{2} \mathrm{H}_{6} \mathrm{O}\right)$ contain combined oxygen.

The oxygen and air necded for the complete combustion of a hydrocarbon fuel are calculated from the hydrogen and carbon contents just as though they existed alone, with due allowance being made for any oxygen that may be contained in the fuel. The combustion of such fuels does not usually occur as the combustion of free hydrogen and solid carbon except possibly when the fuel reaches a high temperature and decomposes before oxygen reaches it. Once complete combustion is attained, regardless of the mechanism involved, the final oxygen requirement is correctly predicted by this method.

As an example consider the combustion of a fuel having a weight composition of 0.46 carbon, 0.23 hydrogen, and 0.31 oxygen. Since $\frac{32}{12} \mathrm{lb}$ of $\mathrm{O}_{2}$ is required for the complete combustion of a pound of C to $\mathrm{CO}_{2}$ and $\frac{1}{2}-\mathrm{lb} \mathrm{O}_{2}$ is needed for one pound of $\mathrm{H}_{2}$ to $\mathrm{H}_{2} \mathrm{O}$, the total $\mathrm{O}_{2}$ required for one pound of fuel is $\left(\frac{32}{12} \times 0.46\right)+\left(\frac{16}{2} \times 0.23\right)=3.07$. Because one pound of fuel contains 0.31 lb of $\mathrm{O}_{2}$, only $3.07-0.31=2.76$ of $\mathrm{O}_{2}$ must be supplied for each pound of fuel completely burned and this represents $\frac{2.76}{0.23}=12.1 \mathrm{lb}$ of air per pound of fuel. The products of combustion from one pound of fuel completely burned contain $\frac{44}{12} \times 0.46=1.69 \mathrm{lb}$ $\mathrm{CO}_{2}, \frac{18}{2} \times 0.23=2.07 \mathrm{lb} \mathrm{H}_{2} \mathrm{O}$ vapor, and $12.1 \times 0.77=9.3 \mathrm{lb} \mathrm{N} 2$.

If 1.5 times as much air had been supplied, the air fuel ratio would have been $1.5 \times 12.1$ or 18.2 lb of air per pound of fuel and this would be referred to as 50 per cent excess air. For a fuel that contains oxygen, the excess air is reckoned on the basis of the air that must be supplied and not upon the basis of the oxygen that enters in the reaction. When such excess air is supplied the extra oxygen appears in the products of combustion along with the extra nitrogen. For the present example, $0.23 \times 18.2=4.19 \mathrm{lb} \mathrm{O}_{2}$ would be supplied and since only $2.76 \mathrm{lb} \mathrm{O}_{2}$ is needed for combustion, $4.19-2.76=1.43 \mathrm{lb}$ of free oxygen per pound of fuel would appear in the calculated products of combustion along with $2.07 \mathrm{lb} \mathrm{H}_{2} \mathrm{O}$ vapor and $0.77 \times 18.2=13.82 \mathrm{lb} \mathrm{N}_{2}$.
6.4. Incomplete Combustion. If complete combustion of a fuel is to result, each fucl particle must come in contact with enough oxygen molecules to complete its burning and do it at a temperature above its ignition point. When a solid fuel such as coal burns in the fuel bed of a furnace, the heat released as the primary air passes through the fuel drives off vapors at the surface that must be burned by secondary air so directed as to produce turbulence and thorough mixing. Similarly when a finely divided spray of liquid fuel is injected into a furnace or engine, adequate mixing with the combustion air is essential to complete burning.

Incomplete combustion of a fuel may result from a deficiency of air supplied or it may occur when there is adequate or excess air but poor mixing. An extreme and very obvious case of incomplete burning occurs when a carbon fuel burns with a great deficiency of air, poor mixing, or when the flame strikes a cool surface before combustion is complete and solid carbon is deposited. A more common but less apparent form of the incomplete combustion of carbon occurs when both $\mathrm{CO}_{2}$ and CO appear in the products of combustion. This indicates that although all the carbon molecules made contact with enough oxygen to form CO , only part of the CO was burned to $\mathrm{CO}_{2}$ and the failure to convert all the CO to $\mathrm{CO}_{2}$ represents a loss of a part of the available heating value of the fuel.

The incomplete combustion of hydrogen or very stable hydrocarbon gases results in unburned fuel in the products of combustion but the incomplete burning of more complex hydrocarbon fucls may give a variety of results depending on combustion temperatures and conditions. When coal is burned with mild air deficiency or poor air mixing the result is chiefly reflected by the presence of carbon monoxide. Incomplete burning of liquid hydrocarbon fuels in internal-combustion engines, on the other hand, results in the presence of methane $\left(\mathrm{CH}_{4}\right)$ and free hydrogen $\left(\mathrm{H}_{2}\right)$ in addition to carbon monoxide (CO). Roughly, for each

7 mols of CO in such an exhaust sample there may be as much as 1 mol of $\mathrm{CH}_{4}$ and 3 mols of $\mathrm{H}_{2}$. This means that the usual assumption that incomplete combustion is reflected only by the presence of CO in the products of combustion may be seriously in error, particularly when liquid hydrocarbon fuels are being burned under unfavorable conditions.

It is usual to assume that where there is an oxygen deficiency all the $\mathrm{H}_{2}$ is completely burned and that the oxygen shortage is absorbed by C that is burned only to CO instead of to $\mathrm{CO}_{2}$. This may be illustrated by assuming the fuel of the example in article 6.3 to be burned with 11.0 lb of air per pound of fuel, instcad of the 12.1 lb needed for complete combustion. The $0.23 \times 11=2.52 \mathrm{lb}$ of $\mathrm{O}_{2}$ supplied, together with 0.31 lb $\mathrm{O}_{2}$ from the pound of fuel gives 2.84 lb available. The $0.23 \mathrm{lb} \mathrm{H}_{2}$ uses $8 \times 0.23=1.84 \mathrm{lb} \mathrm{O}_{2}$ for complete combustion to $\mathrm{H}_{2} \mathrm{O}$, leaving 2.84$1.84=1.0$ of $\mathrm{O}_{2}$ for the carbon. To burn 0.46 lb C to CO requires $\frac{1}{1} \frac{6}{2} \times 0.46=0.614 \mathrm{lb}$ of $\mathrm{O}_{2}$, leaving $1.00-0.614=0.386 \mathrm{lb}$ of $\mathrm{O}_{2}$ which is not enough to burn all the CO to $\mathrm{CO}_{2}$. For each pound of C in CO burned to $\mathrm{CO}_{2}, \frac{16}{12} \mathrm{lb}$ of $\mathrm{O}_{2}$ is needed, which means that $\frac{0.386}{16 / 12}=0.29$ lb of C is completely burned to $\mathrm{CO}_{2}$ and $0.46-0.29=0.17 \mathrm{lb}$ of C is incompletely burned to carbon monoxide. The products of combustion may be calculated as in article 6.3 and they will contain CO in addition to the usual $\mathrm{CO}_{2}, \mathrm{H}_{2} \mathrm{O}$, and $\mathrm{N}_{2}$.
6.5. Heat of Combustion. The heat of combustion of a fuel is determined experimentally by burning a sample and absorbing the heat released in the water bath of a calorimeter. The heating values of solid and nonvolatile liquid fuels are usually determined in oxygen-bomb calorimeters, whereas gaseous and volatile liquid fuels are generally tested in continuous calorimeters of the Junkers type. Since these experimental heating values are affected by the burning conditions they can be compared only when reduced to the same conditions.

In a bomb calorimeter a small quantity ( 1 gram or less) of heavy liquid or finely divided solid fuel is ignited in a steel bomb by an electric fuse wire and burns in oxygen at 300 lb per sq in. pressure. The heat released is absorbed by approximately 2000 cc of water in which the bomb is immersed and by the metal parts of the calorimeter. The combustion heat released results in a temperature rise of from 3 to $6 \mathrm{deg} F$.

Heat losses from the water to its surroundings are prevented or minimized by various insulating schemes, so that after corrections for the fuse wire heat and the formation of nitric and sulphuric acid are made, the heating value may be calculated.

The combustion takes place at constant volume, so no external work is done and the initial and final temperatures do not differ greatly. Since
the atmosphere in the bomb is saturated with water vapor at the outsct and at the end of combustion, practically all the water vapor resulting from combustion is condensed and its latent heat of vaporization is recovered. The heating value so determined in a bomb calorimeter is very nearly equal to the actual potential chemical energy released during combustion and is known as the higher heating value.

Gaseous and volatile liquid fucls are burned at a measured rate in the Junkers type continuous calorimeter and the combustion heat is transferred to cooling water that is circulated in the jacket. The rates of flow are adjusted so the products of combustion leave at the entering temperature of the air and fuel. If the conditions are such that none of the water vapor from combustion is condensed out of the exhaust gases, and it carrics its latent heat of vaporization away from the calorimeter, the heating value found is the lower heating value. Under usual conditions of use, a part of this water vapor from the hydrogen of the fucl does condense in the calorimeter and is collected and measured. Under this condition where a part but not all the water leaves the calorimeter as vapor, the heating value determined is neither the higher nor the lower value, but it can be corrected to the lower heating value by subtracting the latent heat of vaporization of the moisture condensed from the heat absorbed in the calorimeter.

The difference between the higher and lower heating values for a fuel that has 0.12 hydrogen by weight and is burned at constant pressure in a Junkers calorimeter is the latent heat of vaporization of the $9 \times 0.12=$ 1.08 lb of water vapor that is formed. Since the latent heat varies more than 30 Btu per lb between 40 and 100 deg F , this difference is not constant, but is usually calculated at 68 deg F which in this case would result in $1.08 \times 1053.8=1142 \mathrm{Btu}$ per lb of fuel.

The trend in American practice is generally toward the use of the lower heating value in efficiency calculations. It is based on the argument that it is no fault of a furnace or engine that it is not convenient or possible for it to cool the products of combustion to a low enough temperature to recover the latent heat of vaporization of the water vapor formed during combustion.

Table $6 \cdot 3$ shows generally accepted heating values of the various fuel elements.

These heating values are accurate when the various fucl clements exist separately, but when the heat of combustion of a hydrocarbon fuel is calculated as the sum of the heating values of its constituent part, the results are not very dependable. This is illustrated in Table $6 \cdot 4$ for four fuels.

Such calculated heating values are inaccurate because they ignore the heat of formation of the molecules that must be destroyed before the

TABLE 6•3

| Reaction |  |  |
| :--- | :--- | :--- |
| C | to | $\mathrm{CO}_{2}$ |
| C | to | CO |
| CO | to | $\mathrm{CO}_{2}$ |
| S | to | $\mathrm{SO}_{2}$ |
| HI | to | $\mathrm{H}_{2} \mathrm{O}$ |

Heating Value
14,150 Btu per lb C
3,960 Btu per lb C
4,367 Btu per lb CO
3,890 Btu per lb S
61,000 Btu per $\mathrm{lb} \mathrm{H}_{2}$
(higher heating value)

TABLE 6.4

| Fuel | Experimental <br> Higher Heating Value Btu per Lb | Calculated Higher Heating Value Btu per Lb |
| :---: | :---: | :---: |
| Methane ( $\mathrm{CH}_{4}$ ) | 23,670 | $\left(\frac{1}{12} \times 14,150\right)+\left(\frac{4}{18} \times 61,000\right)=25,850$ |
| Ethyl alcohol ( $\mathrm{C}_{2} \mathrm{H}_{6} \mathrm{O}$ ) | 12,820 | $\left(\frac{2}{4} 4 \times 14,150\right)+\left(\frac{4}{4} 8 \times 61,000\right)=15,330$ |
| Acetylene ( $\mathrm{C}_{2} \mathrm{H}_{2}$ ) | 21,590 | $\left(\frac{2}{2} \frac{1}{6} \times 14,150\right)+\left(\frac{2}{26} \times 61,000\right)=17,780$ |
| Octane ( $\mathrm{C}_{8} \mathrm{H}_{18}$ ) | 20,580 | $\left(\frac{9}{19^{8}} \times 14.150\right)+\left(\frac{18}{114} \times 61,000\right)=21,540$ |

carbon and hydrogen are available for combustion. The calculated value for ethyl alcohol is 2510 Btu per lb high because that much energy is necessary to tear apart the $\mathrm{C}_{2} \mathrm{H}_{6} \mathrm{O}$ molecule. For acetylene on the other hand, the calculated value is 3810 Btu per lb low, which means that the destruction of the unstable $\mathrm{C}_{2} \mathrm{H}_{2}$ molecule releases that much heat.

These facts appear reasonable when it is remembered that during the fermentation process that forms alcohol, heat is given off and this heat must be again supplied to destroy the molecules. On the other hand, heat must be supplied during the formation of acetylene, and this heat is again given off when the molecules break up.

There are many specialized equations for calculating the heating values of the various fuels that take into account the heat of formation of the molecules. Chief among these is DuLong's equation for coal which takes advantage of the fact discussed in article $6 \cdot 2$ that the oxygen in coal comes predominately from the water of crystallization. For each pound of $\mathrm{O}_{2}$ in coal there must then be $1 / 8 \mathrm{lb}$ of $\mathrm{H}_{2}$ in water of crystallization. This means that an amount of hydrogen equal to $1 / 8 \mathrm{lb}$ of oxygen is not available for combustion and gives off no heat. The equation for the higher heating value of coal is, then,

$$
\text { Higher heating value }=14,150 \times \mathrm{C}+61,000\left[\mathrm{H}-\frac{\mathrm{O}}{8}\right]
$$

This equation gives quite reliable values for the heating value of coal.

The incomplete combustion losses discussed in article 6.4 may be illustrated from Table 6.3. Since each pound of C burned to $\mathrm{CO}_{2}$ releases $14,150 \mathrm{Btu}$ and each pound of C burned incompletely to CO releases but 3960 Btu, it follows that each pound of C that is burned to CO instead of $\mathrm{CO}_{2}$ represents an incomplete combustion loss of $14,150-3960=$ $10,190 \mathrm{Btu}$. When the heats of combustion of hydrogen and hydrocarbons are considered, it is casy to appreciate the magnitude of the heat losses that are represented by even small quantities of unburned hydrogen and hydrocarbons in the products of combution.
6.6. Analysis of Products of Combustion. The ideal case of complete combustion when only the theoretical minimum amount of air is supplied is rarely met in practice. Ordinarily excess air (ranging in amount from a few to several hundred per cent of the theoretical) is supplied and even then imperfect mixing and poor burning result in some incomplete combustion. The usual products of combustion contain both CO and $\mathrm{O}_{2}$ in addition to the $\mathrm{N}_{2}$ and $\mathrm{CO}_{2}$ and $\mathrm{H}_{2} \mathrm{O}$.

Combustion cannot be studied quantitatively as it occurs, but by analyzing a representative sample of the products of combustion, it is possible to find out much about what happens during burning. A volumetric analysis of products of combustion on a dry or water-vapor-free basis may be made with an Orsat analyzer.

A sample (usually 50 cc ) of the gasen is collected over water at room temperature in a buret and its volume measured at atmospheric pressure. It is then exposed to a potassium hydroxide solution that absorbs the $\mathrm{CO}_{2}$. After the loss in volume is measured the sample is then passed through alkaline pyrogallate to absorb the $\mathrm{O}_{2}$ and finally through acid cuprous chloride to absorb the CO , the decrease in volume being determined each time. If the temperature remains constant, the partial pressure exerted by the saturated water vapor remains constant and the volumetric analysis so determined is on a dry or moisture-free basis.

Any $\mathrm{SO}_{2}$ present is absorbed with the $\mathrm{CO}_{2}$ and this causes an error if much sulphur is present in the fuel. Care must be exercised to pass the gas through the solutions in the correct order, for the pyrogallate will absorb both $\mathrm{CO}_{2}$ and $\mathrm{O}_{2}$ and the cuprous chloride solution will remove both $\mathrm{O}_{2}$ and CO.

If gasoline with a weight composition of $\mathrm{C}=0.84$ and $\mathrm{H}_{2}=0.16$ is burned with theoretical air, $\left(0.84 \times \frac{32}{12}\right)+(0.16 \times 8)=3.52 \mathrm{lb} \mathrm{O}_{2}$, or 15.3 lb of air is needed for each pound of fuel. This is referred to as an air-fuel ratio of 15.3 . Since 15.3 lb of air contains $0.77 \times 15.3 \mathrm{lb}$, or $\frac{0.77 \times 15.3}{28}=0.421 \mathrm{mols}$ of $\mathrm{N}_{2}$, and 0.84 lbC produces $\frac{0.84 \times 44}{12} \mathrm{lb}$, or $\frac{0.84 \times 44}{44 \times 12}=\frac{0.84}{12}=0.07 \mathrm{mols}$ of $\mathrm{CO}_{2}$, the theoretical dry volumetric
exhaust gas analysis would be $\frac{0.07}{0.07+0.421}=0.143 \mathrm{CO}_{2}$ and $\frac{0.421}{0.07+0.421}$ $=0.857 \mathrm{~N}_{2}$.

A typical Orsat analysis of the exhaust gases of an internal-combustion engine burning gasoline with a rich mixture is

$$
\mathrm{CO}_{2}=0.116 \quad \mathrm{CO}=0.03 \quad \mathrm{O}_{2}=0.014 \quad \mathrm{~N}_{2}=0.84
$$

Being a volumetric analysis, this means that each mol of dry exhaust gas contains 0.116 mols of $\mathrm{CO}_{2}, 0.03$ mols of $\mathrm{CO}, 0.014$ mols of $\mathrm{O}_{2}$, and 0.84 mols of $\mathrm{N}_{2}$.

Since the only source of nitrogen is from the combustion air, 0.84 mols of $\mathrm{N}_{2}$ represents $\frac{0.84 \times 28}{0.77}=30.5 \mathrm{lb}$ of air furnished for each mol of dry exhaust gas. Assuming that all the carbon of the fuel was burned, $12 \times 0.03=0.36$ of C is present in the 0.03 mols of CO and $12 \times 0.116=$ 1.39 lb of C in the 0.116 mols of $\mathrm{CO}_{2}$. This gives a total carbon weight of $1.39+0.36=1.75$ and since the fuel is 0.84 carbon by weight, $\frac{1.75}{84}=$ 2.08 lb of fuel and 30.5 lb of air were used to produce a mol of dry exhaust gas. This represents an air-fuel ratio of $\frac{30.5}{2.08}=14.7$.

The accuracy of this deduction may be checked by assuming that all the hydrogen of the fuel was burned to water. The 30.5 lb of air furnished per mol of gas contained $0.23 \times 30.5=7.0 \mathrm{lb}$ of oxygen. There was $0.014 \times 32=0.45 \mathrm{lb}$ of free oxygen, $0.03 \times 16=0.48 \mathrm{lb}$ of $\mathrm{O}_{2}$ in the CO and $0.116 \times 32=3.71 \mathrm{lb} \mathrm{O}_{2}$ in the $\mathrm{CO}_{2}$. This accounts for $0.45+0.48+3.71=4.64 \mathrm{lb}$ of $\mathrm{O}_{2}$, leaving $7.0-4.64=2.36 \mathrm{lb}$ of $\mathrm{O}_{2}$ that must have united with $\mathrm{H}_{2}$ to form $\mathrm{H}_{2} \mathrm{O}$. For each pound of $\mathrm{O}_{2} 1 / 8 \mathrm{lb}$ $\mathrm{H}_{2}$ is required, which means that $\frac{2.36}{8}=0.295 \mathrm{lb} \mathrm{H}_{2}$ must have been in the fuel burned to produce 1 mol of dry exhaust gas. Since the fuel is $0.16 \mathrm{H}_{2}$ by weight, there must have been $\frac{0.295}{0.16}=1.85 \mathrm{lb}$ of fuel furnished. This gives an air-fuel ratio of $\frac{30.5}{1.85}=16.5$, which does not check very closely with the results found on the carbon basis.

This could have been caused by several things. The most likely cause is the presence of unburned hydrogen or hydrocarbons in the exhaust, but an air leak in the sampling could account for it. It is always possible that the fuel composition is not accurate, but this possibility can be eliminated by finding the weight of fuel from the weights of C and $\mathrm{H}_{2}=$
$1.75+0.295=2.04 \mathrm{lb}$ which would correspond to an air-fuel ratio of 30.5 $\overline{2.04}=14.9$. This method must always be resorted to when the exact composition of the fuel is not known.

From the Orsat analysis of the products of combustion it is relatively simple to calculate the loss in heating value caused by the incomplete combustion of carbon. Since a mol of CO contains the same weight of C as a mol of $\mathrm{CO}_{2}$ it follows from the exhaust analysis that for cach 0.116 lb of C burned to $\mathrm{CO}_{2}$ there is 0.03 lb incompletely burned to CO . This means that a fractional part 0.03
$\frac{0.03}{0.03+0.116}=0.205$ of the carbon present is burned to CO. Since $0.8 t$ of the fuel is carbon and each pound of C burned to CO represents a loss of $10,190 \mathrm{Btu}$, this corresponds to a loss of $0.205 \times 0.84 \times 10,190=$ 1750 Btu per lb of fuel.


Fig. 6.2. Theoretical exhaust gas composition.

When coal with a low volatile or hydrocarbon content is being burned, the combustion analysis based on an Orsat analysis is quite reliable. However, as mentioned in article 6.4, when a high volatile coal or petroleum fuel is incompletely burned the heating value loss by the unburned hydrogen and hydrocarbons in the stack or exhaust gases may, and very often does, greatly exceed that due to carbon monoxide.

Combustion analysis equipment does exist that will determine the presence of hydrogen and hydrocarbons quantitatively, but such equipment is complicated and slow to use and consequently is not often found in industrial use.

Combustion control in power plants is commonly accomplished through the use of indicating and recording " $\mathrm{CO}_{2}$ meters." These instruments depend for their indications upon the fact that the rate of heat loss from a hot wire to the products of combustion in an enclosed space is dependent upon the $\mathrm{CO}_{2}$ content. Since this rate of loss is also affected by the presence of hydrogen and hydrocarbons as well as by other variables, such instruments should be frequently checked by an Orsat analysis.

Once such a meter is calibrated for a given fucl and furnace, the $\mathrm{CO}_{2}$ indication may be used as a satisfactory indication of air-fuel ratio and the combustion conditions. The theoretical dry exhaust gas analysis for a hydrocarbon fuel is shown in Figure 6.2. Such curves are cal-
culated on the assumption of complete combustion of the hydrogen and ideal mixing. Consequently, at low air-fuel ratios, they do not give accurate results, but once the actual curves for a given fuel and furnace have been established, the $\mathrm{CO}_{2}$ content may be used as a satisfactory indication of combustion conditions.

## Suggested Reading

Fuels
Barnard, Ellenwood, and Hirshfeld, Heat Power Engineering. Part II, Chapter XXVII.
Severns and Degler, Steam, Air, and Gas Power. Pages 81-92.
Fuels and Combustion
Craig and Anderson, Steam Power and Internal Combustion Engines. Chapter III. Gaffert, Steam Power Stations. Chapter VII.

Fuel Calorimeters and Flue-Gas Analyzers
Severns and Degler, Steam, Air, and Gas Power. Pages 93-110.

## CHAPTER 7

## HEAT TRANSFER

7•1. Modes of Heat Transfer. The transfer of heat from one medium to another is one of the most basic problems involved in industrial equipment and the transfer occurs under a great variety of conditions. Whether it involves the transfer of heat from hot furnace gases to water or vapor in boiler tubes, the cooling of an electrical machine or the condensation of vapor in air conditioning equipment, all can be broken down into some combination of five modes of transferring heat. A logical approach to the subject is through the study of the basic principles of conduction, convection, radiation, evaporation, and condensation.

Heat conduction through matcrials is essentially a molecular phenomenon. In stagnant gases and vapors some molecules from a hightemperature region, by virtue of indiscriminant motion and collisions, diffuse into a low-temperature region, and a corresponding number of molecules with a low kinetic energy content do the reverse. In this way energy is carried through the gas.

In solids where the molecules vibrate about fixed positions, molecules with a high energy content transmit some of their energy to adjacent lower-temperature molecules with less violent motion. There is also evidence that particularly in metals, the passage of free electrons through the lattice of the molecules is responsible for some of the transmission of energy, for there is more than an accidental relationship between the electrical and thermal conductivities of metals.

Since the molecules of liquids are not as free as in gases, or in fixed arrangements as in solids, it is difficult to conjure a satisfactory molecular picture. It is usually believed that in stagnant liquids heat is transmitted by both types of molecular activity. A vapor with a distinct odor will quickly diffuse throughout the still air of a room, which is an indication of a rapid molecular exchange. On the other hand, a colored liquid placed in undisturbed water diffuses very slowly, which indicates that the close molecular spacing of liquids greatly reduces the molecular freedom.

True conduction of heat in liquids, gases, and vapors occurs only when thin layers of fluid are involved. When an appreciable temperature
difference exists through any but very small bodies of fluid, resulting differences in density produce convection currents that result from the warmer fluid rising and the cooler part descending. As soon as such a mass motion of fluid is present, energy is transmitted or transported by it and the effect of the molecular migration and vibration of conduction is decreasingly significant.

When this mass movement is produced wholly by differences in fluid density the result is known as free or natural convection, and when additional fluid circulation is induced by mechanical means, forced convection results. At all times, true conduction heat transfer occurs through a stagnant layer of fluid that adhercs to or wets the surface, but as the distance from the surface increases, mass motion of the fluid is the predominating influence. Thus, although conduction is a function only of the fluid involved, heat convection depends in addition upon the condition of fluid motion, which is affected by many things.

All heated objects emit heat energy in a form called radiation and this radiant energy is known to be transmitted through space to reappear as heat energy in any opaque object that absorbs it. It is known to be released by the electrons of the atoms and to have a wavelength and frequency dependent upon the material and its temperature. Its intensity can be measured and it has been identified as a small frequency band in the broad radiation phenomena ranging in frequency from radio waves, through light waves to cosmic rays.
There exists, however, no satisfactory picture of its exact form as it travels through space or of its means of transmission. Its existence is obvious in the sun's heat, but it is also present in reduced magnitude in practically all heat transfer.

The transfer of heat by evaporation and condensation approaches what might well be thought of as heat transportation. Large quantities of water vapor evaporate into the air in one section of the country and absorb the necessary latent heat of vaporization. When condensation takes place from this same air-vapor mixture as rain in another section of the country, this latent heat of vaporization is released.
A refrigerant vaporizes at a low temperature and absorbs its latent heat, thus cooling its surroundings. After being raised to a higher temperature and corresponding saturation pressure in a compressor, it is condensed and gives up its latent heat of vaporization at a higher temperature.

Evaporation is the most readily controlled method of heat loss available to the human body, and combinations of successive evaporation and condensation are widely used methods of heat transportation in industrial processes.

7•2. Systems of Units. It is no accident that dimensions of machines are given in inches, dimensions of houses in feet, and distances on maps in miles. In each a unit of length that results in convenient numbers has come into common use. Each branch of the physical sciences has developed its own set of units more or less independently and it is not surprising that in the aggregate they are not consistent. For example, some of the literature on heat transfer comes from the ficld of aerodynamics where the second is the convenient unit of time and some comes from the heating and ventilation literature where the hour is the most desirable time unit. Many physical constants used in the subject are in physics absolute English or cgs units and others are in the enginecring or gravitational units.

In the absolute system of units the concepts of time, distance, and mass are taken as fundamental and in the gravitational or engineering system time, distance, and force are considered fundamental. The contention is that whereas time, distance, and force can be experienced, mass is a completely derived and intangible concept that cannot be experienced by the physical senses. The net practical effect is that the engineering unit of force, the pound, is 32.2 times as large as the absolute English force unit, the poundal. Also, the engineering unit of mass, the slug, is 32.2 times as large as the absolute pound mass.

Since the subject of heat transfer draws on many sources for data it is not surprising that, at present, many conflicting systems of units appear in the literature. Although it would simplify the presentation to consider arbitrarily one system of units to the exclusion of all others, it is considered more important to develop an understanding of the systems of units in current use and the ability to transfer from one system to another.
$7 \cdot 3$. Thermal Conductivity of Gases. Although heat is conducted through all materials, the basic principles involved can be most easily developed for the simpler molecular state of gases and the results then extended to cover the more complex molecular states of dense vapors, liquids, and solids. The travel of molecules from one point to another was credited in article 7.1 with the conduction of heat through a stagnant gas when a temperature difference exists. The rate at which molecules move from one point to another in a gas is expressed quantitatively by its viscosity.

The absolute viscosity $\mu$ of a gas may be defined as the mass per second of molecules that travel a unit of distance through a unit area. This means that the mass per sccond that would travel a distance $l$ normal to a surface of area $a$ is

$$
\text { Mass per second transferred }=\frac{\mu a}{l}
$$

Most values of viscosity in the current literature are in the absolute cgs viscosity the poise. This unit expresses the grams mass per second of gas molecules that move one centimeter through an area of one square centimeter. It is usually desirable to convert such values of viscosity to the engineering system of units, and such a conversion of units is made using equation $7 \cdot 1$.
Since the engineering force unit, the pound, is 445,000 times as large as the cgs absolute force unit, the dyne, the engineering mass unit, the slug, is $\frac{445,000}{12 \times 2.54}=14,600$ times as large as the cgs gram mass. In general, the simplest method of converting the units of an equation is first to write it in the original units as

$$
\frac{\text { Mass }_{\mathrm{grams}}}{\text { Second }}=\frac{\mu_{\mathrm{porqe}}}{} \frac{U_{\mathrm{qq}} \mathrm{~cm}}{l_{\mathrm{cm}}}
$$

and then to imagine what would be done if the numbers to be used were in the new units. This would be

$$
\frac{\text { Mass }_{\text {grams }}}{\text { Sccond }}=\frac{\mu_{\text {poise }} a_{\mathrm{sq}}-\frac{\mathrm{ft}}{} \times(12 \times 2.54)^{2}}{l_{\mathrm{ft}}}
$$

Then to change the mass from grams to slugs,

$$
\frac{\text { Mass }_{\text {qlugs }}}{\text { Second }}=\frac{\text { Mass }_{\text {grams }}}{14,600 \text { second }}=\frac{\mu_{\text {ponse }} \times 12 \times 2.54}{14,600} \frac{a_{\text {sq }} \mathrm{ft}}{l_{\mathrm{ft}}}
$$

Written directly in the new units, equation $7 \cdot 1$ would be

$$
\frac{\text { Mass }_{\mathrm{slugs}}}{\text { Second }}=\frac{\mu_{\mathrm{eng}} a_{\mathrm{sq} \mathrm{ft}}}{l_{\mathrm{ft}}}
$$

This means that the cgs viscosity, $\mu_{\text {poise }}$, must be multiplied by $\frac{12 \times 2.54}{14,600}=0.00209$ to obtain the viscosity in engineering units, $\mu_{\text {eng }}$.

Since the specific heat at constant volume $c_{v}$ is the internal kinetic energy change when a pound of gas changes one degree Fahrenheit in temperature, the energy change for a slug of gas would be $c_{v} g$ for each degree Fahrenheit change in temperature. If a temperature difference of $d t$ degrees Fahrenheit exists in a distance of $d x$ feet and if all molecules at one temperature had the same kinetic energy the rate of heat conduction through an area $a$ would be

$$
\begin{align*}
& q_{\mathrm{Btu} \text { per sec }}=c_{v} g \times \text { mass per sec } \times d t \\
& q_{\mathrm{Btu} \text { per sec }}=c_{v} g \frac{\mu a}{d x} d t=c_{v} g \mu a \frac{d t}{d x}
\end{align*}
$$

Naturally, the higher-velocity molecules that have experienced favorable impacts are the ones making the greatest headway at any instant, with the result that the average energy content of the molecules involved in transferring heat is some factor $\epsilon$ times the average energy of all the molecules at any instant. As a result, equation $7 \cdot 6$ must be corrected to read

$$
q_{\mathrm{Btu} \text { per sec }}=\epsilon c_{v} g \mu a \frac{d t}{d x}
$$

Practically all the kinctic energy of a single atom molecule is due to translational velocity, whereas in complex molecules a large portion of the kinetic energy is in atomic vibration and molecular spin. It is not surprising to find that the variations in molecular energy, represented by the factor $\epsilon$ in Table 7•1, decrease as the molecules become more complex.

TABLE 7-1

| Molecular | Energy |
| :--- | :---: |
| Variation |  |
| Type of Gas | Energy Ratio є |
| Monatomic | 2.45 |
| Diatomic | 1.90 |
| Triatomic | 1.70 |
| Complex molecules | 1.30 |

Since the hour is the most convenient time unit in heat transfer work, equation $7 \cdot 7$ may be changed from seconds to hours, or

$$
q_{\text {Btu per hr }}=3600 \epsilon c_{v} g \mu a \frac{d t}{d x}
$$

It is common practice to replace $3600 g_{\epsilon} c_{v} \mu$ by the thermal conductivity of the gas, designated by $k$, giving

$$
q_{\mathrm{Btuphhr}}=k a \frac{d t}{d x}
$$

which is known as Fourier's equation of heat conduction. This makes the thermal conductivity a physical concept that parallels viscosity in that it is the Btu per hour conducted through a square foot of area with a temperature gradient of one degree Fahrenheit per foot.

Because building wall thicknesses are usually given in inches, the values of conductivity from heating and ventilation literature are in Btu per hour per square foot for a temperature gradient of a degree Fahrenheit per inch. Such values are twelve times as large as conductivities based on the consistent use of the foot as a unit of length.

Table 7.2 gives values of viscosity $\mu$ in enginecring units (equation $7 \cdot 5$ ) of thermal conductivity $k$ and check values of ( $3600 g \epsilon c_{v} \mu$ ), for air, hydrogen, carbon dioxide, and saturated steam.

TABLE 7-2
Thermal Conductivity and Viscosity of Gases
$\mu=$ slugs per second per square foot per foot thickness
$k=$ Btu per hour per square foot per degree per foot

|  | Temperature, Degrees Fahrenheit | 50 | 100 | 200 | 300 | 400 | 500 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Air | $\mu \times 10^{6}$ | 0.370 | 0.396 | 0.442 | 0.485 | 0.526 | 0.564 |
|  | $k$ | 0.0140 | 0.0151 | 0.0171 | 0.0190 | 0.0208 | 0.0224 |
|  | $3600 g_{\in} c_{v} \mu$ | 0.0138 | 0.0149 | 0.0157 | 0.0180 | 0.0204 | 0.0224 |
| $\mathrm{H}_{2}$ | $\mu \times 10^{6}$ | 0.198 | 0.202 | 0.230 | 0.250 | 0.271 | 0.282 |
|  | $k$ | 0.097 | 0.104 | 0.117 | 0.132 | 0.146 | 0.160 |
|  | $3600 g \epsilon c_{v} \mu$ | 0.108 | 0.111 | 0.126 | 0.136 | 0.149 | 0.156 |
| $\mathrm{CO}_{2}$ | $\mu \times 10^{6}$ | 0.290 | 0.310 | 0.360 | 0.400 | 0.450 | 0.500 |
|  | $k$ | 0.0084 | 0.0092 | 0.0109 | 0.0126 | 0.0143 | 0.0163 |
|  | $3600 g \epsilon c_{v} \mu$ | 0.00894 | 0.0098 | 0.0120 | 0.0143 | 0.0172 | 0.0199 |
| Saturated steam | $\mu \times 10^{6}$ | 0.200 | 0.209 | 0.271 | 0.330 | 0.450 | 0.676 |
|  | $k$ | 0.0100 | 0.0111 | 0.0133 | 0.0154 | 0.0175 | 0.0197 |
|  | $3600 g_{\epsilon} c_{v} \mu$ | 0.0130 | 0.014 | 0.0193 | 0.0292 | 0.053 | 0.117 |

Table 7.2 indicates that the values of conductivity as predicted from the kinetic theory as $3600 g \epsilon c_{v} \mu$ check quite well with the experimental values of $k$ for gases, are fair for low-pressure saturated steam, but break down completely for high-pressure steam where the molecular condition varies so radically from that assumed in the kinetic theory. When it is remembered that experimental difficulties in the determination of gaseous conductivities result in values that are not dependable closer than 5 per cent, the check with theory is quite satisfactory.

The progressive increase in both the viscosity and conductivity of gases as the temperature increases is consistent with the molecular concepts. Since it was shown that the absolute temperature of a gas is proportional to the square of the mean velocity of translation of the molecules, it would not be expected that the viscosity or molecular exchange would vary directly with the molecular velocity because the number of impacts would also change, but it is quite logical for the viscosity and thermal conductivity of a gas to increase with increased temperature.

7-4. Thermal Conductivity of Solids and Liquids. Since there is no molecular freedom in solids and heat is transmitted chiefly by the little understood atomic and molecular vibrations, the conductivity of solids does not appear to be simply related to their other properties. Also, the conduction equation 7 -9 cannot be derived for solids as it was for gases, but the fact that equation 7.9 also applies for solids must be looked upon as a purely experimental conclusion.

In general, the thermal conductivity of pure metals is higher than that of alloys and the values for pure metals are practically constant whereas those for alloys tend to increase with temperature. Grain structure affects conductivity directly since annealed metals have higher values than similar hardened metals.

Practically all good heat insulators with low conductivity are porous, cellular, fibrous, or laminated materials, and their conductivity, in general, increases with density, moisture content, and with increased temperature. Values of conductivity for some common metals are given in Table $7 \cdot 3$ and those for typical insulating materials in Table 7•4.

TABLE $7 \cdot 3$
Thermal Conductivity of Metals
$k=$ Btu per hour per square foot per degree per foot

| Temperature, <br> Degrees Fahrenheit | 50 | 100 | 200 | 300 | 400 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Aluminum | 121 | 122 | 125 | 128 | 131 |
| Brass | 62.5 | 65.0 | 71.0 | 77.0 | 85.0 |
| Cast iron | 29.2 | 28.5 | 26.8 | 23.0 | 18.5 |
| Copper | 222.0 | 221.0 | 220.0 | 218.0 | 217.0 |
| Lead | 20.0 | 20.0 | 20.0 | 20.0 | 20.0 |
| Nickel | 34.0 | 35.0 | 36.5 | 37.6 | 34.2 |
| Steel, $1 \%$ carbon | 25.0 | 25.0 | 25.0 | 25.0 | 25.0 |

TABLE $7 \cdot 4$
Thermal Conductivity of Insulating Materials
$k=$ Btu per hour per square foot per degree per foot

|  |  | 0.022 |  |
| :--- | :--- | :--- | :---: |
| Asbestos | $0.046-0.065$ | Paper | 0.83 |
| Brick | $0.125-0.42$ | Porcelain | 0.10 |
| Concrete | $0.41-1.25$ | Press board | $0.029-0.039$ |
| Cork board | 0.028 | Rock wool | $0.083-0.125$ |
| Earth | $0.084-0.37$ | Rubber (solid) | $0.17-0.67$ |
| Glass | $0.35-0.48$ | Sand | Wood (across grain) |
| Ice | $1.2-1.3$ | W.058-0.12 |  |

Before continuing with a consideration of the thermal conductivity of liquids it is desirable to develop the concept of viscosity from another and more commonly used point of view. If a gas stream is flowing smoothly and there is a velocity gradient or velocity difference between adjacent layers, each exchange of molecules takes a higher-velocity molecule from the fast-moving layer and trades it for a lower-velocity molecule from the more slowly moving layer. The effect of this exchange is to slow down the faster-moving layer and to speed up the slower one, just as though mechanical friction existed between the two layers.

If two layers are separated by a distance $d x$ between centers and one is flowing with a velocity $d v$ greater than the other, the mass per second of molecules exchanged between them through a contact area $a$ is $\mu a / d x$, as explained in equation $7 \cdot 1$. The equation of accelerated motion, Force $=$ mass $\times$ acceleration, or $F=m \frac{d v}{d t}$ may be written as

$$
F=\frac{m}{d t} d v
$$

Thus, the equivalent force between the two layers may be written as the product of the mass per second exchanged between them and the difference in velocity, or

$$
F=\frac{\mu a}{d x} d v=\mu a \frac{d v}{d x}
$$

This equation for viscous flow, which was first formulated from the experimental study of liquid flow, has led to the definition of viscosity as the dragging or frictional force in pounds per square foot of area between two layers of liquid when a velocity gradient of one foot per second per foot exists. It implies that the retarding force of one layer upon another is due to friction of molecules sliding over each other. This mechanical picture of viscosity long aided in concealing the true significance of the viscosity of a gas where the retarding effect of one layer on another is due entirely to free molecular interchange between layers.

The sliding friction between two solid surfaces, on the other hand, depends on the mutual attractive forces between molecules. These forces increase greatly as the molecules become more closely spaced, and actual welding occurs when by heat and pressure the molecules of the two pieces are brought closely enough together to re-form the original bonds between molecules.

From this point of view the retarding force between liquid layers may be visualized as partly due to the exchange of free molecules between layers but chiefly caused by the fact that the mutual attractive forces
between the closcly spaced liquid molecules result in faster-moving molecules tending to draw slower-moving adjacent molecules along with them. This leads to the conclusion that whereas the viscosity of a gas is a measure of molecular travel and consequently is directly connected with its thermal conductivity the viscosity of a liquid is more a measure of mutual molecular attractive forces and need have no connection with its thermal conductivity. It also makes it possible to understand why the viscosity of a gas increases with temperature, and the viscosity of liquids as shown in Table 7.5 decreases rapidly as the temperature increases. As the kinetic energy of the liquid molecules increases, they are less susceptible to the attractive forces of other molecules.

TABLE 7.5
Viscosity of Liguids ( $\mu \times 10^{5}$ )
$\mu=$ slugs per second per aquare foot per foot

| Temperature, Degrees, Fahrenheit | 50 | 100 | 150 | 200 | 300 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Water | 3.2 | 1.80 | 1.10 | 0.70 | 0.40 |
| Gasoline (sp. gr. $=0.748$ ) | 1.6 | 1.1 | 0.80 | 0.60 | . |
| Lubricating oll (SAE 30) | $\ldots$. | 200 | 72 | 33 | 7.2 |

It seems logical to view a liquid as an intermediate condition between a solid and a gas. In a solid a fixed molecular arrangement exists and heat is conducted by the effects of atomic and molecular vibration. In a gas there is complete molecular freedom and heat is conducted by molecular travel. In the liquid, molecular travel is restricted by close spacing and heat is transmitted both by the molecular travel of the gas and the molecular vibration of the solid. There is not such a wide

TABLE 7-6
Thermal Conductivity of Liguids
$k=$ Btu per hour per square foot per degree per foot

| Temperature, Degrees Fahrenheit | 50 | 100 | 150 | 200 |
| :--- | :---: | :---: | :---: | :---: |
| Water | 0.33 | 0.35 | 0.37 | 038 |
| Lubricating oil | 0.097 | 0.096 | 0.095 | 0.094 |
| Transil oil | 0.078 | 0.076 | 0.075 | 0.074 |

difference in liquid conductivities as in those of both gases and solids, and, as shown by Table $7 \cdot 6$, some increase with temperature and some decrease.
7.5. Heat Conduction. Fourier's conduction equation $7 \cdot 9$ which applies to solids and to stagnant layers of gases, liquids, and vapors should be written

$$
\begin{equation*}
q=-k a \frac{d t}{d x} \tag{7•12}
\end{equation*}
$$

The minus sign is necessary since heat is transferred in the direction of decreasing temperature and the direction of the heat flow $q$ is inherently contrary in sign to the temperature gradient $d t / d x$. For a time after a temperature difference is applied across any material, heat is stored and all the heat that enters one side does not leave at the other. This transient period varies from a few seconds for thin metal sheets to many hours for thick insulating walls. After steady-state conditions are established all the heat that enters one side leaves the other. Many times the duration of this transient period is of importance, but most significant heat transfer problems are concerned with continuous operation or steady-state conditions. For such sustained conditions, the total rate of heat flow $q$ of equation $7 \cdot 12$ may be considered constant.
Most frequently it is sufficiently accurate to use an average value of the conductivity $k$ for the temperature range involved. The greatest error of such an assumption occurs when large temperature differences exist across stagnant layers of gas, but seldom is further refinement justified.

When heat is flowing through plane surfaces, the area $a$ is constant and if a temperature drop $t_{1}-t_{2}=\Delta t_{1-2}$ occurs in a thickness $l$, the rate of heat flow $q$ may be found from equation $7 \cdot 12$ by integrating,

$$
\begin{equation*}
\int_{x=0}^{x=l} q d x=-\int_{t=t_{1}}^{t=\iota_{2}} k a d t \tag{7•13}
\end{equation*}
$$

When $q, k$, and $a$ are considered as constants this gives

$$
\begin{equation*}
q=-\frac{k a}{l}\left(t_{2}-t_{1}\right)=\frac{k a}{l}\left(t_{1}-t_{2}\right)=\frac{k a}{l} \Delta t_{1-2} \tag{7•14}
\end{equation*}
$$

If a temperature difference of 70 deg F exists between the two surfaces of an 8 -in. brick wall having an estimated conductivity ( $k=0.20$ ) Btu per hr per sq ft per deg per ft, the rate of heat loss through an area of one square foot is

$$
q=\frac{0.20 \times 1}{\frac{8}{12}} \times 70=21 \mathrm{Btu} \text { per } \mathrm{hr}
$$

When heat flows through several layers of different materials in series as in an insulated refrigerator wall, the flow $q$ is common to all layers,
and the total temperature drop is the sum of all the individual drops, or

$$
\Sigma \Delta t=\frac{q l_{1}}{a k_{1}}+\frac{q}{a} \frac{l_{2}}{k_{2}} \cdots=\frac{q}{a}\left(\frac{l_{1}}{k_{1}}+\frac{l_{2}}{k_{2}} \cdots\right)
$$

The rate of heat flow through such a compound wall is

$$
q=\frac{1}{\frac{l_{1}}{k_{1}}+\frac{l_{2}}{k_{2}} \cdots} a \Sigma \Delta t
$$

It has been found convenient to define the conductance $U$ of such a wall as

$$
U=\frac{1}{\frac{l_{1}}{k_{1}}+\frac{l_{2}}{k_{2}} \cdots}
$$

and the conduction equation becomes

$$
q=U a \Sigma \Delta t
$$

The conductance $U$ which defines the characteristics of one or more definite thicknesses of material in the units Btu per square foot per degree Fahrenheit should not be confused with the conductivity $k$ which describes the thermal properties of a material in the units Btu per hour per square foot per degree per foot.

When heat is conducted radially through the lagging on a stcam pipe or through the insulation of an electric cable, the variation of the area $a$ through which the heat flows must be considered. For the example illustrated in Figure $7 \cdot 1$ where $q$ Btu per hour


Fig. 7-1. Radıal heat conduction. flows radially through insulation having conductivity $k$, the temperature drop across the thickness $d r$ is given by equation $7 \cdot 12$ as

$$
\begin{equation*}
q=-k a \frac{d t}{d x}=-k 2 \pi r L \frac{d t}{d r} \tag{7•19}
\end{equation*}
$$

where $L$ is the length of insulation in feet. If the temperature at the inside surface is $t_{1}$ and of the outside is $t_{2}$,

$$
\frac{q}{2 \pi k L} \int_{R_{1}}^{R_{2}} \frac{d r}{r}=-\int_{t_{1}}^{t_{2}} d t
$$

or

$$
t_{1}-t_{2}=\Delta t_{1-2}=\frac{q}{2 \pi k L} \log _{\epsilon} \frac{R_{2}}{R_{1}}
$$

If an electric power cable has a conductor radius of 0.7 in . and 0.4 in . thick insulation and its power loss is 10.45 watts or 35.7 Btu per hr per ft length, the temperature drop through the insulation may be calculated from equation $7 \cdot 21$. Using the conductivity of the paper as $k=0.022$ from Table 7.4,

$$
\Delta t_{1-2}=\frac{q}{2 \pi k L} \log _{\epsilon} \frac{R_{2}}{R_{1}}=\frac{35.7 \times 2.3 \log _{10} \frac{1.1}{0.7}}{2 \pi \times 0.022 \times 1}=117 \mathrm{deg} \mathrm{~F}
$$

Several radial layers of different materials would be handled in a method similar to that used in equation $7 \cdot 15$. There are many specialized cases of transient and steady-state flow but the steady-state conduction through flat and concentric materials cover most ordinary problems met in practice.

7•6. Heat Convection. The film theory of heat convection, developed by Langmuir and Rice between 1910 and 1920, was the original analysis of the exchange of heat by convection between a fluid and a surface. This theory assumed that the entire temperature drop takes place in a stagnant layer of fluid that adheres to the surface through which heat must pass by conduction, and outside this stagnant layer complete turbulence was assumed to exist. Subsequent investigations of this boundary layer have verified the existence of a fluid layer that adheres to the surface and in which the motion is smooth or laminar. The fluid velocity, which is zero at the surface, increases outward until a point is reached where the fluid flow is turbulent, and this transition is quite abrupt. The actual conditions at the surface are in no way as simple and clean-cut as assumed in the original theory, for there are actually convection currents inside this boundary layer that was assumed to be completely stagnant; however, the general method of analysis originally used by Langmuir is still accepted as the best approach to the problem of heat convection.
7.7. Dimensional Analysis. At some time or another each division of knowledge progresses slowly from a pure art to an exact science. In an art each individual result is reached by trial-and-error experience and in a science the cumulative experience in the subject is so well and definitely formulated that results may be accurately predicted in advance. It is convenient to express these well-proved experimental conclusions in the form of mathematical equations that are usually simple in form because, as a rule, only one or two variables are con-
sidered. For example, in mechanics the average results of experimental experience are summarized in the statement that, for most materials, stress is approximately proportional to deformation. Also, as stated in article $2 \cdot 5$, experimental results for gases are expressed in the statement, that when the temperature is held constant, the volume varies inversely as the pressure.

When the factors that affect any result become greater in number, as in the conductance of a fluid film in heat convection, the job of expressing experimental results in convenient mathematical form is made much easier by the method of dimenszonal analysis. Dimensional reasoning is premised on the assumption that the most fundamental form of variation for any variable is exponential, and upon the fact that in any true equation the units must balance, that is, if square foot occurs on one side, square foot must also be present on the other.

This may be illustrated by assuming that the correct form of the equation of motion of a freely falling body is to be worked out from experiment. Suppose that in the light of existing experimental knowledge it seems reasonable to expect that the distance traveled by a falling body depends upon the time $t$, the acceleration of gravity $g$, and the mass of the object $m$. According to the principles of dimensional analysis, the most probable form of this equation would be assumed as

$$
s=K g^{a} t^{b} m^{c}
$$

where $K$ is a pure number or constant and $a, b$, and $c$ are exponents, all unknown.

If three basic concepts are accepted as fundamental units in any subject, all other concepts may be expressed in terms of the three. In most dimensional studies in mechanics mass $m$, length $l$, and time $t$ are used as fundamental and all others are expressed in terms of them. In this way $g$, which has the units feet per second per second would become

$$
\frac{\mathrm{ft}}{\sec ^{2}}=\frac{l}{t^{2}}=l t^{-2}
$$

The units of equation $7 \cdot 22$ would then be written

$$
l=l^{1} m^{0} t^{0}=\left(l t^{-2}\right)^{a} t^{b} m^{c}
$$

Since the unit $m$ must balance on the two sides of the equation its exponents must be equal, or $c=0$. For the unit $t, 0=-2 a+b$ or $b=2 a$. For $l, 1=a$, which gives $b=2 a=2$. Equation $7 \cdot 22$ may then be written

$$
s=K g^{1} t^{2} m^{0}=K g t^{2}
$$

If this form of an equation agrees well with experimental results, it would be accepted and the best average value for $K$ would be evaluated. Dimensional analysis contains no magic and it can often give erroneous results. Some important variables might be overlooked in the original assumptions, and it is always possible that the effect of some variable on the result is such that it is not accurately expressed by an exponential variation.

This method of approach is used in analyzing experimental results of heat convection, and the units involved are listed in Table 7-7. In addi-

TABLE 7•7
Dimensional Units

| Concept | Symbol | Units | Dimensions |
| :---: | :---: | :---: | :---: |
| Distance | $l$ | ft | $l$ |
| Mass | $m$ | slugs | $m$ |
| Time | $t$ | seconds | $t$ |
| Heat | H | Btu | H |
| Temperature | $T$ | deg F | $T$ |
| Velocity | $v$ | ft per sec | $t^{-1}$ |
| Acceleration | $d v / d t$ or $g$ | ft per sec per sec | $t^{-2}$ |
| Force | F | mass $\times$ accel. $=1 \mathrm{~b}$ | $m l^{-2}$ |
| Unit pressure | $P$ | pounds per sq ft | $m l^{-1} t^{-2}$ |
| Density | $\rho$ | slugs per cu ft | $m^{-3}$ |
| Viscosity | $\mu$ | slugs $/ \mathrm{sec} / \mathrm{sq} \mathrm{ft} / \mathrm{ft}$ | $m l^{-1} t^{-1}$ |
| Heat flow | $q$ | Btu per sec | $\mathrm{Ht}^{-1}$ |
| Conductivity | $k$ | Btu/see $/ \mathrm{sq} \mathrm{ft} / \mathrm{deg} / \mathrm{ft}$ | $H l^{-1} t^{-1} T^{-1}$ |
| Conductance | $U$ | $\mathrm{Btu} / \mathrm{sec} / \mathrm{sq} \mathrm{ft} / \mathrm{deg}$ | $\mathrm{Hl}^{-2} t^{-1} \mathrm{~T}^{-1}$ |
| Specific heat | $c_{v}$ or $c_{p}$ | Btu/slug/deg * | $\mathrm{Hm}^{-1} \mathrm{~T}^{-1}$ |

* Defined in mass units; previously used in weight units.
tion to the basic units $m, l$, and $t$ just discussed, two basic thermal units, heat quantity, $H$, and temperature, $T$, are added. To overcome the conflict in time units discussed in article $7 \cdot 2$, the second is used throughout. This means that the previously discussed values of conductivity $k$ and conductance $U$ must be divided by 3600 before use in a dimensional equation.
7.8. Forced Convection. As pointed out in article 7.5, forced convection refers to the cases where the velocity over the surface is almost completely determined by the mechanically induced flow and is very little affected by natural convection currents. Analysis and experiment have shown the boundary layer of article 7.5 to be dependent for any object upon the fluid velocity $v$, the size of the object designated by a
length $l$, upon the fluid density $\rho$, and the fluid viscosity $\mu$. Since the film conductance $U$, article $7 \cdot 4$, depends upon the type of boundary layer and in addition upon the conductivity $k$ of the fluid, the method of dimensional analysis is to assume

$$
U=\text { function of } v, l, \rho, \mu, \text { and } k
$$

Assuming that each factor enters exponentially (article 7.6) makes it possible to write this equation as

$$
U=K v^{a} l^{b} \rho^{d} \mu^{c} k^{f}
$$

Written in the units of Table 7•7, this becomes

$$
\begin{equation*}
m^{0} I I^{1} l^{-2} t^{-1} T^{-1}=\left(l t^{-1}\right)^{a} l^{l}\left(m l^{-3}\right)^{d}\left(m l^{-1} t^{-1}\right)^{e}\left(H l^{-1} t^{-1} T^{-1}\right)^{f} \tag{7•27}
\end{equation*}
$$

For the units of $m$

$$
0=d+e \quad \text { or } \quad d=-e
$$

For II $\quad 1=f$ or $f=1$.
For $l \quad-2=a+b-3 d-e-f$.
For $t \quad-1=-a-e-f$ or $a=-e, \quad$ since $f=1$.
For $T \quad-1=-f$ or $f=1$.
This duplicates the conclusions from $l$ and results in four useful equations and five unknown quantities, so one exponent must be left unknown. Since it usually proves to be good tactics, the exponent of $\mu$, which is $\rho$, will be left unknown. This gives

$$
d=-e \quad a=-e \quad f=1 \quad \text { and } \quad b=-1-e
$$

If these values are substituted into the original equation $7 \cdot 26$, the result is

$$
U=K v^{-\epsilon} l^{-1-c} \rho^{e} \mu^{e} k
$$

which is usually written

$$
\begin{equation*}
\frac{U l}{k}=K\left(\frac{v l \rho}{\mu}\right)^{-e} \tag{7•29}
\end{equation*}
$$

If the Langmuir-Rice concept of a stagnant film of definite thickness is accepted, the film conductance $U$ would become the fluid conductivity divided by the boundary layer thickness, $k / x$. The left-hand side of equation $\mathbf{7} \cdot 29, \mathrm{Ul} / \mathrm{k}$, which is usually known as Nusselt's number, would become but $l / x$, the ratio of the significant dimension $l$ of the object to the film thickness $x$. The difficulty introduced by different time units, the hour and the second, discussed in connection with Table 7.7 is minimized in Nusselt's number $U l / k$. Since the ratio $U / k$ is independent of
time units, the units of $U$ found from equation $7 \cdot 29$ will depend upon the time units used for $k$.

In a similar manner, the right-hand function $v l_{\rho} / \mu$, which is customarily known as Reynolds number, also has physical significance. Since acceleration $\frac{d v}{d t}$ may be written as $\frac{d v}{d s} \frac{d s}{d t}$ or as $v \frac{d v}{d s}$, the inertia force on an element of fluid has the units of volume times density times acceleration equals $l^{3} \rho \frac{v^{2}}{l}$ equals $l^{2} \rho v^{2}$. The viscous force on a surface has the units $\mu l^{2} \frac{v}{l}=\mu l v$ (equation $7 \cdot 11$ ) and the ratio of inertia force to viscous force in the fluid has the units $\frac{l^{2} \rho v^{2}}{\mu l v}=\frac{\rho l v}{\mu}$, which are the units of Reynolds number. Equation $7 \cdot 29$ may then be interpreted as saying that the ratio of the significant dimension of an object to the thickness of the stagnant fluid film about it depends only upon the ratio of inertia to viscous forces in the fluid flowing about it. When the object is a wire with flow across it or a pipe with flow inside it or around the outside, the diameter is the significant dimension $l$. When it is a plane surface with flow along it, the dimension $l$ of equation $7 \cdot 29$ is the length parallel to the flow.

When an exponential equation such as equation $7 \cdot 29$ is plotted on $\log -\log$ paper it is a straight line with a slope $e$, and an intercept equal to the constant $K$. Experimental work from a variety of sources has borne out the form of equation $7 \cdot 29$ as fundamental for all fluids under a variety of conditions. The best current experimental values of the exponent $-e$ and of the constant $K$ for common cases are shown in Table 7•8.

## TABLE $7 \cdot 8$

Forced Convection Constants

| Type of Flow | Exponent -e | Constant $K$ |
| :---: | :---: | :---: |
| Across small wires and pipes * | 0.41 | 0.764 |
| Across large wires and pipes $\dagger$ | 0.585 | 0.282 |
| Parallel to plane surfaces | 0.75 | 0.055 |
| Inside pipes when the total distance the fluid temperature rise are small | 0.80 | 0.028 |
| $* \text { For } \frac{v d \rho}{\mu} 250 \text { or lees. }$ | ore than 250. |  |

Suppose, for example, it is desired to predict the rate of heat loss by forced convection from a $2-\mathrm{in}$. outside diameter steam pipe, 10 ft long, if its surface temperature is 240 deg F and atmospheric pressure air at

60 deg F is blown across it at a velocity of 30 ft per sec. For an average temperature of 150 deg F Table $7 \cdot 2$ gives the viscosity of air as $0.419 \times 10^{-6}$ slugs per sec per sq ft per ft . The density $\rho$ may be found from the gas equation $P V=W R T$ as $\rho=\frac{W}{g V}=\frac{P}{g R T}$.

$$
\rho=\frac{14.7 \times 144}{32.2 \times 53.3 \times(460+150)}=0.00202 \text { slugs per cu ft }
$$

In equation 7.29 the dimension $l$ is the pipe diameter of $\frac{2}{12} \mathrm{ft}$ and Table $7 \cdot 8$ gives $-e=0.585$ and $K=0.282$. This gives

$$
\begin{gathered}
\frac{U l}{k}=K\left(\frac{v l \rho}{\mu}\right)^{-e}=0.282\left(\frac{30 \times \frac{12}{12} \times 0.00202}{0.419 \times 10^{-6}}\right)^{0585} \\
\frac{U l}{k}=0.282(24,100)^{0585}=0.282 \times 372=105
\end{gathered}
$$

The value of $k$ for air at 150 deg F from Table $7 \cdot 2$ is $k=0.0161 \mathrm{Btu}$ per hr per sq ft per deg per ft , and since $l=$ diameter $=\frac{2}{12}, U=$ $\frac{105 \times 0.0161}{2 / 12}=10.1 \mathrm{Btu}$ per hr per sq ft per deg. For a temperature difference of $240-60=180 \operatorname{deg} F$ and a total surface area of $10 \times \pi \times \frac{2}{12}=5.24 \mathrm{sq} \mathrm{ft}$, the rate of heat loss by forced convection $=q=U a \Delta t=10.1 \times 5.24 \times 180=9500$ Btu per hr. If $k$ has been used in second units, $q$ would have been in Btu per second.
7.9. Free Convection. When the convection of heat between a fluid and a surface is "free" or "natural," the only velocity present is that caused by density differences in the fluid. As heat is conducted to the fluid near the surface, the resulting fluid expansion reduces its density and its velocity increases. As it rises, both its temperature and velocity increase but the temperature difference between the fluid and surface decreases. As a result, the film conductance for free convection is dependent upon many variables among which the temperature coefficient of expansion is very important. Because the thermal expansion of liquids is not as conveniently expressed as that of gases and since most problems in free convection involve gases, the subject will be developed only for gases.

The temperature coefficient of expansion of a gas may be measured by the rate of change of density with respect to temperature. From the gas equation $\rho=\frac{P}{g R T}$ and

$$
\begin{equation*}
\frac{\partial \rho}{\partial T}=-\frac{P}{g R T^{2}}=-\frac{\rho}{T} \tag{7•30}
\end{equation*}
$$

This is written as a partial rather than a total differential since only the variation with respect to temperature is being considered. Absolute temperature $T$ is used in free convection instead of Fahrenheit temperature $t$ because the gas equation is involved.

The usual dimensional analysis of free convection assumes that the film conductance $U$ is a function of cight different variables: the viscosity, $\mu$; the characteristic height of the surface, $l$; the mass specific heat at constant pressure, $c_{p}$; the acceleration of gravity, $g$; the mass density, $\rho$; the temperature difference between the surface and the ambient gas, $\Delta T$; the expansion cocfficient, $\frac{\partial \rho}{\partial T}$; and the thermal conductivity, $k$. This would be expressed as

$$
\begin{equation*}
U=K \mu^{a} l^{b} c_{p}^{c} g^{d} \rho^{\rho}(\Delta T)^{f}\left(\frac{\partial \rho}{\partial T}\right)^{g}(k)^{h} \tag{7•31}
\end{equation*}
$$

Writing this equation dimensionally,

$$
\frac{H}{t L^{2} T}=\left(\frac{M}{L t}\right)^{a} L^{b}\binom{M}{M T}^{c}\left(\frac{L}{t^{2}}\right)^{d}\left(\frac{M}{L^{3}}\right)^{e} T^{f}\left(\frac{M}{L^{3} T}\right)^{g}\left(\frac{H}{t L^{2} T / L}\right)^{h}
$$

Equating the exponents of each dimensional concept gives:
$\left.\begin{array}{lrl}\text { For } H & 1 & =c+h . \\ \text { For } t & -1 & =-a-h-2 d . \\ \text { For } L & -2 & =-a+b+d-3 e-3 g-h . \\ \text { For } M & 0 & =a-c+e+g . \\ & \text { For } T & -1\end{array}\right)=-c+f-g-h . \quad$.

Since there are eight variables and only five possible equations, variables $c, d$, and $f$ will be left unknown. Thus,

$$
\begin{align*}
& a=c-2 d \\
& b=-1+3 d \\
& e=2 d-f  \tag{7•34}\\
& g=f \\
& h=1-c
\end{align*}
$$

Substituting these results in equation $7 \cdot 31$ gives

$$
U=K(\mu)^{c-2 d} l^{-1+3 d} c_{p}^{c} g^{d} \rho^{2 d-f}(\Delta T)^{f}\left(\frac{\partial \rho}{\partial T}\right)^{f}(k)^{1-c}
$$

Collecting terms,

$$
\frac{U l}{k}=K\left(\frac{\mu c_{p}}{k}\right)^{c}\left(\frac{g l^{3} \rho^{2}}{\mu^{2}}\right)^{d}\left(\frac{\partial \rho}{\partial T} \frac{\Delta T}{\rho}\right)^{f}
$$

In consistent mass units, $k=\epsilon \epsilon_{v} \mu$ from equation $7 \cdot 8$ and since $c_{p} / c_{v}$ is the adiabatic exponent $\gamma$, the first term of equation $7 \cdot 36$ reduces to

$$
\left(\frac{\mu c_{p}}{k}\right)^{c}=\left(\frac{\mu c_{p}}{\epsilon \epsilon_{v} \mu}\right)^{c}=\left(\frac{\gamma}{\epsilon}\right)^{c}
$$

If the value of $\frac{\partial \rho}{\partial T}$ from cquation $7 \cdot 30$ is combined with the third term of equation $7 \cdot 33$, the result is

$$
\begin{equation*}
\left(\frac{\partial \rho}{\partial T} \frac{\Delta T}{\rho}\right)^{f}=\left(\frac{\rho}{T} \frac{\Delta T}{\rho}\right)^{f}=\left(\frac{\Delta T}{T}\right)^{f} \tag{7•38}
\end{equation*}
$$

The final dimensional form of the free convection equation $7 \cdot 33$ is

$$
\begin{equation*}
\frac{U l}{k}=K\left(\frac{\gamma}{\epsilon}\right)^{c}\binom{g l^{3} \rho^{2}}{\mu^{2}}^{d}\left(\frac{\Delta T}{T}\right)^{f} \tag{7•39}
\end{equation*}
$$

This is rather complicated for regular use, and attempts at simplification have resulted in the use of the very arbitrary expedient that $c=d=f$. This works out with a fair degree of accuracy, not because it is theoretically true, but more because $\gamma / \epsilon$ is a function only of the type of gas being used, and the gases involved are usually diatomic. Also, for the usual temperature range encountered, $d$ and $f$ are very close to the same value. The final free convection equation is, then,

$$
\begin{equation*}
\frac{U l}{k}=K\left(\frac{\rho^{2} g l^{3} \gamma \Delta T}{\epsilon \mu^{2} T}\right)^{d} \tag{7•40}
\end{equation*}
$$

As in forced convection, the gas properties $\left(\rho=\frac{P}{g R T}\right)$ and $\mu$ are evaluated at the average temperature of the surface and the ambient gas, $\Delta T$ is the temperature difference, and $T$ is used as the average absolute temperature of the gas film. The dimension $l$ is the height of a plane surface, but is the diameter for a horizontal pipe or wire.

Such an equation as 7.40 should become a straight line on log-log graph paper and the slope of the curve should be the exponent $d$ when Nusselt's number, $U l / k$, is plotted against the free convection modulus $\frac{\rho^{2} g l^{3} \gamma \Delta T}{\epsilon \mu^{2} T}$. Experimental results give such a log-log curve that is not precisely a straight line, which indicates that the dimensional equation $7 \cdot 40$ is not quite adequate to express all the variations in free convection. This difficulty is met by breaking the experimental curve into two approximate straight line sections, and the values of the constant $K$ and exponent $d$ are given in Table 7•9.

## TABLE $7 \cdot 9$

## Free Convection Constants

| Condition | Exponent $d$ | Constant $K$ |
| :--- | :---: | :---: |
| For convection modulus less than 1000 | 0.25 | 0.55 |
| For convection modulus greater than 1000 | 0.33 | 0.13 |

Free convection calculations may be illustrated by considering the pipe of the example in article 7.8 in still air. For this

```
average \(\quad \rho=0.00202\) slugs per cubic foot
            \(g=32.2\) fect per second per second
            \(l=\frac{2}{12}\) foot
for air \(\quad \gamma=1.4\)
    \(\Delta T=240-60=180\)
for air \(\quad \epsilon=1.9 \quad\) (see Table 7.1)
average \(\mu=0.419 \times 10^{-6}\) slugs per second per square foot per
    foot
```

average $T=\frac{240+60}{2}+460=610$ degrecs Rankine
$\frac{U l}{k}=0.13\left(\frac{\rho^{2} g l^{3} \gamma \Delta T}{\epsilon \mu^{2} T}\right)^{0.33}$
$=0.13\left(\frac{(0.00202)^{2} \times 32.2 \times\left(\frac{2.2}{12}\right)^{3} \times 1.4 \times 180}{1.9 \times\left(0.419 \times 10^{-6}\right)^{2} \times 610}\right)^{0.33}$
$\frac{U l}{k}=0.13(755,000)^{033}=0.13 \times 91=11.8$
$U=\frac{11.8 k}{l}=\frac{11.8 \times 0.0161}{\frac{2}{12}}=1.14 \mathrm{Btu}$ per hr per sq ft per deg F

The corresponding rate of heat loss, $q=U a \Delta t=1.14 \times 5.24 \times 180$ $=1070 \mathrm{Btu}$ per hr as compared with 9500 Btu per hr when the air velocity is 30 ft per sec.

7•10. Radiation. Heated objects radiate energy at many different wavelengths and the amount of energy of any wavelength depends on the temperature and the type of surface. The range of wavelengths between 0.4 and 0.7 microns (one micron is a thousandth of a millimeter) is visible light and heat. Shorter wavelengths are ultraviolet light and longer ones are invisible or infrared heat.

Not all bodies absorb all wavelengths of energy equally well, and a theoretical black body is one that absorbs radiation of all frequencies completely. A black body may be approached by painting a surface
with lampblack. The radiation at the various wavelengths for a black body is shown in Figure $7 \cdot 2$ for three temperatures. The total energy radiated per unit area at any temperature is the area under the curve and this energy has been found experimentally to vary as the fourth power of the absolute temperature.

The effectiveness of any surface as a source of radiation is expressed as its emissivity $e$ which is defined as the ratio of its intensity of radiation to that of a black body at the same temperature. Its absorption coefficient, which is its ability to absorb radiation as compared to a


Fig 72 Radiation intensity. black body has been found experimentally to be the same as its emissivity. Approximate values of emissivity are given in Table $7 \cdot 10$ for some of the common materials.

TABLE 710
Radiation Emissivity of Matcrials

| Surface | $c$ | Surface | $e$ |
| :---: | :---: | :---: | :---: |
| Aluminum (metal) | $005-016$ | Ice | 063 |
| Aluminum (paint) | 065 | Lamphlack | 096 |
| Brick (red) | 0 26-0 38 | Plasteı wall | 091 |
| Cast aron (oxidized) | 0 63-0 73 | Steel (polshed) | 0 07-0 40 |
| Concrete | 063 | Steel (oxidized) | 079 |
| Copper (polished) | 0 04-0 17 | Surface soll | 038 |
| Glass | 090 | Water | 095 |

The experimentally determined total rate of radiation from any surface is

$$
\begin{equation*}
q=0.174 e\left(\frac{T}{100}\right)^{4} \text { Btu per hr per sq ft } \tag{7•41}
\end{equation*}
$$

where $T$ is the surface temperature in degrees absolute Fahrenheit or Rankine.

Since this radiant energy passes out in every direction from each particle, if the exchange of radiation between two small surfaces is desired the radiation from each surface that strikes the other must be
calculated; however, most engineering problems involve cither the radiation of a small object in a large space or the radiation exchange between large surfaces close together.

When a small object radiates in a large space, all its radiation is ultimately absorbed by the enclosure. The absorption occurs regardless of the emissivity of the surface of the enclosure, because there is little chance of any of the radiation that leaves the small object ever being reflected back to it, notwithstanding the number of reflections before it is absorbed by the enclosure which has the properties of a black body. If the absolute temperature of the object is $T_{1}$ it loses heat by radiation at the rate $0.174 e_{1}\left(\frac{T_{1}}{100}\right)^{4}$.

If the walls of the enclosure are at a temperature $T_{2}$ each square foot of surface of the object reccives radiation at the rate $0.174\left(\frac{T_{2}}{100}\right)^{4}$ from the wall, since an enclosure radiates as a black body. Of this amount, the object absorbs $0.174 e_{1}\left(\frac{T_{2}}{100}\right)^{4}$ and reflects the remainder to be re-absorbed by the enclosure. The net loss of heat by radiation from a small object with temperature $T_{1}$ and emissivity $e_{1}$ in a large enclosure having a wall temperature $T_{2}$ is

$$
q=0.174 c_{1}\left[\left(\frac{T_{1}}{100}\right)^{4}-\left(\frac{T_{2}}{100}\right)^{4}\right]
$$

This is commonly known as the Stefan-Boltzmann radiation equation.
The other important case is that of two large parallel surfaces so close together that all the radiation that leaves one must strike the other. If the hotter surface has an emissivity $e_{1}$ and temperature $T_{1}$ and the other has emissivity $e_{2}$ and temperature $T_{2}$, the radiation from the first is $0.174 e_{1}\left(\frac{T_{1}}{100}\right)^{4}$. When this strikes the other surface, an amount $e_{2} 0.174 c_{1}\left(\frac{T_{1}}{100}\right)^{4}$ is absorbed and the remaining

$$
\left[\left(1-c_{2}\right) 0.174 e_{1}\left(\frac{T_{1}}{100}\right)^{4}\right]
$$

is reflected and strikes surface number one. Of this amount

$$
\left[e_{1}\left(1-e_{2}\right) 0.174 e_{1}\left(\frac{T_{1}}{100}\right)^{4}\right]
$$

is absorbed and

$$
\left[\left(1-\iota_{1}\right)\left(1-\rho_{2}\right) 0.174 c_{1}\left(\frac{T_{1}}{100}\right)^{4}\right]
$$

is reflected again to strike body number two.
If these successive absorptions and reflections are followed through for the radiation of both bodies, the result will be the net radiation exchange expressed by the geometric series,

$$
{ }_{1} q_{2}=0.174\left[\left(\frac{T_{1}}{100}\right)^{4}-\left(\frac{T_{2}}{100}\right)^{4}\right]\left[e_{1} \rho_{2}+\epsilon_{1} \rho_{2}\left(1-e_{1}\right)\left(1-e_{2}\right), \text { ctc. }\right]
$$

The sum of such a series is

$$
{ }_{1} q_{2}=\frac{e_{1} \rho_{2}}{e_{1}+e_{2}-e_{1} c_{2}} 0.174\left[\binom{T_{1}}{100}^{4}-\binom{T_{2}}{100}^{4}\right]
$$

When $c_{1}$ and $e_{2}$ are close to unit this may be approximated by

$$
\begin{equation*}
{ }_{1} q_{2}=0.174 e_{1} e_{2}\left[\binom{T_{1}}{100}^{4}-\binom{T_{2}}{10 \overline{0}}^{4}\right] \tag{7•44}
\end{equation*}
$$

which is usually used as the radiation exchange between large parallel surfaces.

The problem of determining the true temperature of a heated gas in a container with cool walls is typical of the manner in which radiation often enters into heat transfer calculations. If a glass thermometer placed in the gas indicates a temperature of 300 deg F when the walls of the container are at 200 deg F , it means that the thermometer is losing heat to the walls by radiation. It must pick up this heat from the gas by convection, which means that the temperature it indicates must be less than the true gas temperature.

The bulb of the thermometer with an emissivity $e_{1}=0.90$ (Table $7 \cdot 10$ ) loses heat to the walls by radiation (equation $7 \cdot 42$ ).
Radiation loss $=0.174 \times 0.90\left[\left(\frac{460+300}{100}\right)-\left(\frac{460+200}{100}\right)^{4}\right]$
$=227 \mathrm{Btu}$ per hr per sq ft
If the gas velocity is such as to result in a film conductance of 10.0 Btu per hr per sq ft per deg (example article $7 \cdot 8$ ), the temperature difference between the thermometer and the gas would have to be $\frac{227}{10}=22.7$ deg F lower than the true gas temperature in order to receive by convection from the gas the heat it loses to the walls by radiation. The true gas temperature would then be $300+22.7=322.7 \mathrm{deg} \mathrm{F}$ and the error in the thermometer reading would be 22.7 deg F.

## Suggested Reading

Heat Transmission
Severns, Heating, Ventilating, and Air Conditioning Fundamentals. Pages 38-54. Greene, Heating, Ventilating and Air Conditioning. Pages 50-75.
Barnard, Ellenwood, and Hirshfeld, Heat Power Engineering. Part II, Chapter XXVI.
Dimensional Analysis
Croft, Thermodynamics, Fluid Flow and Heat Transmission. Pages 8-28.
Radiation
Stoever, Applied Heat Transmission. Chapter II.
Jakob and Hawkins, Elcments of Heat Transfer and Insulation. Chapter XII.

## CHAPTER 8

## COMPRESSION OF GASES

8.1. Compressors. Reciprocating or piston-type compressors are usually used when moderate volumes of high-pressure gases are desired and centrifugal blowers and fans are used to deliver large volumes at low pressures. The cylinders of reciprocating compressors range from 3 in . to 3 ft in diameter with strokes varying from 4 to 40 in . Exeept for some small units, compressors are ordinarily double-acting, both faces of the piston being active.

In the double-acting reciprocating compressor illustrated by Figure $8 \cdot 1$ gas is drawn into the left-hand intake valve as the rotation of the crank moves the piston to the right. This is shown on the pressurevolume indicator diagram as the intake stroke. The intake valve, which on the intake stroke was held open against a light spring pressure by the slightly reduced pressure in the cylinder, closes as the piston starts the return stroke and the pressure of the trapped gas rises progressively. The compression stroke, shown on the indicator diagram of Figure 8.1, terminates when the pressure in the cylinder slightly exceeds the pressure in the receiver tank and the spring-loaded exhaust valve is forced open.

During the discharge portion of the stroke high-pressure gas is moved into the recciver and this continues until the end of the stroke. For mechanical reasons, the piston must not quite reach the cylinder head and it is also necessary to have some space around the intake and discharge valves. This volume of high-pressure gas that remains in the cylinder space at the end of the discharge stroke is known as the clearance volume and it varies from 1 per cent of the cylinder volume or piston displacement in large machines to as much as 8 per cent in small ones.

As the piston starts its return stroke to the right, Figure 8•1, the discharge valve closes and the clearance air re-expands until the cylinder pressure is down to suction pressure, at which time the intake valve opens for a new charge to enter and another cycle starts. The fraction of the return stroke that is consumed by the re-expansion of the clearance air increases as the receiver pressure is increased, with a corresponding decrease in the portion of the stroke left for intake.

The cylinder casting is usually cored and water is circulated around it and the discharge passages to remove as much of the heat of compression as possible. In some small units, heat is lost from metal fins


Fig. 8•1. Reciprocating compressor.


Fig. 8.2. Two-stage compressor.
on the outside of the cylinder to the air stream that is forced over them, usually by a simple fan built into the flywheel. Lubrication of the piston, piston rings, and valves is accomplished by lubricating oil that is drawn in sparingly with the intake gas in double-acting machines and
by splash lubrication from the crankease below the piston in small single-acting units.

Compressors usually require relatively heavy flywheds, since the variations in the gas pressure on the piston and the inertia of the reciprocating parts would otherwise produce large instantaneous speed fluctuations during each revolution. For receiver prestures below 80 lb per sq in. simple single-cylinder units are customarily used, but for higher pressures it is necessary to use multi-stage compresors, as illustrated in Figure 8.2, to prevent excessive temperature and reduce the compression work. If the discharge temperature becomes too high there is some danger of an explovion of carbonized oil and combustible dust often present in the intake gas. By cooling the partially compressed gas between stages in a water-cooled intercooler the final temperature may be controlled and the total compression work reduced.
8.2. Indicator Cards. With the aid of inducators, pressure-volume charts or indicator cards are taken for actual empressors and such a card is illustrated by Figure $8 \cdot 3$. In an indicator the instantancous pressure in the cylinder is transmitted to a small piston that deforms a calibrated spring, and as it deflects the spring it causes a pencil or stylus to move vertically. From a reducing motion on the compressor cross-head a drum with paper on it is caused to rotate in proportion to the piston travel. The resulting graph that is drawn as the stylus presses against the moving paper is the pressure-volume history of the compression cycle in the cylinder. A spring scale of 100 lb per sq in. per in. means that a cylinder pressure of that amount will produce a onc-inch


Fig. 8 3. Indicator card. vertical travel of the stylus point.

Indicator cards yield much useful information about the operation of a compressor. As indicated by Figure 8.3, the size and shape of the hatched area above the receiver pressure line reflects the amount of exhaust-valve pressure loss just as the area below the atmospheric pressure line measures intake valve losses. If the temperature of the reexpanded clearance gases at atmospheric pressure 1 and that of the compression gas at 2 were the same as that of the supply, the ratio of the lengths $\left[\frac{1-2}{4-5}\right]$ would be the volumetric efficiency. Strictly defined, the volumetric efficiency is the ratio of the actual weight of gas taken in per stroke to the weight that would be taken if the full piston displacement were filled at suction pressure and temperature. The valuc of volu-
metric efficiency as found from the indicator card is just an approximation that may be several per cent in error.

The area of an indicator card may be determined by mechanical integration with a planimeter and when this area is divided by the card length, 4-5, the result is the mean ordinate. When this mean ordinate is multiplied by the scale of the indicator spring, the result is the average unit net pressure acting on the piston during the working stroke, and this is known as the mean effective pressure or mep. When this mean effective pressure is multiplied by the net piston area, the result is the average force acting on the piston throughout the stroke. The full piston area is effective on the head end, but the piston rod area must be subtracted when the crank end is being considered. When this average force on the piston is multiplied by the stroke of the compressor the result is the work done on the gas per stroke. When this work per stroke is multiplied by the strokes per minute and this power is changed to horsepower, it is known as the indicated horsepower or i hp of the machine.
8.3. Compressor Cycles. The usual theoretical analysis of compressor cycles assumes a cylinder without clearance for the sake of simplification. Such an assumption amounts to adopting the point of view that it is the fault of the actual


Fig. 8-4. Theoretical indicator cards. machine that it must have a clearance volume. However, particularly in large compressors, the actual effect of clearance on the total work done is small because most of the compression work done on the clearance gas is regained upon re-expansion.

Such theoretical indicator cards are shown by Figure 8.4 for three types of compression. The isothermal or constant-temperature compression is shown as 1-2 and the adiabatic compression by 1-4. These two compressions were discussed in article 4.4. The polytropic, 1-3, represents any compression between an isothermal and an adiabatic, where some cooling is accomplished but not enough to prevent a temperature rise. Since an isothermal compression on a $P-V$ diagram is represented by $P V=$ constant and an adiabatic (equation $4 \cdot 25$ ) by $P V^{\gamma}=$ constant, it is reasoned by analogy that any compression between these limits may be represented by the polytropic equation $P V^{n}=$ constant, where $n$ is known as the polytropic exponent with values between 1 and $\gamma$.

Actually, for an expansion or compression to follow such an equation, heat would have to be added or conducted away at precisely the correct rate at the different points along the stroke. Even though this rarely happens, compresion and expansion curves for actual machines are fitted surprisingly well by the general equation $P V^{n}=$ constant.

The value of the exponent $n$ may be found from actual indicator cards by scaling off values of absolute pressure and total volume, including clearance and plotting on log-log graph paper. Values for actual air compressors for which $\gamma=1.4$ vary from $n$ equals 1.35 for high-speed machines to values as low as $n$ equals 1.15 for small low-speed compressors where cylinder cooling is unusually effective.

The discussion of article 4.4 was concerned only with the compression portion of the cycle, but the entire cycle of a compressor may be analyzed in terms of the general encrgy equation,

$$
d Q=c_{v} d T+\frac{1}{J} P d V^{\prime}+{ }_{J}^{1} V^{\prime} d I^{\prime}+\frac{1}{J g} v d v
$$

In the analysis of high-pressure compressors kinetic energy of flow is usually neglected, which means that the acceleration work term $\frac{1}{J g} v d v$ is assumed to be zero.

Isothermal Compression. Since the indicator diagram, Figure 8.4, is referred to the absolute zero of pressure, the entering gas must be credited with the flow work that would be required to move a volume $V_{1}$ from zero pressure to a region at pressure $P_{1}$, or ${ }_{J}^{1} P_{1} V_{1}$ Btu. This is represented by area $1,6,7,8,9$. As developed in article $4 \cdot 4$ in equations $4 \cdot 16$ to $4 \cdot 20$, the total compression work is

$$
\frac{P_{1} V_{1}}{J} \log _{\epsilon} \frac{V_{2}}{V_{1}} \text { or } \frac{W R T_{1}}{J} \log _{\epsilon} \frac{P_{1}}{P_{2}}
$$

This is represented in Figure $8 \cdot 4$ by area $9,1,2,8$. The absolute flow work involved in moving the compressed volume $V_{2}$ into the receiver is $-\frac{P_{2} V_{2}}{J}$, the minus sign being an indication that it is work done on the gas.

The total externally supplied work for the entire cycle is

$$
\text { Work per cycle }=\frac{P_{1} V_{1}}{J}+\frac{W R T_{1}}{J} \log _{\epsilon} \frac{P_{1}}{P_{2}}-\frac{P_{2} V_{2}}{J}
$$

Since for an isothermal change $P_{1} V_{1}$ equals $P_{2} V_{2}$, the work per cycle turns out to be merely
Isothermal work per cycle $=\frac{W R T_{1}}{J} \log _{\epsilon} \frac{P_{1}}{P_{2}}=-\frac{W R T_{1}}{J} \log _{\epsilon} \frac{P_{2}}{P_{1}}$
the negative sign indicating the fact that it is work done on the gas. During such a cycle it is assumed that no heat is extracted during suction or discharge and an amount

$$
-\frac{W R T_{1}}{J} \log _{\epsilon} \frac{P_{2}}{P_{1}}
$$

must be removed during compression to maintain constant temperature.
Adiabatic Compression. It was shown in equations $4 \cdot 21$ to $4 \cdot 26$ that during an adiabatic compression the work done on the gas is equal but opposite in sense to the gain in internal energy. This would mean that for the adiabatic cycle of Figure 8.4,

$$
\begin{equation*}
\text { Cycle work }=\frac{P_{1} V_{1}}{J}-W^{r}\left(T_{4}-T_{1}\right)-\frac{P_{4} V_{4}}{J} \tag{8.4}
\end{equation*}
$$

This may be written as
Cycle work $=-W c_{\nu}\left(T_{4}-T_{1}\right)-\frac{W R}{J}\left(T_{4}-T_{1}\right)$

$$
=-W c_{p}\left(T_{4}-T_{1}\right)
$$

Also for such a compression, from equation $4 \cdot 26$,

$$
\frac{T_{4}}{T_{1}}=\left(\frac{P_{4}}{P_{1}}\right)^{(\gamma-1) / \gamma}
$$

Polytropic Compression. During a polytropic compression, Figure 8.4, the temperature changes and heat is conducted away, so the energy equation becomes

$$
d Q=W c_{v} d T+\frac{P d V}{J}
$$

For a polytropic, $P_{1} V_{1}^{n}=P_{3} V_{3}^{n}=P V^{n}$ or

$$
P=\frac{P_{1} V_{1}^{n}}{V^{n}}=P_{1} V_{1}^{n} V^{-n}
$$

By combining equations 8.7 and 8.8 and by integrating between limits 1 and 3 ,

$$
\begin{gather*}
d Q=W c_{v} d T+\frac{P_{1} V_{1}^{n}}{J} V^{-n} d V \\
{ }_{1} \Delta Q_{3}=W c_{v}\left(T_{3}-T_{1}\right)+\frac{P_{1} V_{1}^{n}}{J(1-n)}\left(V_{3}^{1-n}-V_{1}^{1-n}\right) \tag{8•10}
\end{gather*}
$$

As $P_{1} V_{1}^{n}=P_{3} V_{3}^{n}$, equation $8 \cdot 10$ may be reduced to

$$
{ }_{1} \Delta Q_{3}=W c_{v}\left(T_{3}-T_{1}\right)+\frac{1}{J(1-n)}\left(P_{3} V_{3}-P_{1} V_{1}\right)
$$

Since $P_{3} V_{3}=W R T_{3}$ and $P_{1} V_{1}=W R T_{1}$,

$$
{ }_{1} \Delta Q_{3}=W c_{v}\left(T_{3}-T_{1}\right)-\frac{W R}{J(n-1)}\left(T_{3}-T_{1}\right)
$$

When volume is eliminated between the general gas equation and the polytropic $P-V$ equation, as was done for the adiabatic in equations $4 \cdot 25$ and $4 \cdot 26$, the result is

$$
\begin{equation*}
\frac{T_{3}}{T_{1}}=\left(\frac{P_{3}}{P_{1}}\right)^{(n-1) / n} \tag{8•13}
\end{equation*}
$$

The two terms of equation 8.12 could be combined into a simple one, but it is worth while to evaluate them separately so changes in internal energy and comprestion work done may be identified individually.

When the flow work terms are combined with the compression work for a polytropic,
Polytropic cycle work

$$
\begin{align*}
& ={ }_{J}^{1} P_{1} V_{1}-W R\left(T_{3}-T_{1}\right)  \tag{8•14}\\
& J(n-1) \\
& { }_{J}^{1} P_{3} V_{3} \\
& =-{ }_{J}^{W R}\left(1+\frac{1}{n-1}\right)\left(T_{3}-T_{1}\right)=-\frac{W R n}{(n-1)}-\frac{\left(T_{3}-T_{1}\right)}{J}
\end{align*}
$$

The following numerical comparison of isothermal, adiabatic, and polytropic compression cycles for air initially at 14.7 lb per sq in . absolute pressure and 80 deg F will aid in visualizing the differences involved. The receiver pressure will always be assumed to be 100 lb per sq in . gage and the polytropic exponent $n=1.3$ will be used.

TABLE $8 \cdot 1$
Comparison of Compression Cycles

|  | Final <br> Temperature | Btu per Pound <br> Extracted During <br> Compression | Cycle Work <br> Btu per Pound |
| :---: | :---: | :---: | :---: |
| Cycle | Degrees Fahrenheit | 76.0 | 76.0 |
| Isothermal | 80 |  |  |
| Polytropic |  |  |  |
| $n=1.3$ | 406 | 0 | 102.0 |
| Adiabatic | 502 |  |  |

Two-Stage Compression. The ideal indicator cards for a two-stage compressor with no clearance or losses are shown in Figure 8.5.

The polytropic work in the low-pressure cylinder, represented by area $1,2,6,7$, is

$$
\text { Low-pressure work }=\frac{W R n}{J(1-n)}\left(T_{2}-T_{1}\right)
$$

High-pressure work (area $3,4,5,6)=\frac{W R n}{J(1-n)}\left(T_{4}-T_{3}\right)$
The usual limit for intercooling is for $T_{3}=T_{1}$ and the work is usually balanced between the two cylinders. For the same exponents $n$ and equal work,

$$
T_{2}-T_{1}=T_{4}-T_{1}
$$

or

$$
\frac{T_{2}}{T_{1}}=\frac{T_{4}}{T_{1}}
$$

But

$$
\frac{T_{2}}{T_{1}}=\left(\frac{P_{2}}{P_{1}}\right)^{(n-1) / n} \text { and } \frac{T_{4}}{T_{1}}=\left(\frac{P_{4}}{P_{3}}\right)^{(n-1) / n}=\left(\frac{P_{4}}{P_{2}}\right)^{(n-1) / n}
$$

This results in the relationship

$$
\frac{P_{2}}{P_{1}}=\frac{P_{4}}{P_{2}} \quad \text { or } \quad P_{2}=\sqrt{P_{1} P_{4}}
$$

This gives the correct interstage pressure to balance the high- and lowpressure work. The theoretical saving in total compression work resulting from two-stage compression


Fig. 8.5. Two-stage compression cycle. instead of single-stage is illustrated by the shaded area in Figure 8.5. This full gain is not actually realized, because a two-stage compressor has higher mechanical friction losses than a singlestage machine. Since the air is admitted to high- and low-pressure cylinders at the same temperature, $T_{1}=T_{3}$, the low-pressure cylinder must be the larger by the pressure ratio $P_{2} / P_{1}$, if they are to handle the same weight of air at the same rpm.

If 80 deg F atmospheric pressure air is compressed to 100 lb per sq in. gage or 114.7 lb per sq in. absolute pressure in an ideal two-stage com-
pressor having a polytropic exponent $n=1.3$ in both cylinders, the ideal intercooler pressure is

$$
P_{2}=\sqrt{P_{1} P_{4}}=\sqrt{14.7 \times 114.7}=41 \mathrm{lb} \text { per sq in. abs or }
$$

26.3 lb per sq in. gage

The temperature at the end of compression is the same in both cylinders,

$$
T_{2}=T_{4}=T_{1}\left(\frac{P_{2}}{P_{1}}\right)^{(n-1) / n}=540\left(\frac{41}{14.7}\right)^{0.3 / 1.3}
$$

$$
=684 \operatorname{deg} F \text { abs }=224 \operatorname{deg} F
$$

The heat extracted during compression in each cylinder is

$$
Q=W\left(c_{v}+\frac{R}{J(1-n)}\right)\left(T_{2}-T_{1}\right)=-8.2 \text { Btu per } \mathrm{lb}
$$

The heat taken out in the intercooler to reduce the air to its original temperature is

$$
Q=-W c_{p}\left(T_{2}-T_{3}\right)=-34.5 \text { Btu per lb }
$$

The total work done in each cylinder is

$$
\text { Work }=\frac{W R n}{J(1-n)}\left(T_{2}-T_{1}\right)=-42.8 \text { Btu per lb }
$$

which means that the total work for the two cylinders is -85.6 Btu per lb.

If a single-stage compression is used, the temperature at the end of compression as given in Table $8 \cdot 1$ for the same conditions is 406 deg F and the cycle work is 97.2 Btu per lb or 1.13 times as much as is required for a two-stage cycle.
8.4. Compressor Performance. ${ }^{1}$ The important performance characteristics of a reciprocating compressor are its gas-handling capacity which depends upon its size, speed, and volumetric efficiency; its compression efficiency as measured by the indicated work and the ideal cycle work; its mechanical efficiency which measures friction and rotation losses; and the horsepower required to drive it.

Gas-handling capacity is usually measured at intake temperature and pressure conditions. This is the reason why an air compressor is rated in cubic feet per minute of "free air." This can be evaluated approximately from the speed, displacement, and the volumetric efficiency as estimated from indicator cards. A more accurate determination requires the measurement of the intake gas which is difficult to do because of the pulsating nature of the suction. In general, reduced cylinder
cooling, increased speed, and higher receiver pressures are accompanied by reduced volumetric efficiency and lower air-handling capacity.

The compression efficiency is measured by the ratio of the ideal cycle compression work per pound to the indicated work per pound as determined from actual indicator cards. There is no uniformity in reporting compression efficiency. Sometimes it is referred to an isothermal cycle and other times the adiabatic is used. It is very important when referring to compression efficiency to be sure to designate which ideal cycle is involved. Using an isothermal cycle amounts to saying that it is the fault of the compressor that complete cooling is not accomplished and to use the adiabatic as a reference implies that no cooling is expected and that the machine should be credited with any cooling savings that are made.
Since the efficiency of any machine is the ratio of output to input, the mechanical efficiency of a compressor is the ratio of indicated horsepower to input horsepower required to drive it. The determination of input horsepower by test requires the use of a dynamometer, whereas the indicated horsepower may be determined readily for a machine while it is in actual operation. The mechanical efficiency of large well-designed machines is usually between 0.90 and 0.95 , but in small high-speed machines it may easily be as low as 0.75 .
8.5. Centrifugal Blowers. In a centrifugal blower, which consists of a bladed wheel rotating in a casing as shown in Figure 8.6, the gas enters axially near the impeller shaft and the action of centrifugal force creates a higher pressure in the casing. In addition, the gas is accelerated to a higher velocity as it passes through the blades of the impeller and this energy may be converted to pressure by the orderly deceleration of the gas in guide vanes sometimes placed in the casing or in the volute or scroll casing of simpler machines.

Both the centrifugal pressure and the kinetic energy of the gas leaving the wheel vary as the square of the rotative speed, and both are increased by a greater impeller blade length. Low-speed fans with short impeller blades develop pressures of the order of a few ounces per square inch, but multiple-stage high-speed blowers are built for pressures of 25 to 30 lb per sq in.

Blowers and fans are used for ventilation and air conditioning, to supply combustion air to boiler furnaces, blast furnaces, etc., and for such applications as the pneumatic conveying of material such as powdered coal, grain, and cement.
8.6. Blower Cycles and Performance. Since a blower is a continu-ous-flow machine instead of a "batch" type such as a compressor, there is really no blower diagram that exactly corresponds to the indicator
card. It is possible, however, to represent the overall history of a pound of gas as it passes through a blower on a pressure-volume diagram.
Such a diagram is shown in Figure 8.7 for gas that enters a blower at conditions $P_{1}, V_{1}$, and $T_{1}$ and leaves at $P_{2}, V_{2}$, and $T_{2}$. It is, of course, impossible to represent the effect of kinetic energy changes on such a diagram which shows only compression and flow work. In lowpressure machines the change in volume during compression is so small


Fig. 8-6. Centrifugal blower.


Fig. 8.7. Pressure-volume for a blower.
as to be negligible, and it is customary to ignore this as well as the internal energy change.

If it is assumed that the blower picks up the gas with no initial velocity, the acceleration work or the increase in kinetic energy of flow from the general energy equation for $W$ pounds of gas is

$$
\text { Change in kinetic energy }=\frac{W}{g} \int_{0}^{v} v d v=\frac{W v^{2}}{2 g} \mathrm{ft}-\mathrm{lb}
$$

The previously discussed approximate flow work is

$$
\text { Flow work }=\left(P_{2}-P_{1}\right) V_{1}=\Delta P V_{1} \mathrm{ft}-\mathrm{lb}
$$

The effect of neglecting compression work and approximating the flow work is to overestimate the actual sum of these two by an amount represented by the hatched area in Figure 8.7.

For this assumption which is satisfactory for low-pressure work, the blower is credited with delivering an amount of energy,

$$
\text { Work done }=\Delta P V_{1}+\frac{W v^{2}}{2 g} \mathrm{ft}-\mathrm{lb}
$$

For one cubic foot, since $W / g V$ is the mass density $\rho$,

$$
\text { Work done }=\Delta P+\frac{\rho v^{2}}{2} \cdots \mathrm{ft}-\mathrm{lb} \text { per cu } \mathrm{ft}
$$

where $v$ is the leaving velocity in feet per second, $\rho$ is slugs per cubic foot, and $\Delta P$ is pounds per square foot. For a blower delivering gas with a velocity $v$ feet per second through a discharge pipe of area $A$ and imparting a pressure increase of $\Delta P$, the power would be

$$
\begin{equation*}
\dot{\text { Power }}=\Delta P v A+\frac{\rho v^{2}}{2} v A=\Delta P v A+\frac{\rho v^{3} A}{2} \mathrm{ft}-\mathrm{lb} \text { per sec } \tag{8•24}
\end{equation*}
$$

With the blower input known from a dynamometer test, the blower efficiency, power output divided by input, may be found.

The approximate method that neglects volume and temperature changes is satisfactorily accurate for low-pressure fans but is too inaccurate for high-pressure machines. Since no attempt is made to cool the gases in a blower, the ideal compression cycle would be the adiabatic and it would be calculated just as it was for the compressor in article $8 \cdot 3$. One practical difficulty met when the pressure ratio of equation $8 \cdot 6$ is near unity is that of accurately calculating the final temperature $T_{2}$ from the adiabatic equation,

$$
\frac{T_{2}}{T_{1}}=\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma-1) / \gamma}
$$

It is often convenient to expand the value of $\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma-1) / \gamma}$ by the binomial theorem and use the first two terms,
$\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma-1) / \gamma}=\left[1+\left(\frac{P_{2}}{P_{1}}-1\right)\right]^{(\gamma-1) / \gamma} \approx 1+\frac{\gamma-1}{\gamma}\left(\frac{P_{2}}{P_{1}}-1\right)$
In addition to the adiabatic temperature rise, the rotational losses of the impeller and other turbulence losses cause an additional increase in temperature. The difference between the actual leaving temperature and the theoretical adiabatic value may be used as a measure of such losses, provided the heat losses from the blower casing may be considered negligible. For a high-pressure blower, where compression work and temperature rise are considered, the total work for $W$ pounds from equations 8.5 and 8.20 is

$$
\text { Theoretical work }=\left[W c_{p}\left(T_{2}-T_{1}\right) J+\frac{W v^{2}}{2 g}\right] \text { foot-pounds }
$$

Just as for compressors it is customary to report the gas-handling capacity of blowers in terms of cubic feet per minute at intake conditions. The usual performance data for a centrifugal blower, Figure
8.8 , consist of curves of blower pressure, output or air horsepower, input horsepower, and blower efficiency.


Fig. 8.8. Centrifugal blower characteristics.
8.7. Air Flow Measurement. The measurement of the flow of gases such as air is usually accomplished by causing it to speed up to pass through a calibrated restriction such as a nozzle or orifice. Since it is usually desirable to keep the resulting pressure drop small to prevent energy losses, the general flow or energy equation $8 \cdot 1$ may be used with the assumptions that the temperature and specific volume do not change and that no heat has time to be conducted to or from the flowing gas. For such the energy equation becomes

$$
\begin{equation*}
0=V d P+\frac{W}{g} v d v \tag{8•27}
\end{equation*}
$$

If the air is drawn from a large space where the initial velocity may be considered negligible, the integration of equation 8.27 for constant volume gives

$$
\begin{equation*}
0=\int_{P_{1}}^{P_{2}} V d P+\frac{W}{g} \int_{0}^{V_{2}} v d v \quad \text { or } \quad V\left(P_{2}-P_{1}\right)+\frac{W}{2 g} v_{2}^{2}=0 \tag{8•28}
\end{equation*}
$$

Since $\frac{W}{g V}$ is the mass density $\rho$,

$$
P_{1}-P_{2}=\Delta P=\rho \frac{v_{2}^{2}}{2}
$$

This relationship between the pressure drop through an orifice or nozzle and the resulting velocity is the usual basis for gas measurement.

When a sharp-edged intake orifice, Figure 8.9, is used, the full belocity and pressure drop of equation $8 \cdot 29$ are not yet set up in the plane


Fig. 8.9. Gas measurement.
of the orifice, but occur at the throat of the jet further down stream. At this throat of the stream, the cross-sectional area of the jet is much smaller than the orifice area and there are always present certain inevitable turbulence and friction losses that cause the throat velocity to be slightly less than that predicted by equation $8 \cdot 29$. These two cffects arc taken care of in flow calculations by using a discharge coefficient $c_{d}$ so defined that

$$
\begin{equation*}
\text { Rate of discharge }=c_{d} A v \tag{8•30}
\end{equation*}
$$

where $A$ is the orifice area and $v$ is the calculated throat velocity. When such an orifice is installed in a pipe or plenum chamber with a diameter at least four times the orifice diameter, $c_{d}=0.61$ may be assumed for air. When the orifice is placed at the entrance of a smaller pipe, a special calibration for $c_{d}$ is usually necessary.

When a nozzle with a properly designed approach section is used, the throat flows full, and there is no problem of jet contraction. The result is that the only experimental correction needed is for the slight amount of turbulence loss present and $c_{d}$ may be assumed to be 0.96 to 0.99 for almost any type of intake installation.

## REFERENCE

1. Ingersoll Rand Co., Compressed Air Data (detailed information).

## Suggested Reading

Compressed Air and Compressor Types
Severns and Degler, Steam, Air, and Gas Power. Chapter XV.
Compression and Expansion of Gascs
Emswiler, Thermodynamics. Chapters XI and XII.
Air Measurement and Centrifugal Fans
Severns, Hcating, Ventilating, and Air Comlitioning Fundamentals. Pages 277-297
Centrifugal Fans
Gafrert, Steam Power Stations. Pages 386-393.

## CHAPTER 9

## UTILIZATION OF HEAT

9.1. Uses of Heat. Some engineering problems are concerned only with the characteristics of gases and vapors, others only with the combustion of fuels or with the transfer of heat, and in others gases and vapors are compressed for industrial uses. The heat released by the combustion of fuels is stored as thermal energy in the hot products of combustion and it is in this form that it must be utilized. Where it is used directly for heating purposes, at least one transfer to another medium is involved, and the transportation of thermal energy is usually accomplished by the flow of the gas, vapor, or liquid that absorbs it to the point of use.

In a warm-air heating system the heat from the products of combustion is more or less completely transferred to air that is circulated about the furnace by a fan or blower, with the warmed air subsequently flowing to the space to be heated. A double transfer of heat is involved in industrial applications where the heat from the furnace gases is used to generate steam which flows to another part of a plant where it condenses and again transfers the heat to a liquid or gas to be warmed. In such applications the complete combustion of the fuel, the reduction of unintentional heat losses and the complete transfer of the heat from one medium to another are the things of prime importance.

When the heat from fuels is used to produce power this is accomplished by the expansion of gases and vapors. When an internal-combustion engine is used, the fuel is burned directly in the working cylinder and the products of combustion expand in the cylinder, doing work as the piston moves and the gases expand. In steam power plants all the various functions that are accomplished in the cylinder of the internalcombustion engine are performed in different pieces of equipment. The fuel is burned in a furnace and the heat of the products of combustion is transferred to steam in a boiler and superheater, with the final conversion to work being accomplished as the high-pressure steam expands through a turbine or engine.
9.2. Reversible Heat Engine Cycles. Since thermal energy is converted to mechanical work by the expansion of gases and vapors that in-
crease in volume as they expand, such a conversion cannot be complete. This fact and the concept of entropy were developed in article 3.8 where the maximum possible conversion efficiency was also discussed. The next step is to investigate the requirements that must be met by an engine cycle that will convert this maximum possible portion of thermal energy into uscful work.

It was in 1824 that Sadi Carnot first presented the idea of a reversible cycle as the most efficient one possible. Such a cycle may be used as either a heat engine or as a heat pump. Heat may be absorbed by a gas or vapor from a high-temperature source, part of it converted to work by expansion, and the remainder rejected to the cooling medium at the lower temperature. Such a reversible cycle, when operated backwards, would be able to absorb the same amount of heat previously rejected at the lower temperature, and by having an amount of compression work done on it equal to that delivered by it during expansion, the original amount of heat can be delivered back to the initial condition.


Fig. 9.1. Carnot cycle

This is an ideal limit that may be approached but never completely reached by actual cycles. For its accomplishment, heat would have to be transferred with a zero temperature difference, for otherwise it could not be accomplished with equal ease in either direction. It is true that if a large enough surface area is provided and sufficient time is allowed, heat can be transferred from one medium to another with a very small temperature difference, but it can never be transferred where there is no temperature difference.

A reversible compression or expansion would have to be an adiabatic change with perfect insulation so no heat is conducted to or from the cylinder. In addition, it would have to be done slowly and without turbulence, for otherwise it could not be performed in the reverse direction. Such a reversible adiabatic change would be done at constant entropy, since $d Q$ and consequently $\frac{d Q}{T}=d s$ would be zero.

Carnot's ideal reversible closed cycle is shown by Figure $9 \cdot 1$ on a pressure-volume chart. Heat, ${ }_{1} Q_{2}$, is added at constant temperature $T_{1}=T_{2}$, with the piston moving from volume $V_{1}$ to $V_{2}$. The entropy increase during this portion of the expansion is $\int_{1}^{2} \frac{d Q}{T_{1}}=\frac{{ }_{1} Q_{2}}{T_{1}}$. The expansion continues as the adiabatic (2-3) until the lowest cooling tem-
perature available, $T_{3}$, is reached. Since $d Q=0$ for this portion of the stroke the entropy change is zero.

The return of the working medium to the initial condition is started in the Carnot cycle by the compression 3-4 performed at constant temperature $T_{3}=T_{4}$. During this operation an amount of heat, $\int_{3}^{4} d Q={ }_{3} Q_{4}$, is conducted to the cooling medium, and this corresponds to a decrease in entropy of $\int_{3}^{4} \frac{d Q}{T_{3}}=\frac{{ }_{3} Q_{4}}{T_{3}}$. The volume $V_{4}$ is such that the reversible adiabatic (4-1) will return the gas or vapor in the cylinder to the original starting condition at (1).
During this cycle an amount of heat ${ }_{1} Q_{2}$ is added at temperature $T_{1}$, a net amount of useful work represented by the work area $1,2,3,4$ is performed and heat $\left({ }_{3} Q_{4}\right)$ is rejected to the cooling medium at the temperature $T_{3}$. Since the entropy is constant during each of the adiabatic changes $2-3$ and 1-4, it must follow that the entropy increase during the isothermal expansion 1-2 when ${ }_{1} Q_{2}$ is added at temperature $T_{1}$ must be equal to the decrease in entropy during the compression $3-4$, where the heat conducted away is ${ }_{3} Q_{4}$ at the temperature $T_{3}$. Thus,

$$
\frac{\text { Heat added }}{T_{2}}=\frac{\text { Heat rejected }}{T_{3}}
$$

From equation 9.1
Work done $=$ Heat added - Heat rejected $=$ Heat added $\left(1-\frac{T_{3}}{T_{2}}\right) \quad[9 \cdot 2]$
This shows that the reversible Carnot cycle has an overall efficiency equal to the maximum possible value at which thermal energy can be converted to mechanical work.

9-3. Entropy as a Coordinate. Pressure-volume diagrams such as Figure $9 \cdot 1$ and Figure $4 \cdot 1$ are useful for showing the mechanical concepts of pressure excrted and volume occupied, but they are not adapted to showing the thermal conditions of a gas or vapor. Since entropy is defined by $d s=\frac{d Q}{T}$, it is very useful to show such cycles as the ones of Figure $9 \cdot 1$ on the temperature-entropy diagram shown in Figure 9.2.

In the Carnot cycle the heat added at the upper temperature $T_{1}$ is $\int_{1}^{2} d Q=\int_{1}^{2} T_{1} d s$ and this is shown in Figure 9.2 as the area $1,2,3,5,6,4$. The heat rejected during the compression $3-4$ is $\int_{3}^{4} d Q=$
$\int_{1}^{2} T_{3} d s$, which is shown by the area $3,5,6,4$. The useful work done by the cycle, which is the difference between the heat added and the heat rejected, is shown by the area $1,2,3,4$.

Temperature-entropy diagrams are equally useful for showing the effects of deviations from ideal cycles. If, in the Carnot cycle just discussed, heat had been lost by conduction from the cylinder during the expansion $2-3$, it would be represented on a $T-s$ diagram by some line $2-3^{\prime}$. The heat input to the cycle is the same as before, but the useful


Fig. 9.2. Temperature-entropy diagram. expansion work done is reduced by the area $2,3^{\prime}, 3$. The heat conducted away during expansion is $5^{\prime}, 3^{\prime}, 2,3,5$ and the total heat discharged to the cooling medium is represented by area $6,4,3^{\prime}, 2,3,5$.

When any change in condition of a gas or vapor, such as constant pressure or constant volume, is shown on a tempera-ture-entropy diagram the relationship between the available and unavailable portions are immediately discernible as areas and this is the real uscfulness of such diagrams. It is usually a very good idea, when analyzing any cycle, to use both $P-V$ and $T-s$ diagrams.
9.4. Practical Limitations. When fuels are burned, the calculated amount of air for complete combustion is the theoretical amount to be supplied. If less is supplied, incomplete combustion losses occur and when excess air is added the thermal energy released is distributed to a correspondingly greater weight of exhaust gases. In actual practice incomplete mixing and insufficient burning time available may necessitate the use of large quantities of excess air to avoid prohibitive incomplete burning losses and at other times excess fuel may be deliberately used to obtain a maximum output from an engine or furnace even though a reduced efficiency is involved.

To prevent losses when heat is transferred from a hot medium to another that is being heated, the transfer should be complete. However, as a heat source such as the products of combustion is cooled to a few hundred degrees Fahrenheit, the temperature difference causing heat transfer is reduced and the rate of heat transfer becomes smaller. It is usually more economical to permit some of the heat to leave with the products of combustion than to make the equipment investment that would be required to make such a transfer of heat complete. For this
same reason it is not economically feasible to approach even closely the ideal reversible process of transferring heat with a negligible temperature difference in a heat engine cycle.

A reversible expansion or compression is more nearly feasible than a reversible heat transfer operation, especially when gases or vapors expand through a nozzle to create a high-velocity jet as in turbines. The time involved is so short that heat losses are practically negligible and in properly designed nozzles, turbulence losses are small.

An extreme example of heat transfer during compression is that of air compression discussed in article 8.3. There cooling water is circulated in the cylinder jacket with the intention of removing a maximum amount of heat and it reaches values as high as one-third of the compression work.

In internal-combustion engines some heat must be removed by conduction from the high-temperature combustion gases to avoid overheating the pistons and cylinder walls and make lubrication possible. This heat loss has the general order of magnitude of one-tenth the heat of the products of combustion. This represents a deviation from the ideal reversible or constant entropy expansion that is dictated by mechanical necessity.

In the reversible cycle the expansion is continued until the expanding gases reach the lowest available temperature but this is not desirable in an actual enginc. Long before such a temperature is reached the pressure of the expanding gases would be so low that thermal and mechanical losses would consume most of the additional work that might be obtained by further expansion. Consequently, in reciprocating or piston type heat engines, the expansion is not carried to completion before the gases are released to the exhaust. In steam turbines and other types of continuous flow machines these limitations are not present and complete expansion is usually carried out.

This list of deviations from the ideal reversible cycle in no wise detracts from the value of the available and unavailable energy concepts previously developed. The fact that in actual heat engines practical considerations result in deviations from the ideal cycle makes it even more important to have the reversible Carnot cycle as an unvarying basis of comparison.

## Sugaested Reading

[^1]
## CHAPTER 10

## INTERNAL-COMBUSTION ENGINES

10.1. Types of Engines. Early internal-combustion engines were not practically successful because the gas pressures in the cylinder were very low and the output of a given sized engine was quite small. The mixture of air and fuel was usually ignited at atmospheric pressure, and this shortcoming was not overcome until the air-fuel mixture was precompressed to a fairly high pressure before ignition occurred. The result of this compression was to increase greatly the average working pressure during the expansion stroke. The pre-compression idea is incorporated in all present-day internal-combustion engines.

Engines are classified as four-stroke cycle or two-stroke cycle engines, depending on whether four or two strokes are required to complete one cycle of events. In the four-stroke cycle engine the first stroke as the piston moves outward in the cylinder serves to draw an air or air and fucl charge into the cylinder. During the return stroke the charge is compressed and exists at a high pressure in the clearance space at the end of this second stroke. After the charge is ignited it expands, forcing the piston out again and doing work during the third or expansion stroke. During the fourth or final stroke the gases are forced out as the piston returns and the engine is then ready for another cycle. The four strokes of this cycle are illustrated by Figure $10 \cdot 1$.

In the simple two-stroke cycle shown in Figure 10.2 the ignited mixture expands behind the piston on the down stroke $a$ and at the same time the air or combustion mixture in the crankcase is pre-compressed to a pressure that is slightly above atmospheric. This continues until the piston reaches the by-pass from the crankcase when the exhaust valve opens and the expanded gases flow out. As the by-pass is uncovered after the piston passes, the pre-compressed gas in the crankcase flows into the cylinder, tending to purge the burned gases and fill the cylinder with a fresh charge.
This continues until when, on the up stroke $b$, the piston reaches the by-pass port, the exhaust valve closes and the by-pass is closed off. During the remainder of the up stroke the fresh mixture above the piston is compressed and new air or gas mixture is drawn into the crankcase


Fig. 10•1. Four-stroke cycle.


Fia. 10-2. Two-stroke cycle.
through the intake valve which opens after the piston passes the by-pass port.

The advantage of a two-stroke cycle over the four-stroke cycle is that it makes possible a working stroke each revolution for each cylinder instead of requiring two revolutions to yield one working stroke. Its chief disadvantage lies in the difficulty of purging all the burned gases from the cylinder in the short time available.

Internal-combustion engines are also classified in terms of the method utilized to ignite the air-fuel mixture. In the spark ignation type, this is accomplished by an electric spark across the gap of a spark plug in the cylinder. In compression igntion engines only air is present in the cylinder during the compression stroke, and the compression is continued until the air temperature is above the ignition point of the fuel to be used. At the end of the compression the liquid fuel is injected into the cylinder in the form of an atomized spray and burning results. Much higher compression pressures are possible in the compression ignition engines than in the spark ignition type. Since combustion cannot occur until the fuel is injected, the limit of spontancous combustion of the air-fuel mixture that controls pressures in spark ignition engines is not present.

Engines are also designated by the number of cylinders such as 4, 6, 8, 9,14 , as well as by the cylinder arrangement, such as in-line, V , or radial. Minor classifications designate engines in terms of the valve location and operation and other mechanical details that in no wise change the principle of operation.

The size of an engine cylinder is designated by the cylinder diameter or bore, stated first, followed by the length of stroke, both given in inches such as $41 / 2$ in. by $43 / 4 \mathrm{in}$. The volume of gas in the cylinder of an engine at the start of compression is the piston displacement, $p d$, plus the clearance volume $c$. The volume of gas at the end of the compression stroke is the clearance volume. This ratio of initial to final volume is known as the compression ratio $r$ of the engine, or

$$
r=\frac{\text { Piston displacement }+ \text { clearance }}{\text { Clearance }}=\frac{p d+c}{c}
$$

The compression ratio, which is much greater in compression ignition engines than in spark ignition ones, is a very useful basis for the classification of engines. Thus, an engine having a clearance volume one-fifth of the piston displacement has a compression ratio of

$$
r=\frac{p d+\frac{1}{5} p d}{\frac{1}{5} p d}=\frac{1+\frac{1}{5}}{\frac{1}{5}}=\frac{6}{1} \quad \text { or } \quad 6 \text { to } 1
$$

10.2. Gasoline Engines. The function of the carburetor of a gasoline engine is to meter the fuel and atomize it in the air going to the engine. It must furnish a proper mixture of fucl in air over a wide range of loads and temperatures and be adapted to sudden changes in running conditions, Only a portion of fuel is vaporized in the carburetor, the remainder being vaporized later in the cylinder. The richest air-vapor mixture that will burn is 8 to 1 by weight and 20 to 1 is the leanest combustible mixture.

In the gasoline engine, the entire fucl charge is in the cylinder when ignition occurs and since no further external control can be exercised over the burning process, it is essential to know what things influence the burning process. After combustion starts in a stagnant mixture in an enclosed space, the flame front starts traveling through the mixture.

The burned and hot gases behind the flame front tend to expand and compress the unburned portion. The temperature of the unburned portion is raised by its compression and by radiation from the burning gases. The expansion of the burned gases tends to blow the flame front forward at an increasing velocity. This flame velocity is of the order of 15 to 30 ft per sec in a nonturbulent mixture.
High-speed engines would be impossible with the flame velocities attained in stagnant mixtures. The residual velocity and turbulence caused by the piston motion during compression, combined with the turbulence that is caused by the expansion of the burning gases in a properly designed combustion space, produce combustion rates many times greater than those realized in stagnant gases. Without turbulence, the entire surface of the combustion space would be covered by a stagnant layer of mixture that would be cooled so rapidly that it would never burn. High turbulence sweeps this mixture from the walls and causes it to be burned. High turbulence is essential to complete and rapid burning.

If the unburned portion of the mixture receives heat faster than it transmits it to the surrounding metal, the critical pressure and temperature of the mixture may be reached. At this critical condition of spontaneous ignition, the entire unburned portion ignites spontancously and sets up destructively high-pressure waves in the combustion space. This phenomenon is known as detonation and represents one of the most serious problems of high-speed gasoline engines.

The tendency to detonate is aggravated by increased compression ratio because pressures and temperatures are increased, and the amount of dilution from burned gases contained in the clearance space is decreased. The greater the distance of flame travel, the more chance for detonation, which puts a premium on small-bore engines and centrally located spark
plugs. A combustion chamber design that subjects the last part of the mixture that is burned to the rapid cooling of a large exposed surface reduces detonation.

The detonating characteristics of different gasolines vary widely and the relative "knock" or detonation ratings of fuels are expressed as their octane ratings. Since octane $\left(\mathrm{C}_{8} \mathrm{H}_{18}\right)$ is better in this respect than gasolines, and heptane $\left(\mathrm{C}_{7} \mathrm{H}_{16}\right)$ is worse than any gasolines, the octane number of a gasoline is the percentage of octane in a mixture of octane and heptane that gives the same detonation tendencies as the fuel in question.

The comparison is made in a standard C.F.R. engine (Cooperative Fuel Research), and the intensity of detonation is measured by the bounce imparted to a pin resting on a diaphragm when detonation occurs. Octane " 70 " indicates a gasoline that gives the same intensity of detonation in the C.F.R. engine as is given by a mixture of 70 per cent octane and 30 per cent heptane.

There is an appreciable time delay between ignition and the time when enough fuel has been burned to produce a rapid pressure rise. For this reason, ignition occurs before the end of compression, and this ignition advance is as much as 40 deg of crank angle for high-speed engines. To give the burned gases time to escape from the cylinder and to prevent an excessive back pressure on the exhaust stroke, the exhaust valves are opened before the end of the expansion stroke. This angle of lead is often 45 deg before bottom dead center in automotive engines.

Since the pressure in the cylinder during the suction stroke is less than atmospheric, it has been found that the weight of charge taken into the cylinder is increased by leaving the intake valves open for as much as 45 deg after the end of the suction stroke. Similarly, the exhaust valves do not close until 10 deg after the end of the exhaust stroke to aid in clearing burned gases from the cylinder.
10.3. Oil Engines. In the true Diesel cycle, the fucl is burned at constant pressure, but in most modern oil engines, the method of fuel injection results in some constant-volume burning. The oil engine differs from the gasoline engine chiefly in the fact that the fuel is injected into the cylinder at the end of compression and by varying the rate of injection, some external control of the burning process can be exercised. At the end of compression, the air pressure is approximately 500 lb per sq in., and the temperature about 1000 deg F. The fuel is forced in at a pressure of 2000 to 3000 lb per sq in. through a nozzle that gives a finely atomized spray that penetrates throughout the combustion space.

Once it is in the combustion space, the manner in which the fuel burns may vary widely. If sufficient oxygen is locally available to each drop-
let of liquid hydrocarbon fuel as it reaches its ignition temperature, normal burning may be expected. However, if the fuel reaches its dissociation temperature without contacting the oxygen for combustion, it is believed to break down into hydrogen, methane, and carbon. Once this hydrogen mixes with the proper amount of air, it is thought to detonate, following which the methane burns and the carbon remains largely unburned.

This detonation which results in a sudden pressure rise, carbon deposit, and noisy running is known as Diesel "bump." It was originally believed to be a property of a Diesel fuel, but it is equally dependent on the engine-operating conditions and the fuel-injection system. In addition to


Fig. 10-3. Otto cycle events.


Fig. 10-4. Ideal Otto cycle.
the requirements of proper fuel distribution in the combustion space, the precision required of fuel-injection pumps and nozzles may be illustrated by the fact that for each firing stroke of a 5 in . by 7 in . cylinder of a Diesel engine a maximum of 0.0003 lb of fuel is injected in atomized form. This represents a sphere of oil $1 / 8 \mathrm{in}$. in diameter.
10.4. The Ideal Otto Cycle. The pressure-volume indicator diagram for a four-stroke Otto cycle engine is shown by Figure 10.3. In the Otto cycle engine the fucl-air mixture is ignited at the end of compression and constant-volume burning is assumed. In actual engines this burning period extends over several degrees of crank angle. The pressure in the cylinder of an actual engine during the exhaust stroke is higher than atmospheric and the pressure during intake or suction stroke is less than atmospheric by the amount necessary to produce the required gas velocity through the valves.

The compression and expansion are usually assumed to be reversible adiabatic or constant entropy operations. However, in an actual engine it is necessary to water-jacket the cylinder and remove heat with cooling water to protect the metal parts from overheating and to insure satisfactory lubricating oil films.

The resulting ideal Otto cycle shown in Figure 10.4 is usually analyzed by assuming that the medium in the cylinder is air and that its specific
heat is constant. During the constant entropy compression 1-2 the compression work done is numerically equal to the increase in internal energy. The thermal input during the constant-volume combustion 2-3 is $W c_{v}\left(T_{3}-T_{2}\right)$ and during the constant entropy expansion $3-4$, the expansion work done is accompanied by a corresponding decrease in internal energy. The gases in the cylinder at the end of the expansion stroke are at a high temperature $T_{4}$, and the thermal energy discharged with the exhaust gases is the same as the amount of heat that would have to be removed to cool the gases at constant volume from $T_{4}$ to $T_{1}$.

Since the useful work done is the difference between the input and the losses, the efficiency is

$$
\text { Efficiency }(e)=\frac{W c_{2}\left(T_{3}-T_{2}\right)-W c_{n}\left(T_{4}-T_{1}\right)}{W c_{v}\left(T_{3}-T_{2}\right)}
$$

The compression 1-2 and the expansion 3-4 are both assumed to be adiabatic changes, so

$$
\begin{align*}
& T_{1}=T_{2}\left(\frac{V_{2}}{V_{1}}\right)^{\gamma-1} \\
& T_{4}=T_{3}\left(\frac{V_{3}}{V_{4}^{\prime}}\right)^{\gamma-1}
\end{align*}
$$

Since $\frac{V_{3}}{V_{4}}=\frac{V_{2}}{V_{1}}$, equation $10 \cdot 2$ combined with equations $10 \cdot 3$ and $10 \cdot 4$ results in

$$
e=\frac{\left(T_{3}-T_{2}\right)-\left(T_{3}-T_{2}\right)\left(\frac{V_{2}}{V_{1}}\right)^{\gamma-1}}{T_{3}-T_{2}}=1-\left(\frac{V_{2}}{V_{1}}\right)^{\gamma-1}
$$

The volume ratio $V_{1} / V_{2}$ is the compression ratio, so the ideal Otto cycle efficiency may be written,

$$
e=1-\left(\frac{1}{r}\right)^{\gamma-1}
$$

This expression for the Otto cycle efficiency is based on the assumption that $c_{v}$ and $\gamma$ are constant around the entire cycle, which is not accurate. During compression the cylinder contains a mixture of gasoline vapor, air, and products of combustion from the clearance volume at a temperature around 500 deg F , for which the average value of $c_{v}=$ about 0.175 applies and $\gamma=1.4$. At the end of burning the cylinder is filled with a mixture of $\mathrm{CO}_{2}, \mathrm{~N}_{2}$, and water vapor at a temperature above 3000 deg F and for which $c_{v}$ is about 0.25 and $\gamma$ is less than 1.3.

From this it is apparent that to assume $c_{v}$ and $\gamma$ constant is a very crude approximation. Such a cold-air standard using $\gamma=1.4$ gives re-
sults that are entirely too high and it is not a satisfactory quantitative analysis because it ignores the actual properties of the mixture in the engine. In spite of such shortcomings it is useful for pointing out the qualitative effect of increased compression ratio on the efficiency of an Otto cycle.

A rather arbitrary expedient known as the hot-air standard efficiency is to use a value of $\gamma=1.3$ in equation $10 \cdot 6$. The efficiency so calculated is reasonably close to the accurate values obtained by the use of gas tables that will be discussed in article $10 \cdot 6$.
10.5. The Ideal Diesel Cycle. In the Otto cycle engine variations in load are met by varying the pressure in the cylinder during the intake


Fig. 10-5. Ideal Diesel cycle.


Fig. 10-6. Effect of late burning.
stroke by means of a throttle valve between the carburetor and the cylinder. This changes the weight of mixture taken in and consequently the compression and expansion pressures. At all loads the burning of the mixture is assumed to be at constant volume at the end of the compression stroke, and the ideal Otto cycle analysis is the same for all loads.

In the ideal Diesel cycle shown in Figure 10.5, the analysis is complicated by the fact that burning is assumed to occur at constant pressure and variations in load are met by changing the duration of the burning period. This not only affects the location of point 3 , Figure $10 \cdot 5$, but it also changes the composition of the combustion mixture in the cylinder during expansion, since the variable amount of fuel injected into a constant amount of air causes a variation in air-fuel ratio.

The ratio of the gas volume in the cylinder at the end of fuel injection $V_{3}$ to the clearance volume $V_{2}$ is known as the cut-off ratio. This cut-off ratio $V_{3} / V_{2}$ usually varies from 1.25 at light loads to about 2.50 at heavy loads. As a result, the ideal Diesel cycle efficiency varies with engine load and is not a fixed value, depending only on the compression ratio as it is in the ideal Otto cycle.

In actual fuel-injection, compression-ignition engines the early portion of the burning occurs essentially at constant volume and the remainder
at conditions some place between constant pressure and constant temperature. Since there is no single ideal cycle that covers all cases, it is advantageous to analyze the so-called Diesel cycle in terms of the instantaneous efficiency for the fuel as it burns at any time during the injection period.

For any fuel that is burned at constant volume at the beginning of the expansion stroke in an engine having a clearance volume that is a fractional part $c$ of the piston displacement, the ideal efficiency from equation 10.6 is

$$
e=1-\left(\frac{1}{r}\right)^{\gamma-1}=1-\left(\frac{c}{1+c}\right)^{\gamma-1}
$$

since the compression ratio $r=\frac{1+c}{c}$.
The instantaneous efficiency of utilization for heat released by fuel burned at any point a fraction $x$ of the stroke after expansion starts may be found by the method illustrated by Figure $10 \cdot 6$. If it is imagined that no fuel is injected until the air has re-expanded to point 3, the result would be the indicator card $1,3,4,5$ and the efficiency would be the same as that of an ideal Otto cycle with a compression ratio,

$$
\begin{align*}
& \text { Equivalent compression ratio }=\frac{V_{1}}{V_{3}}=\frac{1+c}{c+x} \\
& \text { Instantaneous efficiency }=1-\left(\frac{c+x}{1+c}\right)^{\gamma-1}
\end{align*}
$$

In this way the ideal efficiency for the fuel burned at any point during expansion may be calculated.

## Diesel Cycle Example

For an engine with a clearance volume 0.08 times the piston displacement, the hot-air standard efficiency for the fuel burned at the outset of expansion is found from equation $10 \cdot 7$ as

$$
e=1-\left(\frac{0.08}{1.08}\right)^{13-1} \cdot=0.541
$$

If the cut-off ratio is $2, \frac{c+x}{c}=2$, or the last fuel is burned at a fraction of the expansion stroke, $x=2 \times 0.08-0.08=0.08$. The efficiency of utilization of this last fuel burned is

$$
e=1-\left(\frac{c+x}{1+c}\right)^{\gamma-1}=1-\left(\frac{0.16}{1.08}\right)^{13-1}=0.437
$$

The average efficiency for any particular cycle would depend upon the way in which the fuel is assumed to be burned.

Equation 10.9 reflects a very important characteristic of the Diesel cycle. At light load, fuel is injected for a short period only and a small value of $x$ indicates a high efficiency. As the load is increased, the duration of the burning period increases, and the average cycle efficiency becomes progressively lower.
10.6. Cycle Analysis Using Gas Tables. It was assumed in the analysis of the ideal Otto cycle in article 10.4 that gas specific heats at high temperatures are constant and that the expansion mixture had the properties of cold air. These assumptions do not have to be made if the gas table calculation methods of article 4.5 are used. The use of such tables has been discussed but it may be well to carry through an illustrative example.

## Example of Otro Cycle Efficiency Calculations Using Gas Tables

If it is assumed that the ideal cycle of Figure 10.4 is for a compression ratio of $\frac{V_{1}}{V_{2}}=\frac{7}{1}$, the reversible adiabatic or constant entropy compression 1-2 may be calculated as shown by Figure 4-2. For such a compression ratio the dilution of the compression mixture with burned products of combustion will not be serious, and the compression mixture may be considered as air starting at an initial temperature at 1 of, say, 110 deg F or 570 deg R .

From Table 4•2, at 570 deg R , the internal energy $u_{1}=5.1 \mathrm{Btu}$ per lb . The entropy change along a constant temperature compression from $V_{1}$ to $V_{2}$, Figure $4 \cdot 4$, is from equation $4 \cdot 31$,

$$
\Delta s_{T_{1} z^{\prime}}=\frac{R}{J} \log _{\epsilon} \frac{V_{2}}{V_{1}}=-\frac{53.3}{778} \log _{\epsilon} 7=-0.1332 \text { Btu per lb per deg R }
$$

If the entropy at 2 , Figure $4 \cdot 4$, is to be the same as that at 1 , there must be an increase in entropy during the constant-volume change $2^{\prime}-2$, Figure $4 \cdot 4$, equal to the entropy decrease along the constant-temperature change 1-2'.

$$
s_{v_{2}}=s_{v_{2}}+0.1332=s_{v_{570}}+0.1332=0.0093+0.1332=0.1425
$$

From Table $4 \cdot 2, T_{2}=1221 \mathrm{deg} \mathrm{R}$ and the internal energy $u_{2}$, Figure $10 \cdot 4$, is 119.2 Btu per lb.

Since the work done by the piston on the air during the constant entropy compression in an engine cylinder is numerically equal to the change in internal energy, $u_{2}-u_{1}=119.2-5.1=114.1$ Btu per lb.
During the constant-volume burning 2-3, Figure $10 \cdot 4$, the fuel and air are burned to products of combustion. For an air-fuel ratio of 18 to 1, corresponding to about 20 per cent excess air, Table $4 \cdot 9$, and a lower heating of 18,800 Btu per lb of fuel, the internal energy $u_{3}$ per pound of combustion mixture is greater than that before burning at 2 by an amount $\frac{18,800}{19}=988$ Btu per lb .

The lower heating value of the fuel must be used in this calculation because the constants for the combustion mixture in Table $4 \cdot 11$ do not include the vaporization of the moisture in the combustion mixture. Thus the internal energy per pound of combustion mixture is $988+119.2=1107.2 \mathrm{Btu}$ per lb and from Table $4 \cdot 9$, the temperature at the end of combustion $T_{3}$ is 5049 deg R .

The temperature $T_{3}$, calculated in this manner, is too high because at temperatures in excess of 4000 deg $R$, the dissociation of carbon dioxide is significant, and this factor is not considered in Table 4.9 .

The constant entropy expansion 3-4, Figure 10•4, may be calculated as illustrated in Figure 4.4 by using a constant-temperature expansion from $V_{3}$ to $V_{4}$, and a constant-volume change from $T_{3}$ to $T_{4}$. The entropy increase during the constant-temperature expansion from $V_{3}$ to $V_{4}$ is

$$
\Delta s_{T}=\frac{R}{J} \log _{\epsilon} \frac{V_{4}}{V_{3}}=\frac{53.8}{778} \log _{\epsilon} 7=0.1332
$$

This means that during the constant-volume cooling from $T_{3}=4922 \mathrm{deg} \mathrm{R}$ to $T_{4}, s_{v}$ must decrease by 0.1332 if the expansion $3-4$ is to be at constant entropy.

$$
s_{v_{4}}=s_{v_{3}}-0.1332=0.5008-0.1332=0.3676
$$

From Table 4.9, $T_{4}=2994 \mathrm{deg} \mathrm{R}$ and $u_{4}=553.9$ Btu per lb.
The internal encrgy change during expansion which is equal to the total work done on this piston is

$$
u_{3}-u_{4}=1107.2-553.9=553.3 \text { Btu per } \mathrm{lb}
$$

The net work done per cycle is the difference between the expansion work and the compression work, or

$$
\text { Output }=553.3-114.1=439.2 \text { Btu per } \mathrm{lb}
$$

The value of the efficiency depends upon whether the higher or lower heating value of the fuel is used. Using the higher heating value amounts to saying that the cycle is at fault because the combustion gases are not expanded to such a temperature that the moisture formed by the combustion of hydrogen in the fuel is condensed. Using the lower heating value amounts to saying that the cycle should not be expected to expand to such a low temperature.

$$
\begin{aligned}
& \text { Efficiency for lower heating value }=\frac{439.2}{988}=0.445 \\
& \text { Efficiency for higher heating value }=\frac{439.2}{1083}=0.406
\end{aligned}
$$

The corresponding cold-air standard Otto cycle efficiency for a compression ratio of 7 is

$$
1-\left(\frac{1}{7}\right)^{1.4-1}=0.448
$$

and the hot-air standard value is

$$
1-\left(\frac{1}{7}\right)^{1.3-1}=0.358
$$

In a similar manner the efficiency for a Diesel cycle or any modification of it may be calculated. In the Diesel cycle the compression is the same as in the Otto cycle, but the combustion is at constant pressure instead of at constant volume, and the expansion is through a volume ratio less than that of the compression.

Too much significance should not be placed on the maximum temperatures calculated for ideal cycles because delayed burning, heat losses, and dissociation cause them to be much lower.
10.7. Internal-Combustion Engine Performance. The final output or brake horsepower of an engine is found by measuring the output torque with a brake or dynamometer and determining the rotative speed or rpm. The input is found by weighing the rate of fuel consumption and knowing the heating value of the fuel used. This ratio of brake output to thermal input, correct in units, is known as the brake thermal efficiency. Another method of reporting efficiency that is not quite so exact but which is commonly used is the pounds of fuel required for each horse-power-hour delivered. In modern engines using liquid hydrocarbon fuels this fuel consumption is usually between 0.4 and 0.6 lb of fuel per brake horsepower-hour.

Such an efficiency includes both the mechanical and thermal losses and whenever possible it is desirable to separate the losses. The thermal losses are represented by the difference between the thermal input and the work done on the piston by the gases during each cycle is represented in Figure 10.3 by the difference between the area under the expansion curve and that under the compression curve, or area a,b,c,d. From this area it is possible to find an average gas pressure which, acting on the piston throughout one stroke, would account for the net work done. This pressure is known as the indicated mean effective pressure (imep). The indicated horsepower (i hp) of the engine may then be calculated from the net work per cycle and the number of working strokes per minute for the engine under consideration. Each cylinder of a four-stroke cycle engine gives one working stroke for each two revolutions and one working stroke per revolution in a two-stroke cycle engine.

The difference between the energy accounted for by the indicated horsepower and the thermal input of the fuel represents the thermal losses. The ratio of indicated work to thermal input is known as the indicated thermal efficiency. The difference between the indicated horse-
power and the brake horsepower is the friction horsepower and the mechanical efficiency is the ratio of brake to indicated horsepower. The friction horsepower may be broken up into the mechanical friction and the pumping loss. The pumping loss, which is the work done in moving the exhaust gases from the cylinder and that necessary to draw the fresh charge in, is represented by the area a,e,f,g, Figure $10 \cdot 3$. This pumping loss is small in low-speed engines with adequate valve passages but increases in higher-speed engines. Since gasoline engines are throttled to reduced loads by restricting the intake to the engine, this lowers the suction pressure along a,g,f and increases the pumping loss.

Since accurate indicator cards are difficult to obtain for high-speed engines, the indicated horsepower is often found from the brake horscpower by adding the friction horsepower as obtained by using a dynamometer to drive the engine with the fuel shut off. Such a test does not exactly reproduce the friction losses present in an engine under load conditions, but it is a fairly satisfactory substitute.

A widely used concept is that of an equivalent gas pressure that is thought of as acting on the piston during the working stroke, this pressure being sufficient to account for the brake horsepower. This concept of a brake mean effective pressure (bmep) is used because in many tests only the brake output is determined, the friction and indicated horsepower being unknown. This would mean that the ratio, bmep/imep, would be cqual to the mechanical efficiency.

When maximum economy of operation is desired of an engine, the airfucl mixture used is on the lean side to insure the complete combustion of all the fuel. However, when the maximum possible amount of power is desired from a given engine, excess fuel is used to insure the complete utilization of all the oxygen in the cylinder charge. It is highly desirable to take in a maximum amount of new air each stroke during each intake stroke if the power developed by a given engine is to be a maximum. The weight of the new charge taken in during the suction stroke is reduced by the necessary expansion of the hot, high-pressure gases in the clearance space and by heat conduction from the hot metal of the engine.

This means that an engine with a low-compression and large clearance volume will take in a smaller charge of new air than one with a highcompression ratio and correspondingly small clearance volume. This effect is measured quantitatively by the volumetric efficiency which is defined as the ratio of the actual weight of air taken in per suction stroke to the weight that would be handled if the entire piston displacement could be filled with air at the intake pressure and temperature.

## Example of Internal-Combustion Engine Performance

Performance calculations may be illustrated by considering the test results for a six-cylinder, 3 in . by 4 in . (bore and stroke), 3000 rpm , four-stroke cycle gasoline engine that has a compression ratio of seven to one. Dynamometer tests resulted in a brake horsepower of 58 , mechanical efficiency of 0.78 , and a fuel consumption of 0.59 lb of fuel per brake horsepower-hour. Exhaust gas analysis gave an average air-fucl ratio of 15.1 lb of air per lb of fuel and the higher heating value of the fuel used was 20,600 Btu per lb .

## Brake Thermal Efficiency

The thermal input per brake horsepower-hour is $0.59 \times 20,600=12,130 \mathrm{Btu}$ and since one brake horsepower-hour is $\frac{550 \times 3600}{778}=2545$ Btu the brake thermal efficiency is $\frac{2545}{12,130}=0.21$.

## Indicated Thermal Efficiency

For a brake thermal efficiency of 0.21 and a mechanical efficiency of 0.78 , the indicated thermal efficiency is $\frac{0.21}{0.78}=0.27$ as compared with a value of 0.35 that was calculated for this same compression ratio, using the ideal cycle and the gas tables.

## Brake Mean Effective Pressure

For a brake mean effective pressure equal to $P$, the work done per expansion stroke is $\frac{\pi}{4}(3)^{2} \times P \times \frac{4}{12}$; the number or working strokes per minute is $3000 \times \frac{8}{2}=9000$. The brake horsepower is

$$
58=\frac{\pi}{4}(3)^{2} P \frac{4}{12} \frac{9000}{33,000}
$$

or $P=$ brake mean effective pressure is 90.5 lb per sq in.

## Volumetric Efficiency

If it is assumed that the engine tests were run at an air temperature of 70 deg F and 14.7 lb per sq in., 1 cu ft of air weighs 0.073 lb , one full piston displacement of air would weigh $\frac{\pi}{4} \frac{(3)^{2} \times 4}{1728} \times 0.073=0.0012 \mathrm{lb}$. At 100 per cent volumetric efficiency the engine would use $\frac{8}{2} \times 3000 \times .0012=10.8 \mathrm{lb}$ of air per minute or 649 lb per hour. Since the engine uses $58 \times 0.59=34.2 \mathrm{lb}$ of fuel per hour at an air-fuel ratio of 15.1 , it actually uses 516 lb of air per hour. This represents a volumetric efficiency of $\frac{516}{649}=0.795$.

Suggested Reading<br>Internal-Combustion Engines and Engine Details<br>Norris and Therkelsen, Heat Power. Chapters II and III.

Internal-Combustion Engines
Severns and Degler, Steam, Air, and Gas Power. Chapter XVI.
Gas Engine Cycles
Moyer, Calderwood, and Potter, Elements of Engineering Thermodynamics. Chapter IV.

## CHAPTER 11

## STEAM ENGINES

11.1. Steam Power. The oil engine with its fuel-injection pump and cooling system is a complete power plant. Similarly, the gasoline engine with its carburetor, ignition system, and cooling equipment, forms a self-contained unit. In both engines the fuel is burned directly in the cylinder and the products of combustion are the working medium.

In a steam plant the various operations are separated. The fuel is burned in a furnace and the heat of the products of combustion is transferred to the steam in the boiler and superheater. The steam then goes to an engine or turbine where it expands and does work, the steam being discharged into a condenser where it is condensed. This condensate then enters the boiler-feed pump and is pumped through a feed-water heater into the boiler.

In smaller plants some of these elements may be missing but in large plants all are present together with many auxiliarics such as condenser pumps, forced-draft fans for the furnace, and fuel-handling equipment. Steam power is consequently best adapted to larger power plants. Many small plants exist in ships and locomotives, but steam is best adapted to the development of large concentrated blocks of power.

Steam has several advantages as a medium for the development of power. Water, which is universally available, has a small volume and may be pumped into a boiler with a minimum amount of work. After heat is added and the water is vaporized to steam, a large volume of vapor is available for expansion in an engine or turbine. Low back pressures may be maintained by discharging the expanded steam into a condenser where it is condensed and the condensate is then returned to the boiler. The pressure that is maintained in the condenser affects the portion of the energy of the steam that is converted into useful work and this pressure is dependent upon the temperature of the cooling water available to the condenser.

The energy of steam may be converted into mechanical work in either of two ways. It may be expanded in an engine cylinder and do work on the moving piston. Such an engine is a "batch" type machine in which the steam is carried through a complete cycle in individual batches.

Steam may also be expanded through the nozzles of a turbine after which the kinetic energy of the steam jet is transformed into useful work by the blading of the rotating turbine wheel. Such a "continuous flow" machine as a turbine finds its most economical application in large installations and because of its weight and bulk the reciprocating-piston type engine finds its greatest application in the smaller sized units.
11.2. Vapor Charts. The first step in the study of steam equipment is a ready understanding of the properties of steam and a familiarity with


Fig. 11-1. Temperature-entropy diagram for steam.
steam tables and steam charts. In addition to the pressure-volume diagram of steam shown by Figure $4 \cdot 1$ and discussed in article $9 \cdot 3,{ }^{*}$ diagrams that indicate entropy changes are very useful.

The temperature-entropy $T-s$ chart of Figure $11 \cdot 1$ and the Mollier diagram or enthalpy-entropy $h-s$ diagram of Figure $11 \cdot 2$ are quite useful in aiding with the visualization of vapor operations. A reversible adiabatic or constant entropy change becomes a vertical line on each diagram, and since vaporization of the liquid at any pressure occurs at the corresponding saturation temperature, lines of constant pressure and of constant temperature coincide in the mixture region.

On the temperature-entropy diagram, constant-pressure lines are discontinuous in the superheat region, but on the Mollier or enthalpy-

[^2]entropy diagram it is the constant-temperature lines that change at the saturation line. Both diagrams show constant-quality lines in the mixture region, but as a rule, neither type of diagram shows volume lines.

There are many problems that may be solved graphically on largescale charts, but even when vapor tables are used it is always very helpful to sketch the operations involved on at least two types of diagrams.


Entropy (s)
Fig. 11-2. Enthalpy-entropy or Mollier diagram.
11.3. Steam Calorimeters. When the temperature and pressure of steam are measured, if the temperature should be above the saturation value corresponding to the pressure, the steam is known to be superheated and its properties may be found in the superheated steam tables. However, if the measured temperature should be equal to the saturation value, the condition of the steam would not be definitely known, because more or less moisture might be present. If the moisture should amount to more than 3 or 4 per cent of the mixture by weight it is usually determined by mechanically separating it from the steam in a separating calorimeter. In such an instrument the moisture that is centrifugally separated from the steam is measured and the amount of dry steam flowing is found either by a flow meter or by being condensed and weighed.

Since most so-called saturated steam comes from the boiler with less than 3 per cent moisture, its quality is ordinarily determined with a throttling calorimeter such as illustrated in Figure 11.3.

When steam at a quality $x_{1}$ and pressure $P_{1}$ is expanded through the insulated nozzle of the calorimeter and comes to rest or moves with a very low velocity in the cup, its temperature is indicated by the thermometer. This free expansion, which occurs with no loss or gain of heat from the surroundings, is represented on the Mollier diagram of Figure 11.4 by the constant enthalpy line $1-2$. If the initial pressure and quality $P_{1}$ and $x_{1}$ are such that the final temperature $T_{2}$ as found by the calorimeter thermometer is in the superheat region, the final and the


Fig. 11-3. Throtthing calorimeter.


Fig. 11-4. Throttling on an $h$-s diagram.
initial enthalpy may be evaluated, and the initial quality $x_{1}$ may be calculated.

The final enthalpy $h_{2}$ may be interpolated from superheat tables in terms of the pressure and temperature or it may be calculated as the enthalpy of saturated vapor at the pressure $P_{2}$ (usually atmospheric) plus the enthalpy of superheat. Since the specific heat at constant pressure $c_{p}$ of atmospheric pressure steam near saturation is quite constant at 0.47 , the enthalpy at the condition 2 is,

$$
h_{2}=h_{g 2}+0.47\left(t_{2}-t_{\text {sat } 2}\right)
$$

where $h_{g 2}$ is the enthalpy of saturated steam and $t_{\mathrm{sat}} 2$ is the saturation temperature at pressure $P_{2}$. Since the throttling process of the throttling calorimeter is a constant enthalpy change,

$$
h_{1}=h_{2}=h_{f 1}+x_{1} h_{f g 1}=h_{g 2}+0.47\left(t_{2}-t_{\text {sat } 2}\right)
$$

From equation $11 \cdot 1$ it is possible to calculate the initial quality $x_{1}$ of the wet steam provided the initial moisture content is not too great for the final condition to be in the superheat region.

An idea of the lowest quality at which a throttling calorimeter can be used successfully may be obtained by considering wet steam at 100 lb per
sq in . absolute passing through a calorimeter to an atmospheric back pressure. If 1 deg of superheat is assumed to be the minimum amount that can be definitely detected by the calorimeter thermometer, the limiting conditions would then be defined by

$$
t_{2}=213 \operatorname{deg} \mathrm{~F} \quad t_{\mathrm{sat} .2}=212 \operatorname{deg} \mathrm{~F}
$$

and for $P_{1}=100 \mathrm{lb}$ per sq in.

$$
h_{f 1}=298.40 \quad h_{f g 1}=888.8 \quad h_{82}=1150.4
$$

thus,

$$
298.40+x_{1} 888.8=1150.4+0.47(213-212)
$$

and $x_{1}=0.96$ which means that under these conditions, a throttling calorimeter could not be used when the moisture exceeds 4 per cent by weight.
11.4. Operation of Steam Engines. The simplest form of a steam engine is the slide-valve type illustrated in Figure 11-5. In slow-speed reciprocating boiler-feed pumps and like applications where ruggedness and simplicity are more important than high efficiency, the valve remains in the position shown in Figure 11.5 until the piston is near the head, or in this case, the left-hand end of the cylinder. During this entire stroke, steam is admitted to the right-hand or crank-end side of the piston through the intake port, and steam is exhausted from the head end of


Fig. 11-5. Slide-valve steam engine. the cylinder through that discharge port. When the piston is near the left end of the cylinder a lug on the piston rod actuates a bell crank which moves the valve completely to the right-hand position and the return stroke is started.

This results in steam being admitted at constant pressure throughout the stroke and this means poor economy, since no expansion of the steam occurs. In such steam engines variations in load are accommodated by reducing the pressure of the steam admitted to the cylinder by throttling it through a valve.

In small steam engines with flywheels the valve is actuated by an eccentric on the crankshaft. This eccentric is usually designed and adjusted to admit steam for 0.4 to 0.6 of the stroke, after which the steam then in the cylinder is permitted to expand during the remainder of the stroke. In simpler engines this eccentric setting is fixed, and constant and load variations are met by throttling the steam to a lower pressure
before it is admitted to the cylinder. The throttle valve is controlled by the weighted fly-ball governor of the engine.

A decrease in speed causes the governor to open the throttle valve and thus increase the pressure of the steam admitted to the cylinder and restore the speed to its initial valuc. Such throttling governing is not economical but its appeal lies in its mechanical simplicity.

In more highly developed steam engines an inertia governor on the flywheel rotates the valve eccentric to vary the point of cut-off and thus meet variations in load by changing the portion of the stroke during which full-pressure steam is admitted. At light loads steam is admitted for only a small portion of the stroke and at heavier loads steam is cut off later in the stroke and the amount of expansion is reduced.

If an engine had no losses, maximum efficiency would be realized with a cut-off carly enough to permit the complete expansion of the steam to the back pressure against which it must be exhausted. In an actual engine with losses it does not pay to expand the steam to such a low pressure because the friction and thermal losses would be greater than the work done by the steam during the low-pressure part of the expansion. For this reason the expansion is incomplete in actual engines because cut-off occurs so late that at the end of the expansion stroke when the exhaust valves are opened, the pressure of the steam in the cylinder is greater than the back pressure against which it must be exhausted. This means that energy is lost as the steam expands freely through the exhaust valves. There is also a corresponding loss during admission in the form of a pressure drop to give a high steam velocity through the intake valve port.

The greatest loss that occurs in the slide-valve engine is due to initial condensation. During the cxhaust stroke low temperature, wet steam passes out through the valve ports and around the valves, cooling the metal. When high-pressure steam is admitted for the next stroke, it must flow over these same cooled metal surfaces and heat them. This results in the condensation of some of the steam admitted, and the heat thus stored in the metal is later picked up and carried away by the exhaust steam at the end of expansion. The net result is the same as if some of the incoming steam were allowed to blow right past the cylinder to the exhaust.

The initial condensation that takes place inside the cylinder is reduced somewhat by the compression of the clearance steam. Just before the end of the exhaust stroke both the exhaust and intake valves are closed, and the steam so trapped in the clearance space is compressed and its temperature and that of the cylinder head and piston are raised by this compression. This results in a reduced condensation loss and at the
same time it is a mechanical precaution used to bring the piston to rest by the cushioning effect of the trapped steam.

The Uniflow engine shown in Figure 11.6 was developed to reduce this initial condensation loss. In it the inlet and exhaust ports are separated. by using a long piston, placing the exhaust ports at the center of the cylinder, and causing the piston to act as its own exhaust valve. The duration of admission of steam, which is regulated by the intake valves, is controlled by the governor as in other steam engines.

With this arrangement the metal near the exhaust ports remains at the exhaust steam temperature, and that of the intake ports remains near the intake steam temperature. Thus the initial condensation


Fig. 11•6. Poppet-value Uniflow steam engine.
caused by the alternate heating and cooling of metal in the slide-valve engine is eliminated. This accomplishes a very substantial increase in efficiency and has resulted in use of Uniflow engines in most of the larger reciprocating steam engine applications where economy is a governing consideration.
11.5. Complete Expansion Rankine Cycle. Since a steam cycle involves an engine or turbine, a condenser, a boiler-feed pump, and a boiler, the analysis makes it necessary to follow the working medium from one piece of equipment to another. Since the ideal Carnot cycle (article $9 \cdot 2$ ) is not practical for a vapor cycle, a slightly modified Rankine cycle is commonly used as a basis of comparison in steam plants.

The history of one pound of steam as it passes through this ideal cycle is shown by Figure 11.7b. Steam at condition 1 or $1^{\prime}$ expands at constant entropy through an engine or turbine to the condenser pressure 2 or $2^{\prime}$, which is governed by the temperature of the available condenser cooling water. At this pressure and temperature the exhaust steam is condensed to liquid at 3 and this condensate is then pumped into the boiler at pressure $P_{4}=P_{1}$. Heat must then be supplied to heat the liquid from 3 to 4 and to evaporate it into steam at the initial condition.

The corresponding pressure-volume diagram for the steam in the engine cylinder is shown by Figure $11 \cdot 7 a$. As one pound of liquid is pumped into the boiler and evaporated at constant pressure $P_{1}$, another
pound of steam is forced into the engine and an amount of flow work $\frac{1}{J} P_{1} V_{1}$, shown by area $4,1,5,0$, Figure $11 \cdot 7 a$, is done during admission.

During the constant entropy expansion 1-2 the expansion work done is equal to the change in internal energy $u_{1}-u_{2}$, and this is represented graphically by area $5,1,2,6$. As the steam is moved into the condenser

(a)


Entropy (s)
(b)

Fig. 11.7. Complete expansion Rankine cycle.
during the exhaust stroke the piston returns an amount of flow work $\frac{1}{J} P_{2} V_{2}$, shown by the area $3,2,6,0$. The net amount of work delivered to the piston per pound of stcam is

Rankine output $=\frac{1}{J} P_{1} V_{1}+\left(u_{1}-u_{2}\right)-\frac{1}{J} P_{2} V_{2}=h_{1}-h_{2}$
Since an ideal condenser should condense the exhaust steam at the saturation temperature corresponding to $P_{2}$, the feed pump picks up the condensate with an enthalpy $h_{f 2}$. This means that the total cycle input per pound of steam is

$$
\text { Rankine input }=h_{1}-h_{f 2}
$$

$$
\text { Rankine efficiency }=\frac{\text { Output }}{\text { Input }}=\frac{h_{1}-h_{2}}{h_{1}-h_{f 2}}
$$

For such a complete expansion Rankine cycle the steam condition at 2 or $2^{\prime}$ is calculated from the steam tables as an ideal constant entropy expansion for which $s_{1}=s_{2}$.

A complete expansion Rankine cycle is shown on a $T-s$ diagram in Figure $11 \cdot 8$. For steam that is initially dry and saturated this cycle is
indicated by $1,2,4,5$. A corresponding Carnot cycle is shown by $1,2,3,5$ and the only difference is that in the Carnot cycle condensation is stopped at a condition 3 so determined that a constant entropy compression will


Fig. 11-8. T-s diagram of Carnot and Rankine cycles. end at point 5 on the saturated liquid line. In the Rankine cycle it is found to be more practical to complete the condensation at 4, pump the liquid in the boiler, and then add heat to reach condition 5 . The magnitude of the difference in efficiency between these two cycles can probably be best illustrated by working out numerical examples.

## Example of a Complete Expansion Rankine Cycle

For dry saturated steam at an initial pressure $P_{1}=100 \mathrm{lb}$ per sq in. absolute and an atmospheric condenser pressure, $h_{1}=1186.6$ Btu per lb , and $s_{1}=1.6022$. For the expansion 1-2,

$$
\begin{aligned}
s_{1}=s_{2} & =s_{f 2}+x_{2} s_{f g} \\
1.6022 & =0.3119+1.4446 x_{2}
\end{aligned}
$$

or

$$
\begin{aligned}
x_{2}=0.893 \text { and } h_{2} & =h_{2}+x_{2} h_{f 2} \\
h_{2}=180.0-0.892 \times 970.2 & =1046.0 \text { Btu per } \mathrm{lb}
\end{aligned}
$$

Useful work $=h_{1}-h_{2}=1186.6-1046.0=140.6$ Btu per lb
Energy supplied $=h_{1}-h_{f 2}=1186.6-180.0=1006.6$ Btu per lb
Heat absorbed in the condenser $=h_{2}-h_{f 2}=1046.0-180=866.0 \mathrm{Btu}$ per lb .
The cycle efficiency is

$$
e=\frac{h_{1}-h_{2}}{h_{1}-h_{f 2}}=\frac{140.6}{1006.6}=0.140
$$

## Example of Ideal Carnot Cycle

For the example previously worked out, $T_{1}=460+327.83=787.83 \mathrm{deg} \mathrm{R}$, which is the temperature at which heat was added. The condenser temperature at which heat is discharged is

$$
T_{2}=460+212=672.0 \mathrm{deg} \mathrm{R}
$$

The Carnot efficiency is

$$
e=\frac{T_{1}-T_{2}}{T_{1}}=\frac{787.83-672.0}{787.83}=0.147
$$

as compared with 0.140 for the Rankine cycle operating between the same limits.
11.6. Incomplete Expansion Cycles. The complete expansion cycle of Figure $11 \cdot 7 a$ is realized in actual engines only at very light loads in cut-off governed engines. The pressures near the end of the expansion stroke are so low that the output of an engine would little more than overcome friction losses. Over most of the operating range of an engine steam is admitted during a greater portion of the stroke, with the result that the pressure at the end of the expansion stroke is somewhat above the condenser pressure. Such an incomplete expansion eycle is shown by Figure 11.9 and it is somewhat less efficient than the ideal complete expansion cycle. The corresponding history of a pound of steam as it goes through the cycle is illustrated by Figure $11 \cdot 10$.


Fig. 11.9. Incomplete expansion Rankine cycle.


Fig. 11 10. Incomplete expansion cycle on an $h$-s diagram.

The free expansion that occurs when the exhaust valve opens and the steam rushes into the condenser from the higher release pressure is represented by the throt tling process $2-3$, and the accompanying entropy growth is an indication of the increase in unavailable energy that results.

The output of such a cycle is the flow work during admission $\frac{1}{J} P_{1} V_{1}$, plus the expansion work $u_{1}-u_{2}$, minus the exhaust flow work $\frac{1}{J} P_{3} V_{3}$ $=\frac{1}{J} P_{3} V_{2}$.

$$
\begin{gather*}
\text { Output }=\frac{1}{J} P_{1} V_{1}+u_{1}-u_{2}-\frac{1}{J} P_{3} V_{2} \\
\text { Output }=u_{1}+\frac{1}{J} P_{1} V_{1}-\left(u_{2}+\frac{1}{J} P_{2} V_{2}\right)-\frac{1}{J} P_{3} V_{2}+\frac{1}{J} P_{2} V_{2} \\
\text { Output }=h_{1}-h_{2}+\frac{V_{2}}{J}\left(P_{2}-P_{3}\right)
\end{gather*}
$$

As before, the ideal expansion is defined by $s_{1}=s_{2}$ and the Rankine input is

$$
\begin{gather*}
\qquad \text { Input }=h_{1}-h_{f 3} \\
\text { Incomplete expansion efficiency }=\frac{h_{1}-h_{2}+\frac{V_{2}}{J}\left(P_{2}-P_{3}\right)}{h_{1}-h_{f 3}}
\end{gather*}
$$

## Example of an Incomplete Expansion Rankine Cycle

For dry saturated steam at $P_{1}=100 \mathrm{lb}$ per sq in. gage pressure expanded to an absolute release pressure of 30 lb per sq in., and exhausted to atmospheric pressure,

$$
s_{1}=s_{2}=s_{f 2}+x_{2} s_{f 8} ; 1.6022=0.3680+x_{2} \cdot 1.3310
$$

$$
x_{2}=0.926 \quad h_{2}=h_{f 2}+x_{2} h_{f^{2} 2}=218.73+0.926 \times 945=1093.73 \text { Btu per } \mathrm{lb}
$$

$$
V_{2}=V_{f 2}+x_{2} V_{f_{k} 2}=0.01698+0.926 \times 13.728=12.737 \mathrm{cu} \mathrm{ft} \text { per } \mathrm{lb}
$$

$$
h_{f 3}=180 \text { Btu per } \mathrm{lb} \quad h_{1}=1186.6 \mathrm{Btu} \text { per } \mathrm{lb}
$$

$$
\frac{1186.6-1093.73+\frac{(30-14.7) 144 \times 12.737}{778}}{1186.6-180}=0.128
$$

In very simple pumping engines admission at full boiler pressure is continued throughout the stroke as shown in Figure 11•11, and such an engine operates on a nonexpansive cycle. For such a cycle the efficiency is

$$
\text { Efficiency }=\frac{\frac{1}{J}\left(P_{1} V_{1}-P_{2} V_{2}\right)}{h_{1}-h_{f 2}}=\frac{\frac{V_{1}}{J}\left(P_{1}-P_{2}\right)}{h_{1}-h_{f 2}}
$$

For the same 100 lb per sq in. steam pressure and atmospheric exhaust used in the previous examples, the efficiency of the nonexpansive cycle is

$$
\text { Efficiency }=\frac{144(100-14.7) 4.426}{778(1186.6-180.0) 778}=0.069
$$

The various examples may then be summarized:

| Carnot cycle efficiency | 0.147 |
| :--- | :--- |
| Complete expansion efficiency | 0.140 |
| Incomplete expansion efficiency | 0.128 |
| Nonexpansive efficiency | 0.069 |

All these efficiencies are, of course, calculated for low-pressure and temperature steam and atmospheric exhaust. When high-pressure superheated steam is used with a low condenser pressure, all the efficiencies are greater, but the relative values are essentially the same.
11.7. Throttling Loss. When the output of an engine is governed by throttling the steam to a lower pressure before admitting it to the engine the entropy growth during throttling reflects a decrease in the energy available for conversion to mechanical work. The effects of such a throttling process are illustrated on the Mollier diagram of Figure 11•12. If steam initially at condition 1 is expanded at constant entropy to the back pressure $P_{0}$ the ideal available energy is $h_{1}-h_{2}$. However, if it is first throttled to pressure $P_{2}$ at 3 before expansion to the back pressure at 4 , the available energy is reduced to $h_{3}-h_{4}$, which is less than $h_{1}-h_{2}$.


Fig. 11•11. Nonexpansive cycle.


Fig. 11•12. Throttling loss.

## Example of Throttling Loss

If saturated steam is at 100 lb per sq in . absolute pressure as in the complete expansion Rankine cycle example, it was found that $h_{1}-h_{2}=1186.6-1046.0$ $=140.6 \mathrm{Btu}$ per lb . If the steam is first throttled at constant enthalpy to an admission pressure of $P_{2}=50 \mathrm{lb}$ per sq in. absolute, it is superheated with $h_{1}=h_{2}$. The resulting temperature from superheat tables is $305.4 \operatorname{deg} \mathrm{~F}$ and $s_{3}=1.6751$.

Since $s_{3}=s_{4}=s_{f 4}+x_{4} s_{f_{k}}$

$$
\begin{gathered}
x_{4}=\frac{1.6751-0.3134}{1.4446}=0.944 \\
h_{4}=180+0.944 \times 970.2=1096 \text { Btu per lb }
\end{gathered}
$$

The work done is $h_{1}-h_{4}=90.6 \mathrm{Btu}$ per lb as compared with 140.6 Btu per lb for no throttling.
11.8. Steam-Engine Performance. The actual work done on the piston of an engine by the steam is less than that calculated from the corresponding ideal cycle by an amount equal to the thermal losses, and the previously mentioned initial condensation loss is predominant. The indicated horsepower is customarily determined from indicator cards, the brake horsepower from brake or dynamometer tests, and the rate of steam consumption is found by condensing and weighing the exhaust steam.

The best measure of the degree of perfection of any engine is a comparison of its performance with that of the corresponding ideal cycle operating under the same conditions. The indicated engine efficiency is the ratio of the indicated work per pound of steam to the work done by a pound of steam from the ideal cycle. The brake engine efficiency is a corresponding ratio of brake work per pound of steam to the ideal cycle work.

The overall or brake thermal efficiency of a steam engine is calculated as the ratio of brake output to Rankine cycle input per pound of steam, and the indicated thermal efficiency is found in a similar way. Another, but a rather indefinite method of expressing the economy is in terms of the pounds of steam used per brake horsepower hour, and this is known as the water rate. It is not a definite method of measuring engine efficiency since initial stcam pressures and temperatures, as well as the back pressures maintained by condensers, vary so much in practice. It is usual practice to express this water rate in pounds of dry steam, because any moisture initially in the steam makes a negligible contribution to the work done by the steam.

Low condenser pressures are very important if steam engines are to have high efficiencies. There is almost as much available energy in the expansion of steam from atmospheric pressure to an absolute pressure of one inch of mercury as when saturated steam expands from 100 lb per sq in. to atmospheric pressure. The use of high initial pressures and temperatures and very low condenser pressures increases the difference between intake and exhaust temperatures and makes initial condensation a more serious loss.

During the period when reciprocating steam engines were the chief source of power, this loss was reduced by breaking up the total expansion in as many as three and four steps. In such multiple expansion engines the temperature range in any one cylinder was much less and condensation was materially reduced.

## Suggested Reading

## Steam Engines

Gaffert, Steam Power Stations. Chapter II.

## Steam Engine Details

Severns and Degler, Steam, Air, and Gas Power. Pages 239-284.

Special Steam Engines<br>Craig and Anderson, Steam Power and Internal Combustion Engines. Chapter XI.<br>Steam Calorimeters and Steam Charts<br>Barnard, Ellenwood, and Hirshfeld, Heat Power Engineering. Part I, Pages 301-308.

## CHAPTER 12

## NOZZLES

12.1. Jet Formation. Even though a nozzle is but a specially shaped passage through which high-pressure gas or vapor is expanded to create a high-velocity jet, it serves to transform part of the random thermal or molecular energy into controlled and directed energy that can later be converted into useful mechanical work. When a gas or vapor has no flow velocity the average molecular velocity is the same in all planes of motion, but once flow starts, the average velocity of the molecules in the direction of flow is greater than that in the direction counter to the flow. This means that a molecule moving in the direction of flow will experience less difficulty due to collisions than one attempting to move counter to the flow, and by such a mechanism the molecular velocity in the flow direction is increased.

This increase in the molecular kinetic energy in the direction of flow is accompanied by a decrease in the kinetic energy in the plane normal to the flow, which is reflected by decreased pressure along the jet. The curved approach scction shown by the nozzle in Figure $12 \cdot 1$ is the general shape that has been found experimentally to promote jet formation with the least turbulence and disturbance.

The simple nozzle shown by the solid lines in Figure $12 \cdot 1$ is made up of a curved approach section and a parallel throat section. The velocity of the jet in the parallel throat section is increased by a reduction of the back pressure against which the discharge occurs. In this way the throat pressure may be decreased until the flow velocity reaches the average molecular velocity (article 2.2), usually referred to as the acoustic velocity. Once this velocity is reached, the mechanism of molecular impact in the direction of flow is incapable of causing a greater jet velocity, because the molecules are no longer colliding with those ahead. Consequently, any additional reduction in the back pressure cannot produce a further increase in the throat velocity or change the throat pressure.

If such a simple nozzle were used to discharge gas or vapor into a region of low pressure, the jet would leave the nozzle throat at a greater pressure and it would consequently spread out and become turbulent.

The diverging nozzle section shown by broken lines in Figure $12 \cdot 1$ must be added when a nozzle is to be used with a low back pressure. The action of such a diverging section in increasing the flow velocity above the maximum attainable in a parallel section is illustrated schematically in Figure $\mathbf{1 2 \cdot 2}$. A molecule that strikes the nozzle wall with only a transverse velocity $v$ normal to the flow direction, rebounds from the wall with a greatly reduced transverse velocity and a flow component that depends on the angle of the wall. It is in this manner that the flow velocity of a jet can be increased above the acoustic value and the pressure reduced in a diverging nozzle section.


Fig. 12•1. Nozzle shape.


Fig. 12.2. Diverging nozzle section.

There are many factors involved in nozzle design that must be determined experimentally but there are also many important conclusions that may be reached analytically. Even though a nozzle is a simple appearing passage that does not move, it accomplishes the very important function of reducing the amount of random and uncontrolled molecular motion and increasing the portion of the molecular energy that is directed and controlled in the form of a high-velocity jet.
12.2. Ideal Nozzle Equations. The principle of conservation of energy may be written for a gas or vapor as it passes through a nozzle,

$$
\begin{gather*}
\text { Heat conducted }=\underset{\text { Energy }}{\text { Internal }}+\underset{\text { Work }}{\text { Compression }}+\underset{\text { Work }}{\text { Flow }}+\underset{\text { Work }}{\text { Acceleration }} \\
d Q=d u+\frac{1}{J} P d V+\frac{1}{J} V d P+\frac{1}{J g} v d v
\end{gather*}
$$

Likewise, it can also be stated that the pounds per second passing any point in the nozzle is constant for steady flow, or

$$
\begin{equation*}
\delta A v=\text { Constant } \tag{12•2}
\end{equation*}
$$

where $\delta=$ density in pounds per cubic foot at any point, and $A=$ nozzle area in square feet at any point.

For an ideal nozzle with no heat conducted to or from the gas or vapor, $d Q=0$, equation $12 \cdot 1$ may be written in terms of enthalpy as

$$
0=d u+\frac{1}{J} d(P V)+\frac{1}{J g} v d v=d h+\frac{1}{J g} v d v
$$

For vapors and high-temperature gases, the enthalpy change must be found from tables, but for low-temperature gases it may be calculated. For gases,

$$
d h=c_{p} d T
$$

If the flow through the nozzle is assumed to be nonturbulent and reversible, the entropy is constant. This condition must be solved from tables when vapors are involved, but for gases $P V^{\gamma}=$ constant or when this is combined with the general gas equation, the temperature at any point is given by

$$
T=T_{1}\left(\frac{P}{P_{1}}\right)^{(\gamma-1) / \gamma}
$$

where the subscript 1 refers to a known initial condition. The density at any condition must be found from tables when vapors are involved, but may be calculated from the general gas equation in other cases. For gases,

$$
\delta=\frac{w}{V}=\frac{P}{R T}
$$

Since the concept of absolute temperature refers to a gas at rest when the average molecular velocities are the same in all three planes of motion, it must be redefined for application to high-velocity jets. It cannot be considered as the reading that would be recorded by a stationary thermometer introduced in the strcam, but must be thought of as the temperature that would be indicated by a thermometer moving with the jet. In this way, the gas equations derived for stationary gases may be applied to gases moving at high velocity with unequal components of molecular velocity as long as the temperatures and pressures used are relative to the moving


Fig. 12 3. Expanding nozzle. stream.
12.3. Gas Nozzles. For a gas flowing through the nozzle of Figure $12 \cdot 3$, equations $12 \cdot 3$ and $12 \cdot 4$ may be combined to give

$$
c_{p} d T+\frac{1}{J g} v d v=0
$$

If this equation is integrated between point 1 , where the temperature is $T_{1}$ and the velocity is zero, and point 2 in the nozzle throat, where the relative temperature is $T_{2}$ and the velocity is $v_{2}$,

$$
\int_{T_{1}}^{T_{2}} c_{p} d T+\frac{1}{J g} \int_{0}^{v_{2}} v d v=0
$$

or

$$
c_{p}\left(T_{2}-T_{1}\right)+\frac{1}{2 J g} v_{2}=0
$$

Solving for the throat velocity,

$$
v_{2}=\left[2 J g c_{p}\left(T_{1}-T_{2}\right)\right]^{3 / 2}
$$

Since the throat temperature $T_{2}$ is not usually known, it may be eliminated by using equation 12.5 to obtain

$$
T_{2}=T_{1}\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma-1) / \gamma}
$$

By combining equations $12 \cdot 9$ and $12 \cdot 10$,

$$
v_{2}=\left[2 J g c_{p} T_{1}\left(1-\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma-1) / \gamma}\right)\right]^{1 / 2}
$$

Since the density of the gas at the throat condition 2 is

$$
\delta_{2}=\frac{P_{2}}{R T_{2}}=\frac{P_{2}}{R T_{1}\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma-1) / \gamma}}=\frac{P_{1}}{R T_{1}}\left(\frac{P_{2}}{P_{1}}\right)^{1 / \gamma}
$$

The rate of discharge in pounds per second for a throat area $A_{2}$ square feet may be found from equations $12 \cdot 11$ and $12 \cdot 12$ as

$$
\begin{gather*}
\delta_{2} A_{2} v_{2}=A_{2} \frac{P_{1}}{R T_{1}}\left(\frac{P_{2}}{P_{1}}\right)^{1 / \gamma}\left[2 J g c_{p} T_{1}\left[1-\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma-1) / \gamma}\right]\right]^{1 / 2} \\
\delta_{2} A_{2} v_{2}=A_{2}\left(\frac{2 J g c_{p} P_{1}^{2}}{R^{2} T_{1}}\left[\left(\frac{P_{2}}{P_{1}}\right)^{2 / \gamma}-\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma+1) / \gamma}\right]\right)^{3 / 2}
\end{gather*}
$$

The critical pressure ratio $P_{2} / P_{1}$ at which the velocity $v_{2}$ at the nozzle throat reaches the acoustic velocity (article $12 \cdot 1$ ) corresponds to the value that results in a maximum discharge rate $\left(\delta_{2} A_{2} v_{2}\right)$. For a given set of initial conditions $P_{1}$ and $T_{1}$, this critical pressure ratio that results in maximum discharge may be found by equating the first dif-
ferential of equation $12 \cdot 14$ to zero. Since $\left(\delta_{2} A_{2} v_{2}\right)^{2}$ will be maximum when $\delta_{2} A_{2} v_{2}$ is maximum,

$$
\frac{d\left[\left(\frac{P_{2}}{P_{1}}\right)^{2 / \gamma}-\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma+1) / \gamma}\right]}{d\left(\frac{P_{2}}{P_{1}}\right)}=0
$$

This results in

$$
\text { Critical }\left(\frac{P_{2}}{P_{1}}\right)=\left(\frac{2}{\gamma+1}\right)^{\gamma /(\gamma-1)}=\left(\frac{2}{2.4}\right)^{14 / 04}=0.53
$$

Combining this with equation $12 \cdot 11$, gives

$$
\text { Critical } v_{2}=\left[2 J g c_{p} T_{1} \frac{(\gamma-1)}{(\gamma+1)}\right]^{3 / 2}
$$

This means that the throat velocity and rate of discharge will progressively increase as the pressure against which the nozzle discharges is decreased. This will continue until, for diatomic gases,

$$
\frac{P_{2}}{P_{1}}=0.53
$$

Further decrease in the discharge pressure will not succeed in reducing the throat pressure below this value, since the flow velocity in the parallel throat section cannot exceed the acoustic velocity given by equation $12 \cdot 17$.

When a nozzle is to operate with a lower back pressure the expanding section, Figure $12 \cdot 3$, must be added and the velocity in this section will exceed the critical value. The discharge conditions at point 3 may be calculated by writing equations $12 \cdot 11$ and $12 \cdot 12$ between 1 and 3 to give

$$
v_{3}=\left[2 J g c_{p} T_{1}\left[1-\left(\frac{P_{3}}{P_{1}}\right)^{(\gamma-1) / \gamma}\right]\right]^{3 / 2}
$$

and

$$
\delta_{3}=\frac{P_{1}}{R T_{1}}\left(\frac{P_{3}}{P_{1}}\right)^{1 / \gamma}
$$

The proper angle of divergence for such an expanding section cannot be calculated but has been found experimentally to be between 10 and 14 deg. With a larger angle there is danger of the flow breaking away from the nozzle wall, thus causing turbulence losses, and with smaller angles the nozzle becomes too long and the friction losses are too great. The
throat and discharge areas $A_{2}$ and $A_{3}$ may be calculated from the continuous flow, equation $12 \cdot 2$, and the values of velocity and density at each point.

## Example of Gas Nozzle Calculations

For an ideal gas nozzle discharging 100 lb per sq in. gage pressure, 80 deg F air into a region atmospheric pressure, the initial pressure $P_{1}=(100+14.7) \times$ $144 ; P_{3}=14.7 \times 144 \mathrm{lb}$ per sq ft absolute, and the throat pressure $P_{2}=0.53 P_{1}$ $=60.8 \times 144 \mathrm{lb}$ per sq ft . The throat velocity may be found from either equation $12 \cdot 11$ or equation $12 \cdot 17$ as

$$
\begin{gathered}
v_{2}=\left[2 J g c_{p} T_{1} \frac{(\gamma-1)}{(\gamma+1)}\right]^{7 / 2}=\left[2 \times 778 \times 32.2 \times 0.241 \times 540 \times \frac{0.4}{2.4}\right]^{1 / 2} \\
v_{2}=1042 \mathrm{ft} \mathrm{per} \mathrm{sec}
\end{gathered}
$$

The density at the throat is

$$
\delta_{2}=\frac{P_{1}}{R T_{1}}\left(\frac{P_{2}}{P_{1}}\right)^{1 / \gamma}=\frac{114.7 \times 144}{53.3 \times 540}(0.53)^{1 / 44}=0.362 \mathrm{lb} \text { per cu ft }
$$

The discharge velocity is

$$
\begin{aligned}
& v_{3}=\left[2 J g c_{p} T_{1}\left(1-\left(\frac{P_{3}}{P_{1}}\right)^{(\gamma-1) / \gamma}\right)\right]^{3 / 2}= \\
& \quad\left[2 \times 778 \times 32.2 \times 0.241 \times 540\left(1-\left(\frac{14.7}{114.7}\right)^{0.4 / 1.4}\right)\right]^{7 / 2} \\
& v_{3}=1723 \mathrm{ft} \text { per sec } \\
& \delta_{3}=\frac{P_{1}}{R T_{1}}\left(\frac{P_{3}}{P_{1}}\right)^{1 / \gamma}=\frac{114.7 \times 144}{53.3 \times 540}\left(\frac{14.7}{114.7}\right)^{1 / 14}=0.132 \mathrm{lb} \text { per cu ft }
\end{aligned}
$$

Since $\delta_{2} A_{2} v_{2}=\delta_{3} A_{3} v_{3}$,

$$
\frac{A_{3}}{A_{2}}=\frac{\delta_{2} v_{2}}{\delta_{3} v_{3}}=\frac{0.362 \times 1042}{0.132 \times 1723}=1.66
$$

which is the ratio of discharge area to throat area for such an ideal nozzle.
The relative throat temperature $T_{2}$ that would be read by a thermometer traveling with the stream is found from equation $12 \cdot 10$ as

$$
T_{2}=T_{1}\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma-1) / \gamma}=540 \times 0.833=450 \mathrm{deg} \mathrm{abs}
$$

or -10 deg F. Obviously, a stationary thermometer introduced into the throat stream would indicate no such temperature.

12•4. Steam Nozzles. Nozzle calculations for steam or other vapors differ from gas calculations only in that vapor tables and not gas equations are used to evaluate the properties. For steam, equation 12.3 may be integrated for the nozzle of Figure 12.3 to give

$$
\begin{equation*}
\int_{h_{1}}^{h_{2}} d h+\frac{1}{J g} \int_{0}^{v_{2}} v d v=0 \tag{12•20}
\end{equation*}
$$

or

$$
h_{2}-h_{1}+\frac{1}{2 J g} v_{2}^{2}=0
$$

which gives

$$
v_{2}=\left[2 J g\left(h_{1}-h_{2}\right)\right]^{3 / 2}=223.7\left(h_{1}-h_{2}\right)^{3 / 2}
$$

Similarly,

$$
v_{3}=223.7\left(h_{1}-h_{3}\right)^{3 / 2}
$$

For a reversible expansion, the entropy is constant, or

$$
s_{1}=s_{2}=s_{3}
$$

The density may be calculated from the specific volume from the vapor tables, or

$$
\begin{equation*}
\delta_{2}=\frac{1}{V_{2}} \quad \text { and } \quad \delta_{3}=\frac{1}{V_{3}} \tag{12.25}
\end{equation*}
$$

Even though steam does not conform to the ideal gas equations, the critical pressure ratio at which the acoustic velocity is reached in the nozzle throat may be calculated approximately from equation $12 \cdot 16$. For a constant entropy expansion of moderate pressure and temperature superheated steam, the pressure and volume conditions are predicted fairly well by using $\gamma=1.31$ in the gas equation, $P V^{\gamma}=$ constant. For a reversible expansion of wet steam a value of $\gamma=1.13$ does quite well. Based on these values of $\gamma$,

$$
\text { Critical }\left(\frac{P_{2}}{P_{1}}\right) \text { for superheated steam }=\left(\frac{2}{2.31}\right)^{131 / 031}=0.545
$$

and

$$
\text { Critical }\left(\frac{P_{2}}{P_{1}}\right) \text { for wet steam }=\left(\frac{2}{2.13}\right)^{1.13 / 013}=0.584
$$

Steam expanding from the superhcat region into the wet steam region is especially complicated and will be discussed in article $12 \cdot 5$.

## Example of a Steam Nozzle

Assuming an initial condition of saturated steam at a pressure $P_{1}=100 \mathrm{lb}$ per sq in. absolute, $h_{1}=1186.6 \mathrm{Btu}$ per lb and $s_{1}=1.6022$. If the back pressure is set at $P_{3}=80 \mathrm{lb}$ per sq in. absolute, the throat pressure will be the same, for $P_{2} / P_{1}$ will be greater than 0.584 . For a constant entropy expansion, $s_{1}=s_{2}$ $=s_{f 2}+x_{2} s_{f k 2}$; or $1.6022=0.4532+1.167 x_{2} ; x_{2}=0.985$. This gives $h_{2}=h_{f 2}$ $+x_{2} h_{f g 2}=281.9+0.985 \times 900.5=1169.4$ Btu per lb. Also $V_{2}=V_{f 2}+$ $x_{2} V_{f g_{2}}=0.0175+0.985 \times 5.452=5.398 \mathrm{cu} \mathrm{ft}$ per lb.

For these values,

$$
v_{2}=223.7 \sqrt{h_{1}-h_{2}}=930 \mathrm{ft} \text { per sec }
$$

and the pounds per second per square foot of nozzle throat area is

$$
\frac{W}{a_{2}}=\frac{u_{2}}{V_{2}}=\frac{930}{5.398}=172 \mathrm{lb} \text { per sec per sq } \mathrm{ft}
$$

In a similar manner Table $12 \cdot 1$ has been calculated for the same initial saturated steam at $P_{1}=100 \mathrm{lb}$ per sq in. absolute.

TABLE $12 \cdot 1$
Flow of Steam Through an Ideal Nozzle

| $P_{3}$ | $P_{2}$ | $x_{2}$ | $x_{3}$ | $v_{2}$ | $v_{3}$ | $W / a_{2}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 90 | 90 | 0.993 | 0.993 | 557 | 557.0 | 114.2 |
| 80 | 80 | 0.985 | 0.985 | 930 | 930.0 | 172.0 |
| 70 | 70 | 0.975 | 0.975 | 1210 | 1210.0 | 201.0 |
| 60 | 60 | 0.965 | 0.965 | 1442 | 1442.0 | 208.0 |
| 58 | 58 | 0.963 | 0.963 | 1480 | 1480.0 | 208.5 |

Throat conditions do not change for lower back pressures.

|  |  |  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 50 | 58 | 0.963 | 0.952 | 1480 | 1685 | 208.5 |
| 40 | 58 | 0.963 | 0.942 | 1480 | 1885 | 208.5 |
| 30 | 58 | 0.963 | 0.927 | 1480 | 2160 | 208.5 |
| 20 | 58 | 0.963 | 0.907 | 1480 | 2470 | 208.5 |
| 14.7 | 58 | 0.963 | 0.893 | 1480 | 2650 | 208.5 |

The results of the example of Table $12 \cdot 1$ indicate that for saturated steam at an initial condition of 100 lb per sq in . absolute, the critical throat velocity is reached with a back pressure of 58 lb per sq in . Further reduction of the back pressure cannot increase the throat velocity above the critical acoustic value of 1480 ft per sec or increase the rate of dis-
charge of steam. When accompanied by a properly designed diverging nozzle section, a further reduction in back pressure does increase the final velocity $v_{3}$ of the leaving steam. Since the rate of discharge is the same at all sections throughout the length of a nozzle, the necessary area may be calculated for any pressure condition.
12.5. Supersaturation in Steam Nozzles. The method used to calculate the quality and enthalpy at any point during the constant entropy expansion of steam in a nozzle assumes that equilibrium has been estab-


Fig. 12-4. Section of $h-s$ diagram showing supersaturation limit (Wilson line).
lished. This same assumption is made in all vapor tables and charts, but in the absence of dust particles or other nuclei that promote drop formation the steam may have passed from the nozzle before condensation starts. When such an unstable condition, known as supersaturation, occurs, steam that would normally be wet continues to act as superheated steam. As a consequence the steam conditions and flow velocity as predicted in the usual way are in error.

Careful experimental work ${ }^{1}$ indicates that when steam expands continuously through a nozzle, normal condensation does not occur until the region corresponding to about $41 / 2$ per cent moisture on a Mollier diagram is reached. This condensation region, represented by the "Wilson line," is more accurately defined as being parallel to the saturated steam line on an $h-s$ diagram but approximately 60 Btu below it, as shown by Figure 12•4.

When a nozzle uses steam in this unstable region, the final steam condition after a constant entropy expansion may be calculated with reason-
able accuracy by considering the supersaturated steam to act as a gas having an adiabatic exponent $\gamma=1.31$ and a gas constant $R=85.6$. The final temperature may be found from equation 12.5 as

$$
T_{2}=T_{1}\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma-1) / \gamma}
$$

The specific volume may be found from the gas equation and the velocity by integrating equation $12 \cdot 3$. Since for a gas,

$$
d u=c_{v} d T=\frac{c_{v}}{R} d(P V)
$$

equation $12 \cdot 3$ becomes

$$
\begin{equation*}
0=\left(\frac{c_{v}}{R}+\frac{1}{J}\right) d(P V)+\frac{1}{J g} v d v \tag{12.27}
\end{equation*}
$$

Since

$$
\begin{gather*}
\left(\frac{c_{v}}{R}+\frac{1}{J}\right) J= \\
\frac{c_{v}+\frac{R}{J}}{\frac{R}{J}}=\frac{c_{p}}{c_{p}-c_{v}}=\frac{c_{p} / c_{v}}{\frac{c_{p}}{c_{v}}-1}=\frac{\gamma}{\gamma-1}  \tag{12•29}\\
0=\frac{\gamma}{\gamma-1} d(P V)+\frac{1}{g} v d v
\end{gather*}
$$

By integrating equation $12 \cdot 29$,

$$
0=\int_{P_{1} V_{1}}^{P_{2} V_{2}} \frac{\gamma}{\gamma-1} d(P V)+\frac{1}{g} \int_{0}^{V_{2}} v d v
$$

or

$$
\frac{v_{2}^{2}}{2 g}=\frac{\gamma}{\gamma-1}\left(P_{1} V_{1}-P_{2} V_{2}\right)
$$

## Example of Supersaturation

For saturated steam at 100 lb per sq in . absolute pressure expanding through a nozzle to a throat pressure of 60 lb per sq in., from the steam tables $V_{1}=4.432$ cu ft per lb and $T_{1}=327.8+460=787.8 \mathrm{deg}$ R. The final temperature after a constant entropy expansion is found from equation 12.26 as

$$
T_{2}=T_{1}\left(\frac{P_{2}}{P_{1}}\right)^{(\gamma-1) / \gamma}=787.8\left(\frac{60}{100}\right)^{(131-1) / 131}=697 \operatorname{deg} \mathrm{R} \text { or } 237 \mathrm{deg} \mathrm{~F}
$$

Had the normal condensation assumed in Table $12 \cdot 1$ taken place the temperature would have been the saturation value corresponding to 60 lb per sq in . or 292.7 deg F.

The specific volume at the throat as calculated from the gas equation is

$$
V_{2}=\frac{R T_{2}}{P_{2}}=\frac{85.6 \times 697}{60 \times 144}=6.9 \mathrm{cu} \mathrm{ft} \mathrm{per} \mathrm{lb}
$$

The throat velocity from equation $12 \cdot 31$ is

$$
\begin{gathered}
v_{2}=\left[\frac{2 g \gamma}{\gamma-1}\left(P_{1} V_{1}-P_{2} V_{2}\right)\right]^{1 / 2} \\
v_{2}=\left[\frac{2 \times 32.2 \times 1.31}{1.31-1}(100 \times 144 \times 4.43-60 \times 144 \times 6.9)\right]^{1 / 2}
\end{gathered}
$$

$v_{2}=1080 \mathrm{ft}$ per sec as compared with 1442 ft per sec calculated in Table $12 \cdot 1$. The corresponding discharge rate per square foot of nozzle throat is

$$
\frac{W}{a_{2}}=\frac{v_{2}}{V_{2}}=\frac{1080}{6.9}=1.57 \mathrm{Jb} \text { per sec per sq } \mathrm{ft}
$$

of area instead of the 208.5 calculated when normal condensation is assumed.
This illustrates that when a nozzle is used to expand steam below the saturation condition, both the energy conversion and rate of discharge as calculated using the assumption of equilibrium and normal condensation are high. In an actual nozzle this condensation does not occur until after the steam has passed from the nozzle, and a loss in available energy due to shock pressure waves occurs when the condensation does take place.
12.6. Nozzle Performance. The theoretical characteristics of steam nozzles may be illustrated graphically by Figure $12 \cdot 5$.

Experimental results for actual nozzles check the theoretical conclusions quite satisfactorily. Such items as nozzle wall friction, turbulence, and improper nozzle shape cause the actual steam velocity to be less than the theoretical value by from 1 to 2 per cent. This ratio, $\frac{\text { actual velocity }}{\text { theoretical velocity }}$, is known as the velocity coefficient of the nozzle and it is usually 0.95 to 0.99 .

The actual rate of discharge of a nozzle may be higher than the theoretical value in high-velocity nozzles. This may be the result of friction in the nozzle producing a density increase sufficient to offset more than the velocity decrease, and at other times it may be the result of initial moisture in the steam. Such drops of water do not travel at as high a velocity as the steam, thus they do not absorb their proportionate part
of the kinetic energy and the dry steam attains a higher velocity than the calculated value. In general, there exists a closer check between theory and practice in nozzles than in many other phases of heat power.

## REFERENCE

1. A.S.M.E. Transactions, April, 1939, page 177.

## Suggested Reading

The Flow of Fluids in Nozzles
Emswiler, T'hermodynamics, Chapter XVIII.
Flow of Fluids
Moyer, Calderwood, and Potter, Elements of Engineering Thermodynamics. Chapter IX.

Flow in Nozzles
Croft, Thermodynamics, Fluid Flow and Heat Transmission. Pages 133-138.

## Steam Nozzles

Keenan, Thermodynamics. Pages 136-147.

## CHAPTER 13

## TURBINES

13.1. Turbine Operation. In a turbine, high-pressure gas or vapor is first expanded through stationary nozzle passages and the resulting high-velocity vapor is directed into the buckets or blading on a rotating wheel where the kinetic energy is reduced as the jet does work on the moving blades. This is illustrated by Figure $13 \cdot 1$.

The gencral idea is to reduce the vapor velocity to as low a value as possible in the blading and to accomplish this with the least amount of


Fig. 13 1. Nozzles and buckets of an impulse turbine.
turbulence. The pressure drop that may be used in any one set of nozzles is limited by the permissible bucket velocity, and for this reason a turbine may have from one to as many as thirty stages. Each stage is made up of its set of stationary nozzles and moving buckets with the steam passing successively from one stage to the next.

In a pure impulse turbine the decrease in pressure and increase in velocity occur only in the stationary blading or nozzles whereas in the reaction turbine there is a pressure drop in both the stationary and moving blades. In both types of turbines the blading length must increase progressively in the lower-pressure stages to accommodate the increased volume of steam that must be handled, and it is this volume of steam
that must be passed through the last stage that limits the capacity of most turbines.
13.2. Simple Impulse Stages. A simple single-stage impulse turbine and the corresponding velocity vector diagram are illustrated by Figure $13 \cdot 2$. In such a turbine the absolute velocity $v$ at which the steam leaves the nozzle is shown at the nozzle angle $\alpha$. If the absolute bucket velocity is subtracted vectorially, the resulting vector is the relative velocity with which the steam enters the blading. Since the buckets of such a turbine are symmetrical, the relative leaving velocity vector may be drawn. To


Fig. 13 2. Simple impulse stage.
find the absolute velocity with which the steam leaves the wheel it is only necessary to add the absolute bucket velocity $r \omega$ to the relative leaving velocity.

The entering and leaving blade angle $\beta$ determines the shape of the blading and it may be seen from the vector diagram that the initial component of the steam velocity that is normal to the wheel $(v \sin \alpha)$ is not altered. This means that the nozzle angle $\alpha$ has a very important influence on the performance of a turbine stage. The smallest possible absolute final steam velocity realized when $2 r \omega=v \cos \alpha$ is $v \sin \alpha$. A small nozzle angle, $\alpha$, would correspond to a low leaving velocity and high efficiency, but this is limited by the fact that it is this same component of velocity normal to the wheel that governs the amount of steam that may be passed through the turbine. It would be expected that the nozzle angles in the lower-pressure stages of a turbine where steam volumes are large would be greater than those in the high-pressure end of the unit. In practice, nozzle angles vary from 10 to 30 deg.
For an ideal nozzle discharging $W$ pounds per second of stcam at a velocity $v$, the stage input is $\frac{W}{2 g} v^{2}$ foot-pounds per second. The tan-
gential force exerted on the blading may be found from the motion equation $F=M a$, by rearranging it,

$$
F=M a=M \frac{d v}{d t}=\frac{M}{d t} d v=\text { Mass per sec } \times \text { Change in velocity }
$$

For a discharge of $W$ pounds per second, the mass flowing per second is $W / g$. The change in absolute tangential steam velocity produced by the buckets is the initial tangential absolute velocity ( $v \cos \alpha$ ) minus the final tangential component of absolute velocity which is shown by Figure $13 \cdot 2$ to be $-(v \cos \alpha-2 r \omega)$. The change in velocity is

$$
v \cos \alpha-[-(v \cos \alpha-2 r \omega)]=2(v \cos \alpha-r \omega)
$$

The tangential force on the wheel is

$$
F=\frac{W}{g} 2(v \cos \alpha-r \omega)
$$

The power delivered in foot-pounds per second for a wheel velocity $r \omega$ is

$$
\text { Stage output }=F r \omega=\frac{2 W r \omega}{g}(v \cos \alpha-r \omega)
$$

This results in a general expression for wheel efficiency of

$$
\text { Wheel efficiency }=\frac{\frac{2 W}{g} r \omega(v \cos \alpha-r \omega)}{\frac{W}{2 g} v^{2}}
$$

which has a maximum value when $r \omega=\frac{v \cos \alpha}{2}$ of

$$
\begin{equation*}
\text { Maximum efficiency }=\frac{v \cos \alpha \times v \cos \alpha}{v^{2}}=\cos ^{2} \alpha \tag{13•4}
\end{equation*}
$$

For the impulse turbine the same result may also be reached from an energy point of view.

$$
\begin{equation*}
\text { Input energy per pound }=\frac{1}{2 g} v^{2} \tag{13•5}
\end{equation*}
$$

Leaving energy lost per $\mathrm{lb}=\frac{1}{2 g}\left[v^{2} \sin ^{2} \alpha+(v \cos \alpha-2 r \omega)^{2}\right]$

Leaving energy lost per $\mathrm{lb}=\frac{1}{2 g}\left[v^{2} \sin ^{2} \alpha+v^{2} \cos ^{2} \alpha\right.$

$$
\left.-4 r \omega v \cos \alpha+4(r \omega)^{2}\right]
$$

Leaving energy lost per $\mathrm{lb}=\frac{1}{2 g}\left[v^{2}-4 r \omega v \cos \alpha+4(r \omega)^{2}\right]$
Output per pound $=$ input - losses

$$
=\frac{1}{2 g}\left[v^{2}-v^{2}+4 r \omega v \cos \alpha-4(r \omega)^{2}\right]
$$

This gives as the expression for wheel efficiency,

$$
\text { Efficiency }=\frac{4 r \omega v \cos \alpha-4(r \omega)^{2}}{v^{2}}
$$

which is the same as equation $13 \cdot 3$ found on a power and force basis.
The important conclusions about a simple impulse stage are that for maximum efficiency the tangential wheel velocity should be half the tangential component of the steam velocity, and that a small nozzle angle results in a high wheel efficiency.
13.3. Double-Velocity Stages. When it is desirable to have such a large pressure drop through the first stage nozzles of a turbine that the bucket speed cannot be increased to the value for maximum efficiency, it is often necessary to use a double-velocity impulse or Curtin stage.

The velocity diagram for such a stage, shown in Figure 13•3, is the equivalent of two simple impulse stage diagrams with the addition of an intermediate set of stationary blades to reverse the direction of the steam between the two rows of moving buckets.

Maximum efficiency is realized when the absolute leaving velocity is a minimum or when $v \cos \alpha-4 r \omega=0$ or when $r \omega=\frac{v \cos \alpha}{4}$. This means that such a stage requires only half as high a bucket velocity as is needed in a simple impulse stage.

The change in absolute velocity taking place in the first row of moving buckets is $v \cos \alpha-[-(v \cos \alpha-2 r \omega)]=(2 v \cos \alpha-2 r \omega)$. In the second row of moving blades is $(v \cos \alpha-2 r \omega)-[-(v \cos \alpha-4 r \omega)]=$ $2 v \cos \alpha-6 r \omega$. For the condition of maximum efficiency, $r \omega=\frac{v \cos \alpha}{4}$, the change in velocity in the first row of buckets is $2 v \cos \alpha-\frac{v \cos \alpha}{2}=$ $\frac{3}{2} v \cos \alpha$. In the second row, the change in velocity is $2 v \cos \alpha-\frac{6}{4} v \cos \alpha$ $=\frac{1}{2} v \cos \alpha$. This means that at this operating condition three-fourths
of the total stage work is done in the first row of blades and one-fourth in the second row of moving blades.

Although such a double-velocity stage has the same value of theoretical maximum efficiency as a simple impulse stage, the higher steam velocities that are used usually result in lower values of actual efficiency. In spite of this, however, such stages are widely used when a large pressure drop in one stage is needed.
13.4. Reaction Stages. The usual reaction turbine actually utilizes both impulse and reaction. As a rule, the stationary and moving blades


Fig. 13.3. Double-velocity stage.


Fig. 13•4. Reaction stage.
are similar and the passages are so designed that there is a pressure drop and a velocity increase in each set of blades.

As indicated in Figure 13.4, this means that the entering velocity $v_{1}$ relative to the blades is the same in each set and the relative leaving velocity $v_{2}$ is common. Consequently the absolute entering velocity for the stationary blades is $v_{1}$ at the angle $\alpha$ and the absolute leaving velocity is $v_{2}$ at the angle $\beta$. For moving blades the absolute entering velocity is $v_{2}$ and the absolute leaving velocity is $v_{1}$. The change in absolute tangential velocity in the moving blades is

$$
\begin{align*}
& \Delta v=v_{2} \cos \beta-\left[-\left(v_{1} \cos \alpha\right)\right]=v_{2} \cos \beta+v_{2} \cos \beta-r \omega \\
& \Delta v=2 v_{2} \cos \beta-r \omega \tag{13•9}
\end{align*}
$$

For a flow of $W$ pounds per second the power delivered to the moving blades is

$$
\text { Power }=\frac{W}{g} \Delta v r \omega=\frac{W}{g} r \omega\left(2 v_{2} \cos \beta-r \omega\right)
$$

For a given tangential velocity $\left(v_{2} \cos \beta\right)$ this will be a maximum when

$$
r \omega=v_{2} \cos \beta
$$

which corresponds to an absolute leaving velocity $v_{1}$ that is normal to the wheel.

The corresponding maximum power delivered to the moving blades is, from equation $13 \cdot 10$,

$$
\begin{equation*}
\text { Power }=\frac{W}{g}\left(v_{2} \cos \beta\right)^{2} \tag{13•12}
\end{equation*}
$$

Since no power is delivered to the stationary blades, the increase in kinetic energy in them corresponds to the thermal energy conversion in the converging nozzle-shaped passage.

$$
\text { Energy converted in stationary blades }=\frac{W}{2 g}\left(v_{2}^{2}-v_{1}^{2}\right) \quad[13 \cdot 13]
$$

However,

$$
\begin{align*}
v_{1}^{2} & =\left(v_{2} \sin \beta\right)^{2}+\left(v_{2} \cos \beta-r \omega\right)^{2} \\
v_{1}^{2} & =v_{2}^{2}-2 v_{2} r \omega \cos \beta+(r \omega)^{2}
\end{align*}
$$

Thus, energy converted in stationary blades $=$

$$
\frac{W}{2 g}\left[2 v_{2} r \omega \cos \beta-(r \omega)^{2}\right]
$$

For the condition of maximum efficiency, $v_{2} \cos \beta=r \omega$, this energy conversion becomes

$$
\text { Conversion in stationary blades }=\frac{W}{2 g}\left(v_{2} \cos \beta\right)^{2}
$$

If this is compared with equation $13 \cdot 12$, it may be seen that it is exactly half of the power delivered to the moving blades. Since the absolute velocity with which the steam leaves the moving blades is the same as the absolute velocity with which it comes to the stationary blades, the sum of the energy converted into velocity in the stationary and moving blades goes to do work on the moving blades. This means that at the operating condition of maximum efficiency half of the total conversion from thermal energy occurs in the stationary and half in the moving blades.

For this maximum output of $\frac{W}{g}\left(v_{2} \cos \beta\right)^{2}$ foot-pounds per second the corresponding input is greater by the initial kinetic energy delivered to the stage,

$$
\begin{equation*}
\text { Stage input }=\frac{W}{g}\left(v_{2} \cos \beta\right)^{2}+\frac{W}{2 g} v_{1}^{2} \tag{13•17}
\end{equation*}
$$

From equation $13 \cdot 14$ for the condition of maximum efficiency when $r \omega=$ $v_{2} \cos \beta$,

$$
v_{1}^{2}=v_{2}^{2}-2\left(v_{2} \cos \beta\right)^{2}+\left(v_{2} \cos \beta\right)^{2}=v_{2}^{2}-\left(v_{2} \cos \beta\right)^{2}
$$

The efficiency, which is the ratio of output to input becomes

$$
\begin{gather*}
\text { Maximum cfficiency }=\frac{\frac{W}{g}\left(v_{2} \cos \beta\right)^{2}}{\frac{W}{g}\left(v_{2} \cos \beta\right)^{2}+\frac{W}{2 g}\left[v_{2}^{2}-\left(v_{2} \cos \beta\right)^{2}\right]} \\
\text { Maximum efficiency }=\frac{2(\cos \beta)^{2}}{1+(\cos \beta)^{2}}
\end{gather*}
$$

A comparison of equation $13 \cdot 20$ with equation $13 \cdot 4$ indicates that the reaction stage is more efficient than an impulse stage with the same nozzle angle. The reaction stage entering steam velocity can be only half that used in an impulse stage, however, and this has the disadvantage of requiring more stages for a given pressure range in a turbine.
13.5. Turbine Layout. The tangential blade velocities used in turbines vary from 300 ft per sec in small turbines to a maximum value of 1250 ft per sec in large ones. For such blade velocities the most efficient steam velocities would lie between 600 and 2500 ft per sec in simple impulse stages and half that much for reaction wheels.

From the nozzle equations of Chapter 12, an enthalpy drop of only $\Delta h=\frac{v^{2}}{(223.7)^{2}}$ or 1.8 Btu per lb of steam would produce a velocity of 300 ft per sec and a drop of 125 Btu per lb would be required to produce a steam velocity of 2500 ft per sec. A reversible or constant entropy expansion of saturated 100 lb per sq in . steam to atmospheric pressure represents an enthalpy drop of 143 Btu per lb and a similar expansion of 1200 lb per sq in., 900 deg F steam to a back pressure of one inch of mercury represents an enthalpy drop of 590 Btu per lb. This illustrates the need for dividing turbines into stages to attain maximum efficiency.

Double-velocity stages or Curtis wheels are commonly used in small turbines or as the first stage of a large turbine. If such a stage is used with a blade velocity of 600 ft per sec, the steam velocity would be above 2400 ft per sec. This would represent an enthalpy drop in the nozzles of

$$
\Delta h=\frac{(2400)^{2}}{(223.7)^{2}}=115 \text { Btu per } \mathrm{lb}
$$

This would represent a pressure drop in the nozzles from 1200 lb per sq in. and 900 deg F to a pressure of about 475 lb per sq in. Since the steam does not enter the casing until it has passed through the first-stage nozzles, this would mean that the use of a double-velocity first-stage wheel reduces the pressure for which the turbine casing is designed to 475 lb per sq in. It also would materially reduce the additional number of simple stages needed to complete the expansion.

As the steam expands through a turbine, its volume increases and the blade length must be increased to pass the increased volume. The only


Fig. 13.5. Variation of turbine blade length. other way of passing a greater volume is to increase the nozzle angles and thus increase the component of steam velocity parallel to the turbine axis. Since an increased nozzle angle decreases the ideal stage efficiency it is preferable to increase the blade length, resulting in the type of turbine layout illustrated by Figure 13.5.
As the blades become longer, the variation in $r \omega$ along the blades makes it necessary to vary the entrance angle at different points along both moving and stationary blades. With blade lengths limited structurally to about 18 in . on $3600-\mathrm{rpm}$ wheels and to about 36 in . on $1800-\mathrm{rpm}$ turbines, and tip speeds limited to 1250 ft per sec, the maximum amount of steam that can be passed through a turbine is limited by the last-stage wheel.

Although the exact method used to divide a total energy drop into the various stages varies with designers and conditions, a reasonably representative proportion results if the energy drop is made equal in all stages, except when such special arrangements as Curtis double-velocity wheels are used.
13.6. Turbine Losses and Performance. In the foregoing discussion of the stage efficiency of turbines, only the leaving velocity loss was considered, and the turbulence and friction losses always present in nozzles and blades were ignored. In a single-stage turbine, these friction losses pass out to the exhaust in the form of heat. In a multi-stage turbine, the friction and turbulence losses of one stage are available in the form of heat for expansion in the next stage.

Some of these losses occur in the nozzles or stationary blading, and some of them take place in the moving blades, but it is usual to consider that at the lower pressure in any stage the friction losses of that stage
reheat the steam at constant pressure. This is probably best shown on a Mollier enthalpy-entropy diagram as shown in Figure $13 \cdot 6$.

If a three-stage turbine with no losses operates between $P_{1}$ and $P_{4}$, the path of the steam through it would be represented by the constant entropy line $a b c d$, the heat converted into work would be $h_{a}-h_{d}$ and the cycle efficiency would be

$$
\text { Cycle efficiency }=\frac{h_{a}-h_{d}}{h_{a}-h_{f d}}
$$

If, however, the losses are considered, the pressure drop in the first stage is from $a$ to $b$, and then the losses of the stage reheat the steam at constant pressure to $b^{\prime}$. This procedure is repeated in each succeeding stage with the result that the actual path of the steam is $a b^{\prime} c^{\prime} d^{\prime}$. The actual heat converted into work is $h_{a}-h_{d^{\prime}}$. The thermal efficiency of the turbine is
Thermal efficiency $=\frac{h_{a}-h_{d^{\prime}}}{h_{a}-h_{f d}} \quad[13 \cdot 22]$


Fig. 13 6. Turbine reheat diagram.

The comparison of the actual turbine and the theoretical cycle gives

$$
\begin{equation*}
\text { Engine efficiency }=\frac{h_{a}-h_{d^{\prime}}}{h_{a}-h_{d}} \tag{13.23}
\end{equation*}
$$

The individual stage efficiencies in well-designed turbines are as high as 0.83 , a Curtis double-velocity stage has an efficiency of 0.71 or 0.72 , and the excessive moisture in the wet steam of the last stages causes the stage efficiency there to drop as low as 0.70 . ' A first-stage efficiency of 0.70 in Figure 13.6 would mean that the reheat $h_{b}-h_{b^{\prime}}$ would be 0.3 of the stage heat drop $h_{a}-h_{b}$.

## Example of Turbine Performance

A small four-stage, $3600-\mathrm{rpm}$ turbine uses steam at 300 lb per sq in. absolute and 600 deg F . It is so designed that the pressure after the double-velocity first stage is 50 lb per sq in . absolute, after the next impulse stage the pressure is 15 lb per sq in . absolute, after the next 3 lb per sq in ., and at the end of the last stage it is 1.5 in . of mercury.

At the initial steam condition in Figure $13 \cdot 7$ the enthalpy $h_{1}=1313.3 \mathrm{Btu}$ per lb and the entropy is $s_{1}=1.6256$. For the constant entropy expansion of the ideal turbine to the back pressure $P_{5}=1.5$ of Hg .

$$
\begin{aligned}
& s_{1}=s_{f 5}+x_{5} s_{f_{g}} \text { or } 1.6256=0.1147+x_{5} 1.8877 \\
& x_{5}=0.80 \quad h_{5}=59.72+0.80 \times 1040.8=891.7 \text { Btu per lb }
\end{aligned}
$$



Fig. 13.7. Mollier diagram solution of turbine problem.
The ideal heat converted to work is

$$
\begin{gathered}
h_{1}-h_{5}=1313.3-891.7=421.6 \\
\text { Cycle efficiency }=\frac{h_{1}-h_{5}}{h_{1}-h_{f 5}}=\frac{1313.3-891.7}{1313.3-59.7}=0.337
\end{gathered}
$$

In a frictionless nozzle of stage one, for a constant entropy expansion to $P_{2}=50 \mathrm{lb}$ per sq in.,

$$
8_{1}=s_{f 2}+x_{2} s_{f g 2} \quad \text { or } \quad 1.6256=0.4111+x_{2} 1.2469
$$

$$
x_{2}=0.974 h_{2}=249.98+0.974 \times 923.5=1148.8 \text { Btu per } \mathrm{lb}
$$

The energy drop converted into velocity in stage one nozzles $=h_{1}-h_{5}=$ $1313.3-1148.8=164.5$ Btu per lb , which means a steam velocity $=$ $223.7 \sqrt{164.5}=2870 \mathrm{ft}$ per sec. For a nozzle angle of 15 deg , for maximum efficiency, $v \cos \alpha=4 r w$, or the blade velocity $r w$ should be 692 ft per sec, which means a correct wheel diameter of 3.66 ft for the Curtis wheel.

A stage efficiency of 0.7 would mean that $0.7\left(h_{1}-h_{2}\right)$ or $0.7 \times 164.5=$ 115 Btu per lb is converted into work in stage one, and the remaining $164.5-115=49.5$ Btu goes to reheat the steam at constant pressure from 2 to $2^{\prime}$.

$$
h_{2^{\prime}}=h_{2}+49.5=1148.8+49.5=1198.3 \text { Btu per } \mathrm{lb}
$$

The resulting temperature is 330 deg F and the entropy $s_{2^{\prime}}=1.69$.
The constant entropy expansion from $2^{\prime}$ to $3^{\prime \prime}$ in the stationary blading of stage two results in $h_{3^{\prime \prime}}=1107 \mathrm{Btu}$ which represents an energy drop

$$
h_{2^{\prime}}-h_{3^{\prime \prime}}=1198.3-1107=91.3 \text { Btu per } \mathrm{lb}
$$

A second-stage efficiency of 0.8 means a conversion of $0.8 \times 91.3=73.0$ Btu per lb into work in stage two, and reheating to

$$
h_{3^{\prime}}=1107+18.3=1125.3 \text { Btu per } \mathrm{lb} \quad \text { and } \quad s_{3^{\prime}}=1.719
$$

Similarly in stage three, the heat converted into work with a stage efficiency of 0.8 is $(1125.3-1021.0) 0.8=83.4 \mathrm{Btu}$ per lb and

$$
h_{4^{\prime}}=1041.9 \text { Btu per lb and } s_{4^{\prime}}=1.743
$$

Using a fourth-stage efficiency of 0.7 because of the moisture present, the energy converted into work in stage four is $(1041.9-957.0) 0.7=59.4 \mathrm{Btu}$ per lb and

$$
h_{5^{\prime}}=982.5 \text { Btu per } \mathrm{lb} \quad s_{5^{\prime}}=1.79
$$

and the quality is 0.887 .
In the actual turbine $h_{1}-h_{5^{\prime}}=1313.3-982.5=330.8$ Btu per lb is converted into work.

Thermal efficiency $=\frac{h_{1}-h_{5^{\prime}}}{h_{1}-h_{f 5}}=\frac{330.8}{1253.6}=0.264$
Engine cfficiency $=\frac{h_{1}-h_{5^{\prime}}}{h_{1}-h_{5}}=\frac{330.8}{421.6}=0.785$
Since one horsepower $=\frac{33,000 \times 60}{778}=2545 \mathrm{Btu}$ per hr, the water rate of the turbine should be

$$
\frac{2545}{h_{1}-h_{5^{\prime}}}=\frac{2545}{330.8}=7.7 \mathrm{lb} \text { steam per } \mathrm{hp}-\mathrm{hr}
$$

or

$$
\frac{7.7}{0.746}=10.3 \mathrm{lb} \text { per kw-hr }
$$

## Suggested Reading

## Steam Turbines

Gaffert, Steam Power Stations. Chapter III.
Craic and Anderson, Steam Power and Internal Combustion Engines. Pages 344-394.
Severns and Degler, Steam, Arr, and Gas Power. Pages 287-314.
Keenan, Thermodynamics. Pages 125-135; 158-169.

## CHAPTER 14

## CONDENSERS

14•1. Condenser Requirements. The expansion of steam to pressures below that of the atmosphere is possible only when it can be discharged into an enclosure and condensed. The importance of maintaining a low back pressure for a turbine exhaust may be emphasized by the fact that there is approximately as much available encrgy in the expansion of steam from atmospheric pressure to a back pressure of 1.5 in . of mercury as there is in the expansion of 600 deg F 300 lb per sq in. steam to atmospheric pressure.

The lower limit of the pressure that may be maintained in a condenser is the saturation pressure (from vapor tables) corresponding to the temperature of the cooling medium available. Although air is used as a condenser cooling medium in small refrigerators and under special conditions in steam plants, water is the cooling medium of chicf importance. The purity and availability of condenser cooling water at hand have a very definite influence on power plant design. Where a river or other body of water is available the power cost of pumping the cooling water through the condenser is the chief item that must be considered and a very low back pressure might prove to be economically feasible.

Where the supply is limited, the cooling water must be reused by cooling it by evaporation in an outside spray pond or cooling tower, where the heat required to evaporate a part of it is withdrawn from the remainder of the cooling water. In such a case it would probably not prove feasible to attempt such a very low condenser pressure. The magnitude of the amount of cooling water required may be visualized by considering the fact that approximately 1000 Btu must be absorbed by the condenser cooling water for each pound of steam condensed. Since it is usually impractical to have much more than a $25 \operatorname{deg} \mathrm{~F}$ temperature rise as the cooling water passes through the condenser, it means that as much as 40 lb of cooling water must be used for each pound of steam condensed.

Condensation of exhaust steam is accomplished in jet condensers by spraying the cooling water directly in contact with the steam. Such condensers have the advantage of being compact but their use is prohibited when the cooling water is impure and would have to be treated before it could be used as feed water for a boiler. The cooling water also contains
considerable entrained air that would have to be pumped from the condenser, which makes it a bit difficult to maintain a good condenser vacuum.

In surface condensers, the cooling water flows inside thin-walled metal tubes, and the exhaust steam is discharged into the condenser shell around the tubes as illustrated by Figure 14.1. The fact that the condensed steam and cooling water are not mixed in surface condensers makes them much better adapted to impure cooling water, low pressures, and variable load operation. Small surface condensers are


Fig. 14.1. Small surface condenser. equipped with a single pump, known as a wet vacuum pump, which removes both the condensate and noncondensable gases. Since the condenser opcrates at sub-atmospheric pressures, air leaks are inevitable and this air must be removed if a good vacuum is to be maintained. This makes air pumps, either centrifugal or the steamjet ejector type necessary on large condensers. The presence of air in a condenser not only raises the back pressure against which the turbine must exhaust, but also retards heat transfer by "blanketing" the tubes in the portions of the condenser that are less actively swept by the exhaust steam.
14.2. Surface Condenser Heat Transfer Rates. In a surface condenser the heat from the condensing steam is transmitted by convection or film conduction from the steam to the outside tube wall, by conduction through the metal of the tube wall and by convection inside tube wall to the cooling water flowing through it.

In Chapter 7 the general equation for the forced convection film conductance $U$ in Btu per hour per square foot per degree Fahrenheit was developed as

$$
U=\text { Constant } \times \frac{k}{l}\left(\frac{v l_{\rho}}{\mu}\right)^{-e}
$$

where $k=$ conductivity of the fluid in Btu per hour per square foot per degree Fahrenheit per foot.
$v=$ fluid velocity in feet per second.
$\rho=$ density of fluid in slugs per cubic foot.
$\mu=$ fluid viscosity.
$-e=$ an exponent.
$l=$ significant dimension of the surface.

Since the flow of the water is parallel to the tube wall and condenser tubes do not vary widely in size, the effect of changes in $l$ may be ignored. Since the temperature of cooling water is usually fairly constant, variations of $\rho$ and $\mu$ may be neglected, and a large amount of experimental work has shown that the water film conductance $U_{w}$ may be expressed as

$$
U_{w}=270\left(v_{w}\right)^{0.8} \text { Btu per hr per sq ft per deg } \mathrm{F}
$$

if the tube surface area is evaluated on the outside (the usual method) or as

$$
U_{w}=300\left(v_{w}\right)^{0.8}
$$

if the tube surface area is evaluated on the inside.
When using a tube wall thickness of 0.05 in. and a metal conductivity of 70 Btu per hr per sq ft per deg per ft the tube conductance

$$
U_{t}=\frac{k}{X}=\frac{70}{\frac{0.05}{12}}=16,800 \text { Btu per hr per sq ft per } \operatorname{deg} \mathrm{F}
$$

The temperature drop between the steam and the outside of the tubes is affected by more variables than on the inside. The extreme variation in outside film conductance between clean, new tubes and old, fouled tubes was difficult to explain until it was discovered that two types of condensation are possible. When the tubes are clean and bright, the condensed steam does not wet the surface because the cohesive forces of the molecules cause the water to form in drops that quickly enlarge and drop off. When the tubes are corroded or contaminated by a scale or film, the adhesive forces cause the water to wet the tube surface and cover it with a stagnant layer of water through which heat must pass by conduction.

The net effect is a very high steam film conductance $U_{s}$ when tubes are new and clean so that drop condensation occurs and a much reduced conductance with contaminated tubes when film condensation takes place. This problem is complicated further by the fact that the film of oxides and deposit on the tube surfaces when they are dirty offers an additional resistance to heat flow.

The best data now available indicate that if a clean tube is thoroughly swept by steam and drop condensation occurs, $U_{s}$ may reach a value of 3000 Btu per hr per sq ft per deg $F$, but in condensers where the tubes are in banks that are not all swept by steam and there is considerable entrained air to retard heat transfer, a maximum safe value of $U_{8}$ for drop condensation is $U_{s}=1000 \mathrm{Btu}$ per hr per sq ft per deg F is seldom exceeded.

Under very favorable conditions with contaminated tubes and film condensation $U_{s}$ may reach 800 to 1000 Btu per hr per sq ft per deg F , but for actual conditions in a badly fouled condenser, $U_{s}$ drops as low as 250 .

In a new condenser with clean tubes, the oxide film and scale conductance $U_{f}$ is very large and causes a negligible temperature drop. In a badly fouled condenser, $U_{f}$ may drop as low as 2000 Btu per hr per sq ft per deg F.

When $q$ Btu per hour per square foot is being transferred from steam to water, the temperature drop from the steam to the tube is $\Delta t_{s}=\frac{q}{U_{s}}$. The drop across the film or scale, $\Delta t_{f}=\frac{q}{U_{f}}$; the temperature drop through the tube wall $\Delta t_{t}=\frac{q}{U_{t}}$, and the drop between the inside tube surface and the cooling water $\Delta t_{w}=\frac{q}{U_{w}}$. The total temperature drop between steam and water,

$$
\begin{aligned}
& \Sigma \Delta t=\Delta t_{s}+\Delta t_{f}+\Delta t_{t}+\Delta t_{w} \\
& \Sigma \Delta t=q\left[\frac{1}{U_{s}}+\frac{1}{U_{f}}+\frac{1}{U_{t}}+\frac{1}{U_{w}}\right]
\end{aligned}
$$

or the overall conductance is

$$
\begin{equation*}
U=\frac{q}{\Sigma \Delta t}=\frac{1}{\frac{1}{U_{s}}+\frac{1}{U_{f}}+\frac{1}{U_{t}}+\frac{1}{U_{w}}} \tag{14•4}
\end{equation*}
$$

## Example

For new tubes and drop condensation, with a water velocity of 7 ft per sec,

$$
\begin{aligned}
& U_{w}=270 \times(7)^{08}=1,280 \text { Btu per hr per sq ft per deg } F \\
& U_{t}=16,800 \text { " " " " " " " " " } \\
& \begin{array}{lcccccc}
U_{f}= & \infty & \text { " } & \text { " } & \text { " } & \text { " } & \text { " } \\
U_{\mathrm{s}}= & 1,000 & \text { " } & \text { " } & \text { " } & \text { " } & \text { " } \\
\end{array}
\end{aligned}
$$

The overall conductance,

$$
U=\frac{1}{\frac{1}{1000}+\frac{1}{\infty}+\frac{1}{16,800}+\frac{1}{1280}}=543 \text { Btu per hr per sq ft per deg } F
$$

For badly fouled tubes and film condensation, using $U_{f}=2000, U_{s}=250$,

$$
U=\frac{1}{\frac{1}{250}+\frac{1}{2000}+\frac{1}{16,800}+\frac{1}{1280}}=187 \text { Btu per hr per sq ft per deg } F
$$

The choice of the proper cooling water velocity to use in any particular case is essentially an economic problem. As illustrated by Figure 14•2, an increase in this flow velocity results in a higher rate of heat transfer but it also results in an increased pressure loss that must be overcome by the circulating pump. These factors, as well as the availability of cooling water, are among the things that influence the final choice of cooling water velocity.
14.3. Log-Mean Temperature Difference. The cooling water enters the tubes of a condenser at a low temperature and leaves at a somewhat higher


Fig. 14-2. Condenser heat transfer. value, whereas the temperature of the condensing steam on the outside of the tubes is essentially constant at a value near the saturation temperature corresponding to the condenser pressure. Although the overall heat conductance $U$ is very nearly constant throughout the length of the tube, it does not follow that the arithmetic average temperature difference should be used when calculating the rate of heat transfer.


Fig. 14.3. Temperature variation along a condenser tube.
The theoretically correct temperature difference to be used in any case may be established by considering the heat transfer to the condenser tube illustrated by Figure $14 \cdot 3$.* The rate at which the fluid in a length

[^3]$d x$ of tube receives heat from the condensing steam is $U(\pi D d x)\left(T_{0}-T\right)$ and this must be equal to the rate at which it stores heat
$$
\left[\frac{\pi D^{2}}{4} d x \delta c \frac{d T}{d t}\right]
$$
where $\delta=$ cooling medium density .
$c=$ its specific heat.
$t=$ time.
$$
U \pi D d x\left(T_{0}-T\right)=\frac{\pi D^{2}}{4} d x \delta c \frac{d T}{d t}
$$

Separating variables to integrate between the limits of Figure 14•3,

$$
\int_{0}^{t} d t=\frac{D \delta c}{4 U} \int_{T_{1}}^{T_{2}} \frac{d T}{T_{0}-T}
$$

Integrating,

$$
t=\frac{D \delta c}{4 U} \log _{\epsilon}\left(\frac{T_{0}-T_{1}}{T_{0}-T_{2}}\right)
$$

which is the length of time the cooling medium must take to flow through the tube if it is to be heated from $T_{1}$ to $T_{2}$.

If $\Delta T$ is defined as the effective temperature difference causing heat transfer, the average rate of transfer to the element of fluid in the length of tube $d x$ is $U(\pi D d x) \Delta T$. This average rate multiplicd by the time $t$ involved (from equation $14 \cdot 7$ ) must give the total heat reccived by the element as it passes through the tube.

$$
(U \pi D d x \Delta T) \frac{D \delta c}{4 U} \log _{\epsilon}\left(\frac{T_{0}-T_{1}}{T_{0}-T_{2}}\right)=\frac{\pi D^{2}}{4} d x \delta c\left(T_{2}-T_{1}\right)
$$

Solving equation $14 \cdot 8$ for the effective temperature difference $\Delta T$,

$$
\begin{equation*}
\Delta T=\frac{T_{2}-T_{1}}{\log _{\epsilon}\left(\frac{T_{0}-T_{1}}{T_{0}-T_{2}}\right)} \tag{14•9}
\end{equation*}
$$

Since the temperature of the cooling fluid does not vary linearly along the tube, the average temperature difference between the cooling water and the condensing steam, $T_{0}-\frac{T_{1}+T_{2}}{2}$ is not very accurate. In its stead the effective temperature difference from equation $14 \cdot 9$, which is known as the log-mean temperature difference should be used in calculating the heat transfer to condenser tubes.

## Example

The difference between the arithmetic mean temperature difference and the log-mean value may be illustrated by considering an example where the cooling water enters at 80 deg F and leaves at 110 deg F with a condensing steam temperature of 125 deg F

$$
\begin{aligned}
& \text { Arithmetic mean difference }=T_{0}-\frac{T_{1}+T_{2}}{2}=125-\frac{80+110}{2}=30 \mathrm{deg} \mathrm{~F} \\
& \qquad \text { Log-mean difference }=\frac{T_{2}-T_{1}}{\log _{\epsilon} \frac{T_{0}-T_{1}}{T_{0}-T_{2}}}=\frac{110-80}{\log _{\epsilon} \frac{125-80}{125-110}}=27.4 \mathrm{deg} \mathrm{~F}
\end{aligned}
$$

Although the log-mean temperature difference is theoretically more accurate than the simpler arithmetic value, the fact that overall values of conductance (Figure $14 \cdot 2$ ) must be estimated between wide limits causes common use of the simpler but less accurate arithmetic mean. Consequently, it is important to determine the basis used in reporting any data obtained from the literature of the subject.
14.4. Condenser Performance. The thermal load on a condenser is found as the difference between the enthalpy $h$ of the steam entering it and the enthalpy of the condensate removed by the condenser pump. It is rare that the information available gives the exact quality of the steam entering the condenser for this would be difficult information to determine directly by experiment. Usually, it is necessary to subtract from the initial enthalpy of the steam the known or estimated heat converted into work by the turbine or engine.

To be consistent with the ideal Rankine cycle, a theoretical condenser should deliver the condensate at the saturation temperature corresponding to the condenser pressure. Although this assumption is usually made, it is not uncommon for a condenser to undercool the condensate to some lower temperature.

When, owing to leaks or other causes, appreciable air and other noncondensable gases are in a condenser, the total pressure is the sum of the partial pressure of the steam and that of the gases. Consequently to assume the steam temperature to be the saturation value corresponding to the total pressure is to overestimate the actual steam temperature promoting heat transfer. Conversely, readings of both steam temperature and total condenser pressure, when checked against a steam table, may be used to detect the presence of excessive amounts of air.

In proportioning a condenser for any application the intelligent choice of cooling water velocity and tube diameter requires a careful economic study and a wide knowledge of structural requirements. The estimation
of a reasonable and safe value of overall conductance $U$ requires both a knowledge of the condenser arrangement and some knowledge of probable operating conditions. Once these factors have been established, the other items of the condenser design may be calculated as illustrated in the following example.

## Example

Proportion a small condenser for a $50-\mathrm{hp}$ steam turbine that uses 100 lb per sq in . absolute pressure saturated steam and has a water rate of 20 lb of steam per $\mathrm{hp}-\mathrm{hr}$. The cooling water is available at 70 deg F and it is deemed advisable to have it leave the condenser 20 deg F cooler than the incoming steam. The condenser is to maintain an absolute pressure of 3 in . of mercury and the tubes are to be 0.5 outside diameter and 0.4 in . inside. It is estimated that the water velocity should be 4 ft per sec and it is considered safe to anticipate an overall conductance $U$ of 300 Btu per hr per sq ft per deg F based on outside tube area and the arithmetic mean temperature difference.

## Condenser Load

Since one horsepower-hour = 2545 Btu , a water rate of 20 lb of steam per horsepower-hour means that $\frac{2545}{20}=127.2 \mathrm{Btu}$ per lb of steam is converted into work by the turbine. The initial enthalpy per pound of steam as found from the steam tables is 1186.6 Btu which means that the enthalpy of the steam entering the condenser is $1186.6-127.2=1059.4$ Btu per lb . For condensate leaving at the saturation temperature corresponding to an absolute pressure of 3 in . of mercury, the leaving enthalpy is 83 Btu per lb . This means that the condenser must absorb $1059.4-83.0=976.4$ Btu per lb of steam. For $50 \times 20=1000 \mathrm{lb}$ of steam per hour, $1000 \times 976.4=976,400 \mathrm{Btu}$ per hr must be transferred to the cooling water in the condenser.

## Cooling Water Required

Since the cooling water enters at 70 deg F and is to leave 20 deg F cooler than the steam temperature of 115.1 deg F corresponding to the condenser pressure, it leaves at $115.1-20=95.1 \mathrm{deg} F$ and each pound absorbs $95.1-70$ $=25.1 \mathrm{Btu}$. This means that $\frac{976,400}{25.1}=38,900 \mathrm{lb}$ per hr or $\frac{38,900}{62.4}=625 \mathrm{cu} \mathrm{ft}$ per hr must be circulated.

## Number of Tubes in Parallel

One tube 0.4 in . in inside diameter with a cooling water velocity of 4 ft per sec will pass $\frac{\pi(0.4)^{2} \times 4 \times 3600}{4 \times 144}=12.6 \mathrm{cu} \mathrm{ft}$ per hr. To handle 625 cu ft per hr will require $\frac{615}{12.6}=49.5$ or 50 tubes to be used in parallel.

## Surface Area

The arithmetic mean temperature difference is $115.1-\frac{95.1+70}{2}=31.4$ deg F. To transfer $976,400 \mathrm{Btu}$ per hr with a $U=300$ would require a surface area of $A=\frac{q}{U \Delta t}=\frac{976,400}{300 \times 31.4}=103.5 \mathrm{sq} \mathrm{ft}$.

## Tube Length

For an outside tube diameter of $0.5 \mathrm{in} ., 103.5 \mathrm{sq} \mathrm{ft}$ of surface area would correspond to a total tube length of $\frac{103.5}{\pi .5}=790 \mathrm{ft}$. Using a two-pass con12
denser as shown in Figure $14 \cdot 1$ with two banks of 50 tubes each, the necessary tube length is $\frac{790}{2 \times 50}=7.9 \mathrm{ft}$.

## Sugaested Reading

Condensers and Auxiliaries
Gaffert, Steam Power Stations. Chapter IV.
Condensing Equipment
Craig and Anderson, Steam Power and Internal Combustion Engines. Chapter XIII.

## Steam Condensers

Severns and Degler, Steam, Air, and Gas Power. Chapter XIII.

## Heat Transfer Equipment

Stoever, Applied Heat Transmission. Chapter VI.

## CHAPTER 15

## BOILERS

15.1. Boiler Development. Early steam boilers were hand-fired and the hot gases passed through the tubes. The water surrounded the tubes and low-pressure wet steam was generated. As steam pressures increased, water tube boilers of the type illustrated in Figure $15 \cdot 1$ re-


Fig. 15•1. Horizontal drum boiler.
placed the older fire tube boilers, automatic stokers were developed and superheaters were added.

As forced draft was added and furnace temperatures increased, it was found advantageous to place an economizer between the last bank of boiler tubes and the stack. Passing the incoming boiler feed water through the economizer after it leaves the boiler feed pump and before
it reaches the boiler reduces the temperature of the products of combustion leaving the unit and thereby increases the efficiency. With the development of feed-water heating by extracting partially expanded steam from the prime mover, the demands on economizers were reduced and stack temperatures increased. This resulted in the addition of air pre-


Fig. 15.2. Pulverized-coal-fired steam generator.
heaters, Figure $15 \cdot 2$, to recover some of this waste by using it to preheat the combustion air.

The use of fuel oil and powdered coal made it necessary to protect the firebrick of the furnace walls by lining the inside of the walls with water tubes as shown in Figure 15•2. This, plus the addition of forced circulation in boilers to replace natural circulation together with high-velocity circulation of the hot gases over the boiler tubes, has increased the rate of heat transfer in some units to several hundred thousand Btu per hour per square foot of surface as contrasted with several thousand Btu per hour per square foot common in older type boilers.
15.2. Boiler Heat Transfer. The flame temperature in the furnace of a boiler depends upon the type of fuel used, the quantity of combustion air, and the amount of heat that is transferred by radiation to the water-cooled furnace walls and to the boiler and superheater tubes that receive direct radiation from the flame. In actual furnaces between 0.30 and 0.50 of the heat released during the combustion of the fuel is absorbed by radiation from the flame. The result is that the products of combustion leave the combustion space at temperatures between 1500 and 3000 deg F depending upon the furnace design and the rate of combustion.

The hot gases then pass over the boiler and convection superheater tubes where heat is transferred by convection. After that the furnace gases lose additional heat to the feed water in the economizer and to the combustion air in the air preheater. The products of combustion finally leave in the stack at a temperature between 250 and 700 deg F , and it is important that the leaving stack gas temperature be kept as low as possible if the thermal losses to the stack are to be kept low.

The rate of heat transfer by radiation from the flame to the exposed boiler tube surface may be calculated from the radiation equation,

$$
q=0.174 e_{f} e_{s}\left[\left(\frac{T_{f}}{100}\right)^{4}-\left(\frac{T_{s}}{100}\right)^{4}\right]
$$

where $q=$ Btu per hour per square foot.
$e_{f}=$ flame emissivity.
$e_{s}=$ surface emissivity.
$T_{f}=$ flame temperature.
$T_{s}=$ surface temperature.
The flame emissivity, $e_{f}$, depends upon the amount of carbon dioxide and water vapor present in the flame and upon the flame density and thickness. It varies from 0.25 to 0.95 depending upon conditions, whereas the value of $e_{8}$ depends upon the shape and condition of the combustion space but is usually quite high. The entire subject of flame radiation involves many variables and is difficult to generalize.

As the hot gases sweep over the banks of boiler and superheater tubes, heat is transferred by convection to these surfaces. The gas film conductance varies with the composition, temperature, and velocity of the products of combustion, and with the size and arrangement of the tubes.

The fundamental forced convection equation,

$$
U=\operatorname{Constant} \times \frac{k}{d}\left(\frac{v d \rho}{\mu}\right)^{n}
$$

may be written

$$
U=\frac{\text { Constant }}{d^{1-n}}(v \rho)^{n} \frac{k}{\mu^{n}}
$$

Since the heat conductivity $k$ of gases varies as the product of the specific heat and viscosity

$$
k \propto \mu c_{v}
$$

Equation $15 \cdot 3$ then becomes

$$
U=\frac{\text { Constant }}{d^{1-n}}(v \rho)^{n} c_{v}(\mu)^{1-n}
$$

The viscosity of gases varies as the absolute temperature to some exponent, and for the diatomic gases such as nitrogen,

$$
\mu \propto T^{0.76}
$$

If this is substituted into equation $15 \cdot 5$

$$
U=\frac{\text { Constant }}{d^{1-n}}(v \rho)^{n} c_{v} T^{0.76(1-n)}
$$

For banks of tubes, the product of velocity times mass density $v \rho$ is best replaced by the mass velocity $G$ which is defined as the pounds of gas per second per square foot of net opening between tubes at the most restricted point.

The results of tests made at the usual operating conditions in boilers give a valuc of $n=0.585$ and a constant of 2.28 . The final convection equation is

$$
U=2.28 \frac{c_{v} G^{0.585} T^{0.315}}{d^{0.415}}
$$

Using an average value of $c_{v}=0.18$,

$$
U=0.41 \frac{G^{0.585} T^{0.315}}{d^{0.415}}
$$

The absolute temperature $T$ is used as the average film temperature. The conductance $U$ is in Btu per hour per square foot per degree Fahrenheit and $d$ is the outside tube diameter in feet. The mass velocity $G$ varies from 0.1 to 10 lb of gas per sec per sq ft of opening, depending on the rate of fuel combustion and the boiler design. The resulting film conductance varies from 2 to 100 Btu per hr per sq ft per deg F .

The temperature drop through the steel tube walls is governed by the conductance $U_{t}=\frac{k}{x}$, where $k$ is the conductivity of the tube material $=$

25 Btu per hr per sq ft per deg per ft and $x$ is the tube wall thickness in feet. Any scale on the inside of the tube or slag on the outside will reduce this value materially.

The transfer of the heat from the inside of the boiler tubes to the boiling water is affected by many things. If the fluid has a weak tendency to wet the inside tube surface, the bubbles of stcam that are generated form as shown in Figure 15•3.

The result is that such bubbles are sluggish in disengaging themselves from the surface and grow to considerable size bcfore they rise. With a large portion of the heating surface thus blanketed with bubbles of super-


Heat transfer to boiling water.
Fig. 15•3.
Fig. 15-4.
heated steam, most of the heat must be transmitted through a steam film of low conductance.

If the liquid wets the surface easily, bubbles of steam form as shown in Figure 15.4. They cover a very small area of the surface, are formed rapidly, and disengage themselves readily. As a consequence most of the heat is transferred from the surface to the liquid instead of to the steam as when the large bubbles are formed. Anything that is favorable to the formation of the small bubble vaporization results in a higher film conductance. A rough surface is favorable to the formation of small bubbles by furnishing many nuclei for their formation. Entrained air in the water has a similar effect for the same reason. Increased circulation velocities and high rates of heat transfer also cause this type of boiling and increase the film conductance. For smooth surfaces and no forced circulation, the film conductance of boiling water varies directly with the rate of heat transfer and may be expressed approximately by

$$
U=200+0.043 \frac{q}{A}
$$

$U=$ Btu per hour per square foot per degree Fahrenheit.
$\frac{q}{A}=$ heat transfer in Btu per hour per square foot.
The film conductance for the transfer of heat from the inside of the superheater tubes to the superheated steam flowing inside varies in the
same manner as shown in equation $15 \cdot 5$. The best experimental results for the turbulent flow of vapors in a pipe give ( $n=0.8$ ) and since $\mu$ of steam varies as $T^{1.1}$, for steam in superheater tubes,

$$
U=\text { Constant } \frac{c_{v} G^{0.8} T^{0.22}}{d^{0.2}}
$$

Since for the range of superheat temperatures, $T^{0.22}$ varies but little, $d^{0.2}$ does not change much for the entire range of tube sizes, and $c_{v}$ does not vary widely in the superheat region, it is sufficiently accurate to calculate the steam film conductance from

$$
U=8.0 x^{0.8}
$$

where $G$ varies from about 1 to 60 lb per see per sq ft of tube cross section.

## Example of Boiler Heat Transfer

## Maximum Flame Temperature

Coal having a heating value of 14,000 Btu per lb is burned with sufficient excess air to form 15 lb of products of combustion per pound of coal in a furnace where 0.4 of the heat of combustion is lost by radiation during burning. This means that $0.6 \times 14,000=8400 \mathrm{Btu}$ are available to heat the 15 lb of gases. Using an average $c_{p}=0.29$ for the products, the temperature rise $=\frac{8400}{15 \times 0.29}$ $=1930 \mathrm{deg} \mathrm{F}$, giving a flame temperature of $70+1930=2000 \mathrm{deg} \mathrm{F}$.

## Heat Transfer by Radiation

If the radiant heat from the 2000 deg F flame is transferred to a water wall in the furnace, the surface of the metal will reach such a temperature that the heat received by radiation will be transferred by conduction through the metal and by convection to the boiling water in the wall. For a water temperature of 400 deg F , it will be estimated that the outside metal temperature is 520 $\operatorname{deg} \mathrm{F}$. For such a temperature and 2000 deg F flame temperature, the maximum rate of heat transfer by radiation from equation $15 \cdot 1$, using $e_{f}=e_{s}=1$ is

$$
q=0.174\left[\left(\frac{2460}{100}\right)^{4}-\left(\frac{980}{100}\right)^{4}\right]=62,200 \text { Btu per hr per sq } \mathrm{ft}
$$

For such a rate of transfer, the conductance to the water from equation $15 \cdot 10$ is

$$
U_{w}=200+0.043 \times 62,200=28,700 \text { Btu per hr per sq ft per deg } \mathrm{F}
$$

The conductance of the $1 / 2$ in. thick metal for a conductivity $k=25$ is $\frac{25}{1 / 24}=600$ $=U_{t}$ Btu per hr per sq ft per $\operatorname{deg} \mathrm{F}$

The overall conductance,

$$
U=\frac{1}{\frac{1}{U_{t}}+\frac{1}{U_{w}}}=\frac{1}{\frac{1}{600}+\frac{1}{28,700}}=522 \text { Btu per sq ft per hr per deg } \mathrm{F}
$$

It is here interesting to note that the conductance of the metal wall is the governing factor.
The temperature drop from water to outside metal surface $=\frac{62,200}{522}=120$ deg $F$, which checks the assumption made and establishes the rate of heat transfer to the radiation surface at $62,200 \mathrm{Btu}$ per hr per sq ft .

## Heat Transfer by Convection to Superheater

For a mass velocity over the tubes of $G=4 \mathrm{lb}$ per sq ft per sec, a tube diameter of 1.5 in . and an estimated mean gas film temperature of 1500 deg F or 1960 deg absolute, the gas film conductance from equation 15.9 becomes

$$
U_{\mathrm{g}}=\frac{0.41(4)^{0585}(1960)^{0315}}{\left(\frac{1.5}{12}\right)^{0415}}=27.3 \text { Btu per hr per sq ft per deg } \mathrm{F}
$$

For a tube wall thickness of $1 / 4$ in., the tube conductance,

$$
U_{t}=\frac{25}{1 / 48}=1200 \text { Btu per hr per sq ft per deg F }
$$

Assuming a mass velocity of steam $G$ in the tubes of 20 lb per sq ft per sec, the steam film conductance from equation $15 \cdot 12$ is

$$
U_{s}=8.0(20)^{0}=88 \text { Btu per hr per sq ft per deg } \mathrm{F}
$$

The overall conductance,

$$
U=\frac{1}{\frac{1}{U_{\varepsilon}}+\frac{1}{U_{t}}+\frac{1}{U_{s}}}=\frac{1}{\frac{1}{27.3}+\frac{1}{1200}+\frac{1}{88}}=20.5 \text { Btu per sq ft per hr per deg F }
$$

For a gas temperature of 2000 deg F and a superheated steam temperature of $600 \mathrm{deg} F$, the rate of heat transfer,

$$
q=\left(t_{g}-t_{s}\right) U=20.5 \times 1400=28,700 \text { Btu per hr per sq ft }
$$

## Heat Transfer by Convection to Boiler Tubes

Assuming the same gas film conductance, $U_{g}=27.3$ as in the superheater example and the same tube conductance $U_{t}=1200$, and calculating the boiling water film conductance from equation $15 \cdot 10$ for an assumed $q=42,000 \mathrm{Btu}$ per hr per sq ft .

$$
U_{w}=200+0.043 \times 42,000=2000 \text { Btu per hr per sq ft per deg } F
$$

The overall conductance,
$U=\frac{1}{\frac{1}{U_{g}}+\frac{1}{U_{t}}+\frac{1}{U_{w}}}=\frac{1}{\frac{1}{27.3}+\frac{1}{1200}+\frac{1}{2000}}=26.3$ Btu per sq ft per hr per deg F
For a gas temperature of 2000 deg F and a steam temperature of 400 deg F, the rate of heat transfer to boiler tubes by convection is

$$
q=26.3 \times(2000-400)=42,000 \text { Btu per hr per sq ft of tube area }
$$

## Summary of Rates of Heat Transfer

Radiation to water wall $62,000 \mathrm{Btu}$ per hr per sq ft
Convection to superheater 28,700 " " " " " "
Convection to boiler tubes 42,000 " " " " " "
It is apparent from the number of assumptions involved in the previous example that there is much about heat transfer calculations for steam generators that requires actual design experience. However, the basic principles are quite well understood and the results sought may be very simply stated. It is desirable to build a combustion chamber that will. promote complete burning of the fuel, to transfor as much of the heat as is economically feasible from the hot products of combustion to the steam, and to have a proper relationship between the heat transferred to the water wall, boiler tubes, superheater, economizer, and air preheater. It is also very desirable to attain such a balance that the steam pressure and temperature will not vary too much under fluctuating load conditions.

This last point may be illustrated by considering the characteristics of radiant and convection superheaters. In a convection superheater the outside gas film conductance, which is the chief limitation to heat transfer, does not increase in proportion to the increase in the rate of stcam flow as the load on the boiler becomes greater. As a result, the final steam temperature produced by a convection superheater tends to decrease as the load on the boiler is increased. A radiant superheater has a reverse tendency because the increased flame temperature that accompanies a higher combustion rate tends to increase the rate of heat transfer more rapidly than the steam flow inside the superheater tubes is increased. Consequently, it is possible, by using the proper balance between convection and radiant superheater surface, to produce a combination that will give an essentially constant superheat temperature for varying loads.
15.3. Boiler Losses. If the portion of the heat released by the combustion of fuel in a steam generator that is usefully transferred to the
steam is to be maintained at a maximum value, it is necessary to understand and analyze the various thermal losses that occur.

The first possibility of loss would be the failure to burn a part of a solid fuel and this is usually known as the ashpit loss. In a coal-fired boiler this may take the form of unburned or incompletely burned coal that drops to the ashpit.

The next possibility is that of incomplete combustion of the fuel and this is usually reflected by the presence of carbon monoxide in the stack gases.
The most obvious loss is the sensible heat that passes to the stack in the hot products of combustion. Another, but less obvious stack loss is represented by the latent heat of vaporization of the water vapor present in the stack gases. For convenience it is customary to divide the stack losses into the sensible heat in the dry stack gases and the heat loss due to water vapor.

The losses duc to conduction, radiation, and convection from the boiler setting cannot be evaluated directly but usually must be determined by a process of elimination. Once the heat supplied by the fuel, the heat absorbed by the steam, and the losses previously discussed have been evaluated, the difference is referred to as radiation and miscellaneous losses.

Since the principles of combustion have been considered previously, the evaluation of the various losses can probably be best illustrated by following through the calculations for a small hand-fired boiler in which the losses are exaggerated.

## Example of Boller Losses

## Test Data

Weight analysis of coal as fired: moisture $=0.016$; ash $=0.062 ; \mathrm{H}_{2}=0.057$; $\mathrm{O}_{2}=0.042 ; \mathrm{C}=0.823$.
Higher heating value of fuel $=14,300 \mathrm{Btu}$ per lb .
Ashpit refuse $=0.10 \mathrm{lb}$ per lb of coal fired.
Dry volumetric analysis of stack gases: $\mathrm{CO}_{2}=0.10 ; \mathrm{O}_{2}=0.08 ; \mathrm{CO}=0.01$; $\mathrm{N}_{2}=0.81$.
Stack gas temperature $=500 \mathrm{deg}$ F. Boiler-room air $=80 \mathrm{deg} \mathrm{F}$ and 50 per cent humidity.

## Combustible Loss to the Ashpit

If the combustible material that reaches the ashpit comes from incompletely burned coal, it is safe to say that the unburned portion is all carbon. For such a condition the difference between the refuse per pound of coal and the ash per pound of coal, or $0.10-0.062=0.038 \mathrm{lb}$ gives the pound of unburned carbon per pound of coal. This means a loss of $0.038 \times 14,150=537$ Btu per
lb which is an efficiency loss of $\frac{537}{14,300}=0.0376$ for a heating value of 14,300 Btu per lb of coal.

If the ashpit refuse contains an appreciable amount of unignited coal that drops through the grate, the unburned part of the fuel contains hydrocarbons, and the foregoing assumption cannot be made. In such a case, the best method is to determine the actual heating value of the refuse in a bomb calorimeter.

In the absence of such a calorimeter test, the maximum possible value of the ashpit loss may be calculated by assuming that all the combustible in the pit comes from coal that drops through the grate without being ignited at all. If a fractional part $x$ of each pound of coal is burned completely, $0.062 x$ pounds of pure ash results. The remainder of the 0.10 lb of refuse would be the $(1-x)$ pound of unburned coal, or

$$
0.1=0.062 x+(1-x) \quad \text { or } \quad x=0.96 \mathrm{lb}
$$

This means that 0.04 lb of unburned coal dropped through the grate for each pound of coal used.

For a heating value of 14,300 , this represents a loss of $0.04 \times 14,300=572$ IStu per lb of coal. This is a loss of 0.04 as compared with a minimum possible value of 0.038 previously found, so the difference is very slight.

## Combustible Loss in the Stack Gases

The only combustible stack loss that is detected by the usual Orsat analysis of the products of combustion is that due to the CO content. When volatile coal or fuel oil is being burned, a very substantial loss is often caused by unburned hydrocarbons in the flue gases. The presence of such hydrocarbons can be detected only by special methods of analysis and they are often undetected.

The only combustible loss in the stack gases in this example is due to the CO in the dry volumetric stack gas analysis. Since for each mol of CO there are 10 mols of $\mathrm{CO}_{2}$ and since there is the same weight of carbon in a mol of $\mathrm{CO}_{2}$ as in a mol of CO , it follows that one-eleventh of the carbon burned forms CO. Since each pound of coal contains 0.823 lb of carbon and 0.038 of C go unburned to the ashpit (first assumption), $\frac{1}{\mathrm{I}_{1}}(0.823-0.038)=0.0713 \mathrm{lb}$ of carbon are burned to CO instead of to $\mathrm{CO}_{2}$. This represents a heating value loss of $0.0713(14,150-3960)=726$ Btu per lb of coal, or an efficiency loss of $\frac{726}{14,300}=0.051$.

## Sensible Heat Loss in the Dry Stack Gases

In each mol of dry exhaust gases there are $0.10 \times 44=4.4 \mathrm{lb}$ of $\mathrm{CO}_{2}$; $0.01 \times 28=0.28 \mathrm{lb}$ of $\mathrm{CO} ; 0.08 \times 32=2.56 \mathrm{lb}$ of $\mathrm{O}_{2} ;$ and $0.81 \times 28=22.7$ lb of $\mathrm{N}_{2}$. There are $12 \times 0.10=1.2 \mathrm{lb}$ of C in the $\mathrm{CO}_{2}$ and $12 \times 0.01=0.12$ lb of C in the CO . In each pound of coal there are 0.823 lb of C , of which $(0.823-0.038)=0.785 \mathrm{lb}$ are burned to $\mathrm{CO}_{2}$ and CO . Since $1.2+0.12=$ 1.32 lb of carbon must be burned to form a mol of dry exhaust gases, the corre-
sponding pound of coal produces $\frac{1.32}{0.785}=1.683$ per mol of dry gases. For each pound of coal used, $\frac{4.4}{1.683}=2.61 \mathrm{lb}$ of $\mathrm{CO}_{2}$ are formed, $\frac{0.28}{1.683}=0.167 \mathrm{lb}$ of CO; $\frac{2.56}{1.683}=1.522 \mathrm{lb}$ of free $\mathrm{O}_{2}$; and $\frac{22.7}{1.68 \overline{3}}=13.48 \mathrm{lb}$ of $\mathrm{N}_{2}$.

The heat lost in the dry exhaust gases is the amount that would be recovered if the gases were cooled at constant pressure from the stack temperature of 500 deg F to the boiler-room temperature of 80 deg F . Using average values of $c_{p}$ to evaluate these losses,

$$
\begin{aligned}
& \text { Loss in } \mathrm{CO}_{2}=2.61 \times 0.22(500-80)=241.0 \\
& \text { Loss in CO }=0.167 \times 0.25(500-80)=17.5 \\
& \text { Loss in } \mathrm{O}_{2}=1.522 \times 0.219(500-80)=140.0 \\
& \text { Loss in } \mathrm{N}_{2}=13.48 \times 0.25(500-80)=1415.0 \\
& \text { Total loss per pound of coal fired }=\overline{1813.5} \mathrm{Btu}
\end{aligned}
$$

This represents an efficiency loss of $\frac{1813.5}{14,300}=0.127$ caused by the sensible heat in the drystack gases.

## Water Vapor Losses in Stack Gases

The water vapor in the stack comes from the vapor in the combustion air, from the moisture content of coal, from the water of crystallization of the coal and from the combustion of the hydrogen of the hydrocarbons of the coal. The moisture in the combustion air enters as vapor and needs only to be superheated, but the moisture from the water of crystallization, from the mechanically contained moisture, and from the combustion of the hydrogen must not only be superheated but also be furnished with the latent heat of vaporization.

Since 13.48 lb of $\mathrm{N}_{2}$ is involved in the combustion of each pound of coal, $\frac{13.48}{0.77}=17.5 \mathrm{lb}$ of dry air is used for each pound of coal fired. Since one pound of dry air at 80 deg F and 50 per cent humidity is accompanied by 0.0112 lb of moisture, the 17.5 lb of dry combustion air is accompanied by 0.195 lb of water vapor. The moisture in each pound of coal is 0.016 lb and the moisture corresponding to the hydrogen content is $9 \times 0.057=0.513 \mathrm{lb}$. This means that the water vapor loss per pound of coal is the sum of the latent heat of vaporization of 0.529 lb of moisture and the heat required to superheat $0.529+0.195=0.724 \mathrm{lb}$ of moisture from 80 deg F to 500 deg F . Thus, the approximate water vapor loss per pound of coal may be calculated by using $c_{p}=0.44$ and $h_{f g}=1040 \mathrm{Btu}$ per lb.

Moisture loss $=0.724 \times 0.44 \times 420+1040 \times 0.529$.
Moisture loss $=683 \mathrm{Btu}$ per lb .
This represents an efficiency loss of $\frac{683}{14,300}=0.048$.

| Combustible to ashpit | 0.038 |
| :--- | :--- |
| Incomplete combustion | 0.051 |
| Sensible heat to stack gases | 0.127 |
| Moisture loss to stack gas | $\underline{0.048}$ |
| $\quad$ Total losses | 0.264 |

This represents a boiler heat transfer efficiency of 0.74 which would probably be reduced to 0.70 by radiation. These results are representative of a small hand-fired boiler and it should be remembered that in large boilers the efficiency may be as high as 0.88 .
15.4. Boiler Performance. The rating or output of a boiler is dependent upon the capacity of the furnace for burning fuel and releasing heat and upon the amount and effectiveness of the active heat transfor surface. The size and shape of the combustion space and the combustion rate vary widely. The highest rates of combustion are obtained with oil and powdered-coal firing and the lowest with coal burned on grates. The combustion rate for stoker furnaces varies from 6 to 40 lb of coal per hr per sq ft of grate area, depending upon the type of coal used and the draft available.

If perfect mixing could be realized in a furnace, only the air theoretically required for complete combustion would need to be supplied. In actual boilers with imperfect mixing of the air, the stack losses are a minimum when between 0.30 and 0.60 of excess air is supplied. When the excess air is too low, the stack losses caused by incomplete combustion increase, and if too much air is supplied, the stack losses due to the heat carried away in the products of combustion is increased.

The usual method of stating the capacity of steam-generating units is to state the rate of heat absorption in Btu per hour or in pounds of steam evaporated per hour. The first method is the more concise. The meaning of the second method of stating the pounds of steam evaporated per hour is dependent upon the steam pressure and temperature and the feed-water temperature and is consequently not an exact method of giving the rating of a unit.

There are several older methods of rating boilers that are now obsolete. Years ago a boiler horsepower was represented by 10 sq ft of heatabsorbing area. Later, a boiler horscpower was defined as the evaporation of 30 lb of steam at 70 lb per sq in. with 100 deg F feed water. Later, this was changed to its equivalent of evaporating 34.5 lb of steam per hr from and at 212 deg F . This unit was used because it represented the pounds of steam necessary to develop a horsepower in the engines used then. The normal rating was assumed to be developed when 34.5 lb of
steam was evaporated from and at 212 deg F for each 10 sq ft of heating surface. With modern improvements in furnaces and boilers, ratings of several times "normal" are obtained on this basis, and this has caused it to be replaced by a more rational method of rating.

The efficiency of a steam-generating unit is the ratio of the heat absorbed by the water and steam passing through it, divided by the heat supplied in the fuel burned in the same time. If the total steam output of the boiler is used, the result is the gross efficiency. In plants that have many boiler accessories such as draft fans, coal-handling equipment, boiler-feed pumps, and ash-handling equipment, it is often desirable to find the net efficiency. When this is desired it is necessary to subtract the net amount of heat delivered to the auxiliaries from the output of the boiler.

Steam-Generating Units
Gaffert, Steam Powcr Stations. Chapter VIII.
Barnard, Ellenwood, and Hirshfeld, Heat Power Engineering. Part II, Chapter XXX.
Steam Power Boilers
Craig and Anderson, Stcam Power and Internal Combustion Engines. Chapter IV. Heating Boilers

Severns, Heating, Ventılating, and Air Conditioning Fundamenalals. Chapter VII.

## CHAPTER 16

## POWER PLANT CYCLES

16.1. Steam Plant Cycles. The simplest possible steam power plant consists only of a boiler and engine with a simple feed pump for forcing water into the boiler. The amount of additional equipment that is added in any case is usually determined by economic considerations. The gain in efficiency and the accompanying reduction in the fuel cost of pro-


Fig. 16.1. Steam plant flow diagram.
ducing power must be great enough to justify the equipment investment. This results in a great variety of steam plant layouts but at the present time the majority of installations involve at least the minimum elements shown schematically in Figure 16•1. Large central stations often have several stages of feed-water heating for which steam is extracted from the turbine at as many points.

The theoretical analysis of the various cycles ignores such important items as pressure drop in piping and superheater tubes and heat losses from piping and equipment. Such analyses are very useful, however, as
standards of comparison and as means of understanding the principles involved.
16.2. The Rankine Power Plant Cycle. The Rankine plant cycle, shown on the $h-s$ diagram of Figure 16.2 differs from the Rankine engine cycle of article 11.5 only in that it considers the boiler-fecd work. As in the engine cycle the theoretical output of the prime mover is


Fig. 16.2. Rankine power plant cycle.
$h_{1}-h_{2}$ and the heat discharged to the condenser is $h_{2}-h_{3}=h_{2}-h_{f 2}$. The total boiler-feed work is,

$$
\text { Boiler-feed work }=\frac{P_{1}-P_{2}}{J} V_{f 2} \text { Btu per } \mathrm{lb}
$$

The use of the volume of liquid pumped as $V_{f 2}$ is very slightly in error, as strictly, the specific volume actually changes slightly with increased pressure.

The Rankine power plant cycle input is ( $h_{1}-h_{f 2}$ ) - boiler-feed work, since the enthalpy increase in the feed pump does not have to be supplied in the thermal input to the boiler. The net output of the plant is ( $h_{1}-h_{2}$ ) - boiler-feed work, since the prime mover must furnish the mechanical power to drive the feed pumps. Thus,

Rankine plant cycle efficiency $=\frac{\left(h_{1}-h_{2}\right)-\frac{\left(P_{1}-P_{2}\right)}{J} V_{f 2}}{\left(h_{1}-h_{f 2}\right)-\frac{\left(P_{1}-P_{2}\right)}{J} V_{f 2}}$
Since the liquid volume $V_{f 2}$ is approximately 0.018 cu ft per lb , the boilerfeed work will not amount to 1 Btu per lb until

$$
\frac{P_{1}-P_{2}}{J} V_{f 2}=1 \quad \text { or } \quad P_{1}-P_{2}=\frac{1 \times 778}{144 \times 0.018}=300 \mathrm{lb} \text { per sq in. }
$$

Since this term is so small it is very commonly ignored in the calculation of the Rankine plant cycle efficiency, except for very high pressures.

Increasing either the initial superheat temperature or boiler pressure increases the efficiency of a Rankine cycle. The maximum steam temperature is limited by the safe working temperature for available metals and this limit is now about 1000 deg F but it has been rising steadily with the improvement in the physical properties of metals at high temperatures.

As the steam pressures are raised, either the tube and drum-wall thickness must be increased or stronger metals developed, which means that increases in working pressures are dependent upon metallurgical developments, but not quite to the same extent as are steam temperatures. Pressures of 1200 and 1400 lb per sq in. have been safely and economically used for several years.

An idea of the increase in ideal efficiency that may be realized from an increase in temperature may be had from the fact that increasing the steam temperature of a 350 lb per sq in . plant from the saturation temperature of 430 deg F to a temperature of 900 deg F , raises the ideal cycle from 0.31 to 0.35 . Since an increase in initial temperature will reduce the moisture content of the steam in the last stages of the turbine, the actual efficiency increase is usually greater than the ideal value.

If, on the other hand, the pressure of 900 deg F steam is raised from 350 to 1400 lb per sq in., the ideal efficiency of the plant is raised from 0.35 to 0.41 .
16.3. Regenerative or Extraction Cycles. The greatest single energy loss in the Rankine steam plant cycle is the latent heat of vaporization of the exhaust steam that is transferred to the cooling water in the condenser. The regenerative or steam extraction cycle refers to the idea of extracting portions of the steam at various pressures as it expands through the turbine and mixing it with the feed water from the condenser to heat the feed water before it enters the boiler. A single extraction cycle is shown in Figure 16.3.


Fig. 16.3. Extraction cycle.

In this ideal regenerative cycle with one extraction stage, a pound of steam enters the turbine at condition 1. After it has expanded to some pressure $P_{2}$, a fractional part $y$ is extracted for heating feed water and the remaining $1-y$ pounds continues to expand in the turbine to the condenser pressure $P_{3}$. The $(1-y)$ pounds of steam that passes completely through the turbine and is condensed at the saturation temperature corresponding to $P_{3}$ is pumped by a feed pump from the pressure $P_{3}$ to $P_{2}$ where it is mixed with the $y$ pounds of steam bled from the
turbine at pressure $P_{2}$. If just enough stcam is bled to heat the $1-y$ pounds of condensate from $3^{\prime}$ to $2^{\prime}$,

$$
y\left(h_{2}-h_{f 2}\right)=(1-y)\left(h_{f 2}-h_{f 3}\right)
$$

The ideal cycle output per pound of steam, neglecting boiler-feed pump work is

$$
\begin{gather*}
\text { Cycle output }=h_{1}-h_{2}+(1-y)\left(h_{2}-h_{3}\right)  \tag{16•4}\\
\text { Cycle input }=h_{1}-h_{f 2} \\
\text { Extraction cycle efficiency }=\frac{h_{1}-h_{2}+(1-y)\left(h_{2}-h_{3}\right)}{h_{1}-h_{f 2}}
\end{gather*}
$$

The number of points at which it proves economical to bleed steam from the turbine to heat feed water varies widely with conditions. Most standard turbines in the larger sizes are equipped for steam extraction for feed-water heating at three points, and some plants have been constructed with four stages of heating. As indicated by Figure 16•1, such a feed-water heater must be preceded by a feed pump which raises the pressure above the saturation value corresponding to the temperature of the feed water leaving the heater, for otherwise vaporization would occur.
16.4. Reheat Cycles. Excessive moisture in the last stages of a turbine causes serious efficiency losses and blade erosion. To reduce this


Fig. 16.4. Reheat cycle. trouble, the stcam is sometimes extracted from the turbine after it has expanded part way through and resuperheated before continuing the expansion in the turbine. Such a reheat cycle has approximately the same ideal efficiency as the straight Rankine cycle, so any increase in turbine efficiency that results from the reduction in moisture is a net gain.
This ideal cycle is shown in Figure $16 \cdot 4$ where the steam is expanded from 1 to 2 in the turbine, then is extracted and reheated to the initial temperature at the extraction pressure $P_{2}$. The actual temperature to which the stcam is ever reheated is dependent upon conditions, but it should be high enough to avoid appreciable moisture content in the last stage of the turbine.

After reheating to $2^{\prime}$, the steam is returned to the turbine and the expansion continued to the condenser pressure $P_{3}$. The output, neglecting boiler-feed work is

$$
\begin{gather*}
\text { Cycle output }=\left(h_{1}-h_{2}\right)+\left(h_{2^{\prime}}-h_{3}\right)  \tag{16.7}\\
\text { Cycle input }=\left(h_{1}-h_{f 3}\right)+\left(h_{2^{\prime}}-h_{2}\right) \\
\text { Reheat cycle efficiency }=\frac{\left(h_{1}-h_{2}\right)+\left(h_{2^{\prime}}-h_{3}\right)}{\left(h_{1}-h_{f 3}\right)+\left(h_{2^{\prime}}-h_{2}\right)}
\end{gather*}
$$

The reheat cycle is not used as widely as the extraction cycle because it usually involves a large expenditure for equipment. Also, the volume of the steam is very great at the reheat pressure and it is difficult to return it to the reheat coils in the steam generator without incurring excessive pressure losses.

## Example of Rankine Plant Cycle

The effects of the modifications of the Rankine plant cycle may be illustrated by calculations for an initial steam condition of 350 lb per sq in . and 600 deg F with a condenser pressure of 2 in . of mercury.

$$
h_{1}=1309.4 \text { Btu per } \mathrm{lb} \quad s_{1}=1.6059
$$

For an ideal reversible expansion,

$$
\begin{aligned}
& s_{1}=s_{2}=s_{f 2}+x_{2 s_{f 2}}=1.6059=0.1316+x_{2} 1.8468 \\
& x_{2}=0.80 \quad h_{2}=h_{f 2}+x_{2} h_{f 82}=69.10+0.8 \times 1035.7 \\
& h_{2}=898.1 \text { Btu per lb } h_{f 2}=69.1 \text { Btu per lb }
\end{aligned}
$$

Rankine cycle efficiency $=\frac{h_{1}-h_{2}}{h_{1}-h_{f 2}}=\frac{1309.4-898.1}{1309.4-69.1}=0.332$

## Example of Extraction Cycle

For the same initial and final conditions, an extraction cycle uses an extraction point at $P_{2}=100 \mathrm{lb}$ per sq in. (Figure $16 \cdot 3$ ). For the expansion 1 to 2 , $s_{1}=s_{2}=1.6059$.

From the superheated tables, $t_{2}=333 \mathrm{deg} \mathrm{F}$ and $h_{2}=1190.7$ and $h_{f 2}=$ 298.33 Btu per lb . To find the fraction $y$ of the steam that must be extracted for feed-water heating,

$$
\begin{gathered}
y\left(h_{2}-h_{f 2}\right)=(1-y)\left(h_{f 2}-h_{f 3}\right) \\
y(1190.7-298.33)=(1-y)(298.33-69.1) \quad y=0.204 \\
\text { Efficiency }=\frac{\left(h_{1}-h_{2}\right)+(1-y)\left(h_{2}-h_{3}\right)}{h_{1}-h_{f 2}} \\
\text { Efficiency }=\frac{(1309.4-1190.7)+0.796(1190.7-898.1)}{1309.4-298.33}=0.346
\end{gathered}
$$

## Example of Reheat Cycle

For the same conditions and reheating at 100 lb per sq in . absolute to the initial temperature of 600 deg F (Figure 164 ), a reheat cycle has

$$
\begin{gathered}
h_{1}=1309.4 \quad h_{2}=1190.7 \quad h_{2^{\prime}}=1327.9 \\
s_{2^{\prime}}=1.7569=s_{3}=0.1316+x_{3} 1.8468 \quad x_{3}=0.88 \\
h_{3}=60.1+0.88 \times 1035.7=981.1 \\
\text { Efficiency }=\frac{\left(h_{1}-h_{2}\right)+\left(h_{2^{\prime}}-h_{3}\right)}{\left(h_{1}-h_{f_{3}}\right)+\left(h_{2^{\prime}}-h_{2}\right)} \\
\text { Efficiency }=\frac{(1309.4-1190.7)+(1327.9-981.1)}{(1309.4-69.1)+(1327.9-1190.7)}=0.337
\end{gathered}
$$

This ideal reheat cycle has essentially the same efficiency as the regular Rankine cycle and accomplishes a reduction in final moisture content from 020 to 0.12 , which would result in an increased turbine efficiency in the last stages.
16.5. Binary Vapor Cycles. The binary cycle is made up of a regular Rankine cycle using a medium which condenses at high enough tem-


Fig. 16.5. Mercury-vapor-steam plant.
perature that its condenser may act as a steam boiler to operate a regular steam cycle "under it." It may thus be considered as a steam producer which, for a given amount of fuel, produces almost as much steam as in a
regular steam cycle, and has as a by-product the additional power produced by the second vapor.

The requirements for the characteristics of the second vapor are so exacting that to date mercury-vapor-steam plants similar to the one illustrated by Figure 16.5 are the only binary vapor plants in commercial use.

A typical mercury-vapor plant would be one in which the mercury vapor leaves for the turbine at 140 lb per sq in . absolute and 958 deg F and leaves the turbine at 1.6 lb per sq in . absolute and 489.6 deg F . Saturated steam leaves the mercury condenser-steam boiler unit at 425 lb per sq in. and 450 deg F to go to the steam superheater in the furnace. Steam at 425 lb per sq in. and 760 deg F goes to the steam turbine and leaves the turbine for the steam condenser at a pressure of one inch of mercury absolute.

## Example of Mercury-Vapor-Steam Cycle

From the $h$-s diagram for the mercury portion of the previously described cycle, shown in Figure 16.6, the mercury vapor leaves the ideal mercury turbine at 2 with an enthalpy $h_{2}=107.60$. It leaves the mercury con-


Binary vapor cycle.
Fig. 16.6.


Fig. $16 \cdot 7$.
denser-steam boiler with the enthalpy of liquid $h_{f 2}=17.07$ Btu per lb . The feed water enters the condenser-boiler with the heat of liquid shown by the steam cycle, Figure 16.7, as $h_{f 5}=47.05 \mathrm{Btu}$ per lb and it leaves for the steam superheater in the furnace with the enthalpy of saturated steam,

$$
h_{4}=1204.6 \text { Btu per lb }
$$

The result is that for each pound of mercury passing around the cycle there circulates

$$
\frac{h_{2}-h_{f 2}}{h_{4}-h_{f 5}}=\frac{107.6-17.07}{1204.6-47.05}=0.0775 \mathrm{lb} \text { of steam }
$$

The combined output of the two ideal turbines per pound of mercury is
Output $=1\left(h_{1}-h_{2}\right)+0.0775\left(h_{3}-h_{5}\right)$.
Output $=(150.47-107.6)+0.0775(1393.9-902.0)$.
Output $=42.87+38.1=80.97 \mathrm{Btu}$.
The input to the mercury is $\left(h_{1}-h_{f 2}\right)=150.47-17.07=133.4$ Btu and the input to superheat the steam is $0.0775\left(h_{3}-h_{4}\right)=0.0775(1393.9-1204.6)=$ 14.7 Btu. For the total input per pound of mercury of $133.4+14.7=148.1$ Btu, the ideal binary cycle efficiency is

$$
\frac{80.97}{148.1}=0.545
$$

as compared with a value of 0.346 found in the extraction steam cycle example.

## Sugaested Reading

Steam Power Plants
Barnard, Ellenwood, and Hirshfeld, Heat Power Engincering. Part II, Chapter XLIX.
Analysis of Cycles
Emswiler, Thermodynamics. Chapter IX.
High-Pressure and Binary Vapor Cycle Boilers
Gaffert, Steam Power Statoons. Chapter IX.
Choice of Cycles
Gaffert, Steam Power Stations. Chapter XXI.

## CHAPTER 17

## REFRIGERATION

17.1. Mechanical Refrigeration. The purpose of a mechanical refrigeration system is to absorb heat at a low temperature and discharge it at a higher temperature. Since the unaided exchange of heat is always in a reverse direction, the equipment illustrated by Figure $17 \cdot 1$, or its equivalent, is required to transport heat from a lower to a higher temperature.

In such a system high-pressure liquid expands through the throttling valve into the evaporator where it absorbs its latent heat of vaporization at a low temperature corresponding to the pressure maintained there. This low-pressure vapor then passes to the compressor where its pressure is raised to a value that has a saturation temperature corresponding to that of the condenser cooling medium to which the heat must be delivered. After the latent heat of vaporiza-


Fig. 17-1. Mechanical refrigeration cycle. tion is transferred to the condenser cooling medium, the high-pressure liquid passes to the accumulator which equalizes the flow in the system and from which it passes again to the expansion valve.

In this way heat is absorbed to vaporize a liquid at a low temperature, compression work is done on the vapor in a compressor, and an amount of heat equal to that absorbed plus the compression work done is discharged to the condenser at a higher temperature in the condenser.

The rate at which heat is removed from the region of low temperature is the measure of capacity of a refrigerator. This may be expressed in Btu per minute or second, but the ratings of commercial machines are usually expressed in tons of refrigeration. A ton of refrigeration represents the removal in twenty-four hours of the heat equivalent of melting 2000 lb of ice. Since the latent heat of fusion of ice is 143.3 Btu per lb, a ton of refrigeration in twenty-four hours would mean the removal of heat at the rate of approximately 200 Btu per minute.

17-2. Refrigerants. The properties sought in a refrigerant are low working pressures, frecdom from corrosive action and deterioration of lubricants, stability, nontoxic and noninflammable vapors, and low cost. The latent heat of the vapor should be large and the volume of the vapor should be small to keep down the compressor displacement.

Carbon dioxide, $\mathrm{CO}_{2}$, is harmless but must be operated between 400 and 1400 lb per sq in. pressure. Sulphur dioxide operates between 10 and 70 lb per sq in . and its chief disadvantage is its corrosive action in the presence of any water in the system. Ammonia, $\mathrm{NH}_{3}$, may be operated between 30 and 180 lb per sq in . and has long been a widely used refrigerant. Methyl chloride, $\mathrm{CH}_{3} \mathrm{Cl}, \mathrm{CCl}_{2} \mathrm{~F}_{2}$, commonly known as "freon" or " $\mathrm{F}-12$," ethyl chloride, $\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{Cl}$, and many others are used commercially.
Water vapor can be used satisfactorily when the lower temperature need not go below 40 deg F , but it has the disadvantage of operating at extremely low pressures, which means that large volumes of vapor must be handled. Such a refrigerator operating between 40 and 100 deg F would have a low pressure of 0.1217 lb per sq in. and an upper pressure of 0.9487 lb per $\mathrm{sq} \mathrm{in}$. water vapor at the lower pressure is 2445 cu ft per lb, centrifugal compressors are commonly used. To maintain such pressures very tight condensers and equipment are necessary to prevent air leaks.
17.3. Vapor Compression Cycles. In the vapor refrigeration cycle shown by Figure $17 \cdot 2$ the lower pressure $P_{1}$ is the saturation value cor-


Fig. 17-2. Vapor refrigeration cycle. responding to the temperature of the evaporator, and the upper pressure $P_{2}$ is determined by the temperature at which the condenser operates. The condition of the refrigerant as it leaves the evaporator and enters the compressor may vary from dry vapor $1^{\prime}$ to vapor of such low quality 1 that it is dry, saturated vapor at the end of the constant entropy compression, 1-2. The path $1^{\prime}-2^{\prime}$ is referred to as a dry compression cycle and 1-2 as a wet compression cycle.

The heat absorbed in the evaporator is shown as $h_{1}-h_{4}$ or $h_{1^{\prime}}-h_{4}$ and since $3-4$ is a pure throttling process, $h_{4}=h_{3}=h_{f 2}$. The ideal compression work is $h_{2}-h_{1}$ done at constant entropy. The heat discharged to the condenser is $h_{2}-h_{3}$ or $h_{2}-h_{f 2}$, and this must be the sum of heat absorbed in the evaporator and the compression work done.

The heat absorbed in the evaporator $h_{1}-h_{f 2}$ is known as the refrigeration effect and the ratio of the refrigeration effect to the compression
work done is known as the coefficient of performance. Thus for one pound of refrigerant,

$$
\begin{align*}
& \text { Refrigeration effect }=h_{1}-h_{4}=h_{1}-h_{f 2} \\
& \text { Compression work }=h_{2}-h_{1} \\
& \text { Cocfficient of performance } \beta=\frac{h_{1}-h_{f 2}}{h_{2}-h_{1}}
\end{align*}
$$

Since the coefficient of performance $\beta$ is the ratio of refrigeration cffect to the work done, one ton of refrigeration or the removal of 200 Btu per minute would require an ideal mechanical input of

$$
\text { Ideal input horsepower per ton }=\frac{200}{\beta} \frac{J}{33,000}
$$

For the refrigeration effect of $h_{1}-h_{f 2}$ per lb , a ton of refrigeration would require the circulation of $\frac{200}{h_{1}-h_{f 2}} \mathrm{lb}$ per minute, and the size of the compressor needed would be established by the fact that at the suction of the compressor, one pound of refrigerant has a volume of $V_{1}$ or $V_{1}$, cubic feet, depending on the cycle used.

Since the dry compression cycle extends into the superheat region, heat is not extracted at constant temperature in the condenser as it is in the wet compression cycle. This causes the wet compression cycle to be slightly more efficient than dry compression, but since actual compressors give better performance when no liquid is present, the dry compression cycle is most used.

The properties of refrigerants are often shown on pressure-enthalpy $P-h$ diagrams similar to the one illustrated by Figure $17 \cdot 3$. The dry compression cycle shown for $\mathrm{F}-12$ indicates the use of such a chart.

## Example of Dry Compression Cycle Using Freon

The cycle shown on Figure $17 \cdot 3$ is for an evaporator temperature of 20 deg F and pressure of 36 lb per sq in. and a condenser pressure of 151 lb per sq in. and a corresponding saturation temperature of 110 deg F . The refrigeration effect per pound $h_{1^{\prime}}-h_{4}=80.5-33.7=46.8 \mathrm{Btu}$ per lb and the compression work per pound, $h_{2^{\prime}}-h_{1^{\prime}}=91.8-80.5=11.3 \mathrm{Btu}$ per lb. This corresponds to a performance coefficient of $\frac{46.8}{11.3}=4.15$, and an ideal horsepower per ton of

$$
\frac{200}{4.15} \times \frac{778}{33,000}=1.13
$$

For each ton of refrigeration, $\frac{200}{46.8}=4.27 \mathrm{lb}$ per minute of freon must be circulated, and since the specific volume of saturated vapor at the compressor intake condition 1 is 1.14 cu ft per lb , the compressor must handle $4.27 \times 1.14=$ 4.88 cu it of vapor per minute.


Fig. 17-3. $P-h$ diagram for Freon refrigeration cycle.

## Example of Wet Compression Cycle Using Ammonia ${ }^{1}$

For a wet compression ammonia cycle (Figure 17-2) operating between the temperature range of the previous example the evaporator pressure would be 48.21 lb per sq in. absolute and the condenser pressure would be 247.0 lb per sq in.

$$
s_{2}=1.1566 \quad s_{f_{1}}=0.1437 \quad s_{f_{11}}=1.1532
$$

since

$$
\begin{aligned}
s_{1} & =s_{2} 1.1566=0.1437+x_{1} 1.1532 \\
x_{1} & =0.878 \\
h_{1} & =64.7+0.878 \times 553.1=550.7 \text { Btu per } \mathrm{lb} \\
V_{1} & =0.878 \times 5.910=5.20 \mathrm{cu} \text { ft per } \mathrm{lb} \\
h_{2} & =633.7 \text { and } h_{f 2}=167.0 \text { Btu per lb }
\end{aligned}
$$

Refrigeration per pound $=h_{1}-h_{f 2}=550.7-167.0=383.7$ Btu per lb.
Ideal compression work $=h_{2}-h_{1}=633.7-550.7=83.0$ Btu per lb.
Performance coefficient $=\begin{gathered}383.7 \\ 83.0\end{gathered}=4.62$.
Ideal horsepower per ton $=\frac{200}{4.62} \times \frac{778}{33,000}=1.02$.
For each ton of refrigeration, $\frac{200}{h_{1}-h_{f 2}}=\frac{200}{383.7}=0.522 \mathrm{lb}$ per minute of ammonia must be handled. For a specific volume $V_{1}=5.20 \mathrm{cu} \mathrm{ft} \mathrm{per} \mathrm{lb}$, ideal compressor would have to handle $5.20 \times 0.522=2.72 \mathrm{cu} \mathrm{ft}$ per minute of vapor.
17.4. Refrigeration Applications. The effect of the losses that occur in an actual compressor but are not considered in the ideal cycle is to increase the amount of compression work that is required to produce a given refrigeration effect. For example, if the actual compressor of a refrigeration unit is only half as efficient as the ideal cycle and the ideal coefficient of performances is 4 , the compression work is 24 or $1 / 2 \mathrm{Btu}$ per Btu of refrigeration. In the ideal cycle, one Btu of refrigeration requires $1 / 4 \mathrm{Btu}$ of compression work and $11 / 4 \mathrm{Btu}$ is discharged to the condenser. In the actual unit, $1 / 2 \mathrm{Btu}$ of work is done and $11 / 2 \mathrm{Btu}$ is discharged to the condenser.

In actual refrigerators the evaporator temperature must be somewhat lower than that of the space being cooled in order to obtain a rapid rate of heat transfer. For the same reason the condenser temperature must be somewhat higher than that of the condenser cooling medium and both of these factors reduce the coefficient of performance.

Although refrigerators are designed for the purpose of cooling, theoretically at least, a reversed refrigerator could be an efficient method of heating a building. This could be accomplished by placing the evaporator out-of-doors and the compressor and condenser in the space to be heated. Heat would be absorbed by the evaporating refrigerant in the evaporator at an outside temperature of perhaps 20 deg F. After the compression, heat could then be given up to the space being heated by the condenser, operating at 70 or 80 deg F .

For a coefficient of performance of, say, 6.0, and a compressor efficiency of 0.75 , one Btu from the low temperature outside, $\frac{1 / 6}{0.75}=0.222$ Btu of compression work, could deliver 1.222 Btu inside.

There are a large number of special applications of refrigeration principles including the "flash freczing" of ice. If the evaporator pressure in a water-vapor refrigerator is reduced to 0.08854 lb per sq in., which is
the saturation pressure corresponding to 32 deg F , part of the entering water will evaporate and obtain its heat of vaporization by freezing the remaining part. This "flash freezing" results in a snow ice that is useful in refrigeration. If of cach pound of water that enters with an enthalpy $h_{f 1}$, the portion $z$ that evaporates at 32 deg F will cool all the water to 32 deg F and freeze $1-z$ pounds of it. For a latent heat of fusion of 143.3 Btu per lb it is possible to calculate the part $z$ that must evaporate to freeze what remains.

In the past the ammonia absorption system was widely used, but with the development of mechanical refrigeration its use has become more limited. Water is capable of absorbing large quantities of ammonia vapor, and will absorb more at a low temperature than at a high one. In the absorption refrigerator, the ammonia from the refrigerator or expansion coils is absorbed by water in the externally cooled absorber tank. This strong solution is pumped into a generator tank where heat is applied and ammonia vapor is driven off and the weak solution returns by gravity to the absorber tank to be cooled.

The ammonia vapor that is driven off passes through a cooler or condenser where it is liqucfied. This higher-pressure liquid ammonia passes through an expansion valve and is vaporized in the expansion or refrigerator coil, and then passes to the absorber tank to start another cycle.
17.5. Steam Jet-Water Vapor Refrigeration. When water vapor is used as a refrigerant, the large volume of vapor that must be handled prohibits the use of reciprocating-piston type compressors and for this application, centrifugal compressors are commonly used. Since centrifugal compressors are less efficient than reciprocating machines they are often replaced by steam-jet pumps such as shown in Figure 17.4. The latent heat required for the evaporation in the evaporator is furnished by the water in the evaporator and this cools it. The low-pressure vapor is picked up by the steam-jet pump, and as the velocity of the vapor is reduced in the booster, the pressure increases to that in the condenser. The condenser pressure is determined by the temperature of the cooling water available. An auxiliary jet pump (not shown) is necessary to remove the air that enters the condenser by leakage and is brought in by the warmed water entering the evaporator.

Figure 17.5 illustrates the steam-jet refrigeration cycle on an $h-s$ diagram. When one pound of steam expands through the steam nozzle from pressure $P_{1}$ to the evaporator pressure $P_{2}$ at constant entropy, the enthalpy drop $h_{1}-h_{2}$ Btu per pound is converted into kinetic energy of flow $\frac{1}{2 J g} v_{2}^{2}$ Btu per pound. In the entrance of the booster this highvelocity jet picks up or carries along with it $y$ pounds of dry saturated
steam (point 6) from the evaporator. The heat to evaporate this steam $y\left(h_{6}-h_{5}\right)$ must be given up by the remaining water in the evaporator, which cools it.


Fig. 17 4. Steam jet-water vapor refrigerator
These two quantitics of steam are usually assumed to mix at constant total momentum and after mixing exist at condition 3, Figure $17 \cdot 5$. A constant momentum mixing of 1 lb with a velocity $v_{2}$ with $y$ pounds with no velocity results in $1+y$ pounds with a velocity $v_{3}$, such that

$$
\frac{1}{g} v_{2}+\frac{y}{g} \times 0=\frac{(1+y)}{g} v_{3}
$$

or

$$
\begin{equation*}
v_{3}=\frac{1}{1+y} v_{2} \tag{17•5}
\end{equation*}
$$



Fig. 17-5. Steam-jet refrigeration cycle.

There is a corresponding decrease in kinetic energy of the jet of

$$
\begin{gather*}
\Delta K E=\frac{1}{2 g J} v_{2}^{2}-\frac{(1+y)}{2 g J} v_{3}^{2}=\frac{1}{2 g J}\left[v_{2}^{2}-\frac{(1+y) v_{2}^{2}}{(1+y)^{2}}\right] \\
\Delta K E=\frac{v_{2}^{2}}{2 J g} \frac{y}{(1+y)} \mathrm{Btu} \tag{17•6}
\end{gather*}
$$

Since $\frac{v_{2}^{2}}{2 J g}=h_{1}-h_{2}$, this may be written

$$
\begin{equation*}
\Delta K E=\left(h_{1}-h_{2}\right) \frac{y}{1+y} \tag{17•7}
\end{equation*}
$$

This decrease in jet kinetic energy represents directed energy that goes back to random heat energy during the mixing process. This energy goes to evaporate moisture at the evaporator temperature $T_{2}$ which means an increase in entropy of $\Delta K E / T_{2}$. From this, the total entropy of the $1+y$ pounds at point 3 is the sum of the initial entropy plus the entropy increase that results from the mixing, or

$$
1 \times s_{1}+y s_{6}+\frac{\left(h_{1}-h_{2}\right)}{T_{2}} \frac{y}{(1+y)}=(1+y) s_{3}
$$

In an ideal booster the mixture at point 3 is decelerated and brought to rest at constant entropy, resulting in $s_{3}=s_{4}$. Equation 17.8 may be written

$$
s_{1}+y s_{6}+\frac{\left(h_{1}-h_{2}\right)}{T_{2}} \frac{y}{(1+y)}=(1+y) s_{4}
$$

The one pound of high-pressure steam at 1, Figure $17 \cdot 5$, was initially at rest and the $y$ pounds of vapor picked up in the evaporator had no initial velocity. Since the mixture


Fig. 17.6. Graphical solution for steam jet-water vapor cycle. is finally at rest in the condenser and in ideal equipment there are no thermal losses, the total initial enthalpy must be equal to the final value.

$$
1 \times h_{1}+y h_{6}=(1+y) h_{4} \quad[17 \cdot 10]
$$

The simultaneous solution of equations $17 \cdot 9$ and $17 \cdot 10$ will give the pound $y$ of vapor that will be removed by one pound of steam flowing through the nozzle. Since there is no simple relationship between $s_{4}$ and $h_{4}$, a graphical solution must be used. Such a solution is illustrated by Figure 17•6.

## REFERENCE

1. "Tables of Thermodynamic Properties of Ammonia," Bureau of Standards Circular 142.

## Suggested Reading

Refrigcration Principles
Sparks, Theory of Mechanical Refrigeration. Chapter III.
Refrigeration and Refrigcrants
Severns, Heating, Ventilating, and Air Conditioning. Pages 428-450.
Absorption Refrigeration
Sparks, Theory of Mechanical Refrigeration. Chapter IX.
Refrigeration Machinery
Barnard, Ellenwood, and Hirshfeld, Heat Power Enginecring. Part III, Pages 1098-1143.

## PROBLEMS

## Chapter 1

### 1.1. Pressure Units

Pressures are measured in pounds per square inch, in inches of water, and in inches of mercury, and a standard atmospheric pressure is 14.7 lb per sq in., 29.9 in . of mercury, and 34 ft of water. Pressures measured above that of the atmosphere are called gage pressures and the gage pressure plus the atmospheric pressure, both expressed in the same units, is called the absolute pressure.
(a) Convert a gage pressure of 30 in . of water to an absolute pressure expressed in inches of mercury and in pounds per square inch.
(b) What is meant by the expression "a 28 -in. vacuum referred to a 30 -in. barometer"? Convert this to absolute pressure expressed in pounds per square inch.

## 1-2. Inclined Manometer

Low pressures are often measured by using an inclined manometer, commonly called a draft gage. Such a manometer has one leg made of $1_{10}$-in. inside diameter glass tubing inclined at, an angle of 12 deg above the horizontal. The other leg is vertical and is $1 / 2$-in. inside diameter. The manometer uses oil with a specific gravity of 0.83 . When a pressure of 1 in . of water is applied to the manometer, how far will the oil move in the inclined leg? All the oil that moves out of the inclined tube must go into the $1 / 2-\mathrm{in}$. diameter vertical leg thereby changing the liquid level in it.

## 1•3. Energy Units

A $4000-\mathrm{lb}$ automobile traveling 50 mph is brought to rest by applying the brakes. If there is a total of 100 lb of steel in the brake drums and shoes, what would be the temperature rise of the metal if any heat dissipated to the air during the stopping time is neglected? The average specific heat of steel is 0.12 Btu per lb per deg F .

### 1.4. Thermal Units

If the heating value of coal is 14,000 Btu per lb of coal, what is it when expressed in calories per gram of coal?

### 1.5. Heating Costs

With power at 2 cents a $\mathrm{kw}-\mathrm{hr}$, what is the cost of heating 100 gal of water from 40 to 190 deg F ?

### 1.6. Power Units

In a hydroelectric plant where the water level is 300 ft above the plant, how many cubic feet per second of water must flow through the turbine to deliver $30,000 \mathrm{kw}$ at the switchboard if the turbine efficiency is 0.82 and the efficiency of the generator is 0.91 .

## 1•7. Energy Conversion

If a man can develop a useful output of $1 / 3 \mathrm{hp}$ when working hard, and his waste heat loss is 1200 Btu per hr , what is his efficiency as a heat engine?

### 1.8. Automobile Fuel Consumption

An automobile with a transmission efficiency of 0.85 has an engine that uses 0.6 lb of gasoline with a specific gravity of 0.77 per hr per hp developed. If the automo215
bile has a total rolling and air resistance of 140 lb at 48 mph , calculate the miles per gallon of fuel at this speed.

### 1.9. Airplane Fuel Costs

A $2200-\mathrm{lb}$ airplane has a total air resistance of 490 lb at 180 ft per sec air speed. The propeller has an efficiency of 0.78 and the engine has an overall efficiency of 0.26 . The engine uses gasolme with a heat content of $18,800 \mathrm{Btu}$ per lb , the specific gravity of the fuel is 0.76 and it costs 14 cents a gallon. When the airplane has an air speed of 180 ft per sec and is bucking a $30-\mathrm{mph}$ head wind, find the fuel cost in cents per ground mile. Find the fuel cost per ground mile when traveling at the same air speed but with a $30-\mathrm{mph}$ tail wind.

### 1.10. Power Cost

A steam plant uses coal having a heating value of 13,000 Btu per lb , costing $\$ 4.80$ per ton. If the furnace and boiler efficiency is 0.78 , the turbine efficiency is 0.21 and the generator efficiency is 0.81 , calculate the tuel cost in cents per kilowatthour delivered to the switchboard.

## 1•11. Vaporization of Liquid Air

In order to shrink the aluminum impeller of an air blower, so a reinforcing ring may be placed on it, the wheel is immersed in liquid air at -318 deg $F$. If the wheel weighs 5 lb and has a specific heat of 0.22 Btu per lb per deg F , how many pounds of liquid air will be vaporized if the wheel is initially at $80 \mathrm{deg} F$ and if 91.5 Btu are required to vaporize one pound of liquid air?

### 1.12. Thermal Efficiency of a Cupola Furnace

Approximately 600 lb of coke having a heating value of $12,500 \mathrm{Btu}$ per lb is used to melt a ton of pig iron. The iron must be heated from 70 deg F to 2700 deg F with an average specific heat of 0.19 Btu per lb per deg F , and the melting requires 60 Btu per lb of iron. If the useful output is considered to be only the heat that goes to the iron and the heat released by the oxidation of silicon and carbon from the pig iron is ignored, what is the thermal efficiency of such a furnace?

### 1.13. Annual Heating Cost

The heating season in different localities of the United States is reported in "de-gree-days." This represents a summation of the degrees difference between indoor and outdoor temperatures times the days duration of each temperature difference. This ranges from 2000 to 8000 deg-days in different localities.

In a section where the heating season is 5300 deg-days, a seven-room house has a heat loss of 1835 Btu per hr per deg F difference between indoor and outdoor temperature. If the furnace has an efficiency of 0.7 and uses 14,000 Btu per $\mathrm{lb}, \$ 10.00$ per ton coal, calculate the annual heating cost.

## Chapter 2

### 2.1. Molecular Energy

Calculate the kinetic energy of translation of the molecules in 1 lb of air considered in the example of article $2 \cdot 2$.

### 2.2. Variation of Molecular Velocity

Reasoning from the expression for molecular velocity (equation 2.2) and the gas equation (2.3), is the molecular velocity affected by changing the gas pressure if the temperature remains constant?

### 2.3. Gas Equations

The "gage pressure" in a 16 in . by 7 in . airplane tire at sea level where atmospheric pressure is 14.7 lb per sq in. and the temperature is 70 deg F is 20 lb per sq in. What will be the gage pressure in the same tire at an altitude where the barometer is 20 in . of mercury and the temperature is zero deg F?

## 2•4. Gcneral Gas Equation

If an air receiver tank which has a volume of 39.1 ru ft , is filled with air at 80 lb , per sq in. gage and 150 deg F , how many pounds of air are in the receiver? What will be the pressure gage reading after the air has cooled to 70 deg F?

## 2•5. Density of Gas

What is the weight of a cubic foot of dry air on a day when the temperature is 96 deg F and the barometric pressure is 30.14 in . of mercury?

### 2.6. Aneroid Barometer

Ancroid barometers are made that are sensitive enough to measure the pressure change when they are lifted from the floor to a table top. To what pressure change must the syphon bellows of such an instrument respond if it is to react to a $1-\mathrm{ft}$ change in elevation when the air temperature is $70 \mathrm{deg} F$ and the total barometer reading is 30 in . of mercury?
2.7. Variation of Air Pressure with Altitude

The decrease in air pressure, $-d p$, in a vertical distance $d h$ is equal to the weight of a column of air 1 sq ft in area and $d h$ feet high. Express $d p$ in terms of pressure $p$ and absolute temperature $T$. If $T$ is $460+60=520$ deg R at sea level where $h$ is zero, and decreases 3.5 deg per 1000 ft of altitude (this from test results), express the temperature $T$ at any height $h$. Combine this with the first equation and integrate for the equation between pressure $P$ and altitude $h$, evaluating the constant of integration for sea-level pressure corresponding to 14.7 per sq in., expressed in pounds per square foot.

From this equation, calculate the barometer reading at $20,000 \mathrm{ft}$ altitude.
If this temperature gradient of -3.5 deg per 1000 ft persisted throughout the outer atmosphere, which it does not beyond $40,000 \mathrm{ft}$, how thick would the earth's atmosphere have to be to account for the sea-level barometer reading?

### 2.8. Buoyancy of Gases

The lifting force of a balloon depends upon the difference between the weight of the gas and that of the air it displaces, and the bag is usually loose enough that the internal and external pressures are essentially equal. What weight and volume of helium will be required to lift a $1000-\mathrm{lb}$ load at sea level and 70 deg F temperature? What lift will it exert if the balloon rises quickly to $25,000 \mathrm{ft}$ altitude where barometric pressure is 11 in . of mercury and the air temperature is -30 deg F if the helium remains at 70 deg F ? What will it be when the helium has cooled to -30 $\operatorname{deg} \mathrm{F}$ ?

### 2.9. Natural Draft

The pressure difference that causes the gases to flow in a natural draft chimney results from the difference in weight between a column of the chimney gas and the weight of a similar column of outside air. On a day when the barometer reading is standard and the outside air is at 70 deg F the chimney gases which may roughly be considered as having the properties of nitrogen, have an average temperature of 250 deg F. Calculate the pressure difference producing gas flow in pounds per square foot, pounds per square inch, and in inches of water, if the stack is 150 ft high.

### 2.10. Centrifugal Pressure from a Blower

The impeller of a centrifugal blower, rotating an annulus of air at an angular velocity of $\omega$ radians per second produces a greater air pressure at the outer radius that is greater than that at the center by the cumulative effect of centrifugal force. For an element at any radius $r$ subtended by the angle $d \theta$ and having a radial depth $d r$ the volume is $r d \theta d r l$, where $l$ is the width of the blower normal to the wheel.

If this air is at a temperature ( $T$ average), express the weight of the element in terms of the volume, pressure, temperature, and the gas constant $R$. Next, express the centrifugal force on this element. This centrifugal force is balanced by the increase in pressure $d p$ acting inward over the area rdol.

When the centrifugal force is equated to the restraining or balancing force, the resulting equation relating the pressure $P$ and the radius $r$ can be integrated for the total centrifugal pressure increase if the temperature is considered constant and the intake pressure at the center where $r=0$ is assumed to be atmospheric.

From the equation thus developed, calculate the centrifugal pressure increase that occurs in a supercharger with a $10.5-\mathrm{in}$. outside diameter impeller rotating at 21,000 rpm , if the average air temperature is 180 deg F and the entering pressure at the center of the wheel is 14.7 lb per sq in.

### 2.11. Sublimation of Dry Ice

In the absence of a source of a high-pressure fluid, a vessel must be tested to destruction. If the tank has a volume of 40 cu ft and it is estimated that it will fail at a gage pressure of 300 lb per sq in., how many pounds of solid carbon dioxide which exists at -110 deg $F$ at atmospheric pressure must be sealed in the vessel to give a 300 lb per sq in. gage pressure when the temperature reaches 70 deg F ?

## 2•12. Gas Composition

The volumetric analysis of a gas is $\mathrm{H}_{2}=0.48 ; \mathrm{O}_{2}=0.06 ; \mathrm{CO}=0.38 ; \mathrm{N}_{2}=0.05$; $\mathrm{CH}_{4}=0.03$. Using the mol method, change this to a weight composition.

## 2•13. Effect of Humidity on the Gas Constant of Air

When air contains 0.03 lb of water vapor and 0.02 lb of carbon dioxide per pound of dry air, what is the correct value of the gas constant $R$ to use?

### 2.14. Vacuum Pump Performance

A vacuum pump with a 2 -in.-diameter piston and a 2 -in. stroke completely fills with air at whatever pressure exists in the tank at any time. How long will be required to reduce the pressure in the 2 -cu- ft container to 0.01 lb per sq in. if the temperature remains at 80 deg F , the initial tank pressure is 14.7 lb per sq in . and the pump operates at 300 strokes a minute?

## Chapter 3

## 3•1. Energy Changes

In terms of the general energy equation trace the energy changes that occur when high-pressure air is released into a lower-pressure tank by opening a valve.

### 3.2. Internal Energy Change

A test chamber in which rotating wheels are given overspeed tests is 10 ft in diameter and 6 ft deep. At the beginning of a test the air in it is at 80 deg F and 14.7 lb per $s q$ in. The test chamber is sealed and during a test the pressure rises to 19.2 lb per sq in. How many foot-pounds of friction work was done during the test, if it is assumed that no heat is transferred to the walls?

### 3.3. Expansion Work

A 30 -caliber rifle shoots a $0.30-\mathrm{in}$. diameter bullet weighing 0.30 oz from a barrel 30 in . long. If it leaves with a velocity of 2660 ft per sec, what must be the average gas pressure as it expands in the barrel? Ignore the effects of friction and rotation of the bullet.

### 3.4. Expansion Work

An air drill has a piston 3 in . in diameter that moves through a $4-\mathrm{in}$. stroke. If 80 lb per sq in . gage-pressure air is admitted behind the piston throughout the stroke, how many pounds of air are in the cylinder at the end of each stroke, when the exhaust valve opens, if the air is at 70 deg F at that point? How much total work is done by each pound of air admitted to the air drill and how much useful work is done?

### 3.5. Warm-Air Furnace

A residence having a heat loss of $138,000 \mathrm{Btu}$ per hr is heated by a warm-air furnace. The air enters the furnace shell from the return duct at $70 \operatorname{deg} F$ and leaves at 180 deg F to go to the various rooms. How many cubic fect per minute of 70 deg F atmospheric-pressure air must be circulated?

### 3.6. Heating a Gas

In the air preheater in a power plant $10,000 \mathrm{lb}$ of air per hr are heated from 80 to 400 deg F before being delivered to the furnace, and the air passes through the preheater at essentially constant pressure. Calculate the Btu per hour that go to increase the internal energy of the air and explain what becomes of the difference.

## 3•7. Energy of Gases

Air is picked up from rest at 70 deg F in a boiler room by a fan and given a velocity of 100 ft per sec. It then passes through a heater at constant pressure, emerging at a temperature of 400 deg F . How much energy is delivered by the fan in Btu per pound of gas? Using Table $3 \cdot 1$ and the energy equation, find the heat added in Btu per pound and explain what becomes of this heat.

## 3•8. Constant-Volume Heating

A closed metal tank is designed to be safe when subjected to an internal pressure of 100 lb per sq in . absolute. It is used to hold compressed air and is filled at 80 lb per sq in. absolute and 60 deg F . The tank stands in the sun and its contents attain a temperature of 125 deg F . Assuming that the tank will not expand with pressure and temperature changes, will the design pressure be exceeded? What will be the increase in internal energy of the air in the tank?

### 3.9. Expansion Work

A constant-pressure commercial gas-storage tank contains $60,000 \mathrm{cu} \mathrm{ft}$ of methane gas at a temperature of 65 deg F and a pressure of 30 lb per sq in . gage. The gas temperature is increased to 90 deg $F$ by the heat of the sun. Calculate the volume increase and the external work done as the gas expands. How many pounds of gas must be removed from the tank after heating to maintain the original volume of $60,000 \mathrm{cu} \mathrm{ft}$ ?

## 3•10. Energy Concepts

In most problems equilibrium is assumed throughout all portions of a body of gas. However, during sudden changes one portion may be instantaneously much hotter or cooler than the remainder. A crude equivalent of local burning in an engine cylinder may be had by visualizing a small, completely flexible bag of air having a volume of 1 cuft placed in a vessel of, say, 10 cu ft and the vessel sealed. Imagine the entire contents initially at a constant temperature and pressure, and then assume the air in the bag to be instantaneously heated to a higher temperature, no
heat being added to the surrounding air. Assuming that not enough time has elapsed to accomplish any appreciable transfer of heat, explain just what happens and trace all energy exchanges that occur.

## 3•11. Heating a Gas

One pound of air at atmospheric pressure and 70 deg F is placed in each of three containers. One container is completely rigid, one completely flexible, and the third is assumed to stretch under increased pressure. The temperature of the air in the flexible container is increased by 100 deg F and the pressure, of course, remains at atmospheric. When the air in the rigid container is heated through 100 deg F its volume remains constant and its pressure increases. The volume of the third container is assumed to vary directly with absolute pressure, that is, for an internal pressure of two atmospheres its volume would be twice its initial value, or

$$
V=V_{1} \frac{P}{P_{1}}
$$

Evaluate the internal energy change in each of the three cases for a 100 deg F temperature rise. Evaluate the expansion work done by the gas in the completely flexible tank. For the semi-flexible contaner, using the expression for the volume at any pressure and the gencral gas equation, integrate for the expansion work done on the container by the gas, and find the final pressure and volume. Since the pressure outside the tank is always atmospheric, the work done against outside pressure by the tank may be cvaluated from the outside pressure and the increase in volume.

What is the difference between the expansion work done on the tank by the gas and that done against the external pressure by the tank? Explain what becomes of the difference between them.

## 3•12. Enthalpy Change in a Jet

Air originally at 100 lb per sq in. gage and 500 deg F expands through a nozzle and comes out at a high velocity and at 80 deg F . Evaluate the enthalpy change, using specific heat values from Table $3 \cdot 1$. If no heat is conducted away during this expansion and no turbulence is present, what is the final jet velocity if the initial flow velocity was negligible?

## 3•13. Mol Specific Heats

Evaluate the translational kinetic energy, vibrational kinetic energy, and external expansion work per degree Fahrenheit per mol for oxygen and for hydrogen at 2000 deg R. Explain the reasons for any differences.

## 3•14. Unavailable Energy

A partial mechanical equivalent of a gas that increases in volume as it does expansion work is involved in the idea of hoisting a load with a flexible cable. Suppose a $100-\mathrm{lb}$ load is to be lifted 10 ft by a cable that is initially 10 ft long but that stretches elastically 10 per cent under a $100-\mathrm{lb}$ load. As the upper end of the cable moves up, the load does not begin to rise until the upper end has moved far enough to build up a $100-\mathrm{lb}$ tension in the cable. All the work done up to that point is stored as elastic energy in the cable. When the load has been lifted 10 ft and the cable is unhooked, it suddenly contracts to its original length and the elastic strain energy stored in it is lost in internal friction.

Calculate the hoisting efficiency of such an arrangement. (It will be well to discontinue the analogy at this point, for such analogies are dangerous and misleading if carried too far.)

## 3•15. Available Energy

Evaluate the entropy change when air is heated at constant volume from 80 deg F to 500 deg F and repeat for constant-pressure heating. Calculate the fraction of the heat added in each case that is available for mechanical work with a lowest available cooling temperature of 60 deg F . Explain the physical reasons for any difference.

## 3•16. Entropy Change Due to Mixing

One pound of air at 600 deg F is mixed at constant pressure with one pound of air initially at 200 deg F .
(a) Using specific heats from Table $3 \cdot 1$, calculate the final temperature after mixing.
(b) Find the decrease in entropy of the pound initially at $600 \mathrm{deg} F$, the increase in entropy of the pound initially at 200 deg F , and the net entropy increase of the system.
(c) For a lowest available temperature of $60 \mathrm{deg} F$, find the decrease in available energy that results from the mixing and explain the physical meaning of this.

## 3•17. Entropy Change During Freezing

Calculate the entropy change as 1 lb of water initially at 32 deg F is frozen to ice, the latent heat of fusion being 144 Btu per lb .

Calculate the entropy change as 10 lb of water is heated from 32 deg F to 100 deg F .
Calculate the final temperature when 1 lb of ice is put into 10 lb of water at 100 $\operatorname{deg} F$.

Calculate the entropy above 32 deg F for 11 lb of water at this temperature.
Is the total entropy of the system before and after mixing constant, and if it is why should it be? If it does not prove to be constant what is the explanation?

## Chapter 4

### 4.1. Volume of a Vapor

Using the general gas equation and the value of $R$ for water vapor, calculate the specific volume in cubic feet per pound for steam at 14.7 lb per sq in. absolute and 380 deg F , and compare the result with the exact value from the steam tables.

## 4•2. Low-Pressure Water Vapor

By treating low-pressure superheated steam as a gas, calculate the volume per pound of superheated vapor for 5 lb per sq in. absolute pressure at 200 and at 500 $\operatorname{deg} \mathrm{F}$. Calculate also the change in enthalpy between these two temperatures by assuming it to be a gas with a constant specific heat. Compare these values with those found from the steam tables.

## 4•3. Internal Energy Changes

Using the steam tables, calculate the internal energy for steam at 1200 lb per sq in. pressure and 1000 deg F and also at 1 lb per sq in. and 1000 deg F . Explain the reason for the difference.

Calculate the volume of a pound of steam at each of these conditions using the general gas equation and the theoretical value of the gas constant $R$ and compare with actual values from the steam tables.

### 4.4. Steam Tables

A steam engine cylinder contains 2 cu ft of wet steam at 80 lb per sq in . absolute pressure and a quality of 0.75 . Find the steam temperature, pounds of dry steam, pounds of moisture, total enthalpy and total internal energy.

### 4.5. Vapor Tables

(a) Steam comes from a boiler at 120 lb per sq in . absolute pressure and a quality of 0.93 . Find the temperature, volume, enthalpy, and internal energy per pound.
(b) If it is superheated to 650 deg F at the same pressure, find the new volume, enthalpy, and internal energy, together with the Btu per pound added in the superheater.

### 4.6. Constant-Volume Hcating

A steam mixture, originally at 20 lb per sq in . absolute pressure and 0.20 quality is heated at constant volume to a pressure of 100 lb per sq in . absolute. Find the final steam condition and the heat added per pound.

### 4.7. Steam Tables

A low-pressure heating system supplies 95 per cent quality steam to the radiators at a temperature of 100 deg F . If the internal volume or the steam space of a radiator is 2 cu ft calculate:
(a) The pressure at which the vapor is supplied.
(b) The pounds of dry steam in the radiator.
(c) The pounds of moisture in the radiator.
(d) The total heat content and the total internal energy of the mixture in the radiator.

### 4.8. Evaporative Cooling

The cooling water from a condenser reaches a spray tower at 120 deg F and there it is cooled to 70 deg F by evaporation. Using the average value of the enthalpy of vaporization for this range, calculate the fractional part of each pound that must evaporate to cause the remainder to leave at 70 deg F.

### 4.9. Evaporation of Water

From a feed-water heater containing water at 250 lb per sq in . absolute pressure and 400.95 deg F , water escapes to a region at 14.7 lb per sq in. If no transfer of heat occurs what will be the quality of the steam formed?

### 4.10. Steam Calculations

A 450 lb per sq in. absolute boiler, equipped with a superheater is supplied with feed water at saturation temperature. The steam leaves the boiler steam drum with a quality of 98 per cent and enters the superheater. It leaves the superheater at 750 deg F . If the boiler generates $87,000 \mathrm{lb}$ of steam per hr , calculate:
(a) Heat added in the boiler per hour.
(b) Heat added in the superheater per hour.
(c) Tons of coal burned per hour, if $10,000 \mathrm{Btu}$ per lb of coal are used to generate the steam.

## 4•11. Throtling

Steam at 120 lb per sq in. gage pressure and 0.96 quality is throttled to 30 lb per sq in. gage. Find the final steam condition, the increase in volume per pound, and the increase in unavailable energy for a cooling temperature of 70 deg F .

## 4-12. Throttling Calorimeter

Wet steam is expanded in a throttling calorimeter from a pressure of 160 lb per sq in. absolute to an exhaust pressure of 14.5 lb per sq in . absolute. The temperature of the superheated steam in the calorimeter is 284 deg F. Calculate the initial quality of the wet steam using the superheat tables and also by assuming $c_{p}$ for superheated steam as 0.47 .

## 4•13. Quality of Steam

From the size and speed of a steam engine and its rate of steam consumption it is estimated that the volume per pound of wet steam at the end of the expansion stroke is 5.8 cu ft per lb when the pressure is 40 lb per sq in. gage. Find the temperature, quality, and enthalpy of the steam. If dry saturated steam enters the engine at 120 lb per sq in. gage, what is the decrease in enthalpy of the steam passing through the engine?

### 4.14. Entropy of Vapors

(a) Using $c_{p}$ of water as 1.0 , calculate the entropy of liquid at 100 lb per sq in . absolute and compare with $s_{f}$ from the steam tables.
(b) Using $h_{f g}$ at 100 lb per sq in . absolute from the steam tables, calculate $s_{f g}$ and compare with $s_{f_{g}}$ from the tables.
(c) At 100 lb per sq in. absolute in the superheat tables, calculate the mean $c_{p}$ of superheated steam between saturation and $500 \mathrm{deg} F$. Using this value of $c_{p}$, calculate the change in entropy between saturation and 500 deg F and check it against steam table values.
(d) What portion of the heat added in each of $a, b$, and $c$ is available with a low temperature of 32 deg F ?

## 4•15. Entropy Change

Calculate the increase in entropy and the increase in unavailable energy that accompanies the throttling process of problem 4.9 using a lowest available temperature of 70 deg F .

### 4.16. Air Chamber

It is necessary for single-cylinder liquid pumps to have an air chamber with a volume about eight times as great as the cylinder capacity connected to the discharge line to aid in smoothing out pressure variations. Such an "air dome" contains 2 cu ft of 60 deg F air when the pressure is at its minimum value of 120 lb per sq in . gage. If the maximum surge pressure reached is 200 lb per sq in . gage, what is the final volume of the air and the amount of energy stored in it if the compression is assumed to be isothermal? Solve also assuming adiabatic compression.

## 4•17. Atmospheric Temperature Variation

If it is assumed that a large mass of air that rises to a higher altitude from a sealevel temperature of 60 deg F and a barometer of 29.92 in . of mercury without mixing with surrounding air masses, what would its temperature be at an altitude of $25,000 \mathrm{ft}$ where the barometer is 11 in . of mercury?

What would account for the decrease in internal energy and what could cause the temperature so calculated to be other than the value of $-30 \operatorname{deg} F$ that is the accepted value at that altitude?

## 4•18. Adiabatic Compression

If the air is compressed adiabatically from 14.7 lb per sq in . absolute and 80 deg F to 80 lb per sq in. gage pressure find:
(a) The final temperature.
(b) The change in volume per pound.
(c) The compression work in Btu per pound of air.
(d) The change in internal energy per pound.
4.19. Isothermal Compression

Solve problem 4.18 for isothermal compression.

### 4.20. Compression Pressure

In a Diesel engine, air initially at $150 \mathrm{deg} F$ and 14.7 lb per sq in. absolute pressure is compressed until its volume is one-sixteenth of its initial value. Using gas tables, find the final temperature assuming an adiabatic compression. Find the final pressure, the compression work done per pound of air, and the internal energy change.

## 4•21. The Energy Equation

The energy equation in the form of equation $3 \cdot 15$ may be combined with equation 3.21 to give a velocity-temperature equation. This equation when integrated for an adiabatic equation may be combined with the adiabatic $P-T$ equation $4 \cdot 23$ to give a relationship between pressure change and velocity change during an ideal adiabatic expansion.

From such an equation calculate the theoretical velocity that would result when 500 deg F air at rest in a tank at 30 lb per sq in . absolute pressure expands through an opening to atmospheric pressure.

## 4•22. Exhaust Gas Turbine

The exhaust gases leave an internal-combustion engine at 40 lb per sq in. absolute pressure and 1200 deg F and enter the exhaust gas turbine of a supercharger unit. If an ideal constant entropy expansion occurs in the turbine, what will be the exhaust temperature when a pressure of 16 lb per sq in . absolute is reached? Find the expansion work done per pound of exhaust gas. Use gas tables.

### 4.23. Use of Gas Tables

In an engine air is compressed at constant entropy from 14.7 lb per sq in . and 140 deg $F$ to 1200 deg R. Using gas tables, find the final pressure.

## 4-24. Gas Table Calculations

Solve problem 4.21 using the gas tables instead of the gas equations.

### 4.25. Expansion of Hydrogen

Solve problems 4.21 and 4.24 using hydrogen instead of air.

## Chapter 5

## 5•1. Gas Mixtures

A sample of dry flue gas has the following volume analysis: $\mathrm{CO}_{2}, 11.00$ per cent; $\mathrm{CO}, 0.90$ per cent; $\mathrm{O}_{2}, 6.90$ per cent; $\mathrm{N}_{2}, 81.20$ per cent. For this mixture, calculate the percentage weight composition, the gas constant $R$, and $c_{v}$ at zero $\operatorname{deg} F$.

### 5.2. Gas Mixtures

The volumetric analysis of an illuminating gas is: $\mathrm{H}_{2}=0.48 ; \mathrm{O}_{2}=0.06 ; \mathrm{CO}=$ $0.38 ; \mathrm{N}_{2}=0.05 ; \mathrm{CH}_{4}=0.03$. Calculate the weight composition, and the fraction of the total pressure exerted by each gas.

## 5•3. Gas Mixtures

The exhaust gas from a furnace has a dry volume composition of 14 per cent $\mathrm{CO}_{2}$, 9 per cent $\mathrm{O}_{2}$, and 77 per cent $\mathrm{N}_{2}$. Calculate the percentage composition by weight and the values of $R$ and $c_{v}$ for the mixture.

### 5.4. Exhaust-Gas Composition

Verify the value of $R$ given for the products of combustion of Table 4•11. Using this composition and Tables 4.2 to $4 \cdot 8$, check the values of $u$ and $s_{v}$ given in Table 4.9 at 1000 deg R.

### 5.5. Clearance Gases in an Engine

At the end of the exhaust stroke in an internal-combustion engine, the clearance space is filled with the combustion mixture at atmospheric pressure and 1500 deg F . At the end of the suction stroke, when the new air charge, initially at 80 deg F , has been drawn in at atmospheric pressure, the temperature of this mixture is 200 deg F . Assuming no heat loss or gain from the cylinder walls, calculate the fraction of the final mixture by weight that is air and the weight composition of the final mixture.

## 5•6. Fuel Mixing

The fuel for a gas furnace is furnished in the form of methane at 70 deg F and atmospheric pressure that is mixed with 13 cu ft of air at atmospheric pressure and 300 deg F for each cubic foot of methane. Calculate the temperature of the mixture and the density in pounds per cubic foot.

## 5•7. Natural Gas

Natural gas that is 0.92 methane, 0.055 nitrogen, and 0.025 ethane is distributed in containers at a pressure of 1500 lb per sq in . gage. How many pounds would be in a 4 ft long container that is 5 in . inside diameter if the temperature is 80 deg F ?

## 5•8. Vapor Mixture

What is the air-fuel ratio when air is saturated with gasoline vapor at 40 deg F ? (Approximate gasoline vapor as octane $\mathrm{C}_{8} \mathrm{H}_{18}$ to calculate volume.) If the air and liquid gasoline are supplied at 60 deg F , how much external heat must be supplied per pound of mixture?

## 5•9. Dew Point Calculation

Calculate the dew point temperature for the combustion mixture of Table 4.11. At what pressure would condensation start if the temperature is held at 160 deg F ?

## 5•10. Gas and Vapor Mi.rtures

A room 20 ft by 24 ft by 12 ft is filled with saturated air at 78 deg F and at a total barometric pressure of 30.1 in . of mercury. Find the pressure exerted by the water vapor and by the air, the pounds of dry air, and the pounds of water vapor present.

## 5•11. Air Density

Compare the weight of 1 cu ft of dry air at 70 deg F and 14.7 lb per sq in . absolute with the weight of an equal volume of saturated mixture of air and water vapor at the same temperature and total pressure.

## 5•12. Dew Point

The exhaust gases from a furnace have a dry volumetric analysis of $0.14 \mathrm{CO}_{2}$, $0.09 \mathrm{O}_{2}$, and $0.77 \mathrm{~N}_{2}$. If this furnace is in a location where the barometric pressure is 22.0 in . of mercury and the dew point temperature of these gases is found to be $142 \mathrm{deg} . \mathrm{F}$ find the pounds of water vapor that accompany a pound of the dry gases. What would be the dew point temperature of the same mixture if the barometer reading had been 29.92 in . of mercury?

## 5•13. Condensation

(a) A cold-water pipe through which water at 50 deg F is flowing, passes through a room in which the temperature is 73 deg F . Calculate the humidity of the air in the room at which the pipe will begin to "sweat."
(b) Air initially at 14.7 lb per sq in. absolute 80 deg F and 60 per cent humidity is cooled at constant total pressure to 45 deg F . How much water will be precipitated per 1000 cu ft of the original air?

### 5.14. Moisture Condensation

An air compressor handles 2000 cu ft per hr of room air that is at 75 deg F and 70 per cent humidity and compresses it to 80 lb per sq in. gage pressure. If it cools to 75 deg F in the receiver, how many pounds per hour of moisture will be condensed?

## 5•15. Atmospheric Moisture

For air that has a dry bulb temperature of 70 deg F and a wet bulb reading of 60 deg F, calculate:
(a) Pounds of vapor per pound of dry air for saturation at 70 deg F .
(b) Actual pounds of vapor per pound of dry air and the percentage humidity.
(c) Dew point temperature.
(d) Total enthalpy per pound of dry air.

## 5•16. Atmospheric Moisture

For air having a dry bulb temperature of 83 deg F and a wet bulb reading of 67 $\operatorname{deg} F$, calculate the grains of moisture per pound of dry air, the dew point temperature and the enthalpy, using steam tables.

## 5-17. Effect of Altitude on Atmospheric Moisture

If a standard psychrometric chart based on a 29.92 in. barometer is used where the barometer reading is 20 in . of mercury, what errors in the dew point temperature moisture content, enthalpy and percentage humidity will result for a dry bulb temperature of 80 deg F and a wet bulb temperature of 65 deg F ?

## 5•18. Saturation of Air

If air that is initially saturated with water vapor at 100 deg F is exposed to gasoline vapor until equilibrium is reached, still at $100 \mathrm{deg} F$, will moisture be condensed? Will a pound of such saturated air contain more gasoline vapor at saturation than air that is initially dry?

## 5•19. Moisture Evaporation

Below what percentage humidity of the air at 80 deg F will moisture evaporate from a lake that is at 65 deg F ? What will happen if air with a higher moisture content comes into contact with the lake water?

### 5.20. Relative Humidity and Percentage Humidity

The percentage humidity is defined as the ratio of the moisture weight per pound of dry air to the pounds necessary to saturate the air at that temperature. The relative humidity is defined as the ratio of the partial exerted by the moisture to the pressure it exerts at saturation. Work out carefully the pounds of moisture per pound of dry air and the dew point temperature for 100 deg F air that has 65 per cent humidity and also for a relative humidity of 0.65 .

## 5•21. Vapor Pressure of Ice

Ice at zero deg $F$ exerts a vapor pressure of 0.0383 in . of mercury. How much ice will be formed from the moisture that is condensed from 100 cu ft of saturated 70 deg $F$ air when it is cooled to zero deg $\mathbf{F}$ ?

### 5.22. Evaporation or Condensation?

Explain the phenomenon commonly referred to as "steam rising" from a lake on a chilly morning when the sun first comes up.

### 5.23. Fog

Can the fogs of the northwest coast of the United States be related to warm ocean currents off shore? Why is it always stated that a fog "settled down"?

### 5.24. Frost

Does a white frost on the ground in the fall or spring necessarily mean that the atmospheric temperature has reached 32 deg F during the night? How did the ancient Egyptians manufacture ice?

## Chapter 6

### 6.1. Combustion of Carbon and Hydrogen

What weight of air is required to burn a pound of carbon to carbon dioxide? a pound of carbon to carbon monoxide? a pound of hydrogen to water? Convert weights of air to cubic fect of air at $70 \mathrm{deg} F$ and 14.7 lb per sq in. absolute.
6.2. Combustion of Hydrocarbons

For the complete combustion of methane gas $\left(\mathrm{CH}_{4}\right)$ calculate:
(a) Mols of $\mathrm{O}_{2}$ per mol of gas.
(b) Pounds of air per pound of gas.
(c) Cubic feet of air per cubic foot of gas.

### 6.3. Combustion of Gases

The volumetric analysis of a natural gas is $\mathrm{CH}_{4}=0.90, \mathrm{C}_{2} \mathrm{H}_{6}=0.08$, and $\mathrm{H}_{2}=$ 0.02 . Find the cubic feet of air required for the complete combustion of a cubic foot of gas and the pounds of $\mathrm{N}_{2}, \mathrm{H}_{2} \mathrm{O}$, and $\mathrm{CO}_{2}$ formed from each pound of gas.
6.4. Combustion of Hydrocarbons

For the complete combustion of ethyl alcohol $\left(\mathrm{C}_{2} \mathrm{H}_{6} \mathrm{O}\right)$ calculate:
(a) The pounds of $\mathrm{O}_{2}$ needed per pound of fuel.
(b) The pounds of air needed per pound of fuel.
(c) The pounds of each of the constituents of the exhaust gases per pound of fuel burned.
6.5. Combustion of a Cas

The volumetric analysis of an illuminating gas is: $\mathrm{H}_{2}=0.48 ; \mathrm{CO}=0.38 ; \mathrm{CH}_{4}=$ $0.03 ; \mathrm{O}_{2}=0.06 ; \mathrm{N}_{2}=0.05$. Calculate:
(a) The theoretical volumetric air-tuel ratio.
(b) The $d r y$ volumetric exhaust gas analysis for 50 per cent excess air.
(c) The value of $R$ for the $d r y$ exhaust gases and 50 per cent excess air.
6.6. Combustion of Coal

The weight analysis of coal is:

| Moisture | 0.035 | Oxygen | 0.06 |
| :--- | :--- | :--- | :--- |
| Ash (inert) | 0.08 | Nitrogen | 0.015 |
| Carbon | 0.75 | Sulphur | 0.01 |
| Hydrogen | 0.05 |  |  |

Calculate the pounds of air per pound of coal for the complete combustion, and the higher heating value in Btu per pound.

### 6.7. Incomplete Combustion

Hexane $\left(\mathrm{C}_{6} \mathrm{H}_{14}\right)$ is burned with an air-fuel ratio of 14 to 1. Calculate:
(a) Loss in heating value per pound of fuel due to incomplete combustion.
(b) Wet volumetric exhaust gas analysis.

### 6.8. Delayed Burning

Liquid fuel that is 0.85 carbon and 0.15 hydrogen by weight is dissociated into hydrogen and methane, in equal volumes, and free carbon by excessive temperature before it contacts the oxygen necessary for combustion. Later the hydrogen and
methane burn, but the free carbon goes unburned. Find the fraction of the carbon that does not burn and the fraction of the total heating value that is not realized.

## 6•9. Heating Value of a Gas

Calculate the higher and lower heating values for the gas of problem $6 \cdot 5$, expressed in Btu per cubic foot measured at 60 deg F and 14.7 lb per sq in. pressure.

## 6•10. Temperature of Combustion

If it is desired to have the products of combustion from the burning of fuel that is 0.86 carbon and 0.14 hydrogen by weight leave at 1800 deg F when there are no heat losses during combustion, what air-fuel ratio must be used? Solve this using gas tables.

## 6•11. Burning Using Liquid Air

If liquid air is to be used to burn gasoline that is 0.855 carbon and 0.145 hydrogen by weight, what maximum temperature may be reached if 20 per cent excess air is used? Use the gas tables, neglect dissociation and heat loss and consider the fact that liquid air exists at -318 deg F and has a latent heat of vaporization of 91.5 Btu per lb.

### 6.12. Incomplete Combustion

A petroleum fuel that is 0.86 carbon and 0.14 hydrogen by weight is burned with 12.5 lb of air per lb of fuel. Find the fraction of the carbon burned to CO instead of $\mathrm{CO}_{2}$ and the pounds of each exhaust gas constituent formed per pound of fuel burned. 6.13. Properties of Coal

A West Virginia coking coal has a proximate analysis: moisture $=0.014$; volatile $=0.221$; fixed carbon $=0.667$; and ash $=0.098$ by weight. Its ultimate analysis is: sulphur $=0.015$; hydrogen $=0.05$; carbon $=0.777$; nitrogen $=0.015$; oxygen $=0.045$; and ash $=0.098$ by weight.
(a) Assuming that all oxygen in the coal comes from either the mechanically contained moisture or the water of crystallization, calculate the amount of water of crystallization from the oxygen of the ultimate analysis and the mechanical moisture of the proximate analysis.
(b) Assuming the volatile of the proximate analysis to be composed of hydrocarbons, water of crystallization, and nitrogen, calculate the amount of hydrocarbons in a pound of cual and the hydrocarbon composition of the volatile.
(c) Calculate the theoretical pound of air per pound of coal.
(d) Calculate the theoretical higher and lower heating values.

6-14. Incomplete Burning
When coal having a low volatile content is incompletely burned, it is true that the chief fuel element in the products of combustion is carbon monoxide and any ordinary Orsat analysis is dependable. However, when fuel oil is burned under unfavorable conditions there are many fuel elements in the exhaust gases and because an Orsat analysis detects only the carbon monoxide, calculations from it are inaccurate. This is illustrated by the complete dry volumetric analysis of the stack gases from a small household oil burner burning fuel oil with the following analysis:

Fuel Oil Weight Analysis: $\mathrm{C}=0.845 ; \mathrm{H}_{2}=0.13 ; \mathrm{N}_{2}=0.01 ; \mathrm{S}=0.005$.
Stack Gas Analysis by Volume: $\mathrm{CO}_{2}=0.1210 ; \mathrm{C}_{2} \mathrm{H}_{4}=0.0035 ; \mathrm{O}_{2}=0.035 ; \mathrm{CO}$ $=0.0510 ; \mathrm{H}_{2}=0.0070 ; \mathrm{CH}_{4}=0.0060 ; \mathrm{N}_{2}=0.7980$.
(a) Calculate the analysis that would have been given by an Orsat, considering only $\mathrm{CO}_{2}, \mathrm{O}_{2}, \mathrm{CO}$, and $\mathrm{N}_{2}$.
(b) Calculate the air-fuel ratio, per cent theoretical air, and loss in heating value per pound of fuel on the Orsat basis.
(c) Using the complete gas analysis, calculate the air-fuel ratio, the per cent theoretical air, and loss in heating value per pound of fuel.

## 6•15. Exhaust Gas Analysis

The dry volumetric exhaust gas analysis of a gasoline engine burning fuel with a weight analysis of $\mathrm{C}=0.85$ and $\mathrm{H}_{2}=0.15$ is: $\mathrm{CO}_{2}=0.116 ; \mathrm{O}_{2}=0.025 ; \mathrm{CO}=$ $0.015 ; \mathrm{N}_{2}=0.844$. Calculate the air-fuel ratio and the incomplete combustion loss per pound of fuel by accounting for the C and also by accounting for the $\mathrm{H}_{2}$.

## 6•16. Dew Point Temperature of Exhaust Gases

For $\mathrm{C}_{8} \mathrm{H}_{18}$ burned with 14 lb of $d r y$ air per pound of fuel, calculate the wet volumetric exhaust gas analysis and the dew point temperature at which moisture would start to condense from the products of combustion.

### 6.17. Air-Fuel Ratio

When a fuel of unknown composition is burned, the dry volumetric exhaust gas analysis is: $\mathrm{N}_{2}=0.85 ; \mathrm{CO}_{2}=0.12 ; \mathrm{CO}=0.02 ;$ and $\mathrm{O}_{2}=0.01$. Calculate the airfuel ratio, the fuel composition, and loss due to incomplete combustion of carbon per pound of fuel.

## 6•18. Range of an Airplane

An airplane, flying at its most economical speed, has a total drag that is always $1 / 11$ of its total weight, is equipped with an engine that develops 1 bhp -hr from 0.40 lb of fuel that has a heating value of $18,000 \mathrm{Btu}$ per lb and has a propeller with an efficiency of 0.83 . If the airplane's gross weight at take-off is one-third fuel, what is its maximum range in miles? If the heating value of the fuel is reduced by one-half and the engine develops a horsepower-hour from the same number of Btu as with the original fucl, is the range reduced by one-half?

## 6•19. Exhaust Gas Analysis

An automobile engine is adjusted for ideal complete combustion of a fuel that has the same composition as octane. If one of the 6 cylinders is not firing and the fuel from that cylinder does not burn in the exhaust manifold, in what way would this be reflected by a $\mathrm{CO}_{2}$ indicator?

### 6.20. Calorimeter Error

An analysis of the exhaust gases from a Junkers type calorimeter burning pure methane indicates that the fucl is being burned with 300 per cent excess air. If the exhaust gases leave 10 deg F warmer than the entering gas and air, what per cent error in the higher heating value of the fuel would result?

## Chapter 7

## 7•1. Viscosity of Liquids

Plot curves of the viscosity against temperature on a $\log -\log$ scale for air, water, and lubricating oil and explain why they have different tendencies.

## 7-2. Heat I'ransfer Units

Develop a multiplying factor for converting values of thermal conductivity $k$ from the units Btu per hour per square foot per degree Fahrenheit per foot to the units watts per square inch per degree centigrade per inch.

## 7-3. Thermal Conductivity of Gas Mixtures

From the discussion of gas mixtures, thermal conductivity, and viscosity of gases, attempt to reason out the way in which the properties of each constituent of a gas mixture affect the thermal conductivity of the mixture. Test your theory on the
following case. At 50 deg F , a mixture that is $0.83 \mathrm{H}_{2}$ and $0.17 \mathrm{CO}_{2}$ by weight has a $k=0.068$ and one that is $0.37 \mathrm{H}_{2}$ and $0.63 \mathrm{CO}_{2}$ has a $k=0.025$. Table $7 \cdot 2$ gives $k$ for all $\mathrm{H}_{2}$ and for all $\mathrm{CO}_{2}$.

### 7.4. Thermal Conductivity of Gases

Calculate the heat loss in Btu per hour per square foot through a stagnant air film 0.02 in . thick, for a temperature of 100 deg F on one side and 200 deg F on the other.

### 7.5. Conductivity of Metals

On a curve sheet using thermal conductivity as the abscissa and electrical conductivity as the ordinate, spot points representing the metals of Table 7•3 and see if any general conclusion can be drawn.

## 7-6. Viscosity Units

Convert the absolute viscosity of air at 50 deg F from the slug, second, foot units to the gravitational or engineering system using pounds, hours, and feet.

### 7.7. Heat Conduction

A cold-storage room has a wall made of 4 in . of face brick, 8 in . of concrete, $11 / 2 \mathrm{in}$. of cork and $1 / 2 \mathrm{in}$. of cement plaster. Starting with the conduction equation, calculate the heat conducted through 1 sq ft when the inside temperature is 40 deg F and the outside is 100 deg F .

### 7.8. Heat Conduction Through Building Roof

The ceiling of a house is made up of $7 / 8 \mathrm{in}$. of dry wood and $1 / 2 \mathrm{in}$. thickness of press board. The roof which has a 45 deg slope is made of $1 / 4 \mathrm{in}$. of low-grade rubber roofing on a $2-\mathrm{in}$. thick, dry concrete slab. The outdoor temperature is zero $\operatorname{deg} F$ and the inside temperature under the ceiling is $75 \mathrm{deg} F$. Calculate the attic temperature and the rate of heat loss per square foot of ceiling area.

### 7.9. Radial Conduction

Derive the expression for the heat loss per foot of pipe when a pipe of radius $R_{1}$ is covered by an insulating material of outside radius of $R_{2}$ and conductivity of $K$ when on the outside surface of the insulation there is a thin stagnant air film having a conductance of $U$ if the pipe surface temperature is $T_{1}$ and the air temperature is $T_{2}$.

## 7•10. Radial Conduction

Steam from a 600 lb per sq in. absolute and 700 deg F boiler is carried by a 10 -in. wrought-iron pipe (outside diameter of 10.75 in .) to a high-pressure turbo-generator unit. The steam main runs lengthwise through a small control room 8 ft long and 6 ft wide. The main is covered with asbestos insulation 3 in . thick. If the outside surface temperature of the pipe is 690 deg F and the outside temperature of the asbestos insulation is 120 deg $F$, calculate the heat loss per hour from the steam main to the room. (Use $K$ for wrought iron as 34.5.)

## 7•11. Steam-Pipe Insulation

The stagnant air film at the surface of an 8 -in. diameter steam pipe has a conductance $U$ of 1.5 Btu per hr per sq ft per deg F . If the steam pipe is at $240 \mathrm{deg} F$ and the air is at 75 deg F , compare the heat loss from a $100-\mathrm{ft}$ length of bare pipe with the loss when it is lagged with a $11 / 2 \mathrm{in}$. thick layer of insulation having a $k=0.1$ Btu per hr per sq ft per deg F per ft, if it is assumed that the value of the air film $U$ at the outside surface of the lagging is the same as it is at the surface of the bare pipe.

## 7•12. Radial Heat Conduction

The steam header in a building is nominal $11 / 2-\mathrm{in}$. steel pipe that has an inside diameter of 1.61 in . and an outside diameter of 1.90 in . It has a $11 / 4 \mathrm{in}$. thick covering of asbestos and the steam temperature inside is 240 deg F. For a temperature of 90 deg F at the outside of the covering, calculate the heat loss in Btu per hour per foot length of pipe.

## 7-13. Dimensional Analysis

Using dimensional analysis, assume the centrifugal force $F$ on a whirling mass to be a function of the mass $m$, the radius $r$, and the angular whirling speed $\omega$ and solve for the expression for centrifugal force (angle in radians is are/radius).

## 7-14. Dimensional Analysis

Using dimensional analysis, solve for the general expression for the unit pressure $p$ developed by a centrifugal blower, assuming the pressure to depend on the mass density $\rho$ of the air, the blower radius $R$, and the rotative speed of the blower $\omega$ in radians per second.
7•15. Dimensional Analysis
Using dimensional analysis, assume that the rate of discharge of oil through a pipe for a steady viscous flow ( $Q=$ cubic feet per second) is a function of the pressure drop per foot length ( $P / L=$ pounds per square foot per foot length), the pipe radius ( $R=$ feet), and the viscosity $\mu$ of the oil and derive the dimensional equation for $Q$.

### 7.16. Heat Conrection

Tests on a heated steel plate in still air gave a heat loss of 720 Btu per hr per sq ft by convection when the plate temperature was $480 \mathrm{deg} F$ and the air temperature was 80 deg F . When tested in an air stream having a velocity of 90 ft per sec under the same temperature difference, the heat loss by convection was found to be 6960 Btu per hr per sq ft.

Using the conductivity of the arr in the stagnant film as the average between the plate and the free air, calculate the thickness of the equivalent stagnant air film in each case.
7•17. Forced Convction
(a) For water at an average temperature of 100 deg F flowing in a $3 / 4-\mathrm{in}$. diameter tube at a velocity of 6 ft per sec, calculate the film conductance $U$ when the tube wall has a temperature of 200 deg F .
(b) Find the temperature rise of the water that occurs in a $10-\mathrm{ft}$ tube.
(c) Would there be a greater or smaller temperature rise in a smaller tube if all other conditions remained the same?
7•18. Forced Convection
A No. " 0 " bare electrical conductor having a diameter of 0.325 in . and an electrical resistance of 0.10 ohms per 1000 ft is exposed to a cross wind of 30 ft per sec. If the gas temperature is $50 \mathrm{deg} F$ what current in amperes will result in a copper temperature of $150 \mathrm{deg} F$ if only forced convection is considered. Solve for air cooling and also for hydrogen cooling.
7-19. Free Convection
A steam radiator which is 30 in . high has 42 sq ft of surface area, an inside steam temperature of 240 deg F , and an outside air temperature of 70 deg F . How many Btu per hour will be lost to the room air by free convection?
7-20. Free Convection
Solve problem $7 \cdot 18$ for free convection.

## 7•21. Free Convection

Find the pounds per hour of 15 lb per sq in . absolute pressure saturated steam that would be condensed by the free convection loss from a 14 -section, single-column, 32 in . high steam radiator that has a total surface area of 35 sq ft . Assume the metal temperature to be equal to the steam temperature and the room air temperature to be 70 deg F .

## 7-22. Radiation

Find the pounds of steam per hour that would be condensed by the radiation from the steam radiator of problem $7 \cdot 21$ if it is painted aluminum and if the full surface area is assumed to be effective in radiation.

## 7•23. Radiation and Convection

Plot curves of the black body radiation and free convection heat losses per square foot from a 2 -ft-high surface against surface temperature. Extend the curves to a surface temperature of 250 deg F and find the value of emissivity that will cause the radiation loss to be practically equal to free convection through this temperature range.

## 7-24. Combined Radiation, Conduction, and Convection

Considering all modes of heat transfer involved, find the outside surface temperature and heat loss per square foot for a vertical furnace wall. It is made up of 3 in . of fire brick on the inside and 4 in . of common brick covered with aluminum paint. The inside of the firebrick is at 1600 deg F and the outside room air is at 70 deg F . The firebrick, $k=0.70$; the common brick, $k=0.4$ Btu per hr per sq ft per deg F per ft ; and the furnace wall is 3 ft high.

## 7-25. Radiation Correction for a Thermometer

When a thermometer is used to measure air temperature in an enclosure, the reading is affected by radiation from the enclosure walls. If the walls are cooler than air, the thermometer bulb transmits heat to the walls by radiation, and this heat must be absorbed from the surrounding air by convection. The thermometer bulb will have to be cooler than the air and will consequently read low.

Air leaving a heater passes through a large pipe having a wall temperature of 80 $\operatorname{deg} \mathrm{F}$. A thermometer introduced into the air stream indicates a temperature of 300 deg F. Assuming the film conductance around the thermometer bulb to be 2.0 Btu per hr per sq ft per deg F and the emissivity of the bulb to be $e=0.6$, calculate the true air temperature.

## Chapter 8

## 8-1. Indicator Cards

Indicator cards from a 5 in . by 4 in . $500-\mathrm{rpm}$ air compressor are 2.9 in . long. The area of the head end card is 1.30 sq in. and that of the crank end is 1.37 sq in ., using a 60 lb per sq in. per in. indicator spring. If the piston rod diameter is $7 / 8 \mathrm{in}$., calculate head and crank end mep and the total ihp.

## 8-2. Adiabatic Compression

During the compression stroke in a Diesel engine air initially at atmospheric pressure and $150 \mathrm{deg} F$ is compressed through a ratio of 16 to 1 , that is, the final volume is one-sixteenth of the initial volume. If this compression is done adiabatically calculate:
(a) The final pressure.
(b) The final temperature.
(c) The external work done per pound.
(d) The change in internal energy per pound of air.
(e) The heat extracted per pound of air.
8.3. Isothermal Comprcssion

Solve problem 8.2 using an isothermal compression.

### 8.4. Polytropic Compression

Solve problem $8 \cdot 2$ for a polytropic compression that corresponds to an exponent. ( $n=1.31$.)

### 8.5. Polytropic Compression

Although the usual method of obtaining the polytropic exponent is by a mathematical analysis of a pressure-volume indicator diagram, it can be found from pressure and temperature readings. If a compression starts at atmospheric pressure and 70 deg F and if the final pressure is found to be 100 lb per $\mathrm{sq} \mathrm{in}$. final temperature 320 deg F , calculate the value of the compression exponent $n$. Also find the work done per pound of air and the heat extracted.

### 8.6. Single-Stage Air Compression

For a single-stage air compressor taking air at 100 deg F and 15 lb per sq in. absolute pressure and compressing it to 75 lb per sq in . gage, calculate the temperature at the end of compression, the heat extracted during compression, and the net cycle work per pound of air compressed:
(a) For adiabatic compression.
(b) For isothermal compression.
(c) For polytropic compression $P V^{1.25}=$ constant.

### 8.7. Two-Stage Air Compression

A two-stage air compressor takes air at 15 lb per sq in . absolute pressure and 90 deg $F$ and delivers it at 100 lb per sq in . gage pressure. If the work is the same in the high- and low-pressure cylinders, the air is cooled back to 90 deg F in the intercooler and both cylinders compress with a polytropic exponent $n=1.32$, calculate:
(a) The correct intermediate pressure.
(b) The temperature at the end of each compression.
(c) The heat extracted during each compression per pound of air.
(d) The heat extracted in the intercooler per pound of air.
(e) The horsepower to compress 1000 cu ft of free air per minute.

### 8.8. Air Compressor Cooling

The aim of this problem is to compare single-stage compression with cooling during compression with two-stage compression with interstage cooling, but no cylinder cooling. Assume air at atmospheric pressure and 70 deg F compressed in two adiabatic stages to 90 lb per sq in. gage and cooled back to 70 deg F in the intercooler. Calculate the total cycle work per pound and then find the value of polytropic exponent $n$ that would correspond to the same cycle work with a single-stage compression. Compare the final temperature in the two cases.

### 8.9. Compression Efficiency

If the idea of a polytropic compression is extended to cover the case of turbulent energy losses during compression it may be used to analyze high-speed blower and supercharger cycles where the temperature rise during compression is greater than the adiabatic rise corresponding to the compression ratio used. This may be done by using equations $8 \cdot 5,8 \cdot 6,8 \cdot 13$, and $8 \cdot 15$. If there is no cooling and the actual cycle work per pound is 1.25 times the adiabatic work for the same pressure ratio, this would correspond to a compression efficiency of 0.80 on an adiabatic basis.

By equating the adiabatic work from equation 8.5 to 0.80 of the polytropic compression work from equation $8 \cdot 15$ and eliminating $T_{3}$ and $T_{4}$ by using equations $8 \cdot 13$ and $8 \cdot 6$, a final equation involving only the pressure ratio $\frac{P_{4}}{P_{1}}=\frac{P_{3}}{P_{1}}$ results. For pressure ratios from 1 to 4 plot a curve of $n$ corresponding to an 0.80 compression efficiency. The final and most useful curve would be that of the temperature ratio $T_{3} / T_{4}$ plotted against pressure ratio. From a family of such curves plotted for various efficiencies once the actual final temperature $T_{3}$ is found from test results, the corresponding adiabatic temperature $T_{4}$ for the same pressure ratio and the same initial temperature $T_{1}$ could be calculated. From this temperature ratio $T_{3} / T_{4}$ and the pressure ratio, the value of compression efficiency on an adiabatic basis could be read directly from the curves.

Complete such a curve for 0.80 efficiency and see if it can be plotted on semi-log or $\log -\log$ paper in such a way as to become a straight line.

## 8-10. Compressor Performance

A single-stage air compressor handles 1000 cu ft per minute of atmospheric pressure, 80 deg F air, and delivers it to a rcceiver at 80 lb per sq in . gage pressure. Its volumetric efficiency is 0.72 , its compression efficiency on an isothermal basis is 0.85 and its mechanical efficiency is 0.90 . If it rotates at 350 rpm , what must be its piston displacement and what horsepower is required to drive it?

## 8-11. Volumetric Efficiency

An air compressor with a clearance volume that is 0.07 times the piston displacement operates between atmospheric pressure and 80 lb per sq in. gage pressure. The polytropic compression exponent is 1.22 and the re-expansion curve has an exponent of 1.12 .

Find the volume at the end of the re-expansion of the clearance air and its temperature at that time. If it is assumed that the cylinder at the end of the intake stroke is filled with air at this temperature and atmospheric pressure, what is the volumetric efficiency?

## 8-12. Centrifugal Blower

A $2500-\mathrm{rpm}$ centrifugal air blower takes 1000 cu ft per minute at 70 deg F and 14.7 lb per sq in. and delivers it with a static pressure of 4 in . of water in a $7-\mathrm{in}$. diameter discharge pipe. If the blower input horsepower is 1.6 , calculate the blower efficiency.

## 8•13. Centrifugal Blower

An air blower handles 500 cu ft per minute of 14.7 per sq in. 80 deg F air and delivers it in a $5-\mathrm{in}$. diameter discharge pipe at a static pressure increase of 11.0 in . of water at an efficiency of 0.40 . Calculate the air horsepower and input horsepower.

### 8.14. Supercharger

A gear-driven supercharger takes air at an absolute pressure of 11 in . of mercury and -30 deg F at an altitude of $25,000 \mathrm{ft}$ and delivers it to an engine at 14 lb per sq in. absolute pressure. The supercharger has a compression efficiency of 0.70 referred to an adiabatic compression and the engine uses 15 lb of fuel and 7 lb of air per hp -hr.

Calculate the temperature at which the air is delivered to the engine, and the fraction of the engine's output that is used for supercharging.

### 8.15. Air Measurement

If the pressure drop across a 3.5 -in.-diameter, sharp-edged intake orifice is 5 in . of water, calculate the cubic feet per minute of 14.7 lb per sq in. absolute, 70 deg F
air flowing through it. What pressure drop would occur if an intake nozzle is used for the same flow?

### 8.16. Piston Effect of a Train in a Tunnel

A locomotive moving through a tunnel at 60 miles per hr fills 0.6 of the crosssectional area of the tunncl. Considering the air to be at 14.7 lb per sq in. pressure and 70 deg F and assuming that no air is forced out the end of the tunnel, what must be the approximate difference in pressure between the front and rear of the locomotive, expressed in inches of water?

## 8-17. Air in a Diesel Enginc Fuel-Injection Line

A Diesel engine fuel-injection pump has a total displacement of 0.01 cu in. and the injection valve will not open until an oil pressure of 2000 lb per sq in. gage is reached. Approximately what length of the $\frac{1}{1} 0-\mathrm{in}$. diameter fuel line must be full of air at atmospheric pressure to make it impossible for the injection valve to open?

### 8.18. Pressure Limit of an Air Compressor

If it is assumed that power and mechanical strength need not be considered, what would limit the maximum air pressure against which a compressor can deliver air?

## Chapter 9

## 9•1. Entropy of Gases

(a) Show isothermal, adiabatic, and polytropic compressions on a temperatureentropy diagram.
(b) Show the corresponding expansion curves.

## 9-2. Carnot Cycle

Derive the Carnot cycle efficiency by using the energy equation and integrating for the heat added between 1 and 2, Figure 9•1, and the heat discharged from 3 to 4.

### 9.3. The Carnot Engine

A Carnot cycle is to be performed in a cylinder with a maximum volume, $V_{3}$, Figure $9 \cdot 1$, of 1 cu ft , the lower temperature is 70 deg F , the upper temperature is 800 deg F , and the pressure $P_{3}$, Figure $9 \cdot 1$, is atmospheric. Show the $P-V$ diagram of a Carnot cycle, indicating all pressures and volumes at points 1, 2, 3, and 4, Figure 9•1.

## 9•4. Reversible Cycle Efficiency

Assuming 70 deg F as the lowest available temperature, plot curves of power cost in cents per kilowatt-hour against the temperature at which heat is supplied for both coal and oil as fuel, assuming the conversion at the ideal reversible cycle efficiency. Plot curves from a supply temperature from 500 to 3000 deg $F$ for both coal at $\$ 5.50$ a ton with a higher heating value of $13,500 \mathrm{Btu}$ per lb and for fuel oil with a specific gravity of 0.80 , a higher heating value of $20,200 \mathrm{Btu}$ per lb and a cost of 5.5 cents per gal.

### 9.5. Simple Cycle Efficiency

Calculate the efficiency of a cycle that is accomplished by filling a cylinder with atmospheric pressure, 70 deg F air which is then heated at constant volume to a final temperature of 800 deg F . After being heated the air is expanded at constant entropy to atmospheric pressure. Find the pressure at the end of heating, the temperature at the end of expansion, the cycle efficiency, and the output work per pound of air. Show this cycle on a temperature entropy diagram and point out its shortcomings.

### 9.6. Availability of Energy

One pound of air is compressed adiabatically from 80 deg F and 14.7 lb per sq in . absolute pressure to 200 lb per sq in . gage pressure. It is then heated at constant volume to 3000 deg F . If this air is then expanded adiabatically to its initial volume, calculate the portions of the heat that are available and unavailable at each point, considering the initial heat content as zero.

### 9.7. Available Energy

In a steam plant 8 lb of steam are formed per pound of coal fired and 14.5 lb of products of combustion are formed for each pound of coal fired. All the products of combustion pass over a convection superheater and are cooled from 1200 to 750 deg F. All the steam enters the superheater at 300 lb per sq in. absolute and a quality of 0.99 .
(a) Assuming the furnace gases to have the properties of nitrogen, calculate the temperature at which the superheated steam leaves.
(b) Calculate the decrease in available energy caused by the transfer of heat from the high-temperature gases to the lower-temperature steam, assuming the lowest available temperature to be 65 deg F .

## 9-8. Carnot Cycle Using Steam

A Carnot cycle using steam operates between temperatures $T_{2}$ and $T_{1}$ (Figure $4 \cdot 1$ ). The upper temperature is the saturation value corresponding to 100 lb per sq in. absolute pressure, and the lower temperature corresponds to atmospheric pressure. Heat is added from $A$ to $B$ and a constant-entropy expansion from $B$ terminates at the lower temperature, $T_{1}$. Heat is extracted during the condensation at temperature $T_{1}$ and starting at the proper quality a constant entropy compression ends at the liquid line at $A$.
(a) Calculate the quality at which this compression must start, using steam table.
(b) Find the cycle efficiency from the steam tables by evaluating the heat added and the heat discharged.
(c) Compare this result with that evaluated from equation $9 \cdot 2$.

### 9.9. Entropy Change During a Polytropic Change

Explain how to use the gas tables to evaluate an entropy change during a polytropic expansion.

Evaluate the increase in entropy per pound of air expanding polytropically from 100 lb per sq in . absolute pressure and 1000 deg R to a pressure of $20 \mathrm{lb} \mathrm{per} \mathrm{sq} \mathrm{in}$. absolute. Show this expansion on a temperature-entropy diagram. Use a value of $n=1.26$.

## Chapter 10

## 10•1. Compression Ratio

The compression ratio of the $31 / 4 \mathrm{in}$. by $41 / 2 \mathrm{in}$. cylinder of a C.F.R. fuel-testing engine is varied by moving the entire cylinder up or down with respect to the piston top dead center position. Through what distance must it be moved to increase the compression ratio from 5 to 8 ?

### 10.2. Compression Pressure

For an adiabatic compression what is the ratio of final to initial pressure in an engine with a clearance volume of 0.086 times the piston displacement?

## 10-3. Burning Time and Flame Velocity

The spark plugs in an 8 -cylinder, $3800-\mathrm{rpm}, 4$-stroke cycle, valve-in-head automobile engine are placed in the side walls of the $33 / 8-\mathrm{in}$. diameter combustion spaces.

The indicator diagram shows burning to be completed in 10 deg of crank angle. Find the total burning time in seconds and the average flame velocity in feet per second.
10.4. Piston Velocity

What is the average piston velocity in a $33 / 8 \mathrm{in}$. by $35 / 8 \mathrm{in}$. 3700 -rpm engine? What is piston acceleration at top dead center if the ratio of connecting rod length to crank radius is 3.6 ? What gas pressure on the piston would be required to give the piston this acceleration if the piston weighs 14 oz ?

## 10-5. Octane Number

Instead of using pure octane and heptane for commercial testing, less expensive secondary reference fuels are used. In what proportions should two reference fuels of octane 90 and 60 be blended to produce a fuel of octane 77 ?

## 10•6. Ideal Otto Cycle

Derive the expression for the ideal Otto cycle efficiency by evaluating the useful work done by integrating for the net work area $3,2,4,1$, Figure $10 \cdot 4$, instead of using the method of equation $10 \cdot 2$.

### 10.7. Otto Cycle Efficiency

Assuming a higher heating value of $19,800 \mathrm{Btu}$ per lb of fuel used, plot a curve of fuel consumption in pounds of fuel per horsepower-hour for an ideal hot-air standard Otto cycle against compression ratio, extending the curve from $r=4$ to $r=12$.

## 10•8. Diesel Cycle Efficiency

For a Diesel engine having a clearance volume of 8 per cent of the piston displacement, plot a curve showing the ideal hot-air standard efficiency at which the last fuel burned is utilized. Extend the curve from a cut-off ratio of 1 to a value of 2.5.

### 10.9. Ideal Diescl Cycle

Derive the expression for the efficiency of the ideal Diesel cycle with constantpressure burning. This expression involves the compression ratio and the cut-off ratio.
10•10. Methods of Cycle Analysis
The products of combustion in an engine cylinder exist at $3500 \mathrm{deg} R$ at the end of burning. They are then assumed to be expanded reversibly and adiabatically through a volume ratio of 6 . Calculate the final temperature and expansion work done in Btu per pound for the following assumptions:
(a) Cold-air standard, $R=53.3, \gamma=1.4$.
(b) Hot-air standard, $R=53.5, \gamma=1.3$.
(c) Using gas tables but assuming it to be air.
(d) Using the combustion-mixture gas table.

In parts (a) and (b) it may be convenient to evaluate $c_{v}$ from $c_{v}=R / J(\gamma-1)$. Prove this to be true before using it.

## 10•11. Effect of Mixture on Engine Power

Proposition. Using the incomplete combustion assumptions of article 6.4 for a fuel with a weight composition of 0.86 C and $0.14 \mathrm{H}_{2}$, it is possible to show that theoretically the maximum heat released per pound of oxygen used occurs when just enough air is supplied to result in 17.7 per cent of the carbon of the fuel being burned to CO instead of $\mathrm{CO}_{2}$.
(a) Prove the proposition to be true or false.
(b) In the light of these results explain why an actual gasoline engine delivers maximum power at an air-fuel ratio of approximately 12.5 to 1 instead of at the ideal complete combustion air-fuel ratio of approximately 15.2 to 1 .

## 10•12. Diesel Engine Performance

A 5 in . by 7 in ., 2 -cylinder, 4 -stroke cycle Diesel engine with clearance volume equal to 7 per cent of piston displacement develops 15 hp at 600 rpm at a fuel consumption of 0.6 lb of $19,000 \mathrm{Btu}$ per lb fuel for each horsepower-hour.
(a) Calculate the compression ratio of the engine.
(b) Calculate the brake thermal efficiency.
(c) Calculate the brake mean effective pressure.

## 10•13. Otto Gas Engine

The $53_{8}^{3}$ in. by 10 in . Otto gas engine has a compression ratio of 4.85 and operates at 250 rpm . It it burns methane $\left(\mathrm{CH}_{4}\right)$ with theoretical air, what would be the ideal indicated horsepower and ideal indscated mean effective pressure if it operated at the hot-air cycle efficiency with a volumetric efficiency of 70 ?

## 10-14. Specific Output of Airplane Engine

An airplane engine delivers 1 bhp -hr for each 0.44 lb of fuel having a higher heating value of 19,400 Btu per lb at an ar-fuel tatio of 14.1 to 1 . It has an estimated volumetric efficiency of 0.80 , a mechameal efficiency of 0.84 , and weighs 0.76 lb per b hp output. Find the indicated horsepower per cubic inch of piston displacement and the bmep for an average piston speed of 3000 ft per minute.

## 10•15. Diestl C'ycle Efficiency

An ideal Diesel cycle has a compression ratio of 13.53 to 1 , uses a fuel with a higher heating value 20,500 and a lower heating value $18,800 \mathrm{Btu}$ per lb ) with an air-fuel ratio of 20 to 1 . Using the air table for the compression and combustion-mixture table of article 4.5 for the entire combustion and expansion portions of the ideal Diesel cycle, calculate the ideal cycle efficiency tor a rut-off rator of 2 to 1 . (Assume intake temperature $=110 \mathrm{deg} F$ and intake pressure $=14.7 \mathrm{lb}$ per sq in.)

## 10•16. Gasoline Engine

Calculate the probable brake horsepower for a $3 \frac{1}{4} \mathrm{in}$. by $37 / 8$ in., 8 -cylinder, 4 stroke cycle, 3000 -rpm automobile engine having a compression ratio of 6 . Assume a mechanical efficiency of 0.70 , volumetric efficiency of 0.65 , an air-fuel ratio of 15.2 to 1 , and a lower heating value for the gasoline of $18,000 \mathrm{Btu}$ per lb to calculate the probable brake thermal efficiency and pounds of fuel per brake horsepower based on the hot-air standard cycle efficiency.

## 10•17. Effect of Air Density on Engine Output

If thermal efficiency and fuel mixture are constant, the indicated horsepower of an internal-combustion engine varies directly as the air density in the cylinder at the beginning of the compression stroke. This density varies as the outside air pressure and approximately as the inverse square root of the absolute external temperature. If it were not for the heating of the mixture during intake it would vary inversely as the absolute temperature, but changing this to the square root has been found experimentally to account for this heating effect with satisfactory accuracy.

If the friction horsepower is assumed to remain constant, which is not precisely true, what will be the reduction in brake horsepower of an unsupercharged engine at an altitude of $20,000 \mathrm{ft}$ where the barometer reading is 13.75 in . of mercury and the air temperature is -12.3 deg F ? The full-load mechanical efficiency of the engine at sea-level pressure and 59 deg F air temperature is 0.83 .

## 10-18. Engine Cooling

If the engine of the preceding problem is supercharged, it may be assumed that the heat loss from the cylinders for satisfactory cooling must be the same at 20,000 -
ft altitude as at sea level. Referring to the basic forced convection heat trausfer equations, decide what change in velocity of the cooling air over the cylinders will be necessary between sea level and $20,000 \mathrm{ft}$ altitude to insure the same cylinder temperature for an air-cooled engine.

## Cilapter 11

### 11.1. Steam Tables

Find the quality or superheat, enthalpy, entropy, and internal energy for 1 lb of steam at 200 lb per sq in . absolute when it occuphes a volume of (a) 2.00 cu ft and (b) 3.00 cu ft .

## 11-2. Properties of Stcam

Using the Moller diagram and superheated steam tables evaluate the volume per pound of steam at various pressures between 500 lb per sq 1 n . and 1 lb per sq in . absolute at a constant temperature of 500 deg F . Show this on the same $P-V$ diagram with a calculated isothermal passmg through 500 lb per sq in . pressure and 500 deg $F$ that is based on the assumption that superheated steam values may be predicted by the gas equations. By how much does the actual superheated steam isothermal fail to be checked by the corresponding gas equation values?

## 11-3. Steam Charts

On pressure-volume, temperature-entropy, and ent halpy-entropy diagrams, sketch rurves of constant pressure, constant volume, constant entropy, constant temperature, and constant enthalpy changes for steam, including both wet steam and superheat regions.

### 11.4. Constant Entropy Expansion of Superheated Steam

Using steam tables evaluate pressure and volume per pound at various pressures for a constant entropy expansion of steam from 100 lb per sq in. absolute pressure and 500 deg F. Evaluate the exponent $n$ in $P^{P} V^{n}=$ constant, and find an average value of $n$ to fit the constant entropy expansion.

## 11•5. Constant Entropy Expansion of Wet Steam

Evaluate an average value of $n$ to fit the equation $P V^{n}=$ constant to a constant entropy expansion of 100 lb per sq in . saturated steam expanding to a moisture content of 10 per cent.

### 11.6. Steam Mixing

What is the final condition of the mixture when 10 lb of superheated steam at 100 lb per sq in. absolute and 500 deg F is mixed with 8 lb of sat urated water at the same pressure and 327.81 deg F? Find the initial and final total entropy and explain the significance of the change. Show the initial condition of the steam and the water and the final condition of the mixture on $P-V, h-s$, and $T-s$ diagrams.

### 11.7. Throttling

Show a throttling process for a superheated vapor on $P-V, h-s$, and $T-s$ diagrams.

### 11.8. Throttling Calorimeter

On a day when a barometer reading is 30.04 in . of mercury, 120 lb per sq in. gage pressure wet steam when passed through a throttling calorimeter results in a calorimeter temperature of 238 deg F. Calculate the initial steam quality, enthalpy, internal energy, entropy, and specific volume.

### 11.9. Steam Calorimeter

Steam at 116 lb per sq in . gage pressure passes through a throttling calorimeter and the calorimeter temperature is 240 deg F with a barometric pressure of 30.20 in. of mercury. Deriving equations used, calculate the initial steam quality.

## 11•10. Steam-Engine Condensation

A 12 in . by 16 in., single-cylinder, 300-rpm, double-acting, slide-valve steam engine that uses steam initially at 130 lb per sq in . absolute and 0.985 quality has such a high initial condensation that at cut-off occurring at 0.4 stroke the quality is 0.85 , the pressure remaining constant. How many pounds of steam per hour is used by the engine?

### 11.11. Rankine Cycle

Compare the complete expansion Rankine cycle efficiency and ideal pounds of steam per kilowatt-hour for an initial steam condition of 350 lb per sq in . absolute pressure and 540 deg $F$ and a condenser pressure at atmospheric with that for the same initial steam conditions and a condenser pressure of 1 in . of mercury absolute.

## 11•12. Complete Expansion Rankine Cycle

An ideal complete expansion engine with no clearance uses 130 lb per sq in. absolute steam at a quality of 0.98 , and exhausts to atmospheric pressure.
(a) Calculate the necessary point of cut-off to permit complete expansion.
(b) Calculate the cycle efficiency and ideal water rate in pounds of steam per horsepower-hour.

## 11•13. Nonexpansion Rankine Cycle

For the steam conditions of problem $11 \cdot 12$, calculate the cycle efficiency and ideal water rate for a nonexpansion cycle.

### 11.14. Incompletc Expansion Rankine Cycle

For the steam conditions of problem 11•12, calculate the cut-off for an ideal incomplete expansion cycle to have a release pressure of 30 lb per sq in. absolute.

### 11.15. Boiler-Feed Pump

If a boiler that operates at 125 lb per sq in. gage pressure has its feed water delivered by an ideal nonexpansive engine with an atmospheric pressure exhaust, how many pounds of boiler steam, assumed to reach the engine with 0.98 quality, must be used to pump a pound of water into the boiler?

Since the steam tables do not give the volume of liquid at atmospheric pressure and 150 deg F it must be approximated. When doing this it is important to realize that temperature has a much greater effect on liquid volume than pressure.

### 11.16. Incomplete Expansion Rankine Cycle

The steam in the cylinder of an ideal steam engine at cut-off is at 120 lb per sq in. absolute pressure and has a quality of 0.98 . The release pressure is 30 lb per sq in . absolute and the condenser pressure is 5 lb per sq in. absolute. Find the quality of the steam at release, the ideal cycle efficiency, the ideal pounds of steam required per horsepower-hour, and the quality of the steam when it is in the condenser.

## 11•17. Incomplete Expansion with Throttling

Steam at 150 lb per sq in. absolute pressure and a quality of 0.985 is throttled to 100 lb per sq in. absolute and admitted to an ideal incomplete expansion Rankine engine. If the release pressure is 25 lb per sq in. absolute and the exhaust is atmospheric pressure, calculate the necessary cut-off point, the cycle efficiency, and the ideal water rate in pounds of dry steam per horsepower-hour.

### 11.18. Steam-Engine Performance

A steam engine using steam at 150 lb per sq in. absolute pressure and 0.975 -quality steam operates with a 0.25 -stroke cut-off and the steam expands with a polytropic exponent of $n=1.18$ as found from the indicator card.
(a) Neglecting clearance and assuming no condensation during admission, calculate the pressure and quality at release.
(b) Calculate the heat transferred from the cylinder during expansion, expressing it as Btu per pound of steam.
(c) If it is an atmospheric-exhaust engine compare its efficiency with that of a complete expansion Rankine cycle for the same steam conditions.

## 11-19. Steam Compressor

In a plant having available only 120 lb per sq in. gage-pressure steam at 0.985 quality, a small quantity of 250 lb per sq in. gage-pressure steam is needed for process work and it is estimated that it would be cheaper to compress the low-pressure steam than to install a separate steam generator.

For an ideal compression calculate the input work per pound of steam compressed, the ideal horsepower for 100 lb of steam per hr and the cubic feet per minute of steam at inlet conditions.

## 11-20. Double-Expansion Steam Engine

Steam at 300 lb per sq in. absolute pressure and 500 deg F is to be expanded to an absolute pressure of 5 in . of mercury in two ideal stages. Using the Mollier diagram find the correct intermediate pressure to balance the work between the high- and low-pressure cylinders and the condition of the steam as it enters the low-pressure cylinder. Find the correct ratio of low-pressure to high-pressure cylinder volume and the ideal pounds of steam per horsepower-hour.

## Chapter 12

### 12.1. Diverging Nozzle Analogy

An elastic ball moves between two walls, each similar to Figure 12•2, with an initial velocity of 100 ft per sec and equal components in the transverse and longitudinal directions. If each wall diverges at an angle of 8 deg , calculate the two components of velocity after the third elastic impact of the ball as it rebounds along the passageway. What fraction of its total kinetic energy is changed from the transverse direction to the axial with each impact?

### 12.2. Nozzle Equations

Derive the nozzle equations of article $12 \cdot 3$, including all the mathematical steps. 12•3. Nonexpanding Gas Nozzle

The oxygen tip of a welding torch is $1 / 8-\mathrm{in}$. inside diameter and is supplied with oxygen at 70 deg F and 13 lb per sq in . gage pressure. Calculate the velocity of the oxygen as it leaves the nozzle and also find its temperature and density. Calculate the discharge in pounds per hour.
12.4. Air Nozzle

An air nozzle is to be supplied with 100 lb per sq in. gage pressure, 80 deg F air by a compressor that handles 5000 cu ft per minute of atmospheric pressure 70 deg F air. The nozzle is to discharge the compressor output with an atmospheric back pressure. Calculate the throat and discharge velocities and nozzle areas and the length of the diverging portion of the nozzle if the total included angle of the nozzle walls is 14 deg.

### 12.5. Air Jet

Calculate the pounds of air per second that would be discharged through a nonexpanding nozzle $1 / 2 \mathrm{in}$. in diameter discharging against atmospheric pressure when the initial air pressure is 10 lb per sq in. gage, and when it is 25 lb per sq in. gage, its initial temperature being 80 deg F .

## 12•6. Velocity Measurement

If a small impact or Pitot tube is pointed into a stream of high-velocity gas, the absolute pressure indicated by a gage attached to the tube will be the sum of the static pressure of the stream and impact pressure that results from bringing the gas to rest adiabatically. The energy equation and the adiabatic equations may be written between the point in the moving stream where the pressure is $P_{1}$, the temperature is $T_{1}$, the velocity is $v_{1}$, and the impact tube where the velocity is zero, the pressure is $P_{2}$, and the temperature is $T_{2}$. Thus it $P_{1}, P_{2}$, and either $T_{1}$ or $T_{2}$ are known, it is possible to solve for the velocity, $v_{1}$.

Derive such an equation assuming $T_{2}$ to be known and solve for the velocity of an air stream flowing at atmospheric pressure $P_{1}$ if the impact tube gage pressure is 32 in . of mercury and the temperature of the air as brought to rest in the impact tube is 206 deg F. Calculate the temperature, $T_{1}$, of the moving stream.

### 12.7. Air Diffuser

When high-velocity air from the impeller of a supercharger is discharged from the wheel at a high velocity it enters a diverging passageway known as a diffuser, where it is slowed down to a lower velocity before it is discharged into the casing. Thus, a diffuser is a nozzle in reverse.

Using the basic nozzle equations calculate the ideal final pressure and temperature of the air if it enters a diffuser at $220 \mathrm{deg} \mathrm{F}, 900 \mathrm{ft}$ per sec, and at 21 lb per sq in . absolute if its velocity is reduced to 300 ft per sec in the diffuser. This problem requires a trial-and-error solution, and may be solved using either gas tables or gas equations.

## 12•8. Nozzle Discharge

For a nonexpanding nozzle discharging to atmospheric pressure, plot a curve of pounds per second of steam discharge per square inch of nozzle throat area against the initial steam pressure at which steam is supplied. Assume the supplied steam always to be dry and saturated and extend the curve to an initial steam pressure of 100 lb per sq in. absolute.

### 12.9. Steam Nozzle Layout

A full expanding nozzle has a throat diameter of 0.6 in . and the expanding portion has a total included angle of 10 deg. Dry saturated steam at 120 lb per sq in. absolute pressure flows through it to an atmospheric back pressure. Calculate the discharge in pounds per second, the necessary discharge diameter, and the length of the expanding section.

## 12•10. Nozzle Discharge

Steam at a gage pressure of 120 lb per sq in. and a quality 0.985 is throttled to 90 lb per sq in. gage pressure and admitted to a nonexpanding nozzle having a throat diameter of 0.60 in . Calculate the critical throat gage pressure and the corresponding maximum discharge in pounds per second.

### 12.11. Steam Nozzle

The first stage nozzles of a steam turbine are supplied with 350 lb per sq in. absolute pressure, 600 deg F steam, and are to discharge steam at 2180 ft per sec. Find the ideal discharge pressure using the Mollier diagram and check the accuracy using
steam tables. Specify the type of nozzle needed and ralculate the total nozzle throat area needed in a $5000-\mathrm{hp}$ turbine that has a water rate of 7.6 lb of steam per hp -hr.

### 12.12. Supersaturation

Calculate the percentage error involved in neglecting the effect of supersaturation on discharge velocity and weight discharged for a turbine nozzle that takes steam at 20 lb per sq in. absolute pressure and 260 deg F and discharges to a pressure of 8 lb per sq in. absolute.

## 12•13. Nozzle Condensation

Assume that steam expands in a nozzle in a supersaturated condition until it is halfway between the saturation line and Wilson line, Figure $12 \cdot 4$, at which time the normal condensation suddenly occurs. Attempt to reason out the physical picture of the sequence of events that will follow.

## 12•14. Nozzle Losses

What back pressure must be maintained with a nozzle using 160 lb per sq in. absolute pressure, 440 deg F steam if a leaving velocity of 1200 ft per sec is desired and the nozzle energy efficiency of 0.92 ? What will be the leaving temperature of the steam? What would be the leaving velocity for an ideal nozzle between the same pressures?

### 12.15. Metering Nozzle

A rectangular nozzle ${ }_{6}{ }_{2} \mathrm{in}$. by ${ }^{5}{ }_{16} \mathrm{in}$. is used to meter air that enters it from a large pipe at 90 lb per sq in . gage pressule and 72 deg F . When the pressure drop across the nozzle is 27 in . of water, how many pounds per minute of air are flowing if the calibration of the nozzle indicates a velocity coefficient of 0.96 ? Would the assumption of constant density be accurate enough in this case? (See article 8.7.)

## 12•16. Acoustic Velocity

From the nozzle equations of article $12 \cdot 3$ express the acoustic velocity in the throat of a nozzle opetating at the critical pressure ratio in terms of the gas constants and the initial temperature $T_{1}$. Express it also in terms of the throat temperature $T_{2}$. Is there a physical explanation for the fact that the pressure does not enter into the expression for critical or acoustic velocity?

## 12-17. Turbine Blade T'emperature

For 1200 lb per sq in. absolute pressure, 1000 deg F steam expanding through the first stage nozzles of a turbine to a pressure of 650 lb per sq in., calculate the ideal discharge velocity and the temperature that would be indicated by a thermometer traveling with the steam. If the first stage turbine blades have a velocity one-half that of the steam and in the same direction, the maximum temperature of the blades would be that resulting when the steam at the condition as it leaves the nozzle is decelerated to half that velocity at constant entropy. Derive the necessary equations and calculate that temperature using steam tables or the Mollier diagram.

## Chapter 13

### 13.1. Thrust of a Jet

Using the concepts of article 13.1 find the reartion force on the nozzle when air at 80 lb per sq in . gage and 500 deg F expands through a full expanding nozzle to a back pressure of atmospheric pressure. (The thrust to create a fluid jet is the same as the force required to stop it.)

### 13.2. Jet Propulsion

If the products of combustion of a rocket leave it at a relative velocity of 4000 ft per sec, plot a curve of the thrust in pounds produced per pound per second of products of combustion discharged against the velocity of the rocket.

## 13•3. Turbine Velocity Diagram

A simple impulse turbine stage which operates with a steam velocity of 1000 ft per sec and a nozzle angle of $181 / 2 \mathrm{deg}$, has a bucket pitch diameter of 4 ft and turns 1800 rpm . Solve both analytically and graphically for the entrance angle of the buckets, the relative entering velocity, and the absolute leaving steam velocity. For each pound per second of steam, calculate the force on the blading, the torque and horsepower developed. Calculate the stage efficiency by using the input and output and also by using the input and the leaving energy.

## 13•4. Effect of Turbine Blade Friction

Excessive friction losses in the blading of the turbine stage of problem $13 \cdot 3$ causes the relative velocity at which the steam leaves the bucket to be 0.8 of the relative entering velocity. Draw the resulting velocity vector diagram, calculate each item asked for in problem $13 \cdot 3$ and in addition the side force on the wheel for each pound per second of steam.

### 13.5. Turbine Blading Angles

If the turbine wheel of problem $13 \cdot 3$ has blading with a root diameter of 3 ft 3 in . and a tip diameter of 4 ft 6 in ., what is the proper entrance angle at the blade root and at the tip? If the leaving angle of the entire blade is constant at the value that is correct for the root section, what is the difference in magnitude and direction of the absolute leaving velocity between the blade root and tip?

### 13.6. Turbine Steam Flow

Suppose the blading of the turbine stage of problem $13 \cdot 3$ to be 6 in . long and to be spaced so the distance between blades is 1 in ., measured normal to the relative entering velocity of the steam. If the steam leaves the nozzles at 70 lb per sq in . absolute pressure and 380 deg F , find the pounds per second passing through one opening between blades. Calculate the total force on the blade in pounds.

### 13.7. Impulse Turbine Stage

Derive all equations involved and plot curves of ideal stage efficiency and stage horsepower and torque for each pound of steam used per second against wheel rpm. Assume a steam velocity of 1500 ft per sec, a nozzle angle of 15 deg , and a wheel diameter designed to give maximum efficiency at 1800 rpm .

### 13.8. Variable-Speed Turbine

For the solution of problem 13.7 to be correct the blade angle would have to be different at each operating condition, but in actual practice this is not true. Suppose the blade angle to be fixed at the proper value for $1800-\mathrm{rpm}$ operation and assume that at any other operating speed only the component of the relative entering velocity that is parallel to this blade angle is useful. Recalculate the curves for problem 13.7. New equations must be derived to fit these assumptions.

### 13.9. Single-Stage Gas Turbine

The nozzles of a single-stage gas turbine are set at an angle of 20 deg and expand products of combustion from fuel oil burned with 20 per cent excess air from 100 lb per sq in. gage pressure and 1400 deg $F$ to a back pressure of atmospheric pressure.

If the wheel is to run at $20,000 \mathrm{rpm}$, what is its proper pitch diameter and the ideal horsepower developed per pound per second of gas?

### 13.10. Turbine Runaway Speed

A turbine consists of a single double-velocity, impulse-stage wheel with a pitch diameter of 24 in . The nozzles are at 18 deg. The initial steam is at 120 lb per sq in. absolute, dry and saturated, and the turbine exhausts to atmospheric pressure. Such a turbine is governed by throttling the steam to a lower pressure before admission to the nozzles.

If at no load the governor fails to operate, what maximum runaway rpm would be reached if the wheel did not fail first? Neglect wheel windage and friction and after finding this top speed explain what is happening to the steam in each set of moving blades. This entire analysis will, of course, assume that the biading angles are proper for this condition of operation which is not accurate. An actual turbine could not reach the speed so calculated because of the turbulence of the stcam passing improperly through the blading.

### 13.11. Curtis Double-Velocity Impulse Stage

A simple atmospheric-exhaust turbine has a single-pressure, double-velocity stage Curtis wheel 4 ft in diameter and turns 1800 rpm . Saturated steam at 80 lb per sq in. gage enters the nozzles and is discharged at an angle of $161 / 2$ deg into the atmospheric pressure of the casing.
(a) Draw the vector diagram and determine the correct entrance angles for the two sets of blades.
(b) Calculate the turbine horsepower per pounds of steam per second, ignoring friction.
(c) Calculate the stage efficiency.
(d) Calculate the ideal water rate.

### 13.12. Reaction Turbine Stage

In a symmetrical reaction turbine stage the steam leaves the stationary blading with an absolute velocity of 1200 ft per sec at an angle of 16 deg . The moving blades are on an $1800-\mathrm{rpm}$ wheel with a pitch diameter of 5 ft .
(a) Calculate the absolute velocity with which the steam leaves the moving blades and enters the stationary ones.
(b) Calculate the necessary energy drop in Btu per pound across the stationary blades and across the moving blades.
(r) Calculate the stage horsepower for one pound per second of steam.
(d) Calculate the stage efficiency.

### 13.13. Turbine Efficiency

A small single-stage impulse turbine uses 120 lb per sq in. gage pressure, 0.985 quality steam and at rated load the steam is throttled to 105 lb per sq in. gage pressure before being admitted to the nozzles. If the back pressure is 5 in . of mercury absolute and the steam is exhausted with 10 deg F of superheat, what is the stage efficiency and the water rate in pounds of steam per horsepower-hour?

### 13.14. Preliminary T'urbine Layout

The high-pressure end of a turbine is to use steam at 1000 lb per sq in . absolute pressure and 850 deg F and discharge it at a pressure as near 350 lb per sq in . absolute pressure as will fit a full number of stages. The turbine is to turn at 3600 rpm and is to have a Curtis double-velocity, first-stage wheel to reduce the casing pressure to 650 lb per sq in.
(a) Ignoring losses, find the diameter of this first-stage wheel to give maximum efficiency for the pressure drop from 1000 lb per sq in . to 650 lb per sq in . using a nozzle angle of 16.5 deg .
(b) Ignoring all losses and assuming all succeeding stages to have this same wheel diameter, calculate the number of simple impulse stages to reduce the pressure to approximately 350 lb per sq in., designing each stage for maximum efficiency and 16.5 deg nozzle angles.
(c) From the Mollier chart, find the various stage pressures.
(d) Assuming 0.65 efficiency for the first stage and 0.76 for each of the others, lay out the steam path on a Mollier chart for the stage pressures found in ( $r$ ).
(e) Calculate the cycle efficiency, thermal efficiency, turbine efficiency, and water rate in pounds per horsepower-hour.

### 13.15. Exhaust-Gas Turbine

A $1000-\mathrm{hp}$ internal-combustion engine is equipped with an exhaust-gas-driven turbo-supercharger. The engine uses 0.5 lb of fuel per b hp -hr with an air-fuel ratio of 15 to 1 and the exhaust gaves leave at 30 lb per sq in. gage pressure and 1600 deg $R$ at an altitude where the barometric pressure is 10 in . of mercury.
(a) What ideal horsepower is available in the gases and what would be the final gas temperature? (Assume Table 4.9 to apply to the exhaust gases.)
(b) What would be the theoretical rpm for a single-stage wheel with a $14-\mathrm{in}$. pitch diameter if the nozzles are set at 20 deg ?
(c) Suppose a turbine of two stages is used and one-half of the available pressure drop is used in each stage, find: (1) The horsepower developed in the first stage with a stage efficiency of 0.70 . (2) Assuming the losses of stage one to rehent the gases at constant pressure between stages, calculate the horsepower developed in the second stage if it has a stage efficiency of 0.70 . (3) Compare the actual developed horsepower with the ideal available horsepower.

### 13.16. Turbine Steam Consumption

A throttle governed single-stage impulse turbine has five 0.60 -in.-diameter nozzles and exhausts at atmospheric pressure. If it is supplied with 130 lb per sq in . absolute pressure, dry saturated steam, plot a curve of ideal total pounds of steam used per hour, against ideal horsepower output. Solve by assuming various nozzle pressures after throttling and find the corresponding ideal rate of steam flow and ideal work per pound of steam. Extend the curve from maximum output to about one-fourth load and include a curve of nozzle pressure plotted against horsepower.

## Chapter 14

### 14.1. Condenser Air

The presence of air in a condenser resulted in the temperature of the air-vapor mixture in the condenser being 100 deg F when the condenser pressure was 2.15 in . of mercury. For such conditions how many cubic feet of mixture must be withdrawn from the condenser for each pound of air removed and how many pounds of vapor accompany such a pound of air?

### 14.2. Condenser Air Remonal

When the open boiler-feed water heater of a plant is not in use, the $50 \mathrm{deg} F$ atmospheric pressure boiler feed water contains as much as 0.05 absorbed air by volume. This air passes through the boiler and turbine into the condenser where it must be removed by the reciprocating vacuum pump. From the volume of water discharged and the number of working strokes per minute of the pump, it was estimated that at the end of each suction stroke the pump cylinder was one-sixth filled with water and five-sixths with air-vapor mixture.

If it is assumed that in the pump none of the air is absorbed in the water and the mixture is at a temperature of 100 deg F , what is the condenser pressure? What would be the condenser pressure if no air were present?

## 14-3. Jet Condenser

Steam from a turbine enters a jet condenser at a pressure of 1.5 in . of mercury (absolute) and a quality of 0.887 . How many pounds of 70 deg F spray water per pound of steam must be used to maintain that condenser pressure? Find the approximate condenser pressure that would result if half that much spray water is used, assuming the enthalpy of the entering steam to remain the same as before, which, of course, it would not.

### 14.4. Cooling Tower

Where condenser circulating water is cooled in a spray tower the water cascades down over baffles and air is blown upward through it. The limiting condition would be for the air to leave the top saturated with vapor at the incoming water temperature and for the cooling water to leave the bottom at a temperature approaching the wet bulb temperature of the incoming air. In such a spray tower where the air enters the bottom at 85 deg F and 65 deg F wet bulb temperature, the condenser water enters the top at 115 deg F. Assume the limiting conditions to be satisfied and calculate the pounds of dry air that must enter per pound of cooling water entering and the fraction of each pound of entering condenser water that is lost by evaporation.

## 14-5. Condenser Heat Transfer

Assuming very good condenser conditions (Figure 14-2), find the temperature at which the cooling water will leave a 10 ft long tube with a $1 / 2-\mathrm{in}$. inside diameter if the steam temperature is 110 deg F and the cooling water enters at 70 deg F and flows 4 ft per sec? All other conditions being the same, what would the leaving temperature be if the tube is 1 in . inside diameter?

### 14.6. Condenser Heat Transfer

Tests of a condenser having $0.80-\mathrm{in}$. outside diameter tubes with 0.04 in . thick walls gave an overall conductance $U=420 \mathrm{Btu}$ per hr per sq ft per deg F with a water velocity of 5 ft per sec when new, and this dropped to $U=180$ for the same conditions when the tubes had fouled. From this calculate the steam film conductance for new tubes and the combined steam film and scale conductance for the fouled tubes.

### 14.7. Condenser Performance

A 15 -kw turbogenerator using 90 lb of saturated 110 lb per sq in . gage steam per kw -hr discharges its exhaust steam into a surface condenser at an absolute pressure of 5 in. of mercury. The condenser has a double pass and has a total of 1966 -ft tubes, 0.50 -in. inside diameter, and $0.625-\mathrm{in}$. outside diameter.

The cooling water enters at $60 \mathrm{deg} F$ and leaves at 110 deg F and the condensate leaves at 105 deg F. The combined electrical and mechanical efficiency of the unit is 0.70 .
(a) Find the Btu per hour absorbed by the condenser.
(b) Find the pounds of cooling water per hour and the water velocity in the tubes.
(c) Calculate the overall conductance in Btu per hour per square foot per degree Fahrenheit.
(d) Calculate the steam film conductance.

### 14.8. Condenser Layout

A steam turbine uses 9 lb of 280 lb per sq in. absolute pressure 500 deg F steam per hp-hr and develops $30,000 \mathrm{hp}$ with a condenser pressure of 2 in . of mercury ab-
solute. Cooling water enters at 70 deg F and is to leave 9 deg F cooler than the saturation temperature of the steam in the condenser. Cooling water flows through the $1-\mathrm{in}$. inside diameter, 0.05 in. thick-walled tubes at 3.0 ft per sec and fair maintenance is assumed. Assuming a double-pass condenser calculate the square feet of condenser surface required per turbine horsepower and the total number and length of tubes required. Find the cubic feet per hour of cooling water used.

### 14.9. Condenser Size

If in problem 14.8 the water velocity in the tubes is raised to 6 ft per sec, all other conditions remaining the same, calculate the number of feet of condenser tubes that could be saved and find the new condenser dimensons.

## 14•10. Condenser Variables:

With a water velocity of 2 ft per sec through a 1 -in. inside diameter condenser tube 12 ft long, the pressure drop is 0.2 lb per sq in ., and this pressure loss varies as the square of the water velocity. The water is assumed to enter at 60 deg F , the steam temperature is assumed as 105 deg F , and the conductance of the steam film is assumed constant at 1000 Btu per hr per sqft per deg F . Plot the following curves against the water velocity using values from 1 to 12 ft per sec:
(a) Overall conductance.
(b) Leaving-water temperature.
(c) Log-mean temperature difference.
(d) Btu per hour exchanged in the tube.
(c) Btu removed per pound of cooling water.
(f) Pressure drop in pounds per square inch.
(g) Pumping horsepower.
(h) Btu exchanged per horsepower-hour of pumping work.

## 14-11. Condenser Relief Valve

If the condenser of problem 14.8 is to be equipped with a relief valve that will allow the full steam load to be blown to the atmosphere it the pump fails, what must be the approximate net area of the safety valve opening if the condenser pressure is not to exceed 20 lb per sq in . gage?

### 14.12. Window Condensation

Test results indicate that when the humidity of the 70 deg F air in a room is 0.40 , condensation will start on single pane glass windows when the outside air is 35 deg F. Can this be checked by using heat transfer and humidity theory?

## Chapter 15

### 15.1. Flame Temperature

When no heat is lost from a flame the maximum temperature reached is governed by the tendency of the products of combustion to dissociate into fuel elements at higher temperatures. Usually, however, the rate of heat loss is large enough to prevent the flame temperature from reaching a value where dissociation is important. Assuming no dissociation, calculate the maximum flame temperature that would result from the combustion of a fuel oil that is 0.76 carbon and 0.14 hydrogen by weight if it is burned with 20 per cent excess air in a furnace where the heat loss by radiation during combustion is one-third of the heat released. (Assume that the one-third heat loss is based on the lower heating value for if the higher heating value were used the heat of vaporization of the water vapor would have to be considered.) Use Table $3 \cdot 11$ and an initial air temperature of 80 deg F.

### 15.2. Air-Fuel Ratio

If the fuel of problem $15 \cdot 1$ is to be burned in the combustion chamber of a gas turbine, what air-fuel ratio must be used to maintain a gas temperature of 1600 deg F if radiation loss is neglected?

## 15•3. Convection Super heater

A convection superheater is made of $1.75-\mathrm{in}$. outside diameter tubes with $3 / 16 \mathrm{in}$. thick walls. At a light load the average steam temperature is 500 deg F , the mass flow of the steam is $\left(i_{s}=10 \mathrm{ll}\right.$ per sec per sq ft of tube area, the average gas temperature is $2000 \mathrm{deg} F$, and the rate of gas flow is $G_{g}=3 \mathrm{lb}$ per see per sqft of net area between tubes. If the load on the boiler in doubled, $G_{s}$ will be doubled, and since the fuel consumption will increase in a like proportion, $a_{r}$ will be approximately doubled. Calculate the rate of heat transfer per square foot of surtace for both cases, assuming the same steam and gas temperatures, and from this predict what effect increased load has on temperature rise in a superheater if the gas temperature is not changed.

### 15.4. Convection Superhcater

In a 350 lb per sq in . absolute boiler, steam with a quality of 0.98 goes to the superheater tubes and leaves the superheater at 600 deg F . When the boiler output is $30,000 \mathrm{lb}$ of steam per hour, 3000 lh of coal an hour are burned with 16 lb of air per pound of coal. The net gas passage area between the superheater tubes is 7 sq ft and the gases are at 2200 deg F average temperature as they pas, over the tubes.

If the superheater consists of $181.75-\mathrm{in}$. outside diameter, $3_{16}$-in. walled tubes in parallel, calculate the necessary tube length, if all the heat transter to the tubes is by convection.

## 15•5. Steam-Gcnerator Heat Transser

The feed water enters the economizer of a 350 lb per sq in . absolute pressure steamgenerating unit (Figure $15 \cdot 2$ ) at 100 deg F and from there enters the boiler drum at 400 deg F. It leaves the boiler to enter the superheater with 8 per cent moisture and leaves the superheater at 650 deg F. If the unit has an efficiency of 0.85 , what fraction of the heat of the fuel is transferred in the economizer, the boiler tubes, and in the superheater?

### 15.6. Radiant Heat Transfer

In a powdered coal-fired boiler the furnace wall firebrick are protected by water tubes known as a water wall. With an average flame temperature of 3000 deg F , a water-wall metal thickness of 0.3 in . and an average water temperature of 350 deg $F$, calculate the rate of heat transfer per unit area by radiation. (A trial-and-error method of determining the metal surface temperature must be used.)

## 15•7. Tube Bank Air Cooler

A rectangular duct has a staggered tube bank made up of $0.5-\mathrm{in}$. outside diameter, 0.4 -in. inside diameter brass tubes spaced 1 in . apart across the duct and ten rows deep in the direction of flow. Water at an average temperature of 45 deg $F$ flows 4 ft per sec through the tubes. The air has a velocity of 10 ft per sec in the unobstructed duct ahead of the tubes and is at an average temperature of $85 \operatorname{deg} \mathrm{~F}$ as it passes over the tube bank.

Using the condenser heat transfer data from Chapter 14 to find the water film conductance and the boiler heat transfer equations from article $15 \cdot 2$ to evaluate the gas film conductance, calculate the average overall conductance.

What temperature drop would be expected as the air flows through such a tube bank that is ten rows deep if the air pressure is atmospheric?

### 15.8. Boiler Losses

Coal having an analysis $\mathrm{C}=0.82, \mathrm{H}_{2}=0.06, \mathrm{O}_{2}=0.04$, moist ure $=0.02$, ash $=$ 0.06 is fired in a furnace and the weight of ashpit refuse removed is one-tenth the weight of the coal fired. The boiler-room air is at 75 deg F and 0.60 humidity and the stack gases are at 480 deg F . The dry stack gas analysis is $\mathrm{CO}=0.025, \mathrm{O}_{2}=$ $0.04, \mathrm{~N}_{2}=0.815$. Calculate all losses and the boiler efficiency.

### 15.9. Stack Gas Loss

Evaluate the dry stack gas loss for the example of article $15 \cdot 3$ using the gas tables of article 4.5 and determine the accuracy of the approximation used in the example.

## 15•10. Effect of Excess Air

Assuming the stack gases from a small oil-fired furnace to leave at $600 \mathrm{deg} F$ with a boiler-room temperature of $80 \mathrm{deg} F$, calculate the approximate change in the dry stack gas loss that accompanies each 10 per cent of excess air used.

## 15-11. Boiler Performance

 -The feed water is pumped into a 1200 lb per sq in. absolute boiler at $450 \mathrm{deg} F$ and the superheated steam leaves at 800 deg $F$. The boiler has a full-load efficiency of 0.84 and 3 tons of $14,100 \mathrm{Btu}$ per lb coal is burned each hour. Calculate:
(a) Boller output in Btu per hour.
(b) Pounds of steam per hour.
(c) Pounds of steam per pound of coal.
(d) Boiler horsepower developed.

15•12. Effect of Stack Temperature
In a small heating boiler using a fuel oil that is 0.86 carbon and 0.13 hydrogen by weight, calculate the drop in efficiency that would accompany a rise from 500 to $600 \mathrm{deg} F$ in the stack gas temperature if 20 per cent excess ar is used.
15•13. Combustion Experiment
Weigh a candle, determine the rate of weight lose as the candle burns, and estimate the volume of active flame. Estimate the Btu per hour released per cubic foot of active flame and compare this with the rate of combustion in an oil-burning heating furnace where 2 gal per hr of fuel oil is burned in a space 30 in . in diameter and 36 in. high.

## 15•14. Bagasse Furnace

In many sugar mills sugar cane fiber known as bagasse is burned to generate process steam and such installations represent the extreme in moisture loss to the stack, since bagasse may be as much as 50 per cent moisture. A test of such a plant (Transactions A.S.M.E., Vol. 61) gave the following results:

Fuel, 0.48 moisture by weight
Dry fuel analysis by weight
$\mathrm{H}_{2}=0.0580 ; \quad \mathrm{C}=0.4631 ; \quad \mathrm{O}_{2}=0.4582 ; \quad \mathrm{N}_{2}=0.0026$
Ash $=0.0181$
Heating value of dry bagasse $=8300$ Btu per lb
Test data
Incoming air temperature $=140 \mathrm{deg} F$
Flue gas temperature $=485 \mathrm{deg} \mathrm{F}$
Heat absorbed by steam per pound of fuel as fired $=2850 \mathrm{Btu}$
Stack gas volumetric analysis
$\mathrm{CO}_{2}=0.1645 ; \quad \mathrm{O}_{2}=0.0375 ; \quad \mathrm{CO}=0.0016$
Calculate the overall efficiency, the pounds of air supplied per pound of fuel fired, and show the probable heat balance in as much detail as the test data will permit.

15•15. Cooling of Exhaust Pipe
Using the heat transfer data of Chapters 7 and 15, calculate for the following conditions the probable outside surface temperature of an engine exhaust pipe that has a 4 -in. inside diameter and a wall thickness of 316 in.: the exhaust gases in the pipe are from gasoline burned with 20 per cent excess air and they have a velocity of 600 ft per sec, a temperature of 910 deg $F$, and a pressure of 15 lh per sq in. gage; the propeller slip-stream air which flows over the outside of the exhaust pipe is at a temperature of $-10 \operatorname{deg} \mathrm{~F}$, is at a pressure of 19 in . of mercury and has a velocity of 220 mph .

## Chapter 16

## 16•1. Rankine Cycle Efficiency

Find the percentage error that results from ignoring boiler-feed work when calculating the Rankine cycle efficiency for an initial steam condition of 750 lb per sq in. absolute and 800 deg F and a condenser pressure of 1 in . of mercury.

### 16.2. Extraction ('ycle

A steam plant operating on 450 lb per sq in. absolute pressure 600 deg F steam extracts steam for feed-water heating at 100 lb per sq in. absolute, and the condenser pressure is 1 in . of mercury absolute. Calculate the theoretical amount to be bled, the cycle efficiency and simple Rankine cycle efficiency for the same supply steam and condenser conditions.

## 16•3. Single-Stage Extraction ('ycle

For a steam condition of 350 lb per sq in. absolute and 600 deg F and a 2 -in. mercury absolute condenser pressure, plot a curve of the fraction of the steam bled and the cycle efficiency against the pressure at which the steam is bled, using a single extraction cycle.

### 16.4. Extraction Cycle

For an initial steam condition of 400 lb per sq in. absolute and $700 \mathrm{deg} F$ and a condenser pressure of 2 in . of mercury absolute, an extraction cycle bleeds steam for feed-water heating at 100,40 , and 10 lb per sq in . absolute. Calculate the fraction of the steam bled at each point, the cycle efficiency, and the water rate in pounds of steam per horsepower-hour.

### 16.5. Reheat Extraction Cycle

For the same initial steam and condenser conditions as in problem 16.4 steam is bled for feed-water heating and the rest of the steam is reheated to 600 deg F and then returned to the turbine to complete the expansion. Calculate the fraction extracted and the cycle efficiency.

### 16.6. High-Pressure Cycle

A high-pressure cycle is superimposed on the cycle of problem 16.4 and it operates between 1400 lb per sq in. and 800 deg F . The steam is reheated to 600 deg F before being returned to the low-pressure cycle. Calculate the improvement in efficiency and the fraction of the total work done in the high-pressure unit.

### 16.7. Mercury Steam Cycle

In a plant similar to that of Figure 16.5, the mercury vapor is generated at 180 lb per sq in. absolute and the mercury turbine exhaust at 1.8 lb per sq in . absolute. Steam is generated at 500 lb per sq in . absolute and 800 deg F , and the steam condenser is maintained at an absolute pressure of 1 in . of mercury. Steam is bled for feed-water heating at 90 lb per sq in. absolute.
(a) Find the pounds of steam circulated per pound of mercury.
(b) Find the fraction of the steam bled.
(c) Find the fraction of the total work that is from the mercury turbine.
(d) Find the cycle efficiency and the ideal Btu supplied per kilowatt-hour delivered.

|  | proplrties of mebcury |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Pressure | $h_{f}$ | $h_{g}$ | sf | $s_{g}$ | Temperature |
| 200 | 36.09 | 151.10 | 0.04058 | 0.11939 | 999.6 deg F |
| 1.8 | 17.37 | 143.42 | 0.02486 | 0.15654 | 497.7 deg F |

### 16.8. Plant Efficiency

A boiler that is 0.80 efficient generates steam at 400 lb per sq in . absolute and 600 $\operatorname{deg} \mathrm{F}$. Between the boiler and turbine there is a 20 lb per sq in. pressure drop and negligible heat loss. It then expands to 90 lb per sq in. in the upper stages of the turbine with a turbine efficiency of 0.8 .

At this point steam is bled for feed-water heating and the rest of the steam goes to the re-superheater. It reaches the re-superheater at 80 lb per sq in. and is heated to 600 deg F and is returned to the turbme with a negligible heat drop but at 70 lb per sq in . pressure. It then expands through the remaining stages of the turbine with a turbine efficiency of 0.7 to the pressure of 2 in . of mercury in the turbine exhaust. The condenser pressure is $1^{1} \frac{1}{2} \mathrm{in}$. of mercury. If the plant uses coal having a heating value of $14,300 \mathrm{Btu}$ per lb , calculate the plant efficiency and the pounds of coal per turbine horsepower-hour. (Ignore boiler-feed work.)
10.9. Effect of Load on Plant Efficiency

In a small plant when the steam flow was 5000 lb per hr the steam reached the turbine at 1500 lb per sq in. and at 760 deg F . At a load of $20,000 \mathrm{lb}$ per hr the superheat temperature was 920 deg F and the pressure dropped to 1370 lb per sq in. At 5000 lb per hr steam was bled for feed-water heating at $400 \mathrm{deg} F$ and at 550 deg F at $20,000 \mathrm{lb}$ per hr. At the lighter load the condenser pressure was 2 in . of mercury and at the higher load this rose to 3 in . What was the effect of load on the ideal cycle efficiency and water rate?
16•10. Continuous Extraction C'yrle
For the initial and final steam conditions of problem $16 \cdot 2$ calculate the approximate cycle efficiency that would result from continuous extraction or an infinite number of contact feed-water heaters. The first extraction would occur when, in Figure $16 \cdot 3$, the expanding steam reaches the saturation temperature corresponding to $P_{1}$. One possible method would be to plot a curve of ( $h_{2}-h_{f 2}$ ) against temperature, using the Mollier diagram; and another curve of sperific heat of liquid against temperature, from $3^{\prime}$ to $1^{\prime}$, Figure $16 \cdot 3$.

If these curves can be expressed by simple equations in terms of temperature, an integration for the work done in the turbine between the upper and lower extraction temperatures may be possible. Careful thought may reveal other and better methods of attack.

## Chapter 17

### 17.1. Vapor-Cycle Refrigeration

Calculate the coefficient of performance, horsepower per ton, and the cubic feet per minute of vapor at admission conditions that must be handled for an ideal watervapor refrigerator operating between 40 deg F and 100 deg F :
(a) For wet compression.
(b) For dry compression.
17.2. Ammonia-Vapor Refrigeration

PROPERTIES OF SATURATED AMMONIA

| Temp. <br> deg F | $P$ lb per <br> sq in. | $V_{g}$ | $h_{f}$ | $h_{f-g}$ | $h_{g}$ | $s_{f}$ | $s_{g}$ |
| :---: | :---: | ---: | :---: | :---: | :---: | :---: | :---: |
| -10.0 | 23.74 | 11.500 | 32.1 | 576.4 | 608.5 | 0.0738 | 1.3558 |
| +10.0 | 38.51 | 7.304 | 53.8 | 561.1 | 614.9 | 0.1208 | 1.3157 |
| +70.0 | 128.8 | 2.312 | 120.5 | 508.6 | 629.1 | 0.2537 | 1.2140 |
| +100.0 | 211.9 | 1.419 | 155.2 | 477.8 | 633.0 | 0.3166 | 1.1705 |

PROPERTIFS OF SUPERILEATED AMMONIA VAPOR

$$
(P=125 \mathrm{lb} \text { per } \mathrm{sq} \text { in. })
$$

| Trmp $\operatorname{deg} \mathrm{F}$ | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 1,50 | 160 | 170 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $V$ | 2392 | 2.461 | 2.528 | 2.593 | 2.657 | 2.719 | 2750 | 2840 | 2900 | 2.9 .78 | 3.016 |
| h | 630.0 | 637.2 | 64.4.0 | 650.7 | 657.1 | 66.3.5 | 669.7 | 67.5 .8 | 681.8 | 687.8 | 693.7 |
| $s$ | 1.2189 | 1.2322 | 1.2448 | 1.2568 | 1.2682 | 1.2792 | 1.2899 | 130 HO 2 | 1.3102 | 1.3199 | 1.3294 |

$$
\left(l^{\prime}=130 \mathrm{lb} \text { per } \mathrm{sq} \mathrm{in} .\right)
$$

| Temp $\operatorname{deg} \mathrm{F}$ | 70 | 80 | 90 | 100 | 110 | 120 | 130 | 140 | 150 | 160 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $V$ |  | 2.355 | 2.421 | 2.481 | 2546 | 2606 | 2.665 | 2.724 | 2.781 | 2.838 |
| $h$ |  | 636.0 | 643.0 | 6497 | 6.563 | 662.7 | 668.9 | 675.1 | 681.2 | 687.2 |
| $s$ |  | 1.2260 | 1.2388 | 1.2509 | 1.2625 | 1.2736 | 1.2843 | 1.2947 | 1.3048 | 1.3146 |

Calculate the coefficient of performance horsepower per ton of refrigeration and the cubic feet per minute of vapor at intake condition for an ideal ammonia-vapor refrigerator operating between +10 deg F and 70 deg F for:
(a) Wet compression.
(b) Dry compression.

### 17.3. Ice-Freczing Cost

An ammonia refrigerator that is 0.7 as efficient as the ideal cycle is used to freeze ice, using power that costs 5 cents a kw-hr. It absorbs heat from the brine of the freezing tank at $+10 \operatorname{deg} \mathrm{~F}$, and the spray-cooled condenser operates at 70 deg F . Calculate the power cost per hundred pounds of ice frozen if the water to be frozen is put in the cans at 70 deg F . Assume a dry compression cycle.

### 17.4. Reversed Refrigerator

Except for the equipment investment involved, an economical method of house heating would be to place the expansion coil of a refrigerator outdoors, absorb heat at the low temperature there, and by placing the condenser inside, discharge both the heat absorbed and the compression work inside. The evaporator would have to
operate at a temperature below the outside temperature and the condenser above the indoor temperature to promote heat transfer from the coils used.

For an evaporator operating at $36 \mathrm{deg} F$ and a condenser at 80 deg F if a dry compression water-vapor machine is used that has a centrifugal compressor unit with an overall efficiency of 0.60 , what maximum price per kilowatt-hour could be paid for power and enable it to compete with a conventional furnace operating at 0.70 efficiency and using 13,500 Btu per $\mathrm{lb}, \$ 10.00$ a ton coal?

### 17.5. Flash Freezing of Ice

If 70 deg F water is introduced into an evaporator maintained at the pressure corresponding to 32 deg F as described in article $17 \cdot 4$, what fraction of it must evaporate to freeze the remainder? Find the horsepower input per ton of refrigeration to an ideal dry compression water-vapor machine to remove this vapor if the condenser is maintained at 100 deg F . Find the intake cubic feet per minute of vapor that must be handled per ton of refrigeration.

## 17•6. Water-Vapor Refrigeration

An ideal water-vapor refrigerator operates with an evaporator temperature of 40 $\operatorname{deg} \mathrm{F}$ and a condenser temperature of 80 deg F and the water to be cooled enters the evaporator at 80 deg F .
(a) If a steam jet and booster using 100 lb per sq in. dry saturated steam is used, calculate the pounds of high-pressure steam used per pound evaporated in the evaporator, and the pound of steam per minute per ton of refrigeration.
(b) If an ideal Rankine engine using steam at the same condition as in (a) with an atmospheric exhaust drives an ideal centrifugal compressor to replace the jet and booster, calculate the pounds of high-pressure steam used per pound evaporated and the pounds of steam per minute per ton of refrigeration.

### 17.7. Refrigerator Operation

For the refrigeration system shown in Figure $17 \cdot 1$ and for the one in Figure $17 \cdot 4$ analyze the sequence of events that result when:
(a) The heat load to the evaporator is increased.
(b) The condenser cooling water increases in temperature.
(c) The expansion valve opening is increased.
(d) The expansion valve opening is reduced.
(e) The rpm of the compressor (Figure 17•1) is increased or the steam pressure to the steam jets (Figure 17.4) is increased.

### 17.8. Air Cycle Refrigeration

An air compressor and an air engine are to be used to deliver 60 deg F air into a room on a day when the outside air temperature is 100 deg F . In the ideal system 100 deg F air is compressed adiabatically to some higher pressure and then cooled at that pressure to the outside temperature. It then goes to an air engine where it is expanded at constant entropy to atmospheric pressure and discharged into the room. The compression pressure must be so chosen that the air will enter the room at 60 deg F . The ideal work will be the difference between the compression work done and that regained during the expansion in the air engine. If the room is being maintained at 75 deg F , the cooling effect will be the heat absorbed as the air warms from $60 \mathrm{deg} F$ to 75 deg F at constant pressure.

For the conditions outlined, calculate the proper compression pressure, the horsepower required per ton of refrigeration, and the cubic feet per minute of air that must enter the compressor for each ton of refrigeration.

## 17•9. Dccp Refrigeration

The principle of ideal low-temperature refrigeration may be illustrated by the following case.

Atmospheric pressure 70 deg F air is compressed in a multi-stage centrifugal compressor to 100 lb per sq in. absolute pressure and cooled to 70 deg F in a heat exchanger. Two-thirds of each pound of $70 \mathrm{deg} \mathrm{F}, 100 \mathrm{lb}$ per sq in. air is then expanded at constant entropy in an air turbine to atmospheric pressure. This low-temperature air is then used in an indirect heat exchanger to cool the remaining $1 / 3 \mathrm{lb}$ of 100 lb per sq in. air, it usually being assumed that this exchange is complete; that is, the high- and low-temperature air approach the same temperature.

The refrigerated $1 / 3 \mathrm{lb}$, of air is then expanded in a turbine at constant entropy to atmospheric temperature. Culculate the temperature at which this air will leave the turbine and find the not, work required to produce this $1 / 3 \mathrm{lb}$ of low-temperature air assuming adiabatic compression and expansion in each and assuming the physical constants of air at atmospheric temperature to apply at low temperatures.
17•10. Joule-Thompson Effect
Write a short summary of the application of the Joule-Thompson effect in the Linde type liquid air machine, explaining the principle involved. See Thermodynamics by Keenan or other references.

## 17-11. Dry Ice

Summarize a commercial method of producing solid carbon dioxide, paying particular attention to the principles involved.

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[^0]:    * Superior numbers indicate references listed at the close of each chapter.

[^1]:    Available Energy
    Emswiler, Thermodynamics. Chapter XVII.
    Barnard, Ellenwood, and Hirshfeld, Heat Power Engineering. Part I, Chapter V.

[^2]:    * It is recommended that articles $4 \cdot 1,4 \cdot 2$, and $4 \cdot 3$ be reviewed at this point.

[^3]:    *This section uses $t$ for time and consequently to avoid duplication uses the symbol of absolute temperature $T$ to represent degrees Fahrenheit.

